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JET SCHEMES OF NON-ISOLATED HYPERSURFACE SINGULARITIES

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JET SCHEMES OF NON-ISOLATED HYPERSURFACE SINGULARITIES

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TABLE OF CONTENTS

LIST OF SYMBOLS

• $V(f_1,\dots,f_n)$	Vanishing set of polynomials $f_1,\dots,f_n$
• $\mathcal{O}_{p,X}$	Localization of X at p
• $SingX$	Singular locus of X
• $NP(f)$	Newton polygon of f
• $DNP(f)$	Dual Newton polygon of f
• $J_0(S)$	0^{th} jet scheme of surface S
• $J_m(S)$	m^{th} jet scheme of surface S

LIST OF FIGURES

ABSTRACT

In this thesis, we construct the jet schemes of some non-isolated hypersurface singularities in $\mathbb{C}^3$ and some isolated surface singularities in $\mathbb{C}^4$ which appear as the normalisation of our non-isolated singularity. We want to determine the jet scheme structure in these.

The singularities we consider are called rational triple singularities. They are of 9 types. In this work, we focus on the jet scheme structure of three of them, called E_{60}, E_{70} and E_{07}. We construct their jet graphs and toric embedded resolution.

The study differs from the existing results in the literature as it is about the case of non-isolated hypersurfaces and surface which is not monomial neither determinantal variety.

ÖZET

Biz bu tezde, $\mathbb{C}^3$'de yaşayan izole olmayan bazı hiperyüzey tekilliklerinin ve onların normalleşmesi halinde görünen $\mathbb{C}^4$'de yaşayan bazı izole yüzey tekilliklerinin jet şemalarını inşa ettik.

Dikkate aldığımız tekillikler, rasyonel üçlü tekillikler olarak adlandırılır. Onlar dokuz türlüdür. Biz bu çalışmada E_{60}, E_{70} ve E_{07} olarak adlandırılan, onların üç tanesinin jet şeması yapısına odaklandık. Onların jet grafiklerini ve torik gömülü çözümlerini inşa ettik.

Çalışma izole olmayan hiperyüzeylerle ve ne tek terimli ne de determinantal varyete olmayan yüzeylerle ilgili olduğu için, literatürde varolan sonuçlardan farklıdır.

1. INTRODUCTION

Let k be an algebraically closed field of characteristic 0. Let X be an affine variety over k. The m^{th} jet scheme of X, $J_m(X)$, is the set of all m-jets on X. We have $J_0(X) := X$ and $J_1(X) := TX$ where TX is total tangent space of X. If X is a smooth variety over k with dimension n, then $J_m(X)$ is k^{mn}-bundle. When X is singular, one may study on X by considering different types of singularity in X. For example, when X is of type A_n, D_n, E_6, E_7 or E_8, the jet scheme structure of X is given in (Mourtada, 2013); also when X is defined by a monomial ideal and when X has a determinental singularity, the jet scheme structure of X is given in (Yuen, 2006).

In this thesis, we construct the jet schemes of some non-isolated hypersurface singularities in $\mathbb{C}^3$ and some isolated surface singularities in $\mathbb{C}^4$ which appear as the normalisation of our non-isolated singularity. We want to determine the jet scheme structure in these.

The singularities we consider here are called rational triple singularities. They are of 9 types. In this work, we focus on the jet scheme structure of three of them, called E_{60}, E_{70} and E_{07}. We construct their jet graphs and toric embedded resolution.

We first recall some basic definitions and properties in algebraic geometry to use in the following sections.

In Chapter 3, we define the regular subdivision of a cone and more generally of a set of cones, called a fan.

In Chapter 4, we recall the ADE singularities of hypersurfaces and we introduce jet scheme structure of these singularities. Following (Mourtada, 2013), we construct explicitly the jet schemes of the singularity of type E_7.

In Chapter 5, we consider some hypersurfaces having one dimensional singular locus, which are called non-isolated forms of the rational triple singularities in $\mathbb{C}^3$. We present an explicit construction of non-isolated forms of types E_{60}, E_{70} and E_{07}.

In Chapter 6, we construct the jet schemes of surface defined by one of the equations given in (Tyurina, 1968). We call these the isolated forms of rational triple points. Then we compare the results of this chapter with the results obtained in Chapter 5.

2. PRELIMINARIES

Let k be a field. The ring of polynomials in one variable x over k is the set of elements in the form

$$f(x) = c_n x^n + c_{n-1}x^{n-1} + \ldots + c_0$$

where $c_i \in k$ for all i, $0 \leq i \leq n$ and $n \geq 0$. Such a ring is denoted by $k[x]$. Similarly, the ring of polynomials in several variables $x_1, \ldots x_n$ over k is the set of elements in the form

$$f(\mathbf{x}) = \sum_{\alpha} \mathbf{c}_\alpha \mathbf{x}^\alpha$$

where $\alpha = (\alpha_1, \ldots \alpha_n)$ and $\mathbf{x}^\alpha = x_1^{\alpha_1} \ldots x_n^{\alpha_n}$; it is denoted by $k[x_1, \ldots, x_n]$.
When we permit to have infinite sum as

$$f(x) = c_0 + c_1 x + \ldots + c_n x^n + \ldots$$

the set of such elements is called the ring of formal power series and denoted it by $k[[x]]$. Hence, we have $k[x_1, \ldots, x_n] \subset k[[x_1, \ldots, x_n]]$.

2.1 Ideals in $k[x_1, \ldots, x_n]$

Let $k = \mathbb{C}$ and $R = \mathbb{C}[x_1, \ldots x_n]$. An ideal I in R is a nonempty subset of R which is closed under addition in I and multiplication by the elements of R.
The ideal $I = \{0\}$ and $I = R$ are called the trivial ideals in R. Any ideal I in R is generated by a finite set of elements in R and denoted by

$$< f_1, \ldots f_r > = \{g_1 f_1 + \ldots + g_r f_r \ \mid \ g_i \in R, \ \ 1 \leq i \leq r\}$$

Here $f_1, \ldots, f_r$ are called generators of I.

Exemple 2.1.1. Consider the ideals $I_1 =< x, x^2y, y >$ and $I_2 =< x, y >$. Note that I_1 and I_2 are the same subsets in R, so $I_1 = I_2$. Hence, an ideal can be defined by different set of generators.

Theorem 2.1.2. (Hilbert Basis Theorem)*Every polynomial ideal in R is finitely generated.*

Let I and J be two ideals in R. The sum of $I =< f_1, \ldots f_r >$ and $J =< g_1, \ldots g_s >$ is defined as

$$I + J = \{f + g \ \mid \ f \in I, g \in J\}$$

In fact, we have $I + J =< f_1, \ldots f_r, g_1, \ldots g_s >$. The intersection $I \cap J$ is defined as

$$I \cap J = \{h \ \mid \ h \in I \ \ and \ \ h \in J\}$$

The product $I \cdot J$ is defined as

$$I.J = \{\sum f_i g_j \ \mid \ i = 1, \ldots, r \quad j = 1, \ldots, s\}$$

Exemple 2.1.3. Let $I = J =< x, y >$ in $\mathbb{C}[x, y]$. The sum of ideals is $I + J =< x, y >$, the intersection is $I \cap J =< x, y >$ and the product is $I.J =< x^2, xy, y^2 >$.

Definition 2.1.4. *If each generator of an ideal I in R is monomial, it is called monomial ideal.*

Definition 2.1.5. *Let I be an ideal in R. The radical of I, $\sqrt{I}$, is the set*

$$\{x \in R \mid x^n \in I \ \ n > 0\}$$

If $I = \sqrt{I}$, then I is called a radical ideal.

Remark 2.1.6. *Every prime ideal is a radical ideal.*

Definition 2.1.7. *An ideal I of a ring R is a primary ideal if for any $x, y \in R$, $xy \in I$ and $x \notin I$ imply that $y \in \sqrt{I}$.*

Definition 2.1.8. *A primary decomposition of an ideal I in R is defined as follows:*

$$I = \bigcap_{i=1}^{n} p_i$$

where every p_i is primary.

Exemple 2.1.9. Consider an ideal $I =< xy, x^3 - x^2, x^2y - xy >$. A primary decomposition of I is $< x > \cap < x - 1, y > \cap < x^2, y >$.

Exemple 2.1.10. Consider $I =< x^2, xy >$.It can be written as finite intersection of primary ideals:

$$I =< x > \cap < x^2, y >$$

or

$$I =< x > \cap < x^2, xy, y^2 >$$

Remark 2.1.11. *The primary decomposition of an ideal is not unique.*

2.2 Affine Varieties

Consider the map

$$V : \mathbb{C}[x_1, \ldots, x_n] \longrightarrow \mathbb{C}^n$$

which is associated to the zero set in $\mathbb{C}^n$ of each polynomial in $\mathbb{C}[x_1, \ldots x_n]$. That is we have

$$f \longmapsto V(f)$$

where $V(f) = \{(a_1, \ldots, a_n) \in \mathbb{C}^n \ \mid \ f(a_1, \ldots, a_n) = 0\}$.
More generally, V sends each ideal I in R to the zero set of each of its elements, means for $I =< f_1, \ldots, f_r >$;

$$V(I) = \{(a_1, \ldots, a_n) \in \mathbb{C} \ \mid \ f_i(a_1, \ldots, a_n) = 0 \ \ 1 \leq i \leq r\}$$

Definition 2.2.1. *The set $V(I)$ is called an affine variety in $\mathbb{C}^n$.*

Theorem 2.2.2. *An affine variety $V(I) = X$ is irreducible if and only if its defining ideal $I(X)$ is a prime ideal.*

Let $I =< f_1, \dots f_r >$ be an ideal in R. Consider two elements f and g in R. We say that f is equivalent to g modulo I if $f - g \in I$. This is written as $f \equiv g \ mod(I)$.

This relation is symmetric, reflexive and transitive, so it is an equivalence relation on R.

Definition 2.2.3. *The set of equivalence classes modulo I is called the quotient ring, which is denoted by R/I.*
An element $g \in R$ which is equivalent to f_i modulo I takes the same values on $V(I)$.

Assume that I is a radical ideal in R. The quotient ring R/I is called the coordinate ring of $Y := V(I)$ and denoted by $\mathbb{C}[Y]$. Theorem 2.10 implies that $\mathbb{C}[Y]$ is an integral domain.

The dimension of an affine variety equals the dimension of its coordinate ring which is the Krull dimension defined as the maximum $d \in \mathbb{N}$ such that there exist a chain of prime ideals

$$P_0 \subsetneq P_1 \subsetneq \dots \subsetneq P_d$$

of length d in R. We denote it by $d = dimY$ for $Y = V(I)$.

The number $n - d$ is called the codimension of the affine algebraic variety Y in $\mathbb{C}^n$. Note that an hypersurface in $\mathbb{C}^n$ is of codimension 1.

2.3 Singularities of Varieties

We are dealing here with the surface in $\mathbb{C}^n$. A surface is a 2 dimensional affine variety in $\mathbb{C}^n$.

Let $f \in \mathbb{C}[x_1, \dots, x_n]$. Let S be a hypersurface in $\mathbb{C}^n$ defined by f; it means

$$S := V(f) = \{(a_1, \dots, a_n) \in \mathbb{C}^n | \ \ f(a_1, \dots, a_n) = 0\}$$

Definition 2.3.1. *A point $p = (a_1, \dots, a_n) \in S$ is said to be singular or (singularity of S) if $f(p) = 0$ and*

$$\frac{\partial f}{\partial x_i}(p) = 0 \qquad \forall i, \ i = 1, \dots, n$$

Exemple 2.3.2. Consider the hypersurface defined by $f(x, y, z) = x^2 + y^3 + yz^3$ in $\mathbb{C}^3$. $f(0, 0, 0) = 0$ and

$$\frac{\partial f}{\partial x} = 2x = 0$$

$$\frac{\partial f}{\partial y} = 3y^2 + z^3 = 0$$

$$\frac{\partial f}{\partial z} = 3yz^2 = 0$$

Hence the point $(0, 0, 0)$ is singularity of f.

More generally, let $I \subset \mathbb{C}[x_1, \dots, x_n]$ be an ideal and S be a surface in $\mathbb{C}^n$ defined by I, means

$$S := V(I) = \{(a_1, \dots, a_n) \in \mathbb{C}^n | \;\; f(a_1, \dots, a_n) = 0 \;\; \forall f \in I\}$$

Definition 2.3.3. *A point $p \in S$ is said to be singular if the rank of the Jacobien matrix of I equals $n-2$, means*

$$rk(\frac{\partial f_i}{\partial x_j})(p) = n - 2$$

where $i = 1, \dots, m$, $j = 1, \dots, n$ and $I =< f_1, \dots, f_m >$.
If p is not a singular point of S then it is called a non-singular or a smooth point of S.

Exemple 2.3.4. Let us consider the ideal $I =< f_1, f_2, f_3 >$ where

$$f_1(x, y, z, w) = z^2 - yw + y^3 = 0$$

$$f_2(x, y, z, w) = zw - x^2y = 0$$

$$f_3(x, y, z, w) = w^2 - y^2w - x^2z = 0$$

The ideal I defines a surface $S := V(I)$ in $\mathbb{C}^4$. The surface S has an isolated singularity since the singular locus is only the origin. We want to examine the singular point of S; so the Jacobien matrix

$$Jac(I) = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial w} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial w} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} & \frac{\partial f_3}{\partial w} \end{pmatrix} = \begin{pmatrix} 0 & -w + 3y^2 & 2z & -y \\ -2xy & -x^2 & w & z \\ -2xz & -2yw & -x^2 & 2w - y^2 \end{pmatrix}$$

has rank 2 at a smooth point of S and has rank 3 at singular point.

Exemple 2.3.5. Consider the hypersurface defined by

$$f(x, y, z) = x^2 + y^3 + yz^3$$

in $\mathbb{C}^3$. It has a singular point at the origin.

Definition 2.3.6. *Let S be a surface in $\mathbb{C}^n$. A singular point $p \in S$ is called an isolated singularity if $S \backslash \{p\}$ is smooth; in other words, p has a neighborhood U in S such that $U \backslash \{p\} \cong \mathbb{C}^2$.*

Exemple 2.3.7. In the example 2.3.5. has an isolated singularity at the origin.

Exemple 2.3.8. Consider the hypersurface defined by

$$f(x, y, z) = z^3 + y^3z + x^2y^2$$

in $\mathbb{C}^3$. It has singularity along $(x, 0, 0)$. Such a singularity is called non-isolated singularity .

2.4 Newton Polygon

Let $f \in \mathbb{C}[x_1, \ldots, x_n]$ with $f(x_1, \ldots, x_n) = \sum\limits_{\alpha} c_\alpha \mathbf{x}^\alpha$ where $\mathbf{x}^\alpha = x_1^{\alpha_1} \ldots x_n^{\alpha_n}$ and $\alpha = (\alpha_1, \ldots, \alpha_n) \in \mathbb{R}^n$. The support of f is the set

$$supp(f) := \{\alpha \in \mathbb{R}^n | c_\alpha \neq 0\}$$

Exemple 2.4.1. Consider the example 2.3.2. The support of $f(x, y, z)$ is

$$supp(f) = \{(2, 0, 0), (0, 3, 0), (0, 1, 3)\}$$

Definition 2.4.2. *The closure in $\mathbb{R}^n$ of the convex hull of the set*

$$\bigcup_{\alpha \in Supp(f)} (\alpha + \mathbb{R}^n_{\geq 0})$$

is called the Newton polygon of f. We will denoted it by $NP(f)$.

Exemple 2.4.3. Consider the example 2.3.2. The $NP(f)$ is

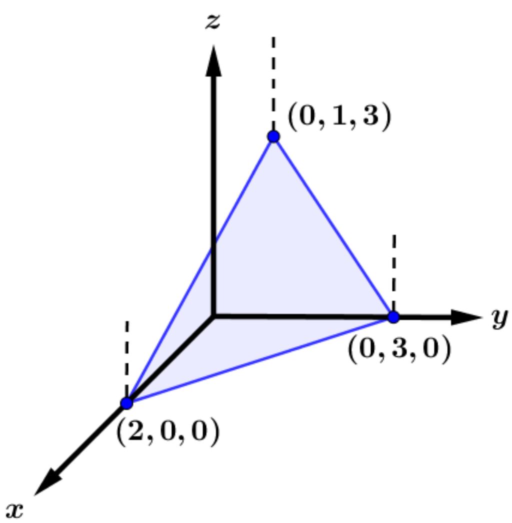

Figure 2.1: $NP(f)$

Exemple 2.4.4. Let us consider $f(x,y,z) = z^3 + y^3z + x^2y^2$ in $\mathbb{C}^3$. The $NP(f)$ is

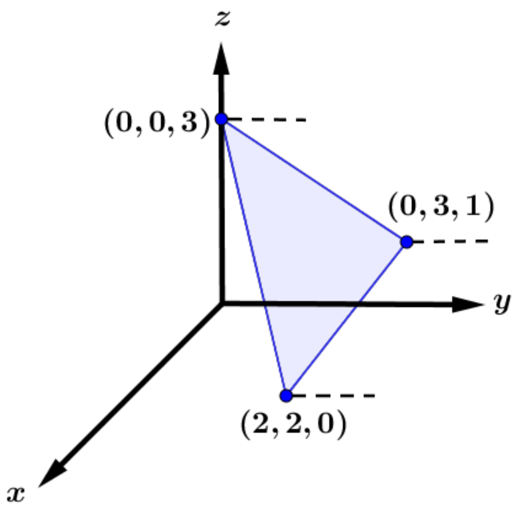

Figure 2.2: $NP(f)$

Definition 2.4.5. *The closure in $\mathbb{R}^n$ of the convex hull of the set*

$$\{v \in \mathbb{R}^n | < v, w > \geq 0 \quad \forall w \in NP(f)\}$$

is called dual Newton polygon of f. We will denoted it by DNP(f).

Exemple 2.4.6. Consider hypersurface defined by $f(x,y,z) = x^2 + y^3 + yz^3$ in $\mathbb{C}^3$. The support set is

$$supp(f) = \{(2,0,0), (0,3,0), (0,1,3)\}$$

The DNP(f) is

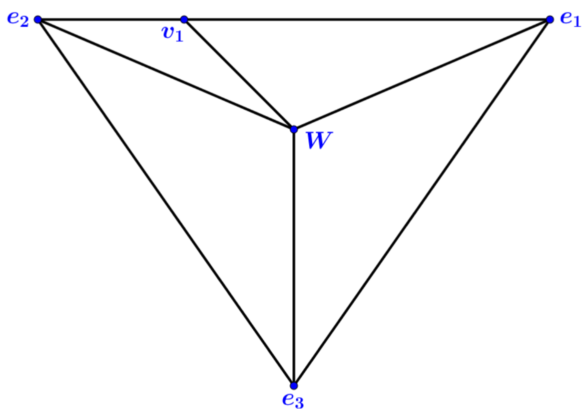

Figure 2.3: $DNP(f)$

where $e_1 = (1,0,0)$, $e_2 = (0,1,0)$, $e_3 = (0,0,1)$, $v_1 = (1,2,0)$ and $W = (9,6,4)$.

3. RESOLUTİON OF HYPERSURFACES

A *cone* σ in $\mathbb{R}^3$ is

$$cone(v_1, \dots, v_n) = \{\sum_{i=1}^{n} a_i v_i \mid \ v_i \in \mathbb{R}^3, a_i \in \mathbb{R}_{\geq 0}\}$$

The vectors $v_1, \dots, v_k$ are called the generators of σ. A *cone* σ is called regular if $det(v_1, \dots, v_k) = 1$ such that $det(v_1, \dots, v_k) := gcd(det(M_1), \dots, det(M_l))$ where M_i is the minor of the matrix $(v_1, \dots, v_k)$.

Definition 3.0.1 *For every compact faces F_i of $NP(f)$ in $\mathbb{C}^n$, if the system*

$$\frac{\partial F_i}{\partial x_j} = 0, \quad \textit{where } 0 \leq j \leq n$$

has no solution in $\mathbb{C}^*$, then f is a non-degenerate singularity.
Any *cone* can become regular by a suitable subdivision.

Theorem 3.0.2. *A non-degenerate singularity can be resolved by using Newton polygon.*

3.1. Regular Subdivision (of Newton Polygon)

A vector v in $\mathbb{R}^3$ is called integral vector if each component v_i of v is a positive integer where $v = (v_1, v_2, v_3)$. Let σ be a *cone* in $\mathbb{R}^3$ generated by two integral vectors $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$. If $det(\sigma) = 1$ then σ is regular.

If $det(\sigma) = d$ and $d \geqslant 2$, we check whether there exists $d_1 \in \mathbb{N}^*$ such that the quotient

$$v^1 := \frac{v + d_1 u}{d}$$

is an integral vector. If such d_1 exists it is not unique. Take the smallest value of d_1. If $d_1 = 1$, then the cone $\sigma = \sigma_1 \cup \sigma_2$ becomes regular where $\sigma_1 =< u, v^1 >$ and $\sigma_2 =< v^1, v >$.

Definition 3.1.1. *A fan $\sum$ in $\mathbb{R}^3$ is a finite collection of convex cones σ_i such that*
(i) Each face of any σ_i is also a cone in $\sum$.
(ii) Any intersection of two σ_i in $\sum$ is a face of each cone.

Note that, after subdivision σ becomes a fan in $\mathbb{R}^3$ and each σ_i is regular cone.
If the smallest $d_1 \neq 1$ then we check whether there exists $d_2 \in \mathbb{N}^*$ such that the quotient

$$v^2 := \frac{v + d_2 v^1}{d_1}$$

is an integral vector. If $d_2 = 1$, the regular subdivision of σ

is given by $\sigma = \sigma_1 \cup \sigma_2 \cup \sigma_3$ where $\sigma_1 =< u, v^1 >$, $\sigma_2 =< v^1, v^2 >$ and $\sigma_3 =< v^2, v >$. Note that, σ is a fan in which each σ_i is a regular cone.

If $d_2 \neq 1$, then we return to check the appropriate quotient. We continue until we find a regular subdivision of σ_i.
The regular subdivision of a fan consists of doing regular subdivision of each cone in the fan.

Remark 3.1.2. *In the sequel, we will draw the $DNP(f)$ by considering its intersection with the plane $x + y + z = 1$ to make easier the drawing.*

3.2. Isolated Hypersurface Singularity and Regular Subdivision

In this section, we will consider an hypersurface in $\mathbb{C}^3$ which has an isolated singularity at the origin and its corresponding Newton polygon. We will find the dual of the Newton polygon, which is a fan, and its regular subdivision. Such a subdivision will lead us to obtain the minimal resolution graph of the singularity at hand.

Exemple 3.2.1. In the example 2.13 (case E_7 singularity), the dual Newton polygon of $f(x, y, z)$ in $\mathbb{C}^3$ has 3 cones as follows:

$$\sigma_1 =< u_1, u_2, u_3, u_4 >$$

$$\sigma_2 =< v_1, v_2, v_3 >$$

$$\sigma_3 =< w_1, w_2, w_3 >$$

where $e_2 = u_1 = (0, 1, 0)$, $u_2 = v_2 = w_2 = (9, 6, 4)$, $u_3 = v_1 = (1, 2, 0)$, $e_3 = u_4 = w_3 = (0, 0, 1)$ and $e_1 = v_3 = w_1 = (1, 0, 0)$.
When we consider the intersection of the subdivided $DNP(f)$ and the plane $x+y+z = 1$, we obtain the subdivided $DNP(f)$ as follows:

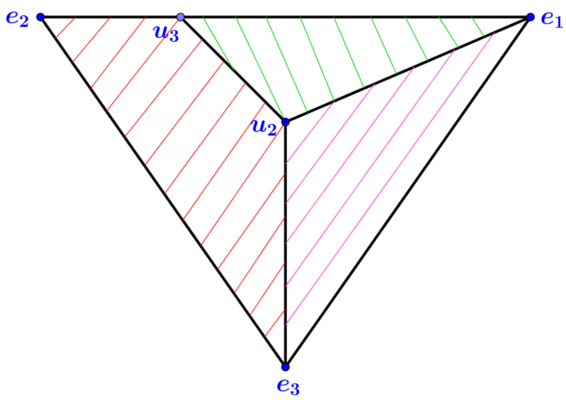

Figure 3.1: E_7a

We compute the regular subdivision of $\sigma_{ij} =< u_i, u_j >$, $\tau_{ij} =< v_i, v_j >$ and $\delta_{ij} =< w_i, w_j >$ for all i, j. Let us check σ_{12}:
$d = det(\sigma_{12}) = \begin{pmatrix} 0 & 1 & 0 \\ 9 & 6 & 4 \end{pmatrix} = gcd(9, 4) = 1$, hence σ_{12} is regular. Let us check σ_{13}:

$d = det(\sigma_{13}) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix} = gcd(1) = 1$, hence σ_{13} is regular. Let us check σ_{14}:
$d = det(\sigma_{14}) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = gcd(1) = 1$, hence σ_{14} is regular. Let us subdivide σ_{23} into regular cones:
$d = det(\sigma_{23}) = \begin{pmatrix} 9 & 6 & 4 \\ 1 & 2 & 0 \end{pmatrix} = gcd(12, 4, 8) = 4$. Consider the sequence of quotients:

$$v^1 := \frac{u_2 + d_1.u_3}{d} = \frac{(9, 6, 4) + 3.(1, 2, 0)}{4} = (3, 3, 1)$$

$$v^2 := \frac{u_2 + d_2.v^1}{d_1} = \frac{(9, 6, 4) + 2.(3, 3, 1)}{3} = (5, 4, 2)$$

$$v^3 := \frac{u_2 + d_3.v^2}{d_2} = \frac{(9, 6, 4) + 1.(5, 4, 2)}{2} = (7, 5, 3)$$

Now let us subdivide σ_{24} into regular cones:
$d = det(\sigma_{24}) = \begin{pmatrix} 9 & 6 & 4 \\ 0 & 0 & 1 \end{pmatrix} = gcd(9, 6) = 3$. Consider the sequence of quotients:

$$v^4 := \frac{u_2 + d_1.u_4}{d} = \frac{(9, 6, 4) + 2.(0, 0, 1)}{3} = (3, 2, 2)$$

$$v^5 := \frac{u_2 + d_2.v^4}{d_1} = \frac{(9, 6, 4) + 1.(3, 2, 2)}{2} = (6, 4, 3)$$

For the regular subdivision of $\tau_{ij} =< v_i, v_j >$, we already checked τ_{12}. Let us subdivide τ_{13} into regular cones:
$d = det(\tau_{13}) = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix} = gcd(2) = 2$. Consider the quotient:

$$v^6 := \frac{v_1 + d_1.v_2}{d} = \frac{(1, 2, 0) + 1.(1, 0, 0)}{2} = (1, 1, 0)$$

Let us subdivide τ_{23} into regular cones:
$d = det(\tau_{23}) = \begin{pmatrix} 9 & 6 & 4 \\ 1 & 0 & 0 \end{pmatrix} = gcd(6, 4) = 2$. Consider the quotient:

$$v^7 := \frac{v_2 + d_1.v_3}{d} = \frac{(9, 6, 4) + 1.(1, 0, 0)}{2} = (5, 3, 2)$$

To compute the regular subdivision of δ_{ij} we need to check δ_{12}, δ_{13} and δ_{23}. We already check δ_{12} and δ_{23} cases. Let us check δ_{13}:
$d = det(\delta_{13}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = gcd(1) = 1$, hence δ_{13} is regular.

We will denote $u_2 = W$. The additional vectors obtained above are denoted in Figure E_7b and it becomes:

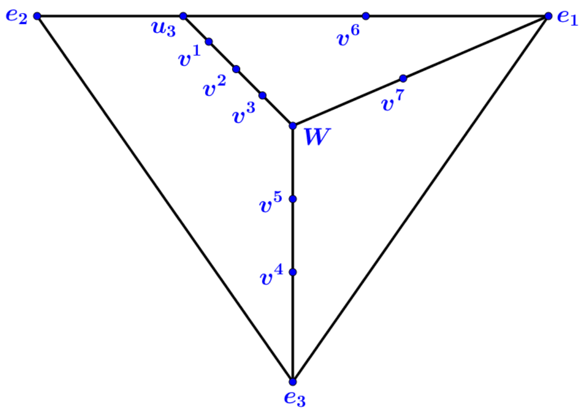

Figure 3.2: E_7b

This subdivision gives us the following graph which is the minimal resolution graph of singularity E_7:

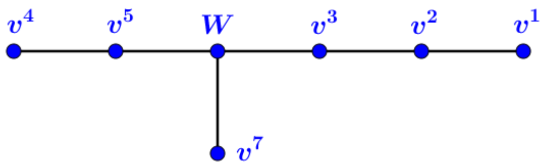

Figure 3.3: E_7m

We should check whether all possible two dimensional cones in Figure E_7b is regular. For this, we should compute the regular subdivision of the cones $< u_i, v^j >$, $< u_i, v_j >$, $< u_i, w_j >$, $< v_i, v^j >$, $< v_i, w_j >$ and $< w_i, v^j >$ for all i, j. Let us check: $< v^5, u_1 >$:

$d = det(\begin{pmatrix} 6 & 4 & 3 \\ 0 & 1 & 0 \end{pmatrix}) = gcd(6, 3) = 3$. Consider the sequence of quotients:

$$v^8 := \frac{v_5 + d_1.u_1}{d} = \frac{(6, 4, 3) + 2.(0, 1, 0)}{3} = (2, 2, 1)$$

$$v^9 := \frac{v_5 + d_2.v^8}{d} = \frac{(6, 4, 3) + 1.(2, 2, 1)}{2} = (4, 3, 2)$$

Let us check $< v^4, v_3 >$:
$d = det(\begin{pmatrix} 3 & 3 & 3 \\ 1 & 0 & 0 \end{pmatrix}) = gcd(2, 2) = 2$. Consider the quotient:

$$v^{10} := \frac{v_4 + d_1.v_3}{d} = \frac{(3, 2, 2) + 1.(1, 0, 0)}{2} = (2, 1, 1)$$

Let us check $< v^6, v^7 >$:
$d = det(\begin{pmatrix} 1 & 1 & 0 \\ 5 & 3 & 2 \end{pmatrix}) = gcd(2, 2, 2) = 2$. Consider the quotient:

$$v^{11} := \frac{v_6 + d_1.v^7}{d} = \frac{(1, 1, 0) + 1.(5, 3, 2)}{2} = (3, 2, 1)$$

Let us check $< v^8, u_4 >$:
$d = det(\begin{pmatrix} 2 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}) = gcd(2,2) = 2$. Consider the quotient:

$$v^{12} := \frac{v_8 + d_1.u_4}{d} = \frac{(2,2,1) + 1.(0,0,1)}{2} = (1,1,1)$$

Hence the dual Newton polygon in Figure E_7b becomes as the following with all the obtained vectors. We will called this regularised dual Newton polygon.

