RISK-ADJUSTED JOINT OPTIMIZATION OF BASE-STOCK LEVELS AND COMPONENT ALLOCATION IN AN (RE) ASSEMBLE-TO-ORDER SYSTEM WITH RETURNS

(BİR (YENİDEN) SİPARİŞ-ÜZERİNE-MONTAJ SİSTEMİNDE GERİ DÖNEN ÜRÜNLERİN PARÇA TAHSİSİ VE RİSK-AYARLI BİRLEŞİK TEMEL-STOK SEVİYESİ OPTİMİZASYONU)

by

Eda BİLİCİ, B.S.

Thesis

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

MASTER OF SCIENCE
in
INDUSTRIAL ENGINEERING
in the
INSTITUTE OF SCIENCE AND ENGINEERING
of
GALATASARAY UNIVERSITY

June 2012

RISK-ADJUSTED JOINT OPTIMIZATION OF BASE-STOCK LEVELS AND COMPONENT ALLOCATION IN AN (RE) ASSEMBLE-TO-ORDER SYSTEM WITH RETURNS

(BİR (YENİDEN) SİPARİŞ-ÜZERİNE-MONTAJ SİSTEMİNDE GERİ DÖNEN ÜRÜNLERİN PARÇA TAHSİSİ VE RİSK-AYARLI BİRLEŞİK TEMEL-STOK SEVİYESİ OPTİMİZASYONU)

by

Eda BİLİCİ, B.S.

Thesis

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

MASTER OF SCIENCE

Date of Submission : May 25, 2012
Date of Defense Examination : June 15, 2012

Supervisor : Asst. Prof. Dr. M. Ebru ANGÜN
Committee Members : Asst. Prof. Dr. S. Şebnem AHISKA
Prof. Dr. Necati ARAS

ACKNOWLEDGMENTS

First and foremost I would like to express my sincere gratitude to my supervisor, Asst. Prof. Dr. M. Ebru Angün for her patience, motivation, and guidance throughout my thesis. Without her supervision and knowledge this study would not have been successful.

Besides my supervisor, I would like to thank the members of my thesis committee: Prof. Dr. Necati Aras and Asst. Prof. Dr. S. Şebnem Ahıska, for their time, interest, encouragement, and insightful comments.

Lastly, I would like to thank my family members, my parents and my brother. They were always supporting me and encouraging me with their best wishes. Without my family's encouragement, I would never have been able to finish the degree. Thank you.

Eda Bilici
Galatasaray University
June 2012

TABLE OF CONTENTS

LIST OF SYMBOLS

i	: Number of components, $i = 1, \ldots, m$.
j	: Number of products, $j = 1, \ldots, n$.
t	: Time periods, $t = 0, 1, 2, \ldots$.
α	: Significance level, $\alpha \in (0,1)$.
μ	: Mean vector.
λ	: Parameter that can be tuned for a tradeoff between minimizing on average and risk control.
η	: Upper bound on random second-stage cost.
A_{it}	: Total amount of replenishment for component i in period t, $i = 1, \ldots, m$.
b_{ij}	: Usage rates of component i to manufacture unit demand of product j, $i = 1, \ldots, m$ and $j = 1, \ldots, n$.
b'_{ij}	: Usage rates of component i to remanufacture unit demand of product j, $i = 1, \ldots, m$ and $j = 1, \ldots, n$.
D_{it}	: Total demand for component i in period t, $i = 1, \ldots, m$.
I_{it}	: Inventory level of component i at the end of period t, $i = 1, \ldots, m$.
L_i	: Replenishment lead time of component i, $i = 1, \ldots, m$.
P_{jt}	: Demand stream for product j in period t, $j = 1 \ldots, n$.
Q_{it}	: Total amount of component i disassembled from cores of quality level 3 in period t, $i = 1, \ldots, m$.
R_{jt1}	: Amount of cores that belongs to the quality level 1 in period t for product j, $j = 1 \ldots, n$.
R_{jt2}	: Amount of cores that belongs to the quality level 2 in period t for product j, $j = 1 \ldots, n$.
R_{jt3}	: Amount of cores that belongs to the quality level 3 in period t for product j, $j = 1 \ldots, n$.
R_{jt4}	: Amount of cores that belongs to the quality level 4 in period t for product j, $j = 1 \ldots, n$.
S_i	: Base-stock level (order-up-to level) for component i, $i = 1, \ldots, m$.
w_j	: Response time window for product j, $j = 1 \ldots, n$.
q_j	: Unit penalty cost for product j, $j = 1 \ldots, n$.

x^{r}_{jl}	: Remanufactured amounts of product *j* with response time window *l*, $j = 1\ldots, n$ and $l = 0, 1, \ldots, w_j$.
x^{m}_{jl}	: Manufactured amounts of product *j* with response time window *l*, $j = 1\ldots, n$ and $l = 0, 1, \ldots, w_j$.
$\tilde{\delta}_j$	: Realizations of multivariate normally distributed random variables for product *j*, $j = 1\ldots, n$.
$\tilde{\rho}_j$	: Fractions of cores that fall into the four quality levels for product *j*.
σ_{ij}	: Covariance of two random variables, X_i and X_j, $i = 1, \ldots, m$ and $j = 1\ldots, n$.
$\tilde{R}_{j1}$	: Amount of cores that belongs to the quality level1, for product *j*, $j = 1\ldots, n$.
$\tilde{R}_{j2}$	: Remanufacturable amount of cores that belongs to the quality level 2, for product *j*, $j = 1\ldots, n$.
$Q(\mathbf{S}, \varepsilon)$	: Second-stage cost.
$D_i[s,t]$	: Total demand amount for component *i* from period *s* through period *t*, $i = 1, \ldots, m$.
$A_i[s,t]$	: Total replenishment amount for component *i* from period *s* through period *t*, , $i = 1, \ldots, m$.
$Q_i[s,t]$	: Total disassembled amount for component *i* from period *s* through period *t*, , $i = 1, \ldots, m$.
$\mathbf{c} = (c_1, \ldots, c_m)^T$	: Vector of procurement costs per unit of the *m* components.
$\mathbf{q} = (q_1, \ldots, q_n)^T$	: Vector of unit shortage costs per unit *n* products.
$\mathbf{S} = (S_1, \ldots, S_m)^T$	: Vector of base-stock levels of the *m* components.
$\mathbf{u} = (u_1, \ldots, u_n)^T$	: Vector of shortage amounts of the *n* products.
$\boldsymbol{\xi}$	: Random vector with realizations as ξ.
ω	: Random event.
$\Re$	: Real numbers.
q	: Second-stage objective vector $(q^T y)$.
p	: Probability of a random elements.
E	: Expectation operator of an optimality cut in the L-shaped method.
D	: Left-hand side vector of feasibility (optimality) cut in the L-shaped method.
b	: First-stage right hand side .
A	: First-stage matrix.

N	: Number of random elements.
T_s	: A matrix that consists of the coefficient of the first-stage decision variables.
S	: Feasibility sets.
X	: First-stage feasible set $(x \in X)$.
Y	: Second-stage feasible set $(y \in Y)$.
c	: First-stage objective or real vectors.
d	: Right-hand side of feasibility cut in the L-shaped method, a demand or a real vector.
l	: Index or lower bound on a variable.
r	: Index for the feasibility cuts.
s	: Scenario or index for the optimality cuts.
v	: Number of iterations.
W	: Recourse matrix.
h	: Right-hand side in second-stage .
σ_s^v	: Vector of simplex multipliers.
x_s^v	: Optimal solution.
π_s^v	: Vector of simplex multipliers obtained at the optimal solution of (4.5).
$\sum$	: Covariance matrix.

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

ATO	Assemble-to-Order
CTO	Configure-to-Order
CVaR	Conditional Value at Risk
FCFS	First-Come First-Served
iid	Independently and Identically Distributed
LP	Linear Programming
MDKP	Multidimensional Knapsack Problem
PC	Personal Computer
RATO	Reassemble-to-Order
SAA	Sample Average Approximation
SP	Stochastic Programming
VaR	Value at Risk

ABSTRACT

In recent years, product recovery business that is concerned with all activities related to regaining materials and value added out of used products has been receiving growing attention for various reasons such as consumer awareness, economic incentives, environmental concerns and legal pressure. Product recovery can be performed in many ways; e.g. remanufacturing, reconditioning, recycling, and refurbishing of products. Remanufacturing is one of the highly important fields of product recovery. In the context of our study, it refers to the process through which used products (also called cores or returns) are brought to as good as new condition by inspecting their components, and performing repairing, replacing, restoring operations and/or updating them with new specifications when necessary.

Remanufacturing differentiates from brand-new production basically in two aspects. One of the major differences between remanufacturing and new product manufacturing systems lies in the supply side which is to take back used products before the end of their useful life cycle due to the fact that manufacturers would like to retain some fraction of the original manufactured value besides the value of the extracted and refined material. Another main issue encountered by the manufacturers is the uncertainty in the timing, quantity and quality of product returns which may affect the cost of remanufacturing considerably, and may result in several challenging issues in the (re)assembly stage. To cope with such uncertainties, firms involve take-back campaigns and leasing agreements to reduce the uncertainty in the timing and quantity of returns.

Many firms have been looking for ways to decrease their response times to the market because the pressure for serving customers speedily and the impact of product obsolescence increases. One way to deal with the aforementioned issues is to adopt an assemble-to-order (ATO) manufacturing strategy and/or its variations (i.e., reassemble-to-order (RATO), configure-to-order (CTO) etc.) instead of employing a traditional make-to-stock system. In an (R)ATO system, the inventories are held at component or

part levels, which substantially reduces the inventory holding costs. This (R)ATO system further increases customer satisfaction through decreasing response times to the demands and increasing fill rates (i.e., the fraction of demands satisfied from on-hand inventory to the total demands).

In this dissertation, a hybrid manufacturing and remanufacturing system in which a remanufacturing plant operates along with a manufacturing facility in fulfilling customer demand is analyzed. We consider a multi-component, multi-product, periodic-review (re) assemble-to-order system (ATO) that uses an independent base-stock policy for inventory replenishment of the components. Initially, end-of-lease cores are returned at the beginning of each period. Since the quality of cores are random, they are sorted, tested and graded into four pre-specified quality levels regarding their conditions. Then, the random, jointly and continuously distributed demands for the products are realized. Since some components, parts and modules are common to several products, they can be pooled together and then allocated to the respective products in accordance with their quality levels at the (re)assembly stage. In our problem, partial fulfillment is not allowed. The system quotes a predetermined response time window for each product, and it penalizes if the demand is not satisfied within this time window.

We model this problem through a risk-averse, two-stage stochastic programming problem, where the first stage decisions are the base stock levels for all components, and the second stage decisions are the allocations of components to different products. In order to control the manufacturers' risk of losses/costs, a risk measure such as conditional value-at risk (CVaR) is incorporated into the model. Risk adjustment is modeled through a chance constraint, which is then replaced by a CVaR constraint. We assume that the joint distribution of the random data is known, therefore, we approximate the problem through the sample average approximation (SAA) method, and we solve this approximation through the L-shaped method. We further analyze the validity of our results and present some encouraging numerical results.

RESUME

Suite à la sensibilisation des consommateurs, à sanctions légales et à la sensibilisation du peuple à l'environnement les activités de recyclage deviennent de plus en plus importantes. Ces activités consistent à regagner les matériaux et aux profits obtenus à partir des produits utilisés. C'est grâce à la reproduction, à la réparation, au renouvellement et grâce au recyclage qu'on obtient un profit du produit. La reproduction est le point le plus important au niveau d'avoir un profit du produit. A notre travail la reproduction contient le contrôle des morceaux des produits utilises (les retours ou les produits retournes aux producteurs), la réparation, le changement, le renouvellement et/ou au moment nécessaire à l'aide des nouvelles spécifications l'actualisation et mis à l'état neuf de ces morceaux.

La reproduction se différencie de la nouvelle production par deux points : La différence la plus importante entre la reproduction et la nouvelle production est au point de l'approvisionnement, parce que les producteurs reprennent les produits utilises avant la date d'expiration; ainsi ils ont dans leurs mains une partie du produit original et en même temps avec l'amélioration des morceaux utilisés ils essaient de gagner des profits. La gestion du temps de retour des produits, leurs quantités et l'imprécision de leurs qualités ont une grande influence sur la reproduction et créent des problèmes au moment du remontage. Pour écarter les imprécisions de la gestion du temps de retour des produits et l'imprécision de leurs qualités, les firmes font une campagne de récupération des produits et établissent des contrats de location.

A cause de la pression de service rapide aux clients, le risque que le produit ne soit plus a la mode, plusieurs firmes essaient de minimaliser le délai de réponse à la demande des clients. La meilleure façon de traiter le problème est, au lieu de suivre le système classique qui consiste à la production liée aux stocks; est de suivre la stratégie de montage sur commande : MSC, et/ou ses variations: par exemple remontage sur commande : RMSC, configuration sur commande: CSC. En suivant le système de remontage sur commande on ne fait que l'inventaire des composants et des pièces, ainsi

on minimalise le prix de revient de l'inventaire. Le système de remontage sur commande, en diminuant le délai de réponse aux demandes et en augmentant le pourcentage de réponse aux demandes (c'est-a-dire la fraction de réponse aux demandes d'après les inventaires), augmente la satisfaction des clients.

