STOCHASTIC ADAPTIVE RECEDING HORIZON CONTROLLERS

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ABSTRACT

In this thesis, the deterministic, stochastic and stochastic adaptive control possibilities based on the method of receding horizon is examined. The receding horizon method assumes a fixed horizon length for feedback law calculation at each step. Therefore, the feedback law is optimal in one-step-ahead manner and the feedback gain is constant. The other advantages are of not having to choose the state penalization matrix and of replacing the solution of Riccati equation by a linear one. We alleviated some problems associated with the practical use of this method, such as calculation time and singular state transition matrices by some fast algorithms and non-zero set points by modification of the basic equations.

Modelling the system in state space innovations representation or transforming it to this form if it is not modelled in innovations form originally, solves the problem of state reconstruction under noise effects. The overall design enjoys the separation property, that is, of having a separate design for control and estimation parts.

In the case of some unknozn parameters in the system equations, our controller works using the state estimates, found by utilizing the parameter estimates, in the control law, and parameter estimates, found by using the state esti-

mates, in the feedback gain calculation. This controller with this enforced certainty equivalence property enjoys many favorable characteristics such as refraining from the use of Riccati equation in control, matrix update equations for state and parameter estimation uncertainties, external perturbation signals to secure stability, and trial and error procedures in the choice of state penalization matrices. Moreover, the method is general enough to control with any prescribed control strength, multi-input, multi-output systems under noise effects, modelled in difference equation form with multi-parameter uncertainty.

CONTENTS

INTRODUCTION

We may roughly distinguish three phases in the history of automatic control.

In the deterministic phase, the system equations, system inputs which are either designed by the control specialist and/or disturbance effects of the environment were assumed to be known. This assumption, with the additional assumption of linearity of equations describing the behavior of the system results in a huge number of effective techniques of control known under the names classical (s-domain) and modern (state-space) methods. Even the systems that could not be modelled as linear, were each fitted with specific techniques of control, some of which were the direct extentions of the ones in linear theory.

Then control specialists realized that it is more realistic to model some external disturbances or some unknown parameters of the system as random variables because they did not yield to easy deterministic equations for their behavior. The probability theory and the theory of stochastic processes were ready to use and the control specialists made good use of them by adopting them to their needs. So became popular the theory of stochastic control. But the problem was not completely solved yet, because to use the theory of random processes, one had to know in advance, the statistical characteristics of the random variables and then the remaining work was to use the techniques of deterministic system design and place the designed fixed structures in the appropriate places afterwards.

The fast growth in the capabilities of digital computing instruments, the popularity of "cybernetics" led the control specialists to think of machines that tune themselves according to the control needs, that is, adapt their behavior when a parameter of the system or an external disturbance on the system changes. In this continuing phase of adaptivity, all the existing adaptive controllers can be placed somewhere in-between the stochastic control and the truly adaptive control mechanisms, since there is no method that controls a system while the system operates without any a-priori information about that system.

In this work, our approach will follow the main trend in the history of automatic control. We take up a control method for deterministic systems (Chapter 1). First, we design several algorithms to render the method easily implementable, extend its use to some situations that may arise in practice (Chapter 2). We next place the system in a stochastic setting and extend our method to allow for the case where random disturbances with known statistics are acting on the system (Chapter 4). Then we modify our algorithm (Chapter 6) attaching to it the quality of adaptibility so that it can control stochastic systems with unknown parameters. In each case, sufficient supporting theory and practical simulations are provided to verify the workability of the method.

In Chapter 1, we have introduced our deterministic control problem: Having a linear, time-invariant, sampled-data system with some relation between the states of the system and the input to the system, find the best input which alters the states while satisfying some other operating economy requirement specific to the type of the controller used. The control is closed-loop such that we design our control action to be based on system states. Confinement to discrete-time is favored by digital computer implementations.

In Chapter 2, the existing computational technique is given and our novel approaches are presented with justifications. The approaches we propose combine computational simplicity and improved rate of convergence. Moreover, the extension to the control of systems with singular transition matrices of states and non-zero set points for the controlled variables is made. For the control with singular state transition matrix we have developed another fast algorithm. These results are utilized in Chapter 6.

In Chapter 3, we develop stochastic state models which we use in the following chapters. Developing sound models is necessary for effective analysis and synthesis of stochastic systems.

In Chapter 4, the state estimation and the stochastic version of our controller are introduced. The noise effects on the system and the measurements are present, but knowing the statistics of these noises, we find the best estimate of the states and base our control action on these estimates instead of the true but unknown states.

In Chapter 5, we make an introductory treatment of identification methods which estimate certain system parameters using the knowledge of system structure, deterministic input, measured output and noise statistics. Next, identifiability of systems operating (inherently or forcefully) under closed-loop is examined to be used in the next chapter. Then we make an introduction to adaptive control problem in z-domain.

In Chapter 6, state space adaptive control techniques are mentioned and our novel approach is presented with theoretical verifications and simulated examples. Given the system structure, the statistics of the noise acting on the system and input/output measurements, we wish to control a stochastic system whose some parameters are unknown.

In Chapter 7, some other adaptive estimation schemes are introduced which are not used in this work because of their impracticality as experienced by the author, but present possibilities of use. Some on-line techniques for estimation of noise characteristics are also given.

CHAPTER 1

DETERMINISTIC PROBLEM

1.1. INTRODUCTION

In this chapter, we will start out with the formulation and the solution of the control problem for linear discrete time systems with quadratic criterion. This method involves a trial and error procedure criterion selection. To systematize and simplify the procedures of linear quadratic formulation, we will then present the receding horizon control concept and heuristically show that its solution can be obtained by considering a spècial limiting case of the general linear quadratic problem. Then a mathematically rigorous derivation of the solution is given and existence conditions for the solution are proved.

1.2. LINEAR QUADRATIC CONTROL

Let us consider the linear discrete-time system

$$x(k+1) = A(k)x(k) + B(k)u(k), \quad x(o) \text{ known} \quad k=0,\ldots,N \quad (1.1)$$

which can be controlled via the variable

$$z(k) = D(k)x(k) \tag{1.2}$$

then the problem of determing the input sequence u(k), k=0,...,N-1, such that an a-priori chosen criterion of performance

$$\sum_{k=0}^{N-1} \left[z^T(k+1)Q_z(k+1)z(k+1) + u^T(k)R(k)u(k)\right] + \\ + x^T(N)Q_F x(N) \tag{1.3}$$

is minimized is called the discrete time deterministic optimal control problem. Here Q_z is a positive semi-definite matrix to penalize the deviation in the controlled variable, R is the positive definite control energy penalization matrix, and Q_F is a positive semi-definite matrix used to penalize the final state deviation from the desired value. If all the matrices in the above formulation are constant, then the problem is called the time-invariant discrete-time linear optimal regulator problem.

For the general case where the matrices are allowed to be time-varying, the optimal input choice is given by

$$u(k) = -F(k)x(k), \quad k=0,...,N-1 \tag{1.4}$$

and the gain sequence is given by

$$F(k) = \{R(k) + B^T(k)[Q_x(k+1) + P(k+1)]B(k)\}^{-1}B^T(k) \cdot [Q_x(k+1) + P(k+1)]A(k) \qquad (1.5)$$

In the above equality, the inverse always exists and

$$Q_x(k) = D^T(k)Q_z(k)D(k) \qquad (1.6)$$

P(k) is found by

$$P(k) = A^T(k)[Q_x(k+1) + P(k+1)][A(k) - B(k)F(k)] \qquad (1.7)$$

Starting with the terminal condition $P(N) = Q_F$ and running backwards.

