PORTFOLIO OPTIMIZATION WITH SYSTEMIC RISK AND LIQUIDITY RISK FACTORS

(SİSTEMİK VE LİKİDİTE RİSK FAKTÖRLERİYLE PORTFÖY OPTİMİZASYONU)

by

Fatih ALALİ, M.S.

Thesis

Submitted in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

in

INDUSTRIAL ENGINEERING

in the

GRADUATE SCHOOL OF SCIENCE AND ENGINEERING

of

GALATASARAY UNIVERSITY

May 2019

This is to certify that the thesis entitled

PORTFOLIO OPTIMIZATION WITH SYSTEMIC AND LIQUIDITY RISK FACTORS

prepared by **Fatih ALALİ** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy in Industrial Engineering** at the **Galatasaray University** is approved by the

Examining Committee:

Assoc. Prof. A. Çağrı TOLGA (Supervisor)
Department of Industrial Engineering
Galatasaray University -------------------------

Assoc. Prof. Emre ALPTEKİN
Department of Industrial Engineering
Galatasaray University -------------------------

Assoc. Prof. Sezi Çevik ONAR
Department of Industrial Engineering
İstanbul Technical University -------------------------

Prof. Dr. H. Ziya ULUKAN
Department of Industrial Engineering
Galatasaray University -------------------------

Prof. Dr. Cengiz KAHRAMAN
Department of Industrial Engineering
İstanbul Technical University -------------------------

Date: -------------------------

ACKNOWLEDGEMENTS

The completion of this thesis would not have been possible without the support of my family and my advisor. My wife, my mother and my sister were always there for me and respected the time that I spent for my thesis while I was also working full-time. Implicitly, there are countless hours, silent days, sleepless nights in this thesis.

My advisor, A. Cagri Tolga always supported me during the toughest times at my job and gave me courage to continue and brought out best in me. Also, Thesis Committee Members, Emre Alptekin and Sezi Çevik Onar had encouraged me by giving constructive comments during my study.

I also would like to thank you to all my teachers, whether academicals or professional, that helped me to achieve my strong background in professional and academical life. It would have been impossible to complete this thesis while I was working without having an established background.

Finally, I should admit that I am very proud of what I have done, and this thesis transformed the way I approach the problems in finance and risk.

May 2019
Fatih ALALİ

TABLE OF CONTENTS

LIST OF SYMBOLS

1/N	: Naively diversified portfolios, equally weighted portfolios.
ES	: Expected Shortfall
MCDM	: Multi-Criteria Decision Making
MTMA	: Markov Transition Matrix Approach
MVP	: Minimum Variance Portfolio
OP	: Observation Period (Number of Quarters)
P1	: %1 Percentile
P10	: %10 Percentile
P25	: %25 Percentile
P5	: %5 Percentile
Ret	: Return
SR	: Sharpe Ratio
VaR	: Value-at-Risk

TABLE OF FIGURES

LIST OF TABLES

ABSTRACT

Evolution of portfolio optimization is expected in the direction that satisfied the needs of traders by answering the current status of global markets. In the past decade, previously unknown new risk types that are needed to be taken into consideration in the portfolio optimization process has emerged. These risk types are systemic risk and liquidity risk. Even without the economic new landscape, continuous improvement of portfolio optimization results is an ongoing challenge of both academicians and practitioners. My original contribution to knowledge is by evaluating new risk measures from portfolio optimization perspective, analyzing portfolio optimization results in detail based on empirical data and developing a new methodology, Markov Transition Matrix Approach for return estimation. In addition to this, I adapted an emerging multi-criteria decision-making methodology, TODIM, to portfolio optimization.

In the wake of Global Financial Crisis of 2007-2009, a lot of academical, regulatory and professional work has been focused on identifying the weaknesses in the financial system. As financial sector became more integrated by the support of technology and globalization, it has been found that the interconnectedness in the system would create a domino effect based on the recent costly financial crisis experience.

It is clear that new problems can't be solved with the old tools that caused the new problems. In this regard, systemic risk is defined as a risk type. Numerous systemic risk measures have been proposed to identify and monitor the interconnectedness in the financial system.

Systemic risk doesn't affect only financial institutions but also all companies in the financial sector including hedge funds, investors etc. In this regard, portfolio optimization problems need to take care of the systemic risk by incorporating systemic risk parameters in the problem. Also, another crucial rarely addressed risk is the liquidity risk. Likewise, liquidity risk remained one of the critical risks during the financial crisis.

In this thesis, in first section, I summarized the purpose of the thesis along with different parts of the thesis. I also provided the connection between different sections of the thesis to facilitate the readability of the thesis.

In the second section, as part of the literature review, I analyzed various systemic risk and liquidity risk measures and assessed these risk measures from portfolio optimization angle. I put forward the liquidity and systemic risk measures that may be useful for portfolio optimization problem.

In third section, I analyzed most common risk measures and return measures in terms of predictability. In essence, all portfolio optimization problems with naïve return and risk estimates rely on a simplistic assumption: "Past is the best predictor of the future". I tested this assumption empirically based on US Equity Data from 2000 to 2016 and I also tested this assumption based on different horizon from 1 quarter to 12 quarters. I also tested this assumption for different measures such as return, variance, skewness, kurtosis, value-at-risk (VaR), expected shortfall (ES). I also tested multivariate relationship between measures. For instance, I tried to answer if past kurtosis data could be a predictor of future return. On a high level I found that, past return is a very poor predictor of future return. However, past risk is a good predictor of future risk. The level of the predictive power varies across the risk measures. I'll examine in detail the results of this phase in the thesis.

In the fourth section, I tried to assess the performance of a simple portfolio optimization by running 36.000 different portfolio optimization problems by using empirical US Equity Data from 2000 to 2016. The motivation behind this phase was to maximize the efficiency of the portfolio optimization problem by simply solving the problem with right constraints. As an output of this phase, I found out the most efficient portfolio optimization problems without changing the input estimation methods. Naïve return and risk (variance, covariance) estimation techniques are used. In fourth section, as foreseen in the second section, I found that optimal portfolios with aggressive return targets have lower risk-adjusted returns. Also, long-short portfolios yield lower risk-adjusted returns compared to long-only portfolios. Moreover, minimum variance portfolios (MVP) dominate equally-weighted portfolios based on empirical data. These results brought us to our final phase. It is clear that too much power is given to return predictions by either setting aggressive return targets or allowing short-selling the risk-adjusted returns of portfolios decreases. Improving return estimation and incorporating these estimates into portfolio optimization problems is one of the key challenges in the literature. I devised a new methodology for this in the fourth phase.

In the fifth section, Markov Transition Matrix Approach (MTMA) is devised to estimate returns for portfolio optimization. In short, this methodology relies on using past data efficiently by allocating each equity to a bin and then predicting returns for each bin by using the state transition probabilities of each bin. State transition probabilities are calculated based on empirical data. This methodology produces higher risk-adjusted results compared to optimal portfolios based on naïve return estimate inputs when return target is aggressive. This methodology combines both academical and practitioners' methodologies by incorporating the momentum and mean-reversal effect implicitly. From this angle, this is the unique methodology that can incorporate conflicting effects in the same time based on empirical data. What is more, this methodology may be used with alternative portfolio optimization methodologies and configurations. All the investor has to is to use point estimates of this methodology.

In six section, I utilized an emerging multi-criteria decision-making methodology, TODIM, (Portuguese acronym for interactive and multicriteria decision-making) for portfolio allocation. The main parameters to study in this section is the input weights and relevant inputs for decision making. I adapted the study to offer portfolio allocation weights rather than offering a single winner stock. To my knowledge, this is the first study incorporating TODIM for portfolio allocation.

In conclusion section, the results, implications of this thesis and further academical work that can be done is summarized. The main conclusion would be summarized as below: "Improvement of return estimation methodologies is an infinite journey that will last forever as the dynamics of the markets and investor philosophies will not stay same forever. What can be done resides on the efficient usage of data. This thesis approaches the return estimation from this angle by utilizing the data more efficiently for portfolio optimization problems. However, the same Markov Transition Approach methodology could be adapted for different purposes as well. Another improvement angle would be utilizing TODIM method for portfolio allocation to mitigate risk of noise in estimates or incorrect risk and return estimates, form more diversified portfolios providing better risk-adjusted returns than equally weighted portfolios"

Keywords: Portfolio optimization, TODIM, Markov Transition Matrix Approach, Systemic risk, Liquidity risk

ÖZET

Portföy optimizasyonunun, küresel piyasaların mevcut durumuna cevap vererek borsacıların (trader) ihtiyaçlarını karşılayan bir doğrultuda gelişmesi beklenmektedir. Son on yılda, portföy optimizasyon sürecinde daha önceden bilinmeyen yeni risk türleri ortaya çıktı. Bu risk türleri, sistemik risk ve likidite riskidir. Ekonomik yeni gelişmeler olmadan bile, portföy optimizasyonun sürekli geliştirilmesi akademisyenler ve profesyoneller için devam eden bir zorluktur. Literatüre olan özgün katkım, portföy risk optimizasyon perspektifinden yeni risk ölçütlerini değerlendirmek, portföy optimizasyon sonuçlarını ampirik verilere göre detaylı olarak analiz etmek ve yeni bir metodoloji olan Markov Geçiş Matrisi Yaklaşımı geliştirmektir. Buna ek olarak, ortaya çıkan çok kriterli bir karar verme metodolojisi olan TODIM'i portföy optimizasyonuna uyarladım.

2007-2009 dönemi Küresel Finansal Krizin ardından, finansal sistemdeki zayıflıkları belirlemek için akademik, düzenleyici ve mesleki çalışmalara ağırlık verildi. Finansal sektör, teknoloji ve küreselleşmenin desteğiyle bütünleştikçe, sistemdeki birbirine bağlılığın, son zamanlarda yaşanan maliyetli mali kriz deneyimine dayanan bir domino etkisi yaratacağı keşfedildi.

Yeni sorunların, bu sorunlara neden olan eski araçlar ile çözülemediği açıktır. Bu bağlamda, sistemik risk yeni bir risk türü olarak tanımlanır. Finansal sistemdeki birbirine bağlılığı belirlemek ve izlemek için çok sayıda sistemik risk ölçütü önerilmiştir.

Sistemik risk yalnızca finansal kuruluşları değil aynı zamanda finansal korunma fonları, yatırımcılar da dahil olmak üzere finansal sektördeki tüm şirketleri etkilemektedir. Bu bağlamda, portföy optimizasyon problemleri, sistemik risk parametrelerini probleme dahil ederek sistemik riski göze almak zorundadır. Ayrıca, nadiren ele alınan bir diğer önemli risk, likidite riskidir. Ayrıca, likidite riski, finansal kriz sırasında yaşanan kritik risklerden biri olmayı sürdürmüştür. Bu tezde, birinci aşamada, çeşitli sistemik risk ve likidite risk ölçümlerini analiz ettim ve bu risk önlemlerini portföy optimizasyon açısından değerlendirdim. Portföy optimizasyon problemi için faydalı olabilecek likidite ve sistemik risk ölçütlerini öne sürdüm.

Bu tezde, ikinci bölümde tezin amacını tezin farklı bölümleriyle birlikte özetledim. Ayrıca, tezin okunabilirliğini kolaylaştırmak için tezin farklı bölümleri arasındaki bağlantıyı da detaylandırdım.

Üçüncü bölümde, en sık görülen risk ölçütlerini analiz ettim ve öngörülebilirlik bakımından önlemleri değerlendirdim. Özünde, naif getiri ve risk tahminleriyle birlikte tüm portföy optimizasyon problemleri basit bir varsayıma dayanıyor : "Geçmiş, geleceğin en iyi tahmincisidir". Bu varsayımı ampirik olarak 2000'den 2016'ya kadar olan ABD Borsa Verilerine dayanarak test ettim ve aynı zamanda bu varsayımı 1 çeyrekten 12 çeyreğe kadar farklı ufuklara dayalı olarak test ettim. Ayrıca, bu varsayımı, getiri, varyans, çarpıklık, kurtosis, riske maruz değer (VaR), beklenen düşüş (Expected Shortfall) gibi farklı ölçütler için test ettim. Ölçümler arasındaki çok değişkenli ilişkiyi de test ettim. Örneğin kurtoz verilerini geçmişi, gelecekteki dönüşün bir öngörüsü olabilir mi? Geçmiş getirinin gelecekteki getirinin çok zayıf bir öngörüsü olduğunu buldum. Bununla birlikte, geçmiş risk gelecekteki risklerin iyi bir belirleyicisi olduğunu teyit ettim. Tahmini gücün seviyesi, risk ölçütleri arasında değişmektedir. Tezin bu aşamasının sonuçlarını ayrıntılı olarak inceleyeceğim.

Dördüncü bölümde, sistemik risk ve likidite risk ölçümlerine bakılmaksızın, 2000'den 2016'ya kadar ampirik ABD Borsa Verileri kullanarak 36.000 farklı portföy optimizasyon problemi yürüterek basit bir portföy optimizasyonunun performansını değerlendirmeye çalıştım. Bu aşamanın arkasındaki motivasyon, problemi doğru kısıtlamalarla çözerek portföy optimizasyon sorununun verimliliğini en üst düzeye çıkarabilmekti. Bu fazın bir çıktısı olarak, girdi tahmin yöntemlerini değiştirmeden en verimli portföy optimizasyon problemlerini buldum. Naif getiri ve risk (varyans, kovaryans) tahmin teknikleri kullanıldı. Dördüncü bölümde, üçüncü bölümde öngörüldüğü gibi, agresif getiri hedefleri olan optimal portföylerin, risk düzeltilmiş getirilerin daha düşük olduğunu keşfettim. Ayrıca, uzun ufuklu portföyler, yalnızca uzun vadeli portföylere kıyasla daha düşük risk ayarlamalı getiri getirdiğini gördüm. Buna ek olarak, minimum varyanslı portföyler, ampirik verilere dayalı olarak eşit ağırlıklı portföylerden daha iyi performans gösterdiğini teyit ettim. Bu sonuçlar bizi bir sonraki bölümümüze getirdi. Açık oturumda, agresif geri dönüş hedefleri belirleyerek veya portföylerin riske ayarlı getirilerinin kısa sürede satılmasına izin vererek öngörüler dönmek için çok fazla güç verildiğinde azaldığı açıktır. Geri dönüş tahmininin iyileştirilmesi ve bu tahminlerin portföy optimizasyon problemlerine dahil edilmesi, literatürdeki en önemli zorluklardan biridir. Bunun için beşinci bölümde yeni bir metodoloji geliştirdim.

Beşinci bölümde, portföy optimizasyonu için getirileri tahmin etmek için Markov Geçiş Matrisi Yaklaşımı (MTMA) tasarlanmıştır. Kısacası, bu metodoloji, her hisseyi bir bölmeye atayarak geçmiş verilerin verimli bir şekilde kullanılmasına ve her bölmenin durum geçiş olasılıklarını kullanarak her bölmenin getirilerini tahmin etmesine dayanmaktadır. Markov geçiş olasılıkları ampirik verilere dayanarak hesaplanır. Bu metodoloji, getiri hedefi agresif olduğunda naif dönüş tahmin girişlerine dayalı optimal portföylere kıyasla daha yüksek risk ayarlamalı getiriler üretir. Bu metodoloji, akademik ve profesyonellerin yöntemlerini, ivme (momentum) ve ortalama ters (reversal) etkisini örtüşerek birleştirir. Bu açıdan, çelişkili etkileri içererek empirik veriye dayanan benzersiz metodolojidir. Dahası, bu metodoloji alternatif portföy optimizasyon metodolojileri ve yapılarıyla ile kullanılabilir. Yatırımcının tek yapması gereken şey, bu metodolojinin getiri tahminlerini kullanmaktır.

Altıncı bölümde, portföy tahsisi için yeni ortaya çıkan çok kriterli bir karar verme metodolojisi olan TODIM (Portekizce etkileşimli ve çok kriterli karar vermenin kısaltması) kullandım. Bu bölümde çalışılacak ana parametreler, girdi ağırlıkları ve karar verme için ilgili girdilerdir. TODIM metodolojisini tek bir kazanan hisse senedi önermek yerine portföy ağırlıkları sunacak şekilde uyarladım. Bildiğim kadarıyla, bu portföy tahsisi için TODIM kullanımını içeren ilk çalışma özelliğini taşımaktadır.

Sonuç bölümünde, bu tezin sonuçları, sonuçları ve yapılacak daha akademik çalışma özetler. Temel sonuç şu şekilde özetlenebilir: "Getiri tahminleme metodolojilerinin geliştirilmesi sonsuza dek süren sonsuz bir yolculuk olup piyasaların dinamikleri ve yatırımcı felsefeleri sonsuza dek aynı kalmayacaktır. Yapılacabilecek ek şeyler verinin etkin kullanımı üzerineder. Bu açıdan, bu tez, portföy optimizasyon problemleri için veriyi daha etkin bir şekilde kullanarak getiri tahminini Markov geçiş yöntemiyle portföy optimizasyonuda kullanmaktadır. Buna ek olarak, TODIM methodu adapte edilerek, getiri ve risk tahminlerindeki gürültünün (noise) etksini azaltarak, eşit ağırlıklı portföylere göre daha iyi riske duyarlı getiri üreten ve diğer portföylere göre daha fazla çeşitlendirmiş portföyler üretilmesini sağlayacak metodoloji geliştirilmiştir.

Anahtar sözcükler: Portföy optimizasyonu, TODIM, Markov Geçiş Matrisi yaklaşımı, Sistemik risk, Likidite riski

1. INTRODUCTION

This thesis is focused on portfolio optimization problems taking into account recent events including financial crisis of 2007-08. Output of a portfolio optimization can be improved by two different angles: 1) Improving inputs of portfolio optimization problem 2) Improving portfolio optimization process. In this regard, I divided my efforts into 5 parts:

1) Improving inputs of portfolio optimization problem:
    a. Analysis of new risk measures as an input for portfolio optimization problem: Based on the recent financial crisis, I researched the potential systemic risk measures and liquidity risk measures and their integratability for portfolio optimization problem. Second section provides a detailed research and literature review on this aspect of the thesis.
    b. Analysis of traditional risk and return inputs for portfolio optimization problem: In general, most portfolio optimization studies rely on return and risk inputs based on historical data only and in most of the cases these inputs are only sample estimates derived from historical data. This generalization is not only relevant for portfolio optimization but also for risk applications in other fields such as credit risk management as well. In this part, I analyzed the reliability of risk and return inputs on portfolio
    c. optimization based on S&P 500 data. Third section of the thesis is devoted to this analysis.
    d. Improving return estimation process with Markov Transition Matrix Approach: After quantitatively demonstrating the unreliability of return estimates based on pure historical data, which is advocated in a lot of

studies, I proposed a new method for calculating return estimates, Markov Transition Matrix Approach

2) Improving portfolio allocation process:
    a. Performance analysis of traditional portfolio optimization problem: In order to improve the performance of a portfolio optimization problem, one should first start with analyzing the out-of-sample performance of a classical portfolio optimization problem. A detailed analysis on this aspect is managed in forth section.
    b. Adaptation of TODIM method for portfolio allocation: I adapted an emerging MCDM methodology, TODIM, for portfolio allocation purposes. To my knowledge, this study is the first study adapting TODIM method for portfolio allocation purposes. Sixth section provides all the details regarding this study.

2. LITERATURE REVIEW

Literature review section is divided into three to focus on different contributing parts of this thesis. Systemic risk measures, liqudidity risk measures and portfolio optimization subsections will provide more details as the fundamental starting point of this thesis.

2.1. Literature Summary on Systemic Risk Measures

Broadly used systemic risk definition is “any set of circumstances that threatens the stability of or public confidence in the financial system” (Billio et al, 2012) Systemic risk has two main elements: shocks and propagation mechanisms. There are two types of shocks: idiosyncratic and systematic. Idiosyncratic shocks are caused by change of the price of a single institution while systemic shocks are caused by the comovements of prices of multiple institutions at the same time. Propagation is the transmission of the shock to the other markets, institutions and sectors and as a result effect to the economy. Propagations may occur through two main channels: (i) domino effects (ii) imperfect information. As a result, the cause of propagations may both be irrational and rational. Propagations may lead to serious economic crisis. In a fragile financial system, propagations may happen easily and quickly. Nowadays, propagations may occur rapidly through financial markets. The effect of the propagations is widely knowns as spillover effects in the current academical literature.

Accurate and timely measurement of systemic risk measures is one of the top priorities in the agenda of regulators and government. Due to the complex nature of systemic risk, it is hard to gauge the systemic risk behavior with one single measure. Different systemic risk measures are needed in order to monitor and measure systemic risk from different angles. Based on the requirement and possible action, frequency of calculation of systemic risk measures may change from intraday to quarterly periods.

One of the challenges of measuring systemic risk is that it generally builds in times of stable economy, low volatility and emerge during crisis. So, systemic risk may rise meanwhile even the current market data could show no sign. This is one of the challenges and also reasons of measuring systemic risk. Regulators, banks and other financial institutions could see no sign of a crisis risk in market data even though it is just behind the door.

Another notion that is introduced after the crisis is Too Big to Fail Institutions which corresponds to institutions that create systemic risk in the event of a default or credit rating change. Some systemic risk measures are also used to identify the Too Big to Fail Institutions with a quantitative perspective. There is also a debate on which type of institutions are causing systemic risk. Banks are clearly contributing to systemic risk but also insurance companies, hedge funds and even other non-financial companies may contribute to systemic risk.

Bisias et al. (2012) has provided an extensive overview of current systemic risk measures. Different grouping of systemic risk measures can be established. Systemic risk measures can be divided into two different groups based on regulatory emphasis: systemic risk measures based on micro prudential analytics and systemic risk measures based on macroprudential analytics. Systemic risk measures can also be categorized based on institution types: securities and commodities, banks, insurance and pensions, and general applications. For the sake of keeping the focus on the thesis, I will invest more time on banks and general applications with a deeper focus on CoVaR. Although liquidity measures can be categorized as a subsegment of securities and commodities, I will investigate liquidity risk measures in a different section apart from systemic risk measures.

A growing literature apart from the categories mentioned in the paragraph above is related to agent-based modelling. Agent-based modelling is an area of modelling traders, institutions as agents with interactions between each other as a system and computing risk measures. Agent-based modelling requires simulation techniques and computational power. Although I won't go into detail on the agent-based modelling, it would worth mentioning for further research.

Since some of the systemic risk measures may require additional disclosures by financial institutions, they are not currently applicable as an input to portfolio optimization. However, there is a growing pressure on financial institution for additional and more frequent disclosure. In five years, we may have richer data for detecting systemic risk for both taking regulatory action and executing trading strategies based on the data. Dodd-Frank Act will clearly set new standards for disclosure by systematically important financial institutions (SIFIs) and has given authority to Office of Financial Research (OFR) to induce SIFIs to provide the necessary information.

As we expect the number of systemic risk measures grow with new disclosure rules, optimization of selection of systemic risk measures both by regulators and traders could be an area for further research in future. IMF's paper (2009) also highlight that we need to tighten data gap between countries, establish rules for transparency and more frequent disclosure. I believe that as multinational institutions put forth these kinds of medications to financial system, regulators will require new data disclosure standards. One of the key sign is the new BCBS 239 Principles for Effective Risk Data Aggregation and Risk Reporting document issued by Basel Committee for Banking Supervision. In near future, regulators, researchers and traders will see a surge in financial data on institution level. This surge in data will improve applicability of this thesis as well.

A Survey of Systemic Risk Measures article clearly summarizes the rich amount of systemic risk measures are created. Since we have a rich amount of well-defined systemic risk measures, I don't spend time to create new systemic risk measures. I will rather try to analyze these systemic risk measures from portfolio optimization perspective.

Data requirement for portfolio optimization is a critical factor to identify the right systemic risk measures for portfolio optimization. Since portfolio optimization models are used by traders, I will have to use systemic risk measures that can be calculated based on public data. Another factor to consider is the frequency of data. Weekly/daily data could be more reliable to use for portfolio optimization. Based on the data requirements, the systemic risk factors worth investigating could be summarized as below:

- Conditial Value at Risk (CoVaR)
- Co-Risk
- Crowded Currency Trades
- Marginal and Systemic Expected Shortfall
- Multivariate Density Estimator
- Option iPoD
- Principal Components – Absorption Ratio

2.1.1. Conditional Value at Risk (CoVaR)

CoVaR measure is proposed by Adrian et al. (2011). CoVaR is defined as the difference between the conditional value at risk of the financial system conditional on an institution being in distress and the CoVaR conditional on the median state of the institution. This difference, ΔCoVaR, indicates the contribution of an institution to the overall systemic risk.

CoVaR calculations have been proposed and examined for different markets. Arias et al (2010) and Bjarnadottir (2012) focused on Columbian and Swedish markets respectively. One of the crucial comments for this thesis was that CoVaR is a systemic risk indicator that is easy to interpret, does not need complicated data set and can be used with other risk indicators.

There are various methods to estimate CoVaR. Quantile regressions, GARCH models, time-varying second moments, Bayesian methods are widely used methods for estimating CoVaR. Quantile regression method is used in Adrian and Brunnermeir's (2011) paper.

Quantile regression, first introduced by Koenker et al (1978), is based on calculating regression coefficients by dividing the sample space to the quantiles of the independent variable. The motivation behind quantile regression is that dependent variables affect may change based on the independent variables quantile. In risk management sense, independent variables effect may increase or decraese in case of extreme events.

As mentioned before, systemic risk measures should be forward-based looking based. In most of the occasions, conventional risk measures may not indicate a systemic risk just before the crisis. Adrian and Brunnermeier (2011) proposed forward-looking version of CoVaR, referred as the forward CoVaR. The forward CoVaR could also be used in portfolio optimization. Since conventional risk measures may be misleading for portfolio optimization in a forward-looking sense, forward CoVaR could be a more robust parameter.

One possible drawback of CoVaR is that it is fully based on statistics as traditional risk measures do. It may also capture the common exposure of institutions to exogenous macroeconomic factors.

There is also Exposure-ΔCoVaR measure presented in Adrian and Brunnermeier (2011)'s paper which calculates risk from an opposite view. Exposure-ΔCoVaR is related to the risk of an institution given that a financial crisis occurred.

With simpler adaptations to CoVaR, similar risk measures can be calculated as well. An important measure could be CoES, stands for conditional expected loss CoES.

Another important finding of Adrian and Brunnermeier's (2011) paper is that size of an institution may mislead traders on the contribution of an institutions to systemic risk. Also, two year ahead forward-ΔCoVaR calculations was able to capture one third of the cross sectional variation of realized ΔCoVaR during the crisis. This finding is very important for portfolio optimization purposes. As traditional risk measures may not present a forward-looking view, forwad-ΔCoVaR may remedy this problem.

2.1.2. Co-Risk

Co-Risk measure is documented in International Money Fund's (2009) paper. Co-Risk measure is also based on market data which is also positive for portfolio optimization purposes. One main assumption of using a risk measure based on market data is that market is efficient. For portfolio optimization purposes, we assume market is efficient beforehand because of the return data so that using co-risk measure won't add a new assumption to our model.

One of the strong points that co-risk is relying on is that co-movements of financial institutions' risk measures do not possess a linear pattern, they increase more than proportionally with the increase in the level of risk. In order to capture the non-linearity different methods have been used including extreme value theories and quantile regressions. In Co-Risk method, quantile regression is used.

In International Money Fund's (2009) paper, CDS spread data is used as market data. By using quantile regression, dependencies of risks between institutions are measured based on the extreme movements of CDS spreads of the institutions. U.S Yield Curve, general risk premium, LIBOR spread, liquidity squeeze factor, implied volatility index (VIX) are the independent variables proposed in International Money Fund's (2009) paper. By adding a specific instituttion i's CDS spread to these independent variables and estimating institution j's CDS spread with quantile regression technique on 95th quantile, one can translate institution i's contribution to the risk of institution j in case of an event that causes distress to institution i.

