DETERMINING THE VALUE OF A PRODUCT SUBSTITUTION STRATEGY FOR A STOCHASTIC MANUFACTURING/REMANUFACTURING SYSTEM

(BİR STOKASTİK ÜRETİM/YENİDEN ÜRETİM SİSTEMİNDE ÜRÜN İKAME STRATEJİSİNİN DEĞERLENDİRİLMESİ)

by

Emre KÜRTÜL, B.S.

Thesis

Submitted in Partial Fulfillment
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Committee Members : Prof. Dr. Russell E. KING
Assoc. Prof. Dr. M. Ebru ANGÜN

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ABSTRACT

With the increasing environmental consciousness in the last decades, people are now more aware that Earth's resources are finite. These scarce resources cause a risk for the manufacturers since their costs fluctuate unexpectedly. As a result big manufacturing firms always look for ways to decrease their dependence on resources and reduce the risks. Moreover governments put into action some regulations in order to limit the utilization of natural resources and to reduce the gas emissions. Both the increasing costs of using virgin materials and the limiting regulations on their usage make the manufacturers investigate the product recovery options. In this study we consider remanufacturing as the recovery option and analyze an infinite horizon periodic-review inventory control problem for a hybrid manufacturing/remanufacturing system with product substitution where demand and returns are stochastic. In this system, a remanufactured item is considered to have an inferior value from the customer's viewpoint and thus has a lower selling price than a new item, which leads to a segmented market for manufactured and remanufactured items. The manufacturer considers the use of a one-way product substitution strategy, according to which the demand for remanufactured items is satisfied by new items at the reduced price, if the remanufactured item inventory runs out of stock and the new item inventory is positive. With this strategy, the firm aims to decrease the risk of stock-outs and increase the customer satisfaction. The problem is formulated as a discrete-time Markov Decision Process in order to determine the value of product substitution. The profitability of using the product substitution strategy is investigated through a numerical study based on real data from an automobile parts manufacturer.

RESUME

Avec l'augmentation dans la conscience environnementale les gens sont plus informés sur la pénurie des ressources terrestres. Ces ressources limitées entraînent un risque pour les fabricants car leurs coûts fluctuent de façon inattendue. C'est pourquoi grandes entreprises de fabrication cherchent toujours d'autres moyens afin de réduire leur dépendance sur les ressources et réduire les risques. Par ailleurs quelques gouvernements dans le monde mettent en œuvre des règlements afin de limiter l'utilisation des ressources naturelles et de réduire les émissions. Les coûts croissants de l'utilisation de matériaux vierges et les règlements qui limitent leur usage font les fabricants étudier les options de récupération du produit. Dans cette étude, nous considérons la refabrication (remanufacturing) comme l'option de récupération et analysons le problème de contrôle des stocks pour un système de fabrication / refabrication hybride ou on utilise la substitution de produits et la demande et les rendements sont stochastiques. Dans ce système, un élément remanufacturé est considéré comme ayant une valeur inférieure du point de vue du client et a donc un prix de vente inférieur à celui d'un nouveau produit, ce qui conduit à un marché segmenté des articles manufacturés et remanufacturés. La firme considère l'utilisation d'une stratégie de substitution selon laquelle la demande de produits remanufacturés est satisfaite par de nouveaux articles au prix réduit, si l'inventaire de l'article remanufacturé est en rupture de stock et le stock de l'article nouveau est positif. De cette façon, l'entreprise vise à réduire le risque de ruptures de stock et augmenter la satisfaction du client. Le problème est formulé comme un processus de décision Markovien à temps discret pour déterminer la valeur de substitution du produit. La rentabilité de l'utilisation de la stratégie de substitution est étudiée à travers une étude numérique basée sur des données réelles provenant d'une entreprise qui fabrique des pièces d'automobile.

ÖZET

Son yıllarda artan çevre bilinciyle birlikte, insanlar dünya kaynaklarının sınırlılığı konusunda daha fazla farkındalığa sahiptirler. Bununla birlikte kısıtlı kaynakların maliyetlerinin zaman içinde dalgalanması üreticiler için de riskleri beraberinde getirmektedir. Bu nedenle büyük üreticiler risklerini azaltmak için bu kaynaklara olan bağlılıklarını azaltma çabasındadırlar. Kaynakların kısıtlı olmasının yanı sıra bazı hükümetlerin kaynak kullanımını düzenlemek ve karbon emisyonlarını azaltmak için yürürlüğe koyduğu yasalar da üreticileri ürünlerin yeniden kazanılması konusunda teşvik etmektedir. Bu çalışmada yeniden üretim incelenecek olup, stokastik talep ve geri dönüşlerin olduğu melez üretim/yeniden üretim sistemine ilişkin bir stok kontrol problemi ele alınmıştır. Bu sistemde yeniden üretilen ürünlerin müşteri gözünde yeni ürünlere göre daha düşük değerde olduğu ve bu nedenle farklı satış fiyatlarıyla farklı pazarlara hitap ettiği varsayılmıştır. Üretici tek yönlü bir ürün ikame stratejisi kullanacak olup bu sayede yeniden üretilmiş ürüne olan talebi yeni ürünü düşük fiyattan satarak karşılayabilmeyi planlamaktadır. Bu stratejiyle birlikte stoksuz kalma olasılığını azaltıp müşteri memnuniyetini arttırmak hedeflenmektedir. Bu problem kesikli Markov karar süreci olarak modellenip çözülerek ürün ikame stratejisinin önemi değerlendirilmiştir. İkame stratejisinin karlılığı bir otomobil yedek parça firmasından alınan gerçek verilere dayanan sayısal bir çalışma yapılarak incelenmiştir.

1 INTRODUCTION

Product recovery has received increasing attention by manufacturers, since reprocessing the returned items for reuse is proved to generate reasonable economic benefits when the recovery process is controlled properly. The awareness about the product recovery has increased significantly recently, mainly because the environmental regulations in some countries force the manufacturers to consider end of life options for their products. A common example of these regulations is take-back obligations after usage (Fleischmann et al. 2002). Furthermore, environmental consciousness of the public is rising with the extensive research about the climate change reported by the media, and this is putting pressure on manufacturers to make their processes more environmentally friendly (Ilgin & Gupta 2010). Another consideration is the significant increase in disposal costs. The waste, if not recovered, has to be incinerated or landfilled for disposal, which decreases significantly the disposal capacity. This has forced the governments to put regulations in place in order to restrict further usage of this capacity. Such regulations, in turn, put pressure on companies to consider the total life cycle of their products and manufacture them accordingly. (Gungor & Gupta, 1999)

Thierry et al. (1995) explain different product recovery options. They analyze the treatment options for returned products in three main categories. First is the direct reuse option where the returns are assumed not to need any treatment before they are resold. Second category is the product recovery allowing the returned products to pass through different processes before they could be used again as a whole or in parts. Last option is that the returns are disposed using incineration or landfilling. According to Thierry et al. (1995) product recovery options differ from each other by the point the returns join the integrated reverse supply chain process. These options are repair, refurbishing, remanufacturing, cannibalization and recycling. Cannibalization and recycling are the recovery options where the returns are disassembled or decomposed into their components, and either none or only a small portion of these components are used in order to manufacture similar product. Especially, in recycling, the materials are induced into their elemental structure in order to be used in different applications. However for the other options, the components are sought to be used maximally. In repair and refurbishing, the objective is to return the products to working order where the quality of the resulted product is generally less than the newly manufactured ones. In these options the returns are generally not disassembled into all their modules, instead they go through a general inspection and basic treatment. However, in remanufacturing, the returned products are disassembled into their modules and the remanufactured parts are usually assumed to have the same quality with the new ones.

In this study, remanufacturing is considered as the recovery option. Remanufactured goods are usually considered to be “like new”. Examples for such products are aircraft or automobile engines, aviation equipment, medical equipment, office furniture, machine tools, copiers, electronics equipment, toner cartridges, cellular telephones, single-use cameras, etc. (Thierry et al., 1995; Fleischmann et al., 1997; United States Environmental Protection Agency, 1997; Toktay et al., 2000). When the remanufactured items are considered to be identical to the new products, they are sold for the same price to the same market. However in real life, this assumption might not be valid, because some remanufactured products could have an inferior value in customer’s viewpoint and companies might use these remanufactured products to serve a different market using a different pricing. This segmented market approach considers different demand streams and separate handling of inventories for different types of products and examples of its use exist in the literature for products such as upgraded computers, retread tires, reconditioned photocopiers, overhauled automobile engines and spare parts (Ayres et al., 1997; Ferrer, 1997a; Ferrer, 1997b) .

In case of a segmented market for manufactured and remanufactured products, the manufacturer may use a stockout- based one-way substitution strategy, according to which the demand for remanufactured items can be satisfied by new items if the remanufactured item inventory runs out of stock and the new item inventory is positive. This strategy is usually adopted in the systems where stock-out risk of remanufactured items is higher than the stock-out risk of new items, and this is mainly because remanufacturing capacity is constrained by the returns. The opposite way of substitution is usually not considered since customers who demand new items would not like to be offered instead an inferior valued item.

We consider an inventory problem for a manufacturing/remanufacturing system, motivated by a real case at the Turkish subsidiary of an international company that concentrates its business on producing and selling automotive spare parts. Their customers are major automobile manufacturers and they have agreements which bound the company to provide certain spare parts within certain time limits. These obligations and concerns on losing potential customers for remanufactured items make the company to consider using a one-way product substitution strategy. According to this substitution strategy, the demand for remanufactured items can be satisfied using the manufactured items (i.e. new items) if the remanufactured item inventory runs out of stock and the manufactured item inventory is positive. In this case, the new item is offered at the discounted price of a remanufactured item.

