COMPONENT SUBSTITUTION IN MASS CUSTOMIZATION ENVIRONMENT

(KİTLESEL ÖZELLEŞTİRME ORTAMINDA BİLEŞEN İKAMESİ)

by

Ufuk BAHÇECİ, M.S.

Thesis

Submitted in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

in

INDUSTRIAL ENGINEERING

in the

INSTITUTE OF SCIENCE AND EGINEERING

of

GALATASARAY UNIVERSITY

January 2012

COMPONENT SUBSTITUTION IN MASS CUSTOMIZATION ENVIRONMENT

(KİTLESEL ÖZELLEŞTİRME ORTAMINDA BİLEŞEN İKAMESİ)

by

Ufuk BAHÇECİ, M.S.

Thesis

Submitted in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

Date of Submission : December 9, 2011

Date of Defense Examination : January 5, 2012

Supervisor : Assoc. Prof. Dr. Orhan FEYZİOĞLU

Committee Members : Assoc. Prof. Dr. Temel ÖNCAN

Prof. Dr. İlker TOPÇU (ITU)

Assist. Prof. Dr. Müjde EROL GENEVOIS

Assoc. Prof. Dr. Gülgün KAYAKUTLU (ITU)

Acknowledgements

This thesis would not have been possible without the support of my family. At the first place, I would like to show my gratitude to my parents Güner Bahçeci and Işık Bahçeci, for guiding and supporting me throughout my life. I would like to thank to my supervisor Prof. Orhan Feyzioğlu for his supervision and patience. I would also like to thank to my advisors Prof. Temel Öncan and Prof. Ş. İlker Bilbil, for their precious time and comments. My sincere thanks go to my jury members: Prof. İlker Topçu, Prof. Müjde Erol Genevois and Prof. Gülgün Kayakutlu. Lastly, it is a pleasure for me to thank all those who have continually supported me spiritually.

In closing, I remember my mother Güner Bahçeci again who I miss and will never forget.

Ufuk BAHÇECİ
İstanbul, 09/12/2011

Table of Contents

Acknowledgements ii

Table of Contents iii

List of Symbols vi

List of Figures vii

List of Tables ix

Abstract xii

Résumé xv

Özet xviii

1 Introduction 1

2 Literature Survey 4

2.1 Mass Customization 4

2.2 Modularity 8

2.3 Component Commonality and Substitution 11

2.4 Literature Survey of Inventory Planning Models 17

3 Selected Stochastic Programming Approach 20

3.1 Two-Stage Stochastic Linear Programming Problems with Fixed Recourse 21

3.2 Conditional Value at Risk 26

4 Assemble-to-Order with Substitution in Mass Customization Environment . . . 29

4.1 Motivation 29

4.2 Problem Formulation with Expected Cost Objective 31

4.3 Demand Structure 33

4.3.1 Demand Scenarios 33
4.3.2 Methods for the Scenario Generation 35
4.4 General Case 36
4.4.1 Generalized Network Formulation with Equality Constraints . . . 36
4.4.2 Pure Network Formulation with Proportionality Constraints 41
4.5 Subcontracting Case 41
4.5.1 Generalized Network Formulation with Equality Constraints . . . 41
4.5.2 Pure Network Formulation with Proportionality Constraints 44
4.6 Spot Market Case 44
4.6.1 Generalized Network Formulation 44
4.6.2 Pure Network Formulation 45
4.7 Problem Formulation with CVaR Objective 45

5 Theoretical Analysis of Inventory Allocation Subproblems and Development of Efficient Solution Methods 47
5.1 Disconnected Subnetworks with Proportional Flow Requirements 47
5.1.1 Minimum Cost Network Flow Problem Variants 49
5.1.2 Reformulation from Complicating Constraints to Complicating Variables 51
5.1.3 Structure of the Basis 51
5.1.4 A Special Primal Simplex Algorithm 55
5.2 Bipartite Structure 60

6 Computational Results 65
6.1 Summary of the Solution Method 65
6.2 The Model Parameters 66
6.3 CPU Time Performance of the SPSA 66
6.4 The Analysis of the Stochastic Model 70
6.4.1 The Performance Metrics 70
6.4.2 The Effect of the Variation in Shortage and Substitution Costs . . 71
6.4.3 The Effects of Increasing the Number of Components and Modules 73

7 Conclusion and Further Suggestions 77

Works Related to this Thesis 81

References 82

Appendix A 94

Appendix B 110

Appendix C 123

Biographical Sketch 128

List of Symbols

ASR: “ Average Supply Ratio ”

CVaR: “ Conditional Value at Risk ”

EEV: “ Expected Result of Using the Expected Value Problem ”

EV: “ Expected Value Problem ”

EVPI: “ Expected Value of Perfect Information ”

LPS: “ Linear Programming Solver ”

MCNF: “ Minimum Cost Network Flow ”

MCPFD: “ Minimum Cost Proportional Flow Problem with Disconnected Subnetworks ”

MNF: “ Manufacturing Network Flow ”

RP: “ Here-and-Now Solution ”

SPSA: “ Special Primal Simplex Algorithm ”

VaR: “ Value at Risk ”

VSS: “ Value of Stochastic Solution ”

WS: “ Wait-and-See Solution ”

GLPK: “ GNU Linear Programming Kit ”

List of Figures

4.1 Mass customization via modular assembly 30
4.2 Geographical warehouse clusters 31
4.3 Demand decomposition tree 34
4.4 Subgraph associated with the allocation of the first component 37
4.5 Subgraph associated with the allocation of the second component 38
4.6 Subgraph associated with the allocation of the dummy component 39
4.7 Graph associated with two modules and two components with equal flow arcs emphasized 40
5.1 Modeling generalized networks with linear gain/loss 49
6.1 Integration of L-Shaped algorithm with SPSA 65
6.2 Special primal simplex algorithm 65
6.3 SPSA/LPS CPU time ratios when increasing the number of modules . . . 69
A.1 ASR for scenario [50-250][0.4-0.6] 94
A.2 ASR for scenario [50-250][0.5-0.5] 94
A.3 ASR for scenario [50-250][0.25-0.75] 95
A.4 ASR for scenario [100-200][0.4-0.6] 95
A.5 ASR for scenario [100-200][0.5-0.5] 96
A.6 ASR for scenario [100-200][0.25-0.75] 96
A.7 ASR for scenario [150-150][0.4-0.6] 97
A.8 ASR for scenario [150-150][0.25-0.75] 97
A.9 CVaR/RP for scenario [50-250][0.4-0.6] 98
A.10 CVaR/RP for scenario [50-250][0.5-0.5] 98
A.11 CVaR/RP for scenario [50-250][0.25-0.75] 99
A.12 CVaR/RP for scenario [100-200][0.4-0.6] 99
A.13 CVaR/RP for scenario [100-200][0.5-0.5] 100
A.14 CVaR/RP for scenario [100-200][0.25-0.75] 100
A.15 CVaR/RP for scenario [150-150][0.4-0.6] 101

A.16 CVaR/RP for scenario [150-150][0.25-0.75] 101
A.17 EVPI for scenario [50-250][0.4-0.6] 102
A.18 EVPI for scenario [50-250][0.5-0.5] 102
A.19 EVPI for scenario [50-250][0.25-0.75] 103
A.20 EVPI for scenario [100-200][0.4-0.6] 103
A.21 EVPI for scenario [100-200][0.5-0.5] 104
A.22 EVPI for scenario [100-200][0.25-0.75] 104
A.23 EVPI for scenario [150-150][0.4-0.6] 105
A.24 EVPI for scenario [150-150][0.25-0.75] 105
A.25 VSS for scenario [50-250][0.4-0.6] 106
A.26 VSS for scenario [50-250][0.5-0.5] 106
A.27 VSS for scenario [50-250][0.25-0.75] 107
A.28 VSS for scenario [100-200][0.4-0.6] 107
A.29 VSS for scenario [100-200][0.5-0.5] 108
A.30 VSS for scenario [100-200][0.25-0.75] 108
A.31 VSS for scenario [150-150][0.4-0.6] 109
A.32 VSS for scenario [150-150][0.25-0.75] 109

List of Tables

2.1 Literature survey for the most relevant inventory planning models 19
5.1 Literature survey for the unbalanced transportation problem 61
5.2 Performance analysis of Dantzig Wolfe decomposition method 63
5.3 Model variants and solution procedures 64
6.1 SPSA/LPS CPU time ratios for the low safety stock ($SS^o = 100\ \forall o$) case 68
6.2 SPSA/LPS CPU time ratios for the moderate safety stock ($SS^o = 150\ \forall o$) case 68
6.3 SPSA/LPS CPU time ratios for the high safety stock ($SS^o = 200\ \forall o$) case 69
6.4 The effects of increasing the number of components 75
6.5 The effects of increasing the number of modules 76
B.1 ASR values for low shortage and low substitution costs (F,V) 110
B.2 VSS values for low shortage and low substitution costs (F,V) 110
B.3 EVPI values for low shortage and low substitution costs (F,V) 110
B.4 CVaR/RP values for low shortage and low substitution costs (F,V) 111
B.5 ASR values for low shortage and low substitution costs (V,F) 111
B.6 VSS values for low shortage and low substitution costs (V,F) 111
B.7 EVPI values for low shortage and low substitution costs (V,F) 111
B.8 CVaR/RP values for low shortage and low substitution costs (V,F) 112
B.9 ASR values for low shortage and low substitution costs (V,V) 112
B.10 VSS values for low shortage and low substitution costs (V,V) 112
B.11 EVPI values for low shortage and low substitution costs (V,V) 112
B.12 CVaR/RP values for low shortage and low substitution costs (V,V) 113
B.13 ASR values for low shortage and high substitution costs (F,V) 113
B.14 VSS values for low shortage and high substitution costs (F,V) 113
B.15 EVPI values for low shortage and high substitution costs (F,V) 113
B.16 CVaR/RP values for low shortage and high substitution costs (F,V) 114
B.17 ASR values for low shortage and high substitution costs (V,F) 114

B.18 VSS values for low shortage and high substitution costs (V,F) 114
B.19 EVPI values for low shortage and high substitution costs (V,F) 114
B.20 CVaR/RP values for low shortage and high substitution costs (V,F) 115
B.21 ASR values for low shortage and high substitution costs (V,V) 115
B.22 VSS values for low shortage and high substitution costs (V,V) 115
B.23 EVPI values for low shortage and high substitution costs (V,V) 115
B.24 CVaR/RP values for low shortage and high substitution costs (V,V) 116
B.25 ASR values for high shortage and low substitution costs (F,V) 116
B.26 VSS values for high shortage and low substitution costs (F,V) 116
B.27 EVPI values for high shortage and low substitution costs (F,V) 116
B.28 CVaR/RP values for high shortage and low substitution costs (F,V) 117
B.29 ASR values for high shortage and low substitution costs (V,F) 117
B.30 VSS values for high shortage and low substitution costs (V,F) 117
B.31 EVPI values for high shortage and low substitution costs (V,F) 117
B.32 CVaR/RP values for high shortage and low substitution costs (V,F) 118
B.33 ASR values for high shortage and low substitution costs (V,V) 118
B.34 VSS values for high shortage and low substitution costs (V,V) 118
B.35 EVPI values for high shortage and low substitution costs (V,V) 118
B.36 CVaR/RP values for high shortage and low substitution costs (V,V) 119
B.37 ASR values for high shortage and high substitution costs (F,V) 119
B.38 VSS values for high shortage and high substitution costs (F,V) 119
B.39 EVPI values for high shortage and high substitution costs (F,V) 119
B.40 CVaR/RP values for high shortage and high substitution costs (F,V) 120
B.41 ASR values for high shortage and high substitution costs (V,F) 120
B.42 VSS values for high shortage and high substitution costs (V,F) 120
B.43 EVPI values for high shortage and high substitution costs (V,F) 120
B.44 CVaR/RP values for high shortage and high substitution costs (V,F) 121
B.45 ASR values for high shortage and high substitution costs (V,V) 121
B.46 VSS values for high shortage and high substitution costs (V,V) 121
B.47 EVPI values for high shortage and high substitution costs (V,V) 121
B.48 CVaR/RP values for high shortage and high substitution costs (V,V) . . . 122

C.1 Results for low shortage and low substitution costs (F,V) 123
C.2 Results for low shortage and low substitution costs (V,F) 123
C.3 Results for low shortage and low substitution costs (V,V) 124
C.4 Results for low shortage and high substitution costs (F,V) 124
C.5 Results for low shortage and high substitution costs (V,F) 124
C.6 Results for low shortage and high substitution costs (V,V) 125
C.7 Results for high shortage and low substitution costs (F,V) 125
C.8 Results for high shortage and low substitution costs (V,F) 125
C.9 Results for high shortage and low substitution costs (V,V) 126
C.10 Results for high shortage and high substitution costs (F,V) 126
C.11 Results for high shortage and high substitution costs (V,F) 126
C.12 Results for high shortage and high substitution costs (V,V) 127

Abstract

An optimal inventory acquisition-allocation problem for the assemble-to-order manufacturing systems is investigated in this study. When the products are assembled based on the customer orders, it is well-known that the number of end product configurations can be very large in practice. This makes the purchase of the right amount of components at the right time to initiate the assembly a very cumbersome task. This observation is in fact the motivation of this thesis. Moreover, as the component purchasing lead-times are generally much longer than the assembly lead-times, a two stages decision process can be conceived. At the first stage, component purchase quantities are to be decided on with respect to the exactly unknown (or stochastic) total end product demands and user preferences (or market proportions for the component options). At the second stage, all the characteristics of the customer demand are known with certainty, and thus, component inventories should be appropriately allocated to satisfy this demand. This decision process is formulated as a two-stage stochastic programming model with fixed recourse by taking into account the concepts of mass customization, component substitution, product modularity and shortage. The proposed model is interesting as it uniquely covers diverse practical aspects of the problem.

According to different viewpoints and assumptions made, several variants of the base model are developed. May be the most relevant one aims to prevent the excessive money loss associated with the erroneously held component inventories. To model the behavior of the risk-averse decision-maker, the conditional value-at-risk of the recourse actions is considered when taking the first stage component purchasing decisions. The outcomes of this approach are compared with the decisions of the risk-neutral decision-maker by formulating and solving the associated expected value model.

The introduced two-stage stochastic programming models are solved with the adapted L-Shaped method. The optimum solutions of these models are found by this method after

adding numerous feasibility and/or optimality cuts to the master (first stage decision) problems. Given that these cuts are generated by solving the related (second stage decision) subproblems many times, an efficient solution technique for those subproblems must be existent to solve the overall problem in a reasonable amount of time. It is shown that the optimal inventory allocation model can be converted to a minimum cost proportional flow model with disconnected networks, and hopefully, a fast executing exact solution algorithm can be devised after a careful study of the theoretical aspects of this later model. In particular, it is aimed to benefit from the underlying network flow structure and modular decomposition as much as possible with this primal simplex algorithm.

Various computational experiments are later carried out to provide insights into these inventory planning problems. First, several experiments are designed to study the effects of product shortage and component substitution costs based on a simple product consisting of two modules and two components per module. Different demand scenario sets are considered and the solutions are evaluated according to some predetermined performance metrics such as the average supply ratio, the expected value of perfect information, and the value of the stochastic solution. Secondly, the effects of increasing the number modules and the number of components within each module are investigated to reveal the impact of the end product structure on the mentioned performance metrics. Based on the experimental results, the product shortage costs, the component substitution costs, the uncertainty of total demand and market proportions for the component options, the number of components and modules in the end product structure are all identified as the key factors affecting the performance metrics. Furthermore, these key factors interact with each other in general, so that different combinations may lead to different outcomes on the performance metrics under study.

The average supply ratio reveals how a firm reacts to the demand uncertainty by means of stock levels. In general, high shortage costs lead to a situation where out-of-stock is less desirable, and then more inventory is held to satisfy the customers. This effect is more significant when the uncertainty on the total demand quantity is high. When the shortage costs are low, the stock levels may decrease while substitution costs increase, since the out-of-stock situation would be more acceptable in this case. This effect is more significant when the uncertainty of market proportions for the component options is high. On the other hand, when the shortage costs are high, the stock levels may increase while substitution costs increase because the use of original components would be more favorable in this case.

The expected value of perfect information corresponds to the price of the option to know the complete and accurate information about the future. Within this context, it enables a decision-maker to assess how much he/she would spend for an initiative to reduce the demand uncertainty. Based on the findings, the shortage and substitution costs significantly affect the expected value of perfect information and their effects is more visible when the uncertainty of total demand and market proportions for the component options are high. Late differentiation strategy for component options can be a solution to deal with the uncertainty of market proportions. More generally, a need for accurate demand forecast is apparent and a detailed market survey can help to improve this situation. Similarly, a firm has to be ready to pay more when dealing with the demand uncertainty for the end products having complex structures. In an assemble-to-order manufacturing setting, an effort to reduce component lead times can result to a decrease in the number of components or modules that an inventory planner has to deal with.

Finally, it is observed that the risk of being out-of-stock is underestimated by a risk neutral decision-maker. In that case, it should be wiser to invest in the customer retention or loyalty programs. The risk of a high loss increases when the substitution costs increase, and the increase is sharper as the uncertainty of market proportions for the component options is high. This increase can be attributed to diminution of the risk pooling effect of substitution.

Résumé

Un problème d'acquisition et de répartition optimal des stocks pour un système de montage sur commande est étudié dans cette recherche. Lorsque les produits sont montés selon les commandes des clients, il est bien connu que le nombre de configurations de produits finals peut être très élevé dans la pratique. Cela rend l'achat de la bonne quantité de composantes au bon moment pour initier l'assemblage une tâche très difficile. Cette observation est en fait la motivation de cette thèse. Par ailleurs, comme les délais pour l'achat des composants sont généralement beaucoup plus longs que les délais d'assemblage, un processus de décision en deux étapes peut être conçu. À la première étape, les quantités d'achat de composants doivent être décidées en considérant l'incertitude à propos de la demande totale (demande stochastique) des produits finals et des préférences des utilisateurs (ou les proportions de sélection des composantes). À la seconde étape, toutes les caractéristiques de la demande de la clientèle sont connues avec certitude, et donc, les stocks des composantes doivent être bien répartis pour satisfaire cette demande. Ce processus de décision est formulé comme un modèle de programmation stochastique à deux étapes avec recours fixe en prenant en compte les concepts de personnalisation de masse, substitution de composants, modularité et pénurie du produit final. Le modèle proposé est intéressant car il inclut uniquement divers aspects pratiques du problème.

Selon les différents points de vue et les suppositions basées, plusieurs variantes du modèle de base sont développées. Peut être la plus pertinente a pour but de prévenir la perte excessive de capitale associée aux stocks des composantes détenus par erreur. Pour modéliser l'attitude préventive du décideur à l'égard du risque, la valeur conditionnelle à risque des actions de recours est considérée lors de la prise des décisions d'achat des composantes de la première étape. Les résultats de cette approche sont comparés avec les décisions du décideur indifférant à l'égard du risque en formulant et résolvant le modèle de valeur attendue associé.

Les modèles de programmation stochastique en deux étapes introduits sont résolus avec une adaptation de la méthode L-Shaped. Les solutions optimales de ces modèles sont trouvées par cette méthode après l'ajout des nombreuses coupes de faisabilité et/ou d'optimalité aux problèmes maîtres (décision première étape). Étant donné que ces coupes sont formées à plusieurs reprises par la résolution des sous-problèmes liés (décision de deuxième étape), une technique de résolution efficace pour les sous-problèmes doit être existante pour résoudre le problème global dans une durée raisonnable. Il est montré que le modèle de répartition optimal des stocks peut être transformé en un modèle de flux proportionnelle à coût minimum avec des réseaux déconnectés. Et heureusement, un algorithme de résolution exacte et rapide est conçu après une étude approfondie des aspects théoriques de ce dernier modèle. En particulier, on vise à bénéficier de la structure du réseau de flux impliquée et de la décomposition modulaire autant que possible avec cet algorithme primal du simplexe.

Diverses analyses numériques sont effectuées pour fournir des indications sur ces problèmes de planification des stocks. Tout d'abord, plusieurs expériences sont conçues pour étudier les effets des coûts de pénurie de produits et des coûts de substitution des composantes pour un produit simple qui se compose de deux modules et deux composantes par module. Différents scénarios de demande sont considérés et les solutions sont évaluées en fonction des métriques de performances prédéterminées comme la valeur moyenne du taux d'approvisionnement, la valeur attendue de l'information parfaite, et la valeur de la solution stochastique. Deuxièmement, les effets de l'augmentation du nombre de modules et du nombre de composantes au sein de chaque module sont étudiés pour révéler l'impact de la structure du produit final sur les métriques de performance mentionnées. Sur la base des résultats expérimentaux, les coûts de pénurie de produits, les coûts de substitution des composantes, l'incertitude de la demande totale et l'incertitude des préférences de la clientèle, le nombre de composants et de modules dans la structure du produit final sont tous identifiés comme des facteurs clés qui influent sur les métriques de performance. En outre, ces facteurs clés interagissent les uns avec les autres en général, afin que les différentes combinaisons peuvent conduire à des résultats différents sur les métriques de performance étudiées.

La valeur moyenne du taux d'approvisionnement révèle comment une firme réagit à l'incertitude de la demande en ajustant les niveaux de stock. En général, les coûts de pénuries élevées conduisent à une situation où la rupture de stock est moins souhaitable, et puis plus d'inventaire est tenue pour satisfaire les clients. Cet effet est plus significatif lorsque l'incertitude sur la quantité de la demande totale est élevée. Lorsque les coûts de pénurie sont faibles, les niveaux de stock peuvent diminuer alors que les coûts de substitution augmentent, puisque la situation de rupture de stock serait plus acceptable dans ce cas. Cet effet est plus important lorsque l'incertitude des préférences de la clientèle est élevé. D'autre part, lorsque les coûts de pénurie sont élevés, les niveaux de stock peuvent augmenter alors que les coûts de substitution augmentent parce que l'utilisation de composantes originales serait plus favorable dans ce cas.

La valeur attendue de l'information parfaite permet à un décideur d'évaluer combien il/elle dépenserait pour une initiative visant à réduire l'incertitude de la demande. Basé sur les résultats, les coûts de pénurie et de substitution affectent de manière significative la valeur attendue de l'information parfaite, et leurs effets sont plus visibles lorsque l'incertitude de la demande totale et des préférences de la clientèle sont élevés. La stratégie de différenciation retardée peut être une solution pour faire face à l'incertitude des préférences de la clientèle. Plus généralement, un besoin de prévision de la demande précise est apparent et une étude de marché détaillée peut aider à améliorer cette situation. De même, une entreprise doit être prête à payer plus lorsqu'il s'agit de l'incertitude de la demande pour les produits finis ayant des structures complexes. Dans un système de montage sur commande, un effort pour réduire les délais d'approvisionnement des composantes peut entraîner une diminution des nombres de composants ou de modules que doit traiter un responsable de la programmation d'approvisionnement.

Enfin, il est observé que le risque de rupture de stock est sous-estimé par un décideur indifférant à l'égard du risque. Dans ce cas, il devrait être plus prudent d'investir aux programmes de fidélisation de la clientèle. Le risque d'une perte élevée augmente lorsque les coûts de substitution augmentent, et cette augmentation est plus nette lors que l'incertitude des préférences de la clientèle est élevé. Cette augmentation peut être attribuée à la diminution de l'effet de mise en commun des risques par la substitution.

Özet

Bu çalışmada siparişe göre montaj yapılan üretim sistemleri için bir eniyi envanter tedariği ve tahsisi problemi incelenmiştir. Pratikte iyi bilinmektedir ki ürünlerin müşterilerin siparişlerine uygun montajı yapıldığı zaman üretilebilecek nihai ürün çeşidi oldukça fazladır. Tabii bu da montajı başlatabilmek için gerekli doğru sayıda bileşenin doğru zamanda satın alınma işlemini önemli ölçüde zorlaştırmaktadır. Bu gözlem aslında bu tezin çıkış noktasını oluşturmaktadır. Buna ilaveten, bileşen tedarik sürelerinin montaj sürelerine kıyasla daha uzun olması iki aşamalı bir karar sürecine neden olur. Birinci aşamada kesin olarak bilinmeyen (veya stokastik) toplam nihai ürün talebi ve müşteri tercihleri (veya bileşen tercih oranları) dikkate alınarak tedarik edilecek bileşen miktarına karar verilmelidir. İkinci aşamada ise tüm müşteri talebinin unsurlarının kesin olarak bilindiği düşünülerek bu talebi karşılamak üzere eldeki bileşen envanterinin uygun biçimde tahsisi istenmektedir. Bu karar süreci iki aşamalı doğrusal stokastik programlama modeli ile ifade edilmiştir, ve bu modele kitlesel özel üretim, bileşen ikamesi, ürün modülerliği ve noksanlığı kavramlarına ilişkin unsurlar dahil edilmiştir. Probleme ilişkin uygulamadan gelen çok sayıda detayı kapsamasından dolayı önerilen modelin karar vericiler için yol gösterici olacağı düşünülmektedir.

Farklı bakış açıları ve varsayımlar göz önüne alınarak temel modelin çeşitleri geliştirilmiştir. Bunlar arasından belki de en dikkat çekici olanı yanlış miktarda tutulan bileşen envanteri nedeniyle oluşan aşırı sermaye kaybını engellemeyi amaçlayan modeldir. İlk aşamadaki bileşen tedariği kararları alınırken, riskten kaçınan karar vericinin davranışını modelleyebilmek için ikinci aşama maliyetlerinin koşullu riske maruz değerinin enaza indirilmesi düşünülmüştür. Bu yaklaşımın sonuçları, riske duyarsız karar vericinin davranışını ifade eden bir beklenen değer modeli oluşturularak ve çözülerek karşılaştırılmıştır.

Önerilen iki aşamalı stokastik programlama modelleri yeniden düzenlenmiş L-Shaped yöntemi ile çözülmüştür. Bu modellerin eniyi çözümleri ana (ilk aşama kararları) prob-

lemlere çok sayıda olurluluk ve/veya eniyilik kesmeleri eklenerek bulunmaktadır. Bu kesmelerin de ilişkin alt problemlerin (ikinci aşama kararları) gene çok defa çözülmesiyle elde edildiği düşünülürse, tüm problemin makul bir süre içinde çözülebilmesi için bu alt problemleri etkin biçimde çözecek bir teknik var olmalıdır. Eniyi envanter tahsisi modelinin ayrık ağlı en düşük maliyetli orantılı akış modeline çevrilebileceği gösterilmiştir. Bu sayede de bu ikinci modelin teorik unsurlarının analizi doğrultusunda göreceli olarak hızlı çalışan bir çözüm algoritması geliştirilebilmiştir. Bu temel simpleks algoritması var olan ağ akış yapısından ve modüler parçalamadan olabildiğince yararlanmaktadır.

Bu envanter planlama problemleri hakkında daha ayrıntılı bilgiler sunabilmek için deneysel hesaplamalar yapılmıştır. Öncelikle iki modül ve modül başına iki bileşenden oluşan basit bir ürün yapısı düşünülerek, ürün noksanlığı ve bileşen ikamesi maliyetlerinin etkilerini araştırmak üzere deneyler tasarlanmıştır. Farklı talep senaryoları dikkate alınmış ve sonuçlar ortalama tedarik oranı, tam bilginin beklenen değeri ve stokastik çözümün değeri gibi performans ölçütlerine dayanarak değerlendirilmiştir. Bunu müteakiben, nihai ürün yapısının bu belirtilen performans ölçütlerine etkisini belirlemek üzere modül sayısının ve her modüldeki bileşen sayısının arttırılmasının etkisi incelenmiştir. Deneysel sonuçlara göre ürün noksanlığı maliyeti, bileşen ikamesi maliyeti, toplam talebin ve müşteri tercihlerinin belirsizliği, nihai ürünün montajını yapmada gerekli bileşen ve modül sayıları etkenlerinin hepsi anılan performans ölçütlerini etkileyen ana unsurlardır. Ayrıca bu ana etkenler birbirleri ile etkileşim halindedir ve farklı karmalar performans ölçütlerini farklı biçimde etkilemektedir.

Ortalama tedarik oranı bir firmanın talep belirsizliğine karşı stok sevilerini düzenleyerek nasıl tepki verdiğinin bir ölçütüdür. Genel olarak yüksek noksanlık maliyetleri stoksuz kalmayı tahammül edilemez kılmaktadır, ve müşteri talebini karşılamak üzere daha çok envanter tutmayı teşvik etmektedir. Bu etki toplam talep miktarındaki belirsizlik arttıkça daha belirgin hale gelmektedir. Noksanlık maliyetleri düşük olduğunda, ikame maliyetleri yüksek ve müşteri tercihlerinin belirsizliği fazla olsa bile stok seviyeleri azalmaktadır çünkü bu durumda stoksuz kalmak daha kabul edilebilir olmaktadır. Noksanlık maliyetleri yüksek iken ikame maliyetlerini arttığında stok seviyelerini de arttırma ihtiyacı doğmaktadır çünkü bu durumda orijinal bileşen kullanımı öne çıkmaktadır.

Tam bilginin beklenen değeri, geleceğe dair tam ve doğru bilgi edinmek için ne kadar istekli olmamız gerektiğinin bir ölçütüdür. Bu bağlamda, karar verici talepteki belirsizliği azaltmak üzere bir girişimde bulunursa buna ne kadar çaba harcaması gerektiğini değerlendirmede yardımcıdır. Elde edilen bulgulara göre noksanlık ve ikame maliyetleri bu ölçütü önemli ölçüde etkilemektedir ve bu etki toplam talebin ve müşteri tercihlerinin belirsizliği fazla olduğunda daha da artmaktadır. Müşteri tercihlerindeki belirsizlikle başa çıkmak üzere geciktirilmiş farklılaştırma stratejisi bir çözüm olabilir. Daha genel biçimde talep tahmininin doğruluk derecesini arttırma ihtiyacı belirgindir ve ayrıntılı bir pazar araştırması bunu sağlamada yardımcı olabilir. Benzer şekilde bir şirketin, karmaşık yapıya sahip nihai ürünlerin talebindeki belirsizlikle başa çıkabilmesi için daha fazla yatırım yapmaya hazır olması gereklidir. Siparişe uygun olarak montaj yapılan bir üretim sistemi için, bileşen tedarik zamanlarının kısaltılması envanter planlamasında göz önüne alınması gereken bileşen ve modül sayılarının azaltılmasında yardımcı olacaktır.

Son olarak stoksuz kalma riskinin riske duyarsız karar verici tarafından hafife alındığı tespit edilmiştir. Bu durumda müşteri sadakati veya tutundurma programlarına yatırım yapılması akılcı olacaktır. İkame maliyetleri arttıkça yüksek sermaye kaybıyla karşılaşma riski artmaktadır ve müşteri tercihlerindeki belirsizlik arttıkça bu risk daha da büyümektedir. Riskteki bu artış ikamenin sağladığı risk paylaşımının azalmasına bağlanabilir.

1 Introduction

Today, the quality requirements of customers is more difficult to satisfy than it is in the past. As a result, customer needs should be well understood, and innovative products and services targeting those needs should be developed. Indeed, it is very difficult to conceptualize a single type of product or service that fits all needs of all the customers. Even if it was possible to achieve it, it will be very expensive from customer perspective since it has to contain the needs of others too. Thus, products and services should be customized with an eye on the associated costs. At this point, the concept of mass customization aims to provide the flexibility of customization while price, quality, and speed dimensions of the competition are preserved.

The mass customization is accepted as an important dimension of business competition in the new age of economy. This fact is observable from both the increasing number of academic papers and the various field applications in manufacturing and service industries. According to Kumar et al.[1], over one thousand publications about mass customization topic are cited in several journals and magazines. Indeed, the mass customization seems to remain on the focus of this endeavour thanks to the growing expectations of customers on the price, quality, speed and customization issues.

In manufacturing industries, the analysis of customer's value perception is central to any design of product variety by customization. This analysis results often into a product structure where this variety is offered to customers by a modular design. In such a situation, inventory management is a hard task because a large number of stock items resulting from modular combination of options and having stochastic demand has to be planned. Hence, firms have to revise their production strategies considering the dimensions of competition.

From economies of scale viewpoint, make-to-stock production strategy is efficient in terms of production costs per unit and it offers great availability for the stocked products. However, its drawback is that it incurs financial losses because of the large amount of capital invested in the production process to ensure this product availability. On the other hand, make-to-order production strategy promises cost savings by reducing the amount of capital investment in production process because it is initiated when customer demand is received. Nevertheless, this strategy has also a drawback, it mostly decreases product availability by increasing customer order lead times. At this point, assemble-to-order production strategy promises efficiencies in both capital investment and product availability issues, in particular when the raw material lead times are greater than the time required for the assembly. In this strategy, production is still initiated when the customer demand is received, however, a large amount of raw material needs to be purchased before starting assembly process. This makes the production lead time much shorter than it is for the make-to-order strategy. Because of the advantages gained, a modular product architecture is an enabler of this strategy.

On the other hand, substitution is an another interesting concept that may lead to higher expected profits in demand scenarios with high variance and negative correlation [2]. It allows a need for a component requirement to be fulfilled by use of another component when the former is out of stock. Hence, it may affect optimum design of product variety and optimum component stock quantities. Therefore, it is possible to say from managerial perspective that assemble-to-order strategy should be developed with substitution.

In the assemble-to-order strategy, the decision variables may be classified into two stages because the component lead times are in general much greater than the assembly lead times. The first stage decision variables consist of component purchase quantities which have to be calculated with respect to stochastic product demand. At the second stage, where demand realization occurs, the allocation of components to products has to be determined. This decision process results into a two stage stochastic programming model with fixed recourse. This large-scale linear programming problem is interesting from both computational perspective and stochastic performance measures involved.

Assemble-to-order models are particularly important when dealing with the inventory planning problem of modular products in mass customization. Currently, mass customization and modularization strategies are adopted by several industries including automotive, apparel, footwear, and consumer electronics but not limited to. Thus, the developed models can be used as analytic tools to determine the appropriate levels of component inventories in those industries.

The thesis is organized as follows. An extensive literature survey is given in Chapter 2 and the findings of this survey motivated us to propose several assemble-to-order models including modularity, shortage, and substitution concepts under stochastic total demand and stochastic market proportions for the component options to fill an important gap in the literature. These models are practically relevant as they cover all of those concepts whose importance is stressed. The topics of conditional value at risk (CVaR) and two stage stochastic linear programming are briefly introduced in Chapter 3. Next, the planning of assemble-to-order with substitution under mass customization is studied in Chapter 4. Several models with different assumptions including general, subcontracting and spot market cases are investigated. The models with risk neutral and risk-averse objectives are represented. In Chapter 5, the mathematical properties of the assemble-to-order with substitution problem is analyzed theoretically. The networks with gains and losses, and the Hitchcock transportation problem are discussed. In particular, a special algorithm for the modular networks with proportional flows is proposed. Based on this analysis, the relationship between several models is also given in Chapter 5 and the related solution methods are summarized. In Chapter 6, first the performance of the special primal simplex algorithm is discussed. Later, the sensitivity analysis is carried out to understand the effects of shortage and substitution costs on the stochastic performance metrics. The effects of increasing the number of components and modules are also discussed based on the same stochastic performance metrics. Finally, Chapter 7 contains some concluding remarks and perspectives.

