A MIXED INTEGER LINEAR PROGRAMMING MODEL FOR THE COMBINED AUDIT SCHEDULING PROBLEM

(BİRLEŞİK DENETİM ÇİZELGELEME PROBLEMİ İÇİN BİR KARMA TAM SAYILI DOĞRUSAL PROGRAMLAMA GÖSTERİMİ)

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LIST OF SYMBOLS

ACS	: Ant Colony Sytems
AHP	: Analytic Hierarchy Process
AON	: Activity on Node
ASP	: Audit Scheduling Problem
BILP	: Binary Integer Linear Programming
B&B	: Branch and Bound
B&C	: Branch and Cut
B&P	: Branch and Price
CASP	: Combined Audit Scheduling Problem
CPM	: Critical Path Method
DGCA	: Directorate General of Civil Aviation
DP	: Dynamic Programming
EASA	: European Aviation Safety Agency
EF	: Earliest Finish
ES	: Earliest Start
FAA	: Federal Aviation Administration
FIFO	: First Come First Served
FRCPSP	: Flexible Resource Constrained Project Scheduling Problem
GA	: Genetic Algortihms
ILP	: Integer Linear Programming
ISO	: International Organization for Standardization
LF	: Latest Finish
LP	: Linear Programming
LS	: Latest Start
MILP	: Mixed Integer Linear Programming
MJSF	: Minimum Job Slack First
MS	: Management System
MSRCPSP	: Multi-Skill Resource Constrained Project Scheduling Problem
OHSAS	: Occupational Health and Safety Assessment Series
PERT	: Project Evaluation and Review Technique
PSP	: Project Scheduling Problem
RAMSES	: Randomized Mode Selection and Scheduling
RCPSP	: Resource Constrained Project Scheduling Problem
RCMPSP	: Resource Constrained Multi-Project Scheduling Problem
SA	: Simulated Annealing
SAE	: Society of Aautomative Engineers
TS	: Tabu Search

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ABSTRACT

In this thesis, we suggest a Mixed Integer Linear Programming (MILP) for the Combined Audit Scheduling Problem (CASP). This problem is faced by many medium and large-sized organizations and the importance of problem is increased day by day with increasing number of management system standards (MSS). CASP is a kind of Resource Constrained Multi-Project Scheduling Problem (RCMPSP) and requires that assigning auditors to audits according to their skills for few management systems where audits are grouped into engagements for each organizational function or process. We assume that each audit has same processing steps based on ISO 19011:2018 Guidelines for auditing management syste and show how to construct an Activity On Network (AON) for CASP. The proposed model includes tailor-made constraints for the CASP. The model minimizes firstly throughput time of each engagement and after arrival time of each task. An illustrative real-life problem has been introduced and a feasible solution is attained in limited time by this model. Finally, the results are demonstrated with a Gantt chart and discussed.

ÖZET

Bu tezde, Birleşik Denetim Çizelgeleme Problemi (BDÇP) için bir karma tam sayılı doğrusal programlama (KTDP) sunduk. Bir çok orta ve büyük ölçekli organizasyon bu problemle yüzleşmektedir ve problemin önemi, artan Yönetim Sistemi Standartları (YSS) sayısı ile günden güne artmaktadır. BDÇP bir Kaynak Kısıtlı Çoklu Proje Çizelgeleme Problem (KKÇPÇP) türüdür ve bir kaç yönetim sistemi için organizasyonun fonksiyonu veya sürecine göre programlara guruplandırılan denetimlere, denetçileri yeteneklerine uygun bir şekilde atamayı gerektirir. Bu programlarda, her bir denetim ISO 19011:2018 yönetim sistemleri denetim kılavuzuna göre aynı işlem adımlarına sahip olduğunu varsaydık ve BDÇP için Düğüm Üzerinde Faaliyet (DÜF) ağının nasıl kurulacağını gösterdik. Önerilen model BDÇP için özel kısıtlar içermektedir. Model ilk önce her bir programın verim süresini daha sonra her bir görevin başlama zamanını en aza indirmektedir. Örnek bir gerçek hayat problemi ortaya gösterildi ve sınırlı bir sürede bu modelle uygulanabilir bir çözüme ulaşıldı. En sonunda, sonuçlar Gantt şemasıyla gösterildi ve yorumlandı.

1. INTRODUCTION

Today, organizations have several certificate registrations for standards that are issued by voluntary organizations such as International Standards Organization (ISO), Society of Automotive Engineers (SAE) or regulatory organizations such as Directorate General of Civil Aviation (DGCA) of Turkey, Federal Agency of America (FAA) and European Aviation Safety Agency (EASA). Registered organizations carry out internal audits to control compliance with the requirements of these management systems (MSs) or regulations.

Management system audits are verifying and validating processes, products or services of organization for purpose of assurance. An auditor who is qualified in MSs examines the processes and products of an organization (auditee) systematicaly and detects improvement opportunities. After the findings of auditor, the auditee takes preventive or corrective actions. Basically, an audit can be classied into two groups. These are internal audit and external audit. Internal audit is executed by the auditor of organization. On the other hand, external audit is performed by other auditor who is employed by certification institutions or customer. Auditing is quite vital for MSs such as ISO 9001, AS 9110, ISO 14001, ISO 45001, ISO 27001 or regulations such as DGCA / FAA / EASA Part-145. Organizations implement numerous MSs. Furthermore, the departments must also be audited internally in fixed time periods. Furthermore, internal audits have to be applied to the departments periodically.

An audit report includes information about the effectiveness of each accountable organization’s function or relevant process of the organization whether it is functioning properly in conformance of the audit criteria.Therefore, the audit process is a feedback tool for the organization to continuously improve itself in discipline of MSs e.g. customer satisfaction (ISO 10002), continuously improvement (ISO 9001), on time delivery (AS

9110), environmental protection (ISO 9001), health and safety (ISO 45001) information security (ISO27001) or in compliance of regulations e.g. airplan maintenance management (DGCA / FAA / Part 145) and safety management (ICAO Annex 19). The effectiveness of these feedbacks and auditor performance are bound up with adequate planning and scheduling of audits which consists of determining the audit duration, sequence of audits, arrival and due date of audits and auditor assignments.

When an organization has a few of these standards and regulations, separately planning and realizing audits is inefficient in terms of resource utilization. The skills of multi-qualified auditors are not efficiently used and many common subjects of MSs are audited over and over again. Luckily, numerous audits in different disciplines can be combined through generic audit guidelines such as ISO 19011: 2018 and (Karapetrovic and Willborn, 2000). Combining and scheduling different MS's audits together provides common control of compliance in related disciplines and using resources more efficiently for organizations

In many organizations, there is a quality department or a similar one. These departments employ several qualified and experienced auditors who are capable of processing a variety of audit tasks. These auditors generally are assigned to engagements, a set of audits which will be performed by the auditee in a yearly planning horizon. This is a typical audit scheduling, which is a planning process. The audit scheduling process consists of the determination of audit tasks that will be processed, the assignment of auditors to audit task, and the scheduling of all these activities (Dodin & Chan, 1991). The previous studies (Balachandran & Zoltners, 1981; Dodin & Chan, 1991; Dodin & Elimam, 1997) focused on only accounting management disciplines, but now organizations planning audits for several MSs together (Simon et al., 2014; Hoy and Foley, 2015; Bernardo et al., 2015; Savino & Batbaatar, 2015; Domingues et al., 2016; Bernardo et al., 2018; Nunhes et al, 2018). Thus, the classical audit scheduling problem has evolved into multidisciplinary perspective because of efficiently using resources and some other reasons that mentioned in Section 1.3. Therefore, we have the CASP which deals with simultaneously scheduling audits of different MSs and/or regulations and assigning single-qualified or multi-qualified auditor to audit tasks. The CASP may become intractable with the lack of experienced and qualified auditors similar to the case of audit scheduling problem (ASP). The proliferation of new standards and regulations makes CASP crucial more than ever. Therefore, if it is feasible, different audit types need to be systematically combined and optimized for a common schedule and resource assignment.

Each audit can be treated as a project and several audits can be combined on a major project network. Therefore, ASP and CASP can be considered as a multi-project scheduling under limited resources. This problem is called as the well-known resource constrained project scheduling problem (RCPSP). The methods of project and production scheduling can also be applied to audit scheduling problem (Dodin & Chan, 1991). Here, resources correspond to auditors who are skilled labor (Drexl, 1991). In the simplest term, several auditors have to be assigned to a few projects and in these projects; they are assigned specific tasks according to their skills.

1.1 Combining Audits

ISO statistics[1] show that the number of standards implemented is increased from 2015 to 2016 (Figure 1.1). The organization certificated in several MSs that has still applied for new standards. The number of MSs' certificates in an organization does not increase the number of audits by itself but also the size of the organization has a great impact.

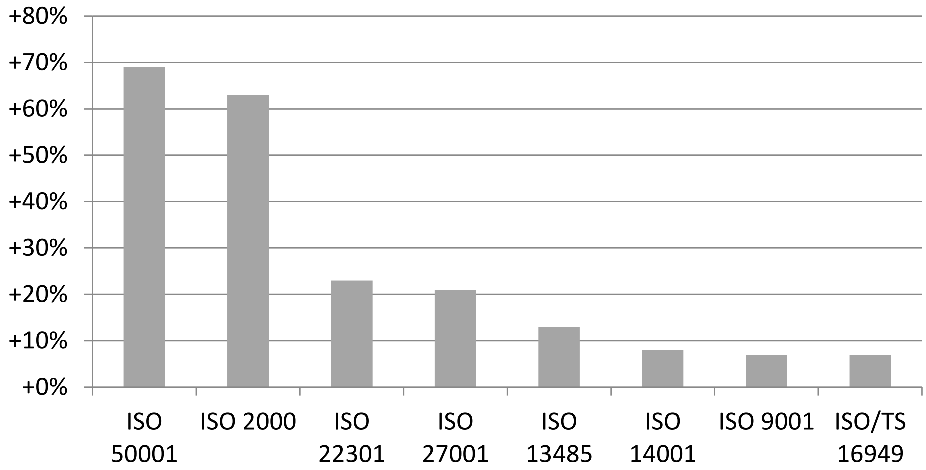

Figure 1.1: The change percent in the number of certificated organizations from 2015 to 2016

As organizations grow, their processes become complex. However, it may still be easy to audit the processes of such an organization if it is audited through only quality requirements (ISO 9001). When the diversity of requirements is increased with the complexity of processes, the scheduling of unrelated or uncombined audits becomes more complex; this is the case when the organization is audited through, for example, environmental (ISO 14001) and information security requirements (ISO 27001) in addition to quality requirements. Both organizations process complexity and number of MSs determine numbers of audits required. Suppose that an organization has processes as in Table 1.1. In case of uncombined audit, the number of audits increases as the number of the MSs increases as Figure 1.2 for the case of the organization showed in Table 1.1.

