A MULTICRITERIA DECISION MAKING APPROACH TO HUMAN RESOURCES PERFORMANCE EVALUATION

(İNSAN KAYNAKLARI PERFORMANS DEĞERLENDİRMESİNE ÇOK ÖLÇÜTLÜ KARAR VERME YAKLAŞIMI)

by

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LIST OF SYMBOLS

A&D	:	Agreement and Disagreement
AAS	:	Analitik Ağ Süreci
AHP	:	Analytic Hierarchy Process
ANP	:	Analytic Network Process
CF	:	Customer Feedback
CI	:	Choquet Integral
CnI	:	Consistency Index
CR	:	Consistency Ratio
CRPC	:	Customer Related Performance Criteria
ÇÖKV	:	Çok Ölçütlü Karar Verme
DM	:	Decision Maker
E	:	Experience
EL	:	Educational Level
FMP	:	Factors Moderating Performance
FPPS	:	First-Past-the-Post System
HR	:	Human Resources
HRM	:	Human Resources Management
HPWP	:	High Performance Work Practices
JR	:	Junior
KSA	:	Knowledge, Skills and Abilities
M&A	:	Mergers and Acquisitions
MA	:	Marketing Ability
MACBETH	:	Measuring Attractiveness by a Categorical Based Evaluation TecHnique
MC	:	Managing the Change
MCDM	:	Multi-Criteria Decision Making

MnP	:	Management Politics
MP	:	Marketing Politics
MRS	:	Majority representation System
NCG	:	New Customer Gain
NCP	:	Number of Customer in Portfolio
PE	:	Performance Evaluation
PI	:	Performance Improvement
PM	:	Performance Management
PO	:	Preference Order
POF	:	Public Offering
PVS	:	Preferential Voting System
RBS	:	Repeated Ballot System
RC	:	Relations with Colleague
RI	:	Random Index
RRPC	:	Relations Related Performance Criteria
RUM	:	Relations with Upper Management
S	:	Sales
SBS	:	Second Ballot System
SCF	:	Social Choice Function
SCPO	:	Social Choice Preference Order
SHRM	:	Strategic Human Resources Management
SoP	:	Social Power
SP	:	Salary Politics
SR	:	Senior
SRE	:	Sales Report Efficiency/Effectiveness
SRPC	:	Sales Related Performance Criteria
SS	:	Salary - Satisfaction
VAR	:	Value Added to Reputation

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ABSTRACT

During last ten years, great majority of all the papers about human resources management and performance focused on the human resources as department in an enterprise, personnel selection and the organization's performance, but not on the workers or employees' performance evaluation – PE. According to this, it wouldn't be bold to say there is a gap in the employees' performances evaluation field. Of course in a conventional way, there are methods and tools to do that task but we have to ask ourselves if these methods are objective enough, if they are completely able to compare effectively the employees, how they manage to assess simultaneously the tangible and intangible factors and if they take the interdependencies / interactions in consideration. In this study, trying to handle the tangible and intangible factors simultaneously, it is specially focused on the last question, i.e. to consider effectively the interdependencies / interactions occurring between those factors. Hence this will lead us to the effective performance evaluation of the employees.

The aim of the proposed model is, by developing and improving the writer's previous works, to present a study on the employees' performance evaluation, a field that has not been highly considered in the literature of MCDM, and to bring a clearly higher level of objectivity to the performance evaluation for the employees working for a same task. Hence the MCDM perspective, added in the previous work to the human resources field to be more efficient at handling the tangible and intangible data simultaneously, will be improved by taking into account the dependence or interaction between criteria and alternatives.

In MCDM methods, most of the time there is the assumption of "independence of the criteria / sub-criteria / alternatives". Because of that, people usually have a tendency to construct independent criteria or criteria that are supposed to be so, which is

practically erroneous in real life problems and will lead to errors in judgments. To be able to do such assumption, one must be sure that there is no possible relation or in other words interdependence between criteria or alternatives according to the data at hand, because the final decision may seriously change with this assumption. In this study, the dependence of the criteria will be considered and analyzed very carefully.

Two methods, which will be presented in this study for the purpose of dealing with the interactions, handle different case of dependencies or interactions between criteria. One of the methods, namely the Analytic Network Process – ANP, is one of the most popular MCDM tools which is able to handle the interdependencies among criteria / alternatives and considers the dependencies like one criterion having an effect on another such as the aerodynamic and the fuel consumption of a car in case of selection of car problem. Or the example of a teacher and two criteria to evaluate his performance: the knowledge on the course he/she is teaching and the teaching ability. So the interdependence to analyze here could be the effect of basic knowledge on the ability of teaching.

The other method, namely the Choquet Integral – CI, is a fuzzy integral and considers the interactions between *k* out of *n* criteria of the problem, which is called the *k*-additivity property. The interaction value (positive or negative) of the *k* criteria under consideration will determine if these criteria must be satisfied at the same time or only one of them satisfied is enough to be considered as a successful or good performing unit. For the sake of simplicity and the applicability to the real life situations as effectively as possible, in this study, the 2-additive CI will be used. Hence, with the explanation given above, a positive interaction between two criteria will show that both of them have to be satisfied and a negative one will show that it's enough that one of the criteria be satisfied by an alternative in order to be considered successful. In other words, these cases can be considered as "conjunctive" or "disjunctive" relations between criteria. It has to be noted that in these kinds of interdependencies / interactions, the relations between criteria and alternatives are not mentioned. This is just the relations between criteria.

As, in both methods which will be presented in this study, the decision makers - DM(s) will give their preference orders most of the time, it can be a wise approach to apply the Social Choice Functions – SCFs to the alternatives rankings when the situation will be a group decision. Therefore, for group decision cases, it is proposed to use an appropriate SCF, once the ranking of alternatives for each Decision Maker – DM is found. By doing this, a suitable ranking for the alternatives will be found in order to satisfy the individually found rankings the most.

The proposed performance evaluation method is as follows: First of all, the model according to the organization's requirements from its group of employees will be built after the meetings made with the DMs. The criteria and the sub-criteria will clearly be defined. Then, in order to handle the interdependencies / interactions, the model will be evaluated using ANP or CI or with a combination of these two according to the nature of the relations existing between the criteria, the sub-criteria and the alternatives of the model. Once the results for each DM are obtained, the SCFs will be used in order to find the best suitable ranking of the employees to this group of DMs.

RESUME

Au long de ces dix dernières années, la grande majorité de tous les articles à propos de ressources humaines et performance est focalisée sur les ressource humaines étant un département de l'entreprise, la sélection du personnel et la performance de l'organisation mais pas sur l'évaluation de performance des employés. Selon ce fait, il ne sera pas trop étonnant de dire qu'il y a un manque dans le domaine de l'évaluation de performance des employés. Biensur que dans une manière conventionnelle, il y a des méthodes et des outils pour ce faire mais on doit se demander si ces méthodes sont assez objectives, si elles sont complètement capable de comparer effectivement les employés, comment elles arrivent a évaluer les facteurs quantitatifs et qualitatifs simultanément et si elles prennent les dépendances et les interactions en considération. Dans ce travail, tout en gérant à s'occuper des facteurs quantitatifs et qualitatifs simultanément, la dernière question est spécialement mise sous l'analyse, c'est à dire considérer effectivement les dépendances et les interactions existant entre ces facteurs. Par conséquent ce fait apportera une évaluation de performance effective.

Le but du modèle proposé est de présenter une étude sur l'évaluation de performance des employés, un domaine qui n'est pas vraiment considérés dans la littérature, en développant les études précédentes de l'auteur et d'apporter un niveau d'objectivité clairement supérieure à l'évaluation de performance des employés travaillant pour la même tâche. Par conséquent la perspective de MCDM ajoutée au domaine de ressources humaines dans les études précédentes pour pouvoir analyser les facteurs quantitatifs et qualitatifs simultanément, sera améliorée en prenant la dépendance et les interactions entre les critères en considération.

Dans les méthodes de MCDM, la plupart du temps, il y a l'hypothèse de l'indépendance des critères, sous-critères et les alternatifs. A cause de ce fait, les gens

ont souvent l'intention de construire des critères indépendantes ou bien des critères qui sont sensés de l'être ce qui est pratiquement faux dans les problèmes de la vie actuelle et du coup ce qui causera des erreurs. Pour être capable de faire une telle hypothèse, il faut être sure qu'il n'y a pas de relations possibles entre les critères ou les alternatifs selon les données en question car la décision finale peut sérieusement changer avec cette hypothèse. Dans ce travail, la dépendance des critères sera considérée et analysée très attentivement.

Deux méthodes qui seront présentées et proposées dans ce travail afin de gérer les dépendances ou les interactions, gèrent différent type de dépendance et interaction. Une des méthodes, nommé « Analytic Network Process » – ANP, est capable de gérer les dépendances entre les critères, les sous-critères et les alternatifs. Les dépendances comme celle de l'aérodynamique et de la consommation d'essence d'une voiture ou bien comme celle du savoir et de la compétence d'explication d'un professeur sont le sujet de cette méthode.

L'autre méthode, nommée l'intégrale de Choquet – CI, est un intégrale flou considérant les interactions entre *k* critères parmi les *n* critères du problème en question, ce qui est appelé la propriété de *k*-additivité. La valeur d'interaction (positive ou négative) de ces *k* critères en question déterminera si ces critères doivent être satisfaits en même temps ou bien seulement un d'eux doit être satisfait afin d'être considéré comme un employé bien performant. Pour la raison de simplicité et l'applicabilité à la vie actuelle au maximum possible, dans ce travail, l'intégrale de Choquet 2-additive sera utilisée. Par conséquent, avec l'explication donnée la dessus, une interaction positive entre deux critères montrera que ces deux critères doivent être satisfaits en même temps et une interaction négative montrera qu'il est suffisant de satisfaire un de ces deux critères pour être bien performant. En d'autres termes, ces cas peuvent être considérés comme « conjonctive » et « disjonctive » relations entre les critères. Il doit être noté que dans ce type de relations, les relations sont toujours entre les critères eux-mêmes.,

Comme, dans toutes les deux méthodes proposées dans ce travail, les décideurs donneront leurs avis et leurs ordres de préférences, il peut être une approche rigoureux d'utiliser les Fonctions de Choix Social – SCF pour les rangements des alternatifs quand il est le cas d'une décision de groupe. Donc pour les cas de décision de groupe, il est proposé d'utiliser les SCFs une fois que le rangement des alternatifs pour chaque décideur est trouvé. En faisant ça, un rangement approprié sera trouvé afin de satisfaire l'ensemble des rangements individuellement trouvés.

La méthode d'évaluation proposée est comme suit : premièrement, le modèle, selon les exigences de l'entreprise de ses employés, sera formé après les réunions faites avec les décideurs. Les critères et les sous-critères seront clairement définis. Puis, le modèle sera traité avec ANP ou CI ou bien une combinaison de ces deux selon la nature des relations existantes entre les critères, les sous-critères et les alternatifs du problème. Une fois que les résultats pour chaque décideur sont trouvés, les SCFs seront utilisées afin de trouver le résultat qui représentera les décisions individuelles au mieux possible.

ÖZET

Son on yılda insan kaynakları yönetimi ve performans hakkında yayımlanan makalelerin büyük bir çoğunluğu, firmalardaki insan kaynakları bölümü, personel seçimi ve organizasyon performansı üzerinde yoğunlaşıp, çalışanların performans ölçümü konusuna gereken önemi vermemektedir. Buna göre, çalışanların performans değerlendirmesi alanında bir boşluk olduğunu söylemek yanlış olmayacaktır. Tabii ki bu işlemi gerçekleştirebilmek adına yöntemler ve araçlar mevcuttur. Lakin bu yöntemlerin yeteri kadar nesnel olup olmadıkları, çalışanların performanslarını etkin bir şekilde değerlendirip değerlendiremedikleri, nicel ve nitel unsurları eşzamanlı olarak değerlendirip değerlendiremedikleri ve bunları yaparken bağımlılıkları dikkate alıp almadıkları sorulması gereken sorular olarak karşımıza çıkmaktadır. Bu çalışmada nicel ve nitel unsurları eşzamanlı olarak değerlendirilmesinin üzerine gidilirken özellikle bağımlılıklar üzerinde odaklanılacaktır. Bu da bizi etkin bir performans değerlendirmesine götürecektir.

Önerilen performans değerlendirme modelinin amacı, yazarın önceki çalışmalarını geliştirerek, çalışanların performans değerlendirmesine katkıda bulunmaktır. Önceki çalışmalarda önerilen Çok Ölçütlü Karar Verme – ÇÖKV yaklaşımı, ölçütler, alt ölçütler ve alternatifler arasındaki bağımlılıklar ve etkileşimler dikkate alınarak geliştirilecektir.

ÇÖKV yöntemlerinde, ölçütlerin bağımsızlığı varsayımı vardır. Bu yüzden, uygulayıcılar, bağımsız ölçütler seçme veya seçtikleri ölçütlerin öyle olduklarını var sayma eğiliminde olurlar ki bu da yanlı değerlendirmelere sebep olmaktadır. Gerçek hayatta bu ciddi yanılgılara yol açabilmektedir. Bu şekilde bir varsayım yapabilmek için, uygulamayı yapan kişinin, ölçütler, altölçütler ve alternatifler arasında olası bir

bağımlılığın olmadığından emin olması gerekmektedir. Aksi takdirde sonuç bu varsayım yüzünden değişik çıkma riskine sahiptir.

Bu çalışmada sunulacak olan iki yöntem, ölçütler arasında olabilen farklı tipte ilişkiler ile ilgilenmektedir. Analitik Ağ Süreci – AAS, bir ölçütün diğer bir ölçüt veya alternatif ile arasındaki bağımlılık ile ilgilenmektedir. Bir öğretmenin performans değerlendirmesinde performans boyutları olarak incelenecek olan bilgi düzeyinin öğretmenin öğretme kapasitesi üzerindeki etkisi veya bir arabanın aerodinamik yapısı ile benzin sarfiyatı üzerindeki etkisi örneklerinde olduğu gibi.

Diğer yöntem, Choquet integrali – CI, bulanık integraldir ve modelin *n* adet ölçütünden *k* tanesi arasındaki etkileşimi ile ilgilenir. Bu etkileşim değeri, başarılı olmak adına bu *k* adet ölçütün herbirinden aynı anda başarılı olmanın veya tek tek herhangi birinden başarılı olmanın yeterli olup olmayacağının göstergesidir. Buna *k*-toplamalı CI denir. Önerilen yöntemin kullanıcılar açısından kolay anlaşılabilir ve gerçek hayata olabildiğince yakın kılınabilir olması için, çalışmada 2-toplamalı CI kullanılacaktır. Yukarıda verilen açıklama ile iki ölçüt arasındaki pozitif etkileşim değeri, bir alternatifin başarılı olabilmesi adına bu iki ölçütten de başarılı olması gerektiğini, diğer yandan negatif etkileşim değeri de bu iki ölçütten sadece birinden başarılı olsa yeterli olacağını gösterecektir. Burada sözü edilen etkileşim ölçütler arası etkileşimden öteye gitmemektedir.

Sözü edilen iki yöntemde de karar vericilerin öncelik sıralamaları dikkate alınacak olması, çalışmada grup kararı sözkonusu olduğunda, toplumsal seçim fonksiyonlarının kullanımının önerilmesinin akılcı olacağı fikrini gündeme getirmiştir. Böylece, her karar verici için çalışanların performanslarına göre sıralamasının elde edilmesinden sonra, karar verici grubunun ortak kararı olabilecek en uygun sıralamanın elde edilmesi gerçekleştirilebilecektir.

Önerilen performans değerlendirmesi yöntemi şu şekilde işlemektedir: İlk olarak, firmanın çalışanlarından beklentilerine göre modelin oluşturulması, karar vericilerle yapılacak toplantılarda gerçekleştirilecek ve ölçütler açıkça belirlenecektir. Daha sonra modelin yapısı göz önünde bulundurularak, AAS veya CI veya ikisinin bir kombinasyonunun kullanımına karar verilecek ve her karar verici için çalışanların performanslarına göre sıralaması elde edildikten sonra da toplumsal tercih fonksiyonlarının kullanımıyla bir sonuca ulaşılacaktır.

1. INTRODUCTION

During last ten years, great majority of all the papers about human resource management – HRM and performance focused on the human resources – HR as department in an enterprise, personnel selection and the organization's performance, but not on the workers or employees' PE. Some of the works in the mentioned areas were:

- The connection between HRM (as a source of competitive advantage) and the organizational performance [1, 2, 3 and 4],
- The need to develop effective personnel selection mechanism to find the most suitable talents to the organizations. Optimizing HR allocation problems with respect to organizational requirements and jobs classification [5, 6 and 7],
- Determination of the salary and benefit for the applicants based on their qualifications [8],
- The main contributions of HR dimensions for the environmental management in a company [9].

A relatively detailed review of the publications on human performance in the last decade can be seen in Appendix A. According to this review, it wouldn't be bold to say there is a gap in the employees' PE field. Of course in a conventional way, there are methods and tools to do that task but the questions to ask ourselves are:

- Are these methods objective enough?
- Are they completely able to compare effectively the employees?
- How they manage to assess simultaneously the tangible and intangible factors?
- Do they take the interdependencies / interactions in consideration?

Traditional PEs are subjective and one-sided and therefore ineffective in providing honest and objective feedback. 360-Degree feedback assessment process offers a better way. Regardless of the position you hold in a company, it is important to understand how others (boss, team, peers and customers) perceive your effectiveness, strengths and weaknesses as a leader [10]. The 360-degree PE, which in the last decade became very popular around the world, is a system that evaluates the employees' performance through an open feedback system. A number of evaluators including the superiors, co-workers, customers and subordinates of the employee, with their immediate superior are evaluating the employee. Hence the employee observes his/her PE from many different perspectives. From that point of view, if the feedbacks are objective as they are expected to be, the PE may be effective. The success of this PE system is based on the organizational environment. It requires a mature organizational structure with intense communication [11]. A combination of classical management with old-fashioned supervisors will not match with this system and therefore the concerned parties don't trust the system. On the other hand, organizations failing in following the changes and innovations will also fail in implementing the 360-degree PE due to the changes and stress in management or leadership [12]. Even if they don't fail in implementing, the process itself will be far from reliable.

On the other hand, in this last ten years, only works on HR including MCDM tools are the following papers and they are mostly focused on personnel allocation:

- Using AHP together with linear programming for HR allocation problem [6],
- Using ANP for a PE model for project managers [13],
- Using ordered weighted average (OWA) aggregation operators for the personnel selection problem [14],
- Using a multi-criteria ordinal ranking model for personnel allocation among branches of a large-scale commercial bank [5].

There are three very frequent situations which make us think that PEs may have errors: The first is the fact that a great percentage of the users do their evaluations clearly above the average, which is called the generosity error. The managers accepted they are generous in their judgments because, they think giving a negative feedback will affect face to face relations and they want to motivate and help to advance their employees. They also confessed that they decreased their evaluations in order to teach something to the employee or to say to him/her that it's time to quit. To avoid this kind of political attitudes, the high level management has to take the PE in serious and transmit this to the lower levels. The second is the fact of not being able to observe the expected performance differences of a group in the evaluations. And the third is the situation in which the evaluations of a person on different performance dimensions show a strong correlation.

The aim and the originality of the proposed model are, by developing and improving the writer's previous works [15], to present a study on the employees' PE, a field that has not been highly considered in the literature of MCDM, and to bring a clearly higher level of objectivity to the PE for the employees working for the same task. Hence the MCDM perspective, added in the previous work to the HR field to be more efficient at handling the tangible and intangible data simultaneously, will be improved by taking into account the dependence or interaction between criteria and alternatives.

In MCDM methods, most of the time there is the assumption of "independence of the criteria / sub-criteria / alternatives". Because of that, people usually have a tendency to construct independent criteria or criteria that are supposed to be so, causing some bias effect in evaluation [16] and which is practically erroneous in real life problems. To be able to make such assumption, one must be sure that there is no possible relation or in other words interdependence between criteria or alternatives according to the data at hand, because the final decision may seriously change with this assumption. In this study, the dependence of the criteria will be considered and analyzed very carefully.

Two methods, which will be presented in this study for the purpose of dealing with the dependencies, handle different case of dependencies or interactions between criteria. One of the methods proposed by Thomas Saaty [17], namely the Analytic Network Process – ANP, one of the most popular MCDM tools which is able to handle the interdependencies among criteria / alternatives, considers the dependencies like one criterion having an effect on another such as the aerodynamic and the fuel consumption of a car in case of selection of car problem. Or the example of a teacher and two criteria to evaluate his performance: the knowledge on the course he/she is teaching and the teaching ability. So the interdependence to analyze here could be the effect of basic knowledge on the ability of teaching.

The DM(s), in ANP provides his preferential information according to his/her expertise on the subject, by giving pairwise comparisons in order to find the relative importance of the criteria, the alternatives' relative importance for the qualitative criteria and the addition of the interdependencies to the model. The quantitative data on alternatives is also needed in order to find the relative importance of the alternatives according to the quantitative data. Then the collection of all these information into a supermatrix will be realized and the limiting power of this supermatrix will provide the solution and the final ranking of the considered alternatives. Therefore ANP can treat problems having complex interrelationships among factors (dependencies and feedbacks) so that it can handle the complexities of real-world problems for making societal, governmental, and corporate decisions [18, 19, 20, 21 and 22].

The ANP approach has been successfully used in many areas such as business and management [23 – 31], location selection [32, 33], supplier selection [34, 35], social themes [36 – 40] and manufacturing [18, 41 – 44], decision contexts or used to predict sports outcomes [45], telecommunication [46 – 48], construction [49] and economic turns [50].

The other method, namely the Choquet Integral – CI, which has been introduced in the fuzzy measure community by Murofushi and Sugeno [51], is a fuzzy integral proposed by Gustave Choquet [52] and considers the interactions between k out of n criteria of the problem, which is called the k-additivity property. The interaction value (positive or negative) of the k criteria under consideration will determine if these criteria must be satisfied at the same time or only one of them satisfied is enough to be considered as a successful or good performing unit. For the sake of simplicity and the applicability to the real life situations as effectively as possible, in this study, the 2-additive CI will be used. Hence, with the explanation given above, a positive interaction between two criteria will show that both of them have to be satisfied and a negative one will show that it's enough that one of the criteria be satisfied by an alternative in order to be considered successful. In other words, these cases can be considered as "conjunctive" or "disjunctive" relations between criteria. It has to be noted that in these kinds of interdependencies / interactions, the relations between criteria and alternatives are not mentioned. This is just the relations between criteria.

The CI approach has been used in many areas such as risk management [53], telecommunication [54], transportation [55], flexible job-shop scheduling [56], monitoring the industrial performance [57], measuring the service quality of e-stores [58], supplier chain management [59] and new product development [60].

The basic matter in CI, is to define the weight of each elementary performance expression in relation to all other contributions to the overall performance, i.e. the Shapley parameters and the interaction parameters of any pair of performance criteria. There are several methods to do that but in this study, one of them will be proposed: determination of the weights by Measuring Attractiveness by a Categorical Based Evaluation TecHnique – MACBETH, a multi-criteria decision analysis approach which has been proposed by Bana e Costa [61].

The DM(s), in this step of CI method, provides his preferential information again according to his/her expertise on the subject, but this time by giving a pre-order of the criteria and alternatives with respect to criteria in order to find the relative importance of the criteria and the alternatives as well as the interactions between criteria.

As, in both method which will be presented in this study, the DM(s) will give their preference orders most of the time, it can be a wise approach to apply the Social Choice Functions – SCFs to the alternatives rankings when the situation will be a group decision. Therefore, for group decision cases, it is proposed to use an appropriate SCF, once the ranking of alternatives for each DM(s) is found. By doing this, a suitable ranking for the alternatives will be found in order to satisfy the individually found rankings the most. The use of SCFs is also chosen because of hybrid utilization of two different methods (ANP and CI). SCFs will provide an opportunity to compare the results found with two methods on a same basis. This will also provide an easy way to the group decision process for MACHBETH integrated in CI.

Section 2 of this study is about Human Resources Performance Evaluation. The general information about HR and PE will be given in this section. Section 3 and 4 will be explaining in detail the ANP and CI methods respectively. Section 5 of the study is dedicated to explain the voting, the preference voting system, the necessity of SCFs, some most widely used SCFs and finally the agreement and disagreement concept. After the methodology part where how the proposed model will work is explained, at the end of the study, in section 7, an application will be presented in order to see the use of the proposed methods. As a conclusion of the study, the advantages, disadvantages of the proposed methods compared to the current methods for human PE as well as the contributions of the methods to the human PE will be discussed in details with decision makers and will be reported here in the study.

2. HUMAN RESOURCES PERFORMANCE EVALUATION

2.1. General Information

PE is one of the most important fields to analyze for the continuity of an organization. It's been recently emphasized and systematized in business because the importance of human resources – HR in creating a high performance organization is understood [11]. Being also one of the most central HR practices in organizations due to its critical relations with selection, compensation, training and other employment practices [62], it can be applied in individuals' level and also in inter-organizational level. But in any case, the purpose is to see in which situation the individual or the organization is. If the situation is under the requested level, then the purpose is to find the means to increase the performance and to improve the situation. Otherwise, the purpose is to take precautions to keep this acquired level of performance. It is a mechanism by which the organization holds the individuals responsible for their behavior. In other words, the PE process provides inputs for almost all Human Resource Management – HRM functions and the outputs form the objective data in decision and applications [11]. Evaluation has been demonstrated to increase performance and effectiveness [63].

PE is a process including activities that determine an employee's efficiency through examination of his/her work, activities, weaknesses, strengths, competences and deficiencies [64]. Although its importance is not ignored, PE is not considered as a system in many corporations. Hence a suitable PE system with the corporate strategy and culture could not be formed. A highly significant role in a PE system is played by the employees' trust to the system which directly influences the success of a PE system.

HR, although it can be the most difficult to control expense of an organization, can be central ingredients affecting organizational performance [65]. For this challenge, a research area named Strategic Human Resource Management – SHRM, devoted to understand the effects of HRM practices on organization's outcomes [66 – 67], has been arisen. Boxall et al. [68] divides SHRM into the following three major subfields:

- Micro HRM, covering the sub-functions of HR policy and practice.
- SHRM, covering the overall HR strategies and trying to measure their impacts on performance and
- International HRM, covering HRM in companies operating across national boundaries.

In their study, considered by many as the earliest SHRM paper, Tichy et al. [69] argued that:

- HR activities have a major influence on individual performance and therefore productivity and organizational performance,
- the cycle of HR activities is interdependent and
- effective strategic management requires effective HRM.

2.2. High Performance Work Practices

HR practices that SHRM theorists consider performance enhancing are known as high-performance work practices – HPWPs [70] including training, employee participation, flexible work arrangements [65, 70], increasing the employees' knowledge, skills and abilities (KSAs) and encouraging employees to leverage their KSAs for organizational benefit [71, 72] which result in greater job satisfaction, lower employee turnover, higher productivity and better decision making hence improve organizational performance [73]. According to the study made by Combs et al. [74], a statistical aggregation of 92 studies supported the hypothesis of the use of HPWPs is positively related to the organizational performance. Furthermore, the study supported the hypothesis of the positive relationship between HPWPs and organizational performance being larger for manufacturing organizations than for service organizations because of the following four reasons:

- Manufacturing jobs often involve complex and potentially dangerous machinery whereas service organizations are less reliant on.
- Manufacturing organizations rely more on their HR system to deliver KSAs and motivation than service organizations do.
- In manufacturing, product quality is determined by employees, processes, equipments and managers whereas in service organizations, this additionally depends also on the interaction between customers and employees.
- Some HPWPs appears to be better aligned with manufacturing work.

Using relatively large samples of companies and individuals from different sectors, the studies [75 – 80] showed and supported the HR practices have a direct effect on organizational performance, business strategy moderates the relationship between HR practices and organizational performance and successful companies had increased HR involvement in strategic decisions and formalized HR practices.

2.3. Performance Evaluation

2.3.1. The Use of Performance Evaluation

When we look from a historical perspective to business PE, we see that, PE has been used to take management / personal decisions. PE provided very useful data to determine the salaries, to promote or to fire people. But we don’t have to think that these evaluations are the only ones to consider while taking the management decisions. For example salary increase is also related with the situation of the market in which the company is.

A research shows that PE information is used specially in four fields: decisions requiring inter-personal comparisons (salary determination, promotion etc.), personal comparison (feedback, personal educational need etc.), decisions orientated to the continuation of the system (target determination, human force planning etc.) and documentation.

Other than the management use, PEs are also used in order to inform the employees about their performances. Whereas the purpose in the management use is to evaluate the performance of the employee by comparing him/her to the others doing similar work and to find his/her position. The purpose of feedback evaluations is to find employee's position according to predefined standards and to define where he/she works good/bad and do what is necessary.

2.3.2. Bad Performing Employees

The reasons lying under the fact that "the employees don't work properly, efficiently or don't perform as their best" are various. All the articles and books mentioning about motivation, underline the importance of "knowing why to do it". Actually there could be no manager who would do the mistake of saying "No questions! Do as you are told! That's why you are paid!". But, unfortunately, very little percentage of them do the effort of answering this question – "why?" – before it's been asked. Hence, a lot of employees work badly because they don't know the reason why they work. Even if you maintain surveillance, they will do their work just because you've asked them to do so. But you have to keep in mind that you can't keep a continuous surveillance. So before assigning a job to the employees, it would be wise to explain them why they have to do this job. And when it's a long and delicate job you have to convince them about the advantages in the long term. If you want to evolve their attitude, you have to explain them the problems and the goal to achieve, discuss the solutions and show the advantages in case of success and the disadvantages in the opposite case.

Most industrial psychologists agree that individual performance does change over time, yet few performance measurement models directly address this aspect of job performance [81]. Typical performance of an individual represents the level of performance that usually achieved by this individual and maximal performance is the performance level that is achieved by the individual in case of a high motivation while doing the job. Sackett et al. [82] specified three defining characteristics for maximum performance:

- individuals are aware of being evaluated,
- individuals are aware of and have accepted instructions to maximize their efforts and
- performance is measured a short duration of time to ensure that attention remains focused on the accepted goal to maximize performance.

Bailey [83] argued that employees often perform below their potential because they have arbitrary use of their talent and time. The reasons for them to work have to be strong enough to make them work even if the supervisor turns around and stop watching. In other words, the employees must be motivated to leverage their KSAs. HPWPs such as performance appraisal, internal promotion policies, security, flexible work hours and schedules can help to increase motivation of the employees [65, 70, 72 and 84]. It has been demonstrated by several studies that to encourage the employees to participate in organizational efforts is important in increasing organizational performance [85 – 87]. The employee performance can remarkably increase with the application of the appropriate HR practices [88].

Gong et al. [89], in order to advance the understanding of the link between systems of HR practices and firm performance, focused on affective and continuance commitment to a firm, where affective commitment refers to an emotional attachment to a firm and continuance commitment refers to the tendency to stay in a firm because of the potential loss or costs associated with leaving the firm [90 and 91]. Continuance commitment, as studies frequently show, is considered undesirable due to its negative relationship with (or being unrelated to) job performance [92]. On the other hand, researches suggest that affectively committed employees have lower tardiness rates and absenteeism, higher task performance and they are ready to help others [93 and 94].

2.3.3. Effective HRM

In order to survive in the competitive world, the globalization of the economy makes it more important for the enterprises the process of adaptation, evolution and development. These factors involve a lot of revision in order to evolve its internal functioning and assure its development and evaluation is an important part of this revision and has a strong influence on both internal functioning and development. To develop itself, the enterprise needs activity, dynamism, innovation and motivation from employees. It's always said that a motivated employee takes initiatives. But the reciprocal is also true: an employee who takes initiatives gets motivated.

In the perspective of an effective HRM, the enterprise will let the process of evaluation develop itself. Such an enterprise has to build a system which answers the needs of the enterprise as well as those of the individuals. This means the system,

- guarantees enough of objectivity,
- presents credible stakes of progression and career,
- simplifies the organization functioning and
- attaches importance to development.

Evaluation can be applied in three directions in the context of animation and management of a team: the service, the competence and the potential of the employee. The evaluation of the service is about determining what the employee did, and this evaluation will be used in the rewarding phase. The evaluation of the competence is about what the employee can do, and this evaluation will be used in assigning tasks or change of work. The evaluation of the potential is about what the employee will be able to do, and this evaluation can be used in career management or in promotions.

To avoid psychological phenomena and answer the expectations of the employees, the evaluation process has to insure a sufficient level of objectivity and only a limited level of subjectivity [95]. Very often, the impossibility of being perfectly objective drives the evaluators to be perfectly subjective which will lead to the mistrust to their legitimacy. Therefore, in order to satisfy these two conditions, the evaluator should keep separate the evaluation of service, competence and potential, because they don't insure the same level of objectivity. This separation will allow well understanding of their role in the process of global evaluation and their impact on the decision of the management.

In addition to these, the evaluation process has to be explicit and organized. Therefore it will lay on criteria defined beforehand; it will be elaborated by maintenance and will allow a connection between the evaluation realized and the perspectives of career evolution and rewards offered. Such system will give a chance to the evaluation in order to be concrete, transparent and to satisfy the conditions of sufficient objectivity and limited subjectivity. Also this will offer significant behavioral control over the evaluated employees but the higher this control, the greater the cost. The criteria definition is the most important stage of a PE system as it is directly related of the eventual success of a system. It has to be kept in mind that the system effectiveness is also a function of putting the outcomes into practice.

2.3.4. Erroneous Performance Evaluations

The PEs may contain errors. The errors can be made by the evaluator, the evaluated or can be caused by the corporate atmosphere or the chosen method. In any case, this will cause a decrease in the trust to the evaluation system. These errors can be collected under the following names and cases:

- Halo effect: one factor has disproportionate influence on the others.
- Horning effect: evaluation according to the weaknesses.
- Cluster tendency: avoiding very high or low values.
- Tolerance effect: overrating.
- Toughness effect: underrating.
- Similarity effect: favoring an individual having similar background to that of the evaluator.
- Prejudice against the individual.

- Evaluation performed according to recent performance.

The reasons of the errors can be strategic, psychological and sociological. A supervisor may underrate a very good performing member of the team in order to keep this member in his team and not loose it because of a promotion after a favorable evaluation. The exact opposite version of this one may also happen and a member can be overrated in order to get rid of him/her by a promotion to another department. The responsible of a team may have the legitimate desire of select the people he wants to work with. This desire is valid at the group level also. In each team, there exist a cooptation which relies on a community of ideas, convictions, values, principles and a pressure of conformity which will influence the group to reject a member according his/her culture or values. In fact, between what is operationally desirable and culturally acceptable, the group or its responsible may choose the second term over the first.

Evaluation files filled by the managers generally end up in dusty folders on a shelf where no one come and consult them. The information lying down in those files are so vague and so little reliable that no one will try to refer them to take the minimal decision about personnel and surely about promotions or increase of salaries. Finally, this end of year ritual of PE is just a waste. This decreases the credibility of the one performing the evaluations. Whereas the direction of the enterprise thinks that they are taking the valuable time from the people who can do better things instead of these evaluations. Have you been in situations like that? Have you ever seen that your performance is evaluated in such way that didn't help you very much to improve it? What does your current system of performance management bring to you? Nothing? Not that much?

2.4. Performance Management - PM

PM is a process of continuous connection that has been settled as a partnership between the employee and his supervisor. It's an investment which will allow you, with complete confidence, to let your employees do their job. This implies having a clear definition and precise objectives including:

- the principal works expected from the employee,
- the way in which the work of the employee shows contribution to the global objectives of the enterprise,
- what we hear from “doing his work good” in a concrete way,
- the way that the employee and the supervisor can collaborate to support, improve or develop the actual performances of the employee,
- the way in which the performances will be measured,
- common desire to resist and eliminate the obstacles on the way of the performance.

