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MULTIPRODUCT, MULTIFACILITY, MULTIPERIOD DETERMINISTIC PRODUCTION PLANNING

by

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ABSTRACT

This thesis deals with a multiproduct, multifacility, multiperiod and deterministic production planning problem. The designed linear programming model is applied to TEKERSAN JANT SANAYİİ A.Ş., which produces different types of wheels on different production facilities. The objective is minimization of the total cost consisting of production, stock holding, surplus and idle time costs while complying with the constraints denoting satisfaction of the demand, full utilization of the production facilities, and satisfaction of production-stock balances at the assembly centers. The required inputs to the model are the demand levels and characteristics of the items to be manufactured, the cost coefficients of the objective function, and the capacities and available working times of the production facilities. Allocation of different products to the production facilities for each time interval in the planning horizon, utilizations of the production centers, starting and ending inventories and the production levels of the products for all time intervals are the main outputs obtained by this model. The different cases the model can handle may be summarized as: (a) cases where there exist starting inventories for any one of the products; (b) cases where there exist demands with production priority; and (c) cases where there occur any type of decisional or structual change after a fixed point of time within the planning horizon. Finally, in this model there are no restrictions on the number of products, production facilities or on the length of the planning horizon.

ÖZET

Bu tezde, çok ürünlü, çok tezgahlı, çok dönemli ve gerekirci bir üretim planlama problemi incelenmiştir. Geliştirilen doğrusal programlama modeli, değişik tipte jantları değişik tezgahlar üzerinde üreten TEKERSAN JANT SANAYİİ A.Ş.'ne uygulanmıştır. Amaç, istemin bütünüyle karşılanması, tezgahların en iyi şekilde kullanılması ve montaj hatlarında üretim-stok dengesinin kurulması kısıtları sağlanırken; üretim, envanter, atıl kapasite ve artık maliyetlerinin enküçüklemmesidir. Model için gerekli olan girdiler, ürünlerin istem düzeyleri ve özellikleri, amaç işlevi maliyet katsayıları ile tezgahların kapasite ve olası çalışma süreleridir. Modelden elde edilen çıktılar; değişik ürünlerin her zaman aralığında tezgahlara göre dağılımı, tezgahların kapasite kullanım oranları ve ürünlerin her dönem için başlangıç, bitiş envanterleri ve üretim düzeyleridir. Bu model tarafından çözümlenebilecek değişik durumlar şu şekilde özetlenebilir: (a) herhangi bir ürünün başlangıç envanterinin verildiği durum; (b) üretim önceliği taşıyan istemin verildiği durum; ve (c) planlama süresi içinde yapısal veya kararsal bir değişikliğin gerçekleştiği sabit bir andan sonraki durum. Modelin diğer bir özelliği ise tezgah ve ürünlerin sayısında ve planlama süresinin uzunluğunda hiç bir kısıt olmamasıdır.

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1. INTRODUCTION

1.1. Concept of Production Planning :

Production planning is the activity of establishing production goals over some future time period with the objective of planning the optimal use of resources to meet production requirements or taking advantage of potential sales opportunities.

As input, production planning uses the following kinds of information :

- Current inventory levels
- Current backlog positions
- Forecasts of future demand
- Current work-in-process
- Current work force levels
- Capacity of each production center
- Material availability
- Production standards
- Cost standards and selling prices
- Management policies

This information is gathered and analyzed periodically to develop production plans. The output from this activity may take a variety of forms, specifying the following types of information for each period of the planning horizon :

-Quantities of each product to be produced

-Quantities of a given product to be produced by each of several alternative processes

-Quantities of each product to be produced by a given process

-Target inventory levels by products

-Work force levels

-Overtime,additional shifts,unused capacity,etc.

-Quantities of material and semifinished products to be transported between stages in a multistage production system

-Subcontracted plans

-Purchased material requirements

The decisions made in production planning affect several classes of costs and revenues:

-Production costs (fixed and variable)

-Production rate change costs

-Capacity change costs

-Inventory holding costs

-Customer service and shortage losses

-Procurement costs

Production planning decisions affect inventory levels.Plans may all be building up inventories in slack periods for use in later peak periods.They may create large in-process inventories because of scheduled batch sequence changes between operations. They may specify different production rates for successive operations,resulting in an inventory between them.The problem to be solved by the production plan is one of trading of the cost of inventories against the value of inventories.

Failure to meet customer demand on time may result in tangible loss of profit or expediting costs and intangible loss of goodwill.Also,the inability of salesmen to make competitive delivery promises may result in no order at all.There are two ways that production planning and control actions result in these losses.The first is the failure to coordinate production with demand,which results in shortages and late deliveries.The second is the generation of excessive idle time,which results in the actual output being less than the rated output.If the lost production could be sold,the company would have a shortage loss because of poor efficiency.

Production planning also determines the ability to economically acquire materials used in production and it affects how much finished or semifinished product must be procured from outside sources.

Finally,in the longer run,production planning decisions will interact with plans to change capacity by the acquisition or disposal of plant and equipment.If these alternatives are available,the appropriate economic factors must be identified and estimated.

1.2. Need of a Company for a Production Planning System:

The management functions to be performed by the production system of a typical manufacturing company may be summarized as:

-Sales forecasting

-Production planning

-Materials ordering

-Production scheduling

-Distribution planning

The sales forecasting function must produce both long and short-term projections of sales for each item sold.The forecasts become information input to the production planning and distributio planning management functions.The impact of these forecast errors are immediately felt in terms of unplanned excesses of inventory or unplanned shortages and possibly lost of sales.

The production planning function must provide the anticipated dates and run quantities for each item at the factory over the planning horizon. The horizon itself must be long enough to permit early order of raw-materials and supplies, unless the lead time for ordering raw-materials is so long that the company keeps large inventories of these materials at all times. The production plan also calculates target levels for system wide inventories. As part of regular planning process, the management of this function may determine the need to expand capacity or improve facility capability. Inputs to the planning pfocess include forecasts,sales,capacity availabilities,rawmateria availability,production efficiencies and equipment requirements.

The plans become input information to the production-scheduling, materials-ordering and distribution planning management functions Errors in production planning arise because of poor forecasting, too little or too much production scheduled relative to actual customer demand, sufficient capacity at the plant is not available to produce all the targeted amounts because of poor planning efficiency and equipment utilization factors,down-time of equipment at the plant is higher than anticipated owing to equipment break-downs or labour shortages, and raw-materials supplies are insufficient to produce the items because of suppliers' inabilities to ship on time or due to erroneous procurement decisions.

The materials-ordering function must purchase supplies, that is, determine when and how much to purchase, monitor the status of purchase orders outstanding and control the inventory levels of raw-materials held by the company. Inputs to this function are the production plan,a bill of materials for each item,current raw-materials invetory levels,current production schedules,minimum order sizes and shipping options.

The production-scheduling function must develop daily schedules at the factory.The information inputs are the producti plan, the availibility of raw-materials,the amount and location of work-in-process within the factory,the available work-force and the status of machinery. Notification of the availibility of finished product each day must be sent to production planning function.Notification of withdrawals of raw-materials must be se to the materials-ordering function.

The distribution-planning function must monitor inventories throughout the system so that at each location adequate stocks are held in anticipation of forecasted demand.The input data for the function in question are the production plan,the production schedule,up-to-date inventory-stock status and forecasted demand. The information must be available by item and for each stocking point.

In a company where the production system,which unavoidably includes the inventory system in its boundaries,operates badly, as evidenced by high inventories of some items,too frequent stockouts of other items,a large number of revisions in scheduled production quantities,frequent shortages in raw-materia and supplies and high transportation and warehousing costs due to poor forward planning although the capacity is highly exceedi the demand,a production planning system is definitely needed.

2. GENERAL INFORMATION ABOUT THE SELECTED COMPANY

2.1. Introduction :

The company studied in this thesis is Tekersan Jant Sanayii A.Ş.,which produces different types of wheels each having differer number of parts.There also exist many different production centers.The production capacity presently exceeds the actual demand,possibly meaning satisfaction of demand,which is the most important management policy, will be fulfilled any way, but this may not be true always.

The company's basic policy is maximizing the sales, optimizing the raw-materials inventory and work-force levels, minimizing the work-in-process stock levels and increasing the capacity utilization.Therefore a production planning system in all its detailes is desired to be designed providing reliable information flow between the management functions mentioned in Chapter 1.The aim of this production system is to answer the following questions considering all the economical and technolog constraints and maximizing the total revenue at any given time:

-Current situation

-When to produce which product at which amount

-How long to continue producing that product on the line.

2.2. Company's History:

The company is one of the biggest among its few competitor in the specific field of products,namely wheels.The feasibility studies,layout planning and all other technical and economical analyses where completed in 1978,so 1978 become the year of establishment of the company.The management is in Istanbul,Turk The plant is placed on the Istanbul-Eskişehir highway,at the 230th km from İstanbul and 20 km from Bilecik.The plant is also near to the railway facilities.The plant was built on a land of 100,000 m^2 with a covered area of 10,500 m^2.Construction works were completed in late 1979.Imported machinery and equipment started to arrive during 1979 followed immediately by local assembly.In early 1980,the production in the factory had partly begun with approximately 20% capacity utilization.After a trial period it is planned that the factory starts normal production with almost 50% and in four years capacity utilization may increase up to 100% mainly depending on the automotive sector in Turkey,generally.

The company was established originally to produce commerc and agricultural wheels.However,the high capacity and modern machines and die-making capabilities enables it to produce all kinds of other wheels,including automobile wheels.370,000 industrial wheels are to be produced in one shift.Increasing number of shifts as well as a shift to lighter wheels such as automobile wheel,will increase production capacity.

The today's conditions of Turkey's automotive sector are absolutely valid for this company.The production of few differe products and the low level of capacity utilization are the reflection of the bottlenecks in this sector of Turkey.

2.3. Products:

The company is manufacturing wheels.The product can mainly be classified in two groups:wheels for trucks and wheels for tracktors.Each group can further be devided into subgroups as front wheels and back wheels.Since the sizes and the combinatio of the parts comprising the wheel's body differ from one type to another,the company,in fact,has ten different kinds of products.One subproduct of the wheel,the disc,is also sold separately.So,in total,eleven different products are produced by this company.

Detailed information about the products of the company is given in Table 1.

2.4. Raw-materials:

The main raw-materials for producing the different types of wheels are as follows:

-Steel profile:The main raw-material for manufacturing the truck's wheels are specially prepared and they are imported from Germany,Italy or Spain.

-Coil:These,which is mainly used for manufacturing tracktors'

CODE	NAME	EXPLANATION
1) 6.5x20	Otosan Fiat	Front wheel for truck
2) 6.5x20	Otosan Ford	Back wheel for truck
3) 7x20	BMC	Back wheel for truck
4) W10x24	BMC	Back wheel for tractor
5) W13x28	Fiat 480	Back wheel for tractor
6) 4x16	BMC	Front wheel for tractor
7) 3.62x19	Fiat 480	Front wheel for tractor
8) 11x28	Burtrak	Front wheel for tractor
9) 4x16	Burtrak	Back wheel for tractor
10) j5x5x14		Front wheel for tractor
11) 13x28		Disc

TABLE 1. Present list of products of Tekersan

wheels are obtained from Ereğli Demir Çelik Fabrikası.

-Metal sheet:They are mainly used for producing discs,and are available from the local markets.The most important types used by the company are 9 mm,6,5 mm,and 11 mm thick metal sheets.

-Wire electrods:They are used while assembling the rims and discs

-Secondary production materials:The most important one among these is paint.Primer and finishing paint are the two types used.They are obtained mainly from İstanbul,Ankar or İzmir.Chemical materials,nuts and bolts are other items used during production.

2.5. Production Centers:

The production centers in the factory are mainly the production lines and assembly centers.Parts are produced on six production lines and assembled in three main and one subsidiary assembly centers.

The production lines are fully automatic.The number and size of each machine making up a line are determined so that the optimum working cycle for the whole line is obtained and excess work-in-process stocks in the line are avoided.

Some information about the production lines and assembly centers are given in Table 2.

Metal sheet cutting line precedes the tracktor,disc and bracket lines.The lines except the metal sheet cutting line are working in a parallel manner.All the production lines are precedin the assembly centers.Stock places for the work-in-process elements are available in front of the assembly centers.There are also subsidiary production facilities such as painting shop,quality control department and warehouses for raw-materials and final products.

2.6. Production Characteristics:

2.6.1. Production characteristics of truck's wheels:

Production of a truck's wheel starts by manufacturing rim, disc,side and lockring separetely.To manufacture the rim,first

NAME	EXPLANATION
1) Production lines	
-Metal sheet cutting line	Metal sheets are cut to size.
-Side and lockring line	Side and lockrings for truck's wheels are manufactured.
-Truck line	Rims for truck's wheels are manufactured.
-Disc line	Discs for truck's and tractor's wheels are manufactured.
-Tractor line	Rims for tractor's wheels are manufactured.
-Bracket line	Brackets for tractor's back wheels are manufactured.
2) Assembly Centers	
-First assembly center	Rim and disc of a truck are assembled.
-Second assembly center	Side and lockring are assembled to rim or to rim+disc of a truck
-Third assembly center	Rim and disc of a tractor are assembled.
-Fourth assembly center (Subsidiary center)	Bracket and rim of a tractor are assembled.

TABLE 2. Existing Production Centers of Tekersan

drawn steel profiles are cut.Then these profiles are bended in a circular bending machine.The rounded rim's ends are faced to each other by a special press and the ends are welded by a special welding machine.After the welding operation,surplusses of welds are cleaned and the welded places are cleared. In order to bring the rim to the desired finak specification, it goes to squeezing and expanding machines. Holes for tire valves are drilled on the rim. If the wheel is going to have a disc, it is sent to the assembly center. Disc is manufactured from metal sheets. Depending on the metal sheet's thickness and the size of the disc different presses are available. Discs are first cut to the necessary outer diameter. All holes are drilled. The form to the disc is given by one or two operations at the mold. After the final calibrations have been completed, the disc is sent to the assembly center. At the assembly center, disc is put into the rim and then they are welded at the contour or at the places where the form allows. After drilling the holes the rim with the disc is inspected many times. Side and lockrings are manufactured seperately and their production is similar to that of the rim. Since the ends of the lockring are left open, no welding takes place. Side and lockring are then added to the rim and the production of the truck's wheel is finished after the final quality control.

2.6.2. Production characteristics of tractor's wheel:

The necessary raw-materials for tractor wheel production are coil and sheet metal. After the coil and sheet metal are cut to the required lengths, they are bent in a circular bending machine. Their ends are welded in a special welding machine. After the cleaning operations, a form is given to the sheet metal on a profiling machine. The final calibrations on the size of the rim are done. Before the rim is sent to the assembly center, holes are

PRODUCTION CENTER	INPUT	OUTPUT	NEXT CENTER
Metal sheet cutting line	Metal sheet	Sheet	Bracket,disc and tractor lines
Side and lockring line	Profile	Side and lockring	Painting shop and 2.assembly center
Truck line	Profile	Truck's rim	Painting shop and 1.assembly center
Disc line	Sheet	Truck's, Tractor's disc	Painting shop and 1.assembly center
Tractor line	Sheet	Tractor's rim	Painting shop and 3. and 4. assembly centers
Bracket line	Sheet	Bracket	4.assembly center
1.assembly center	Truck's rim and disc	Truck's rim + disc	2.assembly center
2.assembly center	Side and lockring truck's rim+disc truck's rim	Truck's rim + side and lockring truck's rim+disc +side and lockring	Warehouse of truck's wheel
3.assembly center	Tractor's rim + bracket, rim,disc	Tractor's rim+disc+ bracket, rim+disc	Warehouse of tractor's wheel
4.assembly center	Tractor's rim, bracket	Tractor's rim + bracket	3.assembly center

TABLE 3. Flow relations of the production centers

drilled in.All tractor's wheels contain a disc.The discs are manufactured similar to truck's discs.At the assembly center, the disc of tractor's front wheel is welded to the rim; whereas the disc of tractor's back wheel is screwed to the rim by the help of brackets. Tractor wheels do not contain side- and lock-rings. The wheel is painted and sent to final quality control center.

2.7. Input and Output Relations of the Production Centers:

Although it can be said that most of the lines are working in a parallel manner, there is a strick flow of materials between the lines, assembly centers, stocking places and warehouses.

Flow of materials between the production centers are given in Table 3. The layout of the plant and flow relations of the production centers are shown in Figure 1.

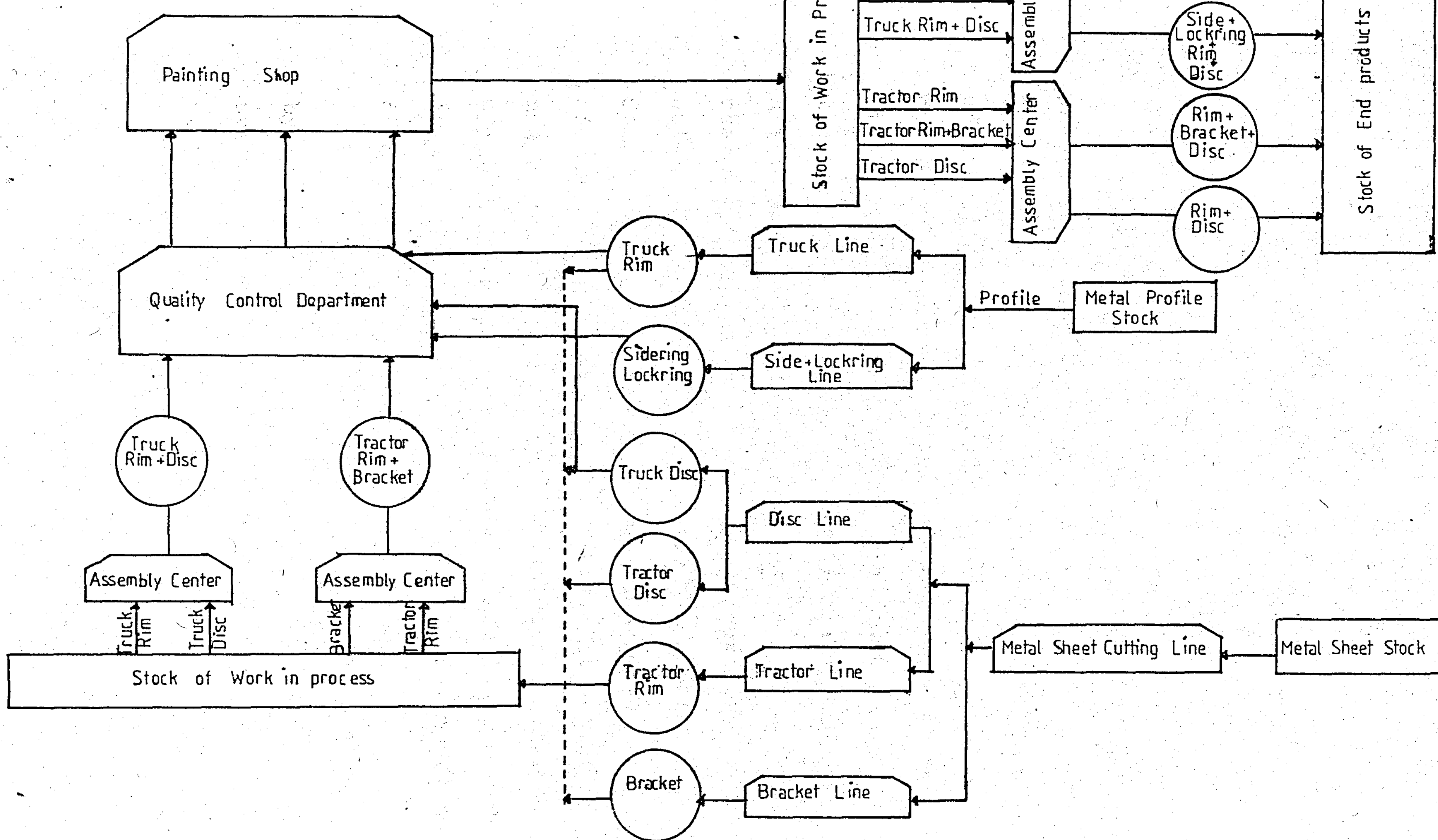

Figure 1 : Layout and Flow of Materials in the Plant

3. LITERATURE SURVEY

3.1. Introduction :

In the literature, there exist many models related to production planning problems. Most of them are generated for specific types of manufacturing systems. As a concequence of the different requirements set by the managements of these different manufacturing systems the emphasis is set on one of the management functions, which must be performed all together to build up the production planning system, or on a very specific and detailed problem of the firm. Thus it is nearly impossible to find a model presented in the literature which can directly applicable to the problem in question. Some of the existing models can be used to solve subproblems of the wheel company and then they will be incorporated to a master model to give a final solution. Others can be used after some extentions and changes are done. A study of these models in the conclusion that a new model has to be developed for the case in hand.

In this chapter, some of the models will be presented by specifying a general view of their features and their applicability to the specific problem of the wheel company.

3.2. Models for Production Planning Problems :

3.2.1. Parametric Production Planning (1,4)[x] :

The main objective of this model is to optimize the work-force level and the production rate of the plant. The procedure involves structuring a logical set of decision rules, which take the current status information and a forecast of requirements over the lead time and compute changes to be made. These rules are a function of few parameters, whose optimal values are determined through a search procedure involving simulation against historical demand and use of the actual cost structure of the organization. The resulting rules are used each period on a moving horizon basis to plan production and work-force for the next period.

