A JOINT ASSORTMENT PLANNING AND SHELF SPACE ALLOCATION OPTIMIZATION FOR A SUPERMARKET CHAIN IN TURKEY

(TÜRKİYE'DEKİ BİR SÜPERMARKET ZİNCİRİ İÇİN ORTAK BİR ÇEŞİT PLANLAMA VE RAF PAYI DAĞITMA OPTİMİZASYONU)

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ABSTRACT

A retailer's assortment is the set of products carried at each store at each point in time. Given the fixed store space and limited financial resources, assortment planning requires a tradeoff between how many different categories to carry, how many stock keeping units (SKUs) to carry in each category and how much inventory does the retailer stock of each SKUs. Another problem a retailer has to face is the shelf space allocation, where the retailer decides on facings and replenishments based on space elasticity effects, limited shelf space, and operational restocking constraints.

This study models a joint assortment planning and shelf space allocation problem. The assortment planning sub-problem consists of introducing special products into the assortment. The tradeoff between these two sub-problems, namely assortment planning and shelf space allocation, can be explained as follows. If these special products are included, they are to be placed on the shelves that were previously occupied by the standard products. The reduction in the shelf space occupied by the standard products can result in the decrease of the demands for these products. On the other hand, the introduction of the special products can increase the demand for the standard products through cross-selling. Hence, the value of a special product is not just assessed according to the direct sales of the product itself, but also its value depends on the cross-selling effect that it creates to the standard products.

The value of any product as well as its cross-selling effect is estimated using the real sales data of one year of the biggest supermarket chain of Turkey. We have one year of shopping basket details according to the fidelity card of 600 customers of a specific supermarket. We use multiple linear regression, more specifically the ordinary least squares estimation techniques, to estimate the value of each category and the cross-selling effect between two categories.

The problem is formulated through an integer nonlinear programming problem. The nonlinear objective function maximizes the total revenue of products, where the linear part consists of the revenue per meter of the individual products, whereas the nonlinear part considers the cross-selling effects between the pairs of the products. As constraints, we consider limited shelf space and the groups of categories that have to be allocated at adjacent shelves. We also consider box constraints, namely upper and lower bounds on the shelf space that can be allocated to each category. The integer decisions allocate shelf space to each category and include or not the special products into the assortment.

We derive a greedy heuristic procedure that can be used easily by practitioners. To compare the performances of our heuristic with one of the most used nonlinear integer programming solvers, namely Dicopt, we solved 100 medium-scale test problems by GAMS 23.7 (using Dicopt) and our heuristic. Our heuristic solved all of the test problems while GAMS solved 78 out of 100 test problems within 17 minutes time limit; i.e., if GAMS did not provide a solution within 17 minutes, we stopped the optimization procedure. For each problem, our heuristic outperformed GAMS in terms of the optimal objective value as well as the execution time. The optimal objective value found by our heuristic is, on average, 14% higher than the optimal objective value found by GAMS. The average execution time of our heuristic is less than one quarter of the average execution time of GAMS.

We contribute to the literature by considering assortment planning and shelf space allocation jointly. We estimate the cross-selling effects through the linear regression techniques. The cross-selling is a well-known concept in the marketing literature, but to the best of our knowledge, how to estimate cross-selling effect has not been mentioned in that literature. We also derive a simple heuristic to solve our problem that can be applied by users who are not familiar with the optimization methods.

ÖZET

Bir perakendecinin ürün çeşidi her mağazasında her zaman aralığında taşıdığı ürün miktarıdır. Kısıtlı finansal kaynaklar ve sınırlı mağaza alanı düşünüldüğünde, çeşit planlama beraberinde bir takım sorular getirir: Perakendeci kaç farklı kategoriden ürün bulunduracak, her bir kategoride kaç farklı ürün bulunduracak, bu ürünlerden ne kadar stoklayacak, vs. Perakendecinin yüzleştiği bir diğer problem ise ürün önyüzü belirleme ve boşalan rafı tekrar doldurma kararlarını verdiği raf payı dağıtımı problemidir.

Bu çalışma ortak bir çeşit planlama ve raf payı dağıtma problemini modeller. Çeşit planlama alt problemi özel ürünlerin pörtföye alınıp alınmama kararını verir. Bu iki problem arasındaki ilişki şu şekilde açıklanabilir. Eğer özel ürünler portföye alınırsa, standart ürünlerin yer aldığı raf alanından pay alacaklar. Standart ürünlerin raf alanındaki azalma, onlara ait talebi düşürebileceği gibi, özel ürünlerin portföye alınması sonucu oluşan çapraz satış etkisi sonucu standart ürünlerin satışını artırabilir. Böylece özel bir ürünün değeri sadece kendi satışları ile değil aynı zamanda standart ürünler için yarattığı çapraz satış etkisi ile ölçülmektedir. Bir ürünün kendi değeri ve başka ürünlerle olan çapraz satış etkisi, Türkiye'nin en büyük perakendecisinden alınan 600 müşterinin bir yıllık sepet bilgileri doğrultusunda çoklu regresyon teknikleri kullanılarak tahmin edilmiştir.

Problem doğrusal olmayan tamsayılı programlama ile modellendi. Doğrusal olmayan amaç fonksiyonu ürünlerin kendi değerlerini ve çapraz satış etkilerini beraber göz önüne alarak toplam getiriyi en büyüklemektir. Kısıt olarak kısıtlı raf alanı, ürünlerin beraber atanması, ürünlerin alt ve üst önyüz sınırları dikkate alındı. Tam sayılı karar değişkenleri standart ürünlerin raf payı miktarını belirlerken, özel ürünlerin portföye alınıp alınmayacağı kararını verir.

Problemin çözümü için açgözlü sezgisel bir yöntem geliştirildi. 100 orta ölçekli test örneği hem sezgisel yöntem ile hem de GAMS 23.7 (Dicopt çözücüsü kullanılarak) ile çözüldü. Sezgisel yöntem tüm test problemlerini çözerken, GAMS yalnızca 78 tanesini 17 dakikalık zaman limitinde çözebildi. Her bir problem için sezgisel yöntem GAMS e göre amaç fonksiyonu değeri ve çözüm süresi kriterleri için daha iyi sonuç verdi. Sezgisel yöntem GAMS'e göre amaç fonksiyonu değeri için ortalama %14 yüksek değer bulurken, çözüm süresi göz önüne alındığında GAMS'in çözüm süresinin ortalama dörtte birinden daha kısa sürede sonuç verdi. Bu çaışmanın literatüre katkısı, çeşit belirleme ve raf payı dağıtımı problemlerini beraber göz önüne almak, çapraz satış etkilerini tahminleme yöntemi ve oluşturulan sezgisel yöntem olarak özetlenebilir.

1. INTRODUCTION

Assortment means the variety of products that a store carries. Since there exist lots of products in a retail store, accurate and effective assortment planning is one of the most important problems for retailers. Shelf space is the area which is reserved for display of products and is a scarce resource for retailers. Shelf space allocation is the distribution of appropriate amount of shelf space among different products, categories, brands. The allocation of shelf space is also called a planogram, which is used for visual representations of a store's products or services. Planograms are to show exact location and number of facings of each item on the shelves. Due to the limited shelf space, assortment and shelf space decisions jointly affect the total profit and/or customer satisfaction for retailers.

A retailer’s assortment is the set of products carried at each store at each point in time. Given the fixed store space and limited financial resources, assortment planning requires a tradeoff between the following factors:

- How many different categories does the retailer carry?
- How many stock keeping units (SKUs) does the retailer carry in each category?
- How much inventory does the retailer stock of each SKUs?

Another problem a retailer has to face is the shelf space allocation. Shelf space is the area which is reserved for display and stocking of products. The retailer decides on facings and replenishments based on space elasticity effects, limited shelf space, and operational restocking constraints. Due to the limited shelf space, assortment and shelf space decisions jointly affect the total profit and/or customer satisfaction for retailers.

1.1. Problem Description

This study defines a joint shelf space allocation and assortment planning problem. The problem is divided into two sub-problems. The assortment planning sub-problem consists of introducing special products into the assortment. The addition of these special products can cause a decrease in the sales of standard products because of the reduction of space for these products; on the other hand it can increase the sales of standard products through cross-selling opportunities. The trade-off between these two sub-problems, namely assortment planning and shelf space allocation, can be explained as follows. If these special products are included, since the shelf space capacity is limited they are to be placed on the shelves that were previously occupied by the standard products. The reduction in the shelf space occupied by the standard products can result in the decrease of the demands for these products. On the other hand, the introduction of the special products contributes the revenue through its own sales and also the increase in the demand for the standard products through cross-selling. Hence, the value of a special product is not just assessed according to the direct sales of the product itself, but also its value depends on the cross-selling effect that it creates to the standard products. Furthermore, some products have to be allocated adjacently due to customer habits and/or preferences while some products cannot be allocated adjacently because of regulations. In this problem adjacency requirements of products are taken into account.

In our problem, the categories are taken into account as products. Since the categories differ from each other, substitution is not considered. We have two groups of categories, each having four standard products such as pasta, spice, soup, and flour. As special products, we have diabetic category. Each group of products has to be assigned adjacently to an aisle because of customer habits and/or preferences. Special products can be assigned with both groups of categories.

Briefly we consider the problem to decide which products to offer and how much shelf space to allocate to that product while satisfying the adjacency, upper and lower bounds of products and capacity constraints. The objective is to develop a model that maximizes the total revenue of products consisting of revenue of the individual products and the cross-selling effects between the pairs of products.

1.2. Research Objectives and Primary Contributions

This study aims to develop a model that decides on assortment planning and shelf space allocation for a retailer under adjacency, capacity and availability constraints in order to maximize total revenues. The contributions of this study can be stated as follows:

- We propose a nonlinear integer programming model to jointly optimize shelf space allocation and assortment.
- We derive a simple heuristic for practitioners, who have no knowledge of optimization.
- We estimate the cross-selling effects through linear regression techniques using real-life data.

1.3. Methodology of the Research

Customer Relationship Management (CRM) activities and customer basket information have become very popular these days. Shopping basket details of customers enable to estimate cross-selling effects of products. A specific store of a leading supermarket chain in Turkey is chosen. In order to estimate the per meter revenues of products as well as the cross-selling effects between products; one year of shopping basket details of customers of that store are examined. Therefore, we apply linear regression techniques to the shopping basket sales data of customers to estimate per meter revenue of products as well as the cross-selling effect between product pairs. We also perform t-test to find out which estimates are statistically significant.

Once these revenues and effects are determined, the model solves the assortment planning and shelf space allocation problem of the retail store. We solve the problem in GAMS 23.4 using the solver Dicopt. We provide numeric results and perform sensitivity analyses to illustrate the effects of cross-selling on our results. We also derive a simple greedy heuristic to solve the problem. The solutions of the proposed model and the heuristic are discussed.

