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FUZZY LINMAP METHOD FOR SUPPLIER SELECTION PROBLEM

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FUZZY LINMAP METHOD FOR SUPPLIER SELECTION PROBLEM

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LIST OF SYMBOLS

AHP	: Analytic Hierarcy Process
ANP	: Analytic Network Process
CBR	: Case-Based Reasoning
DEA	: Data Envelopment Analysis
DM	: Decision Maker
DMU	: Decision Making Unit
ED&T	: Engineering, Design & Testing
FNIS	: Fuzzy Negative Ideal Solution
FPIS	: Fuzzy Positive Ideal Solution
JIT	: Just-In-Time
LINMAP	: Linear Programming Technique for Multi-Dimensional Analysis of Preference
MADM	: Multi-Attribute Decision Making
MCDM	: Multi-Criteria Decision Making
OEM	: Original Equipment Manufacturer
OR	: Operations Research
PAB	: Passenger Airbag
PIS	: Positive Ideal Solution
RFQ	: Request for Quotation
TARR	: Time Adjusted Rate of Return
TOPSIS	: Technique for Order Preference by Similarity to Ideal Solution

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ABSTRACT

Purchasing operations are discussed more in the recent years. The reason is its effect on companies' profitability. Companies' aim is to make money while buying. Thus, supplier selection process is gained importance. Generally, supplier selection is carried out by comparing the cost metrics, e.g. piece price, investment, logistic. This comparison has become insufficient with the globalization and strategic sourcing approaches.

A supplier selection problem of an automotive company is reviewed in this study. The typical evaluation criteria of this company is time adjusted rate of return (TARR) which is calculated by comparing with the current price. However, the decision makers could not decide by only TARR. Qualitative criteria, e.g. reliability and future localization opportunity for the suppliers, is brought to the table.

Therefore, the decision process needs to be standardized and to include both qualitative and quantitative criteria. In this study, fuzzy LINMAP is proposed and applied on the supplier selection process. Since the method allows to use both crisp and linguistic inputs, it suits to the problem.

The study is organized as follows: the problem and method is basically introduced in the first section. In the second section, the literature review regarding supplier selection is summarized. The method, fuzzy LINMAP is explained in the third section. The business case is analyzed by the method in the fourth section. In the last section, results are reviewed.

ÖZET

Satınalma operasyonları son yıllarda daha çok tartışılmaktadır. Sebebi, firmaların karlılığı üzerinde etkisidir. Firmaların amacı satın alırken kazanmaktır. Bu sebeple, tedarikçi seçimi problmei önem kazanmıştır. Genellikle, tedarikçi seçimi maliyet parametreleri, birim fiyat, yatırım, nakliye gibi, kıyaslanarak gerçekleştirilir. Küreselleşme ve stratejik satınalma yaklaşımları ile bu kıyaslama yetersiz kalmıştır.

Bu çalışamada bir otomotiv firmasının tedarikçi seçim problemi incelenmiştir. Bu firmanın klasik değerlendirme kriteri güncel fiyatla kıyaslanarak hesaplanan getiri oranıdır. Fakat karar verici sadece getiri oranı ile karar verememiştir. Niteliksel kriterler, güvenilirlik ve gelecek yerlileştirme fırsatları gibi, gündeme gelmiştir.

Dolayısıyla, karar verme prosesinin standartlaştırılması ve hem nicel hem nitel kriterlerin girdi olarak kullanılması gerekmektedir. Bu çalışmada, bulanık doğrusal programlama tedarikçi seçim problemine uygulanmıştır. Metot nitel ve nicel verileri kullanmayı sağladığı için problem için uygundur.

Çalışmanın organizasyonu şu şekildedir: ilk bölümde problem ve metot üzerine giriş yapılmıştır, ikinci bölümde tedarikçi seçimi üzerine literatür taraması özetlenmiştir, bulanık doğrusal programlama metodu üçüncü bölümde incelenmiştir, dördüncü bölümde problem bulanık doğrusal programlama yaklaşımı ile modellenmiştir. Son bölümde sonuçlar incelenmiştir.

1. INTRODUCTION

Automotive industry with its more than 200 years of past is one of the leading sectors in the world economy. During the last decades, its globalization is accelerated as a result of construction of overseas facilities (Domansky, 2006). As the automobile industry becomes more global, supply chain network is also extended. Besides, an automobile consists of thousands parts which means every original equipment manufacturer (OEM) has business with lots of suppliers.

While OEMs get more dependent on suppliers, purchasing decisions become more significant. The great variety of sourced products has made procurement operations an increasingly important function, and the complexities of sourced products are now greater than ever. Purchasing strategies and operations management are main indicators on profitability. In a global, complex and fuzzy sourcing environment, the procurement operations are more critical. Although cost was a typical key decision making factor in the classic sourcing decisions, sourcing decision includes capabilities, business processes, etc. in today's complex sourcing environment. Hence, cost is not a primary decision-making factor. Every sourcing decision is unique and consists of interrelated factors**.**

The factors affecting the initial purchasing decision are listed below:

- More people involved: increase in outsourcing, spreading of purchasing function,
- Larger set of options: Internet, globalization of trade,
- More transparency required: Government regulations,
- Larger set of criteria: Environmental concerns,

- Higher speed required: Changing customer preferences,
- More severe consequence of poor decisions

These factors require a more standardized and clear approach to purchasing decision-making, especially regarding the supplier selection (Carter et al., 1998). The methods of contemporary operations research (OR) provide supportive techniques in dealing with the complexity of the decisions. These techniques are multi criteria decision making, mathematical programming, data mining techniques and problem structuring approaches. OR methods could enhance the effectivity of the purchasing decisions by:

- Aiding the purchaser in solving the right problem, e.g. refraining from dropping a supplier when the delivery problems are actually caused by feeding the supplier with outdated information;
- Aiding the purchaser in taking more and relevant alternatives criteria into account when making purchasing (management) decisions, e.g. more long-term considerations when deciding on make-or-buy;
- Aiding the purchaser to more precisely model the decision situation, e.g. dealing specially with intangible factors and group decision making.
- Enabling automated and faster computation and analysis of decision making information, e.g. data on suppliers found on the internet
- Enabling more efficient storage of purchasing decisions making process and access to this information in future cases, e.g. saving files that contain criteria-structures for supplier evaluation;
- Eliminating redundant criteria and alternatives from the decision or evaluation process, e.g. in extensive and expensive suppliers audit programs;
- Facilitating more efficient communication about and justification of the outcome of decision-making processes, e.g. when reporting to management or suppliers (Boer et al., 2001)

A decision maker (DM) is faced with the problem of selecting, evaluation or ranking alternatives that are characterized by multiple, usually conflicting, attributes in multiple attribute decision-making (MADM) problems (Hwang, Chen, & Hwang, 1992). LINMAP is a MADM method and is based on pair-wise comparisons of alternatives given by decision makers and generates the best compromise alternative as the solution that has the shortest distance to the positive ideal solution (PIS) (Srinivasan & Shocker, 1973). Since most of the MADM problems include both quantitative and qualitative attributes that use imprecise data and human judgments, crisp values are insufficient (Hwang et al., 1992; Su, 2011). In MADM problems, fuzzy set theory is well suited to deal with such decision problems (Ross, 2004; Van Laarhoven & Pedrycz, 1983; Y. M. Wang & Parkan, 2005; L. Zadeh, 1965). The fuzzy LINMAP method (Albayrak, 2008; Albayrak & Erensal, 2006, 2009; Bereketli, Genevois, Albayrak, & Ozyol, 2011; D. F. Li, 2008; D. F. Li, Chen, & Huang, 2010; D. F. Li & Sun, 2007; D. F. Li & Yang, 2004) is a linear programming model based consistency and inconsistency indices of the preferences given by decision maker. According to the concept of fuzzy and technique for order preference by similarity to ideal solution (TOPSIS), the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS) are defined (C. T. Chen, 2000). By solving the linear programming problem, FPIS, the weights of attributes and the distance of each alternative from the FPIS are calculated. According to the increasing order of these distances, the best alternative is obtained and the ranking order of all alternatives is determined.

As stated above, there are several supportive decision methods in supplier selection. In Fig. 1. 1., the supplier selection process with the supportive tools is shown.

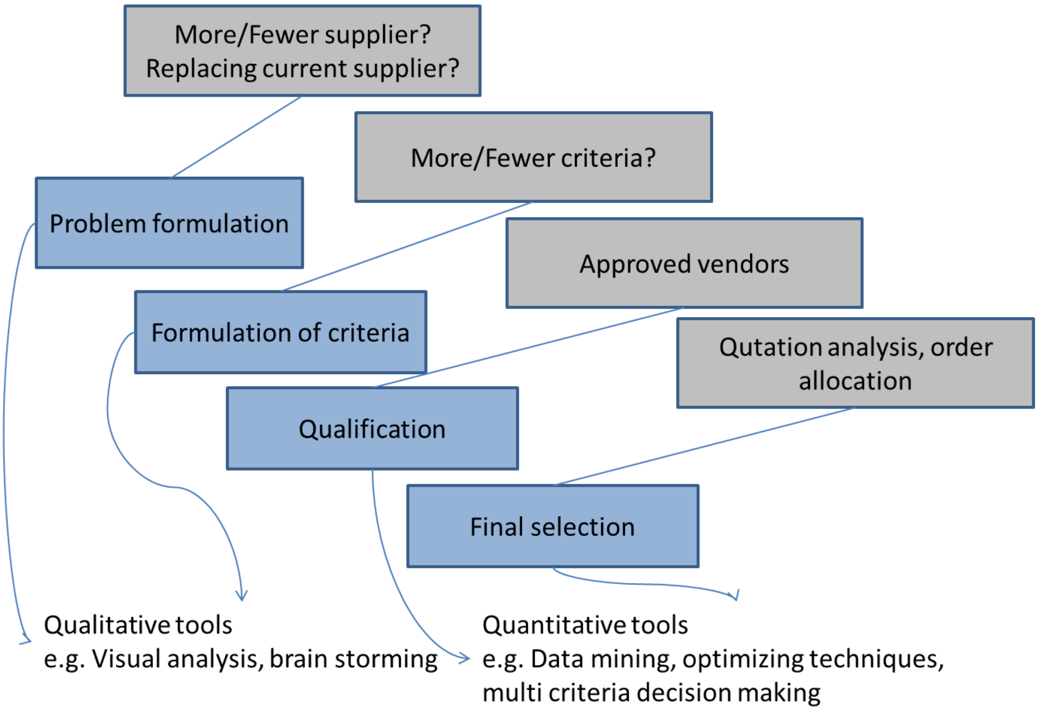

Figure 1.1: Supplier Selection Process & Methods

In automotive industry, sourcing strategy is reviewed at the beginning of every project. If the sourcing strategy is determined as market test, the first step is to select the candidate suppliers. Market test is the process of evaluating the suppliers by using both quantitative and qualitative criteria. The process is generally conducted between at least 3 suppliers. The candidate suppliers are selected by purchasing and product development departments. Reliability, business links with other OEMs, geographical location, previous experience, etc. are taken into consideration while choosing the suppliers for the market test.