We aim here to analyze the inventory control problem of a periodically reviewed hybrid manufacturing/remanufacturing system with product substitution where demands for manufactured and remanufactured items as well as returns are stochastic. We formulate this problem as a discrete time Markov Decision Process (MDP) where states of the system are the manufactured, remanufactured and returned item inventories and the decisions are manufacturing and remanufacturing amounts. The infinite horizon MDP problem is solved to determine the optimal decisions for each state for both no substitution and substitution cases, and the profitability of using the substitution strategy is evaluated through numerical experimentations.

There have been some efforts to solve the inventory control problem for a stochastic hybrid manufacturing/ remanufacturing system under product substitution. While Inderfurth (2004) and Bayındır et al. (2007) find optimal policies for a single period problem, Bayındır et al. (2005) consider a multi-period problem with a predetermined inventory strategy and find the optimal values of policy parameters. However to the best of our knowledge, no study exists in the literature, which finds the optimal inventory policy analytically or numerically for the multi-period periodic-review inventory problem.

The remainder of this thesis is organized as follows: Chapter 2 provides a literature survey on different approaches to solve the inventory problem of hybrid manufacturing/remanufacturing systems with a special focus on the studies that consider the use of product substitution in a recoverable manufacturing system. MDP formulations and the proposed solution methodology are given in chapters 3 and 4 respectively. In chapter 5, a numerical study is done based on a real data from an automotive spare parts manufacturer. Finally, chapter 6 concludes the study and mentions future work.

2 LITERATURE SURVEY

As product recovery becomes a valuable concept in industrial practice because of the environmental consciousness and legislation of some countries, it has started to gain more attention from the researchers. Inventory control problem for single product hybrid manufacturing/remanufacturing systems are analyzed by several authors. The literature on such inventory problems differ in several aspects and they can be categorized as deterministic versus stochastic models with respect to customer demand and returns, finite horizon versus infinite horizon models, periodic review versus continuous review models, models with/without disposal option, etc.

In this chapter, a commonly used classification in the literature is adopted. According to this classification product recovery models are categorized as deterministic models versus stochastic models. Moreover, stochastic models are broken down into to two parts: periodic review models and continuous review models.

2.1 Deterministic Models

In deterministic models, all the parameters such as cost parameters, demand and return rates, etc. are assumed to be known with certainty. Furthermore, we can analyze such models in two categories: static and dynamic models. While for the static models demand and return rates do not vary over time, for dynamic models, they change over time.

Schrady (1967) introduces a static inventory control model for recoverable systems which is derived from the well-known economic order quantity (EOQ) model. Schrady (1967) considers a system with two inventory types (serviceable and recoverable products) and aims to minimize the cost that consists of fixed cost for external order and linear holding costs. In their model, demand, return and lead times are deterministic. Capacities of stocking points are not considered, thus there is no disposal option

Underthese assumptions a three-parameter control policy, which consists of an order quantity for external procurement, a batch size for recovery option and the number of identical batches that should follow every external procurement order, is proposed. Mabini et al. (1992) and Richter (1996) develop further EOQ type models. Mabini et al. (1992) try to minimize total cost in a system with purchases and repairs by determining optimal repair and purchase quantities for a single item case. They also extend their research for a multiple-item case where items share a common repair capacity. Richter (1996) presents a two-stage model with two workshops. In the first shop it is assumed that new production and repair of items appear and in the second shop items leave the system with a constant demand rate or they are disposed. At the end of each period the items left in the second shop is either disposed or brought back to first shop for a repair. In such system, Richter develop optimal repair and purchase quantities for cost minimization. More recently, Konstantaras (2010) develop a new EOQ model where backlogging is allowed. In his research he fixed the quantity of either manufacturing or remanufacturing in a period and tried to find the optimal quantity for the other operation. El Saadany and Jaber (2011) also use EOQ approach in their work, however their decision variable is the ratio of remanufacturing to newly manufacturing rather than the quantities. Also they point out that in the literature the returns are assumed to be recovered as whole units, however in general the items could be disassembled. Thus they insert subassembly processes to their model and they assume multiple recoverable stocks and one serviceable stock. A mathematical programming model is introduced and solved with different parameter values. It is shown that according to numerical examples 65% of the time pure manufacturing or remanufacturing policies are optimal for a system with subassemblies and zero leadtimes. Chung and Wee (2011) try a new approach by analyzing product life span. A closed loop system is considered with dependable static demand and return where products are assumed to be deteriorating in time. They try to find the optimal quantities of manufacturing and remanufacturing as well as the times a product can be remanufactured before it is disposed, in order to minimize the costs.

Aside the models with static demand and return, there is also some work in literature where these rates are dynamic, but known deterministically throughout the planning horizon. The variants of Wagner/Whitin algorithm have been developed to solve dynamic recoverable inventory problems. For instance Richter and Sombrutzki (2000) study a pure reverse Wagner/Whitin model, where only the remanufacturing option is available to meet the demand. Furthermore they extend their research to another model where new manufacturing is also considered as an alternative production option to meet the demand. In a later work, Richter and Weber (2001) incorporate variable manufacturing and remanufacturing costs into the model and investigate how the existence of a disposal option for used products would affect the solution. There also exists some work in literature approaching the single product recovery problem as a dynamic optimization problem. The work by Kleber et al. (2002) is one of them where demand and return are said to be dynamic because of the seasonality effects. They try to investigate the difference between the seasons where demand is more than the return and the seasons where the amount of return surpasses the amount of return. Moreover they introduce multiple remanufacturing options with different demands, because of the variable product quality or market segmentation. This leads to one recoverable stock and multiple serviceable stocks. The dynamic cost minimization problem is solved using Pontryagin's Maximum Principle under the assumptions of no backorders and zero leadtimes and an optimal policy is provided. Kiesmüller (2003) also uses dynamic programming approach, but he considers the basic single-item two-stocking point model. Optimal manufacturing, remanufacturing and disposal rates are provided for systems with or without backorders and for different leadtime scenarios. Hedjar et al. (2005) add deterioration to the model. They assume that deterioration occurs with a rate in the serviceable inventory and items return back to recoverable inventory for later reprocessing. They use receding horizon control and propose different solutions for both periodic and continuous-review cases. Pineyro and Viera (2010) also consider a deterministic model however they assume that newly manufactured and remanufactured products are not identical, thus they suggest a model with two serviceable stocks. Moreover, they adopt a one-way product substitution strategy according to which they satisfy the demand of remanufactured products with newly manufactured ones. They formulate a mixed integer program (MIP) for cost minimization problem to find order up to levels for both product types and use a tabu search algorithm to solve this NP-Hard problem.

2.2 Stochastic recoverable inventory models

In real cases, demand and returns are generally not known with certainty because of the volatility of the markets, hence they are represented using a stochastic distribution. Although the stochastic nature adds complexity to the problem, it should be taken into account in order to represent better the real world cases. Regarding the stochastic recoverable inventory control problems, two streams of contributions can be found in the literature: one includes periodic-review models and the other includes continuous-review models.

Two main approaches are observed in the literature regarding inventory control of stochastic recovery models. First approach, which has been seldom used, is to determine analytically the optimal control policy using dynamic programming tools. Another approach consists of finding optimal or near optimal parameter values for predetermined control policies with the use of techniques such as Markov Decision Processes, heuristics, simulation, system dynamics etc. Even though the latter approach has the drawback of considering pre-determined policies that are not necessarily optimal, it is commonly used in the literature for both periodic and continuous review systems. Moreover, generally in these studies, the results are not compared to the optimal solutions, thus no strong evidence for solution quality is provided.

2.2.1 Periodic-review models

Periodic review models have received attention recently in the context of product recovery. This is mainly because these models are known as more adequate to use in practical situations. However the majority of the work on periodic-review models investigates the parameter values for predetermined control policies that are not guaranteed to be optimal. One of the earliest papers considering such approach is by Kiesmüller (2003). This paper considers a PULL policy, named as (*S*, *M*) policy for a stochastic hybrid manufacturing/remanufacturing system with two stocking points without a disposal option and with different lead time cases for manufacturing and remanufacturing operations where *S* denotes the order up to level and *M* denotes the remanufacture up to level. Instead of using one inventory position for production and remanufacturing decisions they define different inventory positions and provide a comparative analysis between these two approaches. They calculate the optimal parameter values for (*S*, *M*) policy using grid search and simulation. Similarly Mahadevan et al. (2003) search the optimal parameter values for a predetermined (*R*, *M*) PUSH policy where they consider review period (R) and manufacturing up to level (M) as their control parameters and they use heuristics in order to find these values. Teunter et al. (2004) examine several control policies used in previous work and propose a new policy called Separate PULL strategy. They compare all these policies numerically for different parameter values and show that separate PULL strategy is the best policy in most cases. Zhou et al. (2006) also consider a type of PULL strategy with ten control parameters. They use a system dynamics approach and try to find optimal parameter values in order to optimize six different objective rates. Takahashi et al. (2007) develop the basic hybrid manufacturing/ remanufacturing model further by adding a decomposition process to the system. They assume that the returned items are decomposed into raw materials or half finished product parts. As a result they try to optimize a system with three stocking points which are the serviceable stock, the parts stock and the materials stock. This system is modeled as a Markov chain and parameter values for two predetermined policies are examined. More recently, Nenes et al. (2010) formed a model with three different inventory types by considering inspection in their model. They assume that returns are inspected first and they are either selected to remanufacture or go directly to the serviceable stock if they can satisfy certain quality standards. In such system, they propose seven different control policies and compare these policies for 48 scenarios with different parameter values. As a result they determine better policies for different market states considering the average costs. Xin (2010) considers an EOQ policy for Poisson distributed demand and return. He assumes that the remanufactured products are sold with a discount rate and use this rate to calculate the lost sales cost. Mathematical expressions to calculate optimal order quantity and optimal handling period of remanufacturables are derived, and numerical experiments are performed.