Cette thèse analyse la coopération entre une firme de reproduction et une firme de production de nouveaux produits. Production hybride et système de reproduction sont également analysés. Multi pièce, multi produit, les suivis périodiques du système de l'inventaire de montage sur commande sont des sujets d'étude de ce travail. L'étude du renouvellement de l'inventaire des morceaux utilise le niveau de stock de base indépendant. Premièrement au début de chaque période les produits utilisés dont les contrats sont finis, reviennent aux producteurs. Les qualités des produits utilises étant aléatoires, les produits retournés, testés et classifiés, sont gradues d'après quatre niveaux déterminés à l'avance. Ensuite réalisent pour les produits les demandes aléatoires combines. On réunit les composants, morceaux et modules pareils pour plusieurs produits ; et par rapport à leurs degrés de qualités, pendant le période de de montage sont attribués aux produits. Notre problème ne donne pas l'autorisation aux réponses pour les demandes partielles. Le système fixe une date précise pour la réponse aux clients et la demande non satisfaite dans ces périodes est pénalisée.

Nous modelons le problème avec l'aide du critère d'éviter les risques et avec une programmation d'excès de stock a deux étapes. Le premier est la détermination des stocks fondamentaux de tous les morceaux et la deuxième est l'attribution des morceaux aux différents produits. Pour pouvoir conjecturer les pertes et les prix de revient des producteurs ; un critère de valeur de risque dépendant des conditions (VRDC) est ajoute au modèle. L'adaptation de risque est formulée avec une contrainte probabiliste et ensuite est changée avec VRDC. La distribution conjointe des données aléatoires étant supposée connue le problème est valorisé avec la moyenne d'exemplification et cette approche est résolue avec méthode de forme L. En outre l'analyse de la validité des résultats nous donne des résultats numériques encourageant.

ÖZET

Tüketici bilinci, ekonomik teşvikler, çevreye olan farkındalık, ve yasal yaptırımlar gibi sebeplerden, malzemelerin yeniden kazanılması ve kullanılmış ürünlerden elde edilecek değerlere ilişkin tüm aktiviteleri kapsayan ürün kazanım faaliyetlerine olan ilgi son yıllarda büyüyerek artmıştır. Ürün kazanımı, örneğin; ürünlerin yeniden üretimi, onarımı, geri dönüşümü, ve yenilenmesi gibi birçok şekilde yapılabilmektedir. Yeniden üretim, ürün kazanımında en önemli alanlardan biridir. Çalışmamızda yeniden üretim, kullanılmış ürünlerin (iadeler veya üreticiye geri dönen ürünler olarakta tabir edilmektedir) parçalarının kontrol edilmesi, onarılması, değiştirilmesi, yenilenmesi operasyonlarını ve/veya gerektiğinde yeni spesifikasyonlarla güncellenerek yenisi kadar iyi duruma getirilmesini içermektedir.

Yeniden üretim, yepyeni üretimden temelde iki açıdan farklılık gösterir. Yeniden üretim ve yeni ürün üretimi arasındaki en önemli farklardan biri tedarik bölümündedir; çünkü, üreticiler kullanılmış ürünleri kullanım sürelerinin tamamlanmadan geri alarak orjinal ürünün bir bölümünü elinde bulundurmanın yanısıra üründe kullanılan malzemelerin iyileştirilmesi ile de değer sağlamak istemektedirler. Üreticilerin karşılaştığı yeniden üretimi ciddi bir şekilde etkileyen, ve sonucunda (yeniden) montaj safhasında güçlük yaşanmasına sebep olan bir diğer önemli konu ise geri dönen ürünlerin zamanlaması, miktarı ve kalitesindeki belirsizliklerdir. Geri dönen ürünlerin zamanlamasında ve miktarlarındaki belirsizlikleri gidermek için firmalar, ürünleri geri-alma kampanyaları ve kiralama anlaşmaları yapmaktadırlar.

Müşteriye hızlı hizmet verme baskısı, ve ürünün modasının geçme etkisinin hızla artmasından dolayı birçok firma pazarda talebe cevap verme sürelerini azaltmanın yollarını aramaktadırlar. Sözü edilen konu ile başa çıkmanın yollarından biri klasik stoğa-üretim sistemi yerine sipariş-üzerine-montaj (SÜM) üretim stratejisinin ve/veya onun varyasyonlarının (örn., yeniden sipariş-üzerine-montaj (YSÜM), sipariş-üzerine-yapılandırma (SÜY) vs.) kullanılmasıdır. Bir (Y)SÜM sisteminde, envanterler bileşenler ve parçalar seviyesinde tutulmaktadır, böylece envanter tutma maliyeti de

önemli ölçüde azalmaktadır. (Y)SÜM sistemi, taleplere cevap verme sürelerini azaltarak ve talep karşılama oranlarını (bir diğer deyişle, toplam talebin eldeki envanterden karşılanma fraksiyonunu) arttırarak müşteri memnuniyetini yükseltmektedir.

Bu tezde, müşteri taleplerini karşılamada bir yeniden üretim tesisinin, yeni ürün üretim tesisi ile birlikte faaliyet göstermesini kapsayan bir melez üretim ve yeniden üretim sistemi analiz edilmektedir. Çalışmada, çoklu parça, çoklu ürün, periyodik takip edilen (yeniden) sipariş-üzerine-montaj (SÜM) envanter sistemi, parçalara ilişkin envanterin yenilenmesinde bağımsız temel stok seviyesini kullanarak incelemekteyiz. İlk olarak, her bir periyodun başında sözleşmesi biten kullanılmış ürünler üreticiye geri gelmektedir. Kullanılmış ürünlerin kalitelerinin rassal olmasından ötürü, test edilen, sınıflanan geri dönen kullanılmış ürünler önceden belirlenmiş olan dört adet kalite seviyesine göre derecelendirilirler. Ardından, ürünler için birleşik sürekli dağılan rassal talepler gerçekleşir. Bazı bileşenler, parçalar ve modüller birçok üründe ortak olması sebebiyle, hepsi bir araya getirilir ve sırasıyla ürünlere kalite seviyeleri ile ilişkili olarak (yeniden) montaj safhasında tahsis edilirler. Problemimizde, kısmi talep karşılamaya izin verilmemektedir. Sistemde her bir ürün için önceden karar verilmiş müşteriye cevap verme süresi belirtilir, ve belirlenen bu süre içerisinde karşılanamayan talep cezalandırılmaktadır.

Biz, bu problemi riskten kaçınma ölçütlü, iki-aşamalı stokastik programlama aracılığı ile modellemekteyiz. İlk aşama kararı, tüm parçaların temel stok seviyelerinin belirlenmesi ve ikinci aşama kararı da parçaların farklı ürünlere tahsis edilmesidir. Üreticinin, kayıplarının/maliyetlerinin riskini ölçümlemek için şarta bağlı-risk-değeri (CVaR) gibi bir risk ölçütü modele eklenmiştir. Risk uyarlaması, bir olasılıksal kısıt ile formule edilip, ardından CVaR kısıtı ile değiştirilmektedir. Rassal verilerin ortak dağılımının bilindiği varsayılmaktadır, böylece problem örneklem ortalaması yaklaşımı (SAA) ile değerlendirilmekte, ve bu yaklaşım L-şekilli metot ile çözümlenmektedir. İlave olarak, sonuçların geçerliliği analiz edilip ortaya teşvik edici sayısal sonuçlar konmaktadır.

1 INTRODUCTION

Remanufacturing is the process where a used product is disassembled and its modules, components and parts are recovered, processed, and used in the production of new products. The reuse of product returns can be very profitable, especially for the high-tech products that have quite long product life cycles [1]. For instance, the characteristic life cycle of a computer chip is 80,000 hours for which only 20,000 hours are used; therefore, that chip can still be economically used for 60,000 hours in some other products through, say, remanufacturing; see [2]. This high level recovery option is broadly found for industrial products such as photocopiers, computers, cellular phones, aviation equipments, vehicle engines, telecommunication, and medical equipments [3].

Economic incentives, ethical responsibilities, environmental concerns, legislation, market share and brand protection are some of the principal reasons why many firms engage in remanufacturing activities (e.g., [4], [5], and [6]). From this point of view, we consider a remanufacturing plant that operates along with a manufacturing facility in meeting customer order. These systems are also known as *hybrid manufacturing and remanufacturing systems* in the literature. Remanufacturing a used product, in general, is less costly than manufacturing a new one [7]. However, remanufacturing has some general attributes, which complicate the supply chain and production, i.e., uncertainty in the timing, quantity, and quality of returns, compared to manufacturing. At this point, leasing which is an ownership-based relationship will be a viable approach that aids to manage the return processes. [8] discusses factors that complicate management and planning of supply chain functions in a recoverable manufacturing system, including the uncertainty in timing and quantity of cores, and the uncertainty in quality of cores. It is found that lease agreements and take-back arrangements provide for greater certainty regarding the return time and quantity of cores; see the Xerox Europe case study in [8]. Nevertheless, the quality of cores is difficult to predict. In a remanufacturing system, components and modules can typically differentiate in quality so that some modules have higher quality level and longer residual life than others. Under such conditions, reassembly policy and inventory management are more complicated than traditional assembly systems because the manufacturer must jointly manage inventories across variable quality modules besides different types of modules.

In our model, end-of-lease cores at the beginning of each period are returned. Since the quality of cores is random; they are tested, sorted, and graded into four pre-specified quality levels. Then, production decisions are made based on these quality levels, see Figure 1.1.

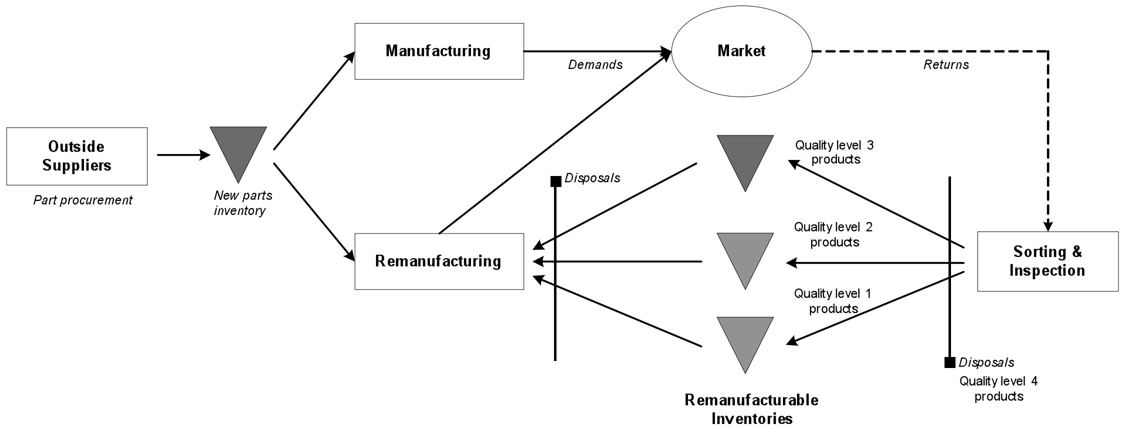


Figure 1.1. Hybrid manufacturing and remanufacturing system.

In this dissertation, we consider a multi-component, multi-product, periodic review inventory problem for *a hybrid (re)assemble-to-order system,* and we jointly analyze assemble-to-order (ATO) and reassemble-to-order (RATO) inventory strategies. In the ATO inventory strategy, products are designed around interchangeable items and modules. A firm makes and keeps only the modules and the main components in inventory, however, final products that may have common components, are assembled only after customer orders are realized such as Dell Computer (e.g., [9] and [10]). ATO systems can be found in the personal computer (PC) industry, in which customers are offered a wide range of product configurations and specifications such as processors, hard drives, memory cards, and other components (DVD, modem etc.) with a reasonably short response time after the demands are realized. In the RATO inventory strategy, on the other hand, returns are not remanufactured to stock and have chosen to reassemble the required components and items per each customer order. The remanufactured inventories are held at the component and part level. For instance, Xerox uses a reassemble-to-order policy, due to the Xerox's products are based on modular design principles. This enables the firm to configure a remanufactured product at the reassembly stage based on customer needs and preferences [8]. These strategies are mainly advantageous to firms that have significant component replenishment lead times, and the assembly times of products are relatively short.

In our system, partially order service, in which a customer order may be only partially accepted, yet guaranteed to be met eventually is not allowed. For instance, Amazon.com which is a e-commerce company, sells goods over the Internet. Their product portfolio includes DVDs, music CDs, softwares, video games, electronics, furnitures etc. If there is no available stock on their hand regarding the ordered product, they may split customer orders into multiple shipments (i.e., the rest of the items may be shipped from different fulfillment centers). In other words, if customer orders include five products, and only if three of them are available in stock, then they will satisfy the orders with available ones, and remaining orders are supplied to customer within a predetermined time. However, if the remaining orders are not available in the other fulfillment centers, these orders will be rejected at the beginning. We do not employ a partial fulfillment in our model. That is, customer orders are satisfied within their response window times if all items/parts requested are available in inventory. Our system quotes a predetermined response time window for each product, and it penalizes if the demand is not met within this time window. Firms whose inventory model is a make-to-stock system, commonly employ partially order services, in addition, firms can also use this model to evaluate customer impatiance.