The required set of inputs contains the following elements :

-Current inventory level
-Current work-force level
-Current production rate
-Demand during the previous periods and the current demand leve
-Required man-hour of labour to produce one unit of product
-Number of units to be producable at a specific work-force level

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x) Numbers in paranthesis refer to Reference List.

-Production cost per unit

-work-force and work-force change costs

-Inventory holding costs per unit per period

The work-force level of a period is defined as a linear function of the actual work-force level of the last period and the ideal work-force level of that period, which in terms is a linear function of the expected demand and the difference of ideal and actual inventory levels. The production level is expressed as a function of the ideal production rate and work-force level in that period. The ideal production level is similarly defined as the ideal work-force level to be a function of the expected demand and the difference of the ideal and actual inventory levels.

The derivation of the relations for the work-force level and the production level requires the use of some parameters. A search method,where the response for any set of parameters values determined by computing the costs of production,work-force,work-force changes and inventories,is used to select the best values of these parameters.

The main outputs obtained by this model can be summarized as follows :

-The production levels,concequently the quantities of the products to be produced in each period

-The work-force levels.

The model using the information of the plant at the present time gives information about the work-force levels and changes in production rate and in work-force levels during the planning horizon. Although this information is required by the management of the wheel company, they are not the most important and the single questions to be answered. Hence, this model can only be used partly to organize the work-force level in the plant.

3.2.2. Multiproduct, Multiperiod, Resource Constrained Problem with Process Selection Decisions Considering Work-Force Level (2) :

This is a linear programming model considering production and work-force levels of a multiproduct and multiperiod case. The objective is minimization of total cost subject to linear constraints denoting the satisfaction of demand, balancing the work-force levels, minimization of inventory level and minimization of costs due to lost sales.

The required inputs to be specified for the LP problem are as follows :

-Number of man hours required to produce a unit

-Cost of man hour labour on regular time and an overtime in each time period

-Unit variable cost, exclusive of labour of each product by each process in each period
-Cost to increase or decrease the work-force level by one man hour in each period.

The results obtained by this model are summarized as follows :

-Optimum work-force level in each period
-Increase or decrease in work-force level between two periods
-Overtime or undertime scheduled in each period
-Quantity of each product produced by each process in each time interval, i.e. an optimum production schedule.

By analyzing the outputs, it could be concluded that this model is applicable, but the technology used and the complex structur of the wheel company makes the use of this model, as it is, impossible. So without any extentions this model is not applicable directly.

3.2.2. A Goal Programming Approach to Aggregate Planning of Production and Work-Force (3) :

This model presents a goal programming approach to the problem of scheduling aggregate production. It answers the following questions; given forecasted demands what should period production rates be in order to minimize the sum of payroll, hiring and layoff, over and idle time, inventory and stockout and production rate change costs over a specified length of the planning horizon? The main inputs to be specified to obtain these results are the demand level of each period and the number of man hour labour required to produce one unit. The temms in the objective function are all quadratic and the model presented is a linearization method based on goal programming approach and the problem then becomes a linear model to be solved.

The terms of the objective function, each denoting one type of cost, are nonlinear. All these terms are expressed as differences. The goal programming approach described in this model is based on the idea that each of the quadratic cost term becomes zero when the expressions denoting the differences, are zero. Therefore the minimization of each quadratic cost temr can be thought of as a constraint and formulated as a constraint.

A linear approximation is done on the original objective function. The coefficients of the slack varibles, which are added to the constraints, in the objective function must be selected in such a way to give adequate cost approximation to the original quadratic terms they represent. This can be done by fitting linear segments to each of the quadratic functions involved. The desired cost coefficients of these variables are given by the slopes of the linear segments.

Although theoretically models could be formulated with any number of such linear segments the size of the LP model increases greatly with the number of such segments. The original constraint, defining the ending inventory level at the end of any period be the starting stock plus the production minus demand in that time period must also be included in the constraint set. By assignig positive coefficients to the slack variables in the objective function, the effect is to penalize deviations from the desired goals.

Goal programming approach is only applicable to the theoretical level of modelling. For relatively low degree of nonlinearity in objective function, this model provides an efficient and effective solution while for higher nonlineari it will be inappropriate.

3.2.4. Economic Lot Size Determination in Multistage Assembly Systems (5) :

This model is developed to consider optimal lot size problems for multistage systems with any number of predecess but with only one successor. Demand is assumed to be constan over an infinite horizon with instantaneous production. The objective is the choise of a lot size for each facility whic minimizes average per period production and inventory holdin costs, where at each facility production costs consist of a set-up cost and possibly a linear component and inventory holding costs are linear subject to all technical constraint and the constraints of satisfying all demand with no back-orders.The inputs to be specified for this model are :

-The predeđence relation of the production facilities
-Set up cost at each facility
-Linear per unit installation inventory holding cost
-Number of units required at each stage to make up one final product.

In a multistage system, the production of endproducts require completion of a number of operations. A stage might consist of an operation such as procurement of raw-materials, fabrication of parts or assembly with no capacity constraint. A fixed sequence of operations is assumed. The output of the final stage serves the customer demand. Output of each stage may be stored if a necessity arises. No stockouts are permitted. The aim of this model is to determine the economic lot size in a multistage system. Dynamic programming technique is used to solve the problem. If all the lot sizes in the system were equal, then given the assumption of instantaneous production, inventory would be held on the final stage. However, if the lot sizes between the two succeding stages are different, then stock is created at one of the stages and an average level of such inventory is a function of the lot sizes of both stages. Since demand is constant,such an average inventory level is easily obtained. Thus,the total holding and set-up cost for each stage is obtained.The sum of total cost incurred in each stage, summed up over all stages,give the total cost of the system. By using dynamic algorithm, solution proceeds from the raw-material stage to the final stage, which is the total cost at that stage and minimum possible total cost up to that stage. At each stage the total cost , defined by the recurrance relation,is computed for all possible lot sizes.

The assumption set by the model above, that each stage must have only one successor, makes the model inapplicable without any revisions on the structur of the plant. The use of dynamic programming features would be an advantage of this model, since it would imply the use of large number of stages which is the case in the problem specified.

3.2.5. A Deterministic, Multiproduct, Multifacility Production and Inventory Model (6) :

This model considers concave production costs which can depend on the production in several different facilities and piecewise concave inventory costs. Backlogging of unsatisfied demand is permitted. Object is to determine a production schedule that specifes how much each facility in the network should produce so that the total cost is minimized. Production and inventory holding costs are the two main items of the objective function. The facility receices inputs from raw-materials and one or more facilities,then in each period manufactures a specific product on its own production line. This product is stored in inventory until neede either to satisfy market requirements or to supply the other facilities.

The main inputs to be specified in order to obtain the optimal production schedule at the minimum inventory level and at the minimum total cost are as below :

-Number of periods in the planning horizon

-Number of facilities in the plant

-The market requirements for each facility in each period

-The precedence relations in the plant

-The amount of time required to complete production in each facility

-The number of units required from the product coming from preceeding facility to produce one unit of a product at each facility

-Backlog limit for each facility.

The outputs obtained are summarized as :

-The amount of production completed in each period by each facility

-The starting and ending inventory levels at each facility.

Total demands on each facility in a period is defined to be the market requirements and the sum of the requirements of all facilities following this one. The ending inventory level is the difference of the amount produced and the demand. The problemthen is minimization of the costs such as a production and inventory holding costs satisfying the constraints of fulfilling the total requirements of each facility in each period.

To determine an optimal production schedule first starts by defining the set of all feasible schedules,which is called the dominant set. It is constructed based on dynamic programming algorithm. Starting from the final facility which fulfills only the market requirements all feasible schedules are determined by considering the facility on hand and all facilities prior to it. Thus the dominant set for the entire N facilitites acyclic network will be determined After the dominant set has been constructed, the optimal vector, the production schedule, will be obtained by exhaustive enumaration of the cost of all vectors in the dominant set. The most important disadvantage of this presente model is enormous long computation time of the dominant set for a large and complicated acyclic network. For cases, where facilities are only paralell or only serial, then using dynamic algorithm, it is possible to solve larger problems in an acceptable time span.

For large values of n and N, where n is the number of periods and N is the number of facilities, the model's application will be nearly impossible. But for pure paralell and serial systems, algorithms are derived for better and faster solutionsusing dynamic programming. Since the structur of the plant in hand is neither parallel nor serial, the application of the algorithm is not possible.

3.2.6. Computer Algorithms for Production Planning Problems :

There exist mainly two computer algorithms in the literature focusing on line balancing, one of the aspects to be considered within the scope of the production planning problems. Both of the algorithms generate feasible solutions and choose the best one among the alternate solutions by setting some rules.

The first of these computer algorithms is COMSOAL, Computer Method of Sequencing Operations for Assembly Lines(7). The basic methodology of COMSOAL is based on generation of fairly large number of feasible solutions to the line balance problem by biased sampling. Alternate solutions to the particular line balance problem are then based on the best solution generated.

The basic inputs for COMSOAL may be listed as follows:

-The precedence relations

-The total number of tasks

-The working times of the tasks

Another important input is the specification of the rule, according which a task can be assigned to a station. The rule may be neglected and the specification of the task is done at random. Another rule is weighing the tasks that fit in proportion to task time. A possible rule is to give those tasks that have a larger number of followers greater probability of being assigned than the tasks with a small number of followers.

The general procedure of COMSOAL starts by tabulating the total number of tasks which have immediately preceed each given task. Then all tasks with no preceeding task are listed in the available list. In the fit list all tasks which have times no greater than the time left available at station being assigned work, are transfered. Either at random or by using of the set rules, an assignment is now made. The assigned task is eliminated from all lists and the required updates are accomplished. This routine continues until all stations are fully assigned. As a solution is completed its station number is compared to the station count of the previous best sequence .If there is an improvement, the new solution is stored in memeory and the old one is discarded.

If after the allocation of the production lines, which are assumed to be automatic, line balancing is required, this computer program will be applicable and very efficient.

The second computer algorithm available is BALANCE; an assembly line balancing problem (8). The objective of this computer program is to find out either the minimum required number of work stations given a desired output rate or to find out the minimum cycle time given the maximum allowable production line length, that is the number of work stations. As in the previous computer algorithms,the required input in general are the followings:

-The work elements

-The working times of these work elements

-The precedence relations of these work elements

-The maximum number of work stationsor the minimum required number of work stations.

The ideal situation, perfect balance, is that in which each station on the assembly line exactly the same amount of work as every other station. The length of the production line is number of stations and cycle time is the maximum value of the total work element times assigned to any one station.

This computer program assignes work elements to stations determines final cycle time and calculates the balance delay, where the delay is defined to be the ratio of total idle time to total work time.

The computer program BALANCE may be used for different heuristics of line balancing. The selection of an element from the available list can be based on either its largest or smallest times or first or next available ones or it is made at random.

As in the previous computer algorithm, BALANCE could be used after the scheduling of the production lines is completed.

3.2.7. Optimal Production Planning by a Gradient Technique(9)

A gradient technique is used to solve a production planning problem with known salės forecasting. The function to be minimized is the sum of the costs derived from holding inventories, stockouts and deviations of production rate of the plant. This technique has the advantage of being able to investigate problems withaalarge number of state variables and can also be extended to muliproduct, multifacility operations with complex interconnections.

One advantage of this technique is that it does not have any dimensionality difficulty. The disadvantages of the model are summarized as follows:

- It can not conveniently handle problems with inequality constraints on state variables
- Convergence to an extreme which is not absolute maximum or minimum may occur in applying this technique.
- The convergence rate may be very slow during the last part of the iteration where the current values are near to the optimum.

However, iterative techniques have been devised for solving problems with state variable inequality constraints by the gradient technique. The problem of convergence to relative optimum can be overcome by ısing different starting points.In order to obtain the following results for a production scheduling problem:

- The production rate of the plant for each period
- The inventory levels of each product foe each period

-The overall utilization of the plant,
the inputs below have to be specified :
-The forecasts of the sales
-The precedence relations of the facilities
-The locations and the capacities of the stock-holding places
-Current production rate of aech facility
-Current inventory level of the products
-Final conditions, the predetermined inventory level of the product at some stock-holding places.

A typical dynamic system in production planning can be represented by a differential equations. They are function of time, of production rate, which is to be choosen optimally and of the state variables, which are defined to be the inventorylevels at that time. The differential equations will be approximated by difference equations. To use the gradient technique, an ideal initial approximation for the control variable, the production rate, must be estimated. In order to examine the influnce of the control variable on the state variable, on the state of the plant and to obtain the direction of the steepest descent, an iterative procedure is formulated with the estimated values of the control variables.

The algorithm presented here, in this model considers a single product and a single facility case.

It is not possible to apply the model without any extentions to multiproduct and multifacility problems as in the real problem on hand.

3.2.8. Line Balancing for Mixed Model Assembling (10) :

Mixed model assembly occurs when more than one model of the same general product are intermixed on one assembly line. The amount of work required to assemble units can vary from model to model creating an uneven flow of work along the line. This model is a procedure of adopting a single model line balancing technique to mixed model schedules. It also introduces a sequencing procedure for determining the order in which models are to flow down to utilize the lines.

Because the line balancing and sequencing procedure consider a wide variety of factors, they are applicable to many types of assembly lines. Considering of an efficient mixed-model assembly line entail the solution to two separate but related problems;line balancing and model sequencing.

The inputs that must be specified for the line balancing and model sequencing are the followings :

- The number and characteristics of the work stations
- The number of operators required per labour group which is comprised of work stations with one or more operators manning each station.

-The number of units per model to be assembled in the total time duration within a time interval

-The inefficiency penalties: The various models to be assembled will sequenced to control the effect of inefficiencies. If a penalty in TL/minute is associated with each station, it is possible to compute the total cost of inefficiencies, like idleness or work efficiency, resulting from scheduling a unit of a given model in sequence. At the end of this model the optimum production schedule which minimizes the total inefficiencies costs is obtained.

Line Balancing : It is a procedure of assigning work to assembly operators in such a way as to balance the work assignments among the operators and to minimize tne number of operators required. In balancing a mixed model line, it seems to be possible to consider each model independently and consequently to balance the work among operators for each separate model. This procedure reduces the larger problem to a number of smaller single product balancing problems. A method to solve the balancing problem of a mixed model assembly line is one that considers the total schedule for a whole shift and assigns work elements to operators on a shift basis rather than on a cycle time basis as is done on single product lines. Because each element is assigned to only one operator, the time and cost is reduced and the general efficiency of work on a mixed model line should be improved.

Model sequencing : Such a problem occurs in industries which desire to keep several models in production rather than to produces batches of each intermittently for inventory. The amount of work required to assemble units can vary from model to model creating an uneven flow of work along the line. Due to the complexity in assigning this work to the operators there is a tendency to have an oversupply of operators to meet the uneven flow of work. Hence, in the objective of sequencing procedure is to determine the ordering in the flow of models which allows the optimum utilization of the assembly line operators. This is based on the computation of penalty costs of inefficiencies resulting from sequencing the various models. The procedure assumes that each operator begins processing the first unit as soon as it enters his station and that this movement about his station can be measured throughout the time. The model, whichoperation time in each station are less than the corresponding station times in, is launched first and the total inefficiency cost is calculated. This procedure is repeated for all combinations of sequences of models.

3.2.9. Estimating the Output of a Serial Production System (11) :

The aim of this model is to estimate the output of a production line with one surge or with muliple surges. The model is only applicable to serial lines. The machines making up the lines may differ and they may have general failure distributions. The model uses the idea of couppling the serial production line elements before and after the surge and analyze their up and down times.

The main outputs obtained by this model are the numbers capacities and locations of the buffers. With the specified numbers and locations of the buffers, the best expected output of the production lines is obtained. The required inputs for this model are summarized as follows :

- The production rates of each production line
- The failure distribution of each production line
- The precedence relations of the production lines
- The possible capacities of the buffers
- The possible locations of the buffers.

The model is based on the case where there exist N machi and a single surge and then it is extended to the case where there exist N machines and M surges.

For the case, where there is only a single surge, the system can be thought as to be two coupled systems, an input and an output devise. The machines with the minimum production rate among the machines making up the input or the output devise is choosen to be the representative one for those devises respectively. Of interset in the system is the expected output as a function of surge capacity. It is assumed that the surge protects the devise with slower production rate from failures of the devise with faster production rate. Focusing on the devise with the minimum production rate provides a mechanism for computing the system's availability, which is defined to be: In addition to the time the input devise is down because of its own failures, how much time is it forced down because it is blocked. Thus, the system is up if and only if the input devise is up. An assumption that simplifies the calculation of the system's availability is that the model precludes simultaneous repairs of input and output devises and repair priority is always given to the input devise. The effect of the surge on the availability of the system is to increase the uptimes and reduce the downtimes of the output devise as seen by the input devise. Thus, the system availabilaty is found by computing equivalent up- and downtimes of the output devise.

The approach for N machines and M surges system is heuristically modify the methodology for N machines and a single surge case. The method involves solving a series of N machines single surge problem. First the production rate of the system is set to be equal to the rate of the first machine, then to the rate of the last machine and finally to the rate of an intermediate machine. For each case the expected output is evaluated and the best is choosen.

In this model the production lines are coupled in a serial manner. This assumption by itself makes the model inapplicable to the plant in guestion. Though it may be applied to the serial parts to obtain partial results. Another problem arising in application of the model will be the existinglayout of the plant, namely the number and capacities of the surges are already specified, so the optimization of them will not be needed.

3.2.10. <u>A Class of Deterministic Production Problem (12):</u>

A deterministic production planning problem with limited backlogging, inventory and production capacity constraints is considered. The model also includes cost function which is neither concave nor convex. A characteristic of the extreme points is provided and an algorithm is suggested for the problem with equal production capacities.

The distinguishing features of the proposed model can be listed as follows :

-Production cost functions are composite functions, where one is a nondecreasing concave function and the other is a

nonnegative piecewise linear, convex and continous function.
-Backlogging is permitted only for some fixed periods.
-Inventories are constrained.

The objective function minimizes the sum of production, inventory holding and backlogging costs over the entire planning horizon. The constraints are denoting the followings:
-The amount of production and the starting level of inventories in each period must be equal to the amount of market requirements and amount backlogged in the last period makes up the ending inventories for that time period.
-There exist upper limits for stock and backlog limits.
-Inventory levels must be zero for the first and last period.

With this formulation the model is presented as a network with gains on some arcs. This presentation and the characteristics of the cost function an optimal solution is guaranteed. The algorithm requires solving a shortest route problem.

The model presented above is based on the assumption that all production facilities have the same capacity whereas the capacities differ by large amounts in the real case to be solved.It is not designed for multiproduct case and the output obtained does not provide the way of scheduling the production facilities by the different products. Therefore the direct application of this model is not possible.

3.2.11. Network Models for Production Scheduling Problems with Convex Cost and Batch Processing (13) :

The model considers the problem of producing N products on M identical facilities where each product is produced in batches. The planning horizon is made up of H equal length production periods where each facility is capable of producing one batch of any product during a production period. Total cost can be expressed as the sum of all production costs which is a cumulation of all costs caused by production of batches from each product type in that period and in all periods and on all facilities. The aim is to determine a production schedule minimizing total cost. The required inputs to achieve this aim are the followings:

- The known demands of all products in each period
- The size of batches of each product in each period
- The production, inventory carrying and backorder cost coefficients.

The production occurs in batches of fixed size for each product, which implies the restriction that the number of batches from the endproducts to be an integer. Under these assumptions, this problem can be modelled as a minimum cost network flow problem with convex arc cost and integer capacities.

The model including any number of products, in general has H+HN+2 nodes and H+2HN arcs. In such a network each arc corresponds to the number of products produced in batches in each period. Upper and lower bounds on the total number of batches of all products produced in each period can be imposed by putting appropriate upper and lower bounds on the flow in the arcs. Hence, the inventory levels of each product at the end of the planning horizon can either be fixed at specified values by setting the corresponding lower or upper bounds equal to each other or allowed to fluctuate between specified bounds.

Flows in arcs for all products and periods represent the number of batches of the products in any period. Hence, the production cost is assigned for each unit of flow on arc. The relevant inventory/backorder costs must also be included. First, the possible inventory levels for each period and each product is calculated, then the relevant cost is calculated So to each arc the costs are assigned. Solving the minimum cost network flow problem, the optimum schedule is obtained.

This model has the advantage over all other methods presenting the production planning problem as a network, that it does nor require any restriction in the number of predecessors and successors. Bur this model considers the problem of producing N products on M identical facilities, which is not acceptable. The above presented model could only be applied, if applied, to some oftheparts of the plant and a system must be designed to incorporate the results to a master model.

3.2.12. The Effect of Interstage Buffer Storage on the Output of two Unreliable Production Units in Series with different Production Rates (14) :

This model is designed to see the effect of interstage buffer storage on the output of a two stage production line so the emphasis is put to a very special point of production planning problem. The output of the system will be effected by the capacity of the buffer and the capacities of the buffers will be effected by the specifications of the two production units. The specifications of the production units are :

-The production rate of each production unit

-The repair rate of each production unit

-The breakdown rate of each production unit.

The question to be answered by this model is that how the average production rate of the whole line depends on the capacity of the buffer. To answer this question, regeneration points are defined. The state, where the first production unit is down, the second unit is running and the inventory level in the buffer is zero is called regeneration point. The time between two such points is called a cycle. The output rate of the production line is the expected production used per cycle divided by the expected duration of a cycle. In order to increase the output rate, it is desired to find out an optimal buffer capacity. Thereby the expected cost,which consists of the production, stock holding and the cost incurred being in one of the states, should be minimized. This model gives the details of derivation of the expected cost, in general, which turns out to be a first order linear system of differential equations.

