MULTI-OBJECTIVE DECISION MAKING APPROACH FOR NUCLEAR POWER PLANT INSTALLATION

(NÜKLEER ENERJİ TESİSİ KURULUMU İÇİN ÇOK AMAÇLI KARAR VERME YAKLAŞIMI)

by

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Thesis

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

MASTER OF SCIENCE
in
INDUSTRIAL ENGINEERING
in the
GRADUATE SCHOOL OF SCIENCE AND ENGINEERING
of
GALATASARAY UNIVERSITY

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September 2020

This is to certify that the thesis entitled

MULTI-OBJECTIVE DECISION MAKING APPROACH FOR NUCLEAR POWER PLANT INSTALLATION

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ACKNOWLEDGEMENTS

Foremost, I would like to thank Prof. Dr. Ziya Ulukan, Research Director of this project, for his invaluable help and for the time he has dedicated to me. I express my sincere gratitude for the quality of his teaching, his advice and his undeniable interest in all students. It is a great honor for me to be his student and to work as his assistant.

Also, I would not forget to thank to all staff of Galatasaray University for the tremendous work. They create the most favorable conditions for the conduct of our studies.

I present my sincere thanks to my family. My parents, all my relatives and friends have accompanied, helped, supported and encouraged me throughout my university life and throughout the realization of my project.

ESRA ÇAKIR

September 2020

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LIST OF SYMBOLS

Abbreviations

AHP Analytic Hierarchy Process

ANP Analytic Network Process

AOA Activity-On-Arc

AON Activity-On-Node

BÇAADP Bulanık Çok Amaçlı Ağırlıklı Doğrusal Programlama

BÇADP Bulanık Çok Amaçlı Doğrusal Programlama

CPM Critical Path Method

DM Decision Maker

ETKB Enerji ve Tabii Kaynaklar Bakanlığı

FMOLP Fuzzy Multi-Objective Linear Programming

FMWOLP Fuzzy Multiple Weighted-Objective Linear Programming

gCeq gram Carbon-equivalent

GERT Graphical Evaluation and Review Technique

GP Goal Programming

Gtoe Billion of tonnes of oil equivalent

GWe GigaWatt-electric

IAEA International Atomic Energy Agency

IEA International Energy Agency

IIASA International Institute for Applied System Analysis

kWh KiloWatt-hour

LESS Less Cost Estimating and Scheduling

MCDM Multiple Criteria Decision Making

MJ Mega Joule

MOLP Multi-Objective Linear Programming

MWe Megawatts electric

NEA Nuclear Energy Agency

NP-hard Non-deterministic Polynomial time - hardness

NWIA Nearest Weighted Interval Approximation

OECD Organisation for Economic Co-operation and Development

PCM Pairwise Comparison Matrix

PDM Precedence Diagramming Method

PEP Project Evaluation Procedure

PERT Project Evaluation and Review Technique

TOE Tonne of Oil Equivalent

TPA Two Phase Approach

TTB Türk Tabipleri Birliği

TWh TeraWatt-hour

Nomenclature

(i, j) sequence of nodes, j will be processed after i is processed

$\bar{M}_f(A)$	f-weighted mean of fuzzy number A
¢	cent
λ	membership degree of the objective
λ_k	membership degree of k^{th} objective
$\mu_A(x)$	membership function of fuzzy number A
C_L^f	nearest lower weighted point approximation $(NLWPA)$
C_U^f	nearest upper weighted point approximation $(NUWPA)$
D_{ij}	normal duration time for activity (i,j)
d_{ij}	shortest duration time for activity (i,j)
E_j	start time for node i
EF	early finish time
ES	early start time
F	required completion time for the project
i	node number, $i = 1,2,...,N$
j	node number, $j = 1,2,...,N$
k	objective $k = 1,2,...,N$
$K_{D_{ij}}$	direct cost of activity (i,j) under normal time
l	penalty cost per unit time
LF	late finish time
LS	late start time
NIS	negative ideal solution
$p_{i,j}^{+,-}$	deviation variable for activity (i,j)

PIS	positive ideal solution
$q_{i,j}^{+,-}$	deviation variable for activity (i, j)
$s_{i,j}$	crashing cost per unit time for activity (i, j)
T_{ij}	duration time for activity (i, j)
TC	total cost for the project
TP	total duration time for the project
TR	total crash time for the project
w_k	weight of objective k
Y_{ij}	crash time for activity (i, j)

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ABSTRACT

Due to the increase in energy demand, many countries suffer from energy poverty because of insufcient and expensive energy supply. Plans to use alternative power like nuclear power for electricity generation are being revived among developing countries. Decisions for installation of power plants need to be based on careful assessment of future energy supply and demand, economic and nancial implications and requirements for technology transfer. Since the problem involves many vague parameters, a fuzzy model should be an appropriate approach for dealing with this problem. This study develops a Fuzzy Multi-Objective Linear Programming (FMOLP) model for solving the nuclear power plant installation problem in fuzzy environment. FMOLP approach is recommended for cases where the objective functions are imprecise and can only be stated within a certain threshold level. The proposed model attempts to minimize total duration time, total cost and maximize the total crash time of the installation project. By using FMOLP, the weighted additive technique can also be applied in order to transform the model into Fuzzy Multiple Weighted-Objective Linear Programming (FMWOLP) to control the objective values such that all decision makers target on each criterion can be met. The optimum solution with the achievement level for both of the models (FMOLP and FMWOLP) are compared with each other. FMWOLP results in better performance as the overall degree of satisfaction depends on the weight given to the objective functions. Nuclear power plant installation problem for Turkey demonstrates the feasibility of applying the proposed models in real world cases.

Keywords: Project Management · Nearest Weighted Interval Approximation Method · Goal Programming · Fuzzy Multi-Objective Linear Programming · Nuclear Power Plant

ÖZET

Enerji talebindeki artış nedeniyle, birçok ülke yetersiz ve pahalı enerji arzından dolayı enerji yoksulluğu ile karşı karşıyadır. Elektrik üretimi için nükleer güç gibi alternatif güç kullanma planları, gelişmekte olan ülkeler arasında yeniden canlanmaktadır. Enerji santrallerinin kurulumuna ilişkin kararlar, gelecekteki enerji arzının ve talebin dikkatli bir şekilde değerlendirilmesine, ekonomik ve finansal sonuçlara ve teknoloji transferine yönelik gereksinimlere dayanmalıdır. Birçok belirsiz parametreyi içerdiğinden, bu problemle başa çıkmak için bulanık bir model uygun bir yaklaşımdır. Bu çalışma, bulanık ortamda nükleer santral kurulum problemini çözmek için Bulanık Çok Amaçlı Doğrusal Programlama (BÇADP) modelini geliştirmektedir. BÇADP yaklaşımı, amaç fonksiyonlarının kesin olmadığı durumlarda ve sadece belirli eşik seviyesinde belirtilebilen durumlar için önerilir. Önerilen model, toplam proje süresini, toplam maliyeti en aza indirmeye ve toplam erkene çekme süresini en üst düzeye çıkarmayı hedeflemektedir. BÇADP kullanarak, tüm etkenlerin her bir kritere göre hedeflenmesi için objektif değerlerin kontrol edilmesi amacıyla; modelin Bulanık Çok Amaçlı Ağırlıklı Doğrusal Programlamaya (BÇAADP) dönüştürülmesi için ağırlıklı model de uygulanabilir. Her iki model için (BÇADP ve BÇAADP) başarı seviyesi ile optimum çözüm birbiriyle karşılaştırılmıştır. BÇAADP, genel memnuniyet derecesinin amaç fonksiyonlarına verilen ağırlığa bağlı olmasından dolayı, daha iyi performansla sonuçlanır. Uygulama sahası olarak seçilen Türkiye için nükleer santral kurulum probleminde, önerilen modellerin uygulanabilirliği gösterilmektedir.

Anahtar Kelimeler: Proje Yönetimi · En Yakın Ağırlıklı Aralık Yaklaşımı Yöntemi · Hedef Programlama · Bulanık Çok Amaçlı Doğrusal Programlama · Nükleer Güç Santrali

1 INTRODUCTION

This chapter clarifies the context of the research and author's motivation. Furthermore, the objectives and contributions of the study are highlighted. The organization of this thesis is detailed and the forthcoming chapters are briefly presented.

1.1 Context and Motivations

In order to stabilize the growth of countries and the need for electricity, renewable, non-renewable and nuclear energy must be used. Nuclear energy is one of the most practical ways to meet energy demands. That is why nuclear power plant installation seems to be an important subject for the country's development. As a developing country, Turkey's energy production and energy consumption is continuously increasing. Therefore, the country should meet this increasing demand with more domestic production and accordingly reduce the dependence on foreign countries by making new investments in energy. Turkey is largely dependent on fossil fuels such as petroleum, natural gas and coal which make up a significant part of energy intake. Renewable and nuclear energy resources can be considered as some of the most effective solutions for Turkey as clean and sustainable energy. They have fewer effects on the environment as compared to fossil energy resources. It is also important to note that nuclear and renewable energy resources reduce CO_2 emissions and assist in protecting the environment. "Nuclear energy is seen by government as an important way of diversifying energy types, cheap, sustainable, eco-friendly and reducing this dependence" (Udum, 2010). But the construction of a nuclear power plant is a very complex project management task that requires a huge investment, time and attention. Project scheduling is a helpful tool for monitoring and controlling activities in large-scale projects. With the project scheduling, the critical activities of the project and the completion period are calculated from the periods of these activities by looking at the relations with the precursors. The first step in project scheduling is to demonstrate the project as a whole with a network diagram, in line with the premise of the activities with respect to each other. Thanks to the network diagram, it is easy to follow what activity is coming before or after. After this step, the critical path of the project must be identified and the completion period calculated. The need for effective methods of management and control of large and complex projects has led to the emergence of two basic techniques in this area. These are Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT) and network diagrams are used at the basis of both techniques.

Because of the nature of project management, uncertainties are very high and fuzzy multi-objective decision making methods can be a useful tool. Zimmermann's linear programming methods and the fuzzy set theory of Bellman and Zadeh combine to "achieve effective results in project planning and develop strategies that are more relevant to the objectives" (Zimmermann, 1978; Bellman and Zadeh, 1970). Fuzzy set theory is a method that offers a solution for the model where the individuality and uncertainty are at a high level. This theory has allowed the real world to be expressed mathematically, so that the definite boundaries created by classical mathematics are overcome and uncertainty takes place in the decision-making process. The widespread use of fuzzy set theory in almost every area of science and technology has broadened the scope of its classical operations research studies with new expansions to decision making in industrial systems. This theory is widely used in navigation research, linear programming, nonlinear programming, goal programming, dynamic programming, transport models, game theory and network problems. In many studies, fuzzy logic has been included in mathematical modeling. For example, decisions for installation of power plants need to be based on careful assessment of future energy supply and demand, economic and nancial implications and requirements for technology transfer. Since the problem involves many vague parameters, a fuzzy model should be an appropriate approach for dealing with this problem.

This study develops a fuzzy multi-objective linear programming (FMOLP) model for solving the nuclear power plant installation problem in fuzzy environment. FMOLP approach is recommended for cases where the objective functions are imprecise and can only be stated within a certain threshold level. The proposed model attempts to minimize total duration time, total cost and maximize the total crash time of the installation project. By using FMOLP, the weighted additive technique can also be applied in order to transform the model into Fuzzy Multiple Weighted-Objective Linear Programming (FMWOLP) to control the objective values such that all decision makers target on each criterion can be met. The optimum solution with the achievement level for both of the models (FMOLP and FMWOLP) are compared with each other. FMWOLP results in better performance since the degree of the overall satisfaction does not simply impersonate the membership degree of the worst objective. The application demonstrates the feasibility of applying the proposed models to nuclear power plant installation problem.

1.2 Aims and Objectives

The establishment of a nuclear power plant is a sensitive issue for the society. The construction decision is taken as a result of strict inspections, and the implementation phase is very difficult. Since it is a very large scale project, time and cost balance should be established well. Based on this problem, this thesis aims to provide cost and time balance in the management phase of a large scale project, and it proposes a fuzzy multi-objective decision making approach. The proposed model brings a new approach to existing models already present in the literature. For the first time, two subjects such as FMOLP-FMWOLP and project planning are combined on nuclear power plant subject. Nearest interval approximation methodology and goal programming methodology are used to generate the weights of the objective functions without the need for consistency check.

Accordingly, the objectives of this thesis are to:

- Propose a nuclear power plant installation project decision-making framework;
- Implement CPM-PERT in nuclear power plant installation project;
- Express project network as a linear programming model with algebraic linear expressions which is describing the objective function and constraints;
- Set total time, total crash time and total cost as objective functions and establish fuzzy multi-objective linear programming model;

- Add opinions of decision makers as weights of objectives to established fuzzy multiple weighted-objective linear programming model;
- Observe the effect of decision makers on the model comparing the two models;
- Choose the less costly, shorter nuclear power plant installation project plan.

1.3 Original Contributions

A new approach to existing techniques is introduced in the proposed model. The main contribution of this study is the use of Nearest Interval Approximation and goal programming to obtain the importance weight of the objectives in FMWOLP models. Therefore, deriving the weights of criteria even from inconsistent fuzzy comparison matrix would be possible with nearest interval approximation and goal programming approach. These techniques are combined with the CPM-PERT model used in project management. The new combined models are applied for the first time in a nuclear power plant installation project data. As a result of the study, duration of some activities are shortened and the total cost is reduced by taking into account the time-cost balance. The two applied models are compared and by making a less costly planning, the feasibility of the proposed models is shown in a nuclear power plant installation project.

In addition, this study contributes to the literature with a conference paper (Çakır and Ulukan, 2019) and an article (Çakır and Ulukan, 2020) .

1.4 Thesis Organization

This thesis consist of four main part and eleven chapters. As in Figure 1.1, the first part is introduction of the study. This part explains context of research, author's motivation, aims and objectives of thesis and layout of the study. The second part details the content of thesis. All techniques required for application are introduced. Details of the technique and literature review are given on each subject. At the end of this part, two models are proposed with all the techniques introduced. The third part is experimental. A detailed research is included in the field of application. Then, the hybrid models presented in the second part are applied. In the fourth section, the general structure of the study is evaluated and the results are commented on. The results of the two models are compared. Suitability of methods and limitations of the study are discussed. The ideas are put forward for future research.

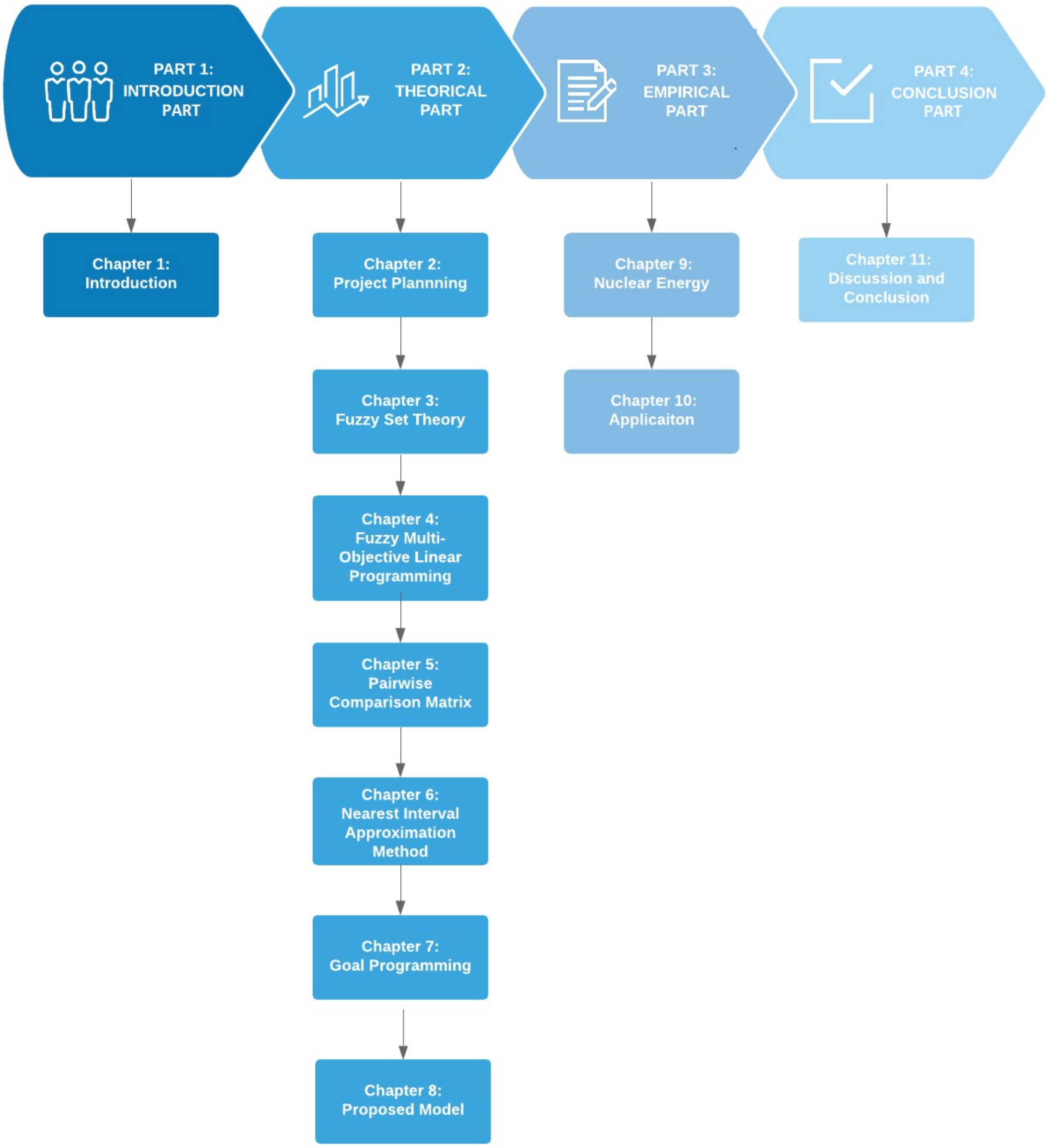


Figure 1.1: Thesis Organization.

The remainder of the thesis is organized as chapters as follows:

In Chapter 2, techniques such as Gantt Chart, PERT and CPM used in project planning are examined. These techniques are compared with each other and their use in project management is shown. Project compression and crashing cost are mentioned. At the end of the chapter, publications on this subject are listed.

In Chapter 3, fuzzy set theory is examined. Triangular and trapezoidal fuzzy numbers are defined and their operators are shown. Also, comparison of fuzzy numbers is included. At the end of the chapter, publications on this subject are listed.

In Chapter 4, fuzzy multi-objective linear programming model is examined. Firstly, linear programming structure is examined and multi-objective programming is included. By adding fuzzy numbers to these models, fuzzy multi-objective linear programming and fuzzy multiple weighted-objective linear programming models are examined. At the end of the chapter, publications on this subject are listed.

In Chapter 5, pairwise comparison matrices are examined to compare fuzzy statements of decision makers. Methods to aggregate fuzzy matrices are investigated. The method for aggregating pairwise comparison judgments of decision makers is shown. At the end of the chapter, publications on this subject are listed.

In Chapter 6, nearest interval approximation method is examined and its usage with triangular and trapezoidal fuzzy numbers is given. At the end of the chapter, publications on this subject are listed.

In Chapter 7, goal programming is defined. General structure of model is mentioned and formulation is given for weighted goal programming. At the end of the chapter, publications on this subject are listed.

In Chapter 8, using the methods examined in the previous titles, a new model is proposed. The stages of the model are explained in detail. It consists of two models. Firstly, fuzzy multi objective linear programming model (FMOLP) is shown. By adding the judgments of decision makers, first model becomes fuzzy multiple weighted-objective linear programming (FMWOLP) model. Judgments of decision makers are included in the model using nearest interval approximation method and goal programming.

In Chapter 9, nuclear energy is introduced. Information is given about its history, impact on the environment and the economy, contribution to the energy sector, risks and costs. Status of nuclear power in Turkey are examined. At the end of the chapter, publications on this subject are listed.

In Chapter 10, the proposed model is applied to the planned nuclear power plant project data to be held in Turkey. Firstly, project management techniques are used, and then the proposed model is applied. At the end of the application, two different results are evaluated for FMOLP and FMWOLP models.

In Chapter 11, according to the determined purposes and method, an overview of the study is given and the results are interpreted. Suitability of methods and limitations of the study are discussed. The ideas are given for future research.

2 PROJECT PLANNING

"Project planning is a part of project management, which relates to the use of schedules such as the Gantt charts to plan and subsequently report progress within the project environment" (Kerzner, 2003). Project planning can be done manually or by the use of project management software. In addition to many methods used in project management, the most prevalent methods are PERT and CPM. Both methods emerged towards the end of the 1950s. PEP, LESS, GERT and PDM can be listed as other methods. Project planning techniques are generally used in production and distribution projects planning, site selection, resource management, financial planning problems, construction sector and research and development projects. "The use of PERT and CPM is more common than others" (Spinner, 1997).

Some tools used since the beginning of 1960s have facilitated the discovery of both methods. One of the known tool is the Gantt chart, which is widely used today. This chart, which first appeared in 1910 to control production, received the name of Henry L. Gantt. The Gantt chart consists of interdependent bars.

"In 1958, it was envisaged to develop a more advanced methodology for the management of the Polaris project jointly conducted by the United States Navy Special Projects Office and the Booz, Allen and Hamilton consulting firm" (Stevenson, 1993). This project included the first nuclear powered submarine capable of guided missiles. The presence of about 3000 subcontractors who made various parts of the submarine was one of the factors that made the administration of the project difficult. The person who developed the method in this project where PERT was used for the first time.

Another method developed in the same year is the CPM, which is very similar to PERT but is separated from it by some features."The CPM was first used in the construction and maintenance of chemical plants in 1958. E.I. du Pont de Nemours used this method first time. Remington Rand revealed the CPM" (Star, 1971).

Project management techniques are based on certain rules:

- The project consists of a series of consecutive or parallel activities;
- The project ends with the completion of all of these activities;
- There is a certain sequence of operations and the technical priorities are laid down;
- The duration of the activities is known or predicted in advance. Once an activity starts, it continues uninterrupted until it is finished;
- An activity can not start from itself until all of its previous activities have come to an end. If the activity is not on the critical path, there is no obligation to start immediately, the start can be delayed for a while;
- Every project has a beginning node and an end node.

These rules set out the basics of project management techniques.

2.1 Project Planning Techniques

"One of the most important areas of project management is the scheduling of the project" (Soltani and Haji, 2007). "Scheduling emerges as an important factor that determines the effectiveness and efficiency of the project in the planning phase and includes the decisions of management" (Paksoy, 2007). Today, due to the increase of product variety, product life span is shortened and each project deals with different activities and periods of activity. For this purpose, "it is very important to plan, manage and schedule processes as far as possible without uncertainty" (Kökçam and Engin, 2010).

In order to achieve time-cost goals, resources available should be used for specific purposes and to implement the necessary activities. In other words; "scheduling problem is the scheduling of allocated resources in order to balance the total cost and the completion time of the project" (Ke and Liu, 2010; Zhang and Chen, 2012; Liu, 2009). In the project schedule; "determination of the priority and sequence relations among the activities, determination of budget and resource constraints to be allocated to each activity and calculation of the duration of each activity takes place" (Kurt, 2006). For this purpose, project scheduling activities are primarily focused on daily working hours, weekly working days, starting with the preparation of the work schedule. "It is estimated how long the activities and the schedule of the project is prepared with the contribution of the activity schedule, which is based on the priorities and the periods of activity during the process" (Kökçam and Engin, 2010). Many of these problems are NP-hard problems. In the scheduling literature," $P \infty |prec|$ are classified as scheduling problems with an *"infinite"* parallel machine or source exposed to priority constraints expressed in C_{max} notation" (Pinedo, 2012).

"The purpose of scheduling projects; determining what activities are critical to ensure that the project is completed on time, determining what activities can be delayed for longer without delaying the completion period of the project when necessary, determining when the activities will start, how much money should be spent on the project at any moment" (Kurt, 2006; Kolaylıoğlu, 2006), whether it is worth the extra expenditure to accelerate some activities.

"Three approaches are generally used in the scheduling of activities in the projects; Gantt chart, CPM, PERT" (Kurt, 2006; Soltani and Haji, 2007; Kökçam and Engin, 2010). The common aim of these approaches is to minimize the total duration of the project. Comparison table of Gantt chart, CPM and PERT according to some criteria is shown in Table 2.1 (Cleland, 1990). "In addition, PEP, LESS, GERT and PDM can also be used in this area and especially modeling of business processes" (Spinner, 1997). In all of these approaches, non-fuzzy conditions, i.e. deterministic decision environments, are mentioned. In fuzzy approaches, fuzzy project scheduling method is used.

Table 2.1: Comparison of Gantt chart, CPM and PERT according to some criteria.

Criterion	Gantt Chart	CPM	PERT
Applicability	There may be an error between measurements during short operating periods. There is no clear method of demonstrating interaction relationships.	It establishes the relationships between work order and activities. The accuracy of the estimation reduces the margin of error.	The relationship between work order and activities is as in CPM. It's too complicated for small projects. The presence of three time estimates increases the accuracy of the results.
Reliability	The existence of a single time estimate for each activity prevents errors that can arise from excessive complexity. It depends on the judgment of its reliability estimator. Small insecurities in large projects affect the status of the project.	Finding a single time estimate for each activity prevents errors from excessive complexity. The sum of small insecurities in the activities of a large project can affect decisions in determining the status of the whole project.	Probabilistic time estimates are more accurate than a single time estimate. Obtaining three estimates for each activity requires more information and may result in extra errors.
Execution	It is the easiest to understand among all systems. It is very difficult to implement in the control of transactions where time standards are not available.	Graphical representation of business sequences and relationships between activities is preferred by managers of complex projects. It is difficult to explain the system to unfamiliar ones. The complexity of programming can intimidate the customer.	Graphical representation of business sequences and relationships between activities is preferred by managers of complex projects. A complete PERT system is complex and difficult to implement. It can scare users and customers for the first time.
Simulation ability	There is no clear competence.	It is perfect for evaluating alternative plans considering time-cost-resource issues in a computerized application. Requires a computer for all projects except very small projects.	It is perfect for evaluating alternative plans considering time-cost-resource issues in a computerized application. Requires a computer for all projects except very small projects.
Update status	It is easy to update the charts periodically without the use of the computer, unless there are significant program changes. Graphics may need to be redrawn due to inability to update existing schemes.	The qualification is good. Activities are clearly defined and time estimates can be obtained if necessary. Even for moderately complex projects, programs require computer use.	Events are clearly defined and can be obtained when necessary. Estimates of activity times are time consuming and calculation of expected times requires computer use.
Flexibility	It can also be used to estimate resource requirements. If there are often changes of viewpoint, many graphs may need to be completely reconstructed.	Networking can be changed to reflect different perspectives in computerized applications. It is used to estimate resource requirements if drawn on a time scale. Even for projects with medium complexity, programming is done by computer.	As the project changes, network and new time estimates change immediately to reflect changes. It can be used to estimate resource requirements when drawn on a time scale. Even for moderately complex projects, programs require computer use.
Cost	Data collection and processing is relatively inexpensive. It can be cheap if the current charts are up to date and cheap materials are used. As the graphics are not flexible, program changes take time and require high-cost new graphics.	It can significantly reduce overall project costs through better planning and control. A significant amount of data and computer use is required to use CPM as a planning and status reporting tool. Therefore, the cost is quite high.	It can significantly reduce overall project costs through better planning and control. It requires more data and calculation than all other systems. Therefore, the cost of the system is high.

2.1.1 Gantt Chart

One of the useful techniques in project planning is the Gantt charts. "It is a visual representation of a chart with the simplest expression. This method is developed by Henry L. Gantt during World War I" (Shtub et al., 1994). In the Gantt charts, when the time is shown in the horizontal axis, the tasks are shown vertically to the left of the fiddle. On the schematic, the horizontal bars indicate the duration of the task. The milestones (turning points) are usually expressed by a single point or a diamond quadrilateral - triangles are also used. Connections between tasks are indicated by vertical dashed lines or connected arrows. Arrows point the order in which the tasks will be completed for the continuity of the project.

The Gantt charts are used in four main areas of industry:

- Schedules of man and instrument registration;
- Project schedules;
- Installation diagrams;
- Development schemes.

"In addition to being extremely simple and useful, the Gantt chart has some shortcomings. One of the biggest shortcomings of this method is that all logical connections between activities can not be shown. So, these diagrams do not show how a delay in one phase will affect the other phase" (Albayrak, 2001). Besides this, only limited amount of control activities can be carried out besides planning. This is just the percentage of completion of the activities in the project. Apart from this, it is not possible to reach report information such as which activities should be completed on time, the weight of each activity during the project period. If there is a change in the application of any work, the entire project must be redrawn. As a result, "bar diagrams are nevertheless extremely simple to use for projects consisting of a limited number of activities and the Gantt chart is an effective planning, programming, control and reporting tool" (Goodman and Love, 1980).

However, as the complexity of projects increases, the Gantt chart is insufficient to show details, so methods are needed to provide more detailed imaging and operation. Thus, networking methods have been developed to address the complexity of the project.

2.1.2 Project Evaluation and Review Technique - PERT

When the historical development of PERT is examined, it is seen that the Gantt charts are the basis for PERT. However, the Gantt chart is a method with a number of deficiencies, because it shows a degree of relation between the phases of the project. This is why PERT is a more advanced form of the Gantt chart.

PERT determines the duration of each activity using some probability distributions, rather than dealing with fixed periods of activity. In this method, uncertainties in the project are dealt with by using statistical methods that will estimate for three periods. These periods are the most optimistic (a), the most pessimistic (b) and the most probable (m). The aim of PERT is to find the average and variance of each activity and the probability distribution of the whole process. The information obtained in this regard provides management planning knowledge, which is used to evaluate the feasibility of the project.

The PERT Three-Estimate Approach

The three estimates are defined for each activity as follows:

- **Most likely estimate** (m) **:** "estimate of the most likely value of the duration"
- **Optimistic estimate** (o) **:** "estimate of the duration under the most favorable condition"s
- **Pessimistic estimate** (p) **:** "estimate of the duration under the most unfavorable conditions"

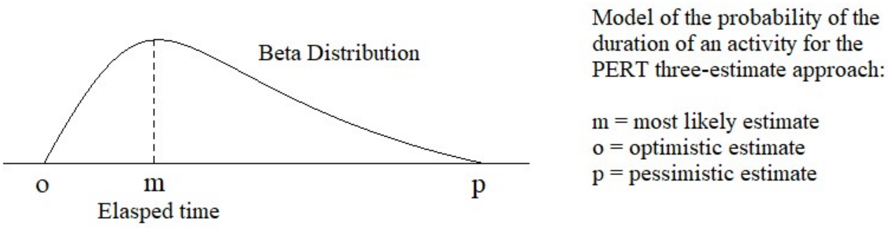


Figure 2.1: Model of the probability distribution of the duration.

The intended location of these three estimates with respect to the probability distribution is shown in Figure 2.1. Thus, the optimistic and pessimistic estimates are meant to lie at the extremes of what is possible, whereas the most likely estimate provides the highest point of the probability distribution. PERT also assumes that the form of the probability distribution is a beta distribution (which has a shape as Figure 2.1) in order to calculate the mean (μ) and variance (σ^2) of the probability distribution. For most probability distributions such as the beta distribution, essentially the entire distribution lies inside the interval between $(\mu - 3\sigma)$ and $(\mu + 3\sigma)$. Therefore, an approximate formula for σ^2 is

$$\sigma^2 = \left(\frac{p-o}{6}\right)^2 \tag{2.1.1}$$

Similarly, an approximate formula for μ is

$$\mu = \frac{o+4m+p}{6} \tag{2.1.2}$$

2.1.3 Critical Path Method - CPM

"The Critical Path Method was developed by Kelly and Walker in 1957 to assist in the construction and maintenance of the chemical plant in Dupont" (Kelly, 1957; Kelly and Walker, 1959). CPM assumes that the durations of activities are definite. In the CPM, each process is represented by an arrow (generally in arrow diagrams). Each process starts with a node point and ends at another node point. In fact, "CPM is a process that determines when jobs should be done through the priority associations between activities" (Levine, 2002).

Over time, CPM rationing has introduced a complex set of computer software that is equipped with a number of new options and offers functions that meet the different needs of users. Most of today's software used it as a network planning and control tool and they compute according to CPM principles.The interaction between activities must be under continuous control, as any delay in activity can affect other related activities and perhaps can change the completion period of the project. Apart from this, there are activities that are used to indicate significant turning points of a project. "This turning point is called the target date or intermediate term (milestone) and it usually represents the completion time of a group of activities" (Westland, 2006). The execution times of these target dates must also be closely monitored in order for the project to run without interruption.

Generally, the CPM method is similar to the PERT, but unlike PERT, CPM is a deterministic method in which it is assumed that operating periods are known precisely. In fact, this also creates the weakness of the CPM. It is not realistic in today's world that there are many uncertainties to assume that the periods of activity are known. "But it is a very convenient method for repetitive projects that have already been done. Because, it is assumed that there are no mistake in estimating the time in such projects" (Shtub et al., 1994).

2.2 Scheduling a Project with PERT-CPM

Assume that there is a sample project. Table 2.2 shows the activity codes, predecessors and activity durations. Perform the CPM-PERT procedure on this sample project.

Table 2.2: Activity list for sample project.

Activity Code	Immediate Predecessors	Duration (Week)
1	-	4
2	-	7
3	1	8
4	1	3
5	2	9
6	3	5
7	3	2
8	4,5,6	6
9	2	5

Project Network

A network used to represent a project is a project network. It consists of a number of "nodes" and a number of "arcs" that lead from some node to another.

A network diagram is a visual representation of a project's schedule. Well-known complements to network diagrams include the "PERT" and "Gantt charts". A network diagram in project management is useful for planning and tracking the project from beginning to finish. It represents a project's critical path as well as the scope of the project.

A good network diagram is a clear and concise graphic representation of a project. There are three types of information to describe a project.

1. "Activity information"
2. "Precedence relationship"
3. "Time information"

There are two different ways to draw the project network.

- "First one is the Activity-On-Arc (AOA) project network, where each activity is represented by an arc. A node is used to separate an activity from each of its immediate predecessors. The sequencing of the arcs thereby shows the precedence relationships between the activities. The original versions of PERT and CPM use AOA project networks."
- "The second one is the Activity-On-Node (AON) project network, where each activity is represented by a node. The arcs are used to show the precedence relationships between the activities. In particular, the node for each activity with immediate predecessors has an arc coming in from each of these predecessors. When drawing AOA networks, "dummy" activities are used where necessary."

Accordingly, AON and AOA project network drawings of the sample project are as shown in Figure 2.2 and Figure 2.3.

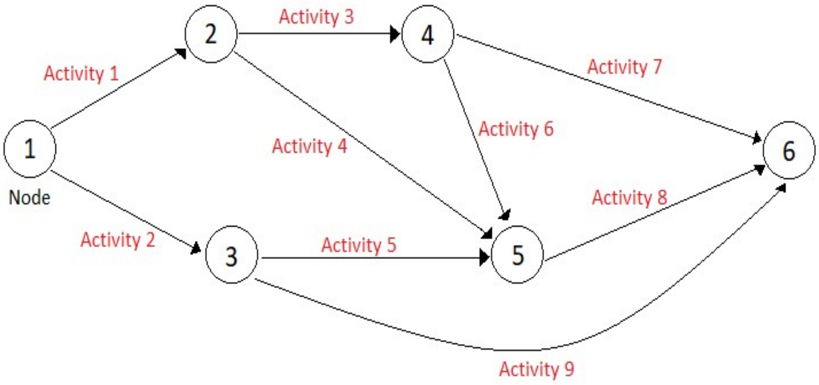


Figure 2.2: The AOA project network for sample project.

