INVENTORY ANALYSIS FOR A MANUFACTURING/REMANUFACTURING SYSTEM UNDER PRODUCT SUBSTITUTION

(ÜRÜN İKAMESİ OLAN BİR ÜRETİM VE YENİDEN ÜRETİM SİSTEMİ İÇİN ENVANTER ANALİZİ)

by

Fethullah GÖÇER, B.S.

Thesis

Submitted in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE

in

INDUSTRIAL ENGINEERING

in the

INSTITUTE OF SCIENCE AND ENGINEERING

of

GALATASARAY UNIVERSITY

Jan 2014

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Date of Submission : Jan 3, 2014

Date of Defense Examination : Jan 21, 2014

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ACKNOWLEDGEMENTS

I would like to express my sincere gratitude and appreciation to my thesis supervisor Assistant Professor S. Şebnem Ahiska for her guidance, kind advices, encouragement, courtesy and patience throughout this study. I am grateful to her valuable comments and constructive suggestions regarding my thesis.

I would also like to appreciate Dr. Russell E. King for the time, guidance and effort he set aside to help where we need his expertise.

Finally, I would like to extend thanks to my family, especially to my parents Kudret and Cuma Göçer for their support and encouragement to pursue my desire. Without the help, encouragement and love of my family, this work could not ever been possible.

Jan 2014
Fethullah GÖÇER

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LIST OF SYMBOLS

M	Manufacture
R	Remanufacture
Y	Returns
I_u	Recovered Item Inventory
I_m	Manufactured Item Inventory
I_r	Remanufactured Item Inventory
X_m	Manufactured Item Demand
X_r	Remanufactured Item Demand
f	Substitution
d_m	Decision to Manufacture
d_r	Decision to Remanufacture
S	State
p_m	Unit Price for Manufactured Product
l_m	Unit Lost Sales Cost for Manufactured Products
p_r	Unit Price for Remanufactured Product
l_r	Unit Lost Sales Cost for Remanufactured Products
s_m	Setup Cost for Manufacturing
k	Unit Disposal Cost for Used Products
s_r	Setup Cost for Remanufacturing
DSP	Disposal Amount for Current Period
c_m	Unit Manufacturing Cost
LS_m	Lost Sales of Manufactured Items for Current Period
c_r	Unit Remanufacturing Cost
LS_r	Lost Sales of Remanufactured Items for Current Period
h_m	Unit Holding Cost per Period for Manufactured Product
BO_m	Backordered Demand of Manufactured Items for Current Period

h_r	Unit Holding Cost per Period for Remanufactured Product
h_u	Unit Holding Cost per Period for Used Product
BO_r	Backordered Demand of Rem. Items for Current Period
b_m	Unit Backordering Cost per Period for Manufactured Product
b_r	Unit Backordering Cost per Period for Remanufactured Product
Sub.	Substitution
Prod	Product
Rem.	Remanufactured
Opt.	Optimal
O-W	one-way
T-W	two-way
S.P.	Single Product
T.P.	Two Products
M.P.	Multiple Products
Stch.	Stochastic
Det.	Deterministic
R.S.N.	Remanufactured Item Substitute New Item
S.E.A.	New and Remanufactured Items Substitute Each Other
A.G.A.N.	As Good As New
R.I.V.	Remanufactured Items has Inferior Value than New Items
S	Single Period
M	Multiple Period
C.R.	Continuous Review
L.O.P	Less Than One Period
O.P.	One Period
N.O.	Near Optimal
O.	Optimal
N/M	Not Mentioned
T_m	Target Value for Manufactured Item Inventory
T_r	Target Value For Remanufactured Item Inventory
T_s	Secondary Target Value for Remanufactured Item Inventory ($T_s < T_r$)

T_{m_max}	Maximum Value for Manufactured Item Inventory
g_r	Remanufacturing Amount Required for the Target Remanufactured Item Inventory
g_m	Manufacturing Amount Required for the Target Manufactured Item Inventory
g_s	Supplemental Manufacturing Amount Required in case of Used Item Shortage
C_u	Underage Cost
C_o	Overage Cost
CF	Critical Fractile
GS	Greedy Search
D1	Distance-1 Local Search

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ABSTRACT

We consider the inventory control problem for an infinite-horizon stochastic hybrid manufacturing /remanufacturing system with product substitution under stochastic demand and returns. Remanufactured and manufactured products are considered as two different products, having different costs and selling prices as well as separate demand streams. Remanufactured products have a higher stock out risk than manufactured products because the remanufacturing capacity is mainly dependent on the amount of returns available for remanufacture. One way to cope with the stock-out issue for remanufactured products is to use a downward substitution strategy, which allows a manufactured product (i.e. higher value item) to be substituted for a remanufactured product (i.e. lower value item) in case the latter runs out of stock. We formulate this problem as Markov Decision Process in order to determine the optimal manufacturing and remanufacturing decisions under product substitution, and through numerical experimentation, we investigate the effects of stochastic demand/return distributions on the profitability of the substitution strategy. The optimal policy determined by MDP has a complicated structure which is very hard to be represented using a few control parameters. We propose several simple-structured intuitive heuristic policies that are easy to implement in practice. Then, we develop several heuristic search methodologies to determine the parameter values for these policies. We evaluate the performance of these techniques with respect to the solution quality as well as computational time.

RÉSUMÉ

On considère le problème de control d'inventaire pour un système hybride de fabrication/refabrication à horizon infini avec la substitution de produit à l'existence de demande et produits retournés stochastiques. Les produits fabriqués et refabriqués sont considérés comme deux produits différents, ayant des coûts et des prix de vente différents ainsi que les flux de demande distincts. Les produits refabriqués ont un risque de rupture de stock supérieur à celui des produits fabriqués parce que la capacité de refabrication dépend principalement de la quantité des produits retournés disponibles pour la refabrication. Une façon de résoudre le problème de rupture de stock pour les produits refabriqués est d'utiliser une stratégie de substitution downward, ce qui remplace un produit refabriqé (c'est à dire le produit de la valeur plus basse) par un produit fabriqué (c'est à dire le produit de la valeur plus haute) au cas où les produits refabriqués sont en rupture de stock. On formule ce problème comme le processus de décision de Markov (MDP) afin de déterminer les décisions de fabrication et refabrication sous la substitution de produit, et par l'expérimentation numérique on étudie les effets des distributions stochastiques des demandes et des produits retournés sur la rentabilité de la stratégie de substitution. La politique d'inventaire optimale déterminée par le modèle de MDP a une structure complexe qui est très difficile d'être représentée à l'aide de quelques paramètres de contrôle. On propose des politiques d'inventaire heuristiques intuitifs à structure simple qui sont faciles à utiliser dans la pratique. Ensuite, on développe des méthodes heuristiques pour déterminer les valeurs des paramètres de ces politiques. On évalue la performance de ces techniques par rapport à la qualité de la solution ainsi que le temps de calcul.

ÖZET

Ürün ikame stratejisinin kullanıldığı bir stokastik melez üretim/yeniden üretim sistemine ilişkin stok kontrol problemi ele alınmıştır. Bu sistemde ürünlere ilişkin talepler ve geri dönen ürünler stokastik dağılıma sahiptir. Yeni ve yeniden üretilmiş ürünler, farklı maliyetlere ve farklı satış fiyatlarına sahip ve ayrı müşteri kitlelerine hitap eden iki farklı ürün olarak kabul edilmektedir. Yeniden üretim kapasitesi büyük oranda yeniden üretim için gerekli olan geri dönen ürün miktarına bağlı olduğundan yeniden üretilmiş ürünler yeni üretilmiş ürünlere oranla daha yüksek stok dışı kalma riskine sahiptir. Yeniden üretilmiş ürünlerin stok dışı kalma sorunu ile başa çıkmanın yollarından biri, bu ürün stoğunun tükenmesi durumunda, yeni üretilmiş ürünle (yani, yüksek değerli ürünle) yeniden üretilmiş ürünün (düşük değerli ürünün) ikame edilmesine izin verilmesi anlamına gelen, aşağı yönlü ikame stratejisini kullanmaktır. Bu problem, ürün ikamesi durumunda en iyi üretim ve yeniden üretim kararlarını belirlemek için Markov karar süreci (MKS) olarak formüle edilmiştir, ve sayısal deneylerle stokastik talep ve geri dönen ürünlerin dağılımlarının ikame stratejisinin karlılığı üzerine etkileri incelenmiştir. MKS tarafından belirlenen optimum politika birkaç kontrol parametresi kullanarak temsil edilmesi çok zor olan karmaşık bir yapıya sahiptir. Bu çalışmada, pratikte uygulaması kolay olan basit yapılı birkaç sezgisel politika önerilmiştir. Ayrıca, bu politikaların parametrelerinin değerlerini belirleyen birkaç sezgisel arama yöntemi geliştirilmiştir. Bu yöntemlerin performansı, hem çözüm kalitesi hem de çözüm süresi dikkate alınarak değerlendirilmiştir.

1. INTRODUCTION

In the traditional way of producing an item, the manufacturers use only virgin raw materials and parts during the manufacturing process. Once the products' ownership is transferred to the customers, it is usually the customer's responsibility to return or dispose the products at the end of their usable life. However, nowadays, more and more manufacturers are collecting back their products from customers after usage or at the end of their life due to both environmental regulations and concerns as well as the potential economic benefits of product recovery. Product recovery, especially remanufacturing, can substantially reduce the resource consumption and waste disposal, which consequently results in savings in material, energy and disposal costs.

Remanufacturing can be profitable for the producers in several other ways. For manufacturers such as Xerox, HP, Bosch, and Cummins, etc., remanufactured item sales has decreased the sales rate of the new item if offered in the similar markets (Debo et al., 2006), which consequently decreased the consumption of natural resources. During the early years of the remanufacturing operations, manufacturers gave importance to only savings in costs. Later, when many governments made environmental laws and regulations, many manufacturers started to incorporate product recovery activities into their manufacturing systems where a significant portion of production uses recovered material. As product returns increase, the profitability of operating hybrid recoverable manufacturing systems increase (Robotis et al., 2005). While manufacturers often consider remanufacturing as an obligation forced by government regulations, in recent years, they have also realized that customers may also prefer remanufactured products for the price advantage as well as environmental awareness.

In this thesis, we consider the inventory control problem for a stochastic hybrid manufacturing /remanufacturing system with product substitution in an infinite-horizon stochastic demand and return settings. Remanufactured products may have an inferior value from customers' point of view and may be seen as not good as new products, which lead to different selling prices and different demand streams for manufactured and remanufactured products and thus creates a segmented market. Remanufacturing capacity is mainly limited by the amount of returns which is usually not under the control of the manufacturer. The stock-out situation for remanufactured products can be coped with a substitution strategy according to which the remanufactured product demand is satisfied using new products.

In most studies on hybrid manufacturing/remanufacturing systems, the manufactured and remanufactured items are assumed to be alike; therefore they are stored in the same serviceable inventory and have a common demand stream. In some cases though, the perceived quality of a remanufacturing item may have an inferior value from customers' point of view. Therefore, a customer may be willing to pay more for the new item than for the remanufactured one. In this case, a remanufactured item is seen as not good as a new item, which creates a segmented market among manufactured and remanufactured items. When manufactured and remanufactured items are non-identical, product substitution may be done in case of any stock-out situations of manufactured/ remanufactured inventories. The substitution type varies according to the process whether it is driven by customers or manufacturers. Manufacturer-driven substitution strategy is usually a 'downward substitution', which is a one-way substitution according to which a higher-value item is substituted for a lower-value item in case the inventory for lower-value item runs out of stock. This strategy is commonly used by automotive spare part manufacturers for example for injectors and engine starter (Ahiska et al., 2013a). Similarly, an 'upward substitution' refers to the case where a customer who demands a newly manufactured product ends up accepting a remanufactured product. Customer driven process is known as two-way substitution, according to which when a customer's first-choice product turns out to be out-of-stock, he/she ends up accepting to buy another alike product within same category (Huang et al., 2011).

In this study, we investigate how the profitability of the substitution strategy is affected as the means of stochastic manufactured item and remanufactured item demand and used item return distributions vary. Our research extends the earlier researches by solving to optimality a periodically reviewed stochastic hybrid manufacturing and remanufacturing system under product substitution. The optimal inventory policies are determined using Markov decision process (MDP). An optimal policy found by MDP is a list of optimal manufacturing/ remanufacturing decisions for every system state, which may be a very long list considering there may be thousands of states even for moderate-size problems. This list of decisions, although optimal, does not provide a direct insight into the structure of optimal policy and it may not be considered practical to use. Therefore, we propose several easy-to-implement simple-structured inventory policies to control the hybrid system under product substitution strategy. We develop two heuristic search algorithms to determine the values of the control parameters for these policies, and we evaluate the performance of the proposed inventory policies and algorithms using real data for three products produced by a spare parts manufacturer.

The rest of this thesis is organized as follows. In section 2, we briefly discuss the existing studies related to the inventory control problem of hybrid manufacturing and remanufacturing systems focusing on the product substitution and inventory policies. Section 3 describes the problem under consideration and provides the discrete-time Markov Decision Process formulation of the problem. Section 4 includes the numerical experiments and results for demand and return distributions on profitability of substitution. The base-case data for these experiments are collected from an automotive spare parts manufacturer. In section 5, three easy-to-implement heuristic inventory policies are proposed for the hybrid system under substitution. Then, two heuristic search algorithms are described for determining the control parameters of the inventory policies. The performances of the inventory policies determined by the algorithms are compared using real data for three products produced by the spare part manufacturer. Finally, we conclude our study in section 6 and mention the limitations of this study and possible directions for future work.

2. LITERATURE REVIEW

In this chapter, we review the studies in the literature related to the inventory control of the hybrid systems. The problem of inventory control for the hybrid manufacturing and remanufacturing systems recently has gained a growing interest in the literature. Instead of just selling the products and waiting for them to be wasted, many producers have started to take back the used items after usage and recover them through several product recovery options such as remanufacturing due to environmental concerns as well as the political obligations and economic benefits. The potential benefits of the remanufacturing led to many different research areas and gave encouragement to new researchers. The inventory control problems are described as systems having two types of supply modes in order to satisfy the customer demand: manufacturing of new items and remanufacturing of returned items. The production planning and control of new and remanufactured items focuses on the effective utilization of resources in order to satisfy customer demand in an efficient manner. There are mainly three types of inventory in the hybrid systems where the new and remanufactured items are not considered having the same quality, which consist of new products, returned products and remanufactured product inventories.

In this thesis, we categorize the studies in the literature regarding inventory models for recoverable manufacturing systems in several ways. One categorization is deterministic models versus stochastic models. In deterministic models, we have considered the stationary and dynamic demands while the stochastic models are organized into two categories: periodic review and continuous review. All of the above classifications are also being assessed according to whether or not product substitution is considered in the hybrid system. We briefly review the findings and suggestions in the existing literature in the following sections.

2.1 No Substitution Cases

The continuous-review version of the inventory problem with product returns has received a lot of attention. Kenne et al. (2012) deal with manufacturing and remanufacturing policies that minimize the holding or backlog costs using optimal control theory based on a stochastic dynamic program. They find optimal control decisions in order to manage the serviceable inventory in their model. Aras et al. (2004) implement a continuous- review base stock policy in their model. Their study is focused on the stochastic nature of product returns and in particular, the variability in the condition of the returns. They also use another approach in order to evaluate the impact of quality-based categorization of returned products and the incorporation of returned product quality in the remanufacturing and disposal decisions and conclude that usually putting higher priority to higher quality returns in remanufacturing is a better strategy. Padakala (2008) solves the continuous-review base stock policy inventory problem in a manufacturing and remanufacturing hybrid system managing a fleet of products in service that are condition monitored at discrete intervals. This hybrid system is defined as a product-based service scenario and demand occurs one at a time, remanufacturing leadtimes are known, remanufacturable products are pulled 'one at a time' into the remanufacturing process and demand is approximated with a continuous distribution. According to this policy, when a product is out of service, another product 'as good as new' is offered to the customer from the inventory. As soon as a replacement is made in the inventory, a product is pulled into the remanufacturing process to replenish the serviceable inventory. Dungbo et al. (2006) consider a hybrid production system simultaneously satisfying the customer demand with both manufactured and remanufactured products. Remanufactured products are considered as good as new products therefore they are kept in the same serviceable inventory. They formulate the problem as Markov quasi-birth-death (QBD) processes to find optimal inventory control policy. Korugan and Cadirci (2008) worked on the single-stage hybrid production system with random returns under a pull-type production control policy. They have developed a stochastic model for four pull control policies which are Kanban, Base stock, Generalized Kanban and Extended Kanban control systems. Their aim is to compare the performance of these policies in a hybrid manufacturing and remanufacturing system using continuous time Markov chains.

There are also some other studies which take into account no set up costs and zero or identical lead times for manufacturing and remanufacturing. Van Der Laan and Teunter (2006) analyze the optimal policy structure for the joint manufacturing and remanufacturing inventory system where remanufacturing is the cheaper alternative with unit products and demand returns. There is no setup, holding and backorder cost with non-zero manufacturing and remanufacturing lead times. They establish a closed form formula to calculate optimal or near optimal policy parameters for push and pull remanufacturing policies. Kiesmüller (2003) address inventory control problem for stochastic hybrid manufacturing and remanufacturing systems where manufacturing and remanufacturing have different leadtimes. He approaches this stochastic problem using a pull policy and improves the system performance by a new approach for the control of stochastic system.

Most of the studies regarding both newly manufactured and remanufactured products assume that both products have equivalent value and therefore customers are regarded as being indifferent to which product to buy. Since remanufactured and new items are considered to have equal value from customer's point of view, inventory of both items are kept in the same serviceable stocking point, therefore coordination of the remanufactured and manufactured products are a necessity since the manufacturing and remanufacturing decisions affect both the same serviceable stocking point. Several studies have been carried out to find policies that minimize the cost associated with periodic review stochastic hybrid systems. Arifoglu and Ozekici (2010) evaluate a periodic review single product inventory model with fixed finite supplier capacity and random yield in a random environment to identify the optimal policy using partially observed Markov decision process. They consider the problem in three cases regarding the planning horizon such as single period, multiple periods and infinite number of periods in order to show that a state dependent modified inflated base stock policy is optimal. Chou (2013) conducts a case study about an inventory system under periodic review where two types of products share a common hardware platform with different installation cost, different selling price and different customer demand while source for the products comes from two different types of software. Their optimal policy is order-up-to policy in each scenario under one and multiple periods.

When no substitution is considered, there are abundant source of knowledge about the periodic review policies of hybrid manufacturing and remanufacturing problems in recent literatures. Wang et al. (2011) provide a hybrid manufacturing and remanufacturing systems with stochastic demand and return for products with a short life cycle. They find optimum policy to minimize the total cost of hybrid systems by means of theoretical analysis and numerical experiments. Nakashima et al. (2004) follow a Markov decision process to find optimal control problem of a remanufacturing system by considering two types of inventories: the actual product inventory at the factory and virtual inventory at the hands of the customers. They have obtained the optimal production policy by using the Markovian policy iteration method. Kiesmüller and Scherer (2003) have shown a numerical example by investigating a fast algorithm to provide a method for exact computation of the optimal periodic policy parameters for a stochastic one product recovery system. Despite the availability of high technology modern computer system, solving dynamic demands and returns is quite time consuming therefore they provide two different approximations to reduce computational time. These two approximations are based on two different heuristics. One heuristic uses value-function approximation in the dynamic programming problem while the other is based on deterministic model approximations.