The more the line is unbalanced the less buffer capacity is needed. The difference between the net production rates of the two production units can be caused by a difference in production and in repair rates. The effect of a buffer on the line production rate will be different in each of the cases.

In any production planning problem, there are the problems of storage and an inventory system must be designed to have a complete solution procedure. This model creates a look into the effect of an interstage buffer storage. Because of the size and the complexity of the plant, this model is also not possible to be applied directly. Based on these ideas, a new model should be designed to fulfill all the requirements set by the management of the wheel company.

3.3. Conclusion :

The purpose of analyzing the models presented in the literature is to gain a view how the problems similar to the one in question are approached. Since each problem has its own characteristics, most of the models are designed for special cases; thus a direct applicationof them will only be then possible, if both problems have exactly the same specifications or if the assumptions to be done to convert the case studied to the model's specifications will be acceptable. Since the characteristics of a production planning model are layout of the plant, specifications of production facilities, the types of products manufactured and the sequence of operations the products must go through, the model must be dependent on the special case. All these imply that the design of a new model which fulfills all the requirements desired is needed. The model designed specially for Tekersan Jant Sanayii A.Ş. is explained in the next chapters in detail.

4.METHODOLOGY

4.1. Introduction :

"It is paradoxical, that a few of these techniques in the literature have been implemented in real manufacturing companies. Only rarely have companies been able to adopt the approaches that have appeared in the scientific literature.And usually when methods have been accepted,the adopted approaches are simplified approximations to the original technique. " (15)

Since each company in the real life has its own characteristics and since the production plan must give answers to the specific demanded questions,it is much more advantageous from the viewpoint of the company to design a new production plan which fits to the special characteristics of the company than to apply a model from the literature giving only approximate answers to the problems. With this in mind,a new production planning is designed and implemented for the case in hand.

4.2. Modelling :

The first step in modelling is to determine the inputs and outputs of the model.Then comes the real modelling studies. Using the inputs,if the model fulfills the set of requirements, economic and efficiency analyses are performed.The model with valid results involving these analyses will be the one applied to the company.

In the following sections, the steps in the evolution of the production planning model are explained.

4.2.1. <u>Inputs :</u>

The inputs to the production planning of a deterministic, multiproduct,multifacility and multiperiod problem are classified in three groups; the ones that are specified by the management, others that are there because of the physical structure of the plant and the third group which is derived by means of calculations by other departments.

One of the most important inputs that is specified by the management is the length of the planning horizon and the individual time intervals. After some discussions the planning horizon is set to be one month and the time interval one day; so there are 20 time intervals in the planning horizon. An optimum running time of the model,the ability of getting the results of short term changes immediately and the effect of minimizing the work-in-process stocks are the factors that led the management to the setting of the planning horizon to be one month and the individual intervals to be one day.

Another input that was determined by the management is the level of demand of the wheel's types,which are assumed to be deterministic.The sale forecasts and economic analyses in the market determine these levels,which become the minimum production levels.Beside the production levels of the wheels,the management determined the different types of wheels to be produced depending on the sales contracts and on the situation of the market.

The following belong to the second group of inputs, the inputs arising from the physical structure of the problem:

-The number of components of each wheel and their names.

-The parts that are produced on each line.

-The possible places of stocks in the plant and their sizes.

-The amount of each part produced in one period on each line, i.e. the capacity of each line.

-Maximum usable times of the lines in one period.

-Number of production facilities, i.e. production lines, assembly centers, painting shop and quality control centers.

-Precedence relations, on which lines should the parts be produced in order to be operated on the line in question,

The elements of the third group of the inputs are the cost coefficients. They, mostly are calculated by means of cost accounting.

Production cost gives the cost of producing each part. Stockholding cost gives the cost of holding one processed part during one period of time in the stock. Each idle period of the line incurs idle cost of that line. Another cost coefficient is the surplus cost. Each unit processed that exceeds the actual demand is stored and so a surplus cost is incurred. Rate of change of production level, change of work-force levels, change of some production levels all incur some costs to the model.

The inputs of the first and second group make up the constraints to the model. By considering these constraints feasible solutions are obtained. But there exist a solution among all possible feasible solutions, which best fits to the decisions of the management, i.e. the optimum solution. This solution is selected by the help of the objective function which is created by considering the elements of the third group of inputs.

4.2.2. Outputs :

The outputs, that are obtained are summarized mainly in two groups;

Outputs obtained for each time interval:

-The allocation of the products to the production centers and amount of production realized

-The utilization of the production centers

-The starting and ending inventory levels of each product.

Outputs obtained for the entire planning horizon:

-The overall utilization of the production centers

-The total amount of production realized from each product.

With these outputs, the management expects from the productio in an efficient way so as to meet the interval target quantities. Some of the other outputs are:

-When are the raw-materials to be ordered and in what quantities?

How are the shortages removed, if there exists any?

-During each time interval, what are the starting and ending inventary levels for all parts at every stocking point within the system?

-During the entire planning horizon, what is the required work-force level? What is the allocation of the work-force among the periods and among the production centers? How efficient is the utilization of the work-force in the whole planning horizon?

4.2.3. Constraints:

The group of constraints related to the technology and the structure of the plant form the main part of total constraints. The explanations of all the constraints are given below:

-Demands must be satisfied: Since the aim of the management is to increase the utilization of the factory, production must be increased. The produced items must be sold and availibilities for further sales must be searched at any moment. For the time being, this production planning problem must force the demand to be satisfied. Namely, the minimum total production level for each product must be greater or equal to the demand of that product during the planning horizon.

-Each line is specified to produce one type of product. But there exists different sizes of each type of a product which are operated on the same line. So the plan to be designed determines for each line on which part to work during the whole planning horizon. At any time, on a specific line, only and only one part can be produced.

-Each line has a maximum available time to be utilized. This maximum time is calculated for each line by considering the regular maintenance, repair and regular down-times. The calculated times are deducted from the planning horizon and available times for each line are specified,

-Precedence relations: Different number of parts make up a final product. These parts must be produced according to a specific sequence of lines. In order for a part on a line to be produced the parts prior to that must have already been produced prior to that specific time interval. This must be true for all parts of the final products and for any time interval. This constraints are valid for all lines except the assembly centers.

-An assembly operation can only be realized if the necessary two or more parts to be assembled have already been produced prior to that time. This constraint must be repeated for all assembly centers, for all parts to be assembled on these centers and for all time intervals.

-As mentioned before, stocks are only available at assembly centers. Since the cycle time of production lines prior to assembly center and that of the assembly center itself are not equal and the working frequencies differ, there arises stocks of parts at these stock points. The difference in working frequencies of any two production centers determine the level of ending inventories at any time interval. By specifying the starting inventory of some parts, the working frequencies can be regulated exagenously.

4.2.4. Objective Function :

The definition of an objective function causes many discussions between the departments, by these discussions many objective function definitions arise. Either all the objective functions are considered and a multiobjective formulation is realized or all terms are defined in terms of one unit, for example in terms of cost, and a single objective function is obtained. In this case, all terms are defined by monetary units and the minimization of total cost is defined to be the actual objective function. The individual cost items are given below:

- Production cost: It is a variable cost, which is assumed to be linearly increasing by an increase in production level.
- Stock-holding cost: It is also a variable cost defined as the cost per period per unit.
- Surplus cost: Each unit produced exceeding the actual demand is stored in the warehouse of final products incurring a surplus cost.
- Idle time cost of the lines: For each time unit a line is not utilized, a cost is incurred. One of the aims, as mentioned before, is to increase the capacity utilization and this explains

the importance of this item of objective function.

If at any instance importance of an item increases, then a weight could be given to that item and, in this way, the best solution among the alternatives may be obtained.

In the sections above, facts and informations that must be considered while designing the model are explained. In the following sections, the steps involved in the evolution of the model are listed.

4.3. Mixed Integer Programming Method for Production Planning Problem of a Deterministic, Multiproduct and Multifacility Plant:

By analyzing the inputs, required outputs, the constraints and the characteristics of a mixed integer programming, it seams that this solution procedure fits best to this specific problem.

4.3.1. Definition of the Variables:

m= subscript denoting the endproducts.

n= subscript denoting the part of the endproduct.

j= subscript denoting the production center.

t= subscript denoting the time interval.

PL= set of the production lines.

AC= set of the assembly centers.

$X_{mnjt} = \begin{cases} 1, & \text{if the nth part of the mth endproduct is assigned to production center j for the time interval t.} \\ 0, & \text{otherwise.} \end{cases}$

Tl_{mnj} = the number of units of the nth part of the mth endproduct produced on line j during one time interval.

PC_{mn} = Production cost per period per unit of nth part of the mth product,

STC_{mn} = stockholding cost per period per unit of the nth part of the mth endproduct.

$TMAX_j$ = maximum available time of line j.

D_{mn} = demand of nth part of the mth endproduct.

ST_{mnt} = ending inventory level of the nth part of the mth endproduct for the time interval t.

CIT_j = cost of line j when it is idle during time interval t.

SC_{mn} = surplus cost per each extra unit of nth part of the mth endproduct.

4.3.2. Assumptions:

Without loosing the generality of the problem, the following assumptions are made to get a better overlook to the problem.

- The quality control department is not considered. It is assumed that inspection is realized immediately after the manufacturing of the parts and the time consumed during the inspection is included to the production time of that part.
- The painting shop is assumed to have infinite capacity and it causes no bottleneck throughout the plant.
- It is assumed that no stockout of raw-materials arises in the entire plant horizon; i.e. there will be no time the production

centers will be idle because there is no raw-material.

-Stock variables defined in the model only consider the inventory levels of the work-in-process items.The inventory levels of the end-product are not of importance initially;the warehouse for the end-products are at another place of the plant.

4.3.3. Mathematical formulation of mixed integer programming (MIP) model:

Objective function is minimization of the sum of the total costs combining the production,stockholding,surplus and idle time costs.

$$\min z = \sum_m \sum_n PC_{mn} \sum_j \sum_t X_{mnjt}\, Tl_{mnj} +$$
$$\sum_m \sum_n \sum_t STC_{mnt}\, ST_{mnt} +$$
$$\sum_m \sum_n SC_{mn} \left(D_{mn} - \sum_j \sum_t X_{mnjt}\, Tl_{mnj} \right) +$$
$$\sum_j CIT_j \left(TMAX_j - \sum_m \sum_n \sum_t X_{mnjt} \right)$$

subject to:

(1) Demand must be satisfied:

$$\sum_j \sum_t X_{mnjt}\, Tl_{mnj} \geqslant D_{mn} \qquad \text{for all m and n}$$

(2) At any time interval,each line can only be assigned to one part:

$$\sum_m \sum_n X_{mnjt} \leqslant 1 \qquad \text{for all j and t}$$

(3) In the entire planning horizon, each production center can only be used by its maximum availabe time :

$$\sum_{m}\sum_{n}\sum_{t} X_{mnjt} \leq TMAX_j \qquad \text{for all } j$$

(4) The assigning order of parts to production lines must be according to prededence relations :

$$X_{mnjt} \leq \sum_{s=1}^{t-1} X_{m,n-1,k,s} \qquad \text{for all } m,n,t/t=1,j\varepsilon PI$$

(5) The precedence relation at the assembly centers must also be taken into account :

$$2\, X_{mnjt} \leq \sum_{s=1}^{t} X_{m,n-1,k,s} + \sum_{s=1}^{t} X_{m,n-2,h,s}$$

$$\text{for all } m,n,t/t=1,j\varepsilon AC$$

where k and h are determined by specifying m and n-1 or m and n-2,respectively.

(6) The difference between the working cycles of the production lines and the assembly centers increases or decreases the stock levels.The number of parts that are assembled depends on the characteristics of the product.Ending inventory level of parts are given by the following constraints.The number of constraints for each endproduct is identical to the number of parts assembled at that assembly center:

$$ST_{m,n-i,t} = ST_{m,n-i,t-1} + \sum_{s=1}^{t-1} X_{m,n-i,k,s} - \sum_{s=1}^{t-1} X_{m,n,j,s}$$

for all $m,t,j \in AC$

i in the above equation denotes the number of parts making up the nth part of the mth endproduct.

(7) Nonnegativity constraints:

$$ST_{mnt} \leq 0 \qquad \text{for all } m,n,t$$

$$X_{mnjt} = \begin{cases} 0 \\ 1 \end{cases} \qquad \text{for all } m,n,j \text{ and } t$$

4.3.4. Size of the MIP model:

The number of products, their parts, production lines and the assembly centers, need of getting the results as soon as possible at any arbitrary moment, availibility of obtaining the effects that appear by decision changes immediately force the formulation of the model in such a way that it has to be solved by the computer. The reasonably short usage time indicates the efficiency of the model. The efficiency of the formulation, the size and characteristics (such as linearity or nonlinearity of constraints and/or objective function) of the model determine the usage time of the computer. From this point of view, the size of the model is analyzed in detail to judge whether to use it, i.e. whether it is economic and efficient.

-NUMBER OF VARIABLES:

1) X_{mnjt} is the main decision variable. It only takes the values of zero or one. Total number of this (0,1) variable is determined by the subscripts of m,n,j,t.

m takes values from 1 to 11.

n takes values depending on m. For each m,n differs at the widest range from 1 to 7.

j is a dependent subscript. By specifying m and n, j is already specified. j can take values from 1 to 10.

t takes values from 1 to 20 (working days in a month).

By analyses of the characteristics of the endproducts and the number of time intervals, the number of (0,1) variables, X_{mnjt}, comes out to be 1300.

2) ST_{mnt} is the second variable of the model. It takes integer values greater or equal to 0. This variable denotes the stock level of the work-in-process parts and there exist 1000 of them.

-NUMBER OF CONSTRAINTS:

1) First constraint is repeated for all m and n. Since in total the number of combinations of m and n is 65, there exist 65 of these constraints.

2) Since the second constraint is repeated for all j and t, there exist 200 of this second constraint.

3) The third constraint, giving an upper bound for using the product centers, is repeated for 10 times.

4) This constraint gives the precedence relation for the assembly

lines and there are 323 of them.

5) This constraint gives the precedence relation for the assembly centers.There are 532 of them.

6) The sixth constraint determines the stock levels of the parts and there exist 1000 of these.

-NUMBER OF OBJECTIVE FUNCTIONS:

The objective is to minimize the total costs and there is only one objective function in the start of formulation.

So in total,there exist

2300 variables

2130 constraints

1 objective function.

The details for obtaining these amounts are given in Tables 4,5 and 6 also in Figure 2 .

The above formulated and explained model is a mixed integer programming model,which will be solved on a computer.One of the best known and most effective solution technique for such problems having only a finite number of solutions is "Branch and Bound" method.The fundamental idea of this technique is to partition the set of all feasible solutions into several subsets and a lower bound (for minimizing objective function) is determined for each of the subsets.Then subsets which have a lower bound greater than the current upper bound are eliminated from further consideration. The solution time of Branch and Bound technique increases exponentially as the number of variables increases and linearly

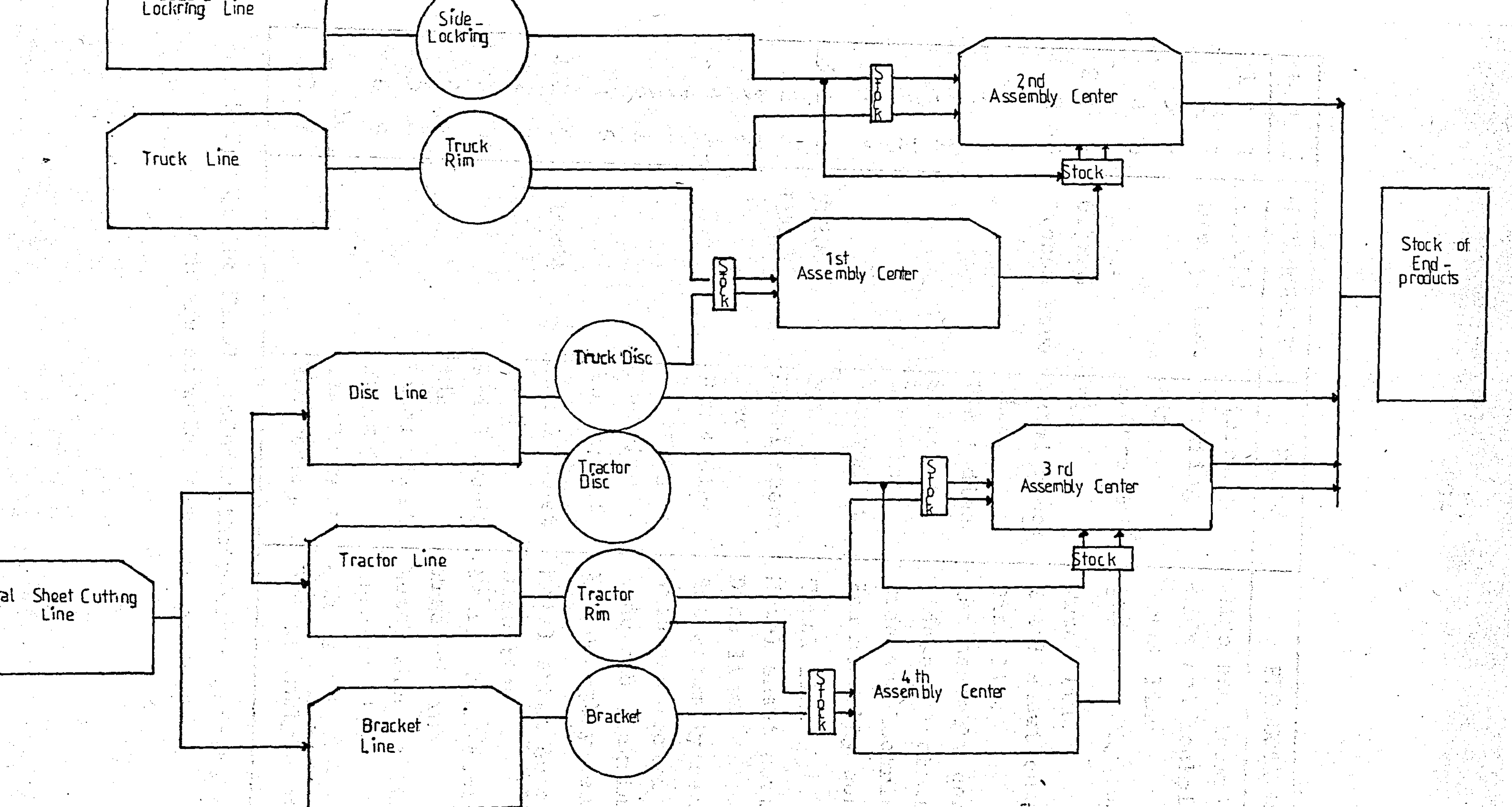

Figure 2 : Layout and Flow of Materials for MIP Formulation

m	n	j	ENDPRODUCTS	PARTS	PRODUCTION CENTER
1	1	1	6.5x20	Sidering	Side and lockring line
	2	1	Otosan Fiat	Lockring	Side and lockring line
	3	2		Rim	Truck line
	4	6		Endproduct	2nd assembly center
2	1	2	6.5x20	Rim	Truck line
	2	8	Otosan Ford	Metal sheet	Metal sheet cutting line
	3	3		Disc	Disc line
	4	5		Rim + disc	1st assembly center
	5	1		Sidering	Side and lockring line
	6	1		Lockring	Side and lockring line
	7	6		Endproduct	2nd assembly center
3	1	2	7x20	Rim	Truck line
	2	8	BMC	Metal sheet	Metal sheet cutting line
	3	3		Disc	Disc line
	4	5		Rim + disc	1st assembly center
	5	1		Sidering	Side and lockring line
	6	1		Lockring	Side and lockring line
	7	6		Endproduct	2nd assembly center
4	1	8	W10x24	Metal sheet	Metal sheet cutting line
	2	3	BMC	Disc	Disc line
	3	8		Metal sheet	Metal sheet cutting line
	4	4		Rim	Tractor line
	5	8		Metal sheet	Metal sheet cutting line
	6	9		Bracket	Bracket line
	7	10		Rim+Bracket	4th assembly center
	8	7		Endproduct	3rd assembly center

TABLE 4: The final products, their parts and the production centers on which they are produced

m	n	j	ENDPRODUCTS	PARTS	PRODUCTION CENTER
5	1	8	W13x28	Metal sheet	Metal sheet cutting line
	2	3	Fiat 480	Disc	Disc line
	3	8		Metal sheet	Metal sheet cutting line
	4	4		Rim	Tractor line
	5	8		Metal sheet	Metal sheet cutting line
	6	9		Bracket	Bracket line
	7	10		Rim+bracket	4th assembly center
	8	7		Endproduct	3rd assembly center
6	1	8	4x16	Metal sheet	Metal sheet cutting line
	2	3	BMC	Disc	Disc line
	3	8		Metal sheet	Metal sheet cutting line
	4	4		Rim	Tractor line
	5	7		Endproduct	3rd assembly center
7	1	8	3.62x19	Metal sheet	Metal sheet cutting line
	2	3	Fiat 480	Disc	Disc line
	3	8		Metal sheet	Metal sheet cutting line
	4	4		Rim	Tractor line
	5	7		Endproduct	3rd assembly center
8	1	8	11x28	Metal sheet	Metal sheet cutting line
	2	3	Burtrak	Disc	Disc line
	3	8		Metal sheet	Metal sheet cutting line
	4	4		Rim	Tractor line
	5	7		Endproduct	3rd assembly center
9	1	8	4x16	Metal sheet	Metal sheet cutting line
	2	3	Burtrak	Disc	Disc line
	3	8		Metal sheet	Metal sheet cutting line
	4	4		Rim	Tractor line
	5	7		Endproduct	3rd assembly center

TABLE 4: continued

m	n	j	ENDPRODUCTS	PARTS	PRODUCTION CENTER
10	1	8	J5x5x14	Metal sheet	Metal sheet cutting line
	2	3		Disc	Disc line
	3	8		Metal sheet	Metal sheet cutting line
	4	4		Rim	Tractor line
	5	7		Endproduct	3rd assembly center
11	1	8	13x28	Metal sheet	Metal sheet cutting line
	2	3		Disc	Disc line

TABLE 4: continued

CODE	NAME	OUTPUT
1	Side and lockring line	Side or lockring
2	Truck line	Truck's rim
3	Disc line	Disc
4	Tractor line	Tractor's rim
5	1st assembly center	Truck's rim + disc
6	2nd assembly center	(disc+side and lockring) or (disc+rim+side and lockring)
7	3rd assembly center	(rim+disc) or (rim+bracket+disc)
8	Metal sheet cutting line	Metal sheet
9	Bracket line	Bracket
10	4th assembly center	Rim + bracket

TABLE 5: The production centers and their outputs

ENDPRODUCTS	1	2	3	4	5	6	7	8	9	10	11
NUMBER OF PARTS	4	7	7	8	8	5	5	5	5	5	2

TABLE 6: Maximum number of parts of each endproduct

as the number of constraints increases which implies increase in occupation of the computer.The use of this formulation with 2300 variables and 2130 constraints on the computer will be inefficient and ineconomical as it is evident.

