# ANALYSIS OF INVENTORY POLICIES UNDER IMPERFECT SUPPLY

(GÜVENİLİR OLMAYAN TEDARİKÇİ VARLIĞINDA STOK POLİTİKALARININ ANALİZİ)

by

**Faruk AKIN, B.S.**

**Thesis**

Submitted in Partial Fulfillment

of the Requirements

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May 2014
Faruk AKIN

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**List of symbols**

| | |
|---|---|
| $S$ | Current state of the system |
| $I$ | Inventory level at the beginning of the period |
| $J$ | The unreliable supplier's status at the beginning of the period |
| $I_{min}$ | Minimum inventory level for the retailer |
| $I_{max}$ | Maximum inventory level for the retailer |
| $k_u$ | Order quantity placed on the unreliable supplier |
| $k_r$ | Order quantity placed on the reliable supplier |
| $A_s$ | The set of alternative order quantities |
| $W$ | Transition probability matrix |
| $\alpha$ | Probability for the unreliable supplier from the up state to the up state |
| $\beta$ | Probability for the unreliable supplier from the down state to the up state |
| $S'$ | Next state of the system |
| $I'$ | Inventory level at the end of the period |
| $J'$ | The unreliable supplier's status at the end of the period |
| $P_{SS'}^{k}$ | Probability of going from one state to another following decision $k$ |
| $c_u$ | Unit purchasing cost charged by the unreliable supplier |
| $c_r$ | Unit purchasing cost charged by the reliable supplier |
| $f_a$ | Administrative cost per order from either supplier |
| $f_t$ | Transportation cost per order from either supplier |
| $h$ | Holding cost per unit held at the end of period |
| $b$ | Backordering cost per unit |
| $l$ | Lost sales cost per unit |
| $B$ | Backordered amount at the end of period |
| $L$ | Lost sales amount at the end of period |
| $D$ | Demand |
| $d$ | Demand for the period |

| | |
|---|---|
| $C$ | Cost for the period |
| $\delta$ | Cost for order placed on the reliable supplier/supplier 1 |
| $\gamma$ | Cost for order placed on the unreliable supplier/supplier 2 |
| $q_S^k$ | The expected period cost for state $S$ following alternative $k$ |
| $P_D(d)$ | Probability mass function of demand |
| $s_r$ | Reorder point for the reliable supplier |
| $S_r$ | Order-up-to level for the reliable supplier |
| $s_u$ | Reorder point for the unreliable supplier |
| $S_u$ | Order-up-to level for the unreliable supplier |
| $c_1$ | Unit purchasing cost charged by supplier 1 |
| $c_2$ | Unit purchasing cost charged by supplier 2 |
| $k_1$ | Order quantity placed on supplier 1 |
| $k_2$ | Order quantity placed on supplier 2 |
| $W^1$ | Transition probability matrix for supplier 1 |
| $W^2$ | Transition probability matrix for supplier 2 |
| $J_1$ | Supplier 1's status at the beginning of the period |
| $J_2$ | Supplier 2's status at the beginning of the period |
| $J_1'$ | Supplier 1's status at the end of the period |
| $J_2'$ | Supplier 2's status at the end of the period |
| $f_{1a}$ | Administrative cost per order from supplier 1 |
| $f_{1t}$ | Transportation cost per order from supplier 1 |
| $f_{2a}$ | Administrative cost per order from supplier 2 |
| $f_{2t}$ | Transportation cost per order from supplier 2 |
| $\alpha_1$ | Probability for supplier 1 from the up state to the up state |
| $\beta_1$ | Probability for supplier 1 from the down state to the up state |
| $\alpha_2$ | Probability for supplier 2 from the up state to the up state |
| $\beta_2$ | Probability for supplier 1 from the down state to the up state |
| $s_j$ | Reorder point below which orders are made to both suppliers simultaneously |
| $s_1$ | Reorder point for supplier 1 |
| $S_1$ | Order-up-to level for supplier 1 |
| $s_2$ | Reorder point for supplier 2 |
| $S_2$ | Order-up-to level for supplier 2 |

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## ABSTRACT

We study a single-product, stochastic, periodic-review inventory problem in which a retailer uses a dual sourcing strategy to cope with potential supply disruptions.A new fixed cost structure is introduced consisting of basic and secondary fixed costs where the former is incurred whenever an order is placed (e.g. administrative costs) and the latter is incurred only when an order is filled (e.g. transportation costs). We consider two cases: In the first case, the retailer can place orders on two suppliers: one supplier is perfectly reliable but offers a high unit price, and the other offers a lower unit price but is unreliable following a Markovian availability structure.The unreliable supplier alternates between "up" and "down" states where his current supply state depends on his supply state in the previous period. The inventory control problem under consideration is modeled as an infinite-horizon discrete-time Markov decision process in order to find the optimal ordering decisions and solved using a variant of Howard's policy iteration method. Through numerical experiments, the structure of the optimal ordering policies are found be following one of the following four cases: case 1: order only from the unreliable supplier; case 2: order simultaneously from both suppliers or only from the unreliable supplier depending on the inventory level; case 3: order from one or the other but not both suppliers simultaneously; and case 4: order only from the reliable supplier.The $(s, S)$ -like policies characterize the above-mentioned optimal structures due to the existence of fixed cost structure. In the second case, the retailer adopts again a dual sourcing strategy, but this time both suppliers are unreliable following a Markovian availability structure. The two suppliers may differ in their reliability levelsand unit purchasing costs,but they have the same lead time characteristics as in the first model, i.e. order placed on one of the suppliers at the beginning of a period arrives at the end of the period if his status stays up during the period. Through numerical experiments, the structure of optimal ordering policies are found to be one of the following three cases: case 1: first, order from both suppliers

simultaneously and then order only from the less reliable supplier depending on the inventory level; case 2: first, order from both suppliers simultaneously and then order only from the more reliable supplier depending on the inventory level; case 3:first, order from both suppliers simultaneously, then order from a single supplier at a time in the following order:first order only from the more reliable supplier, and then order only from the less reliable supplier depending on the inventory level. Ordering from both suppliers simultaneously has a more complicated structure than the simple $(s, S)$ policy, howeverwhen it is optimal to order from a single supplier, orders are placed according to an$(s, S)$ policy.

**RÉSUMÉ**

Nous étudions un problème de stocks stochastique à revue périodique dans lequel un détaillant utilise unestratégie duale d'approvisionnement pour faire face aux interruptions d'approvisionnement. On introduit une nouvelle structure de coût fixe qui est constituée du coût fixe primaire et du coût fixe secondaire où le premier est engagé chaque fois qu'une commande est passée (par exemple, les frais d'administration) et le dernier est engagé seulement si l'ordre est reçu (par exemple les frais de transport). Nous considéronsdeux cas pour ce problème. Dans le premier cas, le détaillant a deux fournisseurs pour passer des commandes: l'un est parfaitement fiable, mais offre un prix élevé, et l'autre propose un prix plus bas, mais n'est pas fiable en suivant une structure de disponibilité de Markov. L'état du fournisseur qui n`est pas fiable s'alterne entre "disponible" et "pas disponible", et son état de disponibilité dans la période actuelle dépend de son état de disponibilité dans la période précédente. On formule ce problème comme un processus de décision de Markov à temps discret et à l'horizon infini afin de trouver les décisions des commandes optimales, et on le résout en utilisant une variante de la méthode d'itération de la politique de Howard.En faisant des expériences numériques, les politiques des commandes optimales sont trouvées et caractérisées. Les résultats montrent que la structure des politiques de commande suit un de ces quatre cas: cas 1: Passer la commande seulement au fournisseur faible; cas 2: Passer la commande aux deux fournisseurs en même temps ou seulement à un fournisseur, ce qui dépends du niveau des stocks; cas 3: commander àl'un ou l'autre mais pas aux deux fournisseurs en mêmes temps; et cas 4: Passer la commande seulement aufournisseur fiable. Dans le deuxième cas, le distributeur adopteune nouvelle stratégie de double source, mais cette fois tous les deux fournisseurs ne sont pas fiables. Il peut y avoirdes différences dans leurs niveaux de fiabilité et de coûts d'achat unitaires, mais les deux fournisseurs ont les mêmes caractéristiques de temps de livraison, donc les commandes

faites à un fournisseur au début d'une période arriventà la fin de la période, si l'état de celui-ci reste disponible au cours de la période.

**ÖZET**

Bu çalışmada olası tedarik aksamalarıyla başa çıkmak adınaiki tedarikçi kullanma stratejisi izleyen bir perakendeci için, güvenilir olmayan ortamda, periyodik olarak kontrol edilen ve tek üründen oluşan bir stok problemiincelenmiştir. Probleme birincil ve ikincil sabit sipariş maliyetlerinden oluşmak üzere yeni bir sabit sipariş maliyet yapısı tanıtılmıştır ki; bunlardan ilki her bir sipariş verilişinde gerçekleşir (örneğin siparişi başlatmak için gerekli olan idari maliyetler), ikincisi ise sadece sipariş edilen miktar perakendeciye ulaştığı takdirde gerçekleşir (örneğin siparişin ulaştırılması esnasında gerçekleşen taşıma maliyetleri). Söz konusu problem için iki ayrı durumdikkate alınmıştır. Birinci durumdaperakendecinin sipariş verebileceği;biri tamamen güvenilir fakat pahalı, diğeri ise güvenilir olmayan fakat ilkine göre daha düşük fiyat sunan iki tedarikçisi bulunmaktadır. Güvenilir olmayan tedarikçinin durumu "açık" veya "kapalı" olarak ifade edilmektedir, şöyle ki; tedarikçi ancak açık olduğu durumda sipariş kabul edebilmekte ve sipariş aldıktan sonra uygunluk durumu kapalıya değiştiğinde ise alınan sipariş tamamen iptal edilmektedir. Güvenilir olmayan tedarikçinin mevcut durumu bir önceki periyottaki uygunluk durumuna bağlıdır. Söz konusu stok problemi, en iyi sipariş kararlarını bulabilmek için sonsuz planlama dönemli ve kesik zamanlı bir Markov karar süreci olarak formüle edilmiş ve Howard'ın politika iteleme metodunun bir versiyonu kullanılarak çözülmüştür. Sayısal deneyler sonucunda yukarıda ifade edilen sabit sipariş maliyetleri altında, en iyi sipariş kararları bulunmuş ve en iyisatışpolitikasıyapılarınınmevcut stok durumuna bağlı olarak aşağıda açıklanan dört yapıdan birine uyduğu gözlemlenmiştir: durum 1: Sadece güvenilir olmayan tedarikçiye sipariş vermek; durum 2: Ya iki tedarikçiye eş zamanlı olarak sipariş vermekya da sadece güvenilir olmayan tedarikçiyesipariş vermek; durum 3: Eş zamanlı olmamak koşuluyla ya güvenilir tedarikçiye ya da güvenilir olmayan tedarikçiye sipariş vermek; durum 4:Sadece güvenilir tedarikçiye sipariş vermek.Sabit sipariş maliyetinin varlığındanötürü $(s, S)$ tipi politikalar yukarıda sözü

edilen yapıları karakterize etmektedir. Kurduğumuz ikinci modelde ise perakendecinin sipariş verebildiği her iki tedarikçi de tam güvenilir olmayan biryapıya sahiptir. Sayısaldeneyler sonucunda, en iyi sipariş kararları bulunmuş ve en iyi satış politikası yapılarının aşağıda açıklanan üç yapıdan birine uyduğu gözlemlenmiştir: durum 1: Mevcut stok durumuna bağlı olarak öncelikle her iki tedarikçiye de eş zamanlı olarak sipariş vermek ve ardından sadece daha az güvenilir olan tedarikçiye sipariş vermek; durum 2: Mevcut stok durumuna bağlı olarak öncelikle her iki tedarikçiye de eş zamanlı olarak sipariş vermek ve ardından sadece daha fazla güvenilir olan tedarikçiye sipariş vermek; durum 3: Mevcut stok durumuna bağlı olarak öncelikle her iki tedarikçiye de eş zamanlı olarak sipariş vermek ve ardından sadece daha fazla güvenilir olan tedarikçiye sipariş vermek, son olarak da sadece daha az güvenilir olan tedarikçiye sipariş vermek. Her iki tedarikçiye de ayı anda yapılan siparişlerin yapısı basit $(s, S)$ tip politikalardan daha karmaşık olmakla birlikte, tek bir tedarikçiye yapılan siparişler $(s, S)$ tip politikalara uymaktadır.

# 1. INTRODUCTION

Sourcing from low-cost offshore suppliers has become the main strategy of many companies while developments in technology and transportation help companies organize supply chains. Although more and more companies tend to use those low-cost suppliers, there are still high chances of supply disruptions that may endanger the sourcing networks. Supply disruption can be defined as an unplanned situation that makes the supplier become completely or partially unavailable to receive and/or process the orders. Supply disruptions have large impacts on the entire supply chain because they may stop the operation of the entire supply chain (Huang et al., 2012).

Supply disruptions may arise from several reasons. Atasoy et al. (2013) classify the reasons for disruptions into two groups: *unpredictable disruptions*, which may be caused by disasters, terrorist attacks and accidents etc. and *predictable disruptions* like capacity restrictions and scarcity of some resources at the supplier. Arreola-Risa & DeCroix (1998) categorize the disruptions as *process-related* where the supply may become unavailable due to breakdowns, transportation disruptions or a strike and as *market related*, in which the supplier may give preference to big companies at the expense of small companies. According to Speier et al. (2011), supply chain disruptions may result from either *unintentional causes* such as accidents or natural disasters or *intentional causes* such as theft, contamination/sabotage, or a terrorist attack. Schmitt & Snyder (2012) state that disruptions sometimes can be a planned part of a supplier-retailer relationship based on contracted material availability. If, for example, a supplier promises an 80% material availability in their contract with a retailer, then the retailer can anticipate that its supply will be unavailable up to 20% of the time.

The following disaster example is from a report by Oxford Economics for the Airbus company and shows how a natural disaster can cause the disruption of the supply globally: When a volcano in Iceland erupted in 2010, ashes rising into the sky interrupted the air traffic over Europe and caused the cancellation of flights across Europe affecting global travel, trade and business. It also disrupted supply routes around the globe, which was acutely felt by exporters and companies who rely on imported inputs. The disruption to air services, for example, was reported to lead to suspensions in car production at BMW and Nissan plants in Germany, USA and Japan due to shortages of airfreighted components such as pressure sensors.

To prevent the interruption of the business during the supply disruptions, firms can employ a range of strategies (Chen et al., 2012). In case of any disruption, manufacturers may employ *supplier diversification* to reduce the risk and dependencies that arise due to reliance on a single supplier (Swaminathan & Shanthikumar, 1999). Apart from the dangers of disruptions, a single source can establish closer contacts with the supplier, in some cases these contacts extend as far as synchronizing their production delivery schedules to reduce inventory. However, relying on one supplier is risky, and often might not provide the lowest costs for the product. It may also lead to the loss of technological thrust for the supplier (Ganeshan et al., 1999). To avoid the disadvantages of single sourcing, companies are increasingly moving towards a supplier base of more than one supplier (Anupindi & Akella, 1993) where they use secondary or multiple suppliers to maintain a desirable service level, reduce customer service time and costs. Dual or multiple sourcing strategies are particularly very useful to retailers for newly launched products, which undergo several changes and updates during their early stage of life cycle (Giri, 2011). Multi-sourcing is an attractive alternative because it may create a competition among suppliers that can force them to provide faster delivery (Arda & Hennet, 2006). Having suppliers in different geographic locations can also reduce the impacts of supply disruptions caused by disasters or other source of disruptions occurring in one geographic location (Veeraraghavan & Scheller-Wolf, 2008).

