SPREADSHEET HEURISTIC FOR VOLATILE DEMAND ENVIRONMENTS TO SOLVE THE JOINT REPLENISHMENT PROBLEM

(TOPLU SİPARİŞ PROBLEMİ ÇÖZÜMÜNDE DEĞİŞKEN TALEP İÇİN HESAP TABLOSU YÖNTEMİ)

by

Buket TÜRKAY, B.S.

Thesis

Submitted in Partial Fulfillment

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Buket Türkay
July, 2013

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LIST OF SYMBOLS

i	1,2,3...,n, a product index
n	number of items ordered from a single supplier
D_i	average demand for product i (TL /units/week)
σ	standard deviation of demand forecast errors during one unit of time
z	multiplier of σ (determines the service level)
h_i	the holding cost for one unit of product i (TL/unit/week)
s_i	the minor ordering cost of product i incurred when product i is included in a group replenishment (TL/order)
S	the major ordering cost associated with a replenishment involving one or more products (TL/order)
Q_i	order quantity for product i, a decision variable (units)
T	replenishment interval or basic cycle time
T_i	the cycle time between placing consecutive orders of item i in weeks
k_i	the integer number of T intervals that the replenishment quantity of item i will last (decision variable)
m	integer number decided by decision maker.
C_o	total ordering cost per week
C_c	total carrying cost per week
TC	total cost per week

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ABSTRACT

A proper replenishment strategy is a critical enabler in the success of increased revenue, net profits and customer service. Inventory management requires constant and careful evaluation of external and internal factors and control through planning and review. In order to control the inventory, one major requirement is to provide efficient replenishment technique such as jointly replenishment of products. Searching for efficient replenishment techniques is a common and usually a mandatory topic for the organizations.

We consider inventory systems with multiple products in the presence of volatile demand and jointly incurred order setup cost. In this thesis, a new adaptation of spreadsheet heuristic for volatile environment is presented. The simplicity of application of spreadsheet method and its efficiency enables us to consider its modified version for the joint replenishment problem under volatile demand. The principle of the procedure is to find a balance between the replenishment and holding cost for jointly replenished items.

The real business data is conducted to evaluate the performance of the heuristic. The study shows that the proposed algorithm performs well in comparison with well known RAND heuristic for the numerical data. Additionally, the importance of using volatile demand strategy over deterministic strategy is also highlighted with a calculation. The proposed strategy gives higher customer service level which means lower unmet customer demand. Owing to the simplicity and effectiveness of the proposed algorithm, we believe that it can be applicable in volatile demand environments.

Keywords: Inventory management, Spreadsheet heuristic for volatile demand, Joint replenishment problem.

RESUME

Une stratégie propre de réapprovisionnement est une des clés du succès pour augmenter des revenues, du profit net, et de la satisfaction des clients. Pour bien gérer le stock, on a besoin de mesurer fréquemment l'évaluation des facteurs internes et externes, de les planifier, de les contrôler suivant le planning et de les réviser. Une technique efficace de réapprovisionnement est nécessaire pour gestion de stocks et donc est très importante pour les organisations.

Dans le système de stock, il s'agit de multi produits, de demande volatil, et de cout total de réapprovisionnement. Dans ce travail, la nouvelle adaptation de "spreadsheet heuristic" dans l'environnent volatil est présentée. La méthode de "spreadsheet heuristic" est efficace et facile à appliquer c'est pourquoi on considère le problème de réapprovisionnement dans un environnent volatil. L'objectif est de trouver une balance entre le cout de réapprovisionnement et le cout de possession.

La data réelle est utilisé pour évaluer la performance de heuristique. Les résultats montrent que la performance de notre heuristique est mieux que la performance de la meilleure heuristique dans la literature pour les données numériques. En outre, on montre que la stratégie volatil est mieux que la stratégie déterministe.

La stratégie volatil proposée dans ce travail rend mieux service pour les clients. Comme l'algorithme proposé est simple et efficace, il peut être utilisé dans les environnements de la demande volatils.

Mot clés : Gestion de stock, Spreadsheet heuristic pour demande volatil, Problème de réapprovisionnement.

ÖZET

Doğru bir ikmal politikası, gelirlerin, net karlılığın ve müşteri memnuniyetinin arttırılmasındaki başarının en kritik faktörüdür. Envanter yönetimi, dış ve iç bileşenlerin sürekli ve itinalı bir şekilde ölçümlenmesini, planlama ve inceleme yoluyla kontrol edilmesini gerektirir. Envanter kontrolünün en önemli gereksinimi, toplu sipariş tekniği gibi etkin bir ikmal politikası oluşturmaktır. Etkin bir ikmal tekniği arayışı, organizasyonlar için yaygın ve zorunlu bir konu haline gelmiştir.

Çalışmada, değişken talep ve toplu sipariş maliyetlerini içeren çoklu ürünlü envanter sistemi incelenmiştir. Bu tezde, değişken talep ortamında hesap tablosu sezgisel yaklaşımının yeni bir uyarlaması sunulmuştur. Hesap tablosu sezgisel metodunun kolay uygulanabilirliği ve etkinliği, onun değişken talep altında toplu sipariş problem için uyarlanmasına sebebiyet vermiştir. Yöntemin prensibi, toplu ikmal edilen ürünler için sipariş maliyeti ve elde tutma maliyeti arasındaki optimal dengeyi bulmaktır.

Uyarlanan sezgisel yöntemin performansının test edilebilmesi için, yöntem gerçek bir işletme verisi üzerinde uygulanmıştır. Yapılan çalışmalar, uygulamada kullanılan data seti için önerilen algoritmanın RAND yönteminden daha iyi sonuçlar verdiğini göstermiştir. Ek olarak, rastgele olmayan strateji yerine değişken stratejinin kullanılmasının önemi bir hesaplama ile vurgulanmıştır. Önerilen değişken talep yöntemi ile, rassal olmayan yöntemden daha yüksek müşteri servis seviyesine ulaşılmıştır. Sunulan algoritma, sadeliği ve etkinliği sayesinde değişken talep ortamlarında uygulanabilir bir çözüm tekniği niteliğindedir.

Anahtar kelimeler: Envanter yönetimi, Değişken talep için hesap tablosu yöntemi, Toplu sipariş problemi.

1. INTRODUCTION

Replenishment policies are highly important in inventory management area. In the literature and practice, there are many types of inventory models dealing with multi-product environments (Aksoy & Erenguc, 1988). The objective of an inventory model is generally finding the trade-off between holding cost and replenishment cost.

Holding cost is the associated of storing inventory or assets that remain unsold. Holding cost includes storage cost for products in stock, taxes, insurance as well as opportunity cost that money could be deployed elsewhere. Additionally, cost of being damaged over time, obsolescence, wages and salaries paid to personnel to handle inventory, warehouse space cost are also in holding cost.

Ordering cost is the cost associated in preparing and processing purchase orders as well as receiving and inspecting purchased products. In the replenishment process, ordering cost incurred in every purchasing phase. Ordering process has two cost components:

1. Major ordering cost which is independent of the number of replenished products. It includes the cost of preparing the order, bookkeeping cost, transportation mean's fixed cost and other handling cost associated with generating an order.
2. Minor ordering cost which depends on the products in the order. It includes freight cost as volume based and any special cost incurred by the item. Minor ordering cost is charged for individual items in the order.

In inventory management, joint replenishment is referred as a family of items replenished together in order to share a common setup cost. Substantial cost savings is reasonable in terms of high major ordering cost. Joint replenishment strategy is widely accepted by both academic and many manufacturing companies.

A proper replenishment strategy is a critical enabler in the success of increased revenue, net profits and customer service. Inventory management requires constant and careful evaluation of external and internal factors and control through planning and review. In order to control the inventory, one major requirement is to provide efficient replenishment techniques such as jointly replenishment of products. Searching for efficient replenishment techniques is a common and usually a mandatory topic for the organizations.

Joint replenishment strategy is also important for companies in order to reduce purchase transaction cost using a common supplier for multi product replenishment. Since the 1980s, many manufacturing companies have been reducing their supplier bases. For example, Xerox reduced its supplier base in early 1980s from 5000 to 400 (Burt, 1989), Texas Instruments reduced its Maintenance, Repair and Operating (MRO) suppliers from 5000 to 750 between 1998 and 2000 (Pantumsinchai, 2000), Merck reduced its total global supplier base from 40,000 in 1992 to fewer than 10,000 in 1997 (Genna, 1997), IBM reduced the number of its suppliers to only 50 for 85% of its requirements (Carbone, 1999) and Sun Microsystems reduced its supplier base to 40 for 90% of its requirements (Carbone, 1996). Among other things, reduction of the supplier base helps companies decrease their inventory holding, transportation and purchasing costs by giving them the capability of jointly replenishing multiple items from common suppliers (Tanrıkulu, 2006).

Sourcing from one supplier is not a requirement for jointly replenishing multiple items. Companies may lead to combine different items into a single delivery. This strategy allows the joint procurement of multiple products from different suppliers located closely, and helps companies consolidate smaller shipment into more efficient larger shipments. For example, commercial vehicle producer MAN, Ankara plant successfully reduced its inbound transportation cost and component inventory by consolidating its shipments from various component manufacturers located in close proximity in Northwestern Turkey. (Bostancı et al., 2005)

The classic joint replenishment problem includes deterministic and uniform demand, no shortage allowed, no quantity discount and holding cost is linear. In deterministic problems, each items' average demand values are taken into account. Due to not considering demand variation, there may be some mistakes or missing interpretation for future decisions for deterministic inputs. Manufacturers generally ignore the variation in demand and assign the same inventory targets for the products same average. However, joint replenishment is viable for volatile demand environments since demand variances can also be considered to model the problem.

The objective of this article is present a new adaptation of the spreadsheet heuristic for volatile demand environment in joint replenishment. Real world data is used and tested in order to measure the effectiveness of the proposed method.

The numerical data constructed in the application belongs to a worldwide known consumer electronic company and an effective joint replenishment heuristic is used to minimize the total costs. High technology industries have several unique characteristics in terms of supply chain management. First and foremost, technology products have short life cycles and high rate of obsolescence. Due to high level demand uncertainty, technology companies tend to have low inventory targets. Inventory shortage is another reason for uncertainty. In order to balance between high inventory level and shortage, an effective replenishment strategy is required.

The organization of the dissertation has the following outline. In Section 2 we give related literature. Section 3 describes inventory management, inventory replenishment, the joint replenishment problem and methodologies that constitute the proposed framework. The steps and details of the proposed solution procedure are given in Section 4. The heuristic is then tested and results are compared with well known heuristic in Section 5. Finally, Section 6 concludes the study.

2. LITERATURE REVIEW

Joint replenishment problem has been studied by many researchers. Since early work of Starr and Miller (1962) and Shu (1971), joint replenished items problem has received considerable attention. Because of the existing of major ordering cost, there can be obtained substantial cost savings by using group replenishment. The savings are more significant in higher major cost environments.

Studies can be split into two types in terms of input: deterministic and stochastic problems. To mention a few, Brown (1967) has suggested a simple heuristic procedure, and Goyal (1974) has proposed a more systematic but lengthy procedure which results in the optimal solution. Silver (1976) achieved near optimal results with a simple procedure which was later modified by Goyal and Belton (1979) and also by Kaspi and Rosenblatt (1983). Atkins and Iyogun (1988) have considered the case where demand varies over time by extending the Silver-Meal (1973) heuristic. Furthermore, they suggested a lower bound on the cost by allocating the family ordering cost to the various products (Eynan and Kropp, 1998).

The problem of determining the frequency of packaging set-ups for joint replenished items is akin to the problem of determining the economic ordering frequency of items procured from a single supplier, which is discussed in Goyal (1973). He proposed upper and lower bound for multiple values of basic cycle time of each item. With his method optimum solutions were obtained for 95% of problems.

In Silver's (1976) study a simple method for determining the basic cycle time and item multiples are proposed. The result of his method outweighs over Brown's (1967) and has 0.2% penalty compared with best solution by search. However, the iterative method is dependent on the chosen item as the first item.

Goyal and Belton (1979) proposed an improvement of Silver (1976) by modifying the first step of his method. Since the improvement was adding major cost to determine lowest multiplier of basic cycle time, they obtained equally good and better results to all problems examined.

The modified version of Silver (1976) algorithm was suggested by Kaspi and Rosenblatt (1983). In order to improve the algorithm, it is suggested that once the value of basic cycle time is obtained, it should be used to recalculate the values of the multiple of basic cycle time for each item. This modification is desirable, since Silver's algorithm depends on selecting the first item. The average improvement for this algorithm was about 3.3% for 3000 problems compared to only 0.29% improvement for the Goyal and Belton modification.

Goyal (1988) improved the algorithm by determining the first initial estimate of basic cycle time. The results obtained from 80 sample test problems, the heuristic method outperformed the previous methods.

In the study presented by Kaspi (1991), the solution procedure is the modification of Goyal (1988). He illustrated that Goyal's approach can converge to a local minimum if multiple integer values are not recalculated. His modification dominated Goyal's results in three measures, the number of optimal solutions, average error and maximum error.