2.1.3. Crowded Currency Trades

A measure for crowded currency trades is proposed by Pojarliev and Levich (2011). Crowded trading strategies are seen as one of the major triggers of a crisis. Crowdedness measures should try to estimate how many people will try to run away simultaneously from a burning home. This measure has also a liquidity risk aspect. When a majority of fund managers share the same trading strategy, an event that causes shift in their opinions may induce currency fund managers try to trade against their previous strategies. If a majority of the fund managers react in the same time, there will be nobody to take the opposite side of the trade. This may lead to huge price drops and with the mark-to-market accounting may generate big losses in funds balance sheets. A sudden drop in funds value may trigger a financial crisis as funds try to liquidate their positions.

Importance of detecting crowded trades is evident. A measure for monitoring crowded trades could add significant foresight to regulators policies for detecting a severe potential crisis. Before Pojarliev and Levich’s paper, there was no measure to capture the crowdedness of a trade. Bojarliev and Levich measures crowdedness based on the daily data on currency funds by estimating currency funds trading strategy with multifactor models. Factors are defined four key strategies in currency trading: Carry factor, Trend factor, value factor and currency volatility factor. Crowdedness is measures based on funds that have significant positive or negative exposure to these factors.

Data is based on Deutsche Bank FXSelect trading platform. Data availability of fund returns and survivorship bias could limit the applicability of crowdedness measures for portfolio optimization purposes. However, increased regulatory oversight may lead to frequent disclosure in future and data that may be used for crowding detection may increase in future.

2.1.4. Marginal and Systemic Expected Shortfall

Marginal and Systemic Expected Shortfall measures are proposed by Acharya, Pedersen, Philippon and Richardson (2010). Two separate but related measures are proposed for detecting systemic risk: Systemic Expected Shortfall (SES) and Marginal Expected Shortall.

Marginal Expected Shortfall is defined as the return of each firm during the %5 worst days of the market. Acharya, Pedersen, Philippon and Richardson (2010) found that

MES and leverage predict each firm's contribution to a crisis while firm-level risks such as VaR, expected loss, or volatility have nearly no explanatory power.

With Marginal Expected Shortfall measure, a financial institution's price behavior in a normal tail event in the market can be analyzed. Assuming that a systemic event could happen once or twice in a decade, a financial institution's price behavior can be inferred from Marginal Expected Shortfall measure by using extreme value theory. Systemic Expected Shortfall is the price behavior of the financial institution during a systemic event.

Systemic Expected Shortfall has three main components: Leverage, marginal expected shortfall and an adjustment term. It is found that Systemic Expected Shortfall is a solid predictor of emerging risks during the financial crisis of 2007-08.

Based on the Acharya, Pedersen, Philippon and Richardson's (2010) paper, Marginal Expected Shortfall and leverage ratios can be used in portfolio optimization. Calculation simplicity of Marginal Expected Shortfall and leverage ratios makes them a solid candidate as an input to portfolio optimization calculations. Systemic Expected Shortfall could also be used for portfolio optimization purposes but its drawbacks are the computational effort and model risk.

2.1.5. Multivariate Density Estimator

A multivariate density estimator is proposed by Segoviano and Goodhart (2009). Segoviano and Goodhart (2009) defined the banking system as a portfolio of banks and calculates the multivariate density distribution. Publicly available information from 2005 to the beginning of October 2008 is used in this article for calculating systemic risk. Dependence between banks are modeled with a non-parametric copula approach named as Consistent Information Multivariate Density Optimizing Copula function, CIMDO.

Since distress dependence is based on a tail event it cannot be captured by correlation coefficients. CIMDO enables a reduced form or nonparametric approach to model copulas that captures default dependence dynamically at different points of economic cycle. As CIMDO is method for only calculating dependence between institutions, default probability of individual institutions should also be estimated with other measures. Structural Methods, Credit Default Swaps or Out of the Money Option Prices can be used for calculating individual probability of defaults. CIMDO framework can be coupled with various probability of default estimation methods.

CIMDO method is based on minimum cross-entropy approach for calculating a posterior multivariate distribution. Prior density is updated with empirical information via a set of constraints. As new empirical information come out, distribution is updated.

An advantage of this method is that it can also cover non-banking financial institutions. This could be a practical advantage in portfolio optimization process. Disadvantage of this method is that it requires complex calculations for modelling dependence process.

2.1.6. Option iPoD

The probability of default implied by option prices based on entropy, Option-iPoD (individual probability of default), is proposed by Capuano (2008) as calculating probability of default of individual stocks. Yet, this measure is not adapted to calculate contribution of systemic risk to the probability of default.

One of the biggest advantages of this method is that it doesn’t rely on specific distributional assumptions. The principle of maximum entropy and the related minimum cross-entropy enables to deduce the probability distribution of the asset.

Disadvantages of this method is that it relies on option pricing data. In emerging markets, options are generally not liquid enough to calculate systemic risk measures frequently. Secondly, the complex mathematical calculations may impede the robustness of such a systemic risk measure. Third disadvantage of this method is that it relies on single stock options and there is no guidance on how to infer the systemic risk contribution to the default probability.

In future, if option contract become more liquid and there will be progress on inferring relationship between systemic risk and implied probability of default, this measure may be an input for portfolio optimization.

2.1.7. Principal Components – Absorption Ratio

Absorption ratio, proposed by Kritzman, Li, Page and Rigbon (2011), is introduced as a measure of implied systemic risk. Absorption ratio is defined as the fraction of total variance of a set of asset returns explained by a fixed number of eigenvectors. Authors also showed that changes in absorption ratio is closely related to significant declines in U.S Stock prices.

As absorption ratio goes higher, systemic risk increases. This proposition is intuitive. If higher fraction of asset returns is explained by limited factors, it will mean that asset returns will be more tied to each other. Although one cannot expect simply the factors will cause decline immediately, it will signify market fragility. There are also instances where stocks perform well during absorption ratio increases but is clear that stocks perform worst when absorption ratio spikes.

For portfolio optimization purposes, absorption ratio can be calculated separately for stocks traded in separate countries or indices. Kritzman, Li, Page and Rigbon (2010) also examined a shifting from %100 equity to %100 bond portfolio strategy during higher absorption ratio. Absorption ratio could also be embedded in a goal programming algorithm as the first constraint. During higher absorption ratio period, higher bond allocation could be done in portfolio optimization process. Bond allocation can be forced with an additional constraint. During lower absorption period, exactly same logic applies with a lower bond allocation constraint.

2.2. Literature Summary on Liquidity Risk Measures

Liquidity has different definitions depending on the contexts. Liquidity may be referred as the capability to maintain the balance between the inflows and outflows over time for banks. Basel Committee on Banking Supervision (2000) defined liquidity in a similar manner as these definitions may not be very relevant from the traders perspective. Second definition of liquidity provided by Vento and Ganga (2009) is that the measure of capability of a financial firm to turn an asset into cash quickly, without capital loss or interest penalty. The latter definition makes more sense from portfolio optimization perspective. In academic literature, the former definition may be referred as funding liquidity risk while the latter definition may be referred as market liquidity risk. In this sense, we will be focusing on market liquidity risk. Source of the two different liquidity risk types will be explored in the next paragraph.

As highlighted in Brunnermeier (2018), “funding liquidity risk takes three forms: 1) margin/haircut risk, the risk that margins and haircuts will change 2) rollover risk, the risk that it will be more costly or impossible to roll over short-term borrowing; and 3) redemption risk, or the risk that demand depositors of banks withdraw funds.” Funding liquidity risk may damage the bank only when the assets be liquidated at fire-sale prices to create funding.

Although market liquidity risk and funding liquidity risks are related to different dynamics they interact with each other. In this respect, Brunnermeier and Pedersen (2009) proposed a model that shows how tightened funding liquidity may lower market liquidity. In summary, when funding liquidity is tight, brokers become more reluctant to finance positions of hedge funds and market makers by increasing haircuts and margins on illiquid assets. In return, this may dry up the liquidity in the market and when the liquidity is lower, volatility may increase with small trades. Brokers and clearing houses respond to volatility increase by increasing margins and haircuts and this may strengthen the vicious cycle of funding liquidity risk and market liquidity risk. This example shows how funding liquidity may trigger market liquidity. There could be other reasons that may cause market to enter such a vicious cycle. For instance, a negative market shock could decrease the prices of substantially. As a result of this decrease, value-at-risk methods used by brokers and clearing houses may produce higher margin results. Based on the results clearing houses and brokers may want additional collateral from hedge funds and other traders which in return dry up the funding liquidity in the market. From this step, a similar vicious cycle could be repeated. In summary, deteriorating conditions in funding liquidity risk and market liquidity is seen simultaneously and the deteriorating conditions may be triggered by a liquidity shock, systemic event, temporary demand/supply shock or all together. This cycle is also defined as liquidity spirals by Brunnermeier and Pedersen (2009). A graphical illustration of liquidity spiral is given in Figure 2.1:

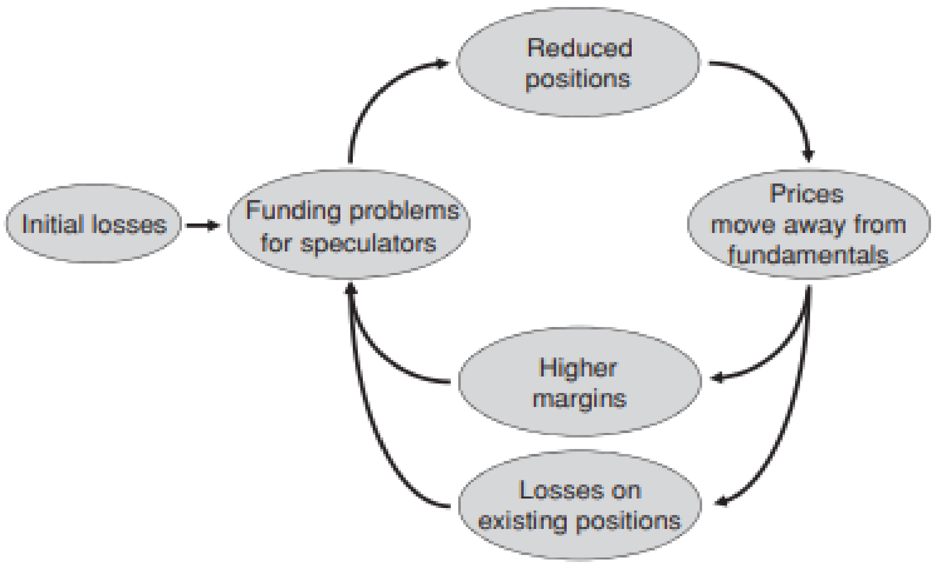

Figure 2.1: Liquidity Spirals (Brunnermeier and Pedersen, 2009)

Another important finding of Brunnermeier and Pedersen (2009) is that the markets may have two equilibriums stated: (i) illiquid equilibrium (ii) liquid equilibrium. Another potential output of this thesis may be modelling switching between these states with Markov Chains based on the current state of the market. Using this estimation as an input to portfolio optimization process may add value to the portfolio optimization results. Illiquid equilibrium is also referred as the period when "flight to quality" phenomenon happens in the market. In this period, bid-ask spreads and trading volume gap, or in other words liquidity gap, between safe instruments with lower volatility and high risk instruments with higher volatility widens. Such a liquidity dry up happens very fast and occurs in high correlation within securities sharing similar characteristics. From portfolio optimization perspective, it is important to take into account the securities data during these two equilibriums. If historical data doesn't cover the illiquid equilibrium, results may suggest having more long positions in illiquid stocks. On the other hand, if historical data doesn't cover the liquid equilibrium, results may suggest having less long positions in illiquid stocks. Traders may want to keep the balance of their portfolio that is suitable for both states in terms of portfolio optimization or if this is not applicable, traders may want to foresee the switching between the states. Systemic risk factors may help estimating the switching between those two equilibrium states since a systemic risk event may trigger liquidity spiral as well.

Importance of liquidity risk is re-emphasized after the global financial crisis of 2007-08. As highlighted in Basel Committee's Principles for Sound Liquidity Risk Management and Supervision paper "In advance of the turmoil, asset markets were buoyant and funding was readily available at low cost. The reversal in market conditions illustrated how quickly liquidity can evaporate and that illiquidity can last for an extended period of time". The fractal nature of liquidity risk may lead to modelling complexities but one should admit that a portfolio optimization process that lacks liquidity risk may lead to suboptimal portfolio decisions.

Liquidity cost is composed of broker fees and bid-ask spreads in a narrower sense but costs related to search for a counterparty and costs related to delay of execution should also be taken into account when calculating liquidity cost. Liquidity risk can be defined as risk of increase of liquidity cost. Liquidity risk may be divided into two categories: Endogenous liquidity risk and exogenous liquidity risk. Endogenous liquidity risk may increase with the size of the portfolio. As portfolio size increases, portfolio traders may not be considered as price takers in the market in case of an unwinding a large position. In case of a forced liquidation, traders may face with widened bid-ask spreads. One can see numerous examples of forced liquidation and widened bid-ask spreads in case of a systemic event. So, systemic events may increase endogenous liquidity risks indirectly. Exogenous liquidity risk is related to market characteristics and it is independent of portfolio size. For example, market of government bonds, G7 currencies are typically characterized by high trading volumes and small bid-ask spreads. When a systemic event occurs, market uncertainty arises and this causes to widening of bid-ask spreads. The degree of this widening effect may depend on the market characteristic. Bid-ask spreads of equity and derivatives generally increase more than those of government bonds in case of a systemic event. One of the motivations of this thesis is to explore the liquidity risk from different angles such as liquidity risk as a consequence of systemic risk and liquidity risk as an independent factor. Exogenous liquidity risk will be examined and measured in detail while endogenous liquidity risk will not be considered as an input for the sake of the thesis complexity. Endogenous liquidity risk may also be prevented by putting some limits to avoid concentrations on specific securities.

Based on the traditional portfolio view, risk should be compensated with expected return. According to the traditional portfolio view, there could be some unattractive investment vehicles. They may offer lower expected return with a medium risk. On the other hand, these kinds of unattractive investment may be liquidated very quickly. 1998 Russian Crisis has shown that liquidity should be one of the main criteria for asset allocation. Portfolio optimization process should take into account liquidity risk factor as an input. With liquidity risk factors as an input, unattractive investments may become attractive because of their high liquidity. Bangia et al. (2002) have shown that ignoring the liquidity risk may lead the underestimation of market risk by as much as %25-30 in emerging markets. In case of portfolio allocation this underestimation may lead to huge variations in portfolio optimization output and thus to a suboptimal portfolio decision. Also, endogenous liquidity risk is applicable to a narrower audience that hold larger position while exogenous liquidity risk is appliacble to anyone that holds any position.

Pastor et al. (2001) has shown that liquidity risk is related to expected return differences that are not explained by stocks' betas, market cap, P/E. Acharya and Pedersen (2005) also proposed three liquidity risk measures designed as a covariance of two factors. These measures are (i) covariance of market illiquidity with stock illiquidity (ii) covariance of stock returns with market illiquidity (iii) covariance of stock illiquiditiy with market returns. Each illiquidity factor could have different effects on pricing and portfolio optimization. Acharya and Pedersen (2005) also found that liquidity-adjusted capital asset pricing model (CAPM) explains the data better than the standard CAPM.

Incorporating liquidity measures may create extra complexity to the portfolio optimization process. It be should analyzed when to include liquidity risk measures into portfolio optimization process based on the market states. When systemic risk measures signal an increase in systemic risk, liquidity risk may be more important that they should be added as an input to portfolio optimization process. Because liquidation will be costlier when liquidity is low and systemic risk is high. On the other hand, liquidity risk factors could be as an input in portfolio optimization model regardless of market state. For instance, Bangia et al. (2002) assumed that in adverse market environments extreme events in returns and extreme events in spreads happen concurrently. This assumption may help us to develop a model that aggregates systemic risk and liquidity risk in a single equation.

Bangia et al. (2012) concludes that liquidity risk may be more important in emerging market securities. So, one can expect that inclusion of liquidity risk to portfolio optimization process may not add significant value and it may not compensate the increase of complexity in the portfolio optimization model.

As clearly mentioned in the paragraph above, another challenge of this thesis is to understand relation between systemic risk and liquidity risk. One should be able to extract liquidity risk part that is independent of systemic risk so that an optimization model with independent parameters could be created. Another approach could be to structure two-stage optimization model. In the first stage, a model that estimates current systemic risk can be constructed. Depending on the level of systemic risk, whether including liquidity risk or not could be decided as mentioned in the paragraphs above. As clearly indicated in Chordia et al. (2001), the most significant independent variable in determining the liquidity status of the market is the down-market variable. Spreads increase dramatically in down markets, but one can’t see the inverse effect during up

markets. As we expect a down market systemic event, we should incorporate liquidity risk factors into our analysis.
One of the bitter market experience on liquidity risk management is Long-Term Capital Management (LTCM) example in 1998. LTCM had long positions in less liquid assets and short positions in more liquid assets. After the Russian debt crisis, less liquid assets prices decreased sharply and this triggered liquidation with big losses. Jorion (1999) examined in detail the failure of Long-Term Capital Management Fund. Long-Term Capital Management Fund had inefficient practices in risk management including assuming a normal distribution for calculating VaR and considering a short horizon for calculating volatility and correlations. While these inefficiencies are related to risk management assumptions, they may also have acted incorrectly when things started to get worse. They sold high liquid assets and kept less liquid assets in their portfolio when things got worse. The reasoning behind this was to justify risk-return profile of the fund. As summary, LTCM example is a good example of how things can go wrong when one doesn't consider liquidity factors and rely on static covariance measures in portfolio optimization process. In parallel, Jorion (1999) also stated that "Traditional risk management models ignore asset and funding liquidity. When positions are relatively large and leveraged, it is important to account for price impact of forced sales."

Based on the data requirements, the liquidity risk measures worth investigating could be summarized as below:

- Noise as Information for Illiquidity: Hu, Xing, Pan and Wang (2013)
- Equity Market Liquidity: Khandani and Lo (2007)
- Exogenous Spread Approach: Bangia et al (2002)

2.2.1. Noise as Information for Illiquidity

Noise measure as information for illiquidity is proposed by Hu, Xing, Pan and Wang (2013). Hu, Pan and Wang holds that noise may increase if liquidity decreases. As irrational trades may affect price formation process more steeply in illiquid market status, noise will be much more visible in illiquid market states. Big price deviations from fundamental values may happen rarely when liquidity is high. That is why noise is considered measure of liquidity.

One main assumption of this measure is the presence of the arbitrage capital in the related market. As arbitrage capital is limited in the emerging markets, noise as information liquidity may not be considered a solid input for portfolio optimization. However, it may reflect the current liquidity status of the market. Current liquidity of the market could be calculated with noise measure. Relationship of returns of each equity and current liquidity status could be calculated with regression or correlation coefficient. However, this method could introduce additional parameter complexity to the portfolio optimization process. A more robust method such as quantile regression can be used for identifying the impact of liquidity to returns of equities. Another disadvantage of this model is the complexity of detection of the deviation from fundamental values. While bonds fundamental prices could be easily identified with level of interest rates, equities' fundamental prices can't be calculated with a single risk factor such as interest rates.

2.2.2. Equity Market Liquidity

Khandani and Lo (2007) assumed that price of a security changes occur due to two reasons: (i) arrival of new information about the security's value (ii) temporary supply/demand imbalances. While the first factor can be modeled by random walk process, second factor can be modeled as a mean-reversion process. Second factor is directly related to liquidity of the market. If one can manage to attribute changes of security prices to each factor, it may provide an insight about the liquidity status of the market.

When the market is illiquid, mean-reversion process may occur slower and this may lead to losses of market making strategies. We can imitate market making strategy based on mean-reversion process (buying the losing shares and selling the winning shares) and calculate the profitability throughout a period. Based on the output of the strategy we can track the liquidity status of the market.

The first method proposed by Hu, Xing, Pan and Wang (2013) may be more applicable to bonds while the second method proposed by Khandani and Lo (2007) may be applied for equity portfolios as well.

2.2.3. Exogenous Spread Approach

One of the main assumptions of market risk is that traders may buy/sell securities on market price. Market price is generally defined as mid-price between bid-ask spread. Liquidity cost can be defined as the between bid-ask spread divided by two. In this sense, liquidity risk may seem as negligible compared to market risk. However, in turbulent periods liquidity risk may increase in a manner that cannot be modeled with normal distribution.

Historical bid-ask spreads may be used and a certain quantile (99.9, 99 or 95) can be used for calculating liquidity risk. This method is relevant for only exogenous liquidity risk. For unwinding large positions quoted bid-ask spreads may not be relevant. Due to this fact, distribution on bid-ask spreads can’t be used for calculating endogenous liquidity risk.

For the sake of robustness of the portfolio optimization model, covariance between liquidity risk and systemic risk of a stock can be assumed as 1. Using this assumption, liquidity risk and systemic risk measures can be aggregated by summing up in a single equation.

2.3. Literature Summary on Portfolio Optimization

Literature summary on portfolio optimization is divided into three subsections: (i) Modern Portfolio Theory (ii) Drawbacks of Modern Portfolio Theory (iii) Risk Measures used as an input in Portfolio Optimization

2.3.1. Modern Portfolio Theory

Harry Markowitz introduced the Portfolio Theory also known as Modern Portfolio Theory in 1952 (Markowitz, 1952). In 1990, he won the Nobel Prize in Economic Sciences for the Theory.

Markowitz formulated the portfolio problem as a trade-off between return and risk of a portfolio of assets. He proved the fundamental theorem of mean variance portfolio theory, (Fabozzi, 2002) namely holding constant variance, maximize expected return, and holding constant expected return minimize variance. Based on these two principles, efficient frontier can be formed as a combination of expected risk and return. Investors could choose his or her preferred portfolio as a point in efficient frontier, depending on individual risk return preferences. (Gruber, 1997)

Efficient Frontier contribution is made by James Tobin and Harry Markowitz in 1958. Efficient Frontier is a curve drawn on a graphic with return on y axis and risk on x axis. The theory tells us that for every additional expected return, additional risk needs to be taken and there is no arbitrage on this. However, some combination of securities may not lie on efficient frontier. In other words, for certain investments, there may be alternatives with same expected return but less risk. In this case, these investments are not lying on efficient frontier. To summarize, efficient frontier resembles the maximum expected return for each risk level. That is why, it resembles a curve. Generally, a single equity doesn’t rely on efficient frontier due to lack of diversification. However, same level of return can be achived by combining equities and thereby helping investor moving to closer to Efficient Frontier.

Portfolio Theory shows that risks can be reduced by diversification and hence heping investors make investments residing on Efficient Frontier. However, some risks such as systematic risks cannot be eliminated with diversification as every equity in a investment universe carry a certain exposure to country risk or stock market risk; thereby weakening the practical usage of the theory. To better reflect these types of common risks, the capital asset pricing model was proposed by William Sharpe in 1970 in his book, "Portfolio Theory and Capital Markets".

What modern theory has taught us is that we need to be concerned with risk as well as return in examining performance. Although risk was always an important dimension, there wasn't any formula of framework that integrates the return decisions with its implication for risk. After modern portfolio theory, several criteria emerged for assessing performance of portfolio with regards to its return and risk.

Early studies suggested a variety of evaluation techniques, including the Sharpe ratio, the Treynor ratio, Jensen alpha and Friend, Blume and Crockett's use of randomly generated passive portfolios of the same risk. Each of these studies evaluated the portfolio performance from different risk measure perspectives. Some used total risk (Friend, Blume and Crockett, 1970; Sharpe 1966) as the correct measure of risk. Others (Treynor 1965; Jensen, 1968 and 1969; Friend, Blume and Crockett, 1970) used beta as the correct measure of risk. (Gruber, 1997)

The most popular portfolio evaluation measures such as Sharpe ratio, Treynor ratio, Jensen Alpha do not take into account the systemic risk that the investor took. These ratios are based on beta and standard deviation. Beta can't be considered as a well-established systemic risk measure currently. Beta is just an average covariance, while other systemic risk measures capture tail dependence. If the new portfolio evaluation measure will be used for portfolio evaluation purposes in trading desks, it may also balance the systemic risk taken by an institution.

2.3.2. Drawbacks of Modern Portfolio Theory

First of all, it may be critical to highlight that the modern portfolio theory is a normative theory. A normative theory is one that describes a standard or norm of behavior that investors should pursue in constructing a portfolio, in contrast to a theory that is actually followed. Considering just mean return and variance of return of a portfolio is, of course, a simplification as additional moments that might more completely describe the distribution of portfolio returns. In this regard, Kraus et al (1976) suggested alternative portfolio theories integrating more moments such as skewness or kurtosis to portfolio allocation process. Nevertheless, mean-variance theory has remained the cornerstone of modern portfolio theory due its simplicity despite alternatives. (Gruber, 1997) There are numerous risk measures proposed for fixing this drawback. Proposed risk measures will be summarized in next section.

Modern portfolio theory was established to find the optimum portfolio theory for an investor focusing on return distributions over a single period. One of the critical theoretical problems that has been investigated is how the single-period problem should be modified if the investor's true problem is multi-period in nature. This problem is analysed by Hakansson (1970,1974), Fama (1970), Merton and Samuelson(1992) and Mossin (1969) under various assumptions. The papers suggest that the multi-period problem can be solved as a sequence of single-period problems under several sets of reasonable assumptions. However, the optimum portfolio would be different from that selected if only one period is examined. The difference arises because of the differences between utility functions for sigle period and multi-period applications. (Gruber, 1997)

2.3.3. Risk Measures used as an input in Portfolio Optimization

One of the basic assumptions of the modern portfolio theory is that returns are normally distributed, and investors care only about mean, variance and covariance parameters. However, empirical data shows that returns are nonnormally distributed and they generally exhibit fat tails. New risk measures taking into account tail risk has been proposed by academicians. Nonetheless, there is no widely agreed unique risk measure for calculating tail risk.

Risk measures can be divided into two categories: two-sided and downside risk measures. While two-sided risk measures are calculated based on the up and down movements of asset downside risk measures cover only down movements. Standard deviation, variance, mean-absolute deviation are among the most known two-sided risk measures. Semivariance, value-at-risk, conditional value-at-risk, in other words expected shortfall, are among the most known downside risk measures. As mentioned in Fabozzi et al. (2007), although downside risk measures may seem more intuitive from risk management point of view, they are generally harder to compute and aggregate in the portfolio context. Also, downside risk measures use only a portion of the original data – maybe even just the tail of the empirical distribution – and this may lead the increase of estimation errors. I will examine these risk measures in detail in the next paragraphs.

Value-at-Risk attracted great attention after the Basel Accord. Disadvantages of value-at-risk are it is not an additive risk measure and it gives little information about the distribution of potential losses. It is based on only a point quantile of loss distribution. Expected shortfall provides more information about the nature of the loss after the specified quantile, generally measured as %90, %95 or %99. Mean-Expected Shortfall and Mean-Variance optimization problems generate the same output when assumption of normality holds.

Main inputs for portfolio optimization are return and risk. While candidates as a return input is less controversial in portfolio optimization theory, we see different inputs as a risk measure in portfolio optimization models. The most popular choice as a risk measure to portfolio optimization models are standard deviation. Covariance is the most popular choice for modelling dependence between stocks in portfolio optimization process. One of the biggest drawbacks in portfolio optimization process is parameter instability. Chance relationships may occur, and it may cause unreliable covariance estimates.

Recent crisis has shown that covariance may not explain the comovement of securities during the crisis. The independent stocks based on the covariance may behave dependently during the crisis. Liquidity risk and systemic risk measures may help us to explain this phenomenon and this will help us to form an outperforming portfolio when a systemic event occurs. In order to form such a portfolio we should be able to calculate systemic risk and liquidity risk measures.

Another critical aspect of the portfolio optimization process will be to weight the risk factors based on the future expectations. When we do not expect the markets to move into a systemic risk state we shouldn't use systemic risk factors with higher weights. Thus, we may model the market states with a Markov chain

3. EMPIRICAL ANALYSIS OF RISK AND RETURN MEASURES

3.1. Data Preparation

Portfolio Optimization methodology relies on minimizing risk for a target return or maximizing return for a target risk given that inputs on equity level are provided. Portfolio Optimization methodology developed by Markowitz requires that correlations, variances and returns of each equity are fed as input.

