PRELIMINARY DESIGN AND ANALYSIS OF SUBMERGED TUNNELS

THESIS

MUSTAFA TEZÇAKAR

BOĞAZİÇİ UNIVERSITY
Civil Engineering Department

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PRELIMINARY DESIGN AND ANALYSIS OF SUBMERGED TUNNELS

by

Mustafa TEZÇAKAR

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Dr. Hüsamettin Alper
(Thesis Supervisor)

Dr. Sahim Tekeli

Dr. Ahmet Ceranoğlu

ABSTRACT

A submerged tunnel, attached to the bottom by steel cables is analyzed using semi-analytical finite element methods. The tunnel with an elliptic cross-section with three circular tunnels inside is considered for which two computer programs are developed. The former one being a plane stress analysis program is used for the analysis of the structure under the effect of the hydrostatic forces and own weight while the second one performs analysis for the lateral current forces. The cross-section is divided into linear triangular elements then the strains together with the stresses are evaluated for each element separately. Furthermore, the principal stresses and their respective directions are obtained for design purposes.

ÖZET

Deniz dibine çelik kablolarla bağlanıp suya batırılmış olan tünel (tüp geçit) yarı analitik sonlu elemanlar metodu kullanılarak analiz edilmektedir. İçinde üç adet dairesel tüneli bulunduran ve elliptik bir keside sahip olan yapı için iki bilgisayar programı geliştirilmiştir. Bunlardan ilki bir düzlem gerilme analiz programı olup yapının hidrostatik basınç yükleri ve zatî yükü altında analizini yaparken ikincisi yatay akıntı kuvvetleri ve trafik yükü altında analiz yapmaktadır. Kesit öncelikle üçgen elemanlara bölünmekte sonra gerilmelerle birlikte birim deformasyonlar bütün elemanlar için ayrı ayrı bulunmaktadır. Bundan başka asal gerilmeler ve onlara ilişkin açı değerleri de dizayn gayesiyle elde edilmektedir.

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LIST OF SYMBOLS

B	Buoyancy force exerted on the body
W	Weight of the body
γ_c	Unit weight of concrete
V_c	Volume of the concrete
γ_w	Unit weight of water
V_T	Total Volume
$\lvert k \rvert$	Stiffness matrix
$\lvert B \rvert$	Strain shape function
$\lvert D \rvert$	Elasticity matrix
$\{q\}$	Nodal displacements
$\{r\}$	External nodal force
x	Reaction at the left end
y	Reaction at the right end
p^{ℓ}	Pressure at the left end
p^{r}	Pressure at the right end
u^{ℓ}, v^{ℓ}, w^{ℓ}	} ℓth displacement components in respective coordinate directions
$\{\varepsilon\}$	Strain vector
N_i	Displacement shape function
A	Area of Triangular element
a	Span of the tunnel

CHAPTER 1

INTRODUCTION

1.1. GENERAL

A sea strait is a channel connecting two basins of different properties where one may find a two-layer current system with the layers flowing in opposite directions. The Bosphorus and the Strait of Gibraltar could be cited as examples.

Frequently the circumstances are not suitable for transportation across the strait. A famous example in the world is the Messina strait which separates Sicily Island and Southern Italy. It has a width of 3 km and is susceptible to very strong winds, in addition to highly destructive earthquakes. Considering these conditions, a submerged tunnel was designed to provide transportation across the strait (1). The structure itself is placed at a certain height from the sea bottom so that earthquake effects are minimized. Moreover, it is tied to the bottom by steel cables to stabilize the structure against current forces.

In many physical problems the situation is such that the geometry and material properties do not vary along one coordinate direction. However, the loads may still exhibit a variation in that direction, preventing the use of such simplifying assumptions as plane stress or plane strain analysis instead of a full three dimensional treatment. In such cases it is still possible to consider a substitute problem, not involving the particular coordinate (along which the properties do not vary), and to synthesize the final results from a series of such simplified solutions (2). The method known as semi-analytical finite element processes uses Fourier series expansion with orthogonal functions.

1.2. OBJECT AND SCOPE

This study deals with the analysis of a submerged tunnel crossing the Bosphorus. An elliptic cross-section with three circular tunnels was selected for analysis. The structure is massive thus an advanced finite element technique is required for proper analysis. Two computer programs are developed and used to investigate the effects of submergence depth and current velocity on element stresses and deformations under the action of loads.

CHAPTER 2

PRELIMINARY CROSS-SECTION SELECTION

2.1. GENERAL

This chapter deals with the choice of a shape and various dimensions. It was assumed that three tunnels, two of which are for highways and the other for railways, would be needed. An advantage provided by these openings is that they reduce the weight so that the tunnel can float. While some considerations are taken into account a full design procedure is not intended.

2.2. SELECTION OF CROSS-SECTION AND DEPTH

From the fluid dynamics point of view an elliptic section with three circular tunnels inside seems to best suit the specific needs of the problem. Considering a two-lane highway a radius of 6 m. was assumed to be sufficient for the circular tunnels. Trial and error attempts for encasing these circular tunnels in a suitable section without increasing the weight of the body led to an elliptic section with major diameter twice the minor diameter.

The required condition for the structure as to be unaffected by the surface navigation and the existing waves should be considered in choosing a depth. In this study, the structure is assumed to be submerged to a depth of 50 meters from the sea surface.

2.3. DETERMINATION OF CROSS-SECTIONAL DIMENSIONS

The tunnel is desired to float while moored by steel cables to the sea bottom. Therefore, buoyancy forces must be equal to or larger than the weight of the body, thus causing tensile stresses in the mooring cables. For the cross-sectional shape shown in Figure 1, the above explanation leads to an equation of the form

$$B \geq W \tag{2.1}$$

in which B and W denote the buoyancy force and the weight per unit length of the body. The previous equation can be written as

$$\gamma_w V_T \geq \gamma_c V_c \tag{2.2}$$

where γ_c and γ_w are the unit weights of concrete and water, respectively; V_c = the concrete volume and V_T = the total volume.

Considering a unit thickness and recalling that the area of an ellipse is $\frac{\pi c T}{4}$, where c,T are defined in Figure 1, one can write

$$1.03\ A_T \geq 2.2\ A_c \tag{2.3}$$

$1.03\ A_T \geq 2.2\ (A_T - 3 \times \pi \times 36)$ from which $A_T \leq 637.98\ m^2$ taking the major radius as 20 m which is clear in Figure 1

$$\pi \times 20 \times \frac{T}{2} \leq 637.98 \qquad T \leq 20.30\ m.$$

Assuming T as 20 m. will both satisfy the above inequality and also simplify the numerical calculations.

2.4. LOADS AND MATERIAL PROPERTIES

For design purposes the load carrying capacity of the materials will be taken as (3)

Concrete:	28 day cylinder strength 30 MN/M^2
Reinforcing steel:	Deformed bars yield stress = 410 MN/M^2
Structural Steel :	Yield stress = 240 MN/M^2
Post-tensioning :	Ultimate strength = 1720 MN/M^2

Four different types of loads need to be considered traffic load, weight, hydrostatic pressure and hydrodynamic pressure due to existing currents. The traffic load transferred to the main body by the wheels of moving vehicles, is considered as a point load and hence treated accordingly in the program. Conservatively, a load of 10 tons per wheel was assumed for the analysis.

The direction and velocity of the current in the Bosphorus changes with respect to depth (4). The current values presented in Table 1 are averages of 15 minute metering from anchored ships. Values have been corrected for ship swing. At the point under consideration maximum depth was 91 m. and the wind speed 3 knots from the direction N 045°E. A constant velocity distribution of 1m/sec is assumed over the depth.

TABLE 1. FLOW CHARACTERISTICS MEASURED AT BEBEK (Lat 41° 04'29°N - Long. 29°03'08°E) Bebek (Ist. Sta.28) 9.5.1974

Depth (m)	Density σ_t	Current Direction (deg)	Current Speed (m/sn)	Temperature (°C)
0	13.59	151	1.11	10.73
5	13.76	169	1.09	10.51
10	13.89	168	1.02	10.38
15	14.11	186	0.95	10.15
20	15.20	169	0.82	9.24
25	19.49	189	0.55	9.84
30	21.16	068	0.08	10.23
35	24.02	010	0.42	11.37
40	26.02	015	0.52	12.80
45	26.33	010	0.39	13.76
50	26.35	030	0.51	13.94
55	26.36	023	0.47	13.98
60	26.36	023	0.26	14.13
65	26.36	032	0.51	14.73
70	26,36	034	0.59	14.36
75	26.36	038	0.31	14.36
80	26.36	037	0.41	14.42
85	26.36	027	0.48	14.44

The hydrodynamic pressure distribution on the circumference of an ellipse is approximated in Schlichting (5) as follows

$$\frac{\text{drag per unit length}}{\frac{1}{2}\,\rho\,U_o^2 \cdot T} < 0.10 \quad \text{for } \frac{c}{T} = 2.0$$

where ρ = density of sea water and U_o = the free stream velocity. The transition point from laminar to turbulent boundary layers occurs at

$$\frac{X_{crit}}{\ell'} = 0.28 \quad \text{for} \quad R = \frac{U_\infty \ell}{\nu} = 10^7$$

in which X_{crit} = critical length, ℓ = length of the ellipse parallel to the flow, U_∞ = undisturbed upstream velocity, ν = kinematic viscosity of water and $2\ell'$ = circumference of the ellipse. The Reynolds number for the chosen conditions is

$$R = \frac{100 \times 4000}{0.01} = 4 \times 10^7$$

Schlichting (5) has presented curves for shear stress and pressure distribution on elliptic cylinders as shown in Figures 2 and 3. By interpolation and extrapolation in these curves, shear stresses and pressure distribution on the circumference of the ellipse can be obtained. Values obtained for the ellipse used in this study are presented in Table 2.

