CALCULATION OF THE HALF-LIVES FOR ^{117,119,121}Te ISOTOPES BY PYATOV'S METHOD

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CALCULATION OF THE HALF-LIVES FOR $^{117,119,121}Te$ ISOTOPES BY PYATOV'S METHOD

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I certify that in my opinion the thesis submitted by ZOHRA ALI AHMED ALWASI titled "CALCULATION OF THE HALF-LIVES FOR $^{117,119,121}Te$ ISOTOPES BY PYATOV'S METHOD" is fully adequate in scope and in quality as a thesis for the degree of Master of Science.

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“I declare that all the information within this thesis has been gathered and presented in accordance with academic regulations and ethical principles and I have according to the requirements of these regulations and principles cited all those which do not originate in this work as well.”

ZOHRA ALI AHMED ALWASI

ABSTRACT

M. Sc. Thesis

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In this study, the collective spin-isospin excitations have been investigated for Te-117, Te-119 and Te-121 isotopes by using the proton-neutron quasiparticle random phase approximation. The total Hamiltonians of the Pyatov-Salamov method and the schematic model have been defined in the particle-particle and the particle-hole quasiparticle spaces. The equations of motion have been solved by founding the eigenvalues and eigenfunctions for both methods. The half-lives of the allowed Gamow-Teller transitions have been calculated with the obtained reduced matrix elements. The numerical calculations have been executed with Fortran77 code. The calculated results of the Pyatov-Salamov method and the schematic model have been compared with the experimental values. Because of the calculated half-lives by Pyatov-Salamov method are in good agreement with the experimental values, the importance of the restoration term in this method has been emphasized.

Key Words : GT transitions, Pyatov-Salamov method, pn-QRPA, odd-mass nuclei.

Science Code : 202.1.108

ÖZET

Yüksek Lisans Tezi

^{117,119,121}Te İZOTOPLARININ YARI-ÖMÜR SÜRELERİNİN PYATOV YÖNTEMİ İLE HESAPLANMASI

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Bu çalışmada, Te-117, Te-119 ve Te-121 izotopları için kollektif spin-isospin uyarılmaları proton-nötron kuazi parçacık rastgele faz yaklaşımı kullanılarak incelendi. Pyatov-Salamov ve şematik model toplam Hamiltonyenleri parçaçık-parçacık ve parçacık-boşluk kuaziparçacık uzaylarında tanımlandı. Her iki model için özdeğer ve özfonksiyonlar bulunarak hareket denklemleri çözüldü. Elde edilen indirgenmiş matris elemanları ile izinli Gamow-Teller geçiş yarı ömür süreleri hesaplandı. Nümerik hesaplamalar Fortran77 kodu ile yapıldı. Pyatov-Salamov ve şematik model sonuçları deneysel değerler ile karşılaştırıldı. Pyatov-Salamov yöntemi ile hesaplanan yarı ömür süreleri deneysel değerler ile oldukça uyumlu olduğu için bu yöntemdeki restorasyon teriminin önemi vurgulandı.

Anahtar Kelimeler : GT geçişleri, Pyatov-Salamov metodu, pn-QRPA, tek kütleli çekirdekler.

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SYMBOLS

β : beta particles
ℓ : orbital angular quantum number
j : total angular momentum quantum number
s : spin
p : proton
n : neutron
e^+ : positron
e^- : electron
v : neutrino
$\bar{v}$: anti neutrino
C : coulomb
Q : decay energy
sq : single particle
sqp : single quasi particle

ABBREVITIONS

F : Fermi
GT : Gamow-Teller
GTR : Gamow-Teller Resonance
IAR : Isobar Analog Resonance
ISR : Ikeda Sum Rule
WS : Woods-Saxon Potential
PM : Pyatov-Salamov Method
SM : Schematic Model
TDA : Tamm Dancoff Approximation

RPA : Random Phase Approximation

QRPA : Quasiparticle Random Phase Approximation

pn-QRPA : proton neutron-Quasiparticle Random Phase Approximation

H : Hamiltonian operator

H_{sqp} : The single quasiparticle Hamiltonian

h_0 : The restoration term

h_{pp} : The particle-particle interaction term

h_{ph} : The particle-hole interaction term

V_c : Coulomb potential

V_{ls} : Spin orbital potential

V_1 : The average area potential isovector fraction

s_{β^+} : Transition strength of β^+

s_{β^-} : Transition strength of β^-

$B_{GT}^{\pm}$: Beta decay strength distributions

$G_{1\mu}^{\pm}$: Gamow-Teller operator

E_p : Proton energy

φ : Free particle wave function

Ψ_i : Initial wave function

Ψ_s : Final states wave function

μ : Reduced mass

ω_i : Gamow-Teller 1^+ states energy

χ_{ph} : Particle-hole interaction parameter

χ_{pp} : Particle-particle interaction parameter

$\hat{\tau}$: Isospin operator

ε : Single particle energy

$\hat{a}^+$: Quasiparticle production operator

$\hat{a}$: Quasiparticle annihilation operator

d_{np} : Reduced matrix elements

γ_ρ : The strength parameter of effective interaction

Ψ_{np}^i : Amplitude of the quasi-boson wave function

φ_{np}^i : Amplitude of the quasi-boson wave function

Te : Tellurium

PART 1

INTRODUCTION

The weak interaction is one of the four fundamental forces in nature and it plays a key role in many astrophysical processes. The first theory of weak interaction has been developed by Enrico Fermi [1] after Wolfram Pauli postulated the existence of the neutrino [2].

The iso-scalar giant quadrupole resonance with spin-parity $J^{\pi} = 2^{+}$ and the iso-scalar giant monopole resonance with spin-parity $J^{\pi} = 0^{+}$ were well established in 1980. The iso-scalar (T=0) modes and iso-vector modes (T=1) are vibrations in which protons and neutrons move in phase and in opposite phase, respectively. The giant electric dipole resonance with spin-parity $J^{\pi} = 1^{-}$ is very strongly excited by photon absorption. Nucleons with spin up and spin down may move either in phase (spin-scalar S=0 modes) or out of phase (spin-vector S=1 modes) [3]. The collective spin modes provide direct information on the spin and spin-isospin dependent effective interactions in the medium. These effective forces are presented on the microscopic level by the residual particle-hole (*ph*) interaction. The spin excitations are the weak, the strong and the electromagnetic interactions. In the weak interaction, it is the axial vector current and which induces the Gamow-Teller (GT) transitions of nuclear beta (β) decay. The Gamow-Teller transitions are represented by the spin-isospin operator $g_A \sigma \tau_{\pm}$, where g_A, σ and τ are the axial vector coupling constant, the spin operator and the isospin operator, respectively [4]. Examining these transitions gives about information on the spin-isospin properties of nuclei. The (*p*,*n*) charge-exchange experiments that is to say the spin-isospin correlations in nuclei started in 1980 [5-8]. These experiments have shown the existence of very collective spin-isospin modes in nuclei. This collective mode is interpreted as the giant Gamow-Teller resonance (GTR) [9] and also, this collective mode was predicted as a theoretical by Ikeda, Fujii and Fujita in 1963 [10]. First experimental studies of the Gamow-Teller resonance were observed in 45 MeV with Zr-90 (*p*,*n*) experiment by

Doering et al. [11], and in different low-energy with Zr-90 (^3He, t) experiments by Galonsky et al. and Ovazza et al. [12,13].

The Gamow-Teller excitations have a great importance to investigate installation of nuclear, moreover the GT strength distribution in case of comparatively low energies of great importance for comprehending the physical processes such as nucleosynthesis stellar and supernova formation [14]. Therefore, investigating the properties of the nuclear structure is very important for the calculation of the β-decay rates. The rates of weak interaction in nuclei far from the stability valley provide good information about the estimation of their β-decay properties. Firstly, the GT beta decay transition rates in medium and heavy nuclei were calculated using the random phase approximation (RPA) by Halbleib and Sorensen [15]. The RPA model is the independent particle model and the solutions of the RPA equations are more confusing than those of the Tamm Dancoff Approximation (TDA). In the RPA the residual interaction is diagonalized within the model space of 1particle-1hole (*1p-1h*) excitations. The quasi random phase approximation (QRPA) model contains both particle-particle (*p-p*) and particle-hole (*p-h*) matrix elements [16]. The properties of the Gamow-Teller resonance for Bi-208 isotope were studied using the TDA method by Colo *et al.* and the GTR energy was founded between the range of 18-24 MeV [17]. Dang *et al.* were calculated the GTR energy at 16.6 MeV for Bi-208 isotope using *2p-2h* configurations [18].

Why is important Tellurium (Te) isotopes? To understand the production of the heaviest elements in the universe and to test nucleosynthesis models one may search the isotopic and elemental abundance patterns in meteorites, the solar photosphere and metal-poor halo stars. All elements heavier than the iron group can be produced by neutron capture reactions. These reactions occur on timescales that are slow (*s*) or rapid (*r*) and they are known as the *s*-process and *r*-process. The solar system *r*-process distribution was calculated as the residual in the solar system isotopic abundance distribution by Roederer *et al* [19]. Also, nuclei with neutron numbers $N = 50, 82$ and 126 were reduced neutron-capture cross sections, giving rise to peaks in the abundance distributions. The *s*- and *r*- processes must be able to reproduce the nucleosynthetic yields. Stars are rich laboratories to study the nucleosynthesis of heavy elements. Tellurium is the heaviest element whose solar system *r*-process production can be calculated largely based on experimental nuclear data. The first observations of tellurium produced by the *r*-process operating in the early Galaxy provide critical data for validating *r*-process models with the least amount of nuclear physics uncertainty. The experimental half-lives on *r*-process calculations are important to support the observed solar system *r*-process abundances in stars [20]. Tellurium is predominantly produced in the main component of the *r*-process between the rare earth elements.

In this thesis, the nuclear β-decay properties in odd mass tellurium nuclei are investigated by the microscopic model proton–neutron Quasiparticle Random Phase Approximation (pn-QRPA). The vacuum of the pn-QRPA involves correlations that affect quantitative and even qualitative properties of electromagnetic and beta decays. The pn-QRPA provides the Ikeda sum rule for Gamow-Teller transitions, as did the RPA and the QRPA [21].

As is known, the pairing potential is not commutative with the Gamow-Teller operator. The broken supersymmetry property of the electric dipole resonances, the magnetic dipole resonances, the isobar analogue resonances, the Gamow-Teller resonances between nucleons must be restored. The restoration method was developed by Pyatov and Salamov in 1977 [22]. The broken supersymmetry property of the pairing interaction in the nuclear part of total Hamiltonian was restored by Pyatov-Salamov method (PM). In this study, the Gamow-Teller $J^{\pi} = 1^{+}$ states were investigated with the restoration method. The influence of the restoration on the GTR in odd mass ^{117,119,121}Te isotopes was examined with the pn-QRPA method using the separable residual GT efficient interactions in the particle- hole (*ph*) and particle-particle (*pp*) channels. The same calculations were studied with schematic model (SM). The schematic model Hamiltonian does not contain the restoration term. The effect of the restoration term on the β-decay half-lives for some odd mass Te isotopes is first time investigated.

The thesis is organized as follows: The Gamow-Teller decay properties and the general information about Fermi beta decay theory are given briefly in section 1 and in section 2, respectively. In Section 3, the details of the restoration by using pn-QRPA model are investigated. In Section 4, the obtained results in both the Pyatov-

Salamov method and the schematic model are compared with the experimental data. The main conclusions are also summarized in Section 5.

PART 2

FERMI BETA THEORY

In 1934, Fermi was able to develop a theory considered successful to understand the β-decay depending on Pauli's neutrino hypothesis [1]. This theory is known as the Fermi theory of beta decay. In this theory, Fermi assumes that there are four fermions interact with each other directly, from this interaction we can interpret beta decay of a neutron by linear coupling of neutron with an electron, a neutrino (which determined later to be antineutrino) and a proton, moreover, Fermi interaction was considered the introduction to the theory for the weak interaction that occurs between the proton-neutron and electron-antineutrino is intercede by virtual W^{-}boson [23].

2.1. BETA DECAY

Beta particles are electrons or positrons. When beta decay occurs in a nucleus, one of the protons or neutrons is transformed into the other. β-decay changes both Z and N by one unit and A remains constant. The importance of β- decay appears that it can be used in studies the case of weak interactions and for definition of the corresponding interaction parameters. The basic beta decay processes are given [24]:

a) the β^{-} decay

$$n \rightarrow p + e^{-} + \bar{\nu} \tag{2.1}$$

b) the β^{+} decay

$$p \rightarrow n + e^{+} + \nu \tag{2.2}$$

c) the electron capture (ε)

$$p + e^{-} \rightarrow n + \nu \tag{2.3}$$

The beta decay is very important to calculate the constant of weak interaction. The known formula for the Hamiltonian of weak interaction is given

$$H_\beta = \sum_{i=1}^{5}\{(\bar{\psi}_p O_i \psi_n)(\bar{\psi}_e \acute{O}_i(G_i + \acute{G}_i\gamma_5)\psi_v) + c.c.\} \tag{2.4}$$

The formula of Hamiltonian includes 20 real constants, but the numeral constants is reduced to 5 constants if the stability of H_β is under time reversal is supposed and the two-component theory of neutrino is used. When the theory of universal weak interactions is utilized the number of constants is reduced, by this theory it can be concluded that the β- decay Hamiltonian comprise only vector and axial vector terms, then the Hamiltonian is written as [25]

$$H_\beta = \frac{G_V}{\sqrt{2}}\sum_{\mu=1}^{4}\{(\bar{\psi}_P\gamma_\mu(1 + \lambda_{\gamma 5})\psi_n)(\bar{\psi}_e\gamma_\mu(1 + \gamma_5)\psi_v) + c.c.\} \tag{2.5}$$

where $\lambda = G_A/G_V,\ \ \gamma_4 = \beta,\ \ \gamma_m = -i\beta\alpha_m,\ \ \alpha_m = \begin{pmatrix} 0 & \sigma_m \\ \sigma_m & 0 \end{pmatrix},\ m = 1,2,3.$

The interaction Hamiltonian H_β is rewritten as the sum of the Fermi and Gamow-Teller terms,

$$H_\beta = \frac{G_V}{\sqrt{2}}\left\{\sum_i \tau^{+(i)}(\gamma_\mu)_i\left(\bar{\psi}_e(r)\gamma_\mu(1 + \gamma_5)\psi_v(r)\right) + c.c.\right\}$$
$$+\frac{G_A}{\sqrt{2}}\left\{\sum_i \tau^{+(i)}(\gamma_\mu\gamma_5)_i(\bar{\psi}_e(r)\gamma_\mu(1 + \gamma_5)\psi_v(r)) + c.c.\right\}. \tag{2.6}$$

The G_V and G_A are the vector and axial vector interaction constants. The matrix elements of the beta transition between the initial and final nuclear states are equal to

$$M_\beta = \langle f|H_\beta|i\rangle = M_V + M_A \tag{2.7}$$

where M_V and M_A are the Fermi and Gamow-Teller matrix elements.

When the neutrino and electron emitted with anti-parallel spin, coupling to ($S = 0$) and $\Delta J = 0$, this case is known as Fermi decay. In this situation, the allowed approximation is ($\ell = 0$) and $\Delta I = |I_i - I_f| = 0$. If the spins of electron and neutrino are parallel ($S = 1$), this is the Gamow-Teller decay and $\Delta J = 0, \pm 1$. The selection rules for allowed beta decay are given

$$\Delta I = 0, 1 \quad \Delta\pi(\text{ parity change}) = \text{no.} \tag{2.8}$$

A forbidden decay occurs when the initial and final states have opposite parities. Forbidden decays with $l = 1$ are called first forbidden decays and like the allowed decays have Fermi and Gamow-Teller types. The selection rules for first-forbidden decays are

$$\Delta I = 0, 1, 2 \quad \Delta\pi \text{ (parity change)} = \text{yes.} \tag{2.9}$$

The selection rules in ΔI and parity change for the allowed and the first forbidden beta decay transitions given in Table 2.1.

Table 2.1. Selection rules of the allowed and first forbidden beta transitions [25].

Classification	Matrix Elements	$\pi_i \pi_s$	Selection Rules
A	$\langle 1 \rangle$ $\langle \sigma \rangle$	1 1	$\Delta I = 0$ $\Delta I = 0, \pm 1$ except $0 \to 0$
1	$\langle r \rangle$ $\langle \sigma\gamma_5 \rangle$ $\langle \gamma_5 \rangle$ $\langle \sigma r \rangle$ $\langle \sigma \mathrm{x} \mathrm{r} \rangle$ B_{ij}	-1 -1 -1 -1 -1 -1	$\Delta I = 0, \pm 1$ except $0 \to 0$ $\Delta I = 0, \pm 1$ except $0 \to 0$ $\Delta I = 0$ $\Delta I = 0$ $\Delta I = 0, \pm 1$ except $0 \to 0;\ 1/2 \to 1/2$ $\Delta I = 0, \pm 1$ except $0 \to 0;\ 1/2 \to 1/2;\ 0 \to 1$
1*	B_{ij}	-1	$\Delta I = 0, \pm 2$ except $0 \to 0;\ 1/2 \to 1/2;\ 0 \to 1$

2.2. THE NUCLEAR MODEL

The shell model has extreme importance, which provides explanations for the atomic structure and describes non-interacting particles that are moving in joint potential well, this potential is formed by all particles of nucleus. The shell model agrees with Pauli principle, where the shells were filled with electrons in order to compound the energy, thus an inert core of filled shells and some numeral of valence electrons are gotten, then this model suppose that the valence electrons determine the atomic properties. The advantage of atomic shell theory is the existence of spatial orbits, because it is salutary to describe atomic in terms of spatial orbits of the electrons [26]. The single-particle shell model that is to say the model of independent particles is too simple to depict the nuclear structure accurately. It is applicable to description of the nuclear properties. The nuclear forces own a short extent and they are very strong and essentially attractive [27].