From now on, we will only consider the case of time-invariant matrices since this helps to reduce the size of our problem and besides it is a good model approximation to most phenomena we encounter in practice.

If the system is both stabilizable and detectable, then we have the solution of Equation 1.7 converging to a steady-state solution $\bar{P}$ as $N \to \infty$ for any initial P(N). The resulting steady-state optimal control gain is constant and when applied to the system, it stabilizes the system asymptotically. This control also minimizes the criterion for all initial P(N).

Different choices of Q_z, R and Q_F will mean differing degrees of importance attachment to the value of control and the value of deviations from the desired state throughout the time of operation. Therefore, the choice of these quantities is a subjective matter and by no means a simple problem. Selection is based on the designer's experience and a trial and error approach. Some rules of thumb in selecting these weighting matrices are as follows [1]:

1. Generally, Q_z, R and Q_F are all chosen as diagonal matrices. This facilitates the penalization of the specific components of the state and control vectors individually among themselves and the relative penalization of the state and control vectors.
2. The larger the value of Q_F chosen, the larger will be the resultant feedback gains near the terminal time.
3. The larger Q_z, the larger will be the feedback gain and faster the time the state perturbations

are reduced to small values.

4. The larger R, the smaller the gain matrix and slower the system.
5. Penalization of time derivatives of state variables can be done to reduce overshoot.
6. The weighting matrices can be selected as an upper bound to the effects of second derivative matrices if the linear system to be controlled is obtained via linearization of a nonlinear model.

To systematize the procedure of linear quadratic control design, Thomas and Barraud [2] have proposed "the receding horizon control" method to compute the state feedback optimal controls without specifying the state penalization matrices. This method entails the solution of a linear difference equation simpler to solve than Equation 1.5 over a pre-selected horizon time.

1.3. RECEDING HORIZON CONTROL

In this section, we will only deal with the discrete-time version, continuous formulation is given in Appendix A.

Given the discrete-time linear time-invariant system described by

$$x(k+1) = Ax(k) + Bu(k), \quad A \text{ is non-singular} \tag{1.8}$$

the performance index

$$J = \frac{1}{2} \sum_{k=0}^{N-1} u^T(k)Ru(k) \tag{1.9}$$

and the equality constraint

$$x(N) = 0 \tag{1.10}$$

if the system is controllable and N (which is called the horizon time) is bounded below by $N_o = n-r+1$

n = dimension of the state

r = rank of B

then the control vector at the initial stage that minimizes the performance index in Equation 1.9, satisfying the constraints in Equations 1.8 and 1.10 are given by

$$u(o) = -R^{-1}B^T A^{-T} W^{-1}(o)x(o) \tag{1.11}$$

and W(o) is the matrix which is the solution to

$$W(k) = A^{-1}W(k+1)A^{-T} + (A^{-1}B)R^{-1}(A^{-1}B)^T, \quad W(N) = 0 \tag{1.12}$$

at the instant k=0, or W(o) can be written in the explicit form

$$W(o) = \sum_{k=1}^{N} A^{-k} BR^{-1}B^{T}A^{-k^{T}} \tag{1.13}$$

We apply the constant gain found above for the zeroth stage, throughout the horizon length, so

$$u(k) = -R^{-1}B^{T}A^{-T}W^{-1}(o)\ x(k) \tag{1.14}$$

Validity of the above equations can easily be demonstrated in the following manner. In the usual quadratic criterion for constant weighting matrices,

$$\sum_{k=0}^{N-1} \left[z^{T}(k+1)Q_{z}z(k+1) + u^{T}(k)Ru(k)\right] + x^{T}(N)Q_{F}x(N) \tag{1.15}$$

If we let Q_F to take arbitrarily large values and not penalize the state throughout the stages, Equation 1.15 becomes Equation 1.9 and the corresponding Riccati equation is

$$P(k) = A^{T}\{P(k+1) - P(k+1)B\left[B^{T}P(k+1)B+R\right]^{-1}B^{T}P(k+1)\}A \tag{1.16}$$

which should be initialized with large values of P(N).

If we assume invertibility of P(k) and using matrix inversion lemma

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}$$

we obtain

$$P^{-1}(k) = W(k) = A^{-1}W(k+1)A^{-T} + (A^{-1}B)R^{-1}(A^{-1}B)^{T},$$
$$W(N) = 0 \qquad (1.12)$$

A property of prime importance is that the closed-loop system that results from the application of the receding horizon control law is asymptotically stable, see [3].

Let us now prove more rigorously what we have proposed. Using the state equation and the equality constraint, one can write

$$0 = x(N) = A^{N}x(o) + [A^{N-1}B,...,AB,B]\begin{bmatrix} u(o) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix} \qquad (1.17)$$

so let

$$x(o) = \tilde{F}u \qquad (1.18)$$

where

$$\tilde{F} = -[A^{-1}B, \ldots, A^{-N}B], \quad \text{dimension } \tilde{F} = n x(N x m)$$
$$u = [u^T(o), \ \ldots, \ u^T(N-1)]^T$$
$$m = \text{dimension } u$$

Using the positive definite R, we form (Nxm)x(Nxm) matrix

$$R_N = \begin{bmatrix} R & & 0 \; 0 \\ & \ddots & \\ 0 & & R \end{bmatrix} \qquad (1.19)$$

which is also positive definite.

R_N can be factored as

$$R_N = L_N \, L_N^T \qquad (1.20)$$

where L_N is non-singular.

Let

$$\bar{u} = L_N^T u, \quad \bar{F} = \tilde{F} L_N^{-T} \qquad (1.21)$$

so that we can write

$$x(o) = \bar{F}\bar{u} \qquad (1.22)$$

The unique solution in least squares sense of this equation is given by

$$\bar{u}^* = (\bar{F})^+ x(o) \qquad (1.23)$$

where $(.)^+$ denotes the pseudo-inverse of a matrix. $\bar{u}^*$ is the vector which secures the minimum value of $||\bar{u}||^2$ that also minimizes $||x(o) - \bar{F}\,\bar{u}||^2$.

In the case we are interested in, Equation 1.22 provides the advantage that for $Nm \geq n$, we have $||x(o) - \bar{F}\bar{u}|| = 0$. Moreover,

$$||\bar{u}||^2 = u^T R_N u = \sum_{k=0}^{N-1} u^T(k) R u(k) \qquad (1.24)$$

So, in this way, we have satisfied all three constraints corresponding to the state equation and the equality constraint which we used in Equation 1.17 and the performance criterion in Equation 1.24. Thus we conclude that Equation 1.23 constitutes the solution to our problem.