[1] URL: https://www.iso.org/the-iso-survey.html

Each new MSs would add several new audits to existing ones as many as applicable processes. Organization in this situation also would need more qualified auditors. This diversity of the MSs also adds new constraints on schedule that auditors are assigned to different audits in a time horizon if only if they are qualified in the relevant MSs.

Table 1.1: Proliferation of audit number with additional MSs

An organization's generic functions	**Number of Units**	**Management Systems**					**Number of Audits**
		ISO 9001	ISO 14001	ISO 45001	ISO 2700 1	ISO 5500 1	
Finance and Accounting	15	x	O	x	x	o	45
Human Resource	21	x	X	x	x	o	84
Information Technology	16	x	O	x	x	x	64
Facility Maintenance	25	x	O	x	o	x	75
Production Planning	23	x	X	x	x	x	115
Operations Management	172	x	X	x	x	x	860
Procurement and Logistic	29	x	X	x	x	x	145
Marketing and Sales	19	x	O	x	x	o	57
Individual count		320	245	320	295	265	
Cumulative count		320	565	885	1180	1445	
x: MSs is effective to accountable organization functions **o: MSs is not effective to accountable organization functions**							

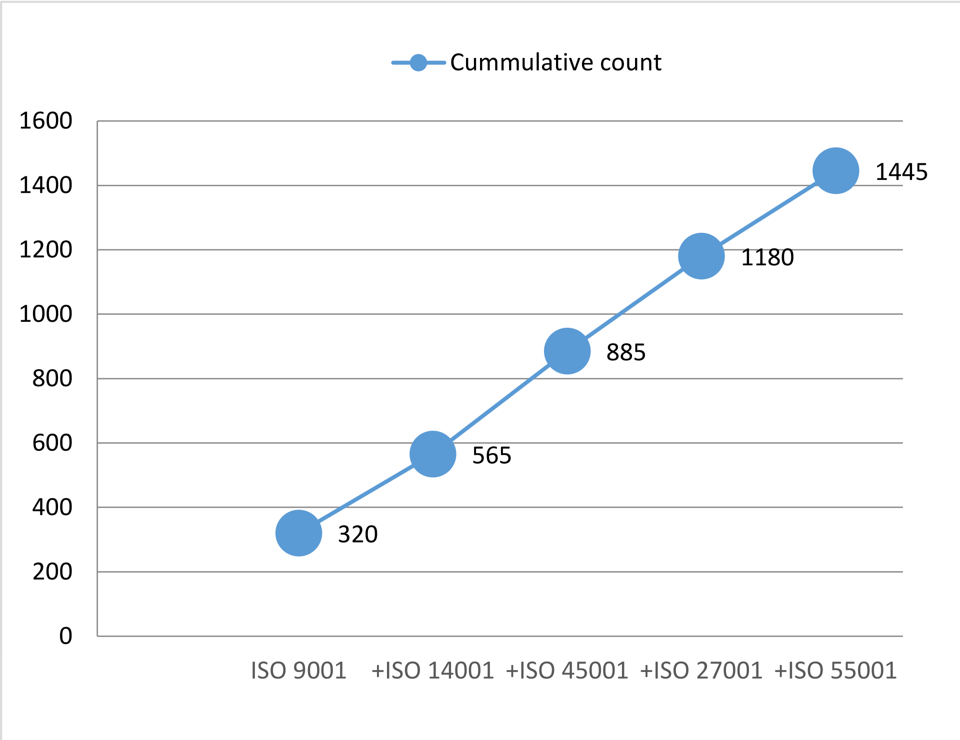

Figure 1.2: Proliferation of audit number with additional MSs

The increasing number of MSs enforces companies to combine several audits in order to optimize their resources (Kraus & Grosskopf, 2008; Karapetrovic & Walter Willborn 2001). Luckily, it is feasible to conduct simultaneously audits for more than one MSs under the same guidelines. A combined audit is performed by a single auditee under diverse disciplines e.g. quality (ISO 9001) and environment (ISO14001). Combined audits are also known as integrated audits in practice with a little difference where MSs are combined. However, the principles of a combined and integrated audit are slightly different. Organizations combine audits instead of conducting them separately, and hence, integrating annual audit program of MSs. (Karapetrovic & Willborn, 1998; Wilkinson & Dale, 1999; Douglas & Glen, 2000; Zutshi & Sohal, 2005; Bernardo et al., 2010; Simon et al., 2011; Simon et al., 2014; Hoy & Foley, 2015; Bernardo et al., 2015; Savino & Batbaatar, 2015).

The integration degree of audits changes from organization to organization (Bernardo et al., 2010; Simon et al., 2011). The subjects of audit combined by an organization can be summarized as bellow:

- Different audit teams: different auditor teams conduct different MSs' audits.
- Different audit plans: audits are conducted different times for same auditee for different MSs.
- Different audit reports: different reports are produced for different MSs audits.
- Different audit results: audit findings are evaluated against different criteria
- Different audit guidelines: different procedures and processes are used.
- Different execution methodology: different audit techniques (e.g. process-based, requirement-based) are implied for different MSs.

19011:2018 - Guidelines for auditing MSs provides a generic system to conduct a single or combined audit. It considers the needs of combining multiple MSs audits and stated that the number of new standards is increased.

1.2 Conducting an Audit

According to ISO 19011, an audit is a systematic, independent and documented process in order to obtain objective evidence and to evaluate this evidence objectively. Audits criteria in this definition are based on requirements of MSs and expectations of stakeholders such as agreements or regulations. An audit is systematic because it has interrelated activities and takes evidence as an input and produce findings as output. It must be carried out independently to report findings objectively. Auditors report their findings to auditee who takes corrective and preventive action and to other stakeholders e.g. top management, customer, and legal authorities.

An audit can be classified according to what to audit. A product audit is the assessment of a final product, subpart or a service to investigate whether it conforms to specifications before delivery to a customer. A process audit verifies that processes are performed in predermined limits and in compliance to establihed requirements in documents such as procedures, work instructions and contracts. Such an audit controls the conformance to specified requirements such as due dates, task steps, appropriate tools, skills, or physical attributes such as pressure and temperature. Auditors examine inputs of the process such as manpower, materials, machines, methods, meusuremets and environment and outputs of the process such as the delivered product or service specifications for performance of process.

Figure 1.3: The audit process

Each audit requires unique activities however each audit has the same phases. These phases are generic for all MS and constitute the audit process. We will call this audit process flow as audit tasks in order to avoid confusion with ISO 19011:2018 that also describes the audit leaders and auditor responsibilities in audit conducting phase.

However, an audit process flow is designed when auditors and audit leader realize audit activities and their responsibilities. An audit process flow includes tasks that are assigned to an auditor. These tasks are preparing to audit, open meeting, executing, closing meeting and reporting as depicted in Figure 1.3 and they are executed in this order. ISO 19011:2018 audit activities for conducting an audit is compared with individual audit tasks and depicted how they coincide in Table 1.2. An audit process consists of tasks which are the same for all MSs when conducting an audit.

Table 1.2: ISO 19011:2011 Audit Tasks & ISO 19011:2018 Audit Activities

The audit process	ISO 19011:2018 audit guidelines
Preparation: After audit assigments to the auditor, she collects information about auditee activites in accordance with the audit criteria. Most of the information included in documents, such as procedures, work instructions, standards, process maps, organization chart etc. Auditor can contact to auditee and request information. In this process step an auditor may prepare a checklist and evaluate risky points to check.	6 Conducting an audit 6.1 General 6.2 Initiating audit 6.2.1 General 6.2.2 Establishing contact with auditee 6.2.3 Determining feasibility of audit 6.3 Preparing audit activities 6.3.1 Performing review of documented information 6.3.2 Audit planning 6.3.3 Assigning work to audit team 6.3.4 Preparing documented information for audit
Opening meeting: An auditor attends the scheduled meeting to collaborate and inform auditee for critical points, request extra information about processes will be audited and determine a sample plan.	6.4 Conducting audit activities 6.4.1General 6.4.2 Assigning roles and responsibilities of guides and observers 6.4.3 Conducting opening meeting

The audit process	ISO 19011:2018 audit guidelines
Execution: Auditor checks the requirements of MSs, controls the specification of the process, assesses operations, examine confirmity and existence of all related documents, methods, tools, environment, resources, sofwares etc. and collects evidence to show whether the processes conform to the standards.	6.4.4 Communicating during audit 6.4.5 Audit information availability and access 6.4.6 Reviewing documented information while conducting audit 6.4.7 Collecting and verifying information 6.4.8 Generating audit findings 6.4.9 Determining audit conclusions
Closing meeting: Auditor reports his findings, verified and validates the findings, agree with auditee for due dates and provides feedback for improvement opportunities.	6.4.10 Conducting closing meeting
Reporting: Auditor reports all validated findings to the auditee and other related parties, transfer findings to software to track findings status takes information about preventive and corrective actions from auditee.	6.5 Preparing and distributing audit report 6.5.1 Preparing audit report 6.5.2 Distributing audit report
Follow-up: Auditor checks whether preventive and corrective action taken is effective. (This step is excluded from audit scheduling tasks, because the processing time of this task is negligible, and this task's arrival time is unpredictable which depends on when auditee summits her responds.)	6.6 Completing audit 6.7 Conducting auidit follow-up

Audits can be conducted process or functional base. A horizontal audit (process- based) is an audit of one process, such as training or corrective action, across several departments. A vertical audit (functional-based) is an audit of several processes, such as testing, test equipment, test status, and nonconformance's, within one department (Russell, 2012).

