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FINITE ELEMENT MODEL FOR BRICK WALLS POST-TENSIONED BY EXTERNAL STEEL BARS

THESIS

Khalil Nijem

BOĞAZİÇİ UNIVERSITY
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BY

KHALIL NIJEM

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FINITE ELEMENT MODEL FOR BRICK

WALLS POST-TENSIONED BY EXTERNAL STEEL BARS

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ABSTRACT

A finite element model was developed to study the behavior of masonry walls externally post-tensioned by vertical steel bars under lateral loading in the plane of the wall. Material non-linearity of mortar and non-linearity due to failure of mortar joints and splitting of bricks are taken into consideration. Both the cracking pattern and post-cracking behavior can be studied. Comparison with analysis of unreinforced masonry wall was done. It is concluded that the model gives realistic prediction about the behavior of walls since analysis results conform to the expected behavior, in that, with post-tensioning the cracking strength, stiffness and post-cracking load capacity increased, and under service loads tensile stresses doceasead significantly.

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ÖZET

Düşey çelik çubuklara dışardan ön gerilme verilmiş tuğla duvarların, duvar düzleminde yatay yükler altında analizi için bir sonlu elemanlar modeli geliştirilmiştir. Harcın malzemesel lineer olmayan davranışı ile tuğlaların ve tuğla-harç birleşme yüzeylerinin kırılmasından doğan lineer olmayan davranış göz önüne alınmıştır. Modelle gerek çatlama şekilleri, gerek çatlak sonrası davranış incelenebilmektedir.

Analiz sonuçları beklenilen davranış ile uyum içinde olduğundan, modelin duvarların davranışı hakkında gerçekçi tahminler yapabildiği sonucuna varılmıştır. Şöyle ki, öngerilme duvarların çatlama mukavemetini, rijitliğini ve çatlak sonrası yük kapasitesini artırmakta ve çalışma yükleri altında çekme gerilmelerini önemli ölçüde azaltmaktadır.

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LIST OF SYMBOLS

[C]	=	consistent damping matrix
[D]	=	material property matrix
$[D]_b$	=	material property matrix for uncracked brick element
$[D]_{bc}$	=	symbolic matrix for cracked part of brick element
[D']	=	unit joint stiffness matrix
D'_n	=	joint stiffness in the normal direction
D'_s	=	joint stiffness in the tangential direction
{d}	=	nodal displacement vector
$\{\dot{d}\}$	=	nodal velocity vector
$\{\ddot{d}\}$	=	nodal acceleration vector
E	=	Young's Modulus of elasticity
E'	=	instantaneous normal elastic modulus for the mortar
$\{F_{ext}\}$	=	external nodal force vector
$\{F\}_p$	=	nodal pseudoload vector due to cracking
F_n	=	normal force
F_s	=	shear force
{f}	=	initial end load vector
[G]	=	strain matrix
G	=	shear modulus of elasticity
G'	=	instantaneous shear modulus of elasticity
[k]	=	element stiffness matrix
[K]	=	system stiffness matrix
$[k]_b$	=	stiffness matrix for uncracked brick element
$[k]_{bc}$	=	stiffness matrix for the cracked portion of brick

$[k]_{jnt}$ = joint stiffness matrix

$[k]_{rec}$ = rectangular element stiffness matrix

$[k]_{st}$ = element stiffness matrix for steel

$[M]$ = consistent mass matrix

$[N]$ = shape function matrix

$\{P\}$ = vector of force per unit length

$\{P_b\}$ = vector of body forces or distributed forces

P_n = normal force per unit length

P_s = shear force per unit length

$[S]$ = stress matrix

T = wall thickness

t = mortar thickness

$\{u\}$ = element generic displacement vector

U_e = strain energy stored in the element

W_e = work done by external forces on the element

$\{w\}$ = relative displacement vector

w_n = normal relative displacement

w_s = tangential relative displacement

$\{\sigma\}$ = stress vector

$\{\sigma\}_{bc}$ = stress vector of the cracked portion of bric

$\{\sigma\}_b^p$ = principal stress vector for brick

$\{\sigma\}_{bc}^p$ = principal stress vector for the cracked bric

σ_n = normal stress

σ_s = tangential stress

$\{\epsilon\}$ = strain vector

ϵ_n = normal strain

ϵ_s = tangential strain

$[\Delta]$ = operator matrix of derivatives

π = system potential energy

π_e = element potential energy

μ = Poisson's ratio

1. INTRODUCTION

1.1 GENERAL

In both developed and developing countries, masonry elements are largely in use especially as partition elements, ret- ng walls, and culverts. In countries where reinforced concret industry is not very advanced, the use of masonry ranges from 20%–70%, and sometimes goes up to 90% in some rural ares. Design and construction in these areas depend largely upon experience inherited as well as gained, rather than theory or code. In fact, it has been found out that most catastrophic failures in eartquakes have been of such unreinforced masonry buildings. On the other hand, if masonry were properly reinforced and confined, quite large deformations can be accommodated with some cracking but without collapse.(7)

Prestressed brick walls are highly recommended in areas of high seismic action in place of unreinforced walls.

Although the mechanical properties of materials improve with increasing rate of application of load, a member may fail because of fatigue at a stress smaller than the yield point of the matirial under many repetitions of loading.[8] A crack forms at a point of high stress concentration. As stress is repeated the crack slowly spreads, until the member ruptures without measurable yield. Fatigue strength of a material may be improved by prestressing in such a way as to introduce favorable internal stresses.

Bricks as well as mortar are basicly compressive materials with low and unreliable tensile strengths. Prestressing plies a precompression to the wall or member which reduces or eleminates undesirable tensile stresses that would otherwise be present. Moreover, cracking can be minimized if not avoided under service loads. Deflections, as well, may be limited to an acceptable value. However, in order to specify seismic forces properly, the behavior of prestressed walls under lateral loads must be studied, especiallythe post cracking behavior. The behavior of such walls is complicated by the presence of such factors as the nonhomogeniety of the system, the low stren of bricks in tension, the nonlinear properties of bricks and mortar, and the influence of creep and shrinkage.

Considerable research has been done to investigate experimentally the behavior of masonry under typical lateral and vertical loadings[5,6,15]. Meli[7] has investigated unreinforced, reinforced and precompressed masonry walls under stati and dynamic loadings experimentally. Page[10] has formulated a finite element procedure based on a joint element adopted fro

Goodman and Taylor(3). He has assumed that failure can occur only through the weaker joints. He has not accounted for any rupture or cracking occurring in brick elements. This is generally true for unreinforced masonry walls. While it was shown experementally that when precompressive forces are high or when brick elements are relatively weak, crack lines pass through brick elements(4,7).

1.2 OBJECT AND SCOPE

In this study, taking some experimental results(4) into consideration, a finite elementmodel is developed for the analysis of masonry walls post-tensioned by external, vertical steel bars under lateral loads in the plane of the wall. The complications mentioned above, except creep and shrinkage, are taken into consideration. The influence of creep and shrinkage would require a more elaborate analysis where the time factor is taken into consideration. The model developed here accounts for the failure of bricks by splitting as well as the failure of joints.

The model predicts the cracking and post-cracking behavior, and the deformational and stress charecteristics of masonry walls. Because the non-linear behavior of masonry is load path dependent, an incremental solution was adopted. At each increment, an iterative procedure is followed to check for both cracking and crack propagatin. Three types of finite elements are used in the model. For bricks, a rectangular plain stress element with eight degrees of freedom is used. The joint element adopted in the model is a line element first develope by Ngo and Scordellis[9] for the analysis of reinforced concrete. It was adopted by Goodman and Taylor[3] for modeling rock and later by Page[10] for the analysis of masonry walls. For steel a bar element with two degrees of freedom is used.

In this study, the effect of post-tensioning is investigated by comparing the behavior of an unreinforced and a post-tensioned masonry wall of the same dimensions. Results of non-linear and linear analyses are also compared.

The model proposed here can be used for future research to identify and analayze the effect of different parameters on the capacity of post-tensioned walls. Some of these parameters that influence the behavior of masonry are post-tensioning stre area of steel tendons, spacing and number of tendons, and the dimensions of wall panels.

2. PROPERTIES AND BEHAVIOR OF

MASONRY

The behavior and general properties of masonry have been discussed in detail previously(4,5,7,10,11,15). Some relevant properties will be mentioned here to furnish a background for the mathematical model.

In the following sections, first the general behavior of masonry is discussed, and then the corresponding properties of brick and mortar are briefly stated.

2.1 MASONRY

Masonry is a nonhomogeneous two phase material of relatively elastic bricks linked together with an inelastic motar matrix. At high stresses masonry starts to behave nonlinearly as a result of the nonlinear force-deformation characteristics of the joints under shear and compression, the local failure and slip that occurs in the joints, and the splitting failure of bricks. At the same time, redistribution of stress occurs which leads to a loss of bond between mortar and bricks; thus localized failure occurs which propagates to other elements as the load is increased.

Failure of post-tensioned or prestressed masonry panels usually occurs either in joints, in bricks, or a combination of both. Failure in joints occurs when a tensile or shear bond strength criterion is violated. Failure in bricks, however, occurs by splitting when the tensile stress of bricks in one of the principal directions exceeds the tensile strength of brick. This is caused by the differential lateral deformation of the stiffer brick and the relatively more flexible mortar matrix.

In addition to the effect of prestressing, the performance and durability of masonry depend upon other factors such as the quality of materials, that is, of bricks, mortar and post-tensioning steel. They also depend on the design, workmanship and methods followed in manufacturing and construction. In addition, the strength of a masonry panel depends largely on the ratio of compressive to shear forces. It is found that with higher compressive stresses the capacity of walls to carry lateral load after cracking increases with increasing the number of prestressing tendons(4). The increase in cracking load due to precompression has been found to be approximately 40% of the total vertical load applied. However, although precompression on the wall causes an increase in strength, for high vertical stresses, behavior tends to change to a brittle shear failure[7]. It has also been found out that the strength and deformation characteristics of masonry depend highly on the orientation of stress, that is, τ/σ ratio, defining a friction coefficient independent of the type of mortar[5,6,7]. It should be noted also that for walls with low precompression or low vertical reinforcement, failure is governed by yielding of reinforcement which is similar to an underreinforced concrete beam[4,7].

Some of the properties of the component materials that affect the behavior of masonry are discussed briefly in the following two sections.

2.2 BRICKS

The properties of bricks produced in different localities vary even for bricks from the same batch, depending upon workmanship and the clays and shales used in manufacturing. This causes a scatter of results of the mechanical properties obtained. Bricks have been found to exhibit elastic brittle behavior. They are not necessarily homogeneous or isotropic[10]. The properties of bricks which affect the performance of walls are the compressive and tensile strengths, and water absorption and saturation coefficients. Properties that affect the bond between mortar and brick are the rate of absorption when laid and the surface texture of bricks[11].

2.3 MORTAR

The strength of mortar is usually less than that of bricks. Its stress-strain relationships for compression and shear are non-linear which is responsible for the non-linear behavior of masonry before cracking. Mortar has a low tensile but high compressive and shear strengths which are functions of the bond strength between mortar and brick, and the superimposed compressive force.

The bond strength is usually the critical factor for precompressed or post-tensioned masonry. Test results of masonry bonds give a wide range of results which are due to the large numbers of variables, some of which are hard to detect or control, such as flow of mortar, elapsed time between spreading mortar and placing brick in contact with it, pressure or tapping applied to joint during forming, texture of brick surface, and other factors which have not been identified.(11) Bond strength is found to increase with the flow increase. Thus loss of water from mortar due to evaporation after mixing decreases the flow which in turn reduces the tensile bond strength. Workmanship exerts a large influence on this. This effect is one of the reasons that experimental results obtained are so much scattered.

3. FINITE ELEMENT MODEL

The masonry wall is assumed to be a continuous non-elastic medium. The stresses and strains are continuous all through the wall except for the cracked parts of the system. In order to be able to represent the equations of equilibrium of the system in the form of linear simultaneous equations, this continuous medium is idealized into a mathematical model consisting of a finite number of elements interconnected by a finite number of nodal points. Finite elements are introduc as a means of a mathematical model to convert a continuous physical system into a discontinuous discrete system such tha the total potential energy of both the original continuum and the mathematical model are the same.

3.1 FORMULATION AND SOLUTION OF FINITE ELEMENT EQUATIONS - GENERAL PROCEDURE

Finite element method is basicly minimizing the total potential energy of the system with respect to nodal deformations in an affort to find the configuration satisfying equilibrium conditions. In mathematical notation, the potential energy π of an element can be expressed as

$$\pi = U_e + (-W_e) \qquad (3.1)$$

where, U_e = internal strain energy of the element

W_e = work done by external forces

The potential energy of the system is assumed to be equal to the sum of potential energies of the elements, that is,

$$\pi = \sum_{i=1}^{n} \pi_{ei}$$

where, π = total potential energy of thesystem of n finite elements

Expressing the displacements at any point within the element as functions of the nodal displacements, and minimizing the total potential energy, N simultaneous equations are obtained

$$\frac{\partial \pi}{\partial d_i} = 0 \qquad i=i,2,\ldots,N \qquad (3.3)$$

where N is the number of independent nodal deformations in the global system.

The resulting governing equation of motion in matrix notation will be of the following form

$$[K]\{d\} + [C]\{d\} + [M]\{d\} = \{F_{ext}\} - \sum\{f\} \qquad (3.4)$$

where, $[K]$ = System stiffness matrix

$\{d\}$ = nodal displacement vector

$[C]$ = consistent damping matrix

$\{\dot{d}\}$ = nodal velocity vector

$[M]$ = consistent mass matrix

$\{\ddot{d}\}$ = nodal acceleration vector

$\{F_{ext}\}$ = nodal external force vector

$\sum\{f\}$ = initial load vectors including initial strain initial stress, temperature loads, body forces, and/or edge loads

For the solution of statics problems the equations takes the form

$$[K]\{d\} = \{F_{ext}\} - \sum\{f\} \tag{3.4a}$$

The solution procedure is initiated by defining the deformations at each point within a certain element as a function of the nodal deformations of that element, that is,

$$\{u\} = [N]\{d\} \tag{3.5}$$

where $\{u\}$ = element generic displacement vector

$[N]$ = shape functions matrix

$\{d\}$ = element **nodal displacement vector**

The strains inside the element are found as

$$\{\epsilon\} = [\Delta]\{u\} \tag{3.6}$$

where $\{\epsilon\}$ = element strain vector

$[\Delta]$ = operator matrix relating the strains to deformatior within an element

Substituting equation (3.5) in equation (3.6) one gets

$$\{\epsilon\} = [\Delta][N]\{d\}$$

Defining $[G] = [\Delta][N]$, the strains are expressed as

$$\{\epsilon\} = [G]\{d\} \tag{3.7}$$

where $[G]$ = strain matrix relating strains to nodal deformati of the element.

Defining the material matrix, $[D]$, to relate the stresses to strains within the element, stresses are obtained as

$$\{\sigma\} = [D]\{\epsilon\} \tag{3.8}$$

where $\{\sigma\}$ = stress vector.

Substituting the value of $\{\epsilon\}$ from equation (3.7) into equation (3.8), and defining the stress matrix $[S] = [D][G]$, stresses within an element are obtained by

$$\{\sigma\} = [S]\{d\} \tag{3.9}$$

Strain energy U_e stored in an element is defined as

$$U_e = \frac{1}{2}\int_V \{\epsilon\}\{\sigma\} dV \tag{3.10}$$

Substituting the values of $\{\epsilon\}$ and $\{\sigma\}$, the potential energy becomes

$$U_e = \frac{1}{2}\int_V \{d\}^T [G]^T [D][G]\{d\}\, dV \tag{3.11}$$

Defining $[k] = \int_V [G]^T [D][G]\, dV$, the stored potential energy is expressed as

$$U_e = \frac{1}{2}\{d\}^T [k]\{d\} \tag{3.12}$$

where $[k]$ = element stiffness matrix.

For statics problems, the external work can be generally expressed in the following manner

$$W_e = \int_V \{u\}^T \{P_b\}\, dV + \oint_S \{u\}^T \{P_s\}\, dS + \{d\}^T \{F_{ext}\} \tag{3.13}$$

where $\{P_b\}$ = vector of body forces or distributed forces within the element

$\{P_s\}$ = vector of edge loads

and, $\{F_{ext}\}$ = vector of discrete external nodal forces.

Following a simillar procedure to the one described above for the strain energy, external work energy can be obtained. Substituting the results in equations (3.1), (3.2), and equation (3.3), equation (3.4a) is obtained.

More detailed discussion of the formulation of the Finite Elements equations can be found in the literature.[12,14 A list of the basic equations of the Finite Element method is presented in Appendix I.

The non-homogeneous nature of the structure presents no problem for the finite element method, as each element can have a different material property. The stiffness matrices of each component material can be calculated seperately. The global stiffness matrix of the whole structure can be generated using the code number technique[13]. Using the same technique, the global force vector of the system can be generated. Now, the global nodal deformations can be solved for

$$\{d\} = [K]^{-1}(\{F_{ext}\} - \sum\{f\}) \qquad (3.14)$$

Back substituting in equations (3.7) and (3.8) or (3.9 strains and stresses within each element can be obtained.

3.2 MASONRY WALL MODEL

For analyzing masonry walls, a plane stress problem is simulated. The wall panel is divided into discrete finite elements. Subdivision of the wall is presented in Appendix II.