A case of particular interest to us is when F is of maximum rank, because then the pseudo-inverse is known explicitly. This corresponds to the case where (A,B) is controllable and $N \geq n-r+1$. In such a situation, the pseudo-inverse becomes

$$L_N^{-T}\bar{u} = L_N^{-T} L_N^{-1} \tilde{F}^T (\tilde{F} L_N^{-T} L_N^{-1} \tilde{F}^T)^{-1} x(o) =$$

$$R_N^{-1} \tilde{F}^T (\tilde{F} R_N^{-1} \tilde{F}^T)^{-1} x(o) \qquad (1.25)$$

defining

$$\begin{bmatrix} u(o) \\ \vdots \\ u(N-1) \end{bmatrix} = \begin{bmatrix} -F(o) \\ \vdots \\ -F(N-1) \end{bmatrix} x(o) \qquad (1.26)$$

and combining the above equations to get

$$u(o) = -R^{-1}(A^{-1}B)^T \left[\sum_{k=1}^{N} A^{-k} BR^{-1}B^T A^{-k^T} \right]^{-1} x(o) \qquad (1.27)$$

So for every k,

$$u(k) = -R^{-1}(A^{-1}B)^T \left[\sum_{k=1}^{N} A^{-k}BR^{-1}B^T A^{-k^T} \right]^{-1} x(k)$$

$$= -R^{-1}(A^{-1}B)^T W^{-1}(o)x(k)$$

which is Equation 1.14. Instead of summing, one may recursively compute W(o) which can be found by Equation 1.12.

Existence of $W^{-1}(o)$ for controllable systems and $N \geq n-r+1$ can be guaranteed by showing that W(o) is positive definite. Let us invert the direction of iteration as we do in computer programming.

$$W(k+1) = A^{-1}W(k)A^{-T} + (A^{-1}B)R^{-1}(A^{-1}B)^T, \; W(o) = 0 \qquad (1.28)$$

from Equation 1.13, we obtain

$$W(n-r+1) = \sum_{k=1}^{n-r+1} (A^{-k}B)R^{-1}(A^{-k}B)^T \qquad (1.29)$$

factoring out $A^{-(n-r+1)}$

$$W(n-r+1) = A^{-(n-r+1)}[B, AB, \ldots, A^{n-r}B]\, R_N^{-1} \begin{bmatrix} B^T \\ (AB)^T \\ \vdots \\ (A^{n-r}B)^T \end{bmatrix} A^{-(n-r+1)^T} \qquad (1.30)$$

Since A is invertible, and the system is controllable with R_N positive definite, W(n-r+1) is positive definite.

For

$$W(i+1) = \sum_{k=1}^{i+1} (A^{-k}B)R^{-1}(A^{-k}B)^T = \sum_{k=1}^{i} (A^{-k}B)R^{-1}(A^{-k}B)^T + (A^{-(i+1)}B)R^{-1}(A^{-(i+1)}B)^T$$

$$= W(i) + A^{-(i+1)}BR^{-1}(A^{-(i+1)}B)^T \qquad (1.31)$$

The rightmost term is positive semidefinite, therefore $W(i+1) \geq W(i)$ which was positive definite by the above reasoning for $i=n-r+1$, therefore $W(i) \geq W(n-r+1) > 0$, which was to be proved. Since W(i) is positive definite for $i \geq n-r+1$, so is P(i) positive definite for $N-i \geq n-r+1$.

1.4. COMMENT

In this chapter, we started with general linear quadratic control problem and showed the difficulty associated with choosing the cost criterion. To alleviate this difficulty, we have presented receding horizon control method. In the next chapter, we will be introducing our novel approaches and extensions.

CHAPTER 2

COMPUTATIONAL TECHNIQUES, NOVELTIES AND EXTENSIONS

2.1. INTRODUCTION

In this chapter, we will present the existing computational techniques for the discrete receding horizon control problem and present our simpler and faster algorithms. Then, we will extend the use of this control method to singular state transition matrices and non-zero set points.

After introduction of the existing most effective method of Thomas and Barraud, based on successive matrix decompositions, we propose two fast algorithms to calculate the feedback gains for the receding horizon control method. This controller has been introduced to systematize and simplify the procedures of linear quadratic control problem. To maintain this spirit of the method we will propose faster and simpler algorithms.

Then we will extend the results of the previous chapter by removing the constraint that the state transition matrix should be invertible. This is made possible by reconsidering the original Riccati equation in the problem solution, which leads to another fast computational scheme that does not require the invertibility of the state transition matrix but another regularity condition which we will prove to hold true in all cases.

The last part consists of the extension of the results to non-zero equality constraints on the final state which is the case most frequently met in practice.

2.2. THE FEEDBACK GAIN CALCULATION

It is quite possible to calculate the gain matrix by using Equation 1.12 in a recursive manner for a fixed horizon length chosen by the designer. Or one can sum up the terms as in Equation 1.13. To conserve symmetry and positive definiteness of W(k) in the course of iterations, while using lower precision arithmetic, Thomas and Barraud [2] proposed the following algorithm:

1) Determine the lower triangular P' by Cholesky factorization, such that

$$P'P'^T = R \qquad (2.1)$$

2) Determine the lower triangular L' and upper triangular U such that

$$L'U = A \tag{2.2}$$

the next two steps are devoted to calculate a C' matrix such that

$$C'C'^T = A^{-1}BR^{-1}(A^{-1}B)^T \tag{2.3}$$

which is positive semi-definite, and

$$C'C'^T = A^{-1}BR^{-1}(A^{-1}B)^T = A^{-1}BP'^{-T}P'^{-1}(A^{-1}B)^T$$

$$= (A^{-1}BP'^{-T})(A^{-1}BP'^{-T})^T$$

3) Determine a V such that

$$(L'U)V = B \tag{2.4}$$

without explicit matrix inversion but by solving two linear algebraic equations successively. Note that V corresponds to $A^{-1}B$.

4) Determine a C' so that

$$P'C'^T = V^T \tag{2.5}$$

the following steps are repeated for all k,

5) Solve for D' as in Step 3, for

$$(L'U)D' = S(k) \qquad (2.6)$$

S(k) corresponds to the square root of W(k).

6) Determine T by the Householder transformation, so that

$$T \begin{bmatrix} D'^T \\ \hline C'^T \end{bmatrix} = \begin{bmatrix} S^T(k+1) \\ \hline 0 \end{bmatrix} \quad \text{with } S(1)=0 \qquad (2.7)$$

7) Find X' in

$$(P'P'^T)X' = V^T \qquad (2.8)$$

8) Find F(o) which is the feedback gain, as a solution to linear algebraic equation

$$S(N)S^T(N)\ F^T(o) = -X'^T \qquad (2.9)$$

It is the author's belief that the receding horizon control should be given the emphasis it deserves as a sub-optimal control law mainly to be used together with parameter adaptive methods of synthesis, therefore this specific controller should benefit more from the techniques of numerical analysis. We will, hence, propose two doubling algorithms which not only solve our problem in a quicker manner but much easier to understand and implement than the other techniques.

To compute the value of W(o) in Equation 1.12, one can either use

ALGORITHM I.

$$a(k+1) = a^2(k), \quad a(o) = A$$
$$b(k+1) = a(k)b(k)a^T(k) + b(k), \; b(o) = BR^{-1}B^T$$
$$W(o) = a^{-1}(L)b(L)a^{-T}(L) \quad \text{where } 2^L \leq N \qquad (2.10)$$

or

ALGORITHM II.

$$a(k+1) = a^2(k), \; a(o) = A^{-1}$$
$$b(k+1) = a(k)b(k)a^T(k) + b(k), \; B(o) = (A^{-1}B)R^{-1}(A^{-1}B)^T$$
$$W(o) = b(L), \quad 2^L \leq N \qquad (2.11)$$

VERIFICATION. It is a simple exercise to iterate simultaneously with the algorithms described here and with Equation 1.28 comparing the results as one continues.