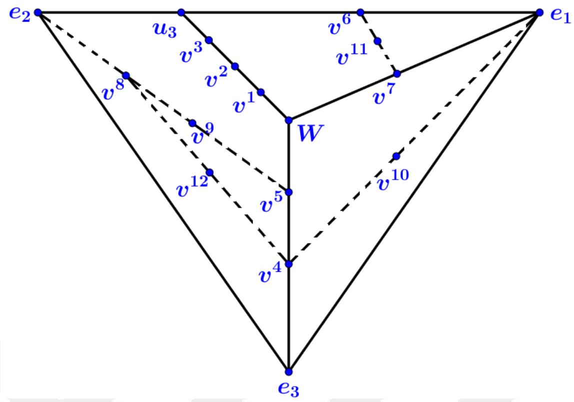

Figure 3.4: E_7c

Hence in our case we obtain the vectors which are $W = u_2 = v_2 = w_2 = (9,6,4)$, $u_3 = v_1 = (1,2,0)$, $v^1 = (3,3,1)$, $v^2 = (5,4,2)$, $v^3 = (7,5,3)$, $v^4 = (3,2,2)$, $v^5 = (6,4,3)$, $v^6 = (1,1,0)$, $v^7 = (5,3,2)$, $v^8 = (2,2,1)$, $v^9 = (4,3,2)$, $v^{10} = (2,1,1)$, $v^{11} = (3,2,1)$ and $v^{12} = (1,1,1)$.

Remark 3.2.2. *We will compare all the vectors marked in Figure E_7c with the vectors in the jet graph Figure 4.1.*

3.3. Non-Isolated Hypersurface Singularity and Regular Subdivision

In this section, we will consider an hypersurface in $\mathbb{C}^3$ which has a non-isolated singularity at the origin and its corresponding Newton polygon. We will find the dual of the Newton polygon and its regular subdivision to obtain the minimal resolution graph of the singularity at hand. For this we will follow (Altıntaş & Çevik & Tosun, 2016).

Exemple 3.3.1. Consider E_{60} singularity given by the equation:

$$f(x,y,z) = z^3 + y^3z + x^2y^2 = 0$$

in $\mathbb{C}^3$. The DNP(f) has three cones as follows:

$$\sigma_1 =< u_1, u_2, u_3, u_4 >$$

$$\sigma_2 =< v_1, v_2, v_3, v_4 >$$

$$\sigma_3 =< w_1, w_2, w_3 >$$

where $e_1 = u_1 = w_1 = (1,0,0)$, $e_2 = u_2 = (0,1,0)$, $u_3 = v_2 = (0,3,2)$, $u_4 = v_1 = w_2 = (5,4,6)$, $e_3 = v_3 = (0,0,1)$ and $v_4 = w_3 = (1,0,2)$.

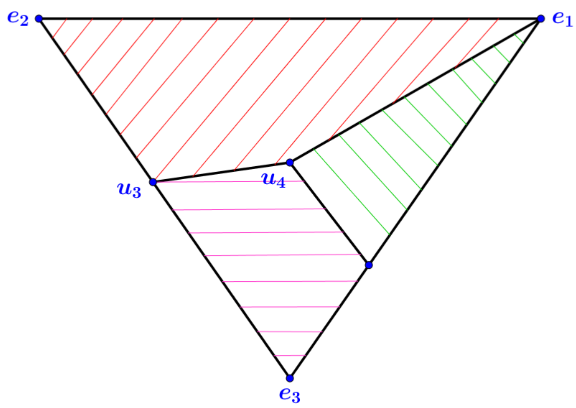

Figure 3.5: $E_{60}a$

We compute the regular subdivision of $\sigma_{ij} =< u_i, u_j >$, $\tau_{ij} =< v_i, v_j >$ and $\delta_{ij} =< w_i, w_j >$ for all i, j. Let us check σ_{12}:

$d = det(\sigma_{12}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = gcd(1) = 1$, hence σ_{12} is regular. Let us check σ_{13}:

$d = det(\sigma_{13}) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 3 & 2 \end{pmatrix} = gcd(3, 2) = 1$, hence σ_{13} is regular. Let us check σ_{14}:

$d = det(\sigma_{14}) = \begin{pmatrix} 1 & 0 & 0 \\ 5 & 4 & 6 \end{pmatrix} = gcd(4, 6) = 2$. Consider the quotient:

$$v^1 := \frac{u_1 + d_1.u_4}{d} = \frac{(1,0,0) + 1.(5,4,6)}{2} = (3,2,3)$$

Let us check σ_{23}:

$d = det(\sigma_{23}) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 3 & 2 \end{pmatrix} = gcd(2) = 2$. Consider the quotient:

$$v^2 := \frac{u_2 + d_1.u_3}{d} = \frac{(0,1,0) + 1.(0,3,2)}{2} = (0,2,1)$$

Let us check σ_{24}:

$d = det(\sigma_{24}) = \begin{pmatrix} 0 & 1 & 0 \\ 5 & 4 & 6 \end{pmatrix} = gcd(5, 6) = 1$, hence σ_{24} is regular. Let us check σ_{34}:

$d = det(\sigma_{34}) = \begin{pmatrix} 0 & 3 & 2 \\ 5 & 4 & 6 \end{pmatrix} = gcd(15, 10, 10) = 5$. Consider the sequence of quotients:

$$v^3 := \frac{u_3 + d_1.u_4}{d} = \frac{(0,3,2) + 3.(5,4,6)}{5} = (3,3,4)$$

$$v^4 := \frac{u_3 + d_2.v^3}{d_1} = \frac{(0,3,2) + 1.(3,3,4)}{3} = (1,2,2)$$

For the regular subdivision of $\tau_{ij} =< v_i, v_j >$, we already checked τ_{12}. Let us check τ_{13}:

$d = det(\tau_{13}) = \begin{pmatrix} 5 & 4 & 6 \\ 0 & 0 & 1 \end{pmatrix} = gcd(5, 4) = 1$. Hence τ_{13} is regular. Let us check τ_{14}:

$d = det(\tau_{14}) = \begin{pmatrix} 5 & 4 & 6 \\ 1 & 0 & 2 \end{pmatrix} = gcd(4, 4, 8) = 4$. Consider the sequence of quotients:

$$v^5 := \frac{v_1 + d_1.v_4}{d} = \frac{(5,4,6) + 3.(1,0,2)}{4} = (2,1,3)$$

$$v^6 := \frac{v_1 + d_2.v^5}{d_1} = \frac{(5,4,6) + 2.(2,1,3)}{3} = (3,2,4)$$

$$v^7 := \frac{v_1 + d_3.v^6}{d_2} = \frac{(5,4,6) + 1.(3,2,4)}{2} = (4,3,5)$$

Let us check τ_{23}:
$d = det(\tau_{23}) = \begin{pmatrix} 0 & 3 & 2 \\ 0 & 0 & 1 \end{pmatrix} = gcd(3) = 3$. Consider the quotient:

$$v^8 := \frac{v_2 + d_1.v_3}{d} = \frac{(0,3,2) + 1.(0,0,1)}{3} = (0,1,1)$$

Let us check τ_{24}:
$d = det(\tau_{24}) = \begin{pmatrix} 0 & 3 & 2 \\ 1 & 0 & 2 \end{pmatrix} = gcd(3,2,6) = 1$, hence τ_{24} is regular. Let us check τ_{34}:
$d = det(\tau_{24}) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 2 \end{pmatrix} = gcd(1) = 1$, hence τ_{34} is regular.
For the regular subdivision of δ_{ij}, we already check δ_{12} and δ_{23} cases. Let us check δ_{13}:
$d = det(\delta_{13}) = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 2 \end{pmatrix} = gcd(2) = 2$. Consider the quotient:

$$v^9 := \frac{w_1 + d_1.w_3}{d} = \frac{(1,0,0) + 1.(1,0,2)}{2} = (1,0,1)$$

When we consider the intersection of subdivided $DNP(f)$ and the plane $x+y+z=1$, we obtain the subdivided $DNP(f)$. We will denote $u_4 = W$. The additional vectors obtained above are denoted in Figure $E_{60}b$ and it becomes:

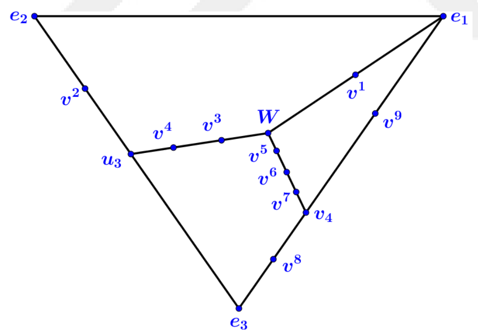

Figure 3.6: $E_{60}b$

This subdivision gives us the following graph which is the minimal resolution graph of the singularity E_{60}:

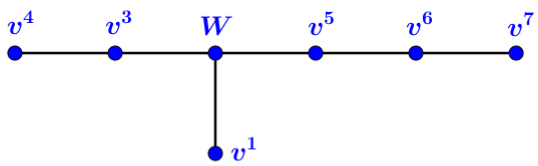

Figure 3.7: $E_{60}m$

We should whether all possible two dimensional cones in the Figure $E_{60}b$ is regular. For this, we should compute the regular subdivision of the cones $< u_i, v^j >$, $< u_i, v_j >$, $< u_i, w_j >$, $< v_i, v^j >$, $< v_i, w_j >$ and $< w_i, v^j >$ for all i, j. Let us check $< u_1, v^6 >$

$d = det(\begin{pmatrix} 1 & 0 & 0 \\ 3 & 2 & 4 \end{pmatrix}) = gcd(2,4) = 2$. Consider the quotient:

$$v^{10} := \frac{u_1 + d_1.v^6}{d} = \frac{(1,0,0) + 1.(3,2,4)}{2} = (2,1,2)$$

Let us check $< u_1, v^4 >$

$d = det(\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 2 \end{pmatrix}) = gcd(2,2) = 2$. Consider the quotient:

$$v^{11} := \frac{u_1 + d_1.v^4}{d} = \frac{(1,0,0) + 1.(1,2,2)}{2} = (1,1,1)$$

Let us check $< v_3, v^3 >$:

$d = det(\begin{pmatrix} 0 & 0 & 1 \\ 3 & 3 & 4 \end{pmatrix}) = gcd(3,3) = 3$. Consider the sequence of quotients:

$$v^{12} := \frac{v_3 + d_1.v^3}{d} = \frac{(0,0,1) + 2.(3,3,4)}{3} = (2,2,3)$$

$$v^{13} := \frac{v_3 + d_2.v^{12}}{d_1} = \frac{(0,0,1) + 1.(2,2,3)}{2} = (1,1,2)$$

Hence the dual Newton polygon in Figure $E_{60}b$ becomes the regularised dual Newton polygon of E_{60} as the following with all the obtained vectors.

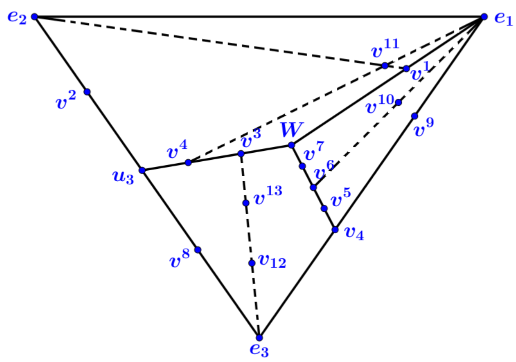

Figure 3.8: $E_{60}c$

Hence in our case, we have the vectors $u_3 = v_2 = (0,3,2)$, $W = u_4 = v_1 = w_2 = (5,4,6)$, $v_3 = (0,0,1)$, $v_4 = (1,0,2)$, $v^1 = (3,2,3)$, $v^2 = (0,2,1)$, $v^3 = (3,3,4)$, $v^4 = (1,2,2)$, $v^5 = (2,1,3)$, $v^6 = (3,2,4)$, $v^7 = (4,3,5)$, $v^8 = (0,1,1)$, $v^9 = (1,0,1)$, $v^{10} = (2,1,2)$, $v^{11} = (1,1,1)$, $v^{12} = (2,2,3)$ and $v^{13} = (1,1,2)$.

Remark 3.3.2. *We will compare all the vectors marked in Figure $E_{60}c$ with the vectors in the jet graph Figure 5.1.*

Exemple 3.3.3. Consider E_{70} singularity given by the equation:

$$f(x,y,z) = z^3 + x^2yz + y^4 = 0$$

in $\mathbb{C}^3$. The DNP(f) has three cones as follows:

$$\sigma_1 = < u_1, u_2, u_3, u_4 >$$
$$\sigma_2 = < v_1, v_2, v_3 >$$
$$\sigma_3 = < w_1, w_2, w_3, w_4 >$$

where $e_1 = u_1 = w_1 = (1,0,0)$, $e_2 = u_2 = (0,1,0)$, $u_3 = v_2 = (0,2,1)$, $u_4 = v_1 = w_2 = (5,6,8)$, $v_3 = w_3 = (0,1,3)$ and $e_3 = w_4 = (0,0,1)$.

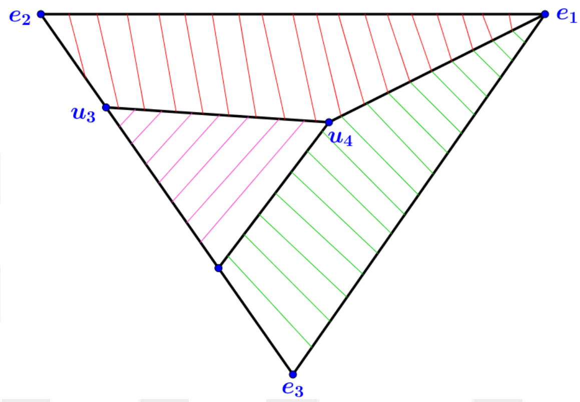

Figure 3.9: $E_{70}a$

We will denote $u_4 = W$. When we consider the intersection of subdivided $DNP(f)$ and the plane $x+y+z=1$, we obtain the subdivided $DNP(f)$ as follows:

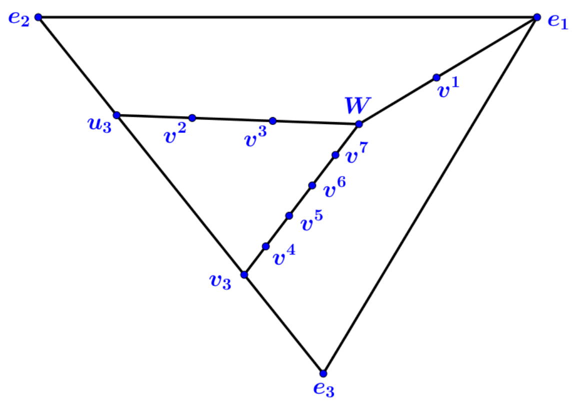

Figure 3.10: $E_{70}b$

where $v^1 = (3,3,4)$, $v^2 = (1,2,2)$, $v^3 = (3,4,5)$, $v^4 = (1,2,4)$, $v^5 = (2,3,5)$, $v^6 = (3,4,6)$ and $v^7 = (4,5,7)$.
This subdivision gives us the following graph which is the minimal resolution graph of the singularity E_{70}:

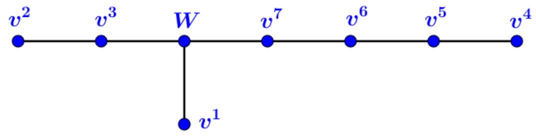

Figure 3.11: $E_{70}m$

In the subdivided DNP given in Figure $E_{70}b$, the only cones which still need to be subdivided are $< u_1, v^2 >$, $< u_2, v^6 >$, $< w_4, v^1 >$ which give the vectors $v^8 = (1,1,1)$, $v^9 = (2,3,4)$, $v^{10} = (2,2,3)$ and $v^{11} = (1,1,2)$. Hence the regularized $DNP(f)$ in this case is:

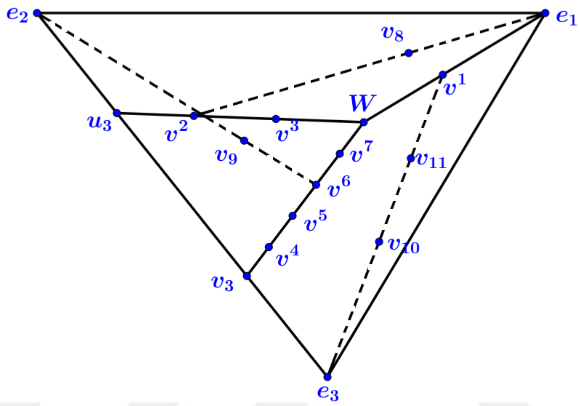

Figure 3.12: $E_{70}c$

Hence in our case we obtain the vectors $u_3 = v_2 = (0,2,1)$, $W = u_4 = v_1 = w_2 = (5,6,8)$, $w_3 = v_3 = (0,1,3)$, $v^1 = (3,3,4)$, $v^2 = (1,2,2)$, $v^3 = (3,4,5)$, $v^4 = (1,2,4)$, $v^5 = (2,3,5)$, $v^6 = (3,4,6)$, $v^7 = (4,5,7)$, $v^8 = (1,1,1)$, $v^9 = (2,3,4)$, $v^{10} = (2,2,3)$ and $v^{11} = (1,1,2)$.

Remark 3.3.4. *We will compare all the vectors marked in Figure $E_{70}c$ with the vectors in the jet graph Figure 5.4.*

Exemple 3.3.5. Consider E_{07} singularity given by the equation:

$$f(x,y,z) = z^3 + y^5 + x^2y^2 = 0$$

in $\mathbb{C}^3$. The DNP(f) has three cones as follows:

$$\sigma_1 =< u_1, u_2, u_3, u_4 >$$

$$\sigma_2 =< v_1, v_2, v_3 >$$

$$\sigma_3 =< w_1, w_2, w_3 >$$

where $e_1 = u_1 = w_1 = (1,0,0)$, $e_2 = u_2 = (0,1,0)$, $u_3 = v_2 = (0,3,2)$, $u_4 = v_1 = w_2 = (9,6,10)$ and $e_3 = v_3 = w_3 = (0,0,1)$.

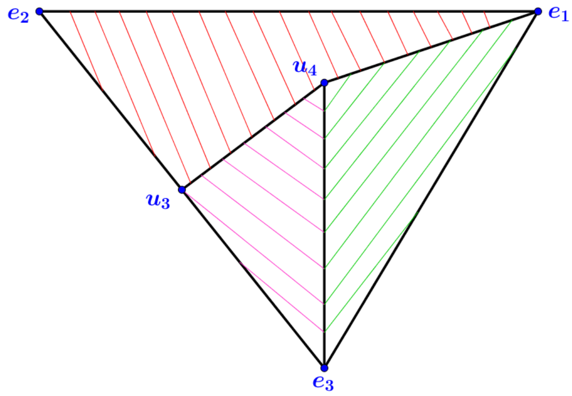

Figure 3.13: $E_{07}a$

When we consider the intersection of subdivided $DNP(f)$ and the plane $x+y+z=1$, we obtain the subdivided $DNP(f)$. We will denote $u_4 = W$.

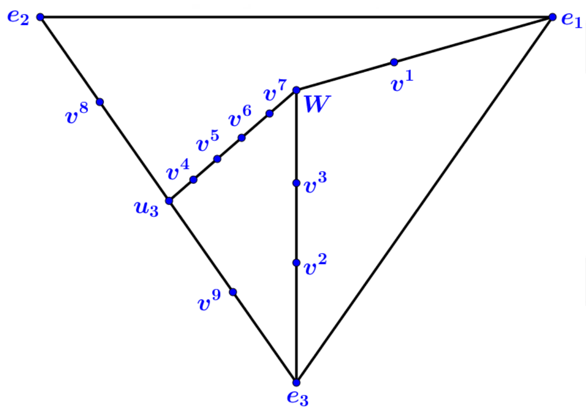

Figure 3.14 $E_{07}b$

where $v^1 = (5,3,5)$, $v^2 = (3,2,4)$, $v^3 = (6,4,7)$, $v^4 = (1,2,2)$, $v^5 = (3,3,4)$, $v^6 = (5,4,6)$, $v^7 = (7,5,8)$, $v^8 = (2,1,2)$ and $v^9 = (3,2,3)$,. This subivision gives us the following graph which is the miniml resolution graph of the singularity E_{07}:

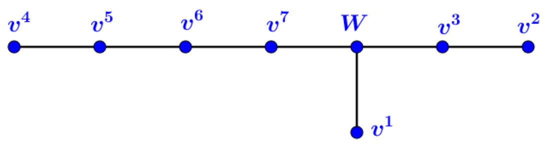

Figure 3.15: $E_{07}m$

In the subdivided $DNP(f)$ given in Figure $E_{07}b$, the only cones which still need to be subdivided are $< u_1, v^3 >$, $< u_2, v^1 >$, $< v_4, v^5 >$ and $< v^{14}, v_3 >$ which give the vectors $v^{10} = (1,1,1)$, $v^{11} = (2,2,3)$, $v^{12} = (1,1,2)$, $v^{13} = (0,2,1)$, $v^{14} = (0,1,1)$ and $v^{15} = (4,3,5)$. Hence the regularized $DNP(f)$ in this case is:

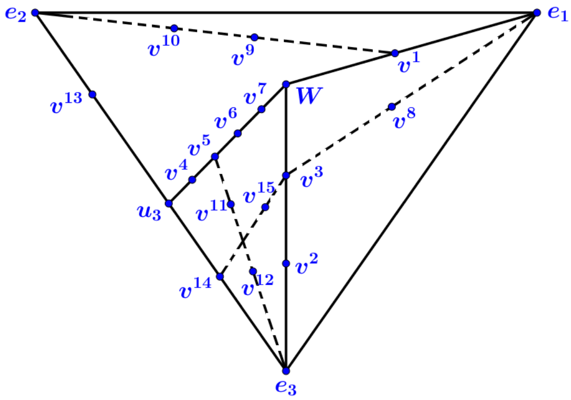

Figure 3.16: $E_{07}c$

Hence in our case we obtain the vectors $u_3 = v_2 = (0,3,2)$, $u_4 = v_1 = w_2 = (9,6,10)$, $v^1 = (5,3,5)$, $v^2 = (3,2,4)$, $v^3 = (6,4,7)$, $v^4 = (1,2,2)$, $v^5 = (3,3,4)$, $v^6 = (5,4,6)$, $v^7 = (7,5,8)$, $v^8 = (2,1,2)$, $v^9 = (3,2,3)$, $v^{10} = (1,1,1)$, $v^{11} = (2,2,3)$, $v^{12} = (1,1,2)$, $v^{13} = (0,2,1)$, $v^{14} = (0,1,1)$ and $v^{15} = (4,3,5)$.

Remark 3.3.6. *We will compare all the vectors marked in Figure $E_{07}c$ with the vectors in the jet graph Figure 5.7.*

4. JET SCHEMES OF ADE SINGULARITIES

In this chapter, we will consider the jet scheme structure of an hypersurface. We will find the jet schemes of an hypersurface with isolated singularity at the origin which is one of the hypersurface of ADE-singularities.

4.1 Hypersurfaces with Isolated Singularity

Let X be an hypersurface in $\mathbb{C}^n$ defined by $f(x_1, \ldots, x_n)$. Let $\mathbb{C}[[t]]$ be the ring of formal power series in t. Consider the morphism

$$\varphi : \mathbb{C}[x_1, \ldots, x_n]/ < f > \longrightarrow C[[t]]$$

defined by

$$\varphi(x_i) = x_{i,0} + x_{i,1}t + x_{i,2}t^2 + x_{i,3}t^3 + \ldots$$

with

$$f(x_1(t), x_2(t)), \ldots, x_n(t)) = F_0 + tF_1 + t^2F_2 + \ldots = 0$$

The Spec of each of rings gives:

$$\gamma : Spec\mathbb{C}[[t]] \longrightarrow X$$

This morphism defines a parametrized curve $\gamma(t)$ on X.

Definition 4.1.1. *The parametrized curve $\gamma(t)$ on X is called an arc.*
The space of arcs on X, denoted by $J_\infty(X)$, is the set of all arcs on X and is given by

$$J_\infty(X) = Spec\frac{\mathbb{C}[x_{1,0}, x_{1,1}, \ldots, x_{n,0}, x_{n,1}, \ldots]}{< F_0, F_1, \ldots, F_m >}$$

Now let us consider the morphism

$$\varphi_m : \mathbb{C}[x_1, \ldots, x_n]/ < f > \longrightarrow C[[t]]/ < t^{m+1} >$$

defined by

$$\varphi_m(x_i) = x_{i,0} + x_{i,1}t + x_{i,2}t^2 + \ldots + x_{i,m}t^m \qquad mod(t^{m+1})$$

so we have:

$$\gamma_m : Spec(\mathbb{C}[[t]]/ < t^{m+1} >) \longrightarrow X$$

An m^{th} jet on X is an arc given by the morphism γ_m.

Remark 4.1.2. *The set of all m jets on X, denoted by $J_m(X)$, forms a scheme structure.*

An m^{th} jet scheme of X is

$$J_m(X) = Spec\frac{\mathbb{C}[x_{1,0}, \dots, x_{n,0}, x_{1,1}, \dots x_{n,1}, \dots, x_{n,m}]}{< F_0, F_1, \dots, F_m >}$$

The ideal $I_m =< F_0, F_1, \dots, F_m >$ is said to be the defining ideal of $J_m(X)$.

Remark 4.1.3. *The m^{th} jet scheme of X, $J_m(X)$ is irreducible if its defining ideal $I_m =< F_0, F_1, \dots, F_m >$ is prime.*

Exemple 4.1.4. Consider an hypersurface X in $\mathbb{C}^2$ defined by $f(x, y) = xy$.

For m=0, we have

$$f(x_0, y_0) = x_0y_0 = F_0$$

The 0^{th} jet scheme of X is given by

$$J_0(X) = Spec\frac{\mathbb{C}[x_0, y_0]}{< F_0 >}$$

For m=1, we have

$$f(x_0 + x_1t, y_0 + y_1t) = x_0y_0 + t(x_0y_1 + x_1y_0) = F_0 + F_1t$$

The 1^{th} jet scheme of X is given by

$$J_1(X) = Spec\frac{\mathbb{C}[x_0, x_1, y_0, y_1]}{< F_0, F_1 >}$$

$$\vdots$$

For m, we have

$$f(x_0 + \dots + x_mt^m, y_0 + \dots + y_mt^m) = F_0 + tF_1 + \dots + t^mF_m$$
$$= x_0y_0 + t(x_0y_1 + x_1y_0) + \dots + t^m(x_0y_m + \dots + x_my_0)$$

The m^{th} jet scheme of X is given by

$$J_m(X) = Spec\frac{\mathbb{C}[x_0, \dots, x_m, y_0, \dots y_m]}{< F_0, F_1, \dots, F_m >}$$

Remark 4.1.5. *(i) The 0^{th} jet scheme of X, $J_0(X)$ is equal to X.*
(ii) The 1^{th} jet scheme of X, $J_1(X)$ is equal to total tangent space of X.

Let us assume $X = \mathbb{C}^n$ be a smooth variety. Consider the morphism

$$\varphi_m : \mathbb{C}[x_1, \dots, x_n] \longrightarrow C[[t]]/ < t^{m+1} >$$

The m^{th} jet scheme of X is given by

$$J_m(X) = Spec\mathbb{C}[x_{1,1}, \dots, x_{1,m}, x_{2,1}, \dots x_{2,m}, \dots, x_{n,m}]$$

Hence we obtain $J_m(X) = \mathbb{C}^{nm}$. Consider the projection map

$$\pi_m^{m-1} : J_m(\mathbb{C}^n) \longrightarrow J_{m-1}(\mathbb{C}^n)$$

This map is surjective and it is induced by the inclusion

$$\mathbb{C}[x_{1,1}, \ldots, x_{1,m}, \ldots, x_{n,1}, \ldots, x_{n,m}] \hookrightarrow \mathbb{C}[x_{1,1}, \ldots, x_{1,m-1}, \ldots, x_{n,1}, \ldots, x_{n,m-1}]$$

Let us denote $Sing(X)$ for the singular locus of X. We define easly the projection map

$$\pi_m : J_m(X) \longrightarrow J_0(X) = X$$

In the case of X has singularity, the projection map π_m composes of $\pi_m^{-1}(Sing(X))$ and $\pi_m^{-1}(Reg(X))$ where $Reg(X)$ is the set of smooth points of X. We will interest in the irreducible components of $\pi_m^{-1}(Sing(X))$. In order to find defining ideal of $J_m(X)$, we need to look all of F_i, $0 \leq i \leq m$. Since every step contains the previous one we will write F_m for $J_m(X)$ and we will take into consideration all F_j where $0 \leq j \leq m$.