A typical element division is shown in Fig.-1. The joint elements are linked to the neighboring brick elements at the nodes. Since the joint elements are thin, same coordinates are assigned for pairs of nodes. For example, nodes 1 and 4 have the same coordinates, and nodes 2 and 3 share

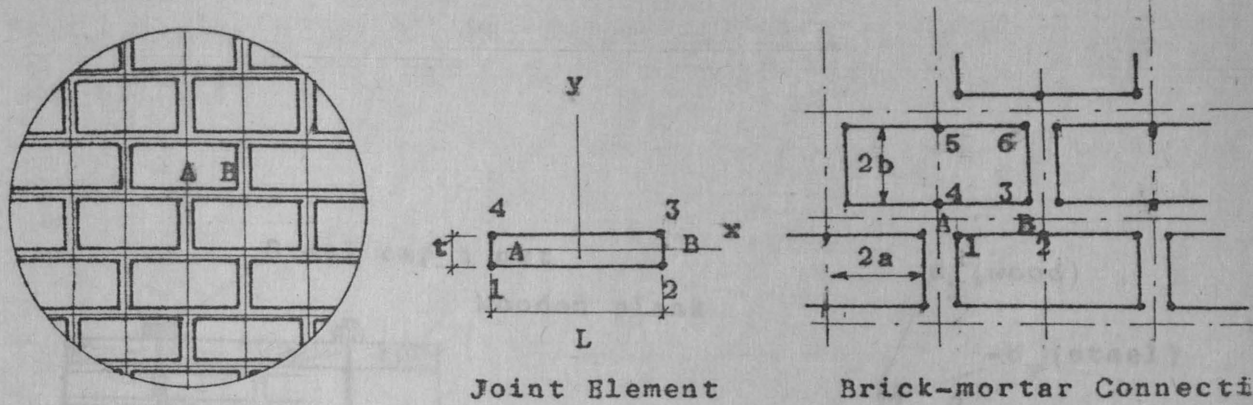


Fig.-1 Typical element division

the same coordinates. The joint thickness t is used only in calculating the joint element properties. The dimensions of the brick should be corrected by the appropriate proportion of joint thickness.

In the experiments done by Hacım(4), post-tensioning was provided by tensioning external vertical steel tendons. Vertical loads on the wall applied by the tendons were distributed to the upper edge of the wall with the help of a wooden plank (Fig.-2). To simulate the experimental problem, relative stiff wood elements are linked to the wall at the top. In Fig.-3, node 1 of the wood element and node 2 of the steel element share the same coordinates. To satisfy continuity, the two nodes are assumed to share the same deformations in the x and y-directions. But, while generating the global stiffness matrix, they are assumed to have opposite deformations in the y-direction. Thus, a tensile force applied

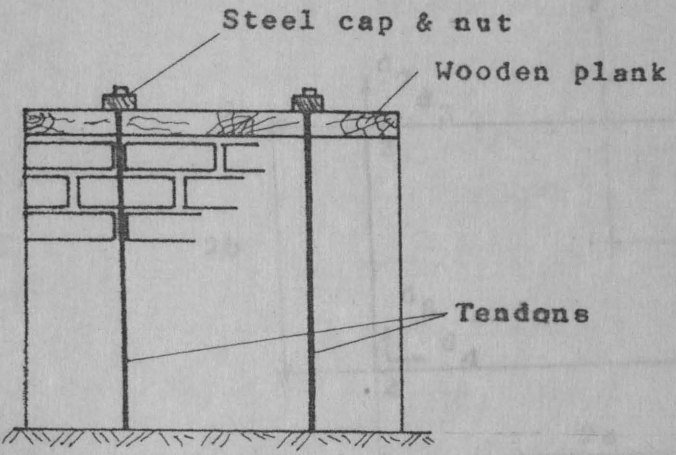


Fig.-2 Typical Post-tensioned Wall

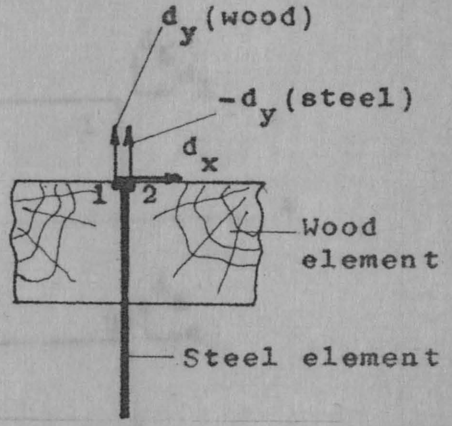


Fig.-3 Steel-wood connecti

to the steel element will, at the same, time constitute a compressive force on the wood element, and thus, a compressive force on the wall.

3.3 MATERIAL PROPERTIES AND THE CORRESPONDING STIFFNESS MATRICES

3.3.1 BRICKS

The bricks are considered isotropic and elastic material. The variability of brick properties and anisotropy is neglegted in the analysis. Average values of 68460 kg/cm^2 for Young's modulus, E, and 0.167 for Poisson's ratio, μ, are used[10]. Rectangular plane stress elemen: with isotropic elastic properties is used. The rectangular element suits the rectangular shape of the bricks. A typical rectangular element is shown in Fig.-4.

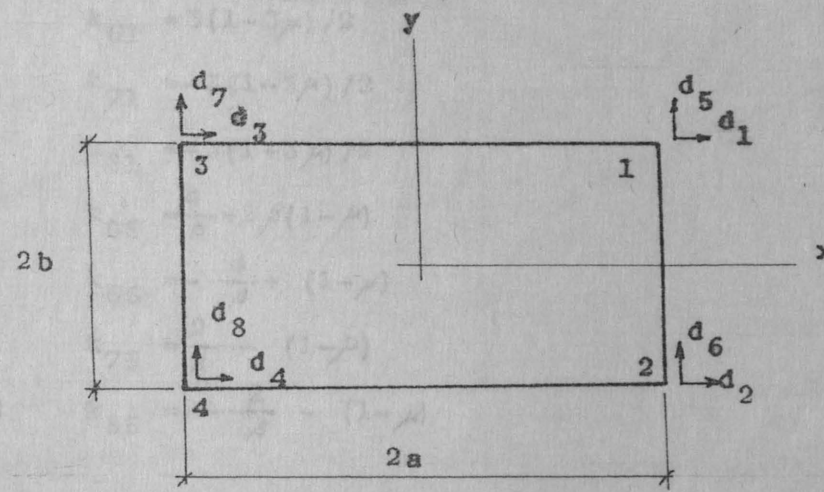


Fig.-4 Typical rectangular finite element

Fig.. typical rectangular finite element

Defining the a. ratio $\beta = \frac{b}{a}$, the shiffness matrix for an element of thicknesst, be obtained as follows,(14)

$$[k]_{rec} = \begin{bmatrix} k_{11} & & & & & & & \\ k_{21} & k_{11} & & & & & & \\ k_{31} & k_{41} & k_{11} & & \text{SYMMETRIC} & & & \\ k_{41} & k_{31} & k_{21} & k_{11} & & & & \\ k_{51} & k_{71} & k_{61} & k_{81} & k_{55} & & & \\ k_{61} & k_{81} & k_{51} & k_{71} & k_{65} & k_{55} & & \\ k_{71} & k_{51} & k_{81} & k_{61} & k_{75} & k_{85} & k_{55} & \\ k_{81} & k_{61} & k_{71} & k_{51} & k_{85} & k_{75} & k_{65} & k_{55} \end{bmatrix} \frac{Et}{12(1-\cdots}$$

where,

$$k_{11} = 4\beta + 2\,(1-\mu)/\beta$$

$$k_{21} = -2\beta - 2\,(1-\mu)/\beta$$

$$k_{31} = -4\beta + 2\,(1-\mu)/\beta$$

$$k_{41} = -2\beta - (1-\mu)/\beta$$

$$k_{51} = 3(1+\mu)/2$$

$$k_{61} = 3(1-3\mu)/2$$

$$k_{71} = -3(1-3\mu)/2$$

$$k_{81} = -3(1+3\mu)/2$$

$$k_{55} = \frac{4}{\beta} + 2\beta(1-\mu)$$

$$k_{65} = -\frac{4}{\beta} + (1-\mu)$$

$$k_{75} = \frac{2}{\beta} - (1-\mu)$$

and

$$k_{85} = -\frac{2}{\beta} - (1-\mu)$$

3.2 MORTAR (JOINT ELEMENTS)

Due to the non-linear behavior of mortar, its force-deformation characteristics in shear and in compression should be determined experimentally on masonry walls after the average properties of bricks are known. The joint model is expressed as a function of two parameters defining the behavior of the mortar in the directions perpendicular (normal) to the joint and parallel (shear) to the joint. Figures 5 and 6 show the results obtained by Page[10] for the behavior of mortar in compression and shear respectively.

The idea of using joint elements was first introduced into the finite element method by Ngo and Scordellis[9] for the analysis of reinforced concrete members. Later special linkage elements were developed and used in the analysis of jointed rocks[3], and masonry[10].

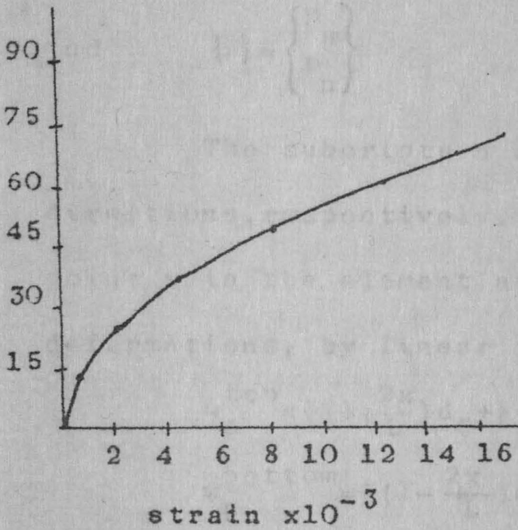


Fig.-5 Stress-strain curve for mortar in compression

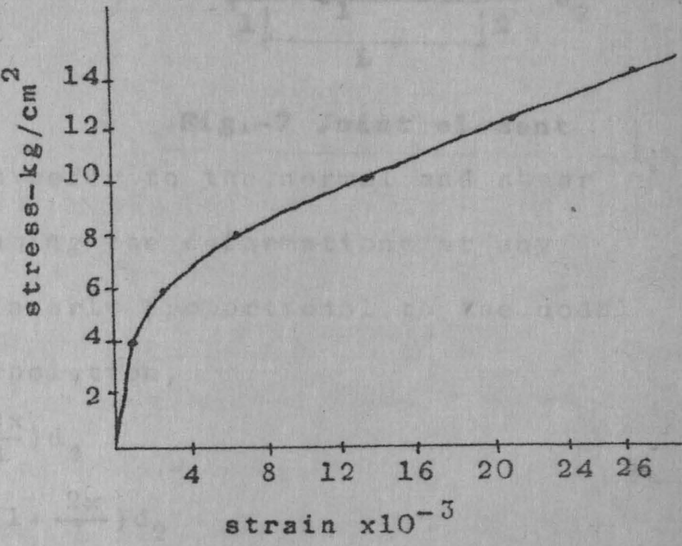


Fig.-6 Stress-strain curve for mortar in shear

The joint element is a line element of a length L and thickness t. The potential energy stored per unit length is expressed as the applied force per unit length multiplied by the deformations. For a joint of length L, the strain energy U_e is obtained as,

$$U_e=\frac{1}{2}\int_{-L/2}^{+L/2}\{W\}^T\{P\}dx \tag{3.15}$$

where, $\{W\}$= relative displacement vector

$\{P\}$= vector of force per unit length

The joint element is assumed to deform only in the shear and normal directions (x and y in the local coordinates of Fig.-7). Therefore,

$$\{w\} = \begin{Bmatrix} \Delta w_s \\ \\ \Delta w_n \end{Bmatrix} \begin{Bmatrix} w_s^{top} - w_s^{bottom} \\ \\ w_n^{top} - w_n^{bottom} \end{Bmatrix}$$

and $\{P\} = \begin{Bmatrix} P_s \\ P_n \end{Bmatrix}$

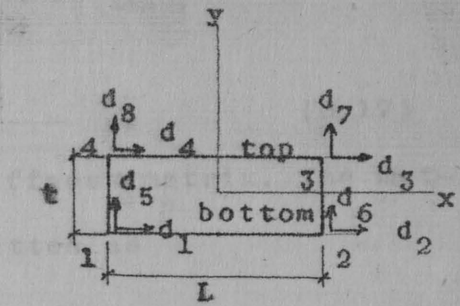


Fig.-7 Joint element

The subcripts n and s refer to the normal and shear directions, respectively. Assuming the deformations at any point x in the element are linearly proportional to the nodal deformations, by linear interpolation,

$$w_s^{top} = \tfrac{1}{2}(1+\frac{2x}{L})d_3 + \tfrac{1}{2}(1-\frac{2x}{L})d_4$$

$$w_s^{bottom} = \tfrac{1}{2}(1-\frac{2x}{L})d_1 + \tfrac{1}{2}(1+\frac{2x}{L})d_2$$

Defining $A=(1-\frac{2x}{L})$ and $B=(1+\frac{2x}{L})$, then

$$w_s = w_s^{top} - w_s^{bottom} = \tfrac{1}{2}\begin{bmatrix} -A & -B & B & A \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix}$$

Similarly,

$$w_n = w_n^{top} - w_n^{bottom} = \tfrac{1}{2}\begin{bmatrix} -A & -B & B & A \end{bmatrix} \begin{Bmatrix} d_5 \\ d_6 \\ d_7 \\ d_8 \end{Bmatrix}$$

Defining $\begin{bmatrix} G \end{bmatrix} = \begin{bmatrix} -A & -B & B & A & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -A & -B & B & A \end{bmatrix}$

the vector of relative displacements can be expressed as

$$\{w\} = \tfrac{1}{2}\,[G]\{d\} \qquad (3.16)$$

where $\{d\}$ = vector of nodal deformations.

The vector of force per unit length is related to the relative deformatins by,

$$\{P\} = [D']\{w\} \tag{3.17}$$

where $[D']$ is called the unit joint stiffness matrix. The matrix of material properties can be written as

$$[D'] = \begin{bmatrix} D'_s & 0 \\ 0 & D'_n \end{bmatrix}$$

where D'_s and D'_n are joint stiffnesses per unit length in the tangential and normal directions, respectively.

To understand the meaning of unit joint stiffness, consider a joint of length L and unit width. Apply a normal force F_n and get the normal relative deformation w_n. If w_n is plotted against the applied force per unit length F_n/L, the slope of the curve will give unit normal stiffness for the joint. Similarly, in a direct shear test, if the relative shear deformation w_s is plotted against the applied shear force per unit length F_s/L, the slope of the curve will give unit shear stiffness for the joint.

In other words,

$$F_n/L = D'_n w_n \quad \text{and} \quad F_s/L = D'_s w_s \tag{3.18}$$

If a normal stress σ_n is applied to a wall of thickness T and length L, the total normal force F_n is expressed as

$$F_n = \sigma_n TL$$

and the force per unit length as

$$F_n/L = \sigma_n T \tag{3.19}$$

From Hooke's law

$$\sigma_n = E\epsilon_n \tag{3.19}$$

where E = Young's modulus

and ϵ_n = normal strain.

The normal strain ϵ_n can be obtained by

$$\epsilon_n = w_n/t \tag{3.21}$$

where t = thickness of mortar.

Substituting (3.18) and (3.20) in (3.19), one gets

$$F_n/L = \frac{ET}{t} w_n \tag{3.22}$$

Comparing equations (3.18) and (3.22), D'_n can be written as

$$D'_n = \frac{ET}{t} \tag{3.23a}$$

Similarly, D'_s can be determined as

$$D'_s = \frac{GT}{t} \tag{3.23b}$$

where G = shear modulus of elasticity.

For non-linear stress-strain relation of mortar, D'_n and D'_s will become as follows

$$D'_n = \frac{E'T}{t} \tag{3.24a}$$

and

$$D'_s = \frac{G'T}{t} \tag{3.24b}$$

where E' = instantaneous normal elastic modulus

instantaneous shear elastic modulus.

Substituting equations (3.16) and (3.17) in equation (3.15), the stored strain energy becomes

$$U_e = \frac{1}{2}\int_{-L/2}^{L/2} \{d\}^T [G]^T [D'][G]\{d\}\, dx \tag{3.25}$$

Defining the matrix $[H] = [G]^T [D'][G]$, and performing the triple matrix multiplication, $[H]$ becomes

$$[H] = \begin{bmatrix} D'_sA^2 & D'_sAB & -D'_sAB & -D'_sA^2 & 0 & 0 & 0 & 0 \\ D'_sAB & D'_sB^2 & -D'_sB^2 & -D'_sAB & 0 & 0 & 0 & 0 \\ -D'_sAB & -D'_sB^2 & D'_sB^2 & D'_sA^2 & 0 & 0 & 0 & 0 \\ -D'_sA^2 & -D'_sAB & D'_sAB & D'_sA^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & D'_nA^2 & D'_nAB & -D'_nAB & -D'_nA^2 \\ 0 & 0 & 0 & 0 & D'_nAB & D'_nB^2 & -D'_nB^2 & -D'_nAB \\ 0 & 0 & 0 & 0 & -D'_nAB & -D'_nB^2 & D'_nB^2 & D'_nAB \\ 0 & 0 & 0 & 0 & -D'_nA^2 & -D'_nAB & D'_nAB & D'_nA^2 \end{bmatrix}$$

...rying out the necessary integrations, the terms in the matrix are obtained:

$$\int_{-L/2}^{L/2} A^2 dx = \int_{-L}^{L/2} (1 - \frac{2x}{L})^2 dx = \frac{4}{3} L$$

$$\int_{-L/2}^{L/2} B^2 dx = \int_{-L/2}^{L/2} (1 + \frac{2x}{L})^2 dx = \frac{4}{3} L$$

$$\int_{-L/2}^{L/2} AB dx = \int_{-L/2}^{L/2} (1 - \frac{4x^2}{L^2}) dx = \frac{2}{3}$$

The strain energy can now be expressed as

$$U_e = \tfrac{1}{2} \{d\}^T [k]_{jnt} \{d\}$$

where

$\frac{\partial \pi_e}{\partial d_i} = 0$ for i=1,2,...,8, the equilibrium equations for the element are obtained as

$$[k]_{jnt}\{d\} - \{F\} = 0$$

$$\text{or } [k]_{jnt}\{d\} = \{F\} \tag{3.27}$$

Here $[k]_{jnt}$ is the joint stiffness matrix. It should be noted that the element stiffness matrix for the mortar is a line element derived in the local coordinate system. While the local and global coordinates coincide for horizontal joint elements, the stiffness matrix for vertical joints should be transformed to the global doordinate system using a special transformation matrix before assembling the system stiffness matrix.