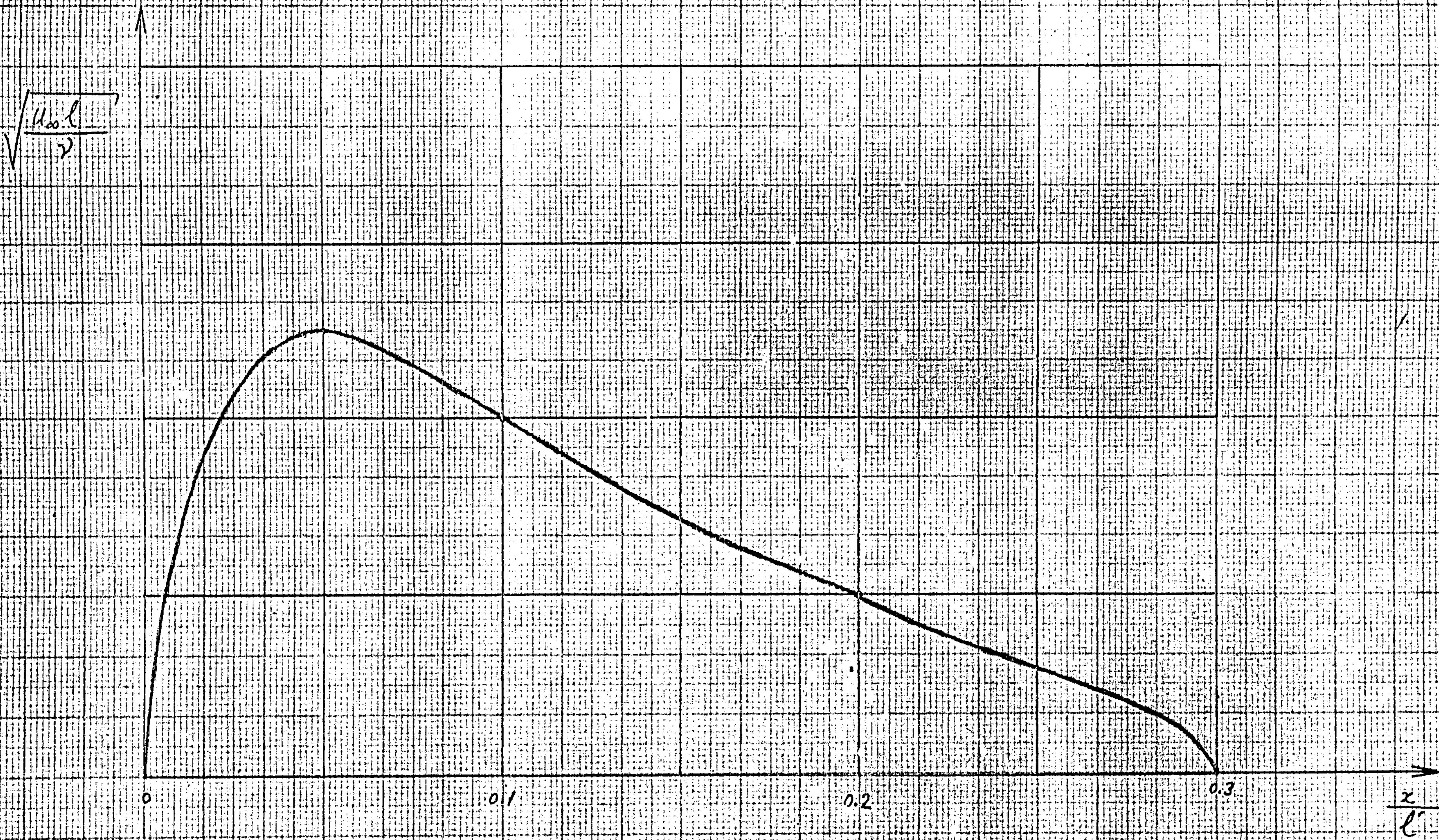


Fig 2 Shear stress distribution

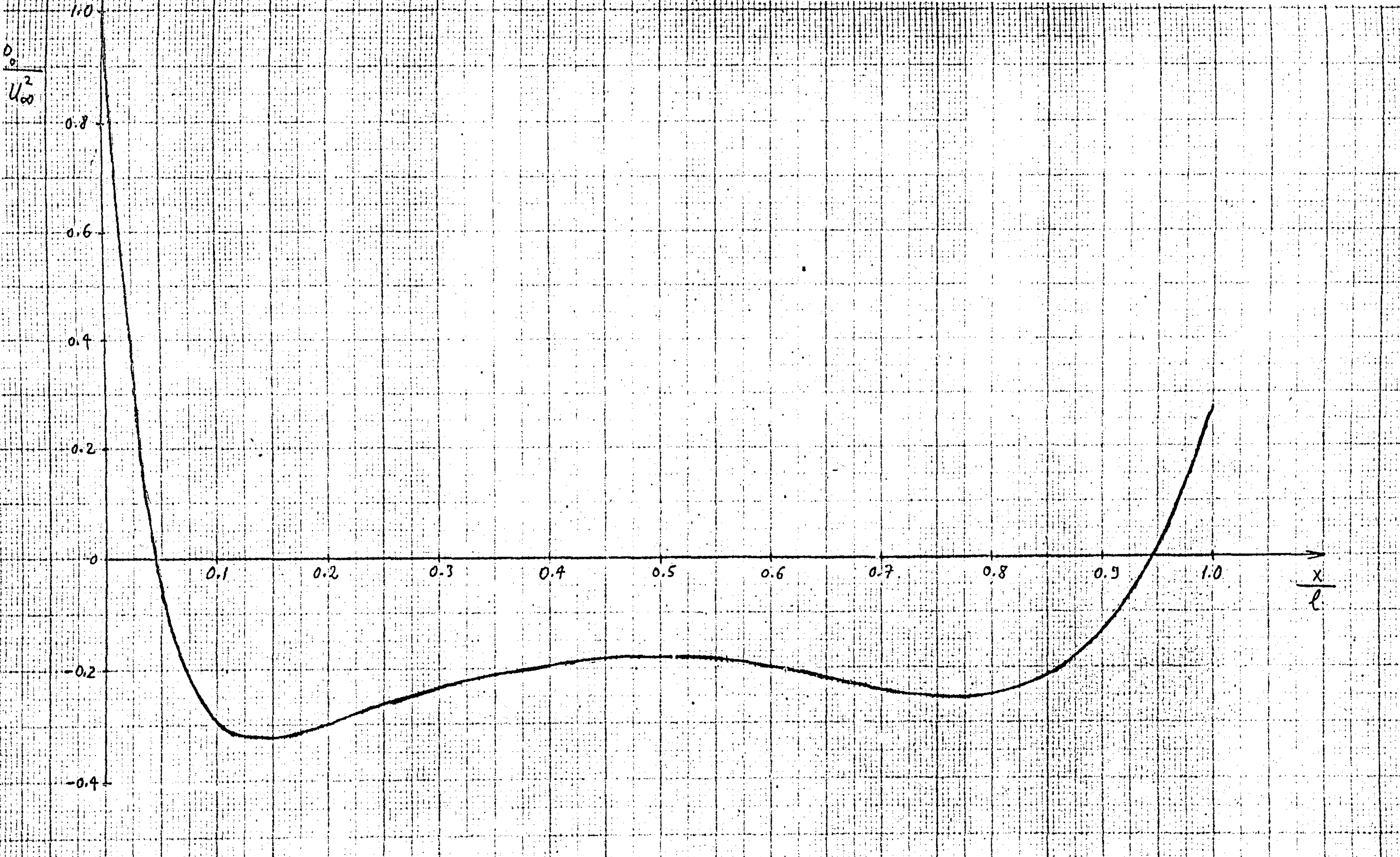


Fig 3 Pressure distribution on an aerofoil $l = 5.83'$

TABLE 2. PRESSURE AND SHEAR STRESS DISTRIBUTION ON THE PERIPHERY OF THE SECTION

Joint No.	Pressure t/m^2	τ_o t/m^2	Joint No.	Pressure t/m^2
1	10	0	34	3.3
2	9.9	4	40	3.2
3	9.5	5	46	3.1
4	8.4	5.2	50	4.0
8	6	5	63	4.1
10	5	4.5	66	4.5
13	4.7	4.2	69	4.7
15	4.2	4	71	4.4
17	4.1	3.7	73	4.1
19	4.0	3.7	75	5.2
21	3.9	3	77	5.3
23	3.8	2.9	79	5.5
25	3.7	2	81	5.6
27	3.6	2	86	5.7
29	3.5	1	87	5.8
31	3.4		88	5.9
			89	6.0

CHAPTER 3

METHOD OF ANALYSIS

3.1. GENERAL

The basic finite element equation for the element is

$$|k|\{q\} = \{r\} \tag{3.1}$$

where $|k|$ = the element stiffness matrix, $\{q\}$ = the nodal displacement vector and $\{r\}$ is the nodal load vector.

The stiffness matrix is evaluated by the following equation:

$$|k| = \int_V |B|^T |D| |B| dv \tag{3.2}$$

in which $|B|$ = strain shape function matrix and $|D|$ = elas ticity matrix.

Two types of analyses are carried out: plane str analysis under hydrostatic and dead loads and three dimensional analysis under current and traffic loads. The results are then superposed. Since the structure to be analyzed is prismatic, i.e., the geometry and material pr perties do not change along the length but only the loads change, the three dimensional finite element formulation is based on the use of orthogonal functions. For the plane stress analysis the linear triangle is used. Since the derivation of the stiffness matrix for this element is available in the literature it will not be repeated here.

3.2. PLANE STRESS ANALYSIS

The linear triangular element used for the plane stress analysis of the cross-section is shown in Figure 5 together with the nodal deformation numbers. The mesh used is shown in Figure 8. Since the cross-section and the loading considered are symmetrical with respect to the vertical axis through the centroid, only half of it needs to be considered. The nodal load vector {r} is obtained by transferring the distributed hydrostatic load on the periphery to statically equivalent nodal loads.

Hydrostatic pressure acts perpendicular to the surface considered and increases linearly with depth. Considering two adjacent nodes on the circumference of the ellipse, vertical equilibrium of forces in Figure 4 gives

$$x + y = \frac{(p^{\ell} + p^{r})\,\ell}{2} \qquad (3.3)$$

in which p^{ℓ} = pressure at the left node, p^{r} = pressure at the right node, ℓ = distance between the nodes; x and y =

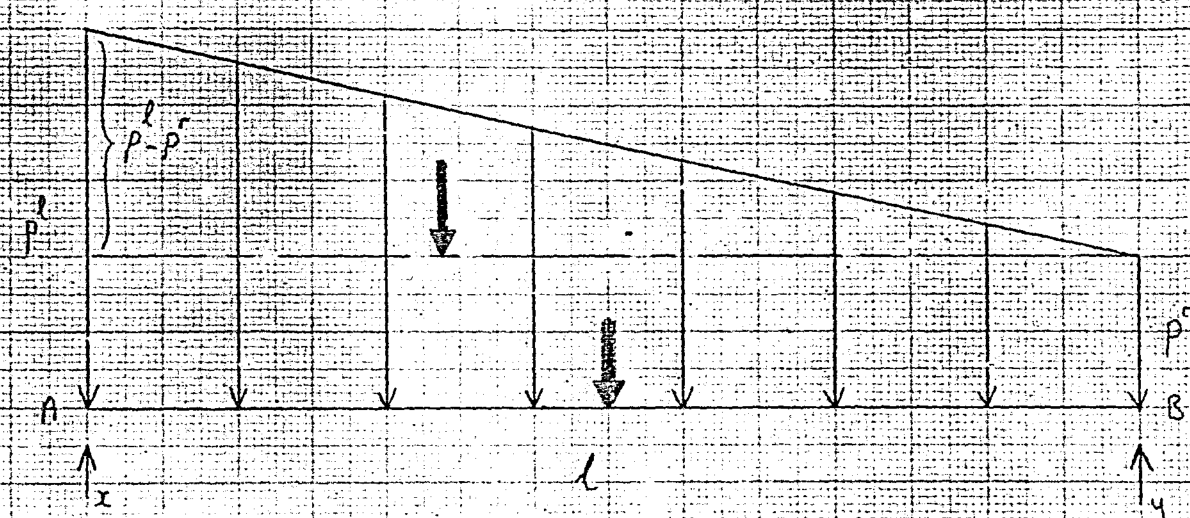


Fig 4 Transformation of hydrostatic pressure to equivalent nodal forces

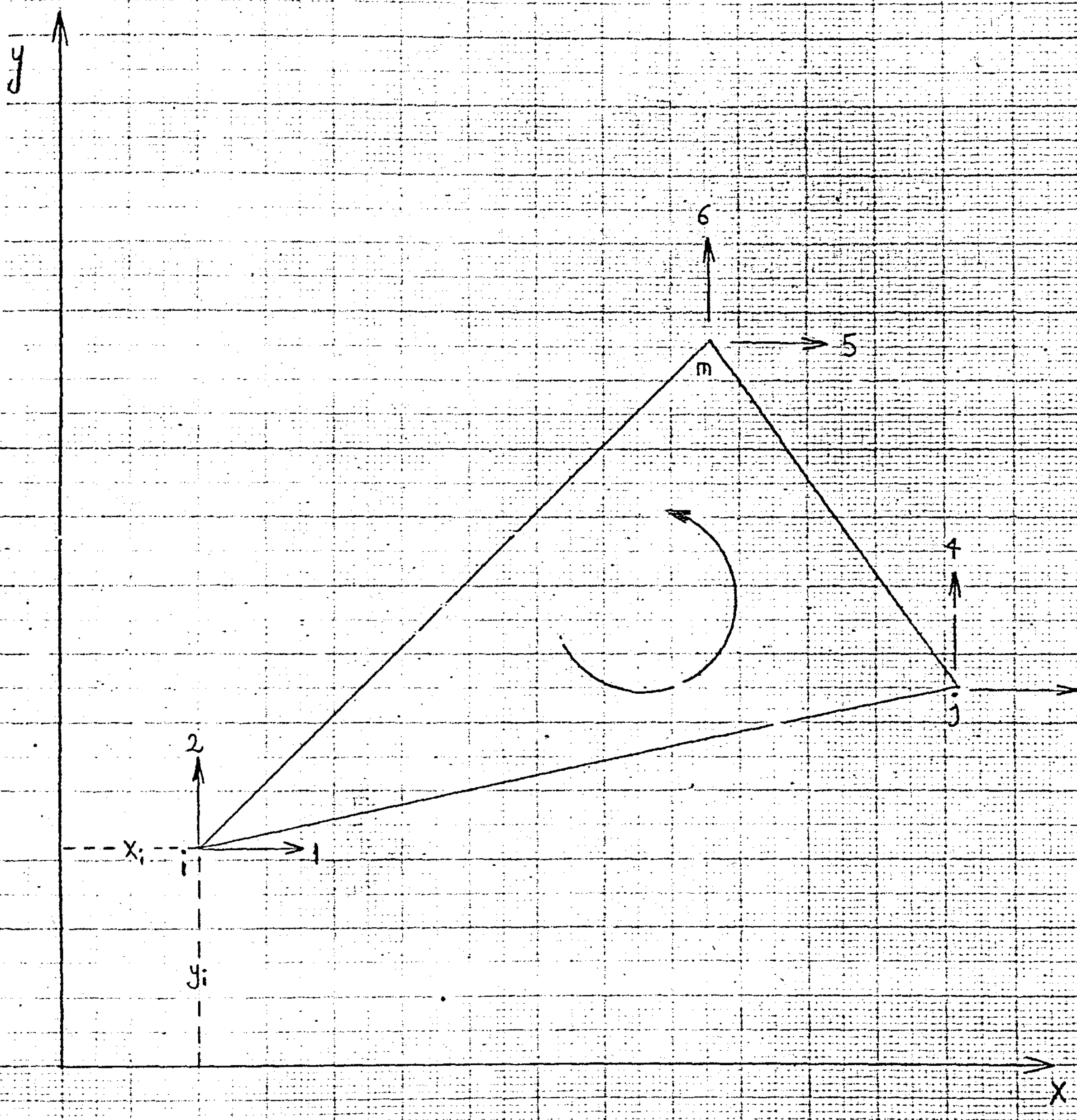


Fig 5 Element used for plane stress analysis

the reactions at the left and right ends, respectively. Now taking moments with respect to point A,

$$p^r \ell \frac{\ell}{2} + \frac{(p^\ell - p^r)\ \ell}{2} \cdot \frac{1}{3}\ell - y\ell = 0 \qquad (3.4)$$

from which

$$y = \frac{\ell}{6}(p^\ell + 2p^r) \qquad (3.5)$$

Taking moments with respect to point B.

$$p^r \ell \frac{\ell}{2} + \frac{(p^\ell - p^r)\ \ell}{2} \frac{2}{3}\ell - x\ell = 0 \qquad (3.6)$$

Solving for x

$$x = \frac{\ell}{6}(p^r + 2\ p^\ell) \qquad (3.7)$$

The above results for x and y can be verified by inserting them into Equation (3.3). The statically equivalent nodal loads are -x and -y.

3.3. SEMI-ANALYTICAL FINITE ELEMENT ANALYSIS

The linear triangular element was employed in the three dimensional stress analysis also. The degrees of freedom defined at each node as seen in Figure 6 are nine in this case. The mesh shown in Figure 9 was used for the analysis.

