HUB COVERING FLOW PROBLEM WITH HUB UTILIZATION

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LIST OF SYMBOLS

HLP **:** Hub Location Problem
λ_{max} **:** Lambdamax
O-D **:** Origin-Destination
α **:** Inter-hub Discount Rate
USApHMP-Q: Quadratic Uncapacitated Single Allocation p-Hub Median Problem
GRASP : Greedy Randomized Adaptive Search Procedure
***p*HCP** : *p*-Hub Center Problems
***p*HMP** : *p*-Hub Median Problems
***p*-HMP-S** : *p*-Hub Median Problem with Single Allocation
***p*-HMP-M** : *p*-Hub Median Problem with Multiple Allocation
GA : Genetic Algorithm
TS : Tabu Search
SA : Simulated Annealing
BB : Branch &Bound
LP :Linear Programming
MIP : Mixed Integer Programming
BS : Binary Search
VNS :Variable Neighbourhood Search
CAB : Civil Aeronautics Board
CHLP :Cycle Hub Location Problem
MINLP **:**Mixed Integer Nonlinear Programming
SC : Supply Chain
NSGA-II **:** Non-dominated Sorting Genetic Algorithm II
HCFP **:** Hub Covering Flow Problem
r : Coverage Radius

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ABSTRACT

Hub location problems occupy an important place among transportation problems. In such problems, each node in the network including hubs can be the origin or destination point of a transport demand. What makes hubs different from the other nodes is that they also act as transit points, that is, they enable the combined demand moved from different origin nodes to be sent by separating according to destination nodes. The existence of hubs simplifies the structure of the transport network. At the same time, hubs ensure better use of the capacity of transportation vehicles and decreases the unit transportation cost due to the possibility of transporting large volumes between hubs. Hub location problems are frequently encountered in the design of cargo distribution, airline passenger transport and telecommunication networks. In this study, hub-covering problem, which is a special hub location problem, is emphasized. The basic assumption for this type of problems is that some nodes in the network cannot be directly or indirectly connected due to distance, time or cost constraints. Capacity utilization and transportation costs are generally ignored in the mathematical models developed for the hub-covering problem. To fill this gap in the literature, a deterministic multiple assignment mixed integer optimization model has been developed. How and to what extent the change in different model parameters affects the best solution for the proposed model was examined by numerical experiments using a benchmark data set. As a result, it was revealed how important it is to consider the aforementioned elements in the hub-covering problem and practical insights have been provided.

ÖZET

Taşıma problemleri arasında Ana Dağıtım Üssü (ADÜ) yerleşim problemleri önemli bir yer işgal etmektedir. Bu türdeki problemlerde ADÜ'ler de dahil ağdaki her düğüm bir taşıma talebinin çıkış veya varış noktası olabilir. ADÜ'leri diğerlerinden farklı kılan ise bu düğümlerin geçiş noktası vazifesi de görmeleri, yani farklı çıkış düğümlerinden taşınarak birleştirilen talebin varış düğümlerine göre ayrıştırılarak gönderilebilmesini sağlamalarıdır. ADÜ'lerin varlığı taşıma ağının yapısını basitleştirmektedir. Aynı zamanda ADÜ'ler hem taşıma araçlarının kapasitesinin daha iyi kullanılmasını, hem de ADÜ'ler arası büyük hacimlerde taşıma imkânı bulunduğundan birim taşıma maliyetinin düşmesini sağlarlar. ADÜ yerleşim problemleriyle sıklıkla kargo dağıtım, havayolu yolcu taşımacılığı ve telekomünikasyon ağlarının tasarımında karşılaşılır. Bu çalışmada özel bir ADÜ yerleşim problemi olan ADÜ kapsama problemi üzerinde durulmuştur. Bu türdeki problemlerin temel varsayımı mesafe, süre veya maliyet kısıtı nedeniyle ağdaki bazı düğümler arasında doğrudan veya dolaylı bağlantı kurulamayacağıdır. ADÜ kapsama problemi için geliştirilen matematiksel modellerde kapasite kullanımı ve taşıma maliyetleri unsurları genellikle göz ardı edilmiştir. Yazındaki bu boşluğu doldurmak amacıyla determinist çoklu atamalı karışık tam sayılı eniyileme modeli geliştirilmiştir. Önerilen model için farklı model parametrelerindeki değişimin en iyi çözümü nasıl ve ne derecede etkilediği kıyaslama veri kümesi kullanılarak yapılan sayısal deneylerle incelenmiştir. Sonuçta anılan unsurların ADÜ kapsama probleminde dikkate alınmasının ne derece önemli olduğu ortaya konmuş ve pratik çıkarımlar sunulmuştur.

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1. INTRODUCTION

By virtue of advanced technology and changed regulatory, hub networks took a noticeable role in last years for many sectors such as networks of airline passenger transportation, emergency treatment service, telecommunication, express delivery service, postal, urban traffic and trucking systems. Hubs can be categorized in many different ways. For example, a terminal or a transit stop for many routes stands for a hub in urban traffic networks; a warehouse or a facility located at the center implies a hub in trucking systems; both a transit point and a geographical area which is having vast amount of passengers exceeding a specific level symbolizes a hub in airline transportation systems; a server both to receive, to process and to send information represents a hub in telecommunication systems; a center in which switching and sorting operations are done means a hub in express delivery service networks. Hub-and-spoke systems share common features even if these hubs serve in various industries with different concepts. One of these features is that hub facilities are points and are utilized in order to consolidate, to disseminate, to switch, to transship as well as to sort demand flows. The second one is that in order to send to lots of O-D pairs, relatively small number of arcs and indirect connections are allowed in hubs. Two features decrease the number of connections needed in network. Not only a simplified network structure but also reduced construction costs are the results of fewer connections. Inter-hub links have a lower cost for unit transportation that is expressed as a discount rate δ, when compared with other links through a consolidated and disseminated demand flows. A hub-and-spoke system is shown in Figure 1.1. Nodes i and j are assigned to hub node k and m and inter-hub links are shown by bold lines for the sake of clearly.

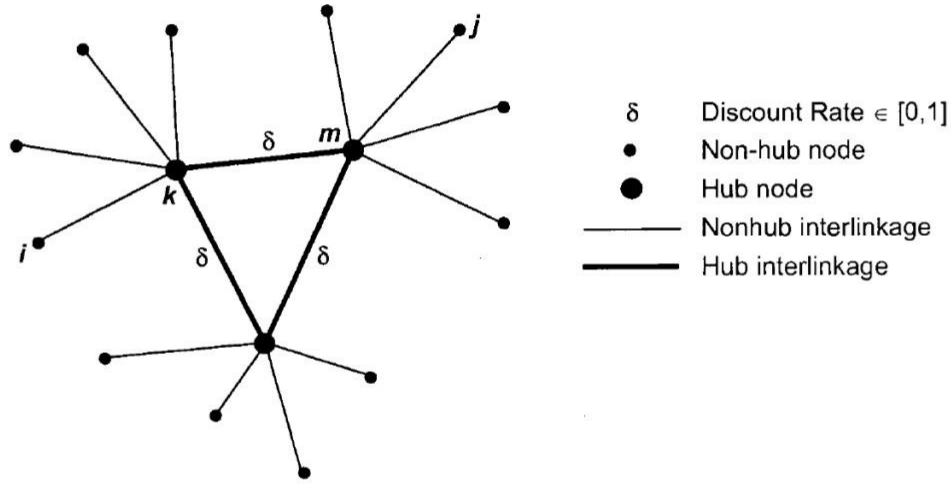

Figure 1.1: Hub-and-spoke network (Wu, 2006, p.2)

There are various Hub Location Problems (HLP) in the literature that it requires to be classified in 8 main categories such as: *a*) solution domain, *b*) criterion, *c*) source determining the number of hubs to locate, *d*) the number of hub nodes, *e*) hub capacity, *f*) the cost of locating hub nodes, *g*) the allocation of a non-hub node to hub nodes, *h*) the cost of connecting non-hub nodes to hub nodes. Domain is for whether there is a connected or discrete or continuous network. Criterion is either mini-sum in which the total cost incurred by locating hub nodes and allocation of non-hub nodes to hub nodes is minimized, or mini-max in which the maximum transportation cost from origin nodes to destination nodes is minimized. Source which is determining the number of hubs to locate is either exogenous which means that the number of hubs to locate is known or endogenous which means that the number of hubs to locate is not known at the beginning but is obtained as a result of solution. The number of hubs can be single or multiple. Hubs have capacity or not. No-cost, fixed cost and variable cost are different forms of costs not only for locating hub nodes but also for connecting non-hub nodes to hub nodes features. Single and multiple allocation are for allocation of a non-hub node to hub node feature.

Concentrated through hubs in the HLPs that is substantial feature of the HLPs serves a good transportation between the nodes. For example, in case of presence of a single hub in network, the network will need $2(n\text{-}1)$ pairs to connect the O-Ds instead of $n(n\text{-}1)$ which is the case of not presence of a hub in the network. Thus, this is a clear evidence of that hubs decrease the number of links in structure of transportation network (Farahani et al., 2013). In addition, one another beneficial effect is about economies of scale since there are flows concentrated among hubs. On the top of that, the transportation cost between hubs is arranged to be decreased by a certain discount factor. In brief, the economies of scale as a result of concentrated flows and smaller number of links and smaller investment for network are advantages of hub networks. However, investments for hubs, demand for switching, operations for transshipment and organizing of operations and movements for greater distance and also requirement of greater time are disadvantages of hub networks.

Many researchers focused on the location theory because of increasing commercial activities in the developing world. HLP is emerged as one of the developing research areas in location theory. HLP is both a facility location and location-allocation problem consisting of selection of hubs at first and then of allocation of demand hubs to previously selected hubs. Although there are some studies solely focusing on allocation part, both must be taken into consideration as the optimal solution is affected by the locations of hubs. Briefly, HLPs can be divided in two steps such as to select hub location for p nodes within n nodes, and to allocate demand points. HLPs structures also can be divided in two with respect to the allocation of demand points to the hubs, either single or multiple allocation.

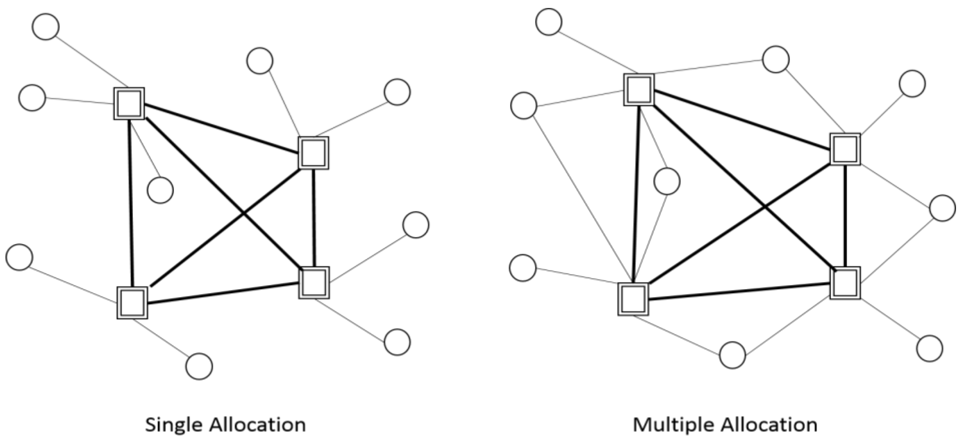

Figure 1.2: Network structures for single and multiple allocation (Yildirim, 2013)

This thesis is organized as follows. In Chapter 2, we provide a literature review about hub location problems in terms of their types, problem environments, solution approaches and applications. In Chapter 3, we first mention three different types of hub covering models and then give information about our proposed hub covering flow problem model with two formulations. Chapter 4 contains data, model implementation and the results based on the computational analysis of our mathematical model mentioned. Finally, Chapter 5 includes conclusion and insights.

2. LITERATURE REVIEW: HUB LOCATION PROBLEM

A brief about network design before keep going to literature of HLP is nice to be explained. Forsgren and Prytz (2006) expressed the important problem classes of network design in many ways such as minimum cost multicommodity flow, uncapacitated network design-fixed charge, capacitated network design, network loading problem, topology constraints, routing constraints, multiperiod problems, hierarchical network design and survivability. The question in minimum cost multicommodity flow problem is how to send a number of commodities via network with minimum cost subject to capacity constraints of links. In the uncapacitated network design-fixed charge which is a fundamental network design problem the case whether the replacement of capacity constraints with a cost for utilizing the arc is examined. Capacitated network design is examined if a fixed cost pls capacity constraints in the arc are added to the minimum cost multicommodity flow problem. The question in network loading problem is what the capacity levels of arcs are, from given set of capacity levels, since that which links can be used has already been decided. Topology constraints may be imposed to have a certain type such as ring-structures, tree structures or more specific structure. Routing constraint represents the routing in the network may be forced in a more complex manner than in capacity levels. Multiperiod problems do not include timescale. Hierarchical network design consists of several levels; on the top there is a backbone network as so to refer different criteria; for the lowest level, a local network within either a company or a private home can be given as example; for the intermediate level, a range of various network levels can be examined. Survivability means to construct network that are robust in case of a link of node failure.

In the literature, the location-allocation problem has been issued with the assumption that sources are solely interacted with its destinations which are assigned to them. Therefore, the source is not allowed to connect with neither the other non-hubs nor the destinations for other hubs. This is very limiting result of that assumption in many diversifications of applications (i.e. communication satellites and the airline industry). However, that the location-allocation problem considering interaction between the sources is equivalent to the hub location problem is stated by O'Kelly (1986). Then, the term source and destination are interchanged with hub and spoke, respectively. In detail, the location of hubs and the allocation of spokes to hubs are considered in the hub location problem. Thus, the level of interaction in between hubs is calculated by the amount of flow in between spokes.

2.1. Types of HLPs

This problem can be classified as either continuous or discrete. If hubs can be located anywhere in the plane, it is called continuous, otherwise, if there are finite number of points for hub location, it is called discrete. In most cases, assumptions for finding the number of hubs to be located are relaxed since the problem is already quite complex even though the number of hubs is exogenous.

2.1.1. Continuous Hub Location Problem

In many surveys, the continuous hub location problems that are concerned with the locating hub facilities on a plane instead of the nodes of a network. This continuous type problem is first represented by O'Kelly (1986a). In his study, that the single hub location problem in a plane reduces to the classical Weber least cost location problem is showed. In two-hub case, a procedure given by Ostrech (1975) to solve the two-center location-allocation problem is adopted to solve it. In two-center problem, Ostrech displayed that the optimal solution must be among a finite number of $n(n-2)/2$ of non-overlapping partitions. Picking a pair of nodes, passing a line through them and rotating the line slightly to divide the nodes in two groups result in a partition. O'Kelly (1986a) pointed a drawback of using results found by Ostrech (1975) to solve the two-hub problem, that the optimal allocation pattern could be found by one of the non-overlapping will never be examined. This issue is further discussed in Aykin (1988) in detail. In addition, one another drawback of O'Kelly's (1986a) study is that it cannot be applicable to solve problems including more than two hubs, unless serious computational efforts are provided.

Aykin and Brown (1992) suggested a heuristic approach in order to solve the continuous hub location problem with more than two hubs, in other words, *p*-hubs. The heuristic based on the alternate location and allocation heuristics in Cooper (1964), figures out the location and allocation phases individually and iteratively. They tested two different versions of heuristic with respect to the original alternate location and allocation heuristic. That the loss of applying the original alternate location and allocation heuristic to HLP can be calculated as 11.4%. Then, authors made some modifications on their heuristic with the aim of solving the HLP on sphere. To represent the earth's surface by a sphere is more accurate in particular applications of more planetary attributes (i.e. international travel or defense issues). One may refer to O'Kelly (1986a, 1992b), Aykin (1988,1995b), Campbell (1993), O'Kelly and Miller (1991) and Aykin and Brown (1992) for more details.

2.1.2. Discrete Hub Location Problems

Discrete hub location problems are mentioned by Campbell (1994): "Hub location problems can be viewed as embedded in an undirected network $N = (V, A)$, where the set of nodes, or vertices, of the network $V = \{v_1, v_2, ..., v_q\}$ correspond to origins/destinations and potential hub locations. Thus, hubs are restricted to be located at a subset of the vertices. Associated with link $(a, b) \in A$, which connects vertices v_a and v_b, is a non-negative weight $d(a, b) = d(b, a)$ representing its length. This may correspond to travel distance, time, cost or some other attribute. Define C_{ab} to be the length of the shortest path between nodes a and b. The cost for movement on the path from origin i to destination j via hubs at nodes k and m, in that order, is $C_{ik} + \alpha C_{km} + C_{mj}$, where α is the discount factor for the inter-hub transportation. If $k = m$, then there is no inter-hub transportation. Associated with each O-D pair (i, j) is a non-negative weight representing the flow from i to j."

