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GRADUATE SCHOOL OF SCIENCE AND ENGINEERING

A MULTI-STAGE NEW PRODUCT DEVELOPMENT USING TYPE-2 FUZZY NUMBERS IN A REAL OPTION VALUATION

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(GERÇEK OPSİYONLAR DEĞERLEMESİNDE TİP-2 BULANIK SAYILAR KULLANILARAK ÇOK AŞAMALI YENİ ÜRÜN GELİŞTİRME)

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This is to certify that the thesis entitled

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LIST OF SYMBOLS

S : Stock price / Present value of a project's operating cash flows

X : Exercise price / Investment costs

T : Time to expiry / Length of time the decision may be deferred

r_f : Risk-free interest rate / Time value of money

σ^2 : Standard deviation of returns on stock / Risk of the Project

r : Constant risk-free rate (discount factor)

$N(d_1)$: Probability that the option will expire in the money

$N(d_2)$: Risk adjusted probability that the option will be exercised

u : Upper multiplier

$\tilde{\tilde{u}}$: Upper multiplier (type-2)

$\tilde{u}_H$: Upper multiplier (high volatility / type-1)

$\tilde{u}_L$: Upper multiplier (low volatility / type-1)

d : Down multiplier

$\tilde{\tilde{d}}$: Down multiplier (type-2)

$\tilde{d}_H$: Down multiplier (high volatility / type-1)

$\tilde{d}_L$: Down multiplier (low volatility / type-1)

p : Probability to move the upper state

q : Probability to move the down state (1-p)

V_0 : Value at the beginning of time (t=0)

V : Value of the project at time t

V_u : Value of the project at the upper state (t+1)

V_d : Value of the project at the down state (t+1)

C : Call option value

C_u : Call option value at the upper state

C_d : Call option value at the down state

$\mu_{\tilde{A}}$: Membership function of a type-1 fuzzy set $\tilde{A}$

$\tilde{A}$: Type-1 fuzzy set

x : Cluster element

$\tilde{\tilde{A}}$: Type-2 cluster

c : Center of the membership function

d : Width of the membership function

x : Input vector

$\tilde{A}_U$: Type-2 fuzzy set, upper bound

$\tilde{A}_L$: Type-2 fuzzy set, lower bound

$a_{1, \ldots, 4}$: Reference points

$\tilde{\tilde{\sigma}}$: Volatility (type-2)

$\tilde{\sigma}_H$: High volatility (type-1)

$\tilde{\sigma}_L$: Low volatility (type-1)

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ABSTRACT

Introducing new products is a key factor for the success and survival of companies. Managers need to examine every possibility that can occur during the life span of their new product and focus on the right strategy. Even then, launching a new product is highly risky due to uncertainties of the market and competitors.

The chosen bank is a Netherland origin multinational bank that got into the Turkish market in 2008 and the product of the project is a saving account that supports the bank's vision. The main challenge for this bank is the adjustment of this new market and its vagueness.

Traditional discounted cash flow models have some major disadvantages, and they under estimate the value of investments. This study presents a proposal of real option evaluation through fuzzy logic for a new product development project in a retail banking market with Type-2 fuzzy sets. A multi-stage new product development is used to have a better understanding of the long term success of the project.

One of the real options evaluation techniques is binomial lattice model. This technique is mostly suitable for managerial flexibility decisions. In this study different from the literature Binomial Lattice method and Type-2 fuzzy sets are combined to cope with uncertainties of the practice. Finally, a numerical analysis is presented to demonstrate the compound option pricing under fuzzy environment.

Keywords—*Real Options; Fuzzy Type-2 Sets; Multi-Stage Project Evaluation; New Product Development*

ÖZET

Piyasaya yeni ürünler sürmek şirketler için başarı ve sürdürülebilirliklerini sağlama yönünde anahtar bir faktördür. Yöneticiler yeni ürünlerinin yaşam döngüsünde başlarına gelebilecek her tür olasılığı incelemek ve doğru stratejiye odaklanmak zorundadırlar. Her türlü olasılığın değerlendirilmesi halinde dahi, yeni bir ürünün piyasaya sürülmesi piyasadaki ve rakip firma stratejilerindeki belirsizlikler nedeniyle yüksek risk taşır.

Seçilen banka 2008 yılında Türkiye pazarına giren Hollanda kökenli çok uluslu bir bankadır ve projedeki ürün bankanın vizyonunu destekleyen bir yatırım hesabıdır. Banka için en büyük risk bu yeni piyasa ve belirsizliklerine uyum göstermektir.

Geleneksel nakit akış teknikleri çok büyük dezavantajlara sahiptir ve bu teknikler yatırımın gerçek değerini ölçümleyemezler. Bu çalışma bireysel bankacılık sektöründeki yeni ürün geliştirme projesinin gerçek opsiyonlar ve tip-2 bulanık sayılar ile değerlendirilmesi konusunda bir öneri getirir. Projenin uzun soluklu başarısının daha iyi ölçümlenmesi için ise çok aşamalı bir yeni ürün geliştirme metodu kullanılmıştır.

Gerçek opsiyon tekniklerinden biri de binomial ağaç modelidir. Bu teknik esnek yönetsel kararlar için oldukça uygundur. Çalışmada, uygulamadaki belirsizlikler ile baş etmek için, literatürden farklı olarak, binomial lattice metodu ile tip-2 bulanık sayılar birlikte kullanılmıştır. Son olarak, bulanık sayılarla bileşik opsiyon fiyatlama örneklendirilmek amacıyla sayısal bir örnek tanıtılmıştır.

Anahtar Sözcükler—*Gerçek Opsiyonlar; Tip-2 Bulanık Sayılar; Çok Aşamalı Değerleme; Yeni Ürün Geliştirme*

1. INTRODUCTION

Capital investment decision-making is crucial for long-term success of the corporations. Introducing new products is critical for survival of companies and making of a decision to invest is a multi-stage interactive process which includes certain stages such as; idea generation, business analysis, testing and commercialization. Especially for retail banking, it is highly risky due to the amount of money, reputation and credibility.

Retail banking refers to the customer-oriented services offered by commercial banks. These services include saving accounts, mortgages, loans and investment products. Most commercial bank’s goal is to reach a wider consumer base. Launching new products to the target audience serves this purpose if they have right marketing, strategy and timing.

The commercial bank, subject to this project, is a Netherland origin multinational bank with diverse international activities, founded in 1991. Bank got into the Turkish market in 2008, and still continues its activities successfully. As a foreign new player in the field, the adjustment remains a challenge for the company and brings uncertainty.

Managers need to examine every possibility that can occur during the life span of their new products and focus on the right strategy. Even then, launching a new product is highly risky due to uncertainties of the market and competitors. Once new product is on the market, customer experiences and feedbacks are great ways to develop new ideas and going further.

Bank considers releasing a new saving account, which support their vision. This new product has a high interest rate for customers; it requires the use of their deposit accounts and credit cards. Fluctuation in the market, entrance of new competitors and similar products make it hard to see the profit of the product, and to decide the strategy.

These uncertainties are almost impossible to be represented by crisp values and traditional methods are inadequate to solve real life problems. Our study touches a field that very few academicians have explored. Practitioners excessively used the net present value (NPV) and discounted cash flow (DCF) techniques to evaluate project investments. Given that, the real world is characterized by change, uncertainty and competitive interactions, these methods are ineligible to have the best perspective.

Traditional discounted cash flow models have some major disadvantages, and they under estimate the value of investments. While these traditional techniques make implicit assumptions, the real options theory focus on managerial flexibility and uncertainty of real world management decisions. Many aspects of investment decision-making problems are similar to real options. Businesses usually have the option to make an investment but they are not obliged to do so but they can defer, expand, abandon, or put-on-hold a project.

As Real Options, Fuzzy Logic has been widely used to provide framework to deal with uncertain project parameters or for handling the lack of certainty in data. Unlike the crisp parameters used in real options theory, fuzzy real option evaluations allow fuzzy parameters and practitioners can detect objectively real options embedded in projects.

Hsien-Chung Wu (2007) considered fuzzy interest rate, fuzzy volatility and fuzzy stock price to generate the pricing boundaries of the European call and put options using Black and Scholes formula. In this manner, Young-Chan Lee and Seung-Seok Lee (2011) used a fuzzy approach to evaluate RFID investments. They chose Black and Scholes' method to handle supply chain decisions with real options embedded inside. And Shiu-Hwei Ho and Shu-Hsien Liao (2011) used fuzzy binomial approach for investment project valuation in an uncertain environment.

The structure of a compound option is an option on another. The exercise payoff of a compound option includes the value of the other option. This type of combined options has more than one expiration date and strike price.

This study presents a proposal of real option evaluations through fuzzy logic. The proposed model is a framework that combines real options and decision tree analyses.

These techniques are performed in a multi-stage approach because of managerial reasons of the bank and upper-lower bounds are traced for a better understanding of the possibilities. Finally, a numerical analysis presents the compound option pricing under fuzzy environment.

Binomial evaluation method is mixed with type-2 fuzzy set theory to answer decision making of a new product launch in retail banking market. Binomial tree method shows the future possible results explicitly. The most invincible characteristic of the binomial method is the flexibility of the variables that allows the practitioners to change the direction anytime during the life of the options. This quality makes the binomial tree method eminently convenient for real options.

The research methodology will be quantitative in nature. A discrete-time, real options valuation model will be developed. Binomial tree structure will allow graphically illustration of the stochastic process of the underlying risky asset. Project will be divided into two stages in order to observe the success of the new product. And the essential variable of options, volatility, is decided as the main uncertainty of the project.

This thesis is formed of nine chapters and organized as follows. In first chapter, we introduce the value of our research; we highlight our research goals and outputs that we try to obtain. In chapter 2, we will conduct a research for Real Options theory and review the literature of both real options and type-2 fuzzy numbers. In chapter 3, several definitions in financial options are given and the concept is explained with figures.

In chapter 4, we present real options theory, types of real options, show differences and similarities to financial options. Real options valuation methods and the comparison between these techniques will be analyzed. In chapter 5, we introduce fuzzy set theory and its extensions. In chapter 6, our approach which combines these two methods will be presented with details and Fuzzy type-2 sets will be applied to Binomial Lattice. In chapter 7, the model adopted will be applied to evaluate real options of a new product development project. In chapter 8, compound option model will be demonstrated with fuzzy variables.

In chapter 9, we will discuss the methodology used in our application and its limitations. In the final chapter, chapter 10, we conclude the study, evaluate our findings, our approach and suggest future research directions.

The aim of this thesis is to fill the gap in the literature in this field and provide managers a simple tool to evaluate projects with a real options perspective. A Dutch commercial bank is chosen to observe the process of a new product development project and real options embedded in this problematic are discovered with a structured methodology.

The objective can be cited as; application of real option valuation in the retail banking market by utilizing an efficient model, binomial lattice method, in order to determine net present value of a potential new product development project, and combination of this technique with type-2 fuzzy sets to avoid deviations of the variables in the framework.

2. LITERATURE REVIEW

Several studies, like the traditional Discounted Cash Flow (DCF) methods and the Real Option Valuation technique, have been made to evolve better understanding and to sustain decision making. However, they are in reality inaccurate when examining projects with high levels of uncertainty.

The most widely utilized investment valuation methods, discounted cash flow (DCF) and net present value (NPV) analysis are inadequate when evaluating real-life management problems. (Dixit & Pindyck, (1994); Hoesli, Jani, & Bender, (2006); Copeland & Antikarov, (2003)). Practitioners' knowledge of the investment determines the expected value of each alternative.

Trigeorgis and Manson (1987) underlines that when the businesses use traditional NPV methods to make decision, their theories are based on the assumption that the estimated future cash flows are certain. Therefore if uncertainty exists, the NPV or DCF cannot estimate the management flexibility of changes.

Myers (1984), Pindyck (1991), and Trigeorgis (1993) all underline that Net Present Value technique ignores flexibility of real market investment opportunities. Dixit and Pindyck (1994) claim that these valuation techniques are less accurate; when the cost of the investment is partially or completely irreversible or timing of the investment is flexible.

Even before the concept of Real Options was stated, some economists like Roberts and Weitzman pointed out that in multi-stage decision making, even with a negative cash flow, it may be worthwhile to start an investment. This flexibility does add significant value to potential projects and this value grows as the volatility of potential project cash flows grows. (Copeland & Antikarov, (2003). It reflects synergies that NPV misses. (Galli & Armstrong, 1999)

NPV of the real asset investment = NPV of estimated cash flows + option values.

In the literature, there is a wide diversity of valuation models that attempt to find the value of an investment. Real options valuation method is one of them, they are originated from financial options. S.C. Myers, T.E. Copeland, A.K. Dixit and R.S. Pindyck, L. Trigeorgis, A.J. Triantis, T.A. Luehrman and M. Amram can be called the gurus of the topic of Real Options.

Avinash Dixit and Robert Pindyck (1994) have done a lot of research in this area and known by their book: “Investment under Uncertainty". Thomas Copeland (2003) is also an expert in financial decision-making. In his book, “Real-Options: A Practioner's Guide", he studies many valuation methods and analyze all types of Real-Options. Lenos Trigeorgis is again a great contributor, an advocate and a pioneer to the Real-Options theory and practice.

Copeland and Antikarov (2003), Schwartz and Trigeorgis (2003), Dixit and Pindyck (1994) and Brennan and Trigeorgis (2000) introduce comprehensive studies on this subject. Copeland and Antikarov (2003) present a computer based model that helps valuating real options. Dixit and Pindyck study switch options to reach optimal solution and the others give several examples on the topic.

Common features in capital investment projects are uncertainty, irreversibility and flexibility. Real options theory has been used in many industries, Lander and Pinches (1998) summarize these application areas as natural resources, corporate strategy, R&D projects, real estate and others (Table 1).