Second step is to send out the request for quotation (RFQ) letters with technical details. Each supplier is expected to give their quotations according to the program timing. In case of necessity, meetings are organized with the supplier, the product development team and the purchasing. It is vital to have quotations from the suppliers for exactly the same design.

Third step is to collect the quotations, and set up meetings with the suppliers in order to understand their quotation and determine further cost reduction opportunities. It is aimed to provide the final quotations in this step. Figure 1. 2., purchasing steps are summarized.

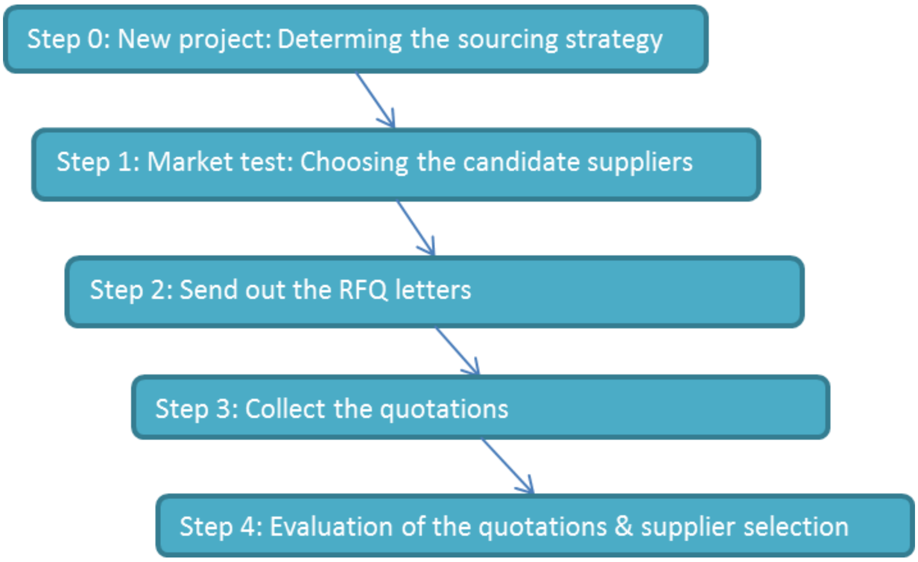

Figure 1.2: The Purchasing Steps

After creating the business case between the suppliers, the final step is to select the supplier. The most complex step of the purchasing function is choosing the best supplier.

The study is organized as follows: The literature on the supplier selection problem is reviewed in chapter 2. The approaches for supplier evaluation and selection are mentioned, the previous studies are briefly summarized. In chapter 3, the proposed model, fuzzy LINMAP is presented. The approach is applied on a problem in automotive industry, and reviewed comprehensively in chapter 4. The results are reported and discussed in chapter 5.

2. LITERATURE REVIEW

The critical objective of the purchasing department is to procure right product at the right cost in the right quantity with the right quality at the right time from the right source. This requires executive effective decisions concerning supplier selection and evaluation (Ware et al., 2012). In addition, supplier selection involves several conflicting criteria that are imprecise. The criteria includes many factors both quantitative, i.e. cost, financial status, delivery performance, and qualitative, i.e. reliability, reputation. Thus, it is a typical MCDM problem.

Lately, the effectiveness of the supplier selection has received considerable attention in global market. Selecting the most advantageous supplier is an opportunity to establish an effective supply chain system. It enables organizations increasing profit, while reducing the cost. Thus, there are various studies on supplier selection in the literature. Both method and criteria are reviewed by the researchers. Some of the most frequently use approaches are MADM techniques, mathematical programming models, artificial intelligence methods, fuzzy logic approaches and integrated approaches.

According to Boer et al. (2001), there are four steps in supplier selection process:

1. Problem definition
2. Decision criteria formulation
3. Pre-qualification of potential suppliers
4. Making a final choice

2.1. Supplier Selection Criteria

Although the primary quantitative indicator of the supplier selection is generally cost, it is not sufficient to evaluate the suppliers by the cost. Quality, delivery performance, reliability, facility location, logistic costs, technology, service are also critical factors affecting supplier selection process. A list of the criteria is shown in Table 2.1 (Dickson, 1966). Criteria are grouped according to importance by Dickson: extreme importance, considerable importance, average importance, and slight importance. Performance related features are ranked at the top, the least important criteria is reciprocal arrangements.

Table 2.1: Supplier Selection Criteria

Rank	Factor	Evolution
1	Quality	Extreme importance
2	Delivery	
3	Performance history	
4	Warranties and claim policies	
5	Production facilities and capacity	Considerable importance
6	Price	
7	Technical capability	
8	Financial position	
9	Procedural compliance	
10	Communication system	
11	Reputation and position in industry	
12	Desire for business	
13	Management and organization	

Table 2.1: Supplier Selection Criteria *(continued)*

Rank	Factor	Evolution
14	Operating controls	Average importance
15	Repair services	
16	Attitude	
17	Impression	
18	Packaging ability	
19	Labor relations record	
20	Geographical location	
21	Amount of past business	
22	Training aids	
23	Reciprocal arrangements	Slight importance

According to Dickson's study, the most important criteria are listed as a result of a survey between purchasing managers in North America. Although a ranking is given by Dickson, the ranking is principally dependent on the industry. If the supplier selection process is held on a low profitable industry, then cost is one of the most important criteria. However, Dickson's study is the first to clarify and group the supplier selection criteria. The criteria of supplier selection are also studied by Ellram (1990), Weber et al. (1991), Krause et al. (2000) and Birch (2001). According to Ellram, the traditional criteria, such as cost, quality, delivery reliability, etc., are not sufficient in developing strategic partnerships with suppliers. Thus, four categories of qualitative criteria are suggested: (1) financial issues, (2) organizational culture and strategy, (3) technology, (4) miscellaneous factors. Weber et. al. reviewed the articles since 1966, the criteria are classified and ranked according to the articles. According to Weber's study, the criteria are classified regarding to Just-In-Time (JIT) manufacturing strategies. As a result, price, quality and delivery performance are ranked as extremely important criteria.

According to Ho et al, the criteria used for supplier selection is listed with its frequency in Table 2.2.

Table 2.2: The criteria used for supplier selection, frequency

Criteria	Number of articles (between 2000-2009)
Quality	68
Delivery	64
Price/Cost	63
Manufacturing capability	39
Service	35
Management	25
Technology	25
Research and development	24
Finance	23
Flexibility	18
Reputation	15
Relationship	3
Risk	3
Safety and environment	3

The most popular criteria among the supplier selection criteria are quality, delivery and price/cost. Some of the quality related attributes are listed below:

- Net rejections
- Continuous improvement
- Six-sigma
- Total quality management
- Quality planning

Some of the delivery related attributes are listed below:

- Compliance with due date
- Delivery delays
- Geographical location
- On-time delivery
- Delivery lead time

Some of the price/cost related attributes are listed below:

- Unit cost
- Ordering cost
- Logistics cost
- Manufacturing cost
- Competitiveness of cost

Based on the literature, it is clarified that the traditional cost oriented approach is no longer suitable for the supplier selection problem.

In this study, the proposed model is applied on a supplier selection problem of an automotive company. The criteria are preferred regarding the industry, the commodity and the previous ranking studies.

2.2. Supplier Selection Process

In industrial companies, purchasing holds at least 50% of total turnover which is a significant amount (Telgen, 1994). In addition, several developments, i.e. globalization, internet, changing customer preferences, lead to a new organization forms that involve more decision-makers. Thus, a more standardized and systematic approach on the purchasing process is required.

Boer et al. (1998) proposed a 4-step-model for the supplier selection process. The steps are reviewed regarding two purchasing situations:

- New task situation: New product, new suppliers – no historical data,
- Modified rebuy: New product/known suppliers or existing product/new suppliers, historical data available (Faris et al., 1967)

The first step is to define the problem. If the commodity is new, the problem is whether to use a supplier or not. If the case is modified rebuy, there is a repeating decision and the options are choosing more, fewer or other suppliers.

The second step is to formulate the criteria. For the new task situation the criteria is entirely new. Because, there is neither historical data nor previously used criteria available. If the situation is modified rebuy, it is possible to use previous experience. As stated in Chapter 1, Dickson (1966) identified 23 attributes by surveying purchasing managers. The study is a reference for the following papers on the supplier selection.

The third step is pre-qualification of potential suppliers which is the process of sorting acceptable suppliers and reducing the number of candidate suppliers.

The final step is choosing the suitable and most competitive supplier. There are various approaches in order to identify best supplier in the literature which is reviewed in section 2.3.

2.3. Analytical Methods

In the literature, the articles focused on both deterministic and non-deterministic analytical methods: i.e. mathematical programming, multi-attribute decision making

(MADM) approaches, stochastic/fuzzy integer programming and fuzzy/stochastic multi-objective programming for supplier selection problem.

2.3.1. Data envelopment analysis

Data envelopment analysis (DEA) is an analytical tool for measuring and evaluating performance (Cooper et al., 2004). DEA is first introduced by Charnes, in 1978. Operations research, economics and management science are the fundamentals of DEA. It is a mathematical programming approach to provide a relative efficiency assessment for a group of decision making units (DMUs) which convert multiple inputs to multiple outputs. Recent articles are listed Table 2.3.