4.2 ADE Singularities

The hypersurfaces defined in $\mathbb{C}^3$ by one of the following equations are called ADE-hypersurfaces ($n \in \mathbb{N}$) :

$$A_n : xy - z^{n+1} = 0$$
$$D_n, n \geq 4 : z^2 - x(y^2 + x^{n-2}) = 0$$
$$E_6 : z^2 + y^3 + x^4 = 0$$
$$E_7 : x^2 + y^3 + yz^3 = 0$$
$$E_8 : z^2 + y^3 + x^5 = 0$$

These hypersurfaces have an isolated singularity at the origin in $\mathbb{C}^3$, they are called ADE-singularities.

4.3 Jet Schemes of an Hypersurface of type E_7

Let X be an hypersurface of type E_7 in $\mathbb{C}^3$. We know that X is defined by

$$f(x, y, z) = x^2 + y^3 + yz^3$$

in $\mathbb{C}[x, y, z]$. We write

$$f(x_0+x_1t+\ldots+x_mt^m, y_0+\ldots+y_mt^m, z_0+\ldots+z_mt^m) = F_0+tF_1+\ldots \mod (t^{m+1}) \quad (4)$$

For m=0, we have

$$F_0(x_0, y_0, z_0) = x_0^2 + y_0(y_0^2 + z_0^3)$$

• This says that $x_0 = y_0 = 0$. Hence the ideal is $I_0 =< x_0, y_0 >$. This corresponds to the vector $v_1^0 = (1, 1, 0)$ with codimension 2 in $\mathbb{C}^3$. $J_0(E_7)$ is given by

$$J_0(E_7) = Spec \frac{\mathbb{C}[x_0, y_0, z_0]}{< F_0 >}$$

For m=1, (4) gives;

$$F_1(x_0,\ldots,z_1)=x_0x_1+y_1z_0^3+y_0(y_0y_1+z_0^2z_1)$$

• Over the ideal I_0, we obtain two possible ideals $I_{11}=<x_0,y_0,z_0>$ and $I_{12}=<x_0,y_0,y_1>$. These correspond to the vectors $v_1^1=(1,1,1)$ and $v_2^1=(1,2,0)$ respectively. Each of which of codimension is 3. $J_1(E_7)$ is given by

$$J_1(E_7)=Spec\frac{\mathbb{C}[x_0,y_0,z_0,x_1,y_1,z_1]}{<F_0,F_1>}$$

For m=2, (4) gives;

$$F_2(x_0,\ldots,z_2)=x_0x_2+x_1^2+y_0(y_0y_2+y_1^2+z_0^2z_2+z_0z_1^2)+z_0^2(y_1z_1+y_2z_0)$$

• Over the ideal I_{11} we obtain the ideal $I_{21}=<x_0,y_0,z_0,x_1,>$. The corresponding vector $v_1^2=(2,1,1)$ is of codimension 4.
• Over the ideal I_{12} we obtain two possible ideal $I_{22}=<x_0,y_0,y_1,x_1,z_0>$ and $I_{23}=<x_0,y_0,y_1,x_1,y_2>$ with corresponding vector $v_2^2=(2,2,1)$ and $v_3^2=(2,3,0)$ respectively. Each of which of codimension is 5. $J_2(E_7)$ is given by

$$J_2(E_7)=Spec\frac{\mathbb{C}[x_0,\ldots,z_2]}{<F_0,F_1,F_2>}$$

For m=3, (4) gives;

$$F_3(x_0,\ldots,z_3)=x_0x_3+x_1x_2+\ldots+y_1^3+\ldots+y_3z_0^3$$

• Over the ideals I_{21} and I_{22} we obtain the ideal $I_{31}=<x_0,y_0,z_0,x_1,y_1>$. The corresponding vector $v_1^3=(2,2,1)$ is of codimension 5.
• Over the ideals I_{23}, we obtain two possible ideals $I_{32}=<x_0,y_0,y_1,x_1,y_2,z_0>$ and $I_{33}=<x_0,y_0,y_1,x_1,y_2,y_3>$ with corresponding vector $v_2^3=(2,3,1)$ and $v_3^3=(2,4,0)$. Each of which of codimension is 6. $J_3(E_7)$ is given by

$$J_3(E_7)=Spec\frac{\mathbb{C}[x_0,\ldots,z_3]}{<F_0,F_1,F_2,F_3>}$$

Remark 4.3.1. *The m^{th} jet scheme of E_7 is given by*

$$J_m(E_7)=Spec\frac{\mathbb{C}[x_0,y_0,z_0,\ldots,z_m]}{<F_0,F_1,\ldots,F_m>}$$

for all m.

For m=4, (4) gives;

$$F_4(x_0,\ldots,z_4)=x_0x_4+x_1x_3+\ldots+y_1^2y_2+\ldots+y_4z_0^3$$

• Over the ideal I_{31}, we obtain the ideal $I_{41}=<x_0,y_0,z_0,x_1,y_1,x_2>$. The corresponding vector $v_1^4=(3,2,1)$ is of codimension 6.
• Over the ideal I_{32}, we obtain the ideal $I_{42}=<x_0,y_0,y_1,x_1,y_2,z_0,x_2>$. The corresponding vector $v_2^4=(3,3,1)$ is of codimension 7.
• Over the ideal I_{33}, we obtain two possible ideals $I_{43}=<x_0,y_0,y_1,x_1,y_2,y_3,x_2,z_0>$ and $I_{44}=<x_0,y_0,y_1,x_1,y_2,y_3,x_2,y_4>$ with corresponding vectors $v_3^4=(3,4,1)$ and $v_4^4=(3,5,0)$ respectively. Each of which of codimension is 8.

Remark 4.3.2. *The vectors (a, b, c) in $\mathbb{R}^3$ corresponding to an ideal defining $J_m(X)$ which is irreducible is called weight vector. It is obtained by the number of appearances of x, y and z in the ideal independently their subscript.*

For m=5, (4) gives;

$$F_5(x_0, \ldots, z_5) = x_0x_5 + x_1x_4 + \ldots + y_1y_2^2 + \ldots + y_5z_0^3$$

• Over the ideal I_{41}, we obtain two possible ideals $I_{51} =< x_0, y_0, z_0, x_1, y_1, x_2, z_1 >$ and $I_{52} =< x_0, y_0, z_0, x_1, y_1, x_2, y_2 >$ with corresponding vectors $v_1^5 = (3, 2, 2)$ and $v_2^5 = (3, 3, 1)$ respectively. Each of which of codimension is 7.
• Over the ideal I_{42}, we obtain the ideal I_{52}. Over the ideal I_{43}, we obtain the ideal $I_{53} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, z_0 >$. The corresponding vector $v_3^5 = (3, 4, 1)$ is of codimension 8.
• Over the ideal I_{44}, we obtain two possible ideals $I_{54} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, z_0 >$ and $I_{55} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, y_5 >$ with corresponding vectors $v_4^5 = (3, 5, 1)$ and $v_5^5 = (3, 6, 0)$ respectively. Each of which of codimension is 9.

For m=6, (4) gives;

$$F_6(x_0, \ldots, z_6) = x_0x_6 + x_1x_5 + \ldots + y_2^3 + \ldots + y_6z_0^3$$

• Over the ideal I_{51}, we obtain the ideal $I_{61} =< x_0, y_0, z_0, x_1, y_1, x_2, z_1, x_3, y_2 >$. The corresponding vector $v_1^6 = (4, 3, 2)$ is of codimension 9.
• Over the ideal I_{52}, we obtain two possible ideals I_{61} and $I_{62} =< x_0, y_0, z_0, x_1, y_1, x_2, y_2, x_3, y_3 >$. The corresponding vector of I_{62} is $v_2^6 = (4, 4, 1)$ with codimension 9.
• Over the ideal I_{53}, we obtain the ideal I_{62}.
• Over the ideal I_{54}, we obtain the ideal $I_{63} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, z_0, x_3 >$. The corresponding vector $v_3^6 = (4, 5, 1)$ is of codimension 10.
• Over the ideal I_{55}, we obtain two possible ideals $I_{64} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, y_5, x_3, z_0 >$ and $I_{65} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, y_5, x_3, y_6 >$. The corresponding vectors $v_4^6 = (4, 6, 1)$ and $v_5^6 = (4, 7, 0)$ are of codimension 11.

For m=7, (4) gives;

$$F_7(x_0, \ldots, z_7) = x_0x_7 + x_1x_6 + \ldots + y_2^2y_3 + \ldots + y_7z_0^3$$

• Over the ideal I_{61}, we obtain the ideal $I_{71} =< x_0, y_0, z_0, x_1, y_1, x_2, z_1, x_3, y_2 >$. The corresponding vector $v_1^7 = (4, 3, 2)$ is of codimension 9.
• Over the ideal I_{62}, we obtain the ideal $I_{72} =< x_0, y_0, z_0, x_1, y_1, x_2, y_2, x_3, y_3, z_1 >$. The corresponding vector $v_2^7 = (4, 4, 2)$ is of codimension 10.
• Over the ideal I_{63}, we obtain the ideal $I_{73} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, z_0, x_3 >$. The corresponding vector $v_3^7 = (4, 5, 1)$ is of codimension 10.
•Over the ideal I_{64}, we obtain the ideal $I_{74} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, y_5, x_3, z_0 >$. The corresponding vector $v_4^7 = (4, 6, 1)$ is of codimension 11.

For m=8, (4) gives;

$$F_8(x_0, \ldots, z_8) = x_0x_8 + x_1x_7 + \ldots + y_2y_3^2 + \ldots + y_8z_0^3$$

• Over the ideal I_{71}, we obtain the ideal $I_{81} =< x_0, y_0, z_0, x_1, y_1, x_2, z_1, x_3, y_2, x_4 >$. The corresponding vector $v_1^8 = (5, 3, 2)$ is of codimension 10.
•Over the ideal I_{72}, we obtain the ideal $I_{82} =< x_0, y_0, z_0, x_1, y_1, x_2, y_2, x_3, y_3, z_1, x_4 >$. The corresponding vector $v_2^8 = (5, 4, 2)$ is of codimension 11.
• Over the ideal I_{73}, we obtain two possible ideals $I_{83} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, z_0, x_3, x_4, z_1 >$ and $I_{84} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, z_0, x_3, x_4, y_5 >$ with corresponding vectors $v_3^8 = (5, 5, 2)$ and $v_4^8 = (5, 6, 1)$. Each of which of codimension is 12.
• Over the ideal I_{74}, we obtain the ideal I_{84}.

For m=9, (4) gives;

$$F_9(x_0, \ldots, z_9) = x_0x_9 + \ldots + x_4x_5 + y_0^2y_9 + \ldots + y_3^3 + y_0z_0^2z_9 + \ldots + y_9z_0^3$$

• Over the ideal I_{81} and I_{82} , we obtain the ideal $I_{91} =< x_0, y_0, z_0, x_1, y_1, x_2, y_2, x_3, y_3, z_1, x_4 >$. The corresponding vector $v_1^9 = (5, 4, 2)$ is of codimension 11.
•Over the ideal I_{83}, we obtain the ideal $I_{92} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, z_0, x_3, x_4, z_1 >$. The corresponding vector $v_2^9 = (5, 5, 2)$ is of codimension 12.
• Over the ideal I_{84}, we obtain two possible ideals $I_{93} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, z_0, x_3, x_4, z_1, y_5 >$ and $I_{94} =< x_0, y_0, y_1, x_1, y_2, y_3, x_2, y_4, z_0, x_3, x_4, y_5, y_6 >$ with corresponding vectors $v_3^9 = (5, 6, 2)$ and $v_4^9 = (5, 7, 1)$. Each of which of codimension is 13.

In the same way;

For m=10, (4) gives;

$$F_{10}(x_0, \ldots, z_{10}) = x_0x_{10} + x_1x_9 + \ldots + y_0^2y_{10} + \ldots + y_3^2y_4 + y_0z_0^2z_{10} + \ldots + y_{10}z_0^3$$

• Over the ideal I_{91}, we obtain two possible ideals I_{101} and I_{102} with corresponding vectors $v_1^{10} = (6, 4, 3)$ and $v_2^{10} = (6, 5, 2)$. Each of which of codimension is 13.
• Over the ideal I_{92}, we obtain the ideal I_{102}.
• Over the ideal I_{93}, we obtain the ideal I_{103}. The corresponding vector $v_3^{10} = (6, 6, 2)$ is of codimension 14.

For m=11, (4) gives;

$$F_{11}(x_0, \ldots, z_{11}) = x_0x_{11} + x_1x_{10} + \ldots + y_0^2y_{11} + \ldots + y_3y_4^2 + y_0z_0^2z_{11} + \ldots + y_{11}z_0^3$$

• Over the ideal I_{101}, we obtain the ideal I_{111}. The corresponding vector $v_1^{11} = (6, 4, 3)$ is of codimension 13.
• Over the ideal I_{102} we obtain two possible ideals I_{112} and I_{113} with corresponding vectors $v_2^{11} = (6, 5, 3)$ and $v_3^{11} = (6, 6, 2)$. Each of which of codimension is 14.
• Over the ideal I_{103}, we obtain the ideal I_{113}.

For m=12, (4) gives;

$$F_{12}(x_0, \ldots, z_{12}) = x_0x_{12} + x_1x_{11} + \ldots + y_0^2y_{12} + \ldots + y_4^3 + y_0z_0^2z_{12} + \ldots + y_{12}z_0^3$$

• Over the ideal I_{111} and I_{112}, we obtain the ideal I_{121}. The corresponding vector $v_1^{12} = (7, 5, 3)$ is of codimension 15.

• Over the ideal I_{131}, we obtain two possible ideals I_{122} and I_{123} with corresponding vectors $v_2^{12} = (7, 6, 3)$ and $v_3^{12} = (7, 7, 2)$. Each of which of codimension is 16.

For m=13, (4) gives;

$$F_{13}(x_0, \ldots, z_{13}) = x_0x_{13} + x_1x_{12} + \ldots + y_0^2y_{13} + \ldots + y_4^2y_5 + y_0z_0^2z_{13} + \ldots + y_{13}z_0^3$$

• Over the ideal I_{121}, we obtain the ideal I_{131}. The corresponding vector $v_1^{13} = (7, 5, 3)$ is of codimension 15.
• Over the ideal I_{122}, we obtain the ideal I_{132}. The corresponding vector $v_2^{13} = (7, 6, 3)$ is of codimension 16.

For m=14, (4) gives;

$$F_{14}(x_0, \ldots, z_{14}) = x_0x_{14} + x_1x_{13} + \ldots + y_0^2y_{14} + \ldots + y_4y_5^2 + y_0z_0^2z_{14} + \ldots + y_{14}z_0^3$$

• Over the ideal I_{131}, we obtain two possible ideals I_{141} and I_{142} with corresponding vectors $v_1^{14} = (8, 5, 4)$ and $v_2^{14} = (8, 6, 3)$. Each of which of codimension is 17.
• Over the ideal I_{132}, we obtain ideal I_{142}.

For m=15, (4) gives;

$$F_{15}(x_0, \ldots, z_{15}) = x_0x_{15} + x_1x_{14} + \ldots + y_0^2y_{15} + \ldots + y_5^3 + y_0z_0^2z_{15} + \ldots + y_{15}z_0^3$$

• Over the ideal I_{141}, we obtain the ideal I_{151}. The corresponding vector $v_1^{15} = (8, 6, 4)$ is of codimension 18.
• Over the ideal I_{142}, we obtain two possible ideals I_{152} and I_{153}. The corresponding vector of I_{153} is $v_2^{15} = (8, 7, 3)$ with codimension 18.

For m=16, (4) gives;

$$F_{16}(x_0, \ldots, z_{16}) = x_0x_{16} + x_1x_{15} + \ldots + y_0^2y_{16} + \ldots + y_5^2y_6 + y_0z_0^2z_{16} + \ldots + y_{16}z_0^3$$

• Over the ideal I_{151}, we obtain the ideal I_{161}. The corresponding vector $v_1^{16} = (8, 6, 4)$ is of codimension 18.

For m=17, (4) gives;

$$F_{17}(x_0, \ldots, z_{17}) = x_0x_{17} + x_1x_{16} + \ldots + y_0^2y_{17} + \ldots + y_5y_6^2 + y_0z_0^2z_{17} + \ldots + y_{17}z_0^3$$

• Over the ideal I_{161}, we obtain the ideal I_{171}. The corresponding vector $v_1^{17} = (9, 6, 4)$ is of codimension 19.

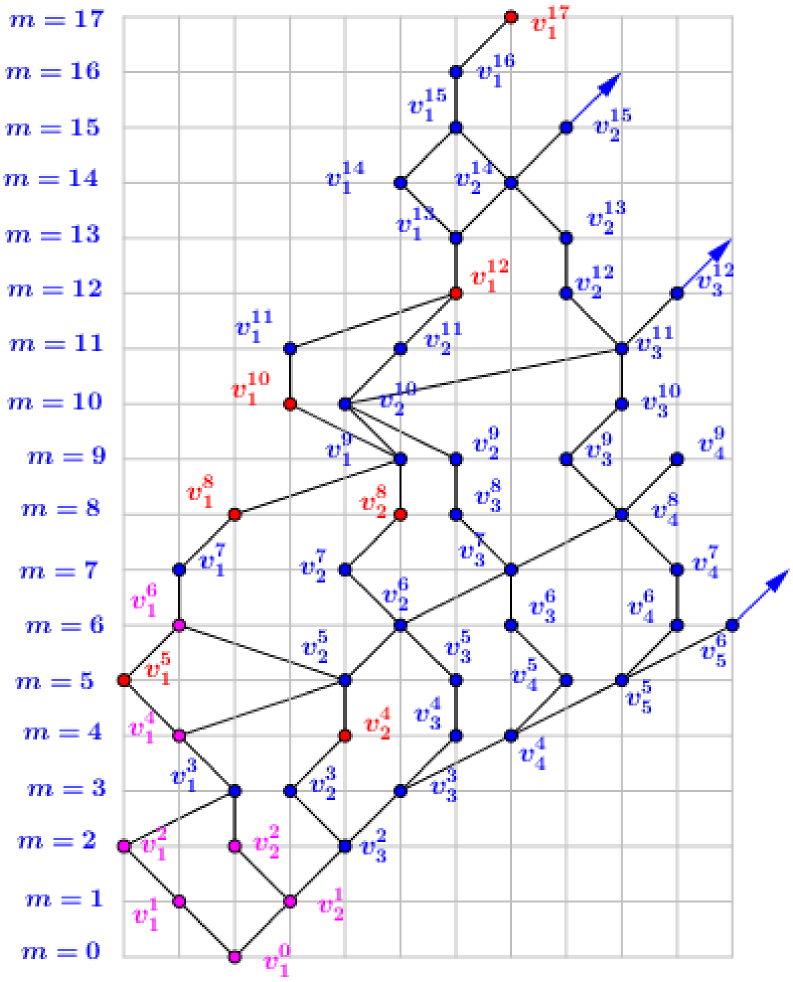

Figure 4.1: Jet graph of E_7

$v_1^0 = (1,1,0)$ $\quad v_1^1 = (1,1,1)$ $\quad v_2^1 = (1,2,0)$ $\quad v_1^2 = (2,1,1)$ $\quad v_2^2 = (2,2,1)$
$v_3^2 = (2,3,0)$ $\quad v_1^3 = (2,2,1)$ $\quad v_2^3 = (2,3,1)$ $\quad v_3^3 = (2,4,0)$ $\quad v_1^4 = (3,2,1)$
$v_2^4 = (3,3,1)$ $\quad v_3^4 = (3,4,1)$ $\quad v_4^4 = (3,5,0)$ $\quad v_1^5 = (3,2,2)$ $\quad v_2^5 = (3,3,1)$
$v_3^5 = (3,4,1)$ $\quad v_4^5 = (3,5,1)$ $\quad v_5^5 = (3,6,0)$ $\quad v_1^6 = (4,3,2)$ $\quad v_2^6 = (4,4,1)$
$v_3^6 = (4,5,1)$ $\quad v_4^6 = (4,6,1)$ $\quad v_5^6 = (4,7,0)$ $\quad v_1^7 = (4,3,2)$ $\quad v_2^7 = (4,4,2)$
$v_3^7 = (4,5,1)$ $\quad v_4^7 = (4,6,1)$ $\quad v_1^8 = (5,3,2)$ $\quad v_2^8 = (5,4,2)$ $\quad v_3^8 = (5,5,2)$
$v_4^8 = (5,6,1)$ $\quad v_1^9 = (5,4,2)$ $\quad v_2^9 = (5,5,2)$ $\quad v_3^9 = (5,6,2)$ $\quad v_4^9 = (5,7,1)$
$v_1^{10} = (6,4,3)$ $\quad v_2^{10} = (6,5,2)$ $\quad v_3^{10} = (6,6,2)$ $\quad v_1^{11} = (6,4,3)$ $\quad v_2^{11} = (6,5,3)$
$v_3^{11} = (6,6,2)$ $\quad v_1^{12} = (7,5,3)$ $\quad v_2^{12} = (7,6,3)$ $\quad v_3^{12} = (7,7,2)$ $\quad v_1^{13} = (7,5,3)$
$v_2^{11} = (7,6,3)$ $\quad v_1^{14} = (8,5,4)$ $\quad v_2^{14} = (8,6,3)$ $\quad v_1^{15} = (8,6,4)$ $\quad v_2^{15} = (8,7,3)$
$v_1^{16} = (8,6,4)$ $\quad v_1^{17} = (9,6,4)$

Note that $v^1 = v_2^4$, $v^2 = v_2^8$, $v^3 = v_1^{12}$, $v^4 = v_1^5$, $v^5 = v_1^{10}$, $v^7 = v_1^8$ and $W = v_1^{17}$ in Figure E_7c.

Definiion 4.3.3. *This graph is called the jet graph of the singularity.*

Remark 4.3.4. *We stop the process when we obtain the weight vector W.*

Remark 4.3.5. *Here and below the vectors represented with red correspond to the vectors given in E_7b, the vectors represented in pink also correspond to the vectors given in E_7c and the rest are obtained as the supplemented vectors.*

Remark 4.3.6. *The minimal resolution graph of singularity E_7 is as follow*

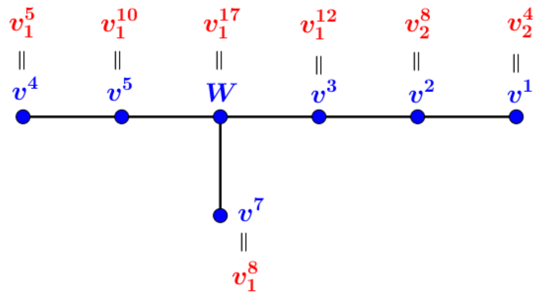

Figure 4.2: Minimal resolution graph of E_7

Remark 4.3.7. *So the Figure E_7c becomes*

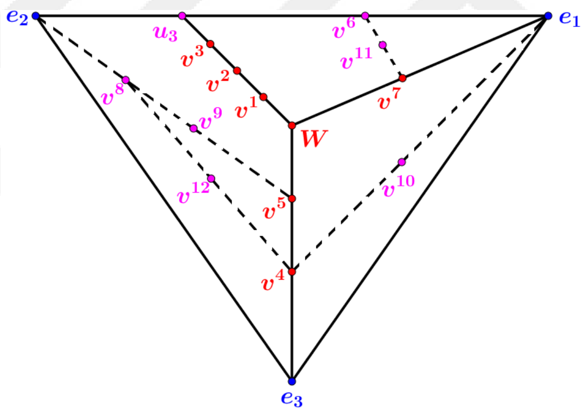

Figure 4.3: E_7d

where $u_3 = v_2^1$, $v^6 = v_1^0$, $v^8 = v_2^2$, $v^9 = v_1^6$, $v^{10} = v_1^2$, $v^{11} = v_1^4$ and $v^{12} = v_1^1$.

Theorem 4.3.8. *(Mourtada, 2013) Let X be an hypersurface in $\mathbb{C}^3$ of type E_7. For $m \geq 17$, the number of irreducible components of $J_m(E_7)$ equals the number of exceptional curves in the minimal resolution of the singularity.*

Theorem 4.3.9. *(Mourtada & Plenat, 2015) The set of weight vectors corresponding to m^{th} jets of E_7 give a canonical toric minimal embedded resolution of the singularity.*

5. JET SCHEMES OF NON-ISOLATED HYPERSURFACE SINGULARITIES

In this chapter, we will consider some hypersurfaces having one dimensional singular locus, which are called non-isolated forms of the rational triple singularities of surfaces in $\mathbb{C}^4$.

5.1 Hypersurfaces with Non-Isolated Singularity

Let X be an hypersurface in $\mathbb{C}^3$. Assume that X is defined by one of the following equations:
$A_{k-1,\ell-1,m-1}$, $k, \ell, m \geq 1$.

- $k \geq \ell \geq m$,
$$z^3 + xz^2 - (x + y^k + y^\ell + y^m)y^k z + y^{2k+\ell} = 0,$$
- $k = \ell < m$,
$$z^3 + (x - y^k)z^2 - (x + y^k + y^m)y^k z + y^{2k+m} = 0.$$

$B_{k-1,m}$, $k \geq 2$, $m \geq 3$.

- $m = 2\ell$,
$$z^3 + xz^2 - (y^{k+1} + y^\ell)y^k z - xy^{2k+1} = 0,$$
- $m = 2\ell - 1$,
$$z^3 + (x - y^{\ell-1})z^2 - y^{2k+1}z - xy^{2k+1} = 0.$$

$C_{k-1,\ell+1}$, $k \geq 1$, $\ell \geq 2$,
$$z^3 + xz^2 - \ell x^{\ell-1}y^{2k}z - (x^\ell + y^2)y^{2k} = 0.$$

D_{k-1}, $k \geq 1$,
$$z^3 + (x + y^{2k})z^2 + (2xy^k - y^2)y^k z + x^2y^{2k} = 0.$$

$E_{6,0}$,
$$z^3 + y^3 z + x^2y^2 = 0,$$

$E_{0,7}$,
$$z^3 + y^5 + x^2y^2 = 0,$$

$E_{7,0}$,
$$z^3 + x^2yz + y^4 = 0.$$

F_{k-1}, $k \geq 1$,
$$z^3 + (x + y^{2k})z^2 + 2xy^{2k}z + (x^2 + y^3)y^{2k} = 0.$$

H_n, $n \geq 1$,

- $n = 3k - 1$;
$$z^3 + x^2y(x + y^{k-1}) = 0,$$
- $n = 3k$;
$$z^3 + xy^kz + x^3y = 0,$$
- $n = 3k + 1$;
$$z^3 + xy^{k+1}z + x^3y^2 = 0.$$

Definition 5.1.1. *These singularities are called rational triple singularities, studied in (Tyurina, 1968).*
Such an hypersurface X has its singular locus along the x-axis; so they have non-isolated singularities. In this chapter, we are interested in understanding the jet schemes of these hyprsurfaces X. For this let us consider:

$$f(x_0+x_1t+\ldots+x_mt^m, y_0+\ldots+y_mt^m, z_0+\ldots+z_mt^m) = F_0+tF_1+\ldots \mod (t^{m+1}) \quad (5)$$

Our aim is to describe the jet graph and the toric embedded resolution of an hypersurface of one of these types. The construction process is similar to ADE-cases. In order to determine whether an m^{th} jet scheme is irreducible, we will determine the defining ideal of each jet $J_m(X)$.