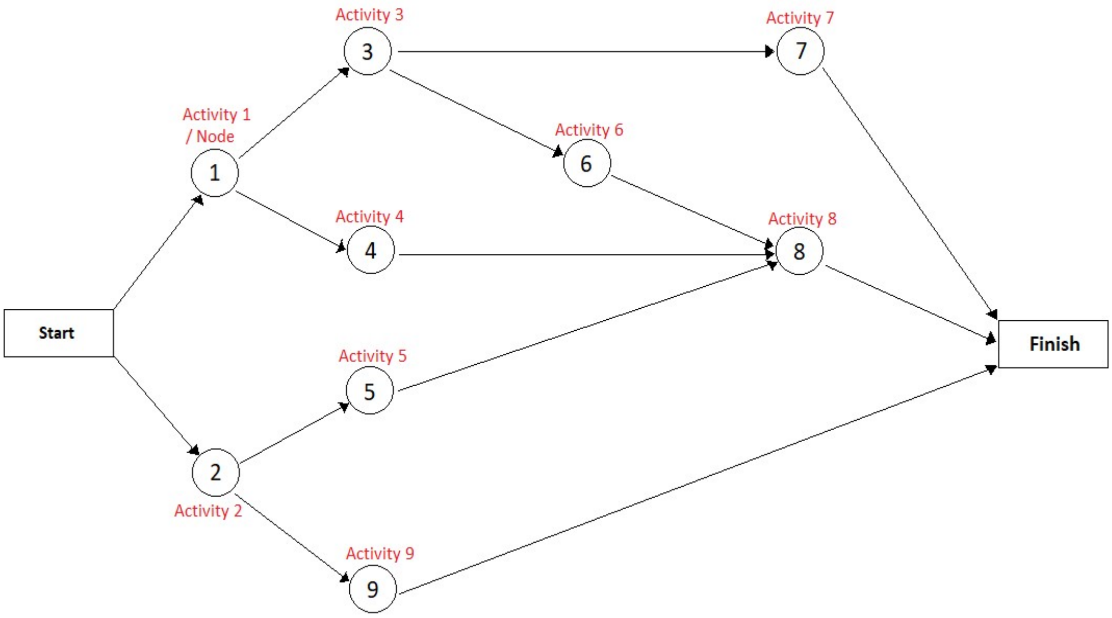


Figure 2.3: The AON project network for sample project.

Gantt Chart

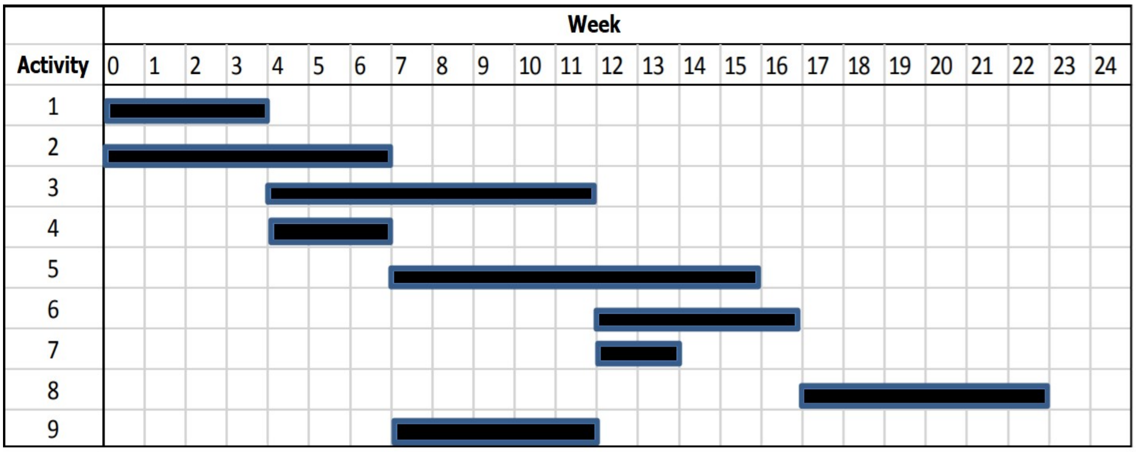
The Gantt chart displays a visual image of the project schedule and shows when the activities start and how long some activities may be delayed without affecting the project. The Project Manager can use the Gantt chart to see which activities can follow the project and what activities are behind the schedule.

While the Gantt chart is being drawn, the activities are placed in order (as in the project network) and placed on the table for the duration of time (adhering to priority orders).

This demonstration is an effective scheme for finding the critical path, seeing the total project duration and showing the locations of the activities.

The Gantt chart created for the sample project is in Table 2.3. According to this diagram, the total duration of the sample project is 23 weeks.

Table 2.3: Gantt chart for sample project.



Critical Path

A path through a project network is one of the routes following the arcs from the "*START*" node to the "*FINISH*" node for AON networks. The length of a path is the sum of the (estimated) durations of the activities on the path.

The five paths through the sample project network in Figure 2.3 are given in Table 2.4, along with the calculations of the lengths of these paths. The path lengths range from 12 weeks up to 23 weeks for the longest path.

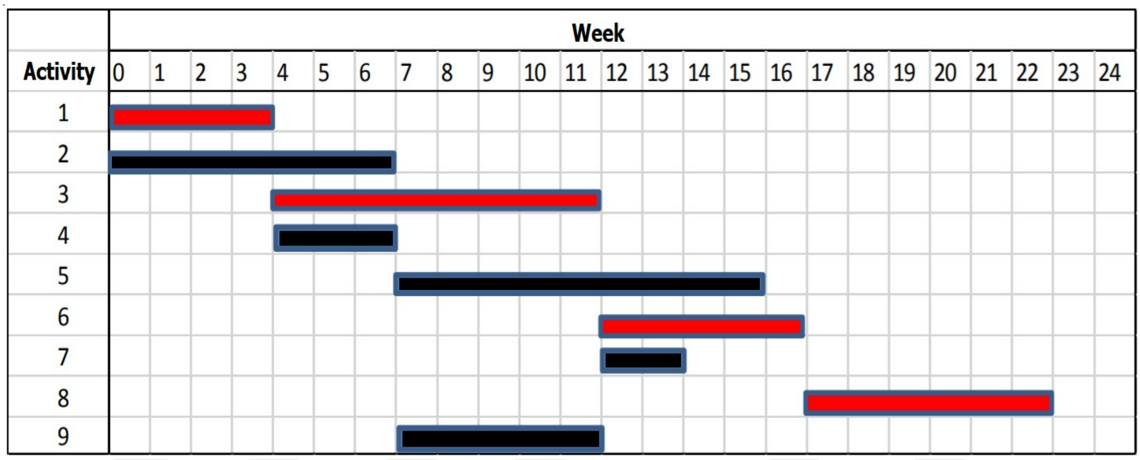
Table 2.4: The paths and path lengths through network of sample project.

Path	Length
"START → 1 → 3 → 7 → FINISH"	4+8+2 = 14 weeks
"START → 1 → 3 → 6 → 8 → FINISH"	4+8+5+6 = 23 weeks
"START → 1 → 4 → 8 → FINISH"	4+3+6 = 13 weeks
"START → 2 → 5 → 8 → FINISH"	7+9+6 = 22 weeks
"START → 2 → 9 → FINISH"	7+5 = 12 weeks

The (estimated) project duration equals the length of the longest path through the project network. This longest path is called the **critical path**.

Thus, for the sample project, the critical path is "$START \rightarrow 1 \rightarrow 3 \rightarrow 6 \rightarrow 8 \rightarrow FINISH$" and (estimated) project duration is 23 weeks. Also, the critical path can easily seen in the Gantt chart as in Table 2.5.

Table 2.5: Gantt chart with critical path for sample project.



2.2.1 Scheduling Individual Activities

The starting and finishing times of each activity (if no delays occur anywhere in the project) are called the earliest start time and the earliest finish time of the activity. These times are represented by the symbols;

ES : "Early Start time for a particular activity"

EF : "Early Finish time for a particular activity"

where

EF = "ES + (estimated) duration of the activity"

Rather than assigning calendar dates to these times, it is conventional instead to count the number of time periods from when the project started. Thus, starting time for project is "0".

The earliest start time of an activity is equal to the largest of the earliest finish times of its immediate predecessors. In symbols,

ES = "largest EF of the immediate predecessors"

The latest start time for an activity is the latest possible time that it can start without delaying the completion of the project (so the "FINISH" node still is reached at its earliest finish time), assuming no subsequent delays in the project. The latest finish time has the corresponding definition with respect to finishing the activity. In symbols,

LS: "Late Start time for a particular activity"

LF: "Late Finish time for a particular activity"

where

LS = "LF - (estimated) duration of the activity"

The latest finish time of an activity is equal to the smallest of the latest start times of its immediate successors. In symbols,

LF = "smallest LS of the immediate successors"

These terms are located next to nodes as in Table 2.6.

Table 2.6: The presentation of the earliest and late start-finish times of a node.

ES	LS
EF	LF

The result is shown in Figure 2.4 when the time of each node is calculated by adhering to these rules.

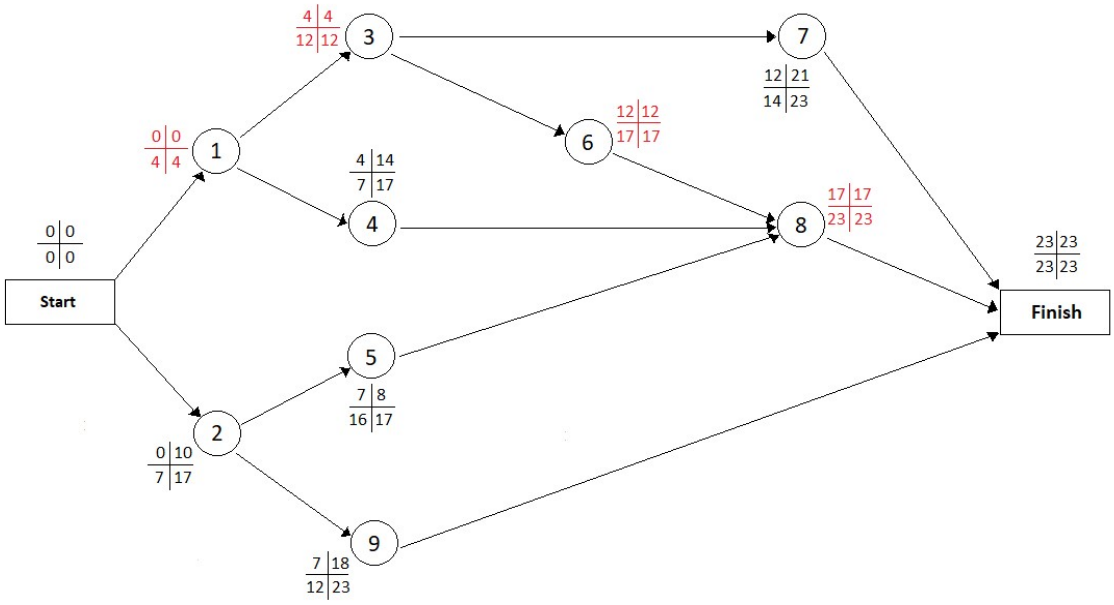


Figure 2.4: The sample project network with the earliest and latest start-finish times.

2.2.2 Identifying Slack in the Schedule

The slack for an activity is the difference between its latest finish time and its earliest finish time. In symbols,

"Slack = LF - EF = LS - ES"

Each activity with zero slack is on a critical path through the project network such that any delay along this path delays project completion. Thus, the critical path is "$START \rightarrow 1 \rightarrow 3 \rightarrow 6 \rightarrow 8 \rightarrow FINISH$" as in Table 2.7 by calculating the slacks.

Table 2.7: Slack for activities of sample project.

Activity	Slack (LF - EF)	On Critical Path ?
1	0	Yes
2	10	No
3	0	Yes
4	10	No
5	1	No
6	0	Yes
7	9	No
8	0	Yes
9	11	No

2.3 Project Compression - Crashing

The first question that comes to mind after determining the critical path of the project is that the duration of the project can not be shortened. The reduction of the duration of the whole project is called crashing. In many projects, the project managers agree to pay a certain price for early termination of the project.

Project crashing is a process that can be used in the management when the program is running behind schedule. Project crashing can be necessary when:

- "The program planning has been inaccurate"
- "When there have been unforeseen events which have caused delays, such as defects being discovered"
- "If the client has requested that the project (or a section of the project) is completed earlier than previously specified. For example, if there has been an extension of time, but the client still wishes to achieve the original completion date (this is generally referred to as acceleration)"

The aim of crashing is to achieve the maximum decrease in schedule for minimum additional cost. Activities can be crashed by:

- "Addressing productivity issues being experienced by the current resources and trying to find ways of increasing their efficiency"
- "Increasing the assignment of resources on the critical path activities. These could be internal resources or subcontracted resources"
- "Adopting different techniques. This might include off-site prefabrication, extra scaffolding, temporary weatherproofing and so on"
- "Overlapping activities"
- "Working longer hours"

- "Additional supervision"
- "Changing in design or specifications"
- "Reduction in scope"
- "Early procurement of items"

2.3.1 Value and Cost of Crashing

There is a cost to crash an activity. The project must be suitable for an activity to finish early. Managers should make a decision in this regard. Crashing an activity in general means working creatively for that activity. "Many of strategies will necessarily lead to some additional costs being incurred, or cost uncertainty. Whilst the same number of tasks need to be performed, they are condensed into a shorter period and so are likely to require more resources. In addition, crashing costs may be higher due to time pressures, incomplete information and the complexity of managing the interfaces between elements. A greater number of variations are also likely than on a traditional contract" (Castle, 2019). It is important to be clear whether it is the client or the contractor that will bear these additional costs.

There are several risks attached to project crashing. "As resources are typically focused on the critical path activities, there is a possibility that non-critical paths can also be affected. Quality, safety and compliance should not be affected as a result of the critical path being crashed. Another risk is that new resources may not be as productive as existing resources, because they may be unfamiliar with the project" (Castle, 2019). This may occur that the size of the contingency is increased. Project crashing should be resisted if:

- "It threatens the integrity of the works, or compromises health and safety"
- "It is no longer cost effective to continue"
- "It causes another path to become critical"
- "Time reduction is no longer realistically achievable"

2.4 Literature review on project management

Publications on project management are listed in Table 2.8.

Table 2.8: Literature review on project management.

Author	Year	Title	Type
Ansoff et al.	(1976)	"From Strategic Planning to Strategic Management"	Book
Pinto and Slevin	(1988)	"Project Success: Definitions and Measurement Techniques"	Article
Kumar et al.	(1994)	"Fuzzy systems and neural networks in software engineering project management"	Article
Hapke et al.	(1994)	"Fuzzy project scheduling system for software development"	Article
Keil	(1995)	" Pulling the Plug: Software Project Management and the Problem of Project Escalation"	Article
Mon et al.	(1995)	"Application of fuzzy distributions on project management"	Article
Munns and Bjeirmi	(1996)	"The role of project management in achieving project success"	Article
Shipley et al.	(1997)	"BIFPET methodology versus PERT in project management: fuzzy probability instead of the beta distribution"	Article
Turner	(1999)	"The handbook of project-based management: improving the processes for achieving strategic objectives"	Book
Kuchta	(2001)	"Use of fuzzy numbers in project risk (criticality) assessment"	Article
Slyeptsov and Tyshchuk	(2003)	"Fuzzy temporal characteristics of operations for project management on the network models basis"	Article
Wang and Liang	(2004)	"Project management decisions with multiple fuzzy goals"	Article
Dweiri and Kablan	(2006)	"Using fuzzy decision making for the evaluation of the project management internal efficiency"	Article
Wei et al.	(2007)	"A comprehensive supply chain management project selection framework under fuzzy environment"	Article
Chen and Huang	(2007)	"Applying fuzzy method for measuring criticality in project network"	Article
Zeng et al.	(2007)	"Application of a fuzzy based decision making methodology to construction project risk assessment"	Article

Author	Year	Title	Type
Longa and Ohsato	(2008)	"Fuzzy critical chain method for project scheduling under resource constraints and uncertainty"	Article
Schwalbe	(2008)	"Information Technology Project Management, Reprint"	Book
Liang	(2009)	"Fuzzy multi-objective project management decisions using two-phase fuzzy goal programming approach"	Article
Ruuska et al.	(2009)	"Dimensions of distance in a project network: Exploring Olkiluoto 3 nuclear power plant project"	Article
Abdelgawad and Fayek	(2010)	"Risk Management in the Construction Industry Using Combined Fuzzy FMEA and Fuzzy AHP"	Article
Liang	(2010)	"Applying fuzzy goal programming to project management decisions with multiple goals in uncertain environments"	Article
Nieto-Morotea and Ruz-Vilab	(2011)	"A fuzzy approach to construction project risk assessment"	Article
Burke	(2013)	"Project management: planning and control techniques"	Book
Aaltonen et al.	(2015)	"Stakeholder Dynamics during the Project Front-End: The Case of Nuclear Waste Repository Projects"	Article
Walker	(2015)	"Project management in construction"	Book
Heagney	(2016)	"Fundamentals of project management"	Book
Case and Stylios	(2016)	"Fuzzy Cognitive Map to model project management problems"	Paper
Prascevic and Prascevic	(2017)	"Application of fuzzy AHP for ranking and selection of alternatives in construction project management"	Article
Nguyen et al.	(2018)	"The Application of Fuzzy Analytic Hierarchy Process (F-AHP) in Engineering Project Management"	Paper
Hamzeh and Mousavi	(2019)	"A New Fuzzy Approach for Project Time Assessment Under Uncertain Conditions"	Paper
Fayek	(2020)	"Fuzzy logic and fuzzy hybrid techniques for construction engineering and management"	Article
Alizdeh and Saeidi	(2020)	"Fuzzy project scheduling with critical path including risk and resource constraints using linear programming"	Article

3 FUZZY SET THEORY

Due to increased complexity of the developments in science and technology, the decision processes became unclear and have gained a difficult characteristic. Hence, the concepts and methods of probability theory used to study uncertainty and these methods have been re-examined and criticized in the 1960s. In the light of these criticisms, "the work of developing methods that can be used as a substitute for probability theory has been intensified" (Tuş, 2006). The definition of fuzzy sets is made in the original article "Fuzzy Sets" published in 1965 by Lotfi A. Zadeh, who made his first studies on fuzzy sets and he was regarded as the theorist of this subject (Zadeh, 1965). He defined a fuzzy set as "a class of objects with a continuum of grades of membership". In other words, "a set is described as fuzzy if there is no sharp boundary between those elements (or objects) which belong to the associated class and those which do not" (Bellman and Zadeh, 1970). "A crisp set allows only full membership or no membership at all, whereas fuzzy sets allow for partial membership. It offers a unifying framework for modelling various types of information, ranging from precise numerical, interval-valued data, to linguistic knowledge with a stress on semantics" (Delgado et al., 1997).

A fuzzy set is a type of set that has elements with different memberships, that is, membership degrees. "A fuzzy set can be characterized by a membership function that assigns membership values to each of the elements from 0 to 1. Members who are not included in the set are assigned membership values of 0 and those who are included in the set are assigned membership values of 1. Uncertain elements are assigned values between 0 and 1 according to the uncertainty state" (Bellman and Zadeh, 1970). However, "there is no such thing as an indefinite element in definite set theory" (Altaş, 1999).

This membership level can be measured by a function that attempts to define uncertainty. This function transforms the elements of a fuzzy set $\widetilde{A}$ into a real value in the range [0,1] and assumes $f_{\widetilde{A}(x)} \in [0,1]$, x as a non-empty set. A fuzzy set $\widetilde{A}$ in x is customized with the membership function $\widetilde{A} : X \to [0,1]$. For $\forall\ X \in x$; the membership level of x is defined as $f_{\widetilde{A}(x)}$.

"The greatest contribution of fuzzy set theory is the ability to represent uncertain information in a large amount of ambiguous and inconsistent data. If decision makers are inconsistent and uncertain data, they can benefit from fuzzy set theory. They may also represent potential risk in analytic models through the use of fuzzy data" (Nachtmann and Needy, 2001).

3.1 Fuzzy Numbers

Fuzzy numbers are expressed as a convex, normalized fuzzy set with limited continuous membership function and defined in real numbers. "Since fuzzy sets are defined by membership functions, fuzzy numbers are the same concepts as their own membership functions. Hence, there are as many fuzzy number types as membership function type" (Karakaşoğlu, 2008).

3.1.1 Triangular Fuzzy Numbers

"A triangle fuzzy number is represented by (l,m,u). For a fuzzy event, the l,m and u parameters represent the smallest possible value, the most expected value, and the largest possible value, respectively" (Alonso and Lamata, 2006). In the Figure 3.1, a fuzzy triangle is given as an example.

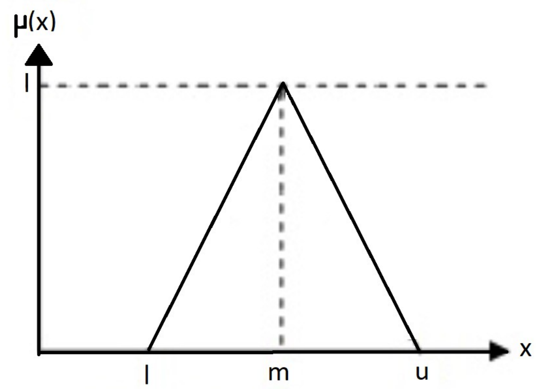


Figure 3.1: Triangular fuzzy number (l,m,u).

"A linear representation of a triangular fuzzy number can be defined by the following membership function as the left and right sides" (Zadeh, 1965):

$$\mu(x/M) = \begin{cases} 0, & x < l, \\ \dfrac{x-l}{m-l}, & l \leq x \leq m, \\ \dfrac{u-x}{u-m}, & m \leq x \leq u, \\ 0, & x > u \end{cases} \tag{3.1.1}$$

Triangular Fuzzy Numbers Operations

Factors to consider when handling triangular fuzzy numbers:

- "The result of the addition and subtraction of triangular fuzzy numbers is again a triangular fuzzy number"
- "Multiplication, division and inverse operations with triangular fuzzy numbers do not always give a triangular fuzzy number as a result"
- "Maximum or minimum operations with triangular fuzzy numbers do not always give a triangular fuzzy number as a result"

However, the results of these operations can be regarded as approximately triangular fuzzy numbers.

1. **Addition:** $\widetilde{A} + \widetilde{B} = (l_A + l_B,\ m_A + m_B,\ u_A + u_B)$.
2. **Extraction:** $\widetilde{A} - \widetilde{B} = (l_A - u_B,\ m_A - m_B,\ u_A - l_B)$.

3.1.2 Trapezoidal Fuzzy Numbers

"Another shape of fuzzy number is trapezoidal fuzzy number. This shape is originated from the fact that there are several points whose membership degree is maximum (α = 1)" (Alsawy and Hefny, 2013).

A trapezoidal fuzzy number $\widetilde{A}$ can be defined as $\widetilde{A} = (a_1, a_2, a_3, a_4)$ The membership function of this fuzzy number is interpreted as Figure 3.2.

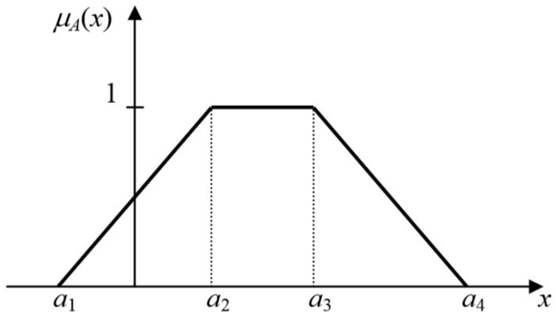


Figure 3.2: Trapezoidal fuzzy number $\widetilde{A} = (a_1, a_2, a_3, a_4)$.

α -cut interval for this shape is written a follows:

$$\widetilde{A}_\alpha = [(a_2 - a_1)\alpha + a_1, \ -(a_4 - a_3)\alpha + a_4], \quad \forall \alpha \in [0,1] \tag{3.1.2}$$

When $a_2 = a_3$, the trapezoidal fuzzy number coincides with triangular one.

"A linear representation of a triangular fuzzy number can be defined by the following membership function" (Zadeh, 1965):

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4, \\ 0, & x > a_4 \end{cases} \tag{3.1.3}$$

Trapezoidal Fuzzy Numbers Operations

Factors to consider when handling trapezoidal fuzzy numbers:

- "Addition and subtraction between fuzzy numbers become trapezoidal fuzzy number"
- "Multiplication, division and inverse need not be trapezoidal fuzzy number"

- "Max and Min of fuzzy number is not always in the form of trapezoidal fuzzy number"

In many cases, "the operation results from multiplication or division are approximated trapezoidal shape. As in triangular fuzzy number, addition and subtraction are simply defined and multiplication and division operations should be done by using membership functions" (Lee, 2005).

1. **Addition:** $\widetilde{A}+\widetilde{B}=(a_1+b_1,\ a_2+b_2,\ a_3+b_3,\ a_4+b_4)$.
2. **Extraction:** $\widetilde{A}-\widetilde{B}=(a_1-b_4,\ a_2-b_3,\ a_3-b_2,\ a_4-b_1)$.

3.2 Defuzzification and Comparison of Fuzzy Numbers

"Defuzzification is the last operation of the fuzzy system. It is the whole process of transforming fuzzy information into definite numbers" (Karataş, 2011).

"A ranking function is a function $g: F(R) \to R$, which maps each fuzzy number into the real line, where a natural order exists. Asady and Zendehnam proposed a defuzzification using minimization of the distance between two fuzzy numbers" (Asady and Zendenam, 2007). If $\tilde{A}=(a,b,c)$ be a triangular fuzzy number, then distance minimization of a fuzzy number $\tilde{A}$ (denoted by $M(\tilde{A})$) is defined as follows:

$$M(\tilde{A})=\tfrac{1}{4}(a+2b+c)$$

This ranking function has the following properties.
Property 1: If $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers, then

$$\begin{aligned} M(\tilde{A})>M(\tilde{B}) \quad & iff \quad M(\tilde{A})\succ M(\tilde{B}) \\ M(\tilde{A})<M(\tilde{B}) \quad & iff \quad M(\tilde{A})\prec M(\tilde{B}) \\ M(\tilde{A})=M(\tilde{B}) \quad & iff \quad M(\tilde{A})\approx M(\tilde{B}) \end{aligned}$$

Property 2: If $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers, then

$$M(\tilde{A} \oplus \tilde{B}) = M(\tilde{A}) + M(\tilde{B})$$

3.3 Literature Review on Fuzzy Set Theory

Publications on fuzzy numbers are listed in Table 3.1.

Table 3.1: Literature review on fuzzy set theory.

Author	Year	Title	Type
Zadeh	(1965)	"Fuzzy sets"	Article
Roy	(1968)	"Classement et choix en présence de points de vue multiples (la méthode ELECTRE)"	Article
Zadeh	(1975)	"The concept of a linguistic variable and its applications to approximate reasoning"	Article
Hwang and Masud	(1979)	"Multiple Objective Decision Making — Methods and Applications: A Stateof-the-Art Survey"	Book
Lowen	(1980)	"Convex fuzzy sets"	Article
Wang-jin	(1983)	"Operations on fuzzy ideals"	Article
Yager	(1986)	"On The Theory of Bags"	Article
Atanassov	(1986)	"Intuitionistic Fuzzy Sets"	Article
Kaleva	(1987)	"Fuzzy differential equations"	Article
Gerstenkorn and Mańko	(1991)	"Correlation of intuitionistic fuzzy sets"	Article
Bustince and Burillo	(1995)	"Correlation of interval-valued intuitionistic fuzzy sets"	Article
Chang	(1996)	"Applications of the extent analysis method on fuzzy AHP"	Article
Suh	(1998)	"Axiomatic Design Theory for Systems"	Article
Chen et al.	(2001)	"Introduction to Fuzzy Sets, Fuzzy Logic, and Fuzzy Control Systems"	Article
Kahraman et al.	(2006)	"A fuzzy optimization model for QFD planning process using analytic network approach"	Article
Garibaldi et al.	(2008)	"Nonstationary Fuzzy Sets"	Article

Author	Year	Title	Type
Lin and Wu	(2008)	"A causal analytical method for group decision-making under fuzzy environment"	Article
Rodriguez et al.	(2009)	"Hesitant Fuzzy Linguistic Term Sets for Decision Making"	Article
Tseng	(2010)	"Implementation and performance evaluation using the fuzzy network balanced scorecard"	Article
Kahraman and Kaya	(2010)	"A fuzzy multi criteria methodology for selection among energy alternatives"	Article
Torra	(2010)	"Hesitant fuzzy sets"	Article
Xu	(2010)	"Choquet integrals of weighted intuitionistic fuzzy information"	Article
Zhu et al.	(2012)	"Dual Hesitant Fuzzy Sets"	Article
Calabrese et al.	(2013)	"Using Fuzzy AHP to manage Intellectual Capital assets: An application to the ICT service industry"	Article
Onar et al.	(2014)	"Strategic Decision Selection Using Hesitant fuzzy TOPSIS and Interval Type-2 Fuzzy AHP: A case study"	Article
Atanassov	(2017)	"Type-1 Fuzzy Sets and Intuitionistic Fuzzy Sets"	Article
Li and Zeng	(2017)	"Distance Measure of Pythagorean Fuzzy Sets"	Article
Ullah et al.	(2018)	"Similarity Measures for T-Spherical Fuzzy Sets with Applications in Pattern Recognition"	Article
Alcantud and Torra	(2018)	"Decomposition theorems and extension principles for hesitant fuzzy sets"	Article
Wang et al.	(2019)	"Similarity Measures of q-Rung Orthopair Fuzzy Sets Based on Cosine Function and Their Applications"	Article
Bi et al.	(2019)	"Two Classes of Entropy Measures for Complex Fuzzy Sets"	Article
Garg and Rani	(2019)	Some results on information measures for complex intuitionistic fuzzy sets	Article
Egrioglu et al.	(2020)	"Picture fuzzy time series: Defining, modeling and creating a new forecasting method"	Article
Zhou et al.	(2020)	"A New Divergence Measure of Pythagorean Fuzzy Sets Based on Belief Function and Its Application in Medical Diagnosis"	Article
Shukla et al.	(2020)	"Veracity handling and instance reduction in big data using interval type-2 fuzzy sets"	Article

4 FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING

In this chapter, fundamental notions and methods of multi-objective linear programming (MOLP) incorporating with fuzzy numbers are explained in detail.

4.1 Linear Programming

$$
\begin{array}{ll}
\textit{Minimize} & c_1x_1 + c_2x_2 + ... + c_nx_n \\
\textit{subject to:} & a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\
 & a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\
 & \quad . \qquad . \qquad ... \qquad . \qquad . \\
 & \quad . \qquad . \qquad ... \qquad . \qquad . \\
 & a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m \\
\textit{and} & x_1 \geq 0,\ x_2 \geq 0, ...,\ x_n \geq 0
\end{array}
\tag{4.1.1}
$$

where the b_i's, c_i's and a_i's are fixed real constants and the x_i's are real numbers to be determined. It is always assumed that each equation has been multiplied by minus unity, if necessary, so that each $b_i \geq 0$.

In more compact vector notation, this standard problem becomes

$$
\begin{array}{ll}
\textit{Minimize} & c^T x \\
\textit{subject to:} & \mathbf{Ax = b} \text{ and } x \geq 0
\end{array}
\tag{4.1.2}
$$

"Here x is an n-dimensional column vector (called *decision variables*), c^T is an n-dimensional row vector, **A** is an mxn matrix, and **b** is an m- dimensional column vector. the vector inequality $x \geq 0$ means that each component of x is non-negative" (Luenberger, 1989).

"It is easy to convert constraints from one form to another" (Vanderbei, 1997). For example, an inequality constraint

$$a_1x_1 + a_2x_2 + ... + a_nx_n \leq b \tag{4.1.3}$$

can be converted to an equality constraint by adding a non-negative variable, w, which is called a *slack variable*

$$a_1x_1 + a_2x_2 + ... + a_nx_n + w = b \qquad w \geq 0 \tag{4.1.4}$$

On the other hand, an equality constraint

$$a_1x_1 + a_2x_2 + ... + a_nx_n = b \tag{4.1.5}$$

can be converted to inequality form by introducing two inequality constraints

$$\begin{aligned} a_1x_1 + a_2x_2 + ... + a_nx_n \leq b \\ a_1x_1 + a_2x_2 + ... + a_nx_n \geq b \end{aligned} \tag{4.1.6}$$

4.2 Multi-Objective Programming

"The problem to optimize multiple conflicting objective functions simultaneously under given constraints is called Multi-Objective Programming problem and can be formulated as the following $vector - minimization$ problem" (Nehi and Hajmohamadi, 2012):

$$\left.\begin{aligned} &\textit{Minimize } f(x) \triangleq (f_1(x), f_2(x), ..., f_k(x))^T \\ &\textit{subject to: } x \in X \triangleq \{x \in R^n | g_j \leq 0,\ j = 1, ...m\} \end{aligned}\right\} \tag{4.2.1}$$

where $f_1(x), f_2(x), ..., f_k(x)$ are k distinct objective functions of the decision vector x, $g_1(x), ..., g_m(x)$ are m inequality constraints and X is the feasible set of constrained decisions.

If the notion of optimally for Single-Objective Linear Programming is applied to this Multi-Objective Linear Programming, complete optimal solution occurs.

Definition: Complete optimal solution x^* is said to be a complete optimal solution if and only if there exist $x^* \in X$ such that $f_i(x^*) \leq f_i(x),\ i = 1, ..., k\ \ \forall x \in X$.

4.3 Fuzzy Multi-Objective Linear Programming

In 1978, Zimmermann extended his Fuzzy Linear Programming approach to the following Multi-Objective Linear Programming problem with k linear objective functions $z_i(\mathbf{x}) = \mathbf{c_i x}, \ i = 1, ..., k$

$$\left.\begin{array}{ll} Minimize\ \mathbf{z(x)} \triangleq (z_1(\mathbf{x}), z_2(\mathbf{x}), ..., z_k(\mathbf{x}))^T \\ subject\ to: \quad \mathbf{Ax} \le \mathbf{b},\ \mathbf{x} \ge \mathbf{0} \end{array}\right\} \tag{4.3.1}$$

where $\mathbf{c_i} = (c_{i1}, ..., c_{in}),\ i = 1, ..., k,\ \mathbf{x} = (x_1, ..., x_n)^T,\ \mathbf{b} = (b_1, ..., b_m)^T$ and $A = [a_{ij}]$ is an mxn matrix (Zimmermann, 1978, 1976).

For each of the objective functions $z_i(\mathbf{x}) = \mathbf{c_i x},\ i = 1, ..., k$, of this problem, assume that the decision maker (DM) has a fuzzy goal such as "the objective function $z_i(\mathbf{x})$ should be substantially less than or equal to some value p_i". Then the corresponding linear membership function $\mu_i^L(z_i(\mathbf{x}))$ is defined as:

$$\mu_i^L(z_i(\mathbf{x})) = \begin{cases} 0, & z_i(\mathbf{x}) \ge z_i^0 \\ \dfrac{z_i(\mathbf{x}) - z_i^0}{z_i^1 - z_i^0}, & z_i^0 \ge z_i(\mathbf{x}) \ge z_i^1 \\ 1, & z_i(\mathbf{x}) \le z_i^1 \end{cases} \tag{4.3.2}$$

where z_i^0 or z_i^1 denotes the value of the objective function $z_i(\mathbf{x})$ such that the degree of membership function is 0 or 1 respectively.

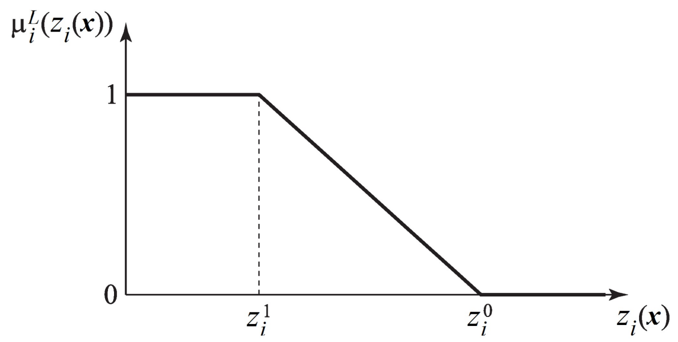


Figure 4.1: Linear membership function.