The remainder of the study is organized as follows: Section 2 focuses on related literature survey about both shelf space allocation and assortment planning. In Section 3, we present the mathematical model as nonlinear integer programming problem. In the fourth section, the proposed greedy heuristic is explained. In section 5, the numerical results and sensitivity analysis are presented. Finally in Section 6, we give conclusions and a few issues that will be considered as future research areas.

2. LITERATURE SURVEY

2.1. Assortment Planning

Kök et al. (2008) briefly review literature about assortment planning.

Category captainship occurs when a retailer let a leading manufacturer make strategic decisions about category and assortment selection in order to increase sales in the category. The number of categories offered by retailers are augmented therefore the management of these categories become difficult for retailers that's why they need may category captainship. A retailer can benefit the category captainship if it believes that a manufacturer's capability is more than the capability of retailer itself. This also leads to a better understanding the need of consumers by the manufacturers. Kurtuluş et al. (2014) first consider a benchmark model that the retailer makes the strategic growth decisions on category and makes an assortment selection by itself. Then they consider a captainship model wherein the retailer let the captain to develop a marketing strategy to grow the sales of category. Here, the retailer challenges with setting a target to the captain because the retailer doesn't know the capability of captain. And the captain makes an assortment selection in return for a target sales level. In this model, they let the captain to exert category expending and share shifting. In the paper, they consider multiple manufacturers selling different products with different attractiveness to consumers in order to avoid the opportunistic behavior of the captain and competitive exclusion. By this assumption, they do not just focus on optimizing the breadth of category but also optimizing the appeal of category as well. Multinomial Logit (MNL) demand model is considered. Each manufacturer offers one product with a different attractiveness. Developing a marketing strategy which includes consumer education programs, advertisement campaigns and designing programs can affect the total category demand. The higher capability, the lower cost for stimulating the category demand is considered. Retailer's net profit margin from all products is assumed to be the same. So the prices for products are proportioned positively by the net profit margin of manufacturers. Retailer's operational costs (replenishment and inventory costs) are also considered in the model. Benchmark model first decides the total appeal of the category and then determines an assortment set is closest to the total appeal. The retailer's demand stimulating effort is increases in the margin and capability, decreases in the operational costs. In the captainship model, captain is more capable to stimulate demand than the retailer perceived capability of captain. The captain's profit is decreasing in the assortment appeal because the captain loses its market share in order to offer a higher appeal. This target sales given by the retailer is to make a pressure to the captain to include the products by other manufacturers. The captain determines the assortment sets that meet the target sales, and chooses the assortment set with the minimum appeal so that it can have more market share. The retailer offers a target sales level to a leading manufacturer, if the manufacturer accepts to become a captain, it selects an assortment set, if not retailer selects an assortment set as in benchmark model. Two types of target selection equilibrium are proposed. First, separating equilibrium in which the retailer sets an aggressive target sales level such that H type manufacturer accepts the offer while L type manufacturer rejects it. In the case of rejection, the retailer cannot take the superior advantage of L type manufacturer. By this equilibrium, the retailer doesn't let the captain to benefit from opportunistic behavior. Second, pooling equilibrium in which the retailer offers a conservative target such that both manufacturers accept to become a captain. In this case, L type of manufacturer cannot make an opportunist behavior while an H type of manufacturer can benefit from opportunistic behavior. The retailer sets a conservative target if the retailer is certain about captain's capability and the retailer sets an aggressive target if the retailer is uncertain about captain's capability. As an extension, the retailer asks the manufacturers to make a promise for the category sales, and selects the one with the highest value. They identify three factors that affect the breadth and the appeal of the assortment under category captainship: retailer's perceived capability of captain, the captain's true capability and the heterogeneity of product set. The first two factors play a role on the appeal of the assortment and the third factor plays a role on the breadth of the assortment. The paper characterizes the impact of these three factors on both the appeal and the breadth of the category under category captainship and also the decisions of the retailer while setting target sales level for the captain in terms of these factors. They identify conditions where category captainship can be beneficial for i) retailer, ii) captain in expense to the non-captain manufacturers and iii) both the captain and the non-captain manufacturers.

Talebian et al. (2014) have focused on incorporating active demand learning into dynamic joint assortment and price optimization. They considered product families rather than the products and the product families are assumed to be highly differentiated. Consequently, the demands for these product families are independent and no substitution between product families occurs. The randomness in the problem is caused by the uncertainty about the market size for a product family. The retailer is assumed to have an initial belief about this size, and as more sales data become available, the estimates for the market sizes are updated through Bayesian updates. Furthermore, the demands are influenced by the current prices, and they are determined through the known price-response functions. The authors consider a finite planning horizon (perishable products), and formulate the problem through a stochastic dynamic optimization problem, which maximizes revenues subject to a shelf space constraint; the decisions are products to be included in the assortment and prices. They analyze three policies, namely no-learning, passive learning, and active learning. For the no-learning policy, it is assumed that there is no uncertainty about the market size, so that prices and assortment are determined for once for the whole planning horizon. In the passive learning policy, the new sales information is used to update the estimates of the market sizes, and this policy results in fixed prices and dynamic assortment. The active learning policy, however, uses price markdowns to learn more about the market sizes so that it results in dynamic prices and assortment. They find that both passive and active learning policies are effective in increasing revenues and recapturing loss of revenue due to the lack of full information. Moreover, the value of learning increases with a longer sales horizon, less shelf space, and higher belief uncertainty.

In their recent work Katsifou et al. (2014) consider a joint assortment, price, and inventory level optimization problem. They consider product categories hence substitution is not considered. They consider heterogeneous customers, namely, loyal and non-loyal customers, and they consider standard and special products. The demands for these two types of products are modeled through MNL models separately for loyal and non-loyal customers, which capture customers' preferences and price sensitivity as well as the store choice process and the cross-selling effect. The loyal customers consider purchasing standard products, whereas non-loyal customers look for special products; there are, however, also opportunities for cross-selling. The standard products are carried for a longtime period by the retailer. Hence, a periodic review order-up-to inventory policy is used for such products. The special products are ordered once at the beginning of the period with no replenishment opportunities. Hence, they use a newsboy-like model. They maximize the nonlinear expected net profit subject to a shelf space constraint. The decisions are assortment decisions for standard and special products, inventory levels for these products, and their prices. The resulting problem is a nonlinear mixed integer programming problem. Their model is profitable when the cross-selling is possible. Also, they illustrate that the special products are primarily intended to increase store traffic, whereas standard products aim to increase profit.

Alptekinoğlu & Grasas (2014) study how a retailer makes an optimal assortment under return policy. Apart from the literature they consider both product assortment and returns into account. Nested MNL model is used for consumer choice demand model. In the first stage consumers make purchase and in the second stage they keep or return the product with random utilities. In the case of a return, the retailer refunds the product to the customer with a refund fraction. Products are horizontally differentiated. Products only differ by their attractiveness. All types of costs are identical. Products with high attractiveness are called as popular products and products with low attractiveness are called as eccentric products. Popular products have higher demand and lower return. Eccentric products have lower demand and higher return. In the model the eccentric products can be beneficial due to the refund fraction. They consider refund amount as a single aspect of return policy. They consider both make to order and make to stock as operational environments. Under make to order strategy, the retailer does not stock the products, the procurement of a product occurs after the realization of demand by the consumer. Under make to stock strategy, the retailer determines the quantity of products before the consumers make their purchase decisions. Pre purchase heterogeneity depending on advertising, packing and post purchase heterogeneity depending on experimental factors as aesthetic fit and feel are considered for demand. They assume that unsatisfied consumers return. They ignore that these consumers can resell their products. They show that if the retailer offers low refund for the returned products, the optimal assortment contains a mix of the most popular and most eccentric products. If the refund is high, the optimal assortment consists of most popular products.

Honhon & Seshadri (2013) model an assortment planning problem for one period considering consumer driven, dynamic, stock out based substitution and random proportions of each consumer's type. Each customer belongs to a consumer type which corresponds to a list of products he is willing to buy in decreasing order of preference. They use MNL model for demand estimation. Under fixed proportion of demand model, the distribution of the number of customers who visit the store in one period and the distribution of customer preferences in the population is assumed to be known. Only uncertainty is the demand in this model. Apart from the literature they propose random proportion of demand model in which proportions of customers of each type who visit the store is a random variable with a binomial distribution. The authors consider a product category of n products. The amount of inventory of each product is determined at the beginning of each period and the inventory left at the end of the period is salvaged. Inventory cannot be replenished during the period. Optimal expected profit under fixed proportions is always greater or equal to the optimal expected profit under random proportions. They find an upper bound to the random proportion model by solving fixed proportion model. They use the difference in sales of one product up to the first stock out epoch between these models and this provides a lower bound to the random proportions model.

Sauré & Zeevi (2013) propose a dynamic assortment planning model that trades off between information collection (exploration) and revenue maximization (exploiting). Pricing is not treated in the model. They assume that prices of products are fixed during selling season. Capacity constraint that limits the size of the assortment is considered. Each customer has a random utility for the products in the assortment and purchases the product with maximum utility. They ignore inventory replenishment, assortment sequencing and switching costs. They ignore stock out based substitution. They assume that there is no priori information about consumer preferences and the assortment planning is done according to observed sales. They propose assortment policies in order to learn demand. Objective is to minimize the expected revenue loss because of non-anticipating assortment policy. They show that exploration can be restricted at most order N assortments. The assortment that single sale profit maximizes is used as a lower bound on expected revenues generated by any admissible policy. A product's frequency of purchase provides information on the performance of the same products in other assortments. Mean utilities are calculated according to observed consumer purchase decisions at the beginning and then mean utilities are reconstructed by those probabilities. For an assortment, they call a product as potentially optimal if the mean utility of this product changes the mean utilities of products in the assortment and as strictly suboptimal if not. By this definition, they consider only worthy policies of assortment. An ideal policy should offer suboptimal assortments to at most order T log customers. These assortments include both potentially optimal and strictly suboptimal products. The products are only differentiated by their profit margin so any product with a margin less than a specific value becomes strictly suboptimal product. Assortment sets for exploration can be determined by adding at least one potentially optimal product. The idea behind is to limit the exploration phase of strictly suboptimal products.