As discrete hub location type is more realistic than continuous one, it received more attention in many works in literature. The researchers addressed on finding heuristic solution approaches solving the discrete HLP, under two assumptions given below:
-the number of hubs (*p*) is exogenous/given
-each spoke has to be assigned to just one hub.

HLPs which satisfy these two conditions are called as discrete *p*-HLPs in simple terms in early literature. The discrete hub location models can be classified in four categories (Alumur and Kara, 2008). These categories are:

- The *p*-hub median problems (*p*HMP),
- The hub location problems with fixed costs,
- The p-hub center problems (*p*HCP) and
- The hub covering problems that involve the *p*-hub maximal covering problem and hub set covering problem.

The rest of this chapter is devoted to the first three categories. As the subject of this study is a hub covering problem, the next chapter will be entire dedicated to the literature survey of the last category. Figure 2.1 provides the derivation of problems in detail.

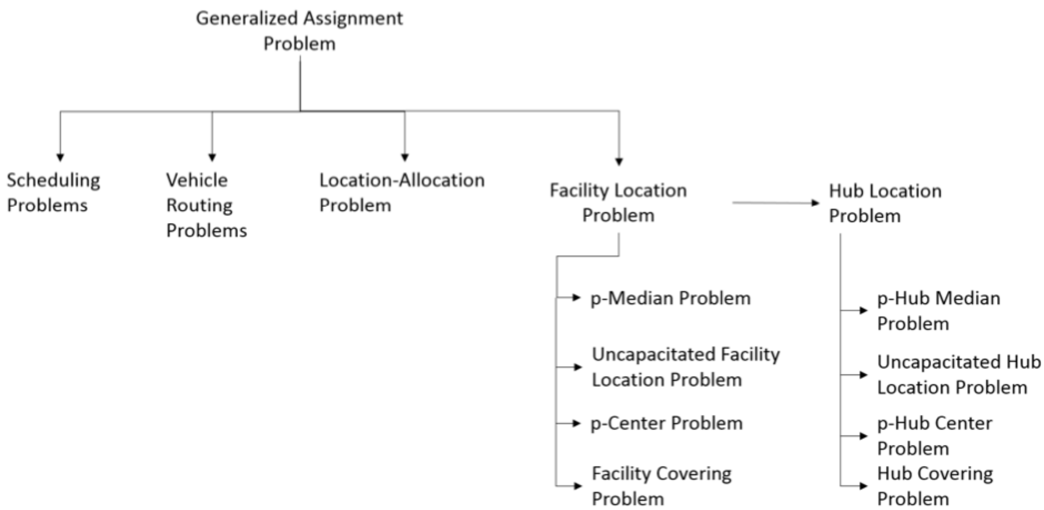

Figure 2.1: Development of HLP (Yildirim, 2013)

Each class name is correlative to a classical facility location problem. For example, the *p*-Hub Median Problem is also named with the *p*-Hub Location Problem. Yet, from this point on, the names and the notations given above are used to refer various HLP classes.

- ***p*-Hub Median Problem (*p*-HMP)**

O'Kelly (1987) is the first researcher to formulate the *p*-hub median problem as a quadratic integer programming. To locate the hubs and allocate non-hub nodes to hubs with the aim of minimizing the total cost in the network is *p* hub median models' objective. A *p*-hub median in O'Kelly (1987) is the first hub-and-spoke network model without restrictions of the hubs number ever formulated. The USApHMP-Q, uncapacitated single allocation *p*-hub median problem and *Q* is for quadratic, problem was formulated in the paper:

$$\min \quad \sum_i \sum_j W_{ij} \left(\sum_k Z_{ik} C_{ik} + \sum_m Z_{jm} C_{jm} + \alpha \sum_k \sum_m Z_{ik} Z_{jm} C_{km}\right) \qquad (2.1)$$

s.t.

$$(n - p + 1)\, Z_{kk} - \sum_i Z_{ik} \geq 0 \qquad \text{for all } k \qquad (2.2)$$

$$\sum_k Z_{ik} = 1 \qquad \text{for all } i \qquad (2.3)$$

$$\sum_k Z_{kk} = p \qquad (2.4)$$

$$Z_{ik} \in \{0,1\} \qquad \text{for all } i, k \qquad (2.5)$$

Z_{ik} is a variable and equals to one as node *i* is, if and only, assigned to hub *k* and equals to zero in contrary situation. Z_{kk} has a specific situation in which that parameter equals to one only if node *k* is a hub (O'Kelly, 1987). The other parameters that defined in paper are:

W_{ij} : the number of units of flow from node *i* to node *j*, ,

C_{ij}: the transportation cost of a unit of flow from node *i* to node *j*,

W_{ii} = 0 and C_{ii}=0 by assumption,

p: the total number of hubs to set,

n: the total number of cities to link.

Eq. (2.1) represents the total cost of assigning a node for outgoing and incoming flows and also the costs of their interactions in hubs which are given inside the brackets, once units of flows are transferred from originated node i to hub k and from hub k to hub m and from hub m to destination hub j. Eq.(2.2) represents that it is impossible to assign a node to a hub if a hub is not opened at that location. Moreover, it ensures that there can be at most *n-p+1* nodes to be assigned to a certain hub. Eq. (2.3) provides that there is only one hub to be assigned by each node. Eq. (2.4) ensures that there are p hubs to be opened. Eq. (2.5) represents whether node i is assigned to hub k or not.

O'Kelly (1987) also formulated the objective function differently by means of defining O_i and D_i, which are the total amount of flow launching from node *i*, and the total amount of flow that terminates in node i, in turn.

$$\text{Min} \sum_i \sum_k Z_{ik} C_{ik} \left(O_i + D_i\right) + \sum_j \sum_m Z_{jm} \sum_i \sum_k Z_{ik} (\alpha W_{ij} C_{km}) \qquad (2.6)$$

Since both objective functions are in quadratic form, solving large instances of this model can become very complicated. O'Kelly stated that this problem is NP-hard and suggested two enumeration-based heuristics with the aim of providing a solution. In each heuristic, all possible combinations of p-hub are taken into consideration. In first heuristic, the nearest hub is chosen for allocation while in second one, the first and the second nearest hub are chosen for allocation. As a result, that the second heuristic gives a tighter upper bound on objective function with respect to the first one is concluded.

Klincewicz (1991, 1992) proposed different heuristic approaches for p-hub median problem. Klincewicz (1991) included two solution approaches such that the first is based on single and double heuristics and the second is based on clustering. A multi-criteria assignment procedure which considers not only the distance from spoke *s* to hub but also the flow between *s* and the other spokes of the network while assigning a spoke, to allocate spokes to hubs is suggested by the author. In Klincewicz (1992), tabu search and a greedy randomized adaptive search procedure (GRASP) are used to solve pHMP. Both are developed from artificial intelligence techniques in order to solve combinatorial optimization problems. The author discussed that the use of sophisticated assignment rules is not necessary and recommended to use the distance-based assignment rule that is proposed previously (O'Kelly, 1987). In addition, Aykin (1990) studied this assignment issue. The heuristic approaches in Klincewicz (1991, 1992) not only need less computational effort when compared to the enumeration-based heuristics in O'Kelly (1987) and also are used to solve problems with large size.

Campbell (1991a,1991b) developed the more general assignment of nodes to multiple hubs. In these studies, the second assumption mentioned previously does not hold. Here, letters S and M are used in problem notation and stand for single and multiple allocation respectively. Yet, these letters can be located either at the beginning or at the end of problem notation. For instance, *p*HMP-S denotes the p-Hub Median Problem with single allocation. The fact that a solution for *p*HMP-M provides a lower bound for *p*HMP-S is showed by Campbell (1991a). This is rational since the total number of possible assignments for *p*HMP-M is less than that for *p*HMP-S. Thus, to solve *p*HMP-M and use its solution as a starting point for p-HMP-S is not very difficult. Putting this idea into practice, the author elaborated two heuristic approaches. Campbell (1991b) studied to present the mathematical formulation for each class of HLP mentioned above. Also, both multiple and single allocation cases are considered in the study. Moreover, the linear programming equivalent of cases are given even though they are quadratic programming problems.

Table 2.1: Studies on pHMP-S

Year	Authors	Model
1987	O'Kelly	The first quadratic model, HEUR1 and HEUR 2
1994	Campbell	The first linear integer formulation
1996	Skorin-Kapov et al.	A mixed 0-1 integer model, TS
1996	O'Kelly et al.	A model for exact solution, effect of α.
1996	Ernst and Krishnamoorthy	A mixed integer formulation, SA, BB algorithm
2001	Ebery	A mixed integer formulation for p=2 and 3
2009	Yaman	Hierarchical hub network design
2012	Yaman and Elloumi	Star *p*HMP

Table 2.2: Studies on pHMP-M

Year	Authors	Model
1992	Campbell	The first linear integer programming formulation
1994	Campbell	A new formulation with flow threshold and fixed cost
1996	Skorin-Kapov et al.	A new mixed-integer formulation, enumeration search tree
1998a	Ernst and Krishnamoorthy	A new formulation with LP relaxation
1998b	Ernst and Krishnamoorthy	BB algorithm based on shortest path
1999	Sasaki et al.	1-stop multiple allocation pHMP, BB method, greedy-type heuristic
2004	Boland et al.	Preprocessing technique, tightening constraints
2009	Campbell	Two new models, maximum traveling time constraint

- **The Hub Location Problem with Fixed Costs**

This type of HLP and pHMPs have common features but also two great distinctions. First one is that many pHMP formulations do not take the fixed costs for opening the hubs into consideration since they focused to consider the number of hubs to open. However, the hub locations problems with fixed costs include these costs in the objective function. The second distinction is that the number of hubs to be opened is not a fixed number but should be low as much as possible due to incurring costs.

As there are that much similarities between them, modifying models of pHMP can give most of the formulations of HLP with fixed costs. Also, the difference between them require following two modifications:

- the objective function has to include the fixed costs defined with F_k.
- Eq. (3) should be disregarded from the model as the number of hubs to open is not an exogenous parameter anymore.

If these two modifications are applied for O'Kelly (1987) in O'Kelly (1992), they will be same. Similarly, Campbell (1994) has the modifications for pHMPs that are issued in the same paper.

Table 2.3: Studies on uncapacitated fixed cost HLP-S

Year	Authors	Model
1992	O'Kelly	Quadratic integer programming formulation
1994	Campbell	First linear formulation
1998	Abdinnour-Helm and Venkataramanan	New quadratic integer formulation, BB, multi-commodity
1998	Abdinnour-Helm	GA, TS, shortest method and heuristic
2005	Topcuoglu et al.	GA
2007	Cunha and Silva	Hybrit GA
2007	Chen	New hybrid method with SA, tabu list

Table 2.4: Studies on uncapacitated fixed cost HLP-M

Year	Authors	Model
1994	Campbell	The first linear integer model
1996	Klincewicz	Dual-ascent and dual-adjustment based BB
2002	Mayer and Wagner	Hublocater
2004	Hamacher et al.	Polyhedral, facet-defining
2004	Boland et al.	Preprocessing procedure, tightening constraints
2007	Canovas et al.	Dual-ascent based heuristic

Table 2.5: Studies on capacitated fixed cost HLP

Year	Authors	Model
1994	Aykin	BB, a heuristic method
1999	Ernst and Krishnamoorthy	Mixed integer programming, BB
2000	Ebery et al.	New mixed integer formulation, LP-based BB
2003	Sasaki and Fukushima	1-stop HLP, BB
2005	Labbé	BB, polyhedron studies
2008	de Costa et al.	Bi-criteria approach to minimize total and service time

- **p-Hub Center Models**

*p*HCPs are another variation of hub location problem differing in way of its minimax objective function. 3 types of HCP are defined by Campbell (1994). First one's goal is to minimize the maximum cost among each O-D pair, while second one's goal is to minimize the maximum cost on any connection either origin-hub or hub-hub or hub-destination. And final one's goal is to minimize the maximum cost in all either hub-origin or hub-destination pair. For both the single and multiple allocation versions of these three objective functions, simple formulations of pHCP are presented in this part of literature review.

First type objective function proposed for both single and multiple allocation model is:

$$\text{minmax}_{ijkm}\ \{X_{ijkm}, C_{ijkm}\}.$$

Second type objective function proposed for both single and multiple allocation model is:

$$\text{minmax}_{ijkm}\ \{\max\ (C_{ik}, C_{mj}, \alpha C_{km}) X_{ijkm}\}$$

Third type objective function proposed for both single and multiple allocation model is:

$$\text{minmax}_{ijkm}\ \{\max\ (C_{ik}, C_{mj}) X_{ijkm}\}$$

Although Campbell (1994) defined three types of pHCP, some contributions in which many of them are familiar to

Table 2.6: Studies on pHCP

Year	Authors	Model
1994	Campbell	Fixed integer model, three type p-center problem
1999	Kara and Tansel	New mixed integer programming, linearization of models of Campbell
2000	Pamuk and Sepil	Single-reallocation heuristic
2003	Hamacher and Meyer	BS algorithm
2005	Ernst et al.	New mixed integer formulation for single allocation, two integer programming for multiple allocation, shortest path-based BB
2008	Meyer et al.	Two-phase algorithm, shortest path-based BB

2.2. Problem Environments

As HLP includes strategic decisions for long-term, its certain parameters (i.e. flows, costs and distances) can alter with the time. Therefore, to consider the problem in an uncertain environment is meaningful. Two main environments in literature to deal with uncertainty are randomness and fuzziness.

To handle uncertainty as randomness (i.e. stochastic HLP) is one main subtopic. Marianov and Serra (2003) attempted to model hubs as M/D/c queuing systems and to formulate a linear mixed integer programming with chance-constraints. Sim et al. (2009) both stated stochastic *p*HCP and also presented a chance-constrained programming with constraint of service-level. Yang et al. (2011) extended the problem by considering discrete random travel time. Alumur et al. (2012) focused on a comprehensive model not only single but also multiple allocations with random demands and set-up costs. Hult et al. (2014) improved exact solution approaches that are based on reduction of variable and also a separation algorithm in order to solve uncapacitated single allocation case.

A fuzzy programming approach to model dynamic virtual hub location problem is employed by Taghipourian et al. (2012). A fuzzy possibilistic bi-objective model for hub covering problem considering production facilities, time horizon and transporter vehicles is established by Ghodratnama et al. (2013). A fuzzy p-hub center problem in which the travel times are characterized by normal fuzzy vectors first proposed by Yang et al. (2013a). Yang et al. (2013b) continued to present a risk aversion formulation through adopting value-at-risk criterion in the function of objective.

2.3. Solution Approaches for HLPs

To cope with different types of HLPs, miscellaneous solution algorithms have been suggested. In this part, related articles are examined, several of represented solution approaches are listed. Although most of HLPs are modeled as a network location problem, there are some studies done in discrete and continuous domains. Note that some notations are given in Table 1 to better understand the various kinds of HLPs.

Table 2.7: Notations for different types of HLPs

Capacity of hub node	**Allocation of nodes**	**Type of HLP**	**Number of hub nodes**
Capacitated (C)	Single allocation (SA)	Median (M)	Single (1)
Uncapacitated (U)	Multiple Allocation (MA)	Center (T)	More than one (P)
		Covering (V)	
		Set Covering (SV)	
		Maximum Covering (MV)	

When solving instances of HLPs, exact algorithms are more practical when the problem size is small. Larger instances in HLPs require to be solved either by heuristic procedures or by meta-heuristic procedures although small hub problems can be solved by integer programming optimization approaches. Large-sized instances can be handled by specialized exact methods such as benders decomposition and branch-and-price methods. Yet, as a matter of fact, development of meta-heuristics has a great advantageous and serves many real-life applications. Thus, optimal or near optimal solutions can even be obtained in less computational time. In this section, studies in last 10 years which make use of exact optimization methods and heuristics for finding solution of HLP are presented in Tables 2.8-2.9.

2.4. Applications of HLPs

Since O'Kelly (1987) formulated HLP as quadratic integer programming by choosing hubs and their assignments, more and more attentions of researchers who are from operations research, transportation, geography, network design, telecommunications, regional science, economics and etc., are directed to this field. Campbell and O'Kelly (2012) is suggested for further details. Although applications of HLP are mostly encountered in air and road transportation systems, we expose all application areas as much as possible in this section Table 2.10 shows grouped papers with their related application areas. The most recent ones are also discussed briefly.