Figure 4.1 illustrates the graph of the possible shape of the linear membership function. "Using such linear membership functions $\mu_i^L(z_i(\mathbf{x})),\ i = 1, ..., k$ and following the fuzzy decision of Bellman and Zadeh" (Bellman and Zadeh, 1970), the original Multi-Objective Linear Programming problem can be interpreted as:

$$\left.\begin{aligned} &Maximize \ \min_{i=1,\ldots,k} \mu_i^L(z_i(\mathbf{x})) \\ &subject\ to: \quad \mathbf{Ax} \leq \mathbf{b},\ \mathbf{x} \geq \mathbf{0} \end{aligned}\right\} \tag{4.3.3}$$

"Zimmermann first proposed fuzzy approach named as $max - min$ operator to solve MOLP" (Zimmermann, 1978). It focuses on the maximizing the minimum membership degree. By introducing the auxiliary variable λ, it can be reduced to the following conventional Linear Programming problem

$$\left.\begin{aligned} &Maximize \quad \lambda \\ &subject\ to: \quad \lambda \leq \mu_i^L(z_i(\mathbf{x})),\ i = 1,2,\ldots,k \\ &\qquad\qquad\quad \mathbf{Ax} \leq \mathbf{b},\ \mathbf{x} \geq \mathbf{0} \end{aligned}\right\} \tag{4.3.4}$$

By assuming the existence of the optimal solution $\mathbf{x}^{io}$ of the individual objective function minimization problem under the constraints defined by

$$\min_{x \in X} z_i(\mathbf{x}),\ i = 1,2,\ldots,k, \tag{4.3.5}$$

"Zimmermann suggested a way to determine the linear membership function $\mu_i^L(z_i(\mathbf{x}))$" (Zimmermann, 1978). To be more specific, using the individual minimum

$$z_i^{min} = z_i(\mathbf{x}^{io}) = \min_{x \in X} z_i(\mathbf{x}),\ i = 1,2,\ldots,k, \tag{4.3.6}$$

together with

$$z_i^m = max(z_i(\mathbf{x}^{1o}),\ldots,z_i(\mathbf{x}^{i-1,o}),z_i(\mathbf{x}^{i+1,o}),\ldots,z_i(\mathbf{x}^{ko})),\ i = 1,2,\ldots,k, \tag{4.3.7}$$

He determined the linear membership function as in Equation (4.3.2) by choosing $z_i^1 = z_i^{min}$ and $z_i^0 = z_i^{min}$. For this membership function, it can be easily shown that if the optimal solution of Equation (4.3.3) or Equation (4.3.4) is unique, it is also a Pareto optimal solution of the Multi-Objective Linear Programming problem.

"In the case where not only fuzzy goals but also fuzzy constraints exist, using linear membership functions for fuzzy constraints, similar discussion can be made. Zimmermann called the fuzzy decision the minimum operator and for other aggregation patterns than the minimum operator, he considered the product fuzzy decision" (Zimmermann, 1987). He called the product fuzzy decision the product operator and proposed using the product operator. In this case, the problem to be solved becomes:

$$\left.\begin{array}{r} \textit{Maximize } \prod_{i=1}^{k} \mu_i^L(z_i(\mathbf{x})) \\ \textit{subject to: } \mathbf{Ax} \leq \mathbf{b},\ \mathbf{x} \geq \mathbf{0} \end{array}\right\} \tag{4.3.8}$$

"Unfortunately, with the product operator, even if the linear membership functions is used, the objective function of this problem becomes a non-linear function and hence, the Linear Programming Method cannot be applied" (Dantzig, 1961).

"In 1981, by considering the rate of increased membership satisfaction need not always be constant as in the case of the linear membership function proposed by Zimmermann, Leberling introduced special non-linear functions and showed that the resulting Non-Linear Programming problem can be equivalently converted to a conventional Linear Programming problem" (Leberling, 1981).

"As another extension of the linear membership function of Zimmermann, in 1981, Hannan proposed a different approach from Leberling" (Hannan, 1981a). For each of the objective functions of the multi-objective Linear Programming problem, assuming that the DM could specify the degree of membership for several values of $z_i(\mathbf{x})$, he introduced the piecewise linear membership function. "By adopting the piecewise linear membership function to represent the fuzzy goal of the DM for the Multi-Objective Linear Programming problem together with the fuzzy decision of Bellman and Zadeh, the problem to solved can be converted to the ordinary Linear Programming problem" (Bellman and Zadeh, 1970).

However, "suppose that the interaction with the DM establishes that the first membership function should be linear, the second hyperbolic, the third piecewise linear and so forth" (Sakawa et al., 1984). "In such a situation, following the fuzzy decision of Bellman and Zadeh, the resulting problem becomes a Non-linear Programming problem and cannot be solved be by a Linear Programming Method" (Bellman and Zadeh, 1970; Dantzig, 1961).

"In 1983, to quantify the fuzzy goals of the DM by eliciting the corresponding membership functions, Sakawa proposed using five types of membership functions: linear, exponential, hyperbolic, hyperbolic inverse and piecewise linear functions" (Sakawa, 1983). "Through the use of these membership functions including non-linear ones, the fuzzy goals of the DM are quantified. Then following the fuzzy decision of Bellmann and Zadeh, the problem becomes a Non-linear Programming problem" (Bellman and Zadeh, 1970). However, "it can be reduced to a set of linear inequalities if some variable fixed. Based on this idea Sakawa proposed a new method combining the use of the Bisection Method and the Linear Programming Method" (Dantzig, 1961; Sakawa, 1983).

4.4 Fuzzy Multiple Weighted-Objective Linear Programming

It has been proved that $max-min$ operator approach possesses some good properties. However, the efficiency of the solution yielded by $max-min$ operator is not guaranteed. "Li and Zhang developed minimum operator with adding a second phase, named as Two Phase Approach (TPA)" (Li et al., 2006). In the second phase, the purpose is to improve the degrees of memberships by assigning weights to objectives which are obtained from the first phase. The second phase as:

$$\begin{aligned} &Maximize \quad \lambda = \textstyle\sum_{k=1}^{N} w_k \lambda_k \\ &subject\ to{:}\ \ \mu_i^L(z_i(\mathbf{x})) \geq \lambda_k \quad \forall k = 1,2,...,N \\ &\qquad\qquad\ \ \lambda_k \in [0,1] \qquad\qquad \forall k = 1,2,...,N \\ &\qquad\qquad\ \ x \in X,\ \textstyle\sum_{k=1}^{K} w_k = 1,\ w_k \geq 0 \end{aligned} \tag{4.4.1}$$

where w_k is the weight of k^{th} objective and $\mu_i^L(z_i(\mathbf{x}))$ is membership degree of k^{th} objective that is obtained from first phase.

4.5 Literature Review on Fuzzy Multi-Objective Linear Programming

Publications on FMOLP are listed in Table 4.1.

Table 4.1: Literature review on FMOLP.

Author	Year	Title	Type
Kelly and Walker	(1961)	"Management models and industrial applications of linear programming"	Book
Zimmermann	(1976)	"Description and Optimization of Fuzzy Systems"	Article
Siskos and Hubert	(1983)	"Multi-criteria analysis of the impacts of energy alternatives: A survey and a new comparative approach"	Article
Chen and Hwang	(1992a)	"Fuzzy Multiple Attribute Decision Making"	Book
Carlsson and Fullér	(1996)	"Fuzzy multiple criteria decision making: Recent developments"	Article
Triantaphyllou and Lin	(1996)	"Development and evaluation of five fuzzy multiattribute decision-making methods"	Article
Opricovic	(1998)	"Multicriteria Optimization of Civil Engineering Systems"	Phd Thesis
Zopounidis and Doumpos	(2002)	"Multicriteria Decision Aid Classification Methods"	Book
Phruksaphanrat and Ohsato	(2004)	"Linear coordination method for fuzzy multi-objective linear programming problems with convex polyhedral membership functions"	Article
Amid et al.	(2006)	"Fuzzy multi-objective linear model for supplier selection in a supply chain"	Article
Araz et al.	(2007)	"An integrated multicriteria decision-making methodology for outsourcing management"	Article
Onar and Ateş	(2008)	"A fuzzy model for operational supply chain optimization problems"	Article
Tuzkaya et al.	(2010)	"An integrated fuzzy multi-criteria decision making methodology for material handling equipment selection problem and an application"	Article
Wu and Chen	(2011)	"The ELECTRE multicriteria analysis approach based on Atanassov's intuitionistic fuzzy sets"	Article

Author	Year	Title	Type
Trivedi and Singh	(2017)	"A hybrid multi-objective decision model for emergency shelter location-relocation projects using fuzzy analytic hierarchy process and goal programming approach"	Article
Adalı and Işık	(2017)	"The multi-objective decision making methods based on MULTI-MOORA and MOOSRA for the laptop selection problem"	Article
Liu et al.	(2018a)	"Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making"	Article
Liu et al.	(2018b)	"Partitioned Heronian means based on linguistic intuitionistic fuzzy numbers for dealing with multi-attribute group decision making"	Article
Tavana et al.	(2019)	"A fuzzy multi-objective multi-period network DEA model for efficiency measurement in oil refineries"	Article
Kayapınar and Erginel	(2019)	"Designing the airport service with fuzzy QFD based on SERVQUAL integrated with a fuzzy multi-objective decision model"	Article
Tayebikhorami et al.	(2019)	"A fuzzy multi-objective optimization approach for treated wastewater allocation"	Article
Dong and Wan	(2019)	"A new method for solving fuzzy multi-objective linear programming problems"	Article
Stanojević and Stanojević	(2019)	"On Fuzzy Solutions to a Class of Fuzzy Multi-objective Linear Optimization Problems"	Paper
Lua et al.	(2020)	"Integrated forward and reverse logistics network design for a hybrid assembly-recycling system under uncertain return and waste flows: A fuzzy multi-objective programming"	Article
Nguyen et al.	(2020)	"Multi-objective decision-making and optimal sizing of a hybrid renewable energy system to meet the dynamic energy demands of a wastewater treatment plant"	Article

5 PAIRWISE COMPARISON MATRIX

In this chapter, the structure of the pairwise comparison matrix is discussed. Methods of collecting judgments from decision makers, creating comparison matrices and processing are examined.

5.1 Aggregation of Judgments

The results of decisions could impact many people or project in numerous ways. Each of people or project stages could have completely different preferences for the results. Rather than letting a complex decision be made by a single person or group, taking ideas from all individuals or experts who will represent them can reduce the risk. However, moving from one decision maker to a multiple decision maker introduces a good deal of complexness. The matter is not any longer the choice of the foremost most popular various among the non-dominated solutions in step with one individual's preference structure. "The analysis should be extended to account for the conflicts among different interest teams have different objectives, goals, criteria and so on. They sometimes have disagreements among themselves. The disagreements come back from the variations in their subjective evaluations of the choice issues, caused by the differences in data and/or the variations in personal or group objectives, goals and criteria" (Zhou, 1996).

5.2 Construction of Pairwise Comparison Matrix

The pairwise comparison Method has been broadly used to handle the abstract and target judgments about subjective or potentially quantitative criteria in MCDM, particularly in the AHP and ANP. "The inclination relations in the PCMs are filled in by the leader judgments and exhibited utilizing diverse estimation scales, for example, ratio scale, geometric scale and logarithmic scale etc." (Saaty, 1977; Lootsma, 1989; Ishizaka et al., 2010). "The judgments might be conflicting or potentially fragmented in light of the cut-off points of leaders' aptitude and abilities or the unpredictability of the choice issues and different methodologies and models are proposed to deal with these issues" (Benítez et al., 2011; Ergu et al., 2011; Kou et al., 2014). "To assess the level of inconsistency in a PCM, distinctive consistency files have been proposed and thought about" (Brunelli et al., 2013). "The weights of criteria and the positioning of options are frequently judged through the need weights got from a PCM, in this way numerous approaches have been proposed to get the need weights from a PCM" (Kou et al., 2014; Cavallo and D'Apuzzo, 2011).

Saaty proposed two versions of the scale, described in Table 5.1. "The first one is used for objects which are clearly different and uses values from 1 to 9. The other one is used for only slightly different objects, for which most evaluations would concentrate between 1 and 2. In this situation Saaty suggested to use values from the interval $1.1 - 1.9$" (Saaty, 2008) .

Table 5.1: Saaty's fundamental scale (Saaty, 2008).

Intensity of Importance	**Definition of Importance**	**Explanation**
1	"Equal"	"Both activities contribute equally to the objective"
2	"Weak or slight"	"Intermediate importance between 1 and 3"
3	"Moderate"	"Experience and judgment slightly favor activity *i* over *j*"
4	"Moderate plus"	"Intermediate importance between 3 and 5"
5	"Strong"	"Experience and judgment slightly favor activity *i* over *j*"
6	"Strong plus"	"Intermediate importance between 5 and 7"
7	"Very strong or demonstrated"	"Activity *i* is favored very strongly over *j*; its dominance demonstrated in practice"
8	"Very, very strong"	"Intermediate importance between 7 and 9"
9	"Extreme"	"The evidence favoring activity *i* over *j* is of the highest possible order of affirmation"
1.1 - 1.9	"When all compared activities are close: a decimal is added to 1 to show their difference as appropriate"	"A better alternative way of assigning small decimals is to compare two close activities with other widely contrasting ones, favoring the larger one little over the smaller one when using the 1-9 values"
Reciprocals of above	"If activity *i* has one of the above non-zero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*"	"A logical assumptions"

Assume in a group decision making circumstance the group comprises of *m* people and the gather choice issue has n components. In case the PCMs are made independently by each person within the choice gather, m nxn PCMs should be get; each pairwise: comparison framework comes about in one need vector. This need vector comprises of m elements for the need weights and is inferred by utilizing prioritization strategies. "Two concepts have been included. One is the pairwise comparison matrix (A), which is the result of pairwise comparison. Each component in framework A records the relative inclination of one component over another component. The other concept is need vector, which is inferred from network A and records the relative weight of one choice component over the n choice components. With the objective of getting the totaled gather need vector in intellect, the aggregation is based on the person pairwise comparison frameworks inside a group" (Zhou, 1996). The first one is used in this thesis. The aggregation strategies are worked on a group of PCMs. An totaled bunch PCM is gotten from the operation. At that point, the totaled bunch need vector is inferred from the aggregated gather PCM by utilizing the prioritization method.

5.2.1 Representation of Pairwise Comparison Matrix for Group Judgment Aggregation

"Suppose there is a decision group of m people, each of whom has a pairwise comparison matrix A_i defined over n decision elements, where i stands for member i in the group and $i = 1,..,m$. Considering the judgments are made separately by each member, the judgments of the group can be represented by a vector of m-components, where each component is an nxn pairwise comparison matrix. Let $\{A_i\} = (A_1, A_2, ..., A_m)$ be the vector" (Zhou, 1996). Each A_i can be represented as:

$$A_i = \begin{bmatrix} \{a_{11}\}_i & \{a_{12}\}_i & ... & \{a_{1n}\}_i \\ \{a_{21}\}_i & \{a_{22}\}_i & ... & \{a_{2n}\}_i \\ . & . & ... & . \\ . & . & ... & . \\ \{a_{n1}\}_i & \{a_{n2}\}_i & ... & \{a_{nn}\}_i \end{bmatrix} \tag{5.2.1}$$

where $\{A_{jk_i}\}$ denotes a pairwise comparison regarding decision elements j and k $(j,k = 1,2,...,n)$ judged by person i in the group.

5.2.2 Geometric Mean for Aggregating Pairwise Comparison Judgments

"Aczel et al. proposed a functional equation approach to aggregate the ratio judgments" (Aczel and Saaty, 1973; Aczél and Alsina, 1987, 1986). "Let us suppose that the numerical judgments $x_1, x_2, ..., x_m$ given by m people lie in a continuum (interval) P of positive numbers so that P may contain $x_1, x_2, ..., x_m$ as well as their powers, reciprocals and geometric means, etc. The aggregating function $f(\,)$ will map $\{P_m\}$ into a proper interval J and $f(x_1, x_2, ..., x_m)$ will be called the result of the aggregation for the judgments $x_1, x_2, ..., x_m$" (Zhou, 1996). The function $f(\,)$, which should satisfy the Separability condition, Unanimity condition and Reciprocal condition, is the geometric mean as follows:

$$f(x_1, x_2, ..., x_m) = (x_1 x_2 ... x_m)^{\frac{1}{m}} \tag{5.2.2}$$

Now apply this equation to the aggregation problem defined. Since x_i is a ratio judgment, so is $\{a_{jk_i}\}$. Therefore, Equation (5.2.2) can be directly applied to the aggregation problem.

Approach A: "The approach A is to derive $\bar{A}$ from $\{A_i\}$. By applying Equation (5.2.2) to every element of the pairwise comparison matrix $\{A_i\}$" (Cho and Cho, 2008), the following expression is obtained:

$$\bar{A} = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & ... & \bar{a}_{1n} \\ \bar{a}_{21} & \bar{a}_{22} & ... & \bar{a}_{2n} \\ . & . & ... & . \\ . & . & ... & . \\ \bar{a}_{n1} & \bar{a}_{n2} & ... & \bar{a}_{nn} \end{bmatrix} = \begin{bmatrix} (\prod_{i=1}^{m}\{a_{11}\}_i)^{\frac{1}{m}} & (\prod_{i=1}^{m}\{a_{12}\}_i)^{\frac{1}{m}} & ... & (\prod_{i=1}^{m}\{a_{1n}\}_i)^{\frac{1}{m}} \\ (\prod_{i=1}^{m}\{a_{21}\}_i)^{\frac{1}{m}} & (\prod_{i=1}^{m}\{a_{22}\}_i)^{\frac{1}{m}} & ... & (\prod_{i=1}^{m}\{a_{2n}\}_i)^{\frac{1}{m}} \\ . & . & ... & . \\ . & . & ... & . \\ (\prod_{i=1}^{m}\{a_{n1}\}_i)^{\frac{1}{m}} & (\prod_{i=1}^{m}\{a_{n2}\}_i)^{\frac{1}{m}} & ... & (\prod_{i=1}^{m}\{a_{nn}\}_i)^{\frac{1}{m}} \end{bmatrix} \tag{5.2.3}$$

Approach B: "As an alternative to approach A, the aggregated group priority vector $\bar{V}$ can be obtained from the priority vector of each person in the group. The priority vector for each individual V_i of the group is derived from A_i" (Zhou, 1996). Approach B can be summarized as follows:

$$\begin{gathered} V_i = f(A_i) = (\{\upsilon_1\}_i, ..., \{\upsilon_n\}_i),\ i = 1, ..., m \\ \bar{V} = (\bar{\upsilon}_1, \bar{\upsilon}_2, ..., \bar{\upsilon}_n) = ((\prod_{i=1}^{m}\{\upsilon_1\}_i)^{\frac{1}{m}}, (\prod_{i=1}^{m}\{\upsilon_2\}_i)^{\frac{1}{m}}, ..., (\prod_{i=1}^{m}\{\upsilon_n\}_i)^{\frac{1}{m}}) \end{gathered} \tag{5.2.4}$$

5.3 Literature Review on Pairwise Comparison Matrix

Publications on pairwise comparison matrix are listed in Table 5.2.

Table 5.2: Literature review on pairwise comparison matrix.

Author	**Year**	**Title**	**Type**
Saaty	(1980)	"The Analytic Hierarchy Process"	Book
Laarhoven and Pedrycz	(1983)	"A fuzzy extension of Saaty's priority theory"	Article
Buckley	(1985a)	"Fuzzy hierarchical analysis"	Article
Chang	(1996)	"Applications of the extent analysis method on fuzzy AHP"	Article
Deng	(1999)	"Multicriteria analysis with fuzzy pairwise comparison"	Article
Csutora and Buckley	(2001)	"Fuzzy hierarchical analysis: the Lambda-Max method"	Article
Mikhailov	(2003)	"Deriving priorities from fuzzy pairwise comparison judgements"	Article
Choo and Wedley	(2004)	"A common framework for deriving preference values from pairwise comparison matrices"	Article
Zeng et al.	(2007)	"Application of a fuzzy based decision making methodology to construction project risk assessment"	Article
Kahraman and Kaya	(2010)	"A fuzzy multi-criteria methodology for selection among energy alternatives"	Article
Mikhailov	(2014)	"A fuzzy approach to deriving priorities from interval pairwise comparison judgements"	Article
Rezaei et al.	(2014)	"Supplier selection in the airline retail industry using a funnel methodology: Conjunctive screening method and fuzzy AHP"	Article
Ramík	(2015)	"Isomorphisms between fuzzy pairwise comparison matrices"	Article
Kubler et al.	(2017)	"Knowledge-based consistency index for fuzzy pairwise comparison matrices"	Paper
Krejčí	(2017a)	"Additively reciprocal fuzzy pairwise comparison matrices and multiplicative fuzzy priorities"	Article
Krejčí	(2017b)	"Fuzzy eigenvector method for obtaining normalized fuzzy weights from fuzzy pairwise comparison matrices"	Article
Kubler et al.	(2018)	"Measuring inconsistency and deriving priorities from fuzzy pairwise comparison matrices using the knowledge-based consistency index"	Article

Author	Year	Title	Type
Krejčí	(2018)	"Obtaining fuzzy priorities from additive fuzzy pairwise comparison matrices"	Article
Cavallo	(2019)	"Coherent weights for pairwise comparison matrices and a mixed-integer linear programming problem"	Article
Mousavi	(2019)	"A new interval-valued hesitant fuzzy pairwise comparison–compromise solution methodology: an application to cross-docking location planning"	Article
Liu et al.	(2020)	"Deriving priorities from pairwise comparison matrices with a novel consistency index"	Article
Cavallo and D’Apuzzo	(2020)	"Preservation of preferences intensity of an inconsistent Pairwise Comparison Matrix"	Article
Ramik	(2020a)	"Pairwise Comparison Matrices in Decision-Making"	Article
Ramik	(2020b)	"Pairwise Comparisons Matrices with Fuzzy and Intuitionistic Fuzzy Elements in Decision-Making"	Article

6 NEAREST INTERVAL APPROXIMATION METHOD

Ranking fuzzy numbers plays a crucial role in fuzzy decision making problems. So, deriving the final efficiency and powerful ranking are useful. In recent years, "several ranking strategies are introduced by researchers; a number of these ranking strategies have been compared and reviewed by Bortolan and Degani" (Bortolan and Degani, 1985). Some different strategies use statistical techniques such as simulation and hypothesis and quadratic fuzzy regression. "Yager and Filev planned a ranking methodology with parameterized valuation functions" (Yager, 1981; Yager and Filev, 1994). "Tran and Duckstein proposed a weight operate that represents the decision maker's attitude" (Tran and Duckstein, 2002). "Asady and Zendenam planned the ranking of fuzzy numbers by sign distance" (Asady and Zendenam, 2007). "Grzegorzewski suggested a new interval approximation operator, which is the best one with respect to a certain measure of distance between fuzzy numbers" (Grzegorzewski, 2002).

6.1 Nearest Interval Approximation of Triangular and Trapezoidal Fuzzy Numbers

Definition 1: "A fuzzy number $\widetilde{A} = (a,b,c)$ is called a triangular fuzzy number if its membership function $\widetilde{A}(x)$ has the following form" (Rani et al., 2016):

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \end{cases} \tag{6.1.1}$$

Definition 2: "A fuzzy number $\widetilde{A} = (a,b,c,d)$ is called a trapezoidal fuzzy number if its membership function $\widetilde{A}(x)$ has the following form" (Rani et al., 2016) :

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \dfrac{d-x}{d-c}, & c \leq x \leq d \end{cases} \tag{6.1.2}$$

Definition 3: "For two arbitrary fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ with $\gamma-cuts$ $[\widetilde{A}]_\gamma = [\underline{a}(\gamma), \bar{a}(\gamma)]$ and $[\widetilde{B}]_\gamma = [\underline{b}(\gamma), \bar{b}(\gamma)]$ respectively, the quantity" (Saeidifar, 2011):

$$d(\widetilde{A}, \widetilde{B}) = \left[\int_0^1 (\underline{a}(\gamma) - \underline{b}(\gamma) + \int_0^1 (\bar{a}(\gamma) - \bar{b}(\gamma))^2 d\gamma\right]^{\frac{1}{2}} \tag{6.1.3}$$

Definition 4: "Let $\widetilde{A}$ be a fuzzy number with $[\widetilde{A}]_\gamma = [\underline{a}(\gamma), \bar{a}(\gamma)]$ and $f(\gamma) = (\underline{f}(\gamma), \bar{f}(\gamma))$ be a weighting function.The nearest $f = (\underline{f}, \bar{f})$ weighted interval approximation of $\widetilde{A}$ is defined as

$$NWIA_f(\widetilde{A}) = [C_L^f, C_U^f] = \left[\int_0^1 \underline{f}(\gamma)\underline{a}(\gamma)d\gamma, \int_0^1 \bar{f}(\gamma)\bar{a}(\gamma)d\gamma\right] \tag{6.1.4}$$

where C_L^f is the nearest lower weighted point approximation $NLWPA_{\underline{f}}(\widetilde{A})$ and C_U^f is the nearest upper weighted point approximation $NUWPA_{\bar{f}}(\widetilde{A})$ of fuzzy number $\widetilde{A}$" (Saeidifar, 2011).

Theorem 1: "Let $\widetilde{A}$ be a fuzzy number with $[\widetilde{A}]_\gamma = [\underline{a}(\gamma), \bar{a}(\gamma)]$ and $f(\gamma) = (\underline{f}(\gamma), \bar{f}(\gamma))$ be a weighting function. Then, the interval $NUWPA_{\bar{f}}(\widetilde{A}) = [NLWPA_{\underline{f}}(\widetilde{A}), NUWPA_{\bar{f}}(\widetilde{A})]$ is the nearest weighted interval approximation to fuzzy number $\widetilde{A}$" (Izadikhah, 2012).

Definition 5: "Let $\widetilde{A}$ be a fuzzy number with $[\widetilde{A}]_\gamma = [\underline{a}(\gamma), \bar{a}(\gamma)]$ and $f(\gamma) = (\underline{f}(\gamma), \bar{f}(\gamma))$ be a weighting function" (Saeidifar, 2011). The f-weighted mean of $\widetilde{A}$ is defined as

$$\bar{M}_f(\widetilde{A}) = \int_0^1 \frac{\underline{f}(\gamma)\underline{a}(\gamma) + \bar{f}(\gamma)\bar{a}(\gamma)}{2} d\gamma \tag{6.1.5}$$

In fact, $\bar{M}_f(\widetilde{A})$ is the weighting mean of fuzzy number $\widetilde{A}$

$$\begin{aligned} \bar{M}_f(\widetilde{A}) &= \frac{1}{2}\int_0^1 (\underline{f}(\gamma)\underline{a}(\gamma) + \bar{f}(\gamma)\bar{a}(\gamma))d\gamma \\ &= \frac{\int_0^1 \underline{f}(\gamma)\underline{a}(\gamma)d\gamma + \int_0^1 \bar{f}(\gamma)\bar{a}(\gamma)d\gamma}{\int_0^1 \underline{f}(\gamma)d\gamma + \int_0^1 \bar{f}(\gamma)d\gamma} \end{aligned} \tag{6.1.6}$$

Therefore, the following theorems is obtained.

Theorem 2: "Let $\widetilde{A}$ be a fuzzy number with $[\widetilde{A}]_\gamma = [\underline{a}(\gamma), \bar{a}(\gamma)]$ and $f(\gamma) = (\underline{f}(\gamma), \bar{f}(\gamma))$ be a weighting function.Then $\bar{M}_f(\widetilde{A})$ is the nearest f-weighted point approximation to fuzzy number $\widetilde{A}$ which is unique" (Saeidifar, 2011).

Theorem 2 shows that "$\bar{M}_f(\widetilde{A})$ is the nearest f-weighted point approximation to fuzzy number $\widetilde{A}$ which is unique. The nearest f-weighted point to fuzzy number $\widetilde{A}$ belongs to support function and this theorem is a new and interesting justification for the definition of the weighted mean of a fuzzy number" (Saeidifar, 2011).

Theorem 3: "Let $\widetilde{A},\widetilde{B}$ be two fuzzy numbers, let $f(\gamma) = (\underline{f}(\gamma),\bar{f}(\gamma))$ be a weighting function and let $\lambda \in \mathbb{R}$"$(Saeidifar,\ 2011)$. Then,

$$NLWPA_{\underline{f}}(\widetilde{A}+\widetilde{B}) = NLWPA_f(\widetilde{A}) + NLWPA_f(\widetilde{B})$$

$$NLWPA_{\underline{f}}(\lambda\widetilde{A}) = \lambda NLWPA_{\underline{f}}(\widetilde{A})$$

$$\bar{M}_f(\widetilde{A}+\widetilde{B}) = \bar{M}_f(\widetilde{A}) + \bar{M}_f(\widetilde{B})$$

$$\bar{M}_f(\lambda\widetilde{A}) = \lambda\bar{M}_f(\widetilde{A})$$

Corollary 1: "Let $\widetilde{A} = (a,b,c,d)$ be a trapezoidal fuzzy number and let $f(\gamma) = (\underline{f}(\gamma),\bar{f}(\gamma))$ be a weighting function" (Saeidifar, 2011). Then,

1. For $f(\gamma) = (1,1)$,

$$NWIA_f(\widetilde{A}) = \left[\frac{a+b}{2},\frac{c+d}{2}\right] \qquad \bar{M}_f(\widetilde{A}) = \frac{a+b+c+d}{4} \tag{6.1.7}$$

2. For $f(\gamma) = (2\gamma,2\gamma)$,

$$NWIA_f(\widetilde{A}) = \left[\frac{a+2b}{3},\frac{2c+d}{3}\right] \qquad \bar{M}_f(\widetilde{A}) = \frac{a+2(b+c)+d}{6} \tag{6.1.8}$$

3. For $f(\gamma) = (n\gamma^{n-1},n\gamma^{n-1}), n \in \mathbb{N}$

$$NWIA_f(\widetilde{A}) = \left[\frac{a+nb}{n+1},\frac{nc+d}{n+1}\right] \qquad \bar{M}_f(\widetilde{A}) = \frac{a+n(b+c)+d}{2n+2} \tag{6.1.9}$$

4. For $f(\gamma) = (n\gamma^{n-1},m\gamma^{m-1}), n,m \in \mathbb{N}$

$$NWIA_f(\widetilde{A}) = \left[\frac{a+nb}{n+1},\frac{mc+d}{m+1}\right] \qquad \bar{M}_f(\widetilde{A}) = \frac{a+nb}{2n+2} + \frac{d+mc}{2m+2} \tag{6.1.10}$$

Example 1: "Let $\tilde{A} = (3,7,8,13)$ be a trapezoidal fuzzy number and also let $f_1(\alpha) = (2\alpha, 2\alpha)$ and $f_2(\alpha) = (4\alpha^3, 4\alpha^3)$ be two weighting functions" (Izadikhah, 2012). Then the nearest weighted intervals to $\tilde{A}$ is as follows:

$$NWIA_f(\widetilde{A}) = \left[\frac{17}{3}, \frac{29}{3}\right] \quad NWIA_f(\widetilde{A}) = \left[\frac{31}{5}, 9\right]$$

The graphical representation is given in Figure 6.1.

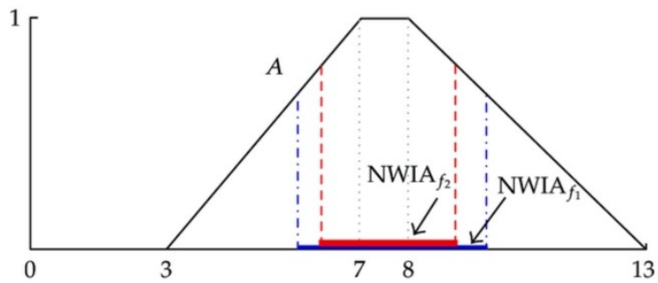


Figure 6.1: Trapezoidal fuzzy number and its interval approximation.

Note: Set b = c to use these equations in triangular fuzzy numbers.

Example 2: "Let $\widetilde{A} = (3,4,7)$ be a triangular fuzzy number and also let $f_1(\alpha) = (2\alpha, 2\alpha)$ and $f_2(\alpha) = (4\alpha^3, 4\alpha^3)$ be two weighting functions" (Izadikhah, 2012). Then the nearest weighted intervals to $\tilde{A}$ is as follows:

$$NWIA_f(\widetilde{A}) = \left[\frac{11}{3}, 5\right] \quad NWIA_f(\widetilde{A}) = \left[\frac{19}{5}, \frac{23}{5}\right]$$

The graphical representation is given in Figure 6.2.

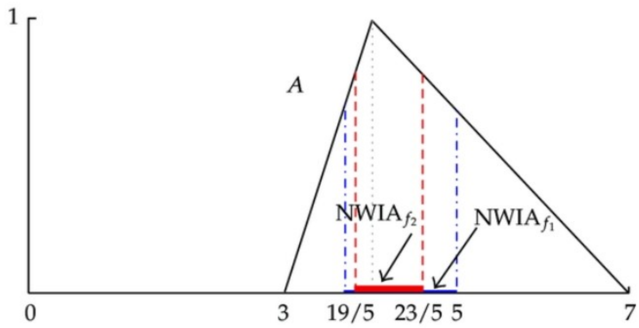


Figure 6.2: Triangular fuzzy number and its interval approximation.

Corollary 2: "Let $\widetilde{A} = (a,b,c,d)$ be a trapezoidal fuzzy number and let $f(\gamma) = (n\gamma^{n-1}, m\gamma^{m-1})$, $n,m \in \mathbb{N}$ be a weighting function" (Izadikhah, 2012). Then, for $m,n \to \infty$

$$NWIA_f(\widetilde{A}) = \left[\frac{a+nb}{n+1}, \frac{mc+d}{m+1}\right] \to [b,c] \qquad \bar{M}_f(\widetilde{A}) \to \frac{b+c}{2} \tag{6.1.11}$$

"The above corollary shows that, for large values m and n, the interval $[b,c]$ and the point $\frac{b+c}{2}$ are the nearest weighted interval and point to the trapezoidal fuzzy number" (Saeidifar, 2011), respectively as in Figure 6.3.

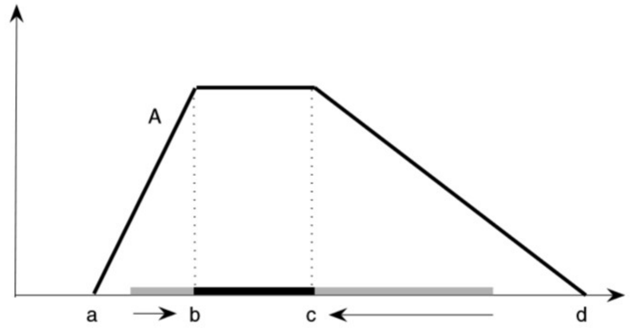


Figure 6.3: Fuzzy number $\widetilde{A}$ and its intervals.

Hereby, ranking method by the weighting mean of a fuzzy number is as follows:

Definition 6: "For two fuzzy numbers $\widetilde{A}, \widetilde{B}$ and the weighting function f, the ranking of A and B by $\bar{M}_f(\widetilde{A})$ is defined" (Saeidifar, 2011), i.e.,

$$\begin{aligned}
\bar{M}_f(\widetilde{A}) > \bar{M}_f(\widetilde{B}) \quad &\text{if and only if} \quad \bar{M}_f(\widetilde{A}) \succ \bar{M}_f(\widetilde{B}) \\
\bar{M}_f(\widetilde{A}) < \bar{M}_f(\widetilde{B}) \quad &\text{if and only if} \quad \bar{M}_f(\widetilde{A}) \prec \bar{M}_f(\widetilde{B}) \\
\bar{M}_f(\widetilde{A}) = \bar{M}_f(\widetilde{B}) \quad &\text{if and only if} \quad \bar{M}_f(\widetilde{A}) \sim \bar{M}_f(\widetilde{B})
\end{aligned}$$

Then, the order $\preceq$ and $\succeq$ are formulated as:

$$\begin{aligned}
\widetilde{A} \preceq \widetilde{B} \quad &\text{if and only if} \quad \widetilde{A} \prec \widetilde{B} \quad \text{or} \quad \widetilde{A} \sim \widetilde{B} \\
\widetilde{A} \succeq \widetilde{B} \quad &\text{if and only if} \quad \widetilde{A} \succ \widetilde{B} \quad \text{or} \quad \widetilde{A} \sim \widetilde{B}
\end{aligned}$$

6.2 Literature Review on Nearest Interval Approximation

Publications on nearest interval approximation are listed in Table 6.1.

Table 6.1: Literature review on nearest interval approximation.