5.2 Jet Schemes of an Hypersurface of type E_{60}

Let X be an hypersurface of type E_{60} in $\mathbb{C}^3$. We know that X is defined by

$$f(x, y, z) = z^3 + y^3z + x^2y^2$$

in $\mathbb{C}[x, y, z]$. Hence $(x, 0, 0)$ is the singular locus of X. Let us apply the equality (5):

For m=0, we have
$$F_0(x_0, y_0, z_0) = z_0^3 + y_0^2(y_0z_0 + x_0^2)$$
• This says that $z_0 = y_0 = 0$ or $z_0 = x_0 = 0$. Hence the ideals are $I_{01} =< y_0, z_0 >$ and $I_{02} =< x_0, y_0 >$. These correspond to the vectors $v_1^0 = (0, 1, 1)$ and $v_2^0 = (1, 0, 1)$. Each of which of codimension is 2. Hence $J_0(E_{60})$ is given by

$$J_0(E_{60}) = Spec\frac{\mathbb{C}[x_0, y_0, z_0]}{< F_0 >}$$

For m=1, (5) gives;

$$F_1(x_0, \ldots, z_1) = z_0^2z_1 + y_0(y_0y_1z_0 + y_0^2z_1 + x_0x_1y_0 + x_0y_1)$$

• Over the ideal I_{01}, we obtain the ideal $I_{11} =< y_0, z_0 >$. The corresponding vector $v_1^1 = (0, 1, 1)$ is of codimension 2.
• Over the ideal I_{02}, we obtain two possible ideal $I_{12} =< x_0, z_0, z_1 >$ which is over the generic point and $I_{13} =< x_0, y_0, z_0 >$ which is over the singular point. We have the

ideal I_{12} with corresponding vector $v_2^1 = (1,0,2)$. It is of codimension 3. Hence $J_1(E_{60})$ is given by

$$J_1(E_{60}) = Spec\frac{\mathbb{C}[x_0, y_0, \ldots, z_1]}{< F_0, F_1 >}$$

For m=2, (5) gives;

$$F_2(x_0, \ldots, z_2) = z_0^2 z_2 + z_0 z_1^2 + y_0^3 z_2 + y_0^2 y_1 z_1 + \ldots + x_0^2 y_1^2 + x_1^2 y_0^2 + x_0 x_2 y_0^2$$

• Over the ideal I_{11}, we obtain two possible ideals $I_{21} =< y_0, y_1, z_0 >$ and $I_{22} =< x_0, y_0, z_0 >$. The corresponding vectors $v_1^2 = (0,2,1)$ and $v_2^2 = (1,1,1)$ are of codimension 3.
• Over the ideal I_{12}, we obtain the jets which project on the regular axis included in the variety. Hence this branch continuous with the vector $(1,0,2)$. Hence $J_2(E_{60})$ is given by

$$J_2(E_{60}) = Spec\frac{\mathbb{C}[x_0, y_0, \ldots, z_2]}{< F_0, F_1, F_2 >}$$

Remark 5.2.1. *In the same way, the m^{th} jet scheme of E_{60} is given by*

$$J_m(E_{60}) = Spec\frac{\mathbb{C}[x_0, y_0, z_0, \ldots, z_m]}{< F_0, F_1, \ldots, F_m >}$$

for all m.
For m=3, (5) gives;

$$F_3(x_0, \ldots z_3) = z_0^2 z_3 + z_0 z_1 z_2 + z_1^3 + y_0^3 z_3 + \ldots + y_1^3 z_0 + x_0^2 y_0 y_3 + \ldots + x_1 x_2 y_0^2$$

• Over the ideal I_{21}, we obtain the ideal $I_{31} =< y_0, y_1, z_0, z_1 >$. The corresponding vector $v_1^3 = (0,2,2)$ is of codimension 4.
• Over the ideal I_{22}, we obtain the ideal $I_{32} =< x_0, y_0, z_0, z_1 >$. The corresponding vector $v_2^3 = (1,1,2)$ is of codimension 4.

For m=4, (5) gives;

$$F_4(x_0, \ldots z_4) = z_0^2 z_4 + \ldots + z_1^2 z_2 + y_0^3 z_4 + \ldots y_1^2 y_2 z_0 + x_0^2 y_0 y_4 + \ldots + x_2^2 y_0^2$$

• Over the ideal I_{31}, we obtain two possible ideals $I_{41} =< y_0, y_1, z_0, z_1, y_2 >$ and $I_{42} =< y_0, y_1, z_0, z_1, x_0 >$. The corresponding vectors $v_1^4 = (0,3,2)$ and $v_2^4 = (1,2,2)$ are of codimension 5.
• Over the ideal I_{32}, we obtain two possible ideals I_{42} and $I_{43} =< x_0, y_0, z_0, z_1, x_1 >$. The corresponding vector of I_{43}, $v_3^4 = (2,1,2)$ is of codimension 5.

For m=5, (5) gives;

$$F_5(x_0, \ldots z_5) = z_0^2 z_5 + \ldots + z_1 z_2^2 + y_0^3 z_5 + \ldots y_1 y_2^2 z_0 + x_0^2 y_0 y_5 + \ldots + x_2 x_3 y_0^2$$

• Over the ideal I_{41}, we obtain the ideal $I_{51} =< y_0, y_1, z_0, z_1, y_2 >$. The corresponding vector $v_1^5 = (0,3,2)$ is of codimension 5.

• Over the ideal I_{42}, we obtain the ideal $I_{52} =< y_0, y_1, z_0, z_1, x_0 >$. The corresponding vector $v_2^5 = (1,2,2)$ is of codimension 5.
• Over the ideal I_{43}, we obtain the ideal $I_{53} =< x_0, y_0, z_0, z_1, x_1, z_2 >$. The corresponding vector $v_3^5 = (2,1,3)$ are of codimension 6.

In the same way;
For m=6, (5) gives;

$$F_6(x_0, \dots z_6) = z_0^2 z_6 + \dots + z_2^3 + y_0^3 z_6 + \dots y_2^3 z_0 + x_0^2 y_0 y_6 + \dots + x_3^2 y_0^2$$

• Over the ideal I_{51}, we obtain two possible ideals $I_{61} =< y_0, y_1, z_0, z_1, y_2 >$. The corresponding vector $v_1^6 = (0,3,2)$ is of codimension 5.
• Over the ideal I_{52}, we obtain the ideal I_{62}. The corresponding vector $v_2^6 = (1,2,2)$ is of codimension 5.
• Over the ideal I_{53}, we obtain two possible ideals I_{63} and I_{64}. The corresponding vectors $v_3^6 = (2,2,3)$ and $v_4^6 = (3,1,4)$ are of codimension 7 and 8 respectively. The equation of I_{64} is toric so its jets are irreducible and it is continue with the vector $(3,1,4)$.

For m=7, (5) gives;

$$F_7(x_0, \dots z_7) = z_0^2 z_7 + \dots + z_2^2 z_3 + y_0^3 z_7 + \dots y_2^2 y_3 z_0 + x_0^2 y_0 y_7 + \dots + x_3 x_4 y_0^2$$

• Over the ideal I_{61}, we obtain two possible ideals I_{71} and I_{72}. The corresponding vectors $v_1^7 = (0,4,3)$ and $v_2^7 = (1,3,3)$ is of codimension 7.
• Over the ideal I_{62}, we obtain the ideals I_{72} and I_{73}. The corresponding vector of I_{73}, $v_3^7 = (2,2,3)$ is of codimension 7.
• Over the ideal I_{63}, we obtain the ideal I_{73}.

For m=8, (5) gives;

$$F_8(x_0, \dots z_8) = z_0^2 z_8 + \dots + z_2 z_3^2 + y_0^3 z_8 + \dots y_2 y_3^2 z_0 + x_0^2 y_0 y_8 + \dots + x_4^2 y_0^2$$

• Over the ideal I_{71}, we obtain two possible ideals I_{81} and I_{82}. The corresponding vectors $v_1^8 = (0,5,3)$ and $v_2^8 = (1,4,3)$ are of codimension 8.
• Over the ideal I_{72}, we obtain two possible ideals I_{82} and I_{83}. The corresponding vector of I_{83}, $v_3^8 = (2,3,3)$ is of codimension 8.
• Over the ideal I_{73}, we obtain two possible ideals I_{83} and I_{84}. The corresponding vector of I_{84}, $v_4^8 = (3,2,3)$ is of codimension 8.
For m=9, (5) gives;

$$F_9(x_0, \dots z_9) = z_0^2 z_9 + \dots + z_3^3 + y_0^3 z_9 + \dots y_3^3 z_0 + x_0^2 y_0 y_9 + \dots + x_4 x_5 y_0^2$$

• Over the ideal I_{81}, we obtain the ideal I_{91}. The corresponding vector $v_1^9 = (0,5,4)$ is of codimension 9.
• Over the ideals I_{82}, we obtain the ideal I_{92}. The corresponding vector $v_2^9 = (1,4,4)$ is of codimension 9.
• Over the ideal I_{83}, we obtain the ideal I_{93}. The corresponding vector $v_3^9 = (2,3,4)$ is of codimension 9.

• Over the ideal I_{84}, we obtain the ideal I_{94}. The corresponding vector $v_4^9 = (3,2,4)$ is of codimension 9 and at the same time the vector $(3,2,3)$ continue since it is smooth so it is irreducible.

For m=10, (5) gives;

$$F_{10}(x_0,\dots z_{10}) = z_0^2 z_{10} + \dots + z_3^2 z_4 + y_0^3 z_{10} + \dots y_3^2 y_4 z_0 + x_0^2 y_0 y_{10} + \dots + x_5^2 y_0^2$$

• Over the ideal I_{91}, we obtain two possible ideals I_{101} and I_{102}. The corresponding vectors $v_1^{10} = (0,6,4)$ and $v_2^{10} = (1,5,4)$ are of codimension 10.
• Over the ideal I_{92}, we obtain two possible ideals I_{102} and I_{103}. The corresponding vector of I_{103}, $v_3^{10} = (2,4,4)$ is of codimension 10.
• Over the ideal I_{93}, we obtain two possible ideals I_{103} and I_{104} The corresponding vector of I_{104}, $v_4^{10} = (3,3,4)$ is of codimension 10.
• Over the ideal I_{94}, we obtain the ideal I_{104} and at the same time the vector $(3,2,4)$ continue since it is smooth so it is irreducible.

For m=11, (5) gives;

$$F_{11}(x_0,\dots z_{11}) = z_0^2 z_{11} + \dots + z_3 z_4^2 + y_0^3 z_{11} + \dots y_3 y_4^2 z_0 + x_0^2 y_0 y_{11} + \dots + x_5 x_6 y_0^2$$

• Over the ideal I_{101}, we obtain the ideal I_{111}. The corresponding vector $v_1^{11} = (0,6,4)$ is of codimension 10.
• Over the ideal I_{102}, we obtain the ideal I_{112}. The corresponding vector $v_2^{11} = (1,5,4)$ is of codimension 10.
• Over the ideal I_{103}, we obtain the ideal I_{113}. The corresponding vector $v_3^{11} = (2,4,4)$ is of codimension 10.
•Over the ideal I_{104}, we obtain the ideal I_{114}. The corresponding vector $v_4^{11} = (3,3,4)$ is of codimension 10.

For m=12, (5) gives;

$$F_{12}(x_0,\dots z_{12}) = z_0^2 z_{12} + \dots + z_4^3 + y_0^3 z_{12} + \dots y_4^3 z_0 + x_0^2 y_0 y_{12} + \dots + x_6^2 y_0^2$$

• Over the ideal I_{111}, we obtain the ideal I_{121}. The corresponding vectors $v_1^{12} = (0,6,4)$ is of codimension 10.
• Over the ideal I_{112}, we obtain the ideal I_{122}. The corresponding vector $v_2^{12} = (1,5,4)$ is of codimension 10.
• Over the ideal I_{113}, we obtain the ideal I_{123}. The corresponding vector $v_3^{12} = (2,2,4)$ is of codimension 10.
• Over the ideal I_{114}, we obtain the ideal I_{124}. The corresponding vector $v_4^{12} = (3,3,4)$ is of codimension 10.

For m=13, (5) gives;

$$F_{13}(x_0,\dots z_{13}) = z_0^2 z_{13} + \dots + z_4^2 z_5 + y_0^3 z_{13} + \dots y_4^2 y_5 z_0 + x_0^2 y_0 y_{13} + \dots + x_6 x_7 y_0^2$$

• Over the ideal I_{121}, we obtain two possible ideals I_{131} and I_{132}. The corresponding vectors $v_1^{13} = (0,7,5)$ and $v_2^{13} = (1,6,5)$ are of codimension 12.

• Over the ideal I_{122}, we obtain two possible ideals I_{132} and I_{133}. The corresponding vector of I_{133}, $v_3^{13} = (2, 5, 5)$ is of codimension 12.
• Over the ideal I_{123}, we obtain two possible ideals I_{133} and I_{134}. The corresponding vector $v_4^{13} = (3, 4, 5)$ is of codimension 12.
• Over the ideal I_{124}, we obtain two possible ideals I_{134} and I_{135}. The corresponding vector $v_5^{13} = (4, 3, 5)$ is of codimension 12.

For m=14, (5) gives;

$$F_{14}(x_0, \ldots z_{14}) = z_0^2 z_{14} + \ldots + z_4 z_5^2 + y_0^3 z_{14} + \ldots y_4 y_5^2 z_0 + x_0^2 y_0 y_{14} + \ldots + x_7^2 y_0^2$$

• Over the ideal I_{131}, we obtain two possible ideals I_{141} and I_{142}. The corresponding vectors $v_1^{14} = (0, 7, 5)$ and $v_2^{14} = (1, 7, 5)$ are of codimension 12 and 13 respectively.
• Over the ideal I_{132}, we obtain two possible ideals I_{142} and I_{143}. The corresponding vector of I_{143}, $v_3^{14} = (2, 6, 5)$ is of codimension 13.
• Over the ideal I_{133}, we obtain two possible ideals I_{143} and I_{144}. The corresponding vector of I_{144}, $v_4^{14} = (3, 5, 5)$ is of codimension 13.
• Over the ideal I_{134}, we obtain two possible ideals I_{144} and I_{145}. The corresponding vector of I_{145}, $v_5^{14} = (4, 4, 5)$ is of codimension 13.
• Over the ideal I_{135}, we obtain the ideal I_{145} and at the same time the vector $(4, 3, 5)$ continue since it is smooth so it is irreducible.

For m=15, (5) gives;

$$F_{15}(x_0, \ldots z_{15}) = z_0^2 z_{15} + \ldots + z_5^3 + y_0^3 z_{15} + \ldots y_5^3 z_0 + x_0^2 y_0 y_{15} + \ldots + x_7 x_8 y_0^2$$

• Over the ideal I_{141}, we obtain the ideal I_{151}. The corresponding vector $v_1^{15} = (0, 8, 6)$ is of codimension 14.
• Over the ideal I_{142}, we obtain the ideal I_{152}. The corresponding vector $v_2^{15} = (1, 7, 6)$ is of codimension 14.
• Over the ideal I_{143}, we obtain the ideal I_{153}. The corresponding vector $v_3^{15} = (2, 6, 6)$ is of codimension 14.
• Over the ideal I_{144}, we obtain the ideal I_{154}. The corresponding vector $v_4^{15} = (3, 5, 6)$ is of codimension 14.
• Over the ideal I_{145}, we obtain the ideal I_{155}. The corresponding vector $v_5^{15} = (4, 4, 6)$ is of codimension 14.

For m=16, (5) gives;

$$F_{16}(x_0, \ldots z_{15}) = z_0^2 z_{16} + \ldots + z_5^2 z_6 + y_0^3 z_{16} + \ldots y_5^2 y_6 z_0 + x_0^2 y_0 y_{16} + \ldots + x_8^2 y_0^2$$

• Over the ideal I_{151}, we obtain two possible ideals I_{161} and I_{162}. The corresponding vectors $v_1^{16} = (0, 9, 6)$ and $v_2^{16} = (1, 8, 6)$ are of codimension 15.
• Over the ideal I_{152}, we obtain two possible ideals I_{162} and I_{163}. The corresponding vector of I_{163}, $v_3^{16} = (2, 7, 6)$ is of codimension 15.
• Over the ideal I_{153}, we obtain two possible ideals I_{163} and I_{164}. The corresponding vector of I_{164}, $v_4^{16} = (3, 6, 6)$ is of codimension 15.
• Over the ideal I_{154}, we obtain two possible ideals I_{164} and I_{165}. The corresponding vector of I_{165}, $v_5^{16} = (4, 5, 6)$ is of codimension 15.
• Over the ideal I_{155}, we obtain two possible ideals I_{165} and I_{166}. The corresponding vector of I_{166}, $v_6^{16} = (5, 4, 6)$ is of codimension 15.

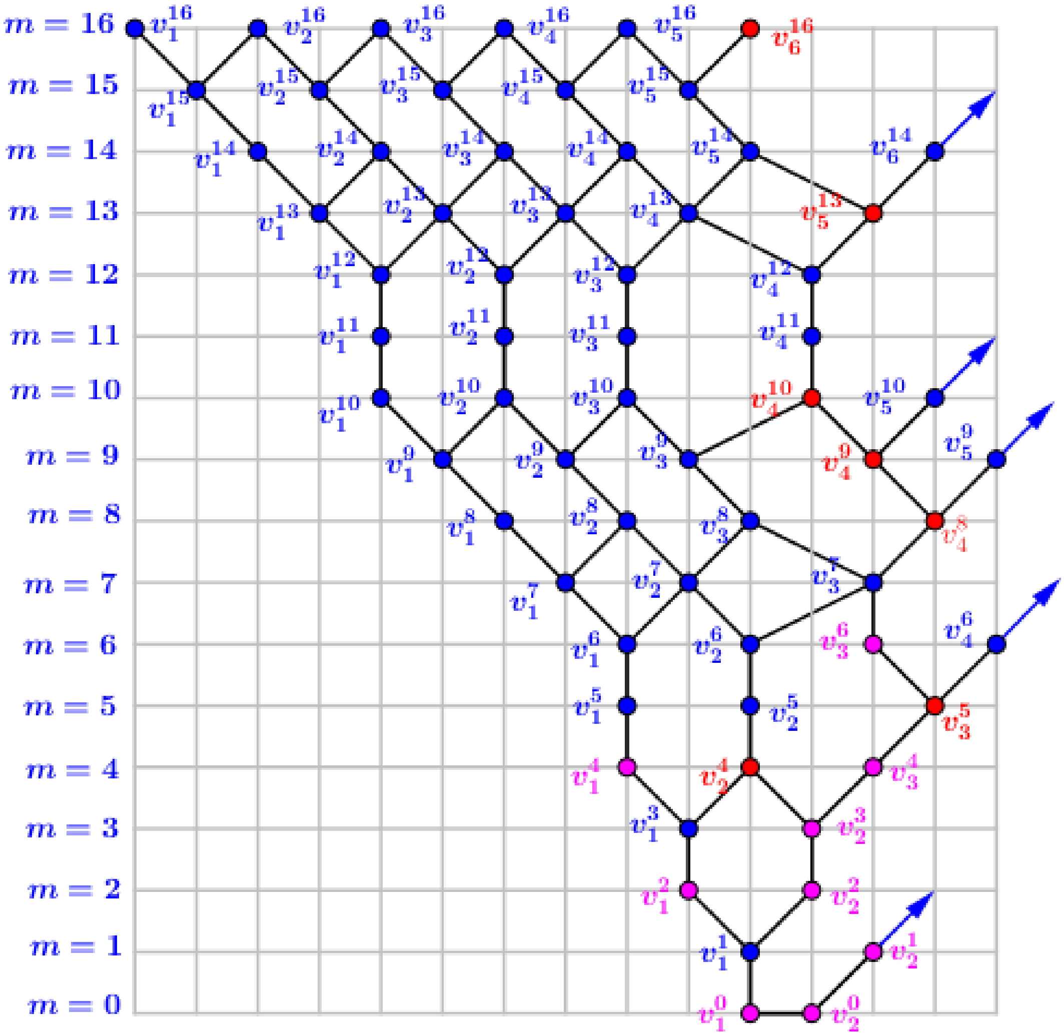

Figure 5.1: Jet graph of E_{60}

$v_1^0 = (0,1,1)$	$v_2^0 = (1,0,1)$	$v_1^1 = (0,1,1)$	$v_2^1 = (1,0,2)$	$v_1^2 = (0,2,1)$
$v_2^2 = (1,1,1)$	$v_1^3 = (0,2,2)$	$v_2^3 = (1,1,2)$	$v_1^4 = (0,3,2)$	$v_2^4 = (1,2,2)$
$v_3^4 = (2,1,2)$	$v_1^5 = (0,3,2)$	$v_2^5 = (1,1,2)$	$v_3^5 = (2,1,3)$	$v_1^6 = (0,3,2)$
$v_2^6 = (1,2,2)$	$v_3^6 = (2,2,3)$	$v_4^6 = (3,1,4)$	$v_1^7 = (0,4,3)$	$v_2^7 = (1,3,3)$
$v_3^7 = (2,2,3)$	$v_1^8 = (0,5,3)$	$v_2^8 = (1,4,3)$	$v_3^8 = (2,3,3)$	$v_4^8 = (3,2,3)$
$v_1^9 = (0,5,4)$	$v_2^9 = (1,4,4)$	$v_3^9 = (2,3,4)$	$v_4^9 = (3,2,4)$	$v_5^9 = (3,2,3)$
$v_1^{10} = (0,6,4)$	$v_2^{10} = (1,5,4)$	$v_3^{10} = (2,4,4)$	$v_4^{10} = (3,3,4)$	$v_5^{10} = (3,2,4)$
$v_1^{11} = (0,6,4)$	$v_2^{11} = (1,5,4)$	$v_3^{11} = (2,4,4)$	$v_4^{11} = (3,3,4)$	$v_1^{12} = (0,6,4)$
$v_2^{12} = (1,5,4)$	$v_3^{12} = (2,4,4)$	$v_4^{12} = (3,3,4)$	$v_1^{13} = (0,7,5)$	$v_2^{13} = (1,6,5)$
$v_3^{13} = (2,5,5)$	$v_4^{13} = (3,4,5)$	$v_5^{12} = (4,3,5)$	$v_1^{14} = (0,8,5)$	$v_2^{14} = (1,7,5)$
$v_3^{14} = (2,6,5)$	$v_4^{14} = (3,5,5)$	$v_5^{14} = (4,4,5)$	$v_6^{14} = (4,3,5)$	$v_1^{15} = (0,8,6)$
$v_2^{15} = (1,7,6)$	$v_3^{15} = (2,6,6)$	$v_4^{15} = (3,5,6)$	$v_5^{15} = (4,4,6)$	$v_1^{16} = (0,9,6)$
$v_2^{16} = (1,8,6)$	$v_3^{16} = (2,7,6)$	$v_4^{16} = (3,6,6)$	$v_5^{16} = (4,5,6)$	$v_6^{16} = (5,4,6)$

Remark 5.2.2. *Here and below the vectors represented with red correspond to the vectors given in $E_{60}b$, the vectors represented in pink also correspond to the vectors given in $E_{60}c$ and the rest are obtained as the supplemented vectors.*

Remark 5.2.3. *The minimal resolution graph of singularity E_{60} is as follow*

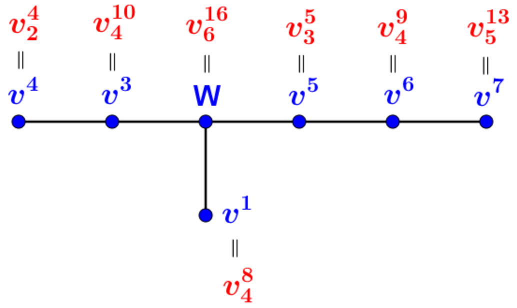

Figure 5.2: Minimal resolution graph of E_{60}

Remark 5.2.4. *The minimal toric embedded resolution graph of E_{60} is as follow*

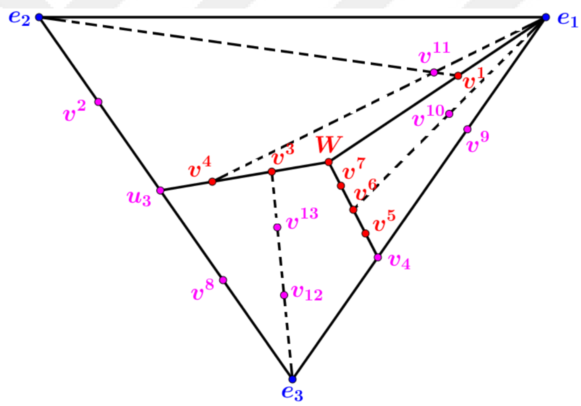

Figure 5.3: $E_{60}d$

where $v^2 = v_1^2$, $v^8 = v_1^0$, $v^9 = v_2^0$, $v^{10} = v_3^4$, $v^{11} = v_2^2$, $v^{12} = v_3^6$, $v^{13} = v_2^3$, $v_4 = v_2^1$ and $u_3 = v_1^4$.

Proposition 5.2.5. *Let X be an hypersurface in $\mathbb{C}^3$ of type E_{60}. For $m \geq 16$, the number of irreducible components of $J_m(E_{60})$ equals the number of exceptional curves on the minimal resolution of the singularity.*

Proposition 5.2.6. *The set of weight vectors corresponding to m^{th} jets of E_{60} give a canonical toric minimal embedded resolution of the singularity.*

5.3 Jet Schemes of an Hypersurface of type E_{70}

Let X be an hypersurface of type E_{70} in $\mathbb{C}^3$. We know that X is defined by

$$f(x, y, x) = z^3 + x^2yz + y^4$$

in $\mathbb{C}[x, y, z]$. Hence $(x, 0, 0)$ is the singular locus of X.

Remark 5.3.1. *The m^{th} jet scheme of E_{70} is given by*

$$J_m(E_{70}) = Spec\frac{\mathbb{C}[x_0, y_0, z_0, \ldots, z_m]}{< F_0, F_1, \ldots, F_m >}$$

for all m.

Let us apply the equality (5):

For m=0, we have

$$F_0(x_0, y_0, z_0) = z_0^3 + y_0(x_0^2 z_0 + y_0^3)$$

• This says that $y_0 = z_0 = 0$. Hence the ideal is $I_0 =< y_0, z_0 >$. The corresponding vector $v_1^0 = (0, 1, 1)$ is of codimension 2.

For m=1, (5) gives;

$$F_1(x_0, \ldots, z_1) = z_0(z_0 z_1 + x_0^2 y_1) + y_0(x_0 x_1 z_0 + x_0^2 z_1 + y_0^2 y_1)$$

• Over the ideal I_0, we obtain the ideal $I_1 =< y_0, z_0 >$. The corresponding vector $v_1^1 = (0, 1, 1)$ is of codimension 2.

For m=2, (5) gives;

$$F_2(x_0, \ldots, z_2) = z_0^2 z_2 + z_0 z_1^2 + x_0 x_2 y_0 z_0 + \ldots + x_0^2 y_1 z_1 + y_0^3 y_2 + y_1^2 y_0^2$$

• Over the ideal I_1, we obtain three possible ideals $I_{21} =< y_0, z_0, y_1, >$, $I_{22} =< y_0, z_0, z_1, >$ and $I_{23} =< y_0, z_0, x_0 >$. The corresponding vectors $v_1^2 = (0, 2, 1)$, $v_2^2 = (0, 1, 2)$ and $v_3^2 = (1, 1, 1)$ are of codimension 3.

For m=3, (5) gives;

$$F_3(x_0, \ldots, z_3) = z_0^2 z_3 + z_0 z_1 z_2 + z_1^3 + \ldots + y_0^3 y_3 + y_0^2 y_1 y_2 + y_0 y_1^3$$

• Over the ideal I_{21}, we obtain the ideal $I_{31} =< y_0, y_1, z_0, z_1 >$. The corresponding vector $v_1^3 = (0, 2, 2)$ is of codimension 4.
• Over the ideal I_{22}, we obtain three possible ideals I_{31}, $I_{32} =< y_0, z_0, z_1, z_2 >$ and $I_{33} =< y_0, z_0, z_1, x_0 >$. The corresponding vector $v_2^3 = (0, 1, 3)$ and $v_3^3 = (1, 1, 2)$ are of codimension 4.
• Over the ideal I_{23}, we obtain the ideal I_{33} with the corresponding vector $v_3^3 = (1, 1, 2)$.
For m=4, (5) gives;

$$F_4(x_0, \ldots, z_3) = z_0^2 z_4 + \ldots + z_1^2 z_2 + x_0 x_4 y_0 z_0 + \ldots x_0^2 y_0 z_4 + y_0^3 y_4 + \ldots + y_1^4$$

• Over the ideal I_{31}, we obtain three possible ideals $I_{41} =< y_0, z_0, y_1, z_1, y_2 >$, $I_{42} =< y_0, z_0, y_1, z_1, z_2 >$ and $I_{43} =< y_0, z_0, y_1, z_1, x_0 >$. The corresponding vectors $v_1^4 = (0, 3, 2)$, $v_2^4 = (0, 2, 3)$ and $v_3^4 = (1, 2, 2)$ are of codimension 5.
• Over the ideal I_{32}, we continue the vector $(0, 1, 3)$.
• Over the ideal I_{33}, we obtain the ideal I_{43}.

For m=5, (5) gives;

$$F_5(x_0,\ldots,z_5)=z_0^2z_5+\ldots+z_1z_2^2+x_0x_5y_0z_0+\ldots x_0^2y_0z_5+y_0^3y_5+\ldots+y_1^3y_2$$

• Over the ideal I_{41}, we obtain three possible ideals $I_{51}=<y_0,z_0,y_1,z_1,y_2,y_3>$, $I_{52}=<y_0,z_0,y_1,z_1,y_2,z_2>$ and $I_{53}=<y_0,z_0,y_1,z_1,y_2,x_0>$. The corresponding vectors $v_1^5=(0,4,2)$, $v_2^5=(0,3,3)$ and $v_3^5=(1,3,2)$ are of codimension 6.
• Over the ideal I_{42}, we obtain three possible ideals I_{52}, $I_{54}=<y_0,z_0,y_1,z_1,z_2,z_3>$ and $I_{55}=<y_0,z_0,y_1,z_1,z_2,x_0>$. The corresponding vectors $v_4^5=(0,2,4)$ and $v_5^5=(1,2,3)$ are of codimension 6.
• Over the ideal I_{43}, we obtain the ideal $I_{56}=<y_0,z_0,y_1,z_1,x_0>$ The corresponding vector $v_4^5=(1,2,2)$ is of codimension 5.