DERIVATION OF THE ALGORITHMS

Let us consider the forward iteration of Equation 1.28, which for convenience is rewritten below

$$W(k+1) = A^{-1}\left[W(k) + BR^{-1}B^T\right]A^{-T}, \quad W(o) = 0 \qquad (2.12)$$

So our objective is to find W(N) knowing W(o). We will decompose W(k) in such a manner that

$$W(k) = X(k)\ Y^{-1}(k) \qquad (2.13)$$

so $W(k+1) = X(k+1)Y^{-1}(k+1)$. One can easily verify that, partitioning W(k) in this fashion such that

$$\begin{bmatrix} X(k+1) \\ Y(k+1) \end{bmatrix} = \begin{bmatrix} A^{-1} & A^{-1}BR^{-1}B^T \\ 0 & A^T \end{bmatrix} \begin{bmatrix} X(k) \\ Y(k) \end{bmatrix} = \mathbb{H} \begin{bmatrix} X(k) \\ Y(k) \end{bmatrix} \qquad (2.14)$$

with $X(o) = 0$ and $Y(o) = 1$, is equivalent to using Equation 2.12.

A special property of matrix $\mathbb{H}$ will be given to aid in our derivation:

SYMPLECTICITY [4],[5]: A 2nx2n matrix Z is symplectic if

$$Z^T JZ = J \quad \text{where} \quad J = \begin{bmatrix} 0 & -I_n \\ I_n & 0 \end{bmatrix}$$

I_n is the n dimensional identity matrix. Any power of a symplectic matrix is symplectic which easily follows from the definition. Another property is that if the matrix is written as

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

where Z_{11} is assumed to be non-singular for the time being,

$$Z_{22} = Z_{11}^{-T} + Z_{21}Z_{11}^{-1}Z_{12} \qquad (2.15)$$

When we apply this new concept to our problem, we see that $\mathbb{H}$ is symplectic and if it is written as

$$\mathbb{H} = \begin{bmatrix} a^{-1}(o) & a^{-1}(o)b(o) \\ 0 & a^{T}(o) \end{bmatrix} = \mathbb{H}(o) \qquad (2.16)$$

then

$$\mathbb{H}(k) = \begin{bmatrix} a^{-1}(k) & a^{-1}(k)b(k) \\ 0 & a^{T}(k) \end{bmatrix} \qquad (2.17)$$

We next use the fact that $\mathbb{H}(k+1) = \mathbb{H}^2(k)$, since by squaring one can calculate in a recursive manner for integer powers of 2, to obtain Algorithm I. Taking X(o) = 0, and Y(o) = 1,

$$\begin{bmatrix} X(2^k) \\ Y(2^k) \end{bmatrix} = \mathbb{H}(k) \begin{bmatrix} X(o) \\ Y(o) \end{bmatrix} = \begin{bmatrix} a^{-1}(k)b(k) \\ a^{T}(k) \end{bmatrix} \qquad (2.18)$$

and finally

$$W(2^k) = X(2^k)Y^{-1}(2^k) = a^{-1}(k)b(k)a^{-T}(k) \qquad (2.19)$$

To obtain the second algorithm, form $\mathbb{H}'$ as

$$\mathbb{H}' = \begin{bmatrix} a(o) & b(o)a^{-T}(o) \\ 0 & a^{-T}(o) \end{bmatrix} = \mathbb{H}'(o) \qquad (2.20)$$

then

$$\mathbb{H}'(k) = \begin{bmatrix} a(k) & b(k)a^{-T}(k) \\ 0 & a^{-T}(k) \end{bmatrix} \qquad (2.21)$$

Since

$$\mathbb{H}'(k+1) = \mathbb{H}'^{2}(k)$$

we obtain Algorithm II with initial conditions:

$$\begin{bmatrix} X(2^k) \\ Y(2^k) \end{bmatrix} = \mathbb{H}'(k) \begin{bmatrix} X(o) \\ Y(o) \end{bmatrix} = \begin{bmatrix} b(k)a^{-T}(k) \\ a^{-T}(k) \end{bmatrix} \qquad (2.22)$$

so

$$W(2^k) = X(2^k)Y^{-1}(2^k) = b(k) \qquad (2.23)$$

COMMENTS

1. Since R is chosen as a diagonal matrix in practice, the only matrix inversions take place in the last step of the first algorithm when calculating the feedback gains. This prevents propagation of errors due to inversion in the first step as in Algorithm II. However, it may be desirable to work with the inverse of the state transition matrix rather than itself and Algorithm II provides

this flexibility. Algorithm I is similar to Kleinman's algorithm, but his is defined in a different context and no derivation is given [6].

2. If the system to be controlled is continuous, one can try one of the following alternatives as arise in the Riccati equation solution:

a) One may discretize the system at the outset and then proceed to find the optimal feedback gain.

b) One can calculate exp($\mathbb{H}_c$) where $\mathbb{H}_c$ is the continuous time Hamiltonian matrix and use the discrete doubling algorithm.

c) Pose a discrete-time problem by using a bilinear transformation and then use the above algorithms.

d) Try the method based on the sign of the Hamiltonian [4].

3. Square root forms of the above algorithms are possible to maintain positive definiteness and numerical accuracy but with the price of more computations.

2.3. THE EXAMPLE OF DIGITAL POSITION CONTROLLER AND DISCUSSIONS

Let us consider the digital positioning system described in Kwakernaak and Sivan [7]. The discrete-time state and measurement equations are

$$x(k+1) = \begin{bmatrix} 1 & .08 \\ 0 & .631 \end{bmatrix} x(k) + \begin{bmatrix} .003 \\ .063 \end{bmatrix} u(k) \qquad (2.24)$$

$$z(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

with the performance index

$$\sum_{k=0}^{N-1} \left[z^2(k+1) + \rho u^2(k) \right] \qquad (2.25)$$

where $\rho = 2x10^{-5}$. This system is to be controlled by means of a digital computer. If we find the receding horizon control feedback gains for this example and compare with the corresponding gains found via the solutions of Riccati equation or the solutions of the deadbeat control problem we arrive at the following results:

1. The optimal feedback gains as found by Riccati equation comes to a steady state value after 6,7 stages.

If we use the constant feedback gains found by receding horizon and linear quadratic approaches, the following closed-loop eigenvalues for the two controllers are respectively $.57 \pm j.06$ and $.23 \pm j.32$. One can deduce that the closed-loop system as regulated by the receding horizon controller as compared with linear quadratic optimal regulator is slower but much less oscillatory. This is due to the constant penalization of the state along the trajectory in linear quadratic regulator which results in a larger feedback gain matrix and faster response time of the closed loop system. Since in receding horizon control cost criterion, the state is not penalized at all for the intermediate stages, for long horizon lengths, it seems as if we had heavy penalization of the input energy with respect to state and this results in a smaller feedback gain matrix, slower closed-loop system with much less control energy spent.

2. For short horizon lengths as represented by those near the minimum value $N_o=n-r+1$, the terminal time is always very near and so the feedback matrix is always large. Comparison of receding horizon for N=2 with a deadbeat controller shows that the feedback gains are identical $F = (158.75, 17.35)$. Hence for short horizon lengths, the receding horizon controller acts like a deadbeat controller.