4.4. Linear Programming Method for Production Planning Problem of a Deterministic,Multiproduct,Multiperiod and Multifacility Plant :

Starting from the point of reducing the size,namelyddecreasin the number of variables and the number of constraints, some assumptions are taken,the definition of variables and parameters are altered in such a way that the model become more applicable. By doing these changes,the formulation become more efficient and economic to be used on computer,since the size is reduced and the output is better clarified.

4.4.1.Assumptions :

The aim of the assumptions below is to enable a general overlook to the problem,to prevent undesired details and still obtain the decided and foreseen output without loosing the generality of the problem.New assumptions are explained below, previous assumptions listed in Section 4.3.2. still hold.

-The two parts of truck's wheels are side and lockring.They are both processed on the same line.There exists only small differences (two or three operations) in the manufactoring of them.The use of one means unavoidably the use of the other part. It is assumed that they are produced together.The production time of them together is the sum of their production times.

-Bracket line and the 4th assembly center are subsidiary production centers.Bracket line,actually is one part of the metal sheet cutting line.The wastes of the metal sheet cutting line are used to produce the brackets,which enables the attachment of the disc of the tractor's back wheel to the rim.In the 4th assembly center,the bracket is welded to the rim;this actually is one of the operations realized in the 3rd assembly center. So it is assumed now,that the bracket line is included into the metal sheet cutting line,i.e. bracket line is ignored and the assembly operation realizing the welding of bracket to the rim is included into the tractor line.Hence,the output of tractor line is assumed to be rim + bracket or rim for back or front tractor's wheel,respectively;i.e. the 4th assembly center is also ignored.

-Although production planning and inventory systems are not to be seperated from each other,the inventory system uses the result of the former as input to design the optimum delivery times, delivered amounts and inventory levels.This mathematical model is to plan production;the inventory model will be designed after the results of the production planning are obtained.So the departments relevant to inventory system are ignored.It is assumed that metal sheet cutting line (including bracket line) belongs more to inventory part rather than to the production planning system,so it is ignored.Another reason why it is no more to be consædered is that it causes no bottleneck at all unless there is a defficiency in the raw-materials.Also,it has been assumed that there will be no stockout in raw-materials.

-The assumption that only stock levels of work-in-process elements are of interest still holds.

At this stage,the products,their parts,the lines,the assembly centers and the layout of the departments relevant to this study are given in Table 7,8 and 9,also Figure 3.

4.4.2. Definition of the variables:

-Subscripts:

m = Subscript denoting the endproduct ($m = 1,\ldots,11$)

n = Subscript denoting the parts of the endproducts.It differs from product to product.In the widest range n goes from 1 to 5.

j = Subscript denoting production centers. ($j = 1,\ldots,7$)

t = Subscript denoting the time intervals. ($t = 1,\ldots,20$)

-Decision variables:

X_{mnjt} = Variable denoting the percentage of t'th time interva production center j is allocated for production of part n of m'th endproduct.It takes values between 0 and 1 and it is continous.

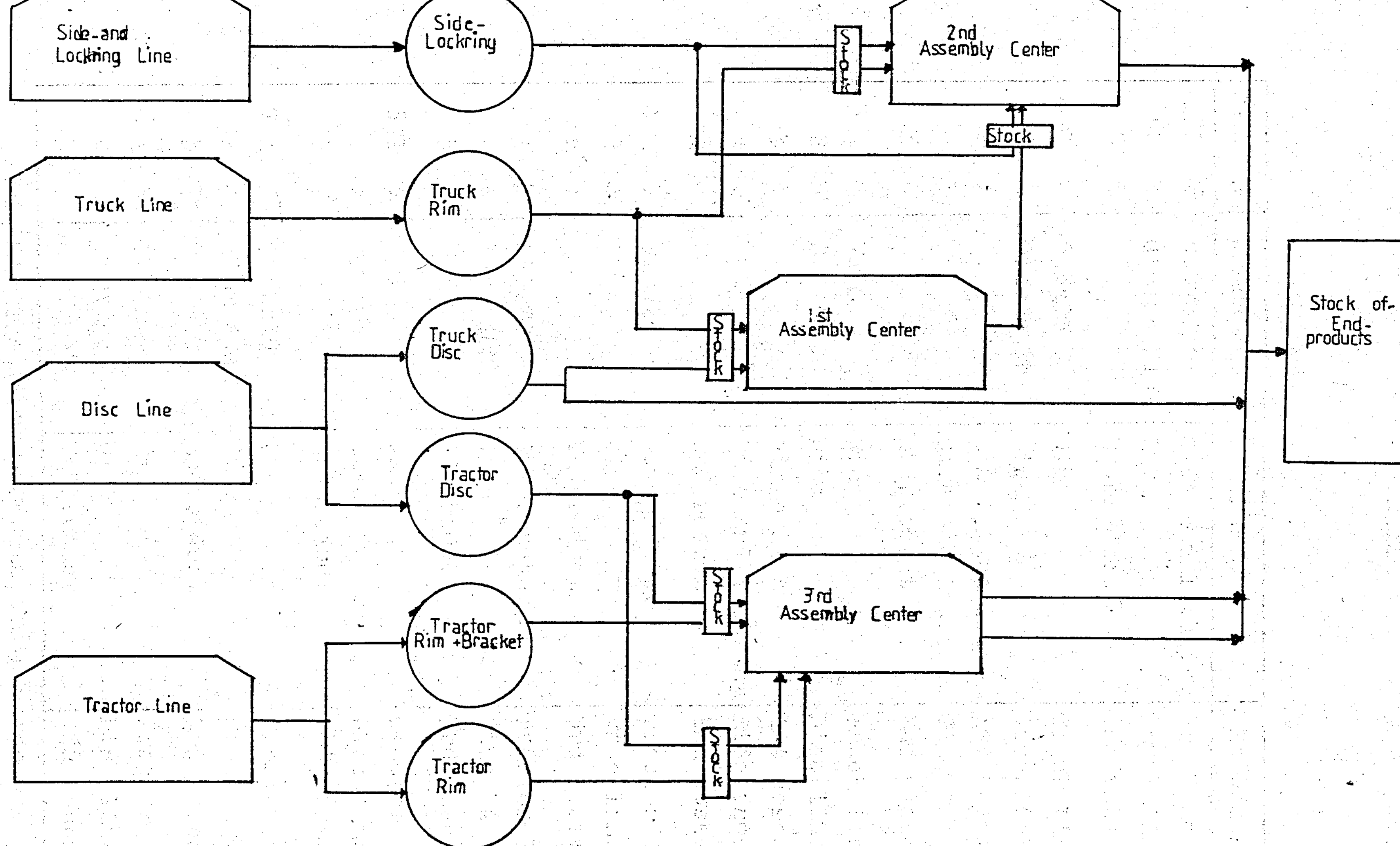

Figure 3 : Lay-out and Flow of Materials for LP Formulation

m	n	j	ENDPRODUCTS	PARTS	PRODUCTION CENTER
1	1	1	6.5x20	Side and lockring	Side and lockring line
	2	2	Otosan Fiat	Rim	Truck line
	3	6		Endproduct	2nd assembly center
2	1	2	6.5x20	Rim	Truck line
	2	3	Otosan Ford	Disc	Disc line
	3	5		Rim + disc	1st assembly center
	4	1		Side and lockring	Side and lockring line
	5	6		Endproduct	2nd assembly center
3	1	2	7x20	Rim	Truck line
	2	3	BMC	Disc	Disc line
	3	5		Rim + disc	Ist assembly center
	4	1		Side and lockring	Side and lockring line
	5	6		Endproduct	2nd assembly center
4	1	3	W10x24	Rim+bracket	Tractor line
	2	4	BMC	Disc	Disc line
	3	7		Endproduct	3rd assembly center
5	1	3	W13x28	Rim+bracket	Tractor line
	2	4	Fiat 480	Disc	Disc line
	3	7		Endproduct	3rd assembly center
6	1	3	4x16	Rim	Tractor line
	2	4	BMC	Disc	Disc line
	3	7		Endproduct	3rd assembly center
7	1	3	3.62x19	Rim	Tractor line
	2	4	Fiat 480	Disc	Disc line
	3	7		Endproduct	3rd assembly center
8	1	3	11x28	Rim	Tractor line
	2	4	Burtrak	Disc	Disc line
	3	7		Endproduct	3rd assembly center

TABLE 7: The final products, their parts and production centers on which they are produced for LP model

m n j	ENDPRODUCTS	PARTS	PRODUCTION CENTER
9 1 3	4x16	Rim	Tractor line
2 4	Burtrak	Disc	Disc line
3 7		Endproduct	3rd assembly center
10 1 3	J5x5x14	Rim	Tractor line
2 4		Disc	Disc line
3 7		Endproduct	3rd assembly center
11 1 3	13x28	Disc	Disc line

TABLE 7: continued

CODE	NAME	OUTPUT
1	Side and lockring line	Side and lockring
2	Truck line	Truck's rim
3	Disc line	Disc
4	Tractor line	Tractor's (rim) or (rim + bracket)
5	1st assembly center	Truck's rim + disc
6	2nd assembly center	(Disc + side and lockring) (Disc+rim+side and lockrin
7	3rd assembly center	(Rim + disc) or (rim+bracket+disc)

TABLE 8: The production centers and their outputs

ENDPRODUCTS	1	2	3	4	5	6	7	8	9	10	11
NUMBER OF PARTS	3	5	5	3	3	3	3	3	3	3	1

TABLE 9: Maximum number of parts of each endproduct

ST_{mnt} = Variable denoting the ending inventory level of part n belonging to m'th endproduct in time interval t. It takes values greater than 0.

-Input data:

D_{mn} = Data denoting the demand of n'th part of m'th endproduct. The values of demand levels of the parts are identical to demand level of their endproduct.

Tl_{mn} = Data denoting the number of n'th part of m'th endproduct if it is produced the whole time interval.

PC_{mn} = Cost of producing one unit of n'th part of m'th endproduct. The costs are identical for all time intervals.

STC_{mn} = Cost of holding one unit of n'th part of m'th endproduct in stock for one period.

SC_{mn} = Cost of each extra unit of n'th part of m'th endproduct.

CIT_j = Cost incurred when production center j is left idle for one time interval.

$TMAX_j$ = Maximum time production center j can be utilized in the entire planning horizon.

MN = Total number of items produced in the whole plant. It has the value of 35.

ML = Set denoting the assembly centers consisting of the elements $ML = \{5,6,7\}$.

PL = Set denoting the production lines consisting of the elements PL = $\{1,2,3,4\}$.

H(M,N)= Set denoting the production center j on which only and only the n'th part of m'th endproduct can be be produced.

A(M,J)= Set denoting the part n belonging to endproduct m and is produced on production center j.

NN(M) = Set giving the maximum number of parts of m'th endproduct.

Detailed information about all above mentioned variables, parameters and data is given in Appendix 1.

4.4.3. Mathematical formulation of LP model:

The objective function is minimization of total cost as in the former formulation.It contains such elements as production, stockholding,surplus and idle time costs which may be summerized as:

$$\min z = \sum_{m=1}^{11} \sum_{n=1}^{NN(m)} PC_{mn} \sum_{j=1}^{7} \sum_{t=1}^{20} X_{mnjt}\, T1_{mn} +$$

$$\sum_{m=1}^{11} \sum_{n=1}^{NN(m)} \sum_{t=1}^{20} STC_{mn}\, ST_{mnt} +$$

$$\sum_{m=1}^{11} \sum_{n=1}^{NN(m)} SC_{mn} \left(\sum_{j=1}^{7} \sum_{t=1}^{20} X_{mnjt}\, T1_{mn} - D_{mn} \right) +$$

$$\sum_{j=1}^{7} CIT_{j} \left(TMAX_{j} - \sum_{m=1}^{11} \sum_{n=1}^{NN(m)} \sum_{t=1}^{20} X_{mnjt} \right)$$

subject to:

(1)The first and the most important constraint is the satisfactio of demand of all products which implies the satisfaction of demand of all parts making up final products.This constraint repeats itself for 35 items produced in the plant during the entire planning horizon:

$$\sum_{j=1}^{7} \sum_{t=1}^{20} X_{mnjt} \; TI_{mn} \geq D_{mn} \qquad \text{for } m = 1,\ldots,11 \quad n = 1,\ldots,NN(m)$$

(2)At any time interval,each line can be at most 100 % utilized, i.e. some of time segments during which any one part that can be operated on a specific production center cannot be greater than 1.

$$\sum_{m=1}^{11} \sum_{n=1}^{NN(m)} X_{mnjt} \leq 1 \qquad \text{for } j= 1,\ldots,7 \quad t= 1,\ldots,20$$

As the ranges of subscripts j and t indicate,this constraint repeats itself 140 times.

(3)Stock and production balance for the assembly centers is explained in the following constraints.Production at any of the assembly centers can only be realized if the necessary parts do exist in sufficient amounts.As input the parts of all the endproducts which are produced on each of the assembl centers are given,namely n=A(m,j) where m ranges from 1 to 11 and j is an element of the set of ML. Knowing the assembly center j, the endproduct m and its part n being assembled, the necessary two parts, making up n, are the parts (n - 1) and (n - 2). It is also known how many of the assembled items (n'th part of m'th product) can be obtained in any certain time increment if the assembly center, j ML, is allocated for that part, namely by Tl_{mn} x percentage of that time interva

Under these conditions;

a) If there exists necessary amounts of (n - 1)st and (n - 2)nd parts in stock in the beginning of the time interval t, that specific assembly center can be fully assigned to production of n'th part of the m'th endproduct.

b) If there does not exist enough of either parts, the stock levels of parts (n - 1) and (n - 2) and the difference in the production rates of the assembly center and production lines in question determine the percentage of time for allocation to produce the n'th part of the m'th endproduct.

$$ST_{m,n-1,t} = ST_{m,n-1,t-1} + X_{m,n-1,k,t}\, Tl_{m,n-1} - X_{mnjt}\, Tl_{mn}$$

$$ST_{m,n-2,t} = ST_{m,n-2,t-1} + X_{m,n-2,h,t}\, Tl_{m,n-2} - X_{mnjt}\, Tl_{mn}$$

$$\text{for}\quad \begin{aligned} m &= 1,\ldots,11 \\ j &\in ML \\ n &= A(m,j) \\ t &= 1,\ldots,20 \\ k &= H(m,n-1) \\ h &= H(m,n-2) \end{aligned}$$

This constraint repeats itself for all work-in-process parts existing in the plant for all time intervals in the entire planning horizon. Hence, there are 480 of them.

(4) Nonnegativity constraints are given by:

$$0 \leq X_{mnjt} \leq 1 \qquad \text{for } m = 1,\ldots,11;\ n = 1,\ldots,NN(m);\ j = H(m,n);\ t = 1,\ldots,20$$

$$ST_{mnt} \leq 0 \qquad \text{for } m = 1,\ldots,11;\ n = 1,\ldots,NN(m);\ t = 1,\ldots,20$$

4.4.4. <u>Size of the LP problem:</u>

-NUMBER OF VARIABLES:

700 of X_{mnjt}

480 of ST_{mnt}

1180 continous nonnegative variables

-NUMBER OF CONSTRAINTS:

35 of 1st constraints

140 of 2nd constraints

480 of 3rd constraints

---

655 linear constraints

-NUMBER OF OBJECTIVE FUNCTIONS:

1 linear objective function
to be minimized

The use of this model is now more efficient and economic, since the size is more compact, the variables are nonnegative and continous and the type of the constraints does not differ much. So the steps in designing the final model is explained.

5. SOLUTION PROCEDURE

5.1. Introduction:

One of the points which was considered during the design phase of modelling was that the solution will be obtained by a computer,namely UNIVAC 1106.In order to obtain efficient, practical and economic use of the computer,some additional efforts are done such as reducing the size of the problem, eliminating the integer variables and insisting on linearity.

There exists many algorithms to solve LP models on computer. FMPS (Functional Mathematical Programming Systems) is one of those methods.Due to easiness of manipulation,efficient allocatio of memory space and an acceptable running time FMPS was decided to be used for this study.FMPS requires the input data in a very specific form.In order to avoid the high consumption of data cards,to enable an easy way of updating input and to obtain a general form of inputs to this specific and other similar problems,an INPUT CREATION PROGRAM was prepared.Using the output of this program,FMPS was run.Finally the results of the FMPS routine was translated into a form on which the answers of the questions specified by the management could be found immediately

In order to prepare the decided output formusing the raw data gathered from the factory or obtained by means of some calculations the following programs were used:

-INPUT CREATION PROGRAM

-FMPS PROGRAM

-RESULTS READING PROGRAM

-OUTPUT PROGRAM

In the following sections detailed information such as input,formulation and output are clarified.

5.2. INPUT CREATION PROGRAM:

The first program to be prepared is an alternative to define the input data for FMPS.The conventional way is to enter the data by means of cards containing 80 columns each according to the specific format statements in the program.By this conventional method,about 5000 data cards are consumed.To update any value of an already specified input requires new data cards each time. To economize the consumption of data cards,to enable ease of updating and,most importantly,to generalize the specific problem, an INPUT CREATION PROGRAM was prepared.The first objective of this program is to create a file on which the input data is in the form required by FMPS and the second objective is to check whether a feasible solution is possible with the givens by comparing the capacity of the plant with the production to be satisfied.If the total amount of production exceeds the capacity of the plant,no further calculations take place and the reason of this termination is printed out.

The following information is specified by this program:

-A row number is given to the objective function and to each constraint.

-The characteristics of each row is specified,whether it is an equality or inequality.

-Which variable is in which row and the coefficient in that row is specified.

-The right hand side values of each row are specified.

-The upper and/or lower bound for each variable are specified.

5.2.1. <u>Input to the INPUT CREATION PROGRAM:</u>

Afore mentioned inputs are the raw data obtained from the factory.There are also some which are obtained after some assumptions and some calculations.The way they are obtained and their values are further explained in Chapter 6.

The total number of products,their maximum number of parts, the number and capacities of the production lines and assembly centers,the production sequence of the parts on these production facilities ,the length of the planning horizon,coefficients of the objective function and the constraints of the variables,the scheduled and unscheduled demand and the initial stock levels of the parts and endproducts are the main inputs specified for this program.Furthermore there arise the need of specification of other variables denoting the number of constraints for some of the variables,basicly for the variables denoting the assembly operations.Since there is not a single regularity how the constrai for these variables change, they must be defined exagenously. Beside the assembly operations, initial stock levels have the same problem, hence, they are also defined exagenously.

5.2.2. Main sections and output of INPUT CREATİON PROGRAM:

There are mainly four sections of this program:

- -Rows section
- -Columns section
- -Right hand side section
- -Bounds section

ROWS SECTION

The first section of INPUT CREATİON PROGRAM specifies the types of each row, types of the objective function and all constraints. The specification of the types are defined by:

N : no constraint (objective function)

G : a greater than or equality constraint

E : an equality constraint

L : a less than or equality constraint

In this section, the first identification of the problem, namely a row-wise identification, is realized.

COLUMNS SECTION

In the second section, a column-wise identification is done. The actual values of the matrix elements in terms of column vectors are defined. The matrix elements must be specified by columns, that is, when one variable is given, all other nonzero elements of that variable in other rows must be entered before another variable is mentioned.Zero entries should not be specified.In order to write the coefficient values either "E" or "F" format can be used.

RIGHT HAND SIDES SECTION

In this third section,the right hand side values of the constraints are specified.The sequence of rows defined in the first section must be identical to the sequence of values of right hand sides.Zero entries should not be specified.The first 35 constraints force the model to satisfy demands of parts and endproducts.Therefore the right hand sides are identical to D(m,n) respectively.The 140 constraints after the 35 constraints force the utilization of the production facilities to be 100 % or less,so the right hand sides of these constraints are 1.The rest of the constraints are equalities,therefore the rest of the right hand side vector is set to be equal to zero.