Our presented inventory problem is defined through *risk-adjusted (averse)* settings that the objective is to minimize the total expected cost/loss of the production system. This mathematically refers to a stochastic optimization problem with chance constraints [11]. The risk-aversion is formulated through a probabilistic constraint, which is then replaced by a Conditional Value-at-Risk (CVaR) constraint (see, e.g.[12]). [13] describes a Conditional Value-at-Risk (CVaR), coherent risk measure, as the expected value of tail distributions of returns or losses. Coherent risk measures meet stochastic dominance conditions and cause a convex optimization problem [14].

To address our inventory problem, we formulate a two-stage stochastic program through a risk-adjusted setting, where the first-stage decisions are the base-stock levels for all components, and the second-stage decisions are the allocations of components to different products. We solve the first stage of the problem to determine the base-stock (order-up-to level) policy of the components for inventory replenishments by exploiting a Monte Carlo simulation based technique called *sample average approximation* (SAA) method, which approaches the expected objective function of the stochastic program with a sample average estimation on a number of randomly generated scenarios; see [15] and we solve this approximation through the *L-shaped* method that is an exact mathematical technique, and the main idea of it is to approximate the non-linear term in the objective (i.e., the recourse function). We assume that the replenishment lead time of each component can be variable for different components. In the second-stage, we determine component allocation decisions according to the on-hand inventory and the received orders. Some components are common to several products, thus component allocation decision is used to determine the amounts of components to be allocated to each product. The growing popularity of fast serving and mass customization in production systems has led to a new direction of study in fields of component commonality and ATO systems (see, e.g., [16] and [17]).

In this study, multi-component, multi-product, periodic review hybrid (re)assemble-to-order inventory system is considered and defined through a risk-adjusted manner. We model this system as a two-stage stochastic programming problem. The risk-aversion is formulated through a probabilistic constraint, which is then replaced by a CVaR constraint. We assume that the joint distribution of the random data is known, thus we approximate the problem through the SAA method, and we solve this approximation through the L-shaped method. We solve the model using CPLEX using MATLAB as its interface and report the results for different data instances.

The rest of the study is organized as follows. We present a review of the related literature and the contibutions of this study in Section 2, we present a motivating example, the description of the system and our risk-adjusted two-stage optimization problem in Section 3. We define the Sample Average Approximation and the L-shaped methods in Section 4. We present our numerical results in Section 5. Finally, we summarize our contributions and discuss future research possibilities in Section 6.

2 LITERATURE REVIEW

This review is structured as follows. First, in Section 2.1, we shortly discuss the quality uncertainties in remanufacturing systems. Section 2.2, continues with the related literature on quality uncertainties in hybrid production systems. In Section 2.3, we present the related literature of the (re) assemble-to-order inventory policies with performance measures. Finally, in Section 2.4, we point out contributions of this study to the existing literature.

2.1 Literature Review on Quality Uncertainty in Remanufacturing

There is a growing body of literature addressing core quality uncertainties and quality based categorizations for remanufacturing systems. Firms include quality based categorization in which returns are categorized with respect to their qualities, i.e., specification and functionality variations, in their production environment to have a key competitive advantage in lowering costs, to facilitate remanufacturing and disposal decisions of a manufacturer.

[18] considers production decisions of a remanufacturer under core quality uncertainties. In this study, cores are graded with respect to their conditions. After learning the product's quality level, the firm decides to either remanufacture the product or sell it as-is at a lower price to get some revenue from on-hand stock. This study shows that the choice of optimal product combination is a critical decision for a manufacturer because it may determine the long-term profitability of the firm. [19] studies the issue of core quality overestimation and analyze its impact on the remanufacturer's profitability. We refer the reader to [20] and [21] for more information on these systems.

Acquisition and sorting policies under return quality uncertainties are some of the other important focuses in remanufacturing. [22] analyzes the optimal acquisition and sorting policies for remanufacturers that encounter variable core conditions.

[23] considers the acquisition and remanufacturing decisions for situations with multiple discrete quality categories under core quality uncertainties.

In the majority of the published papers, graded cores are completely disassembled and stocked as components/parts at a remanufacturable inventory. We depart from the existing literature in that not all of our graded used products are stored only as parts or components. After required sorting and grading operations, cores are stocked as products and parts separately according to their quality levels. Only the cores that belong to the third class, are totally disassembled and stored as parts/items. If there is no available component in this level to use, at this stage component procurement decision will be made for a certain type of product(s).

2.2 Literature Review on Quality Uncertainty in Hybrid Production Systems

In hybrid production systems, where simultaneous remanufacturing of used products and manufacturing of new ones are considered, firms initially tend to satisfy market orders from remanufactured products since cores can be remanufactured at a lower cost than the initial manufacturing cost. If there is no available stock on hand, then they will fulfill the demand with the manufactured ones.

As proved in many researches, such an approach affects the profitability of companies significantly. A considerable number of studies (e.g., [24], [25], [26], [27], [28] and [29]) cope with cases in which both used and new products are used to meet the product demand.

Quality uncertainty in returns and quality based categorization are also investigated under hybrid manufacturing and remanufacturing context. [30] studies the issue of stochastic nature of the returned products and analyze the conditions under which quality-based categorization is most cost effective in a joint production system. [31] focuses on a product recovery system in a hybrid manufacturing/remanufacturing environment. In this study, the optimal recovery and production policies are assessed and the effects of various sources of uncertainty are analyzed.

2.3 Literature Review on (Re) Assemble-to-Order Systems

Assemble-to-order (ATO) manufacturing system deals with multiple demand classes in which many different types of customized products are produced using common components, items or modules and share the component inventories. ATO enhances the probability of meeting a customized demand in terms of time and low cost. In the reassemble-to-order (RATO) system, on the other hand, returns are not remanufactured to an inventory and have chosen to (re)assemble the required components and items per each customer order. The remanufactured inventories are held at the component and part level.

One of the other commonly used inventory strategy, which is configure-to-order (CTO) system, is a special case of the assemble-to-order system, and it is widely employed especially in the electronic products industry. In the CTO strategy, components are divided into parts, and the customer chooses components from those parts [32]. According to [33], the CTO strategy allows customers to select a finished product by choosing a customized set of components that go into the product. In this study, the aim of CTO policy is defined as the minimization of the expected inventory investment subject to fulfilling the service requirement for each product family.

Component allocation rules are considered in production systems to provide an effective inventory management. If the component allocation decisions are poorly managed which leads to the excessive on-hand inventories, the expected service level may not be achieved. According to [34], the replenishment decisions and the component allocation decisions should simultaneously be taken into consideration to lower the base stock levels and to deliver better service to customers.

In our inventory model, product returns are classified into four different quality classes that are determined by the quality of each module for remanufacturing. The manufacturer must jointly manage inventories across variable quality modules as well as different types of modules. However, these policies and inventory control are more complicated than traditional assembly systems due to the wide variety of items, products and variable quality modules.

A large body of literature within the context of ATO system focuses on base-stock policies under some allocation rules, for instance, the first-come first-served (FCFS) basis is used in continuous review models and the fixed priority rule is used in periodic-review (discrete-time) models. In the majority of these studies, order fullfillment performances are also incorporated into the multi-component ATO systems. [10] who considers order fulfillment performance measures, incorporating the probability of satisfying a customer demand within a specified time window for a multi-component assemble-to-order inventory system with stochastic leadtimes. Demands are met on a *FCFS* basis, and unsatisfied ones are backlogged. In this study, *partial fullfillment* is allowed that can be applicable in case of customer impatiance, e.g. to avoid the risk of losing customer. [17] studies a single product periodic-review ATO model under base-stock control. They assume multivariate normal distributions for demand and constant lead times for component replenishments. [9] analyzes the order-based backorders in a continuous review, multi-component base-stock inventory system with constant lead times and multiple demand classes. Order-based backorder refers to the average number of customer demands that are not yet completely met, perceived as an important measure to demonstrate customer satisfaction.

In some papers, heuristics are proposed for component allocations. [35] analyzes a periodic-review model for joint order fullfillment probability (the probability of satisfying all orders that arrive in a period within a pre-established time limit) in a multi-component inventory system. Demands follow a multivariate normal distribution. In this study, *equal fractile heuristic* is employed in terms of component allocation rule. [16] studies a multi-component inventory control problem with component commonality and correlated end product orders in any period. In this research, component allocation is based on the *fair shares allocation rule* that a fraction of the available stock is allocated to the different demands in case of shortages of item stocks. Apart from them, we do not consider *fair shares* policies for component allocation. We optimize our inventory problem, whereas they use heuristics to solve their model. [34] considers an ATO system that uses the independent base-stock policy for inventory replenishment. They model a two-stage stochastic integer program in order to determine the optimal replenishment rule and the optimal component allocation policy in an ATO system. They incorporate performance measures into their model, in which the long-run average reward ratio is used to diminish the *Type-II service level*, also called as the fill rate, for identical reward rates. The component allocation problem is formulated as a general *multidimensional knapsack problem (MDKP),* and an order-based heuristic is proposed to solve this problem. In this paper, the sample average approximation method is also presented in order to determine the optimal order-up-to levels, and compared with two variations of the *equal fractile heuristic*.

2.4 Contributions to the Literature

Our study differs from the existing literature in the following ways:

- This study considers the joint optimization of the base-stock levels and component allocation in case there are cores of uncertain quality.
- The problem is considered in a risk-adjusted manner by considering the so-called conditional value-at-risk constraint.

The first item was also considered in [34], yet they analyzed this item for ATO systems without considering core quality uncertainties and in a risk- neutral enviroment. Our study differs from them in that we have a risk adjusted approach in a hybrid manufacturing and remanufacturing environment. We consider return (input) quality uncertainties, and lastly we do not allow a partial fulfillment in our research.

To the best of our knowledge, the problem of inventory replenishment and component allocation in a (R)ATO system in a risk-averse environment has not been addressed in the literature. Thus, our study is the first that includes the combination of all these aforementioned subjects.

3 PROBLEM DEFINITION AND FORMULATION

3.1 Motivating Example and the Model Description

We consider a firm that produces brand-new products and remanufactures cores into as "good" as new ones to meet market orders. In the remanufacturing process, used products are returned to the producer at the end of their leasing periods and remanufactured. In this system, products are constituted based on an (re)assemble-to-order inventory system and the orders are met based on a first-come first-served (FCFS) inventory commitment rule. Finished product is assembled to order from a set of components. That is to say, no serviceable goods inventory is kept for any finished product, while each component has its own inventory, replenished from a supplier following an order-up-to (base-stock) level. The cores are sorted into four different quality classes and after the demand is realized, cores of the quality class 1 are refurbished and cores of quality class 2 are remanufactured, and only if there is no available stock on hand for remanufacturing, manufacturing decision will be made for a certain type of product(s). From consumer's point of view, remanufactured products are perceived as good as manufactured products in terms of both price and quality.

In the remanufacturing plant, different quality levels could be processed, yet the variation in core qualities may lead to a higher processing time and cost. We refer the reader to [18] for more information who describe remanufacturing operations at ReCellular for quality grading and remanufacturing processes. We analyze a joint (re) assemle-to-order inventory system and the problem of interest is analyzed in two stages.

In the two-stage stochastic programming method for optimization under uncertainty, the decision parameters are partitioned into two sets, and this approach is described in the study of [36] as follows. The first stage parameters are those that have to be determined prior to the actual realization of the random variables. Then, once the realizations of the random variable occur, the second stage or recourse variables are decided to make further design or operational system advancements. The aim is to select the first stage decision variables in a way that the sum of first stage costs and the expected value of the random second stage costs is minimized.

A standard formulation of the two-stage stochastic program is as follows [36, 37, 38]:

$$\underset{x \in X}{\text{Min}} \left\{ g(\mathrm{x}) := c^T x + \mathrm{E}\left[Q(x, \xi(\omega)) \right] \right\}, \tag{3.1}$$

where

$$Q(x, \xi) := \min_{y \in Y} \left\{ q^T y : W y \geq h - T x \right\} \tag{3.2}$$

is the optimal value and $\xi := (q, T, W, h)$ represents the potential random vector. It is supposed that some (or all) of the components/parts of $\xi(\omega)$ are random, and the expectation in (3.1) is taken with respect to the probability distribution of $\xi(\omega)$ that is assumed to be known. Problem (3.1), with variables $x \in R^{n_1}$, form the first stage that entails to be determined before a realization of $\xi(\omega)$ occurs, and problem (3.2), with variables $y \in R^{n_2}$, form the recourse for given first stage decision x and realization ξ of the random data. In our model, in the first stage, the base-stock levels for all components, and in the second stage, the allocation of components to different products are decided. In addition, we incorporate a response time window, which is a performance criterion and measures the time to meet customer orders into our system.

To illustrate the model, we provide some numerical data from the existing research of [33] and we demonstrate our model on this small instance, for details, see, Section 5.1. Since the size of this instance is small, we take the number of Monte Carlo sample as three for a simple example. The return time and quantity of the used products are considered as deterministic (i.e., leasing contract etc.). In our model, we have the following assumptions: (i) The joint distribution of demands follows a multivariate normal distribution; (ii) The expected processing times, manufacturing and remanufacturing costs are known; (iii) The manufactured products do not differentiate from remanufactured ones.