For m=6, (5) gives;

$$F_6(x_0,\ldots,z_6)=z_0^2z_6+\ldots+z_2^3+x_0x_6y_0z_0+\ldots x_0^2y_0z_6+y_0^3y_6+\ldots+y_1^2y_2^2$$

• Over the ideal I_{53}, we obtain the ideal $I_{61}=<y_0,z_0,y_1,z_1,y_2,x_0,z_2>$. The corresponding vector $v_1^6=(1,3,3)$ is of codimension 7.
• Over the ideal I_{55}, we obtain the ideal $I_{62}=<y_0,z_0,y_1,z_1,z_2,x_0>$. The corresponding vector $v_2^6=(1,2,3)$ is of codimension 6.
• Over the ideal I_{56}, we obtain the ideal I_{62} wit the corresponding vector $v_2^6=(1,2,3)$.

For m=7, (5) gives;

$$F_7(x_0,\ldots,z_7)=z_0^2z_7+\ldots+z_2^2z_3+x_0x_7y_0z_0+\ldots x_0^2y_0z_7+y_0^3y_7+\ldots+y_1^2y_2y_3$$

• Over the ideal I_{61}, we obtain the ideal $I_{71}=<y_0,z_0,y_1,z_1,y_2,x_0,z_2>$. The corresponding vector $v_1^7=(1,3,3)$ is of codimension 7.
• Over the ideal I_{62}, we obtain three possible ideals I_{71}, $I_{72}=<y_0,z_0,y_1,z_1,x_0,z_2,z_3>$ and $I_{73}=<y_0,z_0,y_1,z_1,x_0,z_2,x_1>$. The corresponding vectors $v_2^7=(1,2,4)$ and $v_3^7=(2,2,3)$ are of codimension 7.

In the same way;
For m=8, (5) gives;

$$F_8(x_0,\ldots,z_8)=z_0^2z_8+\ldots+z_2z_3^2+x_0x_8y_0z_0+\ldots x_0^2y_0z_8+y_0^3y_8+\ldots+y_2^4$$

• Over the ideal I_{71}, we obtain three possible ideals I_{81}, I_{82} and I_{83}. The corresponding vectors $v_1^8=(1,4,3)$, $v_2^8=(1,3,4)$ and $v_3^8=(2,3,3)$ are of codimension 8.
• Over the ideal I_{72}, we obtain the ideal I_{82} with the corresponding vector $v_2^8=(1,3,4)$.
• Over the ideal I_{73}, we obtain the ideal I_{83} with the corresponding vector $v_3^8=(2,3,3)$.

For m=9, (5) gives;

$$F_9(x_0,\ldots,z_9)=z_0^2z_9+\ldots+z_3^3+x_0x_9y_0z_0+\ldots x_0^2y_0z_9+y_0^3y_9+\ldots+y_2^3y_3$$

• Over the ideal I_{81}, we obtain the ideal I_{91}. The corresponding vector $v_1^9=(1,4,4)$ is of codimension 9.

• Over the ideal I_{82}, we obtain three possible ideals I_{91}, I_{92} and I_{93}. The corresponding vectors $v_2^9 = (1,3,5)$ and $v_3^9 = (2,3,4)$ are of codimension 9.
• Over the ideal I_{83}, we obtain the ideal I_{93} with the corresponding vector $v_3^9 = (2,3,4)$ is of codimension 9.

For m=10, (5) gives;

$$F_{10}(x_0,\dots,z_{10}) = z_0^2 z_{10} + \dots + z_3^2 z_4 + x_0 x_{10} y_0 z_0 + \dots x_0^2 y_0 z_{10} + y_0^3 y_{10} + \dots + y_2^2 y_3^2$$

• Over the ideal I_{91}, we obtain three possible ideals I_{101}, I_{102} and I_{103}. The corresponding vectors $v_1^{10} = (1,5,4)$, $v_2^{10} = (1,4,5)$ and $v_3^{10} = (2,4,4)$ are of codimension 10.
• Over the ideal I_{92}, we obtain three possible ideals I_{102}, I_{104} and I_{105}. The corresponding vectors $v_4^{10} = (1,3,6)$ and $v_5^{10} = (2,3,5)$ are of codimension 10.
• Over the ideal I_{93}, we obtain the ideal I_{106}. The corresponding vector $v_6^{10} = (2,3,4)$ is of codimension 9.

For m=11, (5) gives;

$$F_{11}(x_0,\dots,z_{11}) = z_0^2 z_{11} + \dots + z_3 z_4^2 + x_0 x_{11} y_0 z_0 + \dots x_0^2 y_0 z_{11} + y_0^3 y_{11} + \dots + y_2 y_3^3$$

• Over the ideal I_{103}, we obtain the ideal I_{111}. The corresponding vector $v_1^{11} = (2,4,4)$ is of codimension 10.
• Over the ideal I_{105}, we obtain the ideal I_{112}. The corresponding vector $v_2^{11} = (2,3,5)$ is of codimension 10.
• Over the ideal I_{106}, we obtain three possible ideals I_{111}, I_{112} and I_{113}. The corresponding vector $v_3^{11} = (3,3,4)$is of codimension 10.

For m=12, (5) gives;

$$F_{12}(x_0,\dots,z_{12}) = z_0^2 z_{12} + \dots + z_4^3 + x_0 x_{12} y_0 z_0 + \dots x_0^2 y_0 z_{12} + y_0^3 y_{12} + \dots + y_3^4$$

• Over the ideal I_{111}, we obtain the ideal I_{121}. The corresponding vector $v_1^{12} = (2,4,5)$ is of codimension 11.
• Over the ideal I_{112}, we obtain the ideal I_{121} with the corresponding vector $v_2^{11} = (2,4,5)$.
• Over the ideal I_{113}, we obtain the ideal I_{122}. The corresponding vector $v_2^{12} = (3,4,5)$ is of codimension 12.

For m=13, (5) gives;

$$F_{13}(x_0,\dots,z_{13}) = z_0^2 z_{13} + \dots + z_4^2 z_5 + x_0 x_{13} y_0 z_0 + \dots x_0^2 y_0 z_{13} + y_0^3 y_{13} + \dots + y_3^3 y_4$$

• Over the ideal I_{121}, we obtain three possible ideals I_{131}, I_{132} and I_{133}. The corresponding vectors $v_1^{13} = (2,5,6)$, $v_2^{13} = (2,4,6)$ and $v_3^{13} = (3,4,5)$ are of codimension 12.
• Over the ideal I_{122}, we obtain the ideal I_{133} with the corresponding vector $v_3^{13} = (3,4,5)$.

For m=14, (5) gives;

$$F_{14}(x_0,\ldots,z_{14}) = z_0^2z_{14} + \ldots + z_4z_5^2 + x_0x_{14}y_0z_0 + \ldots x_0^2y_0z_{14} + y_0^3y_{14} + \ldots + y_3^2y_4^2$$

• Over the ideal I_{131}, we obtain the ideal I_{141}. The corresponding vector $v_1^{14} = (2,5,6)$ is of codimension 13.
• Over the ideal I_{132}, we obtain three possible ideals I_{141}, I_{142} and I_{143}. The corresponding vectors $v_2^{14} = (2,4,7)$ and $v_3^{14} = (3,4,6)$ are of codimension 13.
• Over the ideal I_{133}, we obtain the ideal I_{144}. The corresponding vector $v_4^{14} = (3,4,5)$ is of codimension 12.

For m=15, (5) gives;

$$F_{15}(x_0,\ldots,z_{15}) = z_0^2z_{15} + \ldots + z_5^3 + x_0x_{15}y_0z_0 + \ldots x_0^2y_0z_{15} + y_0^3y_{15} + \ldots + y_3y_4^3$$

• Over the ideal I_{143}, we obtain the ideal I_{151}. The corresponding vector $v_1^{15} = (3,4,6)$ is of codimension 13.
• Over the ideal I_{144}, we obtain the ideal I_{151} with the corresponding vector $v_1^{15} = (3,4,6)$.

For m=16, (5) gives;

$$F_{16}(x_0,\ldots,z_{16}) = z_0^2z_{16} + \ldots + z_5^2z_6 + x_0x_{16}y_0z_0 + \ldots x_0^2y_0z_{16} + y_0^3y_{16} + \ldots + y_4^4$$

• Over the ideal I_{151}, we obtain the ideal I_{161}. The corresponding vector $v_1^{16} = (3,5,6)$ is of codimension 14.

For m=17, (5) gives;

$$F_{17}(x_0,\ldots,z_{17}) = z_0^2z_{17} + \ldots + z_5z_6^2 + x_0x_{17}y_0z_0 + \ldots x_0^2y_0z_{17} + y_0^3y_{17} + \ldots + y_4^3y_5$$

• Over the ideal I_{161}, we obtain three possible ideals I_{171}, I_{172} and I_{173}. The corresponding vectors $v_1^{17} = (3,6,6)$, $v_2^{17} = (3,5,7)$ and $v_3^{17} = (4,5,6)$ are of codimension 15.

For m=18, (5) gives;

$$F_{18}(x_0,\ldots,z_{18}) = z_0^2z_{18} + \ldots + z_6^3 + x_0x_{18}y_0z_0 + \ldots x_0^2y_0z_{18} + y_0^3y_{18} + \ldots + y_4^2y_5^2$$

• Over the ideal I_{171}, we obtain the ideal I_{181}. The corresponding vector $v_1^{18} = (3,6,7)$ is of codimension 16.
• Over the ideal I_{172}, we obtain three possible ideals I_{181}, I_{182} and I_{183}. The corresponding vectors $v_2^{18} = (3,5,8)$ and $v_3^{18} = (4,5,7)$ are of codimension 16.
• Over the ideal I_{173}, we obtain the ideal I_{183} with the corresponding vector $v_1^{18} = (4,5,7)$.
For m=19, (5) gives;

$$F_{19}(x_0,\ldots,z_{19}) = z_0^2z_{19} + \ldots + z_6^2z_7 + x_0x_{19}y_0z_0 + \ldots x_0^2y_0z_{19} + y_0^3y_{19} + \ldots + y_4y_5^3$$

• Over the ideal I_{183}, we obtain the ideal I_{191}. The corresponding vector $v_1^{19} = (4,5,7)$ is of codimension 16.

For m=20, (5) gives;

$$F_{20}(x_0,\ldots,z_{20}) = z_0^2 z_{20} + \ldots + z_6 z_7^2 + x_0 x_{20} y_0 z_0 + \ldots x_0^2 y_0 z_{20} + y_0^3 y_{20} + \ldots + y_5^4$$

• Over the ideal I_{191}, we obtain the ideal I_{201}. The corresponding vector $v_1^{20} = (4,6,7)$ is of codimension 17.

For m=21, (5) gives;

$$F_{21}(x_0,\ldots,z_{21}) = z_0^2 z_{21} + \ldots + z_7^3 + x_0 x_{21} y_0 z_0 + \ldots x_0^2 y_0 z_{21} + y_0^3 y_{21} + \ldots + y_5^3 y_6$$

• Over the ideal I_{201}, we obtain the ideal I_{211}. The corresponding vector $v_1^{21} = (4,6,8)$ is of codimension 18.

For m=22, (5) gives;

$$F_{22}(x_0,\ldots,z_{22}) = z_0^2 z_{22} + \ldots + z_7^2 z_8 + x_0 x_{22} y_0 z_0 + \ldots x_0^2 y_0 z_{22} + y_0^3 y_{22} + \ldots + y_5^3 y_6$$

• Over the ideal I_{211}, we obtain three possible ideals I_{221}, I_{222} and I_{223}. The corresponding vectors $v_1^{22} = (4,7,8)$, $v_2^{18} = (4,6,9)$ and $v_3^{18} = (5,6,8)$ are of codimension 19.

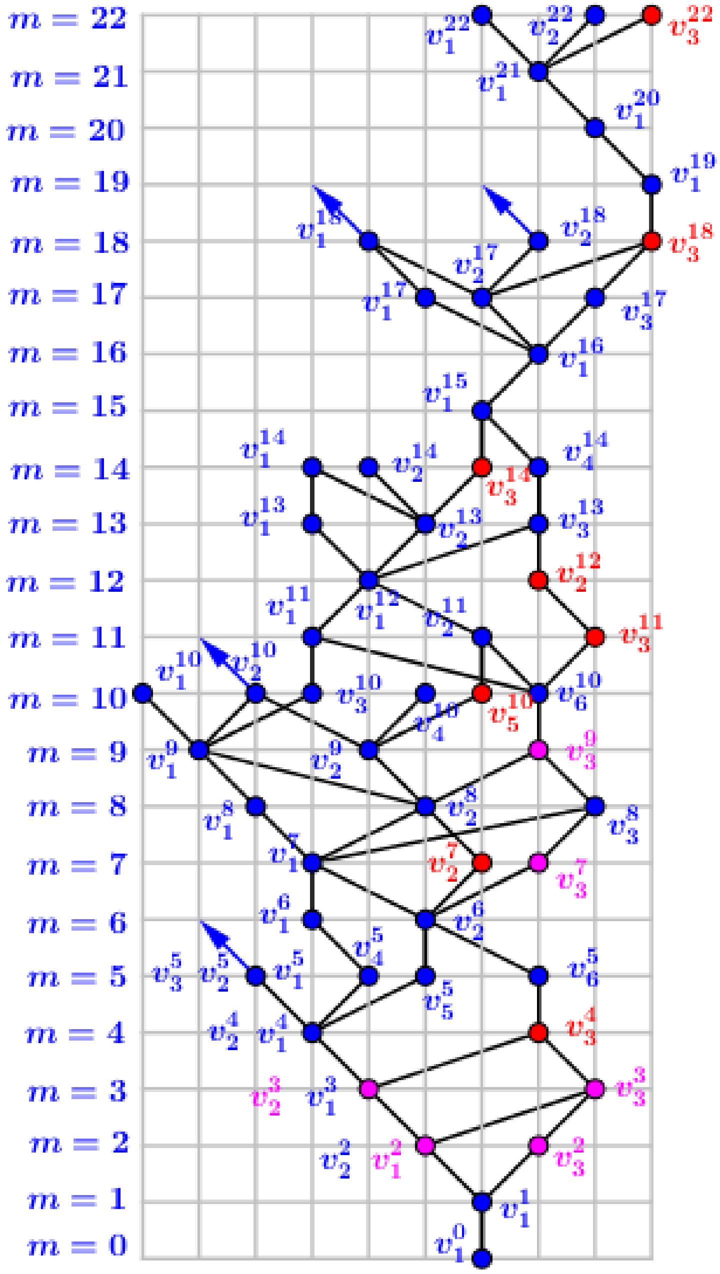

Figure 5.4: Jet graph of E_{70}

$v_1^0 = (0,1,1)$	$v_1^1 = (0,1,1)$	$v_1^2 = (0,2,1)$	$v_2^2 = (0,1,2)$	$v_3^2 = (1,1,1)$
$v_1^3 = (0,2,2)$	$v_2^3 = (0,1,3)$	$v_3^3 = (1,1,2)$	$v_1^4 = (0,3,2)$	$v_2^4 = (0,2,3)$
$v_3^4 = (1,2,2)$	$v_1^5 = (0,4,2)$	$v_2^5 = (0,3,3)$	$v_3^5 = (1,3,2)$	$v_4^5 = (0,2,4)$
$v_5^5 = (1,2,3)$	$v_6^5 = (1,2,2)$	$v_1^6 = (1,3,3)$	$v_2^6 = (1,2,3)$	$v_1^7 = (1,3,3)$
$v_2^7 = (1,2,4)$	$v_3^7 = (2,2,3)$	$v_1^8 = (1,4,3)$	$v_2^8 = (1,3,4)$	$v_3^8 = (2,3,3)$
$v_1^9 = (1,4,4)$	$v_2^9 = (1,3,5)$	$v_3^9 = (2,3,4)$	$v_1^{10} = (1,5,4)$	$v_2^{10} = (1,4,5)$
$v_3^{10} = (2,4,4)$	$v_4^{10} = (1,3,6)$	$v_5^{10} = (2,3,5)$	$v_6^{10} = (2,3,4)$	$v_1^{11} = (2,4,4)$
$v_2^{11} = (2,3,5)$	$v_3^{11} = (3,3,4)$	$v_1^{12} = (2,4,5)$	$v_2^{12} = (3,4,5)$	$v_1^{13} = (2,5,6)$
$v_2^{13} = (2,4,6)$	$v_3^{13} = (3,4,5)$	$v_1^{14} = (2,5,6)$	$v_2^{14} = (2,4,7)$	$v_3^{14} = (3,4,6)$

$v_4^{14} = (3,4,5)$	$v_1^{15} = (3,4,6)$	$v_1^{16} = (3,5,6)$	$v_1^{17} = (3,6,6)$	$v_2^{17} = (3,5,7)$
$v_3^{17} = (4,5,6)$	$v_1^{18} = (3,6,7)$	$v_2^{18} = (3,5,8)$	$v_3^{18} = (4,5,7)$	$v_1^{19} = (4,5,7)$
$v_1^{20} = (4,6,7)$	$v_1^{21} = (4,6,8)$	$v_1^{22} = (4,7,8)$	$v_2^{22} = (4,6,9)$	$v_3^{22} = (5,6,9)$

Remark 5.3.2. *Here and below the vectors represented with red correspond to the vectors given in $E_{70}b$, the vectors represented in pink also correspond to the vectors given in $E_{70}c$ and the rest are obtained as the supplemented vectors.*

Remark 5.3.3. *The minimal resolution graph of singularity E_{70} is as follow*

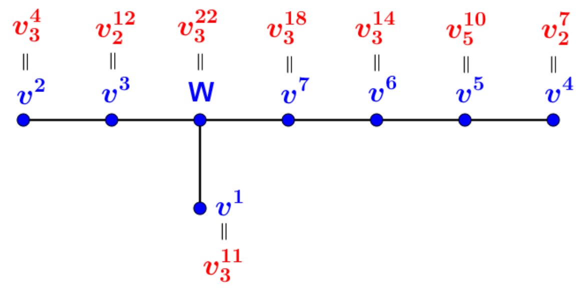

Figure 5.5: Minimal resolution graph of E_{70}

Remark 5.3.4. *So the Figure $E_{70}c$ becomes*

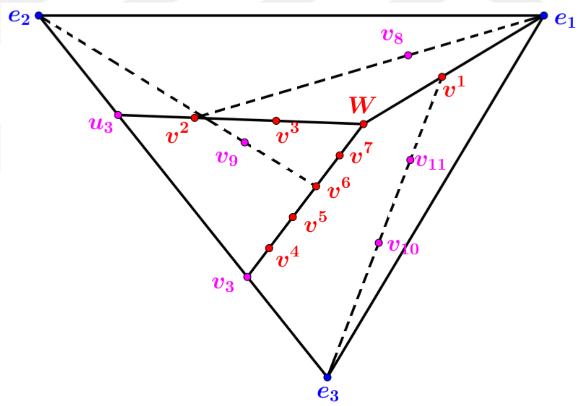

Figure 5.6: $E_{70}d$

where $u_3 = v_1^2$, $v_3 = v_2^3$, $v_8 = v_3^2$ $v_9 = v_3^9$, $v_{10} = v_3^7$ and $v_{11} = v_3^3$.

Proposition 5.3.5. *Let X be an hypersurface in $\mathbb{C}^3$ of type E_{70}. For $m \geq 22$, the number of irreducible components of $J_m(E_{70})$ equals the number of exceptional curves on the minimal resolution of the singularity.*

Proposition 5.3.6. *The set of weight vectors corresponding to m^{th} jets of E_{70} give a canonical toric minimal embedded resolution of the singularity.*

5.4 Jet Schemes of an Hypersurface of type E_{07}

Let X be an hypersurface of type E_{07} in $\mathbb{C}^3$. We know that X is defined by

$$f(x,y,x) = z^3 + y^5 + x^2y^2$$

in $\mathbb{C}[x,y,z]$. Hence $(x,0,0)$ is the singular locus of X.

Remark 5.4.1. *The m^{th} jet scheme of E_{07} is given by*

$$J_m(E_{07}) = Spec\frac{\mathbb{C}[x_0, y_0, z_0, \ldots, z_m]}{< F_0, \ldots, F_m >}$$

for all m.
Let us apply the equality (5):

For m=0, we have

$$F_0(x_0, y_0, z_0) = z_0^3 + y_0^2(y_0^3 + x_0^2)$$

• This says that $y_0 = z_0 = 0$. Hence the ideal $I_0 =< y_0, z_0 >$. The corresponding vector $v_1^0 = (0, 1, 1)$ is of codimension 2.

For m=1, (5) gives;

$$F_1(x_0, \ldots, z_1) = z_0^2 z_1 + y_0(y_0^3 y_1 + x_0 x_1 y_0 + x_0^2 y_1)$$

• Over the ideal I_0, we obtain the ideal $I_1 =< y_0, z_0 >$. The corresponding vector $v_1^1 = (0, 1, 1)$ is of codimension 2.

For m=2, (5) gives;

$$F_2(x_0, \ldots, z_2) = z_0(z_0 z_2 + z_1^2) + y_0(y_0^2 y_1^2 + y_0^3 y_2 + x_0 x_2 y_0 + x_1^2 y_0 + x_0^2 y_2) + y_1(x_0^2 y_1 + x_0 x_1 y_0)$$

Over the ideal I_1, we obtain two possible ideals $I_{21} = (y_0, y_1, z_0)$ and $I_{22} = (x_0, y_0, z_0)$. The corresponding vectors $v_1^2 = (0, 2, 1)$ and $v_2^2 = (1, 1, 1)$ are of codimension 3.

For m=3, (5) gives;

$$F_3(x_0, \ldots, z_3) = z_0^2 z_3 + \ldots + z_1^3 + y_0^4 y_3 + \ldots + y_0^2 y_1^3 + x_0 x_3 y_0^2 + \ldots + x_0^2 y_0 y_3$$

• Over the ideal I_{21}, we obtain the ideal $I_{31} =< y_0, y_1, z_0, z_1 >$. The corresponding vector $v_1^3 = (0, 2, 2)$ is of codimension 4.
• Over the ideal I_{22}, we obtain the ideal $I_{32} =< x_0, y_0, z_0, z_1 >$. The corresponding vector $v_2^3 = (1, 1, 2)$ is of codimension 4.

For m=4, (5) gives;

$$F_4(x_0, \ldots, z_4) = z_0^2 z_4 + \ldots + z_1^2 z_2 + y_0^4 y_4 + \ldots + y_0 y_1^4 + x_0 x_4 y_0^2 + \ldots + x_0^2 y_0 y_4$$

• Over the ideal I_{31}, we obtain two possible ideals $I_{41} =< y_0, y_1, z_0, z_1, y_2 >$ and $I_{42} =< y_0, y_1, z_0, z_1, x_0 >$. The corresponding vectors $v_1^4 = (0, 3, 2)$ and $v_2^4 = (1, 2, 2)$ are of codimension 5.
• Over the ideal I_{32}, we obtain two possible ideals I_{42} and $I_{43} =< x_0, y_0, z_0, z_1, x_1 >$. The corresponding vector of I_{43}, $v_3^4 = (2, 1, 2)$ is of codimension 5.

For m=5, (5) gives;

$$F_5(x_0, \ldots, z_5) = z_0^2 z_5 + \ldots + z_1 z_2^2 + y_0^4 y_5 + \ldots + y_1^5 + x_0 x_5 y_0^2 + \ldots + x_0^2 y_0 y_5$$

• Over the ideal I_{41}, we obtain the ideal $I_{51} =< y_0, y_1, z_0, z_1, y_2 >$. The corresponding vector $v_1^5 = (0, 3, 2)$ is of codimension 5.
• Over the ideal I_{42}, we obtain the ideal $I_{52} =< y_0, y_1, z_0, z_1, x_0 >$. The corresponding vector $v_2^5 = (1, 2, 2)$ is of codimension 5.
• Over the ideal I_{43}, we continue with the vector $(2, 1, 2)$.

For m=6, (5) gives;

$$F_6(x_0, \dots, z_6) = z_0^2 z_6 + \dots + z_2^3 + y_0^4 y_6 + \dots + y_1^4 y_4 + x_0 x_6 y_0^2 + \dots + x_0^2 y_0 y_6$$

• Over the ideal I_{51}, we obtain the ideal $I_{61} =< y_0, y_1, z_0, z_1, y_2 >$. The corresponding vector $v_1^6 = (0, 3, 2)$ is of codimension 5.
• Over the ideal I_{52}, we obtain the ideal $I_{62} =< y_0, y_1, z_0, z_1, x_0 >$. The corresponding vector $v_2^6 = (1, 2, 2)$ is of codimension 6.

In the same way;
For m=7, (5) gives;

$$F_7(x_0, \dots, z_7) = z_0^2 z_7 + \dots + z_2^2 z_3 + y_0^4 y_7 + \dots + y_1^3 y_2^2 + x_0 x_7 y_0^2 + \dots + x_0^2 y_0 y_7$$

• Over the ideal I_{61}, we obtain two possible ideals I_{71} and I_{72}. The corresponding vector $v_1^7 = (0, 4, 3)$ and $v_2^7 = (1, 3, 3)$ are of codimension 7.
• Over the ideal I_{62}, we obtain two possible ideals I_{72} and I_{73}. The corresponding vector of I_{73}, $v_3^7 = (2, 2, 3)$ is of codimension 7.

For m=8, (5) gives;

$$F_8(x_0, \dots, z_8) = z_0^2 z_8 + \dots + z_2 z_3^2 + y_0^4 y_8 + \dots + y_1^2 y_2^3 + x_0 x_8 y_0^2 + \dots + x_0^2 y_0 y_8$$

• Over the ideal I_{71}, we obtain two possible ideals I_{81} and I_{82}. The corresponding vectors $v_1^8 = (0, 5, 3)$ and $v_2^8 = (0, 5, 3)$ are of codimensoin 8.
• Over the ideal I_{72}, we obtain two possible ideals I_{82} and I_{83}. The corresponding vector of I_{83}, $v_3^8 = (2, 3, 3)$ is of codimensoin 8.
• Over the ideal I_{73}, we obtain two possible ideals I_{83} and I_{84}. The corresponding vector of I_{84}, $v_4^8 = (3, 2, 3)$ is of codimensoin 8.

For m=9, (5) gives;

$$F_9(x_0, \dots, z_9) = z_0^2 z_9 + \dots + z_3^3 + y_0^4 y_9 + \dots + y_1 y_2^4 + x_0 x_9 y_0^2 + \dots + x_0^2 y_0 y_9$$

• Over the ideal I_{81}, we obtain the ideal I_{91}. The corresponding vector $v_1^9 = (0, 5, 4)$ is of codimensoin 9.
• Over the ideal I_{82}, we obtain the ideal I_{92}. The corresponding vector $v_2^9 = (1, 4, 4)$ is of codimensoin 9.
• Over the ideal I_{83}, we obtain the ideal I_{93}. The corresponding vector $v_3^9 = (2, 3, 4)$ is of codimensoin 9.
• Over the ideal I_{84}, we obtain the ideal I_{94}. The corresponding vector $v_4^9 = (3, 2, 4)$ is of codimensoin 9.

For m=10, (5) gives;

$$F_{10}(x_0,\ldots,z_{10}) = z_0^2 z_{10} + \ldots + z_3^2 z_4 + y_0^4 y_{10} + \ldots + y_2^5 + x_0 x_{10} y_0^2 + \ldots + x_0^2 y_0 y_{10}$$

• Over the ideal I_{91}, we obtain two possible ideals I_{101} and I_{102}. The corresponding vectors $v_1^{10} = (0,6,4)$ and $v_2^{10} = (1,5,4)$ are of codimensoin 10.
• Over the ideal I_{92}, we obtain two possible ideals I_{102} and I_{103}. The corresponding vector of I_{103} $v_3^{10} = (2,4,4)$ is of codimensoin 10.
• Over the ideal I_{93}, we obtain two possible ideals I_{103} and I_{104}. The corresponding vector of I_{104} $v_4^{10} = (3,3,4)$ is of codimensoin 10.
• Over the ideal I_{94}, we obtain the ideal I_{104}.

For m=11, (5) gives;

$$F_{11}(x_0,\ldots,z_{11}) = z_0^2 z_{11} + \ldots + z_3 z_4^2 + y_0^4 y_{11} + \ldots + y_2^4 y_3 + x_0 x_{11} y_0^2 + \ldots + x_0^2 y_0 y_{11}$$

• Over the ideal I_{101}, we obtain the ideal I_{111}. The corresponding vector $v_1^{11} = (0,6,4)$ is of codimensoin 10.
• Over the ideal I_{102}, we obtain the ideal I_{112}. The corresponding vector $v_2^{11} = (1,5,4)$ is of codimensoin 10.
• Over the ideal I_{103}, we obtain the ideal I_{113}. The corresponding vector $v_3^{11} = (2,4,4)$ is of codimensoin 10.
• Over the ideal I_{104}, we obtain the ideal I_{114}. The corresponding vector $v_4^{11} = (3,3,4)$ is of codimensoin 10.

For m=12, (5) gives;

$$F_{12}(x_0,\ldots,z_{12}) = z_0^2 z_{12} + \ldots + z_4^3 + y_0^4 y_{12} + \ldots + y_2^3 y_3^2 + x_0 x_{12} y_0^2 + \ldots + x_0^2 y_0 y_{12}$$

• Over the ideal I_{111}, we obtain two possible ideals I_{121} and I_{122}. The corresponding vectors $v_1^{12} = (0,7,5)$ and $v_2^{12} = (1,6,5)$ is of codimensoin 12.
• Over the ideal I_{112}, we obtain two possible ideals I_{122} and I_{123}. The corresponding vector of I_{123}, $v_3^{12} = (2,5,5)$ is of codimensoin 12.
• Over the ideal I_{113}, we obtain two possible ideals I_{123} and I_{124}. The corresponding vector of I_{124}, $v_4^{12} = (3,4,5)$ is of codimensoin 12.
• Over the ideal I_{114}, we obtain two possible ideals I_{124} and I_{125}. The corresponding vector of I_{125}, $v_5^{12} = (4,3,5)$ is of codimensoin 12.