2.4. SINGULAR STATE TRANSITION MATRICES

In this section, we extend our results to the case where the state transition matrix of the system to be controlled is singular. Singular transition matrices can naturally arise in practice as can be the case, for example, with the "blood pressure regulator" system of Chapter 6, which is originally modelled as a stochastic difference equation [8] or with sampled continuous time systems with time delays [9].

Let us consider Equation 1.16 and rewrite it here with order of indices changed:

$$P(k+1) = A^T\{P(k) - P(k)B[B^TP(k)B + R]^{-1}B^TP(k)\}A \quad (2.26)$$

where

$$P(o) = \lim_{\beta\to\infty} \beta I$$

by using matrix inversion lemma, one can write

$$P(k+1) = A^T[P^{-1}(k) + BR^{-1}B^T]^{-1}A \quad (2.27)$$

multiplying by P(k) together with its inverse gives

$$P(k+1) = A^TP(k)[I + BR^{-1}B^TP(k)]^{-1}A \quad (2.28)$$

where the expression in the brackets is obviously positive definite hence invertible. Since we have $P(o) = \lim_{\beta\to\infty} \beta I$, using Equation 2.28,

$$P(1) = \lim_{\beta\to\infty} A^T.\beta I[I+ BR^{-1}B^T\cdot\beta I]^{-1}A = A^T(BR^{-1}B^T)^{-1}A \tag{2.29}$$

which coincides with the solution of Equation 1.12, and the invertibility is guaranteed in the cases given in Chapter 1. Since $P(1)$ is bounded, $P(2), P(3), \ldots$, etc. are bounded. We do not use $P(o)$ in our calculations, since $N_{min}=n-r+1 = 1$ for $n=1$ and $r=1$. Hence all $P(k)$ for $k\geq 1$ are defined properly.

Let us now partition $P(k)$ as

$$P(k) = Y(k)X^{-1}(k) \tag{2.30}$$

and

$$\begin{bmatrix} X(k+1) \\ Y(k+1) \end{bmatrix} = \begin{bmatrix} A^{-1} & A^{-1}BR^{-1}B^T \\ 0 & A^T \end{bmatrix} \begin{bmatrix} X(k) \\ Y(k) \end{bmatrix} \text{ with } X_o=0, Y_o=I$$

$$= \mathbb{H} \begin{bmatrix} X(k) \\ Y(k) \end{bmatrix} \tag{2.31}$$

we obtain

$$P(k+1) = Y(k+1)X^{-1}(k+1) = A^TY(k)[X(k) + BR^{-1}B^TY(k)]^{-1}A \tag{2.32}$$

we right multiply inside and left multiply outside of the brackets with $X^{-1}(k)$ to obtain Equation 2.28. Use of this $\mathbb{H}$ matrix will give us the first fast algorithm

$$a(k+1) = a^2(k), \quad a(o) = A$$
$$b(k+1) = b(k) + a(k)b(k)a^T(k), \quad b(o) = BR^{-1}B^T$$

for the discrete Riccati equation in 2.28. The feedback gain is found as

$$F = R^{-1}B^TA^{-T}P(N)$$

where

$$P(N) = a^T(L')b^{-1}(L')a(L'), \; 2^{L'} \leq N \qquad (2.33)$$

let us write P(N) in terms of P(N-1) from Equation 2.28

$$P(N) = A^TP(N-1)\left[I + BR^{-1}B^TP(N-1)\right]^{-1}A \qquad (2.34)$$

and substituting into Equation 2.33

$$F = -R^{-1}B^TA^{-T}A^TP(N-1)\left[I + BR^{-1}B^TP(N-1)\right]^{-1}A$$

$$= -R^{-1}B^TP(N-1)\left[I + BR^{-1}B^TP(N-1)\right]^{-1}A \qquad (2.35)$$

$$= -R^{-1}B^T\left[P^{-1}(N-1) + BR^{-1}B^T\right]^{-1}A \qquad (2.36)$$

writing

$$P(N-1) = a^T(L)a^{-1}(L)a(L) \quad \text{for } 2^L \leq N-1$$

and substituting into Equation 2.36

$$F = -R^{-1}B^T[a^{-1}(L)b(L)a^{-T}(L) + BR^{-1}B^T]^{-1}A \quad (2.37)$$

manipulating a bit to get

$$F = -R^{-1}B^Ta^T(L)[b(L) + a(L)b(o)a^T(L)]^{-1}a(L)A \quad (2.38)$$

so the algorithm becomes

ALGORITHM III.

$$a(k+1) = a^2(k) \qquad a(o) = A$$
$$b(k+1) = b(k) + a(k)b(k)a^T(k), \quad b(o) = BR^{-1}B^T$$
$$F = -R^{-1}B^Ta^T(L)[b(L) + a(L)b(o)a^T(L)]^{-1}a(L)A, \quad 2^L \leq N-1$$

Since this last algorithm does not involve the inverse of A, we can effectively use it for singular state transition matrices. Let us now prove for the sake of completeness that the inverse of the expression in the brackets exists. To do that, we will first show b(L) is positive definite for $2^L \geq n-r+1$, and then show that the following b(k)'s are all greater than or equal to b(L) hence positive definite also. Since the other expression in the brackets is positive semi-definite, positive definiteness of b(L) ensures invertibility. Use of $2^L \leq N-1$ in the expressions will oblige the minimum choice of N to $N_{min}+1$, to secure invertibility. This does not restrict the application of the algorithm, since it is designed for high N anyway.

Let us first write explicitely what b(k) are:

$$b(o) = BR^{-1}B^T$$

$$b(1) = BR^{-1}B^T + ABR^{-1}B^TA^T$$

$$b(2) = BR^{-1}A^T + \ldots + A^3BR^{-1}B^TA^{3^T}$$

$$b(3) = BR^{-1}B^T + \ldots + A^7BR^{-1}B^TA^{7^T}$$

so the general expression for b(i) is

$$b(i) = \sum_{k=0}^{2^i-1} A^k BR^{-1}B^T(A^k)^T \qquad (2.39)$$

We are interested in b(i) for $2^i \geq n-r+1$, so first let $2^i = n-r+1$,

$$b(i) = \sum_{k=0}^{n-r} A^kBR^{-1}B^TA^{k^T} \qquad (2.40)$$

and write

$$b(i) = [B, AB, \dots, A^{n-r}B] \underbrace{\begin{bmatrix} R^{-1} & & & 0 \\ & \cdot & & \\ & & \cdot & \\ 0 & & & R^{-1} \end{bmatrix}}_{n-r+1 \text{ times}} \begin{bmatrix} B \\ AB \\ \vdots \\ (A^{n-r}B)^T \end{bmatrix} \qquad (2.41)$$

Due to controllability of the system and positive definiteness of R^{-1}, $b(i) > 0$. Consider

$$b(i+1) = \sum_{k=0}^{i+1} A^k B R^{-1} B^T A^{k^T} = \sum_{k=0}^{i} A^k B R^{-1} B^T A^{k^T}$$

$$+ A^{i+1} B R^{-1} B^T A^{i+1^T} \qquad (2.42)$$

Since the right-most term is positive semi-definite and $b(i) > 0$, we obtain $b(i+1) \geq b(i) > 0$ for $2^i \geq n-r+1$. We will use this algorithm in Chapter 6 with system 4.