Table 2.8: Exact solution algorithms in HLPs (in last 10 years)

Problem	Article	Solution algorithm	Efficiency (# of nodes)	# of hubs
C-MA-p-HLP	Gelareh and Pisinger (2011)	Mixed Integer Prog.	15	–
	Alumur et al.(2016)	Mixed Integer Prog.	15-25	–
C-SA-p-HLP	Correia, Nickel, and Saldanha-da-Gama (2010a)	Mixed Integer Prog.	50	–
	Correia, Nickel and Saldanha-da-Gama (2010b)	Linear Prog.	–	–
	Kratica et al. (2011)	Mixed Integer Prog.	–	–
	de Camargo and Miranda (2012)	Generalized Benders decomposition method	100	20
	Taghipourian et al. (2012)	Fuzzy Integer Linear Programming	20	4–7
	Alumur et al.(2016)	Mixed Integer Prog.	15-25	–
U-MA-p-HLP	Contreras, Cordeau, and Laporte (2011c)	Enhanced Bender decomposition method	500	–
	Gelareh and Nickel (2011)	Bender decomposition method	50	20
	Vasconcelos, Nassi, and Lopes (2011)	Integer Prog.	12	–
	Vidovic et al. (2011)	Mixed Integer Prog.	–	–
	Alumur et al. (2012)	Stochastic Prog.	25	4
U-SA-p-HLP	Contreras, Fernandez, and A Marin (2010)	Mixed Integer Prog.	25	8
	Lin (2010)	Integer Linear Prog.	–	–
U-SA-1-HLP	Alumur, Nickel, et al. (2012)	Stochastic Prog.	25	5
U-MA-M-p-HLP	Garcia, Landete, and A Marin (2012)	Integer Prog.-Branch and Cut	200	190
U-SA-M-p-HLP	Puerto, Ramos, and Rodriguez-Chia (2011)	Mixed Integer Prog.	20	10
U-MA-T-p-HLP	Yaman and Elloumi (2012)	Mixed Integer Prog.	70	20
U-SA-T-p-HLP	Yaman and Elloumi (2012)	Mixed Integer Prog.	50	10

Table 2.9: Heuristics and meta heuristic solution algorithms for HLPs (in last 10 years)

Problem	Article	Solution Algorithm	Efficiency (# of nodes)	#of hubs
C-SA-p-HLP	Lin and Lee (2010)	Lagrangian relaxation	–	–
	de Camargo, Miranda, and Ferreira (2011)	Outer approximation/benders decomposition method	200	–
	Contreras, Diaz, and Fernandez (2011)	Branch and price – Lagrangian relaxation	200	–
C-MA-M-p-HLP	Lin, Lin, and Chen (2012)	Genetic algorithm	–	–
C-SA-V-p-HLP	Mohammadi, Jolai, and Rostami (2011)	Imperialist competitive algorithm and genetic algorithm	70	–
U-MA-p-HLP	Gelareh, Nickel, and Pisinger (2010)	MILP – Lagrangian decomposition method	20	7
	Contreras et al. (2011a)	Monte Carlo simulation-based algorithm/benders decomposition method	50	–
U-SA-p-HLP	Han (2010)	Integer programming – Tabu search	50	11
	Catanzaro, Gourdin, Labbe, and Ozsoy (2011)	Branch and cut	20	–
U-MA-M-p-HLP	Cetiner, Sepil, and Sural (2010)	Iterative heuristic	81	6
	Ishfaq and Sox (2011)	Tabu search	100	6
	Ishfaq and Sox (2012)	Tabu search	25	5
U-SA-M-p-HLP	Ilić, Urošević, Brimberg, and Mladenovic (2010)	General variable neighborhood search	1000	20
U-MA-V-p-HLP	Karimi and Bashiri (2011)	Heuristic algorithms	37	6
U-SA-V-p-HLP	Karimi and Bashiri (2011)	Heuristic algorithms	37	6

Table 2.10: Application areas of HLPs

Application Area	Paper
Airlines and airports:	Toh et al. (1985), Shaw (1993), Aykin (1995), Jaillet et al. (1996), Bania et al. (1998), Sasaki et al. (1999), Martin and Roman (2003), Adler and Hashai (2005), Ozger and Oktal (2009), Eiselt and Marianov (2009), Kawasaki (2012), Davari et al. (2013), Ozger and Oktal (2013).
Transportation and handling problems:	Don et al. (1995), Lumsdenk et al. (1999), Aversa et al. (2005), Baird et al. (2006), Cunha and Silva (2007), Yaman et al. (2007), Eiselt (2007), Verma et al. (2017), Li et al. (2019), Danijela (2019), Vural and Aygun (2019), Carman et al. (2019), Msakni et al. (2020), Park and Kim (2020).
Post delivery services and fast delivery packing companies:	Kuby et al. (1993), Krishnamoorthy et al. (1994), Ernst and Krishnamoorthy (1996), Ebery et al. (2000).
Telecommunication systems and massage delivery networks:	Lee et al. (1996), Klincewicz (1998), Carello et al. (2004), Bollapragada et al. (2006), Contreras and Fernandez (2012).
Emergency services:	Hakimi (1964), Berman et al. (2007), Chen et al. (2013), Zhang et al. (2017), Rostami et al. (2018)
Chain stores in supply chain	Marufuzzaman and Eksioglu (2014), Roni et al. (2017), Razmi and Rahmanniya (2019), Fakhrzad et al. (2019).
Perishable Food sector and environment:	Esmizadeh and Bashiri (2014), Etemadnia et al. (2015), Musavi and Bozorgi-Amiri (2017).
Green environment	Maiyar and Thakkar (2019), Dukkanci et al. (2019), Parsa et al. (2019).

In airlines and airports, Ozger and Oktal (2013) modeled constrained choices when establishing cargo hub and its spoke networks. They improved MILP model which is introducing additional constraints to the traditional model of uncapacitated multiple allocation hub location problem. Then, this developed model is tested empirically. According to test results, the major factors effecting hub location along with the cost of airline movements are aircraft range and trip cost, runaway availability and cargo traffic continuity of an airport. Davari et al. (2013) dealt with an incomplete hub-covering network design problem in which the exact locations of demands are not know and are estimated as fuzzy variables. They modified and earlier model in the HL literature in order to focus on the uncertainity of problem, also designed an efficient simulation embedded Variable Neighbourhood Search (VNS). The CAB dataset is used for its performance testing. Kawasaki (2012) focused on the scheduling effect on the demand side and the number of passengers traveling between each city pair. That the hub city is not always selected such that the number of rim passengers is minimized is shown by the study. Also, after additional simulation analyses showed that the probability of choosing a not preferable hub city is small.

In transportation and handling problems, Park and Kim (2020) presented a real-world hub-and-spoke allocation problem and its mathematical model. Moreover, they introduced the parcel classification system currently implemented by a courier company. Assigning each group of destination spokes a unique code is stated as a critical issue. Moreover, a good solution in reasonable time is obtained by the suggested algorithm. Msakni et al. (2020) studied different network designs for a linear shipping company. In their study, a feeder network is served to connect a major European port with local port. Models for both design and their solutions based on realistic dataset are included and they provide a discussion of which network design offering better cost. Carman et al. (2019) proposed an integrated model simultaneously taking into consideration of cargo flight network design and the fleet routing selection for the air cargo transportation. The comparison of two transportation modes that are the direct transportation mode in point-to-point networks and the transshipment mode in hub-and-spoke networks is provided. A swarm-intelligence-based algorithm is benefitted in order to solve optimization problem, its computational results displayed that the transportation cost can significantly decreased by proper setting of hub and transshipment route selection in an air cargo.

In telecommunication sector, cycle hub location problem (CHLP) seeks to locate p hub facilities connected by means of a cycle, as well as to route flows between pair of nodes through the cycle-star networks that minimize the total cost. The CHLP is also useful in modelling applications where large setup costs on the link and reliability requirements make cycle topologies an outstanding network architecture. Carello et al. (2004) dealt with HLP and their network presented two different nodes, access nodes and transit nodes. While access nodes represent source and destination of traffic demands but are not able to be directly connected, the transit nodes do not have their own traffic demands but can both collect traffics belonging to access nodes and route them through the network. To decide number and positions of transit nodes in order to guarantee satisfying capacity constraints is the problem. Thus, a local search approach is suggested and based on such local search 1, different metaheuristics have been developed. Bollapragada et al. (2006) presented a quantitative model of telecommunication network installation via companies. Moreover, they aimed both to maximize the expected demand coverage subject to a budget constraint on hub installation and technological constraints on demand coverage via hubs installed. They improved a practical greedy heuristic based on the budgeted maximum-coverage problem. In general, a data-dependent performance guarantee is developed. Kim and O'Kelly (2009) presented a new HLP, which is named reliable *p*-hub location problem and which its focus is to maximize network performance in terms of reliability by locating hubs for delivering flows amid city nodes. They formulated two sub-models such as *p*-hub maximum reliability and p-hub mandatory dispersion. The first one showed how optimal HL can be determined under different reliability conditions on both hubs and inter-hub links, the latter take the dispersion of hub facilities in hub network design into consideration in order to avoiding the excessive concentration of interaction flows from particular hub facilities. Contreras and Fernandez (2012) first introduced this problem in the context of general network design problems. In addition to the network design and assignment decisions concerned, CHLP takes additional routing decisions into consideration and addresses to the minimization of the total flow cost between many node pairs. In telecommunication network design case, electronic equipment such as concentrators, multiplexors and switches correspond to hub facilities, while data packages routed over a variety of physical media, such as coper cables, fiber-optic cables and telephone lines or through the air by using satellite channels are demand flows. A general architecture of these networks comprised a number of tributary networks connecting nodes to hubs and a backbone network interconnecting the hubs. Backbone links have higher capacities and route larger volumes of flow as compared to tributary links, in general, due to the configuration of their networks. Thus, a discount on the costs of using backbone link is considered instead of the cost of a tributary link. A cycle-star topology may be chosen since it provides an alternative path between every pair of hubs if a link does not succeed. Klincewicz (1998) is suggested to review for more details about design of telecommunications hub networks.

In emergency service, optimal configuration of emergency response resources is considered as crucial in order to mitigate the disaster and to protect public health and safety. Chen et al. (2013) addressed to formulate the problem of configuring disaster response resources between a set of candidate hubs, and then took as p-hub center problem minimizing the maximum travel time from hubs to demand hubs. The authors proposed a formulation based on two-stage stochastic programming, in order to handle the uncertainty of travel time, and benefited from real data of Yunnan province in China. Zhang et al. (2017) employed uncertainty theory to focus on the location problem of emergency service facilities under uncertainty. They first offered the location set covering problem in uncertain environment, later, investigated the maximal covering location problem in an uncertain environment. Finally, a case study illustrated the ideas of uncertain models. Rostami et al. (2018) studied reliable single allocation HLP under hub breakdowns such as disasters or strikes, developed a nonlinear two-stage formulation for this problem. Moreover, they designed a branch-and-cut framework based on Bender decomposition. The solution instances for much bigger than those solved so far in the literature are obtained.

In transportation sector, in particular in the design of rapid transit systems, the location of hub cycles arises in public transportation planning. By locating a circular rapid transit line (or hub cycle) such as a subway, a tram or an express bus lane, most of network planners may be interested in studying the effect of extending an already used public transportation network in a metropolitan region. The Moscow Underground, the Melbourne Circular Tram Line and some of the Montreal bus lines can be given as examples of such circular lines. Subway, tram, or bus stations where an alternation of mode of transportation is usually possible correspond to hub facilities, while bus stops, taxi stations or urban districts correspond to non-hub nodes. Users travelling between O-D pairs is the representation of demand flow, and to improve the network's total efficiency is the goal. Moreover, the discount factor is for the use of a faster transport technology connecting hubs nodes. In certain situations, both due to the reliability requirements and because it offers an alternative path reducing the travel time for some pairs of O-D, a circular line may be preferred.

In supply chain, Razmi and Rahmanniya (2019) presented a p-median hub model that aims to achieve both efficient and effective distribution while designing distribution network. Their decision variable is type of hub, since hubs have various capacities and various establishing costs. As result if computational numerical experiments, it is noticed that reducing in service level lead to decrease in establishment cost and to increase in transportation cost. Moreover, that decrease in factor which represents the economics of scale in distribution stage in comparison with the decrease in factor which represents the economics of scale between hubs has greater effect on reducing value of objective function. Fakhrzad et al. (2019) proposed an integrated model for HLP in multi-location, multi-period, multi-commodity (3M) three echelon SC and formulated this problem as a MIP model, then used GAMS to solve it. A new algorithm for re-formulation is offered to transfer into MILP, since the developed model is a MINLP and NP-hard. In addition, a new heuristic is improved to reach a solution in a reasonable time. CAB dataset is used to prove the applicability and the benefits of the proposed model. Marufuzzaman and Eksioglu (2014) aimed to design a cost-efficient and reliable SC networks for biomass delivery that its supply is seasonal to biofuel plants. This SC can cope with the biomass supply fluctuations with the help of the dynamic intermodal HL model. As their suggestion of MINLP is NP-hard, they needed to develop a rolling horizon algorithm to solve the problem. The performance of the algorithm is tested on a case study using data from the southeast region of US. Then, that a near-optimal solution of large-scale problem is provided by this proposed algorithm in a reasonable time is concluded. Roni et al. (2017) is also related with a multi-objective, hub-and-spoke model to design and manage biofuel supply chains.

In perishable food sector, some goods need to be heated or cooled at regular intervals and a hub center are required for this service. Musavi and Bozorgi-Amiri (2017) optimized scheduling and sequencing of the vehicles at hubs while considering the environmental conservation to design a sustainable supply chain. Their model as a multi-objective MILP optimizes not only the total transportation costs but also the freshness and quality during the delivery, the total carbon emissions of vehicles to provide the sustainability desire of environment as well. They suggested an adopted NSGA- II meta-heuristic in order to solve the NP-hard problem. Etemadnia et al. (2015) presented a MILP model for finding optimal hub locations in a national logistics system. They aimed to design an optimal HL network to serve food consumption markets by efficient connections along with production sites. As the possible hub-node combination are abundant within a national logistics system, they developed a heuristic solution algorithm in order to reduce computational costs. Their first finding is that model is sensitive to the distance over which commodities are allowed to travel using land transportation. Second finding is about hub capacity constraints. That maximum land shipping distances are held constant at 200 miles while minimum number of hub capacity is increased up to 100,000 tons is issued. Then, not surprisingly, there is a decrease by over 60 percent in the optimal number of hubs. Thus, the demand for air transportation will be decreased. Esmizadeh and Bashiri (2014) considered to develop a hierarchical hub network system with refreshing operation in the network. The first level includes a complete network connecting the centrak hubs while the second level includes a form of star networks connecting the remaining hubs to central hubs and third level includes demand nodes connecting hubs and central hubs in a star form. In their study, different level hubs provide refreshment operations for those goods whose delivery time exceeds the freshness time limit. Thus, the total cost of the network and spoilage rate in network is reduced. The CAB dataset is used in computational studies to illustrate the proposed method that is performing better than classical approaches for perishable goods.

In green environmental, Maiyar and Thakkar (2019) studied a green multi-objective transportation problem considering wastages is formulated. Moreover, they developed a multi-period MINLP embedded in hub-and-spoke network. Before benchmarked with NSGA- II, MOPSODE is used while solving the problem. They tested the model for various sizes and configurations in the problem. They aimed to learn the impact of varying hub location, its capacity level and the wastage threshold. Dukkanci et al. (2019) introduced the green hub location problem and considered vehicle speed and payload to estimate fuel consumption. For this purpose, they improved a nonlinear formulation model by using second order cone programming and perspective cuts. Then, they conducted an extensive computational study on CAB and TR datasets. Parsa et al. (2019) introduced a new mitigation measures in response to the rapid growth of environmental problems related to air transportation including emissions and noise. They offered a multi-objective MIP model and utilized several methodologies to determine the best design. The results of their computations displayed that using the new measure can costeffectively decrease the projected cumulative CO2 emission relative to the traditional model that is based on minimizing only the total cost of flow and opening hubs.

3. HUB COVERING MODELS

To cover all demand and to minimize either the number of hubs or the cost for opening are needed in hub covering problems. As in the p-hub center problems, 3 types of hub covering problems are stated by Campbell (1994). In first type, if the total distance from node i to j by hub k and m are smaller than a certain distance value, the O-D is covered. In second type, if the cost on all links do not exceed a certain cost value, an O-D pair is covered. In third type, if the origin-first hub and second hub-destination links do not exceed a certain difference values, the O-D pair is covered. A first basic formulation for the single allocation hub covering problem is provided by Campbell (1994). In his formulation, V_{ijkm} represents binary variable which equals to 1 if the hub k and m can cover the origin-destination pair (i, j) and F_k stands for the cost of opening a hub in k. The USASCP-1L model tends to minimize the total cost of opening hubs. Similar to the single allocation hub covering problem, a formulation of the multiple allocation version of hub covering problem is suggested by Campbell (1994). The objective function of UMASCP-1L is identical to the one in USASCP-1L.