The PM is not

- a system imposed to the employees by the managers,
- a conspiracy in order to make them work more,
- a process applied only in case of bad performances,
- an annual ritual in order to fill out forms at the end of the year [96].

If you apply correctly the PM by investing the necessary time and by implementing the cooperation relations with your personnel, it can:

- Decrease the need to have an eye on everything on any time,
- Help you save time: you help your employees to be more self sufficient by insuring yourself that they have what it takes to have good decisions,
- Prevent the misunderstandings between employees by determining who is responsible of what,
- Make employees see their weaknesses and strengths,
- Decrease the risk of missing the crucial information when you need it,
- Decrease the number of errors and the risk of seeing them repeat,
- Help a supervisor in coaching and mentoring employees more effectively by strengthening the supervisor – subordinate relation and interaction,
- Develop the supervising skills,

- Facilitate the teamwork.

From the employees' point of view, an efficient PM could help them to solve the following problems affecting their life:

- Not knowing if they are doing a good job and if they are in the right path.
- Not having any idea of the level of authority they have.
- Not having any sign of appreciation of their success.
- Not having any opportunity of acquire new competencies.
- Noticing the supervisor unhappy of the work of the employee but applying the "ostrich policy".
- Not being able to take any decision no matter how elementary it is.
- To be in a constant surveillance and interventions of the supervisor.
- Missing the necessary material in order to work well.

However, the employees have to see and understand the point of the PE. If they don't understand the process or they don't see the point of doing this, they will take this process as something imposed by the upper management and the more they will ask themselves questions that will not have answers the more they will have doubts about the process, which is not the ideal atmosphere to build a process relying on cooperation and partnership. It's your duty to prepare and inform the employees. The employees have to know:

- the importance of PM,
- utility of the process for them, for you as well as for the enterprise,
- your general philosophy (cooperation, auto-evaluation system etc.),
- the way of happening of the performance planning meetings,
- the kind of information they'll have to provide,
- the kind of questions they'll have to answer,
- the way in which the decisions will be made in these meetings,
- the level of flexibility of the objectives and the assigned works,
- the kind of preparation that they'll have to assume,

- how the annual balance will happen?
- what will happen in case of disagreement?
- how will the evaluations affect the salaries?

Even though they differ considerably from one country to another, the laws settle the rules for hiring and firing the personnel and you have to obey these rules. If you want to fire an employee for the bad performances of his/hers, he/she can attack you by stating that the real reason of this is a sort of discrimination (age, sex, ethnicity etc.) or that he/she hasn't been noticed and you didn't give him/her a chance to improve him/herself. So, at the court, you may be forced to establish that the performance problems are the real reasons of your action. A good PM allows to collect and preserve the problems related to the performances with a correct chronology and to notify the employees whereas telling them how to improve themselves. This will be a non-discussable prove in the eyes of the authorities.

A PM system must be and keep being practical. If the process becomes heavy, then nobody will want to use it and it will not be useful anymore. If a good system solves a number of problems, then a bad one will create some new ones. Let alone the waste of time and other resources, bad systems have the great risk of not providing the necessary information which will cause the managers confront with very difficult situations and be inefficient to solve the important performance problems. Therefore, the system must:

- Be as simple and as less bureaucratic as possible,
- Require a minimum investment of time,
- Offer a maximum level of comfort,
- Answer the needs of the managers, the employees and the enterprise.

As the main role of the PM is to help the enterprise, its different units and its each member to focus on the same target, you have to define as clearly as possible the objectives that the enterprise wants to achieve and the ways to achieve them. It's a big advantage for the employees to know exactly in what their work show contribution.

This will reinforce their interest and motivation. The following documents will help you to establish the links between the individual objectives and the enterprise's objectives:

- General strategic plans of the enterprise,
- Operational planning of the year,
- Operational and strategic plan for the departments,
- PE files of each employee for the previous year.

The cycle of PM starts with the planning and ends with evaluation. But the element increasing the efficiency is what happens between these two stages: continuous communication. Long ago, the professional world was clearly more stable. An employee could do the same work for years in a repeating way. Evolution rhythm was way calmer. But this is not the case anymore. Modern enterprise is in constant transformation. The needs because of competitive world push the enterprise to improvise continuously. The work is more complex and its rhythm has an increasing speed. One of the advantages of the continuous communication is to give the process a flexibility, dynamism and optimal adaptability. It will allow adapting ourselves better to the changes. Hence, the purpose of the continuous communication is to insure that, day by day, everyone receives the information that they need to improve themselves.

Even with the best intentions of the world, some managers create situations where their approach of communication provoke negative effects because the employees have the feeling of being constantly under control in scholar way or because they fear from the fact that if they will talk honestly they will receive punishment or extra work. In order to prevent this problem,

- Underline the "we/us": "How can **we**…?" "When will **we**…?" etc.
- Never ask intimidating questions, always questions that you need in order to solve the current problem,
- Make sure your employees well understood what you want, what you need,

- Don’t focus only on the problems. Try to see also the success. The employees need to know what they did well and not only what they need to improve,
- Encourage your employees to evaluate their own work and their own progress.

The performance of an individual at work is a function of the following factors:

- Abilities, capacities,
- Experience,
- Goals and values,
- Energy,
- Rewards,
- Personality.

Abilities are inherited whereas capacities are learned. Abilities are the indicators for potential, even though they affect the performance, they don’t insure it. From the childhood, we discover that some kids are better in, for example, drawing or singing or running than other kids. Then, schools, universities and colleges attempt to assess differences in these abilities. It’s quite normal that an experienced employee outperforms the novice.

In normal conditions, the performance will increase with experience wherever the knowledge, skill and practice are relevant to job. Here “normal conditions” term is preferred because as in abilities, the experience may contribute to the performance but it’s merely a guide to potential.

Of all the factors affecting the performance, a person’s motivation is the best performance predictor. Because of the fact that the humans are purposive, we choose to behave in particular ways. We select goals and try to achieve them. And the outcome of our behavior in order to do this is called motivation. Motivation is a process requiring definition of goals, decision making and energy canalization into achieving goals. There is a high degree of consensus about the more frequently cited goals that individuals try to satisfy at work:

- Comfort,
- Structure,
- Relationships,
- Recognition and status,
- Power,
- Autonomy, creativity, growth

Deciding to expend energy involves a choice and an assessment of the total energy available. Some people have more energy than others. Some people spare a higher proportion of their energy to their careers than others. We know that some people expend energy in short bursts while others sustain much longer bursts. That's why some rise to the middle management and fade while others goes towards the top management. Some fade when they are 35, some 45, some 70. Some researches show that this is related to the time orientation of the individual or how far ahead he/she can think and plan.

As a conclusion of work and work hard, organizations offer extrinsic and intrinsic rewards to the employees. Extrinsic rewards include salary, bonuses, commission payments etc. whereas intrinsic ones include satisfying other goals, lifestyle, comfort, status, public acclaim etc. And the individuals are motivated by these rewards that are available. Finally it is a choice with their abilities, experience and goals whether they will work.

Personality has not been shown by industrial psychologists to be a very helpful predictor of performance but this doesn't mean it is useless. It is more an admission that, at this stage of development, the personality trait literature is not very helpful in predicting performance.

3. ANALYTIC NETWORK PROCESS – ANP

3.1. General Information

ANP is a generalization of Saaty's Analytical Hierarchy Process (AHP), which is one of the most widely used multi-criteria decision support tools. AHP is limited to relatively static and unidirectional interactions with little feedback among decision components and alternatives [38]. Many real life decision problems cannot be structured as a hierarchy, because of the fact that they involve the interaction and dependence of higher level elements in a hierarchy on lower level elements. So the hierarchy becomes more like a network (See Figure 3.1 where a loop means an inner dependence). On this context, ANP and its supermatrix technique can be considered as an extension of AHP that can handle a more complex decision structure [17, 97], as the ANP framework has the flexibility to consider more complex inter-relationships (outer-dependence) among different elements.

AHP, as known, incorporates both qualitative and quantitative approaches to a decision problem [98]. It is also capable of capturing the tangible and intangible aspects of relative criteria that have some bearing on the decision making process [17], but AHP cannot deal with interconnections and inner-dependence between decision factors in the same level. This is because an AHP model is structured in a hierarchy in which no horizontal links are allowed. In other words, AHP can only be applied to a hierarchy that assumes unidirectional relation between decision levels. In fact, this weakness can be overcome by using the advance multi-criteria making technique, which is ANP. So, ANP is very useful in these kinds of situations providing a general framework without the assumptions of independence of higher-level elements from lower ones, or independence on the same level [34].

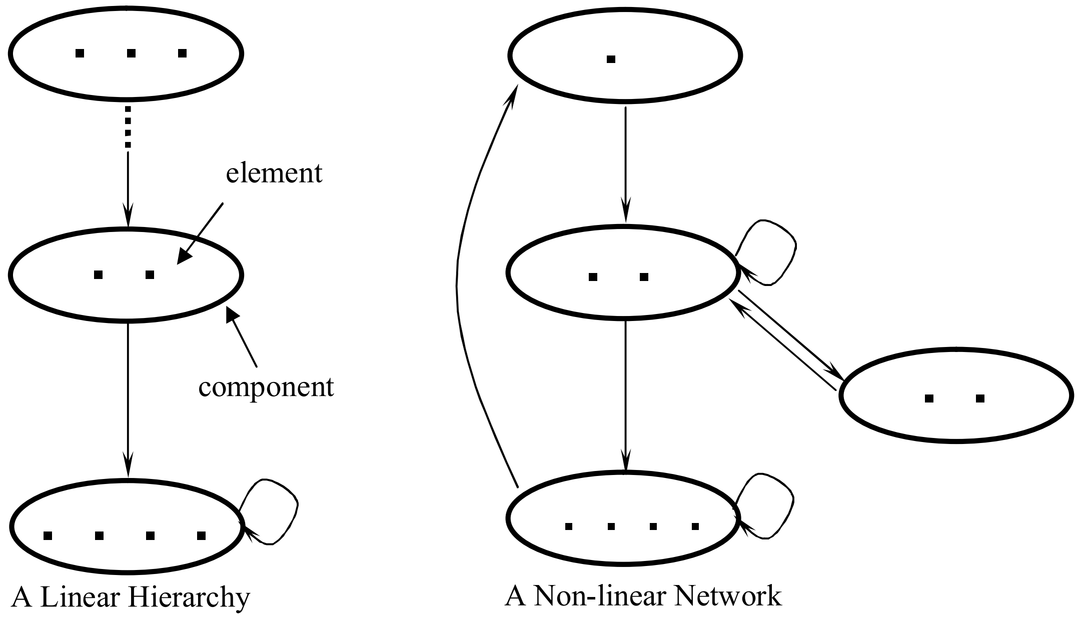

Figure 3.1. Structural difference between linear and nonlinear network

Thus, ANP consists of three parts: the first part is the control hierarchy for the network of the criteria and sub-criteria, the second part is a network of influences among the elements and clusters, and the third is the feedback between the various clusters and elements within a cluster. Therefore, ANP is a more powerful technique in modeling complex decision environments than AHP because it can be used to model very sophisticated decisions involving a variety of interactions and dependencies [26, 99] that exist in real-world problems.

3.2. Pairwise Comparisons, Eigenvectors and Consistency

In ANP, relative priorities are established in the same way that it is done in AHP. The qualitative aspects are weighted through pairwise comparisons using the fundamental scale given in Table 3.1.

Table 3.1. Saaty's Fundamental Scale

Value	Definition	Explanation
1	Equally important	Two decision elements have equal influence on the superior decision element.
3	Moderately more important	One decision element has moderately more influence than the other.
5	Strongly or essentially more important	One decision element has strongly more influence than the other.
7	Very strong or demonstrated importance	One decision element has very strongly more influence than the other.
9	Extremely more important	One decision element has extremely more influence than the other.
2, 4, 6, 8	Intermediate values of judgment	
Reciprocals		If *v* is the judgment value when *i* is compared to *j*, then *1/v* is the judgment value when *j* is compared to *i*.

Using the ratings given in Table 3.1, the pairwise comparison matrices $A=(a_{ij})$ are formed as seen below, in order to calculate the relative priorities of the elements forming these matrices in further steps:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \text{ where } a_{ij} = 1/a_{ji} \quad \forall i, j = 1,\ldots,n \text{ and } a_{ii} = 1 \ \forall i = 1,\ldots,n$$

If the matrix *A* wouldn't contain errors and the judgments were perfectly consistent, then:

$$a_{ik} \,.\, a_{kj} = a_{ij} \quad \forall i, j, k = 1,\ldots,n \tag{3.1}$$

Therefore all the elements in this matrix could be expressed as follows:

$$a_{ij} = w_i / w_j \quad \forall i, j = 1, \ldots, n \tag{3.2}$$

And this would yield to the following equality:

$$\begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} = n \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} \tag{3.3}$$

If the judgments are not perfectly consistent, the previous equality becomes $Aw = \lambda_{\max} w$, where $\lambda_{\max}$ is the principal eigenvalue of A. In other words, A is consistent if and only if $\lambda_{max}=n$ and as Saaty demonstrated, it turns out that the inequality $\lambda_{max} \geq n$ is always true.

The solution can be found by raising the matrix to a sufficiently large power (the Power Method), then performing the column normalization to obtain the relative priority vector $w = (w_1, w_2, \ldots, w_n)$. The process is stopped when the difference between the k^{th} and $k+1st$ power of the matrix is smaller than a predetermined small value.

An easy way to get an approximation of the relative priority vector is to make a column normalization of the matrix A and then take the arithmetic mean of the rows. Hence:

$$w_i = \sum_{j=1}^{n} \left[a_{ij} \Big/ \sum_{k=1}^{n} a_{kj} \right] \Big/ n \qquad \forall i = 1, \ldots, n \tag{3.4}$$

and

$$\lambda_{\max} = \frac{1}{n}\sum_{i=1}^{n}\frac{\sum_{j=1}^{n} a_{ij}.w_j}{w_i} \tag{3.5}$$

It has to be underlined that for important application; only the eigenvector derivation procedure has to be used because approximations can lead to a wrong ranking of the alternatives.

The consistency index – CnI of a comparison matrix is given by:

$$CnI = \frac{\lambda_{\max} - n}{n-1} \tag{3.6}$$

And the consistency ratio – CR is obtained by comparing the CnI value with the random inconsistency – RI values given in the Table 3.2. The judgments in the comparison matrix are said to be consistent and therefore the relative priority vector estimation is accepted if CR value is less than 10%. When greater values are found, the comparison matrix i.e. the judgments in the matrix need to be revised.

Table 3.2 RI Values for Different Size *n* of the Comparison Matrices

n	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***13***	***14***	***15***
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Using these values, the CR value is calculated as follows:

$$\text{CR} = \text{CnI} / \text{RI} \tag{3.7}$$

3.3. Dependence, Feedback and Networks

As stated before, many real life decision problems cannot be structured as hierarchies because of the interactions and dependences of higher level elements on lower ones. Feedback enables factoring the future into the present to determine what has to be done to attain a desired future [17].

To deal with the complexity, the decision makers choose to use simple hierarchic structures consisting of a goal, criteria and alternatives. Yet decisions obtained from a network can be significantly different than the ones obtained from a simple, moreover a complex hierarchy. It is not smart to avoid the complexity artificially and hope for the correct results which will reflect the reality. To deal with complex networks will surely take more time and effort but will give us more "healthy" results.

There are two types of influence/dependence: outer and inner. In outer dependence, one compares the influence of elements in a component / cluster on elements of another component / cluster with respect to a control criterion. In inner dependence, one compares the influence of elements in a group on each one.

3.4. Supermatrix

Considering these dependences, the priorities derived from pairwise comparison matrices are placed into a supermatrix. The supermatrix represents the influence priority of an element on the left of the matrix on an element on the top of the matrix with respect to a particular control criterion. Every element of a component doesn't need to impact an element in another component. Thus those who don't impact are given a zero value for their contribution.

Assuming n components, C_j where $j = 1, \ldots, n$, with each one having n_j elements, the supermatrix will be as follows:

$$W = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1n} \\ W_{21} & W_{22} & \cdots & W_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ W_{n1} & W_{n2} & \cdots & W_{nn} \end{bmatrix} \end{array}$$

$$\text{where} \quad W_{ij} = \begin{bmatrix} w_{i1}^{j_1} & w_{i1}^{j_2} & \cdots & w_{i1}^{j_{n_j}} \\ w_{i2}^{j_1} & w_{i2}^{j_2} & \cdots & w_{i2}^{j_{n_j}} \\ \vdots & \vdots & \ddots & \vdots \\ w_{in_i}^{j_1} & w_{in_i}^{j_2} & \cdots & w_{in_i}^{j_{n_j}} \end{bmatrix} \quad \forall i, j = 1, \ldots, n$$

Here, $w_{in_i}^{j_{n_j}}$ represents the impact of the n_i^{th} element of the component i on the n_j^{th} element of the component j. Therefore, each column in the matrix W_{ij} is a principal eigenvector that represents the impact of all the elements in the i^{th} component on each of the elements in the j^{th} component.

The resulting matrix needs to be stochastic, i.e. the columns have to sum up to one, in order to continue the calculations and obtain meaningful limiting results. It's necessary to compare the components themselves to ensure that. The pairwise comparisons of the components are made with respect to each of the components or according to some attribute presented in a separate control hierarchy for that system. The resulting priorities are used to weight the column vectors of the supermatrix previously obtained. Hence the resulting supermatrix is column stochastic.

The overall priorities of each element of each cluster is given by the solution to:

$$\lim_{k \to \infty} W^{2k+1} \tag{3.8}$$

Now what is desired, if it exists, is the limiting priority of impact of each element on every other element. It has to be noted that if the matrix is positive or if it becomes positive after raising it to some power; it turns out that a unique answer can be obtained. But when no power of the matrix is strictly positive, there may not be a unique limit as in oscillating powers of the matrix where different limits are obtained as shown in the following example:

Example: Let

$$W = \begin{bmatrix} 0 & W_{12} & 0 \\ 0 & 0 & W_{23} \\ W_{31} & 0 & 0 \end{bmatrix}$$

Then,

$$W^2 = \begin{bmatrix} 0 & 0 & W_{12}W_{23} \\ W_{23}W_{31} & 0 & 0 \\ 0 & W_{31}W_{12} & \end{bmatrix}$$

$$\text{and } W^3 = \begin{bmatrix} W_{12}W_{23}W_{31} & 0 & 0 \\ 0 & W_{23}W_{31}W_{12} & 0 \\ 0 & 0 & W_{31}W_{12}W_{23} \end{bmatrix}$$

And hence,

$$W^{3k} = \begin{bmatrix} (W_{12}W_{23}W_{31})^k & 0 & 0 \\ 0 & (W_{23}W_{31}W_{12})^k & 0 \\ 0 & 0 & (W_{31}W_{12}W_{23})^k \end{bmatrix}$$

$$W^{3k+1} = \begin{bmatrix} 0 & (W_{12}W_{23}W_{31})^k W_{12} & 0 \\ 0 & 0 & (W_{23}W_{31}W_{12})^k W_{23} \\ (W_{31}W_{12}W_{23})^k W_{31} & 0 & 0 \end{bmatrix}$$

$$W^{3k+2} = \begin{bmatrix} 0 & 0 & (W_{12}W_{23}W_{31})^k W_{12}W_{23} \\ (W_{23}W_{31}W_{12})^k W_{23}W_{31} & 0 & 0 \\ 0 & (W_{31}W_{12}W_{23})^k W_{31}W_{12} & 0 \end{bmatrix}$$

Here it's visible that there is not a single limiting answer but W raised to powers, passes through three different cyclic forms. For a limiting outcome, the average of these three limit matrices can be taken.

3.5. ANP Procedure

The outline of ANP steps as follows:

i. Describe the decision problem in detail with objectives, criteria, sub-criteria.
ii. Determine the general network of components / clusters and the elements within the clusters.
iii. Determine all the inter and inner-dependencies that exist in the decision problem. After this step, the network of the decision problem will also be found.
iv. Build the supermatrix by performing the pairwise comparisons, prioritization and define the weights of the criteria and the sub-criteria while considering the inter-dependencies between them.

v. Perform pairwise comparison on clusters. The derived weights will be used to find the weighted supermatrix.

vi. Perform consistency analysis of all the pairwise comparisons, made by the experts or decision makers, in order to make the necessary changes if there is any inconsistency above the allowed limit.

vii. Rate the alternatives according all the criteria and sub-criteria.

viii. Find the weighted supermatrix.

ix. Compute and find the limit supermatrix from which the overall score for the alternatives is retrieved and make the final decision as to choose the best alternative or to obtain the final ranking of the alternatives.

4. CHOQUET INTEGRAL

4.1. Measures and Integral

In this section, X will be assumed as a finite set with n elements, and its power set, i.e. the family of all subsets of X will be denoted by 2^X. The term "family" is used to represent a set of sets.

4.1.1. Set Functions

Definition 4.1.: A function ξ defined on a family of sets is called a set function.

The set function ξ defined on 2^X is said to be:

- additive if:

  $\xi(A \cup B) = \xi(A) + \xi(B)$ for every pair of disjoint sets A and B of X.
  Here, note that if ξ is additive, then $\xi(\emptyset) = 0$ since $\xi(\emptyset) = \xi(\emptyset) + \xi(\emptyset)$.

- monotone if:

  $\xi(A) \leq \xi(B)$ for every pair of sets A and B of X such that $A \subset B$

- normalized if:

  $\min\{\xi(A) \mid A \subset X\} = 0$ and $\max\{\xi(A) \mid A \subset X\} = 1$

Proposition 4.1.: If ξ defined on 2^X is non-negative and additive, then ξ is a monotone set function.

Demonstration: Let $A \subset B \subset X$. Then $\xi(B)$ can be expressed as follows:

$$\xi(B) = \xi(A \cup (B \setminus A))$$

Since $A \subset B$, $A \cap (B \setminus A) = \emptyset$ and with the additive property of ξ following equality can be found:

$$\xi(B) = \xi(A \cup (B \setminus A)) = \xi(A) + \xi(B \setminus A)$$

And as ξ is non-negative, $\xi(A) \geq 0$ for all subset A of X. And hence:

$$\xi(B) = \xi(A) + \xi(B \setminus A) \geq \xi(A) \text{ as } B \setminus A \text{ is also a subset of } X.$$

So, it is proven that ξ is a monotone set function.

Since X is a finite set, an additive set function ξ can be expressed as

$$\xi(A) = \sum_{x \in A} \xi(\{x\}) \text{ for } A \subset X.$$

Definition 4.2.: For a set function ξ defined on 2^X and A a subset of X, the restriction ξ_A of ξ on A is defined as

$$\xi_A(B) = \xi(A \cap B) \text{ for all } B \subset X .$$

Definition 4.3.: For a set function ξ defined on 2^X such that $\xi(\emptyset) = 0$, its conjugate set function, $\overline{\xi}$, is defined as:

$$\overline{\xi}(A) = \xi(X) - \xi(A^c) \text{ for all } A \subset X \text{ with } A^c \text{ the complement of the set } A.$$

By definition, it can be seen that $\overline{\xi}(\emptyset) = 0$ and $\overline{\xi}(X) = \xi(X)$.

4.1.2. Measures

Definition 4.4.: A non-negative additive set function defined on 2^X is a measure on X. A measure measures the size of a set. A signed measure on X is an additive set function defined on 2^X. A normalized measure is called a probability measure.

According to this definition, the relation between signed measures, measures and probability measures can be seen on the Figure 4.1.

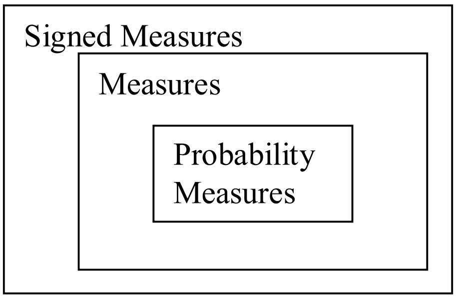

Figure 4.1. Signed Measures, Measures and Probability Measures

Example 4.1.: Let X be a finite set of objects and v_x and m_x be respectively the volume and the mass of each object x. Then the set functions V: $2^X \rightarrow \boldsymbol{R}_+$ which measures the volume of each subset A of X and M: $2^X \rightarrow \boldsymbol{R}_+$ which measures the mass of each subset A of X represented below are measures on X.

$$V(A) = \sum_{x \in A} v_x$$

$$M(A) = \sum_{x \in A} m_x$$

On the other hand, if each object x is electrified with q_x coulombs, then the set function Q: $2^X \rightarrow \boldsymbol{R}_+$, as represented below, which measures the quantity of electricity of each subset A of X will be a signed measure on X.

$$Q(A) = \sum_{x \in A} q_x$$

Definition 4.5.: Let *m* be a signed measure on *X*. A subset *N* of *X* is called *m*-null (or simply null) set if $m(M) = 0$ whenever $M \subset N$.

The properties of a null set are as follows:

- The empty set is a null set.
- A null set is of measure zero.
- If *m* is non-negative, i.e. a measure, then a set of measure zero is a null set.
- A subset of a null set is a null set.
- A union of null sets is a null set.

4.1.3. Integrals

Definition 4.6.: Let *m* be a signed measure on *X* and *f* (see Figure 4.2. for the graph of f) a function on *X*. The integral $\int f(x)dm(x)$ of *f* (see Figure 4.3.) with respect to *m* is defined as:

$$\int f(x)dm(x) = \int fdm = \sum_{x \in X} f(x).m(\{x\}) \tag{4.1}$$

Let $A \subset X$ and let 1_A be the indicator of *A*. The integral $\int_A f(x)dm(x)$ over *A* is defined as follows:

$$\int_A f(x)dm(x) = \int_A fdm = \int_A f1_A dm = \sum_{x \in A} f(x)m(\{x\}) \tag{4.2}$$

$$1_A = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$$

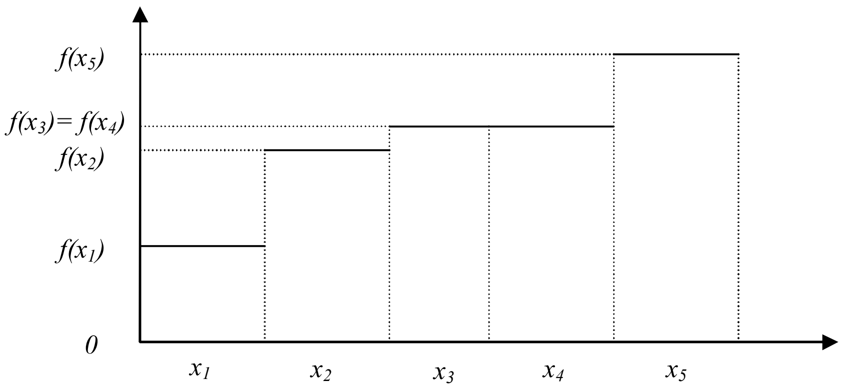

Figure 4.2. Graph of *f*

$$\int fdm = (1) + (2) + (3) + (4) + (5)$$

$$(i) = f(x_i).m(\{x_i\}) \text{ with } i=1,\ldots,5$$

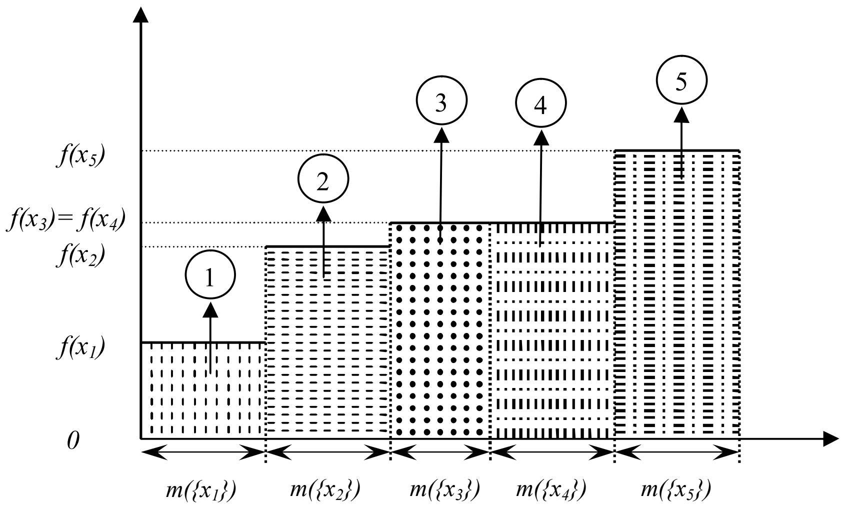

Figure 4.3. Representation of the integral of *f* defined as in equation (4.2)

Example 4.2.: Let the density of each object x of the Example 4.1 be $f(x)$ g/cm^3. Then the integral of f with respect to V is equal to the total mass of X. And for every $A \subset X$:

$$M(A) = \int_A fdV$$

Proposition 4.2.: Let m be a signed measure on X and $a, b, a_1, a_2, \ldots, a_n$ be real numbers, f and g functions on X and $A, A_1, A_2, \ldots, A_n$ subsets of X. Then:

(i) $$\int (af + bg)dm = a\int fdm + b\int gdm$$

(ii) $$\int \sum_{i=1}^{n} a_i 1_{A_i} dm = \sum_{i=1}^{n} a_i m(A_i)$$

(iii) $$\int fdm \leq \int gdm \text{ with } m \text{ a measure and } f \leq g$$

According to *(ii)* of the above proposition, note that every function f on X can be presented as:

$$f = \sum_{i=1}^{n} a_i 1_{A_i}$$

$$f = \sum_{x \in X} f(x) 1_{\{x\}} \tag{4.3}$$

$$f = \sum_{i=1}^{n} a_i 1_{\{x|\mathrm{f(x)=a_i}\}} \tag{4.4}$$

$$f = \sum_{i=1}^{n} (a_i - a_{i-1}) 1_{\{x|\mathrm{f(x)\geq a_i}\}} \tag{4.5}$$

For a function f represented as in (4.2), its integral is given in Figure 4.3. For a function f represented as in (4.4), the integral is given as follows in the equation (4.6) and it's represented in Figure 4.4:

$$\int f dm = \sum_{i=1}^{n} a_i m(\{x \mid \mathrm{f(x)} = \mathrm{a_i}\}) \qquad (4.6)$$

For a function represented as in (4.5), the integral is given as follows in the equation (4.7) and it's represented in Figure 4.5:

$$\int f dm = \sum_{i=1}^{n} (a_i - a_{i-1}).m(\{x \mid \mathrm{f(x)} \geq \mathrm{a_i}\}) \qquad (4.7)$$

$\int f dm =$ ① + ② + ③ + ④

ⓘ $= a_i.m(\{x|f(x)=a_i\})$ with $i=1,\dots,4$

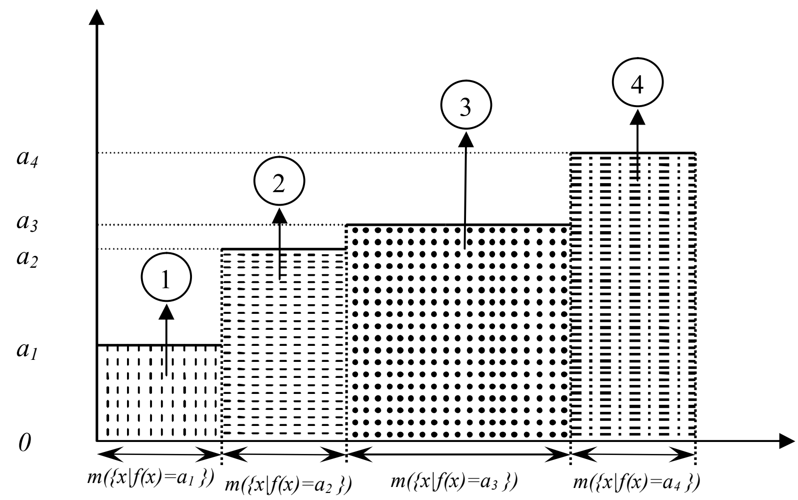

Figure 4.4. Representation of the integral of *f* defined as in equation (4.6)

$$\int f dm = (1) + (2) + (3) + (4)$$

$$(i) = (a_i - a_{i-1}).m(\{x|f(x) \geq a_i\}) \text{ with } i=1,\dots, 4$$

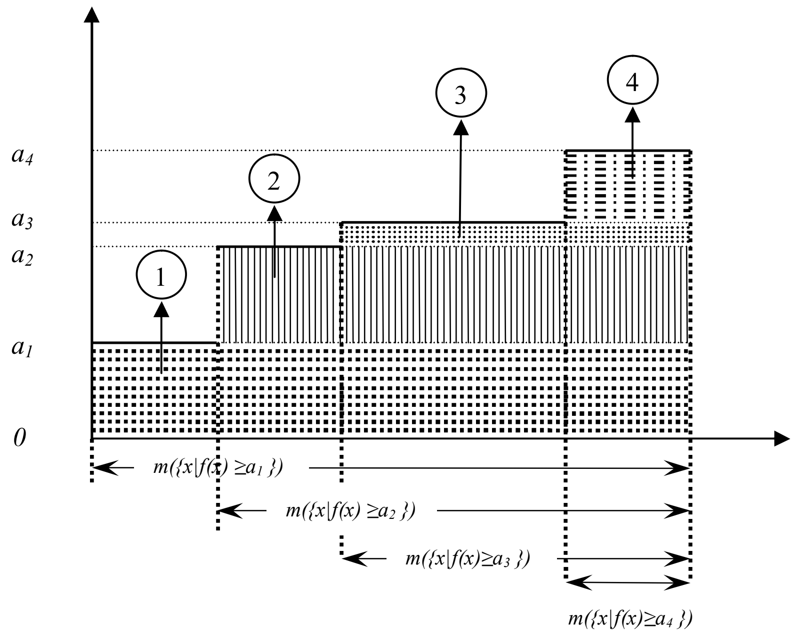

Figure 4.5. Representation of the integral of *f* defined as in equation (4.7)

4.2. Fuzzy Measures and the Choquet Integral

4.2.1. Fuzzy Measures

A (monotonic) fuzzy measure on X is a monotone set function μ defined on 2^X which vanishes at the empty set. A non-monotonic (or signed) fuzzy measure is a set function defined on 2^X which vanishes at the empty set.

Note that a fuzzy measure is not necessarily a measure. Because a fuzzy measure is non-negative since $\mu(A) \geq \mu(\emptyset) = 0$ for every A of X, but not necessarily additive. The main characteristic of a fuzzy measure is non-additivity, so that a fuzzy measure is also called non-additive measure. The relation between non-monotonic fuzzy measures, signed measures, fuzzy measures and measures can be seen on the following figure:

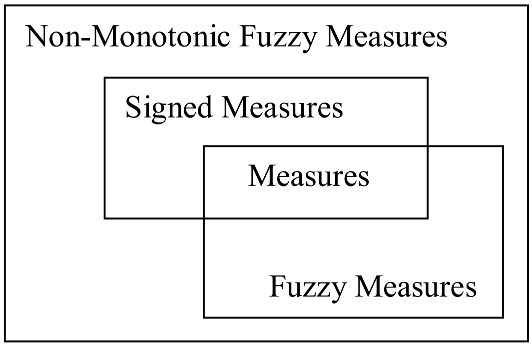

Figure 4.6. Families of set functions on a finite set X

A fuzzy measure μ is said to be

- Additive if $\mu(A \cup B) = \mu(A) + \mu(B)$ when $A \cap B = \emptyset$
- Super additive if $\mu(A \cup B) \geq \mu(A) + \mu(B)$ when $A \cap B = \emptyset$
- Sub additive if $\mu(A \cup B) \leq \mu(A) + \mu(B)$ when $A \cap B = \emptyset$

It can be observed that if a fuzzy measure μ is additive, it will be enough to define the n coefficients $[\mu(\{x_1\}), \mu(\{x_2\}), \ldots, \mu(\{x_n\})]$. In the general case, it is necessary to define 2^n coefficients corresponding to all subsets of X. (Here it can be assumed that X is the set of criteria and $\mu(A)$ represents the importance weight of the criteria set A)

Example 4.3.: Let X be the set of all workers in a workshop, A and B be two disjoint set of workers working separately and μ be a set function defined on 2^X which gives the production quantity per hour of a set of workers. Suppose that every group works in the most efficient way. It's visible that μ is monotone and vanishes at the empty set, and hence it is a fuzzy measure. μ is not necessarily additive because if the workers of the sets A and B work separately, then $\mu(A \cup B) = \mu(A) + \mu(B)$. But, since generally the workers interact on each other, the equality may not always hold and either $\mu(A \cup B) < \mu(A) + \mu(B)$ in case of incompatibility between the operation of two groups or $\mu(A \cup B) > \mu(A) + \mu(B)$ in case of the effective cooperation of members of $A \cup B$ can occur.