2 Literature Survey

2.1 Mass Customization

From a broader view, mass customization means to provide customized products or services through flexible processes in high volumes and at reasonably low costs [3]. Strategies similar to the mass customization have been applied in the past. Alizon et al. [4] compare Ford's approach to current mass customization approach, and they argue that a kind of mass customization was also implemented by Ford through specialized manufacturers which tailored the cars with respect to the customer requirements. According to Davis [5], mass customization is a new idea in marketing that could lead to significant changes in business style. The evolution of business strategies from mass production to mass customization is explained well by Pine [6]. Kumar [7] states that mass customisation is a research field that has achieved maturity. Nonetheless, mass customization has a growing number of application areas and it is still being examined from multiple perspectives: Chen and Hao [8] underline the necessity of applying the concepts of mass customization into the design of service delivery system to cope with the efficiency and customization dilemma. From manufacturing perspective, Dean et al. [9] develop a new manufacturing resource planning method for mass customization by analyzing the data of customer demands and manufacturing resource requirements. From organizational perspective, Huang et al. [10] show that organic structures support the mass customization strategy. However, they argue that this fact is significant for customizers only at the design or fabrication stages, not at the assembly or delivery stages.

There are several levels and strategies related to mass customization. Da Silveira et al. [3] list eight generic levels of mass customization:

- Design,

- Fabrication,
- Assembly,
- Additional custom work,
- Additional services,
- Package and distribution,
- Usage,
- Standardization.

Pine [11] cites five necessary directions in a mass customization implementation:

- Service customization around standard products,
- Mass production of products that can be later modified,
- Point of delivery customization,
- Quick response,
- Component modularization.

For a manufacturing firm with design, fabrication, assembly and distribution stages in its value chain, Lampel and Mintzberg [12] classify mass customization strategies into five categories:

- *Pure standardization* where all the design, fabrication, assembly and distribution stages are standardized,
- *Segmented standardization* where the customization starts from the distribution stage,
- *Customized standardization* where the customization starts from the assembly stage,

- *Tailored customization* where the customization starts from the fabrication stage,
- *Pure customization* where all stages are customized.

Gilmore and Pine [13] identify four distinct approaches to mass customization:

- *Collaborative* where there is a dialog process between designers and customers,
- *Adaptive* where standard products are customized to individual needs during use,
- *Cosmetic* where standard products are differentiated by packaging,
- *Transparent* where the customization of products is indiscernible from viewpoint of customers.

Amaro et al. [14] develop a taxonomy including 11 types of non make-to-stock companies based on the degree of customization, company responsibility and activities performed after order acceptance. In particular, they propose two new labels: "versatile manufacturing company" following customization by individual order and "repeat business customizer" following customization by contract. Duray et al. [15] differentiate between the design, fabrication, assembly, and use stages in the value chain of mass customizers and the authors classify them into four categories by points of customer involvement and modularity type:

- *Fabricators* where both the points of customer involvement and modularity are in the design and fabrication stages,
- *Involvers* where the point of modularity is in the assembly and use stages,
- *Modularizers* where the point of customer involvement is in the assembly and use stages,
- *Assemblers* where both the points of customer involvement and modularity are in the assembly and use stages.

Furthermore, the authors argue that the use of modularity in the assembly and use stages can lead to higher performance for the values of return on investment, profit margin, and market share.

The design of product variety is accepted as a strategic decision leading to success in the mass customization. The analysis of product quality in relation to mass customization, that is the value perception of customer by this variation is central while deciding on the product variety. A method to capture and describe information for product configuration is proposed by Feng et al. [16]. It is based on the general requirement concept which covers the dimensions of time, space, and character. In their method, the time dimension deals with the whole product life cycle including marketing, design, production, supply, and service stages. The space dimension is concerned with the global supply chain including customers, manufacturers, suppliers, distributors. Finally, in the character dimension, the domains of product, cooperation, technology, and management are considered.

Merle et al. [17] define the five perceived benefits of mass customization from a consumer viewpoint:

- *Utilitarian value* to show the gap between consumer needs and product features,
- *Uniqueness* to show to what degree the consumers feel that they are individually targeted,
- *Self-expressiveness* to show to what degree the consumer personality is expressed,
- *Hedonic value* to show the consumer pleasure,
- *Creative achievement* to show to what degree consumers feel that they are creative.

Trade-offs exist between product customization and manufacturing costs, and delivery lead times. However these trade-offs can be reduced at the assembly level by the use of modularization [18].

2.2 Modularity

Kumar [19] sees the modularity as the essence of mass customization. Standardized modules can be manufactured with mass production techniques that allow to reduce costs [15]. Similarly, Piller [20] gives some examples of mass customization and in particular, gives a successful case of company having modular product range and employing assemble-to-order production system. Modularity is a well investigated topic in the literature and the current trend indicates that it will remain popular in the future [21]. Ulrich and Tung [22] classify the modularity into six categories:

- *Component sharing* modularity: several products use a common component to satisfy some functional requirement,
- *Component swapping* modularity: different functionalities are provided by the use of interchangeable components,
- *Cut-to-fit* modularity: products required at a wide range of sizes are obtained from a common product having a larger size,
- *Mix* modularity: various mixes of components can lead to different functionalities,
- *Bus* modularity: the use of several number of components is allowed in the bus architecture,
- *Sectional* modularity: a different requirement can be addressed by rearranging a set of components.

Hsuan [23] points out the component, module, sub-system and system levels of modularization. The author evaluates the relationship between several modularization levels and interface constraints including the factors of interface compatibility, component customization, value inputs, and supplier-buyer interdependence. The author argues that the component level of modularization are associated with low interface constraints. Ernst and Kamrad [24] define rigid, postponed, modularized and flexible supply chain structures as follows:

- *Rigid* with low postponement and low modularization,
- *Postponed* with high postponement and low modularization,
- *Modularized* with low postponement and high modularization,
- *Flexible* with high postponement and high modularization.

Furthermore, the authors argue that there is a relationship between undesired vertically integrated supply chain structure and low modularization. Brun and Zorzini [25] analyze the relationship between product features and supply chain structures and they argue that flexible supply chain structures are associated with complex and highly customized products, postponed supply chain structures with simple but highly customized products, and modularized supply chain structures with complex but lowly customized products. Duray [26] answers the question of how the point of customer involvement and modularity influence the decision process in a production system. The author argues that modularity influences the supplier integration, stock levels, and inventory planning. While, the point of customer involvement influences only the inventory planning decisions. Later, Howard and Squire [27] find that modularity helps to increase the collaboration between a buyer and its supplier with relationship specific assets and information sharing.

In the *modular product design*, there is a dedicated *module* for each function of the product and each module has several *variants* (or *options*) such that a customer can choose a configuration according to his/her preferences for each function [1]. The complexity of manufacturing processes can be reduced by use of the modular product design concept [1]. As an another important result, the mass customization strategy is supported by the modular product design concept [28].

The product variety design is positively influenced by the modular design. Following this idea, Jose and Tollenaere [29] claim that modularity and standardization are enablers of the platform strategy, where the use of a standard module between different products corresponds to a platform. The authors search for the platform concept in the literature while focusing on the modular and platform methods for product family design. They conclude that an efficient product family design allows easy modifications to differentiate between products and uses maximum number of common components while preserving the distinctiveness between products. Later, Jiao et al. [30] provides a review on the research activities in product family design and platform-based product development. The authors underline the importance of the holistic view. Similarly, Antonio et al. [31] conduct a literature survey on the impacts of modular product design on several competitive dimensions including price, quality, customer service, flexibility, and delivery. According to their empirical study, modular product design can not enhance each competitive dimensions simultaneously, and in particular, it can not deal with the price and quality dimensions.

In the literature, there are several papers that deal with the design of modular products under optimization viewpoint. Fujita [32] simultaneously design multiple products by determining the contents of modules and their combinations under fixed modular product architecture. For this purpose, the author distinguishes between optimal attribute assignment and optimal module combination aspects of design of product variety. The author also gives the following examples: design of television receiver circuits for optimal module combination and aircraft design for optimal attribute assignment. In some situations, it is feasible to produce only a small number of products from a large set of product configurations. Then, it is common to have one product with higher options to substitute for the demand of non produced one with lower options. For this purpose, Briant and Naddef [33] define the diversity management problem that consists of choosing an optimal subset of configurations that will be produced to meet all the demand. The authors model the problem as a large-scale integer linear problem and use Lagrangian relaxation to reduce its size. In an another work, the trade-off between labor costs and transportation costs is investigated by da Cunha et al. [34] when determining the optimum portfolio of modules to be supplied from a distant manufacturing site. In their model, a module consists of a subset of basic components, and the final product is assembled from a subset of modules when a customer order is received. Furthermore, the authors assume that the probability of demand for each of the final products is known. They solve an integer programming model which takes into account the mean final assembly time constraint to determine the optimum portfolio of modules with heuristic algorithms.

Li et al. [35] consider the product platform customization problem. In this problem, a customer requirement is matched by configuring an optimal product variant based on the given product platform. The authors distinguish three product platform customization approaches: scalable, configurable, and adaptive. In the scalable approach, the configuration of product variant is done by stretching or shrinking the common parameters of the product platform. In the configurable approach, swapping standard modules is preferred without changing their parameters. In the adaptive approach, both the scalable and configurable approaches are valid. Finally, the authors propose a heuristic method to solve the adaptive product platform customization problem.

The modularity and commonality concepts are investigated together in many studies. Fixson [36] reviews the modularity and commonality based on the dimensions of subject, effect, and research method. The titles of product, process, organization, and innovation take part in the subject dimension. The dimension of effect consists of product performance/quality, product variety, costs, and time. Finally theory building, concepts and frameworks, process modeling, mathematical modeling, simulation, experiments, empirical studies, case studies, and reviews are in the dimension of research method. In particular, the author notices that the research method dimension is based on mathematical models for the study of commonality while it relies on the frameworks and small n-case studies for the study of modularity. Hence, the lack of operationalization of modularity is apparent [36].

2.3 Component Commonality and Substitution

The *inventory management* is a critical issue for mass customization. The component commonality and substitution play an important role in this context. In particular, substitution is on the focus of this research.

The component commonality is a widely adopted concept and its risk pooling effect is well known. Moreover within commonality, there is an opportunity to sell more of the product with higher margin when the component stock level is not enough to meet all the demand even for perfectly correlated product demands. This fact is designated as "revenue-maximization option" effect by Van Mieghem [37]. There are many studies concerning the effects of commonality in the literature. However, these effects are not known completely in multistage systems with multiple products and multiple common items [38].

Hillier [39] studies the commonality within a simple assembly model with service level constraints. In this model, two final products are considered, each of which uses two components. The author finds that commonality can be useful in the single-period models, but also argues that it may not be useful in the multiple-period models. Ma et al. [40] consider a multi-period, multi-product, and multi-stage assembly model with random end product demands. The authors conclude that the savings from using common components continuously from the first stage to a late stage in an assembly system depend on the component cost structure, replenishment lead times, and the assembly times. Chod et al. [41] study the commonality and product pricing issues in their assembly model. The authors argue that the optimal prices of two products are positively correlated when the commonality between is high.

From an optimization perspective, Jönsson et al. [42] incorporates the commonality concept to an assemble-to-order inventory problem under budget constraint and solves it with the scenario aggregation approach. In a modular end-product structure, Paul and Vakharia [43] determine the optimal stock quantities of component options under a given service level. In their study, two distinct sources of uncertainty are defined: stochastic total demand and unknown market proportions for the component options. However, component substitution is not included in their model. Nonas [44] proposes a gradient based search method to determine the optimum stock quantities in an inventory model with random demand drawn from a distribution with joint density function. In this model, three products can share any number of common components.

Substitution within the inventory management context has received much attention from many scholars. The component substitution can offer significant cost savings particularly when the demands of components highly variate and are negatively correlated [2]. In the production process, it allows the demand of a component to be fulfilled by use of another component when the former is out of stock. Hence, it may affect the optimum design of product variety and the optimum stock quantities. Particularly, two types of substitution are encountered in the literature: Two ways substitution and one way/downward substitution. In the two ways substitution, one component can substitute for another, and vice versa. However, in the one way substitution, only components with higher degree value can substitute for ones with lower degree value.

The one way substitution problem is investigated by many authors. In particular, special solution methods are developed exploiting the one way substitution property. Chen [45] determines the substitution policy in an inventory model with one way substitution under optimization viewpoint. Hsu and Bassok [46] consider a multiproduct single-period inventory model where there is one raw material input from which end products are obtained according to some given random yields. Moreover, the demand for end products is also random and one way substitution is possible between end products. Then, the authors develop a greedy algorithm exploiting the one way substitution property for the subproblems resulting from the decomposition of their main inventory problem. Bassok et al. [47] also consider a single period inventory model with multiple products and one way substitution. In their model, production or purchase orders are placed before the demand realization, but the allocation of inventory is done when the demand is known. First, the authors propose an optimal greedy inventory allocation policy, then they argue that it is more beneficial to consider substitution at the ordering stage instead of not considering it at this stage. They observe that this effect is more significant when the demand variability is high, the substitution costs are low, the profit margins are low, the salvage values are high, and the products have about same prices and costs. Hale [48] considers an inventory model with two simple end products having normally distributed demands. Each end product is assembled from two of four distinct components. For this end product structure, there is one-way substitution relationship between two specific components. Then, the author solves this model by numerical integration. As a result of this study, substitution is showen to substantially reduce costs. Axsater [49] develops an approximate technique to deal with the order policy evaluation in an inventory model with multiple products, Poisson demand and one way substitution. Rao et al. [50] develop heuristic algorithms to solve a two-stage stochastic integer program dealing with a single-period inventory problem in which multiple products, setup costs, one way substitution, and random demands are considered. The authors underline the cost advantages of considering substitution. Hsu et al. [51] propose two dynamic lot size models with multiple products by taking into account setup costs, production costs, and inventory holding costs and one of them includes unit conversion costs for one way substitution. To solve both models, the authors develop dynamic programming based algorithms which work in polynomial time under fixed number of products assumption. They also propose a heuristic algorithm. Li et al. [52] deal with an uncapacitated production planning problem with multiple products and one way substitution. In their model, the demand of a product type can be met by manufacturing new products or remanufacturing used products. The authors propose both dynamic programming and approximate approaches to solve their problem by taking into account manufacturing, remanufacturing, setup, inventory holding, and substitution costs.

The one way substitution problem is also interesting from managerial perspective. A two-echelon distribution system with multiple products having normally distributed demand is studied by Weng [53]. The product with highest degree is used as the joint buffer stock in this system. The author argues that it is possible to benefit more from this joint buffer stock particularly when the service levels are high. Inderfurth [54] considers a single-period inventory model with new and used products having stochastic demands and returns respectively. The author assumes that new products have a higher degree, and there is a one way substitution relationship between new and used products. Furthermore, the author takes into account different manufacturing and remanufacturing lead times, and this leads to different points in time for the execution of manufacturing and remanufacturing decisions, and different optimal policies. Tibben-Lembke and Bassok [55] develop a two-stage stochastic programming model to deal with an inventory problem in which both the generic product and the regular products are used to meet the demand. There is one way substitution relationship between the generic product and the regular products. The generic product has a higher degree and it is more expensive. The substitution can take place when the demand is known. Based on their model, the authors argue that it is possible to increase the profits by the use of a generic product. Bayindir et al. [56] consider an inventory model with a common capacity constraint and stochastic demand for the products. In their model, the brand new and retreated tires are manufactured and remanufactured to meet the demand, respectively. The brand new tires have higher degree than the retreated tires, and one way substitution is allowed. However, this substitution can be accepted by customers according to some given probability when a stock-out situation occurs. Based on the results of their computational study, the authors conclude that the common capacity constraint has influence on the optimal inventory levels according to the different capacity requirements and production costs of the tire types. Karakul and Chan [57] develop a single-period inventory model with a new and an existing products. The one way substitution is possible in a shortage situation and the new product has a higher degree than the existing product. Moreover, the authors assume that the demand models of products are linear functions of the new product price and have a noise term following a general discrete demand distribution. The authors seek to maximize the profit obtained from the inventory model by determining the price of the new product and the inventory levels of both products. Finally, the authors conclude that substitution can lead to both a higher profit and a higher optimal price of the new product.

The two ways substitution problem is cited extensively in the literature. Although there are no solution algorithms as fast as for the one way substitution problem, several optimization approaches are developed to deal with the two ways substitution problem. Balakrishnan and Geunes [58] develop a component requirements planning model for a two-stage manufacturing system with multiple products and component substitution. The authors call their model as the "requirements planning with substitutions" problem and they develop a dynamic programming based solution method to solve it by taking into account setup, production, conversion, and holding costs. However, they do not consider the projection of product shortages on the consumption of components required for the assembly. Later, Geunes [59] reformulates the requirements planning with substitutions problem as an uncapacitated facility location problem to solve it effectively. Nagarajan and Rajagopalan [60] study an inventory model with substitutable products by considering the revenue of selling products, purchasing costs, inventory holding costs, and shortage costs. The authors assume that only a fixed proportion of the customers can accept the substitution in a stock-out situation. Then, they propose heuristic approaches to find the optimal inventory levels. The authors argue that their heuristic approach is effective particularly at low levels of substitution, low levels of aggregate demand variation, and high service levels.

It is possible to derive several managerial implications from the two ways substitution problem. Mishra [61] studies an assemble-to-order model with multiple products, stochastic product demand, and stochastic component yield to analyze both component commonality and component substitution. The author argues that the approach of substituting the expected values for random variables in the model is not an effective approach. Moreover, the author finds that it is possible to benefit more from commonality when the demand variates highly and correlates negatively. However, the modularity concept is not considered. Smith and Agrawal [62] consider an inventory model with multiple products and negative binomial demand distribution to study the effects of substitution. In their model, the substitution occurs according to some given probabilities for each product when a stock out occurs. The authors conclude that the optimal assortment size can be reduced by considering substitution in case of products with different profit margins. Mahajan and Van Ryzin [63] develop an inventory model with multiple products and substitution based on customer choices. The authors assume that customers seek for utility maximization. Then, they propose a stochastic gradient algorithm to solve their model. Finally, the authors conclude that substitution has an effect of increasing the optimum inventory levels of popular products and decreasing the optimum inventory levels of unpopular products. Iravani et al. [64] formulate an assemble-to-order problem with substitution as a continuous-time Markov chain model where customers can accept substitutions for missing products according to some given probabilities. The authors argue that customer preferences can have influence on the inventory levels. Agrawal and Smith [65] develop a 0-1 mixed integer linear programming model to find the optimal assortment in an inventory model with multiple products, substitution, and complementary products. The authors consider the holding costs, stockout costs, and fixed costs of including a product in the assortment. The authors underline the effect on profits of considering substitution when determining the optimal assortment. Furthermore, the authors argue that the lack of a small subset of products that can be used to meet most of the demand may decrease profits. Denton and Gupta [66] develop a two-stage stochastic integer programming model to determine the optimum inventory levels of semi-finished products which are produced to stock in order to reduce order lead times when customer orders are received. The authors propose a heuristic method to solve their model and conclude that more accurate demand information can lead to higher profits. Thomas and Warsing [67] consider a service parts inventory model in which service parts are stocked at both the assembly and component levels. The authors formulate a singe-period stochastic linear programming model to solve their model by taking into account the assembly, disassembly, holding, and shortage costs. The substitution is enabled by allowing the assembly/disassembly option for the service parts. The authors argue this option can lead to significant cost savings. Ganesh et al. [68] argue that substitution can reduce the value of information sharing within a supply chain and they explain this fact by the risk-pooling effect of substitution. Yang and Schrage [69] study the risk pooling effect of substitution, and in particular they reveal the conditions under which the inventory levels are increased. The authors link this unexpected effect to the use of right skewed demand distributions and to the case of partial substitution where only some of the customers can accept the substitution.

2.4 Literature Survey of Inventory Planning Models

There are many studies concerning inventory planning models. However, Wazed et al.[70] notice the lack of research related to any holistic model to study the uncertainty issues in multi-period, multiple products, and multi-stage environments for manufacturing resources planning. Nonetheless, inventory planning models that are the most relevant to this study are summarized in Table 2.1. They are evaluated according to which concepts investigated: commonality, substitution, modularity, mass customization, risk neutral objective, risk-averse objective, and service level constraints. The first four concepts are already discussed. On the other hand, the last tree concepts classify the inventory planning models by their objectives from a viewpoint of risk. In a risk neutral case, all the prospective scenarios are taken into account by decision makers. However, the worst case scenarios are heavily considered in a risk-averse approach. In the inventory planning models seeking to satisfy some given service levels, the service rate constraints are generally used to ensure that the demand is satisfied. The works in Table 2.1 are listed in a chronological order to expose the current research trends.

It can be deduced from Table 2.1 that substitution and risk neutral decision making are the most addressed concepts. In contrast, only recently there has emerged a trend toward the risk-averse models. Moreover, mass customization is not well investigated in these models despite its popularity. Another result that can be derived from Table 2.1 is that the commonality, substitution, modularity, and mass customization concepts are integrated only in a few inventory planning models.

This study focuses on developing and studying models that are capable to deal with inventory planning decisions in a mass customization environment. Based on the findings of our literature survey, these models integrate all the concepts of modularity, assemble-to-order strategy, product shortage, and component substitution. Meanwhile, stochastic demand can be studied by considering two sources of uncertainty: stochastic total demand and stochastic market proportions for the component options. On the other hand, there is a lack of studies that take into account the risk-averse approach and the mass customization together. This study also addresses this issue.

In this research, several mathematical models on inventory planning in mass customization having special network structure are investigated. The generalized network structure is taken into account and it is also shown that the extra decomposability provided by the modular product structure has a great impact to improve solution times for large instances. Chapter 3 introduces two stage stochastic linear programming problems and a risk-averse approach before discussing those models.

Table 2.1: Literature survey for the most relevant inventory planning models

Authors	Commonality	Substitution	Modularity	Mass cust.	Risk neutral	Risk averse	Service level
Rajaram and Tang [2]		X			X		X
Van Mieghem [37]	X		X		X		
Hillier [39]	X		X				X
Ma et al. [40]	X		X	X			X
Chod et al. [41]	X		X	X	X		
Jönsson et al. [42]	X		X		X		
Paul and Vakharia[43]			X	X			X
Nonas [44]	X		X		X		
Hsu and Bassok [46]		X			X		
Bassok et al. [47]		X			X		
Axsater [49]		X					X
Rao et al. [50]		X			X		
Hsu et al. [51]		X			X		
Li et al. [52]		X			X		
Weng [53]		X					X
Inderfurth[54]		X			X		
Tibben-Lembke and Bassok [55]		X			X		
Bayindir et al. [56]		X			X		
Karakul and Chan [57]		X			X		
Balakrishnan and Geunes [58]		X	X	X	X		
Nagarajan and Rajagopalan [60]		X			X		
Smith and Agrawal [62]		X			X		
Mahajan and Van Ryzin [63]		X			X		
Denton and Gupta [66]		X			X		
Thomas and Warsing [67]		X	X		X		
Yang and Schrage [69]		X			X		X
Xu and Chen [71]						X	
Borgonovo and Peccati [72]						X	
Zhang et al. [73]						X	

3 Selected Stochastic Programming Approach

Uncertainty is an important aspect in many optimization models and stochastic programming is the branch of mathematical programming to deal with uncertainty. Unlike the deterministic optimization problems which assume that all model parameters are known precisely, stochastic programming allows to deal with the uncertainty of some model parameters. Consequently, the application scope of stochastic programming is larger than the deterministic programming.

Each probabilistic outcome or scenario is considered explicitly in stochastic programming approach. An alternative way would be formulating a single scenario which considers the expected data. But, this approach has drawbacks. First, the optimum solution derived from the expected data is not guaranteed to be feasible for all prospective scenarios. Furthermore, this solution may not be optimum although it may be feasible. For this, it is vital to check both feasibility and optimality of a decision with respect to all different scenarios in stochastic programming.

Within the context of stochastic programming, there are many models proposed in the literature with deterministic constraints and an objective where all the probabilistic outcomes are considered in the form of expectation. An alternative approach is to formulate problems with a deterministic objective and probabilistic constraints. These probabilistic or chance constraints are assumed to hold with some probability or reliability level [74]. Indeed, it is possible to formulate a mathematical equivalence between these two approaches for some problems [75]. In this study, we consider only stochastic programming models with a probabilistic objective and deterministic constraints. Indeed, we are not limited only to expectation-type objectives, it is also possible to consider quantile objectives [76].

Stochastic programming models are classified according to their characteristics. They can be linear or nonlinear, use stochastic data having discrete or continuous probability density functions, or contain single or multiple stages.

In each stage, new decisions are taken given the past decisions, and resolved uncertainty, and by taking into account future scenarios. As an example, consider the two-stage case. The first-stage decision in a two-stage stochastic program is taken by considering the probabilistic future scenarios. Then a recourse action is made at the second stage when the uncertainty is resolved by taking into account the first stage decision. Hence, the optimal decision consists of the first-stage decision and the collection of recourse decisions at the second stage.

Finally, it is possible to characterize stochastic programming models from a risk perspective. Krokhmal et al. [77] define the risk as "a quantitative expression of a system of attitudes, or preferences with respect to a set of random outcomes". Thus, a high value of a risk measure has to be associated with a high danger of being exposed to high cost values in a cost minimization problem [78]. In stochastic programming, the expected or quantile value of the recourse actions may be included in the objective when taking the first-stage decision. They represent risk-neutral and risk-averse objectives, respectively. In particular, the expectation-type and CVaR-type objectives are used in inventory problems [71, 72, 73, 79], as the risk-neutral and risk-averse cases, respectively.

3.1 Two-Stage Stochastic Linear Programming Problems with Fixed Recourse

The general formulation of a two-stage stochastic linear programming problem with fixed recourse (see [74]; [80]) is given in (3.1)-(3.6):

$$\min \ z = \mathbf{c}^T\mathbf{x} + E_{\tilde{\mathbf{q}},\tilde{\mathbf{T}},\tilde{\mathbf{d}}}[Q(\mathbf{x}, \tilde{\mathbf{q}}, \tilde{\mathbf{T}}, \tilde{\mathbf{d}})] \tag{3.1}$$

$$\text{s.t.} \ \mathbf{A}\mathbf{x} = \mathbf{b} \tag{3.2}$$

$$\mathbf{x} \geq 0 \tag{3.3}$$

and

$$Q(\mathbf{x}, \tilde{\mathbf{q}}, \tilde{\mathbf{T}}, \tilde{\mathbf{d}}) = \min \ \tilde{\mathbf{q}}^T \mathbf{y} \tag{3.4}$$

$$\text{s.t.} \ \ \tilde{\mathbf{T}}\mathbf{x} + \mathbf{W}\mathbf{y} = \tilde{\mathbf{d}} \tag{3.5}$$

$$\mathbf{y} \geq 0 \tag{3.6}$$

At the first stage of this model, the objective is to minimize the current costs plus the expected value of future recourse actions. $\mathbf{x} \in \mathbb{R}^C$ is the vector of the first stage decision variables, and $\mathbf{c}$ is the vector of costs. Then, the uncertainty is resolved, and some recourse actions are taken at the second stage. $\mathbf{y} \in \mathbb{R}^{C'}$ is the vector of the second stage decision variables or recourse actions, and $\tilde{\mathbf{q}}$ is the vector of costs. $\mathbf{W}$ and $\tilde{\mathbf{T}}$ are defined in the recourse problem (3.4)-(3.6), and they correspond to the "fixed recourse matrix" and "technology matrix", respectively [74]. Here, the meaning of the fixed recourse is that the marginal effects of recourse actions are deterministic. Note that $\tilde{\mathbf{q}}$ and $\tilde{\mathbf{T}}$ are stochastic whereas $\mathbf{W}$ is deterministic. If $\tilde{\mathbf{q}}$ and $\tilde{\mathbf{T}}$ are known, the formulation (3.7)-(3.12) including recourse problems that differ only in the right-hand side value $\tilde{\mathbf{d}}$ is obtained.

$$\min \ \ z = \mathbf{c}^T\mathbf{x} + E_{\tilde{\mathbf{d}}}[Q(\mathbf{x}, \tilde{\mathbf{d}})] \tag{3.7}$$

$$\text{s.t.} \ \ \mathbf{A}\mathbf{x} = \mathbf{b} \tag{3.8}$$

$$\mathbf{x} \geq 0 \tag{3.9}$$

and

$$Q(\mathbf{x}, \tilde{\mathbf{d}}) = \min \ \mathbf{q}^T \mathbf{y} \tag{3.10}$$

$$\text{s.t.} \ \ \mathbf{T}\mathbf{x} + \mathbf{W}\mathbf{y} = \tilde{\mathbf{d}} \tag{3.11}$$

$$\mathbf{y} \geq 0 \tag{3.12}$$

There exist several solution methods to solve two-stage stochastic linear programming problems. Their mathematical properties has been investigated by many researchers and are well known. One of the key properties of the problem (3.7)-(3.12) lies in that the recourse problems (3.10)-(3.12) can be solved separately for each scenario if the first stage decision vector $\mathbf{x}$ is known. Another important property that leads to the development of computationally efficient solution methods is that $Q(\mathbf{x}, \tilde{\mathbf{d}})$ is polyhedral and convex with respect to $\mathbf{x}$. Hence, $\mathbf{x}$ can be thought of as complicating variables in linear programming problem (3.7)-(3.12).

The linear programming models with complicating variables or complicating constraints can benefit from the well-known decomposition techniques such as Benders decomposition [81] or Dantzig-Wolfe decomposition [82], respectively. Both techniques solve the original problem iteratively by introducing a master problem and one or more subproblems. Benders decomposition approximates the original problem by adding cuts to the master problem whereas Dantzig-Wolfe decomposition relies on column generations for the master problem. Since a linear programming model with complicating variables can be easily converted to one with complicating constraints by using the duality concept, Benders decomposition and Dantzig-Wolfe decomposition can be used alternatively to solve the original model with complicating variables and its dual with complicating constraints, respectively. Indeed, it is possible to derive the Benders decomposition procedure from the Dantzig-Wolfe decomposition procedure by applying the latter to the dual of the original model considered by Benders decomposition [83]. Lagrangian decomposition is an alternative technique to deal with complicating constraints. The original problem is relaxed by removing the complicating constraints. The difference between Lagrangian and Dantzig-Wolfe decomposition techniques lies in the master problem: Dantzig-Wolfe decomposition technique seeks optimization over the convex combinations of relaxed problem solutions whereas Lagrangian decomposition technique updates the Lagrange multipliers used to penalize violations of the complicating constraints [84]. Indeed, it is possible to derive the master problem of Dantzig-Wolfe decomposition from the dual of the master problem of Lagrangian decomposition [85]. The motivation behind the use of these techniques lies in that they allow to solve the main problem in a distributed context or they lead to a simplified subproblem structure following the decomposition of the main problem.

The L-shaped method is a widely used decomposition technique to solve two-stage stochastic linear programming problems. It is originally proposed by Van Slyke and Wets [86]. Similar to Benders decomposition, an approximation of (3.7)-(3.12) is solved in L-shaped method based on cut generations in the master problem which is extended at each iteration by adding feasibility and optimality cuts. L-shaped method can be seen as a specialized version of the Benders decomposition [87]. Indeed, L-shaped method extends Benders decomposition by considering explicitly the feasibility of the recourse problems [74]. As it is case for the Benders and Dantzig-Wolfe decomposition techniques, it is also possible to derive a column generation-based alternative method from L-Shaped method by use of the duality. However, if the number of decision variables is greater than the number of constraints in the first stage, L-shaped method is advantageous in computational terms [74]. The basis factorization methods can also be considered when solving two-stage stochastic linear programming problems. However, they do not offer advantage over L-Shaped method [74]. Alternatively, the Level decomposition method [88] can be used to solve two-stage stochastic linear programming problems. This method relies on an approximative version of the constrained level method [89]. In particular, it offers improved stability. Indeed, it is also possible to improve the performance of the L-shaped method. In particular, the master problem size of the L-Shaped method can be controlled by adding a quadratic regularizing term objective and so determining the active constraints within the master problem [87]. This is called the regularized decomposition method and is finitely convergent [90]. The augmented Lagrangian decomposition is another method which is effective particularly for dealing with convex multistage stochastic programs [87]. Among alternative methods, the simplicity of the L-shaped method is noteworthy. Therefore, we consider the L-shaped method when dealing with two-stage stochastic linear programming problems. An algorithm for the L-shaped method is provided by Birge and Louveaux [74] and it consists of the following steps:

I. $\rho = 0,\ \tau = 0,\ \mu = 0.$

II. $\mu = \mu + 1$. Solve the master linear programming problem (3.13)-(3.17)

$$
\begin{aligned}
&\min && z = \mathbf{c}^T\mathbf{x} + \theta && && (3.13)\\
&\text{s.t.} && \mathbf{A}\mathbf{x} = \mathbf{b}, && && (3.14)\\
& && \mathbf{g}_i\mathbf{x} \geqslant \alpha_i, && i = 1, ..., \rho, && (3.15)\\
& && \mathbf{h}_i\mathbf{x} + \theta \geqslant \beta_i, && i = 1, ..., \tau, && (3.16)\\
& && \mathbf{x} \geqslant 0, \quad \theta \in \mathbb{R}. && && (3.17)
\end{aligned}
$$

Let us denote the values of variables at the optimal solution as $\mathbf{x}^\mu$ and θ^μ. Note that if there was no constraint (3.16) defined in the master problem (3.13)-(3.17), then θ^μ would not be considered in the computation of $\mathbf{x}^\mu$ and then $\theta^\mu = -\infty$.

III. Let us denote the possible realizations of $\tilde{\mathbf{d}}$ as $\mathbf{d}^k$ where $k = 1, ..., K$. Solve the linear programming problem (3.18)-(3.20) for $k = 1, ..., K$

$$
\begin{aligned}
&\min && \bar{\Omega} = \mathbf{u}^T\mathbf{v}^+ \ + \ \mathbf{u}^T\mathbf{v}^- && (3.18)\\
&\text{s.t.} && \mathbf{W}\ \mathbf{y}_k + \mathbf{I}\ \mathbf{v}^+ \ - \ \mathbf{I}\ \mathbf{v}^- \ = \ \mathbf{d}^k - \mathbf{T}\mathbf{x}^\mu \ : \mathbf{e}^\mu, (dual\ variable) && (3.19)\\
& && \mathbf{y}_k \geqslant 0,\ \mathbf{v}^+ \ \geqslant 0,\ \mathbf{v}^- \ \geqslant 0, && (3.20)
\end{aligned}
$$

where $\mathbf{u}^T = (1, ..., 1)$, $\mathbf{I}$ is identity matrix, and $\mathbf{v}^+$ and $\mathbf{v}^-$ are auxillary variables. If for some k, we have $\bar{\Omega} > 0$, then (3.21) and (3.22) are defined to form a *feasibility cut*:

$$\mathbf{g}_{\rho+1} = (\mathbf{e}^\mu)^T\mathbf{T} \tag{3.21}$$

and

$$\alpha_{\rho+1} = (\mathbf{e}^\mu)^T\mathbf{d}^k. \tag{3.22}$$

Let $\rho = \rho + 1$ and add the new feasibility cut $\mathbf{g}_\rho\mathbf{x} \geqslant \alpha_\rho$ to the constraint set (3.15). Return to Step II. If $\bar{\Omega} = 0$ for all k, then proceed to Step IV.