Let $[T]$ be the transformation matrix that relates the displacements and forces in the global coordinate system X-Y to the displacements and forces in the local system x-y. Then,

$$\{d\}_{xy} = [T]\{d\}_{XY} \tag{3.28}$$

and,

$$\{F\}_{xy} = [T]\{d\}_{XY} \tag{3.29}$$

Substituting equations (3.28) and (3.29) in equation (3.27) and solving for $\{F\}_{XY}$ one gets

$$\{F\}_{XY} = [T]^{-1}\left[k_{jnt}\right]_{xy}[T]\{d\}_{XY}$$

Therefore,

$$\left[k_{jnt}\right]_{XY} = [T]^{-1}\left[k_{jnt}\right]_{xy}[T]$$

But since $[T]$ is an orthogonal matrix

$$[T]^{-1} = [T]^{T}$$

Thus,

$$\left[k_{jnt}\right]_{XY} = [T]^{T}\left[k_{jnt}\right]_{xy}[T] \tag{3.30}$$

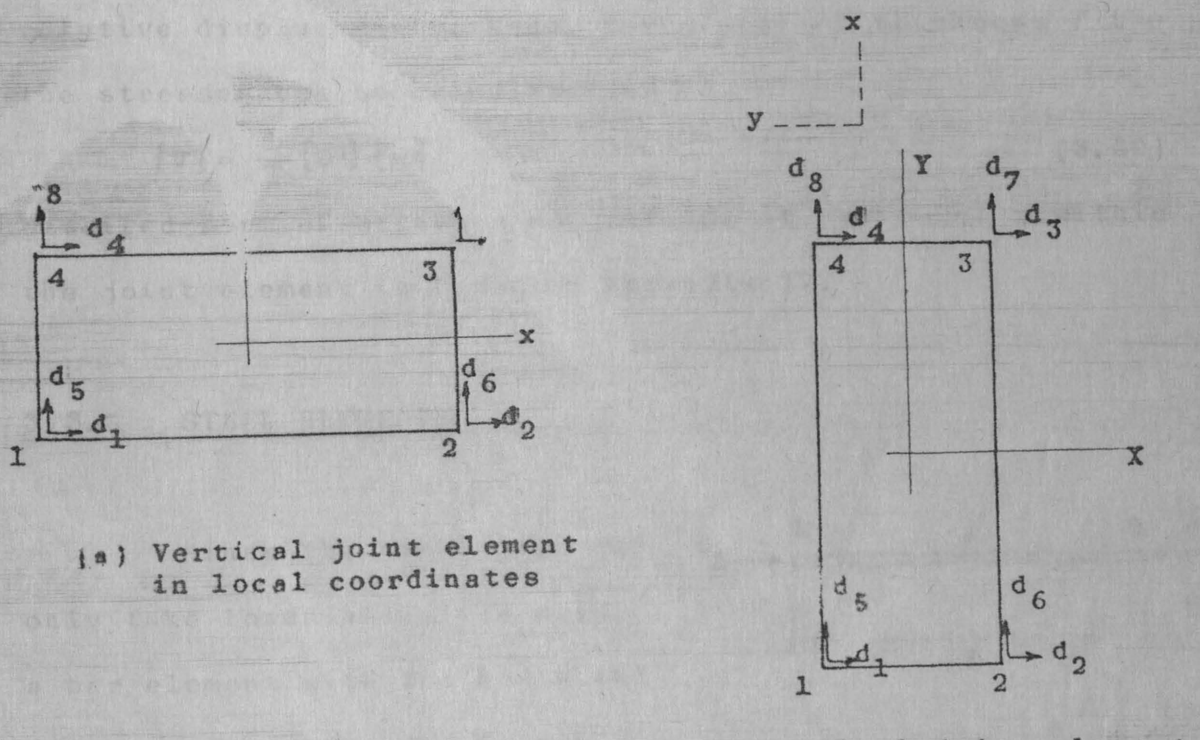
Figure 8(a) shows a vertical element in the local coordinates, while figure 8(b) shows it in the global coordinates. Comparing the two figures, and i:

$$\{d\}_{xy} = [T]\{d\}_{XY}$$

[T]matrix can be written as follows:

$$[T] = \begin{bmatrix} 0 & [T_1] \\ [-T_1] & 0 \end{bmatrix}$$

where
$$[T_1] = \begin{bmatrix} 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$



(a) Vertical joint element in local coordinates

(b) Vertical joint element in global coordinates

Fig.-8 Vertical joint element transformation

Therefore, for vertical joint elemnts, the stiffness matrix is generated in the local coordinates, and using equation (3.30) is transformed to global coordinates. Equations

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(3.28) and (3.29) are used in a similar manner. The stiffness matrix of a vertical element in global coordinates is found in Appendix III.

The stresses and strains at any point x within the element can be found in the usual manner. But it should be noted here that the strain matrix [G] used in the formulation of the joint stiffness relates relative displacements to nodal displacements. Thus the strain vector for a mortar of thickness t is

$$\{\epsilon\} = \frac{1}{2t}[G]\{d\} \qquad (3.31)$$

Equation (3.17) relates force per unit length to relative displacements. Thus, for a wall of thickness T the stresses can be calculated as

$$\{\sigma\} = \frac{1}{T}[D']\{w\} \qquad (3.32)$$

Detailed form of stresses and strains at any point x within the joint element is found in Appendix IV.

3.3.3 STEEL ELEMENTS

Since the steel bar can only take loads along its axis, a bar element with two nodes and two degrees of freedom is used (Fig.-9).

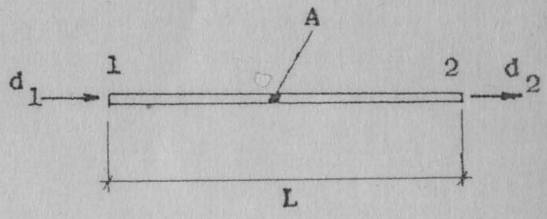

Fig.-9 Typical steel element

Following the usual manner, the element stiffness matrix for steel is found as

$$[k]_{st} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \frac{EA}{L}$$

where, E= Young's modulus for steel

A= Cross sectional area of the bar

L= Length of the bar.

3.3.4 WOOD ELEMENTS

For the wooden plank, rectangular plain stress elements similar to the ones discussed in section 3.4.1 for brick elements are used, but with appropriate mechanical properties for wood. Similar to the bricks anisotropy and non-linearity were neglected. Average values of 125830 kg/cm^2 and 0.3 were used for Young's Modulus and Poisson's ratio, respectively.

4. SOLUTION PROCEDURE

A realistic analysis for internal stress distribution and postcracking behavior of masonry walls should include several complexities such as :

(a) Non-homogeniety of the construction

(b) Non-linearity in the behavior of mortar

(c) The influence of progressive cracking under increasing load

(d) The effect of bond between mortar and bricks

(e) The effect of post-tensioning.

An incremental finite element program is used in the analysis to account for the progressive cracking behavior. At each load level, an iterative solution is used to account for the non-linear behavior of materials. At each iteration both mortar and brick elements are checked for the violation of their respective failure criteria. For the case of mortar joint failure, residual stiffness properties are obtained and the stiffness matrix is updated. As for bricks, nodal psuedo-loads are applied at the nodes of the cracked brick element to account for the stresses to be restrained because of the crack. These are discussed in more detail in the following sections.

4.1 MORTAR FAILURE

Depending on the relative magnitudes of normal and shear stresses, mortar fails either by tensile bond failure or shear bond failure. In tensile bond failure, due to high tensile stresses and low shear stresses, the failure occurs by seperation of joints, while in shear bond failure, failure occurs under a combination of shear and compressive stresses. A joint failure envelope based on experimental results was proposed by Page(10). This failure envelope is shown in Fig.-10.

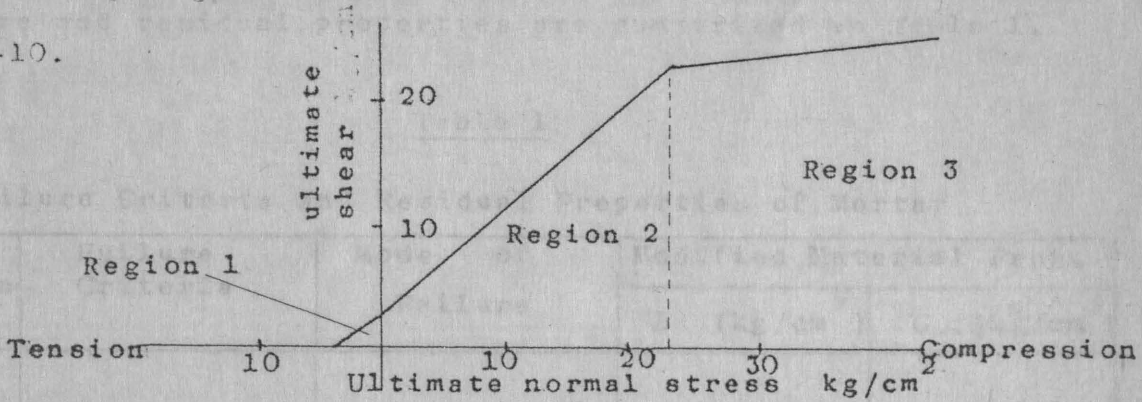

Fig.-10 Mortar failure envelope

Page has produced the following equations for each region in the envelope:

Region 1: $\tau_u = 0.66\sigma_{nu} + 2.31$

Region 2: $\tau_u = 0.87\sigma_{nu} + 2.31$

Region 3: $\tau_u = 0.11\sigma_{nu} + 20.08$

where τ_u = ultimate shear stress in kg/cm^2

and σ_{nu} = ultimate normal stress in kg/cm^2

In each iteration, each joint is checked for failure and appropriate modification[10] to the material properties is applied, and the problem is solved another time. Forexample if the faliure criterion of region 1 is violated, tensile bond failure is assumed. The joint can no more carry any shear or normal stresses. If the failure criterion of regions 2 or 3 is violated shear bond failure is assumed. The joint, in this case, can sustain its normal strength, but it loses some of its strength in shear depending on the magnitude of the normal stress. The shear strength, in this case, is due to the frictional resistance of the goint. For instance, in region 2, and depending on the normal stress; the shear modulus will be $G = 10.868\sigma_{nu}$. In region 3, a constant value of $G = 254.1\ kg/cm^2$ is assumed. The above mentioned modes of failure and residual properties are summurized in Table 1.

Table 1

Failure Criteria and Residual Preperties of Mortar

Region	Fuilure Criteria	Mode of Failure	Modified Material Prop.	
			E (kg/cm^2)	G (kg/cm^2)
1	$\tau_u = 0.66\sigma_{nu} + 2.31$	Tensile bond	0	0
2	$\tau_u = 0.87\sigma_{nu} + 2.31$	Shear bond	No change	$10.868\sigma_{nu}$
3	$\tau_u = 0.11\sigma_{nu} + 20.1$	Shear bond	No change	254.1

4.2 BRICK FAILURE

Cracking in bricks is assumed to occur when one of the principal stresses of the element exceeds the tensile strength of the brick. Thus, in the analysis, the stresses are calculated from nodal displacements for all brick elements and then from these stresses the principal stresses are determined. If the principal stress in any direction for a certain brick element exceeds the ultimate tensile strength, the element considered to have cracked normal to that direction. As brick are assumed incapable of sustaining tension more than the tensile strength, the excess tension is removed. These excess tensile stresses are converted into nodal pseudoloads. During the next cycle, these restraining nodal forces are applied to the structure so as to distribute the excess tensile stresses to the adjacent elements. The method to apply pseudoloads at the nodes was first developed by Colville and Abbasi(1) to be used in the analysis of reinforced concrete. The method is adopted here to account for cracking in brick elements.

The brick element in equilibrium is assumed elastic only in the uncracked part. The potential energy of an element is discribed as

$$\pi_b = U_b - U_{bc} + (-W_e) \qquad (4.1)$$

where U_b = strain energy of uncracked brick element

U_{bc} = strain energy of the cracked part of the brick element

W_e = external work

If the strains are functions of nodal deformations, U_b, U_{bc} and W_e can be written as follows:

$$U_b = \tfrac{1}{2}\{d\}^T(\int_{v_b}[G]^T[D]_b[G]\,dV)\{d\}$$

$$U_{bc} = \tfrac{1}{2}\{d\}^T(\int_{v_{bc}}[G]^T[D]_{bc}[G]\,dV)\{d\}$$

$$W_e = \{d\}^T\{F\}$$

where $\{d\}$ = nodal displacement vector

$[G]$ = strain matrix relating strains to nodal deformations

$[D]_b$ = material property matrix for uncracked brick

$[D]_{bc}$ = symbolic matrix for the cracked part which need not to be explicitly defined

$\{F\}$ = external nodal force vector

v_c = volume of uncracked brick element

v_{bc} = volume of the cracked part of the brick element

Let $[k]_b = \int_{v_b}[G]^T[D]_b[G]\,dV$ = stiffness matrix for uncracked brick element

and $[k]_{bc} = \int_{v_{bc}}[G]^T[D]_{bc}[G]\,dV$ = stiffness matrix for the cracked portion.

Then the potential energy functional can be written as

$$\Pi_e = \tfrac{1}{2}\{d\}^T[k]_b\{d\} - \tfrac{1}{2}\{d\}^T[k]_{bc}\{d\} - \{d\}^T\{F\} \qquad (4.2)$$

Minimizing Π_e to obtain equilibrium configuration by differentiating with respect to nodal deformations

$\frac{\partial \Pi_e}{\partial d_i} = 0$ for i=1,2,...,8, the governing equilibrium equations for the cracked brick becomes

$$[k]_b\{d\} = \{F\} + [k]_{bc}\{d\} \qquad (4.3)$$

Now, nodal pseudoload vector, due to cracking, $\{F\}_p$, can be defined as

$$\{F\}_p = [k]_{bc}\{d\}$$

that is

$$\{F\}_p = (\int_{V_{bc}} [G]^T [D]_{bc} [G] dV)\{d\} \quad (4.4)$$

Recalling that the stress vector in equation (3.9) is defined as

$$\{\sigma\} = [D][G]\{d\}$$

the pseudoload vector becomes

$$\{F\}_p = \int_{V_{bc}} [G]^T \{\sigma\}_{bc} dV \quad (4.5)$$

where $\{\sigma\}_{bc}$ = stress vector of the cracked portion of the brick element.

In the analysis, each brick element is subdivided into nine subregions as shown in Fig.-11. Each subdivision is assumed to have a constant stress distribution. In the $(i-1)^{th}$ iteration stresses in each subregion are computed by

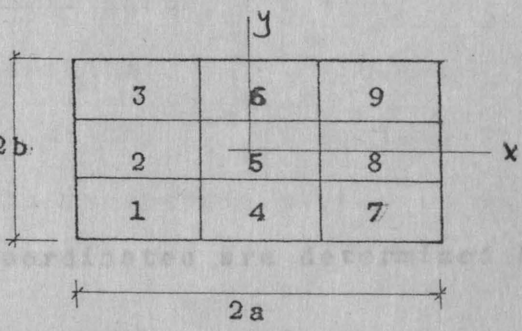

Fig.-11 Subdivision of brick element

$$\{\sigma\} = [D]_b [G]\{d\}$$

Using Mohr's circle, the principal stresses are computed. The principal stress vector will have the form

$$\{\sigma\}_b^p = \begin{Bmatrix} \sigma_{max}^p \\ \sigma_{min}^p \\ 0 \end{Bmatrix}$$

where σ_{max}^p and σ_{min}^p are maximum and minimum principal stresses respectively.

The principal stresses are then compared with the tensile strength f_t of the brick. Three possibilities exist:

(i) $\sigma^p_{max} < f_t$, that is no cracking. In this case, the principal stress vector for the cracked part will be

$$\{\sigma\}^p_{bc} = 0$$

(ii) $\sigma^p_{max} \geq f_t$ but $\sigma^p_{min} < f_t$. In this case, cracking in a direction perpendicular to σ^p_{max} is assumed and

$$\{\sigma\}^p_{bc} = \begin{Bmatrix} \sigma^p_{max} \\ 0 \\ 0 \end{Bmatrix}$$

(iii) $\sigma^p_{max} \geq f_t$ and $\sigma^p_{min} \geq f_t$

In this case, cracking in both directions is assumed, and

$$\{\sigma\}^p_{bc} = \begin{Bmatrix} \sigma^p_{max} \\ \sigma^p_{min} \\ 0 \end{Bmatrix}$$

The stresses in global coordinates are determined by

$$\{\sigma\}_{bc} = \left[T_o\right]\{\sigma\}^p_{bc}$$

where $\left[T_o\right]$ is a transformation matrix defined as

$$\left[T_o\right] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta \\ \sin\theta\cos\theta & -\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

and θ is the angle that maximum principal stress makes with the global X-axis.

Pseudoloads can now be calculated by substituting $\{\sigma\}_{bc}$ in equation (4.5).