Of course, the output quality of a methodology cannot be better than input quality. In other words, “Garbage In Garbage Out (GIGO)”. In a plain portfolio optimization problem, risk and return measures are calculated based on the historical data. However, the size of the historical data that will be used for calculating these inputs remains as a parameter to be determined by the user. What is more, the investment horizon is also critical. Depending on the investment horizon, the choice of the data size may change. For instance, if the investor plans to allocate the portfolio for 3 months without rebalancing, data length that will be used for risk and return input estimation should be different than a similar optimal portfolio that will remain constant for 12 months without rebalancing.

In this section, I try to analyze the fundamental inputs and alternative inputs of portfolio optimization methodology. I test these parameters simultaneously based on different investment and observation horizons. I define observation horizon as the length of the historical data to be used. It is defined in terms of quarters. The same methodology is applied for the investment horizon reciprocally.

For analyzing these inputs, S&P 500 daily return data that is provided by QuantQuote is used. I believe that for the sake of the initial code development and analysis S&P 500 data will be sufficient. Although the data covers the date range from the beginning of

1998 to 2016 December, I focused on the 10 years of data from the beginning of 2000 to the end of 2009. As this period includes both upturns and downturns in the economic cycle, it could provide a solid base for initial analysis. Our analysis requires the daily price data to be complete for each individual security. There are 425 securities that have complete price data. I also added sector information based on the manual checks from Yahoo to get the sector data. There were also sectoral changes that is caused by mergers and acquisitions. We have done manual interventions to clean the data.

SAS programming language is used to get the input data in the right format for the analysis. We have totally 1,808,278 rows of daily return and volume data. Sector information is added to the data. A view of daily data format is given in Table 3.1:

Table 3.1: Data Format of Equity Prices

Equity	Date	Open	High	Low	Close	Volume	Return (%)
A	18-Nov-99	42.2076	46.382	37.4581	39.1928	43981813	0
A	19-Nov-99	39.8329	39.8885	36.9293	37.6251	11390201	-4
A	22-Nov-99	38.3208	40.0091	37.1613	39.9442	4654716	6.1637
A	23-Nov-99	39.4247	40.4729	37.3375	37.5138	4268903	-6.0845
A	24-Nov-99	37.2262	38.9052	37.1056	38.0889	3602367	1.533
A	26-Nov-99	37.9219	38.4971	37.8013	38.2095	1332198	0.3166
A	29-Nov-99	38.0332	39.369	37.6251	38.9052	3038788	1.8208
A	30-Nov-99	38.9609	39.8329	37.9776	39.1371	2968070	0.5961
A	1-Dec-99	39.1371	40.2967	38.8496	40.1204	2198051	2.5124
A	2-Dec-99	40.5842	41.7438	40.0648	41.5119	2308009	3.4683
A	3-Dec-99	41.6881	42.3839	41.1037	41.6881	2308440	0.4245
A	6-Dec-99	41.9757	43.0796	41.92	42.3839	1713596	1.6691

Return is calculated based on the input data with the formula given below:

$$\text{Return}(t) = \ln[\text{Close}(t) / \text{Close}(t-1)] \tag{3.1}$$

As the raw input data for individual securities are in separate 500 excel sheets. I developed a SAS code for merging individual excel sheets into one single SAS dataset. We call this code “Input Data Merger Code”. The code is given in appendix.

A view of the data in industry format is given in Table 3.2:

Table 3.2: Equity and Sector Code Mapping

Equity	Industry Code
A	Health Care
AA	Materials
AAPL	Information Technology
ABC	Health Care
ABT	Health Care
ACE	Financials
ACT	Health Care
ADBE	Information Technology
ADI	Information Technology
ADM	Consumer Staples
ADP	Information Technology
ADSK	Information Technology
AEE	Utilities
AEP	Utilities
AES	Utilities
AET	Health Care

Sectoral overview of equities that have sector mappings is given in Table 3.3:

Table 3.3: Number of Equities in a Sector

Sector	Number of Equities
Consumer Discretionary	64
Consumer Staples	36
Energy	36
Financials	72
Health Care	45
Industrials	55
Information Technology	55
Materials	25
Telecommunications Services	5
Utilities	29

All sectors have enough data for analysis with the exception of Telecommunication Services. However, I will try to analyse the Telecommunication Services data as well.

Industry classification is based on the Global Industry Classification Standard (GICS). There are two levels in the industry classification. I focus on the first level of classification to keep as much data as we have in one class.

3.2. Code Development

Prerequisite of the all analysis codes is the data quality. In the first step we merged the input data in a common data format. Although the current data is only US data I can use the same code for merging the data from other markets as well.

After merging the input data, we can start calculating basic risk and return factors. I first wrote the code for finding the eligible securities that have complete daily data during the period analysed. We call this code "Finding Eligible Securities" code. The code is given in appendix. As the initial period to be analysed is from the beginning of 2000 to the end of 2009, we found 435 eligible securities. I wrote the code in a parametric format. If I change the analysis period, the system will recalculate eligible securities automatically.

After finding the eligible securities we can calculate the risk and return factors for the subperiods in the analysis range. We define subperiods as yearly periods. As a result, we have 10 subperiods in our analysis. (2000, 2001, … 2009)

I will do analysis from a traders' perspective that allocate his portfolio based on the portfolio optimization steps below:

1) At the beginning oe each year, he calculates risk and return measures, covariance matrices based on the previous year data.

2) He/she runs portfolio optimization process based on the input data calculated in Step 1.

3) When the year ends, he recalculates the same risk and return measures for the past year and reallocates his portfolio.

Although this strategy may not be a complete reallife example, it could help us to understand similar strategies that rely on historical data. The strategy mentioned above could be successful if the past data can predict future returns and risk measures. I will do an empirical analysis that tests the validity of this assumption. Analysis involves 2 steps of calculation:

1) Calculating mean return, median return, variance, skewness, kurtosis, VaR(%99, 1 year), VaR(%95, 1 year), VaR (%90, 1 year), VaR (%75, 1 year), P (%25 percentile, 1 year) P10 (%10 percentile, 1 year), P5 (5% percentile, 1 year), P1 (%1 percentile, 1 year), Expected Shortfall(%1,1 year), Expected Shortfall(%5,1 year), Expected Shortfall(%10,1 year), Expected Shortfall(%25,1 year), for each security and each year. A sample of calculation of these measures are provided in appendix. Percentile measure will help us to analyze the effect of historical return spikes over the future returns. This will lead to the change of sign in the correlation matrix. We will have a two-dimensional matrix with 435 rows for each security and 170 columns. 170 (17*10) columns include 17 risk factors calculated for each year in the sample data.

2) Calculating correlations between each column. We will have two-dimensional matrix with 170 columns and 170 rows. Correlations on sector level are also calculated as well.

This correlation matrix would reveal empirical insights about risk and return factors. When the correlation is close to zero between consecutive years for the same factor, it will help us to conclude that there is no predictive power of the historical data for such risk factor. Based on the correlation matrix results, we can analyse both the relationship between factors over time as well as relationship between past values of the risk factor to the future values of the risk factor. If past data and future data are uncorrelated for a specific factor it would be nonsense to use the risk factor for portfolio optimization purposes.

3.3. Application

I calculate risk and return measures based on the formulas (3.2) and (3.3) below for each security and each year:

$P(i,j,k)$: Percentile at level %i for security j in year k.
$x_{j,k}$: Daily return of security j in year k.

$$i = prob\left(X_{j,k} \geq P(i,j,k)\right) = \int_{P(i,j,k)}^{+\infty} f\left(x_{J,k}\right) dx \qquad (3.2)$$

I calculate $P(i,j,k)$ for:

$i \in (0.01, 0.05, 0.10, 0.25, 0.5{,}0.75, 0.90, 0.95, 0.99)$

$j \in (1{,}2, \ldots{,}425)$

$k \in (2000{,}2001, \ldots 2009)$

All value-at-risk and percentile measures are computed according to equation (3.2).

Empirical distributions are used for $f\left(x_{J,k}\right)$ based on the historical data provided. Based on eq. (3.2) we calculated VaR(%99, 1 year), VaR(%95, 1 year), VaR (%90, 1 year), VaR (%75, 1 year), P (%25 percentile, 1 year) P10 (%10 percentile, 1 year), P5 (5% percentile, 1 year), P1 (%1 percentile, 1 year), median return for each security and each year.

Expected shortfall risk measure belongs to spectral risk measures family. Expected shortfall is gaining popularity in the last years with Basel's adoption of Expected Shortfall in Market Risk as outlined in the paper "Fundamental review of the trading book: A revised market risk framework". In previous years, Value-at-Risk was used for regulatory market risk capital calculation. It is proven that Expected Shortfall is more robust than Value-at-Risk Measures.

Based on these results and arguments, I included Expected Shortfall measures based on difference confidence intervals. Expected Shortfall Calculation formula is given below:

$$ES_{\alpha,k} = \frac{1}{\alpha} \int_0^{\alpha} VaR_{y,k},(X) d_y \qquad (3.3)$$

ES_{α} = Expected shortfall at % α confidence level

$VaR_y(X)$ = Value-at-Risk at confidence level y for Security X.

I calculate $ES_{\alpha}$for:

$\alpha \in (0.01, 0.05, 0.10, 0.25)$

X $\in$ (1,2, ...,425)

$k \in (2000,2001, \ldots 2009)$

Mean return, variance, skewness and kurtosis are calculated in the same manner for each security and year based on the empirical data. Based on these calculations, I can form a two-dimensional matrix of risk and return measures for each year where columns are risk and return measures and rows are securities as demonstrated below:

$$X_k = \begin{pmatrix} \alpha_{1,1} & \cdots & \alpha_{13,1} \\ \vdots & \ddots & \vdots \\ \alpha_{1,425} & \cdots & \alpha_{13,425} \end{pmatrix} \qquad (3.4)$$

$k \in (2000,2001, \ldots 2009)$

I appended two-dimensional matrixes for calculating correlation of risk and return measures between risk and return factors calculated based on each year data as shown below:

$$A = (\; X_{2000} \quad X_{2001} \;\ldots\; X_{2009} \;) \qquad (3.5)$$

Matrix A has 425 rows for each equity and 170 columns for risk and return factors calculated for each year. Columns of Matrix A are given in the below:

$$A = (\,Mean_{2000} \quad Median_{2000}\ Kurtosis_{2000}\ \ldots\ VaR(0.99)_{2009}) \tag{3.6}$$

$Mean_{2000}$ column consists of mean daily return of 425 equities for year 2000. I calculate a correlation matrix based on columns of matrix A. Correlation matrix has 170 columns and 170 rows.

I also applied similar analysis on a sector basis. To elaborate, equities belong to a certain sector, such as Materials, are included in all calculations and other equities are excluded. I repeated the same exercise to analyse different behaviour and predictability levels across sectors.

Results of the wide correlation matrix are very wide that the full version can't be shared in the appendix. However, a sample of the covariance matrix for the years 2000, 2001, 2000-2001 will be shared in the appendix. This wide correlation matrix includes the statistics below:

1. Correlation of risk and return factors in the same year: This kind of statistics doesn't help us a lot for portfolio optimization perspective. Traders need to understand the future behaviour of risk and return factors based on the historical data. So, the correlation risk and return factors in the same year provide only information regarding the current year.

2. Correlation of the same risks or return factor between years: These statistics help us to understand whether the risk factors of securities move in the same direction over time. If they move in the same direction, I can reliably use such risk factors.
3. Correlation of different risk and returns factors between years: These statistics help us to reveal interaction between different risk and return factors over time. For example, if there was a positive correlation between past year variance and current year return, I could conclude that more risky securities could generate higher returns on average.

In addition to yearly calculations I also added a section for calculations on 3-year frequency. I divided the data into 3 3-year periods:

- 2000-2002
- 2003-2005
- 2006-2008

So, overall the total risk factor covariance matrix is reduced to 3 from 9. Result of these calculations will help us to understand whether optimizing portfolio on 3 years basis may be more efficient than optimizing portfolio on a yearly basis or not.

3.4. Detailed Results

In this subsection, results of correlation matrices are explored. I focused on both univariate and multivariate correlations through time to understand the dependency between risk and return measures. Average correlation between risk factors through time is given in appendix.

3.4.1. Predictability of Return

Return estimation could be considered as the major obstacle in portfolio optimization. In this subsection, I focused on the correlation on future return with current risk and return measures. In order to have an overperforming portfolio allocation it is not critical to estimate return of a single equity precisely. To highlight this view, I provided the example in our conference paper below:

Assume that variance of security A is 5 and variance of security B is 2 for the previous year. We define variance vector as $[5\ 2]^T$ *for the previous year. In portfolio optimization problems I incorporate next years' variance vector into the optimization problem. What I propose is that, even if we can't forecast next periods' variances for each individual security, there is a high probability that A will be riskier than B in next year given that yearly variance vector is positively correlated with the subsequent years' variance vector. In this case, portfolio optimization will still be efficient even if* *the future periods variances are unknown. As illustrated in this example, if we look at the correlations between years and analyse finding, we can form more efficient portfolio based on the out-of-sample data, which is important for traders.*

In this regard, the first aspect that I would like to analyze the past return-future return relationship. Overall the average correlation is -0.07. Although this implies almost no correlation, on a yearly basis, strong positive and negative correlation values are seen as demonstrated in Figure 3.1:

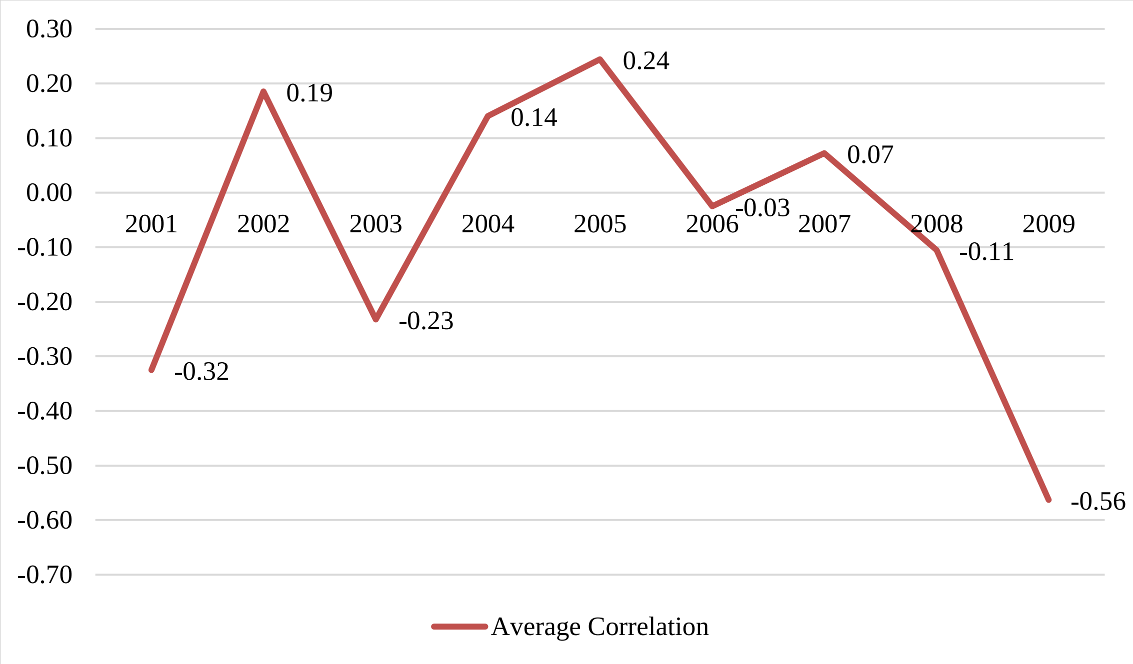

Figure 3.1: Historical Return – Future Return Correlation

From the figure 3.1, it can be shown that the correlation varies over time, it can be regarded mostly as a noise factor, but with the exception of the last year. Moreover, in any of these years strong positive correlation is not occurred. This fact alone can be presented as a strong argument against using historical return directly as an input in portfolio optimization. Similar argument can be supported for correlations between past and future returns calculated over 3 years. On sector level, average correlation in years varies between 0.03 and -0.12, which implies there is no correlation between past overperformers and future overperformers in a sector. Figure 3.2 demonstrating average correlation of past and future return within sector reveals interesting insights that will be detailed in the subsequent paragraphs:

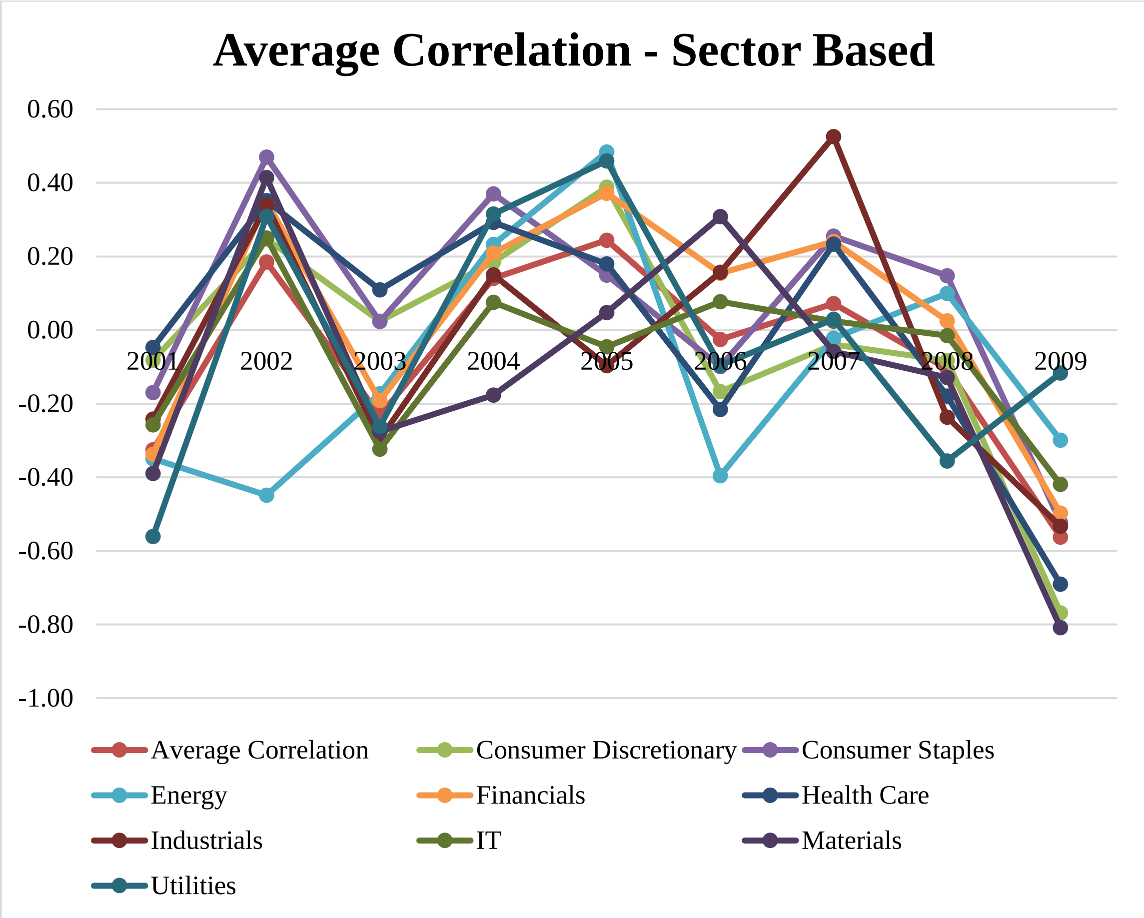

Figure 3.2: Sector Based Average Correlation on a Yearly Basis

Correlation within sector have a common movement pattern. The average correlation within sectors are moving in parallel. I would like to focus on the strategic year of 2009 that could be regarded as the jump back year after 2008 crisis. Except Utility and Energy sectors, all sector had strong negative correlation. Utility and Energy sectors could be regarded as crisis-resistant sectors and these sectors may have less speculative movements compared to other sectors. In those sectors, speculative traders may have more positions that may needed to be liquidated in the crisis and this may have pulled back the prices to very low levels and once the crisis effect is weakened in 2009, the prices may have reached the normal levels. From portfolio optimization perspective, it might have a strong implication. It is very critical to not allocate portfolios, particularly for crisis-nonresistant sectors, based on historical returns after the crisis. Even, the traders may assign returns by multiplying by minus one if they anticipate a mean-reversion effect. For the pre-crisis years I don’t see any notable movement.

3.4.2. Predictability of Variance

Variance is the conventional risk measure of portfolio optimization problem. In the original Markowitz's paper (1952, 1959), variance was suggested due to its aggregability on portfolio level with simple formula given below:

$$\sigma_p^2 = \rho_{a,b}\, \sigma_a\, \sigma_b$$

$$\sigma_p^2 = \sum_{i<j}^{j=n} \rho_{i,j}\, \sigma_i\, \sigma_j + \sum_{i=1}^{i=n} \sigma_i^2$$

First formula is given for simple case of two equities and second formula is used for generalized cases with multiple equities.

I followed the same methodology in variance case. Average correlation between past years variance and future years variance is 0.80. This implies that historical variance could be used as an input for portfolio optimization problem. What is more, historical variance has strong correlation with other risk measures and upside measures as well. The average correlations are given in Table 3.4:

Table 3.4: Correlation Between Historical and Future Risk Measures

Risk Measure	Correlation
Future P10	0.82
Future P5	0.81
Future VaR	0.80
Future ES (%75)	0.79
Future VaR(%90)	0.79
Future VaR (%75)	0.78
Future ES (%90)	0.77
Future VaR (%95)	0.77
Future ES (%95)	0.75
Future P1	0.73
Future VaR (%99)	0.71
Future ES (%99)	0.68
Future Return	0.14
Future Skewness	0.13
Future Kurtosis	-0.01
Future Median	-0.07

Overall, variance could be good predictor for both future upside returns and tail risk measures. The predictive power decreases as related statistics move closer to the tail. It can be concluded that even the focus is on variance during conventional portfolio optimization, it may also reduce other risk measures as well because of the correlation between historical variance and future risk measures. However, this correlation shall be treated with caution due to the correlation effect and non-convex nature of Value-at-Risk measures. Another point worth mentioning is that minimizing variance could also reduce upside returns as well, which is not desirable by traders.

Yearly evolution of correlation between historical variance and future variance is given in Figure 3.3:

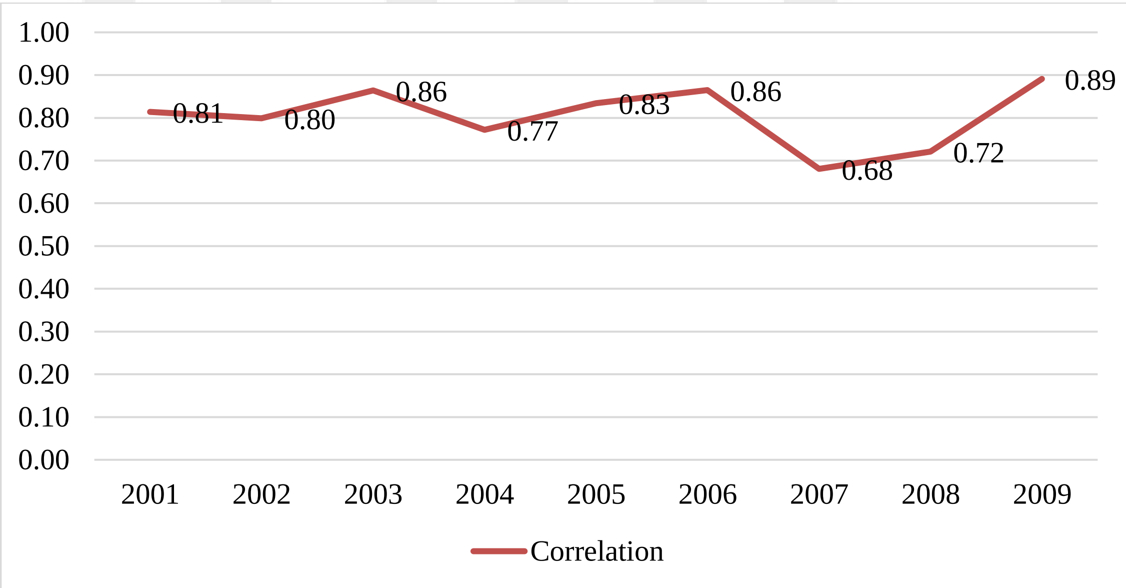

Figure 3.3: Correlation of Historical and Future Variance on a Yearly Basis

Although average correlation between future variance and historical variance varies between years, historical correlation can always be treated as a strong predictor of future variance.

Correlations between past and future variances calculated over 3 years is 0.65 and it is relatively lower compared to correlation between past and future variance calculated over one year. Thus, it can be concluded that the predictive power decreases as investment horizon increases and using longer historical data can't prevent this decrease. The average correlation between 3 years of historical variance and 3 years of future risk measures are given in Table 3.5:

Table 3.5: Correlation between Risk Measures on a 3-yearly basis

Risk Measure	**Correlation**
Future VaR (%75)	0.75
Future P25	0.72
Future P10	0.72
Future VaR (%90)	0.69
Future ES (%75)	0.68
Future P5	0.67
Future VaR (%95)	0.66
Future Variance	0.65
Future ES (%90)	0.65
Future ES (%95)	0.63
Future P10	0.61
Future VaR (%99)	0.58
Future ES (%99)	0.57
Future Skewness	0.03
Future Kurtosis	-0.05
Future Median	-0.06

The relative predictive power decreases when the investment horizon extends to 3 years. Overall, the same tendency of decreased predictive power as risk measure gets closter to tail is detected in 3 years of horizon as well.

The average correlation between future variance and historical variance within the sector is slightly lower. However, the predictive power of historical variance is still strong. The comovement of correlation within years is not detectable. The sector level correlation graphic is given in Figure 3.4:

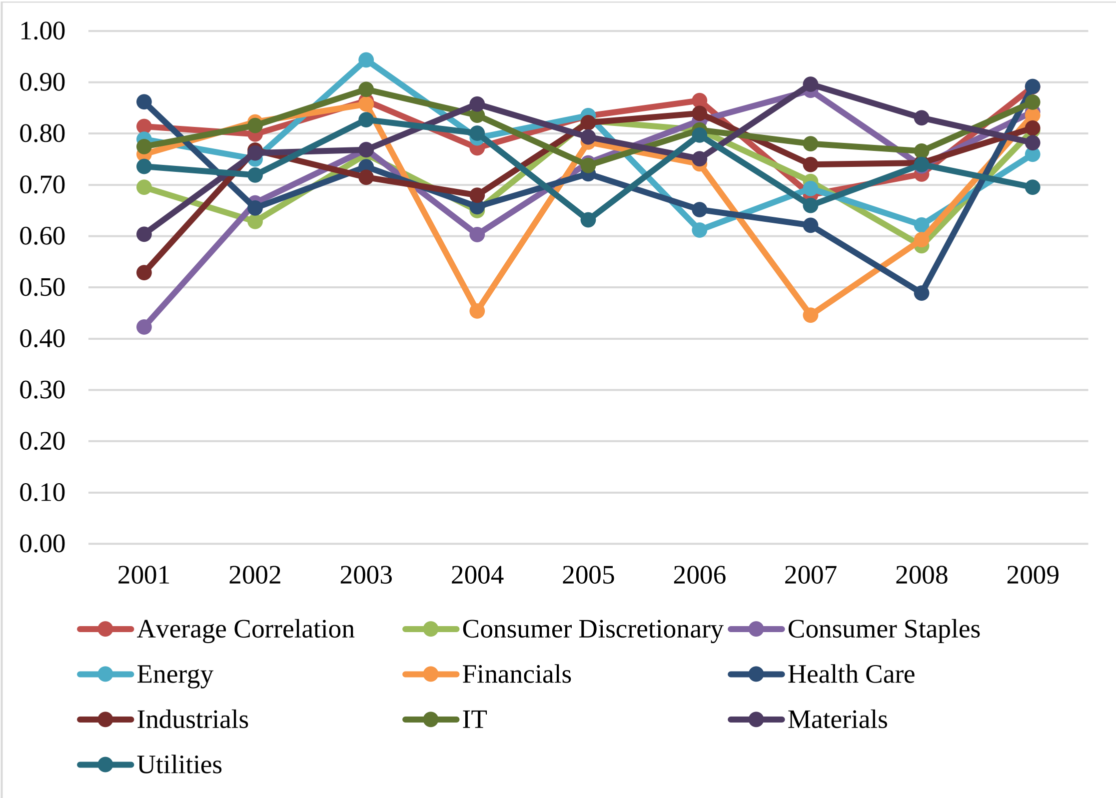

Figure 3.4: Correlation of Hisorical Variance and Future Variance on Sector Level

The average predictive power of historical variance is highest in IT sector and lowest in Financials and Health Care sectors but the difference between the sectors with highest predictive power and lowest predictive power is relatively low.