BOUNDS SECTION

The fourth and last section is an optional section.This section imposes limits on the values that variables can take. If none of the variables are bounded,this section is omitted, in FMPS this will be translated in a way that the limits are between 0 and + .The sequence of identification of bounds for columns must match the sequence of column section.Lower or upper bounds,fixed value and free variable are the types that can be used in this section.

In the specific problem defined above,all X_{mnjt} variables have an upper boud of 1.The program calculates the lower bounds according to the unscheduled demand.The reason for lower bounds is that it must be satisfied as soon as possible.The mentioned demand is compared with the capacity of production centers on which the relevant part of the required endproduct can be produced. This comparison specifies how many time intervals should that variable have a lower bound.

At the end of all of these four sections,there must be an indication for FMPS showing that the program is finished.

The program prepares all information explained above.The output of it is a file on computer which will be the input file for FMPS.The name of this file is INCREPR.

5.3. FMPS PROGRAM:

FMPS is the actual program which solves the LP model.The input for FMPS as mentioned before is prepared by the INPUT CREATION PROGRAM and is read from the file INCREPR.After the reading of the input data solution of the problem starts.If the problem is solvable,i.e. if there exists a feasible solution, if the problem is bounded or if the inverse of the matrix is available,the solution will be prepared.The solution is loaded to a file,called SOLFİLE.

SOLFILE is prepared in three sections:

-Identifier Section

-Section 1-Rows

-Section 2-Columns

IDENTIFIER SECTION

It gives the operating mode,optimal objective function value and objective function weight (maximization or minimization) and how many iterations have been done to obtain the optimal solution.

SECTION 1 - ROWS

It contains information on the selected rows in the matrix. The report contains ten columns of information.Table 10 describes each of the 10 columns of this part.

SECTION 2 - COLUMNS

It contains information on the selected columns in the matrix.The report contains eight columns of information,Table 11 describes each of the 8 columns of this part.

There does not exist any exagenous input data to be fed to FMPS.The file created by the first program,INCREPR,is sufficient.

5.3.1. Run Stream and Data Deck Organization:

Data deck organization is as follows:

```
&RUN,E      INAL,111-15-202,TEZ          .CANAN INAL.
&ASG,AX     INCREPR.
&USE        7,INCREPR.
&DELETE,C   SOLFILE.
&ASG,U      SOLFILE.
&USE        11,SOLFILE.
&ASG,AX     QFMPSxFMPS.
&XQT        QFMPSxFMPS.MEDFMPSABS
```

COLUMN	HEADING	DESCRIPTION OF INFORMATION IN COLUMN
1	NUMBER	Row number (including objective function row)
2	ROW	Row name
3	AT	Status of the row BS : Slack variable of that row is basic and feasible xx : Slack variable of that row is basic and infeasible EQ : Row is initially in equality form and artificial slack variable added to that row is nonbasic UL : Row at upper limit LL : Row at lower limit
4	ACTIVITY	The activity of the row.For type rows,the row activity = RHS + slack activity;for or = type rows,row activity = RHS - slack activity
5	SLACK ACTIVITY	The value of the slack activity added to that row
6	LOWER LIMIT	Lowest value that row activity may be
7	UPPER LIMIT	Highest value that row activity may be
8	DUAL ACTIVITY	Value of dual variable corresponding to that row
9	INPUT COST	The objective function coefficient (in initial tableau) of slack variable of that row (slack price,if specified during input)
10	REDUCED COST	Coefficient of slack variable corresponding to that row at optimal tableau (shadow price)

TABLE 10 : SECTION 1 - ROWS output description

COLUMN	HEADING	DESCRIPTION OF INFORMATION IN COLUMN
1	NAMUBER	Variable number
2	NAME	The name of the variable
3	AT	Status of the variable BS : Variable in basis and feasible xx : Variable in basis and infeasible FR : Variable basic and free EQ[a]: Variable fixed and nonbasic UL[a]: Variable nonbasic at upper limit LL[a]: Variable nonbasic at lower limit AO[b]: Alternative optima
4	ACTIVITY	The optimal value of the variable
5	INPUT COST	The objective function coefficient of that variable in the original tableau
6	LOWER LİMİT	The given lower limit of that variable
7	UPPER LIMIT	The given upper limit of that variable
8	REDUCED COST	The objective function coefficient of that variable at the optimal tableau

TABLE 11 : SECTİON 2 - COLUMNS output description

a : Note that although that variable may have nonzero value,it is considered as nonbasic.This is because of upper bound technique utilized by FMPS.

b : The objective function coefficient of that nonbasic variable is zero at the optimal tableau.FMPS gives only one of the optimal solutions.

```
C       Define page heading
         TITLE FMPS-LP MODE xxxxxPRODUCTION PLANNING PROBLEMxxxxx
C       Setting FMPS operating mode as linear programming mode
         CALL ENTER(LP)
C       Specify the mode the input is fed into by INCREPR file
         CALL ATTACH('INCREPR',7,CARD,INONLY)
C       Specify the mode the output is desired,to file SOLFILE
         CALL ATTACH('SOLFILE',11,FORTRAN,NEW)
C       Initialize no feasible solution interrupt.If this occurs
        during optimization phase,control passes to statement
        number 100.
         ASSIGN 100 TO KNFS
C       Initialize unbounded solution interrupt
         ASSIGN 200 TO KUBS
C       Initialize matrix inversion interrupt.Whenever matrix
        inversion is required during optimization,control passes
        to statement number 300
         ASSIGN 300 TO KINV
C       Initialize name of the problem as PROPLAN
         APBNAME = 'PROPLAN'
C       Initialize name of data deck as WHEEL
         ADATA = 'WHEEL'
C       Initialize name of the objective function as OBJ
         AOBJ = 'OBJ'
C       Initialize name of right hand side vector as RHS
         ARHS = 'RHS'
C       Initialize name of bound row as BOUND,if bounds on variables
        will be specified
         ABOUND = 'BOUND'
C       Initialize the output format as E
         AFORMAT = 'E'
C       Specify the type of objective function (minus 1 for
        maximization,plus 1 for minimization)
```

```
      FOBJWT = 1.0
C     Letting FMPS to load LP matrix from file INCREPR which is
      assigned
      CALL INPUT(FILE,'INCREPR')
C     Display the original matrix in tableau form
      CALL OUTPUT(BYROWS)
C     Solve the LP matrix
      CALL OPTIMIZE
C     Display the solution to a file,SOLFILE
      CALL SOLUTION(FILE,'SOLFILE')
      STOP
C     Whenever inversion interrupt occurs during optimization
      phase,print the current matrix and return to optimize
  300 CALL INVERT
      RETURN
C     When no feasible solution occurs,display the infeasible
      rows and terminate
  100 CALL OUTPUT(BYROWS,ROWS,LISTI)
      STOP
C     When unbounded solution occurs,display the unbounded
      column and terminate
  200 CALL OUTPUT(BYCOLS,COLS,LISTU)
      STOP
      END
&FIN
```

5.4. RESULTS READING PROGRAM:

As mentioned in the last section,the output of FMPS contains 3sections.The answers to questions

-when to produce which part

-what should the amount of that item be

-how is the utilization of the production centers

-what are the starting and ending inventories of the work-in-process

-total production of the endproducts

-day by day situation of the plant

are obtained by the help of SECTION 2,which gives the optimal values of the variables,X_{mnjt} and ST_{mnt}.However,the optimal values,by themselves are not sufficient to answer the specified information.Therefore,two programs are prepared.The first takes the SECTION 2-COLUNMS part of FMPS solution and load these values to a file named SOLFILE1.This file is in the form :

Variable ==> Optimal value of the variable

The second program written checks the values which are in form X.XXXXXX ± YYY and converts them into X.XXXXXX ± YY .So the final values from FMPS are carried to a file named OUTPUTPR containing all the information required to derive the answers.

5.5. OUTPUT PROGRAM:

This program aims to prepare the output in the form desired and set by the management.It uses the OUTPUTPR file,the result of the previous program and data specified exageneusly.The required input for this program may be summerized as follows:

-the file OUTPUTPR containing the variables and their optimal values,which are obtained by FMPS package and reorganized by the other program.

-the set of inputs,which is identical to those specified in the first program,contains the products,their parts,production facilities,length of planning horizon,the precedence relations of facilities,demand,unscheduled demand and initial stock levels. -The set of input,which is defined for this specific program,is the specification of the endproducts and their parts,namely the names of all the parts making up the final products.

Output of this program is realized in three sections.The first is the allocation of the production facilities by the parts in each time interval,the second is the total production realized in the entire planning horizon and the last is the utilization of the production facilities in the whole planning horizon.

Detailed information about the variables denoting the input, their values and the way of their specification is given in Appendix 1;information about the form of the output is given in Appendix 2.

6. DATA COLLECTION

6.1. Introduction :

The aim of this study, which includes the analysis of the plant, the formulation of the required output and the design of the model that fits to the case, is to obtain an applicable solution method for a real plant. Therefore, the importance of the data collection procedure is evident. In this chapter, the collection of the data and the adaptation of the raw data to the model are explained.

6.2 Procedure :

In order to obtain a complete solution procedure to the existing problems of the plant different types of input data are required. The first group contains the elements that build up the actual model; examples for the first group are objective function, the coefficients and bounds of all the variables and right hand side values of the constraints. The second group contains the ones that must be received continuously to see any instantaneous situation of the plant and analyze the future with the given con conditions at that point of time. The values of input are either obtained directly, as the raw data, or calculated after some assumptions. The data related to the technology of the plant which belongs to the first group is taken directly. Details of the first group may be given as :

-H(m,n), the matrix denoting the production facility j on which the n'th part of the m'th endproduct is produced. (Table 12)

-A(m,j), the matrix denoting the part n, which belongs to m'th endproduct and is produced on the production center j. (Table 13)

N \ M	1	2	3	4	5	6	7	8	9	10	11
1	1	2	2	3	3	3	3	3	3	3	3
2	2	3	3	4	4	4	4	4	4	4	0
3	6	5	5	7	7	7	7	7	7	7	0
4	0	1	1	0	0	0	0	0	0	0	0
5	0	6	6	0	0	0	0	0	0	0	0

TABLE 12 : H(m,n) matrix

J \ M	1	2	3	4	5	6	7	8	9	10	11
1	1	4	4	0	0	0	0	0	0	0	0
2	2	1	1	0	0	0	0	0	0	0	0
3	0	2	2	1	1	1	1	1	1	1	1
4	0	0	0	2	2	2	2	2	2	2	0
5	0	3	3	0	0	0	0	0	0	0	0
6	3	5	5	0	0	0	0	0	0	0	0
7	0	0	0	3	3	3	3	3	3	3	0

TABLE 13 : A(m,j) matrix

A(m,j) and H(m,n) are two matrices.They both are obtained by analyzing the flow of materials,parts and endproducts in the plant. The 0 values in the matrices meam that there does not exist any relation between the two entries of the matrices.

-IC(j),the array denotes the capacities of the production facilities.The elements give the number of parts to be produced on the respective center during one period of time.(Table 14) The determination of the capacities is done in the following manner: There exists many different machines making up each production center.From some of these machines there are more than one.Each machine has a specific capacity,number of items produced per day.The capacity of the machine with the minimum rate of production is choosen as the capacity of the production center.The capacity is desired as the number of units per day, thus, there is no need for any transformation.

-The demand of the endproduct is perhaps the most important input to the model.The values are obtained from the sales department. After market research is done and some contracts are realized, the demand,i.e. the minimum production level of the endproducts are determined.The forecasts are done on annual basis.It is assumed that there do not exist any fluctuations among the months within a year,thus demand for a month is just 1/12 of annual values which are given in Table 15.

The complicated part of the data collection procedure is the calculation of the cost coefficients.The production cost per each

PRODUCTION CENTER	CAPACITY (number of units/day)	COST OF IDLE TIME OF PROD. CENTER (TL/day)
1. PRODUCTION LINES		
Side and Locking Line	1800	31484
Truck Line	880	94834
Disc Line	840	29090
Tractor Line	560	46718
2. ASSEMBLY CENTERS		
1st Assembly Center	880	4834
2nd Assembly Center	880	14504
3rd Assembly Center	560	4834

TABLE 14: Capacity and Idle Time Costs of the Production Facilities

ENDPRODUCTS	CODE	DEMAND units/month
1. Truck wheel		
6.5x20 O.Fiat	1	167
6.5x20 O.Ford	2	2917
7x20 BMC	3	1250
2. Tractor wheel		
4x16 BMC	6	417
3.62x19 F.480	7	833
11x28 Burtrak	8	417
4x16 Burtrak	9	417
j5x5x14	10	4167
W10x24 BMC	4	417
W13x28 F.480	5	833
3. Special parts		
13x28	11	1250

TABLE 15: Demands of the endproducts

unit manufactured,stock holding cost per unit per time interval, the surplus cost per each unit exceeding the sales capacity of that planning horizon and cost incurred for every time interval in which a production center is left idle are the actual cost coefficients needed for the formulation.

The first cost item,incurred every time a unit is manufactured is a variable cost.It increases linearly as the number of produced items increases.The components of the production cost are the followings:

-Raw-material cost which is the amount required of that raw-material per item times the unit cost of respective raw-materials.Bill of the materials and unit costs are given in detail in Table 16 and Table 17,respectively.

-The depreciation cost of the production centers is also included in the production costs.The machines making up the production center have a depreciation cost which is allocated in percentages to the units produced on that machines.

-The overhead costs are also taken into account in the production costs.

The production cost of a unit does not include any item that is not directly related to production such as fixed costs,labour costs and financial costs (tax,interest and return of capital).

The stock holding cost is considered in two parts:

-The cost incurred every time a work-in-process element is left in stock for one or more time intervals,called the stock holding cost.

-The cost incurred every time an endproduct exceeding the sales capacity is stored in the warehouse for final products,called the surplus cost.

Raw-material expences are about 40% of production costs.The raw-material costs,on the other hand,are about 65% of the production costs.The difference,namely 25%,should be the stock holding costs of these raw-materials.The specification of surplus cost follows the decisions taken by the management.To have a high level of

FINAL PRODUCTS -MATERIAL		6.5x20 O.Fiat	6.5x20 O.Ford	7x20 BMC	W10x24 BMC	W13x28 F.480	4x16 BMC	3.62x19 F.480	11x28 Burtrak	4x16 Burtrak	J5x5x14	13x28
ofile	kg	33.24	33.28	33.81					54.40	12.44		
ts,Bolts	unit				4.00				6.00			
re Electrod	kg		0.70	0.28			0.56	0.54	0.27	0.56	0.58	
eventing gas	kg		0.01	0.05			0.10	0.10		0.10	0.32	
imer	kg	0.36	0.61	0.65	0.85		0.33		0.96	0.33	0.36	
nal Paint	kg	0.30	0.50	0.53	0.70		0.28		0.80	0.28	0.30	
emicals	kg	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	
lled Sheet	kg				16.83	29.10	7.44	6.06			13.88	
eet 9mm	kg				20.03		6.41	8.92				29.92
eet 6.5mm	kg				1.67	2.31						
eet 11mm	kg		20.91	19.23								
tal Weight	kg	33.90	56.00	54.60	40.10	31.40	15.00	15.60	56.40	13.70	15.50	29.92

TABLE 16: Bill of Materials

RAW-MATERIAL	UNIT	UNIT PRICE (TL/UNIT)
Profile	kg	70.oo
Nuts,bolts	unit	15.oo
Wire electrod	kg	250.oo
Preventing gas	kg	180.oo
Primer	kg	195.oo
Final painting	kg	290.oo
Chemicals	kg	160.oo
Rolled sheet	kg	48.0o
Sheet 9mm	kg	53.oo
Sheet 6.5mm	kg	55.oo
Sheet 11mm	kg	55.oo

TABLE 17: Unit prices of the raw-materials (1980)

surplus inventory is undesired,since this event causes some unplanned expenditures such as holding some amount of unusable tied up capital and extra cost of preventing the endproducts frpm any damages.But to assign a monetary term which exactly specifies all above mentioned elements is nearly impossible. For these reasons,the surplus cost is assumed to be 1 for all endproducts.By this way it is desired to prevent unlimited increase in stock level,although it is known that this value does not reflect the real cost incurred.

The values of production,stock holding and surplus costs are given in detail in Table 18.

The last cost coefficient is the cost incurred when a peoduction center remains unallocated.Value for this cost coefficient is calculated in three steps:

-Each production center can produce parts belonging to some of the endproducts.Idle time cost of a production center changes

CODE	ENDPRODUCTS	PARTS	COSTS Pro-duc-tion(a)	Stock-hol -ding(b)	Sur-plus (b)
1	6.5x20	Side&lockring	773	193	-
	O.Fiat	Rim	1995	499	-
		Endproduct	452	-	1
2	6.5x20	Rim	1995	499	-
	O.Ford	Disc	1316	329	-
		Rim+disc	731	183	-
		Side&lockring	773	193	-
		Endproduct	244	-	1
3	7x20	Rim	2140	535	-
	BMC	Disc	1222	306	-
		Rim+disc	707	177	-
		Side&lockring	668	167	-
		Endproduct	236	-	1
6	4x16	Rim	620	155	-
	BMC	Disc	487	122	-
		Endproduct	617	-	1
7	3.62x19	Rim	553	139	-
	F.480	Disc	623	156	-
		Endproduct	469	-	1
8	11x28	Rim	1925	481	-
	Burtrak	Disc	2297	574	-
		Endproduct	933	-	1
9	4x16	Rim	778	195	-
	Burtrak	Disc	507	127	-
		Endproduct	558	-	1

TABLE J8: Production,stockholding,surplus costs of all products

(a) TL / unit

(b) TL / unit / day

CODE	ENDPRODUCTS	PARTS	COSTS		
			Pro-duc-tion(a)	Stock hol-ding(b)	Sur-plus (b)
10	J5x5x14	Rim	496	124	-
		Disc	591	148	-
		Endproduct	719	-	1
4	W10x24	Rim+bracket	1076	269	-
	BMC	Disc	1299	325	-
		Endproduct	834	-	1
5	W13x28	Rim+bracket	1671	418	-
	F.480	Disc	282	71	-
		Endproduct	171	-	1
11	13x28	Disc	1846	-	1

TABLE 18: continued

from endproduct to endproduct, since the demand levels, which are assumed to be the minimum production levels, differ from type to type. From the matrix A(m,j) the nonzero elements specify the number of parts of the endproducts to be produced on each production center.

-The second step in determination of idle time costs is to calculate the idle time costs depending on the endproducts, since the allocated capacity of each center changes from product to product:

$$\text{Idle time cost of j producing part n of m'th endproduct} = \left\{ \frac{\text{Fixed cost incurred}}{\text{Allocated capacity of j producing n of m}} - \frac{\text{Fixed cost incurred}}{\text{Total capacity of production center j}} \right\}$$

where fixed cost incurred (TL/planning horizon) consists of the maintenance and deprèciation costs of the machines making up the centers; total capacity of the production center j denotes total number of units producable per planning horizon and the allocated capacity of center j producing part n of m'th endproduct denotes the number of units to be produced in the planning horizon, which is equal to the demand level of that endproduct. This cost is calculated for all production centers and for all (m,n) pairs produced on that production center.

-The third step is the actual calculation of this cost coefficient. Every time a unit is produced on the production center j, a corresponding idle time cost per unit is incurred. This can be formulated as below:

$$\text{Idle time cost of production center j/unit} = \sum_{m \in A(m,j)} \left\{ \begin{matrix}\text{Idle time cost of}\\ \text{j producing part}\\ \text{n of m'th endprd.}\end{matrix} \times \frac{\text{Amount produced of m(n)}}{\text{Total production on j}} \right\}$$

To express idle time cost per each time interval, namely per each day, the weighted average above is multiplied by the average production levels per day.

$$\text{Idle time cost of j per day} = \left\{ \begin{matrix}\text{Idle time cost}\\ \text{of j per unit}\end{matrix} \times \frac{\text{Average production}}{\text{time interval (day)}} \right\}$$

Values of idle time costs of the lines and assembly centers are given in Table 14. By using the above reasoning the cost coefficients for the lines are calculated. However, the assembly

M	N	C(M,N) OF X_{mnjt}	C1(M,N) OF ST_{mnt}
1	1	1359916	193
	2	1501166	499
	3	383806	
2	1	585406	499
	2	1646710	329
	3	1143576	183
	4	1284316	193
	5	210766	
3	1	493006	535
	2	1769510	306
	3	1060856	177
	4	1241116	167
	5	203726	
4	1	1062070	325
	2	555842	269
	3	462766	
5	1	207790	71
	2	889052	418
	3	91486	
6	1	379990	122
	2	300482	155
	3	341246	
7	1	494230	156
	2	262962	139
	3	258366	
8	1	1900390	574
	2	1031282	481
	3	518206	
9	1	396790	127
	2	4308962	195
	3	308206	
10	1	467350	148
	2	231042	124
	3	398366	
11	1	1522390	

TABLE 19: Objective function;coefficients of the variables

centers do not allow to obtain such coefficients because of their physical structure. The capacity of the assembly centers, production costs incurred, and idle time costs are partitioned among the centers on the work-force level, depreciation of the machines and time required to finish the part.

Knowing the values of production, stock holding and surplus costs and the cost incurred when facilities are left idle, the objective function coefficients of the variables X_{mnjt} and ST_{mnt} are obtained by multiplying out the summations in the objective function. Values are given in Table 19.