2.2.1. The Woods Saxon Potential

The Woods Saxon (WS) potential is so appropriate option for the one body potential. It is a spherical symmetric, finite depth potential, the parameters were chosen to fit of nuclear single particle energies and nuclear radii. The WS potential is sum of a spin independent central potential, a spin orbit potential, the Coulomb potential and the following

$$V(r) = V_o(r) + V_{so}(r)\vec{\ell}.\vec{s} + V_c(r) \tag{2.10}$$

The equipotential surface which (r =R_0) matches to the half of it at the nuclear center, the potential consists of two parts; the central part and the spin-orbit coupling term are given

$$V(r) = -\frac{V_o^{N,Z}}{1 + exp[(1/a)(r-R_0)]}, \tag{2.11}$$

$$V_{ls}(r) = -\zeta\frac{1}{r}\frac{dV(r)}{dr}(ls) \tag{2.12}$$

where V_o^N and V_o^Z are the selection parameters.

$$\left.\begin{aligned} V_o^N &= V_o\left[1-o.63\,\frac{N-Z}{A}\right] \\ V_o^Z &= V_o\left[1+0.63\frac{N-Z}{A}\right] \end{aligned}\right\} \tag{2.13}$$

and $V_o = 53MeV$, $R_o = r_o A^{1/3}$, $r_0 = 1.24 \times 10^{-13} cm.$

The value of the surface thickness is $a = 0.63 \times 10^{-13} cm$, and ζ which represents the spin orbit coupling strength $\zeta = 0.263\{1 + 2[(N - Z)/A]\}(10^{-13}cm)^2$, where the parameters are sufficiently steady for spherical nuclei within broad range of the atomic numbers A, when the proton levels are calculated the term of the Coulomb potential must be added to the potential, disregarding the impact of the surface [25]

$$V_c(r) = \frac{(Z-1)e^2}{r}\begin{cases} \frac{3r}{2R_o} - \frac{1}{2}(r/R_o)^3, & r \leq R_o \\ 1, & r > R_o \end{cases} \tag{2.14}$$

In the case where nuclei with an excess of neutrons, the protons will sense a stronger potential more than the neutrons, since the medium proton-neutron potential is much stronger than the average neutron-neutron or proton-proton potential, so it is taken that

$$V_p = V_o + \frac{(N-Z)}{A}V_1 \quad \text{(for protons)} \tag{2.15}$$

and

$$V_n = V_o + \frac{(N-Z)}{A}V_1 \quad \text{(for neutrons)} \tag{2.16}$$

Nuclei in which protons are not equal to neutrons r_0 and a_0 could also be a little different for proton and neutron, thus six parameters can have the spin-independent potential (and even more), the value of parameters is selected to give an overall accounting of the watched single particle energies, the *rms* charge radii, and the electron scattering form factors.

Figure 2.1. shows the comparison between the Woods-Saxon potential, oscillator potentials and square potential. The Woods-Saxon potential has a flatter bottom and corresponds to the medium case between oscillator and square well [25].

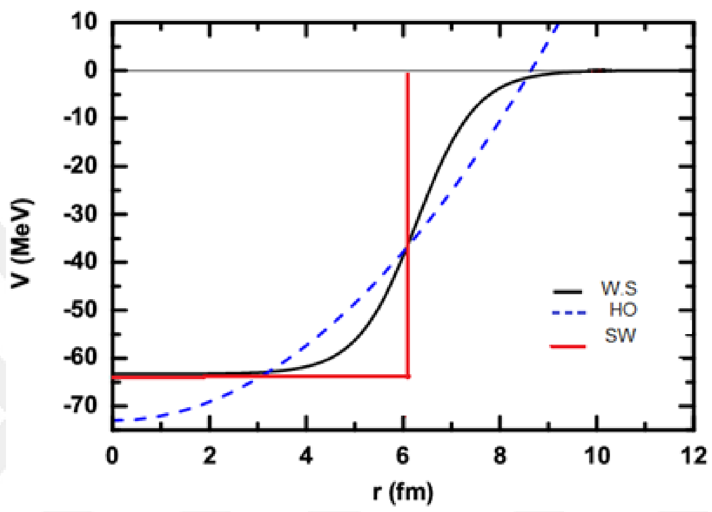

Figure 2.1. The curve of some nuclear potentials [11].

PART 3

THEORETICAL FORMALISM

In this section, the solutions of the pn-QRPA equation, the total Hamiltonians of the Pyatov-Salamov method and the schematic model were investigated.

3.1. THE pn-QRPA EQUATION

Just as the RPA and QRPA equations, the pn-QRPA equation can be derived by using the equations of motion method. It may be briefly given the RPA and the QRPA equations as the following

- ✓ The RPA excitation operator and the RPA matrix equation are

$$Q_\omega^\dagger = \sum_{ab}\left[X_{ab}^\omega A_{ab}^\dagger(JM) - Y_{ab}^\omega \tilde{A}_{ab}(JM)\right], \tag{3.1}$$

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} = E_\omega \begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} \tag{3.2}$$

and the matrix containing A and B is not Hermitian, and the RPA eigenvalue problem is *non*-Hermitian.

- ✓ In a similar way, the QRPA excitation operator and the QRPA matrix equation are

$$Q_\omega^\dagger = \sum_{a\le b}\left[X_{ab}^\omega A_{ab}^\dagger(JM) - Y_{ab}^\omega \tilde{A}_{ab}(JM)\right], \tag{3.3}$$

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} = E_\omega \begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix}. \tag{3.4}$$

The matrix elements are independent of M, the X and Y amplitudes must be independent of M. The QRPA equation comprise a *non*-Hermitian eigenvalue problem. Secondly, the pn-QRPA equation was analyzed. The basic excitation is

$$|\omega\rangle = Q_{\omega}^{\dagger}|pnQRPA\rangle \tag{3.5}$$

where the pn-QRPA phonon creation operator is

$$Q_{\omega}^{\dagger} = \sum_{pn}\left[X_{pn}^{\omega}A_{pn}^{\dagger}(JM) - Y_{pn}^{\omega}\tilde{A}_{pn}(JM)\right] \tag{3.6}$$

and $|pnQRPA\rangle$ denotes the pn-QRPA vacuum. The pn-QRPA matrix equation can be expressed

$$\begin{pmatrix} A & B \\ -B^{*} & -A^{*} \end{pmatrix}\begin{pmatrix} X^{\omega} \\ Y^{\omega} \end{pmatrix} = E_{\omega}\begin{pmatrix} X^{\omega} \\ Y^{\omega} \end{pmatrix} \tag{3.7}$$

and as in the case of the QRPA, A is Hermitian and B is symmetric. The eigenvalue problem is *non*-Hermitian. As is known, the RPA, the QRPA and the pn-QRPA equations have very complicated solutions. The fundamental pn-QRPA equations were expressed in this chapter. Details of the RPA, the QRPA and the pn-QRPA formalism see in From Nucleons to Nucleus / Concepts of Microscopic Nuclear Theory written by Jouni Suhonen [28].

3.2. SOLUTIONS OF THE pn-QRPA EQUATION FOR THE GAMOW-TELLER TRANSITIONS

3.2.1. Hamiltonian

The total Hamiltonians was obtained in regard to the Pyatov-Salamov method and the schematic model. The super symmetry ownership of the terms, inclusive pairing interaction in the nuclear part of the total Hamiltonian was restored employing the Pyatov-Salamov method. The schematic model Hamiltonian for Gamow-Teller excitations in the quasiparticle representation is usually given as

$$H_{SM} = H_{sqp} + h_{ph} + h_{pp} \tag{3.8}$$

where H_{sqp} is the single quasi particle Hamiltonian and represents as

$$H_{sqp} = \sum_{\tau,jm} \varepsilon_{j_\tau} \alpha^{\dagger}_{j_\tau m_\tau} \alpha_{j_\tau m_\tau} \qquad (\tau = n, p) \tag{3.9}$$

The h_{pp} and h_{ph} are the Gamow-Teller effective interactions in the *pp* and *ph* channels, respectively. The effective interaction constants were fixed from the experimental value of the Gamow-Teller resonance energy and β-decay log*ft* values between the low energy states of daughter and parent nuclei. The broken commutativity of the remaining part due to the shell model mean field approximation was restored by adding an effective interaction term as follows

$$\left[H_{SM} - \left(h_{ph} + h_{pp}\right) - \left(V_1 + V_c + V_{ls} + h_0\right), G^{\pm}_{1\mu}\right] = 0 \tag{3.10}$$

or

$$\left[\mathrm{H_{sqp}} - \mathrm{V_1} - \mathrm{V_c} - \mathrm{V_{ls}} + \mathrm{h_0}, \mathrm{G^{\pm}_{1\mu}}\right] = 0 \tag{3.11}$$

where V_1 , V_c , V_{ls} and $\mathrm{h_0}$ are the isovector potential, the Coulomb potential, the spin-orbital potential and the restoration term, respectively.

The effective interaction h_0 term is found as following

$$h_0 = \sum_{\rho=\pm} \frac{1}{4\gamma_\rho} \sum_{\mu=0,\pm 1} \left[\left(H_{sqp} - V_1 - V_c - V_{ls}\right), G^{\rho}_{1\mu}\right]^{\dagger} \times \left[H_{sqp} - V_1 - V_c - V_{ls}, G^{\rho}_{1\mu}\right] \tag{3.12}$$

The strength parameter of the effective interaction term is found from the commutation condition in eq. (3.11)

$$\gamma_\rho = \frac{\rho(-1)^\mu}{2}\left\langle 0\right| \left[\left[H_{sqp} - V_1 - V_c - V_{ls}, G^{\rho}_{1\mu}\right], G^{\rho}_{1\mu}\right] \left|0\right\rangle. \tag{3.13}$$

Thus, the total Hamiltonian of the system according to Pyatov-Salamov method is given that

$$H_{PM} \;=\; H_{sqp} +\; h_{ph} + h_{pp} + h_0. \tag{3.14}$$

3.2.2. The pn-QRPA Equation In Even Mass Nuclei

A system of nucleons in a spherical symmetric average field with pairing forces was considered here and the corresponding quasiparticle Hamiltonian of the system is expressed as eq. (3.9), where ε_{j_τ} is single quasiparticle energy of the nucleon with angular momentum j_τ. Also, $\alpha^{\dagger}_{j_\tau m_\tau}$ and $\alpha_{j_\tau m_\tau}$ are the quasiparticle creation and annihilation operators. The Gamow-Teller operator in the quasiparticle space according to quasiboson parameters is given as the following [29].

$$G^{-}_{np} \;=\; \sum_{np}\left[b_{np}C^{\dagger}_{np}(\mu) + (-1)^{1+\mu} b_{np} C_{np}(-\mu)\right], \tag{3.15}$$

and

$$G^{\dagger}_{1\mu} \;=\; \left[G^{-}_{1\mu}\right]^{\dagger} \tag{3.16}$$

where $C_{np}(\mu)$ is the quasiboson annihilation operator and $C^{\dagger}_{np}(\mu)$ is the quasiboson creation operator.

$$C^{\dagger}_{np}(\mu) = \sqrt{\frac{3}{2j_n+1}}\sum_{m_n,m_p}(-1)^{j_p-m_p}\left\langle j_p m_p 1_\mu \middle| j_n m_n\right\rangle \alpha^{\dagger}_{j_n m_n}\alpha^{\dagger}_{j_p -m_p} \tag{3.17}$$

and

$$C_{np}(\mu) \;=\; \left[C^{\dagger}_{np}(\mu)\right]^{\dagger} \tag{3.18}$$

The operators here accept the next commutation rule in the quasiboson approximation and are given as

$$\left[C_{np}(\mu)C^{\dagger}_{n'p'}(\mu')\right] = \delta_{nn'}, \delta_{pp'}, \delta_{\mu\mu'}, \qquad \left[C_{np}(\mu)C_{n'p'}(\mu')\right] = 0 \tag{3.19}$$

The h_0, h_{ph}, and h_{pp} effective interactions in quasiboson space are rewritten in the following formulas

$$h_0 = \sum_{npn'p'\mu\rho} \frac{1}{2\gamma\rho} E^{\rho}_{n\rho} E^{\rho}_{n'\rho'} \left[C_{np}(\mu) + \rho(-1)^{1+\mu} C^{\dagger}_{np}(-\mu)\right] \times \left[C^{\dagger}_{n'p'}(\mu') + \rho'(-1)^{1+\mu'} C_{n'p'}(-\mu')\right], \tag{3.20}$$

$$h_{ph} = 2\chi_{ph} \sum_{npn'p'\mu'} \left[\bar{b}_{np} C^{\dagger}_{np}(\mu) + (-1)^{1+\mu} b_{np} C_{np}(-\mu)\right] \times \left[\bar{b}_{np} C^{\dagger}_{np}(\mu) + (-1)^{1+\mu} b_{np} C_{np}(-\mu)\right] \tag{3.21}$$

and

$$h_{pp} = -2\chi_{pp} \sum_{npn'p'\mu'} \left[d_{np} C^{\dagger}_{np} - (-1)^{1+\mu} \bar{d}_{np} C_{np}(-\mu)\right] \times \left[d_{n'p'} C_{n'p'}(\mu') - (-1)^{1+\mu} \bar{d}_{n'p'} C^{\dagger}_{n'p'}(-\mu')\right] \tag{3.22}$$

where b, $\bar{b}$, d, and $\bar{d}$ are the reduced matrix elements.

The eigenvalues and eigenfunctions of the total Hamiltonian in eq. (3.14) were solved within the framework of the pn-QRPA. The i^{th} excited GT 1^+ states in odd-odd nuclei are treated as the phonon excitations in the pn-QRPA model and depicted by:

$$|i\rangle = Q^{\dagger}_i(\mu)\,|0\rangle = \sum_{np}\left[\psi^i_{np} C^{\dagger}_{np}(\mu) - (-1)^{1+\mu} \varphi^i_{np} C_{np}(-\mu)\right]|0\rangle \tag{3.23}$$

where the term $Q_i^{\dagger}(\mu)$ is the pn-QRPA phonon creation operator, $|0\rangle$ is the phonon vacuum which matches with the ground state of an even-even nucleus and accomplish $Q_i(\mu)|0\rangle = 0$ for all i ψ_{np}^{i} and φ_{np}^{i} . They represent quasiboson amplitudes. Supposing that the phonon operators comply the following commutation relations

$$\langle 0|\left[Q_i(\mu), Q_j^{\dagger}(\mu')\right]|o\rangle = \delta_{ij}\delta_{\mu\mu'} \tag{3.24}$$

and the next orthonormalization condition was obtained for amplitudes ψ_{np}^{i} and φ_{np}^{i}

$$\sum_{np}\left[\psi_{np}^{i}\psi_{np}^{i'} - \varphi_{np}^{i}\varphi_{np}^{i'}\right] = \delta_{ii'}. \tag{3.25}$$

From the pn-QRPA equation of motion, the energies and wavefunctions of the GT 1^{+} states are procured

$$\left[H_{PM}, Q_i^{\dagger}(\mu)\right]|0\rangle = \omega_i Q_i^{\dagger}(\mu)|0\rangle \tag{3.26}$$

where ω_i represents the energy of the GT 1^{+} states revolving in the neighboring odd-odd nuclei. It was considered that these energies are counted over the ground state of the parent nuclei. The next liner equations are acquired for the ψ_{np}^{i} and φ_{np}^{i} amplitudes

$$\sum_{np}\left[\left(\rho_{npn'p'} - \omega_i\delta_{nn'}\delta_{pp'}\right)\psi_{np}^{i} - (-1)^{1+\mu}\eta_{npn'p'}\varphi_{np}^{i}\right] = 0,$$

$$\sum_{np}\left[\eta_{npn'p'}\psi_{np}^{i} - (-1)^{1+\mu}\left(\rho_{npn'p'} - \omega_i\delta_{nn'}\delta_{pp'}\right)\varphi_{np}^{i}\right] = 0. \tag{3.27}$$

The following double commutators are resolved to calculate every ρ and η matrixes

$$\rho_{npn'p'} = \left[C_{n'p'}(\mu), \left[H_{PM}, C_{np}^{\dagger}(\mu)\right]\right] \tag{3.28}$$

and

$$\eta_{npn'p'} = \left[C_{n'p'}(\mu), \left[H_{PM}, C^{\dagger}_{np}(-\mu)\right]\right]. \tag{3.29}$$

The next expressions are obtained for these matrixes

$$\rho_{npn'p'} = \varepsilon_{np}\delta_{nn'}\delta_{pp'} + \sum_{\rho}\frac{1}{2\gamma_{\rho}}E^{\rho}_{np}E^{\rho}_{n'p'}$$
$$+2\chi_{ph}\left(\bar{b}_{n'p'}\bar{b}_{np} + b_{n'p'}b_{np}\right) 2\chi_{pp}\left(\bar{d}_{n'p'}\bar{d}_{np} + d_{n'p'}d_{np}\right) \tag{3.30}$$

$$\eta_{npn'p'} = (-1)^{\mu}\left\{\sum_{\rho}\frac{\rho}{2\gamma_{\rho}}E^{\rho}_{np}E^{\rho}_{n'p'}\right\}$$
$$+2\chi_{ph}\left(\bar{b}_{n'p'}b_{np} + b_{n'p'}\bar{b}_{np}\right) - 2\chi_{pp}\left(\bar{d}_{n'p'}d_{np} + d_{n'p'}\bar{d}_{np}\right) \tag{3.31}$$

The roots of secular equation are the energies ω_i and given as

$$\begin{vmatrix} \sum\left(\rho_{npn'p'} - \omega_i\delta_{nn'}\delta_{pp'}\right) & \sum\eta_{npn'p'} \\ \sum\eta_{npn'p'} & \sum\left(\rho_{npn'p'} - \omega_i\delta_{nn'}\delta_{pp'}\right) \end{vmatrix} = 0. \tag{3.32}$$

ψ^{i}_{np} and φ^{i}_{np} amplitudes from eqs. (3.30) and (3.31) were found.