A good example for dual sourcing strategy is the one adopted by Zara. Zara procures products from two distinctive markets. Less expensive products are outsourced from Asian facilities with a long lead time to get competitive prices. But the majority of Zara's designs are produced in costly European and North African factories with a short lead time in order to pursue the perishable fashion. Similarly, Hewlett-Packard purchases electronic components from both contract markets and spot market (Song et al., 2014).

In this study, we consider an infinite-horizon, single-product, periodic-review inventory system for a retailer who uses a dual sourcing strategy to cope with potential supply interruptions. We analyze the ordering policies for two cases: In the first case, one supplier is completely reliable in the sense that whenever an order is placed on that supplier at the beginning of a period, it arrives with certainty at the end of the period. Other supplier is unreliable, by which we mean that he alternates between "up" and "down" states. Transitions from one state to another occur according to a two-state Markov process. Orders may be done to the unreliable supplier only when his current status is up, and if his status happened to go down during the period then any outstanding order is canceled. When the unreliable supplier is down, the retailer's only option is to source from the reliable supplier. Fixed costs of ordering are considered along with the unit purchasing, holding, backordering and lost sales costs. Unit purchasing cost charged by the reliable supplier is more expensive than that charged by the unreliable supplier. The fixed ordering cost consists of two components: *administrative* and *transportation* costs. Administrative cost occurs every time the retailer places an order, however transportation cost occurs if the order is delivered to the retailer. Since the reliable supplier will ship the ordered amount by the end of period with certainty, both administrative and transportation costs are paid to the reliable supplier whenever an order is made. The administrative cost is incurred for the unreliable supplier for each order placed, whether or not he makes a delivery. The transportation cost for the unreliable supplier is incurred only when the ordered amount is delivered. Through numerical experimentations, we investigate the optimal ordering strategies under this fixed ordering cost structure.

In the second case, we analyze the same system for the retailer operating with two unreliable suppliers who may differ in unit purchasing costs as well as their reliability levels. We assume that the two suppliers follow a Markovian supply availability structure. Both suppliers have the same lead time characteristics, i.e. order placed on a supplier at the beginning of a period arrives at the end of the period depending the availability of that supplier. Numerical experimentations are done in order to explore different ordering policies that can be optimal for the retail system.

The problem under consideration is formulated as a Markov decision problem (MDP) in order to determine the optimal ordering policy, which is a list of the optimal action to follow (i.e. the optimal order quantities from both suppliers) in each possible state of the system. While optimal, the MDP policy does not provide managerial insight into the structure of the policy. In this study, we follow the MDP-based characterization methodology that has been used by Ahiska et al. (2013) in order todefine a generalizable ordering policy in a structured way using a few control parameters.In this methodology, first the optimal decisions for each system state are found using the MDP for a set of problem instances, and then, through careful observation of MDP solutions, candidate policy structures are identified, which mimic the optimal decisions in each state. Finally,the performance of the characterized policy is evaluated by comparing its cost to the cost of the optimal policy found by MDP.

We extend thestudy of Ahiska et al. (2013) by investigating the effects of different components of fixed ordering costs on the optimal ordering policy. We then build another model under the same assumptions, where we explore the optimal ordering policies for a retailer who works with two unreliable suppliers.

The rest of the thesis is organized as follows: In Section 2, a review of workon inventory models under supply disruptions is presented. In Section 3, we give problem descriptions, MDP formulations and policy characterizations for the two cases we consider for the unreliable supply problem. In the first case, the retailer works with one reliable and one unreliable supplier while in the second case, the retailer works with two unreliable suppliers.Section 4 gives concluding remarks and suggests directions for further research.

## 2. LITERATURE REVIEW

There is a significant amount of studies on inventory models that consider supply disruptions. These studies differ in several characteristics such as the sourcing strategy considered (a single supplier vs. two or more suppliers or backup/emergency suppliers), the timing of the inventory control (periodic-review vs. continuous-review inventory models), the nature of the product demand (deterministic stationary or non-non-stationary vs. stochastic), the supply lead time (deterministic vs. stochastic lead time) and the modeling of supply uncertainty. Supply uncertainty may take several different forms, as explained below (Ahiska et al., 2013).

***Random durations of on/off periods*** refer to stochastic durations of the supplier's *on* and *off* periods (Parlar&Berkin, 1991; Parlar& Perry, 1995; Gupta, 1996; Gürler&Parlar 1997; Parlar, 1997; Arreola-Risa&DeCrois 1998;Parlar, 2000; Ross et al., 2008; Li & Chen, 2010; Sajadifar&Pourghannad, 2011). Mohebbi (2003, 2004)andSilbermayr&Minner (2013) assume that a switch to *off* state affects only the acceptance of a replenishment order in the future and does not affect the order (if any) that has already been accepted while Mohebbi&Hao (2008) assume that in case of an interruption during the processing of an order, the process restarts when the supplier becomes available again. Weiss & Rosenthal (1992),Qi et al. (2009) and Sargut& Qi (2012) study continuous-review inventory problems where both retailer and supplier may go through random durations of *on* and *off* periods.Qi (2013) assumes that when the primary supplier's random length of *off* period exceeds the maximum waiting time, then the retailer places an order to a backup supplier, which is expensive but reliable.

***Unreliable delivery time*** means that delivery lead time of a supplier has a stochastic nature (Sculli, 1981; Ganeshan, 1999; Mohebbi, 2003;Li & Chen, 2010; Yumei et al. 2011; Silbermayr&Minner, 2013; Sajadifar&Pourghannad, 2011). Ramasesh et al.

(1991) examine equal order splitting between two suppliers who differ only in fixed costs and stochastic lead times. They state that when the uncertainties are high and ordering costs are low, dual sourcing is more cost effective than single sourcing.Kelle& Miller (2000) consider order splitting between two suppliers, which have random lead times with different characteristics.Ryu& Lee (2003) consider a model where orders are split between two suppliers who differ in unit purchasing costs and delivery lead times, which is stochastic and can be reduced at a cost. Arts et al. (2009) study a single product, periodic-review inventory problem facing stochastic demand with two supply options having different lead times, which are modeled as both stochastic and deterministic. Arda&Hennet (2006) study an inventory system with Poisson demand and multiple suppliers, which differ only in random lead times. They show that sending orders to several suppliers has an economic advantage rather than to single supplier.

***Random yield*** means that when an order is placed, either the supplier delivers a random fraction of the ordered quantity or only a fraction of the delivered order is defect free (Parlar& Perry, 1995; Gurnani et al., 2000; Iakovou et al., 2010; Zhu & Fu, 2012; Xanthopoulos et al., 2012; Kouvelis and Li, 2013). Gerchak&Parlar (1990), Henig&Gerchak (1990), Parlar& Wang (1993), Chopra et al. (2007) and Giri (2011) assume that the received amount may also be larger than what is ordered. Moinzadeh& Lee (1989) and Bassok&Akella (1991) study inventory systems with stochastic demand and one supplier where orders may arrive in two shipments. Anupindi&Akella (1993) analyze an inventory system with stochastic demand and two suppliers having random yield. They develop three models: in the first one, each supplier delivers the ordered quantities in the current period or nothing. In the second one, each supplier delivers a random fraction of the ordered quantity in the current period, and the undelivered portion is canceled. In the third one, the undelivered quantity is delivered in the next period. The optimal policy they find has three cases: order from both suppliers; order only from the less expensive supplier or order nothing.Dada et al. (2007) consider a single-season newsvendor procurement problem with multiple suppliers, where any given supplier is either perfectly reliable or unreliable in the sense that it only delivers an amount less than ordered. They show that although reliability affects how much is ordered from a selected supplier, cost generally takes precedence over reliability when it comes to selecting suppliers in the first place.Warsing et al. (2013) study a periodicreview inventory system where the single unreliable supplier, each order being a Bernoulli trial, delivers the current order and any accumulated backorders at the end of the current period with a given probability.

***Supply with Bernoulli-nature*** means that when an order is placed on the supplier, he makes either a full delivery or nothing at all (Atasoy et al., 2012). Özekici&Parlar (1999) consider a periodic-review inventory model with an unreliable supplier where all parameters are affected by a randomly changing environment, and the supplier is either available making deliveries with zero lead time, or unavailable delivering nothing at all. Swaminathan&Shanthikumar (1999) consider both single and multiple period inventory systems, respectively, with stochastic demand where there are two alternative suppliers with different unit costs and reliabilities, which deliver all of the ordered quantity or nothing.Serel (2008)considers a single-period inventory problem, in which there is a reliable main supplier and a second supplier that has an *all-or-nothing* availability and is available according to a Bernoulli probability to deliver the orders immediately. Yan & Liu (2009) study a periodic review inventory model in the presence of fixed ordering costs with a fast supplier which can make instant deliveries and a slow supplier who makes deliveries one period later in full or deliver nothing. Fadıloğlu et al. (2008) and Tajbakhsh et al. (2010) study a specialized form of *Bernoulli supply nature* with multiple suppliers where each unit delivered by suppliers has a constant probability that it is either acceptable or not, independent of the order quantity. Schmitt & Snyder (2012) consider both a system with one unreliable supplier and a system with one unreliable and a second perfectly reliable but more expensive supplier. The retailer must reserve capacity at the reliable supplier and pays an additional reservation cost per unit. The retailer either receives the ordered quantity or nothing when the unreliable supplier is disrupted.

***Markovian supply availability structure***means that current supply state (e.g. available or not available) depends on the supply state in the previous period. Ahiska et al. (2013) analyze policy characterizations for a single product, periodic-review and infinite-horizon inventory system under setup costs with one reliable and one unreliable supplier where the latter's availability in the current period depends on his status in the previous period. Parlar et al. (1995) address a periodic-review, finite-horizon inventory model, with setup costs, where the probability that an order placed now is filled in full depends on whether the supply was available in the previous period.

## 2.1. Inventory Systems under Deterministic Demand

Most studies dealing with deterministic demand develop EOQ-type models that consider fixed ordering costs.Parlar&Berkin (1991) analyze an EOQ-type inventory problem and observe that if the random duration time of unavailability approximates to zero, then their model reduces to the classical EOQ model.Parlar& Perry (1995) study a continuous-review inventory model whose objective is to determine the reorder point, the order quantity in *on* state, and how long to wait before the next order if the system is in *off*state.Parlar& Perry (1996) consider a continuous-review inventory model where they apply $(Q, r)$-type policy if the supplier is in *on* state, and $(s, S)$-type policy if the supplier is in *off* state. They show that as the number of suppliers becomes large, the model reduces to the classical EOQ model. Gürler&Parlar (1997) study a continuousreview inventory problem with two unreliable suppliers, which reduces to a standard EOQ model when at least one of the suppliers is always available. Parlar (2000) examines an inventory system operated according to a $(Q, r)$ policy, which is of a classical EOQ modelwith deterministic demand and zero lead timeexcept for the supply interruptions. They find optimal values of the reorder point and order quantity. Fadıloğlu et al. (2008) and Tajbakhsh et al. (2010) study EOQ modelsand find that sole sourcing is always optimal in the EOQ models with binomial yield. Sajadifar&Pourghannad (2011) study a continuous-review inventory problem based on a $(Q, r)$type policy facing Poisson demand with two unreliable suppliers. They show that using two suppliers provides reduction in the expected total cost.

Other works dealing with periodically reviewed models are the following: Güllü et al. (1999) study a periodic review inventory system and provide a stochastic dynamic programming formulation and show the optimality of the order-up-to levels. Kelle& Miller (2001) study a single-item inventory model that can be used for both continuous and periodic review ordering policies and show that uneven split lowers the stockout risk when compared to an even split if the two suppliers have different lead time characteristics. Atasoy et al. (2012) study a periodic-review inventory problem with an unreliable supplier and show that when there is *advanced supply information,* optimal policy is characterized as a state dependent $(s, S)$ policy. Schmitt & Snyder (2012) consider a periodic-review inventory problem that operates under a base-stock policy. They show that when disruption risk high, the optimal strategy is to stock multiple periods of demand.

## 2.2. Inventory Systems with Stochastic Demand Controlled by aPredefined Policy

Systems operating under stochastic demand and controlled by a predefined policy seek optimal or near-optimal parameters for the optimal policy using exact or heuristic methods. Most of these works are controlled by a continuous-review $(Q, r)$ policy (Moinzadeh& Lee, 1989; Parlar, 1997), which is easy to implement and analytically tractable. Gupta (1996) considers a continuous-review, $(Q, r)$-type policy inventory problem where they develop two cases of the model, in the first of which the lead time is zero and if the supplier is *off* when an order is placed, then the order becomes pending until the supplier returns*on*, and there is no restriction on the orders pending. In the second case, the lead time is a constant number and only one order is allowed to be pending when the supplier is *off.* Ganeshan et al. (1999) study a continuous-review inventory system according under a $(Q, r)$-type policy where there are two suppliers; one reliable the other unreliable. Although placing an order to just unreliable supplier is more expensive than placing an order to just reliable supplier, the unreliable supplier offers a price discount to offset this disadvantage. They provide a heuristic that determines the amount of discount to make order splitting a worthwhile policy and the fraction of the order of the suppliers.Mohebbi (2003, 2004) and Mohebbi&Hao (2008) consider continuous-review inventory systems under $(Q, r)$-type policy with Poisson demand and an unreliable supplier, who has a stochastic lead time and whose states switch between *on* and *off.*

## 2.3. Inventory Systems with Stochastic Demand Investigating the Structure of the Optimal Policy

Some of these works assume zero fixed ordering cost.Swaminathan&Shanthikumar (1999) show that when demand is continuous, it is never optimal to order alone from the more expensive (and more reliable) supplier. When demand is discrete, they provide conditions under which when it is optimal to order alone from the more expensive supplier.Tomlin (2006) studies a periodic-review and infinite-horizon inventory system with two suppliershaving equal lead times: one who is unreliable and cheap and another who is reliable but more expensive. They formulate an infinite-state discretetime Markov process where the states indicate whether the unreliable supplier is *up* or *down* in a period, in which he makes full delivery or not respectively, and for how many periods he has been down since the last up state. The reliable supplier is capacity constrained, but may have volume flexibility. They show that a supplier's percentage uptime and the nature of the disruptions (e.g., frequent but short versus rare but long) are the key determinants of the optimal strategy.Veerarahavan&Scheller-Wolf (2008) examine a periodic-review and infinite-horizon inventory system whit two suppliers having different lead times. They show that dual-index policy (carrying two inventory positions one for the regular supplier, and one for the expedited supplier) is nearly optimal and outperforms single sourcing.

Federgruen& Yang (2011) analyze a periodic-review and finite-horizon inventory problem for a firm having a set of potential suppliers, each with specific yield and price characteristics. They formulate the problem as a dynamic programming and show that orders are placed not according to the classical base stock policy, but to the "maximum ordering inventory level" in each period such that orders are placed if and only if the starting inventory is below that threshold.Yeo & Yuan (2011) study a periodic-review inventory system with supply uncertainty where demands reserved in the previous period are supposed to be fulfilled in the current period, but open to cancellation. They find the optimal replenishment policy when lead time is zero and there exists no fixed ordering cost.Kouvelis& Li (2013) study a periodic-review inventory problem an unreliable supplier and characterize optimal decisions on the cycle order size, the emergency order size, and the way to split orders between slow and emergency suppliers.

There are also studies that assume positive fixed ordering cost. Parlar et al. (1995) analyze a periodic-review, finite-horizon inventory model, with setup costs, where a basic setup cost is incurred whenever an order is placed, and a secondary setup cost is incurred only when an order is filled. They prove that the optimal policy is of an $(s, S)$type where the reorder level $s$ depends on the state of the supplier in the previous period, while the order-up-to level $S$ does not. They point out that an exact procedure to compute the optimal values of the control parameters $s$ and $S$ could be very hard to develop. Chen et al. (2012) examine a multi-period inventory system consisting of two suppliers with fixed ordering costs. The regular supplier can supply the entire quantity ordered unless unexpected disruption occurs. The reliable backup supplier has a limited capacity, and is used in case of any disruption. They show that the optimal policy is characterized partially to have a state-dependent structure.