Author	Year	Title	Type
Chanas	(2001)	"On the interval approximation of a fuzzy number"	Article
Grzegorzewski	(2002)	"Nearest interval approximation of a fuzzy number"	Article
Abbasbandy and Asady	(2004)	"The nearest trapezoidal fuzzy number to a fuzzy quantity"	Article
Ban	(2006)	"Nearest Interval Approximation of an Intuitionistic Fuzzy Number"	Article
Abbasbandy and Amirfakhrian	(2006a)	"The nearest approximation of a fuzzy quantity in parametric form"	Article
Abbasbandy and Amirfakhrian	(2006b)	"The nearest trapezoidal form of a generalized left right fuzzy number"	Article
Nasibov and Peker	(2008)	"On the nearest parametric approximation of a fuzzy number"	Article
Ban and Coroianu	(2012)	"Nearest interval, triangular and trapezoidal approximation of a fuzzy number preserving ambiguity"	Article
Coroianu et al.	(2013)	"Nearest piecewise linear approximation of fuzzy numbers"	Article
Izadikhah et al.	(2014)	"Extending TOPSIS in fuzzy environment by using the nearest weighted interval approximation of fuzzy numbers"	Article
Ren and Wang	(2017)	"An approach for solving a fuzzy bilevel programming problem through nearest interval approximation approach and KKT optimality conditions"	Article
Ahmadian et al.	(2016)	"Nearest Interval-Valued Approximation of Interval-Valued Fuzzy Numbers"	Article
Sharma and Aggarwal	(2018)	"Solving Fully Fuzzy Multi-objective Linear Programming Problem Using Nearest Interval Approximation of Fuzzy Number and Interval Programming"	Article
Ren and Luo	(2020)	"A novel distance of intuitionistic trapezoidal fuzzy numbers and its-based prospect theory algorithm in multi-attribute decision making model"	Article
Li et al.	(2020)	"On product of positive L-R fuzzy numbers and its application to multi-period portfolio selection problems"	Article

7 GOAL PROGRAMMING

Goal programming (GP) is a method that is applied to linear programming problems that have many goals or objectives. Decision-makers are required to rank their objectives by their importance and to set a target value that they want to achieve for each goal. Then, "the appropriate solutions are found by minimizing the deviations between the actual values and the target values. GP is used to manage conflicting goals" (Leung et al., 2003). "It is an important technique for decision making problems where the decision maker aims to minimize the deviation between the achievement of goals and their aspiration levels" (Azmi and Tamiz, 2010). In other words, "it is the most widely used multi-objective technique in management science because of its inherent flexibility in handling decision-making problems with several conflicting objectives and incomplete or imprecise information" (Romero, 1991, 2004; Chang, 2007). When uncertain goals exist, the fuzzy GP, which is implemented by applying the fuzzy set theory to goal programming, is used.

"The GP technique was first described by Charnes and is described as a version of the Linear Programming technique" (Charnes, 1955). It was also defined as a technique for optimizing the objective function, under certain constraint equations, as close as possible to the goals. "It has been shown that Non-solution Linear Programming problems can be solved by setting the deviations (positive, negative, or both) that occur in each objective function and minimizing them by putting these deviation variables into the achievement function" (Charnes and Cooper, 1961). In addition, "prioritization and weighting methodologies of the GP are included. In 1965, Ijiri has made the GP method more usable than the existing solutions" (Ijiri, 1965). It was aimed to make it easier to identify, measure, weight and prioritize sub-objectives by dividing the main management objectives into related sub-objectives. "The GP method's weighting and prioritization methodologies are then combined to formulate one or more formulations with one or more sub-goals" (Wu and Coppins, 1981). "In 1968, Contini adapted GP to uncertainty situations and Jääskeläinen adapted it to mass production planning" (Contini, 1968; Jääskeläinen, 1976). "A lot of application area have been accomplished through Lee's book "Goal Programming For Decision Analysis" and the computer programs developed" (Lee, 1972; Lee and

Jääskeläinen, 1971). Therefore, it is stated that the generalized GP technique is a practical and healthy tool for modeling and solving multi-objective mathematical programming problems.

7.1 Structure of Goal Programming

- **Objectives** can be defined as the way in which the criteria are oriented in the direction of the decision makers' desires (Evren and Ülengin, 1992). In other words; the goals express the results that decision-makers want to achieve from the solution of the problem. The objectives may be in the same direction, or in opposite directions.
- In a decision making process, the system of interest is carefully observed and parameters are determined that can control their value and affect the performance of the system. These parameters are under the control of the managers and are called **decision variables** (Koçak, 2007). Decision variables are unknown variables whose value is to be determined.
- **Deviation variables** that indicate how above or below the specified target is reached. The deviation of i from the goal in a positive direction is indicated as d_i^+; the deviation of i from the goal in a negative direction is indicated as d_i^-. At least one of the two deviation variables will be zero so that both positive and negative deviations can not occur at the same time.
- Technological, structural or **system constraints** are constraints that are developed for the probing and which must be fully provided in the GP models and that are not allowed to deviate (Öztürk, 2007).
- **Goal constraints** that the decision maker wants or needs to reach are passed as goal constraints to the GP model. These constraints have a more flexible structure than system constraints (Alp, 2008).
- The functions that have the smallest deviations from the goal of each objective are called **success functions** (Alp, 2008).

- **The objective function** is the function obtained by weighing all successive functions in the model or by writing their precedence levels together and their general summation.

7.2 Formulation of Goal Programming

Goal programming is one of the first techniques specially designed for solving Multi-Objective Optimization problems. In this method, the decision maker has to set goals for each objective that is desired to be achieved. These values include probing as additional constraints. Then, the absolute deviations of the objective function from the specified objective are tried to be minimized.

$$\begin{aligned} Min\ Z &= \sum_{k=1}^{q}\sum_{i=1}^{m} P_k + (d_i^- + d_i^+) \\ &\sum_{j=1}^{n} a_{ij}x_j + d_i^- - d_i^+ = b_i \quad i = 1,2,...,m \\ &x_j, d_i^-, d_i^+ \geq 0 \qquad\qquad i = 1,2,...,m,\ j = 1,2,...,n \end{aligned} \tag{7.2.1}$$

7.3 Weighted Goal Programming

"In the weighted GP technique, a single objective function is transformed into the weighted sum of the functions representing the goals of the problem" (Rossdy, 2010). Suppose that the goal i of a GP model with n goals is given as follows:

$$Min\ G_i,\ i = 1,2...,n \tag{7.3.1}$$

The combined objective function used in the weighting method

$$Min\ z = w_1G_1 + w_2G_2 + ... + w_nG_n \tag{7.3.2}$$

where $i = 1,2,...,n$ are positive weights that reflect the decision maker's preferences regarding the relative importance of each goal. "For example, for all i, w_i = 1 indicates that all goals have equal weight. The determination of the specific values of these weights is a subjective matter" (Rossdy, 2010). As a matter of fact, "the complex sophisticated analytic procedures developed in the literature have always been based on subjective evaluations" (Taha, 2000). In the following years, "some researchers have studied the fuzzy priorities of problem formulation and fuzzy goals in the field of fuzzy GP and they have developed solution proposals for them" (Tiwari et al., 1987). Most of the researchers have used minimization operators in fuzzy GP formulations to reach fuzzy decisions that imply fuzzy goals and constraints with their maximum memberships values.

7.4 Literature Review on Goal Programming

Publications on Goal programming are listed in Table 7.1.

Table 7.1: Literature review on Goal programming.

Author	**Year**	**Title**	**Type**
Dantzig	(1948)	"Programming in a Linear Structure"	Paper
Simon	(1957)	"Models of Man"	Book
Charnes and Cooper	(1961)	"Management Models and the Industrial Applications of Linear Programming"	Book
Lee	(1972)	"Goal Programming for Decision Analysis"	Book
Yaghoobi and Tamiz	(1979)	"A short note on the relationship between goal programming and fuzzy programming for vectormaximum problems"	Article
Lin	(1980)	"A Survey of Goal Programming Applications"	Article
Hannan	(1981b)	"On fuzzy goal programming"	Article
Schniederjans and Kwak	(1982)	"An Alternative Method for Solving Goal Programming Problems: A Reply"	Article
Markowski and Ignizio	(1983)	"Theory and Properties of the Lexicographic Linear Goal Programming"	Article
Ignizio	(1985)	"An Algorithm for Solving the Linear Goal-Programming Problem by Solving Its Dual"	Article
Crowder and Sposito	(1987)	"Comments on "An Algorithm for Solving the Linear Goal-Programming Problem by Solving Its Dual""	Article
Rifai	(1996)	A "Note on the Structure of the Goal-Programming Model: Assessment and Evaluation"	Article

Author	Year	Title	Type
Kuwano	(1996)	"On the fuzzy multi-objective linear programming problem: Goal programming approach"	Article
Tamiz and Jones	(1996)	"Goal Programming and Pareto Efficiency"	Article
Parra et al.	(2001)	"A fuzzy goal programming approach to portfolio selection"	Article
Yaghoobi and Tamiz	(2006)	"On improving a weighted additive model for fuzzy goal programming problems"	Article
Winston	(2004)	"Operations Research: Applications and Algorithms"	Book
Sharma and Sharma	(2006)	A "Multi-Objective Decision-Making Approach For Mutual Fund Portfolio"	Article
El-Wahed et al.	(2006)	"Interactive fuzzy goal programming for multi-objective transportation problems"	Article
Hu et al.	(2007)	"A fuzzy goal programming approach to multi-objective optimization problem with priorities"	Article
Jafari et al.	(2008)	"An Optimal Model using Goal Programming for Rice Farm"	Article
Tamiz and Jones	(2010)	" Practical Goal Programming. International Series in Operations Research & Management Science"	Book
Orumie and Ebong	(2011)	"An Alternative Method of Solving Goal Programming" Problem	Article
Alp et al.	(2011)	"Using Linear Goal Programming in Surveying Engineering for Vertical Network Adjustment"	Article
Nabendu and Manish	(2012)	"A Goal Programming Approach to Rubber Plantation Planning in Tripura"	Article
Jayaraman et al.	(2017)	"A fuzzy goal programming model to analyze energy, environmental and sustainability goals of the United Arab Emirates"	Article
Pandey et al.	(2017)	"A fuzzy goal programming approach for selecting sustainable suppliers"	Article
Mokhtari and Hasani	(2017)	"A multi-objective model for cleaner production-transportation planning in manufacturing plants via fuzzy goal programming"	Article
Hossain and Hossain	(2018)	"Application of interactive fuzzy goal programming for multi-objective integrated production and distribution planning"	Article
Subali et al.	(2018)	"Time and cost optimization using fuzzy goal programming"	Paper

Author	Year	Title	Type
Zamanian et al.	(2019)	"A Fuzzy Goal-Programming Model for Optimization of Sustainable Supply Chain by Focusing on the Environmental and Economic Costs and Revenue: A Case Study"	Article
Anukokila et al.	(2019)	"Goal programming approach for solving multi-objective fractional transportation problem with fuzzy parameters"	Article
Javid et al.	(2020)	"Multi-objective flexibility-complexity trade-off problem in batch production systems using fuzzy goal programming"	Article
Arani and Sadegheih	(2020)	"A Fuzzy Goal Programming for Dynamic Cell Formation and Production Planning Problem Together with Pricing and Advertising Decisions"	Article
Sun	(2020)	"Green and Reliable Freight Routing Problem in the Road-Rail Intermodal Transportation Network with Uncertain Parameters: A Fuzzy Goal Programming Approach"	Article

8 PROPOSED MODEL

In this section, a new approach to existing models is introduced in the light of literature review. The features added to the existing models are explained in the relevant sections.

8.1 Fuzzy Multi-Objective Linear Programming (FMOLP)

For the project management problem, a fuzzy multi-objective linear programming model is proposed (Kang et al., 2016). The steps are as follows:

8.1.1 Multi-Objective Linear Programming Model

By constructing balance between the time and the cost, a compromise project implementation plan is found. 3 objectives are considered: least total project cost, shortest total project period and longest total crash time.

First Objective Function: Minimize the total project cost TC.

$$Min\ TC = \sum_i \sum_j K_{D_{ij}} + \sum_i \sum_j Y_{ij} s_{ij} + [l * Max\{0, E_N - TP\}] \tag{8.1.1}$$

Equation (8.1.1) is the first objective function. The purpose is to minimize the total cost (TC) to be spent to complete the project. This cost includes the direct cost ($K_{D_{ij}}$), the crashing cost (s_{ij}) and the penalty cost (l) (i.e. the cost to be incurred if the project is delayed during the normal period).

Second Objective Function: Minimize the total project duration time TP.

$$Min\ TP = E_N - E_1 \tag{8.1.2}$$

Equation (8.1.2) is the second objective function. This equation indicates the difference between the start and end times of the project, i.e. the total duration of the project.

Third Objective Function: Maximize the total crash time TR.

$$Max\ TR = \sum_i \sum_j Y_{ij} \tag{8.1.3}$$

Equation (8.1.3) is the third objective function. This equation gives the sum of the crash times of each activity in the project.

The constraints are:

$$E_i + T_{ij} \leq E_j \quad \forall i, j \tag{8.1.4}$$

$$T_{ij} + Y_{ij} = D_{ij} \quad \forall i, j \tag{8.1.5}$$

$$Y_{ij} + d_{ij} \leq D_{ij} \quad \forall i, j \tag{8.1.6}$$

$$E_1 = 0 \tag{8.1.7}$$

$$E_N \leq F \tag{8.1.8}$$

and all variables are non-negative.

Equation (8.1.4) shows that the sum of the start time (E_i) and the process time (T_{ij}) of activity i should be equal to or smaller than the start time (E_j) of activity j.

Equation (8.1.5) is the sum of the process time (T_{ij}) and the crashing time (Y_{ij}) between i and j activities equal to the planned normal duration (D_{ij}) of the (i, j) node.

Equation (8.1.6) shows that the sum of the crashing time (Y_{ij}) and the shortest possible process time (d_{ij}) of the node could be smaller than the planned normal process time (D_{ij}) of the node. That is, the process time (D_{ij}) of a node should not go below the shortest specified process time (d_{ij}).

Equation (8.1.7) assumes that the initial moment of the first activity (E_1) is zero.

Equation (8.1.8) ensures that the last activity is completed before or on the required project completion time (F).

8.1.2 Positive and Negative Ideal Solutions

Calculate the positive ideal solution (PIS) and the negative ideal solution (NIS) of the three objective functions.

$$TC^{PIS} = Min\ TC \quad , \quad TC^{NIS} = Max\ TC \tag{8.1.9}$$

Equation (8.1.9) assumes that positive ideal solution of total cost is minimization of total cost and negative ideal solution of total cost is maximization of total cost.

$$TP^{PIS} = Min\, TP \quad , \quad TP^{NIS} = Max\, TP \tag{8.1.10}$$

Equation (8.1.10) assumes that positive ideal solution of total time is minimization of total time and negative ideal solution of total time is maximization of total time.

$$TR^{PIS} = Max\, TR \quad , \quad TR^{NIS} = Min\, TP \tag{8.1.11}$$

Equation (8.1.11) assumes that positive ideal solution of total crash time is maximization of total crash time and negative ideal solution of total crash time is minimization of total crash time.

So, the membership function for each of the three objective functions is established as follows:

$$\lambda_{TC} = \begin{cases} 1, & TC \leq TC^{PIS} \\ \dfrac{TC^{NIS} - TC}{TC^{NIS} - TC^{PIS}}, & TC^{PIS} < TC \leq TC^{NIS} \\ 0, & TC > TC^{NIS} \end{cases} \tag{8.1.12}$$

$$\lambda_{TP} = \begin{cases} 1, & TP \leq TP^{PIS} \\ \dfrac{TP^{NIS} - TP}{TP^{NIS} - TP^{PIS}}, & TP^{PIS} < TP \leq TP^{NIS} \\ 0, & TP > TP^{NIS} \end{cases} \tag{8.1.13}$$

$$\lambda_{TR} = \begin{cases} 1, & TR \geq TR^{PIS} \\ \dfrac{TR - TR^{PIS}}{TR^{NIS} - TR^{PIS}}, & TR^{NIS} \leq TR < TR^{PIS} \\ 0, & TR < TR^{NIS} \end{cases} \tag{8.1.14}$$

8.1.3 Fuzzy Programming Method

After the membership function values are obtained, apply "the fuzzy programming method proposed by Zimmermann to add auxiliary variable λ, which might take into account the three objective functions at the same time" (Zimmermann, 1978). The initial FMOLP problem is remodeled into a crisp single-goal linear programming problem. By maximizing λ, a compromise resolution is obtained.

$$Max\ \lambda \tag{8.1.15}$$

subject to:

$$\lambda \leq \lambda_{TP} \tag{8.1.16}$$

$$\lambda \leq \lambda_{TC} \tag{8.1.17}$$

$$\lambda \leq \lambda_{TR} \tag{8.1.18}$$

$$E_i + T_{ij} \leq E_j \quad \forall i, j \tag{8.1.19}$$

$$T_{ij} + Y_{ij} = D_{ij} \quad \forall i, j \tag{8.1.20}$$

$$Y_{ij} + d_{ij} \leq D_{ij} \quad \forall i, j \tag{8.1.21}$$

$$E_1 = 0 \tag{8.1.22}$$

$$E_N \leq F \tag{8.1.23}$$

and all variables are non-negative.

The time constraints in Eq(8.1.19), Eq(8.1.20), Eq(8.1.21) and Eq(8.1.23) are accepted to be crisp numbers.

8.1.4 FMOLP Model Result

The results include the whole project cost, the whole project period time and also the total crash time and therefore, the three objectives are satisfied.

8.2 Fuzzy Multiple Weighted-Objective Linear Programming (FMWOLP)

The decision makers could take into account that the importance of each objective is completely different. That is, some objectives are additional vital than others. First, the weights of the objectives should be determined by the management. A FMWOLP model is made for project management. The relative weights of the multiple objectives are calculated by the nearest interval approximation and goal programming and with using obtained weights, the FMOLP will be created to reach best TC, TP, TR values.

A importance level should be determined for each of the objectives in the project. For this purpose, the project experts are asked for their views. First of all, the experts should be selected and then they are asked to evaluate the objectives in accordance with the linear programming model. A PCM is used for this. The decision makers are expected to compare the objectives in pairs. Fuzzy numbers are used in comparison because, decision-makers may not be able to make a definite decision, or they may not be able to make certain rankings (Zimmermann, 1978).

A PCM where all its elements are fuzzy numbers is considered as follows:

$$\tilde{A} = \begin{bmatrix} (a_{11}^L, a_{11}^M, a_{11}^U) & (a_{12}^L, a_{12}^M, a_{12}^U) & (a_{13}^L, a_{13}^M, a_{13}^U) \\ (a_{21}^L, a_{21}^M, a_{21}^U) & (a_{22}^L, a_{22}^M, a_{22}^U) & (a_{23}^1, a_{23}^2, a_{23}^3, a_{23}^4) \\ (a_{31}^L, a_{31}^M, a_{31}^U) & (a_{32}^1, a_{32}^2, a_{32}^3, a_{32}^4) & (a_{33}^L, a_{33}^M, a_{33}^U) \end{bmatrix} \tag{8.2.1}$$

where $\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^U)$ is a triangular fuzzy number, $\tilde{a}_{ij} = (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4)$ is a trapezoidal fuzzy number. $\tilde{A}$ is reciprocal, if the following conditions are satisfied (Chen and Hwang, 1992b; Ramík and Korviny, 2011).

$$\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^U) \quad implies \quad \tilde{a}_{ji} = (\frac{1}{a_{ij}^U}, \frac{1}{a_{ij}^M}, \frac{1}{a_{ij}^L}) \quad \forall i, j = 1, ..., n. \tag{8.2.2}$$

$$\tilde{a}_{ij} = (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4) \quad implies \quad \tilde{a}_{ji} = (\frac{1}{a_{ij}^4}, \frac{1}{a_{ij}^3}, \frac{1}{a_{ij}^2}, \frac{1}{a_{ij}^1}) \quad \forall i, j = 1, ..., n. \tag{8.2.3}$$

In this study, the numbers suggested by the Saaty (Section 5.2) are expanded. A comparison matrix consisting of triangular-trapezoidal fuzzy numbers can be used, if decision makers choose a value between the fuzzy values. In other words, there is no obligation to create a decision matrix consisting only of triangular fuzzy numbers. For this, this study suggests using the characteristics in Table 8.1.

Table 8.1: Characteristic function of the fuzzy numbers.

Fuzzy Number	Characteristic Function	
$\tilde{1}$	(1, 1, 1)	
$\tilde{x}$	$(x-1,\ x,\ x+1)$	$for\ x = 2, ...8$
$\tilde{x}.5$	$(x-1.5,\ x-0.5,\ x+0.5,\ x+1.5)$	$for\ x = 2, ...8$
$\tilde{9}$	(7, 9, 9)	

8.2.1 Integrated Fuzzy Pairwise Comparison Matrix

By collecting the fuzzy judgment matrices from all decision makers, "this matrices can be aggregated by using the fuzzy geometric mean method of Buckley" (Buckley, 1985b,a). The aggregated triangular fuzzy numbers of n decision makers' judgment in an certain case $\tilde{u}_{ij} = (l_{ij},\ m_{ij},\ u_{ij})$ is

$$\tilde{u}_{ij} = \left(\prod_{i=1}^{n} \tilde{a}_{ijk} \right)^{\frac{1}{n}} \tag{8.2.4}$$

where $\tilde{a}_{ijk}$ is the relative importance in form of triangular fuzzy numbers of the k^{th} decision makers' view and n is the total number of decision makers.

8.2.2 Importance Weights Using The Nearest Interval Approximation and Goal Programming

- **The Nearest Interval Approximation** "The nearest weighted possibilistic interval approximation is an interval operator of a fuzzy number. First, an f-weighted distance quantity on the fuzzy numbers is introduced and then the Interval Approximations for a fuzzy number is obtained" (Izadikhah, 2012).

  Definition 7: "A weighting function is a function as $f = (\underline{f}, \bar{f}) : ([0,1],[0,1]) \rightarrow (R,R)$ such that the functions are non-negative, monotone increasing and satisfies the following normalization condition" (Saeidifar, 2011): $\int_0^1 \underline{f}(\alpha)d\alpha = \int_0^1 \bar{f}(\alpha)d\alpha = 1$

  Definition 8: "Let $\tilde{A}$ be a fuzzy number with $\widetilde{A}_\alpha = [\underline{a}(\alpha), \bar{a}(\alpha)]$ and $f(\alpha) = (\underline{f}(\alpha), \bar{f}(\alpha))$

being a weighted function" (Saeidifar, 2011). Then the interval

$$NWIA_f(\widetilde{A}) = \int_0^1 \underline{f}(\alpha)\underline{a}(\alpha)d\alpha = \int_0^1 \bar{f}(\alpha)\bar{a}(\alpha)d\alpha \tag{8.2.5}$$

is the nearest weighted interval approximation to fuzzy number $\tilde{A}$.

"The function $f(\alpha)$ can be understood as the weight of the interval approximation; the property of monotone increasing of function $f(\alpha)$ means that the higher the cut level is, the more important its weight is in determining the interval approximation of fuzzy numbers" (Izadikhah, 2012). In applications, the function $f(\alpha)$ can be chosen according to the actual situation.

Corollary 3: "Let $\tilde{A} = (a,b,c)$ be a triangular fuzzy number and let $f(\alpha) = (n\alpha^{n-1}, n\alpha^{n-1})$ be a weighting function" (Izadikhah, 2012) . Then,

$$NWIA_f(\widetilde{A}) = \left[\frac{a+nb}{n+1}, \frac{nb+c}{n+1}\right] \tag{8.2.6}$$

- **Goal Programming**

  In the conventional case, "if a PCM A be reciprocal and consistent then the weights of each criterion are simply calculated as $w_i = \frac{a_{ij}}{\sum_{k=1}^{n} A_{kj}}$, $i = 1,...,n$. In the case of inconsistent matrix, it must obtain the importance weights w_i, $i = 1,...,n$ such that $a_{ij} = \frac{w_i}{w_j}$ or equivalently $a_{ij}w_j - w_i = 0$. Therefore, in the case of uncertainty, for deriving the weights of criteria from inconsistent fuzzy comparison matrix, the next procedure is followed" (Izadikhah, 2012).

Firstly, by Equation 8.2.6, each fuzzy element $\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^M, a_{ij}^U)$ of the PCM is converted to the nearest weighted interval approximation $\bar{a_{ij}} = [\bar{a}_{ij}^L, \bar{a}_{ij}^U]$. Hence, the fuzzy PCM $\tilde{A}$ is converted to an interval PCM $\bar{A}$. After, the weight vector w_i, $i = 1,...,n$ is calculated such that $\bar{a}_{ij}^L \leq \frac{w_i}{w_j} \leq \bar{a}_{ij}^U$; therefore, $\bar{a}_{ij}^L wj \leq w_i \leq \bar{a}_{ij}^U wj$ is obtained. Hence, here is the deviation variables p_{ij}^-, p_{ij}^+ and q_{ij}^-, q_{ij}^+ which lead to

$$\bar{a}_{ij}^L w_j - w_i + p_{ij}^- - p_{ij}^+ = 0 \tag{8.2.7}$$

$$-\bar{a}_{ij}^L w_j + w_i + q_{ij}^- - q_{ij}^+ = 0 \tag{8.2.8}$$

where deviation variables p_{ij}^-, p_{ij}^+ and q_{ij}^-, q_{ij}^+ are non-negative real numbers but cannot be positive at the same time, that is, $p_{ij}^- p_{ij}^+ = 0$ and $q_{ij}^- q_{ij}^+ = 0$. Here, the GP method is applied. "It is desirable that the deviation variables p_{ij}^{+} and q_{ij}^{+} are kept to be as small as possible, which leads to the following GP model" (Izadikhah, 2012):

$$Min \sum_{i=1}^{n}\sum_{j=1}^{n}(p_{ij}^{+}+q_{ij}^{+}) \tag{8.2.9}$$

subject to:

$$\bar{a}_{ij}^{L}w_j - w_i + p_{ij}^{-} - p_{ij}^{+} = 0 \tag{8.2.10}$$

$$-\bar{a}_{ij}^{L}w_j + w_i + q_{ij}^{-} - q_{ij}^{+} = 0 \tag{8.2.11}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{8.2.12}$$

$$w_i, p_{ij}^{-}, p_{ij}^{+}, q_{ij}^{-}, p_{ij}^{+} \geq 0$$

By solving this model the optimal weight vector $W = (w_1, ..., w_n)$ which shows the importance of each criterion is obtained. "These weights can be used in the process of solving a multiple criteria decision-making problem. Also, these weights show which criterion is more important than others. For ranking of these criteria, rank 1 is assigned to the criterion with the maximal value of w_i and so forth, in a decreasing order of w_i. The proposed method is able to derive the weights of criteria when the elements of the PCM are fuzzy in any form of triangular or trapezoidal fuzzy numbers" (Izadikhah, 2012).

This GP model used in practice is always feasible. Here is the little evidence for its feasibility.

Consider $\widehat{W} = (\hat{w}_1, ..., \hat{w}_n)$ which has the condition $\sum_{i=1}^{n} \hat{w}_i = 1,\ \hat{w}_i \geq 0,\ i = 1, ..., n.$ Then, deviation variables are defined as:

$$\hat{p}_{ij}^{-} = max\left\{-(\bar{a}_{ij}^{L}\hat{w}_j - \hat{w}_i) \quad , \ 0\right\} \tag{8.2.13}$$

$$\hat{p}_{ij}^{+} = max\left\{(\bar{a}_{ij}^{L}\hat{w}_j - \hat{w}_i) \quad , \ 0\right\} \tag{8.2.14}$$

$$\hat{q}_{ij}^{-} = max\left\{-(-\bar{a}_{ij}^{U}\hat{w}_j + \hat{w}_i) \quad , \ 0\right\} \tag{8.2.15}$$

$$\hat{q}_{ij}^{+} = max\left\{(-\bar{a}_{ij}^{U}\hat{w}_j + \hat{w}_i) \quad , \ 0\right\} \tag{8.2.16}$$

It is clear that $(\hat{W},\ \hat{p}_{ij}^{-},\ \hat{p}_{ij}^{+},\ \hat{q}_{ij}^{-},\ \hat{q}_{ij}^{+})$ is a feasible solution.

8.2.3 FMWOLP Model

The overall aim is to maximize the $\lambda_{TC}, \lambda_{TP}, \lambda_{TR}$ which is the satisfaction rating, using the weights obtained. Thus, the best possible TC, TP, TR values are obtained, with the objectives of minimizing the total project cost, minimizing the total project duration time and maximizing the total crash time.

$$Max\ \lambda = w_{TC} * \lambda_{TC} + w_{TP} * \lambda_{TP} + w_{TR} * \lambda_{TR} \tag{8.2.17}$$

subject to:

$$\lambda_{TP} \leq \frac{TP^{NIS} - TP}{TP^{NIS} - TP^{PIS}} \tag{8.2.18}$$

$$\lambda_{TC} \leq \frac{TC^{NIS} - TC}{TC^{NIS} - TC^{PIS}} \tag{8.2.19}$$

$$\lambda_{TR} \leq \frac{TR - TR^{PIS}}{TR^{NIS} - TR^{PIS}} \tag{8.2.20}$$

$$E_i + T_{ij} \leq E_j \quad \forall i, j \tag{8.2.21}$$

$$T_{ij} + Y_{ij} = D_{ij} \quad \forall i, j \tag{8.2.22}$$

$$Y_{ij} + d_{ij} \leq D_{ij} \quad \forall i, j \tag{8.2.23}$$

$$E_1 = 0 \tag{8.2.24}$$

$$E_N \leq F \tag{8.2.25}$$

and all variables are non-negative.

where w_{TC}, w_{TP} and w_{TR} are the normalized importance weights for the total project cost, the total project duration time and the total crash time, respectively; λ_{TC}, λ_{TP}, λ_{TR} are the degrees of satisfaction for the total project cost, the total project duration time and the total crash time, respectively.

8.2.4 FMWOLP Model Result

The results include the whole project cost, the whole project period time, and also the total crash time and therefore the three objectives are satisfied.

Briefly, the steps of the proposed methodology are given in Figure 8.1.

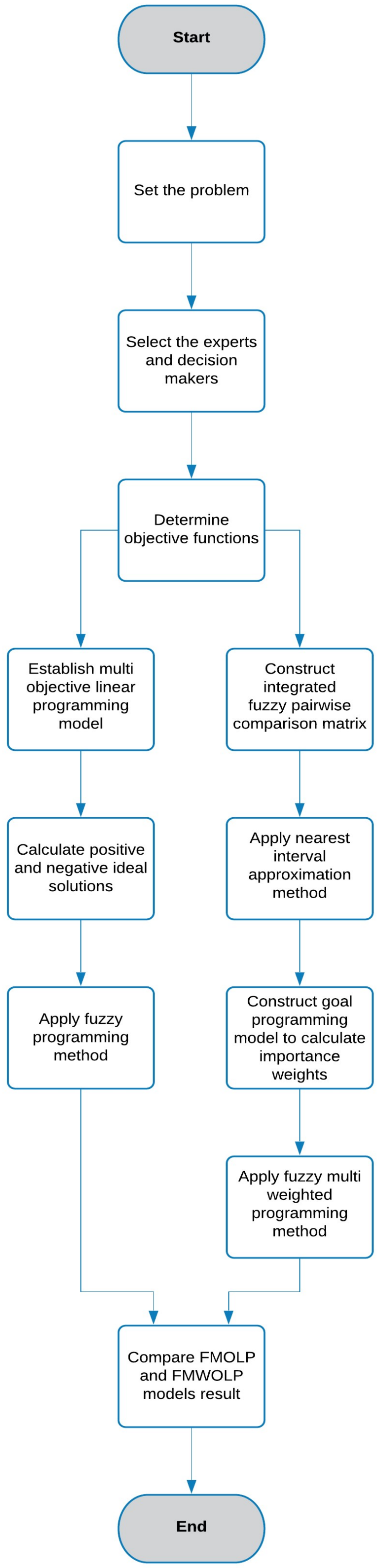


Figure 8.1: Flowchart of the proposed methodology.

9 NUCLEAR ENERGY

In parallel with the growth in the industrialization and urbanization of developing countries, the human population is increasing rapidly. Although the energy demand per capital has increased significantly, the demand for energy in every field has increased so fast. In the world, there is no new and big fuel reserves for use in energy production; petroleum reserves are assumed to be depleted in 2050, natural gas reserves in 2070 and Coal reserves in 2170. "This assumption has increased the interest in energy production and countries with insufficient resources in energy production have turned to new energy production alternatives" (ETKB, 2014a).

Sources or fuels used in energy production in the world:

- "Thermal Resources (Coal, Oil and Natural Gas)"
- "Nuclear Sources (Uranium, Thorium and Plutonium)"
- "Hydraulic Resources (Seas, Lakes and Rivers)"
- "Other Resources (Solar, Wind, Hydrogen, Geothermal)"

From these sources; thermal and nuclear resources are consumable energy sources, hydraulic and other sources are renewable energy sources.

In energy production; thermal power plants that operate with fossil fuels such as coal, oil and natural gas leave many harmful gases into the environment and atmosphere. "These harmful gases are carbon dioxide (CO_2), nitrogen oxides (NO_x) and sulfur dioxide (SO_2). (CO_2) gas causes the greenhouse effect (warming of the atmosphere). (NO_x) and (SO_2) gases cause acid rain. Acid rain threatens the natural life of the lakes and negatively impacts the vegetation, forests, agricultural areas" (ETKB, 2014a).

Nuclear power plants are the plants that produce energy intensively by providing the least damage to the environment and providing continuous energy production at reasonable prices. They are, in contrast to what is believed, environmentally friendly and energy efficient. Nuclear power plants are intensive energy sources. "1 *kg* of nuclear fission (the disintegration of the nuclear fuel radioactive material (U235)) yielded 90 * 106 *MJ* of energy, while 14-19 *MJ* from 1 (kg) of lignite, 45-46 *MJ* from petroleum, 121 *MJ* from Hydrogen and 39 *MJ* from 1 m^3 of natural gas are obtained" (ETKB, 2014a).

9.1 History of Nuclear Energy

In 1934, physicist Enrico Fermi saw that the atoms that had emerged after bombing uranium atoms with neutrons were much smaller atoms than uranium and recognized the potential of the nuclear division reaction. In 1942, using uranium and control bars in a similar way to today's nuclear power plants, he created the first controlled, self-sustaining nuclear power generation system. After seeing the power and potential of this new technology, the United States performed the first nuclear weapons test in New Mexico, July 1945. With these developments, the 1950s and 1960s were the years in which nuclear power plants were spreading rapidly. "The amount of energy generated by the division of one uranium atom is 10 million times the energy generated by the combustion of one coal atom, in other words, the energy that can be obtained from half a kilogram of uranium can produce the same energy as millions of liters of oil" (ETKB, 2014a).

However, on March 28, 1979, an accident caused by a faulty valve at the Three Mile Island nuclear plant in the United States revealed how dangerous the nuclear could be. The fact that the nuclear fuel could not be cooled sufficiently could cause significant damage to the central workers and the environment and nature. On the other hand, the subject of the use of used fuels has become the soft belly of nuclear energy for years. The following years, technological developments, new and safer designs, new accidents, concerns about global warming; nuclear energy was reflected in the years of tides. Some periods of nuclear energy were considered as a hope, some periods should be rid of as soon as possible. The timeline of nuclear energy developments is shown in Table 9.1.

Table 9.1: Timeline of nuclear energy developments (Rhodes, 1986; Weinberg, 1994; Cooke, 2009).

1895	"Roentgen discovers X-rays."
1896	"Becquerel discovers rays emitted spontaneously from uranium salts."
1898	"The Curies identify two radioactive nuclides, coin term "radioactive"."
1899	"Rutherford distinguishes alpha and beta radiation and discovers half-life."
1909	"Rutherford discovers that most mass is concentrated in a small nucleus."
1920	"Rutherford theorizes a "neutron"."
1932	"Chadwick identifies neutrons."
1938	"Hann and Strassman split uranium atoms with neutrons, Meitner and Frisch explain what's happening and name it "fission"."
1939	"Fermi and Szilard measure neutron multiplication, conclude that a nuclear chain reaction is possible."
1939	"Szilard, Wigner and Teller convince Einstein to sign a letter warning Roosevelt of possibility of nuclear weapons. Roosevelt authorizes creation of Advisory Committee on Uranium, begins US nuclear bomb effort."
1942	"Fermi achieves first nuclear chain reaction."
1945	"The world's first weapon test, the Trinity shot, is successful. Atomic bombs Little Boy and Fat Man dropped on Japanese cities, Hiroshima and Nagasaki. Up to 240,000 people died. Japan surrenders unconditionally, ending WWII."
1951	"EBR-I reactor is the first to generate electricity in Argo, ID."
1953	"Eisenhower gives Atoms for Peace speech launching civilian program."