3.2.3. Matrix Elements Of $\beta^{\pm}$ Transition In Even Mass Nuclei

One of the distinctive quantities for the GT 1^{+} states revolving in the neighboring odd-odd nuclei is the GT transition matrix elements, by using the next expressions it can be easy to calculate the $0^{+} \rightarrow 1^{+}$ β^{-} and β^{+} transition matrix elements

$$M^{i}_{\beta^{-}}(0^{+} \rightarrow 1^{+}_{i}) = \left\langle 1^{+}_{i}, \mu\left|G^{-}_{1\mu}\right|0^{+}\right\rangle = \left\langle 0\left|\left[Q_i(\mu), G^{-}_{1\mu}\right]\right|0\right\rangle$$
$$= -\sum_{np}\left(\psi^{i}_{np}b_{np} + \varphi^{i}_{np}\bar{b}_{np}\right), \tag{3.33}$$

$$M^{i}_{\beta^{+}}(0^{+} \rightarrow 1^{+}_{i}) = \left\langle 1^{+}_{i}, \mu\left|G^{+}_{1\mu}\right|0^{+}\right\rangle = \left\langle 0\left|\left[Q_i(\mu), G^{+}_{1\mu}\right]\right|0\right\rangle$$
$$= \sum_{np}\left(\psi^{i}_{np}\bar{b}_{np} + \varphi^{i}_{np}b_{np}\right) \tag{3.34}$$

Also, the $\beta^{\pm}$ reduced matrix elements are set as

$$B_{GT}^{(\pm)}(\omega_i) = \sum_\mu \left|M_{\beta^\pm}^i(0^+ \rightarrow 1_i^+)\right|^2 \qquad (3.35)$$

and the $\beta^\pm$ transition strengths ($S^\pm$) must achieve the Ikeda sum rule (ISR) [30]

$$S^\pm = \sum_i B_{GT}^{(\pm)}(\omega_i), \qquad (3.36)$$

and

$$\mathrm{ISR} = S^{(-)} - S^{(+)} \cong 3(\mathrm{N} - \mathrm{Z}). \qquad (3.37)$$

3.2.4. The pn-QRPA Equation In Odd Mass Nuclei

In this part, the Gamow-Teller collective effective interaction are discussed in odd A mass nuclei. One particle and one hole nuclei allow of the simplest possible theoretical description of their states. The structure of one-particle nuclei within the simple mean field approximation is the following. One-proton states $|\nu\rangle$ and one-neutron states $|k\rangle$ are described as

$$|\nu\rangle = \mathrm{C}_\nu^\dagger|\mathrm{core}\rangle, \qquad |k\rangle = \mathrm{C}_\mathrm{k}^\dagger|\mathrm{core}\rangle \qquad (3.38)$$

where $|\mathrm{core}\rangle$ is the core with its Fermi level at some magic number. The operator $\mathrm{C}_\nu^\dagger$ and $\mathrm{C}_\mathrm{k}^\dagger$ create a proton and a neutron in single particle orbital above the magic shell gap. These nuclei are odd-mass or odd-A nuclei. The strength parameters of nuclei with odd-neutron (k) and with odd-proton (ν) are as follows

$$(\gamma_\rho)^n = -\sum_{np} E_{np}^\rho b_{np}^\rho + \frac{1}{2j_k + 1}\sum_p K_{kp}^\rho d_{kp}^\rho \qquad (3.39)$$

and

$$(\gamma_\rho)^p = -\sum_{np} E_{np}^\rho b_{np}^\rho - \frac{1}{2j_\nu + 1}\sum_n K_{n\nu}^\rho d_{n\nu}^\rho \qquad (3.40)$$

where E and K are the reduced matrix elements.

Firstly, the nucleus consisting of odd-neutron investigated. The wavefunction of this one-quasiparticle in the pn-QRPA method is selected as follows

$$|\,j_k m_k\rangle^n = \left(\Omega^{+}_{j_k m_k}\right)^n |0\rangle = \left[N^n_{ik}\alpha^{+}_{j_k m_k} + \sum_{i\nu\mu} R^{k\nu}_{ni}\langle j_\nu m_\nu 1\mu | j_k m_k\rangle \hat{Q}^{+}_i(\mu)\alpha^{+}_{j_\nu m_\nu}\right]|0\rangle \qquad (3.41)$$

where N^n_{ik} , $R^{k\nu}_{ni}$ and n are one-quasiparticle amplitude of the wavefunction, one particle + one-phonon amplitude and the particle number, respectively. One-phonon is composed of three quasiparticle.

The N^n_{ik} and $R^{k\nu}_{ni}$ amplitudes satisfy normalization condition from the commutation condition below,

$$\left[\Omega^n_{j_k m_k}, \Omega^{+n'}_{j_{k'} m_{k'}}\right] = \delta_{j_k j_{k'}}\delta_{m_k m_{k'}}\delta_{nn'} \qquad (3.42)$$

$$N^n_{ik}N^{n'}_{ik} + \sum_{i\nu} R^{k\nu}_{ni}R^{k\nu}_{n'i} = \delta_{nn'} \qquad (3.43)$$

Thus, the equation of motion the pn-QRPA may be written as follow

$$\left[\hat{H}, \left(\Omega^{+}_{j_k m_k}\right)^n\right]|0\rangle = \omega^k_n \left(\Omega^{+}_{j_k m_k}\right)^n |0\rangle \qquad (3.44)$$

After some mathematical operations, the secular equation for the ω^k_n energies is found as follows

$$\omega^k_n - E_{jk} = \frac{3}{2j_k + 1}\sum_{i,j_\nu} \frac{\left(\sum_\rho \frac{1}{2(\gamma_\rho)^n} q^\rho_i K^\rho_{k\nu}\right)}{\omega^k_n - E_{j\nu} - \omega_i} \qquad (3.45)$$

Quasiparticle + phonon amplitudes for each energy value are found by the following

$$R_{ni}^{k\nu} = \sqrt{\frac{3}{2j_k+1}}\frac{\sum_{\rho}\frac{1}{2(\gamma_\rho)^n} q_i^\rho K_{np}^\rho}{\omega_n^k - E_{j\nu} - \omega_i} N_{ik}^n \tag{3.46}$$

$$N_{ik}^n = \frac{1}{\sqrt{Z(\omega_n^k)}} \tag{3.47}$$

The $Z(\omega_n^k)$ normalization coefficient are calculated with eq. (3.43).

$$Z(\omega_n^k) = 1 + \frac{3}{2j_k+1}\sum_{i\nu}\left(\frac{\sum_{\rho}\frac{1}{2\gamma_\rho^n} q_i^\rho K_{np}^\rho}{\omega_n^k - E_{j\nu} - \omega_i}\right)^2 \tag{3.48}$$

Necessary expressions for odd proton nucleus are given as below for the wavefunction, the motion of equation, the secular equation, the normalization condition and the amplitudes of wavefunction, respectively.

$$|\, j_\nu m_\nu \rangle^n = (\Omega^+_{j_\nu m_\nu})^n |0\rangle = \left[N_{i\nu}^n \alpha^+_{j_\nu m_\nu} + \sum_{ik\mu} R_{ni}^{k\nu} \langle j_k m_k 1\mu | j_\nu m_\nu \rangle \hat{Q}_i^+(\mu) \alpha^+_{j_k m_k} \right] |0\rangle \tag{3.49}$$

$$\left[\hat{H}, (\Omega^+_{j_\nu m_\nu})^n \right] |0\rangle = \omega_n^k (\Omega^+_{j_\nu m_\nu})^n |0\rangle \tag{3.50}$$

$$\omega_n^k - E_{j\nu} = \frac{3}{2j_\nu+1}\sum_{i,jk}\frac{\left(\sum_{\rho}\frac{1}{2(\gamma_\rho)^p} q_i^\rho K_{k\nu}^\rho\right)}{\omega_n^k - E_{jk} - \omega_i} \tag{3.51}$$

$$N_{ik}^n N_{ik}^{n'} + \sum_{i\nu} R_{ni}^{k\nu} R_{n'i}^{k\nu} = \delta_{nn'} \tag{3.52}$$

$$R_{ni}^{k\nu} = (-1)^{jk+j\nu} \sqrt{\frac{3}{2j_\nu + 1}} \frac{\sum_\rho \frac{1}{2(\gamma_\rho)^p} q_i^\rho K_{k\nu}^\rho}{\omega_n^k - E_{jk} - \omega_i} N_{i\nu}^n \tag{3.53}$$

$$N_{i\nu}^n = \frac{1}{\sqrt{Z(\omega_n^\nu)}} \tag{3.54}$$

3.2.5. Matrix Elements Of $\beta^{\pm}$ Transition In Odd Mass Nuclei

The $\beta^{\pm}$ decays were investigated between odd mass nuclei. The reduced beta decay probability is given that

$$B(j_k \to j_\nu, \beta^-) = \frac{1}{(2j_k + 1)} \left| \left\langle j_\nu \left\| \hat{G}_{1\mu}^- \right\| j_k \right\rangle \right|^2 . \tag{3.55}$$

The beta decay transitions are two kinds for independent quasiparticle model.

- ✓ the beta decay transitions with odd nucleon: state non-changing the number of pair.
- ✓ the beta decay transitions with a nucleon in core: state changing the number of pair.

These transitions are investigated separately as explained in the below sections.

3.2.5.1. State Non-Changing The Number Of Pair

The negative beta decay transition matrix element $\left| j_k m_k \right\rangle^n \xrightarrow{\beta^-} \left| j_\nu m_\nu \right\rangle^n$, from a neutron state $j_k m_k$ to proton state $j_\nu m_\nu$ are calculated as follow

$$M(j_k \to j_\nu, \beta^-) = \left\langle j_\nu m_\nu \left| \hat{G}_{1\mu}^- \right| j_k m_k \right\rangle \tag{3.56}$$

$$M\left(j_k \to j_\nu, \beta^-\right) = \left\langle 0 \left| N^n_{i\nu} \alpha_{j_\nu m_\nu} + \sum_{ik'\mu'} R^{\nu k'}_{ni'} \left\langle j_{k'} m_{k'} 1\mu' \middle| j_\nu m_\nu \right\rangle \hat{Q}_{i'}(\mu') \alpha_{j_{k'} m_{k'}} \right| \hat{G}^-_{1\mu} \right|$$

$$N^n_{ik} \alpha^+{}_{j_k m_k} + \sum_{i''\nu'\mu''} R^{\nu' k}_{ni''} \left\langle j_{\nu'} m_{\nu'} 1\mu'' \middle| j_k m_k \right\rangle \hat{Q}^+{}_{i''}(\mu'') \alpha^+{}_{j_{\nu'} m_{\nu'}} \left| 0 \right\rangle \tag{3.57}$$

This matrix element consists of four terms:

The NN term is

$$\Rightarrow N^n_{i\nu} N^n_{ik} d_{k\nu} \sqrt{\frac{3}{2j_k + 1}} \left\langle j_\nu m_\nu 1\mu \middle| j_k m_k \right\rangle, \tag{3.58}$$

The RR term is

$$\Rightarrow \sqrt{\frac{3}{2j_k + 1}} \left\langle j_\nu m_\nu 1\mu \middle| j_k m_k \right\rangle \sqrt{(2j_k + 1)(2j_\nu + 1)} \times$$

$$\sum_{i'k'\nu'} (-1)^{j\nu + jk'} R^{\nu k'}_{ni'} R^{k\nu'}_{ni'} \bar{d}_{k'\nu'} \begin{Bmatrix} j_{k'} & 1 & j_\nu \\ j_k & 1 & j_{\nu'} \end{Bmatrix} \tag{3.59}$$

The RN term is

$$\Rightarrow \sum_i (-1)^{j\nu - jk} M^i_{\beta^-} R^{\nu k}_{ni} N^n_{ik} \sqrt{\frac{2j_\nu + 1}{2j_k + 1}} \left\langle j_\nu m_\nu 1\mu \middle| j_k m_k \right\rangle \tag{3.60}$$

The NR term is

$$\Rightarrow \sum_i M^i_{\beta^+} N^n_{i\nu} R^{\nu k}_{ni} \sqrt{\frac{2j_k + 1}{2j_\nu + 1}} \left\langle j_\nu m_\nu 1\mu \middle| j_k m_k \right\rangle. \tag{3.61}$$

Thus, the reduced matrix element for state non-changing the number of pair is

$$\left\langle j_{\nu}\left\|\hat{G}^{-}\right\|j_{k}\right\rangle=\sqrt{3}d_{k\nu}N^{n}_{i\nu}N^{n}_{ik}+$$

$$\sqrt{3(2j_{k}+1)(2j_{\nu}+1)}\sum_{k'\nu'}(-1)^{j\nu+jk'}R^{\nu k'}_{ni'}R^{k\nu'}_{ni'}\bar{d}_{k'\nu'}\begin{Bmatrix} j_{k'} & 1 & j_{\nu} \\ j_{k} & 1 & j_{\nu'} \end{Bmatrix}+$$

$$\sum_{i}(-1)^{j\nu-jk}M^{i}_{\beta^{-}}R^{\nu k}_{ni}N^{n}_{ik}\sqrt{(2j_{\nu}+1)}+\sum_{i}M^{i}_{\beta^{+}}N^{n}_{i\nu}R^{\nu k}_{ni}\sqrt{(2j_{k}+1)} \qquad (3.62)$$

which is found by Wigner Eckart theorem [31].

$$\left\langle j_{\nu}m_{\nu}\left|\hat{G}^{-}_{1\mu}\right|j_{k}m_{k}\right\rangle=\frac{\left\langle j_{\nu}m_{\nu}1\mu|j_{k}m_{k}\right\rangle}{\sqrt{2j_{k}+1}}\left\langle j_{\nu}\left\|\hat{G}^{-}\right\|j_{k}\right\rangle \qquad (3.63)$$

3.2.5.2. State Changing The Number Of Pair

The beta decay transition matrix element $\left|j_{k}m_{k}\right\rangle^{n}\xrightarrow{\beta^{+}}\left|j_{\nu}m_{\nu}\right\rangle^{n}$, for an odd neutron state $j_{k}m_{k}$ changing the number of pair are calculated as following

$$M(j_{k}\rightarrow j_{\nu},\beta^{+})=\left\langle j_{\nu}m_{\nu}\left|\hat{G}^{+}_{1\mu}\right|j_{k}m_{k}\right\rangle \qquad (3.64)$$

$$M(j_{k}\rightarrow j_{\nu},\beta^{+})=\langle 0|N^{n}_{i\nu}\alpha_{j_{\nu}m_{\nu}}+\sum_{ik'\mu'}R^{\nu k'}_{ni'}\langle j_{k'}m_{k'}1\mu'|j_{\nu}m_{\nu}\rangle\hat{Q}_{i'}(\mu')\alpha_{j_{k'}m_{k'}}\left|\hat{G}^{+}_{1\mu}\right|$$

$$N^{n}_{ik}\alpha^{+}{}_{j_{k}m_{k}}+\sum_{i''\nu'\mu''}R^{\nu'k}_{ni''}\langle j_{\nu'}m_{\nu'}1\mu''|j_{k}m_{k}\rangle\hat{Q}^{+}{}_{i''}(\mu'')\alpha^{+}{}_{j_{\nu'}m_{\nu'}}|0\rangle \qquad (3.65)$$

The matrix element is composed of four terms and every term was separately considered.