The authors in (Rooderkerk et al., 2013) model the sales of the different stock keeping units; their approach is not based on stock keeping units but on attributes of the stock keeping units. This enables them to keep the model parsimonious. The sales model also considers substitution between SKUs, and cannibalization. Retail assortment selection problem is a constrained expected net profit maximization problem at the store level, where the expected net profit is obtained through the included SKUs to the assortment. The constraints are given by the shelf space constraint, the number of SKUs in an assortment has an upper bound because of the administrative and handling costs, from one period to the next the assortment change should not be drastic and should have an upper bound, inclusion of a very (in)expensive unit in an assortment can (decrease) increase the price level of that assortment, which is undesirable. There is also a constraint on the price increases and decreases. The decisions are whether to include or not SKU k in the assortment (0-1 decision variables). They optimize the assortment alone, and the assortment and the price jointly using very large neighborhood search heuristics. They contribute to the literature in the following ways: using an attribute-based approach to handle large sets of items, accounting for similarity effects, controlling for the marketing mix during estimation and optimization, accounting for assortment and price endogeneity, optimizing for each store separately, optimizing assortments and prices jointly.

McElreath & Mayorga (2012) propose a locational choice model that customer choice is differentiated both vertically and horizontally. Apart from the literature, they show that they find the optimal assortment for this problem. Horizontal differentiation refers variety attributes such as size or color while vertically differentiation refers quality attributes such as higher resolution for digital cameras all else being equal. In the model, multiple quality levels exist where higher quality products provide higher profits. The retailer pays a fixed cost for a product included in the assortment and a unit cost for each product sold. Number of items in the assortment is constrained. The range of the size of the assortment is constrained by the maximum and minimum locations on the attribute space for products to be placed in. The retailer must chooses a product line which is well matched with the customer preferences under cost constraints. The retailer operates make to order environment. The probability of an item being chosen depends not only it's location but also depends on the locations of other products in the assortment. They assume that customer preferences for horizontal attribute follow a unimodal distribution. There exist two vertical attributes namely low and high quality levels. Customer preferences are homogeneous for the quality of the products. Customers' utility depends on the price of the product and the distance between the quality of product's attribute and the customer preference. For each product, a first choice interval is determined. First choice interval consists of the locations of customers who will choose this product for as their first choice. They use a dynamic programming for the location of the first product in the assortment and a line search for determining the optimal assortment. The first product in the assortment is determined by its first choice interval and its profit. The products are added to the assortment according to the same logic and also depending on the products that are already chosen for the assortment.

Honhon & Pan (2012) model an assortment planning problem of a category with vertically differentiated products. Each product is characterized by a quality level. Quality level can be seen as a combination of multiple characteristics. Inventory levels are ignored therefore substitution is not considered. The objective is to maximize utility. Utility is a linear function is increasing in quality and customer valuation of quality and is decreasing in price. They assume that the retailer knows the customer valuation but does not know each customers quality valorization. A fixed cost incurs to include a product in the assortment and a variable cost incurs per product sold. Quality levels, fixed and variable costs of products are exogenously determined. They consider two different scenarios. In the first scenario, they assume that selling prices are fixed. Here the retailer only decides the set of products to be included in the assortment. It is assumed that manufacturers' suggested retail prices are taken into account for retailer. In this case optimal assortment is a function of customer valuation and is solved by shortest path problem. They show that a dominated product is never included in the assortment with the product that dominates it while a dominated product can be included in the assortment instead of the product that dominates it. In the second model, they determine the optimal assortment and the selling prices of the products in the assortment. They show that dominated products can be included in the assortment. This is due to pricing the dominated products high enough to increase the expected profit so that no customer buys them. They propose a supplementary case for this scenario. They assume that fixed costs are zero. In this case the optimal assortment does not depend on the distributions of customer valuation.

Kurtuluş & Nakkas (2011) propose a game theoretic model where there are multiple manufacturers selling their products through one retailer. The main motivation under category captainship is the belief that category captain has better information about the consumer preferences. Retailer needs category captainship because the captain provides consumer insights and help the retailers increase traffic into the category. On the other hand, the retailers face with competitive exclusion under category captainship.

Competitive exclusion is reducing the variety of category. Apart from the literature they assume that retail prices are fixed and the assortment planning is considered as a decision variable. They find that competitive exclusion happens when the category captain has private information about the consumers and ability to drive additional traffic into the category. They model the problem as there are multiple manufacturers. Each manufacturer sells one product. Retailer makes it's assortment in the first stage. The customer arrives at the store and decides to buy a product or not to buy anything. Products are identical. The selling prices of products are the same. The manufacturers offer the products to the retailer at the same price. So profit margin for each product is equal. They assume an operational cost for carrying a variety of products in the assortment. They use a MNL consumer choice model. Consumers make a purchase decision according to the attraction of a product. Attraction may be defined as function of advertising, price, and reputation of company, service given during and after purchase. They assume that the attraction of products are equal and either high or low with probabilities α and (1- α). They compare 2 models. In the first model, retailer selects the assortment by itself and in the second one the retailer delegates the assortment selection to a captain in return for a target profit. They focus on the size of the assortment and how the size changes by the category captains. The retailer sets a target profit to the category captain. If the category captain accepts the challenge, assortment decision is made by the captain. If not, the retailer makes the assortment decisions by itself. Moreover they consider a model where target sales instead of target profit set by the retailer to the category captain; they show that competitive exclusion doesn't occur. In this case, the retailer doesn't need to share its sensitive information with the captain.

Kök & Xu (2011) study assortment planning and pricing for a product category with heterogeneous products from two brands. They use Nested MNL model because of heterogeneous product groups. They model two hierarchical choice processes. First, brand primary model in which customers first choose which brand to buy and then a product type within that brand. In type primary model, customers choose a product type first and then one of the brand of that particular product type. They consider two management regimes. In centralized management, category manager makes the assortment and pricing decisions for maximizing the total category profit. In decentralized management, two independent brand managers makes assortment and pricing decisions in order to maximize their own brand profit. In the model, they consider one product category with two brands offering one product per product type. They assume that products are horizontally differentiated with homogenous quality therefore the procurement cost is equal for each product. In addition to procurement cost, there is an operational cost which reflects the optimized inventory cost. Inclusion of a product to the assortment has two sides. It generates more sales and makes the assortment more attractive. On the other hand, it cannibalizes the demand for existing products, reducing their revenue and lowering their operational efficiency. Optimal assortment has to trade off the two sides of product inclusion. The brand primary model is more suitable for the categories within horizontally differentiated products and strong brand loyalty. The type primary model suits with the categories where there are vertically differentiated products. The brand primary model: Interbrand heterogeneity is higher than intrabrand heterogeneity. Centralized management: The markup across all products that are offered by both brands is equal. The competition is between the whole category (between brands and product types). Popular product types with higher expected demand will be priced niche product types with low expected demand. Decentralized management: the markups are identical for product types within a brand. The competition is between brands because after the consumer makes his brand choice, the product types within the brand are owned by the same brand manager. Popular product types with higher expected demand will be priced niche product types with low expected demand. A brand with a higher market share can afford charging higher prices. Optimal assortment set is always in the popular assortment set of brands. Both brands' assortment includes the most popular products types in the brands. The type primary model: Inter product type heterogeneity is higher than intra product type heterogeneity. Centralized management: The competition is between the whole category (between brands and product types). The markup is identical for all product types of both brands. Optimal assortment contains a set of most popular product types offered by both brands. A more popular product type should be offered by more brands. Decentralized management: the markup is different across products and brands. A constant higher margin for monopoly products and constant lower margin for the product types in direct competition with the other brands. The optimal assortment set of a brand contains the most popular products that the other brand does not offer. Both brands assortment together offers the most popular products. Optimal assortment set is in the popular assortment set that the product types are offered at least by one brand. The breadth of the optimal assortment is in the popular assortment set. The depth (number of brands) of the optimal assortment is increasing in the popularity of the product type. The optimal pricing structure in centralized management is the same for both hierarchical models because the competition is within the category as whole. Price competition is more detrimental to profitability than is the assortment competition.

Rodríguez & Aydın (2011) consider a configurable product with a required component and an optional component. Each component has certain variants. Customers purchase a component according to their utility for each component. But if they purchase both components, the attraction of new configurable product has to be considered. An optional component can be purchased if only a required component has chosen as well. The purchase decision depends on both the prices and the assortments of required and optional components. One component's assortment influences another assortment's demand. The demands for these components are complementary. If the retailer chooses to decrease the variety of a component, the demand for other components can decrease. The customer may not find its ideal component because of lower variety and may choose to search for the ideal configurable product elsewhere. The price of the configurable product is the sum of each component's price. They define effective profit margin as a function of selling price, unit cost, underage (stock out) and overage (holding) cost, service level and demand variability. Higher quality variants (or out of ordinary colors) have higher gross margins. They use MNL demand model. Apart from the literature, they study a joint pricing and assortment selection problem. They also propose an assortment with two different components with complementary demands. They model the problem as selection of assortment for components, their prices and stock levels for one period. The fixed cost of carrying a variant is equal for all variants of the given component. Service levels of all variants are equal. According to MNL model, components will be priced by having the same profit margin. Optimal margin of optional component will be zero as well. They also propose a model in which the customers purchase the components sequentially. The customers first decide to buy a required component and if they buy any, they then decide to buy an optional component.

The paper, (Honhon et al, 2010) consider a joint assortment and inventory planning problem. They consider n substitutable products, a single cycle newsvendor profit function, and dynamic substitution (stock out-based substitution). Customer types are modeled through a sequence of products in the order of decreasing preferences. The fixed proportions of each customer types are assumed to be known, and the demand is assumed to be a continuous random variable with a known density and a known mean. They reformulate the problem as a dynamic optimization problem where the value function measures the maximum expected profit that can be obtained from the remaining customers given the current assortment. They show that the value function is a convex non-increasing function of the demand and has a piecewise structure where each piece corresponds to a different assortment. They provide an algorithm to obtain the optimal assortment and inventory levels when demand is continuous and the proportions of customers of each type are constant. Furthermore, they show that the algorithm is a good heuristic procedure that performs better than the other methods when demand is discrete and the proportions of customers of each type are random. The authors recommend using their algorithm if any of the following conditions hold: the mean demand is large, the degree of substitutability is high, the population is homogeneous, or prices and/or costs vary across products.

Yücel et al. (2009) consider a joint product assortment planning with substitution and multi-product inventory management problem. The authors do not distinguish between the stock out-based substitution and the assortment-based substitution; however, different from the literature, they consider multi-level substitution. Furthermore, they consider a single inventory cycle. They assume that each product is supplied by a single supplier, so that product assortment planning decisions also correspond to the supplier selection decisions; however, a supplier can supply more than one product. The random demand for products is modeled through a joint discretely distributed random vector. The decisions are on the tactical level which products should be ordered from suppliers (also the supplier selection) and on the operational level the optimal ordering quantities to maximize the expected net profit. The costs include ordering cost per order, fixed cost of supplier selection, purchasing cost, inventory holding cost, cost incurred because of poor quality products, and customer substitution cost. The constraints are shelf space limitation and ordering quantity quotas. They build a mixed integer programming problem. They also examine the effects of substitution cost, supplier selection cost, shelf space limitation and demand variability on the decisions and on the net profit (performance measure).