In Section 5.1 and Section 5.2, test problems from the study of [33] and test problems from the real-life (actual data) are presented, respectively. Then, computational results are given to provide more insight into our model.

3.2 The System

We consider a hybrid assemble-to-order (ATO) and reassemble-to-order (RATO) system with m components, indexed by $i = 1, \ldots, m$, and n products, indexed by $j = 1, \ldots, n$. The following sequence of events is typical for the system for every period t, $t = 0, 1, 2, \ldots$. At the beginning of a period, the inventory position of each component is reviewed, and the component replenishment orders are placed according to the inventory policy. After the receipt of the replenishment for earlier orders and update of the inventory positions of the components, end-of-lease products (cores) are returned. These cores are subject to be tested, graded, and sorted into a number of quality levels, so that the random amounts of cores, which fall into each quality level are revealed. Then, random orders for different products arrive through lease agreements.

Each component i operates under a periodic-review, independent base-stock policy with the base-stock (order-up-to level) policy for component i denoted by S_i. That is, if at the beginning of a period, the inventory position of component i is less than S_i, the system orders up to S_i; otherwise, it does not order. Furthermore, the ordering decisions for component i are made on the basis of the inventory position of component i only. Such a policy is in general not optimal, but it has been adopted in analysis and in practice due to its simplicity and minimal requirement of system-wide information. Moreover, the replenishment lead time of component i, denoted by L_i, is a constant integer which is an integer multiple of the review interval. These lead times can be different for different components.

The lease agreements for the n products enable us to consider the return time and quantity of the cores as deterministic. However, the quality of the cores is difficult to predict; hence, they are tested, graded, and sorted into, say, four quality levels; see the Xerox Europe case study in [8]. The cores of quality level 1 (i.e., best cores) are unused products requiring only minor servicing, and the cores of quality level 2 are in good condition, but they require some of the components to be replaced during the remanufacturing process. Moreover, the cores of quality level 3 are in good condition, but not economically fit for remanufacturing. Therefore, these cores are disassembled, and after being repaired, some of their components enter the reusable parts inventory. Finally, the cores of quality level 4 are immediately disposed off. For product *j*, the cores that are classified into four quality levels in period *t* are denoted by the vector of the random variables $(R_{jt1}, R_{jt2}, R_{jt3}, R_{jt4})(j = 1, \ldots, n)$, and the sum $R_{jt1} + R_{jt2} + R_{jt3} + R_{jt4}$ equals the deterministic amount of cores for product *j* returned in period *t*. For product *j*, $(R_{jt1}, R_{jt2}, R_{jt3}, R_{jt4})$ are correlated in the same period, but are independently and identically distributed (iid) vectors across different periods. Furthermore, the random amounts $R_{jt1}, R_{jt2}, R_{jt3}$ and R_{jt4} are independent of the random demands for product *j* in the same period.

The major barrier to the success of remanufacturing has been the misperception among some customers that remanufactured products are inferior to brand-new ones. In this study, however, we assume no market segmentation between remanufactured and manufactured products. In other words, the remanufactured products are perceived as good as manufactured ones, and hence, for product *j* there is a single demand stream. This demand stream for product *j* in period *t* is denoted by the random variable P_{jt}, where $(P_{1t}, P_{2t}, \ldots, P_{nt})$ are correlated in the same period, but are iid vectors across different periods.

Each brand-new product is assembled from multiple units of a subset of *m* components, and each core of quality level 2 is remanufactured by replacing a pre-specified number of components. Let b_{ij} and b'_{ij} denote the usage rates of component *i* to manufacture and to remanufacture unit demand of product *j*, respectively, where $b_{ij} \geq b'_{ij}$. The system quotes a response time window, w_j, for product *j*. This time window is pre-specified and fixed for every product type by the system. We assume that the system is penalized by a unit penalty, q_j, if a demand for product *j* cannot be filled within w_j periods after its arrival. Furthermore, a demand for product *j* is considered to be filled if that demand is allocated b_{ij} or b'_{ij2} units of component *i*; in other words, the product is really a product (incomplete if any of its components is missing), and partial shipment is not allowed. The system uses the following order to fill the demands: first, the cores of quality level 1, then the cores of quality level 2, and finally the brand-new products. This order is reasonable because the production times and component requirements increase in the same order.

The problem of interest is analyzed in two stages. The first-stage decisions are the optimal base-stock levels S_i for i = 1, …, m, and these decisions are taken without observing the realizations of the random demands for the n products. After the cores are tested and graded, and customers' demands are received, the second-stage decisions, namely the amounts of inventories to be allocated to the unfilled demands, are made in each period. We assume that the inventories are allocated to the unfilled demands subject to a first-come first-served (FCFS) inventory commitment rule. Under the FCFS rule, the allocation problem is concerned with the demands for different product types that occur in the same period. The FCFS rule enhances analytical tractability, and has been adopted by [16], [34], and [35].

Now, we introduce new random variables, which depend on the joint random demands $(P_{1t}, P_{2t}, \ldots, P_{nt})$ for the n products and the joint random amounts of the cores that fall into four quality levels $(R_{jt1}, R_{jt2}, R_{jt3}, R_{jt4})$. These new random variables will be used to derive an equation for the inventory on-hand that simplify the formulation of our model in the next section. For i = 1, …, m, j = 1, …, n, and t = 0, 1, …

D_{it} be the total demand for component i in period t

$$\text{i.e., } D_{it} = \sum_{j=1}^{n} \left[b'_{ij2} \min\left\{ (P_{jt} - R_{jt1})^{+}, R_{jt2} \right\} + b_{ij} (P_{jt} - R_{jt1} - R_{jt2})^{+} \right]$$

Q_{it} be the total amount of component i disassembled from cores of quality level 3 in period t with b'_{ij3} being rate of disassembled component i from product j.

$$\text{i.e., } Q_{it} = \sum_{j=1}^{n} b'_{ij3} R_{jt3}$$

A_{it} be the total amount of replenishment for component i in period t.

I_{it} be the inventory level (i.e., on-hand inventory minus backlog) of component i at the end of period t.

where, for any two random variables X and Y, $(X-Y)^{+} = \max\{X-Y, 0\}$. We further denote the total demand, the total replenishment, and the total disassembled amount for component i $(i = 1, \ldots, m)$ from period s through period t inclusive by $D_i = [s,t]$, $A_i = [s,t]$, and $Q_i = [s,t]$ respectively, where

$$D_i[s,t] = \sum_{u=s}^{t} D_{iu} \qquad A_i[s,t] = \sum_{u=s}^{t} A_{iu} \qquad Q_i[s,t] = \sum_{u=s}^{t} Q_{iu}$$

Whenever $s > t$, $D_i[s,t] \equiv 0$, $A_i[s,t] \equiv 0$, and $Q_i[s,t] \equiv 0$.

Now, we derive an equation for the inventory on hand. Assume that k is a nonnegative integer such that $k \le L_i$ for any lead time L_i. Later, we use k as an index of response time windows for all products. Because each component is operated under an independent base-stock level S_i, based on Hadley and Whitin [39], the following equation for the inventory level at the end of period $t + k$ can be written

$$I_{i,t+k} = S_i - D_i[t+k-L_i, t+k] + Q_i[t+k-L_i, t+k] \qquad i = 1, \ldots, m \tag{3.3}$$

Furthermore, using balance equations and FCFS inventory commitment rule, the inventory level at the end of period $t + k$ is related to the one at the end of period $t - 1$ as follows

$$I_{i,t+k} = I_{i,t-1} + A_i[t, t+k] - D_i[t, t+k] + Q_i[t, t+k] \tag{3.4}$$

Substituting (3.4) and (3.3), we reach the following result

$$I_{i,t-1} + A_i[t, t+k] + Q_i[t, t+k] = S_i - D_i[t+k-L_i, t-1] + Q_i[t+k-L_i, t+k] \tag{3.5}$$

Note that $I_{i,t-1} + A_i[t, t+k] + Q_i[t, t+k]$ is the net inventory level at the end of period $t + k$ after having received all replenishment orders and having disassembled all repairable component i from cores of quality level 3, but before allocating any inventory to the demands realized after period $t - 1$. Furthermore, because of the FCFS rule, if the amount $S_i - D_i[t+k-L_i, t-1] + Q_i[t+k-L_i, t+k]$ is positive, that inventory will be committed to the demands $(P_{1t}, P_{2t}, \ldots, P_{nt})$ of period t before any demands of the subsequent periods. Now, suppose that the response time windows for the n products can be ordered as $w_1 \leq w_2 \leq \ldots \leq w_n$. Then, the on-hand inventory of component i to be committed to $(P_{1t}, P_{2t}, \ldots, P_{nt})$ for $k = 0,1, \ldots, w_n$ is given by

$$\left(S_i - D_i[t+k-L_i, t-1] + Q_i[t+k-L_i, t+k]\right)^+ \tag{3.6}$$

Before presenting the formulation, we assume the following: the longest response window w_n does not exceed the shortest of the lead times L_i; i.e., $w_n \leq \min_{1 \leq i \leq m} L_i$. This is a plausible assumption because if there exists any product j for which the lead time of component i satisfies $L_i < w_j$, that component i can be replenished to fill the orders of product j before its response time window w_j. Hence, the component i will not be considered in the allocation problem for product j.

We consider an infinite horizon, and we shall focus on stationary random data; i.e., $(P_{1t}, P_{2t}, \ldots, P_{nt})$ and $(R_{jt1}, R_{jt2}, R_{jt3}, R_{jt4})$ are invariant in distribution over time, and hence denoted by $(P_1, P_2, \ldots, P_n)$, and $(R_{j1}, R_{j2}, R_{j3}, R_{j4})$. Moreover, the on-hand inventory level of component i in (3.6) becomes

$$\left(S_i - D_i[L_i - k] + Q_i[L_i + 1]\right)^+ \tag{3.7}$$

3.3 Risk-Adjusted Two-Stage Stochastic Programming Formulation

We consider the following problem with a chance constraint:

$$\min_{S=(S_1,\ldots,S_m)^T \in \Re^m} \quad c^T \mathbf{S} + \mathrm{E}\left[Q(\mathrm{S}, \varepsilon)\right]$$

$$\text{s.t.} \quad \mathrm{Prob}\left\{Q(\mathbf{S}, \varepsilon) \le \eta\right\} \ge 1 - \alpha \tag{3.8}$$

$$\mathbf{S} \ge \mathrm{S}_{\text{safe}}$$

where $S_{safe} = z_\alpha \, \sigma_D \sqrt{L_i}$. α is significance level where $\alpha \in (0,1)$. z_α is the α-quantile of the standard normal distribution. For a realization $\tilde{\varepsilon} = \left(\tilde{P}_1, \ldots, \tilde{P}_n, \tilde{R}_{11}, \ldots, \tilde{R}_{n4}\right)$ of ε , the second-stage cost $Q(\mathbf{S}, \tilde{\varepsilon})$ is given by

$$\min \quad q^T u$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \sum_{l=0}^{k} b'_{ij2} \, x^r_{jl} \le \left(S_i - \tilde{D}_i[L_i - k] + \tilde{Q}_i[L_i + 1]\right)^+ \tag{3.9a}$$

$$\sum_{j=1}^{n} \sum_{l=0}^{k} b_{ij} \, x^m_{jl} \le \left(S_i - \tilde{D}_i[L_i - k]\right)^+ \tag{3.9b}$$

for $k = 0, 1, \ldots, w_n$ and $i = 1, \ldots, m$

$$\sum_{l=0}^{w_j} \left(x^r_{jl} + x^m_{jl}\right) + u_j = \left(\tilde{P}_j - \tilde{R}_{j1}\right)^+ \text{ for } j = 1, \ldots, n \tag{3.9c}$$

$$\sum_{l=0}^{w_j} x^r_{jl} \le \tilde{R}_{j2} \quad \text{for } j = 1, \ldots, n \tag{3.9d}$$

$$x^r_{jl} \ge 0, \; x^m_{jl} \ge 0, \; u_j \ge 0 \text{ for } l = 0, \ldots, w_j \text{ and } j = 1, \ldots, n.$$

In the following, we define the notation in (3.8) and (3.9). $\mathbf{c} = (c_1, \ldots, c_m)^T$ is vector of procurement costs per unit of the m components, $\mathbf{S} = (S_1, \ldots, S_m)^T$ is vector of base-stock levels of the m components, and η is upper bound on random second-stage cost

$Q(\mathbf{S}, \varepsilon)$. The first-stage here-and-now decisions are the base-stock levels $(S_1, ..., S_m)$, and these decisions are made before observing the random data. Furthermore, $\mathbf{q} = (q_1, ..., q_n)^T$ is vector of shortage costs per unit of n products, and $\mathbf{u} = (u_1, ..., u_n)^T$ is vector of shortage amounts of the n products. The second-stage wait-and-see decisions are the remanufactured and manufactured amounts x_{jl}^r and x_{jl}^m of product j $(j = 1, \ldots, n)$ respectively, within its response time window $l = 0, 1, \ldots, w_j$, and the shortage amounts u_j. The second-stage decisions are made after observing the random demands and the random amounts of cores that fall into each of the four quality levels. Moreover, (3.9a) and (3.9b) imply that the amount of component i used for remanufacturing and manufacturing within response time windows cannot exceed its on-hand inventory level respectively; (3.9c) implies that the total remanufactured and manufactured amounts of product j within its response time window w_j plus the shortage amount has to be equal to the net demand $(\tilde{P}_j - \tilde{R}_{j1})$ for product j, because the cores of quality level 1 (i.e., $\tilde{R}_{j1}$) are ready to fill the net demand for product j after only minor servicing. Furtheromore, (3.9d) implies that the total remanufactured amounts of product j within w_j cannot exceed the remanufacturable amount $\tilde{R}_{j2}$ for product j. Additionally, in case all penalty costs are $q_j = 1$, the objective function in (3.9) divided by the sum of the expected demands for all n products equals the expected average no-fill rate (i.e., the complement of fill rate with respect to one).