In this thesis, we aim to find optimal policy characterizations for a retailer that operates a single-item, periodic-review and infinite-horizon stochastic inventory system where he adopts a dual sourcing strategy. We analyze two cases for dual sourcing: In the first case, there are two suppliers, one is completely reliable and the other is unreliable but provides a lower unit purchasing cost. In the second case, both suppliers are unreliable which may have different unit purchasing costs and reliability levels but have same lead time characteristics. We model these problems as a Markov decision process in order to find the optimal ordering amounts from both suppliers in every system state. Then, by observing carefully these optimal decisions, we determine both the structure of the optimal policy (i.e. what control parameters are needed) and the optimal values of the control parameters.

# 3. PROBLEM DESCRIPTION AND MDP FORMULATIONS

We consider an inventory problem for a retailer who adopts a dual sourcing strategy to cope with potential supply disruptions. We analyze two cases of the problem. In the first case, the retailer has two suppliers, one being completely reliable and the other unreliable in the sense that his availability in the current period depends on his availability in the previous period. In the second case, the retailer has two unreliable suppliers, who may differ in their unit costs, fixed ordering costs and reliability levels. We formulate both cases as infinite-horizon discrete-time Markov decision processes, and in order to find the optimal ordering decisions, we use a variant of Howard's policy iteration method. The two cases are presented in details in the following sections.

## 3.1. The Model with One Reliable and One Unreliable Supplier

In this section, we consider a model for a retailer that operates a single product, periodic-review and infinite-horizon inventory system where he can place orders to two suppliers: one is perfectly reliable with an expensive unit purchasing cost while the other is unreliable but charges a lower unit purchasing cost. The retailer may place orders on either or both suppliers depending on the current inventory level and the availability of the unreliable supplier. The unreliable supplier follows a Markovian availability structure and alternates between "up" and "down" states, where his current supply state depends on his supply state in the previous period. An order placed on the reliable supplier at the beginning of a period arrives with certainty at the end of the period. The unreliable supplier accepts orders only if his status is in the up state, and makes the delivery at the end of the period if he doesn't go down during the period. However, if the unreliable supplier's status goes down after an order is accepted, then the order is cancelled. For each order made there are two types of fixed ordering costs. First, an *administrative cost* per order is incurred. This represents a fixed cost to initiate an order, and it is incurred whether the order is delivered or not. The second part of the fixed ordering cost is the *transportation cost*per order, which is incurred only if a delivery is made. Apart from the fixed cost, the retailer incurs a unit purchasing cost for products that are actually delivered.

We assume that demand in any period is stationary, independent and identically distributed. We further assume that all cost parameters of the system are stationary, i.e. they do not vary over time. On-hand inventory at the end of a period (i.e. each unit carried over to the next period) incurs a holding cost. Each unit demanded is met with a unit from inventory if there is stock on-hand. Any unsatisfied demand is backordered, to be met in future periods, and incurs a backordering cost for each period the demand is backordered. It is assumed that there is a maximum limit on the amount to be backordered, and any unmet demand beyond that limit is lost and incurs a unit lost sales cost for the retailer. Defining such a limit serves as a lower bound on the inventory and enables the problem to be modeled as a finite-state MDP and solved within reasonable amount of time (Ahiska et al., 2013).

The aim of the retailer is to find an optimal replenishment strategy (i.e., how much and when to order from each supplier), which minimizes the total expected cost per period. The total cost consists of the following: administrative and transportation fixed ordering costs; unit purchasing cost, assumed to be linear with ordered units, where the unit purchasing cost of the unreliable supplier is less than that of the reliable supplier; holding cost, which is linear in the quantity of units held at the end of the period in inventory; backorder cost, which is linear in the quantity of units backordered; and lost sales cost, which is linear in the quantity of lost sales.

Events in a period occur in the following order:

1. The state of the system (the inventory level and unreliable supplier status) is observed at the beginning of period.
2. Ordering decisions are made.
3. Demand occurs during the period and is filled (if possible) from beginning of period inventory as it occurs.

4. The orderplaced to the reliable supplier (if any) is received at the end of period.
5. The status of the unreliable supplier is checked at the end of period.
6. The orderplaced to the unreliable supplier (if any) is received or canceled depending on the supplier's status.
7. Ordering costs, holding cost and backordering/lost sales costs are incurred.

We formulate the problem as a discrete-time Markov decision process in order to find the optimal ordering policy, i.e. the optimal ordering quantities from both suppliers. Puterman (1994) defines a Markov decision process as a stochastic model consisting of decision epochs (i.e. periods in our problem), in which the system occupies a state. A decision maker observes the system in state $s \in S$, and may choose action $a$ from a set of allowable actions in state $s, A_s$, where both $S$ and $A_s$ are assumed not to vary over time. As a result of choosing action $a \in A_s$ in state $s$ at decision period $t$, the decision maker receives an immediate reward, $r_t(s, a)$, and the system state at the next decision period is determined by a transition probability matrix that defines the probability of going from one state to another under a selected decision. A policy is thus determined by the decision maker specifying the decision rule to be used at all decision periods, which provides the decision maker with a prescription for action selection in any possible future system state. The MDP model for this system adopted from Ahiska et al. (2013) is described in sections 3.1.1-3.1.3.

### 3.1.1. State Variables, Decision Variables and Alternatives

The system is defined by two random variables: the retailer's inventory level at the beginning of the period and the availability status of the unreliable supplier. The system is denoted by $S = (I, J)$ where the retailer's inventory level $I$ is limited between $I_{min}$ and $I_{max}$ (i.e.$I_{min} \leq I \leq I_{max}$) and the unreliable supplier's availability status $J$ is either 0 or 1, depending on his status in the previous period being up or down, respectively. Note that $I_{min} > -\infty$ and $|I_{min}|$ represents the maximum amount to backorder. This limit can be adjusted to represent any behavior of the retailer in case of a stock-out such as full backordering, partial backordering or no backordering. $I_{max}$represents the inventory capacity of the retailer where $I_{max} < \infty$.

We define $k_u$ as the quantity of the order placed on the unreliable supplier and $k_r$ as the quantity of the order placed on the reliable supplier and show these order quantities as a vector denoted by $k = (k_u, k_r)$. For state $S = (I, J)$, the set of alternative order quantities, $A_s$, is determined considering the storage capacity of the retailer and the supply state of the unreliable supplier as:

$$A_s = \begin{cases} (k_u, k_r) \text{ such that } k_u \geq 0,\ k_r \geq 0, \text{ and } k_u + k_r \leq I_{max} - I & \text{if } J = 0 \\ (k_u, k_r) \text{ such that } k_u = 0, \text{ and } 0 \leq k_r \leq I_{max} - I & \text{if } J = 1 \end{cases} \tag{3.1}$$

### 3.1.2. State Transitions and Transition Probabilities

State transitions of the model are defined by two Markov processes, the status of the unreliable supplier and inventory position at the beginning of a period, the former of which is independent of the latter.

We define a transition probability matrix $W$, according to which the status of the unreliable supplier changes from state $i$ to state $j$ in one period. We assume that the transition probabilities are stationary, i.e. they do not change at every period. The two probabilities, $\alpha$ and $\beta$, define these transitions. The value of $\alpha$ is defined as the probability that the status of the unreliable supplier will be up at the end of the period given that his status is up at the beginning of the period, while $\beta$ is defined as the probability that the status of the unreliable supplier will be up at the end of the period given that his status at the beginning of the period is down.

$$W = \begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} \alpha & 1-\alpha \\ \beta & 1-\beta \end{bmatrix} \end{array} \tag{3.2}$$

Demand, $D$, in each period is stochastic, independent and identically distributed but stationary and occurs according to a specified distribution, and is denoted as $P_D(d) = P[\text{Demand during a period is } d \text{ units}]$.

Suppose that the system occupies the state $S = (I, J)$, the retailer's ordering decision is $k = (k_u, k_r)$ and the demand during the period, $D$, is $d$, then the next state of the system $S' = (I', J')$ is described as follows depending on the current status of the unreliable supplier, $J$:

- If $J = 0$, i.e. the status of the unreliable supplier is up at the beginning of the period, then

$$J' = \begin{cases} 0 & \text{if the unreliable supplier remains up through the period} \\ 1 & \text{if the unreliable supplier goes down during the period} \end{cases} \tag{3.3}$$

$$I' = \begin{cases} \max\{I - d, I_{min}\} + k_u + k_r & \text{if the unreliable supplier remains up through the period} \\ \max\{I - d, I_{min}\} + k_r & \text{if the unreliable supplier goes down during the period} \end{cases} \tag{3.4}$$

  Note that the quantity ordered to the unreliable supplier, $k_u$, is canceled if his status happened to go down during the period.

- If $J = 1$, i.e. the status of the unreliable supplier is down at the beginning of the period, then

$$J' = \begin{cases} 0 & \text{if the unreliable supplier returns to up by the end of the period} \\ 1 & \text{if the unreliable supplier stays down during the period} \end{cases} \tag{3.5}$$

$$I' = \max\{I - d, I_{min}\} + k_r \tag{3.6}$$

  Note that when the unreliable supplier's status is down at the beginning of the period, the retailer can place an order only on the reliable supplier.

$P_{SS'}^{k}$ is defined as the probability that the system makes a transition to state $S'$ (i.e. $S' = (I', J')$) at the end of a period following decision $k = (k_u, k_r)$ when its state at the beginning of the period was $S = (I, J)$. The values of $P_{SS'}^{k}$ can be put into four groups based on the status of the unreliable supplier as given in Table 3.1.

Table 3.1 State transitions and probabilitiesfrom one state to another following an alternative

| Current state unreliable supplier status, $J$ | Next state unreliable supplier status,$J'$ | Next state inventory, $I'$ | Transition probability[a], $P_{SS'}$ |
|---|---|---|---|
| 0 | 0 | $\max\{I - d, I_{min}\} + k_u + k_r$ | $W_{00}P_D(d) = \alpha P_D(d)$ |
| 0 | 1 | $\max\{I - d, I_{min}\} + k_r$ | $W_{01}P_D(d) = (1-\alpha)P_D(d)$ |
| 1 | 0 | $\max\{I - d, I_{min}\} + k_r$ | $W_{10}P_D(d) = \beta P_D(d)$ |
| 1 | 1 | $\max\{I - d, I_{min}\} + k_r$ | $W_{11}P_D(d) = (1-\beta)P_D(d)$ |

[a] In case there are more than one event that makes the system state transition from $S$ to $S'$, the probabilities of all those events are summed for the value of $P_{SS'}$ .

Table 3.2 Notation

| Parameter | Description |
|---|---|
| $c_u$ | Unit purchasing cost charged by the unreliable supplier |
| $c_r$ | Unit purchasing cost charged by the reliable supplier |
| $f_a$ | Administrative cost per order placed to either supplier |
| $f_t$ | Transportation cost per order received from either supplier |
| $h$ | Holding cost per unit held at the end of period |
| $b$ | Backordering cost per unit |
| $l$ | Lost sales cost per unit |
| $B$ | Backordered amount at the end of period |
| $L$ | Lost sales amount at the end of period |

### 3.1.3. Objective Function for the Model

The aim of the retailer in this model is to minimize the total expected cost per period whose inputs are administrative and transportation costs per order, unit purchasing costs, unit holding cost, unit backordering cost and unit lost sales cost. We assume that the unit purchasing charged by the unreliable supplier is less than that of the reliable supplier ($c_u < c_r$). The notation provided in Table 3.2 is used for the cost function.

Given that the system is in state $S = (I, J)$, alternative $k = (k_u, k_r)$ is followed, and the demand during the period is $D = d$, the cost for the period is calculated as:

$$C(S,k,d,J') = \delta(k_r) + \gamma(k_u) + h[I']^+ + bB + lL \tag{3.7}$$

where the terms represent the costs of the orders placed to the reliable and unreliable suppliers, inventory holding cost, backordering cost and lost sales cost, respectively. These costs are calculated using equations (3.8)-(3.12).

$$\delta(k_r) = \begin{cases} f_a + f_t + c_r k_r & \text{for} k_r > 0 \\ 0 & \text{for} k_r = 0 \end{cases} \tag{3.8}$$

$$\gamma(k_u) = \begin{cases} f_a + f_t + c_u k_u & \text{for} k_u > 0 \text{and} J' = 0 \\ f_a & \text{for} k_u > 0 \text{and} J' = 1 \\ 0 & \text{for} k_u = 0 \end{cases} \tag{3.9}$$

$$[I']^+ = max\{I', 0\} \tag{3.10}$$

$$B = \begin{cases} -max\{I - d, I_{min}\} & \text{if} I - d < 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.11}$$

$$L = \begin{cases} I_{min} - (I - d) & \text{if} I - d < I_{min} \\ 0 & \text{otherwise} \end{cases} \tag{3.12}$$

Consider a state of the system $S = (I, J)$. The expected period cost for state $S$ when following alternative $k$, $q_S^k$ is computed as follows:

$$q_S^k = \sum_d P_D(d) \sum_{J'=0}^{1} W_{JJ'} C(S,k,d,J') \tag{3.13}$$

The above-described MDP model is solved using a variant of Howard's policy iteration method in order to find the optimal policy. Howard's policy iteration method consists of two phases: value determination phase and policy iteration phase. In the value determination method, the relative values and gain is calculated for a fixed policy. In the policy improvement phase, a better policy is found using the relative values found in the first phase. Morton (1971) introduces a fixed policy approximation, which eliminates the need for matrix inversion to solve linear equations in Phase 1of Howard's approach. The algorithm that solves the MDP model (i.e. finds the optimal policy) is provided in the Appendix A.

### 3.1.4. Policy Characterization Approach

When the MDP model described in the previous section is solved, we obtain a list of the optimal decisions corresponding to all possible states of the system for given input parameters. Ahiska et al. (2013) have characterized this list of the optimal decisions into easily implemented policies that have a few control parameters. They propose a policy characterization approach, which successfully finds both the structure of the characterized policy and the optimal values. The percentage deviation of the cost attained by following the characterization from the optimal cost of the MDP is used as a measure to evaluate the performance of the characterization.

Related with the policy characterization, some studies use direct observation approach: Hodgson et al. (1987) use direct observation approach to characterize optimal routing policies for automated guided vehicles systems. Ahiska& King (2010a, 2010b) use direct observation to determine inventory policy characterizations for a manufacturing/ remanufacturing system.More recently, Ahiska& King (2014) have used neural networks to develop a functional relationship between the input cost parameters and thepolicy parameters for the same recoverable system problem.

Ahiska et al. (2013)used the following approach to find a characterization for the inventory problem under consideration:

1. Determine the optimal policy, i.e. optimal values of order quantities $k_u$ and $k_r$, for each system state by solving the MDP model for a given scenario of input parameters.

2. By carefully observing the list of the optimal decisions for the recurrent states only, determine what control parameters should be used and their corresponding values, which form the characterized policy. For the retailer system under consideration, the appropriate control parameters turn out to be the following:

- The order-up-to levels of the reliable supplier ($S_r$) and the unreliable supplier ($S_u$), for which the estimated values to be used in the characterized policies,are calculated as shown below:

$$S_r = I^{max}_{k_r>0} + k_r \tag{3.14}$$

$$S_u = I^{max}_{k_u>0} + k_u \tag{3.15}$$

Here $I^{max}_{k_r>0}$ and $I^{max}_{k_u>0}$ represent the highest inventory levels for which the order quantities from reliable supplier ($k_r$) and unreliable supplier ($k_u$) are positive in the optimal policy, respectively.