The authors in (Mantrala et al., 2009) address the variety, depth and service level aspects of product assortment planning. Variety is number of categories, depth is number of SKUs within a category and service level is number of individual items of a particular SKU. Product Assortment Planning (PAP) requires three aspects: first consumer perception and preferences, second retailer supply constraints, and environmental factors. Consumer perception and preferences consist of a choice set of consumer (consumers' desire for flexibility, consumer preferences instability, global versus local utility, too much choice, actual versus perceived variety) and consumer search cost and substitution behavior. Purchase occasion is usually separate from the consumption occasion. Retailers need to offer flexibility in order to meet the changing goals, needs of consumers or social situations. Assortment flexibility also leads consumers to learn about the possible choices in the set, to experience them. Economists mostly assume that consumer preferences are stable but in real life consumer preferences changes over time and instable. Consumers may choice a product which responds to her global utility instead of local utility such as buying a memorial item. On the other hand large assortments affect negatively the customers. They may get frustrated or overwhelmed because of too much choice. Key factor is here to offer more attributes instead of more alternatives. Actual and perceived variety is not the same and depends on the assembling of the assortment (the organization of the assortment and relative symmetry in the frequencies of items. A consumer may be willing to go another store even if she finds an acceptable product. Retailers cannot offer a 100% service level. Out of stock situations help retailers to identify substitutable products for consumers. Physical space is the most important constraint for a retailer. Physical dimensions of an item affect the shelf space allocated to this item. The more shelf space allocated to an item, the more attracts consumers' attention. Average demand and variability of demand also affects the number of SKUs carried in the store. Retailers should make decisions about higher versus lower service levels for specific items. Finally delivery cycle and case pack size also affect the space allocated to the items. If case pack size is small and the delivery cycle is short, then retailers may decide to hold fewer inventories in the store. Market positioning, format choice and the ratio of private labels and national labels also affect the assortment combination in terms of quality, price levels and brands. There exist finally environmental aspects. The same product category is offered by different retail formats. The retailers decide which shopping trip to respond: stock up or quick trips. Economic and environmental conditions change and retailers must respond these issues such as ecologically friendly products or green depots. Lifestyle trends and consumer profiles are changing as well. For example feeding behavior and aging of population can affect the assortment. The location of a store requires customization of assortment specifically to that area. The typical PAP starts with long term planning steps which determines the breadth/variety of a store's assortment by defining categories and subcategories. Retailer then passes to short term step and determines the demand forecasts, sales-margin-turnover goals, space allocations and inventory investments. Then the decisions of variety-depth-service level must be done. An assortment planning should offer demand forecasts, both top down and bottom up, both for old and new items. The assortment planning should customize the retail assortment at the store level and develop an attribute based approach instead of product focused approach. Thanks to attribute based approach, new items demand can be predicted from the attributes existing and it enables to handle with the large sets of products.

2.2. Shelf Space Allocation

Hübner & Kuhn (2012) briefly review literature about shelf space allocation. Tsao et al. (2014) propose a model for category shelf space allocation under trade allowance of manufacturers. Manufacturers pay trade allowances for category promotions of the retailer. They assume that the retailer has a promotion budget. The retailer assigns the promotion budget and the shelf space to the categories under the manufacturer's trade allowance. They use Stackelberg game for modeling the negotiation between the retailer and manufacturers where retailer is the leader and manufacturers are followers. They extend the model in (Irion, Lu, Al-Khayyal, & Tsao, 2012) by proposing trade allowance, trade promotions and Stackelberg game for the negotiation. Apart from the model proposed by (Irion, Lu, Al-Khayyal, & Tsao, 2012), the authors create the demand by considering space elasticity, cross space elasticity, promotion elasticity and cross promotion elasticity. They use MNL model for demand and they add promotion and cross promotion elasticity to the demand function. They introduce a promotion budget constraint with a budget limit and a promotion control constraint with upper/lower bounds on the promotion level. The upper/lower bounds of the promotion level of products depend on the magnitude of manufacturers allowances. The magnitude of manufacturers allowances changes the promotional budget of the retailer. The retailer is the leader who suggests to each manufacturer a supply package. The supply package is a function of number of facings of a product and the promotion level of the particular product. Manufacturers who follow the retailer decide to pay a trade allowances on the supply package proposed by the retailer. The retailer finds equilibrium for the assortment decision, shelf space allocation and promotion level for each product by considering the manufacturers' trade allowance payment.

The paper (Gilland & Heese, 2013) determines how much shelf space to allocate to each product in order to satisfy the aggregate demand for a retailer. They consider only two substitutable products because of computational complexity added by sequence of customer arrivals. A customer decides to buy (if any) based on product availability at the time of arrival. The sequence of customer arrivals changes the quantity of products on the shelf in case of purchasing. A customer decides to buy (if any) based on product availability at the time of arrival. They consider two substitutable products. Dynamic stock out based substitution is taken account. The sequence of customer arrivals changes the quantity of products on the shelf in case of purchasing. They consider only two products because of computational complexity added by sequence of customer arrivals. They define price and cost for each product and also a cost for purchasing non preferred product (substitution cost) and a cost for not purchasing any product (stocked out cost). The available shelf space for the product category is exogenous. All inventory kept in the shelf space and total number of units that can fit into the shelf is fixed. Each product occupies the same amount of shelf space. Retailer uses all available shelf space. They assume that customer types are random factions of total demand. They consider two customer types. They model the problem in three possible solutions. First, n^{th} arriving customer finds the preferred product on the shelf and makes a purchase. Second, n^{th} arriving customer finds only the substitute product available. Third, n^{th} arriving customer finds neither product available on the shelf. They show that retailer should allocate more shelf space o the products with higher profitability, higher stock out cost, and higher substitution cost. They characterize the effect of dynamic stocked out based substitution on retailer's profitability.

Leng et al. (2013) assume there exist multiple retailers. They change an amount of shelf space to increase accessibility without opening new stores. They assume that two retailers do not sell the same products. The products are neither substitutable nor complementary. If not there will be a competition between them. Unlikely two retailers use this strategy in order to increase profit. When two retailers exchange shelf space, their customers may incur lower travel costs and two retailers may increase their prices (so their profits) without losing customers. Space exchange strategy is beneficial if the products are more accessible (after the space exchange, some customers would prefer to go to another retailer's store because it is closer to these customers). There is a tradeoff between lower travel cost and higher prices of products for the customers for this strategy's success. In the model, optimal prices of products are determined according to given host and guest space of retailers. They find the nash equilibrium pricing and space allocation decisions. They assume two retailers. Each retailer sells one product different from other retailer's product. They use hotelling model in which retailers 1 and 2 are located at two end points of a linear city. All consumers are uniformly distributed along the city. There is an availability constraint for products. Number of products that can be stocked at the store is limited. First, the optimum pricing and maximum profit is determined without a space exchange strategy. Second, two retailers decide to change shelf space. They first assume that two retailers have sufficiently large space to sell their products and there assumed to be no space (capacity) constraints. Then the problem is solved under space constraint. The retailers host space in his own store and his guess space in other retailer's store are given. They analyze if this strategy is profitable for both retailers or not.

Irion et al. (2012) propose a shelf space allocation optimization model. Shelf space as an integer number of facing is allocated to a product. Single product category is considered. Cross and space elasticities are taken into account. Products are restocked after the number of units on the shelves is zero. There exists no backroom space for inventory. Cost is assumed to be a function of replenishment cost, inventory cost and restocking cost. A fixed cost is also considered for the products to be introduced in the assortment. Total available shelf space is considered. The proposed model is nonlinear. They use a piece wise linearization technique to reformulate the problem as a mixed integer linear programming. The model is extended by introducing pricing as marketing variable. In this case, the problem deals with facing and pricing decisions. The problem with a warehouse space is also modeled in which the number of products that are not placed on the shelf because of the limited space is stocked in the warehouse. In this case total warehouse space has to be taken into account. In the main model, it is assumed that a product is restocked as soon as it stocks out. They extend the model by considering a specific restocking time for all products. In this case, substitution effects are considered by introducing substitution probability between products. They define bottom up approach as assigning shelf space to individual products. The amount of shelf space allocated to a product category is determined before the assigning shelf space to individual products. In the top down approach, the store shelf space is allocated to product categories. In this paper, bottom up approach is used for allocation decisions. These two hierarchical approaches are linked in the following paper.

Murray et al. (2012) propose a model for inventory management decisions in consideration of stochastic demand for each product. There exist multiple products. The demand for a product is a function of the price of that particular product and the prices of all other substitutable and complementary products. They take into account own and cross price effects. Objective is to maximize profit by setting optimal prices and ordering quantities for each product. Ordering quantities can only take integer values.

There are restrictions on selling prices which the retailer must set a price from the set of allowable selling prices because of competitive market. There also exist upper and lower bound on the procurement quantities for each product. A budget is taken into account to limit the amount spend on product procurement. Space limitations are considered to set a maximum volume of goods that can be stored. Apart from the procurement cost, there's also a holding cost of unsold products. The model solves the problem by determining optimal procurement quantities for each given prices in the set of allowable prices.

Yapıcıoğlu & Smith (2012) propose a model to allocate departments in a racetrack configuration. In racetrack layout departments are lined up around the outer rim and within the center space. A racetrack aisle separates the outer rim from the center space and provides one or more entry/exits. The store consists of departments, the racetrack aisle and entry/exit aisle. The racetrack is the main travel path for customers. The racetrack is also treated as a department with area allocation and revenue generation. They assume that there exists a single racetrack. The area of the aisle is assumed to generate revenue because it valorizes the display area and increases shopping pleasure. Department sizes and shapes are constrained. Adjacency requirements of departments are taken into account. The objective is revenue maximization and adjacency satisfaction among departments. The revenue of a department is defined as a function of its area and its exposure to the aisle network. The area of department depends on two effects namely area effect which is the area allocated to the department and store effect which is the location of the department within the store. The store area is divided into three zones namely high/medium/low traffic zones. Departments are classified in three groups as high/medium/low impulse purchase departments. The revenue of the department depends on the deviation of the department’s actual location from its ideal location. Two departments are considered adjacent if they share a common edge. Departments who are separated only by an aisle are considered as adjacent. A matrix is presented in order to identify the adjacency rankings among departments. A two stage approach is proposed to solve the problem. First, NLP assigns departments to the areas. Second a Tabu search determines the exact locations and territories of the departments.