1954	"USS Nautilus launches, the first nuclear powered submarine."
1954	"Obninsk reactor in the Soviet Union becomes the first commercial nuclear power."
1957	"Shipppingport reactor begins operation, first commercial nuclear power."
1974	"French Prime Minister Messmer launches huge nuclear power program in response to oil crisis."
1979	"Three Mile Island reactor suffers a partial melt down. Reaction largely contained."
1986	"EBR-II reactor demonstrates that advanced, sodium cooled reactors can passively shut down without backup systems."
1986	"Chernobyl reactors suffers a large power excursion resulting in the release of large amounts of radiation. 50+ firefighters die, up to 400 civilians estimated to die of early cancer."
1994	"Megatons to Megawatts program started, turns 20,000 nuclear weapons into electricity. By 2000, 10% of US electricity comes from dismantled Russian warheads."
2004	"After decades of electricity generation with no deaths in th US, a Nuclear Renaissance is discussed, with talks of more reactor builds to offset carbon emission."
2004	"75% of France's electricity is nuclear."
2011	"Four reactors of Fukushima Daiichi lose backup generators due to tsunami and suffer core meltdowns, hydrogen explosions."
2013	"Voyager-I enters interstellar space after travelling the solar system for 36 years. It is powered by a Plutonium-238 radio-isotopic thermal generator."

9.2 Nuclear Power Plants in the World

According to data from the International Atomic Energy Agency, 450 nuclear reactors are operating in 31 countries around the world (May 22, 2018). "In 2017, around 11% of the world's electricity production was supplied from nuclear power plants with 2477 TWh. The construction of 59 nuclear reactors in 18 countries continues" (IAEA, 2018).

99 nuclear reactors are operating in the USA. "In 2017, 20% of electricity generation of USA was supplied from nuclear power plants. Two nuclear reactors are under construction" (IAEA, 2018).

37 nuclear reactors are in operation in Russia. "Russia meets approximately 17% of its electricity generation from nuclear power plants and continues the construction of 6 nuclear reactors" (IAEA, 2018).

39 nuclear reactors operating in the People's Republic of China; 18 nuclear reactors are under construction. "China receives 3,5% of its electricity from nuclear energy. China has taken important steps at localization of nuclear power plants. China has its own nuclear power plant design and it also markets its own design reactors to the international market" (IAEA, 2018).

In the United Kingdom, 15 nuclear reactors are in operation and 20% of the generated electricity is supplied from nuclear power plants. "UK plans to build a total of 16 GW of new nuclear power plants according to its policy published in 2006" (IAEA, 2018).

58 nuclear reactors are in operation in France. "72% of electricity generation is provided by nuclear power plants. One nuclear reactor is under construction" (IAEA, 2018).

In summary, there are 435 nuclear plants in 31 countries. These countries are shown on the world map in Figure 9.1 and their numbers are shown in Figure 9.2 (ETKB, 2014a).

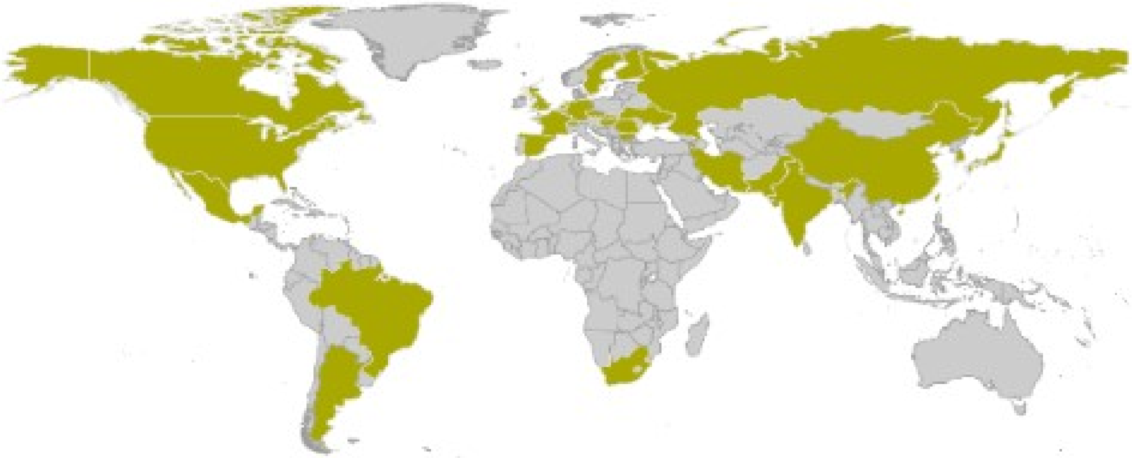


Figure 9.1: Countries with nuclear power plants in the world (ETKB, 2014a).

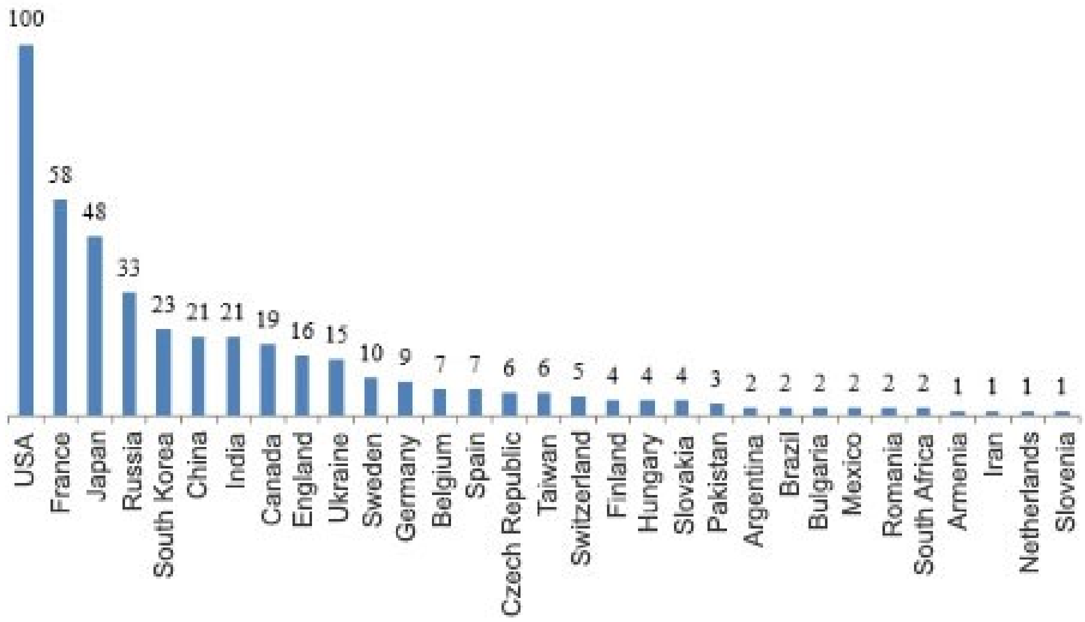


Figure 9.2: Countries with nuclear power plants and number of power stations (ETKB, 2014a).

72 nuclear power plants are under construction in 16 countries in 2014, Figure 9.3 and Figure 9.4 show the distribution of these power plants and countries.

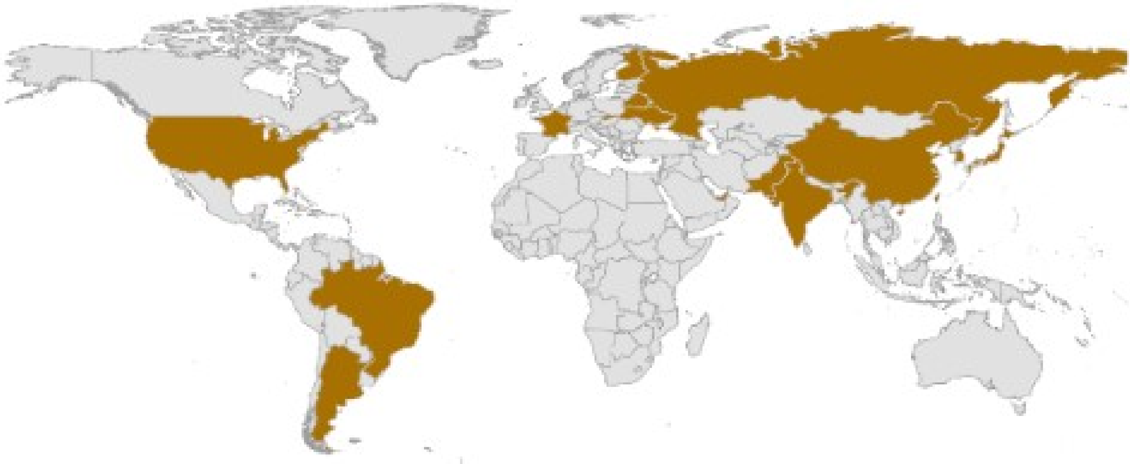


Figure 9.3: Countries that build nuclear power plants (ETKB, 2014a).

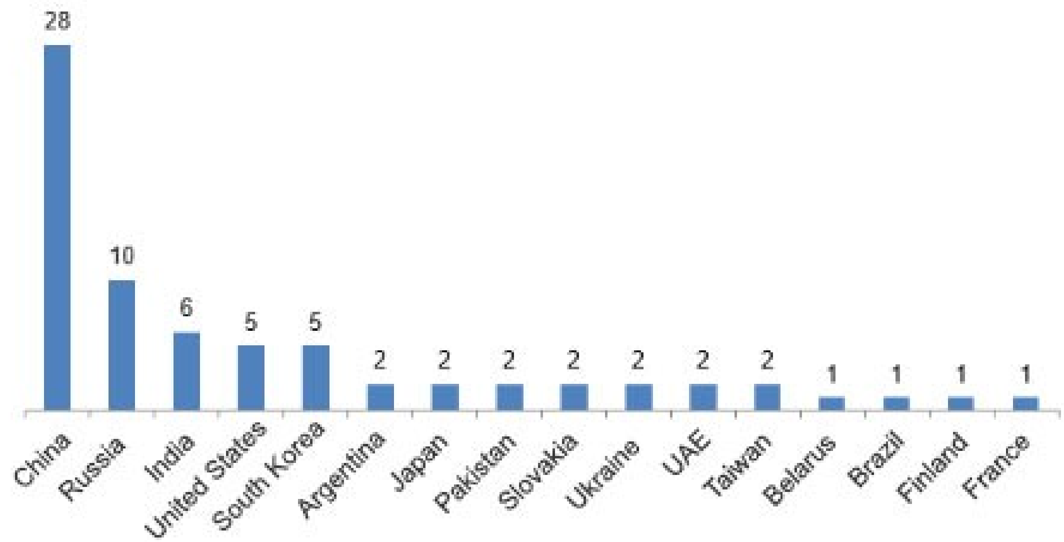


Figure 9.4: Number of nuclear power plants under construction by country (ETKB, 2014a).

According to March 2013 data, the number of nuclear power plants operating and under construction in the world and the share of nuclear energy in countries' electricity production are shown in Table 9.2 .

Table 9.2: Numbers of the world's nuclear power plants being in-service and under construction and nuclear energy's share in the electricity production of the countries (ETKB, 2014a).

Countries	Number of nuclear power plants in-service	Number of nuclear power plants under construction	Share of nuclear energy in electricity production (%)
USA	100	5	%19,0
France	58	1	%73,3
Japan	48	2	-[1]
Russia	33	10	%17,5
South Korea	23	5	%30,4
India	21	6	%3,5
China	21	28	%2,1
Canada	19		%15,3
United Kingdom	16		%18,1
Ukraine	15	2	%43,6
Sweden	10		%38,1
Germany	9		%15,5
Spain	7		%20,5
Belgium	7		%51,0
Czech Republic	6		%35,3
Taiwan	6	2	%18,4
Switzerland	5		%35,9
Finland	4	1	%33,3
Hungary	4		%50,7
Slovakia	4	2	%51,7
Pakistan	3	2	%4,4
Argentina	2	2	%4,4
Brazil	2	1	%2,8
Bulgaria	2		%30,7
Mexico	2		%4,6
Romania	2		%19,4
South Africa	2		%5,7
Armenia	1		%29,2
İran	1		%0,6
The Netherlands	1		%4,4
Slovenia	1		%33,6
The United Arab Emirates		2	
Belarus		1	
TOTAL	435	72	World-wide %13,5

Due to economic and population growth, world energy demand will continue to increase as shown in the Figure 9.5 (Bos et al., 1992). The vast majority of this increase will take place in developing countries. Because these countries strive to increase the living standards of their growing populations. In 1998, IIASA and the World Energy Council concluded that worldwide energy demand would likely increase by 1.5-3.0 times, by 2050 . Figure 9.5 shows the development in global primary energy consumption per energy resource, and a possible scenario for future developments.

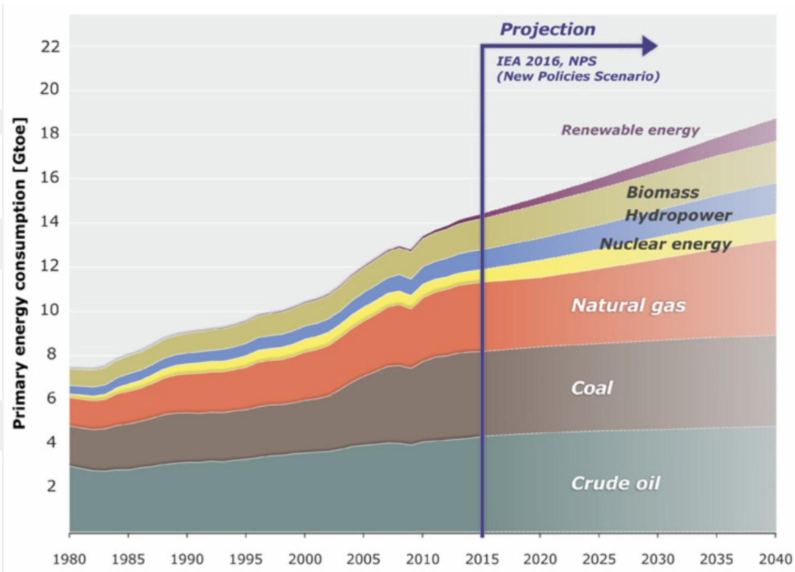


Figure 9.5: Development in global primary energy consumption per energy resource, and a possible scenario for future developments (IEA, 2016).

Today, meeting the growing needs of society without affecting future generations is an important challenge.

9.3 Sustainable Development and Nuclear Energy

Energy is an important component of policies related to sustainable development. Because energy is vital for human activity and economic growth. The current view that today's energy supply technologies are not sustainable and this situation is now becoming increasingly dominant. Therefore, nuclear energy plays an important role in determining the future of energy source in the energy supply markets.

Sustainability is traditionally discussed in 3 different dimensions. These are environmental, social and economic aspects.

9.3.1 Environmental Impact

The environmental sustainability of a given material is defined by the conditions such as the availability of reserves and the direct effects on the environment.

Availability of Resources

Uranium is widely distributed in the crust and oceans. There are more uranium atoms in the world than silver atoms. As of the beginning of 2001, estimated conventional uranium sources (known and undiscovered) have a total volume of 16 million tonnes. Considering the current usage rate, it can be said that there are about 250 years of resources.

In addition, there are also sources where unconventional, i.e. uranium, is present in low concentrations and can also be obtained as a by-product of uranium. The amount of these non-conventional uranium sources in phosphate deposits is around 22 million tons. "It is also known that around 4000 million tons of uranium is found in sea water. Research shows that uranium can also be separated in seawater, but this can only be carried out on a laboratory scale. The cost of this process is still about 5-10 times higher than those obtained from normal uranium mines" (ETKB, 2014a).

In the long term, however, whether natural uranium will be sufficient will depend on reactor technologies and adopted fuel cycle strategies. Re-processing of used fuel from existing light-water reactor technologies will, in principle, reduce uranium demand by 10-15%. Moreover, in the future, if fast reactors are included in the commercial reactors, the fuel efficiency will increase. If fast reactors can replace existing reactors, 50 times more efficient use of existing uranium sources can be achieved with the re-processing of spent fuel.

Some other advanced technologies that are currently only imaginable will be able to provide future use of thorium. In particular, India, with its large thorium reserves, is currently working on an experimental scale to transform thorium into uranium. In fact, it is not possible to see nuclear energy as having limited resources.

Direct Environmental Impact

Nuclear energy is one of the few sources of energy that pollute the air or cause almost no greenhouse gases. "All nuclear fuel cycle stages, including uranium mining and nuclear power plant construction, result in the release of 2.5 to 6 grams of carbon into the atmosphere per kWh energy produced. This is approximately equal to the estimated carbon emissions resulting from the use of renewable energy sources such as wind, hydroelectricity and solar energy. In addition, as a result of the use of natural gas known as the cleanest among the fossil sources, the amount of carbon to be released is less than 20-75 times" (IAEA, 2000).

Therefore, nuclear energy is one of the energy production methods that are compatible with the purpose of limiting the emission of carbon to the atmosphere. In OECD countries alone, nuclear power plants prevent the emission of 1200 million tons of carbon dioxide (CO_2) gas per year. If all nuclear power plants in the world are replaced with fossil fuel power plants, CO_2 emissions from the world energy sector will increase by about 8%.

Nuclear energy also prevents the release of gases and dust, such as sulfur and nitrogen oxides, which cause local air pollution. These gases and dusts cause acid rains and respiratory tract diseases.

The amount of solid waste produced per unit of electricity produced by nuclear energy is much lower than any other fossil fuel source. In fact, "the amount of solid waste generated by nuclear power generation is at the same level as the amount of solid waste generated by renewable energy sources such as solar energy" (IAEA, 2000).

On the other hand, the nuclear production capacity needs to expand considerably so that nuclear energy can contribute to the prevention of excessive global warming. Nowadays, nuclear energy meets only a fraction of the world's electricity generation, and electricity generation is only a sub-branch of the energy sector. In the light of current estimates, if the installed nuclear energy capacity has reached 10 times by 2100, the share of 7% among today's primary energy sources will increase to 25%. and only about 15% of the cumulative carbon emissions expected during this time period. however, it is also important to note that if the increase in nuclear capacity is based on existing technologies, a significant increase in the volume of radioactive wastes will also occur.

In a nutshell, nuclear energy is one of the options that can contribute to meeting the projected increase in world energy demand, and in doing so it does not cause carbon emissions, which are almost the cause of almost no greenhouse gases. However, in order to be efficient and acceptable, there is a need for advanced design reactor technologies and strategies for recycle of fuel. During this century, the commercial reactor fleet of existing light-water reactors must be replaced by advanced technologies, including fuel recycling, such as fast-reacting reactors. Such a change requires considerable investment. However, this investment need is not much more than the investment to be made to other energy production strategies in order to meet the increasing energy need to limit global warming. It should also be noted that fast-productive reactor technology has not yet been commercialized.

Life Cycle of Waste

High-level wastes can remain radioactive for long periods of time, even if their volume is small. Research on deep geological storage areas has been carried out over the last few decades. The expert opinion is that they do not have any technical obstacles to their very high standards of reliability. Although there are some developments on this issue, especially in Finland and the USA, a final storage area has not yet been put into operation. Hence, the final storage of high-level wastes is already a threat to the sustainability of nuclear energy.

The research and development work on advanced fuel cycles and the processing of spent fuel wastes produces promising results for reducing the volumes of wastes and the times they need to be finally stored. However, it can be seen that the results of these studies cannot become available on a commercial scale for the next few decades.

9.3.2 Social Dimension

Technical Infrastructure and Unemployment

Nuclear energy has some special features arising from the scientific and technological developments of the 20^{th} century. The high cost requirement of nuclear facilities is largely related to science and technology. The security of nuclear technology needs to be maintained and continually improved.

The nuclear industry employs highly trained, skilled, educated people compared to many other energy and manufacturing industries. Skilled people, although vulnerable to political interference, are important social capitals. Skilled people form the basis for continuous performance improvements that the industry needs.

Sustainability of nuclear energy can be achieved through the existence of a complex and expensive infrastructure, including skilled human resources. This complex and expensive infrastructure is the core of social capital. If this infrastructure is lost, it is very difficult to replace it cheaply and quickly.

Side effects

Providing, maintaining and improving the technical and intellectual infrastructure to be created to support nuclear energy also brings many benefits to society. As with other very advanced technologies, nuclear energy has played an important role in the development of new materials, techniques and skills in the past. These development activities had beneficial side effects to many other sectors such as medicine, manufacturing, public health and agriculture, and provided significant economic benefits.

Social Issues

All energy technologies tend to be a source of social anxiety and can even become a cause of conflict and conflict. In the case of nuclear power, all concerns focus on security, disarmament and final storage of waste. Coal has its own history of deep conflict and social conflict. Petrol also has an important history of disagreement, both internationally. Even the use of renewable energy sources has become subjected to careful scrutiny in recent days, and their forced installation and demanding enormous fields lead to people's opposition. Large hydroelectric projects face opposition on a global scale, and this is due to their social and environmental impacts that leave large areas under water (ETKB, 2014a).

9.3.3 Economics of Nuclear Energy

Nuclear energy is characterized by low production cost, high initial investment cost, insensitivity to changes in fuel prices, long operating life and considerable regulatory costs. Due to the high investment costs, the decision to make new nuclear power plants is significantly dependent on public policies.

One of the differences between nuclear power and other power generation methods is that some expenditure items, which are considered as external costs for other energy sources, are considered as internal costs in nuclear energy.

Elements of Nuclear Production Cost

The investment costs include the construction of the power plant, the renovation work carried out during certain periods. Operating-maintenance costs mainly cover personnel, education, safety and low / medium level radioactive waste management costs.

Fuel costs include all activities related to the fuel cycle. These activities include the purchase, conversion, enrichment of uranium, the production of fuel bundles, pretreatment of spent fuel, storage, re-processing according to open-loop principles, and the storage of high effluents from re-processing plants.

Electricity generation costs are generally divided into three main categories as investment, operation-maintenance and fuel.

Investment costs include design, construction, renewal and disassembly costs when the plant is at a certain age. The final component consists of all costs incurred from the closure of the power plant to the evacuation of the power plant site in accordance with national policy. The cost of the investment includes the costs of managing the radioactive and other wastes produced during disassembly until they are disposed of. To cover these costs, construction and operation phases are added to the licensing authority for the provision of compulsory licenses.

Operating-maintenance costs include all cost items that are not considered as investment and fuel costs. All activities related to the listed below are considered as operation and maintenance costs:

1. "Business and staff"
2. "Staff education"
3. "Physical security"
4. "Occupational health and Safety"
5. "Management and disposal of operational wastes"

In addition, the costs incurred by the periodical maintenance and inspection activities (which require the plants to be disabled) are included in this scope. Since the investment costs are substantially fixed after construction, operation and maintenance costs are the most important opportunity to reduce the costs of an existing plant.

Fuel costs include the disposal costs of high-level waste or spent fuel from the purchase, conversion and enrichment of uranium, fuel production, spent fuel operations, reprocessing, transportation or re-processing. Nuclear power plants are relatively insensitive to fuel price fluctuations, as opposed to fossil-fueled power plants, as fuel costs account for only 20% of nuclear-based electricity generation costs (ETKB, 2014a).

Long Term Financial Risks and Responsibilities

The decision to build a nuclear plant or to operate an existing nuclear power plant carries more commercial risk compared to other energy sources. The reasons are listed below:

- "The long planning process and long operating life offer more potential for long-term changes and it may affect the revenues of the plant positively or negatively"
- "The high fixed-cost component resulting from high initial investment cost creates greater vulnerability to short-term fluctuations in market conditions"

- "The strong regulatory framework (licensing authority's legislation and audits) raises the possibility of reducing the flexibility of operations and demanding changes to comply with the new legislation that could have a negative impact on costs"
- "Disassembly and long-life waste disposal costs include uncertainties"
- "While non-nuclear power plants may sell or trade their existing stocks under unfavorable economic conditions, this cannot be achieved in practice for nuclear power plants (for example, a gas-fired power plant may sell or buy gas in the market)"

While the costs of disassembly of the nuclear power plant and the disposal costs of the discharged wastes are high, they are greatly reduced due to the long operating life of the plant and constitute only a small component of the total life-cycle costs.

Considering the long service life of the plants, there is uncertainty about the future costs. One of the most important uncertainties is the regulatory legislation based on the licensing authority, which is changing and possibly becoming more stringent. Naturally, there are new costs incurred in compliance with the new legislation. Therefore, part of the appropriation allocated for dismantling the plant is used for these purposes as uncertainty allowance (ETKB, 2014a).

Comparative Costs of Electricity Generation

Compared with nuclear energy, natural gas-fired power plants have low initial investment and high fuel costs. Coal fuel is at the middle level in both cost items. In general, fuel costs constitute a relatively large proportion of the production costs of fossil fuel plants. Therefore, fossil fuel plants are sensitive to fuel price fluctuations.

Renewable energy sources such as wind and hydraulic energy are similar to nuclear energy with high initial investment and low production costs per unit of generated energy.

It is possible to say that recent nuclear power costs and initial investment costs for many existing power plants are already significantly amortized, and that existing nuclear power plants are competitive worldwide.

External Costs

One of the differences between nuclear power and other electricity generation methods is that some of the cost items, which are not included in the cost of other electricity sources. The external costs are considered as internal cost in nuclear energy and included in the cost calculations. The most important thing is the cost of radioactive waste management and disposal. This cost item is also taken into account when determining the price of nuclear electricity in the market.

The technology used in nuclear energy requires high level expertise and meticulous work. It should be taken into account that any minor deficiencies or misunderstandings that are based on long-term, high-cost, complex and high technology may cause irreparable or very difficult damages. Every step of the project should be strictly adhered to accordingly. Starting from the location selection of the project, the licensing and implementation in the planning, construction, operation and termination processes must be at the highest level of security and quality requirements at every stage.

Project planning and operation require very intensive, complex and detailed technological application and analysis. There is a need for experts who have the knowledge and experience of national and international knowledge, practices and experience to meet this and keep up with it. Every stage of implementation should be carried out in accordance with nationally accepted standards in accordance with internationally recognized standards.

9.4 Nuclear Energy in Turkey

Nuclear power operations in order to meet the energy supply have become widespread in the world and Turkey has taken action in this regard. Nuclear energy is preferred because it is an alternative to other energy sources, it has low fuel costs, reduces dependence on foreign sources and it is a clean energy type. In Turkey, construction work for nuclear power plants are carried out by Ministry of Energy and Natural Resources (ETKB), Turkish Atomic Energy Authority (TAEK), Electricity Generation Company (EGC), Turkey Electricity Transmission Company and the Mining Technical Exploration General Directorate. In addition, TAEK realizes the technical management and coordination of nuclear power plant investments. Nuclear energy program in Turkey is called Nuclear Technology and Energy Development Project. "TAEK has determined that the provinces such as Mersin-Akkuyu, Sinop-İnceburun and Thrace (Tekirdağ-Edirne), Adana and Ankara are the provinces where nuclear power plants can be established" (Turan, 2006). A brief history of nuclear power development in Turkey is given in Table 9.3.

Table 9.3: Development of nuclear power in Turkey (Temurçin, 2003).

Years	Nuclear Energy Experiences
1956	Establishment of the Atomic Energy Commission
1957	Membership to the International Atomic Energy Agency (UAEA)
1960	The year in which nuclear power experiences began
1972	Establishment of the Department of Nuclear Energy
1972	Selection of Mersin-Akkuyu site as first plant location
1980	Selection of Sinop - Inceburun site as second plant location
1984	Membership in the OECD Nuclear Energy Agency (NEA)
1986	Suspension of nuclear power plant operations due to Chernobyl nuclear power plant accident
1996	Opening of Akkuyu project tender (1400-2800 MW power)
2004	TAEK has announced three nuclear reactors of 5000 MWe will be made
2010	Russia and Turkey has signed the agreement "Joint Declaration on Cooperation in Nuclear Power Plant in Turkey"
2017	Limited Work Permit was received from TAEK for the first unit of Akkuyu
2018	Limited Work Permit was received from TAEK for the second unit of Akkuyu

"The first nuclear power plant in Turkey, was constructed in Mersin-Akkuyu in 2013 with $ 20 billion in costs by Russia. The first unit of the nuclear power plant was expected to be operational in 2018 and the final reactor is expected to be operational in 2021. It is stated that the life of these power plants will be 60 years" (TTB, 2014). Japan will complete the nuclear plant to be established in Sinop. Turkey in 2020 is expected to be a total installed power of 109.218 MWe.

Turkey's energy production and energy consumption is continuously increasing. Therefore, the country should meet this increasing demand with more domestic production and accordingly reduce the dependence on foreign countries by making new investments in energy. The share of electricity production in nuclear power plants in Turkey is expected to be 8% by 2020 and 20% by 2030. Turkey's primary energy production and consumption targets are shown in Table 9.4.

Table 9.4: Turkey's primary energy production and consumption targets (x1000 TOE) (Kömürcü and Filiz, 2009).

	Energy Production			Energy Consumption		
	2010	**2015**	**2020**	**2010**	**2015**	**2020**
Coal	5092	5109	4755	17282	26864	48156
Lignite	18001	24190	32044	18001	24190	32044
asphaltite	301	301	301	301	301	301
Oil	1573	1069	693	41184	50420	60918
Natural gas	235	213	229	37192	44747	51536
Hydroelectric	4903	7060	9419	4903	7060	9419
Geothermal	2080	3166	4914	2080	3166	4914
Wind	421	571	721	421	571	721
Sun	495	605	862	495	605	862
Wood	3383	3075	3075	3383	3075	3075
Plant-animal waste	1034	926	850	1034	926	850
Nuclear	**0**	**8.229**	**8.229**	**0**	**8.229**	**8.229**
Total	**37.516**	**54.514**	**66.094**	**126.274**	**170.154**	**222.424**

"According to the Ministry of Energy and Natural Resources Planning, between 2000 and 2020, the ratio of domestic energy production to consumption will decrease from 34.2 percent to 25.3 percent" (Serteller, 2006). The decrease in the ratio of production to consumption indicates the importance of producing electricity from nuclear energy in meeting the country's increasing energy demand.

9.5 Literature Review on Nuclear Power Plant

Publications on nuclear power plant are listed in Table 9.5.

Table 9.5: Literature review on nuclear power plant.

Author	Year	Title	Type
Chang et al.	(1995)	"Development of the On-Line Operator Aid System OASYS Using A Rule-Based Expert System and Fuzzy Logic for Nuclear Power Plants"	Article
Fantoni and Mazzola	(1996)	"Multiple-Failure Signal Validation in Nuclear Power Plants Using Artificial Neural Networks"	Article

Author	Year	Title	Type
Reifman	(1997)	"Survey of Artificial Intelligence Methods for Detection and Identification of Component Faults in Nuclear Power Plants"	Article
Liberatore	(2002)	"Project schedule uncertainty analysis using fuzzy logic"	Article
Hines and Davis	(2005)	"Lessons learned from the U.S. nuclear power Plant on-line monitoring programs"	Article
Zhang et al.	(2006)	"Fuzzy dynamic programming method for progress adjustment of project mangement"	Article
Srividya et al.	(2007)	"Fuzzy AHP in prioritizing feeders for maintenance in nuclear power plants"	Paper
Rao et al.	(2007)	"Test interval optimization of safety systems of nuclear power plant using fuzzy-genetic approach"	Article
Kahraman and Kaya	(2010)	"A fuzzy multicriteria methodology for selection among energy alternatives"	Article
Costa et al.	(2011)	"An efficient Neuro-Fuzzy approach to nuclear power plant transient identification"	Article
Ekmekçioglu et al.	(2011)	"A Fuzzy Multi-Criteria SWOT Analysis: An Application to Nuclear Power Plant Site Selection"	Article
Wang et al.	(2011)	"Managing construction risks of AP1000 nuclear power plants in China"	Article
Guimarães et al.	(2011a)	"Fuzzy uncertainty modeling applied to AP1000 nuclear power plant LOCA"	Article
Guimarães et al.	(2011b)	"Fuzzy methodology applied to Probabilistic Safety Assessment for digital system in nuclear power plants"	Article
Rastogi and Gabbar	(2013)	"Fuzzy-Logic-Based Safety Verification Framework for Nuclear Power Plants"	Article
Hellström et al.	(2013)	"Project governance and path creation in the early stages of Finnish nuclear power projects"	Article
Purba	(2014)	"Fuzzy probability on reliability study of nuclear power plant probabilistic safety assessment: A review"	Article
Erol et al.	(2014)	"Fuzzy MCDM framework for locating a nuclear power plant in Turkey"	Article
Shin et al.	(2016)	"Comparison of Risk Assessment for a Nuclear Power Plant Construction Project Based on Analytic Hierarchy Process and Fuzzy Analytic Hierarchy Process"	Article
Erdoğan and Kaya	(2016)	"A combined fuzzy approach to determine the best region for a nuclear power plant in Turkey"	Article
Korobkin and Kolodenkova	(2016)	"Evaluation of the feasibility of the project to create control and management systems for nuclear power plants using fuzzy cognitive modeling"	Book

Author	Year	Title	Type
Shahi et al.	(2018)	"The development of nuclear power plants by means of modified model of Fuzzy DEMATEL and GIS in Bushehr, Iran"	Article
Sperry and Jetter	(2019)	"A Systems Approach to Project Stakeholder Management: Fuzzy Cognitive Map Modeling"	Article
Islam et al.	(2019)	"Modified Fuzzy Group Decision-Making Approach to Cost Overrun Risk Assessment of Power Plant Projects"	Article
Wu et al.	(2020)	"A two-stage decision framework for inland nuclear power plant site selection based on GIS and type-2 fuzzy PROMETHEE II: Case study in China"	Article
Zhao et al.	(2020)	"Diagnosis of operational failures and on-demand failures in nuclear power plants: An approach based on dynamic Bayesian networks"	Article
Abro et al.	(2020)	"Ageing Analysis of Power Cable used in Nuclear Power Plant"	Article
Kim et al.	(2020)	"Fuzzy Based Risk Assessment for Decommissioning Concrete Bioshield Structures in Nuclear Power Plants: Structural Risks and Worker Safety"	Article

10 APPLICATION

This chapter evaluates two models FMOLP and FMWOLP by using data of a nuclear power plant installation project in Turkey. Using the model presented in the previous chapter, three main objective functions have been identified for the installation project such as; total cost, total time and total crash time. These objectives are modeled by FMOLP. The satisfaction degrees are determined for all objectives and these values were maximized with the established model. The goal is to achieve the lowest cost, the minimum time and the highest crash time optimization. The same problem is then used by weighting objective functions. First, with the help of decision makers, nearest interval approximation and goal programming techniques are used for criterion weighting. Then, FMWOLP model is established. The highest satisfaction is obtained. The results of the two models are evaluated.

10.1 Case Study

Nuclear energy technology is an energy production method in the form of electricity generation from turbine connected generator and steam turbine rotation using the heat energy produced by the division of certain heavy atoms such as uranium and plutonium. It was first developed in the 1940s and after the World War II it was used for commercial electricity production.

In the early years of nuclear power, it was predicted that the energy needs of mankind in the coming thousands of years could be met with the help of nuclear energy and it would be very cheap in the future. On the other hand, it was observed that this was not the case over time, and it was understood how difficult it would be to operate the high-radiation systems. As a result of accidents, nuclear power plant conditions have been tightened, making their design and construction more expensive. Following new regulations, safety measures and strict licensing terms, nuclear energy has become more expensive and risky investment. In many countries, the problem of nuclear waste, which is not resolved due to political reasons and problems of interest with people's acceptance, has limited the spread of nuclear power plants. Following the massive nuclear stagnation in the western world in the 1990s, the Fukushima accident hampered the nuclear renaissance and re-enactment for the post-2010 period. Many countries have changed or questioned their future nuclear predictions.