The second group of inputs are received any time there arise a decisional change or other changes from the physical situation of the plant. Following are such inputs:

1. Decisional changes :

-Adding and/or eliminating any product,production facility

-Increasing or decreasing the capacity of production facilities

-Changes in any of the cost coefficients

-Changes in demand of the endproducts

-Accepting any unscheduled demand

2. Input reflecting the physical situation of the plant :

-Initial stock level of the parts and endproducts

-Level in the raw-material stocks

-Availability of the production facilities.

7. RESULTS

7.1. Introduction:

After the model has been designed and the actual data has been gathered, the model is run with the given inputs, results are obtained and outputs are analyzed. Beside the run with the initial inputs, some different runs are obtained to check the flexibility of the model and the ability of handling different cases. The factors determining the different cases are the values of inputs like scheduled demand that must be satisfied in the entire planning horizon, level of unscheduled demand arising at any point of time in the planning horizon that must be satisfied as soon as possible, the starting inventory level of work-in-process elements and of final products. The other input values do not change unless there arise a decisional change or a change in the physical outlook of the plant. Some sensitivity analyses are also done to interpret the output of the model.

The results obtained in these different runs can be read from the computer outputs of the last program, which interprets the solution derived from the FMPS program.

FMPS forms an equality system from the input matrix by adding a nonnegative slack variable to each L, E and N rew, and subtracting a nonnegative slack variable from each G row. The slack variables added to the input rows are called positive slack variables and those subtracted from input rows are called negative slack variables.The positive and negative notation for slack variables only reflect a relationship of a slack variable to the input row. The resultant matrix vector for a positive slack variable is a positive unit vector,zeros in all rows,except the row,the slack variable is added to,which contains plus one.The resultant matrix vector for a negative slack variable is a negative unit vector. Slack prices are the objective function coefficients for the slack variables.If no price is input for those variables the slack price is assumed to be zero.In the following discussion, c_j refers to the objective function coefficient for variable j.

The dual π vector is computed by :

$$\pi = - \text{FOBJWT} \times c_B \times B^{-1}$$

where FOBJWT is the objection function weight (-1 for maximization and +1 for minimization)

c_B is the row vector of objective function coefficients of basic variables

B^{-1} is basis invert.

The reduced cost for variable j, d_j is computed by :

$$d_j = \text{FOBJWT} \times c_j + \pi \times a_j$$

where c_j is the objective function coefficient for variable j

a_j is the matrix vector for variable j(a positive unit vecto for positive slack variables and a negative unit vector for negative slack variables.

For maximization or minimization, at optimality, d_j is ≥ 0 for nonbasic, lower bounded variable at lower bound, is ≤ 0 for nonbasic upper bounded variable at upper bound and is $= 0$ for all basic variables. Fixed variables may have positive or negative reduced costs at optimality. The following discussion assumes the preceeding deffinition of terms. It assumes no basis change over the range of change of an item.

-The change in objective function value Δz, for a change in the activity of variable j, ΔX_j, is:

$$\Delta z = FOBJWT \times \Delta X_j \times d_j$$

-At optimality, for a nonbasic variable to enter the basis, its objective function coefficient, c_j, must be changed by $+d_j$ for maximization or $-d_j$ for minimization.

-For a positive (negative) slack variable row j with dual activity $_j$ and slack price c_j, the slack variable reduced cost, d_j, is

$$d_j = +\pi_j + FOBJWT \times c_j$$

$$(d_j = -\pi_j + FOBJWT \times c_j)$$

-The change in the objective function value z, for a change in cost coefficient for variable j, Δc_j with activity X_j is

$$\Delta z = \Delta c_j \times X_j$$

-The change in objective function value, Δz, for a change in RHS for row j, ΔRHS_j, with dual activity π_j is :

$$\Delta z = -FOBJWT \times \Delta RHS_j \times \pi_j$$

-At optimality, for a negative slack variable j with slack price c_j, the dual activity for row containing slack variable j, π_j is constrained as :

$$\pi_j \leq FOBJWT \times c_j$$

-At optimality, for a positive slack variable j with slack price c_j, the dual activity for row containing slack variable j, π_j is constrained as :

$$\pi_j \geq -FOBJWT \times c_j$$

The dual activity at optimality, for row j with basic slack variable equals $-FOBJWT \times c_j$ for positive slack rows and equals $FOBJWT \times c_j$ for negative slack rows.

By analyzing the reduced costs appearing in the last column of the FMPS output, the change in the objective function can be predicted by changing either the right hand side of any constraint or increasing the value of any variable by one unit.

Change in the objective function can be achieved in the following manner and the magnitude of the change is calculated as follows:

-Increasing the right hand side of the greater or equality constraints:

A slack variable is subtracted from each constraint of this type to obtain an equality system.The reduced costs corresponding to these constraints are either positive,i.e. the slack variable is at its lower bound,0 in the optimum solution,or the reduced cost is equal to zero,i.e. the slack variable has a positive value If the reduced cost corresponding to any of the constraints of the first type (constraints from 1 to 35) is positive,it means that the objective function will worsen (increase for minimization or decrease for maximization) By that amount when the right hand side is increased by one unit.In this problem,the first type of constraints force the production to be at least the demand level. If in the optimal solution the slack variable is equal to zero, it means that the production is realized just as it is required, the constraint is satisfied as an equality.Increasing the right hand side by one unit forces the production of one more unit.So the production and stock holding costs of this unit increases the objective function,but the deduction of production time of this unit from the total idle time of that production center decreases the objective function value.By assembly operations,the stock holding costs of the two appropriate parts are also deducted from the optimal value.All these increases and decreases are given in a compact way by the reduced cost.

-Increasing the right hand side of the less than or equality constraints:

A slack variable is added to each constraint to form an equality system.The reduced cost corresponding to these constraints are either zero,which means that the value of the slack activity is positive in the optimal solution,the constraints are not satisfied at their upper bounds,or negative meaning an increase of the right hand sides by one unit will better the objective function by that amount. In this specific problem,the second type of the constraints (from 36 to 175) denotes the utilization of the production centers. If in the optimal solution a slack variable belonging to one of these constraints is greater than zero,meaning there exists that amount of idle time for that production center in that time interva there will be no change in the objective function by increasing the right hand side.The reduced cost corresponding to such constraints is zero.

-Increasing the right hand side of an equality constraint:

The third type of constraints are equality constraints denoting the production and inventory balance for the assembly centers.The increase of their right hand sides will decrease the ending invento of the parts but increase the ending inventory of the respective endproduct.So the objective function value will be reduced by STC_{mn},i.e. stock holding cost of n'th part of the m'th endproduct and increase by the surplus cost of the m'th endproduct.

-At optimality,the reduced cost of a nonbasic lower bounded variable at its lower bound is positive,it is negative for an upper bounded variable at its upper bound.For all basic variables the resulting reduced cost is zero.

7.2. First Run and Results :

By the first run it is aimed to reflect the initial situation of the plant. This is the initial situation of the plant when starting the production the first time.The way of obtaining and the values of the required input have already been given in the previous chapter.Since the activity newly starts in the wheel company,there is no way to have nonzero values for the initial stock levels of parts and endproducts.Since in the beginning of the first production,there has not been arrived any unscheduled demand;therefore the lower bounds for all variables are zero.

The results obtained in this run with the above mentioned conditions are given in detail in Appendix II.

- The objective function value for the optimal solution which is obtained after 677 iterations is 49,988,293.00 TL.
- The production level for each endproduct is the actual demand level;namely the constraints which exist to force the program to fulfill the demand are satisfied at their lower limits.
- It can be seen from the optimal solution that in order to obtain this minimum cost solution,the capacity of all the production facilities has been sufficient;i.e. none of the facilities cause a bottleneck.But,the third line is the one which most likely will cause a bottleneck if higher level of production is required,since the percentage utilization of this line is higher than the others.

The percentage utilization of the production facilities in the optimal solution is shown in Table 20.

PRODUCTION CENTER	PERCENTAGE UTILIZATION OF PRODUCTION CENTER
Side&Lockring Line	12.04%
Truck Line	24.63%
Disc Line	76.89%
Tractor Line	66.94%
1st Assembly Center	23.68%
2nd Assembly Center	24.63%
3rd Assembly Center	66.94%

TABLE 20 :Percentage Utilization of the Production Centers in the first Run

-The optimal production schedules are such that there does not exist any ending inventory of the work-in-process elements in any time interval.

-Total CPU time required to obtain these results is 8.51:828 minutes.

If the FMPS output is analyzed in detail,the possible changes in the objective function values can be foreseen. These changes can be achieved when the right hand sides of the corresponding constraints are increased by one unit.The amount of change of the objective function is given by the reduced cost.The reason for the reduced cost values read from the FMPS are explained below for some constraints of the first type chosen at random.

NUMBER OF CONSTRAINT	REDUCED COST	REASON	
1	948.4999	PC of 11	+773
		STC of 11	+193
		CIT of 1	- 17.49
			948.51
3	856.4999	PC of 34	+707
		STC of 34	+167
		CIT of 1	- 17.49
			856.51
29	282.3749	PC of 43	+834
		SC of 43	+ 1
		STC of 42	-325
		STC of 41	-369
		CIT of 7	- 8.63
			282.37
35	439.3749	PC of 103	+719
		SC of 103	+ 1
		STC of 102	-124
		STC of 101	-148
		CIT of 7	- 8.63
			439.37

Since the reduced costs corresponding to the second type of constraints (36 to175) are all zero, there will be no change in the objective function value by changing the right hand sides of these constraints.

The reduced costs of the constraints of the third type (176 to 655) are all negative meaning that an increase in the right hand sides of these constraints will cause a decrease of the amount of reduced costs in the objective function.The increase of the right hand sides is reflected as a decrease of the ending inventories,so the objective function values decrease by the stock holding costs of the respective parts.Some examples are explained below:

NUMBER OF CONSTRAINTS	REDUCED COST	REASON STC OF mn
176-195	-193	193 of 11
216-235	-167	167 of 34
316-335	-306	306 of 32
616-635	-183	183 of 23

7.3. Second Run and Results:

The aim of this run is to analyze the situation of the plant,if there exists unscheduled demand on some final products, namely on the first three endproducts and some initial stock level for some parts.Given input is explained in Table 21 and in Table 22.

The changes caused by this additional input data compared with the first run are as follows:

-The setting of lower bounds on some variables because of the existance of unscheduled demand should be realized.Since uunscheduled demand is given for products one,two and three, the variables X_{111t},X_{122t},X_{136t},X_{212t},X_{223t},X_{235t},X_{241t},X_{256t}, and ,X_{312t},X_{323t},X_{335t},X_{341t},X_{356t} will have a lower bound for some t depending on the values of unscheduled demand and capacities of production centers on which the corresponding parts are

PRODUCTS	1	2	3	4	5	6	7	8	9	10	11
DEMAND	167	2917	1250	417	833	417	833	417	417	4167	1250
UNSCHEDULED DEMAND	50	124	47	-	-	-	-	-	-	-	-
AMOUNT TO BE PRODUCED	217	3041	1297	417	833	417	833	417	417	4167	1250

TABLE 21: The Scheduled and Unscheduled Demand and the total amount to be produced for the second run.

PARTS	11	21	23	33	61	92	102
INITIAL STOCK	15	110	1	1	11	15	30.

Table 22: The Initial Stock level of the parts for the second run.

produced.The lower bound values of the variables are specified in Table 23.The calculations of the lower bounds follow the reasoning given in the introduction part of this chapter.

X_{mnjt}	LOWER BOUND
X_{1111}	.0278
X_{1221}	.0568
X_{1361}	.0568
X_{2121}	.1409
X_{2231}	.1476
X_{2351}	.1409
X_{2411}	.0689
X_{2561}	.1409
X_{3121}	.0534
X_{3231}	.0559
X_{3351}	.0534
X_{3411}	.0261
X_{3561}	.0534

TABLE 23: The lower bound values of X_{mnjt} for second run

-The right hand sides of ten constraints out of the first 35 constraints are increased from the level of the demand to the total amount of production which is the sum of scheduled and unscheduled demand specified as in Table 21 and 22.

-Right hand sides of the stock-production balance constraints corresponding to pairs mn,such as 11,21,23,31,61,92,and 102, are increased to IS_{mn} values from zero.

All these changes are realized by the first program,INCREPR. The results obtained by the second run are given in Appendix III.

-The objective function value obtained after 743 iteration is 51,170,340.00 TL.Time requirement for this run is 9.51:828 minutes.

-The optimal production level of the final products is the minimal possible production level,i.e. the required level.Thus, the constraints existing to enforce the satisfaction of the minimum production level are fulfilled at their lower bounds.

-With all the changes in the givens,none of the production centers cause a bottleneck.The percentage utilizations in the whole planning horizon is explained in Table 24.

PRODUCTION CENTER	PERCENTAGE UTILIZATION OF PRODUCTION CENTER
Side&Lockring Line	12.65%
Truck Line	21.51%
Disc Line	73.99%
Tractor Line	65.67%
Ist Assembly Center	22.93%
2nd Assembly Center	22.03%
3rd Assembly Center	63.07%

TABLE 24: The percentage utilization of production centers in the second run

As in the first run,the disc line is the highest utilized production center.

-By obtaining the optimum solution by this run,there arise some final inventory on some parts which are given in Table 25.

m n	TIME PERIOD	STOCK LEVEL
1 1	20	15
2 1	20	110
2 3	20	1
3 3	20	1
6 1	20	11
9 2	20	15
10 2	220	30

TABLE 25: The ending inventory levels for parts (m,n) in the optimal solution for the second run

The existance of these ending stock levels in 20th time interval for the above parts means production off the same amounts has been taken place in the 20th time interval,which implies that keeping the lines unallocated incures more cost to the objective function than holding stock of parts for one period.

7.4. Third Run and Results:

The additional input specified in this run is the initial stock of the final products besides unscheduled demand on all products and initial stock of some parts.The amount of scheduled and unscheduled demand,initial stock of the endproducts and initial stock of the parts are given in table 26 and in table 27. The existance of unscheduled demand causes the increase in total required production in the entire planning horizon and forces

PRODUCTS	1	2	3	4	5	6	7	8	9	10	11
DEMAND	167	2917	1250	417	833	417	833	417	417	4167	1250
UNSCHEDULED DEMAND	10	10	5	5	10	15	10	5	10	5	5
INITIAL STOCK	7	-	-	9	4	-	-	-	5	5	5
AMOUNT TO BE PRODUCED	170	2927	1255	417	839	432	843	422	422	4167	1250

TABLE 26: The Scheduled and Unscheduled Demand, Initial Stock and total amount to be produced for the third run.

PARTS	11	12	31	61	82
INITIAL STOCK	15	110	1	1	11

TABLE 27: The Initial Stock Level of the parts for the third run.

X_{mnjt}	LOWER BOUND
X_{1111}	.0056
X_{1221}	.0114
X_{1361}	.0114
X_{2121}	.0114
X_{2231}	.0118
X_{2351}	.0114
X_{2411}	.0056
X_{2561}	.0114
X_{3121}	.0057
X_{3231}	.0059
X_{3351}	.0057
X_{3411}	.0028
X_{3561}	.0057
X_{4131}	.0059
X_{4241}	.0089
X_{4371}	.0089
X_{5131}	.0119
X_{5241}	.0179
X_{5371}	.0179

X_{mnjt}	LOWER BOUND
X_{6131}	.0179
X_{6241}	.0268
X_{6371}	.0268
X_{7131}	.0119
X_{7241}	.0179
X_{7371}	.0179
X_{8131}	.0059
X_{8241}	.0089
X_{8371}	.0089
X_{9131}	.0119
X_{9241}	.0179
X_{9371}	.0179
X_{10131}	.0059
X_{10241}	.0089
X_{10371}	.0089
X_{11131}	.0059

TABLE 28: Lower bounds for the variables for the third run

the production of these amounts as soon as possible by setting the lower bounds.Since the unscheduled demand for all products implicitely for all parts makink up the endproducts compared with the capacity of the relevant production centers is less,implies that lower bounds of the variables in question exist only and only for the first periods.The values of the lower bounds set by the INCREPR are given in Table 28.

-The right hand sides of all the first type constraints are increased from demand levels to total production levels which are given in Table 26.

-Right hand sides of the stock-production balance equations corresponding to pairs mn,such as 11,12,31,61 and 82,are increased to 15,110,1,1 and 11 respectively. (Table 27)

The allocation of the production centers in each time intervals,the starting and final inventory levels,the activities taking place at the production centers and the utilization of the production centers are given in detail in the computer output of OUTPUTPR in Appendix IV.

-The optimal solution is obtained after 660 iterations.The resulting optimum objective function value is 50,264,592.00 TL.

-Except the first product all other products are produced at their required lower limits.The amount exceeding the required level is 15 units,and this is the level of initial stock of the first part of the first product.On the other hand the initial stock of the second part of the same endproduct is 110 units; so if leaving the production centers idle would have fewer effect on cost then stock holding or surplus items .Then the first,second and sixth production centers will be allocated in such a way that total production will be 170 units and during

PRODUCTION CENTER	PERCENTAGE UTILIZATION OF PRODUCTION CENTER
Side&Lockring Line	12.09%
Truck Line	24.73%
Disc Line	77.20%
Tractor Line	67.30%
1st Assembly Center	23.76%
2nd Assembly Center	24.81%
3rd Assembly Center	77.20%

TABLE 29: Percentage utilization of the production centers for the third run

the time the 15 units are producable the relevant production center would be left idle.

-The overall utilization of the production centers in percentages is as in Table 29.As it is implied by the comparison of the capacities of the production centers,it is evident that the third production center,namely the disc line is always the one with the highest utilization percentage.

-The amount of parts that must be stored in some time intervals by the optimum production scheduling with the specified conditions are given in Table 30.

-Total CPU time required to obtain these results is 10:40.530 minutes.

m n	TİME PERIOD	STOCK LEVEL
1 2	20	95
3 1	20	1
6 1	20	1
8 2	20	11

TABLE 30: Ending inventory levels for parts in the third run

7.5. Fourth Run and Results:

In all the three runs it comes out,as it should be,that the disc line is the one that will cause the first bottle-neck,if total amount to be produced increases.It is desired to get an optimum feasible solution where disc line is 100% utilizied by this run.All the givens,scheduled and unscheduled demand and initial stock levels,are as in the last case.

The change done in the program,in INCREPR is to set the constraints (from 76 to 95) equal to one;they were less than or equality type of constraints stricting the program to fulfill everything in regular time (use of overtime is not allowed). The optimal solution could be obtained after 888 iterations.The minimum cost incurred by this solution is increased from 50,264,592.00 TL to 51,985,195.00 TL.

-Production level of the endproducts is at the minimum level which is "Demand+Unscheduled Demand-Initial Stock" of the final products.

m n	j	TIME PERIOD	STOCK LEVEL
1 1	1	20	15
2 1	2	20	110
2 3	3	20	1
3 3	3	20	1
5 1	3	16	483
5 1	3	17	1323
5 1	3	18	2163
5 1	3	19	3003
5 1	3	20	3830
6 1	3	20	11
9 2	4	20	15

TABLE 31: The ending inventory level of the parts in the fourth run

-This additional condition,with respect to the last case,squeezes the required lower limit of production to first periods (except the tenth product,its lower limit is achieved at the end of the 16th time interval) and the amount produced in the last four time intervals are all work-in-process elements which increase the ending inventories.The reason for the existing high level of ending inventories is based on the condition that the disc line should be utilized 100% throughout the planning horizon.It should be noted that all high stock levels are accumulated in the third, namely disc line.The ending inventory levels are figured out in Table 31.

PRODUCTION CENTER	PERCENTAGE UTILIZATION OF PRODUCTION CENTER
Side&Lockring Line	12.09%
Disc Line	100.00%
Truck Line	24.73%
Tractor Line	67.30%
1st Assembly Center	23.76%
2nd Assembly Center	24.73%
3rd Assembly Center	67.30%

TABL TABLE 32: Percent utilization of the production centers in the fourth run

-The percent of idle time for each production center in each time interval is given in comparison with the last run in Table 33.The overall utilization is figured out in Table 32, although the values are the same as in the last case the utilizatio of the production centers (and also the idle times) differ very much from time interval to time interval in both cases.

-Total CPU time required to obtain this optimum solution is 11:23.267 minutes.

INTERVAL

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3rd RUN	75.2	100	100	100	54.4	100	100	53.3	100	100	100	100	84.4	53.3	53.3	100	100	100	84.4	100
4th RUN	75.4	100	84.4	100	84.4	100	53.3	84.4	84.4	84.4	84.4	61.7	84.4	84.4	93.0	100	100	100	100	99.
3rd RUN	60.2	100	100	100	6.1	100	100	4.6	100	100	100	100	68.2	4.6	4.6	100	100	100	68.2	89.
4th RUN	60.7	100	68.2	100	68.2	100	4.6	68.2	68.2	68.4	68.2	21.7	68.2	68.2	85.8	100	100	100	100	87.
3rd RUN	0	0	0	33.3	0	33.3	33.3	0	33.3	33.3	33.3	33.3	0	0	0	33.3	33.3	70.7	0	85
4th RUN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3rd RUN	126	0	100	0	97.5	0	0	100	0	0	0	0	0	100	100	0	0	56.4	0	98
4th RUN	8	0	0	0	0	44.6	100	0	0	0	0	73.0	0	0	0	36.3	100	100	100	99.
3rd RUN	68.7	100	100	100	6.1	100	100	4.6	100	100	100	100	68.18	4.55	4.55	100	100	100	68.2	100
4th RUN	67.5	100	68.2	100	68.2	100	4.6	68.2	68.2	68.2	68.2	21.7	68.2	68.2	85.8	100	100	100	100	99.
3rd RUN	47.6	100	100	100	6.1	100	100	4.55	100	100	100	100	68.2	4.6	4.6	100	100	100	68.2	100
4th RUN	47.9	100	68.2	100	68.2	100	4.6	68.2	68.2	68.2	68.2	21.7	68.2	68.12	85.8	100	100	100	100	100
3rd RUN	0	0	100	0	97.5	0	0	100	0	0	0	0	0	100	100	0	0	56.4	0	100
4th RUN	0	0	0	0	0	44.6	100	0	0	0	0	73.0	0	0	0	36.3	100	100	100	100

TABLE 33: The percentage of idle times of production centers in each time interval in comparison of the third and forth runs.