Irion et al. (2011) consider both top down and bottom up approach. They assume that the output of the top down approach (the amount of shelf space allocated to a category) becomes a constraint for the bottom up approach (total shelf space for a given product category). They propose a model that links these two approaches to allocate shelf space. They divide the problem into two sub problems. In the first step, store model is used to allocate shelf space to product categories. In the second step, product category model is used for individual products within a category to be allocated to the given shelf space of the category. Cross elasticities among products within a product category is considered. The demand of a product depends on the total amount of shelf space allocated to the specific product category because of impulse buyers (the more they see the product category the more they buy the products within the category). Demand of a product is assumed to be independent of the amount of shelf space allocated to another product category. Product categories are considered independent and unlinked units. Different (profit, shelf space) pairs for each product category are determined for the product category model. Then the store model determines the optimal amount of shelf space assigned to each product category in order to maximize the sum of expected product category profits. Finally each product category model is solved with the given optimal shelf space of the category. The model finds hopefully near optimal solutions.

Russell & Urban (2010) propose a model to allocate product families to the shelves and to determine shelf location of each product family. Individual products are grouped as product families according to brand or label. Each item has a margin, a facing length and maximum and minimum number of facings. Minimum number of facings is positive numbers so all of the items are included in the assortment set. Since the costs are not considered each profitable item will be located to the shelf. Product family integrity is considered. Each individual item is stacked together on one shelf while a product family may span several shelves. If a product family spans more than one shelf space, the items in that product family has to maintain a rectangular physical presence e.g uniform column. In this case a product family also has to be located to the adjacent shelves. Each item is measured relative to the lower left corner of the shelf. They assume that sales tend to be quadratic in both horizontal and vertical dimension. Objective function is to maximize the profit and is represented in a quadratic formulation. They extended the model by considering assortment based substitution. They let minimum number of facings can take zero value where the items with zero facing are not taken into the assortment. The lost sales of an item that is not in the assortment is ignored. They also proposed a second model by dividing a large shelf into smaller artificial shelf sections in order to solve the larger problem sizes. In this model, they do not specify the location of the products. The model assigns the items and their number of facings to a discrete partition of shelf. In this case, vertical alignments of product families located above or below each other on different shelf partitions are not enforced as in the first model. They propose a greedy heuristic to solve the problem. In both models, the problem is not solved to optimality.

Gajjar & Adil (2010) model the problem as if there exist given number of products to allocate to given number of shelves. For each product a length, profit function, minimum and maximum number of facings are defined. Space elasticity of products is considered. However they ignore cross product elasticity. They assume that products can be placed on shelves in any combination. A product can be allocated to more than one shelf. There is no constraint on units of products to be placed adjacently. For each shelf a length is defined as well. Demand of a product is a function of number of facings allocated to the product and space elasticity of the product. They aim to find number of facings of all products in the shelves in order to maximize the total profit.

Yang & Chen (1999) assume that there are five aspects of shelf space allocation as fixture location, product category location, item location within categories, off shelf display, and Point of Sales (POS) promotional support. They model the allocation problem as knapsack problem. There are m shelves with a given length. A facing length is defined for each item. Objective is to maximize the total profit. Demand function is a MNL model. They take into account space and cross space elasticity and other marketing variables e.g price, advertisement, promotion in the demand structure. Space elasticity is defined as the ratio of relative change in unit sales to relative change in shelf space. Cross space elasticity is the ratio of relative change in unit sales to locate 2 different items together in the shelf depending on the products relation e.g being independent, complementary, and substitutable. The gross margin of an item is equal to the unit price minus buying cost, ordering cost and holding cost. They assume that the gross margin of a product is linear to its unit margin. The total profit of an item is calculated as the gross margin times demand. They define four constraints as capacity constraints of shelves, lower and upper bounds of facings of each item, availability constraints of each item that the demand of an item cannot exceed the supply of that item. The model is nonlinear so they propose an alternative model in order to make the model more applicable. In the alternative model, they ignore the availability of product constraint. They change the objective function by identifying the profit per facing of an item. The total profit of a product is the profit per facing times number of facing. By this modification, the problem is turned into an integer programming model. They propose a multi stage solving procedure. First, the optimum shelf space is allocated to the product categories. The space allocated to the product categories become capacity constraints for individual products in the 2nd stage where individual products are allocated. Event thought the solution in each stage is optimum; the solution in the end is probably suboptimal.

Borin et al. (1994) formulate the problem as constrained optimization problem. The model decides product assortment and the space allocated to the products in the assortment. Allocating zero space to a product means eliminating the product from the assortment. Space is assumed to be number of facings allocated to an SKU. The minimum number of facings is given for each product. The space allocated to an item must meet at least its case pack quantity. Product's market share is considered in two components namely uncompromised and compromised demand. Uncompromised demand is a function of consumer preference for SKUs, in store merchandising e.g shelf space, display, advertising and product availability. Compromised demand is used to account for both short term and long term effects of assortment based and stocked out based substitution. Objective is to maximize the category's return on inventory. Total return is assumed to be a function of retailer's cost of total shelf inventory. Demand is composed of four factors namely unmodified demand, modified demand, acquired demand and stock out demand. The first three factors have a positive effect on an item's sale while the last one represents a loss of sales. Unmodified demand is the sales of an SKU under perfectly stocking policy. Modified demand is considered as in-store merchandising e.g promotion, pricing, advertising and also as a shelf space allocated to the product and shelf space allocated to its competitors. Acquired demand is the opportunity cost of a stock out or elimination of an item from the assortment. Stock out demand is the cost for an item that is temporarily stocked out. They propose a simulated annealing algorithm to solve the problem but not optimally.

3. MATHEMATICAL MODEL

This section introduces the mathematical model to determine the optimal shelf space allocation and the assortment planning. Before presenting the model, we explain the constraints and the assumptions of the problem. The categories are taken into account as products in our problem.

3.1. Assumptions

3.1.1 Adjacencies among categories

In practice, some products are allocated adjacently due to customer habits and/or preferences (e.g chocolates and cookies) while some products must be allocated separately because of regulations (e.g food and detergent categories). We consider adjacency requirements among products due to customer habits and preferences in order to provide a comfortable shopping experience. While allocating products, we deal with which products to allocate and how much shelf space to allocate those products. However we do not consider the ordering of products allocated in the aisle.

3.1.2 Shelf capacity

Since the aisles are fixed in a store, the capacity of an aisle is limited. Capacity of aisles can differ from each other due to physical reasons. We study to either include special products into the assortment or not under limited capacity constraints.

3.1.3 Upper and lower bounds of categories

Each category reflects a basic need. We set upper and lower bounds on the size of shelf space that can be allocated to the categories. Retailers need to provide 100% service level by satisfying each basic need which refers to standard categories. These bounds are set logically so that each standard category is guaranteed to be offered.

3.2. Nonlinear Integer Programming Model

Before presenting our model, we give below the notation used in the rest of this study.

I Set of aisles indexed by i and j

K Set of all products indexed by k and l

C_m Set of products in category m

N Set of categories indexed by m

p_k Revenue per meter of product k

p_{kl} Revenue per meter of the cross-selling effect between the pairs of the products k and l

l_k Lower bound on the meters allocated to product k

u_k Upper bound on the meters allocated to product k

s_i Capacity of aisle i in meters

x_{ik} Shelf size in meters allocated to product k in aisle i, nonnegative, integer decision variables

θ_{im} Binary logical decision variables

The optimization problem is as follows:

$$\text{Maximize} \quad \sum_{i\in I}\sum_{k\in K}\left(p_k x_{ik} + \sum_{j\in I}\sum_{l\in K} p_{kl} x_{ik}\, x_{jl}\right) \tag{1a}$$

$$\text{Subject to} \quad \sum_{k\in C_m} x_{ik} - \theta_{im} s_i \le 0 \quad i \in I, m \in N \tag{1b}$$

$$\sum_{j\in I: j\neq i}\sum_{k\in C_m} x_{jk} + \theta_{im}\left(\sum_{j\in I: j\neq i} s_j\right) \le \sum_{j\in I: j\neq i} s_j \quad i \in I, m \in N \tag{1c}$$

$$\sum_{k \in K} x_{ik} \leq s_i \; i \in I \tag{1d}$$

$$\sum_{i \in I} x_{ik} \geq l_k \; k \in K \tag{1e}$$

$$\sum_{i \in I} x_{ik} \leq u_k \; k \in K \tag{1f}$$

$$x_{ik} \in Z^+, \theta_{im} \in \{0,1\} \tag{1}$$

where Z^+ is the set of nonnegative integers.

The decision variables, the constraints, and the objective function in formulation (1) can be explained as follows.

Constraints (1b) and (1c) ensure adjacencies requirements among products. The 0-1 decision variable θ_{im} takes value one if category m of products is assigned to aisle i, and zero otherwise. Then, (1b) implies that the total meters allocated to category m in aisle i cannot exceed the capacity of that aisle. The constraint (1c) implies that all products of the same category m cannot be allocated in other aisles than aisle i. In brief, constraints (1b) and (1c) make sure that all products in the same category m are assigned in the same aisle i. Furthermore, the constraint (1d) sets shelf capacity requirements. The total shelf space of the products that are allocated to aisle i cannot exceed the capacity of that aisle. Moreover, the constraints (1e) and (1f) ensure lower and upper bound requirements. These constraints imply that the total size in meters allocated to product k is within its lower and upper bounds. Finally, the objective function (1a) maximizes the sum of the total revenues of all products, including the individual revenue per product and the revenue due to the cross-selling effect between the pairs of products.

The problem (1) is a nonlinear integer programming problem. Decision variables x_{ik} are the size of shelf space allocated to the product k in aisle i and are defined as nonnegative integer decision variables. Logical decision variables θ_{im} are defined as binary numbers. All constraints in (1) are linear in the decision variables x_{ik} and θ_{im}.

The nonlinear objective function maximizes the total revenues of products, where the linear part consists of the revenue per meter of the individual products p_k, whereas the nonlinear part considers the cross-selling effects between the pairs of the products p_{kl}.

Furthermore, we consider two types of products, namely standard products that are in the current assortment and special products. Then, the decisions x_{ik} for all standard products determine the shelf space allocation for these products. By taking $l_k = 0$ and $u_k = 1$ for the special products, the decisions x_{ik} determine whether to include or not these products in the current assortment.

4. GREEDY HEURISTIC

The problem in (1) can be solved by any nonlinear integer programming solver such as Dicopt, Baron, etc. We, however, also derive a heuristic procedure that can be used easily by practitioners, who have no knowledge of optimization. Below, we explain the steps of this heuristic. We use the same notation as in the mathematical model in (1).

Initialization: If the shelf space capacities s_i $i \in I$ are different, create a set D of dummy aisles of the same cardinality; i.e., the sets I and D have the same number of elements. All dummy aisles have the same shelf space capacity s_d , and it is equal to the maximum capacity of the real aisles; i.e., $s_d = \max_{i \in I} s_i$.