Today, about 16 countries provide at least 25 percent of their electricity needs from nuclear energy. The use of nuclear power in Turkey has been on the agenda since the 1970s. In order to use nuclear technology cheaply and safely, many conditions and serious infrastructure problems should be examined. In recent years, nuclear plant installation in Turkey has become an important issue and research on the subject has increased.

The nuclear power plant installation case is used for demonstrate the effectiveness of the proposed model. The fundamental development information of installation nuclear power plant in Turkey are shown in Table 10.1 (IAEA, 2012; ETKB, 2014b, 2017). Under normal conditions, the finish time is 20 years.

Table 10.1: Construction data for nuclear power plant project.

Activity Code	Activity	Duration (Year)	Crash Time (Year)	Cost (₺)	Crash Cost (₺)
(a)	Pre-project	3	1	33,210,601.6	4,427,765.3
(b)	Decision-making	7	3	349,090,909.091	23,271,072.19
(c)	Project management	16	7	628,363,636.36	18,616,557.75
(d)	Preparation of site infrastructure	4	1	768,000,000	68,261,811.76
(e)	Detailed design engineering	6	2.5	698,181,818.18	53,191,022.14
(f)	Equipment and component	5	2	5,026,909,090.91	446,804,586.04
(g)	Construction, erection and installation	5	2	6,004,363,636.36	533,683,255.55
(h)	Commissioning and plant organization	4	1.5	488,727,272.72	52,127,201.704

10.2 Scheduling the Project with CPM/PERT

10.2.1 Project Network

The predecessors and times of the activities of the nuclear power plant project are shown in Table 10.2.

Table 10.2: Activity list for nuclear power plant project.

Activity Code	Immediate Predecessors	Duration (Year)
a	-	3
b	a	7
c	a	16
d	b	4
e	b	6
f	b	5
g	d	5
h	d,e,f	4

As shown in Section 2.2, the AOA project network is drawn as in Figure 10.1 by utilizing the activity list of the project. Thus, activities connected to each other were visualized.

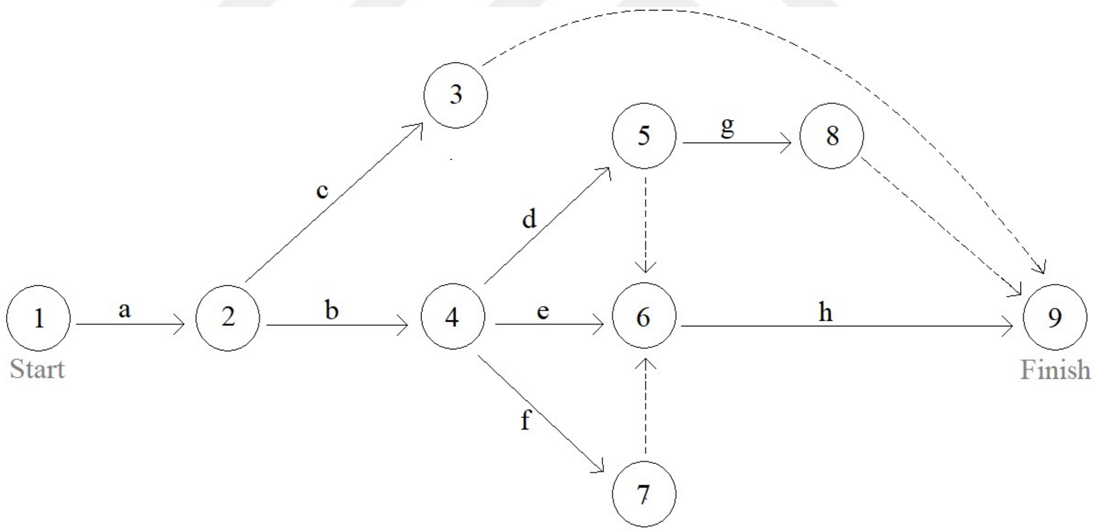
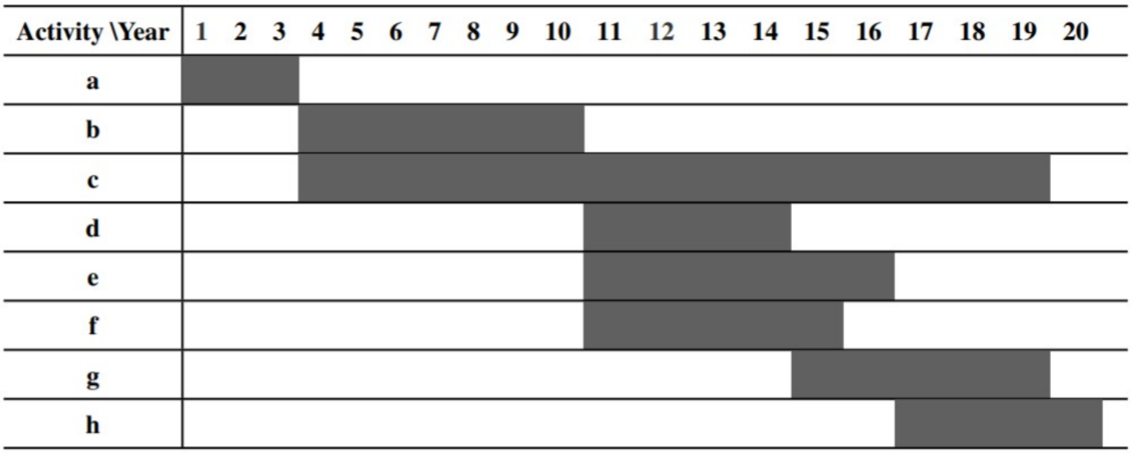


Figure 10.1: The AOA project network for nuclear power plant project.

10.2.2 Gantt Chart

The Gantt chart is drawn as in Table 10.3 by utilizing the project network and the given times. The duration of the activities and their locations in the project are clearly demonstrated by this scheme. The total project duration is 20 years.

Table 10.3: Gantt chart for nuclear power plant project.



10.2.3 Critical Path

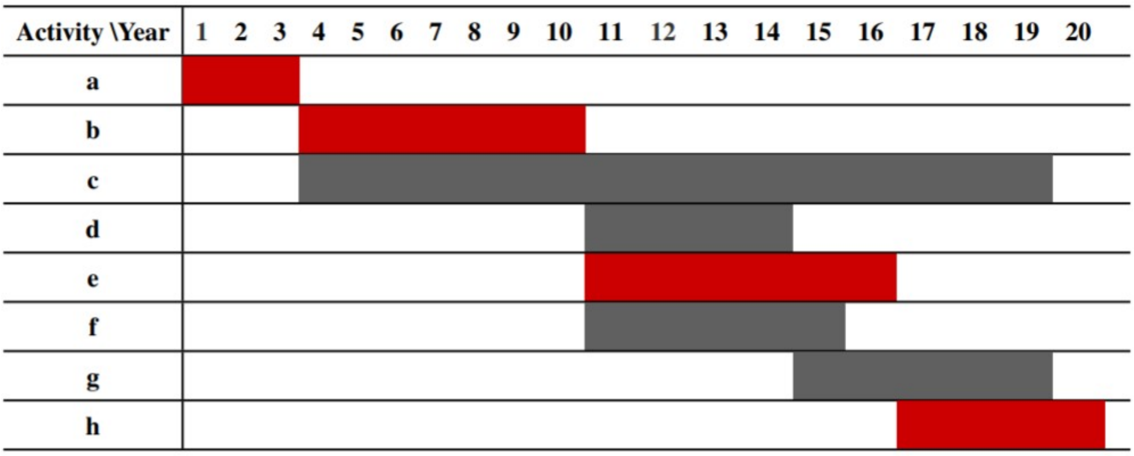
The five different paths through the nuclear power plant installation project network in Figure 10.1 are given in Table 10.4, along with calculation of the lengths of these paths. The paths lengths range from 18 years up to 20 years.

Table 10.4: The paths and path lengths through network of the nuclear power plant project.

Path	Length
"START → a → c → FINISH"	3+16 = 19 years
"START → a → b → d → g → FINISH"	3+7+4+5 = 19 years
"START → a → b → d → h → FINISH"	3+7+4+4 = 18 years
"START → a → b → e → h → FINISH"	3+7+6+4 = 20 years
"START → a → b → f → h → FINISH"	3+7+5+4 = 19 years

Hence, the critical path is ”$START \to a \to b \to e \to h \to FINISH$” and project duration is 20 years as in Table 10.4. The critical path is illustrated in the Gantt chart as in Table 10.5.

Table 10.5: Gantt chart with critical path for nuclear power plant project.



10.2.4 Scheduling Individual Activities

The earliest and latest start-finish times of each activity as in Section 2.2.1 are as shown in Figure 10.2.

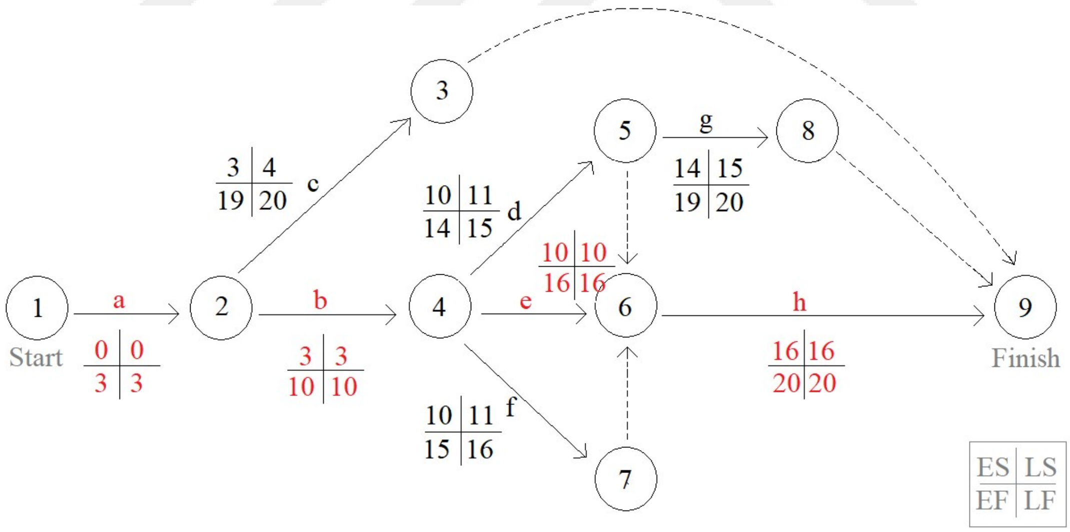


Figure 10.2: The earliest and latest start-finish times of each activity for nuclear power plant project.

10.2.5 Identifying Slack in the Schedule

Each activity with zero slack is on a critical path. Any delay along this path will delay project completion. Thus, the critical path is "$START \rightarrow a \rightarrow b \rightarrow e \rightarrow h \rightarrow FINISH$" as in Table 10.6. So, the critical path is found again by calculating the slacks.

Table 10.6: Slacks of activities of nuclear power plant project.

Activity	Slack (LF - EF)	On Critical Path ?
a	0	Yes
b	0	Yes
c	1	No
d	1	No
e	0	Yes
f	1	No
g	1	No
h	0	Yes

10.3 Fuzzy Multi-Objective Linear Programming

Each project network can be expressed as a linear programming model with algebraic linear expressions which is describing the objective function and constraints.

10.3.1 Multi-Objective Linear Programming Model

In this section, the nuclear power plant installation project is transformed into a linear programming model, taking into account the time and cost balance. The project network in Figure 10.2 and the data in Table 10.1 are used. The model has three objectives at the same time. Minimum total cost $(MinTC)$, minimum total project time $(MinTP)$ and maximum crash time $(MaxTR)$. To achieve these goals, based on Equations (8.1.1)-(8.1.8) , the MOLP model is constructed as follows:

$$Min\ TC = (33,210,601.6 + 4,427,765.3\ Y_{12}) + (349,090,909.091 + 23,271,072.19\ Y_{24}) + (628,363,636.36 + 18,616,557.75\ Y_{23}) + (768,000,000 + 68,261,811.76\ Y_{45}) + (698,181,818.18 + 53,191,022.14\ Y_{46}) + (5,026,909,090.91 + 446,804,586.04\ Y_{47}) + (6,004,363,636.63 + 553,683,255.55\ Y_{58}) + (488,727,272.72 + 52,127,201.704\ Y_{69})$$

$$Min\ TP = E_9 - E_1$$

$$Max\ TR = Y_{12} + Y_{23} + Y_{24} + Y_{45} + Y_{46} + Y_{47} + Y_{58} + Y_{69}$$

subject to:

$$E_1 + T_{12} = E_2$$
$$E_2 + T_{23} = E_3$$
$$E_2 + T_{24} = E_4$$
$$E_4 + T_{45} = E_5$$
$$E_4 + T_{46} = E_6$$
$$E_4 + T_{47} = E_7$$
$$E_5 + T_{58} = E_8$$
$$E_6 + T_{69} = E_9$$
$$T_{12} + Y_{12} = 3$$
$$T_{23} + Y_{23} = 16$$
$$T_{24} + Y_{24} = 7$$
$$T_{45} + Y_{45} = 4$$
$$T_{46} + Y_{46} = 6$$
$$T_{47} + Y_{47} = 5$$
$$T_{58} + Y_{58} = 5$$
$$T_{69} + Y_{69} = 4$$
$$Y_{12} - 1 \leq 0$$
$$Y_{23} - 7 \leq 0$$
$$Y_{24} - 3 \leq 0$$
$$Y_{45} - 1 \leq 0$$
$$Y_{46} - 2.5 \leq 0$$
$$Y_{47} - 2 \leq 0$$
$$Y_{58} - 2 \leq 0$$
$$Y_{69} - 1.5 \leq 0$$

and all variables are non-negative.

Note that the reason for the equality of constraints in Equation (8.1.1) is due to the assumption that there is no gap between activities. The linear programming code of this model on online linear programming solver (Zwols and Sierksma, 2015) is in Appendix A.

10.3.2 Positive and Negative Ideal Solutions

The positive and negative ideal solutions of objectives represent the best and worst values that objectives can achieve. The PIS and the NIS values of the three objectives are calculated by applying the Equations (8.1.9)-(8.1.11). The values are shown in Table 10.7. For example, since TP is aimed at minimization, the PIS value of the objective TP is 12 years and the NIS value of the objective TP is 20 years. Conversely, since TR is aimed at maximization, the PIS value of the objective TR is 20 years and the NIS value of the objective TR is zero.

Table 10.7: The PIS and the NIS values of the three objectives.

Objective Function	**Min TC**	**Min TP**	**Max TR**	**(PIS, NIS)**
TC (₺)	13,996,846,965.491	16,199,790,853.687	16,441,809,704.457	(13,996,846,965.491 ; 16,441,809,704.457)
TP (Year)	20	12	12	(12 ; 20)
TR (Year)	0	18.5	20	(20 ; 0)

Based on the values of the PIS and NIS, the membership functions of the objective functions are established using Equations (8.1.12)-(8.1.14) as follows:

$$\lambda_{TC} = \begin{cases} 1, & TC \leq 13,996,846,965.491 \\ \dfrac{16,441,809,704.457 - TC}{16,441,809,704.457 - 13,996,846,965.491}, & 13,996,846,965.491 < TC \leq 16,441,809,704.457 \\ 0, & TC > 16,441,809,704.457 \end{cases}$$

$$\lambda_{TP} = \begin{cases} 1, & TP \leq 12 \\ \dfrac{20 - TP}{20 - 12}, & 12 < TP \leq 20 \\ 0, & TP > 20 \end{cases}$$

$$\lambda_{TR} = \begin{cases} 1, & TR \geq 20 \\ \dfrac{0 - TR}{0 - 20}, & 0 \leq TR < 20 \\ 0, & TR < 0 \end{cases}$$

10.3.3 Fuzzy Programming Method

The auxiliary variable λ is added to linear programming model with the membership values for the objective functions. The membership degree of each objective has a value between 0 and 1. The fact that this value is close to 1 indicates its success to the aim. For example, if the λ_{TP} equals 1, the TP becomes 12. This value is the positive ideal solution (*PIS*) of TP, that is, the best value desired. Therefore, it is desirable that these λ values be as close to 1 as possible.

By applying Equations (8.1.15)-(8.1.18), the initial FMOLP problem is remodeled into a crisp single-goal linear programming problem. This model maximizes/minimizes values of TC, TP and TR by maximizing the overall satisfaction degree λ, depending on the constraints.

$$Max\ \lambda$$

subject to:

$$\lambda \leq \frac{16,441,809,704.457 - TC}{16,441,809,704.457 - 13,996,846,965.491}$$

$$\lambda \leq \frac{20 - TP}{20 - 12}$$

$$\lambda \leq \frac{0 - TR}{0 - 20}$$

$$TC = (33,210,601.6 + 4,427,765.3\ Y_{12}) + (349,090,909.091 + 23,271,072.19\ Y_{24}) +$$
$$(628,363,636.36 + 18,616,557.75\ Y_{23}) + (768,000,000 + 68,261,811.76\ Y_{45}) +$$

$$(698,181,818.18+53,191,022.14\,Y_{46})+(5,026,909,090.91+446,804,586.04\,Y_{47})+$$
$$(6,004,363,636.63+553,683,255.55\,Y_{58})+(488,727,272.72+52,127,201.704\,Y_{69})$$
$$TP = E_9 - E_1$$
$$TR = Y_{12}+Y_{23}+Y_{24}+Y_{45}+Y_{46}+Y_{47}+Y_{58}+Y_{69}$$

$$E_1 + T_{12} = E_2$$
$$E_2 + T_{23} = E_3$$
$$E_2 + T_{24} = E_4$$
$$E_4 + T_{45} = E_5$$
$$E_4 + T_{46} = E_6$$
$$E_4 + T_{47} = E_7$$
$$E_5 + T_{58} = E_8$$
$$E_6 + T_{69} = E_9$$
$$T_{12} + Y_{12} = 3$$
$$T_{23} + Y_{23} = 16$$
$$T_{24} + Y_{24} = 7$$
$$T_{45} + Y_{45} = 4$$
$$T_{46} + Y_{46} = 6$$
$$T_{47} + Y_{47} = 5$$
$$T_{58} + Y_{58} = 5$$
$$T_{69} + Y_{69} = 4$$
$$Y_{12} - 1 \leq 0$$
$$Y_{23} - 7 \leq 0$$
$$Y_{24} - 3 \leq 0$$
$$Y_{45} - 1 \leq 0$$
$$Y_{46} - 2.5 \leq 0$$
$$Y_{47} - 2 \leq 0$$
$$Y_{58} - 2 \leq 0$$
$$Y_{69} - 1.5 \leq 0$$

and all variables are non-negative.

The linear programming code of this model is in Appendix B.

10.3.4 FMOLP Model Result

By using online linear programming solver (Zwols and Sierksma, 2015), the results shows that $\lambda_{TC} = 0.802$, $\lambda_{TP} = 1$ and $\lambda_{TR} = 0.8$. Other results are shown in Table 10.8. Activities (a), (b), (c), (d), (e) and (h) should be crashed. Thus, the duration time (TP) for the activities will reduce from the original 20 years, as shown in Table 10.8, to 12 years. As a result, total cost (TC) is 14.480.834.021,227 ₺, total project duration time (TP) is 12 years and the total crash time (TR) is 16 years. The resulting degree of the problem (λ=0.8) appears to be a fairly high value.

Table 10.8: Results of FMOLP model.

Activity Code	Duration (Year)	Crash Time (Year)	Crash Cost (₺)
(a)	2	1	4,427,765.3
(b)	4	3	69,813,216.57
(c)	9	7	130,315,904.25
(d)	3	1	68,261,811.76
(e)	3.5	2.5	132,977,555.35
(f)	5	0	-
(g)	5	0	-
(h)	2.5	1.5	78,190,802.556
		TR = 16	**Total Crash Cost = 483,987,055.786**

10.4 Fuzzy Multiple Weighted-Objective Linear Programming

The same case applies to the proposed FMWOLP model. The main difference in both models is that the weights of the objectives are taken into account in the FMWOLP model. Five decision makers are selected and an integrated fuzzy PCM is generated.

10.4.1 Collecting Data from Decision Makers

Five decision makers gave priority to three objectives according to the rules set out in Table 8.1. The fuzzy judgment matrices of DMs are as follows:

DM 1	TC	TP	TR
TC	(1,1,1)	(4,5,6)	(6,7,8)
TP	$(\frac{1}{6},\frac{1}{5},\frac{1}{4})$	(1,1,1)	(3,4,5,6)
TR	$(\frac{1}{8},\frac{1}{7},\frac{1}{6})$	$(\frac{1}{6},\frac{1}{5},\frac{1}{4},\frac{1}{3})$	(1,1,1)

DM 3	TC	TP	TR
TC	(1,1,1)	$(\frac{1}{8},\frac{1}{7},\frac{1}{6})$	$(\frac{1}{6},\frac{1}{5},\frac{1}{4})$
TP	(6,7,8)	(1,1,1)	(2,3,4,5)
TR	(4,5,6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3},\frac{1}{2})$	(1,1,1)

DM 2	TC	TP	TR
TC	(1,1,1)	(5,6,7)	(7,8,9)
TP	$(\frac{1}{7},\frac{1}{6},\frac{1}{5})$	(1,1,1)	(1,2,3,4)
TR	$(\frac{1}{9},\frac{1}{8},\frac{1}{7})$	$(\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{1}{1})$	(1,1,1)

DM 4	TC	TP	TR
TC	(1,1,1)	(7,8,9)	(6,7,8)
TP	$(\frac{1}{9},\frac{1}{8},\frac{1}{7})$	(1,1,1)	(2,3,4,5)
TR	$(\frac{1}{8},\frac{1}{7},\frac{1}{6})$	$(\frac{1}{5},\frac{1}{4},\frac{1}{3},\frac{1}{2})$	(1,1,1)

DM 5	TC	TP	TR
TC	(1,1,1)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$
TP	(2,3,4)	(1,1,1)	(1,1,1,1)
TR	(2,3,4)	(1,1,1,1)	(1,1,1)

10.4.2 Integrated Fuzzy Pairwise Comparison Matrix

As shown in Section 8.2.1, by collecting the fuzzy judgment matrices from all decision makers, integrated fuzzy PCM is constructed in Table 10.9.

Table 10.9: Integrated fuzzy pairwise comparison matrix.

	TC	TP	TR
TC	1	(1.34, 1.62, 1.99)	(1.6, 1.92, 2.35)
TP	$(\frac{1}{1.99},\frac{1}{1.62},\frac{1}{1.34})$	1	(1.64, 2.35, 2.99, 3.59)
TR	$(\frac{1}{2.35},\frac{1}{1.92},\frac{1}{1.6})$	$(\frac{1}{3.59},\frac{1}{2.99},\frac{1}{2.35},\frac{1}{1.64})$	1

10.4.3 Importance Weights

The degrees of importance for three objectives are found in two steps. First step is to convert integrated fuzzy PCM in Table 10.9 to interval approximation PCM in Table 10.10 by using the nearest interval approximation. The second step is the construct the GP model. As stated in Section 8.2.2, this method can be used to derive weight from consistent and inconsistent matrices.

10.4.3.1 Nearest Interval Approximation

Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number and let $f(a) = (na^{n-1},\ na^{n-1})$ be a weighted function.

$$\text{NWIA } f(A) = \left[\frac{a+nb}{n+1}, \frac{nb+c}{n+1}\right]$$

Set n = 5 for high sensitivity (Izadikhah, 2012) and apply the formula accordingly. So,

$$f(a) = (5a^4, 5a^4)$$

$$\text{NWIA } f(A) = \left[\frac{a+5b}{6}, \frac{5b+c}{6}\right]$$

Using the above statements, the integrated fuzzy PCM in Table 10.9 is converted to interval approximation PCM in Table 10.10. Thus, the decisions are ready for use in the GP model.

Table 10.10: Interval approximation pairwise comparison matrix.

	TC	**TP**	**TR**
TC	[1.0 1.0]	[1.573 1.6812]	[1.86 1.992]
TP	[0.598 0.638]	[1.0 1.0]	[2.231 3.09]
TR	[0.505 0.538]	[0.325 0.456]	[1.0 1.0]

10.4.3.2 Goal Programming

The GP model is constructed to derive weights of three objectives from the above interval approximation PCM based on the Equations (8.2.9)-(8.2.12) as folllows:

$$Min\ p_{12}^{+} + q_{12}^{+} + p_{13}^{+} + q_{13}^{+} + p_{21}^{+} + q_{21}^{+} + p_{23}^{+} + q_{23}^{+} + p_{31}^{+} + q_{31}^{+} + p_{32}^{+} + q_{32}^{+}$$

subject to:

$$
\begin{gathered}
1.573w_2 - w_1 + p_{12}^- - p_{12}^+ = 0 \\
w_1 - 1.6812w_2 + q_{12}^- - q_{12}^+ = 0 \\
1.86w_3 - w_1 + p_{13}^- - p_{13}^+ = 0 \\
w_1 - 1.992w_3 + q_{13}^- - q_{13}^+ = 0 \\
0.598w_1 - w_2 + p_{21}^- - p_{21}^+ = 0 \\
w_2 - 0.638w_1 + q_{21}^- - q_{21}^+ = 0 \\
2.231w_3 - w_2 + p_{23}^- - p_{23}^+ = 0 \\
w_2 - 3.09w_3 + q_{23}^- - q_{23}^+ = 0 \\
0.505w_1 - w_3 + p_{31}^- - p_{31}^+ = 0 \\
w_3 - 0.538w_1 + q_{31}^- - q_{31}^+ = 0 \\
0.325w_2 - w_3 + p_{32}^- - p_{32}^+ = 0 \\
w_3 - 0.456w_2 + q_{32}^- - q_{32}^+ = 0 \\
w_1 + w_2 + w_3 = 1 \\
w_i,\ p_{ij}^-,\ p_{ij}^+,\ q_{ij}^-,\ q_{ij}^+ \geq 0
\end{gathered}
$$

By solving this model on online linear optimization solver application (Zwols and Sierksma, 2015), the weights of objective functions are obtained as $w_{TC} = 0.467288$, $w_{TP} = 0.2981297$, $w_{TR} = 0.2345823$. The linear programming code of this model is in Appendix C.

10.4.4 FMWOLP Model

By adding the weights of objectives to the linear programming model, it is aimed to change the membership values of the objectives depending on their weights. The overall satisfaction degree λ is maximized. However, in this weighted model, the membership degrees of the objectives are included in the increase in different rates. For example, the membership value of the highest weighted objective increases more than others. This increases the λ of the highest weighted objective more and it makes the value of the objective come closer to its *PIS* value. In other words, the weights given according to the opinions of the DMs decide which objective should be closer to its aim in the model. Therefore, based on Section 8.2.3, the FMWOLP model is formulated as follows:

$$Max\ \lambda = 0.467288\lambda_{TC} + 0.2981297\lambda_{TP} + 0.2345823\lambda_{TR}$$

subject to:

$$\lambda_{TC} \leq \frac{16,441,809,704.457 - TC}{16,441,809,704.457 - 13,996,846,965.491}$$

$$\lambda_{TP} \leq \frac{20 - TP}{20 - 12}$$

$$\lambda_{TR} \leq \frac{0 - TR}{0 - 20}$$

$$TC = (33,210,601.6 + 4,427,765.3\ Y_{12}) + (349,090,909.091 + 23,271,072.19\ Y_{24}) +$$
$$(628,363,636.36 + 18,616,557.75\ Y_{23}) + (768,000,000 + 68,261,811.76\ Y_{45}) +$$
$$(698,181,818.18 + 53,191,022.14\ Y_{46}) + (5,026,909,090.91 + 446,804,586.04\ Y_{47}) +$$
$$(6,004,363,636.63 + 553,683,255.55\ Y_{58}) + (488,727,272.72 + 52,127,201.704\ Y_{69})$$
$$TP = E_9 - E_1$$
$$TR = Y_{12} + Y_{23} + Y_{24} + Y_{45} + Y_{46} + Y_{47} + Y_{58} + Y_{69}$$

$$E_1 + T_{12} = E_2$$
$$E_2 + T_{23} = E_3$$
$$E_2 + T_{24} = E_4$$
$$E_4 + T_{45} = E_5$$
$$E_4 + T_{46} = E_6$$
$$E_4 + T_{47} = E_7$$
$$E_5 + T_{58} = E_8$$
$$E_6 + T_{69} = E_9$$
$$T_{12} + Y_{12} = 3$$
$$T_{23} + Y_{23} = 16$$
$$T_{24} + Y_{24} = 7$$
$$T_{45} + Y_{45} = 4$$
$$T_{46} + Y_{46} = 6$$
$$T_{47} + Y_{47} = 5$$

$$T_{58} + Y_{58} = 5$$
$$T_{69} + Y_{69} = 4$$
$$Y_{12} - 1 \leq 0$$
$$Y_{23} - 7 \leq 0$$
$$Y_{24} - 3 \leq 0$$
$$Y_{45} - 1 \leq 0$$
$$Y_{46} - 2.5 \leq 0$$
$$Y_{47} - 2 \leq 0$$
$$Y_{58} - 2 \leq 0$$
$$Y_{69} - 1.5 \leq 0$$

and all variables are non-negative.

The linear programming code of this model is in Appendix D.

10.4.5 FMWOLP Model Result

After solving FWMOLP model, the results show that $TC = 0.83$, $TP = 1$ and $TR = 0.75$. Other results are shown in Table 10.11. Activities (a), (b), (c), (e) and (h) should be crashed. Thus, the duration time for the activities will reduce from the original 20 years, as shown in Table 10.11, to 12 years. As a result, total cost (TC) is 14,412,572,209.517 ₺, total project duration time (TP) is 12 years and the total crash time (TR) is 15 years. The resulting degree of the problem (λ = 0.86) is a fairly high value to raise membership values.

Table 10.11: Results of FMWOLP model.

Activity Code	Duration (Year)	Crash Time (Year)	Crash Cost (₺)
(a)	2	1	4,427,765.3
(b)	4	3	69,813,216.57
(c)	9	7	130,315,904.25
(d)	4	0	-
(e)	3.5	2.5	132,977,555.35
(f)	5	0	-
(g)	5	0	-
(h)	2.5	1.5	78,190,802.556
		TR = 15	**Total Crash Cost = 415,725,244.026**

11 DISCUSSION AND CONCLUSION

The impact of FMOLP and FMWOLP models on the nuclear power plant project has been extensively studied in this thesis. This section begins with a general evaluation of the study and is followed by a discussion of the application results. Different areas are proposed for future studies after the evaluation of limitations.

11.1 Thesis Overview

In order to stabilize the growth of countries and the need for electricity, renewable, non-renewable and nuclear energy must be used. Nuclear energy is one of the most practical way to meet energy demands. That is why nuclear power plant installation seems to be an important subject for the country's development. In this study, two approaches are proposed to enhance the utilization of FMOLP for solving nuclear power plant installation problem in Turkey. The proposed method uses fuzzy logic approach to make decisions for determination of objective weights. A fuzzy mathematical model is proposed for the project and weighted fuzzy logic for the same problem is re-modeled. Three main objective functions have been identified for the plant installation such as; total cost, total time and total crash time. These objectives are modeled by FMOLP. The satisfaction degrees are determined for all objectives and these values are maximized with the established model. The goal is to achieve the lowest cost, the minimum total time and the highest crash time optimization. The same problem is then used by weighting objective functions. First, with the help of decision makers, nearest interval approximation and goal programming techniques are used for criterion weighting. Then, FMWOLP model is established. The highest satisfaction is obtained. The results of the two models are evaluated. It is shown that the model using weighting gives better results for all objectives than the first model. The main contribution of this study to the existing literature is the use of nearest interval approximation and goal programming to obtain the importance weight of the objectives in FMWOLP models.

11.2 Results and Discussion

According to FMOLP model, activities (a), (b), (c), (d), (e) and (h) should be crashed. Thus, TP reduces from the original 20 years to 12 years. TC is 14,480,834,021.227 ₺ and TR is 16 years. According to FMWOLP model, activities (a), (b), (c), (e) and (h) should be crashed. TC is 14,412,572,209.517 ₺, TP is 12 years and TR is 15 years. Based on data from DMs, TC becomes more important than other objectives. Therefore, TC is reduced by the change in activity times, although TP does not change. The change in TR does not affect TP since activity (d) is not in the critical path. The comparison of the two models are shown in Table 11.1.

Table 11.1: Results of FMOLP and FMWOLP models.

	λ	λ_{TC}	λ_{TP}	λ_{TR}	**TC (₺)**	**TP (Year)**	**TR (Year)**
FMOLP	0.8	0.802	1	0.8	14,480,834,021.277	12	16
FMWOLP	0.86	0.83	1	0.75	14,412,572,209.517	12	15

λ is the overall satisfaction degree of the model and λ_{TC}, λ_{TP} and λ_{TR} are membership function of three objectives. These values can be considered as the level of success. For example, λ_{TP} equals 1 in both models. This means that the objective of minimizing the total project time has been at the highest level. According to PIS and NIS values of TP, the total duration can be minimum 12 years. Since this objective is achieved in the linear programming models, the λ_{TP} is equal to 1. Unlike, λ_{TC} and λ_{TR} value are not equal to 1 in both models. This means that the objectives of minimizing TC and maximizing TR could not be achieved at the highest level. They can still be minimized and maximized, respectively. However, since the model has three objective functions at the same time, these values are normal. If all values were met in the highest way, all lambdas would be 1 and there was no need to build a model to balance time and cost. If these values are close to 1, it shows how close the model is to the aim.

The TC and TP objective functions aim at minimization. Therefore, the increase in membership values λ_{TC} and λ_{TP} causes the value of TC and TP to decrease. On the contrary, the TR objective function aims at maximization and the decrease in the membership level causes the value of TR to decrease.

While the total project time TP is the same, the FMWOLP model crashes less activity and prevents the additional cost of 68,261,811.76 ₺. So, in nuclear power plant installation project, FMWOLP model presents a less costly project plan in the same total project time TP with FMOLP model, because overall satisfaction degree (λ) increases in FMWOLP model and thus, the satisfaction degree of TC (λ_{TC}) increases and the total cost aimed at minimization is reduced.

Consequently, by combining project planning techniques and multi-objective decision making models, considering the time-cost balance, this study succeeded in shortening the total project time and also reducing the total cost with the help of the weights determined by the opinions of the decision-makers in the nuclear power plant installation project. The methods used enable us to reach the goals, which shows that the techniques are suitable for application.

11.2.1 Sensitivity Analysis

While applying the nearest interval approximation method in Section 10.4.3.1, the parameter n is set as $n = 5$. In this section, the change of the parameter n between 1 and 10 is observed and its effect on the result is examined.

As the parameter n increases, the ranges of nearest interval values become narrower. Thus, more precise data emerges. Depending on the change in the parameter n , interval approximation PCMs are given in Table 11.2.