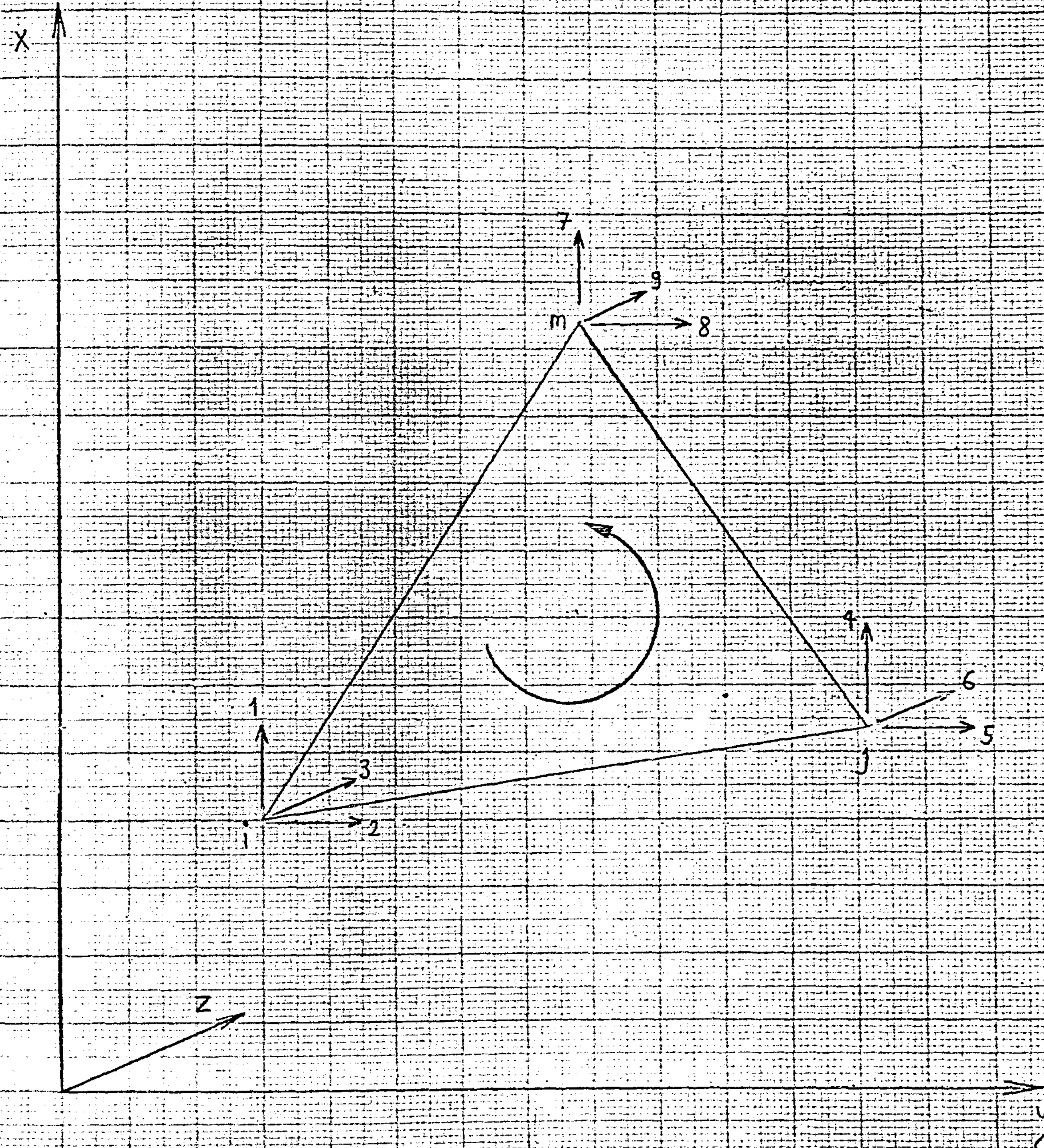


Fig 6 Element used for three dimensional analysis.

Subdividing into finite elements in the xy plane the ℓth displacement components can be prescribed in the x,y,z directions as

$$u^{\ell} = |N_i, N_j, N_m| \operatorname{Sin} \frac{\ell\pi z}{a} \begin{vmatrix} u_1 \\ u_2 \\ u_3 \end{vmatrix}^{\ell} \quad (3.8)$$

$$v^{\ell} = |N_i, N_j, N_m| \operatorname{Sin} \frac{\ell\pi z}{a} \begin{vmatrix} v_1 \\ v_2 \\ v_3 \end{vmatrix}^{\ell} \quad (3.9)$$

$$w^{\ell} = |N_i, N_j, N_m| \operatorname{Cos} \frac{\ell\pi z}{a} \begin{vmatrix} w_1 \\ w_2 \\ w_3 \end{vmatrix}^{\ell} \quad (3.10)$$

Letting $\gamma = \frac{\ell\pi z}{a}$ derivatives of u,v and w are obtained as

$$\frac{\partial u}{\partial x} = \operatorname{Sin}\gamma\left(\frac{\partial N_i}{\partial x} u_i + \frac{\partial N_j}{\partial x} u_j + \frac{\partial N_m}{\partial x} u_m\right) \quad (3.11)$$

$$\frac{\partial v}{\partial y} = \operatorname{Sin}\gamma\left(\frac{\partial N_i}{\partial y} v_i + \frac{\partial N_j}{\partial y} v_j + \frac{\partial N_m}{\partial y} v_m\right) \quad (3.12)$$

$$\frac{\partial w}{\partial z} = -\frac{\ell\pi}{a} \operatorname{Sin}\gamma(N_i w_i + N_j w_j + N_m w_m) \quad (3.13)$$

$$\frac{\partial u}{\partial y} = \operatorname{Sin}\gamma\left(\frac{\partial N_i}{\partial y} u_i + \frac{\partial N_j}{\partial y} u_j + \frac{\partial N_m}{\partial y} u_m\right) \quad (3.14)$$

$$\frac{\partial v}{\partial x} = \operatorname{Sin}\gamma\left(\frac{\partial N_i}{\partial x} v_i + \frac{\partial N_j}{\partial x} v_j + \frac{\partial N_m}{\partial x} v_m\right) \quad (3.15)$$

$$\frac{\partial v}{\partial z} = \frac{\ell\pi}{a} \operatorname{Cos}\gamma(N_i v_i + N_j v_j + N_m v_m) \quad (3.16)$$

$$\frac{\partial w}{\partial y} = \text{Cos}\gamma\left(\frac{\partial N_i}{\partial y} w_i + \frac{\partial N_j}{\partial y} w_j + \frac{\partial N_m}{\partial y} w_m\right) \tag{3.17}$$

$$\frac{\partial w}{\partial x} = \text{Cos}\gamma\left(\frac{\partial N_i}{\partial x} w_i + \frac{\partial N_j}{\partial x} w_j + \frac{\partial N_m}{\partial x} w_m\right) \tag{3.18}$$

$$\frac{\partial u}{\partial z} = \frac{\ell\pi}{a} \text{Cos}\gamma(N_i u_i + N_j u_j + N_m u_m) \tag{3.19}$$

The above expressions are substituted in Equation (3.20), and matrix $|B|$ is obtained as follows:

$$\{\varepsilon\} = \begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{vmatrix} = \begin{vmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial w/\partial z \\ \partial u/\partial y + \partial v/\partial x \\ \partial v/\partial z + \partial w/\partial y \\ \partial w/\partial x + \partial u/\partial z \end{vmatrix} \tag{3.20}$$

$$= \left[\begin{array}{ccc|ccc|ccc}
\frac{\partial N_i}{\partial x}\text{Sin}\gamma & 0 & 0 & \frac{\partial N_j}{\partial x}\text{Sin}\gamma & 0 & 0 & \frac{\partial N_m}{\partial x}\text{Sin}\gamma & 0 & 0 \\
0 & \frac{\partial N_i}{\partial y}\text{Sin}\gamma & 0 & 0 & \frac{\partial N_j}{\partial y}\text{Sin}\gamma & 0 & 0 & \frac{\partial N_m}{\partial y}\text{Sin}\gamma & 0 \\
0 & 0 & -N_i\frac{\ell\pi}{a}\text{Sin}\gamma & 0 & 0 & -N_j\frac{\ell\pi}{a}\text{Sin}\gamma & 0 & 0 & -N_m\frac{\ell\pi}{a}\text{Sin}\gamma \\
\frac{\partial N_i}{\partial y}\text{Sin}\gamma & \frac{\partial N_i}{\partial x}\text{Sin}\gamma & 0 & \frac{\partial N_j}{\partial y}\text{Sin}\gamma & \frac{\partial N_j}{\partial x}\text{Sin}\gamma & 0 & \frac{N_m}{\partial y}\text{Sin}\gamma & \frac{N_m}{\partial x}\text{Sin}\gamma & 0 \\
0 & \frac{\ell\pi}{a}N_i\text{Cos}\gamma & \frac{\partial N_i}{\partial y}\text{Cos}\gamma & 0 & \frac{\ell\pi}{a}N_j\text{Cos}\gamma & \frac{\partial N_j}{\partial y}\text{Cos}\gamma & 0 & \frac{\ell\pi}{a}N_m\text{Cos}\gamma & \frac{\partial N_m}{\partial y}\text{Cos}\gamma \\
\frac{\ell\pi}{a}N_i\text{Cos}\gamma & 0 & \frac{\partial N_i}{\partial x}\text{Cos}\gamma & \frac{\ell\pi}{a}N_j\text{Cos}\gamma & 0 & \frac{\partial N_j}{\partial x}\text{Cos}\gamma & \frac{\ell\pi}{a}N_m\text{Cos}\gamma & 0 & \frac{\partial N_m}{\partial x}\text{Cos}\gamma
\end{array}\right]
\begin{vmatrix} u_i \\ v_i \\ w_i \\ u_j \\ v_j \\ w_j \\ u_m \\ v_m \\ w_m \end{vmatrix}$$

$$\longleftarrow |B|_i \longrightarrow \longleftarrow |B|_j \longrightarrow \longleftarrow |B|_m \longrightarrow$$

(3.21)

The shape function. N at node is is given as (2)

$$N_i = \frac{1}{2A}(a_i + b_i x + c_i y) \tag{3.22}$$

where

$$a_i = x_j y_m - x_m y_j \tag{3.23}$$

$$b_i = y_j - y_m = y_{jm} \tag{3.24}$$

$$c_i = x_m - x_j = x_{mj} \tag{3.25}$$

and

$$A = \frac{1}{2} \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{vmatrix} \tag{3.26}$$

Then matrix $|B|$ becomes

$$|B| = \left[\begin{array}{ccc|c|c} \frac{b_i}{2A}\sin\gamma & 0 & 0 & & \\ 0 & \frac{c_i}{2A}\sin\gamma & 0 & & \\ 0 & 0 & -N_i\frac{\ell\pi}{a}\sin\gamma & & \\ \frac{c_i}{2A}\sin\gamma & \frac{b_i}{2A}\sin\gamma & 0 & & \\ 0 & \frac{\ell\pi}{a}N_i\cos\gamma & \frac{c_i}{2A}\cos\gamma & & \\ \frac{\ell\pi}{a}N_i\cos\gamma & 0 & \frac{b_i}{2A}\cos\gamma & & \end{array} \right] \tag{3.27}$$

$\longleftarrow |B|_i \longrightarrow \longleftarrow |B|_j \longrightarrow \longleftarrow |B|_m$

Zienkiewicz (2) gives the elasticity matrix $|D|$ as

$$|D| = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ & 1-\nu & \nu & 0 & 0 & 0 \\ & & 1-\nu & 0 & 0 & 0 \\ & & & \frac{1-2\nu}{2} & 0 & 0 \\ & & & & \frac{1-2\nu}{2} & 0 \\ \text{Symmetric} & & & & & \frac{1-2\nu}{2} \end{bmatrix} \quad (3.28)$$

Details of the derivation of the 9×9 stiffness matrix is presented in the Appendix.

3.4. STEEL CABLES

The steel cables used for tying the tunnel to the sea bottom are modeled as truss bars. Denoting the direction cosines of centroidal axis of a truss element as ℓ,m,n the element stiffness matrix in the gobal coordinate system is given as (6)

$$|k|_{xyz} = \frac{AE}{L} \begin{bmatrix} m^2 & mn & -m^2 & -mn \\ mn & n^2 & -mn & -n^2 \\ -m^2 & -mn & m^2 & mn \\ -mn & -n^2 & mn & n^2 \end{bmatrix} \quad (3.29)$$

in which A,L,E are the cross-sectional area, the length and the Young modulus of steel, respectively. The cable is assumed to be tied to node 35 with coordinates 3.9 and 4.1 as shown in Figure 7 and Figure 8. The steel cables are assumed to be inclined 30° with the horizontal. The reason for choosing the angle as small as 30° and not a