Step 1, Feasibility check: Compute the lower bound in meters to be allocated to each category m; i.e., compute $\sum_{k \in C_m} l_k$. If there exists at least one aisle i for which the aisle capacity $s_i \geq \sum_{k \in C_m} l_k$ for each category $m \in N$, then the problem is feasible, and go to Step 2. Otherwise, the problem is infeasible, hence stop the heuristic procedure.

Step 2, Initial allocations and updates: For each category m, allocate $\sum_{k \in C_m} l_k$ meters in a dummy aisle d. For all products in category m, update the allocated meters $x_{dk} \leftarrow l_k$ and update the upper bounds $u_k \leftarrow u_k - l_k$. If for some product k in category m the updated upper bound u_k becomes zero, remove this product from the category. Repeat this step for all categories.

Step 3, Optimality check: Compute for each product k, $\Delta_k = p_k + \sum_{d \in D} \sum_{l \in K} p_{kl}\, x_{dl}$. This Δ_k is the marginal benefit of increasing the shelf space allocated to product k by one meter. If all $\Delta_k \leq 0$, stop the heuristic procedure, because it is not profitable to increase the current shelf space allocation for any product. Otherwise, select the product k with the biggest Δ_k.

Step 4, Make pairs of categories and aisles: Sort in descending order both $\sum_{k \in C_m} x_{dk}$ and s_i. Make pairs of $\{\sum_{k \in C_m} x_{dk}, s_i\}$ for each category m and aisle i, where for each pair $s_i \geq \sum_{k \in C_m} x_{dk}$ is satisfied.

Step 5, Allocate and update: If $s_i = \sum_{k \in C_m} x_{dk}$ for a pair, remove this category m from the dummy aisle. Allocate this category to aisle i. Remove category m and aisle i from the sets of categories and aisles, respectively.

Step 6, Allocate and update: Increase the shelf space allocation of product k by one *meter; i.e.,* $x_{dk} \leftarrow x_{dk} + 1$. *Update its upper bound* $u_k \leftarrow u_k - 1$.

Step 7, Check extra stopping criteria: In addition to the stopping criterion in Step 3, the heuristic procedure stops when one of the following two criteria is satisfied:

- The set of categories or the set of aisles becomes empty.
- Upper bounds u_k for all products become zero. If the heuristic procedure is stopped, remove all categories from the dummy aisles and assign them to aisles in their pairs $\{\sum_{k \in C_m} x_{dk}, s_i\}$ (already determined in Step 4).

Otherwise, go to Step 3. This finishes our discussion of the steps of the heuristic.

5. MODEL RESULTS

In this section, we first present the supermarket case. We give the results of estimations for per meter revenues of each category and the cross-selling effects between categories. Furthermore we compare the results found both in GAMS and our heuristic. We perform sensitivity analyses to illustrate the effects of cross-selling on our results. Then, we enlarge the problem size to test if GAMS is able to solve the problem in a reasonable execution time as well as our heuristic.

5.1. Supermarket Case

We set adjacency requirements by defining groups of categories. We study two groups of categories. Each group of categories has four standard products, which must be allocated into the same aisle. Category 1 includes spices, soup mix, flour, cake mix and category 2 includes ketchup, pasta, grains and sauces. As special products, we have diabetic category. We assume that diabetic category can be assigned with both groups of categories. Since the categories differ from each other, substitution is not considered.

For the maximization objective, the values of any product as well as its cross-selling effect are estimated using the real sales data of one year of the largest supermarket chain in Turkey. We have one year of shopping basket details according to the fidelity card of 600 customers of a specific supermarket. We sum the basket expenses of each customer monthly, and finally we have 5.958 different customer-basket data.

We use multiple linear regression, more specifically the ordinary least squares estimation techniques, to estimate the per meter revenue for category and the per meter cross-selling effects between the pairs of categories. We also perform t - test to find out the statistically significant p_k and p_{lk} at significance level α =5%.

5.1.1 Parameter Estimation

The per meter revenues of each category is given in the Table 5.1. Pasta has the biggest revenue among all categories. Pasta is followed by cake mix, grains, spices, sauces, soup mix, flour, ketchup and lastly diabetic. Diabetic has the smallest revenue. Since diabetic products are becoming popular, diabetic category is considered as a special category.

Table 5.1 : Per meter revenues for categories

Category (k)	Per meter revenue (p_k)
Spices	35.1890
Soup mix	20.6279
Flour	13.6612
Cake mix	42.0354
Ketchup	8.9362
Pasta	60.0973
Grains	39.0736
Sauces	25.6088
Diabetic	5.6918

The cross-selling effects for category 1 are shown in Table 5.2. The cross-selling effect between flour and soup has biggest positive value among all entries in Table 5.2. Spices have a significant negative impact on the sales of both soup mix and cake mix.

Table 5.2 : Per meter cross-selling effects for category 1

Per meter revenue (p_{lk})	Spices	Soup mix	Flour	Cake mix	Diabetic
Spices		-3.2557	-0.6113	-3.2134	0.3847
Soup mix			3.8507	0	0
Flour				0	0.6766
Cake mix					0
Diabetic					

The cross-selling effects for category 2 are shown in Table 5.3. The cross-selling effect between ketchup and pasta has a significant positive value among all entries in Table 5.3. Diabetics have a negative impact on the sales of ketchup and grains.

Table 5.3 : Per meter cross-selling effects for category 2

Per meter revenue (p_{lk})	Ketchup	Pasta	Grains	Sauces	Diabetic
Ketchup		6.1359	-1.1923	0.9792	-0.8843
Pasta			0	0	0
Grains				0.7068	-0.7719
Sauces					0.6349
Diabetic					

The cross-selling effects between category 1 and category 2 are shown in Table 5.4. Pasta has a significant negative effect on the sales of both soup mix and flour while the pairs pasta - spices and soup mix - grains support each other's sales.

Table 5.4 : Per meter cross-selling effects between category 1 and category 2

Per meter revenue (p_{lk})	Ketchup	Pasta	Grains	Sauces	Diabetic
Spices	1.1238	3.5344	0	0	0.3847
Soup mix	-1.3838	-5.9543	3.5246	-1.5135	0
Flour	2.4902	-5.467	-1.8801	2.1816	0.6766
Cake mix	0	0	0	0	0
Diabetic	-0.8843	0	-0.7719	0.6349	0

Table 5.5 shows upper and lower bounds for the categories. As we can understand from lower bounds, the assortment decision occurs for the diabetic category ($l_k = 0$). For the other products where lower bounds are greater than zero, shelf space allocation decisions are in question.

Table 5.5 : Upper and lower bounds for categories

Category (k)	Upper bound (u_k)	Lower bound (l_k)
Spices	3	1
Soup mix	3	1
Flour	3	1
Cake mix	5	1
Ketchup	3	1
Pasta	5	2
Grains	5	2
Sauces	3	1
Diabetic	1	0

We consider two aisles with capacities eight and ten meters. We add the constraint "if diabetic category is included in the assortment, it will be allocated with either category 1 or category 2" to the formulation (1) as seen in equation (1g).

$$\sum_{i \in I} x_{ik} \leq 1 \quad k \in \{Diabetic\} \tag{1g}$$

5.1.2 Solution in GAMS

Using the data in Tables 5.1 through 5.5, and the data of the capacities of aisles, we solve the problem in GAMS 23.4 using the solver Dicopt. The optimal solution is given in Table 5.6 with the optimal objective value equal to 720.189.

Table 5.6 : Optimal allocation in meters of the formulation (1)

Category 1	Optimal allocation	Category 2	Optimal allocation
Spices	3	Ketchup	2
Soup mix	1	Pasta	5
Flour	1	Grains	2
Cake mix	3	Sauces	1
Diabetic	0	Diabetic	0

For the estimated values of p_k and p_{lk}, category 1 is allocated to the aisle with capacity 8 while category 2 is allocated to the aisle with capacity 10. For category 1, shelf size allocated to spices is equal to its upper bound while shelf size allocated to soup mix and flour is equal to their lower bounds. For category 2, pasta is allocated to the number of shelf size equal to its upper bound while grains and sauces are allocated to the number of shelf equal to their lower bounds. The diabetic category is not found profitable to be included in the assortment.

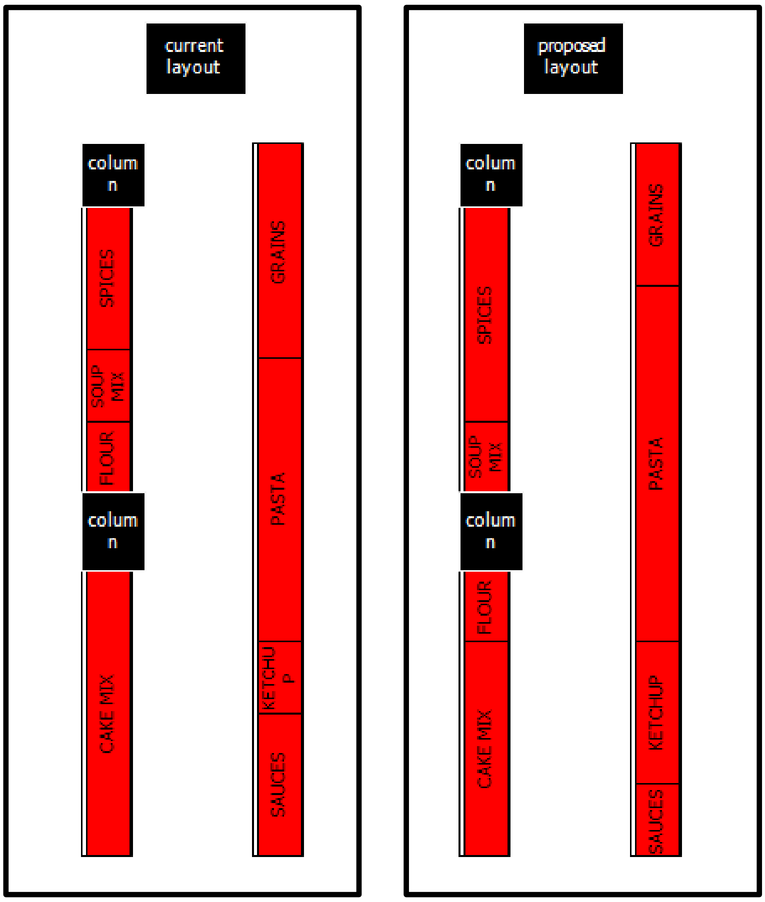

Figure 5.1 : Current versus proposed layout

Figure 5.1 shows the current layout and the proposed layout through our model. Table 5.7 shows the current allocation and the optimal allocation of categories in meters. According to our solution, the aisles of the categories are not changed. In category 1, the space allocated to spices is augmented by 1 meter while the space allocated to cake mix is reduced by 1 meter. In category 2 the spaces allocated to both pasta and ketchup are the augmented by 1 meter, while the space allocated to grains and sauces are both reduced 1 meter.