Table 1 Real option topics and areas of application (Lander and Pinches, 1998)

Author(s)	Pub. year	Sector
Titman	1985	Real Estate
Brennan and Schwartz	1985	Natural Resource Valuation
Paddock et al.	1988	Natural Resource Valuation
Bowman and Hurry	1993	Corporate Strategy
Kumaraswamy	1996	R&D
McGrath	1997	Corporate Strategies
Childs and Triantis	1999	R&D
Kogut and Kulatilaka	2001	Corporate Strategies
Dias	2004	Natural Resource Valuation

New product introduction constitute 36.2 % of researches (Table 2). This application area is well suited to Real Option approach with the lack of information on the market and on competitors. Vagueness and high risk are the main properties of new product development projects and also by the reason that it requires high investment cost at the beginning of the project.

Table 2 Application fields of Real Options (Lander and Pinches, 1998)

Application fields	Used by (%)
New product introduction	36.2 %
Research & Development	27.8 %
Mergers or acquisitions	22.1 %
Foreign investment	9.6 %
Other	4.3%
Total	**100 %**

In literature, there exist various types of type-2 fuzzy membership functions, i.e. gaussian, triangular, trapezoidal, etc. and type-2 fuzzy logic has been used in problems such as clustering and classification to provide a simple tool that allows resolution of uncertainty in complex problems.

Table 3 Type-2 fuzzy systems in clustering and classification

Author(s)	Pub. Year	Domain	Comparison with type-1	Type-2
(Sharma & Bajaj)	2010	Vehicle classification	Yes	Uncertainty and imperfection of data
(Hosseini et al.)	2010	Computer aided detection	Yes	Imprecision in image processing
(Wu & Mendel)	2010	Vehicle classification	Yes	Uncertainty in information
(Yu et al.)	2009	Classification and forecasting	No	Uncertainty in data
(Chua & Tan)	2008	Automotive classification	Yes	Uncertainty in classification
(Zhang et al.)	2007	Clustering	No	Uncertainty in clustering
(Liang & Mendel)	2000	Classification	Yes	Uncertainty in classification

Various real option valuation methods appear in the literature to address valuation problems under uncertainty.

Carlsson and Fuller (2002) combine possibility theory and fuzzy real option valuation to evaluate options. Thavaneswaran (2009) present a volatility model with fuzzy sets theory. Guerra et al. (2007) uses the Black and Scholes method to study option pricing with fuzzy numbers and conduct a sensitivity analysis.

Nowak and Romaniuk (2010) present a technique for option pricing based on stochastic analysis using fuzzy numbers. Thiagarajah et al. (2006) introduce the Black-Scholes model with quadratic adaptive fuzzy numbers.

Liu and Ouyang (2003) generate a decision making model that is base on real options, with this model they try to conduct a strategic project investment process. They discuss the best investment opportunities and based on the results they evaluate real options, expand or abandon the project.

Li, Qu and Feng (2003) use a two stages investment decision making model with real options techniques. They try to estimate the flexible and optimal investment value in present market risk. This approach provides great support for short-term investment decision-making.

Black–Scholes method is used by Benaroch and Kauffman (1999) to evaluate projects for a electronic banking network in New England. Copeland and Antikarov (2001) choose again real option valuation techniques to figure out which online sales method is the best for the American brand who manufacture system recovery software.

Cherubini (1997) utilizes a fuzzified version of the Black and Scholes model. Ghaziri et al. (2000) present artificial intelligence as a way to price options and use fuzzy logic. Zmeskal (2001) apply Black-Scholes for a European call option by using the input data in a form of fuzzy numbers.

Carlsson and Fuller (2001) study Real Options with fuzzy logic where present values of expected cash flows and costs are represented by trapezoidal fuzzy numbers. In their paper called “A fuzzy approach to real option valuation” they try to find the optimal exercise time of the option and how long an investment can be postponed.

Collan et al., (2009) introduce a technique that evaluate and analyze Real Options, cost and revenues are also presented as fuzzy numbers to estimate expected future pay-off. In their paper Carlsson and Fuller study a model for valuing real options for R&D projects. The future cash flows and expected costs are presented by trapezoidal fuzzy numbers.

In real world, a capital investment project can be split into many stages. It reduces risks of the project. Childs and Triantis (1999) presented a model of dynamic R&D investment which underlines the interactions across projects.

In the real market, due to market fluctuations and human errors, information cannot be collected precisely. The literature on option pricing under the fuzzy environment studies the European option and based on the Black-Scholes formula. For example, Yoshida (2003) uses fuzzy logic in a stochastic model for European options. Wu (2007) transforms the Black-Scholes formula to a fuzzy model by using interest rate, volatility, and stock price as fuzzy numbers.

3. FINANCIAL OPTIONS

Option is a type of futures contract that gives the buyer the right to buy or sell a financial instrument at a future date. Strike Price or exercise price is the agreed future price that an asset can be bought or sold and the price paid to gain this right is called Option Premium.

Options can be classified in two types, American or European options. The difference between these two is in the possibility of the time of exercising the option. While European Option gives the buyer the right to exercise the option only at the agreed upon date, American Option, gives the buyer the right to exercise the option at any time before the expiration date. American Options are the most common ones to be traded on the exchanges. On the other hand, European Options are easier to analyze. In both cases, the exercise is irreversible.

Since option is a right but not an obligation, the option holder can decide not to exercise the right and let the option expire. There are two types of options; call options and put options. A Call Option is used to hedge the risk against the price of an instrument going up and a Put Option is used to hedge the risk against the price of an instrument going down.

Two sides are present in every option agreement. On the one side there is an financier, who has taken the long position which means he has bought the option. And on the other, there is another financier, who has taken a short position who has sold the option. The profit or loss for the writer is the reverse of that for the purchase of the option (Hull, 2010).

In this research a lease contract will be discussed and this contract can be seen as a European option. A lease contract, for say five years, will expire five years from now. This term is fixed and the renewal contract (the option) can only be exercised at the specific date five years from now, when the contract expires. The new contract will only be valid from the expiration date, if renewal is the case. This is the same with a European option where it is only possible to exercise the option at expiration date.

2.1. Call Options

A call option is a type of option who gives the option buyer the right to buy the underlying asset at a predetermined fixed price. If at the expiration date, the value of this underlying asset is less than the strike price, the option is not exercised by its owner and expires. The option holder loses the premium paid at the beginning of the deal. In the same manner, if the value of the underlying asset is greater than the strike price, the option is exercised and the option holder gets the profit.

Gross profit in this deal is the difference between the underlying asset value and the exercise price. The net profit on the investment is the difference between the gross profit and the price paid for the call initially. This is illustrated in the figure below

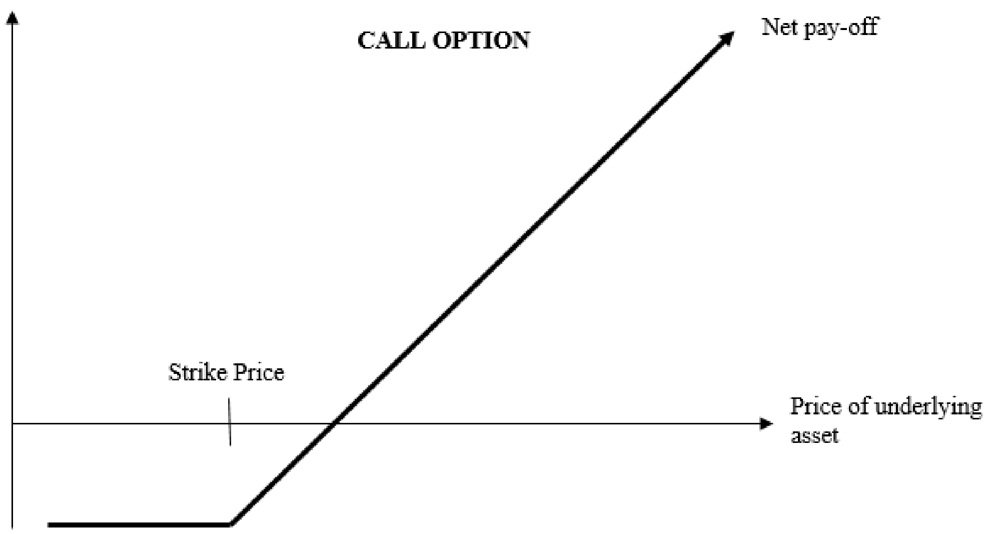

Figure 1 Pay-off Diagram of a Call Option

2.2. Put Options

A put option is a type of option who gives the option buyer the right to sell the underlying asset at a predetermined fixed price. The buyer pays a price called "option premium" for this right. If the price of the underlying asset is greater than the strike price, the option will not be exercised by its owner; if not the put option will be exercised.

A put option gets a negative pay off if the value of the underlying asset passes the strike price. The gap between the strike price and the value of the underlying asset is called the gross pay off in the case where the asset value is less than the strike price. It can be summarized by the figure below Fig.2.

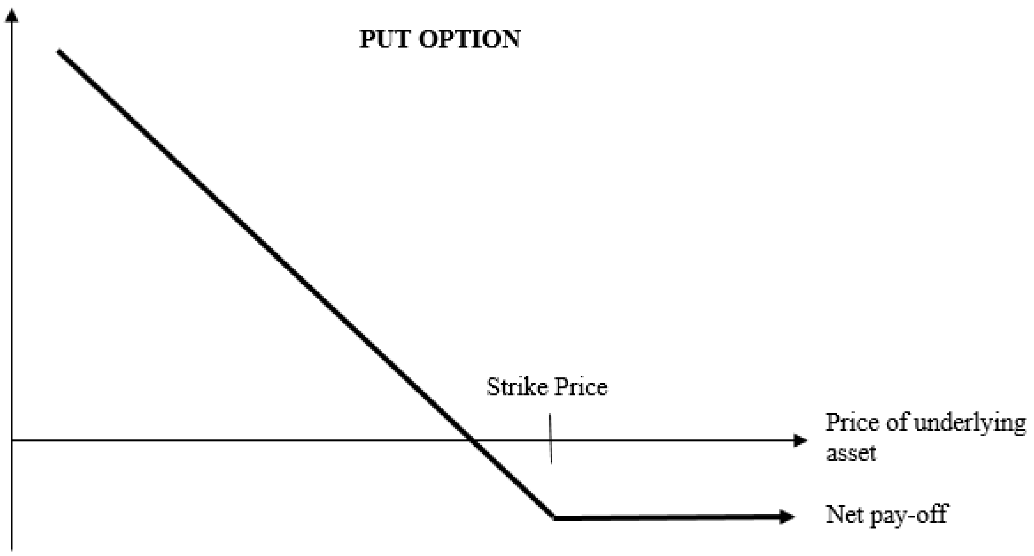

Figure 2 Pay-off Diagram of a Put Option

In the world of finance, several methods are available to appraise options. Because the value of an option depends on a number of different variables in addition to the value of the underlying asset, options are difficult and complex to value. There are many different pricing models that are used to value options.

Financial option valuation is based on several important principles. The first important principle is that it must be assumed that there are no possibilities of arbitrage. No arbitrage possibilities mean that an investor does not have the possibility to create a positive cash flow without paying an extra risk premium for this. In case an arbitrage possibility occurs this possibility is immediately used. Because the arbitrage possibility is over asked an instant correction of the price occurs. The price is now in line with the risk. Only in the situation when arbitrage possibilities do not occur, the price of an option is equal to the costs of the alternative portfolio.

The second important principle, namely the assumption that a company is capable of composing a perfect hedged alternative portfolio on the financial markets. This hedge can be created by buying a number of shares in combination with a loan against the risk free interest rate. The combination of the shares and the loan has the same pay off as the option. This results in that the price of the option will be equal to the costs to create this hedge (Trigeorgis, 1999).

Another important assumption is risk neutral valuation. The formula relating option prices to stock prices remains the same. Valuation with risk neutral approach has two properties; the expected return on a stock is the risk free interest rate and the discount rate used for the expected payoff on an option the risk free interest rate.

In finance there are several methods that are being used to value options. These are the Black-Scholes model, the binomial options pricing model, the Monte Carlo option model, the Finite difference methods for option pricing and a few more. The first two, the Black-Scholes model and the binomial options pricing model, are the most common and well known, and will therefore be discussed in the following paragraphs.

4. REAL OPTION THEORY

The “Real Options” term was initially used by Myers (1977) in corporate planning. Main Idea behind this term is treating investment decisions as exercising a financial option, but with real assets. Real options are usually embedded in investment projects and opportunity to invest is very similar to a call option.

In contrast to financial options, real options are not traded on the financial markets. Real options can be described as opportunities that the management in the future holds on to. In financial options this opportunity is the right to sell or buy a common stock for a predetermined price. In real options, the opportunity is the possibility to delay an investment or the possibility to launch a new product.

3.1. Definitions

Myers (1977) revealed that Real Options are investment opportunities for companies. It is a right but not an obligation and its value is liable on the uncertain price of underlying assets.

Dixit and Pindyck (1994) introduce Real Options as managerial opportunities to obtain real assets. Real Option investments can be described as irreversible investments in an unpredictable market. Businesses with opportunities to invest / grow are very similar to a financial call option.

Copeland and Antikarov (2001) define Real-Options as the right, but not the obligation, to act at a predetermined cost called the exercise price, for a predetermined period of time, which is the life of the option.

According to Triantis (2000), Real Options are opportunities to defer an investment until the unknown market condition is clear enough to take an action and invest.

Panayi and Trigeorgis (1998) describe Real Options as rights to exchange assets for a predetermined value or price. Trigeorgis (1993) states that option-based techniques give managerial flexibility to companies.

Amram and Kulatilaka (2000) point out that Real Option are extensions of financial option models only with real assets. And it gives decision makers a tool to make strategic decisions.

3.2. Real Options

The three factors cited below affect Real Options value and the amount of managerial flexibility at the same time. More managerial flexibility assures more valuable Real Options (Copeland & Antikarov, 2001).

1. Irreversibility
2. Uncertainty of future rewards
3. Timing or staging

Real Options are essential to make proper capital budgeting decisions and to design optimal decision-making path. Real Option methods are more useful if the market has at least one of those properties cited above.