At each load level, the iteration process is started by setting $\{\sigma\}_{bc} = 0$ for all brick elements other than those already had cracked at previous load levels. Pseudoloads are calculated as mentioned above, and the system is solved another time until convergence is obtained.

4.3 CONVERGENCE CRITERIA

After the stiffness properties of mortar elements have been updated in the $(k-1)^{th}$ iteration, and after the pseudoloads have been calculated and applied at the nodes, the system is analyzed again. New deformations are obtained and compared with $(k-1)^{th}$ deformations. If the change in deformations is smaller than a preset value, the iteration process is terminated. Absolute Deviation Test[2] is used in this study to check convergence at the k^{th} iteration. That is, for n unknown deformations, the system is assumed to have converged if

$$\frac{1}{n} \sum_{i=1}^{n} \frac{d_i^k - d_i^{k-1}}{d_i^k} \leq \epsilon$$

where ϵ is a small positive preassigned number. Two values of ϵ were used in the study, 0.0001 and 0.01. Using a value of $\epsilon = 0.01$ is found to be more practical because the number of iterations are decreased with no significant effect on accuracy.

Moreever, if the number of iterations exceeds a certain preassigned number, the analysis is assumed to diverge and the system is considered to reach its ultimate load. The maximum number of iterations used in this study was 20.

4.4 COMPUTER PROGRAM

The method of analysis is divided into two major parts. The first part is an incremental process to allow for cracking to occur in each load level. The second is an iterative procedure that allows for redistribution of stresses and propogation of cracks at the same load level.

Two methods were used in modifying the material characteristics of mortar elements. The first was to use the instantaneous tangent to the stress-strain diagrams to determine D_n' and D_s' defined in equations (3.24a) and (3.24b). This shown in Fig.-12.

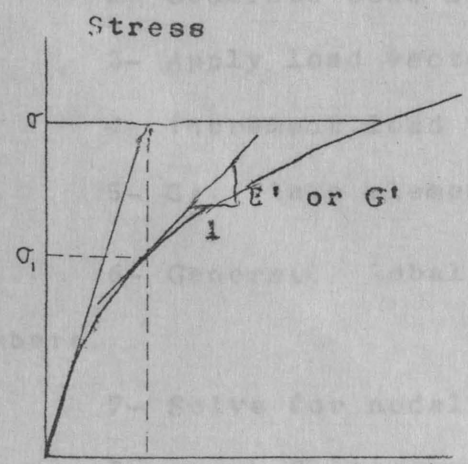

Fig.-12 Instantaneous tangent modulus

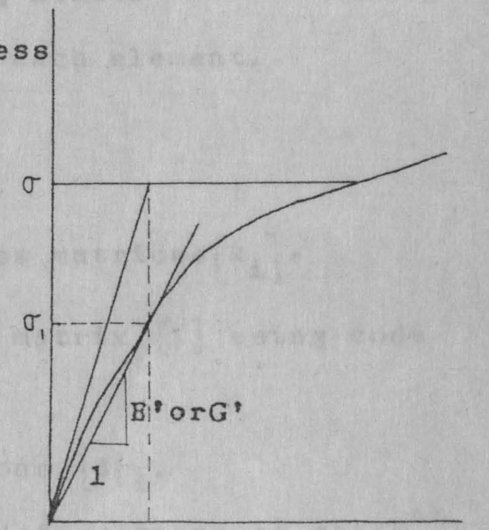

Fig.-13 Instantaneous secant modulus

At a certain stress level σ the actual stress σ_1 and the actual strain ϵ_1 are determined from the corresponding stress-strain diagrams of Fig.-5 and Fig.-6. The slope of the stress-strain diagram at the actual stress and strain is taken as the modulus to be used for further approximations. In the second method the slope of the secant (Fig.-13) at the actual strain and strain was used for further approximations. In both cases, the initial elastic mudulii were used. It was found out that although both methods reached almost the same results, using the tangent moduli converged in a time of 5-10% less than that when the secant moduli were used. Thus, the instantaneous tangent moduli were used in later stages of the study.

The algorithm of analysis used in the study can be summarized in the following steps:

1- Read in general mechanical properties, geometrical properties, external and prestressing loads.

2- Generate code numbers for each element.

3- Apply load vector.

4- Increment load vector.

5- Calculate element stiffness matrices $[k_i]$.

6- Generate global stiffness matrix $[K]$ using code numbers.

7- Solve for nodal deformations $\{d\}_i$.

8- Check for convergence of deformations at the i^{th} iteration. If yes go to step 10, otherwise go to step 9.

9- Check if number of iterations exceeded the pre-assigned maximum number. If yes, the analysis is terminated, otherwise update mortar stiffness matrix and go to step 6.

10- Check for failure.

(a) If failure criteria for brick element is violated, apply pseudoloads.

(b) If failure criteria for mortar element is violated, residual stiffness properties are allocated and element stiffness matrix is updated.

If any failure has been detected, mode and position of failure is printed and the problem is sòlved again by going to step 6; othrewise the analysis procedes to the next step.

(11) If the external load level has not reached the preassigned maximum load, the load is incremented and steps 5 through 10 are repeated; otherwise, the final stress distribution is printed out and the analysis is terminated.

A general flow chart of the algorithm described above is shown in Fig.- 14.

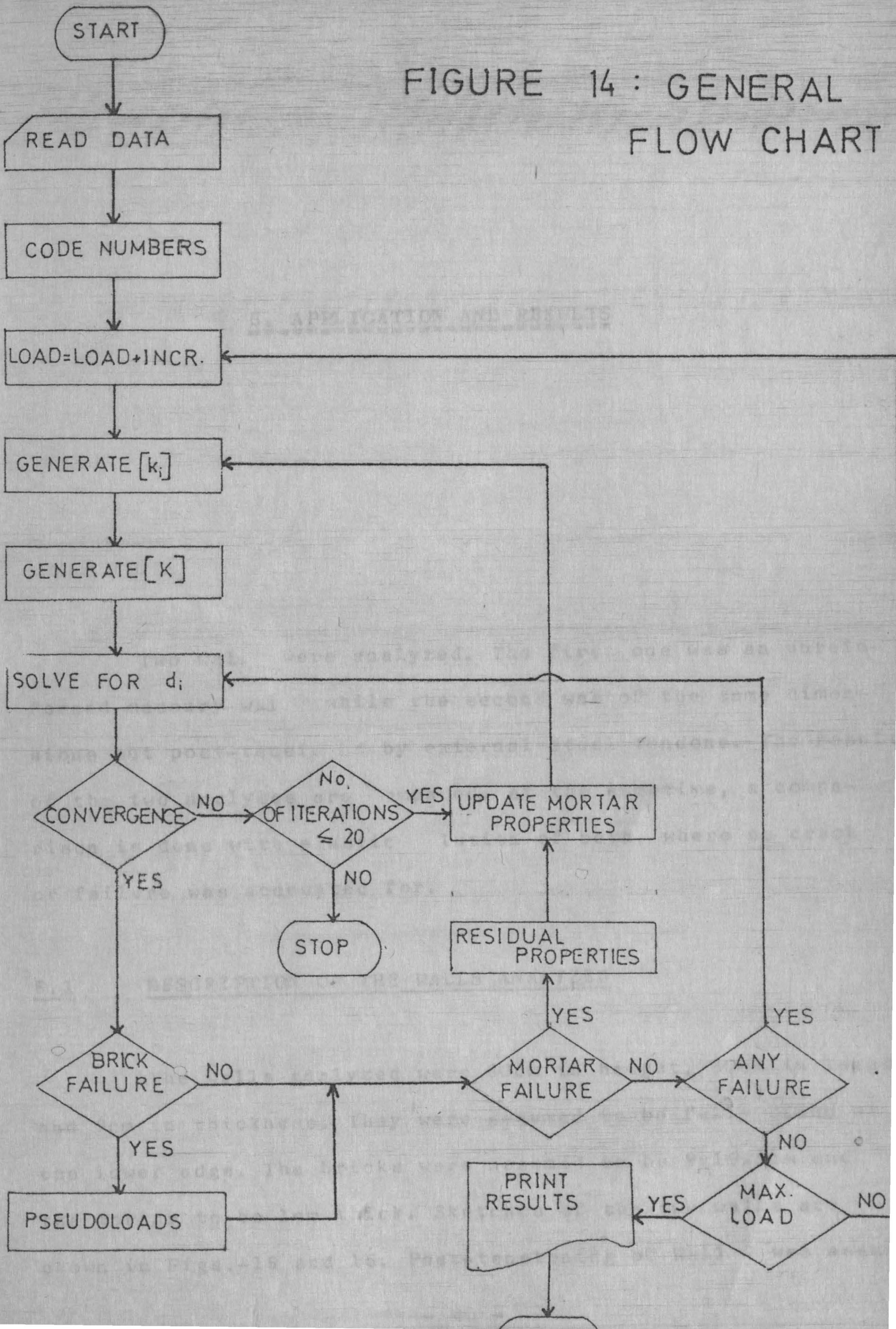

FIGURE 14 : GENERAL FLOW CHART

5. APPLICATION AND RESULTS

Two walls were analyzed. The first one was an unreinforced masonry wall while the second was of the same dimensions but post-tensioned by external steel tendons. The result of the two analyses are compared. At the sametime, a comparison is done with elastic solution of both, where no crack or failure was accounted for.

5.1 DESCRIPTION OF THE WALLS ANALYZED

The walls analyzed were 60cm in height, 80cm in lengt and 9cm in thickness. They were assumed to be fully fixed at the lower edge. The bricks were assumed to be 9x19x5cm and the mortar to be 1cm thick. Sketches of the two walls are shown in Figs.-15 and 16. Post-tensioning of Wall 2 was assu

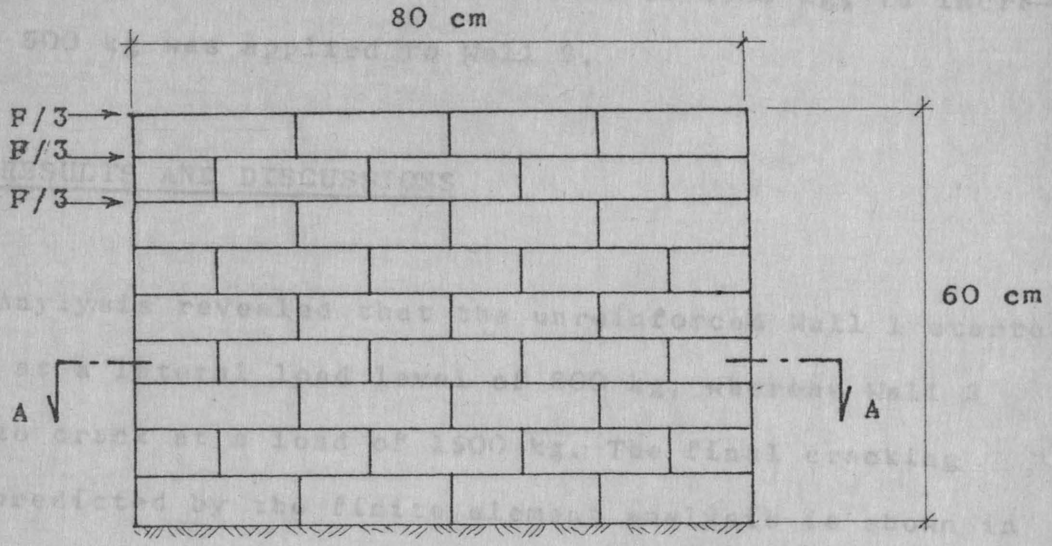

Fig.-15 Unreinforced Wall 1

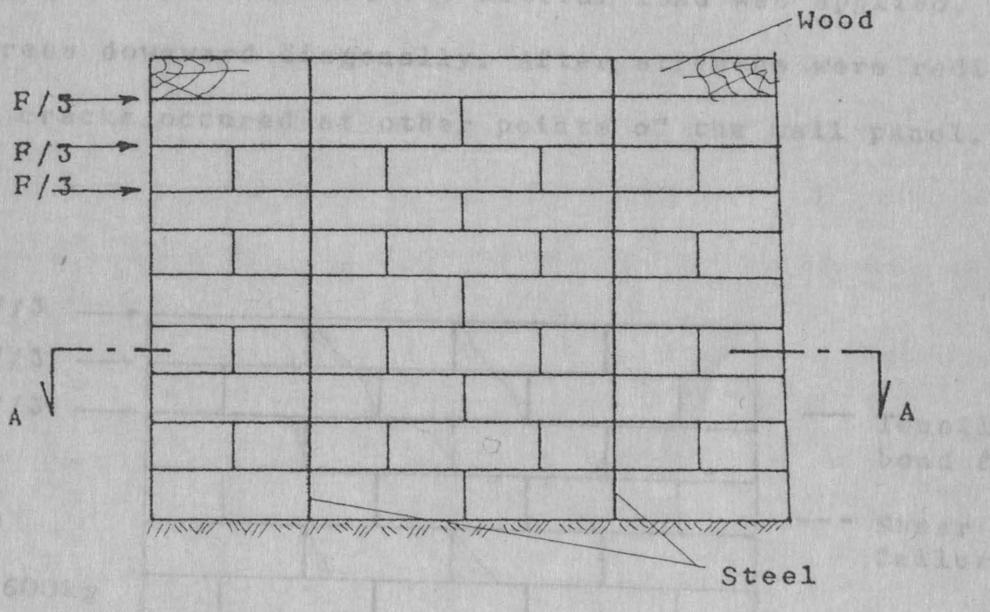

Fig.-16 Post-tensioned Wall 2

to be done by four external 14mm diameter bars. A lateral load of magnitude F was applied on the two walls as shown in the figures Fig.-15 and Fig.-16. Finite element discretization is presented in Appendix II.

Wall 2 was post-tensioned by a load of 600 kg. The lateral load applied to Wall 1 was of 200 kg increments up to a maximum load of 2000 kg. A maximum load of 4000 kg, in increments of 500 kg was applied to Wall 2.

5.2 RESULTS AND DISCUSSIONS

Anylysis revealed that the unreinforced Wall 1 started to crack at a lateral load level of 800 kg, whereas Wall 2 started to crack at a load of 1500 kg. The final cracking pattern predicted by the finite element analysis is shown in figures Fig.-17 and Fig.-18 for walls 1 and 2, respectively.

In the post-tensioned Wall 2, cracking is predicted to start at the point where the lateral load was applied, and progress downward diagonally. After stresses were redistributed, cracks occured at other points of the wall panel.

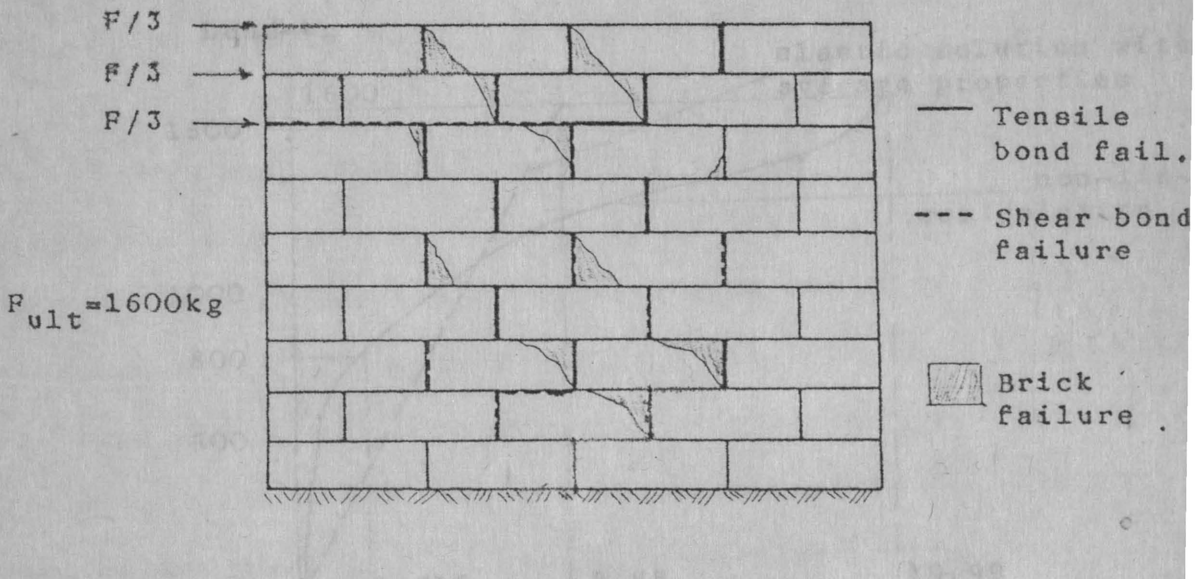

Fig.- 17 Cracking pattern for Wall 1

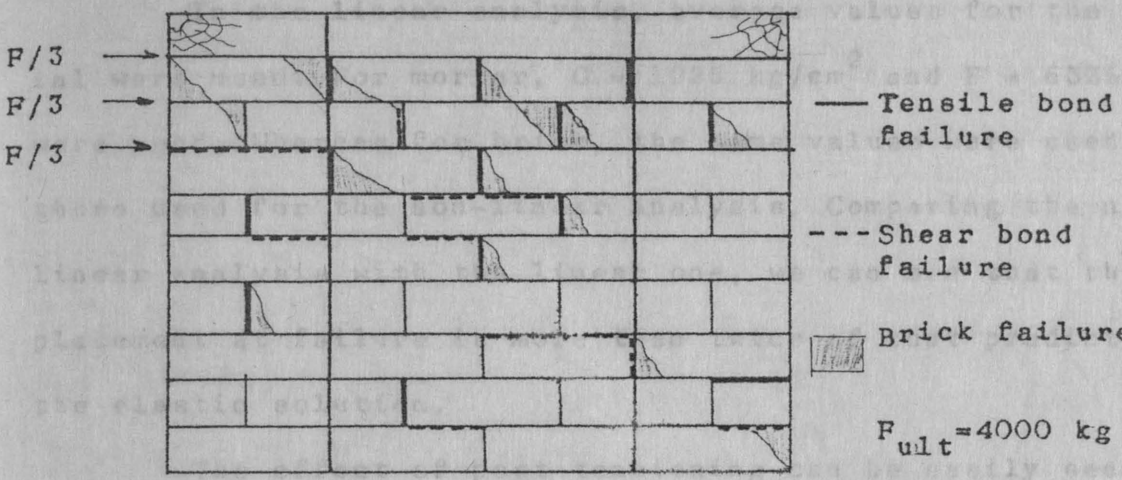

Fig.- 18 Cracking pattern for Wall 2

It should be noted here that Wall 1 ceased to converge at a load of 1600 kg. This can be easily seen from the force-displacement diagram in Fig.-19. The analysis predicted cracking to start at a load level of 800 kg and continue until the system diverges at load level 1600 kg. It is noted that