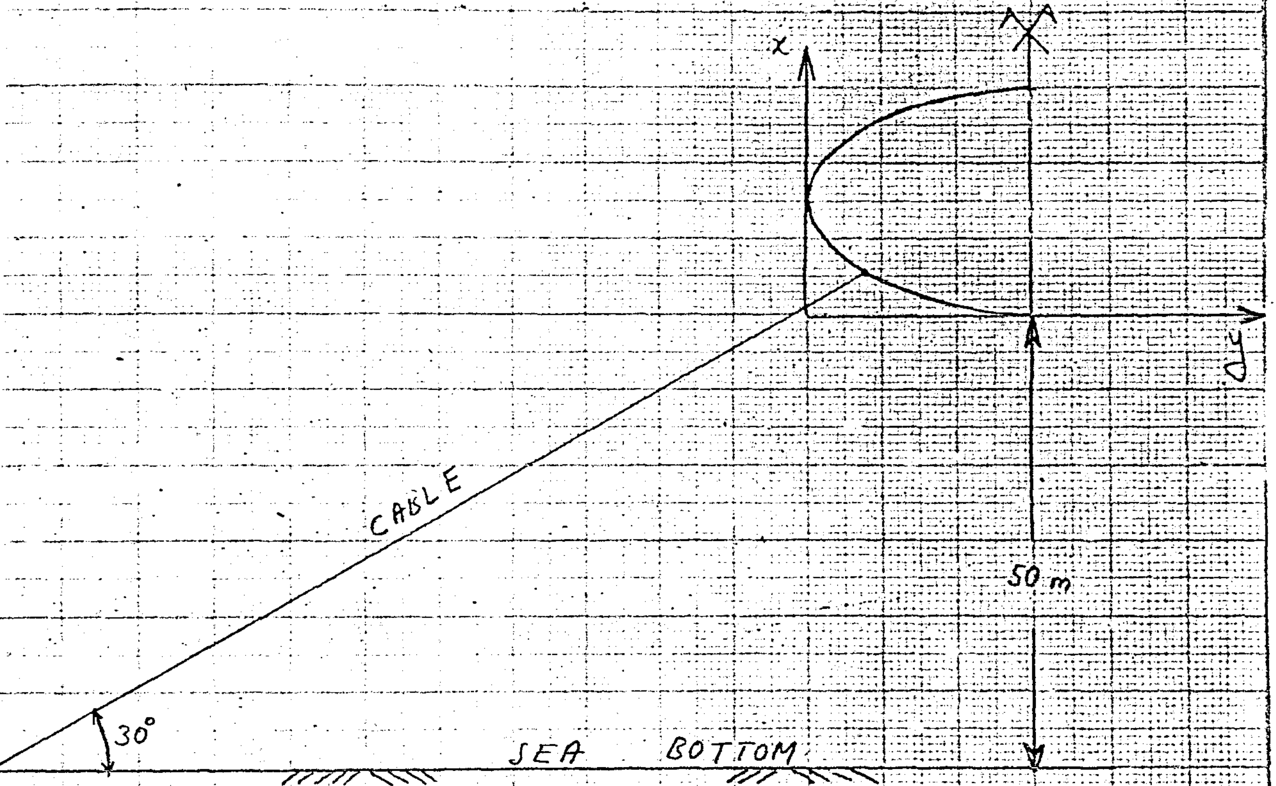
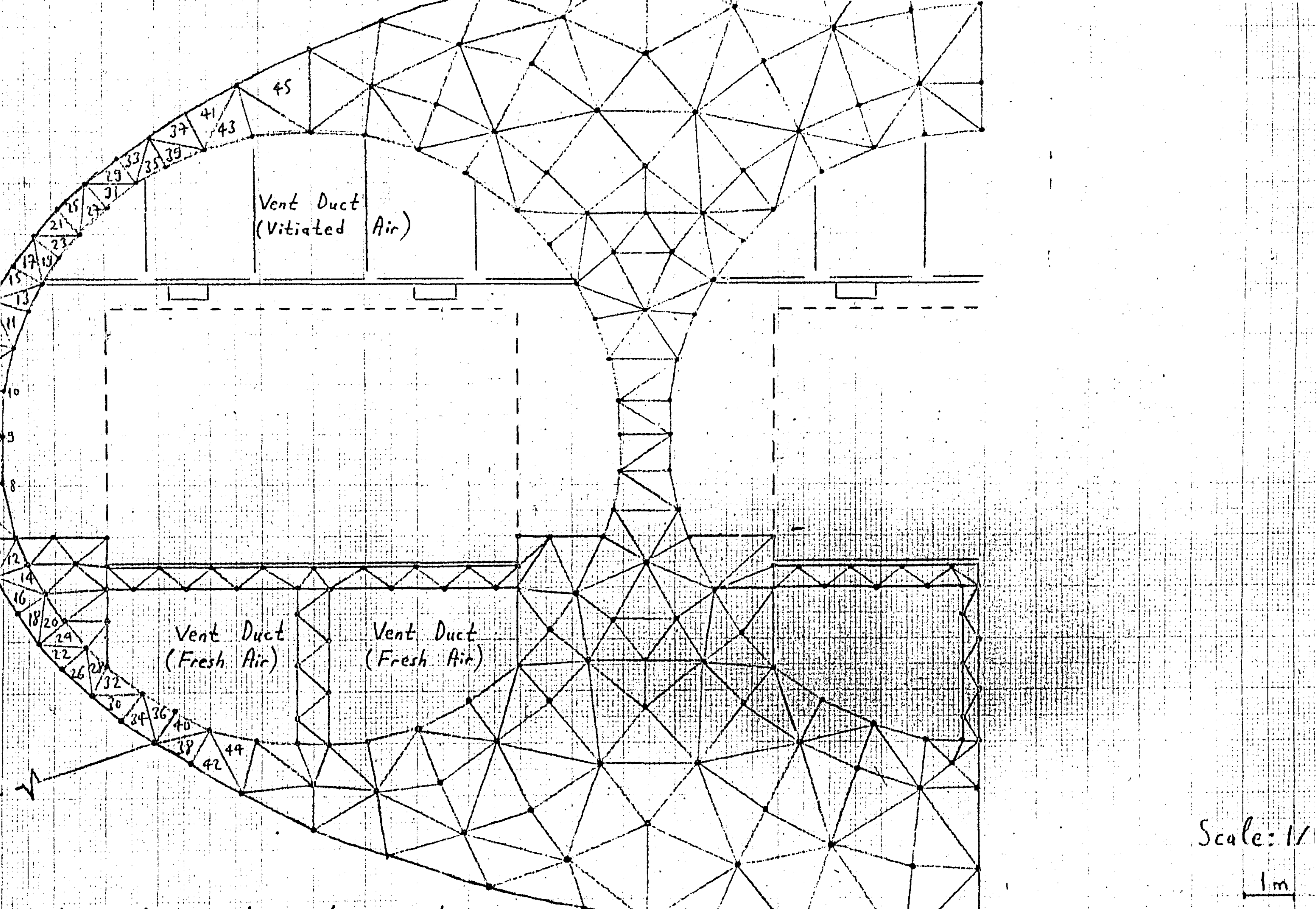


Fig 7 General layout of the structure



Mesh used for plane stress analysis

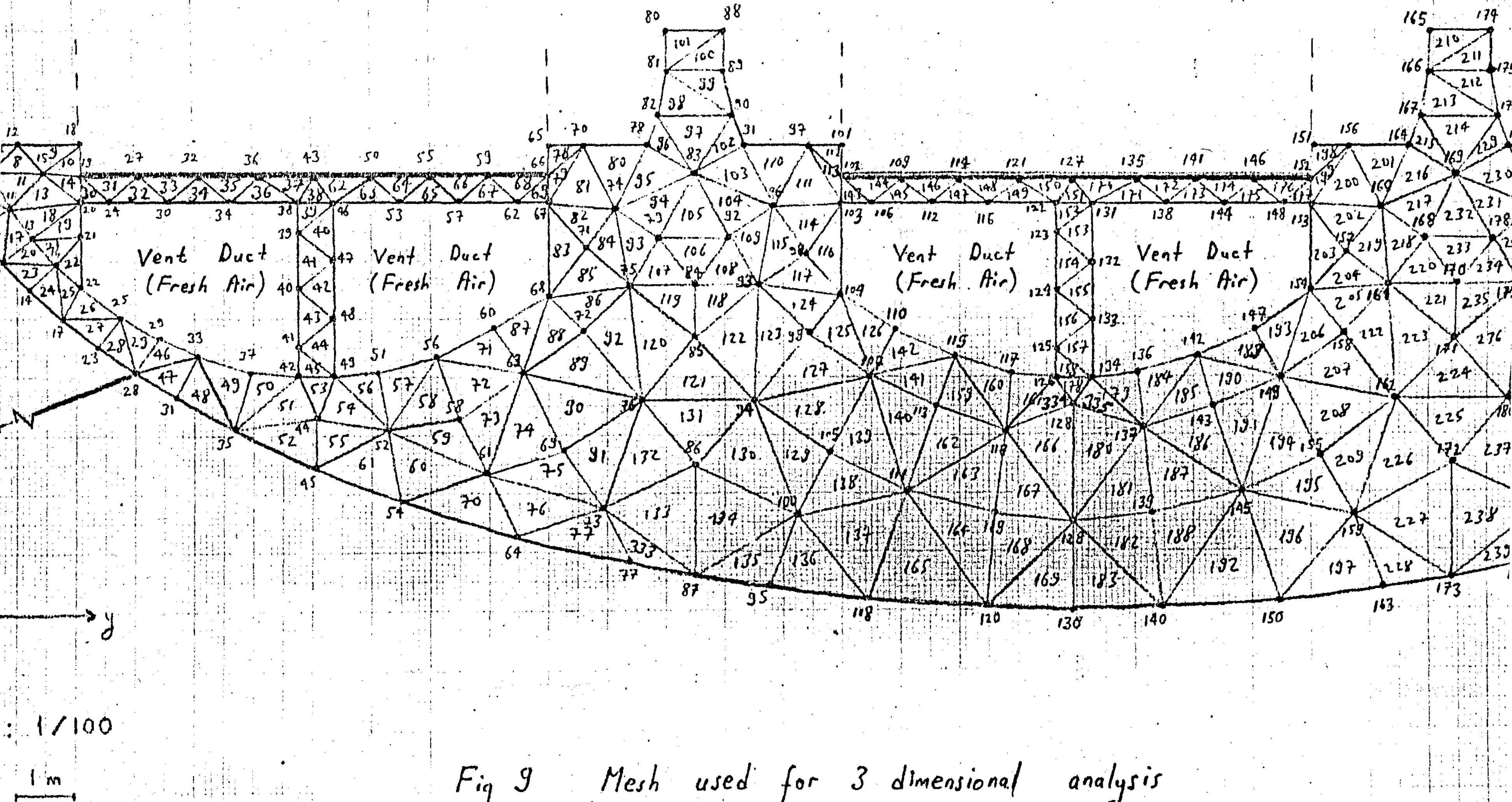


Fig 9 Mesh used for 3 dimensional analysis

larger value depends largely upon the stability problem of the structure under the effect of current loads.

3.5. ELEMENT STRESSES AND STRAINS

Having solved the unknown displacements at the nodes by Gauss elimination it is now possible to find the strains and stresses at each element.

3.5.1. Plane Stress Analysis

For the plane stress case one has

$$\{\varepsilon\} = |B|\{q\} = |B_i, B_j, B_m| \begin{vmatrix} \{q\}_i \\ \{q\}_j \\ \{q\}_m \end{vmatrix} \qquad (3.30)$$

where

$$|B|_i = \frac{1}{2A} \begin{bmatrix} b_i & 0 \\ 0 & c_i \\ c_i & b_i \end{bmatrix} \qquad (3.31)$$

Thus the relation $\{\varepsilon\} \equiv |B|\{q\}$ can be expressed as

$$\begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{vmatrix} = \frac{1}{2A} \begin{bmatrix} b_i & 0 & b_j & 0 & b_m & 0 \\ 0 & c_i & 0 & c_j & 0 & c_m \\ c_i & b_i & c_j & b_j & c_m & b_m \end{bmatrix} \begin{vmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_m \\ v_m \end{vmatrix} \qquad (3.32)$$

from which one can write

$$\varepsilon_x = \frac{1}{2A} (b_i u_i + b_j u_j + b_m u_m)$$
$$\varepsilon_y = \frac{1}{2A} (c_i v_i + c_j v_j + c_m v_m) \qquad (3.33)$$
$$\gamma_{xy} = \frac{1}{2A} (c_i u_i + b_i v_i + c_j u_j + b_j v_j + c_m u_m + b_m v_m)$$

The relation between stress and strain is

$$\{\sigma\} = |D|\{\varepsilon\} \qquad (3.34)$$

where $\{\sigma\}$ is the stress vector and $|D|$ is the elasticity matrix. Writing the previous equation componentwise

$$\begin{vmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{vmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-x}{2} \end{bmatrix} \begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{vmatrix} \qquad (3.35)$$

Hence

$$\sigma_x = \frac{E}{1-\nu^2} (\varepsilon_x + \nu\varepsilon_y) \qquad (3.36)$$

$$\sigma_y = \frac{E}{1-\nu^2} (\nu\varepsilon_x + \varepsilon_y) \qquad (3.37)$$

$$\tau_{xy} = \frac{E}{1-\nu^2} \frac{1-\nu}{2} \gamma_{xy} = \frac{E}{2(1+\nu)} \gamma_{xy} \qquad (3.38)$$

in which E is the elastic modulus and ν is the Poisson's ratio.

3.5.2. Semi-Analytical Analysis

For the three dimensional stress analysis matrix $|B|$ takes the following form:

$$|B| = \frac{1}{2A}\left[\begin{array}{ccc:c:c} b_i\text{Sin}\gamma & 0 & 0 & \uparrow & \uparrow \\ 0 & c_i\text{Sin}\gamma & 0 & & \\ 0 & 0 & -N_i'\frac{\ell\pi}{a}\text{Sin}\gamma & & \\ c_i\text{Sin}\gamma & b_i\text{Sin}\gamma & 0 & |B|_j & |B|_m \\ 0 & \frac{\ell\pi}{a}N_i'\text{Cos}\gamma & c_i\text{Cos}\gamma & & \\ \frac{\ell\pi}{a}N_i'\text{Cos}\gamma & 0 & b_i\text{Cos}\gamma & \downarrow & \downarrow \end{array}\right] \tag{3.39}$$

Expressing Equation (3.30) as

$$\{\varepsilon\} = |B|\{q\} \tag{3.40}$$

or componentwise

$$\begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{vmatrix}_{6\times1} = |B|_{6\times9} \begin{vmatrix} u_i \\ v_i \\ w_i \\ u_j \\ v_j \\ w_j \\ u_m \\ v_m \\ w_m \end{vmatrix}_{9\times1} \tag{3.41}$$

and inserting $|B|$ matrix from the previous equation one finds

$$\varepsilon_x = \sum_{i,j,m} b_i\text{Sin}\gamma u_i \tag{3.42}$$

$$\varepsilon_y = \sum c_i v_i \text{Sin}\gamma \tag{3.43}$$

$$\varepsilon_z = \sum - N_i' \frac{\ell\pi}{a} w_i \text{Sin}\gamma \tag{3.44}$$

$$\gamma_{xy} = \Sigma\ (c_i u_i \text{Sin}\gamma + b_i v_i \text{Sin}\gamma) \tag{3.45}$$

$$\gamma_{yz} = \Sigma\ (\frac{\ell\pi}{a} N_i' v_i \text{Cos}\gamma + c_i w_i \text{Cos}\gamma) \tag{3.46}$$

$$\gamma_{zx} = \Sigma\ (\frac{\ell\pi}{a} N_i' u_i \text{Cos}\gamma + b_i w_i \text{Cos}\gamma) \tag{3.47}$$

where

$$\gamma = \frac{\ell\pi z}{a} \quad \text{and} \quad N_i' = 2AN_i$$

Rearranging,

$$\varepsilon_x = \text{Sin}\gamma(b_i u_i + b_j u_j + b_m u_m) \tag{3.48}$$

$$\varepsilon_y = \text{Sin}\gamma(c_i v_i + c_j v_j + c_m v_m) \tag{3.49}$$

$$\varepsilon_z = -\frac{\ell\pi}{a} \text{Sin}\gamma|(a_i+b_i x+c_i y)w_i + (a_j+b_j x+c_j y)w_j + (a_m+b_m x+c_m y)\, w_m| \tag{3.50}$$

$$\gamma_{xy} = \text{Sin}\gamma(c_i u_i + b_i v_i + c_j u_j + b_j v_j + c_m u_m + b_m v_m) \tag{3.51}$$

$$\gamma_{yz} = \text{Cos}\gamma\{\frac{\ell\pi}{a}|(a_i+b_i x+c_i y)v_i + (a_j+b_j x+c_j y)v_j + (a_m+b_m x+c_m y)v_m| + c_i w_i + c_j w_j + c_m w_m\} \tag{3.52}$$

$$\gamma_{zx} = \text{Cos}\gamma\{\frac{\ell\pi}{a}|(a_i+b_i x+c_i y)u_i + (a_j+b_j x+c_j y)u_j + (a_m+b_m x+c_m y)u_m| + b_i w_i + b_j w_j + b_m w_m\} \tag{3.53}$$