4.2.2. Choquet Integral - CI

The CI is an extension of the ordinary integral and the most natural fuzzy integral. A fuzzy measure is generally non-additive. Hence the right hand sides of (4.1), (4.5), (4.6) are generally different from each other. The right hand side of (4.6) is the most appropriate to the integration with respect to (non-monotonic) fuzzy measures, and this is CI [100].

Definition 4.7.: Let μ be a non-monotonic fuzzy measure on X and f a function on X with range $\{a_1, a_2, \ldots, a_n,\}$ where $a_1 \leq a_2 \leq \cdots \leq a_n$. The CI $(C)\int f(x)d\mu(x)$ or simply $(C)\int fd\mu$ of f with respect to μ is defined as follows:

$$(C)\int fd\mu = \sum_{i=1}^{n}(a_i - a_{i-1}).\mu(\{x \mid f(x) \geq a_i\}), \text{ where } a_0 = 0 \qquad (4.8)$$

The CI with $a_1 > 0$ is the same as the one represented on Figure 4.5. When $a_1 < 0$ then the situation is shown in Figure 4.7.

$$\int f dm = \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4}$$

$$\textcircled{i} = (a_i - a_{i-1}).\ \mu(\{x|f(x) \geq a_i\}) \text{ with } i=1,\ldots, 4$$

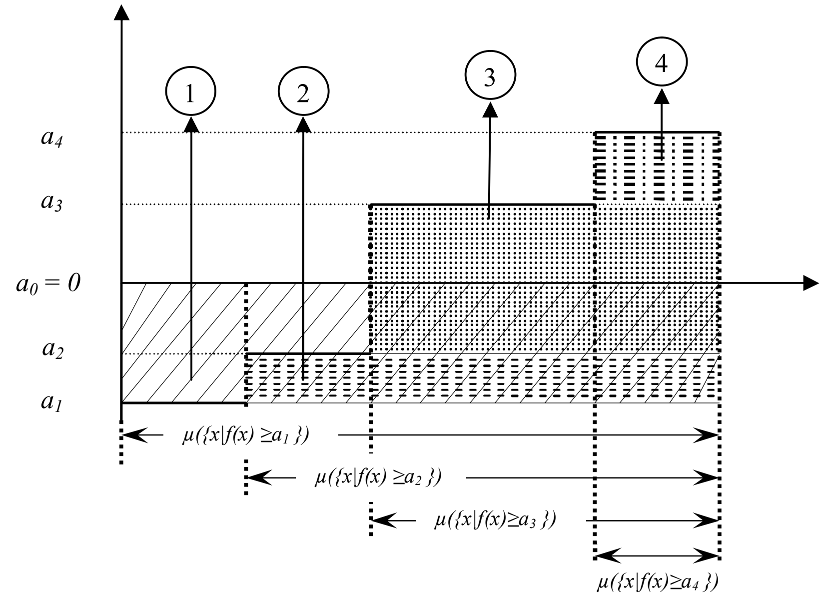

Figure 4.7. The CI of *f*

Example 4.4.: (Continued from Example 4.3.). Let $X = \{x_1, x_2, \ldots, x_n\}$. Each worker x_i works $f(x_i)$ hours a day from the opening hour. Without loss of generality, it can be assumed that $f(x_1) \leq f(x_2) \leq \cdots \leq f(x_n)$. Then, for $i \geq 2$, $f(x_i) - f(x_{i-1}) \geq 0$ and

$$f(x_i) = f(x_1) + [f(x_2) - f(x_1)] + [f(x_3) - f(x_2)] + \cdots + [f(x_i) - f(x_{i-1})]$$

Working hours of all the workers can be aggregated in the following way: First, the group X with n workers works $f(x_1)$ hours. The next group $X \backslash \{ x_1\} = \{ x_2, x_3, \ldots, x_n\}$ works for $f(x_2) - f(x_1)$ hours, then the next group $X \backslash \{ x_1, x_2 \} = \{ x_3, x_4, \ldots, x_n\}$ works for $f(x_3) - f(x_2)$ hours and so on. So the last worker x_n works for $f(x_n) - f(x_{n-1})$ hours.

Therefore, since a group $A \subset X$ produces $\mu(A)$ products in one hour, the total production number per day will be:

$$
\begin{aligned}
& f(x_1).\mu(X) \\
& +\left[f(x_2) - f(x_1)\right].\mu(X \setminus \{x_1\}) \\
& +\left[f(x_3) - f(x_2)\right].\mu(X \setminus \{x_1, x_2\}) \\
& +\cdots \\
& +\left[f(x_n) - f(x_{n-1})\right].\mu(x_n\}) \\
& = \sum_{i=1}^{n}\left[f(x_i) - f(x_{i-1})\right].\mu(\{x_i, x_{1+1}, \ldots, x_n\})
\end{aligned}
$$

where $f(x_0)=0$. This is the CI of f with respect to μ.

Proposition 4.3.: Let f and g be functions on X and $A \subset X$. The CI has the following properties:

(i) $(C)\int 1_A d\mu = \mu(A)$

(ii) If μ is a fuzzy measure and $f \leq g$, then

$$(C)\int f d\mu \leq (C)\int g d\mu$$

(iii) If a is a non-negative real number and b is a real number, then

$$(C)\int (af + b) d\mu = a.(C)\int f d\mu + b.\mu(x)$$

(iv) $(C)\int(-f)d\mu = -(C)\int fd\overline{\mu}$

(v) $(C)\int(-f)d\mu = -(C)\int fd\mu$ for all functions *f* on *X* iff $\overline{\mu} = \mu$.

(vi) $(C)\int fd\mu = (C)\int f^{+}d\mu - (C)\int f^{-}d\overline{\mu}$

where $f^{+}(x) = \max\{f(x), 0\}$ and $f^{-}(x) = \max\{-f(x), 0\}$.

(vii) If *a* is a real number, then

$$(C)\int fd(a.\mu) = a.(C)\int fd\mu$$

(viii) If μ and v are fuzzy measures on *X* such that $\mu \leq v$ and $\mu(X) = v(X)$, then for all function *f* of *X*

$$(C)\int fd\mu \leq (C)\int fdv$$

(ix) If *N* is a null set and if *f(x)* = *g(x)* for all $x \notin X$ then

$$(C)\int fd\mu = (C)\int gd\mu$$

4.2.3. Möbius Transformation and *k*-order additivity

Definition 4.8.: Let μ be a set function on *X*. The Möbius transform of μ is a set function on *X* defined by

$$m(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} \mu(B), \quad \forall A \subseteq X \tag{4.9}$$

The transformation is invertible and μ can be recovered from m by

$$\mu(A) = \sum_{B \subseteq A} m(B), \quad \forall A \subset X \tag{4.10}$$

Here, note that surely, any set of 2^n coefficients $\{m(T) \mid T \subseteq N\}$ could not be the Möbius representation of a fuzzy measure. The following boundary and the monotonicity conditions must be ensured [16]:

$$\begin{cases} m(\emptyset) = 0 \\ \sum_{T \subseteq N} m(T) = 1 \\ \sum_{T: i \in T \subseteq S} m(T) \geq 0, \quad \forall S \subseteq N, \quad \forall i \in S \end{cases} \tag{4.11}$$

Since a fuzzy measure defined on a set of n elements requires 2^n real coefficients for its definition, k-additive measures have been introduced by Grabisch [101] in order to decrease the exponential complexity of fuzzy measures in practical applications.

Definition 4.9.: A fuzzy measure μ is said to be k-order additive or k-additive if its Möbius transform $m(A) = 0$ for any subset A of X such that $|A| > k$, and there exists at least one subset A of X with exactly k elements such that $m(A) \neq 0$.

Therefore, *k*-additive measures can be represented by a limited set of coefficients, at most $\sum_{i=1}^{k}\binom{n}{i}$ coefficients.

4.2.4. Interaction between Criteria

Definition 4.10.: Let μ be a fuzzy measure on *X*. The Shapley Index for every $i \in X$ is defined by

$$\Phi_i = \sum_{K \subset X \setminus \{i\}} \frac{(n-|K|-1)!\,|K|!}{n!}\left[\mu(K \cup \{i\}) - \mu(K)\right] \tag{4.12}$$

Note that $\sum_{i=1}^{n}\Phi_i = \mu(X)$ as a basic property of the Shapley value.

The Shapley value of μ is the vector $\Phi(\mu) = [\Phi_1, \Phi_2, \ldots, \Phi_n]$. The Shapley index can be interpreted as a kind of the weighted average value of the marginal contribution of element *i* alone in all coalitions. In other words, the Shapley value represents a true sharing of the total amount $\mu(X)$.

The Shapley value is a fundamental concept in game theory expressing the power index [102]. Analogy with the multi-criteria decision making can be made as follows: *X* being the set of criteria, $\mu(X)$ has the maximal value, being one by convention. The Shapley index expresses the relative importance of a single criterion into the decision problem. The fact that the Shapley value for the criterion, Φ_i, is different than $\mu(\{i\})$ is a proof for the interaction of the criteria.

Considering a pair $\{i, j\} \subset N$, the quantity $m(\{i, j\}) = \mu(\{i, j\}) - \mu(\{i\}) - \mu(\{j\})$ seems to define the degree of interaction between i and j. The difference will be zero when there will be no interaction, will have a positive value if there is a synergy effect and will have a negative value when there is a negative interference between *i* and *j*. But to see the proper interaction between *i* and *j*, not only $\mu(\{i\})$, $\mu(\{j\})$ and $\mu(\{i,j\})$ but also all the subsets containing *i* and *j* should be considered. That is $\forall K \subset N \setminus \{i, j\}$:

$$\mu(K \cup \{i, j\}) - \mu(K \cup \{i\}) - \mu(K \cup \{j\}) + \mu(K) \tag{4.13}$$

Murofushi and Soneda [103], based on considerations of multiattribute utility theory, have proposed the following definition to reflect the above discussion.

Definition 4.11.: Let μ be a fuzzy measure on *X*. The interaction index between the elements *i* and *j* of *X* is defined by:

$$I_{ij} = \sum_{K \subset N \setminus \{i,j\}} \frac{(n-|K|-2)!|K|!}{(n-1)!} \left[\mu(K \cup \{i, j\}) - \mu(K \cup \{i\}) - \mu(K \cup \{j\}) + \mu(K)\right] \tag{4.14}$$

This definition can be enlarged to any coalition as done by Grabisch [101] as follows:

$$I(A) = \sum_{K \subset N \setminus A} \frac{(n-|K|-|A|)!|K|!}{(n-|A|+1)!} \sum_{C \subset A} (-1)^{|A \setminus C|} \mu(C \cup K) \tag{4.15}$$

If for any $A \subset N \setminus \{i\}$, $\mu(A \cup \{i\}) = \mu(A) + \mu(\{i\})$ than *i* is said to be a dummy criterion.

If μ is a *k*-additive measure on *X*. Then,

(i) $I(A) = 0$ for every $A \subset X$ with $|A| > k$

(ii) $I(A) = m(A)$ for every $A \subset X$ with $|A| = k$

Similarly, as in $m(\{i, j\})$, if I_{ij} is positive (resp. negative) then the interaction between i and j is said to be positive (resp. negative).

4.2.5. The 2-order model

In this section, the focus will be on the 2-order additivity which seems to be the most interesting in practical applications since it allows modeling the interactions between criteria while remaining simple.

It requires only $n + \binom{n}{2} = \frac{n(n+1)}{2}$ coefficients to define the fuzzy measure as follows:

$$\mu(i) = m(i), \quad i \in N$$

$$\mu(\{i, j\}) = m(\{i\}) + m(\{j\}) + m(\{i, j\}), \quad \{i, j\} \subseteq N \tag{4.16}$$

The other coefficients are given by:

$$\mu(S) = \sum_{i \in S} m(i) + \sum_{\{i,j\} \subseteq S} m(\{i, j\}), \quad S \subseteq N, \quad |S| \geq 2 \tag{4.17}$$

Let $t_i, \quad i = 1,\ldots,n$ be the scores on the criteria. By using only the interaction index, it is possible to express CI in the case of 2-additive measures as follows:

$$C_\mu(t_1,\ldots,t_n) = \sum_{I_{ij}>0}(t_i \wedge t_j)I_{ij} + \sum_{I_{ij}<0}(t_i \vee t_j)\left|I_{ij}\right| + \sum_{i=1}^{n} t_i(\Phi_i - \frac{1}{2}\sum_{j\neq i}\left|I_{ij}\right|) \qquad (4.18)$$

With $\Phi_i - \frac{1}{2}\sum_{j\neq i}\left|I_{ij}\right| \geq 0, \quad \forall i = 1,\ldots,n$

Note that the CI for 2-additive measures can be decomposed in a conjunctive, a disjunctive and an additive part, corresponding respectively to positive, negative interactions and the Shapley value [100]. In the CI framework:

- Positive values of I_{ij} implies a conjunctive behavior between *i* and *j*. i.e. simultaneous satisfaction of both criteria *i* and *j* is significant for the global score.
- Negative values of I_{ij} implies a disjunctive behavior between *i* and *j*. i.e. the satisfaction of either *i* or *j* is sufficient to have a significant effect on the global score.
- The Shapley value acts as a weight vector in a weighted arithmetic mean. This represents the linear part of CI.

In the 2-order case, the Shapley indices are assumed to be zero for the subsets of at least three elements. Therefore, in terms of the Möbius transformation the CI becomes:

$$C_\mu(t) = \sum_{i\in N} m(i)t_i + \sum_{\{i,j\}\subseteq N} m(\{i,j\})(t_i \wedge t_j), \quad x \in \Re^n \qquad (4.19)$$

On the other hand, the conditions given in (4.11) for the coefficients $m(\emptyset)$, $m(i)(i \in N)$, $m(\{i,j\})(i,j \in N)$ to define a fuzzy measure become

$$\begin{cases} m(\emptyset) = 0 \\ \sum_{i \in N} m(i) + \sum_{\{i,j\} \subseteq N} m(\{i,j\}) = 1 \\ m(i) \geq 0, \quad \forall i \in N \\ m(i) + \sum_{j \in T} m(\{i,j\}) \geq 0, \quad \forall i \in N, \quad \forall T \subseteq N \setminus \{i\} \end{cases} \tag{4.20}$$

4.2.6. Identification of Fuzzy Measures and Weights with MACBETH

In CI, to define the weight of each elementary performance expression in relation to all other contributions to the overall performance, i.e. the Shapley parameters and the interaction parameters of any pair of performance criteria is one of the most important matters. If the DMs are able to provide a fuzzy measure for their criteria set, it is possible to define the Shapley indices and interaction values using the formula (4.12) and (4.14). Or if, using their experience and expertise, they are able to provide the weights of their criteria and information about interactions (being strongly positive, positive, null, negative or strongly negative) then in this case also the interaction values can be retrieved using some other simplifying approaches such as Berrah et al. [57] used. There are several methods such as the one mentioned above and also definition with linear programming etc. to do that but in this study, one of them will be proposed: determination of the weights by MACBETH.

4.2.6.1. MACBETH

Measuring Attractiveness by a Categorical Based Evaluation TecHnique – MACBETH is a multi-criteria decision analysis approach which has been proposed by Bana e Costa [61, 104 – 106]. MACBETH has been used in various fields such as an individual's career choice [107], evaluation and comparison of the technical performance of on-board hydrogen storage technologies [108], politics [109], supply chain management [110], earthquake risk mitigation [111].

The method requires only qualitative judgments about differences of value to help an individual or a group in quantifying the relative attractiveness of the elements of a finite set A and to associate a real number $v(x)$ to each element x of A [112].

Let X be the finite set of elements (alternatives) with at least two elements and J the group of DMs who want to compare the relative attractiveness of these elements. Here, it is assumed that the DM or each DM is able to rank the elements of X either directly or through pairwise comparisons. Each DM is first asked to provide a judgment about the relative attractiveness of two elements at a time to retrieve the ordinal judgment. Then secondly, he/she is asked to provide a qualitative judgment on the difference of attractiveness of those two elements if they are not equally attractive. In order to ease the process, the following six semantic categories of difference of attractiveness (or a succession of these if the DM hesitates) are offered to the DMs as possible answers:

- Very weak – VW
- Weak – W
- Moderate – M
- Strong – S
- Very strong – VS
- Extreme – E

The principle of the method is to transform the qualitative data, which is always available due to the human expertise and which is collected from the DMs, into the quantitative data. But in the performance aggregation procedure, the elementary performance values must respect the commensurability requirement and be coherent with the chosen aggregation operator, generally the weighted mean. The MACBETH method, presents a procedure to transform qualitative preferences into coherent quantified elementary and aggregated performances. To solve the inter-criteria commensurability problem, it is sufficient to determine, for all interval scales, two common reference points namely the *good* situation and the *neutral* situation with the performance values 1 and 0 respectively.

4.2.6.2. Preferential Information

4.2.6.2.1. TYPE I Information

This is the information given by the DM about the relative attractiveness of two elements. The responses that DM gives help us to construct the following three binary relations on X:

- P = $\{(x, y) \in X \times X :$ x is more attractive than y$\}$
- I = $\{(x, y) \in X \times X :$ x is not more attractive than y and y is not more attractive than x$\}$
- ? = $\{(x, y) \in X \times X :$ x and y are not comparable in terms of their attractiveness$\}$

4.2.6.2.2 TYPE I+II Information

Once the TYPE I information is collected, $\{P, I, ?\}$ about X, the DM is asked to judge the difference of attractiveness between x and y, $\Delta att(x, y)$, in the form d_s, for all $(x, y) \in P$. Here, the d_s with $s = 1, \ldots, 6$, is representing the six semantic categories of difference of attractiveness mentioned in (4.2.6.2.1) with d_1 being "very weak" and d_6 "extreme".

By doing the previous procedure, the relations C_{st} $(s, t \in \aleph, 1 \le s \le t \le 6)$ are obtained as follows:

$$C_{st} = \{ (x, y) \in P \mid \Delta att(x, y) \text{ is "} d_s \text{ to } d_t \text{"} \}$$

According to this definition, C_{ss} will be simply called C_s. Hence,

$$C_s = \{ (x, y) \in P \mid \Delta att(x, y) \text{ is } d_s \}$$

Therefore they allow constructing the structure {*P, I, ?, P^e*} about *X* where P^e is an asymmetric relation on *P* as follows:

$$(x, y) P^e (z, w) \quad when \quad \Delta att(x, y) \succ \Delta att(z, w)$$

4.2.6.3. Numerical Representation of Preferential Information

4.2.6.3.1. Type I Scale

Having the Type I information about *X*, a Type I scale on *X* relative to {*P, I*} is a function $v : X \to \Re$ satisfying the following condition:

Condition 1: $\forall x, y \in X, [xPy \Rightarrow v(x) > v(y)] \wedge [xIy \Rightarrow v(x) = v(y)]$

Here, $Sc_I = \{ v : X \to \Re \mid v$ is a Type I scale on *X* relative to {*P, I*}}.

4.2.6.3.2. Type I+II Scale

Having the Type I+II information about *X*, a Type I+II scale on *X* relative to {*P, I, ?, P^e*} is a function $v : X \to \Re$ satisfying the following condition:

Condition 2: $\forall x, y, z, w \in X, \left[(x, y)P^e(z, w) \Rightarrow v(x) - v(y) > v(z) - v(w)\right]$

Here, Sc_{I+II}={ $v : X \to \Re \mid v$ is a Type I+II scale on *X* relative to {*P, I, P^e*}}.

4.2.6.4. Consistency – Inconsistency

Type I information about *X* is:

- Consistent when $Sc_I \neq \phi$
- Inconsistent when $Sc_I = \phi$

Type I+II information on *X* is:

- Consistent when $Sc_{I+II} \neq \phi$
- Inconsistent when $Sc_{I+II} = \phi$

Let $X = \{x, y, z\}$, when $Sc_I = \phi$, that means there is no function Type I scale on *X*. Hence there is no possible ranking for $x, y, z \in X$. For example one of the followings occur:

$$xIy; \quad yIz \quad \wedge \quad xPz$$
$$xPy; \quad yPz \quad \wedge \quad zPx$$

When $Sc_{I+II} = \phi$, that means the judgment is inconsistent in either one of the following two ways:

- Sub-Type A: Conflict between Type I information and P^e that makes the simultaneous satisfaction of Condition 1 and 2 impossible. Such as:

- $[xPy; \quad yPz; \quad xPz] \ \wedge \ (y,z)P^e(x,z)$
- $[xPy; \quad yPz; \quad xPz] \ \wedge \ (x,y)P^e(x,z)$
- $[xIy; \quad yPz; \quad xPz] \ \wedge \ (x,z)P^e(y,z)$
- $[xIy; \quad zPy; \quad zPx] \ \wedge \ (z,x)P^e(z,y)$

- Sub-Type B: No conflict between Type I information and P^e but at least one conflict exists inside P^e that makes satisfaction of Condition 2 impossible. Such as:

  - $[xPy; \quad xPw; \quad xPz; \quad yPz; \quad yPw; \quad wPz]$ with

$$\left.\begin{array}{l}(x,y)\in C_1\\(w,z)\in C_2\end{array}\right\} v(w)-v(z)>v(x)-v(y) \quad (1)$$

$$\left.\begin{array}{l}(x,w)\in C_3\\(y,z)\in C_2\end{array}\right\} v(x)-v(w)>v(y)-v(z) \quad (2)$$

There is a contradiction between the equations (1) and (2), hence the inconsistency.

4.2.6.5. Consistency Test

4.2.6.5.1. Pre-Test of the Preferential Information

Property 4.1: Let $X^* \subset X$; if $\forall x \in X^*, \exists y \in X^* \mid xPy$, then $\exists x_1, x_2, \ldots, x_p \in X^* \mid x_1Px_2P\ldots Px_pPx_1$ (cycle).

Pre-test seeks a permutation $\varphi : \aleph_{1;\mathrm{n}} \rightarrow \aleph_{1;n} \mid \forall i,j \in \aleph_{1;n}, (i>j \Rightarrow a_{\varphi(i)}(notP)a_{\varphi(j)})$ where $\aleph_{1;n} = \{x \in \aleph \mid 1 \le x \le n\}$.

PRETEST:

i. $s \leftarrow n$

ii. Find a_i among $a_1,\ldots,a_s \mid \forall j=1,\ldots s \quad j \neq i \quad a_i(notP)a_j$

If $\exists a_i$ then go to *(iii)*. If not, $Sc_I = \phi$. FALSE. FINISH.

iii. Permute a_i and a_s

iv. $s \leftarrow s-1$

If $s=1$ then TRUE. FINISH.

If not, go to *(ii)*.

4.2.6.5.2. Consistency Test for TYPE I Information

Let's suppose that PRETEST detected that there is no cycle within P and the elements of X were numbered as $\forall i, j \in \aleph_{1;n}, (i > j \Rightarrow a_i(notP)a_j)$.

In case of incomplete Type I information, i.e. $? \neq \phi$, the following LP-test$_I$ is considered with variables $x_1, x_2, \ldots, x_n$ where d_{min} is a positive constant, the variables x_j represent the numbers $v(a_j)$ that satisfy the Condition 1 and the objective function is arbitrary:

$$
\begin{aligned}
&\min x_1 \\
&st \\
&x_i - x_j \geq d_{\min} \quad \forall (a_i, a_j) \in P \\
&x_i - x_j = 0 \quad \forall (a_i, a_j) \in I \quad i \neq j \\
&x_i \geq 0 \quad \forall i \in \aleph_{1;n}
\end{aligned}
\tag{4.21}
$$

It should be noted that it's trivial that $Sc_I \neq \phi \Leftrightarrow$ LP-test$_I$ is feasible.

On the other hand, in case of complete Type I information, i.e. $? = \phi$, DIR-test$_1$ is considered based on the following proposition allowing one to verify whether $P \cup I$ is a complete preorder on *X*.

Proposition 4.4.: If $\forall i, j \in \aleph_{1;n}$ with *i*<*j*, $(a_i, a_j) \in P \cup I$ then, $P \cup I$ is a complete preorder on *X* if and only if $\forall i, j \in \aleph_{1;n}$ with *i*<*j* we have:

$$a_i P a_j \Rightarrow \begin{cases} \forall s \leq i, \forall t \geq j, a_s P a_t \\ \exists s \mid i \leq s \leq j-1 \quad \wedge \quad a_s P a_{s+1} \end{cases}$$

4.2.6.5.3. Consistency Test for TYPE I+II Information

For this purpose, a LP-test$_{I+II}$ based on the following lemma is used:

Lemma 4.1.: Let $v: X \rightarrow \Re$. v satisfies Condition 1 and 2 if and only if there exists *Q* "thresholds" $0 < \sigma_1 < \sigma_2 < \ldots < \sigma_Q$ that satisfies the following three conditions.

Condition 3: $\forall x, y \in I \quad v(x) = v(y)$

Condition 4: $\forall i, j \in \aleph_{1;Q}$ with $i \leq j \quad \forall (x; y) \in C_{ij} \quad \sigma_i < v(x) - v(y)$

Condition 5: $\forall i, j \in \aleph_{1;Q}$ with $i \leq j \quad \forall (x; y) \in C_{ij} \quad v(x) - v(y) < \sigma_{j+1}$

LP-test$_{I+II}$ is as follows:

$$
\begin{aligned}
&\min x_1 \\
&st \\
&\quad x_p - x_r = 0 \quad \forall (a_p;a_r) \in I \quad p<r \\
&\quad \sigma_i + d_{\min} \le x_p - x_r \quad \forall i;j \in \aleph_{1;Q} \quad i \le j \quad \forall (a_p;a_r) \in C_{ij} \\
&\quad x_p - x_r \le \sigma_{j+1} - d_{\min} \quad \forall i;j \in \aleph_{1;Q-1} \quad i \le j \quad \forall (a_p;a_r) \in C_{ij} \\
&\quad d_{\min} \le \sigma_1 \\
&\quad \sigma_{i-1} + d_{\min} \le \sigma_i \quad \forall i \in \aleph_{2;Q} \\
&x_i \ge 0 \quad \forall i \in \aleph_{1;n} \\
&\sigma_i \ge 0 \quad \forall i \in \aleph_{1;Q}
\end{aligned}
\tag{4.22}
$$

Here note that it's trivial that $Sc_{I+II} \neq \phi \Leftrightarrow$ LP-test$_{I+II}$ is feasible.

Example 4.5.: Let $X = \{a_1, a_2, a_3, a_4\}$ and the DM gave the following judgments:

- $P = \{(a_1,a_2),(a_1,a_3),(a_2,a_3),(a_3,a_4)\}$
- $(a_1,a_2) \in C_1;\quad (a_1,a_3) \in C_4;\quad (a_2,a_3) \in C_2;\quad (a_3,a_4) \in C_2$
- In addition to the judgments above, the DM confirms his judgments and also judges that $a_2 P a_4 \quad \wedge \quad (a_2,a_4) \in C_3$.

In this case, the LP-test$_{I}$ is feasible as the judgments are compatible with a ranking but according to the LP-test$_{I+II}$ the following constraints have to be satisfied:

$$
\left.
\begin{array}{l}
\left.\begin{array}{l} \sigma_1 < x_1 - x_2 < \sigma_2 \\ \sigma_2 < x_3 - x_4 < \sigma_3 \end{array}\right\} \Rightarrow x_1 - x_2 < x_3 - x_4 \\
\sigma_2 < x_2 - x_3 < \sigma_3 \\
\left.\begin{array}{l} \sigma_3 < x_2 - x_4 < \sigma_4 \\ \sigma_4 < x_1 - x_3 < \sigma_5 \end{array}\right\} \Rightarrow x_2 - x_4 < x_1 - x_3
\end{array}
\right\} \Rightarrow Contradiction
$$

$$
0 < \sigma_1 < \sigma_2 < \sigma_3 < \sigma_4 < \sigma_5 < \sigma_6
$$

Hence, LP-test$_{I+II}$ is not feasible and therefore the judgments are inconsistent. To solve this inconsistency, the suggestions to the DM could be as follows noting that any modification of "$(a_2, a_3) \in C_2$" would not eliminate the inconsistency:

- Replace $(a_1, a_3) \in C_4$ with $(a_1, a_3) \in C_3$ or
- Replace $(a_2, a_4) \in C_3$ with $(a_2, a_4) \in C_4$ or
- Replace $(a_3, a_4) \in C_2$ with $(a_3, a_4) \in C_1$ or
- Replace $(a_1, a_2) \in C_1$ with $(a_1, a_2) \in C_2$

4.2.6.6. The MACBETH Procedure

The MACBETH Procedure consists in four main steps [114]:

- Context definition.
- Identification of the objective, criteria and alternatives.
- Quantify in parallel:
  - the vector of elementary expressions. (Step 2)
  - the weights of the weighted arithmetic mean (WAM). (Step 3)
- Calculate the aggregated performance associated to different situations (alternatives).

The verification of judgments' consistency is made in the second and third steps. The elementary performance expression step is made in two stages. In the first one, the DM is asked to determine the preferences of the alternatives for each criterion *i* of the context and in the following stage, the DM is asked to express the strength of the judgments he provided in the previous stage.

4.2.6.6.1. Elementary Performance Expression Determination

Let p_i^k be the performance expression of the k^{th} alternative for criterion *i*. Suppose the DM prefers for criterion *i* the alternative *k* to the alternative *l*, therefore, it will mean:

$A^k \succ A^l \Leftrightarrow p_i^k > p_i^l$. And if the DM finds the two alternatives are equivalent for the criterion *i*, then $A^k \approx A^l \Leftrightarrow p_i^k = p_i^l$.

In addition to that information, DM will characterize the strength of his judgments with a level of strength that can take values from one to six (from the least to the most strong level) according to the six semantic categories of difference of attractiveness explained in section (4.2.6.1) and zero for a null strength. This level will be denoted with *h*. Therefore, if the DM prefers for criterion *i* the alternative *k* to the alternative *l*, with a strength *h*, then this will give the following equation where α is a coefficient necessary to meet the condition p^k and $p^l \in [0;1]$:

$$A^k \succ^h A^l \Leftrightarrow p_i^k - p_i^l = h\alpha \tag{4.23}$$

Example 4.6.: Suppose that the DM gives the following preferences and the strength of preferences for three alternatives according to some criteria:

$$Good \succ^3 A^3 \succ^2 A^2 \succ^1 A^1 \succ^2 Neutral$$

Therefore, the following system of independent equations can be retrieved:

$$\begin{cases} p^{Good} - p^3 = 1 - p^3 = 3\alpha \\ p^3 - p^2 = 2\alpha \\ p^2 - p^1 = \alpha \\ p^1 - p^{Neutral} = p^1 - 0 = 2\alpha \end{cases}$$

Hence the following results of the elementary performance expressions are defined along interval scales defined on the interval [0; 1] in a commensurate way [114]:

$$p^1 = \frac{1}{4}, \quad p^2 = \frac{3}{8}, \quad p^3 = \frac{5}{8}, \quad \left(\alpha = \frac{1}{8}\right)$$

4.2.6.6.2. The WAM Weights Determination

As for the elementary performance expressions, in order to do that, MACBETH proposes to consider some particular and possibly fictive situations, *S*, in which are associated the elementary expression vectors so that the aggregated performance expression is reduced simply to $p^i_{Ag} = w_i$ where p^i_{Ag} is the aggregated performance from the vector where only $p_i = 1$ and all other $p_j = 0$ with $j \neq i$. The DM will give the preference relations and their strenghts that each one of them will be as follows and all together they will provide us a system of *n* independent equations:

$$p^i_{Ag} - p^g_{Ag} = h\alpha = w_i - w_g \tag{4.24}$$

Example 4.7.: Suppose that the DM provided the following information

$$S^{(0,1,0)} \succ^{\text{moderate}} S^{(1,0,0)} \succ^{strong} S^{(0,0,1)} \succ^{weak} S^{(0,0,0)}$$

Hence the following system of equations and the WAM weights can be found:

$$\begin{cases} p_{Ag}^{(0,0,1)} - p_{Ag}^{(1,0,0)} = 3\alpha = w_2 - w_1 \\ p_{Ag}^{(1,0,0)} - p_{Ag}^{(0,1,0)} = 4\alpha = w_1 - w_3 \\ p_{Ag}^{(0,1,0)} - p_{Ag}^{(0,0,0)} = 2\alpha = w_3 \\ w_1 + w_2 + w_3 = 1 \end{cases}$$

$$w_1 = \frac{6}{17}, \quad w_2 = \frac{9}{17}, \quad w_3 = \frac{2}{17}, \quad \left(\alpha = \frac{1}{17}\right)$$

4.2.6.6.3. The Aggregated Performance

The aggregated performance of the alternative situation *k* is calculated as follows:

$$p_{Ag}^{k} = \sum_{i=1}^{n} w_i p_i^k \tag{4.25}$$

4.2.6.7. MACBETH & 2-additive Choquet Integral

In this section, we will explain how to find the weight of each elementary performance expression in relation to all other contributions to the overall performance, i.e. the Shapley parameters v_i and the interaction parameters I_{ij} of any pair of performance criteria. In the case of performance expression, the 2-additive CI expression given in (4.18) can be represented as follows [57]:

$$p_{Total} = \sum_{i=1}^{n} v_i p_i - \frac{1}{2} \sum_{\substack{\{i,j\} \\ i \neq j}} I_{ij} \left| p_i - p_j \right| \tag{4.26}$$

where $v_i - \frac{1}{2}\sum_{j=1}^{n}\left|I_{ij}\right| \geq 0 \quad \forall i \in [1,n]$ and $j \neq i$

The elementary performance expressions are defined as it is explained in the section (4.2.6.6.1). So the CI parameters have to be defined. In order to do that, the DM is asked to provide preferential information on the criteria and the couples of criteria including the strength of the preferences. This information will help to build a system of equations with the Shapley and the Interaction parameters as variables.

As in section (4.2.6.6.2.), in the situations, *S*, where only one $p_i = 1$ and all others are equal to zero, the aggregated performance expression will be as follows:

$$p^i_{Ag} = v_i - \frac{1}{2}\sum_{\substack{j=1 \\ j \neq i}}^{n} I_{ij} \tag{4.27}$$

Note that if there is no interaction between criteria, $I_{ij} = 0 \quad \forall i, j \in \aleph_{1;n}$ and therefore, $p^i_{Ag} = v_i$, in other words WAM weights.