Amram and Kulatilaka (2000) worked on a list of criteria that defines under which conditions the real options method is efficient:

1. If there is a probable investment decision.
2. If vagueness of decision is large and it is rational to wait for more information.
3. If the value of the opportunity seems to be higher in future

4. If vagueness of decision is large enough to make consider flexibility. The real options approach can value investments in flexibility.
5. If strategy and motivation can be updated during the project

Table 4 Real Options usage by industry (Amram and Kulatilaka, 2000)

	Number of participants	Using Real-Options
Beverages	3	0
Energy	25	11
Finance	31	2
Food processing	9	0
Health care	26	4
Manufacturing	57	3
Publishing	5	0
Retail	44	0
Technology	36	13
Transportation	12	1
Wholesale	9	0
Utilities	22	6
Total	**279**	**40**

A project consist different types of real options. Table 4 presents the usage of Real Options by industry. The diversity in the businesses indicates that in almost every project, decision making can be made by sorting real options embedded inside.

Recognizing the types of real options in a study is fundamental before identifying appropriate valuation method. Copeland and Antikarov (2003) argue that the most common types are the construct, abandon, defer, expand, compound, switch and rainbow options. Trigeorgis (1993) claims staged investments (sequential) options as one of the basic real options.

Table 5 Types of Real Options

Real Option Type	Financial Equivalent	Description
Option to construct	Call Option	The right to execute the project
Option to abandon	Put Option	The right to terminate or sell project
Option to defer	Call Option	The right to delay a project until better market information is available
Option to expand	Call Option	The right to increase the scale of a project
Compound option	-	An option that is contingent on the execution of another option
Switching option	-	An option to switch from one product type to another
Rainbow option	-	An option which has more than one source of uncertainty

Trigeorgis (1993) groups real options into the following categories:

- **Option to defer:**

This option appears when the time of investment is flexible. This means timing of the project can be changed in another word it can be optimized. If the interest rate can be accepted as constant over time and the future cash flow is not evident then it is possible to postpone an investment. The optimum situation for this type of option is the time when NPV increases as interest rates decrease.

- **Time-to-build option:**

Investments can be made step by step in many stages, decision can be corrected or be changed with the arrival of new information. Especially in R&D and NPD projects, multi-stage investment is favorable due to unpredictable market conditions.

- **Option to alter the operating scale:**

This option gives the decision maker to react upon a changing market. If the market condition is unfavorable for the company they can scale down operations or expand otherwise. This option type is usually chosen when the company wants to enter a new market or develop a new product.

- **Option to abandon (put option):**

If the market conditions require selling the project then the decision maker uses the option to abandon. Salvage value is the only value that can be regained after selling the project and it can be included the project’s cash flows.

- **Option to switch:**

This classical option contains the possibility to switch inputs and outputs according to changing market conditions.

- **Growth option:**

Growth options are useful when the project doesn’t seem to be advantageous at the time when the project begins considering the cash flow and investment costs but it can be become valuable in the future. R&D projects are good examples for this type of options.

- **Multiple interacting options:**

Multiple interacting options are combinations of all types of real options listed above.

Amram and Kulatilaka (1999) define the timing, exit, operating, growth, flexibility, staging and learning options and Trigeorgis (2005) identifies another Real Option type;

- **Corporate growth options:**

This option is very likely to the expand option. The value of these projects may not be measured by their initial cash flow. Investments in technological projects can be valued by this strategic option.

3.3. Real vs Financial Options

Real options are very likely to financial options by their characteristics who give them the right to buy or sell an asset at a certain price. But there are important differences between the both options. The parameters on which the value depends are the same with both options.

A big difference between financial options and real options is the character of the underlying asset. In contrast to financial options, the underlying asset with real options is physical. With financial options it is more about the difference between the potential and the strike price. As with real options, the environment is more complex than with financial options. In general, management sufficiency, market developments and technological developments will have a greater influence on the value.

The concept of time is another difference. The timeline with financial options is clear, there is a certain expiration date of the option. As with real options, this timeline is not clear as such.

With financial options the price of the underlying asset is always clear and available, because shares are always traded every day. For real options the value of the underlying asset is not always clear.

Trigeorgis (1996) outlines the specific characteristics of real options, listed as follows:

- They are not only one investor’s belongings
- They are not suitable to negotiate. The option holder either exercise the option of gives up
- They give the investor a right of preemptive investment.
- They are mostly composed.

Factors who specify the value of an option were introduced by Black and Scholes in their paper published in 1973. These factors are: the underlying asset price, exercise price, time to maturity, risk-free rate and volatility of the underlying asset (Table 6).

Table 6 Correspondence between finance option and real option (Black and Scholes, 1973)

Financial Option	Variable	Real Option
Stock price	S	Present value of a project's operating cash flows
Exercise price	X	Investment costs
Time to expiry	T	Length of time the decision may be deferred
Risk-free interest rate	r_f	Time value of money
Standard deviation of returns on stock	σ^2	Risk of the project

The stock price (S) is the value of the underlying asset. Its real market equivalent is the expected present value of cash flows of an investment opportunity.

The exercise price (X) is the price agreed in advance at which the option can be exercised. Its equivalent in Real Options is the present value of all fixed costs expected over the lifetime of the investment opportunity.

Uncertainty (σ^2) is a useful to measure the unpredictable future asset price movements. The real market equivalent is now in relation to the cash flows associated with the asset.

Time to expiry (T) is the period of time where the option can be exercised. Its equivalent is the period for which the investment opportunity is valid.

The risk-free interest rate (r_f) is the yield of a riskless security with the same maturity as the duration of the option.

One of the major differences between financial options and real options is how to handle private risk. The underlying assets of financial options are traded market assets, and market risk is the major source of risk among all financial options. Private risk can be treated simply as errors. The underlying assets of real options, however, are usually non-traded assets that do not have market equivalent.

Table 7 Comparison between Financial Option and Real Option (Copeland and Antikarov, 2001)

Characteristics	Financial Options	Real Options
Maturity	Short, usually in months	Long, usually in years
Underlying asset	Traded stocks, with comparables and pricing information	Not traded project free cash flow, proprietary in nature, with no explicit market comparables
Management manipulation	Value does not change due to individual management assumptions or actions	Value has to do with individual management assumptions and actions
Competition and market effect	Irrelevant to pricing	Direct drivers of value

Private risks cannot be hedged. The other difference is the effect of management and competition. Financial options on the same underlying asset and the same maturity date are identical. They are widely held to be market efficient. A single transaction usually does not affect the pricing of financial options, neither does management or competition. Real options, on the other hand, are lumpy or one-of-the-kind in nature. Exercise of real options by management can have profound impact on the underlying asset value.

3.3.1. Option Pricing Models

Practitioners and academics use three methods to solve real option problems. One of the most applied methods is the "Black-Scholes". Model employs Geometric Brownian Motion in a continuous time process. Monte Carlo simulation is a useful tool for conducting scenario analysis. And finally Binomial Lattices use discrete time dynamics. It can be utilized to price American options and provides an early exercise opportunity to the option holder.

3.3.2. Black – Scholes Formula

The Black-Scholes model is presented in 1973 by economists; Fischer Black, Myron Scholes and Robert Merton in a paper "The Pricing of Options and Corporate Liabilities" It was designed to value European options, which were dividend-protected.

The models' assumptions are listed below:

- All options are European which means they can only be exercised at their expiration date
- During the life of the European option, dividends are not paid out
- Fluctuation in the market cannot be predicted
- No commissions
- The risk-free rate and volatility of the underlying asset are constant values
- Follows a lognormal distribution; and returns on the underlying are normally distributed.

The model itself can be written as:

Value of call; $$C = SN(d_1) - e^{-rt}N(d_2) \tag{4.1}$$

Where;

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \tag{4.2}$$

$$d_2 = d_1 - \sigma\sqrt{t} \tag{4.3}$$

where;

S = Current value of the underlying asset

K = Strike price of the option

t = Time to expiration of the option

r = Riskless interest rate corresponding to the life of the option

σ^2 = Variance in the ln(value) of the underlying asset

This formula can be split into two parts; $SN(d_1)$, the first part, multiplies the price by the change in the call premium in relation to a change in the underlying price. It shows the expected benefit to acquire the asset. The second part, $e^{-rt}N(d_2)$, provides the current value of paying the exercise price upon expiration.

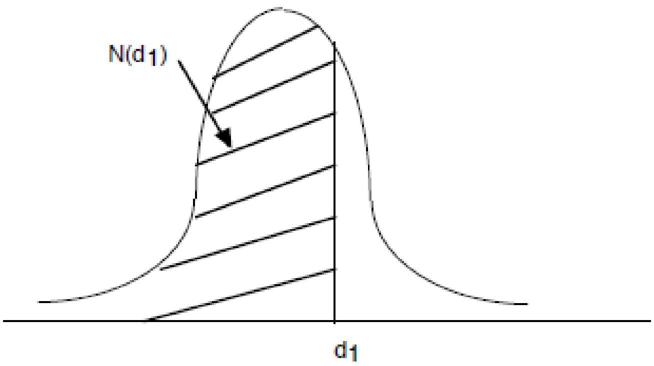

Figure 3 Probability that the option will expire in the money

3.3.3. Monte Carlo Simulation

One of the main Real Option valuation techniques is the Monte Carlo Simulation method. This method measures the options value by randomly simulating hundreds of possible scenarios for uncertain variables.

This method is originally named after the casinos in Monte Carlo, this simulation method invented after chance games. It is now widely used to simulate stochastic processes by sampling large quantity of random outcomes. Monte Carlo simulation randomly generates values for uncertain variables to simulate a real-life model. It is a computationally complex technique, yet easy to model and understand.

Usually, Monte Carlo simulation is used as an intermediate step to estimate volatility of the project. The steps followed in the combined approach are to:

1. Identify risk drivers;
2. Estimate the probability distribution of each risk driver using historical data or subjective estimates;
3. Build present value model;
4. Define input variables with the possible range of value and a probability distribution
5. MS Excel spreadsheet equipped with Monte Carlo simulation tools;
6. Define correlations among the risk variables;
7. Define forecast variables., e.g., rate of return for the project;
8. Run the simulation a thousand times;
9. Read the outputs of the forecast variables and their volatility distributions; and
10. Use the outputs as input variables to build the event tree

3.3.4. Binomial Lattice Method

This widely known option valuation method evaluates real options by creating binomial trees. In every node, "up" and "down" values present possible value of the underlying asset for the time period of time. A binomial lattice can be seen as a decision tree with probabilities on every branch. At the end of each period, the price of the underlying asset can increase or decrease with specific probabilities, different in each branch.

In Binomial trees, the price of an asset is assumed to either move up, by an up multiplier u, or move down, by a down multiplier d, in a single time period, with $u > 1$ and $d < 1$.

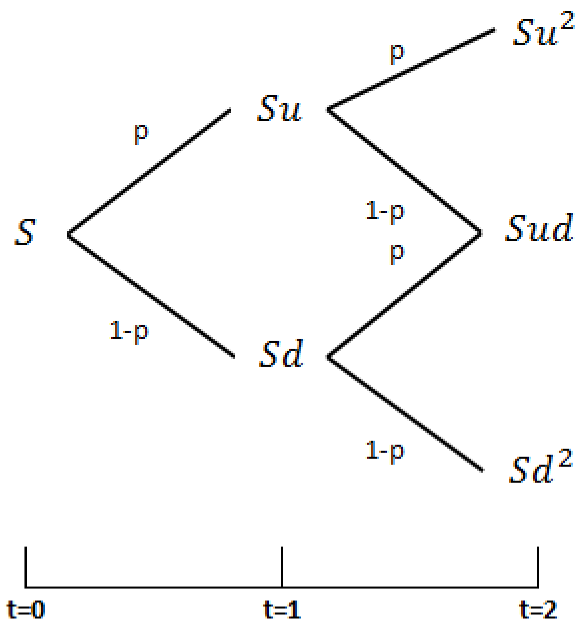

Figure 4 Binomial tree with probabilities

Fig. 4 illustrates a stock price fluctuation based on variables S, u, d, and p. S is the current stock price; it moves up with the probability p, become Su and down to Sd with probability $1-p$ at the end of the period.

Up and down multipliers are;

$$u = e^{\sigma\sqrt{\Delta t}} \quad (4.4)$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u} \quad (4.5)$$

Where σ = volatility of the underlying asset

Binomial tree modeling, as shown in Cox et al. (1979), is a discrete time model who suppose some assumptions on a perfect market. In this model, the market value of the call option (C) and the underlying (S) tracks a binomial process.
These perfect market assumptions are;

- Options are traded in a perfect market.
- There is a continuum of stock prices.
- Stock prices fluctuate randomly with normal or lognormal probability distribution.
- There are no arbitrage opportunities.
- A risk-free interest rate is used to measure future cash flows.

Let V_u represent the value of the project in the up state at the end of the first period with probability p and V_d represent the value of the project in the down state with probability q. One period binomial tree of project value is;

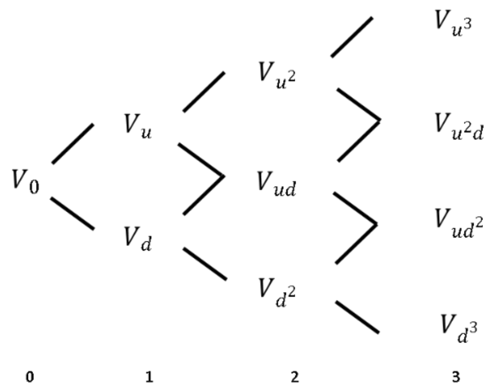

Figure 5 Fluctuation of the value of the project with up and down multipliers

V → V_u, with probability p

V → V_d, with probability q

The present value of an option to exercise C, is determined by the rational exercise rule where C_u represent the value of the call option at the end of the first period for the up state value of the project, and C_d represent the value of the call option for the down state. One period binomial tree of Call Option is;

C → $C_u = \max[0, V_u - X]$, with probability p

C → $C_d = \max[0, V_d - X]$, with probability q

with probabilities p and q, respectively;

$$p = \frac{r-d}{u-d} \tag{4.6}$$

$$1 - p = \frac{u-r}{u-d} \tag{4.7}$$

Then equation can be rewritten to solving option values:

$$C = \frac{C_u - C_d}{u-d} + \frac{uC_d - dC_u}{(u-d)r} \tag{4.8}$$

$$= [pC_u + (1-p)C_d]/r \tag{4.9}$$

where r is the discount factor equaling 1+ r_j and r_j is the risk-free rate of interest over the life of the option.