Having obtained the strains one can now obtain the stresses from the relationship

$$\begin{vmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{vmatrix} = |D| \begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{vmatrix} \quad (3.54)$$

Using Equation (3.28) stress compnents are determined as:

$$\sigma_x = \xi \, |(1-\nu)\varepsilon_x + \nu\varepsilon_y + \nu\varepsilon_z| \quad (3.55)$$

$$\sigma_y = \xi \, |\nu\varepsilon_x + (1-\nu)\varepsilon_y + \nu\varepsilon_z| \quad (3.56)$$

$$\sigma_z = \xi \, |\nu\varepsilon_x + \nu\varepsilon_y + (1-\nu)\varepsilon_z| \quad (3.57)$$

$$\tau_{xy} = \frac{E}{2(1+\nu)} \gamma_{xy} \quad (3.58)$$

$$\tau_{yz} = \frac{E}{2(1+\nu)} \gamma_{yz} \quad (3.59)$$

$$\tau_{zx} = \frac{E}{2(1+\nu)} \gamma_{zx} \quad (3.60)$$

where

$$\xi = \frac{E}{(1+\nu)(1-2\nu)} \quad (3.61)$$

3.6. DESCRIPTION OF PROGRAM AND THE FLOWCHART

Once the sketch and numbering of the mesh is completed the user can proceed to the preparation of data for the program. To perform the analysis of submerged tunnels two programs are developed. The first one is a plane stress finite element program obtained from Dr. Alper which analyzes the tunnel under the effect of hydrostatic pressure considering own weight. The second one is again a finite element program but modified to perform three-dimensional stress analysis by semi-analytical finite element methods using orthogonal functions.

While obtaining the system stiffness matrix only half of the matrix above the diagonal needs to be generated since the element stiffness matrices are symmetric. Moreover, all the non-zero coefficients in the system stiffness matrix are confined within a band whose width can be calculated before the generation of the matrix. Thus in the computer programs only the storage of the elements within the upper half of the band width is necessary. Meanwhile the reader is referred to Table 3 for the definition of the variables used for data storage.

In the main program the number and area of the cables are read by the computer. Then five call statements refer to the relevant subroutines for the necessary evaluations. The first subroutine numbers the deformations

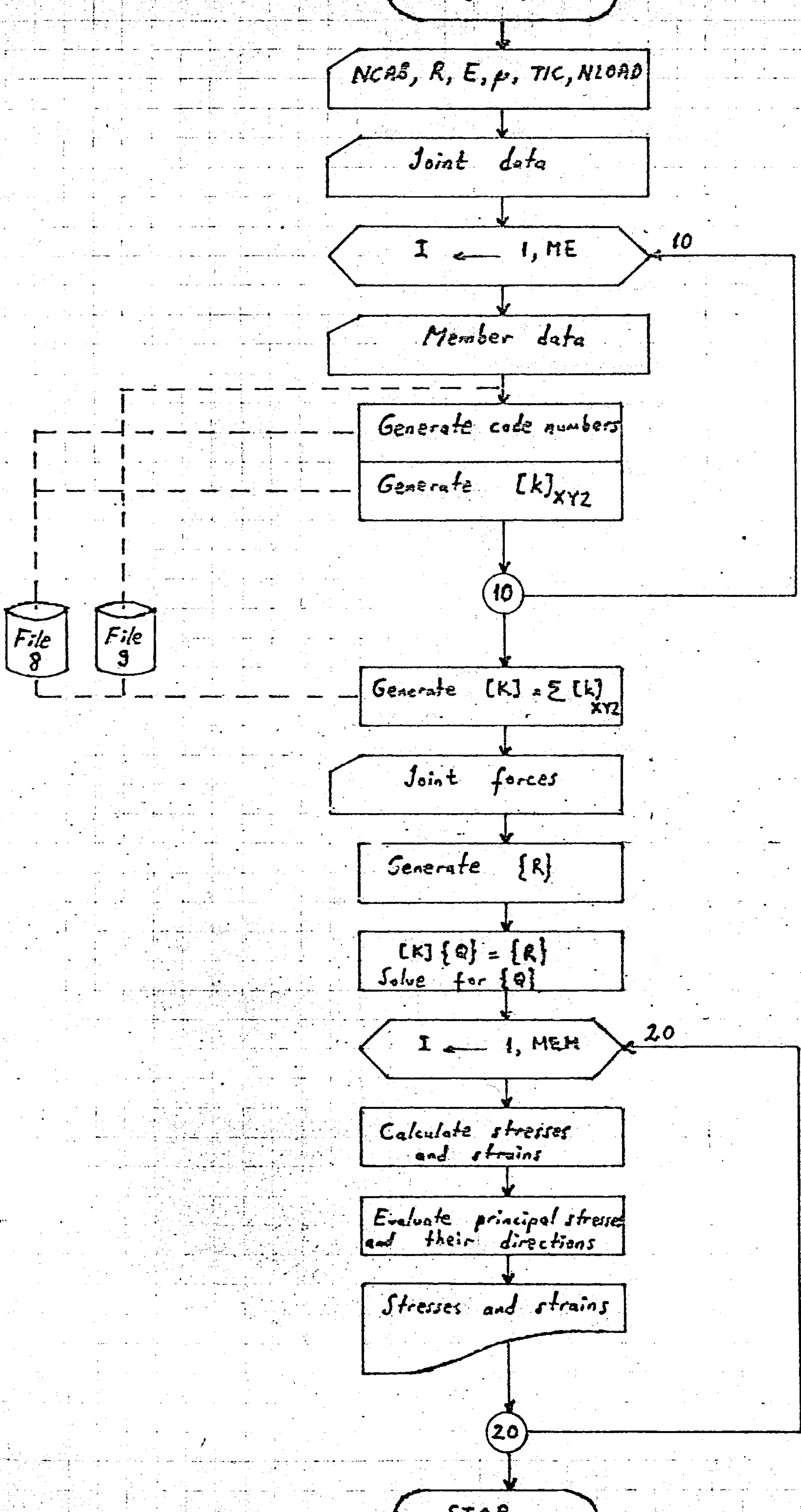
TABLE 3. VARIABLES USED IN COMPUTER PROGRAMMING

Variable Name (Dimensions)	Description
TIT(20)	Title for the Project
DM(6)	Values of deformations for an element (maximum is 9)
JDEF(800)	Deformation numbers of the nodes in order
DEF(2)	Number of deformations at a joint (max is 3)
ST(21)	The lower triangular part of $\|k\|_{9\times9}$ listed in columwise order (max is 45)
NODE(400,3)	Element node numbers in order
X(300)	The apsissas of all the nodes
Y(300)	The ordinates of all the nodes
S(20000)	Structure stiffness vector
NCODE(6)	Code numbers of an element (maximum is 9)
COR(800)	The coordinates of all the joints
V(3)	Number of joints of the finite element used
NE(33)	Number of joints on the periphery of the ellipse
FNE(33)	Forces normal to the periphery of the ellipse
FNEY(33)	Horizontal force components
FNED(33)	Vertical force components
FTE(15)	Forces tangent to the circumference of the ellipse
A(6)	The lower triangular part of $\|\sigma\|_{3\times3}$ listed in rowwise order.
B(3)	Eigenvalue vector
R(3,3)	Modal matrix
TIC	Thickness of the element
PS	Print option (if it is not zero all the stiffness matrices are printed)
POISON	Poisson's ratio
MS	Number of total deformations of an element
STRN	Variable used for assigning the appropriate title
E	Young Modulus of Elasticity
MST	Number of deformations at a joint

TABLE 3 (Continued)

Variable Name (Dimensions)	Description
JBAND	Band width of the structure stiffness matrix
NLOAD	Number of load cases
ME	Element number under consideration
N	Number of unknowns
NHEP	Dimension of the vector {S}
TOL	Tolerance compared with STRN for title purposes
NJ	Number of joints
NJT	Number of nodes of the finite element used
PI	The value of $\pi = 3.14$
NCAB	Number of cables
R	Cross sectional area of one cable
KAPE	Dimension of the modal matrix $\|R\|$ when considered as a unidimensional array
MV	Option for the evaluation of the modal matrix $\|R\|$
S80	Variable used to denote the end of data

and stores into an array while the coordinates are also stored to a vector, before the control is transferred to the calling program. The second subroutine develops the elements of the elasticity matrix according to the type of the problem as plane stress or plane strain whichever is the case. The joint numbers for all of the elements are stored into an array followed by a print of bandwidth of the system stiffness matrix. Then the following subroutine establishes the code numbers for each element separately. Joining the element stiffness matrices in another subroutine and solving them for the nodal deformations we come up with the last subroutine which evaluates the strains, stresses, principal stresses and their directions for all of the elements under consideration.


NCAS, R, E, μ, TIC, NLOAD
Joint data
I ← 1, ME
10
Member data
Generate code numbers
Generate [k]XYZ
File 8
File 9
10
Generate [K] = Σ [k]XYZ
Joint forces
Generate {R}
[K] {Q} = {R}
Solve for {Q}
I ← 1, MEM
20
Calculate stresses and strains
Evaluate principal stresses and their directions
Stresses and strains
20
STOP

CHAPTER 4

RESULTS AND DISCUSSIONS

The order or the numbering of the elements is not crucial; however, the order of the nodes will strongly influence the amount of memory usage and as a general rule, the numbering should be such as to minimize the nodal difference for each element (maximum node number nimus minimum node number). In a finite element solution of any problem, about eighty percent of the time is spent during the solution of the algebraic equations for the unknown quantities, and since the solution time is approximately proportional with the square of the band width it is desirable to minimize the band width as much as possible. The band width of the system stiffness matrix depends upon the largest difference between any two external node numbers for a single element. Therefore, special care must be given to the numbering of the nodes in order to minimize both the solution time and the storage requirements for the system stiffness matrix.

It may be noted that the stresses, obtained from the computer solution, do not exceed the allowable values for none of the elements within the section. The thickness of the concrete reduces to its minimum value, which is about 85 cm, to the left and right of the section creating the most critical part to be considered under the effect of the loads. Even this region does not exhibit any problem from the point of view of the allowable stresses. For instance the maximum stress occurring at triangle 30 comes out to be 120 t/m^2 for 50 m depth and 1 m/sec current velocity and 1500 t/m^2 for the same depth but 3 m/sec current velocity. Thus, even under the worst condition we have about 50% safety assuming the strength of concrete to be equal to 3000 t/m^2.

As the current velocity changes from 1 m/sec to 3 m/sec with increment one the maximum stress, occurring at the element mentioned previously comes out to be 134.4, 239.7, 345.1 t/m^2 for the three respective velocities under the effect of current loads.

The deformations found at each joint are in good agreement with the nodal forces existing there. Since the direction of the deformation under the effect of the forces can be judged intuitively, verification of the results affirms their correctness. For the three dimensional analysis obtained at the midspan, longitudinal deformations at all nodes are obtained as zero, as expected, due to the symmetry of the structure and the loads with respect to the midspan. This observation also supports the correctness of the results.

CHAPTER 5

CONCLUSIONS

1. The structure, floating 50 meters below the water surface can successfully resist the hydrostatic pressure and the existing current.

2. The dominant parameter affecting the behavior of the structure is the submergence depth. As the depth increases, both the stresses and deflections also increases.

3. The second important parameter is the existing current velocity which is about 1 m/sec. If it increases, the stresses also increase as explained numerically in the previous chapter. Larger current velocity will result with greater hydrodynamic pressure and shear stresses on the surface of the body increasing the stresses and nodal deformations within the body.

4. Despite the submerged tunnel seems feasible as far as stresses are concerned, it has to be cast as hundred meter parts on the site and join them in the water using latest developed welding techniques.