Table 2.3: Data envelopment analysis in the literature

Authors	**Area of application**	**Criteria**
Braglia and Petroni (2000)	Bottling machines and packaging lines manufacturing	Profitability, quality, delivery performance, management capabilities,
Liu et al. (2000)	Agricultural and construction equipment manufacturing	Quality, price, delivery perormance, location
Forker and Mendez (2001)	Electronic components manufacturing	Quality, product/service desing, process management
Narasimhan et al. (2001)	Telecommunications industry	Quality, price, delivery, cost reduction,
Talluri and Baker (2002)	Supply Chain management	Delivery performance, accounting performance, logistic cost

Table 2. 3: Data envelopment analysis in the literature *(continued)*

Authors	**Area of application**	**Criteria**
Talluri and Narasimhan (2004)	Telecommunications industry	Quality, price, delivery, cost reduction
Garfamy (2006)	Manufacturing - supplier evaluation and management accounting	Manufacturing cost, quality cost, input technology, aftersales service
Ross et al. (2006)	Communications industry	Delivery performance, quality
Saen (2006)	Nuclear power industry	Cost, technological know-how
Seydel (2006)	Consumer products manufacturing	Price, quality, lead time, quantity, delivery,
Talluri et al. (2006)	Pharmaceutical industry	Quality, delivery, price
Saen (2007)	Supply Chain management	Logistic cost, supplier reputation, accounting
Wu et al. (2007)	Aviation electronics manufacturing	Revenue,satisfaction, cost, judgement

2.3.2 Mathematical programming models

The aim is to select supplier by maximizing/minimizing the objective function subject to buyer/supplier constraints in the mathematical programming models (Deshmukh, 2011). Linear programming is applicable where the objective function and constraints are linear. Linear programming is a practical method and easy to use with commercial solvers. Non-linear programming avoids linearity for objective function and constraints. Both linear programming and non-linear programming approaches consider single objective function. Non-linear functions are represented by integer-programming approaches as well. Goal programming and multi-objective programming enables to consider multiple objectives. Recent literature on linear programming, non-linear programming, integer programming and goal programming are listed in Table 2.4.

Table 2.4: Mathematical programming models in the literature

Approach	**Authors**	**Area of application**	**Criteria**
Linear Programming	Talluri and Narasimhan (2003)	Pharmaceutical industry	Price, quality, delivery
Linear Programming	Talluri and Narasimhan (2005)	Telecommunications industry	Quality, price, delivery, cost reduction performance
Linear Programming	Ng (2008)	Agricultural and construction equipment manufacturing	Supply variety, quality, distance, delivery, price
Binary Integer Linear Programming	Talluri (2002)	Pharmaceutical industry	Price, quality, delivery
Mixed Integer Linear Programming	Hong et al.(2005)	Agricultural industry	Delivery, quality, price, quantity
Mixed Integer Nonlinear Programming	Ghodsypour and O'Brien (2001)	Hypothetical case	Price, ordring cost, perfect rate, on-time delivery, capacity
Goal Programming	Karpak et al. (2001)	Hydraulic gear pump manufacturing	Product cost, quality of castins urchased, delivery reliability of castings purchased

Table 2.4: Mathematical programming models in the literature *(continued)*

Approach	**Authors**	**Area of application**	**Criteria**
Multi-objective Programming	Narasimhan et al. (2006)	Personal computer manufacturing	Direct cost, indirect-coordination cost, quality, delivery reliability, complexity of supply arrangement
Multi-objective Programming	Wadhwa and Ravindran (2007)	Hypothetical case	Price, lead time, rejects

2.3.3 Case-based reasoning

Case-based reasoning is a problem solving method by noticing new problem's similarity with prior problems and adapting old solutions to the new circumstances. The quality of the case-based reasaoning approach depends on the experience, the ability to understand new problem in terms of previous problems, the ability to adaptation.

Table 2.5: Case-based reasoning in the literature

Approach	**Authors**	**Area of application**	**Criteria**
Case-based reasoning	Choy and Lee (2002); (2003); (2004); (2005)	Consumer products manufacturing	Delivery, shipment quality, product price, manufacturing capability, customer service, management commitment, product development

2.3.4 Analytic hierarchy process

Thomas L. Saaty developed the anlytic hierarchy process in the 1970s. The analytic hierarchy process enables to structure a complex, multi-person, and multi-attribute problem hierarchically. AHP uses pairwise comparison, measures the relative importance of attributes to determine best alternative. Its difficulty is to determine suitable weight and order of each alternative (Liu et al., 2004). However, it is stable, flexible and also user-friendly. The recent literatures on the analytic hierarchy process are listed in Table 2. 6.

Table 2.6: AHP approaches in the literature

Authors	**Area of application**	**Criteria**
Akarte et al. (2001)	Automobile castings	Maximum casting size, minimum section thickness, testing facilities, quality certification, total casting cost, sample delivery time
Muralidharan et al. (2002)	Bicycles manufacturing	Quality, delivery, price, technical capability, financial position, facility, flexebility, service
Chan (2003)	Manufacturing	Cost, quality, design capability, manufacturing capability, technical capability, technological capabilty,
Chan and Chan (2004)	Semiconductor industry	Cost, delivery, flexibility, innovation, quality, service
Liu and Hai (2005)	Furniture industry	Quality, responsiveness, discipline, delivery, financial, management, technical capability, facility

Table 2.6: AHP approaches in the literature *(continued)*

Authors	**Area of application**	**Criteria**
Chan et al. (2007)	Airline industry	Cost, satisfaction of supplier, quality, R&D, financial issues, technological issues, safety,
Hou and Su (2007)	Printer manufacturing	Quality, cost, technology, production capability, R&D, delivery & location, performance & service

2.3.5 Analytic network process

Analytic network process (ANP) which is an extended version of analytic hierarcy process is also developed by Saaty. It is especially used in the risk and uncertainty studies. ANP enables to structure more complex, interdependent relationships among the attributes. The recent studies are listed in Table 2. 7.

Table 2.7: ANP approaches in the literature

Authors	**Area of application**	**Criteria**
Sarkis and Talluri (2002)	High technology metal-based manufacturing	Culture, technology, relationship, cost, quality, time, flexibility
Bayazit (2006)	Hypothetical case	Flexibility, on-time delivery, price, delivery lead-time, quality, market share, personnel capability, financial capability

Table 2.7: ANP approaches in the literature

Authors	**Area of application**	**Criteria**
Gencer and Gürpinar (2007)	Electronic industry	Facility location, number of working years, references, service capability, communication capability, organization structure,

2.3.6 Fuzzy-set theory

If the linguistic variables are used to assess the criteria, fuzzy-set theory is proposed to deal with the supplier selection problem (Chen et al., 2006). The method is suitable in which some of the parameters are fuzzy in nature. The applications in the literature are listed in Table 2. 8.

Table 2.8: Fuzzy-set theory approaches in the literature

Authors	**Area of application**	**Criteria**
Chen et al. (2006)	High-technology manufacturing	Profitability of supplier, relationship closeness, technological capacity, conflict resolution
Sarkar and Mohapatra (2006)	Hypothetical case	Price, quality, delivery lead time, production facilities and capacity, management and organization, technological capabilty, etc.
Florez-Lopez (2007)	Hypothetical case	Cost reduction effort, delivery delays, price, reliability, responsiveness,

Most of the literature on supplier selection problem is based on classical multi-criteria decision methods. However, supplier selection process includes imprecise and qualitative considerations. Thus, the decision making process becomes more complex and the classical methods cannot effectively provide the best optimum solution. The objective of this thesis is to propose fuzzy LINMAP which is a fuzzy integrated approach for supplier selection. Constructing a fuzzy LINMAP model enables to include both crisp and vague criteria in the supplier selection problem. The following section, fuzzy LINMAP is reviewed.

3. METHODOLOGY

The classical multi criteria decision making (MCDM) methods are based on crisp numbers. Yet, crisp numbers is not sufficient as describing the problem in most cases. The criteria could be both quantitative and qualitative. Thus, the MCDM problem is structured by crisp, fuzzy and/or linguistic data. A linguistic variable is a variable whose values are linguistic terms (Zadeh, 1975). The concept of linguistic variable is very useful when dealing with situations which are too complex and/or not well defined to be reasonably described in conventional quantitative expressions (Zimmermann, 1991).

3.1 Basic Concepts

A fuzzy number m is a special fuzzy subset on the set R of the real numbers which satisfy the following conditions:

(1) There exists a $x_0 \in R$ so that the degree of its membership$\mu_m(x_0) = 1$;
(2) Membership function $\mu_m(x)$ is left and right continuous.

Generally, a fuzzy number m can be written as

$$\mu_m(x) = \begin{cases} L(x)(l \le x \le m) \\ R(x)(m \le x \le r) \end{cases} \quad (3.1)$$

where $L(x)$is an increasing function of $x \in [l, m]$ and right continuous,$0 \le L(x) \le 1$; $R(x)$is a decreasing function of $x \in [m, r]$ and left continuous, $0 \le R(x) \le 1$. m is called a mode of m, and l and r are called the low and upper limits of m, respectively. This kind of fuzzy numbers is often called L–R fuzzy numbers (Li, 2003).

Let $m = (l, m_1, m_2, r)$ be a trapezium fuzzy number, where the membership function μ_m of m is

$$\mu_m(x) = \begin{cases} \frac{x-l}{m_1-l} \; (l \leq x < m_1) \\ 1 \; (m_1 \leq x \leq m_2) \\ \frac{r-x}{r-m_2} \; (m_2 < x \leq r) \end{cases} \tag{3.2}$$

The closed interval $[m_1, m_2]$ is the mode of m. l and rare the low and upper limits of m, respectively.

A trapezium fuzzy number $m = (l, m_1, m_2, r)$ is reduced to a real number m if $l = m_1 = m_2 = r$. Conversely, a real number m can be written as a trapezium fuzzy number $m = (m, m, m, m)$.

If $m_1 = m_2$ then $m = (l, m, r)$ is called a triangular fuzzy number, where $m = m_1 = m_2$. In other words, a triangular fuzzy number has the following membership function

$$\mu_m(x) = \begin{cases} \frac{x-l}{m-l} \; (l \leq x < m) \\ \frac{r-x}{r-m} \; (m < x \leq r) \end{cases} \tag{3.3}$$

So a triangular fuzzy number is a special case of a trapezium fuzzy number.

Similarly, a triangular fuzzy number $m = (l, m, r)$ is reduced to a real number m if $l = m = r$. Conversely, a real number m can be written as a triangular fuzzy number $m = (m, m, m)$.

$m = (l, m_1, m_2, r)$ is called a positive trapezium fuzzy number if $l \geq 0$ and one of l, m_1, m_2 and r is nonzero. Furthermore, $m = (l, m_1, m_2, r)$ is called a normalized positive trapezium fuzzy number if it is a positive trapezium fuzzy number and$l \geq 0, r \leq 1$.