The aggregated performance expression of the situations, *S*, where only one $p_i = 0$ and all others are equal to one (which means that all the criteria except one is satisfied simultaneously) will be as follows:

$$p^i_{Ag} = 1 - v_i - \frac{1}{2}\sum_{\substack{j=1 \\ j \neq i}}^{n} I_{ij} \tag{4.28}$$

The aggregated performance expression of the situations, *S*, where only two elementary performance expressions are equal to one $p_i = 1$ and $p_j = 1$ and all others are equal to zero (which means only the criteria *i* and *j* are satisfied simultaneously) will be as follows:

$$p_{Ag}^{i,j} = v_i + v_j - \frac{1}{2}\left(\sum_{k \in \aleph_{1;n} | p_k = 0} I_{ik} + \sum_{k \in \aleph_{1;n} | p_k = 0} I_{jk} \right) \tag{4.29}$$

Example 4.8.: Suppose that a decision making problem with four criteria has to be solved and the DM provided the following information about situations:

$$S^{(1,1,1,0)} \succ_{\text{moderate}} S^{(1,1,0,1)} \succ_{very\_strong} S^{(1,0,1,1)} \succ_{weak} S^{(0,1,1,1)} \succ_{strong} S^{(1,1,0,0)} \succ_{\text{moderate}} S^{(1,0,0,1)}$$

$$S^{(0,1,1,0)} \succ_{extreme} S^{(0,1,0,1)}$$

$$S^{(1,0,0,0)} \succ_{strong} S^{(0,0,0,1)} \succ_{very\_weak} S^{(0,1,0,0)} \succ_{\text{moderate}} S^{(0,0,1,0)} \succ_{\text{moderate}} S^{(0,0,0,0)}$$

Hence the following system of equations with the Shapley and Interaction parameters is obtained:

$$\begin{cases}
p_{Ag}^{(1,1,1,0)} - p_{Ag}^{(1,1,0,1)} = 3\alpha = v_3 - v_4 - \frac{1}{2}\left[I_{14} + I_{24} - I_{13} - I_{23}\right] \\
p_{Ag}^{(1,1,0,1)} - p_{Ag}^{(1,0,1,1)} = 5\alpha = v_2 - v_3 - \frac{1}{2}\left[I_{13} + I_{34} - I_{12} - I_{24}\right] \\
p_{Ag}^{(1,0,1,1)} - p_{Ag}^{(0,1,1,1)} = 2\alpha = v_1 - v_2 - \frac{1}{2}\left[I_{23} + I_{24} - I_{13} - I_{14}\right] \\
p_{Ag}^{(0,1,1,1)} - p_{Ag}^{(1,1,0,0)} = 4\alpha = v_3 + v_4 - v_1 - \frac{1}{2}\left[I_{12} - I_{23} - I_{24}\right] \\
p_{Ag}^{(1,1,0,0)} - p_{Ag}^{(1,0,0,1)} = 3\alpha = v_2 - v_4 - \frac{1}{2}\left[I_{14} + I_{23} - I_{12} - I_{34}\right] \\
p_{Ag}^{(0,1,1,0)} - p_{Ag}^{(0,1,0,1)} = 6\alpha = v_3 - v_4 - \frac{1}{2}\left[I_{13} + I_{24} - I_{14} - I_{23}\right] \\
p_{Ag}^{(1,0,0,0)} - p_{Ag}^{(0,0,0,1)} = 4\alpha = v_1 - v_4 - \frac{1}{2}\left[I_{12} + I_{13} - I_{24} - I_{34}\right] \\
p_{Ag}^{(0,0,0,1)} - p_{Ag}^{(0,1,0,0)} = \alpha = v_4 - v_2 - \frac{1}{2}\left[I_{14} + I_{34} - I_{12} - I_{23}\right] \\
p_{Ag}^{(0,1,0,0)} - p_{Ag}^{(0,0,1,0)} = 3\alpha = v_2 - v_3 - \frac{1}{2}\left[I_{12} + I_{24} - I_{13} - I_{34}\right] \\
p_{Ag}^{(0,0,1,0)} - p_{Ag}^{(0,0,0,0)} = 3\alpha = v_3 - \frac{1}{2}\left[I_{13} + I_{23} + I_{34}\right] \\
p_{Ag}^{(1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4
\end{cases}$$

Associated matrix for this system of equations and its resolution is given below:

$$\begin{bmatrix}
0 & 0 & 1 & -1 & 0 & 0{,}5 & -0{,}5 & 0{,}5 & -0{,}5 & 0 & -3 \\
0 & 1 & -1 & 0 & 0{,}5 & -0{,}5 & 0 & 0 & 0{,}5 & -0{,}5 & -5 \\
1 & -1 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & -0{,}5 & -0{,}5 & 0 & -2 \\
-1 & 0 & 1 & 1 & -0{,}5 & 0 & 0 & 0{,}5 & 0{,}5 & 0 & -4 \\
0 & 1 & 0 & -1 & 0{,}5 & 0 & -0{,}5 & -0{,}5 & 0 & 0{,}5 & -3 \\
0 & 0 & 1 & -1 & 0 & -0{,}5 & 0{,}5 & 0{,}5 & -0{,}5 & 0 & -6 \\
1 & 0 & 0 & -1 & -0{,}5 & -0{,}5 & 0 & 0 & 0{,}5 & 0{,}5 & -4 \\
0 & -1 & 0 & 1 & 0{,}5 & 0 & -0{,}5 & 0{,}5 & 0 & -0{,}5 & -1 \\
0 & 1 & -1 & 0 & -0{,}5 & 0{,}5 & 0 & 0 & -0{,}5 & 0{,}5 & -3 \\
0 & 0 & 1 & 0 & 0 & -0{,}5 & 0 & -0{,}5 & 0 & -0{,}5 & -3 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ I_{12} \\ I_{13} \\ I_{14} \\ I_{23} \\ I_{24} \\ I_{34} \\ \alpha \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$v_1 = .332 \quad v_2 = .268 \quad v_3 = .195 \quad v_4 = .205 \qquad \alpha = .018$

$I_{12} = .155 \quad I_{13} = .027 \quad I_{14} = .082 \quad I_{23} = .173 \quad I_{24} = -.009 \quad I_{34} = .082$

5. SOCIAL CHOICE THEORY

5.1. Voting

Voting is a group decision making method in a democratic society which expresses the will of the majority. It is also an MCDM process whenever a voter is about to select a candidate as the candidates are judged according to their capabilities, honesty, trustworthiness, political stance etc. The voter summarizes those criteria in his/her mind in order to form a utility function then decides according to that function. Hence it can be briefly said that a democratic voting process is a group decision making method under multi criteria.

When the voter has only one vote whereas there are many candidates, this is called a non-ranked voting system. This method is perfectly satisfactory when there are only two candidates and the winner is simply the one who has the majority of the votes, simple majority.

However this method is quite unreliable when there are three or more candidates. If one candidate were to be selected from many candidates, two systems exist in order to decide the winner: the First-Past-The-Post System – FPPS and the Majority Representation System – MRS. With the FPPS, the candidate who has the greatest total votes is selected as winner. However, with the MRS, a candidate must have the absolute majority in order to be declared as the winner. Otherwise, the Repeated Ballot System – RBS or the Second Ballot System – SBS is used to find the winner. With the RBS, the voting starts with many candidates from whom, after each ballot the hopeless ones withdraw in favor of those who have the chance to success. This procedure is repeated until one candidate has the absolute majority of the votes. With the SBS, after the first ballot, unless a candidate has the absolute majority, a second ballot is realized for two candidates who had been the highest in the first ballot.

The most naive approach to the elections is to say that the candidate who gets the most votes wins. But with a more detailed look to the methods cited above, it has to be asked whether or not those systems represent the people's will. The cases, given by Dodgson [114], in the following example, demonstrate the injustices which may occur with those systems.

Example 5.1. Consider 11 voters who will vote for 4 candidates a, b, c, and d, by representing their preference order - PO.

In Case 1 presented in Table 5.1., although the candidate *a* is considered the best by three voters and 2nd by all the rest, the candidate *b*, who is selected the best by four voters however the worst by all other seven, is selected according to the FPPS. In Case 2 presented in Table 5.2., no candidates other than *a* and *b* are defined as the best one and *b* would win this voting by the absolute majority although the candidate *a* is considered the best by five voters and second by the rest of them and *b* is considered best by six and the worst by all others.

Table 5.1. Case 1

	Voters										
PO	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***
1	a	a	a	b	b	b	b	c	c	c	d
2	c	c	c	a	a	a	a	a	a	a	a
3	d	d	d	c	c	c	c	d	d	d	c
4	b	b	b	d	d	d	d	b	b	b	b

Table 5.2. Case 2

	Voters										
PO	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	b	b	b	b	b	b	a	a	a	a	a
2	a	a	a	a	a	a	c	c	c	d	d
3	c	c	c	d	d	d	d	d	d	c	c
4	d	d	d	c	c	c	b	b	b	b	b

In Case 3 presented in Table 5.3., no candidate has the majority of the votes and the candidate *a* could be the most generally acceptable one to win this voting as he/she was not put lower than second place as opposed to the candidate *c* and *d* who were put last by four voters each and *b* by three voters. However, by the SBS, the candidates *a* and *d* are eliminated after the first ballot as the candidates *b* and *c* are the two candidates to have the highest number of votes.

Table 5.3. Case 3

	Voters										
PO	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	b	b	b	c	C	c	C	d	d	a	a
2	a	a	a	a	A	a	A	a	a	b	d
3	d	c	d	b	B	b	D	c	b	d	c
4	c	d	c	d	D	d	B	b	c	c	b

5.2. Preferential Voting System – PVS

As previously seen with Dodgson's examples, since the non-ranked voting system may result in the selection of the least popular candidate, a method of voting that allows the voter to indicate his/her order of preference for the candidates is needed. By doing that, the voter will not only define the best candidate but will also define the ranking of the candidates according to him/her. This is called the preferential voting which was first proposed by Chevalier de Borda in a paper he wrote in 1770 but not published until 1784 for unknown reasons [115].

The voting procedure is simple in practice nonetheless after polling is completed, the problem is to aggregate the individual preferences in order to form a social choice. This problem has been the subject of many discussions. In 1875, Condorcet discovered the "paradox of voting" stating that social choice processes based on a principle of majority rule can cause the cyclical ranking of the candidates even if all the voters has a transitive ranking of the candidates as presented in the following example.

Example 5.2. Consider three voters *A*, *B* and *C* who gave their preferences on three candidates *x*, *y* and *z* as follows (the notation $x\ P\ y$ will be used when a voter prefers *x* over *y*):

A: $x\ P\ y\ P\ z$
B: $y\ P\ z\ P\ x$
C: $z\ P\ x\ P\ y$

If this committee were to rank the candidates by a majority rule, then it would rank *x* over *y*, *y* over *z* and *z* over *x*, hence the circular preference among the candidates even though the voters don't have such circular preference.

5.3. Social Choice Function – SCF

A SCF is a mapping which assigns a nonempty subset of the potential feasible subset to each ordered pair consisting of a potential feasible subset of alternatives and a schedule of profile of voter's preferences [116].

In order to generate a single preference order for the society a set of operations must be made on the preferences of the individuals forming that society. In general terms, the problem is to define "fair" methods for amalgamating individual choices to yield a social decision. Although there are many different SCFs, in this section only three of them, namely Condorcet's, Borda's and Copeland's will be presented in order to be used in the proposed model of performance evaluation.

5.3.1. Condorcet's Function

Condorcet Principle is to select the candidate who beats every other candidate under simple majority when such candidate exists. Whenever there is no simple majority along with the cyclical majorities, Condorcet's function measures the worst a candidate does against others. Hence the function is as follows and the greater the function value the better the candidate:

$$f_C(x) = \min_{y \in A \setminus \{x\}} \#(i : xP_i y) \tag{5.1}$$

Example 5.3. Consider the following preference orders made by 60 voters for three candidates *a*, *b* and *c*.

23 votes	:	$a\ P\ b\ P\ c$
17 votes	:	$b\ P\ c\ P\ a$
2 votes	:	$b\ P\ a\ P\ c$
10 votes	:	$c\ P\ a\ P\ b$
8 votes	:	$c\ P\ b\ P\ a$

Therefore:

$$\#(i : aP_i b) = 33 \quad \#(i : bP_i a) = 27$$
$$\#(i : aP_i c) = 25 \quad \#(i : cP_i a) = 35$$
$$\#(i : bP_i c) = 42 \quad \#(i : cP_i b) = 18$$

And finally, as $f_C(b) > f_C(a) > f_C(c)$, the social preference ordering of the candidates according to Condorcet's function is "*b P a P c*".

5.3.2. Borda's Function

The Borda score of a candidate *x* is equivalent to the sum of the number of voters who have preferred *x* to others [117]. Therefore, Borda's function is as follows and the greater the function value the better the candidate:

$$f_B(x) = \sum_{y \in A \setminus \{x\}} \#(i : xP_i y) \tag{5.2}$$

Example 5.4. Considering the preference orders given in the previous example, Borda scores for the candidates *a*, *b* and *c* are 58, 69 and 53 respectively. Hence the social preference ordering of the candidates according to Borda's function is "*b P a P c*".

5.3.3. Copeland's Function

The Copeland score of a candidate *x* is equivalent to the difference between the number of candidates that *x* has strict simple majority over and the number of candidates who have strict simple majorities over *x*. Therefore the Copeland's function is as follows and the greater the function value the better the candidate:

$$f_{CP}(x) = \#(y : y \in A \quad \wedge \quad xPy) - \#(y : y \in A \quad \wedge \quad yPx) \tag{5.3}$$

Example 5.5. Consider 5 candidates namely *a*, *b*, *c*, *d* and *e* and 12 voters. The preference orders or the voters are as follows:

4 votes	:	$a\ P\ b\ P\ c\ P\ d\ P\ e$
3 votes	:	$d\ P\ b\ P\ c\ P\ e\ P\ a$
2 votes	:	$c\ P\ a\ P\ d\ P\ b\ P\ e$
2 votes	:	$e\ P\ a\ P\ d\ P\ c\ P\ b$
1 votes	:	$b\ P\ c\ P\ e\ P\ a\ P\ d$

Hence:

$$
\begin{array}{ll}
\#(i : aP_i b) = 8 & \#(i : bP_i a) = 4 \\
\#(i : aP_i c) = 6 & \#(i : cP_i a) = 6 \\
\#(i : aP_i d) = 9 & \#(i : dP_i a) = 9 \\
\#(i : aP_i e) = 6 & \#(i : eP_i a) = 6 \\
\#(i : bP_i c) = 8 & \#(i : cP_i b) = 4 \\
\#(i : bP_i d) = 5 & \#(i : dP_i b) = 7 \\
\#(i : bP_i e) = 10 & \#(i : eP_i b) = 2 \\
\#(i : cP_i d) = 7 & \#(i : dP_i c) = 5 \\
\#(i : cP_i e) = 10 & \#(i : eP_i c) = 2 \\
\#(i : dP_i e) = 9 & \#(i : eP_i d) = 3
\end{array}
$$

So the Copeland's function values for the candidates and their ranking will be:

$$f_{CP}(a) = 1 \quad f_{CP}(b) = 0 \quad f_{CP}(c) = 1 \quad f_{CP}(d) = 1 \quad f_{CP}(e) = -3$$

$$a \quad I \quad c \quad I \quad d \quad P \quad b \quad P \quad e$$

For conclusion, at this voting, the worst candidate turned out to be the candidate *e* and the candidates *a*, *c* and *d* are tied for the best candidate.

5.4. Agreement – Disagreement

There are functions used to find the agreement or disagreement measures for a possible ranking of the candidates. The ranking may be found by a SCF given above or another type of method. As in SCFs, there are many methods for that purpose however in this section only two of them, namely Cook and Seiford's and Kemeny's functions will be detailed in order to be used in the study.

5.4.1. Cook and Seiford's Function

Cook and Seiford [118] introduce a distance function as a measure of agreement or disagreement between rankings. Then the consensus ranking is defined in order to minimize the total absolute distance (disagreement).

Definition 5.1. Let r_{ij} represent the rank given to the candidate *j* (where $j = 1,\ldots,m$) by the voter *i* (where $i = 1,\ldots,n$). In this case, the candidate's disagreement (distance) from the consensus ranking *k* (where $k = 1,\ldots,m$) will be:

$$d_{jk} = \sum_{i=1}^{n} \left| r_{ij} - k \right| \tag{5.4}$$

Therefore, for each possible ranking for each candidate, a d_{jk} value will be computed and *m*x*m* distance values will be obtained. The purpose is to find the ranking of candidates which will minimize the sum of the distances. This can be achieved by solving the following assignment problem:

$$\begin{aligned} \min \quad & \sum_{j=1}^{m}\sum_{k=1}^{m} d_{jk} x_{jk} \\ st \quad & \\ & \sum_{j=1}^{m} x_{jk} \qquad k = 1,\ldots,m \\ & \sum_{k=1}^{m} x_{jk} \qquad j == 1,\ldots,m \\ & x_{jk} = \{0;1\} \end{aligned}$$

Example 5.6. Considering the preference orders given in Example 5.3, with the given notations, we will have $i = 1,\ldots,60,\quad j = a,b,c$ and $k = 1,2,3$.

For the candidate *a*, the distances if *a* is assigned 1st, 2nd and 3rd are d_{a1}, d_{a2} and d_{a3} respectively and they are calculated as follows:

$$d_{a1} = \sum_{i=1}^{60} |r_{ia} - 1| = 23.|1-1| + 17.|3-1| + 2.|2-1| + 10.|2-1| + 8.|3-1| = 62$$

$$d_{a2} = \sum_{i=1}^{60} |r_{ia} - 2| = 23.|1-2| + 17.|3-2| + 2.|2-2| + 10.|2-2| + 8.|3-2| = 48$$

$$d_{a3} = \sum_{i=1}^{60} |r_{ia} - 3| = 23.|1-3| + 17.|3-3| + 2.|2-3| + 10.|2-3| + 8.|3-3| = 58$$

Similarly d_{b1}, d_{b2}, d_{b3}, d_{c1}, d_{c2} and d_{c3} are calculated and all those values can be collected as shown in the matrix below:

j \ k	1	2	3
a	62	48	58
b	51	29	69
c	67	43	53

By applying the Hungarian Algorithm to this problem, it can be found out that the ranking $a\ P\ b\ P\ c$ corresponding to the minimum distance of $d = 62 + 29 + 53 = 144$ will be chosen as the best ranking of the candidates.

5.4.2. Kemeny's Function

The purpose of the Kemeny's function is to find the maximum total amount of agreement between the consensus ranking s and n voters' preference orders for the candidates.

Definition 5.2. Let $L = (l_{ij})_{m\times m}$ be the ranking matrix with

$$l_{ij} = \begin{cases} 1 & \text{if } \mathrm{a}_i \text{ is preferred to } \mathrm{a}_j \\ 0 & \text{if they are tied} \\ -1 & \text{if } \mathrm{a}_j \text{ is preferred to } \mathrm{a}_i \end{cases} \qquad i, j = 1, \ldots, m \tag{5.5}$$

Definition 5.3. Let m_{ij} be the number of individuals who prefer a_i to a_j and m^*_{ij} the number of individuals who are indifferent between a_i and a_j. Then the proportion matrix M is defined as follows:

$$M_{ij} = \frac{m_{ij} + m_{ij}^{*}}{n}, \quad i \neq j$$
$$M_{ii} = 1/2 \tag{5.6}$$

Definition 5.4. Let $E = (e_{ij})_{m \times m}$ be the translated election matrix, $E = M - M^{t}$. Note that the elements e_{ij} of the matrix represent the difference between the proportion of voters preferring a_i to a_j and that of voters preferring a_j to a_i.

Then, by taking a possible linear order of the candidates, *L*, and $< E , L >$ the simple inner product of *E* and *L*, the Kemeny's function is as follows:

$$f_K = \max_{L} < E, L >= \max_{L} \sum_{\{i,j\}} l_{ij}.e_{ij} \tag{5.7}$$

Example 5.7. Considering the preference orders given in Example 5.3, and the solution found by applying Condorcet's function, *i.e.* $b\ P\ a\ P\ c$, then *M*, *E* and *L* matrices will be as follows:

$$M = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1/2 & 33/60 & 25/60 \\ 27/60 & 1/2 & 42/60 \\ 35/60 & 18/60 & 1/2 \end{bmatrix} \end{array}$$

$$E = M - M^{t} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 6/60 & -10/60 \\ -6/60 & 0 & 24/60 \\ 10/60 & -24/60 & 0 \end{bmatrix} \end{array}$$

$$L = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} \end{array}$$

Therefore, < *E* , *L* > = 16/60. Similarly, it's possible to calculate this value for different possible rankings that can be generated. That is:

$$\begin{aligned} aPbPc &\Rightarrow\ < E, L >= 40/60 \\ cPaPb &\Rightarrow\ < E, L >= -16/60 \\ aIbPc &\Rightarrow\ < E, L >= 28/60 \\ aPcPb &\Rightarrow\ < E, L >= -56/60 \\ bPcPa &\Rightarrow\ < E, L >= 56/60 \end{aligned}$$

For this set of voters, the maximum value of < *E* , *L* > is obtained by the ranking *b P c P a*. Meaning this ranking gives the maximum agreement with the individuals' preference orders.

6. METHODOLOGY

The proposed method is, as it is mentioned before, to make an approach to the human PE from a MCDM perspective while considering the interactions between criteria. The vital implication of MCDM methods, i.e. the independence of the criteria, can be a seriously limiting property while doing that. Either the assumption of the independence or trial to arrange independent criteria will occur. Then the constructed model will be not as close as it is expected to be to the real life situation and therefore the obtained solution may be different than it should be (i.e. from the situation when those interactions are taken into account).

For the applications which will be presented in this study, first of all, the model will be built after the reunions made with the DMs. Criteria, sub-criteria, alternatives for the model will clearly be defined. Then, in order to eliminate this difficulty of "criteria independence", the model will be evaluated using ANP or CI or with a combination of these two as a MCDM tool according to the adaptability of one of these tools to the model (See Figure 6.1).

Once the model is built and the relations between criteria are defined, the first thing to do is to decide the method to use. This is not an arbitrary choice. If there is an outer-dependence between sub-criteria, then this is something to be analyzed with ANP because of the simple fact that the CI cannot handle two elements that are connected to two different points. In this case two sub-criteria in question belong to two different criteria. Hence, these dependencies will be handled with ANP. Sub-criteria belonging to the same cluster can be analyzed in order to define the conjunctive and disjunctive behavior between them. If there is not a relation of this kind, then CI has to be used in order to find the interaction values. In case of no interaction, then the relations will be handled with ANP.

After handling the sub-criteria, the upper level, i.e. the criteria, has to be taken in consideration. A preference ranking of the criteria given by the DMs will define the conjunctive / disjunctive behavior between those. If there is not any interaction of this kind between criteria, then the model will be solved with ANP. The final aggregation will be made and a ranking will be obtained. If there are conjunctive / disjunctive behavior between criteria, then the Shapley indices and the interaction values will be used including the weights of the sub-criteria and the alternatives' individual performance values for each of those sub-criteria in order to perform the final aggregation.

To resume this,

- Outer dependencies between sub-criteria + no conjunctive / disjunctive behavior between them + No conjunctive / disjunctive behavior between criteria = ANP.
- No outer dependencies between sub-criteria + conjunctive / disjunctive behavior between them + conjunctive / disjunctive behavior between criteria = CI.
- Other than those situations = Hybrid ANP & CI.

In the use of CI method, if the experts, in other words DMs, are able to provide a fuzzy measure for their criteria set, it is possible to define the Shapley indices and interaction values using the formula (4.12) and (4.14). Or if, using their experience and expertise, they are able to provide the weights of their criteria and information about interactions (being strongly positive, positive, null, negative or strongly negative) then in this case also the interaction values can be retrieved using some other simplifying approaches such as Berrah et al. [57] used. Thus, there is only definition of individual performance values of the alternatives according to those criteria to do. Here it has to be noted that for the quantitative criteria, a simple normalization procedure will be enough to have the values. On the other hand, in order to define the performance values for the qualitative ones, one can use a tool such as AHP or MACBETH.

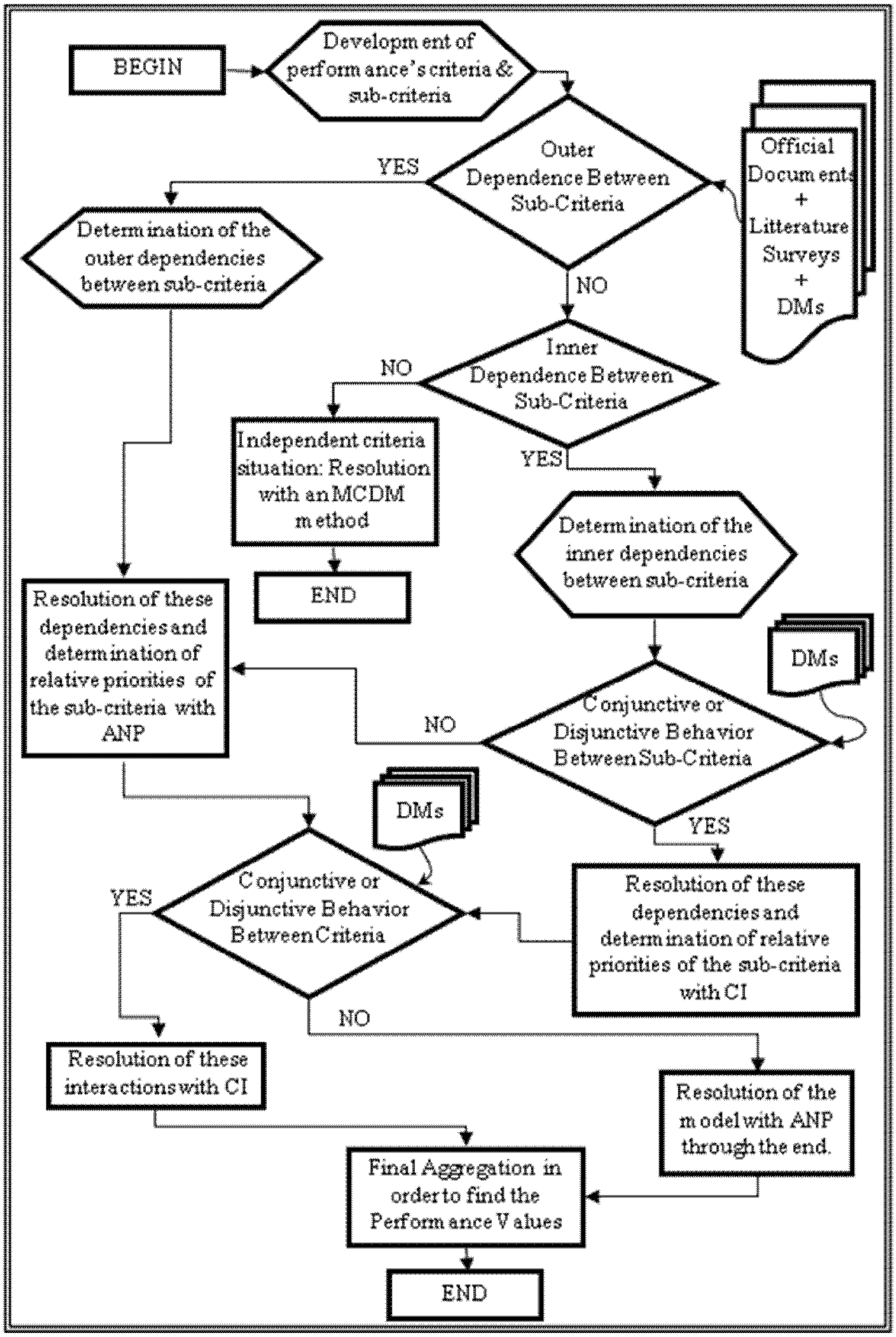

Figure 6.1 Flowchart of the proposed method

However, according to Grabisch [101], fuzzy measures are not easy to handle in a practical problem, although they constitute a flexible tool for modeling the importance of coalitions. There are two reasons:

- Defining a fuzzy measure on a set *X* of *n* elements requires the definition of 2^n positive real coefficients, i.e. for each subset of *X*, satisfying some constraints, which will be very complex if *n* goes beyond, say 8. Therefore, a practitioner will either use simply an additive fuzzy measure with a poor modeling or he will try the full complexity of fuzzy measures and in this case he lacks tools for handling them properly.
- Even for low values of *n* it will be difficult to find values for all subsets of *X*. Most of the time, an expert will be able to guess the values $\mu(A)$ with $A \subset X$ for singletons and for pairs but not for all the subsets. Reciprocally if a fuzzy measure is given, nobody can tell what it means in terms of behavior in decision making. Hence, fuzzy measures can be intuitively appealing but on the other hand, they remain difficult to interpret and understand.

In cases where DMs are unable to provide such info, then in order to define the Shapley indices and the interaction values, the use of MACBETH defined in paragraph (4.2.6.1) is proposed (Determination of individual performance values of the alternatives according to problem's criteria remaining the same). It is also possible to define those parameters using linear programming as shown in the study made by Marichal and Roubens [16]. In that case the procedure is done with the knowledge of a partial ranking over a reference set of alternatives, the set of criteria and the set of interactions between pairs of criteria. It is also mentioned before that there are some other simplifying approaches such as Beraah et al. [57] have used.

The choice between those methods is arbitrary however simplifying approaches will not be proposed in this study in order to find those parameters in a more scientific way. MACBETH or linear programming can be interchangeably proposed as more accurate ways for that purpose. However, equation systems of MACBETH can be easier to build, understand and solve than the constraints and objective function of linear programming for a DM working in the private sector who may not know about mathematical programming but still have the knowledge of high school maths.

While using MACBETH, again, if the DMs are able to give all the pairwise comparisons of the criteria including a pre-rank of those or the comparisons of the alternatives with respect to each criterion including a pre-rank for each criterion, the consistency study has to be done and in case of inconsistencies the DMs has to be warned in order to modify the necessary judgments. For the DMs who are having difficulties in this kind of judgments, requiring those comparisons from them will make the proposed method much more complicated and a lot less user-friendly for them as much as the result could be more "healthy". However, it is highly recommended to obtain those comparisons if the DMs have no difficulty to give in order to approach to the real life situation as much as possible.

The ranking of the alternatives/candidates will be found for each DM with the proper method for the model in hand. Then to define the best ranking representing those individual rankings, SCF theory represented in Section 5 will be used.

7. APPLICATION

7.1. Choquet Integral Case

7.1.1. General Information

In order to see the use of the proposed method, the application of this study will be made in a law firm which is a partnership founded in 2002.

As a general formation of law firms, in this one also, there are three main levels for the lawyers as you can see in the following figure:

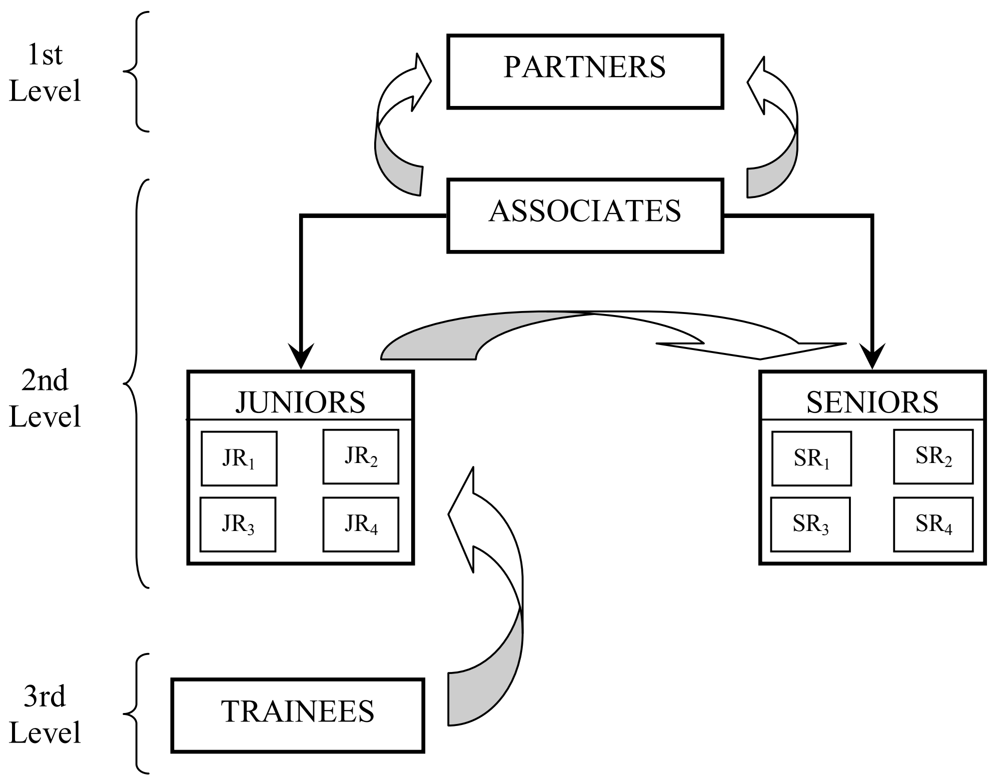

Figure 7.1. General Formation of a Law Firm

A trainee hired by the law firm, according to his/her performance, goes up to the associate level which can be broken down into two main categories, namely juniors – JRs and seniors – SRs. Between the lowest level of JR and the highest level of SR, there is eight level of experience. An associate who has achieved the highest level of SR, again according to his/her performance, is promoted to the level of partner. At this moment, the firm has nine associates from which six are JRs, three are SRs and there are four partners.

The firm is active mainly in Mergers & Acquisitions – M&A and Public Offerings – POFs. In M&A there are three main tasks to perform, namely the due diligence performed by JRs and trainees, contract drafting by SRs and partners and negotiation by SRs respectively. However the POFs are a little bit different. After the due diligence exercise, performed again by the JRs and trainees, there is the preparation of the offering documents performed by partners and SRs. Then there is a procedure performed by the investment banks to the issuer whose securities are to be offered. In connection with the offering process and in order for the banks to close the transaction, the partner in charge issues a legal opinion which, among others, covers Turkish law matters and issues relating to the issuer.

7.1.2. Performance Criteria

In the application, the yearly performance of the JRs working for the firm will be evaluated. Each JR is assigned to a number of ten to fifteen projects per year. To evaluate the performance of those JRs, first the criteria have to be defined. The three DMs, i.e. the partners of TAD, are asked to cite those criteria. After a meeting with the DMs, they agreed on the following five criteria:

- Working Performance – Ability to Use Capacity (C_1)
- Contribution to Colleagues – Team Work Ability (C_2)
- Ability to comment (C_3)
- Ability to Manage the Workload (C_4)
- Client's Feedback (C_5)

7.1.3. Performance Evaluation with Choquet Integral

7.1.3.1. Identification of Criteria Weights and Their Interactions with MACBETH

7.1.3.1.1. Preferential Ranking of the Decision Makers

The DMs were asked to give their preferential rankings on the criteria and groups of criteria and then they expressed their preferences. This stage was followed by asking the strength of their preference ranking. Below, you can see the preference rankings and the strength of those preferences for the three DMs with the same notations used in Section 3:

For the DM_1 the ranking of the situations was as follows:

$$S^{(1,0,0,0,1)} \approx S^{(1,0,1,0,0)} \approx S^{(0,0,1,0,1)} \succ^{VS} S^{(0,0,1,1,0)} \succ^{VW} S^{(0,0,0,1,1)}$$
$$\succ^{VW}$$
$$S^{(0,1,0,0,1)} \succ^{VW} S^{(1,1,0,0,0)} \succ^{W} S^{(1,0,0,1,0)} \succ^{M} S^{(1,0,0,0,0)} \succ^{M} S^{(0,1,1,0,0)}$$
$$\succ^{M}$$
$$S^{(0,0,1,0,0)} \succ^{VS} S^{(0,0,0,0,1)} \succ^{W} S^{(0,1,0,1,0)} \succ^{W} S^{(0,0,0,1,0)} \succ^{M} S^{(0,1,0,0,0)} \succ^{S} \text{"0"}$$

For the DM_2 the ranking of the situations was as follows:

$$S^{(1,0,0,1,0)} \succ^{VW} S^{(1,0,0,0,1)} \succ^{VW} S^{(1,0,1,0,0)} \succ^{VW} S^{(1,1,0,0,0)} \succ^{VW} S^{(0,0,0,1,1)}$$
$$\succ^{VW}$$
$$S^{(0,0,1,1,0)} \succ^{VW} S^{(0,1,1,0,0)} \succ^{VW} S^{(0,1,0,1,0)} \succ^{VW} S^{(0,1,0,0,1)} \succ^{VW} S^{(0,0,1,0,1)}$$
$$\succ^{VW}$$
$$S^{(1,0,0,0,0)} \succ^{VW} S^{(0,0,0,1,0)} \succ^{VW} S^{(0,0,0,0,1)} \succ^{VW} S^{(0,0,1,0,0)} \succ^{VW} S^{(0,1,0,0,0)} \succ^{M} \text{"0"}$$

For the DM_3 the ranking of the situations was as follows:

$$S^{(0,0,1,0,1)} \succ^{M} S^{(0,0,1,1,0)} \succ^{W} S^{(0,1,1,0,0)} \approx S^{(1,1,0,0,0)} \approx S^{(1,0,1,0,0)}$$
$$\succ^{M}$$
$$S^{(0,0,0,1,1)} \approx S^{(0,1,0,0,1)} \succ^{W} S^{(1,0,0,1,0)} \approx S^{(1,0,0,0,1)} \succ^{M} S^{(0,1,0,1,0)}$$
$$\succ^{W}$$
$$S^{(0,0,1,0,0)} \succ^{S} S^{(0,1,0,0,0)} \approx S^{(0,0,0,0,1)} \succ^{M} S^{(1,0,0,0,0)} \approx S^{(0,0,0,1,0)} \succ^{M} \text{"0"}$$

7.1.3.1.2. Equation Systems and Calculation of Criteria Weights and Interactions

Here the criteria weight and interaction values for each DM will be calculated using the MACBETH procedure. From the preferential rankings and strength values provided by the DMs in the previous section, using the equations (4.27), (4.28) and (4.29) given in section (4.2.6.7), the systems of equations which can be analyzed in Appendix B can be retrieved for each DM.