3.4. Comparison of Methods

The most fundamental European call option valuation method is the Black-Scholes formula, which was introduced by Fischer Black, Robert Merton, and Myron Scholes in the early 1970s. Black-Scholes formula exposes meaningful results in both financial and real options. However, since the Black-Scholes method requires a fixed expiration date (European options), it loses its capability to answer real options' flexibility where exercise of an option at any time of the options life is possible (American options).

Table 8 Comparison of real option valuation methods

Methods	Advantages	Potential Issues
Black-Scholes	Uses risk-free rate Is easy to use Provides quick estimate	Restrictive to one European put and call on non-dividend paying stocks Explanation is difficult due to closed-form formula
Binomial Approximation	Can be used with all types real of options Uses risk-free rate In each branch, decisions can be made clearly	Timing of exercise may not yield optimal project value Unable to handle "path-dependent" options
Simulation	Relaxes replicating portfolio Models path-dependent cash flows	Requires modeling effort Difficult to debug, verify, and validate complex models

Another valuation model is the Binominal Lattice technique. It's developed by Cox, Ross, and Rubinstein (1979). The binomial option valuation method has lots of advantages over other valuation techniques. The binominal option valuation model, like Black-Scholes model, is based on the risk-neutral argument. So the model doesn't require risk-adjusted discount rates.

Another approach to real options valuation is the Monte Carlo simulation method. This technique measure the options value by randomly simulating hundreds of possible scenarios for uncertain variables.

4. FUZZY LOGIC SYSTEMS

Fuzzy set was presented by Zadeh (1965, 1978) to handle non-statistical uncertainty of data and information processing. A fuzzy expert system uses fuzzy sets and fuzzy logic to deal with problems that occur when the data is uncertain. Fuzzy set theory is able to describe linguistically a particular phenomenon.

Fuzzy Logic shows all possible solutions of a particular situation. Then the program or the user can choose the appropriate alternative who suits the best to the problem. This logic provides flexibility to the system. In Fuzzy logic all data and rules are fuzzy sets.

Some of the essential characteristics of fuzzy logic, designed to mathematically represent uncertainty and vagueness, relate to the following (Zadeh, 1992):

- In fuzzy logic, approximate reasoning exists
- In fuzzy logic, everything explains with degrees
- In fuzzy logic, knowledge is based on fuzzy interpretation of variables.
- In fuzzy logic, every logical system can be fuzzified.
- In fuzzy logic, decision making can be made with uncertain information.

There are different sources of uncertainty in the evaluation of projects.

- Measurement uncertainty, error on observed quantities.
- Process uncertainty, dynamic randomness.
- Model uncertainty, wrong specification of the model

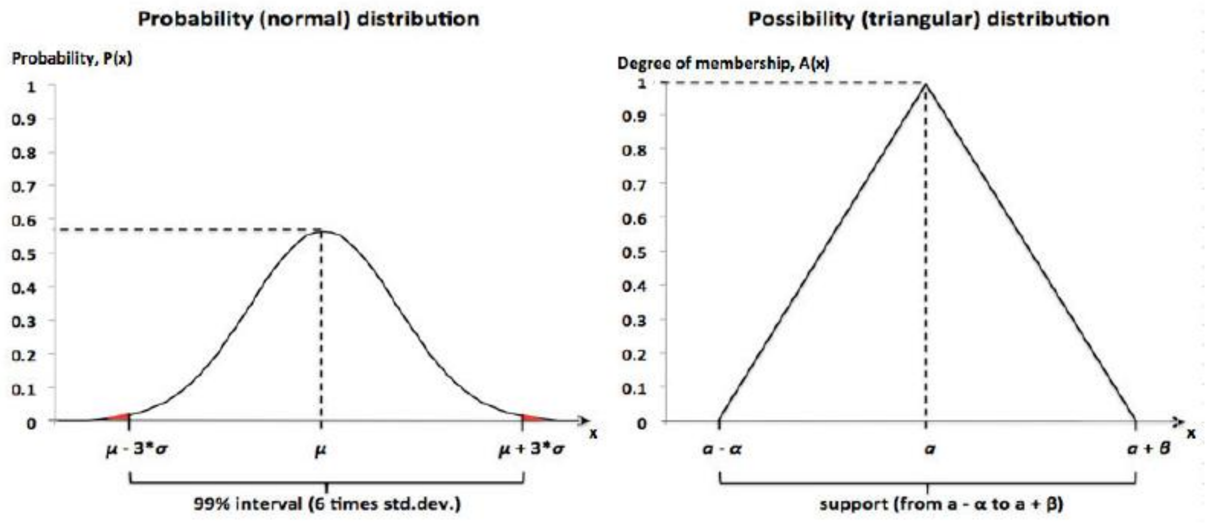

Figure 6 Normal Probability and Possibility Distribution

4.1. Fuzzy Set Theory

Fuzzy set was introduced by Zadeh to deal with vagueness of data and information processing. In this logic, each component is mapped to [0,1] by membership function.

$$\mu_{\tilde{A}}: X \rightarrow [0,1]$$

where [0,1] presents real numbers between 0 and 1 (including 0 and 1).

It is defined that a fuzzy set i.e. a type-n fuzzy set, with n=1,2,3,…, (n-1), n if its membership function line up over fuzzy set of type n-1.

While or type-1 fuzzy sets, membership functions are precise and ranges over [0, 1], for type-2 fuzzy sets, membership functions that are themselves fuzzy.

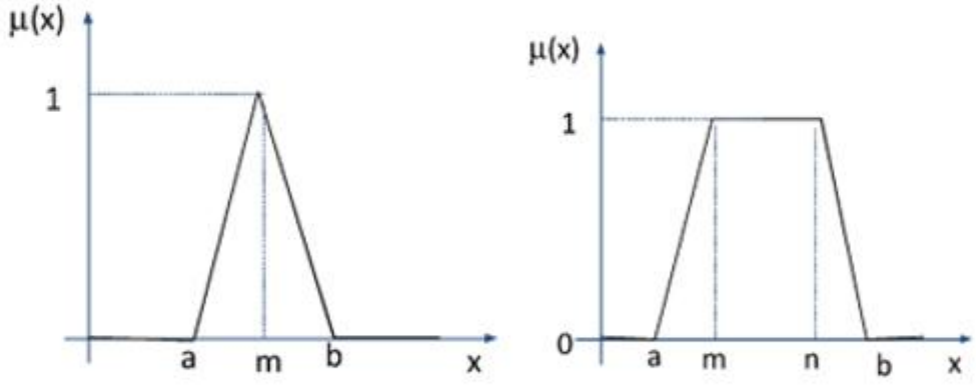

Figure 7 Triangular and trapezoidal membership functions

i) Type-1 Fuzzy Numbers

A type-1 fuzzy set, $\widetilde{A}$, with a single variable $x \epsilon X$, may be represented as:

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) \mid \quad \forall x \epsilon X\} \tag{5.1}$$

Fuzzy set $\widetilde{A}$ can also be defined as:

$$\widetilde{A} = \int_{x \epsilon X} \mu_{\widetilde{A}}(x) / x \tag{5.2}$$

where $\int$ denotes union over all admissible x.

A type-1 Gaussian membership function, $\mu_{\widetilde{A}}(x)$, is a two-dimensional membership function which has no vagueness associated with it.

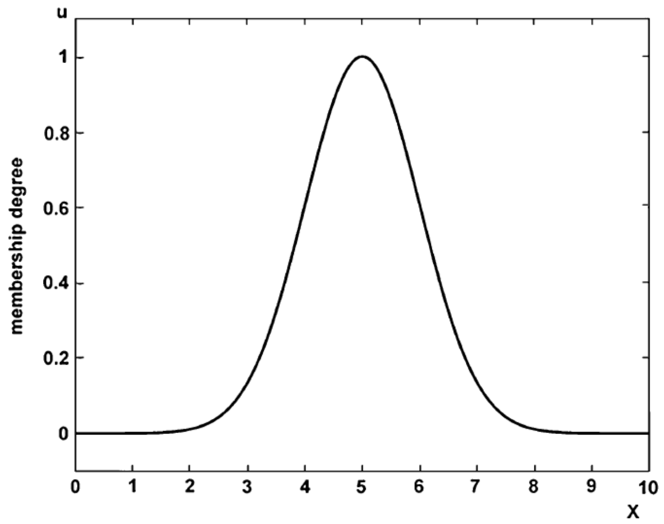

Figure 8 An example of type 1 membership function

ii) Type-2 Fuzzy Numbers

Type-2 fuzzy sets were introduced by Zadeh as an extension of type-1 fuzzy sets. Type-2 fuzzy sets are used for modeling uncertainty and imprecision in a better way.

A type-2 fuzzy set, $\tilde{A}$, may be represented as;

$$\tilde{\tilde{A}} = \left\{ \left((x,u), \mu_{\tilde{\tilde{A}}}(x,u) \right) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1] \right\} \quad (5.3)$$

where $\mu_{\tilde{\tilde{A}}}(x,u)$ is the type-2 fuzzy membership function in which $0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1$.

$\tilde{\tilde{A}}$, type-2 fuzzy set can be represented as follows;

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u) \quad (5.4)$$

where $J_x \subseteq [0,1]$ and $\iint$ denotes union over x and u. And type-1 membership functions form the upper and the lower membership functions of an interval type-2 fuzzy set.

Mendel and Liang introduce a new concept that allows the characterization of a type-2 fuzzy set with a superior and inferior membership functions. These two functions are themselves are type-1 fuzzy set membership functions an the interval between these superior and inferior type-1 fuzzy functions represent the footprint of uncertainty (FOU), which is used to define a type-2 fuzzy set.

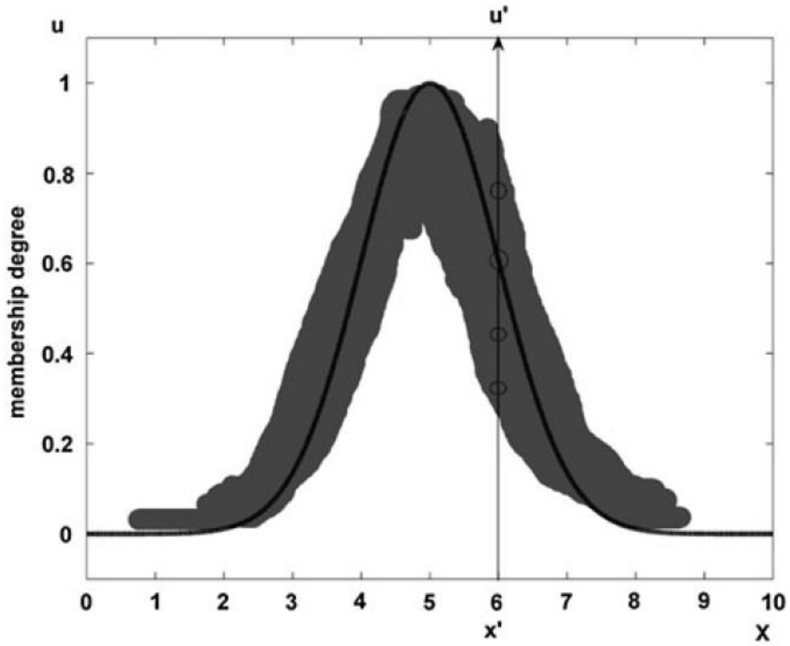

Figure 9 Blurred type-1 membership function

In this thesis, we utilized interval type-2 fuzzy to deal with uncertainty of real world problems. Interval representation of fuzzy sets is easy to use and implement.

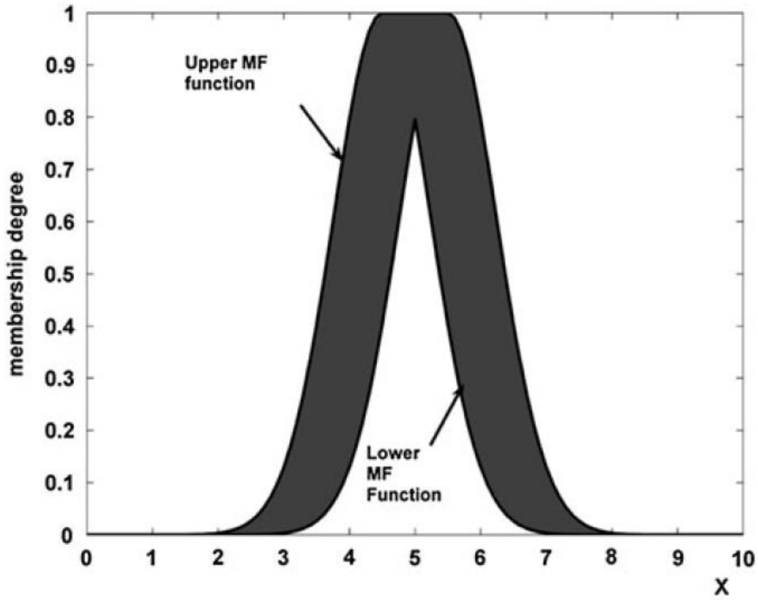

Figure 10 Interval type-2 membership function

Triangular type-2 fuzzy sets with uncertain width is shown

$$\tilde{\mu}(x) = \begin{Bmatrix} 1 - \frac{|x-c|}{d} & if\ c-d < x < c+d \\ 0 & else \end{Bmatrix} \quad (5.6)$$

where *c* and *d* are the center and the width of the membership function, *x* is the input vector.