A more useful approach for a single product recovery system would be generation of optimal control policy structures. This approach is rarely used in literature because of the solution complexity. However, it has gained some attention in recent years since it provides better solution quality. To the best of our knowledge, the work by Simpson (1978) is the earliest work where the optimal control policy structure is derived analytically. This paper considers a finite-horizon repairable inventory problem with two stocking points and a three-parameter optimal control policy structure is generated under a zero lead time assumption using backward dynamic programming technique. In a more comprehensive work, Inderfurth (1997) addresses a product recovery problem with or without stock keeping of returned items and derive optimal policies using stochastic dynamic programming. For the cases without stock keeping he examines both equal and unequal lead time scenarios and derives optimal policy structures. However for the stock keeping case, he generates an optimal policy only for the case where lead times are equal and there is no setup cost. He adds that if there exist fixed costs for procurement and remanufacturing, these are not necessarily optimal. Kiesmüler and Scherer (2003) consider the optimal structures provided by Inderfurth (1997) and they propose one exact and two heuristic methods to find the values of policy parameters for the cases with and without stock keeping and equal lead times.

More recently, Ahiska and King (2010a) have considered a similar hybrid manufacturing/remanufacturing system. The inventory problem is modeled for both zero and non-zero lead time cases as a Markov Decision Process (MDP).Several cost scenarios are formed using the data given by Kiesmüler and Scherer (2003) as base-case scenario, and optimal policies are determined. MDP provides optimal manufacturing and remanufacturing amount decisions for each system state, which are then characterized into practical structured optimal or near optimal policies that are easy to implement. Numerical experimentation shows that the characterized policies are very close to optimal if not optimal. They take their research further in Ahiska and King (2010b) by making a product life cycle analysis. Considering the typical patterns of demand and return rates of a remanufactured product over its life cycle, considered also previously by Van der Laan and Salomon (1997a), they characterize different inventory policies for different stages of the product's life cycle and they find that appropriate policy structures are strongly dependent on the existence of setup costs for manufacturing and remanufacturing operation. A performance comparison between the newly developed policies and the PULL policy considered by Van der Laan and

Salomon (1997b) is also provided. Recently, Flapper et al. (2012) also use MDP for their problem. However their problem is different than the one considered by Ahiska and King (2010a) in the sense that they consider a system where returns go directly to the serviceable stock, because they are assumed to be already in good condition, i.e they do not need any reprocessing for reuse. They propose optimal policies for the cases where advance return and demand information are available due to warranty returns and buy back contract returns etc., and where demand and return occur unannounced. Hsueh (2011) also considers the different stages of the product life cycle. However in this paper, it is assumed that the demand and return rates are not constant during a stage. Moreover he considers a correlation between demand and return rates. Under these assumptions, optimal policies are proposed for each stage of the product life cycle.

There also exist some papers in literature with different approaches to the periodic-review inventory control of hybrid systems. For instance Foul et al. (2007) incorporate the item deterioration concept to their model, which is a generally ignored concept. They assume both remanufacturable and serviceable items are subject to deteriorate, and provide optimal policies for the two cases where the deterioration rates are known and unknown. DeCroix (2006) changes the model slightly by assuming that the procurement of the items occurs in multiple stages. The newly manufactured products are assumed to arrive to this process in the first stage, however the product recovery can take place in any stage. Thus, he compares the cases ‘upstream product recovery’, where recoverable items join the system in the first stage, and ‘downstream product recovery’, where items join the system in any other stage. He models this system as a two-stage dynamic program and finds optimal policies for both cases. Zhou et al. (2011) consider a system with different return qualities, where the returns of all type of used items are stochastic. Furthermore they assume that remanufactured items have the same quality as the new items thus there are multiple recoverable stocking points but only one serviceable inventory. They aim to find the decisions of manufacturing, remanufacturing and disposal amounts that minimize the cost. Solving the dynamic optimization problem provides an optimal policy for the case of identical lead times. For the case of non-identical lead times, a heuristic is developed and the performance of the heuristic is compared with the optimal cost obtained by dynamic program.

Furthermore they consider the sales dependent product returns case with identical lead times and they discuss that a heuristic to solve this problem is also needed, however it is not yet developed.

2.2.2 Continuous-review models

The earliest work on a continuous review model with product recovery is by Heyman (1977) who considers a single item recovery system with no fixed costs and no lead times. It is assumed that the returns are either disposed or repaired upon arrival; thus, there is no recoverable items inventory. Under these assumptions, the disposal level is optimized, which is defined as the serviceable inventory at which returned products are disposed of. Muckstadt and Isaac (1981) develop a similar model, but they disregard the disposal option of returned items. Under the assumptions of fixed outside procurement costs, non-zero but equal lead times for procurement and repair and Poisson distributed return and demand, they consider predetermined (r, Q) policy where r is the serviceable stock position at which an order of Q items is placed. They extend their results to two-echelon system where several retailers satisfy demand and a warehouse provides products to these retailers.

Van der Laan et al. (1995) propose a three parameter (s_m, Q_m, Q_r) PUSH and a four parameter (s_m, Q_m, s_r S_r) PULL policy for the inventory control of a manufacturing/ remanufacturing hybrid production system. In PUSH policy, remanufacturing decision depends on the recoverable inventory and when it reaches to a certain level (Q_r) the products are remanufactured no matter the serviceable stock amount. However in PULL policy the main objective is to remanufacture recovered items as late as possible, thus the remanufacturing decision is made when serviceable inventory is under s_r. They search for the optimal parameter values for these pre-determined policies. Even though these policies are not guaranteed to be optimal, this approach is easier to implement and therefore widely used in practice. Van der Laan et al. (1996) extend the model of Van der Laan et al. (1995) by implementing disposal option to the model. In their model, there is fixed cost for outside procurement while there is no fixed cost for remanufacturing. They consider deterministic lead time for outside procurement and stochastic remanufacturing lead time. Three control policies are considered, namely (s_p,Q_p,s_d,N), (s_p,Q_p,s_d) and (s_p,Q_p,N), which differ from each other only with respect to disposal control rule. They form a queuing system for remanufacturing by considering c parallel machines and assume that if all of the c machines are busy, the new arrivals of recoverable items are disposed. The items can be also disposed according to the rules given by the pre-determined policies. In order to calculate expected cost for pre-determined policies, they employ an analytical procedure that requires formulating and solving a continuous-time Markov chain model with two state variables: serviceable inventory position and the stock of returned items in the remanufacturing facility. They use an enumerative search procedure to find the optimal parameter values for the policies under consideration. In a further study, Van der Laan and Salomon (1997) consider a model with backorders and disposals where customer demand and returns are correlated and production and remanufacturing have non-zero deterministic lead times. They develop PUSH and PULL policies similar to ones proposed by Van der Laan et al. (1995). These policies though have an additional parameter controlling the disposal decision. Comparisons between the systems with and without disposals and between PUSH and PULL policies are made through numerical experiments. Moreover they provide a life cycle analysis, determining the different demand and return rates for different stages of a product’s life cycle. Van der Laan et al. (1999a, 1999b) consider similar models with Van der Laan and Salomon (1997). The main aim of Van der Laan et al. (1999a) is to investigate the impact of lead times. They consider both deterministic and stochastic lead times. The effect of variability and duration of lead times are observed for PUSH and PULL policies. They model the system under consideration as a continuous-time Markov chain and use an enumerative search procedure to find the optimal control parameters (or, optimal cost) for PUSH and PULL policy. Similarly, Van der Laan et al. (1999b) present an exact methodology that requires an extensive enumerative search to optimize PUSH and PULL control strategies. They compare the effectiveness of these strategies through numerical studies and conclude that PULL strategy gives better results for the majority of times. They also make a comparison between this system and the traditional production system controlled by the (s,S) inventory policy and show the benefits of manufacturing/remanufacturing hybrid system. In another study where pre-determined control policies are discussed, Van der

Laan and Teunter (2006) consider three policies namely PUSH, simple PULL and general PULL in a system where remanufacturing is less expensive than manufacturing. Their system includes set up costs, holding costs and backordering costs. They develop simple closed form expressions for calculating near-optimal parameter values for these policies, and in an extensive numerical study, they evaluate the performance of the proposed heuristics by comparing their costs to the costs associated with optimal parameter settings for policy structures under consideration. Inderfurth and van der Laan (2001) discuss that if the inventory position of remanufacturable products is determined in a proper way, than the performance of the policy can be improved considerably. They consider a 4-parameter PUSH policy and try to control the inventory position by treating remanufacturing lead time as a decision variable rather than a constant. In conclusion they propose that by varying this lead time, i.e. by changing the definition of the inventory position, system cost can be influenced positively.