3.4.3. Predictability of Other Risk Measures

After the financial crisis, practitioners, academicians and regulators started focus on tail risk measures as it became more evident that returns are non-normal and there may be huge risks in the tail due. Institutions and individuals are more exposed to tail risks as new complex products such as options and futures are issues and became widely used.

The increased sophistication of financial products along with more frequent turbulent periods which may be attributed to globalization has called for attention of academicians and regulators. Variance is not sufficient to differentiate riskiness of portfolios in terms of tail risk. The details for the weak points of variance as a risk measure is provided in literature review section.

In this subsection I focus on two popular risk measure Value-at-Risk and Expected Shortfall in terms of predictive power. As there are multiple ways to extend portfolio optimization structure in order to take into account tail risk measures. However, if predictive power of historical tail risk measures is not enough for future, implementation of portfolio optimization with tail risk measures wouldn't be successful.

The correlation between historical risk measure results and future risk measure results is given in Table 3.6:

Table 3.6: Correlation between Risk Measures on a Yearly Basis

Correlation	Future VaR (%75)	Future Variance	Future VaR (%99)	Future VaR (%95)	Future VaR (%90)	Future ES (%75)	Future ES (%90)	Future ES (%95)	Future ES (%99)
Historical VaR (%75)	0.77	0.76	0.66	0.74	0.76	0.76	0.73	0.70	0.63
Historical Variance	0.78	0.80	0.71	0.77	0.79	0.79	0.77	0.75	0.68
Historical VaR (%99)	0.70	0.73	0.64	0.70	0.72	0.72	0.70	0.68	0.62
Historical VaR (%95)	0.78	0.79	0.70	0.77	0.80	0.79	0.77	0.74	0.67
Historical VaR (%90)	0.79	0.79	0.69	0.77	0.80	0.79	0.76	0.74	0.66
Historical ES (%75)	0.79	0.80	0.70	0.78	0.80	0.80	0.77	0.75	0.67
Historical ES (%90)	0.77	0.79	0.69	0.76	0.78	0.78	0.76	0.74	0.67
Historical ES (%95)	0.74	0.77	0.68	0.73	0.75	0.76	0.74	0.72	0.65
Historical ES (%99)	0.66	0.70	0.61	0.66	0.67	0.68	0.66	0.65	0.60

As given in table 3.6, it can be concluded that the predictive power reduces as risk measure gets closer to tail. For instance, the tail measures that could be predicted with highest confidence are Expected Shortfall (%75) and VaR (%90). On the other hand, investors are more interested in tail events which can be captured by either VaR (%99) or ES (%99). Still historical data may be useful for future tail risk measures. Interestingly, in order to predict ES(%99) and VaR(%99), historical variance seems to be most appropriate risk measure based on the average correlation. This could be because of instability historical tail risk measures since there are less number of observations in the tail. Using longer investment horizon and observation horizon could be a remedy for this by increasing number of observations due to longer horizon. In this regard, the correlation between historical risk measures and future risk measures calculated over 3 years of observation and investment horizon is given below:

Table 3.7: Correlation between Risk Measures on a 3-Yearly Basis

Correlation	Future VaR (%75)	Future Variance	Future VaR (%99)	Future VaR (%95)	Future VaR (%90)	Future ES (%75)	Future ES (%90)	Future ES (%95)	Future ES (%99)
Historical VaR (%75)	0.78	0.64	0.57	0.68	0.71	0.69	0.65	0.61	0.54
Historical Variance	0.75	0.65	0.58	0.66	0.69	0.68	0.65	0.63	0.57
Historical VaR (%99)	0.71	0.63	0.57	0.63	0.65	0.65	0.63	0.61	0.56
Historical VaR (%95)	0.78	0.67	0.61	0.69	0.72	0.71	0.67	0.65	0.59
Historical VaR (%90)	0.79	0.68	0.61	0.70	0.73	0.72	0.68	0.65	0.59
Historical ES (%75)	0.78	0.67	0.60	0.69	0.71	0.70	0.67	0.64	0.58
Historical ES (%90)	0.75	0.66	0.59	0.67	0.69	0.69	0.66	0.64	0.58
Historical ES (%95)	0.73	0.64	0.58	0.65	0.67	0.67	0.64	0.62	0.57
Historical ES (%99)	0.65	0.58	0.52	0.57	0.59	0.60	0.57	0.56	0.53

Increasing observation horizon reduces predictive power for tail risk measures as well in parallel to what I’ve also detected in variance subsection.

Overall it can be concluded that there is a positive correlation between historical tail risk measures and future risk measures. However, the correlation effect diminishes as risk measure moves closer to tail. It could be because of instability of the results of tail risk measures due to the lower number of observations. Even so, increasing number of observations may not help as it is tested in a longer observation horizon.

3.5. Main Findings

Based on the detailed analysis of results, summary of findings are shared below:

1. Historical yearly mean return almost has no predictive power for future mean return. For 5 years we see weak negative correlation and for 4 years I calculate weak positive correlation. Average correlation for each consecutive year is -0.06.

2. Historical kurtosis may provide limited insight for future kurtosis measures. There is weak positive correlation between past kurtosis and future kurtosis for each consecutive year analyzed. Average correlation for each consecutive year is 0.27. On a sectoral basis average correlation of kurtosis is reduced to 0.20. However, we see variability of correlation between different sectors. For instance, average correlation for Materials sector is 0.42.

3. Historical skewness has no predictive power over future return and historical return has no predictive power over future skewness. This fact is also true on a sectoral and three-year basis.

4. Historical percentile measures have strong predictive power over next years' percentile measures for the same level of percentile. I see the same decay behavior when the percentile decreases as we approach to the tail of the return distribution. For P10, correlation between past year and current year is 0.80, while the correlation decreases to 0.63 for P1. Same level of decay is seen for Expected Shortfall basis as well. Correlation for Expected Shortfall (%75) is 0.72 and correlation decays to 0.51 for Expected Shortfall (%99)

5. Expected Shortfall (%75,1 year) is very correlated with next years' variance, Value-at-Risk (%90) and Expected Shortfall (%75). In terms of correlation it has stronger correlation than Value-at-Risk measures however Variance also has the same level of strong correlation with risk measures. This could be explained the factor that Variance and Expected Shortfall uses much more data in calculations whereas Value-at-Risk is calculated based on one point. This fact may increase the sensitivity of Value-at-Risk to the input data and that may reduce its predictivity.

6. On 3 years basis calculations, correlations decrease. So, one-year based portfolio models are more efficient in terms of predictability.

7. Sector based correlation matrixes imply that the correlations become much lower than index level-based correlation matrixes. This result shows that within a sector there is much variability on a yearly basis. This demonstrates the fact that one shouldn't decide first on the sector allocations and decide within sector allocations based on historical performance.

8. Within the sector-based correlation matrices, Information Technology correlation matrix shows the strongest correlation for predictability for the next years. In Information Technology Matrix, Variance, Expected Shortfall (%75) and Value-at-Risk(%90) is the most predictive risk factors for the next years' Variance and Value-at-Risk (%90).

9. After the crisis years, it is very crucial to not use historical returns for portfolio allocation especially for crisis-non-resistant sectors. There is strong negative correlation between past and future returns except for Energy and Utility sectors, which are regarded as crisis-resistant sectors.

Overall, one can conclude that one of the essential parameters, variance, of portfolio optimization can be predicted with historical data. On the other hand, based on the empirical data, historical return data does not have predictive value. What is more, there is no historical risk measure that may also add value to out-of-time return estimations process.

In the next section, I will analyse the implications of this result by calculating optimum portfolios and analysing their performance.

4. EMPIRICAL PERFORMANCE OF PORTFOLIO OPTIMIZATION METHODOLOGY

In the previous section, I analyzed predictive value of potential risk measures and returns for portfolio optimization. In this section, I analyzed the out-of-sample results of portfolio optimization problem based on different configurations. Although portfolio optimization problem may have a simple structure, there are a lot of parameters to decide before solving the portfolio optimization problem such as (1) size of the historical data to be used for input estimation (2) the length of the investment period (3) the extent of short-selling constraint (4) the extent of concentration constraint. (5) the return target.

Before improving portfolio optimization problem results by applying different methodologies, which will be the focus of fifth section, it is important to understand the results of a simple portfolio optimization problem to detail the weaknesses and strengths. Then, it is also crucial to identify the optimal portfolio optimization structure with the best risk-adjusted returns.

In this regard, this section focuses on portfolio optimization results in detail from an empirical point of view based on US Equity data.

4.1. Data Preparation

In fourth section, infrastructure for different portfolio optimization problems is formed in SAS platform, also the automation of calculations is done. Commercial QuantQuote S&P 500 data is used in this section for al analysis. The scope of the analysis is extended to the end of 2016 stock data.

I solved 50 different portfolio optimization problems by differentiating upper bounds, lower bounds and return targets then calculating return and risk factors based on different periods. Based on these breakdown criteria, a total of 36.000 portfolio optimization are solved and then performance results on out-of-sample data for 12 different investment periods are calculated. Overall, I calculated 432.000 out-of-sample results (36.000 portfolio optimization output * 12 investment periods). This is one of the most extensive empirical study of portfolio optimization results based on recent data.

4.2. Automation and Parametrisation of the Process

This subsection aims to compare the performance of different portfolio optimization methods on out-of-time empirical U.S. S&P 500 Equity data. Differentiation of these portfolio optimization methods is structured by changing the following parameters: Investment horizon, risk and return parameters estimation horizon, return constraints, upper and lower bounds on a single stock allocation.

There are many parameters to decide on portfolio optimization. In this section, I focused on several parameters in the portfolio optimization. I am trying to mimic a trader who would like to use the portfolio optimization problem. As there are lots of parameters to decide I focus on the variance minimization problem with upper bound (UB), lower bound (LB), return constraints (Ret). Also, the observation period and investment period will be parametric in this optimization problem. Both observation period and investment period will range from 1 quarter to 12 quarters independently.

QuantQuote S&P 500 daily data between 1 January 1999 and 31 December 20016 is used. Overall, 17 years of daily S&P stock return data is used for the application. There are total of 462 stocks that have available data between those dates. So, the optimization problem will focus on the data of 462 stocks.

In order to detail the portfolio optimization types that will be covered, I'll start with the brief simple formulation of a portfolio optimization problem that we'll work on.

$$Min\ f(x) = w^T\ \Sigma\ \text{w st}$$

$$\sum w_i\ = 1$$

$$LB\ \leq w_i\ \leq \text{UB}$$

for i=1,..,462

$$\sum w^T\ *\ \mu \geq \text{Ret}$$

(4.1)

f(x) is variance of portfolio as in portfolio optimization, we are trying to minimize variance or standard deviation. W is the vector for optimum portfolio weights with 464 rows, w^T is transposed version of vector w. Σ is the calculated covariance matrix based on the observation period used that will also range between 1 quarter to 12 quarters. μ is the observed daily return vector with 1x464 dimensions based on the same observation period of covariance matrix.

In this section, I'll specifically focus on different values of LB, UB and Ret.LB is the parameter for lower bound of investment each stock. LB could be 0 or -0.99. So, it is basically a parameter that will decide whether short-selling is allowed or not. UB is the parameter for Upper Bound of investment on each stock. Upper bound will take values of 5,10 and 99. The effect of concentration constraints will be measured. It will help the investor to avoid overconcentration issues. Ret is the target return for the portfolio optimization problem. Yearly return target will range between -%99 to %5 and it will get the values of %99, %1, %3, %5 and %7. So, it will help investor to find different points in the efficient frontier. I could select more points on target return. However, due to the performance constraints I focused on a limited target return set.

4.3. Application of the Process

Application of the problem includes the steps below:

1) Filtering the input data depending on the portfolio creation date. (The simulated portfolio creation date starts from 1.1.2002 till 1.1.2017 and will increase quarterly. There are 60 simulated current dates. I will use hereafter "PCD" as the acronym of portfolio allocation date.)

2) Input calculation for portfolio optimization problem depending on the duration of the observation period. For the sake of comparability, I used empirical risk and return inputs without any adjustment. (There are 12 Observation period types ranging from 1 quarter to 12 quarters. I will refer Observation Period as "OP" hereafter. Based on the observation period value, input data before the current date will be used.)
   a) Return calculation
   b) Variance/covariance matrix calculation

3) Solving portfolio optimization problem for 50 different optimization configurations (The list of portfolio optimization configurations covered are listed in the appendix section. Each configuration has an acronym. Acronym formulation is given in appendix.

4) Running out-of-sample performance calculation depends on the duration of the investment period. (Similar to observation period, there are 12 investment period types ranging from 1 quarter to 12 quarters. I will refer Investement Period as "IP" in this section) The statistics below are calculated for out-of-sample results. The details for the calculation of these will be provided in the subsequent section.

   a) Return
   b) Standard Deviation
   c) VaR(%99, 1 Day)

d) VaR(%95,1 Day)
e) VaR (%90, 1 Day)
f) Sharpe Ratio
g) Turnover
h) Effective N (Inverse HHI)
i) Kurtosis
j) Skewness

Long-short portfolios can be constructed by both buying securities to have upside exposure and selling securities to gain from the downside movements. For long-short portfolios it is tricky to calculate out-of-sample performance results and comparability between long-short portfolios and long only portfolios may be lost on return and risk level. Because the denominator, invested amount, for return and risk calculations is subject to interpretation for long-short portfolios. However, Sharpe ratio could help us to level the playing field for all portfolio formulations. In this regard, I focus on Sharpe ratio to compare the performance. Comparing Sharpe Ratios could also be regarded as a common methodology for comparing performance of the portfolio construction methodologies, based on our literature review that is summarized in the second section.

In summary, I formulated 50 different portfolio optimization types by changing target return, upper bound and lower bound parameters. I fed variance, covariance and return parameters; based on 60 different starting dates and 12 different observation period lengths. Overall, 720 input datasets are calculated (60 portfolio creation date * 12 observation period lengths). Fifty different portfolio optimization configurations are solved for each input dataset summing to 36,000 optimization problems. I calculated out-of-sample performance of each optimization problem for various investment horizons. A total of 36,000*12=432,000 out-of-sample results are calculated. I should also note that some out-of-sample results are blank, since the out-of-sample data exists till the end of 2016. I disregarded blank out-of-sample results in our analysis.

Optimization problem results with the formulation below will be detailed in Appendix:

ConfigurationCode_PCD_OP

For example, *LB_M99_UB_99_Ret_5_1_4* is decomposed as:

Configuration Code*: LB_M99_UB_99_Ret_5 (formulation is given in appendix.)*

Portfolio Creation Date: 1 (1 Jan 2002)

Obseravation Period: 4 (4 quarters)

In summary, *LB_M99_UB_99_Ret_5_1_4* is the result of an optimization problem for a trader who is trying to form a portfolio on 1st January 2002 by using the input data for the last 4 quarters. He formulates portfolio optimization problem with no concentration constraints, no short-selling restriction and a yearly return target of %5. Based on the results of this problem, 12 different out-of-sample performance results are calculated. Duration of out-of-sample results also range from 1 quarter to 12 quarters. 36,000 different portfolio optimization problems are solved in the same manner using the same formulation and then out-of-sample results are calculated.

This application will help me to analyze the joint effect of (i) observation period (ii) investment/rebalancing period (iii) return constraints (iv) short-selling limits (v) concentration limits on the portfolio optimization problem that has risk and return inputs calculated from empirical input data. I consider the number of iterations sufficiently large to analyze results and deduce conclusions from results. Also, the empirical data includes both upturns and downturns of U.S. Equity Market.

SAS/OR® Non-Linear Programming Solver (NLP Solver) is used to avoid non-invertible matrix problem that is caused by lower number of observations to calculate covariance matrix. This may be particularly a concern when observation period is short. SAS uses Interior Point Algorithm to solve this particular problem.

It took 3 weeks to finish whole process in a fast personal computer. Also, another challenging aspect of solving such a high number of portfolio optimization problem is the automation work that helped me to eliminate operational risks. This also led to a very code-intensive work. I shared the sample codes written in SAS in the appendix. I used SAS Base, SAS OR libraries and SAS Macro code language for the application part of this section. I believe that this study is one of the most data-intensive and code-intensive empirical portfolio optimization work in the literature.

After solving optimization output I calculated out-of-sample performance results for different investment periods. I used standard Sharpe ratio for the comparison of results.

The formula is given below:

$$Sharpe\ Ratio = \frac{Return\ of\ the\ Optimal\ Portfolio - Risk - free\ Rate}{Standard\ Deviation\ of\ \ the\ Optimal\ Portfolio} \tag{4.2}$$

4.4. Detailed Results

In order to calculate performance, I wrote parametric performance calculation code that takes portfolio optimization output as input and run this input on the future data. The duration of the future period is parametric so that different investment periods could be analysed from 1 quarter to 12 quarters.

4.4.1. Effect of Short-Selling Constraint

Based on the out-of-sample-results I found that short-selling constraint is the most impactful contributor to the results of portfolio optimization problem among all constraints and parameters analyzed.

Optimal portfolios with short selling constraint yield better results than equally weighted, 1/N, portfolios whereas optimal portfolios without short selling constraint are dominated by equally weighted portfolios. The results based on different lower bounds are given in the Table 4.1:

Table 4.1: Performance Results based on Different Short-Selling Constraints

LB	Sharpe Ratio
0	1.66
1/N Portfolio	1.44
5%	1.03
10%	0.91
20%	0.86
99%	0.23

This table explains one of the most fundamental discussion in portfolio optimization literature regarding whether naïve strategies yield better results than optimal portfolios created with Markowitz theorem. Depending on the usage of short selling constraint one can deduce different outcomes.

4.4.2. Effect of Concentration Constraint

Concentration constraint should be one of the preferred constraints in a portfolio optimization problem from practitioner perspective. Risk diversification is an essential requirement of a portfolio allocation process. However, portfolio optimization problem implicitly takes into account concentration risk by aiming to minimize risk. Depending on the correlation matrix data, optimal portfolio should be well diversified. However, in certain cases, depending on the outlier data the portfolio optimization problem may yield an optimal portfolio that includes concentrated stocks. Especially, these results are more pronounced in portfolios without short-selling constraint.

For long-only portfolios I find the effect of short selling constraint very limited. This result implies that result of portfolio optimization problem is not concentrated most of the time and concentration constraint may be a slack constraint especially for investment universes with more than 100 stocks. The results are given in Table 4.2:

Table 4.2: Performance Results based on Different Concentration Constraints

UB	Sharpe Ratio
1/N Portfolio	1.44
5%	1.64
10%	1.65
99%	1.68

As portfolios without concentration constraint yield slightly better results, I drilled down the results based on different angles. From return constraint perspective I find the similar result trend for concentration constraint as given in the Table 4.3:

Table 4.3: Performance Results based on Different Concentration and Return Constraints

Ret	UB	Sharpe Ratio
1%	5%	1.64
1%	10%	1.66
1%	99%	1.68
3%	5%	1.63
3%	10%	1.65
3%	99%	1.67
5%	5%	1.62
5%	10%	1.64
5%	99%	1.66
7%	5%	1.65
7%	10%	1.64
7%	99%	1.67
1/N Portfolio	1/N Portfolio	1.44
MVP	5%	1.67
MVP	10%	1.68
MVP	99%	1.70

Another angle that I would like to analyze is observation period. As observation period gets shorter the probability of finding extreme return results increases. In this respect, even the main goal of portfolio optimization is diversification, extreme return inputs may still cause overconcentration in certain stocks. In this case, the question would be whether overconcentration would help or not. The results based on different observation periods are given in Table 4.4:

Table 4.4: Performance Results based on Different Concentration Constraints and Observation Period

OP	UB	Sharpe Ratio
1	5%	1.50
1	10%	1.57
1	99%	1.69
2	5%	1.64
2	10%	1.66
2	99%	1.69
3	5%	1.59
3	10%	1.60
3	99%	1.64
4	5%	1.55
4	10%	1.55
4	99%	1.54
5	5%	1.62
5	10%	1.60
5	99%	1.65
6	5%	1.59
6	10%	1.59
6	99%	1.61
7	5%	1.63
7	10%	1.63
7	99%	1.64
8	5%	1.70
8	10%	1.71
8	99%	1.72
9	5%	1.76
9	10%	1.76
9	99%	1.78
10	5%	1.72
10	10%	1.72
10	99%	1.73
11	5%	1.69
11	10%	1.70
11	99%	1.71
12	5%	1.71
12	10%	1.72
12	99%	1.73

From the results it can be seen that as observation period gets shorter, it would make sense to invest in an overconcentrated portfolio. This could be attributed to the momentum effect. Momentum effect is the tendency of recent outperformers to remain as outperformers in the future. A better representation of this results is given in the graphic below. The Figure 4.1 shows the Sharpe Ratio performance gap between portfolios with %5 concentration constraint and no concentration constraint for each observation period.

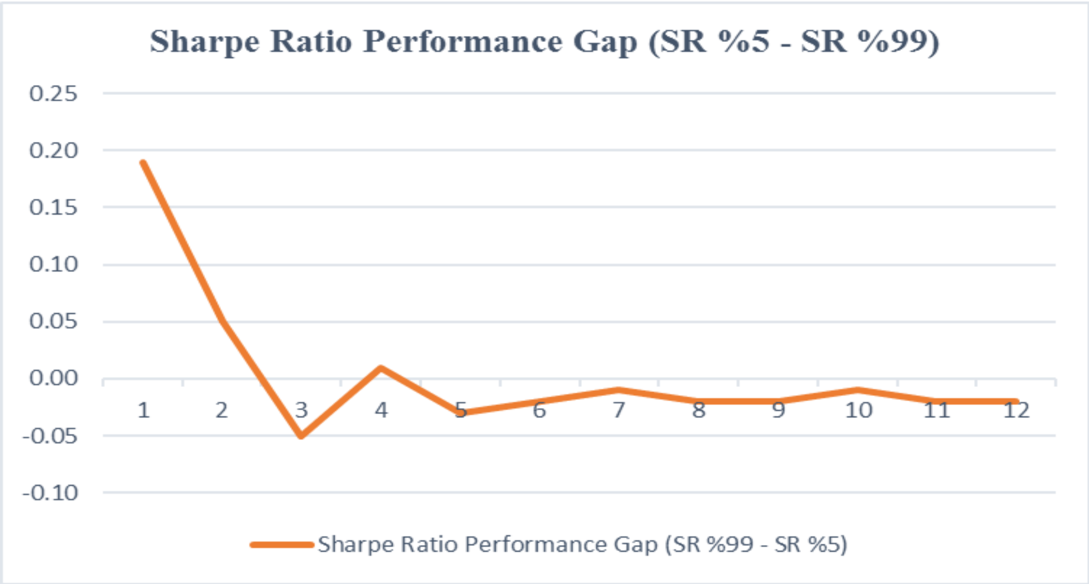

Figure 4.1: Sharpe Ratio Performance Gap between Portfolios with no concentration constraint and %5 concentration constraints

In Figure 4.1, SR %5 stands for portfolios with %5 concenstration constraint, while SR %99 stands for portfolios with no constraint. The graphic demonstrates that when observation period is shorter than 3 quarters, it makes sense not to add concentration constraint.. However, if observation period is longer than 4 quarters, it would be a better decision to add concentration constraint from Sharpe Ratio perspective. This result could be causes by the decay of momentum effect in the long-run.

4.4.3. Effect of Return Target

The primary focus of a trader minimizing risk for a certain level of return. In this regard, in this subsection I analyzed the effect of adding return target to portfolio optimization problem and sensitivity of the out-of-sample results to return constraint. High level results are given in Table 4.5:

Table 4.5: Performance Comparison based on Different Return Constraints

Ret	Sharpe Ratio
1%	1.66
3%	1.65
5%	1.64
7%	1.65
1/N Portfolio	1.44
MVP	1.68

Independent of return target, optimal long-only portfolios always outperformed 1/N naively diversified portfolios. However, results imply that minimum variance portfolios outperform portfolios with return target. Consequently, adding return target doesn't make sense.

Another question would be whether setting return target higher is whether helps achieving higher returns with higher risk or not. The results below in Table 4.6 shed light on this question:

Table 4.6: Detailed Performance Comparison based on Return Constraint

Ret	Sharpe Ratio	Average Return	Average Standard Deviation
1%	1.66	12.69%	10.86%
3%	1.65	12.57%	10.85%
5%	1.64	12.49%	10.84%
7%	1.65	12.63%	10.84%
1/N Portfolio	1.44	18.87%	18.30%
MVP	1.68	12.95%	10.89%

Interestingly, minimum variance portfolios yield higher returns than portfolios with return target and there is not any major difference between the results of optimal portfolios with different return targets. Another notable aspect of this result is the performance of 1/N naively diversified portfolios. 1/N portfolios yield higher returns but in expense of higher risk. It can be concluded that 1/N portfolios are not diversified enough. Thereby, I can conclude that taking into account covariance matrix and variance measures help us decrease the risk. This result is parallel with the results of the previous section (Empirical Analysis of Risk and Return Factors). There is no correlation between historical return and future return. On the other hand, there is strong correlation between historical risk measures and future risk measures. As a result of this, using empirical data for return and risk measures may help decreasing risk but it won't have a positive effect on return. The existence of the fifth section (Estimating Return with Markov Transition Matrix Approach) is due to this fact. I'll try to improve the predictability of return input by using Markov Transition Matrix Approach.

4.4.4. Effect of Observation Period

Although it is not directly observable as a parameter in the portfolio optimization problem, setting observation period is one of the key decisions in portfolio allocation problems. Observation period could be defined as the length of historical data that will be used for calculating return and risk inputs. Using longer historical data may help to smooth out outlier behaviors at the expense of ignoring short-term movements. The Table 4.7 below demonstrates results for long-only portfolios with different observation periods on a quarterly basis:

Table 4.7: Performance Comparison for Long-only portfolios with Different Observation Period

OP	Sharpe Ratio	Average Return	Average Standard Deviation
1	1.59	12.27%	11.51%
2	1.65	12.86%	11.24%
3	1.61	11.99%	10.65%
4	1.54	11.44%	10.61%
5	1.62	12.37%	10.69%
6	1.60	12.41%	10.75%
7	1.63	12.66%	10.82%
8	1.71	13.07%	10.81%
9	1.77	13.61%	10.85%
10	1.73	13.38%	10.85%
11	1.70	13.06%	10.96%
12	1.72	13.29%	11.02%

Based on the results, I can conclude that using longer empirical data can produce higher average risk-adjusted. For long only portfolios, longer empirical data may provide very modest improvements in Sharpe ratio and this improvement stops after 9 quarters of empirical data threshold. As Sharpe ratio has two components, return and standard deviation these results for each component can also be analyzed.

From risk perspective, using even 2 quarters of data may be sufficient for diversification. For return estimation, using at least 2 years of data may yield better results. However, usage of 2 years of data instead of 2 quarters of data may produce similar results for risk measures. In conclusion, using at least 2 years of data for risk and return estimation is optimal. This conclusion creates a new question: "What should we do if the stock has limited data?". I'll suggest a new approach to solve this problem in the next section (Estimating Return with Markov Transition Matrix Approach).