3.2 Linguistic variable

A linguistic variable is a variable whose values are linguistic terms. The concept of linguistic variable is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions. For example, the ratings of alternatives on qualitative attribute "reliability" could be expressed using linguistic variables such as "very low," "low," "medium," "high," "very high," etc. Such linguistic values can also be represented using positive trapezium fuzzy numbers. For example, "very low," "low," "medium," "high" and "very high" can be represented by positive trapezium fuzzy numbers $(0, 0.1, 0.2, 0.3)$, $(0.1, 0.2, 0.3, 0.4)$, $(0.3, 0.4, 0.5, 0.6)$, $(0.5, 0.6, 0.7, 0.8)$ and $(0.7, 0.8, 0.9, 1)$, respectively.

3.3 Distance between two trapezium fuzzy numbers

Let $m = (m_1, m_2, m_3, m_4)$ and $n = (n_1, n_2, n_3, n_4)$ be two trapezium fuzzy numbers. Then the vertex method is defined to calculate the distance between them as follows:

$$d(m, n) = \sqrt{\frac{1}{6}[(m_1 - n_1)^2 + 2(m_2 - n_2)^2 + 2(m_3 - n_3)^2 + (m_4 - n_4)^2]} \quad (3.4)$$

which is proved to be metric. Equation 3.4 is a simple method to calculate the distance between two trapezium fuzzy numbers.

If both m and n are real numbers then the distance measurement $d(m, n)$ is identical to the Euclidean distance. In fact, suppose that both $m = (m_1, m_2, m_3, m_4)$ and $n = (n_1, n_2, n_3, n_4)$ are two real numbers and let $m_1 = m_2 = m_3 = m_4 = m$ and $n_1 = n_2 = n_3 = n_4 = n$. The distance measurement $d(m, n)$ can be calculated as

$$d(m,n) = \sqrt{\frac{1}{6}[(m_1 - n_1)^2 + 2(m_2 - n_2)^2 + 2(m_3 - n_3)^2 + (m_4 - n_4)^2]}$$

$$d(m,n) = \sqrt{\frac{1}{6}[(m - n)^2 + 2(m - n)^2 + 2(m - n)^2 + (m - n)^2]}$$

$$d(m,n) = \sqrt{(m - n)^2} = |m - n| \quad (3.5)$$

Furthermore, two trapezium fuzzy numbers m and n are identical if and only if the distance measurement$d(m, n) = 0$.

3.4. Normalization method

Suppose there are n possible alternatives $x_1, x_2, \ldots, x_n$ from which the decision maker has to choose on the basis of m attributes $f_1, f_2, \ldots, f_n$, both quantitative and qualitative (Li, 2003). Suppose that the rating of alternative $x_j (j = 1, 2, \ldots, n)$ on attribute f_i $(i = 1, 2, \ldots, m)$ given by the decision maker is a trapezium fuzzy number$f_{ij} = (a_{ijl}, a_{ijm_1}, a_{ijm_2}, a_{ijr})$. Hence, a fuzzy multiattribute decision making problem can be concisely expressed in matrix format as follows:

$$F = \begin{pmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & \vdots & \ddots & \vdots \\ f_{m1} & f_{m2} & \cdots & f_{mn} \end{pmatrix} \quad (3.6)$$

which is referred to as a fuzzy decision matrix usually used to represent the fuzzy multi-attribute decision making problem.

Since the physical dimensions and measurements of the m attributes are different, so the fuzzy decision matrix F needs to be normalized. Tthe following normalization formula is used in this study:

$$r_{ij} = \left(\frac{a_{ijl}}{a_{ir}^{max}}, \frac{a_{ijm_1}}{a_{im_2}^{max}}, \frac{a_{ijm_2}}{a_{im_1}^{max}} \Lambda 1, \frac{a_{ijr}}{a_{il}^{max}} \Lambda 1\right) (i \epsilon F^1) \quad (3.7)$$

$$r_{ij} = \left(\frac{a_{il}^{min}}{a_{ijr}}, \frac{a_{im_1}^{min}}{a_{im_2}}, \frac{a_{im_2}^{min}}{a_{im_1}} \Lambda 1, \frac{a_{ir}^{min}}{a_{ijl}} \Lambda 1\right) (i \epsilon F^2) \quad (3.8)$$

Where F^1 and F^2 are the set of benefit attributes and cost attributes, respectively, and

$$a_{il}^{max} = \max\{a_{ijl} | f_{ij} = (a_{ijl}, a_{ijm_1}, a_{ijm_2}, a_{ijr}), j = 1,2, \ldots, n\}$$
$$a_{il}^{min} = \min\{a_{ijl} | f_{ij} = (a_{ijl}, a_{ijm_1}, a_{ijm_2}, a_{ijr}), j = 1,2, \ldots, n\}$$
$$a_{im_1}^{max} = \max\{a_{ijm_1} | f_{ij} = (a_{ijl}, a_{ijm_1}, a_{ijm_2}, a_{ijr}), j = 1,2, \ldots, n\}$$
$$a_{im_1}^{min} = \min\{a_{ijm_1} | f_{ij} = (a_{ijl}, a_{ijm_1}, a_{ijm_2}, a_{ijr}), j = 1,2, \ldots, n\}$$
$$a_{im_2}^{max} = \max\{a_{ijm_2} | f_{ij} = (a_{ijl}, a_{ijm_1}, a_{ijm_2}, a_{ijr}), j = 1,2, \ldots, n\}$$
$$a_{im_2}^{min} = \min\{a_{ijm_2} | f_{ij} = (a_{ijl}, a_{ijm_1}, a_{ijm_2}, a_{ijr}), j = 1,2, \ldots, n\}$$
$$a_{ir}^{max} = \max\{a_{ijr} | f_{ij} = (a_{ijl}, a_{ijm_1}, a_{ijm_2}, a_{ijr}), j = 1,2, \ldots, n\}$$
$$a_{ir}^{min} = \min\{a_{ijr} | f_{ij} = (a_{ijl}, a_{ijm_1}, a_{ijm_2}, a_{ijr}), j = 1,2, \ldots, n\} \quad (3.9)$$

Denote r_{ij} by $r_{ij} = r_{ijl}, r_{ijm_1}, r_{ijm_2}, r_{ijr}$ for any $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. All r_{ij} are trapezium fuzzy numbers. Furthermore, all $r_{ij} \epsilon [0, 1]$ ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$ i.e., each r_{ij} is a normalized positive trapezium fuzzy number.

Using equations 3.7 and 3.8, equation 3.6 can be transformed into the following normalized positive trapezium fuzzy number decision matrix:

$$R = (r_{ij})_{mxn} = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix} \quad (3.10)$$

3.5 Fuzzy LINMAP model and method

Let $R_j = (r_{1j}, r_{2j}, \ldots, r_{mj})^T$ express a normalized positive trapezium fuzzy number vector for alternatives $x_j (j = 1, 2, \ldots, n)$, where $r_{ij} = (r_{ijl}, r_{ijm_1}, r_{ijm_2}, r_{ijr})$ $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$ is a normalized positive trapezium fuzzy number. Suppose that the fuzzy positive ideal solution be $a^* = (a_1^*, a_2^*, \ldots, a_m^*)^T$ which is unknown a priori and needs to determine, where $a_i^* = (a_{il}^*, a_{im_1}^*, a_{im_2}^*, a_{ir}^*)(i = 1, 2, \ldots, m)$ is a positive trapezium fuzzy number on attribute f_i .

Using equation 3.4 the square of the weighted Euclidean distance between the alternative $R_j = (r_{1j}, r_{2j}, \ldots, r_{mj})^T$ and the FPIS $a^* = (a_1^*, a_2^*, \ldots, a_m^*)^T$ can be calculated as

$$S_j = \sum_{i=1}^{m} w_i [d(r_{ij}, a_i^*)]^2 \quad (3.11)$$

S_j can be written explicitly as

$$S_j = \sum_{i=1}^{m} \frac{w_i}{6} \begin{bmatrix} (a_{ijl} - a_{il}^*)^2 + 2(a_{ijm_1} - a_{im_1}^*)^2 \\ +2(a_{ijm_2} - a_{im_2}^*)^2 + (a_{ijr} - a_{ir}^*)^2 \end{bmatrix} \quad (3.12)$$

where $w = (w_1, w_2, \dots, w_m)^T$ is a weight vector which is unknown a priori and needs to determine.

Assume that the decision maker gives the preference relations between alternatives by $\Omega = \{(k, j)|x_k \geq x_j, (k, j = 1, 2, \dots, n)\}$ from his/her knowledge and experience, where the symbol "≥" is a preference relation given by the decision maker. $x_k \geq x_j$ means that either the decision maker prefers the alternative x_k to x_j or the decision maker is indifferent between x_k and x_j . If the weight vector $w = (w_1, w_2, \dots, w_m)^T$ and the fuzzy positive ideal solution $\mathrm{a}^* = (\mathrm{a}_1^*, \mathrm{a}_2^*, \dots, \mathrm{a}_\mathrm{m}^*)^\mathrm{T}$ are chosen by the decision maker already, then using equation 3.11 the decision maker can calculate the square of the weighted Euclidean distance between each pair of alternative (k, j) ∈ Ω and the fuzzy positive ideal solution $\mathrm{a}^* = (\mathrm{a}_1^*, \mathrm{a}_2^*, \dots, \mathrm{a}_\mathrm{m}^*)^\mathrm{T}$ as follows:

$$S_k = \sum_{i=1}^{m} w_i [d(r_{ik}, a_i^*)]^2 \quad (3.13)$$

$$S_j = \sum_{i=1}^{m} w_i [d(r_{ij}, a_i^*)]^2 \quad (3.14)$$

For each pair of alternatives (k, j) ∈ Ω, the alternative x_k is closer to the FPIS than the alternative j if $S_j \geq S_k$. So the ranking order of alternatives x_k and x_j determined by S_j and S_k based on (w, a^*) is consistent with the preference given by the decision maker. Conversely, if $S_j < S_k$, then (w, a^*) is not chosen properly since it results in that ranking order of alternatives x_k and x_j determined by S_j and S_k based on (w, a^*) is inconsistent with the preferences given by the decision maker. Therefore, (w, a^*) should be chosen so that the ranking order of alternatives x_k and x_j determined by S_j and S_k is consistent with the preference provided by the decision maker.