In order to define deterministic model, Mabini et al. (1992) have used some variants of economic order quantity model (EOQ) if demands and returns are stationary. They have suggested the substitution policy as the control policy and their assumptions were about deterministic demand that is supplied from purchased item and subsequent demand that is satisfied from remanufactured items. Koh et al. (2002) propose a model for the inventory system in which demands are stationary and can be satisfied by recovered product or new products. They assume that demand is deterministic and known, remanufactured products are as good as new products and it is economically more efficient to remanufacture the items rather than producing new ones. Richter and

Sombrutzki (2000) use some variants of Wagner/Whitin algorithm for the case where demands and returns are deterministic and dynamic. Dobos and Richter (2006) investigate an integrated production recycling system using a deterministic EOQ model. They developed this model to examine a pre-determined production inventory policy in order to show that the inventory holding cost function can be minimized and non-EOQ related disposal, production, recycling and repurchasing costs are minimal.

To solve the stochastic inventory models for the recoverable manufacturing systems, most of the time the optimal or near optimal values for the parameters of a predetermined reasonable, but not necessarily optimal, inventory policy structures are found by developing either heuristic or exact methodologies. Nenes et al. (2010) have considered alternative policies for an ordering and remanufacturing system for a real case where both demands and returns for new and returned products are stochastic. Although the returned items are usually cheaper compared to the procurement of new items, unfortunately the quality and the quantity of returned items are highly stochastic. They have described each of the policies by implementing an illustrative example and investigating the economic outcomes for all policies in which inventories are kept in three different types: good, remanufacturables and uninspected items inventories compared to traditional inventories of having only two types: good products inventories and remanufacturable items inventories. Aras and Gurbuz (2007) formulated a base-case model and two extensions to determine optimal quality levels and prices in a hybrid manufacturing/remanufacturing system.

2.2 Substitution Cases

In section 2.1, we reviewed studies where manufactured and remanufactured items are assumed to be alike therefore there is no need to do a substitution among them in order to satisfy customers' needs. In reality though, it is very common that a substitution might occur among manufactured and remanufactured items since some of the remanufactured products may have an inferior value from customers point of view and might be seen as not good as new items which creates a segmented market. Inventory models with product substitution can be divided into two main categories. First category is one way substitution according to which a product with a higher value can substitute for a lower- value product. One way substitution means that suppliers are the ones that make the decision to substitute or not on behalf of customer. In second category, customers are allowed to substitute the product of their choice in two ways: either customers' decisions are based on maximizing the profit gained with stochastic customer appearance so that the attitude of the customers can influence the substitution or products are substituted according to some probability in two product or multi product cases thus the total product demand evaluated at the end of the period and inventory level is found out by evaluating the substitution's effect on demand.

More recently, researchers have considered options involving one way product substitution among hybrid manufacturing and remanufacturing inventories under single period stochastic model settings. Kaya (2010) considers partial substitution of newly manufactured and remanufactured products using stochastic demand functions to determine the optimal production quantities for optimal incentive determination problem using three different business models in a single period newsvendor setting. In the first model, they consider remanufactured products apart from the original products. Second model considers the equivalent values for both original and remanufactured products and third model considers the original and remanufactured products differently in the market while demands are partially substitutable. Jin et al. (2007) have characterized a threshold for whether to offer remanufactured products in addition to new products and to substitute them for remanufactured products by investigating the profitability of offering remanufacturing products. Robotis et al. (2005) explains remanufactured products in a stochastic multi-product inventory/production management context of newsvendor problem with downward substitution. They consider the use of used products in secondary markets either by selling fraction of them to developing markets or investing in the remanufacturing of these used products. Inderfurth (2004) deals with hybrid manufacturing and remanufacturing problems in the context of stochastic inventory control problems for single-state single-period optimal policy analysis such that leadtimes are deterministic with substitutable products. He presents a hybrid system that simultaneous manufacturing and remanufacturing occurs and they are not directly connected if remanufactured item has a lower selling price in the market. One way substitution is considered for remanufactured items in the area of hybrid systems with an independent demand streams for manufactured and remanufactured items. Remanufactured items are usually assessed as lower class than newly manufactured items from customers' point of view therefore consumer profile and market strategy of remanufactured items can vary compared to newly manufactured items. When there are different markets for manufactured and remanufactured items without product substitution, it is foreseeable to think these processes should be controlled independently from each other. Conversely, when there exist a product substitution among hybrid systems and both are using the common resources, it is a necessity to coordinate manufacturing and remanufacturing decisions. One-way substitution policy is also discussed by Dutta and Chakraborty (2010) as single period inventory model in a fuzzy environment. They study the benefits of substitution for newsboy problem under uncertainty in customer demand to demonstrate one way substitution policy creates higher profits. They propose a search technique for finding the optimum using fuzzy logic while their fuzzy model considers the newsboy problem without holding cost, salvage value, etc. Huang et al. (2013) study multi product newsboy problem with shortage penalty cost and partial product substitution in single period inventory model. They have conducted a numerical experiment to characterize the unique Nash equilibrium by illustrating the impact of product substitution. Gurler and Yilmaz (2010) consider a single period newsboy type inventory problem with two substitutable perishable products in a two level supply chain, consisting of a retailer and a manufacturer and introduce three cases with one-way substitution accompanied with no returns, full returns and full return with one product and no return with the other.

Inventory systems with one way substitutions are also analyzed as a base stock policy with periodic. Deflem and Nieuwenhuse (2013) studied one-way substitution in a newsvendor problem. They presented two item periodic inventory systems to minimize the total cost per period and found out that order-up-to levels should be set for both a single period and infinite horizon. Rao et al. (2002) investigate a single period multi product inventory problem with stochastic demand, setup cost for production and one-way product substitution in the downward direction as a two-stage, mixed integer, stochastic program. They present a model, properties and an effective solution methodology that exploits the problem structure. Bassok et al. (1999) study a single period periodic review multiproduct inventory model with stochastic demands, proportional revenues and costs, substitution, and arbitrary starting inventory. They first discuss a two-stage profit maximization formulation for the multiproduct substitution problem and show that a greedy allocation policy is optimal.

The dynamics of the hybrid manufacturing and remanufacturing systems can be modeled as continuous time. Bayindir et al. (2005) consider segmented markets for manufactured and remanufactured products with stockout-based one-way substitution with continuous review inventory control policy. They construct Markovian model of a steady state probability in order to show that remanufacturing is profitable under one-way substitution policy. Manufactured and remanufactured products face independent Poisson demand. They solve this Markovian model by means of matrix geometric techniques through a computational study. Their reason for applying one-way product substitution among manufactured and remanufactured products is not to lose the customers' goodwill so have a higher loyalty with the hope of getting more benefits later. In another study of Bayindir et al. (2007) include a capacity constraint to the same analysis and in addition to above findings point out that under finite production capacity, lower capacity requirements mostly utilize the remanufacturing to optimality. Liu and Lee (2007) investigate two part inventory systems by continuous time Markov process using multi-item base-stock policy where unidirectional substitutions are allowed. They propose three policies for a spare part inventory management where one way substitutions are allowed.

Manufacturing and remanufacturing processes are complex systems which provide significant economic, environmental, and social benefits therefore the coordination between them, more importantly; the management of their inventories has been studied immensely and discussed over the past decade. The majority of hybrid systems with product substitution is dealing with these problems under deterministic conditions. Pineyro and Viera (2010) provides an NP-hard economic lot-sizing problem and find an optimal or near optimal solution using metaheuristic Tabu-search procedure for such a problem where two independent demand streams exist in which remanufactured items can also be satisfied by new products but not vice versa. They have assumed that the demands and returns are deterministic and known over the finite planning horizon. Li et al. (2006) propose a dynamic program in order to minimize manufacturing, remanufacturing, holding and substitution costs by finding optimal solution to uncapacitated multi-product production planning problem which has a time varying demands in a finite time horizon with no disposal or backlog. They develop an optimization model to formulate this problem, and then propose dynamic programming approach to derive the optimal solution. After that, they use these optimality conditions and the dynamic programming approach to obtain near-optimal solutions for the general problem by developing an approximate solution procedure. Hsu et al. (2010) consider a finite horizon two multi-product dynamic lot size problem on deterministic models with one-way product substitution. They have developed a heuristic as well as a dynamic programming algorithm to solve the problems in polynomial time. One way product substitution is formulated into two variants of multi-product dynamic lot size problem as one for the substitution with conversion and the other for the substitution without conversion.

There is also another line of research done for product substitution. Inventory management models that allow two-way substitutability is another stream of research area that enable consumers to substitute among products within the same category. Tan and Karabati (2013) present a customer driven substitution for an inventory management problem in retail setting. Their proposal is to use order-up-to levels to maximize the profit with fixed review period through a computational method. Stavrulaki (2011) models a single-period, two-product, stochastic-demand environment and demonstrate how the optimality conditions expand the traditional newsvendor solution, to capture the marginal revenue influences of demand substitution and demand stimulation and also consider two heuristic policies, one that ignores demand stimulation and another that ignores product substitution, to gauge when it would be most beneficial to use the optimal policy. Nagarajan and Rajagopalan (2008) explore a two way substitution due to stockouts on the inventory decisions in both single-period and multiperiod scenarios. The authors show that for single-period case optimal inventory level can be computed and also heuristics based on the decoupled inventory policy perform well under realistic conditions.

There are studies that consider substitution among the products with different quality levels. Korugan and Cevahir (2006) study the production control and inventory management on the basis of substitution by developing a heuristic to simplify the decision making process with random production yields. They have constructed a model that provides threshold values for substitution. Their model concludes that upward substitution is valid only for boundary states while downward substitution is valid for both boundary and non-boundary states where a line equation determines if substitution is made or not for non-boundary states. Yaman (2009) considers the two-item uncapacitated lot-sizing problem with one-way product substitution. The author's aim is to propose a minimum cost production and substitution plan for the two items in each period. Korugan and Gupta (2001) study a hybrid system that satisfies the demand for a certain type of product with either new items or remanufactured items. They find optimal switching functions for substitution decisions using a Markov decision process and define several control policies and compare them with respect to the expected total cost function of the system. Helvacioglu et al. (2009) develop a method to determine the optimal order up-to levels in the presence of stock-out based substitution. They compute the expected sales, inventory levels of each product, number of substitutions between all product pairs, service levels achieved for each product, and service level achieved by the system.

Evaluation of the articles about manufacturing and remanufacturing systems with substitution is summarized in table 2.1 and their basic features are compared to the fundamental features of our research. Not all features will apply in all articles presented, and not all articles need to include all positive features of ours, so this is not a score card. The features are intended to help us see the similarity and differences of the existing studies with our work. Table 2.1 provides a framework for investigating the aspects of a hybrid manufacturing/remanufacturing with product substitution, whether it is an article in a journal or paper presented in a conference proceeding; or any other source.

We have used abbreviations in order to make the demonstration of the literature comparison table more manageable. The notation used for the table is described below:

Subs.	Substitution	S-P	Single Period
Prod.	Product	M-P	Multiple Period
Rem.	Remanufactured	*O.P.*	One Period
Opt.	Optimal	*C.R.*	Continuous Review
O-W	One-Way	*N.O.*	Near Optimal
T-W	Two-Way	*O.*	Optimal
S.P.	Single Product	*L.O.P.*	Less than One Period
T.P.	Two products	*R.S.N.*	Remanufactured Item Substitute New Item
M.P.	Multiple Products	*S.E.A.*	New & Remanufactured Items Substitute Each Other
Stch.	Stochastic	*R.I.V.*	Remanufactured Items has Inferior Value than New Items
Det.	Deterministic	*A.G.A.N.*	As good as New
N/M	Not Mentioned		

Table 2.1: Literature Comparison Table

Author(s)	Subs. Type	Prod. Type	Demand Type	Subs. Style	Rem.	Period	Lead Type	Opt.
Our research	*O-W*	*S.P*	*Stc.*	*R.S.N.*	*R.I.V*	*M-P*	*O.P.*	*N.O.*
Korugan & Gupta (2001)	*T-W*	*S.P.*	*Stc.*	*R.S.N.*	*A.G.A.N*	*N/M*	*N/M*	*N.O.*
Korugan (2004)	*T-W*	*S.P.*	*Stc.*	*S.E.A*	*A.G.A.N*	*N/M*	*N/M*	*N.O.*
Inderfurth (2004)	*O-W*	*T.P.*	*Stc.*	*R.S.N*	*R.I.V.*	*S-P*	*Det.*	*O.*
Robotis et al. (2005)	*O-W*	*T.P*	*Stc.*	*R.S.N*	*Two Different*	*S-P*	*N/M*	*O.*
Bayindir et al. (2005)	*O-W*	*S.P.*	*Stc.*	*R.S.N*	*R.I.V.*	*C.R.*	*Stc.*	*O.*
Li et al. (2006)	*O-W*	*M.P.*	*Det.*	*R.S.N*	*A.G.A.N*	*M-P*	*L.O.P*	*N.O.*
Li et al. (2007)	*O-W*	*T.P.*	*Det.*	*R.S.N*	*A.G.A.N*	*M-P*	*L.O.P*	*O.*
Bayindir et al. (2007)	*T-W*	*S.P.*	*Stc.*	*S.E.A*	*R.I.V.*	*S-P*	*L.O.P*	*O.*
Jin et al. (2007)	*O-W*	*S.P.*	*N/M*	*R.S.N*	*R.I.V*	*S-P*	*N/M*	*O.*
Pineyro & viera (2010)	*O-W*	*S.P.*	*Det.*	*R.S.N*	*R.I.V.*	*M-P*	*zero*	*N.O.*
Kaya (2010)	*T-W*	*M.P*	*Stc.*	*S.E.A*	*A.G.A.N & R.I.V*	*S-P*	*N/M*	*O.*
Ahiska et al. (2013a)	*O-W*	*S.P*	*Stc.*	*R.S.N.*	*R.I.V.*	*M-P*	*O.P.*	*O.*

2.3 Inventory Policy Characterization Cases

There are several control policies proposed in the literature for the control of manufacturing and remanufacturing system where manufactured and remanufactured items are considered to have the same quality and stored in the same serviceable inventory. We briefly describe here some of these policies. Mahadevan et al. (2003) use *R* – remanufacturing policy, which is a periodic review, push policy, for the inventory system with different stocking points for serviceable and returned goods. In this system, there are holding and backorder costs but no set-up costs are considered. Fleischmann et al. (2003) consider a periodic (*s*,*S*) policy that remanufactures up to ‘S’ and halt the operations until inventory comes down to ‘s’ and restarts the remanufacturing again when the inventory goes below s. A fixed order cost, holding and backorder costs are considered. Jia et al. (2010) apply the periodic review remanufacture-to-stock (push) and remanufacture-to-order (pull) inventory policies in a system where there are different quality categorizations for returned items, and there is a fixed unit holding cost in their research. Teunter et al. (2004) explain push and pull inventory policies in different aspects. They provide detailed explanations for standard push and pull policies as well as lead time-adjusted push policy and separate pull policy. Inventory policies for a periodic review inventory system with different stocking options also are discussed in Gharote et al. (2007) who consider one-parameter inventory policy with re-order level (*S*) and two-parameter inventory policy with manufacturing re-order level (S_m) and remanufacturing and disposal re-order level (S_d). In this system, there is a backorder cost, holding cost and disposal cost but no set-up costs are considered. Inderfurth and Kleber (2013) have recently studied the heuristic procedures for parameter determination. They have showed that their myopic newsvendor approach to determine the order-up-to levels for remanufacturing and extra production performs fairly well. They consider a periodic review, finite horizon, stochastic demands and returns with backorders and no holding or disposal costs.

There is also another study that needs careful attention in our review. Other than earlier studies, Ahiska et al. (2013) discuss multi-period periodic-review inventory control problem for a hybrid manufacturing/remanufacturing system with product substitution to find the optimal inventory policies for both with and without one-way product substitution using discrete-time Markov Decision Process. They assume stochastic demands and returns with no setup cost and one period lead time for manufacturing and remanufacturing operations.

The earlier studies Ahiska & King (2010a, 2010b) also need some attention. Ahiska & King (2010a) determine the optimal or near-optimal inventory policy characterizations for a recoverable system through analysis of Markov Decision Process without pre-specifying the structure of the inventory policies beforehand. Single product recoverable manufacturing system with setup costs and different lead time cases for manufacturing and remanufacturing inventory optimization is considered while customer demands are satisfied through either regular production (manufacturing) of new items or remanufacturing of returned items. In another study, Ahiska and King (2010b) investigate the inventory policies over the life cycle of a remanufacturable product. They also perform a performance comparison of the proposed policies with a PULL strategy. Finally, they illustrate the importance of frequent policy revision numerically over the product life cycle.

This literature survey on inventory control of hybrid manufacturing and remanufacturing systems in the context of product substitution reveals that key aspects of policy characterization have been, for the most part, overlooked by researchers. In the thesis, we aim to find easy-to-implement heuristic policies having a few control parameters that are near-optimal for the inventory control problem for a periodically reviewed stochastic hybrid manufacturing and remanufacturing system under product substitution using a Markov decision analysis. To the best of our knowledge, this study has not yet been done in the literature, so our aim here is to determine optimal or near optimal policy characterizations with practical structure through a case study regarding an automotive spare parts manufacturer.

3. PROBLEM DESCRIPTION AND MDP FORMULATIONS

We consider three stocking points, the recoverable (i.e. used items) inventory, the remanufactured items inventory and the manufactured items inventory, for the inventory management of hybrid manufacturing/remanufacturing system. In this system, remanufactured products have an inferior value from customers' point of view and they are not regarded as good as new items. Therefore, there exists a segmented market among manufactured and remanufactured items. Remanufactured items have lower selling price and different customer profile. Customers who have image concerns prefer newly manufactured items. Customers who prefer the price advantage, demand remanufactured items. In real-world situations, demand is not certain, i.e. has a stochastic nature. For manufactured or remanufactured products, excess inventory may acquire a huge keeping cost. Manufacturer must find a common ground between incurring lost sales or keeping excess inventory in order not to lose customers goodwill. To reduce lost sales, manufacturers may prefer to consider a stock-out based substitution. This paper considers such a problem where manufactured and remanufactured items have different quality and prices, and downward substitution is used to reduce lost sales for remanufactured items. We assume in our model that unacceptable items for remanufacturing operations are identified prior to the inclusion in recoverable inventory. Our model considers one period lead time for manufacturing and remanufacturing operations with disposal option of returned items if recoverable inventory is full and backordering of manufactured item demands which are allowed up to a certain level. If the demand is still not met, then it is lost.

3.1. Problem description

Figure 3.1 illustrates the hybrid manufacturing/remanufacturing system under product substitution that is considered in this paper. The incoming returned items are disposed only if recovered item inventory is full, otherwise they are stored to be used later for remanufacturing operation. After manufacturing and remanufacturing operations, items are stored in manufactured item inventory and remanufactured item inventory, accordingly. If there are demands for the manufactured and remanufactured items during some period, the corresponding inventory levels diminish. The manufacturing and remanufacturing decisions that in turn affect the inventory levels are the decision variables of this problem. If any stock-out situations occur for the remanufactured inventory, in order to satisfy remanufactured item demand, substitution of products is considered, according to which a manufactured product is offered to the customer demanding a remanufactured product for the price of the remanufactured product (i.e. the discounted price) if the manufactured item inventory is positive.