Table 11.2: Interval approximation pairwise comparison matrices according to the change in the parameter n.

n=1	**TC**	**TP**	**TR**
TC	[1.0 1.0]	[1.48 1.805]	[1.76 2.135]
TP	[0.56 0.681]	[1.0 1.0]	[1.995 3.29]
TR	[0.473 0.572]	[0.306 0.517]	[1.0 1.0]

n=2	**TC**	**TP**	**TR**
TC	[1.0 1.0]	[1.526 1.743]	[1.813 2.063]
TP	[0.579 0.660]	[1.0 1.0]	[2.113 3.19]
TR	[0.489 0.555]	[0.316 0.487]	[1.0 1.0]

n=3	TC	TP	TR
TC	[1.0 1.0]	[1.55 1.712]	[1.84 2.027]
TP	[0.588 0.649]	[1.0 1.0]	[2.172 3.14]
TR	[0.497 0.547]	[0.320 0.471]	[1.0 1.0]

n=4	TC	TP	TR
TC	[1.0 1.0]	[1.564 1.694]	[1.856 2.006]
TP	[0.594 0.643]	[1.0 1.0]	[2.208 3.11]
TR	[0.502 0.542]	[0.323 0.462]	[1.0 1.0]

n=5	TC	TP	TR
TC	[1.0 1.0]	[1.573 1.6812]	[1.86 1.992]
TP	[0.598 0.638]	[1.0 1.0]	[2.231 3.090]
TR	[0.505 0.538]	[0.325 0.456]	[1.0 1.0]

n=6	TC	TP	TR
TC	[1.0 1.0]	[1.58 1.673]	[1.874 1.981]
TP	[0.601 0.636]	[1.0 1.0]	[2.248 3.076]
TR	[0.507 0.535]	[0.326 0.452]	[1.0 1.0]

n=7	TC	TP	TR
TC	[1.0 1.0]	[1.585 1.666]	[1.88 1.974]
TP	[0.603 0.633]	[1.0 1.0]	[2.261 3.065]
TR	[0.509 0.534]	[0.327 0.448]	[1.0 1.0]

n=8	TC	TP	TR
TC	[1.0 1.0]	[1.589 1.661]	[1.88 1.967]
TP	[0.604 0.631]	[1.0 1.0]	[2.271 3.056]
TR	[0.510 0.532]	[0.328 0.446]	[1.0 1.0]

n=9	TC	TP	TR
TC	[1.0 1.0]	[1.592 1.657]	[1.888 1.963]
TP	[0.606 0.630]	[1.0 1.0]	[2.279 3.05]
TR	[0.511 0.531]	[0.329 0.444]	[1.0 1.0]

n=10	TC	TP	TR
TC	[1.0 1.0]	[1.595 1.654]	[1.89 1.959]
TP	[0.607 0.629]	[1.0 1.0]	[2.285 3.044]
TR	[0.512 0.53]	[0.329 0.442]	[1.0 1.0]

According to the applied model in Section 10.4.3.2, the goal programming model is constructed for the above interval approximation PCMs. By solving these models on online linear optimization solver application (Zwols and Sierksma, 2015), the weights of objective functions are obtained as in Table 11.3. As the parameter n increases, w_{TP} decreases, w_{TC} and w_{TR} increase very slowly.

Table 11.3: Weights of objective functions according to the change in the parameter n.

	w_{TC}	w_{TP}	w_{TR}
n=1	0.4652496	0.316835	0.2179155
n=2	0.4662589	0.3077309	0.2260102
n=3	0.4667793	0.3029398	0.2302809
n=4	0.4669614	0.3002562	0.2327824
n=5	0.467288	0.2981297	0.2345823
n=6	0.4671161	0.2970858	0.2357981
n=7	0.4673802	0.2958517	0.2367681
n=8	0.4674233	0.2949441	0.2376326
n=9	0.4674155	0.2944717	0.2381128
n=10	0.4674067	0.2939988	0.2385945

For each weight group, the FMWOLP model is rebuilt. Comparative data of the FMWOLP model for the change in the parameter n are given in Table 11.4.

Table 11.4: Results of FMWOLP model according to the change in the parameter n.

FMWOLP	λ	λ_{TC}	λ_{TP}	λ_{TR}	**TC (₺)**	**TP (Year)**	**TR (Year)**
n=1	0.8664133	0.83	1	0.75	14,412,572,209.517	12	15
n=2	0.8642179	0.83	1	0.75	14,412,572,209.517	12	15
n=3	0.8630617	0.83	1	0.75	14,412,572,209.517	12	15
n=4	0.8624054	0.83	1	0.75	14,412,572,209.517	12	15
n=5	0.8618999	0.83	1	0.75	14,412,572,209.517	12	15
n=6	0.8616252	0.83	1	0.75	14,412,572,209.517	12	15
n=7	0.8613377	0.83	1	0.75	14,412,572,209.517	12	15
n=8	0.8611143	0.83	1	0.75	14,412,572,209.517	12	15
n=9	0.8609956	0.83	1	0.75	14,412,572,209.517	12	15
n=10	0.8608766	0.83	1	0.75	14,412,572,209.517	12	15

As the parameter n increases, overall satisfaction degree λ is slightly decreasing. However, λ_{TC}, λ_{TP} and λ_{TR} do not change. Because small changes in weight values do not affect the main model. But, small variations in weights change the overall lambda value slightly. Finally, the value (5) for the parameter n used in nearest interval approximation method in application proved to be valid because its change do not affect the result.

11.2.2 Consistency of Matrices

As mentioned in the Section 8.2.2, the combined nearest interval approximation and goal programming method is able to derive weights from fuzzy pairwise comparison matrices even if the matrices are inconsistent. In this section, the data used in the study and the results of the method are examined.

Firstly, the consistency ratios of the matrices of five decision makers used in the application are calculated by Saaty's method. He developed an "index based on the principal eigenvalue of the matrix of pairwise comparisons to measure inconsistency" (Saaty, 1980).

"Let $A = a_{ij}$ be a reciprocal matrix of order n. Let λ_{max} be the principal eigenvalue of A. It is known that A is reciprocal and $a_{ij}a_{jk} = a_{ik},\ for\ all\ i, j, k$, that is, A is a consistent matrix, then $\lambda_{max} = n$, and that in general $\lambda_{max} \geq n$" (Saaty, 1980) . Since the trace of the matrix is equal to the sum of its eigenvalues,

$$CI = \frac{\lambda_{max} - n}{n-1} \tag{11.2.1}$$

Saaty defined the consistency ratio (CR) as:

$$CR = \frac{CI}{RI} \tag{11.2.2}$$

where random consistency index (RI) is the average value of consistency index (CI) for random matrices using the Saaty scale.

According to these formulas, the consistency ratios of the matrices used in the Section 10.4 are given in Table 11.5.

Table 11.5: Consistency ratios of matrices used in Section 10.4.

	DM1	**DM2**	**DM3**	**DM4**	**DM5**	**Interval approximation PCM**
CR	12.9%	7.7%	10.0%	19.0%	0.0%	2.0%

As in the table, the consistency ratios of decision makers' pairwise comparison matrices are between 0.0% and 19.0%. According to the method of Saaty, values over ten percent ($CR \geq 10.0\%$) are considered inconsistent. However, the CR of the interval approximation PCM generated from the integrated fuzzy PCM is 2.0%. In this case, the combined nearest interval approximation and goal programming method actually generates weights from a consistent interval approximation PCM. But even if this matrix was inconsistent, the method could derive weight (Izadikhah, 2012).

11.3 Limitations

Nuclear power plant installation is a controversial issue. It is the structure that societies frequently oppose but it is politically needed. Thanks to its high energy generation capacity, it is essential for future generations. However, it is a very risky project considering past accidents and their terrible consequences. For this reason, there are many procedures, approvals, social acceptance and error-free stages. In this study, only project planning steps consisting of technical issues and deterministic costs are used. The controversial stages of the nuclear power plant have not been applied to the techniques. Therefore, the study includes only the structure aimed at balancing the cost and time. It does not carry social, political and psychological elements. With the impact of all other external factors, the course of the project may change, new costs may arise, but it is difficult to accommodate all these elements in a single study.

Another limitation is reaching decision makers and their ideas. It was not easy to find experts who know the nuclear power plant projects and could give their statements with fuzzy numbers. In addition, these ideas may change over time and lose their validity in the future. Similarly, the techniques used may become invalid because of new procedures and steps of nuclear power plant installation project in the future. This study dealt with a project in the conditions of the period it was written. It created a project plan by establishing the time-cost balance with fuzzy multi objective linear programming model and its new version that combines nearest interval approximation and goal programming methods.

11.4 Further Research

For further research, there are a number of recommendations. Firstly, FMOLP and FMWOLP can be extended so as to work with hesitant fuzzy sets. Another type of integration of multi objective decision making methods such as interval type-2 fuzzy sets, intuitionistic fuzzy sets, pythagorean fuzzy sets, spherical fuzzy sets and neutrosophic fuzzy sets can also be proposed. Secondly, non-linearity in membership functions can be considered and new techniques should be tried in weight calculation since the function structure changes. The proposed model can be applied to different installation projects, such as shipbuilding, waste facility, recycling buildings etc. Finally, the results from these new integrations can be compared with this thesis.

REFERENCES

Aaltonen, K., Kujala, J., Havela, L., and Savage, G. (2015). Stakeholder dynamics during the project front-end: The case of nuclear waste repository projects. *Project Management Journal*, 46(6):15–41.

Abbasbandy, S. and Amirfakhrian, M. (2006a). The nearest approximation of a fuzzy quantity in parametric form. *Applied Mathematics and Computation*, 172(1):624–632.

Abbasbandy, S. and Amirfakhrian, M. (2006b). The nearest trapezoidal form of a generalized left right fuzzy number. *International Journal of Approximate Reasoning*, 43(2):166–178.

Abbasbandy, S. and Asady, B. (2004). The nearest trapezoidal fuzzy number to a fuzzy quantity. *Applied Mathematics and Computation*, 156(2):381–386.

Abdelgawad, M. and Fayek, A. R. (2010). Risk management in the construction industry using combined fuzzy fmea and fuzzy ahp. *Journal of Construction Engineering and Management*, 136(9).

Abro, S. H., Shah, S. A. A., Alaboodi, A. A., and Shoaib, T. (2020). Ageing analysis of power cable used in nuclear power plant. *Mehran University Research Journal of Engineering and Technology*, 39(1).

Aczel, C. and Saaty, T. L. (1973). Procedures for synthesizing ratio judgments. *Journal of Mathematical Psychology*, 27:93–102.

Aczél, J. and Alsina, C. (1986). On synthesis of judgments. *Socio-Econ. Planning Sci.*, 20:333–339.

Aczél, J. and Alsina, C. (1987). Synthesizing judgements: A functional equations approach. *Mathematical Modelling*, 9:311–320.

Adalı, E. A. and Işık, A. T. (2017). The multi-objective decision making methods based on multimoora and moosra for the laptop selection problem. *Journal of Industrial Engineering International*, 13:229–237.

Ahmadian, A., Senu, N., Salahshour, S., and Suleiman, M. (2016). Nearest interval-valued approximation of interval-valued fuzzy numbers. *MALAYSIAN JOURNAL OF MATHEMATICAL SCIENCES*, 10:325–336.

Albayrak, B. (2001). *Proje Yönetimi ve Proje Danışmanlığı, Birinci Baskı*. Beta Yayınevi, İstanbul.

Alcantud, J. C. R. and Torra, V. (2018). Decomposition theorems and extension principles for hesitant fuzzy sets. *Information Fusion*, 41:48–56.

Alizdeh, S. and Saeidi, S. (2020). Fuzzy project scheduling with critical path including risk and resource constraints using linear programming. *International Journal of Advanced Intelligence Paradigms*, 16(1).

Alonso, J. and Lamata, M. (2006). Consistency in the analytic hierarcy process: A new approach. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 14(4):445–459.

Alp, S. (2008). Doğrusal hedef programlama yönteminin otobüsle kent içi toplu taşıma sisteminde kullanılması. *Istanbul Ticaret Üniversitesi Fen Bilimleri Dergisi*, 13:73–91.

Alp, S., Yavuz, E., and Ersoy, N. (2011). Using linear goal programming in surveying engineering for vertical network adjustment. *International Journal of the Physical Sciences*, 6:1982–1987.

Alsawy, A. A. and Hefny, H. A. (2013). On uncertain granular numbers. *International Journal of Computer Applications*, 62(18):20–27.

Altaş, H. (1999). Bulanık mantık: Bulanıklılık kavramı. *Enerji, Elektrik, Elektromekanik-3e, Bileşim Yayıncılık A.Ş.*, 62:80–85.

Amid, A., Ghodsypour, S. H., and O'Brien, C. (2006). Fuzzy multiobjective linear model for supplier selection in a supply chain. *International Journal of Production Economics*, 104(2):394–407.

Ansoff, H., Declerk, R., and Hayes, R. (1976). *From Strategic Planning to Strategic Management.* John Wiley and Sons, New York.

Anukokila, P., Radhakrishnan, B., and Anju, A. (2019). Goal programming approach for solving multi-objective fractional transportation problem with fuzzy parameters. *RAIRO Operations Research*, 53(1):157 – 178.

Arani, S. D. and Sadegheih, A. (2020). A fuzzy goal programming for dynamic cell formation and production planning problem together with pricing and advertising decisions. *International Journal of Industrial Engineering Production Research*, 31(1):13–34.

Araz, C., Ozfirat, P. M., and Ozkarahan, I. (2007). An integrated multicriteria decision-making methodology for outsourcing management. *Comput Oper Res*, 34(12):3738–3756.

Asady, B. and Zendenam, A. (2007). Ranking of fuzzy numbers by distance minimization. *Applied Mathematical Modeling*, 31:2589–2598.

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Set Syst*, 20:87–96.

Atanassov, K. T. (2017). Type-1 fuzzy sets and intuitionistic fuzzy sets. *Algorithms*, 10(3).

Azmi, R. and Tamiz, M. (2010). A review of goal programming for portfolio selection. *New Developments in Multiple Objective and Goal Programming*, 638):15–33.

Ban, A. I. (2006). Nearest interval approximation of an intuitionistic fuzzy number. *Computational Intelligence, Theory and Applications*, pages 229–240.

Ban, A. I. and Coroianu, L. (2012). Nearest interval, triangular and trapezoidal approximation of a fuzzy number preserving ambiguity. *International Journal of Approximate Reasoning*, 53(5):805–836.

Bellman, R. and Zadeh, L. (1970). Decision making in a fuzzy environment. *Management Science*, 17(4):141–164.

Benítez, J., Delgado-Galvána, X., Izquierdo, J., and Pérez-García, R. (2011). Achieving matrix consistency in ahp through linearization. *Applied Mathematical Modelling*, 35(9):4449–4457.

Bi, L., Zeng, Z., Hu, B., and Dai, S. (2019). Two classes of entropy measures for complex fuzzy sets. *Mathematics*, 7(1).

Bortolan, G. and Degani, R. (1985). A review of some methods for ranking fuzzy subsets. *Fuzzy Sets and Systems*, 15:1–19.

Bos, E., Vu, M. T., Leven, A., and Bulatao, R. A. (1992). *World Populalion Projections 1992-1993*. Johns Hopkins University Press, Baltimore.

Brunelli, M., Canal, L., and Fedrizzi, M. (2013). Inconsistency indices for pairwise comparison matrices: a numerical study. *Annals of Operations Research*, 211(1):493–509.

Buckley, J. J. (1985a). Fuzzy hierarchical analysis. *Fuzzy Sets and Systems*, 17(3):233–247.

Buckley, J. J. (1985b). Ranking alternatives using fuzzy numbers. *Fuzzy Sets and Systems*, 15(1):21–31.

Burke, R. (2013). *Project management: planning and control techniques*. New Jersey, USA.

Bustince, H. and Burillo, P. (1995). Correlation of interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 74(2):237–244.

Çakır, E. (2019). Fuzzy modm approach for nuclear. Last accessed 14.11.2019: `https://github.com/esrckr/Fuzzy-MODM-approach-for-nuclear-/tree/master`.

Çakır, E. and Ulukan, Z. (2019). Fuzzy multi-objective decision making approach for nuclear power plant installation. In *Intelligent and Fuzzy Techniques in Big Data Analytics and Decision Making*, pages 1258–1265. Springer-Verlag.

Çakır, E. and Ulukan, Z. (2020). Fuzzy multi-objective decision making approach for nuclear power plant installation. *Journal of Intelligent & Fuzzy Systems,*, Pre-press. DOI:`10.3233/JIFS-189101`.

Calabrese, A., Costa, R., and Menichini, T. (2013). Using fuzzy ahp to manage intellectual capital assets: An application to the ict service industry. *Expert Syst Appl*, 40(9):3747–3755.

Carlsson, C. and Fullér, R. (1996). Fuzzy multiple criteria decision making: Recent developments. *Fuzzy Set Syst*, 78(2):139–153.

Case, D. M. and Stylios, C. D. (2016). Fuzzy cognitive map to model project management problems. In *2016 Annual Conference of the North American Fuzzy Information Processing Society (NAFIPS)*, pages 1–6.

Castle, K. (2019). Project crashing. Last accessed 10.08.2019: `https://www.designingbuildings.co.uk/wiki/Project_crashing`.

Cavallo, B. (2019). Coherent weights for pairwise comparison matrices and a mixed-integer linear programming problem. *Journal of Global Optimization*, 75:143–161.

Cavallo, B. and D'Apuzzo, L. (2020). Preservation of preferences intensity of an inconsistent pairwise comparison matrix. *International Journal of Approximate Reasoning*, 116:33–42.

Cavallo, B. and D'Apuzzo, L. (2011). Deriving weights from a pairwise comparison matrix over an alo-group. *Soft Computing*, 16(2):353–366.

Chanas, S. (2001). On the interval approximation of a fuzzy number. *Fuzzy Sets and Systems*, 122(2):353–356.

Chang, C. T. (2007). Efficient structures of achievement functions for goal programming models. *Asia-Pacific Journal of Operational Research*, 24(6):755–764.

Chang, D. Y. (1996). Applications of the extent analysis method on fuzzy ahp. *Eur J Oper Res*, 649-655:95(3).

Chang, S. H., Kang, K. S., Choi, S. S., Kim, H. G., Jeong, H. K., and Yi, C. U. (1995). Development of the on-line operator aid system oasys using a rule-based expert system and fuzzy logic for nuclear power plants. *Nuclear Technology*, 112(2):266–294.

Charnes, A. (1955). Optimal estimation of executive compensation by linear programming. *Management Science*, 2(1):138–151.

Charnes, A. and Cooper, W. (1961). *Management Models and Industrial Applications of Linear Programming*. Wiley, New York.

Chen, C. T. and Huang, S. (2007). Applying fuzzy method for measuring criticality in project network. *Information Sciences*, 177(12)A fuzzy approach to construction project risk assessment:2448–2458.

Chen, G., Pham, T. T., and Boustany, N. M. (2001). Introduction to fuzzy sets, fuzzy logic, and fuzzy control systems. *ASME. Appl. Mech. Rev*, 54(6):B102–B103.

Chen, S. J. and Hwang, C. L. (1992a). *Fuzzy Multiple Attribute Decision Making*. Springer, Verlag.

Chen, S. J. and Hwang, C. L. (1992b). Fuzzy multiple attribute decision making methods. *Lecture Notes in Economics and Matematical Systems*, 375:298–486.

Cho, Y. and Cho, K. (2008). A loss function approach to group preference aggregation in the ahp. *Computers Operations Research*, 35(3):884 – 892.

Choo, E. U. and Wedley, W. C. (2004). A common framework for deriving preference values from pairwise comparison matrices. *Computers Operations Research*, 31(6):893–908.

Cleland, D. I. (1990). *Project Management: Strategic Design and Implementation*. TAB Professional and Reference Books, USA.

Contini, B. (1968). A stochastic approach to goal programming. *Operations Research*, 16(3):576–586.

Cooke, S. (2009). *In Mortal Hands: A Cautionary History of the Nuclear Age*. Bloomsbury.

Coroianu, L., Gagolewski, M., and Grzegorzewski, P. (2013). Nearest piecewise linear approximation of fuzzy numbers. *Fuzzy Sets and Systems*, 233:26–51.

Costa, R. G., Mol, A. C. A., Carvalho, P. V. R., and Lapa, C. M. F. (2011). An efficient neuro-fuzzy approach to nuclear power plant transient identification. *Annals of Nuclear Energy*, 38(6):1418–1426.

Crowder, L. J. and Sposito, V. A. (1987). Comments on "an algorithm for solving the linear goal-programming problem by solving its dual". *The Journal of the Operational Research Society*, 38:335–340.

Csutora, R. and Buckley, J. J. (2001). Fuzzy hierarchical analysis: the lambda-max method. *Fuzzy Sets and Systems*, 120:181–195.

Dantzig, G. B. (1948). Programming in a linear structure. Washington DC.

Dantzig, G. B. (1961). *Linear Programming and Extensions*. Princeton University Press, New Jersey.

Delgado, M. R., Gomez-Skennata, A. F., and Martin, F. (1997). Rapid prototyping for fuzzy models. *Fuzzy Model Identification: Selected Approaches*, pages 121–161.

Deng, H. (1999). Multicriteria analysis with fuzzy pairwise comparison. *International Journal of Approximate Reasoning*, 21(3):215–231.

Dong, J. and Wan, S. (2019). A new method for solving fuzzy multi-objective linear programming problems. *Iranian Journal of Fuzzy Systems*, 16(3):145–159.

Dweiri, F. T. and Kablan, M. M. (2006). Using fuzzy decision making for the evaluation of the project management internal efficiency. *Decision Support Systems*, 42(2):712–726.

Egrioglu, E., Bas, E., Yolcu, U., and Chene, M. (2020). Picture fuzzy time series: Defining, modeling and creating a new forecasting method. *Engineering Applications of Artificial Intelligence*, 88.

Ekmekçioglu, M., Kutlu, A. C., and Kahraman, C. (2011). A fuzzy multi-criteria swot analysis: An application to nuclear power plant site selection. *International Journal of Computational Intelligence Systems*, 4(4):583–595.

El-Wahed, A., Waiel, F., and Lee, S. M. (2006). Interactive fuzzy goal programming for multi-objective transportation problems. *Omega*, 34(2):158–166.

Erdoğan, M. and Kaya, I. (2016). A combined fuzzy approach to determine the best region for a nuclear power plant in turkey. *Applied Soft Computing*, 39:84–93.

Ergu, D., Kou, G., Peng, Y., and Shi, Y. (2011). A simple method to improve the consistency ratio of the pairwise comparison matrix in anp. *European Journal of Operational Research*, 213(1):246–259.

Erol, I., Sencer, S., Özmen, A., and Searcy, C. (2014). Fuzzy mcdm framework for locating a nuclear power plant in turkey. *Energy Policy*, 67:186–197.

ETKB (2014a). Dünyada nükleer güç santralleri. Last accessed 15.12.2019: `https://nukleer.enerji.gov.tr/tr-TR/Sayfalar/Dunyada-Nukleer-Guc-Santralleri`.

ETKB (2014b). Nükleer santraller ve Ülkemizde kurulacak nükleer santrale İlişkin bilgiler. *Nükleer Enerji Proje Uygulama Dairesi Başkanlığı*, Yayın no: 1.

ETKB (2017). Nükleer güç santralleri ve türkiye. *Nükleer Enerji Proje Uygulama Dairesi Başkanlığı*, Yayın no: 2.

Evren, R. and Ülengin, F. (1992). *Yönetimde Çok Amaçlı Karar Verme*. I.T.Ü. Matbaası, İstanbul.

Fantoni, P. F. and Mazzola, A. (1996). Multiple-failure signal validation in nuclear power plants using artificial neural networks. *Nuclear Technology*, 113(3):368–374.

Fayek, A. R. (2020). Fuzzy logic and fuzzy hybrid techniques for construction engineering and management. *Journal of Construction Engineering and Management*, 146(7).

Garg, H. and Rani, D. (2019). Some results on information measures for complex intuitionistic fuzzy sets. *International Journal of Intelligent Systems*, 34(10):2319–2363.

Garibaldi, J. M., Jaroszewski, M., and Musikasuwan, S. (2008). Nonstationary fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 1072-1086:16(4).

Gerstenkorn, T. and Mańko, J. (1991). Correlation of intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 44(1):39–43.

Goodman, L. and Love, R. N. (1980). *Project Planning and Management: An Integrated Approach*. Pergamon Press, New York.

Grzegorzewski, P. (2002). Nearest interval approximation of a fuzzy number. *Fuzzy Sets and Systems*, 130(3):321–330.

Guimarães, A. C. F., Lapa, C. M. F., Filho, F. F. L. S., and Cabrala, D. C. (2011a). Fuzzy uncertainty modeling applied to ap1000 nuclear power plant loca. *Annals of Nuclear Energy*, 38(8):1775–1786.

Guimarães, A. C. F., Lapa, C. M. F., and Moreira, M. L. (2011b). Fuzzy methodology applied to probabilistic safety assessment for digital system in nuclear power plants. *Nuclear Engineering and Design*, 241(9):3967–3976.

Hamzeh, A. M. and Mousavi, S. M. (2019). A new fuzzy approach for project time assessment under uncertain conditions. In *2019 15th Iran International Industrial Engineering Conference (IIIEC)*, pages 76–80.

Hannan, E. (1981a). Linear programming with multiple fuzzy goals. *Decision Sciences*, 6 (3):235–248.

Hannan, E. L. (1981b). On fuzzy goal programming. *Decision Sciences*, 12:522–531.

Hapke, M., Jaszkiewicz, A., and Slowinski, R. (1994). Fuzzy project scheduling system for software development. *Fuzzy Sets and Systems*, 67(1):101–117.

Heagney, J. (2016). *Fundamentals of project management*.

Hellström, M., Ruuska, I., Wikström, K., and Jåfs, D. (2013). Project governance and path creation in the early stages of finnish nuclear power projects. *International Journal of Project Management*, 31(5):712–723.

Hines, J. W. and Davis, E. (2005). Lessons learned from the u.s. nuclear power plant on-line monitoring programs. *Progress in Nuclear Energy*, 46(3-4):176–189.

Hossain, S. and Hossain, M. (2018). Application of interactive fuzzy goal programming for multi-objective integrated production and distribution planning. *International Journal of Process Management and Benchmarking*, 8(1):35–58.

Hu, C. F., Teng, C. J., and Li, S. Y. (2007). A fuzzy goal programming approach to multi-objective optimization problem with priorities. *Eur J Oper Res*, 176(3):1319–1333.

Hwang, C. L. and Masud, A. S. M. (1979). *Multiple Objective Decision Making — Methods and Applications: A Stateof-the-Art Survey*. Springer.

IAEA (2000). Sustainable energy development. *IAEA Bulletin*, 42(2).

IAEA (2012). Project management in nuclear power plant construction: Guidelines and experience. *IAEA Nuclear Energy Series*, NP-T-2.7.

IAEA (2018). *IAEA Annual Report for 2018*. Agency's Statute.

IEA (2016). *World Energy Outlook 2016*. IEA/OECD, Paris.

Ignizio, J. P. (1985). An algorithm for solving the linear goal-programming problem by solving its dual. *Journal of operational Research Society*, 36:507–515.

Ijiri, Y. (1965). *Management Goals and Accounting for Control*. North-Holland Publishing Company, Amsterdam.

Ishizaka, A., Balkenborg, D., and Kaplan, T. (2010). Influence of aggregation and measurement scale on ranking a compromise alternative in ahp. *Journal of the Operational Research Society*, 62:700–710.

Islam, M. S., Nepal, M. P., and Skitmore, M. (2019). Modified fuzzy group decision-making approach to cost overrun risk assessment of power plant projects. *Journal of Construction Engineering and Management*, 145(2).

Izadikhah, M. (2012). Deriving weights of criteria from inconsistent fuzzy comparison matrices by using the nearest weighted interval approximation. *Hindawi Publishing Corporation Advances in Operations Research*.

Izadikhah, M., Saeidifar, A., and Roostaee, R. (2014). Extending topsis in fuzzy environment by using the nearest weighted interval approximation of fuzzy numbers. *Journal of Intelligent Fuzzy Systems,*, 27(6):2725–2736.

Jääskeläinen, V. (1976). *Linear Programming and Budgeting*. Petrocelli-Charter, New York.

Jafari, H., Koshteli, R., and Khabiri, B. (2008). An optimal model using goal programming for rice farm. *Applied Mathematical Sciences*, 2:1131–1136.

Javid, N., Damghani, K., Makui, A., and Abdi, F. (2020). Multi-objective flexibility-complexity trade-off problem in batch production systems using fuzzy goal programming. *Expert Systems with Applications*, 148.

Jayaraman, R., Liuzzi, D., Colapinto, C., and Malik, T. (2017). A fuzzy goal programming model to analyze energy, environmental and sustainability goals of the united arab emirates. *Annals of Operations Research*, 251:255–270.

Kahraman, C., Ertay, T., and Buyukozkan, G. (2006). A fuzzy optimization model for qfd planning process using analytic network approach. *Eur J Oper Res*, 171(2):390–411.

Kahraman, C. and Kaya, I. (2010). A fuzzy multicriteria methodology for selection among energy alternatives. *Expert Syst Appl*, 37(9):6270–6281.

Kaleva, O. (1987). Fuzzy differential equations. *Fuzzy Sets and Systems*, 24(3):301–317.

Kang, H., Lee, A. H. I., and Huang, T. (2016). Project management for a wind turbine construction by applying fuzzy multiple objective linear programming models. *MDPI*.

Karakaşoğlu, N. (2008). *Bulanık Çok Kriterli Karar Verme Yöntemleri ve Uygulama*. Pamukkale Üniversitesi Sosyal Bilimler Enstitüsü, Denizli.

Karataş, (2011). *Bulanık Faaliyet Tabanlı Maliyetleme Yaklaşımı ve Bir Uygulama*. Süleyman Demirel Üniversitesi Sosyal Bilimler Enstitüsü, Isparta.

Kayapınar, S. and Erginel, N. (2019). Designing the airport service with fuzzy qfd based on servqual integrated with a fuzzy multi-objective decision model. *Total Quality Management Business Excellence*, 30(13-14):1429–1448.

Ke, H. and Liu, B. (2010). Fuzzy project scheduling problem and its hybrid intelligent algorithm. *Applied Mathematical Modelling*, 34:301–308.

Keil, M. (1995). Pulling the plug: Software project management and the problem of project escalation. *MIS Quarterly*, 19(4):421–447.

Kelly, J. E. (1957). Computers and operations research in roadbuilding.

Kelly, J. E. and Walker, M. R. (1959). *Critical-Path Planning and Scheduling: An Introduction*. Mauchly Associates, Inc., Ambler, Pa.

Kelly, J. E. and Walker, M. R. (1961). *Management models and industrial applications of linear programming*. Wiley, New York.

Kerzner, H. (2003). *Project Management: A Systems Approach to Planning, Scheduling, and Controlling*. Wiley, 8th edition.

Kim, B., Lee, J., and Ahn, Y. (2020). Fuzzy based risk assessment for decommissioning concrete bioshield structures in nuclear power plants: Structural risks and worker safety. *Appl. Sci.*, 10.

Kolaylıoğlu, O. (2006). İnşaat sektöründe proje yönetimi ve proje yöneticisi. Yayınlanmamış Yüksek Lisans Tezi, Izmir,Dokuz Eylül Üniveristesi Sosyal Bilimler Enstitüsü.

Korobkin, V. V. and Kolodenkova, A. E. (2016). *Evaluation of the feasibility of the project to create control and management systems for nuclear power plants using fuzzy cognitive modeling*. AO Kontsern Rosehnergoatom, Russian Federation.

Kou, G., Ergu, D., and Shang, J. (2014). Enhancing data consistency in decision matrix: adapting hadamard model to mitigate judgment contradiction. *European Journal of Operational Research*, 236(1):261– 271.

Koçak, H. (2007). *Dogrusal Programlama Karar Verme ve Modeller Sunum Notlar*.

Krejčí, J. (2017a). Additively reciprocal fuzzy pairwise comparison matrices and multiplicative fuzzy priorities. *Soft Computing*, 21,:3177–3192.

Krejčí, J. (2017b). Fuzzy eigenvector method for obtaining normalized fuzzy weights from fuzzy pairwise comparison matrices. *Fuzzy Sets and Systems*, pages 26–43.

Krejčí, J. (2018). Obtaining fuzzy priorities from additive fuzzy pairwise comparison matrices. *IMA Journal of Management Mathematics*, 29(3):297–323.

Kubler, S., Derigent, W., Voisin, A., Robert, J., and Traon, Y. L. (2017). Knowledge-based consistency index for fuzzy pairwise comparison matrices. pages 1–7.

Kubler, S., Derigent, W., Voisin, A., Robert, J., Traon, Y. L., and Viedma, E. H. (2018). Measuring inconsistency and deriving priorities from fuzzy pairwise comparison matrices using the knowledge-based consistency index. *Knowledge-Based Systems*, 162:147–160.

Kuchta, D. (2001). Use of fuzzy numbers in project risk (criticality) assessment. *International Journal of Project Management*, 19(5):305–310.

Kumar, S., Krishna, B. A., and Satsangi, P. S. (1994). Fuzzy systems and neural networks in software engineering project management. *Applied Intelligence*, 4:31–52.

Kurt, O. (2006). Proje planlama ve programlama teknikleri ve İnşaat sektörü ait bir uygulaması. Yayınlanmamış Yüksek Lisans Tezi, Antalya, Akdeniz Üniversitesi Sosyal Bilimler Enstitüsü.

Kuwano, H. (1996). On the fuzzy multi-objective linear programming problem: Goal programming approach. *Fuzzy Set Syst*, 82(1):57–64.

Kökçam, H. and Engin, O. (2010). Solving the fuzzy project scheduling problems with meta-heuristic methods. *Journal of Engineering and Natural Sciences*, Sigma,28:86–101.

Kömürcü, M. I. and Filiz, M. H. (2009). Türkiye'nin enerji kaynakları ve Çevre, sürdürülebilir kalkınma ve temiz enerji kaynakları. *VII. Ulusal Temiz Enerji Sempozyumu*, pages 12–24.

Laarhoven, P. J. M. V. and Pedrycz, W. (1983). A fuzzy extension of saaty's priority theory. *Fuzzy Set Syst*, 11(1–3):199–227.

Leberling, H. (1981). On finding compromise solution in multi-criteria problems using the fuzzy min-operator. *Fuzzy Sets and Systems*, 6:105–228.

Lee, K. H. (2005). *First Course on Fuzzy Theory and Applications*. Springer-Verlag, Berlin.

Lee, S. M. (1972). *Goal Programming for Decision Analysis*. Auerbach Publishers, Philadelphia.

Lee, S. M. and Jääskeläinen, V. (1971). Goal programming: Management's math model. *Industrial Engineering*, 3(1):30–35.

Leung, S., Wu, Y., and Lai, K. (2003). Multi-site aggregate production planning with multiple objectives: A goal programming approach. *Production Planning Control*, 15(5):425–436.

Levine, H. A. (2002). *Practical Project Management: Tips, Tactics and Tools*. John Wiley Sons,Inc., New York.

Li, D. and Zeng, W. (2017). Distance measure of pythagorean fuzzy sets. *International Journal of Intelligent Systems*, 33(2):348–361.

Li, X., Jiang, H., Guo, S., Ching, W., and Yu, L. (2020). On product of positive l-r fuzzy numbers and its application to multi-period portfolio selection problems. *Fuzzy Optimization and Decision Making volume*, 19:53–79.

Li, X. Q., Zhang, B., and Li, H. (2006). Computing efficient solutions to fuzzy multiple objective linear programming problems. *Fuzzy Sets and System*, 157:1328–1332.

Liang, T. (2009). Fuzzy multi-objective project management decisions using two-phase fuzzy goal programming approach. *Computers Industrial Engineering*, 57(4):1407–1416.

Liang, T. F. (2010). Applying fuzzy goal programming to project management decisions with multiple goals in uncertain environments. *Expert Systems with Applications*, 37(12):8499–8507.

Liberatore, M. J. (2002). Project schedule uncertainty analysis using fuzzy logic. *Project Management Journal*, 33(4):15–22.

Lin, C. J. and Wu, W. W. (2008). A causal analytical method for group decision-making under fuzzy environment. *Expert Syst Appl*, 34(1):205–213.

Lin, W. T. (1980). A survey of goal programming applications. *Omega*, 8:115–117.

Liu, B. (2009). *Theory and Practice of Uncertain Programming*. UTLAB, China, 3rd edition.

Liu, F., Zou, S. C., and Li, Q. (2020). Deriving priorities from pairwise comparison matrices with a novel consistency index. *Applied Mathematics and Computation*, 374.

Liu, P., Liu, J., and Chen, S. M. (2018a). Some intuitionistic fuzzy dombi bonferroni mean operators and their application to multi-attribute group decision making. *Journal of the Operational Research Society*, 69:1–24.

Liu, P., Liu, J., and Merigób, J. M. (2018b). Partitioned heronian means based on linguistic intuitionistic fuzzy numbers for dealing with multi-attribute group decision making. *Applied Soft Computing*, 62:395–422.

Longa, L. D. and Ohsato, A. (2008). Fuzzy critical chain method for project scheduling under resource constraints and uncertainty. *International Journal of Project Management*, 26(6):688–698.

Lootsma, F. (1989). Conflict resolution via pairwise comparison of concessions. *European Journal of Operational Research*, 40:109–116.

Lowen, R. (1980). Convex fuzzy sets. *Fuzzy Sets and Systems*, 3(3):291–310.

Lua, S., Zhu, L., Wang, Y., Xie, L., and Su, H. (2020). Integrated forward and reverse logistics network design for a hybrid assembly-recycling system under uncertain return and waste flows: A fuzzy multi-objective programming. *Journal of Cleaner Production*, 243.