Fleischmann et al. (2002) consider a basic single-echelon system where returns are assumed to be in the same quality with the new products and the returns go directly to the serviceable stock. They model this system as a continuous time Markov chain with a single state variable (inventory position) and use steady state probabilities to calculate the long run average cost in order to show the optimality of conventional (s,Q) policy. Further they propose a procedure to determine optimal parameter values for the (s,Q) policy and make a numerical study in order to investigate the effect of varying the ratio of return to demand on the optimal costs. In a later work Fleischmann and Kuik (2003) use the same model where the returns go directly to the serviceable stock, but this time they assume the demand can also be negative, in consequence of dominance of the returns. They show that this model can be easily transformed into the model where the demand is non-negative and prove the optimality of (S, s) policy using the benefits of Markov decision processes in order to calculate average cost. Through a numerical study, the effect of return flow on the system costs (fixed cost, holding cost and backordering cost) is investigated.

The work by Bayındır et al. (2005) is one of the few studies considering quality differences between newly produced products and remanufactured ones and they assume the products have different prices. Because of the quality and price differences they store the two types of products in different stocking points and they allow one–way product substitution, similar to our assumption in this paper. However they do not consider a recoverable items stock as it is assumed that a sufficient return flow is maintained allowing to reach remanufacture up to level at all times. They solve a continuous-time Markov model to find the expected profit for a given order-up-to policy for both production options. They determine the optimal policy parameters using complete search. They investigate the profitability of remanufacturing and the one-way substitution strategy through a computational analysis. More recently, Kenné et al. (2012) propose a system with three stocking points where newly manufactured and remanufactured items are again handled separately, and also returns are stocked. They consider a closed-loop supply chain model assuming deterministic demand and return rates are in correlation. Although they assume constant demand and return, they add stochasticity to the model by considering random failures and repairs of machines. Their objective is to propose a manufacturing/remanufacturing policy that would minimize the total cost. By considering the production rates of the manufacturing and the remanufacturing as decision variables they develop optimality conditions using the optimal control theory based on stochastic dynamic programming.

In this paper we consider a hybrid manufacturing/remanufacturing production system where there exist three stocking points; one for recoverable inventory and two separate serviceable inventories for manufactured and remanufacturing products. This work is inspired by a real case in automotive spare parts industry where the manufactured and remanufactured products, though having the same quality in terms of functionality, are perceived to have a different value by the customers. Hence, the two products have different demand streams and prices although remanufactured products are considered as good as the new ones. Another observation is that, in such market, lost sales costs can be huge mainly because of the contracts forcing both the automotive firms and the part manufacturers to pay fines for each day while the vehicles on warranty period are waiting for service. Therefore, the use of a production substitution strategy might be preferred in such systems. However only a small number of customers of newly manufactured products seem to accept remanufactured products. For this reason, we only consider one-way substitution that allows remanufactured product demand to be satisfied from the manufactured products stock when needed for the same price as a remanufactured one.

In most work on hybrid manufacturing/remanufacturing systems found in literature newly manufactured and remanufactured products are assumed to have same quality and same price, and thus they are both kept in a one common serviceable stock. Since selling different units with the same price does not have any effect on revenue, these papers consider a cost minimization problem rather than profit maximization. To our knowledge, models with multiple recoverable or serviceable inventories are developed by few researchers, e.g. see Kleber et al. (2002), Inderfurth (2004), Bayındır et al. (2005), Bayındır et al. (2007), Pineyro and Viera (2010), El Saadany and Jaber (2011) and Kenné et al. (2012). Inderfurth (2004), Bayındır et al. (2005), Bayındır et al. (2007), Pineyro and Viera (2010), and Kenné et al. (2012) consider the case where remanufactured and newly manufactured items are handled in separate inventory. Kenné et al. (2012) consider the multiple serviceable inventories because holding and lost sales costs are different for remanufactured and manufactured items. They formulate a cost minimization problem. Inderfurth (2004) and Bayındır et al. (2007) both consider a profit maximization problem and product substitution, however they use expected values of demand and return streams to solve the problem for a single period. Pineyro and Viera (2010) consider a profit maximization problem for deterministic demand and return rates. Bayındır et al. (2005) has the most similarity with our work in terms of stocking points, product substitution and solution methodology. However, in contrary to our paper, their model does not include a recoverable inventory as they assume sufficient return flow is available when it is needed to remanufacture. Furthermore, they consider a pre-determined order up to level strategy for manufacturing and remanufacturing and find the optimal values of the parameters for this strategy using complete search.

In this study, we consider the inventory control problem for a periodically reviewed manufacturing/remanufacturing system under product substitution. The problem is formulated as an infinite-horizon Markov decision process (MDP) model and solved to find the optimal manufacturing/remanufacturing decisions that maximize the profit. Our aim is to determine the value of production substitution when the optimal inventory policy is followed. The numerical experimentations done for this purpose are based on a real case regarding an automotive spare parts manufacturer.

3 PROBLEM DESCRIPTION AND MDP FORMULATIONS

The manufacturing/remanufacturing system under consideration has three stocking points: the recoverable inventory, the remanufactured parts inventory and the newly manufactured parts inventory. The recoverable inventory consists of the used core parts that are collected back from customers for remanufacturing. After remanufacturing, the used core items become like new, i.e. the remanufactured and newly manufactured parts are considered to have same quality. However, as a sales strategy, they are sold for different prices. Since remanufacturing a used core item is significantly less expensive than manufacturing a new item, this sales strategy sounds profitable. This system is depicted in Figure 3.1.

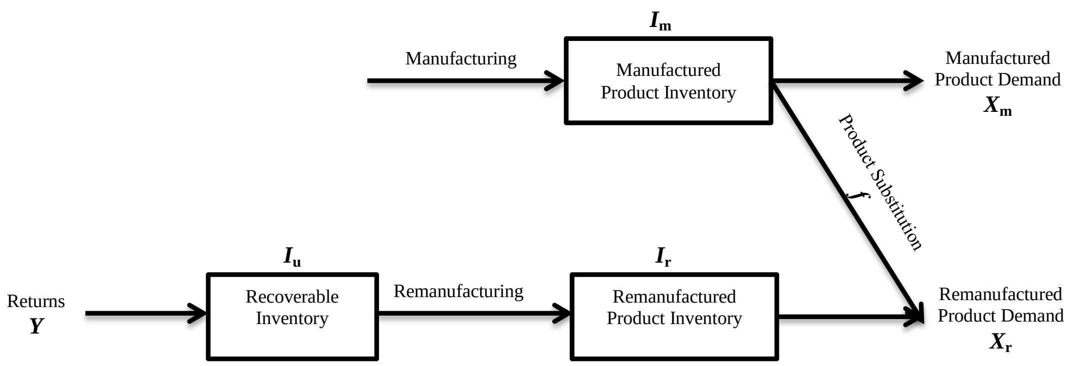

Figure 3.1: Hybrid manufacturing/remanufacturing system under product substitution

In this system, the customers are classified into two groups: type I customer always buys newly manufactured parts because of image concerns. Type II customer prefers remanufactured parts for the price advantage.

The customers of the company we are analyzing are big international Original Equipment Manufacturer (OEM) firms, including, but not limited to, Ford Otosan, Mercedes Benz Turkey, Tofaş (Fiat Turkey). The customer satisfaction is veryimportant, thus lost sales and backorder costs are assumed to be very high. In order to guarantee high customer satisfaction, the company is considering implementing a product substitution strategy where they would sell newly manufactured products at a discounted price to satisfy the demand of type II customer who wants to buy remanufactured products in case there is no remanufactured item available in stock. In order to evaluate this product substitution strategy, we formulate the inventory control problem for this manufacturing/remanufacturing system as a Markov Decision Process and determine numerically the value of this strategy for several production scenarios. The manufacturing and remanufacturing processes do take approximately the same time, so the lead times for both processes are assumed to be one period.

3.1 State Space

The state of the system in a period, denoted by S, is represented by three variables I_u , I_r , and I_m which are the inventory levels of used (i.e. recoverable), remanufactured and manufactured items respectively. These inventory levels are bounded as follows.

$$I_r^{min} \leq I_r \leq I_r^{max} \tag{3.1}$$

$$I_m^{min} \leq I_m \leq I_m^{max} \tag{3.2}$$

$$0 \leq I_u \leq I_u^{max} \tag{3.3}$$

I_j^{min} means that backordering of the demand is allowed up to $-I_j^{min}$ for j=r,m if $I_j^{min} < 0$.

3.2 Decision Space

In this system we have to make the decisions of how many units to manufacture (d_m), and to remanufacture (d_r). For each system state, we find the feasible values for (d_m, d_r) decisions as follows:

A feasible remanufacturing decision d_r is bounded by the capacities of remanufacturing process and the remanufactured product inventory, and it is dependent on the inventory levels of recoverable and remanufactured products. As a result we find that d_r can take following values: $d_r = 0,1,\ldots,d_r^{max}$ where d_r^{max} is calculated by Eq. 3.4.

$$d_r^{max} = min\{I_u, R_{max}, I_r^{max} - I_r\} \tag{3.4}$$

With a similar logic d_m takes the values: $d_m = 0,1,\ldots,d_m^{max}$ where d_m^{max} is calculated with the Eq. 3.5.

$$d_m^{max} = min\{I_m^{max} - I_m, M_{max}\} \tag{3.5}$$

In these equations R_{max} and M_{max} are the capacities of the remanufacturing and manufacturing processes, respectively.