IV. Solve the linear programming problem (3.23)-(3.25) for $k = 1, ..., K$

$$\min \quad \Omega = \mathbf{q}^T \mathbf{y}_k \tag{3.23}$$

$$\text{s.t.} \quad \mathbf{W}\ \mathbf{y}_k = \mathbf{d}^k - \mathbf{T}\mathbf{x}^\mu \ : \mathbf{o}_k^\mu, (dual\ variable) \tag{3.24}$$

$$\mathbf{y}_k \geqslant 0. \tag{3.25}$$

Let us define

$$\mathbf{h}_{\tau+1} = \sum_{k=1}^{K} p_k (\mathbf{o}_k^\mu)^T \mathbf{T} \tag{3.26}$$

and

$$\beta_{\tau+1} = \sum_{k=1}^{K} p_k (\mathbf{o}_k^\mu)^T \mathbf{d}^k. \tag{3.27}$$

Here p_k is the probability of the k^{th} realization. If $\theta^\mu \geqslant \Omega^\mu = \beta_{\tau+1} - \mathbf{h}_{\tau+1}\mathbf{x}^\mu$, then $\mathbf{x}^\mu$ is an optimal solution. Else, $\tau = \tau + 1$, add the optimality cut $\mathbf{h}_\tau \mathbf{x} + \theta \geqslant \beta_\tau$ to the constraint set (3.16), and return to Step II.

3.2 Conditional Value at Risk

Pritsker [91] defines Value at Risk (VaR) as “the largest loss in portfolio value that would be expected to occur due to changes in market prices over a given period of time in all but a small percentage of circumstances”. To illustrate this, 95%-VaR is the largest loss in all but 5% of circumstances. Pritsker [91] also gives an alternative description of VaR in which it is “the amount of capital the firm would require to absorb its portfolio losses in all but a small percentage of circumstances”. VaR can be efficiently calculated based on the normally distributed data but this is not the case when non-normal and discrete distributions are used because of the lack of sub-additivity and non-convexity [92]. To overcome these difficulties, CVaR is defined as an another risk measure based on the risk measure of VaR. CVaR corresponds to the conditional expected loss under the condition that it exceeds VaR [92]. CVaR can be viewed as the mean excess loss, the mean shortfall, or the tail VaR for continuous distributions, whereas it is the weighted average of VaR and the losses strictly exceeding VaR for discrete distributions [92].

The definition and calculation of CVaR ([92]; [93]; [94]) are briefly introduced in the next. Let $f(\mathbf{x}, \tilde{\mathbf{d}}) \in \mathbb{R}$ stands for the cost experienced as a function of a decision vector $\mathbf{x} \in \mathbb{R}^C$, and a random vector $\tilde{\mathbf{d}} \in \mathbb{R}^N$. $p(\tilde{\mathbf{d}})$ is the probability density function of $\tilde{\mathbf{d}}$. The probability of $f(\mathbf{x}, \tilde{\mathbf{d}})$ being smaller than ζ is denoted by $F(\mathbf{x}, \zeta)$ and it can be calculated as given in (3.28).

$$F(\mathbf{x}, \zeta) = \int_{f(\mathbf{x}, \tilde{\mathbf{d}}) \leqslant \zeta} p(\tilde{\mathbf{d}}) d\tilde{\mathbf{d}}. \tag{3.28}$$

The VaR and $CVaR$ values for the cost incurred with decision $\mathbf{x}$ and probability level $\alpha \in (0, 1)$ are denoted by $VaR_\alpha(\mathbf{x})$ and $CVaR_\alpha(\mathbf{x})$ and are given as

$$VaR_\alpha(\mathbf{x}) = \min\{\zeta \in \mathbb{R} : F(\mathbf{x}, \zeta) \geqslant \alpha\}, \tag{3.29}$$

$$CVaR_\alpha(\mathbf{x}) = (1 - \alpha)^{-1} \int_{f(\mathbf{x}, \tilde{\mathbf{d}}) \geqslant VaR_\alpha(\mathbf{x})} f(\mathbf{x}, \tilde{\mathbf{d}}) p(\tilde{\mathbf{d}}) d\tilde{\mathbf{d}}, \tag{3.30}$$

respectively. It is possible to reformulate $CVaR_\alpha(\mathbf{x})$ as given in (3.31)-(3.32).

$$CVaR_\alpha(\mathbf{x}) = \min_{\zeta \in \mathbb{R}} L_\alpha(\mathbf{x}, \zeta) \tag{3.31}$$

where

$$L_\alpha(\mathbf{x}, \zeta) = \zeta + (1 - \alpha)^{-1} \int_{\tilde{\mathbf{d}} \in \mathbb{R}^N} \max(f(\mathbf{x}, \tilde{\mathbf{d}}) - \zeta, 0) p(\tilde{\mathbf{d}}) d\tilde{\mathbf{d}}. \tag{3.32}$$

Furthermore, $L_\alpha(\mathbf{x}, \zeta)$ and $CVaR_\alpha(\mathbf{x})$ are convex if $f(\mathbf{x}, \tilde{\mathbf{d}})$ is convex. Thus, minimizing $CVaR_\alpha$ is equivalent to minimizing $L_\alpha(\mathbf{x}, \zeta)$, or

$$\min_{\mathbf{x} \in \mathbb{R}^C} CVaR_\alpha(\mathbf{x}) \equiv \min_{(\mathbf{x}, \zeta) \in \mathbb{R}^C \times \mathbb{R}} L_\alpha(\mathbf{x}, \zeta). \tag{3.33}$$

The function $L_\alpha(\mathbf{x}, \zeta)$ can be approximated by the function $\tilde{L}_\alpha(\mathbf{x}, \zeta)$ when the number of scenarios is finite:

$$\tilde{L}_\alpha(\mathbf{x}, \zeta) = \zeta + (1 - \alpha)^{-1} \sum_{k=1}^{K} z_k \max(f(\mathbf{x}, \mathbf{d}^k) - \zeta, 0) \tag{3.34}$$

where z_k are probabilities of scenarios $\mathbf{d}^k$, $k \in \mathcal{K}$. Note that if the cost function $f(\mathbf{x}, \tilde{\mathbf{d}})$ is linear in $\mathbf{x}$, then the function $\tilde{L}_\alpha(\mathbf{x}, \zeta)$ is convex and piecewise linear. Therefore, the problem given in (3.35) is an approximation of (3.33).

$$\min_{(\mathbf{x},\zeta)\in\mathbb{R}^C\times\mathbb{R}} \quad \tilde{L}_\alpha(\mathbf{x},\zeta) \tag{3.35}$$

Furthermore, it is possible to linearize the function $\tilde{L}_\alpha(\mathbf{x},\zeta)$ if the auxiliary variables η_k, $k = 1, ..., K$ and a set of linear constraints are introduced. This is shown in (3.36)-(3.38) which is equivalent to (3.35).

$$\min_{(\mathbf{x},\zeta)\in\mathbb{R}^C\times\mathbb{R}} \zeta + (1-\alpha)^{-1}\sum_{k=1}^{K} z_k\eta_k \tag{3.36}$$

$$\text{s.t.} \quad \eta_k \geqslant f(\mathbf{x},\mathbf{d}^k) - \zeta \qquad k = 1, ..., K, \tag{3.37}$$

$$\eta_k \geqslant 0 \qquad k = 1, ..., K. \tag{3.38}$$

4 Assemble-to-Order with Substitution in Mass Customization Environment

4.1 Motivation

Nowadays, modular assembly is an important mass customization strategy adopted in principle by many competing companies. Generally, the component stocks are held instead of the end product stocks so that they are assembled to form an end product and to meet the customer demand in this strategy. Furthermore, it is often the case to obtain end products with different functionalities by adopting the component swapping modularity. Illustrative example given in Figure 4.1 shows the case where six end product configurations can be formed from three options available in the first module and two options available in the second module. Note that the total number of end product configurations may become large quickly if the options are various. Thus, a wide range of customer requirements can be addressed with mass customization. On the other hand, mass production of standardized components helps to reduce operating costs.

In modular assembly, two stage decision making is common since the lead time required to acquire components are longer than the time required for their assembly. At the first stage, one has to decide how many components to purchase or manufacture without knowing the customer demand realization. At the second stage, the problem of how to allocate available components to end product assemblies to meet the customer demand is faced.

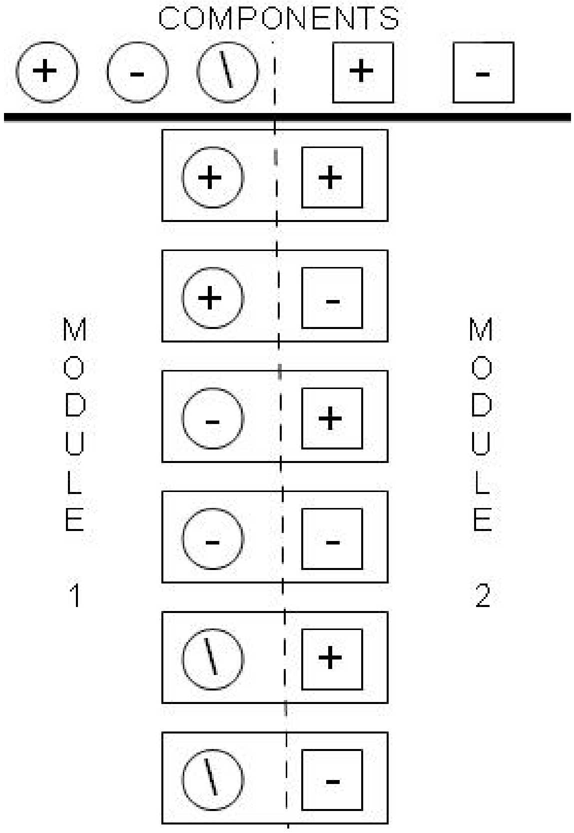


Figure 4.1: Mass customization via modular assembly

Now, another interesting frame similar to previously discussed one will be investigated. In Figure 4.1, the components in a given module are differentiated according to their features. Under current global economic conditions, companies may work with many suppliers dispersed around the world as a necessity of the global competition. This fact gives another criterion to classify the components. Thus, the geographical location where the components are manufactured is also a major cost driver. In Figure 4.2, we aim to illustrate this case. The component in the first module can be supplied by two suppliers associated with different geographical locations (Loc 1 and 2), and the component in the second module can be supplied by three suppliers associated with different geographical locations (Loc 3, 4 and 5). Consequently, six possible distribution warehouse locations can be determined with respect to suppliers proximity as shown in Figure 4.2. It is also possible to create structures where both features and origins of components are taken into account.

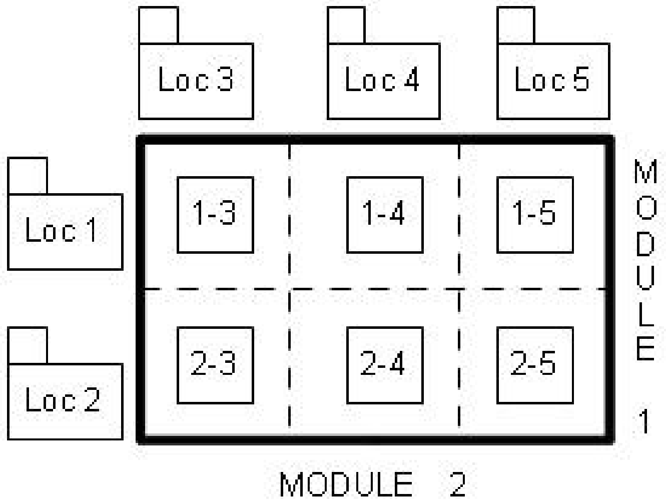


Figure 4.2: Geographical warehouse clusters

4.2 Problem Formulation with Expected Cost Objective

For the modular assembly problem investigated in this study, it is assumed that each end product configuration is assembled from a fixed number of *modules*, and each module is made of a predetermined amount of *substitutable components*. Although it is possible to relax this assumption within the setting that will be presented in the next, we will keep it for notational simplicity. Accordingly, each customer demand involves a collection of components, one for each module.

In the *traditional* assemble-to-order strategy, when a customer order is received, the assembly process is triggered and all components required for the assembly are retrieved from the store. If any component is out-of-stock then it is impossible to immediately start the assembly, so the customer order is not satisfied or delayed. In the assemble-to-order strategy with *component substitution*, however, there is the option to substitute a component with another when it is out-of-stock. This action incurs some direct and indirect costs. From the customer perspective, the substitution may lead to a dissatisfaction which may also affect potential customers. From the manufacturer perspective, this substitution may require further processing of the components or an upgrade, so this cost is related to the component converting costs. Therefore, the decision variables can be grouped into two. The first stage variables correspond to the purchase or manufacture quantities of components and the second stage variables are related to allocation of these components to the customer orders when they are realized. At the first stage purchasing or manufacturing costs incur, while at the second stage the component allocation and holding costs, and also the product shortage costs are relevant. We assume that the allocation costs include the cost of substitution.

Based on this brief introduction, a mathematical model aiming to reduce the total cost of the mass customization is given. The model parameters are as follows:

M: total number of modules,
C: total number of components,
N: total number of products,
K: total number of scenarios,
c_i^o: purchasing cost of component *i* of module *o*,
p_{ij}^o: allocation cost of component *i* of module *o* to product *j*,
$\tilde{d}_j$: stochastic demand of product *j*,
$\tilde{\mathbf{d}} = (\mathbf{d}^1, \mathbf{d}^2, ..., \mathbf{d}^K)$: stochastic demand vector of products,
h_i^o: holding cost of component *i* of module *o*,
s_j: shortage cost of product *j*,
a_{ij}^o: needed amount of component *i* of module *o* to the assembly of product *j*. The first stage variables are:

x_i^o: purchased quantity of component *i* of module *o*,

and the second stage variables are:

y_{ij}^o: allocated quantity of component *i* of module *o* to the assembly of product *j*,
e_i^o: unused quantity of component *i* of module *o*,
u_j: shortage quantity for product *j*.

Finally we define the following sets:

$\mathcal{R}_j^o$: The set of substitutable components for module o that can be selected to manufacture product j,

$\mathcal{U}_i^o$: The set of products that can be assembled from component i of module o.

$$\min \sum_{o,i} c_i^o x_i^o + E_{\tilde{\mathbf{d}}}[Q_1(\mathbf{x}, \tilde{\mathbf{d}})] \tag{4.1}$$

$$\text{s.t. } x_i^o \geq 0 \qquad \forall o, \forall i \tag{4.2}$$

and

$$Q_1(\mathbf{x}, \tilde{\mathbf{d}}) = \min \sum_{o,i} h_i^o e_i^o + \sum_j s_j u_j + \sum_{o,i} \sum_{j \in \mathcal{U}_i^o} p_{ij}^o y_{ij}^o \tag{4.3}$$

$$\text{s.t. } \sum_{j \in \mathcal{U}_i^o} y_{ij}^o + e_i^o = x_i^o \qquad \forall o, \forall i \tag{4.4}$$

$$\sum_{i \in \mathcal{R}_j^o} \frac{y_{ij}^o}{a_{ij}^o} + u_j = \tilde{d}_j \qquad \forall o, \forall j \tag{4.5}$$

$$y_{ij}^o, u_j, e_i^o \geqslant 0 \qquad \forall o, \forall i, \forall j \tag{4.6}$$

The objective function of the first stage in (4.1) minimizes the sum of component purchasing costs and the expected cost of the second stage of a given demand scenario. Constraints in (4.2) assure the non-negativity of the purchased quantities. The objective function in (4.3) of the second stage is defined for a given demand realization $\tilde{\mathbf{d}}$ and we aim to minimize the sum of component holding and allocation costs, and product shortage costs. Constraints in (4.4) make sure that only in-stock components are used. Constraints in (4.5) are for the allocation of components according to the customer demand. Finally, constraints in (4.6) are for the non-negativity of the second stage's decision variables.

4.3 Demand Structure

4.3.1 Demand Scenarios

As previously mentioned, it is assumed that a product is assembled from a certain number of modules and there exist several substitutable components that can be selected for each module. However a specific product is made up from a unique component selection.

Then, it is possible to represent the end products by the leaf nodes of a multilevel tree where each level corresponds to a module. In this tree structure, a conditional choice probability is determined for each branch. The demand of a specific product j can be determined by multiplying preference rates along its own path on the tree. Note that the demand structure taking into account the aggregate product demand variance and the stochastic market proportions for the module options is also adopted by others [43].

To illustrate, let us consider a product family with two modules and two components in each module. The aggregated demand of the product family is equal to 1000. Then, it is possible to determine specific end product configuration demands as shown in Figure 4.3.

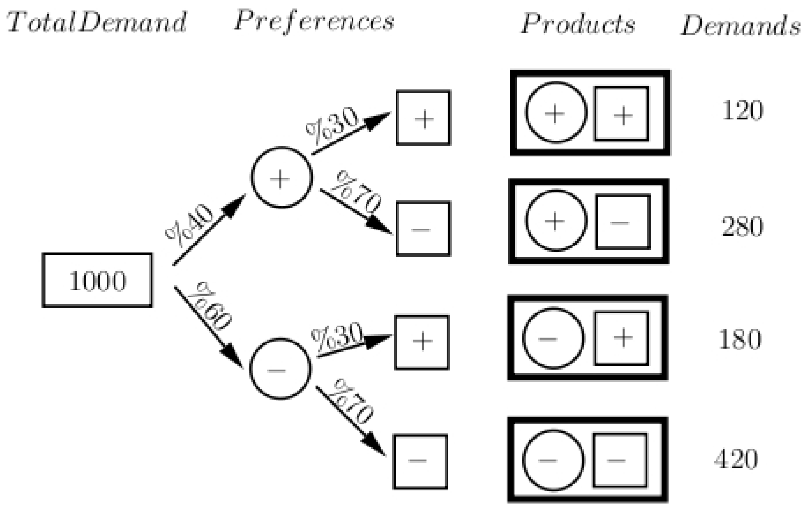


Figure 4.3: Demand decomposition tree

Given the total demand of the product family and the preference rates on each branch of the demand decomposition tree, it's possible to describe a demand scenario for all products. Note that a single demand scenario describes a deterministic model. If the preference rates can take different values, then it is possible to generate several demand decomposition trees having different leaves. Accordingly, each tree derived from possible values of these preference rates corresponds to a new demand scenario. So, the model consisting of the all possible demand scenarios is stochastic. The total number of scenarios can be calculated by multiplying the numbers of possible preference rate combinations over all modules. For example, if the total number of modules is M and there exist 2 possible component options per module, then the total number of scenarios can be at most 2^M.

The total demand for the product family may fluctuate, so its different realizations will correspond also to new scenarios. Continuing from the previous example, if there are D possible total demand realizations, then the total number of scenarios can be at most $D \times 2^M$.

4.3.2 Methods for the Scenario Generation

In the stochastic programming framework, the values of some parameters vary by taking into account the uncertainty about the future. If this uncertainty spans a single period then we may assume that these parameters follow some probability distributions. In the multiple period case, these parameters can be described by stochastic processes. In this thesis, we deal only with the former case but the multi-period case can be also elaborated in a future study.

The stochastic parameters may be derived from a data set of the past observations and they may also include some previsions. If their statistical properties are already known, for example, they can follow some continuous probability distributions. In this case, one approach is to approximate the continuous probability distributions by their discrete counterparts in order to be able to use efficient solution methods developed for stochastic programming. If the stochastic parameters are assumed to follow discrete probability distributions, it is possible to create scenarios with respect to the all possible values of these parameters.

In a typical scenario generation procedure, it is possible to observe the steps of model assumption, historical data collection, parameter estimation, discretization of the distributions and sampling [95]. Indeed, today's information systems tools are really advanced and the investments for these systems are increasing. Di Domenica et al. [96] explain how to integrate such systems with the decision support systems based on stochastic programming. The main scenario generation methods include sampling, simulation, statistical approaches and hybrid methods [97]. However, the quality of the generated scenarios does not depend on how well the distributions are approximated, but stability and absence of bias are the most important requirements [98]. Thus, in-sample stability, out-of-sample stability and absence of bias requirements are defined, but in-sample stability is used in practice because it does not involve the calculations with the true distribution [98].

4.4 General Case

4.4.1 Generalized Network Formulation with Equality Constraints

Let us consider the linear programming model in (4.3)-(4.6). If each variable u_j is replaced by the variable u_j^o, then (4.5) can be replaced by (4.9) and (4.10), and thus, the model in (4.3)-(4.6) can be transformed to the model in (4.7)-(4.11).

$$
\begin{aligned}
Q_2(\mathbf{x}, \tilde{\mathbf{d}}) = \min \quad & \sum_{o,i} h_i^o e_i^o + \sum_{j} s_j u_j + \sum_{o,i} \sum_{j \in \mathcal{U}_i^o} p_{ij}^o y_{ij}^o && && (4.7)\\
\text{s.t.} \quad & \sum_{j \in \mathcal{U}_i^o} y_{ij}^o + e_i^o = x_i^o && \forall o, \forall i && (4.8)\\
& \sum_{i \in \mathcal{R}_j^o} \frac{y_{ij}^o}{a_{ij}^o} + u_j^o = \tilde{d}_j && \forall o, \forall j && (4.9)\\
& u_j = u_j^1 = \dots = u_j^M && \forall j && (4.10)\\
& y_{ij}^o, u_j, u_j^o, e_i^o \geqslant 0 && \forall o, \forall i, \forall j && (4.11)
\end{aligned}
$$

The linear programming model in (4.7)-(4.11) can be represented by a generalized network flow model with equality constraints. For the sake of simplicity, consider a model with two modules having two components per module and let us to set the a_{ij}^o values to one. Then, Figures 4.4, 4.5, 4.6, and 4.7 illustrate the resulting network flow structure. It is not difficult to observe that this network has the potential to be further decomposed by modules. The only obstacle against this is the equality constraints in (4.10).

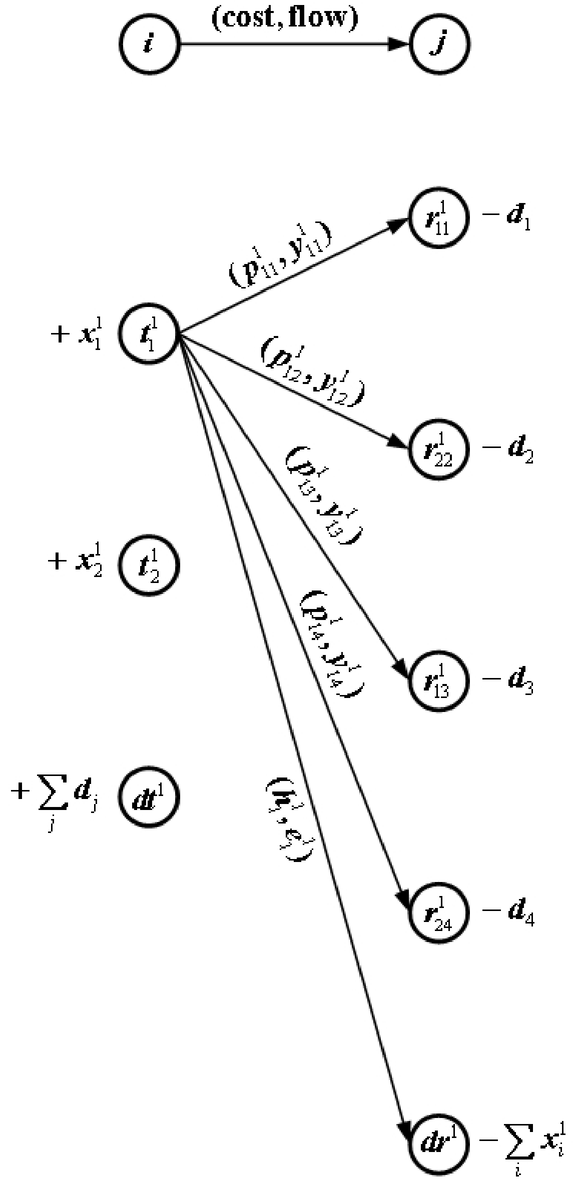


Figure 4.4: Subgraph associated with the allocation of the first component

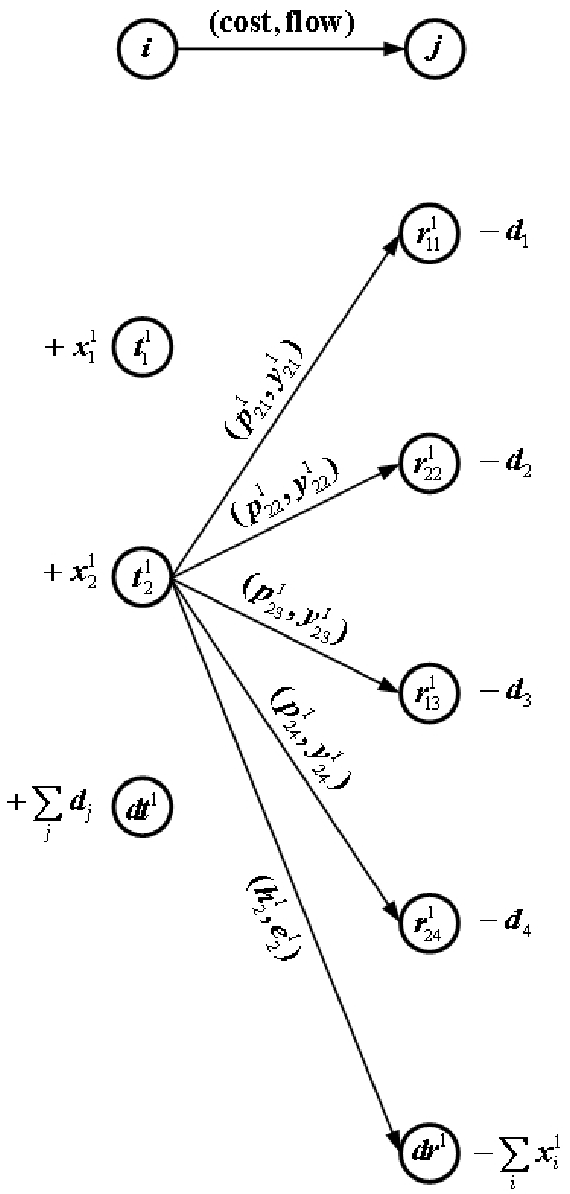


Figure 4.5: Subgraph associated with the allocation of the second component

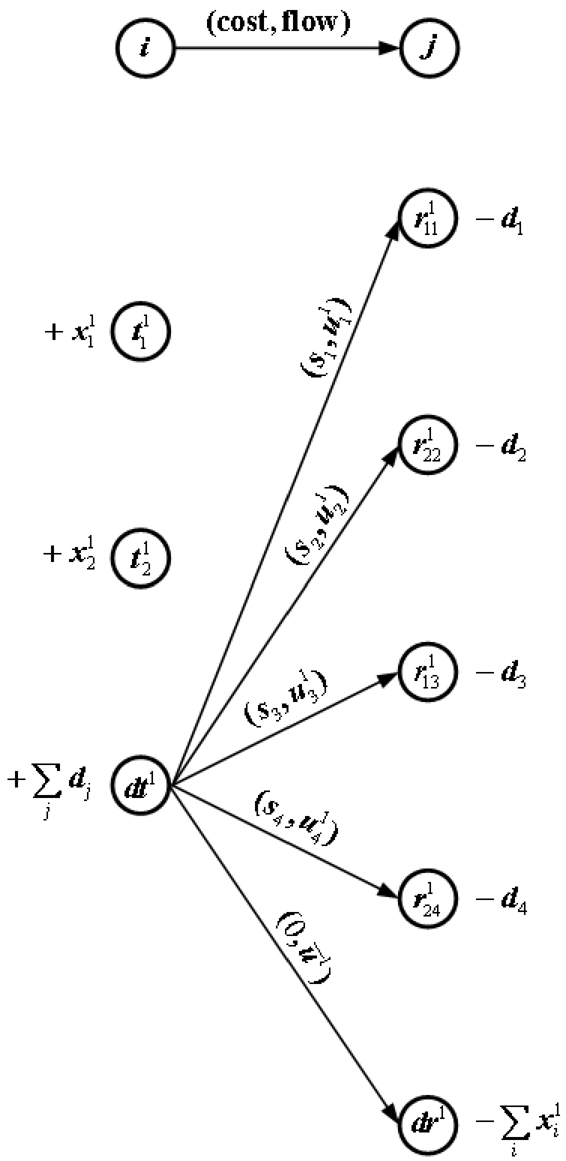


Figure 4.6: Subgraph associated with the allocation of the dummy component

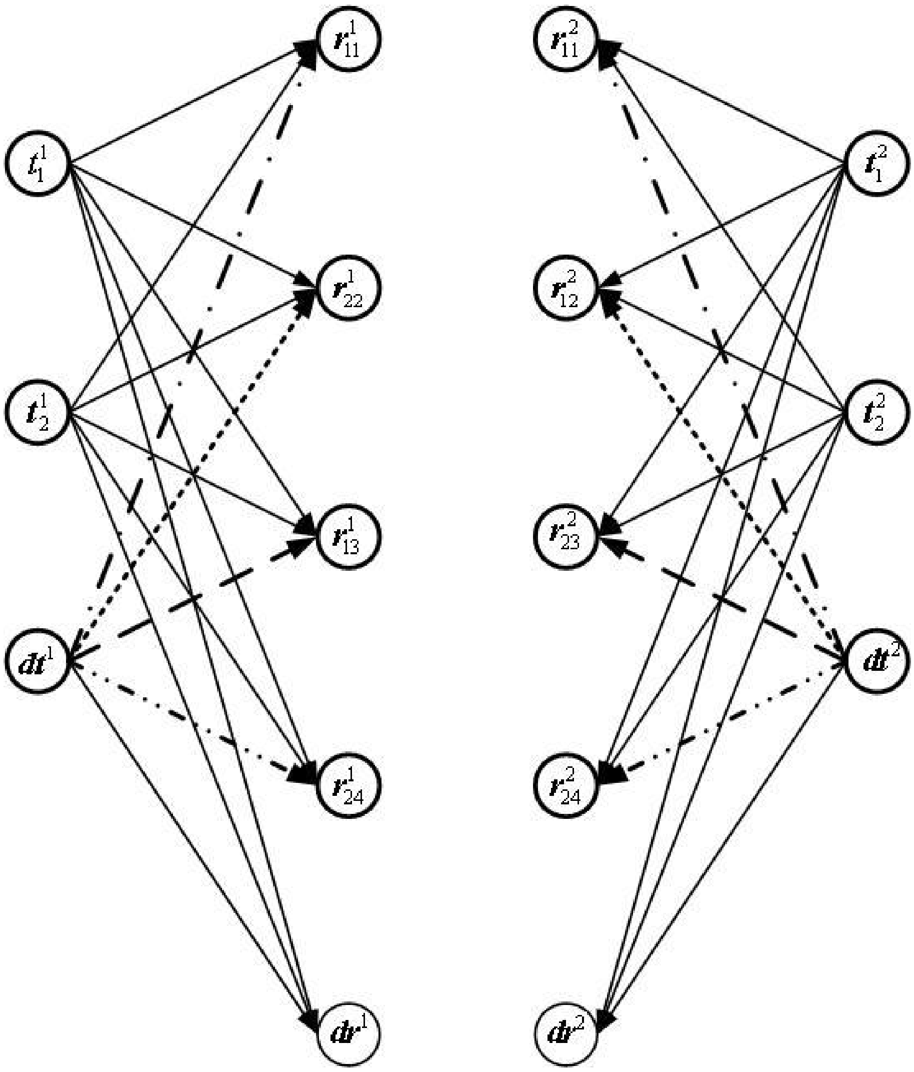


Figure 4.7: Graph associated with two modules and two components with equal flow arcs emphasized

In Figures 4.4, 4.5, 4.6, and 4.7, t_i^o's and dt^o's are for the component supply nodes and the dummy component supply nodes, respectively. Similarly, r_{ij}^o's and dr^o's denote the product demand nodes and the dummy product demand nodes, respectively. The auxiliary variables $\bar{u}^o$ in Figure 4.6 are used to connect the dummy component supply nodes and the dummy product demand nodes to conserve the flow balance constraint. In Figure 4.7, the equal flow constraints are illustrated by dashed lines with different styles, each of which corresponds to some equal flow requirement. Finally, we will note here that only first module of the network is illustrated in Figures 4.4, 4.5, and 4.6. This is because the interpretation of the second module is similar. However, there is a small exception in the use of the costs s_j: their values are set to s_j in one module, but they have to be equal to zero in other modules.