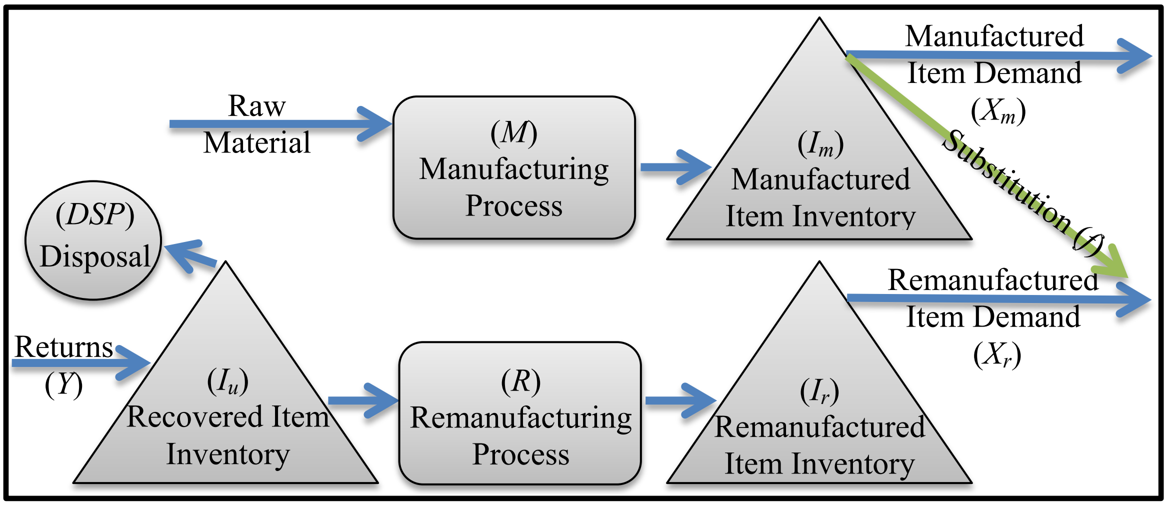

Figure 3.1: Hybrid manufacturing/remanufacturing system under product substitution

This problem is formulated as a discrete-time Markov Decision Process (MDP) in order to find the optimal inventory policy under product substitution strategy. The model is solved using a variant of Howard's policy iteration method Howard (1960) where the fixed policy successive approximation method Morton (1971) is used for computational efficiency.

3.2. MDP formulations

An MDP model is a stochastic sequential-decision model that is defined by a set of system states, a set of decisions to make, an immediate reward function and a transition probability matrix that defines the probability of going from one state to another in one transition under a selected decision. The MDP model formulated for this problem is briefly described below. You can also see the solution algorithm for the infinite horizon MDP model in appendix A.

3.2.1. State Space

The state of the system in a period, denoted by S, is represented by three variables I_u , I_r , and I_m which are the inventory levels of used (i.e. recoverable), remanufactured and manufactured items respectively. These inventory levels are bounded as $I_m^{min} \leq I_m \leq I_m^{max}$, $I_r^{min} \leq I_r \leq I_r^{max}$ and $0 \leq I_u \leq I_u^{max}$. I_j^{min} means that backordering of the demand is allowed up to $-I_j^{min}$ for $j=r,m$ if $I_j^{min} < 0$.

3.2.2. Decision Space

In this system we have to make the decisions of how many units to manufacture (d_m), and to remanufacture (d_r). For each system state, we find the feasible values for (d_m, d_r) decisions as follows.

A feasible manufacturing decision d_m is bounded by the capacity of manufacturing process, and it is dependent on the inventory level of manufactured products and maximum allowed manufactured item stock. As a result we find that d_m can take following values: $d_m = 0,1, \dots, d_m^{max}$, where d_m^{max} is calculated as $d_m^{max} = min\{I_m^{max} - I_m, M_{max}\}$. A feasible remanufacturing decision follows a similar logic with the added constraint that it cannot exceed the available recoverable inventory, thus

$d_r = 0,1, \ldots, d_r^{max}$where d_r^{max} is calculated as $d_r^{max} = min\{I_u, R_{max}, I_r^{max} - I_r\}$. In these equations R_{max} and M_{max} are the capacities of the remanufacturing and manufacturing processes, respectively.

3.2.3. State Transition and Transition Probabilities

Given that the current state is S=(I_u, I_r, I_m), the manufacturing and remanufacturing decisions are d_r and d_m, and manufactured item demand (X_m), remanufactured item demand (X_r) and returns (Y) take the values x_m ,x_r and y, respectively, the next state will be $S' = (I_u', I_r', I_m')$ where I_u', I_r' and I_m' are calculated as follows.

The inventory level for used items decreases by the amount of used items sent into the remanufacturing process and increases by the amount of used items that are returned, but cannot exceed the used item storage capacity, as shown below.

$$I_u' = min\{I_u - d_r + y, I_u^{max}\} \quad (1)$$

The inventory levels for manufactured and remanufactured items at the end of the current period do not only depend on current inventories, demand for corresponding items and manufacturing and remanufacturing decisions, but also on the product substitution strategy. According to this strategy, if some of the demand for remanufactured items cannot be satisfied from remanufactured item stock because the remanufacturing item runs out of stock, then it is met from the manufactured item stock if there is manufactured item left in stock after satisfying first the demand for manufactured items. In this case, the manufactured item is sold at the remanufactured item price (i.e. a discounted price) to the customer who demanded a remanufactured item. No explicit cost associated with substitution is considered other than the opportunity cost of selling the manufactured item at the discounted price rather than at its original price. Under this strategy, the amount of remanufactured item demand satisfied from new item stock, i.e. the amount of substitution, denoted by f, is formulated using the following reasoning.

Clearly, if $I_r \geq x_r$ (no shortage for remanufactured items) or if $I_m \leq x_m$ (no manufactured item left in stock after satisfying demand for manufactured items), no product substitution will occur (f=0). In this case, the amount of remanufactured item demand that remains unsatisfied, denoted by l, is $l=\max\{x_r-I_r, 0\}$. On the other hand, if $I_r<x_r$ (i.e. there is a shortage of x_r-I_r remanufactured items) and if $I_m>x_m$, then there are I_m-x_m items left in manufactured item stock that can be used to deal with the remanufactured item shortage. In this case, the amount of substitution is $f=\min\{I_m-x_m, x_r-I_r\}$ and the amount of remanufactured item demand that remains unsatisfied after product substitution occurs is $l=\max\{x_r-I_r-f, 0\}$. General formulations for f and l that cover all the 'if' conditions defined in this paragraph can be formed as: $f=[\min\{I_m-x_m, x_r-I_r\}]^+$ and $l=[x_r-I_r-f]^+$ where $[x]^+=\max\{x,0\}$.

The substitution amount f and unsatisfied remanufactured item demand l being defined as above, the inventory levels for manufacturing and remanufacturing items at the beginning of next period are formulated as:

$$I'_m = max\{I_m - x_m - f, I_m^{min}\} + d_m \qquad (2)$$

$$I'_r = max\{I_r - x_r, -l, I_r^{min}\} + d_r \qquad (3)$$

The state transitions under *no substitution* can be simply obtained by setting f=0 in the formulations above.

The transition probability from S to S' under decision (d_m, d_r), represented by $P(S, S', (d_m, d_r)$ equals the sum of the probabilities of occurrence for demands and returns, (x_m, x_r, y), that lead to transition from S to S' under the decision (d_m, d_r), as indicated below.

$$P\big(S, S', (d_m, d_r)\big) = \sum_{(x_m, x_r, y) \in A_{S \to S'}^{(d_m, d_r)}} P(X_m = x_m, X_r = x_r, Y = y) \qquad (4)$$

where $A_{S\to S'}^{(d_m,d_r)}$ is the set of the values of demand for manufactured and remanufactured items and the returns $(x_m, x_r, y\ )$ that make the system transition from state S to state S' under decision(d_m, d_r).

3.2.4. Reward Function

The reward function for this problem represents the expected profit per period. It is defined by the total revenue obtained from the products minus the total cost including manufacturing and remanufacturing cost, holding costs for different stocking points, backordering cost, lost sales cost and disposal cost. The following notation is used for the reward function.

p_m : unit price for manufactured product
p_m : unit price for remanufactured product
s_m : Setup cost for manufacturing
s_r : setup cost for remanufacturing
$\underline{c}_{\underline{m}}$: unit manufacturing cost
c_r : unit remanufacturing cost
h_m : unit holding cost per period for manufactured product
h_r : unit holding cost per period for remanufactured product
h_u : unit holding cost per period for used product
b_m : unit backordering cost per period for manufactured product
b_r : unit backordering cost per period for remanufactured product
l_m : unit lost sales cost for manufactured products
l_r : unit lost sales cost for remanufactured products
k : unit disposal cost for used products
DSP : disposal amount for current period
LS_m : lost sales of manufactured items for current period
LS_r : lost sales of remanufactured items for current period
BO_m : backordered demand of manufactured items for current period
BO_r : backordered demand of remanufactured items for current period

Given that the system is in state S, the decisions d_r and d_m are made, demand for manufactured and remanufactured items are x_m and x_r units, respectively, and y units of return occur, the profit is calculated as:

$$Profit\big(S,(d_r,d_m),(x_m,x_r,y)\big) = p_r(Q_r+f)+p_mQ_m-\begin{bmatrix}\delta(d_r)+\gamma(d_m)+h_r[I_r']^{+}+h_m[I_m']^{+}+h_u\,I_u' \\ +b_mBO_m+b_rBO_r+l_mLS_m+l_rLS_r+kDSP\end{bmatrix} \tag{5}$$

where Q_r and Q_m represent the amounts of remanufactured and manufactured items sold for their corresponding prices, respectively.

$$Q_r=\begin{cases}x_r & if\ x_r<I_r\\ max\{I_r,0\} & otherwise\end{cases} \tag{6}$$

$$Q_m=\begin{cases}x_m & if\ x_m<I_m\\ max\{I_m,0\} & otherwise\end{cases} \tag{7}$$

$$\delta(d_r)=\begin{cases}s_r+c_rd_r & for\ d_r>0\\ 0 & for\ d_r=0\end{cases} \tag{8}$$

$$\gamma(d_m)=\begin{cases}s_m+c_md_m & for\ d_m>0\\ 0 & for\ d_m=0\end{cases} \tag{9}$$

$$BO_m=\begin{cases}-max\{I_m-x_m,I_m^{min}\} & if\ I_m<x_m\\ 0 & otherwise\end{cases} \tag{10}$$

$$BO_r=\begin{cases}l & if\ \ l\leq -I_r^{min}\\ -I_r^{min} & otherwise\end{cases} \tag{11}$$

$$LS_m=\begin{cases}I_m^{min}-(I_m-x_m) & if\ I_m-x_m<I_m^{min}\\ 0 & otherwise\end{cases} \tag{12}$$

$$LS_r = \begin{cases} 0 & if \;\; l \leq -I_r^{min} \\ l + I_r^{min} & otherwise \end{cases} \tag{13}$$

$$DSP = \begin{cases} I_u - d_r + y - I_u^{max} & if \; I_u - d_r + y > I_u^{max} \\ 0 & otherwise \end{cases} \tag{14}$$

Then the expected profit in a given period is calculated as:

$$E\big[Profit\big(S,(d_r,d_m)\big)\big] = \sum_{x_m}\sum_{x_r}\sum_{y} P(X_m = x_m, X_r = x_r, Y = y)\, Profit\big(S,(d_r,d_m),(x_m,x_r,y)\big) \tag{15}$$

3.3 Solution methodology

In this chapter, we use Markov decision process to determine the optimal inventory policies for the periodically-reviewed stochastic hybrid manufacturing and remanufacturing system with and without product substitution. We investigate the effects of demand and return distributions on the profitability of substitution through numerical experiments (Gocer et al. 2014). The optimal inventory policy found by MDP is a list of optimal manufacturing and remanufacturing decisions for all system states, which may be a very long list since even a moderate-size inventory problem would have thousands of states. Therefore, this list of decisions is not practical for implementation. We propose in this study three easy-to-implement policies that are defined using two to three control parameters, and we develop two heuristic algorithms to determine the values of control parameters for these policies. Finally, we evaluate the performance of these policies with respect to the optimal policy for three products produced by an automotive spare part manufacturer. To the best of our knowledge, no existing work has studied the proposed policies in this study.

4. EFFECTS OF DEMAND/RETURN DISTRIBUTIONS ON PROFITABILITY OF SUBSTITUTION

Using Markov decision analysis for a periodically reviewed stochastic hybrid manufacturing and remanufacturing system under product substitution, we analyze the inventory control problem to find the optimal inventory policies for both the substitution and no substitution cases. One objective of this paper is to assess the profitability of the substitution strategy. For the numerical evaluation of our model, we first generate a large set of scenarios to display the improvement made in profit and then evaluate the effect of substitution's profitability by mean or standard deviation of demand and return distributions, later we analyze how the profitability is effected by coefficient of variation which is changing the means for the return distributions and keeping same standard deviations or vice versa.

4.1. Design of Numerical Experiments

For the numerical experimentation, we consider a product produced by an international automotive spare part manufacturer. Due to privacy concerns, the data is scaled and the identity of the firm is kept anonymous. Due to the vigorous competition in the sector, over the last few years the firm noticed that the lost sales due to stock-outs of remanufactured products were resulting in loss of customers and damage to the image of the firm in the market. Hence, customer satisfaction is very important, and in order to guarantee a high level of customer satisfaction, the company is considering a stock-out based substitution strategy. The product for which we evaluate the substitution strategy is an 'engine starter' which is a type of electric motor. This product family was among the firm's first production, and a better service level for this product is considered to be prestigious by the manufacturer Ahiska et al. (2013a).

The unit selling prices for the manufactured (i.e. new) and remanufactured engine starter are 68.39€ and 51.85€, respectively, and the unit manufacturing and remanufacturing costs are 22.74€ and 17.46€. The manufacturer tolerates the backordering of the manufactured item demand up to a certain level (i.e. I_m^min<0) while backordering of the remanufactured item demand is not allowed (i.e. I_r^min=0) due to the risks associated with receiving returns when needed. If some remanufactured item demand remains unsatisfied after the substitution is done, then this demand is lost. Unit backordering cost for manufactured product per period is calculated as 20% of its unit price while unit lost sales cost (cost of goodwill loss) for both manufactured and remanufactured products are calculated as 25% of the corresponding unit price.

The annual holding costs for manufactured and remanufactured items are calculated as 20% of the corresponding unit cost, and the holding cost for a used item is considered to be half of the holding cost for a remanufactured item. The lead times for manufacturing and remanufacturing are both one period. No set up costs exists for either production option.

4.2. Results

In this section, we analyze numerically the profitability of using the downward substitution strategy under different demand/return distributions.

We design the first set of experiments in order to investigate how the profitability of product substitution strategy is affected as the means of the demand and return distributions change. In this set of experiments, we use bounded discrete stochastic distributions with three different shapes for the manufactured and remanufactured item demands and used item returns, which are uniform, normal, and right skewed as the distribution shapes are shown in figure 4.1. The mean of each different-shape stochastic distribution is assigned three different values: low, medium and high, as shown in Table 4.1.

Table 4.1: Mean values for different distributions.

Distribution shape	Mean		
	High	Medium	Low
Uniform (Uni)	2.00	1.50	1.00
Normal (Nrm)	2.51	2.00	1.50
Right skewed (RS)	1.20	1.05	0.54

In all, 27 combinations of the three means are created by assigning the three different levels of the mean of the distribution for manufactured item demand ($E[X_m]$), remanufactured item demand ($E[X_r]$) and used item returns ($E[Y]$). These 27 combinations coupled with the three distribution shapes yield a total of 81 scenarios. For each scenario, the optimal expected profits per period for the hybrid system under substitution and no substitution strategies are determined by solving the MDP as defined in the previous section.

The % improvements in profit gained by using the substitution strategy over no substitution case are reported in Table 4.2. We make the following observations: When the mean of remanufactured item demand is at least as much as the mean of returns ($E[X_r] \geq E[Y]$), the substitution strategy results in additional profit for the manufacturer. Among the 54 scenarios where $E[X_r] \geq E[Y]$, the highest improvement in profit was 85%. When returns are substantially higher than the remanufactured item demand (i.e. $E[X_r] < E[Y]$), the use of substitution is not economically justified. It caused loss of profit but only up to 3% among the 27 scenarios we considered (see Table 4.2). Further experimentation (not shown here) reveals that if the average returns exceed the demand but at a lower level than the amounts shown in Table 4.1, substitution is still profitable.

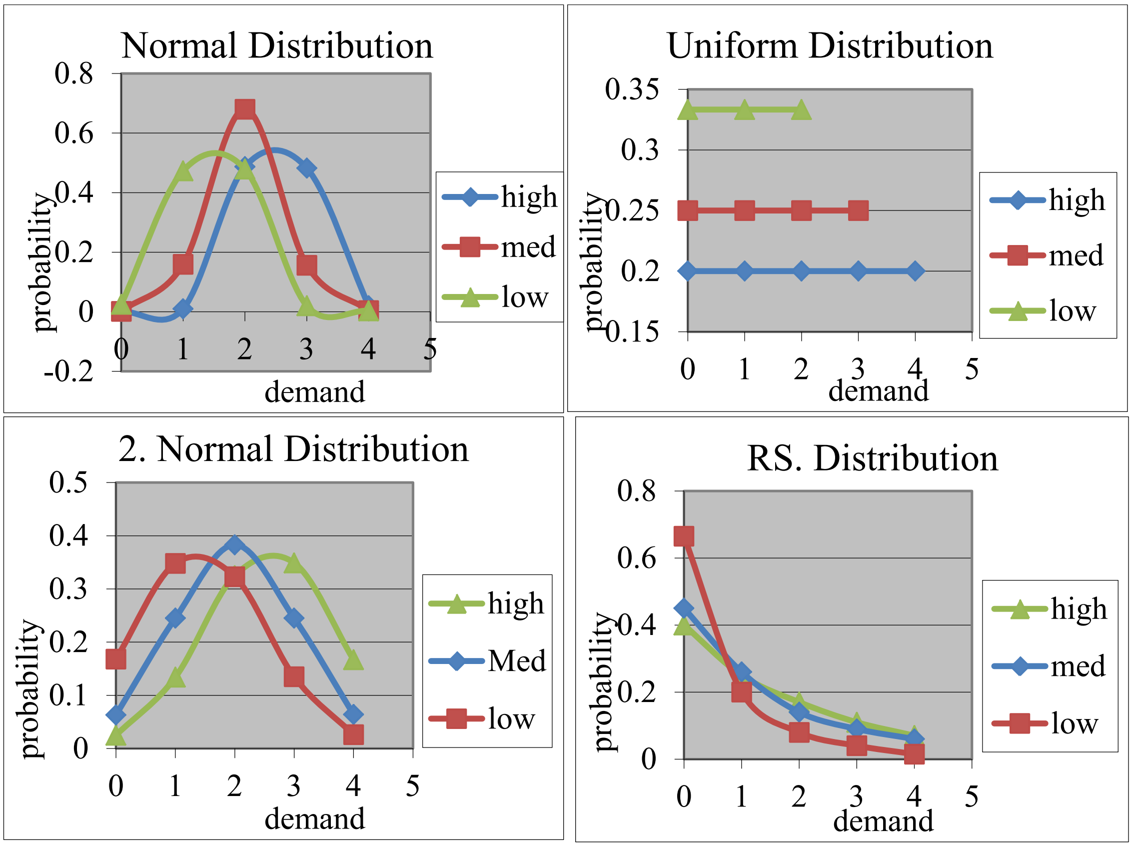

Figure 4.1: Distribution types for Normal, Uniform, Right Skewed shapes.

Clearly, substitution results in a higher improvement in profit when the expected remanufactured item demand gets higher and/or the expected return gets lower. For representative results supporting this comment, see figure 4.4, which plots the % improvements in profit by substitution for nine scenarios with the low level of mean manufactured item demand and the Normal shaped distribution, and the mean of remanufactured item demand and returns as low, medium and high. As the ratio of the mean remanufactured item demand to the mean returns increases from lowest ($E[X_r]$=low, $E[Y]$=high) to highest ($E[X_r]$=high, $E[Y]$=low), the percent change of firm's profit when the product substitution strategy is used increases from -1.1% to 39.9%.