3.3 State Transition and Transition Probabilities

Given the current state is S=(I_u, I_r, I_m), the manufacturing and remanufacturing decisions are d_r, and d_m, and demands for manufactured and remanufactured items, X_m and X_r, and return, Y take the values x_m ,x_r and y, respectively, the next state will be $S' = (I_u', I_r', I_m')$ where the calculation of I_u', I_r' and I_m' are respectively given below.

I_u' is calculated easily by considering the fact that the inventory level for used items decreases by the amount of used items sent into the remanufacturing process and increases by the amount of used items that are returned, but can not exceed the used item storage capacity, as shown by Eq. 3.6.

$$I_u' = min\{I_u - d_r + y, I_u^{max}\} \tag{3.6}$$

Unlike inventory level of used items, the inventory levels for manufactured and remanufactured items at the end of current period do not only depend on current inventories, demand for corresponding items and manufacturing and remanufacturing decisions, but also on the product substitution strategy. Hence we should first describe clearly the product substitution strategy employed in this system.

A one-way product substitution strategy is considered, which allows new items to be sold at a discounted price in order to satisfy Type II customer demand. The product substitution strategy considered here is described in detail as follows: If some of the demand for remanufactured items cannot be satisfied from remanufactured item stock, then it can be met from new item (i.e. manufactured item) stock if there is new item left in stock after satisfying all the demand for manufactured items. Under this strategy, the amount of remanufactured item demand satisfied from new item stock, i.e. the amount of substitution, denoted by f, is formulated using the following reasoning:

Clearly, if $I_r \geq x_r$ (no shortage for remanufactured items) or if $I_m \leq x_m$ (no manufactured item left in stock after satisfying demand for manufactured items), no product substitution will occur (f=0). In this case, the amount of remanufactured item demand that remains unsatisfied, denoted by l, is $l=\max\{x_r-I_r, 0\}$.

On the other hand, if $I_r<x_r$ (i.e. there is a shortage of x_r-I_r remanufactured items) and if $I_m>x_m$, then there are $I_m- x_m$ items left in manufactured item stock that can be used to deal with the remanufactured item shortage. In this case, the amount of substitution is $f=\min\{I_m-x_m, x_r-I_r\}$ and the amount of remanufactured item demand that remains unsatisfied after product substitution occurs is $l=\max\{x_r-I_r-f, 0\}$.

General formulations for f and l that cover all the 'if' conditions defined above can be formed as:

$$f = [\min\{I_m - x_m, x_r - I_r\}]^+ \quad (3.7)$$

$$l = [x_r - I_r - f]^+ \quad (3.8)$$

where $[x]^+=\max\{x,0\}$.

The substitution amount *f* and unsatisfied remanufactured item demand *l* being defined as above, the inventory levels for manufacturing and remanufacturing items at the beginning of next period are formulated as:

$$I_m^{'} = max\{I_m - x_m - f, I_m^{min}\} + d_m \tag{3.9}$$

$$I_r^{'} = max\{I_r - x_r, -l, I_r^{min}\} + d_r \tag{3.10}$$

The value of product substitution can be analyzed under the following 2 cases regarding the backordering of remanufactured items:

Case 1:

The backordering of demand for remanufactured items is not allowed (i.e. $I_r^{min} = 0$). Since there is uncertainty about when returns will occur, customers are assumed to be not willing to wait. In this case, the amount of demand for remanufactured items that remains unsatisfied after product substitution is done, *l*, becomes lost sales for remanufactured items (BO_r=0, LS_r=*l*).

Case 2:

The backordering of demand for remanufactured items is allowed up to a certain level, which is $-I_r^{min}$ where $-\infty < I_r^{min} < 0$. In this case, if $l \leq -I_r^{min}$, then backordered amount for remanufactured item is $B_r = l$ and no lost sales for remanufacturing items will occur (i.e. LS_r=0) . On the other hand, if $l > -I_r^{min}$, then $BO_r = -I_r^{\min}$ and $LS_r = l + I_r^{\min}$

The general formulations for B_r and L_r, which cover both cases described above are given below:

$$BO_r = min\{l, -I_r^{min}\} \quad (3.11)$$

$$LS_r = [l + I_r^{min}]^+ \quad (3.12)$$

$$BO_r = min\{l, -I_r^{min}\}$$

$$LS_r = [l + I_r^{min}]^+$$

Figure 3.2 gives general steps for calculating the substitution amount and the backordered and lost demand for remanufactured items.

In short,

If $I_r \geq x_r$ or if $I_m \leq x_m$ (no product substitution)

f=0, l=max{x_r-I_r, 0)

If $I_r < x_r$ and $I_m > x_m$ (there is product substitution)

f=min{I_m-x_m , x_r-I_r }, l=max{x_r-I_r-f, 0}

(If I_m- $x_m \geq x_r$-I_r

f= x_r-I_r, l=0 (Hence, BO_r=0, LS_r=0)

else

f= I_m-x_m, l=x_r-I_r-f ($B_r = min\{l, -I_r^{min}\}$, $LS_r = \max\{l + I_r^{min}, 0\}$))

In general terms,

f=[min{I_m-x_m, x_r-I_r }]$^+$, l=[x_r-I_r-f]$^+$

$BO_r = min\{l, -I_r^{min}\}$, $LS_r = [l + I_r^{min}]^+$

where $[x]^+$=max{x,0}

Figure 3.2: General description for calculating substitution amount, backordered and lost demand for remanufactured items

The transition probability from S to S' under decision (d_m, d_r), represented by $P(S, S', (d_m, d_r)$ equals the sum of the probabilities of occurrence for demands and returns, (x_1, x_2, y), that lead to transition from S to S' under the decision (d_m, d_r), as indicated by equation 3.13.

$$P\left(S, S^{\prime}, (d_m, d_r)\right) = \sum_{(x_m, x_r, y) \in A_{S \to S^{\prime}}^{(d_m, d_r)}} P(X_m = x_m, X_r = x_r, Y = y) \tag{3.13}$$

where $A_{S \to S^{\prime}}^{(d_m, d_r)}$ is the set of the values of demand for manufactured and remanufactured items and the returns (x_m, x_r, y) that make the system transition from state S to state $S^{\prime}$ under decision(d_m, d_r).

3.4 Reward Function

The reward function for this problem represents the expected profit per period. It is defined by the revenue obtained from the products minus the total cost including manufacturing and remanufacturing cost, holding costs for different stocking points, backordering cost, lost sales cost and disposal cost. The following notation is used for the reward function.

p_m: unit price for manufactured product
p_r: unit price for remanufactured product
s_m: Setup cost for manufacturing
s_r: setup cost for remanufacturing
c_m: unit manufacturing cost
c_r: unit remanufacturing cost
h_m: unit holding cost per period for manufactured product
h_r: unit holding cost per period for remanufactured product
h_u: unit holding cost per period for used product
b_m: unit backordering cost per period for manufactured product
b_r: unit backordering cost per period for remanufactured product
l_m: unit lost sales cost for manufactured products
l_r: unit lost sales cost for remanufactured products
k: unit disposal cost for used products
DSP: disposal amount for current period
LS_r: lost sales of remanufactured items for current period

LS_m: lost sales of newly manufactured items for current period

BO_r: backordered demand of remanufactured items for current period

BO_m: backordered demand of newly manufactured items for current period

Given the system is in state S, the decisions d_r and d_m are made, x_m, x_r units of demand for manufactured and remanufactured items and y units of return occur, the profit is calculated as:

$$Profit\big(S,(d_r,d_m),(x_m,x_r,y)\big) = p_r(Q_r+f)+p_m(Q_m) - \begin{bmatrix} \delta(d_r)+\gamma(d_m)+h_r\big[I_r^{\prime}\big]^+ + h_m\big[I_m^{\prime}\big]^+ + h_u I_u^{\prime} \\ +b_rBO_r+b_mBO_m+l_mLS_m+l_rLS_r+kDSP \end{bmatrix} \tag{3.14}$$

where:

Q_r and Q_m represent the amounts of remanufactured and manufactured items sold for their corresponding prices, respectively.

Cost parameters associated with the profit function of Eq. 3.14 are calculated by using equations 3.15 through 3.23.

$$Q_r = \begin{cases} x_r & if\ x_r < I_r \\ \max\{I_r,0\} & otherwise \end{cases} \tag{3.15}$$

$$Q_m = \begin{cases} x_m & if\ x_m < I_m \\ \max\{I_m,0\} & otherwise \end{cases} \tag{3.16}$$

$$\delta(d_r) = \begin{cases} s_r + c_r d_r & for\ d_r > 0 \\ 0, & for\ d_r = 0 \end{cases} \tag{3.17}$$

$$\gamma(d_m) = \begin{cases} s_m + c_m d_m & for\ d_m > 0 \\ 0, & for\ d_m = 0 \end{cases} \tag{3.18}$$

$$BO_m = \begin{cases} -max\{I_m - x_m, I_m^{min}\} & if\ I_m < x_m \\ 0 & otherwise \end{cases} \tag{3.19}$$

$$BO_r = min\{l, -I_r^{min}\} \tag{3.20}$$

$$LS_m = \begin{cases} I_m^{min} - (I_m - x_m) & if\ I_m - x_m < I_m^{min} \\ 0 & otherwise \end{cases} \tag{3.21}$$

$$LS_r = \left[l + I_r^{min}\right]^+ \tag{3.22}$$

$$DSP = \begin{cases} I_u - d_r + y - I_u^{max} & if\ I_u - d_r + y > I_u^{max} \\ 0 & otherwise \end{cases} \tag{3.23}$$

Then the expected profit to occur in a given period is calculated as:

$$E\left[Profit\left(S,(d_r,d_m)\right)\right] = \sum_{x_m}\sum_{x_r}\sum_{y} P(X_m = x_m, X_r = x_r, Y = y) * \ Profit\left(S,(d_r,d_m),(x_m,x_r,y)\right) \tag{3.24}$$

4 SOLUTION METHODOLOGY

Consider an *N* state Markov process with a transition probability matrix *P* and a reward matrix *R*. In such a system when we allow the system to make transitions for an infinite period of time, if this system is completely ergodic, then the total earnings of this system g is only dependent of limiting state probabilities (π_i), which are independent of the starting state, and immediate rewards for all states (q_i). In this case, the gain is calculated as follows:

$$g = \sum_{i=1}^{N} \pi_i q_i \tag{4.1}$$

In a sequential decision problem, decisions add another dimension to this process with rewards. A policy matrix, consisting of alternative decisions that can be made in each state, is also formed and the problem can be solved by formulating and solving multiple Markov processes, each corresponding to a given policy. Among these Markov processes, the one with the best gain is selected, and the policy used to achieve that gain is the optimal policy. However as the number of states and number of alternative decisions increase, the number of possible policies increases exponentially. Even for a modest problem with 20 states and 20 decisions, there exist 20^{20} policies and it is infeasible to calculate gains for each policy and select the best one.