As in section (4.2.6.8), let A be the matrix of the coefficients for the equation system above, V the column matrix of variables, namely the Shapley and interaction parameters and α, and e_i the column vector where all the elements except i^{th}, which is equal to one, is null. Therefore, the matrix operation below has to be solved:

$$A \times V = e_{16}$$

Hence the matrix operations for DM_1, DM_2 and DM_3 respectively are as presented in Appendix C. And finally the resolution of matrix operations gives the criteria weights and the interaction between criteria as follows respectively in Table 7.1 and 7.2.

Table 7.1. Criteria weights according to the DMs

	v_1	v_2	v_3	v_4	v_5
DM₁	.217	.108	.232	.144	.299
DM₂	.406	.118	.138	.220	.118
DM₃	0	0	.5	0	.5

In Table 7.1, it can observed that the most important criterion for DM_1 is the "Client's Feedback" with a weight of .299 whereas the least important is "Contribution to Colleagues – Team Work Ability" with a weight of .108. For DM_2 the most important criterion is "Working Performance – Ability to Use Capacity" with a quite dominant weight of .406 whereas the least important criteria turn out to be "Contribution to Colleagues – Team Work Ability" and "Client's Feedback" with equal weight of .118. An interesting case occurs for DM_3 for whom the evaluation procedure turns out to be based on two equally important criteria namely "Ability to Comment" and "Client's Feedback".

In Table 7.2, for the DM_1 it can be observed that there is no interaction between the second and third criteria as I_{23} is null. On the other hand, having a negative interaction value for C_1 & C_3, C_1 & C_4 and C_2 & C_4 shows that for this DM, there is a disjunctive behavior between criterion couples *{1; 3}*, *{1; 4}* and *{2; 4}*. i.e. an alternative can be successful only in one of the criteria among the considered couple of criterion and considered successful overall. However for the criterion couples *{1; 2}*, *{1; 5}*, *{2; 5}*, *{3; 4}*, *{3; 5}* and *{4; 5}* positive interaction values imply a conjunctive behavior between those couples. i.e. for an alternative to be considered successful overall from one of those couples, it has to be successful from both criteria. Similar observation can be made for DM_2 and conclude that a disjunctive behavior between criteria couples *{2; 4}*, *{2; 5}*, *{3; 4}*, *{3; 5}* and *{4; 5}* and a conjunctive behavior between *{1; 2}*, *{1; 3}*, *{1; 4}*, *{1; 5}* and *{2; 3}*. For DM_3 from the fact that the only interaction, and a positive one, existing is the one between C_3 & C_5 and null interaction values for all other criteria couples, it can be concluded that an alternative should be successful in both C_3 & C_5 and be considered successful overall. The interaction value being equal to one shows a full complementarity between those criteria.

Table 7.2. Interaction between criteria according to the DMs

	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}
DM₁	.0206	-.0206	-.031	.031	0	-.0103	.1443	.0825	.0928	.124
DM₂	.0588	.0588	.0588	.0588	.0588	-.0647	-.0647	-.0235	-.147	-.0235
DM₃	0	0	0	0	0	0	0	0	1	0

7.1.3.2. Identification of Alternatives' Performance with Respect to the Criteria

After defining criteria weights and interactions between criteria, now the turn is to define the performance values of the alternatives with respect to the problem's criteria. The alternatives, i.e. six JRs will be called A, B, C, D and E. The DMs were asked to give a preorder of the alternatives according to each of the criteria with the strength of their evaluation.

The preorders and the strength of judgments given by DM_1:

$$\begin{array}{ll} C_1 \Rightarrow & good \succ^1 A \approx C \succ^{VW} B \succ^S E \succ^{VW} D \succ^M F \succ^1 neutral \\ C_2 \Rightarrow & good \succ^1 C \approx D \succ^M B \approx A \succ^{VS} E \approx F \succ^1 neutral \\ C_3 \Rightarrow & good \succ^1 A \approx C \succ^{VW} B \succ^S E \succ^{VW} D \succ^M F \succ^1 neutral \\ C_4 \Rightarrow & good \succ^1 C \succ^M A \succ^W B \succ^S E \succ^{VW} D \succ^M F \succ^1 neutral \\ C_5 \Rightarrow & good \succ^1 C \succ^S A \approx B \succ^M D \succ^{VW} E \approx F \succ^1 neutral \end{array}$$

The preorders and the strength of judgments given by DM_2:

$$\begin{array}{ll} C_1 \Rightarrow & good \succ^1 C \succ^S A \succ^M B \succ^{VW} D \succ^S F \succ^{VW} E \succ^1 neutral \\ C_2 \Rightarrow & good \succ^2 A \approx B \approx C \approx D \succ^W E \approx F \succ^1 neutral \\ C_3 \Rightarrow & good \succ^1 A \approx C \succ^W B \approx D \succ^S E \approx F \succ^1 neutral \\ C_4 \Rightarrow & good \succ^1 A \approx C \succ^{VW} B \approx D \succ^M E \approx F \succ^1 neutral \\ C_5 \Rightarrow & good \succ^2 A \approx B \approx C \succ^W D \succ^S E \approx F \succ^1 neutral \end{array}$$

The preorders and the strength of judgments given by DM_3:

$$
\begin{aligned}
C_1 &\Rightarrow \quad good \succ^1 A \approx B \succ^M C \succ^M D \succ^M E \succ^M F \succ^1 neutral \\
C_2 &\Rightarrow \quad good \succ^1 A \succ^M B \succ^M E \succ^M D \succ^M C \succ^M F \succ^1 neutral \\
C_3 &\Rightarrow \quad good \succ^1 A \succ^{VW} B \succ^{VW} C \succ^W D \succ^S E \approx F \succ^1 neutral \\
C_4 &\Rightarrow \quad good \succ^1 D \succ^W A \succ^W B \approx E \succ^W C \approx F \succ^1 neutral \\
C_5 &\Rightarrow \quad good \succ^1 B \succ^W A \approx C \approx D \succ^W E \approx F \succ^1 neutral
\end{aligned}
$$

From the preferential information given above, using the equation (4.23), the equation systems and solutions can be found as shown in Appendix D.

Now the aggregated performance value for each alternative can be calculated using the formula given in (4.26). The performance values for the alternatives according to the criteria and their aggregated performance values are calculated and gathered in Table 7.3.

From Table 7.3 it can be seen that for DM_1 and DM_2 C is clearly the best candidate with A placed as second but for DM_3 C is placed third after A and B who were placed first and second respectively. The worst candidate for DM_1 is by far the candidate F and for DM_2 it's the candidate E and finally for DM_3, E and F are nominated by far as the worst performing candidates.

Therefore the ranking of the alternatives according to each DM will be as follows:

$$
\begin{aligned}
DM_1 &\Rightarrow C \succ A \succ B \succ D \succ E \succ F \\
DM_2 &\Rightarrow C \succ A \succ B \succ D \succ F \succ E \\
DM_3 &\Rightarrow A \succ B \succ C \succ D \succ E \approx F
\end{aligned}
$$

Table 7.3. Aggregated performance of the alternatives according to DMs

	Alt.	C_1	C_2	C_3	C_4	C_5	p_{Ag}
DM_1	***A***	0,909	0,6	0,909	0,733	0,5	0,782
	B	0,818	0,6	0,818	0,6	0,5	0,709
	C	0,909	0,9	0,909	0,933	0,9	0,912
	D	0,364	0,9	0,364	0,267	0,2	0,428
	E	0,455	0,1	0,455	0,333	0,1	0,335
	F	0,091	0,1	0,091	0,067	0,1	0,094
	Alt.	C_1	C_2	C_3	C_4	C_5	p_{Ag}
DM_2	***A***	0,667	0,6	0,875	0,833	0,778	0,740
	B	0,467	0,6	0,625	0,667	0,778	0,588
	C	0,933	0,6	0,875	0,833	0,778	0,851
	D	0,4	0,6	0,625	0,667	0,556	0,547
	E	0,067	0,2	0,125	0,167	0,111	0,124
	F	0,133	0,2	0,125	0,167	0,111	0,145
	Alt.	C_1	C_2	C_3	C_4	C_5	p_{Ag}
DM_3	***A***	0,929	0,941	0,9	0,625	0,5	0,9
	B	0,929	0,765	0,8	0,375	0,833	0,833
	C	0,714	0,235	0,7	0,125	0,5	0,7
	D	0,5	0,412	0,5	0,875	0,5	0,5
	E	0,286	0,588	0,1	0,375	0,167	0,167
	F	0,071	0,059	0,1	0,125	0,167	0,167

7.1.3.3. Identification of a Social Choice Preference Order – SCPO

7.1.3.3.1. SCPO with Condorcet's Function

In order to find the values of the function given in formula (5.1) for each candidate, $\#(i : xP_iy)$ is counted for each candidate couple. Therefore:

$$\#(i:AP_iB)=3 \quad \#(i:BPA)=0 \quad \#(i:CPA)=2 \quad \#(i:DPA)=0 \quad \#(i:EPA)=0 \quad \#(i:FPA)=0$$

$$\#(i:AP_iC)=1 \quad \#(i:BPC)=1 \quad \#(i:CPB)=2 \quad \#(i:DPB)=0 \quad \#(i:EPB)=0 \quad \#(i:FPB)=0$$

$$\#(i:AP_iD)=3 \quad \#(i:BPD)=3 \quad \#(i:CPD)=3 \quad \#(i:DPC)=0 \quad \#(i:EPC)=0 \quad \#(i:FPC)=0$$

$$\#(i:AP_iE)=3 \quad \#(i:BPE)=3 \quad \#(i:CPE)=3 \quad \#(i:DPE)=3 \quad \#(i:EPD)=0 \quad \#(i:FPD)=0$$

$$\#(i:AP_iF)=3 \quad \#(i:BPF)=3 \quad \#(i:CPF)=3 \quad \#(i:DPF)=3 \quad \#(i:EPF)=1 \quad \#(i:FPE)=1$$

And the Copeland Function values for the candidates are:

$$f_C(A)=1 \quad f_C(B)=0 \quad f_C(C)=2 \quad f_C(D)=0 \quad f_C(E)=0 \quad f_C(F)=0$$

Hence the final ranking will be:

$$C \succ A \succ B \approx D \approx E \approx F$$

7.1.3.3.2. SCPO with Borda's Function

Using the function (5.2) and the numbers counted in the previous section, the Borda's function values for the candidates are as follows:

$$f_B(A)=13 \quad f_B(B)=10 \quad f_B(C)=13 \quad f_B(D)=6 \quad f_B(E)=1 \quad f_B(F)=1$$

Hence the final ranking will be:

$$A \approx C \succ B \succ D \succ E \approx F$$

7.1.3.3.3. SCPO with Copeland's Function

Using the function (5.3) and the numbers counted in the previous section, the Copeland's function values for the candidates are as follows:

$$f_{CP}(A)=3 \quad f_{CP}(B)=1 \quad f_{CP}(C)=5 \quad f_{CP}(D)=-1 \quad f_{CP}(E)=-5 \quad f_{CP}(F)=-5$$

Hence the final ranking will be:

$$C \succ A \succ B \succ D \succ E \approx F$$

7.1.3.4. Agreement and Disagreement

7.1.3.4.1. Agreement and Disagreement with Cook and Seiford's Function

Calculating d_{jk} value for each candidate j where k = 1,...,6 using the formula (5.4) will give:

$$\begin{array}{llllll}
d_{A1}=2 & d_{A2}=1 & d_{A3}=4 & d_{A4}=7 & d_{A5}=10 & d_{A6}=2 \\
d_{B1}=5 & d_{B2}=2 & d_{B3}=1 & d_{B4}=4 & d_{B5}=7 & d_{B6}=10 \\
d_{C1}=3 & d_{C2}=3 & d_{C3}=4 & d_{C4}=10 & d_{C5}=13 & d_{C6}=16 \\
d_{D1}=9 & d_{D2}=6 & d_{D3}=3 & d_{D4}=0 & d_{D5}=3 & d_{D6}=6 \\
d_{E1}=13 & d_{E2}=10 & d_{E3}=7 & d_{E4}=4 & d_{E5}=1 & d_{E6}=2 \\
d_{F1}=14 & d_{F2}=11 & d_{F3}=8 & d_{F4}=5 & d_{F5}=2 & d_{F6}=1
\end{array}$$

Therefore, by applying the Hungarian Algorithm to the following matrix will give the optimal result:

j \ k	***1***	***2***	***3***	***4***	***5***	***3***
A	2	1	4	7	10	2
B	5	2	1	4	7	10
C	3	3	4	10	13	16
D	9	6	3	0	3	6
E	13	10	7	4	1	2
F	14	11	8	5	2	1

The optimal solution is found after the subtraction of the smallest number of each row from each element of that row:

j \ k	***1***	***2***	***3***	***4***	***5***	***3***
A	1	**0**	3	6	9	1
B	4	1	**0**	3	6	9
C	**0**	0	1	7	10	13
D	9	6	3	**0**	3	6
E	12	9	6	3	**0**	1
F	13	10	7	6	1	**0**

And the final ranking is:

$$C \succ A \succ B \succ D \succ E \succ F$$

7.1.3.4.2. Agreement and Disagreement with Kemeney's Function

The L matrix given in (5.5) will be as follows for the three rankings found with Condorcet, Borda and Copeland's functions. They will be noted L_C, L_B and L_{CP} respectively.

$$L_C = \begin{bmatrix} 0 & 1 & -1 & 1 & 1 & 1 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \qquad L_B = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ -1 & 0 & -1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 & 1 \\ -1 & -1 & -1 & -1 & 0 & 0 \\ -1 & -1 & -1 & -1 & 0 & 0 \end{bmatrix}$$

$$L_{CP} = \begin{bmatrix} 0 & 1 & -1 & 1 & 1 & 1 \\ -1 & 0 & -1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 & 1 \\ -1 & -1 & -1 & -1 & 0 & 0 \\ -1 & -1 & -1 & -1 & 0 & 0 \end{bmatrix}$$

The proportion matrix M defined in (5.6) and E matrix defined in Definition 5.4 will be as follows:

$$M = \begin{bmatrix} 1/2 & 1 & 1/3 & 1 & 1 & 1 \\ 0 & 1/2 & 1/3 & 1 & 1 & 1 \\ 2/3 & 2/3 & 1/2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1/2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1/2 & 2/3 \\ 0 & 0 & 0 & 0 & 1/3 & 1/2 \end{bmatrix} \qquad E = \begin{bmatrix} 0 & 1 & -1/3 & 1 & 1 & 1 \\ -1 & 0 & -1/3 & 1 & 1 & 1 \\ 1/3 & 1/3 & 0 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 & 1 \\ -1 & -1 & -1 & -1 & 0 & 1/3 \\ -1 & -1 & -1 & -1 & -1/3 & 0 \end{bmatrix}$$

Therefore, the simple inner product of E and L matrices will be,

$$< E, L_C > = 46/3 \quad < E, L_B > = 74/3 \quad < E, L_{CP} > = 76/3$$

It can be observed that the closest ranking out of the three rankings found using SCFs is the one found with Copeland's Function. Note that the maximum inner product value that can be found out of any ranking combination of the candidates is 26 and can be achieved with the ranking found with Cook and Seiford's function. Hence the ranking found with Cook and Seiford's function can be chosen as the best one.

7.2. Analytic Network Process Case

7.2.1. General Information

To analyze the use of the proposed method, the performance of a set of employees working in a company in medical sector will be evaluated.

The company produces infusion sets, i.e. the equipment to inject serum from the bottle to the patient's arm. In the production area, there are six different task stations that you can see below, hence, six different groups of employees to do these tasks. But the calculations for only one of the following groups, Assembling Flashtube – Adaptor – AF in which five employees are working, will be detailed to show how the model works:

i. Assembling Needle - Adaptor – "AI"
ii. Assembling Flashtube - Adaptor – "AF"
iii. Assembling Needle + Flash tube - Pipe E5 – "EF"
iv. Assembling of Dropping group - Pipe E5 – "ED"
v. Putting the set in the bag and Sealing – "S"
vi. Packaging – "K"

7.2.2. Network and Inter-dependencies

We want to determine the best performing employee. For this company, this determination depends on the productivity (C1), the absence (C2), the hygiene (C3), the education level (C4) and the characteristic properties (C5) of the employee.

The measurement of workers' productivity (labor productivity) is important for productivity improvement. Labor productivity shows how well the labor force has been used. The productivity of the employee is being measured with the following two ratios: "Value Added/Total Work Hours" (C11) and "Value Added/Salaries & Wages" (C12). The absence of the employee can be without any given reason (AWOL) (C21), in case of sickness (C22), in case of an official leave (C23) or by being late to work (C24). As the company performs a very delicate task concerning human life, hygiene is of a great importance in this company and it can be evaluated by considering both personal hygiene (C31) and following the rules about the hygiene inside and outside the production area (C32). The education level also plays an important rule in the performance evaluation of the employee and it is affected by the personal experience (C42) of the employee as well as his/her scholar evolution (C41). And finally the characteristic properties of the employee are evaluated with respect to his/her social relationships (C51), responsibility feeling (C52), reliability (C53) and the ability of empathy (C54).

The Figure 7.2 shows how the problem's network is formed and Figure 7.3 is a screenshot of the SuperDecisions software used to solve the model. All the sub-criteria are connected to each one of the five employees working for the task AF. The inter and inner-dependencies among the criteria and can be seen from this figure also. All the sub-criteria belonging to C2, C3 and C4 are affecting the productivity ratios in C1. Because the education level (especially the experience of the employee) and the absence of any kind will affect the value added as well as any non-hygienic situation. In addition to that, a non-hygienic situation will have a serious damage on the final product causing to restart from the beginning or the destruction of the product.

The scholar evolution of a person as well as his/her reliability and the ability of empathy will have an effect on the person's willing of following the rules. And also an employee who is eager to follow the rules is becoming reliable in the managers' opinion.

The higher the responsibility feeling of the employee, the less AWOL he/she will do or the less he/she will be late or the less he/she will take official leave every now and then. These dependencies are concluded by a statistical analyze using the data that the company was keeping during last 15 years of all its employees they have been hiring.

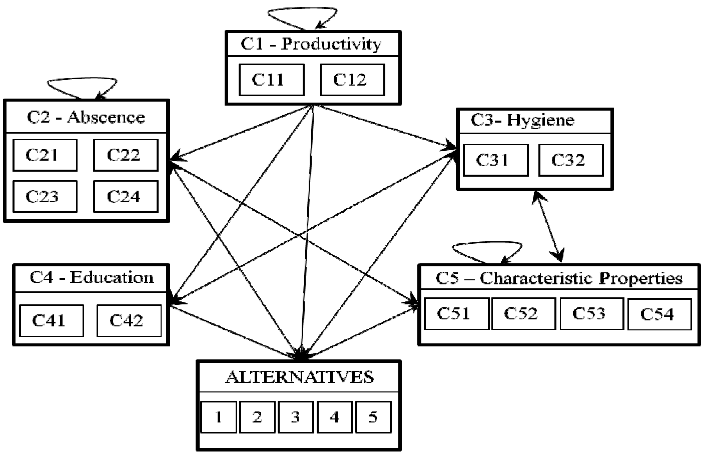

Figure 7.2. The Network Model of the Problem

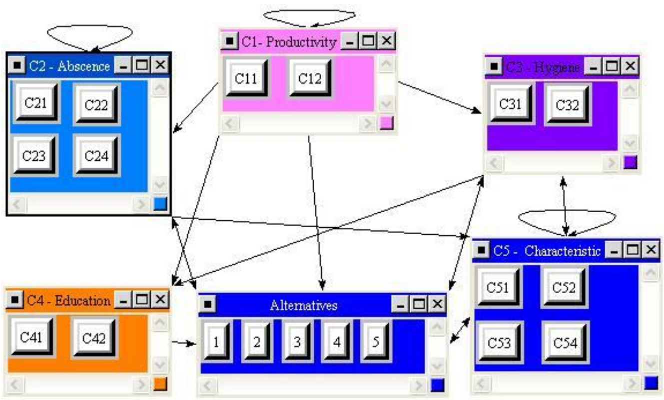

Figure 7.3. The Network Model of the Problem with SuperDecisions Software

7.2.3. Pairwise Comparisons

Here, only a couple of the main comparisons have been shown to give an idea about the procedure. The 1-9 scale of Saaty [119] given in Table 3.1 has been used.

The relative importance weights of the components can be seen on each comparison matrix. The consistency analyses of those comparisons are also made. In Table 7.4, the pairwise comparisons of the alternatives with respect to the sub-criterion C54 is represented. The CR for this matrix is found to be 0.02 which is above the allowed limit. According to the weights, A1 with the relative importance value of 0.54, is the best performing employee for this sub-criterion. One such pairwise comparison with respect to all of the problem's sub-criteria has been made.

Table 7.4 Pairwise comparison of alternatives with respect to sub-criteria C54

C54	**A1**	**A2**	**A3**	**A4**	**A5**	*w*
A1	1	3	9	5	7	0.54
A2	0.33	1	5	2	3	0.21
A3	0.11	0.2	1	0.33	0.33	0.04
A4	0.2	0.5	3	1	2	0.12
A5	0.14	0.33	3	0.5	1	0.08

Cluster comparisons also have to be made. Table 7.5 shows one of the cluster comparisons, the cluster comparisons with respect to C1. A pairwise comparison among the clusters that are connected from C1 has been made and has been presented in this table. The CR of these comparisons is 0.02 which also proves that the comparisons are consistent enough. According to the relative importance values, C3 with the relative importance value of 0.49 and C4 with 0.05 have the maximum and minimum effects on C1 and 0.05 respectively.

Table 7.5 Pairwise comparison of clusters with respect to C2

C1	**Alt.**	**C1**	**C2**	**C3**	**C4**	*w*
Alt.	1	2	0.5	0.25	3	0.14
C1	0.5	1	0.33	0.2	2	0.09
C2	2	3	1	0.33	5	0.23
C3	4	5	3	1	7	0.49
C4	0.33	0.5	0.2	0.14	1	0.05

Using the relative importance vectors obtained from these cluster comparison matrices, the cluster matrix is formed as shown below in Table 7.6.

Table 7.6 Cluster Matrix

	Alt.	*C1*	*C2*	*C3*	*C4*	*C5*
Alt.	0	0.137	0.443	0.122	1	0.637
C1	0	0.085	0	0	0	0
C2	0.258	0.232	0.169	0	0	0
C3	0.637	0.496	0	0	0	0.105
C4	0	0.05	0	0.32	0	0
C5	0.105	0	0.387	0.558	0	0.258

After all the pairwise comparisons have been made, the "Super Decisions" software is used to construct the unwieghted, weighted supermatrices and the limit supermatrix as shown in Appendix E.

7.2.4. Concluding Remarks

Normalizing the values taken from the limit supermatrix, we can see the relative priorities are, .283, .257, .129, .149, .182 for A1, A2, A3, A4 and A5 respectively.

Hence the best performing employee of the task AF is the employee number one. And the worst performing is the employee number three. Therefore, with respect to the relative priorities, we can form the final ranking of the alternatives as follows:

$$A1 \succ A2 \succ A5 \succ A4 \succ A3$$

If we were to ignore the interdependencies between the clusters, and apply the AHP to our problem, the weights for the criteria, w_{C_i}, and the relative priorities of the alternatives, w_{A_i}, would be as seen on the Table 7.7 and hence, the final ranking would be as given below:

Table 7.7 Criteria weights and relative priorities for AHP

i	1	2	3	4	5
w_{C_i}	0.2105	0.0496	0.4918	0.0754	0.1727
w_{A_i}	0.257	0.306	0.146	0.155	0.136

$$A2 \succ A1 \succ A4 \succ A3 \succ A5$$

As we can clearly see, ignoring the inter-dependencies caused not only the change in the ranking of the alternatives but also a change of the best and worst performing alternatives.

The analysis of this problem requires a model capable of consisting all the criteria and the relationships between them. Here we presented a real life MCDM problem with interactions and wanted to underline the importance of considering the inter-dependencies between the criteria and alternatives which are clearly effective in the decision making process and have a great influence on the final decision of the problem.

7.3. ANP & CI Hybrid Case

7.3.1. General Information

Finally, in order to see explicitly how the proposed hybrid model works, in this section, a PE model for marketing employees will be presented. The case study has been made in a pharmaceutical company for a specific region where they are active and for which there are four assigned employees. Three DMs will be evaluating the performance of those employees over a period of three months.

7.3.1.1. Model's Criteria and Sub-Criteria

DMs agreed on three main performance criteria group containing altogether eight sub-criteria which are listed as follows:

- Sales Related Performance Criteria – SRPC
  - Sales Quantity – S
  - Sales Reports Efficiency/Effectiveness – SRE
- Customer Related Performance Criteria – CRPC
  - Customer Feedback – CF
  - Value Added to Reputation – VAR
  - New Customer Gain – NCG
  - Number of Customers in Portfolio – NCP
- Relations Related Performance Criteria – RRPC
  - Relations with Upper Management – RUM
  - Relations with Colleagues – RC

7.3.1.2. Factors Moderating the Performance – FMP

Other than criteria, to be able of evaluate the performance, the factors effecting the performance have to be taken in consideration also. These factors are called in this study as "moderating factors". For example, the experience of an employee is not a PE criterion; however it is a factor that will surely affect his/her performance in marketing. The criteria of the PE model presented in this section are moderated by the following factors:

- Experience – E: This is representing the experience level of the employee.
- Marketing Ability – MA: The MA represents how well an employee can use his talents in order to make a product to be accepted by the consumer.
- Salary / Satisfaction – SS: This factor represents the level of happiness and satisfaction of the employee because of the opportunities as well as the salary offered to him/her.

- Social Power – SoP: The SoP of an employee shows how well, out of his MA, he/she can socialize and find ways to communicate with customer.
- Educational Level – EL: The EL can be academic or the education on the field the employee is working in such as seminars, conferences etc.
- Marketing Politics – MP: Represents the MP of the company according to different variables and practices of the marketing.
- Management Politics – MnP: Represents the general MnP of the company.
- Salary Politics – SP: Represents the SP of the company including the promotions, commissions, bonuses etc.
- Ability of Managing the Changes – MC: This shows the ability of the company to adapt itself to the changes occurring in the market, economy, environment etc.

7.3.2. Relations between Sub-Criteria and Moderating Factors

7.3.2.1. Relations for the SRPC

Under the SRPC, sales and sales reports efficiency/effectiveness are reciprocally affecting each other. Sales are affected from all the sub-criteria that are under CRPC. Customer feedback and value added to reputation will affect the management's and other potential customers' behavior towards the employee, and he/she will have more or less sales related to the values of those criteria. On the other hand, if the employee has a big potential to gain new customers and hence have a larger customer portfolio, then he/she is assumed to have bigger sale numbers. Sales are also directly affected by the employee's experience, marketing ability, salary/satisfaction and educational level. A more experienced employee with high marketing ability will be more able to get to the customer and sell the product. Salary/satisfaction level will trigger the will of the employee to work better and to sell more. A high level of social power, if it's well combined with marketing ability and using the tools coming along with the educational level, will allow the employee to convince the customer to come around even if he/she doesn't have the idea of buying the product. The marketing politics of the company have also one of the major influences on sales. The right marketing politics combined with the ability of managing the changes will allow the product and the employees reach the customer and be accepted. Managing the changes is crucial if a company wants to keep up in today's competitive world. With appropriate management and salary politics, an employee will be better motivated in order to work better for the company and hence it will affect the sales.

An employee's sales reports efficiency/effectiveness based on a sale, will depend on the feedback of the customer in question. He/she may be more efficient while preparing the reports of a customer having a better feedback about him/her. From the same point of view, new customer gain and number of customers in portfolio will affect the sales reports efficiency/effectiveness. The more an employee will have a customer; he/she will be more efficient in his/her reports in order to keep them in his/her portfolio. An experienced employee, of course with his/her educational level, will surely know how to be efficient in his/her reports, without forgetting that his/her salary/satisfaction level will seriously change this situation. Ergo, the sales politics of the company have a role in that too. The management politics of the company will set the rules on the preparation of the sales report and the limitations should be well defined.

7.3.2.2. Relations for the CRPC

The sales and number of customers in portfolio of an employee which will be resulted from his/her marketing ability and social power, will have an effect on the customer feedback of a customer whose opinion will change accordingly. An experienced employee, using his/her marketing ability, educational level and social power, will know how to impress a customer. In addition to that, if the employee is satisfied with his work and salary, his/her advantage will be indestructible. Of course the marketing and management politics and ability of managing the changes of the company will define the employee's limitations in his/her actions within the relations with customers. A marketing employee can be only as effective as the marketing politics and ability of managing the changes of the company. If the marketing strategy is wrong and the company has a poor level of ability of managing the changes then the customer won't show any interest to the product, hence the customer feedback will be affected.

The value added to reputation by an employee will be affected by his sales and the efficiency/effectiveness of his/her sales reports. The customers' feedback on this employee, new customers in his portfolio and the number of customers currently in his portfolio will show the value added to the company's reputation. An employee satisfied with his job and salary; by using his/her experience, marketing ability, social power and educational level, will do everything possible in order for the company he/she works for gets a better reputation. Again his/her actions on this purpose will be limited by company's marketing and management politics and the ability of managing the changes.

To be efficient/effective for an employee in sales, will help him/her to gain new customers for his portfolio. Good customer feedbacks, the value added to reputation by him/her and the number of customers in his portfolio will modify the other customers' or potential customers' opinions. It is quite obvious that the experience, marketing ability, social power and educational level of the employee will seriously affect his/'her potential of gaining new customers. On the other hand, a good level of job and salary satisfaction will encourage the employees' behaviors in order to gain new customers for the company he/she works for. A customer needs to be reached the way he/she likes and to be satisfied on the product. Hence the importance of the company's marketing and management politics and the ability of managing the changes. The relations with upper management can give the employee the opportunity to take initiatives and make special arrangements in order to gain new customers. These will be most importantly defining factors for a customer choosing this company. Correct salary politics and also the bonuses and promotions will be affecting the satisfaction of the employee hence the employee will be encouraged or discouraged for the purpose of gaining new customers.

The number of customers in the employee's portfolio is an important yet not a guaranteed number. It will directly change with the gain of new customers due to the sales and be changing with the signals sent us with the customers' feedbacks and the value added to the reputation of the company. As in new customer gain, this sub-criterion will be affected from the experience, the marketing ability, the salary/satisfaction, the social power and the educational level of the employee and the marketing, management and salary politics and the ability of managing the changes of the company.

7.3.2.3. Relations for the RRPC

Good sales and efficient/effective sales reports will be highly welcomed by the upper management and the employees showing this quality will be the ones preferred by the upper management and therefore will have better relationships with the upper management. But this is not enough all alone. The CRPC are also important for that matter. The customer feedbacks will show if the employee was able to sell the products momentarily. Loss of customers or lack in new customers will make the upper management unsatisfied because of the employees' performance even though the employee is able to sell. The loss in the value added to reputation will soon decrease the sales and affect the sustainability of those. If an employee has good relationships with his/her colleagues and helps to improve the positive atmosphere at work which is encouraging others to work more efficiently, the upper management will be eager to keep good relations with this employee in order to keep him in their company. Also, an experienced employee with good education, marketing ability and social power is a priceless asset that a company wouldn't like to lose. Therefore, good relations with the upper management will be triggered by these qualities of the employee. Marketing policy and ability of managing the changes will cause decrease in sales and make the employee look bad and hence this will affect the relations with upper management. The management and salary policies will directly affect these relations because they will modify the satisfaction of the employee.

Customer feedbacks are very important data which are not only related to the employees. These data can include very important information about region, changes etc. By collecting those data and sharing with friends in order to make the sales grow and managing the changes will surely affect the relations with colleagues. Marketing and management politics and the ability to manage changes along with the relations with upper management, will motivate the employees and modify the work environment of them, for instance by encouraging the teamwork. Team work and coordination between colleagues are also important in adding value to the reputation. In all these sharing and coordination, the experience, marketing ability, social power and educational level of the employee owns importance from the point of view of the sustainability of the company by helping relatively new employees develop and become stronger in what they do for a living. Company's salary politics are very important for the harmony of the work environment. A complete transparent politic will require a righteous salary system otherwise the differences on the salaries, if not righteous, will cause gossiping and affect badly the relations between colleagues regardless how satisfied they are with their salary and work.

7.3.2.4. Relations for the FMP

With each new customer added in his/her portfolio, the employee's experience will grow. The marketing ability, social power and educational level also will help him/her in order to move to a new experience level. Similarly, the marketing ability will grow as the employee confronts new challenges with each new customer added to portfolio. His experience and new techniques added with the education also will help him improving his marketing ability. As the employee observes the value he/she is adding to the company's reputation, his/her confidence will be encouraged and he/she will be open to take new actions maybe risks in order to make sale and this can increase his/her marketing ability.

All the sub-criteria under SRPC and CRPC and the relations with upper management have a direct effect on the salary and satisfaction of the employee. Good performances occurring in those criteria will allow to the employee better work conditions, increase in his salary, bonuses. Therefore his/her satisfaction level will increase. As all these conditions and payments are products of management and salary politics, it won't be bold to say that the company plays an important role in its employees' job satisfaction.

Companies will generally look for an experienced employee who trained himself in his/her education, with a good level of marketing ability and social power. These will increase the demand of such employee. Hence his/her salary and satisfaction will somehow be granted.
Adding new customers in the portfolio and a significant number of customers in the portfolio will assure the self-confidence of an employee. He/She will find the courage to explore new frontiers. With the growing experience and courage of his/hers, the social power will also increase. Educational level will increase his/her communication skills by means of language, techniques and methods. Hence the social power will be increased.

If the company is eager to adapt itself to changes and agrees to the need of constant personal development of its employees, then it will give the possibility to do that. Sometimes the company may not sponsor such concept. Then in this case, the employee's salary takes the lead in order to create such possibility.

Marketing and salary politics of the company will be primarily modified with the sales quantity. For salary politics, this will define the bonus scaling. On the other hand, for marketing politics, changes in sales quantity should be carefully analyzed along with the customer feedbacks. The changes in the customer number, if there isn't any other reason such as economic crisis etc. should be a warning for the company to modify the marketing politics. However it should be noted here that these modifications depend on the ability of managing the change and management politics of the company. Each marketing and management politics, because of different types of investments, target groups and expectancies, requires different type of salary politics.