Kaspi and Rosenblatt (1991) is proposed very effective and well known algorithm in the literature called as RAND. It calculates a lower and an upper bound for replenishment interval. These bounds are divided into m equally spaced values. Iteratively, these values are used to apply Silver's improved heuristic. RAND approach promises successful outcomes over all previous methods. In all three measures, number of optimal solutions, average error and maximum error, the RAND is superior to the others and is almost as good as the optimal solution obtained through enumeration. (Kaspi and Rosenblatt, 1991)

Goyal and Deshmukh (1993) introduced a tighter lower bound and this reduces the range of basic cycle time. The results of testing the modified RAND on 48,000 randomly generated problems shows that it identified the optimal solution for 39,239 of the problems compared to 37,903 of the RAND without the modification for m = 10.

Van Eijs (1993) argued that for problems with low values of the major ordering cost relative to the minor ordering cost, the strict cyclic policies may result in higher total cost than the cyclic policies. Van Eijs (1993) proposed an algorithm which like Goyal (1974) algorithm requires computing an upper bound on the total cost which is updated at each iterative step of the algorithm. (Khouja and Goyal, 2008)

Horst and Paradalos (1995) suggested applying Lipschitz optimization with a dynamic constant to improve the feasible solution. They tested 2400 randomly generated problems for distribution similar Ben-Daya and Hariga (1995) problems and found that the running time of the algorithm increases linearly in the number of products. The running time decreases in the major set up cost and the required precision.

Viswanathan (1996) suggested an algorithm which iteratively improves the bounds of basic cycle time. Since improving the bounds on basic cycle time requires iterative computations of integer multiple of cycle time for each value, the iterative process is carried out only as long as the improvement in the bounds is some predetermined value. The performance of the algorithm relative to Goyal's algorithm improves as the problem size increases. Goyal's algorithm is faster when the major ordering cost is small. Compared to Van Eijs's algorithm, the proposed algorithm required significantly less cost enumerations. These enumerations were almost 10-folds less when the major ordering cost is small whereas these improvements were very small when the major ordering cost is large. (Khouja and Goyal, 2008)

The study of tighter bounds on basic cycle time is proposed by Fung and Ma (2001) when the major ordering cost is small. The modified bounds were evaluated on 4200 test problems, not compared total cost values with other algorithms.

Viswanathan (2002) indicated that bounds proposed by Fung and Ma do not guarantee an optimal solution and corrected the bounds. However, the experiment showed that Viswanathan (1996) algorithm is faster than corrected algorithm of Fung and Ma's.

A development is presented by Porras and Dekker (2006) and compared their results with best results reported in Viswanathan (2002). As the ratio of major ordering cost to minor ordering cost decreases, the advantage shifts in favour of the modified algorithm.

Nilsson et al (2007) proposed a recursion procedure as spreadsheet technique for the joint replenishment problem. In this study, deterministic model is presented and tested the results according to an extensive template. Their testing results perform well and outperform older models. Although their heuristic gives a higher average and maximum error it produces a lot more optimal solutions. In our paper, we illustrate that spreadsheet method can also yield substantial savings for stochastic models.

Evolutionary algorithm is also applied to solve joint replenishment problem. Khouja et al. (2000) implemented genetic algorithm and compared results with RAND method. Each chromosome represents the integer multipliers of basic cycle time and one point crossover performed. Authors conducted several experiments with different parameters and selected best values. In order to test the performance, same distribution used for two algorithms. For 1200 randomly generated problems, genetic algorithm reached same solution as RAND for 63% problems, under-performed 35%, and outperformed 1%. Olsen (2005) applied to joint replenishment problem using direct grouping strategy. The proposed algorithm outperformed RAND method in only small subset of problems.

In recent years, researchers have paid attention to the stochastic models. Stochastic models consider volatility in demand. In that environments, the coordination and control is more difficult and obviously these systems are more costly. There are two main policies for stochastic models: Periodic replenishment policy and can-order policy.

In can-order policy, each product has three variable, must order level s_i, can order level c_i, up to order level S_i. Any item's inventory drops its s_i level, it should be replenished to bring it to up-to level S_i with the items whose inventory level below c_i. Thus, there may be substantial cost saving opportunities while products are jointly replenished.

The can-order policy was first introduced by Balintfy (1964). Assuming no lead time and identical items he calculates a can-order policy. This policy is continuous review joint ordering and suitable for computer controlled systems. Any item's inventory level drops under its must order level, an order is triggered. Simultaneously, other items' inventory positions are also checked. The items whose inventory positions are below can order levels can also be replenished with the same order. The policy seems simple, but it is difficult to derive cost expressions analytically.

The (Q,S) policy is one of the proposed algorithms for continuous review control policies. It is first suggested by Renberg and Planche (1967) for Poisson demand process for products. Under the (Q,S) policy, aggregate inventory level of all products is monitored and when it reaches a certain level, Q, all products are replenished to the base levels. The (Q,S) policy performs well in problems for which the major ordering cost is high or when products have similar demand and cost parameters. (Viswanathan, 1997)

Silver (1974) relaxes these restrictive assumptions and introduces the principle of decomposition: from time to time an item *i* is faced with an opportunity of discount replenishment, namely when another item reaches its must-order level and places an order. Assuming this process of discount opportunities is independent of item *i*, the multi-item inventory problem can be decomposed into several single-item inventory problems, each with occasional opportunities for discount replenishments, and solved by successive iterations. For item *i* the discount opportunity process is generated by the order placements of all items but item *i*. (Melchiors, 2002)

Federgruen et al. (1984) suggested a can-order policy with semi Markov decision model and use decomposition approach. The algorithm is based on compound Poisson demand and proposes a heuristic method using a policy iteration algorithm to find the control parameters. The approach simplifies the analysis since the problem becomes n independent single item process.

Moreover, Zheng (1994), in a theoretical paper, proved that if the discount opportunity process is Poisson then the can-order policy is optimal. After the single-item problems for each item have been solved, the rate at which discount opportunities are generated is calculated and used in the next iteration. The procedure stops when the optimal policies are unchanged. (Melchiors, 2002)

Cheung and Lee (2002) study is also on the (Q,S) policy, but in a setting with single warehouse and multiple retailers. The policy works similarly with in an inventory system with a single retailer multiple items. In this multi retailer case, an order is triggered when a total of Q units are demanded in all retailers. After an order is triggered, inventory positions of the retailers are all raised up to their maximum levels S. Cheung and Lee (2002) analyze the model exactly in a setting where the warehouse uses the (Q,R) policy for its inventory control. They also propose a new model applying the same policy in which the stocking positions of the retailers can be rebalanced while unloading the items and find a lower and an upper bound for this model. (Tanrıkulu, 2006)

On the other hand, Evans (1967) modelled periodic review policy and inventory systems with multiple products, random demands and a finite planning horizon. He developed the form of the optimal policy for multi-product control for such a system. More recent studies mostly are concentrated on periodic-review and single-product systems with production-capacity constraints. For example, Florian and Klein (1971) and De Kok et al. (1984) characterized the structure of the optimal solution to a multi-period, single-item production model with a capacity constraint. (Choi et al, 2005)

Atkins and Iyogun (1988) proposed periodic replenishment models for Poisson demand. In this policy, (T, M_i), at the same review interval point T, every products should be replenished to bring its M_i level. After that policy, they suggested a modified method

MP (T, M_i). Products belong to base set replenished to bring its M_i level. Others are replenished to bring its M_i level in every k_iT interval. Atkins and Iyogun (1988) tested the performance of can-order policy, MP (T, M_i) policy and (Q,S) policy. Numerical findings indicate that (Q,S) and MP policies have comparable performance. The (Q,S) policy outperforms the can-order policy for problems with high ordering cost, small number of products, and low shortage costs. The can-order policy outperforms the (Q,S) policy for problems with small ordering costs. (Khouja and Goyal, 2008)

Eynan and Kropp (1998) proposed a periodic review heuristic for multi item model for volatile demand environment. The input of the model is normally distributed demand values and corresponding residuals. Throughout the study, demand is stationary and forecast errors are normally distributed. Safety stock is calculated in the model in order to consider forecast errors. The holding cost of the safety stock of products is added to the total cost of classical joint replenishment problem cost function. The authors use the heuristic suggested by Silver (1967) and its improved version by Kaspi and Rosenblatt (1983). In their heuristic, the product with the smallest independent cycle time needs to replenished most frequently. Therefore, their method may easily be referred as RAND method in volatile demand. Their results perform well compared with optimal solutions of problems.

Qu et al. (1999) suggested a method modified periodic review inventory policy in an integrated inventory transportation system with travelling-salesman problem approach. In their problem, the central supplier serves geographically dispersed retailers. The objective of the problem is optimal route and inventory plan to meet every retailer's demand on time. In the study, there is a fixed cost for each stopover of truck in addition to the classical joint replenishment problem. The authors decomposed the problem into classical joint replenishment problem and travelling salesman problem in order to solve this combined model.

Melchiors (2002) provided an improvement to the can-order policy using a compensation approach. The author observed that when the major ordering cost is high, the must-order inventory levels of the can-order policy, s_i, are low (Federgruen et al., 1984). This is because the product which is being considered for ordering has to absorb 100% of the major ordering cost. However, other products, especially those with low inventory levels, will benefit from the order. Therefore, when the problem is decomposed into n problems, a product i placing the order must be compensated by the expected value of the benefit provided to other products by this discounted ordering opportunity. Using variations on the 12 products problem introduced by Atkins and Iyogun (1988), Melchiors found that the compensation approach outperforms other methods for computing the can-order policy on problems where the can-order policy outperforms the periodic review policy. Johansen and Melchiors (2003) extended the compensation approach of Melchiors (2002) by approximating the discount opportunities by a Bernoulli process with outcome 1 if a discount order opportunity occurs and 0 otherwise. The authors used three 12 products examples with the first one based on the example introduced by Atkins and Iyogun (1988). Demand for each product follows a Bernoulli process with positive demand probability proportional to the standard example. The performance of the extended compensation can-order policy was compared to the P (m, M) and the can-order policy of Federgruen et al. (1984). The new policy has a cost advantages that can reach 15% for problems with high demand variability. (Khouja and Goyal, 2008)

In the studies of stochastic multi echelon systems, Gürbüz at al. (2004) focus on the supplier which gives service multiple identical retailers. Replenishment system is triggered when the total demand Q is observed or a retailer’s inventory level drops to its minimum stock level. Thus, retailer’s inventory positions are raised their order up to level via joint replenishment process. The author tested its heuristic results with (Q,S) policy, periodic review order up to policy and can order policy. The numerical results indicated that proposed policy outperforms other policies.

Lee and Chew (2005) developed a dynamic joint replenishment algorithm for products with stochastic demand. Product demands are independent and auto-correlated. Past demand information can be used to obtain better estimates of future demand. The minimum time unit is assumed to be 1. The basic period is an integer multiple of the minimum time unit and this basic period can change depending on demand estimates.

Since the conditional mean (conditioned on past observations) of the demand is dynamic, a more effective inventory policy should be able to take account of the dynamic conditions. At each review point, a new conditional mean and variance of demands for all the products are calculated, then Atkins and Iyogun (1988) method is used to compute T, M_i, and k_i, the marginal cost saving from delaying the order of each product (MC_i), based on the savings in the expected holding cost vs. the additional minor ordering cost and expected shortage cost, is computed, and if $MC_i \geq 0$ then ordering product i is delayed, else product i is replenish to level M_i. The proposed algorithm was compared to the static MP (T, M_i) policy in a 12-product simulation. The proposed method provides substantial savings as demand variability and the correlation coefficients increase. (Khouja and Goyal, 2008)

In Minner and Silver (2005) study, a multi product inventory replenishment problem with Poisson demand is analyzed. The aggregate inventory level is restricted by a common budget or space limitation. The authors use semi-Markov decision problem formulation and several heuristic for finding the replenishment quantities. The study is developed under zero replenishment lead time assumption and continuous inventory review policy. In their model, when a product's inventory level drops to zero, its inventory is replenished subject to the budget or space constraint. The main contribution of the study is the development of a dynamic heuristic to determine replenishment quantities given the inventory levels of all other products. Their heuristic provides good performance, especially for larger problems which makes it very promising in applications of practical size.

Choi et al. (2005) addressed the problem that stochastic inventory models for multiple items with both equal and unequal replenishment intervals under limited warehouse capacity. They proposed three efficient and intuitive heuristics and these heuristic provide optimum replenishment quantities in case of equal intervals. The numerical comparison of the heuristic solutions to the optimal solutions shows that the heuristics yield high quality solutions.