Table 5.7 : Current and optimal allocation of the categories

	Category	Current allocation	Optimal allocation
Category 1	Spices	2	3
	Soup mix	1	1
	Flour	1	1
	Cake mix	4	3
Category 2	Ketchup	1	2
	Pasta	4	5
	Grains	3	2
	Sauces	2	1

5.1.3 Iterations and Solution in Our Heuristic

The greedy heuristic is solved in 9 iterations which can be seen in Table 5.8. In iteration 1, all of the categories are allocated to the number of shelf space equal to their lower bounds. The objective value at iteration 1 is calculated as 343.86. Pasta has the biggest marginal benefit Δ_k, which is equal to 58.347, of increasing the shelf space allocated to that category by one meter. The heuristic allocates pasta during iterations 2, 3 and 4. The objective value increases from 402.21 to 518.903. At iteration 4, the number of shelf space allocated to pasta has reached to its upper bound. So spices with the biggest $\Delta_k = 46.905$ are chosen to be allocated at iterations 5 and 6. The objective value becomes 612.712 at the end of iteration 6. Since the number of shelf space allocated to spices reached to its upper bound, ketchup is chosen to be allocated with a marginal benefit of 42.689. Ketchup is allocated in iteration 7 and the total space allocated to ketchup is equal to its upper bound. Furthermore the total number of shelf space allocated to category 2 reached to 10 which is the capacity of one of the aisles. The heuristic cannot allocate any further space to category 2. At iterations 8 and 9, cake mix is allocated with a marginal benefit of 32.396. At the end of iteration 9, the total number of shelf space allocated to the category 1 is equal to 8. The category 1 is allocated to aisle with capacity 8. The final objective function is found as 720.189. Diabetic category is not taken into the assortment. The objective function value and the optimal allocations found through the greedy heuristic are equal to the ones found in GAMS.

Table 5.8 : Iterations, objective value, category to allocate, marginal benefit of category

	Iteration number								
Product k	1	2	3	4	5	6	7	8	9
Spices	1	1	1	1	2	**3**	3	3	3
Soup mix	1	1	1	1	1	1	1	1	1
Flour	1	1	1	1	1	1	1	1	1
Cake mix	1	1	1	1	1	1	1	2	3
Diabetic	0	0	0	0	0	0	0	0	0
Ketchup	1	1	1	1	1	1	**2**	2	2
Pasta	2	3	4	**5**	5	5	5	5	5
Grains	2	2	2	2	2	2	2	2	2
Sauces	1	1	1	1	1	1	1	1	1
Diabetic	0	0	0	0	0	0	0	0	0
Objective	343.86	402.21	460.56	518.90	565.81	612.71	655.4	687.79	720.19
k	Pasta	Pasta	Pasta	Spices	Spices	Ketchup	Cake mix	Cake mix	
Δ_k	58.347	58.347	58.3463	46.905	46.905	42.689	32.396	32.396	

For this case, the heuristic solved the problem to optimality which can be seen through explicit enumeration. Table 5.9 shows five following best solutions found by explicit enumeration.

Table 5.9 : Optimal allocations and objective value of five best solutions found through explicit enumeration

Category k	Optimal allocation	Optimal allocation	Optimal allocation	Optimal allocation	Optimal allocation
Spices	3	1	1	2	2
Soup mix	1	1	1	1	1
Flour	1	1	1	1	1
Cake mix	3	5	5	4	4
Diabetic	0	0	0	0	0
Ketchup	1	2	1	2	1
Pasta	5	5	5	5	5
Grains	3	2	3	2	3
Sauces	1	1	1	1	1
Diabetic	0	0	0	0	0
Objective	717.7348	714.6315	714.4236	714.1975	712.8658

5.1.4 Sensitivity Analysis

We perform sensitivity analysis to observe how the optimal objective value changes as the cross-selling effect changes between two categories.

First we observe the cross-selling effect between pasta and spices. The estimated value of the cross-selling effect between pasta and spices is 3.5344. We change the cross-selling effect from -6.5344 to 6.5344. The change in the objective value is piecewise linear as can be seen in Figure 5.2. Optimal objective value ranges between 664.29 and 765.19.

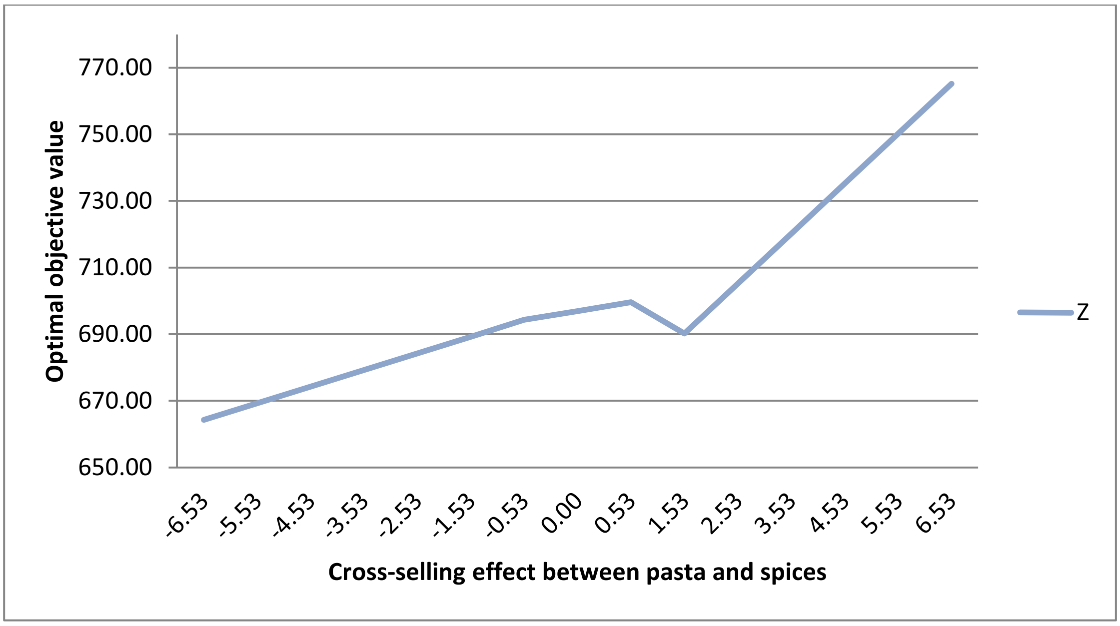

Figure 5.2 : Sensitivity of the objective with respect to the cross-selling effect

Second we observe the effect of increasing capacity of an aisle on the optimal allocation of categories which can be seen in Table 5.10. The columns under optimal allocation represent the shelf size allocated to categories in the order of spices, soup mix, flour, cake mix and diabetic for category 1; in the order of ketchup, pasta, grains, sauces and diabetic for category 2. We set the capacity of aisles equal to the minimum sum of the lower bounds of category 1 and category 2. Then we increase the capacity of an aisle one by one and observe optimal allocation and the objective value.

We start with capacities equal to 6. Both categories are allocated. Then we increase the capacity of an aisle to 7. Category 2 is allocated to the aisle with the capacity 7 while category 1 is allocated to the aisle with capacity 6. The capacity of the aisle is increased one by one until the capacity is equal to 16 which is the sum of upper bounds of the category 2. Each time category 2 is allocated to the aisle with the biggest capacity.

Table 5.10 : Optimal allocation and objective value while increasing the capacity of one aisle

Capacity		Optimal allocation		Optimal objective
Aisle 1	Aisle 2	Category 1	Category 2	Z
6	6	(1, 1, 1, 3, 0)	(1, 2, 2, 1, 0)	421.507
6	7	(3, 1, 1, 1, 0)	(1, 3, 2, 1, 0)	481.881
6	8	(3, 1, 1, 1, 0)	(1, 4, 2, 1, 0)	547.296
6	9	(3, 1, 1, 1, 0)	(1, 5, 2, 1, 0)	612.711
6	10	(3, 1, 1, 1, 0)	(2, 5, 2, 1, 0)	655.399
6	11	(3, 1, 1, 1, 0)	(3, 5, 2, 1, 0)	698.087
6	12	(3, 1, 1, 1, 0)	(3, 5, 3, 1, 0)	735.936
6	13	(3, 1, 1, 1, 0)	(3, 5, 4, 1, 0)	773.784
6	14	(3, 1, 1, 1, 0)	(3, 5, 5, 1, 0)	811.631
6	15	(3, 1, 1, 1, 0)	(3, 5, 5, 2, 0)	844.38
6	16	(3, 1, 1, 1, 0)	(3, 5, 5, 3, 0)	877.129

Third we look at the change in the optimal allocation of categories while increasing the capacity of both aisles one by one which can be seen in Table 5.11. We set the capacity of aisles equal to the sum of the lower bounds of category 1 and 2 which is equal to 4 and 6 respectively. We see that diabetic category is included into the assortment and allocated with the category 1 to aisle 1 when its capacity is equal to 11. Then we observe that the total shelf size of the category 1 is reached at its limit when the capacity of the aisle is equal to 11. Even though the capacity of aisle is increased, the total space allocated to category 1 remains the same. The space allocated to spices, cake mix and diabetic are equal to their upper bounds while soup mix and flour are allocated one meter each because of cross-selling effects. On the other hand, the space allocated to category 2 is equal to the sum of its upper bound. Ketchup, pasta, grains and sauces are allocated equally to their upper bounds.

Table 5.11 : Optimal allocation and objective value while increasing the capacity of both aisles

Capacity		Optimal allocation		Optimal objective
Aisle 1	Aisle 2	Category 1	Category 2	Z
4	6	(1, 1, 1, 1, 0)	(1, 2, 2, 1, 0)	343.864
5	7	(2, 1, 1, 1, 0)	(1, 3, 2, 1, 0)	442.045
6	8	(3, 1, 1, 1, 0)	(1, 4, 2, 1, 0)	547.296
7	9	(3, 1, 1, 2, 0)	(1, 5, 2, 1, 0)	645.106
8	10	(3, 1, 1, 3, 0)	(2, 5, 2, 1, 0)	720.19
9	11	(3, 1, 1, 4, 0)	(3, 5, 2, 1, 0)	795.272
10	12	(3, 1, 1, 5, 0)	(3, 5, 3, 1, 0)	865.515
11	13	(3, 1, 1, 5, 1)	(3, 5, 4, 1, 0)	905.78
12	14	(3, 1, 1, 5, 1)	(3, 5, 5, 1, 0)	942.856
13	15	(3, 1, 1, 5, 1)	(3, 5, 5, 2, 0)	976.239
14	16	(3, 1, 1, 5, 1)	(3, 5, 5, 3, 0)	1009.622

We conclude that even there is more shelf space available; the algorithm cannot allocate more shelf space to a category because of the negative cross-selling effects between products which can reduce the optimal objective value.