7.6. Runs with no Feasible Solutions:

Mainly there are two types of runs;the first group contains the runs with the scheduled demands as it is specified previously and different values for unscheduled demand,initial stock levels for the parts and final products.The second group of runs contains runs with multiple level of demand with different values of initial stock and unscheduled demand.For these runs,it has not been possible to obtain any feasible solution.The values of input and reasons for infeasibilities are explained in the following sections.

7.6.1. Fifth run and results:

The input data specified for this run,namely the level of unscheduled demand and initial stock of parts are given in detail in Table 34 and Table 35.If there exists unscheduled demand on some products,the INCREPR program sets lower bounds on the necessary variables during required time intervals,in order to satisfy the production of these amounts in the first periods as soon as possible.While setting the lower bounds,the products are considered one by one.The resulting lower bounds on the variables are given in table 36.After determining the lower bounds,they are combined in the constraints defining the line utilizations. In this run,when the lower bounds of the variables corresponding to the parts of different endproducts which are produced on the same line are combined to see the utilization of that line in

PRODUCTS	1	2	3	4	5	6	7	8	9	10	11
DEMAND	167	2917	1250	417	833	417	833	417	417	4167	1250
UNSCHEDULED DEMAND	100	300	250	1700	300	450	183	150	500	634	1179
AMOUNT TO BE PRODUCED	267	3217	1500	2117	1133	867	1016	567	917	4801	2429

TABLE 34: The Scheduled and Unscheduled Demand and total amount to be produced for the fifth run.

PARTS	11	21	23	33	61	92	102
INITIAL STOCK	15	110	1	1	11	15	30

TABLE 35: The Initial Stock level of the parts for the fifth run.

X_{mnjt}	LOWER BOUND
X_{1111}	.053
X_{1221}	.114
X_{1361}	.114
X_{2121}	.341
X_{2231}	.357
X_{2351}	.341
X_{2411}	.159
X_{2561}	.341
X_{3121}	.284
X_{3231}	.298
X_{3351}	.284
X_{3411}	.133
X_{3561}	.284
X_{4131}	1.000
X_{4241}	1.000
X_{4371}	1.000
X_{4132}	1.000
X_{4242}	1.000
X_{4372}	1.000
X_{4133}	.023
X_{4243}	1.000
X_{4373}	1.000
X_{4134}	.000
X_{4244}	.035
X_{4374}	.035

X_{mnjt}	LOWER BOUND
X_{5131}	.341
X_{5241}	.535
X_{5371}	.535
X_{6131}	.536
X_{6241}	.803
X_{6371}	.803
X_{7131}	.218
X_{7241}	.326
X_{7371}	.326
X_{8131}	.170
X_{8241}	.268
X_{8371}	.268
X_{9131}	.595
X_{9241}	.893
X_{9371}	.893
X_{10131}	.755
X_{10241}	1.000
X_{10371}	1.000
X_{10132}	.000
X_{10242}	.148
X_{10372}	.148
X_{11131}	1.000
X_{11132}	.404

TABLE 36: Lower bounds of the variables for the fifth run

the specific time interval,reasons for the infeasibility become immediately evident.The utilization of the production centers, disc line,tractor line and third assembly center,for the first and second time intervals causes the infeasibility (by setting the bounds into the following equations,the violation of the constraints are obtained).

$$X_{223t} + X_{323t} + X_{413t} + X_{513t} + X_{613t} + X_{713t} + X_{813t} + X_{913t} + X_{1013t} + X_{1113t} \leq 1$$

$$X_{424t} + X_{524t} + X_{624t} + X_{724t} + X_{824t} + X_{924t} + X_{1024t} \leq 1$$

$$X_{437t} + X_{537t} + X_{637t} + X_{737t} + X_{837t} + X_{937t} + X_{1037t} \leq 1$$

for $t = 1,2$

On the other hand,if the total amount of production for the endproducts is analyzed,another point of infeasibility also becomes clear.The second to 11th products must pass through the disc line.The total number of units that can be produced by the disc line in all 20 time intervals are 16,800 whereas the desired amount of products two to eleven are 18,564.Hence, even if the lower bounds of the variables are released,i.e. the production of unscheduled demand is spread out into the entire planning horizon,there is no way to obtain a feasible solution with this given input data.

7.6.2. Sixth run and results:

The input data for this run, scheduled and unscheduled demands of the endproducts and initial stock levels of some parts are figured out in Table 37 and Table 38. In this run, total amounts passing the side and lockring line are 6,134 units, whereas total capacity is 37,600 units. Total capacity of the truck line is 17,6oo units through which 6,134 units have to pass. 13,938 units must be produced on the disc line which has a total capacity of 16,800 units. 8,521 units must be produced on the tractor line and third assembly center, their capacities are 11,200 units each. 17,600 units are the capacities of the first and second assembly centers and amounts to be produced on are 4,167 and 6,134 units, respectively. By analyzing these amounts, it can be seen that the satisfaction of these amounts is possible from the point of the capacity of the plant. But the unscheduled demand of the first, forth and sixth products causes the infeasibility again. The lower bounds of the variables due to unscheduled demand, calculated and set by INCREPR are given in Table 39. By setting these lower bounds into the corresponding constraints, the violation of them become evident:

$$X_{2231} + X_{3231} + X_{4131} + X_{5131} + X_{6131} + X_{7131} + X_{8131} + X_{9131} + X_{10131} + X_{11131} \leq 1$$

$$X_{4241} + X_{5241} + X_{6241} + X_{7241} + X_{8241} + X_{9241} + X_{10241} \leq 1$$

PRODUCTS	1	2	3	4	5	6	7	8	9	10	11
DEMAND	167	2917	1250	417	833	417	833	417	417	4167	1250
UNSCHEDULE DEMAND	1800	-	-	460	-	560	-	-	-	-	-
AMOUNT TO BE PRODUCED	1967	2917	1250	877	833	977	833	417	417	4167	1250

TABLE 37: The Scheduled and Unscheduled Demand and total amount to be produced for the sixth run.

PARTS	11	12	41	71	72	81	82
INITIAL STOCK	169	169	417	837	834	418	418

TABLE 38: The Initial Stock Level of the parts for the sixth run.

X_{mnjt}	LOWER BOUND
X_{1111}	.957
X_{1221}	1.000
X_{1361}	1.000
X_{1112}	.000
X_{1222}	1.000
X_{1362}	1.000
X_{1113}	.000
X_{1223}	.045
X_{1363}	.045
X_{4131}	.548
X_{4241}	.821
X_{4371}	.821
X_{6131}	.667
X_{6241}	1.000
X_{6371}	1.000

TABLE 39: The lower bounds of the variables for the sixth ru

$$X_{4371} + X_{5371} + X_{6371} + X_{8371} + X_{9371} + X_{10371} \leq 1$$

even if all the products except the fourth and sixth are not produced in the first time interval,it is not possible to have a feasible solution when overtime is not allowed for the disc and tractor lines and the third assembly center.With two shifts only for these production centers a feasible solution can be obtained for this problem.

In the last three runs, the reason for the infeasibility is always the same, namely the number of units to be produced due to unscheduled demand which must be produced on the same production center exceed the actual capacity of that center in some time intervals. In order to obtain feasible solutions, even if there exists unscheduled demand on the products to be produced on the same facility, a revision is made on the program. The sum of lower bounds of the variables, X_{mnjt}, belonging to the same constraint, utilization of the production facility j in period t, are checked; at the moment the sum exceeds one, meaning that the production facility is filled up for that time interval, the remaining lower bounds of the variables are shifted one period and lower bounds in that period are set equal to zero for all variables. It could still be impossible to obtain feasible solutions for these runs, but the infeasibilities come from other points such as exceeding the capacity of some productio centers (for fifth and seventh runs) or the unbalance of the assembly centers (for the sixth run). With this extension, it is guaranteed to have always an optimal solution independent of how high the level of unscheduled demand is unless this level does not exceed the total capacity of the plant.

7.6.3. Seventh rmn and results:

By this run, it is desired to see what happens if the level of scheduled demand becomes some multiples of the initial case, by holding the unscheduled demand level and initial stock level at zero.Demand is taken successively three,four and five times as the initial level and infeasible solutions are obtained in each case,since the capacity of the plant is not sufficient to satisfy that level of production.The disc line first and then the tractor line and finally the third assembly center cause bottleneck,hence,the reason of infeasibility.Obviously,there is no way to obtain some feasible solutions,if at the same time the level of unscheduled demand is increased.

7.7. Sensitivity Analyses:

Four different sensitivity analyses on the existing model are done to investigate the effect on the optimal solution provided by simplex method,if the parameters take on other possible values.By these analyses answers to the following questions are sought:

- What can be the maximum production level of each product seperately?
- What can be the maximum production level of all products simultaneously satisfying the initial demand or having no initial demands?
- What kind of a production scheduling will result,if a second shift is added to some of the production centers?

The formulations of these different runs,the changes to be done in the program and results obtained are explained in the following sections.

7.7.1. Eighth run and results:

By this run,an answer is obtained to the following question: What is the maximum possible increase in production level of any product satisfying the initial demand level of all final products? This run gives the maximum production level of one and only one product.By obtaining the maximum production le vels of all products seperately,it is not possible to conclude that the maximum available levels for all products are these amounts simultaneously.With this run,the maximum production level,DEL1 of the first product,6.5x20 Otosan Fiat,is obtained.The formulation required is as follows:

$$\max z = DEL1$$

subject to:

1. The demand for all products and their parts must be satisfied.

$$\sum_{j} \sum_{t} X_{mnjt}\, T1_{mn} \geqslant D_{mn} \quad \text{for all } m,n/m=1,n=1,2,3$$

2. The demand of the first product and its three parts must be satisfied and the maximum production level DEL1 is to be specified.

$$\sum_{t} X_{111t}\, T1_{11} - DEL1 \geqslant D_{11}$$

$$\sum_{t} X_{122t}\, T1_{12} - DEL1 \geqslant D_{12}$$

$$\sum_t X_{136t}\, TI_{13} - DEL1 \geq D_{13}$$

3. The second type of constraints,denoting the utilizations of the production centers in each time interval and the third type of constraints,denoting the stock-production balance at assembly centers remain unchanged in this formulation.

These changes are all done in the first computer program, INCREPR.The objective function is changed to maximization of production;therefore all the objective function coefficients of the variables are changed.A new variable,DEL1 is introduced.The constraints in which this new variable exists are figured out and the coefficients of this variable is entered in the constraints in question.All these changes are organized and input to the INCREPR.7:29.021 minutes and 347 iterations are required to obtain the optimal solution.

-The objective function value comes out to be 13,266,which is the maximum production level of the first product while the demand level of all other products are satisfied.

-The demand and the realized production level of the products are as follows: The maximum increase in production of the first product is 13,266 units;the amounts to satisfy the demand of this product,167 units,are also produced.So im total 13,433 units are produced from the first product.Beside this,production levels of products 7,9,and 10 are also increased by 287,143 and 39 units,respectively.All other products except the first, seventh,nineth and tenth are produced at their minimum production levels,namely at their initial demand levels.

PRODUCTION CENTER	PERCENTAGE UTILIZATION OF PRODUCTION CENTER
Side&Lockring Line	48.88%
Truck Line	100.00%
Disc Line	61.00%
Tractor Line	71.00%
1st Assembly Center	24.00%
2nd Assembly Center	100.00%
3rd Assembly Center	71.16%

TABLE 40: Percent utilization of the production centers for the eighth run

-The utilizations of the production centers are given in Table 40. The values are the avarage utilization values in the entire planning horizon.The first product has three parts.The capacities of the production centers on which these parts are produced restricts the value of DEL1.The first part is produced on the first line with a capacity of 1,800 units per time interval. The second line is produced on the truck line with a capacity of 880 units per time interval and the third part,which is the endproduct,is assembled on the second assembly center with a capacity of 880 units per time interval.The truck line and the second assembly center are both fully utilized and the first line is 48.88% utilized.

7.7.2. Ninth run and results:

The immediate question arising from the results of the last run is: what happens if a second shift is added to the truck line and/or to the second assembly center?

If a shift is added only to one of them,it is evident that there will not be a change in the value of the objective function and in the production scheduling.An overtime of the length same as the regular time is added to the truck line;after 344 iterations (4:09.149 minutes) the optimal solution,which is equivalent to the previous one is obtained.The maximum possible amount that can be produced from the first product satisfying the initial demand of all other products is again 13,266 units in the entire planning horizon.The first line is 48.88% utilized.The regular time of the truck line is fully utilized,but the overtime is left totally idle.The second assembly center is also fully utilized.

For completeness,a shift is added to both of the production centers,truck line and second assembly center.The formulation that will give the desired information is as follows:

The objective function is to maximize the total amount of production of the first product

$$\max z = DEL1$$

subject to:

1. The minimum production level of the products is the initial demand level.

$$\sum_j \sum_t X_{mnjt} \, T1_{mn} \geqslant D_{mn} \qquad \text{for all } m,n \;/\; m=1, n=1,2,3$$

2. The first product must be produced to fulfill the demand and to maximize DEL1.

$$\sum_t X_{111t} \, T1_{11} - DEL1 \geqslant D_{11}$$

$$\sum_t X_{122t} \, T1_{12} - DEL1 \geqslant D_{12}$$

$$\sum_t X_{136t} T1_{13} - DEL1 \geqslant D_{13}$$

3. All the production centers except the second and the sixth can be at maximum 100% utilized.

$$\sum_m \sum_n X_{mnjt} \leq 1 \qquad \text{for all } j \;/\; j \neq 2,6 \text{ and } t$$

4. Overtime is allowed for the second and sixth production centers.

$$\sum_m \sum_n X_{mn2t} \leq 2 \qquad \text{for all } t$$

$$\sum_{m} \sum_{n} X_{mn6t} \leq 2 \qquad \text{for all } t$$

5.The third type of constraints,denoting the stock-production balance at the assembly centers remain unchanged in this formulation.

-The optimal solution under the given conditions is obtained after 284 iterations (3:o2.126 minutes).The objective function is 17,433 units.In the last three runs,the number of iterations to obtain the optimal solution is decreased gradually from the first to the third run.If the restrictions are stricter,the number of iterations increases and the objective function value decreases (for maximimization) . The third run has the least number of restrictions (second and sixth production centers are both allowed to use overtime) and it required the least number of iterations (284) and the maximum objective function (13,433 units).

-Beside the first product,(DEL1 = 17,433 units),excess production arise in products 4,5 and 10 by 143,287 and 39 units,respectively. So in total,17,600 units from the first,560 units from the fourth,1,120 units from the fifth and 4,206 units from the tenth product are produced.All others are satisfied at their initial demand levels.

-In order these amounts to be produced,the production centers are utilized as shown in Table 41.If the utilization of the production centers is analyzed beside the other results,it may

PRODUCTION CENTER	PERCENTAGE UTILIZATION OF PRODUCTION CENTER
Side&Lockring Line	60.46%
Truck Line	123.67%
Disc Line	76.35%
Tractor Line	71.16%
1st Assembly Center	23.67%
2nd Assembly Center	123.67%
3rd Assembly Center	71.16%

TABLE 41: Percentage utilization of the production centers for the ninth run

lead to following conclusion: It is possible to increase the objective function by utilizing the first,second and sixth production centers.Disregarding all other factors and viewing only the capacities,the objective function may increase up to 3o,866 units by using the second and sixth production centers 200% and the first 85.74%.This is an impossible result,since in the optimal solution the variables X_{1361} to X_{13620} have already reached their upper bound values with a reduced cost of 880 units.This implies that an increase in the values of the variables each by one unit,will cause an increase in the objective function by Δz,which is calculated as:

$$\Delta z = FOBJWT \times \Delta X_j \times d_j$$
$$= (-1) \times (+1) \times (-880)$$
$$= 880$$

The constraint,denoting that the amount of the production of the third part belonging to the first endproduct,must satisfy the initial demand and meanwhile satisfy the maximum production level is one of the second type of constraints,in order to make it an equality equation a slack variable is subtracted:

$$\sum_t X_{136t} - DEL1 - SLACK\ VARIABLE = 167$$

In the optimal solution,the slack variable has a positive value. If in the activity of the slack variable a reduction by one unit occurs,this will effect the objective function as:

$$\Delta z = FOBJWT \times \Delta X_j \times d_j$$
$$= (-1) \times (-1) \times (.999)$$
$$= 1$$

i.e. decrease in the values of slack variables by one unit, which means an increase of the demand of that product by one unit,will increase the objective function by one unit.

7.7.3. Tenth run and results:

By this run, the maximum possible increases in the production levels of all products, DEM_m, simultaneously are desired. A minimum production level for the products is not specified. Objective function is the sum of the total production which is to be maximized. The formulation then becomes:

$$\max z = \sum_{m=1}^{11} DEM_m$$

subject to:

$$\sum_{j} \sum_{t} X_{mnjt} \times T1_{mn} - DEM_m \geqslant 0 \qquad \text{for all } m,n$$

The second and third type of constraints, denoting the line utilizations and stock-production balance at the assembly centers, respectively remain the same as the initial case.

If compared with the initial formulation, new variables, DEM_m, are introduced and the objective function is changed from cost minimization to production maximization. The coefficients of the objective function and constraints, and the constraints to which the variables belong must be specified by the input generation program, INCREPR, for this formulation. After all required changes are accomplished, a solution is obtained in 5:25.356 minutes. In the optimum solution, maximum possible increases of the production of all products simultaneously are obtained. The objective function value is 34,400 units.To maximize the objective fmnction,two products,namely,the first and the eleventh are increased from 0 to 17,600 and 16,800 respectively and all others are left at zero level.As mentioned above,the first product has three parts produced on the production centers 1,2,and 6 with 1,800 , 880 , and 880 unit capacities per time interval,the eleventh product has only one part prodmced on line three with capacity of 840 units per time interval.If a 20 period planning horizon is assumed the increases in production levels will be as follows: These two products are choosen.among eleven possibilities,since they have the less number of operations to be accomplished to produce the endproduct.

As a result of the formmlation, $(\max \sum_{m} DEM_m)$,all X_{mnjt} variables are at their lower bounds.The increase of these variables by one unit will cause a decrease in DEM_m variables and thus a decrease of the objective function.This is reflected by the reduced cost of the variables X_{223t} and X_{323t},for all t, which are 880 and 840 respectively,i.e. increase of them for any time interval will be reflected by a decrease of 880 or 840 units of the objective function.

Production centers 1,2 and 6 are used by DEM_1 and 3 is used by DEM_{11}.The first production center,side and lockring line is utilized for an avarage of 48.88% in the whole planning horizon. The production centers 2,3 and 6 are fully utilized and the others are not used at all.

PRODUCTION	1	2	3	4	5	6	7	8	9	10	11
REQUIRED PRODUCTION LEVEL	167	2917	1250	417	833	417	833	417	417	4167	1250
REALIZED PRODUCTION LEVEL	167	2917	1250	417	833	417	833	417	417	4167	1250
DEMm	13266	0	0	0	2670	0	0	0	1029	0	183
TOTAL PRODUCTION	13433	2917	1250	417	3503	417	833	417	1446	4167	1433

TABLE 42 : The required production level, the realized production level and the values of DEMm variables for the elevent run.

7.7.4. The eleventh run and results:

In the last run,there does not exist any minimum required production levels;production level in this run is the initial demand level.The question to which an answer is sought by this run is: satisfying the initial demand,what can be the maximum increases in the production levels of all products simultaneously?

The required production level,the realized production level and the values of DEM_m variables come out (7:25.256 minutes) to be as given in Table 42.

In order to accomplish these amounts the production centers are utilized as follows;the second,third,fourth,sixth and seventh production centers are all fully utilized in the entire planning horizon.The side and lockring line is only 48.88% used,where the second assembly center is used 23.67%.

By analyzing the reduced costs of the slack variables and variables,X_{mnjt} and ST_{mnt},it is concluded that no increase in the objective function value cam be achieved by changing the levels of activities of variables.On the contrary,the increase of slack variables belonging to constraints 56-75 and 76-95 (utilization of lines 2 and 3) by one unit will decrease the objective function value by 880 and 840 units,respectively.

8. DISCUSSION AND CONCLUSIONS

8.1. Introduction:

It is accepted that applicable production planning models are mostly designed for special cases;the reason being to solve specific problems and complications for an existing plant,Although the apperent aim in this study seems to be the satisfaction of the requirements for a wheel company's management,the main purpose was to design a model as general as possible which can cover almost any case for similar production plants.

In this chapter,the features of the model relating to the generality of it,the changes to be done to enable the model applicable to different cases,and the possible extensions that can be included into the boundaries of such a model to obtain a complete system are explained.