4.4.5. Integrated Results Analysis

Decision of adding or configuring one constraint should not be evaluated isolated. In this subsection, I compare the out-of-sample performance results of 50 different portfolio optimization configurations over 60 sub-periods. In this section, I focus only on quarterly investment period, since quarterly investment period provides better risk-adjusted performance based on the all analysis done in the previous section. The description of each configuration is given in the appendix section. The Table 4.8 below summarizes Top 5 and Bottom 5 portfolio optimization configurations in addition to 1/N strategy based on the average out-of-sample Sharpe Ratio for quarterly investment period:

Table 4.8: Integrated Results Analysis for Performance Comparision of Optimal Portfolios based on Empirical Data

Rank	Configuration_Name/IP	Quarterly
1	LB_0_UB_99_Ret_M99	1.84
2	LB_0_UB_10_Ret_M99	1.82
3	LB_0_UB_99_Ret_1	1.81
4	LB_0_UB_99_Ret_7	1.80
5	LB_0_UB_99_Ret_3	1.80
16	CurReturn	1.59
47	LB_M99_UB_99_Ret_3	0.38
48	LB_M99_UB_99_Ret_1	0.33
49	LB_M99_UB_99_Ret_M99	0.27
50	LB_M99_UB_99_Ret_7	0.26
51	LB_M99_UB_99_Ret_5	0.24

Table above demonstrates average Sharpe Ratio based on quarterly investment period and the results are aggregated for each observation period length that is analysed in this study. It is clearly shown that the primary effect on the performance is short-selling constraint. The worst performers don’t have short-selling constraints while top performers always have short-selling constraints. Return constraints and concentration constraints may not be a strong differentiator. As the final step, I take one step closer to the results by adding observation period as a dimension to out-of-sample average quarterly Sharpe Ratio results for the Top 5 and Bottom 5.

Table 4.9: Top 5 and Bottom 5 Optimum Portfolio Configurations in terms of Sharpe Ratio

Rank	Configuration Name	Quarterly OP	Yearly OP	Three Yearly OP
1	LB_0_UB_99_Ret_3	1.83	1.69	1.89
2	LB_0_UB_10_Ret_3	1.69	1.72	1.89
3	LB_0_UB_99_Ret_M99	1.90	1.70	1.88
4	LB_0_UB_10_Ret_M99	1.76	1.72	1.86
5	LB_0_UB_10_Ret_1	1.73	1.70	1.85
16	CurReturn	1.59	1.59	1.59
47	LB_M99_UB_99_Ret_3	-0.28	0.28	0.89
48	LB_M99_UB_99_Ret_1	0.10	0.08	0.86
49	LB_M99_UB_99_Ret_5	-0.34	-0.10	0.83
50	LB_M99_UB_99_Ret_7	-0.47	0.20	0.81
51	LB_M99_UB_99_Ret_M99	-0.55	0.56	0.81
	Average	0.60	1.07	1.39

Based on the Table 4.9 above, three main empirical findings can be summarized: (1) Longer observation periods provide higher Sharpe Ratios. (2) Portfolio with short-selling constraints provide higher risk-adjusted returns than long-short portfolios and performance of 1/N strategy lies between those portfolios. (3) For long-only portfolios, when the observation period is shorter, portfolios without concentration constraints generate higher risk-adjusted returns. This could be explained by momentum effect. Even in some cases, it seems that using shorter observation period could be better than using longer observation period for portfolios without concentration constraints. However, the results are very similar, and difference is not statistically significant. (4) For long-short portfolios, the momentum effect does not work in favor of portfolios without concentration constraints.

4.5. Main Findings

In this section, I solved 36.000 different portfolio optimization problems based on daily S&P U.S. data. I compared 50 different portfolio optimization methods that are differentiated by altering return target, observation period, concentration and short-selling constraints. I can summarize the results of this paper based on the comparison of Sharpe Ratio as a benchmark on out-of-time results.

Among the all parameters perturbed, short-selling constraint is the most critical constraint when empirical data for return and risk estimation is used. Long-only portfolios dominate 1/N strategy (p-value<%14) while long-short portfolios are dominated by 1/N strategy (p-value<%0.1). The level of significance is very high that trading cost may be negligible. For long-only portfolios Sharpe Ratio can be increased by increasing the observation period till 2 years of observation period then it doesn’t make any difference to increase the observation period. For long-short portfolios, it is advised that short-selling constraint should be very limited and observation period should be at least 3 years to smooth out outlier returns.

For long-only portfolios, it is advised not to add concentration constraint when shorter observation period and investment period is preferred to benefit from momentum effect.

For long-short portfolios same principle is not advised. This could imply that momentum effect is weaker for losers.

Returning back to my initial questions, if the trader would like to form long-short portfolios using empirical data is not helpful, trader should find other ways to incorporate information into portfolio optimization process. For long-only portfolios trader can use empirical data of at least 2 years especially for investment horizons longer than two quarters. For quarterly invetment horizon and quarterly observation period for long-only portfolios, it is advised not to add concentration constraint to capture momentum effect.

Based on the detailed analysis in previous subsections, I would like to highlight once more the main take away points below:

1. Short-selling constraint is the most critical constraint when empirical data for return and risk estimation is used. Long-only portfolios dominate 1/N strategy (p-value<%14) while long-short portfolios are dominated by 1/N strategy (p-value<%0.1). The level of significance is very high that trading cost may be negligible.

2. For long-only portfolios Sharpe Ratio can be increased by increasing the observation period till 2 years of observation period then it doesn't make any difference to increase the observation period

3. For long-only portfolios, it is advised not to add concentration constraint when shorter observation period and investment period is preferred to benefit from momentum effect. For long-short portfolios same principle is not advised. This could imply that momentum effect is weaker for losers.

4. If the trader would like to form long-short portfolios using empirical data is not helpful, trader should find other ways to incorporate information into portfolio optimization process. For long-only portfolios trader can use empirical data of at least 2 years especially for investment horizons longer than two quarters. For quarterly

invetment horizon and quarterly observation period for long-only portfolios, it is advised not to add concentration constraint to capture momentum effect.

5. For long-short portfolios, it is advised that short-selling constraint should be very limited and observation period should be at least 3 years to smooth out outlier returns.

6. Naively diversified 1/N portfolios may generate higher returns in the expense of higher risk and based on Sharpe Ratio getting higher return may not be justified and relying on optimal portfolios would yield higher risk-adjusted returns. Another implication of this results would be the related to usage of empirical risk and return inputs. As optimal portfolios yield lower returns independent of return target than 1/N naively diversified portfolios, the usability of empirical historical return should be questioned. In this regard, I'll try to improve the usability of empirical historical return in the next section.

Since estimating return is more important, I will focus on alternative methods for estimating return in the next section rather than improving risk measures by adding systemic risk factors.

5. ESTIMATING RETURN WITH MARKOV TRANSITION MATRIX APPROACH

In this section, I focused on improving return estimation by using a novel method that is based on Markov Transition Matrix approach.

5.1. Estimating Returns with Markov Transition Matrix Approach

Markov Transition Matrix Approach is based on the assumption that a security's current state is the only determinant for the future state. It also assumes that history, or past states, is not important. Only current state is relevant. To my knowledge, Markov Transition Matrix is not used for portfolio optimization problems. In order to use MTMA we need to ask ourselves 3 questions: 1) What defines current state? 2) How we can assign securities to states? 3) How we can calculate transition probabilities.

I tried to answer these questions experimentally based on empirical data of QuantQuote S&P 500 Data. The methodology steps below define the overall process for Markov Transition Matrix Approach for Portfolio Optimization Problem:

1. Calculate risk and return data for the last 6 quarterly periods.

2. Classify securities from individual return perspective first for the recent period and assign (bins) to each security based on a simple sorting algorithm.

3. Calculate the empirical transition probabilities based on the current and previous period for the last 6 periods. This will lead to five empirical transition matrices. I also analysed the results based on different averaging periods. The number of periods used are: 2 (no averaging), 6, 11.

4. Calculate transition probabilities by averaging five empirical transition matrices.

5. Estimate return for the future investment period for each bin based on conditional probability formula.

6. Assign expected return for each security based on the assigned rating (bin) of each equity.

I believe that, this estimation method will reduce the estimation error and improve portfolio optimization results. This methodology shares some commonalities with averaging methods. The differentiation is the usage of bucketed data for each security, not the average of all securities because the global average may not be the best predictor for the return of a security. For covariance matrix empirical results based on the previous quarters covariance matrix will be used.

I used SAS/IML® (Interactive Matrix Language) and SAS Base ® for calculating return input with Markov Transition Approach. SAS/IML processes matrix-type calculations efficiently and our methodology estimation requires matrix level calculation steps. SAS Base handles the fundamental calculations before and after SAS/IML process such as calculating return for each security based on the empirical data for each quarter.

5.2. Automation of the System based on Markov Transition Matrix Approach

In order to calculate results in an automatized manner, I coded Markov Transition Matrix Approach Parametrically and in line with the expected input format of the portfolio optimization problem that is built in the previous sections.

I also leveraged what I had learned based on Empirical Performance of Portfolio Optimization Methodology section. I didn’t use the same 50 portfolio optimization configurations. Instead, I focused on long-only portfolios by using 15 different configurations by differentiating return estimation and upper bounds. I also used 12 quarters and 16 quarters as observation periods as it is clear that shorter observation periods don’t add value. In terms of results that are shown in section, I reported results with 16 quarters of observation period but it is worth highlighting that the results do not deviate too much when 12 quarters of observation period is used for covariance matrix calculation.

Instead of assigning nominal return estimates, I assigned ratios that calculates different points in efficient frontier. The formula for target return is given below:

$$Ret = RetMin * (1 - \alpha) + RetMax * \alpha \tag{5.1}$$

where $0 \leq \alpha < 1$, and Ret is defined as target return is defined as target return. RetMin is defined as the minimum estimated expected return. It is the lowest part of the efficient frontier. RetMax is defined as the maximum estimated expected return. It is the highest part of the efficient frontier. Parameter α helps us to solve portfolio optimization problem for the different parts of efficient frontier. Potential values for α is 0, 0.2, 0.4, 0.6, 0.8. These points are selected to cover each part of efficient frontier proportionately. When $\alpha = 0$, the minimum variance problem is solved. For other values, I go through the potential points of the efficient frontier. Definition of other parameters remained same in this progress period. I solved the portfolio optimization problems based on expected returns estimated with MTMA and naive return estimations based on 12 quarters of historical data. Also, for validating results I compared the output results of minimum variance portfolios as these portfolios don’t rely on return estimations. So far, it is validated that the code is working properly.

I believe that this methodology will bring a new way of thinking for input estimation for portfolio optimization problems. MTMA is easier to form and calculate. It is intuitively simple. It doesn’t increase the solver time since the format of portfolio optimization problem doesn’t change. Only the input calculation duration changes and this duration is relatively short and the effect could be neglected compared to the optimization time. What is more, MTMA can also utilize the power of momentum and mean-reversal effects.

The code for return estimation with MTMA is given in the Appendix section.

5.3. Application

I used QuantQuote S&P 500 daily data between 2000 and 2016. I believe that this period covers both economic upturns and downturns so that our analysis is robust to the variability of economic cycle.

Our equity universe includes 462 equity data that has available data for the period analysed. I divided the equity performance data into quarterly intervals. In order to compare performance results on out-of-time data I always used historical data for input calculation then solve the portfolio optimization problem and calculate performance results based on the out-of-time data.

As in the Estimating Returns with Markov Transition Matrix Approach section I detailed the input estimation part, I'll continue with the structure of the problem. I solved a total of 15 different portfolio optimization configurations for each period. These configurations are created based on the differentiation of the basic parameters of a portfolio optimization problem. These parameters are concentration constraints and return target. The formulation is given below:

$$Min\ f(x) = w^T\ C\ w$$

st.

$$\sum w_i = 1$$

$$LB \le w_i \le UB$$

for i=1,..,462

$$\sum w^T * \mu \ge Ret \tag{5.2}$$

w is the vector for optimum portfolio weights with 462 rows. C is the calculated covariance matrix based on the previous quarterly return data. μ is the calculated daily return vector with Markov Transition Matrix Approach defined in the previous section.

The values assigned for each parameter and interpretation of each parameter is given below:

LB is the lower bound for the investment in an individual stock. LB=0, short selling is not allowed in this application to narrow the focus and ensure comparability with other works in the literature.

UB is the upper bound for the investment in an individual stock. Potential values are: %99, %10, %5. When UB=%99, there is no concentration constraint. As formulated in Jagannathan and Ma (2003), using upper bound for portfolio weights is equal to adjusting variance-covariance matrix by increasing lower correlations.

Ret is the target return of the portfolio optimization problem. Ret is defined based on a more complicated formula by differentiating the parameter α. Potential values for α is 0, 0.2, 0.4, 0.6, 0.8. When α=0, the minimum variance problem is solved. For other values, I go through the potential points of the efficient frontier. The formula for Ret is given in formula in (5.1).

RetMin is defined as the minimum estimated expected return. It is the lowest part of the efficient frontier. RetMax is defined as the maximum estimated expected return. It is the highest point of the efficient frontier. Parameter α helps us to solve portfolio optimization problem for the different parts of efficient frontier.

I believe that the formulation of the problem above is closer to what practitioners are doing based on the recent investment management survey for portfolio construction (Noël, Goltz and Lioui, 2011) except return estimation methodology designed in this paper. Survey results indicate that (1) variance (directly or indirectly by using normal distribution VaR) is still the most popular risk measure (2) covariance/variance matrix is calculated based on sample estimate (3) estimation risk is mostly dealt with imposing weight constraints.

For each configuration, I solved the problem from 2002 to 2016 for each quarter by calculating return and risk inputs. I also used empirical data from 2000 to 2002 to calculate initial risk and return inputs. It helped me to form a stable foundation to compare the results of Markov Transition Matrix Approach based on different circumstances by solving problem over different periods and portfolio optimization configurations that might be applicable for practitioners with various goals. I solved each problem with empirical return inputs and then return estimations with Markov Transition Matrix Approach. A total of 3200 problems are solved. Then based on the results of the optimization problem I calculated out-of-time performance of each portfolio. I used one of the most widely used performance calculation parameter, Sharpe Ratio as the performance comparison criteria.

5.4. Detailed Results

In this section, I analysed the results based on out-of-sample Sharpe Ratio, tail risk measures and compared outputs by using several breakdown criteria such as return target and concentration constraint to distinguish the effect of these parameters.

Before I would like to compare and analyse the results, I would like to note that the configuration description of each problem is given in the appendix section. As a Prefix, I used "TM_Averaging Period" for the results that are based on return estimation with Markov Transition Matrix approach and "ER" for the results that are based on empirical return estimations. "TM_1" means that I used only 1 transition matrix and no averaging is done and TM_5 and TM_10 relies on 5 and 10 empirical transition matrices respectively.

Before diving into detailed results analysis, I first validated our results by comparing Sharpe Ratios quarterly investment period of each methodology when Return constraint is 0. The results for minimum variance portfolios are given in the table 5.1:

Table 5.1: Performance Result for Markov Transition Matrix Approach

Methodology	Return (%)	Std Dev.(%)	Sharpe Ratio
1/N	13.02%	18.16%	1.15
ER	11.02%	11.18%	1.45
TM_10	11.02%	11.18%	1.45
TM_1	11.02%	11.18%	1.45
TM_5	11.02%	11.18%	1.45

First of all, based on the results I can validate our methodology and process. As minimum variance portfolio doesn't rely on return estimates all methodologies yield the same results, as their risk estimates are common. What is more, I can conclude that minimum variance portfolios have superior performance results than equally weighted, 1/N, portfolios. I would also like to test different points of efficient frontier. The results of a very modest return target strategy are given in Table 5.2Table :

Table 5.2: Results for Quarterly Investment Period when Ret=%20

Methodology	Return (%)	Std Dev.(%)	Sharpe Ratio
1/N	13.02%	18.16%	1.15
ER	11.02%	11.18%	1.45
TM_10	11.06%	11.19%	1.46
TM_1	11.05%	11.18%	1.46
TM_5	11.02%	11.18%	1.45

As it is seen in the table above, Ret=%20 constraint is not a binding constraint for almost all portfolios. Minimum variance portfolio can satisfy this constraint in %99 of the cases. Ret=%20 constraint could be regarded as a modest constraint. In other words, return estimations are not very important when return target is not high. In Table 5.3 and 5.4, I compare the results of portfolios with aggressive return target to compare the performance of Markov Transition Matrix methodology with naïve return estimation methods.

Table 5.3: Results for Quarterly Investment Period when Ret=%60

Methodology	Return (%)	Std Dev.(%)	Sharpe Ratio
1/N	13.02%	18.16%	1.15
ER	11.95%	13.46%	1.34
TM_10	10.26%	11.36%	1.34
TM_1	10.57%	11.40%	1.40
TM_5	10.84%	11.39%	1.39

Table 5.4: Results for Quarterly Investment Period when Ret=%80

Methodology	Return (%)	Std Dev.(%)	Sharpe Ratio
1/N	13.02%	18.16%	1.15
ER	14.31%	19.45%	1.20
TM_10	8.73%	12.10%	1.20
TM_1	11.32%	12.12%	1.44
TM_5	10.56%	12.10%	1.34

As it can be seen clearly in the tables above, Transition Matrix methodology outperforms empirical naïve return estimations when return targets are higher. Moreover, Transition Matrix Methodology that is not based on averaging (TM_1) and using the recent transition matrix outperforms Transition Matrix Methods with averaging.

There are various critiques against return estimations. Also, mean-variance optimal portfolios in upper part of the efficient frontier are generally unstable and generate worse results than optimal portfolios in lower parts of the efficient frontier. By using transition matrix estimation methods for return estimation, I can still outperform 1/N portfolio and risk-adjusted returns are still as good as minimum variance portfolio.

I also compared the results of each Transition Methodology when return target is %80 based on different concentration constraints. Concentration constraint doesn't affect results for TM_1 and TM_5. However, there are small performance improvements for TM_10 as given in Table 5.5.5:

Table 5.5: Results for Quarterly Investment Period when Ret=%80 Based on Different Concentration Constraints

Methodology	UB (%)	Return (%)	Std Dev.(%)	Sharpe Ratio
TM_10	5	9.69%	12.31%	1.29
TM_10	10	8.84%	12.09%	1.22
TM_10	99	8.73%	12.10%	1.20

For any constraint, TM_10 methodology still underperforms TM_1. For this study, I can conclude that averaging transition matrices do not add value. However, one can try different binning methodology or change the number of bins to test the robustness of results. Optimization of number of bins and binning methodology is beyond the scope of this thesis.

I also analysed the results from Value-at-Risk perspective; as it is known that tail risk measures are also very critical for practitioners based on survey Noël, Goltz and Lioui (2011). Based on the results it is found that Transition Matrices Portfolios are less exposed to tail risk than Empirical Return Portfolios and Equally Weighted Portfolios and the difference between results are very high. Value at Risk results are not very sensitive to the choice of averaging method as demonstrated in Table 5.6::

Table 5.6: Value-at-Risk Results for Quarterly Investment Period when Ret=%80

Methodology	VaR(1 day,%95)	VaR(1 day,%90)
1/N	-1.85%	-1.41%
ER	-1.90%	-1.43%
TM_10	-1.24%	-0.91%
TM_1	-1.19%	-0.91%
TM_5	-1.22%	-0.89%

Value-at-risk gap narrows down as we set return target less aggressive as can be seen from the Table 5.7:

Table 5.7: Value-at-Risk Results for Quarterly Investment Period when Ret=%60

Methodology	VaR(1 day,%95)	VaR(1 day,%90)
1/N	-1.85%	-1.41%
ER	-1.33%	-1.01%
TM_10	-1.13%	-0.83%
TM_1	-1.12%	-0.84%
TM_5	-1.12%	-0.84%

5.5. Main Findings

In this section, I proposed a new methodology for return estimation in portfolio optimization: Markov Transition Matrix Approach (MTMA). I believe that this methodology may provide: (1) Robust results by using a sophisticated averaging method (2) More realistic return estimations that can embed the effect of mean-reversals intrinsically, which empirical returns estimations or shrinking methods can never do because the starting point for shrinking is the empirical returns. Thus, returns of underperforming securities can't be estimated higher than other securities with shrinking methods. (3) Lower solver time compared to sophisticated sampling or optimization techniques because proposed methodology is based on the simple quadratic optimization methodology. (4) Less reliance on longer historical data. This is useful for securities that are issued recently. The data proposed methodology would require is only a quarterly data on equity level. (5) Intuitively simple estimates as it is based on a transition matrix as an input. (6) Flexibility in term of portfolio optimization techniques selection. User can use this methodology as a return estimator and then plug these estimates into any type of portfolio optimization problem given that the methodology requires a point estimate for returns. (7) More flexible usage than naïve averaging methods, because practitioner can't select target return in naïve averaging methods for each bin.

Based on out-of-sample performance results I can conclude that Transition Matrix Approach improve Sharpe Ratios when set aggressive return target is set compared to empirical return methods. It can also be concluded that MTMA method doesn't cause unstable returns or Sharpe Ratios for the upper part of Efficient Frontier. Besides, optimal portfolios that are based on Markov Transition Matrix methodology are exposed to much less tail risk compared to optimal portfolios based on empirical return. Last but not least, Transition Matrix methodologies that are based on averaging (TM_5, TM_10) are underperformed by Transition Matrix methodologies without averaging (TM_1). This result implies that previous transition matrix is the best predictor of next transition matrix.

I also validated the results of Kritzman et al. (2010): Optimized portfolios, whether empirical returns or transition matrix returns are used, are superior than equally weighted (1/N) portfolios.

Due to data availability issues for systemic risk measures calculation and time constraint, my thesis evolved into improvement of portfolio optimization by suggesting new methodology for portfolio allocation. The suggested methodology is detailed in the next section.

6. PORTFOLIO ALLOCATION WITH TODIM

6.1. Muti-Criteria Decision Making Methodologies for Portfolio Allocation

Portfolio optimizers have several drawbacks: 1) They are sensitive to the noise in the input and they may be viewed as error optimizers rather than objective optimizers. 2) It might take more time to solve a portfolio optimization problem. 3) Optimization algorithms may seem complex and hard to understand for high level management. That is why high-level management may be reluctant to leave power to an algorithm that they might not understand. 4) Optimizers may get more complex when relaxed assumptions are included in the optimization problem such as short-selling constraint, tracking error constraint, turnover constraint etc. 5) It is harder to balance or optimize multiple criteria with optimizers.

To address these issues, various methodologies have been proposed. Robust portfolio optimization methodologies have been proposed to overcome extreme sensitivity to inputs. Multi-criteria decision making (MCDM) methodologies have been proposed to remedy the issues summarized above. MCDM has a promising capability of taking into account multiple criteria in portfolio allocation process. In this respect, various studies incorporated various MCDM methodologies such as genetic algorithms (see Kalayci, Ertenlice, Akyer and Aygoren (2017), combination of MCDM techniqus (see Ho, Tsai, Tzeng & Fang, 2011)

6.2. Portfolio Allocation with TODIM

TODIM method relies on Prospect Theory proposed by Kahneman and Tversky (1979) that explains how people decide in risky circumstances. The main takeaway of the theory is that people have asymmetric responses to gain and losses. To detail, losses

with the same level of gains weight higher absolute value. This response level difference is embedded in TODIM with attenuation factor. I used different levels of attenuation factors to assess the impact of attenuation factors as well.
TODIM methodology for portfolio allocation is detailed in 6 main steps:

(1) Criteria Matrix Calculation: Criteria matrix is the main input of TODIM methodologies and ingredients of the matrix depend on the investor circumstance and preference. In this study, I relied on the most common inputs (Return, standard deviation and correlation) of portfolio allocation to be able to compare the result of TODIM methodology with classical portfolio optimization problem: For each period, I calculated these measures over three different observation periods. (1 quarter, 1 year and 3 years). In total, 9 criteria are calculated (3 different observation measure * 3 different criteria). All these criteria are calculated on sample historical data. Among all these criteria, correlation has a specific treatment due to its multidimensional nature. To detail, while an equity has single empirical return and standard deviation in a period, it has *n* correlations with *n* equities (including its own correlation with itself). To plug correlation into TODIM criteria matrix, average correlation for each equity in the investment universe is calculated with the formula below:

$$\gamma_i = \frac{\left[\left(\sum_{j=1}^{j=n} Corr_{i,j}\right) - 1\right]}{n-1} \tag{6.1}$$

where γ_i is average correlation of each equity, *n* is number of equities in the investment universe and $\boldsymbol{Corr_{i,j}}$ is the sample correlation between equity *i* and *j*. This formula is used to calculate correlation for each equity based on each sample period (3 months, 1 year, 3 years)

The output of this step is matrix **A** -criteria output matrix- with *i* number of rows and *j* number of columns, where *i* is the number of equities in the investment universe and *j* is the number of criteria. Although S&P 500 includes 500 equities, equities with complete data are included. For this reason, *i*=462 and *j*=9 in our study.

(2) Criteria Matrix Transformation: In this step, to manage outliers, row criteria output matrix calculated in the first step, **A**, is transformed into a matrix by binning each element in the matrix into 10 bins. Score of 10 is assigned for the equity that is in top 10% for the related criteria. As it could be well anticipated, equities with lowest average correlation are assigned 10 for the related average correlation criteria, whereas equities with returns in top 10% are awarded 10. On the other hand, equities with highest correlation are assigned 1 for the related average correlation criteria, whereas equities with returns in bottom %10 received 1 as related return criteria score. I'll use both the output of this step, the Ranked Criteria Matrix and Raw Criteria Matrix in the subsequent steps to analyze the effect of the transformation and test whether it has an added value in the process. For the sake of simplicity, both matrices (ranked and row) are referred as **A** in subsequent steps.

(3) Normalized Criteria Matrix Calculation: Normalized Criteria Matrix has *n* rows and *j* columns. In this case, *n* is equal to 462 (number of equities) and *j* is equal to 9 (number of criteria). Number of rows is equal to number of equities in investment universe while number of columns is equal to number of criteria. To make criteria scores comparable, normalization is handled by applying the min-max formula below for each criteria score:

$$N_{i,c} = \frac{A_{i,c} - \min_i A_{i,c}}{\max_i A_{i,c} - \min_i A_{i,c}} \tag{6.2}$$

where $\boldsymbol{A_{i,c}}$ is the score of alternative *i* against criteria *c*. For instance, $\boldsymbol{A_{i,c}}$ could be long-term return of equity *i*. $\mathbf{N}_{i,c}$ is the normalized score of alternative *i* for criteria *c* so that all normalized scores range between 0 and 1. For return criteria, the higher the score the better it is for the portfolio. However, opposite is true for standard deviation and correlation as these criteria are treated as cost in other problems.

To calculate intuitive scores, the formula used for correlation and variance is altered as given below:

$$N_{i,c} = \frac{\max_i A_{i,c} - A_{i,c}}{\max_i A_{i,c} - \min_i A_{i,c}} \tag{6.3}$$

With absolute value operation, all normalized criteria part of alternative matrix range between 0 and 1 and best alternative gets the score of 10 for the related criteria. This step is repeated for each alternative and criteria to calculate all the elements of alternative matrix, $\mathbf{N}_{i,c}$.

(4) Alternative Comparison: In this step, alternatives are compared for each criterion separately quantitatively and then summed over all criteria. Prospect theory comes into play in this step with attenuation factor (AF) in the formula below:

$$CS_{i,j,c} = \begin{cases} \sqrt{\dfrac{x_c(N_{i,c} - N_{j,c})}{\sum_{c=1}^{C} x_c}} & if\ N_{i,c} > N_{j,c} \\ 0 & if\ N_{i,c} = N_{j,c} \\ \dfrac{-1}{AF}\sqrt{\dfrac{x_c(N_{j,c} - N_{i,c})}{\sum_{c=1}^{C} x_c}} & if\ N_{i,c} < N_{j,c} \end{cases} \tag{6.4}$$

$$FS_{i,j} = \sum_{c=1}^{C} CS_{i,j,c} \tag{6.5}$$

where $\boldsymbol{CS_{i,j,c}}$ is criteria score of equity *i* against equity *j* and $\boldsymbol{FS}_{i,j}$ is final comparison score of equity *i* against equity *j*. Another critical input is $\boldsymbol{x_c}$, which is defined as weight of each criteria. In application section, more details regarding quantification of weight of each criterion is provided. The process formulated above is repeated for all alternative pairs and criteria. The result of this process is a score matrix with *n* columns and *n* rows. In this case *n* is equal to 462, the number of alternatives.