In order to be able to manage the changes, the management politics have to be somehow modified. The management politics, on the other hand, are affecting the ability to manage the changes for a company. Necessary changes, such as the organization chart, require the approval of the management politics. Customer feedbacks will be the guide in how a company can manage a change.

7.3.3. Model of the PE

Under the light of the information about relations given in the previous section, the PE model can be seen in the following figure.

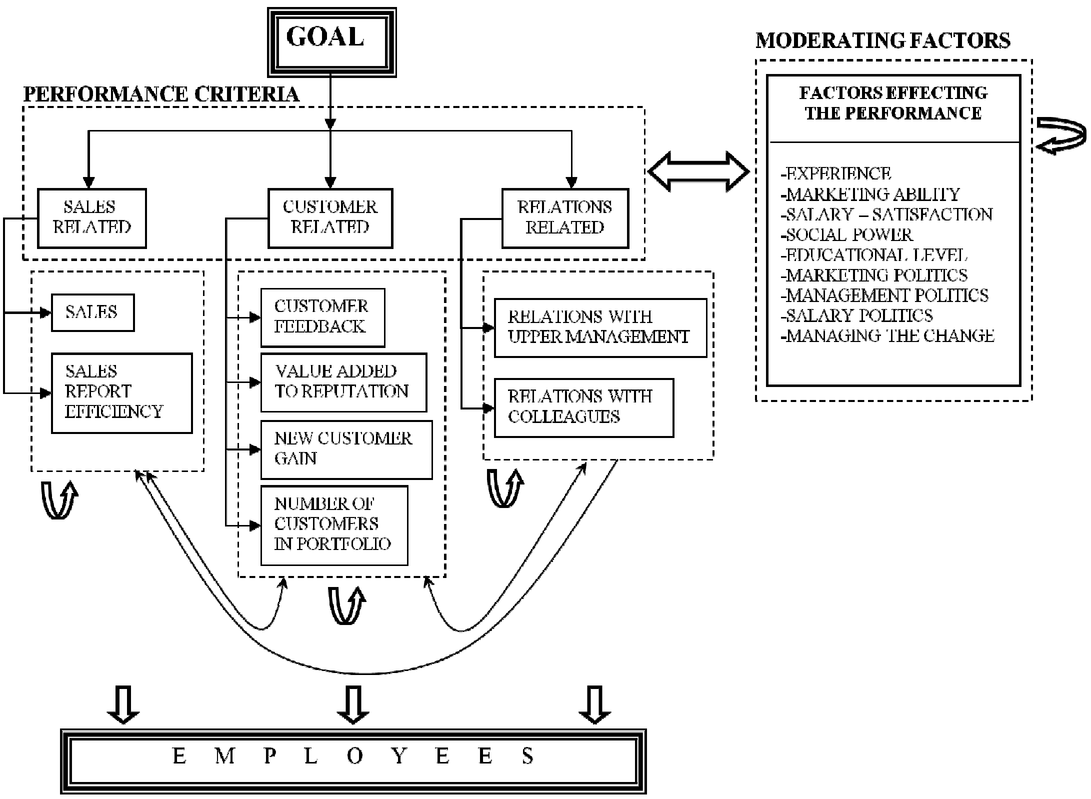

Figure 7.4 PE model for the marketing employees

7.3.4. Relations between Criteria

7.3.4.1. Preferential Rankings According to the DMs

Three DMs gave their preferential rankings for the model’s criteria and groups of criteria as follows:

$$DM_1 \rightarrow C1 \,\&\, C2 \succ^S C1 \,\&\, C3 \succ^M C1 \succ^{VW} C2 \,\&\, C3 \succ^W C2 \succ^M C3 \succ^S "0"$$

$$DM_2 \rightarrow C1 \,\&\, C2 \succ^S C1 \,\&\, C3 \succ^M C1 \succ^M C2 \,\&\, C3 \succ^{VW} C2 \succ^S C3 \succ^W "0"$$

$$DM_3 \rightarrow C1 \,\&\, C2 \succ^S C2 \,\&\, C3 \succ^M C2 \succ^M C1 \,\&\, C3 \succ^W C1 \succ^S C3 \succ^W "0"$$

Hence the situation rankings for each DM will be:

$$DM_1 \rightarrow S^{(1,1,0)} \succ^S S^{(1,0,1)} \succ^M S^{(1,0,0)} \succ^{VW} S^{(0,1,1)} \succ^W S^{(0,1,0)} \succ^M S^{(0,0,1)} \succ^S "0"$$

$$DM_2 \rightarrow S^{(1,1,0)} \succ^S S^{(1,0,1)} \succ^M S^{(1,0,0)} \succ^M S^{(0,1,1)} \succ^{VW} S^{(0,1,0)} \succ^S S^{(0,0,1)} \succ^W "0"$$

$$DM_3 \rightarrow S^{(1,1,0)} \succ^S S^{(0,1,1)} \succ^M S^{(0,1,0)} \succ^M S^{(1,0,1)} \succ^W S^{(1,0,0)} \succ^S S^{(0,0,1)} \succ^W "0"$$

7.3.4.2. Determination of Shapley Indices & Interactions

Weights and interaction values of the criteria for each DM will be calculated using the equations (4.27), (4.28) and (4.29) given in section (4.2.6.7). From the preferential rankings and strength values provided by the DMs in the previous section, the systems of equations, which can be analyzed in Appendix F, can be retrieved for each DM. Matrix operations for these systems of equations are also presented in the same appendix. Resolution of those matrix operations gives the criteria weights and the interaction between criteria as follows for each DM in Table 7.8.

Table 7.8 Criteria weights and interaction values

	v_1	v_2	v_3	I_{12}	I_{13}	I_{23}
DM_1	.528	.333	.139	0	-.056	-.111
DM_2	.583	.306	.111	.111	.056	-.056
DM_3	.343	.559	.098	.216	0	.039

7.3.4.3. Comments

From Table 7.8, it can be observed that SRPC is the most important criterion for DM_1 and DM_2 and CRPC is the most important criterion for DM_3. However, all three DMs have agreed on the least important criterion which is RRPC. For DM_2 and DM_3 an employee must be successful on both SRPC and CRPC in order for him/her to be considered successful but for DM_1 there is no interaction between these two criteria. For DM_1, the satisfaction of either one of SRPC and RRPC is enough in order to be successful but for DM_2 an employee will be successful if he/she satisfies both SRPC and RRPC. On the other hand there is no interaction between these two criteria for DM_3. And finally DM_1 and DM_2 agree on the sufficiency of the satisfaction of either CRPC or RRPC but DM_3 disagree with them by requiring a simultaneous satisfaction of these criteria.

7.3.5. Pairwise Comparisons

As in the application given in Section 7.2, for this model, pairwise comparisons have been made. Three DMs gave their opinion in order to fill the comparison matrices from which relative priority vectors would be extracted in order to build the supermatrix. The software called "SuperDecisions" has been used for that purpose.

7.3.5.1. Pairwise Comparisons for Sub-Criteria

Each node *i*, i.e. sub-criterion or a factor effecting the performance in Figure 7.4, that has an effect on another node *k* under consideration, is compared with other nodes belonging to the same cluster, i.e. criterion or moderating factors in Figure 7.4, as *i* and having an effect on the node *k* in consideration.

In the following table, one example has been shown. This is the comparison matrix filled by DM_3. Sales are modified with all the nodes within the cluster CRPC. Hence, those nodes will be compared in a pairwise manner between them.

After the calculation of the eigenvector of the comparison matrix, the consistency of the DM's judgments is analyzed and controlled whether the consistency ratio – CR is below the acceptable limit of 0.1. On the following example DM's judgments are quite consistent with a CR value of 0.02. The vector *w* on the right hand side of the table represents the relative priorities (weights) of the effects of CF, VAR, NCG and NCP on sales. This vector will be placed in the supermatrix in the column reserved for sales quantity on the four line block reserved for CRPC.

Table 7.9 Comparison matrix for CRPC with respect to Sales

S	CF	VAR	NCG	NCP	*w*
CF	-	5	4	3	.546
VAR	1/5	-	½	1/3	.084
NCG	1/4	2	-	½	.138
NCP	1/3	3	2	-	.232
				CR=	.02

7.3.5.2. Pairwise Comparisons for Clusters

Once the node comparisons are finished, in the same way, the clusters are compared. This time, the calculated eigenvectors will be used in order to find the weighted supermatrix because the supermatrix has to be stochastic in order to calculate the limit supermatrix.

Below, an example of a cluster comparison matrix is presented. All the clusters that are connected from the cluster of RRPC are presented in the matrix and they are compared in a pairwise manner in order to find the relative priority or weights of their effect on RRPC. Again the consistency of DM's judgments is controlled and with a CR value of 0.02 it is observed that his/her judgments are consistent enough.

Table 7.10 Comparison matrix for clusters with respect to RRPC

RRPC	SRPC	CRPC	RRPC	FMP	*w*
SRPC	-	1/3	2	1/5	.114
CRPC	3	-	3	½	.274
RRPC	1/2	1/3	-	1/7	.074
FMP	5	2	7	-	.538
				CR=	.02

7.3.5.3. Supermatrix Formation

After performing all the pairwise comparisons, supermatrices for all three DMs have been built. Unweighted, weighted and limit supermatrices for three DMs are presented in Appendix G. By normalizing the values for the sub-criteria taken from the limit matrices, the following table is formed:

Table 7.11 Sub-Criteria weights for DMs

		DECISION MAKERS		
		DM_1	***DM_2***	***DM_3***
SUB-CRITERIA	***S***	0,764	0,801	0,757
	SRE	0,236	0,199	0,243
	CF	0,251	0,587	0,353
	VAR	0,08	0,020	0,063
	NCG	0,271	0,193	0,381
	NCP	0,398	0,200	0,202
	RUP	0,779	0,934	0,931
	RC	0,221	0,066	0,069

7.3.5.4. Comments

As it can be observed in Table 7.11, for all three DMs, sales are way important than sales reports' efficiency/effectiveness and in the same way relations with upper management is more important than relations with colleagues. However, for the CRPC there are some differences in their judgments. Although they agree on the fact that the value added to reputation is the least important sub-criteria in this cluster, DM_1 thinks that number of customer in portfolio has slightly the leading importance after new customer gain and customer feedback which are practically equally important; DM_2 thinks that customer feedback, by taking the lead in importance, is remarkably more important than new customer gain and number of customer in portfolio which are practically equally important and finally DM_3 thinks that new customer gain and customer feedback are almost of equal importance and has the lead before number of customer in portfolio.

7.3.6. Performance Scores for the Employees

7.3.6.1. Employees' Performance Scores for Each Sub-Criterion

Four employees are going to be evaluated with respect to eight sub-criteria defined for the model (See Figure 7.4). Two of them belong to SRPC from which one, namely the sales quantity, is a quantitative criterion. Four of them belong to CRPC from which again one, namely the number of customer in portfolio, is quantitative. And last two belong to RRPC. For quantitative criteria, the data are collected from the official documents of the company. Sales quantity for E_1, E_2, E_3 and E_4 are respectively 12500, 15000, 10000 and 18000 TL for last three months and numbers of customers in their portfolio are respectively 14, 10, 12 and 18. High values for these criteria are preferred. Hence the employee with the highest number will be considered as the best performing employee for these criteria with a performance value of one. In other words, the normalization for these criteria will be made by the dividing the number of each employee to the highest number between them. On the other hand, for each of the qualitative ones, each DM gave his/her preferential rankings for the four employees under evaluation. The preferential rankings are as follows:

$$SRE \rightarrow \begin{cases} DM_1 \rightarrow Good \succ^{M} E_1 \succ^{VW} E_3 \succ^{W} E_4 \succ^{M} E_2 \succ^{M} Neutral \\ DM_2 \rightarrow Good \succ^{W} E_3 \succ^{VW} E_1 \succ^{M} E_2 \succ^{W} E_4 \succ^{W} Neutral \\ DM_3 \rightarrow Good \succ^{W} E_3 \succ^{VW} E_4 \succ^{VW} E_1 \succ^{M} E_2 \succ^{M} Neutral \end{cases}$$

$$CF \rightarrow \begin{cases} DM_1 \rightarrow Good \succ^{M} E_2 \succ^{VW} E_1 \approx E_3 \succ^{W} E_4 \succ^{M} Neutral \\ DM_2 \rightarrow Good \succ^{M} E_3 \succ^{W} E_1 \succ^{M} E_2 \succ^{W} E_4 \succ^{VW} Neutral \\ DM_3 \rightarrow Good \succ^{S} E_1 \approx E_3 \succ^{W} E_2 \succ^{VW} E_4 \succ^{M} Neutral \end{cases}$$

$$VAR \rightarrow \begin{cases} DM_1 \rightarrow Good \succ^{W} E_3 \succ^{M} E_4 \succ^{S} E_2 \approx E_1 \succ^{M} Neutral \\ DM_2 \rightarrow Good \succ^{W} E_3 \succ^{M} E_2 \succ^{VW} E_4 \succ^{W} E_1 \succ^{W} Neutral \\ DM_3 \rightarrow Good \succ^{M} E_2 \succ^{W} E_3 \succ^{M} E_4 \approx E_1 \succ^{S} Neutral \end{cases}$$

$$NCG \rightarrow \begin{cases} DM_1 \rightarrow Good \succ^{W} E_3 \succ^{M} E_1 \succ^{W} E_4 \succ^{W} E_2 \succ^{VW} Neutral \\ DM_2 \rightarrow Good \succ^{W} E_3 \succ^{M} E_1 \succ^{W} E_2 \succ^{VW} E_4 \succ^{W} Neutral \\ DM_3 \rightarrow Good \succ^{VW} E_1 \succ^{W} E_2 \succ^{M} E_3 \succ^{W} E_4 \succ^{M} Neutral \end{cases}$$

$$RUM \rightarrow \begin{cases} DM_1 \rightarrow Good \succ^{VW} E_4 \succ^{M} E_2 \succ^{S} E_3 \approx E_1 \succ^{W} Neutral \\ DM_2 \rightarrow Good \succ^{M} E_2 \succ^{VW} E_4 \succ^{S} E_1 \succ^{M} E_3 \succ^{M} Neutral \\ DM_3 \rightarrow Good \succ^{VW} E_4 \succ^{M} E_3 \succ^{M} E_2 \approx E_1 \succ^{W} Neutral \end{cases}$$

$$RC \rightarrow \begin{cases} DM_1 \rightarrow Good \succ^{W} E_1 \succ^{M} E_4 \succ^{W} E_2 \succ^{M} E_3 \succ^{M} Neutral \\ DM_2 \rightarrow Good \succ^{W} E_4 \succ^{M} E_1 \succ^{VW} E_3 \succ^{W} E_2 \succ^{S} Neutral \\ DM_3 \rightarrow Good \succ^{M} E_1 \succ^{VW} E_2 \succ^{W} E_4 \succ^{M} E_3 \succ^{S} Neutral \end{cases}$$

The equation systems deducted from those preferential rankings are collected and presented in Appendix H. The resulting performance values for the employees with respect to each sub-criterion are presented in Tables 7.12, 7.13 and 7.14 for DM_1, DM_2 and DM_3 respectively:

Table 7.12 Employees' individual performance values for DM_1

		SUB-CRITERIA							
		S	***SRE***	***CF***	***VAR***	***NCG***	***NCP***	***RUM***	***RC***
EMPLOYEES	1	0,6944	0,75	0,5556	0,25	0,5	0,7778	0,2	0,8462
	2	0,8333	0,25	0,6667	0,25	0,1	0,5556	0,6	0,4615
	3	0,5556	0,6667	0,5556	0,8333	0,8	0,6667	0,2	0,2308
	4	1	0,5	0,3333	0,5833	0,3	1	0,9	0,6154

Table 7.13 Employees' individual performance values for DM_2

		SUB-CRITERIA							
		S	***SRE***	***CF***	***VAR***	***NCG***	***NCP***	***RUM***	***RC***
EMPLOYEES	1	0,6944	0,7	0,5455	0,2	0,5	0,7778	0,4286	0,5833
	2	0,8333	0,4	0,2727	0,5	0,3	0,5556	0,7857	0,3333
	3	0,5556	0,8	0,7273	0,8	0,8	0,6667	0,2143	0,5
	4	1	0,2	0,8182	0,4	0,2	1	0,7143	0,8333

Table 7.14 Employees' individual performance values for DM_3

		SUB-CRITERIA							
		S	***SRE***	***CF***	***VAR***	***NCG***	***NCP***	***RUM***	***RC***
EMPLOYEES	1	0,6944	0,6	0,6	0,3333	0,9091	0,7778	0,2222	0,7692
	2	0,8333	0,3	0,4	0,75	0,7273	0,5556	0,2222	0,6923
	3	0,5556	0,8	0,6	0,5833	0,4545	0,6667	0,5556	0,3077
	4	1	0,7	0,3	0,3333	0,2727	1	0,8889	0,5385

7.3.6.2. Final Aggregation

Using the formula (4.18) or (4.26) with the values given in Tables 7.8, 7.10, 7.12 – 7.14, calculations of the final performance scores for the employees with respect to each DM is shown in Tables 7.15 – 7.17.

Table 7.15 Employees' final performance values for DM_1

<table>
<tr><th colspan="2">Criteria</th><th colspan="2">SRPC</th><th colspan="4">CRPC</th><th colspan="2">RRPC</th><th></th></tr>
<tr><th colspan="2">Criteria weights</th><td colspan="2">0,528</td><td colspan="4">0,333</td><td colspan="2">0,139</td><th>Final Score</th></tr>
<tr><th rowspan="4">EMPLOYEES</th><td>1</td><td colspan="2">0,707216</td><td colspan="4">0,604044</td><td colspan="2">0,3428102</td><td>0,647</td></tr>
<tr><td>2</td><td colspan="2">0,695412</td><td colspan="4">0,435063</td><td colspan="2">0,5693915</td><td>0,6022</td></tr>
<tr><td>3</td><td colspan="2">0,5821252</td><td colspan="4">0,68783</td><td colspan="2">0,2068068</td><td>0,6024</td></tr>
<tr><td>4</td><td colspan="2">0,882</td><td colspan="4">0,609289</td><td colspan="2">0,8371034</td><td>0,799</td></tr>
<tr><th colspan="2">Sub-criteria</th><th>S</th><th>SRE</th><th>CF</th><th>VAR</th><th>NCG</th><th>NCP</th><th>RUM</th><th>RC</th><td></td></tr>
<tr><th colspan="2">Sub-criteria weights</th><td>0,764</td><td>0,236</td><td>0,25</td><td>0,08</td><td>0,271</td><td>0,398</td><td>0,779</td><td>0,221</td><td></td></tr>
<tr><th rowspan="4">EMPLOYEES</th><td>1</td><td>0,694</td><td>0,75</td><td>0,5556</td><td>0,25</td><td>0,5</td><td>0,778</td><td>0,2</td><td>0,8462</td><td></td></tr>
<tr><td>2</td><td>0,833</td><td>0,25</td><td>0,6667</td><td>0,25</td><td>0,1</td><td>0,556</td><td>0,6</td><td>0,4615</td><td></td></tr>
<tr><td>3</td><td>0,556</td><td>0,6667</td><td>0,5556</td><td>0,8333</td><td>0,8</td><td>0,667</td><td>0,2</td><td>0,2308</td><td></td></tr>
<tr><td>4</td><td>1</td><td>0,5</td><td>0,3333</td><td>0,5833</td><td>0,3</td><td>1</td><td>0,9</td><td>0,6154</td><td></td></tr>
</table>

Table 7.16 Employees' final performance values for DM_2

Criteria		***SRPC***		***CRPC***				***RRPC***		
Criteria weights		***0,583***		***0,306***				***0,111***		**Final Score**
EMPLOYEES	1	0,6955144		0,5762685				0,4388102		***0,621***
	2	0,7470733		0,3390949				0,7558416		***0,612***
	3	0,6042356		0,7306651				0,2331562		***0,598***
	4	0,8408		0,7268834				0,722154		***0,783***
Sub-criteria		***S***	***SRE***	***CF***	***VAR***	***NCG***	***NCP***	***RUM***	***RC***	
Sub-criteria weights		0,801	0,199	0,587	0,02	0,193	0,2	0,934	0,066	
EMPLOYEES	1	0,6944	0,7	0,5455	0,2	0,5	0,7778	0,4286	0,5833	
	2	0,8333	0,4	0,2727	0,5	0,3	0,5556	0,7857	0,3333	
	3	0,5556	0,8	0,7273	0,8	0,8	0,6667	0,2143	0,5	
	4	1	0,2	0,8182	0,4	0,2	1	0,7143	0,8333	

Table 7.17 Employees' final performance values for DM_3

Criteria		***SRPC***		***CRPC***				***RRPC***		
Criteria weights		***0,343***		***0,559***				***0,098***		**Final Score**
EMPLOYEES	1	0,6714608		0,7366139				0,259943		***0,651***
	2	0,7037081		0,5785325				0,2546369		***0,570***
	3	0,6149892		0,5569691				0,5384949		***0,568***
	4	0,9271		0,4331299				0,8647224		***0,583***
Sub-criteria		***S***	***SRE***	***CF***	***VAR***	***NCG***	***NCP***	***RUM***	***RC***	
Sub-criteria weights		0,757	0,243	0,353	0,064	0,381	0,202	0,931	0,069	
EMPLOYEES	1	0,6944	0,6	0,6	0,3333	0,9091	0,7778	0,2222	0,7692	
	2	0,8333	0,3	0,4	0,75	0,7273	0,5556	0,2222	0,6923	
	3	0,5556	0,8	0,6	0,5833	0,4545	0,6667	0,5556	0,3077	
	4	1	0,7	0,3	0,3333	0,2727	1	0,8889	0,5385	

Hence the final rankings are as follows:

$$\begin{cases} DM_1 \Rightarrow E_4 \succ E_1 \succ E_3 \succ E_2 \\ DM_2 \Rightarrow E_4 \succ E_1 \succ E_2 \succ E_3 \\ DM_3 \Rightarrow E_1 \succ E_4 \succ E_2 \succ E_3 \end{cases}$$

7.3.7. Application of SCF – Group Decision

7.3.7.1. Social Choice Preference Order – SCPO with Condorcet's Function

In order to find the values of the function given in formula (5.1) for each candidate $\#(i : xP_i y)$ is counted for each candidate couple. Therefore:

$$\begin{array}{ll} \#(i : E_1 P_i E_2) = 3 & \#(i : E_2 P_i E_1) = 0 \\ \#(i : E_1 P_i E_3) = 3 & \#(i : E_3 P_i E_1) = 0 \\ \#(i : E_1 P_i E_4) = 1 & \#(i : E_4 P_i E_1) = 2 \\ \#(i : E_2 P_i E_3) = 2 & \#(i : E_3 P_i E_2) = 1 \\ \#(i : E_2 P_i E_4) = 0 & \#(i : E_4 P_i E_2) = 3 \\ \#(i : E_3 P_i E_4) = 0 & \#(i : E_4 P_i E_3) = 3 \end{array}$$

And the Condorcet Function values for the employees and their ranking according to Condorcet's function are as follows:

$$f_C(E_1) = 1 \quad f_C(E_2) = 0 \quad f_C(E_3) = 0 \quad f_C(E_4) = 2$$

$$E_4 \succ E_1 \succ E_2 \approx E_3$$

7.3.7.2. SCPO with Borda's Function

Using the function (5.2) and the numbers counted in the previous section, the Borda's function values for the employees and employees' ranking according to Borda's function are as follows:

$$f_B(E_1) = 7 \quad f_B(E_2) = 2 \quad f_B(E_3) = 1 \quad f_B(E_4) = 8$$

$$E_4 \succ E_1 \succ E_2 \succ E_3$$

7.3.7.3. SCPO with Copeland's Function

Using the function (5.3) and the numbers counted in the previous section, the Copeland's function values for the employees and employees' ranking according to Copeland's function are as follows:

$$f_{CP}(E_1) = 1 \quad f_{CP}(E_2) = -1 \quad f_{CP}(E_3) = -3 \quad f_{CP}(E_4) = 3$$

$$E_4 \succ E_1 \succ E_2 \succ E_3$$

7.3.7.4. Agreement and Disagreement – A&D

7.3.7.4.1. A&D with Cook and Seiford's Function

Using the formula (5.4), d_{jk} value for each employee – ranking pair is calculated and collected in the following matrix:

d_{jk}	**1**	**2**	**3**	**4**
E_1	2	1	4	7
E_2	6	3	2	3
E_3	8	5	2	1
E_4	2	3	4	7

Hungarian Algorithm is applied to this problem, and it is found out that the following ranking corresponding to the minimum distance of $d = 2 + 1 + 2 + 1 = 6$ will be chosen as the best ranking of the employees:

$$E_4 \succ E_1 \succ E_2 \succ E_3$$

7.3.7.4.2. A&D with Kemeny's Function

Using (5.5), L matrices are built for three rankings found with Condorcet, Borda and Copeland's functions. They are noted L_C, L_B and L_{CP} respectively and presented below:

$$L_C = \begin{bmatrix} 0 & 1 & 1 & -1 \\ -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad L_B = \begin{bmatrix} 0 & 1 & 1 & -1 \\ -1 & 0 & 1 & -1 \\ -1 & -1 & 0 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad L_{CP} = \begin{bmatrix} 0 & 1 & 1 & -1 \\ -1 & 0 & 1 & -1 \\ -1 & -1 & 0 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

On the other hand, the proportion matrix M defined in (5.6) and E matrix defined in Definition 5.4 will be as follows:

$$M = \begin{bmatrix} 1/2 & 1 & 1 & 1/3 \\ 0 & 1/2 & 2/3 & 0 \\ 0 & 1/3 & 1/2 & 0 \\ 2/3 & 1 & 1 & 1/2 \end{bmatrix} \qquad E = M - M^t = \begin{bmatrix} 0 & 1 & 1 & -1/3 \\ -1 & 0 & 1/3 & -1 \\ -1 & -1/3 & 0 & -1 \\ 1/3 & 1 & 1 & 0 \end{bmatrix}$$

Therefore, the simple inner product of E and L matrices are as follows:

$$< E, L_C >= 26/3 \quad < E, L_B >= 28/3 \quad < E, L_{CP} >= 28/3$$

7.3.8. Concluding Remarks

Note that the maximum inner product value that can be found out of any ranking combination of the employees is $28/3$ (the sum of the absolute values of the elements of E matrix) and it is achieved with the ranking found using the Copeland's and Borda's function.

This ranking was also the one found with Cook and Seiford's function. Hence the rankings found with Copeland's and Borda's function can be chosen as the best ones in order to represent the group decision of this company's three DMs.

8. CONCLUSION

At the beginning of this study, problems or gaps in human PE were cited. Concerns about existing methods were:

- Objectivity,
- Effectiveness,
- Simultaneous consideration of tangible and intangible factor and
- Considering relations between factors.

With those concerns, the aims were to:

- Fill the gap in human PE field and
- Therefore to attain the effective human PE,
- Bring a MCDM approach to HRPE,
- Higher level of objectivity,
- Handle the relations between different performance criteria and
- Present a valid HRPE model from an engineering point of view.

In the proposed MCDM approach, vital assumption of MCDM methods, i.e. criteria's independence, was not necessary, because proposed methods were able to handle the relations between criteria. The reason to choose those methods was the fact that they were able to handle different types of relations. By doing that, all kinds of relations would and could be covered without having the need to try constructing independent criteria or suppose that the criteria at hand are independent.

HRPE is most of the time a group process. Especially last decade's popular PE tool, namely the 360-Degree PE system, used by most of the companies in today's world is a group process with which an employee's performance will be evaluated by a group of people including his/her colleagues as well as his/her superiors. Hence, a method which will be able to answer to group decision making situations had to be proposed. Therefore, as a last new approach, SCFs are integrated to the proposed method in order to handle effectively the group decision making. By doing that, a clearly higher level of objectivity has been assured. Because, every DM would have his/her own preference ranking for the employees' performances but the final ranking can be different from each and every one of those rankings. A suitable ranking would be chosen to represent those individually found rankings. If possible, it can be a wise approach to take a group of DMs formed by a colleague, immediate superior and subordinate, department chief and customer in order to have different points of view about the employee's performance. The more different opinion collected from different types of relationships, the more objective the solution will get. Of course this should be done without attaching more importance to one DM than to another. In other words, for instance, a customer's importance of judgments has to be the same of that of a subordinate or a colleague.

Each approach proposed in this study can be used in order to make an efficient HRPE process. Nonetheless, as the methods handle different types of relations between criteria, the choice is not arbitrary. The model of the process has to be built with care and details. Then the decision of approach has to be made after a careful analysis. In Section 6, proposed methodology covers the steps of this procedure. Following the instructions will lead to the choice of the proper approach to do to the model at hand.

Then, in order to handle the interdependencies / interactions, the model will be evaluated using ANP or CI or with a combination of these two according to the nature of the relations existing between the criteria, the sub-criteria and the alternatives of the model. Once the results for each DM are obtained, the SCFs will be used in order to find the best suitable ranking of the employees to this group of DMs.

To conclude the study, the achievements and the originalities of the presented study can be underlined as follows:

- By applying ANP and CI:

    - A MCDM approach has been made to HRPE and therefore tangible and intangible data are handled simultaneously,
    - All possible types of relations between different factors of performance are dealt with,
    - Possible changes and erroneous final decisions are observed with ignorance of the inter-criteria relations,
    - A more efficient and effective way to measure human performance has been offered,

- By applying SCF:

    - A group decision making approach has been added and this brought a clearly higher level of objectivity
    - The ease of use and user friendliness of this approach is detailed during application,

One might point out that the methods sometimes can seem a little difficult. For instance, the comparison matrices and supermatrix operations of ANP and equation systems of CI can be a little frightening compared to simple PE question and answer procedures that are in use today in the companies. However SuperDecisions, mentioned earlier, for ANP is a very helpful and user friendly software which facilitates the application of ANP tremendously. On the other hand, there are softwares like MathLab or advanced calculators which can help solving the equation systems of CI. In anyway, it won't be bold to say that instead of making a PE with a very easy approach and finding erroneous results and making wrong decisions, it is better to make the correct decisions with correct results by applying a more difficult PE procedure.

Companies in which this approach has been tested are, as it can be seen in previous section, from different sectors. First application was in a law firm, the second was in a company from medical sector and the third one was from pharmaceutical sector. The second and third ones are obviously related but the application field was very different as in the second one the focus was on production however at the third it was on the marketing. These choices were made in order to show that the proposed method is applicable in different fields and sectors. All three companies were quite satisfied with the results of the application made on their employees. The members of management who also played the DM role in the applications pointed out following remarks:

- Relative priority / importance concept is way more efficient than individually evaluating the employees,
- It never occurred to them that ignoring inter-criteria relations can have that much effect on the final decision,
- SCF approach is very easy to use and useful tool when it comes to group decision making.
- It takes more time to apply the procedure but this can be easily affordable given the results. However, could be better to have software doing all those processes in once.

Therefore, future works after this study can be:

- Applying the approach in several companies and make them hear about, meet and know the method,
- Designing and building complete software for this new approach and method for human resources performance evaluation.