4.4.2 Pure Network Formulation with Proportionality Constraints

Consider the generalized network model with equality constraints in (4.7)-(4.11) and in particular, the constraints (4.9) and (4.10). If a_{ij}^o's are all equal for $i \in \mathcal{R}_j^o$, then the constraints (4.9) and (4.10) can be changed as the constraints (4.12) and (4.13) with $a_{ij}^o u_j^o = u_j'^o$ and $a_{ij}^o \tilde{d}_j = \tilde{d}_j'^o$. Hence, a pure network formulation with the proportionality constraints (4.13) is obtained:

$$\sum_{i \in \mathcal{R}_j^o} y_{ij}^o + u_j'^o = \tilde{d}_j'^o \qquad \forall o, \forall j, \tag{4.12}$$

$$\frac{u_j'^1}{a_{ij}^1} = ... = \frac{u_j'^M}{a_{ij}^M} \qquad \forall j. \tag{4.13}$$

4.5 Subcontracting Case

4.5.1 Generalized Network Formulation with Equality Constraints

Now, consider the case described in section 4.1 where the components are classified according to the places they are manufactured. In this case, the components are required by distribution warehouses. Note that the product and the distribution warehouse terms are interchangeable from mathematical notation perspective. The product term is kept for the simplicity required when comparing the cases. The single objective can be associated with a vendor managed inventory strategy. Furthermore, we assume that the demand satisfaction proportions are equal for each distribution warehouse. This situation may arise with subcontracting terms in a vendor managed inventory system. It is reasonable to tolerate small stock deficiencies in distribution warehouses by applying some customer policies, however a large amount of out-of-stocks will lead to great business losses without a doubt. Hence, any unbalanced supply plan for the distribution warehouses should be avoided. Note that this common demand satisfaction proportion is also a performance measure for the suppliers. Let $0 \leqslant u \leqslant 1$ be this common demand satisfaction proportion valid for each distribution warehouse. Thus, the model consisting of the (4.1)-(4.6) and the constraints (4.14) is obtained.

$$\frac{u_1}{\tilde{d}_1} = ... = \frac{u_N}{\tilde{d}_N} = u. \tag{4.14}$$

Accordingly, the model in (4.7)-(4.11) can be modified as in (4.15)-(4.19).

$$\begin{aligned}
Q_3(\mathbf{x}, \tilde{\mathbf{d}}) = \min \quad & \sum_{i,o} h_i^o e_i^o + \sum_j s_j \tilde{d}_j u + \sum_{i,o} \sum_{j \in \mathcal{U}_i^o} p_{ij}^o y_{ij}^o && && (4.15) \\
\text{s.t.} \quad & \sum_{j \in \mathcal{U}_i^o} y_{ij}^o + e_i^o = x_i^o && \forall o, \forall i && (4.16) \\
& \sum_{i \in \mathcal{R}_j^o} \frac{y_{ij}^o}{a_{ij}^o} + \tilde{d}_j u = \tilde{d}_j && \forall o, \forall j && (4.17) \\
& 0 \leqslant u \leqslant 1 && && (4.18) \\
& y_{ij}^o, u, e_i^o \geqslant 0 && \forall o, \forall i, \forall j && (4.19)
\end{aligned}$$

If some additional conditions are met, it is possible to further simplify the model given in (4.15)-(4.19). Let us define first the following sets:

$\mathcal{U}_{ir}^o$: The set of products that can substitute component r with component i for module o.
$\mathcal{J}_r^o$: The set of products that need component *r* for module *o*.
$\mathcal{I}_r^o$: The set of substitutable components for component *r* for module *o*.

Note that $\cup_r \mathcal{U}_{ir}^o = \mathcal{U}_i^o$ and $\mathcal{U}_{ir'}^o \cap \mathcal{U}_{ir}^o = \emptyset \quad \forall r, r'$, it is a partition. Moreover, it is assumed that p_{ij}^o's and a_{ij}^o's are equals for $j \in \mathcal{U}_{ir}^o$. Indeed, the equality of p_{ij}^o's for $j \in \mathcal{U}_{ir}^o$ can be realistic in most cases. This is because the transportation costs are affected mainly by the proximity of distribution warehouses to suppliers. The transportation costs of components manufactured in nearby suppliers are negligible compared to the transportation costs of components manufactured in distant suppliers. Thus, we can say that a distant supplier substitutes for a nearby supplier when the latter is not capable to meet the demand. Hence, the above assumption will be valid if transportation costs are based on a substitution hierarchy between suppliers. Finally, it is possible to obtain the model in (4.20)-(4.24) by aggregating over all $j \in \mathcal{J}_r^o$.

$$
\begin{aligned}
Q_4(\mathbf{x}, \tilde{\mathbf{d}}) = \min \quad & \sum_{i,o} h_i^o e_i^o + \sum_j s_j \tilde{d}_j u + \sum_{o,r} \sum_{i \in \mathcal{I}_r^o} p_{ir}^o \sum_{j \in \mathcal{U}_{ir}^o} y_{ij}^o && && (4.20) \\
\text{s.t.} \quad & \sum_r \sum_{j \in \mathcal{U}_{ir}^o} y_{ij}^o + e_i^o = x_i^o && \forall o, \forall i && (4.21) \\
& \sum_{j \in \mathcal{J}_r^o} \sum_{i \in \mathcal{R}_j^o} \frac{y_{ij}^o}{a_{ij}^o} + \sum_{j \in \mathcal{J}_r^o} \tilde{d}_j u = \sum_{j \in \mathcal{J}_r^o} \tilde{d}_j && \forall o, \forall r && (4.22) \\
& 0 \leqslant u \leqslant 1 && && (4.23) \\
& y_{ij}^o, u, e_i^o \geqslant 0 && \forall o, \forall i, \forall j && (4.24)
\end{aligned}
$$

It is possible to reformulate the constraint (4.22) as the constraint (4.25):

$$
\sum_{i \in \mathcal{I}_r^o} \frac{1}{a_{ir}^o} \sum_{j \in \mathcal{U}_{ir}^o} y_{ij}^o + \sum_{j \in \mathcal{J}_r^o} \tilde{d}_j u = \sum_{j \in \mathcal{J}_r^o} \tilde{d}_j \qquad \forall o, \forall r. \tag{4.25}
$$

To obtain a generalized network structure with equality constraints, let us transform the constraint (4.25) to the constraints (4.26) and (4.27).

$$
\begin{aligned}
& \sum_{i \in \mathcal{I}_r^o} \frac{1}{a_{ir}^o} \sum_{j \in \mathcal{U}_{ir}^o} y_{ij}^o + \sum_{j \in \mathcal{J}_r^o} \tilde{d}_j u_r^o = \sum_{j \in \mathcal{J}_r^o} \tilde{d}_j && \forall o, \forall r && (4.26) \\
& u_r^o = u_{r'}^{o'} && \forall o, o', r, r' && (4.27)
\end{aligned}
$$

Let $y_{ir}^{'o}$ be equal to $\sum_{j \in \mathcal{U}_{ir}^o} y_{ij}^o$. Finally, (4.28)-(4.32) are obtained.

$$
\begin{aligned}
Q_5(\mathbf{x}, \tilde{\mathbf{d}}) = \min \quad & \sum_{i,o} h_i^o e_i^o + \sum_j s_j \tilde{d}_j u_1^1 + \sum_{o,r} \sum_{i \in \mathcal{I}_r^o} p_{ir}^o y_{ir}^{'o} && && (4.28) \\
\text{s.t.} \quad & \sum_r y_{ir}^{'o} + e_i^o = x_i^o && \forall o, \forall i && (4.29) \\
& \sum_{i \in \mathcal{I}_r^o} \frac{1}{a_{ir}^o} y_{ir}^{'o} + \sum_{j \in \mathcal{J}_r^o} \tilde{d}_j u_r^o = \sum_{j \in \mathcal{J}_r^o} \tilde{d}_j && \forall o, \forall r && (4.30) \\
& u_r^o = u_{r'}^{o'} && \forall o, o', r, r' && (4.31) \\
& y_{ir}^{'o}, u_r^o, e_i^o \geqslant 0 && \forall o, \forall i, \forall r && (4.32)
\end{aligned}
$$

This is a generalized network formulation with the equality constraints (4.31).

4.5.2 Pure Network Formulation with Proportionality Constraints

If a^o_{ir}'s are equals for $i \in \mathcal{I}^o_r$, then we can let $a^o_{ir} \sum_{j\in\mathcal{J}^o_r} \tilde{d}_j u^o_r = u'^o_r$ and $a^o_{ir} \sum_{j\in\mathcal{J}^o_r} \tilde{d}_j = \tilde{d}'^o_r$. The constraints (4.30) and (4.31) becomes (4.33) and (4.34), respectively as below:

$$\sum_{i\in\mathcal{R}^o_r} y'^o_{ir} + u'^o_r = \tilde{d}'^o_r \qquad \forall o, \forall r \tag{4.33}$$

$$\frac{u'^o_r}{\tilde{d}'^o_r} = \frac{u'^{o'}_{r'}}{\tilde{d}'^{o'}_{r'}} \qquad \forall o, o', r, r' \tag{4.34}$$

The constraints (4.34) are the proportionality side constraints.

4.6 Spot Market Case

4.6.1 Generalized Network Formulation

We now define the following variables and parameters and assume that all end product demands are satisfied but the missing components are supplied with relatively high costs at the second stage. The interpretation of the model parameters and variables is modified as follows:

u^o_j : quantity outsourced of component requirement in module o for product j,
s^o_j : outsourcing cost of component requirement in module o for product j.

Consequently the generalized network model is formulated as in (4.35)-(4.38).

$$Q_6(\mathbf{x}, \tilde{\mathbf{d}}) = \min \quad \sum_{i,o} h_i^o e_i^o + \sum_j s_j^o u_j^o + \sum_{i,o} \sum_{j \in \mathcal{U}_i^o} p_{ij}^o y_{ij}^o \tag{4.35}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{U}_i^o} y_{ij}^o + e_i^o = x_i^o \qquad \forall o, \forall i \tag{4.36}$$

$$\sum_{i \in \mathcal{R}_j^o} \frac{y_{ij}^o}{a_{ij}^o} + u_j^o = \tilde{d}_j \qquad \forall o, \forall j \tag{4.37}$$

$$y_{ij}^o, u_j^o, e_i^o \geqslant 0 \qquad \forall o, \forall i, \forall j \tag{4.38}$$

4.6.2 Pure Network Formulation

Consider the generalized network formulation in (4.35)-(4.38) and in particular, the constraint in (4.37). If a_{ij}^o 's are equals for $i \in \mathcal{R}_j^o$ then let $a_{ij}^o u_j^o = u_j^{'o}$ and $a_{ij}^o \tilde{d}_j = \tilde{d}_j^{'o}$. The constraints in (4.37) can be then transformed to the constraints in (4.39).

$$\sum_{i \in \mathcal{R}_j^o} y_{ij}^o + u_j^{'o} = \tilde{d}_j^{'o} \qquad \forall o, \forall j \tag{4.39}$$

If constraints (4.37) is replaced with the constraints (4.39) in the model Q_6, the pure network formulation will be obtained.

4.7 Problem Formulation with CVaR Objective

A two-stage stochastic linear programming problem with fixed recourse where the only random parameters are on the right-hand sides is a polyhedral risk functional and this holds for the CVaR case [78, 99]. Recall that this problem can be reformulated as an expectation-based two-stage stochastic linear programming problem [78, 99, 100]. Together with the arguments provided in section 3.2, an extended two-stage stochastic linear programming problem given in (4.40)-(4.47) is obtained:

$$\min \quad \sum_{i,o} c_i^o x_i^o + \zeta + (1-\alpha)^{-1} E_{\tilde{\mathbf{d}}}[\hat{Q}(\mathbf{x}, \zeta, \tilde{\mathbf{d}})] \tag{4.40}$$

$$\text{s.t.} \quad \mathbf{x} \in \mathcal{X}, \zeta \in \mathbb{R} \tag{4.41}$$

where

$$E_{\tilde{\mathbf{d}}}[\hat{Q}(\mathbf{x},\zeta,\tilde{\mathbf{d}})] = \sum_{k=1}^{K} z_k \, \hat{Q}(\mathbf{x},\zeta,\mathbf{d}^k) \tag{4.42}$$

and for a given demand realization (we drop index k for simplicity)

$$\hat{Q}(\mathbf{x},\zeta,\tilde{\mathbf{d}}) = \min \eta \tag{4.43}$$

$$\text{s.t.} \quad \sum_{i,o} h_i^o e_i^o + \sum_j s_j u_j + \sum_{i,o} \sum_{j \in \mathcal{U}_i^o} p_{ij}^o y_{ij}^o - \eta \leqslant \zeta \tag{4.44}$$

$$\sum_{j \in \mathcal{U}_i^o} y_{ij}^o + e_i^o = x_i^o \qquad \forall o, \forall i \tag{4.45}$$

$$\sum_{i \in \mathcal{R}_j^o} \frac{y_{ij}^o}{a_{ij}^o} + u_j = \tilde{d}_j \qquad \forall o, \forall j \tag{4.46}$$

$$y_{ij}^o, u_j, e_i^o, \eta \geqslant 0 \qquad \forall o, \forall i, \forall j \tag{4.47}$$

Objective function (4.40) of the problem minimizes the sum of component purchasing cost at the first stage and the CVaR value associated with the α risk parameter. The objective function (4.43) occurring at the second stage consists of the auxiliary variable η . Constraint (4.44) is added into formulation for the linearization. Constraints (4.45) control the availability of the components needed to assemble end products. Constraints (4.46) ensure that the component allocation for each module matches the satisfied end product demand. Constraints (4.47) declare that the second stage variables are all real and nonnegative.

5 Theoretical Analysis of Inventory Allocation Subproblems and Development of Efficient Solution Methods

5.1 Disconnected Subnetworks with Proportional Flow Requirements

Let us consider the stochastic programming problem in (4.1)-(4.6) and focus on the subproblem in (4.3)-(4.6). This is an inventory allocation problem as long as the purchased quantities of components are known. It is possible to reformulate this problem as a MCNF problem with the requirement that the flows on some arcs to be proportional. This is because the components have to be assembled in proportional amounts to obtain a modular end product. Moreover, the network resulting from this reformulation consists of several disconnected subnetworks because of the modular product structure. In other words, they will still have some relationship through the proportional flow requirements on some arcs. Hence, section 5.1 is devoted to study this type of networks.

For a typical network of this kind, let $\mathscr{G}(\mathcal{N}, \mathcal{A})$ be a directed network. Here, $\mathcal{N} = \{1, \cdots, n\}$ denotes the set of nodes and $\mathcal{A} = \{(i,j) : i, j \in \mathcal{N}\}$ is the set of directed arcs. Here, nodes are used to represent both supplies and demands for components used in the assembly of different products. There are two types of nodes: $i \in \mathcal{N}$ is a supply node with $b_i > 0$, and it is called a demand node with $b_i < 0$. Moreover, it is assumed that $\sum_{i \in \mathcal{N}_k} b_i = 0$ for all $k \in \mathcal{K}$.

There are several subnetworks in $\mathscr{G}$ according to the modular structure. For this, let $\mathscr{G}_k(\mathcal{N}_k, \mathcal{A}_k)$ be a subnetwork of $\mathscr{G}$ where $\mathcal{N}_k = \{1, \cdots, n_k\}$ is the set of nodes and $\mathcal{A}_k = \{(i,j) : i, j \in \mathcal{N}_k\}$ is the set of arcs corresponding to subnetwork $k \in \mathcal{K} = \{1, \cdots, m\}$. Indeed, these subnetworks are a partition of $\mathscr{G}$: $\mathscr{G}_k \cap \mathscr{G}_{k'} = \emptyset$ for each $k \neq k'$ and $k, k' \in \mathcal{K}$, and $\cup_{k \in \mathcal{K}} \mathscr{G}_k = \mathscr{G}$. We will note here that each subnetwork subindex k is associated to some module in the modular product structure.

For an arc $(i, j) \in \mathcal{A}$, u_{ij} and c_{ij} are the flow upper bound and unit flow cost, respectively. Since some arc flows are needed to be proportional between them, let $\mathcal{A}'_{sk}$ be the subset of arcs in $\mathcal{A}_k$ and $\mathcal{A}'_s = \cup_k \mathcal{A}'_{sk}$ be the set of all arcs in $\mathcal{A}$ that should have proportional flow for requirement s respectively where $s \in \mathcal{S} = \{1, \cdots, t\}$. Then, let p_{ij} be the proportionality coefficient related to arc $(i, j) \in \mathcal{A}'_s$, $s \in \mathcal{S}$. Note that each proportional flow requirement subindex s is associated to a specific product variant.

Problem P1 where the amount of flows f_{ij} on each arc $(i, j) \in \mathcal{A}$ has to be determined is called minimum cost proportional flow problem with disconnected subnetworks (MCPFD). Correspondingly, it is wanted to determine how to allocate components between products in the subproblem (4.3)-(4.6). Problem P1 can be used equivalently to solve the subproblem in (4.3)-(4.6). In particular, a specific case is illustrated in Figures 4.4, 4.5, 4.6, and 4.7 to show how to switch between these two problems.

$$\text{P1:} \ \min \sum_{(i,j)\in\mathcal{A}} c_{ij} f_{ij} \tag{5.1}$$

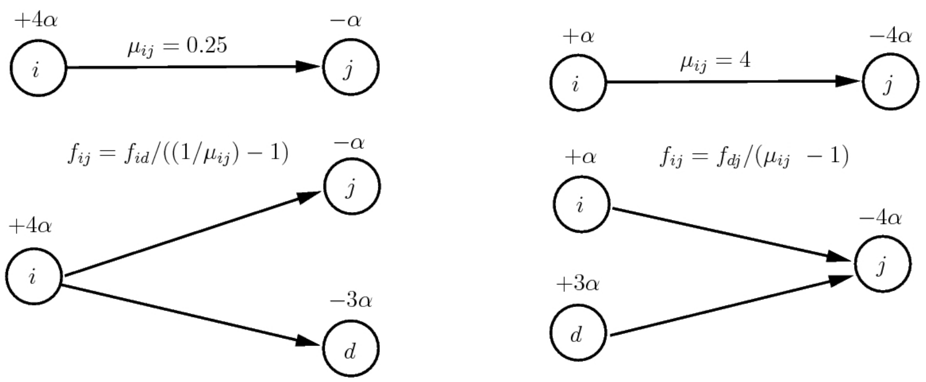
$$\text{s.t.} \sum_{j:(i,j)\in\mathcal{A}_k} f_{ij} - \sum_{j:(j,i)\in\mathcal{A}_k} f_{ji} = b_i \quad i \in \mathcal{N}_k, k \in \mathcal{K}, \tag{5.2}$$

$$f_{ij}/p_{ij} \text{ are all equal} \quad (i, j) \in \mathcal{A}'_s, s \in \mathcal{S}, \tag{5.3}$$

$$0 \leqslant f_{ij} \leqslant u_{ij} \quad (i, j) \in \mathcal{A}_k, k \in \mathcal{K}. \tag{5.4}$$

Constraints (5.2) are defined to satisfy the flow conservation, constraints (5.3) are the proportional flow constraints associated to specific product variants $s \in \mathcal{S}$, and constraints (5.4) are lower and upper bounds for the arc flows. Note that problem P1 is different from the well-known MCNF problem in that it contains the constraints (5.3). There exist efficient algorithms to deal with MCNF such as network simplex algorithm [101]. However, the addition of proportionality constraints complicates the problem and the available solution procedures developed for MCNF can not be applied directly. When the number of nodes is at least an order of magnitude larger than the number of side constraints, it becomes possible to exploit the special network structure [102].

Although problem P1 does not take into account the generalized network models with linear gains and/or losses, they can be easily modeled with a slight modification of MCPFD. Figure 5.1 illustrates how different cases can be transformed appropriately. In that figure, the amount indicated over a node corresponds to its supply or demand, μ_{ij} is the gain/loss factor associated with arc (i, j), and node d is a dummy node.



(a) Example with loss factor ($0 < \mu < 1$) (b) Example with gain factor ($\mu > 1$)

Figure 5.1: Modeling generalized networks with linear gain/loss

However, it would be still useful to transform a generalized network model to a pure network model, if it is possible. Indeed, it is always possible to transform a generalized network flow model with no gain or loss into a pure network flow model [101].

5.1.1 Minimum Cost Network Flow Problem Variants

There are several variants of the MCNF problem with side constraints which have been studied to develop efficient algorithms in computational terms. Based on the relaxation and decomposition techniques, Ali et al. [103] solved the equal flow problem in which selected pairs of arcs are required to have identical flow. The authors used Lagrangian relaxation in their algorithm. Ahuja et al. [104] introduced the simple equal flow problem in which only a single set of arcs is required to have identical flow. The authors developed several algorithms to deal with their problem including the special purpose network simplex algorithm, the parametric simplex algorithm, the combinatorial parametric algorithm, the binary search algorithm, and the capacity scaling algorithm. Calvete [105] introduced the general equal flow problem which extends the simple equal flow problem by allowing multiple sets of arcs to have identical flow. The author proposed a primal simplex algorithm based on the network simplex method. Note that by setting the number of modules and all proportionality coefficients equal to one, it is not difficult to observe that the general equal flow problem is a special case of MCPFD.

Later, Mo et al. [106] considered an integrated manufacturing supply chain where multiple products are manufactured across multiple manufacturing plants by distilling a unique raw material. Similar to [104, 105], the authors presented a modified network simplex method which exploits the special structure of basis. MCPFD problem is also capable to deal with this problem.

In another converging research, Fang and Qi [107] introduced manufacturing network flow (MNF) models. In MNF model, the synthesis of different materials to a single product and the distilling of one material to many different products can be realized. The authors modified the network simplex method according to MNF model and solved a simplified version of their model.

Later, MNF model has attired much attention from several authors. Mo et al. [108] expanded MNF model by incorporating certain features of the ordinary multi-commodity network flow models. Lu et al. [109] studied a MNF model in which the mass balance constraint requiring the total flow in and out of a node to be equal is relaxed. Venkateshan et al. [110] developed a network-simplex-based algorithm based on efficient data structures to solve a MCNF problem formulated on such generalized networks.

At a glance, arcs selected with any purpose in form of a subset can be proportional in MCPFD while only arcs attached to the same node could be proportional in MNF models. Nevertheless, it is possible to transform each one to another [111]. Moreover, MCPFD explicitly takes into account the disconnected network structure.

5.1.2 Reformulation from Complicating Constraints to Complicating Variables

Let $c_s = \sum_{(i,j)\in\mathcal{A}'_s} p_{ij}c_{ij}$ and $u_s = \min_{(i,j)\in\mathcal{A}'_s} u_{ij}/p_{ij}$ for all $s \in \mathcal{S}$. Let $\mathbf{a}_{ij}$ denotes the column associated to arc (i,j) in the node-arc incidence matrix of network $\mathscr{G}$ and a^l_{ij} be the l-th component of vector $\mathbf{a}_{ij}$. Let also $\mathbf{a}_s = \sum_{(i,j)\in\mathcal{A}'_s} p_{ij}\mathbf{a}_{ij}$ for all $s \in \mathcal{S}$ and a^l_s be the l-th component of vector $\mathbf{a}_s$. As $a^i_{ij} = 1$, $a^j_{ij} = -1$, $a^l_{ij} = 0$ for each $l \neq i, j \in \mathcal{N}_k$ and $(i,j) \in \mathcal{A}'_{sk}$ and all the subnetworks are disconnected, it is observed for each subnetwork k that

$$\sum_{l\in\mathcal{N}_k} a^l_s = \sum_{(i,j)\in\mathcal{A}'_{sk}} \sum_{l\in\mathcal{N}_k} a^l_{ij} = 0 \qquad s \in \mathcal{S}. \tag{5.5}$$

Let $\tilde{\mathscr{G}}(\mathcal{N}, \tilde{\mathcal{A}})$ be the network with $\tilde{\mathcal{A}} = \mathcal{A} \setminus \cup_{s\in\mathcal{S}}\mathcal{A}'_s$ and $\tilde{\mathscr{G}}_k(\mathcal{N}_k, \tilde{\mathcal{A}}_k)$ be the network with $\tilde{\mathcal{A}}_k = \mathcal{A}_k \setminus \cup^t_{s=1}\mathcal{A}'_{sk}$ for all $k \in \mathcal{K}$. Then, problem P1 can be transformed into the problem P2 as follows:

$$\text{P2:}\ \min \sum_{(i,j)\in\tilde{\mathcal{A}}} c_{ij}f_{ij} + \sum_{s=1}^{t} c_s f_s \tag{5.6}$$

$$\text{s.t.} \sum_{j:(i,j)\in\tilde{\mathcal{A}}} f_{ij} - \sum_{j:(j,i)\in\tilde{\mathcal{A}}} f_{ji} + \sum_{s=1}^{t} a^i_s f_s = b_i \qquad i \in \mathcal{N}_k, k \in \mathcal{K} \tag{5.7}$$

$$0 \leqslant f_{ij} \leqslant u_{ij} \qquad (i,j) \in \tilde{\mathcal{A}}_k, k \in \mathcal{K} \tag{5.8}$$

$$0 \leqslant f_s \leqslant u_s \qquad s \in \mathcal{S} \tag{5.9}$$

After this reformulation, only problem P2 is considered for further analysis.

5.1.3 Structure of the Basis

It must be noted that if $|\mathcal{S}| = t \geqslant n = |\mathcal{N}|$, then there will be many basic feasible solutions of P2 in (5.6)-(5.9) involving only variables $\{f_1, \cdots, f_t\}$, and the simplex algorithm will pivot between these basic feasible solutions. In turn, this will reduce the efficiency of the proposed algorithm since there will be no possibility to take the advantage of network structure. Therefore, it assumed that $t < n$ holds.

Lemma 5.1. *The rank of the matrix* $\mathbf{A}$ *corresponding to constraints* (5.7) *is equal to* $n - m$.

Proof. The matrix $\mathbf{A}$ has n rows and one column for each arc in $\tilde{\mathscr{A}}$ and one column for each variable f_s, $\mathbf{A} = [\tilde{\mathbf{A}}\ \mathbf{a}_1\ \mathbf{a}_2\ ...\ \mathbf{a}_t]$ where $\tilde{\mathbf{A}}$ is the node-arc incidence matrix of $\tilde{\mathscr{G}}$. First, the maximum rank of $\mathbf{A}$ is $n - m$ because adding all the rows up yields the zero vector for each disjoint subnetwork k corresponding to node set $\mathscr{N}_k$. Furthermore, it is assumed without loss of generality that each network $\tilde{\mathscr{G}}_k$ contains at least one spanning tree since otherwise we can add artificial arcs with sufficiently large costs. This implies that the rank of $\tilde{\mathbf{A}}$ is $\sum_k (n_k - 1) = n - m$ and thus $rank(\mathbf{A}) = n - m$. $\square$

With Lemma 5.1, it is showed that basic feasible solutions of the linear problem P2 consist of $n - m$ basic variables whose corresponding vectors in $\mathbf{A}$ are linearly independent and the rest of the variables are fixed at their lower or upper bound. Following this fact, if none of the variables $\{f_1, \cdots, f_t\}$ are in the basis, then this basis can be represented by an m*-spanning forest* in $\tilde{\mathscr{G}}$ in order to get $n - m$ linearly independent vectors. Otherwise, if r of these variables $\{f_1, \cdots, f_r\}$ are basic, then we should select $n - r - m$ variables $\{f_{ij}, (i,j) \in \tilde{\mathscr{A}}\}$ whose associated vectors in the node-arc incidence matrix $\tilde{\mathbf{A}}$ are linearly independent and also independent of variables $\{f_1, ..., f_r\}$. This latter case can be obtained by removing r arcs from an m-spanning forest in $\tilde{\mathscr{G}}$, which will decompose it into $r + m$ node-disjoint trees $\mathscr{T}_1(\mathscr{N}_1^T, \mathscr{A}_1^T), ..., \mathscr{T}_{r+m}(\mathscr{N}_{r+m}^T, \mathscr{A}_{r+m}^T)$. This collection of trees will again span $\tilde{\mathscr{G}}$ and thus the resulting forest is a $(r + m)$-spanning forest in $\tilde{\mathscr{G}}$ which is denoted as $\mathscr{F}$. Now the structure of the related bases will be analyzed.

Let $\tilde{\mathbf{B}}$ denotes the submatrix of $\tilde{\mathbf{A}}$ associated with the $(r + m)$-spanning forest $\mathscr{F}$ and $\mathbf{a}_1, \cdots, \mathbf{a}_r$ be vectors associated with variables $f_1, \cdots, f_r$. Given that the rank of $\tilde{\mathbf{B}}$ is equal to $n - r - m$, the rank of $\mathbf{B} = [\tilde{\mathbf{B}}\ \mathbf{a}_1\ \mathbf{a}_2\ \cdots\ \mathbf{a}_r]$ is equal to $n - m$ if the vectors in $\tilde{\mathbf{B}}$ with vectors $\mathbf{a}_1, \cdots, \mathbf{a}_r$ are linearly independent. Accordingly, it is provided in the following a suitable condition that guarantees $\mathbf{B}$ is a basis of the problem P2. But before going into the details, it is needed to introduce some additional notation. Lets assume that $\mathscr{T}_1, \cdots, \mathscr{T}_{z_1} \subset \tilde{\mathscr{G}}_1$, $\mathscr{T}_{z_1+1}, \cdots, \mathscr{T}_{z_2} \subset \tilde{\mathscr{G}}_2$ and in general $\mathscr{T}_{z_{(k-1)}+1}, \cdots, \mathscr{T}_{z_k} \subset \tilde{\mathscr{G}}_k$

for all $k \in \mathcal{K}$. Therefore the number of spanning trees in each subnetwork $\tilde{\mathscr{G}_k}$ is equal to $z_k - z_{(k-1)}$ with $z_0 = 0$ and $z_m = r + m$. In a similar fashion, let $\mathcal{N}_1^T = \{1, \cdots, n_1^T\}$, $\mathcal{N}_2^T = \{n_1^T + 1, \cdots, n_2^T\}$, and in general $\mathcal{N}_z^T = \{n_{(z-1)}^T + 1, \cdots, n_z^T\}$ for all $z \in \mathcal{Z} = \{1, \cdots, z_1, z_1 + 1, \cdots, z_2, \cdots, z_{(m-1)} + 1, \cdots, z_m\}$ without loss of generality. Finally, let $\mathbf{D}'$ be the matrix formed by the elements $d_{z,s} = \sum_{l \in \mathcal{N}_z^T} a_s^l$ for all $z \in \mathcal{Z}' = \mathcal{Z} \setminus \{z_1, z_2, \cdots, z_m\}$ and $s \in \mathcal{S}' = \{1, \cdots, r\}$. Note that $|\mathcal{Z}| = r + m$ and $|\mathcal{Z}'| = r$ by definition.

Theorem 5.2. *rank* $(\mathbf{B}) = n - m$ *if and only if rank* $(\mathbf{D}') = r$.

Proof. After column arrangements, the matrix $\mathbf{B}$ can be reexpressed as

$$\mathbf{B} = \left(\begin{array}{c|c|c|c|c|c|c|c}
\mathbf{T}_1 & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{\Delta}_1 \\ \hline
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \hline
\mathbf{0} & \cdots & \mathbf{T}_{z_1} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{\Delta}_{z_1} \\ \hline
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \hline
\mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{T}_{z_{(m-1)}+1} & \cdots & \mathbf{0} & \boldsymbol{\Delta}_{z_{(m-1)}+1} \\ \hline
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \hline
\mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{T}_{z_m} & \boldsymbol{\Delta}_{z_m}
\end{array}\right) \tag{5.10}$$

where $\mathbf{0}$ are matrices of conformal dimensions with all entries equal to zero, $\mathbf{T}_z$ is the node-arc incidence matrix of $\mathscr{T}_z$ and

$$\boldsymbol{\Delta}_z = \begin{pmatrix}
a_1^{n_{(z-1)+1}} & a_2^{n_{(z-1)+1}} & \cdots & a_r^{n_{(z-1)+1}} \\
a_1^{n_{(z-1)+2}} & a_2^{n_{(z-1)+2}} & \cdots & a_r^{n_{(z-1)+2}} \\
\cdots & \cdots & \cdots & \cdots \\
a_1^{n_z} & a_2^{n_z} & \cdots & a_r^{n_z}
\end{pmatrix} \tag{5.11}$$

for all $z \in \mathcal{Z}$. Here, rank$(\mathbf{T}_1) = n_1^T - 1$, rank$(\mathbf{T}_2) = n_2^T - n_1^T - 1$, and in general rank$(\mathbf{T}_z) = n_z^T - n_{(z-1)}^T - 1$ for all $z \in \mathcal{Z}$. As every non-singular square submatrix of the node-arc incidence matrix of a directed network is triangular, the matrix $\mathbf{B}$ can be rewritten as

$$
\mathbf{B}' = \left(\begin{array}{c|c|c|c|c|c|c|c}
\mathbf{T}'_1 & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{\Delta}'_1 \\
\hline
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\hline
\mathbf{0} & \cdots & \mathbf{T}'_{z_1} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{\Delta}'_{z_1} \\
\hline
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\hline
\mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{T}'_{z_{(m-1)}+1} & \cdots & \mathbf{0} & \mathbf{\Delta}'_{z_{(m-1)}+1} \\
\hline
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\hline
\mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{T}'_{z_m} & \mathbf{\Delta}_{z'_m}
\end{array}\right) \tag{5.12}
$$

where

$$
\mathbf{T}'_z = \begin{pmatrix}
0 & 0 & \cdots & 0 \\
\pm 1 & 0 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \pm 1
\end{pmatrix} \tag{5.13}
$$

and

$$
\mathbf{\Delta}'_z = \begin{pmatrix}
d_{z,1} & d_{z,2} & \cdots & d_{z,r} \\
a_1^{n(z-1)+2} & a_2^{n(z-1)+2} & \cdots & a_r^{n(z-1)+2} \\
\cdots & \cdots & \cdots & \cdots \\
a_1^{n_z} & a_2^{n_z} & \cdots & a_r^{n_z}
\end{pmatrix} \tag{5.14}
$$

for all $z \in \mathcal{Z}$. Hence, $\text{rank}(\mathbf{B}) = \sum_{k=1}^{m} \sum_{z=z_{(k-1)}+1}^{z_k} \text{rank}(\mathbf{T}_z) + \text{rank}(\mathbf{D}) = n - r - m + \text{rank}(\mathbf{D})$ where $\mathbf{D}$ is formed by the elements $d_{z,s} = \sum_{l \in \mathcal{N}_z^T} a_s^l$ for all $z \in \mathcal{Z}$ and $s \in \mathcal{S}'$. As we have $0 = \sum_{l \in \mathcal{N}_k} a_s^l = \sum_{l \in \mathcal{N}^T_{z_{(k-1)}+1}} a_s^l + ... + \sum_{l \in \mathcal{N}^T_{z_k}} a_s^l$ for all $k \in \mathcal{K}, s \in \mathcal{S}$ from (5.5), $\text{rank}(\mathbf{B}) = n - m$ holds if and only if $\text{rank}(\mathbf{D}') = r$. □

Therefore, a basic solution to MCPFD consists of an $(r + m)$-spanning forest $\mathscr{F}$ in $\tilde{\mathscr{G}}$ where $r = 0, ..., t$ as well as variables $\{f_1, ..., f_r\}$ verifying that $\text{rank}(\mathbf{D}') = r$. Note that for each $(r + m)$-spanning forest, there are $\binom{t}{r}$ combinations of selecting r variables among $\{f_1, ..., f_t\}$.

Definition 5.3. *An $(r+m)$-spanning forest $\mathscr{F}$ in $\tilde{\mathscr{G}}$ is a good $(r+m)$-forest with respect to the variables $\{f_s\}_{s \in \mathcal{S}'}$ with $\mathcal{S}' \subseteq \mathcal{S}$ and $|\mathcal{S}'| = r$, if rank$(\mathbf{D}') = r$ where $\mathbf{D}'$ is formed* *by the elements* $d_{z,s} = \sum_{l \in \mathcal{N}_z^T} a_s^l$ *for all* $z \in \mathcal{Z}'$ *and* $s \in \mathcal{S}'$ *and* $\mathcal{N}_z^T$ *is the node set of tree* $\mathscr{T}_z$ *in forest* $\mathscr{F}$*.*

Theorem 5.4. *A basic solution of MCPFD is constituted of an* $(r+m)$*-spanning forest* $\mathscr{F}$ *in* $\tilde{\mathscr{G}}$ *where* $r = 0, ..., t$ *and a set of* r *variables* $\{f_s\}_{s \in \mathcal{S}'}$*,* $\mathcal{S}' \subseteq \mathcal{S}$*,* $|\mathcal{S}'| = r$ *verifying that* $\mathscr{F}$ *is a good* $(r+m)$*-forest with respect to* $\{f_s\}_{s \in \mathcal{S}'}$*.*

Proof. It is clear from the preceding developments. □

5.1.4 A Special Primal Simplex Algorithm

In this section, we give in details the main steps required for the special primal simplex algorithm (SPSA) developed to solve problem $P2$.