- The reorder points for the reliable supplier ($s_r$) and the unreliable supplier ($s_u$),which are simply defined as the highest inventory levels below which the respective order quantities are positive in the optimal policy. The estimated values for the reorder points to be used in the characterized policy are therefore calculated as shown below:

$$s_r = I^{max}_{k_r>0} + 1 \tag{3.16}$$

$$s_u = I^{max}_{k_u>0} + 1 \tag{3.17}$$

3. Evaluate the quality of the characterization process by calculating the percentage deviation of the cost of the characterized policyfrom the optimal cost.

Ahiska et al. (2013) find the following policy structures to be optimal by investigating the system under some combinations of the system parameters:

1. When the unreliable supplier is down, orders are made to the reliable supplier according to an $(s_r, S_r)$ policy, i.e. when the inventory level, $I$, is below $s_r$, an order amount that raises the inventory level up to $S_r$is placed to the reliable supplier (i.e. $k_r = S_r - I$).

2. When the unreliable supplier status is up, the ordering structure belong to one of the four cases, described below:
   - Case 1: Order only from the unreliable supplier according to an $(s_u, S_u)$ policy, i.e. when $I < s_u$, an amount that raises the inventory level up to $S_u$ is ordered from the unreliable supplier (i.e. $k_u = S_u - I$).

   - Case 2: Order from the reliable and unreliable suppliers simultaneously or order only from the unreliable supplier depending on the inventory level, as described below:
     When $I < s_r$, first order from the reliable supplier up to $S_r$ (i.e. $k_r = S_r - I$); then, order from the unreliable supplier up to $S_u$, $S_u > S_r$ (i.e. $k_u = S_u - S_r$).
     When $s_r \leq I < s_u$, order from the unreliable supplier up to $S_u$ (i.e. $k_u = S_u - I$).
     When $I \geq s_u$, order nothing.

   - Case 3: Order from either the reliable supplier or the unreliable supplier depending on the inventory level, but never order from both simultaneously, as described below:
     When $I < s_r$, order from the reliable supplier up to $S_r$ (i.e. $k_r = S_r - I$).
     When $s_r \leq I < s_u$, order from the unreliable supplier up to $S_u$ (i.e. $k_u = S_u - I$).

When $I \geq s_u$, order nothing.

- Case 4: Order only from the reliable supplier according to an $(s_r, S_r)$ policy, i.e. when $I < s_r$, an amount that raises the inventory level up to $S_r$is ordered from the reliable supplier (i.e. $k_r = S_r - I$).

We implement the MDP-based characterization approach of Ahiska et al. (2013) to the retail system with different fixed cost structure that we consider in order to investigate how this new fixed cost structure (i.e. *administrative* plus *transportation costs*) affects the optimal policy structures.

### 3.1.5. Computational Results

This section reports the results of a set of numerical experiments that are done in order to see how the changes in several system parameters (e.g. fixed ordering costs, reliability level) affect the optimal ordering policies.

For the experimentation, as the base-case scenario we use the data provided in Ahiska et al. (2013). The demand has some central tendency. A triangular-shaped, discrete distribution with a mean of 5 and a standard deviation of 2.28 represents the demand, for which the probability mass function is given in Table 3.3. The inventory capacity of the retailer is set to 50 units (i.e. $I_{max} = 50$) and the number of units that are backordered cannot exceed 50 (i.e. $I_{min}$ = -50). The upper and lower bounds for the inventory are large enough relative to the mean demand so the retailer's decisions are not restricted by the storage capacity and lost sales are negligible.

Table 3.3 Probability mass function of demand

| $D$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P_D(d)$ | 0.02 | 0.05 | 0.08 | 0.11 | 0.15 | 0.18 | 0.15 | 0.11 | 0.08 | 0.05 | 0.02 |

A set of experiments is created in order to explore different types of optimal ordering policies, if there are any, other than those observed by Ahiska et al. (2013). In the base-case scenario, the following parameters are used: $c_r = 1.5$, $c_u = 1$, $f_a = 2.5$, $f_t = 2.5$,$h = 0.2$, $b = 2$, $l = 20$, $\alpha = 0.5$, $\beta = 1$.

### 3.1.5.1. Increasing One of the Ordering Costs While Other Parameters Are Fixed to a Specified Value

The model of Ahiska et al. (2013) considers only one type of fixed ordering cost that is incurred whenever an order is placed on a supplier. We call this the administrative fixed ordering cost. As stated earlier, we introduce a new fixed cost structure, which consists of administrative and transportation costs. We begin our experiments first by increasing the administrative fixed cost ($f_a$) from 0 to 10 in increments of 1, and keeping all other parameters fixed to their nominal values given above, including the transportation cost ($f_t$). The following observations are made from Table 3.4.a and Figure 3.1.a:

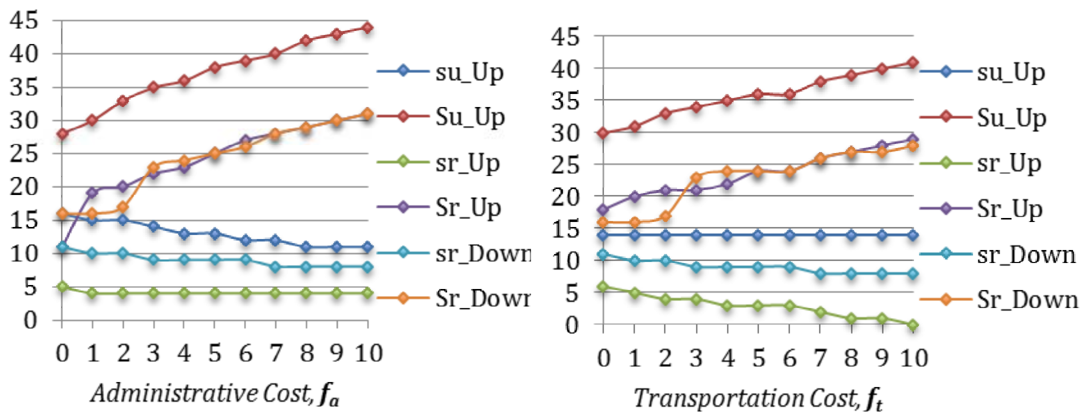
- As the administrative cost ($f_a$) increases, the reorder point ($s_u$) for the unreliable supplier in the up state decreases and the order-up-to level increases ($S_u$), that is, the retailer begins to order less frequently but in larger amounts from the unreliable supplier. The reason is that the cost of "not receiving an order" increases, so the retailer uses less frequently the unreliable supplier by decreasing the reorder point. In addition, since the fixed ordering cost increases, the order quantity increases in order to spread this cost over more units.

- As the administrative cost ($f_a$) increases,the retailer keeps the order frequency from the reliable supplier the same, except for $f_a = 0$(i.e. $s_r$ remains the same for all values of $f_a$except 0). However, it is observed that since the total fixed cost per order increases, the retailer orders in larger quantities from the reliable supplier, i.e. $S_r$ increases.

Table 3.4 Optimal policy control parameters as administrative and transportation costs change, respectively

| (a)Changes in $f_a$ | | | | | | | | (b)Changes in $f_t$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Up State | | | | | Down State | | | Up State | | | | | Down State | |
| $f_a$ | Case | $s_u$ | $S_u$ | $s_r$ | $S_r$ | $s_r$ | $S_r$ | $f_t$ | Case | $s_u$ | $S_u$ | $s_r$ | $S_r$ | $s_r$ | $S_r$ |
| 0 | 2 | 16 | 28 | 5 | 11 | 11 | 16 | 0 | 3 | 14 | 30 | 6 | 18 | 11 | 16 |
| 1 | 3 | 15 | 30 | 4 | 19 | 10 | 16 | 1 | 3 | 14 | 31 | 5 | 20 | 10 | 16 |
| 2 | 3 | 15 | 33 | 4 | 20 | 10 | 17 | 2 | 3 | 14 | 33 | 4 | 21 | 10 | 17 |
| 3 | 3 | 14 | 35 | 4 | 22 | 9 | 23 | 3 | 3 | 14 | 34 | 4 | 21 | 9 | 23 |
| 4 | 3 | 13 | 36 | 4 | 23 | 9 | 24 | 4 | 3 | 14 | 35 | 3 | 22 | 9 | 24 |
| 5 | 3 | 13 | 38 | 4 | 25 | 9 | 25 | 5 | 3 | 14 | 36 | 3 | 24 | 9 | 24 |
| 6 | 3 | 12 | 39 | 4 | 27 | 9 | 26 | 6 | 3 | 14 | 36 | 3 | 24 | 9 | 24 |
| 7 | 3 | 12 | 40 | 4 | 28 | 8 | 28 | 7 | 3 | 14 | 38 | 2 | 26 | 8 | 26 |
| 8 | 3 | 11 | 42 | 4 | 29 | 8 | 29 | 8 | 3 | 14 | 39 | 1 | 27 | 8 | 27 |
| 9 | 3 | 11 | 43 | 4 | 30 | 8 | 30 | 9 | 3 | 14 | 40 | 1 | 28 | 8 | 27 |
| 10 | 3 | 11 | 44 | 4 | 31 | 8 | 31 | 10 | 3 | 14 | 41 | 0 | 29 | 8 | 28 |

- When the unreliable supplier is in the down state, $s_r$decreases and $S_r$ increases; that is the retailer begins to order less frequently but in larger amounts from the reliable supplier since the total fixed cost per order $(f_a + f_t)$increases.




(a) (b)

Figure 3.1Changes in reorder points and order-up-to levels as administrative cost (a) and transportation cost (b) change

In summary, keeping all other parameters fixed at their nominal levels and just increasing the administrative cost of ordering makes the retailer to order less frequently and in larger amounts, as one would expect.

The second set of experiments focuses on the transportation cost ($f_t$). In this set of experiments we increase the transportation cost ($f_t$) from 0 to 10 in increments of 1, keeping all other parameters fixed at their nominal values, including the administrative cost ($f_a$).

The following observations are made from Table 3.4.b and Figure 3.1.b:

- As the transportation cost increases ($f_t$), the reorder point for the unreliable supplier in the up state ($s_u$) is not affected. The reason is that since the transportation cost is only incurred if the order is delivered, the cost of "not receiving an order" is not changed. However, the order-up-to level for the unreliable supplier ($S_u$) increases since cost of "receiving an order" from the unreliable supplier gets more expensive as the transportation cost increases.

- When the unreliable supplier is in the up or down state,as the transportation cost increases, the reorder point for the reliable supplier ($s_r$) decreases and the order-up-to level ($S_r$) increases, i.e. the retailer orders less frequently but in larger amounts from the reliable supplier because the total cost of "receiving an order" ($f_a + f_t$)gets higher as the transportation cost increases.

#### 3.1.5.2. Changes inthe Proportion of the Administrative and Transportation Costs

This set of experiments is created in order to understand the effects of the changes in the proportion of the administrative and transportation costs on the optimal policy structures. We first fix the value of $\beta$ to 1 (see Table 3.5.a) and change the proportions of the administrative and transportation costs while the total fixed cost is fixed to a nominal value (i.e. $f_a + f_t = 5$). In Table 3.5.b, we fix the value of $\beta$ to 0.1. All other parameters are fixed to their nominal values given in Section 3.1.5.

When the total setup cost is fixed, i.e. $f_a + f_t = 5$, and $\beta = 1$, following observations are made from Table 3.5.a and Figure 3.2.a:

- When the unreliable supplier is in the up state, and as $f_t$ increases (and thus $f_a$ decreases), the unreliable supplier reorder point $s_u$ increases and the order up to level, $S_u$ decreases, i.e. the range of inventory levels at which we order from the unreliable supplier increases but the ordering amount decreases due to the reduction in the fixed cost of administration $(f_a)$. That is, "not receiving an order" gets relatively less expensive while the fixed cost of receiving the order $(f_a + f_t)$ remains the same. Consequently, the retailer orders more frequently but in lower amounts from the unreliable supplier.

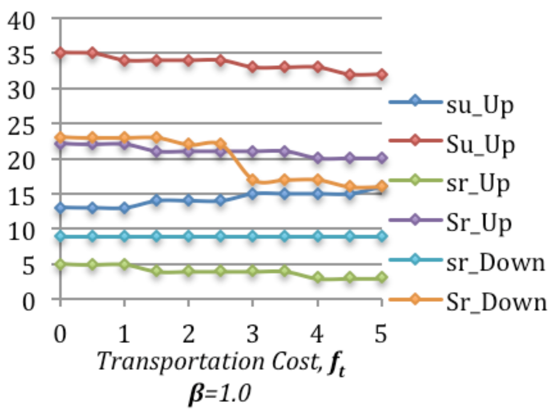
- When the unreliable supplier is in the up state, as $f_t$ increases (and thus $f_a$ decreases), the reliable supplier reorder point $s_r$ decreases and the order up to level, $S_r$ decreases, i.e. the range of inventory levels at which we order from the reliable supplier decreases and the ordering amount decreases. Thus, the retailer orders less frequently and lower amounts from the reliable supplier, relying more on the unreliable supplier.

- Similarly, the order-up-to level for the reliable supplier $(S_r)$ when the unreliable supplier is in the down state decreases as $f_t$ increases (and thus $f_a$ decreases) even though the total fixed cost remains the same. The reason is that ordering from the unreliable supplier in the up state gets cheaper as $f_a$ decreases, so the retailer orders more from the unreliable supplier (i.e. even if the unreliable supplier goes down and the order is canceled as a consequence, the cost that is paid for a canceled order is relatively cheaper). Consequently, the retailer orders less from the reliable supplier when the unreliable supplier is down. Note that when $\beta = 1$, if the unreliable supplier's current state is down, his status will be up next period (with probability equal to 1) and the retailer will be able to order from the unreliable supplier again.

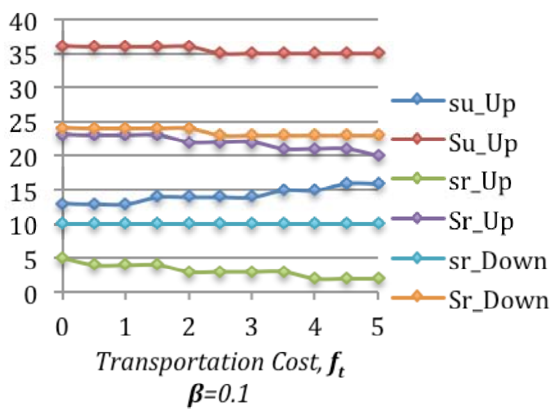
Table 3.5 Optimal policy control parameters as the proportion of administrative and transportation costs changes while the total fixed cost is fixed to a value

| | (a) $\beta = 1.0$ | | | | | | (b) $\beta = 0.1$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Up State | | | | Down State | | Up State | | | | Down State | |
| $f_a - f_t{}^*$ | $s_u$ | $S_u$ | $s_r$ | $S_r$ | $s_r$ | $S_r$ | $s_u$ | $S_u$ | $s_r$ | $S_r$ | $s_r$ | $S_r$ |
| 5.0-0.0 | 13 | 35 | 5 | 22 | 9 | 23 | 13 | 36 | 5 | 23 | 10 | 24 |
| 4.5-0.5 | 13 | 35 | 5 | 22 | 9 | 23 | 13 | 36 | 4 | 23 | 10 | 24 |
| 4.0-1.0 | 13 | 34 | 5 | 22 | 9 | 23 | 13 | 36 | 4 | 23 | 10 | 24 |
| 3.5-1.5 | 14 | 34 | 4 | 21 | 9 | 23 | 14 | 36 | 4 | 23 | 10 | 24 |
| 3.0-2.0 | 14 | 34 | 4 | 21 | 9 | 22 | 14 | 36 | 3 | 22 | 10 | 24 |
| 2.5-2.5 | 14 | 34 | 4 | 21 | 9 | 22 | 14 | 35 | 3 | 22 | 10 | 23 |
| 2.0-3.0 | 15 | 33 | 4 | 21 | 9 | 17 | 14 | 35 | 3 | 22 | 10 | 23 |
| 1.5-3.5 | 15 | 33 | 4 | 21 | 9 | 17 | 15 | 35 | 3 | 21 | 10 | 23 |
| 1.0-4.0 | 15 | 33 | 3 | 20 | 9 | 17 | 15 | 35 | 2 | 21 | 10 | 23 |
| 0.5-4.5 | 15 | 32 | 3 | 20 | 9 | 16 | 16 | 35 | 2 | 21 | 10 | 23 |
| 0.0-5.0 | 16 | 32 | 3 | 20 | 9 | 16 | 16 | 35 | 2 | 20 | 10 | 23 |

*The optimal policy structures for all scenarios above are observed to be case 3, i.e. order either from the reliable or the unreliable supplier, but not simultaneously.