Since total enumeration seems to be infeasible to solve infinite horizon MDP problems, Howard (1960) suggested a two-phase algorithm called the policy iteration method, which is proven to find the optimal policy for infinite horizon MDP problems. In the first phase of this method called *value determination*, the relative values and gain is calculated for a given policy using equation 4.2. In the second phase, named as the *policy improvement* phase, a better policy is searched by using the relative values found in the first phase. These two phases are done iteratively until two consecutive policies found are identical.

$$v_i + g = q_i + \sum_{j=1}^{N} p_{ij} v_j \qquad i = 1,2 \ldots . N \tag{4.2}$$

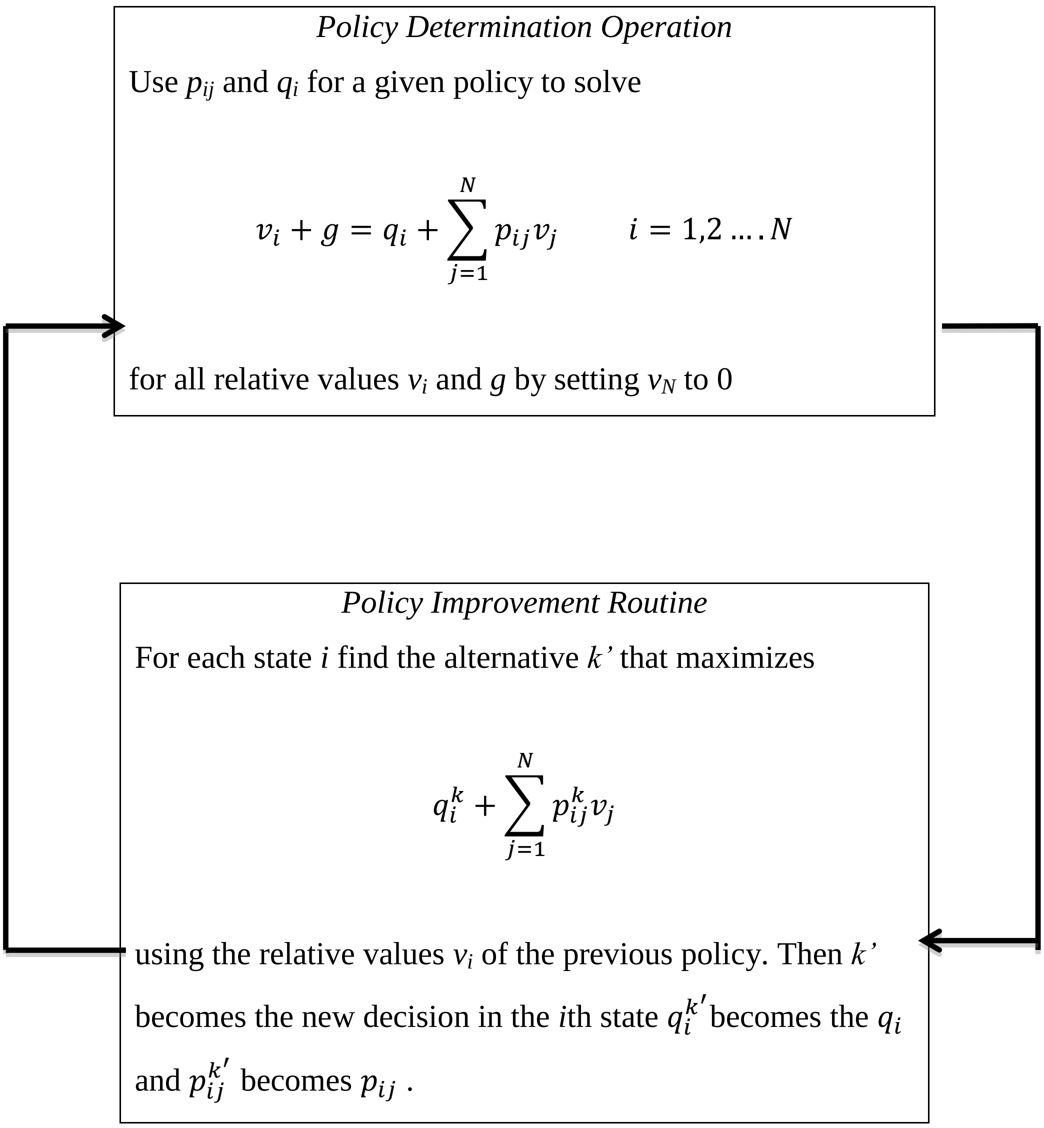

Figure 4.1: Iteration cycle in Howard's two-phase method (Howard, 1960)

Value determination phase of this method requires to solve an *NxN* set of simultaneous linear equations in order to determine the relative values, where *N* is the number of states in the MDP. When the number of states *N* is large, the computational difficulty of solving a very high number of linear equations proves Howard's traditional method is also infeasible for large scale problems.

In order to overcome the computational burden of Howard's method, some new algorithms are proposed throughout the years. One of the first papers on this problem was written by White (1963). He proved that Bellman's method of successive approximations (Bellman, 1957) could be a more efficient algorithm when rewards are discounted for each state i.e when time value of a reward is taken into account. Moreover he proposed minor extensions to (Bellman, 1957) for the case when rewards are supposed to be constant during the problem horizon, thus an average reward value is assumed. Morton (1971) suggested that although White's method is a good alternative for Howard's value determination phase, the convergence rate of successive iterations in this method is small, thus it again requires a great number of operations until the stopping criterion is satisfied. He proposed a new method called fixed policy successive approximation for a better convergence.

In this paper we use Howard's policy iteration algorithm with Morton's fixed policy successive approximation which is proposed by Zaldivar and Hodgson (1975) as a technique for rapid convergence. This method is later used to solve a variety of MDP problems (see e.g. Ding et al., 1988; Ahiska and King, 2010a, 2010b; Ahiska et al, 2013). The algorithm for this method is given below.

N: number of states

k^s: policy chosen in state *s*

q_s^k: expected period profit for state *s* when following alternative *k*

$P_{ss'}^k$: the one-step transition probability from state *s* to state s' when following alternative *k*

S: Set of all possible states

S': Set of states in reach from state s by one-step

Initialization

$k^s \leftarrow arg\max_k\{q_s^k\}$ ***/* for the initialization select the alternative that maximizes the expected profit for each state S */***

$k \leftarrow k^s$

for ∀s∈S $v_s \leftarrow 0$

$i \leftarrow 0$

Phase I. Value Determination

while i<n **do** ***/* n is predetermined number of iterations defined as "cheap iterations" in (Morton, 1971) */ */ n is set to 10 for this paper */***

for s∈S

$$v_s^{'} = q_s^k + \sum_{S^{'}} P_{ss^{'}}^k v_{s^{'}}$$

end

i=i+1

if i==n

$a \leftarrow \sum_S |v_s^{'} - v_N^{'} - v_s|$

end

$v_s \leftarrow v_s^{'} - v_N^{'}$

end

Phase II. Policy improvement

Inputs: $P_{ss^{'}}^k\ q_s^k, v_s, a, \varepsilon.$

Outputs: v_s, k

for $s \in S$

$$k_S' \leftarrow arg\max_k\left\{q_s^k + \sum_{S^{'}} P_{SS^{'}}^k v_{S^{'}}\right\}$$

end

$K^{'} = (k_S^{'})$ ***/*new policy */***

for $s \in S$

$v_S' = q_S^k + \sum_{S'} P_{SS'}^{k_S'} v_{S'}$

$v_S \leftarrow v_S' - v_N'$

end

if $a < \varepsilon$ OR $K == K'$

do break: ***/* stopping criteria are satisfied*/***

else $K \leftarrow K'$ and go to Phase I

end

These two phases of this algorithm is conducted successively until one of the two stopping criteria are satisfied. These conditions can also be described as follows:

1. the relative values found in the value determination phase converge

$$\sum_S |v_S' - v_N' - v_S| < \varepsilon$$

ε is a small number which is implemented to determine convergence. For this paper it is taken as 10^{-8}.

2. the two consecutive policies found in the policy improvement phase are identical.