Table 4.2: Absolute difference and improvement in profit by substitution (%) for different combinations of E[X_m], E[X_r] and E[Y] under different-shape distributions.

Means			Absolute difference between No substitution and Substitution & Improvement in profit by substitution (%)					
E[X_m]	E[X_r]	E[Y]	Uni		Nrm		RS	
			Diff	%Imp	Diff	%Imp	Diff	%Imp
high	high	high	2.63	1.75	-0.01	0.00	2.00	2.29
high	high	med	20.26	15.25	20.90	12.00	6.41	7.63
high	high	low	40.77	37.11	41.64	27.61	27.06	43.84
high	med	high	-1.43	-1.06	-1.36	-0.77	-0.44	-0.53
high	med	med	1.57	1.16	0.01	0.00	1.80	2.18
high	med	low	20.33	17.49	20.49	13.02	20.86	32.81
high	low	high	-1.45	-1.24	-1.27	-0.80	-1.06	-1.63
high	low	med	-1.45	-1.23	-1.26	-0.79	-1.07	-1.64
high	low	low	0.57	0.47	0.03	0.02	0.74	1.11
med	high	high	2.61	2.03	-0.01	0.00	1.99	2.47
med	high	med	20.17	18.23	20.93	13.86	6.40	8.30
med	high	low	40.66	46.42	41.68	32.63	27.07	49.31
med	med	high	-1.43	-1.27	-1.36	-0.89	-0.45	-0.59
med	med	med	1.56	1.38	0.01	0.01	1.79	2.37
med	med	low	20.27	21.58	20.52	15.28	20.85	36.74
med	low	high	-1.45	-1.53	-1.28	-0.95	-1.08	-1.86
med	low	med	-1.45	-1.51	-1.27	-0.94	-1.09	-1.87
med	low	low	0.56	0.57	0.03	0.02	0.73	1.22
low	high	high	2.60	2.44	0.00	0.00	2.00	3.47
low	high	med	20.09	22.73	21.08	16.43	6.42	11.84
low	high	low	40.57	62.10	41.85	39.87	27.11	84.82
low	med	high	-1.43	-1.58	-1.35	-1.04	-0.44	-0.82
low	med	med	1.54	1.70	0.02	0.02	1.81	3.42
low	med	low	20.17	28.14	20.68	18.54	20.90	61.82
low	low	high	-1.45	-2.00	-1.27	-1.13	-1.06	-3.03
low	low	med	-1.45	-1.97	-1.27	-1.12	-1.07	-3.02
low	low	low	0.55	0.73	0.04	0.03	0.74	2.01

We observe from figure 4.2 that profit gets the peak values when return probability of used item and demand of newly manufactured item is lower and demand of remanufactured item is higher or medium as one would expect. As it can be seen from figure 4.2, even though the mean return and demand distributions are different, profitability of substitution for the product behaves similar while profit amount alters. On the other hand, almost non-profit cases occur when return probability gets higher. Furthermore, even when demands are low and returns are high, the loss still remains very minimal, and i.e. the substitution strategy creates very minimal risk. Therefore, we are able to conclude that substitution strategy is favorable since there is more or less profit from doing it so. In that case, manufacturers have to decide whether to use substitution strategy and not to lose customer goodwill or not.

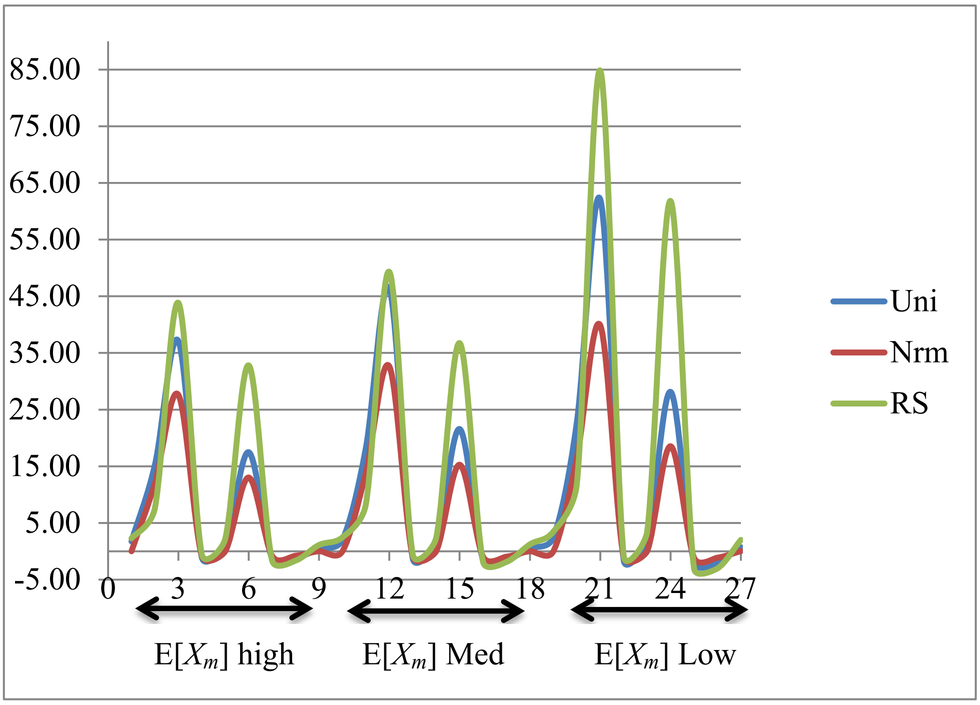

Figure 4.2: %improvement by substitution for 27 scenarios under different distributions.

We design the second set of experiments as bounded discrete stochastic distributions with three different shapes for the manufactured and remanufactured item demands and used item returns, which are totally random distribution for manufactured item, remanufactured demands and returned items. In first random distribution test that we call Random, we create 27 different scenarios as a combination of low, medium and high distribution means. In second random distribution test that we call 3-Random, we again create 27 different scenarios of low, medium and high distribution means but with a higher deviations between means and in third random distribution test that we call 9-Random, we again create 27 different scenarios of low, medium and high distribution means but using 3 different combination of low, medium and high means of distributions as the distribution shapes are shown in figure 4.3 and the mean of each different-shape stochastic distribution shown in Table 4.3.

Table 4.3: Mean values for different distributions.

	Mean		
Distribution shape	High	Medium	Low
Random	1.30	1.00	0.70
3-Random	2.39	1.54	0.66
9-Random	2.43	1.42	0.75
9-Random	3.90	2.40	0.82
9-Random	2.39	1.54	0.66

The % improvements in profit gained by using the substitution strategy over no substitution case are reported in Table 4.4. We make the following observations: We always have an additional profit for the manufacturer for all 81 scenarios as a result of substitution and among those scenarios, the highest with a 250 % when $E[X_r]$ is higher than $E[Y]$ and $E[X_m]$.

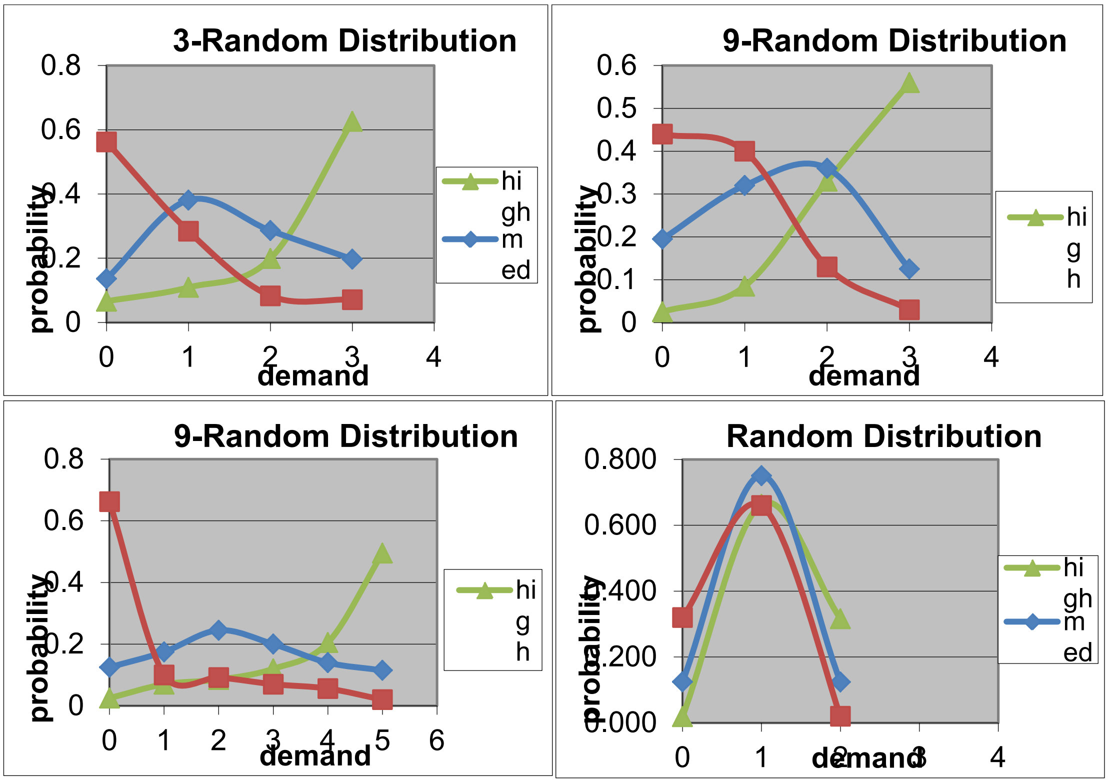

Figure 4.3: Different distribution types for random shapes.

It is also worth noting that the mean of manufactured item demand does not affect the amount of change in profit by substitution. However because the profit of manufacturing process is lower for lower manufactured item demand, a same amount of change in profit by substitution corresponds to a higher percent change of profit over no substitution case as the mean of manufactured item demand decreases. In short, the profitability of product substitution strategy is mainly dependent on the size of remanufactured item demand relative to that of returns.

Table 4.4: Absolute difference and improvement in profit by substitution (%) for different combinations of $E[X_m]$, $E[X_r]$ and $E[Y]$ under different random-shape distributions.

Means			Absolute difference between No substitution and Substitution & Improvement in profit by substitution (%)					
$E[X_m]$	$E[X_r]$	$E[Y]$	Random		3-Random		9-Random	
			Diff	%Imp	Diff	%Imp	Diff	%Imp
high	high	high	0.51	0.51	1.14	0.62	0.00	0.00
high	high	med	12.14	13.77	34.75	23.34	2.41	1.30
high	high	low	24.51	33.03	71.37	66.29	73.50	68.37
high	med	high	0.12	0.13	0.00	0.00	0.13	0.09
high	med	med	0.63	0.70	1.12	0.72	0.11	0.07
high	med	low	12.17	15.60	35.77	30.19	24.18	19.02
high	low	high	0.03	0.04	0.11	0.09	0.13	0.11
high	low	med	0.06	0.08	0.11	0.09	0.11	0.09
high	low	low	0.64	0.80	0.90	0.71	0.48	0.37
med	high	high	0.53	0.61	1.14	0.78	0.00	0.00
med	high	med	12.35	16.60	34.74	31.53	2.41	1.65
med	high	low	24.77	40.97	71.37	103.51	66.62	88.05
med	med	high	0.17	0.23	0.00	0.00	0.18	0.17
med	med	med	0.65	0.86	1.15	0.98	0.18	0.16
med	med	low	12.28	19.11	36.07	45.22	24.43	27.62
med	low	high	0.17	0.26	0.28	0.34	0.22	0.27
med	low	med	0.16	0.24	0.27	0.32	0.23	0.27
med	low	low	0.67	1.01	0.99	1.13	0.56	0.62
low	high	high	0.52	0.72	1.16	1.09	0.00	0.00
low	high	med	12.32	20.21	35.02	50.29	2.44	2.31
low	high	low	24.74	52.60	71.72	252.52	66.98	190.73
low	med	high	0.16	0.26	0.03	0.04	0.23	0.34
low	med	med	0.64	1.02	1.19	1.56	0.23	0.33
low	med	low	12.20	23.99	36.54	93.17	24.76	51.67
low	low	high	0.15	0.29	0.36	0.87	0.26	0.64
low	low	med	0.14	0.27	0.34	0.80	0.27	0.59
low	low	low	0.66	1.25	1.05	2.25	0.60	1.22

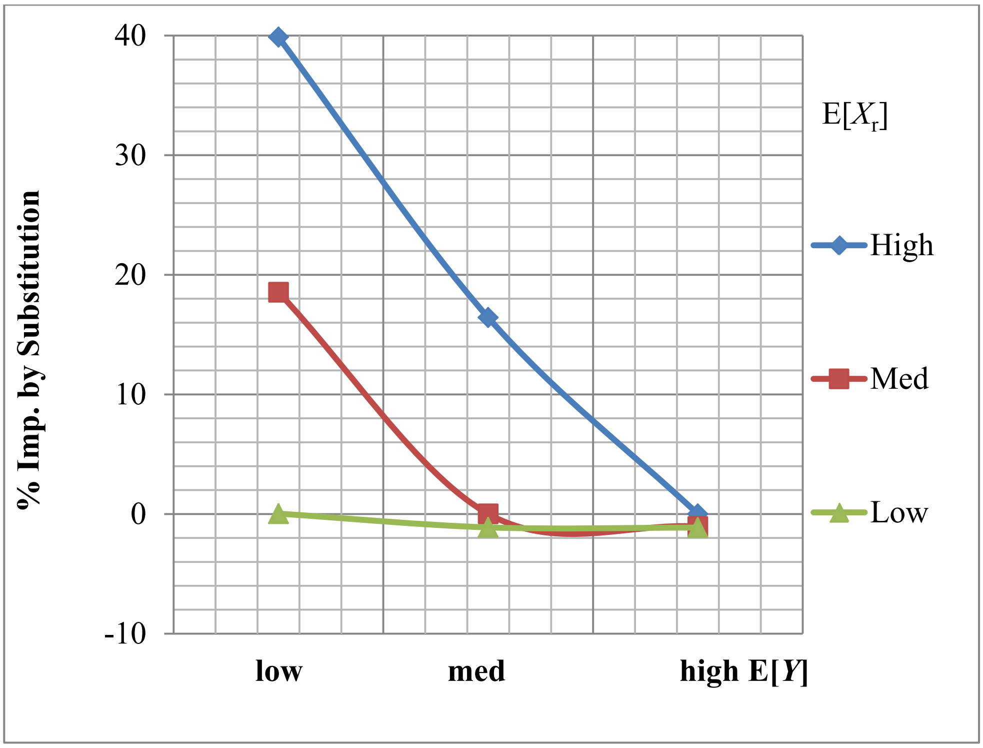

Figure 4.4: % improvement in profit as $E[X_r]$ and $E[Y]$ change (for Normal-shape distribution and low $E[X_m]$).

In order to extend the representative results to support that substitution results in a higher improvement in profit when the expected remanufactured item demand gets higher and/or the expected return gets lower also while manufacturing item demand means are varied, see figure 4.5, which plots the % improvements in profit by substitution for twenty seven scenarios with low, medium and high level of mean manufactured item demand and the Normal shaped distribution, and the mean of remanufactured item demand and returns as low, medium and high.

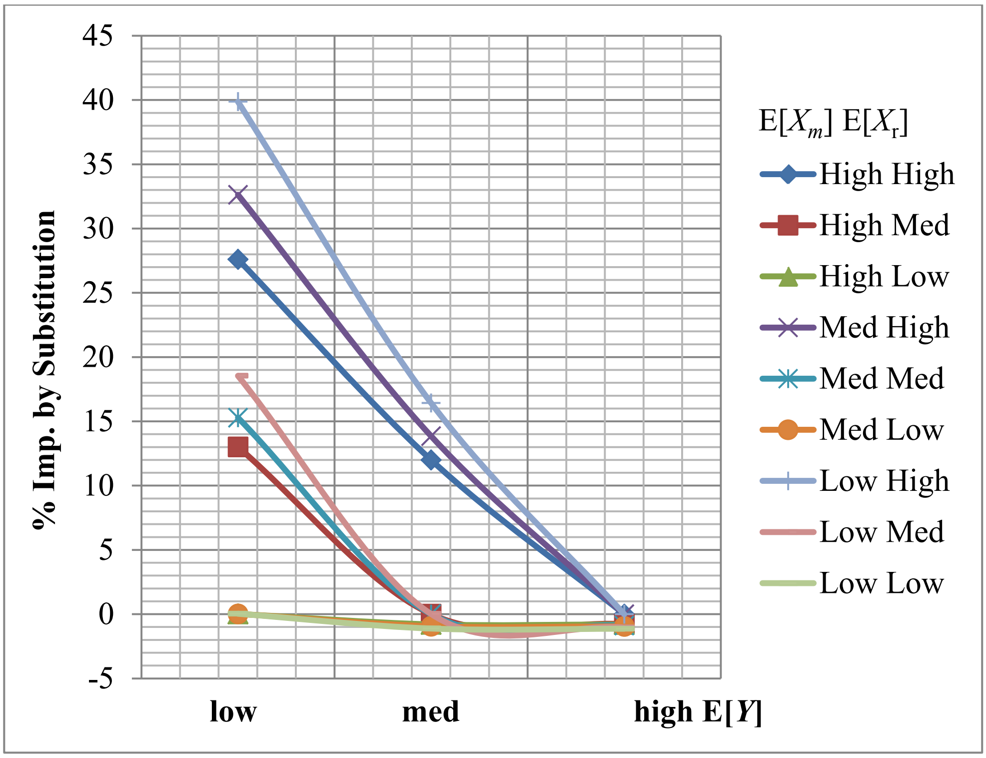

Figure 4.5: % improvement in profit as $E[X_m]$, $E[X_r]$ and $E[Y]$ change (for Normal-shape distribution).

We performed a third set of experiments in order to clearly see how the economic attractiveness of the substitution strategy varies as the return distribution changes. For this purpose, nine different return distributions are created with different coefficients of variations (CVs) ranging from 0.2 to 1.0 with an increment of 0.1, which are plotted in figure 4.6. All the distributions have the standard deviation of 0.5; hence they differ only by their mean, which ranges from 2.5 to 0.5 as CV changes from 0.2 to 1.0. The return distribution with coefficient of variation of 0.6 is also used as the demand distributions for remanufactured and manufactured items in this set of experiments.

Figure 4.7 shows how the expected profits for the hybrid system with/without product substitution change as the mean of the return distribution decreases from 2.5 to 0.5 (or CV increases from 0.2 to 1). The expected profits from the remanufacturing and manufacturing processes are also plotted separately for the no substitution case.
The following observations are made: Recall that in this set of experiments, the CV of remanufacturing item demand distribution was set 0.6. Hence, in all the scenarios with

return distribution's CV<0.6, the mean of return is higher than the mean of remanufactured item demand ($E[Y]>E[X_r]$). When CV<0.6, the use of substitution does not provide substantial additional profit over no substitution case (only around 0.2%) since the amount of returns available are typically sufficient to meet remanufactured item demand. However when CV exceeds 0.6 (i.e. $E[Y]$ goes below $E[X_r]$), a decrease in returns increases the economic attractiveness of product substitution from 0.6% to nearly 28%. Another observation is that when CV<0.6, an increase in CV (i.e. decrease in expected return) results in an increase in remanufacturing process profit while the effect is opposite for CV>0.6.