For m=13, (5) gives;

$$F_{13}(x_0,\ldots,z_{13}) = z_0^2 z_{13} + \ldots + z_4^2 z_5 + y_0^4 y_{13} + \ldots + y_2^2 y_3^3 + x_0 x_{13} y_0^2 + \ldots + x_0^2 y_0 y_{13}$$

• Over the ideal I_{122}, we obtain the ideal I_{131}. The corresponding vector $v_1^{13} = (1,6,5)$ is of codimensoin 12.
• Over the ideal I_{123}, we obtain the ideal I_{132}. The corresponding vector $v_2^{13} = (2,5,5)$ is of codimensoin 12.
• Over the ideal I_{124}, we obtain the ideal I_{133}. The corresponding vector $v_3^{13} = (3,4,5)$ is of codimensoin 12.

• Over the ideal I_{125}, we obtain the ideal I_{134}. The corresponding vector $v_4^{13} = (4,3,5)$ is of codimensoin 12.

For m=14, (5) gives;

$$F_{14}(x_0,\ldots,z_{14}) = z_0^2 z_{14} + \ldots + z_4 z_5^2 + y_0^4 y_{14} + \ldots + y_2 y_3^4 + x_0 x_{14} y_0^2 + \ldots + x_0^2 y_0 y_{14}$$

• Over the ideal I_{131}, we obtain two possible ideals I_{141} and I_{142}. The corresponding vectors $v_1^{14} = (1,7,5)$ and $v_2^{14} = (2,6,5)$ are of codimensoin 13.
• Over the ideal I_{132}, we obtain two possible ideals I_{142} and I_{143}. The corresponding vector of I_{143}, $v_3^{14} = (3,5,5)$ is of codimensoin 13.
• Over the ideal I_{133}, we obtain two possible ideals I_{143} and I_{144}. The corresponding vector of I_{144}, $v_4^{14} = (4,4,5)$ is of codimensoin 13.
• Over the ideal I_{134}, we obtain two possible ideals I_{144} and I_{145}. The corresponding vector of I_{145}, $v_5^{14} = (5,3,5)$ is of codimensoin 13.

For m=15, (5) gives;

$$F_{15}(x_0,\ldots,z_{15}) = z_0^2 z_{15} + \ldots + z_5^3 + y_0^4 y_{15} + \ldots + y_3^5 + x_0 x_{115} y_0^2 + \ldots + x_0^2 y_0 y_{15}$$

• Over the ideal I_{142}, we obtain the ideal I_{151}. The corresponding vector $v_1^{15} = (2,6,6)$ is of codimensoin 14.
• Over the ideal I_{143}, we obtain the ideal I_{152}. The corresponding vector $v_2^{15} = (3,5,6)$ is of codimensoin 14.
• Over the ideal I_{144}, we obtain the ideal I_{153}. The corresponding vector $v_3^{15} = (4,4,6)$ is of codimensoin 14.
• Over the ideal I_{145}, we obtain the ideal I_{154}. The corresponding vector $v_4^{15} = (5,4,6)$ is of codimensoin 15.

For m=16, (5) gives;

$$F_{16}(x_0,\ldots,z_{16}) = z_0^2 z_{16} + \ldots + z_5^2 z_6 + y_0^4 y_{16} + \ldots + y_3^4 y_4 + x_0 x_{16} y_0^2 + \ldots + x_0^2 y_0 y_{16}$$

• Over the ideal I_{151}, we obtain two possible ideals I_{161} and I_{162}. The corresponding vectors $v_1^{16} = (2,7,6)$ and $v_2^{16} = (3,6,6)$ are of codimensoin 15.
• Over the ideal I_{152}, we obtain two possible ideals I_{162} and I_{163}. The corresponding vectors $v_3^{16} = (4,5,6)$ is of codimensoin 15.
• Over the ideal I_{153}, we obtain two possible ideals I_{163} and I_{164}. The corresponding vectors $v_4^{16} = (5,4,6)$ is of codimensoin 15.
• Over the ideal I_{154}, we obtain the ideal I_{164}.

For m=17, (5) gives;

$$F_{17}(x_0,\ldots,z_{17}) = z_0^2 z_{17} + \ldots + z_5 z_6^2 + y_0^4 y_{17} + \ldots + y_3^3 y_4^2 + x_0 x_{17} y_0^2 + \ldots + x_0^2 y_0 y_{17}$$

• Over the ideal I_{162}, we obtain the ideal I_{171}. The corresponding vector $v_1^{17} = (3,6,6)$ is of codimensoin 15.
• Over the ideal I_{163}, we obtain the ideal I_{172}. The corresponding vector $v_2^{17} = (4,5,6)$ is of codimensoin 15.

• Over the ideal I_{164}, we obtain the ideal I_{173}. The corresponding vector $v_3^{17} = (5,4,6)$ is of codimensoin 15.

For m=18, (5) gives;

$$F_{18}(x_0,\ldots,z_{18}) = z_0^2 z_{18} + \ldots + z_6^3 + y_0^4 y_{18} + \ldots + y_3^2 y_4^3 + x_0 x_{18} y_0^2 + \ldots + x_0^2 y_0 y_{18}$$

• Over the ideal I_{171}, we obtain two possible ideals I_{181} and I_{182}. The corresponding vectors $v_1^{18} = (3,7,7)$ and $v_2^{18} = (4,6,7)$ are of codimensoin 17.
• Over the ideal I_{172}, we obtain two possible ideals I_{182} and I_{183}. The corresponding vector $v_3^{18} = (5,5,7)$ si of codimensoin 17.
• Over the ideal I_{173}, we obtain two possible ideals I_{183} and I_{184}. The corresponding vector $v_4^{18} = (6,4,7)$ si of codimensoin 17.

For m=19, (5) gives;

$$F_{19}(x_0,\ldots,z_{19}) = z_0^2 z_{19} + \ldots + z_6^2 z_7 + y_0^4 y_{19} + \ldots + y_3 y_4^4 + x_0 x_{19} y_0^2 + \ldots + x_0^2 y_0 y_{19}$$

• Over the ideal I_{182}, we obtain the ideal I_{191}. The corresponding vector $v_1^{19} = (4,6,7)$ is of codimensoin 17.
• Over the ideal I_{183}, we obtain the ideal I_{192}. The corresponding vector $v_2^{19} = (5,5,7)$ is of codimensoin 17.
• Over the ideal I_{184}, we obtain the ideal I_{193}. The corresponding vector $v_3^{19} = (6,4,7)$ is of codimensoin 17.

For m=20, (5) gives;

$$F_{20}(x_0,\ldots,z_{20}) = z_0^2 z_{20} + \ldots + z_6 z_7^2 + y_0^4 y_{20} + \ldots + y_4^5 + x_0 x_{20} y_0^2 + \ldots + x_0^2 y_0 y_{20}$$

• Over the ideal I_{191}, we obtain two possible ideals I_{201} and I_{202}. The corresponding vectors $v_1^{20} = (4,7,7)$ and $v_2^{20} = (5,6,7)$ are of codimensoin 18.
• Over the ideal I_{192}, we obtain two possible ideals I_{202} and I_{203}. The corresponding vector of I_{203} $v_3^{20} = (6,5,7)$ is of codimensoin 18.
• Over the ideal I_{193}, we obtain the ideal I_{203}.

For m=21, (5) gives;

$$F_{21}(x_0,\ldots,z_{21}) = z_0^2 z_{21} + \ldots + z_7^3 + y_0^4 y_{21} + \ldots + y_4^4 y_5 + x_0 x_{21} y_0^2 + \ldots + x_0^2 y_0 y_{21}$$

• Over the ideal I_{202}, we obtain the ideal I_{211}. The corresponding vector $v_1^{21} = (5,6,8)$ is of codimensoin 19.
• Over the ideal I_{203}, we obtain the ideal I_{212}. The corresponding vector $v_2^{21} = (6,5,8)$ is of codimensoin 19.

For m=22, (5) gives;

$$F_{22}(x_0,\ldots,z_{22}) = z_0^2 z_{22} + \ldots + z_7^2 z_8 + y_0^4 y_{22} + \ldots + y_4^3 y_5^2 + x_0 x_{22} y_0^2 + \ldots + x_0^2 y_0 y_{22}$$

• Over the ideal I_{211}, we obtain two possible ideals I_{221} and I_{222}. The corresponding vectors $v_1^{22} = (5,7,8)$ and $v_2^{22} = (6,6,8)$ are of codimensoin 20.

• Over the ideal I_{212}, we obtain two possible ideals I_{222} and I_{223}. The corresponding vectors $v_1^{23} = (7,5,8)$ is of codimensoin 20.

For m=23, (5) gives;

$$F_{23}(x_0,\ldots,z_{23}) = z_0^2z_{23} + \ldots + z_7z_8^2 + y_0^4y_{23} + \ldots + y_4^2y_5^3 + x_0x_{23}y_0^2 + \ldots + x_0^2y_0y_{23}$$

• Over the ideal I_{222}, we obtain the ideal I_{231}. The corresponding vector $v_1^{23} = (6,6,8)$ is of codimensoin 20.
• Over the ideal I_{223}, we obtain the ideal I_{232}. The corresponding vector $v_2^{23} = (7,5,8)$ is of codimensoin 20.

For m=24, (5) gives;

$$F_{24}(x_0,\ldots,z_{24}) = z_0^2z_{24} + \ldots + z_8^3 + y_0^4y_{24} + \ldots + y_4y_5^4 + x_0x_{24}y_0^2 + \ldots + x_0^2y_0y_{24}$$

• Over the ideal I_{231}, we obtain two possible ideals I_{241} and I_{242}. The corresponding vectors $v_1^{24} = (6,7,9)$ and $v_2^{24} = (7,6,9)$ are of codimensoin 22.
• Over the ideal I_{232}, we obtain two possible ideals I_{242} and I_{243}. The corresponding vector of I_{243}, $v_3^{24} = (8,5,9)$ is of codimensoin 22.

For m=25, (5) gives;

$$F_{25}(x_0,\ldots,z_{25}) = z_0^2z_{25} + \ldots + z_8^2z_9 + y_0^4y_{25} + \ldots + y_5^5 + x_0x_{25}y_0^2 + \ldots + x_0^2y_0y_{25}$$

• Over the ideal I_{242}, we obtain the ideal I_{251}. The corresponding vector $v_1^{25} = (7,6,9)$ is of codimensoin 22.
• Over the ideal I_{243}, we obtain the ideal I_{252}. The corresponding vector $v_2^{25} = (8,6,9)$ is of codimensoin 23.

For m=26, (5) gives;

$$F_{26}(x_0,\ldots,z_{26}) = z_0^2z_{26} + \ldots + z_8z_9^2 + y_0^4y_{26} + \ldots + y_5^4y_6 + x_0x_{26}y_0^2 + \ldots + x_0^2y_0y_{26}$$

• Over the ideal I_{251}, we obtain the ideal I_{261}. The corresponding vector $v_1^{26} = (7,7,9)$ is of codimensoin 23.
• Over the ideal I_{252}, we obtain the ideal I_{262}. The corresponding vector $v_2^{26} = (8,6,9)$ is of codimensoin 23.

For m=27, (5) gives;

$$F_{27}(x_0,\ldots,z_{27}) = z_0^2z_{27} + \ldots + z_9^3 + y_0^4y_{27} + \ldots + y_5^3y_6^2 + x_0x_{27}y_0^2 + \ldots + x_0^2y_0y_{27}$$

• Over the ideal I_{262}, we obtain the ideal I_{271}. The corresponding vector $v_1^{27} = (8,6,10)$ is of codimensoin 24.

For m=28, (5) gives

$$F_{28}(x_0,\ldots,z_{28}) = z_0^2z_{28} + \ldots + z_9^2z_{10} + y_0^4y_{28} + \ldots + y_5^2y_6^3 + x_0x_{28}y_0^2 + \ldots + x_0^2y_0y_{28}$$

• Over the ideal I_{271}, we obtain two possible ideals I_{281} and I_{282}. The corresponding vectors $v_1^{27} = (8,7,10)$ and $v_2^{27} = (9,6,10)$ are of codimensoin 23.

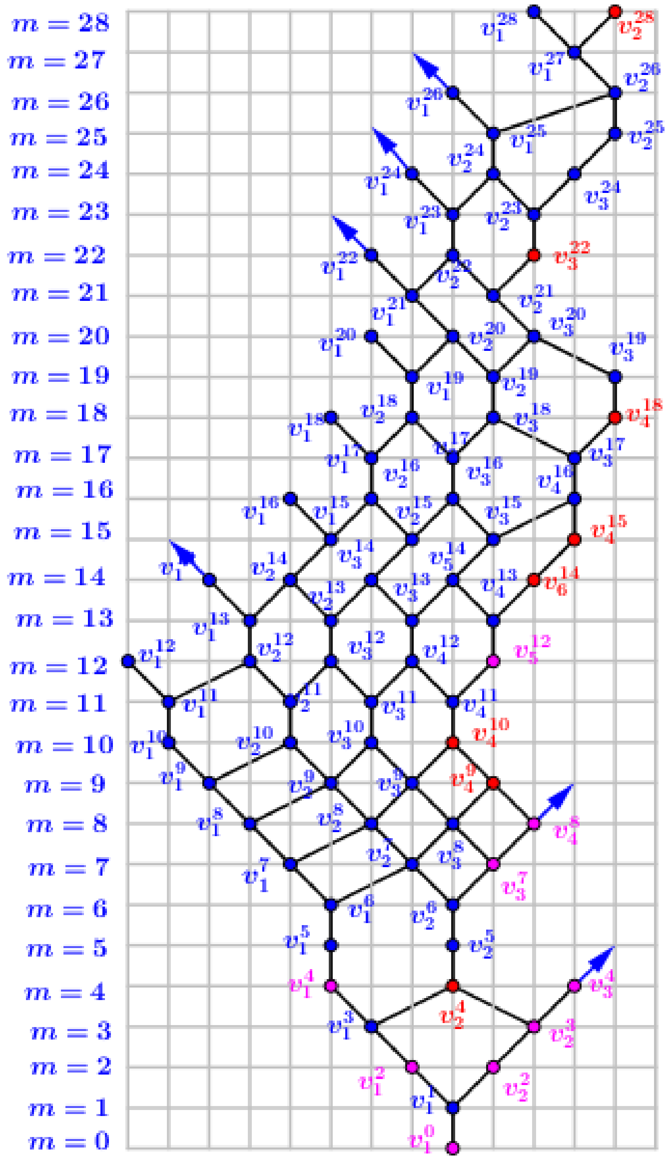

Figure 5.7: Jet graph of E_{07}

$v_1^0 = (0,1,1)$	$v_1^1 = (0,1,1)$	$v_1^2 = (0,2,1)$	$v_2^2 = (1,1,1)$	$v_1^3 = (0,2,2)$
$v_2^3 = (1,1,2)$	$v_1^4 = (0,3,2)$	$v_2^4 = (1,2,2)$	$v_3^4 = (2,1,2)$	$v_1^5 = (0,3,2)$
$v_2^5 = (1,2,2)$	$v_1^6 = (0,3,2)$	$v_2^6 = (1,2,2)$	$v_1^7 = (0,4,3)$	$v_2^7 = (1,3,3)$
$v_3^7 = (2,2,3)$	$v_1^8 = (0,5,3)$	$v_2^8 = (1,4,3)$	$v_3^8 = (2,3,3)$	$v_4^8 = (3,2,3)$
$v_1^9 = (0,5,4)$	$v_2^9 = (1,4,4)$	$v_3^9 = (2,3,4)$	$v_4^9 = (3,2,4)$	$v_1^{10} = (0,6,4)$
$v_2^{10} = (1,5,4)$	$v_3^{10} = (2,4,4)$	$v_4^{10} = (3,3,4)$	$v_1^{11} = (0,6,4)$	$v_2^{11} = (1,5,4)$
$v_3^{11} = (2,4,4)$	$v_4^{11} = (3,3,4)$	$v_3^{11} = (2,4,4)$	$v_4^{11} = (3,3,4)$	$v_1^{12} = (0,7,5)$
$v_2^{12} = (1,6,5)$	$v_3^{12} = (2,5,5)$	$v_4^{12} = (3,4,5)$	$v_5^{12} = (4,3,5)$	$v_1^{13} = (1,6,5)$
$v_2^{13} = (2,5,5)$	$v_3^{13} = (3,4,5)$	$v_4^{13} = (4,3,5)$	$v_1^{14} = (1,7,5)$	$v_2^{14} = (2,6,5)$
$v_3^{14} = (3,5,5)$	$v_4^{14} = (4,4,5)$	$v_5^{14} = (5,3,5)$	$v_1^{15} = (2,6,6)$	$v_2^{15} = (3,5,6)$
$v_3^{15} = (4,4,6)$	$v_4^{15} = (5,4,6)$	$v_1^{16} = (2,7,6)$	$v_2^{16} = (3,6,6)$	$v_3^{16} = (4,5,6)$
$v_4^{16} = (5,4,6)$	$v_1^{17} = (3,6,6)$	$v_2^{17} = (4,5,6)$	$v_3^{17} = (5,4,6)$	$v_1^{18} = (3,7,7)$

$v_2^{18} = (4,6,7)$ $v_3^{18} = (5,5,7)$ $v_4^{18} = (6,4,7)$ $v_1^{19} = (4,6,7)$ $v_2^{19} = (5,5,7)$
$v_3^{20} = (6,5,7)$ $v_1^{21} = (5,6,8)$ $v_2^{21} = (6,5,8)$ $v_1^{22} = (5,7,8)$ $v_2^{22} = (6,6,8)$
$v_3^{22} = (7,5,8)$ $v_1^{23} = (6,6,8)$ $v_2^{23} = (7,5,8)$ $v_1^{24} = (6,7,9)$ $v_2^{24} = (7,6,9)$
$v_3^{24} = (8,5,9)$ $v_1^{25} = (7,6,9)$ $v_2^{25} = (8,6,9)$ $v_1^{26} = (7,7,9)$ $v_2^{26} = (8,6,9)$
$v_1^{27} = (8,6,10)$ $v_1^{28} = (8,7,10)$ $v_2^{28} = (9,6,10)$

Remark 5.4.2. *Here and below the vectors represented with red correspond to the vectors given in $E_{07}b$, the vectors represented in pink also correspond to the vectors given in $E_{07}c$ and the rest are obtained as the supplemented vectors.*

Remark 5.4.3. *The minimal resolution graph of singularity E_{07} is as follow*

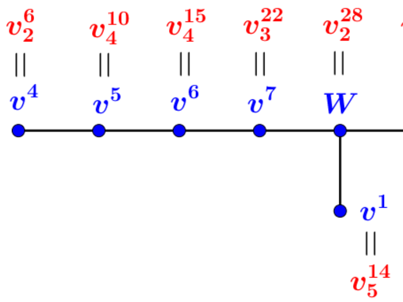

Figure 5.8: Minimal resolution graph of E_{07}

Remark 5.4.4. *So the Figure $E_{07}c$ becomes*

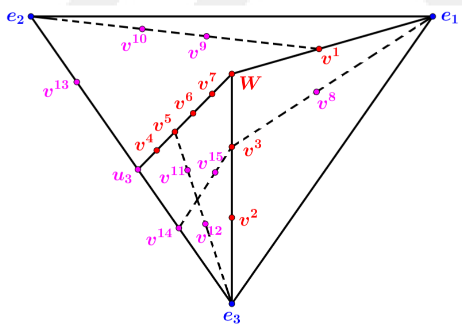

Figure 5.9: $E_{07}d$

where $u_3 = v_1^4$, $v^8 = v_3^4$, $v^9 = v_4^8$, $v^{10} = v_2^2$, $v^{11} = v_3^7$, $v^{12} = v_2^3$, $v^{13} = v_1^2$, $v^{14} = v_1^0$ *and* $v^{15} = v_5^{12}$.

Proposition 5.4.5. *Let X be an hypersurface in $\mathbb{C}^3$ of type E_{07}. For $m \geq 28$, the number of irreducible components of $J_m(E_{07})$ equals the number of exceptional curves on the minimal resolution of the singularity.*

Proposition 5.4.6. *The set of weight vectors corresponding to m^{th} jets of E_{07} give a canonical toric minimal embedded resolution of the singularity.*

6. JET SCHEMES OF ISOLATED SURFACE SINGULARITIES

The rational triple singularities are of the singularities of surfaces in $\mathbb{C}^4$. They are defined by 3 equations in (Tyurina, 1968) and are the normalisation of the hypersufaces given in the beginning of Chapter 5 above. As before, we consider here only the cases E_{60}, E_{70} and E_{07}.

The surface of type E_{60} is defined by:

$$f_1(x,y,z,w) = z^2 - yw + y^3 = 0$$

$$f_2(x,y,z,w) = zw - x^2y = 0$$

$$f_3(x,y,z,w) = w^2 - y^2w - x^2z = 0$$

The surface of type E_{70} is defined by:

$$f_1(x,y,z,w) = z^2 - yw + x^2y = 0$$

$$f_2(x,y,z,w) = zw - y^3 = 0$$

$$f_3(x,y,z,w) = w^2 - x^2w - y^2w = 0$$

The surface of type E_{07} is defined by:

$$f_1(x,y,z,w) = z^2 - yw = 0$$

$$f_2(x,y,z,w) = zw - x^2y - y^4 = 0$$

$$f_3(x,y,z,w) = w^2 - x^2z - y^3z = 0$$

See (Tyurina, 1968) for the rest of RTP-singularities.

6.1 Some Surfaces in $\mathbb{C}^4$

It is defined in a similar way to the case of hypersurfaces. We here assume that X is defined by the ideal $I =< f_1, \ldots, f_k >$. And, consider the morphism

$$\varphi_m : \mathbb{C}[x_1, \ldots, x_n]/ < I > \longrightarrow C[[t]]/ < t^{m+1} >$$

defined by

$$\varphi_m(x_i) = x_{i,0} + x_{i,1}t + x_{i,2}t^2 + \ldots + x_{i,m}t^m \qquad mod(t^{m+1})$$

with

$$f_i(x_{1,0} + x_{1,1}t + \ldots + x_{1,m}t^m, \ldots, x_{n,0} + x_{n,1}t + \ldots + x_{n,m}t^m) = 0 \quad mod(t^{m+1}) \quad (6)$$

for all $1 \leq i \leq k$. We can see (6) in the form

$$F_i^0 + F_i^1 t + \ldots + F_i^m t^m$$

where

$$F_i^j = f_i(x_{1,j}, x_{2,j}, \ldots, x_{n,j})$$

in $Spec\mathbb{C}[x_{1,0}, \ldots, x_{1,m}, \ldots, x_{n,0}, \ldots, x_{n,m}]$. Then the m^{th} jet scheme of X is defined by

$$J_m(X) = Spec\frac{\mathbb{C}[x_1, \ldots, x_n]}{< F_1^0, F_1^1, \ldots, F_1^m, F_2^0, \ldots, F_n^0, \ldots, , F_n^m >}$$

Note that $J_0(X) = X$.

6.2 Jet Scheme of a Surface of type E_{60}

Let us consider the ideal $I =< f_1, f_2, f_3 >$ in $\mathbb{C}[x, y, z, w]$ such that X is the surface E_{60} in $\mathbb{C}^4$. The singular locus is the unique point which is the origin. The map φ_m above is defined as

$$x \longmapsto x_0 + x_1 t + x_2 t^2 + \ldots + x_m t^m$$
$$y \longmapsto y_0 + y_1 t + y_2 t^2 + \ldots + y_m t^m$$
$$z \longmapsto z_0 + z_1 t + z_2 t^2 + \ldots + z_m t^m$$
$$w \longmapsto w_0 + w_1 t + w_2 t^2 + \ldots + w_m t^m$$

such that

$$f_i(x_0 + x_1 t + \ldots + x_m t^m, y_0 + \ldots + y_m t^m, z_0 + \ldots + z_m t^m, w_0 + \ldots + w_m t^m) = 0 \qquad (7)$$

For m=0, (7) gives;

$$F_1^0(x_0, y_0, z_0, w_0) = z_0^2 - y_0 w_0 + y_0^3 = z_0^2 - y_0(w_0 + y_0^2) = 0$$

$$F_2^0(x_0, y_0, z_0, w_0) = z_0 w_0 - x_0^2 y_0 = 0$$
$$F_3^0(x_0, y_0, z_0, w_0) = w_0^2 - y_0^2 w_0 - x_0^2 z_0 = w_0(w_0 - y_0^2) - x_0^2 z_0 = 0$$

• This says that we have $y_0 = z_0 = w_0 = 0$ which is over the generic point. Hence the ideal $I_0 =< y_0, z_0, w_0 >$. The corresponding vector $v_1^0 = (0, 1, 1, 1)$ is of codimension 3. $J_0(E_{60})$ is given by

$$J_0(E_{60}) = Spec\frac{\mathbb{C}[x_0, y_0, z_0, w_0]}{< F_1^0, F_2^0, F_3^0 >}$$

For m=1, (7) gives;

$$F_1^1(x_0, y_0, \ldots, z_1, w_1) = z_0 z_1 - y_0 w_1 - y_{01} w_0 + y_0^2 y_1 = 0$$

$$F_2^1(x_0, y_0, \ldots, z_1, w_1) = z_0 w_1 + z_1 w_0 - x_0 x_1 y_0 - x_0^2 y_1 = 0$$

$$F_3^1(x_0, y_0, \ldots, z_1, w_1) = w_0w_1 - y_0y_1w_0 + y_0^2w_1 - x_0x_1z_0 - x_0^2z_1 = 0$$

• This gives two possible ideals but the only ideal over the generic point is $I_1 = (y_0, z_0, w_0, y_1, z_1)$. The corresponding vector $v_1^1 = (0, 2, 2, 1)$ is of codimension 5. $J_1(E_{60})$ is given by

$$J_1(E_{60}) = Spec\frac{\mathbb{C}[x_0, y_0, \ldots, z_1, w_1]}{< F_1^0, F_2^0, F_3^0, F_1^1, F_2^1, F_3^1 >}$$

For m=2, (7) gives;

$$F_1^2(x_0, y_0, \ldots, z_2, w_2) = z_0z_2 + z_1^2 - \ldots + y_0^2y_2 + y_0y_1^2 = 0$$

$$F_2^2(x_0, y_0, \ldots, z_2, w_2) = z_0w_2 + z_1w_1 + \ldots + x_0x_2y_0 + x_1^2y_0 = 0$$

$$F_3^2(x_0, y_0, \ldots, z_2, w_2) = w_0w_2 + w_1^2 + \ldots + x_1^2z_0 + x_0x_2z_0 = 0$$

• Over the ideal I_1, we obtain two possible ideals $I_{21} = (y_0, z_0, w_0, y_1, z_1, w_1, x_0)$ and $I_{22} = (y_0, z_0, w_0, y_1, z_1, w_1, y_2, z_2)$. The corresponding vector of I_{21}, $v_1^2 = (1, 2, 2, 2)$ is of codimension 7. The corresponding vector of I_{22}, $v_2^2 = (0, 3, 3, 2)$ is of codimension 8. $J_2(E_{60})$ is given by

$$J_2(E_{60}) = Spec\frac{\mathbb{C}[x_0, y_0, \ldots, z_2, w_2]}{< F_1^0, F_2^0, \ldots, F_2^3, F_3^3 >}$$

For m=3, (7) gives;

$$F_1^3(x_0, y_0, \ldots, z_3, w_3) = z_0z_3 + z_1z_2 + \ldots + y_0y_1y_2 + y_1^3 = 0$$

$$F_2^3(x_0, y_0, \ldots, z_3, w_3) = z_0w_3 + z_1w_2 + \ldots + x_0x_1y_2 + x_0^2y_3 = 0$$

$$F_3^3(x_0, y_0, \ldots, z_3, w_3) = w_0w_3 + w_1w_2 + \ldots + x_0x_1z_2 + x_0^2z_3 = 0$$

• Over the ideal I_{21} we obtain ideal $I_{31} = (y_0, z_0, w_0, y_1, z_1, w_1, x_0)$. The corresponding vector $v_1^3 = (1, 2, 2, 2)$ is of codimension 7.
• Over the ideal I_{22} we obtain two possible ideals $I_{32} = (y_0, z_0, w_0, y_1, z_1, w_1, y_2, z_2, x_0)$ and $I_{33} = (y_0, z_0, w_0, y_1, z_1, w_1, y_2, z_2, y_3, z_3)$. The corresponding vector of I_{32}, $v_2^3 = (1, 3, 3, 2)$ is of codimension 9. The corresponding vector of I_{33}, $v_3^3 = (0, 4, 4, 2)$ is of codimension 10. $J_3(E_{60})$ is given by

$$J_3(E_{60}) = Spec\frac{\mathbb{C}[x_0, y_0, \ldots, z_3, w_3]}{< F_1^0, F_2^0, \ldots, F_2^3, F_3^3 >}$$

Remark 6.2.1. *In the same way, the m^{th} jet scheme of E_{60} is given by*

$$J_m(E_{60}) = Spec\frac{\mathbb{C}[x_0, y_0, \ldots, z_m, w_m]}{< F_1^0, F_2^0, \ldots, F_2^m, F_3^m >}$$

for all m.