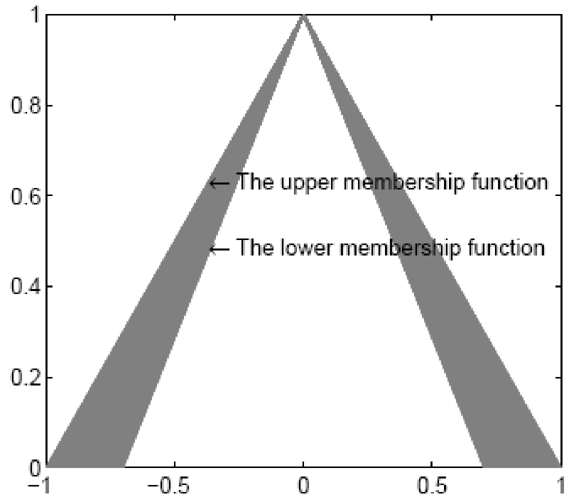

Figure 11 Type-2 fuzzy set with uncertain width

Fig. 12 illustrates type-2 fuzzy set $\tilde{\tilde{A}}$ as follows;

$$\tilde{\tilde{A}} = (\tilde{A}_U, \tilde{A}_L) = \left\{ \left(a_1, a_4, f(\tilde{A}_U) \right), \left(a_2, a_3, f(\tilde{A}_L) \right) \right\} \quad (5.7)$$

where $\tilde{A}_U$ and $\tilde{A}_L$ are type-1 fuzzy sets and the reference points a_1, a_2, a_3, a_4 create $\tilde{\tilde{A}}$, the interval type-2 fuzzy set. $f(\tilde{A}_U)$ and $f(\tilde{A}_L)$ signify the membership values of the elements a_1, a_3, a_2, a_4 in the upper and in the lower membership functions $\tilde{A}_U$ and $\tilde{A}_L$, respectively, where $\tilde{A}_U \in [0,1]$ and $\tilde{A}_L \in [0,1]$.

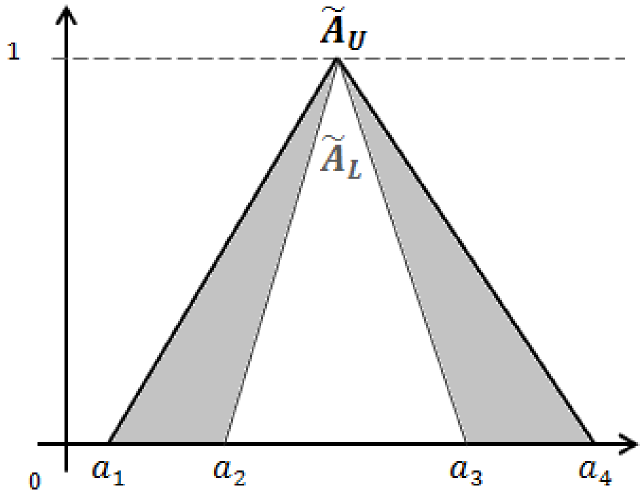

Figure 12 Membership function of an interval type-2 fuzzy number

There exits many uncertainties in an investment project, especially for a new product development in a retail banking market. Identification of the problem and choice of the type of fuzzy system which best suits the problem are highly important before making a decision.

- Definition of input and output variables and their fuzzy values
- Definition of IF-THEN rules
- Choice of Fuzzy method

Definition of these components simplifies the problem and reduces its complexity.

5. APPROACH

For retail banking reaching more customers and more importantly be the main bank that people prefer for their daily routine like payments, savings and transfers is extremely important. To achieve their goals, banks can offer advantageous products or services.

Project Risk	100%	80%	50%	30%
Real Option Value	High	High	Low	0
DCF Value	Negative	Negative	0	Positive

Figure 13 New Product Development Phases with expected risk and project values

ING Bank considers launching a new product, a saving account that offers the customers extra bonus when they spend with their debit cards. Its interest rate is triple the normal saving accounts which can lead the decision maker quit the project if they calculate the present value of the project with traditional discount methods.

Managers have this product to launch in their five year plan. This period of time is for them to observe the market and its needs then take an action when their profit is high. It also gives us the flexibility to choose the best year with optimum profit in next five years.

The following steps, shown in Fig 14, are performed in order to solve this problem;

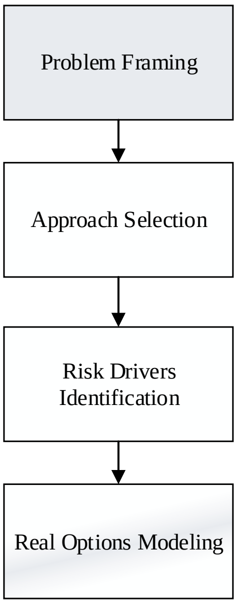

Figure 14 Steps in the analysis

Given that financial markets, in nature, can be described with words instability and uncertainty, predicting customer reactions to this new product from today is very hard. Lots of variables cause these fluctuations and uncertainty; movements in foreign currency, probable new competitors or similar products, changes in interest rates and orientation to real estates. That's the reason why we will take the benefit of fuzzy type-2 interval method for probable income distribution.

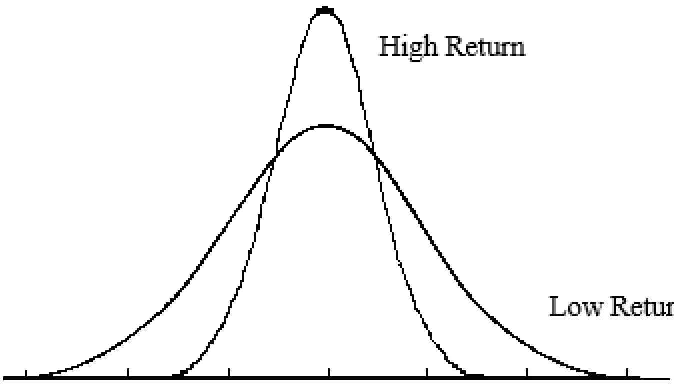

Figure 15 Demonstration of a normal distribution with high and low standard deviations

If we assume the volatility changes with the market fluctuation, we should consider fuzzy numbers, means upper and lower bounds. Let us consider volatility of the underlying asset gets values σ_H and σ_L, which represents upper and lower bounds respectively,

$$\tilde{\tilde{\sigma}} = \{\tilde{\sigma}_H, \tilde{\sigma}_L\} \tag{6.1}$$

Given this interval logic, we can calculate the binomial lattice multipliers as follows;

$$\tilde{\tilde{u}} = \{\tilde{u}_H, \tilde{u}_L\} \tag{6.2}$$

$$\tilde{u}_H = e^{\tilde{\sigma}_H \sqrt{T}} \tag{6.3}$$

with high volatility $\tilde{\sigma}_H$ and $\tilde{u}_L = e^{\tilde{\sigma}_L \sqrt{T}}$ with low volatility $\tilde{\sigma}_L$.

In the same manner;

$$\tilde{\tilde{d}} = \{\tilde{d}_H, \tilde{d}_L\}\ , \tag{6.4}$$

$$\tilde{d}_H = {}^{1}/_{\tilde{u}_H} \text{ with high volatility} \tag{6.5}$$

$$\text{and } \tilde{d}_L = {}^{1}/_{\tilde{u}_L} \text{ with low volatility } \tilde{\sigma}_L. \tag{6.6}$$

Fig. 16 illustrates visualization of the combination binomial real option valuation method and type-2 fuzzy numbers for a one period of time.

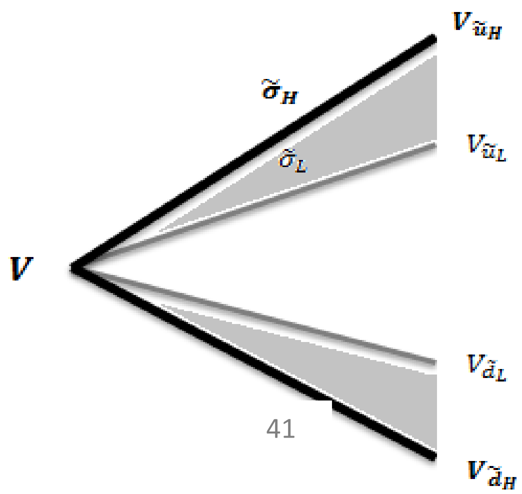

Figure 16 : Interval presentation of a binomial lattice

When firms enter new businesses or make new investments, they have their option to enter the business in stages. This way at each stage they can decide whether to go on to the next stage or not. The bank considers adding other features to this saving account after testing stage if they succeed. This brings again customers interest to the new product.

Copeland and Antikarov's four-step process will be used for valuing real options embedded in this investment decision. The steps can be described as follows;

Step one: Traditional Discounted Cash Flow is created and the present value of the expected cash flows represents the "t equals 0" value of the underlying risky asset.

Step two: Different types of uncertainties are quoted.

Step three: Up and down values are calculated for each node for a certain period of time.

Step four: Payoffs of the binomial decision tree are valued by working backward in time.

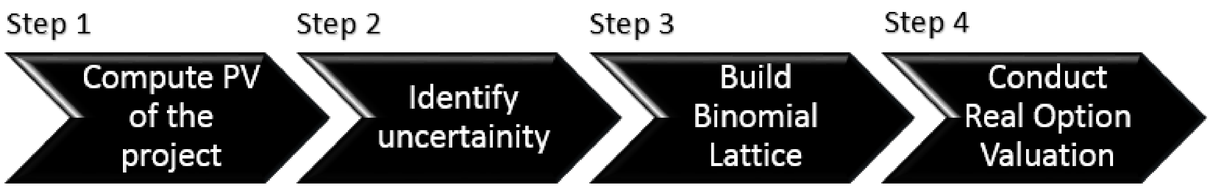

Figure 17 Copeland and Antikarov's four-step process

After those steps the best time to launch the product with the highest net present value is found and the intervals of profit between the high and low volatility of the market are estimated.

According to the Black-Scholes formula, the value of an option depends on the vagueness of returns on the underlying stock. When there are multiple sources of project risk. The variation in cash flows of former projects can be also a good example of the risk of future projects.

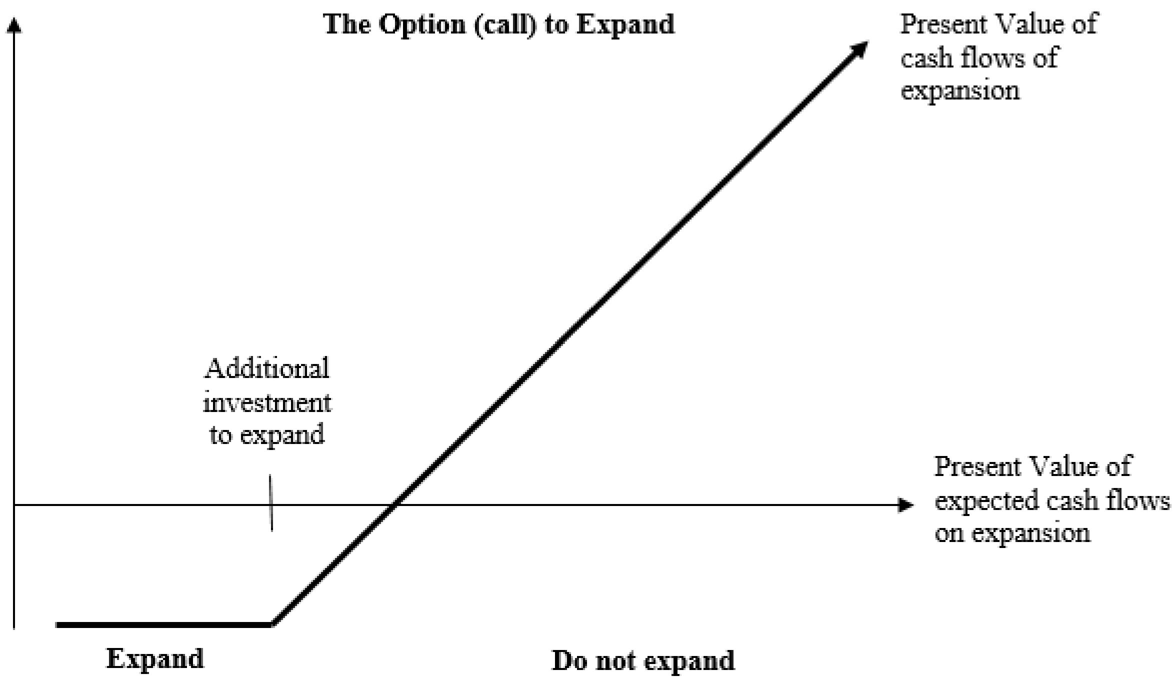

Figure 18 Decision logic of the project

6. APPLICATION

Corporation chosen in this project is a global financial institution, active in over 40 countries in Europe, North and South America, Asia and Australia, offering banking, investment, life insurance and retirement services. Its focus is people and trust. Its strengths are multi-channel distribution strategy and high customer satisfaction levels, as well as offering appealing and easy to understand products.

ING was founded in 1991 in Netherlands and after, bank united much national organization under its roof. ING offers retail banking services in the Netherlands, Belgium, Luxembourg, Poland, Romania, Turkey, India, Thailand and China. Non-retail private banking services are offered in the Netherlands, Belgium, Luxembourg, Switzerland and various countries in Asia and Central Europe.

On June 19, 2007, ING and Oyak Group reached an agreement for the sale of 100% of Oyak Bank's shares to ING Group. On July 7, 2008, ING Bank Turkey started to provide world-class financial services to its individual and corporate customers with the motto “Your money is valuable here”. ING Bank operates with the priority of providing the customers with the right solutions at the right time.

ING Bank Turkey, Deposit Products Marketing Department considers launching a new product, a saving account that accrues high overnight interest, in five years. This account will be closed to deposit but free to withdraw money and no account management fee will be collected from. The only way to save money in this account will be spending money with debit card or giving automatic bill prescription and gain the disposition part. The challenge with this account is the high level of interest rate which is almost triple the regular saving accounts. Main idea behind this product for this bank is to become the primary bank for more customers in the next five years.

Introducing new products on the market is extremely important for continuing success of businesses. They generate employment, economic growth, technological progress, and high standards of living. New product introduction carries significant risk. Yet, without this cycle, failure of companies is inevitable. They are presented to the market after a sequence of stages, beginning with the initial product idea and ending with the release on the market. First of all, information for the product concept is collected, analyzed and developed. For minimizing the level of risk in an uncertain market, it is decided to launch this product on the market in stages where the first stage includes the debit card expenses and the second stage includes transaction and payments.