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APPENDIX – A : Litterature review for the last decade for HRM – HPE - PI

	Year	Reference	Subject
HPE	2000	[120]	Effect of thought suppressions on evaluations.
	2003	[121]	The challenges in identifying and investigating human performance problems.
	2004	[122]	Suggested ways that HR managers can incorporate research results into the design and ongoing evaluation and refinement of ratings and the goal-setting component of performance management systems
	2005	[123]	Self efficacy in human performance
	2006	[124]	PE of collaborative efforts, gaps, interventions.
	2006	[125]	PE of the leaders
	2006	[126]	Employee PI
	2008	[127]	Role of hope in performance
	2008	[128]	Ethnostatistical analysis of performance measurement
	2008	[129]	Performance of individuals from different disciplines forming a team that creates new forms of knowledge and innovative solutions.
	2008	[130]	Human performance
	2009	[131]	Comparing an employee to other employees rather than to use typical “absolute” rating standards.
	2010	[132]	Performance Management Systems in public sector
	2010	[133]	Effect of education on performance
	2010	[134]	Ethics in PE

	Year	Reference	Subject
PERFORMANCE IMPROVEMENT	2003	[135]	Examining the social responsibility of an organization as an ethical imperative in performance improvement:
	2003	[136]	More socially responsible HR practice
	2003	[137]	What's expected from PI professionals.
	2004	[138]	New Metrics for Employee Development
	2005	[139]	Effect of Leadership Style in performance Optimization:
	2006	[140]	Impact of level of perf. On feedback strategy
	2006	[141]	Explores some of the major issues in developing and implementing a competency-based HR development strategy.
	2007	[142]	PI assuming complexity
	2008	[143]	Does objective feedback improve performance?
	2009	[144]	Effectiveness of PI professionals
	2009	[145]	Aviod the negative effects of PI solutions
	2010	[146]	Explores the issues in the development and application of a competency model and provides implications for more precise integration of competencies into HR functions driving PI
	2010	[147]	Effective Professional Development
	2010	[148]	What's expected from PI professionals.
HRM	2004	[149]	HRM
	2009	[150]	HRM

APPENDIX – B : Equation Systems for the Choquet Integral Application

For DM_1

$$\begin{cases}
p_{Ag}^{(1,0,0,0,1)} - p_{Ag}^{(1,0,1,0,0)} = 0 = v_5 - v_3 - \frac{1}{2}\left[I_{13} + I_{25} + I_{45} - I_{15} - I_{23} - I_{34}\right] \\
p_{Ag}^{(1,0,1,0,0)} - p_{Ag}^{(0,0,1,0,1)} = 0 = v_1 - v_5 - \frac{1}{2}\left[I_{12} + I_{14} + I_{35} - I_{13} - I_{25} - I_{45}\right] \\
p_{Ag}^{(0,0,1,0,1)} - p_{Ag}^{(0,0,1,1,0)} = 5\alpha = v_5 - v_4 - \frac{1}{2}\left[I_{15} + I_{25} + I_{34} - I_{14} - I_{24} - I_{35}\right] \\
p_{Ag}^{(0,0,1,1,0)} - p_{Ag}^{(0,0,0,1,1)} = \alpha = v_3 - v_5 - \frac{1}{2}\left[I_{13} + I_{23} + I_{45} - I_{15} - I_{25} - I_{34}\right] \\
p_{Ag}^{(0,0,0,1,1)} - p_{Ag}^{(0,1,0,0,1)} = \alpha = v_4 - v_2 - \frac{1}{2}\left[I_{14} + I_{25} + I_{34} - I_{12} - I_{23} - I_{45}\right] \\
p_{Ag}^{(0,1,0,0,1)} - p_{Ag}^{(1,1,0,0,0)} = \alpha = v_5 - v_1 - \frac{1}{2}\left[I_{12} + I_{35} + I_{45} - I_{13} - I_{14} - I_{25}\right] \\
p_{Ag}^{(1,1,0,0,0)} - p_{Ag}^{(1,0,0,1,0)} = 2\alpha = v_2 - v_4 - \frac{1}{2}\left[I_{14} + I_{23} + I_{25} - I_{12} - I_{34} - I_{45}\right] \\
p_{Ag}^{(1,0,0,1,0)} - p_{Ag}^{(1,0,0,0,0)} = 3\alpha = v_4 - \frac{1}{2}\left[I_{24} + I_{34} + I_{45} - I_{14}\right] \\
p_{Ag}^{(1,0,0,0,0)} - p_{Ag}^{(0,1,1,0,0)} = 3\alpha = v_1 - v_2 - v_3 - \frac{1}{2}\left[I_{14} + I_{15} - I_{24} - I_{25} - I_{34} - I_{35}\right] \\
p_{Ag}^{(0,1,1,0,0)} - p_{Ag}^{(0,0,1,0,0)} = 3\alpha = v_2 - \frac{1}{2}\left[I_{12} + I_{24} + I_{25} - I_{23}\right] \\
p_{Ag}^{(0,0,1,0,0)} - p_{Ag}^{(0,0,0,0,1)} = 5\alpha = v_3 - v_5 - \frac{1}{2}\left[I_{13} + I_{23} + I_{34} - I_{15} - I_{25} - I_{45}\right] \\
p_{Ag}^{(0,0,0,0,1)} - p_{Ag}^{(0,1,0,1,0)} = 2\alpha = v_5 - v_2 - v_4 - \frac{1}{2}\left[I_{15} + I_{35} - I_{12} - I_{14} - I_{23} - I_{34}\right] \\
p_{Ag}^{(0,1,0,1,0)} - p_{Ag}^{(0,0,0,1,0)} = 2\alpha = v_2 - \frac{1}{2}\left[I_{12} + I_{23} + I_{25} - I_{24}\right] \\
p_{Ag}^{(0,0,0,1,0)} - p_{Ag}^{(0,1,0,0,0)} = 3\alpha = v_4 - v_2 - \frac{1}{2}\left[I_{14} + I_{34} + I_{45} - I_{12} - I_{23} - I_{25}\right] \\
p_{Ag}^{(0,1,0,0,0)} - p_{Ag}^{(0,0,0,0,0)} = 3\alpha = v_2 - \frac{1}{2}\left[I_{12} + I_{23} + I_{24} + I_{25}\right] \\
p_{Ag}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5
\end{cases}$$

For DM_2:

$$\begin{cases}
p_{Ag}^{(1,0,0,1,0)} - p_{Ag}^{(1,0,0,0,1)} = \alpha = v_4 - v_5 - \frac{1}{2}\left[I_{15} + I_{24} + I_{34} - I_{14} - I_{25} - I_{35}\right] \\
p_{Ag}^{(1,0,0,0,1)} - p_{Ag}^{(1,0,1,0,0)} = \alpha = v_5 - v_3 - \frac{1}{2}\left[I_{13} + I_{25} + I_{45} - I_{15} - I_{23} - I_{34}\right] \\
p_{Ag}^{(1,0,1,0,0)} - p_{Ag}^{(1,1,0,0,0)} = \alpha = v_3 - v_2 - \frac{1}{2}\left[I_{12} + I_{34} + I_{35} - I_{13} - I_{24} - I_{25}\right] \\
p_{Ag}^{(1,1,0,0,0)} - p_{Ag}^{(0,0,0,1,1)} = \alpha = v_1 + v_2 - v_4 - v_5 - \frac{1}{2}\left[I_{13} + I_{23} - I_{34} - I_{35}\right] \\
p_{Ag}^{(0,0,0,1,1)} - p_{Ag}^{(0,0,1,1,0)} = \alpha = v_5 - v_3 - \frac{1}{2}\left[I_{15} + I_{25} + I_{34} - I_{13} - I_{23} - I_{45}\right] \\
p_{Ag}^{(0,0,1,1,0)} - p_{Ag}^{(0,1,1,0,0)} = \alpha = v_4 - v_2 - \frac{1}{2}\left[I_{14} + I_{23} + I_{45} - I_{12} - I_{25} - I_{34}\right] \\
p_{Ag}^{(0,1,1,0,0)} - p_{Ag}^{(0,1,0,1,0)} = \alpha = v_3 - v_4 - \frac{1}{2}\left[I_{13} + I_{24} + I_{35} - I_{14} - I_{23} - I_{45}\right] \\
p_{Ag}^{(0,1,0,1,0)} - p_{Ag}^{(0,1,0,0,1)} = \alpha = v_4 - v_5 - \frac{1}{2}\left[I_{14} + I_{25} + I_{34} - I_{15} - I_{24} - I_{35}\right] \\
p_{Ag}^{(0,1,0,0,1)} - p_{Ag}^{(0,0,1,0,1)} = \alpha = v_2 - v_3 - \frac{1}{2}\left[I_{12} + I_{24} + I_{35} - I_{13} - I_{25} - I_{34}\right] \\
p_{Ag}^{(0,0,1,0,1)} - p_{Ag}^{(1,0,0,0,0)} = \alpha = v_3 + v_5 - v_1 - \frac{1}{2}\left[I_{23} + I_{25} + I_{34} + I_{45} - I_{12} - I_{14}\right] \\
p_{Ag}^{(1,0,0,0,0)} - p_{Ag}^{(0,0,0,1,0)} = \alpha = v_1 - v_4 - \frac{1}{2}\left[I_{12} + I_{13} + I_{15} - I_{24} - I_{34} - I_{45}\right] \\
p_{Ag}^{(0,0,0,1,0)} - p_{Ag}^{(0,0,0,0,1)} = \alpha = v_4 - v_5 - \frac{1}{2}\left[I_{14} + I_{24} + I_{34} - I_{15} - I_{25} - I_{35}\right] \\
p_{Ag}^{(0,0,0,0,1)} - p_{Ag}^{(0,0,1,0,0)} = \alpha = v_5 - v_3 - \frac{1}{2}\left[I_{15} + I_{25} + I_{45} - I_{13} - I_{23} - I_{34}\right] \\
p_{Ag}^{(0,0,1,0,0)} - p_{Ag}^{(0,1,0,0,0)} = \alpha = v_3 - v_2 - \frac{1}{2}\left[I_{13} + I_{34} + I_{35} - I_{12} - I_{24} - I_{25}\right] \\
p_{Ag}^{(0,1,0,0,0)} - p_{Ag}^{(0,0,0,0,0)} = 3\alpha = v_2 - \frac{1}{2}\left[I_{12} + I_{23} + I_{24} + I_{25}\right] \\
p_{Ag}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5
\end{cases}$$

For DM$_3$:

$$
\begin{cases}
p_{Ag}^{(0,0,1,0,1)} - p_{Ag}^{(0,0,1,1,0)} = 3\alpha = v_4 - v_5 - \frac{1}{2}\left[I_{15} + I_{25} + I_{34} - I_{14} - I_{24} - I_{35}\right] \\
p_{Ag}^{(0,0,1,1,0)} - p_{Ag}^{(0,1,1,0,0)} = 2\alpha = v_4 - v_2 - \frac{1}{2}\left[I_{14} + I_{23} + I_{45} - I_{12} - I_{25} - I_{34}\right] \\
p_{Ag}^{(0,1,1,0,0)} - p_{Ag}^{(1,1,0,0,0)} = 0 = v_3 - v_1 - \frac{1}{2}\left[I_{12} + I_{34} + I_{35} - I_{14} - I_{15} - I_{23}\right] \\
p_{Ag}^{(1,1,0,0,0)} - p_{Ag}^{(1,0,1,0,0)} = 0 = v_2 - v_3 - \frac{1}{2}\left[I_{13} + I_{24} + I_{25} - I_{12} - I_{34} - I_{35}\right] \\
p_{Ag}^{(1,0,1,0,0)} - p_{Ag}^{(0,0,0,1,1)} = 3\alpha = v_1 + v_3 - v_4 - v_5 - \frac{1}{2}\left[I_{12} + I_{23} - I_{24} - I_{25}\right] \\
p_{Ag}^{(0,0,0,1,1)} - p_{Ag}^{(0,1,0,0,1)} = 0 = v_4 - v_2 - \frac{1}{2}\left[I_{14} + I_{25} + I_{34} - I_{12} - I_{23} - I_{45}\right] \\
p_{Ag}^{(0,1,0,0,1)} - p_{Ag}^{(1,0,0,1,0)} = 2\alpha = v_2 + v_5 - v_1 - v_4 - \frac{1}{2}\left[I_{23} + I_{35} - I_{13} - I_{34}\right] \\
p_{Ag}^{(1,0,0,1,0)} - p_{Ag}^{(1,0,0,0,1)} = 0 = v_4 - v_5 - \frac{1}{2}\left[I_{15} + I_{24} + I_{34} - I_{14} - I_{25} - I_{35}\right] \\
p_{Ag}^{(1,0,0,0,1)} - p_{Ag}^{(0,1,0,1,0)} = 3\alpha = v_1 + v_5 - v_2 - v_4 - \frac{1}{2}\left[I_{13} + I_{35} - I_{23} - I_{34}\right] \\
p_{Ag}^{(0,1,0,1,0)} - p_{Ag}^{(0,0,1,0,0)} = 2\alpha = v_2 + v_4 - v_3 - \frac{1}{2}\left[I_{12} + I_{14} + I_{25} + I_{45} - I_{13} - I_{35}\right] \\
p_{Ag}^{(0,0,1,0,0)} - p_{Ag}^{(0,1,0,0,0)} = 4\alpha = v_3 - v_2 - \frac{1}{2}\left[I_{13} + I_{34} + I_{35} - I_{12} - I_{24} - I_{25}\right] \\
p_{Ag}^{(0,1,0,0,0)} - p_{Ag}^{(0,0,0,0,1)} = 0 = v_2 - v_5 - \frac{1}{2}\left[I_{12} + I_{23} + I_{24} - I_{15} - I_{35} - I_{45}\right] \\
p_{Ag}^{(0,0,0,0,1)} - p_{Ag}^{(1,0,0,0,0)} = 3\alpha = v_5 - v_1 - \frac{1}{2}\left[I_{25} + I_{35} + I_{45} - I_{12} - I_{13} - I_{14}\right] \\
p_{Ag}^{(1,0,0,0,0)} - p_{Ag}^{(0,0,0,1,0)} = 0 = v_1 - v_4 - \frac{1}{2}\left[I_{12} + I_{13} + I_{15} - I_{24} - I_{34} - I_{45}\right] \\
p_{Ag}^{(0,0,0,1,0)} - p_{Ag}^{(0,0,0,0,0)} = 3\alpha = v_4 - \frac{1}{2}\left[I_{14} + I_{24} + I_{34} + I_{45}\right] \\
p_{Ag}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5
\end{cases}
$$

APPENDIX – C : Matrix Operations for the Choquet Integral Application

For DM_1:

$$
\begin{bmatrix}
0 & 0 & -1 & 0 & 1 & 0 & -.5 & 0 & .5 & .5 & 0 & -.5 & .5 & 0 & -.5 & 0 \\
1 & 0 & 0 & 0 & -1 & -.5 & .5 & -.5 & 0 & 0 & 0 & .5 & 0 & -.5 & .5 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & .5 & -.5 & 0 & .5 & -.5 & -.5 & .5 & 0 & -5 \\
0 & 0 & 1 & 0 & -1 & 0 & -.5 & 0 & .5 & -.5 & 0 & .5 & .5 & 0 & -.5 & -1 \\
0 & -1 & 0 & 1 & 0 & .5 & 0 & -.5 & 0 & .5 & 0 & -.5 & -.5 & 0 & .5 & -1 \\
-1 & 0 & 0 & 0 & 1 & -.5 & .5 & .5 & 0 & 0 & 0 & .5 & 0 & -.5 & -.5 & -1 \\
0 & 1 & 0 & -1 & 0 & .5 & 0 & -.5 & 0 & -.5 & 0 & -.5 & .5 & 0 & .5 & -2 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & .5 & 0 & 0 & -.5 & 0 & -.5 & 0 & -.5 & -3 \\
1 & -1 & -1 & 0 & 0 & 0 & 0 & -.5 & -.5 & 0 & .5 & .5 & .5 & .5 & 0 & -3 \\
0 & 1 & 0 & 0 & 0 & -.5 & 0 & 0 & 0 & .5 & -.5 & -.5 & 0 & 0 & 0 & -3 \\
0 & 0 & 1 & 0 & -1 & 0 & -.5 & 0 & .5 & -.5 & 0 & .5 & -.5 & 0 & .5 & -5 \\
0 & -1 & 0 & -1 & 1 & .5 & 0 & .5 & -.5 & .5 & 0 & 0 & .5 & -.5 & 0 & -2 \\
0 & 1 & 0 & 0 & 0 & -.5 & 0 & 0 & 0 & -.5 & .5 & -.5 & 0 & 0 & 0 & -2 \\
0 & -1 & 0 & 1 & 0 & .5 & 0 & -.5 & 0 & .5 & 0 & .5 & -.5 & 0 & -.5 & -3 \\
0 & 1 & 0 & 0 & 0 & -.5 & 0 & 0 & 0 & -.5 & -.5 & -.5 & 0 & 0 & 0 & -3 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{12} \\ I_{13} \\ I_{14} \\ I_{15} \\ I_{23} \\ I_{24} \\ I_{25} \\ I_{34} \\ I_{35} \\ I_{45} \\ \alpha \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
$$

For DM_2:

$$
\begin{bmatrix}
0 & 0 & 0 & 1 & -1 & 0 & 0 & .5 & -.5 & 0 & -.5 & .5 & -.5 & .5 & 0 & -1 \\
0 & 0 & -1 & 0 & 1 & 0 & -.5 & 0 & .5 & .5 & 0 & -.5 & .5 & 0 & -.5 & -1 \\
0 & -1 & 1 & 0 & 0 & -.5 & .5 & 0 & 0 & 0 & .5 & .5 & -.5 & -.5 & 0 & -1 \\
1 & 1 & 0 & -1 & -1 & 0 & -.5 & 0 & 0 & -.5 & 0 & 0 & .5 & .5 & 0 & -1 \\
0 & 0 & -1 & 0 & 1 & 0 & .5 & 0 & -.5 & .5 & 0 & -.5 & -.5 & 0 & .5 & -1 \\
0 & -1 & 0 & 1 & 0 & .5 & 0 & -.5 & 0 & -.5 & 0 & .5 & .5 & 0 & -.5 & -1 \\
0 & 0 & 1 & -1 & 0 & 0 & -.5 & .5 & 0 & .5 & -.5 & 0 & 0 & -.5 & .5 & -1 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & -.5 & .5 & 0 & .5 & -.5 & -.5 & .5 & 0 & -1 \\
0 & 1 & -1 & 0 & 0 & -.5 & .5 & 0 & 0 & 0 & -.5 & .5 & .5 & -.5 & 0 & -1 \\
-1 & 0 & 1 & 0 & 1 & .5 & 0 & .5 & 0 & -.5 & 0 & -.5 & -.5 & 0 & -.5 & -1 \\
1 & 0 & 0 & -1 & 0 & -.5 & -.5 & 0 & -.5 & 0 & .5 & 0 & .5 & 0 & .5 & -1 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & -.5 & .5 & 0 & -.5 & .5 & -.5 & .5 & 0 & -1 \\
0 & 0 & -1 & 0 & 1 & 0 & .5 & 0 & -.5 & .5 & 0 & -.5 & .5 & 0 & -.5 & -1 \\
0 & -1 & 1 & 0 & 0 & .5 & -.5 & 0 & 0 & 0 & .5 & .5 & -.5 & -.5 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & -.5 & 0 & 0 & 0 & -.5 & -.5 & -.5 & 0 & 0 & 0 & -3 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{12} \\ I_{13} \\ I_{14} \\ I_{15} \\ I_{23} \\ I_{24} \\ I_{25} \\ I_{34} \\ I_{35} \\ I_{45} \\ \alpha \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
$$

For DM_3:

$$
\begin{bmatrix}
0 & 0 & 0 & 1 & -1 & 0 & 0 & .5 & -.5 & 0 & .5 & -.5 & -.5 & .5 & 0 & -3 \\
0 & -1 & 0 & 1 & 0 & .5 & 0 & -.5 & 0 & -.5 & 0 & .5 & .5 & 0 & -.5 & -2 \\
-1 & 0 & 1 & 0 & 0 & -.5 & 0 & .5 & .5 & .5 & 0 & 0 & -.5 & -.5 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & .5 & -.5 & 0 & 0 & 0 & -.5 & -.5 & .5 & .5 & 0 & 0 \\
1 & 0 & 1 & -1 & -1 & -.5 & 0 & 0 & 0 & -.5 & .5 & .5 & 0 & 0 & 0 & -3 \\
0 & -1 & 0 & 1 & 0 & .5 & 0 & -.5 & 0 & .5 & 0 & -.5 & -.5 & 0 & .5 & 0 \\
-1 & 1 & 0 & -1 & 1 & 0 & .5 & 0 & 0 & -.5 & 0 & 0 & .5 & -.5 & 0 & -2 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & .5 & -.5 & 0 & -.5 & .5 & -.5 & .5 & 0 & 0 \\
1 & -1 & 0 & -1 & 1 & 0 & -.5 & 0 & 0 & .5 & 0 & 0 & .5 & -.5 & 0 & -3 \\
0 & 1 & -1 & 1 & 0 & -.5 & .5 & -.5 & 0 & 0 & 0 & -.5 & 0 & .5 & -.5 & -2 \\
0 & -1 & 1 & 0 & 0 & .5 & -.5 & 0 & 0 & 0 & .5 & .5 & -.5 & -.5 & 0 & -4 \\
0 & 1 & 0 & 0 & -1 & -.5 & 0 & 0 & .5 & -.5 & -.5 & 0 & 0 & .5 & .5 & 0 \\
-1 & 0 & 0 & 0 & 1 & .5 & .5 & .5 & 0 & 0 & 0 & -.5 & 0 & -.5 & -.5 & -3 \\
1 & 0 & 0 & -1 & 0 & -.5 & -.5 & 0 & -.5 & 0 & .5 & 0 & .5 & 0 & .5 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -.5 & 0 & 0 & -.5 & 0 & -.5 & 0 & -.5 & -3 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ I_{12} \\ I_{13} \\ I_{14} \\ I_{15} \\ I_{23} \\ I_{24} \\ I_{25} \\ I_{34} \\ I_{35} \\ I_{45} \\ \alpha \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
$$

APPENDIX – D: Equation Systems for Employees' Individual Performances

For DM_1:

$$C_1 \Rightarrow \left.\begin{array}{l} 1-p^A=\alpha \\ p^A-p^C=0 \\ p^C-p^B=\alpha \\ p^B-p^E=4\alpha \\ p^E-p^D=\alpha \\ p^D-p^F=3\alpha \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^D=4\alpha \\ p^E=5\alpha \\ p^B=9\alpha \\ p^C=10\alpha \\ p^A=10\alpha \\ 1=11\alpha \Leftrightarrow \alpha=\dfrac{1}{11} \end{array}\right\} \begin{array}{l} p^A=10/11 \\ p^B=9/11 \\ p^C=10/11 \\ p^D=4/11 \\ p^E=5/11 \\ p^F=1/11 \end{array}$$

$$C_2 \Rightarrow \left.\begin{array}{l} 1-p^C=\alpha \\ p^C-p^D=0 \\ p^D-p^B=3\alpha \\ p^B-p^A=0 \\ p^A-p^E=5\alpha \\ p^E-p^F=0 \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^E=\alpha \\ p^A=6\alpha \\ p^B=6\alpha \\ p^D=9\alpha \\ p^C=9\alpha \\ 1=10\alpha \Leftrightarrow \alpha=\dfrac{1}{10} \end{array}\right\} \begin{array}{l} p^A=6/10 \\ p^B=6/10 \\ p^C=9/10 \\ p^D=9/10 \\ p^E=1/10 \\ p^F=1/10 \end{array}$$

$$C_3 \Rightarrow \left.\begin{array}{l} 1-p^A=\alpha \\ p^A-p^C=0 \\ p^C-p^B=\alpha \\ p^B-p^E=4\alpha \\ p^E-p^D=\alpha \\ p^D-p^F=3\alpha \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^D=4\alpha \\ p^E=5\alpha \\ p^B=9\alpha \\ p^C=10\alpha \\ p^A=10\alpha \\ 1=11\alpha \Leftrightarrow \alpha=\frac{1}{11} \end{array}\right\} \begin{array}{l} p^A={}^{10}\!/_{11} \\ p^B={}^{9}\!/_{11} \\ p^C={}^{10}\!/_{11} \\ p^D={}^{4}\!/_{11} \\ p^E={}^{5}\!/_{11} \\ p^F={}^{1}\!/_{11} \end{array}$$

$$C_4 \Rightarrow \left.\begin{array}{l} 1-p^C=\alpha \\ p^C-p^A=3\alpha \\ p^A-p^B=2\alpha \\ p^B-p^E=4\alpha \\ p^E-p^D=\alpha \\ p^D-p^F=3\alpha \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^D=4\alpha \\ p^E=5\alpha \\ p^B=9\alpha \\ p^A=11\alpha \\ p^C=14\alpha \\ 1=15\alpha \Leftrightarrow \alpha=\frac{1}{15} \end{array}\right\} \begin{array}{l} p^A={}^{11}\!/_{15} \\ p^B={}^{9}\!/_{15} \\ p^C={}^{14}\!/_{15} \\ p^D={}^{4}\!/_{15} \\ p^E={}^{5}\!/_{15} \\ p^F={}^{1}\!/_{15} \end{array}$$

$$C_5 \Rightarrow \left.\begin{array}{l} 1-p^C=\alpha \\ p^C-p^A=4\alpha \\ p^A-p^B=0 \\ p^B-p^D=3\alpha \\ p^D-p^E=\alpha \\ p^E-p^F=0 \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^E=\alpha \\ p^D=2\alpha \\ p^B=5\alpha \\ p^A=5\alpha \\ p^C=9\alpha \\ 1=10\alpha \Leftrightarrow \alpha=\frac{1}{10} \end{array}\right\} \begin{array}{l} p^A={}^{1}\!/_{2} \\ p^B={}^{1}\!/_{2} \\ p^C={}^{9}\!/_{10} \\ p^D={}^{1}\!/_{5} \\ p^E={}^{1}\!/_{10} \\ p^F={}^{1}\!/_{10} \end{array}$$

For DM_2:

$$C_1 \Rightarrow \left.\begin{array}{l} 1-p^C=\alpha \\ p^C-p^A=4\alpha \\ p^A-p^B=3\alpha \\ p^B-p^D=\alpha \\ p^D-p^F=4\alpha \\ p^F-p^E=\alpha \\ p^E-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^E=\alpha \\ p^F=2\alpha \\ p^D=6\alpha \\ p^B=7\alpha \\ p^A=10\alpha \\ p^C=14\alpha \\ 1=15\alpha \Leftrightarrow \alpha=\frac{1}{15} \end{array}\right\} \begin{array}{l} p^A={}^{10}\!/_{15} \\ p^B={}^{7}\!/_{15} \\ p^C={}^{14}\!/_{15} \\ p^D={}^{6}\!/_{15} \\ p^E={}^{1}\!/_{15} \\ p^F={}^{2}\!/_{15} \end{array}$$

$$C_2 \Rightarrow \left.\begin{array}{l} 1-p^A=2\alpha \\ p^A-p^B=0 \\ p^B-p^C=0 \\ p^C-p^D=0 \\ p^D-p^E=2\alpha \\ p^E-p^F=0 \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^E=\alpha \\ p^D=3\alpha \\ p^C=3\alpha \\ p^B=3\alpha \\ p^A=3\alpha \\ 1=5\alpha \Leftrightarrow \alpha=\frac{1}{5} \end{array}\right\} \begin{array}{l} p^A={}^{3}\!/_{5} \\ p^B={}^{3}\!/_{5} \\ p^C={}^{3}\!/_{5} \\ p^D={}^{3}\!/_{5} \\ p^E={}^{1}\!/_{5} \\ p^F={}^{1}\!/_{5} \end{array}$$

$$C_3 \Rightarrow \left.\begin{array}{l} 1-p^A=\alpha \\ p^A-p^C=0 \\ p^C-p^B=2\alpha \\ p^B-p^D=0 \\ p^D-p^E=4\alpha \\ p^E-p^F=0 \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^E=\alpha \\ p^D=5\alpha \\ p^B=5\alpha \\ p^C=7\alpha \\ p^A=7\alpha \\ 1=8\alpha \Leftrightarrow \alpha=\frac{1}{8} \end{array}\right\} \begin{array}{l} p^A={}^7\!/_8 \\ p^B={}^5\!/_8 \\ p^C={}^7\!/_8 \\ p^D={}^5\!/_8 \\ p^E={}^1\!/_8 \\ p^F={}^1\!/_8 \end{array}$$

$$C_4 \Rightarrow \left.\begin{array}{l} 1-p^A=\alpha \\ p^A-p^C=0 \\ p^C-p^B=\alpha \\ p^B-p^D=0 \\ p^D-p^E=3\alpha \\ p^E-p^F=0 \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^E=\alpha \\ p^D=4\alpha \\ p^B=4\alpha \\ p^C=5\alpha \\ p^A=5\alpha \\ 1=6\alpha \Leftrightarrow \alpha=\frac{1}{6} \end{array}\right\} \begin{array}{l} p^A={}^5\!/_6 \\ p^B={}^4\!/_6 \\ p^C={}^5\!/_6 \\ p^D={}^4\!/_6 \\ p^E={}^1\!/_6 \\ p^F={}^1\!/_6 \end{array}$$

$$C_5 \Rightarrow \left.\begin{array}{l} 1-p^A=2\alpha \\ p^A-p^B=0 \\ p^B-p^C=0 \\ p^C-p^D=2\alpha \\ p^D-p^E=4\alpha \\ p^E-p^F=0 \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^E=\alpha \\ p^D=5\alpha \\ p^C=7\alpha \\ p^B=7\alpha \\ p^A=7\alpha \\ 1=9\alpha \Leftrightarrow \alpha=\frac{1}{9} \end{array}\right\} \begin{array}{l} p^A={}^7\!/_9 \\ p^B={}^7\!/_9 \\ p^C={}^7\!/_9 \\ p^D={}^5\!/_9 \\ p^E={}^1\!/_9 \\ p^F={}^1\!/_9 \end{array}$$

For DM$_3$:

$$C_1 \Rightarrow \left.\begin{array}{l} 1-p^A=\alpha \\ p^A-p^B=0 \\ p^B-p^C=3\alpha \\ p^C-p^D=3\alpha \\ p^D-p^E=3\alpha \\ p^E-p^F=3\alpha \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^E=4\alpha \\ p^D=7\alpha \\ p^C=10\alpha \\ p^B=13\alpha \\ p^A=13\alpha \\ 1=14\alpha \Leftrightarrow \alpha=\frac{1}{14} \end{array}\right\} \begin{array}{l} p^A=13/14 \\ p^B=13/14 \\ p^C=10/14 \\ p^D=7/14 \\ p^E=4/14 \\ p^F=1/14 \end{array}$$

$$C_2 \Rightarrow \left.\begin{array}{l} 1-p^A=\alpha \\ p^A-p^B=3\alpha \\ p^B-p^E=3\alpha \\ p^E-p^D=3\alpha \\ p^D-p^C=3\alpha \\ p^C-p^F=3\alpha \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^C=4\alpha \\ p^D=7\alpha \\ p^E=10\alpha \\ p^B=13\alpha \\ p^A=16\alpha \\ 1=17\alpha \Leftrightarrow \alpha=\frac{1}{17} \end{array}\right\} \begin{array}{l} p^A=16/17 \\ p^B=13/17 \\ p^C=4/17 \\ p^D=7/17 \\ p^E=10/17 \\ p^F=1/17 \end{array}$$

$$C_3 \Rightarrow \left.\begin{array}{l} 1-p^A=\alpha \\ p^A-p^B=\alpha \\ p^B-p^C=\alpha \\ p^C-p^D=2\alpha \\ p^D-p^E=4\alpha \\ p^E-p^F=0 \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^E=\alpha \\ p^D=5\alpha \\ p^C=7\alpha \\ p^B=8\alpha \\ p^A=9\alpha \\ 1=10\alpha \Leftrightarrow \alpha=\dfrac{1}{10} \end{array}\right\} \begin{array}{l} p^A=9/10 \\ p^B=8/10 \\ p^C=7/10 \\ p^D=1/2 \\ p^E=1/10 \\ p^F=1/10 \end{array}$$

$$C_4 \Rightarrow \left.\begin{array}{l} 1-p^D=\alpha \\ p^D-p^A=2\alpha \\ p^A-p^B=2\alpha \\ p^B-p^E=0 \\ p^E-p^C=2\alpha \\ p^C-p^F=0 \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^C=\alpha \\ p^E=3\alpha \\ p^B=3\alpha \\ p^A=5\alpha \\ p^D=7\alpha \\ 1=8\alpha \Leftrightarrow \alpha=\dfrac{1}{8} \end{array}\right\} \begin{array}{l} p^A=5/8 \\ p^B=3/8 \\ p^C=1/8 \\ p^D=7/8 \\ p^E=3/8 \\ p^F=1/8 \end{array}$$

$$C_5 \Rightarrow \left.\begin{array}{l} 1-p^B=\alpha \\ p^B-p^A=2\alpha \\ p^A-p^C=0 \\ p^C-p^D=0 \\ p^D-p^E=2\alpha \\ p^E-p^F=0 \\ p^F-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^F=\alpha \\ p^E=\alpha \\ p^D=3\alpha \\ p^C=3\alpha \\ p^A=3\alpha \\ p^B=5\alpha \\ 1=6\alpha \Leftrightarrow \alpha=\dfrac{1}{6} \end{array}\right\} \begin{array}{l} p^A=1/2 \\ p^B=5/6 \\ p^C=1/2 \\ p^D=1/2 \\ p^E=1/6 \\ p^F=1/6 \end{array}$$

APPENDIX – E : Supermatrices

Unweighted Supermatrix

	1	2	3	4	5	C11	C12	C21	C22	C23	C24	C31	C32	C41	C42	C51	C52	C53	C54
1	0.00	0.00	0.00	0.00	0.00	0.05	0.06	0.22	0.13	0.15	0.25	0.43	0.35	0.10	0.06	0.47	0.13	0.43	0.54
2	0.00	0.00	0.00	0.00	0.00	0.43	0.42	0.11	0.25	0.30	0.13	0.26	0.27	0.22	0.44	0.10	0.35	0.25	0.21
3	0.00	0.00	0.00	0.00	0.00	0.21	0.26	0.22	0.25	0.15	0.25	0.14	0.11	0.06	0.18	0.18	0.11	0.09	0.04
4	0.00	0.00	0.00	0.00	0.00	0.10	0.10	0.22	0.13	0.30	0.25	0.06	0.20	0.45	0.16	0.05	0.05	0.14	0.12
5	0.00	0.00	0.00	0.00	0.00	0.20	0.16	0.22	0.25	0.10	0.13	0.11	0.07	0.17	0.16	0.19	0.35	0.09	0.08
C11	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C12	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C21	0.31	0.32	0.31	0.26	0.26	0.50	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C22	0.12	0.08	0.12	0.19	0.19	0.13	0.13	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C23	0.12	0.12	0.12	0.11	0.11	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C24	0.44	0.47	0.44	0.44	0.44	0.27	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C31	0.25	0.75	0.67	0.20	0.13	0.75	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C32	0.75	0.25	0.33	0.80	0.87	0.25	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
C41	0.00	0.00	0.00	0.00	0.00	0.75	0.25	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
C42	0.00	0.00	0.00	0.00	0.00	0.25	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C51	0.08	0.08	0.08	0.08	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
C52	0.39	0.39	0.39	0.39	0.39	0.00	0.00	1.00	0.00	1.00	1.00	0.00	0.65	0.00	0.00	0.00	0.00	1.00	0.00
C53	0.39	0.39	0.39	0.39	0.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.00	1.00	0.00	0.00
C54	0.14	0.14	0.14	0.14	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00	0.00	1.00	0.00	0.00	0.00

Weighted Supermatrix

	1	2	3	4	5	C11	C12	C21	C22	C23	C24	C31	C32	C41	C42	C51	C52	C53	C54
1	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.12	0.13	0.07	0.11	0.43	0.04	0.10	0.06	0.33	0.09	0.27	0.38
2	0.00	0.00	0.00	0.00	0.00	0.06	0.06	0.06	0.25	0.13	0.06	0.26	0.03	0.22	0.44	0.07	0.25	0.16	0.15
3	0.00	0.00	0.00	0.00	0.00	0.03	0.04	0.12	0.25	0.07	0.11	0.14	0.01	0.06	0.18	0.13	0.08	0.06	0.03
4	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.12	0.13	0.13	0.11	0.06	0.02	0.45	0.16	0.03	0.04	0.09	0.09
5	0.00	0.00	0.00	0.00	0.00	0.03	0.02	0.12	0.25	0.04	0.06	0.11	0.01	0.17	0.16	0.14	0.25	0.06	0.06
C11	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C12	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C21	0.08	0.08	0.08	0.07	0.07	0.12	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C22	0.03	0.02	0.03	0.05	0.05	0.03	0.03	0.00	0.00	0.17	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C23	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C24	0.11	0.12	0.11	0.11	0.11	0.06	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C31	0.16	0.48	0.42	0.13	0.08	0.37	0.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C32	0.48	0.16	0.21	0.51	0.56	0.12	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00
C41	0.00	0.00	0.00	0.00	0.00	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.32	0.00	0.00	0.00	0.00	0.00	0.00
C42	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C51	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29
C52	0.04	0.04	0.04	0.04	0.04	0.00	0.00	0.47	0.00	0.39	0.39	0.00	0.36	0.00	0.00	0.00	0.00	0.26	0.00
C53	0.04	0.04	0.04	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.29	0.00	0.00
C54	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.29	0.00	0.00	0.00

Limit Supermatrix

	1	2	3	4	5	C11	C12	C21	C22	C23	C24	C31	C32	C41	C42	C51	C52	C53	C54
1	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
2	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
3	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
4	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
5	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
C11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C21	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
C22	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
C23	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
C24	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
C31	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
C32	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
C41	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
C42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C51	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
C52	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
C53	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
C54	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03

APPENDIX – F: Equation Systems and Matrix Operations for the Hybrid Application

The system of equation for DM_1:

$$
\begin{cases}
p_{Ag}^{(1,1,0)} - p_{Ag}^{(1,0,1)} = 4\alpha = v_2 - v_3 - \frac{1}{2}\left[I_{13} - I_{12}\right] \\
p_{Ag}^{(1,0,1)} - p_{Ag}^{(1,0,0)} = 3\alpha = v_3 - \frac{1}{2}\left[I_{23} - I_{13}\right] \\
p_{Ag}^{(1,0,0)} - p_{Ag}^{(0,1,1)} = \alpha = 2v_1 - 1 \\
p_{Ag}^{(0,1,1)} - p_{Ag}^{(0,1,0)} = 2\alpha = v_3 - \frac{1}{2}\left[I_{13} - I_{23}\right] \\
p_{Ag}^{(0,1,0)} - p_{Ag}^{(0,0,1)} = 3\alpha = v_2 - v_3 - \frac{1}{2}\left[I_{12} - I_{13}\right] \\
p_{Ag}^{(0,0,1)} - p_{Ag}^{(0,0,0)} = 4\alpha = v_3 - \frac{1}{2}\left[I_{13} + I_{23}\right] \\
p_{Ag}^{(1,1,1)} = 1 = v_1 + v_2 + v_3
\end{cases}
$$

Therefore the matrix operation is as follows:

$$
\begin{bmatrix}
0 & 1 & -1 & .5 & -.5 & 0 & -4 \\
0 & 0 & 1 & 0 & .5 & -.5 & -3 \\
2 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & -.5 & .5 & -2 \\
0 & 1 & -1 & -.5 & .5 & 0 & -3 \\
0 & 0 & 1 & 0 & -.5 & -.5 & -4 \\
1 & 1 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix}
v_1 \\ v_2 \\ v_3 \\ I_{12} \\ I_{13} \\ I_{23} \\ \alpha
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1
\end{bmatrix}
$$