$(S_j - S_k)$ – is defined as an index to measure inconsistency between the ranking order of alternatives x_k and x_j determined by S_j and S_k and the preferences given by the decision maker preferring x_k to x_j as follows

$$(S_j - S_k)^- = \begin{cases} S_j - S_k, (S_j < S_k) \\ 0, (S_j \geq S_k) \end{cases} \quad (3.15)$$

Obviously, the ranking order of alternatives x_k and x_j determined by S_j and S_k based on (w, a^*) is consistent with the preferences given by the decision maker if $(S_j \geq S_k)$. Hence, $(S_j - S_k)^-$ is defined to be 0. On the other hand, the ranking order of alternatives x_k and x_j determined byS_j and S_k based on (w, a^*) is inconsistent with the preferences given by the decision maker if $S_j < S_k$. Hence, $(S_j - S_k)^-$is defined to be $(S_j - S_k)$. Then, the inconsistency index can be rewritten as

$$\left(S_j - S_k\right) = \max\{0, S_j - S_k\} \quad (3.16)$$

Then, a total inconsistency index of the decision maker is defined as

$$B = \sum_{(k,j)\epsilon\Omega}(S_j - S_k)^- = \sum_{(k,j)\epsilon\Omega} \max\{0, S_j - S_k\} \quad (3.17)$$

In a similar way, an index $(S_j - S_k)^+$ to measure consistency between the ranking order of alternatives x_k and x_j determined by S_j and S_k and the preferences given by the decision maker preferring x_k to x_j can be defined as follows:

$$(S_j - S_k)^+ = \begin{cases} S_j - S_k, (S_j \geq S_k) \\ 0, (S_j < S_k) \end{cases} \quad (3.18)$$

This equation mention above can be rewritten as

$$(S_j - S_k)^+ = \max\{0, S_j - S_k\} \tag{3.19}$$

Hence, a total consistency index of the decision maker is defined as

$$G = \sum_{(k,j)\epsilon\Omega}(S_j - S_k)^+ = \sum_{(k,j)\epsilon\Omega} \max\{0, S_j - S_k\} \tag{3.20}$$

To determine(w, a^*), we construct the following mathematical programming as follows:

$$\text{Max}\{\text{G}\} \tag{3.21}$$

$$G - B \geq h$$

$$\sum_{i=1}^{m} w_i = 1$$

$$w_i \geq \varepsilon\,(i = 1, 2, \ldots, m)$$

where $h > 0$ is given by the decision maker a priori and $\varepsilon > 0$ is sufficiently small which ensures that the weights generated are not zero as it may be the case in the LINMAP method (Srinivasan, 1973).

The aim of equation 3.21 is to maximize the total consistency index G of the decision maker under the condition in which the total consistency index G is greater than or equals to the total inconsistency index B by a given value $h > 0$. Using equations 3.15 – 3.20, it follows:

$$G - B = \sum_{(k,j)\epsilon\Omega} \left(S_j - S_k\right)^{+} - \sum_{(k,j)\epsilon\Omega} \left(S_j - S_k\right)^{-}$$

$$= \sum_{(k,j)\epsilon\Omega} \left[\left(S_j - S_k\right)^{+} - \left(S_j - S_k\right)^{-}\right] = \sum_{(k,j)\epsilon\Omega} \left(S_j - S_k\right) \quad (3.22)$$

Combining equations 3.20 and 3.21, mathematical programming can be rewritten as follows:

$$\text{Max}\left\{\sum_{(k,j)\epsilon\Omega} \max\{0, S_j - S_k\}\right\} \quad (3.23)$$

$$\sum_{(k,j)\epsilon\Omega} \left(S_j - S_k\right) \geq h$$

$$\sum_{i=1}^{m} w_i = 1$$

$$w_i \geq \varepsilon\, (i = 1, 2, \dots, m)$$

For each pair of $(k, j)\epsilon\Omega$, let

$$\lambda_{kj} = \max\{0, S_j - S_k\} \quad (3.24)$$

Then for each $(k, j)\epsilon\Omega$,

$$\lambda_{kj} \geq 0 \tag{3.25}$$

$$\lambda_{kj} \geq S_j - S_k \tag{3.26}$$

Thus, equation 3.23 can be transformed into the following mathematical programming

$$\max\{\textstyle\sum_{(k,j)\epsilon\Omega} \lambda_{kj}\} \tag{3.27}$$

$$\sum_{(k,j)\epsilon\Omega} (S_j - S_k) \geq h$$

$$\sum_{i=1}^{m} w_i = 1$$

$$w_i \geq \varepsilon\,(i = 1, 2, \dots, m)$$

$$S_j - S_k + \lambda_{kj} \geq 0 \quad ((k, j)\epsilon\Omega)$$

$$\lambda_{kj} \geq 0 \quad ((k, j)\epsilon\Omega)$$

Using equation 3.12, the following linear programming model can be constructed:

$$\max\{\textstyle\sum_{(k,j)\epsilon\Omega} \lambda_{kj}\} \tag{3.28}$$

$$\sum_{i=1}^{m} w_i \sum_{(k,j)\epsilon\Omega} [(a_{ijl}^2 - a_{ikl}^2) + 2(a_{ijm_1}^2 - a_{ikm_1}^2) + 2(a_{ijm_2}^2 - a_{ikm_2}^2) + (a_{ijr}^2 + a_{ikr}^2)]$$
$$- 2\left[\sum_{i=l}^{m} v_{il} \sum_{(k,j)\epsilon\Omega} [(a_{ijl} - a_{ikl}) + 2\sum_{i=l}^{m} v_{im_1} \sum_{(k,j)\epsilon\Omega} [(a_{ijm_1} - a_{ikm_1}) \right.$$
$$\left. + 2\sum_{i=l}^{m} v_{im_2} \sum_{(k,j)\epsilon\Omega} [(a_{ijm_2} - a_{ikm_2}) + \sum_{i=l}^{m} v_{ir} \sum_{(k,j)\epsilon\Omega} [(a_{ijr} - a_{ikr})\right]$$
$$\geq 6h$$

$$\sum_{i=1}^{m} w_i [(a_{ikl}^2 - a_{ijl}^2) + 2(a_{ikm_1}^2 - a_{ijm_1}^2) + 2(a_{ikm_2}^2 - a_{ijm_2}^2) + (a_{ikr}^2 + a_{ijr}^2)]$$
$$- 2\left[\sum_{i=l}^{m} v_{il} \left(a_{ikl} - a_{ijl}\right) + 2\sum_{i=l}^{m} v_{im_1} (a_{ikm_1} - a_{ijm_1}) \right.$$
$$\left. + 2\sum_{i=l}^{m} v_{im_2} (a_{ikm_2} - a_{ijm_2}) + \sum_{i=l}^{m} v_{ir} (a_{ikr} - a_{ijr})\right] + 6\lambda_{kj} \geq 0$$

$$\sum_{i=1}^{m} w_i = 1$$

$$w_i \geq \varepsilon\,(i = 1, 2, \dots, m)$$

$$\lambda_{kj} \geq 0 \quad ((k,j)\epsilon\Omega)$$

$$v_{il} \geq 0, v_{im_1} \geq 0, v_{im_2} \geq 0, v_{ir} \geq 0,$$

Where

$$v_{il} = w_i a_{il}^*$$

$v_{im_1} = w_i a^*_{im_1}$

$v_{im_2} = w_i a^*_{im_2}$

$$v_{ir} = w_i a^*_{ir} \tag{3.29}$$

w_i , v_{il} , v_{im_1}, v_{im_2} and v_{ir} $(i = 1, 2, \ldots, m)$can be obtained by solving the above linear programming (i.e., equation 3.28) using the Simplex method. Then, the best values of $a^*_{il}, a^*_{im_1}, a^*_{im_2}$ and a^*_{ir} a^*_{ir} are computed using equation 3.29 and are denoted as the trapezium fuzzy number $a^*_i = (a^*_{il}, a^*_{im_1}, a^*_{im_2}, a^*_{ir})$ $(i = 1, 2, \ldots, m)$. Hence the ranking order of the alternative set $X = \{x_1, x_2, \ldots, x_m\}$ is generated based on the increasing order of distances S_j $(j = 1, 2, \ldots, n)$ calculated with equation 3.12.

3.6 Decision process of fuzzy LINMAP method

The fuzzy linear programming model is constructed to solve the weight vector and the FPIS. Hence the ranking order of all alternatives is generated once the distances of alternatives from the FPIS.

To summarize, an algorithm and decision process of the fuzzy multi attribute decision making with fuzzy set approach is given in the following. The approach consists of 10 steps:

- Step 1: The decision maker identifies the evaluation attributes.
- Step 2: The decision maker gives the preference relations between alternatives by $\Omega = \{(k, j) | x_k \geq x_j, (k, j = 1, 2, \ldots, n)\}$.
- Step 3: Choose the appropriate linguistic variables for the linguistic ratings of alternatives on attributes.
- Step 4: Pool the decision maker's opinion to get the linguistic rating f_{ij} of alternative x_j under attribute f_i.

- Step 5: Construct the fuzzy decision matrix F and the normalization positive trapezium fuzzy number decision matrix R.
- Step 6: Construct the linear programming equation 3.28.
- Step 7: Solve equation 3.28 using the Simplex method of the linear programming.
- Step 8: Obtain w_i and $a_i^* = (a_{il}^*, a_{im_1}^*, a_{im_2}^*, a_{ir}^*)$ $(i = 1, 2, \ldots, m)$ using equation 3.29 hence obtain the weight vector $w = (w_1, w_2, \ldots, w_m)^T$ and the fuzzy positive ideal solution $a_i^* = (a_{il}^*, a_{im_1}^*, a_{im_2}^*, a_{ir}^*)^T$
- Step 9: Calculate the distance S_j $j = (1,2,..,n)$ of alternative x_j from the FPIS a_i^* using equation 3.12.
- Step 10: According to the increasing order of the distances S_j $j = (1,2,..,n)$, the best alternative from the alternative set X is determined and the ranking order of all alternatives is generated.

Compared with the LINMAP method equation 3.28 and 3.29 can be used in fuzzy decision-making environments with linguistic ratings. Furthermore, to avoid the situation of $w_i = 0$ as it may be the case in the LINMAP method, the constraints $w_i \geq \varepsilon$ and $\sum_{i=1}^{m} w_i = 1$ are added to equation 3.28. If the fuzzy ratings f_{ik} and f_{ij} (or r_{ik} and r_{ij}) are reduced to the crisp ratings, equations 3.28 and 3.29 are reduced to the linear programming model of the LINMAP method in a crisp environment.