Most of the MSs such as ISO 9001:2015 advice process by process auditing. In process-based audits, the organizational units and sequence of audit for each unit should be determined in parallel with the process. Processes also have precedence relation if a process output is the input of another process (Figure 1.4). This relation between processes can be beneficial when designing audit precedences (Oksana et al., 2017). Note that in both cases an individual audit scope (start to finish) can be confined in an organizational unit such as chieftaincy or a shop achieving process activities.

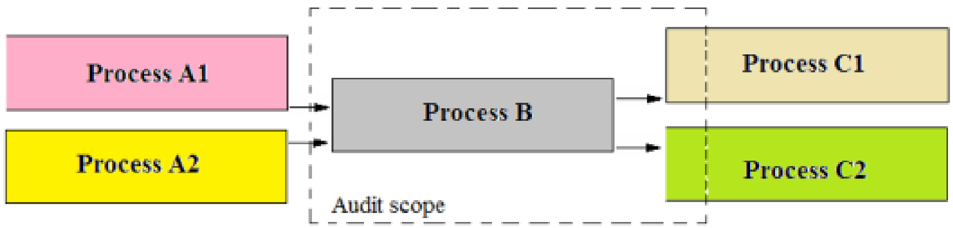

Figure 1.4: Interrelation of processes during the audit

1.3 Conducting Combined Audits

Combining or integrating internal or external audit provides organization optimizing their resources (Karapetrovic & Willborn, 1998; Kraus & Grosskopf, 2008; Salomone, 2008). Audit time can be decreased thanks to the multidisciplinary auditor who has qualifications and competence of auditing different MSs (Kraus & Grosskopf, 2008; Simon et al, 2011).

Karapetrovic and Willborn (1998) investigate the possibilities of harmonizing audits of ISO 9001 and 14001 standards. They state that the process of conducting the quality and environmental audits are very similar. Karapetrovic and Willborn (2000) proposed a broad audit guideline which mention about principles and practices of MS auditing. ISO 19011:2002 is the first generic standard that is issued instead of the well-known standards such as ISO 10011 (quality audit) and ISO 14011 (environmental audit). Latest revision of ISO 19011:2018 is a generic standard providing guidelines for both single and combined MSs' audits. This standard presents a generic guidance for auditing that are about the principles of auditing, managing the audit program and compentance of auditor.

Most of the organizations prefer to integrate audits and the ISO 19011: 2018 standard may facilitate integration of different MSs audits. A new revision of ISO 19011 issued in 2018 includes more detail and requirements about the guidance on conducting an audit. New revision also emphasizes generic nature of management system audit that is a common audit system for the management system. Most of the published MSs of ISO have a common structure. (ISO 19011:2018).

ISO 19011: 2018 uses the term "combined audit" instead of "integrated audit". The integration degree of audits may vary from organization to organization. In order to evaluate the audit integration level, it has been considered four main variables (Bernardo et al., 2010; Simon et al., 2011). Same audit team may audit all standard or selected or different audit teams audit different standards. MSs which can be interrelated, independent or integrated. MSs can be conducted at the same time or different times. Audit plan and report can be separate or combined. With the combination of these variables an audit can be not integrated, partially integrated and fully integrated (Bernardo et al., 2010)

• **Not integrated audits:** Various audit teams conduct audits in different times for audits of the independent MS. Audits have different plans and auditors submit different reports.

• **Partially integrated audits:** A single audit team conduct simultaneous audits, but only for selected and interrelated MSs. Audits have a single audit plan but different audit reports. Organization has also other MSs for which their audits are conducted separately.

• **Fully integrated audits:** Single audit teams conduct audits simultaneously for all MSs; Audits have common audit plans and reports.

Bernardo et al., (2010) have come up with a survey of 435 organizations registered to both the ISO 9001 and ISO 14001 standards in order to determine the levels of integration of audits of MSs. According to the survey, three integration levels are detected which are shown with Figure 1.5:

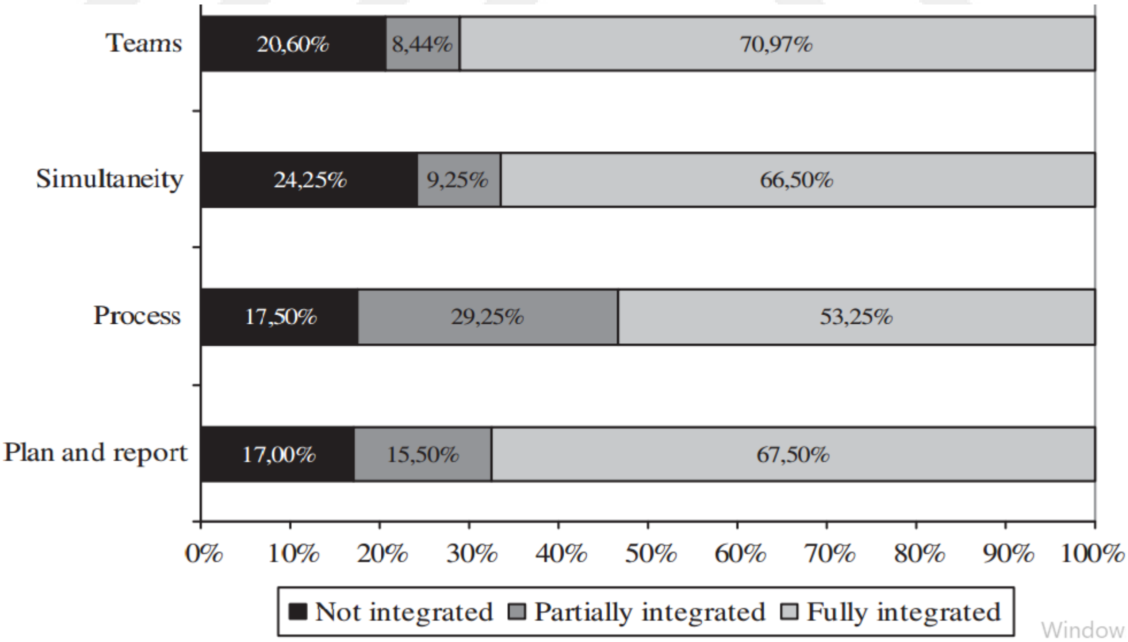

Figure 1.5: Integration aspects of internal audits (adopted from Bernardo et. al,2010)

A survey held among 843 Portuguese Organizations with more than one certified MSs show that these MSs are implemented together. According to survey statistics, 44% of organizations integrated ISO 9001, ISO14001 and OHSAS 18001 (ISO 45001) and 24% organizations integrated ISO 9001 and ISO 14001 (Ribeiro et al.,2017). This means at least 68% of these organizations are scheduling audits for more than one MSs. Another empirical study shows a high level of integration of the internal and external audit process among organizations. Abad et al. (2014) examine statistics of 86 Spanish certified firms (ISO 9001, ISO 14001 and OHSAS 18001) and found that proportion of firms integrating audits are respectively 85% for internal and 79% for external audit. Another empirical study (Moumen & Aoufir., 2017) shows that 74.80% of firms have voted for the better use of audits' results.

2. LITERATURE REVIEW

RCPSP has received enormous attention in literature hence there exists many studies about it. We introduce some survey articles on the RCPSP in Section 2.1. Next, in Section 2.2 we outline modeling approaches for ASP. Lastly, in Section 2.3 meta-heuristics and exact procedures are summarized.

2.1 Review on Resource Constrained Project Scheduling

Project Scheduling Problem (PSP) includes the following components: resources, activities, precedence constraints and performance criteria (Kolisch, 2013). First, resources can be classified according to their consumption type during a period and project they involve. Resources like materials and money are nonrenewable resource and their consumption is unlimited during time or project they involve A resource like manpower, machines, equipment and tools are renewable, their availability is limited during time period (e.g. a day, week or month). Hence, auditors can work only a limited number of periods on the project for a doubly constrained resource.

A project includes a set of activities such as jobs, operations, and tasks. An activity duration may change in accordance with quantity of resource assigned to it. For example, an engagement can be assigned to one auditor with a duration as much as a day or an engagement can be assigned to two identical qualified auditors with a duration as much as a half day. Mode of activity has a trade-off with resource quantity. The mode can be measured as a decreasing linear function of resource usage.

The most common performance measure of an PSP is the minimization of makespan that aims to minimize the completion time of last activity on network thus the completion time of project. Other common performance measures are lateness of job, earliness of job, tardiness of job, and weighted version of these performance measures (Pinedo, 2016). A performance measure can be classified as regular e.g. mean flow time and makespan or non-regular e.g. consumed such as amount of nonrenewable resources (Kolich, 2013).

Pritsker et al. (1969) suggested a Binary Integer Linear Programming (BILP) model for the multi-project scheduling problem under multiple resource constraints. Their formulation includes three different objective function which are to minimize the total throughput time for all projects, to minimize the makespan and the total lateness for all projects. The variables of this formation are x_{ijt} is equal to 1 if job j of project i is completed in period t, 0 otherwise and x_{it} is equal to 1 if all job of project i completed by period t. Their mathematical formulations represent a substantial improvement over the dispatching rules which are first come first served (FIFO) and minimum job slack first

(MJSF). They showed that their formulation requires less variables and constraints than zero-one formulation of Bowman (1959).

Davis (1973) proposed the classification of the project scheduling under fixed resource constraints in two categories according to the type of results which they are produced and characteristic of problem which they are suited. The first procedures are about objective function value, and can be also classified into two categories: heuristic and exact procedures. Heuristics are trying to access a feasible schedule. The heuristics procedures use some priority rules. On the other hand, exact or analytical procedures including mathematical programming produce the optimal schedule. Heuristic procedures may also have divided into two: serial or parallel routines. Optimal procedures are Branch and Bound (B&B), Branch and Cut (B&C), Branch and Price (B&P), Dynamic Programming (DP). Both heuristic and optimal procedures can be classified according to the number of resource types involved in the project. They are one resource type for all jobs (i.e., man-hour, tracks), more than one for project but only one resource per job and lastly more than one resource type per job and project (multi-resource case). The classification of Davis (1973) summarized in Table 2.1. The pioneer researches that are studying Linear Programming (LP) models are also referenced in Davis (1973).