(5) Normalized Alternative Score Calculation: The final step of the original TODIM methodology is calculation of rank ordering of each alternative using the formula below:

$$R_i = \frac{\sum_{j=1}^{N} FS_{i,j} - \min_i \sum_{j=1}^{N} FS_{i,j}}{\max_i \sum_{j=1}^{N} FS_{i,j} - \min_i \sum_{j=1}^{N} FS_{i,j}} \tag{6.6}$$

where $\boldsymbol{R_i}$ is the rank of equity *i*. The outcome of this step is the rank vector with *n* rows has been calculated. Values of elements of Rank vector range between 1 and 0 with most preferred equity having a value of 1 and least preferred equity having a value of 0.

(6) Portfolio Allocation Calculation: This is an additional step to TODIM method to tailor the methodology for portfolio allocation purposes. As the aim of TODIM method is to find the optimum solution, an extra step is needed to find an intuitive solution from portfolio optimization perspective. Because allocating all investment amounts to a single equity wouldn't be an optimal solution, three types of allocation strategies are explored:

- **Weighted Allocation:** Allocation for all equities will be distributed based on the ranks in the portfolio based on the formula below:

$$w_i = \frac{R_i}{\sum_{i=1}^{N} R_i} \tag{6.7}$$

where $\boldsymbol{w_i}$ is weight allocation equity *i*

- **Top *N* Equally Weighted Allocation:** Based on the calculated ranks in step (4), equalt weights are assigned to top *N* equity based on the formula below:

$$w_i = \begin{cases} 0 \; if\ equity\ is\ not\ in\ top\ N \\ \frac{1}{N} if\ equity\ is\ in\ top\ N \end{cases}$$

(6.8)

- **Top *N* Weighted Allocation:** This methodology is the combination of two methodologies listed above. The equities that are not in Top *N* have 0 allocation and for the rest of the equities, formula (6.7) is used to calculate weight allocation.

As detailed in the following section, all three methodologies listed above are compared based on their out-of-sample performance.

6.3. Application: Portfolio Allocation with TODIM

The methodology detailed in the previous section is applied on S&P 500 daily U.S. Equity Data from 2002 to 2016 with sliding windows approach. Sliding step is chosen on a quarterly basis leding to 60 different portfolio allocation dates (15 years * 4 quarters). There are various features to determine when applying TODIM methodology. Among those, factors the most important can be listed as weights of each criterion, attenuation factor and portfolio allocation methodology after applying TODIM. After all, TODIM only provides an output value for each candidate and there should be a connection between this output value and final portfolio weights. In this respect, 440 different configurations are created to find best configurations in terms of performance:

- Different top N ranks such as 50,100, 150 and 462 are used. Except 462, other ranks are divided further by utilizing weighted allocation or equally weighted allocation.
- Different attenuation factors such as 2, 1, 0.5, 0.25 are analyzed.
- The effect of different weights for correlation and variance is investigated.
- Return information is not considered at all in some configurations as it is known that return has noisy estimates.

In total, 440 different configurations are created based on the combination of different values detailed above. 26,400 different portfolio allocations are calculated (440 configurations * 60 periods) to compute out-of-sample results. Calculation of these 26,400 portfolio allocations took 120 hours on a fast-personal computer, 15 seconds per allocation. Out-of-sample results based on Sharpe Ratio is calculated. For Sharpe Ratio calculation, most recent monthly Treasury Bill rate as risk-free rate at portfolio allocation date is utilized.

I utilized SAS IML and Base SAS programs for running all these analysis in an automated manner to minimize operational risk. SAS IML is used for matrix-type calculations while Base SAS is used for other administrative tasks such as data manipulation. The code used in this process is shared in Appendix section Similar code structure can be developed with MATLAB or Stata as well.

6.4. Performance Evaluation

The main difference in this section compared other sections is the utilization of different performance comparison factors or the extension of performance comparison factors in line with referee recommendations. These differences are addition of turnover criteria, addition of inverse Herfindahl Hirschmann Index (HHI) criteria or Effective N criteria and modification of Sharpe Ratio in line with academic best practices, statistical testing of Sharpe Ratio.

Performance evaluation should encompass different success criteria of an investor. A portfolio may perform well, and this can be measured by Sharpe Ratio but in presence of high turnover costs, this performance may be affected significantly. As another example, very concentrated portfolios may perform well in certain time intervals, but this kind of portfolios may possess model risk and they may incur large losses. In summary, an investor with trading cost and concentration risk sensitivity may not decide to invest his or her portfolio to a portfolio with higher risk-adjusted return.

Another critical fact to consider is that Sharpe Ratio is only a point estimator. While a portfolio may seem to be outperforming another portfolio from out-of-sample Sharpe Ratio perspective, this difference may not be statistically significant. In order to assess whether the Sharpe Ratio difference is significant, various statistical testing methodologies are proposed. Among those, parametric test developed by Jobson and Korkie (1981) and corrected by Memmel (2003) is widely used.

The formula for test statistic is given below:

$$z = \frac{SR_i - SR_j}{\sqrt{\left[2 - 2\rho_{i,j} + \frac{\left(SR_i^2 + SR_j^2 - SR_i SR_j\left(1 + \rho_{i,j}^2\right)\right)}{2}\right] / T}} \tag{6.9}$$

where T is the number of observations, $\rho_{i,j}$ is the correlation between portfolios *i* and *j* and SR_i and SR_j are the Sharpe ratios of portfolios *i* and *j*, respectively. Sharpe Ratio.

Sharpe ratio calculation methodology is also modified to be in line with literature. The Same notation of the Sharpe ratio as used in Demiguel, Garlappi and Uppal (2007) is used and same approach is followed for turnover as well. The Sharpe Ratio calculation is handled with the following formula:

$$SR_k = \frac{\hat{\mu}_k}{\hat{\sigma}_k} \tag{6.10}$$

where $\hat{\mu}_k$ is sample mean excess return over the risk-free rate and $\hat{\sigma}_k$ is the sample standard deviation of excess returns.

Turnover is defined as the amount of trading required to rebalance the portfolio to shift portfolio weights to the optimum weights calculated by the relevant portfolio optimization strategy used. Even for equally weighted portfolios turnover cost exists because the price changes in a period would shift the portfolio weight composition away from equally weight. The turnover formula is given below:

$$Turnover_t = \sum_{j=1}^{N} \left| w_{j,t+1} - w_{j,t^+} \right| \tag{6.11}$$

where N is the number of equities, $w_{j,t+1}$ is the required weight for equity j for the next period, and w_{j,t^+} is the weight of the equity at the end of the period subject to price fluctuations. As turnover cost is calculated for each subperiod, in order to compare high level turnover cost, turnover cost is averaged over subperiods to compare different strategies. Theoretical maximum turnover cost would be 200%. (If a portfolio in subperiod t is composed of stock A 100% weight and for the next period the portfolio should include only stock B with 100% weight, the trader should sell 100% of his portfolio (A stock) and then buy 100% of his portfolio (B stock). In total, turnover cost would be calculated based on 200% of his portfolio)

As mentioned above, for comparing portfolios from concentration risk perspective Effective N (Inverse HHI) is used. The formula for Effective N is given below:

$$Effective\ N_t = \frac{1}{\sum_{j=1}^{N} w_{j,t}^2} \tag{6.12}$$

where $w_{j,t}$ is the weight allocated to equity at subperiod t. As is the case with turnover, to arrive at final measure, Effective N is averaged over each subperiod by calculating mean of sample Effective N. Appendix section includes code for Effective N and Turnover cost calculation. Theoretical minimum and maximum for Effective N is 1 (when the portfolio is allocated is only one equity) and N (equally weighted portfolios) respectively. In other words, the more a portfolio is concentrated the lower Effective N and vice versa.

6.5. Out-of-Sample Results

In this section, out-of-sample results of TODIM methodology are compared with most used benchmarks, 1/N equally weighted portfolio and minimum variance portfolios. The comparison is done from risk-adjusted performance, tail risk, concentration risk and turnover cost perspective.

As 440 different configurations have been studied in this section, I divided sample period into two pieces: 40 subperiods for testing and finding the most efficient configuration types and 20 subperiods for comparison of most efficient configuration types with other benchmark portfolio strategies.

The table below summarizes the performance comparison of all TODIM strategies with other benchmark portfolio strategies:

Table 6.1: Performance Comparison between TODIM, 1/N and MVP

Portfolio	Sharpe Ratio	Excess Return (%)	Std Dev. (%)	VaR (95%) (%)	Skewness	Kurtosis	Effective N	Turnover (%)
1/N	0.57	11.51	20.24	2.03	-0.22	1.12	462	19.6
TODIM	0.7	9.94	14.25	1.45	-0.31	1.80	149.87	69.5
MVP	0.84	10.00	11.95	1.19	-0.31	1.92	13.25	85.2

Based on Table 6.1, TODIM method outperforms 1/N portfolio from Sharpe Ratio perspective however performance of TODIM is below MVP from Sharpe ratio perspective. On the other hand, TODIM provides better diversification and less turnover cost then MVP.

Based on detailed TODIM results analysis, outperforming TODIM configurations are elected. The details regarding outperforming TODIM configurations will be provided in main findings subsection.

Table 6.2: Performance Comparison between outperforming TODIM, 1/N and MVP

Portfolio	Sharpe Ratio	Excess Return (%)	Std Dev. (%)	VaR (95%) (%)	Skewness	Kurtosis	Effective N	Turnover (%)
1/N	0.57	11.51	20.24	2.03	-0.22	1.12	462	19.6
TODIM	0.76	10.51	13.83	1.38	-0.27	1.56	123.8	67
MVP	0.84	10.00	11.95	1.19	-0.31	1.92	13.25	85.2

Based on the results shown in the Table 6.2, outperforming TODIM configurations yield statistically better (*p*-value=0.0007) results than the 1/N portfolio, and the difference between MVP and the outperforming TODIM configurations is statistically insignificant (*p*-value=0.14) in terms of the Sharpe ratio.

6.6. Main Findings

In this section, 26,400 portfolio allocation process (440 configurations * 60 subperiods) is completed with TODIM method. Based on the out-of-sample results five main take away is summarized below.

1) The TODIM method can be leveraged as a good filtering and weighting mechanism for portfolio allocation purposes since TODIM configurations that allocate weights only to equities that are in the Top N (i.e., 50, 100, 150 or 200 in our study) rank in terms of their TODIM score provide better results than TODIM configurations that allocate weights to all securities. 2) TODIM configurations without empirical return criteria produce higher risk-adjusted results than other TODIM configurations when the number of equities in the portfolio is low (N=25, 50). This difference can be attributed to noise in return estimates. 3) Investors can make faster portfolio allocation decisions with less impact of estimation errors by using TODIM method as the calculation takes less time. 4) TODIM configurations possess lower concentration risk and consequently offer more diversification and less turnover costs than MVP. 5) TODIM configurations leveraging the four previously described main conclusions deliver statistically higher risk-adjusted returns than 1/N (*p*-value<0.01) and compatible returns with MVP (*p*-value=0.14). The statistical tests are conducted based on the parametric test developed by Jobson and Korkie (1981) and corrected by Memmel (2003).

7. CONCLUSION

Portfolio optimization is one of the key challenges that have constant evolution since it developed by Markowitz. The constant evolution need arises from the endless change in markets, improvement in operations research literature and desire for the increased risk adjusted returns from the industry.

This thesis is unique based on different aspects: 1) It proposes a new methodology for return estimation (MTMA), which is one of the main challenges in portfolio optimization 2) It includes very detailed empirical aspect of current portfolio optimization methodologies. 3) It analyzes the new risk measures from portfolio optimization perspective: systemic risk and liquidity risk. 4) It adapts an emerging MCDM methodology for portfolio allocation purposes.

This thesis itself includes a journey that starts with analyzing S&P 500 QuantQuote data. I tested the quality of key ingredients of portfolio optimization problem: return and risk measures. I validated the result that return may not be predictable based on historical data while historical risk measures could be a good starting point for estimation. Then I applied portfolio optimization problem to empirical data of S&P 500 QuantQuote data and I found out that optimal long-only portfolios outperform naively diversified portfolios. However, long-short portfolios are underperformed by naively diversified portfolios. These results can also be attributed to the unpredictability of return measures. I believe that this part is one of the most data-intensive and code-intensive empirical portfolio optimization works in the literature.

After validating return estimation is one of the most critical inputs in the process, I proposed a new methodology, Markov Transition Matrix Approach (MTMA), to ameliorate portfolio optimization performance. I demonstrated based on empirical data that MTMA yields better results than optimal portfolios with empirical return input when return target is set aggressively. Consequently, I can conclude that MTMA provides better return input when it is needed most, when the return target is aggressive.

This thesis will also pave the way for further studies that may be leveraged on MTMA. MTMA can be used further for other portfolio optimization inputs: variance and correlation. Another alternative way of improving MTMA could be analyzing sensitivity of the results to number of bins or even optimization of number and interval of bins. I believe that the proposed methodology could be used and tested with different equity universes and horizons. Also, this methodology could be improved further by using various binning procedures and different number of bins.

In this thesis, in addition to improving return inputs with MTMA, I also adapted an emerging MCDM technique, TODIM, to improve the portfolio optimization process. Based on the results one can conclude that TODIM can provide better risk-adjusted results than 1/N portfolios and compatible risk-adjusted results with MVP. TODIM delivers more diversified portfolios with lower turnover costs compared to MVP. Further studies can be done mainly on three points: (1) Analysing wider areas of efficient frontier (2) Quantifying the impact of using different calculation methods for risk and return inputs such as shrinkage estimation, principle component analysis for covariance estimation (3) Changing risk calculation measures in the process portfolio optimization problem (4) Adding new risk constraints such as liquidity and systemic risk constraint. Although various researches have been done on these points, a similar empirical big data study could reveal different outcomes.

In summary, based on out-of-sample results, I believe that MCDM and TODIM can be leveraged to improve inputs and process of portfolio optimization respectively.

REFERENCES

Acharya, V. V. (2005). Asset pricing with liquidity risk. *Journal of Financial Economics,* 375-410.

Adrian, T. a. (2011). CoVaR. *National Bureau of Economic Research*.

Arias, M. J.-R. (2010). Applying CoVaR to Measure Systemic Market Risk: the Colombian Case. *IFC Bulletin*.

Bangia, A. D. (2002). Modeling liquidity risk, with implications for traditional market risk measurement and management. *Springer US*, 3-13.

Basel Committee on Banking Supervision. (2000). Sound Practices for Managing Liquidity Risk in Banking Organisation.

Basel Committee on Banking Supervision. (2008). Principles for sound liquidity risk management and supervision.

Basel Committee on Banking Supervision. (2013). BCBS 239 Principles for Effective Risk Data Aggregation and Risk Reporting.

Basel Committee. (2013). *Fundamental review of the trading book: A revised market risk framework. Consultative Document.*

Billio, M., Getmansky, M., Lo, A. W., & Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics, 104*(3), 535-559.

Bisias, D., Flood, M., Lo, A. W., & Valavanis, S. (2012). A survey of systemic risk analytics. *Annual Review of Financial Economics*, *4*, 255-296.

Blume, M. E., Crockett, J., & Friend, I. (1974). Stock ownership in the United States: Characteristics and trends. *Survey of Current Business, 54*(11), 16-40.

Brunnermeier, M. K. (2008). Deciphering the liquidity and credit crunch 2007-08. *National Bureau of Economic Research*, No. w14612.

Brunnermeier, M. K. (2009). Market liquidity and funding liquidity. *Review of Financial studies*, 2201-2238.

Capuano, C. (2008). The Option-iPoD. The Probability of Default Implied by Option Prices Based on Entropy. *IMF Working Paper*.

Chordia, T. R. (2001). Market liquidity and trading activity. *Journal of Finance* , 501-530.

De Bandt, H. (2000). Systemic Risk: A Survey. *European Central Bank Working Paper Series*.

Demiguel, V., Garlappi, L., & Uppal, R. (2007). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *The review of Financial studies, 22*(5), 1915-1953.

F., B. (2012). Implementation of CoVaR, a measure for systemic risk. *Working Paper*.

Fabozzi, F. (2002). The legacy of modern portfolio theory. *The Journal of Investing*, 7-22.

Fabozzi, F. J. (2007). *Robust portfolio optimization and management.* John Wiley & Sons.

Fama, E. F. (1969). The adjustment of stock prices to new information. *International economic review*, 1-21.

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance,* 383-417.

Fama, E. F. (1975). Multiperiod consumption-investment decisions. *Stochastic Optimization Models in Finance,* 389-400.

Friend, I., Blume, M., & Crockett, J. (1970). *Mutual funds and other institutional investors: a new perspective.* McGraw-Hill Companies.

Goodhart, S. a. (2009). Banking Stability Measures. *IMF Working Paper*.

Gruber, E. J. (1997). Modern Portfolio Theory, 1950 to Date. *NYU Finance Working Paper Series.*

Hakansson, N. H. (1974). Convergence to isoelastic utility and policy in multiperiod portfolio choice. *Journal of Financial Economics, 1*(3), 201-224.

Hakansson, N. H. (2011). Optimal investment and consumption strategies under risk for a class of utility functions. *The Kelly Capital Growth Investment Criterion: Theory and Practice*, 91-111.

Ho, W.-R. J., Tsai, C.-L., Tzeng, G.-H., & Fang, S.-K. (2011). Combined DEMATEL technique with a novel MCDM model for exploring portfolio selection based on CAPM. *Expert Systems with Applications, 38*(1), 16-25.

Hu, G. X. (2013). Noise as information for illiquidity. *Journal of Finance*, 2341-2382.

IMF. (2009). Assessing the Systemic Implications of Financial Linkages. *Global Financial Stability Review*.

IMF. (2009). Global Financial Stability Report: Responding to the Financial Crisis and Measuring Systemic Risk. *Working Paper*.

Jagannathan, R., & Ma, T. (2003). Risk reduction in large portfolios: Why imposing the wrong constraints helps. *The Journal of Finance, 58*(4), 1651-1683.

Jensen, M. C. (1968). The performance of mutual funds in the period 1945–1964. *The Journal of finance, 23*(2), 389-416.

Jensen, M. C. (1969). Risk, the pricing of capital assets, and the evaluation of investment portfolios. *The Journal of business, 42*(2), 167-247.

Jobson, J. D., & Korkie, B. M. (1981). Performance hypothesis testing with the Sharpe and Treynor measures. *The Journal of Finance, 36*(4), 889-908.

Jorion, P. (1999). Risk Management and Lessons from Long Term Capital Management. *UC Irvine Graduate School of Management Working Paper*.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47*, 263-292.

Kalaycı, C. B., Ertenlice, O., Akyer, H., & Aygoren, H. (2017). A review on the current applications of genetic algorithms in mean-variance portfolio optimization.". *Pamukkale University Journal of Engineering Sciences, 23*(4).

Khandani, A. a. (2007). What Happened to the Quants in August 2007? *Journal of investment management*, 29-78.

Koenker, R. a. (1978). Regression quantiles. *Econometrica*.

Kraus, A. a. (1976). Skewness preference and the valuation of risk assets. *The Journal of Finance*, 1085-1100.

Kritman, L. P. (2011). Principal components as a measure of systemic risk. *Journal of Portfolio Management*.

Kritzman, M., Page, S., & Turkington, D. (2010). In defense of optimization: the fallacy of 1/N. *Financial Analysts Journal, 66*(2), 31-39.

Levich, P. a. (2011). Detecting crowded trades in currency funds. *Financial Analysts Journal*, 26-39.

Markowitz, H. (1952). Portfolio selection. *The journal of finance, 7*(1), 77-91.

Markowitz, H. (1959). Portfolio Selection. *Cowles Foundation Monograph No. 16.*, 263-74.

Memmel, C. (2003). Performance hypothesis testing with the Sharpe ratio. *Finance Letters, 1*, 21-23.

Merton, R. C., & Samuelson, P. A. (n.d.). *Continuous-time finance.* 1992.

Mossin, J. (1968). Optimal multiperiod portfolio policies. *The Journal of Business, 41*(2), 215-229.

Noël, A., Goltz, F., & Lioui, A. (2011). Practitioner portfolio construction and performance measurement: Evidence from Europe. *Financial Analysts Journal, 67*(3), 39-50.

Pastor, L. a. (2001). Liquidity Risk and Expected Stock Returns. *National Bureau of Economic Research*.

Pedersen, L. H. (2012). Measuring systemic risk. *Centre for Economic Policy Research*.

Sharpe, W. F. (1966). Mutual Fund Performance. *Journal of Business*, 119-138.

Treynor, J. L. (1965). How to rate management of investment funds. *Harvard business review, 45*(1), 63-75.

Vento, G. A. (2009). Bank liquidity risk management and supervision: which lessons from recent market turmoil? *Journal of Money, Investment and Banking*, 79-126.

APPENDICES

Appendix A. CoVaR Calculation

CoVaR is a proxy for the contribution of a specific institutions i’s riskiness to the overall riskiness of a system. To calculate this for a specific institution, riskiness of the system given there is no extreme event occurred for the institution i is compared with riskiness of the system given an extreme event occurred for the institution i. Riskiness of the system is calculated by aggregating CoVaR measures of financial institutions that are representative of financial system. An extreme event is defined as a specific quantile of the return of the institution.

As illustrated in Brunnermeier’s article $CoVaR_q^{j|C(X^{i})}$ is the VaR of the institution j conditional on some event $C(X^i)$ of institution i. Event $C(X^i)$ could be supposed as an event that caused a loss beyond a specific VaR value. In this case, institution i’s contribution to j is denoted by:

$$\Delta CoVaR_q^{j|i} = CoVaR_q^{j|X^i=VaR_q^i} - CoVaR_q^{j|X^i=VaR_{50}^i}$$

Based on the formula above, CoES and Exposure-ΔCoVaR calculations could be easily made.

Appendix B. Input Data Merge Code

```
data pricelist(KEEP= Equity Date Period_Year Open High Low Close Volume _Return
myfilepath);

length myfilepath $256;

length Equity $256;

infile
'C:\Users\turfal\Desktop\Documents\PhD\Data\quantquote_sp500_83986\daily\*.csv'
filename=myfilepath dsd;

input Date yymmdd8. T F Open High Low Close Volume;

format Date date9. Open 10.4 High 10.4 Low 10.4 Close 10.4 Volume 20.;

Equity=upcase(tranwrd(substr(myfilepath,find(myfilepath,'table')+6,11),'.csv',''));

if Equity='BF.B' then Equity='BFB';

Lag_Equity=lag(Equity);

Lag_Close=lag(Close);

if Equity=Lag_Equity then _Return=round((Close/Lag_Close-1)*100,0.0001);

else _Return=0;

Period_Year=Year(Date);

put Period_Year;

run;
```

Appendix C. Finding Eligible Securities Code

```
data  _null_;
set Parameters;

call symputx('VaR_Hist_SR',VaR_Hist_SR);

call symputx('VaR_Hist_LR',VaR_Hist_LR);

call symputx('VaR_Pcnt_High',VaR_Pcnt_High);

call symputx('VaR_Pcnt_Low',VaR_Pcnt_Low);

call symputx('OOS_Range',OOS_Range);

call symputx('Std_Hist_SR',Std_Hist_SR);

call symputx('Std_Hist_LR',Std_Hist_LR);

call symputx('Corr_Hist_SR',Corr_Hist_SR);

call symputx('Corr_Hist_LR',Corr_Hist_LR);

call symput('Samp_Begin',Samp_Begin);

call symput('Samp_End',Samp_End);

run;

Options symbolgen;

/* Calculate date ranges of equities */

data equitypricerange (Keep=equity databegindate dataenddate );

set pricelist;

lag_equity=lag(equity);

lag_date=lag(date);

if _N_=1 then call symputx('_temp_begin_date',Date);

else if equity ne lag_equity then do;

dataenddate=input(put(lag_date,Date9.),Date9.);

equity=lag_equity;

databegindate= input(put(symgetn('_temp_begin_date'),Date9.),Date9.);
```

```
format databegindate Date9.;
format dataenddate Date9.;
call symputx('_temp_begin_date',Date);
end;
if dataenddate ne ' ';
run;
/* based on the range calculate eligible equities */
proc sql;
 create table elig_equity as
 select equity,databegindate,dataenddate
 from equitypricerange
 where databegindate < &Samp_Begin and dataenddate> &Samp_End;
 quit;
proc sql;
create table elig_equity_prices as
select *
from elig_equity,pricelist
where pricelist.equity=elig_equity.equity and Date >= &Samp_Begin and Date<=&Samp_End ;
quit;
```

Appendix D. Risk and Return Factor Calculation Code

```
data elig_equity_prices;
set elig_equity_prices;
Year_Date=YEAR(Date);
run;
proc means data=elig_equity_prices Maxdec=4 noprint;
var _Return;
class Equity Year_Date ;
output out=equity_risk_values
mean=
skewness=
q1=
VAR=
P95=
p99=
kurtosis=
median=
p75=
p1=
p5=
p10=
p90= / autoname;
run;
```

Appendix E. Sample Correlation Between Risk and Return Factors

For the tables below, I will use the abbreviations below:

- Mean_2000: Mean returns during 2000 year for S&P500 Equities included in the empirical analysis.
- Median_2000: Median of returns during 2000 year for S&P500 Equities included in the empirical analysis.
- Skew_2000: Skewness during 2000 year for S&P500 Equities included in the empirical analysis.
- Kurt_2000: Kurtosis during 2000 year for S&P500 Equities included in the empirical analysis.
- Var_2000: Variance during 2000 year for S&P500 Equities included in the empirical analysis.
- Q3_2000: Value-at-Risk (%75, 1 year) during 2000 year for S&P500 Equities included in the empirical analysis.
- P95_2000: Value-at-Risk (%95, 1 year) percentile returns during 2000 year for S&P500 Equities included in the empirical analysis.
- P90_2000: Value-at-Risk (%90, 1 year) percentile returns during 2000 year for S&P500 Equities included in the empirical analysis.
- P99_2000: Value-at-Risk (%99, 1 year) percentile returns during 2000 year for S&P500 Equities included in the empirical analysis.
- P5_2000: %5 percentile returns during 2000 year for S&P500 Equities included in the empirical analysis.
- P1_2000: %1percentile returns during 2000 year for S&P500 Equities included in the empirical analysis.
- P10_2000: %10 percentile returns during 2000 year for S&P500 Equities included in the empirical analysis.

Similar abbreviations apply for 2001 year.