This algorithm is programmed using MATLAB for solving our decision problem. A detailed description of the problem is given and results are discussed in Chapter 5.

5 NUMERICAL STUDY

Automotive manufacturers are responsible for providing specific spare parts to their vehicle owners for a fix period of time known as the protection period, which is generally 8-10 years. This requirement forces in turn the original spare parts manufacturers to keep producing these spare parts during this period. However, as the automotive sector has been improving very fast due to the advances in the technology, the cost of keeping specific spare parts available increases significantly after a few years from the start of the protection period due to the decrease in demand quantities and increase in the cost of maintaining older technologies. These facts motivated some automotive spare part manufacturers to add a remanufacturing process into their production system, which significantly reduces the cost of producing a part due to the savings in materials obtained by using the returned parts instead of virgin materials.

We can analyze the life cycle of a standard spare part, which is in the protection period, in three phases. In the first 2-3 years usually the breakdowns are very limited, thus returns are very low. As a result producers produce only new parts, and usually more than the actual demand in order to sufficiently feed the market for a better return cycle in the future. In the following 4-5 years returns start to accumulate and firms start also using remanufacturing. Finally in the last years of the protection period, generally the demand decreases significantly due to technology change and firms tend to switch solely to remanufacturing.

Here we consider the inventory control problem for the second phase of the spare part life cycle. We analyze the problem through the hybrid manufacturing/remanufacturing system of an existing automotive spare parts manufacturer. Due to privacy concerns, the identity of the firm will be kept anonymous in this study. Remanufacturing efforts started in this firm in 1960s in more developed countries like Germany and Australia mainly to reduce costs for more technological and expensive parts such as injectors, fuel pumps and alternators. However in the last years company also used remanufacturing in order to increase their sales by segmenting remanufactured products in a different market and in order to decrease stock-out situations.

Despite the fact that the used spare parts usually become like new after remanufacturing, due to possible doubts that customers might have regarding the quality of the remanufactured parts, the firm sells a remanufactured part at a lower price than a new part. Since remanufacturing is significantly less expensive than manufacturing, this sales strategy is still profitable. This fact allowed the firm to use remanufactured products to enter another market, where customers are price-sensitive. This is the main motivation behind the firm's decision of implementing remanufacturing in Turkey.

The company's 50 years of experience on remanufacturing shaped their control policies so far. One of their findings about control of this system is that, due to the vigorous competition in the sector, the lost sales due to stock-out situations result in losing customers and damage to the image of the firm in the market. Hence, customer satisfaction is very important, and in order to guarantee a high level of customer satisfaction, the company is considering implementing a stock-out based product substitution strategy according to which a manufactured part is sold at the price of the remanufactured part (i.e. a lower price) to avoid losing the customers of the remanufactured parts in case of a stock-out. This policy was adopted intuitively, because of the firm's determination to maintain good relations with their customer base.

The customs laws and regulations of Turkey forbid the importation of remanufactured or repaired products. This leads the firm to implement remanufacturing process to their manufacturing plant in Turkey. We investigate the profitability of this stock-out based product substitution strategy for three of the products produced by the firm which also have significant sales in Turkish market. The firm is a world leader in diesel injection systems. Hence, two products are chosen from this product family. Product 1 is an "injector nozzle", which allows the pressurized fuel to be injected into the engine. The pressure on these nozzles can get very high, thus these are the parts of injectors that are most vulnerable to breakdowns. Even though they are relatively cheap parts, the fact that their production amounts are large due to frequent breakdowns makes this product worth to analyze. Product 2 is a “common rail injector”, which is one of the latest technologies in the fuel injection systems. Common rail injectors are electrically operated valves which accurately control the quantity of fuel delivered, resulting in fuel efficiency in engines. Product 3 is an “engine starter”, which is a type of electric motor. Electric motors require solid know-how regarding remanufacturing. This product family was among the firm’s first production, and a better service level for this product is considered to be prestigious by the firm. A product substitution strategy might improve service levels for these products.

The demands for manufactured and remanufactured items and the returns over the years 2005-2011 are collected for each product. The demand/return data are then scaled for confidentiality. Table 5.1 includes the probability distributions obtained from the scaled demand and return data for the three products considered and Figures 5.1, 5.2 and 5.3 show the plots of the distributions of the scaled data.

Table 5.1: Probability distributions obtained from the scaled demand and return data for the three products

Level i	Product 1			Product 2			Product 3		
	$P(X_m=i)$	$P(X_r=i)$	$P(Y=i)$	$P(X_m=i)$	$P(Y=i)$	$P(Y=i)$	$P(X_m=i)$	$P(Y=i)$	$P(Y=i)$
0	0.440	0.345	0.714	0.369	0.274	0.714	0.440	0.297	0.762
1	0.512	0.500	0.095	0.476	0.357	0.024	0.500	0.381	0.000
2	0.024	0.131	0.143	0.119	0.274	0.060	0.036	0.286	0.083
3	0.012	0.012	0.012	0.024	0.083	0.071	0.024	0.036	0.071
4	0.000	0.012	0.012	0.012	0.012	0.083			0.060
5	0.012	0.000	0.024			0.048			0.024
Mean	0.66	0.85	0.59	0.83	1.20	0.93	0.64	1.06	0.74