Table 2.1: Constrained-resource scheduling procedure classification

Heuristic Procedures	**Optimal Procedures**
Parallel Allocation Routine Serial Allocation Routine	Linear Programming Enumerative or other
Multi-Resource Models Job-Shop Models Single- Resource Models	

Herroelen et al. (1998) prepare a survey on previous research on optimal solution procedures such as the DP and B&B. Brucker et al. (1999) shows a notation and classification scheme for RCPSP which is compatible to machine scheduling. The scheme consists of resource environment, activity characteristics and objective function. Activity characteristics are processing times, precedence relations etc. Objective function corresponds to some formulas e.g. makespan, tardiness etc. For example, PS|prec|Cmax corresponds to RCPSP which is minimizing the project's makespan and includes precedence and resource constraints. They also review some of the recent exact and heuristic algorithms. Blazewicz et al. (1983) proposed a classification scheme for RCPSP similar to Brucker et al. (1999) and investigate a range of initial results on computational complexity for these problems. Kolisch and Hartmann (2006) summarize heuristics proposed in literature such as X-pass methods, Genetic Algorithms (GA), Tabu Search (TS), Simulated Annealing (SA), Ant Colony Systems (ACS), local search oriented and population-based approaches, forward backward improvements.

Krüger and Scholl (2009) considers the problem of changing the jointly used resources between projects in a multiple projects' environment, and creates a transfer time and influences due dates of projects in reality. They proposed an integer linear programming (ILP) model and priority rule-based heuristic minimizing multi-project duration. The heuristic is based on resource transfer rules and the schedule generation scheme (series and parallel). The experiments tested that transfer times should not be neglected because of multi-project delay.

Naber and Kolisch (2014) investigated interdependency between resources and modeling an MIP for RCPSP. They called this problem a Flexible RCPSP (FRCPSP) in which the resource usage of an activity is adjusted from period to period. They minimize makespan and determine simultaneously the start time, the resource to be used and usage quantity, and the duration of each activity, subject to precedence relationships, limited availability of multiple resources. They classify resources as principal, dependent and independent. Principal resources usage depends on activity and only one principal resource can be assigned to an activity in a time. Usage of dependent resource depends on the usage of principal resource. Independent resource does not depend any usage of resources, but its timing must be synchronized.

In some cases, some projects in portfolio carry higher priority than others. Singh (2014) developed a hybrid procedure with some priority rules and Analytic Hierarchy Process (AHP) for resource constrained multi-project scheduling. Resources are allocated to project activities in combination of activity priority index calculated with best reported dispatching rule and with the priority index of each project determined with AHP.

Beşikci et al. (2015) represent a mathematical formulation and two different GAs for a multi project scheduling problem with multi-mode resource constraints. In the proposed mathematical formulation, a resource is not shared but it is partially dedicated to individual projects with an operating mode. Each project has a due date and activities of project has earliest and latest date. Amount of total resources required, and resources dedicated to projects are constrained with total budget and decision variables along with the classical variable to determine the schedule of the activity. Weighted tardiness values calculated for each project and the objective function minimizes the sum of the total weighted tardiness cost of projects.

Multi-Skill Resource Constrained Project Scheduling Problem (MSRCPSP) considers limited resources with several skills. In this type of problems, the decisions are to assign resources to activities according to their skills. Almeida et al. (2016) investigate a parallel scheduling heuristic for the MSRCPSP.

2.2 Modeling Approach for the Audit Scheduling Problem

Summers (1972) propose an LP model for the ASP. The variables are the actual hours to be worked during planning period by *i*'th auditor on *j*'th audit activity. In this model each auditor has different rate of processing an activity and each activity has different amount of benefit return to audit office which is measured in terms of dollars. The objective function of this model is to maximize audit office benefits multiplication of auditors' activity time and cost of this activity. Summers (1972) considers constraints that limits auditors total working time on engagement, the maximum and minimum time of an activity which can be assigned to an auditor.

Balachandran and Zoltners (1981) developed an MILP model for accounting organization that provide a good basis for ASP. This integer program only assigns auditors to engagements which are compromised of audit tasks e.g. internal control, compliance tests and tax related activities. They use two zero-one variables. The first one is for the combination of auditor *i* and audit task *j*, second one is for the combination of auditor *i* and engagement *k*. They simply assign audit tasks and audit engagement to auditors. Here, auditors are classified according to their experience level and audits tasks are also classified according to industry size, audit operations etc. Thus, specified auditors can be assigned to audit task if their experience level is enough. They developed constraints which ensures that each auditor is neither over nor under-utilized, make sure that every audit-task is completed by one auditor. This model of audit staff assignment provides a good basis for ASP but does not provide a schedule.

Chan and Dodin (1986) model a more realistic audit scheduling by adding new constraints to the formulation devised by Balachandran and Zoltner (1981). They also consider the arrival time of each engagement and can be adjoined into the Project Evaluation and Review Technique (PERT) network by adding a dummy activity at the start of each engagement. Because the decision variable is a combination of the number of alternative auditors, job and period decreasing number of the parameters reduces the number of variables. Therefore, an improvement they made is designing the set of audit tasks for auditors according to their skills. An auditor cannot do all jobs. They establish the set of audit tasks that can possibly be conducted by an auditor and vice versa. Another improvement is that the time period of a job. Time period in this model is discrete and each job time window is limited with earliest start (ES) and latest finish (LF) time similar Pritsker et al. (1969). To do this, a critical path was found on the PERT network. Using ES and LS instead of full time period for each job significantly decreases the number of variables. The objective in this model is minimize the mismatch cost between auditors and tasks plus penalty cost of exceeding the due date of an engagement. An illustrative example with four auditors, two audit engagements and a planning horizon of 12 weeks is solved and an optimal result is depicted with Figure 2.1.

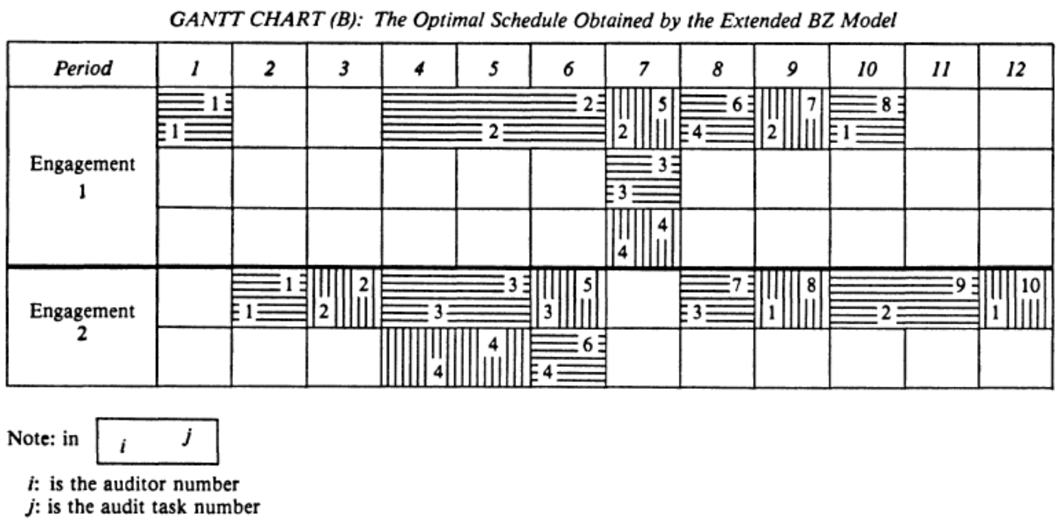

Figure 2.1: Gantt Chart adapted from Chan and Dodin (1986)

Dodin and Chan (1991) use same formulation in Chan and Dodin (1986) but for new objective functions. They implement a real case audit with 2 engagement (projects), totally 19 tasks, 4 auditors in different modes. They solve this problem for objectives of minimizing mismatching cost, minimizing project delay cost and minimizing both mismatching and delay costs.

2.3 Meta-heuristics and exact procedures for the ASP

A lot of procedures are suggested to solve the RCPSP, but priority role-based scheduling is yet the most vital (heuristic) solution method. This is because of a few reasons; first the technique is intuitive and simple to utilize, which makes it appropriate to be utilized inside business applications. Second the procedure is quick as far as the computational exertion which prescribes it to be coordinated inside local search approaches from artificial intelligence. Last, multi-pass usage of the procedure demonstrate the best outcomes reachable by heuristics today (Kolisch,1996)

Drexl (1991) propose a stochastic scheduling heuristic (Monte Carlo) and a hybrid branch and bound/dynamic programming algorithm (exact algorithm) for solving audit scheduling problems. Exact algorithm solves smaller to optimize problems within a reasonable amount of time. The heuristic is more efficient in case of large problems, but it gives approximate results for this time-cost trade-off problems. The idea behind of using Monte Carlo method consists of solving conflicts between jobs which competes for scarce resources. It is comparing available resources for a candidate job that is unscheduled and calculate the maximum opportunity cost if resource assigned to the job.

Dodin and Elimam (1996) extend the ASP by adding new issues such as auditor's travel cost and overlapping audit activities instead of strict precedence relations. They proposed an ILP model. In addition to auditor assignment to a task in a time period variable of Dodin and Chan (1991), they designed a variable which is equal to 1 if only if auditor *i* processes task *k* after *j* and tasks *j* and *k* are assigned to two different audit engagements. With this variable they measure the re-assignment travelling occurred when an auditor transferred from an engagement to another one. The other difference is about overlapping activities. The normal assumption is that an activity can start if all its successors are completed. They find the overlapping quantity by multiplying a ratio less than 1 with processing time. If this ratio is equal to 1 then a strict precedence is used in Critical Path Method (CPM). They provide an example with 2 engagements, 4 auditors and 19 tasks. Tasks durations are changed according to experience of auditor, since they consider the multi-mode RCPSP. In addition to minimizing mismatching and delay costs, now setup cost (auditor's travel cost/time) is one of the objective functions. They present a computational analysis and conclude that the use of exact solution procedures may be inefficient in large problem and the heuristics is more useful for this problem.