Table E.0.1: Correlation Between Risk and Return Factors during 2000

Correlation Matrix/2000 Year	Mean_2000	Skew_2000	Q3_2000	Var_2000	P5_2000	P1_2000	Kurt_2000	Median_2000	P25_2000	P99_2000	P95_2000	P90_2000	P10_2000
Mean_2000	1.00	0.21	0.22	-0.06	0.03	0.02	-0.12	0.66	0.23	0.16	0.14	0.14	0.07
Skew_2000	0.21	1.00	-0.03	-0.03	0.05	0.17	0.07	-0.13	-0.05	0.18	0.06	0.02	0.03
Q3_2000	0.22	-0.03	1.00	-0.89	-0.88	-0.81	0.03	0.36	-0.79	0.81	0.90	0.93	-0.89
Var_2000	-0.06	-0.03	-0.89	1.00	0.95	0.91	0.20	-0.26	0.84	-0.92	-0.95	-0.94	0.93
P5_2000	0.03	0.05	-0.88	0.95	1.00	0.89	0.05	-0.22	0.85	-0.85	-0.92	-0.92	0.95
P1_2000	0.02	0.17	-0.81	0.91	0.89	1.00	0.23	-0.24	0.75	-0.79	-0.85	-0.85	0.86
Kurt_2000	-0.12	0.07	0.03	0.20	0.05	0.23	1.00	-0.14	-0.14	-0.20	-0.05	0.00	-0.02
Median_2000	0.66	-0.13	0.36	-0.26	-0.22	-0.24	-0.14	1.00	0.00	0.24	0.25	0.26	-0.20
P25_2000	0.23	-0.05	-0.79	0.84	0.85	0.75	-0.14	0.00	1.00	-0.74	-0.82	-0.84	0.90
P99_2000	0.16	0.18	0.81	-0.92	-0.85	-0.79	-0.20	0.24	-0.74	1.00	0.89	0.86	-0.83
P95_2000	0.14	0.06	0.90	-0.95	-0.92	-0.85	-0.05	0.25	-0.82	0.89	1.00	0.97	-0.91
P90_2000	0.14	0.02	0.93	-0.94	-0.92	-0.85	0.00	0.26	-0.84	0.86	0.97	1.00	-0.92
P10_2000	0.07	0.03	-0.89	0.93	0.95	0.86	-0.02	-0.20	0.90	-0.83	-0.91	-0.92	1.00

Table E.0.2: Correlation Between Risk and Return Factors during 2000 and 2001

Correlation Matrix/Between 2000 and 2001	Mean_2001	Skew_2001	Q3_2001	Var_2001	P5_2001	P1_2001	Kurt_2001	Median_2001	P25_2001	P99_2001	P95_2001	P90_2001	P10_2001
Mean_2000	-0.32	-0.21	0.00	-0.04	-0.08	-0.10	-0.14	-0.01	-0.06	-0.02	-0.02	-0.02	-0.07
Skew_2000	-0.05	0.03	-0.01	-0.03	-0.02	-0.01	-0.07	-0.08	-0.01	0.06	0.02	0.02	-0.01
Q3_2000	-0.12	-0.40	0.78	-0.78	-0.79	-0.77	0.09	0.24	-0.79	0.69	0.78	0.77	-0.80
Var_2000	0.12	0.40	-0.78	0.81	0.83	0.82	-0.07	-0.21	0.80	-0.75	-0.81	-0.80	0.82
P5_2000	0.10	0.38	-0.79	0.81	0.81	0.79	-0.09	-0.22	0.80	-0.74	-0.81	-0.80	0.82
P1_2000	0.12	0.37	-0.71	0.73	0.74	0.73	-0.09	-0.21	0.73	-0.67	-0.73	-0.72	0.74
Kurt_2000	0.22	0.06	0.09	0.01	0.03	0.08	0.17	0.08	-0.02	-0.01	0.04	0.06	0.00
Median_2000	-0.24	-0.26	0.16	-0.19	-0.23	-0.24	-0.07	0.04	-0.19	0.11	0.14	0.15	-0.21
P25_2000	-0.08	0.27	-0.75	0.74	0.72	0.69	-0.14	-0.25	0.73	-0.70	-0.78	-0.77	0.73
P99_2000	-0.15	-0.41	0.72	-0.77	-0.78	-0.77	0.01	0.19	-0.75	0.70	0.75	0.74	-0.77
P95_2000	-0.12	-0.40	0.78	-0.80	-0.81	-0.80	0.08	0.21	-0.80	0.74	0.79	0.79	-0.81
P90_2000	-0.11	-0.40	0.79	-0.80	-0.82	-0.80	0.10	0.21	-0.81	0.74	0.81	0.80	-0.82
P10_2000	0.05	0.35	-0.80	0.81	0.81	0.78	-0.10	-0.23	0.80	-0.74	-0.82	-0.82	0.82

Table E.0.3: Correlation Between Risk and Return Factors during 2001

Correlation Matrix/2001 Year	Mean_2001	Skew_2001	Q3_2001	Var_2001	P5_2001	P1_2001	Kurt_2001	Median_2001	P25_2001	P99_2001	P95_2001	P90_2001	P10_2001
Mean_2001	1.00	0.32	0.09	0.06	0.17	0.17	-0.01	0.48	0.19	0.04	0.02	0.04	0.15
Skew_2001	0.32	1.00	-0.42	0.39	0.46	0.54	-0.16	-0.24	0.37	-0.20	-0.36	-0.38	0.43
Q3_2001	0.09	-0.42	1.00	-0.93	-0.91	-0.88	0.10	0.40	-0.89	0.82	0.93	0.95	-0.92
Var_2001	0.06	0.39	-0.93	1.00	0.96	0.95	0.09	-0.29	0.93	-0.91	-0.96	-0.96	0.96
P5_2001	0.17	0.46	-0.91	0.96	1.00	0.95	-0.02	-0.27	0.93	-0.85	-0.93	-0.94	0.98
P1_2001	0.17	0.54	-0.88	0.95	0.95	1.00	0.07	-0.27	0.89	-0.83	-0.90	-0.90	0.93
Kurt_2001	-0.01	-0.16	0.10	0.09	-0.02	0.07	1.00	-0.02	-0.11	-0.11	0.06	0.08	-0.06
Median_2001	0.48	-0.24	0.40	-0.29	-0.27	-0.27	-0.02	1.00	-0.17	0.25	0.29	0.32	-0.27
P25_2001	0.19	0.37	-0.89	0.93	0.93	0.89	-0.11	-0.17	1.00	-0.83	-0.92	-0.92	0.95
P99_2001	0.04	-0.20	0.82	-0.91	-0.85	-0.83	-0.11	0.25	-0.83	1.00	0.89	0.88	-0.86
P95_2001	0.02	-0.36	0.93	-0.96	-0.93	-0.90	0.06	0.29	-0.92	0.89	1.00	0.98	-0.94
P90_2001	0.04	-0.38	0.95	-0.96	-0.94	-0.90	0.08	0.32	-0.92	0.88	0.98	1.00	-0.94
P10_2001	0.15	0.43	-0.92	0.96	0.98	0.93	-0.06	-0.27	0.95	-0.86	-0.94	-0.94	1.00

Appendix F. Average Yearly Correlation Between Risk and Return Factors

The same abbreviation methodology in Appendix E section is used for the data demonstrated in this table:

Table F.1: Average Yearly Correlation between Risk Factors

Measure_T/T+1	Mean	Skew	Q1	Var	P95	P99	Kurt	Median	P75	P1	P5	P10	P90	ES25	ES10	ES5	ES1
Mean	-0.07	-0.08	-0.05	0.05	0.05	0.03	-0.06	0.02	0.07	-0.06	-0.08	-0.07	0.05	-0.07	-0.07	-0.06	-0.04
Skew	-0.04	0.03	-0.09	0.07	0.08	0.07	0.01	-0.08	0.06	-0.05	-0.07	-0.06	0.08	-0.07	-0.07	-0.06	-0.06
Q1	-0.13	-0.14	0.77	-0.76	-0.77	-0.68	0.07	0.09	-0.76	0.66	0.74	0.76	-0.78	0.76	0.73	0.70	0.63
Var	0.14	0.13	-0.78	0.80	0.81	0.73	-0.01	-0.07	0.78	-0.71	-0.77	-0.79	0.82	-0.79	-0.77	-0.75	-0.68
P95	0.14	0.13	-0.79	0.80	0.81	0.72	-0.05	-0.07	0.79	-0.70	-0.78	-0.80	0.82	-0.80	-0.77	-0.74	-0.67
P99	0.12	0.12	-0.71	0.73	0.73	0.66	0.01	-0.08	0.70	-0.64	-0.69	-0.71	0.74	-0.72	-0.70	-0.68	-0.62
Kurt	0.01	0.05	0.07	0.00	-0.02	0.04	0.27	-0.06	-0.08	0.00	0.06	0.07	-0.04	0.04	0.02	0.00	-0.04
Median	-0.04	-0.12	0.10	-0.10	-0.10	-0.10	-0.07	0.10	-0.06	0.05	0.06	0.07	-0.10	0.07	0.07	0.06	0.07
P75	0.11	0.08	-0.75	0.74	0.74	0.65	-0.11	-0.02	0.76	-0.65	-0.74	-0.76	0.77	-0.75	-0.72	-0.69	-0.61
P1	-0.15	-0.13	0.70	-0.73	-0.73	-0.67	0.00	0.05	-0.71	0.64	0.70	0.72	-0.74	0.72	0.70	0.68	0.62
P5	-0.15	-0.12	0.78	-0.79	-0.80	-0.72	0.06	0.05	-0.79	0.70	0.77	0.80	-0.81	0.79	0.77	0.74	0.67
P10	-0.14	-0.13	0.79	-0.79	-0.80	-0.71	0.07	0.06	-0.79	0.69	0.77	0.80	-0.82	0.79	0.76	0.74	0.66
P90	0.14	0.12	-0.79	0.79	0.80	0.71	-0.08	-0.06	0.79	-0.69	-0.78	-0.80	0.82	-0.80	-0.77	-0.74	-0.66
ES25	-0.15	-0.13	0.79	-0.80	-0.81	-0.73	0.04	0.07	-0.79	0.70	0.78	0.80	-0.82	0.80	0.77	0.75	0.67
ES10	-0.15	-0.13	0.77	-0.79	-0.79	-0.72	0.02	0.06	-0.78	0.69	0.76	0.78	-0.80	0.78	0.76	0.74	0.67
ES5	-0.15	-0.13	0.74	-0.77	-0.77	-0.70	0.00	0.06	-0.75	0.68	0.73	0.75	-0.78	0.76	0.74	0.72	0.65
ES1	-0.15	-0.13	0.66	-0.70	-0.69	-0.65	-0.04	0.06	-0.67	0.61	0.66	0.67	-0.70	0.68	0.66	0.65	0.60

Appendix G. Portfolio Optimization Types

Table G.1: Portfolio Optimization Types

No	Portfolio Optimization Name	Lower Bound Equity (%)	Upper Bound Equity (%)	Annual Return Target (%)	Description
1	LB_M99_UB_99_Ret_5	-99	99	5	%5 Return Target.No constraints
2	LB_0_UB_99_Ret_5	0	99	5	%5 Return Target.No short sales allowed
3	LB_0_UB_10_Ret_5	0	10	5	%5 Return Target.No short sales allowed and there is also limit for concentration risk
4	LB_0_UB_5_Ret_5	0	5	5	%5 Return Target.No short sales allowed and there is also stricter limit for concentration risk
5	LB_M20_UB_10_Ret_5	-20	10	5	%5 Return Target.Short sales allowed to some extent and there is also limit for concentration risk
6	LB_M20_UB_5_Ret_5	-20	5	5	%5 Return Target.Short sales allowed to some extent and there is also stricter limit for concentration risk
7	LB_M10_UB_5_Ret_5	-10	5	5	%5 Return Target.Short sales are strictly allowed to some extent and there is also stricter limit for concentration risk
8	LB_M10_UB_10_Ret_5	-10	10	5	%5 Return Target.Short sales are strictly allowed to some extent and there is also limit for concentration risk
9	LB_M5_UB_5_Ret_5	-5	5	5	%5 Return Target.Short sales are very strictly allowed to some extent and there is also stricter limit for concentration risk
10	LB_M5_UB_10_Ret_5	-5	10	5	%5 Return Target.Short sales are very strictly allowed to some extent and there is also limit for concentration risk
11	LB_M99_UB_99_Ret_M99	-99	99	-99	Minimum variance portfolio & No constraints
12	LB_0_UB_99_Ret_M99	0	99	-99	Minimum variance portfolio & No short sales allowed
13	LB_0_UB_10_Ret_M99	0	10	-99	Minimum variance portfolio & No short sales allowed and there is also limit for concentration risk
14	LB_0_UB_5_Ret_M99	0	5	-99	Minimum variance portfolio & No short sales allowed and there is also stricter limit for concentration risk

15	LB_M20_UB_10_Ret_M99	-20	10	-99	Minimum variance portfolio & Short sales allowed to some extent and there is also limit for concentration risk
16	LB_M20_UB_5_Ret_M99	-20	5	-99	Minimum variance portfolio & Short sales allowed to some extent and there is also stricter limit for concentration risk
17	LB_M10_UB_5_Ret_M99	-10	5	-99	Minimum variance portfolio & Short sales are strictly allowed to some extent and there is also stricter limit for concentration risk
18	LB_M10_UB_10_Ret_M99	-10	10	-99	Minimum variance portfolio & Short sales are strictly allowed to some extent and there is also limit for concentration risk
19	LB_M5_UB_5_Ret_M99	-5	5	-99	Minimum variance portfolio & Short sales are very strictly allowed to some extent and there is also stricter limit for concentration risk
20	LB_M5_UB_10_Ret_M99	-5	10	-99	Minimum variance portfolio & Short sales are very strictly allowed to some extent and there is also limit for concentration risk
21	LB_M99_UB_99_Ret_1	-99	99	1	%1 Return Target.No constraints
22	LB_0_UB_99_Ret_1	0	99	1	%1 Return Target.No short sales allowed
23	LB_0_UB_10_Ret_1	0	10	1	%1 Return Target.No short sales allowed and there is also limit for concentration risk
24	LB_0_UB_5_Ret_1	0	5	1	%1 Return Target.No short sales allowed and there is also stricter limit for concentration risk
25	LB_M20_UB_10_Ret_1	-20	10	1	%1 Return Target.Short sales allowed to some extent and there is also limit for concentration risk
26	LB_M20_UB_5_Ret_1	-20	5	1	%1 Return Target.Short sales allowed to some extent and there is also stricter limit for concentration risk
27	LB_M10_UB_5_Ret_1	-10	5	1	%1 Return Target.Short sales are strictly allowed to some extent and there is also stricter limit for concentration risk
28	LB_M10_UB_10_Ret_1	-10	10	1	%1 Return Target.Short sales are strictly allowed to some extent and there is also limit for concentration risk
29	LB_M5_UB_5_Ret_1	-5	5	1	%1 Return Target.Short sales are very strictly allowed to some extent and there is also stricter limit for concentration risk
30	LB_M5_UB_10_Ret_1	-5	10	1	%1 Return Target.Short sales are very strictly allowed to some extent and there is also limit for concentration risk
31	LB_M99_UB_99_Ret_3	-99	99	3	%3 Return Target.No constraints
32	LB_0_UB_99_Ret_3	0	99	3	%3 Return Target.No short sales allowed
33	LB_0_UB_10_Ret_3	0	10	3	%3 Return Target.No short sales allowed and there is also limit for concentration risk
34	LB_0_UB_5_Ret_3	0	5	3	%3 Return Target.No short sales allowed and there is also stricter limit for concentration risk

35	LB_M20_UB_10_Ret_3	-20	10	3	%3 Return Target.Short sales allowed to some extent and there is also limit for concentration risk
36	LB_M20_UB_5_Ret_3	-20	5	3	%3 Return Target.Short sales allowed to some extent and there is also stricter limit for concentration risk
37	LB_M10_UB_5_Ret_3	-10	5	3	%3 Return Target.Short sales are strictly allowed to some extent and there is also stricter limit for concentration risk
38	LB_M10_UB_10_Ret_3	-10	10	3	%3 Return Target.Short sales are strictly allowed to some extent and there is also limit for concentration risk
39	LB_M5_UB_5_Ret_3	-5	5	3	%3 Return Target.Short sales are very strictly allowed to some extent and there is also stricter limit for concentration risk
40	LB_M5_UB_10_Ret_3	-5	10	3	%3 Return Target.Short sales are very strictly allowed to some extent and there is also limit for concentration risk
41	LB_M99_UB_99_Ret_7	-99	99	7	%7 Return Target.No constraints
42	LB_0_UB_99_Ret_7	0	99	7	%7 Return Target.No short sales allowed
43	LB_0_UB_10_Ret_7	0	10	7	%7 Return Target.No short sales allowed and there is also limit for concentration risk
44	LB_0_UB_5_Ret_7	0	5	7	%7 Return Target.No short sales allowed and there is also stricter limit for concentration risk
45	LB_M20_UB_10_Ret_7	-20	10	7	%7 Return Target.Short sales allowed to some extent and there is also limit for concentration risk
46	LB_M20_UB_5_Ret_7	-20	5	7	%7 Return Target.Short sales allowed to some extent and there is also stricter limit for concentration risk
47	LB_M10_UB_5_Ret_7	-10	5	7	%7 Return Target.Short sales are strictly allowed to some extent and there is also stricter limit for concentration risk
48	LB_M10_UB_10_Ret_7	-10	10	7	%7 Return Target.Short sales are strictly allowed to some extent and there is also limit for concentration risk
49	LB_M5_UB_5_Ret_7	-5	5	7	%7 Return Target.Short sales are very strictly allowed to some extent and there is also stricter limit for concentration risk
50	LB_M5_UB_10_Ret_7	-5	10	7	%7 Return Target.Short sales are very strictly allowed to some extent and there is also limit for concentration risk

Appendix H. Portfolio Optimization Code

```
Options NoSymbolgen nomlogic nomprint nomfile;

Data _NULL_;
SET Phd.elig_equity;
CALL SYMPUT('EQUITY'||TRIM(LEFT(_N_)),Equity);
Run;

DATA _NULL_;
SET phd.equity_mean_values_merged;
CALL SYMPUTX('VARCOUNT',TRIM(LEFT(_N_)),'g');
RUN;

proc sql noprint;
select Equity into :EQUITY_LIST SEPARATED BY  ","
from PHD.elig_equity;
quit;

%macro writing_macro_variables(variablecount,dataset,correlation_table);
data phd.&dataset (drop=i);
do i=1 to &variablecount;
coefficient=cats(":COEFF_",i,"_1"," - :COEFF_",i,"_462"); output;
end;
run;
%GLOBAL COEFFICIENT_LIST;
%do i=1 %to &variablecount;
%do j=1 %to &variablecount;
%GLOBAL  COEFF_&i._&j.;
%end;
%end;
```

```
proc sql noprint;
select coefficient into :COEFFICIENT_LIST SEPARATED BY  ","
from PHD.&dataset;
quit;

proc sql noprint;
select &EQUITY_LIST into &COEFFICIENT_LIST
from PHD.&correlation_table;
quit;
%mend;

%Macro Data_Preparation(i,j,output);
DATA _NULL_;
SET phd.equity_mean_values_merged;
CALL SYMPUTX('VAR'||TRIM(LEFT(_N_)),Mean_&i._&j,'G');
CALL SYMPUTX('VARCOUNT',TRIM(LEFT(_N_)));
RUN;
%writing_macro_variables(462,coefficient,correlation_&i._&j);
%mend;

%Macro Optimization(BUDGET,RETURN_VALUE,TYPE,a,b,LB,UB,name);
PROC OPTMODEL;
VAR XYZ{1..&VARCOUNT} >= &LB <=&UB;
NUM VAR{1..&VARCOUNT}=[
%DO I = 1 %TO &VARCOUNT;
&&VAR&I
%END;
];
NUM COEFF{1..&VARCOUNT, 1..&VARCOUNT} = [
%DO I = 1 %TO &VARCOUNT;
%DO J = 1 %TO &VARCOUNT;
&&COEFF_&I._&J
```

```
%END;
%END;
];

/* MINIMIZE THE VARIANCE OF THE PORTFOLIO'S TOTAL RETURN */
MINIMIZE F = SUM{I IN 1..&VARCOUNT, J IN
1..&VARCOUNT}COEFF[I,J]*XYZ[I]*XYZ[J];
/* SUBJECT TO THE FOLLOWING CONSTRAINTS */
CON BUDGET: SUM{I IN 1..&VARCOUNT}XYZ[I] = &BUDGET;
CON GROWTH: SUM{I IN 1..&VARCOUNT}VAR[I]*XYZ[I] >=
&RETURN_VALUE;
%IF &TYPE ^= S %THEN %DO;
SOLVE WITH NLP;
/*PRINT XYZ;*/
%END;
%ELSE %DO;
FOR {I IN 1..&VARCOUNT} XYZ[I].LB=-XYZ[I].UB;
performance nthreads=4;
SOLVE WITH NLP;
%END;
/*PRINT XYZ;*/

create data phd.&name._&a._&b from [i]=(1..&VARCOUNT) XYZ[i];

QUIT;
/**/
%MEND;

%Macro Reading_Parameters(Observation);
DATA _NULL_;
SET phd.optimization (firstobs=&Observation obs=&Observation);
CALL SYMPUTX('LB',LB_Equity_Converted,'G');
```

```
CALL SYMPUTX('UB',UB_Equity_Converted,'G');
CALL SYMPUTX('Return',Return_Daily,'G');
CALL SYMPUTX('Name',Real_Name,'G');
RUN;
%mend;

%Macro All_Optimization(beginhorizon,maxhorizon,Start,End);
%do K = &beginhorizon  %to &maxhorizon;
%do L=1 %to 60;/*60 is  defined for solving same problem for 60 different portfolio
creation dates*/
%Data_Preparation(&K,&L,output);
%do M=&Start %TO &end;
%Reading_Parameters(&M);
%Optimization(100000,&Return,A,&K,&L,&LB,&UB,&Name);
%end;
%end;
%end;
%mend;

%All_Optimization(2,12,1,50);
```

Appendix I. Performance Calculation Code

```
data phd.elig_equity;
set phd.elig_equity;
i=_N_;
run;

options mprint symbolgen mlogic;

%MACRO Data_Preparation_For_Perf_Calc(input,i,j,output);
data perf.&output._&i._&j (keep= XYZ Equity);
merge phd.&input._&i._&j phd.elig_equity;
by i;
run;
%mend;

%Macro Data_Prep_Full(input,maxhorizon,output,Start,End);
%do K = 1 %to &maxhorizon;
%do L=1 %to 60; /* 60 parameter is defined for 60 different portfolio creation dates*/
%do M=&Start %TO &end;
%Reading_Parameters(&M);
%Data_Preparation_For_Perf_Calc(&Name,&K,&L,&Name);
%end;
%end;
%end;
%MEND;

%Data_Prep_Full(optimumweights_ss,12,OW_FR30_SS,1,50);

%Macro Performance_Calculation(input,maxhorizon,output);
data phd.&output;
set phd.&input;
```

```
%do i=1 %to 12;
%do J=1 %to 60;
Interval_&i._&J=Interval_1 + 1- &J;
%end;
%end;
run;
%mend;

%Performance_Calculation(elig_equity_prices,12,elig_equity_prices_interval);

proc sort data=phd.elig_equity_prices_interval;
by Equity;
run;

%Macro Merging_Datasets(Period,SubPeriod,Start,End,output);
proc sql noprint;
 select Real_Name
 into :varname&Start.-:varname&End
 from phd.optimization;
 quit;

data phd.elig_equity_prices_interval_&SubPeriod;
set phd.elig_equity_prices_interval(where=(Interval_&Period._&SubPeriod>0 and
Interval_&Period._&SubPeriod<13));
by Equity;
if First.Equity=1 then Cur_Return=1;
Cur_Return=Cur_Return*(1+_Return);
retain Cur_Return;
run;

proc sort data=phd.elig_equity_prices_interval_&SubPeriod;
by Equity;
```

```
run;

data perf.&output (keep=Date Equity Cur_Return Interval_&Period._&SubPeriod
%do K=&Start %to &End; R&&varname&K.._&Period._&SubPeriod %end;)
;
merge phd.elig_equity_prices_interval_&SubPeriod
%do K=&Start %to &End;
perf.&&varname&K.._&Period._&SubPeriod(rename=(XYZ=&&varname&K.._&Peri
od._&SubPeriod))
%end;
;
%do K=&Start %to &End; R&&varname&K.._&Period._&SubPeriod =
&&varname&K.._&Period._&SubPeriod * Cur_Return; %End;
by Equity;
run;

proc sort data=perf.&output;
by Date;
run;

data perf.&output;
set perf.&output;
cur_return=cur_return*100000/462;
run;

data perf.&output._ls;
set perf.&output;
%do K=&Start %to &End; A&&varname&K.._&Period._&SubPeriod = (1.22225 *
abs(R&&varname&K.._&Period._&SubPeriod) +
R&&varname&K.._&Period._&SubPeriod ) / 2.22225;
%end;
run;
```

```
proc means data=perf.&output noprint nway;
Class Date;
output out=perf.&output sum=;
run;

proc means data=perf.&output._ls noprint nway;
Class Date;
output out=perf.&output._ls sum=;
run;

data perf.&output (keep=Date Interval_Final Cur_Return_&Period._&SubPeriod %do
K=&Start %to &End; R&&varname&K.._&Period._&SubPeriod %end;);
set perf.&output._ls;
%do K=&Start %to &End; R&&varname&K.._&Period._&SubPeriod =
(R&&varname&K.._&Period._&SubPeriod-
lag(R&&varname&K.._&Period._&SubPeriod))/abs(lag(A&&varname&K.._&Period._
&SubPeriod));
%End;
Cur_Return_&Period._&SubPeriod=(Cur_Return-
lag(Cur_Return))/abs(lag(Cur_Return));
Interval_Final=Interval_&Period._&SubPeriod./_FREQ_;
run;
%mend;

%Macro Performance_Calculation(Start,End,output,maxhorizon);
%do A = 1 %to 12;
%do B=1 %to 60;
%Merging_Datasets(&A,&B,&Start,&End,&output._&A._&B);
%end;
%end;
%mend;
```

```
%Performance_Calculation(1,50,output,12);

%Macro Perf_Statistics_Calculation(input,Statistics,Interval);

proc means data=perf.&input(where=(Interval_Final<=&Interval))noprint nway;
output out=perf.&input._res_&Interval &Statistics=;

proc transpose data=perf.&input._res_&Interval
out=perf.&input._res_&Interval;
run;

data perf.&input._x_res_&Interval;
set perf.&input._res_&Interval (rename=(Col1=&Statistics._&Interval));
if _NAME_ NOT IN ('_TYPE_','_FREQ_','Date','Interval_Final');
run;

PROC SORT DATA=perf.&input._x_res_&Interval;
by _NAME_;
RUN;

%mend;

%Macro Merging_Statistics(input,maxhorizon,Statistics);
data perf.&input._&Statistics;
merge %do K=1 %to &maxhorizon;  perf.&input._&K
%end;
;
by _NAME_;
run;
%mend;

%Macro Appending_Statistics(output,maxhorizon,Statistics);
```

```
data perf.&output._&Statistics;
length _NAME_ $ 32;
set
%do A=1 %to &maxhorizon;
%do B=1 %to 60;
perf.&output._&A._&B._x_res_&Statistics
%end;
%end;
;
run;

proc sort data=perf.&output._&Statistics;
by _NAME_;
run;
%mend;

%Appending_Statistics(output,12,var);
%Appending_Statistics(output,12,std);
%Appending_Statistics(output,12,p95);
%Appending_Statistics(output,12,p90);
%Appending_Statistics(output,12,p10);
%Appending_Statistics(output,12,p5);
%Appending_Statistics(output,12,p1);
%Appending_Statistics(output,12,q1);
%Appending_Statistics(output,12,kurt);
%Appending_Statistics(output,12,skew);

%MACRO All_Perf_Statistics_Calculation(output,maxhorizon,Statistics);
%do A=1 %to &maxhorizon;
%do B=1 %to 60;
%DO C=1 %to &maxhorizon;
%Perf_Statistics_Calculation(&output._&A._&B,&Statistics,&C);
```

```
%end;
%Merging_Statistics (&output._&A._&B._x_res,&maxhorizon,&Statistics);
%end;
%end;
%Appending_Statistics(&output,&maxhorizon,&Statistics);
%MEND;

%All_Perf_Statistics_Calculation(output,12,Var);
%All_Perf_Statistics_Calculation(output,12,Std);
%All_Perf_Statistics_Calculation(output,12,P95);
%All_Perf_Statistics_Calculation(output,12,P90);
%All_Perf_Statistics_Calculation(output,12,P10);
%All_Perf_Statistics_Calculation(output,12,P5);
%All_Perf_Statistics_Calculation(output,12,P1);
%All_Perf_Statistics_Calculation(output,12,Q1);
%All_Perf_Statistics_Calculation(output,12,Kurt);
%All_Perf_Statistics_Calculation(output,12,Skew);

proc sort data=perf.output_skew;
by _NAME_;
run;

proc sort data=perf.output_kurt;
by _NAME_;
run;

proc sort data=perf.output_mean_deneme;
by _NAME_;
run;

proc sort data=perf.output_p1;
by _NAME_;
```

```
run;

proc sort data=perf.output_p5;
by _NAME_;
run;

proc sort data=perf.output_p10;
by _NAME_;
run;

proc sort data=perf.output_p90;
by _NAME_;
run;

proc sort data=perf.output_p95;
by _NAME_;
run;

proc sort data=perf.output_q1;
by _NAME_;
run;

proc sort data=perf.output_std;
by _NAME_;
run;

proc sort data=perf.output_var;
by _NAME_;
run;
data perf.all_performance_outcome;
merge perf.output_skew perf.output_kurt perf.output_mean_deneme perf.output_p1
perf.output_p5
```

```
perf.output_p10 perf.output_p90 perf.output_p95 perf.output_q1 perf.output_std
perf.output_var;
by _NAME_;
run;
```