5.2. Solutions of Medium Scale Problems

In order to test the solution quality of GAMS and our heuristic in terms of the optimal objective value and execution time of the problem, we enlarge the problem size. We set number of categories to 20 within 5 products. We have 20 aisles. All parameters are created randomly through uniform distribution. Per meter revenues of categories are distributed according to uniform distribution U(-10, 15). Upper and lower bounds of categories are distributed through uniform distribution U(4,10) and U(0,3) respectively. The cross-selling effects between categories are distributed according to uniform distribution U(-5,5). Capacities of aisles are assumed to be between 15 and 25.

We solved 100 medium-scale test problems by both GAMS 23.7 and our heuristic. Our heuristic solved all of the test problems while GAMS solved 78 out of 100 test problems within 17 minutes time limit. For each problem, our heuristic outperformed GAMS in terms of the optimal objective value as well as the execution time which can be seen through APPENDIX, Tables 5.12 and 5.13. Table 5.12 shows the relevant statistics of the solutions found by our heuristic and GAMS for the 100 test problems. The execution time of GAMS is more than 14 times worse than our heuristic, as can be seen in Tables 5.12 and 5.13.

Table 5.12 : Statistical measures of the solutions found in our heuristic and GAMS

	Heuristic		GAMS	
	Execution time(s)	Objective value	Execution time(s)	Objective value
Average	20.30	11,633.86	298.40	9,995.44
Standard Dev.	3.79	1,138.57	403.72727	831.92
Minimum	13.20	8,732.00	38.193	8,224.00
1^{st} Quartile	18.86	10,784.25	58.26725	9,237.75
Median	19.90	11,631.00	74.048	9,994.50
3^{rd} Quartile	20.97	12,436.00	195.3945	10,530.25
Maximum	50.46	14,017.00	1139.654	11,681.00

Table 5.13 shows the statistics of the solutions found by our heuristic and GAMS after excluding the execution time of 22 test problems that GAMS couldn’t solve within 17 minutes. The optimal objective value found by our heuristic is, on average, 14% higher than the optimal objective value found by GAMS. Furthermore, the average execution time of our heuristic is less than one quarter of the execution time of GAMS.

Table 5.13 : Adjusted statistical measures of the solutions found in our heuristic and GAMS

	Heuristic		GAMS	
	Execution time(s)	Objective value	Execution time(s)	Objective value
Average	20.30	11,633.86	86.167	9,995.44
Standard Dev.	3.79	1,138.57	62.01426	831.92
Minimum	13.20	8,732.00	38.193	8,224.00
1^{st} Quartile	18.86	10,784.25	52.7665	9,237.75
Median	19.90	11,631.00	66.8355	9,994.50
3^{rd} Quartile	20.97	12,436.00	87.7035	10,530.25
Maximum	50.46	14,017.00	476.48	11,681.00

6. CONCLUSION

In this study, we derive a nonlinear integer programming problem formulation for the joint shelf space allocation and assortment planning problem. All constraints of the formulation are linear in the decision variables; the objective function, however, has nonlinear terms due to the cross-selling effects between the pairs of products. This type of problems can be solved by standard solvers such as Baron, Dicopt, etc. We also derive a greedy type heuristic procedure that can be used by practitioners with no knowledge of optimization. This heuristic can be very useful when the problem has many integer variables so that the solvers cannot solve the problem within a reasonable amount of time.

In order to compare the performance of our heuristic with standard solvers, we solved 100 medium-scale test problems by both GAMS 23.7 (using Dicopt) and our heuristic. GAMS solved 78 out of 100 test problems within 17 minutes time limit while our heuristic solved all of the test problems. For each problem, our heuristic performed better than GAMS in terms of the optimal objective value and the execution time. The average optimal objective value found by our heuristic is 14% higher than the one found by GAMS. The execution time of our heuristic is, on average, less than one quarter of the execution time of GAMS.

In our study, we allocate some products in the same aisle because of the customer habits and/or preferences. We decide on only which products to allocate and how much shelf space to allocate those products in an aisle. We, however, do not consider the ordering of products allocated in an aisle, which can be considered as a future study.

This study considers a static assortment planning and shelf space allocation problem. In retail sector, new products are introduced frequently while products which perform badly in the market are being taken out of the assortment. Therefore assortment revisions are required within the introduction of new products. Dynamic assortment planning and shelf space allocation where assortment and shelf space decisions change as time elapses can be considered as another future study field.

Neither GAMS nor our heuristic guaranties the optimality. For supermarket case, we showed by explicit enumeration that our heuristic solved the problem to optimality. Although our heuristic offers better solutions than GAMS for medium-scale problems, we do not know the optimal solutions of our problems. Future studies can focus on establishing formal upper bounds for the optimality gaps of our heuristic and work on further improving our heuristic.

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APPENDIX

Test problem number	MATLAB		GAMS	
	Execution time(s)	Objective value	Execution time(s)	Objective value
1	15.35	10,626.00	51.80	9,390.00
2	19.91	9,793.00	52.95	8,957.00
3	21.84	9,717.00	56.99	8,569.00
4	21.49	9,776.00	70.85	8,906.00
5	21.69	10,385.00	-	-
6	22.03	9,896.00	-	-
7	50.46	11,496.00	106.90	10,055.00
8	27.43	9,517.00	-	-
9	24.10	10,814.00	57.86	8,957.00
10	31.96	10,697.00	-	-
11	20.82	10,569.00	-	-
12	19.10	10,807.00	57.53	9,911.00
13	19.22	10,786.00	49.64	9,855.00
14	20.99	10,681.00	-	-
15	20.79	10,645.00	46.22	8,950.00
16	18.40	8,732.00	-	-
17	21.95	10,666.00	44.18	10,016.00
18	20.29	9,562.00	-	-
19	19.74	10,412.00	64.31	8,988.00
20	18.32	10,481.00	-	-
21	18.37	9,495.00	56.84	8,224.00
22	19.60	11,003.00	42.34	9,899.00
23	21.30	11,539.00	51.73	10,158.00
24	21.69	10,386.00	476.48	8,926.00
25	21.15	11,036.00	49.25	9,676.00
26	19.77	11,841.00	48.95	10,221.00
27	19.02	13,244.00	50.78	10,404.00
28	20.67	13,080.00	61.08	11,681.00
29	20.42	12,511.00	41.58	10,741.00
30	20.19	12,433.00	76.70	11,167.00
31	19.89	12,235.00	38.19	11,350.00

32	20.62	12,061.00	60.61	8,771.00
33	20.26	12,349.00	87.20	10,914.00
34	20.56	12,318.00	129.41	9,057.00
35	19.24	11,711.00	49.79	9,919.00
36	19.86	11,767.00	262.81	9,833.00
37	19.88	13,575.00	44.71	11,193.00
38	19.73	11,678.00	70.65	9,756.00
39	19.63	12,078.00	62.54	10,399.00
40	20.99	12,751.00	44.19	10,963.00
41	20.41	13,642.00	59.84	11,463.00
42	20.33	13,891.00	72.19	11,316.00
43	19.78	13,280.00	44.36	10,171.00
44	19.91	14,017.00	-	-
45	20.96	12,617.00	63.37	10,981.00
46	20.40	12,614.00	-	-
47	20.80	12,014.00	49.90	10,073.00
48	20.95	12,273.00	139.64	10,569.00
49	19.51	12,907.00	50.67	11,575.00
50	19.90	12,742.00	-	-
51	18.85	11,027.00	-	-
52	14.24	12,445.00	48.36	10,305.00
53	20.91	11,989.00	50.58	11,595.00
54	20.58	11,334.00	62.20	9,025.00
55	21.64	10,560.00	60.33	8,612.00
56	16.95	12,385.00	58.66	10,811.00
57	21.22	11,621.00	77.25	10,193.00
58	21.52	11,895.00	52.71	9,705.00
59	21.03	11,240.00	194.36	9,527.00
60	21.04	11,437.00	-	-
61	19.38	11,394.00	58.40	10,348.00
62	22.02	11,010.00	87.87	10,414.00
63	13.20	13,476.00	71.81	10,636.00
64	21.58	11,643.00	58.80	10,224.00
65	20.11	12,687.00	75.64	9,173.00
66	20.01	10,779.00	174.28	9,324.00
67	18.09	11,181.00	-	-
68	17.71	11,539.00	85.54	9,770.00
69	18.73	11,561.00	-	-
70	17.44	10,694.00	111.42	9,722.00
71	18.53	12,818.00	164.85	10,878.00
72	18.02	11,574.00	-	-
73	23.40	11,641.00	168.14	9,672.00

74	19.73	12,104.00	198.50	10,070.00
75	20.65	12,033.00	169.46	9,209.00
76	22.11	12,373.00	143.58	9,768.00
77	19.01	11,441.00	-	-
78	23.24	12,265.00	-	-
79	23.66	10,380.00	142.18	9,137.00
80	18.82	11,348.00	-	-
81	17.81	11,550.00	61.51	8,533.00
82	18.76	10,474.00	72.45	9,115.00
83	18.40	11,339.00	69.92	10,093.00
84	19.55	13,167.00	-	-
85	19.68	12,197.00	68.12	11,127.00
86	18.11	12,731.00	-	-
87	18.79	12,929.00	88.18	9,780.00
88	19.63	12,631.00	66.76	10,803.00
89	19.15	13,251.00	153.56	10,015.00
90	19.14	11,852.00	66.91	9,826.00
91	19.94	11,784.00	67.18	9,974.00
92	19.32	13,356.00	77.94	10,204.00
93	18.45	13,842.00	65.96	11,267.00
94	13.95	9,755.00	68.28	9,107.00
95	18.61	10,911.00	67.64	10,128.00
96	18.63	11,350.00	92.01	10,213.00
97	18.86	10,931.00	76.19	9,108.00
98	19.27	12,722.00	117.12	11,213.00
99	20.06	12,174.00	97.20	9,926.00
100	18.43	9,420.00	84.17	9,140.00

BIOGRAPHICAL SKETCH

Can Erdem Özkan was born in Bolu on March 23, 1987. He was graduated from İzzet Baysal Anadolu Lisesi in 2005 and received his B.Sc. degree in Industrial Engineering from Galatasaray University in 2010. He took his degree of M.Sc. in Industrial Engineering from Institut Polytechnique de Grenoble in 2012. Since 2013 he is working at Migros Ticaret A.Ş. in Space Management Unit under Marketing Department. Meanwhile he also studied for degree of M.Sc. in Industrial Engineering from Galatasaray University. This thesis was written in order to fulfill the requirements for his graduation from Galatasaray University under the supervision of Ebru Angün. His research interests assortment planning and shelf space allocation.