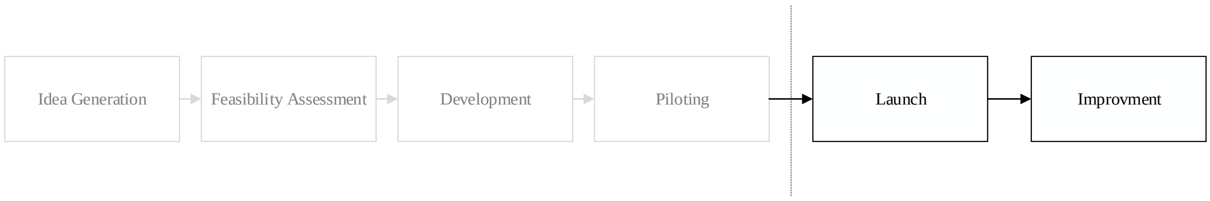

Figure 19 Two stages of the project

Copeland and Antikarov (2000) present a useful four-step process to value real options. A first step is to compute the present value of the project without flexibility. A second step is to model uncertainty. A third step involves the identification of all the options. In the final step the value of the project with flexibility is computed and the real option value is determined.

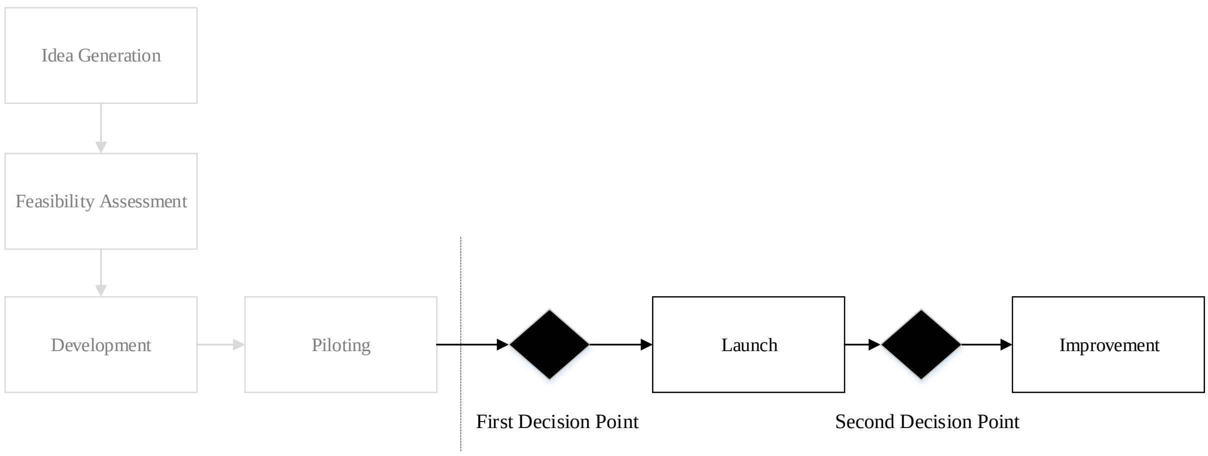

Figure 20 Decision points in the project

Step 1:

The expected present value of the project at T=0, is determined using the traditional DCF method and without considering any managerial flexibility. This requires the estimation of the appropriate risk-adjusted discount rate for the project without options.

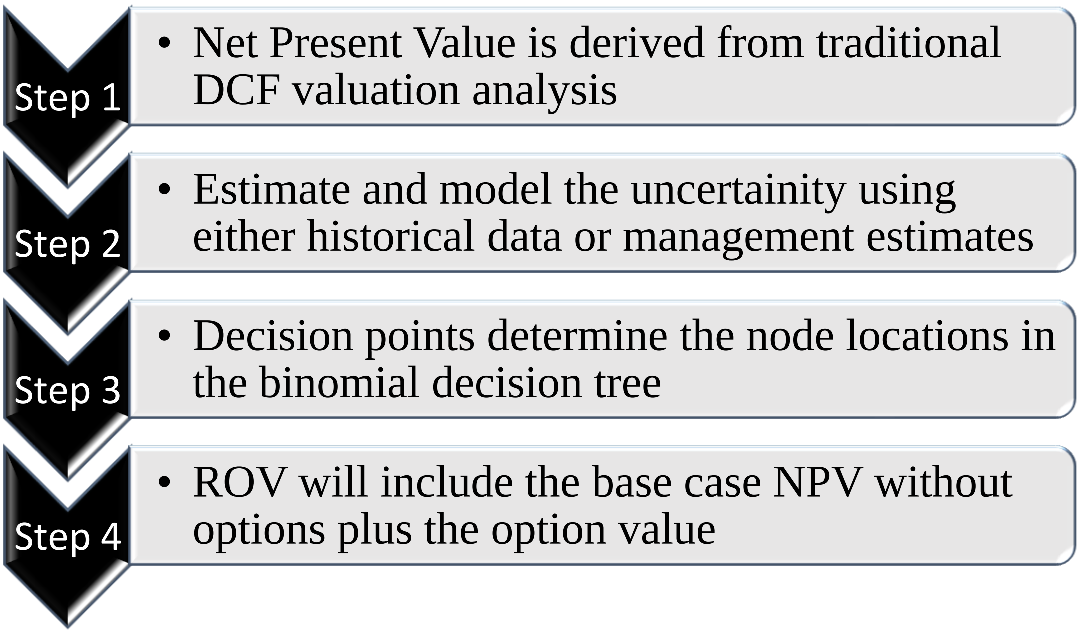

Figure 21 Copeland and Antikarov's proceeding

As a first step, discounted cash flow of these two implementations is created. Launching a new product to a market require initial investment. IT investment requires high initial investment costs and continuous long-term investments, but has significant uncertainty and risk in future cash flow. In retail banking market, early investment include IT development, maintain, marketing, promotion and stuff education costs.

Table 9 Cash flow of the Project in five years in

T =	T = 0	T = 1	T = 2	T = 3	T = 4	T = 5
Initial investment	-4,250,000.00					
Maintain cost		-24,500.00	-24,500.00	-24,500.00	-24,500.00	-24,500.00
Marketing cost		-500,000.00	-500,000.00	-500,000.00	-500,000.00	-500,000.00
Promotion cost		-1,000,000.00	-1,000,000.00	-1,000,000.00	-1,000,000.00	-1,000,000.00
Average income		2,520,000.00	2,520,000.00	2,520,000.00	2,520,000.00	2,520,000.00
		995,500.00	995,500.00	995,500.00	995,500.00	995,500.00
PV		4,151,101.05 YTL				
NPV	-98,898.95					

For the first and principal cash flow, present value and net present value is calculated, respectively ₺ 4,151,101.05 and ₺ -98,898.95 with ₺ 4,250,000.00 of initial investment and a proximate ₺ 1,500,000 yearly expense. For this cash flow, it is presumed that every customer keep average amount of ₺1,000 in their account.

Step 2:

At the next step, fluctuations are determined. Possible changes in the market, interest rates, similar products, entrance of new competitors could affect the expected profit. And profit directly depends on the number of customers which is normally distributed. With a fuzzy type-2 interval perspective, we can say that this distribution could be high or a low distribution due to the vagueness quoted above.

Step 3:

At the third step, the variables of the binomial real option evaluation method are computed with combining the upper and lower bounds of interval type-2 fuzzy sets.

We assume the volatility of the underlying asset can be high $\tilde{\sigma}_H = 30\%$ or low $\tilde{\sigma}_L = 20\%$. The period of time is one year, T=1, and the up and down movements at each node will be calculated with the upper and down multipliers, respectively $\tilde{u}_H = e^{\tilde{\sigma}_H \sqrt{T}} = e^{0.3\sqrt{1}} = 1{,}35$, $\tilde{u}_L = e^{\tilde{\sigma}_L \sqrt{T}} = e^{0.2\sqrt{1}} = 1{,}22$ and $\tilde{d}_H = {}^{1}/_{\tilde{u}_H} = 0{,}74$, $\tilde{d}_L = {}^{1}/_{\tilde{u}_L} = 0{,}82$.

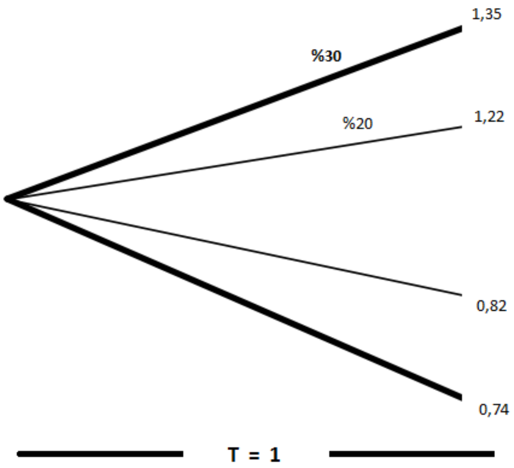

Figure 22 Up and down multipliers of the binomial tree in one period of time

We know the present value of the project is $V_0 =$ ₺ 4,151,101, the project value at the end of the first period will be in between $\tilde{u}_H V_0 =$ ₺5,603,400 and $\tilde{u}_L V_0 =$ ₺5,070,166 with probabilities $(\tilde{p}_H, \tilde{p}_L) =$ (0,56-0,65) or $\tilde{d}_H V_0 =$ ₺3,075,211 and $\tilde{d}_L V_0 =$ ₺ 3,398,634 with probabilities $(\tilde{q}_H, \tilde{q}_L) =$ (0,44-0,35).

Table 10 presents the possible present values of the project for the next 5 years. With a high volatility $\tilde{\sigma}_H = 30\%$, and with upper and down multipliers $(\tilde{u}_H, \tilde{d}_H) = (1{,}35, 0{,}74)$.

Table 11 presents the second possible present value tree of the project for the next 5 years. With a low volatility $\tilde{\sigma}_L = 20\%$, and with upper and down multipliers $(\tilde{u}_L, \tilde{d}_L) = (1{,}22,\ 0{,}82)$.

Table 10 Present Values of the Project for the next five years with a high volatility

T=0	T=1	T=2	T=3	T=4	T=5
V 4.151.101,05 ₺	Vu 5.603.400,32 ₺	Vuu 7.563.799,27 ₺	Vuuu 10.210.061,06 ₺	Vuuuu 13.782.140,85 ₺	Vuuuuu 18.603.944,21 ₺
	Vd 3.075.211,30 ₺	Vud 4.151.101,05 ₺	Vuud 5.603.400,32 ₺	Vuuud 7.563.799,27 ₺	Vuuuud 10.210.061,06 ₺
		Vdd 2.278.172,56 ₺	Vudd 3.075.211,30 ₺	Vuudd 4.151.101,05 ₺	Vuuudd 5.603.400,32 ₺
			Vddd 1.687.711,74 ₺	Vuddd 2.278.172,56 ₺	Vuuddd 3.075.211,30 ₺
				Vdddd 1.250.287,61 ₺	Vudddd 1.687.711,74 ₺
					Vddddd 926.235,84 ₺

TABLE 11 Present Values of the Project for the next five years with a low volatility

T=0	T=1	T=2	T=3	T=4	T=5
V 4.151.101,05 ₺	Vu 5.070.166,27 ₺	Vuu 6.192.715,07 ₺	Vuuu 7.563.799,27 ₺	Vuuuu 9.238.445,29 ₺	Vuuuuu 11.283.862,56 ₺
	Vd 3.398.634,09 ₺	Vud 4.151.101,05 ₺	Vuud 5.070.166,27 ₺	Vuuud 6.192.715,07 ₺	Vuuuud 7.563.799,27 ₺
		Vdd 2.782.566,25 ₺	Vudd 3.398.634,09 ₺	Vuudd 4.151.101,05 ₺	Vuuudd 5.070.166,27 ₺
			Vddd 2.278.172,56 ₺	Vuddd 2.782.566,25 ₺	Vuuddd 3.398.634,09 ₺
				Vdddd 1.865.209,94 ₺	Vudddd 2.278.172,56 ₺
					Vddddd 1.527.104,74 ₺

At the last step, payoffs of the binomial decision tree are calculated by working backward in time. Fig. 23. illustrates a prototype of call option calculation.

Step 4:

In Table 12, call options (net present values) of the projects are introduced. This representation provides us to observe intervals in every step.

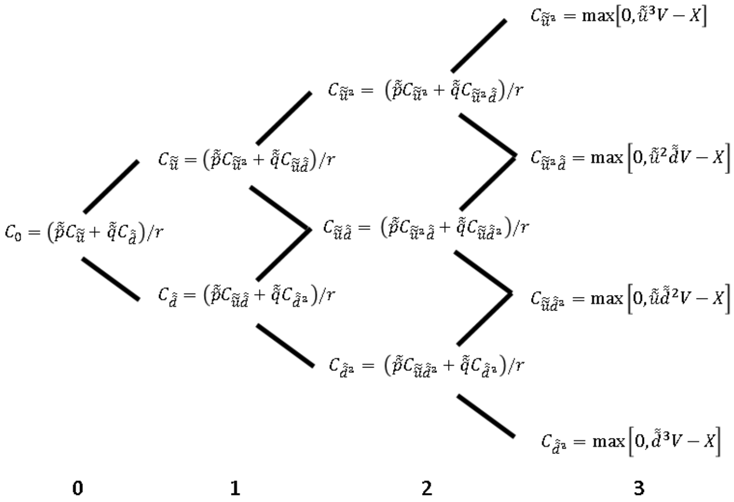

Figure 23 Representation of a call option

TABLE 12 Call option of the Project for both high and Low volatility

T=0	T=1	T=2	T=3	T=4	T=5
C	Cu	Cuu	Cuuu	Cuuuu	Cuuuuu
1.700.139,01 ₺	2.719.449,58 ₺	4.271.138,15 ₺	6.566.371,08 ₺	9.846.955,67 ₺	14.353.944,21 ₺
1.440.352,68 ₺	2.038.658,28 ₺	2.848.194,11 ₺	3.920.109,28 ₺	5.303.260,10 ₺	7.033.862,56 ₺
	Cd	Cud	Cuud	Cuuud	Cuuuud
	725.943,35 ₺	1.260.152,83 ₺	2.157.449,62 ₺	3.628.614,08 ₺	5.960.061,06 ₺
	663.000,97 ₺	1.007.320,76 ₺	1.516.484,40 ₺	2.257.529,89 ₺	3.313.799,27 ₺
		Cdd	Cudd	Cuudd	Cuuudd
		185.572,60 ₺	359.874,70 ₺	697.892,93 ₺	1.353.400,32 ₺
		177.844,82 ₺	296.024,81 ₺	492.736,81 ₺	820.166,27 ₺
			Cddd	Cuddd	Cuuddd
			0,00 ₺	0,00 ₺	0,00 ₺
			0,00 ₺	0,00 ₺	0,00 ₺
				Cdddd	Cudddd
				0,00 ₺	0,00 ₺
				0,00 ₺	0,00 ₺
					Cddddd
					0,00 ₺
					0,00 ₺

$\tilde{\sigma}_H$	0,3
$\tilde{\sigma}_L$	0,2

Comparing these interval values with the initial investment cost, exercise price of the stock, ₺ 4,150,100, we can have an opinion on whether to invest or to abandon. Table 13 shows real options embedded in this project.