Özkaya et al. (2006) proposed (Q, S, T) policy in a single location, N-items setting. This policy functions as follows: a new replenishment is triggered and inventory positions of all of the items are increased up-to their order-up-to points, whenever a total of Q units are demanded or when T time units elapse. In this study, it is shown that the (Q, S, T) policy outperforms the other joint replenishment policies in most of the problem instances considered. The policy achieved a 1.14% average improvement over the next best policy. The new joint replenishment policy is studied and its performance is compared against other policies in a two-echelon setting in Özkaya et al. (2006). (Tanrıkulu, 2006)

The most recent study on stochastic joint replenishment problem in literature is by Wang et al. (2012). In their study, integrated joint replenishment and delivery model under stochastic demand is proposed. They offer an effective and efficient hybrid differential evolution algorithm based on the differential evolution algorithm and genetic algorithm to solve the NP hard problem. The effectiveness and efficiency of the hybrid differential evolution algorithm are verified by benchmark functions and numerical examples. They compare results with another popular evolutionary algorithm, results of numerical examples also indicate that hybrid differential evolution algorithm is faster and the convergence rate is higher.

The main contribution of this thesis to the existing literature is a method that is effective and easy to use for joint replenishment problem for volatile demand environments. In the literature, spreadsheet method is practical and gives substantial solutions. The availability of such a simple procedure encourages the concentration of more complicated inventory problems which requires considering variation in demand. In this paper, we focus on the joint replenishment problem which considers volatility in demand. We propose the modification of spreadsheet algorithm for volatile demand environments and believe that its modification is reasonable in terms of application. Since, considering demand volatility is more appropriate to analyse the business data, implementation of spreadsheet algorithm provides novel and near optimal solutions.

3. INVENTORY MANAGEMENT

Inventory management is an integrated approach to the planning and control of the inventory, throughout the entire network of cooperating organizations from the source of supply to the end user. It is focused on the end-customer demand and aims at improving customer service, increasing product variety, and lowering costs. An inventory policy can possess local or global objectives. In the former case, the inventory policy results from a collection of local policies in which every supply chain actor tends to make decisions on its own inventory solely based on local performance criteria. On the contrary, under a global policy inventory decisions tend to optimize global performance criteria. However, by using effective incentive systems (such as accounting methods, transfer pricing schemes, quantity discount, etc.) every actor's objective can be aligned to that of the supply chain as a whole. Hence, also a collection of local policies can be considered as part of supply chain inventory management approaches. (Giannoccaro et al., 2003)

Inventory management is the key factor to satisfy customer demand as well as to control stock positions for manufacturers. In the past, manufacturers were keen on producing large number of products at the beginning of the selling season. Since adapting demand changes rapidly and managing cash flow is more important, lean manufacturing is well considered. Therefore, replenishment is being preferred on an ongoing basis by manufacturers.

Consumers are demanding greater variety in products, and their preferences are getting harder to predict. As product proliferate and become more susceptible to changing whims, the risk grows that a given product line will have disappointing sales and have to be discounted. But if the manufacturer decides to go lean on inventories, it runs the risk of stock outs, lost sales, and endangered relationship with the chains. (Abernathy et al, 2000)

In terms of retailer side, inventory management involves maintaining a proper combination of ordering, shipping, forecasting and holding cost minimizing. Accurate forecasting methods, identifying inventory requirements, set targets, reporting analysis and monitoring of products moves is fundamental topics of inventory management. The objective is determining the trade-off between the need for product availability against the minimizing the holding cost.

Inventory management plays a critical factor in the success of increased revenue, net profits and customer service. Inventory management requires constant and careful evaluation of external and internal factors and control through planning and review. In order to control the inventory, the major requirement is providing efficient replenishment techniques such as jointly replenishment of products. Searching for efficient replenishment techniques is common and mandatory topic for the organizations.

One of the most important aspects affecting the performance of a supply chain is the management of inventories, since the decisions taken in this respect have a significant impact on material flow time, throughput and availability of products. Particularly interesting is the problem of coordination in the replenishment of multiple products when they share common resources (e.g. same mode of transportation), with the idea of saving fixed costs. (Musalem and Dekker, 2005)

The powerful and precise forecasting methods, analytics functionality and efficient replenishment capabilities help retailers better understand demand environments, satisfy customer order immediately and more accurately manage their inventories throughout the supply chain. In order to increase competitive edge and satisfy the customer demands, an enterprise must adopt an effective multi item inventory strategy for managing its inventory. A popular management strategy for the multi item inventory system is joint replenishment.

3.1. INVENTORY REPLENISHMENT

The goal for establishing an inventory replenishment policy includes maintaining stock on shelves (also known as no stock-outs), avoiding over supply of inventory and minimizing the cost of replenishment. Some of the motivation factors in maintaining inventory is the uncertainty of demand and the possibility of a shortage of supply. The motivating factors in keeping no inventory include the cost of involved in piled up inventory and loss in income from other investments. (Nahmis, 1993)

Inventory replenishment has three basic cost components, holding cost, ordering cost and shortage cost. Holding cost is also known as carrying cost and proportional to the amount of inventory. Various components make up the holding cost of capital tied up in inventory, taxes, insurance, storage space, personnel to handle inventory, damage to inventory, obsolescence and opportunity cost of investment. The lost opportunity to invest the money tied up in inventory makes the major portion of the holding cost. In order to increase the profitability of the business, there are some strategies that can help minimize holding costs in general, and thus increase the net income earned by the company within any given period. One of the most effective ways is to maintain as low an inventory as possible. Carrying a smaller inventory means less expensive warehouse, efficient space utilization and not keeping high inventory.

Ordering cost is the cost of placing an order to the supplier for a number of different products. It consists of major and minor ordering cost. Major ordering cost is a fixed cost which is charged every time one or more items from the family are ordered. This cost is fixed and independent of number and variety of products. Preparing the order, bookkeeping cost and cost of transportation mean can be referred as major ordering cost. There is also an ordering cost of each item in the order. It depends on the item's volume, weight, length and other special handling cost incurred by an item.

Shortage cost is another component of the cost function. When there is not enough inventory to meet customer demand, item is backordered or sale is lost. If the item is backordered, there are additional bookkeeping costs. If the sale is lost, the cost of loss profit and the loss of goodwill occur. Loss sales can damage the brand value and change the customer's brand perception.

Inventory replenishment with accurate demand forecasting is the best opportunity for retail businesses to be demand driven, focusing on customer needs effectively and delivering the products on time. The inventory replenishment has three key points:

1. When should be the reorder
2. How much should be the reordering quantity
3. Creating a purchase order

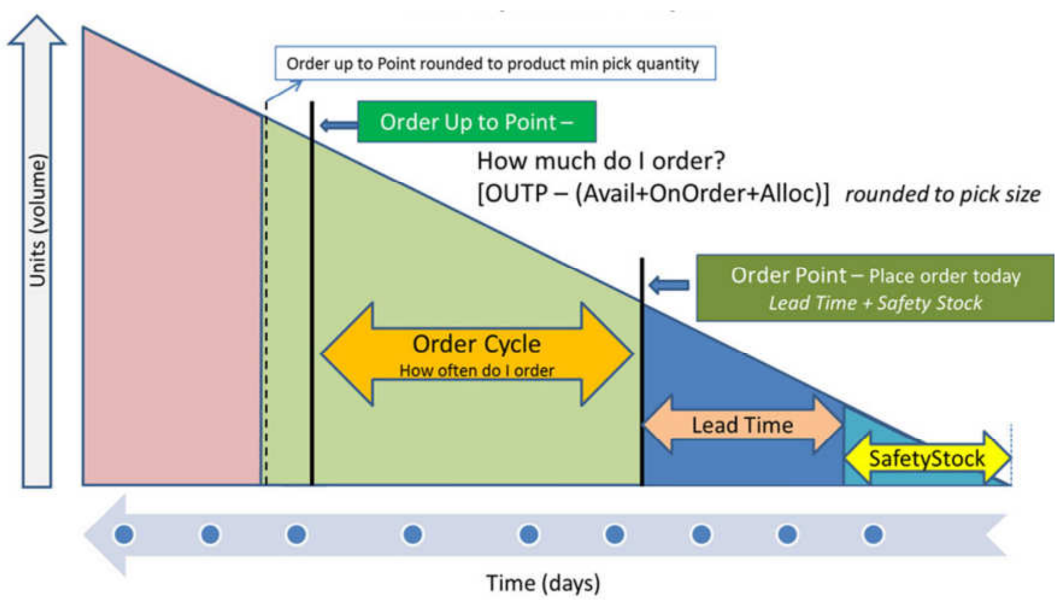

Figure 3. 1 The depletion graph (Data-profits, 2013)

The reorder frequency means the size or amount of an inventory replenishment order. The most profitable businesses know that what product the customer wants, how much they will pay for it, and when they want it. Profitable retailers need only get those products into customer hands with the least amount of expense. While determining reorder frequency, it must be considered the total number of days from placed the replenishment purchase order until the goods are available. Lead time factors include vendor build, preparation and transportation days.

The second question is determining reorder quantity. Retailers need a solution that analyzing past and complex demand forecast algorithms and optimal reorder quantity. Since holding cost is major components of total cost function, businesses are not willing to keep high stock levels. Therefore, intellectual replenishment policies are required in order to manage inventory more effectively.

Creating purchase orders is fundamental in inventory replenishment process. An efficient policy help make the right decision that ensure to get the right inventory replenishment orders placed for the company. Choosing more appropriate supplier, price and quality of purchasing items, special deals and discounts are important factor while creating orders.

These important questions determine organisations' profitability. Retailers and manufacturers who can identify reorder timing and reorder quantity appropriately could achieve success in today's supply chain systems. Therefore, inventory replenishment is very important for inventory optimization and getting optimum profitability.

An inventory replenishment policy should avoid both the problem of oversupply of inventory and the problem of stock outs. The best policy is usually one that takes into account the factors which influence replenishment such as ordering cost and holding cost. The easiest and most fundamental method of inventory replenishment is Economic Ordering Quantity (EOQ) model (Nahmias, 1993). The EOQ model takes into account the trade-off between the cost of ordering and the holding cost. It also forms the basis of more complex models such as joint replenishment. The goal of the EOQ model is derive the optimal number of units of an item to order each time an order is placed. (Olsen, 2002)

In inventory planning, determining safety stock level is essential in order to maintain favourable service level. As the demand variability changes, optimum inventory levels required to guarantee desirable service level. The ability to maintain service level high depends on safety stock level in a supply chain system. The relationship between two parameters is exponential and can be seen in Figure 3.2. In order to guarantee 100% service level in terms of availability, the infinite amount of safety stock is required.

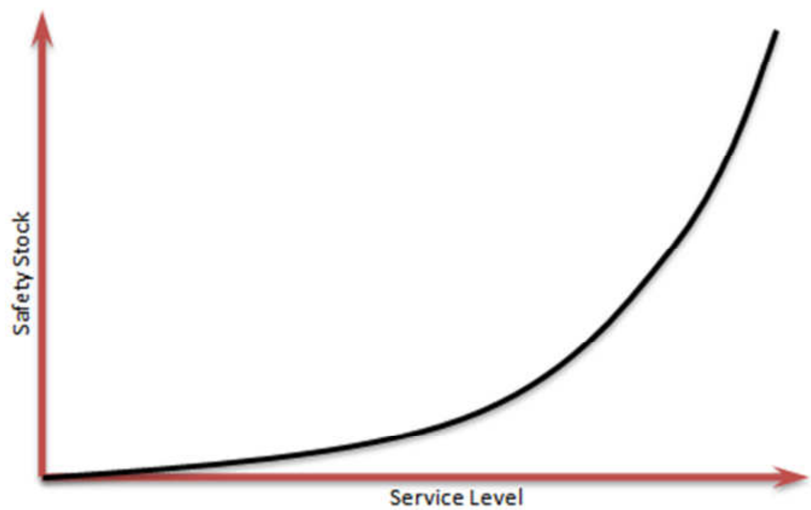

Figure 3. 2 The relationship between safety stock and service level

The trade-off between maintaining high safety stock and obtaining favourable service level is very important in inventory planning. In that way, discriminating products that require high service level and monitoring demand pattern regularly to set new safety stock should be considered within the inventory optimization process.

In order to maintain a proper replenishment policy, historical demand data, supply information and lead time length should be analyzed correctly. Desired service level is another input of the process. Service level depends on the item, its demand, profitability and associative relationship to the other items. An efficient replenishment policy helps determine optimum replenishment interval time, reorder level and order quantity for each item in the process.

Figure 3. 3 Inventory processes (C-coregroup, 2013)

Although inventory replenishment seems directly related to the execution orders (creating orders, transmitting time frame, confirm quantity), it is actually one of the central element of the whole supply chain process. In order to manage inventory, maintain proper product availability and deliver customer needs to the customer, efficient replenishment policy is the key strategy to optimize the system. Therefore, it is a common phenomenon to provide effective replenishment strategy for inventory planning process at most optimal levels.

3.2. NOTATION AND ASSUMPTIONS

The following notation is defined:

i 1,2,3...,n, a product index

n number of items ordered from a single supplier

D_i average demand for product i (TL /units/week)

σ standard deviation of demand forecast errors during one unit of time

z multiplier of σ (determines the service level)

h_i the holding cost for one unit of product i (TL/unit/week)

s_i the minor ordering cost of product i incurred when product i is included in a group replenishment (TL/order)

S the major ordering cost associated with a replenishment involving one or more products (TL/order)

Q_i order quantity for product i, a decision variable (units)

T replenishment interval or basic cycle time

T_i the cycle time between placing consecutive orders of item i in weeks

k_i the integer number of T intervals that the replenishment quantity of item i will last (decision variable)

m integer number decided by decision maker.