4. AN APPLICATION IN AUTOMOTIVE INDUSTRY

In an automotive company that located in Turkey, sourcing strategies for each commodity are determined at the beginning of a mid-cycle project. For the passenger airbag, the sourcing strategy is approved as market test. The details are given below:

- Automotive company: Located in Turkey
- Commodity: Passenger Airbag (PAB) / Restraints
- Sourcing type: Full-service-supplier
- Annual Volume: 60.250
- Project Life Cycle: 5 years

Full-service-suppliers provide engineering and product development service along with the product. In automotive industry, safety critic commodities are sourced to full-service-suppliers. The safety critic commodities are vital elements of an automobile, so supplier's experience is significant.

Annual volume depends on the project. Project life cycle ranges from 5 years to 10 years. A mid-cycle project is minor modified from the base program, so it usually lasts for 5 years. Life cycle of a base program is 10 years.

Although a mid-cycle project is minor modified from the base program, it includes both major and minor modified commodities. The sourcing strategy of minor modified parts is generally carryover which means to continue with the incumbent supplier. The major modified parts go through a market test, especially there is any tool investment needed.

In this application, the commodity is a passenger airbag which is a safety critic product. The project is 5-years-mid-cycle-project; the annual volume is 60.250 units. The RFQ letters are sent out the full-service-suppliers, the quotes are shown in Table 4.1.

Table 4.:. Quotations from the supplies

Supplier	Cost per unit	Tooling cost	ED&T cost	Logistic cost per unit
X1	35.50 €	340,000 €	330,000 €	3.57 €
X2	30.80 €	669,000 €	840,000 €	0.13 €
X3	40.00 €	498,000 €	348,000 €	3.57 €
X4	31.00 €	669,000 €	840,000 €	3.57 €

There are 4 candidate suppliers. X1 is the incumbent supplier, located in Poland. X3 is a new supplier, located in Poland. X2/X4 is a global supplier which has facilities both in Poland and Turkey. X2 is in Turkey, X4 is in Poland. The quotes include cost per unit, tooling cost, engineering design and testing (ED&T) and logistic costs.

Cost per unit is the piece price of a passenger airbag. Tooling cost is the capital investment, i.e. assembly stations, fixtures. ED&T is the payoff of full-service-supplier service. As stated earlier, full-service-suppliers give product development service. In return, they charge all the engineering related work to the customer. The logistic cost contains packaging of the product and transportation between the production facility to customer. The logistic cost is also given per unit.

As seen in the table, the best cost per unit is given by X2 however the worst tooling and ED&T are given by X2 also. The decision cannot be made by only comparing the quotations item to item. The decision maker needs supportive elements.

The decision maker calculates the annual turnover and life-cycle turnover for each supplier. Annual turnover, which is the first year cost of each supplier, is calculated as follows:

Annual Turnover = Tooling Cost + ED&T Cost + Volume x (Cost per Unit + Logistic Cost)

Tooling and ED&T costs are assumed to be paid in the first year, so they are included in the annual turnover. Life-cycle turnover is the life-cycle cost of each supplier and calculated as follows:

Life-cycle Turnover = Tooling Cost + ED&T Cost + Volume x (Cost per Unit + Logistic Cost) x Life-cycle

In this study, volume is 60,250 and life cycle is 5 years. The calculations are shown in Table 4.2.

Table 4.2: The turnovers of each supplier

Suppliers	Annual Turnover	Life-Cycle Turnover
X1	3,023,968 €	12,439,838 €
X2	3,372,533 €	10,826,663 €
X3	3,471,093 €	13,971,463 €
X4	3,591,843 €	11,923,213 €

Considering X1, annual turnover is calculated as below:

$$Annual\ Turnover(X1) = 340{,}000 + 330{,}000 + 60{,}250 * (35.50 + 3.57)$$

Thus, life-cycle turnover of X3 is calculated as:

$$Life - cycle\ turnover(X3) = 498{,}000 + 348{,}000 + 60{,}250 * (40.00 + 3.57) * 5$$

In respect to the annual turnover, X2 is the best candidate. However, X1 is the best supplier according to the life-cycle turnover. Considering X1 is the incumbent supplier, it is questionable to change the supplier for a 1.2mil€ gain in 5 years. Because, X1 is the most experienced supplier on the product.

Lastly, time adjusted rate of return (TARR) is calculated for each supplier in order to support the decision maker and show the best candidate. However, the difference is considerably slight. TARR calculations are given by the finance department to the decision maker. The TARRs are shown in Table 4.3.

Table 4.3: TARR results

Suppliers	TARR
X1	0.70
X2	0.73
X3	0.67
X4	0.65

As a result, an integrated approach is applied on the problem. In this study, fuzzy LINMAP method is proposed and the application steps are reviewed in the following section.

4.1 Fuzzy LINMAP criteria

The decision process of fuzzy LINMAP method is presented in section 3.6. The method is applied on the supplier selection process step by step in this section.

Step 1: The evaluation attributes are identified by the purchasing decision maker. Additional 2 linguistic criteria are determined by the decision maker. The quantitative criteria are listed below:

1. Piece cost (f1): One of the main indicators for purchasing decisions, net product price.
2. Tooling (f2): One of the cost indicators, the investment for tools.
3. ED&T (f3): One of the cost indicators, engineering, design and testing expenses.
4. Logistic costs (f4): Transport and packaging costs.
5. TARR (f5): Time adjusted rate of return, financial assessment of the quotes.

The additional qualitative (linguistic) criteria are listed below:

6. Reliability (f6): Determined by decision maker according to the experience, quality, organizational structure of the supplier.
7. Localization Opportunities (f7): The possibility of building a facility near the automotive company. This also includes tier 2 suppliers.

Table 4.4. shows decision information given by the decision maker, where $x_i$represents supplier and $f_i$represents criteria.

Table 4.4: Decision information given by decision maker

Suppliers \ Criteria	f1	f2	f3	f4	f5	f6	f7
	Piece price (€)	Tooling (€)	ED&T (€)	Logistics (€)	TARR %	Reliability	Localization opportunities
X1	35.50	340,000	330,000	3.57	0.7	H	VH
X2	30.80	669,000	840,000	0.13	0.73	H	M
X3	40.00	498,000	348,000	3.57	0.67	L	VL
X4	31.00	669,000	840,000	3.57	0.65	M	VL

The decision maker rates the supplier's reliability and localization opportunities. Thus, the criteria include linguistic variables.

Step 2: The preference relations between alternative suppliers are obtained. The decision maker prefers X1 to X2 or X3; X2 to X3 or X4. The relations are given below:

$$\Omega = \{(1,3), (1,2), (2,3), (2,4)\}$$

Step 3: The appropriate linguistic variables are chosen (Table 4.1 and Table 4.2).

Table 4.5 The relations between linguistic variables and positive trapezium fuzzy numbers

Linguistic variable	Trapezium fuzzy number
Very high (VH)	(0.7, 0.8, 0.9, 1.0)
High (H)	(0.5, 0.6, 0.7, 0.8)
Medium (M)	(0.3, 0.4, 0.5, 0.6)
Low (L)	(0.1, 0.2, 0.3, 0.4)
Very low (VL)	(0, 0.1, 0.2, 0.3)

Step 4: The appropriate linguistic variables are rated (Table 4.1 and Table 4.2).

Step 5: The fuzzy decision matrix *F* and the normalization positive trapezium fuzzy number decision matrix *R* are constructed.

$$F = \begin{pmatrix} 38 & 30.80 & 40 & 31 \\ 340,000 & 612,000 & 498,000 & 669,000 \\ 330,000 & 525,000 & 348,000 & 840,000 \\ 3.57 & 0.13 & 3.57 & 3.57 \\ 0.70 & 0.73 & 0.67 & 0.65 \\ (0.5,0.6,0.7,0.8) & (0.5,0.6,0.7,0.8) & (0.1,0.2,0.3,0.4) & (0.3,0.4,0.5,0.6) \\ (0.7,0.8,0.9,1.0) & (0.3,0.4,0.5,0.6) & (0.0,0.1,0.2,0.3) & (0.0,0.1,0.2,0.3) \end{pmatrix}$$

Fuzzy decision matrix is transformed into the following the normalized positive trapezium fuzzy number by using equations 3.7 and 3.8. f1, f2, f3, f4, which are the cost attributes, are normalized by using equation 3.8. f5, f6 and f7, which are the benefit attributes, are normalized by equation 3.7.

$$R = \begin{pmatrix} 0.81 & 1.00 & 0.77 & 0.99 \\ 1.00 & 0.56 & 0.68 & 0.51 \\ 1.00 & 0.63 & 0.95 & 0.39 \\ 0.04 & 1.00 & 0.04 & 0.04 \\ 0.96 & 1.00 & 0.92 & 0.89 \\ (0.63,0.86,1.00,1.00) & (0.63,0.86,1.00,1.00) & (0.13,0.29,0.50,0.80) & (0.38,0.57,0.83,1.00) \\ (0.70,0.89,1.00,1.00) & (0.30,0.44,0.63,0.86) & (0.0,0.11,0.25,0.43) & (0.0,0.11,0.25,0.43) \end{pmatrix}$$

4.2 Fuzzy LINMAP model

On step 6, the linear programming is constructed as follows:

There are $n = 4$ alternatives, $x_j (j = 1,2,3,4)$ which are ranked based on $m = 7$ attributes $f_i (i = 1,2, \dots ,7)$.