The NN term is

$$\Rightarrow(-1)^{\mu}N^{n}_{i\nu}N^{n}_{ik}\bar{d}_{k\nu}\sqrt{\frac{3}{2j_{k}+1}}\langle j_{\nu}m_{\nu}1\text{-}\mu|j_{k}m_{k}\rangle \qquad (3.66)$$

The RR term is

$$\Rightarrow (-1)^{\mu}\sqrt{\frac{3}{2j_k+1}}\left\langle j_\nu m_\nu 1-\mu \middle| j_k m_k \right\rangle \sqrt{(2j_k+1)(2j_\nu+1)}\times$$

$$\sum_{i'k'\nu'}(-1)^{j\nu+jk'} R^{\nu k'}_{ni'} R^{k\nu'}_{ni'} d_{k'\nu'} \begin{Bmatrix} j_{k'} & 1 & j_\nu \\ j_k & 1 & j_{\nu'} \end{Bmatrix} \tag{3.67}$$

The RN term is

$$\Rightarrow (-1)^{\mu}\sum_i M^i_{\beta^-} N^n_{i\nu} R^{\nu k}_{ni}\sqrt{\frac{2j_\nu+1}{2j_k+1}}\left\langle j_\nu m_\nu 1-\mu \middle| j_k m_k \right\rangle \tag{3.68}$$

The NR term is

$$\Rightarrow (-1)^{\mu}\sum_i (-1)^{j\nu-jk} M^i_{\beta^+} N^n_{ik} R^{\nu k}_{ni}\sqrt{\frac{2j_\nu+1}{2j_k+1}}\left\langle j_\nu m_\nu 1-\mu \middle| j_k m_k \right\rangle \tag{3.69}$$

Thus, the reduced matrix element in state changing the number of pair is written as follows

$$\left\langle j_k \middle\| \hat{G}^+ \middle\| j_\nu \right\rangle = \sqrt{3}\bar{d}_{k\nu} N^n_{i\nu} N^n_{ik} +$$

$$\sqrt{3(2j_k+1)(2j_\nu+1)}\sum_{k'\nu'}(-1)^{j\nu+jk'} R^{\nu k'}_{ni'} R^{k\nu'}_{ni'} d_{k'\nu'} \begin{Bmatrix} j_{k'} & 1 & j_\nu \\ j_k & 1 & j_{\nu'} \end{Bmatrix} +$$

$$\sum_i (-1)^{j\nu-jk} M^i_{\beta^+} N^n_{ik} R^{\nu k}_{ni}\sqrt{(2j_\nu+1)} + \sum_i M^i_{\beta^-} N^n_{i\nu} R^{\nu k}_{ni}\sqrt{(2j_k+1)} \tag{3.70}$$

The beta decay transition matrix elements from odd proton nucleus to odd neutron nucleus are found by substituting $k \leftrightarrow \nu$ and $\beta^+ \leftrightarrow \beta^-$.

The ft values for the Gamow-Teller $1^+ \rightarrow 0^+$ transitions are calculated by the following formula

$$ft = \frac{D}{(g_A/g_V)^2 B_{GT}^{\pm}(\omega_i)} \tag{3.71}$$

where $D = \frac{2\pi^3\hbar^7 ln2}{g_V^2 m_e^5 c^4} = 6163{,}4\ sec$ and $g_A/g_V = -1{,}26$.

The matrix elements and the solutions of secular equation are broadly given in Appendix A and Appendix B.

PART 4

RESULTS AND CONCLUSIONS

4.1. RESULTS

In this study, the beta decay properties of Te-117, Te-119 and Te-121 isotopes were investigated by using Pyatov-Salamov method and the schematic model in the framework of the pn-QRPA microscopic approximation. The half-lives values for Te-117, Te-119 and Te-121 isotopes were obtained first time in present work with this approximation. The experimental and theoretical information related to Te-117, Te-119 and Te-121 isotopes are presented in Table 4.1. These data were taken from National Nuclear Data Center.

Table 4.1. The general information related to Te-117, Te-119 and Te-121 isotopes [32].

Parent Nucleus	**Parent J^{π}**	**Parent $t_{1/2}$**	**Decay Mode**	**gs-gs Q value (keV)**	**Daughter Nucleus**
${}^{117}_{52}Te$	$1/2^{+}$	62 m	β^{+}	3549	${}^{117}_{51}Sb$
${}^{119}_{52}Te$	$1/2^{+}$	16,05 h	ε	2293	${}^{119}_{51}Sb$
${}^{121}_{52}Te$	$1/2^{+}$	19,17 d	ε	1054	${}^{121}_{51}Sb$

Also, the experimental nuclear structure data of Te-117, Te-119 and Te-121 isotopes are broadly given in Figure 4.1, Figure 4.2. and Figure 4.3. In these figures, level schemes from radioactive decay for Te isotopes with mass number A=117, 119, 121 are presented.

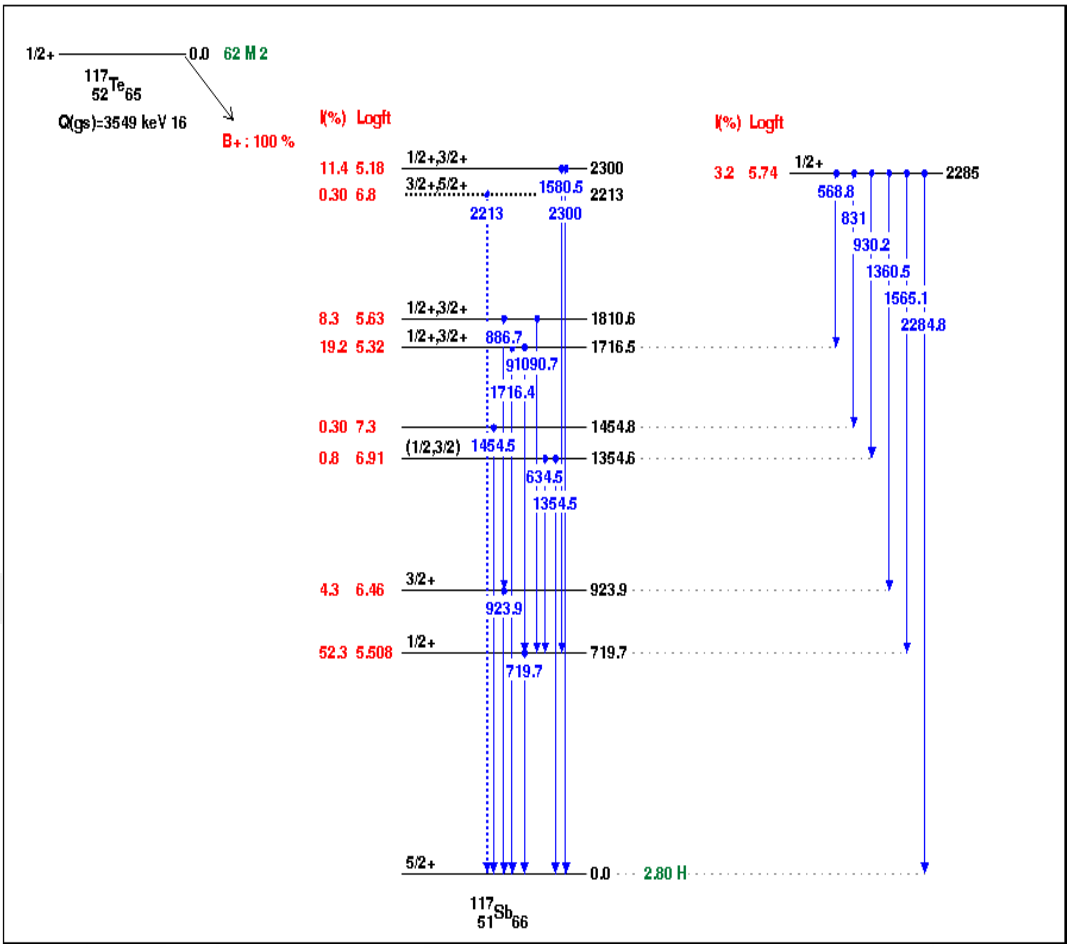

Figure 4.1. Decay scheme of Te-117 isotope [33].

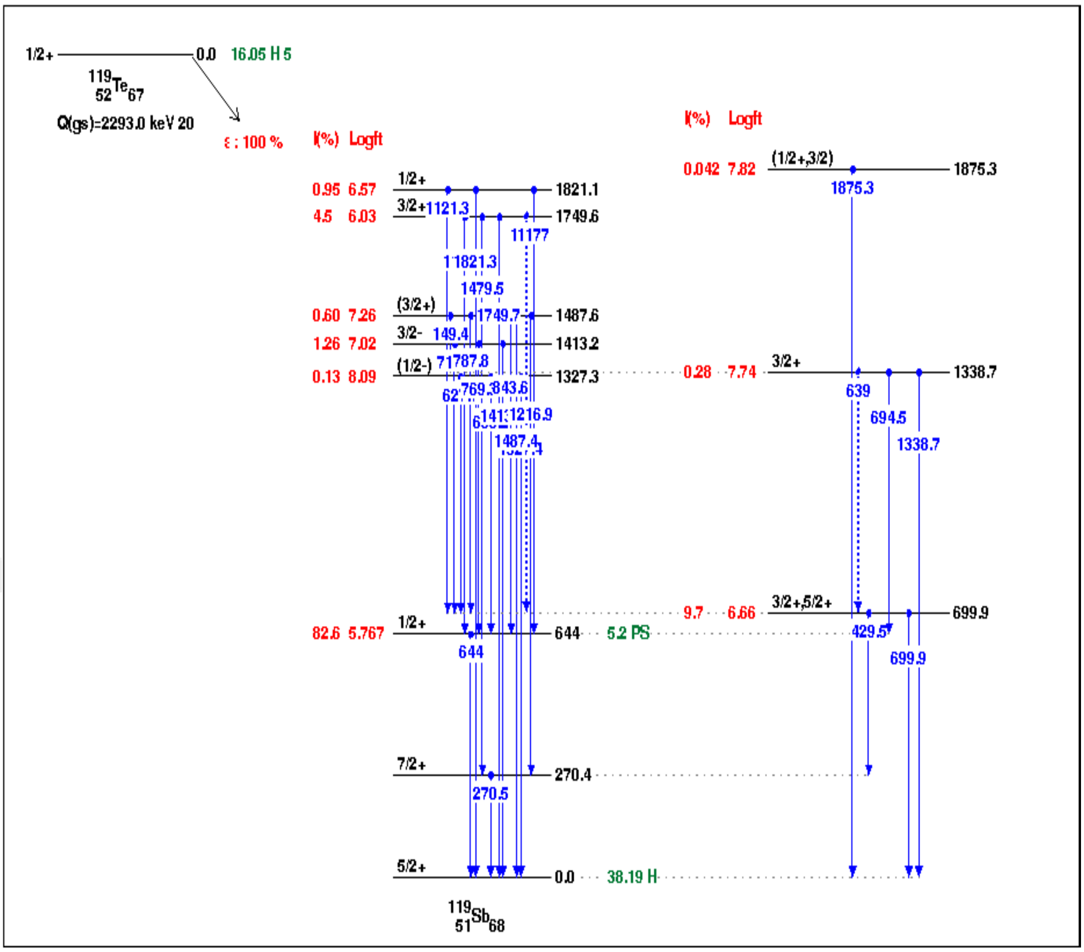

Figure 4.2. Decay scheme of Te-119 isotope [34].

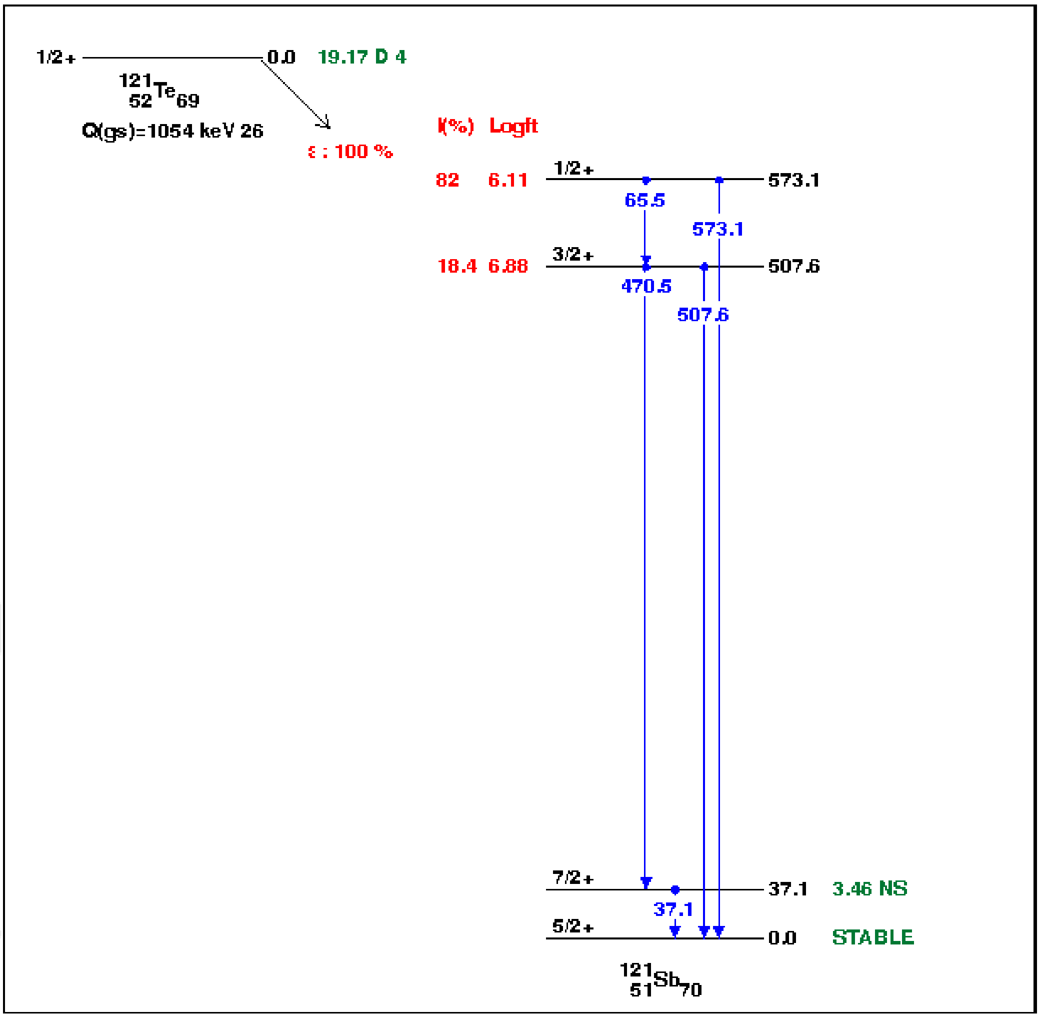

Figure 4.3. Decay scheme of Te-121 isotope [35].

The theoretical calculated half-lives for some odd mass Te isotopes and the fixed values of Gamow-Teller effective interaction parameters were presented in this section. The Woods–Saxon potential with Chepurnov parameterization were used in our calculations which were done in this work. The pair correlation functions of neutron and proton were selected as $C_n = C_p = 12/\sqrt{A}$ for the open-shell nuclei [25]. The basis used in own calculation contains all the neutron-proton transitions which change the radial quantum number n by $\Delta n = 0, 1, 2, 3$.

The χ_{ph} and the χ_{pp} terms are the GT effective interaction parameters in the particle-hole and the particle-particle channels. The χ_{ph} and χ_{pp} parameters in both PM and

SM are used in units of $5.2A^{-0.7}$MeV and $0.52A^{-0.7}$MeV, respectively. The fixed χ_{ph} and χ_{pp} values of Te-117, Te-119 and Te-121 isotopes are presented in Table 4.2.

Table 4.2. The GT effective interaction parameters.

Nuclei	χ_{ph}	χ_{pp}
^{117}Te	1,1	0,25
^{119}Te	1,1	0,25
^{121}Te	1,1	0,25

The beta decay strength distributions for these nuclei were obtained by calculating the reduced matrix elements in Pyatov's restoration method and the schematic model. The *ft* values in eq. (3.71) have been calculated with the calculated these strength distributions. The half-lives of the Gamow-Teller $1^+ \rightarrow 0^+$ transition for PM, SM and the corresponding experimental data are presented in Table 4.3.

Table 4.3. The calculated half-lives values for ^{117,119,121}Te isotopes.

Transition	**The half-lives (*sec*)**		
	PM	**SM**	**Exp. [36]**
$^{117}_{52}Te + e^- \rightarrow {}^{117}_{51}Sb + \nu$	3,48×10^3	3,02×10^3	3,72×10^3
$^{119}_{52}Te + e^- \rightarrow {}^{119}_{51}Sb + \nu$	5,52×10^4	5,17×10^4	5,77×10^4
$^{121}_{52}Te + e^- \rightarrow {}^{121}_{51}Sb + \nu$	15,93×10^5	15,28×10^5	16,56×10^5

The half-live calculations are compared with the recent atomic mass evaluation 2012 data [36], the schematic model and the Pyatov-Salamov method in this Table. The decay half-lives in units *sec* are given in the second, third and fourth columns in Table 4.3. It is clearly seen from Table 4.3 that the calculated half-lives of the schematic model are lower than the corresponding experimental values. The half-lives of Pyatov-Salamov method are (0,2-0,6) units smaller than the experimental data. The calculated half-lives for Te-117, Te-119 and Te-121 isotopes by Pyatov-Salamov method are in good agreement with the experimental values. This case shows that the restoration term plays a more efficient role in beta decay. The restoration term is more effective in heavier isotopes. The *ft* values of beta decay transition become closer to the experimental value when the repulsive particle-particle effective interaction term is taken in consideration for the charge-exchange spin-spin forces.

The numerical calculations in the PM and SM were done with Fortran 77 code written by Prof. Dr. Djavad Salamov and Assoc. Prof. Dr. Necla Cakmak. This code is given in Appendix C.

4.2. CONCLUSIONS

In this thesis, the (*p*,*n*) charge-exchange effective interaction properties in some odd mass Te isotopes have been searched by using two different methods within the framework of the pn-QRPA. The total Hamiltonian containing an effective interaction term coming from the restoration according to Pyatov-Salomov method and the Hamiltonian of the schematic model without restoration have been solved by the pn-QRPA model. Later, the eigenvalues and eigenfunctions of the total Hamiltonian have been found in both methods. Thus, the effect of the restoration term on the Gamow-Teller transition strength distributions has been investigated for Te-117, Te-119 and Te-121 isotopes by giving a comparison of the results of both methods. The calculation results show the significance of the restoration of the broken supersymmetry between total Hamiltonian operator and Gamow-Teller operator. As a result, the restoration leads to a decrease in the probability of the positive beta decay transition.