The formulation (3.8) provides a risk-averse approach to the problem; i.e., it minimizes the random second-stage cost $Q(\mathbf{S}, \varepsilon)$ on average, while controlling its upper limit η for different realizations of the random data. A well-known problem of such a formulation is that chance constraints usually define non-convex feasible sets. It was suggested in Rockafellar and Uryasev [40] to replace chance constraints by conditional value-at-risk constraints, where the Conditional Value-at-Risk of a random variable Z at significance level α is defined as

$$\mathrm{CV@R}_{\alpha}[Z] := \inf_{t \in \Re} \left\{ t + \alpha^{-1} \mathrm{E}[Z - t]^{+} \right\}. \tag{3.10}$$

It was further shown in [40] that (3.10) is a convex conservative approximation to its corresponding chance constraint; i.e., the feasible set defined by $\text{CV@R}_\alpha[Z] \le \eta$ is contained in the feasible set defined by $\text{Prob}[Z \le \eta] \ge 1-\alpha$. Therefore, in our analysis, we will replace the chance constraint in (3.8) by its corresponding CV@R_α constraint.

Ignoring the chance constraint in (3.8), the formulations in (3.8) and (3.9) satisfy the well-known *relatively complete recourse* assumption; i.e., given any feasible first-stage solution $(S_1, \ldots, S_m)$, there exists a feasible second-stage solution $(x^r_{jl}, x^m_{jl}, u_j,$ $(j = 1, \ldots, n$ and $l = 0, \ldots, w_j)$ for almost every (a.e.) realization of ε. To see this, consider the worst-case situation in which a feasible solution with $S_i \ge S_{i,safe}$ for each component *i* for (3.8) is given, but the right-hand-sides in (3.9a) and (3.9b) are all zero; i.e., there is no on-hand inventory for any component *i*. Then, for a.e. realization of ε, $x^r_{jl} = 0$, $x^m_{jl} = 0$, and $u_j = \left(\widetilde{P}_j - \widetilde{R}_{j1}\right)^+$ for $j = 1, \ldots, n$ and $l = 0, \ldots, w_j$ constitutes a feasible solution for (3.9). However, the chance constraint and consequently the CV@R_α constraint can make (3.8) infeasible. Therefore, we relax the CV@R_α constraint as follows. Let

$$\rho_\lambda[Q(S,\varepsilon)] := (1-\lambda)\,\text{E}[Q(S,\varepsilon)] + \lambda\,\text{CV@R}_\alpha[Q(S,\varepsilon)] \tag{3.11}$$

be a real-valued function of the random variable $Q(\mathbf{S},\varepsilon)$, where $\text{E}[Q(\mathbf{S},\varepsilon)]$ is assumed to be well-defined and finite. In (3.11), $\lambda \in [0,1]$ is a parameter that can be tuned for a tradeoff between minimizing on average and risk control. Using (3.10) and (3.11), we reformulate the first-stage problem (3.8) as follows, which we will use throughout the study:

$$\min_{\mathrm{S} \ge \mathrm{S}_{\text{safe}},\ t \in \Re} \mathbf{c}^T\mathbf{S} + \lambda t + \text{E}\{V(\mathbf{S},\varepsilon)\} \tag{3.12}$$

where $V = (\mathbf{S},\varepsilon) = (1-\lambda)\,Q(\mathbf{S},\varepsilon) + \lambda\alpha^{-1}[Q(\mathbf{S},\varepsilon) - t]^+$. Now, the second-stage objective function in (3.9) is given by $(1-\lambda)\,\mathbf{q}^T\mathbf{u} + \lambda\alpha^{-1}\left[\mathbf{q}^T\mathbf{u} - t\right]$. By introducing a new variable v such that $v \ge \mathbf{q}^T\mathbf{u} - t$ and $v \ge 0$, the formulation in (3.9) becomes

$$\min \quad (1-\lambda)\mathbf{q}^T\mathbf{u}+\lambda\alpha^{-1}v \tag{3.13}$$

$$\text{s.t.} \quad \sum_{j=1}^{n}\sum_{l=0}^{k} b'_{ij2}x^{r}_{jl} \le \left(S_i - \widetilde{D}_i[L_i - k] + \widetilde{Q}_i[L_i + 1]\right)^{+}$$

for $k = 0,1, ...,w_n$ and $i = 1, ..., m$

$$\sum_{j=1}^{n}\sum_{l=0}^{k} b_{ij}x^{m}_{jl} \le \left(S_i - \widetilde{D}_i[L_i - k]\right)^{+}$$

for $k = 0,1, ...,w_n$ and $i = 1, ..., m$

$$\sum_{l=0}^{w_j}\left(x^{r}_{jl} + x^{m}_{jl}\right) + u_j = \left(\widetilde{P}_j - \widetilde{R}_{j1}\right)^{+} \text{ for } j = 1,\ldots,n$$

$$\sum_{l=0}^{w_j} x^{r}_{jl} \le \widetilde{R}_{j2} \quad \text{for } j = 1,\ldots,n$$

$$\mathbf{q}^T\mathbf{u} - v \le t$$

$$x^{r}_{jl} \ge 0,\ x^{m}_{jl} \ge 0,\ u_j \ge 0,\ v \ge 0 \ \text{ for } l = 0, ..,w_j \text{ and } j = 1, ..., n.$$

We assume that we can sample from the joint distributions of $(P_1, P_2, \ldots, P_n)$ and $(R_{j1}, R_{j2}, R_{j3}, R_{j4})$ for $j = 1,\ldots,n$ through Monte Carlo simulation and solve the problems in (3.12) and (3.13) through the sample average approximation method combined with the L-shaped algorithm.

4 SAMPLE AVERAGE APPROXIMATION AND THE L-SHAPED METHODS

In this section, based on [36], [38], and [41], we give further details on the Sample Average Approximation method, and the L-shaped algorithm.

4.1 Sample Average Approximation Method

The main objective of Sample Average Approximation (SAA) method for solving stochastic programs is as follows [36, 41].

Initially, a sample $\xi^1, \ldots, \xi^N$ of N realizations of the random vector $\xi(\omega)$ is generated, therefore, the expected value function $\mathrm{E}[Q(x, \xi(\omega))]$ is approximated by the sample average function $N^{-1} \sum_{n=1}^{N} Q(x, \xi^n)$. The obtained sample average approximation

$$\underset{x \in X}{\mathrm{Min}} \left\{ \hat{g}_N(x) := c^T x + N^{-1} \sum_{n=1}^{N} Q(x, \xi^n) \right\} \tag{4.1}$$

of the stochastic program (3.1) is then solved by a deterministic optimization method.

4.2 L-Shaped Method

The L-shaped method which is a decomposition technique, is helpful to solve problems that have the form of a master (main) problem and many subproblems. This technique consists of a number of iterations, and combined solution of it approaches in a finite number of iterations to the optimum.

Except for the last iteration, each iteration adds one or many new constraints to the main problem that limits the set of feasible values for the related parameters. They are called *feasibility* or *optimality cuts*; the optimality cuts cut-off solutions that cannot be optimal. As the iterations proceed, the upper bound increases for a minimization problem. When the upper and lower bound difference is less than the predefined tolerance level, the process terminates and shows the best feasible solution found.

The essential idea of the L-shaped method is to approximate the nonlinear term in the objective. The fundamental concept of this approach is that, since the nonlinear objective term involves a solution of all second-stage recourse linear programs, we would like to avoid many function assessments for it. Hence, that term is used to build a master problem in *s,* yet only the recourse function is precisely assessed as a subproblem.

The following extensive form of a stochastic program is considered with *S* realizations, and p_s be the probability that the s^{th} realization takes place:

$$\begin{aligned} \min \quad & c^T(S,t) = \sum_{s=1}^{S} p_s q_s^T y_s \\ \text{s.t.} \quad & A(S,t) = b, \\ & T_s(S,t) + W y_s = h_s, \qquad s = 1,\ldots\ldots,S. \\ & S \geq S_{safe}\, y_s \geq 0, t \in \Re \end{aligned} \tag{4.2}$$

The above-stated structure of this extensive form has a L-shape and this algorithm is defined by [38] as follows.

Step 0. Initialization: Set $r = s = v = 0$, where *r* is the index for the feasiblity cuts, *s* is the index for the optimality cuts, and *v* is the number of iterations.

Step 1. Set $v = v + 1$. Solve the following master LP (4.3), where $\theta = \mathrm{E}[v(s,\xi)]$. We add a lower bound t= -50,000,000, because otherwise the optimal objective value would be $-\infty$.

$$\begin{aligned} \min \quad & c^T S + \lambda t + \theta \\ \text{s.t.} \quad & t \geq -50{,}000{,}000 \end{aligned} \tag{4.3}$$

$$S \geq S_{safe}\, t \in \Re, \theta \in \Re$$

$$D_l(S,t) \geq d_l, \qquad l = 1, \ldots\ldots, r,$$

$$E_l(S,t) + \theta \geq e_l, \qquad l = 1, \ldots\ldots, s',$$

$$S \geq S_{safe}\ \theta \in \Re,\ t \in \Re$$

Let (x^v, θ^v) be an optimal solution. If no constraint on θ^v is present in the above formulation, θ^v is set equal to $-\infty$ and is not assessed in the computation of S^v and t^v.

Step 2. For each scenario $s = 1, \ldots, S$. Solve the following subproblem.

$$\min \qquad w' = e^T v^+ + e^T v^- \tag{4.4}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \sum_{l=0}^{k} b'_{ij2} x^r_{jl} + v^+_{jl} - v^-_{jl} = \left(S_i - \widetilde{D}_{i,s}[L_i - k] + \widetilde{Q}_{i,s}[L_i + 1]\right)^+$$

for $k = 0, 1, \ldots, w_n$ and $i = 1, \ldots, m$

$$\sum_{j=1}^{n} \sum_{l=0}^{k} b_{ij} x^m_{jl} + v^+_{jl} - v^-_{jl} = \left(S_i - \widetilde{D}_{i,s}[L_i - k]\right)^+$$

for $k = 0, 1, \ldots, w_n$ and $i = 1, \ldots, m$

$$\sum_{l=0}^{w_j} \left(x^r_{jl} + x^m_{jl}\right) + u_j = \left(\widetilde{P}_{j,s} - \widetilde{R}_{j1,s}\right)^+ \text{ for } j = 1, \ldots, n$$

$$\sum_{l=0}^{w_j} x^r_{jl} + v^+_{jl} + v^-_{jl} = \widetilde{R}_{j2,s} \quad \text{for } j = 1, \ldots, n$$

$$\mathbf{q}^T \mathbf{u} - v \leq t$$

$$x^r_{jl} \geq 0,\ x^m_{jl} \geq 0,\ u_j \geq 0,\ v \geq 0,\ v^+ \geq 0,\ v^- \geq 0, \text{ for } l = 0, .., w_j \text{ and } j = 1, \ldots, n.$$

in which $e^T = (1, \ldots, 1)$ until for some *s,* the optimal value $w' > 0$. In this case, σ^v_s is the vector of simplex multipliers and define

$$D_{r+1} = \left(\sigma^v_s\right)^T T_s$$

and

$$d_{r+1} = \left(\sigma_s^v \right)^T h_s$$

to generate a feasibility cut, and set $r = r+1$. T_s is a matrix which consists of the coefficient of the first-stage decision variables, namely S_i and t, and h_s is a vector that consists of the realizations of the random right-hand sides in (4.4). That is, $\widetilde{D}_{i,s}$, $\widetilde{Q}_{i,s}$, $\widetilde{P}_{j,s}$, $\widetilde{R}_{j1,s}$ and $\widetilde{R}_{j2,s}$. Add the resulting feasibility constraint to the master problem and return to Step 1. If for all s, $w' = 0$, go to Step 3.

Note that because for our problem, the relatively complete recourse problem is satisfied, we skip step 2 in our application.