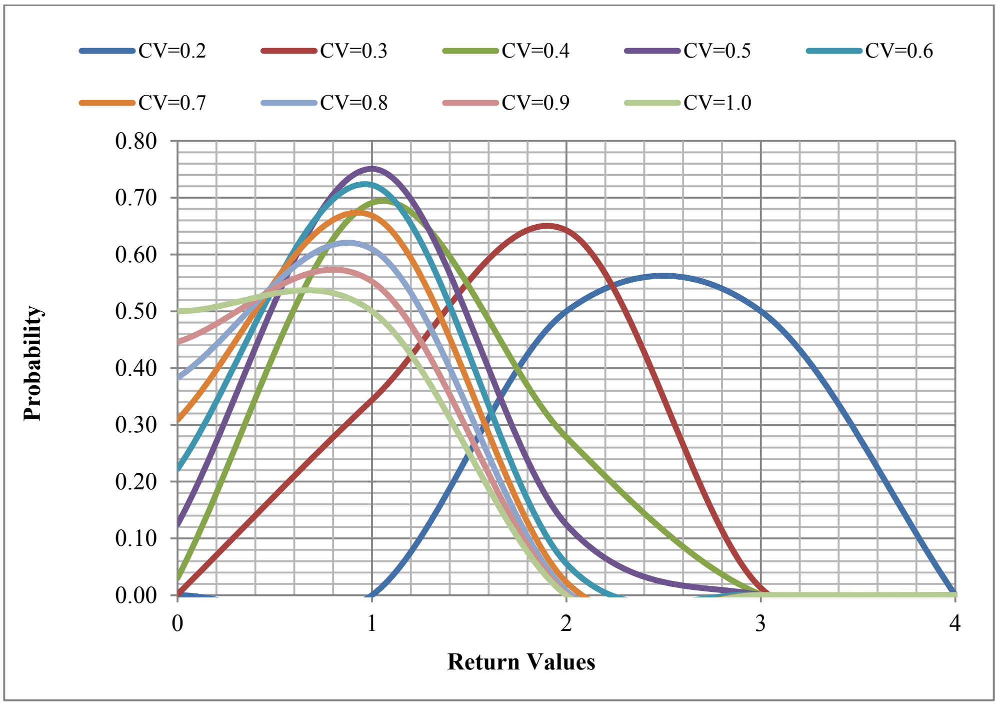

Figure 4.6: The return distributions with different coefficient of variations (CVs).

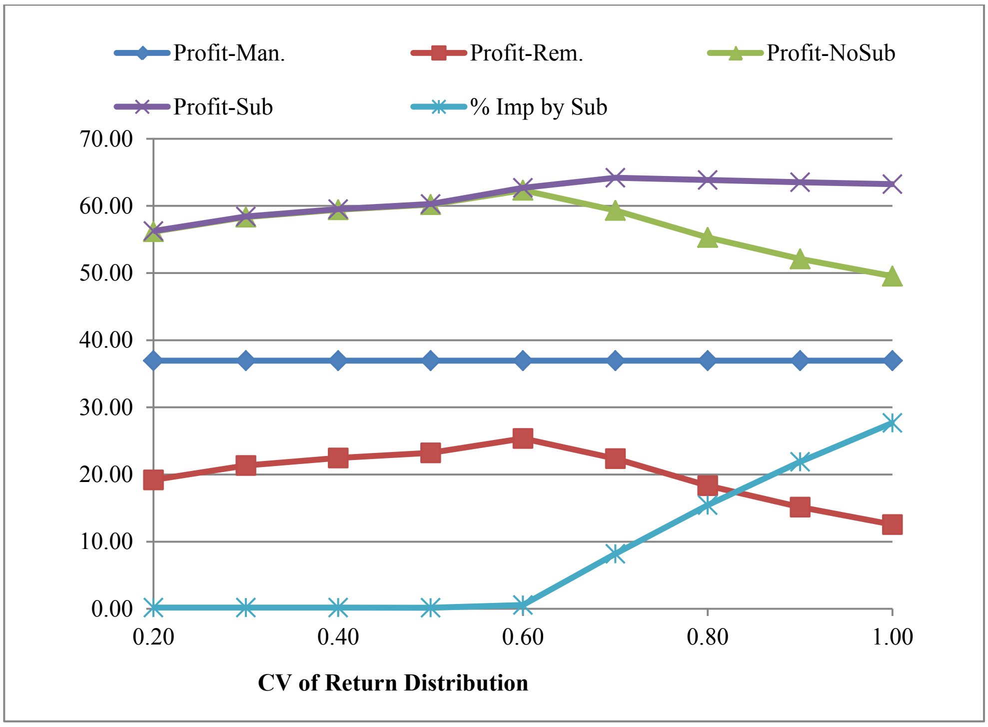

Figure 4.7: The expected profits under different CVs.

This can be explained as follows: For CV<0.6, the expected remanufacturing amount (consequently, the sales revenue for remanufactured items and the remanufacturing cost) remains unchanged as expected returns decrease because the returns are sufficient to meet the remanufactured item demand and the expected remanufacturing amount is just as much as remanufactured item demand. In this case the increase in profit for remanufacturing process is explained by the significant amount of savings obtained in disposal cost since less disposal is needed as returns get lower (see figure 4.8). For CV>0.6 (i.e. returns are not sufficient to meet all remanufactured item demand), a decrease in expected return decreases the profit for remanufacturing process because in this case sales revenue from remanufactured items decreases and the lost sales cost increases (see figure 4.8).

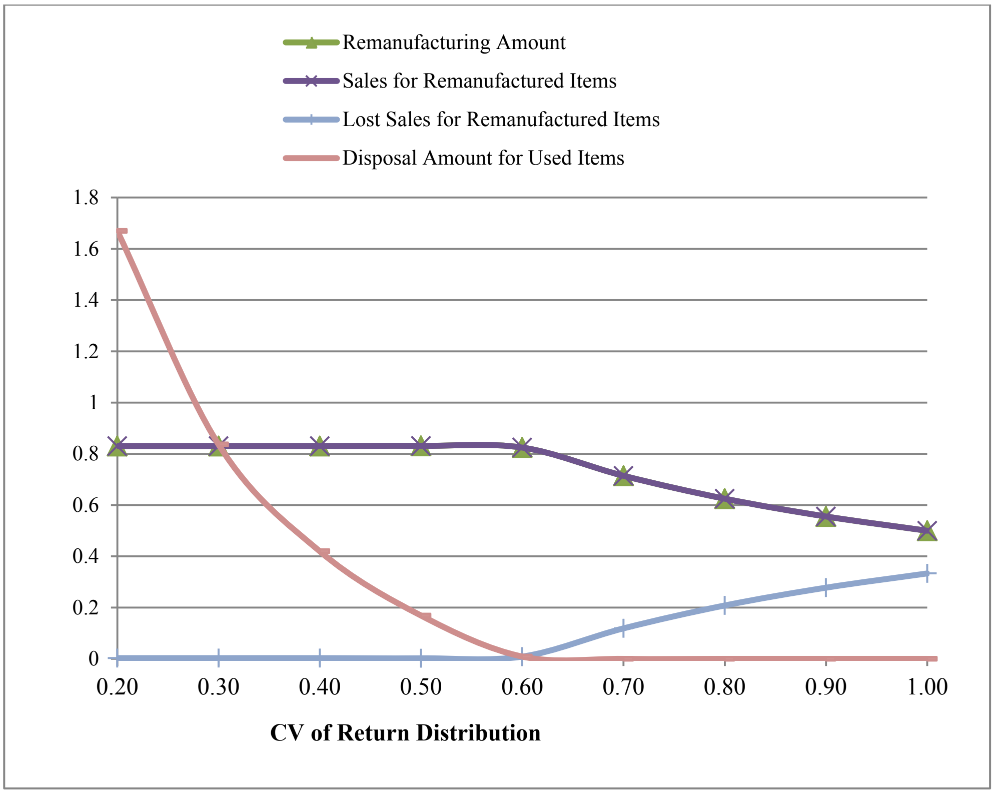

Figure 4.8: Expected values for remanufacturing amount, sales/lost sales for remanufactured items and disposal amount for used items for the no substitution case under different return CVs.

We executed a fourth set of experiments as creating seven different return distributions with same mean of 2 and different coefficients of variations (CVs) ranging from 0.2 to 0.8 with an increment of 0.1 and three different remanufactured item demand distributions with means of 1.5, 2 and 2.5 and same coefficient of variation of 0.4, which are displayed in appendix b. All the distributions for returns have the mean of 2; hence they differ only by their standard deviation, which ranges from 0.2 to 1.4 as CV changes from 0.2 to 0.8. The return distribution with coefficient of variation of 0.4 is also used as the demand distributions for remanufactured and manufactured items in this set of experiments.

Table 4.5: The comparison of different deviations of distribution with a same mean

	CofV E[Y][X_r]	0.20	0.30	0.40	0.50	0.60	0.70	0.80
Man	>	89.78	89.78	89.78	89.78	89.78	89.78	89.78
	=							
	<							
Rem	>	44.88	44.89	44.91	44.94	44.98	45.03	45.09
	=	64.74	64.64	64.49	64.27	64.00	63.66	63.27
	<	61.27	61.25	61.22	61.17	61.11	61.04	60.95
Total NoSub	>	134.66	134.67	134.69	134.71	134.75	134.81	134.87
	=	154.52	154.42	154.27	154.05	153.78	153.44	153.05
	<	151.05	151.03	150.99	150.95	150.89	150.82	150.73
Sub	>	134.78	134.79	134.81	134.84	134.88	134.94	135.00
	=	155.01	155.00	154.97	154.93	154.88	154.83	154.76
	<	171.41	171.40	171.39	171.38	171.35	171.31	171.26
Diff	>	0.13	0.13	0.13	0.13	0.13	0.13	0.13
	=	0.50	0.57	0.70	0.88	1.11	1.38	1.71
	<	20.36	20.37	20.40	20.43	20.46	20.49	20.52
%Imp	>	0.09	0.09	0.10	0.10	0.10	0.10	0.10
	=	0.32	0.37	0.45	0.57	0.72	0.90	1.12
	<	13.48	13.49	13.51	13.53	13.56	13.59	13.61

Table 4.5 shows the numerical results of how the expected profits for the hybrid system with/without product substitution change as the standard deviation of the return distribution increases from 0.2 to 1.4 (or CV increases from 0.2 to 0.8) while keeping the same mean for the distributions. We made the following observations: Recall that in this set of experiments, the CV of remanufacturing item demand distribution was set 0.4. and three different set of remanufactured item demand distribution mean is used to see the effect when E[Y] is greater '>', equal '=' or lower '<' than E[X_r]. Hence, in all the scenarios with the mean of returns are higher, equal or lower, there is an improvement made as a result of substitution. Moreover, it is seen that changing the standard deviation of the distribution makes very minimal effect on the improvement. When CV increases from 0.2 to 0.8, the improvement increases only 0.01%, 0.58% and 0.11% for the mean of returns are higher, equal or lower, respectively. For CVs ranging

from 0.2 to 0.8, expected values for remanufacturing only situation is also depicted in figure 4.9

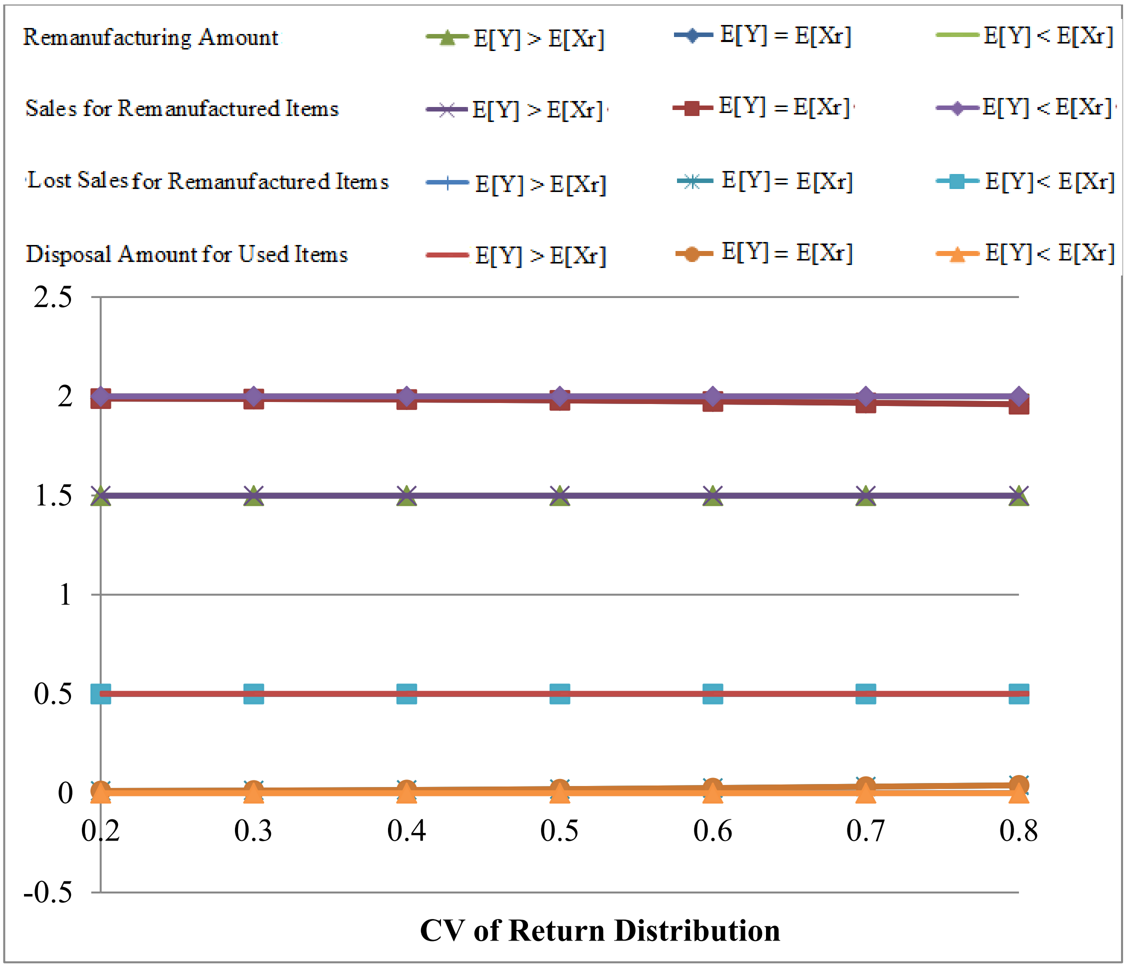

Figure 4.9: Expected values for remanufacturing amount, sales/lost sales for remanufactured items and disposal amount for used items for the no substitution case under different return CVs where mean values of return is greater, equal or lower than remanufactured item demand.

We have analyzed effect of standard deviation of the distribution for remanufactured item demand on improvement in profit by substitution. We are aware that a change in E[Xm] does not significantly affect the additional profit by substitution in terms of absolute value. Therefore we created another normal distribution (Nrm2) with a higher standard deviation than first normal distribution (Nrm) and check the cases where non-profit cases occur which are generally where the return item probability is higher or equal to remanufactured item probability (E[Xr]>=E[Y]). As a result, we have seen that as the standard deviation gets higher values, non-profitability diminishes, in other words profitability increases. You can see the effect of standard deviation of normal distribution in Table 4.6.

Table 4.6: The comparison of different deviations of distribution.

Probabilities			Absolute difference between No substitution and Substitution & Improvement in profit by substitution (%)			
E $[X_m]$	E $[X_r]$	E $[Y]$	Nrm		Nrm2	
			Diff	Imp%	Diff	Imp%
high	med	high	-1.36	-0.8	-1.28	-0.7
high	low	high	-1.27	-0.8	-1.22	-0.7
high	low	med	-1.26	-0.8	-1.25	-0.7
med	med	high	-1.36	-0.9	-1.27	-0.8
med	low	high	-1.28	-1.0	-1.22	-0.9
med	low	med	-1.27	-0.9	-1.24	-0.9
low	med	high	-1.35	-1.0	-1.26	-0.9
low	low	high	-1.27	-1.1	-1.20	-1.0
low	low	med	-1.27	-1.1	-1.21	-1.0

An extended analysis is performed to examine the impact of coefficient of variation on Remanufacture only situation. While keeping same standard deviation, we have altered the mean value of normal distribution with coefficient of variation differs from 0.25 to 1 (CofV= 0.25, 0.267, 0.286, 0.308, 0.4, 0.5, 0.667, 0.8, 1). You can see the created 10 different distribution cases for returned items in Figure 4.10. For these 10 different cases, we have tested remanufacture only case.

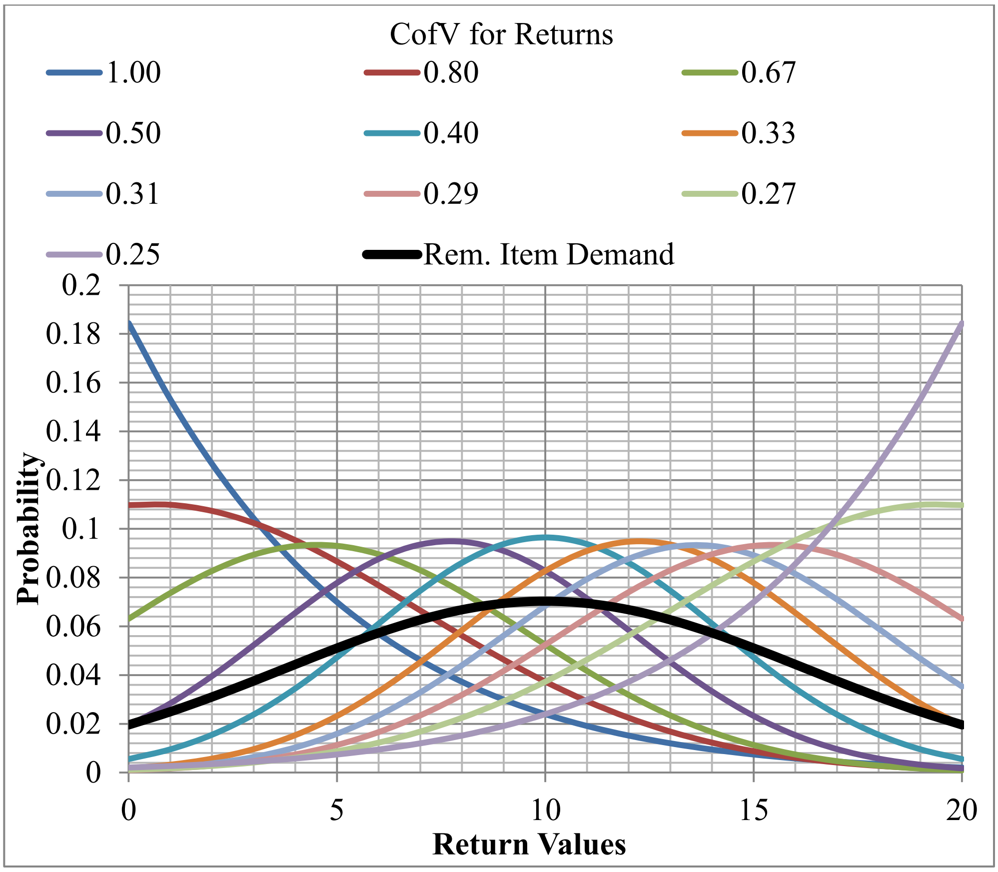

Figure 4.10: Return distributions with different coefficient of variations.

Considering the full set of results in Figure 4.11, we observe that profit increases as the coefficient of variation for used item return distribution gets equals to the coefficient of variation for remanufactured item demand distribution and then it starts to decrease. As it can be seen from figure 4.11, after CV=0.33, an increase in CV (i.e. decrease in expected return) results in a decrease in expected profit. Therefore we result that peak profit for remanufacture only situation is the place where used item return and remanufactured item demand distribution means get equal.