Table 13 Real Options Embedded in this Project

Delay	Delay	Delay EXPAND	Delay EXPAND	EXPAND	EXPAND
	Delay	Delay	Delay	Delay	Delay EXPAND
		Delay Abandon	Delay Abandon	Delay Abandon	Delay Abandon
			Abandon	Abandon	Abandon
				Abandon	Abandon
					Abandon

Launching a new product, exercise the option, is getting profitable after the fourth year when the volatility is low and it's profitable after the second year when the volatility is high. When we put these two option trees in the same picture to value intervals, we can clearly see that the soonest profitable launch can be made at the fourth year of the project.

And the same process is carried for the next implementation phase. For the second discounted cash flow, present value and net present value is calculated respectively ₺6,900,000 and ₺3,700,000, with ₺3,250,000 of initial investment and approximate ₺1,700,000 yearly expenses.

Table 14 Cash flow of the development

	T = 0	T = 1
Initial investment	-3,250,000.00	
Maintain cost		-25500
Marketing cost		-600000
Promotion cost		-1100000
Average income		3400000
		1674500
PV		6,982,439.69 YTL
NPV	3,732,439.69	

For each high and low volatilities of the underlying asset $\tilde{\sigma}_H = 30\%$ and low $\tilde{\sigma}_L = 20\%$ respectively, upper multiepliers are $\tilde{u}_H = e^{\tilde{\sigma}_H \sqrt{T}} = e^{0.3\sqrt{1}} = 1{,}35$ and $\tilde{u}_L = e^{\tilde{\sigma}_L \sqrt{T}} =$

$e^{0.2\sqrt{1}} = 1{,}22$ and down multipliers $\tilde{d}_H = {}^{1}/_{\tilde{u}_H} = 0{,}74$, $\tilde{d}_L = {}^{1}/_{\tilde{u}_L} = 0{,}82$ again for the one year period of time, T=1.

The project value at the end of the first period will be in between $\tilde{u}_H V_0 =$ ₺5,603,400 and $\tilde{u}_L V_0 =$ ₺5,070,166 with the present value, $V_0 =$ ₺ 4,151,101. Probabilities are $(\tilde{p}_H, \tilde{p}_L) = (0{,}56\text{-}0{,}65)$ for the up state or in between $\tilde{d}_H V_0 =$ ₺3,075,211 and $\tilde{d}_L V_0 =$ ₺ 3,398,634 with probabilities $(\tilde{q}_H, \tilde{q}_L) = (0{,}44\text{-}0{,}35)$ in the down state.

Table 15 Project values

T=0	T=1
V 6,982,439.69 ₺	Vu 8,528,371.10 ₺
	Vd 5,716,738.11 ₺

T=0	T=1
V 6,982,439.69 ₺	Vu 9,425,307.71 ₺
	Vd 5,172,718.55 ₺

And finally call option values are calculated by computing backward again. For this second and last stage of the investment, it is seen that addition of new features can be made right away or it can wait for another year.

Table 16 Call Option Values for the Second Stage of the Project

T=0	T=1
C 3.973.180,43 ₺ 3.973.180,43 ₺	Cu 6.175.307,71 ₺ 5.278.371,10 ₺
$\tilde{\sigma}_H$ 0,3 $\tilde{\sigma}_L$ 0,2	Cd 2.466.738,11 ₺ 1.922.718,55 ₺

T=0	T=1
EXPAND	EXPAND
	Abandon

Applying structured binomial real options valuation technique and the flexibility of interval type-2 fuzzy sets proved the success of this new product launch in the retail banking market even the vagueness occurs in volatility. The road map for particularly this product is to wait for four years before making an investment and add other features of the product, saving account right away. This way the managers reach what they planned for the year 2020 without losing times and making benefits.

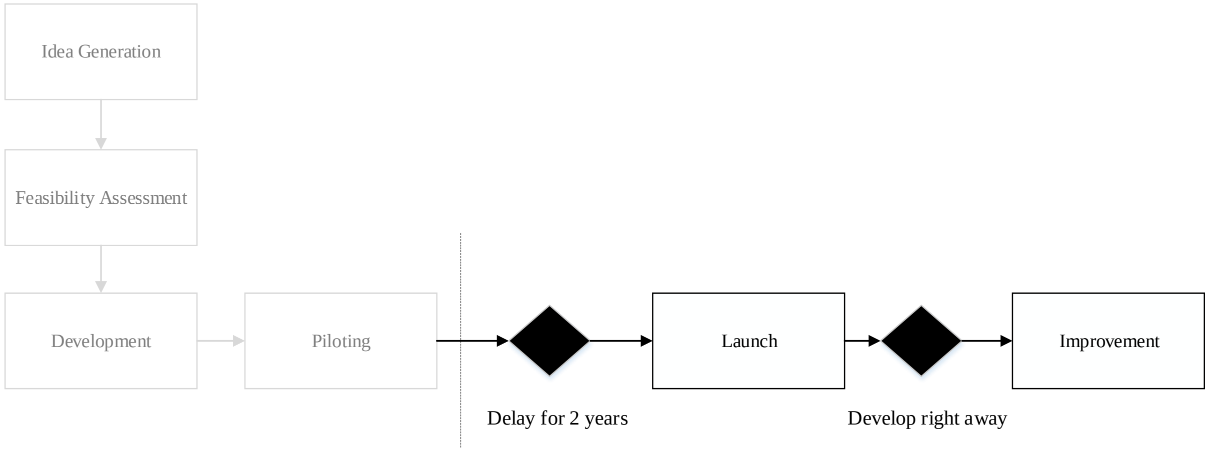

Figure 24 Managerial decisions via Real Option Valuation

7. COMPOUND OPTION PRICING MODEL

There are many stages in a new product development project and the decision maker has to determine whether to exercise the option or to postpone it. This process can be explained by compound options.

Compound options accept other options as underlying assets. A compound option has several expiration dates and several strike prices. Let K_i be the present value of investment cost (strike price) for stages $\mathrm{i} = 1,2,..\mathrm{n}$ and S be the present value of the project return (price of the underlying asset) after market introduction.

In this work, we will be talking about two call options with two expiration dates. If an investor buys an option at time T=0, on the first expiration date T_1, the option holder has the right to buy a new option with the strike price K_1. And the second option gives the investor the right to buy the underlying asset with the strike price K_2 at time T_2.

The pricing formula for compound option ant time T=0 is;

$$C = SN_2(d_1, d_2, \rho) - K_2 e^{-rT_2} N_2(d_3, d_4, \rho) - K_1 e^{-rT_1} N(d_3) \qquad (8.1)$$

where

$$d_1 = \frac{\ln(S/S_*) + \left(r + (1/2)\sigma^2\right)T_1}{\sigma\sqrt{T_1}} \qquad (8.2)$$

$$d_2 = \frac{\ln(S/K_2) + \left(r + (1/2)\sigma^2\right)T_2}{\sigma\sqrt{T_2}} \qquad (8.3)$$

$$d_3 = d_1 - \sigma\sqrt{T_1} \qquad (8.4)$$

$$d_4 = d_2 - \sigma\sqrt{T_2} \qquad (8.5)$$

$$\rho = \sqrt{\frac{T_1}{T_2}} \qquad (8.6)$$

r is the risk-free interest rate, $N(x)$ is the standard normal distribution function, $N_2(x, y, \rho)$ is the bivariate standard normal distribution function with x and y as upper limits and ρ as the correlation coefficient between the two variables, and S_* is the unique solution of the equation;

$$xN\left(d + \sigma\sqrt{T_2 - T_1}\right) - K_2 e^{-r(T_2 - T_1)} N(d) = K_1 \tag{8.7}$$

where

$$d = \frac{\ln\left(\frac{x}{K_2}\right) + \left(r - \left(\frac{1}{2}\right)\sigma^2\right)(T_2 - T_1)}{\sigma\sqrt{T_2 - T_1}} \tag{8.8}$$

7.1. Compound Option Pricing Under Fuzzy Environment

In the real financial market, due to market fluctuations and human errors, some parameters such as the interest rate and volatility sometimes cannot be recorded or collected precisely. The imprecise information and the fluctuation of the financial market from time to time, it is almost impossible to assume that the volatility σ is constant. So we replace σ by fuzzy number $\tilde{\sigma}$ to get the formula for this compound option under fuzzy environment.

Let the interest rate and the volatility be fuzzy numbers, S^* is the unique solution of the equation;

$$xN\left(d^* + \tilde{\tilde{\sigma}}\sqrt{T_1 - T_2}\right) - K_2 e^{-r(T_1 - T_2)} N(d^*) = K_1 \tag{8.9}$$

where

$$d^* = \frac{\ln(x/K_2) + \left(r - (1/2)[\tilde{\tilde{\sigma}}]^2\right)(T_1 - T_2)}{\tilde{\tilde{\sigma}}\sqrt{T_1 - T_2}} \tag{8.10}$$

$$\tilde{\tilde{\sigma}} = [\tilde{\sigma}_H, \tilde{\sigma}_L] \tag{8.11}$$

with $\tilde{\sigma}_H$ and $\tilde{\sigma}_L$ are the higher and lower values of $\tilde{\tilde{\sigma}}$, respectively.

Under fuzzy environment, the call option value $\tilde{\tilde{C}}$ is a type-2 fuzzy number.

The compound option price $\tilde{\tilde{C}}$ is;

$$\tilde{\tilde{C}} = SN_2\left(\tilde{\tilde{d}}_1, \tilde{\tilde{d}}_2, \rho\right) - K_2 e^{-rT_2} N_2\left(\tilde{\tilde{d}}_3, \tilde{\tilde{d}}_4, \rho\right) - K_1 e^{-rT_1} N\left(\tilde{\tilde{d}}_3\right) \qquad (8.12)$$

where

$$\tilde{\tilde{d}}_1 = \frac{\ln(S/S^*) + \left(r + (1/2)[\tilde{\tilde{\sigma}}]^2\right)T_1}{\tilde{\tilde{\sigma}}\sqrt{T_1}} \qquad (8.13)$$

$$\tilde{\tilde{d}}_2 = \frac{\ln(S/K_2) + \left(r + (1/2)[\tilde{\tilde{\sigma}}]^2\right)T_2}{\tilde{\tilde{\sigma}}\sqrt{T_2}} \qquad (8.14)$$

$$\tilde{\tilde{d}}_3 = \tilde{\tilde{d}}_1 - \tilde{\tilde{\sigma}}\sqrt{T_1} \qquad (8.15)$$

$$\tilde{\tilde{d}}_4 = \tilde{\tilde{d}}_2 - \tilde{\tilde{\sigma}}\sqrt{T_2} \qquad (8.16)$$

with $\tilde{\tilde{\sigma}} = [\tilde{\sigma}_H, \tilde{\sigma}_L]$

The fuzzy set of $\tilde{\tilde{C}}$ may be denoted as $\tilde{\tilde{C}} = [\tilde{C}_H, \tilde{C}_L]$ and $\tilde{C}_H$ and $\tilde{C}_L$ can be calculated as the following theorem.

$$\tilde{\tilde{C}} = [\tilde{C}_H, \tilde{C}_L] = \begin{bmatrix} \left(SN_2\left(\tilde{d}_1^H, \tilde{d}_2^H, \rho\right) - K_2 e^{-rT_2} N_2\left(\tilde{d}_3^H, \tilde{d}_4^H, \rho\right) - K_1 e^{-rT_1} N\left(\tilde{d}_3^H\right)\right), \\ \left(SN_2\left(\tilde{d}_1^L, \tilde{d}_2^L, \rho\right) - K_2 e^{-rT_2} N_2\left(\tilde{d}_3^L, \tilde{d}_4^L, \rho\right) - K_1 e^{-rT_1} N\left(\tilde{d}_3^L\right)\right) \end{bmatrix} \qquad (8.17)$$

where

$$\tilde{\tilde{d}}_1 = \left[\tilde{d}_1^H, \tilde{d}_1^L\right] \qquad (8.18)$$

$$\tilde{d}_1^H = \frac{\ln(S/S^*) + \left(r + (1/2)[\tilde{\sigma}_H]^2\right)T_1}{\tilde{\sigma}_H\sqrt{T_1}} \qquad (8.19)$$

$$\tilde{d}_1^L = \frac{\ln(S/S^*) + \left(r + (1/2)[\tilde{\sigma}_L]^2\right)T_1}{\tilde{\sigma}_L\sqrt{T_1}} \qquad (8.20)$$

$$\tilde{\tilde{d}}_2 = \left[\tilde{d}_2^H, \tilde{d}_2^L\right] \tag{8.21}$$

$$\tilde{d}_2^H = \frac{\ln(S/K_2) + \left(r + (1/2)[\tilde{\sigma}_\mathrm{H}]^2\right)T_2}{\tilde{\sigma}_\mathrm{H}\sqrt{T_2}} \tag{8.22}$$

$$\tilde{d}_2^L = \frac{\ln(S/K_2) + \left(r + (1/2)[\tilde{\sigma}_\mathrm{L}]^2\right)T_2}{\tilde{\sigma}_\mathrm{L}\sqrt{T_2}} \tag{8.23}$$

$$\tilde{\tilde{d}}_3 = \left[\tilde{d}_3^H, \tilde{d}_3^L\right] \tag{8.24}$$

$$\tilde{d}_3^H = \tilde{d}_1^H - \tilde{\sigma}_\mathrm{H}\sqrt{T_1} \tag{8.25}$$

$$\tilde{d}_3^L = \tilde{d}_1^L - \tilde{\sigma}_\mathrm{L}\sqrt{T_1} \tag{8.26}$$

$$\tilde{\tilde{d}}_4 = \left[\tilde{d}_4^H, \tilde{d}_4^L\right] \tag{8.27}$$

$$\tilde{d}_4^H = \tilde{d}_2^H - \tilde{\sigma}_\mathrm{H}\sqrt{T_2} \tag{8.27}$$

$$\tilde{d}_4^L = \tilde{d}_2^L - \tilde{\sigma}_\mathrm{L}\sqrt{T_2} \tag{8.28}$$

7.2. Application of Compound Option Pricing to NPD Project

In this section, some numerical demonstration of compound option pricing under fuzzy environment is made.