(a)




(b)

Figure 3.2Changes in order-up-to levels and reorder points for both suppliers when the total ordering cost is fixed to a nominal value and $\beta$ changes from 1 to 0.1

When the total setup cost is fixed, i.e. $f_a + f_t = 5$, and $\beta = 0.1$, following observations are made from Table 3.5.b and Figure 3.2.b:

- It is observed that when the unreliable supplier is in the up state; as $f_t$ increases (and thus $f_a$ decreases), the reorder points $s_u$ increases and $s_r$ decreases, i.e. the range of inventory levels at which we order from the unreliable supplier increases due to the reduction in the fixed cost of administration $(f_a)$(though not as much as when $\beta = 1$). The reason is thatthe fixed cost of administration $(f_a)$ the retailer pays even when he does not receive the order decreases, that is, “not receiving an order” gets relatively less expensive as described above. Similarly, the retailer orders less frequently and lower amounts from the reliable supplier, relying more on the unreliable supplier.

- The order-up-to level for the reliable supplier $(S_r)$ when the unreliable supplier is in the down state stays approximately the same (as can be seen in Figure 3.2.b) as the portion of the total fixed cost for the administrative and transportation costs change, unlike the order-up-to level $(S_r)$ in the down state when $\beta$ was equal to 1. The reason is that if the unreliable supplier’s current state is down, then his next state is most likely to be down again (since $\beta = 0.1$) and the retailer will most probably not be able to place an order on him in the next period. So, the retailer continues to order from the reliable supplier in the same order frequency and amount in the down state.

#### 3.1.5.3. Optimal Policy Structures under Different Reliability Levels and Different Fixed Ordering Costs

This set of experiments is designed to gain additional insight into how different reliability levels for the unreliable supplier affect the optimal policy when there are two types of fixed ordering costs and whether or not the optimal policy structure is different than the policies described in Section 3.1.4. For that purpose, test scenarios are created for $\alpha \in [0.1, 0.5, 0.75, 0.95]$ as the values of $f_a$ and $f_t$ range from 0 to 10 in increments of 0.5, while other parameters are fixed to $c_r = 1.5$, $c_u = 1$, $\beta = 1.0$, $h = 0.2$, $b = 2$ and $l = 20$.

It should be noted that the optimal policies are not different than the characterized policies found by the MDP-based characterization approach that are described in Section 3.1.4. The following observations are made based on the plots in Figure 3.3 for when the unreliable supplier is up at time of ordering:

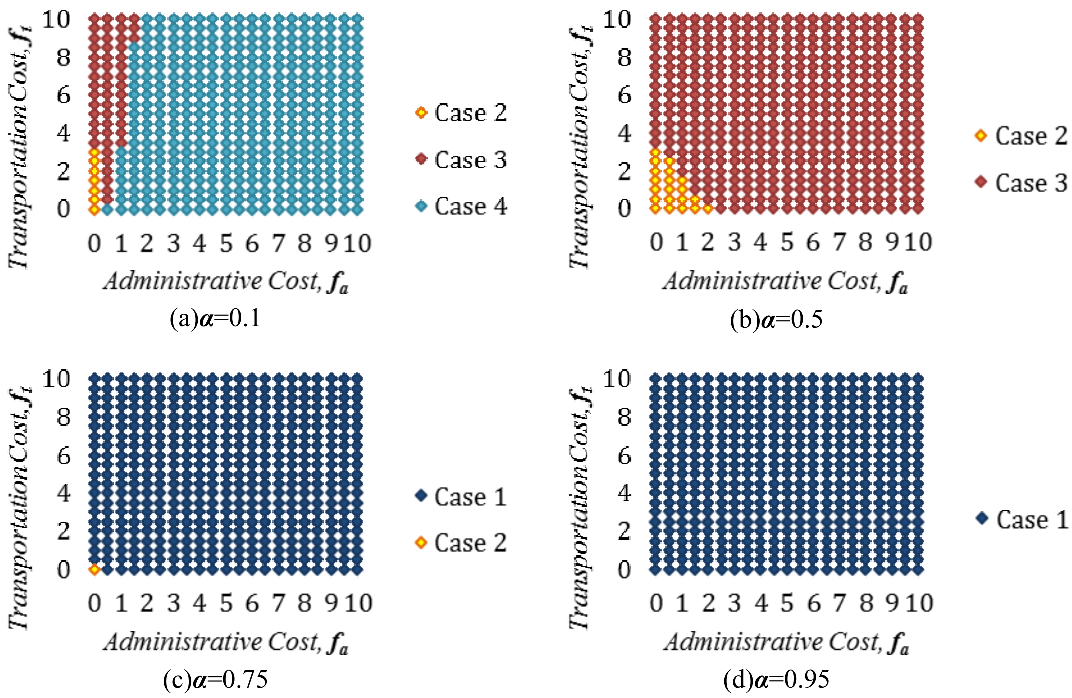



Figure 3.3 Optimal policy structures when α=0.1, 0.5, 0.75 and 0.95, respectively

- When $\alpha = 0.1$ (see Figure3.3.a), the unreliable supplier is almost completely unreliable and in this case, the dominating structure is case 4 policy. For the zero administrative cost (i.e. $f_a = 0$), when the transportation cost increases the optimal policy structure changes from case 2 to case 3. The reason is that as the transportation cost increases, so does the total cost of ordering, the retailer does not want to pay the total fixed cost of ordering twice. For a fixed transportation cost, when the administrative cost increases, the retailer tends to leave case 3 and follows case 4 policy in order to avoid the high cost of not receiving an order, because the unreliable supplier will most probably go down after the order is placed, since $\alpha = 0.1$.

- When $\alpha = 0.5$ (see Figure 3.3.b), as both types of the fixed costs increase, the retailer avoids using both suppliers simultaneously (i.e. leaves case 2) and orders from either the unreliable supplier or the reliable supplier (i.e. case 3 policy) since he does not want to pay the total fixed cost of ordering twice. Using only the reliable supplier (i.e. case 4) is not seen as an optimal policy structure when the unreliable supplier is equally to be up or down in the next period when his current status is up.

- When $\alpha = 0.75$(see Figure 3.3.c), except for the point where the total fixed ordering cost is $0$, the optimal policy structure is case 1 (i.e. ordering only from the unreliable supplier) for every combination of the administrative and transportation fixed costs. Since the probability of receiving an order placed to the unreliable supplier is high, ordering only from the unreliable supplier is optimal. The changes in administrative or transportation costs do not affect the optimal policy structure, but the reorder points and order-up-to levels may change.

- When $\alpha = 0.95$ (see Figure 3.3.d), the optimal policy structure for all combinations of administrative and transportation costs is case 1 policy. Since the unreliable supplier is regarded as almost completely reliable, the changes in both administrative and transportation costs do not affect the optimal policy structure. Because of his lower unit purchasing cost, the retailer places orders only on the unreliable supplier.

### 3.2. The Model with Two Unreliable Suppliers

In this section, we consider an extension of the previous model and assume that the retailer now deals with two unreliable suppliers who may be in up or down states independent of each other. The retailer may place orders on either or both suppliers depending on the current inventory level and availabilities of both suppliers. The suppliers follow a Markovian availability structure and alternate between up and down states, where one supplier's current supply state depends on his supply state in the previous period. When one supplier is in the down state, no orders can be placed on that supplier. Furthermore, any order placed on one supplier when he is in the up state is cancelled when that supplier's status goes down during the period. A supplier accepts orders only if his current status is up. If both suppliers are in the down state at the same time, then the retailer cannot order until either or both suppliers return to the up state. The lead time for both suppliers is one period, i.e. an order placed on one supplier at the beginning of a period arrives at the end of the period if concerning supplier does not go down during the period.

We apply the same cost structure to this model as in the first model. The parameters $c_1$ and $c_2$are associated with the unit purchasing costs charged by supplier 1 and supplier 2, respectively. The parameters$f_{1a}$ and $f_{2a}$ are the administrative fixed costs associated with suppliers 1 and 2, respectively, while $f_{1t}$ and $f_{2t}$ are the transportation fixed costs for suppliers 1 and 2, respectively. All other cost parameters $h$, $b$, $l$ and descriptions of $B$ and $L$ have the same interpretation as in Section 3.1.and the demand in each period is again stochastic, stationary, independent and identically distributed.

Events in each period occur in the following order:

1. The state of the system (the inventory level and the states of both suppliers) is observed at the beginning of the period.
2. Ordering decisions are made.
3. Demand occurs during the period and is filled (if possible) from beginning of period inventory as it occurs.
4. The states of the suppliers are checked at the end of the period.

5. Ordersplaced on the suppliers (if any) are received or canceled depending on theirstatus.
6. Ordering costs, holding cost and backordering/lost sales costs are incurred.

### 3.2.1. State Variables, Decision Variables and Alternatives

The problem is formulated as an infinite-horizon discrete-time Markov decision process to find the optimal ordering policy. The system is defined by three random variables, the retailer's inventory level at the beginning of the period, and the availability statuses of supplier 1 and supplier 2. The system is denoted by $S = (I, J_1, J_2)$ where the retailer's inventory level $I$ is limited between $I_{min}$ and $I_{max}$ (i.e.$I_{min} \leq I \leq I_{max}$) and the availability statuses of the suppliers are either up or down (i.e. 0 or 1, respectively) depending on their statuses in the previous period. The value $k_1$ is defined as the order quantity placed with supplier 1 and $k_2$ is defined as the order quantity placed with supplier 2. We show these order quantities as a vector denoted by $k = (k_1, k_2)$. For state $S = (I, J_1, J_2)$, the set of alternative order quantities, $A_s$, is determined considering the storage capacity of the retailer and the supply states of both suppliers as:

$$A_s = \begin{cases} (k_1, k_2) & \text{such that} k_1 \geq 0, k_2 \geq 0, k_1 + k_2 \leq I_{max} - I & \text{if} J_1 = 0,\ J_2 = 0 \\ (k_1, k_2) & \text{such that} k_1 = 0,\ 0 \leq k_2 \leq I_{max} - I & \text{if} J_1 = 1,\ J_2 = 0 \\ (k_1, k_2) & \text{such that} k_2 = 0,\ 0 \leq k_1 \leq I_{max} - I & \text{if} J_1 = 0,\ J_2 = 1 \\ (k_1, k_2) & \text{such that} k_1 = 0, k_2 = 0 & \text{if} J_1 = 1,\ J_2 = 1 \end{cases} \tag{3.18}$$

### 3.2.2. State Transitions and Transition Probabilities

Three independent Markov processes are considered: the retailer's inventory position at the beginning of the period, the status of supplier 1 and the status of supplier 2. These Markov processes determine the state transitions of the model. $W_{ij}^1$and$W_{ij}^2$ represent the one-step transition probability matrices of supplier 1 and supplier 2, respectively, from state $i$ to state $j$ in one period. The transition probabilities, which are assumed to be stationary, are described in Table 3.6:

Table 3.6 Transition probabilities for both suppliers

| | *Description* |
|---|---|
| $\alpha_1$ | Probability that supplier 1 stays in the up state from one period to the next |
| $\beta_1$ | Probability that supplier 1 transitions from down to up from one period to the next |
| $\alpha_2$ | Probability that supplier 2 stays in the up state from one period to the next |
| $\beta_2$ | Probability that supplier 2 transitions from down to up from one period to the next |

Therefore, we have the corresponding transition probability matrices as shown below:

$$W^1 = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} \alpha_1 & 1-\alpha_1 \\ \beta_1 & 1-\beta_1 \end{bmatrix} \end{array} \tag{3.19}$$

$$W^2 = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} \alpha_2 & 1-\alpha_2 \\ \beta_2 & 1-\beta_2 \end{bmatrix} \end{array} \tag{3.20}$$

Suppose that the system at the beginning of a period is in state $S = (I, J_1, J_2)$, a decision $k = (k_1, k_2)$ is made and demand during the period, $D$, is $d$, then the next state of the system $S' = (I', J_1', J_2')$ can be described as follows:

- $J_1 = 0$: i.e. the current status of supplier 1 is up.
  $J_2 = 0$: i.e. the current status of supplier 2 is up. Then

$$J_1' = \begin{cases} 0 & \text{if supplier 1 remains up through the period} \\ 1 & \text{if supplier 1 goes down during the period} \end{cases}$$

$$J_2' = \begin{cases} 0 & \text{if supplier 2 remains up through the period} \\ 1 & \text{if supplier 2 goes down during the period} \end{cases}$$

$$I' = \begin{cases} \max\{I - d, I_{min}\} + k_1 + k_2 & \text{if both suppliers remain up through the period} \\ \max\{I - d, I_{min}\} + k_2 & \text{if sup.1 goes down and sup.2 remains up during the period} \\ \max\{I - d, I_{min}\} + k_1 & \text{if sup.1 remains up and sup.2 goes down during the period} \\ \max\{I - d, I_{min}\} & \text{if both suppliers go down during the period} \end{cases} \tag{3.21}$$

Note that the order of $k_1$ and/or $k_2$ units is canceled when the corresponding supplier status goes from up to down.

- $J_1 = 1$: i.e. the current status of supplier 1 is down.
  $J_2 = 0$: i.e. the current status of supplier 2 is up. Then

$$J_1' = \begin{cases} 0 & \text{if supplier 1 returns to up by the end of the period} \\ 1 & \text{if supplier 1 stays down during the period} \end{cases}$$

$$J_2' = \begin{cases} 0 & \text{if supplier 2 remains up through the period} \\ 1 & \text{if supplier 2 goes down during the period} \end{cases}$$

$$I' = \begin{cases} \max\{I - d, I_{min}\} + k_2 & \text{if sup. 2 remains up through the period} \\ \max\{I - d, I_{min}\} & \text{if sup. 2 goes down during the period} \end{cases} \tag{3.22}$$

Note that no order is placed on supplier 1 since his status is down at the beginning of the period.

- $J_1 = 0$: i.e. the current status of supplier 1 is up.
  $J_2 = 1$: i.e. the current status of supplier 2 is down. Then

$$J_1' = \begin{cases} 0 & \text{if supplier 1 remains up through the period} \\ 1 & \text{if supplier 1 goes down during the period} \end{cases}$$

$$J_2' = \begin{cases} 0 & \text{if supplier 2 returns to up by the end of the period} \\ 1 & \text{if supplier 2 stays down during the period} \end{cases}$$

$$I' = \begin{cases} \max\{I - d, I_{min}\} + k_1 & \text{if sup. 1 remains up through the period} \\ \max\{I - d, I_{min}\} & \text{if sup. 1 goes down during the period} \end{cases} \tag{3.23}$$

Note that no order is placed on supplier 2 since his status is down at the beginning of the period.