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APPENDIX

DERIVATION OF |K|

Carrying out the triple matrix product $|B|^T|D||B|$ one finds $\frac{\partial^3|k|}{\partial x \partial y \partial z}$ for which the element in the first column and row will be

$$\frac{c}{2A}\left|(1-\nu)b_i^{\,2}\mathrm{Sin}^2\gamma + \left(\frac{1-2\nu}{2}\right)c_i^{\,2}\mathrm{Sin}^2\gamma + \left(\frac{1-2\nu}{2}\right)\frac{\ell^2\pi^2}{a^2}N_i'^{\,2}\mathrm{Cos}^2\gamma\right|$$

where

$$c = \frac{E}{2A(1+\nu)(1-2\nu)}$$

Now one can derive the 45 different elements of stiffness matrix $|k|$ making use of the familiar expression

$$\int_0^a \mathrm{Sin}^2\frac{\ell\pi z}{a}\,dz = \int_0^a \mathrm{Cos}^2\frac{\ell\pi z}{a}\,dz = \frac{a}{2}$$

when $\ell = 1,2,\ldots$

$$k_{11} = \frac{c}{2A}\left\{\left|(1-\nu)b_i^{\,2} + \left(\frac{1-2\nu}{2}\right)c_i^{\,2}\right|\iiint \mathrm{Sin}^2\gamma\,dx\,dy\,dz\; +\right.$$

$$\left. + \left(\frac{1-2\nu}{2}\right)\frac{\ell^2\pi^2}{a^2}\iiint_v (2AN_i)^2\mathrm{Cos}^2\gamma\,dx\,dy\,dz\right.$$

$$= \frac{c}{2A}\left\{\left|(1-\nu)b_i^{\,2} + \left(\frac{1-2\nu}{2}\right)c_i^{\,2}\right|\frac{a}{2}A + \left(\frac{1-2\nu}{2}\right)\frac{\ell^2\pi^2}{a^2}\frac{a}{2}4A^2\frac{A}{6}\right.$$

$$\iint N_i^{\,2}\,dx\,dy = \frac{2!}{4!}2A = \frac{4A}{24} = \frac{A}{6}$$

$$\iint N_i\,dx\,dy = \frac{1}{3!}\,2A = \frac{A}{3}$$

$$= \frac{c}{2A} \cdot \frac{Aa}{2} \left| (1-\nu)b_i^{\,2} + \left(\frac{1-2\nu}{2}\right)c_i^{\,2} + \left(\frac{1-2\nu}{2}\right) \frac{\ell^2\pi^2}{a^2}\, 2\, \frac{A^2}{3} \right|$$

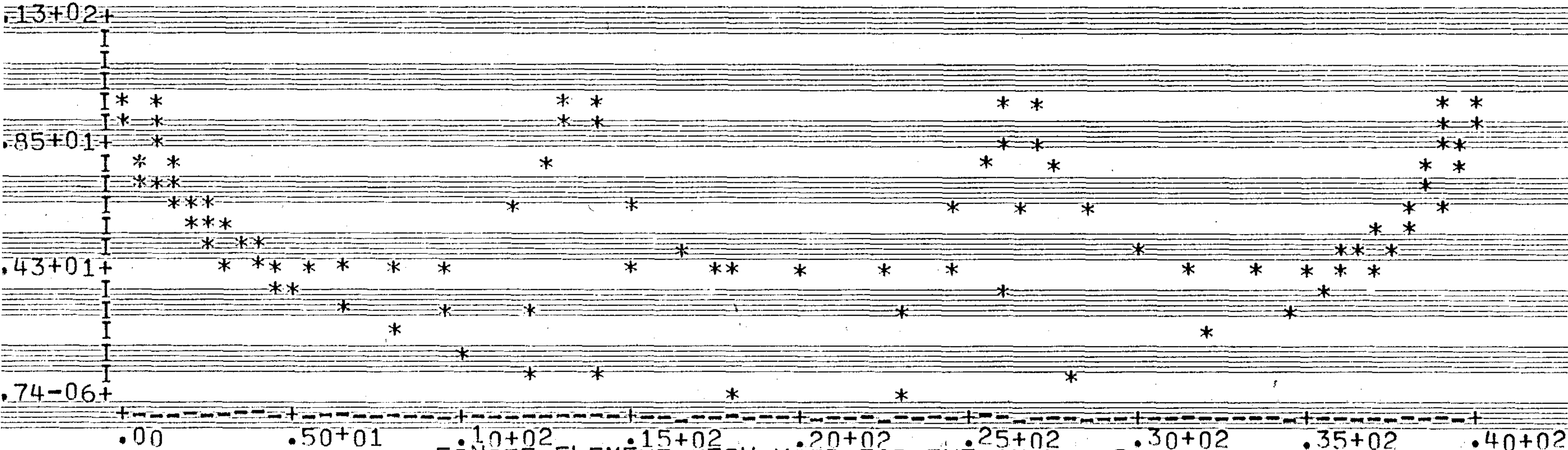
Thus k_{11} will be

$$k_{11} = S\,\frac{\alpha}{2}\left|\gamma b_i^{\,2} + \beta\left(c_i^{\,2} + \frac{2\ell^2\pi^2}{3\alpha^2}\right)\right|$$

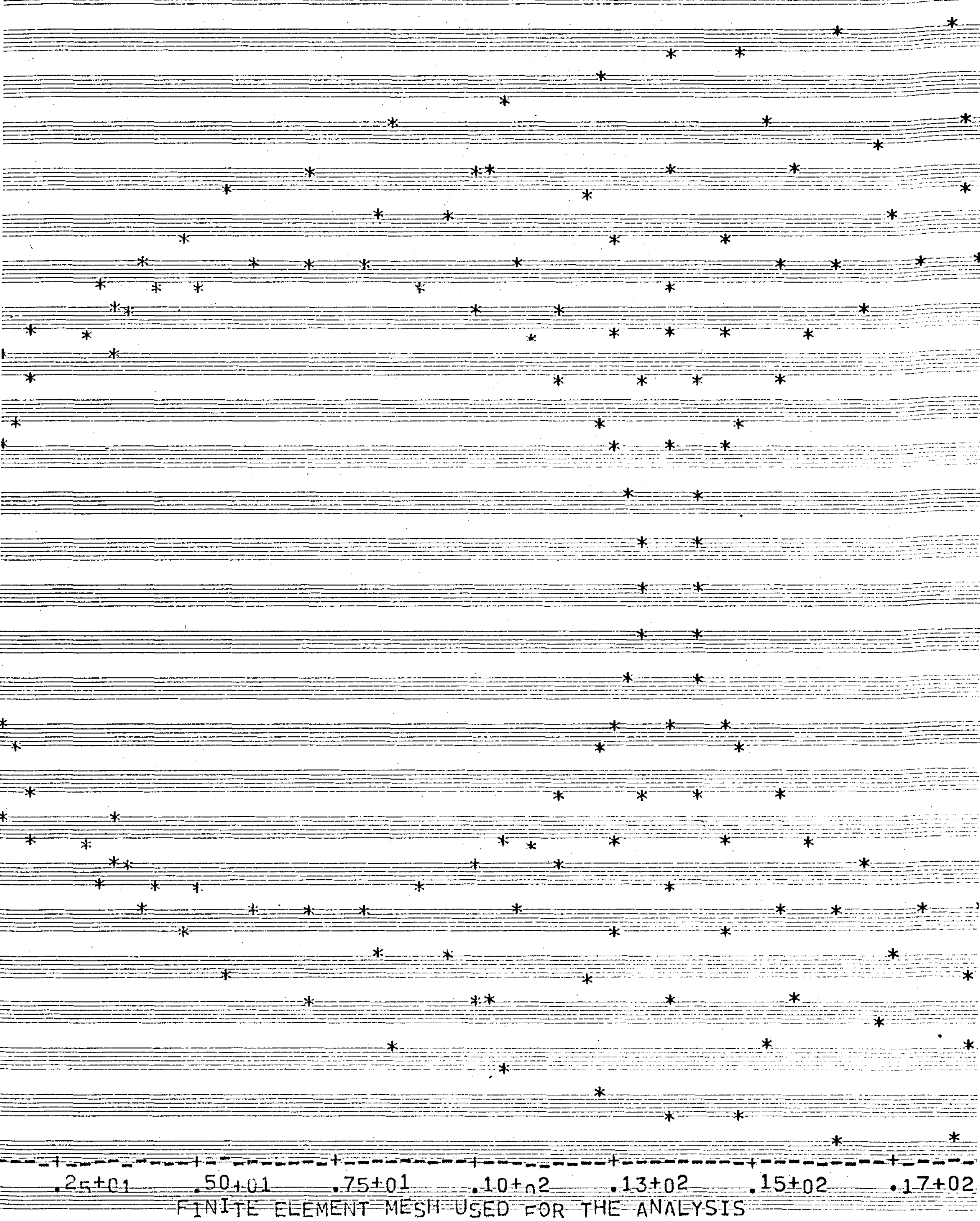
where

$$\frac{a}{A} = \alpha \qquad \frac{1-2\nu}{2} = \beta$$

$$1-\nu = \gamma \qquad S = \frac{E}{4(1+\nu)(1-2\nu)}$$



FINITE ELEMENT MESH USED FOR THE ANALYSIS



FINITE ELEMENT MESH USED FOR THE ANALYSIS

ER*TARKAN(1).FNT

```
      DIMENSION TIT(20),DM(9),JDEF(800),DEF(3),ST(45),NODE(400
      DIMENSION S(20000),LL(801),Z(20000),NCODE(9),COR(800),X(
      COMMON/GHJ/TIC,PS,JDEF,POISON,MS,COR,STRN,E,MSS,MST
      COMMON/GNR/NCODE,JBAND,NLOAD,ME,ST,N,NHEP
      COMMON/DMAT/D11,D22,D33,D12,D21,D13,D23,D31,D32,D44,D55,
      COMMON/OTK/TOL,N8,NJ,COL,ROW,NJT,PI
      EQUIVALENCE (S,Z)
      DEFINE FILE 8(600,70,V,LV)
      DEFINE FILE 9(95,5,V,LY)
      DATA N8,MST,MS,MAXS,NJT,TOL,PI/8,3,9,20000,3,1E-10,3.141
      READ(5,6) NCAB,R
    6 FORMAT(I3,F10.0)
    7 CALL DAT(TIT,DEF,NLOAD,N,X,Y)
      CALL DATTRI(NODE,S,R)
      MS=9
      CALL GENER(TIT,MSS,JDEF,MST,S,MS,MAXS,N8,X,Y)
      CALL GSEL(JBAND,N,NLOAD,LL,Z,NHEP,JDEF,NJ,MST,E,R)
      CALL STRESS(S,JDEF,COR,NBJ,MSS,MST,DM,LL,MS,NCAB,X,Y,NOD
      GO TO 7
      END
```

R*TARKAN(1).DAT

```
      SUBROUTINE DAT(TIT,DEF,NLOAD,N,X,Y)
      COMMON/OTK/TOL,N8,NJ,COL,ROW,NJT,PI
      COMMON/GHJ/TIC,PS,JDEF,POISON,MS,COR,STRN,E,MSS,MST
      DIMENSION TIT(20),COR(800),JDEF(800),DEF(3),X(200),Y(20
      L=0
      N=0
      NJ=0
      MSS=MS*(MS+1)/2
      READ(5,2,END=500) (TIT(J),J=1,20)
    2 FORMAT(20A4)
      PRINT 21,(TIT(J),J=1,20)
   21 FORMAT(1H1,20X,20A4/10X,110(,-,)///)
      READ(5,14) E,POISON,TIC,NLOAD,COL,ROW,STRN,PS
   14 FORMAT(3F10.0,I10,4F10.0)
      IF(STRN.GT.TOL) GO TO 20
      WRITE(6,407)
  407 FORMAT(30X,43(,*,)/30X,,*   FINITE ELEMENT PLANE STRESS
     **,/30X,43(,*,)////)
      GO TO 10
   20 WRITE(6,408)
  408 FORMAT(30X,43(,*,)/30X,,*   FINITE ELEMENT PLANE STRAIN
     **,/30X,43(,*,)/////)
   10 WRITE(6,404) E,POISON,TIC,NLOAD
  404 FORMAT(10X,,MODULUS OF ELASTICITY =,,F11.2/10X,,POISONS
     *      =,,F7.2/10X,,STANDARD THICKNESS     =,,F7.2/10X,,NUMB
     * CASES  =,,I5//37X,,  JOINT DATA,/22X,,JOINT        Y
     *         D1        D2        D3          S80,/22X,57(,*,))
  100 READ(4,XJ,YY,XX,(DEF(J),J=1,MST),S80
    4 FORMAT(3F5.0,4(1X,A1))
      J=XJ
      X(J)=YY
      Y(J)=XX
      NJ=NJ+1
      K=(J-1)*2+1
      COR(K)=YY
      COR(K+1)=XX
      DO 52 I=1,MST
      L=L+1
      IF(DEF(I).EQ.,S,) GO TO 53
      N=N+1
      JDEF(L)=N
      GO TO 52
   53 JDEF(L)=0
   52 CONTINUE
      PRINT 9,J,COR(K),COR(K+1),JDEF(L-2),JDEF(L-1),JDEF(L),S8
    9 FORMAT(22X,I3,3X,2(F8.2,2X),6X,3(I3,4X),3X,A1)
      IF(S80.NE.,S,) GO TO 100
      WRITE(6,1) NJ,N
    1 FORMAT(///1X,,NUMBER OF JOINTS  =,,I3/1X,,NUMBER OF UNKN
      CALL GRAPH4(8.,3.,89,X,Y)
      WRITE(6,22)
   22 FORMAT(30X,,FINITE ELEMENT MESH USED FOR THE ANALYSIS,/3
      RETURN
  500 CALL EXIT
      END
```