3.1. p-Hub Maximal Covering Models

This type models are not to intend to cover all nodes but strive to maximize the demand included by a prearranged maximal number of hubs. However, it is not as predicted, since the hub covering models mentioned in above, attempt to decrease the number of hubs in which all demands can be covered. It is noticeable that *p*-hub maximal covering models are classified as hub covering problems in the literature, although their objective function and constraints are the same as in a *p*-hub median problem. (Campbell, 1994) max $\sum_i \sum_j \sum_k \sum_m W_{ij} X_{ijkm} V_{ijkm}$ (3.1)

s.t.

$\sum_k Y_k = p$ (3.2)

$\sum_k \sum_m X_{ijkm} = 1$ for all *i, j* (3.3)

$X_{ijkm} \leq Y_k$ for all *i, j, k, m* (3.4)

$X_{ijkm} \leq Y_m$ for all *i, j, k, m* (3.5)

$Y_k \in \{0,1\}$ for all *i, k* (3.6)

$0 \leq X_{ijkm} \leq 1$ for all *i, j, k, m* (3.7)

X_{ijkm}: variable indicating the proportion of flow routed by hubs

Y_k: binary variable whether a hub presents in *k*

W_{ij}: the number of unit flow from nodes *i* to *j*

V_{ijkm}: binary parameter whether the O-D pair is covered by the hubs *k* and *m*

The opening of *p* hubs is ensured by Eq. (3.2). All the flow is assured to be routed by the hubs by the Eq. (3.3). Flow from node *i* to *j* is passing thru hubs *k* and *m* is only allowed if hubs *k* and *m* are opened by means of Eqs. (3.4) and (3.5). Whether hub *k* is opened or not is provided by Eq. (3.6). The proportion of flow from node *i* to *j* passing thru hubs *k* and *m* must be between 0 and 1. This is guaranteed by Eq. (3.7).

3.2. Hub Set Covering Location Problem

This type model is a particular case of hub covering location model. Model's assumptions are familiar to median-*p* hub model excluding that the number of hubs is not known and that a fixed cost of hub location is incorporated in the model. Variables and parameters of the model are:

X_k: binary variable if hub is opened in node *k*

Z_{ijkm}: variable indicating the proportion of flow routed by hubs

F_k: fixed hub opening cost for candidate node *k*

V_{ijkm}: binary parameter equals 1 if hubs *m* and *k* cover origin-destination pair *i*, *j*

$$\min \quad \sum_k F_k X_k \tag{3.8}$$

s.t.

$$Z_{ijkm} \leq X_k \quad \text{for all } i, j, k, m \tag{3.9}$$

$$Z_{ijkm} \leq X_m \quad \text{for all } i, j, k, m \tag{3.10}$$

$$\sum_k \sum_m V_{ijkm} Z_{ijkm} \geq 1 \quad \text{for all } i, j \tag{3.11}$$

$$X_k \in \{0,1\} \quad \text{for all } k \tag{3.12}$$

$$0 \leq Z_{ijkm} \leq 1 \quad \text{for all } i, j, k, m \tag{3.13}$$

Eq. (3.8) represents the objective function minimizing the total hub location costs. Opening of hub *k* and *m* restricts the binary variable controlling whether the amount of flow originated from *i* and destinated to *j* uses candidate hubs *k* and *m* in Eq. (3.9) and Eq. (3.10), respectively. That all of O-D pairs are, at least one time, covered is guaranteed by Eq. (3.11). That the variable if hub is opened in *k* is binary is showed in Eq. (3.12) and that the variable controlling whether the amount of flow originated from *i* and destinated to *j* uses candidate hubs *k* and *m* is showed in Eq. (3.13) (Hekmatfar and Pishvaeel, 2009).

Table 3.1: Studies on hub covering problems (1994-2011)

Years	Authors	Model
1994	Campbell	First integer model, defined three coverage criteria
2003	Kara and Tansel	New integer programming linearizations of Campbell's models
2006	Hamacher and Meyer	BS algorithm, polyhedron studies
2008	Wagner	New formulation, preprocessing procedure
2011	Ernst et al.	New formulation, coverage radius concept β

Hwang and Lee (2012) aimed to locate hubs and to allocate non-hub nodes to the hubs in their model, therefore, hub can maximize the demand covered by deadline travelling time. An integer programming formulation for the new hub covering model is stated for CAB dataset. In addition, two heuristics which are distance-based allocation and volume-based allocation, are applied and their computational results showed that good solutions, for most of instances, are found in relatively reasonable computation time.

Peker and Kara (2015) studied SA*p*HMCP and MA*p*HMCP then, observed that there is only binary coverage in HL literature. Thus, they extended the definition of coverage and introduced a new coverage type which is called as partial coverage. An efficient mixed-integer programming formulation not only for single but also for multiple allocation that can be applied for partial coverage is developed. That both formulations generally perform better than the existing is concluded. Through the decreased number of variables and constraints, optimal or near-optimal solutions for larger dataset are obtained. If partial coverage is available, the coverage percentages are, as expected, increased.

Alinaghian et al. (2017) presented a new robust mathematical model for the multi-product capacitated single allocation hub location problem with maximum covering radius. Their objective is to propose a model minimizing various costs such as establishing hubs, preparing hubs for handling products, shipping. A single product of single node can be allocated at most one hub whereas different products of one node can be allocated to different hubs. Also, the model requires if equipment related to that product is installed in order to allocate a product to hub. To solve the large-scale variants of that problem, a GA-based meta-heuristic algorithm is suggested. After comparison of this heuristic with respect to the exact method and simulated annealing algorithms, respectively, the results displayed a good performance of the proposed algorithm.

Jankovic et al. (2017) studied both USApHMCP and UMApHMCP with binary and partial coverage criteria. A unified MIP formulation that can be applied for two coverage criteria, is suggested for USApHMCP and UMApHMCP. According to results of computational experiments, the superiority of newly formulation for UMApHMCP is examined, thus, it is possible to solve larger number of instances in optimality, in a shorter time. However, the case is not the same for the new formulation for USApHMCP since it performs worse when compared to the existing others in literature. In addition, the authors proposed two variable neighborhood search (VNS). Having looked at the presented experimental results, both heuristics are capable either to reproduce an optimal solution or to find a new best-known solution for benchmark problems.

3.3. Hub Covering Flow Problem

As there is an explicit discrepancy between uncapacitated hub location problem and hub covering problem, Lowe and Sim (2013) suggested the hub covering flow problem (HCFP) in which the total cost of opening hub and not only transporting demand flow but also meeting the coverage specifications are incorporated. Their formulation for single assignment HCFP is grounded on the multiple-commodity flow formulation of UHLP by Ernst and Krihnamoorthy (1996). Their single assignment MILP HCFP is formulated as below:

min $\sum_{k\in N} Z_{kk}F_k + \alpha \sum_{i,k,l\in N} Y_{ikl}c_{kl} + \sum_{i,k\in N} c_{kl}(\mathcal{X}O_i + \delta D_i)Z_{ik}$ (3.14)

s.t.

$Z_{ik} \leq A_{ik}Z_{kk}$ for all $i, k \in N$ (3.15)

$\sum_{k\in N} Z_{ik} = 1$ for all $i \in N$ (3.16)

$O_i Z_{ik} = \sum_{j\in N} W_{ik}Z_{jk} + \sum_{l\in N} Y_{ikl} - \sum_{l\in N} Y_{ilk}$ for all $i, k \in N$ (3.17)

$\sum_{l\neq k,\ l\in N} Y_{ikl} \leq O_i Z_{ik}$ for all $i\ k \in N$ (3.18)

$Y_{ikl} \geq 0$ for all $i, k, l \in N$ (3.19)

$Z_{ik} \in \{0,1\}$ for all $i, k \in N$ (3.20)

Y_{ikl} is variable indicating the amount of flow starting from node *i* that stops first to hub *k* and then *l*. F_k is establishing and operating cost at node *k* annually (Lowe and Sim, 2013). The total annualized fixed cost of opening hubs, the cost of transporting demand through the hub network, and the cost of transporting demand between a node and a hub node, respectively are terms of function of objective function in Eq.(3.14). Eq.(3.15) ensures that node is only capable to be assigned to hub opened at *k* which can cover node *i*. The single assignment is ruled by Eq. (3.16) and Eq. (3.20). The flow conservation constraint in each hub *k* for each commodity *i* is stated in Eq. (3.17). That the solution might present routing flow through links which are not selected for the hub network is eliminated by Eq. (3.18) that is akin to suggestion of Correia et al. (2010) on formulation of UHLP in Ernst and Krishnamoorthy (1996).

Lowe and Sim (2013) noted that to fix the values for certain variables so that there is a reduction in size to speed computation, matrix A_{ik} can be applicable in pre-processing step. The results of the study are:

- Increase in the cost of opening hubs, relative to the cost of transferring demand flow through the network, causes not surprisingly that the number of hubs in the network decreases.
- Hub nodes are prone to be located at or near nodes whose demand flow depending on the profile of the fixed cost values for the hubs, are high.
- Hubs are inclined to be located at the high demand flow nodes if the fixed costs are homogeneous.
- Hubs are prone to be located at nodes close to the high demand flow nodes but with lower fixed cost values if the fixed costs of the hub nodes are strongly correlated to their total demand flow.
- The inefficiencies of not to take the transportation costs into account when designing a network (i.e. HCP) could be significant. The overall cost of establishing and operating the network in HCP could cost %40 more than that in HCFP.

3.4. Capacitated Multiple Allocation Hub Covering Flow Problem

With this thesis, we propose an extended mathematical model to the hub covering flow problem by considering multiple allocation of non-hub nodes to hubs, flow processing capacity of hubs and transport flow capacity of network links. To the best of our knowledge, there is no study covering all these extensions altogether as our model. We studied this subject as taking these concepts into consideration during facility design is crucial for real applications.

3.4.1. Sets, parameters and decision variables

In this context, some important notations for both sections are following:

Sets:

V set of nodes

Parameters:

h_{ij} demand flow originating from node $i \in V$ destined for node $j \in V$

H total amount of flow to be sent

O_i total demand originating from node $i \in V$

D_j	total demand destined to node $j \in V$
w_{ik}	flow capacity of the links connecting nodes $i, k \in V$
Γ_k	flow capacity of hub k$\in V$
f_k	hub opening cost for node $k \in V$
c_{ij}	unit flow cost for the link connecting nodes $i, j \in V$
d_{ij}	length of the link connecting nodes $i, j \in V$
A	node coverage matrix (A_{ij} 1 if node j$\in V$ can be covered by node $i \in V$ and 0 if it is otherwise)
B	path coverage matrix ($B_{ij} = A_{ik}A_{kj}$)
α	inter-hub cost discount factor such that $\alpha \in (0,1)$
λ_{max}	maximum unused capacity ratio for hub $k \in V$

It is not difficult to establish that $O_i = \sum_{j \in V} h_{ij}$, $D_j = \sum_{i \in V} h_{ij}$ and $H = \sum_{i,j \in V} h_{ij}$. Hub opening decision is related with the binary decision variable x_k, which equals to 1 if node k is a hub and 0 otherwise. Variable z_{ik} denotes the amount of flow sent from node i to hub node k, q_{ilj} the amount of flow sent from node i to node j through hub node l, and y_{ikl} the amount of flow sent from node i via hub nodes k and l. Finally, variable λ_k designates the ratio of unused capacity for hub a node k.

3.4.2. Formulation without Capacity Constraints

$$\min \quad \sum_{k \in V} f_k x_k + \alpha \sum_{i,k,l \in V} c_{kl} y_{ikl} + \sum_{i,k \in V} c_{ik} z_{ik} + \sum_{i,l,j \in V} c_{lj} q_{ilj} \tag{3.21}$$

s.t.

$$\sum_{k \in V} A_{ik} z_{ik} = O_i \qquad i \in V, \tag{3.22}$$

$$\sum_{k \in V} B_{ilj} q_{ilj} = h_{ij} \qquad i, j \in V, \tag{3.23}$$

$$\sum_{l \in V} B_{ikj} y_{ikj} + \sum_{j \in V} B_{ikj} q_{ikj} - \sum_{l \in V} B_{ilk} y_{ilk} = A_{ik} z_{ik} \qquad i, k \in V, \tag{3.24}$$

$$\sum_{i \in V} B_{ilj} q_{ilj} \leq D_j x_l \qquad l, j \in V, \tag{3.25}$$

$$A_{ik} z_{ik} \leq O_i x_k \qquad i, k \in V, \tag{3.26}$$

$$z_{ik}, q_{ilj}, y_{ikl} \geq 0, \quad x_k \in \{0,1\}, \qquad i, k, l, j \in V. \tag{3.27}$$

The objective in Eq.(3.21) is to minimize the total cost of opening hubs and routing demand through network links by considering inter-hub flow cost discount factor. Eq.(3.22) ensures that all the demand originating from node $i \in V$ is transported through hubs. Eq.(3.23) guarantees that the demand originating form node $i \in V$ destined for node $j \in V$ is transported through hubs. Eq.(3.24) corresponds to the flow conservation constraints at each hub. Eq.(3.25) and Eq.(3.26) together ensure no demand is transported directly between non-hub nodes. Finally, Eq.(3.27) shows the type of decision variables.

3.4.3. Formulation with Capacity Constraints

$$\sum_{i \in v} z_{ik} + \Gamma_k \lambda_k = \Gamma_k x_k \qquad k \in V \qquad (3.28)$$

$$0 \leq \lambda_k \leq \lambda_{max} \qquad k \in V \qquad (3.29)$$

$$\sum_{i \in v} q_{ilj} \leq w_{lj} \left(1 - x_j \right) + H x_j \qquad l, j \in V \qquad (3.30)$$

$$z_{ik} \leq w_{ik}(1 - x_i) + H x_i \qquad i, k \in V \qquad (3.31)$$

Constraints in Eq.(3.28) restrict the inflow towards any hub up to its capacity. Moreover, the unused capacity ratio of hub $k \in V$ is kept track of by means of the variable λ_k. Eq.(3.29) is to restrict the unused capacity ratio up to a certain predetermined level. Constraints in Eq.(3.30) do not allow an amount of flow to be transported from hub node $l \in V$ to node $j \in V$ surpassing the link capacity w_{lj}. In a similar fashion, constraints in Eq.(3.31) guarantee that the amount of flow on the link connecting node $i \in V$ and hub node $k \in V$ does not exceed the link capacity w_{ik}. It can be inferred that inter-hub links are not capacity constrained in this formulation.

4. COMPUTATIONAL ANALYSIS

We make use of the well-known TR data set in our numerical study. This benchmark network data set is available in OR library[1]. TR data set consists of 81 nodes (cities of Turkey) network and is complete as unit flow costs, hub-opening costs, network links' lengths and flow demands are all provided. Solving one instance of our proposed model to optimality with the original TR data set and our available computational facilities takes considerable amount of time. As we are concerned with the validation of the model through a computational study, we rather preferred to work on a restricted data set. Our approach was to select nodes, which correspond to the most populated 25 cities of Turkey, and to update original parameter tables as given in the Appendix.

Inter-hub cost discount factor α is set to 0.4, 0.6 or 0.8. The hub or *node coverage radius* Δ is obtained by multiplying the *coverage ratio R* with length of the longest link of the network, i.e. $\Delta = R \times max_{ij}\{d_{ij}\}$. R should be selected such that the existing network does not contain disconnected sub-networks. Hence, *R* is set to 0.6, 0.7 or 0.8. Then, each element A_{ij} of the *node coverage matrix* is fixed to 1 if $d_{ij} \leq \Delta$, and 0 otherwise.

Link capacities, w_{ij} for all $i, k \in V$, are not included in the original TR data sets, so we developed a procedure to identify them. First, the model without capacity constraints given in Eq.(3.21)-(3.27) is solved to optimality to obtain optimum link flows. As there is no limit on the amount of flow that can be sent between hubs, we excluded inter-hub flows among the optimum link flows and calculated the average (μ) and the standard deviation (σ) of the remaining link flow values. Finally, assuming that the link flows are normally distributed, all of links' capacities w_{ij} $i, k \in V$ were set equal to $w_P = \mu + \zeta_p \sigma$ where ζ_p is the z-score corresponding to probability p with $p = \{0.70, 0.80, 0.90\}$.

[1] http://people.brunel.ac.uk/~mastjjb/jeb/info.html

Original TR data set does not contain also *hub capacities*, so we designated all Γ_k $k \in V$ equal to a fraction (15%, 20%, …, 65%) of the total demand *H*. Finally, maximum unused capacity ratio λ_{max} for a hub varies from 0.05 up to 1.00 with 0.05 increments.