$\max(\lambda_{13} + \lambda_{12} + \lambda_{23} + \lambda_{24})$;

w_1*(((0.77^2-0.81^2)+ 2*(0.77^2-0.81^2)+ 2*(0.77^2-0.81^2)+ (0.77^2-0.81^2)) + ((1.00^2-0.81^2)+ 2*(1.00^2-0.81^2)+ 2*(1.00^2-0.81^2)+ (1.00^2-0.81^2))+((0.77^2-1.00^2)+ 2*(0.77^2-1.00^2)+ 2*(0.77^2-1.00^2)+ (0.77^2-1.00^2))+((0.99^2-1.00^2)+ 2*(0.99^2-1.00^2)+ 2*(0.99^2-1.00^2)+ (0.99^2-1.00^2)))+w_2*(((0.68^2-1.00^2)+ 2*(0.68^2-1.00^2)+ 2*(0.68^2-1.00^2)+ (0.68^2-1.00^2)) + ((0.56^2-1.00^2)+ 2*(0.56^2-1.00^2)+ 2*(0.56^2-1.00^2)+ (0.56^2-1.00^2))+((0.68^2-0.56^2)+ 2*(0.68^2-0.56^2)+ 2*(0.68^2-0.56^2)+ (0.68^2-0.56^2))+((0.51^2-0.56^2)+ 2*(0.51^2-0.56^2)+ 2*(0.51^2-0.56^2)+ (0.51^2-0.56^2)))+w_3*(((0.95^2-1.00^2)+ 2*(0.95^2-1.00^2)+ 2*(0.95^2-1.00^2)+ (0.95^2-1.00^2)) + ((0.63^2-1.00^2)+ 2*(0.63^2-1.00^2)+ 2*(0.63^2-1.00^2)+ (0.63^2-1.00^2))+((0.95^2-0.63^2)+ 2*(0.95^2-0.63^2)+ 2*(0.95^2-0.63^2)+ (0.95^2-0.63^2))+((0.39^2-0.63^2)+ 2*(0.39^2-0.63^2)+ 2*(0.39^2-0.63^2)+ (0.39^2-0.63^2)))+w_4*(((0.04^2-0.04^2)+ 2*(0.04^2-0.04^2)+ 2*(0.04^2-0.04^2)+ (0.04^2-0.04^2)) + ((1.00^2-0.04^2)+ 2*(1.00^2-0.04^2)+ 2*(1.00^2-0.04^2)+ (1.00^2-0.04^2))+((0.04^2-1.00^2)+ 2*(0.04^2-1.00^2)+ 2*(0.04^2-1.00^2)+ (0.04^2-1.00^2))+((0.04^2-1.00^2)+ 2*(0.04^2-1.00^2)+ 2*(0.04^2-1.00^2)+ (0.04^2-1.00^2)))+w_5*(((0.92^2-0.96^2)+ 2*(0.92^2-0.96^2)+ 2*(0.92^2-0.96^2)+ (0.92^2-0.96^2)) + ((1.00^2-0.96^2)+ 2*(1.00^2-0.96^2)+ 2*(1.00^2-0.96^2)+

(1.00^2-0.96^2))+((0.92^2-1.00^2)+ 2*(0.92^2-1.00^2)+ 2*(0.92^2-1.00^2)+ (0.92^2-1.00^2))+((0.89^2-1.00^2)+ 2*(0.89^2-1.00^2)+ 2*(0.89^2-1.00^2)+ (0.89^2-1.00^2)))+w_6*(((0.13^2-0.63^2)+ 2*(0.29^2-0.86^2)+ 2*(0.50^2-1.00^2)+ (0.80^2-1.00^2)) + ((0.63^2-0.63^2)+ 2*(0.86^2-0.86^2)+ 2*(1.00^2-1.00^2)+ (1.00^2-1.00^2))+((0.13^2-0.63^2)+ 2*(0.29^2-0.86^2)+ 2*(0.50^2-1.00^2)+ (0.80^2-1.00^2))+((0.38^2-0.63^2)+ 2*(0.57^2-0.86^2)+ 2*(0.83^2-1.00^2)+ (1.00^2-1.00^2)))+w_7*(((0.00^2-0.70^2)+ 2*(0.11^2-0.89^2)+ 2*(0.25^2-1.00^2)+ (0.43^2-1.00^2)) + ((0.39^2-0.70^2)+ 2*(0.44^2-0.89^2)+ 2*(0.63^2-1.00^2)+ (0.86^2-1.00^2))+((0.00^2-0.39^2)+ 2*(0.11^2-0.44^2)+ 2*(0.25^2-0.63^2)+ (0.43^2-0.86^2))+((0.00^2-0.39^2)+ 2*(0.11^2-0.44^2)+ 2*(0.25^2-0.63^2)+ (0.43^2-0.86^2)))-2*(v_1l*((0.77-0.81)+(1.00-0.81)+(0.77-1.00)+(0.99-1.00))+v_2l*((0.68-1.00)+(0.56-1.00)+(0.68-0.56)+(0.51-0.56))+v_3l*((0.95-1.00)+(0.63-1.00)+(0.95-0.63)+(0.39-0.63))+v_4l*((0.04-0.04)+(1.00-0.04)+(0.04-1.00)+(0.04-1.00))+v_5l*((0.92-0.96)+(1.00-0.96)+(0.92-1.00)+(0.89-1.00))+v_6l*((0.13-0.63)+(0.63-0.63)+(0.13-0.63)+(0.38-0.63))+v_7l*((0.00-0.70)+(0.39-0.70)+(0.00-0.39)+(0.00-0.39))+2*(v_1m*((0.77-0.81)+(1.00-0.81)+(0.77-1.00)+(0.99-1.00))+v_2m*((0.68-1.00)+(0.56-1.00)+(0.68-0.56)+(0.51-0.56))+v_3m*((0.95-1.00)+(0.63-1.00)+(0.95-0.63)+(0.39-0.63))+v_4m*((0.04-0.04)+(1.00-0.04)+(0.04-1.00)+(0.04-1.00))+v_5m*((0.92-0.96)+(1.00-0.96)+(0.92-1.00)+(0.89-1.00))+v_6m*((0.29-0.86)+(0.86-0.86)+(0.29-0.86)+(0.57-0.86))+v_7m*((0.11-0.89)+(0.44-0.89)+(0.11-0.44)+(0.11-0.44)))+2*(v_1n*((0.77-0.81)+(1.00-0.81)+(0.77-1.00)+(0.99-1.00))+v_2n*((0.68-1.00)+(0.56-1.00)+(0.68-0.56)+(0.51-0.56))+v_3n*((0.95-1.00)+(0.63-1.00)+(0.95-0.63)+(0.39-0.63))+v_4n*((0.04-0.04)+(1.00-0.04)+(0.04-1.00)+(0.04-1.00))+v_5n*((0.92-0.96)+(1.00-0.96)+(0.92-1.00)+(0.89-1.00))+v_6n*((0.50-1.00)+(1.00-1.00)+(0.50-1.00)+(0.83-1.00))+v_7n*((0.25-1.00)+(0.63-1.00)+(0.25-0.63)+(0.25-0.63)))+v_1r*((0.77-0.81)+(1.00-0.81)+(0.77-1.00)+(0.99-1.00))+v_2r*((0.68-1.00)+(0.56-1.00)+(0.68-0.56)+(0.51-0.56))+v_3r*((0.95-1.00)+(0.63-1.00)+(0.95-0.63)+(0.39-0.63))+v_4r*((0.04-0.04)+(1.00-0.04)+(0.04-1.00)+(0.04-1.00))+v_5r*((0.92-0.96)+(1.00-0.96)+(0.92-1.00)+(0.89-1.00))+v_6r*((0.80-1.00)+(1.00-1.00)+(0.80-1.00)+(1.00-1.00))+v_7r*((0.43-1.00)+(0.86-1.00)+(0.43-0.86)+(0.43-0.86))) >= -1;

w_1*((0.81^2-0.77^2)+2*(0.81^2-0.77^2)+2*(0.81^2-0.77^2)+(0.81^2-0.77^2))+w_2*((1.00^2-0.68^2)+2*(1.00^2-0.68^2)+2*(1.00^2-0.68^2)+(1.00^2-0.68^2))+w_3*((1.00^2-0.95^2)+2*(1.00^2-0.95^2)+2*(1.00^2-0.95^2)+(1.00^2-0.95^2))+w_4*((0.04^2-0.04^2)+2*(0.04^2-0.04^2)+2*(0.04^2-0.04^2)+(0.04^2-0.04^2))+w_5*((0.96^2-0.92^2)+2*(0.96^2-0.92^2)+2*(0.96^2-0.92^2)+(0.96^2-0.92^2))+w_6*((0.63^2-0.13^2)+2*(0.86^2-0.29^2)+2*(1.00^2-0.50^2)+(1.00^2-0.80^2))+w_7*((0.70^2-0.00^2)+2*(0.89^2-0.11^2)+2*(1.00^2-0.25^2)+(1.00^2-0.43^2))-2*(v_1l*(0.81-0.77)+v_2l*(1.00-0.68)+ v_3l*(1.00-0.95)+ v_4l*(0.04-0.04)+

v_5l*(0.96-0.92)+ v_6l*(0.63-0.13)+ v_7l*(0.70-0.00)+2*(v_1m*(0.81-0.77)+ v_2m*(1.00-0.68)+ v_3m*(1.00-0.95)+v_4m*(0.04-0.04)+ v_5m*(0.96-0.92)+ v_6m*(0.86-0.29)+ v_7m*(0.89-0.11))+2*(v_1n*(0.81-0.77)+ v_2n*(1.00-0.68)+ v_3n*(1.00-0.95)+v_4n*(0.04-0.04)+ v_5n*(0.96-0.92)+ v_6n*(1.00-0.50)+ v_7n*(1.00-0.25))+v_1r*(0.81-0.77)+v_2r*(1.00-0.68)+ v_3r*(1.00-0.95)+ v_4r*(0.04-0.04)+v_5r*(0.96-0.92)+ v_6r*(1.00-0.80)+ v_7r*(1.00-0.43)) +6*y_13 >= 0;