C_o total ordering cost per week

C_c total carrying cost per week

TC total cost per week

In order to determine a joint replenishment policy, a family of item is purchased from single supplier. Similar to the general joint replenishment problem, the following assumptions are made:

1. There is a fixed cost, S, associated with each order independent of the number of items ordered.
2. There is minor ordering cost, s_i, incurred if item i is included but is independent of the other items included in the order.
3. Shortages are not allowed.

4. There is an infinite horizon time.
5. There are no quantity discounts.
6. The entire order quantity is delivered at the same time. (no lead time)
7. There are no budget constraints on the amount of an order.

3.3. REPLENISHMENT POLICIES

The inventory planning process establishes the optimal inventory levels that must be maintained to meet expected service levels for demand fulfilment. Replenishment process needs to define review period for reordering and reordering quantity. Then, it provides with determining whether an order should be placed for replenishment time point or not.

In the replenishment process, continuous review and periodic review strategies refer to the frequency of monitoring to determine when orders must be placed. Under continuous review process, the inventory level is continuously reviewed and when the stock drops under predetermined safety level, the new order is placed up to replenishment quantity. In the real world, companies start to track their inventory levels and continuous review strategy becomes a common and optimal way to plan for replenishment.

In the periodic review strategy, inventory levels reviewed at specific time frequencies. At this review time point, if the inventory level is under safety stock, the order is placed, otherwise ignored until the next review point. This method is easier than continuous review process and well manageable for manual processes which includes large amount of items.

In supply chain systems, there are two ways of placing an order. Order quantity in a replenishment process can be fixed or determined according to the order up to level. In the first process, in every replenishment pre-defined fix order quantity is placed. Since the order quantity for all replenishment orders is fixed in this method, order day may vary or may be fixed depending on the review method. In the second process, order quantity is determined according to the difference between on hand stock and pre-determined order up to level. The order quantity in this process will differ from one order to another depending on the on-hand quantity on the day of the review.

There are four reordering process options are available in the replenishment policies:

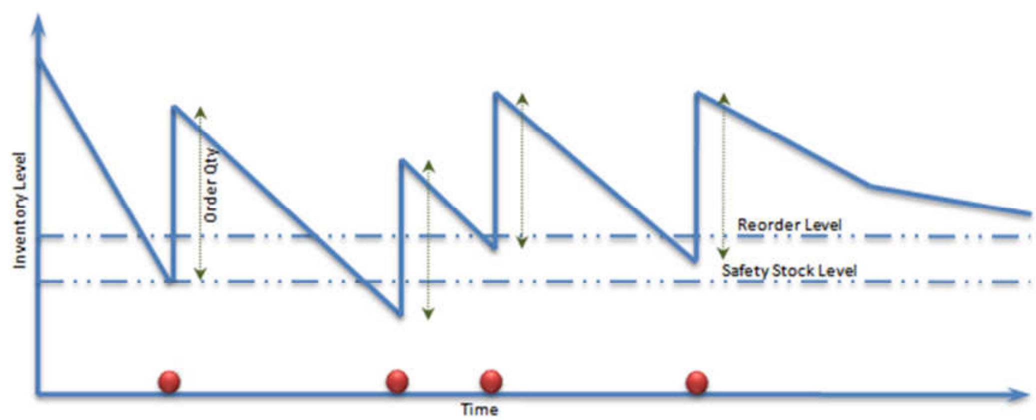

Figure 3. 4 Order point in continuous review system (Supplychainmusings, 2013)

In Figure 3.4, there is a system uses fix order quantity in continuous review policy. The orders are placed when the inventory drops under the pre-determined reorder level. Since the order quantity is fixed, the resulting inventory levels can change according to the starting inventory when an order is created. The advantageous of this system is that the new order is triggered as soon as inventory drops under the reorder level, the stock level rarely drops under the safety stock level.

In Figure 3.5, the system uses fix order quantity in periodic review policy. The bold dots indicate order points, while light dots represent the review point that the inventory point is higher than reorder point and no need to be placed an order. Stable ordering cycle is the benefit of the system, whereas its drawback is that inventory may sometimes fall below the safety stock level.

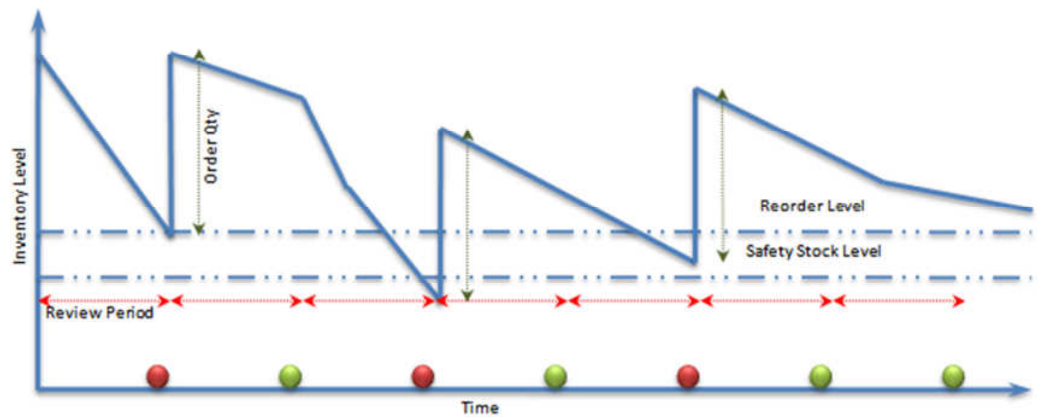

Figure 3. 5 Order point in periodic review system (Supplychainmusings, 2013)

Figure 3.6 shows the system uses order up-to level with continuous review policy. The system is monitored continuously and whenever the inventory drops under reorder level, the new order is placed as soon as possible. Since the system target is reaching pre-defined order up-to level, order quantity varies according to the on hand stock at the review point.

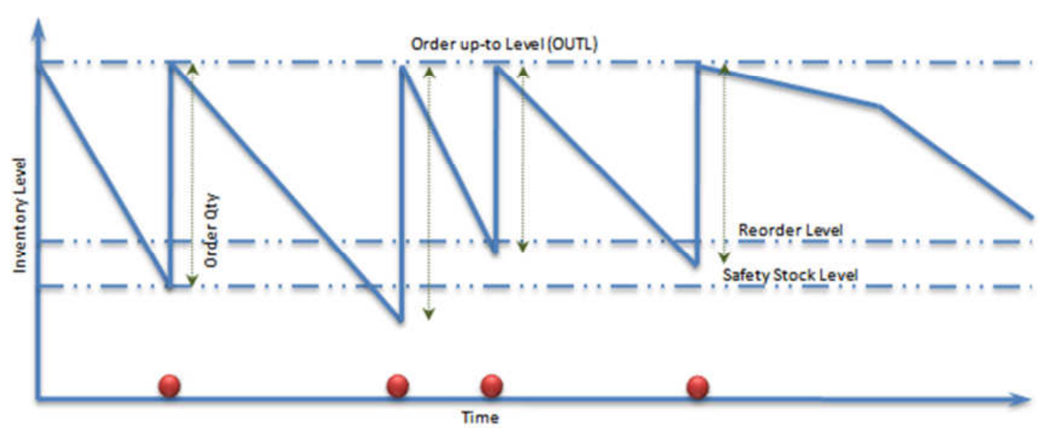

Figure 3. 6 Order up-to level in continuous review system (Supplychainmusings, 2013)

Figure 3.7 represents the system that is order up-to level with periodic review policy. Order quantity changes in every replenishment time points. Order placement is not required in every review point that the inventory level is higher than reorder level.

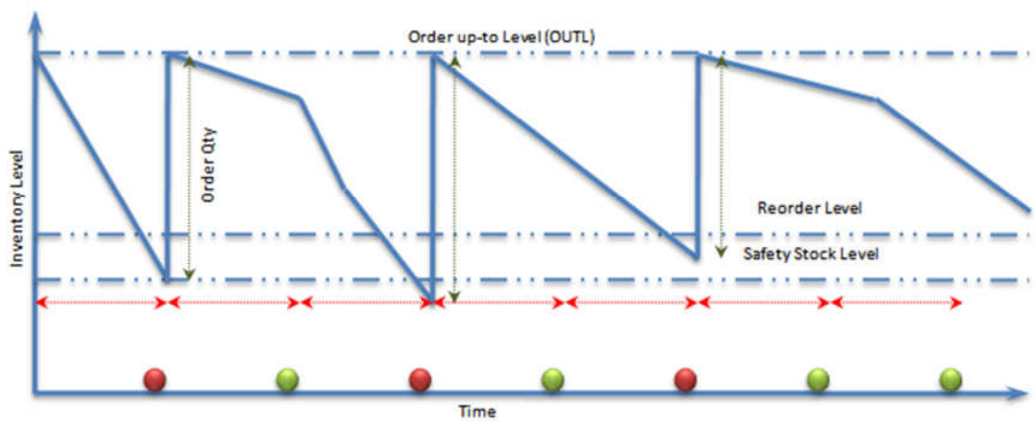

Figure 3. 7 Order up-to level in periodic review system (Supplychainmusings, 2013)

In terms of variety in the order, there is two ways of replenishment polices as independent and joint replenishment. In the independent replenishment policy, every item is replenished individually, whereas in the joint replenishment a family of items replenished mutually at predetermined time points. In this section, independent replenishment and joint replenishment policies are presented with related formulas.

3.3.1. INDEPENDENT REPLENISHMENT

Under independent replenishment, every item is replenished individually. The major ordering cost S is charged in every replenishment which turns with high replenishment cost. The search is to minimize the holding inventory and ordering cost. In deterministic case, average demand has been considered for each product and compute for optimal replenishment cycle for each product.

The ordering cost of n items is as follows:

$$\mathrm{Co} = \sum_{i=1}^{n} (\mathrm{S} + \mathrm{s_i})/\mathrm{T_i} \tag{3.1}$$

The holding cost of n items is as follows:

$$\mathrm{Cc} = \left(\frac{1}{2}\right) \sum_{i=1}^{n} \mathrm{T_i}\, \mathrm{D_i} \mathrm{h_i} \tag{3.2}$$

The total cost of the *i*th item is:

$$TC_i = (\mathrm{S} + \mathrm{s_i})/\mathrm{T_i} \; + \left(\frac{1}{2}\right) \mathrm{D_i} \mathrm{h_i} \mathrm{T_i} \tag{3.3}$$

The total cost of n items, TC is given by:

$$\mathrm{TC} = \mathrm{C_o} + \mathrm{C_c} = \sum_{i=1}^{n} (\mathrm{S} + \mathrm{s_i})/\mathrm{T_i} \; + \left(\frac{1}{2}\right) \sum_{i=1}^{n} \mathrm{T_i}\, \mathrm{D_i} \mathrm{h_i} \tag{3.4}$$

In order to minimize the total cost, optimal order quantity of each item should be determined. The summation of the n optimal costs generates minimal total cost under independent replenishment. The quantity ordered is given by:

$$Q_i = \mathrm{D_i T_i} \tag{3.5}$$

Substituting from (3.5) into the cost formula (3.3), the new equation is:

$$TC_i = \left(\frac{D_i}{Q_i}\right)(S + s_i) + \frac{1}{2}(Q_i h_i) \tag{3.6}$$

To calculate optimum Q_i, we take the derivative with respect to Q_i, the quantity which minimize the total cost is given by:

$$Q_i^* = \sqrt{2(S + s_i)D_i/h_i} \tag{3.7}$$

Since the cycle time T_i is:

$$T_i = \mathrm{Q_i/D_i} \tag{3.8}$$

The optimum cycle time is as follows:

$$T_i^* = \sqrt{2(S + s_i)/(h_i D_i)} \tag{3.9}$$

Substituting (3.9) in (3.3), the minimum total cost for *i*th item is given:

$$TC_i^* = \sqrt{2(S + s_i)(h_i D_i)} \tag{3.10}$$

Thus, the optimum total cost for n items is as follows:

$$TC^* = \sum_i^n \sqrt{2(S + s_i)(h_i D_i)} \tag{3.11}$$

Under independent replenishment strategy, demand fulfilment ratio is high since the products are replenished stand alone. However, it gives high total cost. It is reasonable to think that coordinating orders which include more than one item give substantial cost savings.

3.3.2. JOINT REPLENISHMENT PROBLEM

The JRP encompasses a family of items where there is a major fixed cost for any family replenishment and a minor fixed cost (item-dependent) for each distinct item included in the replenishment. Under the assumption of known level of demand for each item the problem is to select the frequency of family replenishments as well as which items are to be in which family replenishments. As will be seen, it is not straightforward to find the solution that minimizes the total relevant costs. (Nilsson and Silver, 2007)

In the joint replenishment policy, the family of products has a major ordering cost and this cost is independent of the quantity of order. The major ordering cost is fixed and charged at every order for the replenishment group. Thus, fixed replenishment cost is split up by each product in the family in the joint replenishment. It enables to get lower cost than independent replenishment in terms of ordering charges. Furthermore, items are more coordinated due to convenient communication and scheduling in the joint replenishment.