This model is designed for a plant that produces eleven different types of endproducts (each of which is composed of one to five different parts) on four production lines (all of which are working in parallel) and three assembly centers.The raw-materials required for each of the production facilities are taken from the warehouse and it is assumed that shortage of raw-materials will never appear in the total planning horizon. The work-in-process elements are stored in stocks in front of the assembly centers, if the centers are already occupied in that time interval or if any part to be assembled on that assembly center does not exist in the necessary amount in the beginning of that time intàrval.The final products are immediately sent to the warehouse.Objective of this model is to minimize the total cost which comprises of production,stock holding,surplus costs and cost of idle times of production centers in the entire planning horizon.

The presented computer program is still valid even if there arises a change in the structure of the plant,in the number or characteristics of production lines and assembly centers,in the number and configüration of the endproducts,in the objective function (coefficients or type),and/or in the length of the planning horizon.

However,for each of these changes,the program must be updated.Most of these updates are done in the first computer program,i.e. INPUT CREATION PROGRAM.As mentioned in the fourth chapter,thiä program specifies the coefficients and the exact number of constraints in which each variable appears,and this unique way of presentation is a must for FMPS package.It seems initially that this way of specification creates a restriction for general use of the computer program,but to define everything as exagenous variables,whose values are input broadens the scope of application.In the following sections,possible revisions considering the model and corresponding changes in the computer programs are explained seperately.

8.2. Features of the Model:

Before the treatment of different cases which this model can be applied to is analyzed, it will be usefull to summerize the different cases this program can solve, without any revisions to be done, for the specific plant described in Chapter 4.

-First Case: Although the plant has started its activity, the actual production did not yet start. When all machines arrive and actual production starts, then, for this initial case, the model will be the one to be used. There will not be any initial inventory levels for any endproducts and parts, there will not exist any demand that will have a priority to be finished,

-Second Case: When periods after the first are considered, then it is possible that there already exists some parts or endproducts from the previous periods. Then the amounts existing in stock must be imput to the program in order to prevent overproduction.

-Third Case: The production level of this plant has to be parallel to the level of activity in the automotive sector of Turkey. By taking this point into account, the speciality of handling demand with priority is added to the program. In other words, if from any product a specific amount is to be produced as soon as possible, the desired levels of these products are input to the program.

-Fourth Case: If all the above situations occur simultaneously and if all input are specified correctly, there will not arise any problems.On behalf of these,if during a planning horizon,which is set to be one month,any decisional or physical change occurs, this can be incorparated into the model.The existing situation of the plant is analyzed,the available parts and endproducts are specified and it will be possible to see how to continue thereon to achieve the minimum total cost.

8.3. Changes in the Objective Function:

A change in the objective function may occur in two manners: By defining a completely different objective function or by keeping the one as it is but redefining the coefficients.Other objective function alternatives considering a production planning problem may be any one of the followings or any combination of them:

-Profit maximization

-Maximum utilization of the production facilities

-Minimization of idle and/or overtime of production facilities

-Optimization of buffer stock capacities,and locations

-Optimization of raw-material consumption and inventory if there exists alternatives in the use of different type of materials

-Optimization of scheduling of work-force levels among the production facilities and stabilizing employment levels

-Optimizing customer service (delivery performances)

-Minimizing total cost wich is the sum of variable production, procurement and distribution costs,losses associated with shortages,cost of holding inventory,cost of changes in production rate and in work-force levels and cost of changing facility's capacities.

Definig a new objective function among these alternatives may require definition of other variables than existing ones, which means a change in formulation.In such a case,the use of the computer program directly by means of small revisions is not possible at all.On the other hand,if the objective function is kept and the coefficients are changed,C(M,N),coefficients of X_{mnjt} and C1(M,N),coefficients of ST_{mnt} have to be updated.

As in here and in all similar cases,where the values of the coefficients change,the arising updates are only in the data cards.There will not be any revisions in any of the other computer programs.

8.4. Changes in the Number of Products :

Adding or deleting of some products may arise due to policy changes of the decision-maker as a reflection of the market situation or due to facility changes in the plant.In both cases, following updates in the computer program should be made.

The variable MM,denoting the total number of endproducts will be changed by ΔMM. This will cause changes in MN,total number of parts produced and in array NN(M),number of parts of each individual endproduct.The objective function coefficients C(M,N) and C1(M,N) and the matrix TI(M,N),denoting the number of parts to be produced in one time interval must be regulated. The demands D(M),the initial stock levels ISE(M) and unscheduled demand level IUSD(M) are also to be regulated.If there exists inventory levels for deleted endproducts or their parts,these entries should be reduced to zero (in ISE(M),in IS(I)) ; in the reverse case,when a product is added with positive inventory levels,the entries for such items should be inserted into the relevant arrays.The two matrices explaining the precedence relations and the one to one correspondense of production centers and parts to be produced on those production centers,H(M,N) and A(M,J),are also to be arranged.The variables IK,IKK,I3,etc. and IR(M,N),all denoting the number of constraints for corresponding variables,can be arranged after the formulation of the problem is revised by means of this change.

Then the change in the size of the problem will be:

-(ΔMN) : Change in the first type of constraints

-(ΔMM x NML x TT x 2): Change in the third type of constraints

-(ΔMN x TT) : Change in the number of X_{mnjt} variables

-(ΔMN - ΔMM) x (TT) : Change in the number of ST_{mnt} variables

All the variables corresponding to the above mentioned ones in the last program,OUTPUTPR,must also be revised in a parallel manner.

If there arises a change in the number of parts belonging to the final products,which is possible only for the case where new endproducts are added to the program,i.e. where ΔMM > 0,it will be considered exactly in the same manner as described above.If a part is to be produced and to be sold directly,then this part will be assumed as a new product with only one part,and the progra will undergo the same changes as explained above.

8.5. Changes in the Length of Planning Horizon:

The length of planning horizon, i.e. the number of time intervals, is the most effective variable in determining the size of the problem. A change in this variable will not only change the size, but also will cause changes in other variables. The variables in question are mostly the ones specifying the starting number of constraint set (they will increase by an increase in the number of time intervals, ΔTT); these are IK, IKK, IK3, ..., I6 and IR(M,N). Depending on the change, the formulation must be analyzed thoroughly and the values of the above mentioned variables must be input.

The change in the size of the problem will then be:

-JJ x ΔTT : Change in the second type of constraints

-(MN - MM)xΔTT: Change in the third type of constraints

-(MN x ΔTT) : Change in the number of X_{mnjt} variables

-(MN - MM)xΔTT: Change in the number of ST_{mnt} variables

8.6. Changes in the Production Centers:

There may be mainly two types of changes in the production centers. The first type considere all changes in the characteristics of the centers by keeping the structure of the plant fixed. The second type includes all changes in the structure by increasing or decreasing the number of production lines and assembly centers.

The nonstructural changes only require a rearrangement of the values of some variables.A change in the capacity of any production center,namely in array IC(J),in the maximum available time of a center,namely in array TMAX(J),and in the amount of production of m'th product's n'th part on the centers,namely in T1(M,N), are the possible nonstructural changes.These do not require any updates in programs,but only in the values of the above mentioned variables in data cards.The size of the problem is not affected at all.

If the number of production centers differs from the specified values,it means that a structural change is arised.These must be analyzed in two groups: changes in the production lines and changes in the assembly centers.

If the number of production centers is changed,the variable JJ denoting the total number of production centers,is changed by ΔJJ.Since this is a structural change in the plant,it implies the need of updating the entries of the two matrices A(M,J) and H(M,N),both specifying the precedence relations,i.e. the flow of materials in the plant.The updates will be in the form of adding or deleting a row in matrix A(M,J);for H(M,N),it appears as multiple entries for some (M,N) pairs,since the endproducts are all held fixed in this case or equating some entries to zero.The capacity of the production lines in question must also be rearranged either by equating some elements of IC(J) to zero,for the case where some lines are deleted,or adding some new entries,for the case where new lines are added.Respectively the units (M,N) that can be manufactured in one time interval on an appropriate line which is specified as Tl(M,N),must also be updated.As mentioned in the previous chapters,the parts produced on the lines are sent to assembly centers,so a change in the number of these lines causes changes in the matrix IR(M,N),which specifies the number of constraints balancing the stockkand production in assembly centers.The required updates must follow the complete formulation of the problem with the new values.

The change in the size of the problem will be:

$-\Delta$JJ x TT	: Change in the second type of constrai
-(Number of (M,N) passing through ΔJJ) x TT	: Change in the third type of constrain
-(Number of positive entries in H(M,N) - MN)xTT	: Change in the number of X_{mnjt} variabl
-(Number of positive entries in H(M,N) - MN - MM) x TT	: Change in the number of ST_{mnt} variabl

The last possibility to be analyzed is the change in the number of assembly centers when it is assumed that the values of all other fundamental variables such as MM,TT,and the number of production lines remain fixed.Almost the same updates as above are required if the number of assembly centers is changed.JJ will be JJ + ΔJJ,IC(J),Tl(M,N),H(M,N),A(M,J) undergo similar changes, and NML,denoting the number of the assembly centers,will be NML + ΔJJ.

The change in the size of the problem will be:

$-\Delta$JJ x TT	: Change in the second type of constrain
-(Number of endproducts passing ΔJJ) x TT	: Change in the third type of constrain

-(Number of (M,N) pairs to be assembled on ΔJJ) x TT : Change in the number of X_{mnjt} variables

-(Number of positive entries in H(M,N) - MN - MM) x TT :Change in the number of ST_{mnt} variables

8.7. CONCLUSIONS:

As far as the model is considered in its boundaries,under the set of assumptions,and with respect to the desired output,it could be conluded that it is an efficient and a usefull model. The necessary computer time is between 5 and 11 minutes at a total cost of 1,500 TL to 2,500 TL.The flexibility of the model to handle small changes in the existing plamt and in cases similar,in general,to the production of the selected company enlargens the scope of its applicability.

To summarize,the following outputs can be obtained,

- the time interval,further the percentage of that time interval, during which each part is produced on the appropriate production center,
- the percentage idle time of each production center in each time interval,
- the starting inventory level,the amount produced in that time interval and the ending inventory level for each part in each time interval,
- the overall utilizations of the production centers in the entire planning horizon,

- total production level of each endproduct realized in the whole planning horizon,
- possible foreseen changes in the objective function value that can be obtained by changing the right hand sides of the constraints or the activity levels of the variables,
- reasons for the infeasibility,if an infeasible solution is obtained,

with the following inputs:

- the scheduled and unscheduled demand levels for the products,
- the inventory levels of the parts and/or endproducts initially,
- precedence relations in the plant,
- the endproducts and their structural characteristics,
- the characteristics of the production centers,i.e. number and capacity of the production lines and/or assembly centers,
- the cost coefficients,i.e. production cost per item,stock holding cost per item per period,idle time cost of production centers per period and surplus cost per item,

and changes in the objective function,number of products to be produced,length of planning horizon,and characteristics and number of production lines and assembly centers can be handled as described in this chapter.

8.8. Extensions:

The production planning model is only a subsystem of a complete system covering all problems of a manufacturing company. For a facility-wide system, an inventory model organizing the in- and out-flows of raw-materials and all other subsidiary substances must be undoubtedly developed. The optimum inventory level for each item should be determined by this inventory model. This information should be an input to the production planning model and thereon the production planning model must arrange itself to fit the results of the inventory mpdel and vice versa.

A market research program must also precede both the inventory analysis and production planning. By analyzing the output of the production planning model, the capacity utilization of the factory appears in a clear way. If there is underutilization, then in order to increase utilization, a second market research program may be employed to find new products with existing production facilities. Another way of eliminating the lag of utilization of the facilities is to input an extra data, namely the selling prices of different products, so that the model minimizes the idle time by selecting products with highest marginal profit. A work-force level scheduling model may also be developed to control capacity utilization.

As in all factories, the production facilities must undergo maintenance and repair in predetermined periods. According to the plan, the idle periods for each of the facilities are obtainable.

So,in corporation with this information,a maintenance and repair plan may be developed.

A very up-to-date problem of Turkey,as other countries in the world,is the energy crisis.Incorporating all other considerations an energy optimization model could also be designed.

To summarize,an inventory system,a market research program, a maintenance and repair plan,a work-force level scheduling program and an energy optimization model may be some of the possible extensions to the existing model in this study.

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A P P E N D I X 1

User's Manual

User's Manual

The computer program is written in ASC-II FORTRAN. It is planned to be run on UNIVAC 1106/EXEC-8 under the assumption that the FMPS package is available. There exist four computer programs that must be run one after the other. The main input is read from cards and this information is converted via the FMPS package and by the other three programs to the desired results. The prepared computer program can handle different cases that are classified mainly in three groups. The inputs creating the differences in these cases are the initial stock levels and unscheduled demand levels for parts and endproducts.

-First Group : This may be called the initial case. Here, there does not exist any initial stock for the endproducts or for their parts. Besides there is no unit whose production has a higher priority than the others; in other words, the unscheduled demand of all the endproducts is equal to zero.

-Second Group : This case considers all cases after some planning horizons are passed. This means that there exists a starting inventory level of some parts or some endproducts. On the other hand, if the unscheduled demand of some endproducts or of all the endproducts is greater than zero, the program can handle it.

-Third Group : If during a planning horizon any change occurs and it is desired to see the effects of these changes in the rest of the planning horizon, this version of the program is to be used.

The sequence and the way of specifying the input are explained in the following sections.

1) FIRST GROUP

1.1)* First program :

-Purpose: Creating the input file required for FMPS

-Features:

a) The program checks first the feasibility of the problem by comparing the capacity of the plant with the total amount of production to be realized.

b) The program finally checks the feasibility after the lower bounds are specified and reorganizes the calculated lower bounds to guarantee a feasible solution.

c) The program sets the lower bounds only then there exist any unscheduled demand for any one of the endproducts.

-Input Medium : Cards

-Output Medium: File named 'INCREPR'

-Input : The main information that must be specified for this program are the ones required for a Production Planning Model in general. The structur of the plant, capacity of the production centers, types and characteristics of the endproducts, the sequence of operations to be followed, the amuont of production to be realized, initial inventory levels, for the parts and the endproducts and the variables required for programming are the main inputs.

1st Data Card: It is divided into six fields, each being interger and specified by I3.

1. Field: MN denoting the total number of parts produced
2. Field: TT denoting the total number of time intervals in the planning horizon.
3. Field: JJ denoting the total number of production centers
4. Field :MM denoting the total number of endproducts produced
5. Field:NM denoting the maximum number of parts an end-product may have
6. Field: NML denoting the number of the assembly centers

All the variables defined on this data card may take values between 1 and 999.

2nd Data Card: This data card is required for the programming of this problem specifically. It is divided into 10 fields, each of them are integer and specified by I4.

1. & 2. Field: IK and IKK denoting the number of constraints in which X_{mnjt} is inserted, where t=1, j=6, m=1

3. & 4. Fields: IK3 and IK4 denoting the number of constraints in which X_{mnjt} is inserted, where t=1, j=6 and m=2 or 3.

5. & 6. Fields: IKK and IKK1 denoting the number of constraints in which X_{mnjt} is inserted where t=1, j=7 and m=1,2 or 3.

7. & 8. Fields: I3 and I4 denoting the number of constraints in which X_{mnjt} is inserted, where t=1, j=5, m=2

9. & 10. Fields: I5 and I6 denoting the number of constraints in which X_{mnjt} is inserted, where t=1, j=5, m=3.

3rd Data Card: The array specifying the number of parts making up each endproductis given. All the values are integer and is specifed by I1. There are (MMxI1) values on the data card. Each value can be between 1 and 9.

4th Data Card: There are 3 cards in this deck. T1(M,N), the number of units produced from the nth part of the mth endproduct in one time interval are specified. Each of them are real values (F6.0). 13 values are given on each data card.

5th Data Card: The matrix H(M,N), denoting the production center j, on which the nth part of the mth endproduct is produced, is specified. The matrix consists of MM columns and NN rows (11x5). It is represented column by column.

6th Data Card: The matrix A(M,J), denoting the number of part n which belongs to endproduct m and produced on production center j, is specified. The matrix consists of MM columns and JJ rows (11x7). It is represented column by column.

7th Data Card: The array specifying the demand levels of each m'th endproduct is given. There are 11 fields each is specified by F5.0 and they are all real values.

8th Data card: This data card deck is required for the programming of this problem specifically. By the deck, composed of 3 cards, the number of constraints in which ST_{mnt} is inserted(t=1 for all variables), is determined. The matrix IR(M,N) (11x5), is specified column by column. Zero values mean that there does not exist such a constraint.

9th Data Card: This card is divided into two fields, both denoting constants to be used in this program. The format is 2(F3.0,1X).

10th Data Card: This deck is composed of 5 cards. The objective function coefficients of X_{mnjt} variables are specified. Since they are the same for all t's, there exist 35 values. On each data card, there are 7 values and the format of each value is F4.0.

11th Data Card: This deck is composed of 3 cards. The objective function values of ST_{mnt} variables are specified. The value of the variables are the same for all t values and there exist 16 real values. The format is F5.0 .

12th Data Card: The value ISD, denotes the total number of parts which may have an initial stock. It is an integer value and specified by I3. It may take values from 1 to 999.

13th Data Card: The values of initial stocks for the parts IS(I), are specified. There are two cards each having 20 values.

14th Data Card: The level of unscheduled demand IUSD(M), of all products is specified. The values are integer and is specified by I6.

15th Data Card: The array ML(J), denoting the individual assembly centers, is specified. The values are integer and specified by I3.

16th Data Card: The initial stock level of the endproducts ISE(M), is specified. There exist 11 fields on this data card and each is I6.

17th Data Card: IC(J) denoting the capacities of the production facilities are specified. Each field out of 7 fields is specified by I5.

1.2) Second program:

-Purpose: Solving the LP problem by FMPS package

-Input Medium: File 'INCREPR'

-Output Medium: File 'SOLFILE'

Using the input file created by the first program, the solution file 'SOLFİLE' is prepared.

No exageneous input is required for this program.

1.3) Third program :

-Purpose: Transferring the 'SOLFILE' to 'SOLFILE1' and then to 'OUTPUTPR' where only the optimum values of the variables are stored.

-Input Medium : File 'SOLFILE'

-Output Medium: File 'OUTPR'

No exagenous input is required for this program.

1.4) Fourth program:

-Purpose: Creation of the output

-Features:

a) This program prepares an output in the form specified by the management. The allocation of the parts to the production facilities in each time interval is given.

b)The program secondly, calculates the utilization of the production facilities individually in the entire planning horizon.

c) Finally, the program calculates the utilization of the duction facilities in total, the total production realized from all types of the endproducts.

-Input Medium : File 'OUTPR'

-Output Medium: Listing from the printer

-Input : Most of the inputs to be fed to this program are identical to the ones specified for the input creation program.

1st Data Card: This data card is divided into five fields, each of them is integer and the format isI4.

1. Field: The variable MM, denoting the total number of endproducts to be produced is specified.

2. Field: The variable TF, denoting the total number of time intervals in the planning horizon is specified.

3. Field: The variable JF, denoting the total number of production facilities in the plant is specified.

4. Field: The variable MN, denoting the total number of parts to be produced in the entire planning horizon is specified.

5. Field: The variable MNS, denoting the total number of work-in process units in the plant is specified.

2nd Data Card: The array NN(M1), specifying the maximum number of parts making up each endproduct M1, is specified. Each endproduct may have parts between 1 and 9, since each element of the array is specified by the I1 format.

3rd Data Card: There are three cards in this data deck. T1(M,N) the number of units produced from n'th part of the m'th endproduct in one time interval, is specified. Each of them are real values and specified by F6.0. 13 values are given on one data card.

4th Data Card: The matrix A(M,J), denoting the number of part n which belongs to the m'th endproduct and produced on the j'th production facility, is specified. The matrix consists of MM columns and JF rows(11x7). It is represented columnwise.

5th Data Card: There are 3 data cards in this deck.By these cards ENDP(M1), denoting the names of the endproducts are specified. Each name is 16 characters long. There must be five names on one card.

6th Data Card: There are seven cards in this data deck. The names of the parts PART(M1,N1), are specified by these cards. Each name may be at maximum 16 characters long and there exists five names on one card.

7th Data Card: There are two cards. The initial stock levels IS(M1,N1), of the parts and endproducts are specified, each being I4 long. It is a matrix representation which is represented columnwise. There are 20 data values on one data card.

8th Data Card: The array specifying the demand levels of the m'th endproduct is given. There are 11 fields, each of them is a real value with a format of F5.0.

9th Data Card: The level of unscheduled demand for all of the endproducts are given,each being I6.

2) <u>SECOND GROUP</u>

This group considers all cases if any decisional changes occur in the beginning of the planning horizon. Thus, by changing the input data, the solution can be obtained immediately.

3) <u>THIRD GROUP</u>

If any change occurs during a planning horizon, this version is to be used. The steps to be followed to obtain the solution reflecting the changes during the planning horizon are given below:

-Step 1: The listing of 'INCREPR' is obtained.

-Step 2: The required updates must be realized.

a) The coefficients of all variables prior to that time of change(t_o) are deleted from the objective function and from all constraints.

b) The amount of endproducts that have already been manufactured are deducted from the demand level of the corresponding endproduct or input as ISE(M1) variables.

c) The initial inventory level being equal to the ending inventory level at time t_o for parts must be updated.

d) The unscheduled demand for any product, if there exists any, must be updated or otherwise equated to zero.

-Step 3: The FMPS routine and the other two programs must be run.

-Step 4: The required updates for the fourth program, OUTPUT PROGRAM, must be realized. The variables D(M), IUSD(M), IS(M,N) must be updated. Finally, the starting time interval IT, must be specified.