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APPENDIX A.

THE MATRIX ELEMENTS

$$E^{\rho}_{np} = \frac{1}{2}\left\{\varepsilon_{np}\left(\bar{b}_{np} - \rho b_{np}\right) + \bar{k}_{np} + \rho k_{np}\right\}$$

$$P^{\rho}_{np} \equiv \frac{1}{2}\left\{\left(\varepsilon_{n} - \varepsilon_{p}\right)\left(d_{np} + \rho\bar{d}_{np}\right) + \left(k'_{np} + \rho\,\overline{k'_{np}}\right)\right\}$$

$$k_{np} = d_{np} + f_{np} - g_{np}$$

$$d_{np} = \frac{\left\langle j_n \left\| V_c(r)\sigma \right\| j_p \right\rangle}{\sqrt{3}} \vartheta_{j_n} u_{j_p}$$

$$f_{np} = \frac{\left\lfloor j_p\left(j_p + 1\right) - l_p\left(l_p + 1\right) - 3/4 \right\rfloor}{2\sqrt{3}} \left\langle j_n \left\| \left(V^{ls}_1(r)\vec{\sigma}\left(\vec{l}\,\vec{s}\right)\right) \right\| j_p \right\rangle \vartheta_{j_n} u_{j_p}$$

$$g_{np} = \frac{1}{\sqrt{3}} \left\langle j_n \left\| \left(V^{ls}_0(r) - \frac{1}{2} V^{ls}_1(r) \right) (-i) \left(\vec{l} \times \vec{\sigma}\right) \right\| j_p \right\rangle \vartheta_{j_n} u_{j_p}$$

$$\bar{X}_{np} = \frac{X_{np}}{v_{jn} u_{jp}} v_{jp} u_{jn} \left(X = b, d, f, g\right)$$

APPENDIX B.

THE SECULAR EQUATION OF pn-QRPA

$$X_i^+ = \sum_{np} E_{np}^+ \left\{ \begin{array}{l} \dfrac{E_{np}^+ \varepsilon_{np}}{\gamma_+ \left(\omega_i^2 - \varepsilon_{np}^2\right)} X_i^+ + \dfrac{E_{np}^- \omega_i}{\gamma_- \left(\omega_i^2 - \varepsilon_{np}^2\right)} X_i^- + \\ 2\chi_{ph}\left(\dfrac{\bar{b}_{np}}{\omega_i - \varepsilon_{np}} - \dfrac{b_{np}}{\omega_i + \varepsilon_{np}} \right) Y_1^i + 2\chi_{ph}\left(\dfrac{b_{np}}{\omega_i - \varepsilon_{np}} - \dfrac{\bar{b}_{np}}{\omega_i + \varepsilon_{np}} \right) Y_2^i - \\ 2\chi_{pp}\left(\dfrac{\bar{l}_{np}}{\omega_i - \varepsilon_{np}} + \dfrac{l_{np}}{\omega_i + \varepsilon_{np}} \right) Y_3^i - 2\chi_{pp}\left(\dfrac{l_{np}}{\omega_i - \varepsilon_{np}} + \dfrac{\bar{l}_{np}}{\omega_i + \varepsilon_{np}} \right) Y_4^i \end{array} \right\}$$

$$X_i^- = \sum_{np} E_{np}^- \left\{ \begin{array}{l} \dfrac{E_{np}^- \omega_i}{\gamma_+ \left(\omega_i^2 - \varepsilon_{np}^2\right)} X_i^+ + \dfrac{E_{np}^- \omega_i \, \varepsilon_{np}}{\gamma_- \left(\omega_i^2 - \varepsilon_{np}^2\right)} X_i^- + \\ 2\chi_{ph}\left(\dfrac{\bar{b}_{np}}{\omega_i - \varepsilon_{np}} + \dfrac{b_{np}}{\omega_i + \varepsilon_{np}} \right) Y_1^i + 2\chi_{ph}\left(\dfrac{b_{np}}{\omega_i - \varepsilon_{np}} + \dfrac{\bar{b}_{np}}{\omega_i + \varepsilon_{np}} \right) Y_2^i - \\ 2\chi_{pp}\left(\dfrac{\bar{l}_{np}}{\omega_i - \varepsilon_{np}} - \dfrac{l_{np}}{\omega_i + \varepsilon_{np}} \right) Y_3^i - 2\chi_{pp}\left(\dfrac{l_{np}}{\omega_i - \varepsilon_{np}} - \dfrac{\bar{l}_{np}}{\omega_i + \varepsilon_{np}} \right) Y_4^i \end{array} \right\}$$

$$Y_1^i = \sum_{np} \left\{ \begin{array}{l} \dfrac{E_{np}^+}{2\gamma_+}\left(\dfrac{\bar{b}_{np}}{\omega_i - \varepsilon_{np}} - \dfrac{b_{np}}{\omega_i + \varepsilon_{np}} \right) X_i^+ + \dfrac{E_{np}^-}{2\gamma_-}\left(\dfrac{\bar{b}_{np}}{\omega_i - \varepsilon_{np}} + \dfrac{b_{np}}{\omega_i + \varepsilon_{np}} \right) X_i^- + \\ 2\chi_{ph}\left(\dfrac{\bar{b}_{np}^2}{\omega_i - \varepsilon_{np}} - \dfrac{b_{np}^2}{\omega_i + \varepsilon_{np}} \right) Y_1^i + 2\chi_{ph} \bar{b}_{np} b_{np} \dfrac{2\varepsilon_{np}}{\omega_i^2 - \varepsilon_{np}^2} Y_2^i - \\ 2\chi_{pp}\left(\dfrac{\bar{b}_{np}\bar{l}_{np}}{\omega_i - \varepsilon_{np}} + \dfrac{b_{np} l_{np}}{\omega_i + \varepsilon_{np}} \right) Y_3^i - 2\chi_{pp}\left(\dfrac{\bar{b}_{np} l_{np}}{\omega_i - \varepsilon_{np}} + \dfrac{b_{np}\bar{l}_{np}}{\omega_i + \varepsilon_{np}} \right) Y_4^i \end{array} \right\}$$

$$
Y_2^{\mathrm{i}} = \sum_{\mathrm{np}} \left\{ \begin{aligned}
&\frac{E_{\mathrm{np}}^{+}}{2\gamma_{+}}\left(\frac{b_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}-\frac{\bar{b}_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)X_{\mathrm{i}}^{+}+\frac{E_{\mathrm{np}}^{-}}{2\gamma_{-}}\left(\frac{b_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}+\frac{\bar{b}_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)X_{\mathrm{i}}^{-}+ \\
&2\chi_{\mathrm{ph}}\bar{b}_{\mathrm{np}}b_{\mathrm{np}}\frac{2\varepsilon_{\mathrm{np}}}{\omega_{\mathrm{i}}^{2}-\varepsilon_{\mathrm{np}}^{2}}Y_1^{\mathrm{i}}+2\chi_{\mathrm{ph}}\left(\frac{b_{\mathrm{np}}^{2}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}-\frac{\bar{b}_{\mathrm{np}}^{2}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)Y_2^{\mathrm{i}}- \\
&2\chi_{\mathrm{pp}}\left(\frac{b_{\mathrm{np}}\bar{l}_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}+\frac{\bar{b}_{\mathrm{np}}l_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)Y_3^{\mathrm{i}}-2\chi_{\mathrm{pp}}\left(\frac{b_{\mathrm{np}}l_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}+\frac{\bar{b}_{\mathrm{np}}\bar{l}_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)Y_4^{\mathrm{i}}
\end{aligned} \right\}
$$

$$
Y_3^{\mathrm{i}} = \sum_{\mathrm{np}} \left\{ \begin{aligned}
&\frac{E_{\mathrm{np}}^{+}}{2\gamma_{+}}\left(\frac{\bar{l}_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}+\frac{l_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)X_{\mathrm{i}}^{+}+\frac{E_{\mathrm{np}}^{-}}{2\gamma_{-}}\left(\frac{\bar{l}_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}-\frac{l_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)X_{\mathrm{i}}^{-}+ \\
&2\chi_{\mathrm{ph}}\left(\frac{\bar{b}_{\mathrm{np}}\bar{l}_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}+\frac{b_{\mathrm{np}}l_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)Y_1^{\mathrm{i}}+2\chi_{\mathrm{ph}}\left(\frac{b_{\mathrm{np}}\bar{l}_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}-\frac{\bar{b}_{\mathrm{np}}l_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)Y_2^{\mathrm{i}}- \\
&2\chi_{\mathrm{pp}}\left(\frac{\bar{l}_{\mathrm{np}}^{2}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}-\frac{l_{\mathrm{np}}^{2}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)Y_3^{\mathrm{i}}-2\chi_{\mathrm{pp}}\bar{l}_{\mathrm{np}}l_{\mathrm{np}}\frac{2\varepsilon_{\mathrm{np}}}{\omega_{\mathrm{i}}^{2}-\varepsilon_{\mathrm{np}}^{2}}Y_4^{\mathrm{i}}
\end{aligned} \right\}
$$

$$
Y_4^{\mathrm{i}} = \sum_{\mathrm{np}} \left\{ \begin{aligned}
&\frac{E_{\mathrm{np}}^{+}}{2\gamma_{+}}\left(\frac{l_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}+\frac{\bar{l}_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)X_{\mathrm{i}}^{+}+\frac{E_{\mathrm{np}}^{-}}{2\gamma_{-}}\left(\frac{l_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}-\frac{\bar{l}_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)X_{\mathrm{i}}^{-}+ \\
&2\chi_{\mathrm{ph}}\left(\frac{\bar{b}_{\mathrm{np}}l_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}+\frac{b_{\mathrm{np}}\bar{l}_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)Y_1^{\mathrm{i}}+2\chi_{\mathrm{ph}}\left(\frac{b_{\mathrm{np}}l_{\mathrm{np}}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}-\frac{\bar{b}_{\mathrm{np}}\bar{l}_{\mathrm{np}}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)Y_2^{\mathrm{i}}- \\
&2\chi_{\mathrm{pp}}\bar{l}_{\mathrm{np}}l_{\mathrm{np}}\frac{2\varepsilon_{\mathrm{np}}}{\omega_{\mathrm{i}}^{2}-\varepsilon_{\mathrm{np}}^{2}}Y_3^{\mathrm{i}}-2\chi_{\mathrm{pp}}\left(\frac{l_{\mathrm{np}}^{2}}{\omega_{\mathrm{i}}-\varepsilon_{\mathrm{np}}}-\frac{\bar{l}_{\mathrm{np}}^{2}}{\omega_{\mathrm{i}}+\varepsilon_{\mathrm{np}}}\right)Y_4^{\mathrm{i}}
\end{aligned} \right\}
$$

$$
A_{11}X_{\mathrm{i}}^{+}+A_{12}X_{\mathrm{i}}^{-}+A_{13}Y_1^{\mathrm{i}}+A_{14}Y_2^{\mathrm{i}}+A_{15}Y_3^{\mathrm{i}}+A_{16}Y_4^{\mathrm{i}}=0
$$

$$
A_{21}X_{\mathrm{i}}^{+}+A_{22}X_{\mathrm{i}}^{-}+A_{23}Y_1^{\mathrm{i}}+A_{24}Y_2^{\mathrm{i}}+A_{25}Y_3^{\mathrm{i}}+A_{26}Y_4^{\mathrm{i}}=0
$$

$$A_{31}X_i^+ + A_{32}X_i^- + A_{33}Y_1^i + A_{34}Y_2^i + A_{35}Y_3^i + A_{36}Y_4^i = 0$$

$$A_{41}X_i^+ + A_{42}X_i^- + A_{43}Y_1^i + A_{44}Y_2^i + A_{45}Y_3^i + A_{46}Y_4^i = 0$$

$$A_{51}X_i^+ + A_{52}X_i^- + A_{53}Y_1^i + A_{54}Y_2^i + A_{55}Y_3^i + A_{56}Y_4^i = 0$$

$$A_{61}X_i^+ + A_{62}X_i^- + A_{63}Y_1^i + A_{64}Y_2^i + A_{65}Y_3^i + A_{66}Y_4^i = 0$$

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{vmatrix} = 0$$

$$A_{11} = 1 - \frac{1}{\gamma_+} \sum_{np} \frac{\left(E_{np}^+\right)^2 \varepsilon_{np}}{\omega_i^2 - \varepsilon_{np}^2}$$

$$A_{12} = -\frac{\omega_i}{\gamma_-} \sum_{np} \frac{E_{np}^+ E_{np}^-}{\omega_i^2 - \varepsilon_{np}^2}$$

$$A_{13} = -2\chi_{ph} \sum_{np} E_{np}^+ \left(\frac{\bar{b}_{np}}{\omega_i - \varepsilon_{np}} - \frac{b_{np}}{\omega_i + \varepsilon_{np}} \right)$$

$$A_{14} = -2\chi_{ph} \sum_{np} E_{np}^+ \left(\frac{b_{np}}{\omega_i - \varepsilon_{np}} - \frac{\bar{b}_{np}}{\omega_i + \varepsilon_{np}} \right)$$

$$A_{15} = 2\chi_{pp} \sum_{np} E_{np}^+ \left(\frac{\bar{l}_{np}}{\omega_i - \varepsilon_{np}} + \frac{l_{np}}{\omega_i + \varepsilon_{np}} \right)$$

$$A_{16} = 2\chi_{pp}\sum_{np} E^{+}_{np}\left(\frac{l_{np}}{\omega_i - \varepsilon_{np}} + \frac{\bar{l}_{np}}{\omega_i + \varepsilon_{np}}\right)$$

$$A_{21} = \frac{\gamma_-}{\gamma_+} A_{12}$$

$$A_{22} = 1 - \frac{1}{\gamma_-}\sum_{np}\frac{\left(E^{-}_{np}\right)^2 \varepsilon_{np}}{\omega_i^2 - \varepsilon_{np}^2}$$

$$A_{23} = -2\chi_{ph}\sum_{np} E^{-}_{np}\left(\frac{\bar{b}_{np}}{\omega_i - \varepsilon_{np}} + \frac{b_{np}}{\omega_i + \varepsilon_{np}}\right)$$

$$A_{24} = -2\chi_{ph}\sum_{np} E^{-}_{np}\left(\frac{b_{np}}{\omega_i - \varepsilon_{np}} + \frac{\bar{b}_{np}}{\omega_i + \varepsilon_{np}}\right)$$

$$A_{25} = 2\chi_{pp}\sum_{np} E^{-}_{np}\left(\frac{\bar{l}_{np}}{\omega_i - \varepsilon_{np}} - \frac{l_{np}}{\omega_i + \varepsilon_{np}}\right)$$

$$A_{26} = 2\chi_{pp}\sum_{np} E^{-}_{np}\left(\frac{l_{np}}{\omega_i - \varepsilon_{np}} - \frac{\bar{l}_{np}}{\omega_i + \varepsilon_{np}}\right)$$

$$A_{31} = \frac{1}{4\gamma_+\chi_{ph}} A_{13}$$

$$A_{32} = \frac{1}{4\gamma_-\chi_{ph}} A_{23}$$

$$A_{33} = 1 - 2\chi_{ph}\sum_{np}\left(\frac{\bar{b}^2_{np}}{\omega_i - \varepsilon_{np}} - \frac{b^2{}_{np}}{\omega_i + \varepsilon_{np}}\right)$$

$$A_{34} = -2\chi_{ph} \sum_{np} \bar{b}_{np} b_{np} \frac{2\varepsilon_{np}}{\omega_i^2 - \varepsilon_{np}^2}$$

$$A_{35} = 2\chi_{pp} \sum_{np} \left(\frac{\bar{b}_{np} \bar{l}_{np}}{\omega_i - \varepsilon_{np}} + \frac{b_{np} l_{np}}{\omega_i + \varepsilon_{np}} \right)$$

$$A_{36} = 2\chi_{pp} \sum_{np} \left(\frac{\bar{b}_{np} l_{np}}{\omega_i - \varepsilon_{np}} + \frac{b_{np} \bar{l}_{np}}{\omega_i + \varepsilon_{np}} \right)$$

$$A_{41} = \frac{1}{4\gamma_+ \chi_{ph}} A_{14}$$

$$A_{42} = \frac{1}{4\gamma_- \chi_{ph}} A_{24}$$

$$A_{43} = A_{34}$$

$$A_{44} = 1 - 2\chi_{ph} \sum_{np} \left(\frac{b_{np}^2}{\omega_i - \varepsilon_{np}} - \frac{\bar{b}^2{}_{np}}{\omega_i + \varepsilon_{np}} \right)$$

$$A_{45} = 2\chi_{pp} \sum_{np} \left(\frac{b_{np} \bar{l}_{np}}{\omega_i - \varepsilon_{np}} + \frac{\bar{b}_{np} l_{np}}{\omega_i + \varepsilon_{np}} \right)$$

$$A_{46} = 2\chi_{pp} \sum_{np} \left(\frac{b_{np} l_{np}}{\omega_i - \varepsilon_{np}} + \frac{\bar{b}_{np} \bar{l}_{np}}{\omega_i + \varepsilon_{np}} \right)$$

$$A_{51} = -\frac{1}{4\gamma_+ \chi_{ph}} A_{15}$$

$$A_{52} = -\frac{1}{4\gamma_- \chi_{ph}} A_{25}$$

$$A_{53} = -\frac{\chi_{ph}}{\chi_{pp}} A_{35}$$

$$A_{54} = -\frac{\chi_{ph}}{\chi_{pp}} A_{45}$$

$$A_{55} = 1 + 2\chi_{pp} \sum_{np} \left(\frac{\bar{l}_{np}^2}{\omega_i - \varepsilon_{np}} - \frac{l^2_{np}}{\omega_i + \varepsilon_{np}} \right)$$

$$A_{56} = 2\chi_{pp} \sum_{np} \bar{l}_{np} l_{np} \frac{2\varepsilon_{np}}{\omega_i^2 - \varepsilon_{np}^2}$$

$$A_{61} = -\frac{1}{4\gamma_+ \chi_{pp}} A_{16}$$

$$A_{62} = -\frac{1}{4\gamma_- \chi_{pp}} A_{26}$$

$$A_{63} = -\frac{\chi_{ph}}{\chi_{pp}} A_{36}$$

$$A_{64} = -\frac{\chi_{ph}}{\chi_{pp}} A_{46}$$

$$A_{65} = A_{56}$$

$$A_{66} = 1 + 2\chi_{pp} \sum_{np} \left(\frac{l_{np}^2}{\omega_i - \varepsilon_{np}} - \frac{\bar{l}^2_{np}}{\omega_i + \varepsilon_{np}} \right)$$

$$L_1(\omega_i) = \frac{X_i^-}{X_i^+}$$

$$L_2(\omega_i) = \frac{Y_1^i}{X_i^+}$$

$$L_3(\omega_i) = \frac{Y_2^i}{X_i^+}$$

$$L_4(\omega_i) = \frac{Y_3^i}{X_i^+}$$

$$L_5(\omega_i) = \frac{Y_4^i}{X_i^+}$$

APPENDIX C.