Salewski et al. (1997) solves multi-mode ASP under mode identity constraints which is generalization of the multi-mode where the set of all tasks is partitioned into disjoint subsets while all jobs belong to one subset has executed in the same mode. They proposed a tailor-made solution method, randomized mode selection and s
cheduling (RAMSES), which is incorporated with priority rules. By using different priority rules, RAMSES performs assignment of modes to a subset of jobs in one stage, then schedule jobs in the second stage. To measure performance of solution method, they generate test data based on statistical methods. They found that the major factor in determining the size of the problem is the planning horizon. A 13 weeks planning horizon constitutes small problem size which is up to 30 auditors, 95 engagements and 98800 binary variables, a planning horizon of 26 weeks yield medium size problem which is up to 55 auditors, 280 engagements, and 728000 binary variables and lastly 52 weeks planning horizon gives large instances which is with up to 125 auditors, 880 engagements and 5948800 binary variables. In their experimental analysis, under the feasibility, efficiency and acceptance performance measures indicate that some combination of priority rules outperform others. Lastly, their algorithm solved small instances but solve only one large size and under small instances RAMSES may not generate a feasible solution under scarce resources.

3. PROBLEM DEFINITION FOR CASP

CASP is a multi-project resource constrained project scheduling problem where projects demand joint resources simultaneously. CASP has a difference for internal and external audit. In external audit case, the projects arrival times are known in advance but in internal audit case, all projects arrival times are the same. This difference is distinguished in literature as static and dynamic project environments (Dumond & Mabert, 1988). The static environment assumes that all audits are integrated into a super project and scheduled once. However, in the dynamic environment, when scheduled audits are in progress, new audit demands have come from customer and all audits are scheduled again.

A fully integrated audit can be considered as a sequential one thus, we can add a new type to them which is combination simultaneous an integrated. The one we proposed is inherently a simultaneous audit. These audit types with Figure 3.1.

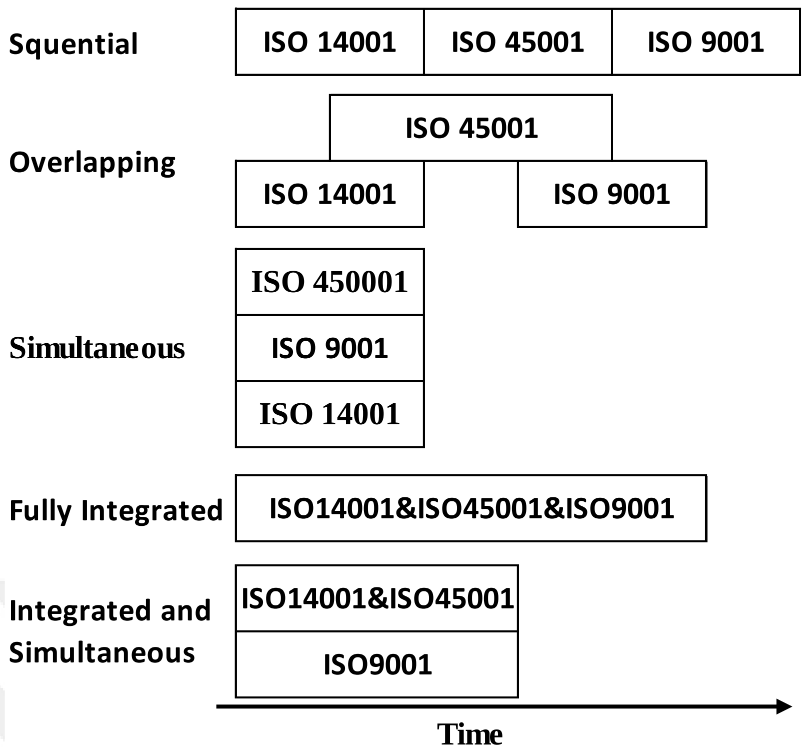

Figure 3.1: Types of combined audits and their relationship on time.

A fully integrated audit consists of a few audits that are performed as a single audit, but the auditor who conducts the integrated audit have all qualification of all MSs covered by the integrated audit or auditors for each MSs jointly conduct the integrated audit. In this case, multiple auditor assignment may require for integrated audit tasks. Lastly, consider the combination of a simultaneous and integrated audit. Assume that an organization has three MSs, integrated two of them (e.g. ISO 14001 and ISO 45001) and manages ISO 9001 separately. Then audits of these MSs can be conducted simultaneously as the last one in Figure 3.1**.** In this study, we design audits as integrated and simultaneous audits. If audits are executed simultaneously, it prevents duplication of audit tasks. For example, an auditor will not check documents of auditee or an auditee will not be disturbed for each audit but once.

CASP can be represented on a project network after determining audit tasks and constructing the relation between them. An audit task can be determined by dividing organization functions or processes into manageable parts. Therefore, audits can be considered as process-based or function-based audits when determining audit tasks. Functions are in organization departments thus it can be divided as in organization chart. On the other hand, processes can be classified according to their hierarchies: such as main processes and sub-processes. A main process includes rutin operational activities of a set of functional departments while a sub-process concentrates on an activity which operated by the subunits of these departments. Processes and functions are divided into manageable sub-parts to determine audits. We will call a manageable process or a function as a unit for convenience. A single audit or combined audits held for units can be called an engagement. An engagement determines which units against to which MSs included to audit scope. Hence, tasks and task relations of CASP determined within an engagement can be represented by an activity networks of a project.

Consider an organization has three MSs which are ISO9001, ISO14001 and ISO45001. Assume that in an engagement, two units belong to a department or two departments separately. These units are responsible for two MSs as shown in Table 3.1**.** Unit 1 is audited under ISO 9001 and ISO 14001. Unit 2 is only audited under standard ISO9001. This engagement can be depicted as a network in Figure 3.2. Also, an organization has 2 auditors; Auditor 1 has the qualification in ISO 14001 and ISO 9001. Auditor 2 has the qualification of ISO 14001 and ISO 45001. Note that for this engagement, even if auditor 2 has qualification of ISO 45001, this engagement is not demanding this skill.

Table 3.1: An engagement scope

Engagement Scope			
MS' Standards	Unit 1	Unit 2	Qualified Auditor
ISO 9001	+	+	Aud.1
ISO14001	+	-	Aud.1, Aud.2
ISO45001	-	-	Aud.2
+: unit is responsible for MS. - : unit is responsible for MS.			

When designing activities network, we consider specific assumptions to CASP. In the literature review section, we showed that all MS audits have the same steps and compatible with ISO 19001:2018. For completeness, we review these audit tasks here again. Each audit has a series of tasks which are respectively preparation, open meeting, execution, and closing meeting and reporting. This order technically entails a precedence relation between tasks. First, the meeting tasks require multi audit assignments if all engagement tasks are not assigned to one multi-qualified auditor. Second a series of tasks for each audit must be assigned to the same auditor. According to these assumptions, the audits in Table 3.1 can be combined and represented by an activity network in Figure 3.2.

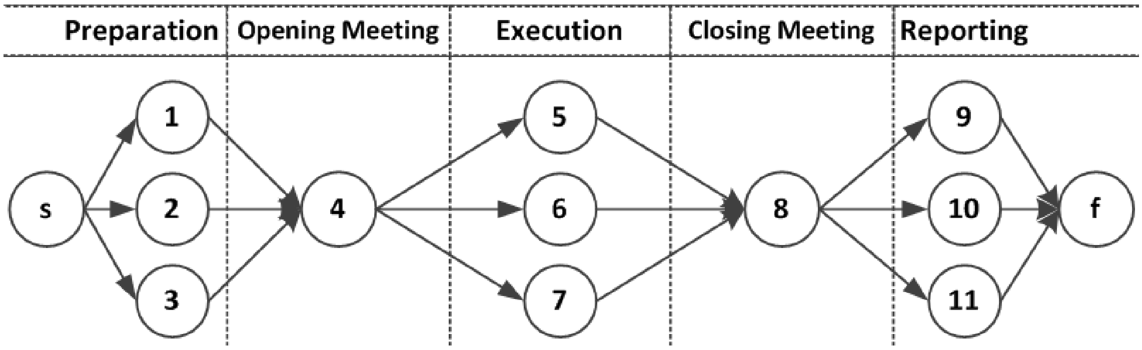

Figure 3.2 Activity network for engagement with 3 audits.

Table 3.2: Process that audited for standards in the case of two engagements.
according to process, standard and audit activities

		Audit Tasks				
Audit No	MS	Preparation	Opening Meeting	Execution	Closing Meeting	Reporting
1	ISO9001	1	4	5	8	9
2	ISO14001	2	4	6	8	10
3	ISO9001	3	4	7	8	11

One can easily observe which audit task belongs to which audit, which MS and what type of task with the help of Table 3.2**.** Always a series of tasks from preparation to reporting creating an audit. Audit 1 has tasks for ISO9001 that consist of the series of tasks 1, 4,5,8 and 9. In same manner, audit 2 has tasks for ISO 14001 are constituted of the series of tasks 2, 4,6,8 and 10. Opening meetings and closing meetings for these audits are common because these audits are planned simultaneously. We call these sets of audits designed in this manner as an engagement with 2 MSs, 2 units and 2 auditors. This is also a single project case of RCPSP.

For multi-project case, engagements are combined in a single network which is called a super-network (Pritsker, 1969), by adding a ‘‘super-source” and a ‘‘super-sink,” where a common resource pool for multiple engagements is taken into account. Thus, all engagements can be combined in an audit program for an organization. This program now can be planned for all units and MSs that organization has. Consider an organization has been certified in 3 MSs and 4 units. Units 1 and 2 are managed by a department, Units 3 and 4 are dealt with another department. Note that a unit is organization process or function. Audits of these units are planned to be executed in two engagements in Table 3.3**.** Each "+" shows an audit in Table 3.3. Combined engagements are depicted by the activities network in Figure 3.3.