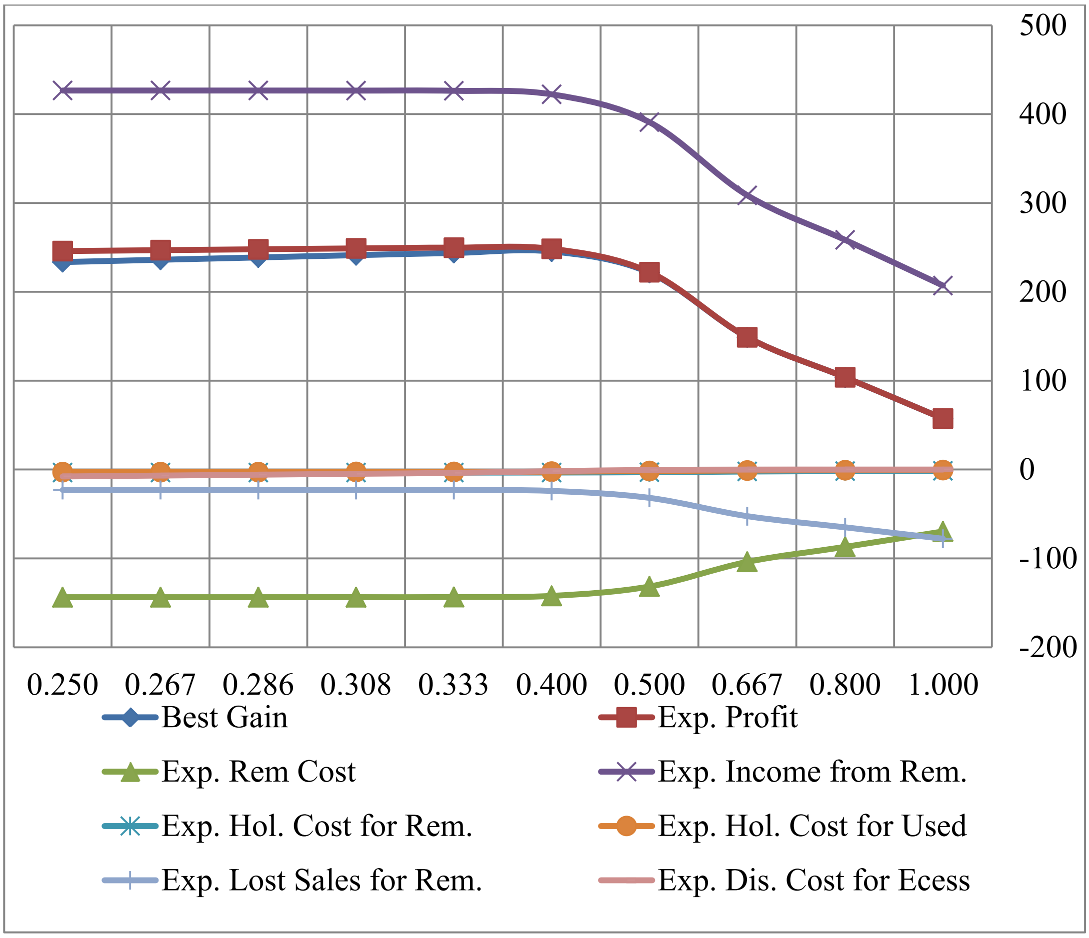

Figure 4.11: Variant of expected values of remanufacture only item under different CVs.

5. INVENTORY POLICY ANALYSIS

In a hybrid manufacturing/remanufacturing system, the manufacturing and remanufacturing decisions should be made after observing the state of the system, i.e. the inventory levels for manufactured, remanufactured and used items. When the MDP model that is described in section 3 is solved for a given scenario, we obtain a list of the optimal manufacturing (d_m) and remanufacturing (d_r) decisions corresponding to all possible states of the hybrid system for that particular scenario. However, the list of optimal decisions that MDP provides does not directly provide intuition into the structure of the optimal policy. In this section, our aim is to find a good characterization of the optimal policy using a few control parameters. We propose several simple-structured intuitive heuristic policies that are easy to implement in practice. Then we develop several heuristic search methodologies to determine the parameter values for these policies. We evaluate the performance of these techniques with respect to the solution quality as well as computational time. As the measure for solution quality, the percentage deviation of the profit of the heuristic policy from the optimal profit found by MDP is considered.

Table 5.1: The means of the scaled demand and return distributions.

	Mean		
Product	X_m	X_r	Y
1	0.66	0.85	0.59
2	0.83	1.20	0.93
3	0.64	1.06	0.74

We evaluated the proposed heuristic policies for three products produced by an automotive spare parts manufacturer. Due to privacy purpose, the identity of the firm will be kept anonymous. The firm is a world leader in diesel injection systems. Hence, two products are chosen from this product family. Product 1 is an "injector nozzle", which allows the pressurized fuel to be injected into the engine. Product 2 is a "common rail injector", which is one of the latest technologies in the fuel injection systems. Product 3 is an "engine starter", which is a type of electric motor. For confidentiality, the demand and return data for these products are scaled. The means of the scaled demand and return distributions are provided in table 5.1. The unit price for manufactured and remanufactured items, the unit manufacturing and remanufacturing costs for the three products are provided in table 5.2. In section 5.1, we introduce three heuristic policies to control the hybrid manufacturing/remanufacturing system under product substitution strategy.

Table 5.2: Unit price/cost information for the manufactured and remanufactured items.

	Product		
Parameter	1	2	3
p_m	20.48	77.94	68.39
c_m	6.11	25.17	22.74
p_r	12.51	57.53	51.85
c_r	3.12	16.36	17.46

5.1. Description of the Proposed Heuristic Policies

In this section, we introduce three heuristic policies that are developed to control the hybrid manufacturing/remanufacturing system where there are no set up costs for manufacturing and remanufacturing operations. Further, we develop two heuristic search techniques to determine the parameter values for the policies: one is a greedy search technique; the other is a distance-1 local search technique.

Among the three policies, one has two parameters, which represent target values for remanufactured and manufactured inventory levels while the other two policies have an additional parameter that limits the amount of additional manufacturing to be done in case the used item inventory falls short to raise the remanufactured item inventory to its target value. The three policies have a common remanufacturing strategy but they differ by their manufacturing strategy. The following notation is used for the description of the policies.

T_m : target value for manufactured item inventory
T_r : target value for remanufactured item inventory
T_s : secondary target value for remanufactured item inventory ($T_s < T_r$)
T_{m_max} : maximum value for manufactured item inventory
g_r : remanufacturing amount required for the target remanufactured item inventory
g_m : manufacturing amount required for the target manufactured item inventory
g_s : supplemental manufacturing amount required in case of used item shortage
d_m : manufacturing decision
d_r : remanufacturing decision
I_u : used item inventory
I_m : manufactured item inventory
I_r : remanufactured item inventory
I_{m_max} : inventory capacity for manufactured items
I_{r_max} : inventory capacity for remanufactured items
M_{max} : manufacturing capacity per period
R_{max} : remanufacturing capacity per period

5.1.1. (T_m, T_r) policy

In the (T_m, T_r) policy, T_m and T_r represent the target values for the manufactured and remanufactured items, respectively. This policy is described below and summarized in Table 5.3. Since the manufacturing decision is dependent on the remanufacturing decision, the remanufacturing decision is made first. If $I_r < T_r$, the required remanufacturing amount to raise the inventory level to T_r is $g_r = T_r - I_r$. However, the amount to remanufacture is restricted by the used item inventory as well as the remanufactured item storage and remanufacturing capacity. Hence, the remanufacturing decision is determined as:

$$d_r = \min\{g_r, I_u, I_{r\_max} - I_r, R_{max}\}.$$

In order to meet the target value for manufactured item inventory T_m, the required manufacturing amount is $g_m = T_m - I_m$. However, if the remanufacturing target cannot be met due to limitations of remanufacturing process (i.e. $d_r < g_r$) then the amount of shortage $(g_r - d_r)$ is met also by manufacturing. Considering the manufacturing and manufactured item storage capacities, the manufacturing decision is determined as:

$$d_m = \min\{g_m + (g_r - d_r), I_{m\_max} - I_m, M_{max}\}.$$

Table 5.3: (T_m, T_r) policy.

Case	**Required man./rem. amounts**	**Inventory Decisions for manufacturing\remanufacturing**
$T_r > I_r$	$g_r = T_r - I_r$	$d_r = \min\{g_r, I_u, I_{r\_max} - I_r, R_{max}\}$
$T_r \leq I_r$	$g_r = 0$	
$T_m > I_m$	$g_m = T_m - I_m$	$d_m = \min\{g_m + (g_r - d_r), I_{m\_max} - I_m, M_{max}\}$
$T_m \leq I_m$	$g_m = 0$	

5.1.2. (T_m, T_r, T_s) policy

In the (T_m, T_r, T_s) policy, three target values are considered: target value for manufactured item inventory, primary target value for remanufacturing item inventory and secondary target value for remanufacturing item inventory in case the primary target cannot be met using remanufacturing. This policy is described below and summarized in Table 5.4.

The remanufacturing decision is determined like in the (T_m, T_r) policy, i.e. if $I_r < T_r$, the required remanufacturing amount to raise the inventory level to T_r is $g_r = T_r - I_r$. However, the amount to remanufacture is restricted by the used item inventory as well as the remanufactured item storage and remanufacturing capacity. Hence, the remanufacturing decision is calculated as:

$$d_r = \min\{ g_r, I_u, I_{r\_max} - I_r, R_{max} \}$$

In order to meet the target value for manufactured item inventory T_m , the required manufacturing amount is $g_m = T_m - I_m$. However, if the primary target for the remanufactured item inventory cannot be met due to limitations of remanufacturing process (i.e. $d_r < g_r$), a secondary target is set, which is aimed to be met with the help of manufacturing. So, if the inventory level that is reached after remanufacturing decision is made is below the secondary target ($T_s > I_r + d_r$), then the shortage amount from the secondary target, i.e. $g_s = T_s - (I_r + d_r)$, is added to the required manufacturing amount. Considering the manufacturing and manufactured item storage capacities, the manufacturing decision is determined as:

$$d_m = \min\{ g_m + g_s, I_{m\_max} - I_m, M_{max} \}$$

Table 5.4: (T_m, T_r, T_s) policy.

Case	Required reman./man./additional man. amounts	Inventory Decisions for manufacturing\remanufacturing
$T_r > I_r$	$g_r = T_r - I_r$	$d_r = \min\{ g_{r,} I_u, I_{r\_max} - I_r, R_{max} \}$
$T_r \leq I_r$	$g_r = 0$	
$T_m > I_m$	$g_m = T_m - I_m$	$d_m = \min\{ g_m + g_s, I_{m\_max} - I_m, M_{max} \}$
$T_m \leq I_m$	$g_m = 0$	
$T_s > (I_r + d_r)$	$g_s = T_s - (I_r + d_r)$	
$T_s \leq (I_r + d_r)$	$g_s = 0$	

5.1.3. (T_m, T_r, T_{m_max}) policy

In (T_m, T_r, T_{m_max}) policy, the parameters T_m and T_r are interpreted like in the previous two policies. T_{m_max} represents the maximum value the manufactured item inventory level can be raised to after the extra manufacturing is done to compensate the insufficient remanufacturing. The manufacturing and remanufacturing decisions under this policy are calculated as shown in table 5.5. The policy operates as follows:

The remanufacturing decision is determined as follows: if $I_r < T_r$, the required remanufacturing amount to raise the inventory level to T_r is $g_r = T_r - I_r$. However, the amount to remanufacture is restricted by the used item inventory as well as the remanufactured item storage and remanufacturing capacity. Hence, the remanufacturing decision is calculated as:

$$d_r = \min\{ g_{r,} I_u, I_{r\_max} - I_r, R_{max} \}$$

In order to meet the target value for manufactured item inventory T_m , the required manufacturing amount is $g_m = T_m - I_m$. However, if this target for the remanufactured item inventory cannot be met due to limitations of remanufacturing process (i.e. $d_r < g_r$), then the shortage amount $g_r - d_r$ is met by doing extra manufacturing as long as the manufactured item inventory does not exceed T_{m_max}. Considering the manufacturing and manufactured item storage capacities, the manufacturing decision is calculated as:

$$d_m = \min\{\, g_m + g_r - d_r,\ T_{m\_max} - I_m, I_{m\_max} - I_m,\ M_{max}\,\}$$

Table 5.5: (T_m, T_r, T_{m_max}) policy.

Case	Required reman./man	manufacturing\remanufacturing decisions
$T_r > I_r$	$g_r = T_r - I_r$	$d_r = \min\{\, g_r, I_u,\ I_{r\_max} - I_r,\ R_{max}\}$
$T_r \le I_r$	$g_r = 0$	
$T_m > I_m$	$g_m = T_m - I_m$	$d_m = \min\{g_m + (g_r - d_r),\ T_{m\_max} - I_m,$ $I_{m\_max} - I_m,\ M_{max}\}$
$T_m \le I_m$	$g_m = 0$	

5.2. Heuristic Methods Proposed to Determine Policy Parameters

In this section, we propose the use of two heuristic search algorithms in order to determine the parameter values for the three inventory policies introduced in section 5.1. Clearly, through the total enumeration (i.e. evaluation of all possible parameter combinations and selection of the best among them) is an exact technique which guarantees to find the optimal parameter values for a given policy. However, this technique is not efficient, because it may require significant computational time when the number of possible parameter combinations is very high. We propose two heuristic algorithms that evaluate significantly less parameter computations than the total enumeration technique, yet they are successful in finding a near-optimal parameter combination, if not optimal. In addition to their advantages of *computational efficiency* and *good solution quality*, both heuristic algorithms are also *simple in concept* and *easy-to-implement*.

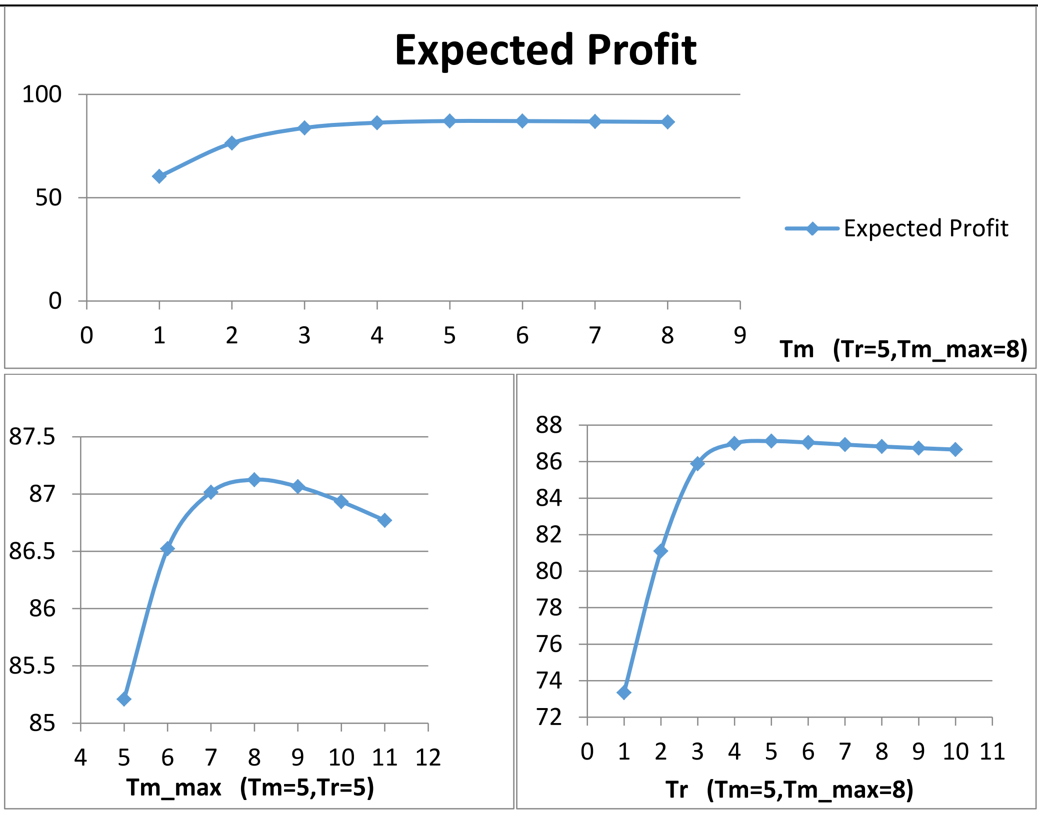

Figure 5.1: The expected profit as a function of one parameter for the (T_m, T_r, T_{m_max}) policy for product 2 when the other two parameters are fixed.

5.2.1. Greedy Search Algorithm

The first heuristic algorithm we propose is a greedy search algorithm (GS) that is based on our observation of the concavity of the profit function. For each heuristic policy for each product, considered a high range of values for the policy parameters and we plotted the expected profit as a function of one policy parameter when the other policy parameters are fixed. Figure 5.1 reports some representative plots for the profit function of the (T_m, T_r, T_{m_max}) policy for product 2. It can be clearly seen from Figure 5.1 that, the expected profit function seems to have a concave shape.

Based on this observation, we designed an algorithm that searches for the best policy parameter combination iteratively where at each iteration, the best value of one parameter for a given inventory policy is searched by increasing or decreasing its current value by one unit (while the values of other parameters are kept fixed) until no more improvement in the profit can be obtained. This process is repeated for every policy parameter at each iteration. The algorithm stops when there is no improvement in profit in the last two consecutive iterations.

The steps of the greedy search (GS) algorithm are the following:

Initialization: Finding an initial policy
In this step, the policy parameters are assigned their initial values. Initial values can be determined either randomly or using a systematic procedure. Two systematic procedures to determine an initial policy are presented later in chapter 5.

Iteration:
For every parameter of the current policy do the following:

1. Increase the current value of the parameter by 1 and calculate the profit of the resulting neighbor policy.
    a. If the profit of the neighbor policy is better than that of current policy, then replace the current value of the parameter with that of the neighbor policy, and go to step 2.
    b. If the profit of the neighbor policy is not better than that of current policy, then decrease the current value of the parameter by 1. Calculate the profit of the neighbor policy. If the neighbor policy is better than the current policy, then replace the current value of the parameter with that of the neighbor policy, and go to step 3. Otherwise, go to step 4.

2. Increase the current value of the parameter by 1 and calculate the profit of the resulting neighbor policy. If the resulting neighbor policy is better than current policy, then repeat step 2. Otherwise, go to step 4.
3. Decrease the current value of the parameter by 1 and calculate the profit of the resulting neighbor policy. If the resulting neighbor policy is better than current policy, then repeat step 3. Otherwise, go to step 4.
4. Stop. The current value for the parameter is its best value at the current iteration.

For instance, for the (T_m, T_r, T_{m_max}) policy, at each iteration, without loss of generality, first the best value for the parameter T_m is determined (i.e. steps 1-3 are done), then the one for T_r is determined and finally the one for T_{m_max} are determined. This reasoning is used also for the (T_m, T_r) and (T_m, T_r, T_s) policies.

Stopping condition.
The algorithm stops when the better policy cannot be found at the last two consecutive iterations. This policy is the best policy found by the greedy search algorithm starting with the given initial policy.

5.2.2. Distance-1 Neighborhood Search Algorithm

The second heuristic algorithm we propose is a distance-1 neighborhood search algorithm. A distance-1 neighbor policy is defined as a policy that is created by increasing or decreasing by 1 unit the value of one parameter of the current policy while the other parameters' values are kept fixed. Hence, for a current policy with three parameters, 6 different neighbor policies can be evaluated this way.

In the distance-1 neighborhood search algorithm, at each iteration, all the distance-1 neighbor policies are evaluated and the best among them is chosen. If the best neighbor is better than the current policy, then this process is repeated. Otherwise, the algorithm stops. The steps of this algorithm are described in details below.

Initialization: Finding an initial policy

In this step, the policy parameters are assigned their initial values. Initial values can be determined either randomly or using a systematic procedure. Two systematic procedures to determine an initial policy are presented later in chapter 5.