All solutions presented in this analysis are obtained by using the CPLEX solver accessed through GAMS 24.9.2. In the subsequent tables given in Tables 4.1-4.6, the horizontal axis corresponds to the hubs' capacities given as a fraction of *H*, and vertical axis is related to maximum unused capacity ratio. Quadrants are also indicated on these tables, and simple statistical inferences about the tables and their quadrants are given beneath each table. Cost values are in thousands.

We first investigate how the optimum number of hubs (oNHs) changes depending on the model parameters. Decrease in the value of inter-hub discount factor α, in other words increase in the cost savings due to the aggregation of flows between hubs, increases oNHs. The average and standard deviation of oNHs given beneath Tables 4.1-4.3 clearly reveal this empirical outcome. This is an expected result as more hubs enables to aggregate more flows. Meanwhile, an interesting observation is that given all combinations of hub capacities (Γ_k) and maximum unused hub capacity ratios (λ_{max}), the smallest oNHs which can be attained does not change depending on the discount factor α. This implies that in some cases, hub capacity related constraints are so restrictive that reduction in the total cost becomes a less important issue. This empirical study also justifies some logical expectations. As for example, oNHs decreases as Γ_k increases or high-capacity hubs are admitted. Meanwhile, oNHs increases as λ_{max} increases or capacity usage constraints are relaxed.

Another outcome is that, as the capacities of links (w_{ik}) become less restrictive, oNHs decreases. Tight link capacities restrict the amount of flow that a non-hub node can send to a single hub node and thus more hubs are needed at the optimum solution to transfer the total flow. Irrespective of link capacities, highest oNHs are observed when hub capacities are low and low capacity utilization is tolerable.

Table 4.1: Optimum number of hubs for α=0.40 and R = 0.60

a4-r6-z7	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	-	-	-	-
0.10	7	5	4	-	3	-	-	-	-	-	-
0.15	7	5	4	-	3	-	-	-	-	-	-
0.20	7	5	5	4	3	3	-	-	-	-	-
0.25	7	5	5	4	3	3	-	-	-	-	-
0.30	8	5	5	4	3	3	3	-	-	-	-
0.35	8	5	5	4	3	3	3	3	-	-	-
0.40	8	6	5	4	4	3	3	3	3	-	-
0.45	9	6	5	5	4	3	3	3	3	3	-
0.50	9	6	5	5	4	4	3	3	3	3	3
0.55	9	6	5	5	4	4	3	3	3	3	3
0.60	9	7	6	5	5	4	4	3	3	3	3
0.65	9	8	6	5	5	4	4	4	3	3	3
0.70	9	8	6	6	5	5	4	4	4	4	3
0.75	9	8	8	6	6	5	4	4	4	4	4
0.80	9	8	8	8	6	6	5	4	4	4	4
0.85	10	8	8	8	8	8	6	5	4	4	4
0.90	10	9	9	8	8	8	8	8	8	6	5
0.95	10	9	9	9	9	9	9	9	8	8	8
1.00	10	9	9	9	9	9	9	9	9	9	9

	Overall	Q1	Q2	Q3	Q4
Mean	5.61	3.00	5.02	7.64	5.06
St.Dev.	2.27	0.00	1.58	1.82	2.21

(*a*) $p = 0.70$

a4-r6-z8	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	-	-	-	-
0.10	7	5	4	-	3	-	-	-	-	-	-
0.15	7	5	4	-	3	-	-	-	-	-	-
0.20	7	5	5	4	3	3	-	-	-	-	-
0.25	7	5	5	4	3	3	-	-	-	-	-
0.30	8	5	5	4	3	3	3	-	-	-	-
0.35	8	5	5	4	3	3	3	3	-	-	-
0.40	8	5	5	4	4	3	3	3	3	-	-
0.45	9	6	5	4	4	3	3	3	3	3	-
0.50	9	6	5	5	4	4	3	3	3	3	3
0.55	9	6	5	5	4	4	3	3	3	3	3
0.60	9	6	5	5	4	4	4	3	3	3	3
0.65	9	7	6	5	4	4	4	4	3	3	3
0.70	9	8	6	6	5	4	4	4	4	3	3
0.75	9	8	6	6	5	4	4	4	4	4	3
0.80	9	8	8	7	6	5	4	4	4	4	4
0.85	10	8	8	8	7	5	5	5	4	4	4
0.90	10	9	9	8	7	7	7	6	5	5	5
0.95	10	9	9	9	8	8	8	8	7	7	7
1.00	10	9	9	9	8	8	8	8	8	8	8

	Overall	Q1	Q2	Q3	Q4
Mean	5.36	3.00	4.98	7.38	4.64
St.Dev.	2.13	0.00	1.58	1.93	1.76

(*b*) $p = 0.80$

a4-r6-z9	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	-	-	-	-
0.10	7	5	4	-	3	-	-	-	-	-	-
0.15	7	5	4	-	3	-	-	-	-	-	-
0.20	7	5	5	4	3	3	-	-	-	-	-
0.25	7	5	5	4	3	3	-	-	-	-	-
0.30	8	5	5	4	3	3	3	-	-	-	-
0.35	8	5	5	4	3	3	3	3	-	-	-
0.40	8	5	5	4	4	3	3	3	3	-	-
0.45	8	6	5	4	4	3	3	3	3	3	-
0.50	9	6	5	4	4	4	3	3	3	3	3
0.55	9	6	5	4	4	4	3	3	3	3	3
0.60	9	6	5	4	4	4	4	3	3	3	3
0.65	9	6	6	4	4	4	4	4	3	3	3
0.70	9	7	6	5	4	4	4	4	3	3	3
0.75	9	7	6	5	5	4	4	4	4	3	3
0.80	9	8	7	5	5	5	4	4	4	4	3
0.85	10	8	8	6	5	5	5	4	4	4	4
0.90	10	9	8	7	7	6	5	5	4	4	4
0.95	10	9	8	8	8	7	6	6	4	4	4
1.00	10	9	8	8	8	7	6	6	5	5	5

	Overall	Q1	Q2	Q3	Q4
Mean	5.00	3.00	4.95	6.92	3.92
St.Dev.	1.95	0.00	1.53	1.97	0.90

(*c*) $p = 0.90$

a4-r6	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	2	-	-	-
0.10	7	5	4	-	3	-	-	2	2	-	-
0.15	7	5	4	-	3	-	-	2	2	-	-
0.20	7	5	5	4	3	3	-	2	2	2	-
0.25	7	5	5	4	3	3	-	2	2	2	2
0.30	8	5	5	4	3	3	3	2	2	2	2
0.35	8	5	5	4	3	3	3	3	2	2	2
0.40	8	5	5	4	3	3	3	3	3	2	2
0.45	8	6	5	4	4	3	3	3	3	2	2
0.50	9	6	5	4	4	4	3	3	3	3	2
0.55	9	6	5	4	4	4	3	3	3	3	3
0.60	9	6	5	4	4	4	4	3	3	3	3
0.65	9	6	6	4	4	4	4	4	3	3	3
0.70	9	6	6	5	4	4	4	4	3	3	3
0.75	9	7	6	5	5	4	4	4	4	3	3
0.80	9	7	7	5	5	5	4	4	4	4	3
0.85	10	7	7	6	5	5	5	4	4	4	4
0.90	10	8	8	6	6	6	5	5	4	4	4
0.95	10	8	8	7	7	7	6	6	4	4	4
1.00	10	8	8	7	7	7	6	6	5	5	5

	Overall	Q1	Q2	Q3	Q4
Mean	4.59	2.28	4.93	6.66	3.92
St.Dev.	1.99	0.46	1.55	1.85	0.90

(*d*) no link capacity

Table 4.2: Optimum number of hubs for α=0.60 and R = 0.60

a6-r6-z7	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	-	-	-	-
0.10	7	5	4	-	3	-	-	-	-	-	-
0.15	7	5	4	-	3	-	-	-	-	-	-
0.20	7	5	5	4	3	3	-	-	-	-	-
0.25	7	5	5	4	3	3	-	-	-	-	-
0.30	7	5	5	4	3	3	3	-	-	-	-
0.35	8	5	5	4	3	3	3	3	-	-	-
0.40	8	5	5	4	4	3	3	3	3	-	-
0.45	8	6	5	4	4	3	3	3	3	3	-
0.50	9	6	5	5	4	3	3	3	3	3	3
0.55	9	6	5	5	4	4	3	3	3	3	3
0.60	9	6	5	5	4	4	3	3	3	3	3
0.65	9	7	5	5	5	4	4	3	3	3	3
0.70	9	7	5	5	5	4	4	4	3	3	3
0.75	9	7	5	5	5	5	4	4	4	3	3
0.80	9	7	7	5	5	5	4	4	4	4	3
0.85	9	7	7	7	6	5	5	4	4	4	4
0.90	10	7	7	7	7	6	5	4	4	4	4
0.95	10	7	7	7	7	6	5	4	4	4	4
1.00	10	7	7	7	7	6	5	4	4	4	4

	Overall	Q1	Q2	Q3	Q4
Mean	4.85	3.00	4.93	6.68	3.68
St.Dev.	1.80	0.00	1.49	1.67	0.62

(*a*) $p = 0.70$

a6-r6-z8	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	-	-	-	-
0.10	7	5	4	-	3	-	-	-	-	-	-
0.15	7	5	4	-	3	-	-	-	-	-	-
0.20	7	5	5	4	3	3	-	-	-	-	-
0.25	7	5	5	4	3	3	-	-	-	-	-
0.30	7	5	5	4	3	3	3	-	-	-	-
0.35	8	5	5	4	3	3	3	3	-	-	-
0.40	8	5	5	4	4	3	3	3	3	-	-
0.45	8	5	5	4	4	3	3	3	3	3	-
0.50	8	5	5	4	4	3	3	3	3	3	3
0.55	9	6	5	5	4	3	3	3	3	3	3
0.60	9	6	5	5	4	4	3	3	3	3	3
0.65	9	6	5	5	4	4	3	3	3	3	3
0.70	9	6	5	5	4	4	4	3	3	3	3
0.75	9	6	5	5	4	4	4	3	3	3	3
0.80	9	7	6	5	4	4	4	4	3	3	3
0.85	9	7	6	5	4	4	4	4	4	3	3
0.90	10	7	6	5	4	4	4	4	4	3	3
0.95	10	7	6	5	4	4	4	4	4	3	3
1.00	10	7	6	5	4	4	4	4	4	3	3

	Overall	Q1	Q2	Q3	Q4
Mean	4.50	3.00	4.90	6.06	3.32
St.Dev.	1.74	0.00	1.48	1.89	0.47

(*b*) $p = 0.80$

a6-r6-z9	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	-	-	-	-
0.10	7	5	4	-	3	-	-	-	-	-	-
0.15	7	5	4	-	3	-	-	-	-	-	-
0.20	7	5	4	4	3	3	-	-	-	-	-
0.25	7	5	5	4	3	3	-	-	-	-	-
0.30	7	5	5	4	3	3	3	-	-	-	-
0.35	7	5	5	4	3	3	3	3	-	-	-
0.40	8	5	5	4	3	3	3	3	3	-	-
0.45	8	5	5	4	4	3	3	3	3	3	-
0.50	8	5	5	4	4	3	3	3	3	3	3
0.55	8	5	5	4	4	3	3	3	3	3	3
0.60	9	5	5	4	4	3	3	3	3	3	3
0.65	9	5	5	4	4	4	3	3	3	3	3
0.70	9	6	5	5	4	4	3	3	3	3	3
0.75	9	6	5	5	4	4	4	3	3	3	3
0.80	9	6	5	5	4	4	4	3	3	3	3
0.85	9	6	5	5	4	4	4	4	3	3	3
0.90	10	6	5	5	4	4	4	4	3	3	3
0.95	10	6	5	5	4	4	4	4	3	3	3
1.00	10	6	5	5	4	4	4	4	3	3	3

	Overall	Q1	Q2	Q3	Q4
Mean	4.35	3.00	4.83	5.72	3.20
St.Dev.	1.67	0.00	1.46	1.87	0.40

(*c*) $p = 0.90$

a6-r6	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	2	-	-	-
0.10	7	5	4	-	3	-	-	2	2	-	-
0.15	7	5	4	-	3	-	-	2	2	-	-
0.20	7	5	4	4	3	3	-	2	2	2	-
0.25	7	5	5	4	3	3	-	2	2	2	2
0.30	7	5	5	4	3	3	3	2	2	2	2
0.35	7	5	5	4	3	3	3	3	2	2	2
0.40	8	5	5	4	3	3	3	3	2	2	2
0.45	8	5	5	4	4	3	3	3	3	2	2
0.50	8	5	5	4	4	3	3	3	3	3	2
0.55	8	5	5	4	4	3	3	3	3	3	2
0.60	9	5	5	4	4	3	3	3	3	3	3
0.65	9	5	5	4	4	3	3	3	3	3	3
0.70	9	5	5	4	4	4	3	3	3	3	3
0.75	9	6	5	4	4	4	4	3	3	3	3
0.80	9	6	5	4	4	4	4	3	3	3	3
0.85	9	6	5	4	4	4	4	3	3	3	3
0.90	10	6	5	4	4	4	4	3	3	3	3
0.95	10	6	5	4	4	4	4	3	3	3	3
1.00	10	6	5	4	4	4	4	3	3	3	3

	Overall	Q1	Q2	Q3	Q4
Mean	4.01	2.25	4.83	5.56	3.10
St.Dev.	1.75	0.44	1.46	1.94	0.36

(*d*) no link capacity

Table 4.3: Optimum number of hubs for α=0.80 and R = 0.60

a8-r6-z7	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	-	-	-	-
0.10	7	5	4	-	3	-	-	-	-	-	-
0.15	7	5	4	-	3	-	-	-	-	-	-
0.20	7	5	4	4	3	3	-	-	-	-	-
0.25	7	5	5	4	3	3	-	-	-	-	-
0.30	7	5	5	4	3	3	3	-	-	-	-
0.35	7	5	5	4	3	3	3	3	-	-	-
0.40	7	5	5	4	3	3	3	3	3	-	-
0.45	7	5	5	4	4	3	3	3	3	3	-
0.50	7	5	5	5	4	3	3	3	3	3	3
0.55	8	5	5	5	4	3	3	3	3	3	3
0.60	8	5	5	5	4	3	3	3	3	3	3
0.65	8	6	5	5	4	4	3	3	3	3	3
0.70	8	6	5	5	5	4	4	3	3	3	3
0.75	8	6	5	5	5	4	4	3	3	3	3
0.80	8	6	5	5	5	4	4	4	3	3	3
0.85	8	6	5	5	5	5	4	4	3	3	3
0.90	8	6	5	5	5	5	4	4	3	3	3
0.95	8	6	5	5	5	5	4	4	3	3	3
1.00	8	6	5	5	5	5	4	4	3	3	3

	Overall	Q1	Q2	Q3	Q4
Mean	4.37	3.00	4.79	5.70	3.24
St.Dev.	1.46	0.00	1.37	1.31	0.43

$(a)\ p = 0.70$

a8-r6-z8	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	-	-	-	-
0.10	7	5	4	-	3	-	-	-	-	-	-
0.15	7	5	4	-	3	-	-	-	-	-	-
0.20	7	5	4	4	3	3	-	-	-	-	-
0.25	7	5	5	4	3	3	-	-	-	-	-
0.30	7	5	5	4	3	3	3	-	-	-	-
0.35	7	5	5	4	3	3	3	3	-	-	-
0.40	7	5	5	4	3	3	3	3	3	-	-
0.45	7	5	5	4	4	3	3	3	3	3	-
0.50	7	5	5	4	4	3	3	3	3	3	3
0.55	8	5	5	4	4	3	3	3	3	3	3
0.60	8	5	5	5	4	3	3	3	3	3	3
0.65	8	5	5	5	4	3	3	3	3	3	3
0.70	8	6	5	5	4	3	3	3	3	3	3
0.75	8	6	5	5	4	3	3	3	3	3	3
0.80	8	6	5	5	4	3	3	3	3	3	3
0.85	8	6	5	5	4	3	3	3	3	3	3
0.90	8	6	5	5	4	3	3	3	3	3	3
0.95	8	6	5	5	4	3	3	3	3	3	3
1.00	8	6	5	5	4	3	3	3	3	3	3

	Overall	Q1	Q2	Q3	Q4
Mean	4.18	3.00	4.79	5.52	3.00
St.Dev.	1.50	0.00	1.37	1.58	0.00