Table 3.3: Process that audited for standards in the case of two engagements

	Engagement 1		**Engagement 2**		**Qualified auditors**
MS	Unit 1	Unit 2	Unit 3	Unit 4	
ISO9001	+	+	+	-	1,2
ISO14001	+	-	-	-	3
ISO45001	-	-	+	+	2,3
+: unit is responsible for MS. -: unit is responsible for MS.					

Table 3.4: Process that audited for standards in the case of two engagements.

in the case of two engagements.

	Audits		**Audit Tasks**				
	Audit No	**MS**	**Preparation**	**Opening Meeting**	**Execution**	**Closing Meeting**	**Reporting**
Engagement 1	1	A: ISO9001	1	4	5	8	9
	2	B: ISO14001	2	4	6	8	10
	3	A: ISO9001	3	4	7	8	11
Engagement 2	4	A: ISO9001	12	15	16	19	20
	5	B: ISO14001	13	15	17	19	21
	6	C: ISO45001	14	15	18	19	22

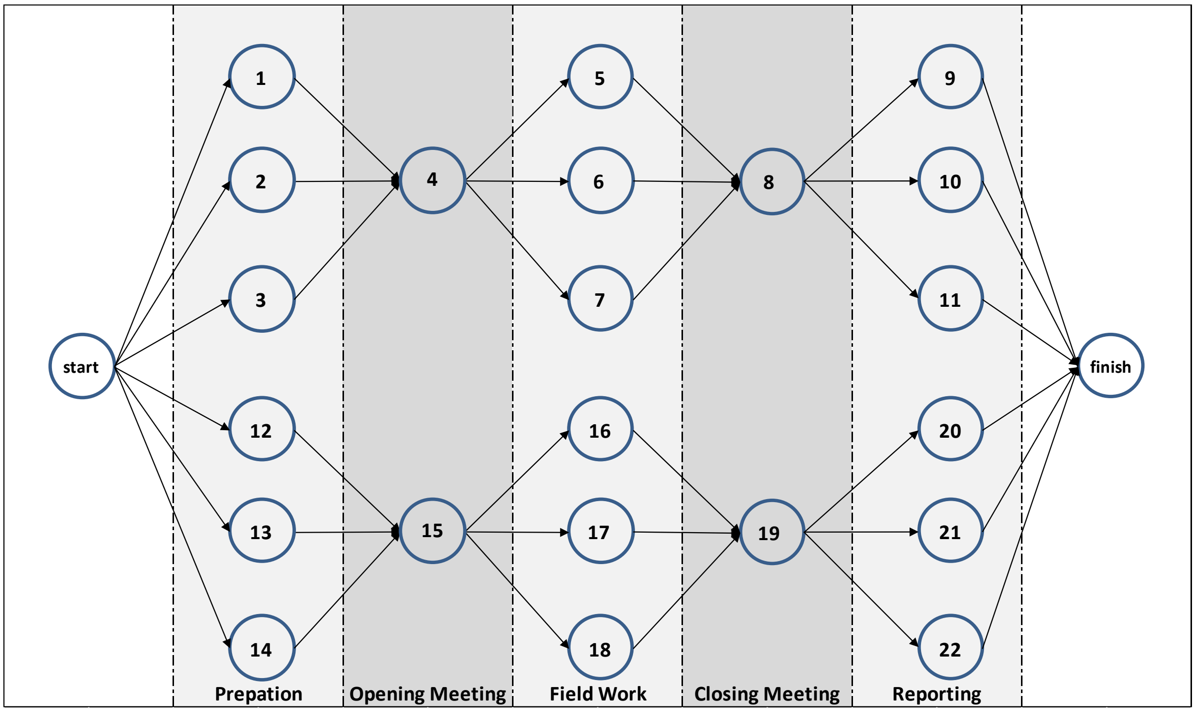

Figure 3.3: Multi-engagement project network

All required parameters for CASP are summarized in Table 3.5 for 2 engagements, 4 units, 3 MSs and 3 auditors. Until now, we think that all units under a department will be audited separately, but in practice it is possible to combine audits of units in an engagement into a single audit. Here, auditor will execute audit for one of unit and then pass to other. For example, in engagement 2 in Table 3.4, both unit 3 and unit 4 are audited from ISO 45001. Hence, it is possible to combine these audit and assume that it is a single audit. Combining audits in this manner may provide some benefits such as less executing time and fewer auditors who have same skills in an engagement.

Table 3.5: Tasks type in project network

Task No	Task Type	MS	Auditor	Task No	Task Type	MS	Auditor
0	start	-	-	12	P	A	1,2
1	P	A	1,2	13	P	C	2,3
2	P	B	1,3	14	P	C	2,3
3	P	A	1,2	15	OM	A, C	1,2,3
4	OM	A, B	1,2,3	16	E	A	1,2
5	E	A	1,2	17	E	C	2,3
6	E	B	1,3	18	E	C	2,3
7	E	A	1,2	19	CM	A,C	1,2,3
8	CM	A,B	1,2,3	20	R	A	1,2
9	R	A	1,2	21	R	C	2,3
10	R	B	1,3	22	R	C	2,3
11	R	A	1,2	23	finish	-	-
P: Preparation, OM: Opening Meeting, E: Execution, CM: Closing Meeting, R: Reporting, A: ISO9001, B: ISO14001, C: ISO45001							

4. AN MILP FORMULATION

In this section we will introduce an MILP formulation for the CASP. Let us define the notation of index and parametres.

Index:

i $= 1,2,\dots,I$ is index number of auditors and I is total number of auditors

j $= 1,2,\dots,J$ is index number of tasks and J is total number of tasks

s $= 1,2,\dots,S$ is index number of series where each series of tasks consists of preparation, opening meeting, execution, closing meeting and reporting task of an audit and S is total number of series

k $= 1,2 \dots,T$ is index number of time-periods and T total number of time-periods

e $= 1,2 \dots,E$ is index number of time-periods and E total number of time-periods

I_j set of auditors who are able to perform task *j*

J_i set of tasks which can be accomplished by auditor *i*

S^p set of preparation task in series of s

S^o opening meeting task in series of s

S^e executing task in series of s

S^c closing task in series of s

S^r reporting task in series of s

A_e set of starting tasks of engagement e

D_e set of finishing tasks of engagement e

Parameters:

LF_j latest finish time for task *j*

EF_j earliest finish time for task *j*

ES_j earliest start time for task *j*

LS_j latest start time of task *j*

p_j processing time for task *j*

$\Phi(j)$ set of tasks which precede task *j*

UT_i maximum available period for an auditor i

LT_i minimum available period for an auditor i

M_{big} a sufficiently big number which is calculated

Note that by definition $LS_j + P_j = LF_j$ and $ES_j + P_j = EF_j$ hold. Next, the binary decision variable for the MILP is defined as follows:

$$x_{ijk} = \begin{cases} 1, & \textit{if task j is assigned to auditor i at the end of period k} \\ 0, & \textit{otherwise} \end{cases}$$

Variable Z stands for objecticve function. This objective function firstly minimizes throughput time of engagements after that minimizes starting time of each tasks. Throughput time of an engagement is equal to finish time minus start time of the engagemet. Tasks are start as soon as possible by sum of finish times of tasks which is dived by sum of latest finish times of tasks.

$$Minimize\ Z = \left(\sum_{i \in I_j} \sum_{j \in A_e} \sum_{k=EF_j}^{LF_j} k.x_{ijk} - \sum_{i \in I_j} \sum_{j \in D_e} \sum_{k=EF_j}^{LF_j} k.x_{ijk} \right) + \left(\sum_{i \in I_j} \sum_{j \in J_i} \sum_{k=EF_j}^{LF_j} k.x_{ijk} \ / \sum_{j \in J_i} LF_j.x_{ijk} \right) \tag{1}$$

subject to

Constraint set (2) ensures that 3ach preparation task is assigned to an auditor.

$$\sum_{k=EF_j}^{LF_j} \sum_{i \in I_j} x_{ijk} = 1 \quad \text{for } j \in S^p \tag{2}$$

Constraint set (3) ensures that preparation tasks $m \in S^p$ and execution tasks $n \in S^e$in a series are assigned to same auditor.

$$\sum_{k=EF_m}^{LF_m} x_{imk} - \sum_{k=EF_n}^{LF_n} x_{ink} = 0 \quad \text{for each } s = 1,2,..,S\ and\ \ i \in (I_m \cap I_n) \tag{3}$$

Constraint set (4) ensures that preparation tasks $m \in S^p$ and reporting tasks $n \in S^r$ in a series are assigned to same auditor.

$$\sum_{k=EF_m}^{LF_m} x_{imk} - \sum_{k=EF_n}^{LF_n} x_{ink} = 0 \quad \text{for each } s = 1,2,..,S \text{ and } i \in (I_m \cap I_n) \tag{4}$$

Constraint set (5) ensures that each opening task $n \in S^o$ in a series are assigned to same auditor who does preparation task $m \in S^p$ in that series.

$$\sum_{k=EF_m}^{LF_m} x_{imk} - \sum_{k=EF_n}^{LF_n} x_{ink} \leq 0 \quad \text{for each } s = 1,2,..,S \text{ and } i \in (I_m \cap I_n) \tag{5}$$

Constraint set (6) ensures that each closing task $n \in S^c$ in a series are also assigned to same auditor who does preparation task $m \in S^p$ in that series.

$$\sum_{k=EF_m}^{LF_m} x_{imk} - \sum_{k=EF_n}^{LF_n} x_{ink} \leq 0 \quad \text{for each } s = 1,2,..,S \text{ and } i \in (I_m \cap I_n) \tag{6}$$

Constraint set (8) ensures that each auditor can process only one task at a given time. Assume v is the completion period of task j, then $EF_j \leq v \leq LF_j$ and $k \leq v \leq k + p(j) - 1$.

$$\sum_{j=1}^{J} x_{ijv} \leq 1 \quad \text{for each } k = 1,2,\dots,T \text{ and } i \in I_j. \tag{7}$$

Constraint set (8) ensures that a successor task can start after its precessedor if either successor or processor task is not opening and closing meeting.