The system of equation for DM_2:

$$
\begin{cases}
p_{Ag}^{(1,1,0)} - p_{Ag}^{(0,1,1)} = 4\alpha = v_1 - v_3 - \frac{1}{2}[I_{23} - I_{12}] \\
p_{Ag}^{(0,1,1)} - p_{Ag}^{(0,1,0)} = 3\alpha = 1 - v_2 - v_1 - \frac{1}{2}[I_{13} - I_{23}] = v_3 - \frac{1}{2}[I_{13} - I_{23}] \\
p_{Ag}^{(0,1,0)} - p_{Ag}^{(1,0,1)} = 3\alpha = 2v_2 - 1 \\
p_{Ag}^{(1,0,1)} - p_{Ag}^{(1,0,0)} = 2\alpha = 1 - v_1 - v_2 - \frac{1}{2}[I_{23} - I_{13}] = v_3 - \frac{1}{2}[I_{23} - I_{13}] \\
p_{Ag}^{(1,0,0)} - p_{Ag}^{(0,0,1)} = 4\alpha = v_1 - v_3 - \frac{1}{2}[I_{12} - I_{23}] \\
p_{Ag}^{(0,0,1)} - p_{Ag}^{(0,0,0)} = 2\alpha = v_3 - \frac{1}{2}[I_{13} + I_{23}] \\
p_{Ag}^{(1,1,1)} = 1 = v_1 + v_2 + v_3
\end{cases}
$$

Therefore the matrix operation is as follows:

$$
\begin{bmatrix}
1 & 0 & -1 & .5 & 0 & -.5 & -4 \\
0 & 0 & 1 & 0 & -.5 & .5 & -3 \\
0 & 2 & 0 & 0 & 0 & 0 & -3 \\
0 & 0 & 1 & 0 & .5 & -.5 & -2 \\
1 & 0 & -1 & -.5 & 0 & .5 & -4 \\
0 & 0 & 1 & 0 & -.5 & -.5 & -2 \\
1 & 1 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix}
v_1 \\ v_2 \\ v_3 \\ I_{12} \\ I_{13} \\ I_{23} \\ \alpha
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1
\end{bmatrix}
$$

The system of equation for DM_3:

$$\begin{cases} p_{Ag}^{(1,1,0)} - p_{Ag}^{(0,1,0)} = 4\alpha = 1 - 2v_2 - \frac{1}{2}\left[I_{13} - I_{12}\right] \\ p_{Ag}^{(0,1,0)} - p_{Ag}^{(1,0,0)} = 3\alpha = v_2 - v_1 - \frac{1}{2}\left[I_{23} - I_{13}\right] \\ p_{Ag}^{(1,0,0)} - p_{Ag}^{(0,1,1)} = 2\alpha = 2v_1 - 1 \\ p_{Ag}^{(0,1,1)} - p_{Ag}^{(1,0,1)} = 3\alpha = v_2 - v_1 - \frac{1}{2}\left[I_{13} - I_{23}\right] \\ p_{Ag}^{(1,0,1)} - p_{Ag}^{(0,0,1)} = 4\alpha = 1 - v_2 - v_3 - \frac{1}{2}\left[I_{12} - I_{13}\right] = v_1 - \frac{1}{2}\left[I_{12} - I_{13}\right] \\ p_{Ag}^{(0,0,1)} - p_{Ag}^{(0,0,0)} = 3\alpha = v_3 - \frac{1}{2}\left[I_{13} + I_{23}\right] \\ p_{Ag}^{(1,1,1)} = 1 = v_1 + v_2 + v_3 \end{cases}$$

Therefore the matrix operation is as follows:

$$\begin{bmatrix} 0 & -2 & 0 & .5 & -.5 & 0 & -4 \\ -1 & 1 & 0 & 0 & .5 & -.5 & -3 \\ 2 & 0 & 0 & 0 & 0 & 0 & -2 \\ -1 & 1 & 0 & 0 & -.5 & .5 & -3 \\ 1 & 0 & 0 & -.5 & .5 & 0 & -4 \\ 0 & 0 & 1 & 0 & -.5 & -.5 & -3 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ I_{12} \\ I_{13} \\ I_{23} \\ \alpha \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

APPENDIX – G: Supermatrices

Unweighted Supermatrix for DM_1

	11	12	21	22	23	24	31	32	41	42	43	44	45	46	47	48	49
11	0	1	1	0,875	0,875	1	0,8	0	0	0	0,75	0	0	1	0	1	0
12	1	0	0	0,125	0,125	0	0,2	0	0	0	0,25	0	0	0	0	0	0
21	0,058	0,731	0	0,109	0,109	0,084	0,277	0,167	0	0	0,167	0	0	0,297	0	0	1
22	0,131	0	0	0	0,309	0,211	0,16	0,833	0	0,109	0,167	0	0	0	0	0	0
23	0,505	0,081	0	0,582	0	0,705	0,467	0	0,143	0,309	0,333	0,833	0	0,163	0	0	0
24	0,306	0,188	1	0,309	0,582	0	0,095	0	0,857	0,582	0,333	0,167	0	0,54	0	0	0
31	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
32	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
41	0,223	0,304	0,262	0,225	0,227	0,228	0,154	0,05	0	0,18	0,356	0,637	0	0	0	0	0
42	0,3	0	0,262	0,318	0,294	0,314	0,04	0,04	0,648	0,168	0,13	0,258	0	0	0	0	0
43	0,055	0,069	0,046	0,049	0,061	0,053	0,087	0,083	0	0	0	0	0,163	0	0	0	0
44	0,15	0	0,113	0,138	0,15	0,135	0,174	0,253	0,122	0,094	0,048	0	0	0	0	0	0
45	0,11	0,42	0,167	0,109	0,11	0,108	0,27	0,206	0,23	0,557	0,278	0,105	0	0	0	0	0
46	0,063	0	0,073	0,07	0,061	0,069	0,106	0,034	0	0	0	0	0	0	0	0,25	0
47	0,025	0,139	0,046	0,022	0,025	0,024	0,051	0,145	0	0	0,083	0	0,297	0,75	0	0,75	1
48	0,044	0,067	0	0,04	0,042	0,039	0,068	0,102	0	0	0,106	0	0	0	0	0	0
49	0,029	0	0,032	0,029	0,031	0,03	0,05	0,088	0	0	0	0	0,54	0,25	1	0	0

Weighted Supermatrix for DM_1

	11	12	21	22	23	24	31	32	41	42	43	44	45	46	47	48	49
11	0	0,258	0,085	0,074	0,063	0,085	0,058	0	0	0	0,068	0	0	0,182	0	0,427	0
12	0,258	0	0	0,011	0,009	0	0,015	0	0	0	0,023	0	0	0	0	0	0
21	0,006	0,077	0	0,036	0,03	0,027	0,048	0,031	0	0	0,048	0	0	0,17	0	0	0,702
22	0,014	0	0	0	0,085	0,069	0,027	0,153	0	0,077	0,048	0	0	0	0	0	0
23	0,053	0,008	0	0,19	0	0,23	0,079	0	0,1	0,217	0,096	0,585	0	0,094	0	0	0
24	0,032	0,02	0,326	0,101	0,16	0	0,016	0	0,602	0,408	0,096	0,117	0	0,31	0	0	0
31	0	0	0	0	0,155	0	0	0,307	0	0	0,5	0	0	0	0	0	0
32	0	0	0	0	0	0	0,284	0	0	0	0	0	0	0	0	0	0
41	0,142	0,194	0,154	0,133	0,113	0,135	0,073	0,025	0	0,054	0,043	0,19	0	0	0	0	0
42	0,191	0	0,154	0,187	0,147	0,185	0,019	0,021	0,193	0,05	0,016	0,077	0	0	0	0	0
43	0,035	0,044	0,027	0,029	0,03	0,031	0,041	0,042	0	0	0	0	0,163	0	0	0	0
44	0,095	0	0,066	0,081	0,075	0,079	0,082	0,129	0,036	0,028	0,006	0	0	0	0	0	0
45	0,07	0,267	0,099	0,064	0,055	0,064	0,128	0,105	0,068	0,166	0,034	0,031	0	0	0	0	0
46	0,04	0	0,043	0,041	0,03	0,041	0,05	0,017	0	0	0	0	0	0	0	0,143	0
47	0,016	0,089	0,027	0,013	0,012	0,014	0,024	0,074	0	0	0,01	0	0,297	0,183	0	0,43	0,298
48	0,028	0,043	0	0,024	0,021	0,023	0,032	0,052	0	0	0,013	0	0	0	0	0	0
49	0,018	0	0,019	0,017	0,016	0,018	0,024	0,045	0	0	0	0	0,54	0,061	1	0	0

Limit Supermatrix for DM_1

	11	12	21	22	23	24	31	32	41	42	43	44	45	46	47	48	49
11	0,045	0,045	0,045	0,045	0,045	0,045	0,045	0,045	0,045	0,045	0,045	0,045	0,045	0,045	0,045	0,045	0,045
12	0,014	0,014	0,014	0,014	0,014	0,014	0,014	0,014	0,014	0,014	0,014	0,014	0,014	0,014	0,014	0,014	0,014
21	0,096	0,096	0,096	0,096	0,096	0,096	0,096	0,096	0,096	0,096	0,096	0,096	0,096	0,096	0,096	0,096	0,096
22	0,031	0,031	0,031	0,031	0,031	0,031	0,031	0,031	0,031	0,031	0,031	0,031	0,031	0,031	0,031	0,031	0,031
23	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,104	0,104
24	0,152	0,152	0,152	0,152	0,152	0,152	0,152	0,152	0,152	0,152	0,152	0,152	0,152	0,152	0,152	0,152	0,152
31	0,032	0,032	0,032	0,032	0,032	0,032	0,032	0,032	0,032	0,032	0,032	0,032	0,032	0,032	0,032	0,032	0,032
32	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009	0,009
41	0,077	0,077	0,077	0,077	0,077	0,077	0,077	0,077	0,077	0,077	0,077	0,077	0,077	0,077	0,077	0,077	0,077
42	0,097	0,097	0,097	0,097	0,097	0,097	0,097	0,097	0,097	0,097	0,097	0,097	0,097	0,097	0,097	0,097	0,097
43	0,025	0,025	0,025	0,025	0,025	0,025	0,025	0,025	0,025	0,025	0,025	0,025	0,025	0,025	0,025	0,025	0,025
44	0,042	0,042	0,042	0,042	0,042	0,042	0,042	0,042	0,042	0,042	0,042	0,042	0,042	0,042	0,042	0,042	0,042
45	0,062	0,062	0,062	0,062	0,062	0,062	0,062	0,062	0,062	0,062	0,062	0,062	0,062	0,062	0,062	0,062	0,062
46	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02
47	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,07
48	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
49	0,114	0,114	0,114	0,114	0,114	0,114	0,114	0,114	0,114	0,114	0,114	0,114	0,114	0,114	0,114	0,114	0,114

Unweighted Supermatrix for DM_2

	11	12	21	22	23	24	31	32	41	42	43	44	45	46	47	48	49
11	0	1	1	0,833	0,167	1	0,833	0	0	0	0,75	0	0	1	0	1	0
12	1	0	0	0,167	0,833	0	0,167	0	0	0	0,25	0	0	0	0	0	0
21	0,553	0,637	0	0,637	0,637	0,637	0,262	0,833	0	0	0,2	0	0	0,648	0	0	1
22	0,04	0	0	0	0,105	0,105	0,055	0,167	0	0,109	0,2	0	0	0	0	0	0
23	0,103	0,105	0	0,258	0	0,258	0,118	0	0,75	0,582	0,522	0,75	0	0,23	0	0	0
24	0,304	0,258	1	0,105	0,258	0	0,565	0	0,25	0,309	0,078	0,25	0	0,122	0	0	0
31	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
32	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
41	0,079	0,224	0,298	0,257	0,222	0,153	0,051	0,191	0	0,649	0,062	0,627	0	0	0	0	0
42	0,144	0	0,191	0,156	0,176	0,191	0,023	0,04	0,072	0	0,086	0,094	0	0	0	0	0
43	0,04	0,114	0,025	0,027	0,023	0,042	0,079	0,04	0	0	0	0	0,122	0	0	0	0
44	0,079	0,042	0,063	0,063	0,127	0,118	0,164	0,318	0,279	0,279	0,176	0	0	0	0	0	0
45	0,072	0,422	0,109	0,077	0,041	0,027	0,108	0,147	0,649	0,072	0,035	0,28	0	0	0	0	0
46	0,27	0	0,141	0,157	0,146	0,145	0,035	0,035	0	0	0	0	0	0	0	0,25	0
47	0,044	0,115	0,05	0,058	0,041	0,049	0,215	0,08	0	0	0,235	0	0,23	0,167	0	0,75	1
48	0,026	0,083	0	0	0,03	0,027	0,133	0,085	0	0	0,405	0	0	0	0	0	0
49	0,247	0	0,122	0,205	0,195	0,248	0,191	0,064	0	0	0	0	0,648	0,833	1	0	0

Weighted Supermatrix for DM_2

	11	12	21	22	23	24	31	32	41	42	43	44	45	46	47	48	49
11	0	0	0	0,234	0,045	0	0,164	0	0	0	0,094	0	0	0	0	0	0
12	0	0	0	0,047	0,224	0	0,033	0	0	0	0,031	0	0	0	0	0	0
21	0,327	0,377	0	0,079	0,076	0,079	0,031	0,123	0	0	0,058	0	0	0,211	0	0	0
22	0,024	0	0	0	0,012	0,013	0,007	0,025	0	0,041	0,058	0	0	0	0	0	0
23	0,061	0,062	0	0,032	0	0,032	0,014	0	0,283	0,22	0,153	0,283	0	0,075	0	0	0
24	0,18	0,153	0	0,013	0,031	0	0,067	0	0,094	0,117	0,023	0,099	0	0,04	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0.10153	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0,026	0,075	0,177	0,153	0,126	0,091	0,031	0,146	0	0,404	0,03	0,39	0	0	0	0	0
42	0,048	0	0,114	0,093	0,1	0,114	0,014	0,03	0,045	0	0,042	0,058	0	0	0	0	0
43	0,013	0,038	0,015	0,016	0,013	0,025	0,049	0,031	0	0	0	0	0,122	0	0	0	0
44	0,026	0,014	0,038	0,038	0,072	0,07	0,101	0,243	0,174	0,174	0,085	0	0	0	0	0	0
45	0,024	0,14	0,065	0,046	0,023	0,016	0,067	0,113	0,404	0,045	0,017	0,174	0	0	0	0	0
46	0,09	0	0,084	0,093	0,083	0,086	0,022	0,027	0	0	0	0	0	0	0	0,198	0
47	0,015	0,038	0,03	0,034	0,024	0,029	0,132	0,061	0	0	0,113	0	0,23	0,089	0	0,595	1
48	0,009	0,028	0	0	0,017	0,016	0,081	0,065	0	0	0,195	0	0	0	0	0	0
49	0,082	0	0,073	0,122	0,111	0,148	0,118	0,049	0	0	0	0	0,648	0,446	1	0	0

Limit Supermatrix for DM_2

	11	12	21	22	23	24	31	32	41	42	43	44	45	46	47	48	49
11	0,064	0	0	0,064	0,064	0	0,064	0,064	0,064	0,064	0,064	0,064	0,064	0	0,064	0	0,064
12	0	0,016	0,016	0,016	0,016	0,016	0,016	0,016	0,016	0,016	0,016	0,016	0,016	0,016	0,016	0,016	0,016
21	0,14	0,14	0,14	0,14	0,14	0,14	0,14	0,14	0,14	0,14	0,14	0,14	0,14	0,14	0,14	0,14	0
22	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005
23	0,046	0,046	0,046	0,046	0,046	0,046	0,046	0,046	0,046	0,046	0,046	0,046	0,046	0,046	0,046	0,046	0,046
24	0,048	0,048	0	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048
31	0,003	0,003	0,003	0,003	0	0,003	0,003	0	0,003	0,003	0	0,003	0,003	0,003	0,003	0,003	0,003
32	2E-04	2E-04	2E-04	2E-04	2E-04	2E-04	0	2E-04	2E-04	2E-04	2E-04	2E-04	2E-04	2E-04	2E-04	2E-04	2E-04
41	0,066	0,066	0,066	0,066	0,066	0,066	0,066	0,066	0,066	0,066	0,066	0,066	0,066	0,066	0,066	0,066	0,066
42	0,035	0,035	0,035	0,035	0,035	0,035	0,035	0,035	0,035	0,035	0,035	0,035	0,035	0,035	0,035	0,035	0,035
43	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012
44	0,033	0,033	0,033	0,033	0,033	0,033	0,033	0,033	0,033	0,033	0,033	0,033	0,033	0,033	0,033	0,033	0,033
45	0,049	0,049	0,049	0,049	0,049	0,049	0,049	0,049	0,049	0,049	0,049	0,049	0,049	0,049	0,049	0,049	0,049
46	0,027	0,027	0,027	0,027	0,027	0,027	0,027	0,027	0,027	0,027	0,027	0,027	0,027	0,027	0,027	0,027	0,027
47	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0,19	0
48	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005
49	0,262	0,262	0,262	0,262	0,262	0,262	0,262	0,262	0,262	0,262	0,262	0,262	0,262	0,262	0	0,262	0,262

Unweighted Supermatrix for DM_3

	11	12	21	22	23	24	31	32	41	42	43	44	45	46	47	48	49
11	0	1	1	0,75	0,333	1	0,75	0	0	0	0,75	0	0	1	0	1	0
12	1	0	0	0,25	0,667	0	0,25	0	0	0	0,25	0	0	0	0	0	0
21	0,546	0,54	0	0,637	0,731	0,731	0,574	0,75	0	0	0,546	0	0	0,714	0	0	1
22	0,084	0	0	0	0,188	0,188	0,108	0,25	0	0,23	0,138	0	0	0	0	0	0
23	0,138	0,163	0	0,258	0	0,081	0,253	0	0,75	0,648	0,232	0,667	0	0,143	0	0	0
24	0,232	0,297	1	0,105	0,081	0	0,065	0	0,25	0,122	0,084	0,333	0	0,143	0	0	0
31	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
32	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
41	0,166	0,435	0,158	0,09	0,14	0,155	0,025	0,042	0	0,54	0,143	0,648	0	0	0	0	0
42	0,263	0	0,303	0,061	0,21	0,222	0,025	0,029	0,54	0	0,049	0,23	0	0	0	0	0
43	0,038	0,148	0,03	0,042	0,03	0,025	0,106	0,021	0	0	0	0	0,109	0	0	0	0
44	0,252	0	0,258	0,344	0,302	0,311	0,221	0,065	0,297	0,297	0,049	0	0	0	0	0	0
45	0,081	0,281	0,112	0,245	0,14	0,074	0,149	0,065	0,163	0,163	0,085	0,122	0	0	0	0	0
46	0,079	0	0,046	0,168	0,051	0,05	0,038	0,218	0	0	0	0	0	0	0	0,25	0
47	0,021	0,046	0,023	0,022	0,019	0,035	0,309	0,309	0	0	0,232	0	0,309	0,833	0	0,75	1
48	0,029	0,09	0	0	0,027	0,019	0,067	0,15	0	0	0,441	0	0	0	0	0	0
49	0,071	0	0,07	0,03	0,08	0,11	0,061	0,101	0	0	0	0	0,582	0,167	1	0	0

Weighted Supermatrix for DM_3

	11	12	21	22	23	24	31	32	41	42	43	44	45	46	47	48	49
11	0	0,105	0,075	0,056	0,018	0,075	0,086	0	0	0	0,157	0	0	0,235	0	0,657	0
12	0,105	0	0	0,019	0,037	0	0,029	0	0	0	0,052	0	0	0	0	0	0
21	0,141	0,139	0	0,101	0,086	0,116	0,157	0,232	0	0	0,313	0	0	0,459	0	0	0,84
22	0,022	0	0	0	0,022	0,03	0,03	0,077	0	0,192	0,079	0	0	0	0	0	0
23	0,036	0,042	0	0,041	0	0,013	0,069	0	0,63	0,544	0,133	0,56	0	0,092	0	0	0
24	0,06	0,077	0,159	0,017	0,01	0	0,018	0	0,21	0,102	0,048	0,28	0	0,092	0	0	0
31	0	0	0	0	0,262	0	0	0,084	0	0	0,109	0	0	0	0	0	0
32	0	0	0	0	0	0	0,074	0	0	0	0	0	0	0	0	0	0
41	0,106	0,277	0,121	0,069	0,079	0,119	0,014	0,026	0	0,087	0,016	0,104	0	0	0	0	0
42	0,168	0	0,232	0,047	0,119	0,17	0,014	0,018	0,087	0	0,005	0,037	0	0	0	0	0
43	0,024	0,094	0,023	0,032	0,017	0,019	0,057	0,013	0	0	0	0	0,109	0	0	0	0
44	0,16	0	0,197	0,263	0,17	0,238	0,119	0,039	0,048	0,048	0,005	0	0	0	0	0	0
45	0,051	0,179	0,086	0,187	0,079	0,056	0,08	0,039	0,026	0,026	0,009	0,02	0	0	0	0	0
46	0,05	0	0,036	0,129	0,029	0,038	0,02	0,132	0	0	0	0	0	0	0	0,086	0
47	0,014	0,029	0,017	0,017	0,011	0,027	0,166	0,188	0	0	0,025	0	0,309	0,102	0	0,257	0,16
48	0,018	0,057	0	0	0,016	0,014	0,036	0,091	0	0	0,048	0	0	0	0	0	0
49	0,045	0	0,054	0,023	0,045	0,085	0,033	0,061	0	0	0	0	0,582	0,02	1	0	0

Limit Supermatrix for DM_3

	11	12	21	22	23	24	31	32	41	42	43	44	45	46	47	48	49
11	0,039	0,039	0,039	0,039	0,039	0,039	0,039	0,039	0,039	0,039	0,039	0,039	0,039	0,039	0,039	0,039	0,039
12	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012
21	0,143	0,143	0,143	0,143	0,143	0,143	0,143	0,143	0,143	0,143	0,143	0,143	0,143	0,143	0,143	0,143	0,143
22	0,026	0,026	0,026	0,026	0,026	0,026	0,026	0,026	0,026	0,026	0,026	0,026	0,026	0,026	0,026	0,026	0,026
23	0,154	0,154	0,154	0,154	0,154	0,154	0,154	0,154	0,154	0,154	0,154	0,154	0,154	0,154	0,154	0,154	0,154
24	0,082	0,082	0,082	0,082	0,082	0,082	0,082	0,082	0,082	0,082	0,082	0,082	0,082	0,082	0,082	0,082	0,082
31	0,043	0,043	0,043	0,043	0,043	0,043	0,043	0,043	0,043	0,043	0,043	0,043	0,043	0,043	0,043	0,043	0,043
32	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003
41	0,067	0,067	0,067	0,067	0,067	0,067	0,067	0,067	0,067	0,067	0,067	0,067	0,067	0,067	0,067	0,067	0,067
42	0,083	0,083	0,083	0,083	0,083	0,083	0,083	0,083	0,083	0,083	0,083	0,083	0,083	0,083	0,083	0,083	0,083
43	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018	0,018
44	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
45	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048	0,048
46	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02
47	0,051	0,051	0,051	0,051	0,051	0,051	0,051	0,051	0,051	0,051	0,051	0,051	0,051	0,051	0,051	0,051	0,051
48	0,008	0,008	0,008	0,008	0,008	0,008	0,008	0,008	0,008	0,008	0,008	0,008	0,008	0,008	0,008	0,008	0,008
49	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,105	0,105

APPENDIX – H: Equation Systems for Employees' Individual Performances

The systems of equation and resolutions for DM_1:

$$SRE \Rightarrow \left.\begin{array}{l} 1-p^1=3\alpha \\ p^1-p^3=\alpha \\ p^3-p^4=2\alpha \\ p^4-p^2=3\alpha \\ p^2-0=3\alpha \end{array}\right\} \left.\begin{array}{l} p^2=3\alpha \\ p^4=6\alpha \\ p^3=8\alpha \\ p^1=9\alpha \\ 1=12\alpha \Leftrightarrow \alpha=\dfrac{1}{12} \end{array}\right\} \begin{array}{l} p^1=3/4 \\ p^2=1/4 \\ p^3=2/3 \\ p^4=1/2 \end{array}$$

$$CF \Rightarrow \left.\begin{array}{l} 1-p^2=3\alpha \\ p^2-p^1=\alpha \\ p^1-p^3=0 \\ p^3-p^4=2\alpha \\ p^4-0=3\alpha \end{array}\right\} \left.\begin{array}{l} p^4=3\alpha \\ p^3=5\alpha \\ p^1=5\alpha \\ p^2=6\alpha \\ 1=9\alpha \Leftrightarrow \alpha=\dfrac{1}{9} \end{array}\right\} \begin{array}{l} p^1=5/9 \\ p^2=2/3 \\ p^3=5/9 \\ p^4=1/3 \end{array}$$

$$VAR \Rightarrow \left.\begin{array}{l} 1-p^3=2\alpha \\ p^3-p^4=3\alpha \\ p^4-p^2=4\alpha \\ p^2-p^1=0 \\ p^1-0=3\alpha \end{array}\right\} \left.\begin{array}{l} p^1=3\alpha \\ p^2=3\alpha \\ p^4=7\alpha \\ p^3=10\alpha \\ 1=12\alpha \Leftrightarrow \alpha=\frac{1}{12} \end{array}\right\} \begin{array}{l} p^1=1/4 \\ p^2=1/4 \\ p^3=5/6 \\ p^4=7/12 \end{array}$$

$$NCG \Rightarrow \left.\begin{array}{l} 1-p^3=2\alpha \\ p^3-p^1=3\alpha \\ p^1-p^4=2\alpha \\ p^4-p^2=2\alpha \\ p^2-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^2=\alpha \\ p^4=3\alpha \\ p^1=5\alpha \\ p^3=8\alpha \\ 1=10\alpha \Leftrightarrow \alpha=\frac{1}{10} \end{array}\right\} \begin{array}{l} p^1=1/2 \\ p^2=1/10 \\ p^3=4/5 \\ p^4=3/10 \end{array}$$

$$RUM \Rightarrow \left.\begin{array}{l} 1-p^4=\alpha \\ p^4-p^2=3\alpha \\ p^2-p^3=4\alpha \\ p^3-p^1=0 \\ p^1-0=2\alpha \end{array}\right\} \left.\begin{array}{l} p^1=2\alpha \\ p^3=2\alpha \\ p^2=6\alpha \\ p^4=9\alpha \\ 1=10\alpha \Leftrightarrow \alpha=\frac{1}{10} \end{array}\right\} \begin{array}{l} p^1=1/5 \\ p^2=3/5 \\ p^3=1/5 \\ p^4=9/10 \end{array}$$

$$RC \Rightarrow \left.\begin{array}{l} 1-p^1=2\alpha \\ p^1-p^4=3\alpha \\ p^4-p^2=2\alpha \\ p^2-p^3=3\alpha \\ p^3-0=3\alpha \end{array}\right\} \left.\begin{array}{l} p^3=3\alpha \\ p^2=6\alpha \\ p^4=8\alpha \\ p^1=11\alpha \\ 1=13\alpha \Leftrightarrow \alpha=\frac{1}{13} \end{array}\right\} \begin{array}{l} p^1=11/13 \\ p^2=6/13 \\ p^3=3/13 \\ p^4=8/13 \end{array}$$

The system of equation and resolutions for DM_2:

$$SRE \Rightarrow \left.\begin{array}{l} 1-p^3=2\alpha \\ p^3-p^1=\alpha \\ p^1-p^2=3\alpha \\ p^2-p^4=2\alpha \\ p^4-0=2\alpha \end{array}\right\} \left.\begin{array}{l} p^4=2\alpha \\ p^2=4\alpha \\ p^1=7\alpha \\ p^3=8\alpha \\ 1=10\alpha \Leftrightarrow \alpha=\frac{1}{10} \end{array}\right\} \begin{array}{l} p^1={}^{7}\!/_{10} \\ p^2={}^{2}\!/_{5} \\ p^3={}^{4}\!/_{5} \\ p^4={}^{1}\!/_{5} \end{array}$$

$$CF \Rightarrow \left.\begin{array}{l} 1-p^3=3\alpha \\ p^3-p^1=2\alpha \\ p^1-p^2=3\alpha \\ p^2-p^4=2\alpha \\ p^4-0=\alpha \end{array}\right\} \left.\begin{array}{l} p^4=\alpha \\ p^2=3\alpha \\ p^1=6\alpha \\ p^3=8\alpha \\ 1=11\alpha \Leftrightarrow \alpha=\frac{1}{11} \end{array}\right\} \begin{array}{l} p^1={}^{6}\!/_{11} \\ p^2={}^{3}\!/_{11} \\ p^3={}^{8}\!/_{11} \\ p^4={}^{9}\!/_{11} \end{array}$$

$$VAR \Rightarrow \left.\begin{array}{l} 1-p^3=2\alpha \\ p^3-p^2=3\alpha \\ p^2-p^4=\alpha \\ p^4-p^1=2\alpha \\ p^1-0=2\alpha \end{array}\right\} \left.\begin{array}{l} p^1=2\alpha \\ p^4=4\alpha \\ p^2=5\alpha \\ p^3=8\alpha \\ 1=10\alpha \Leftrightarrow \alpha=\frac{1}{10} \end{array}\right\} \begin{array}{l} p^1={}^{1}\!/_{5} \\ p^2={}^{1}\!/_{2} \\ p^3={}^{4}\!/_{5} \\ p^4={}^{2}\!/_{5} \end{array}$$

$$NCG \Rightarrow \left.\begin{array}{l} 1-p^3 = 2\alpha \\ p^3 - p^1 = 3\alpha \\ p^1 - p^2 = 2\alpha \\ p^2 - p^4 = \alpha \\ p^4 - 0 = 2\alpha \end{array}\right\} \begin{array}{l} p^4 = 2\alpha \\ p^2 = 3\alpha \\ p^1 = 5\alpha \\ p^3 = 8\alpha \\ 1 = 10\alpha \Leftrightarrow \alpha = \dfrac{1}{10} \end{array}\left.\right\} \begin{array}{l} p^1 = 1/2 \\ p^2 = 3/10 \\ p^3 = 4/5 \\ p^4 = 1/5 \end{array}$$

$$RUM \Rightarrow \left.\begin{array}{l} 1-p^2 = 3\alpha \\ p^2 - p^4 = \alpha \\ p^4 - p^1 = 4\alpha \\ p^1 - p^3 = 3\alpha \\ p^3 - 0 = 3\alpha \end{array}\right\} \begin{array}{l} p^3 = 3\alpha \\ p^1 = 6\alpha \\ p^4 = 10\alpha \\ p^2 = 11\alpha \\ 1 = 14\alpha \Leftrightarrow \alpha = \dfrac{1}{14} \end{array}\left.\right\} \begin{array}{l} p^1 = 3/7 \\ p^2 = 11/14 \\ p^3 = 3/14 \\ p^4 = 5/7 \end{array}$$

$$RC \Rightarrow \left.\begin{array}{l} 1-p^4 = 2\alpha \\ p^4 - p^1 = 3\alpha \\ p^1 - p^3 = \alpha \\ p^3 - p^2 = 2\alpha \\ p^2 - 0 = 4\alpha \end{array}\right\} \begin{array}{l} p^2 = 4\alpha \\ p^3 = 6\alpha \\ p^1 = 7\alpha \\ p^4 = 10\alpha \\ 1 = 12\alpha \Leftrightarrow \alpha = \dfrac{1}{12} \end{array}\left.\right\} \begin{array}{l} p^1 = 7/12 \\ p^2 = 1/3 \\ p^3 = 1/2 \\ p^4 = 5/6 \end{array}$$

The system of equation and resolutions for DM_3:

$$SRE \Rightarrow \left.\begin{array}{l} 1-p^3=2\alpha \\ p^3-p^4=\alpha \\ p^4-p^1=\alpha \\ p^1-p^2=3\alpha \\ p^2-0=3\alpha \end{array}\right\} \left.\begin{array}{l} p^2=3\alpha \\ p^1=6\alpha \\ p^4=7\alpha \\ p^3=8\alpha \\ 1=10\alpha \Leftrightarrow \alpha=\dfrac{1}{10} \end{array}\right\} \begin{array}{l} p^1={}^3\!/_5 \\ p^2={}^3\!/_{10} \\ p^3={}^4\!/_5 \\ p^4={}^7\!/_{10} \end{array}$$

$$CF \Rightarrow \left.\begin{array}{l} 1-p^1=4\alpha \\ p^1-p^3=0 \\ p^3-p^2=2\alpha \\ p^2-p^4=\alpha \\ p^4-0=3\alpha \end{array}\right\} \left.\begin{array}{l} p^4=3\alpha \\ p^2=4\alpha \\ p^3=6\alpha \\ p^1=6\alpha \\ 1=10\alpha \Leftrightarrow \alpha=\dfrac{1}{10} \end{array}\right\} \begin{array}{l} p^1={}^3\!/_5 \\ p^2={}^2\!/_5 \\ p^3={}^3\!/_5 \\ p^4={}^3\!/_{10} \end{array}$$

$$VAR \Rightarrow \left.\begin{array}{l} 1-p^2=3\alpha \\ p^2-p^3=2\alpha \\ p^3-p^4=3\alpha \\ p^4-p^1=0 \\ p^1-0=4\alpha \end{array}\right\} \left.\begin{array}{l} p^1=4\alpha \\ p^4=4\alpha \\ p^3=7\alpha \\ p^2=9\alpha \\ 1=12\alpha \Leftrightarrow \alpha=\dfrac{1}{12} \end{array}\right\} \begin{array}{l} p^1={}^1\!/_3 \\ p^2={}^3\!/_4 \\ p^3={}^7\!/_{12} \\ p^4={}^1\!/_3 \end{array}$$

$$NCG \Rightarrow \left.\begin{array}{l} 1-p^1=\alpha \\ p^1-p^2=2\alpha \\ p^2-p^3=3\alpha \\ p^3-p^4=2\alpha \\ p^4-0=3\alpha \end{array}\right\} \left.\begin{array}{l} p^4=3\alpha \\ p^3=5\alpha \\ p^2=8\alpha \\ p^1=10\alpha \\ 1=11\alpha \Leftrightarrow \alpha=\dfrac{1}{11} \end{array}\right\} \begin{array}{l} p^1={}^{10}\!/_{11} \\ p^2={}^{8}\!/_{11} \\ p^3={}^{5}\!/_{11} \\ p^4={}^{3}\!/_{11} \end{array}$$

$$RUM \Rightarrow \left.\begin{array}{l} 1-p^4=\alpha \\ p^4-p^3=3\alpha \\ p^3-p^2=3\alpha \\ p^2-p^1=0 \\ p^1-0=2\alpha \end{array}\right\} \left.\begin{array}{l} p^1=2\alpha \\ p^2=2\alpha \\ p^3=5\alpha \\ p^4=8\alpha \\ 1=9\alpha \Leftrightarrow \alpha=\dfrac{1}{9} \end{array}\right\} \begin{array}{l} p^1={}^{2}\!/_{9} \\ p^2={}^{2}\!/_{9} \\ p^3={}^{5}\!/_{9} \\ p^4={}^{8}\!/_{9} \end{array}$$

$$RC \Rightarrow \left.\begin{array}{l} 1-p^1=3\alpha \\ p^1-p^2=\alpha \\ p^2-p^4=2\alpha \\ p^4-p^3=3\alpha \\ p^3-0=4\alpha \end{array}\right\} \left.\begin{array}{l} p^3=4\alpha \\ p^4=7\alpha \\ p^2=9\alpha \\ p^1=10\alpha \\ 1=13\alpha \Leftrightarrow \alpha=\dfrac{1}{13} \end{array}\right\} \begin{array}{l} p^1={}^{10}\!/_{13} \\ p^2={}^{9}\!/_{13} \\ p^3={}^{4}\!/_{13} \\ p^4={}^{7}\!/_{13} \end{array}$$

BIOGRAPHICAL SKETCH

The writer, born in 1978 in Istanbul, has finished his high school education in Saint Michel French high school in 1996. Then he completed his B.Sc. on Industrial Engineering in the Industrial Engineering Department of Galatasaray University in 2000 and M.Sc. on Industrial Engineering in Institute of Science and Engineering of Galatasaray University in 2003. His research interests and focus are in the areas of multiple criteria decision making, decision support systems, fuzzy logic and performance evaluation.