$(b)\ p = 0.80$

a8-r6-z9	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	-	-	-	-
0.10	7	5	4	-	3	-	-	-	-	-	-
0.15	7	5	4	-	3	-	-	-	-	-	-
0.20	7	5	4	4	3	3	-	-	-	-	-
0.25	7	5	5	4	3	3	-	-	-	-	-
0.30	7	5	5	4	3	3	3	-	-	-	-
0.35	7	5	5	4	3	3	3	3	-	-	-
0.40	7	5	5	4	3	3	3	3	3	-	-
0.45	7	5	5	4	4	3	3	3	3	3	-
0.50	7	5	5	4	4	3	3	3	3	3	3
0.55	8	5	5	4	4	3	3	3	3	3	3
0.60	8	5	5	4	4	3	3	3	3	3	3
0.65	8	5	5	4	4	3	3	3	3	3	3
0.70	8	5	5	4	4	3	3	3	3	3	3
0.75	8	6	5	4	4	3	3	3	3	3	3
0.80	8	6	5	4	4	3	3	3	3	3	3
0.85	8	6	5	4	4	3	3	3	3	3	3
0.90	8	6	5	4	4	3	3	3	3	3	3
0.95	8	6	5	4	4	3	3	3	3	3	3
1.00	8	6	5	4	4	3	3	3	3	3	3

	Overall	Q1	Q2	Q3	Q4
Mean	4.12	3.00	4.79	5.32	3.00
St.Dev.	1.48	0.00	1.37	1.62	0.00

$(c)\ p = 0.90$

a8-r6	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	7	5	4	-	3	-	-	2	-	-	-
0.10	7	5	4	-	3	-	-	2	2	-	-
0.15	7	5	4	-	3	-	-	2	2	-	-
0.20	7	5	4	4	3	3	-	2	2	2	-
0.25	7	5	5	4	3	3	-	2	2	2	2
0.30	7	5	5	4	3	3	3	2	2	2	2
0.35	7	5	5	4	3	3	3	3	2	2	2
0.40	7	5	5	4	3	3	3	3	2	2	2
0.45	7	5	5	4	3	3	3	3	3	2	2
0.50	7	5	5	4	3	3	3	3	3	2	2
0.55	7	5	5	4	3	3	3	3	3	3	2
0.60	8	5	5	4	3	3	3	3	3	3	3
0.65	8	5	5	4	4	3	3	3	3	3	3
0.70	8	5	5	4	4	3	3	3	3	3	3
0.75	8	5	5	4	4	3	3	3	3	3	3
0.80	8	5	5	4	4	3	3	3	3	3	3
0.85	8	5	5	4	4	3	3	3	3	3	3
0.90	8	5	5	4	4	3	3	3	3	3	3
0.95	8	5	5	4	4	3	3	3	3	3	3
1.00	8	5	5	4	4	3	3	3	3	3	3

	Overall	Q1	Q2	Q3	Q4
Mean	3.81	2.25	4.76	5.14	2.98
St.Dev.	1.53	0.44	1.39	1.58	0.14

(d) no link capacity

A close investigation based on quadrants of Tables 4.1-4.3 exposes that smallest and highest oNHs are observed in the first and third quadrants respectively, when the amount of flow that can be sent on a link is not bounded. When flows on links are restricted, now the fourth quadrant contains smallest oNHs while highest oNHs remain again in the third. This suggests that when high capacity utilization is not a priority, the requirement of low-capacity hubs increases oNHs while the opportunity of opening high-capacity hubs helps to decrease oNHs. Finally, the smallest oNHs are observed when network links have unlimited capacity, hubs' capacities are large and capacity utilization is required to be high.

As a second analysis, we provide how the optimum total cost (z^*) which is the sum of hub opening and transportation costs, changes depending on the model parameters. According to the average and standard deviation values given beneath Tables 4.4-4.6 as discount factor α value decreases, cost savings due to the flow aggregation increase and thus z^* decrease as expected. z^* also decrease when link capacities are less restrictive. This can be easily explained as increasing link capacities allows to send flows from least-cost links.

If we look at the quadrants of Tables 4.4-4.6, highest optimum total costs always occur in the second quadrant. This quadrant corresponds to where hub capacities and maximum unused capacity ratios have the smallest values. Thus, when only low-capacity hubs can be opened, requiring high capacity usage rises the optimum total cost the most. Meanwhile, least total costs are almost always observed in the fourth quadrant where the aforementioned capacities and ratios have the highest values. Another point worth to mention is that fourth quadrants in Tables 4.4-4.6 not only contain the smallest average z^* values but also the smallest standard deviations. The opposite is true for the second quadrant, in other words largest standard deviations of empirical z^* values are observed in this quadrant. Hence relaxing capacity related constraints leads to low and close optimum total cost values, while tighter capacity bounds result in high and dispersed costs.

Table 4.4: Optimum total cost for α=0.40 and R = 0.60

a4-r6-z7	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	3976	3764	3828	-	3752	-	-	-	-	-	-
0.10	3951	3764	3828	-	3739	-	-	-	-	-	-
0.15	3942	3764	3828	-	3735	-	-	-	-	-	-
0.20	3924	3764	3764	3785	3735	3735	-	-	-	-	-
0.25	3906	3764	3711	3749	3735	3708	-	-	-	-	-
0.30	3870	3764	3688	3721	3735	3702	3728	-	-	-	-
0.35	3843	3764	3676	3697	3735	3698	3697	3742	-	-	-
0.40	3827	3751	3669	3676	3696	3695	3673	3708	3749	-	-
0.45	3788	3718	3665	3663	3666	3693	3668	3674	3711	3749	-
0.50	3750	3698	3664	3648	3643	3663	3666	3651	3674	3708	3742
0.55	3734	3691	3664	3638	3634	3629	3666	3641	3648	3669	3697
0.60	3726	3677	3657	3635	3625	3612	3616	3639	3629	3642	3660
0.65	3720	3652	3638	3633	3613	3600	3594	3603	3625	3625	3632
0.70	3718	3621	3629	3613	3606	3596	3581	3576	3590	3614	3620
0.75	3718	3597	3599	3601	3591	3586	3573	3563	3564	3574	3587
0.80	3718	3584	3568	3565	3572	3563	3566	3554	3546	3552	3559
0.85	3653	3581	3551	3533	3525	3535	3535	3537	3540	3537	3540
0.90	3637	3516	3536	3522	3500	3489	3486	3494	3508	3519	3521
0.95	3637	3516	3487	3460	3438	3422	3429	3457	3471	3471	3471
1.00	3637	3516	3487	3460	3438	3422	3411	3409	3409	3409	3409

	Overall	Q1	Q2	Q3	Q4
Mean	3638	3710	3769	3597	3555
St.Dev.	113	31	81	79	78

(a) $p = 0.70$

a4-r6-z8	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	3908	3657	3706	-	3623	-	-	-	-	-	-
0.10	3880	3657	3706	-	3605	-	-	-	-	-	-
0.15	3866	3657	3706	-	3601	-	-	-	-	-	-
0.20	3848	3657	3657	3668	3601	3598	-	-	-	-	-
0.25	3836	3657	3604	3627	3601	3568	-	-	-	-	-
0.30	3801	3657	3583	3600	3601	3563	3589	-	-	-	-
0.35	3773	3657	3573	3576	3601	3558	3558	3608	-	-	-
0.40	3760	3657	3567	3557	3573	3554	3533	3568	3617	-	-
0.45	3747	3630	3565	3549	3545	3552	3528	3534	3572	3617	-
0.50	3706	3606	3565	3545	3525	3539	3524	3502	3534	3568	3608
0.55	3686	3598	3565	3537	3516	3507	3524	3497	3500	3527	3558
0.60	3676	3594	3565	3535	3512	3491	3493	3497	3477	3492	3514
0.65	3669	3587	3548	3534	3509	3484	3473	3481	3474	3469	3481
0.70	3666	3568	3543	3524	3506	3482	3463	3456	3466	3461	3460
0.75	3666	3542	3538	3515	3495	3481	3456	3444	3440	3449	3449
0.80	3666	3530	3513	3507	3483	3466	3454	3437	3427	3428	3435
0.85	3602	3527	3496	3478	3466	3456	3435	3432	3421	3413	3416
0.90	3586	3461	3480	3466	3441	3425	3419	3415	3407	3406	3407
0.95	3586	3461	3431	3403	3378	3357	3361	3381	3394	3394	3394
1.00	3586	3461	3431	3403	3378	3357	3343	3332	3332	3332	3332

	Overall	Q1	Q2	Q3	Q4
Mean	3537	3572	3664	3528	3439
St.Dev.	110	35	97	77	53

(b) $p = 0.80$

a4-r6-z9	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	3864	3591	3628	-	3546	-	-	-	-	-	-
0.10	3836	3591	3628	-	3527	-	-	-	-	-	-
0.15	3816	3591	3628	-	3523	-	-	-	-	-	-
0.20	3800	3591	3591	3590	3523	3520	-	-	-	-	-
0.25	3792	3591	3540	3550	3523	3483	-	-	-	-	-
0.30	3764	3591	3524	3523	3523	3478	3510	-	-	-	-
0.35	3737	3591	3514	3500	3523	3473	3473	3529	-	-	-
0.40	3722	3591	3510	3484	3496	3468	3448	3483	3539	-	-
0.45	3717	3574	3509	3479	3469	3467	3442	3449	3488	3539	-
0.50	3691	3554	3509	3477	3454	3460	3438	3415	3449	3483	3529
0.55	3669	3545	3509	3477	3446	3431	3437	3410	3411	3442	3473
0.60	3656	3543	3509	3477	3442	3419	3416	3409	3382	3402	3429
0.65	3648	3543	3502	3477	3440	3415	3399	3406	3382	3371	3386
0.70	3645	3538	3498	3467	3439	3412	3390	3381	3382	3364	3363
0.75	3644	3524	3495	3461	3437	3412	3384	3370	3364	3364	3350
0.80	3644	3514	3491	3459	3430	3409	3384	3364	3352	3350	3350
0.85	3580	3511	3481	3451	3428	3400	3380	3362	3347	3336	3338
0.90	3564	3446	3462	3442	3414	3395	3375	3359	3347	3335	3329
0.95	3564	3446	3413	3379	3352	3330	3325	3337	3347	3335	3329
1.00	3564	3446	3413	3379	3352	3330	3311	3294	3282	3270	3264

	Overall	Q1	Q2	Q3	Q4
Mean	3477	3490	3600	3493	3365
St.Dev.	114	38	107	79	42

(c) $p = 0.90$

a4-r6	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	3852	3562	3581	-	3499	-	-	3562	-	-	-
0.10	3826	3562	3581	-	3482	-	-	3562	3556	-	-
0.15	3803	3562	3581	-	3478	-	-	3562	3532	-	-
0.20	3787	3562	3562	3543	3478	3476	-	3562	3518	3542	-
0.25	3776	3562	3516	3501	3478	3439	-	3562	3518	3518	3548
0.30	3758	3562	3501	3474	3478	3434	3466	3562	3518	3495	3522
0.35	3730	3562	3491	3451	3478	3429	3428	3485	3518	3481	3496
0.40	3714	3562	3488	3438	3478	3423	3403	3439	3495	3481	3475
0.45	3709	3552	3485	3438	3420	3420	3397	3404	3444	3481	3457
0.50	3686	3530	3485	3434	3406	3408	3391	3370	3404	3439	3453
0.55	3662	3520	3485	3434	3399	3382	3388	3363	3366	3397	3428
0.60	3648	3517	3485	3434	3396	3372	3367	3360	3335	3356	3383
0.65	3640	3517	3480	3434	3394	3368	3351	3356	3333	3320	3340
0.70	3636	3517	3475	3421	3393	3366	3341	3332	3333	3311	3309
0.75	3636	3510	3472	3416	3391	3366	3338	3322	3313	3311	3296
0.80	3636	3501	3458	3415	3384	3359	3338	3316	3303	3298	3295
0.85	3572	3498	3447	3412	3381	3354	3333	3315	3299	3287	3284
0.90	3556	3431	3428	3402	3373	3351	3328	3311	3299	3287	3279
0.95	3556	3431	3378	3339	3311	3284	3278	3289	3299	3287	3279
1.00	3556	3431	3378	3339	3311	3284	3263	3246	3234	3222	3214

	Overall	Q1	Q2	Q3	Q4
Mean	3448	3500	3569	3465	3317
St.Dev.	121	51	118	91	43

(d) no link capacity

Table 4.5: Optimum total cost for α=0.60 and R = 0.60

a6-r6-z7	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	4131	3876	3897	-	3798	-	-	-	-	-	-
0.10	4106	3876	3897	-	3783	-	-	-	-	-	-
0.15	4097	3876	3897	-	3778	-	-	-	-	-	-
0.20	4091	3876	3876	3848	3778	3778	-	-	-	-	-
0.25	4084	3876	3817	3809	3778	3747	-	-	-	-	-
0.30	4078	3876	3794	3800	3778	3740	3770	-	-	-	-
0.35	4056	3876	3779	3785	3778	3736	3735	3786	-	-	-
0.40	4041	3876	3772	3767	3771	3732	3707	3747	3794	-	-
0.45	4035	3874	3769	3758	3750	3730	3702	3709	3751	3794	-
0.50	4019	3861	3768	3747	3730	3730	3698	3682	3709	3747	3786
0.55	3993	3852	3768	3734	3720	3713	3698	3669	3680	3703	3735
0.60	3980	3848	3768	3729	3713	3695	3698	3667	3659	3673	3692
0.65	3970	3847	3768	3727	3704	3682	3679	3667	3652	3655	3663
0.70	3965	3827	3768	3726	3695	3676	3661	3664	3651	3646	3649
0.75	3964	3806	3768	3726	3694	3671	3648	3644	3650	3642	3642
0.80	3964	3797	3755	3726	3694	3664	3642	3629	3627	3635	3642
0.85	3964	3795	3745	3717	3689	3662	3641	3624	3615	3613	3619
0.90	3957	3795	3742	3710	3679	3660	3641	3624	3615	3613	3613
0.95	3957	3795	3742	3710	3678	3656	3638	3624	3615	3613	3613
1.00	3957	3795	3742	3710	3678	3656	3638	3624	3615	3613	3613

	Overall	Q1	Q2	Q3	Q4
Mean	3759	3749	3878	3791	3646
St.Dev.	120	36	117	99	28

(*a*) $p = 0.70$

a6-r6-z8	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	4068	3779	3792	-	3676	-	-	-	-	-	-
0.10	4042	3779	3792	-	3656	-	-	-	-	-	-
0.15	4032	3779	3792	-	3652	-	-	-	-	-	-
0.20	4024	3779	3779	3743	3652	3649	-	-	-	-	-
0.25	4017	3779	3721	3704	3652	3615	-	-	-	-	-
0.30	4011	3779	3702	3695	3652	3609	3639	-	-	-	-
0.35	3999	3779	3689	3676	3652	3603	3603	3659	-	-	-
0.40	3978	3779	3683	3659	3651	3599	3574	3615	3670	-	-
0.45	3972	3779	3680	3654	3641	3596	3568	3576	3619	3670	-
0.50	3967	3779	3679	3649	3625	3596	3564	3539	3576	3615	3659
0.55	3951	3774	3679	3643	3616	3596	3564	3533	3536	3568	3603
0.60	3938	3770	3679	3639	3609	3588	3564	3532	3510	3528	3552
0.65	3927	3769	3679	3636	3603	3577	3564	3532	3506	3502	3515
0.70	3921	3753	3679	3636	3600	3571	3552	3532	3506	3495	3494
0.75	3920	3742	3679	3635	3599	3568	3540	3532	3506	3494	3486
0.80	3920	3742	3676	3635	3599	3567	3536	3519	3506	3494	3486
0.85	3920	3738	3676	3635	3599	3567	3536	3514	3502	3494	3486
0.90	3914	3738	3676	3635	3599	3567	3536	3514	3502	3494	3486
0.95	3914	3738	3676	3635	3599	3567	3536	3514	3502	3494	3486
1.00	3914	3738	3676	3635	3599	3567	3536	3514	3502	3494	3486