$$\sum_{i\in I_j}\sum_{k=EF_m}^{LF_m} k.x_{imk} - \sum_{i\in I_j}\sum_{k=EF_n}^{LF_n}\big(k-p(n)\big).x_{ink} \le 0, \tag{8}$$
$$\text{for all } \ m \in \Phi(n), m \ \ and \ n \ \notin (S^o \cup S^c).$$

Constraint set (9) ensures that a successor task can start after its precessedor if only processor task is opening or closing meeting.

$$\sum_{i\in I_m}\sum_{k=EF_m}^{LF_m} k.x_{imk} - \sum_{k=EF_n}^{LF_n}(k-p(n)-M).x_{ink} - M \le 0, \tag{9}$$
$$\text{for } \ i \in I_n, \quad m \in \Phi(n), \quad m \notin (S^o \cup S^c) \ and \ \ n \in (S^o \cup S^c).$$

Constraint set (10) ensures that a successor task can start after its precessedor if only successor task is opening or closing meeting.

$$\sum_{k=EF_m}^{LF_m} k.x_{imk} - \sum_{i\in I_n}\sum_{k=EF_n}^{LF_n}\big(k-p(n)\big).x_{ink} \ \le 0, \tag{10}$$
$$\text{for } \ i \in I_m, \quad m \in \Phi(n), \quad m \in (S^o \cup S^c) \ and \ \ n \notin (S^o \cup S^c).$$

Constraint set (11) ensures that a meeting task has to be done exactly at the same time period by auditors who are assigned the meeting.

$$\sum_{k=EF_j}^{LF_j} k.x_{i_1jk} - \sum_{k=EF_j}^{LF_j}(M-k).x_{i_2jk} - M \ \le 0 \tag{11}$$
$$\text{for } \ i_1 \in I_j, \quad i_2 \in I_j, \quad i_1 \ne i_2 \ and \ j \in (S^o \cup S^c)$$

Constraint set (12) ensures that work load of an auditor can not exceed availability of the auditor.

$$\sum_{j\in J_i}\sum_{k=EF_j}^{LF_j} p(j).x_{ijk} - UT_i \le 0 \quad \text{for } i \in I_j \tag{12}$$

Constraint set (13) ensures that work load of an auditor can not be less than defined time period.

$$LT_i - \sum_{j\in J_i}\sum_{k=EF_j}^{LF_j} p(j).x_{ijk} \le 0 \quad \text{for } i \in I_j \tag{13}$$

Constraint (14) is non-negativity for the variable Z. Constraints (15) are for the binary restrictions on the x_{ijk} variables.

$$Z \ge 0 \tag{14}$$

$$x_{ijk} \in \{0,1\} \text{ for } i \in I_j; j \in J_i \tag{15}$$

5. ILLUSTRATIVE EXAMPLE

A real-life instance is illustrated for the CASP which consists of 4 engagements where each engagement is designed for a organization department. In this organization ISO9001 with ISO10002 and ISO14001 with ISO45001 are integrated. Audits are planned according to complexity of sub-processes and location of deparments and requirements of MSs. The number of audits planned for each engagement is determined as in Table 5.1. For ease of use, we give letters for MSs e.g letter A stands for ISO9001&ISO10002.

Table 5.1 Number of Audits Planned for Each MS and Department

Engagement No	**Department Name**	**A: ISO9001& ISO10002**	**B: ISO14001& ISO18001**	**C: ISO27001**	**Total**
1	Human Resource	1	1	1	3
2	Sales And Marketing	2	1	1	4
3	Engineering	2	1	1	4
4	Production Planning	3	1	1	5

After audits are determined, an activity on node (AON) graph of audit tasks can be created. The predence matrix for this problem is given at Appendix A. The series of each audit tasks on this network are depicted in Table 5.2.

Table 5.2 Series of Audit Tasks

Engagement No	Audit No	Prepation	Openning Meeting	Executing	Closing Meeting	Reporting
1	1	2	5	6	9	10
1	2	3	5	7	9	11
1	3	4	5	8	9	12
2	4	15	19	20	24	25
2	5	16	19	21	24	26
2	6	17	19	22	24	27
2	7	18	19	23	24	28
3	8	31	35	36	40	41
3	9	32	35	37	40	42
3	10	33	35	38	40	43
3	11	34	35	39	40	44
4	12	47	52	53	58	59
4	13	48	52	54	58	60
4	14	49	52	55	58	61
4	15	50	52	56	58	62
4	16	51	52	57	58	63

The other tasks remain outside of audit task represent dummy tasks where they depicted in Table 5.3.

Table 5.3 Dummy Tasks

Dummy Tasks No	Type of Dummy Task
0	Supper source
1	Predecessor of all tasks of engagement 1
13	Successor of all tasks of engagement 1
14	Predecessor of all tasks of engagement 2
29	Successor of all tasks of engagement 2
30	Predecessor of all tasks of engagement 3
45	Successor of all tasks of engagement 3
46	Predecessor of all tasks of engagement 4
64	Successor of all tasks of engagement 4
65	Supper sink

Processing time of each task is estimated by auditors. Preparation and executing tasks' processing time depends on complexity of audits. However, meeting and reporting tasks have fixed processing times. In this example, one-time period is equal to 4 hours before or after midday for working days.

Table 5.4 Processing Times

Task No	Processing Time	Task No	Processing Time	Task No	Processing Time	Task No	Processing Time
0	0	17	1	34	1	51	2
1	0	18	1	35	1	52	1
2	1	19	1	36	5	53	6
3	2	20	3	37	2	54	5
4	2	21	6	38	5	55	6
5	1	22	1	39	1	56	1
6	2	23	3	40	2	57	6
7	5	24	2	41	1	58	2
8	6	25	1	42	1	59	1
9	2	26	1	43	1	60	1
10	1	27	1	44	1	61	1
11	1	28	1	45	0	62	1
12	1	29	0	46	0	63	1
13	0	30	0	47	2	64	0
14	0	31	2	48	2		
15	1	32	1	49	2		
16	2	33	2	50	1		

The columns in Table 5.5, "ES", "LS", "EF" and "LF" corresponding to the earliest start time, the latest start time, the earliest finish times and the latest finish times respectively. These four parameters are calculated running the CPM algorithm.

Table 5.5 CPM Results

Task No	ES	EF	LS	LF	Task No	ES	EF	LS	LF
0	0	0	47	47	33	0	2	48	50
1	0	0	47	47	34	0	1	49	50
2	0	1	48	49	35	2	3	50	51
3	0	2	47	49	36	3	8	51	56
4	0	2	47	49	37	3	5	54	56
5	2	3	49	50	38	3	8	51	56
6	3	5	54	56	39	3	4	55	56
7	3	8	51	56	40	8	10	56	58
8	3	9	50	56	41	10	11	58	59
9	9	11	56	58	42	10	11	58	59
10	11	12	58	59	43	10	11	58	59
11	11	12	58	59	44	10	11	58	59
12	11	12	58	59	45	11	11	59	59
13	12	12	59	59	46	0	0	47	47
14	0	0	47	47	47	0	2	47	49
15	0	1	48	49	48	0	2	47	49
16	0	2	47	49	49	0	2	47	49
17	0	1	48	49	50	0	1	48	49
18	0	1	48	49	51	0	2	47	49
19	2	3	49	50	52	2	3	49	50
20	3	6	53	56	53	3	9	50	56
21	3	9	50	56	54	3	8	51	56
22	3	4	55	56	55	3	9	50	56
23	3	6	53	56	56	3	4	55	56
24	9	11	56	58	57	3	9	50	56
25	11	12	58	59	58	9	11	56	58
26	11	12	58	59	59	11	12	58	59
27	11	12	58	59	60	11	12	58	59
28	11	12	58	59	61	11	12	58	59
29	12	12	59	59	62	11	12	58	59
30	0	0	48	48	63	11	12	58	59
31	0	2	48	50	64	12	12	59	59
32	0	1	49	50					

Totally 6 auditors are available for these engagements. The first auditor has compatenence to perform tasks of MSs ISO9001, ISO10002, ISO14001 and ISO18001. The other auditors' skills are showed in Table 5.6.

Table 5.6 Auditors' Skills

Auditors	**A: ISO9001& ISO10002**	**B: ISO14001& ISO18001**	**C: ISO27001**
Auditor 1	1	1	0
Auditor 2	1	0	1
Auditor 3	1	1	1
Auditor 4	1	1	0
Auditor 5	0	0	1
Auditor 6	1	0	0

We solved the MILP formulation for 3 hours using Gurobi Optimizer v7.5.2 on a PC with a CPU Core i5 @ 2.3 GHz. A feasible CASP solution is obtained and presented with Gantt chart in Figure 5.1. Note that opening meeting and closing meeting, which are presented with tasks in yellow color, are performed by all assigned auditor at the same time period. The series of tasks which belong to an audit should be assigned to the same auditor. For example, the series of tasks 17, 19, 22, 24 and 27 which belong to audit 6 in Figure 5.1 are assigned to auditor 1. Engagement 1, 2, 3 and 4 are showed with orange, red, green and blue color respectively and throughput time of these engagements are 12, 12, 22 and 22 respectively. When we look the work load of auditors, auditor 6 is the most loaded who has 6 idle periods and auditor 5 is the least loaded who has 13 idle periods. This is a tolerable difference. The difference would be more if we do not set upper and lower bound for auditor capacity usage. Objective function firstly minimize total throughput time and secondly arrive time of tasks. Sum of throughput of engagements 80 time-periods that is 40 days. Tasks also start as soon as possible e.g., task 18 starts at period one instead of period two. If task 18 starts at period two, it does not affect throughput time of engagement 2.