Iteration:

1. Determine all the distance-1 neighbor policies.
2. Calculate the profit for all the neighbor policies.
3. Determine the best neighbor policy.
4. Compare the profit of the best neighbor policy to the profit of current policy.

Stopping condition.

The algorithm stops when the best neighbor found at current iteration is not better than the current policy. Then, the current policy is named as the best policy found by the distance-1 neighborhood search heuristic for the given initial policy.

5.2.3. Methods to Determine an Initial Policy

The heuristic algorithms presented in section 5.2 require the determination of an initial policy to start with. In this section, we present three ways for determining the initial values for the policy parameters: random generation, Newsboy approximation and estimation based on the MDP optimal policy. You can see the optimal policy parameters for the three policies for the three products in table 5.6.

Table 5.6: Optimal parameter values for three policies of the three products.

	Policy 1		**Policy 2**			**Policy 3**		
Product	T_m	T_r	T_m	T_r	T_s	T_m	T_r	T_{m_max}
1	6	2	6	4	1	6	3	7
2	6	3	6	5	2	5	5	8
3	4	3	5	4	1	4	4	7

5.2.3.1. Random Generation

Randomly generating the initial values for the policy parameters is without doubt the simplest way to initialize the algorithm. When the initial solution for a heuristic algorithm (i.e. an algorithm that does not guarantee to find the optimal solution) is determined randomly, a commonly used approach is to run the algorithm multiple times with different randomly generated initial solutions and select the best solution among these multiple runs. Table 5.7 gives an example initial policy parameters randomly generated for the three products and three policies.

Table 5.7: Example for initial parameter values generated randomly.

Policy Parameters	**Product**		
	1	2	3
T_m	7	6	6
T_r	6	7	5
T_s	5	4	7
T_{m_max}	7	8	9

In this study, the heuristic algorithms with randomly generated initial solutions are run 10 times and the best solution among these runs is reported for each of the three heuristic policies and each of the three products. We have used the best results of this 10 random run to compare the initial guesses while forecasting the policy parameters. You can see the percent deviation of forecasted initial policy parameters from optimal policy parameters in table 5.8.

Table 5.8: % Deviation from optimal parameter values for randomly generated initial parameters.

	Policy		
Product	**1**	**2**	**3**
1	-5.818	-3.996	-0.286
2	-2.790	-1.869	-0.262
3	-3.910	-0.363	-1.153

5.2.3.2 Estimation based on optimal policy by MDP

A randomly generated initial solution may be far from the optimal solution, in which case the heuristic algorithm may not be able to find a good quality solution or even if it can, it does so in higher computational time.

Table 5.9: Parameter values estimated based on the optimal policy by MDP.

Policy	**Product**		
Parameters	1	2	3
T_m	6	6	4
T_r	8	9	5
T_s	2	3	3
T_{m_max}	8	9	7

Starting a local search algorithm with an initial solution that is closer to the optimal solution is expected to find a near-optimal or optimal solution in significantly less computational time. Therefore, in order to get a better performance in terms of solution quality and/or computational time for the heuristic algorithms, a better approach than the random generation for initial solution determination may be worth investigating. One approach we propose is to make use of the optimal policy determined by solving the MDP formulations for the inventory control problem of the hybrid manufacturing/ remanufacturing system when estimating initial parameter values for the policy.

Recall that the optimal policy found by MDP is a list of the optimal manufacturing and remanufacturing decisions for every inventory state of the hybrid system, i.e. for every combination of manufactured item, remanufactured item and used item inventories, which form a state of the system, there is a pair of optimal manufacturing and remanufacturing decisions determined by MDP. Considering the meaning of the policy parameters in each heuristic policy, we estimate the initial values for these parameters by observing the optimal policy by MDP as follows.

- Estimation of the value for T_r
  Recall that T_r is the target value for the remanufactured item inventory. In order to estimate the value of T_r, we look at the states where it is optimal to do remanufacturing (i.e. d_r>0). T_r is set to the highest value that the remanufactured item inventory position takes after the remanufacturing decision has been made (i.e. I_r+d_r) among the states where d_r>0.

- Estimation of the value for T_m
  Recall that T_m is the target value set for the manufactured item inventory to meet only the manufactured item demand. On the other hand, the manufacturing is also used to produce the new items that will be used to meet remanufacturing demand in case the remanufacturing target T_r cannot be met under given conditions. Thus, the manufactured item inventory usually contains the new items that will be used to satisfy both manufactured and remanufactured item demand. In order to avoid overestimating the value of T_m, we should look only at the states with positive optimal manufacturing amount (i.e. d_m>0) where it is optimal not to remanufacture (i.e. d_r=0) even though the used item inventory is positive (i.e. I_u>0), because in these states, since no remanufacturing is done, that means the target for remanufactured item inventory is already met; therefore, in these states, the manufacturing is only done to meet the manufactured item demand, no extra manufacturing is done to meet the

remanufactured item demand. T_m is set the highest value the manufactured item inventory takes after the manufacturing decision has been made (i.e. $I_m + d_m$) among those states.

- Estimation of the value for T_{m_max}

  Recall that T_{m_max} is the maximum value the manufactured item inventory can get after making additional manufacturing in case the remanufacturing amount made in a state is not sufficient to meet the target value for the remanufactured item inventory. Therefore, in order to estimate the value of T_{m_max}, we look at the states where it is optimal to manufacture (i.e. $d_m > 0$). T_{m_max} is set the highest value that the manufactured item inventory position gets after the manufacturing decision has been made (i.e. i.e. $I_m + d_m$) among the states where $d_m > 0$.

- Estimation of the value for T_s

  We simply set $T_s = T_{m\_max} - T_m$.

Using this MDP-based estimation approach, the parameter values for the three policies for the three products are determined, which are reported in table 5.9 and the percent deviation of forecasted initial policy parameters from optimal policy parameters are reported in table 5.10.

Table 5.10: % Deviation from optimal parameter values for MDP-based initial parameters.

	Policy		
Product	**1**	**2**	**3**
1	-6.508	-0.749	-0.566
2	-3.561	-0.818	-0.702
3	-3.713	-2.421	-0.173

5.2.3.3. Newsboy-Based Estimation

In this section, we introduce another approach to estimate the initial values for the policy parameters, which uses Newsboy-like formulae. This approach is computationally more efficient than the MDP-based estimation technique, because while in MDP-based estimation technique, the initial values are based on the optimal policy, which requires the solution of an MDP model, in the Newsboy-based estimation approach, the values of the policy parameters are determined using simple formulae, as explained below.

In the multi-period Newsboy model, the underage and overage costs are defined as follows: the underage cost (C_u) is the cost of having one unit of stock-out at the end of a period. If in case of a stock-out, lost sales occur, then the underage cost includes the unit lost sales cost (i.e. penalty cost) and the unit profit lost (i.e. unit selling price-unit cost of production). If in case of stock-out, the unsatisfied demand is backordered, then the underage cost is equal to unit backordering cost. The overage cost (C_o) is the cost of having one unit of excess stock at the end of a period. The overage cost includes the unit holding cost per period.

The critical fractile (CF) in Newsboy model is calculated as the ratio of underage cost to the sum of underage and overage costs as shown below:

$$CF= C_u / (C_u+ C_o)$$

In order to estimate the target value for the manufactured item inventory, i.e. the value of T_m, first the underage and overage cost for manufacturing are determined as follows:

In case of a stock-out for manufactured items, the unsatisfied demand is backordered. Therefore the underage cost is calculated as the sum of the unit backordering cost per period for the manufactured items and the unit profit for manufacturing items, and the overage cost is simply unit holding cost for manufactured items. The critical fractiles calculated to estimate T_m for the three products considered in this study are reported in table 5.11.

$$C_u: \ p_m - c_m + b_m \tag{1}$$

$$C_o : h_m \tag{2}$$

Table 5.11: Critical fractile value to estimate T_m for the three products.

Critical fractile for T_m	**Product**		
	1	2	3
C_u	18.4660	68.3580	59.3280
C_o	0.1018	0.4195	0.3790
CF	0.9945	0.9939	0.9937

In order to estimate the target value for remanufactured item inventory (T_r), the underage and overage costs for the remanufacturing business are calculated as follows: In case of a stock-out for remanufactured items, unsatisfied demand is considered to be lost. Hence, the underage cost is calculated as the sum of unit lost sales cost for the remanufacturing product and the unit profit made by selling a remanufactured product while the overage cost is simply unit holding cost per period for the remanufactured product, as can be seen in equations 3 and 4, respectively. The critical fractile values used to estimate T_r value for the three products are reported in 5.12.

$$C_u: \ p_r - c_r + l_r \tag{3}$$

$$C_o : h_r \tag{4}$$

Table 5.12: Critical fractile value to estimate T_r for the three products.

Critical fractile for T_r	**Product** 1	2	3
C_u	12.5175	55.5525	47.3525
C_o	0.0520	0.2727	0.2910
CF	0.9959	0.9951	0.9939

In order to estimate the value of the secondary target for the remanufactured item (T_s), the underage and overage costs are determined as follows: Recall that the secondary target for remanufactured item inventory is met by doing extra manufacturing. This additional amount manufactured is expected to be sold for the price of remanufactured product to the customers demanding remanufactured products. Hence, the underage cost is calculated as the sum of unit lost sales cost for remanufactured products and the unit profit made by selling a manufactured item for the price of remanufactured item (i.e. $p_r - c_m$) (see equation 5). The overage cost is simply the unit holding cost for manufactured product as shown in equation 6. The critical fractiles calculated to estimate T_s for the three products considered in this study are reported in table 5.13.

$$C_u : \ p_r - c_m + l_r \tag{5}$$

$$C_o : h_m \tag{6}$$

Table 5.13: Critical fractile value to estimate T_s for the three products.

Critical fractile for T_s	**Product** 1	2	3
C_u	9.5275	46.7425	42.0725
C_o	0.1018	0.4195	0.3790
CF	0.9894	0.9911	0.9911

After the critical fractiles are determined, the next step is to determine the cumulative distribution functions for manufactured item demand (F_{Xm}), remanufactured item demand (F_{Xr}) as well as the excess demand (F_{Xr-Y})for remanufactured item that is expected to be satisfied through substitution (i.e. the demand for remanufactured item in excess of returns). The probability mass function for the excess demand for remanufactured items is determined considering the probability mass functions of the remanufactured item demand and used item returns. The cumulative distributions for MD, RD and ED for the three products can be found in Table 5.14.

T_m, T_r and T_s are assigned the smallest values that make the equations 7, 8, and 10 hold, respectively.

$$P \text{ (manufactured item demand} < T_m) \geq \text{critical fractile for } T_m \quad (7)$$

$$P \text{ (remanufactured item demand} < T_r) \geq \text{critical fractile for } T_r \quad (8)$$

$$P \text{ (excess demand for remanufactured item} < T_s) \geq \text{critical fractile for } T_s \quad (9)$$

Table 5.14: Distribution functions for manufactured item demand, remanufactured item demand and excess demand for remanufactured item for the three products.

	Product 1			Product 2			Product 3		
i	$F_{Xm}(i)$	$F_{Xr}(i)$	$F_{Xr-Y}(i)$	$F_{Xm}(i)$	$F_{Xr}(i)$	$F_{Xr-Y}(i)$	$F_{Xm}(i)$	$F_{Xr}(i)$	$F_{Xr-Y}(i)$
0	0.440	0.345	0.514	0.369	0.274	0.466	0.440	0.297	0.461
1	0.952	0.845	0.885	0.845	0.631	0.734	0.940	0.678	0.755
2	0.976	0.976	0.982	0.964	0.905	0.932	0.976	0.964	0.973
3	0.988	0.988	0.991	0.988	0.988	0.991	1.000	1.000	1.000
4	0.988	1.000	1.000	1.000	1.000	1.000			
5	1.000								

Table 5.15: Parameter values estimated by Newsboy-based approach.

Policy parameters	Product		
	1	2	3
T_m	5	4	3
T_r	4	4	3
T_s	3	3	3
T_{m_max}	8	7	6

The value of the parameter T_{m_max} is simply estimated to be $T_{m\_max} = T_s + T_m$. The parameter values calculated for the three products using Newsboy-based estimation approach are reported in table 5.15 and the percent deviation of forecasted initial policy parameters from optimal policy parameters are reported in table 5.16.

Table 5.16: % Deviation from optimal parameter values for Newsboy-based initial parameters.

	Policy		
Product	**1**	**2**	**3**
1	-4.418	-3.874	-0.629
2	-1.459	-2.171	-1.864
3	-1.894	-3.865	-3.435

5.3. Performance Comparison of Heuristic Policies and Methods

In this section, a numerical study is done to compare the performance of the heuristic policies as well as the heuristic algorithms using the data for three products from the automotive spare part manufacturer.

5.3.1 Total Enumeration Results

Total enumeration technique evaluates all possible parameter combinations for a given policy and the parameter combination that gives the highest profit is declared as the optimal parameter combination. For instance, for the (T_m, T_r) policy, each parameter is assigned values from 1 to 20 (20 being the upper bounds for manufactured and remanufactured item inventories), which results in 400 different parameter combinations, i.e. 400 different policies to evaluate. The best among these 400 policies is declared as the optimal (T_m, T_r) policy. However, even though the total enumeration technique guarantees to find the optimal policy, it is a computationally inefficient technique.

Table 5.17: Gains for the three policies for the three products.

Product	Opt.Gain by MDP	Gain for Policies		
		(T_m,T_r)	(T_m,T_r,T_s)	(T_m,T_r,T_{m_max})
1	15.795	15.6406	15.7811	15.7808
2	87.231	86.1548	87.1448	87.1250
3	61.350	59.9081	61.1359	61.2536

Table 5.18: Deviations from optimal gains for the three policies for the three products.

Product	Opt.Gain	% deviation from optimal gain		
		(T_m,T_r)	(T_m,T_r,T_s)	(T_m,T_r,T_{m_max})
1	15.795	-0.978	-0.088	-0.090
2	87.231	-1.234	-0.099	-0.121
3	61.350	-2.350	-0.349	-0.157

The gains of the three heuristic policies calculated using total enumeration technique or heuristic algorithms and their deviations from the optimal gain found by MDP for the three products are reported in Tables 5.17 and 5.18, respectively. According to the results, the best policy seems to be (T_m,T_r,T_{m_max}) policy for product 3, with the smallest deviation from optimal gain, which is 0.157%. For products 1 and 2, the (T_m,T_r,T_s) policy seems to be the best among the three policies, with deviations from optimal gains by 0.088% and 0.099%, respectively.

Table 5.19: Gains of the policies' initial point with different initial procedures for the three products.

Product	**Opt.Gain by MDP**	**Initial Procedure**	**Gain for policies initial point**		
			(T_m,T_r)	(T_m,T_r,T_s)	(T_m,T_r,T_{m_max})
1	15.795	Random	14.731	15.150	15.736
		MDP	14.623	15.663	15.691
		Newsboy	14.950	15.170	15.682
2	87.231	Random	83.751	85.516	86.897
		MDP	83.087	86.432	86.513
		Newsboy	84.898	85.253	85.501
3	61.35	Random	57.566	60.914	60.547
		MDP	57.684	59.656	61.147
		Newsboy	58.773	58.773	59.150

5.3.2. Heuristic Algorithms Results

This section presents the results of the heuristic search algorithms. The gains of the three heuristic policies calculated using different initial procedure's starting point initial solutions and their deviations from heuristics' near optimal gain for Initial procedure's starting point initial solutions found by MDP for the three products are reported in Tables 5.19 and 5.21, respectively. Tables show that even in the worst possible initial state, our initial procedures' starting points are only around 6.5 percent away from heuristic policies' solution and around 7.4 percent away from optimal solution. Table 5.23 presents the percent deviation from optimal profit found by each heuristic algorithm using different initial procedures for each heuristic policy for each product and in order to clarify our findings further, table 5.23 reports the total number of policies (i.e. parameter combinations) evaluated for each heuristic policy by each heuristic algorithm starting with different initial policies for the three products. The detailed information about the parameter values and run times for all the algorithms and the heuristic policies can be found in Appendix C.

Table 5.20: Number of policies evaluated and percent deviation from optimal gain found using different techniques for (T_m, T_r) policy for product 2.

(T_m, T_r) Policy				
	Algorithms	# of Policies Evaluated	% Deviations	Gain
Product 2	Total Enumeration	400	-1.23379	86.15475
	Random Start GS	36	-1.23379	86.15475
	Random Start D1	41	-1.23379	86.15475
	MDP Guess GS	11	-1.23379	86.15475
	MDP Guess D1	23	-1.23379	86.15475
	Newsboy Guess GS	9	-1.63894	85.80133
	Newsboy Guess D1	8	-1.23379	86.15475

The heuristic algorithms, i.e. greedy search (GS) and distance-1 local search (D1) algorithms, whether they start with a randomly generated initial solution or an initial solution determined using MDP-based or Newsboy-based estimation approach, mostly find the optimal values for the parameters of the heuristic policies as reported in Tables 5.17 and 5.18 (i.e. the parameter values determined by total enumeration) for the three products with two exceptions. The gain of (T_m, T_r) policy and its deviation from optimal gain for the second products found by each different technique is reported in table 5.20.

As you can see, there is only a small exception found in greedy search algorithms newsboy formula initiation for the second product of policy one. The other searches and total enumeration deviates from optimal gain by -1.234 for the second product of policy one; however this one exception deviates by -1.639 for newsboy, as it can be seen from table 5.20 and 5.21. Since we use only integer values for parameter values, there is a local optima for the second product of policy one.

Table 5.21: Deviations from near optimal gains of the heuristic policies for different initial procedures for the three products.

Product	Policies	% deviation from heuristic's near optimal gain for Initial procedures					
		(T_m, T_r)		(T_m, T_r, T_s)		(T_m, T_r, T_{m_max})	
	Initial Procedure	Dev. From Opt.	Dev. From Policy	Dev. From Opt.	Dev. From Policy	Dev. From Opt.	Dev. From Policy
1	Random	-6.739	-5.818	-4.081	-3.996	-0.376	-0.286
	MDP	-7.422	-6.508	-0.837	-0.749	-0.656	-0.566
	Newsboy	-5.353	-4.418	-3.959	-3.874	-0.718	-0.629
2	Random	-3.989	-2.790	-1.966	-1.869	-0.383	-0.262
	MDP	-4.750	-3.561	-0.916	-0.818	-0.823	-0.702
	Newsboy	-2.675	-1.459	-2.268	-2.171	-1.983	-1.864
3	Random	-6.168	-3.910	-0.710	-0.363	-1.309	-1.153
	MDP	-5.976	-3.713	-2.762	-2.421	-0.330	-0.173
	Newsboy	-4.200	-1.894	-4.200	-3.865	-3.586	-3.435

The search results show that fastest working algorithm is the greedy search for all three policies. Among the greedy search algorithm, the nearest parameter values guessed through MDP optimal policy. We can see that third product of policy one for the MDP guessed greedy search algorithm only considers seven policies to find the best deviation from optimal gain by -2.350 for the first policy. The total enumeration is, however, considers 400 policy evaluations to find the same result. On the other hand, best resulting deviation which was policy 2 and 3 for the product one as -0.088 and -0.090, respectively, also needs 1900 policy evaluation for second policy and 8000 policy evaluation for third policy. Our heuristic algorithm takes less than fifty steps for maximum policy evaluation on all heuristic procedures.

Table 5.22: %deviation from optimal found by each heuristic algorithm for each heuristic policy for each product.