For m=4, (7) gives;

$$F_1^4(x_0,y_0,\ldots,z_4,w_4)=z_0z_4+z_1z_3+\ldots+y_0y_1y_2+y_1^2y_2=0$$

$$F_2^4(x_0,y_0,\ldots,z_4,w_4)=z_0w_4+z_1w_3+\ldots+x_0x_1y_3+x_0^2y_4=0$$
$$F_3^4(x_0,y_0,\ldots,z_4,w_4)=w_0w_4+w_1w_3+\ldots+x_0x_1z_3+x_0^2z_4=0$$

• Over the ideal I_{31}, we obtain two possible ideals $I_{41}=(y_0,z_0,w_0,y_1,z_1,w_1,x_0,x_1,z_2,w_2)$ and $I_{42}=(y_0,z_0,w_0,y_1,z_1,w_1,x_0,y_2,z_2,w_2)$. The corresponding vector of I_{41}, $v_1^4=(2,2,3,3)$ is of codimension 10 and the corresponding vector of I_{42}, $v_2^4=(1,3,3,3)$ is of codimension 10.
• Over the ideal I_{32}, we obtain the ideal I_{42}.
• Over the ideal I_{33}, we obtain two possible ideals $I_{43}=(y_0,z_0,w_0,y_1,z_1,w_1,y_2,z_2,y_3,z_3,x_0)$ and $I_{44}=(y_0,z_0,w_0,y_1,z_1,w_1,y_2,z_2,y_3,z_3,y_4,z_4,w_2)$. The corresponding vector of I_{43}, $v_3^4=(1,4,4,3)$ is of codimension 12 and the corresponding vector of I_{44}, $v_4^4=(0,5,5,3)$ is of codimension 13.

For m=5, (7) gives;

$$F_1^5(x_0,y_0,\ldots,z_5,w_5)=z_0z_5+z_1z_4+\ldots+y_0y_2y_3+y_1y_2^2=0$$
$$F_2^5(x_0,y_0,\ldots,z_5,w_5)=z_0w_5+z_1w_4+\ldots+x_0x_1y_4+x_0^2y_5=0$$
$$F_3^5(x_0,y_0,\ldots,z_5,w_5)=w_0w_5+w_1w_4+\ldots+x_0x_1z_4+x_0^2z_5=0$$

• Over the ideal I_{41}, we obtain two possible ideals $I_{51}=(y_0,z_0,w_0,y_1,z_1,w_1,x_0,x_1,z_2,w_2,w_3)$ and $I_{52}=(y_0,z_0,w_0,y_1,z_1,w_1,x_0,x_1,z_2,w_2,y_2,w_3)$. The corresponding vectors $v_1^2=(2,2,3,4)$ and $v_2^5=(2,3,3,3)$ are of codimension 11.
• Over the ideal I_{42}, we obtain two possible ideals I_{52} and $I_{53}=(y_0,z_0,w_0,y_1,z_1,w_1,x_0\ y_2,z_2,w_2,y_3,w_3)$. The corresponding vector of I_{53}, $v_3^5=(1,4,3,4)$ is of codimension 12.
• Over the ideal I_{43}, we obtain the ideal I_{53}.
For m=6, (7) gives;

$$F_1^6(x_0,y_0,\ldots,z_6,w_6)=z_0z_6+z_1z_5+\ldots+y_0y_3^2+y_2^3=0$$
$$F_2^6(x_0,y_0,\ldots,z_6,w_6)=z_0w_6+z_1w_5+\ldots+x_0x_1y_5+x_0^2y_6=0$$
$$F_3^6(x_0,y_0,\ldots,z_6,w_6)=w_0w_6+w_1w_5+\ldots+x_0x_1z_5+x_0^2z_6=0$$

• Over the ideal I_{51} and I_{52}, we obtain the ideal $I_{61}=(y_0,z_0,w_0,y_1,z_1,w_1,x_0,x_1,z_2,w_2,w_3,y_2,z_3)$. The corresponding vector $v_1^6=(2,3,4,4)$ is of codimension 13.
• Over the ideal I_{53}, we obtain two possible ideals $I_{62}=(y_0,z_0,w_0,y_1,z_1,w_1,x_0,y_2,z_2,w_2,y_3,w_3,x_1,z_3)$ and $I_{63}=(y_0,z_0,w_0,y_1,z_1,w_1,x_0,y_2,z_2,w_2,y_3,w_3,y_4,z_3,z_4)$. The corresponding vectors are $v_2^6=(2,4,4,4)$ with codimension 14 and $v_3^6=(1,5,5,4)$ with codimension 15.

For m=7, (7) gives;

$$F_1^7(x_0, y_0, \ldots, z_7, w_7) = z_0z_7 + z_1z_6 + \ldots + y_1y_3^2 + y_2^2y_3 = 0$$
$$F_2^7(x_0, y_0, \ldots, z_7, w_7) = z_0w_7 + z_1w_6 + \ldots + x_0x_1y_6 + x_0^2y_7 = 0$$
$$F_3^7(x_0, y_0, \ldots, z_7, w_7) = w_0w_7 + w_1w_6 + \ldots + x_0x_1z_6 + x_0^2z_7 = 0$$

• Over the ideal I_{61}, we obtain two possible ideals $I_{71} = (y_0, z_0, w_0, y_1, z_1, w_1, x_0, x_1, z_2, w_2, w_3, y_2, z_3, x_2, w_4)$ and $I_{72} = (y_0, z_0, w_0, y_1, z_1, w_1, x_0, x_1, z_2, w_2, w_3, y_2, z_3, y_3)$. The corresponding vector of I_{71}, $v_1^7 = (3, 3, 4, 5)$ is of codimension 15 and the corresponding vector of I_{72}, $v_2^7 = (2, 4, 4, 4)$ is of codimension 14.
• Over the ideal I_{62}, we obtain the ideal I_{72}.

In the same way;
For m=8, (7) gives;

$$F_1^8(x_0, y_0, \ldots, z_8, w_8) = z_0z_8 + z_1z_7 + \ldots + y_2y_3^2 + y_2y_3^2 = 0$$
$$F_2^8(x_0, y_0, \ldots, z_8, w_8) = z_0w_8 + z_1w_7 + \ldots + x_0x_1y_7 + x_0^2y_8 = 0$$
$$F_3^8(x_0, y_0, \ldots, z_8, w_8) = w_0w_8 + w_1w_7 + \ldots + x_0x_1z_7 + x_0^2z_8 = 0$$

•We obtain three possible ideals I_{81}, I_{82} and I_{83}. The corresponding vectors $v_1^8 = (3, 3, 5, 6)$, $v_2^8 = (3, 4, 5, 5)$ and $v_3^8 = (2, 5, 5, 5)$ are of codimension 17.

For m=9, (7) gives;

$$F_1^9(x_0, y_0, \ldots, z_9, w_9) = z_0z_9 + z_1z_8 + \ldots + y_2y_3y_4 + y_3^3 = 0$$
$$F_2^9(x_0, y_0, \ldots, z_9, w_9) = z_0w_9 + z_1w_8 + \ldots + x_0x_1y_8 + x_0^2y_9 = 0$$
$$F_3^9(x_0, y_0, \ldots, z_9, w_9) = w_0w_9 + w_1w_8 + \ldots + x_0x_1z_8 + x_0^2z_9 = 0$$

• We obtain two possible ideals I_{91} and I_{92}. The corresponding vectors $v_1^9 = (3, 4, 5, 6)$ and $v_2^9 = (3, 5, 5, 5)$ are of codimension 18.
For m=10, (7) gives;

$$F_1^{10}(x_0, y_0, \ldots, z_{10}, w_{10}) = z_0z_{10} + z_1z_9 + \ldots + y_3^2y_4 + y_3^2y_4 = 0$$
$$F_2^{10}(x_0, y_0, \ldots, z_{10}, w_{10}) = z_0w_{10} + z_1w_9 + \ldots + x_0x_1y_9 + x_0^2y_{10} = 0$$
$$F_3^{10}(x_0, y_0, \ldots, z_{10}, w_{10}) = w_0w_{10} + w_1w_9 + \ldots + x_0x_1z_9 + x_0^2z_{10} = 0$$

• We obtain two possible ideals I_{101} and I_{102}. The corresponding vector of I_{101}, $v_1^{10} = (3, 5, 6, 6)$ is of codimension 20 and the corresponding vector of I_{102}, $v_2^{10} = (4, 4, 6, 7)$ is of codimension 21.

For m=11, (7) gives;

$$F_1^{11}(x_0, y_0, \ldots, z_{11}, w_{11}) = z_0z_{12} + z_1z_{10} + \ldots + y_3^2y_5 + y_3y_4^2 = 0$$
$$F_2^{11}(x_0, y_0, \ldots, z_{11}, w_{11}) = z_0w_{11} + z_1w_{10} + \ldots + x_0x_1y_{10} + x_0^2y_{11} = 0$$

$$F_3^{11}(x_0, y_0, \ldots, z_{11}, w_{11}) = w_0w_{11} + w_1w_{10} + \ldots + x_0x_1z_{10} + x_0^2z_{11} = 0$$

• We obtain two possible ideals I_{111} and I_{112}. The corresponding vectors $v_1^{10} = (4, 5, 6, 7)$ and $v_2^{10} = (4, 4, 6, 8)$ are of codiemnsion 22.

For m=12, (7) gives;

$$F_1^{12}(x_0, y_0, \ldots, z_{12}, w_{12}) = z_0z_{12} + z_1z_{11} + \ldots + y_3y_4y_5 + y_4^3 = 0$$
$$F_2^{12}(x_0, y_0, \ldots, z_{12}, w_{12}) = z_0w_{12} + z_1w_{11} + \ldots + x_0x_1y_{11} + x_0^2y_{12} = 0$$
$$F_3^{12}(x_0, y_0, \ldots, z_{12}, w_{12}) = w_0w_{12} + w_1w_{11} + \ldots + x_0x_1z_{11} + x_0^2z_{12} = 0$$

• We obtain the ideal I_{121}. The corresponding vector $v_1^{12} = (4, 5, 7, 8)$ is of codimension 24.

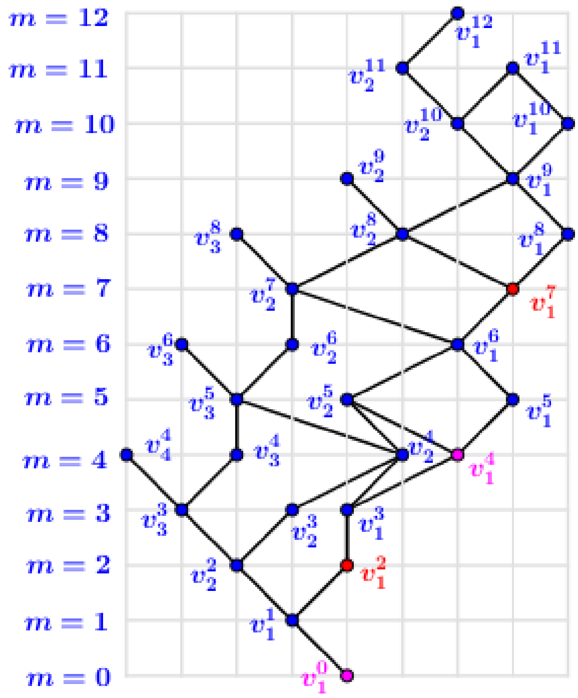

Figure 6.1: Jet graph of isolated E_{60}

Remark 6.2.2. *The weight vectors* $(0, 1, 1)$*,* $(1, 2, 2)$*,* $(2, 2, 3)$ *and* $(3, 3, 4)$ *appeared in* 1^{st}*,* 2^{th}*,* 4^{th} *and* 7^{th} *jet schemes respectively as a projection of* $v_1^0 = (0, 1, 1, 1)$ $v_1^2 = (1, 2, 2, 2)$*,* $v_1^4 = (2, 2, 3, 3)$ *and* $v_1^7 = (3, 3, 4, 5)$*.*

6.3 Jet Scheme of a Surface of type E_{70}

Let us consider the ideal $I =< f_1, f_2, f_3 >$ in $\mathbb{C}[x, y, z, w]$ with E_{70} singularity. The affine variety $V(I)$ defines a surface X of type E_{70} in $\mathbb{C}^4$ and $(0, 0, 0, 0)$ is the only singular point of X. According to the map φ_m above;

For m=0, (7) gives;

$$F_1^0(x_0, y_0, z_0, w_0) = z_0^2 - y_0w_0 + x_0^2y_0 = z_0^2 - y_0(w_0 - x_0^2) = 0$$
$$F_2^0(x_0, y_0, z_0, w_0) = z_0w_0 - y_0^3 = 0$$
$$F_3^0(x_0, y_0, z_0, w_0) = w_0^2 - x_0^2w_0 - y_0^2w_0 = w_0(w_0 - x_0^2 - y_0^2) = 0$$

• This says that $y_0 = z_0 = w_0$ is over the generic point. Hence the ideal $I_0 =< y_0, z_0, w_0 >$. The corresponding vector $v_1^0 = (0, 1, 1, 1)$ is of codimesnion 3. $J_0(E_{70})$ is given by

$$J_0(E_{70}) = Spec\frac{\mathbb{C}[x_0, y_0, z_0, w_0]}{< F_1^0, F_2^0, F_3^0 >}$$

For m=1, (7) gives;

$$F_1^1(x_0, y_0, \ldots, z_1, w_1) = z_0z_1 - y_0w_1 - y_1w_0 + x_0x_1y_0 + x_0^2y_1 = 0$$
$$F_2^1(x_0, y_0, \ldots, z_1, w_1) = z_0w_1 + z_1w_0 - y_0^2y_1 = 0$$
$$F_3^1(x_0, y_0, \ldots, z_1, w_1) = w_0w_1 - x_0^2w_1 - x_0x_1w_0 - y_0^2w_1 - y_0y_1w_0 = 0$$

• Over the ideal I_0, we have two possible ideals but the only ideal over the generic point which is $I_1 = (y_0, z_0, w_0, y_1, w_1)$. The corresponding vector, $v_1^1 = (0, 2, 1, 2)$ is of codimension 5. $J_1(E_{70})$ is given by

$$J_1(E_{70}) = Spec\frac{\mathbb{C}[x_0, y_0, \ldots z_1, w_1]}{< F_1^0, F_2^0, \ldots F_2^1, F_3^1 >}$$

For m=2, (7) gives;

$$F_1^2(x_0, y_0, \ldots, z_2, w_2) = z_0z_2 + z_1^2 + \ldots + x_0x_1y_1 + x_0^2y_2 = 0$$
$$F_2^2(x_0, y_0, \ldots, z_2, w_2) = z_0w_2 + z_1w_1 + \ldots + y_0y_1^2 + y_0^2y_2 = 0$$
$$F_3^2(x_0, y_0, \ldots, z_2, w_2) = w_0w_2 + w_1^2 + \ldots + y_0y_1w_1 + y_0^2w_2 = 0$$

• Over the ideal I_1, we have two possible ideals $I_{21} = (y_0, z_0, w_0, y_1, w_1, z_1, x_0)$ and $I_{22} = (y_0, z_0, w_0, y_1, w_1, z_1, y_2, w_2)$. The corresponding vector of I_{21}, $v_1^2 = (1, 2, 2, 2)$ is of codimension 7 and the corresponding vector of I_{22}, $v_2^2 = (0, 3, 2, 3)$ is of codimension 8. $J_2(E_{70})$ is given by

$$J_2(E_{70}) = Spec\frac{\mathbb{C}[x_0, y_0, \ldots z_2, w_2]}{< F_1^0, F_2^0, \ldots F_2^2, F_3^2 >}$$

For m=3, (7) gives;

$$F_1^3(x_0,y_0,\dots,z_3,w_3)=z_0z_3+z_1z_2+\dots+x_0x_1y_2+x_0^2y_3=0$$
$$F_2^3(x_0,y_0,\dots,z_3,w_3)=z_0w_3+z_1w_2+\dots+y_0y_1y_2+y_0^2y_3=0$$
$$F_3^3(x_0,y_0,\dots,z_3,w_3)=w_0w_3+w_1w_2+\dots+y_0y_1w_2+y_0^2w_3=0$$

• Over the ideal I_{21} we obtain the ideal $I_{31}=(y_0,z_0,w_0,y_1,w_1,z_1,x_0)$. The corresponding vector $v_1^3=(1,2,2,2)$ is of codimension 7.
• Over the ideal I_{22} we obtain two possible ideals $I_{32}=(y_0,z_0,w_0,y_1,w_1,z_1,y_2,w_2,x_0)$ and $I_{33}=(y_0,z_0,w_0,y_1,w_1,z_1,y_2,w_2,y_3,w_3)$. The corresponding vector of I_{32}, $v_2^3=(1,3,2,3)$ is of codimension 9 and the corresponding vector of I_{33}, $v_3^3=(0,4,2,4)$ is of codimension 10. $J_3(E_{70})$ is given

$$J_3(E_{70})=Spec\frac{\mathbb{C}[x_0,y_0,\dots z_3,w_3]}{<F_1^0,F_2^0,\dots F_2^3,F_3^3>}$$

Remark 6.3.1. *In the same way, the m^{th} jet scheme of E_{70} is given by*

$$J_m(E_{70})=Spec\frac{\mathbb{C}[x_0,y_0,\dots z_m,w_m]}{<F_1^0,F_2^0,\dots F_2^m,F_3^m>}$$

for all m.

For m=4, (7) gives;

$$F_1^4(x_0,y_0,\dots,z_4,w_4)=z_0z_4+z_1z_3+\dots+x_0x_1y_3+x_0^2y_4=0$$
$$F_2^4(x_0,y_0,\dots,z_4,w_4)=z_0w_4+z_1w_3+\dots+y_0y_2^2+y_0^2y_4=0$$
$$F_3^4(x_0,y_0,\dots,z_4,w_4)=w_0w_4+w_1w_3+\dots+y_0y_1w_3+y_0^2w_4=0$$

• Over the ideal I_{31} we obtain two possible ideals $I_{41}=(y_0,z_0,w_0,y_1,w_1,z_1,x_0,x_1,z_2,w_2)$ and $I_{42}=(y_0,z_0,w_0,y_1,w_1,z_1,x_0,y_2,z_2,w_2)$. The corresponding vectors $v_1^4=(2,2,3,3)$ and $v_2^4=(1,3,3,3)$ are of codmension 10.
• Over the ideal I_{32} we obtain the ideal I_{42}.
•Over the ideal I_{33} we obtain two possible ideals $I_{43}=(y_0,z_0,w_0,y_1,w_1,z_1,y_2,w_2,y_3,w_3,x_0,y_3,z_2)$ and $I_{44}=(y_0,z_0,w_0,y_1,w_1,z_1,y_2,w_2,y_3,w_3,y_4,z_2,w_4)$. The corresponding vector of I_{43}, $v_3^4=(1,4,3,4)$ is of codimension 12 and the corresponding vector of I_{44}, $v_4^4=(0,5,3,5)$ is of codimension 13.

In the same way;
For m=5, (7) gives;

$$F_1^5(x_0,y_0,\dots,z_5,w_5)=z_0z_5+z_1z_4+\dots+x_0x_1y_4+x_0^2y_5=0$$
$$F_2^5(x_0,y_0,\dots,z_5,w_5)=z_0w_5+z_1w_4+\dots+y_0y_2y_3+y_0^2y_5=0$$
$$F_3^5(x_0,y_0,\dots,z_5,w_5)=w_0w_5+w_1w_4+\dots+y_0y_1w_4+y_0^2w_5=0$$

• Over the ideal I_{41}, we obtain two possible ideals I_{51} and I_{52}. The corresponding vectors $v_1^5 = (2,2,3,4)$ and $v_2^5 = (2,3,3,3)$ are of codimension 1.
• Over the ideal I_{42}, we obtain two possible ideals I_{52} and I_{53}. The corresponding vector of I_{53} is $v_3^5 = (1,4,3,4)$ is of codimension 12.
• Over the ideal I_{43}, we obtain the ideal I_{53}.
• Over the ideal I_{44} we obtain two possible ideals I_{54} and I_{55}. The corresponding vector of I_{54}, $v_4^5 = (1,5,3,5)$ is of codimension 14 and the corresponding vector of I_{55}, $v_5^5 = (0,6,3,6)$ is of codimension 15.

For m=6, (7) gives

$$F_1^6(x_0, y_0, \ldots, z_6, w_6) = z_0z_6 + z_1z_5 + \ldots + x_0x_1y_5 + x_0^2y_6 = 0$$
$$F_2^6(x_0, y_0, \ldots, z_6, w_6) = z_0w_6 + z_1w_5 + \ldots + y_0y_3^2 + y_0^2y_6 = 0$$
$$F_3^6(x_0, y_0, \ldots, z_6, w_6) = w_0w_6 + w_1w_5 + \ldots + y_0y_1w_5 + y_0^2w_6 = 0$$

• Over the ideals I_{51} and I_{52}, we obtain ideal I_{61}. The corresponding vector $v_1^6 = (2,3,4,4)$ is of codimension 13.
• Over the ideal I_{53}, we obtain two possible ideals I_{62} and I_{63}. The corresponding vector of I_{62}, $v_2^6 = (2,4,4,4)$ is of codimension 14 and the corresponding vector of I_{63}, $v_3^6 = (1,5,4,5)$ is of codimension 15.
• Over the ideal I_{55}, we obtain two possible ideals I_{64} and I_{65}. The corresponding vectors of I_{64}, $v_4^6 = (1,6,4,6)$ is of codimension 17 and the corresponding vectors of I_{65}, $v_5^6 = (0,7,4,7)$ is of codimension 18.

For m=7, (7) gives;

$$F_1^7(x_0, y_0, \ldots, z_7, w_7) = z_0z_7 + z_1z_6 + \ldots + x_0x_1y_6 + x_0^2y_7 = 0$$
$$F_2^7(x_0, y_0, \ldots, z_7, w_7) = z_0w_7 + z_1w_6 + \ldots + y_0y_3y_4 + y_0^2y_7 = 0$$
$$F_3^7(x_0, y_0, \ldots, z_7, w_7) = w_0w_7 + w_1w_6 + \ldots + y_0y_1w_6 + y_0^2w_7 = 0$$

• Over the ideal I_{61}, we obtain two possible ideals I_{71} and I_{72}. The corresponding vector of I_{71}, $v_1^7 = (3,3,4,5)$ is of codimension 15 and the corresponding vector of I_{72}, $v_2^7 = (2,4,4,4)$ is of codimension 14.
• Over the ideal I_{62} we obtain the ideal I_{72}.
• Over the ideal I_{63}, we obtain two possible ideals I_{73} and I_{74}. The corresponding vector of I_{73}, $v_3^7 = (2,5,4,5)$ is of codimension 16 and the corresponding vector of I_{74}, $v_4^7 = (1,6,4,6)$ is of codimension 17.

For m=8, (7) gives;

$$F_1^8(x_0, y_0, \ldots, z_8, w_8) = z_0z_8 + z_1z_7 + \ldots + x_0x_1y_7 + x_0^2y_8 = 0$$
$$F_2^8(x_0, y_0, \ldots, z_8, w_8) = z_0w_8 + z_1w_7 + \ldots + y_0y_4^4 + y_0^2y_8 = 0$$
$$F_3^8(x_0, y_0, \ldots, z_8, w_8) = w_0w_8 + w_1w_7 + \ldots + y_0y_1w_7 + y_0^2w_8 = 0$$

• Over the ideal I_{71}, we obtain the ideals I_{81} and I_{82}. The corresponding vectors $v_1^8 = (3,3,5,6)$ and $v_2^8 = (3,4,5,5)$ are of codimension 17.

• Over the ideal I_{72}, we obtain two possible ideals I_{82} and I_{83}. The corresponding vector of I_{83}, $v_3^8 = (2,5,5,5)$ is of codimension 17.
• Over the ideal I_{73}, we obtain the ideal I_{83}.
• Over the ideal I_{74}, we obtain the ideal I_{84} and I_{85}. The corresponding vector of I_{84}, $v_4^8 = (2,6,5,6)$ is of codimension 19 and the corresponding vector of I_{85}, $v_5^8 = (1,7,5,7)$ is of codimension 20.

For m=9, (7) gives;

$$F_1^9(x_0, y_0, \ldots, z_9, w_9) = z_0z_9 + z_1z_8 + \ldots + x_0x_1y_8 + x_0^2y_9 = 0$$
$$F_2^9(x_0, y_0, \ldots, z_9, w_9) = z_0w_9 + z_1w_8 + \ldots + y_0y_4y_5 + y_0^2y_9 = 0$$
$$F_3^9(x_0, y_0, \ldots, z_9, w_9) = w_0w_9 + w_1w_8 + \ldots + y_0y_1w_8 + y_0^2w_9 = 0$$

• In the same, we obtain three possible ideals I_{91}, I_{92} and I_{93}. The corresponding vectors $v_1^9 = (3,4,5,6)$ and $v_2^9 = (3,5,5,5)$ are of codimension 18 and the corresponding vector $v_3^9 = (2,6,5,6)$ is of codimension 19.

For m=10, (7) gives;

$$F_1^{10}(x_0, y_0, \ldots, z_{10}, w_{10}) = z_0z_{10} + z_1z_9 + \ldots + x_0x_1y_9 + x_0^2y_{10} = 0$$
$$F_2^{10}(x_0, y_0, \ldots, z_{10}, w_{10}) = z_0w_{10} + z_1w_9 + \ldots + y_0y_5^2 + y_0^2y_{10} = 0$$
$$F_3^{10}(x_0, y_0, \ldots, z_{10}, w_{10}) = w_0w_{10} + w_1w_9 + \ldots + y_0y_1w_9 + y_0^2w_{10} = 0$$

•We obtain four possible ideals I_{101}, I_{102}, I_{103} and I_{104}. The corresponding vectors $v_1^{10} = (4,4,6,7)$ and $v_3^{10} = (3,6,6,6)$ are of codimension 21. The corresponding vector $v_2^{10} = (3,5,6,6)$ is of codimension 20 and the corresponding vector $v_4^{10} = (2,7,6,7)$ is of codimension 22.

For m=11, (7) gives;

$$F_1^{11}(x_0, y_0, \ldots, z_{11}, w_{11}) = z_0z_{11} + z_1z_{10} + \ldots + x_0x_1y_{10} + x_0^2y_{11} = 0$$
$$F_2^{11}(x_0, y_0, \ldots, z_{11}, w_{11}) = z_0w_{11} + z_1w_{10} + \ldots + y_0y_5y_6 + y_0^2y_{11} = 0$$
$$F_3^{11}(x_0, y_0, \ldots, z_{11}, w_{11}) = w_0w_{11} + w_1w_{10} + \ldots + y_0y_1w_{10} + y_0^2w_{11} = 0$$

• We obtain three possible ideals I_{111}, I_{112} and I_{113}. The corresponding vector $v_1^{11} = (4,5,6,8)$ is of codimension 23. The corresponding vector $v_2^{11} = (4,5,6,7)$ is of codimension 22 and the corresponding vector $v_3^{11} = (3,6,6,6)$ is of codimension 21.

For m=12, (7) gives;

$$F_1^{12}(x_0, y_0, \ldots, z_{12}, w_{12}) = z_0z_{12} + z_1z_{11} + \ldots + x_0x_1y_{11} + x_0^2y_{12} = 0$$
$$F_2^{12}(x_0, y_0, \ldots, z_{12}, w_{12}) = z_0w_{12} + z_1w_{11} + \ldots + y_0y_6^2 + y_0^2y_{12} = 0$$
$$F_3^{12}(x_0, y_0, \ldots, z_{12}, w_{12}) = w_0w_{12} + w_1w_{11} + \ldots + y_0y_1w_{11} + y_0^2w_{12} = 0$$

• We obtain three possible ideals I_{121}, I_{122} and I_{123}. The corresponding vectors $v_1^{12} = (4,5,7,8)$, $v_2^{12} = (4,6,7,7)$ and $v_3^{12} = (3,7,7,7)$are of codimension 24.

For m=13, (7) gives;

$$F_1^{13}(x_0, y_0, \ldots, z_{13}, w_{13}) = z_0 z_{13} + z_1 z_{12} + \ldots + x_0 x_1 y_{12} + x_0^2 y_{13} = 0$$
$$F_2^{13}(x_0, y_0, \ldots, z_{13}, w_{13}) = z_0 w_{13} + z_1 w_{12} + \ldots + y_0 y_6 y_7 + y_0^2 y_{13} = 0$$
$$F_3^{13}(x_0, y_0, \ldots, z_{13}, w_{13}) = w_0 w_{13} + w_1 w_{12} + \ldots + y_0 y_1 w_{12} + y_0^2 w_{13} = 0$$

• We obtain three possible ideals I_{131}, I_{132} and I_{133}. The corresponding vector $v_1^{13} = (5,5,7,9)$ is of codimension 26. The corresponding vectors $v_2^{13} = (4,6,7,8)$ and $v_3^{13} = (4,7,7,7)$ are of codimension 25.