Step 3. For $s = 1, \ldots, S$ solve the LP

$$\min \quad (1-\lambda)\mathbf{q}^T\mathbf{u} + \lambda\alpha^{-1}v \tag{4.5}$$

$$\text{s.t.} \quad \sum_{j=1}^{n}\sum_{l=0}^{k} b'_{ij2}x^r_{jl} \le \left(S_i - \widetilde{D}_{i,s}[L_i - k] + \widetilde{Q}_{i,s}[L_i + 1]\right)^+$$

$$\text{for } k = 0,1,\ldots,w_n \text{ and } i = 1,\ldots,m$$

$$\sum_{j=1}^{n}\sum_{l=0}^{k} b_{ij}x^m_{jl} \le \left(S_i - \widetilde{D}_{i,s}[L_i - k]\right)^+$$

$$\text{for } k = 0,1,\ldots,w_n \text{ and } i = 1,\ldots,m$$

$$\sum_{l=0}^{w_j}\left(x^r_{jl} + x^m_{jl}\right) + u_j = \left(\widetilde{P}_{j,s} - \widetilde{R}_{j1,s}\right)^+ \text{ for } j = 1,\ldots,n$$

$$\sum_{l=0}^{w_j} x^r_{jl} \le \widetilde{R}_{j2,s} \quad \text{for } j = 1,\ldots,n$$

$$\mathbf{q}^T\mathbf{u} - v \le t$$

$$x^r_{jl} \ge 0,\ x^m_{jl} \ge 0,\ u_j \ge 0,\ v \ge 0, \text{ for } l = 0,\ldots,w_j \text{ and } j = 1,\ldots,n.$$

Let π_s^v be the vector of simplex multipliers obtained at the optimal solution of (4.5) Define

$$E_{s'+1} = \sum_{s=1}^{S} p_s \cdot \left(\pi_s^v\right)^T T_s .$$

and

$$e_{s'+1} = \sum_{s=1}^{S} p_s \cdot \left(\pi_s^v\right)^T h_s .$$

Note that because we use sample average approximation method $p_s = 1/N$ for our application, where N denotes the Monte Carlo sample size.

Let $w^v = e_{s'+1} - E_{s'+1}\left(s^v, t^v\right)$. If $\theta^v \geq w^v$, terminate; $\left(s^v, t^v\right)$ is an optimal solution, else set $s' = s' + 1$, add the optimality cut to the master problem, and return to Step 1. Firstly, feasibility cuts are added to determine $\left\{\left(s^v, t^v\right) Q(x) < +\infty\right\}$, and then optimality cuts that are linear approximation to Q on its domain of finiteness are added to the model.

5 COMPUTATIONAL RESULTS

5.1 Test Problems from the Literature

We implement all experiments on a PC with Windows XP, Intel Pentium 4 CPU of 1.60 GHz, and 1.00 GB RAM. Because for now the instances that are detailed below are small, the CPU times are negligible, and hence they are not presented.

The parameter values for our numerical study is taken from the existing research of [33]. In this study, a family of three desktop computers are assembled from a set of 12 different components. Components used in the assembly of a finished product are procured from outside suppliers, and also if available some components are taken form parts/items inventory. The supplier leadtimes are assumed to be deterministic that represents the time required to manufacture the component and ship it from a supplier warehouse to the manufacturing plant. The lead times, the unit acquisition costs and the bill-of-materials structure are given in Table 5.1, and Table 5.2, respectively.

Table 5.1. Components, Lead Times, and Unit Acquisition Costs For the Example Configure-to-Order Systems In [33]

i	Description	Lead time	Unit acquisition cost
1	Base unit	5	215
2	128 MB Card	15	232
3	450 MHz board	12	246
4	500 MHz board	12	316
5	600 MHz board	12	639
6	7 GB disk drive	18	215
7	13 GB disk drive	18	250
8	Preload A	4	90
9	Preload B	4	90
10	CD ROM	10	126
11	Video graphics card	6	90
12	Ethernet card	10	90

We assume that the demands for the three products are multivariate normally distributed with the mean vector (150, 100, 125), the variances (750, 625, 675), and the correlations between the demands are randomly generated from the uniform distribution on (-1, 1). The response window times are first considered as w_1= 1, w_2= 2, w_3= 3 for products 1, 2, and 3. Later, we also consider w_1= w_2= w_3= 0, which enables us to observe immediate fill rates for products 1, 2, and 3. Moreover, the amounts of cores (returned products) are considered as (10, 15, 20) for products 1, 2, and 3. We assume that for each product, the numbers of cores that fall into the quality levels 1, 2, 3, and 4 follow multivariate normal distributions with the following mean vectors and variances: (1200, 1500, 2500, 2200) as the mean vector and (300, 500, 1000, 560) as the variances for the product 1, (3500, 1200, 2200, 1800) as the mean vector and (1000, 600, 1200, 900) as the variances for product 2, and (1500, 1500, 1500, 300) as the mean vector and (900, 900, 900, 125) as the variances for product 3. For each product, the correlations between the quality levels are again randomly generated from the uniform distribution (-1, 1).

Table 5.2. Bill-of-Materials Structure For the Example Configure-to-Order Systems In [33]

i	Description	Product 1	Product 2	Product 3
1	Base unit	1.0	1.0	1.0
2	128 MB Card	1.0	1.0	1.0
3	450 MHz board	1.0	-	-
4	500 MHz board	-	1.0	-
5	600 MHz board	-	-	1.0
6	7 GB disk drive	1.0	0.4	-
7	13 GB disk drive	-	0.6	1.0
8	Preload A	0.7	0.5	0.3
9	Preload B	0.3	0.5	0.7
10	CD ROM	1.0	1.0	1.0
11	Video graphics card	-	0.3	0.6
12	Ethernet card	-	0.2	0.5

We denote the realizations of these multivariate normally distributed random variables by $\tilde{\delta}_1 = (\tilde{\delta}_{1,1}, \tilde{\delta}_{1,2}, \tilde{\delta}_{1,3}, \tilde{\delta}_{1,4})$, $\tilde{\delta}_2 = (\tilde{\delta}_{2,1}, \tilde{\delta}_{2,2}, \tilde{\delta}_{2,3}, \tilde{\delta}_{2,4})$, and $\tilde{\delta}_3 = (\tilde{\delta}_{3,1}, \tilde{\delta}_{3,2}, \tilde{\delta}_{3,3}, \tilde{\delta}_{3,4})$. Note that because for a fixed product j, the sum of the fractions of cores that fall into the four quality levels has to be equal to one, we compute these four fractions from the $\tilde{\delta}_j$

through $\tilde{\delta}_{j,1} / \tilde{\rho}_j$, $\tilde{\delta}_{j,2} / \tilde{\rho}_j$, $\tilde{\delta}_{j,3} / \tilde{\rho}_j$, and $\tilde{\delta}_{j,4} / \tilde{\rho}_j$,where $\tilde{\rho}_j = \tilde{\delta}_{j,1} + \tilde{\delta}_{j,2} + \tilde{\delta}_{j,3} + \tilde{\delta}_{j,4}$. Then, for example, we find the realizations of the cores of quality levels 1, 2, 3, and 4 for product 1 by $10 \times \tilde{\delta}_{1,1} / \tilde{\rho}_1$, $10 \times \tilde{\delta}_{1,2} / \tilde{\rho}_1$, $10 \times \tilde{\delta}_{1,3} / \tilde{\rho}_1$, and $10 \times \tilde{\delta}_{1,4} / \tilde{\rho}_1$, respectively. The penalty costs are $\mathbf{q}^T = (22480, 25200, 33020)$.

We consider λ and α in (3.12) and (3.13) as parameters, and solve these problems for several values of λ and α. We first solve the problems for w_1= 1, w_2= 2, w_3= 3 (instance 1), and then repeat the experiments for w_1= w_2= w_3= 0 (instance 2). We obtain very similar results for both instances, hence we present results in Figures 5.1 and 5.2 only for instance 1. Note that after sampling the demands and the random amounts of cores that fall into each quality levels, the problems in (3.12) and (3.13) are formulated as two linear programming problems, which are then solved through CPLEX 12.2 using MATLAB as its interface.

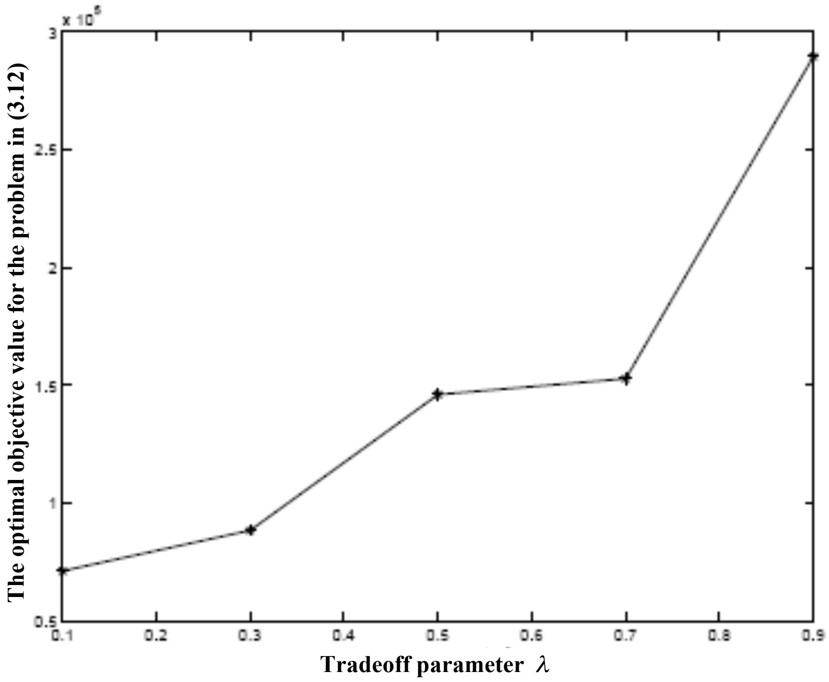


Figure 5.1. Effects of Changing λ on the Optimal Objective Value of the First-Stage Problem In (3.12) : α = 10% and Fixed.

Note that increasing λ or decreasing α would increase the relative importance of the risk adjustment term, and hence would lead to a more conservative system; for α, this can also be seen from the chance constraint in (3.8). Both Figures 5.1 and 5.2 reflect the increase in the conservatism of the system because the optimal objective value of the first-stage problem in (3.12) increases as λ increases in Figure 5.1, and it increases as α decreases in Figure 5.2.

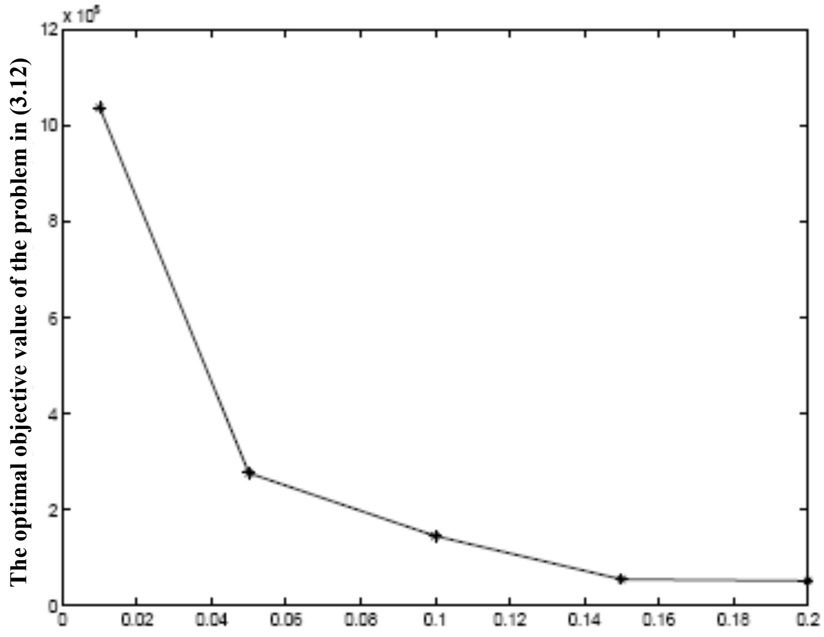


Figure 5.2. Effects of Changing α on the Optimal Objective Value of the First-Stage Problem In (3.12) : $\lambda = 0.5$ and Fixed.

5.2 Test Problems from Real-Life (Actual Data)

The choice of paramater values for our numerical study is based on actual data provided by an International Data Corporation*, in which the data includes the quantity of desktop PCs and portable PCs sold within 2008 and 2011 in Turkey. In order to demonstrate the demand distributions of the acquired data, the input analyzer of Arena 10.0 is used, and the histogram and distribution summaries of demands for five products are presented in Appendix A.

In our numerical analysis, we consider a family of five personal computers, i.e. desktop PCs and portable PCs, are assembled from a set of 12 different components. Some components are common to different products. Components used in the assembly of a finished product are procured from external suppliers, and also if available some components are taken form parts/items inventory that the components of third quality cores take place.

Table 5.3. Average Lead Times, and Unit Acquisition Costs of Each Component.

i	Description	Lead time	Unit acquisition cost
1	Base unit	6	214
2	Processor: 2.2 GHz	15	90
3	Processor: 3.4 GHz	15	117
4	Memory: 2 GB	12	58
5	Memory: 4 GB	12	75
6	Memory: 8 GB	12	105
7	Hard Drive:500 GB	18	100
8	Hard Drive:1 TB	18	130
9	Operating system 1.0	7	186
10	Operating system 2.0	7	249
11	Software *(Common programmes)*	5	180
12	Optical drive	17	47

In this research, like [33], the supplier leadtimes are assumed to be deterministic which represents the time required to manufacture the component and ship it from a supplier warehouse to the production plant. The average lead times, the unit acquisition costs of each component and the bill-of-materials of each product are illustrated in Table 5.3, and Table 5.4, respectively.