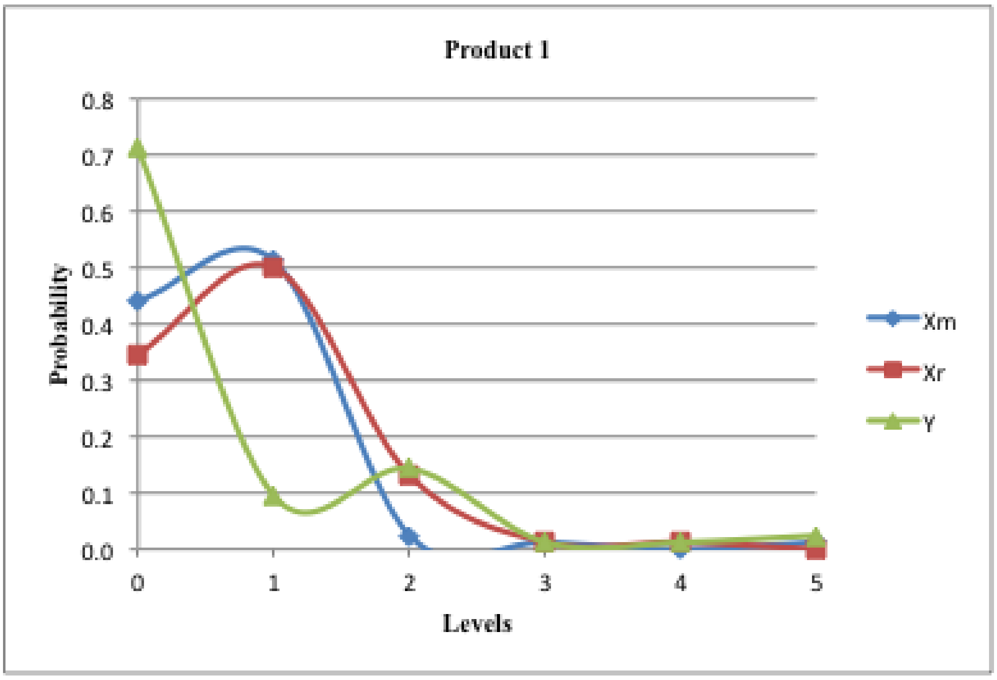

Figure 5.1: Demand and return distributions for Product 1

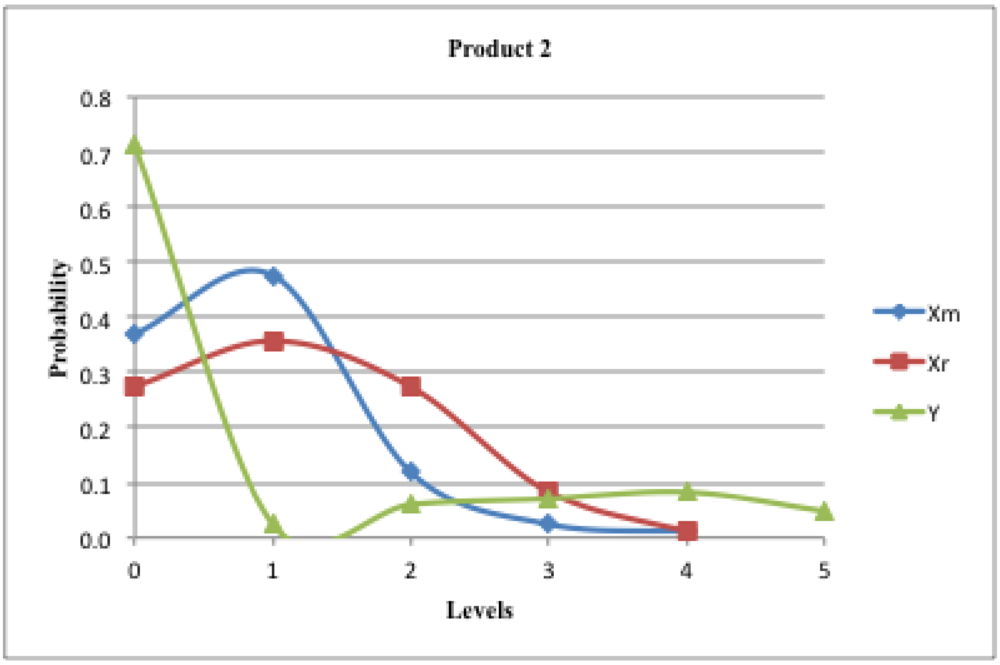

Figure 5.2: Demand and return distributions for Product 2

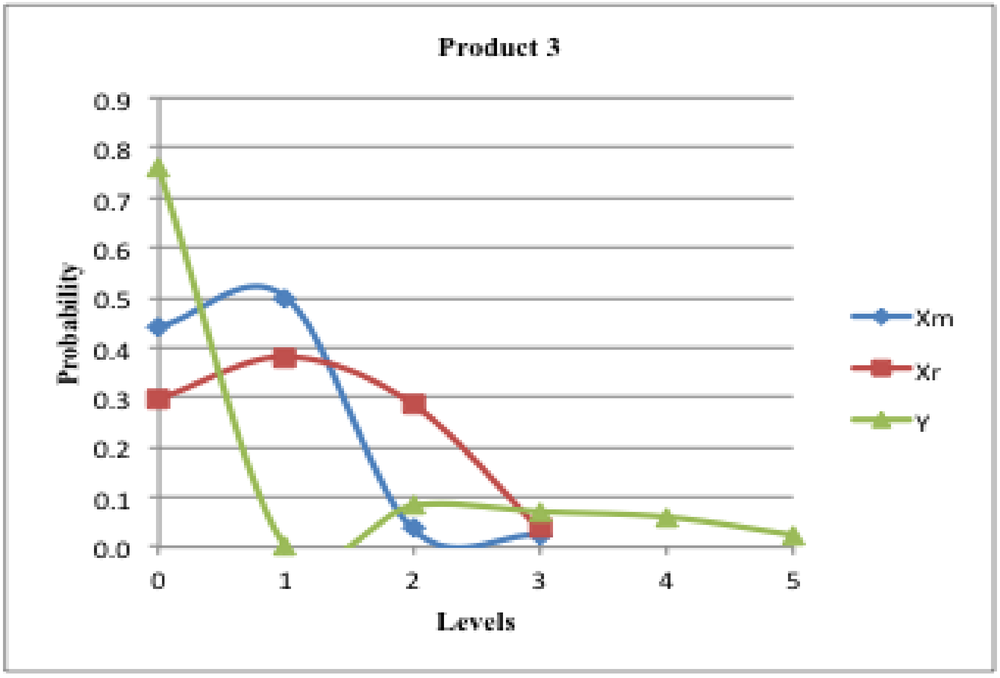

Figure 5.3: Demand and return distributions for Product 3

Backordering of the manufactured item demand is allowed up to a certain level while backordering of the remanufactured item demand is not allowed due to the risks associated with receiving returns when needed. If some remanufactured item demand remains unsatisfied after the substitution is done, then this demand is lost. Unit backordering cost for manufactured product per period is calculated as 20% of its unit price while unit lost sales cost (cost of goodwill loss) for both manufactured and remanufactured products are calculated as 25% of the corresponding unit price. The annual holding costs for unit manufactured and remanufactured items are calculated as 20% of the corresponding unit cost, and the holding cost for a used item is considered to be half of the holding cost for a remanufactured item.

Manufacturing and remanufacturing lead times are both one period. No set up costs exists for either production option. The unit price and cost data (in euros) for the manufactured and remanufactured items regarding the three products are reported in Table 5.2.

The inventory control problem for the firm's manufacturing/remanufacturing system is formulated as a Markov Decision Process as indicated in the previous section, and solved to find the optimal expected profits per period under substitution and no substitution, respectively, for the three products. The value of product substitution for the three products can be clearly seen from Table 5.2. The improvement in expected profit that the firm gains by using the substitution strategy is 6%, 17% and 28% for products 1, 2 and 3, respectively. These results should encourage the firm to use the product substitution strategy since it adds to the firms profit along with improving the service level by reducing the expected lost sales for remanufactured parts.

Table 5.2: Expected profits for substitution and no substitution cases for the three products

Product	p_m	c_m	p_r	c_r	Case	Profit (€/period)	Improvement in Profit
1	20.48	6.11	12.51	3.12	Substitution No Substitution	15.80 14.88	6%
2	77.94	25.17	57.53	16.36	Substitution No Substitution	87.23 74.69	17%
3	68.39	22.74	51.85	17.46	Substitution No Substitution	61.35 48.10	28%

The difference between values of substitution for different products can be explained by two main factors. In order substitution to be profitable, demand for remanufactured products must exceed demand for manufactured products, and there must be sufficient return to satisfy the remanufactured demand. As can be seen from the figures 5.1, 5.2 and 5.3, products 2 and 3 satisfy these conditions and the increase of their profit by substitution is intuitively explainable. However, although Product 1 has similar distributions for new and remanufactured items, and its returns are less than the other products, it is still profitable to use substitution. This is a huge motivation for the firm to implement substitution strategy.

Furthermore in case of a substitution, a manufactured item is sold at the price of a remanufactured item, thus the unit profit made by substituting a remanufactured item with a manufactured item is simply the difference between the unit price of a remanufactured item and the unit manufacturing cost ($p_r - c_m$). Hence, one would expect that the profitability of the substitution strategy is directly affected by both the unit price of a remanufactured item and the unit cost of a manufactured item. In order to see the effects of these parameters on the value of substitution, experimentation is done by changing the p_r / c_m ratio from 0.5 to 2.5 with an increment of 0.5 by using the product 3 data as the base-case scenario. The percent improvements in profit obtained by substitution are reported in Table 5.3.

Table 5.3: % improvement in profit by substitution as p_r/c_m ratio changes for product 3

Scenarios for Product 3	p_r / c_m	p_r	c_m	$p_r - c_m$	Case	Profit (€/period)	Improvement in profit
Base-case	2.28	51.85	22.74	29.11	Substitution	61.35	28%
					No Substitution	48.10	
1	2.50	56.85	22.74	34.11	Substitution	66.64	30%
					No Substitution	51.40	
2	2	45.48	22.74	22.74	Substitution	54.62	24%
					No Substitution	43.91	
3	1.50	34.11	22.74	11.37	Substitution	42.59	17%
					No Substitution	36.42	
4	1	22.74	22.74	0.00	Substitution	30.62	6%
					No Substitution	28.94	
5	0.50	11.37	22.74	-11.37	Substitution	18.86	-12%
					No Substitution	21.45	

We observe from Table 5.3 that as p_r/c_m ratio increases, the percent improvement in profit by substitution increases (from -12% to 30%), as one would expect. On the other hand, an observation which is not so intuitive is that when the p_r/c_m ratio equals 1, i.e. the unit profit by substitution is zero, the use of substitution still increases the overall profit (by 6%). Furthermore, even when p_r/c_m =0.5 (i.e. the unit profit of substitution is negative), the profit still remains positive (18.86€/period), i.e. the substitution is still profitable, although the substitution decreases the overall profit by 12% compared to no substitution case. If the profit is reduced, then it raises the question as to how much profit a company is willing to lose in order to improve customer satisfaction.

The firm usually encounters with stock-out situations and because of their contracts with OEM firms, they have to pay compensations for every day they backorder the customer demand. Hence they are very eager to make decisions based on trustable analysis and our observations on different p_r/c_m ratios give them an important managerial insight for making benefit-cost analysis.

6 CONCLUSION

We consider the inventory control problem for a stochastic periodically reviewed manufacturing/remanufacturing system with one way product substitution. In this system, a remanufactured item is not considered to have the same quality specifications as a manufactured item, and it has a lower selling price and a different customer profile. In case of a stock-out for remanufactured items, a remanufactured item is substituted with a manufactured item in order to avoid losing the customer for the remanufactured item. The problem is formulated as a discrete-time Markov Decision Process in order to find the optimal inventory policies for both with and without product substitution. The profitability of using the product substitution strategy is investigated through a numerical study based on real data from an automobile parts manufacturer. The optimal profits are calculated for three products for both substitution and no substitution cases. Results show that the use of a product substitution strategy improves the profits for all three products. Further experimentation is done to see the effects of changing the ratio of remanufactured item price to the unit manufacturing cost (i.e. p_r/c_m ratio) on the improvement in profit by substitution. As one would expect, as the p_r/c_m ratio increases, the percent improvement in profit that is obtained by using the substitution increases. A less intuitive result is that even when the p_r/c_m ratio is less than 1 (i.e. the unit profit by substitution is negative), the use of substitution can be still profitable although it might not necessarily be as profitable as not using the substitution. These results should encourage the firm to use the product substitution strategy since it mostly increases the firm's profit along with improving the service level by reducing the expected lost sales for remanufactured parts.

As a further work, a comprehensive experimentation can be done to see the effects of several system parameters (such as cost parameters, means of demand/return distributions, etc.) on the value of product substitution. It would be also very useful to perform an analysis to determine easy-to-implement policy structures with a few control parameters which characterize well the optimal inventory policies under product substitution through the means of Markov decision analysis.

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BIOGRAPHICAL SKETCH

Emre Kürtül was born in Gebze on August 23, 1988. He was graduated from Burak Bora Anadolu Lisesi in 2005 and received his B.Sc. degree in Industrial Engineering from Galatasaray University in 2010. Since 2010 he is working through the completion of M.Sc. program in Industrial Engineering at Galatasaray University. This thesis was written in order to fulfill the requirements for his graduation under the supervision of S. Şebnem Ahıska. His research interests include Markov Decision Processes and their applications on hybrid manufacturing systems and inventory control problems.