For m=14, (7) gives;

$$F_1^{14}(x_0, y_0, \ldots, z_{14}, w_{14}) = z_0 z_{14} + z_1 z_{13} + \ldots + x_0 x_1 y_{13} + x_0^2 y_{14} = 0$$
$$F_2^{14}(x_0, y_0, \ldots, z_{14}, w_{14}) = z_0 w_{14} + z_1 w_{13} + \ldots + y_0 y_7^2 + y_0^2 y_{13} = 0$$
$$F_3^{14}(x_0, y_0, \ldots, z_{14}, w_{14}) = w_0 w_{14} + w_1 w_{13} + \ldots + y_0 y_1 w_{13} + y_0^2 w_{14} = 0$$

• We obtain three possible ideals I_{141}, I_{142} and I_{143}. The corresponding vectors $v_1^{14} = (4,7,8,8)$ and $v_2^{14} = (5,6,7,9)$ are of codimension 27. The corresponding vector $v_3^{14} = (5,5,8,10)$ is of codimension 28.

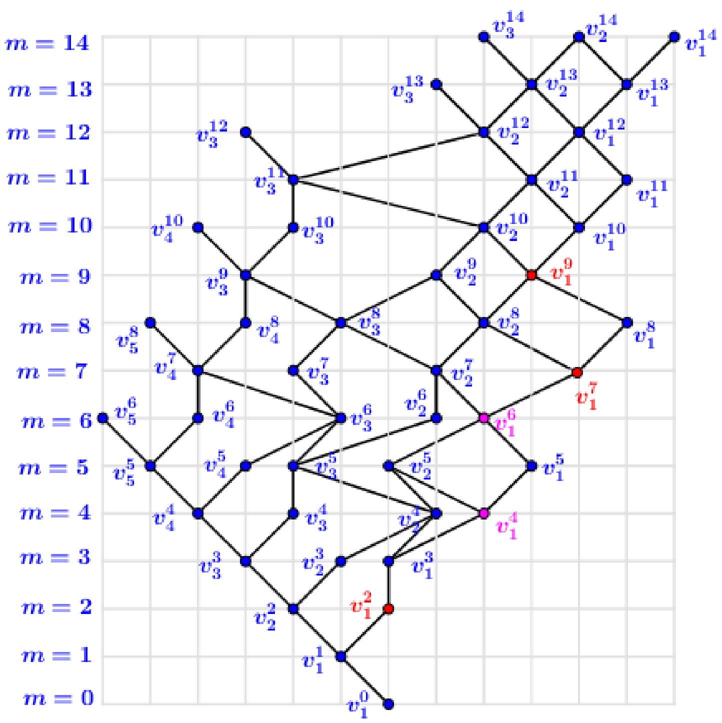

Figure 6.2: Jet graph of isolated E_{70}

Remark 6.3.2. *The weigth vectors* $(1,2,2)$*,* $(2,2,3)$*,* $(2,3,4)$*,* $(3,3,4)$ *and* $(3,4,5)$ *appeared in* 2^{th}*,* 4^{th}*,* 6^{th} 7^{th} *and* 9^{th} *jet schemes respectively as a projection of* $v_1^2 = (1,2,2,2)$*,* $v_1^4 = (2,2,3,3)$*,* $v_1^6 = (2,3,4,4)$ $v_1^7 = (3,3,4,5)$ *and* $v_1^9 = (3,4,5,6)$*.*

6.4 Jet Scheme of a Surface of type E_{07}

Let us consider the ideal $I =< f_1, f_2, f_3 >$ in $\mathbb{C}[x, y, z, w]$ with E_{07} singularity. The affine variety $V(I)$ defines a surface X in $\mathbb{C}^4$ and $(0,0,0,0)$ is the only singular point of X.

Remark 6.4.1. *The* m^{th} *jet scheme of* E_{07} *is given by*

$$J_m(E_{07}) = Spec\frac{\mathbb{C}[x_0, y_0, \dots z_m, w_m]}{< F_1^0, F_2^0, \dots F_2^m, F_3^m >}$$

for all m*.*

According to the map φ_m above;

For m=0, (7) gives;

$$\begin{aligned} F_1^0(x_0, y_0, z_0, w_0) &= z_0^2 - y_0 w_0 = 0 \\ F_2^0(x_0, y_0, z_0, w_0) &= z_0 w_0 - x_0^2 y_0 - y_0^4 = 0 \\ F_3^0(x_0, y_0, z_0, w_0) &= w_0^2 - x_0^2 z_0 - y_0^3 z_0 = 0 \end{aligned}$$

• This says that $y_0 = z_0 = w_0$ is over the generic point. Hence the ideal $I_0 =< y_0, z_0, w_0 >$. The corresponding vector $v_1^0 = (0,1,1,1)$ is of codimension 3.

For m=1, (7) gives;

$$\begin{aligned} F_1^1(x_0, y_0, \dots, z_1, w_1) &= z_0 z_1 - y_0 w_1 - y_1 w_0 = 0 \\ F_2^1(x_0, y_0, \dots, z_1, w_1) &= z_0 w_1 + z_1 w_0 - x_0^2 y_1 - x_0 x_1 y_0 - y_0^3 y_1 = 0 \\ F_3^1(x_0, y_0, \dots, z_1, w_1) &= w_0 w_1 - x_0^2 z_1 - x_0 x_1 z_0 - y_0^3 z_1 - y_0^2 y_1 z_0 = 0 \end{aligned}$$

• Over the ideal I_0, we obtain two possible ideals that one of them is over singularity. The other one $I_1 =< y_0, z_0, w_0, y_1, z_1 >$ is over the generic point. The corresponding vector $v_1^1 = (0,2,2,1)$ is of codimension 5.

For m=2, (7) gives;

$$\begin{aligned} F_1^2(x_0, y_0, \dots, z_2, w_2) &= z_0 z_2 + z_1^2 - y_0 w_2 - y_1 w_1 - y_2 w_0 = 0 \\ F_2^2(x_0, y_0, \dots, z_2, w_2) &= z_0 w_2 + z_1 w_1 + z_2 w_0 + \dots + y_0^2 y_1^2 + y_0^3 y_2 = 0 \\ F_3^2(x_0, y_0, \dots, z_2, w_2) &= w_0 w_2 + w_1^2 - x_0^2 z_2 + \dots + y_0^2 y_1 z_1 + y_0^3 z_2 = 0 \end{aligned}$$

• Over the ideal I_1, we obtain two possible ideals $I_{21} = (y_0, z_0, w_0, y_1, z_1, w_1, x_0)$ and $I_{22} = (y_0, z_0, w_0, y_1, z_1, w_1, z_1, y_2, z_2)$. The corresponding vector of I_{21}, $v_1^2 = (1,2,2,2)$ is of codimension 7. The corresponding vector of I_{22}, $v_2^2 = (0,3,3,2)$

is of codimension 8.

For m=3, (7) gives;

$$F_1^3(x_0, y_0, \ldots, z_3, w_3) = z_0z_3 + z_1z_2 + \ldots + y_2w_1 + y_1w_2 = 0$$
$$F_2^3(x_0, y_0, \ldots, z_3, w_3) = z_0w_3 + z_1w_2 + \ldots + y_0^2y_1y_2 + y_0^3y_3 = 0$$
$$F_3^3(x_0, y_0, \ldots, z_3, w_3) = w_0w_3 + w_1w_2 + \ldots + y_0^2y_1z_2 + y_0^3z_3 = 0$$

• Over the ideal I_{21}, we obtain the ideal $I_{31} = (y_0, z_0, w_0, y_1, z_1, w_1, x_0)$. The corresponding vector $v_1^3 = (1, 2, 2, 2)$ is of codimension 7.
• Over the ideal I_{22}, we obtain two possible ideal $I_{32} = (y_0, z_0, w_0, y_1, z_1, w_1, z_1, y_2, z_2, x_0)$ and $I_{33} = (y_0, z_0, w_0, y_1, z_1, w_1, y_2, z_2, y_3, z_3, w_2)$. The corresponding vector of I_{32}, $v_2^3 = (1, 3, 3, 2)$ is of codimension 9. The corresponding vector of I_{33}, $v_3^3 = (0, 4, 4, 3)$ is of codimension 11.
For m=4, (7) gives;

$$F_1^4(x_0, y_0, \ldots, z_4, w_4) = z_0z_4 + z_1z_3 + \ldots + y_1w_3 + y_2w_2 = 0$$
$$F_2^4(x_0, y_0, \ldots, z_4, w_4) = z_0w_4 + z_1w_3 + \ldots + y_0^2y_1y_3 + y_0^3y_4 = 0$$
$$F_3^4(x_0, y_0, \ldots, z_4, w_4) = w_0w_4 + w_1w_3 + \ldots + y_0^2y_1z_3 + y_0^3z_4 = 0$$

• Over the ideal I_{31} we obtain two possible ideals $I_{41} = (y_0, z_0, w_0, y_1, z_1, w_1, x_0, x_1, z_2, z_3)$ and $I_{42} = (y_0, z_0, w_0, y_1, z_1, w_1, x_0, y_2, z_2, w_2)$. The corresponding vectors $v_1^4 = (2, 2, 3, 3)$ and $v_2^4 = (1, 3, 3, 3)$ are of codimension 10.
• Over the ideal I_{32} we obtain the ideal I_{42}.
• Over the ideal I_{33} we obtain two possible ideals $I_{43} = (y_0, z_0, w_0, y_1, z_1, w_1, y_2, z_2, y_3, z_3, w_2, x_0)$ and $I_{44} = (y_0, z_0, w_0, y_1, z_1, w_1, y_2, z_2, y_3, z_3, w_2, y_4, z_4)$. The corresponding vector of I_{43}, $v_3^4 = (1, 4, 4, 3)$ is of codimension 12 and the corresponding vector of I_{44}, $v_4^4 = (0, 5, 5, 3)$ is of codimension 13.

In the same way;
For m=5, (7) gives;

$$F_1^5(x_0, y_0, \ldots, z_5, w_5) = z_0z_5 + z_1z_4 + \ldots + y_2w_3 + y_3w_2 = 0$$
$$F_2^5(x_0, y_0, \ldots, z_5, w_5) = z_0w_5 + z_1w_4 + \ldots + y_0^2y_1y_4 + y_0^2y_5 = 0$$
$$F_3^5(x_0, y_0, \ldots, z_5, w_5) = w_0w_5 + w_1w_4 + \ldots + y_0^2y_1z_4 + y_0^3z_5 = 0$$

• Over the ideal I_{41} we obtain two possible ideals I_{51} and I_{52}. The corresponding vectors $v_1^5 = (2, 2, 3, 4)$ and $v_2^5 = (2, 3, 3, 3)$ are of codimension 11.
• Over the ideal I_{42} we obtain two possible ideals I_{52} and I_{53}. The corresponding vector of I_{53}, $v_3^5 = (1, 4, 4, 3)$ is of codimension 12.
• Over the ideal I_{43} we obtain the ideal I_{53}.
•Over the ideal I_{44} we obtain two possible ideals I_{54} and I_{55}. The corresponding vector of I_{54}, $v_4^5 = (1, 5, 5, 3)$ is of codimension 14 and the corresponding vector of I_{55}, $v_5^5 = (0, 6, 6, 3)$ is of codimension 15.

For m=6, (7) gives;

$$\begin{aligned}
F_1^6(x_0,y_0,\ldots,z_6,w_6) &= z_0z_6+z_1z_5+\ldots+y_2w_4+y_3w_3=0\\
F_2^6(x_0,y_0,\ldots,z_6,w_6) &= z_0w_6+z_1w_5+\ldots+y_0^2y_1y_5+y_0^2y_6=0\\
F_3^6(x_0,y_0,\ldots,z_6,w_6) &= w_0w_6+w_1w_5+\ldots+y_0^2y_1z_5+y_0^3z_6=0
\end{aligned}$$

• We obtain six possible ideals I_{61}, I_{62}, I_{63}, I_{64}, I_{65} and I_{66}. The corresponding vectors $v_1^6=(3,2,4,5)$, $v_2^6=(2,3,4,4)$, $v_3^6=(2,4,4,4)$, $v_4^6=(1,5,5,4)$, $v_5^6=(1,6,6,4)$ and $v_6^6=(0,7,7,4)$ are of codimension 14, 13, 14, 15, 17 and 18 respectively.

For m=7, (7) gives;

$$F_1^7(x_0,y_0,\ldots,z_7,w_7)=z_0z_7+z_1z_6+\ldots+y_4w_3+y_3w_4=0$$

$$\begin{aligned}
F_2^7(x_0,y_0,\ldots,z_7,w_7) &= z_0w_7+z_1w_6+\ldots+y_0^2y_1y_6+y_0^3y_7=0\\
F_3^7(x_0,y_0,\ldots,z_7,w_7) &= w_0w_7+w_1w_6+\ldots+y_0^2y_1z_6+y_0^3z_7=0
\end{aligned}$$

• We obtain five possible ideals I_{71}, I_{72}, I_{73}, I_{74} and I_{75}. The corresponding vectors $v_1^7=(3,2,4,6)$, $v_2^7=(3,3,4,5)$, $v_3^7=(2,4,4,4)$, $v_4^7=(2,5,5,4)$ and $v_5^7=(1,6,6,4)$ are of codimension 15, 15, 14, 16 and 17 respectively.

For m=8, (7) gives

$$\begin{aligned}
F_1^8(x_0,y_0,\ldots,z_8,w_8) &= z_0z_8+z_1z_7+\ldots+y_5w_3+y_4^2=0\\
F_2^8(x_0,y_0,\ldots,z_8,w_8) &= z_0w_8+z_1w_7+\ldots+y_0^2y_1y_7+y_0^3y_8=0\\
F_3^8(x_0,y_0,\ldots,z_8,w_8) &= w_0w_8+w_1w_7+\ldots+y_0^2y_1z_7+y_0^3w_8=0
\end{aligned}$$

• We obtain five possible ideals I_{81}, I_{82}, I_{83}, I_{84} and I_{85}. The corresponding vectors $v_1^8=(3,3,5,6)$, $v_2^8=(3,4,5,5)$, $v_3^8=(2,5,5,5)$ are of codimension 17. The corresponding vector $v_4^8=(2,6,6,5)$ is of codimension 19 and the corresponding vector $v_5^8=(1,7,7,5)$ is of codimension 20.

For m=9, (7) gives;

$$\begin{aligned}
F_1^9(x_0,y_0,\ldots,z_9,w_9) &= z_0z_9+z_1z_8+\ldots+y_5w_4+y_4w_5=0\\
F_2^9(x_0,y_0,\ldots,z_9,w_9) &= z_0w_9+z_1w_8+\ldots+y_0^2y_1y_8+y_0^3y_9=0\\
F_3^9(x_0,y_0,\ldots,z_9,w_9) &= w_0w_9+w_1w_8+\ldots+y_0^2y_1z_8+y_0^3w_9=0
\end{aligned}$$

• We obtain four possible ideals I_{91}, I_{92}, I_{93} and I_{94}. The corresponding vectors $v_1^9=(4,3,5,7)$ and $v_4^9=(2,6,6,5)$ are of codimension 19. The corresponding vectors $v_2^9=(3,4,5,6)$ and $v_3^9=(3,5,5,5)$ are of codimension 18.

For m=10, (7) gives;

$$F_1^{10}(x_0,y_0,\ldots,z_{10},w_{10})=z_0z_{10}+z_1z_9+\ldots+y_6w_4+y_5w_5=0$$

$$F_2^{10}(x_0, y_0, \ldots, z_{10}, w_{10}) = z_0w_{10} + z_1w_9 + \ldots + y_0^2y_1y_9 + y_0^3y_{10} = 0$$
$$F_3^{10}(x_0, y_0, \ldots, z_{10}, w_{10}) = w_0w_{10} + w_1w_9 + \ldots + y_0^2y_1z_9 + y_0^3z_{10} = 0$$

• We obtain five possible ideals I_{101}, I_{102}, I_{103}, I_{104} andI_{105}. The corresponding vectors $v_1^{10} = (4, 3, 6, 8)$, $v_2^{10} = (4, 4, 6, 7)$ and $v_4^{10} = (3, 6, 6, 6)$ are of codimension 21. The corresponding vector $v_3^{10} = (3, 5, 6, 6)$ is of codimension 20 and $v_5^{10} = (2, 7, 7, 6)$ is of codimension 22.

For m=11, (7) gives;

$$F_1^{11}(x_0, y_0, \ldots, z_{11}, w_{11}) = z_0z_{11} + z_1z_{10} + \ldots + y_6w_5 + y_5w_6 = 0$$
$$F_2^{11}(x_0, y_0, \ldots, z_{11}, w_{11}) = z_0w_{11} + z_1w_{10} + \ldots + y_0^2y_1y_{10} + y_0^3y_{11} = 0$$

$$F_3^{11}(x_0, y_0, \ldots, z_{11}, w_{11}) = w_0w_{11} + w_1w_{10} + \ldots + y_0^2y_1z_{10} + y_0^3z_{11} = 0$$

• We obtain four possible ideals I_{111}, I_{112}, I_{113} and I_{113}. The corresponding vectors $v_2^{11} = (4, 4, 6, 8)$ and $v_3^{11} = (4, 5, 6, 7)$ are of codimension 22. The corresponding vectors $v_1^{11} = (5, 3, 6, 9)$ and $v_4^{11} = (3, 6, 6, 6)$ are of codimension 23 and 21 respectively.

For m=12, (7) gives;

$$F_1^{12}(x_0, y_0, \ldots, z_{12}, w_{12}) = z_0z_{12} + z_1z_{11} + \ldots + y_7w_5 + y_6w_6 = 0$$
$$F_2^{12}(x_0, y_0, \ldots, z_{12}, w_{12}) = z_0w_{12} + z_1w_{13} + \ldots + y_0^2y_1y_{12} + y_0^3y_{13} = 0$$
$$F_3^{12}(x_0, y_0, \ldots, z_{12}, w_{12}) = w_0w_{13} + w_1w_{12} + \ldots + y_0^2y_1z_{11} + y_0^3z_{12} = 0$$

• We obtain four possible ideals I_{121}, I_{122}, I_{123} and I_{123}. The corresponding vectors $v_2^{12} = (4, 5, 7, 8)$, $v_3^{12} = (4, 6, 7, 7)$ and $v_4^{12} = (3, 7, 7, 7)$ are of codimension 24. The corresponding vector $v_1^{12} = (5, 4, 7, 9)$ is of codimension 25.

For m=13, (7) gives;

$$F_1^{13}(x_0, y_0, \ldots, z_{13}, w_{13}) = z_0z_{13} + z_1z_{12} + \ldots + y_7w_6 + y_6w_7 = 0$$
$$F_2^{13}(x_0, y_0, \ldots, z_{13}, w_{13}) = z_0w_{13} + z_1w_{12} + \ldots + y_0^2y_1y_{12} + y_0^3y_{13} = 0$$
$$F_3^{13}(x_0, y_0, \ldots, z_{13}, w_{13}) = w_0w_{13} + w_1w_{12} + \ldots + y_0^2y_1z_{12} + y_0^3z_{13} = 0$$

• We obtain four possible ideals I_{131}, I_{132}, I_{133} and I_{133}. The corresponding vectors $v_1^{13} = (5, 4, 7, 10)$ and $v_2^{13} = (5, 5, 7, 9)$ are of codimension 26. The corresponding vectors $v_3^{13} = (4, 6, 7, 8)$ and $v_4^{13} = (4, 7, 7, 7)$ are of codimension 25.

For m=14, (7) gives;

$$F_1^{14}(x_0, y_0, \ldots, z_{14}, w_{14}) = z_0z_{14} + z_1z_{13} + \ldots + y_8w_6 + y_7w_7 = 0$$
$$F_2^{14}(x_0, y_0, \ldots, z_{14}, w_{14}) = z_0w_{14} + z_1w_{13} + \ldots + y_0^2y_1y_{13} + y_0^3y_{13} = 0$$

$$F_3^{14}(x_0, y_0, \ldots, z_{14}, w_{14}) = w_0w_{14} + w_1w_{13} + \ldots + y_0^2y_1z_{13} + y_0^3z_{14} = 0$$

• We obtain four possible ideals I_{141}, I_{142}, I_{143} and I_{143}. The corresponding vectors $v_2^{14} = (5, 5, 8, 10)$ and $v_3^{14} = (5, 6, 8, 9)$ are of codimension 28. The corresponding vectors $v_4^{14} = (4, 7, 8, 8)$ is of codimension 27 and $v_1^{14} = (6, 4, 8, 11)$ is of codimension 29.

For m=15, (7) gives;

$$F_1^{15}(x_0, y_0, \ldots, z_{15}, w_{15}) = z_0z_{15} + z_1z_{14} + \ldots + y_8w_7 + y_8w_7 = 0$$
$$F_2^{15}(x_0, y_0, \ldots, z_{15}, w_{15}) = z_0w_{15} + z_1w_{14} + \ldots + y_0^2y_1y_{14} + y_0^3y_{15} = 0$$
$$F_3^{15}(x_0, y_0, \ldots, z_{15}, w_{15}) = w_0w_{15} + w_1w_{14} + \ldots + y_0^2y_1z_{14} + y_0^3z_{15} = 0$$

• We obtain four possible ideals I_{151}, I_{152}, I_{153} and I_{153}. The corresponding vectors $v_1^{15} = (6, 4, 8, 12)$ and $v_2^{15} = (6, 5, 8, 11)$ are of codimension 30. The corresponding vectors $v_3^{15} = (5, 6, 8, 10)$ and $v_4^{15} = (5, 7, 8, 9)$ are of codimension 29.

For m=16, (7) gives;

$$F_1^{16}(x_0, y_0, \ldots, z_{16}, w_{16}) = z_0z_{16} + z_1z_{15} + \ldots + y_9w_7 + y_8w_8 = 0$$
$$F_2^{16}(x_0, y_0, \ldots, z_{16}, w_{16}) = z_0w_{16} + z_1w_{15} + \ldots + y_0^2y_1y_{15} + y_0^3y_{16} = 0$$
$$F_3^{16}(x_0, y_0, \ldots, z_{16}, w_{16}) = w_0w_{16} + w_1w_{15} + \ldots + y_0^2y_1z_{15} + y_0^3z_{16} = 0$$

• We obtain three possible ideals I_{161}, I_{162} and I_{163}. The corresponding vectors $v_1^{16} = (6, 5, 9, 12)$, $v_2^{16} = (6, 6, 9, 11)$ are of codimension 32. The corresponding vector $v_3^{16} = (5, 7, 9, 10)$ is of codimension 31.

For m=17, (7) gives;

$$F_1^{17}(x_0, y_0, \ldots, z_{17}, w_{17}) = z_0z_{17} + z_1z_{16} + \ldots + y_9w_8 + y_8w_9 = 0$$
$$F_2^{17}(x_0, y_0, \ldots, z_{17}, w_{17}) = z_0w_{17} + z_1w_{16} + \ldots + y_0^2y_1y_{16} + y_0^3y_{17} = 0$$
$$F_3^{17}(x_0, y_0, \ldots, z_{17}, w_{17}) = w_0w_{17} + w_1w_{16} + \ldots + y_0^2y_1z_{16} + y_0^3z_{17} = 0$$

• We obtain three possible ideals I_{171}, I_{172} and I_{173}. The corresponding vectors $v_2^{17} = (6, 6, 9, 12)$ and $v_3^{17} = (6, 7, 9, 11)$ are of codimension 33. The corresponding vector $v_1^{17} = (7, 5, 9, 13)$ is of codimension 34.

For m=18, (7) gives;

$$F_1^{18}(x_0, y_0, \ldots, z_{18}, w_{18}) = z_0z_{18} + z_1z_{17} + \ldots + y_{10}w_8 + y_9w_9 = 0$$
$$F_2^{18}(x_0, y_0, \ldots, z_{18}, w_{18}) = z_0w_{18} + z_1w_{17} + \ldots + y_0^2y_1y_{17} + y_0^3y_{18} = 0$$
$$F_3^{18}(x_0, y_0, \ldots, z_{18}, w_{18}) = w_0w_{18} + w_1w_{17} + \ldots + y_0^2y_1z_{17} + y_0^3z_{18} = 0$$

• We obtain three possible ideals I_{181}, I_{182} and I_{183}. The corresponding vectors $v_1^{18} = (7, 5, 10, 14)$ and $v_2^{18} = (7, 6, 10, 13)$ are of codimension 36. The corresponding vector $v_3^{18} = (6, 7, 10, 12)$ is of codimension 35.

For m=19, (7) gives;

$$F_1^{19}(x_0, y_0, \ldots, z_{19}, w_{19}) = z_0 z_{19} + z_1 z_{18} + \ldots + y_{10} w_9 + y_9 w_{10} = 0$$
$$F_2^{19}(x_0, y_0, \ldots, z_{19}, w_{19}) = z_0 w_{19} + z_1 w_{18} + \ldots + y_0^2 y_1 y_{18} + y_0^3 y_{19} = 0$$
$$F_3^{19}(x_0, y_0, \ldots, z_{19}, w_{19}) = w_0 w_{19} + w_1 w_{18} + \ldots + y_0^2 y_1 z_{18} + y_0^3 z_{19} = 0$$

• We obtain three possible ideals I_{191}, I_{192} and I_{193}. The corresponding vectors $v_1^{19} = (7, 5, 10, 15)$, $v_2^{19} = (7, 6, 10, 14)$ and $v_3^{19} = (7, 7, 10, 13)$ are of codimension 37.
For m=20, (7) gives;

$$F_1^{20}(x_0, y_0, \ldots, z_{20}, w_{20}) = z_0 z_{20} + z_1 z_{19} + \ldots + y_{11} w_9 + y_{10} w_{10} = 0$$
$$F_2^{20}(x_0, y_0, \ldots, z_{20}, w_{20}) = z_0 w_{20} + z_1 w_{19} + \ldots + y_0^2 y_1 y_{19} + y_0^3 y_{20} = 0$$
$$F_3^{20}(x_0, y_0, \ldots, z_{20}, w_{20}) = w_0 w_{20} + w_1 w_{19} + \ldots + y_0^2 y_1 z_{19} + y_0^3 z_{20} = 0$$

• We obtain three possible ideals I_{201}, I_{202} and I_{203}. The corresponding vectors $v_1^{20} = (7, 7, 11, 15)$ and $v_2^{20} = (8, 6, 11, 15)$ are of codimension 40. The corresponding vector $v_3^{20} = (7, 7, 11, 14)$ is of codimension 39.

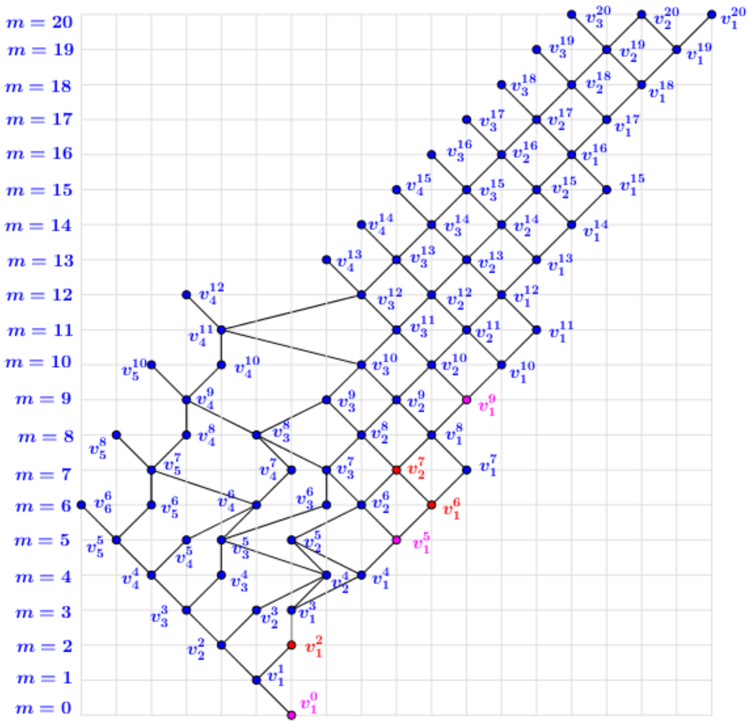

Figure 6.3: Jet graph of isolated E_{07}

Remark 6.4.2. *The weigth vectors* $(0, 1, 1)$, $(1, 2, 2)$, $(2, 2, 3)$, $(3, 2, 4)$, $(3, 3, 4)$ *and* $(4, 3, 5)$ *appeared in* 0^{th}, 2^{th}, 5^{th}, 6^{th}, 7^{th} *and* 9^{th} *jet schemes respectively as a projection of* $v_1^0 = (0, 1, 1, 1)$ $v_1^2 = (1, 2, 2, 2)$, $v_1^5 = (2, 2, 3, 4)$, $v_1^6 = (3, 2, 4, 5)$, $v_1^7 = (3, 3, 4, 5)$ *and* $v_1^9 = (4, 3, 5, 7)$.

7. CONCLUSION

In the literature, the jet schemes of a variety with rational double singularities, a variety with determinantal singularities or a variety defined by a monomial ideal are studied. We are here interested in the case where the variety has a rational triple singularities. This case permits us to study on the jet schemes of non-isolated singularities and also on the singularities which are not complete intersection. All construction in this thesis are new. For next step, we will study on all 9 cases of rational triple singularity.

In this work, we focused on 3 types non-isolated hypersurface singularities in $\mathbb{C}^3$ and their isolated surface singularities in $\mathbb{C}^4$ which appear as the normalise of the non-isolated singularities. When we applied the jet scheme construction to these types of singularities, we investigated a relation between jet graphs of them and their canonical toric minimal embedded resolution graphs.

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