Period	Auditor 1 (A,B)	Auditor 2 (A,C)	Auditor 3 (A,B,C)	Auditor 4 (A,B)	Auditor 5 (C)	Auditor 6 (A)
1	17 B	16 A	34 C	15 A	18 C	48 A
2	50 B		33 B	32 A		
3	19	19		19	19	49 A
4	47 A	21 A	31 A		51 C	
5						
6	52		35	35	52	52
7	22 B		39 C	20 A	23 C	54 A
8	56 B		36 A			
9						
10	24	24		24	24	
11						
12	27 B	26 A		25 A	28 C	55 A
13	53 A		38 B	37 A	57 C	
14						
15						
16						
17						
18						
19	58		40	40	58	58
20			43 B	42 A		
21	59 A		41 A		63 C	61 A
22	62 B	4 C	44 C	3 B		60 A
23						2 A
24		5		5		5
25		8 C		7 B		6 A
26						
27						
28						
29						
30						
31		9		9		9
32						
33		12 C		11 B		10 A

Figure 5.1: Gantt chart for the feasible CASP solution

6. CONCLUSION

Firstly, we discuss how to combine audits in different MSs and defined CASP. We present how to design AON for this problem in a multi-engagement (project) enviroment. We have proposed an MILP formulation for the CASP. In this problem we minimize throughput of engagements and after that minimize starting time of tasks. This objective is quite useful in real life. Problem specific constraints for the CASP are assigning many auditors at the same time to meeting tasks, assigning a series of task to the same auditor where tasks belong to a MS's audit. A real-life problem via a commercial MILP solver is solved. Even if we have not reached to optimal solution because CASP is an NP-hard problem, we obtained sufficient feasible solutions in reasonable computational time. We estimate size of problem for feature works as in Table 6.1

Table 6.1: Problem Size

Features	Small	Medium	Large
Engagements	3	10	18
MSs	3	4	5
Tasks	100	400	800
Auditors	8	20	30
Periods	120	240	480

We have solved small problem with MILP, however, tailor-made algorithms can be developed to solve large-scale problems.

REFERENCES

Abad, J., Dalmau, I., & Vilajosana, J. (2014). Taxonomic proposal for integration levels of management systems based on empirical evidence and derived corporate benefits. *Journal of Cleaner Production*, *78*, 164-173.

Almeida, B. F., Correia, I., & Saldanha-da-Gama, F. (2016). Priority-based heuristics for the multi-skill resource constrained project scheduling problem. *Expert Systems with Applications*, *57*, 91-103.

Balachandran, B. V., & Zoltners, A. A. (1981). An interactive audit-staff scheduling decision support system. *Accounting Review*, 801-812.

Beşikci, U., Bilge, Ü., & Ulusoy, G. (2015). Multi-mode resource constrained multi-project scheduling and resource portfolio problem. *European Journal of Operational Research*, *240*(1), 22-31.

Bernardo, M., Casadesus, M., Karapetrovic, S., & Heras, I. (2010). An empirical study on the integration of management system audits. *Journal of Cleaner Production*, *18*(5), 486-495.

Bernardo, M., Simon, A., Tarí, J. J., & Molina-Azorín, J. F. (2015). Benefits of management systems integration: a literature review. *Journal of Cleaner Production*, *94*, 260-267.

Bernardo, M., Gotzamani, K., Vouzas, F., & Casadesus, M. (2018). A qualitative study on integrated management systems in a non-leading country in certifications. *Total Quality Management & Business Excellence*, *29*(3-4), 453-480.

Blazewicz, J., Lenstra, J. K., & Kan, A. R. (1983). Scheduling subject to resource constraints: classification and complexity. *Discrete Applied Mathematics*, *5*(1), 11-24.

Bowman, E. H. (1959). The schedule-sequencing problem. *Operations Research*, *7*(5), 621-624.

Brucker, P., Drexl, A., Möhring, R., Neumann, K., & Pesch, E. (1999). Resource-constrained project scheduling: Notation, classification, models, and methods. *European Journal of Operational Research*, *112*(1), 3-41.

Chan, K. H., & Dodin, B. (1986). A decision support system for audit-staff scheduling with precedence constraints and due dates. *Accounting Review*, 726-734.

Chang, C. J. (2002). A decision support system for audit staff scheduling of multiple and large-scaled engagements. *Review of Business Information Systems (RBIS)*, *6*(1), 27-40.

Çanakoğlu, E., Muter, İ., & Adanur, O. (2018). Audit Scheduling in Banking Sector. In *Operations Research Proceedings 2016* (pp. 499-505). Springer, Cham.

Davis, E. W. (1973). Project scheduling under resource constraints—historical review and categorization of procedures. *AIIE Transactions*, *5*(4), 297-313.

De Bruecker, P., Van den Bergh, J., Beliën, J., & Demeulemeester, E. (2015). Workforce planning incorporating skills: State of the art. *European Journal of Operational Research*, *243*(1), 1-16.

Dodin, B., & Chan, K. H. (1991). Application of production scheduling methods to external and internal audit scheduling. *European Journal of Operational Research*, 52(3), 267-279.

Dodin, B., & Elimam, A. A. (1997). Audit scheduling with overlapping activities and sequence-dependent setup costs. *European Journal of Operational Research*, 97(1), 22-33.

Domingues, P., Sampaio, P., & Arezes, P. M. (2016). Integrated management systems assessment: a maturity model proposal. *Journal of Cleaner Production*, *124*, 164-174.

Drexl, A. (1991). Scheduling of project networks by job assignment. Management Science, 37(12), 1590-1602.

Dumond, J., & Mabert, V. A. (1988). Evaluating project scheduling and due date assignment procedures: an experimental analysis. *Management Science*, *34*(1), 101-118.

Douglas, A., & Glen, D. (2000). Integrated management systems in small and medium enterprises. *Total Quality Management*, 11(4-6), 686-690.

Herroelen, W., De Reyck, B., & Demeulemeester, E. (1998). Resource-constrained project scheduling: a survey of recent developments. *Computers & Operations Research*, *25*(4), 279-302.

Hoy, Z., & Foley, A. (2015). A structured approach to integrating audits to create organisational efficiencies: ISO 9001 and ISO 27001 audits. *Total Quality Management & Business Excellence*, *26*(5-6), 690-702.

Karapetrovic, S., & Willborn, W. (1998). Integrated audit of management systems. *International Journal of Quality & Reliability Management*, *15*(7), 694-711.

Karapetrovic, S., & Willborn, W. (2000). Quality assurance and effectiveness of audit systems. *International Journal of Quality & Reliability Management*, 17(6), 679-703.

Kolisch, R. (1996). Serial and parallel resource-constrained project scheduling methods revisited: Theory and computation. *European Journal of Operational Research*, *90*(2), 320-333.

Kolisch, R. (2013). *Project scheduling under resource constraints: efficient heuristics for several problem classes*. Springer Science & Business Media.

Kolisch, R., & Hartmann, S. (2006). Experimental investigation of heuristics for resource-constrained project scheduling: An update. *European journal of operational research*, *174*(1), 23-37.

Kraus, J. L., & Grosskopf, J. (2008). Auditing integrated management systems: considerations and practice tips. *Environmental Quality Management*, 18(2), 7-16.

Krüger, D., & Scholl, A. (2009). A heuristic solution framework for the resource constrained (multi-) project scheduling problem with sequence-dependent transfer times. *European Journal of Operational Research*, *197*(2), 492-508.

Moumen, M., & El Aoufir, H. (2017). Quality, safety and environment management systems (QSE): Analysis of empirical studies on integrated management systems (IMS). *Journal of Decision Systems*, *26*(3), 207-228.

Naber, A., & Kolisch, R. (2014). MIP models for resource-constrained project scheduling with flexible resource profiles. *European Journal of Operational Research*, *239*(2), 335-348.

Nunhes, T. V., Vilamitjana, M. B., & Oliveira, O. J. (2018). Guiding principles of integrated management systems: Towards unifying a starting point for researchers and practitioners. *Journal of Cleaner Production*.

Islamova, O. V., & Volkova, R. M. (2017, September). Effectiveness of internal audit of processes in the organization. *Quality Management, Transport and Information Security, Information Technologies* (IT&QM&IS), 2017 International Conference (pp. 425-427). IEEE.

Pinedo, M. L. (2016). Scheduling: theory, algorithms, and systems, fifth edn, Springer New York.

Pritsker, A. A. B., Waiters, L. J., & Wolfe, P. M. (1969). Multiproject scheduling with limited resources: A zero-one programming approach. *Management Science*, *16*(1), 93-108.

Ribeiro, F., Santos, G., Rebelo, M. F., & Silva, R. (2017). Integrated Management Systems: trends for Portugal in the 2025 horizon. *Procedia Manufacturing*, *13*, 1191-1198.

Russell, J. P. (Ed.). (2012). *The ASQ auditing handbook*. ASQ Quality Press.

Salomone, R. (2008). Integrated management systems: experiences in Italian organizations. *Journal of cleaner production*, *16*(16), 1786-1806.

Savino, M. M., & Batbaatar, E. (2015). Investigating the resources for Integrated Management Systems within resource-based and contingency perspective in manufacturing firms. *Journal of Cleaner Production*, 104, 392-402.

Simon, A., Bernardo, M., Karapetrovic, S., & Casadesús, M. (2011). Integration of standardized environmental and quality management systems audits. *Journal of Cleaner Production*, *19*(17-18), 2057-2065.

Simon, A., Yaya, L. H. P., Karapetrovic, S., & Casadesús, M. (2014). An empirical analysis of the integration of internal and external management system audits. *Journal of Cleaner Production*, *66*, 499-506.

Singh, A. (2014). Resource constrained multi-project scheduling with priority rules & analytic hierarchy process. *Procedia engineering*, *69*, 725-734.

Salewski, F., Schirmer, A., & Drexl, A. (1997). Project scheduling under resource and mode identity constraints: Model, complexity, methods, and application. *European Journal of Operational Research*, *102*(1), 88-110.

Summers, E. L. (1972). The audit staff assignment problem: a linear programming analysis. *The Accounting Review*, *47*(3), 443-453.

Wilkinson, G., & Dale, B. G. (1999). Integrated management systems: an examination of the concept and theory. *The TQM Magazine*, 11(2), 95-104.

Zutshi, A., & Sohal, A. S. (2005). Integrated management system: the experiences of three Australian organisations. *Journal of Manufacturing Technology Management*, *16*(2), 211-232.

BIOGRAPHICAL SKETCH

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