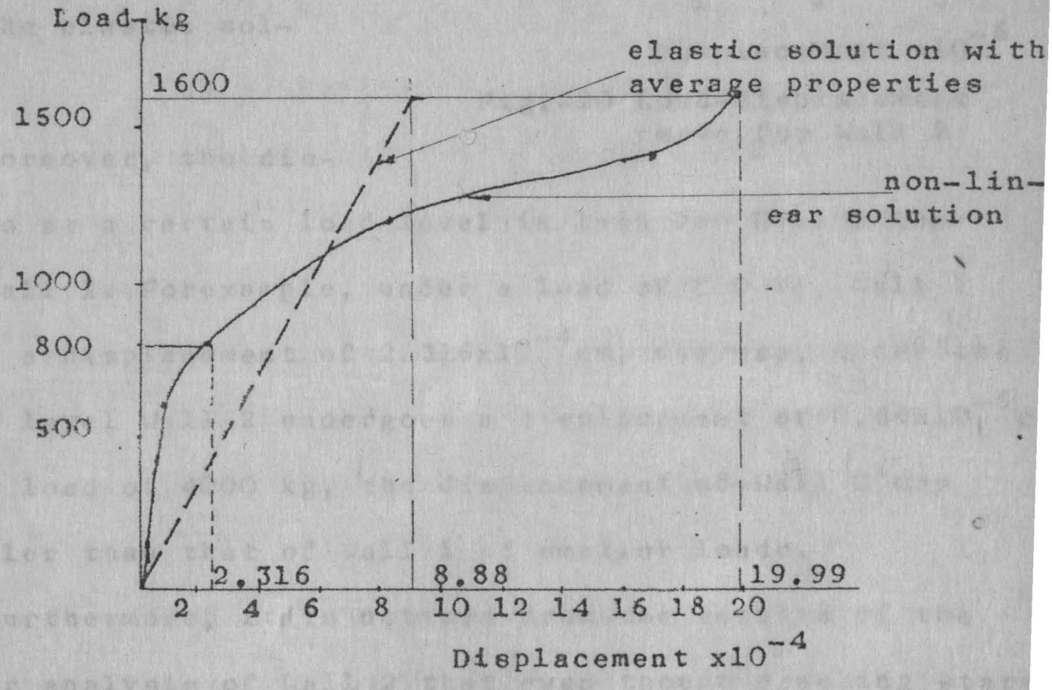

Fig.-19 Load-displacement curve for Wall 1 as the system starts to crack at a load level of 800 kg, the curve starts to flaten as it approaches the ultimate load.

In the linear analysis, average values for the material were used. For mortar, $G = 1225\ kg/cm^2$ and $E = 6335\ kg/cm^2$ were used. Whereas for brick, the same values were used as those used for the non-linear analysis. Comparing the non-linear analysis with the linear one, we can see that the displacement at failure is more than twice of that predicted by the elastic solution.

The effect of post-tensioning can be easily seen from Fig.-20. Wall 2 behaves more or less linearly. But still the displacement predicted by the elastic solution is far less than that determined by the non-linear analysis. The non-linear prediction is 1.58 times more than that of the elastic solution.

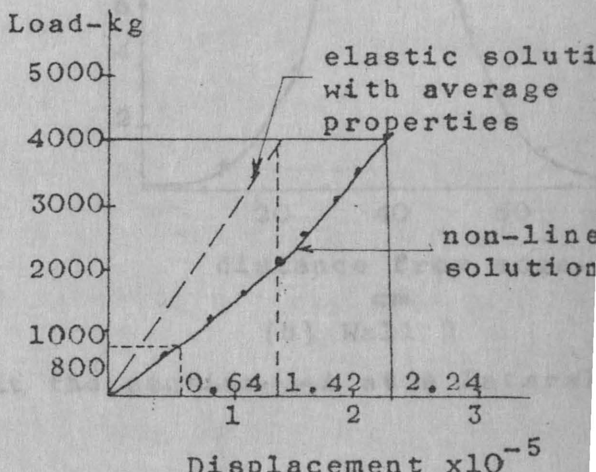

Fig.-20 Load-displacement curve for Wall 2

Moreover, the displacements at a certain load level is less for Wall 2 than that of Wall 1. Forexample, under a load of 800 kg, Wall 1 undergoes a displacement of 2.316×10^{-4} cm, whereas, under the same load level Wall 2 undergoes a displacement of 0.64×10^{-5} cm. Even at a load of 4000 kg, the displacement of Wall 2 was much smaller than that of Wall 1 at smaller loads.

Furthermore, it is noticed from the results of the non-linear analysis of Wall 2 that even though cracking start at a load level of 1500 kg, it can still carry increments of load up to 4000 kg without collapse.

To see the effect of post-tensioning on the distribution of stress, principal tensile stresses at a certain section A-A (in Fig.-15 and Fig.-16) are calculated for both walls at a load level of 200 kg. The result is shown in

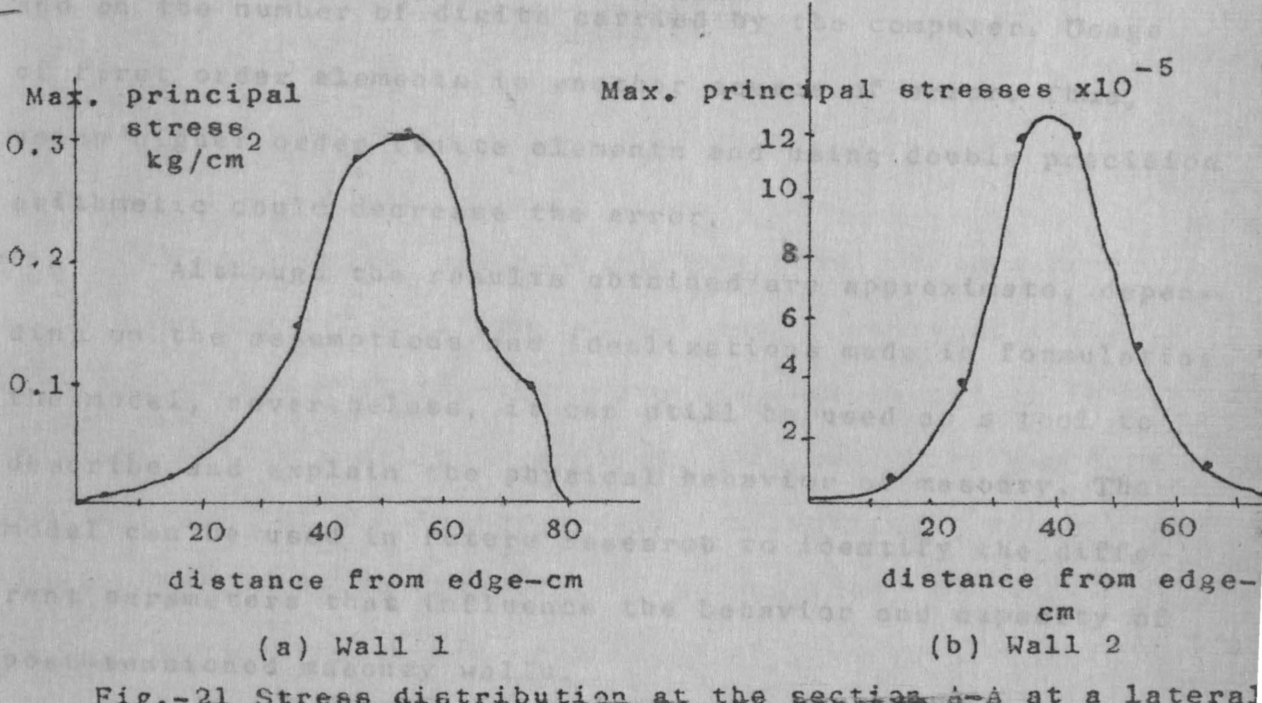

Fig.-21 Stress distribution at the section A-A at a lateral load of 200 kg

Fig.-21 a and b. Comparing the two graphs, one sees that the principal tensile stresses in Wall 2 under a load of 200 kg are of the order of 10^{-4} kg/cm^2. With post-tensioning, the tensile stresses of Wall 1 become almost zero. The 200 kg in this case is analogous to service loads in real structures. Thus, the model predicts that , as expectected, prestressing masonry elements will decrease if not eliminate tensile stresses.

As for the model, it gives a realistic analysis of the behavior of post-tensioned walls, since it takes cracking of bricks into consideration. In particular, it gives a good picture of the post-cracking behavior of masonry.

But still the model has its own shortcomings and limits. One is due to the fact that there are too many calculations to be carried by the computer which introduces the possibility of large round-off errors depending on the number of elements and on the number of digits carried by the computer. Usage of first order elements is another source of error. Thus, using higher order finite elements and using double precision arithmetic could decrease the error.

Although the results obtained are approximate, depending on the assumptions and idealizations made in formulating the model, nevertheless, it can still be used as a tool to describe and explain the physical behavior of masonry. The model can be used in future research to identify the different parameters that influence the behavior and capacity of post-tensioned masonry walls.

It should be noticed that the results obtained in this study are purely mathematical and should be verified experimentally. An attempt was done to compare the results of this study with some experimental results obtained by another research done at Bogazici University (4). Similar qualitative results were obtained as for the cracking pattern and load-deflection relationship of post-tensioned masonry. A quantitative comparison could not be done because the material characteristics used in this study especially for mortar could not be obtained for local mortar.

6. CONCLUSIONS

In this study a finite element model is developed to analyse masonry walls post-tensioned by external steel bars. An incremental-iterative procedure is suggested for the solution of the non-linear problem. This procedure can take into account the non-linear behavior of materials as well as non-linearity due to cracking and propogation of cracks.

Two masonry walls were analyzed, one unreinforced and the other post-tensioned. The following conclusions can drawn from the results of these analyses:

1- In spite of its shortcomings and limits, the model gives realistic and accurate results about the behavior and cracking of post-tensioned masonry walls, evidenced by the following expected observations:

(a) Post-tensioning increased the strength of the masonry wall.

(b) It also increased the capacity of the wall after cracking.

(c) The stiffness of the wall was found to increase with post-tensioning. That is, displacements under a certain load decreased significantly.

(d) Post-tensioning decreased tensile stresses in the wall under service loads.

2- Behavior of post-tensioned masonry is relatively linear and reliable especially under service loads.

3- The model can be used in future research as a tool of analysis to describe and identify the different parameters affecting the behavior and capacity of post-tensioned masonry.

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APPENDIX I

SUMMARY OF FINITE ELEMENT EQUATIONS

1- GENERIC DISPLACEMENTS

$$\{u\} = [N]\{d\} \tag{A.1.1}$$

where $\{u\}$ = generic displacement vector

$[N]$ = shape functions matrix

$\{d\}$ = nodal displacement vector

2- STRAINS

$$\{\epsilon\} = [\Delta]\{u\} \tag{A.1.2}$$

$$\{\epsilon\} = [\Delta][N]\{d\} \tag{A.1.3}$$

If $[G] = [\Delta][N]$ (A.1.4)

then, $\{\epsilon\} = [G]\{d\}$ (A.1.3a)

where $\{\epsilon\}$ = strain vector

$[\Delta]$ = operator matrix of derivatives

$[G]$ = strain matrix

3- STRESSES

$$\{\sigma\} = [D]\{\epsilon\} \tag{A.1.5}$$

$$\{\sigma\} = [D][G]\{d\} \quad (A.1.6)$$

If $[S] = [D][G]$ (A.1.7)

then, $\{\sigma\} = [S]\{d\}$ (A.1.8)

where $\{\sigma\}$ = stress vector

$[D]$ = material matrix

$[S]$ = stress matrix

4- STIFFNESS MATRIX

$$[k] = \int_v [G]^T [D][G] dV \quad (A.1.9)$$

where $[k]$ = element stiffness matrix

5- LOAD VECTORS

(a) Initial Strain Loads:

$$\{f\}_{\epsilon_o} = -\tfrac{1}{2}\int_v [G]^T [D]\{\epsilon_o\} dV \quad (A.1.10)$$

where $\{f\}_{\epsilon_o}$ = initial strain load vector

and $\{\epsilon_o\}$ = initial strain vector

(b) Initial Stress Loads:

$$\{f\}_{\sigma_o} + \tfrac{1}{2}\int_v [G]^T \{\sigma_o\} dV \quad (A.1.11)$$

where $\{f\}_{\sigma_o}$ = initial stress load vector

and $\{\sigma_o\}$ = initial stress vector

(c) Temperature Loads:

$$\{f\}_T = -\tfrac{1}{2}\int_V \alpha \Delta T [G]^T \{D_T\} dV \quad (A.1.12)$$

where $\{f\}_T$ = Temperature load vector

α = material temperature const...

ΔT = temperature change

$\{D_T\}$ = temperature elasticity vector

(d) Body or Distributed Forces:

$$\{f\}_b = -\int_v [N]^T \{P_b\} dV \qquad (A.1.13)$$

where $\{f\}_b$ = nodal body or distributed load vector

$\{P_b\}$ = body or distributed force vector

(e) Edge Loads:

$$\{f_s\} = -\int_s [N]^T \{P_s\} dS \qquad (A.1.14)$$

where $\{f_s\}$ = nodal load vector due to edge loads

and $\{P_s\}$ = edge load vector

6- EQUILIBRIUM EQUATIONS

$$[K]\{d\} + [C]\{\dot{d}\} + [M]\{\ddot{d}\} = \{F_{ext}\} - \sum\{f\} \qquad (A.1.15)$$

where $[K]$ = system stiffness matrix

$[C]$ = co stent damping matrix derived as,

$$[C] = C_o \int_v [N \quad N] dV \qquad (A.1.16)$$

where C_o = material damping constant

$\{\dot{d}\}$ = nodal velocity vector

$[M]$ = consistant mass matrix derived as,

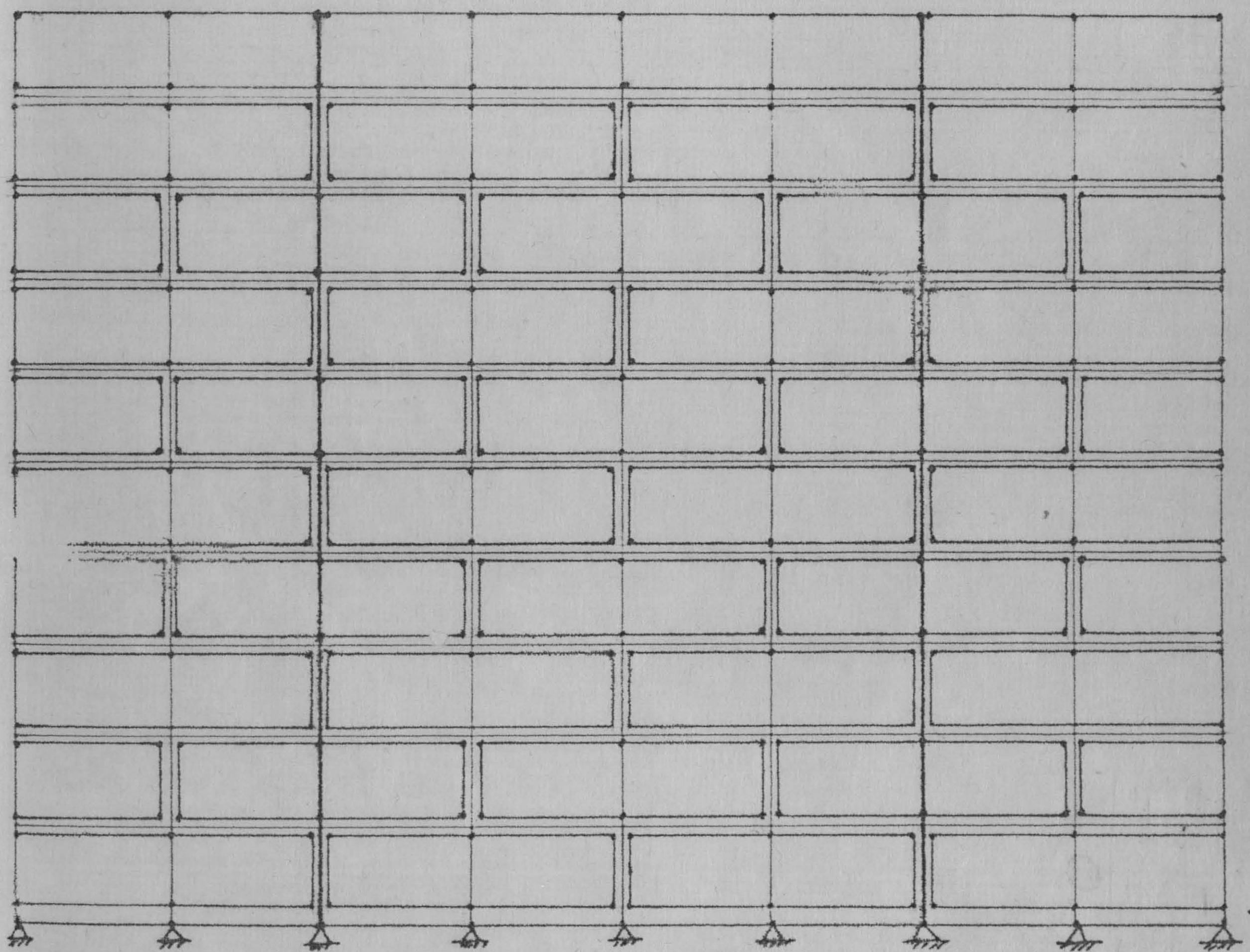
$$[M] = \rho \int_v [N]^T [N] dV \qquad (A.1.17)$$