The joint replenishment problem is usually based on a buyer-only viewpoint with concerning multiple products where economies exist for replenishing products collectively. The problem involves determining a basic replenishment cycle time T and the replenishment interval k_iT for item i, where k_i is an integral number. The objective function of the joint replenishment problem is not convex and typically has several local minima. Optimal algorithms enumerate all the local minimum solutions between a lower bound and an upper bound for T. (Hsu, 2009)

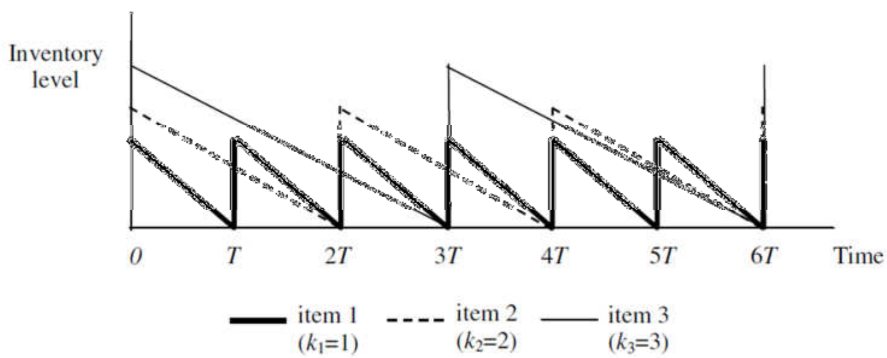

Figure 3. 8 Replenishment cycle for each item (Cha and Moon, 2005)

A joint replenishment is made every T time intervals. However, all items may not be included in each replenishment cycle. Item i is only included every k_iT time intervals. This means that the replenishment of each item is made at every integer multiple (k_i) of the group replenishment time interval (T) as shown in Figure 3.8. This also indicates that k_iT is the cycle time of item i. (Cha and Moon, 2005)

The approach for joint replenishment problem can be classified according to the grouping of item in a replenishment interval. There are two grouping strategies in joint replenishment policy as direct grouping strategy and indirect grouping strategy.

3.3.2.1.DIRECT GROUPING STRATEGY

Under direct grouping strategy DGS, products are divided into a predetermined number of sets and the products within each set are jointly replenished with their set's own cycle time. In direct grouping strategy also known as fixed cycle policy, the n items divided into m groups and each group has own cycle time. The objective is the strategy is to find an optimal grouping for the items and the optimum cycle time for each group in order to total cost function. The notations as follows:

m the number of groups

j the group number

G_j the *j*th group

T_j the cycle time between replenishing items in group *j*.

In the direct grouping strategy total cost function is given by the equation:

$$TC = \sum_{j=1}^{m} \left((S + \sum_{i \in G_j} s_i)/\, T_j + \frac{1}{2} T_j \sum_{i \in G_j} D_i\, h_i \right) \qquad (3.12)$$

By taking the derivative of TC with respect to T_j, the optimal time T_j* is as following:

$$T_j^* = \sqrt{2(S + \sum_{i \in G_j} s_i)/(\sum_{i \in G_j} D_i\, h_i)} \qquad (3.13)$$

After getting T_j* we can find the optimum TC value by substituting in Eq. (3.12)

Rosenblatt (1985) compared a direct grouping method with an indirect grouping method to solve a joint replenishment problem in which the minor ordering cost of an item is dependent on other items which are in the same order. Recall that in the classic JRP the minor ordering cost is independent of other items jointly replenished. Rosenblatt ran experiments using 25 different settings with 50 examples from each setting. He reported that indirect grouping performed better than direct grouping for 60% of the examples. (Olsen, 2005)

3.3.2.2. INDIRECT GROUPING STRATEGY

Under indirect grouping strategy (IGS), replenishment is made at regular time intervals and each product has integer multiple of the regular time interval. The items ordered in the same order share the major ordering cost. The indirect grouping strategy is one type of cyclic policy. It uses common cycle time T and each product has own integer multiple value k_i. The indirect grouping determines the basic cycle time T*, additionally computes each item's integer k_i value that represents the integer multiple of T for item i.

In IGS grouping strategy, formulas are as follows:

$$Q_i = Tk_iD_i \tag{3.14}$$

Total ordering cost of n items is as follows:

$$\mathrm{Co} = \left(\frac{1}{T}\right)\left(S + \textstyle\sum_{\mathrm{i}}^{n} \mathrm{s_i}/\mathrm{k_i}\right) \tag{3.15}$$

The holding cost of n items is as follows:

$$\mathrm{Cc} = \left(\frac{1}{2}\right) T \sum_{i=1}^{n} \mathrm{k_i D_i\, h_i} \tag{3.16}$$

The total cost of n items, TC is given by:

$$TC = C_o + C_c = \left(\frac{1}{T}\right)(S + \textstyle\sum_{i=1}^{n} \mathrm{s_i/k_i}) + \left(\frac{1}{2}\right) T \textstyle\sum_{i=1}^{n} \mathrm{k_i D_i\, h_i} \tag{3.17}$$

The aim of the problem is to minimize the cost function. Therefore, we can take the partial derivative of TC with respect to T. (Assuming a particular set of k_i's is fixed.)

$$T^* = \sqrt{2(S + \textstyle\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{s_i/k_i})/(\textstyle\sum_{i=1}^{n} \mathrm{k_i D_i\, h_i}})\tag{3.18}$$

Substitution of T* into TC formula, gives the minimum total cost:

$$TC^* = \sqrt{2(S + \textstyle\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{s_i/k_i})(\textstyle\sum_{i=1}^{n} \mathrm{k_i D_i\, h_i}})\tag{3.19}$$

Van Eijs et al. (1992) compared the direct and indirect grouping strategies. The results show that IGS outperforms DGS for high major ordering cost because many products can be jointly replenished when using an IGS. Authors also emphasized that performance of the indirect grouping strategy depended on the number of items and the ratio of major ordering cost to the minor ordering cost.

4. PROPOSED MODEL AND SOLUTION PROCEDURES

Due to the nature that the joint replenishment problem is mixed integer nonlinear model, it is NP-hard problem. For large instances of problem, obtaining the optimal solution of problem is prohibitive. In the real world, manufacturers and decision makers require fast and effective way of solution procedures. Enumeration method is difficult in terms of both computational time and optimum programme solvers.

Additionally, when there are certain data sets of data for which the best known algorithm fails to produce optimal results. For joint replenishment problem, Van Eijs et al. (1992) found that the indirect grouping strategy failed to produce optimal results for some data sets where the major ordering cost was small relative to the minor ordering cost. These difficulties arise because there are many factors that need to be considered when modelling the problem and in the real world these factors are difficult to identify. (Olsen, 2002)

When we consider real life manufacturing problem, there are large amount of products to replenish, an effective and rapid solution is needed. It may take long time to calculate the optimal solution by operators. Therefore, there is a need for effective heuristic solution procedures to be discussed in this chapter.

In this chapter, we describe our proposed method "spreadsheet for volatile demand environments" as well as very common method to solve joint replenishment problems, called RAND. We present both algorithms for deterministic and volatile demand environments. In the literature, there is spreadsheet method for deterministic problems, because of its effectiveness and easy to use, we propose spreadsheet method for volatile demand environment problems. In the real world, considering volatility in demand is more appropriate to analyse a data. Therefore, we proposed effective and simple implementation of spreadsheet algorithm for volatile demand environments.

4.1. MODIFIED SPREADSHEET ALGORITHM

For the joint replenishment problem, spreadsheet algorithm is effective and easy to use. The main idea of the spreadsheet heuristic is finding balance between replenishment cost and holding cost. The heuristic is based on Segerstedt's (1999) study, where the method to solve an economic lot scheduling problem (ELSP) with capacity constraint is proposed. The basic assumption is that in an economic order quantity problem, the ratio between replenishment cost and holding cost is equal to one at optimum point. With this logic, Nilsson et al. (2007) proposed a modified version of Segerstedt's (1999) algorithm to be applied to the joint replenishment problem. The closer the ratio is to one, the lower is the cost. Keeping the ratio close to one proved to be a very effective heuristic way to solve joint replenishment problems.

The closer the individual quotients are to one, the better the solution. It is possible to solve JRP by adjusting the quotients to obtain results closer to one. This will be achieved in a two-step heuristic, where the starting solution is where all items are replenished at every time interval (all k-values are set to one). During these steps, simply looking at the quotients and tracking how the total cost changes as the replenishment frequencies (k values) are updated. (Nilsson et al, 2007)

The deterministic case of the problem generates close-to optimum solutions. Based on this finding, our motivation was to explore the heuristic performance in demand volatility and we developed a modified version of the spreadsheet algorithm. First, we will propose the deterministic case of heuristic that developed by Nilsson et al. (2007), after that we present the modified version of the heuristic for the volatile demand environments.

The quotient or the ratio between replenishment cost and holding cost is as follows:

$$q_i = 2s_i / T^2 k_i^2 D_i h_i \tag{4.1}$$

Substitution of T* into the formula, new quotient formula is:

$$q_{i(k)} = (\sum_i^n k_i D_i \, h_i / \, (S + \sum_i^n s_i \, / k_i)) \; s_i / k_i^2 D_i h_i \tag{4.2}$$

The solution procedure of spreadsheet algorithm is the following (Nilsson et al, 2007):

Step 1.Set all k-values to 1 and compute the total cost for the initial solution.

Step 2.Compute quotients Eq. (4.2) and increase the k-value(s) by one for all items with quotients higher than 1.4. Calculate the total cost. Repeat that until all quotients are below 1.4 or the total cost starts to increase. If all quotients are below 1.4 go to step 2 or if the total cost increases, step back one step to the best solution and then go to step 2.

Step 3.Calculate quotients and rank them how far away they are from one. Then individually increase/decrease the k-value of the item with the highest ranking (furthest away from one). Calculate the total cost. Repeat all that until the total cost starts to increase, then step back one step to the best solution and try to adjust the k-value of the item having the second highest ranking. Repeat this step until no more items exist to examine, then go to step 3. Note that all m-values must be >1. If an item has the highest ranking, a quotient below one and m=1, it must be skipped.

Step 4.Final step. Since there are several local optimal solutions from the start solution all the way to the final solution, it is necessary in this heuristic to both change the frequencies together and individually. That is why the heuristic is divided into two steps.

The pseudo code of the algorithm is as follows:

1. Set k_i =1 $\forall i$ Compute total cost $TC_0 = T_k$

2. Compute quotient for each item. If $q_i \geq 1.4$, $k_i^{'} \leftarrow k_i + 1$.
   If $TC_{k'} < TC_0$, then $TC_0 \leftarrow TC_{k'}$. $k_i \leftarrow k_i^{'}$ and repeat step 2.
   Else go to step 3.

3. $l \leftarrow \arg\max(q_i\,, 1/q_i)$ $i \in L$
   If $q_l < 1$ and $k_l = 1$ then $L \leftarrow \frac{L}{\{l\}}$, $L \leftarrow L - 1$ and $l \leftarrow \arg\max(q_i\,, 1/q_i)$ $i \in L$
   $k_i \leftarrow k_i^{'}$, If $q_l > 1$ then $k_l^{'} \leftarrow k_l + 1$ else $k_l^{'} \leftarrow k_l - 1$
   If $TC_{k'} < TC_0$ then $TC_0 \leftarrow TC_{k'}$ and $k \leftarrow k^{'}$ and compute q_i $\forall i$.
   Else $L \leftarrow L/\{l\}$ and $L \leftarrow L - 1$. If $L > 0$ then repeat step 3.

4. Final TC_0 is the best solution.

Application of the spreadsheet heuristic is not only suitable for deterministic models but also for volatile demand models. Because of its simplicity and effectiveness, we modified the heuristic for the volatile demand environments. In the model, there is an average demand and also a standard deviation of demand forecast errors. Based on these inputs, we provide a more proper way of analysing the real world data. Considering only average demand to model the problem, there may be some mistakes or missing interpretation for future decisions.

The inventory demand for items which have the same demand can vary significantly. The manufacturer has to keep a much bigger inventory for big variation product. Although only average demand is taken into account, manufacturers generally ignore the variation and assign the same inventory targets for all SKUs. (Abernathy et al, 2000)

A simple procedure to implement demand volatility modelling for spreadsheet heuristic is indeed appropriate. The simplicity of application of spreadsheet method and its efficiency enables us to consider its modified version for the joint replenishment problem in volatile demand environments.

The total cost function for the joint replenishment problem in volatile demand is presented as follows:

$$TC = \frac{S}{T} + \sum_{i=1}^{n}(s_i\,/k_i)/\,T + \sum_{i=1}^{n}[(D_i\,k_i T h_i/2) + (z_i \sigma_i h_i \sqrt{k_i T}\,)] \tag{4.3}$$

As indicated before, z_i represents service level of item i and σ_i represents standard deviation of demand forecast errors. In the formula, first and second terms are major and minor ordering cost, third term is holding cost and the last term is safety stock.