- $J_1 = 1$: i.e. the current status of supplier 1 is down.
  $J_2 = 1$: i.e. the current status of supplier 2 is down. Then

$$J_1' = \begin{cases} 0 & \text{if supplier 1 returns to up by the end of the period} \\ 1 & \text{if supplier 1 stays down during the period} \end{cases}$$

$$J_2' = \begin{cases} 0 & \text{if supplier 2 returns to up by the end of the period} \\ 1 & \text{if supplier 2 stays down during the period} \end{cases}$$

$$I' = \max\{ I - d, I_{min}\} \tag{3.24}$$

Note that no orders are placed on either supplier since their statuses are down at the beginning of the period.

We define $P_{SS'}^{k}$ to be the one period state transition probability from state $S = (I, J_1, J_2)$ to state $S' = (I', J_1', J_2')$ when the decision $k = (k_1, k_2)$ is made. Based on the statuses of the suppliers, the values of $P_{SS'}^{k}$ are given in Table 3.7.

### 3.2.3. Objective Function for the Model

The aim of the retailer is to minimize the total expected cost per period that includes administrative and transportation costs, unit purchasing cost, holding cost, backordering cost and lost sales cost.

If the system is in state $S = (I, J_1, J_2)$ and the retailer follows alternative $k = (k_1, k_2)$ and demand during the period is $D = d$, then the cost for the period is calculated as follows:

$$C(S, k, d, J_1', J_2') = \delta(k_1) + \gamma(k_2) + h[I']^{+} + bB + lL \tag{3.25}$$

where the terms in the right hand side represent the cost of orders placed on supplier 1 and supplier 2, inventory holding cost, backordering cost and lot sales cost. These costs are calculated using equations (3.26)-(3.30).

$$\delta(k_1) = \begin{cases} f_{1a} + f_{1t} + c_1 k_1 & \text{for} k_1 > 0 \text{and} J_1' = 0 \\ f_{1a} & \text{for} k_1 > 0 \text{and} J_1' = 1 \\ 0 & \text{for} k_1 = 0 \end{cases} \tag{3.26}$$

Table 3.7State transitions and probabilities from one state to another following alternative $\boldsymbol{k} = (\boldsymbol{k_1}, \boldsymbol{k_2})$

| Current state suppliers' statuses, $(J_1, J_2)$ | Next state suppliers' statuses, $(J_1', J_2')$ | Next state inventory, $I'$ | Transition probability*, $P_{SS'}$ |
|---|---|---|---|
| (0,0) | (0,0) | $\max\{I - d, I_{min}\} + k_1 + k_2$ | $W_{00}^1 W_{00}^2 P_D(d) = \alpha_1 \alpha_2 P_D(d)$ |
| | (1,0) | $\max\{I - d, I_{min}\} + k_2$ | $W_{01}^1 W_{00}^2 P_D(d) = (1 - \alpha_1)\alpha_2 P_D(d)$ |
| | (0,1) | $\max\{I - d, I_{min}\} + k_1$ | $W_{00}^1 W_{01}^2 P_D(d) = \alpha_1(1 - \alpha_2) P_D(d)$ |
| | (1,1) | $\max\{I - d, I_{min}\}$ | $W_{01}^1 W_{01}^2 P_D(d) = (1 - \alpha_1)(1 - \alpha_2) P_D(d)$ |
| (1,0) | (0,0) | $\max\{I - d, I_{min}\} + k_2$ | $W_{10}^1 W_{00}^2 P_D(d) = \beta_1 \alpha_2 P_D(d)$ |
| | (1,0) | $\max\{I - d, I_{min}\} + k_2$ | $W_{11}^1 W_{00}^2 P_D(d) = (1 - \beta_1)\alpha_2 P_D(d)$ |
| | (0,1) | $\max\{I - d, I_{min}\}$ | $W_{10}^1 W_{01}^2 P_D(d) = \beta_1(1 - \alpha_2) P_D(d)$ |
| | (1,1) | $\max\{I - d, I_{min}\}$ | $W_{11}^1 W_{01}^2 P_D(d) = (1 - \beta_1)(1 - \alpha_2) P_D(d)$ |
| (0,1) | (0,0) | $\max\{I - d, I_{min}\} + k_1$ | $W_{00}^1 W_{10}^2 P_D(d) = \alpha_1 \beta_2 P_D(d)$ |
| | (1,0) | $\max\{I - d, I_{min}\}$ | $W_{01}^1 W_{10}^2 P_D(d) = (1 - \alpha_1)\beta_2 P_D(d)$ |
| | (0,1) | $\max\{I - d, I_{min}\} + k_1$ | $W_{00}^1 W_{11}^2 P_D(d) = \alpha_1(1 - \beta_2) P_D(d)$ |
| | (1,1) | $\max\{I - d, I_{min}\}$ | $W_{01}^1 W_{11}^2 P_D(d) = (1 - \alpha_1)(1 - \beta_2) P_D(d)$ |
| (1,1) | (0,0) | $\max\{I - d, I_{min}\}$ | $W_{10}^1 W_{10}^2 P_D(d) = \beta_1 \beta_2 P_D(d)$ |
| | (1,0) | $\max\{I - d, I_{min}\}$ | $W_{11}^1 W_{10}^2 P_D(d) = (1 - \beta_1)\beta_2 P_D(d)$ |
| | (0,1) | $\max\{I - d, I_{min}\}$ | $W_{10}^1 W_{11}^2 P_D(d) = \beta_1(1 - \beta_2) P_D(d)$ |
| | (1,1) | $\max\{I - d, I_{min}\}$ | $W_{11}^1 W_{11}^2 P_D(d) = (1 - \beta_1)(1 - \beta_2) P_D(d)$ |

*In case there are more than one event that makes the system state transition from *S* to *S'*, the probabilities of all those events are summed for the value of $P_{SS'}$ .

$$\gamma(k_2) = \begin{cases} f_{2a} + f_{2t} + c_2 k_2 & \text{for} k_2 > 0 \text{and} J_2' = 0 \\ f_{2a} & \text{for} k_2 > 0 \text{and} J_2' = 1 \\ 0 & \text{for} k_2 = 0 \end{cases} \tag{3.27}$$

Note that the order is canceled and only an administrative cost is incurred if the corresponding supplier goes down during the period.

$$[I']^+ = max\{I', 0\} \tag{3.28}$$

$$B = \begin{cases} -max\{I-d, I_{min}\} & \text{If } I-d < 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.29}$$

$$L = \begin{cases} I_{min} - (I-d) & \text{if } I-d < I_{min} \\ 0 & \text{otherwise} \end{cases} \tag{3.30}$$

The expected period cost for state $S = (I, J_1, J_2)$ when following alternative $k = (k_1, k_2)$, $q_S^k$ is computed as follows:

$$q_S^k = \sum_d P_D(d) \sum_{J_1'=0}^{1} \sum_{J_2'=0}^{1} W_{J_1 J_1'}^1 W_{J_2 J_2'}^2 C(S, k, d, J_1', J_2') \tag{3.31}$$

### 3.2.4. Experiments for Policy Characterization

In this section, we perform numerical experiments for the model with two unreliable suppliers in order to determine the different types of ordering policies that can be optimal for the retail system. Weassumethat the suppliers are not identical in terms of reliability levels and cost: supplier 1 is less reliable than supplier 2 but provides a lower unit purchasing cost. In the base-case scenario, the retailer has the same administrative and transportation costs for the orders placed to both suppliers.But since our main concern is the possible effects of the different components of the fixed cost structure, we create scenarios with different combinations of administrative and transportation costs. The parameters for the base-case scenario are set to the following values: $c_1 = 1$, $c_2 = 1.2$, $f_{1a} = 2.5$, $f_{1t} = 2.5$, $f_{2a} = 2.5$, $f_{2t} = 2.5$,$h = 0.2$, $b = 2$, $l = 20$, $\alpha_1 = 0.5$, $\alpha_2 = 0.75$, $\beta_1 = 1$, $\beta_2 = 1$, $I_{max} = 50$, $I_{min} =$ -50. Note that $f_{1a}$ and $f_{2a}$ are the administrative costs for suppliers 1 and 2, respectively; $f_{1t}$ and $f_{2t}$ are the transportation costs for suppliers 1 and 2, respectively. Demand occurs according to a triangular-shaped, discrete distribution with a mean of 5 and a standard deviation of 2.28, whose probability mass function is given in Table 3.3 of Section 3.1.5.

The test scenarios are created by changing the values of administrative and transportation fixed costs one at a time in the base scenario. The values considered for these parameters are provided in Table 3.8. For each test scenario, the MDP-based characterization approach is used to find the optimal policy structure as well as the optimal values of the policy parameters.

Table 3.8 The parameters and their corresponding values considered in the experiments

| Parameters | Ranges of values |
|---|---|
| Administrative cost for supplier 1, $f_{1a}$ | 0-10 in increments of 1 |
| Transportation cost for supplier 1, $f_{1t}$ | 0-10 in increments of 1 |
| Administrative cost for supplier 2, $f_{2a}$ | 0-10 in increments of 1 |
| Transportation cost for supplier 2, $f_{2t}$ | 0-10 in increments of 1 |

We derived the following optimal policy structures based on the experiments carried out:

1. When both suppliers aredown at the beginning of period, no orders can be made to the either supplier.
2. When only one supplier (say, supplier $i$) is up and the other is down, orders are made to supplier $i$ according to an $(s_i, S_i)$ policy, i.e. when the inventory level, $I$, is below $s_i$, an amount that raises the inventory level up to $S_i$ is placed on supplier $i$ (i.e. $k_i = S_i - I$).
3. When both suppliers are up at the beginning of period, three cases are observed:
   - Case 1: First, order from both suppliers simultaneously and then only from the less reliable supplier (supplier 1) depending on the inventory level, as described below:

     When $I < s_j$ (i.e. $s_j$ is the joint reorder point below which orders are made to both suppliers simultaneously), orders are made to both suppliers simultaneouslyaccording to a policy which has a more complicated structure than the simple $(s, S)$ policy.There seem to be multiple order-up-to levels for both suppliers that are dependent on the current inventory level.

     When $s_j \leq I < s_1$, order only from the less reliable supplier (supplier 1) up to $S_1$ (i.e. $k_1 = S_1 - I$).

When $I \geq s_1$, order nothing.

- Case 2: First order form both suppliers simultaneously and then only from the more reliable supplier (supplier 2) depending on the inventory level, as described below:
  When $I < s_j$, orders are made to both suppliers simultaneously according to a policy which has a more complicated structure than the simple$(s, S)$ policy where the order-up-to levels are dependent on the current inventory level.
  When $s_j \leq I < s_2$, order only from the more reliable supplier (supplier 2) up to $S_2$ (i.e. $k_2 = S_2 - I$).
  When $I \geq s_2$, order nothing.

- Case 3: First order form both suppliers simultaneously, then order from a single supplier at a time in the following order: first order only from the more reliable supplier (supplier 2), and then order only from the less reliable supplier (supplier 1) depending on the inventory level, as described below:
  When $I < s_j$, orders are made to both supplierssimultaneously according to a policy which has a more complicated structure than the simple $(s, S)$ policy where the order up to levels are dependent on the current inventory level.
  When $s_j \leq I < s_2$, order only from the more reliable supplier (supplier 2) up to $S_2$ (i.e. $k_2 = S_2 - I$).
  When $s_2 \leq I < s_1$, order only from the less reliable supplier (supplier 1) up to $S_1$ (i.e. $k_1 = S_1 - I$).
  When $I \geq s_1$, order nothing.

Some representing results are provided for cases 1, 2 and 3 in Table 3.9, Table 3.10 and Table 3.11, respectively. Each table gives the optimal MDP solutionsand the optimal policy characterizations that we derived out of them along with the corresponding policy control parameters.

Table 3.9MDP solutions and optimal policy control parameters for case 1 policy when $\boldsymbol{f_{1a}} = \mathbf{0}$*

| Both Suppliers Up | | | | | Supplier 1 Down | | | | | Supplier 2 Down | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $J_1$ | $J_2$ | $I$ | $k_1$ | $k_2$ | $J_1$ | $J_2$ | $I$ | $k_1$ | $k_2$ | $J_1$ | $J_2$ | $I$ | $k_1$ | $k_2$ |
| 0 | 0 | -10 | 24 | 26 | 1 | 0 | -10 | 0 | 36 | 0 | 1 | -10 | 41 | 0 |
| 0 | 0 | -9 | 23 | 26 | 1 | 0 | -9 | 0 | 35 | 0 | 1 | -9 | 40 | 0 |
| 0 | 0 | -8 | 22 | 26 | 1 | 0 | -8 | 0 | 34 | 0 | 1 | -8 | 39 | 0 |
| 0 | 0 | -7 | 21 | 25 | 1 | 0 | -7 | 0 | 33 | 0 | 1 | -7 | 38 | 0 |
| 0 | 0 | -6 | 20 | 25 | 1 | 0 | -6 | 0 | 32 | 0 | 1 | -6 | 37 | 0 |
| 0 | 0 | -5 | 19 | 24 | 1 | 0 | -5 | 0 | 31 | 0 | 1 | -5 | 36 | 0 |
| 0 | 0 | -4 | 19 | 23 | 1 | 0 | -4 | 0 | 30 | 0 | 1 | -4 | 35 | 0 |
| 0 | 0 | -3 | 18 | 23 | 1 | 0 | -3 | 0 | 29 | 0 | 1 | -3 | 34 | 0 |
| 0 | 0 | -2 | 17 | 22 | 1 | 0 | -2 | 0 | 28 | 0 | 1 | -2 | 33 | 0 |
| 0 | 0 | -1 | 16 | 21 | 1 | 0 | -1 | 0 | 27 | 0 | 1 | -1 | 32 | 0 |
| 0 | 0 | 0 | 16 | 20 | 1 | 0 | 0 | 0 | 26 | 0 | 1 | 0 | 31 | 0 |
| 0 | 0 | 1 | 15 | 20 | 1 | 0 | 1 | 0 | 25 | 0 | 1 | 1 | 30 | 0 |
| 0 | 0 | 2 | 14 | 19 | 1 | 0 | 2 | 0 | 24 | 0 | 1 | 2 | 29 | 0 |
| 0 | 0 | 3 | 14 | 18 | 1 | 0 | 3 | 0 | 23 | 0 | 1 | 3 | 28 | 0 |
| 0 | 0 | 4 | 13 | 17 | 1 | 0 | 4 | 0 | 22 | 0 | 1 | 4 | 27 | 0 |
| 0 | 0 | 5 | 13 | 16 | 1 | 0 | 5 | 0 | 21 | 0 | 1 | 5 | 26 | 0 |
| 0 | 0 | 6 | 12 | 15 | 1 | 0 | 6 | 0 | 20 | 0 | 1 | 6 | 25 | 0 |
| 0 | 0 | 7 | 24 | 0 | 1 | 0 | 7 | 0 | 19 | 0 | 1 | 7 | 24 | 0 |
| 0 | 0 | 8 | 23 | 0 | 1 | 0 | 8 | 0 | 18 | 0 | 1 | 8 | 23 | 0 |
| 0 | 0 | 9 | 22 | 0 | 1 | 0 | 9 | 0 | 17 | 0 | 1 | 9 | 22 | 0 |
| 0 | 0 | 10 | 21 | 0 | 1 | 0 | 10 | 0 | 16 | 0 | 1 | 10 | 21 | 0 |
| 0 | 0 | 11 | 20 | 0 | 1 | 0 | 11 | 0 | 15 | 0 | 1 | 11 | 20 | 0 |
| 0 | 0 | 12 | 19 | 0 | 1 | 0 | 12 | 0 | 0 | 0 | 1 | 12 | 19 | 0 |
| 0 | 0 | 13 | 18 | 0 | 1 | 0 | 13 | 0 | 0 | 0 | 1 | 13 | 18 | 0 |
| 0 | 0 | 14 | 17 | 0 | 1 | 0 | 14 | 0 | 0 | 0 | 1 | 14 | 17 | 0 |
| 0 | 0 | 15 | 16 | 0 | 1 | 0 | 15 | 0 | 0 | 0 | 1 | 15 | 16 | 0 |
| 0 | 0 | 16 | 15 | 0 | 1 | 0 | 16 | 0 | 0 | 0 | 1 | 16 | 15 | 0 |
| 0 | 0 | 17 | 14 | 0 | 1 | 0 | 17 | 0 | 0 | 0 | 1 | 17 | 14 | 0 |
| 0 | 0 | 18 | 13 | 0 | 1 | 0 | 18 | 0 | 0 | 0 | 1 | 18 | 13 | 0 |
| 0 | 0 | 19 | 0 | 0 | 1 | 0 | 19 | 0 | 0 | 0 | 1 | 19 | 12 | 0 |
| 0 | 0 | 20 | 0 | 0 | 1 | 0 | 20 | 0 | 0 | 0 | 1 | 20 | 0 | 0 |
| 0 | 0 | 21 | 0 | 0 | 1 | 0 | 21 | 0 | 0 | 0 | 1 | 21 | 0 | 0 |

| Policy Characterization | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Case 1 Policy | | | | | Order Only From Sup. 2 | | Order Only From Sup. 1 | |
| $s_1$ | $S_1$ | $s_2$ | $S_2$ | $s_j$ | $s_2$ | $S_2$ | $s_1$ | $S_1$ |
| 19 | 31 | * | * | 7 | 12 | 26 | 20 | 31 |

*All other parameters are fixed to theirbase-case scenario values.