ρ = material specific density

$\{\ddot{d}\}$ = nodal accelaration vector

APPENDIX II

FINITE ELEMENT DISCRETIZATION OF THE POST TENSIONED MASONRY WALL



Wall	No. of elements	No. of unknowns	Lateral applied load - kg	Post-tension load - kg
Wall 1	183	466	2000	-
Wall 2	193	484	4000	600

APPENDIX III

STIFFNESS MATRIX FOR VERTICAL JOINT ELEMENT

$$[k]_{jnt} = \frac{L}{6}\begin{bmatrix} 2D'_n & -2D'_n & -D'_n & D'_n & 0 & 0 & 0 & 0 \\ -2D'_n & 2D'_n & D'_n & -D'_n & 0 & 0 & 0 & 0 \\ -D'_n & D'_n & 2D'_n & -2D'_n & 0 & 0 & 0 & 0 \\ D'_n & -D'_n & -2D'_n & 2D'_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2D'_s & -2D'_s & -D'_s & D'_s \\ 0 & 0 & 0 & 0 & -2D'_s & 2D'_s & D'_s & -D'_s \\ 0 & 0 & 0 & 0 & -D'_s & D'_s & 2D'_s & -2D'_s \\ 0 & 0 & 0 & 0 & D'_s & -D'_s & -2D'_s & 2D'_s \end{bmatrix}$$

APPENDIX IV

STRESSES AND STRAINS IN JOINT ELEMENT

Relative displacements at any point x withen the joint element of unit length is related to the nodal displacements by the folloeing relation:

$$\{w\} = \tfrac{1}{2}[G]\{d\} \qquad (A.4.1)$$

where $\{w\} = \begin{Bmatrix} w_s \\ w_n \end{Bmatrix}$

w_s = relative shear displacement

w_n = relative normal displacement

$\{d\}$ = nodal displacement vector

and $[G] = \begin{bmatrix} -A & -B & B & A & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -A & -B & B & A \end{bmatrix}$

where $A = (1 - \frac{2x}{L})$ and $B = (1 + \frac{2x}{L})$.

For a mortar of thickness t, the strains can be calculated as $\epsilon_s = w_s/t$ and $\epsilon_n = w_n/t$.

$$\{\epsilon\} = \frac{1}{t}\{w\} \qquad (A.4.2)$$

where $\{\epsilon\} = \begin{Bmatrix} \epsilon_s \\ \epsilon_n \end{Bmatrix}$

ϵ_s = shear strain

ϵ_n = normal strain

t = thickness of mortar

Thus, from equations (A.4.1) and (A.4.2) the strain vector will be

$$\{\epsilon\} = \frac{1}{2t}[G]\{d\} \qquad (A.4.3)$$

In explicit form $\{\epsilon\}$ is expressed as

$$\begin{Bmatrix} \epsilon_s \\ \epsilon_n \end{Bmatrix} = \frac{1}{2t} \begin{Bmatrix} -Ad_1 - Bd_2 + Bd_3 + Ad_4 \\ -Ad_5 - Bd_6 + Bd_7 + Ad_8 \end{Bmatrix}$$

where d_i = i^{th} displacement of the joint element.

Force per unit length was related to therelative displacements (section 3.4.2) by

$$\{P\} = [D']\{w\} \qquad (A.4.4)$$

Thus for a wall of thickness T the stress vector becomes

$$\{\sigma\} = \frac{1}{T}[D']\{w\} \qquad (A.4.5)$$

where $\{\sigma\} = \begin{Bmatrix} \sigma_s \\ \sigma_n \end{Bmatrix}$

σ_s = shear stress

σ_n = normal stress

$[D']$ = material matrix defined in section (3.4.2)

From equations (A.4.1) and (A.4.5) the stress vector becomes,

$$\{\sigma\} = \frac{1}{2T}[D'][G]\{d\} \qquad (A.4.6)$$

In explicit form,

$$\begin{Bmatrix} \sigma_s \\ \sigma_n \end{Bmatrix} = \frac{1}{2T} \begin{Bmatrix} -D_s'Ad_1 - D_s'Bd_2 + D_s'Bd_3 + D_s'Ad_4 \\ -D_n'Ad_5 - D_m'Bd_6 + D_n'Bd_7 + D_n'Ad_8 \end{Bmatrix}$$

```
AIN
**************************************************************************
    THIS PROGRAM ANALYZES MASONRY WALLS UNREINFORCED AS WELL
          AS POST- TENSIONED BY EXTERNAL STEEL BARS
         THE ANALYSIS IS DONE BY FINITE ELEMENTS
**************************************************************************

 COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,2
1),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TYP
2E(300),X(300),Y(300),NCODE(8)
 COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAXS,
1,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUBR
2,RMUWOD,T,THETA,TM,NCRACK
 COMMON LOADND(10,2),DELTA,EPSILN,N3,PSL(72),NPSLD,NRR,NND,NNODE(30
1)
 DIMENSION SS(600),PSLD(8)
 DATA EPSILN,MAXS,KONTR/0.01,20000,1/
 DATA JBAND,NF,NPRINT,PLOAD/3*0,600*0.0/
 DEFINE FILE 10(600,50,V,LV)
 DEFINE FILE 20(600,10,V,IV)

                  READ DATA

 CALL READDT
 DO 60 I=1,ME
 LLD=I
 DO 65 M=1,8
 PSLD(M)=0.0
 WRITE (20,LLD) (PSLD(M),M=1,8)
 RKS=4053.9
 RKN=15338.93
 WRITE (6,1)
 FORMAT (1H1///19X,,ELEMENT NODES AND CORRESPONDING CODE NUMBER,/19
1X,43(,=,)//8X,,ELEMENT,,15X,,NODES,,20X,,CODE NUMBER,/1X,127(,-,)
2)

GENERATE ELEMENT STIFFNESS MATRICES , AND CALCULATE BAND WIDTH
              OF SYSTEM STIFFNESS MATRIX

 DO 10 I=1,ME
 LD=I
 IF (TYPE(I).EQ.1..OR.TYPE(I).EQ.4.) CALL RECSTF
 IF (TYPE(I).EQ.2.) CALL JNTSTF
 IF (TYPE(I).EQ.3.) CALL STLSTF
 CALL JBANDX(I)
 WRITE (10,LD) NC,MS,MSS,(SM(K),K=1,MSS),(NCODE(L),L=1,MS)
 WRITE (6,2) JBAND
 FORMAT(///5X,,BAND WIDTH OF K MATRIX=,,I5)
 NHEP=(N-JBAND)*JBAND+JBAND*(JBAND+1)/2+N
 WRITE (6,6) NHEP
 FORMAT (///5X,,SIZE OF S-MATRIX =,,I6)
 RLOAD=0.0
 DO 20 INCR=1,NRR
 IT=0
 KONTR=1
 NOFAIL=0
 RLOAD=RLOAD+RINCR
```

```
FORMAT (////54X,,LOAD LEVEL=,,G10.4/54X,21(,=,))
DO 50 I=1,N
SS(I)=0.0

ENERATE CODE NUMBERS FOR EACH ELEMENT AND GENERATE
         SYSTEM STIFFNESS MATRIX

CALL GENER

     SOLVE FOR UNKNOWN NODAL DEFORMATIONS

CALL GSEL
IT=IT+1
RNORM=0.
IF(IT.GT.20) GO TO 22

      CHECK FOR CONVEGENCE

DO 30 I=1,N
K2=LL(I+1)+I
IF (S(K2).EQ.0.0) GO TO 30
RMS=(S(K2)-SS(I))/S(K2)
SS(I)=S(K2)
RNORM=RNORM+ABS(RMS)
CONTINUE
RRN=N
DELTA=RNORM/RRN

        CHECK FOR BRICK AND MORTAR FAILURE

NCRACK=0  NO FAILURE IN MORTAR OR BRICK IS DETECTED , OTHERWISE
          NCRACK=1

NCRACK=0
DO 40 I=1,ME
NPSLD=0

NPSLD=0 WHEN NO PSLD IS ADDED, OTHERWISE NPSLD=1

LD=I
IF (TYPE(I).EQ.1.) CALL CHKBRK
IF (TYPE(I).EQ.2.) CALL CHKMOR
CONTINUE
IF (NCRACK.NE.0.OR.DELTA.GT.EPSILN.OR.NPSLD.NE.0) GO TO 21
WRITE (6,5) IT
FORMAT (///5X,,NO. OF ITERATIONS=,,I10)
IF (NOFAIL.EQ.0) WRITE (6,4)
FORMAT (///5X,,NO CRACKING IN BRICKS OR FAILURE IN MORTAR AT THIS
1LOAD LEVEL,)
WRITE (6,1100)
FORMAT(///52X,,EDGE DISPLACEMENTS,,///40X,,NODE NO.,,20X,
1,DISPLACEMENT,)
DO 1001 I=1,NND
II=NNODE(I)
IN=JOX(II)
IX=LL(IN+1)+IN
```

```
      WRITE(6,1002)II,D
      FORMAT (/40X,I5,20X,G12.6)
      CONTINUE

           PRINT FINAL RESULTS

      CALL RESULT
      STOP
      WRITE (6,1000) IT
      CALL RESULT
      FORMAT (//5X,,THE SYSTEM DOES NOT CONVERGE,)
      STOP
      END
```

```
READDT
*******************************************************************

      SOUBROUTINE READDT: READS MECHANICAL AND GEOMETRICAL
                    PROPERTIES OF THE WALL

*******************************************************************

  SUBROUTINE READDT
  COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,2
 1),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TYP
 2E(300),X(300),Y(300),NCODE(8)
  COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAXS
 1,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUBR
 2,RMUWOD,T,THETA,TM,NCRACK
  COMMON LOADND(10,2),DELTA,EPSILN,N3,PSL(72),NPSLD,NRR,NND
 1,NNODE(30)
  INTEGER TITLE(20),STOPRD
  WRITE (6,1)
1 FORMAT (1H1)
  READ(5,2) (TITLE(I),I=1,20)
2 FORMAT (20A4)
  WRITE (6,3) (TITLE(I),I=1,20)
3 FORMAT (//////////24X,20A4/24X,80(,=,))

  READ & PRINT MATERIAL PROPERTIES FOR BRICKS,WOOD,AND STEEL
  FT=TENSILE STRENGTH FOR BRICKS

  READ (5,4) EBR,RMUBR,FT,EWOOD,RMUWOD,ESTEEL
4 FORMAT (8F10.0)
  WRITE (6,5) EBR,RMUBR,EWOOD,RMUWOD,ESTEEL,FT
5 FORMAT (///30X,,YOUNGS MODULUS,,10X,,POISSONS RATIO,/5X,,BRICK,,22
 1X,F10.2,15X,F10.2/5X,,WOOD,,23X,F10.2,15X,F10.2/5X,,STEEL,23X,F10.
 22/////5X,,TENSILE STRENGTH FOR BRICKS=,,G10.4)

  READ DATA FOR MORTAR SHEAR STRESS STRAIN DIAGRAM

  READ (5,6) NS
  READ (5,4) (SHSTRS(I),I=1,NS)
  READ (5,4) (SSTRAN(I),I=1,NS)
  WRITE (6,1)
  WRITE (6,7) (SHSTRS(I),SSTRAN(I),I=1,NS)
7 FORMAT (////20X,,MORTAR SHEAR STRESS-STRAIN RELATIONSHIP,///7X,,ST
 1RESS,,18X,,STRAIN,//(3X,G10.5,12X,G10.5/))
6 FORMAT (16I5)

  READ DATA FOR MORTAR COMPRESSIVE STRESS-STRAIN DIAGRAM

  READ (5,6) MC
  READ (5,4) (COMPST(I),I=1,MC)
  READ (5,4) (COMPSR(I),I=1,MC)
  WRITE (6,8) (COMPST(I),COMPSR(I),I=1,MC)
8 FORMAT (1H1////20X,,MORTAR COMPRESSIVE STRESS-STRAIN RELATIONSHI
 1,///7X,,STRESS,,20X,,STRAIN,/(3X,G10.5,14X,G10.5/))

      READ BRICK, MORTAR AND STEEL THICKNESSES

      T=WALL THICNESS
```

```
      TM=MORTAR THICKNESS
      AR=CROSS SECTIONAL AREA OF STEEL BARS

      READ (5,4) T,TM,AR
      WRITE(6,200) T,TM,AR
200 FORMAT (///20X,,WALL THICKNESS=,,F10.5/20X,,MORTAR THICKNESS,,F10.
   15/20X,,STEEL CROSS SECTION AREA=,,F10.5)

      READ NODAL DATA    ME=# OF ELEMENTS      NJ=# OF JOINTS

      READ (5,6) ME,NJ
      WRITE (6,9) NJ,ME
    9 FORMAT (//////10X,,NUMBER OF JOINTS=,,I5/10X,,NUMBER OF ELEMENTS=,
     1,I5)
      WRITE (6,10)
   10 FORMAT (///30X,,NODAL DATA,/30X,10(,-)///5X,,JOINT,,5X,,XCOOR,,5X
     1,,YCOOR,,5X,, DX ,,5X,, DY ,/1X,127(,=,))
      N=0
      DO 100 I=1,NJ
      READ (5,4) XJN,X(I),Y(I),DX,DY

      XJN=JOINT NUMBER      X(I),Y(I)=X&Y  COORDINATES
      DX,DY=0 IF FREE TO MOVE IN X/Y DIRECTION
      DX,DY=1 IF SUPPORTED IN X/Y DIRECTION

      II=XJN
      IF (II-I) 101,102,101
  101 WRITE(6,11) II,I
   11 FORMAT(///30X,,JOINT NUMBER,,I5,,IS OUT OF ORDER,JOINT NUMBER,,I5
     1,,SHOULD BE READ INSTEAD,)
      STOP
  102 JOX(I)=0
      JOY(I)=0
      IF (DX) 104,103,104
  103 N=N+1
      JOX(I)=N
  104 IF (DY) 100,105,100
  105 N=N+1
      JOY(I)=N
  100 CONTINUE

  NODEST AND NODEWD ARE STEEL AND WOOD NODES THAT SHARE THE SAME POSITI
  STOPRD=1 IF READING SHOULD END

      K=0
  141 READ (5,6) NODEST,NODEWD,STOPRD
      K=K+1
      JOX(NODEST)=JOX(NODEWD)
      JOY(NODEST)=-JOY(NODEWD)
      IF (STOPRD EQ.0) GO TO 141
      K=2*K
      DO 160 I=1,NJ
      IF (JOX(I).GT.0) JOX(I)=JOX(I)-K
      IF (JOY(I).GT.0) JOY(I)=JOY(I)-K
      IF (JOY(I).LT.0) JOY(I)=JOY(I)+K
  160 CONTINUE
      N=N-K
      DO 150 I=1,NJ
  150 WRITE (6,12) I,X(I),Y(I),JOX(I),JOY(I)
   12 FORMAT (I10,2F10.2,2I10/)
```

```
      WRITE (6,20) N
 20   FORMAT (////5X,,NUMBER OF UNKNOWNS=   ,,I5)

                 READ ELEMENT DATA

      EN=ELEMENT NUMBER     D1,D2,D3,D4=NODE NUMBERS OF EACH ELEMENT
      TYPE:  BRICK=1 ;  MORTAR=2;  STEEL=3;    WOOD=4

      WRITE (6,13)
 13   FORMAT (1H1/////40X,,ELEMENT DATA,/40X,12(,-,)///5X,,ELEMENT,,15X,
     1,NODES,,15X,,THICKNESS,,5X,,TYPE,/1X,127(,-,)//)
      N1=0
      N2=0
      DO 110 I=1,ME
      READ (5,4) EN,D1,D2,D3,D4,TYPE(I)
      II=EN
      IF(II-I) 111,112,111
 111  WRITE(6,14) II,I
 14   FORMAT (1H1///30X,,ELEMENT NUMBER,,I5,,IS OUT OF ORDER,ELEMENT NU
     1MBER,,I5,,SHOULD BE READ INSTEAD,)
      STOP
 112  NODE(I,1)=D1
      NODE(I,2)=D2
      NODE(I,3)=D3
      NODE(I,4)=D4
      IF (TYPE(I).NE.3.) WRITE (6,15) I,(NODE(I,J),J=1,4),T
      IF (TYPE(I).EQ.3.) WRITE (6,15) I,(NODE(I,J),J=1,4),AR
 15   FORMAT (/5(5X,I5),F9.4)
      ITYPE=TYPE(I)
      GO TO (113,114,115,116),ITYPE
 113  WRITE (6,16)
 16   FORMAT (1H+,68X,,BRICK,)
      N1=N1+1
      GO TO 110
 114  WRITE (6,17)
 17   FORMAT (1H+,68X,,MORTAR,)
      N2=N2+1
      GO TO 110
 115  WRITE (6,18)
 18   FORMAT (1H+,68X,,STEEL,)
      GO TO 110
 116  WRITE (6,19)
 19   FORMAT (1H+,68X,,WOOD,)
 110  CONTINUE
      WRITE (6,190) N1,N2
 190  FORMAT (//5X,,NUMBER OF BRICK ELEMENTS=  ,,I8//5X,,NUMBER OF MORTA
     1 ELEMENTS =  ,,I8)
      N3=9*N1+N2

      READ EDGE NODES

      READ (5,6) NND
      READ (5,6) (NNODE(I),I=1,NND)

      READ LOAD DATA
      RNODE= NODE AT WHICH LOAD IS ACTING
      PRSLOD=PRESTRESSING LOAD       DIR=1 IFINX,2 IF IN Y

      READ (5,4) RLOAD,R
      NRR=R
```

```
      RINCR=RLOAD/R
      READ (5,6) INCNOD,((LOADND(I,J),J=1,2),I=1,INCNOD)
C***********************************************************************
C
C     PRESTRESS LOAD IS APPLIED AS A COMPREESSIVE LOAD ON THE
C     WOOD ELEMENT
C
  123 READ (5,4,END=130) PRSLOD,RNODE,DIR
      NNR=RNODE
      SAYN=1.
      SAYN1=1.
      IF (DIR.LT.0.) SAYN1=-1.
      IF (DIR.LT.0.) DIR=-DIR
      IDIR=DIR
      GO TO (120,121),IDIR
  120 II=JOX(NNR)
      GO TO 122
  121 II=JOY(NNR)
  122 IF (II.LT.0) II=-II
      IF (II.LT.0) SAYN=-1.
      PLOAD(II)=PLOAD(II)+PRSLOD*SAYN*SAYN1
      GO TO 123
  130 RETURN
      END
```