The aim of the problem is to minimize the cost function. Therefore, we can take the partial derivative of TC with respect to T. (Assuming a particular set of k_i's is fixed.)

$$T^* = \sqrt{2\left(S + \sum_{i=1}^{n}\frac{s_i}{k_i}\right)/\sum_{i=1}^{n} h_i\, k_i (D_i + z_i\sigma_i/\sqrt{k_i T_0})} \tag{4.4}$$

where $T_0 = \sqrt{2\left(S + \sum_{i=1}^{n}\frac{s_i}{k_i}\right)/\sum_{i=1}^{n} h_i\, k_i D_i}$ (4.5)

The solution procedure of modified spreadsheet algorithm for volatile demand problems can be modelled as follows:

1. Set $k_i = 1 \ \forall i$ and compute

$$T_{ini} = \sqrt{2\left(S + \sum_{i=1}^{n} \frac{s_i}{k_i}\right) / \sum_i^n h_i\, k_i (D_i + z_i\sigma_i/\sqrt{k_i T_0})} \tag{4.6}$$

where $T_0 = \sqrt{2\left(S + \sum_{i=1}^{n} \frac{s_i}{k_i}\right) / \sum_{i=1}^{n} h_i\, k_i D_i}$ (4.7)

$$TC_0 = \frac{S}{T_{ini}} + \sum_{i=1}^{n}(s_i/k_i)/\,T_{ini} + \sum_{i=1}^{n}[(D_i\, k_i T_{ini} h_i/2) + (z_i\sigma_i h_i\sqrt{k_i T_{ini}})] \tag{4.8}$$

2. Compute

$$q_i = [\textstyle\sum_i^n k_i(D_i + z_i\sigma_i)h_i/\,(S + \sum_i^n \frac{s_i}{k_i})]\, s_i/\, k_i^2(D_i + z_i\sigma_i)h_i \qquad \forall i \tag{4.9}$$

3. Set $r = 0$

   For items $Q_i > 1.4$, $k_i^{'} \leftarrow k_i + 1$. Compute $T^{'}$ and $TC^{'}$ according to Eq. (4.4) and Eq. (4.5)

   If $TC^{'} < TC_0$, $TC_0 \leftarrow TC^{'}$, $k_i \leftarrow k_i^{'}$.

   Compute new quotient according to Eq. (4.9). Otherwise go to step 4.

4. Set $r = r + 1$

   Find the quotients how far away from one. Sort them in descending order.

   For the furthest quotient, if $q_i < 1$

   If $k_i > 1$, $k_i^{'} \leftarrow k_i - 1$, else look at the second furthest quotient.

   If $q_i > 1$, $k_i^{'} \leftarrow k_i + 1$. Compute $T^{'}$ and $TC^{'}$ according to Eq. (4.4) and Eq. (4.5)

   If $TC^{'} < TC_0$, $TC_0 \leftarrow TC^{'}$, $k_i \leftarrow k_i^{'}$.

   Compute new quotient according to Eq. (4.9). Otherwise go to step 5.

5. In order to guarantee the best solution, try to increase remaining quotients in the order. Stop all items tried. Take minimum value of the total cost.

The appropriate value of the quotient as 1.4, Nilsson et al (2007) tested the performance of the values between 1 and 2 in their study. The largest possible decrease of a quotient, when the *k* value is increased by one will be less than 3/4 of the original value. This will happen when a *k* value is increased from one to two. This means that if a quotient is two or higher an increase in the k value will always gives a lower total cost. Low values are not of interest since too many quotients will be put too low. (Nilsson et al, 2007)

4.2. RAND ALGORITHM

For the joint replenishment problem, RAND method is proposed by Kaspi and Rosenblatt (1991) which is improved heuristic of Silver's. RAND method is based on computing "m" equally spaced values of the fundamental cycle within and lower and upper bound [T_{min}, T_{max}]. Iteratively, these values are applied to Silver's (1976) method to find each product's "k" values. It keeps each interval value's costs and select minimum of them. RAND performed better than all previous algorithms that are not enumerative.

Goyal and Deshmukh (1993) provided a better lower bound for T which further improved the performance of the RAND. A tighter lower bound is helpful because it reduces the range of T from which the "m" equally spaced values are obtained. For the cases where the RAND did not obtain the optimal solution, it was 0.002% from it on average (Khouja et al, 2000).

In the deterministic case of RAND algorithm, there are estimated demands for each product in per calculated cycle (year, month, week). RAND algorithm finds several local optimal solutions from the iterative calculation using different first T values.

Consequently, the best solution is obtained among these local optimum solutions. Therefore, calculating a lower and an upper bound for T is very effective way of obtaining close-optimum solution.

The solution procedure of deterministic RAND algorithm is as follows:

1. Compute

$$T_{\max} = \sqrt{2 * (S + \sum_{i=1}^{n} s_i) / \sum_{i=1}^{n} (D_i h_i)} \tag{4.10}$$

$$T_{\min} = \min(i) \sqrt{2 s_i / D_i h_i} \tag{4.11}$$

2. Divide the range [T_{min}, T_{max}] into “m” equally spaced values of T, (T_1, T_2, ...T_j, T_m). (*m* is to be decided by the decision maker.) Set j=0.

3. Set j=j+1.
   Set q=0.

4. Set q=q+1.
   For T_j, compute “k” values for each product i:

$$k_{i,q}^2 = 2 s_i / (T_j^2 D_i h_i) \tag{4.12}$$

5. Find $k_{i,q}^2$ for each i, where

$$k_{i,q}^* = L \ \text{ If } \ L(L-1) < k_{i,q}^2 \leq L(L+1) \tag{4.13}$$

6. Compute a new cycle time T_j according to

$$T_j = \sqrt{2 * (S + \sum_{i=1}^{n} s_i / k_{i,q}^*) / \sum_{i=1}^{n} D_i h_i k_{i,q}^*} \tag{4.14}$$

7. If q=1 or $k^*_{i,q} \neq k^*_{i,q-1}$ for any i, then go to step 4. At last, compute TC for this (T_j , $k^*_{i,q}$ for all i). If j=m then select (T_j , $k^*_{i,q}$) with the minimum TC. Otherwise, go to step 3.

Note that Silver's modified heuristic (Kaspi and Rosenblatt 1983) is applied in an iterative way until the values of k_i converge. In conducting their simulation experiments, Kaspi and Rosenblatt (1983) found that the major improvement of the algorithm occurs in the first iteration. (Khouja et al., 2000)

On the other hand, RAND algorithm can be also implemented for the volatile demand problems. In this case of the joint replenishment problem, demand of the products is stationary in the mean and there are standard deviations of the forecast errors for each product.

In RAND algorithm for volatile demand problems, the demand is stationary and the forecast errors are normally distributed. This assumption is based on three reasons. First, empirically in many cases the normal distribution provides a better fit to data than most other distributions. Second, particularly if the lead-time is long relative to the "base" forecasting period, forecast errors in many periods are added together, expecting a normal distribution through the Central Limit Theorem is logical. Finally, the normal distribution leads to analytically tractable results (Eynan and Kropp, 1998).

In our study, we are assuming lead time as zero because of delivery time can be negligible when we compare with other time units. Therefore, we generate our solutions according to Eynan and Kropp (1998) algorithm without lead time.

The solution procedure of RAND algorithm for the volatile demand problems is as follows (Eynan and Kropp, 1998):

1. Determine

$$T_i^* = \sqrt{2s_i/h_i\left(D_i + (\frac{z_i\sigma_i}{\sqrt{T_{0i}}})\right)} \tag{4.15}$$

where $T_{0i} = \sqrt{2s_i\,/h_iD_i}$ i=1,....., n. (4.16)

2. Identify the item with the lowest T_i^*. This item will be denoted as item 1 and $k_i = 1$.
3. $T = \sqrt{2(S + s_1)/h_1(D_1 + (z_1\sigma_1/\sqrt{T_0})}$ (4.17)

   where $T_0 = \sqrt{2(S + s_1\,)/h_1D_1}$ (4.18)

4. $k_i = L$ integer such that $\sqrt{L(L-1)} \leq (T_i^*/T) \leq \sqrt{L(L+1)}$ i=2,......, n.
5. $T = \sqrt{2\left(S + \sum_{i=1}^{n}\frac{s_i}{k_i}\right)/\sum_{i=1}^{n} h_i\,k_i(D_i + z_i\sigma_i/\sqrt{k_iT_0})}$ (4.19)

   where $T_0 = \sqrt{2\left(S + \sum_{i=1}^{n}\frac{s_i}{k_i}\right)/\sum_{i=1}^{n} h_i\,k_iD_i}$ (4.20)

6. Repeat steps 4 and 5 as necessary or until the overall total cost value yields bigger than previous iteration’s total cost value.

In the heuristic, safety stock is considered as well as holding cost for demand value. The holding cost of the products, safety stock related forecast errors of these products and replenishment cost form the total cost function for the volatile demand environment. Since RAND is very effective algorithm for the joint replenishment problem under volatile demand, we compare RAND method’s results with proposed spreadsheet algorithm for the real business data and evaluate their effectiveness.

5. NUMERICAL APPLICATION

In this chapter, the real world data was constructed and run in order to find the effectiveness of the proposed algorithm. The given numerical data belongs to a worldwide known consumer electronic company.

High technology industry has several unique characteristics in terms of supply chain management. First and foremost, technology products have short life cycles and high rate of obsolescence. For technology companies, new innovations are developed rapidly and products have short shelf life. In order to increase agility and lower cost, technology companies have low inventory targets. At the same time, they deal with a high level of demand uncertainty. To balance the trade-off between high inventory level and shortage, an effective replenishment strategy is essential for high technology companies.

The consumer electronics market is a highly competitive and volatile market and agility to response the customer is very crucial. To be successful, new products and technologies are introduced continually, enhanced existing products in order to remain competitive and stimulated customer demand for new products. The success of new product introductions is dependent on a number of factors, including market acceptance, the ability to manage the risks associated with product transitions and production ramp issues and the effective management strategy of inventory levels in line with anticipated product demand.

The applied business company orders the items from supplier as produced and there is no need for extra manufacturing process. We assume that products have zero lead time and entire order quantity is delivered at the same time. Table 5.1 shows products' actual demands as weekly basis.

Table 5. 1 Demand data for each item by week

	Demand by quantity					
Sales week	**Item 1**	**Item 2**	**Item 3**	**Item 4**	**Item 5**	**Item 6**
WK09	106	144	192	2126	401	237
WK10	138	95	156	2617	362	130
WK11	94	76	170	1139	194	142
WK12	65	46	211	1282	146	148
WK13	78	85	228	1314	166	262
WK14	69	120	157	1273	147	101
WK15	65	117	144	1395	133	129
WK16	92	123	140	1491	139	98
WK17	102	177	122	1134	170	250
WK18	126	102	131	1055	182	227
WK19	85	90	135	2096	154	342
WK20	78	115	185	1978	126	230
WK21	74	134	190	1646	136	187
Grand total	1172	1424	2161	20546	2456	2483
Average	**90,15**	**109,54**	**166,23**	**1580,46**	**188,92**	**191,00**
St. deviation	**22,88**	**33,07**	**32,86**	**480,23**	**88,08**	**73,51**

We randomly have chosen demand data between week 9 and week 21 which we believe peak season for consumer electronic market. According to the demand data, we assume that demand is stationary and the forecast errors are normally distributed. (As in Eynan and Kropp, 1998) This assumption can be based on some reasons. In many cases, the normal distribution provides a better fit to data than most other distributions. Moreover, since the planning time horizon is infinite, forecast errors are added together, so normal distribution can be expected through the Central Limit Theorem. Finally, the normal distribution leads to analytically tractable results. (Eynan and Kropp, 1998)

In order to test the reliability of the assumption, we utilized from a data fit software called as EasyFit. The forecast errors of each item's distribution graphs are presented in Figure 5.1. According to these results, we can assume that the forecast errors are normally distributed for the applied demand data.