	Overall	Q1	Q2	Q3	Q4
Mean	3656	3619	3781	3718	3518
St.Dev.	140	39	135	116	27

(*b*) $p = 0.80$

a6-r6-z9	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	4031	3723	3722	-	3604	-	-	-	-	-	-
0.10	4006	3723	3722	-	3584	-	-	-	-	-	-
0.15	3995	3723	3722	-	3580	-	-	-	-	-	-
0.20	3987	3723	3722	3674	3580	3577	-	-	-	-	-
0.25	3980	3723	3669	3635	3580	3537	-	-	-	-	-
0.30	3974	3723	3652	3627	3580	3530	3566	-	-	-	-
0.35	3970	3723	3640	3612	3580	3524	3525	3587	-	-	-
0.40	3951	3723	3636	3599	3580	3518	3495	3537	3598	-	-
0.45	3944	3723	3634	3594	3574	3518	3489	3497	3541	3598	-
0.50	3939	3723	3634	3591	3564	3518	3484	3458	3497	3537	3587
0.55	3934	3723	3633	3590	3555	3518	3483	3452	3454	3489	3525
0.60	3923	3723	3633	3590	3548	3518	3483	3451	3420	3442	3473
0.65	3911	3723	3633	3590	3544	3517	3483	3451	3419	3407	3425
0.70	3904	3715	3633	3589	3541	3511	3483	3451	3419	3399	3397
0.75	3903	3704	3633	3589	3540	3509	3478	3451	3419	3399	3388
0.80	3903	3704	3633	3589	3540	3508	3476	3451	3419	3399	3388
0.85	3903	3704	3633	3589	3540	3508	3476	3450	3419	3399	3388
0.90	3898	3704	3633	3589	3540	3508	3476	3450	3419	3399	3388
0.95	3898	3704	3633	3589	3540	3508	3476	3450	3419	3399	3388
1.00	3898	3704	3633	3589	3540	3508	3476	3450	3419	3399	3388

	Overall	Q1	Q2	Q3	Q4
Mean	3594	3543	3725	3677	3436
St.Dev.	157	43	147	129	36

(*c*) $p = 0.90$

a6-r6	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	4023	3698	3696	-	3564	-	-	3598	-	-	-
0.10	3998	3698	3696	-	3545	-	-	3598	3592	-	-
0.15	3987	3698	3696	-	3540	-	-	3598	3566	-	-
0.20	3979	3698	3696	3651	3540	3538	-	3598	3551	3578	-
0.25	3967	3698	3649	3608	3540	3499	-	3598	3551	3551	3584
0.30	3959	3698	3633	3593	3540	3493	3528	3598	3551	3526	3555
0.35	3957	3698	3621	3573	3540	3487	3487	3548	3551	3511	3528
0.40	3948	3698	3616	3562	3540	3480	3458	3499	3551	3511	3505
0.45	3939	3698	3614	3559	3538	3477	3451	3460	3503	3511	3484
0.50	3934	3698	3613	3557	3525	3477	3444	3420	3460	3499	3480
0.55	3929	3698	3613	3556	3517	3477	3441	3412	3416	3452	3480
0.60	3918	3698	3613	3556	3510	3477	3441	3409	3381	3404	3436
0.65	3906	3698	3613	3556	3506	3477	3441	3409	3377	3365	3386
0.70	3899	3698	3613	3556	3503	3473	3441	3409	3377	3353	3351
0.75	3898	3690	3613	3556	3503	3471	3440	3409	3377	3353	3339
0.80	3898	3690	3613	3556	3503	3471	3439	3409	3377	3353	3339
0.85	3898	3690	3613	3556	3503	3471	3439	3409	3377	3353	3339
0.90	3894	3690	3613	3556	3503	3471	3439	3409	3377	3353	3339
0.95	3894	3690	3613	3556	3503	3471	3439	3409	3377	3353	3339
1.00	3894	3690	3613	3556	3503	3471	3439	3409	3377	3353	3339

	Overall	Q1	Q2	Q3	Q4
Mean	3562	3540	3700	3654	3394
St.Dev.	160	45	156	141	38

(*d*) no link capacity

Table 4.6: Optimum total cost for α=0.80 and R = 0.60

a8-r6-z7	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	4278	3984	3959	-	3843	-	-	-	-	-	-
0.10	4247	3984	3959	-	3826	-	-	-	-	-	-
0.15	4238	3984	3959	-	3820	-	-	-	-	-	-
0.20	4230	3984	3959	3909	3820	3820	-	-	-	-	-
0.25	4224	3984	3919	3867	3820	3784	-	-	-	-	-
0.30	4217	3984	3893	3855	3820	3777	3811	-	-	-	-
0.35	4213	3984	3874	3847	3820	3773	3771	3829	-	-	-
0.40	4213	3984	3864	3843	3820	3768	3740	3784	3838	-	-
0.45	4213	3984	3858	3843	3807	3766	3734	3742	3789	3838	-
0.50	4213	3984	3855	3834	3801	3766	3730	3713	3742	3784	3829
0.55	4202	3984	3854	3817	3799	3766	3730	3697	3711	3735	3771
0.60	4187	3984	3854	3808	3786	3766	3730	3695	3688	3704	3723
0.65	4180	3973	3854	3805	3776	3751	3730	3695	3679	3682	3693
0.70	4175	3954	3854	3805	3770	3739	3724	3695	3678	3676	3678
0.75	4175	3947	3854	3804	3769	3736	3706	3695	3678	3674	3674
0.80	4175	3947	3854	3804	3768	3736	3706	3693	3678	3674	3674
0.85	4175	3947	3854	3804	3768	3735	3706	3689	3678	3674	3674
0.90	4175	3947	3854	3804	3768	3735	3706	3689	3678	3674	3674
0.95	4175	3947	3854	3804	3768	3735	3706	3689	3678	3674	3674
1.00	4175	3947	3854	3804	3768	3735	3706	3689	3678	3674	3674

	Overall	Q1	Q2	Q3	Q4
Mean	3840	3788	3970	3914	3693
St.Dev.	158	41	151	148	21

(*a*) $p = 0.70$

a8-r6-z8	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	4218	3896	3860	-	3726	-	-	-	-	-	-
0.10	4187	3896	3860	-	3705	-	-	-	-	-	-
0.15	4177	3896	3860	-	3700	-	-	-	-	-	-
0.20	4170	3896	3860	3810	3700	3698	-	-	-	-	-
0.25	4163	3896	3832	3769	3700	3660	-	-	-	-	-
0.30	4157	3896	3811	3758	3700	3653	3687	-	-	-	-
0.35	4153	3896	3792	3749	3700	3647	3647	3709	-	-	-
0.40	4153	3896	3784	3741	3700	3642	3614	3660	3721	-	-
0.45	4153	3896	3778	3739	3698	3639	3608	3616	3665	3721	-
0.50	4153	3896	3776	3739	3690	3639	3603	3576	3616	3660	3709
0.55	4145	3896	3776	3733	3689	3639	3603	3568	3572	3607	3647
0.60	4130	3896	3776	3727	3688	3639	3603	3567	3541	3562	3590
0.65	4124	3896	3776	3722	3681	3639	3603	3567	3537	3534	3547
0.70	4123	3884	3776	3722	3676	3639	3603	3567	3537	3527	3527
0.75	4123	3877	3776	3722	3676	3639	3603	3567	3537	3526	3523
0.80	4123	3877	3776	3722	3676	3639	3603	3567	3537	3526	3523
0.85	4123	3877	3776	3722	3676	3639	3603	3567	3537	3526	3523
0.90	4123	3877	3776	3722	3676	3639	3603	3567	3537	3526	3523
0.95	4123	3877	3776	3722	3676	3639	3603	3567	3537	3526	3523
1.00	4123	3877	3776	3722	3676	3639	3603	3567	3537	3526	3523

	Overall	Q1	Q2	Q3	Q4
Mean	3736	3665	3879	3838	3559
St.Dev.	182	44	169	161	33

(*b*) $p = 0.80$

a8-r6-z9	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	4186	3848	3794	-	3661	-	-	-	-	-	-
0.10	4156	3848	3794	-	3640	-	-	-	-	-	-
0.15	4145	3848	3794	-	3634	-	-	-	-	-	-
0.20	4137	3848	3794	3745	3634	3632	-	-	-	-	-
0.25	4130	3848	3788	3704	3634	3588	-	-	-	-	-
0.30	4124	3848	3768	3693	3634	3581	3621	-	-	-	-
0.35	4121	3848	3750	3685	3634	3574	3575	3643	-	-	-
0.40	4121	3848	3743	3680	3634	3569	3542	3588	3655	-	-
0.45	4121	3848	3738	3680	3634	3567	3534	3544	3593	3655	-
0.50	4121	3848	3738	3680	3630	3567	3529	3499	3544	3588	3643
0.55	4118	3848	3738	3680	3630	3567	3528	3493	3495	3535	3575
0.60	4103	3848	3738	3680	3630	3567	3528	3492	3457	3482	3517
0.65	4096	3848	3738	3680	3628	3567	3528	3492	3456	3442	3463
0.70	4096	3848	3738	3680	3623	3567	3528	3492	3456	3432	3429
0.75	4096	3843	3738	3680	3623	3567	3528	3492	3456	3432	3423
0.80	4096	3843	3738	3680	3623	3567	3528	3492	3456	3432	3423
0.85	4096	3843	3738	3680	3623	3567	3528	3492	3456	3432	3423
0.90	4096	3843	3738	3680	3623	3567	3528	3492	3456	3432	3423
0.95	4096	3843	3738	3680	3623	3567	3528	3492	3456	3432	3423
1.00	4096	3843	3738	3680	3623	3567	3528	3492	3456	3432	3423

	Overall	Q1	Q2	Q3	Q4
Mean	3676	3595	3828	3797	3476
St.Dev.	200	47	180	169	41

(*c*) $p = 0.90$

a8-r6	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
0.05	4178	3826	3772	-	3627	-	-	3633	-	-	-
0.10	4149	3826	3772	-	3606	-	-	3633	3628	-	-
0.15	4138	3826	3772	-	3600	-	-	3633	3600	-	-
0.20	4131	3826	3772	3724	3600	3598	-	3633	3584	3613	-
0.25	4124	3826	3771	3680	3600	3556	-	3633	3584	3584	3620
0.30	4118	3826	3750	3669	3600	3549	3587	3633	3584	3556	3588
0.35	4115	3826	3736	3661	3600	3542	3543	3609	3584	3541	3558
0.40	4115	3826	3727	3657	3600	3535	3510	3556	3584	3541	3534
0.45	4115	3826	3721	3656	3600	3532	3510	3511	3561	3541	3511
0.50	4115	3826	3718	3656	3600	3532	3496	3467	3511	3541	3506
0.55	4115	3826	3718	3656	3600	3532	3493	3460	3462	3503	3506
0.60	4101	3826	3718	3653	3600	3532	3493	3457	3425	3449	3485
0.65	4095	3826	3718	3653	3598	3532	3493	3457	3421	3408	3431
0.70	4095	3826	3718	3653	3594	3532	3493	3457	3421	3394	3392
0.75	4095	3826	3718	3653	3594	3532	3493	3457	3421	3394	3377
0.80	4095	3826	3718	3653	3594	3532	3493	3457	3421	3394	3377
0.85	4095	3826	3718	3653	3594	3532	3493	3457	3421	3394	3377
0.90	4095	3826	3718	3653	3594	3532	3493	3457	3421	3394	3377
0.95	4095	3826	3718	3653	3594	3532	3493	3457	3421	3394	3377
1.00	4095	3826	3718	3653	3594	3532	3493	3457	3421	3394	3377

	Overall	Q1	Q2	Q3	Q4
Mean	3641	3579	3807	3778	3439
St.Dev.	200	41	189	179	42

(*d*) no link capacity

The general expectation about hub capacity usage is that the optimum total cost falls when it is allowed to fall. It can be detected from tables that this expectation is realized. However, increasing the capacity of hubs do not always reduce z^*. Especially, optimum total cost may increase when high capacity usage is desired for high-capacity hubs. In general, z^* values do not follow a specific trend for a given capacity usage level. The lowest total cost values are observed when network links have unlimited capacity, hubs' capacities are large and capacity utilization is required to be very low.

5. CONCLUSION

This study deals with capacitated multiple allocation hub covering flow problem. The aim is to optimally design and operate hub-and-spoke networks while taking into account hub opening and demand routing costs. It is assumed that a hub covers a node if their distance is less than a predefined value, while the distance between hubs is not restricted. Moreover, flow demand associated with a specific origin-destination node pair must be routed by visiting at least one hub.

Our proposed mathematical model is inspired by the work of Şener (2020) and is formulated as a mixed integer linear program. A benchmark data set well-known from the literature is used to validate the model. A thorough sensitivity analysis is conducted to detect how and to what extent the change in different model parameters affects the best solution. It was shown how important it is to consider the capacity and its utilization and also transport costs in the hub-covering problem. Finally, many practical insights have been provided for the decision makers.

It is not difficult to figure out several future research directions. Our analysis has shown that coverage ratios are not influential on results. This is unexpected as it directly affects the network structure. The analysis can be extended to include more restrictive coverage ratios. The ratio of total hub opening cost to total transportation cost in the optimum solution can play an important role in the results, especially on the number of hubs to locate. Therefore, more analysis can be done on different ratios. Original TR data set and other known data sets such as CAB and AP can be investigated to further generalize obtained results.

As the number of network nodes and links increases, time to find the optimum solution also increases considerably. Hence, heuristic approaches instead of deterministic procedures can be developed to solve the proposed model. As the underlying network for hub covering problems is not complete, every selected hub-nodes set does not produce a feasible solution. Moreover, the addition of capacity related constraints to a mathematical model may render it infeasible in general. These are clearly challenging issues to overcome in developing a solution method.

As the decisions related to the network design problems are of strategic nature and span several future years, they involve a certain level of uncertainty. Therefore, instead of being deterministic, model parameters can be probabilistic of possibilistic. New extended models covering those cases can be derived from the model proposed in this study and solution procedures can be developed accordingly.

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APPENDICES

Appendix A. Distance between cities 25 cities.

		İSTANBUL	ANKARA	İZMİR	BURSA	ANTALYA	ADANA	KONYA	ŞANLIURF	GAZİANTE	KOCAELİ	İÇEL	DİYARBAI	HATAY	MANİSA	KAYSERİ	SAMSUN	BALIKESİ	KAHRAMA	VAN	AYDIN	TEKİRDAĞ	DENİZLİ	SAKARYA	MUĞLA	ESKİŞEHİ
1	İSTANBUL	0	453	561	243	716	939	660	1261	1124	111	932	1363	1130	525	771	734	390	1044	1638	681	132	639	148	780	322
2	ANKARA	453	0	579	382	544	490	258	808	671	342	483	910	681	561	318	414	530	591	1232	603	585	477	305	622	233
3	İZMİR	561	579	0	322	446	900	550	1242	1105	450	892	1418	1091	36	848	993	173	1085	1762	126	505	224	481	225	412
4	BURSA	243	382	322	0	537	837	487	1179	1042	132	829	1281	1028	286	689	745	151	962	1603	442	375	437	159	541	149
5	ANTALYA	716	544	446	537	0	558	322	900	763	605	489	1076	749	428	618	954	510	743	1453	344	848	222	568	313	424
6	ADANA	939	490	900	837	558	0	356	342	205	828	69	518	191	882	333	729	894	185	895	894	1071	768	791	871	688
7	KONYA	660	258	550	487	322	356	0	698	561	549	348	874	547	532	304	640	544	541	1218	542	792	416	512	556	338
8	ŞANLIURFA	1261	808	1242	1179	900	342	698	0	137	1150	411	176	333	1224	490	836	1236	217	553	1236	1393	1110	1113	1213	1030
9	GAZİANTEP	1124	671	1105	1042	763	205	561	137	0	1013	274	313	196	1087	353	725	1099	80	690	1099	1256	973	976	1076	893
10	KOCAELİ	111	342	450	132	605	828	549	1150	1013	0	821	1252	1019	414	660	623	279	933	1527	570	243	528	37	669	211
11	İÇEL	932	483	892	829	489	69	348	411	274	821	0	587	260	874	326	740	886	254	964	833	1064	711	784	802	680
12	DİYARBAKIR	1363	910	1418	1281	1076	518	874	176	313	1252	587	0	509	1400	592	818	1412	369	377	1412	1495	1286	1215	1389	1132
13	HATAY	1130	681	1091	1028	749	191	547	333	196	1019	260	509	0	1073	449	821	1085	176	886	1085	1262	959	982	1062	879
14	MANİSA	525	561	36	286	428	882	532	1224	1087	414	874	1400	1073	0	830	975	137	1067	1744	156	515	206	445	255	394
15	KAYSERİ	771	318	848	689	618	333	304	490	353	660	326	592	449	830	0	449	837	273	914	842	903	716	623	860	540
16	SAMSUN	734	414	993	745	954	729	640	836	725	623	740	818	821	975	449	0	896	645	974	1017	866	891	586	1036	647
17	BALIKESİR	390	530	173	151	510	894	544	1236	1099	279	886	1412	1085	137	837	896	0	1079	1751	293	380	288	310	392	297
18	KAHRAMANMARAŞ	1044	591	1085	962	743	185	541	217	80	933	254	369	176	1067	273	645	1079	0	746	1079	1176	953	896	1056	813
19	VAN	1638	1232	1762	1603	1453	895	1218	553	690	1527	964	377	886	1744	914	974	1751	746	0	1756	1770	1630	1490	1766	1454
20	AYDIN	681	603	126	442	344	894	542	1236	1099	570	833	1412	1085	156	842	1017	293	1079	1756	0	629	126	601	99	483
21	TEKİRDAĞ	132	585	505	375	848	1071	792	1393	1256	243	1064	1495	1262	515	903	866	380	1176	1770	629	0	668	280	728	454
22	DENİZLİ	639	477	224	437	222	768	416	1110	973	528	711	1286	959	206	716	891	288	953	1630	126	668	0	491	145	357
23	SAKARYA	148	305	481	159	568	791	512	1113	976	37	784	1215	982	445	623	586	310	896	1490	601	280	491	0	636	174
24	MUĞLA	780	622	225	541	313	871	556	1213	1076	669	802	1389	1062	255	860	1036	392	1056	1766	99	728	145	636	0	502
25	ESKİŞEHİR	322	233	412	149	424	688	338	1030	893	211	680	1132	879	394	540	647	297	813	1454	483	454	357	174	502	0