The first stage in analyzing multivariate data is computing the mean vector and the variance-covariance matrix. The mean vector consists of the mean of each variable and the variance-covariance matrix consists of the variances of the variables along the main diagonal and the covariances between each pair of variables in the other matrix positions.

Covariance is a measure of how much two variables change or vary together. To demonstrate the relationship between demands and products, we present the covariance matrices in our research. The definitions, notations and formulations of mean vector and variance-covariance matrix are available in Appendix B.

** Confidential data cannot be disclosed.*

Table 5.4. Bill-of-Materials of Each Product.

i	**Description**	***P:***	**1**	**2**	**3**	**4**	**5**
1	Base unit		1	1	1	1	1
2	Processor: 2.2 GHz		1	1	-	-	-
3	Processor: 3.4 GHz		-	-	1	1	1
4	Memory: 2 GB		1	-	-	-	-
5	Memory: 4 GB		-	1	-	1	-
6	Memory: 8 GB		-	-	1	-	1
7	Hard Drive:500 GB		-	1	-	-	-
8	Hard Drive:1 TB		1	-	1	1	1
9	Operating system 1.0		-	1	1	-	-
10	Operating system 2.0		1	-	-	1	1
11	Software *(Common programmes)*		1	1	1	1	1
12	Optical drive		-	1	-	1	1

We consider the set of 15 observations (demand periods), measuring 5 variables (products) is described by its mean vector and variance-covariance matrix as illustrated in Appendix C, and the results are as follows:

$$\sum = \begin{bmatrix} 215.578.429 & 154.121.319 & -99.430.121 & 13.577.471 & -274.639 \\ 154.121.319 & 277.251.123 & -18.875.463 & 17.059.883 & 1.763.616 \\ -99.430.121 & -18.875.463 & 210.154.931 & 8.133.767 & -5.757.537 \\ 13.577.471 & 17.059.883 & 8.133.767 & 12.583.302 & -2.339.957 \\ -274.639 & 1.763.616 & -5.757.537 & -2.339.957 & 2.512.310 \end{bmatrix}$$

It is clear to see that covariance values are both positive and negative in different points, and we can deduce that there are linear dependencies between variables.

We run the program with different penalty costs, and we found out that when these penalty costs are high compared to the acquisition costs, the total estimated objective value can become higher when the parameter λ decreases. Hence, we fixed the penalty costs as 1,5 times of the cost of original products, where the costs of original products are taken as follows: 921,58; 892,76; 932,82; 1012,52; 1042,52. We run the program for 6 different λ values and two different α values, and in Figure 5.3, we obtained the following changes

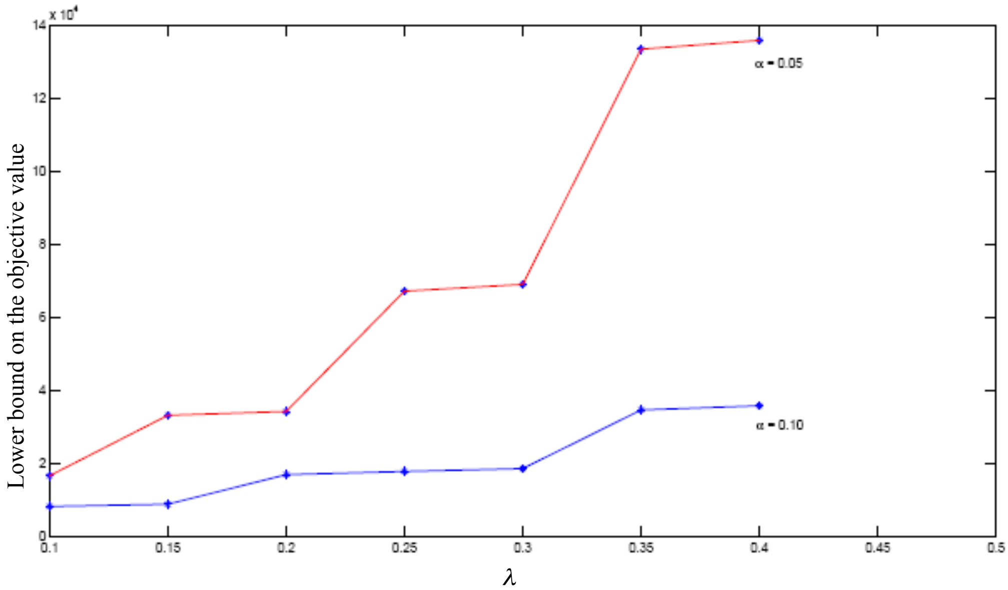


Figure 5.3. Effects of Changing α and λ on the Lower Bound of the Objective Value

When the parameter α decreases, our problem becomes more conservative, yet we cannot say this for λ due to the fact that it highly depends on the parameter values of the λ.

6 CONCLUSIONS

In this dissertation, we consider a multi-component, multi-product, periodic-review (re) assemble-to-order system, and find the joint optimal base-stock levels and component allocation policies in a risk-adjusted environment. We model this problem through a risk-adjusted two-stage stochastic programming problem, where the first stage decisions are the base-stock levels for all components, and the second-stage decisions are the allocations of components to different products. Risk adjustment is achieved through the conditional-value-at-risk constraint. We solve the resulting problem through the sample average approximation combined with the L-shaped method. Our preliminary numerical results are intuitively sound: as we make the system more conservative (by increasing the parameter λ or by decreasing the parameter α), our expected total optimal objective value increases.

Further research should include more numerical results by using different multivariate distributions for demands. The majority of the published studies which consider joint production systems, assume that there is no quality differences between manufactured and remanufactured products that may have an effect on the consumers' preferences. Yet, consumers can be heterogeneous in their willingness to pay and they do not necessarily value remanufactured products as good as brand new ones. Hence, this perception of consumers can lead to market segmentations and multiple demand classes for products. Consideration of segmented markets for manufactured and remanufactured products is a further important issue.

REFERENCES

[1] Inderfurth, K. and Van der Laan, E., "Leadtime Effects and Policy Improvement for Stochastic Inventory Control with Remanufacturing", *International Journal of Production Economics*, **71**, 381−390, (2001).

[2] Geyer, R., Van Wassenhove, L. and Atasu, A., "The Economics of Remanufacturing Under Limited Durability and Finite Product Life Cycles", *Management Science*, **53** (1), 88−100, (2007).

[3] Inderfurth, K., "Optimal Policies in Hybrid Manufacturing/Remanufacturing Systems with Product Substitution", *International Journal of Production Economics*, **90**, 325−343, (2004).

[4] Ferrer, G. and Ayres, R.U., "Analysis: The Impact of Remanufacturing in the Economy", *Ecological Economics*, **32**, 413−429, (2000).

[5] Östlin, J., Sundin, E. and Björkman, M., "Importance of Closed-loop Supply Chain Relationships for Product Remanufacturing", *International Journal of Production Economics*, **115**, 336−348, (2008).

[6] Seitz, M.A. and Peattie, K., "Meeting the Closed-Loop Challenge: The Case of Remanufacturing", *California Management Review,* **46** (2), 74−89, (2004).

[7] Van der Laan, E. and Teunter, R.H., "Simple Heuristics for Push and Pull Remanufacturing Policies", *European Journal of Operational Research*, **175**, 1084−1102, (2006).

[8] Guide, V.D.R., Jayaraman, V. and Linton, J.D., "Building Contingency Planning for Closed-loop Supply Chains with Product Recovery", *Journal of Operations Management*, **21**, 259−279, (2003).

[9] Song, J., "Order-based Backorders and their Implications in Multi-Item Inventory Systems", *Management Science*, **48** (4), 499−516, (2002).

[10] Song, J, Xu, S. and Liu, B., "Order-Fulfillment Performance Measures in an Assemble-to-Order System with Stochastic Leadtimes", *Operations Research*, **47** (1), 131−149, (1999).

[11] Li, P., Arellano-Garcia, H. and Wozny, G., "Chance Constrained Programming Approach to Process Optimization Under Uncertainty", *Computers & Chemical Engineering*, **32**, 25−45, (2008).

[12] Gotoh, J. and Takano, Y., "Newsvendor Solutions via Conditional Value-at-Risk Minimization", *European Journal of Operational Research,* **179**, 80−96, (2007).

[13] Zhang, D., Xu, H. and Wu, Y., "Single and Multi-Period Optimal Inventory Control Models with Risk-Averse Constraints", *European Journal of Operational Research,* **199**, 420−434, (2009).

[14] Ahmed, S., Çakmak, U. and Shapiro, A., "Coherent Risk Measures in Inventory Problems", *European Journal on Operational Research*, **182**, 226−238, (2007).

[15] Shapiro, A., "Analysis of Stochastic Dual Dynamic Programming Method", *European Journal of Operational Research*, **209**, 63−72, (2011).

[16] Agrawal, N. and Cohen, M.A., "Optimal Material Control in an Assembly System with Component Commonality", *Naval Research Logistics*, **48**, 409−429, (2001).

[17] Song, J. and Yao, D.D., "Performance Anaysis and Optimization of Assemble-to-Order Systems with Random Lead Times", *Operations Research*, **50** (5), 889−903, (2002).

[18] Souza, G., Ketzenberg, M. and Guide, V.D.R., "Capacitated Remanufacturing with Service Level Constraints", *Productions & Operations Manage*ment, **11** (2), 231−248, (2002).

[19] Van Wassenhove, L. and Zikopoulos, C.,"On the Effect of Quality Overestimation in Remanufacturing", *International Journal of Production Research*, 1−18, (2009).

[20] Toktay, B., Wein, L. and Zenios, S.,"Inventory Management of Remanufacturable Products", *Management Science*, **46** (11), 1412−1426, (2000).

[21] Guide, V.D.R. and Van Wassenhove, L.N., "Managing Product Returns for Remanufacturing", *Production & Operations Management*, **10** (2), 142−155, (2001).

[22] Galbreth, M. R., and Blackburn, J.D., "Optimal Acquisition and Sorting Policies for Remanufacturing", *Production & Operations Management*, **15** (3), 384−392, (2006).

[23] Teunter, R.H. and Flapper, S.D.P., "Optimal Core Acquisition and Remanufacturing Policies under Uncertain Core Quality Fractions", *European Journal of Operational Research*, **210**, 241−248, (2010).

[24] Ferrer, G. and Swaminathan, J.M., "Managing New and Remanufactured Products", *Management Science*, **52** (1), 15−26, (2006).

[25] Kenné, J., Dejax, P. and Gharbi, A., "Production Planning of a Hybrid Manufacturing-Remanufacturing System under Uncertainty within a Closed-Loop Supply Chain", *International Journal of Production Economics*, **135** (1), 81−93, (2012).

[26] Mukhopadhyay, S.K. and Ma, H., "Joint Procurement and Production Decisions in Remanufacturing under Quality and Demand Uncertainty", *International Journal of Production Economics*, **120**, 5−17, (2009).

[27] Shi, J., Zhang, G. and Sha, J., “Optimal Production Planning for a Multi-Product Closed Loop System with Uncertain Demand and Return”, *Computers & Operations Research,* **38**, 641−650, (2011).

[28] Teunter, R.H. and Vlachos, D., “On the Necessity of a Disposal Option for Returned Items That Can Be Remanufactured”, *International Journal of Production Economics*, **75**, 257−266, (2002).

[29] Wang, J., Zhao, J. and Wang, X., “Optimum Policy in Hybrid Manufacturing/Remanufacturing System”, *Computers & Industrial Engineering*, **60** (3), 411−419, (2010).

[30] Aras, N., Boyaci, T. and Verter, V., “The Effect of Categorizing Returned Products in Remanufacturing”, *IEE Transactions*, **36** (4), 319−331, (2004).

[31] Inderfurth, K., “Impact of Uncertainties on Recovery Behavior in a Remanufacturing Environment”, *International Journal of Physical Distribution & Logistics Management*, **35** (5), 318−336, (2005).

[32] Song, J.S. and Zipkin, P., *Supply Chain Operations: Assemble-to-order Systems*, *Handbooks in OR & MS*, Vol. 11, A.G. de Kok and S.C. Graves, eds., Elsevier, 561−595, (2003).

[33] Cheng, F., Ettl, M., Lin, G., and Yao, D.D., “Inventory-Service Optimization in Configure-to-Order Systems”, *Manufacturing & Service Operations Management*, **4** (2), 114−132, (2002).

[34] Akcay, Y. and Xu, S.H., “Joint Inventory Replenishment and Component Allocation Optimization in an Assemble-to-Order System”, *Management Science*, **50** (1), 99−116, (2004).

[35] Hausman, W.H., Lee, H. and Zhang, A.X., “Joint Demand Fulfillment Probability in a Multi-Item Inventory System with Interdependent Order-up-to Policies”, *European Journal of Operational Research.*, **109**, 646−659, (1998).

[36] Ahmed, S., Shapiro, A. and Shapiro, E., “The Sample Average Approximation Method for Stochastic Programs with Integer Recourse”, *SIAM Journal of Optimization,* **12**, 479−502, (2002).

[37] Kall, P. and Wallace, S.W., *Stochastic Programming*. John Wiley and Sons, Chichester, England, (1994).

[38] Birge, J.R. and Louveaux, F., *Introduction to Stochastic Programming*, Springer, New York, NY, (1997).