2.5. NON-ZERO SET POINTS

Reconsider the linear time-invariant discrete time system

$$x(k+1) = Ax(k) + Bu(k) \qquad (1.1)$$

$$z(k) = Dx(k) \qquad (2.1)$$

Let it be desired that the system is operated about the constant point $Z(k) = Z_o$.

Define

$$U'(k) = u(k) - U_o = \text{the shifted input}$$
$$X'(k) = x(k) - X_o = \text{the shifted state} \quad (2.43)$$
$$Z'(k) = Z(k) - Z_o = \text{the shifted control variable}$$

minimization of

$$\sum_{k=0}^{\infty} \left[Z'^T(k)Q_z Z'(k) + U'^T(k)RU'(k)\right] \quad (2.44)$$

in the act of steering the system states from any initial condition to set point requires the control

$$U'(k) = -\bar{F}\ x'(k) \quad (2.45)$$

where $\bar{F}$ is the steady state feedback gain. Writing Equation 2.46 using original system variables

$$u(k) = -\bar{F}X(k) + U_o' \quad (2.46)$$

so the closed-loop equations become

$$x(k+1) = \bar{A}x(k) + BU_o'$$
$$Z(k) = Dx(k) \quad (2.47)$$

where $\bar{A}$ = A-B$\bar{F}$. Assume that the closed-loop system becomes asymptotically stable at the steady state, the variable which is controlled is

$$\lim_{k\to\infty} Z(k) = H_c(1)U_o' \tag{2.48}$$

where

$$H_c(z) = D(zI - A + B\bar{F})^{-1}B \tag{2.49}$$

is the closed-loop transfer matrix. This implies that zero error in steady state can be achieved with

$$U_o' = H_c^{-1}(1)\dot{Z}_o \tag{2.50}$$

provided that the inverse exists which requires that, first, dimension u = dimension Z, second, the non-zero determinant of $H_c(1)$. But the determinant of $H_c(z)$ can be shown to be equal to the ratio of open-loop zeros to closed-loop poles, so that the transfer function should have non-zero numerator with no zeros at z=1. So the following theorem results:

THEOREM 2.1 [7]:

Consider the system in Equations 1.1 and 1.2, with Z(k) being the controlled variable and dimu=dimZ. Assume any asymptotically stable time-invariant control law as in

Equation 2.46. Let the open loop transfer function be

$$D(zI - A)^{-1}B \tag{2.51}$$

and $H_c(z)$ be the closed loop transfer matrix in Equation 2.49. Then $H_c(1)$ is non-singular and the controlled variable $Z(k)$ can under, steady state conditions, be maintained at any constant set point Z_o by choosing

$$U_o' = H_c^{-1}(1)Z_o \tag{2.50}$$

if and only if open loop transfer matrix has a non-zero numerator polynomial that has no zeros at z=1.

EXAMPLE:

We will apply this result to the system 4 in Chapter 6. Since the discrete receding horizon control law is one which is time-invariant and makes the system asymptotically stable, applying theorem 2.1 to our case where

$$A = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \text{ and } F = -[a^2/b, a]$$

Choosing D as (1,0) since our major aim will be to control x_1, we obtain

$$U_o' = H_c^{-1}(1)Z_o = Z_o/b$$

where Z_o is the desired state. Applying this control will yield in the steady state the desired Z_o for $Z=x_1$.

2.6. CONCLUDING REMARKS

This chapter extends the available results on the receding horizon controller. We simplify and quicken the design procedures by our algorithms, extend the use of the controller to singular state transition matrices and non-zero set points. New results are supported with theoretical verifications and simulated examples.

CHAPTER 3

STOCHASTIC MODELS FOR DYNAMIC SYSTEMS

3.1. INTRODUCTION

Our purpose is to apply the receding horizon control procedures described in the previous chapters to the design of suboptimal control of stochastic and parameter adaptive stochastic systems. So, in this chapter, we will describe various stochastic models that will be suitable for our purposes. First, we scan the deterministic models, next we describe their stochastic counterparts and then we introduce the prediction error formulation that encompasses these and many other stochastic models. We will mainly follow [11] in our treatment.

3.2. DETERMINISTIC MODELS

One form of model that we have been using to describe linear time-invariant discrete-time causal dynamic systems is the state space model where the output is related to the input by

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Eu(k) \tag{3.1}$$

If the initial state is zero, then the z-transform of the output is related to the z-transform of the input by

$$Y^*(z) = H^*(z)U^*(z) \tag{3.2}$$

where

$$H^*(z) = C(zI - A)^{-1}B + E \tag{3.3}$$

Another alternative is to use, the difference equation formulation

$$y(k) + \sum_{d=1}^{d'} f(d)y(k-d) = \sum_{d=0}^{d'} g(d)u(k-d) \tag{3.4}$$

introducing the unit delay operator

$$q^{-1}x(k) = x(k-1) \tag{3.5}$$

so represent Equation 3.4 by

$$f(q)y(k) = g(q)u(k) \tag{3.6}$$

where $f(q)$ and $g(q)$ have obvious definitions. If the systems originally at rest, then the z-transform of $y(k)$ is related to the z-transform of u(k) by

$$F^*(z)Y^*(z) = G^*(z)U^*(z) \tag{3.7}$$

where

$$F^*(z) = \sum_{d=0}^{d'} f(d)z^{d'-d}$$

$$G^*(z) = \sum_{d=0}^{d'} g(d)z^{d'-d} \tag{3.8}$$

are matrix polynomials in z. To secure unique representation of the output by the input, we assume $\det F^*(z) \not\equiv 0$, so Equation 3.7 can be written as

$$Y^*(z) = H^*(z)U^*(z) \tag{3.9}$$

where

$$H^*(z) = [F^*(z)]^{-1}G^*(z) \tag{3.10}$$

$H^*(z)$ will be called the matrix transfer function of the system and the representation in Equation 3.10 as left matrix fraction description (MFD) for $H^*(z)$. It follows that

$$F^*(z) = z^m f(z) \quad \text{and} \quad G^*(z) = z^m g(z) \tag{3.11}$$

so we have

$$H^*(z) = [F^*(z)]^{-1}G^*(z) = f^{-1}(z)g(z) \tag{3.12}$$

and Equation 3.10 is represented as

$$f(z)Y^*(z) = g(z)U^*(z) \tag{3.13}$$

We conclude that the relation between the output and the input is the same whether we use q^{-1} or z^{-1}, and throughout the text, we use them interchangeably to denote unit delay operators.

3.3. COVARIANCE STATIONARY STOCHASTIC MODELS

In this section, we introduce the randomness for the first time. We will associate stochastic processes with arbitrary spectral densities with linear systems driven by white noise. The following two results will establish this:

THEOREM 3.1:

Let $\Phi_y(z)$ be an nxn discrete rational spectral density matrix having full normal rank. Then there exists a unique nxn rational matrix $H^*(z)$ and a unique positive definite real symmetric matrix Φ_n satisfying

i) $\Phi_y(z) = H^*(z)\Phi_n H^{*T}(z^{-1})$

ii) $H^*(z)$ is analytic outside and on the unit circle, that is for $|z| \geq 1$.

iii) $H^{*-1}(z)$ is analytic outside the unit circle, i.e., for $|z| > 1$

iv) $\lim_{z\to\infty} H^{*}(z) = I$

For proof see [10].