1. *Finding the initial basic feasible solution*: If none is conveniently available, the all artificial start method [112, 101, 104] can be used to get a basic feasible solution with artificial variables in the network $\tilde{\mathscr{G}}$. The initial basic feasible solution is constituted by the good m-forest defined by this feasible solution. All other variables are non-basic variables and are equal to their lower bounds or upper bounds.

2. *Computing the values of the basic variables*: From now on we assume that the basis is given by a good $(r+m)$-forest such that $\mathscr{T}_z \subset \mathscr{F}$ for all $z \in \mathcal{Z}$ and the variables $\{f_1, \cdots, f_r\}$. Let $\mathcal{B}$ be the set of arcs $(i,j) \in \tilde{\mathcal{A}}$ such that f_{ij} is a basic variable, and $\mathcal{B}'$ be the set of $s \in \mathcal{S}$ such that f_s is a basic variable. Accordingly, we categorize non-basic variables such that $\mathcal{L} = \{(i,j) \in \tilde{\mathcal{A}} \setminus \mathcal{B} : f_{ij} = 0\}$, $\mathcal{L}' = \{s \in \mathcal{S} \setminus \mathcal{B}' : f_s = 0\}$, $\mathcal{U} = \{(i,j) \in \tilde{\mathcal{A}} \setminus \mathcal{B} : f_{ij} = u_{ij}\}$ and $\mathcal{U}' = \{s \in \mathcal{S} \setminus \mathcal{B}' : f_s = u_s\}$. Finally, we let $\mathcal{V}_z^1 = \{(i,j) \in \mathcal{U} : i \in \mathcal{N}_z^T, j \notin \mathcal{N}_z^T\}$ and $\mathcal{V}_z^2 = \{(i,j) \in \mathcal{U} : i \notin \mathcal{N}_z^T, j \in \mathcal{N}_z^T\}$ for all $z \in \mathcal{Z}'$. Then, the following Theorem guarantees that the values of variables $\{f_1, \cdots, f_r\}$ are solvable.

Theorem 5.5. *The values of basic variables* $\{f_1, \cdots, f_r\}$ *are the solution of the following linear system:*

$$\mathbf{D}'\mathbf{f} = \mathbf{b}' \tag{5.15}$$

where $\mathbf{D}'$ *is previously defined,* $\mathbf{f} = (f_1, \cdots, f_r)^t$ *and* $\mathbf{b}' = (b'_1, \cdots, b'_r)^t$ *with*

$$b'_z = \sum_{l \in \mathcal{N}_z^T} b_l - \left(\sum_{(i,j) \in \mathcal{V}_z^1} u_{ij} - \sum_{(i,j) \in \mathcal{V}_z^2} u_{ij} \right) - \sum_{l \in \mathcal{N}_z^T} \sum_{s \in \mathcal{U}'} a_s^l u_s \qquad z \in \mathcal{Z}' \tag{5.16}$$

Proof. After fixing the values of non-basic variables, each constraints in (5.7) can be reformulated as

$$\sum_{j:(i,j) \in \mathcal{B}} f_{ij} - \sum_{j:(j,i) \in \mathcal{B}} f_{ji} + \sum_{s \in \mathcal{B}'} a_s^l f_s = \hat{b}_l \qquad l \in \mathcal{N} \tag{5.17}$$

where $\hat{b}_l = b_l - \sum_{j:(i,j) \in \mathcal{U}} u_{ij} + \sum_{j:(j,i) \in \mathcal{U}} u_{ij} - \sum_{s \in \mathcal{U}'} a_s^l u_s$. Since u_{ij} vanishes if $i \in \mathcal{N}_z^T$, $j \in \mathcal{N}_z^T$ and $(i,j) \in \mathcal{U}$,

$$\sum_{l \in \mathcal{N}_z^T} \hat{b}_l = \sum_{l \in \mathcal{N}_z^T} b_l - \sum_{\substack{i \in \mathcal{N}_z^T, j \notin \mathcal{N}_z^T, \\ (i,j) \in \mathcal{U}}} u_{ij} + \sum_{\substack{i \notin \mathcal{N}_z^T, j \in \mathcal{N}_z^T, \\ (i,j) \in \mathcal{U}}} u_{ij} - \sum_{l \in \mathcal{N}_z^T} \sum_{s \in \mathcal{U}'} a_s^l u_s \tag{5.18}$$

$$= \sum_{l \in \mathcal{N}_z^T} b_l - \sum_{(i,j) \in \mathcal{V}_z^1} u_{ij} + \sum_{(i,j) \in \mathcal{V}_z^2} u_{ij} - \sum_{l \in \mathcal{N}_z^T} \sum_{s \in \mathcal{U}'} a_s^l u_s = b'_z \tag{5.19}$$

for all $z \in \mathcal{Z}'$. Similar to the proof of Theorem 5.2, the linear system (5.15) can be solved to obtain the value of variables $\{f_1, \cdots, f_r\}$. Hence, the proof is complete. □

The values of basic variables $\{f_1, ..., f_r\}$ affect the requirement of each supply and demand node. Then, the flow values of the remaining arcs in each tree $\mathcal{T}_z$ $z \in \mathcal{Z}$ can be determined by applying the general procedure of the network simplex algorithm.

3. *Computing node potentials for a given basis*: Given a basic feasible solution, it must be verified if it is optimal by calculating node potentials $\boldsymbol{\pi} = (\pi_i : i \in \mathcal{N})$ and taking into account the fact that the reduced cost of each basic variable is zero. In other words, node potentials should be determined such that $c_{ij}^{\boldsymbol{\pi}} = 0$ for all $(i,j) \in \mathcal{B}$ and $c_s^{\boldsymbol{\pi}} = 0$ for all $s \in \mathcal{B}'$ where $c_{ij}^{\boldsymbol{\pi}} = c_{ij} - \pi_i + \pi_j$ for all $(i,j) \in \tilde{\mathcal{A}}$ and $c_s^{\boldsymbol{\pi}} = c_s - \sum_{i \in \mathcal{N}} a_s^i \pi_i$ for all $s \in \mathcal{S}$. The first condition can be satisfied by computing appropriate node potentials as in the network simplex algorithm. If these node potentials also satisfy the second condition then it is done. Otherwise, new node potentials $\tilde{\pi}$ can be calculated such that

$$\tilde{\pi}_i = \begin{cases} \pi_i + \sigma_z & \text{for all } i \in \mathcal{N}_z^T \text{ and } z \in \mathcal{Z}' \\ \pi_i & \text{for all } i \in \mathcal{N}_z^T \text{ and } z \in \{z_1, z_2, \cdots, z_m\} \end{cases} \tag{5.20}$$

where $\boldsymbol{\sigma} = (\sigma_z : z \in \mathcal{Z}')^t$ are obtained by solving the linear system $(\mathbf{D}')^t \boldsymbol{\sigma} = \mathbf{c}^{\boldsymbol{\pi}}$ given $\mathbf{c}^{\boldsymbol{\pi}} = (c_z^{\boldsymbol{\pi}} : z \in \mathcal{Z}')^t$ and $\mathbf{D}'$ previously defined.

Lemma 5.6. *The node potentials $\tilde{\boldsymbol{\pi}}$ given in (5.20) satisfies $c_{ij}^{\tilde{\boldsymbol{\pi}}} = 0$ for all $(i,j) \in \mathcal{B}$ and $c_s^{\tilde{\boldsymbol{\pi}}}$ for all $s \in \mathcal{B}'$.*

Proof. It can be verified that node potentials $\tilde{\boldsymbol{\pi}}$ satisfy $c_{ij}^{\tilde{\boldsymbol{\pi}}} = 0$ for each $(i,j) \in \mathcal{B}$. Then, for all $s \in \mathcal{B}'$, it holds that $c_s^{\tilde{\boldsymbol{\pi}}} = c_s - \sum_{l \in \mathcal{N}} a_s^l \tilde{\pi}_i = c_s^{\boldsymbol{\pi}} - \sum_{z \in \mathcal{Z}'} \sum_{l \in \mathcal{N}_z^T} a_s^l \sigma_z = c_s^{\boldsymbol{\pi}} - \sum_{z \in \mathcal{Z}'} d_{z,s} \sigma_z = c_s^{\boldsymbol{\pi}} - c_s^{\boldsymbol{\pi}} = 0$ by taking into account the linear system given. □

4. *Testing optimality and selecting the entering variable*: Since problem P2 in (5.6)-(5.9) is a linear programming problem, the optimality conditions can be written as

$$c_{ij}^{\boldsymbol{\pi}} \geqslant 0 \;\; (i,j) \in \mathcal{L}, \qquad c_{ij}^{\boldsymbol{\pi}} \leqslant 0 \;\; (i,j) \in \mathcal{U}, \tag{5.21}$$

and

$$c_s^{\boldsymbol{\pi}} \geqslant 0 \;\; s \in \mathcal{L}', \qquad c_s^{\boldsymbol{\pi}} \leqslant 0 \;\; s \in \mathcal{U}'. \tag{5.22}$$

If the given basis satisfies the optimality conditions (5.21) and (5.22), it is optimal and the algorithm terminates. Otherwise, the algorithm selects a non-basic variable f_{ij} with $(i,j) \in \mathcal{L} \cup \mathcal{U}$ violating the condition in (5.21) or a non-basic variable f_s with $s \in \mathcal{L}' \cup \mathcal{U}'$ violating the condition in (5.22) as entering variable according to any usual rules [101].

5. *Selecting the leaving variable*: Suppose that an entering non-basic variable which is equal to its lower bound is selected. Increasing the value of this variable by θ units will necessitate to alter the values of some basic variables to maintain the feasibility. If the value of the entering variable hits its upper bound while the values of the modified basic variables stay between their respective bounds, then the entering variable still remains non-basic. Otherwise, the non-basic variable enters the basis and one of the basic variables will leave at its lower or upper bound. Similar arguments can be made if the entering non-basic variable is initially at its upper bound. Depending on the non-basic variable that enter the basis, three different cases to identify the leaving basic variable will be considered.

Case 1. *The entering variable is* f_{ij} with $i \in \mathcal{N}_z^T$ and $j \in \mathcal{N}_z^T$. In this case, the variable f_{ij} only affects tree $\mathscr{T}_z$. The arc corresponding to this variable is added to $\mathscr{T}_z$ which creates a unique cycle. The amount of flow θ is increased and sent through this cycle until the variable corresponding to one of the arcs of the cycle reaches its upper or lower bound. If this variable is f_{ij}, then it remains non-basic. Otherwise, it enters the basis and one of the basic variables at its lower or upper bound will leave. In either cases, the value of all basic variables corresponding to the remain arcs of the cycle are adjusted with respect to this additional amount of flow.

Case 2. *The entering variable is* f_{ij} with $i \in \mathcal{N}_z^T, j \in \mathcal{N}_{z'}^T$ and $z \neq z'$. Suppose that an additional amount of flow θ is sent through arc (i, j). Then, the demand of tree $\mathscr{T}_z$ decreases in θ units and the demand of $\mathscr{T}_{z'}$ increases in θ units. Therefore, new values of variables $f_1, \cdots, f_r$ are obtained by solving a modification of system (5.15) such that $\mathbf{D}'\mathbf{f} = \bar{\mathbf{b}}'$ where $\bar{\mathbf{b}}' = (b_1', \cdots, b_z' - \theta, \cdots, b_{z'}' + \theta, \cdots, b_r')^t$. Once these values are determined, the value of θ is increased until f_{ij} or one of the basic variables reaches one of its bounds. Then, the arguments presented in Case 1 remains also valid here.

Case 3. *The entering variable is* f_s, where $r < s \leqslant t$. When the value of variable f_s is increased by $\theta \geqslant 0$, the demand of each tree $\mathscr{T}_z$ for which $\sum_{l \in \mathcal{N}_z^T} a_s^l > 0$ should decrease in $\theta \sum_{l \in \mathcal{N}_z^T} a_s^l$, the demand of each tree $\mathscr{T}_z$ for which $\sum_{l \in \mathcal{N}_z^T} a_s^l < 0$ should increase in $\theta \left(-\sum_{l \in \mathcal{N}_z^T} a_s^l\right)$. Therefore, the new values of variables $f_1, ..., f_r$ are determined by solving a modification of system (5.15) such that $\mathbf{D}'\mathbf{f} = \bar{\mathbf{b}}'$ where $\bar{\mathbf{b}}' = \left(b_1' - \theta \sum_{l \in \mathcal{N}_1^T} a_s^l, \cdots, b_r' - \theta \sum_{l \in \mathcal{N}_r^T} a_s^l\right)^t$. Then, the arguments presented in Case 1 remains also valid here.

Now, we can discuss the roles of variable r and parameter m in more details. Note that the steps in Case 1 of the algorithm are identical to ones in the network simplex algorithm. Indeed, if most of the variables f_s remain non-basic, then the execution time of the algorithm speeds up. Because, Case 1 is more likely to occur, and there are not many non-basic arcs connecting distinct trees. When the variable r increases, the basis of the problem becomes more partitioned. Though this offers an improvement in the execution time of the algorithm by dealing with trees smaller in size, the benefit from using the embedded network simplex steps in the algorithm is reduced, because the steps required in Case 2 of the algorithm will be used more frequently. Thus, when the variable r increases, the execution time of the algorithm is longer. In contrast, the high values of the parameter m lead to have a decomposable structure since no arc connecting distinct trees exists in a disconnected network structure. The above situation is of particular interest for the network design. So, one should take into account the possible number of basic variables f_s in the optimal solution. Recently, Wang and Lin [113] have proposed an algorithm with a basis partition technique which uniquely decomposes the basic feasible graph into $p + 1$ trees for solving the minimum distribution cost problem. In their algorithm, p is the total number of D-nodes. Since a D-node can be modeled using a proportional flow subset in MCPFD, their parameter p is the maximum value that can be taken by the variable r in our algorithm (i.e. $r \leq p$). Moreover, their technique requires solving a system of p linear equations, whereas our algorithm requires solving a system of r linear equations. Thus, our algorithm offers a more efficient basis partition technique. Nevertheless, Lu et al. [114] presented an extended cycle method to calculate the reduced costs for non-basic variables when solving the minimum distribution cost problem. Note that in our algorithm, the reduced costs for non-basic variables are calculated based on the node potentials at the cost of solving r linear equations. It is clear that the reduced cost calculation based on the node potentials will be more efficient than one based on the cycle method.

5.2 Bipartite Structure

First, note that the subproblem (4.3)-(4.6) can be restated as a MCNF problem in which a collection of the separated networks is binded by some side constraints associated with the proportional flow requirement. Now, we focus on the number of sources and sinks in this MCNF problem. In the assembly networks with mass customization, it is common to have a great variety of end products (sinks) from a few number of components (sources). Then, the bipartite network in Figure 4.7 becomes unbalanced with respect to the number of sources and sinks. Now, we further assume that the proportional flow requirement is relaxed. Then, the resulting problem is summarized in (5.23)-(5.26). Here, $\mathcal{N}_S$ and $\mathcal{N}_D$ denote the source and sink node sets, respectively. Let $s_i \geqslant 0$ and $d_j \geqslant 0$ denote the supply quantity at source node i and demand quantity at sink node j, respectively. The characteristics of this problem are well investigated. In particular, constraint (5.24) can be considered as the complicating constraint, since the number of sources are assumed to be far less than the number of sinks. There exist several solution procedures including strongly polynomial algorithms to deal with this problem. A literature survey is presented in Table 5.1 where s is the number of sources and d is the number of sinks. Note that the respective complexity of the algorithms given in Table 5.1 is borrowed from Brenner [115].

$$\text{Unb:} \; \min \sum_{(i,j)\in\mathcal{A}} c_{ij} f_{ij} \tag{5.23}$$

$$\text{s.t.} \sum_{j:(i,j)\in\mathcal{A}} f_{ij} = s_i \quad i \in \mathcal{N}_S, \tag{5.24}$$

$$\sum_{i:(i,j)\in\mathcal{A}} f_{ij} = d_j \quad j \in \mathcal{N}_D, \tag{5.25}$$

$$0 \leqslant f_{ij} \qquad (i,j) \in \mathcal{A}. \tag{5.26}$$

Table 5.1: Literature survey for the unbalanced transportation problem

Authors	Title	Complexity $s << d$
Brenner [115]	A faster polynomial algorithm for the unbalanced Hitchcock transportation problem	$O(ds^2(log\, d + s\, log\, s))$
Williams [116]	A treatment of transportation problems by decomposition	$-$
Zemel [117]	An O(n) algorithm for the linear multiple choice knapsack problem and related problems	$O(ds^2 2^s)$
Orlin [118]	A faster strongly polynomial minimum cost flow algorithm	$O(d^2 log\, d(s + log\, d))$
Matsui [119]	A linear time algorithm for the Hitchcock transportation problem with fixed number of supply points	$O((s!)^2 d)$
Kleinschmidt and Schannath [120]	A strongly polynomial algorithm for the transportation problem	$O(d^2 slog\, d)$
Tokuyama and Nakano [121]	Efficient algorithms for the Hitchcock transportation problem	$O(ds^2 log^2 d)$

Since even a few number of components per module can quickly lead to a great variety of end products, an algorithm polynomial in s and linear in d will be interesting. Between strongly polynomial time algorithms, it is remarkable that Brenner's [115] algorithm works in $O(ds^2(log\, d + s\, log\, s))$ time and fits best this purpose. Meanwhile, Williams [116] solves the transportation problem with Dantzig Wolfe decomposition and argues that the proposed algorithm works faster in unbalanced cases. Here, we support this argument and the CPU time performance of this algorithm is analyzed in details. With constraints (5.24) accepted as complicating constraints, the relaxed problem consists of (5.23),(5.25) and (5.26). This relaxed problem has a decomposable structure based on the sink nodes. A multi-thread code is implemented to benefit from this structure. In Table 5.2, we compare the results with those obtained with the network simplex implementation through a set of randomly generated problems. In Table 5.2, the total number of iterations performed by the Dantzig Wolfe decomposition algorithm is also given.

In practice, the Dantzig Wolfe decomposition approach has a linear performance in d and is still polynomial in s for the unbalanced transportation problem. In fact, the relaxed problem obtained by deleting constraints (5.24) from the problem (5.23)-(5.26) is a collection of isolated linear programming problems, each of which has a single equality constraint and has a trivial solution. Thus, this relaxed problem can be solved in $O(ds)$ time. The Dantzig Wolfe decomposition algorithm solves this relaxed problem instead of the problem (5.23)-(5.26) at the cost of extra computational iterations. We denote the number of iterations performed by the Dantzig Wolfe decomposition algorithm as *Iter* in Table 5.2. Furthermore, it is observable from the CPU time results given in Table 5.2 that *Iter* depends mainly on s, but not on d. Thus, it is possible to benefit from a special algebraic structure and in practice, an algorithm polynomial in s and linear in d is obtained. However, when applying this approach to the subproblem (4.3)-(4.6) which includes several unbalanced transportation networks, one needs to deal with the complicating variables corresponding to the side constraints. A nested approach consisting of the Benders and the Dantzig Wolfe decompositions seems to be reasonable only when the number of complicating variables is limited. For example, this solution method may be chosen when unsatisfaction of the customer demand is allowed only for a few number of end products. Otherwise, SPSA should be used to solve the subproblem (4.3)-(4.6). The previously introduced model reformulations and their solution approaches are summarized in Table 5.3.

Table 5.2: Performance analysis of Dantzig Wolfe decomposition method

Network Simplex Algorithm				
		d		
	s	10,000	50,000	100,000
CPU Time (sec.)	2	0.2	1.9	6.5
	4	0.2	2.9	16.7
	5	0.3	3.4	19.1
	7	0.3	3.9	20.4
	10	0.3	5.0	

Dantzig Wolfe Decomposition Algorithm							
		d					
	s	10,000	50,000	100,000	500,000	1,000,000	10,000,000
CPU Time (sec.)	2	0.0	0.1	0.3	0.1	0.3	2.0
	4	0.3	0.2	0.3	1.0	1.0	
	5	0.4	0.4	0.6	1.0	3.0	
	7	0.4	0.8	1.0	2.1	4.9	
	10	0.7	1.1	1.3	6.4		
Number of Iterations	2	11.3	11.4	11.7	10.9	11.6	11.7
	4	42.4	42.3	42.6	41.5	42.4	
	5	61.7	64.9	61.2	64.2	62.0	
	7	110.9	113.3	118.2	114.8	115.4	
	10	209.8	212.1	211.4	213.3		

Table 5.3: Model variants and solution procedures

Model Variant	Gains/ Losses	Assumption	Assumption	Solution Procedure
General Case Gen. Netw. with Equality Const.	Yes	-	-	L-Shaped / SPSA (Gains-Losses)
General Case Pure Netw. with Prop. Const.	No	-	-	L-Shaped / SPSA
Subcontracting Case Gen. Netw. with Equality Const.	Yes	Common Demand Satisfaction Proportion	p_{ij}^{o} 's and a_{ij}^{o} 's are equals for $j \in \mathcal{U}_{ir}^{o}$	L-Shaped / L-Shaped / SPSA (Gains-Losses)
Subcontracting Case Pure Netw. with Prop. Const.	No	Common Demand Satisfaction Proportion	p_{ij}^{o} 's and a_{ij}^{o} 's are equals for $j \in \mathcal{U}_{ir}^{o}$	L-Shaped / L-Shaped / Network Simplex
Spot Market Case Gen. Netw.	Yes	Outsourcing	-	L-Shaped / Dantzig Wolfe
Spot Market Case Pure Netw.	No	Outsourcing	-	L-Shaped / Dantzig Wolfe

6 Computational Results

6.1 Summary of the Solution Method

Before going into the details of the computational results, we would like to briefly expose the solution method of the two stage stochastic linear programming problem given in (4.1)-(4.6). If no upper bounds are defined for the end product shortage quantities, the subproblems (4.3)-(4.6) are always feasible. These subproblems corresponding to different demand scenarios are solved with SPSA each time to add an optimality cut to master problem of L-Shaped method until the optimum component purchase quantities are determined. A flow diagram illustrating the integration of the L-Shaped algorithm with SPSA is provided in Figure 6.1 and the main steps of SPSA are given in Figure 6.2.

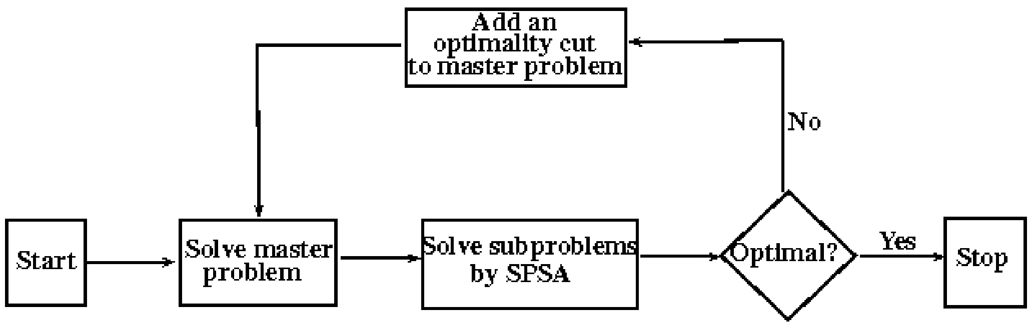


Figure 6.1: Integration of L-Shaped algorithm with SPSA

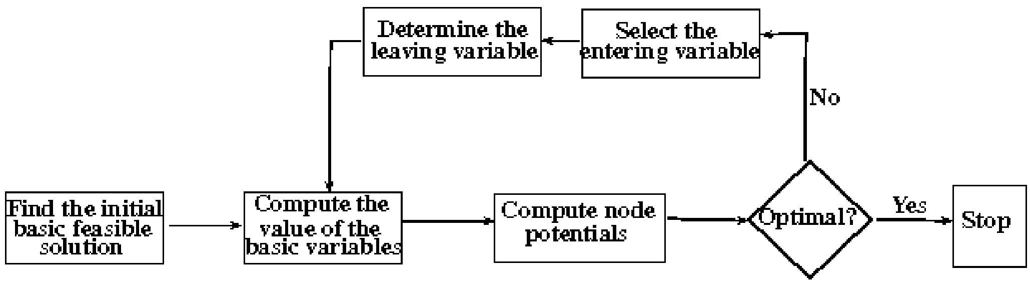


Figure 6.2: Special primal simplex algorithm

6.2 The Model Parameters

The mathematical programming models used in the experiments are based on the models given in (4.1)-(4.6) and (4.40)-(4.47). Accordingly, the purchasing and holding costs for components are fixed to 12 and 0.12, respectively, and the shortage and substitution costs vary between 19-25, and 2-4, respectively. We assumed that the the shortage costs increase linearly with the increase in the number of modules. As for example, if the number of modules is equal to two, then the shortage costs are assumed to vary between 38-50. The number of components in an end product also increases linearly with the number of modules. Hence, it is possible to analyze the effect of increasing the number of modules on several performance metrics. Moreover, it is assumed that the stochastic parameters have discrete probability distributions (see in-sample stability requirement discussed in section 4.3.2). Finally, the risk parameter α associated with the CVaR problem given in (4.40)-(4.47) is assumed to be equal to 0.95.

6.3 CPU Time Performance of the SPSA

In this section, the time performance of the SPSA relative to a primal simplex linear programming solver (LPS) is considered. For this purpose, GNU Linear Programming Kit (GLPK) [122] is used in the computational experiments. The number of basic complicating variables is an important factor when exploiting the underlying network structure. When there are many basic complicating variables, the time performance of the SPSA will be worse. This fact is discussed in section 5.1.4. For the stochastic programming model given in (4.1)-(4.6), complicating variables corresponding to the end product shortage quantities are denoted by u_j. Thus, for each module, additional constraints could be added to the first stage (4.1)-(4.2) of the model to avoid such complicating variables entering the basis at the iterations of the L-Shaped method. An example of such a constraint is below:

$$\sum_i x_i^o \geq SS^o \qquad \forall o \tag{6.1}$$

where SS^o is the safety stock target set for module o. In our experiments, only the total demand scenarios ($d_1 = 100, d_2 = 200$) are considered with high shortage costs ($s_j = 25\ \forall j$) and low substitution costs ($p^o_{ij} = 2\ \forall o, i, j$), and we varied the numbers of components and modules. We also fixed the purchasing costs ($c^o_i = 12\ \forall o, i$) and holding costs ($h^o_i = 0.12\ \forall o, i$) of components.

The experiment results are shown in Tables 6.1, 6.2, and 6.3 by taking into account low, moderate, and high levels of safety stocks, respectively. From the results, the effect of the number of basic complicating variables when exploiting the network structure can be observed. It is clear that the SPSA outperforms the primal simplex LPS in execution times when the number of modules increases as shown in Figure 6.3. This is because SPSA is designed to benefit from modular decomposition. However, there is less evidence to draw the same inference when the number of substitutable components in each module increases. Note that the experiment results are derived from a single thread code implementation. At this point, it is worth to say that the SPSA can also benefit from parallel computing. In fact, the modular product structure can lead to a further decomposition of the subproblems emerging from the stochastic decomposition. Thus, a multi-thread code implementation of the SPSA promises further shortened execution times via today's advanced multi-core CPUs and achievements in distributed computing.

Table 6.1: SPSA/LPS CPU time ratios for the low safety stock ($SS^o = 100\ \forall o$) case

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.460	0.615	0.625	0.548	0.725
2	0.525	0.437	0.335	0.242	0.269
3	0.378				
4	0.272				
5	0.274				
6	0.243				
7	0.242				

Table 6.2: SPSA/LPS CPU time ratios for the moderate safety stock ($SS^o = 150\ \forall o$) case

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.635	0.636	0.512	0.699	0.794
2	0.548	0.393	0.294	0.258	0.256
3	0.338				
4	0.233				
5	0.233				
6	0.190				
7	0.190				

Table 6.3: SPSA/LPS CPU time ratios for the high safety stock ($SS^o = 200\ \forall o$) case

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.761	0.573	0.507	0.439	0.480
2	0.257	0.293	0.274	0.227	0.176
3	0.249				
4	0.168				
5	0.124				
6	0.093				
7	0.090				

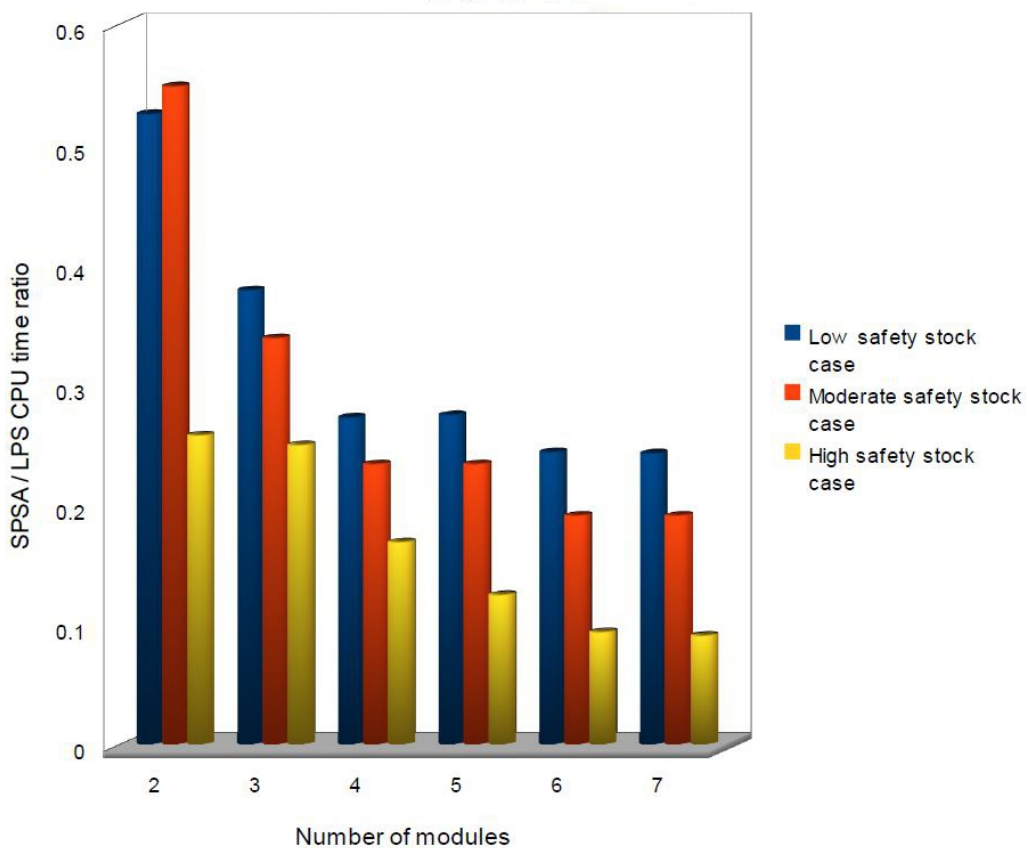


Figure 6.3: SPSA/LPS CPU time ratios when increasing the number of modules

6.4 The Analysis of the Stochastic Model

6.4.1 The Performance Metrics

In this section, the performance metrics used in the analysis of our stochastic model are introduced. The average supply ratio (ASR), the expected value of perfect information (EVPI), the value of stochastic solution (VSS), and the CVaR are used in this analysis. In particular, EVPI and VSS are widely used measures [123, 124] in stochastic programming and they are discussed in details by Birge and Louveaux [74].

In our case, the ASR value denotes the average purchased component quantity per module over the expected total demand quantity of the end products. A low ASR value means that the customer demand is more likely to be met with less component inventory. Indeed, this case may lead to a reduction in inventory costs and decrease in the occurrence of out-of-stock situations. On the other hand, a high ASR value implies that more component inventory is used to meet the customer demand, so an increase in the inventory costs is inevitable.

EVPI represents the price of the option to know the complete and accurate information about the future. Since the two-stage structure of the model given in (4.1)-(4.6) is due to lag of time between the advance purchase of the components and the realization of the end products demand, EVPI can also be considered as the option price to shorten this lag in order to reduce the uncertainty at the time of purchase. A low EVPI value indicates a smaller additional profit when complete information is acquired [125]. To calculate EVPI, the wait-and-see and the here-and-now models corresponding to the recourse problem given in (4.1)-(4.6) are to be solved first. The wait-and-see solution (WS) is defined as

$$WS = E_{\tilde{\mathbf{d}}}[\min_{\mathbf{x}\in\mathbb{R}^C} z(\mathbf{x}, \tilde{\mathbf{d}})], \tag{6.2}$$

while the here-and-now solution (RP) is defined as

$$RP = \min_{\mathbf{x}\in\mathbb{R}^C} E_{\tilde{\mathbf{d}}}[z(\mathbf{x}, \tilde{\mathbf{d}})]. \tag{6.3}$$

Then, it is possible to give the normalized EVPI as:

$$EVPI = \frac{|WS - RP|}{RP}. \tag{6.4}$$

VSS can be viewed as the option price to use the expected information instead of the stochastic information. A low VSS value indicates that the approximation of the stochastic program by the expected value program is a good one [125]. In order to calculate VSS, one needs first to solve the expected value problem (EV) where all random variables are replaced by their expected values:

$$EV = \min_{\mathbf{x} \in \mathbb{R}^C} \; z(\mathbf{x}, \bar{\mathbf{d}}), \tag{6.5}$$

where $\bar{\mathbf{d}}$ denotes the expectation of $\tilde{\mathbf{d}}$. Let $\bar{\mathbf{x}}$ denotes the optimal solution of the expected value problem. Then, the expected value of using the optimal solution of the expected value problem (EEV) can be defined as follows:

$$EEV = E_{\tilde{\mathbf{d}}}[z(\bar{\mathbf{x}}, \tilde{\mathbf{d}})]. \tag{6.6}$$

The normalized VSS is defined as follows:

$$VSS = \frac{EEV - RP}{RP}. \tag{6.7}$$

The last performance metric considered in this study is the CVaR/RP value. CVaR is the problem with an objective of type risk-averse decision maker whereas RP is the problem with an objective of type risk-neutral decision maker. An increase in the CVaR/RP ratio is related to an increase of high loss risk. CVaR/RP is defined as follows:

$$CVaR/RP = \frac{CVaR}{RP} \tag{6.8}$$

6.4.2 The Effect of the Variation in Shortage and Substitution Costs

Our objective is to analyze the effects of the variation in shortage and substitution costs. Different demand scenario sets are included in our experiment plan to better observe the behaviour of the models given in (4.1)-(4.6) and (4.40)-(4.47) under different demand conditions. The variability in both total demand quantity and component preferences are taken into account in the experiments. The respective settings for different scenarios are listed in Appendix A. The experiments are conducted for a model consisting of two modules, and two components per module, and the obtained results are also given in Appendix A.