FORTRAN CODE

```
      DIMENSION NF(2),CNF(2),ANZ(2),CN(2),EEE(500),VN(500),VP(500)
      DIMENSION EN(50,2),NN(50,2),LN(50,2),AJN(50,2),FFN(50,150,2)
      DIMENSION EPSNP(50,2),V3(50,2),U3(50,2),DNCHN(50,2),ANNL(300)
      DIMENSION PRSUMD(2),DELN(2),XLN(2),EFER(2),KCHH(2),AX(150)
      DIMENSION DPN(500),DPNX(500),FPN(500),FPNX(500),BIB(500)
      DIMENSION GPN(500),GPNX(500),EE(500),AAA1(100,100),RON(101)
      DIMENSION AMA(500),AME(500),PSINORM(500),FII(500),PSI(500)
      COMMON/BLOK1/ENPA(500),ENPE(500),EPN(500)
      COMMON/BLOK2/BPN(500),BPNX(500),QPN(500),QPNX(500)
      COMMON/BLOK3/MES,XPH,XPP,XKAPE,XKAPA
      COMMON/BLDD/DET(100,100),DETB(100),DETX(100)
      COMMON/COUL/ROP(101),SIMP(100)
      COMMON/BLOK4/A5,METKA
      COMMON/DET5/D5(5,5)
      COMMON/DET4/D4(4,4)
      OPEN(19,STATUS='OLD',FILE='DAT52116',FORM='FORMATTED')
      OPEN(23,FILE='XADDANK',STATUS='OLD')
      OPEN(99,FILE='first',STATUS='OLD')
C     OPEN(99,FILE='end',STATUS='OLD')
      WRITE(*,*)'METKA AND AV1'
      READ(*,*)METKA,AV1
      WRITE(*,*)'CNN/SQRT(A) AND CNP/SQRT(A)'
      READ(*,*)CNN,CP
      SIMP(1)=1.
      DO 124 IS=1,49
      SIMP(2*IS)=4.
  124 SIMP(2*IS+1)=2.
      SIMP(99)=1.
      READ(19,1112)A,AZ,DEV1,DEV2,DEV3,DEV4,DEV5,DEV6
      READ(19,1112)A,AZ,DEV1,DEV2,DEV3,DEV4,DEV5,DEV6
      WRITE(23,1112)A,AZ,DEV1,DEV2,DEV3,DEV4,DEV5,DEV6
 1112 FORMAT(8F8.3)
      READ(19,*) NF(1),NF(2),CNF(1),CNF(2),RO
```

```
      READ(19,*) ETA,DZET,AKSI,EFARK,EBET
      WRITE(23,*)'NUCLEUS A=',A,'Z=',AZ
      AN=A-AZ
      WRITE(*,*)'AXPH  AND AXPP'
      READ(*,*)AXPH,AXPP
      XPH=AXPH*5.2/A**0.7
      XPP=AXPP*0.52/A**0.7
      IF(METKA.EQ.1) WRITE(23,*)'SM  XPH=',XPH,'XPP=',XPP
      IF(METKA.EQ.0) WRITE(23,*)'SM+PM  XPH=',XPH,
     *'XPP=',XPP
      ANZ(1)=AN
      ANZ(2)=AZ
      DZETA=DZET*(1.+2.*(A-2.*AZ)/A)
      SABIT0=DZETA/AKSI*53.3
      SABITF=SABIT0*2.*ETA*(AN-AZ)/A
      SABITG=SABIT0*(1.-ETA*(AN-AZ)/A)
      SABIT1=SABIT0*2.*ETA*(AN-AZ)/A
      SABITV1=53.3*2.*ETA*(AN-AZ)/A
      CN(1)=CNN/SQRT(A)
      CN(2)=CP/SQRT(A)
      RCUL=RO*(A**0.333333)
      ROCH=1.24*(A**0.333333)
      VCUL=1.5*(AZ-1.)/RCUL*1.44
      SHAG=0.15
      DO 101 KL=1,2
      DELN(KL)=0.
      READ(19,3) KN
      KCHH(KL)=KN
    3 FORMAT(I3)
      NN1=NF(KL)
      WRITE(23,444)
      DO 2 I=1,KN
      READ(19,4) EN(I,KL),NN(I,KL),LN(I,KL),AJN(I,KL)
```

```
      READ(19,5) (FFN(I,J,KL),J=1,100)
 5  FORMAT(10(2X,F8.3))
 2  CONTINUE
      XL1=EN(NF(KL),KL)
66  SUM1=0.
      SUM2=0.
      PRSUMD(KL)=0.
      DO 6 I=1,KN
      B1=SQRT(CN(KL)**2+(EN(I,KL)-XL1)**2)
      EPSNP(I,KL)=B1
      SUM1=SUM1+(AJN(I,KL)+0.5)*(1-EN(I,KL)/B1)
      SUM2=SUM2+(AJN(I,KL)+0.5)/B1
      V3(I,KL)=SQRT(0.5*(1.-(EN(I,KL)-XL1)/B1))
      U3(I,KL)=SQRT(1.-V3(I,KL)**2)
      PRSUMD(KL)=PRSUMD(KL)+(2.*AJN(I,KL)+1.)*V3(I,KL)*V3(I,KL)
      DELN(KL)=DELN(KL)+((2.*AJN(I,KL)+1.)*V3(I,KL))**2*(1-V3(I,KL)**2)
 6  CONTINUE
      DELN(KL)=SQRT(DELN(KL))
      XLN(KL)=(ANZ(KL)-SUM1)/SUM2
      IF(ABS(XLN(KL)-XL1).LT.0.00001) GO TO 67
      XL1=XLN(KL)
      GO TO 66
67  CONTINUE
      EFER(1)=EN(NF(1),1)
      EFER(2)=EN(NF(2),2)
      DO 222 I=1,KN
      DNCHN(I,KL)=0.
      IF(EN(I,KL).LT.EFER(KL)) DNCHN(I,KL)=1.
      IF(I.EQ.NF(KL)) DNCHN(I,KL)=CNF(KL)/(2.*AJN(I,KL)+1.)
      WRITE(23,44) EN(I,KL),V3(I,KL),NN(I,KL),LN(I,KL),AJN(I,KL)
222 CONTINUE
      KCH=1
      EFER(KL)=EN(NF(KL),KL)+0.001
```

```
      WRITE(23,*)PRSUMD(KL),DELN(KL),XLN(KL),CN(KL)
 101  CONTINUE
      ANORN=0.
      ANORP=0.
      RCRN=0.
      RCRP=0.
      DO 1707 I=1,99
      RON(I)=0.
      ROP(I)=0.
      RS=I*SHAG
      AX(I)=RS
      DO 1708 J=1,KCHH(1)
      RON(I)=RON(I)+(2.*AJN(J,1)+1.)*V3(J,1)**2*FFN(J,I,1)**2/RS/RS
1708  CONTINUE
      DO 1709 J=1,KCHH(2)
      ROP(I)=ROP(I)+(2.*AJN(J,2)+1.)*V3(J,2)**2*FFN(J,I,2)**2/RS/RS
1709  CONTINUE
      RCRN=RCRN+SIMP(I)*RON(I)*RS*RS*RS*RS
      RCRP=RCRP+SIMP(I)*ROP(I)*RS*RS*RS*RS
      ANORN=ANORN+SIMP(I)*RON(I)*RS*RS
      ANORP=ANORP+SIMP(I)*ROP(I)*RS*RS
1707  CONTINUE
      ANORN=ANORN*SHAG/3.
      ANORP=ANORP*SHAG/3.
      RCRN=RCRN*SHAG/3.
      RCRP=RCRP*SHAG/3.
      RCRN=SQRT(RCRN/ANORN)
      RCRP=SQRT(RCRP/ANORP)
      WRITE(23,1710) ANORN,ANORP,RCRN,RCRP,TOSP,GAM
1710  FORMAT(2X,3HNN=,F7.3,2X,3HNP=,F6.3,2X,3HRN=,F8.3,2Hfm,2X,
     *3HRP=,F6.3,2Hfm,2X,5HTOSP=,F6.3,2X,5HVCUL=,F9.3,3HMeV)
      WRITE(23,1711) (AX(I),RON(I)/ANORN,ROP(I)/ANORP,I=1,75)
1711  FORMAT(2(5X,3F10.5))
```

```
  4  FORMAT(10X,F15.10,2X,I6,2X,I10,2X,F10.3)
 44  FORMAT(10X,F15.8,2X,F6.2,2X,I10,2X,I10,2X,F10.3)
444 FORMAT (17X, 6H ENERGY, 3X, 15H PARTICLE NUMBER, 3X, 1HN,
11X, 1HL, 9X, 1HJ)
      MES=0
      TOSP=0.
      TOSPA=0.
      TOSPE=0.
      DELEC=0.
      GAM=0.
      XKAPA=0.
      XKAPE=0.
      XKAPAP=0.
      XKAPEP=0.
      EPS1SP=0.
      DHPP1=0.
      DHPP2=0.
      SIFR=0.
      DO 10 I=1,KCHH(1)
      DO 12 J=1,KCHH(2)
      IF(LN(I,1).NE.LN(J,2)) GO TO 12
      ANAK= AJN(I,1)+ LN(I,1)-0.5
      NAK=ANAK+0.1
      ZNAK=(-1.)**NAK
      AJ1=0.5
      AJ2=LN(I,1)
      AJ3=AJN(J,2)
      AJ4=AJN(I,1)
      AJ5=1.
      AJ6=0.5
      AAT=SQRT(6.*(2.*AJ3+1.)*(2.*AJ4+1.))
      AAS=SIXJ(AJ1,AJ2,AJ3,AJ4,AJ5,AJ6)
      UGL1=ZNAK*AAT*AAS
```

```
      ANAK=AJ2+AJ3+AJ1
      NAK=ANAK+0.1
      ZNAK=(-1.)**NAK
      T1=SQRT((2.*AJ3+1.)*(2.*AJ4+1.)*AJ2*(AJ2+1)*(2.*AJ2+1.))
      T2=(AJ4-AJ2)*(AJ4+AJ2+1.)-(AJ3-AJ2)*(AJ3+AJ2+1.)
      UGL3=ZNAK*T1*T2*SIXJ(AJ4,AJ2,AJ1,AJ2,AJ3,1
      UGL2=UGL1*(AJ3*(AJ3+1.)-AJ2*(AJ2+1.)-AJ1*(AJ1+1.))*0.5
      RADS=0.
      RADC=0.
      RADLS=0.
      RADPP=0.
      RADV1=0.
      DO 13 II=1,99
      RAC=II*SHAG
      FR=1./(1.+EXP((RAC-ROCH)/AKSI))
      FFF=FFN(I,II,1)*FFN(J,II,2)*SIMP(II)
      FFFP=FFN(J,II,2)*FFN(J,II,2)*SIMP(II)
      RA=1.5/RCUL-0.5*RAC*RAC/RCUL/RCUL/RCUL
      IF(RAC.GT.RCUL) RA=1./RAC
      RA=FCOUL(RAC)/AZ
      RADS=RADS+FFF
      RADPP=RADPP+FR*FFF
      RADC=RADC+FFF*RA
      RADLS=RADLS+FFF*FR*(FR-1.)/RAC
      RADV1=RADV1+FFF*FR
   13 CONTINUE
      DDNDEL=DNCHN(I,1)-DNCHN(J,2
      MES=MES+1
      VN(MES)=V3(I,1)**2
      VP(MES)=V3(J,2)**2
      BPN(MES)=UGL1*RADS*SHAG/3./SQRT(3.)*U3(J,2)*V3(I,1)
      BPNX(MES)=UGL1*RADS*SHAG/3./SQRT(3.)*V3(J,2)*U3(I,1)
      QPN(MES)=UGL1*RADS*SHAG/3./SQRT(3.)*V3(J,2)*V3(I,1)
```

```
      QPNX(MES)=UGL1*RADS*SHAG/3./SQRT(3.)*U3(J,2)*U3(I,1)
      DPN(MES)=UGL1*(RADC*1.44*(AZ-1)-SABITV1*RADV1*AV1)*SHAG/3./
     *SQRT(3.)*U3(J,2)*V3(I,1)
      DPNX(MES)=UGL1*(RADC*1.44*(AZ-1)-
SABITV1*RADV1*AV1)*SHAG/3./
     *SQRT(3.)*V3(J,2)*U3(I,1)
      FPN(MES)=UGL2*SABITF*RADLS*SHAG/3./SQRT(3.)*U3(J,2)*V3(I,1)
      FPNX(MES)=UGL2*SABITF*RADLS*SHAG/3./SQRT(3.)*V3(J,2)*U3(I,)
      GPN(MES)=UGL3*SABITG*RADLS*SHAG/3./SQRT(3.)*U3(J,2)*V3(I,1)
      GPNX(MES)=UGL3*SABITG*RADLS*SHAG/3./SQRT(3.)*V3(J,2)*U3(I,1)
      EPN(MES)=EPSNP(J,2)+EPSNP(I,1)
      EE(MES)=EPN(MES)
      BBK=DPN(MES)+FPN(MES)-GPN(MES)
      BBKX=DPNX(MES)+FPNX(MES)-GPNX(MES)
      ENPA(MES)=(EPN(MES)*(BPNX(MES)-BPN(MES))+BBKX+BBK)/2.
      ENPE(MES)=(EPN(MES)*(BPNX(MES)+BPN(MES))+BBKX-BBK)/2.
C     ENPA(MES)=(EPN(MES)*(BPNX(MES)-BPN(MES)))/2.
C     ENPE(MES)=(EPN(MES)*(BPNX(MES)+BPN(MES)))/2.
      AMA(MES)=ENPA(MES)
      AME(MES)=ENPE(MES)
      XKAPA=XKAPA-ENPA(MES)*(BPNX(MES)-BPN(MES))/2.
      XKAPE=XKAPE-ENPE(MES)*(BPNX(MES)+BPN(MES))/2.
      KNN1=2.*AJN(I,1)+0.2
      KNN2=2.*AJN(J,2)+0.2
      ANNL(MES)=NN(I,1)*1000000.+NN(J,2)*100000.+LN(I,1)*10000.+
     *KNN1*100+KNN2
      EPS1SP=EPS1SP+BPN(MES)*BPNX(MES)/(EPN(MES)+EBET*0.5)
      TOSPA=TOSPA+BPNX(MES)**2                 !!*BIB(MES)
      TOSPE=TOSPE+BPN(MES)**2                  !!*BIB(MES)
C     READ(99,*)ENPE(MES),ENPA(MES)
      WRITE(23,303) NN(I,1),NN(J,2),LN(I,1),AJN(I,1),AJN(J,2),
     *EPN(MES),BPN(MES),DPN(MES),ENPE(MES),ENPA(MES)
      PSINORM(MES)=0.
```

```
   12 CONTINUE
   10 CONTINUE
C      READ(99,*)XKAPA,XKAPE
  303 FORMAT(2X,3I3,7F9.4)
      WRITE(23,*)'MES=',MES
      TOSPE=TOSPE*3.
      TOSPA=TOSPA*3.
      TOSP=TOSPE-TOSPA
      EPS1SP=3.*EPS1SP
      TOSPEA=TOSPE-TOSPA
      WRITE(23,443) CN(1),CN(2),XLN(1),XLN(2),EPS1SP
  443 FORMAT(2X,3HCN=,F5.3,2X,3HCP=,F5.3,2X,5HLAMN=,F8.4,
     *2X,5HLAMP=,F8.4,2X,7HEPS1SP=,F8.4)
      WRITE(23,*)'XKAPA=',XKAPA,'XKAPAP=',XKAPAP,
     *'XKAPE=',XKAPE,'XKAPEP=',XKAPEP
      WRITE(23,*)'T0SPE=',TOSPE,'TOSPA=',TOSPA,'TOSP=',TOSP
      DO 23 I=1,MES
      DO 24 J=I,MES
      IF(EE(I).LT.EE(J+1)) GO   TO 24
      C=EE(I)
      CC=AMA(I)
      CCC=AME(I)
      EE(I)=EE(J+1)
      EE(J+1)=C
      AMA(I)=AMA(J+1)
      AME(I)=AME(J+1)
      AMA(J+1)=CC
      AME(J+1)=CCC
   24 CONTINUE
   23 CONTINUE
      WRITE(23,100) (EE(I),AMA(I),AME(I),I=1,MES)
 100  FORMAT(3(2X,F10.5))
C      STOP
```

```
      EPSE=0.00001
      EPS=0.000001
      EPS5=0.00001
      EPS6=0.00001
      IM=400
      IER=0
      ITER=40
      SUM=0.
      SUME=0.
      BMM=0.
      BPP=0.
      SUMM=0.
      SUMP=0.
      DELHPP1=0.
      DELHPP2=0.
      EPSE1=0.
      EGTS=0.
      EGTCOUL=0.
      NOME=0
      DELCRPA=0.
      DELCRPB=0.