,APROX

```
***********************************************************************

SUPROUTINE APROX: CALCULATES INSTANTANEOUS TANGENT MODULII

***********************************************************************
   SUBROUTINE APROXK(R,CONT)
   COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,2
  1),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TYP
  2E(300),X(300),Y(300),NCODE(8)
   COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAXS
  1,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUBR
  2,RMUWOD,T,THETA,TM
   DIMENSION XCOOR(10),YCOOR(10)
   XPR=R
   NN=NS
   DO 10 J=1,NN
   YCOOR(J)=SHSTRS(J)
10 XCOOR(J)=SSTRAN(J)
   IF (CONT.EQ.2.) GO TO 21
   NN=MC
   DO 20 J=1,NN
   YCOOR(J)=COMPST(J)
20 XCOOR(J)=COMPSR(J)
21 IF (XPR)  22,23,24
23 SLOPE=YCOOR(2)/XCOOR(2)
   GO TO 35
22 XPR=-XPR
24 DO 30 J=1,NN
   K=J
   IF (XPR.GE.XCOOR(J).AND.K.GE.NN) GO TO 32
   IF (XPR.GE.XCOOR(J).AND.XPR.LT.XCOOR(J+1)) GO TO 31
30 CONTINUE
31 SLOPE=(YCOOR(K+1)-YCOOR(K))/(XCOOR(K+1)-XCOOR(K))
   GO TO 35
32 SLOPE=(YCOOR(K)-YCOOR(K-1))/(XCOOR(K)-XCOOR(K-1))
35 IF (CONT.EQ.1.) RKN=SLOPE*T/TM
   IF (CONT.EQ.2.) RKS=SLOPE*T/TM
   RETURN
   END
```

ANDX

```
**************************************************************************
ROUTINE JBANDX: CALCULATES BAND WIDTH FOR SYSTEM SYSTEM MATRIX

**************************************************************************
SUBROUTINE JBANDX(I)
COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,2
),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TYP
E(300),X(300),Y(300),NCODE(8)
COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAXS
,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUBR
,RMUWOD,T,THETA,TM
DO 10 J=1,NC
IN=NODE(I,J)
NCODE(J)=JOX(IN)
IX=J+NC
NCODE(IX)=JOY(IN)
IF (TYPE(I).EQ.3.) NCODE(J)=JOY(IN)
IF (TYPE(I).EQ.3.) NCODE(IX)=0
CONTINUE
MSM=MS-1
DO 20 J=1,MSM
JP=J+1
IJ=NCODE(J)
IF (IJ) 19,20,21
IJ=-IJ
DO 25 K=JP,MS
IK=NCODE(K)
IF (IK) 24,25,26
IK=-IK
KF=IK-IJ+1
IF (JBAND-KF) 27,25,25
JBAND=KF
CONTINUE
CONTINUE
WRITE (6,1) I,(NODE(I,J),J=1,NC),(NCODE(J),J=1,MS)
FORMAT (1X,5I9,2X,8I10)
RETURN
END
```

```
CSTF
**************************************************************************
BROUTINE RECSTF: GENERATES RECTANGULAR STIFFNESS MATRIX

**************************************************************************
SUBROUTINE RECSTF
COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,2
),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TYP
E(300),X(300),Y(300),NCODE(8)
COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAXS
,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUBR
,RMUWOD,T,THETA,TM
NC=4
MS=8
MSS=36
I=LD
J1=NODE(I,1)
J2=NODE(I,2)
J3=NODE(I,3)
B=(Y(J1)-Y(J2))/2.
A=(X(J1)-X(J2))/2.
E=EBR
RMU=RMUBR
IF (TYPE(I).EQ.1.) A=A-TM/4.
IF (TYPE(I).EQ.1.) B=B-TM/2.
BETA=B/A
IF (TYPE(I).EQ.4.) E=EWOOD
IF (TYPE(I).EQ.4.) RMU=RMUWOD
CO=E*T/12./(1.-RMU*RMU)
SM(1)=4.*BETA-2.*(1.-RMU)/BETA
SM(2)=-2.*BETA-2.*(1.-RMU)/BETA
SM(3)=-4.*BETA+(1.-RMU)/BETA
SM(4)=-2.*BETA-(1.-RMU)/BETA
SM(5)=3.*(1.+RMU)/2.
SM(6)=3.*(1.-3.*RMU)/2.
SM(7)=-3.*(1.-3.*RMU)/2.
SM(8)=-SM(5)
SM(9)=SM(1)
SM(10)=SM(4)
SM(11)=SM(3)
SM(12)=SM(7)
SM(13)=SM(8)
SM(14)=SM(5)
SM(15)=SM(6)
SM(16)=SM(1)
SM(17)=SM(2)
SM(18)=SM(6)
SM(19)=SM(5)
SM(20)=SM(8)
SM(21)=SM(7)
SM(22)=SM(1)
SM(23)=SM(8)
SM(24)=SM(7)
SM(25)=SM(6)
SM(26)=SM(5)
SM(27)=4./BETA+2.*BETA*(1.-RMU)
SM(28)=-4./BETA+BETA*(1.-RMU)
SM(29)=2./BETA-2.*BETA*(1.-RMU)
SM(30)=-2./BETA-BETA*(1.-RMU)
SM(31)=SM(27)
SM(32)=SM(30)
SM(33)=SM(29)
SM(34)=SM(27)
```

```
SM(35)=SM(28)
SM(36)=SM(27)
DO 10 J=1,MSS
SM(J)=SM(J)*CO
RETURN
END
```

```
JNTSTF
************************************************************************

  SUBROUTINE JNTSTF: GENERATES JOINT STIFFNESS MATRIX

************************************************************************
  SUBROUTINE JNTSTF
  COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,
 1),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TY
 2E(300),X(300),Y(300),NCODE(8)
  COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAX
 1,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUB
 2,RMUWOD,T,THETA,TM
  I=LD
  NC=4
  MS=8
  MSS=36
  RK1=RKS
  RK2=RKN
  J1=NODE(I,1)
  J2=NODE(I,2)
  J3=NODE(I,3)
  RLENG=X(J2)-X(J1)
  IF (ABS(RLENG).GT.0.001) GO TO 15
  RK1=RKN
  RK2=RKS
  RLENG=ABS(Y(J3)-Y(J1))
5 DO 10 J=1,36
0 SM(J)=0.
  SM(1)=2.*RK1
  SM(2)=RK1
  SM(3)=-SM(2)
  SM(4)=-SM(1)
  SM(9)=SM(1)
  SM(10)=-SM(1)
  SM(11)=-SM(2)
  SM(16)=SM(1)
  SM(17)=SM(2)
  SM(22)=SM(1)
  SM(27)=2.*RK2
  SM(28)=RK2
  SM(29)=-SM(28)
  SM(30)=-SM(27)
  SM(31)=SM(27)
  SM(32)=-SM(27)
  SM(33)=-SM(28)
  SM(34)=SM(27)
  SM(35)=SM(28)
  SM(36)=SM(27)
  RK=RLENG/6.
  DO 20 J=1,MSS
0 SM(J)=RK*SM(J)
  IF (ABS(X(J2)-X(J1)).GT..001) RETURN
  SM(2)=-SM(1)
  SM(4)=-SM(3)
  SM(10)=SM(4)
  SM(17)=SM(2)
  SM(28)=-SM(27)
  SM(30)=-SM(29)
  SM(32)=SM(30)
  SM(35)=SM(28)
  RETURN
  END
```

.STLSTF

```
********************************************************************************

    SUBROUTINE STLSTF: GENERATES LINE ELEMENT STIFFNESS MATRIX

********************************************************************************
      SUBROUTINE STLSTF
      COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,
     1),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TY
     2E(300),X(300),Y(300),NCODE(8)
      COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAX
     1,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUB
     2,RMUWOD,T,THETA,TM
      NC=2
      MS=2
      MSS=3
      I=LD
      J1=NODE(I,1)
      J2=NODE(I,2)
      RLENG=Y(J2)-Y(J1)
      CO=ESTEEL*AR/RLENG
      SM(1)=1.
      SM(2)=-1.
      SM(3)=1.
      DO 10 J=1,MSS
   10 SM(J)=SM(J)*CO
      RETURN
      END
```

).GENER

```
*************************************************************************************
      SUBROUTINE GENER: GENERATES SYSTEM STIFFNESS MATRIX

*************************************************************************************
      SUBROUTINE GENER
      COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,2
     1),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TYP
     2E(300),X(300),Y(300),NCODE(8)
      COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAXS
     1,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUBR
     2,RMUWOD,T,THETA,TM,NCRACK
      COMMON LOADND(10,2)
      DIMENSION PSLD(8)
      LOC(II,J)=II*MS-II*(II-1)/2-(MS-J)
      IUCGEN(I)=(I-N+JBAND-1)*(I-N+JBAND)/2
      IF (NHEP-MAXS) 10,10,11
 11   WRITE (6,1) NHEP,MAXS
  1   FORMAT (//5X,,PROBLEM SIZE IS TOO LARGE,NHEP=,,I6,,,MAXS=,,I6)
      STOP
 10   DO 20 I=1,NHEP
 20   S(I)=0.
      DO 30 NM=1,ME
      LD=NM
      READ (10,LD) NC,MS,MSS,(SM(K),K=1,MSS),(NCODE(L),L=1,MS)
      LJB=JBAND+1
      NJB=N-JBAND
      DO 31 L=1,MS
      SAYN=1.
      I=NCODE(L)
      IF (I) 12,31,13
 12   SAYN=-1.
      I=-I
 13   IX=(I-1)*LJB-I+1
      IUC=IUCGEN(I-1)
      DO 311 M=1,MS
      SAYN2=1.
      J=NCODE(M)
      IF (J) 14,311,15
 14   SAYN2=-1.
      J=-J
 15   IF (J-I) 311,312,312
312   ID=L
      JD=M
      IF (L-M) 313,313,314
314   ID=M
      JD=L
313   LC=LOC(ID,JD)
      LO=IX+J
      IF (I-NJB-1) 315,315,316
316   LO=LO+IUC
315   S(LO)=S(LO)+SM(LC)
311   CONTINUE
 31   CONTINUE
 30   CONTINUE
      DO 90 I=1,ME
      LD=I
      READ (20,LD) (PSLD(M),M=1,MS)
      READ (10,LD) NC,MS,MSS,(SM(K),K=1,MSS),(NCODE(L),L=1,MS)
      DO 70 M=1,MS
      IN=NCODE(M)
      IF (IN.EQ.0) GO TO 70
```

```
70 CONTINUE
90 CONTINUE
   IF (KONTR.EQ.0) GO TO 39
   WEIGHT=0.
   DO 50 I=1,INCNOD
50 WEIGHT=WEIGHT+LOADND(I,2)
   DO 60 I=1,INCNOD
   NN=LOADND(I,1)
   NUM=JOX(NN)
   RR=LOADND(I,2)
60 PLOAD(NUM)=PLOAD(NUM)+RINCR*RR/WEIGHT
   KONTR=0
39 DO 40 I=1,N
   LO=(I-1)*LJB+JBAND+1
   IF (I-(NJB+1)) 42,42,41
41 LO=LO-IUCGEN(I)
42 S(LO)=S(LO)+PLOAD(I)
40 CONTINUE
   DO 100 I=1,ME
   LD=I
   READ (10,LD) NC,MS,MSS,(SM(K),K=1,MSS),(NCODE(L),L=1,MS)
   READ (20,LD) (PSLD(KK),KK=1,8)
   DO 80 M=1,MS
   IN=NCODE(M)
   IF (IN.EQ.0) GO TO 80
   PLOAD(IN)=PLOAD(IN)-PSLD(M)
80 CONTINUE
100 CONTINUE
   RETURN
   END
```

```
).GSEL
*************************************************************************

 SUBROTINE GSEL: GAUSSIAN ELIMINATION PROCESS

*************************************************************************
      SUBROUTINE GSEL
      COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,2
     1),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TYP
     2E(300),X(300),Y(300),NCODE(8)
      COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAXS
     1,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUBR
     2,RMUWOD,T,THETA,TM
      DIMENSION Z(20000)
      EQUIVALENCE (S,Z)
      JB=JBAND
      NE=N-1
      N1=N+1
      NL=N+1
      NM=0
      JBE=JB-1
      NJJ=N-JBE
      ND=JBE+1
      LL(1)=0
      J2=0
      JCOR=0
      DO 40 I=1,N
      J1=J2+1
      IF(I-NJJ)41,41,42
   41 J2=J1+ND
      GO TO 43
   42 J2=J1+NL-I
   43 DO 100 J=J1,J2
      JCOR=JCOR+1
  100 Z(J)=Z(JCOR)
      J3=J2-1
      JA=J3+J1
      DO 44 K=J1,J3
      J=JA-K
      IF (Z(J)) 50,44,50
   44 CONTINUE
   50 LX=J-J1+1
      IF (LX+1-LL(I)) 51,52,52
   51 LX=LL(I)-1
   52 JT=J3-J1-LX+1
      IF (JT) 40,40,55
   55 JP=J3+1
      DO 56 J=JP,J2
      K=J-JT
   56 Z(K)=Z(J)
      J2=J2-JT
   40 LL(I+1)=LX
      NX=0
      DO 7 I=1,N
      NX=NX+LL(I+1)+1
    7 LL(I+1)=NX-I
      NX=LL(N)+N
      NY=N
      NZ=NX+1
      NT=NZ-NY
      DO 10 K=1,NE
      NBK=LL(K)
      KK=NBK+K
      Q=1./Z(KK)
```

```
      Z(KK)=Q
      IB=K+1
      K2=LL(IB)+K
      IS=K2-NBK-1
      IF (IS-N) 12,11,11
11    IE=N
      IS=NL
      GO TO 17
12    IE=IS
      IF (IB-IE) 22,22,10
22    K1=K2-NM
17    J2=NBK+IS
      IN=IS-IE
      DO 13 I=IB,IE
      KI=NBK+I
      IF (Z(KI)) 14,13,14
14    TA=Q*Z(KI)
      IH=LL(I)-NBK
      DO 15 KJ=KI,J2
      IJ=KJ+IH
      Z(IJ)=Z(IJ)-TA*Z(KJ)
15    CONTINUE
      IF (IN) 18,18,13
18    IH=LL(I+1)+I-K2
      DO 16 KJ=K1,K2
      IJ=KJ+IH
      Z(IJ)=Z(IJ)-TA*Z(KJ)
16    CONTINUE
13    CONTINUE
10    CONTINUE
      KI=NX+1
      Q=1./Z(NX)
      DO 20 J=KI,NZ
20    Z(J)=Q*Z(J)
      DO 38 IX=1,NE
      I=N-IX
      IB=I+1
      NBI=LL(I)
      II=NBI+I
      IC=LL(IB)+I
      Q=Z(II)
      IS=IC-NBI-1
      W=Z(IC)
      IF (IB-IS) 39,39,36
39    DO 37 J=IB,IS
      IJ=NBI+J
      JK=LL(J+1)+J
37    W=W-Z(IJ)*Z(JK)
36    Z(IC)=W*Q
38    CONTINUE
      RETURN
      END
```