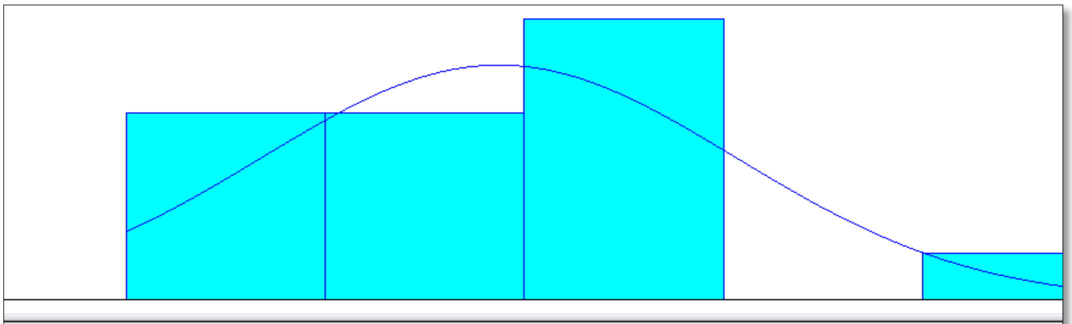
[39] Hadley, G. and Whitin, T.M., *Analysis of Inventory Systems*, Prentice Hall, Englewood Cliffs, NJ, (1963).

[40] Rockafellar, R.T. and Uryasev, S.P. “Optimization of Conditional Value-at-Risk”, *The Journal of Risk*, **2**, 21−41, (2000).

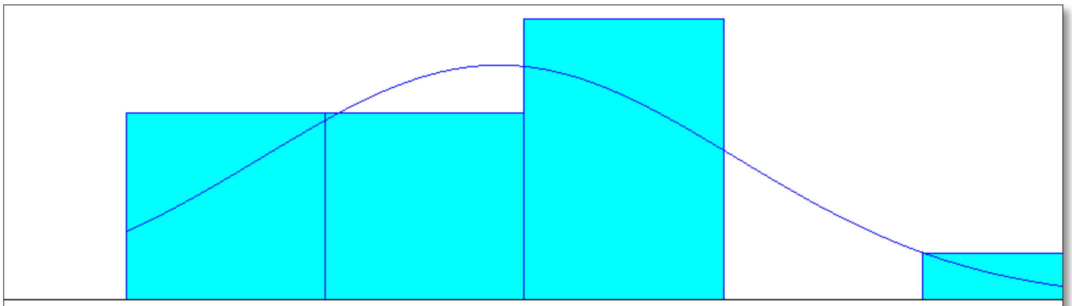
[41] Shapiro, A., Dentcheva, D. and Ruszczyński, A., *Lectures on Stochastic Programming: Modeling and Theory*, SIAM, Philadelphia, (2009).

APPENDICES

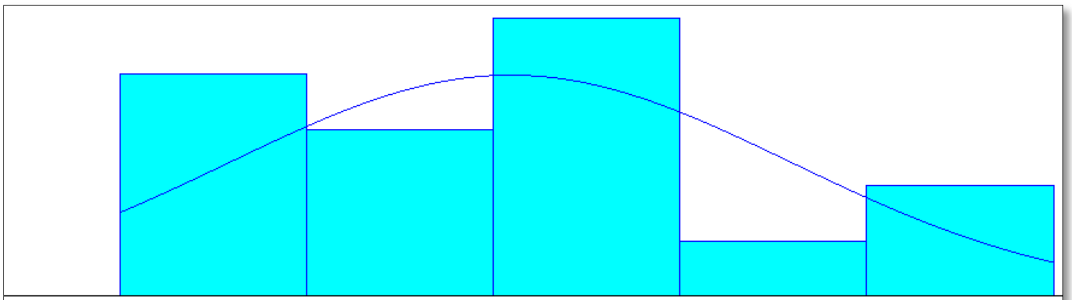
Appendix A. Demand Distribution Histograms of Five Products.



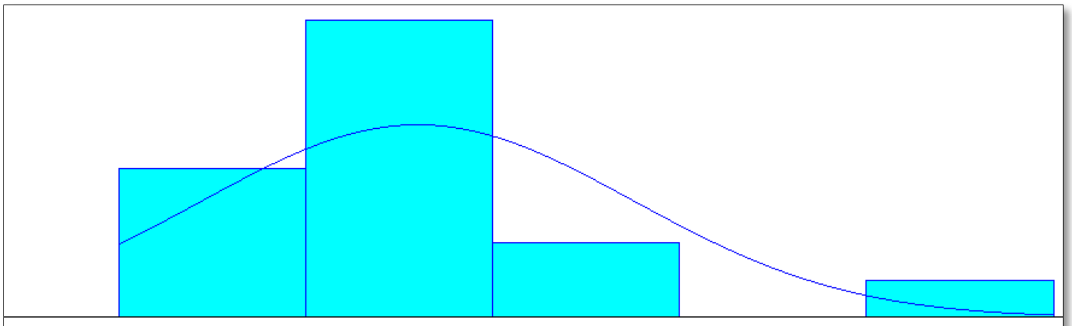
```
        Distribution Summary

Distribution:   Normal
Expression:     NORM(0, 0)
Square Error:   0.042869


Kolmogorov-Smirnov Test
  Test Statistic        = 0.246
  Corresponding p-value > 0.15

        Data Summary

Number of Data Points   = 15
Min Data Value          = 1.78e+004
Max Data Value          = 7.95e+004
Sample Mean             = 4.09e+004
Sample Std Dev          = 1.52e+004

        Histogram Summary

Histogram Range         = 1.78e+004 to 7.95e+004
Number of Intervals     = 5
```



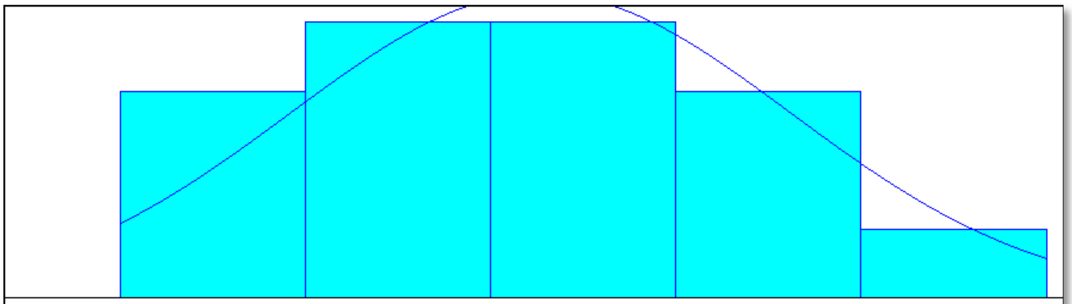
```
        Distribution Summary

Distribution:   Normal
Expression:     NORM(0, 0)
Square Error:   0.042869


Kolmogorov-Smirnov Test
  Test Statistic         = 0.246
  Corresponding p-value > 0.15

        Data Summary

Number of Data Points   = 15
Min Data Value          = 1.78e+004
Max Data Value          = 7.95e+004
Sample Mean             = 4.09e+004
Sample Std Dev          = 1.52e+004

        Histogram Summary

Histogram Range         = 1.78e+004 to 7.95e+004
Number of Intervals     = 5
```



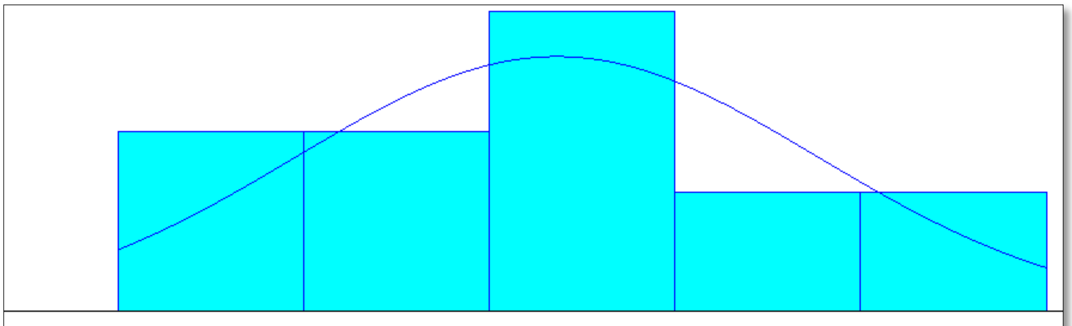
```
        Distribution Summary

Distribution:   Normal
Expression:     NORM(0, 0)
Square Error:   0.035746


Kolmogorov-Smirnov Test
  Test Statistic         = 0.218
  Corresponding p-value > 0.15

        Data Summary

Number of Data Points   = 15
Min Data Value          = 0
Max Data Value          = 5.54e+004
Sample Mean             = 2.32e+004
Sample Std Dev          = 1.72e+004

        Histogram Summary

Histogram Range         = -0.001 to 5.54e+004
Number of Intervals     = 5
```



```
        Distribution Summary

Distribution:   Normal
Expression:     NORM(0, 0)
Square Error:   0.066915


Kolmogorov-Smirnov Test
  Test Statistic        = 0.299
  Corresponding p-value = 0.114

        Data Summary

Number of Data Points   = 15
Min Data Value          = 1.84e+004
Max Data Value          = 8.14e+004
Sample Mean             = 3.85e+004
Sample Std Dev          = 1.5e+004

        Histogram Summary

Histogram Range         = 1.84e+004 to 8.14e+004
Number of Intervals     = 5
```



```
        Distribution Summary

Distribution:   Normal
Expression:     NORM(0, 0)
Square Error:   0.006119


Kolmogorov-Smirnov Test
  Test Statistic        = 0.163
  Corresponding p-value > 0.15

        Data Summary

Number of Data Points   = 15
Min Data Value          = 6.01e+003
Max Data Value          = 1.19e+004
Sample Mean             = 8.67e+003
Sample Std Dev          = 1.64e+003

        Histogram Summary

Histogram Range         = 6.01e+003 to 1.19e+004
Number of Intervals     = 5
```



```
        Distribution Summary

Distribution:   Normal
Expression:     NORM(0, 0)
Square Error:   0.017260


Kolmogorov-Smirnov Test
  Test Statistic        = 0.16
  Corresponding p-value > 0.15

        Data Summary

Number of Data Points   = 15
Min Data Value          = 0
Max Data Value          = 1.26e+004
Sample Mean             = 5.96e+003
Sample Std Dev          = 3.67e+003

        Histogram Summary

Histogram Range         = -0.001 to 1.26e+004
Number of Intervals     = 5
```

Appendix B. Definition, Formulation and Notation of a Variance-Covariance Matrix.

Let $X = (X_1, X_2, \ldots, X_p)$ be a random vector with mean vector $\mu = (\mu_1, \mu_2, \ldots, \mu_p)$. The covariance $\operatorname{cov}(X_i, X_j)$ of the pair (X_i, X_j) is a measure of the linear coupling between these two variables. The covariance of two random variables, X_i and X_j is denoted by σ_{ij} in (B1) $i, j = 1, \ldots, p$ and the covariance matrix is denoted by $\sum$ in (B2).

$$\sigma_{ij} = \operatorname{cov}(X_i, X_j) = E\left[(X_i - \mu_i)(X_j - \mu_j)\right] \qquad \text{(B1)}$$

where $\mu_i = E(X_i)$ and $\mu_j = E(X_j)$ and let,

$$\sum = \sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2p} \\ & & & \\ \sigma_{p1} & \sigma_{p2} & \cdots & \sigma_{pp} \end{bmatrix}. \qquad \text{(B2)}$$

Since $\operatorname{cov}(X_i, X_j) = \operatorname{cov}(X_j, X_i)$, we have $\sigma_{ij} = \sigma_{ji}$. Hence, $\sum$ is symmetric with $(i, j)^{th}$ and $(j, i)^{th}$ elements indicates the covariance between X_i and X_j. Moreover, since $\operatorname{var}(X_i) = \operatorname{cov}(X_i, X_i) = \sigma_{ii}$, the i^{th} diagonal place of $\sum$ includes the variance of X_i. The matrix $\sum$ is called the *Variance-Covariance Matrix* of X_i.

Appendix C. Variance-Covariance Matrix of a Sample Data

$$\sum = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & \sigma_{25} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} & \sigma_{35} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & \sigma_{45} \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & \sigma_{54} & \sigma_{55} \end{bmatrix}$$

BIOGRAPHICAL SKETCH

Eda Bilici was born in Istanbul, Turkey, on August 1, 1986. She received her high school diploma from Istek Bilge Kagan Science High School in 2004, and then she attended the undergraduate program of industrial engineering at Bahcesehir University, Istanbul, Turkey. She received her B.S. degree in industrial engineering in 2009, and then she joined the master program. Eda now works as a Cost Analyst at Abdi Ibrahim Pharmaceuticals, Istanbul, Turkey. Her current research interests include remanufacturing, hybrid production systems, risk measures, and sustainable operations.

The papers that have appeared as full text in referred proceedings of international conferences:

Angun, M. E. and Bilici, E., "Risk-Adjusted Joint Optimization of Base-Stock Levels and Component Allocation in an ATO System with Returns", in *Proceedings of the International MultiConference of Engineers and Computer Scientists 2012*, **2**, 1538–1542, Hong Kong, (2012).