      TOPN=0.
      TOPP=0.
C      XX=0.0
C 7799 CONTINUE
C      WRITE(23,*)XX,FUDIK(XX)
C      WRITE(*,*)XX,FUDIK(XX)
C      PAUSE
C      XX=XX+0.005
C      GO TO 7799
      DO 701 K=1,MES-1
      OM1=EE(K)+EPSE
      OM2=EE(K+1)-EPSE
```

```
      STEP=(OM2-OM1)/IM
      DO 305 J=1,IM-1
      OME1=OM1+(J-1)*STEP
      OME2=OM1+J*STEP
      W1=FUDIK(OME1)
      W2=FUDIK(OME2)
      IF(W1*W2.GT.0.) GO TO 305
      CALL BISEC(OME1,OME2,OMX,W1,W2,EPS,EPSE,ITER,IER)
C       OMX=2.8
      WW=FUDIK(OMX)
      IF(ABS(WW).GT.0.01) GO TO 305
      NOME=NOME+1
      EEE(NOME)=OMX
      ANOR=0.
      BETM=0.
      BETP=0.
C      L1(OME),L2(OME)...L5(OME) LERIN HESAPLANMASI
      IF(METKA.EQ.1) GO TO 7
      DO 111 I=1,5
      DO 112 J1=1,5
      AAA1(I,J1)=DET(I+1,J1+1)
      D5(I,J1)=AAA1(I,J1)
  112 CONTINUE
      DETB(I)=-DET(I+1,1)
  111 CONTINUE
      DD0=DETER5(R)
      DO 115 KK=1,5
      DO 113 I=1,5
      DO 114 J1=1,5
      D5(I,J1)=AAA1(I,J1)
  114 CONTINUE
      D5(I,KK)=DETB(I)
  113 CONTINUE
```

```
      DETX(KK)=DETER5(R)/DD0
  115 CONTINUE
      GO TO 8
    7 CONTINUE
      DO 711 I=1,3
      DO 712 J1=1,3
      AAA1(I,J1)=D4(I+1,J1+1)
      D5(I,J1)=AAA1(I,J1)
  712 CONTINUE
      DETB(I)=-D4(I+1,1)
C     WRITE(*,*)D5(I,1),D5(I,2),D5(I,3)
  711 CONTINUE
      DD0=DETER3(R)
      DO 715 KK=1,3
      DO 713 I=1,3
      DO 714 J1=1,3
      D5(I,J1)=AAA1(I,J1)
  714 CONTINUE
      D5(I,KK)=DETB(I)
  713 CONTINUE
      DETX(KK)=DETER3(R)/DD0
C     WRITE(*,*)DETX(KK),DD0,OMX
  715 CONTINUE
      GO TO 9
    8  CONTINUE
      DO 801 I=1,MES
      EM=OMX-EPN(I)
      EP=OMX+EPN(I)
      PSI(I)=(ENPA(I)/XKAPA/2.+ENPE(I)*DETX(1)/XKAPE/2.+
     *2.*XPH*(BPNX(I)*DETX(2)+BPN(I)*DETX(3))-
     * 2.*XPP*(QPNX(I)*DETX(4)+QPN(I)*DETX(5)))/EM
      FII(I)=-(ENPA(I)/XKAPA/2.-ENPE(I)*DETX(1)/XKAPE/2.+
     *2.*XPH*(BPN(I)*DETX(2)+BPNX(I)*DETX(3))+
```

```
     * 2.*XPP*(QPN(I)*DETX(4)+QPNX(I)*DETX(5)))/EP
      ANOR=ANOR+PSI(I)**2-FII(I)**2
C 816 FORMAT(3(2X,F15.3))
  801 CONTINUE
      GO TO 70
    9 CONTINUE
      DO 7801 I=1,MES
      EM=OMX-EPN(I)
      EP=OMX+EPN(I)
      PSI(I)=(BPNX(I)+BPN(I)*DETX(1)-
     *XPP/XPH*(QPNX(I)*DETX(2)+QPN(I)*DETX(3)))/EM
      FII(I)=-(BPN(I)+BPNX(I)*DETX(1)+
     *XPP/XPH*(QPN(I)*DETX(2)+QPNX(I)*DETX(3)))/EP
      ANOR=ANOR+PSI(I)**2-FII(I)**2
C     WRITE(*,*) ANOR
C     PAUSE
C 816 FORMAT(3(2X,F15.3))
 7801 CONTINUE
  70  CONTINUE
      ASUM=0.
      BM=0.
      BP=0.
      BMK=0.
      DHBM=0.
      DHBP=0.
      BMCOUL=0.
      DOLSAYN=0.
      DOLSAYP=0.
      DO 815 I=1,MES
      PSI(I)=PSI(I)/SQRT(ABS(ANOR))
      FII(I)=FII(I)/SQRT(ABS(ANOR))
      PS=PSI(I)
      FI=FII(I)
```

```
      PSINORM(I)=PSINORM(I)+PS*PS-FI*FI
      DOLSAYN=DOLSAYN+(PS*PS+FI*FI)*VN(I)
      DOLSAYP=DOLSAYP+(PS*PS+FI*FI)*VP(I)
      IF(ABS(PS).GT.0.1.OR.ABS(FI).GT.0.1) WRITE(23,*) PS,FI,ANNL(I)
      BM=BM+PSI(I)*BPN(I)+FII(I)*BPNX(I)
      BP=BP+PSI(I)*BPNX(I)+FII(I)*BPN(I)
      ASUM=ASUM+PSI(I)**2-FII(I)**2
  815 CONTINUE
      WRITE(23,*)'ASUM=',ASUM,DOLSAYN,DOLSAYP
      TOPN=TOPN+DOLSAYN
      TOPP=TOPP+DOLSAYP
      BGTM=BM**2*3.
      BGTP=BP**2*3.
      SUMP=SUMP+BGTP
      SUMM=SUMM+BGTM
      SUME=SUME+BGTM*OMX
      SUM=SUM+BGTM-BGTP
C      EFARK=EEE(1)
      EPSE1=EPSE1+BM*BP/(OMX-EFARK+EBET*0.5)*0.511
      D1=6295./1.254/1.254
      DBM=BGTM
      DBP=BGTP
      FTM=D1/BGTM
      FTP=D1/BGTP
      ALOFTM=(LOG(D1)-LOG(BGTM))/LOG(10.)
      ALOFTP=(LOG(D1)-LOG(BGTP))/LOG(10.)
      WRITE(23,*)'LOG(FTM)=',ALOFTM,'LOG(FTP)=',ALOFTP,'ANORM',ANOR
      WRITE(23,*)'SUMM=',SUMM  ,'SUMP=',SUMP,'EPS-1=',EPSE1*3.
      WRITE(23,*)'FT(-)=',FTM  ,'FT(+)=',FTP
      WRITE(23,802) NOME,OMX-EFARK,WW,BM,BP,BGTM,BGTP,SUM
C      WRITE(99,2802) NOME,OMX,BM,BP
 2802 FORMAT(2X,I3,3(2X,F8.4))
  802 FORMAT(2X,I3,2X,4HOME=,F7.3,2X,3HWW=,F15.4,2X,3HBM=,F8.4,2X,
```

```
     *3HBP=,F8.4,2X,5HBGTM=,F8.3,2X,5HBGTP=,F8.3,2X,6HIKEDA=,F10.4)
C      GELDIM=GELDIM+1.
  305 CONTINUE
C      IF(GELDIM.GT.120)GO TO 702
  701 CONTINUE
      WRITE(23,*)TOPN,TOPP
C  702 CONTINUE
      OMEGTOR=SUME/SUMM-EFARK
      WRITE(23,233) (I,EPN(I),PSINORM(I),ANNL(I),I=1,MES)
  233 FORMAT(2X,I3,2X,F10.5,2X,F8.3,2X,F10.2)
      EPSE1=EPSE1*3.
      WRITE(23,*)'EPSE1=',EPSE1,'EFARK=',EFARK,OMEGTOR
      STOP
      END
      FUNCTION  FUDIK(XX)
      COMMON/BLOK1/ENPA(500),ENPE(500),EPN(500)
      COMMON/BLOK2/BPN(500),BPNX(500),QPN(500),QPNX(500)
      COMMON/BLOK3/MES,XPH,XPP,XKAPE,XKAPA
      COMMON/DET4/D4(4,4)
      COMMON/BLDD/A(100,100),B(100),X(100)
      COMMON/BLOK4/A5,METKA
      DO 22 I=1,6
      DO 23 J=1,6
      A(I,J)=0.
  23  CONTINUE
      B(I)=0.
  22  CONTINUE
      R=0.5
      IF(METKA.EQ.1)GO TO 5
      DO 2 I=1,MES
      EWM=XX-EPN(I)
      EWP=XX+EPN(I)
      A(1,1)=A(1,1)+EPN(I)*ENPA(I)**2/EWP/EWM
```

```
      A(1,2)=A(1,2)+ENPA(I)*ENPE(I)*XX/EWP/EWM
      A(1,3)=A(1,3)+ENPA(I)*(BPNX(I)/EWM-BPN(I)/EWP)
      A(1,4)=A(1,4)+ENPA(I)*(BPN(I)/EWM-BPNX(I)/EWP)
      A(1,5)=A(1,5)+ENPA(I)*(QPNX(I)/EWM+QPN(I)/EWP)
      A(1,6)=A(1,6)+ENPA(I)*(QPN(I)/EWM+QPNX(I)/EWP)
      A(2,2)=A(2,2)+EPN(I)*ENPE(I)**2/EWP/EWM
      A(2,3)=A(2,3)+ENPE(I)*(BPNX(I)/EWM+BPN(I)/EWP)
      A(2,4)=A(2,4)+ENPE(I)*(BPN(I)/EWM+BPNX(I)/EWP)
      A(2,5)=A(2,5)+ENPE(I)*(QPNX(I)/EWM-QPN(I)/EWP)
      A(2,6)=A(2,6)+ENPE(I)*(QPN(I)/EWM-QPNX(I)/EWP)
      A(3,3)=A(3,3)+BPNX(I)**2/EWM-BPN(I)**2/EWP
      A(3,4)=A(3,4)+BPN(I)*BPNX(I)*(1./EWM-1./EWP)
      A(3,5)=A(3,5)+BPNX(I)*QPNX(I)/EWM+BPN(I)*QPN(I)/EWP
      A(3,6)=A(3,6)+BPNX(I)*QPN(I)/EWM+BPN(I)*QPNX(I)/EWP
      A(4,4)=A(4,4)+BPN(I)**2/EWM-BPNX(I)**2/EWP
      A(4,5)=A(4,5)+BPN(I)*QPNX(I)/EWM+BPNX(I)*QPN(I)/EWP
      A(4,6)=A(4,6)+BPN(I)*QPN(I)/EWM+BPNX(I)*QPNX(I)/EWP
      A(5,5)=A(5,5)+QPNX(I)**2/EWM-QPN(I)**2/EWP
      A(5,6)=A(5,6)+QPN(I)*QPNX(I)*(1./EWM-1./EWP)
      A(6,6)=A(6,6)+QPN(I)**2/EWM-QPNX(I)**2/EWP
    2 CONTINUE
      A(2,1)=A(1,2)
      A(3,1)=A(1,3)
      A(3,2)=A(2,3)
      A(4,1)=A(1,4)
      A(4,2)=A(2,4)
      A(4,3)=A(3,4)
      A(5,1)=A(1,5)
      A(5,2)=A(2,5)
      A(5,3)=A(3,5)
      A(5,4)=A(4,5)
      A(6,1)=A(1,6)
      A(6,2)=A(2,6)
```

```
A(6,3)=A(3,6)
A(6,4)=A(4,6)
A(6,5)=A(5,6)
A(1,1)=1.-A(1,1)/XKAPA
A(1,2)=-A(1,2)/XKAPE
A(1,3)=-2.*XPH*A(1,3)
A(1,4)=-2.*XPH*A(1,4)
A(1,5)= 2.*XPP*A(1,5)
A(1,6)= 2.*XPP*A(1,6)
A(2,1)=-A(2,1)/XKAPA
A(2,2)=1.-A(2,2)/XKAPE
A(2,3)=-2.*XPH*A(2,3)
A(2,4)=-2.*XPH*A(2,4)
A(2,5)= 2.*XPP*A(2,5)
A(2,6)= 2.*XPP*A(2,6)
A(3,1)=-A(3,1)/XKAPA/2.
A(3,2)=-A(3,2)/XKAPE/2.
A(3,3)=1.-2.*XPH*A(3,3)
A(3,4)=-2.*XPH*A(3,4)
A(3,5)= 2.*XPP*A(3,5)
A(3,6)= 2.*XPP*A(3,6)
A(4,1)=-A(4,1)/XKAPA/2.
A(4,2)=-A(4,2)/XKAPE/2.
A(4,3)=-2.*XPH*A(4,3)
A(4,4)=1.-2.*XPH*A(4,4)
A(4,5)= 2.*XPP*A(4,5)
A(4,6)= 2.*XPP*A(4,6)
A(5,1)=-A(5,1)/XKAPA/2.
A(5,2)=-A(5,2)/XKAPE/2.
A(5,3)=-2.*XPH*A(5,3)
A(5,4)=-2.*XPH*A(5,4)
A(5,5)=1.+2.*XPP*A(5,5)
A(5,6)=2.*XPP*A(5,6)
```

```
      A(6,1)=-A(6,1)/XKAPA/2.
      A(6,2)=-A(6,2)/XKAPE/2.
      A(6,3)=-2.*XPH*A(6,3)
      A(6,4)=-2.*XPH*A(6,4)
      A(6,5)=2.*XPP*A(6,5)
      A(6,6)=1.+2.*XPP*A(6,6)
      KD=6
      A6=DETER6(R)
C     A5=A(1,1)*A(2,2)-A(1,2)*A(2,1)
C     WRITE(*,*)XX,A5,A6
      FUDIK=A6
      RETURN
    5 CONTINUE
      DO 52 I=1,MES
      EWM=XX-EPN(I)
      EWP=XX+EPN(I)
      A(1,1)=A(1,1)+BPNX(I)**2/EWM-BPN(I)**2/EWP
      A(1,2)=A(1,2)+BPN(I)*BPNX(I)*(1./EWM-1./EWP)
      A(1,3)=A(1,3)+BPNX(I)*QPNX(I)/EWM+BPN(I)*QPN(I)/EWP
      A(1,4)=A(1,4)+BPNX(I)*QPN(I)/EWM+BPN(I)*QPNX(I)/EWP
      A(2,2)=A(2,2)+BPN(I)**2/EWM-BPNX(I)**2/EWP
      A(2,3)=A(2,3)+BPN(I)*QPNX(I)/EWM+BPNX(I)*QPN(I)/EWP
      A(2,4)=A(2,4)+BPN(I)*QPN(I)/EWM+BPNX(I)*QPNX(I)/EWP
      A(3,3)=A(3,3)+QPNX(I)**2/EWM-QPN(I)**2/EWP
      A(3,4)=A(3,4)+QPN(I)*QPNX(I)*(1./EWM-1./EWP)
      A(4,4)=A(4,4)+QPN(I)**2/EWM-QPNX(I)**2/EWP
   52 CONTINUE
      A(2,1)=A(1,2)
      A(3,1)=A(1,3)
      A(3,2)=A(2,3)
      A(4,1)=A(1,4)
      A(4,2)=A(2,4)
      A(4,3)=A(3,4)
```

```
      D4(1,1)=1.-2.*XPH*A(1,1)
      D4(1,2)=-2.*XPH*A(1,2)
      D4(1,3)= 2.*XPP*A(1,3)
      D4(1,4)= 2.*XPP*A(1,4)
      D4(2,1)=-2.*XPH*A(2,1)
      D4(2,2)=1.-2.*XPH*A(2,2)
      D4(2,3)= 2.*XPP*A(2,3)
      D4(2,4)= 2.*XPP*A(2,4)
      D4(3,1)=-2.*XPH*A(3,1)
      D4(3,2)=-2.*XPH*A(3,2)
      D4(3,3)=1.+2.*XPP*A(3,3)
      D4(3,4)=2.*XPP*A(3,4)
      D4(4,1)=-2.*XPH*A(4,1)
      D4(4,2)=-2.*XPH*A(4,2)
      D4(4,3)=2.*XPP*A(4,3)
      D4(4,4)=1.+2.*XPP*A(4,4)
      FUDIK=SERDET(R)
      RETURN
      END
      SUBROUTINE DETER(N,DETA)
      COMMON/BLDD/A(100,100),B(100),X(100)
      DIMENSION G(100,100),T(9999),NS(9999),D(9999)
      DO 200 I=1,N
      DO 150 J=1,N
      G(I,J)=A(I,J)
  150 CONTINUE
  200 CONTINUE
      M=0
      ITOP=0
      IFAKTOR=1
  250 DETG=0
      DO 300 I=1,N
      NS(I)=1
```

```
300 CONTINUE
   NS(N)=0
350 NS(N)=NS(N)+1
   DO 400 J=1,N
   I=N-J+1
   IF(NS(I).LE.N) GO TO 450
   IF(NS(1).GT.N) GO TO 750
   NS(I-1)=NS(I-1)+1
   NS(I)=1
400 CONTINUE
450 DO 550 K=1,N
   IF(K.EQ.N) GO TO 570
   DO 500 L=K,N
   IF(K.EQ.L) GO TO 500
   IF(NS(K).EQ.NS(L)) GO TO 350
500 CONTINUE
550 CONTINUE
570 IF(M.EQ.0) ITOP=ITOP+1