THEOREM 3.2

The output power density spectrum $\Phi_y(z)$ of an aysmptotically stable linear system with transfer function $H^{*}(z)$ driven by a zero mean wide sense stationary process with power density spectrum $\Phi_n(z)$ is

$$\Phi_y(z) = H^{*}(z)\Phi_n(z)H^{*T}(z^{-1})$$

where $z = e^{i\omega}$. For proof see [11].

If we combine these two results, we conclude that a zero mean stochastic process with spectral density $\Phi_y(z)$ may be modelled as the output of a linear system driven by white noise. So the inclusion of randomness in this way, will entail the possibility of representing the output of a system as

$$y(k) = \bar{y}(k) + \eta(k) \tag{3.14}$$

where $\bar{y}(k)$ is the output due to the deterministic input and

$\eta(k)$ is the output of the linear filter with transfer function $H^*(z)$ driven by white noise with covariance Φ_n, so we may model $\eta(k)$ by:

1) MFD form:

$$H^*(z) = [F^*(z)]^{-1}G^*(z) \tag{3.15}$$

with $\lim_{z\to\infty} H^*(z)=I$ and $\det G^*(z) \not\equiv 0 \quad \forall\ |z| > 1$

2) State Space form (SSF):

$$H^*(z) = C(zI - A)^{-1}B + E \tag{2.16}$$

where $E = I$ and H^{*-1} is stable.

Combining both the deterministic and stochastic parts, we have:

1) MFD:

$$y(k) = F_1^{-1}(z)G_1(z)U(k) + F_2^{-1}(z)G_2(z)\varepsilon(k) \tag{3.17}$$

or

$$A(z)y(k) = B(z)U(k) + C(z)\varepsilon(k) \tag{3.18}$$

Here A, B, C, F_1, F_2, G_1 and G_2 are polynomials in z^{-1} and $\{\varepsilon(k)\}$

is a white noise sequence with covariance Σ. The model described by Equation 3.18 is also known under the name ARMAX (auto-regressive-moving average-exogeneous variable).

2) SSF:

$$X_1(k+1) = A_1X_1(k) + B_1u(k)$$
$$X_2(k+1) = A_2X_2(k) + B_2\varepsilon(k)$$
$$y(k) = C_1X_1(k) + D_1u(k) + \eta(k)$$
$$\eta(k) = C_2X_2(k) + \varepsilon(k) \qquad (3.19)$$

or

$$x(k+1) = Ax(k) + Bu(k) + K\varepsilon(k)$$
$$y(k) = Cx(k) + Eu(k) + \varepsilon(k) \qquad (3.20)$$

All the above models may be written in the general form

$$y(k) = H_1(z)u(k) + H_2\varepsilon(k) \qquad (3.21)$$

where $\lim_{z\to\infty} H_2(z) = I$ and $\{\varepsilon(k)\}$ is a white noise sequence with covariance Σ.

3.4. PREDICTION ERROR MODELS (PEM):

In this section, we will describe prediction error models which contain most other stochastic models as subclasses. They will be of use to us when we deal with parameter identification problem. They have the general form:

$$y(k) = f\left[Y_{k-1}, U_k, k\right] + \varepsilon(k) \qquad (3.22)$$

where $Y_{k-1} = \{y_{k-1}, y_{k-2}, \ldots\}$ and $U_k = \{u_k, u_{k-1}, \ldots\}$ and $\varepsilon(k)$ is an innovations sequence with the property

$$E_{\varepsilon(k)|Y_{k-1},U_k}\left[\varepsilon(k)\right] = 0 \qquad (3.23)$$

where $E[.]$ denotes expectation.

We can now show how the other stochastic models can be represented by PEM's. Let us consider the linear system having covariance stationary disturbance

$$y(k) = H_1(z)u(k) + H_2(z)\varepsilon(k) \qquad (3.24)$$

with H_1 and H_2 are stable rational transfer functions. H_2^{-1} stable, $\lim_{z\to\infty} H_2(z) = I$, and $\{\varepsilon(k)\}$ is a white noise sequence with covariance Σ. The properties of $H_2(z)$ allow us to represent $y(k)$ as

$$y(k) = L_1(z)y(k-1) + L_2(z)u(k) + \varepsilon(k) \qquad (3.25)$$

where $L_1(z) = z\,[I - H_2^{-1}(z)]$, $L_2(z) = H_2^{-1}(z)H_1(z)$ and L_1, L_2 are stable transfer functions. Comparison of Equation 3.25 with 3.22 reveals that Equation 3.25 is in PEM form. Let us now consider the linear time-invariant stochastic system

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) + W(k) \\
y(k) &= Cx(k) + Du(k) + V(k)
\end{aligned}
\tag{3.26}
$$

with A stable, (A,C) observable, $\{W(k)\}$ and $\{V(k)\}$ are two uncorrelated white noise sequences with covariances Q_W and Q_V respectively. The output in two components is

$$y(k) = \bar{y}(k) + \eta(k)$$

$\bar{y}(k)$ is modelled as

$$
\begin{aligned}
\bar{x}(k+1) &= A\bar{x}(k) + Bu(k) \\
\bar{y}(k) &= C\bar{x}(k) + Eu(k)
\end{aligned}
\tag{3.27}
$$

and $\eta(k)$ is a zero mean stochastic process having spectral density

$$\Phi_\eta(z) = C(zI - A)^{-1}Q_W(z^{-1}I - A^T)^{-1}C^T + Q_V \tag{3.28}$$

Let us now determine a spectral factorization of $\Phi_\eta(z)$:

$$\Phi_\eta(z) = H^*(z)\Sigma\, H^{*T}(z^{-1}) \tag{3.29}$$

where

$$H^*(z) = C(zI - A)^{-1}K + I \tag{3.30}$$

$$K = APC^T(CPC^T + Q_V)^{-1} \tag{3.31}$$

$$\Sigma = CPC^T + Q_V \tag{3.32}$$

P is the unique positive definite symmetric solution of the algebraic Riccati equation:

$$P = APA^T - APC^T(CPC^T + Q_V)^{-1}CPA^T + Q_W \tag{3.33}$$

This spectral factorization is verified by substitution in Appendix B. Collecting all these equations, we arrive at the conclusion that y(k) has a representation of the form

$$\begin{aligned} \bar{x}(k+1) &= A\bar{x}(k) + Bu(k) \\ \tilde{x}(k+1) &= A\tilde{x}(k) + K\varepsilon(k) \\ y(k) &= C\bar{x}(k) + C\tilde{x}(k) + Du(k) + \varepsilon(k) \end{aligned} \tag{3.34}$$

This model is written letting $\hat{x}(k) = \bar{x}(k) + \tilde{x}(k)$,

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K\varepsilon(k)$$
$$y(k) = C\hat{x}(k) + Eu(k) + \varepsilon(k) \qquad (3.35)$$

The above model which is called the "state space innovations form" is very useful from the viewpoint of control designer, because once this formulation is achieved, estimator design problem becomes trivial. Rearranging,

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K[y(k) - C\hat{x}(k)] \qquad (3.36)$$

Moreover, Equation 3.35 can be expressed as

$$y(k) = H_1(z)u(k) + H_2(z)\varepsilon(k) \qquad (3.37)$$

where

$$H_1(z) = C(zI - A)^{-1}B + E$$
$$H_2(z) = C(zI - A)^{-1}K + I$$

So, one can easily arrive at the PEM form.