We take $\tilde{\tilde{\sigma}} = [\tilde{\sigma}_\mathrm{H}, \tilde{\sigma}_\mathrm{L}] = [0.3, 0.2]$, where $\tilde{\sigma}_\mathrm{H}$ is the high standard deviation and $\tilde{\sigma}_\mathrm{L}$ is the low standard deviation, first expiration date of the initial investment option $T_1 = 5$, second expiration date of the improvement option $T_2 = 6$, risk-free interest rate $r_f = 0.08$.

$$\tilde{\tilde{d}}_1 = \left[\tilde{d}_1^H, \tilde{d}_1^L\right] = [1.25, 1.60] \tag{8.29}$$

$$\tilde{\tilde{d}}_2 = \left[\tilde{d}_2^H, \tilde{d}_2^L\right] = [2.70, 3.74] \tag{8.30}$$

$$\tilde{\tilde{d}}_3 = \left[\tilde{d}_3^H, \tilde{d}_3^L\right] = [0.58, 1.15] \tag{8.31}$$

$$\tilde{\tilde{d}}_4 = \left[\tilde{d}_4^H, \tilde{d}_4^L\right] = [1.96, 3.25] \tag{8.32}$$

$$\rho = 0.91$$

$$\tilde{\tilde{C}} = \left[\tilde{C}_H, \tilde{C}_L\right] = [4{,}922{,}588.72\ ,\ \ 3.545.594.03]$$

This means that the option price will lie in the closed interval $\tilde{\tilde{C}} = \left[\tilde{C}_H, \tilde{C}_L\right] = [4{,}922{,}588.72\ ,\ \ 3.545.594.03]$ with standard deviations $\tilde{\sigma}_H$ and $\tilde{\sigma}_L$, respectively. This interval provides reference for business investors.

If the market conditions support this interval and investor is satisfied with the expected return on the project then decision maker can exercise the option.

8. DISCUSSION

The future market is unpredictable. Decision makers cannot stick to a static strategy for a long time. Many investment opportunities need to be revised in the future to adjust new market conditions. It is almost impossible to take all the right decisions at the beginning of the project.

Uncertainty and flexibility are key determinants of the value of an asset. The traditional valuation based on cash flows has proven inadequate. Because of this, Real option valuation is becoming more and more a critical valuation and decision making tool.

Our study of New Product Development in a Retail Banking market suggests strategic decision-making and use Real Options method in order to make an investment under uncertainty.

The most relevant option embedded in the valuation of New Product Development is a strategic growth option because the capital input is more or less irreversible and the output is subject to uncertainty. Real option analysis also incorporates a learning process that supports multi-stage decision making.

The major difficulty in valuing this investment using Real Option Valuation methods is that there is frequently no direct market information about the underlying asset value and no historical data to deduce important inputs. The discount rate, probabilities, and the expected value of each alternative are determined based on practitioners' knowledge of the investment. This approach is also problematic partly because the value of underlying asset is very hard to estimate in the absence of market information. Finally, it is assumed that the risk-free rate is known and constant. This assumption is almost never true when delay option is the case.

This study also gives a comparison of two methodology; binomial lattice technique with multi-stage real option valuation aspect and compound option method with Black-Sholes formula.

When we compare the results between fuzzy compound real option model and fuzzy binomial option method, we could see there is a great difference in call option values, even in interval values.

This is a possible output because of two reasons; first of all, valuing each option individually and summing these separate option values can overstate the value of a project. This is because; there is a consideration of best option choice at the right time. Project value is measured optimistically.

Second of all, closed form of Black-Sholes formula reacts on the exercise of both options in the project. It cannot properly capture the value investment and it’s hard to track the life time of those options.

9. CONCLUSION

In an uncertain and dynamic global market, managerial flexibility has become essential for companies to take advantage of future investment opportunities. The main focus of this study, in contrast to others in the literature, is to model uncertain information and evaluate the new product development.

The traditional investment evaluation methods based on discounted cash flows have some significant disadvantages: they suppose all decisions are irreversible and they do not take into account the interactions between decisions. The evaluation using real options undertake this disadvantage.

In traditional investment valuation techniques, the only decision criterion is the cash flow of the investment. As a result, a potentially successful investment may be seen less valuable and maybe even rejected. Thus, with a long-term perspective, managers should consider dynamic conditions and after that they should also reckon options to delay, to expand or to abandon the investment, which gives an additional flexibility and value to the project.

Success in new product development still remains critical challenge for growing companies. These types of investments allow companies to gain more customers and to enter other markets in the future. The option to expand, in our new product development case, is used to rationalize taking investments that have a negative net present value.

Under uncertainty, it is cautious to stage an investment. Staging investment gives decision makers flexibility whether to continue to the next stage or to abandon midway. Multi-stage investments may have great option values that can verify making strategic investments despite having a negative cash flow.

This study has presented a type-2 fuzzy binomial valuation application to evaluate new product development project evaluation with real options in an uncertain environment. The binomial tree technique has chosen due to its ease of implementation, and ability of handling with multiple options. The project has divided into two phases so that managers could observe the success of the product launch and decide whether to continue with additional features or to abandon for good. And with the vagueness contained in financial markets, the work is represented with type-2 fuzzy numbers to provide reliable results at the end.

Compound options are options that are written on others, which mean that compound options take standard options as their underlying assets. They are used to hedge risky investments. Considering the uncertainty and vagueness of the real market, a fuzzy pricing formula is introduced for compound option using volatility as fuzzy variable. And results of two techniques are compared.

Our findings support the argument that decision makers either implicitly or explicitly use Real Options. The consequences in this project have shown that, even with negative cash flow, some projects could have benefits for companies with the real options approach. And the fuzzy type-2 interval method ensures to predict possible outcomes when exercising these options.

The main reason to choose type-2 fuzzy sets over type-1 fuzzy sets is the dimension called the footprint of uncertainty which characterizes type-2 fuzzy logic. Despite the immense computational complexities associated with type-2 fuzzy sets, they have great adaptability to real world managerial problems.

Delay the project and consider to launch this new product after two years has a powerful result of this study. At the second stage, implementing new features is appropriate in any time until the end of the lifetime of the project. With these results, updateable feature of the binomial tree evaluation method the management team could saw the effect of their decisions continuously and they have contented. Our results suggest that the options concept could prove to be a fruitful approach to retail banking new product development project.

Real options have been combined with many theories in recent year years. As a future work, multi criteria decision making methods and real options can be examined together to value components of the project. It may provide the decision maker to discover real options.

REFERENCES

Amran M., Kulatilaka N. (1999)., Real Options: Managing Strategic Investment in an Uncertain World, Harvard Business School Press, Boston MA

Black, F. & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. Journal of Political Economy, 81(May-June): 637-659.

Brennan, M.J. and Schwartz, E.S. (1985), "A New Approach to Evaluating Natural Resource Investments," Midland Corporate Finance Journal,3, 1, 37-47

Carlsson C and Fullér R (2001). On possibilistic mean value and variance of fuzzy numbers. Fuzzy Sets Syst. 122: 315–326

Carlsson C and Fullér R (2002). Fuzzy reasoning in decision making and optimization, Studies in Fuzziness and Soft Computing Series, Vol. 82, Springer, Berlin, Heidelberg

Carlsson C, Fullér R and Majlender P (2002). A possibilistic approach to selecting portfolios with highest utility score. Fuzzy Sets Syst. 131: 13–21

Carlsson, C., Fullér, R., Heikkilä, M., Majlender, P., (2007). A fuzzy approach to R&D project portfolio selection. International Journal of Approximate Reasoning 44, 93–105.

Carlsson, C., Fullér, R., (2003). A fuzzy approach to real option valuation. Fuzzy Sets and Systems 139, 297–312.

Carlsson C, Fullér R and Majlender P (2005). On possibilistic correlations. Fuzzy Sets Syst. 155: 425–445

Cherubini U. (1997) , Fuzzy measures and asset prices: Accounting for information ambiguity, Applied Mathematical Finance 4 135–149

Collan M, Fullér R and Mézei J (2009). Fuzzy Pay-off Method for Real Option Valuation. Journal of Applied Mathematics and Decision Systems.

Collan M, Kinnunen J, (2011). A Procedure for the Rapid Pre-acquisition Screening of Target Companies Using the Pay-off Method for Real Option Valuation, Journal of Real Options and Strategy 4(1): 117-141.

Copeland, T., & Antikarov, V. (2003). Real Options: A Practitioner's Guide. New York: Cengage Learning.

Cox, J., Ross, S., & Rubinstein, M. (1979). Option Pricing: A Simplified Approach. Journal of Financial Economics, 7: 229-263.

Dixit, A. K., Pindyck, R.S. (1994): Investment under Uncertainty. Princeton University Press,

Galli, Spe & M. Armstrong. (1999). Comparing Three Methods For Evaluating Oil Projects: Option Pricing,

Ghaziri H., Elfakhani S., Assi J. (2000), Neural networks approach to pricing options, Neural Network World 10 (2000) 271–277.

Guerra M. L., Sorini L. and Stefanini L., (2007) "Parametrized Fuzzy Numbers for Option Pricing," IEEE International Conference on Fuzzy Systems, London, pp. 727-732

Ho S.H., Liao S.H., (2011). A fuzzy real option approach for investment project valuation, Expert Systems with Applications 38: 15296–15302

Hoesli, M., Jani, E., & Bender, A. (2006). Monte Carlo Simulations for Real Estate Valuation. Journal of Property Investment and Finance , 24 (2), 102-122.

Kuchta D. (2001) A fuzzy mode for R&D project selection with benefit: outcome and resource. The Engineering Economist; 46(3):164–80.

Lander, D. M. and Pinches, G. E. (1998). Challenges to the practical implementation of modeling and valuing real options. The Quarterly Review of Economics and Finance, 38, Special Issue, pp. 537-567.

Lee Y.C., Lee S.S., (2011). The valuation of RFID investment using fuzzy real option, Expert Systems with Applications 38: 12195–12201

Myers S. C., (1977), Determinants of Corporate Borrowing, Journal of Financial Economics, 147-76.

Myers S.C., (1984). Finance theory and financial strategy, lnterfaces 14, 126-137.

Myers S. C., Majd S., (1990). Abandonment value and Project Life, Advances in Futures and Options Research, p:1-21.

Nowak P. and Romaniuk M. (2010), "Computing Option Price for Levy Process with Fuzzy Parameters," Vol. 201, No. 16, , pp. 206-210.

Pindyck, R. (1991). Irreversibility, Uncertainty, and Investment. Journal of Economic Literature, 29: 1110-1148.

Wu H.C. (2004), Pricing the European options based on the fuzzy pattern of Black–Scholes formula, Computers and Operations Research 31 1069–1081.

Wu H.C. (2007), Using fuzzy sets theory and Black–Scholes formula to generate pricing boundaries of European options, Applied Mathematics and Computation 185: 136–146

Thavaneswaran A., Appadoo S. S. and Paseka A. (2009), "Weighted Possibilistic Moments of Fuzzy Numbers with Applications to GARCH Modeling and Option Pricing," Mathematical and Computer Modelling, Vol. 49, No. 1-2, pp. 352-368.

Thiagarajah K. Thavaneswaran A. (2006), "Fuzzy Coefficient Volatility Models with Financial Applications," Journal of Risk Finance, Vol. 7, No. 5, , pp. 503-524.

Tolga, C. & Kahraman, C. 2008. Fuzzy Multiattribute Evaluation of R&D Projects Using a Real Options Valuation Model. International Journal of Intelligent Systems, 23: 1153-1176.

Trigeorgis, L. and Manson, S. (1987), „Valuing managerial flexibility", Midland Corporate Finance Journal, vol. 5, no. 1, pp.14-21.

Trigeorgis L. (1996), Real options: Managerial flexibility and strategy in resource allocation, Cambridge, MA: MIT Press,.

Trigeorgis L., (1993), Real options and Interactions with Financial Flexibility, Financial Management, 202-24.

Trigeorgis L. (1993), The Nature of Option Interactions and Valuation of Investments with Multiple Real Options, Journal of Financial and Quantitative Analysis, 309-26.

Trigeorgis L. (1996). Real Options. Managerial Flexibility and Strategy in Resource Allocation, The MIT Press,

Trigeorgis, L. (1998) : Real Options - Managerial Flexibility and Strategy in Resource Allocation. Harvard University.

Wang. X, He. J. Li, S. (2014), "Compound Option Pricing under Fuzzy Environment", Journal of Applied Mathematics, Volume 2014, Article ID 875319

Yoshida, Y. (2003). The valuation of European options in uncertain environment. European Journal of Operational Research, 145(1): 221-229.

Zadeh, L. A. (1965). Fuzzy Sets. Information and Control, 8: 338-353.

Zadeh LA (1978). Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets Syst. 1: 3–28

Zadeh L.A. (2008), Is there a need for fuzzy logic?, Information Sciences, 178, 2751-2779.

Zmeskal Z. (2001), Application of the fuzzy-stochastic methodology to appraising the firm value as a European call option, European Journal of Operational Research 135 303–310.

BIOGRAPHICAL SKETCH

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