When the shortage costs increase, ASR value which is the average purchased component quantity per module over the expected total demand quantity of end products, also increases. This can be explained with the fact that high shortage costs make out-of-stock less desirable, and then more inventory is held to satisfy the customers. This effect is more significant when the interval of uncertainty for the total demand quantity is large.

When the substitution costs increase, ASR value decreases because high substitution costs discourage the planner to meet the customer demand in high demand scenarios when shortage is still a considerable option. This effect is observed with uncertainty of the component preference. On the other hand, note that high substitution costs would lead to an increase in ASR value to meet the customer demand with original components when the shortage is less desirable.

EVPI value depends on both the shortage and substitution costs. When the shortage costs increase, EVPI value increases. It can be observed that this increase becomes more important when the total demand uncertainty is high. A similar effect is observed with the substitution costs. EVPI value increases when the substitution costs increase. This time, it becomes more significant with the high uncertainty of the component preference. Here, it is worth to say that when shortage and substitution costs become higher, it becomes more difficult to fill the gap between the realized demand and the inventory on hand under uncertainty. Thus, when the shortage and substitution costs are high, an accurate forecast of the demand becomes crucial and the firm may decide to invest on information systems to reduce the uncertainty. In that case, EVPI can be used to evaluate how much it would pay for this.

VSS value decreases when the shortage costs increases. This effect is more significant when the total demand uncertainty is high.

CVaR/RP ratio depends on both the shortage and substitution costs. When the shortage costs increase, this ratio decreases. The uncertainty of the total demand affects significantly the decrease. Furthermore, it is observed that ASR value increases in this case. So, the gap between CVaR and RP decreases when the out-of-stock situations become rare. Conversely, this gap will increase in cases where it is more likely to see out-of-stocks. To summarize, we can argue that the risk of out-of-stock position is underestimated by a risk-neutral decision maker when compared to a risk-averse decision maker.

CVaR/RP ratio increases when the substitution costs increase. The degree of uncertainty on the component preference rates affects this increase. In fact the increase can be attributed to the risk pooling effect of the substitution.

6.4.3 The Effects of Increasing the Number of Components and Modules

The number of components and modules are closely related to the end product structure. Although the decision for the end product structure is rather strategic, its impact to the operational area must not be underestimated. Hence, the question of how the structure can affect ASR, EVPI, VSS, and CVaR/RP values is worth to answer. Since the shortage and substitution costs influence these performance metrics, the experiment plan consists of low/high shortage and low/high substitution costs scenarios. Scenario sets including the variability of both total demand quantity and component preference rates are also added to the experiment plan. For the total demand variability, it is assumed to have low/high ($d_1 = 100, d_2 = 200$) total demand quantity scenarios. However, the scenario generation for the different component preference rates is not straightforward. In order to compare the results between the end product structures associated with different number of components, the scenarios need to be generated so that only one component is fully preferred in a prospective scenario. Hence, it would be possible to compare the results of these scenarios because the scenario generation method will be coherent to the variations in the number of components. Table B.1-B.47 show the experiment results based on this scenario generation method. When the number of components in each module is fixed to two and the effects of increasing the number of modules are analyzed, another scenario generation method is implemented where one of two components is preferred with a probability equal to 0.45 or 0.55. The results associated with this case are shown in Table C.1-C.12.

The effects of increasing the number of components and modules are summarized in Tables 6.4 and 6.5, respectively. In these tables, "F" stands for "Fixed" and "V" stands for "Variable". Moreover, a "+" sign indicates an increase whereas a "-" sign indicates a decrease in the performance of the respective measure. First, it is observed that the increase in the number of components has not the same effect as the increase in the number of modules. This is because when new components are added into the end product structure, they can be substituted with already existing components in their own modules. However, when a new module is added into the end product structure, new components can not be substituted with already existing components. Secondly, different demand scenarios have different effects under the same shortage and substitution costs. Thus, the total demand variability and the component preference variability are among the key factors for these performance metrics when increasing the number of components and modules. In particular, almost no effect is observed under the demand scenario with variable total demand quantity and fixed component preference rates.

ASR decreases for scenario with variable total demand quantity, variable component preference rates, and high shortage costs when the number of components increases. This is because more uncertainty on component preference rates is introduced by adding more components.

When the number of modules is increased, ASR value depends on the variability of substitution costs for the scenario with variable total demand quantity, variable component preference rates, and high shortage costs. In particular, ASR increases with low substitution costs whereas it decreases with high substitution costs. Thus, low substitution costs may encourage the planner to meet the customer demand in high demand scenarios when increasing the number of modules.

When the number of components increases, VSS value tends to increase in general for the scenario with variable total demand quantity and variable component preference rates. However, VSS value decreases with high shortage costs when the number of modules increases.

Table 6.4: The effects of increasing the number of components

Shortage Costs	Substitution Costs	Total Demand	Component Preference	ASR	VSS	EVPI	CVaR/RP
low	low	F	V			+	
low	low	V	F				
low	low	V	V		+	+	+
low	high	F	V			+	
low	high	V	F				
low	high	V	V		+	+	+
high	low	F	V			+	
high	low	V	F				
high	low	V	V	-	-+	+	-+
high	high	F	V			+	
high	high	V	F				
high	high	V	V	-	-+	+	+

From the experiment results, it is observable that EVPI value tend to increase when both the number of components and modules increases. Therefore, one has to be ready to pay more for the accurate information when the end product has a complex structure and the demand is uncertain.

CVaR/RP ratio tends to increase for the demand scenario with variable total demand quantity and variable component preference rates when the number of components increases. However, it decreases with high shortage costs when the number of modules increases.

Table 6.5: The effects of increasing the number of modules

Shortage Costs	Substitution Costs	Total Demand	Component Preference	ASR	VSS	EVPI	CVaR/RP
low	low	F	V				
low	low	V	F				
low	low	V	V				
low	high	F	V				
low	high	V	F				
low	high	V	V				
high	low	F	V				
high	low	V	F				
high	low	V	V	+	-	+	-
high	high	F	V				
high	high	V	F				
high	high	V	V	-	-	+	-

7 Conclusion and Further Suggestions

Today, the mass customization is a popular strategy aiming to fill the gap between the mass production efficiency and the customization opportunity. The modularity can be seen as a success factor in a mass customization environment in that it combines the standardization and the flexibility. Moreover, the modularity fits well to the purpose of the assemble-to-order strategy. In this framework, the substitution can be implemented to reduce the costs. This fact is observed and discussed in many studies. An inventory planning model taking into account these popular strategies is studied in this thesis. In particular, the general, subcontracting and spot market cases of the model are considered.

In this model, a lot of end products can be defined, where they are differentiated by the modular component options. Based on the different problem formulations and the assumptions, several solution methods are proposed. In particular, a special algorithm for the modular networks with proportional flows is suggested to deal with the subproblems. This is a primal simplex algorithm which seeks to benefit from the network structure and modular decomposition as much as possible. Its bases are partitioned into the network components and several complicating variables. These complicating variables present the proportional flows.

In the model, some action is taken in the presence of uncertainty at the first stage, then a recourse action is made at the second stage when the uncertainty is resolved. The model is studied from a viewpoint of the risk. The risk-neutral and the risk-averse approaches are studied. The expected and quantile value of the recourse actions are considered when taking the first-stage decisions, respectively. In a risk-neutral case, the decision maker does not differentiate among the prospective scenarios. Then, the model is formulated as a stochastic programming problem. In a risk-averse case, the previous assumption can not be accepted because the decision maker dislikes the risk. The worst case scenarios are heavily considered in a risk-averse approach. So, the CVaR is selected as a risk-averse measure.

Later, the computational experiments are conducted to analyze the stochastic model. First, the experiments are conducted for a model consisting of two modules, and two components per module to study the effects of shortage and substitution costs based on ASR, EVPI, VSS and CVaR/RP performance metrics. The different demand scenario sets are also included into the experiment plan to understand better the behaviour of the investigated model under different conditions. Second, the effects of increasing the number of components and modules on these performance metrics are investigated to reveal the impact of the end product structure on these performance metrics.

Based on the experiment results, the shortage costs, the substitution costs, the demand uncertainty based on both total demand quantity and component preference rates, the number of components and the number of modules in the end product structure are all key factors for these performance metrics. Furthermore, the roles of these key factors are not isolated from each other in general, so that different combinations of them may lead to different effects on the performance metrics under study.

In particular, the ASR value explains how a firm reacts to the demand uncertainty by means of stock levels. In general, high shortage costs lead to a situation where out-of-stock is less desirable, and then more stock is hold to satisfy the customers. This effect is more significant when the uncertainty on the total demand quantity is high. When the shortage costs are low, the stock levels may decrease while substitution costs are increasing because the out-of-stock situation would be more acceptable in this case. This effect is more significant when the uncertainty on the component preference rates is high. On the other hand, when the shortage costs are high, the stock levels may increase while substitution costs are increasing because the use of original components would be more favorable in this case.

The EVPI represents the price of the option to know the complete and accurate information about the future. Thus, it has to be taken into account by a firm to assess how much it would spend for an initiative to reduce the demand uncertainty. Based on our findings, the shortage and substitution costs significantly affect EVPI. These effects are more visible when the uncertainty of total demand and the uncertainty of market proportions for the component options are high. The choice to implement a late differentiation strategy for component options has to be considered to deal with the uncertainty of market proportions for the component options. More generally, a need for a better demand forecast is apparent. A detailed market study can help to improve this situation. On the other hand, a firm has to be ready to pay more when dealing with the demand uncertainty for the end products having complex structures. In an assemble-to-order framework, an effort to reduce the component lead times can result to a decrease in the number of components or modules that an inventory planner has to deal with.

The VSS is the option price to substitute the expected information for the stochastic information. It is observed that the VSS decreases when shortage costs increases. This effect is more significant with high uncertainty of the total demand quantity.

Finally, the risk of out-of-stock is underestimated by a risk-neutral decision maker compared to a risk-averse decision maker. So, the gap between CVaR and RP decreases when there is not a severe out-of-stock situation. Conversely, this gap increases in cases where it is more likely to see out-of-stocks. Therefore, a risk neutral decision maker should have a plan which addresses the customer loyality. On the other hand, the CVaR/RP value increases when the substitution costs increase. This becomes more significant as the uncertainty of the component preference rates is high. This increase is due to the risk pooling effect of substitution.

There are other areas to which this research can be extended. In this thesis, we assumed stochastic end product demand, but not the price elasticity of demand where it is possible to reformulate the demand for an end product as a function of its price. An effort to study the effects of price elasticity in a model including substitution, modularity, and several risk approaches could answer some important questions. On the other hand, single-level bills of materials that include only immediate sub-assemblies or components are considered in this thesis. Consequently, it can be extended to include multi-level bills of materials in order to analyze the effects of the point of substitution in the assembly hierarchy. Another aspect to clarify can be to find the long time effect of substitution. This can be accomplished by implementing multistage stochastic programs to deal with multi-period problems. However, given the fact that the number of scenarios increases exponentially with the number of stages, there is a need to pay more attention when dealing with the demand uncertainty. On the other hand, we studied single manufacturer case. It is possible to extend this research by adding more manufacturers to the model to analyze several supply chain problems. In particular, incorporating procurement lead times would be an interesting research path to follow along with component substitution in a modular product framework. Furthermore, capacity constraints can be added to the model and their effects can be analyzed. Finally, we want to point out the usefulness of SPSA since it can be used to find the solutions of similar network flow problems.

Works Related to this Thesis

- Bahçeci U. and Feyzioğlu O., "Inventory Allocation in Multi-period Multi-echelon Logistics Networks of Modular Products", *Proceedings of the 8th International Conference of Modeling and Simulation, MOSIM'10*, Hammamet / Tunisia, 1021-1030, (2010).
- Bahçeci U. and Feyzioğlu O., "Modüler Ürünlerin Tedarik Ağlarında Çok Dönemli Stok Tahsisi", *Yöneylem Araştırması ve Endüstri Mühendisliği 30. Ulusal Kongresi, YAEM 2010*, İstanbul / Türkiye, (2010).
- Bahçeci U. and Feyzioğlu O., "A network simplex based algorithm for the minimum cost proportional flow problem with disconnected subnetworks", *Optimization Letters*, DOI 10.1007/s11590-011-0356-5, (2011)[1].

[1]The original publication is available at www.springerlink.com: Bahçeci, U. and Feyzioğlu, O., "A network simplex based algorithm for the minimum cost proportional flow problem with disconnected subnetworks", *Optimization Letters*, DOI 10.1007/s11590-011-0356-5, 2011.

References

[1] Kumar, A., Gattoufi, S., Reisman, A., "Mass Customization Research: Trends, Directions, Diffusion Intensity, and Taxonomic Frameworks", *International Journal of Flexible Manufacturing Systems*, 19 (4), 637–665, (2007).

[2] Rajaram, K., Tang, C.S., "The Impact of Product Substitution on Retail Merchandising", *European Journal of Operational Research*, 135 (3), 582–601, (2001).

[3] Da Silveira, G., Borenstein, D., Fogliatto, F.S., "Mass Customization: Literature Review and Research Directions", *Operations Research*, 72 (1), 1–13, (2001).

[4] Alizon, F., Shooter, S.B., Simpson, T.W., "Henry Ford and the Model T: Lessons for Product Platforming and Mass Customization", *Design Studies*, 30 (5), 588–605, (2009).

[5] Davis, S., "From Future Perfect: Mass Customizing", *Planning Review*, 17 (2), 16–21, (1989).

[6] Pine, B.J., *Mass Customization: the New Frontier in Business Competition*, Harvard Business Scholl Press, Boston, (1993).

[7] Kumar, A., "Mass Customization: Manufacturing Issues and Taxonomic Analyses", *International Journal of Flexible Manufacturing Systems*, 19 (4), 625–629, (2007).

[8] Chen, J., Hao, Y., "Mass Customization in Design of Service Delivery System: A Review and Prospects", *African Journal of Business Management*, 4 (6), 842–848, (2010).

[9] Dean, P.R., Xue, D., Tu, Y., "Prediction of Manufacturing Resource Requirements from Customer Demands in Mass-Customisation Production", *International Journal of Production Research*, 47 (5), 1245–1268, (2009).

[10] Huang, X., Kristal, M.M., Schroeder, R.G., "The Impact of Organizational Structure on Mass Customization Capability: A Contingency View", *Production and Operations Management*, 19 (5), 515–530, (2010).

[11] Pine, B.J., "Mass Customizing Products and Services", *Planning Review*, 6–13, (1993).

[12] Lampel, J., Mintzberg, H., "Customizing Customization", *Sloan Management Review*, 38 (1), 21–30, (1996).

[13] Gilmore, J., Pine, B.J., "The Four Faces of Mass Customization", *Harvard Business Review*, 75 (1), 91–101, (1997).

[14] Amaro, G., Hendry, L., Kingsman, B., "Competitive Advantage, Customisation and A New Taxonomy for NonMake-to-Stock Companies", *International Journal of Operations and Production Management*, 19 (4), 349–371, (1999).

[15] Duray, R., Ward, P.T., Milligan, G.W., Berry, W.L., "Approaches to Mass Customization: Configurations and Empirical Validation", *Journal of Operations Management*, 18, 605–625, (2000).

[16] Feng, Y.X., Zheng, B., Tan, J.R., Wei, Z., "An Exploratory Study of the General Requirement Representation Model for Product Configuration in Mass Customization Mode", *International Journal of Advanced Manufacturing Technology*, 40 (7-8), 785–796, (2009).

[17] Merle, A., Chandon, J., Roux, E., Alizon, F., "Perceived Value of the Mass-Customized Product and Mass Customization Experience for Individual Consumers", *Production and Operations Management*, 19 (5), 503–514, (2010).

[18] Squire, B., Brown, S., Readman, J., Bessant, J., "The Impact of Mass Customisation on Manufacturing Trade-offs", *Production and Operations Management*, 15 (1), 10–21, (2006).

[19] Kumar, A., "Mass Customization: Metrics and Modularity", *International Journal of Flexible Manufacturing Systems*, 16 (4), 287–311, (2004).

[20] Piller, F.T., “Observations on the Present and Future of Mass Customization”, *International Journal of Flexible Manufacturing Systems*, 19 (4), 630–636, (2007).

[21] Pandremenos, J., Paralikas, J., Salonitis, K., Chryssolouris, G., “Modularity Concepts for the Automotive Industry: A Critical Review”, *CIRP Journal of Manufacturing Science and Technology*, 1, 148–152, (2009).

[22] Ulrich, K., Tung, K., “Fundamentals of Product Modularity”, in “Proceedings of ASME Winter Annual Meeting Symposium on Issues in Design and Manufacturing Integration”, 73–79, (1991).

[23] Hsuan, J., “Impacts of Supplier-Buyer Relationships on Modularization in New Product Development”, *European Journal of Purchasing and Supply Management*, 5 (3-4), 197–209, (1999).

[24] Ernst, R., Kamrad, B., “Evaluation of Supply Chain Structures through Modularization and Postponement”, *European Journal of Operational Research*, 124 (3), 495–510, (2000).

[25] Brun, A., Zorzini, M., “Evaluation of Product Customization Strategies through Modularization and Postponement”, *International Journal of Production Economics*, 1–16, (2008).

[26] Duray, R., “Mass Customizers' Use of Inventory Planning Techniques and Channel Management”, *Production Planning and Control*, 15 (4), 412–421, (2004).

[27] Howard, M., Squire, B., “Modularization and the Impact on Supply Relationships”, *Management Science*, 27 (11), 1192–1212, (2007).

[28] Kumar, A., “From Mass Customization to Mass Personalization: A Strategic Transformation”, *International Journal of Flexible Manufacturing Systems*, 19 (4), 533–547, (2007).

[29] Jose, A., Tollenaere, M., “Modular and Platform Methods for Product Family Design: Literature Analysis”, *Journal of Intelligent Manufacturing*, 16 (3), 371–390, (2005).

[30] Jiao, J., Simpson, T.W., Siddique, Z., “Product Family Design and Platform-Based Product Development: A State-of-the-Art Review”, *Journal of Intelligent Manufacturing*, 18 (1), 5–29, (2007).

[31] Antonio, K.W.L., Yam, R.C.M., Tang, E., “The Impacts of Product Modularity on Competitive Capabilities and Performance: An Empirical Study”, *International Journal of Production Economics*, 105 (1), 1–20, (2007).

[32] Fujita, K., “Product Variety Optimization Under Modular Architecture”, *Computer Aided Design*, 34 (12), 953–965, (2002).

[33] Briant, O., Naddef, D., “The Optimal Diversity Management Problem”, *IIE Transactions*, 52 (4), 515–526, (2004).

[34] da Cunha, C., Agard, B., Kusiak, A., “Design for Cost: Module-Based Mass Customization”, *IEEE Transactions on Automation Science and Engineering*, 4 (3), 350–359, (2007).

[35] Li, L., Huang, G.Q., Newman, S.T., “A Cooperative Coevolutionary Algorithm for Design of Platform-Based Mass Customized Products”, *Journal of Intelligent Manufacturing*, 19 (5), 507–519, (2008).

[36] Fixson, S.K., “Modularity and Commonality Research: Past Developments and Future Opportunities”, *Concurrent Engineering-Research and Applications*, 15 (2), 85–111, (2007).

[37] VanMieghem, J.A., “Note - Commonality Strategies: Value Drivers and Equivalence with Flexible Capacity and Inventory Substitution”, *Management Science*, 50 (3), 419–424, (2004).

[38] Wazed, M.A., Ahmed, S., Nukman, Y., “Commonality in Manufacturing Resources Planning - Issues and Models: A Review”, *European Journal of Industrial Engineering*, 4 (2), 167–188, (2010).

[39] Hillier, M.S., “Component Commonality in a Multiple-period Inventory Model with Service Level Constraints”, *International Journal of Production Research*, 37 (12), 2665–2683, (1999).

[40] Ma, S., Wang, W., Liu, L., "Commonality and Postponement in Multistage Assembly Systems", *European Journal of Operational Research*, 142 (3), 523–538, (2002).

[41] Chod, J., Pyke, D., Rudi, N., "The Value of Flexibility in Make-to-Order Systems: The Effect of Demand Correlation", *Operations Research*, 58 (4), 834–848, (2010).

[42] Jönsson, H., Jörnsten, K., Silver, E.A., "Application of the Scenario Aggregation Approach to a Two-Stage, Stochastic, Common Component, Inventory Problem with a Budget Constraint", *European Journal of Operational Research*, 68 (2), 196–211, (1993).

[43] Paul, A., Vakharia, A.J., "Requirements Planning for Modular Products", *Naval Research Logistics*, 53 (5), 418–431, (2006).

[44] Nonas, S.L., "Finding and Identifying Optimal Inventory Levels for Systems with Common Components", *European Journal of Operational Research*, 193 (1), 98–119, (2009).

[45] Chen, J., *Substitution and Inspection Models in Production-Inventory Systems*, Ph.D. thesis, Columbia University, (1997).

[46] Hsu, A., Bassok, Y., "Random Yield and Random Demand in a Production System with Downward Substitution", *Operations Research*, 47 (2), 277–290, (1999).

[47] Bassok, Y., Anupindi, R., Akella, R., "Single-Period Multiproduct Inventory Models with Substitution", *Operations Research*, 47 (4), 632–642, (1999).

[48] Hale, W.W., *Component Substitution in Assemble-to-Order Manufacturing*, Ph.D. thesis, University of Minnesota, (2003).

[49] Axsater, S., "Evaluation of Unidirectional Lateral Transshipments and Substitutions in Inventory Systems", *European Journal of Operational Research*, 149 (2), 438–447, (2003).

[50] Rao, U.S., Swaminathan, J.M., Zhang, J., "Multi-product Inventory Planning with Downward Substitution, Stochastic Demand and Setup Costs", *IIE Transactions*, 36 (1), 59–71, (2004).

[51] Hsu, V., Li, C., Xiao, W., "Dynamic Lot Size Problems with One-way Product Substitution", *IIE Transactions*, 37 (3), 201–215, (2005).

[52] Li, Y., Chen, J., Cai, X., "Uncapacitated Production Planning with Multiple Product Types, Returned Product Remanufacturing, and Demand Substitution", *Operations Research*, 28 (1), 101–125, (2006).

[53] Weng, Z., "Risk-pooling Over Demand Uncertainty in the Presence of Product Modularity", *International Journal of Production Economics*, 62 (1-2), 75–85, (1999).

[54] Inderfurth, K., "Optimal Policies in Hybrid Manufacturing/Remanufacturing Systems with Product Substitution", *International Journal of Production Economics*, 90 (3), 325–343, (2004).

[55] Tibben-Lembke, R.S., Bassok, Y., "An Inventory Model for Delayed Customization: A Hybrid Approach", *European Journal of Operational Research*, 165 (3), 748–764, (2005).

[56] Bayindir, Z.P., Erkip, N., Gullu, R., "Assessing the Benefits of Remanufacturing Option under One-way Substitution and Capacity Constraint", *Computers and Operations Research*, 34 (2), 487–514, (2007).

[57] Karakul, M., Chan, L.M.A., "Analytical and Managerial Implications of Integrating Product Substitutability in the Joint Pricing and Procurement Problem", *European Journal of Operational Research*, 190 (1), 179–204, (2008).

[58] Balakrishnan, A., Geunes, J., "Requirements Planning with Substitutions: Exploiting Bill-of-Materials Flexibility in Production Planning", *Manufacturing and Service Operations Management*, 2 (2), 166–185, (2000).

[59] Geunes, J., “Solving Large-Scale Requirements Planning Problems with Component Substitution Options”, *Computers and Industrial Engineering*, 44 (3), 475–491, (2003).

[60] Nagarajan, M., Rajagopalan, S., “Inventory Models for Substitutable Products: Optimal Policies and Heuristics”, *Management Science*, 54 (8), 1453–1466, (2008).

[61] Mishra, A.K., *Optimization Models for Component Commonality in Assemble-to-Order Systems*, Ph.D. thesis, University of Pittsburgh, (1999).

[62] Smith, S.A., Agrawal, N., “Management of Multi-Item Retail Inventory Systems with Demand Substitution”, *Operations Research*, 48 (1), 50–64, (2000).

[63] Mahajan, S., Van Ryzin, G., “Stocking Retail Assortments Under Dynamic Consumer Substitution”, *Operations Research*, 49 (3), 334–351, (2001).

[64] Iravani, S.M.R., Luangkesorn, K.L., Simchi-Levi, D., “On Assemble-to-order Systems with Flexible Customers”, *IIE Transactions*, 35 (5), 389–403, (2003).

[65] Agrawal, N., Smith, S.A., “Optimal Retail Assortments for Substitutable Items Purchased in Sets”, *Naval Research Logistics*, 50 (7), 793–822, (2003).

[66] Denton, B., Gupta, D., “Strategic Inventory Deployment in the Steel Industry”, *IIE Transactions*, 36 (11), 1083–1097, (2004).

[67] Thomas, D., Warsing, D.P., “A Periodic Inventory Model for Stocking Modular Components”, *Production and Operations Management*, 16 (3), 343–359, (2007).

[68] Ganesh, M., Raghunathan, S., Rajendran, C., “The Value of Information Sharing in a Multi-Product Supply Chain with Product Substitution”, *IIE Transactions*, 40 (12), 1124–1140, (2008).

[69] Yang, H., Schrage, L., “Conditions That Cause Risk Pooling to Increase Inventory”, *European Journal of Operational Research*, 192 (3), 837–851, (2009).

[70] Wazed, M.A., Ahmed, S., Nukman, Y., “A Review of Manufacturing Resources Planning Models Under Different Uncertainties: State-of-The-Art and Future Directions”, *South African Journal of Industrial Engineering*, 21 (1), 17–33, (2010).

[71] Xu, M., Chen, F.Y., "Tradeoff Between Expected Reward and Conditional Value-at-Risk Criterion in Newsvendor Models", in "Proceedings of the IEEE International Conference on Industrial Engineering and Engineering Management", 1553–1557, (2007).

[72] Borgonovo, E., Peccati, L., "Financial Management in Inventory Problems: Risk Averse vs Risk Neutral Policies", *International Journal of Production Economics*, 118 (1), 233–242, (2009).

[73] Zhang, D., Xu, H., Wu, Y., "Single and Multi-period Optimal Inventory Control Models with Risk-averse Constraints", *European Journal of Operational Research*, 199 (2), 420–434, (2009).

[74] Birge, J.R., Louveaux, F., *Introduction to Stochastic Programming*, Springer, New York, (1997).

[75] Symonds, G.H., "Chance-Constrained Equivalents of Some Stochastic Programming Problems", *Operations Research*, 16 (6), 1152–1159, (1968).

[76] Kibzun, A.I., Kan, Y.S., *Stochastic Programming Problems with Probability and Quantile Functions*, John Wiley and Sons, Chichester, (1996).

[77] Krokhmal, P., Zabarankin, M., Uryasev, S., "Modeling and Optimization of Risk", *Surveys in Operations Research and Management Science*, 16 (2), 49–66, (2011).

[78] Eichhorn, A., Romisch, W., "Polyhedral Risk Measures in Stochastic Programming", *SIAM Journal on Optimization*, 16 (1), 69–95, (2005).

[79] Ruszczynski, A., Shapiro, A., "Stochastic Programming Models", in "Handbooks in Operations Research and Management Science", volume 10, Elsevier, Amsterdam, 1–64, (2003).

[80] Kall, P., Wallace, S.W., *Stochastic Programming*, John Wiley and Sons, Chichester, (1994).

[81] Benders, J.F., "Partitioning Procedures for Solving Mixed-variables Programming Problems", *Numerische Mathematik*, 4 (1), 238–252, (1962).

[82] Dantzig, G.B., Wolfe, P., "The Decomposition Principle for Linear Programs", *Operations Research*, 8 (1), 101–111, (1960).

[83] Fuller, J.D., Chung, W., "Benders Decomposition for a Class of Variational Inequalities", *European Journal of Operational Research*, 185 (1), 76–91, (2008).

[84] Fuller, J.D., Chung, W., "Dantzig-Wolfe Decomposition of Variational Inequalities", *Computational Economics*, 25 (4), 303–326, (2005).

[85] Conejo, A.J., Castillo, E., Minguez, R., Garcia-Bertrand, R., *Decomposition Techniques in Mathematical Programming*, Springer, Berlin, (2006).

[86] Van Slyke, R.M., Wets, R., "L-shaped Linear Program with Application to Optimal Control and Stochastic Linear Programming", *SIAM Journal and Applied Mathematics*, 17 (4), 638–663, (1969).

[87] Ruszczynski, A., "Some Advances in Decomposition Methods for Stochastic Linear Programming", *Annals of Operations Research*, 85 (0), 153–172, (1999).

[88] Fabian, C.I., Szoke, Z., "Solving Two-stage Stochastic Programming Problems with Level Decomposition", *Computational Management Science*, 4 (4), 313–353, (2007).

[89] Fabian, C.I., "Bundle-type Methods for Inexact Data", *Central European Journal of Operations Research*, 8 (special), 35–55, (2000).

[90] Ruszczynski, A., "A Regularized Decomposition Method for Minimizing a Sum of Polyhedral Functions", *Mathematical Programming*, 35 (3), 309–333, (1986).

[91] Pritsker, M., "Evaluating Value at Risk Methodologies: Accuracy versus Computational Time", *Journal of Financial Services Research*, 12 (2-3), 201–242, (1997).

[92] Krokhmal, P., Palmquist, J., Uryasev, S., "Portfolio Optimization with Conditional Value-at-Risk Objective and Constraints", *Journal of Risk*, 4 (2), 11–27, (2002).

[93] Rockafellar, R.T., Uryasev, S., "Optimization of Conditional Value-at-Risk", *Journal of Risk*, 2 (3), 21–41, (2000).

[94] Rockafellar, R.T., Uryasev, S., "Conditional Value-at-Risk for General Loss Distributions", *Journal of Banking and Finance*, 26 (7), 1443–1471, (2002).

[95] DiDomenica, N., Lucas, C., Mitra, G., Valente, P., "Scenario Generation for Stochastic Programming and Simulation: A Modelling Perspective", *IMA Journal of Management Mathematics*, 20 (1), 1–38, (2009).

[96] DiDomenica, N., Mitra, G., Valente, P., Birbilis, G., "Stochastic Programming and Scenario Generation within a Simulation Framework: An Information Systems Perspective", *Decision Support Systems*, 42 (4), 2197–2218, (2007).

[97] Mitra, S., "A White Paper on Scenario Generation for Stochastic Programming", in "World Wide Web, OptiRisk Systems: White Paper Series", 1–33, (2006).

[98] Kaut, M., Wallace, S.W., "Evaluation of Scenario-Generation Methods for Stochastic Programming", in "World Wide Web, Stochastic Programming E-Print Series", 1–14, (2003).

[99] Dupacova, J., "Risk Objectives in Two-Stage Stochastic Programming Models", *Kybernetika*, 44 (2), 227–242, (2008).

[100] Kunzi-Bay, A., Mayer, J., "Computational Aspects of Minimizing Conditional Value-at-Risk", *Computational Management Science*, 3 (1), 3–27, (2006).

[101] Ahuja, R.K., Magnanti, T.L., Orlin, J.B., *Network Flows: Theory, Algorithms, and Applications*, Prentice Hall, New Jersey, (1993).

[102] Helgason, R.V., Kennington, J.L., "Primal Simplex Algorithms for Minimum Cost Network Flows", in "Handbooks in Operations Research and Management Sciences: Network Models", volume 8, Elsevier, Amsterdam, 85–133, (1995).

[103] Ali, A.I., Kennington, J., Shetty, B., "The Equal Flow Problem", *European Journal of Operational Research*, 36 (1), 107–115, (1988).

[104] Ahuja, R.K., Orlin, J.B., Sechi, G.M., Zuddas, P., "Algorithms for the Simple Equal Flow Problem", *Management Science*, 45 (10), 1440–1455, (1999).

[105] Calvete, H.I., “Network Simplex Algorithm for the General Equal Flow Problem”, *European Journal of Operational Research*, 150 (3), 585–600, (2003).

[106] Mo, J., Qi, L., Wei, Z., “A Manufacturing Supply Chain Optimization Model for Distilling Process”, *Applied Mathematics and Computation*, 171 (1), 464–485, (2005).

[107] Fang, S., Qi, L., “Manufacturing Network Flows: A Generalized Network Flow Model for Manufacturing Process Modelling”, *Optimization Methods and Software*, 18 (2), 143–165, (2003).

[108] Mo, J., Qi, L., Wei, Z., “A Network Simplex Algorithm for Simple Manufacturing Network Model”, *Journal of Industrial and Management Optimization*, 1 (2), 251–273, (2005).

[109] Lu, H., Yao, E., Qi, L., “Some Further Results on Minimum Distribution Cost Flow Problems”, *Journal of Combinatorial Optimization*, 11 (4), 351–371, (2006).

[110] Venkateshan, P., Mathur, K., Ballou, R.H., “An Efficient Generalized Network-Simplex-Based Algorithm for Manufacturing Network Flows”, *Journal of Combinatorial Optimization*, 15 (4), 315–341, (2008).

[111] Bahçeci, U., Feyzioğlu, O., “A Network Simplex Based Algorithm for the Minimum Cost Proportional Flow Problem with Disconnected Subnetworks”, *Optimization Letters*, DOI 10.1007/s11590-011-0356-5, (2011).

[112] Kennington, J.L., Helgason, R.V., *Algorithms for Network Programming*, John Wiley and Sons, New York, (1980).

[113] Wang, I.L., Lin, S.J., “A Network Simplex Algorithm for Solving the Minimum Distribution Cost Problem”, *Journal of Industrial and Management Optimization*, 5 (4), 929–950, (2009).

[114] Lu, H.Y., Yao, E.Y., Zhang, B.W., “A note on a generalized network flow model for manufacturing process”, *Acta Mathematicae Applicatae Sinica-English Series*, 25 (1), 51–60, (2009).

[115] Brenner, U., “A Faster Polynomial Algorithm for the Unbalanced Hitchcock Transportation Problem”, *Operations Research Letters*, 36 (4), 408–413, (2008).

[116] Williams, A.C., “A Treatment of Transportation Problems by Decomposition”, *Journal of the Society for Industrial and Applied Mathematics*, 10 (1), 35–48, (1962).

[117] Zemel, E., “An O(n) Algorithm for the Linear Multiple Choice Knapsack Problem and Related Problems”, *Information Processing Letters*, 18 (3), 123–128, (1984).

[118] Orlin, J.B., “A Faster Strongly Polynomial Minimum Cost Flow Algorithm”, *Operations Research*, 41 (2), 338–350, (1993).

[119] Matsui, T., “A Linear Time Algorithm for the Hitchcock Transportation Problem with Fixed Number of Supply Points”, in “Cooperative Research Report 35(1992), The Institute of Statistical Mathematics, Minami-Azabu, Minato-ku”, 128–138, (1993).