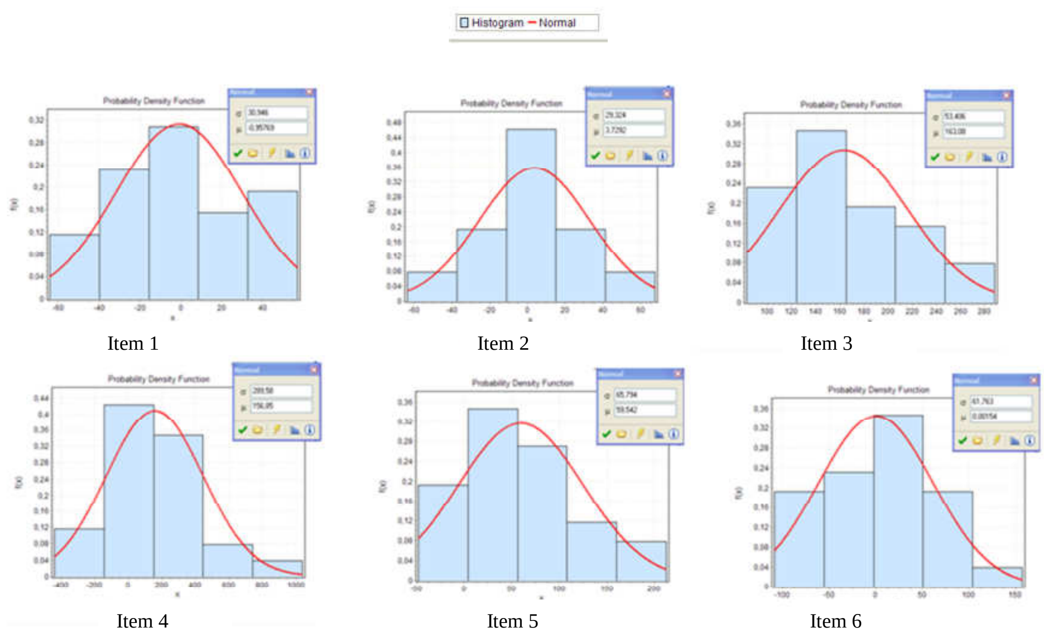

Figure 5. 1 The distribution graph for the forecast errors of each item

The experimentation was carried out through spreadsheet and RAND algorithms for both deterministic and volatile demand cases. At first, we ran RAND and Spreadsheet algorithms for deterministic case. After that, calculation for volatile demand structure was implemented through using standard deviation. We compared the results for the proposed spreadsheet algorithm with well known RAND heuristic for volatile demand cases. In order to calculate the results, we used MATLAB and we coded for RAND (Kaspi and Rosenblatt, 1991), spreadsheet (Nilsson et al., 2007), RAND for the volatile demand (Eynan and Kropp, 1998) and finally proposed spreadsheet algorithm for volatile demand environments.

In order to compare our results with Eynan and Kropp (1998), we used same cost data for minor, major and holding cost. We took the average demand according to our real world data and standard deviation as sigma in the formulas. Service level is taken as 90% as well. The data is indicated as Table 5.2:

Table 5. 2 Data for numerical application

	Item 1	**Item 2**	**Item 3**	**Item 4**	**Item 5**	**Item 6**
Average demand	90,15	109,54	166,23	1580,46	188,92	191,00
St. deviation	22,88	33,07	32,86	480,23	88,08	73,51
Holding cost	0,4	1,0	0,8	0,2	0,8	0,2
Minor repl. cost	1,8	2,0	1,2	3,2	3,1	2,7
Major repl. cost	10					
Service level	90%					

As indicated before, we tested four algorithms, and the deterministic case of the problem is calculated through using average demand. In the RAND algorithm, *m* is taken as 5. In the spreadsheet algorithm, quotient is taken as 1.4. For the volatile demand case of the problem, standard deviation of each item is considered. The results of algorithms are presented in Table 5.3.

Table 5. 3 Numerical results

	Deterministic case		**Volatile demand case**	
	Spreadsheet	**RAND**	**Proposed Spreadsheet**	**Volatile RAND**
TC	**192,99**	**192,99**	**374,82**	**375,14**
T	**0,2251**	**0,2347**	**0,1649**	**0,1598**

According to the Table 5.3, spreadsheet and RAND methods for deterministic case give same results in terms of total cost, whereas in volatile demand case our proposed spreadsheet algorithm outperformed RAND algorithm. The proposed spreadsheet algorithm gives total cost as 374.84, whereas volatile RAND algorithm's is 375.14.

Table 5.4 demonstrates the effectiveness of our spreadsheet method for volatile demand. It can be observed that the first iteration total cost is 376.87 and second iteration gives the result of the heuristic. TC=374.82 and T=0.1649 are obtained with only two iterations through our proposed version.

Table 5. 4 Iteration results for proposed spreadsheet algorithm

Proposed Spreadsheet	**1st iteration**	**2nd iteration**
Item 1	1	1
Item 2	1	1
Item 3	1	1
Item 4	1	1
Item 5	1	1
Item 6	1	2
T	0,1728	0,1649
TC	376,87	374,82

Deterministic algorithms give lower cost than volatile demand algorithms. However, taking average demand into account by itself may give incorrect strategy in terms of customer service rates. Generally, retailers ignore the variation and prefer to use deterministic algorithm. Therefore, we calculated customer service level both deterministic spreadsheet and proposed volatile spreadsheet algorithm. In order to test customer service rate, we used the data between week 22 and week 34 as future demand which follows our previous test data.

Table 5. 5 Demand data for each item by week

	Demand by quantity					
Sales week	**Item 1**	**Item 2**	**Item 3**	**Item 4**	**Item 5**	**Item 6**
WK22	125	117	168	1463	318	203
WK23	88	124	86	1916	335	246
WK24	49	107	101	1670	201	187
WK25	139	107	83	1641	135	203
WK26	73	122	118	1785	286	194
WK27	77	172	127	1863	176	177
WK28	120	87	143	1444	146	196
WK29	72	132	221	1135	117	192
WK30	34	71	276	1183	141	133
WK31	26	147	288	1057	335	128
WK32	147	107	113	1447	156	76
WK33	93	99	135	1320	255	135
WK34	104	129	220	1312	197	256
Grand total	1147	1521	2079	19236	2798	2326
Average	**88,23**	**117,00**	**159,92**	**1479,69**	**215,23**	**178,92**
St. deviation	**38,34**	**25,83**	**69,57**	**279,06**	**80,53**	**49,65**

In order to test customer service rate for both model, we calculated the real demand in the replenishment interval for each item and order quantity in the every replenishment. Since backordering does not exist in our model, the assumption is that if demand is higher than on hand stock, this is considered as lost sale. Replenishment quantity is obtained according to the data between week 9 and week 21, tested on week 22 and week 34 to calculate customer service rate. A part of the calculation for item 1 is presented in Table 5.6 to illustrate the computation:

Table 5. 6 Calculation to find service level

Proposed Spreadsheet		
Average 90,15	St. Deviation 22,88	T 0,1649

SalesDateTime	Demand per week	Demand per rep. interval	Stock	Loss sales/ remaining stock
WK22	125	20,61	14,87	-5,75
		20,61	14,87	-5,75
		20,61	14,87	-5,75
		20,61	14,87	-5,75
		20,61	14,87	-5,75
		20,61	14,87	-5,75
WK23	88	14,90	14,87	-0,04
		14,51	14,87	0,36
		14,51	15,22	0,71
		14,51	15,58	1,07
....				
WK34	104	15,75	92,39	76,64
		17,15	91,50	74,35
		17,15	89,22	72,07
		17,15	86,94	69,79
		17,15	84,65	67,50
		17,15	82,37	65,22
		14,33	80,09	65,76
TTL	1147	**service level**		**96,66%**

The summary of calculation is presented in Table 5.7. For customer service rate, deterministic strategy gives 91.51%, whereas with the volatile demand strategy 95.41% of customer's demand can be fulfilled. Moreover, loss of revenue can be calculated by each item's average lost quantity in a replenishment interval multiplied with its price, since backordering is not allowed in the system. Thus, loss revenue is 588.08 TL in the deterministic method, compared to 195.03 TL for the volatile demand method. This calculation can be interpreted as the difference 393.06 TL will be the charge of unmet customer demand, whereas total cost difference is 181.83 TL by using volatile demand replenishment strategy.

Table 5. 7 Results for service level and loss sales of deterministic and volatile demand algorithms

	Price of item	Service level		Loss sales by quantity		Loss sales by revenue	
		Determistic strategy	Volatility strategy	Determistic strategy	Volatility strategy	Determistic strategy	Volatility strategy
Item 1	4,00 TL	96,17%	96,66%	0,77	0,48	3,08 TL	1,94 TL
Item 2	10,00 TL	92,15%	93,62%	2,06	1,23	20,59 TL	12,28 TL
Item 3	8,00 TL	87,42%	99,92%	4,51	0,02	36,06 TL	0,18 TL
Item 4	20,00 TL	95,06%	98,32%	16,67	3,86	333,47 TL	77,16 TL
Item 5	18,00 TL	84,32%	87,34%	7,70	4,43	138,56 TL	79,67 TL
Item 6	12,00 TL	93,95%	96,59%	4,69	1,98	56,32 TL	23,80 TL
RESULT		91,51%	95,41%	36,40	12,00	588,08 TL	195,03 TL
DIFFER.		3,90%		24,40		393,06	

The proposed spreadsheet algorithm outweighs RAND algorithm for volatile demand case in terms of total cost charges. Therefore, it is logical to use our proposed algorithm as an effective replenishment strategy. Moreover, customer service rate increased from 91.41% to 95.41 through using volatile demand replenishment strategy.

Table 5. 8 Parameter of each item and service level results

		Parameters		Service level		
		WK 9-21	**WK 22-34**	**Deterministic**	**Volatile case**	**Diff**
Item 1	Average	90,15	88,23	**96,17%**	**96,66%**	0,49%
	Deviation	22,88	38,34			
	Dev/Aver.	**25%**	**43%**			
Item 2	Average	109,54	117,00	**92,15%**	**93,62%**	1,47%
	Deviation	33,07	25,83			
	Dev/Aver.	**30%**	**22%**			
Item 3	Average	166,23	159,92	**87,42%**	**99,92%**	12,49%
	Deviation	32,86	69,57			
	Dev/Aver.	**20%**	**44%**			
Item 4	Average	1580,46	1479,69	**95,06%**	**98,32%**	3,26%
	Deviation	480,23	279,06			
	Dev/Aver.	**30%**	**19%**			
Item 5	Average	188,92	215,23	**84,32%**	**87,34%**	3,03%
	Deviation	88,08	80,53			
	Dev/Aver.	**47%**	**37%**			
Item 6	Average	191,00	178,92	**93,95%**	**96,59%**	2,64%
	Deviation	73,51	49,65			
	Dev/Aver.	**38%**	**28%**			

In terms of service level on item basis, we can make some inferences regarding the ratio of deviation to average demand. The highest difference has been seen for item 3, which means that substantial increase on service level can be obtained via using volatile demand replenishment strategy. Item 3 gives 87.42% service level for deterministic strategy whereas 99.92% for volatile demand strategy. Figure 5.1 illustrates the demand chart of item 3. For items whose ratio of deviation to average demand will be higher in the following periods, choosing volatile demand replenishment strategy would be wisely in order to service more customer.

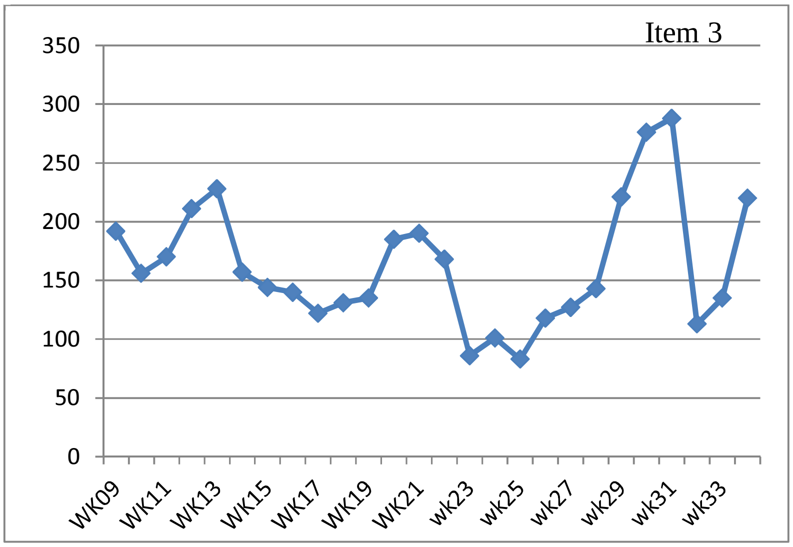

Figure 5. 2 Demand chart of item 3.

In order to evaluate the effectiveness of proposed joint replenishment algorithm, we tested the service level of replenishment of items to be clustered. According to week 9-week 21 data, we can classify items to their ratio of average demand to deviation. For items whose ratio are below 30%, item 1, item 2, item 3 and item 4 can be replenished together. The rest of items can be grouped together. The results are seen in Table 5.9. The first family of items gives total cost 269.07 and the second family does 156.80.

Table 5. 9 Results for replenishment families

Replenishment family	**T**	**TC**
Item (1,2,3,4)	0,1662	269,07
Item (5,6)	0,2783	156,8
Total		425,87

We calculated service level of clustering replenishment and compared the results with joint replenishment service level. Table 5.10 gives the summary of the computation. The replenishment with clustering items gives total cost as 425.87 whereas jointly replenishment's total cost is 374.82. Moreover, higher service level cannot be obtained via clustering items. Service level is better when items are jointly replenished.

Table 5. 10 Service level for joint replenishment versus clustering replenishment

	Service level							
	Item1	**Item2**	**Item3**	**Item4**	**Item5**	**Item6**	**Average**	**TC**
Joint reple.	96,66%	93,62%	99,92%	98,32%	87,34%	96,59%	95,41%	374,82
Clustering	96,53%	93,43%	99,92%	98,31%	85,19%	96,65%	95,00%	425,87
Diff	-0,13%	-0,19%	0,00%	-0,01%	-2,16%	0,06%	-0,40%	51,05

After all the calculations, we can draw some conclusions for our real-world data set. In deterministic case of the problem, RAND and spreadsheet heuristics give same cost result whereas proposed spreadsheet algorithm outperformed RAND in volatile demand environment. The proposed spreadsheet algorithm gives total cost as 374.84 against the volatile RAND algorithm's is 375.14.