.CHKMOR

```
************************************************************************

        SUBROUTINE CHKMOR: CHECKS MORTAR FAILURE

************************************************************************
      SUBROUTINE CHKMOR
      COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,2
     1),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TYP
     2E(300),X(300),Y(300),NCODE(8)
      COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAXS
     1,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUBR
     2,RMUWOD,T,THETA,TM,NCRACK
      COMMON LOADND(10,2),DELTA,EPSILN,N3
      DIMENSION DEF(8),DF(8)
      NREG=4
      NREP=0
      IF (NF.EQ.0) GO TO 1
      DO 80 I=1,NF
      IF (NFAIL(I,1).EQ.LD.AND.NFAIL(I,2).EQ.1) RETURN
   80 CONTINUE
    1 READ (10,LD) NC,MS,MSS,(SM(K),K=1,MSS),(NCODE(L),L=1,MS)
      DO 10 I=1,MS
      DEF(I)=0.
      IN=NCODE(I)
      IF (IN) 10,10,11
   11 IX=LL(IN+1)+IN
      DEF(I)=S(IX)
   10 CONTINUE
      J1=NODE(LD,1)
      J2=NODE(LD,2)
      J3=NODE(LD,3)
      RLENG=X(J2)-X(J1)
      RLENG=ABS(RLENG)
      IF (RLENG.LT.0.00001) GO TO 24
      CO=6./RLENG/T
      RK1=SM(2)*CO
      RK2=SM(28)*CO
      GO TO 23
   24 RLENG=Y(J3)-Y(J1)
      RLENG=ABS(RLENG)
      DF(1)=DEF(6)
      DF(2)=DEF(7)
      DF(3)=DEF(8)
      DF(4)=DEF(5)
      DF(5)=-DEF(2)
      DF(6)=-DEF(3)
      DF(7)=-DEF(4)
      DF(8)=-DEF(1)
      DO 12 I=1,8
   12 DEF(I)=DF(I)
      CO=6./RLENG/T
      RK1=SM(30)*CO
      RK2=SM(4)*CO
   23 SHEAR1=RK1*(DEF(3)-DEF(2))
      SHEAR2=RK1*(DEF(4)-DEF(1))
      SHERAV=ABS(SHEAR1+SHEAR2)/2.
      SHSTRN=SHERAV/RK1
      COMP1=-RK2*(DEF(7)-DEF(6))
      COMP2=-RK2*(DEF(8)-DEF(5))
      COMPAV=(COMP1+COMP2)/2.
      CMSTRN=-COMPAV/RK2
      IF (NPRINT.EQ.1) GO TO 60
      IF (DELTA.GT.EPSILN) GO TO 40
```

```
      IF (COMPAV.LT.-3.5) NREG=1
      IF(COMPAV.LT.-3.5) GO TO 30
      IF (COMPAV.GT.0.) GO TO 21
      IF (COMP1.LE.0.) ULTSH1=0.66*COMP1+2.31
      IF (COMP2.LE.0.) ULTSH2=0.66*COMP2+2.31
      IF (SHEAR1.GE.ULTSH1.OR.SHEAR2.GE.ULTSH2) NREG=1
      GO TO 30
   21 IF (COMPAV.GT.23.38) GO TO 22
      IF (COMP1.LE.23.38) ULTSH1=0.87*COMP1+2.31
      IF (COMP2.LE.23.38) ULTSH2=0.87*COMP2+2.31
      IF (SHEAR1.GE.ULTSH1.OR.SHEAR2.GE.ULTSH2) NREG=2
      GO TO 30
   22 IF (COMP1.GT.23.38) ULTSH1=0.11*COMP1+20.08
      IF (COMP2.GT.23.38) ULTSH2=0.11*COMP2+20.08
      IF (SHEAR1.GE.ULTSH1.OR.SHEAR2.GE.ULTSH2) NREG=3
   30 IF (NREG.EQ.4) GO TO 40
      DO 70 I=1,NF
      IF (NFAIL(I,1).EQ.LD.AND.NFAIL(I,2).EQ.2) GO TO 42
      IF (NFAIL(I,1).EQ.LD.AND.NFAIL(I,2).EQ.3) GO TO 43
   70 CONTINUE
      NOFAIL=1
      NCRACK=1
      NF=NF+1
      IF (NF.GE.N3) GO TO 50
      NFAIL(NF,1)=LD
      NFAIL(NF,2)=NREG
      WRITE (6,8)
    8 FORMAT (//1X,127(,*,)//)
      GO TO (31,32,33),NREG
   31 WRITE (6,7) LD
    7 FORMAT (///5X,,JOINT NO.,,I5,,FAILS,/5X,,MODE OF FAILURE: TENSILE
     1BOND FAILURE,/5X,,REGION OF FAILURE: 1,)
      RKN=0.
      NREP=1
      RKS=0.
      GO TO 34
   32 WRITE (6,2) LD
      NREP=1
   42 RKS=10.868*ABS(COMPAV)
    2 FORMAT (///5X,,JOINT NO.,,I5,,FAILS,/5X,,MODE OF FAILURE : SHEAR B
     1OND FAILURE,/5X,,REGION OF FAILURE : 2,)
      GO TO 40
   33 WRITE (6,3) LD
      NREP=1
   43 RKS=254.1
    3 FORMAT (///5X,,JOINT NO.,,I5,,FAILS,/5X,,MODE OF FAILURE : SHEAR B
     1OND FAILURE,/5X,,REGION OF FAILURE : 3,)
   40 CALL APROXK(CMSTRN,1.)
      IF (NREG.EQ.4) CALL APROXK(SHSTRN,2.)
   34 CALL JNTSTF
      WRITE (10,LD) NC,MS,MSS,(SM(K),K=1,MSS),(NCODE(L),L=1,MS)
      IF (NPRINT.EQ.0.AND.NREG.EQ.4) RETURN
      IF (NPRINT.EQ.0.AND.NREP.NE.1) RETURN
   60 COMP1=-COMP1
      COMP2=-COMP2
      PNMAX=AMAX1(COMP1,COMP2)
      PNMIN=AMIN1(COMP1,COMP2)
      PSHMAX=AMAX1(SHEAR1,SHEAR2)
      PSHMIN=AMIN1(SHEAR1,SHEAR2)
      COMPAV=-COMPAV
      IF (NPRINT.NE.1) WRITE (6,4) PNMAX,PNMIN,COMPAV,PSHMAX,PSHMIN,SHE
     1AV,CMSTRN,SHSTRN
    4 FORMAT (5X,,MAX NORMAL STRESS=,,G14.5,10X,,MIN NORMAL STRESS=,,G1
     1.5,10X,,AVERAGE NORMAL STRESS=,,G14.5/5X,,MAX SHEAR STRESS=,,G14.
     2,11X,,MIN SHEAR STRESS=,,G14.5,10X,,AVERAGE SHEAR STRESS=,,G14.5/
```

```
      IF (NPRINT.EQ.1) WRITE (6,5) LD,PNMAX,PNMIN,COMPAV,PSHMAX,PSHMIN,S
     1HERAV,CMSTRN,SHSTRN
    5 FORMAT (3X,I5,8(1X,G12.6))
      RETURN
   50 WRITE (6,6)
    6 FORMAT (///10X,,THE SYSTEM COLLAPSES UNDER THIS LOAD,)
      STOP
      END
```

```
.CHKBRK
**********************************************************************

  SUBROUTINE CHKBRK: CHECKS FOR BRICK FAILURE

**********************************************************************
   SUBROUTINE CHKBRK
   COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,2
  1),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TYP
  2E(300),X(300),Y(300),NCODE(8)
   COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAXS
  1,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUBR
  2,RMUWOD,T,THETA,TM,NCRACK
   COMMON LOADND(10,2),DELTA,EPSILN,N3
   DIMENSION ST(3,8),SIGMA(3),EP(3),DEF(8)
   IF (DELTA.GT.EPSILN) RETURN
   READ (10,LD) NC,MS,MSS,(SM(K),K=1,MSS),(NCODE(L),L=1,MS)
   DO 10 I=1,MS
   DEF(I)=0.
   IN=NCODE(I)
   IF (IN) 10,10,11
11 IX=LL(IN+1)+IN
   DEF(I)=S(IX)
10 CONTINUE
   J1=NODE(LD,1)
   J2=NODE(LD,2)
   J3=NODE(LD,3)
   A=(X(J1)-X(J3)-TM)/2.
   B=(Y(J1)-Y(J2)-TM)/2.
   AREA=4.*A*B
   D11=EBR/(1.-RMUBR*RMUBR)
   D12=RMUBR*D11
   D33=EBR/2./(1.+RMUBR)
   DO 20 J=1,9
   RJ=J
   II=(J-1)/3
   RII=II
   XX=2./3.*(RII-1.)
   YY=2./3.*(RJ-3.*RII-2.)
   ST(1,1)=D11*B*(1.+YY)
   ST(1,2)=D11*B*(1.-YY)
   ST(1,3)=-ST(1,1)
   ST(1,4)=-ST(1,2)
   ST(1,5)=D12*A*(1.+XX)
   ST(1,6)=-ST(1,5)
   ST(1,7)=D12*A*(1.-XX)
   ST(1,8)=-ST(1,7)
   ST(2,1)=D12*B*(1.+YY)
   ST(2,2)=D12*B*(1.-YY)
   ST(2,3)=-ST(2,1)
   ST(2,4)=-ST(2,2)
   ST(2,5)=D11*A*(1.+XX)
   ST(2,6)=-ST(2,5)
   ST(2,7)=D11*A*(1.-XX)
   ST(2,8)=-ST(2,7)
   ST(3,1)=D33*A*(1.+XX)
   ST(3,2)=-ST(3,1)
   ST(3,3)=D33*A*(1.-XX)
   ST(3,4)=-ST(3,3)
   ST(3,5)=D33*B*(1.+YY)
   ST(3,6)=D33*B*(1.-YY)
   ST(3,7)=-ST(3,5)
   ST(3,8)=-ST(3,6)
   MULTIPLY NODAL DEFORMATIONS BY STRESS MATRIX
```

.PSDLOD

```
******************************************************************************

SUBROUTINE PSDLOD: APPLIES NODAL PSEUDOLOADS DUE TO BRICK CRAKING

******************************************************************************
      SUBROUTINE PSDLOD(J,A,B)
      COMMON COMPSR(10),COMPST(10),JOX(300),JOY(300),LL(550),NFAIL(300,2
     1),NODE(300,4),PLOAD(600),S(20000),SHSTRS(10),SSTRAN(10),SM(36),TYP
     2E(300),X(300),Y(300),NCODE(8)
      COMMON AR,EBR,ESTEEL,EWOOD,FT,GMAX,GMIN,INCNOD,JBAND,KONTR,LD,MAXS
     1,MC,ME,MS,MSS,N,NC,NF,NHEP,NJ,NOFAIL,NPRINT,NS,RINCR,RKN,RKS,RMUBR
     2,RMUWOD,T,THETA,TM,NCRACK
      COMMON LOADND(10,2),DELTA,EPSILN,N3,PSL(72),NPSLD
      DIMENSION PSLD(8),PSLD1(8)
      NDUM=0
      SCCP1=0.
      SCCP2=0.
      IF (NF.EQ.0)GO TO 20
      DO 10 I=1,NF
      IF (LD.EQ.NFAIL(I,1).AND.J.EQ.NFAIL(I,2)) NDUM=1
10    CONTINUE
      IF (NDUM.EQ.1) GO TO 11
20    NF=NF+1
      NOFAIL=1
      NCRACK=1
      NFAIL(NF,1)=LD
      NFAIL(NF,2)=J
11    SCCP1=GMAX
      THETA1=THETA+90.
      IF (GMIN.LT.FT) GO TO 21
      SCCP2=GMIN
      IF (NDUM.EQ.0) WRITE (6,1) LD,J,THETA1,THETA
 1    FORMAT (///5X,,BRICK ELEMENT NO.,,I5,,FAILS IN REGION,'I5,,BY CRAC
     1KING IN BOTH DIRECTIONS AT ANGLES=,,G12.4,,AND,,G12.4)
      GO TO 22
21    IF (NDUM.EQ.0) WRITE (6,2) LD,J,THETA1
 2    FORMAT (///5X,,BRICK ELEMENT NO.,,I5,,FAILS IN REGION,'I5,,BY CRAC
     1KING IN THE DIRECTION OF THETA=,,G12.3)
22    THETA=THETA/180.*22./7.
      IF (NF.GE.N3) GO TO 40
      C1=COS(THETA)*COS(THETA)
      C2=1.-C1
      C3=SIN(THETA)*COS(THETA)
      SCC1=C1*SCCP1+C2*SCCP2
      SCC2=C2*SCCP1+C1*SCCP2
      SCC3=C3*SCCP1-C3*SCCP2
      C1=B*T/9.
      C2=A*T/9.
      II=(J-1)/3
      RII=II
      XXJ1=2./3.*RII-1.
      XXJ2=XXJ1+2./3.
      RJ=J
      YYJ1=(2.*RJ-6.*RII-5.)/3.
      YYJ2=YYJ1+2./3.
      XX=XXJ2-XXJ1
      XXSQ=XXJ2*XXJ2-XXJ1*XXJ1
      YY=YYJ2-YYJ1
      YYSQ=YYJ2*YYJ2-YYJ1*YYJ1
      C3=B/8.*YYSQ*XX*T
      C4=A/8.*XXSQ*YY*T
      PSLD(1)=(C1+C3)*SCC1+(C2+C4)*SCC3
```

```
      PSLD(3)=-(C1+C3)*SCC1+(C2-C4)*SCC3
      PSLD(4)=(C3-C1)*SCC1+(C4-C2)*SCC3
      PSLD(5)=(C2+C4)*SCC2+(C1+C3)*SCC3
      PSLD(6)=-(C2+C4)*SCC2+(C1-C3)*SCC3
      PSLD(7)=(C2-C4)*SCC2-(C1+C3)*SCC3
      PSLD(8)=(C4-C2)*SCC2+(C3-C1)*SCC3
      DO 30 K=1,MS
      M=MS*(J-1)+K
   30 PSL(M)=PSLD(K)
      IF (J.NE.9) RETURN
      DO 50 K=1,MS
      SUM=0.0
      DO 55 M=1,9
      L=MS*(M-1)+K
   55 SUM=SUM+PSL(L)
   50 PSLD(K)=SUM
      READ (20,LD) (PSLD1(M),M=1,MS)
      RNORM=0.0
      DO 60 M=1,MS
      IF (PSLD(M).EQ.0.0) GO TO 60
      RMS=(PSLD(M)-PSLD1(M))/PSLD(M)
      RNORM=RNORM+ABS(RMS)
   60 CONTINUE
      IF (RNORM.LE.0.0001) RETURN
      NPSLD=1
      WRITE (20,LD) (PSLD(M),M=1,MS)
      RETURN
   40 WRITE (6,3)
    3 FORMAT (///5X,,THE SYSTEM COLLAPSES UNDER THIS LOAD,)
      STOP
      END
```

.RESULT

```
*****************************************************************************

      SUBROUTINE RESULT: PRINTS FINAL RESULTS

*****************************************************************************
      COMMON LOADND(10,2),DELTA,EPSILN,N3,PSL(72),NPSLD,NNR,NND
     1,NNODE(30)
      NPRINT=1
      WRITE (6,1)
    1 FORMAT (1H1/////57X,'FINAL RESULTS'/57X,15('='))
      WRITE (6,2)
    2 FORMAT (//46X,'STRESSES AND STRAINS FOR BRICK ELEMENTS'/46X,39('-'
     1)//19X,'STRESSES IN X-Y DIRECTIONS',4X,'STRAINS IN X-Y DIRECTIONS'
     2,9X,'PRINCIPAL STRESSES'/3X,'ELEM. NO.',1X,'REG',5X,'SX',9X,'SY',9
     3X,'SXY',8X,'EX',9X,'EY',9X,'EXY',9X,'1',10X,'2',7X,'THETA',6X,'TMA
     4X'/1X,127('='))
      DO 10 I=1,ME
      LD=I
      IF (TYPE(I).NE.1.) GO TO 10
      CALL CHKBRK
   10 CONTINUE
      WRITE (6,1)
      WRITE (6,3)
    3 FORMAT(//45X,'STRESSES AND STRAINS FOR MORTAR ELEMENTS'/45X,40('=
     1')//23X,'NORMAL STRESSES',30X,'SHEAR STRESSES'/1X,'ELEMENT NO.',5X
     2,'MAX',12X,'MIN',10X,'AVERAGE',10X,'MAX',12X,'MIN',10X,'AVERAGE',8
     3X,'E-NORMAL',7X,'E-SHEAR'/1X,127('='))
      DO 20 I=1,ME
      LD=I
      IF (TYPE(I).NE.2.) GO TO 20
      CALL CHKMOR
   20 CONTINUE
      WRITE (6,1)
      WRITE (6,4)
    4 FORMAT (//48X,'STRESSES AND STRAINS FOR STEEL BARS'/48X,35('-')//3
     17X,'ELEMENT NO',24X,'STRESS',24X,'STRAIN'/1X,127('='))
      DO 30 I=1,ME
      IF (TYPE(I).NE.3.) GO TO 30
      LD=I
      READ (10,LD) NC,MS,MSS,(SM(K),K=1,MSS),(NCODE(L),L=1,MS)
      IF (AR.EQ.0.) SSS=0.
      IF (AR.EQ.0.) DEL=0.
      IF (AR.EQ.0.) GO TO 33
      DO 31 J=1,MS
      SAYN=1.
      DEF(J)=0.
      IN=NCODE(J)
      IF (IN) 32,31,311
   32 IN=-IN
      SAYN=-1.
   11 IX=LL(IN+1)+IN
      DEF(J)=S(IX)
   31 CONTINUE
      DEL=(DEF(2)-DEF(1))*SAYN
      J1=NODE(I,1)
      J2=NODE(I,2)
      RLENG=Y(J2)-Y(J1)
      DEL=DEL/RLENG
      SSS=DEL*ESTEEL
   33 WRITE (6,5) I,SSS,DEL
    5 FORMAT (40X,I5,2(20X,G16.6))
   30 CONTINUE
```

```
      DO 40 I=1,NND
      II=NNODE(I)
      IN=JOX(II)
      IX=LL(IN+1)+IN
      D=S(IX))
40    WRITE (6,7) II,D
 6    FORMAT (//////52X,,EDGE DISPLACEMENTS,//40X,,NODE NO.',20X,
     1,DISPLACEMENT,/128(,=,))
 7    FORMAT (/40X,I5,20X,G12.6)
      RETURN
      END
```