[120] Kleinschmidt, P., Schannath, H., “A Strongly Polynomial Algorithm for the Transportation Problem”, *Mathematical Programming*, 68 (1), 1–13, (1995).

[121] Tokuyama, T., Nakano, J., “Efficient Algorithms for the Hitchcock Transportation Problem”, *SIAM Journal on Computing*, 24 (3), 563–578, (1995).

[122] *GNU Linear Programming Kit*, http://www.gnu.org/software/glpk/, (2011).

[123] Valente, C., Mitra, G., Sadki, M., Fourer, R., “Extending Algebraic Modelling Languages for Stochastic Programming”, *INFORMS Journal on Computing*, 21 (1), 107–122, (2009).

[124] Alem, D.J., Munari, P.A., Arenales, M.N., Ferreira, P.A.V., “On the Cutting Stock Problem Under Stochastic Demand”, *Annals of Operations Research*, 179 (1), 169–186, (2010).

[125] Escudero, L.F., Garin, A., Merino, M., Perez, G., “The Value of the Stochastic Solution in Multistage Problems”, *Top*, 15 (1), 48–64, (2007).

Appendix A

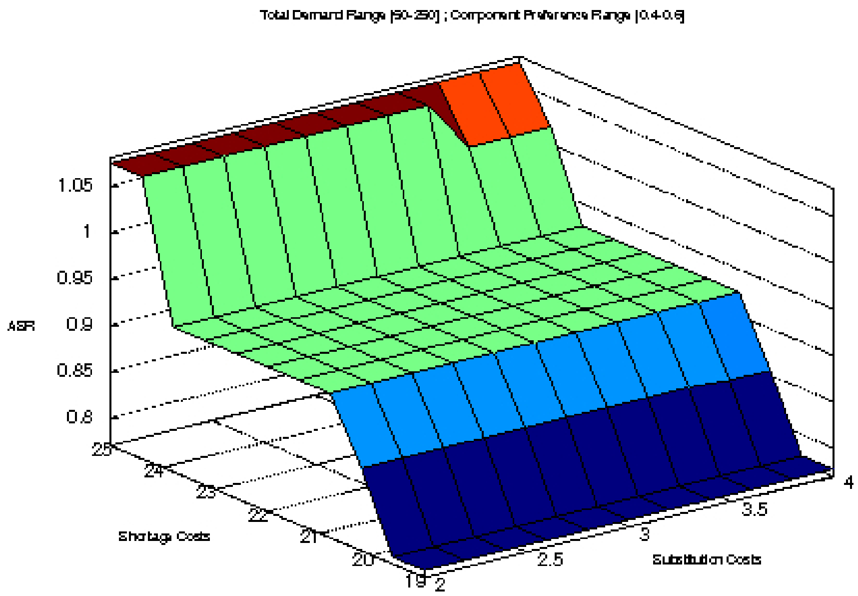


Figure A.1: ASR for scenario [50-250][0.4-0.6]

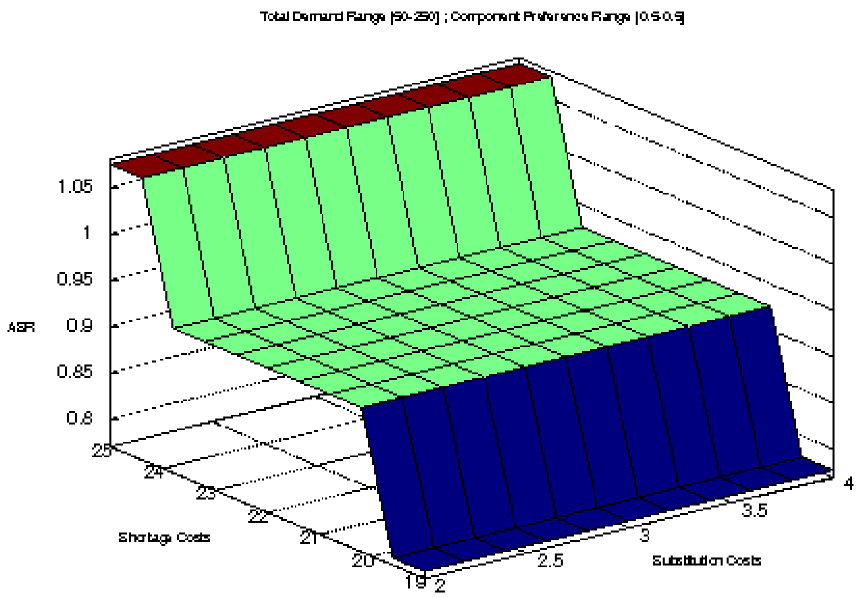


Figure A.2: ASR for scenario [50-250][0.5-0.5]

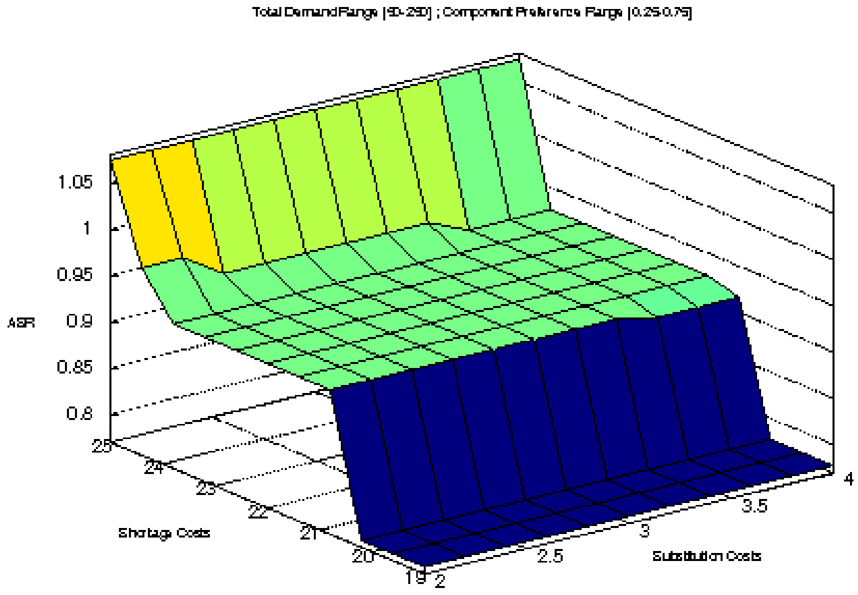


Figure A.3: ASR for scenario [50-250][0.25-0.75]

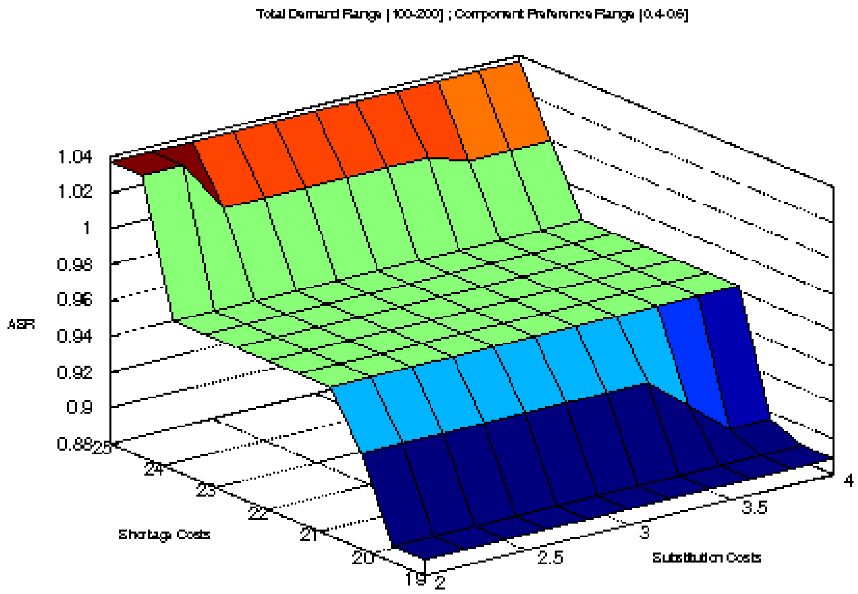


Figure A.4: ASR for scenario [100-200][0.4-0.6]

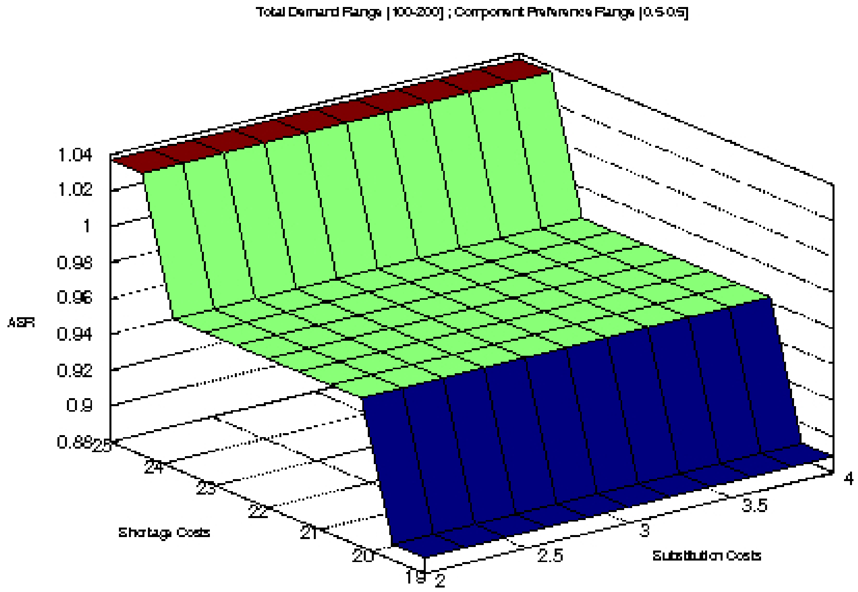


Figure A.5: ASR for scenario [100-200][0.5-0.5]

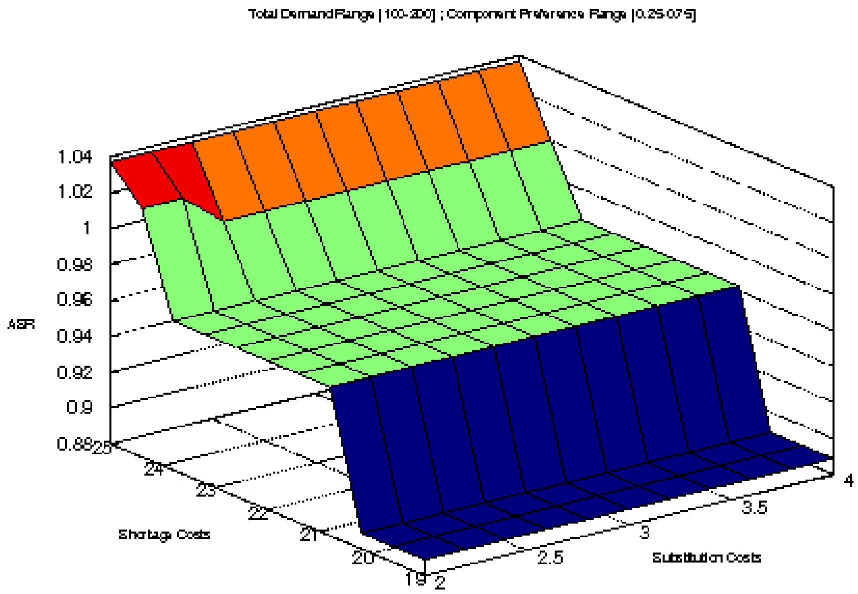


Figure A.6: ASR for scenario [100-200][0.25-0.75]

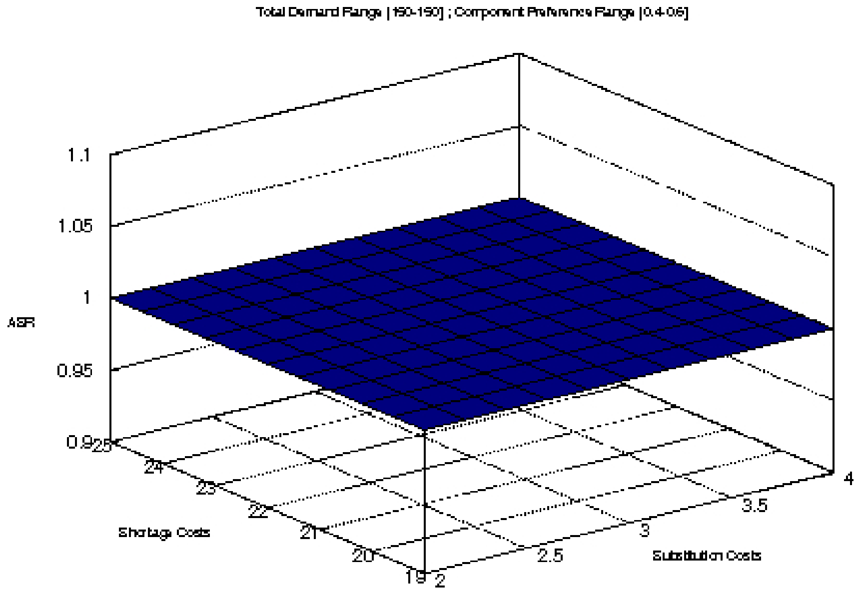


Figure A.7: ASR for scenario [150-150][0.4-0.6]

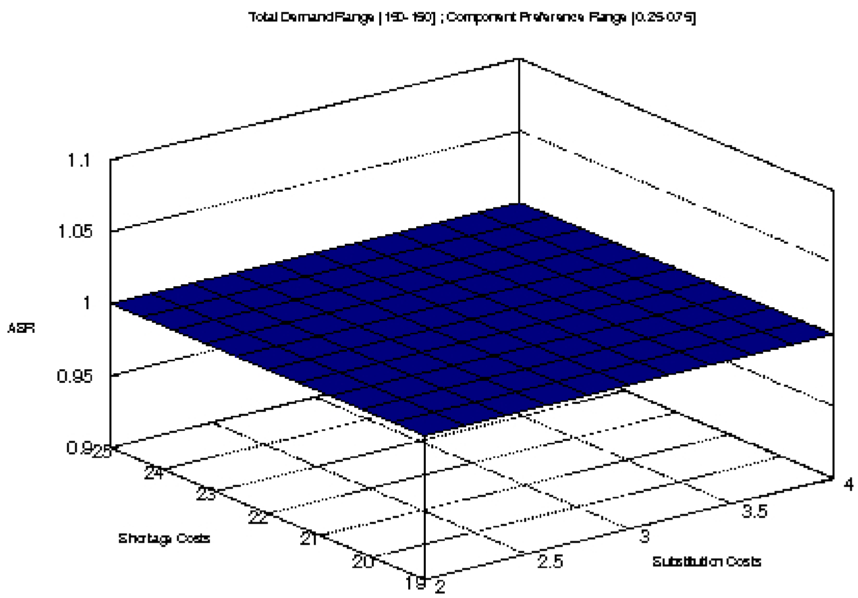


Figure A.8: ASR for scenario [150-150][0.25-0.75]

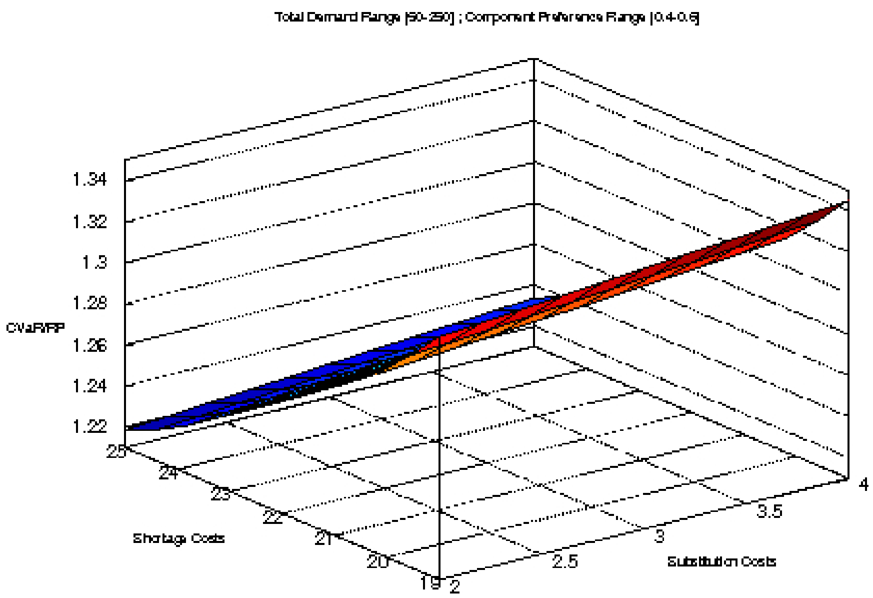


Figure A.9: CVaR/RP for scenario [50-250][0.4-0.6]

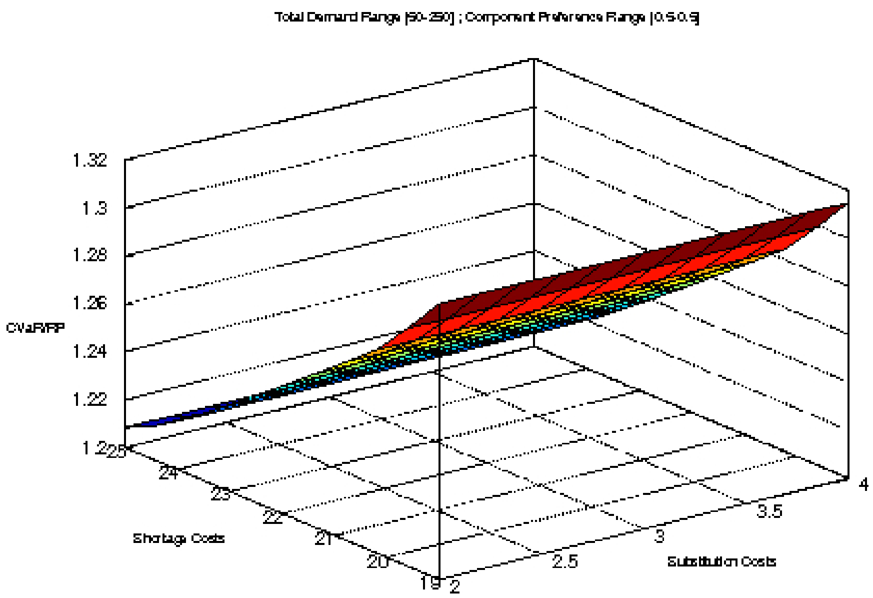


Figure A.10: CVaR/RP for scenario [50-250][0.5-0.5]

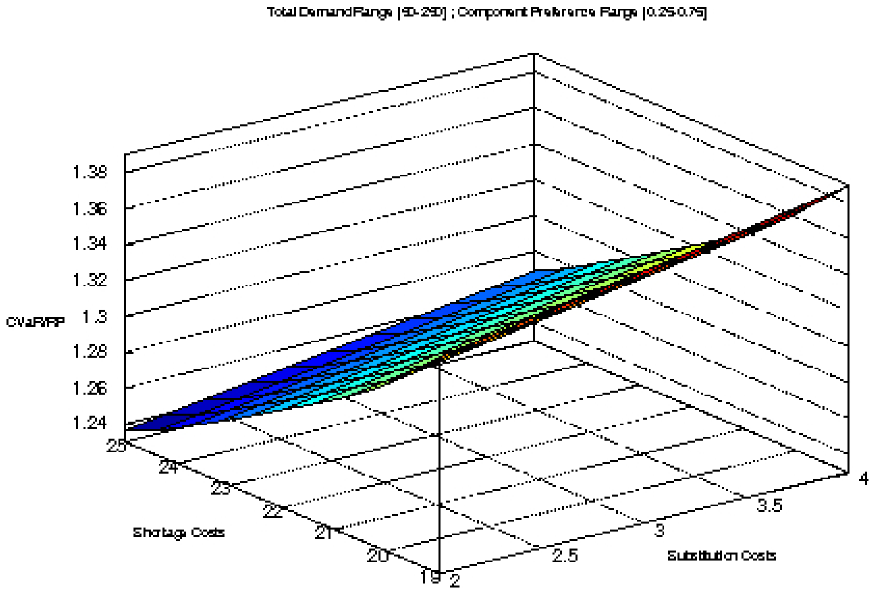


Figure A.11: CVaR/RP for scenario [50-250][0.25-0.75]

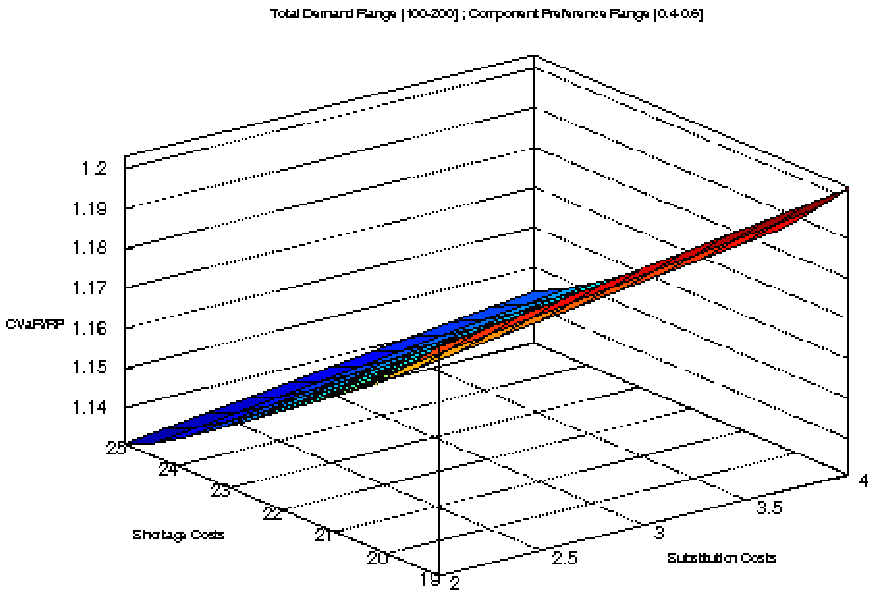


Figure A.12: CVaR/RP for scenario [100-200][0.4-0.6]

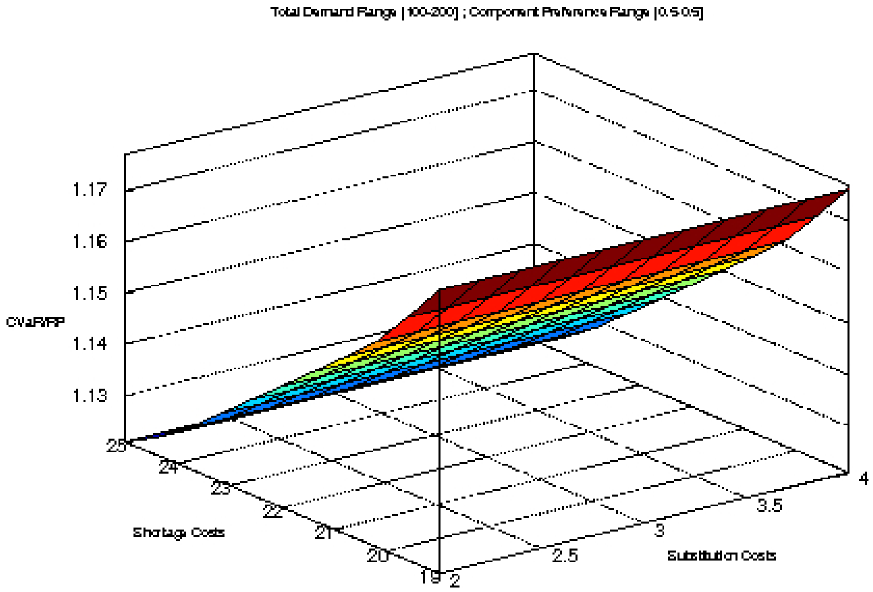


Figure A.13: CVaR/RP for scenario [100-200][0.5-0.5]

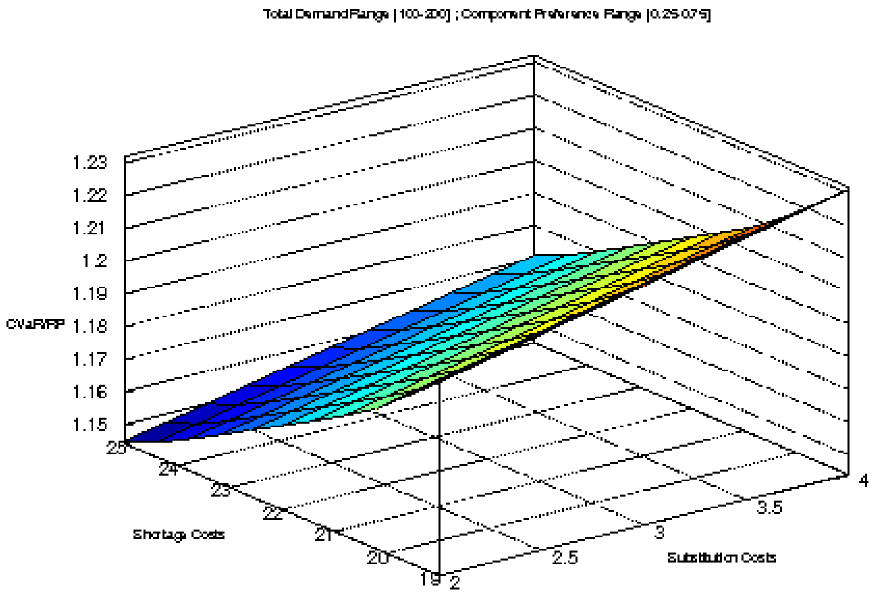


Figure A.14: CVaR/RP for scenario [100-200][0.25-0.75]

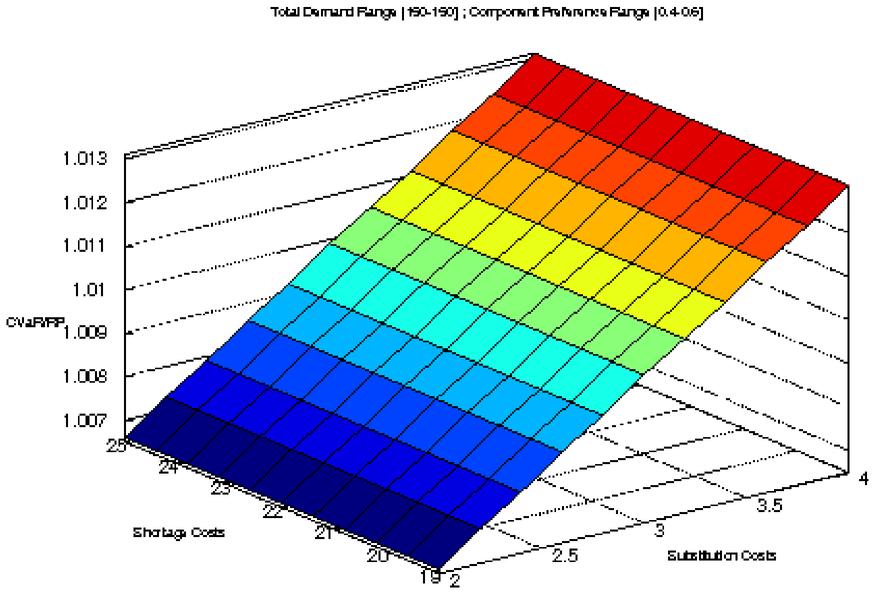


Figure A.15: CVaR/RP for scenario [150-150][0.4-0.6]

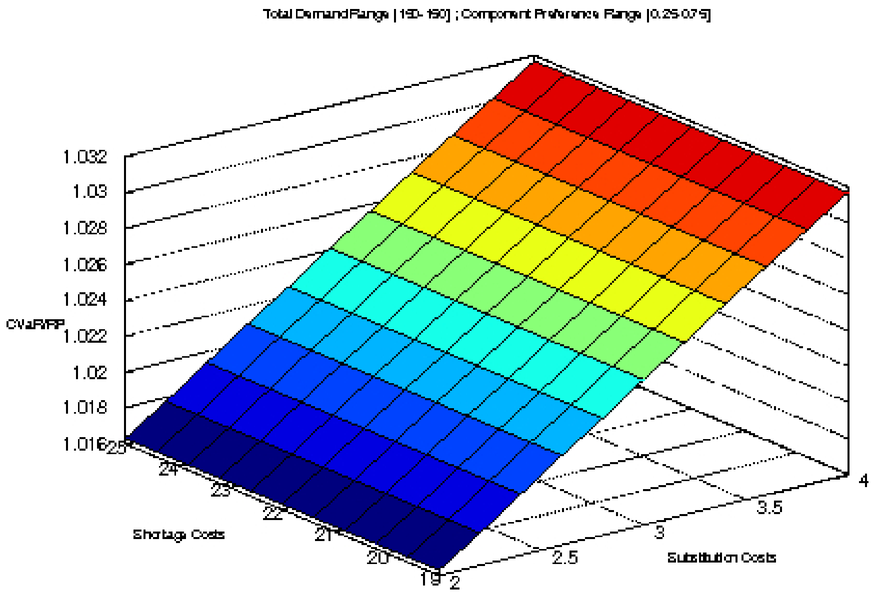


Figure A.16: CVaR/RP for scenario [150-150][0.25-0.75]

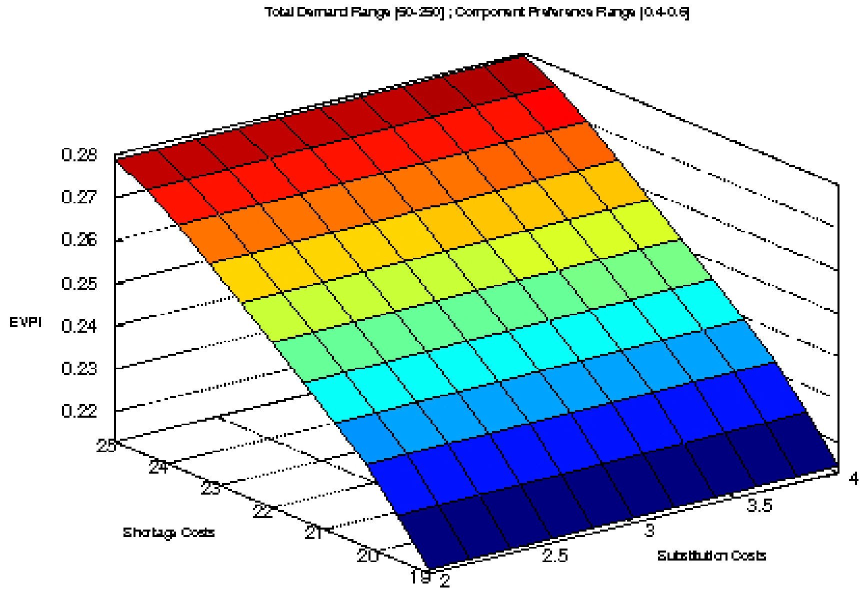


Figure A.17: EVPI for scenario [50-250][0.4-0.6]

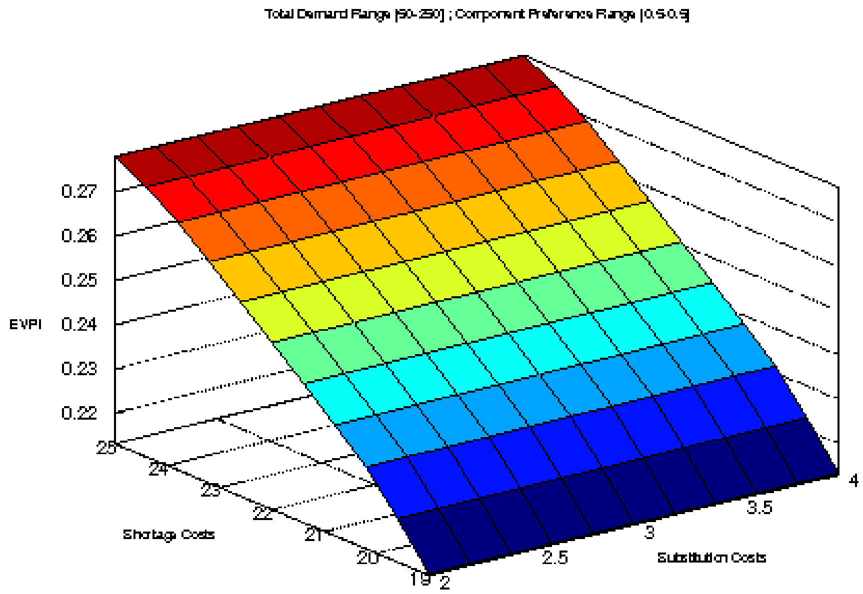


Figure A.18: EVPI for scenario [50-250][0.5-0.5]

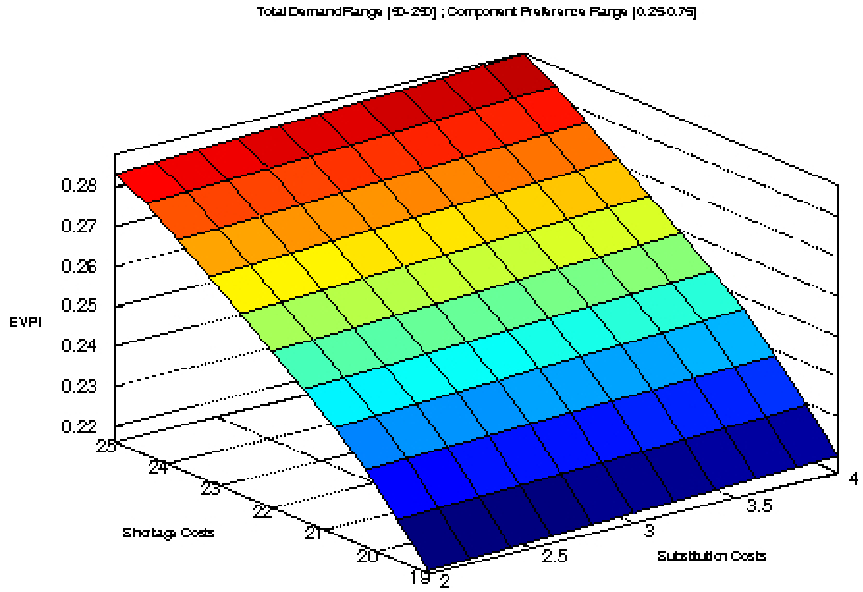


Figure A.19: EVPI for scenario [50-250][0.25-0.75]