According to the customer fulfilment rate, volatile demand replenishment strategy gives total cost as 374.82 TL and 95.41% service level, whereas deterministic strategy gives 91.51%. The loss revenue is 588.08 TL in the deterministic method, compared to 195.03 TL for the volatile demand method. The difference 393.06 TL is the charge of unmet customer demand, whereas total cost difference is 181.83 TL by using volatile demand replenishment strategy. The test results indicate that clustering of items does not provide higher service level. Joint replenishment is well suitable to obtain favourable service levels. Consequently, our proposed spreadsheet algorithm proves that it provides higher service rate with lower cost level.

6. CONCLUSIONS

In this paper, we presented a new adaptation of spreadsheet heuristic for the volatile demand environments. The simplicity of application of spreadsheet method and its efficiency enabled us to consider its modified version for the joint replenishment problem that considers volatility in demand. We offered to consider not only average demand value, but also variability in demand function. Since consumers' preferences are getting harder to predict, the importance of variation is higher than before. For this reason, we thought that if variation in demand is taken into account, it would give more accurate results for the inventory problems.

In order to test and compare the algorithms, the real world data was constructed and ran to find out how effective the proposed algorithm is. The numerical data belongs to a world-wide known consumer electronic company. Because of the short life cycles and high obsolescence of technology products, accurate replenishment policy was highly required in this case. The uncertainty in demand can lead to the inventory's accumulation and also shortages in terms of availability. In order to balance the trade-off between high inventory level and shortage, an effective replenishment strategy was essential for our business problem.

We compared the effectiveness of our proposed algorithm with well known RAND heuristic for joint replenishment problem. In order to code algorithms, we utilized MATLAB. According to the applied business data, both algorithms give same result in deterministic case, whereas proposed spreadsheet algorithm outperformed RAND algorithm for the volatile demand environment.

We also highlighted the importance of using volatile demand strategy with a calculation over deterministic strategy. The customer service rate was computed for deterministic and volatile demand strategies. Proposed strategy gives higher customer service level which means lower unmet customer demand. The charge of unmet customer demand difference between deterministic and volatile demand algorithms is higher than the total cost difference by using volatile demand replenishment strategy. Therefore, choosing volatile demand replenishment strategy is wiser in order to service more customers with a small difference in cost.

In terms of service level on item basis, we also made some inferences regarding the ratio of deviation to average demand. For the items whose ratio of deviation to average demand will be higher in the following periods, volatile demand replenishment strategy enables substantial increase on service level.

Last but not least, we tested the effectiveness of the joint replenishment strategy in terms of service level. For the numerical data, we classified items into two clusters according to their ratios of deviation over average demand. Obviously, clustering replenishment strategy has higher cost than jointly replenishment. More importantly, clustering of items does not provide higher service level. In conclusion, proposed jointly replenishment strategy performs well for the real world data and appropriate for the company's replenishment strategy.

While maintaining an efficient replenishment strategy, it should be highlighted the importance of dynamic recalculation to obtain the proper replenishment interval and k multipliers for each item continuously. Due to the nature of demand volatility, optimal results may vary from period to period. Therefore, heuristic algorithm should be applied in every different period and its results should be recalculated dynamically.

The proposed algorithm could be extended in several directions. For instance, backordering cost could be implemented to the algorithm. In this way, shortage cost could also be considered in addition to the holding costs and the replenishment costs. Furthermore, the concept developed could be applied under dynamic calculation in different time periods. These periods could be determined according to their conditions and characteristics. We believe that future modifications may increase performance.

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APPENDICES

Appendix A- Matlab code for RAND algorithm under deterministic environments

```
function [TC_min] = rand_algorithm(S, s, D, h, m)
k_opt = [];
T_ini = [];
TC=[];
T_max = nthroot(2*(S+sum(s))/sum(D.*h),2);
T_min = min(nthroot(2*s./(D.*h),2));
T_step = (T_max - T_min)/(m-1);
for i = 0:m-1
   T_ini = [T_ini T_min+i*T_step];
end
b=1;
while b<=m
   Tp = T_ini(b);
   k_opt = [];
   k_square = [];
   q=0;
   while q <=1  ||  sum(k_opt(q,:) - k_opt(q-1,:)) ~= 0
      q = q + 1;
      for j = 1:length(s)
         k_square(q,j) = 2*s(j)/(power(Tp,2)*D(j)*h(j));
         %     for L = 0:100
         %        if power(k(q,j),2) > L*(L-1) && power(k(q,j),2)     <= L*(L+1)
         %        k_opt(q,j) = L;
```

```
            %        else
            %        continue
            %        end
            %     end
            L = 1;
            z = 0;
            while z == 0
                if k_square(q,j) <= L*(L+1) && k_square(q,j) >= L*(L-1)
                    z = 1;
                else
                    L = L + 1;
                end
            end
            k_opt(q,j) = L;
        end
        Tp= nthroot(2*((S+sum(s./k_opt(q,:)))/sum(D.*h.*k_opt(q,:))),2);
        TC(b) = (1/Tp)*(S+sum(s./k_opt(q,:)))+(Tp/2)*sum(D.*h.*k_opt(q,:));
        k_sonuc(b,:) = k_opt(q,:);
    end
    b=b+1;
end
[TC_min, ind]  = min(TC);
k_sonuc(ind,:)
TC_min
Tp
```

Appendix B- Matlab code for RAND algorithm under volatile demand environments

```
function [TC_min T_opt, k_son] = rand_volatile(N,D,a,A,h,z,sig)
T=[];
T_ini = [];
k= [];
k_opt = [];
q=0;
for i=1:N
   T0(i)=nthroot(2*a(i)/(h(i)*D(i)),2);
   T(i)=nthroot((2*a(i))/(h(i)*(D(i)+(z*sig(i)/nthroot(T0(i),2)))),2);
end
T_ini=T;
[T_ini,sira] = sort(T_ini,'ascend');
k(sira(1))=1;
T_jrp=nthroot(2*(A+a(1))/(h(1)*(D(1)+(z*sig(1)/nthroot(2*(A+a(1))/h(1)*D(1),4)))),2)
;

for j=1:N-1
   k(sira(1+j))=T(sira(1+j))/T_jrp;
end
for i=1:N
 if round(k(i))==0
   k_opt(i) = 1;
   else
   k_opt(i) = round(k(i));
 end
end

 ini_TC=100000000;
```

```
 while q==0
To=nthroot(2*(A+sum(a./k))/sum(h.*k_opt.*D),2);

T_son=nthroot(2*(A+sum(a./k))/sum(h.*k_opt.*(D+(z*sig/nthroot(k_opt*To,2)))),2);

TC=A/T_son+sum((a./k_opt)/T_son)+sum((T_son*k_opt.*D*h')/2+z*sig.*h*nthroot((k
_opt*T_son),2)');
    if TC< ini_TC;
        ini_TC= TC;
    else
        q=1;
        break;
    end
    for i=1:N
        k(i)=T(i)/T_son;
    if round(k(i))==0
        k_opt(i) = 1;
    else
        k_opt(i) = round(k(i));
    end
    end
 end
TC_min=ini_TC
T_opt=T_son
k_son=k_opt
```

Appendix C- Matlab code for Spreadsheet algorithm under deterministic environments

```
function [TC_min, T_opt, k_son]=spreadsheet(N,D,a,A,h)
k_ini=ones(1,N);
q = 0;
ini_T=nthroot(2*(A+sum(a./k_ini))/(sum(k_ini.*D*h')),2);
ini_TC=A/ini_T+sum(a./(ini_T.*k_ini)+(ini_T*k_ini.*D.*h)/2);
for i= 1:N
   ini_Q(i)=((sum(k_ini.*D*h')/(A+sum(a./k_ini)))*(a(i)/(k_ini(i)*k_ini(i)*D(i)*h(i))));
end
Q = ini_Q;
T = ini_T;
k = k_ini;
while q == 0
for i= 1:N
   if Q(i)>1.4
      k(i)=k(i)+1;
   else
      if sum(Q>1.4) == 0;
         q = 1;
      end
   end
end
   T=nthroot(2*(A+sum(a./k))/(sum(k.*D*h')),2);
   TC=A/T+sum(a./(T.*k)+(T*k.*D.*h)/2);
   if TC< ini_TC;
   ini_TC= TC;
   ini_k = k;
   for i= 1:N
   Q(i)=((sum(k.*D*h')/(A+sum(a./k)))*(a(i)/(k(i)*k(i)*D(i)*h(i))));
```

```
    end
    else
    q = 1;
    end
end
k = ini_k;
while q==1
    temp = abs(1-Q);
    [temp,sira] = sort(temp,'descend');
   b=1;
    while b<=N
       if Q(sira(b))<1
       if k(sira(b))>1
          k(sira(b))=k(sira(b))-1;
          break;
       else
          b=b+1;
       end
       else
       k(sira(b))=k(sira(b))+1;
       break;
       end
    end

    T=nthroot(2*(A+sum(a./k))/(sum(k.*D*h')),2);
    TC=A/T+sum(a./(T.*k)+(T*k.*D.*h)/2);
    if TC< ini_TC;
    ini_TC= TC;
    ini_k = k;
    for i= 1:N
    Q(i)=((sum(k.*D*h')/(A+sum(a./k)))*(a(i)/(k(i)*k(i)*D(i)*h(i))));
    end
```

```
    else
        q=2;
    end
end
TC_min=ini_TC
T_opt=T
k_son=ini_k
```

Appendix D- Matlab code for Spreadsheet algorithm under volatile demand environments

```
function [TC_min, T_opt, k_son]=spreadsheet_volatile(N,D,a,A,h,z,sig)
k_ini=ones(1,N);
T_0=nthroot(2*(A+sum(a./k_ini))/(sum(k_ini.*D*h')),2);
q = 0;
ini_T=nthroot(2*(A+sum(a./k_ini))/(sum(k_ini.*(D+(z*sig)./nthroot((k_ini*T_0),2))*h'
)),2);
ini_TC=A/ini_T+sum((a./k_ini)/ini_T)+sum((ini_T*k_ini.*D*h')/2+z*sig.*h*nthroot((
k_ini*ini_T),2)');
for i= 1:N

ini_Q(i)=((sum(k_ini.*(D+z*sig)*h')/(A+sum(a./k_ini)))*(a(i)/(k_ini(i)*k_ini(i)*(D(i)+
z*sig(i))*h(i))));
end
Q = ini_Q;
T = ini_T;
k = k_ini;
while q == 0
for i= 1:N
   if Q(i)>1.4
      k(i)=k(i)+1;
   else
      if sum(Q>1.4) == 0;
         q = 1;
      end
   end
end
   T=nthroot(2*(A+sum(a./k))/(sum(k.*(D+(z*sig)./nthroot((k*T_0),2))*h')),2);
   TC=A/T+sum((a./k)/T)+sum((T*k.*D*h')/2+z*sig.*h*nthroot((k*T),2)');
```

```
    if TC<= ini_TC;
    ini_TC= TC;
    ini_k = k;
    for i= 1:N
    Q(i)=((sum(k.*(D+z*sig)*h')/(A+sum(a./k)))*(a(i)/(k(i)*k(i)*(D(i)+z*sig(i))*h(i))));
    end
    else
    q = 1;
    end
end

while q==1
    temp = abs(1-Q);
    [temp,sira] = sort(temp,'descend');
   b=1;
    while b<=N
      if Q(sira(b))<1
      if k(sira(b))>1
        k(sira(b))=k(sira(b))-1;
        break;
      else
        b=b+1;
      end
      else
      k(sira(b))=k(sira(b))+1;
      break;
      end
    end
    T=nthroot(2*(A+sum(a./k))/(sum(k.*(D+(z*sig)./nthroot((k*T_0),2))*h')),2);
    TC=A/T+sum((a./k)/T)+sum((T*k.*D*h')/2+z*sig.*h*nthroot((k*T),2)');
```

```
 if TC< ini_TC
    ini_TC= TC;
    ini_k = k;
  for i= 1:N
  Q(i)=((sum(k.*(D+z*sig)*h')/(A+sum(a./k)))*(a(i)/(k(i)*k(i)*(D(i)+z*sig(i))*h(i))));
  end
  else
     q=2;
  end
end
TC_min=ini_TC
T_opt=T

k_son=k
```

BIOGRAPHICAL SKETCH

Buket Türkay, the candidate of Master of Science in Industrial Engineering Department in Galatasaray University, was born in 1988 in Adana. She received her high school degree from Seyhan Anatolian High School. Subsequently, she obtained her Bachelor's degree in Industrial Engineering in Yıldız Technical University in 2010. She was rewarded as "Honor Student" along with her graduation in 2010. Her research interests are stock policies, supply chain management and optimization.