ER*TARKAN(1).DATTRI

```
      SUBROUTINE DATTRI(NODE,S,R)
      COMMON/OTK/TOL,N8,NJ,COL,ROW,NJT,PI
      COMMON/GHJ/TIC,PS,JDEF,POISON,MS,COR,STRN,E,MSS,MST
      COMMON/GNR/NCODE,JBAND,NLOAD,ME,ST,N,NHEP
      COMMON/DMAT/D11,D22,D33,D12,D21,D13,D23,D31,D32,D44,D55,
      DIMENSION ST(45),NODE(400,3),NCODE(9),S(1),JDEF(800),COR
      DIMENSION Y(3),X(3)
      DATA ST,ME,JBAND/45*0.,2*0/
      WRITE(6,36)
   36 FORMAT(1H1,13X,,TRIANGULAR FINITE ELEMENT DATA,/14X,30(,
     *,,TRIANGLE      J1      J2      J3      IP      TNESS      S80,/
C     STRESS PROBLEM, ISOTROPIC MATERIAL
      P=POISON
      P1=1.+P
      P2=1.-P
      P3=(1.-2.*P)/2.
      E=E/(2*P1*P3)
      IF(STRN.LE.0) GO TO 20
C     STRAIN PROBLEM, ISOTROPIC MATERIAL
      E=E/P1
      P=P/P2
      P1=1.-P*P
      P2=1.-P
   20 D11=E*P2
      D12=E*P
      D13=D12
      D21=D12
      D22=D11
      D23=D12
      D31=D12
      D32=D12
      D33=D11
      D44=E*P3
      D55=D44
      D66=D44
  500 READ(5,3) VM,(V(II),II=1,NJT),T,IP,S80
    3 FORMAT(5F5.0,I1,A1)
      ME=ME+1
      IM=VM
      DO 321 II=1,NJT
  321 NODE(IM,II)=V(II)
      IF(T.EQ.0.) T=TIC
      WRITE(6,9) IM,(NODE(IM,IH),IH=1,NJT),IP,T,S80
    9 FORMAT(10X,I5,6X,4(I3,3X),F8.2,8X,A1)
      CALL CODENM(NODE,IM,MST,JDEF,MS,Y,X,COR,JBM,NJT,IP)
      CALL TRISTF(NODE,NCODE,ST,Y,X,A,IM,A1,A2,A3,B1,B2,B3,C1,
     *,R)
      MS=9
      IF(IP.EQ.1) GO TO 11
      WRITE(N8,IM) (ST(K),K=1,MSS),(NCODE(M),M=1,MS),A1,A2,A3,
     *1,C2,C3,A
      W=-2.2/3.*A
      IDIR=2
      WRITE(9,IM) (V(I),I=1,3),IDIR,W
      GO TO 12
   11 WRITE(N8,IM) (ST(K),K=1,MSS),(NCODE(M),M=1,MS)
   12 IF(S80.NE.,S,) GO TO 500
      WRITE(6,40) JBAND,JBM,ME
   40 FORMAT(//5X,,HALF BAND WIDTH=,,I3/5X,,AT TRIANGLE NO =,
     *. OF MEMBERS=,,I3)
      RETURN
      END
```

ER*TARKAN(1).CODENM

```
      SUBROUTINE CODENM(NODE,JM,MST,JDEF,MS,Y,X,COR,JBM,NJT,I
      COMMON/GNR/NCODE,JBAND,NLOAD,ME,ST,N,NHEP
      DIMENSION COR(1),NCODE(9),JDEF(1),Y(3),X(3),NODE(400,3)
      IF(IP.EQ.0) GO TO 1
      NJT=NJT-1
      MS=MS-2
      DO 2 I=5,9
    2 NCODE(I)=0
    1 DO 16 J=1,NJT
      JX=(NODE(JM,J)-1)*MST
      JA=(NODE(JM,J)-1)*2
      Y(J)=COR(JA+1)
      X(J)=COR(JA+2)
      DO 16 L=1,MST
      JZ=(J-1)*MST+L
      IN=JX+L
      NCODE(JZ)=JDEF(IN)
   16 CONTINUE
      MSM=MS-1
      DO 702 J=1,MSM
      JPP=J+1
      IJ=NCODE(J)
      IF(IJ) 42,702,43
   42 IJ=-IJ
   43 DO 703 K=JPP,MS
      IK=NCODE(K)
      IF(IK) 44,703,45
   44 IK=-IK
   45 KE=ABS(IK-IJ)+1
      IF(JBAND-KE) 81,703,703
   81 JBAND=KE
      JBM=JM
  703 CONTINUE
  702 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TRISTF(NODE,NCODE,ST,Y,X,A,IM,A1,A2,A3,B1,B2,
     *3,T,IP,R)
      DIMENSION NODE(400,3),NCODE(9),JDEF(800),COR(800),ST(45)
      COMMON/GHJ/TIC,PS,JDEF,POISON,MS,COR,STRN,E,MSS,MST
      AA(R,S,T,U,V)=(ALPHA/2.*(GAMA*R*S+BETA*(T*U+V)))*S1
      BB(R,S,T,U,V)=(ALPHA/2.*(GAMA*R+BETA*(S*T+U*V)))*S1
      CC(R,S,T,U)=(ALPHA/2.*(POIS*R*S+BETA*T*U))*S1
      DD(R,S)=(L*PI*(BETA*R-POIS*S)/3)*S1
      PS=0.
      IF(IP.EQ.0) GO TO 66
      EL=SQRT((Y(2)-Y(1))**2+(X(2)-X(1))**2)
      EM=(Y(2)-Y(1))/EL
      EN=(X(2)-X(1))/EL
      E=15.*E
      S=R*E/EL
      ST(1)=EM*EM*S
      ST(2)=EM*EN*S
      ST(3)=EN*EN*S
      ST(4)=-ST(1)
      ST(5)=0.
      ST(6)=0.
      ST(7)=-ST(2)
      ST(8)=ST(1)
      ST(9)=-ST(2)
      ST(10)=0.
      ST(11)=0.
      ST(12)=-ST(3)
      ST(13)=ST(2)
      ST(14)=0.
      ST(15)=0.
      ST(16)=ST(3)
      DO 77 I=17,21
   77 ST(I)=0.
      IF(PS.EQ.0) RETURN
      JS=0
      DO 557 I=1,MS
      JB=JS+1
      JS=JB+MS-I
      IF(I.EQ.1) PRINT11,(ST(L),L=JB,JS)
      IF(I.EQ.2) PRINT12,(ST(L),L=JB,JS)
      IF(I.EQ.3) PRINT13,(ST(L),L=JB,JS)
      IF(I.EQ.4) PRINT14,(ST(L),L=JB,JS)
  557 CONTINUE
   11 FORMAT(///5X,4F15.5/)
   12 FORMAT(20X,3F15.5/)
   13 FORMAT(35X,2F15.5/)
   14 FORMAT(50X,F15.5/)
      RETURN
   66 A=ABS((Y(1)-Y(2))*(X(2)-X(3))-(Y(3)-Y(2))*(X(2)-X(1)))/2
      A1=Y(2)*X(3)-Y(3)*X(2)
      A2=Y(3)*X(1)-Y(1)*X(3)
      A3=Y(1)*X(2)-Y(2)*X(1)
      B1=X(2)-X(3)
      B2=X(3)-X(1)
      B3=X(1)-X(2)
      C1=Y(3)-Y(2)
      C2=Y(1)-Y(3)
      C3=Y(2)-Y(1)
      POIS=POISON
      PI=3.1415926
      ALPHA=T/A
      BETA=(1-2.*POIS)/2.
      GAMA=1.-POIS
      S1=E/(4*(1+POIS)*(1-2*POIS))
      XA=(L*PI/ALPHA)**2/3.
      YA=2.*XA
      ST(1)=AA(B1,B1,A1,A1,YA)
      ST(2)=ALPHA/4.*B1*A1
      ST(3)=DD(B1,B1)
      ST(4)=AA(B1,B2,A1,A2,XA)
      ST(5)=CC(B1,A2,B2,A1)
      ST(6)=DD(B2,B1)
      ST(7)=AA(B1,B3,A1,A3,XA)
      ST(8)=CC(B1,A3,B3,A1)
      ST(9)=DD(B3,B1)
      ST(10)=AA(A1,A1,B1,B1,YA)
      ST(11)=DD(A1,A1)
      ST(12)=CC(B2,A1,A2,B1)
```

```
      ST(16)=AA(A3,A1,B3,B1,XA)
      ST(17)=DD(A3,A1)
      ST(18)=BB(YA,B1,B1,A1,A1)
      ST(19)=DD(B1,B2)
      ST(20)=DD(A1,A2)
      ST(21)=BB(XA,A1,A2,B1,B2)
      ST(22)=DD(B1,B3)
      ST(23)=DD(A1,A3)
      ST(24)=BB(XA,A3,A1,B3,B1)
      ST(25)=AA(B2,B2,A2,A2,YA)
      ST(26)=ALPHA/4.*B2*A2
      ST(27)=DD(B2,B2)
      ST(28)=AA(B3,B2,A3,A2,XA)
      ST(29)=CC(A3,B2,B3,A2)
      ST(30)=DD(B3,B2)
      ST(31)=AA(A2,A2,B2,B2,YA)
      ST(32)=DD(A2,A2)
      ST(33)=CC(B3,A2,A3,B2)
      ST(34)=AA(A3,A2,B3,B2,XA)
      ST(35)=DD(A3,A2)
      ST(36)=BB(YA,B2,B2,A2,A2)
      ST(37)=DD(B2,B3)
      ST(38)=DD(A2,A3)
      ST(39)=BB(XA,A3,A2,B3,B2)
      ST(40)=AA(B3,B3,A3,A3,YA)
      ST(41)=CC(B3,A3,B3,A3)
      ST(42)=DD(B3,B3)
      ST(43)=AA(A3,A3,B3,B3,YA)
      ST(44)=DD(A3,A3)
      ST(45)=BB(YA,B3,B3,A3,A3)
      IF(PS.LE.0) RETURN
      JS=0
      DO 558 I=1,MS
      JB=JS+1
      JS=JB+MS-I
      IF(I.EQ.1) PRINT 1,(ST(L),L=JB,JS)
      IF(I.EQ.2) PRINT 2,(ST(L),L=JB,JS)
      IF(I.EQ.3) PRINT 3,(ST(L),L=JB,JS)
      IF(I.EQ.4) PRINT 4,(ST(L),L=JB,JS)
      IF(I.EQ.5) PRINT 5,(ST(L),L=JB,JS)
      IF(I.EQ.6) PRINT 6,(ST(L),L=JB,JS)
      IF(I.EQ.7) PRINT 7,(ST(L),L=JB,JS)
      IF(I.EQ.8) PRINT 8,(ST(L),L=JB,JS)
      IF(I.EQ.9) PRINT 9,(ST(L),L=JB,JS)
  558 CONTINUE
    1 FORMAT(///1X,9F14.4/)
    2 FORMAT(15X,8F14.4/)
    3 FORMAT(29X,7F14.4/)
    4 FORMAT(43X,6F14.4/)
    5 FORMAT(57X,5F14.4/)
    6 FORMAT(71X,4F14.4/)
    7 FORMAT(85X,3F14.4/)
    8 FORMAT(99X,2F14.4/)
    9 FORMAT(113X,F14.4/)
      RETURN
      END
```

```
ER*TARKAN(I).GENER
      SUBROUTINE GENER(TIT,MSS,JDEF,MST,S,MS,MAXS,N8,X,Y)
      COMMON/GNR/NCODE,JBAND,NLOAD,ME,ST,N,NHEP
      DIMENSION NE(33),FHEY(33),X(200),Y(200)
      DIMENSION TIT(20),JDEF(800),NCODE(9),ST(45),S(1),V(4)
      LOC(II,J)=II*MS-II*(II-1)/2-(MS-J)
      IUCGEN(I)=(I-N+JBAND-1)*(I-N+JBAND)/2
      NHEP=(N-JBAND)*JBAND+JBAND*(JBAND+1)/2+N*NLOAD
      IF(NHEP-MAXS)70,70,71
   71 PRINT 72,NHEP,MAXS
   72 FORMAT(///20X,35HPROBLEM SIZE TOO LARGE FOR S  MATRIX,2I
      CALL EXIT
   70 CONTINUE
      DO 62 I=1,NHEP
   62 S(I)=0.
      LJB=NLOAD+JBAND
      NJB=N-JBAND
      DO 9 NM=1,ME
      READ(N8'NM)(ST(M),M=1,MSS),(NCODE(M),M=1,MS)
      DO 8 L=1,MS
      SAYN=1.
      I=NCODE(L)
      IF(I)20,8,22
   20 SAYN=-1.
      I=-I
   22 CONTINUE
      IX=(I-1)*LJB-I+1
      IUC=IUCGEN(I-1)
      DO 77 M=1,MS
      SAYN2=1.
      J=NCODE(M)
      IF(J)30,77,32
   30 SAYN2=-1.
      J=-J
   32 IF(J-I)77,78,78
   78 ID=L
      JD=M
      IF(L-M)122,122,123
  123 ID=M
      JD=L
  122 LC=LOC(ID,JD)
      LO=IX+J
      IF(I-NJB-1)79,79,80
   80 LO=LO-IUC
   79 S(LO)=S(LO)+SAYN*SAYN2*ST(LC)
   77 CONTINUE
    8 CONTINUE
    9 CONTINUE
      WGT=0.
      DO 50 NL=1,NLOAD
C *** OWN WEIGHT OF THE STRUCTURE IS CONSIDERED FIRST AND STOR
      MEM=ME-1
      DO 10 NTR=1,MEM
      READ(9'NTR) (V(JJ),JJ=1,3),IDIR,W
      DO 7 NJT=1,3
      JON=V(NJT)
      IRP=1
      KS=0
      WGT=WGT+W
      JONN=(JON-1)*MS1+IDIR
      NUM=JDEF(JONN)
      SAYN=1.
      I=NUM
      IF(I) 1,7,3
    1 SAYN=-1
      NUM=-NUM
    3 I=NUM
      LO=(I-1)*LJB+JBAND+NL
      IF(I-(NJB+1)) 4,4,5
    5 LO=LO-IUCGEN(I)
    4 S(LO)=S(LO)+SAYN*W
    7 CONTINUE
   10 CONTINUE
      WRITE(3,74) WGT
   74 FORMAT(////5X,'FOTAL WEIGHT OF 1M. LENGTH OF THE STRUCTU
     *0.4,' TONS')
      READ 90,(TIT(J),J=1,20)
   90 FORMAT(20A4)
      PRINT 710,(TIT(J),J=1,20)
  710 FORMAT(////30X,20A4)
      WRITE(3,803) NL
  803 FORMAT(// 25X,'JOINT          DIRECTION        INTENSITY
```

```
     *SKIP     S30',/,15,'FOR NLOAD=',I2,'  JOINT LOADS')
      READ(5,2) (NE(I),I=1,33)
    2 FORMAT(15I4)
      DO 44 I=1,33
   44 FNEY(I)=0.
      DO 45 I=1,32
      K=NE(I)
      L=NE(I+1)
      CF=.02*(Y(K)-Y(L))/2.
      IF(CF.LT.0.) CF=-CF
      FNEY(I)=FNEY(I)+CF
      FNEY(I+1)=FNEY(I+1)+CF
   45 CONTINUE
      IDIR=1
      DO 67 L=1,33
      JON=NE(L)
      DATA IRP,KS/1,0/
      W=FNEY(L)
      WRITE(6,73) JON,IDIR,W,IRP,KS
   73 FORMAT(29X,2(I10,3X),F15.4,2I9)
      JONN=(JON-1)*NSI+IDIR
      NUM=JDEF(JONN)
      SAYN=1.
      I=NUM
      IF(I) 51,67,52
   51 SAYN=-1
      NUM=-NUM
   52 I=NUM
      LO=(I-1)*LJB+JBAND+NL
      IF(I-(NJB+1))791,791,801
  801 LO=LO-IUCGEN(I)
  791 S(LO)=S(LO)+SAYN*W
   67 CONTINUE
   50 CONTINUE
      RETURN
      END
```

```
CR*TARKAN(1).GSEL
      SUBROUTINE GSEL(JBAND,N,NLOAD,LL,Z,NHEP,JDEF,NJ,MST,E,R)
      DIMENSION  LL(1),Z(1),DEF(8),JDEF(1)
      NE=N-1
      JB=JBAND
      N1=N+1
      NM=NLOAD-1
      NL=N+NLOAD
      JBE=JB-1
      NJJ=N-JBE
      ND=JBE+NLOAD
      LL(1)=0
      J2=0
      JCOR=0
      DO 40 I=1,N
      J1=J2+1
      IF(I-NJJ)41,41,42
   41 J2=J1+ND
      GO TO 43
   42 J2=J1+NL-I
   43 DO 1000 J=J1,J2
      JCOR=JCOR+1
 1000 Z(J)=Z(JCOR)
      J3=J2-NLOAD
      JA=J3+J1
      DO 44 K=J1,J3
      J=JA-K
      IF(Z(J))50,44,50
   44 CONTINUE
   50 LX=J-J1+1
      IF(LX+1-LL(I))51,52,52
   51 LX=LL(I)-1
   52 JT=J3-J1-LX+1
      IF(JT)40,40,55
   55 JP=J3+1
      DO 56 J=JP,J2
      K=J-JT
   56 Z(K)=Z(J)
      J2=J2-JT
   40 LL(I+1)=LX
      NX=0
      DO 7 I=1,N
      NX=NX+LL(I+1)+NLOAD
    7 LL(I+1)=NX-I
      NX=LL(N)+N
      NY=N*NLOAD
      NZ=NX+NLOAD
      PRINT 2,  JB,NHEP
    2 FORMAT(///30X,6HJBAND=,I5/30X,6HNHEP =,I5)
      DO 10 K=1,NE
      NBK=LL(K)
      KK=NBK+K
      T=1./Z(KK)
      Z(KK)=T
      IB=K+1
      K2=LL(IB)+K
      IS=K2-NBK-NLOAD
      IF(IS-N)12,11,11
   11 IE=N
      IS=NL
      GO TO 17
   12 IE=IS
      IF(IB-IE)22,22,10
   22 K1=K2-NM
   17 J2=NBK+IS
      IN=IS-IE
      DO 13 I=IB,IE
      KI=NBK+I
      IF(Z(KI))14,13,14
   14 TA=T*Z(KI)
      IH=LL(I)-NBK
      DO 15 KJ=KI,J2
      IJ=KJ+IH
      Z(IJ)=Z(IJ)-TA*Z(KJ)
   15 CONTINUE
      IF(IN)18,18,13
   18 IH=LL(I+1)+I-K2
      DO 16 KJ=K1,K2
      IJ=KJ+IH
      Z(IJ)=Z(IJ)-TA*Z(KJ)
   16 CONTINUE
   13 CONTINUE
```

```
   10 CONTINUE
      KI=NX+1
      T=1./Z(NX)
      DO 20 J=KI,NZ
   20 Z(J)=T*Z(J)
      DO 38 IX=1,NE
      I=N-IX
      IB=I+1
      NBI=LL(I)
      II=NBI+I
      IC=LL(IB)+I
      T=Z(II)
      IS=IC-NBI-NLOAD
      DO 36 K=1,NLOAD
      NK=NLOAD-K
      IK=IC-NK
      W=Z(IK)
      IF(IB-IS)39,39,36
   39 DO37 J=IB,IS
      IJ=NBI+J
      JK=LL(J+1)+J-NK
   37 W=W-Z(IJ)*Z(JK)
   36 Z(IK)=T*W
   38 CONTINUE
      WRITE(6,102)
  102 FORMAT(///40X,'DEFORMATIONS'//)
      WRITE(6,700) NLOAD
  700 FORMAT(5X,'NUMBER OF LOAD CASES=',I2,///)
      WRITE(6,701)
  701 FORMAT(1X,'JOINT',2X,'HOR. DEF',2X,'VER. DEF',3X,'LONG.
      DO 60 I=1,NJ
      DO 63 JIM=1,8
   63 DEF(JIM)=0.
      IDJ=(I-1)*MST
      DO 61 IH=1,MST
      IDJ=IDJ+1
      NNO=IH
      NOM=JDEF(IDJ)
      IF(NOM.LE.0) GO TO 62
      K2=LL(NOM+1)+NOM
      K1=K2-NM
      DO 64 J=K1,K2
      DEF(NNO)=Z(J)
   64 NNO=NNO+MST
      GO TO 61
   62 DO 66 JA=1,NLOAD
      DEF(NNO)=0.
   66 NNO=NNO+MST
   61 CONTINUE
      NNSB=NLOAD*MST
      WRITE(6,733) I,(DEF(JPF),JPF=1,NNSB)
      IF(I.NE.21) GO TO 60
      DT=SQRT((93.7+DEF(1))**2+(54.06+DEF(2))**2)-108.11595
      P=DT/108.11595*R*F
      WRITE(6,1) P
    1 FORMAT(/5X,'CABLE FORCE=',F12.3,' TONS')
   60 CONTINUE
  733 FORMAT(2X,I3,8(F11.7))
      RETURN
      END
```