Table 3.10 MDP solutions andoptimal policy control parameters for case 2 policy when $\boldsymbol{f_{2a} = 0}$*

| Both Suppliers Up | | | | | Supplier 1 Down | | | | | Supplier 2 Down | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $J_1$ | $J_2$ | $I$ | $k_1$ | $k_2$ | $J_1$ | $J_2$ | $I$ | $k_1$ | $k_2$ | $J_1$ | $J_2$ | $I$ | $k_1$ | $k_2$ |
| 0 | 0 | -10 | 25 | 24 | 1 | 0 | -10 | 0 | 36 | 0 | 1 | -10 | 41 | 0 |
| 0 | 0 | -9 | 24 | 23 | 1 | 0 | -9 | 0 | 35 | 0 | 1 | -9 | 40 | 0 |
| 0 | 0 | -8 | 23 | 23 | 1 | 0 | -8 | 0 | 34 | 0 | 1 | -8 | 39 | 0 |
| 0 | 0 | -7 | 23 | 22 | 1 | 0 | -7 | 0 | 33 | 0 | 1 | -7 | 38 | 0 |
| 0 | 0 | -6 | 22 | 21 | 1 | 0 | -6 | 0 | 32 | 0 | 1 | -6 | 37 | 0 |
| 0 | 0 | -5 | 22 | 20 | 1 | 0 | -5 | 0 | 31 | 0 | 1 | -5 | 36 | 0 |
| 0 | 0 | -4 | 21 | 19 | 1 | 0 | -4 | 0 | 30 | 0 | 1 | -4 | 35 | 0 |
| 0 | 0 | -3 | 21 | 18 | 1 | 0 | -3 | 0 | 29 | 0 | 1 | -3 | 34 | 0 |
| 0 | 0 | -2 | 21 | 17 | 1 | 0 | -2 | 0 | 28 | 0 | 1 | -2 | 33 | 0 |
| 0 | 0 | -1 | 20 | 16 | 1 | 0 | -1 | 0 | 27 | 0 | 1 | -1 | 32 | 0 |
| 0 | 0 | 0 | 20 | 15 | 1 | 0 | 0 | 0 | 26 | 0 | 1 | 0 | 31 | 0 |
| 0 | 0 | 1 | 19 | 15 | 1 | 0 | 1 | 0 | 25 | 0 | 1 | 1 | 30 | 0 |
| 0 | 0 | 2 | 19 | 14 | 1 | 0 | 2 | 0 | 24 | 0 | 1 | 2 | 29 | 0 |
| 0 | 0 | 3 | 18 | 13 | 1 | 0 | 3 | 0 | 23 | 0 | 1 | 3 | 28 | 0 |
| 0 | 0 | 4 | 18 | 12 | 1 | 0 | 4 | 0 | 22 | 0 | 1 | 4 | 27 | 0 |
| 0 | 0 | 5 | 0 | 21 | 1 | 0 | 5 | 0 | 21 | 0 | 1 | 5 | 26 | 0 |
| 0 | 0 | 6 | 0 | 20 | 1 | 0 | 6 | 0 | 20 | 0 | 1 | 6 | 25 | 0 |
| 0 | 0 | 7 | 0 | 19 | 1 | 0 | 7 | 0 | 19 | 0 | 1 | 7 | 24 | 0 |
| 0 | 0 | 8 | 0 | 18 | 1 | 0 | 8 | 0 | 18 | 0 | 1 | 8 | 23 | 0 |
| 0 | 0 | 9 | 0 | 17 | 1 | 0 | 9 | 0 | 17 | 0 | 1 | 9 | 22 | 0 |
| 0 | 0 | 10 | 0 | 16 | 1 | 0 | 10 | 0 | 16 | 0 | 1 | 10 | 21 | 0 |
| 0 | 0 | 11 | 0 | 15 | 1 | 0 | 11 | 0 | 15 | 0 | 1 | 11 | 20 | 0 |
| 0 | 0 | 12 | 0 | 14 | 1 | 0 | 12 | 0 | 14 | 0 | 1 | 12 | 19 | 0 |
| 0 | 0 | 13 | 0 | 13 | 1 | 0 | 13 | 0 | 13 | 0 | 1 | 13 | 0 | 0 |
| 0 | 0 | 14 | 0 | 12 | 1 | 0 | 14 | 0 | 12 | 0 | 1 | 14 | 0 | 0 |
| 0 | 0 | 15 | 0 | 0 | 1 | 0 | 15 | 0 | 11 | 0 | 1 | 15 | 0 | 0 |
| 0 | 0 | 16 | 0 | 0 | 1 | 0 | 16 | 0 | 0 | 0 | 1 | 16 | 0 | 0 |
| 0 | 0 | 17 | 0 | 0 | 1 | 0 | 17 | 0 | 0 | 0 | 1 | 17 | 0 | 0 |
| 0 | 0 | 18 | 0 | 0 | 1 | 0 | 18 | 0 | 0 | 0 | 1 | 18 | 0 | 0 |
| 0 | 0 | 19 | 0 | 0 | 1 | 0 | 19 | 0 | 0 | 0 | 1 | 19 | 0 | 0 |
| 0 | 0 | 20 | 0 | 0 | 1 | 0 | 20 | 0 | 0 | 0 | 1 | 20 | 0 | 0 |
| 0 | 0 | 21 | 0 | 0 | 1 | 0 | 21 | 0 | 0 | 0 | 1 | 21 | 0 | 0 |

| Policy Characterization | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Case 2 Policy | | | | | Order Only From Sup. 2 | | Order Only From Sup. 1 | |
| $s_1$ | $S_1$ | $s_2$ | $S_2$ | $s_j$ | $s_2$ | $S_2$ | $s_1$ | $S_1$ |
| * | * | 15 | 26 | 5 | 16 | 26 | 13 | 31 |

*All other parameters are fixed to their base-case scenario values.

Table 3.11MDP solutions andoptimal policy control parameters for case 3 policy when $\boldsymbol{f_{2a} = 3}$*

| Both Suppliers Up | | | | | Supplier 1 Down | | | | | Supplier 2 Down | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $J_1$ | $J_2$ | $I$ | $k_1$ | $k_2$ | $J_1$ | $J_2$ | $I$ | $k_1$ | $k_2$ | $J_1$ | $J_2$ | $I$ | $k_1$ | $k_2$ |
| 0 | 0 | -10 | 25 | 26 | 1 | 0 | -10 | 0 | 39 | 0 | 1 | -10 | 44 | 0 |
| 0 | 0 | -9 | 24 | 25 | 1 | 0 | -9 | 0 | 38 | 0 | 1 | -9 | 43 | 0 |
| 0 | 0 | -8 | 24 | 24 | 1 | 0 | -8 | 0 | 37 | 0 | 1 | -8 | 42 | 0 |
| 0 | 0 | -7 | 22 | 26 | 1 | 0 | -7 | 0 | 36 | 0 | 1 | -7 | 41 | 0 |
| 0 | 0 | -6 | 21 | 25 | 1 | 0 | -6 | 0 | 35 | 0 | 1 | -6 | 40 | 0 |
| 0 | 0 | -5 | 21 | 24 | 1 | 0 | -5 | 0 | 34 | 0 | 1 | -5 | 39 | 0 |
| 0 | 0 | -4 | 20 | 24 | 1 | 0 | -4 | 0 | 33 | 0 | 1 | -4 | 38 | 0 |
| 0 | 0 | -3 | 19 | 23 | 1 | 0 | -3 | 0 | 32 | 0 | 1 | -3 | 37 | 0 |
| 0 | 0 | -2 | 18 | 23 | 1 | 0 | -2 | 0 | 31 | 0 | 1 | -2 | 36 | 0 |
| 0 | 0 | -1 | 18 | 22 | 1 | 0 | -1 | 0 | 30 | 0 | 1 | -1 | 35 | 0 |
| 0 | 0 | 0 | 17 | 21 | 1 | 0 | 0 | 0 | 29 | 0 | 1 | 0 | 34 | 0 |
| 0 | 0 | 1 | 17 | 20 | 1 | 0 | 1 | 0 | 28 | 0 | 1 | 1 | 33 | 0 |
| 0 | 0 | 2 | 16 | 20 | 1 | 0 | 2 | 0 | 27 | 0 | 1 | 2 | 32 | 0 |
| 0 | 0 | 3 | 16 | 19 | 1 | 0 | 3 | 0 | 26 | 0 | 1 | 3 | 31 | 0 |
| 0 | 0 | 4 | 0 | 25 | 1 | 0 | 4 | 0 | 25 | 0 | 1 | 4 | 30 | 0 |
| 0 | 0 | 5 | 0 | 24 | 1 | 0 | 5 | 0 | 24 | 0 | 1 | 5 | 29 | 0 |
| 0 | 0 | 6 | 0 | 23 | 1 | 0 | 6 | 0 | 23 | 0 | 1 | 6 | 28 | 0 |
| 0 | 0 | 7 | 27 | 0 | 1 | 0 | 7 | 0 | 22 | 0 | 1 | 7 | 27 | 0 |
| 0 | 0 | 8 | 26 | 0 | 1 | 0 | 8 | 0 | 21 | 0 | 1 | 8 | 26 | 0 |
| 0 | 0 | 9 | 25 | 0 | 1 | 0 | 9 | 0 | 20 | 0 | 1 | 9 | 25 | 0 |
| 0 | 0 | 10 | 24 | 0 | 1 | 0 | 10 | 0 | 19 | 0 | 1 | 10 | 24 | 0 |
| 0 | 0 | 11 | 23 | 0 | 1 | 0 | 11 | 0 | 18 | 0 | 1 | 11 | 23 | 0 |
| 0 | 0 | 12 | 22 | 0 | 1 | 0 | 12 | 0 | 17 | 0 | 1 | 12 | 22 | 0 |
| 0 | 0 | 13 | 21 | 0 | 1 | 0 | 13 | 0 | 0 | 0 | 1 | 13 | 21 | 0 |
| 0 | 0 | 14 | 20 | 0 | 1 | 0 | 14 | 0 | 0 | 0 | 1 | 14 | 20 | 0 |
| 0 | 0 | 15 | 0 | 0 | 1 | 0 | 15 | 0 | 0 | 0 | 1 | 15 | 19 | 0 |
| 0 | 0 | 16 | 0 | 0 | 1 | 0 | 16 | 0 | 0 | 0 | 1 | 16 | 0 | 0 |
| 0 | 0 | 17 | 0 | 0 | 1 | 0 | 17 | 0 | 0 | 0 | 1 | 17 | 0 | 0 |
| 0 | 0 | 18 | 0 | 0 | 1 | 0 | 18 | 0 | 0 | 0 | 1 | 18 | 0 | 0 |
| 0 | 0 | 19 | 0 | 0 | 1 | 0 | 19 | 0 | 0 | 0 | 1 | 19 | 0 | 0 |
| 0 | 0 | 20 | 0 | 0 | 1 | 0 | 20 | 0 | 0 | 0 | 1 | 20 | 0 | 0 |
| 0 | 0 | 21 | 0 | 0 | 1 | 0 | 21 | 0 | 0 | 0 | 1 | 21 | 0 | 0 |

| Policy Characterization | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Case 3 Policy | | | | | Order Only From Sup. 2 | | Order Only From Sup. 1 | |
| $s_1$ | $S_1$ | $s_2$ | $S_2$ | $s_j$ | $s_2$ | $S_2$ | $s_1$ | $S_1$ |
| 15 | 34 | 7 | 29 | 4 | 13 | 29 | 16 | 34 |

*All other parameters are fixed to their base-case scenario values.

### 3.2.5. Experiments for Effects of the Fixed Cost Structure on the Optimal Policies

The preliminary analysis made in Section 3.2.4 provides insight into the potential optimal policy structures for the model with two unreliable suppliers. Results show that the optimal policy structure can be one of the three cases. In order to study better the behavior of the optimal policy as the administrative and transportation costs for the two suppliers changes, we perform additional experiments which are explained in the following sections.

#### 3.2.5.1. The Effects of Changing the Administrative Cost for the Less Reliable Supplier

When both suppliers are in the up state at the time of ordering, as the administrative cost for the less reliable supplier (i.e. $f_{1a}$) increases (see Table 3.12), the optimal policy structure moves from case 1 policy to case 3 and then to case 2 policies; i.e. as "the cost of not receiving an order" increases, the range of inventory levels that the retailer uses both suppliers simultaneously gets smaller ($s_j$ decreases). If $f_{1a}$ is very small, when the current inventory level justifies the use of single supplier, the use of the less reliable supplier (supplier 1) is optimal (case 1); as $f_{1a}$ increases, the optimal policy takes the form of case 3 policy where the retailer prefers to order from the more reliable supplier when the inventory level is lower (i.e. supplier 2) and then from the supplier 1 when the inventory level is higher. When $f_{1a}$ gets too expensive, the retailer leaves the supplier 1 to avoid the cost of a failed order, and orders only from the more reliable supplier (supplier 2) more frequently and in larger amounts.

When supplier 1 is in the down state at the time of ordering, as the administrative cost for the supplier 1 increases ($f_{1a}$), the reorder point ($s_2$) and the order-up-to level ($S_2$) for the supplier 2 also increase, i.e. the retailer orders more frequently and in larger amounts from supplier 2. The reason is that even though the supplier 1 will return to the up state at the end of the period (remember that $\beta_1 = 1$), the retailer will prefer to use less the supplier 1 when he is up since it is getting more expensive to order from him as

$f_{1a}$ increases, which encourages the retailer to order more from the supplier 2. When the supplier 2 is in the down state, the retailer tends to use supplier 1 less frequently but in larger amounts (note that $s_1$ decreases and $S_1$ increases) since he will be able to place orders on supplier 2 with lower fixed costs in the next period.