<table>
<tr><th rowspan="2">Product</th><th rowspan="2">Policy</th><th>Total</th><th colspan="2">Random</th><th colspan="2">MDP</th><th colspan="2">Newsboy</th></tr>
<tr><th>Enumeration</th><th>GS</th><th>D1</th><th>GS</th><th>D1</th><th>GS</th><th>D1</th></tr>
<tr><td rowspan="3">1</td><td>T_m, T_r</td><td>-0.978</td><td colspan="2">-0.978</td><td colspan="2">-0.978</td><td colspan="2">-0.978</td></tr>
<tr><td>T_m, T_r, T_s</td><td>-0.088</td><td colspan="2">-0.088</td><td colspan="2">-0.088</td><td colspan="2">-0.088</td></tr>
<tr><td>T_m, T_r, T_{m_max}</td><td>-0.090</td><td colspan="2">-0.090</td><td colspan="2">-0.090</td><td colspan="2">-0.090</td></tr>
<tr><td rowspan="3">2</td><td>T_m, T_r</td><td>-1.234</td><td colspan="2">-1.234</td><td colspan="2">-1.234</td><td>-1.638</td><td>-1.234</td></tr>
<tr><td>T_m, T_r, T_s</td><td>-0.099</td><td colspan="2">-0.099</td><td colspan="2">-0.099</td><td colspan="2">-0.099</td></tr>
<tr><td>T_m, T_r, T_{m_max}</td><td>-0.121</td><td colspan="2">-0.121</td><td colspan="2">-0.121</td><td colspan="2">-0.121</td></tr>
<tr><td rowspan="3">3</td><td>T_m, T_r</td><td>-2.350</td><td colspan="2">-2.350</td><td colspan="2">-2.350</td><td colspan="2">-2.350</td></tr>
<tr><td>T_m, T_r, T_s</td><td>-0.349</td><td colspan="2">-0.349</td><td colspan="2">-0.349</td><td colspan="2">-0.349</td></tr>
<tr><td>T_m, T_r, T_{m_max}</td><td>-0.157</td><td colspan="2">-0.157</td><td colspan="2">-0.157</td><td colspan="2">-0.157</td></tr>
</table>

Table 5.23: Number of policies evaluated by each heuristic algorithm for each heuristic policy for each product.

Product		**1**			**2**			**3**		
Policy		**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
Total Enumeration		400	1900	8000	400	1900	8000	400	1900	8000
Random	**GS**	28	30	31	36	25	44	38	26	46
	D1	32	35	38	41	29	45	40	32	45
MDP	**GS**	11	12	19	11	12	26	7	16	8
	D1	23	31	36	23	31	35	11	24	12
Newsboy	**GS**	11	15	15	9	22	29	9	21	49
	D1	13	20	20	8	24	20	8	28	20

These results should encourage the manufacturers to use the proposed heuristic policies to control their system, and use the proposed heuristic algorithms to determine the parameter values of these policies.

6. CONCLUSION AND FURTHER WORK

In this thesis, we analyse a periodically-reviewed stochastic manufacturing/ remanufacturing system where the remanufactured items have an inferior value from customers' point of view compared to manufactured (i.e. new) items. A downward product substitution strategy is employed in case of a stock-out for remanufactured items. The problem is formulated as a discrete-time MDP in order to find the optimal inventory policies for both with and without product substitution. Through a numerical study based on real data for a product produced by an automotive spare part manufacturer, the profitability of substitution is investigated under different demand and return distributions. Results show that the profitability of product substitution strategy is significantly affected by the ratio of the mean of the remanufactured item demand to the mean of returns. As this ratio increases, the value of substitution increases.

Even though the optimal inventory policy for the hybrid manufacturing/remanufacturing system under product substitution can be determined by MDP, this policy has a very complicated structure that is very difficult if not impossible to represent using a few control parameters. Hence, we propose several simple-structured heuristic inventory policies, which are intuitive and easy to implement in practice. Then, we develop two heuristic search methodologies to determine the values of the parameters of these policies. Numerical experimentation show that the proposed search techniques can find optimal or near-optimal parameter values with significant computational savings compared to total enumeration technique. Moreover, the heuristic policies can control the hybrid system under product substitution near-optimally resulting in a profit that deviates from optimal by less than 1%.

6.1 Thesis Contribution

Through a numerical study, the optimal profits are calculated for a product for both substitution and no substitution cases. The results show that the substitution strategy is economically attractive when the expected demand for remanufactured items is at least as much of expected returns, and the improvement in profit by substitution increases significantly as the size of returns decreases relative to the size of remanufactured item demand. Further experimentation is done to see the effects of changing the mean and standard deviation of the distributions for the manufactured/remanufactured item demands and used item returns (i.e. coefficient of variation) on the improvement in profit by substitution. As one would expect, when the mean of remanufactured item demand is at least as much as the mean of returns, the substitution strategy results in additional profit for the manufacturer and when returns are substantially higher than the remanufactured item demand, the use of substitution is not economically justified. Substitution results in a higher improvement in profit when the expected remanufactured item demand gets higher and/or the expected return gets lower. Because the profit of manufacturing process is lower for lower manufactured item demand, a same amount of change in profit by substitution corresponds to a higher percent change of profit over no substitution case as the mean of manufactured item demand decreases. In short, the profitability of product substitution strategy is mainly dependent on the size of remanufactured item demand relative to that of returns.

Furthermore, in this thesis, we provide an inventory analysis of the hybrid system under product substitution. We propose three easy-to-implement inventory policies that have a few control parameters that represent well the optimal inventory policies under product substitution. Two heuristic search methodologies are employed to determine the parameter values for these policies. Results show that the search techniques find near-optimal inventory policies that represent well the optimal inventory policies.

We conclude that these results should encourage the manufacturer to use the product substitution strategy and the proposed heuristic inventory policies to control the hybrid system since the use of substitution may increase significantly the profit and the customer service level by reducing lost sales for remanufactured products.

6.2 Limitations and Future Work

The size of the MDP model (i.e. the number of the inventory states) formulated for the inventory control of the hybrid manufacturing/remanufacturing system under product substitution is significantly affected by the lower and upper bounds on the inventories and very large bounds may lead to a computationally intractable MDP model. In this case, a more efficient solution algorithm than the variant of Howard's policy iteration algorithm may be needed to develop for solving the MDP model or other alternative methodologies that can treat large-scale problems such as simulation may be used.

In this study, we proposed heuristic inventory policies for a hybrid system where manufacturing and remanufacturing processes have no fixed set-up cost. An interesting future work would be to find inventory policies that work well in the existence of positive set-up costs. Opportunities for future work may also include performing extensive experimentation using a broad range of input parameters to better understand the scenarios best suited for substitution and those least suited.

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APPENDICES

Appendix A. The solution algorithm for the infinite horizon MDP model

Howard (1960) developed a policy iteration method for solving infinite horizon MDP problems. It is composed of two phases: value determination phase, where the relative values and gain is calculated for a fixed policy; and policy improvement phase, where a better policy is found using the relative values found in the first phase. These two phases are done iteratively until two consecutive policies found are identical. In the traditional Howard's method, in the value determination phase, it is required to solve an *N*x*N* set of simultaneous linear equations in order to determine the relative values, where *N* is the number of states in the MDP. For large scale problems (i.e. large *N* and/or large number of alternatives per state), solving this set of equations becomes computationally inefficient. Morton (1971) propose the computation of the relative values using fixed policy successive approximation, which eliminates the need for solving linear equations and provides computational efficiency.

The variant of Howard's policy iteration algorithm with fixed policy successive approximation is given below.

N: the number of states

q_S^k: expected period cost for state S when following alternative *k*

$P_{SS'}^k$: the one-step transition probability from state S to state S' when following alternative *k*

Initialization. Any feasible policy can be chosen as the initial policy.

- For simplicity, set the initial policy by selecting the initial alternative k^S for each state *S* that minimizes the expected period cost $q_S^{k^S}$, i.e. $k^S = \arg\min_k \{q_S^k\}$.

Policy $K \leftarrow$ Initial policy

- Set $v_S = 0$ for every state *S*.
- Go to Phase I.

Phase I. Value determination

For the given policy *K*, for a predetermined number of iterations, called as *cheap iterations** in Morton (1971), do the following:

For every cheap iteration,

- For every state *S*, calculate $v'_S = q_S^k + \sum_{S'} P_{SS'}^k v_{S'}$
- If at last cheap iteration, calculate $\sum_S \left| v'_S - v'_N - v_S \right|$ (stopping criterion 1)
- Set $v_S \leftarrow v'_S - v'_N \backslash$

Phase II. Policy improvement

- Given the relative values v=(v_S) from phase I for every state *S*, find the alternative k_S^* that minimizes

$$q_S^k + \sum_{S'} P_{SS'}^k v_{S'}$$

The new policy is $K' = (k_S^*)$

Calculate $v'_S = q_S^{k_S^*} + \sum_{S'} P_{SS'}^{k_S^*} v_{S'}$

Set $v_S \leftarrow v'_S - v'_N$

- Check if the old policy K and the new policy K' are identical (Stopping criterion 2).
- Set $K \leftarrow K'$
- If either stopping criterion is not satisfied, go to Phase I,
- Otherwise Stop, the optimal policy is policy *K* and the optimal cost is v'_N.

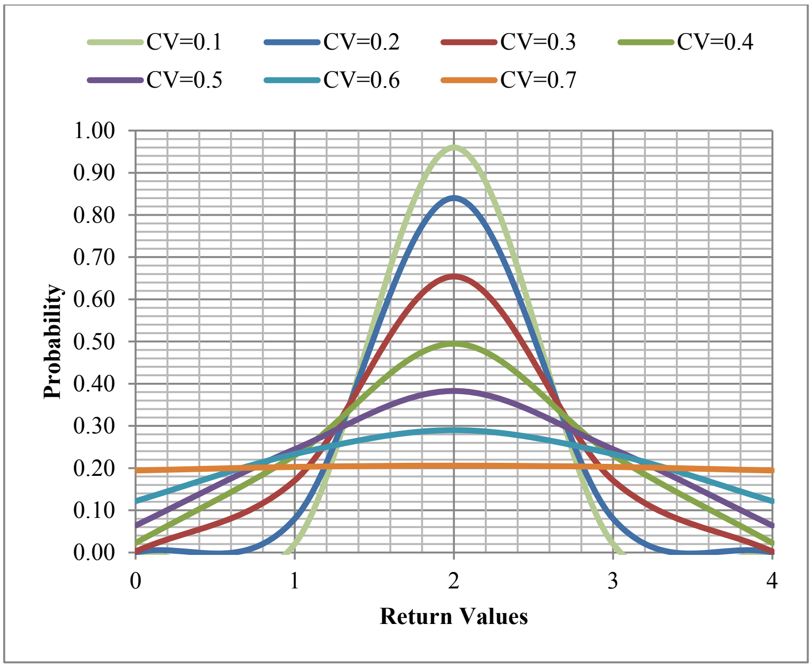
Stopping criteria: The following two conditions must occur to stop the algorithm.

1. the relative values found in the value determination phase converge
   $\sum_{S} \left| v'_S - v'_N - v_S \right| < \varepsilon$
2. the two consecutive policies found in the policy improvement phase are identical[†].

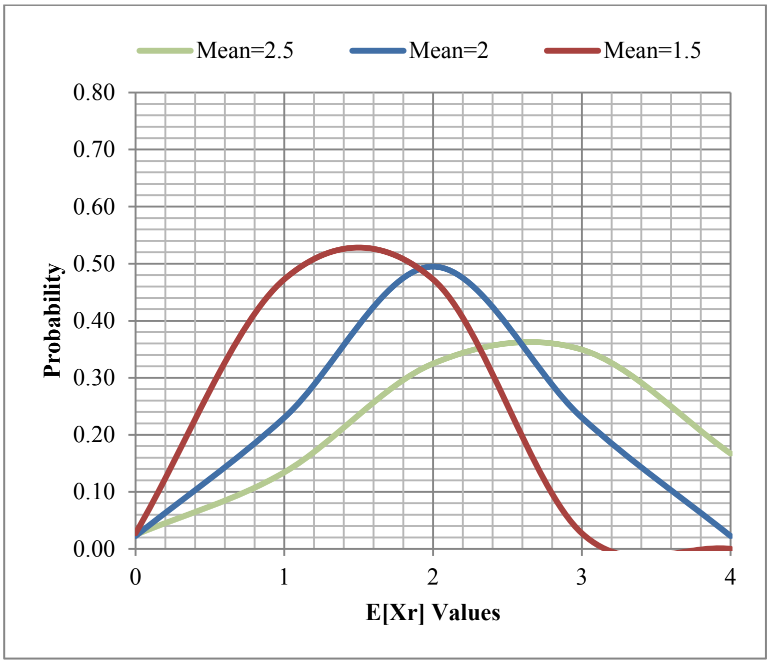
[*]the number of cheap iterations is set to 10 for the experimentation done in this paper.

[†]in practice for very large state spaces stopping criteria 2 may not be satisfied due to computer round off, although this did not occur in our experimentation.

Appendix B. Distribution figures for *E[X_r]* and *E[Y]*



Distributions for Returns (E[Y])



Demand Distributions for Remanufacturing Item (E[Xr])

Appendix C.

Figure C.1: Gain Deviation & Optimal Parameters for Three Policies

Policy 1

Product	T_m	T_r		gain	Deviation	Opt.Gain
1	6	2		15.64057	-0.97781	15.795
2	6	3		86.15475	-1.23379	87.231
3	4	3		59.90813	-2.35024	61.350

Policy 2

	T_m	T_r	T_s	gain	Deviation	Opt.Gain
1	6	4	1	15.78108	-0.08823	15.795
2	6	5	2	87.14484	-0.09878	87.231
3	5	4	1	61.13592	-0.34895	61.350

Policy 3

	T_m	T_r	$T_{m\ max}$	gain	Deviation	Opt.Gain
1	6	3	7	15.78081	-0.08999	15.795
2	5	5	8	87.12503	-0.12148	87.231
3	4	4	7	61.25362	-0.15709	61.350

Figure C.2: Parameter numbers and run times for all algorithms and heuristics.

		Policy 1				
Product	**Algorithm**	Policy	Seconds	Minutes	Hours	Gain
1	Total Enumeration	400	79592.845	1326.547	22.109	15.64057
	Random Start ILS	28	7907.049	131.784	2.196	15.64057
	Random Start NS	32	8492.544	141.542	2.359	15.64057
	Newsboy Guess ILS	11	3882.755	64.713	1.079	15.64057
	Newsboy Guess NS	13	4198.668	69.978	1.166	15.64057
	MDP Guess ILS	11	3090.343	51.506	0.858	15.64057
	MDP Guess NS	23	5082.122	84.702	1.412	15.64057

Product	Algorithm	Policy	Seconds	Minutes	Hours	Gain
2	Total Enumeration	400	52930.560	882.176	14.703	86.15475
	Random Start ILS	36	6982.058	116.368	1.939	86.15475
	Random Start NS	41	9533.167	158.886	2.648	86.15475
	Newsboy Guess ILS	9	2012.643	33.544	0.559	85.80133
	Newsboy Guess NS	8	1881.920	31.365	0.523	86.15475
	MDP Guess ILS	11	2194.919	36.582	0.610	86.15475
	MDP Guess NS	23	4199.268	69.988	1.166	86.15475
3	Total Enumeration	400	29921.429	498.690	8.312	59.90813
	Random Start ILS	38	5408.557	90.143	1.502	59.90813
	Random Start NS	40	6239.167	103.986	1.733	59.90813
	Newsboy Guess ILS	9	1801.352	30.023	0.500	59.90813
	Newsboy Guess NS	8	1417.246	23.621	0.394	59.90813
	MDP Guess ILS	7	925.571	15.426	0.257	59.90813
	MDP Guess NS	11	1315.907	21.932	0.366	59.90813

Policy 2

Product	**Algorithm**	Policy	Seconds	Minutes	Hours	Gain
1	Total Enumeration	1900	15164.900	252.748	4.212	15.78108
	Random Start ILS	28	7183.419	119.724	1.995	15.78108
	Random Start NS	35	8715.346	145.256	2.421	15.78108
	Newsboy Guess ILS	15	3655.841	60.931	1.016	15.78108
	Newsboy Guess NS	20	4815.038	80.251	1.338	15.78108
	MDP Guess ILS	12	2624.461	43.741	0.729	15.78108
	MDP Guess NS	31	6436.216	107.270	1.788	15.78108
2	Total Enumeration	1900	57983.049	966.384	16.106	87.14484
	Random Start ILS	19	3941.504	65.692	1.095	87.14484
	Random Start NS	29	5163.284	86.055	1.434	87.14484
	Newsboy Guess ILS	22	4238.085	70.635	1.177	87.14484
	Newsboy Guess NS	24	5106.802	85.113	1.419	87.14484
	MDP Guess ILS	12	2528.772	42.146	0.702	87.14484
	MDP Guess NS	31	5382.544	89.709	1.495	87.14484
3	Total Enumeration	1900	30903.532	515.059	8.584	61.13592
	Random Start ILS	21	2332.572	38.876	0.648	61.13592
	Random Start NS	32	6866.216	114.437	1.907	61.13592
	Newsboy Guess ILS	21	2417.928	40.299	0.672	61.13592
	Newsboy Guess NS	28	3869.245	64.487	1.075	61.13592
	MDP Guess ILS	16	1701.411	28.357	0.473	61.13592
	MDP Guess NS	24	2255.369	37.589	0.626	61.13592

Product	Algorithm	Policy 3				
		Policy	Seconds	Minutes	Hours	
1	Total Enumeration	8000	40508.216	675.137	11.252	15.78081
	Random Start ILS	31	7711.368	128.523	2.142	15.78081
	Random Start NS	38	6575.623	109.594	1.827	15.78081
	Newsboy Guess ILS	15	39126.282	652.105	10.868	15.78081
	Newsboy Guess NS	20	5167.243	86.121	1.435	15.78081
	MDP Guess ILS	19	5063.147	84.386	1.406	15.78081
	MDP Guess NS	36	8914.456	148.574	2.476	15.78081
2	Total Enumeration	8000	80675.822	1344.597	22.410	87.12503
	Random Start ILS	44	10933.167	182.219	3.037	87.12503
	Random Start NS	45	11873.271	197.888	3.298	87.12503
	Newsboy Guess ILS	29	5063.284	84.388	1.406	87.12503
	Newsboy Guess NS	20	4030.053	67.168	1.119	87.12503
	MDP Guess ILS	26	5210.793	86.847	1.447	87.12503
	MDP Guess NS	35	6345.623	105.760	1.763	87.12503
3	Total Enumeration	8000	41702.491	695.042	11.584	61.25362
	Random Start ILS	46	4852.235	80.871	1.348	61.25362
	Random Start NS	45	4595.523	76.592	1.277	61.25362
	Newsboy Guess ILS	49	5326.855	88.781	1.480	61.25362
	Newsboy Guess NS	20	2516.073	41.935	0.699	61.25362
	MDP Guess ILS	8	865.626	14.427	0.240	61.25362
	MDP Guess NS	12	1360.886	22.681	0.378	61.25362

BIOGRAPHICAL SKETCH

Fethullah Göçer was born in Elbistan, Turkey on December 25, 1984. For his secondary education, he attended Elbistan İHL High school and graduated in 2001. He attended Süleyman Demirel University Technical Education faculty between 2002 and 2003. He started his undergraduate education in Industrial Engineering at University of Toronto, Canada in 2006 and transferred to Girne American University, Northern Cyprus in 2008. He received his Bachelor of Science degree from Girne American University in 2010. Upon graduation, he went abroad for work experience. He has worked as a Site Engineer for a private company in Sierra Leone between 2010 and 2011. He is placed to Kahramanmaraş Sütçü İmam University as research assistant within higher education program of OYP. He is appointed to Galatasaray University for his Master's degree education in Industrial Engineering in 2012. He is currently working as research assistant in the Industrial Engineering Department at Galatasaray University.