ER*TARKAN(1).STRESS

```
      SUBROUTINE STRESS(S,JDEF,COR,NBJ,MSS,MST,DM,LL,MS,NCAB,X
      COMMON/OTK/TOL,N8,NJ,COL,ROW,NJT,PI
      COMMON/GNR/NCODE,JBAND,NLOAD,ME,ST,N,NHEP
      COMMON/DMAT/D11,D22,D33,D12,D21,D13,D23,D31,D32,D44,D55,
      DIMENSION A(6),B(3),R(3,3),NODE(400,3),ST(45),NCODE(9),X
     *0),S(1),JDEF(1),COR(1),DM(1),LL(1)
      LF(JI,NL)=LL(JI+1)+JI-(NLOAD-NL)
      LOC(II,J)=II*MS-II*(II-1)/2-(MS-J)
      NEA=6
      MV=0
      N=3
      KARE=9
      LJB=NLOAD+JBAND
      NJB=N-JBAND
      DO 11 NL=1,NLOAD
      WRITE(6,57) NL
   57 FORMAT(///6X,,NLOAD=,,I2///, THESE ARE THE STRESS RESULT
     *LATED AT MIDSPAN OF THE STRUCTURE,///, TRNGL  EPSX  EPS
     *  GXY   GYZ   GZX   SIGX   SIGY   SIGZ   TXY   TYZ
     *INCIPAL STRESSES  PRINCIPAL DIRECTIONS,/5X,6(2X,,*1E-6,)
      MEM=ME-NCAB
      DO 20 M=1,MEM
      READ(N8,M) (ST(K),K=1,MSS),(NCODE(K),K=1,MS),A1,A2,A3,B1
     *C2,C3,A
      SPN=1000.
      EL=1.
      J1=NODE(M,1)
      J2=NODE(M,2)
      J3=NODE(M,3)
      XG=(X(J1)+X(J2)+X(J3))/3.
      YG=(Y(J1)+Y(J2)+Y(J3))/3.
      C=EL*PI/SPN
      G=C*SPN/2.
      SG=SIN(G)
      CG=COS(G)
      DO 6 K=1,MS
      IK=NCODE(K)
      SAYN=1.
      DM(K)=0.
      IF(IK)21,6,22
   21 IK=-IK
      SAYN=-1
   22 IX=LF(IK,NL)
      DM(K)=S(IX)*SAYN
    6 CONTINUE
      D1=DM(1)
      D2=DM(4)
      D3=DM(7)
      D4=DM(2)
      D5=DM(5)
      D6=DM(8)
      D7=DM(3)
      D8=DM(6)
      D9=DM(9)
      EPSX=SG*(B1*D1+B2*D2+B3*D3)/(2.*A)
      EPSY=SG*(C1*D4+C2*D5+C3*D6)/(2.*A)
      EPSZ=-C*SG*((A1+B1*XG+C1*YG)*D7+(A2+B2*XG+C2*YG)*D8+(A3+
     *G)*D9)/(2.*A)
      GXY=SG*(C1*D1+B1*D4+C2*D2+B2*D5+C3*D3+B3*D6)/(2.*A)
      GYZ=CG*(C*((A1+B1*XG+C1*YG)*D4+(A2+B2*XG+C2*YG)*D5+(A3+B
     *)*D6)+C1*D7+C2*D8+C3*D9)/(2.*A)
      GZX=CG*(C*((A1+B1*XG+C1*YG)*D1+(A2+B2*XG+C2*YG)*D2+(A3+B
     *)*D3)+B1*D7+B2*D8+B3*D9)/(2.*A)
      SIGX=D11*EPSX+D12*EPSY+D13*EPSZ
      SIGY=D21*EPSX+D22*EPSY+D23*EPSZ
      SIGZ=D31*EPSX+D32*EPSY+D33*EPSZ
      TXY=D44*GXY
      TYZ=D55*GYZ
      TZX=D66*GZX
      A(1)=SIGX
      A(2)=TXY
      A(3)=SIGY
      A(4)=TZX
      A(5)=TYZ
      A(6)=SIGZ
      CALL OZDEG(A,R,KARE,NEA,N,MV)
      K=0
      DO 40 I=1,N
      K=K+1
```

```
      WRITE(6,1) M,EPSX,EPSY,EPSZ,GXY,GYZ,GZX,SIGX,SIGY,SIGZ
     *,(B(I),I=1,N),((R(I,J),J=1,N),I=1,N)
    1 FORMAT(I4,1X,6P6F7.1,0P6F7.1,3F7.1,3F7.3,2(/110X,3F7.3
   20 CONTINUE
   11 CONTINUE
      RETURN
      END
```

```
ER*TARKAN(1).OZDEG
      SUBROUTINE OZDEG(A,R,KARE,ME,N,MV)
      DIMENSION A(ME),R(KARE)
    5 RANGE=1.0E-6
      IF(MV-1) 10,25,10
   10 IQ=-N
      DO 20 J=1,N
      IQ=IQ+N
      DO 20 I=1,N
      IJ=IQ+I
      R(IJ)=0.0
      IF(I-J) 20,15,20
   15 R(IJ)=1.0
   20 CONTINUE
   25 ANORM=0.0
      DO 35 I=1,N
      DO 35 J=I,N
      IF(I-J) 30,35,30
   30 IA=I+(J*J-J)/2
      ANORM=ANORM+A(IA)*A(IA)
   35 CONTINUE
      IF(ANORM) 165,165,40
   40 ANORM=1.414*SQRT(ANORM)
      ANRMX=ANORM*RANGE/FLOAT(N)
      IND=0
      THR=ANORM
   45 THR=THR/FLOAT(N)
   50 L=1
   55 M=L+1
   60 MQ=(M*M-M)/2
      LQ=(L*L-L)/2
      LM=L+MQ
   62 IF(ABS(A(LM))-THR) 130,65,65
   65 IND=1
      LL=L+LQ
      MM=M+MQ
      X=0.5*(A(LL)-A(MM))
   68 Y=-A(LM)/SQRT(A(LM)*A(LM)+X*X)
      IF(X) 70,75,75
   70 Y=-Y
   75 SINX=Y/SQRT(2.0*(1.0+(SQRT(1.0-Y*Y))))
      SINX2=SINX*SINX
   78 COSX=SQRT(1.0-SINX2)
      COSX2=COSX*COSX
      SINCS=SINX*COSX
      ILQ=N*(L-1)
      IMQ=N*(M-1)
      DO 125 I=1,N
      IQ=(I*I-I)/2
      IF(I-L) 80,115,80
   80 IF(I-M) 85,115,90
   85 IM=I+MQ
      GO TO 95
   90 IM=M+IQ
   95 IF(I-L) 100,105,105
  100 IL=I+LQ
      GO TO 110
  105 IL=L+IQ
  110 X=A(IL)*COSX-A(IM)*SINX
      A(IM)=A(IL)*SINX+A(IM)*COSX
      A(IL)=X
  115 IF(MV-1) 120,125,120
  120 ILR=ILQ+I
      IMR=IMQ+I
      X=R(ILR)*COSX-R(IMR)*SINX
      R(IMR)=R(ILR)*SINX+R(IMR)*COSX
      R(ILR)=X
  125 CONTINUE
      X=2.0*A(LM)*SINCS
      Y=A(LL)*COSX2+A(MM)*SINX2-X
      X=A(LL)*SINX2+A(MM)*COSX2+X
      A(LM)=(A(LL)-A(MM))*SINCS+A(LM)*(COSX2-SINX2)
      A(LL)=Y
      A(MM)=X
  130 IF(M-N) 135,140,135
  135 M=M+1
      GO TO 60
  140 IF(L-(N-1)) 145,150,145
  145 L=L+1
```

```
      GO TO 50
  160 IF(THR-ANRMX) 165,165,45
  165 IQ=-N
      DO 185 I=1,N
      IQ=IQ+N
      LL=I+(I*I-I)/2
      JQ=N*(I-2)
      DO 185 J=I,N
      JQ=JQ+N
      MM=J+(J*J-J)/2
      IF(A(LL)-A(MM)) 170,185,185
  170 X=A(LL)
      A(LL)=A(MM)
      A(MM)=X
      IF(MV-1) 175,185,175
  175 DO 180 K=1,N
      ILR=IQ+K
      IMR=JQ+K
      X=R(ILR)
      R(ILR)=R(IMR)
  180 R(IMR)=X
  185 CONTINUE
      RETURN
      END
```