w_1*((0.81^2-1.00^2)+2*(0.81^2-1.00^2)+2*(0.81^2-1.00^2)+(0.81^2-1.00^2))+w_2*((1.00^2-0.56^2)+2*(1.00^2-0.56^2)+2*(1.00^2-0.56^2)+(1.00^2-0.56^2))+w_3*((1.00^2-0.63^2)+2*(1.00^2-0.63^2)+2*(1.00^2-0.63^2)+(1.00^2-0.63^2))+w_4*((0.04^2-1.00^2)+2*(0.04^2-1.00^2)+2*(0.04^2-1.00^2)+(0.04^2-1.00^2))+w_5*((0.96^2-1.00^2)+2*(0.96^2-1.00^2)+2*(0.96^2-1.00^2)+(0.96^2-1.00^2))+w_6*((0.63^2-0.63^2)+2*(0.86^2-0.86^2)+2*(1.00^2-1.00^2)+(1.00^2-1.00^2))+w_7*((0.70^2-0.39^2)+2*(0.89^2-0.44^2)+2*(1.00^2-0.63^2)+(1.00^2-0.86^2))-2*(v_1l*(0.81-1.00)+v_2l*(1.00-0.56)+ v_3l*(1.00-0.63)+ v_4l*(0.04-1.00)+ v_5l*(0.96-1.00)+ v_6l*(0.63-0.63)+ v_7l*(0.70-0.39)+2*(v_1m*(0.81-1.00)+ v_2m*(1.00-0.56)+ v_3m*(1.00-0.63)+ v_4m*(0.04-1.00)+ v_5m*(0.96-1.00)+ v_6m*(0.86-0.86)+ v_7m*(0.89-0.44))+2*(v_1n*(0.81-1.00)+ v_2n*(1.00-0.56)+ v_3n*(1.00-0.63)+ v_4n*(0.04-1.00)+ v_5n*(0.96-1.00)+ v_6n*(1.00-1.00)+ v_7n*(1.00-0.63))+v_1r*(0.81-1.00)+v_2r*(1.00-0.56)+ v_3r*(1.00-0.63)+ v_4r*(0.04-1.00)+ v_5r*(0.96-1.00)+ v_6r*(1.00-1.00)+ v_7r*(1.00-0.86)) + 6*y_12 >= 0;

w_1*((1.00^2-0.77^2)+2*(1.00^2-0.77^2)+2*(1.00^2-0.77^2)+(1.00^2-0.77^2))+w_2*((0.56^2-0.68^2)+2*(0.56^2-0.68^2)+2*(0.56^2-0.68^2)+(0.56^2-0.68^2))+w_3*((0.63^2-0.95^2)+2*(0.63^2-0.95^2)+2*(0.63^2-0.95^2)+(0.63^2-0.95^2))+w_4*((1.00^2-0.04^2)+2*(1.00^2-0.04^2)+2*(1.00^2-0.04^2)+(1.00^2-0.04^2))+w_5*((1.00^2-0.92^2)+2*(1.00^2-0.92^2)+2*(1.00^2-0.92^2)+(1.00^2-0.92^2))+w_6*((0.63^2-0.13^2)+2*(0.86^2-0.29^2)+2*(1.00^2-0.50^2)+(1.00^2-0.80^2))+w_7*((0.39^2-0.00^2)+2*(0.44^2-0.11^2)+2*(0.63^2-0.25^2)+(0.86^2-0.43^2))-2*(v_1l*(1.00-0.77)+v_2l*(0.56-0.68)+ v_3l*(0.63-0.95)+ v_4l*(1.00-0.04)+ v_5l*(1.00-0.92)+ v_6l*(0.63-0.13)+ v_7l*(0.39-0.00)+2*(v_1m*(1.00-0.77)+ v_2m*(0.56-0.68)+ v_3m*(0.63-0.95)+ v_4m*(1.00-0.04)+ v_5m*(1.00-0.92)+ v_6m*(0.86-0.29)+ v_7m*(0.44-0.11))+2*(v_1n*(1.00-0.77)+ v_2n*(0.56-0.68)+ v_3n*(0.63-0.95)+v_4n*(1.00-0.04)+ v_5n*(1.00-0.92)+ v_6n*(1.00-0.50)+ v_7n*(0.63-0.25))+v_1r*(1.00-0.77)+v_2r*(0.56-0.68)+ v_3r*(0.63-0.95)+ v_4r*(1.00-0.04)+ v_5r*(1.00-0.92)+ v_6r*(1.00-0.80)+ v_7r*(0.86-0.43))+ 6*y_23 >= 0;

w_1*((1.00^2-0.99^2)+2*(1.00^2-0.99^2)+2*(1.00^2-0.99^2)+(1.00^2-0.99^2))+w_2*((0.56^2-0.51^2)+2*(0.56^2-0.51^2)+2*(0.56^2-0.51^2)+(0.56^2-0.51^2))+w_3*((0.63^2-0.39^2)+2*(0.63^2-0.39^2)+2*(0.63^2-0.39^2)+(0.63^2-0.39^2))+w_4*((1.00^2-0.04^2)+2*(1.00^2-0.04^2)+2*(1.00^2-0.04^2)+(1.00^2-0.04^2))+w_5*((1.00^2-0.89^2)+2*(1.00^2-0.89^2)+2*(1.00^2-0.89^2)+(1.00^2-0.89^2))+w_6*((0.63^2-0.38^2)+2*(0.86^2-0.57^2)+2*(1.00^2-0.83^2)+(1.00^2-1.00^2))+w_7*((0.39^2-0.00^2)+2*(0.44^2-0.11^2)+2*(0.63^2-0.25^2)+(0.86^2-0.43^2))-2*(v_1l*(1.00-0.99)+v_2l*(0.56-0.51)+ v_3l*(0.63-0.39)+ v_4l*(1.00-0.04)+ v_5l*(1.00-0.89)+ v_6l*(0.63-0.38)+ v_7l*(0.39-0.00)+2*(v_1m*(1.00-0.99)+ v_2m*(0.56-0.51)+ v_3m*(0.63-0.39)+ v_4m*(1.00-0.04)+ v_5m*(1.00-0.89)+ v_6m*(0.86-0.57)+ v_7m*(0.44-0.11))+2*(v_1n*(1.00-0.99)+ v_2n*(0.56-0.51)+ v_3n*(0.63-0.39)+ v_4n*(1.00-0.04)+ v_5n*(1.00-0.89)+ v_6n*(1.00-0.83)+ v_7n*(0.63-0.25))+v_1r*(1.00-0.99)+v_2r*(0.56-0.51)+ v_3r*(0.63-0.39)+ v_4r*(1.00-0.04)+v_5r*(1.00-0.89)+ v_6r*(1.00-1.00)+ v_7r*(0.86-0.43)) + 6*y_24 >= 0;

w_1 >= 0.001; w_2 >= 0.001; w_3 >= 0.001; w_4 >= 0.001; w_5 >= 0.001;

w_6 >= 0.001; w_7 >= 0.001;

w_1+w_2+w_3+w_4+w_5+w_6+w_7=1;

v_1l>=0; v_1m>=0; v_1n>=0; v_1r>=0;

v_2l>=0; v_2m>=0; v_2n>=0; v_2r>=0;

v_3l>=0; v_3m>=0; v_3n>=0;v_3r>=0;

v_4l>=0; v_4m>=0; v_4n>=0; v_4r>=0;

v_5l>=0; v_5m>=0; v_5n>=0; v_5r>=0;

v_6l>=0; v_6m>=0; v_6n>=0; v_6r>=0;

v_7l>=0; v_7m>=0; v_7n>=0; v_7r>=0;

$\lambda_{13} \geq 0$; $\lambda_{12} \geq 0$; $\lambda_{23} \geq 0$; $\lambda_{24} \geq 0$;

The equation is solved by using Simplex method of the linear programming by Lingo 14.0. According to preference relations and distance of each alternative from the FPIS, the ranking order of all alternatives are generated.

4.3 Fuzzy LINMAP solution

The optimal solution is obtained as follows:

$$w = (w_1, w_2, w_3, w_4, w_5, w_6, w_7)^T = (0{,}261, 0{,}15{,}0{,}192{,}0{,}148{,}0{,}161{,}0{,}214{,}0{,}175)^{T}$$

And

$$v = (v_1, v_2, v_3, v_4, v_5, v_6, v_7)$$
$$= (0{,}425{,}0{,}496{,}0{,}508{,}0{,}478, 0{,}505(0{,}501{,}0{,}516{,}0{,}498{,}0{,}502), (0{,}513{,}0{,}509{,}518{,}0{,}520))$$

The fuzzy ideal solution (FPIS) is calculated as by using equation 3.29:

$a^* = (a_1^*, a_2^*, a_3^*, a_4^*, a_5^*, a_6^*, a_7^*) =$
(1,876,2,410,2,374,2,215,2,005,(0,416,0,421,0,421,0,425),(0,568,0,574,0574,0,579))

The square of the distance of each supplier from the FPIS a^* is calculated below:

$$S_1 = 5{,}260$$

$$S_2 = 4{,}210$$

$$S_3 = 5{,}544$$

$$S_4 = 4{,}336$$

According to the square of the distance from the FPIS, the ranking order of four passenger airbag suppliers is obtained as follows:

$$x_2 > x_4 > x_1 > x_3$$

The best supplier is X2; the result is in parallel with the TARR calculations. The second supplier is X4 which actually represents another location of X2. The third supplier is X1 which is the incumbent supplier. The least appropriate supplier is X3 which has the least reliability rating also.

X2 is also best in four factors: piece cost, logistics, TARR, and reliability. The results are consistent with the decision maker information.

5. RESULTS AND DISCUSSION

The aim of this study is to construct a decision support system for supplier selection and evaluation. It is insufficient to evaluate suppliers by cost effect in most cases. There are various criteria affecting sourcing decisions. It is aimed to build an integrated method that includes quantitative and qualitative calculations together.

Supply chain management has become a significant issue in real life and in the literature as well in the last decades due to increasing globalization, competition, etc. Moreover, supplier selection and periodical evaluation has ever been important tool for the companies in order to maintain an effective supply chain. Companies tend to cooperate with the best suppliers. Thus, managing suppliers is also becoming more important now, because strategic partnerships are being implemented with vendors to sustain a competitive advantage.

Since supplier selection/purchasing decisions have an effect on profitability, also time is the most critical constraint for the decision process, standardizing the process is very useful for the companies. Selecting the right supplier significantly reduces the total cost, and develops the customer-supplier relationship. Supplier selection problem, which includes multiple conflicting and imprecise criteria, is a type of multi-criteria group decision making problem. Because of vague criteria, the classical MADM methods are not sufficient for supplier selection problem.

The approaches on supplier selection problem are briefly summarized in section 2. In the literature, there are studies on linear programming, non-linear programming, AHP, ANP, integer programming, case-based reasoning, and also fuzzy set theory. In this study, a fuzzy integrated approach, fuzzy LINMAP is proposed to select the best supplier in terms of fuzzy environment. In addition to cost reduction advantage, modelling the problem also provides time-effective purchasing processes.

Fuzzy LINMAP is not restricted to automotive industry supplier selection; the approach could also be applied to the supplier selection of any industry. It should be noted that both every industry and every product should have own evaluating criteria. So, it is important to investigate and identify the product evaluating criteria. Then, the model could be applied for sourcing decision of any other sectors.

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BIOGRAPHICAL SKETCH

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