Luenberger, D. G. (1989). *Linear and Nonlinear Programming*. Second edition.

Markowski, C. A. and Ignizio, J. P. (1983). Theory and properties of the lexicographic linear goal programming. *Large Scale System*, 5:115–121.

Mikhailov, L. (2003). Deriving priorities from fuzzy pairwise comparison judgements. *Fuzzy Sets and Systems*, 134(3):365–385.

Mikhailov, L. (2014). A fuzzy approach to deriving priorities from interval pairwise comparison judgements. *European Journal of Operational Research*, 159:687–704.

Mokhtari, H. and Hasani, A. (2017). A multi-objective model for cleaner production-transportation planning in manufacturing plants via fuzzy goal programming. *Journal of Manufacturing Systems*, 44(1):230–242.

Mon, D. L., Cheng, C. H., and Lu, H. C. (1995). Application of fuzzy distributions on project management. *Fuzzy Sets and Systems*, 73(2):227–234.

Mousavi, S. M. (2019). A new interval-valued hesitant fuzzy pairwise comparison–compromise solution methodology: an application to cross-docking location planning. *Neural Comput Applic*, 31:5159–5173.

Munns, A. K. and Bjeirmi, B. F. (1996). The role of project management in achieving project success. *International Journal of Project Management*, 14(2):81–87.

Nabendu, S. and Manish, N. (2012). A goal programming approach to rubber plantation planning in tripura. *Applied Mathematical Sciences*, 6:6171–6179.

Nachtmann, H. and Needy, K. (2001). Fuzzy activity based costing: A methodology for handling uncertainty in activity based costing systems. *The Engineering Economist*, 46:245–273.

Nasibov, E. N. and Peker, S. (2008). On the nearest parametric approximation of a fuzzy number. *Fuzzy Sets and Systems*, 159(11):1365–1375.

Nehi, H. M. and Hajmohamadi, H. (2012). A ranking function method for solving fuzzy multi-objective linear programming problem. *Annals of Fuzzy Mathematics and Informatics*, 3(1):31–38.

Nguyen, H. T., Safder, U., Nguyen, X. Q. N., and Yoo, C. K. (2020). Multi-objective decision-making and optimal sizing of a hybrid renewable energy system to meet the dynamic energy demands of a wastewater treatment plant. *Energy*, 191.

Nguyen, P. T., Vu, N. B., Nguyen, L. V., Le, L. P., and Vo, K. D. (2018). The application of fuzzy analytic hierarchy process (f-ahp) in engineering project management. In *2018*

IEEE 5th International Conference on Engineering Technologies and Applied Sciences (ICETAS), pages 1–4.

Nieto-Morotea, A. and Ruz-Vilab, F. (2011). A fuzzy approach to construction project risk assessment. *International Journal of Project Management*, 29(2):220–231.

Onar, S., Öztaşi, B., and Kahraman, C. (2014). Strategic decision selection using hesitant fuzzy topsis and interval type-2 fuzzy ahp: A case study. *International Journal of Computational intelligence systems*, 7(5):1002–1021.

Onar, S. Ç. and Ateş, N. Y. (2008). A fuzzy model for operational supply chain optimization problems. *Journal of Multiple-Valued Logic and Soft Computing*, 14, (3-5):355–370.

Opricovic, S. (1998). *Multicriteria Optimization of Civil Engineering Systems*. PhD thesis, Faculty of Civil Engineering, Belgrade.

Orumie, U. C. and Ebong, D. W. (2011). An alternative method of solving goal programming problem. *Nigerian Journal of Operations Research*, 2:68–90.

Öztürk, A. (2007). *Yöneylem Araştırması, Genişletilmiş 9. Baskı*. Ekin Kitabevi, Bursa.

Paksoy, S. (2007). Genetik algoritma ile proje Çizelgeleme. Yayınlanmamış Doktora Tezi, Çukurova Üniversitesi Sosyal Bilimler Enstitüsü.

Pandey, P., Shah, B. J., and Gajjar, H. (2017). A fuzzy goal programming approach for selecting sustainable suppliers. *Benchmarking: An International Journal*, 24(5):1138–1165.

Parra, M., Terol, A., and Uria, M. (2001). A fuzzy goal programming approach to portfolio selection. *Eur J Oper Res*, 133:287–297.

Phruksaphanrat, B. and Ohsato, A. (2004). Linear coordination method for fuzzy multi-objective linear programming problems with convex polyhedral membership functions. *International Journal of Uncertainty, Fuzziness and Knowlege-Based Systems*, 12(3):269–285.

Pinedo, M. (2012). *Scheduling – Theory, Algorithms and Systems*. Springer, New York.

Pinto, J. K. and Slevin, D. P. (1988). Project success: Definitions and measurement techniques. *Project Management Journal*, 19(1):67–72.

Prascevic, N. and Prascevic, Z. (2017). Application of fuzzy ahp for ranking and selection of alternatives in construction project management. *Journal of Civil Engineering and Management*, 23(8):1123–1135.

Purba, J. H. (2014). Fuzzy probability on reliability study of nuclear power plant probabilistic safety assessment: A review. *Progress in Nuclear Energy*, 76:73–80.

Ramik, J. (2020a). Pairwise comparison matrices in decision-making. *Pairwise Comparisons Method*, pages 17–65.

Ramik, J. (2020b). Pairwise comparisons matrices with fuzzy and intuitionistic fuzzy elements in decision-making. *Pairwise Comparisons Method*, pages 125–170.

Ramík, J. (2015). Isomorphisms between fuzzy pairwise comparison matrices. *Fuzzy Optimization and Decision Making volume*, 14:199–209.

Ramík, J. and Korviny, P. (2011). Inconsistency of pairwise comparison matrix with fuzzy elements based on geometric mean. *Fuzzy Sets and Systems*, 161(11):1604–1613.

Rani, B. U., Ganesh, A. H., and Jayakumar, S. (2016). Ranking agro-ecological socio-economic constraints in agricultural using fuzzy ranking method based on radius of gyration of centroids. *Asian Journal of Fuzzy and Applied Mathematics*, 4(3):28–35.

Rao, K. D., Gopika, V., Kushwaha, H. S., Verma, A. K., and Srividya, A. (2007). Test interval optimization of safety systems of nuclear power plant using fuzzy-genetic approach. *Reliability Engineering System Safety*, 92(7):895–901.

Rastogi, A. and Gabbar, H. (2013). Fuzzy-logic-based safety verification framework for nuclear power plants. *Risk Analysis*, 33(6):1128–1145.

Reifman, J. (1997). Survey of artificial intelligence methods for detection and identification of component faults in nuclear power plants. *Nuclear Technology*, 119(1):76–97.

Ren, A. and Wang, Y. (2017). An approach for solving a fuzzy bilevel programming problem through nearest interval approximation approach and kkt optimality conditions. *Soft Computing*, 21:5515–5526.

Ren, H. and Luo, L. (2020). A novel distance of intuitionistic trapezoidal fuzzy numbers and its-based prospect theory algorithm in multi-attribute decision making model. *Mathematical Biosciences and Engineering*, 17(4):2905–2922.

Rezaei, J., Fahim, P. B. M., and Tavasszy, L. (2014). Supplier selection in the airline retail industry using a funnel methodology: Conjunctive screening method and fuzzy ahp. *Expert Syst Appl*, 41(18):8165–8179.

Rhodes, R. (1986). *The Making of the Atomic Bomb*. Simon and Schuster, United States of America.

Rifai, A. K. (1996). A note on the structure of the goal-programming model: Assessment and evaluation. *International Journal of Operations and Production Management*, 16:40–49.

Rodriguez, R. M., Martinez, L., and Herrera, F. (2009). Hesitant fuzzy linguistic term sets for decision making. *Expert Syst Appl*, 36(8):11363–11368.

Romero, C. (1991). *Handbook of Critical Issues in Goal Programming*. Pergamon, Oxford.

Romero, C. (2004). A general structure of achievement function for a goal programming model. *European Journal of Operational Research*, 153:675–686.

Rossdy, M. (2010). A case study on the application of 0-1 goal programming: Nurse scheduling. Master's thesis, Universiti Teknologi Malaysia, Malaysia.

Roy, B. (1968). Classement et choix en présence de points de vue multiples (la méthode electre). *La Revue d'Informatique et de Recherche Opérationelle (RIRO)*, 8:57–75.

Ruuska, I., Artto, K., Aaltonen, K., and Lehtonen, P. (2009). Dimensions of distance in a project network: Exploring olkiluoto 3 nuclear power plant project. *International Journal of Project Management*, 27(2):142–153.

Saaty, T. (1977). A scaling method for priorities in hierarchical structures. *Journal of Mathematical Psychology*, 15:234–281.

Saaty, T. L. (1980). *The Analytic Hierarchy Process*. McGrawHill.

Saaty, T. L. (2008). Relative measurement and its generalization in decision making: Why pairwise comparisons are central in mathematics for the measurement of intangible factors. *The Analytic Hierarchy/Network Process, Review of the Royal Academy of Exact, Physical and Natural Sciences, Series A: Mathematics (RACSAM)*, 102(2):51–318.

Saeidifar, A. (2011). Application of weighting functions to the ranking of fuzzy numbers. *Computers and Mathematics with Applications*, 62:2246–2258.

Sakawa, M. (1983). Interactive computer programs for fuzzy linear programming with multiple objectives. *International Journal of Man-Machine Studies*, 18:489–503.

Sakawa, M., Yumine, T., and Yano, H. (1984). An interactive fuzzy satisficing method for multiobjective nonlinear programming problems. *International Institute for Applied Systems Analysis*.

Schniederjans, M. J. and Kwak, N. K. (1982). An alternative method for solving goal programming problems: A reply. *The Journal of the Operational Research Society*, 33:859–860.

Schwalbe, K. (2008). *Information Technology Project Management, Reprint*. Cengage Learning.

Serteller, N. F. (2006). Türkiye'de kullanılan ve kullanılabilecek olan enerji kaynakları arasında nükleer enerjinin yeri ve önemi. *Dünya Enerji Konseyi Türk Milli Komitesi, Türkiye 10. Enerji Kongresi*, pages 309–313.

Shahi, E., Alavipoor, F. S., and Karimi, S. (2018). The development of nuclear power plants by means of modified model of fuzzy dematel and gis in bushehr, iran. *Renewable and Sustainable Energy Reviews*, 83:33–49.

Sharma, H. P. and Sharma, D. K. (2006). A multi-objective decision-making approach for mutual fund portfolio. *Journal of Business Economics Research*, 4:13–24.

Sharma, U. and Aggarwal, S. (2018). Solving fully fuzzy multi-objective linear programming problem using nearest interval approximation of fuzzy number and interval programming. *International Journal of Fuzzy Systems*, 20:488–499.

Shin, D., Shin, Y., and Kim, G. (2016). Comparison of risk assessment for a nuclear power plant construction project based on analytic hierarchy process and fuzzy analytic hierarchy process. *Journal of Building Construction and Planning Research*, 4(3).

Shipley, M. F., de Korvin, A., and Omer, K. (1997). Bifpet methodology versus pert in project management: fuzzy probability instead of the beta distribution. *Journal of Engineering and Technology Management*, 14(1):49–65.

Shtub, A., Bard, J., and Globerson, S. (1994). *Project Management: Engineering, Technology and Implementation*. Prentice-Hall, New Jersey.

Shukla, A. K., Yadav, M., Kumar, S., and Muhuri, P. K. (2020). Veracity handling and instance reduction in big data using interval type-2 fuzzy sets. *Engineering Applications of Artificial Intelligence*, 88.

Simon, H. A. (1957). *Models of Man*. Wiley Sons, New York.

Siskos, J. and Hubert, P. (1983). Multi-criteria analysis of the impacts of energy alternatives: A survey and a new comparative approach. *Eur J Oper Res*, 13(3):278–299.

Slyeptsov, A. I. and Tyshchuk, T. A. (2003). Fuzzy temporal characteristics of operations for project management on the network models basis. *European Journal of Operational Research*, 147(2):253–265.

Soltani, A. and Haji, R. (2007). A project scheduling method based on fuzzy theory. *Journal of Industrial and Systems Engineering*, 1(1):70–80.

Sperry, R. C. and Jetter, A. J. (2019). A systems approach to project stakeholder management: Fuzzy cognitive map modeling. *Project Management Journal*, 50(6):699–715.

Spinner, M. (1997). *Project Management*. Prentice-Hall, New Jersey.

Srividya, A., Suresh, H. N., and Verma, A. K. (2007). Fuzzy ahp in prioritizing feeders for maintenance in nuclear power plants. In *2007 IEEE International Conference on Industrial Engineering and Engineering Management*, pages 149–153.

Stanojević, B. and Stanojević, M. (2019). On fuzzy solutions to a class of fuzzy multi-objective linear optimization problems. pages 63–76.

Star, M. (1971). *Systems Management of Operations*. Prentice Hal.

Stevenson, W. J. (1993). *Production/Operations Management*. Richard D.Irwin.

Subali, M. A. P., Sarno, R., and Effendi, Y. A. (2018). Time and cost optimization using fuzzy goal programming. pages 471–476.

Suh, N. P. (1998). Axiomatic design theory for systems. *Research in Engineering Design - Theory, Applications, and Concurrent Engineering*, 10(4):189–209.

Sun, Y. (2020). Green and reliable freight routing problem in the road-rail intermodal transportation network with uncertain parameters: A fuzzy goal programming approach. *Journal of Advanced Transportation*.

Taha, H. (2000). *Yöneylem Araştırması (Çev. Ş.A. Baray ve Ş. Esnaf)*. Literatür Yayıncılık, İstanbul.

Tamiz, M. and Jones, D. F. (1996). Goal programming and pareto efficiency. *Journal of Information and Optimization Sciences*, 17:291–307.

Tamiz, M. and Jones, D. F. (2010). *Practical Goal Programming. International Series in Operations Research Management Science*. Springer, New York.

Tavana, M., Khalili-Damghani, K., Arteaga, F. J. S., and Hosseini, A. (2019). A fuzzy multi-objective multi-period network dea model for efficiency measurement in oil refineries. *Computers Industrial Engineering*, 135:143–155.

Tayebikhorami, S., Nikoo, M. R., and Sadegh, M. (2019). A fuzzy multi-objective optimization approach for treated wastewater allocation. *Environmental Monitoring and Assessment*, 191.

Temurçin, K. (2003). Nuclear energy and reality of nuclear energy in turkey in the light of discussions. *Coğrafi Bilimler Dergisi*, 1(2):25–39.

Tiwari, R., Dhamar, S., and Rao, J. (1987). Fuzzy goal programming: An additive model. *Fuzzy Sets and Systems*, 24(1):27–34.

Torra, V. (2010). Hesitant fuzzy sets. *Int J Intell Syst*, 25(6):529–539.

Tran, L. and Duckstein, L. (2002). Comparison of fuzzy numbers using a fuzzy distance measure. *Fuzzy Sets and Systems*, 130(3):331–341.

Triantaphyllou, E. and Lin, C. T. (1996). Development and evaluation of five fuzzy multiattribute decision-making methods. *Int J Approx Reason*, 14(4):281–310.

Trivedi, A. and Singh, A. (2017). A hybrid multi-objective decision model for emergency shelter location-relocation projects using fuzzy analytic hierarchy process and goal programming approach. *International Journal of Project Management*, 35(5):827–840.

Tseng, M. L. (2010). Implementation and performance evaluation using the fuzzy network balanced scorecard. *Comput Educ*, 55(1):188–201.

TTB (2014). *Türk Tabipleri Birliği Halk Sağlığı Kolu Akkuyu Nükleer Güç Santrali Projesi ÇED Raporu Değerlendirmesi*. Türk Tabipleri Birliği Yayınları, Ankara.

Turan, S. (2006). Nükleer enerji: Nükleer santralin konya'ya kurulabilirliği, getirileri ve götürüleri. *Konya Ticaret Odası Araştırma Raporu*, 2006-42/44:1–20.

Turner, J. R. (1999). *The handbook of project-based management: improving the processes for achieving strategic objectives*. McGraw-Hill, London.

Tuzkaya, G., Gulsuna, B., Kahraman, C., and Ozgen, D. (2010). An integrated fuzzy multi-criteria decision making methodology for material handling equipment selection problem and an application. *Expert Syst Appl*, 37, (4),:2853–2863.

Tuş, A. (2006). *Bulanık Dogrusal Programlama ve Bir Üretim Planlamasında Uygulama Örneği*. Pamukkale Üniversitesi Sosyal Bilimler Enstitüsü, Denizli.

Udum, Ş. (2010). Turkey's nuclear comeback. *The Nonproliferation Review*, 17(2):365–377.

Ullah, K., Mahmood, T., and Jan, N. (2018). Similarity measures for t-spherical fuzzy sets with applications in pattern recognition. *Symmetry*, 10(6).

Vanderbei, R. J. (1997). *Linear Programming: Foundations And Extentions*.

Walker, A. (2015). *Project management in construction*.

Wang, P., Wang, J., Wei, G., and Wei, C. (2019). Similarity measures of q-rung orthopair fuzzy sets based on cosine function and their applications. *Mathematics*, 7(4).

Wang, R. C. and Liang, T. F. (2004). Project management decisions with multiple fuzzy goals. *Construction Management and Economics*, 22(10):1047–1056.

Wang, S., Wahab, M. I. M., and Fang, L. (2011). Managing construction risks of ap1000 nuclear power plants in china. *Journal of Systems Science and Systems Engineering*, 20:43–69.

Wang-jin, L. (1983). Operations on fuzzy ideals. *Fuzzy Sets and Systems*, 11(1):31–39.

Wei, C. C., Liang, G. S., and J.Wang, M. J. (2007). A comprehensive supply chain management project selection framework under fuzzy environment. *International Journal of Project Management*, 25(6):627–636.

Weinberg, A. (1994). *The First Nuclear Era*. AIP Press.

Westland, J. (2006). The project management life cycle. *Kogan Page*, page 60.

Winston, W. (2004). *Operations Research: Applications and Algorithms*. Duxbury Press, Pacific Grove.

Wu, M. C. and Chen, T. Y. (2011). The electre multicriteria analysis approach based on atanassov's intuitionistic fuzzy sets. *Expert Syst Appl*, 38(10):12318–12327.

Wu, N. and Coppins, R. (1981). *Linear Programming and Extensions*. McGraw-Hill Inc, New York.

Wu, Y., Liu, F., Huang, Y., Xu, C., Zhang, B., Ke, Y., and Jia, W. (2020). A two-stage decision framework for inland nuclear power plant site selection based on gis and type-2 fuzzy promethee ii: Case study in china. *Energy Science and Engineering*.

Xu, Z. (2010). Choquet integrals of weighted intuitionistic fuzzy information. *Inform Sciences*, 180(5):726–736.

Yager, R. R. (1981). A procedure for ordering fuzzy subsets of the unit interval. *Information Sciences*, 24:143–161.

Yager, R. R. (1986). On the theory of bags. *Int J Gen Syst*, 13(1):23–37.

Yager, R. R. and Filev, D. P. (1994). Parameterized and like or-like owa operators. *International journal of General Systems*, 22:297–316.

Yaghoobi, M. and Tamiz, M. (2006). On improving a weighted additive model for fuzzy goal programming problems. *Int Rev Fuzzy Math*, 1:115–129.

Yaghoobi, M. A. and Tamiz, M. (1979). A short note on the relationship between goal programming and fuzzy programming for vectormaximum problems. *Iranian Journal of Fuzzy Systems*, 2:31–36.

Zadeh, L. (1975). The concept of a linguistic variable and its applications to approximate reasoning. *Inform Sciences*, 8:199–249.

Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, 8:338–352.

Zamanian, M. R., Sadeh, E., Sabegh, Z. A., and Rasi, R. E. (2019). A fuzzy goal-programming model for optimization of sustainable supply chain by focusing on the environmental and economic costs and revenue: A case study. *Advances in Mathematical Finance and Applications*, 4(1):103–123.

Zeng, J., An, M., and Smith, N. (2007). Application of a fuzzy based decision making methodology to construction project risk assessment. *International Journal of Project Management*, 25(6):589–600.

Zhang, J., Li, Y., Zhang, K., and Yang, H. (2006). Fuzzy dynamic programming method for progress adjustment of project management. *Computer Integrated Manufacturing Systems*, 8.

Zhang, X. and Chen, X. (2012). A new uncertain programming model for project scheduling problem. *An International Interdisciplinary Journal*, 15(10):3901–3910.

Zhao, Y., Tong, J., Zhang, L., and Wu, G. (2020). Diagnosis of operational failures and on-demand failures in nuclear power plants: An approach based on dynamic bayesian networks. *Annals of Nuclear Energy*, 138.

Zhou, Q., Mo, H., and Deng, Y. (2020). A new divergence measure of pythagorean fuzzy sets based on belief function and its application in medical diagnosis. *Mathematics*, 8(1).

Zhou, S. (1996). *The Development and Evaluation of Aggregation Methods for Group Pairwise Comparison Judgments*. PhD thesis, Portland State University, Portland.

Zhu, B., Xu, Z., and Xia, M. (2012). Dual hesitant fuzzy sets. *Journal of Applied Mathematics*.

Zimmermann, H. J. (1976). Description and optimization of fuzzy systems. *International Journal of General Systems*, 2:209–215.

Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, 1:45–55.

Zimmermann, H. J. (1987). *Fuzzy Sets Decision- Making and Expert Systems*. Kluwer Academic Publishers, Boston.

Zopounidis, C. and Doumpos, M. (2002). *Multicriteria Decision Aid Classification Methods*. Springer, New York.

Zwols, Y. and Sierksma, G. (2015). Linear and integer optimization: Theory and practice. Last accessed 10.10.2019: `http://www.lio.yoriz.co.uk/`.

APPENDICES

Appendix A

Linear Programming Model Code 1:

Multi-Objective Linear Programming Model (Çakır, 2019)

```
var y12 >= 0;
var y24 >= 0;
var y23 >= 0;
var y45 >= 0;
var y46 >= 0;
var y47 >= 0;
var y58 >= 0;
var y69 >= 0;

var t12 >= 0;
var t23 >= 0;
var t24 >= 0;
var t45 >= 0;
var t46 >= 0;
var t47 >= 0;
var t58 >= 0;
var t69 >= 0;

var e1 >= 0;
var e2 >= 0;
var e3 >= 0;
var e4 >= 0;
var e5 >= 0;
var e6 >= 0;
var e7 >= 0;
var e8 >= 0;
var e9 >= 0;

minimize TP : e9 - e1 ;
```

```
maximize TR : y12 + y23 + y24 + y45 + y46 + y47 + y58 + y69 ;
minimize TC : (33210601.6 + 4427765.3*y12) + (349090909.091 +
              23271072.19*y24) + (628363636.36 + 18616557.75*y23) +
              (768000000 + 68261811.76* y45) + (698181818.18 +
              53191022.14*y46) + (5026909090.91 + 446804586.04*y47) +
              (6004363636.63 + 553683255.55*y58) + (488727272.72 +
              52127201.704*y69);
subject to c11:  e1 + t12 = e2;
subject to c12:  e2 + t23 = e3;
subject to c13:  e2 + t24 = e4;
subject to c14:  e4 + t45 = e5;
subject to c15:  e4 + t46 = e6;
subject to c16:  e4 + t47 = e7;
subject to c17:  e5 + t58 = e8;
subject to c18:  e6 + t69 = e9;

subject to c21:  t12 + y12 = 3 ;
subject to c22:  t23 + y23 = 16;
subject to c23:  t24 + y24 = 7;
subject to c24:  t45 + y45 = 4;
subject to c25:  t46 + y46 = 6;
subject to c26:  t47 + y47 = 5;
subject to c27:  t58 + y58 = 5;
subject to c28:  t69 + y69 = 4;

subject to c31:  y12 - 1 <= 0;
subject to c32:  y23 - 7 <= 0;
subject to c33:  y24 - 3 <= 0;
subject to c34:  y45 - 1 <= 0;
subject to c35:  y46 - 2.5 <= 0;
subject to c36:  y47 - 2 <= 0;
subject to c37:  y58 - 2 <= 0;
subject to c38:  y69 - 1.5 <= 0;

subject to c40:  e9 <= 20;
subject to c41:  e3 <= e9;
end;
```

Appendix B

Linear Programming Model Code 2: FMOLP Model (Çakır, 2019)

```
var y12 >= 0;
var y24 >= 0;
var y23 >= 0;
var y45 >= 0;
var y46 >= 0;
var y47 >= 0;
var y58 >= 0;
var y69 >= 0;

var t12 >= 0;
var t23 >= 0;
var t24 >= 0;
var t45 >= 0;
var t46 >= 0;
var t47 >= 0;
var t58 >= 0;
var t69 >= 0;

var e1 >= 0;
var e2 >= 0;
var e3 >= 0;
var e4 >= 0;
var e5 >= 0;
var e6 >= 0;
var e7 >= 0;
var e8 >= 0;
var e9 >= 0;

var TC >= 0;
var TR >= 0;
var TP >= 0;
var Lambda >= 0;

maximize z: Lambda;
```

```
subject to c01:   Lambda <= (16441809704.457 - TC) /
                            (16441809704.457-13996846965.491);
subject to c02:   Lambda <= (20 - TP) / 8;
subject to c03:   Lambda <= TR / 20;

subject to c04:   (33210601.6 + 4427765.3*y12) + (349090909.091 +
                  23271072.19*y24) + (628363636.36 + 18616557.75*y23) +
                  (768000000 + 68261811.76*y45) + (698181818.18 +
                  53191022.14*y46) + (5026909090.91 +
                  446804586.04*y47) + (6004363636.63 +
                  553683255.55*y58) + (488727272.72 + 52127201.704*y69)
                  = TC;
subject to c05:   y12 + y23 + y24 + y45 + y46 + y47 + y58 + y69 = TR;
subject to c06:   e9 - e1 = TP;

subject to c11:   e1 + t12 = e2;
subject to c12:   e2 + t23 = e3;
subject to c13:   e2 + t24 = e4;
subject to c14:   e4 + t45 = e5;
subject to c15:   e4 + t46 = e6;
subject to c16:   e4 + t47 = e7;
subject to c17:   e5 + t58 = e8;
subject to c18:   e6 + t69 = e9;

subject to c21:   t12 + y12 = 3 ;
subject to c22:   t23 + y23 = 16;
subject to c23:   t24 + y24 = 7;
subject to c24:   t45 + y45 = 4;
subject to c25:   t46 + y46 = 6;
subject to c26:   t47 + y47 = 5;
subject to c27:   t58 + y58 = 5;
subject to c28:   t69 + y69 = 4;

subject to c31:   y12 - 1 <= 0;
subject to c32:   y23 - 7 <= 0;
subject to c33:   y24 - 3 <= 0;
subject to c34:   y45 - 1 <= 0;
```

```
subject to c35:    y46 - 2.5 <= 0;
subject to c36:    y47 - 2 <= 0;
subject to c37:    y58 - 2 <= 0;
subject to c38:    y69 - 1.5 <= 0;

subject to c40:    e9 <= 20;
subject to c41:    e3 <= e9;
end;
```

Appendix C

Linear Programming Model Code 3: Goal Programming Model (Çakır, 2019)

```
var pp12 >= 0;
var pp13 >= 0;
var pp21 >= 0;
var pp23 >= 0;
var pp31 >= 0;
var pp32 >= 0;

var w1 >= 0;
var w2 >= 0;
var w3 >= 0;

var pn12 >= 0;
var pn13 >= 0;
var pn21 >= 0;
var pn23 >= 0;
var pn31 >= 0;
var pn32 >= 0;

var qp12 >= 0;
var qp13 >= 0;
var qp21 >= 0;
var qp23 >= 0;
var qp31 >= 0;
var qp32 >= 0;

var qn12 >= 0;
var qn13 >= 0;
var qn21 >= 0;
var qn23 >= 0;
var qn31 >= 0;
var qn32 >= 0;

minimize goal:   pp12 + pp13 + pp21 + pp23 + pp31 + pp32 +
                 qp12 + qp13 + qp21 + qp23 + qp31 + qp32 ;
```

```
subject to c11:   1.573*w2 - w1 + pn12 - pp12 =  0;
subject to c12:   w1 - 1.6812*w2 + qn12 - qp12 =  0;

subject to c21:   1.86*w3 - w1 + pn13 - pp13 =  0;
subject to c22:   w1 - 1.992*w3 + qn13 - qp13 =  0;

subject to c31:   0.598*w1 - w2 + pn21 - pp21 =  0;
subject to c32:   w2 - 0.638*w1 + qn21 - qp21 =  0;

subject to c41:   2.231*w3 - w2 + pn23 - pp23 =  0;
subject to c42:   w2 - 3.09*w3 + qn23 - qp23 =  0;

subject to c51:   0.505*w1 - w3 + pn31 - pp31 =  0;
subject to c52:   w3 - 0.538*w1 + qn31 - qp31 =  0;

subject to c61:   0.325*w2 - w3 + pn32 - pp32 =  0;
subject to c62:   w3 - 0.456*w2 + qn32 - qp32 =  0;

subject to c70:   w1 + w2 + w3 =  1;
end;
```

Appendix D

Linear Programming Model Code 4: FMWOLP Model (Çakır, 2019)

```
var y12 >= 0;
var y24 >= 0;
var y23 >= 0;
var y45 >= 0;
var y46 >= 0;
var y47 >= 0;
var y58 >= 0;
var y69 >= 0;

var t12 >= 0;
var t23 >= 0;
var t24 >= 0;
var t45 >= 0;
var t46 >= 0;
var t47 >= 0;
var t58 >= 0;
var t69 >= 0;

var e1 >= 0;
var e2 >= 0;
var e3 >= 0;
var e4 >= 0;
var e5 >= 0;
var e6 >= 0;
var e7 >= 0;
var e8 >= 0;
var e9 >= 0;

var TC >= 0;
var TR >= 0;
var TP >= 0;
var LTP >= 0;
var LTR >= 0;
var LTC >= 0;
```

```
maximize Lambda: 0.467288*LTC + 0.2981297*LTP + 0.2345823*LTR;
subject to c01:   LTC <= (16441809704.457 - TC) /
                         (16441809704.457 - 13996846965.491);
subject to c02:   LTP <= (20 - TP) / 8;
subject to c03:   LTR <= TR / 20;

subject to c04:   (33210601.6 + 4427765.3*y12) + (349090909.091 +
                  23271072.19*y24) + (628363636.36 + 18616557.75*y23) +
                  (768000000 + 68261811.76*y45) + (698181818.18 +
                  53191022.14*y46) + (5026909090.91+ 446804586.04*y47)+
                  (6004363636.63 + 553683255.55*y58) + (488727272.72 +
                  52127201.704*y69) = TC;
subject to c05:   y12 + y23 + y24 + y45 + y46 + y47 + y58 + y69 = TR;
subject to c06:   e9 - e1 = TP;

subject to c11:   e1 + t12 = e2;
subject to c12:   e2 + t23 = e3;
subject to c13:   e2 + t24 = e4;
subject to c14:   e4 + t45 = e5;
subject to c15:   e4 + t46 = e6;
subject to c16:   e4 + t47 = e7;
subject to c17:   e5 + t58 = e8;
subject to c18:   e6 + t69 = e9;

subject to c21:   t12 + y12 = 3 ;
subject to c22:   t23 + y23 = 16;
subject to c23:   t24 + y24 = 7;
subject to c24:   t45 + y45 = 4;
subject to c25:   t46 + y46 = 6;
subject to c26:   t47 + y47 = 5;
subject to c27:   t58 + y58 = 5;
subject to c28:   t69 + y69 = 4;

subject to c31:   y12 - 1 <= 0;
subject to c32:   y23 - 7 <= 0;
subject to c33:   y24 - 3 <= 0;
```

```
subject to c34:   y45 - 1 <= 0;
subject to c35:   y46 - 2.5 <= 0;
subject to c36:   y47 - 2 <= 0;
subject to c37:   y58 - 2 <= 0;
subject to c38:   y69 - 1.5 <= 0;

subject to c40:   e9 <= 20;
subject to c41:   e3 <= e9;
end;
```

BIOGRAPHICAL SKETCH

Esra ÇAKIR was born in Istanbul on 1994. In 2012, she started the undergraduate program at Galatasaray University. She received B.S degree in Computer Engineering in 2017 (first in her class) and Industrial Engineering in 2018. In 2018, she was also accepted to the graduate program of Industrial Engineering at Galatasaray University.

In addition to her academic education, she had the chance to work in some big companies such as KoçSistem, InfoTech, Orange Business Services and Turkish Airlines. Since 2019, she has been working as a research assistant in the Department of Industrial Engineering at the Faculty of Engineering and Technology at Galatasaray University.

Publications

- Çakır E., Ziya Ulukan H. (2020) Fuzzy Multi-Objective Decision Making Approach for Nuclear Power Plant Installation. In: Kahraman C., Cebi S., Cevik Onar S., Oztaysi B., Tolga A., Sari I. (eds) Intelligent and Fuzzy Techniques in Big Data Analytics and Decision Making. INFUS 2019. Advances in Intelligent Systems and Computing, vol 1029, pp. 1258–1265. Springer, Cham.
- E. Çakır and H. Ziya Ulukan, "A Fuzzy Logic Programming Environment for Recycling Facility Selection" 2019 11th International Joint Conference on Computational Intelligence (IJJCI), Vienna, Austria, 2019, pp. 367-374.
- E. Çakır and Z. Ulukan, "Dual Hesitant Pythagorean Fuzzy Approach For Sustainable Ecotourism Site Selection" 2019 3th Annual International Conference on Data Science and Business Analytics (ICDSBA), Istanbul, Turkey, 2019.
- E. Çakır and Z. Ulukan, "Interval Type-2 Fuzzy Reliability Modeling for Recycling Facility" 2019 3th International Conference on System Reliability and Safety (ICSRS), Rome, Italy, 2019, pp. 544-548.
- E. Çakır and Z. Ulukan, "A Fuzzy Linguistic Programming for Sustainable Ecotourism Acitivities" 2020 10th Annual Computing and Communication Workshop

and Conference (IEEE-CCWC), Las Vegas, NV, United States of America (USA), 2020, pp. 0121-0126.

- Çakır E., Ulukan Z. (2020) A S-QFD Approach with Bipolar Fuzzy Hamacher Aggregation Operators and Its Application on E-Commerce. In: Lesot MJ. et al. (eds) Information Processing and Management of Uncertainty in Knowledge-Based Systems. IPMU 2020. Communications in Computer and Information Science, vol 1238, pp. 577-587, Springer, Cham.

- Çakır E., Ulukan Z. (2021) A* Algorithm Under Single-Valued Neutrosophic Fuzzy Environment. In: Kahraman C., Cevik Onar S., Oztaysi B., Sari I., Cebi S., Tolga A. (eds) Intelligent and Fuzzy Techniques: Smart and Innovative Solutions. INFUS 2020. Advances in Intelligent Systems and Computing, vol 1197, pp. 302-310, Springer, Cham.

- Çakır E., Ulukan Z. (2021) Bipolar Neutrosophic Fuzzy Dijkstra Algorithm and Its Application. In: Kahraman C., Cevik Onar S., Oztaysi B., Sari I., Cebi S., Tolga A. (eds) Intelligent and Fuzzy Techniques: Smart and Innovative Solutions. INFUS 2020. Advances in Intelligent Systems and Computing, vol 1197, pp. 311-318, Springer, Cham.

- E. Çakır and Z. Ulukan, "An Interval Type-2 Fuzzy Dynamic Approach To Replacement of Server Equipment," 2020 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Glasgow, United Kingdom, 2020, pp. 1-7, DOI:10.1109/ FUZZ48607.2020.9177554.

- E. Çakır and Z. Ulukan, "Sustainable Health Tourism Site Selection with A Fuzzy Linguistic Prolog," The 14th International FLINS Conference on Robotics and Artificial Intelligence. FLINS/ISKE 2020, Cologne, Germany. Developments of Artificial Intelligence Technologies in Computation and Robotics, pp. 1456-1463.

- Çakır, Esra and Ulukan, Ziya. "Fuzzy Multi-Objective Decision Making Approach for Nuclear Power Plant Installation," Journal of Intelligent & Fuzzy Systems, vol. Pre-press, no. Pre-press, 2020, DOI: 10.3233/JIFS-189101.