   KS=0
   DO 650 K=1,N
   DO 600 L=K,N
   IF(NS(K).GT.NS(L)) KS=KS+1
600 CONTINUE
650 CONTINUE
   TERIM=(-1.)**KS
   DO 700 I=1,N
   TERIM=TERIM*G(I,NS(I))
700 CONTINUE
   DETG=DETG+TERIM
   GO TO 350
750 IF(M.EQ.0) GO TO 800
   IFAKTOR=IFAKTOR*M
   D(M)=DETG
```

```
      DETA1=DETA
C     IF(MMETKA.EQ.0) DETA1=0.1
C     IF(MMETKA.EQ.0) WRITE(*,*) DETA
      X(M)=D(M)/DETA1
C     WRITE(*,*) M,'C˜ KOK=',X(M)
  800 IF(M.EQ.0) DETA=DETG
C     WRITE(*,*) 'DETA=',DETA,'DET',M,'=',D(M)
      M=M+1
      IF(M.GT.N) GO TO 900
      DO 850 I=1,N
      G(I,M)=B(I)
      IF(M.EQ.1) GO TO 850
      G(I,M-1)=A(I,M-1)
  850 CONTINUE
      GO TO 250
  900 CONTINUE
C     WRITE(*,*) 'OLMASI GEREKEN OLASILIK SAYISI=',IFAKTOR
      T(1)=0.0
      DO 950 J=1,N
      T(1)=T(1)+A(1,J)*X(J)
  950 CONTINUE
      XBHATA=ABS(T(1)-B(1))
      DO 1050 I=2,N
      T(I)=0.0
      DO 1000 J=1,N
      T(I)=T(I)+A(I,J)*X(J)
 1000 CONTINUE
      FARK=ABS(T(I)-B(I))
      IF(FARK.GT.XBHATA) XBHATA=FARK
 1050 CONTINUE
C     WRITE(*,*) 'MAX ERROR=',XBHATA
      IF(XBHATA.LT.0.001) GO TO 1150
C     DO 1100 I=1,N
```

```
C      WRITE(*,*) T(I),'        /      ',B(I)
C 1100 CONTINUE
C      GO TO 1200
 1150 CONTINUE
C 1200 WRITE(*,*)
C      READ(*,*) K
C      IF(K.EQ.1) GO TO 50
C      STOP
      END
      SUBROUTINE BISEC(X1,X2,X,W1,W2,EPS,EPSE,ITER,IER)
C     SOLVES THE EQUATION W(X)=0 USING A BISECTION METOD
      COMMON/BLOK1/ENPA(500),ENPE(500),EPN(500)
      COMMON/BLOK2/BPN(500),BPNX(500),QPN(500),QPNX(500)
      COMMON/BLOK3/MES,XPH,XPP,XKAPE,XKAPA
C      COMMON/BLDD/A(100,100),B(100),X(100)
      COMMON/BLOK4/A5,METKA
      XA=X1
      XB=X2
      WA=W1
      WB=W2
      IT=0
    1 CONTINUE
      IT=IT+1
      X=(XA+XB)/2.
      WX=FUDIK(X)
C      WRITE(6,*) WA,X-XA,WX,XB-XA,WB
      IF(ABS(WX).LT.EPS) GO TO 10
      IF(IT.GT.ITER) GO TO 20
      AWAX=WA*WX
      IF(AWAX)2,20,4
    2 CONTINUE
C      FUNCTION CHANGES SIGN IN THE LEFT SUBINTERVAL
C      IF(X.EQ.XB) GO TO 90
```

```
      IF(ABS(X-XB).LT.0.000001) GO TO 90
      XB=X
      WB=WX
      GO TO 1
    4 CONTINUE
C       FUNCTION CHANGES SIGGN INTHE RIGHT SUBINTERVAL
C      IF(X.EQ.XA)  GO TO 90
      IF(ABS(X-XA).LT.0.000001) GO TO 90
      XA=X
      WA=WX
      GO TO 1
   10 CONTINUE
      IER=0
      RETURN
   20 CONTINUE
      IER=10
C      WRITE(6,*) AWAX,XA,WA,XB,WB,X,WX
      RETURN
   90 CONTINUE
      IER=20
CBAB    WRITE(6,*) NO CONVERGANCE,STEEP POLE NEAR,X,W=RETURN
      RETURN
      END
       FUNCTION  SIXJ(AJ1,AJ2,AJ12,AJ3,AJ,AJ23)
C      IMPLICIT REAL*8(A-H,O-Z)
       A=ABS(AJ1-AJ2)
       B=AJ1+AJ2
       IF(AJ12.LT.A.OR.AJ12.GT.B) GO TO 3
       A=ABS(AJ2-AJ3)
       B=AJ2+AJ3
       IF(AJ23.LT.A.OR.AJ23.GT.B) GO TO 3
       A=ABS(AJ12-AJ3)
       B=AJ12+AJ3
```

```
IF(AJ.LT.A.OR.AJ.GT.B) GO TO 3
A=ABS(AJ23-AJ1)
B=AJ23+AJ1
IF(AJ.LT.A.OR.AJ.GT.B) GO TO 3
JKK1=AJ1+AJ2-AJ12+0.1
JKK2=AJ1-AJ2+AJ12+0.1
JKK3=-AJ1+AJ2+AJ12+0.1
JKK4=AJ1+AJ2+AJ12+1.1
DABC=0.5*(FAK(JKK1)+FAK(JKK2)+FAK(JKK3)-FAK(JKK4))
JKK1=AJ12+AJ3-AJ+0.1
JKK2=AJ12-AJ3+AJ+0.1
JKK3=-AJ12+AJ3+AJ+0.1
JKK4=AJ12+AJ3+AJ+1.1
DCDE=0.5*(FAK(JKK1)+FAK(JKK2)+FAK(JKK3)-FAK(JKK4))
JKK1=AJ1+AJ-AJ23+0.1
JKK2=AJ1-AJ+AJ23+0.1
JKK3=-AJ1+AJ+AJ23+0.1
JKK4=AJ1+AJ+AJ23+1.1
DAEF=0.5*(FAK(JKK1)+FAK(JKK2)+FAK(JKK3)-FAK(JKK4))
JKK1=AJ2+AJ3-AJ23+0.1
JKK2=AJ2-AJ3+AJ23+0.1
JKK3=-AJ2+AJ3+AJ23+0.1
JKK4=AJ2+AJ3+AJ23+1.1
DBDF=0.5*(FAK(JKK1)+FAK(JKK2)+FAK(JKK3)-FAK(JKK4))
DELTA=DABC+DCDE+DAEF+DBDF
JABDE=AJ1+AJ2+AJ3+AJ+0.1
JACDF=AJ1+AJ12+AJ3+AJ23+0.1
JBCEF=AJ2+AJ12+AJ+AJ23+0.1
JZ1=JABDE
IF(JZ1.GT.JACDF) JZ1=JACDF
IF(JZ1.GT.JBCEF) JZ1=JBCEF
JABC=AJ1+AJ2+AJ12+0.1
JCDE=AJ12+AJ3+AJ+0.1
```

```
      JAEF=AJ1+AJ+AJ23+0.1
      JBDF=AJ2+AJ3+AJ23+0.1
      ASIX=0.
    2 CONTINUE
      IF(JZ1.LT.JABC) GO TO 4
      IF(JZ1.LT.JCDE) GO TO 4
      IF(JZ1.LT.JAEF) GO TO 4
      IF(JZ1.LT.JBDF) GO TO 4
      ASIX=ASIX+(-1.)**JZ1*EXP(FAK(JZ1+1)-FAK(JZ1-JABC)-
     * FAK(JZ1-JCDE)-FAK(JZ1-JAEF)-FAK(JZ1-JBDF)-
     * FAK(JABDE-JZ1)-FAK(JACDF-JZ1)-FAK(JBCEF-JZ1)+DELTA)
    4 JZ1=JZ1-1
      IF(JZ1.LT.0) GO TO 3
      GO TO 2
    3 SIXJ=ASIX
      RETURN
      END
      FUNCTION CLEBSH(AJ1,AM1,AJ2,AM2,AJ,AM)
C       IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 FAK
      REAL*8 J1AM1,J1EM1,J2AM2,J2EM2
      REAL*8 J1AJ2EJ,J1EJ2AJ,J1AJ2AJ,J2EJ1AJ,JAM,JEM
      J1=AJ1+0.1
      M1=AM1+0.1
      J2=AJ2+0.1
      M2=AM2+0.1
      J=AJ+0.1
      M=AM+0.1
      M12=M1+M2
      AM12=AM1+AM2
      AJE12=AJ1-AJ2
      AJA12=AJ1+AJ2
      SUM=0.
```

```
      IF(M12.NE.M) GO TO 3
      IF(AJ.LT.ABS(AJE12).OR.AJ.GT.AJA12) GO TO 3
      AN1=AJ1+AJ2-AJ
      AN2=AJ1-AM1
      AN3=AJ2+AM2
      AN4=AJ-AJ2+AM1
      AN5=AJ-AJ1-AM2
      Z1=AN1
      IF(Z1.GT.AN2) Z1=AN2
      IF(Z1.GT.AN3) Z1=AN3
      IF((AN4+Z1).LT.0.) Z1=ABS(AN4)
      IF((AN5+Z1).LT.0.) Z1=ABS(AN5)
C       IF(AN4.LT.0.) Z1=ABS(AN4)
C       IF(AN5.LT.0.0.AND.ABS(AN5).GT.ABS(AN4)) Z1=ABS(AN5)
      IF(Z1.LT.0.) GO TO 3
      Z=Z1
      JKK=AJ1+AM1+0.1
      J1AM1=FAK(JKK)
      JKK=AJ1-AM1+0.1
      J1EM1=FAK(JKK)
      JKK=AJ2+AM2+0.1
      J2AM2=FAK(JKK)
      JKK=AJ2-AM2+0.1
      J2EM2=FAK(JKK)
      JKK=AJ+AM+0.1
      JAM=FAK(JKK)
      JKK=AJ-AM+0.1
      JEM=FAK(JKK)
      CARP=(J1AM1+J1EM1+J2AM2+J2EM2+JAM+JEM+ALOG(2.*AJ+1.))/2.
      JKK=AJ1+AJ2-AJ+0.1
      J1AJ2EJ=FAK(JKK)
      JKK=AJ1-AJ2+AJ+0.1
      J1EJ2AJ=FAK(JKK)
```

```
      JKK=AJ1+AJ2+AJ+1.1
      J1AJ2AJ=FAK(JKK)
      JKK=AJ2-AJ1+AJ+0.1
      J2EJ1AJ=FAK(JKK)
      D11=J1AJ2EJ+J1EJ2AJ+J2EJ1AJ
      D12=J1AJ2AJ
      DJ1J2J=(D11-D12)/2.
    4 CONTINUE
      JZ=Z
      JAN1=AN1-Z
      JAN2=AN2-Z
      JAN3=AN3-Z
      JAN4=AN4+Z
      JAN5=AN5+Z
      F1=FAK(JZ)
      F2=FAK(JAN1)
      F3=FAK(JAN2)
      F4=FAK(JAN3)
      F5=FAK(JAN4)
      F6=FAK(JAN5)
      NZ=Z
      SUM=SUM+(-1.)**NZ*EXP(DJ1J2J+CARP-(F1+F2+F3+F4+F5+F6))
      Z=Z-1.
      NNK=J-J2+M1-Z
      IF(Z.LT.0.) GO TO 3
      IF((AN1-Z).LT.0.) GO TO 3
      IF((AN2-Z).LT.0.) GO TO 3
      IF((AN3-Z).LT.0.) GO TO 3
      IF((AN4+Z).LT.0.) GO TO 3
      IF((AN5+Z).LT.0.) GO TO 3
      GO TO 4
    3 ACLEB=SUM
      CLEBSH=ACLEB
```

```
      END
      FUNCTION  FAK(K)
      REAL*8 FAK
      KK=K
      DK=KK
      IF(KK.LT.0) GO TO 2
      DKD=0.
      IF(KK.LE.1) GO TO 2
3     DKD=DKD+ALOG(DK)
      KK=KK-1
      DK=KK
      IF(KK.EQ.1) GO TO 2
      GO TO 3
 2    CONTINUE
      FAK=DKD
      END
      FUNCTION DETER6(R)
      COMMON/BLDD/A(100,100),DD(100),DETX(100)
      COMMON/DET5/D5(5,5)
      DETER6=0.
      DO 4 K=1,6
      DO 1 I=1,5
      DO 2 J=1,5
      M=1
      IF(J.LT.K) M=0
      D5(I,J)=A(I+1,J+M)
2     CONTINUE
1     CONTINUE
      DETER6=DETER6+(-1.)**(K+1)*DETER5(R)*A(1,K)
4     CONTINUE
      RETURN
      END
      FUNCTION DETER5(R)
```

```
      COMMON/DET5/D5(5,5)
      COMMON/DET4/D4(4,4)
      DETER5=0.
      DO 4 K=1,5
      DO 1 I=1,4
      DO 2 J=1,4
      M=1
      IF(J.LT.K) M=0
      D4(I,J)=D5(I+1,J+M)
    2 CONTINUE
    1 CONTINUE
      DETER5=DETER5+(-1.)**(K+1)*SERDET(R)*D5(1,K)
    4 CONTINUE
      RETURN
      END
      FUNCTION SERDET(R)
      COMMON/DET4/A(4,4)
      DETA1=(A(1,1)*A(2,2)-A(1,2)*A(2,1))*(A(3,3)*A(4,4)-A(3,4)*A(4,3))
      DETA2=(A(1,3)*A(2,1)-A(1,1)*A(2,3))*(A(3,2)*A(4,4)-A(3,4)*A(4,2))
      DETA3=(A(1,1)*A(2,4)-A(1,4)*A(2,1))*(A(3,2)*A(4,3)-A(3,3)*A(4,2))
      DETA4=(A(1,2)*A(2,3)-A(1,3)*A(2,2))*(A(3,1)*A(4,4)-A(3,4)*A(4,1))
      DETA5=(A(1,4)*A(2,2)-A(1,2)*A(2,4))*(A(3,1)*A(4,3)-A(3,3)*A(4,1))
      DETA6=(A(1,3)*A(2,4)-A(1,4)*A(2,3))*(A(3,1)*A(4,2)-A(3,2)*A(4,1))
      DETA=DETA1+DETA2+DETA3+DETA4+DETA5+DETA6
      SERDET=DETA
C     WRITE(*,*) DETA
      RETURN
      END
      FUNCTION DETER3(R)
      COMMON/DET5/D(5,5)
      DET1=D(1,1)*(D(2,2)*D(3,3)-D(2,3)*D(3,2))
      DET2=D(1,2)*(D(2,1)*D(3,3)-D(2,3)*D(3,1))
      DET3=D(1,3)*(D(2,1)*D(3,2)-D(2,2)*D(3,1))
```

```
      DETER3=DET1-DET2+DET3
      RETURN
      END
      FUNCTION  FCOUL(R)
CXX      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/COUL/ROP(101),SIMP(100)
      SK=0.15
      J=R/SK+0.1
      J1=J+1
      SUM1=0.
      DO 707 I=1,J
      XI=I*0.15
      SUM1=SUM1+XI*XI*ROP(I)*SIMP(I)
  707 CONTINUE
      SUM1=SUM1/R
      SUM2=0.
      DO 708 I=J1,100
      XI=I*0.15
      SUM2=SUM2+XI*ROP(I)*SIMP(I)
  708 CONTINUE
      FCOUL=(SUM1+SUM2)*0.15/3.
      RETURN
      END
```

RESUME

Zohra Ali Ahmed ALWASI was born in Zliten in 1981. After graduating from Sayma High School in 2000, she started to undergraduate program in Almergeb University, Department of Physics in 2005 and she graduated in 2008. Also, she worked as a student laboratory assistant on Physics at Almergeb University between 2011 and 2014. She started to Department of Physics at Karabük University for M.Sc. education. She has completed her M. Sc. degree on 2017 with a thesis on "Calculation of The Half-Lives For ^{117,119,121}Te Isotopes By Pyatov's Method". She presented three posters in some the international nuclear physics conferences.

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