Appendix J. Markov Transition Matrix Code

```
proc rank data=phd.equity_mean_values_merged out=phd.equity_mean_Value_ranks
groups=10; /* 10 is defined for 10 bins or 10  states */
run;

%macro transforming_transitions(input,output,horizon,subperiod);
%let nextperiod=%eval(&subperiod+&horizon);
data &output (keep=prev current horizon period);
set &input;
prev=Mean_&horizon._&subperiod;
current=Mean_&horizon._&nextperiod;
horizon=&horizon;
period=&nextperiod;
run;
%mend;

%macro appending_transitions(input,output);
proc append base=&input data=&output;
run;
%mend;

%macro merging_transitions(input,output,horizon,subperiod);

%do I = 1 %to &horizon;
	%do J=1 %to &subperiod;
%if &I=1 and &J=1 %then %transforming_transitions(&input,&output,&i,&j);
%else %do;%transforming_transitions(&input,a,&i,&j);
%appending_transitions(&output,a);
	%end;
	%end;
%end;
```

```
data &output;
set &output;
if current ne '.';
run;
%mend;

%merging_transitions(phd.equity_mean_Value_ranks,phd.transitions,16,59);

data phd.equity_mean_value_bins (drop=Equity);
set phd.equity_mean_value_ranks;
if _N_=1;
run;

%macro mean_calculation_for_bins(bin,output);
proc iml;
use phd.equity_mean_Value_ranks ;
read all var _NUM_ into ranks [colname=names];
close phd.equity_mean_Value_ranks;
use phd.equity_mean_values_merged ;
read all var _NUM_ into mean [colname=names];
close phd.equity_mean_values_merged;
ranks=ranks+1;
do i=1 to &bin;
rank_bin=ranks#(ranks=i);
sum=rank_bin[+,];
rank_bin=rank_bin#mean;
rank_bin=rank_bin[+,];
rank_bin=rank_bin/sum;
edit phd.equity_mean_Value_bins;
append from rank_bin;
close phd.equity_mean_Value_bins;
end;
```

```
run;
%mend;

%mean_calculation_for_bins(10,output); /*average mean for each bin */

data phd.equity_mean_value_bins;
set phd.equity_mean_value_bins;
if _N_>1;
run;

/*So far transitions are calculated and mean for each state*/

%macro subtransitionmatrix_creation(input,output,horizon);
data phd.transitions_&horizon (drop= horizon);
set phd.transitions;
if horizon=&horizon;
run;

Proc Tabulate data=phd.transitions_&horizon          out=phd.transition_stats ;
        Class prev;
        Class current;
        CLASS period ;
        Table /* Page Dimension */
period ,
/* Row Dimension */
prev,
/* Column Dimension */
RowPctN*
  current / Printmiss                ;
        ;
RUN;
 %do K=1 %to 60;
```

```
data &output._&horizon._&K (keep= prev current PctN_101);
set phd.transition_stats;
if period=&K;
run;

proc transpose data=&output._&horizon._&K  out=&output._&horizon._&K
(drop=_NAME_ prev) ;
by PREV;
var PctN_101;
run;
%end;
%mend;

/*transitionmatrices for each observation period is calculated*/

%macro expected_mean_calculation(pastperiods,horizon,input,output,bins);
%let startperiod=%eval(1+&horizon+&pastperiods);
proc iml;
%do K=2 %to 60;
use phd.transitions_&horizon._&K;
read all var _NUM_ into ranks_&K [colname=names];
close phd.transitions_&horizon._&K;
%end;
/*transition estimation */
%do L=&startperiod %to 60;
%do A=1 %to &pastperiods;
        %LET period= %eval (&L-&A);
        %if &A=1 %THEN sum_ranks_&L=ranks_&L + ranks_&period;
        %else %DO; sum_ranks_&L=sum_ranks_&L+ranks_&period;
   %END;
        ;
%end;
```

```
sum_ranks_&L=sum_ranks_&L/(&A*100);
%end;

/*average transition matrices based on past periods are calculated*/

use phd.equity_mean_value_bins;
read all var _NUM_ into mean_return [colname=names];
close phd.equity_mean_value_bins;
print mean_return;
A={1,2,3,4,5,6,7,8,9,10};

%do L=&startperiod %to 60;
z = j(10,960,0);
%let initial =%eval(&L-1-&pastperiods);
Z[1:10, &initial:&L]=1;
Ave_Ret_Bins_&L     = Z # mean_return;
sum_&L=Ave_Ret_Bins_&L[,+]/&startperiod;
exp_ret_&L=sum_ranks_&L*sum_&L;
A=A || exp_ret_&L;
%end;
print A;

/*expected returns based on transition matrices are calculated for the next periods*/

use phd.equity_mean_Value_ranks ;
read all var _NUM_ into ranks [colname=names];
close phd.equity_mean_Value_ranks;
%let startcolumn=%eval(60*(&horizon-1)+&startperiod);
%let endcolumn=%eval(60*&horizon);
ranks=ranks[1:462,&startcolumn:&endcolumn];
ranks=ranks+1;
print ranks;
```

```
/* rank matrices for the previos periods are formed based on the horizon*/

%let numberofcolumns=%eval(60-&startperiod+2);
A=A[1:10,2:&numberofcolumns]; /*first column is dropped */
print A;
p=ncol(A);
est_ret_mat=j(462,p,0);
%do M=1 %to &bins;
rank_&M=ranks#(ranks=&M)/&M;
A_&M=1/A[&M,];
rank_&M=rank_&M / A_&M;
est_ret_mat=est_ret_mat+rank_&M;
%END;
print est_ret_mat;
est_ret_min=est_ret_mat[><, ];
est_ret_max=est_ret_mat[<>,];
print est_ret_min;
print est_ret_max;
create phd.ERM_est_ret_min from est_ret_min;
append from est_ret_min;
close phd.ERM_est_ret_min;
create phd.ERM_est_ret_max from est_ret_max;
append from est_ret_max;
close phd.ERM_est_ret_max;
create phd.ERM_bin&bins._pastperiod&pastperiods from est_ret_mat;
append from est_ret_mat;
close phd.ERM_bin&bins._pastperiod&pastperiods;
quit;
/* transition estimate multiply with return estimate */
%mend;

%expected_mean_calculation(10,1,phd.transitions,output,10);
```

Appendix K. Markov Transition Matrix Approach Portfolio Optimization Types

As referred in the application section, the portfolio optimization configurations that are analyzed in this paper is given in table below:

Table K.1: Markov Transition Matrix Approach Portfolio Optimization Types

No	Portfolio Optimization Name	Lower Bound Equity (%)	Upper Bound Equity (%)	Alpha (α)
1	LB_0_UB_99_Ret_40	0	99	40
2	LB_0_UB_10_Ret_40	0	10	40
3	LB_0_UB_5_Ret_40	0	5	40
4	LB_0_UB_99_Ret_0	0	99	0
5	LB_0_UB_10_Ret_0	0	10	0
6	LB_0_UB_5_Ret_0	0	5	0
7	LB_0_UB_99_Ret_20	0	99	20
8	LB_0_UB_10_Ret_20	0	10	20
9	LB_0_UB_5_Ret_20	0	5	20
10	LB_0_UB_99_Ret_60	0	99	60
11	LB_0_UB_10_Ret_60	0	10	60
12	LB_0_UB_5_Ret_60	0	5	60
13	LB_0_UB_99_Ret_80	0	99	80
14	LB_0_UB_10_Ret_80	0	10	80
15	LB_0_UB_5_Ret_80	0	5	80

Based on the return estimation methodology I added a prefix for each configuration as described below:

ER_: Empirical return input is used by calculating returns based on the last 16 quarters daily data.

TM_1: Return is estimated based on the transition matrix of last 2 quarters.

TM_5: Return is estimated based on the transition matrix of last 5 quarters.

TM_10: Return is estimated based on the transition matrix of last 11 quarters.

Appendix L. TODIM Methodology Code

```
/* criteria are long term mean (3 years) mid-term mean (1 year)
short-term mean (1 quarter), long term variance, mid-term
variance, short-term variance, long term average correlation,
mid-term correlation, short-term average correlation)*/

/*average correlation per equity*/

%macro
TODIM(period,rank,attenuation,STCor,MTCor,LTCor,STStd,MTStd,LTStd,STRet,MT
Ret,LTRet,Weighted,Ranked,Type);

/*step 1 : Criteria Matrix Calculation*/
%LET perc_attenuation=%sysevalf(&Attenuation/100);

data TODIM_var_criteria(keep=equity VAR_1_&period. VAR_4_&period.
VAR_12_&period.);
set phd.equity_risk_values_merged;
run;

data TODIM_mean_criteria(keep=equity Mean_1_&period. Mean_4_&period.
Mean_12_&period. );
set phd.equity_mean_values_merged_v2;
run;

proc iml;
use phd.correlation_1_&period.;
read all var _NUM_ into correlations_short [colname=names];
avg_corr_short=correlations_short[:,];
avg_corr_short=avg_corr_short`;

use phd.correlation_4_&period.;
read all var _NUM_ into correlations_mid [colname=names];
avg_corr_mid=correlations_mid[:,];
avg_corr_mid=avg_corr_mid`;

use phd.correlation_12_&period.;
read all var _NUM_ into correlations_long [colname=names];
avg_corr_long=correlations_long[:,];
avg_corr_long=avg_corr_long`;

use TODIM_var_criteria;
read all var _NUM_ into variances[colname=names];
stdev=sqrt(variances);

use TODIM_mean_criteria;
```

```
read all var _NUM_ into returns[colname=names];

all_criteria=j(462,9,0);
all_criteria[,1]=avg_corr_short;
all_criteria[,2]=avg_corr_mid;
all_criteria[,3]=avg_corr_long;
all_criteria[,4:6]=stdev;
all_criteria[,7:9]=returns;

if &Ranked=1 then do;
do i=1 to 9;
rank_column=all_criteria[,i];
r=rank(rank_column);
r=floor((r/46.2));
all_criteria[,i]=r;
end;
end;

min_all_criteria=all_criteria[><,];
max_all_criteria=all_criteria[<>,];
dif_all_Criteria=max_all_criteria-min_all_criteria;

normalized_criteria=(all_criteria-min_all_criteria)/dif_all_Criteria;
normalized_Criteria[,1:6]=abs(normalized_Criteria[,1:6]-1); /*reversing normalization
for cor and stdev*/

weight={&STCor &MTCor &LTCor &STStd &MTStd &LTStd &STRet &MTRet
&LTRet};
max_weight=max(weight);
norm_weight=weight/max_weight;
sum_norm_weight=norm_weight[,+];

/*step 2 : dominance degree calculation step 3: overall dominance degree*/

dominance_matrix=j(462,462,0);

do i=1 to 462; /*462 Ai*/
do j=1 to 462; /*462 Aj*/
do c=1 to 9; /* 9 criteria*/
weight=norm_weight[c]/sum_norm_weight;

difference=normalized_Criteria[i,c]-normalized_Criteria[j,c];
if  difference>0 then
dominance_matrix[j,i]=dominance_matrix[j,i]+sqrt(weight*difference);
else if difference<0 then
dominance_matrix[j,i]=dominance_matrix[j,i]+sqrt(weight*difference*-1)*(-
1/&perc_attenuation);
```

```
end;
end;
end;

/*step 4: prospect value calculation Step 5: Ranking*/
sum_dominance_matrix=dominance_matrix[+,];

min_sum_dominance=sum_dominance_matrix[,><];
max_sum_dominance=sum_dominance_matrix[,<>];
dif_sum_dominance=max_sum_dominance-min_sum_dominance;

final_Rank=j(462,1,0);
do i=1 to 462;

final_rank[i,1]=(sum_dominance_matrix[1,i]-
min_sum_dominance)/dif_sum_dominance;

end;

/*step 6: allocation*/

/*weighted allocation to all*/
sum_final_rank=final_rank[+,];
allocation_all_securities=final_rank/sum_final_rank*100000;

/*equally weighted to top N*/
sort_final_Rank=final_rank;
call sort(sort_final_Rank,1,1);

threshold=sort_final_Rank[&rank,];
top_performers=final_rank#(final_rank>threshold);

if &weighted=0 THEN DO;

EW_Top_Performers=(top_performers>0);
Final_Allocation=EW_top_performers*100000/&rank;

END;
ELSE DO;

/*weighted allocation to top N*/
sum_top_performers=top_performers[+,];
Final_Allocation=top_performers/sum_top_performers*100000;

END;


/*portfolio allocation dataset creation*/
```

```
create phd.TODIM_&Period._&Type. from Final_Allocation;
append from Final_Allocation;
close phd.TODIM_&Period._&Type.;
run;
%mend;

/*Automation*/

%Macro Reading_TODIM_Parameters(Observation);
DATA _NULL_;
SET phd.TODIM (firstobs=&Observation obs=&Observation);
CALL SYMPUTX('LTRet',LTRet,'G');
CALL SYMPUTX('MTRet',MTRet,'G');
CALL SYMPUTX('STRet',STRet,'G');
CALL SYMPUTX('LTStd',LTStd,'G');
CALL SYMPUTX('MTStd',MTStd,'G');
CALL SYMPUTX('STStd',STStd,'G');
CALL SYMPUTX('LTCor',LTCor,'G');
CALL SYMPUTX('MTCor',MTCor,'G');
CALL SYMPUTX('STCor',STCor,'G');
CALL SYMPUTX('Rank',Rank,'G');
CALL SYMPUTX('Type',Type,'G');
CALL SYMPUTX('Attenuation',AF,'G');
CALL SYMPUTX('Weighted',Weighted_Num,'G');
CALL SYMPUTX('Ranked',Ranked_Num,'G');
RUN;
%mend;


%Macro all_TODIM(beginperiod,endperiod,Start,End);
%do K = &beginperiod  %to &endperiod;
%do M=&Start %TO &end;
%Reading_TODIM_Parameters(&M);
%TODIM(&K,&rank,&attenuation,&STCor,&MTCor,&LTCor,&STStd,&MTStd,&LT
Std,&STRet,&MTRet,&LTRet,&Weighted,&Ranked,&Type);
%end;
%end;
%mend;

%all_TODIM(1,60,1,440);
```

Appendix M. Turnover and Effective N Calculation Code

```
proc sort data=phd.elig_equity_prices_interval out=phd.elig_equity_prices_by_date
	(keep = Date Equity _Return Interval_1);
	by Date Equity;
run;

%let endperiod=59;
libname debug "C:\Users\turfal\Desktop\Documents\PhD\Datasets\debug";

proc iml;
	use phd.elig_equity_prices_by_date;
	read all var _NUM_ into Equity_Data[colname=names];
	use phd.risk_free;
	read all var _NUM_ into Risk_Free[colname=names];
	Risk_Free_Rates=Risk_Free[,3];
	Risk_Free_Subperiod=Risk_Free[,2];
	Return_Data=Equity_Data[,2];
	Interval_Data=Equity_Data[,3];
	Return_Data_by_Equity=shape(Return_Data,4529,462);
	Interval_Data_by_Equity=shape(Interval_Data,4529,462);
	Return_Data_by_Equity_Trans=Return_Data_by_Equity`;
	Interval_Data_by_Equity_Trans=Interval_Data_by_Equity`;
	Interval_Data_by_Equity_Trans=Interval_Data_by_Equity_Trans[1,];
	free Equity_Data Interval_Data_by_Equity Return_Data_by_Equity;
	print Interval_Data_by_Equity_Trans;
	Interval_Data_by_Equity_Trans[1] = -12;
	do i=2 to 4528;
		if Interval_Data_by_Equity_Trans[i] =
Interval_Data_by_Equity_Trans[i+1] then Interval_Data_by_Equity_Trans[i]=0;
		else Interval_Data_by_Equity_Trans[i] =
Interval_Data_by_Equity_Trans[i]+1;
	end;
	Interval_Data_by_Equity_Trans[4529] = 60;
	print Interval_Data_by_Equity_Trans;

do subperiod=1 to 59; /*horizon*/
		Varname= "perf.HHI_all_" + strip(char(subperiod,4));
		use (Varname);
		read all var _NUM_ into Allocations[colname=names];

		/*storing current allocations for future turnover calculation*/
		Allocation_Wght=Allocations/Allocations[+,];
		Varname= "Cur_All_"  + strip(char(subperiod,4));
		call valset(Varname,Allocation_Wght);

		/*Return Calculation*/
```

```
startpoint=loc(Interval_Data_by_Equity_Trans=subperiod);
startpoint=startpoint+1;
endperiod=min(subperiod+4,60);/*12 for max horizon*/
endpoint=loc(Interval_Data_by_Equity_Trans=endperiod);
total_days=endpoint-startpoint+1;

Return_Data_by_Equity_Trans_S=Return_Data_by_Equity_Trans[,startpoint:endpoint];/*Return of Each Equity*/

do j=2 to total_days;

Return_Data_by_Equity_Trans_S[,j]=Return_Data_by_Equity_Trans_S[,j-1]+Return_Data_by_Equity_Trans_S[,j];
end;

Filtered_Allocation_Results=j(462,4,0);

Return_Data_by_Equity_Trans_S=exp(Return_Data_by_Equity_Trans_S);
Return_Config=j(total_days,ncol(Allocations),0);
Total_Allocation_by_day=j(total_days,ncol(Allocations),0);

do x=1 to ncol(Allocations);
    /* Allocation_Results: Daily Value of Each Equity for Turnover Calculation */
    Allocations_x=Allocations[,x];

    Allocations_Results=Return_Data_by_Equity_Trans_S#Allocations_x;
    Total_Allocation_by_day[,x]=(Allocations_Results[+,])`; /*Total Value per Day*/

    do a=1 to 4;
        endperiod=min(subperiod+a,60);/*12 for max horizon*/

        endpoint=loc(Interval_Data_by_Equity_Trans=endperiod);
        total_days=endpoint-startpoint+1;

        Filtered_Allocation_Results[,a]=Allocations_Results[,total_days];
    end;


    Filtered_Allocation_Results=Filtered_Allocation_Results/Filtered_Allocation_Results[+,];
    Varname= "All_Res_" + strip(char(x,4)) + "_" + strip(char(subperiod,4));
    call valset(Varname,Filtered_Allocation_Results);
end;

do k=1 to total_days;
```

```
                    if k=1 then
                            Return_Config[k,]=Total_Allocation_by_day[k,]/
100000-1;
                    else Return_Config[k,]=(Total_Allocation_by_day[k,]/
Total_Allocation_by_day[k-1,])-1;
            end;


            Varname= "debug.Total_Allocation_by_day_" +
strip(char(subperiod,4));
            create (Varname) from Total_Allocation_by_day[colname=names];
            append from Total_Allocation_by_day;
            close (Varname);

            /*Statistics Calculation per Period*/
            /*Return Calculation per Period*/
            Return_Results=j(4,ncol(Allocations),0);
            Stdev_Results=j(4,ncol(Allocations),0);
            Skewness_Results=j(4,ncol(Allocations),0);
            Kurtosis_Results=j(4,ncol(Allocations),0);
            P95_Results=j(4,ncol(Allocations),0);
            P99_Results=j(4,ncol(Allocations),0);
            P90_Results=j(4,ncol(Allocations),0);
            P1_Results=j(4,ncol(Allocations),0);
            P5_Results=j(4,ncol(Allocations),0);
            P10_Results=j(4,ncol(Allocations),0);
            Annual_Excess_Return_Results=j(4,ncol(Allocations),0);
            Annual_Mean_Results=j(4,ncol(Allocations),0);
            Annual_Stdev_Results=j(4,ncol(Allocations),0);
            Annual_SR_Results=j(4,ncol(Allocations),0);

            do a=1 to 4; /*total subperiod*/
                    /*Statistics Calculation per Period*/
                    endperiod=min(subperiod+a,60);/*12 for max horizon*/
                    endpoint=loc(Interval_Data_by_Equity_Trans=endperiod);
                    total_days=endpoint-startpoint+1;

      Risk_Free_Avg=Risk_Free_Rates[min(subperiod+1,60):min(subperiod+a,60)];
                    Risk_Free_Avg=Risk_Free_Avg[:,];

                    /*return calculation*/

      Return_Results[a,]=(Total_Allocation_by_day[total_days,]/100000)##(1/total_d
ays)-1;
                    Annual_Mean_Results[a,]=Return_Results[a,]*252;
                    Risk_Free_Avg=(Risk_Free_Avg+1)##(1/total_days)-1;
                    Annual_Excess_Return_Results[a,]=(Return_Results[a,]-
Risk_Free_Avg)*252;
```

```
                    /*Other statistics calculation*/
                    Period_Selected=Return_Config[1:total_days,];
                    Stdev_Results[a,]=std(Period_Selected);
                    Annual_Stdev_Results[a,]=Stdev_Results[a,]*sqrt(252);

        Annual_SR_Results[a,]=Annual_Excess_Return_Results[a,]/Annual_Stdev_Res
ults[a,];
                    Skewness_Results[a,]=skewness(Period_Selected);
                    Kurtosis_Results[a,]=kurtosis(Period_Selected);
                    call qntl(Quantile_Results,Period_Selected,0.95);
                    P95_Results[a,]=Quantile_Results;
                    call qntl(Quantile_Results,Period_Selected,0.99);
                    P99_Results[a,]=Quantile_Results;
                    call qntl(Quantile_Results,Period_Selected,0.90);
                    P90_Results[a,]=Quantile_Results;
                    call qntl(Quantile_Results,Period_Selected,0.01);
                    P1_Results[a,]=Quantile_Results;
                    call qntl(Quantile_Results,Period_Selected,0.05);
                    P5_Results[a,]=Quantile_Results;
                    call qntl(Quantile_Results,Period_Selected,0.1);
                    P10_Results[a,]=Quantile_Results;
            end;

            All_Statistics= Return_Results // Stdev_Results // Skewness_Results //
Kurtosis_Results // P99_Results // P95_Results // P90_Results // P10_Results //
P5_Results // P1_Results //
                    Annual_Excess_Return_Results // Annual_Mean_Results //
Annual_Stdev_Results // Annual_SR_Results;
            Varname= "perf.All_Statistics_" + strip(char(subperiod,4));
            create (Varname) from All_Statistics[colname=names];
            append from All_Statistics;
            close (Varname);
    end;

    /*Turnover Calculation per Period*/
    Turnover_Results=j(4,ncol(Allocations),0);

    do subperiod=1 to 59;
            do x=1 to ncol(Allocations);
                    Varname= "All_Res_" + strip(char(x,4)) + "_" +
strip(char(subperiod,4));
                    base=value(Varname);

                    do a=1 to 4;
                            base_sub=base[,a];
                            next_hor=min(subperiod+a,59);
                            Varname= "Cur_All_"  + strip(char(next_hor,4));
```

```
                    next=value(Varname);
                    next_sub=next[,x];
                    trade=abs(base_sub-next_sub);
                    sum_trade=trade[+,];
                    Turnover_Results[a,x]=sum_trade;
                end;
            end;

            Varname= "perf.All_Statistics_" + strip(char(subperiod,4));
            edit (Varname);
                append from Turnover_Results;
                close (Varname);
        end;


/*Configuration based calculations*/
        do subperiod=1 to 59;
do x=1 to ncol(Allocations);


end;
        end;


run;

%macro merging_results(start,end);
        %do B=&start %to &end;

            data perf.All_Statistics_&B.;
                set perf.All_Statistics_&B.;
                Statistics=_N_;
            run;

        %end;

        data perf.All_statistics_all;
            merge
                %do A=&start %to &end;
            perf.All_Statistics_&A.
            %End;
            ;
            by Statistics;
        run;
```

```
%mend;

%merging_results(1,59);

data perf.all_statistics_all_merged(drop=Statistics);
        merge perf.all_statistics_all phd.stats_name;
        by Statistics;
run;

PROC transpose data=perf.All_statistics_all_merged out=perf.All_Statistics_final;
        id Name;
run;

data perf.hhi_results_all(rename=(COL1=HHI));
set perf.hhi_results_all;
run;

proc sort data=perf.all_statistics_final;
by _NAME_;
run;

proc sort data=perf.hhi_results_all(RENAME=(COL1=HHI));
by _NAME_;
run;

data perf.all_statistics_final_;
merge perf.All_Statistics_final perf.hhi_results_all;
by _NAME_;
run;



%MACRO Inv_HHI(horizon,configuration,MVP_configuration,period);

        proc sql noprint;
                select Real_Name
                        into :varname1-:varname&MVP_configuration
                                from phd.optimization;
        quit;

        %do A=1 %to &configuration;
                %do B=1 %to &horizon;

                        data perf.TODIM_&B._&A._hhi
(rename=(COL1=TODIM_&B._&A.) DROP=Equity);
                                set perf.TODIM_&B._&A.;
                                i=_N_;
                        run;
```

```
        %end;
    %end;

    %do A=1 %to &MVP_configuration;
        %do B=1 %to &horizon;

            data perf.&&varname&A.._&period._&B._HHI
(rename=(XYZ=&&varname&A.._&period._&B.) DROP=Equity);
                set perf.&&varname&A.._&period._&B;
                i=_N_;
            run;

        %end;
    %end;

    %do B=1 %to &horizon;

        data perf.HHI_all_&B.;
            merge
                %do A=1 %to &configuration;
                perf.TODIM_&B._&A._hhi
                %End;

                %do C=1 %to &MVP_configuration;
                perf.&&varname&C.._&period._&B._HHI
                %end;
                ;
                by i;
                run;

        data perf.HHI_all_&B.;
        set perf.HHI_all_&B.;
        Cur_Return_&B.=100000/462;
        drop i;
        run;

    %END;
```

```
%MEND;

%Inv_HHI(6,440,1,12);


%macro Inv_HHI_Call(start,end);


%do B=&start %to &end;

proc iml;
use perf.HHI_all_&B.;
read all var _NUM_ into Allocation [colname=names];
Weight = Allocation / Allocation[+,];
sum_weight=Weight[+,];
min_weight=Weight[><,];
max_weight=Weight[<>,];
Weight_sq = Weight#Weight;
Sum_weight_sq = Weight_sq [+,];
Effective_N = 1 / Sum_weight_sq;
create perf.HHI_res_&B. from Effective_N[colname=names];
append from Effective_N;
close perf.HHI_res_&B.;
run;

proc transpose data=perf.HHI_res_&B.
out=perf.HHI_res_&B.;
run;

%end;


data perf.HHI_results_all;
length _NAME_ $25;
set %do B=&start %to &end;
perf.HHI_res_&B.
%end;
;
run;


%mend;

%Inv_HHI_Call(1,60);
```

BIOGRAPHICAL SKETCH

Fatih ALALİ was born in Istanbul on June 15, 1986. He received the B.S. degree in Industrial Engineering from Galatasaray University in 2009. Afterwards, he received an M.S. degree in Financial Engineering from Bogazici University in 2011 as the second ranked student. He worked as Quality Systems Manager in UFO Heating Systems. Then he worked for Yapi Kredi Bank as Business Analyst. After having worked for Yapı Kredi Bank, he had started working as Derivatives Market Specialist in Borsa Istanbul and he worked there for almost 3 years. After leaving Borsa Istanbul, he had worked for SAS Institute in various positions as Princial, Risk Consultancy and Risk Management Business Solutions Manager. At time of completion of this thesis, he has been working for McKinsey as Associate for 2 months.

His areas of interest include risk management, risk analytics, portfolio optimization, data science and financial engineering.

PUBLICATIONS

Journal Articles

Alali, Fatih, and A. Cagri Tolga. "Portfolio Allocation with the TODIM Method." Expert Systems with Applications (2019).

Sensoy, Ahmet, et al. "Effective transfer entropy approach to information flow between exchange rates and stock markets." Chaos, Solitons & Fractals, Vol. 68, 180-185, (2014).

Conference Papers

Alali, Fatih, and Çağrı Tolga, "Covariance Based Selection of Risk and Return Factors for Portfolio Optimization.", World Congress on Engineering. London, (2016). [Best Paper Award of The 2016 International Conference of Financial Engineering]