Appendix B. Flow amounts between cities.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
		İSTANBUL	ANKARA	İZMİR	BURSA	ANTALYA	ADANA	KONYA	ŞANLIURF	GAZİANTE	KOCAELİ	İÇEL	DİYARBAI	HATAY	MANİSA	KAYSERİ	SAMSUN	BALIKESİI	KAHRAMA	VAN	AYDIN	TEKİRDAĞ	DENİZLİ	SAKARYA	MUĞLA	ESKİŞEHİ
1	İSTANBUL	0	694878	584437	368455	298169	320661	380075	250259	222835	209110	286318	236265	217370	218487	183857	209639	186616	173792	152144	164841	108118	147377	131104	124023	122407
2	ANKARA	629407	0	211768	133508	108040	116190	137718	90680	80743	75770	103746	85609	78763	79168	66620	75962	67619	62973	55129	59729	39176	53401	47505	44939	44354
3	İZMİR	524138	209674	0	111178	89970	96757	114685	75514	67239	63097	86394	71291	65590	65927	55477	63257	56310	52441	45908	49740	32624	44470	39560	37423	36935
4	BURSA	324172	129681	109070	0	55645	59843	70931	46704	41586	39025	53434	44093	40566	40775	34312	39124	34827	32434	28394	30763	20177	27504	24467	23146	22844
5	ANTALYA	260724	104299	87722	55304	0	48130	57048	37563	33447	31387	42975	35463	32627	32794	27596	31466	28010	26086	22836	24742	16228	22121	19678	18615	18373
6	ADANA	280943	112387	94525	59593	48225	0	61472	40476	36041	33821	46308	38213	35157	35337	29736	33906	30183	28109	24607	26661	17487	23836	21204	20059	19798
7	KONYA	334738	133907	112625	71003	57459	61793	0	48226	42942	40297	55175	45530	41888	42104	35430	40399	35962	33491	29319	31766	20835	28400	25264	23900	23589
8	ŞANLIURFA	217920	87176	73320	46224	37407	40228	47682	0	27956	26234	35920	29641	27270	27410	23066	26300	23412	21803	19087	20680	13564	18489	16448	15559	15357
9	GAZİANTEP	193578	77438	65131	41061	33228	35735	42356	27889	0	23304	31908	26330	24224	24349	20489	23362	20797	19368	16955	18370	12049	16424	14610	13821	13641
10	KOCAELİ	181439	72582	61046	38486	31145	33494	39700	26140	23276	0	29907	24679	22705	22822	19204	21897	19493	18153	15892	17218	11293	15394	13694	12955	12786
11	İÇEL	250103	100050	84149	53051	42931	46169	54724	36033	32084	30108	0	34018	31297	31458	26472	30184	26869	25023	21906	23734	15567	21220	18877	17857	17624
12	DİYARBAKIR	205484	82201	69136	43587	35272	37933	44961	29605	26360	24737	33870	0	25714	25846	21749	24799	22076	20559	17998	19500	12790	17434	15509	14671	14480
13	HATAY	188741	75503	63503	40035	32398	34842	41298	27192	24213	22721	31110	25672	0	23740	19977	22779	20277	18884	16532	17911	11748	16014	14245	13476	13300
14	MANİSA	189729	75899	63836	40245	32568	35024	41514	27335	24339	22840	31273	25806	23742	0	20082	22898	20383	18983	16618	18005	11809	16097	14320	13546	13370
15	KAYSERİ	159179	63678	53557	33765	27324	29385	34830	22933	20420	19162	26238	21651	19919	20022	0	19211	17101	15926	13942	15106	9908	13505	12014	11365	11217
16	SAMSUN	181906	72769	61204	38585	31225	33580	39802	26208	23336	21898	29984	24742	22763	22880	19254	0	19543	18200	15933	17263	11322	15434	13729	12988	12819
17	BALIKESİR	161607	64649	54374	34279	27740	29833	35361	23283	20732	19455	26638	21981	20223	20327	17105	19504	0	16169	14155	15336	10059	13711	12197	11539	11388
18	KAHRAMANMARAŞ	150335	60140	50581	31889	25806	27752	32894	21659	19286	18098	24780	20448	18813	18909	15912	18144	16151	0	13168	14266	9357	12755	11347	10734	10594
19	VAN	131363	52550	44198	27864	22549	24250	28743	18926	16852	15814	21653	17868	16439	16523	13904	15854	14113	13143	0	12466	8176	11145	9915	9379	9257
20	AYDIN	142482	56998	47939	30223	24458	26302	31176	20528	18278	17152	23485	19380	17830	17922	15081	17196	15307	14255	12480	0	8868	12089	10754	10173	10041
21	TEKİRDAĞ	92997	37202	31290	19726	15963	17167	20348	13398	11930	11195	15329	12649	11638	11697	9843	11224	9991	9304	8145	8825	0	7890	7019	6640	6553
22	DENİZLİ	127195	50883	42796	26980	21834	23480	27831	18325	16317	15312	20966	17301	15917	15999	13463	15351	13665	12726	11141	12071	7917	0	9600	9082	8963
23	SAKARYA	112992	45201	38017	23967	19395	20859	24723	16279	14495	13602	18625	15369	14140	14212	11960	13637	12139	11305	9897	10723	7033	9587	0	8068	7962
24	MUĞLA	106824	42734	35942	22659	18337	19720	23374	15390	13704	12860	17608	14530	13368	13436	11307	12892	11476	10688	9357	10137	6649	9063	8063	0	7528
25	ESKİŞEHİR	105418	42171	35468	22361	18095	19460	23066	15188	13523	12691	17376	14339	13192	13260	11158	12723	11325	10547	9233	10004	6561	8944	7956	7527	0

Appendix C. Fixed linked costs between 25 cities.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
		İSTANBUL	ANKARA	İZMİR	BURSA	ANTALYA	ADANA	KONYA	ŞANLIURF	GAZİANTE	KOCAELİ	İÇEL	DİYARBAI	HATAY	MANİSA	KAYSERİ	SAMSUN	BALIKESİI	KAHRAMA	VAN	AYDIN	TEKİRDAĞ	DENİZLİ	SAKARYA	MUĞLA	ESKİŞEHİ
1	İSTANBUL	0.0000	0.0134	0.0199	0.0136	0.0501	0.0604	0.0362	0.1041	0.1042	0.0109	0.0671	0.1191	0.1072	0.0499	0.0867	0.0725	0.0435	0.1241	0.2219	0.0857	0.0252	0.0908	0.0233	0.1304	0.0556
2	ANKARA	0.0148	0.0000	0.0564	0.0590	0.1038	0.0870	0.0386	0.1842	0.1719	0.0931	0.0960	0.2197	0.1783	0.1461	0.0990	0.1137	0.1616	0.1942	0.4631	0.2082	0.3079	0.1842	0.1324	0.2854	0.1083
3	İZMİR	0.0222	0.0569	0.0000	0.0597	0.1022	0.1920	0.0991	0.3397	0.3395	0.1484	0.2131	0.4119	0.3433	0.0113	0.3156	0.3253	0.0634	0.4274	0.7937	0.0539	0.3192	0.1039	0.2507	0.1262	0.2300
4	BURSA	0.0155	0.0607	0.0609	0.0000	0.1990	0.2884	0.1416	0.5210	0.5172	0.0697	0.3199	0.6000	0.5226	0.1446	0.4153	0.3942	0.0894	0.6129	1.1685	0.2963	0.3832	0.3276	0.1340	0.4820	0.1345
5	ANTALYA	0.0573	0.1076	0.1048	0.2002	0.0000	0.2391	0.1168	0.4946	0.4710	0.4027	0.2346	0.6286	0.4734	0.2691	0.4625	0.6278	0.3755	0.5881	1.3165	0.2867	1.0877	0.2069	0.6036	0.3467	0.4759
6	ADANA	0.0689	0.0899	0.1966	0.2896	0.2386	0.0000	0.1194	0.1747	0.1179	0.5048	0.0307	0.2822	0.1120	0.5153	0.2309	0.4434	0.6115	0.1365	0.7542	0.6907	1.2630	0.6635	0.7692	0.8954	0.7166
7	KONYA	0.0412	0.0397	0.1009	0.1414	0.1159	0.1188	0.0000	0.2989	0.2699	0.2850	0.1301	0.3981	0.2693	0.2610	0.1769	0.3282	0.3125	0.3337	0.8595	0.3512	0.7918	0.3013	0.4244	0.4797	0.2955
8	ŞANLIURFA	0.1195	0.1916	0.3499	0.5264	0.4967	0.1758	0.3023	0.0000	0.1011	0.9055	0.2365	0.1252	0.2518	0.9223	0.4381	0.6555	1.0904	0.2052	0.6018	1.2325	2.1208	1.2380	1.3979	1.6089	1.3844
9	GAZİANTEP	0.1199	0.1792	0.3505	0.5238	0.4741	0.1189	0.2736	0.1013	0.0000	0.8982	0.1777	0.2483	0.1668	0.9223	0.3553	0.6408	1.0917	0.0852	0.8440	1.2336	2.1530	1.2216	1.3803	1.6068	1.3514
10	KOCAELİ	0.0126	0.0972	0.1534	0.0707	0.4059	0.5098	0.2893	0.9088	0.8992	0.0000	0.5661	1.0478	0.9255	0.3777	0.7108	0.5895	0.2994	1.0621	1.9801	0.6874	0.4437	0.7207	0.0557	1.0713	0.3532
11	İÇEL	0.0768	0.0995	0.2188	0.3222	0.2349	0.0308	0.1311	0.2358	0.1767	0.5623	0.0000	0.3589	0.1713	0.5736	0.2539	0.5076	0.6807	0.2101	0.9121	0.7237	1.4094	0.6909	0.8564	0.9261	0.7956
12	DİYARBAKIR	0.1370	0.2288	0.4247	0.6070	0.6320	0.2843	0.4031	0.1254	0.2480	1.0453	0.3604	0.0000	0.4114	1.1218	0.5613	0.6810	1.3245	0.3701	0.4319	1.4974	2.4136	1.5258	1.6181	1.9593	1.6149
13	HATAY	0.1235	0.1860	0.3546	0.5295	0.4767	0.1130	0.2731	0.2525	0.1669	0.9248	0.1723	0.4121	0.0000	0.9329	0.4635	0.7441	1.1044	0.1922	1.1102	1.2480	2.2152	1.2336	1.4215	1.6251	1.3628
14	MANİSA	0.0575	0.1524	0.0116	0.1465	0.2710	0.5199	0.2648	0.9249	0.9226	0.3774	0.5770	1.1235	0.9328	0.0000	0.8533	0.8825	0.1386	1.1613	2.1703	0.1787	0.9010	0.2639	0.6408	0.3882	0.6077
15	KAYSERİ	0.1001	0.1036	0.3269	0.4220	0.4671	0.2337	0.1800	0.4406	0.3565	0.7124	0.2562	0.5638	0.4648	0.8559	0.0000	0.4852	1.0117	0.3535	1.3577	1.1480	1.8836	1.0917	1.0727	1.5604	0.9964
16	SAMSUN	0.0835	0.1187	0.3362	0.3997	0.6327	0.4477	0.3331	0.6578	0.6415	0.5895	0.5110	0.6826	0.7446	0.8832	0.4841	0.0000	0.9486	0.7319	1.2761	1.2208	1.5827	1.1971	0.8846	1.6528	1.0488
17	BALIKESİR	0.0503	0.1691	0.0656	0.0908	0.3791	0.6186	0.3178	1.0964	1.0951	0.3000	0.6866	1.3302	1.1074	0.1390	1.0114	0.9505	0.0000	1.3786	2.5596	0.3940	0.7831	0.4331	0.5241	0.7005	0.5378
18	KAHRAMANMA	0.1435	0.2033	0.4431	0.6234	0.5945	0.1382	0.3398	0.2066	0.0855	1.0653	0.2122	0.3721	0.1929	1.1658	0.3538	0.7342	1.3802	0.0000	1.1683	1.5596	2.5960	1.5407	1.6320	2.0306	1.5864
19	VAN	0.2570	0.4858	0.8244	1.1907	1.3333	0.7653	0.8767	0.6069	0.8492	1.9898	0.9228	0.4351	1.1164	2.1828	1.3615	1.2825	2.5672	1.1704	0.0000	2.9096	4.4614	3.0213	3.0968	3.8937	3.2523
20	AYDIN	0.0991	0.2182	0.0559	0.3016	0.2900	0.7001	0.3578	1.2416	1.2398	0.6901	0.7314	1.5067	1.2537	0.1795	1.1499	1.2256	0.3947	1.5608	2.9065	0.0000	1.4765	0.2149	1.1524	0.2007	0.9920
21	TEKİRDAĞ	0.0293	0.3243	0.3328	0.3920	1.1057	1.2864	0.8107	2.1470	2.1744	0.4476	1.4313	2.4404	2.2362	0.9096	1.8959	1.5966	0.7884	2.6107	4.4783	1.4837	0.0000	1.7458	0.8226	2.2795	1.4537
22	DENİZLİ	0.1052	0.1933	0.1079	0.3340	0.2097	0.6736	0.3075	1.2490	1.2296	0.7245	0.6993	1.5376	1.2411	0.2655	1.0951	1.2036	0.4346	1.5442	3.0226	0.2153	1.7399	0.0000	1.0761	0.3292	0.8213
23	SAKARYA	0.0270	0.1391	0.2609	0.1368	0.6124	0.7820	0.4337	1.4124	1.3913	0.0561	0.8680	1.6329	1.4321	0.6457	1.0776	0.8907	0.5266	1.6380	3.1025	1.1558	0.8210	1.0776	0.0000	1.6512	0.4713
24	MUĞLA	0.1513	0.3001	0.1314	0.4923	0.3520	0.9108	0.4905	1.6266	1.6206	1.0792	0.9392	1.9784	1.6382	0.3913	1.5684	1.6650	0.7043	2.0393	3.9031	0.2014	2.2764	0.3299	1.6522	0.0000	1.3751
25	ESKİŞEHİR	0.0646	0.1139	0.2395	0.1374	0.4832	0.7290	0.3022	1.3998	1.3632	0.3559	0.8070	1.6309	1.3740	0.6127	1.0017	1.0568	0.5408	1.5934	3.2606	0.9956	1.4519	0.8231	0.4717	1.3753	0.0000

Appendix D. Fixed Hub Cost for 25 cities.

1	229.729357	**İSTANBUL**
2	310.437927	**ANKARA**
3	247.333341	**İZMİR**
4	296.378151	**BURSA**
5	326.460914	**ANTALYA**
6	478.957924	**ADANA**
7	385.034994	**KONYA**
8	646.840153	**ŞANLIURFA**
9	493.975979	**GAZİANTEP**
10	499.616196	**KOCAELİ**
11	369.931717	**İÇEL**
12	514.788453	**DİYARBAKIR**
13	530.581113	**HATAY**
14	453.520925	**MANİSA**
15	403.291577	**KAYSERİ**
16	554.392218	**SAMSUN**
17	361.801799	**BALIKESİR**
18	625.805861	**KAHRAMANMARAŞ**
19	566.806027	**VAN**
20	549.690005	**AYDIN**
21	460.064483	**TEKİRDAĞ**
22	346.902826	**DENİZLİ**
23	630.034092	**SAKARYA**
24	447.460135	**MUĞLA**
25	378.539703	**ESKİŞEHİR**

BIOGRAPHICAL SKETCH

Makbule Özge Özler was born in Adana in 1993. She started to Ö.Ç. Bilfen Science High School yet, graduated from Adana College in 2011. After high school graduation, she moved to Istanbul for university education and received her B.S. degree in the Industrial and System Engineering in 2016 from Yeditepe University. After a year trial and error term for master's degree of material science and engineering in ITU, she started again her master in the Industrial Engineering in 2017 at Galatasaray University.