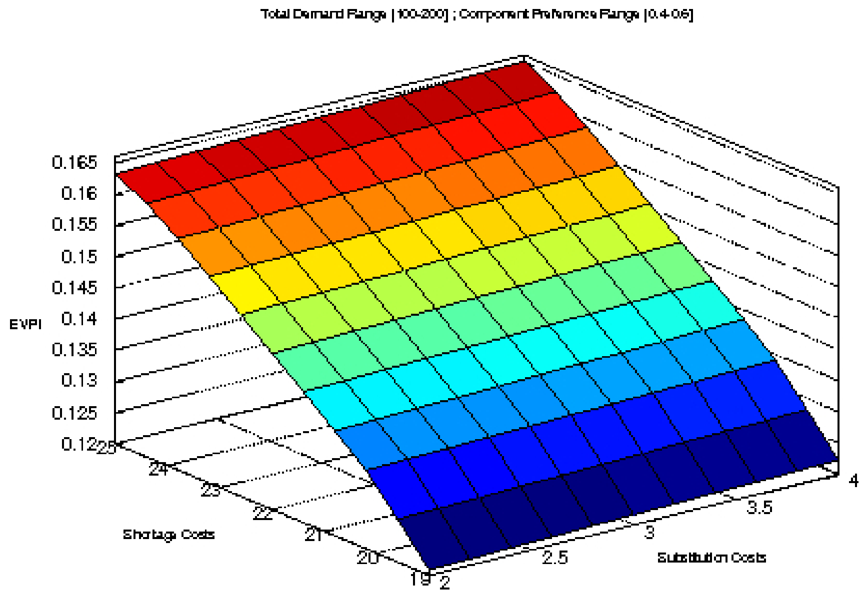


Figure A.20: EVPI for scenario [100-200][0.4-0.6]

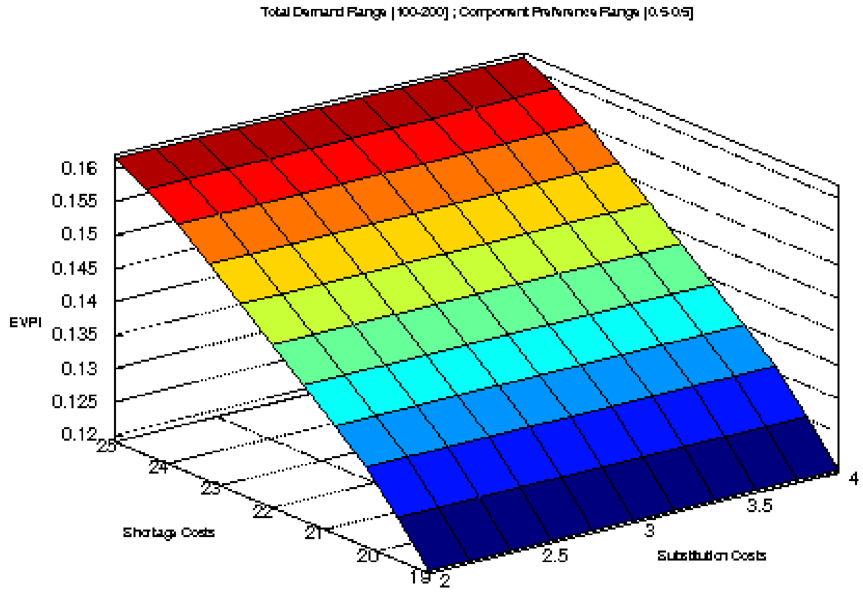


Figure A.21: EVPI for scenario [100-200][0.5-0.5]

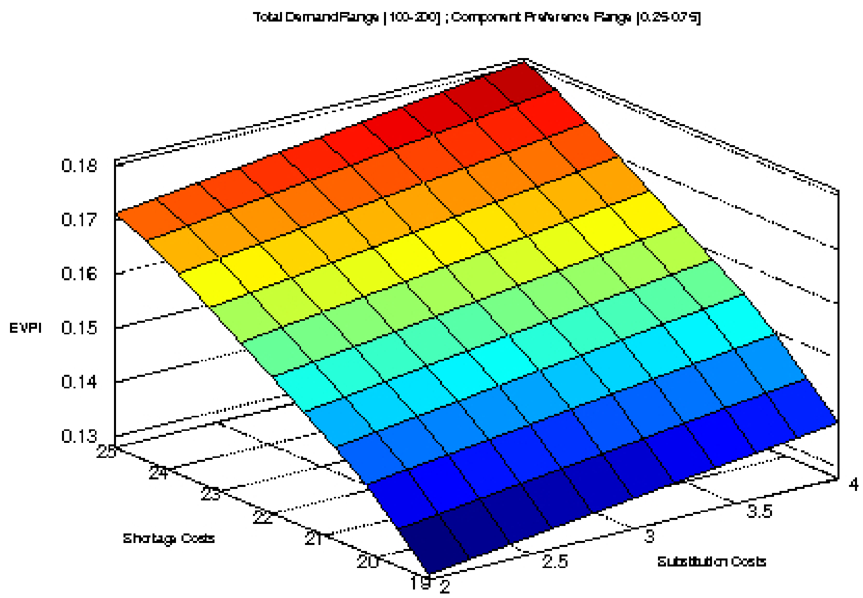


Figure A.22: EVPI for scenario [100-200][0.25-0.75]

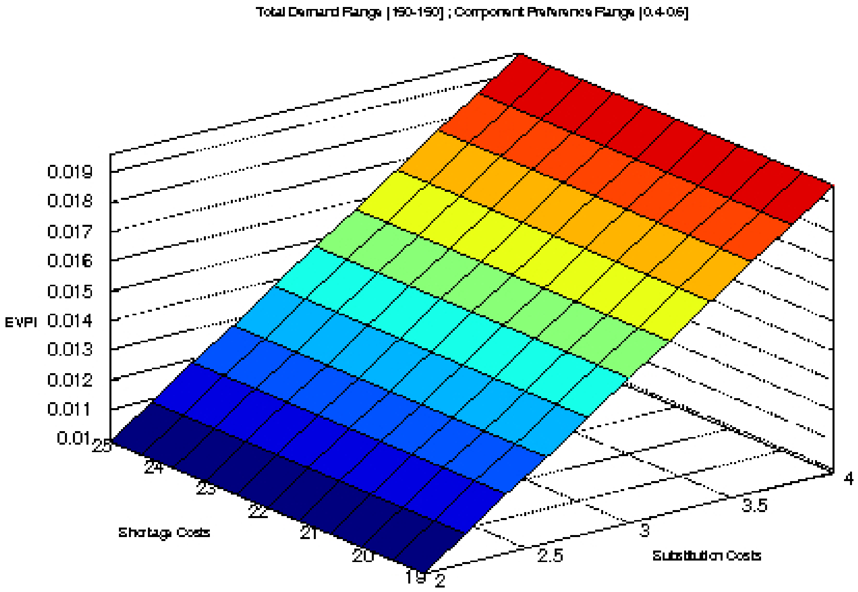


Figure A.23: EVPI for scenario [150-150][0.4-0.6]

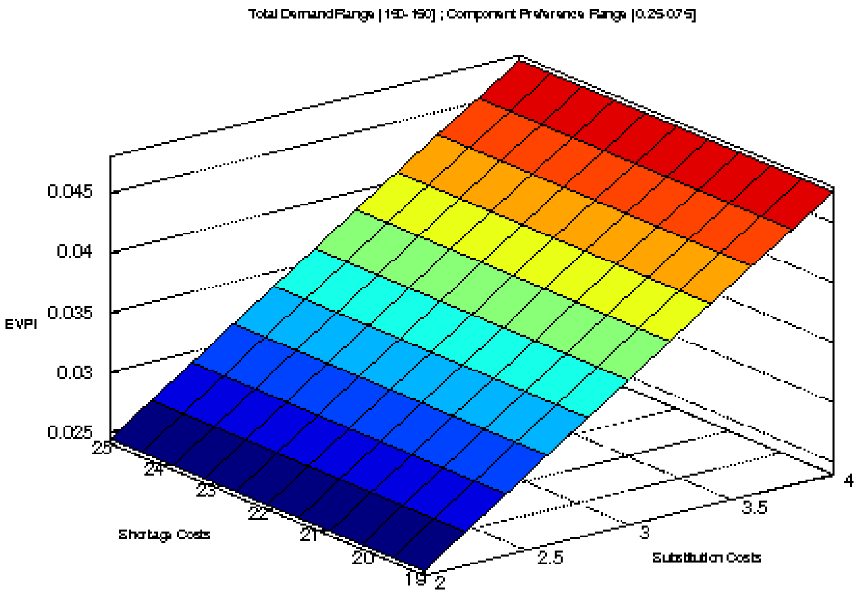


Figure A.24: EVPI for scenario [150-150][0.25-0.75]

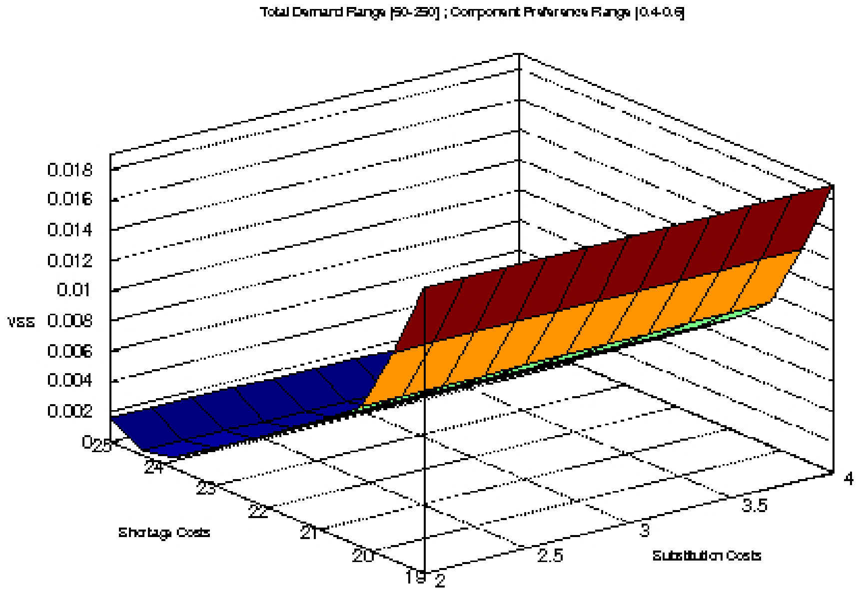


Figure A.25: VSS for scenario [50-250][0.4-0.6]

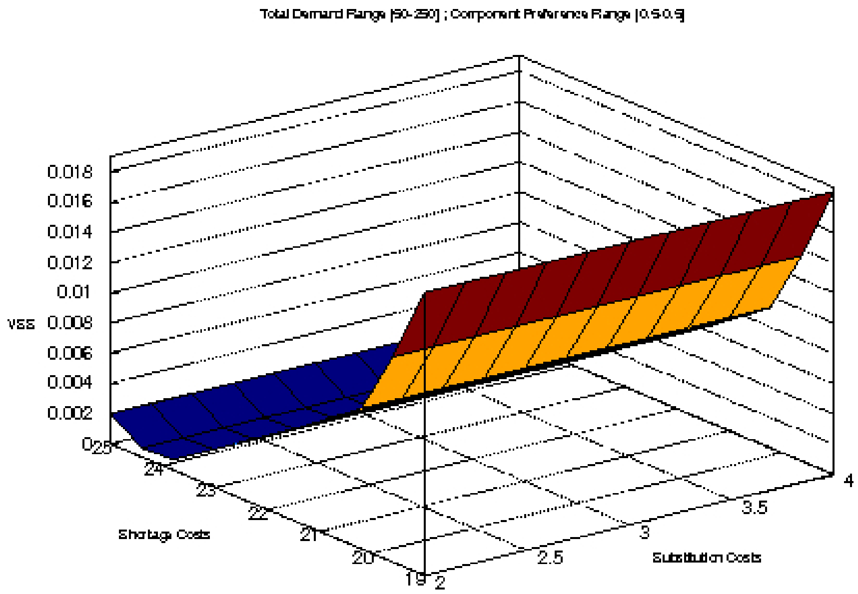


Figure A.26: VSS for scenario [50-250][0.5-0.5]

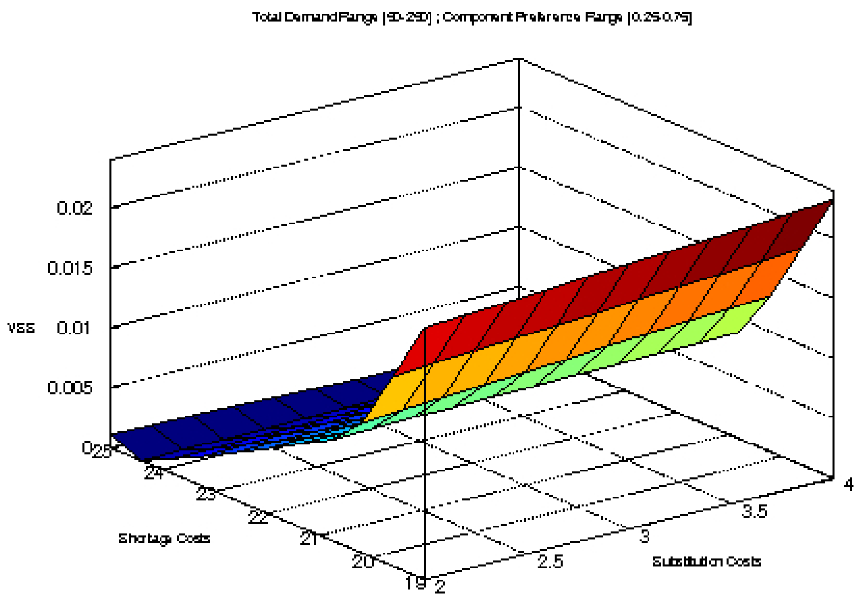


Figure A.27: VSS for scenario [50-250][0.25-0.75]

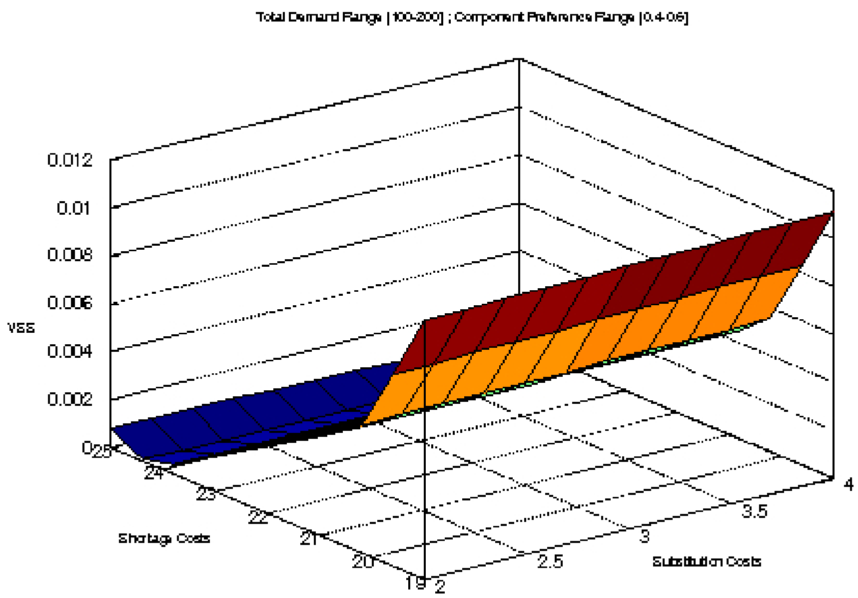


Figure A.28: VSS for scenario [100-200][0.4-0.6]

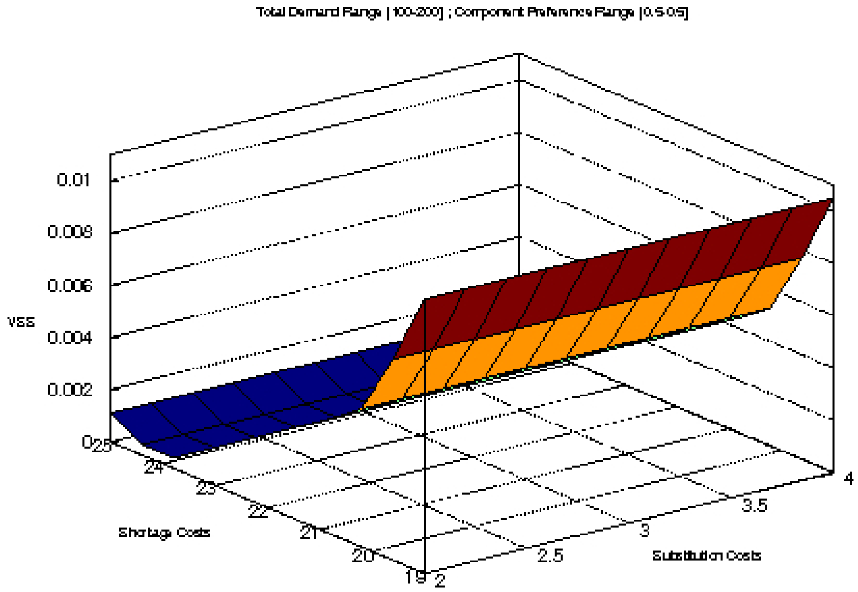


Figure A.29: VSS for scenario [100-200][0.5-0.5]

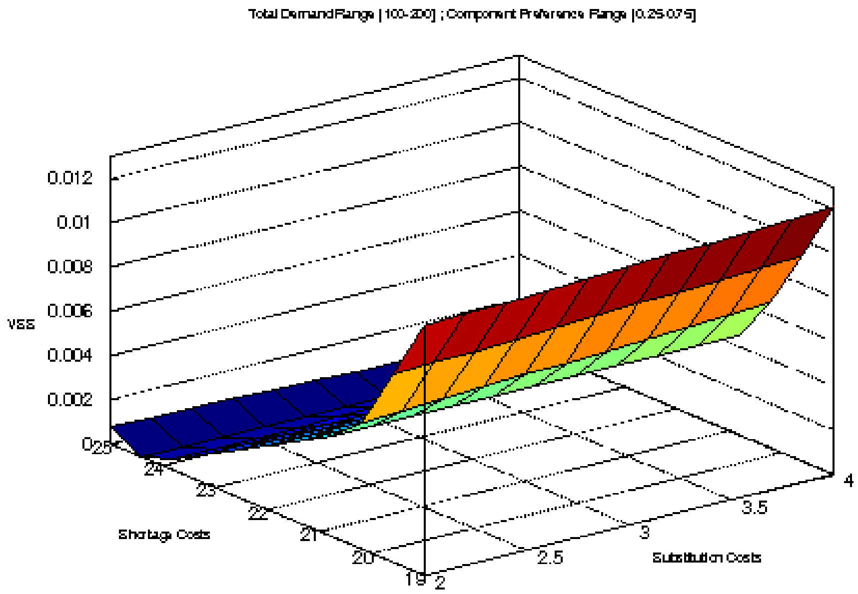


Figure A.30: VSS for scenario [100-200][0.25-0.75]

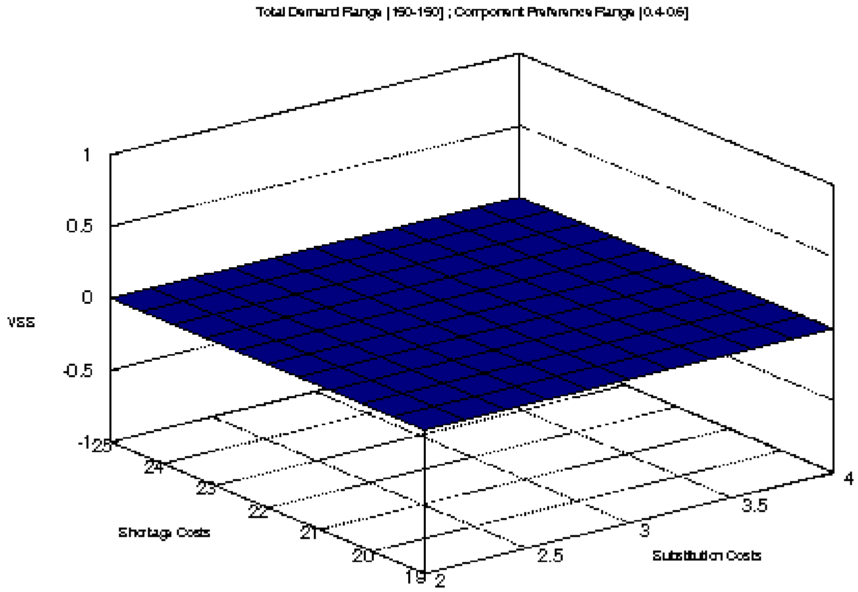


Figure A.31: VSS for scenario [150-150][0.4-0.6]

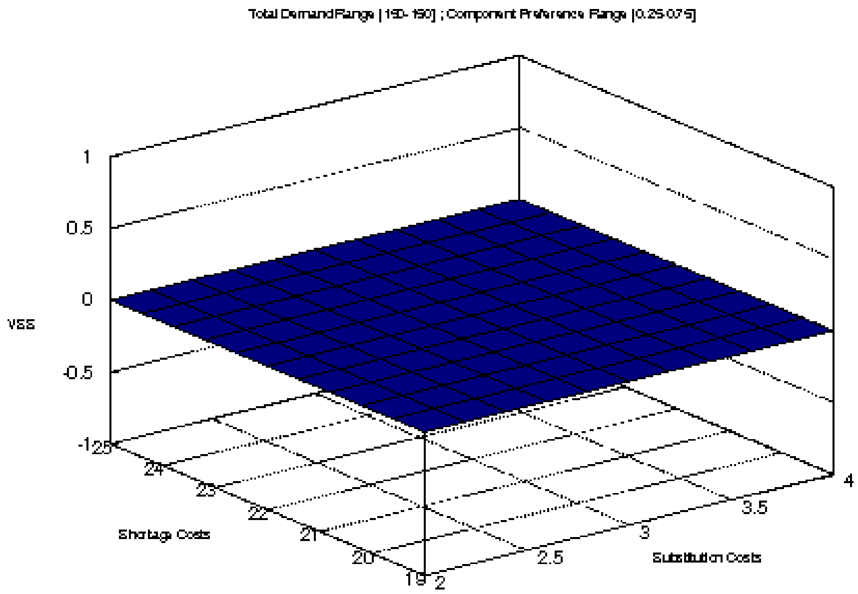


Figure A.32: VSS for scenario [150-150][0.25-0.75]

Appendix B

Full Component Preference Scenario Experiments

Table B.1: ASR values for low shortage and low substitution costs (F,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000

Table B.2: VSS values for low shortage and low substitution costs (F,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.3: EVPI values for low shortage and low substitution costs (F,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	0.0769	0.1000	0.1111	0.1176	0.1220
2	0.0769	0.1000	0.1111	0.1176	0.1220

Table B.4: CVaR/RP values for low shortage and low substitution costs (F,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000

Table B.5: ASR values for low shortage and low substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.6667	0.6667	0.6667	0.6667	0.6667
2	0.6667	0.6667	0.6667	0.6667	0.6667

Table B.6: VSS values for low shortage and low substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.0595	0.0595	0.0595	0.0595	0.0595
2	0.0595	0.0595	0.0595	0.0595	0.0595

Table B.7: EVPI values for low shortage and low substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.1628	0.1628	0.1628	0.1628	0.1628
2	0.1628	0.1628	0.1628	0.1628	0.1628

Table B.8: CVaR/RP values for low shortage and low substitution costs (V,F)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	1.1183	1.1183	1.1183	1.1183	1.1183
2	1.1183	1.1183	1.1183	1.1183	1.1183

Table B.9: ASR values for low shortage and low substitution costs (V,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	0.6667	0.6667	0.6667	0.6667	0.6667
2	0.6667	0.6667	0.6667	0.6667	0.6667

Table B.10: VSS values for low shortage and low substitution costs (V,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	0.0569	0.0634	0.0665	0.0684	0.0696
2	0.0569	0.0634	0.0665	0.0684	0.0696

Table B.11: EVPI values for low shortage and low substitution costs (V,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	0.2000	0.2117	0.2174	0.2208	0.2230
2	0.2000	0.2117	0.2174	0.2208	0.2230

Table B.12: CVaR/RP values for low shortage and low substitution costs (V,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	1.1556	1.1679	1.1739	1.1775	1.1799
2	1.1556	1.1679	1.1739	1.1775	1.1799

Table B.13: ASR values for low shortage and high substitution costs (F,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000

Table B.14: VSS values for low shortage and high substitution costs (F,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.15: EVPI values for low shortage and high substitution costs (F,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	0.1429	0.1818	0.2000	0.2105	0.2174
2	0.1429	0.1818	0.2000	0.2105	0.2174

Table B.16: CVaR/RP values for low shortage and high substitution costs (F,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000

Table B.17: ASR values for low shortage and high substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.6667	0.6667	0.6667	0.6667	0.6667
2	0.6667	0.6667	0.6667	0.6667	0.6667

Table B.18: VSS values for low shortage and high substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.0595	0.0595	0.0595	0.0595	0.0595
2	0.0595	0.0595	0.0595	0.0595	0.0595

Table B.19: EVPI values for low shortage and high substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.1628	0.1628	0.1628	0.1628	0.1628
2	0.1628	0.1628	0.1628	0.1628	0.1628

Table B.20: CVaR/RP values for low shortage and high substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.1183	1.1183	1.1183	1.1183	1.1183
2	1.1183	1.1183	1.1183	1.1183	1.1183

Table B.21: ASR values for low shortage and high substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.6667	0.6667	0.6667	0.6667	0.6667
2	0.6667	0.6667	0.6667	0.6667	0.6667

Table B.22: VSS values for low shortage and high substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.0545	0.0668	0.0727	0.0761	0.0784
2	0.0545	0.0668	0.0727	0.0761	0.0784

Table B.23: EVPI values for low shortage and high substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.2340	0.2552	0.2653	0.2713	0.2752
2	0.2340	0.2552	0.2653	0.2713	0.2752

Table B.24: CVaR/RP values for low shortage and high substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.1915	1.2138	1.2245	1.2308	1.2349
2	1.1915	1.2138	1.2245	1.2308	1.2349

Table B.25: ASR values for high shortage and low substitution costs (F,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000

Table B.26: VSS values for high shortage and low substitution costs (F,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.27: EVPI values for high shortage and low substitution costs (F,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.0769	0.1000	0.1111	0.1176	0.1220
2	0.0769	0.1000	0.1111	0.1176	0.1220

Table B.28: CVaR/RP values for high shortage and low substitution costs (F,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000

Table B.29: ASR values for high shortage and low substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.3333	1.3333	1.3333	1.3333	1.3333
2	1.3333	1.3333	1.3333	1.3333	1.3333

Table B.30: VSS values for high shortage and low substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.0091	0.0091	0.0091	0.0091	0.0091
2	0.0091	0.0091	0.0091	0.0091	0.0091

Table B.31: EVPI values for high shortage and low substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.2519	0.2519	0.2519	0.2519	0.2519
2	0.2519	0.2519	0.2519	0.2519	0.2519

Table B.32: CVaR/RP values for high shortage and low substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.0001	1.0001	1.0001	1.0001	1.0001
2	1.0001	1.0001	1.0001	1.0001	1.0001

Table B.33: ASR values for high shortage and low substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.3333	1.3333	0.6667	0.6667	0.6667
2	1.3333	1.3333	0.6667	0.6667	0.6667

Table B.34: VSS values for high shortage and low substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.0088	0.0021	0.0012	0.0031	0.0043
2	0.0088	0.0021	0.0012	0.0031	0.0043

Table B.35: EVPI values for high shortage and low substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.2817	0.3003	0.3077	0.3103	0.3121
2	0.2817	0.3003	0.3077	0.3103	0.3121

Table B.36: CVaR/RP values for high shortage and low substitution costs (V,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	1.0375	1.0365	1.0385	1.0421	1.0446
2	1.0375	1.0365	1.0385	1.0421	1.0446

Table B.37: ASR values for high shortage and high substitution costs (F,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000

Table B.38: VSS values for high shortage and high substitution costs (F,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.39: EVPI values for high shortage and high substitution costs (F,V)

Modules	Components				
	2	**3**	**4**	**5**	**6**
1	0.1429	0.1818	0.2000	0.2105	0.2174
2	0.1429	0.1818	0.2000	0.2105	0.2174

Table B.40: CVaR/RP values for high shortage and high substitution costs (F,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000

Table B.41: ASR values for high shortage and high substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.3333	1.3333	1.3333	1.3333	1.3333
2	1.3333	1.3333	1.3333	1.3333	1.3333

Table B.42: VSS values for high shortage and high substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.0091	0.0091	0.0091	0.0091	0.0091
2	0.0091	0.0091	0.0091	0.0091	0.0091

Table B.43: EVPI values for high shortage and high substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.2519	0.2519	0.2519	0.2519	0.2519
2	0.2519	0.2519	0.2519	0.2519	0.2519

Table B.44: CVaR/RP values for high shortage and high substitution costs (V,F)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.0001	1.0001	1.0001	1.0001	1.0001
2	1.0001	1.0001	1.0001	1.0001	1.0001

Table B.45: ASR values for high shortage and high substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.3333	0.6667	0.6667	0.6667	0.6667
2	1.3333	0.6667	0.6667	0.6667	0.6667

Table B.46: VSS values for high shortage and high substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.0084	0.0042	0.0102	0.0137	0.0160
2	0.0084	0.0042	0.0102	0.0137	0.0160

Table B.47: EVPI values for high shortage and high substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	0.3093	0.3374	0.3455	0.3502	0.3533
2	0.3093	0.3374	0.3455	0.3502	0.3533

Table B.48: CVaR/RP values for high shortage and high substitution costs (V,V)

	Components				
Modules	**2**	**3**	**4**	**5**	**6**
1	1.0744	1.0798	1.0909	1.0975	1.1018
2	1.0744	1.0798	1.0909	1.0975	1.1018

Appendix C

0.45-0.55 Component Preference Probabilities Scenario Experiments

Table C.1: Results for low shortage and low substitution costs (F,V)

Modules	ASR	VSS	EVPI	CVaR/RP
1	1.0000	0.0000	0.0083	1.0000
2	1.0000	0.0000	0.0083	1.0000
3	1.0000	0.0000	0.0083	1.0000
4	1.0000	0.0000	0.0083	1.0000
5	1.0000	0.0000	0.0083	1.0000

Table C.2: Results for low shortage and low substitution costs (V,F)

Modules	ASR	VSS	EVPI	CVaR/RP
1	0.6667	0.0595	0.1628	1.1183
2	0.6667	0.0595	0.1628	1.1183
3	0.6667	0.0595	0.1628	1.1183
4	0.6667	0.0595	0.1628	1.1183
5	0.6667	0.0595	0.1628	1.1183

Table C.3: Results for low shortage and low substitution costs (V,V)

Modules	ASR	VSS	EVPI	CVaR/RP
1	0.6667	0.0571	0.1647	1.1230
2	0.6667	0.0571	0.1647	1.1230
3	0.6667	0.0571	0.1647	1.1230
4	0.6667	0.0571	0.1647	1.1230
5	0.6667	0.0571	0.1647	1.1230

Table C.4: Results for low shortage and high substitution costs (F,V)

Modules	ASR	VSS	EVPI	CVaR/RP
1	1.0000	0.0000	0.0164	1.0000
2	1.0000	0.0000	0.0164	1.0000
3	1.0000	0.0000	0.0164	1.0000
4	1.0000	0.0000	0.0164	1.0000
5	1.0000	0.0000	0.0164	1.0000

Table C.5: Results for low shortage and high substitution costs (V,F)

Modules	ASR	VSS	EVPI	CVaR/RP
1	0.6667	0.0595	0.1628	1.1183
2	0.6667	0.0595	0.1628	1.1183
3	0.6667	0.0595	0.1628	1.1183
4	0.6667	0.0595	0.1628	1.1183
5	0.6667	0.0595	0.1628	1.1183

Table C.6: Results for low shortage and high substitution costs (V,V)

Modules	ASR	VSS	EVPI	CVaR/RP
1	0.6667	0.0546	0.1667	1.1296
2	0.6667	0.0546	0.1667	1.1296
3	0.6667	0.0546	0.1667	1.1296
4	0.6667	0.0546	0.1667	1.1296
5	0.6667	0.0546	0.1667	1.1296

Table C.7: Results for high shortage and low substitution costs (F,V)

Modules	ASR	VSS	EVPI	CVaR/RP
1	1.0000	0.0000	0.0083	1.0000
2	1.0000	0.0000	0.0083	1.0000
3	1.0000	0.0000	0.0083	1.0000
4	1.0000	0.0000	0.0083	1.0000
5	1.0000	0.0000	0.0083	1.0000

Table C.8: Results for high shortage and low substitution costs (V,F)

Modules	ASR	VSS	EVPI	CVaR/RP
1	1.3333	0.0091	0.2519	1.0001
2	1.3333	0.0091	0.2519	1.0001
3	1.3333	0.0091	0.2519	1.0001
4	1.3333	0.0091	0.2519	1.0001
5	1.3333	0.0091	0.2519	1.0001

Table C.9: Results for high shortage and low substitution costs (V,V)

Modules	ASR	VSS	EVPI	CVaR/RP
1	1.2000	0.0055	0.2546	1.0022
2	1.2000	0.0055	0.2546	1.0022
3	1.2000	0.0055	0.2546	1.0022
4	1.2113	0.0054	0.2546	1.0021
5	1.2662	0.0052	0.2548	1.0019

Table C.10: Results for high shortage and high substitution costs (F,V)

Modules	ASR	VSS	EVPI	CVaR/RP
1	1.0000	0.0000	0.0164	1.0000
2	1.0000	0.0000	0.0164	1.0000
3	1.0000	0.0000	0.0164	1.0000
4	1.0000	0.0000	0.0164	1.0000
5	1.0000	0.0000	0.0164	1.0000

Table C.11: Results for high shortage and high substitution costs (V,F)

Modules	ASR	VSS	EVPI	CVaR/RP
1	1.3333	0.0091	0.2519	1.0001
2	1.3333	0.0091	0.2519	1.0001
3	1.3333	0.0091	0.2519	1.0001
4	1.3333	0.0091	0.2519	1.0001
5	1.3333	0.0091	0.2519	1.0001

Table C.12: Results for high shortage and high substitution costs (V,V)

Modules	ASR	VSS	EVPI	CVaR/RP
1	1.2000	0.0055	0.2546	1.0104
2	1.2000	0.0055	0.2546	1.0104
3	1.2000	0.0055	0.2546	1.0104
4	1.1887	0.0052	0.2548	1.0101
5	1.1558	0.0043	0.2555	1.0092

Biographical Sketch

Ufuk Bahçeci was born December 8, 1980 in Istanbul, Turkey. He finished high school education at Galatasaray High School. In 2003, he earned a diploma in industrial engineering from the University of Galatasaray at Istanbul. In the same year, he joined the Engineering faculty of Galatasaray University where he is currently serving as research assistant in the department of industrial engineering. In 2005, he received the Master degree in industrial engineering from the University of Galatasaray. After having completed his military service, he continued his position at the Engineering faculty of Galatasaray University and started the PhD program in industrial engineering at Galatasaray University.