Table 3.12 Optimal policy characterizations and policy control parameters as the administrative cost for supplier 1($\boldsymbol{f_{1a}}$)increases

| | Both Suppliers Up | | | | | | Sup. 1 Down | | Sup. 2 Down | |
|---|---|---|---|---|---|---|---|---|---|---|
| $f_{1a}$ | Case | $s_1$ | $S_1$ | $s_2$ | $S_2$ | $s_j$ | $s_2$ | $S_2$ | $s_1$ | $S_1$ |
| 0 | 1 | 19 | 31 | * | * | 7 | 12 | 26 | 20 | 31 |
| 1 | 1 | 17 | 32 | * | * | 7 | 13 | 27 | 17 | 32 |
| 2 | 3 | 16 | 33 | 7 | 28 | 6 | 13 | 28 | 16 | 33 |
| 3 | 3 | 14 | 34 | 9 | 29 | 3 | 14 | 29 | 15 | 34 |
| 4 | 2 | * | * | 13 | 30 | 1 | 14 | 30 | 13 | 35 |
| 5 | 2 | * | * | 13 | 30 | -2 | 14 | 30 | 12 | 35 |
| 6 | 2 | * | * | 14 | 31 | -4 | 14 | 31 | 11 | 36 |
| 7 | 2 | * | * | 14 | 31 | -7 | 14 | 31 | 10 | 36 |
| 8 | 2 | * | * | 14 | 31 | -9 | 15 | 31 | 9 | 36 |
| 9 | 2 | * | * | 14 | 31 | -12 | 15 | 31 | 8 | 36 |
| 10 | 2 | * | * | 14 | 32 | -15 | 15 | 32 | 8 | 37 |

#### 3.2.5.2. The Effects of Changing the Transportation Cost for the Less Reliable Supplier

When both suppliers are in the up state at the time of ordering; as the transportation cost for supplier 1 ($f_{1t}$) increases (see Table 3.13), we can see almost the same pattern of change in optimal policy case as the one observed when the administrative cost increases for the same supplier, i.e. as the transportation cost increases, the optimal policy structure switches from case 1 to case 3 and then to case 2. Similarly, when supplier 1 is in the down state the retailer orders more frequently and in larger amounts from supplier 2, and when the supplier 2 is in the down state the retailer orders less frequently but in larger amounts from the supplier 1 in order to take advantage of the lower fixed ordering costs provided by the supplier 2 in the next period.

Table 3.13 Optimal policy characterizations and policy control parameters as the transportation cost for supplier 1 ($\boldsymbol{f_{1t}}$)increases

| | Both Suppliers Up | | | | | | Sup. 1 Down | | Sup. 2 Down | |
|---|---|---|---|---|---|---|---|---|---|---|
| $f_{1t}$ | Case | $s_1$ | $S_1$ | $s_2$ | $S_2$ | $s_j$ | $s_2$ | $S_2$ | $s_1$ | $S_1$ |
| 0 | 1 | 17 | 32 | * | * | 6 | 13 | 28 | 17 | 32 |
| 1 | 3 | 16 | 33 | 7 | 28 | 6 | 13 | 28 | 16 | 33 |
| 2 | 3 | 15 | 34 | 7 | 29 | 5 | 13 | 29 | 16 | 34 |
| 3 | 3 | 15 | 34 | 8 | 29 | 4 | 14 | 29 | 15 | 34 |
| 4 | 3 | 14 | 35 | 10 | 30 | 2 | 14 | 30 | 14 | 35 |
| 5 | 3 | 13 | 35 | 12 | 30 | 1 | 14 | 30 | 14 | 35 |
| 6 | 2 | * | * | 13 | 30 | 0 | 14 | 30 | 13 | 35 |
| 7 | 2 | * | * | 13 | 30 | -1 | 14 | 30 | 12 | 35 |
| 8 | 2 | * | * | 13 | 30 | -2 | 14 | 31 | 12 | 36 |
| 9 | 2 | * | * | 13 | 31 | -4 | 14 | 31 | 11 | 36 |
| 10 | 2 | * | * | 14 | 31 | -5 | 14 | 31 | 11 | 36 |

Table 3.14Optimal policy characterizations and policy control parameters as the administrative cost for supplier 2 ($\boldsymbol{f_{2a}}$)increases

| | Both Suppliers Up | | | | | | Sup. 1 Down | | Sup. 2 Down | |
|---|---|---|---|---|---|---|---|---|---|---|
| $f_{2a}$ | Case | $s_1$ | $S_1$ | $s_2$ | $S_2$ | $s_j$ | $s_2$ | $S_2$ | $s_1$ | $S_1$ |
| 0 | 2 | * | * | 15 | 26 | 5 | 16 | 26 | 13 | 31 |
| 1 | 3 | 14 | 33 | 13 | 27 | 5 | 15 | 27 | 14 | 33 |
| 2 | 3 | 15 | 33 | 9 | 28 | 4 | 14 | 29 | 15 | 33 |
| 3 | 3 | 15 | 34 | 7 | 29 | 4 | 13 | 29 | 16 | 34 |
| 4 | 3 | 16 | 35 | 5 | 30 | 4 | 12 | 30 | 16 | 35 |
| 5 | 1 | 16 | 36 | * | * | 4 | 12 | 30 | 16 | 36 |
| 6 | 1 | 17 | 36 | * | * | 3 | 11 | 33 | 17 | 36 |
| 7 | 1 | 17 | 37 | * | * | 3 | 11 | 31 | 17 | 37 |
| 8 | 1 | 18 | 37 | * | * | 2 | 10 | 32 | 18 | 37 |
| 9 | 1 | 18 | 37 | * | * | 1 | 10 | 32 | 18 | 37 |
| 10 | 1 | 18 | 37 | * | * | 1 | 10 | 32 | 18 | 37 |

Table 3.15 Optimal policy characterizations and policy control parameters as the transportation cost for supplier 2 ($\boldsymbol{f_{2t}}$)increases

| | Both Suppliers Up | | | | | | Sup. 1 Down | | Sup. 2 Down | |
|---|---|---|---|---|---|---|---|---|---|---|
| $f_{2t}$ | Case | $s_1$ | $S_1$ | $s_2$ | $S_2$ | $s_j$ | $s_2$ | $S_2$ | $s_1$ | $S_1$ |
| 0 | 2 | * | * | 14 | 27 | 5 | 15 | 27 | 14 | 32 |
| 1 | 3 | 14 | 33 | 11 | 28 | 5 | 14 | 28 | 15 | 33 |
| 2 | 3 | 15 | 34 | 9 | 29 | 4 | 14 | 29 | 15 | 34 |
| 3 | 3 | 15 | 34 | 7 | 29 | 4 | 13 | 29 | 16 | 34 |
| 4 | 3 | 15 | 35 | 6 | 30 | 4 | 13 | 30 | 16 | 35 |
| 5 | 3 | 16 | 35 | 5 | 30 | 4 | 12 | 30 | 16 | 35 |
| 6 | 1 | 16 | 36 | * | * | 4 | 12 | 30 | 17 | 36 |
| 7 | 1 | 17 | 36 | * | * | 3 | 11 | 31 | 17 | 36 |
| 8 | 1 | 17 | 36 | * | * | 3 | 11 | 31 | 17 | 36 |
| 9 | 1 | 17 | 37 | * | * | 2 | 11 | 32 | 18 | 37 |
| 10 | 1 | 18 | 37 | * | * | 2 | 10 | 32 | 18 | 37 |

### 3.2.5.3. The Effects of Changing the Administrative and Transportation Costs for the More Reliable Supplier

When both suppliers are in the up state at the time of ordering; increases in the administrative and transportation costs for the more reliable supplier (i.e. $f_{2a}$ and $f_{2t}$), move the optimal policy structure form case 2 policy to case 3 and then to case 1 policies (see Table 3.14 and Table 3.15). That is as the administrative or transportation costs increase, the reorder point below which the retailer uses both suppliers simultaneously ($s_j$) decreases and the reorder point for supplier 2 ($s_2$)decreases.The increases in both types of the fixed ordering costs make also the range of inventory levelsat which the retailer uses only the supplier 2 smaller (i.e. $s_2 - s_j$) and then eliminate using only the supplier 2 in favour of taking advantages of the lower unit purchasing and fixed ordering costs offered by the supplier 1. When the supplier 1 is in the down state, the retailer orders less frequently but in larger amounts from supplier 2. When the supplier 2 is in the down state, as the fixed ordering cost for supplier 2 gets higher, the retailer orders more frequently and in larger amounts from the supplier 1 in order to take advantage of the both lower unit purchasing and fixed ordering costs offered by the supplier 1.

# 4. CONCLUSION

In this thesis, we analyze an inventory policy characterization problem for a retailer who operates a single-product, periodic-review and infinite-horizon inventory system to cope with potential supply disruptions. We consider two types of fixed ordering costs. First, an *administrative cost* per order is incurred which represents a fixed cost to initiate an order, and it is incurred whether the order is delivered or not. The second part of the fixed ordering cost is the *transportation cost* which is incurred only if a delivery is made. The aim of this study is to investigate how the new fixed structure affects the optimal policy structures.

Two cases are considered regarding the problem. In both cases, the problem is modeled as infinite-horizon discrete-time Markov decision processes in order to find the optimal ordering decisions and solved using a variant of Howard's policy iteration method. In the first case, one of the two suppliers is completely reliable but offers a high unit purchasing cost while the other is unreliable but offers a less expensive unit purchasing cost. State transitions of the model are defined by two Markov processes, the status of the unreliable supplier and the inventory position at the beginning of a period, both of which is independent of each other. Through numerical experiments for the first model, we observed that the presence of basic and secondary setup costs (i.e. administrative and transportation fixed costs) did not yield a different optimal policy structure than the four structures that are observed by Ahiska et al. (2013). However, it is observed that increasing only the administrative cost while keeping all other parameters fixed to their nominal values leads the retailer to order less frequently from the unreliable supplier as the cost of not receiving an order increases. On the other hand, increasing only the transportation cost leads the retailer to order less frequently from the reliable supplier.When the unreliable supplier is in the down state,as either administrative or transportation cost increases individually, the retailer begins to place orders on the reliable supplier less frequently but in larger amounts in order to spread the increasing total fixed cost over more units.

Another observation from the experiments is related with the changes in proportion of the basic and secondary costs as the total fixed cost remain fixed. As the transportation cost increases (and thusthe administrative cost decreases),the retailer orders in the up state more frequently but in lower amounts from the unreliable supplier and less frequently and in lower amounts from the reliable supplier because not receiving an order gets relatively less expensive.

The last observations from the experiments for the model are related with different reliability levels under different combinations of the fixed ordering costs. When the reliability level of the unreliable supplier is equal to 0.1 (i.e. $\alpha = 0.1$ ),and both administrative and transportation costs are high, the optimal policy is to order only from the reliable supplier. When $\alpha = 0.5$, and as both types of fixed ordering costs increase, the optimal policy structure moves from placing orders on the two suppliers simultaneously to placing orders on a single supplier. Finally, when $\alpha$ is 0.75 and 0.95 (i.e. the unreliable supplier is fairly reliable), increases in both types of the fixed ordering costs do not affect the optimal policy structure, which says to order from only the unreliable supplier.

In the second model, we analyze an extension to the above-mentioned problem where both suppliers areunreliable and differ in terms of reliability levels and cost. Through numerical experiments, the structure of optimal ordering policies are found to be one of the following three cases: case 1: first, order from both suppliers simultaneously and then order only from the less reliable supplier depending on the inventory level; case 2: first, order from both suppliers simultaneously and then order only from the more reliable supplier depending on the inventory level; case 3:first, order from both suppliers simultaneously and then order from the more reliable supplier, and finally order from the less reliable supplier depending on the inventory level. Ordering from both suppliers simultaneously has a more complicated structure than the simple

$(s, S)$policy. However when it is optimal to order from a single supplier, orders are placed according to an$(s, S)$ policy.

As further work for the model with two unreliable suppliers, it is worth investigating whether an$(s, S)$-type policy can be used as a good approximation for the optimal policy for the case where it is optimal to order from both suppliers simultaneously. Furthermore, modeling the problem with both suppliers having random yields or random delivery times and deriving the optimal policy structures can be an extension for the problem.

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## APPENDICES

### Appendix A. The solution algorithm for the infinite horizon MDP model[1]

Howard (1960) developed a policy iteration method for solving infinite horizon MDP problems. It is composed of two phases: value determination phase, where the relative values and gain is calculated for a fixed policy; and policy improvement phase, where a better policy is found using the relative values found in the first phase. These two phases are done iteratively until two consecutive policies found are identical. In the traditional Howard's method, in the value determination phase, it is required to solve an $N$x$N$ set of simultaneous linear equations in order to determine the relative values, where $N$ is the number of states in the MDP. For large scale problems (i.e. large $N$ and/or large number of alternatives per state), solving this set of equations becomes computationally inefficient. Morton (1971) proposes the computation of the relative values using fixed policy successive approximation, which eliminates the need for solving linear equations and provides computational efficiency.

The variant of Howard's policy iteration algorithm with fixed policy successive approximation is given below.

$N$ : the number of states
$q_S^k$:expected period cost for state $S$ when following alternative $k$.
$P_{SS'}^k$ : the one-step transition probability from state $S$ to state $S'$ when following alternative $k$.

[1]The algorithm is taken from Ahiska et al. (2013).

*Initialization:* Any feasible policy can be chosen as the initial policy.

- For simplicity, set the initial policy by selecting the initial alternative$k^S$ for each state $S$that minimizes the expected period cost$q_S^{k^S}$, i.e. $k^S$=$\arg\min_k\{q_S^k\}$.
  Policy $K$ ← Initial policy
- Set $v_S = 0$ for every state $S$
- Go to Phase I.

*Phase I. Value determination:*For the given policy $K$, for a predetermined number of iterations, called as *cheap iterations*[2] in Morton (1971), do the following:

For every cheap iteration,

- For every state $S$, calculate $v_S' = q_S^k + \sum_{S'} P_{SS'}^k v_{S'}$.
- If at last iteration calculate $\sum_S |v_S' - v_N' - v_S|$ (stopping criterion 1).
- Set $v_S \leftarrow v_S' \leftarrow v_N'$.

*Phase I. Policy improvement:*

- Given the relative values $\boldsymbol{v} = (v_S)$ from Phase I for every state $S$, find the alternative $k_S^*$ that minimizes

$$q_S^k = \sum_{S'} P_{SS'}^k v_{S'}$$

  The new policy is $K' = (k_S^*)$.
  Calculate $v_S' = q_S^{k_S^*} + \sum_{S'} P_{SS'}^{k_S^*} v_{S'}$
  Set $v_S \leftarrow v_S' \leftarrow v_N'$

- Check if the old policy $K$ and the new policy $K'$ are identical (stopping criterion 2).
- Set $K \leftarrow K'$.
- If either stopping criterion is not satisfied, go to Phase I.
- Otherwise stop, the optimal policy is policy $K$ and the optimal cost is $v_N'$.

2 The number of cheap iterations is set to 10 for the experimentation done in this thesis.

*Stopping criteria:* the following two conditions must occur to stop the algorithm.

1. The relative values found in the value determination phase converge, with $\sum_S |v'_S - v'_N - v_S| < \varepsilon$.

2. The two consecutive policies found in the policy improvement phase are identical.[3]

[3] In practice for very large state spaces stopping criteria 2 may not be satisfied due to computer roundoff, although this did not occur in this thesis.

## BIOGRAPHICAL SKETCH

**Faruk Akın** was born in Bursa, Turkey. He attended Heybeliada Naval High School between the years 2000 and 2004. He started his undergraduate education in Industrial Engineering Department of Turkish Naval Academy and received his Bachelor of Science degree from there in 2008. He has been serving as an officer in Turkish Navy since 2008. He was accepted to MSc Program in Industrial Engineering at Galatasaray Universityin 2012. He wrote the following paper based on his MSc thesis work:

Faruk Akin, S.SebnemAhiska, Russell E. King, Inventory Policies for an Unreliable Supply Chain under Fixed Ordering Costs,the 44$^{th}$ International Conference on Computers & Industrial Engineering, 2014.