3.5. COMMENTS

In this chapter, which has a transitional character, we have passed from the deterministic realm to the stochastic one. We have introduced important concepts such as representation of a stochastic process, covariance stationary and PEM forms for stochastic models, and innovations representation which we will extensively use in the following chapters.

CHAPTER 4

STOCHASTIC RECEDING HORIZON CONTROLLERS

4.1. INTRODUCTION

This chapter treats the suboptimal terminal control of linear discrete time stochastic systems. Discrete receding horizon concept is used to obtain the solution to the case where perfect measurements of the state are available, and the innovations representation is used to convert the stochastic control problem into one in which perfect measurements of the state are available. The resulting properties of the closed-loop scheme is discussed and the simulations are reported.

4.2. STOCHASTIC RECEDING HORIZON CONTROLLERS

Let us consider the noise corrupted linear time-invariant discrete-time system

$$x(k+1) = Ax(k) + Bu(k)+Kv(k), \qquad k \geq 0 \tag{4.1}$$

and $x(o)$ is a random vector with mean $\bar{x}(o)$ and covariance matrix Q_o. The observed variable is defined as

$$y(k) = Cx(k) + v(k) \tag{4.2}$$

The noise sequence is Gaussian distributed, uncorrelated with $X(o)$ has zero mean and covariance matrix Q_V. We assume that the system is controllable and observable.

We present the stochastic linear discrete-time output feedback receding horizon regulator problem as finding the control vector sequence $u(k)$ in terms of observed variables of the system up to the time instant $k-1$, such that the criterion

$$J = E\{ \sum_{k=0}^{N-1} u^T(k)Ru(k)\} \text{ is minimized} \tag{4.3}$$

subject to the system dynamics, the measurement conditions and the equality constraint

$$E\ \{x(N)\} = 0 \tag{4.4}$$

Here R is positive definite and N is the horizon time lower bounded by N_o=dimensionx - ranku + 1. As in the case of linear quadratic Gaussian problem, the certainty equivalence principle [12] which is a kind of superposition of the control and estimation parts, will be valid since we pose here a subclass of the general linear-quadratic-Gaussian problem. The solution of the stochastic linear discrete-time output feedback receding horizon controller problem is as follows:

The constant gain control sequence is given by

$$u(k) = F\hat{x}(k), \quad k \geq 0 \tag{4.5}$$

where F is the constant feedback gain matrix for the deterministic controller as given by

$$F = -R^{-1}B^T[W(o)A^T]^{-1} \tag{4.6}$$

W(o) is the solution at the zeroth instant of the linear equation

$$W(k) = A^{-1}[W(k+1) + BR^{-1}B^T]A^{-T}, \quad W(N) = 0 \tag{4.7}$$

Moreover, $\hat{x}(k)$ is the minimum mean square linear estimator of x(k) found by utilizing the observations, y(j), $0 \leq j \leq k-1$, such that,

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K[y(k) - C\hat{x}(k)] \quad (4.8)$$

VERIFICATION

Since the certainty equivalence is valid for the general case where the estimator has the equivalent structure and the controller is designed to minimize the cost criterion in Equation 1.15 which is more general, it is still valid for the case where Q_F is arbitrarily large and Q_z is identically equal to the zero matrix. The dynamics of the estimator can be obtained very easily since the system to be controlled is in innovations state space form, so holds Equation 4.8.

REMARKS

1. The orginal stochastic modelling may be done in discrete-time state space form in which case it is easy to transform to an innovations state space form as given in the previous chapter.

2. Since we have uncertainty in the values of the states, we cannot satisfy the final condition exactly. We, instead suppose zero terminal state in the mean. The limit of accuracy, which is the inherent characteristics of any stochastic controller is that the terminal errors are always lower bounded by the estimation errors.

3. The results relating to the asymptotic stability of the deterministic discrete controller can be found in [3]. For the observer, we are using a form that is one-to-one with a steady state Kalman estimator which is asymptotically stable under some weak regularity conditions [7]. Because we have an interconnection of an asymptotically stable observer and a system which is made asymptotically stable by a feedback law, the resultant closed-loop system is asymptotically stable.

4.3. SIMULATION RESULTS AND DISCUSSION

To evaluate this proposed stochastic control algorithm, a series of simulation experiments on several different systems have been performed. The following four systems are considered:

1) $x(k+1) = 2x(k) + u(k) + 1.5v(k)$

$y(k) = x(k) + v(k)$

2) $x(k+1) = 1.1x(k) + u(k) + 2v(k)$

$y(k) = x(k) + v(k)$

3) $x(k+1) = 0.8x(k) + 0.8u(k) + 1.5v(k)$

$y(k) = x(k) + v(k)$

4) $$x(k+1) = \begin{bmatrix} .5 & 0 \\ 0 & .25 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} .2 \\ .2 \end{bmatrix} v(k)$$

$$y(k) = \begin{bmatrix} .5 & .5 \end{bmatrix} x(k) + v(k)$$

The systems may be obtained from the ARMAX model:

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + b_1 u(k-1)$$
$$+ b_2 u(k-2) + v(k) + c_1 v(k-1) + c_2 v(k-2)$$

respectively by:

1) $a_1=2$, $a_2=0$, $b_1=1$, $b_2=0$, $c_1=-.5$, $c_2=0$
2) $a_1=1.1$, $a_2=0$, $b_1=1$, $b_2=0$, $c_1=0.9$, $c_2=0$
3) $a_1=0.8$, $a_2=0$, $b_1=0.8$, $b_2=0$, $c_1=0.7$, $c_2=0$
4) $a_1=.75$, $a_2=.125$, $b_1=1$, $b_2=-.375$, $c_1=-.55$, $c_2=.05$

We took the noises to be white Gaussian with zero mean and variance .25. Changing the variance throughout a certain range did not affect the results much, nor did the change in the initial state estimates. It is evident that the first two describe unstable systems. The experiments are done to test the quality of control and estimation schemes.

EXPERIMENT 1. Efficacy of Estimation: In order to evaluate the relative effectiveness of estimation, three digit estimation accuracy region (the difference between the true state and its estimate is less than .001) is assumed. The first system enters this region in 12, the second 80 and the third in 22, the fourth in 10 steps, never leaving the region. To equalize the effect of feedback as much as possible, the systems are controlled with the same horizon length but of course the response of the closed-loop system depends also on the system structure. The relatively slow convergence of the second system with respect to the others is attributed to its noise transfer function being closest to a non-minimum phase transfer function.

EXPERIMENT 2. Accuracy of Control: For the first three systems, we assumed an accuracy of control criterion as the average state accuracy $(1/50) \sum_{k=1}^{50} \hat{x}(k) < .01$. All three systems controlled with 1-step ahead (N=1) controllers, and they all keep under this value. The accuracy gain is infinite in the first two unstable systems, because if uncontrolled they will diverge to very large values. In the third system, the average state accuracy is four times that of the uncontrolled case. For the fourth system, we used a four-step ahead (N=4) controller and we took an average of the first thousand estimates. For the first state variable, we got .0082, for the second .0052. Even though, the gain in accuracy was not substantial for this system, this was due to the long operating time and the stable eigenvalues of the open-loop system.

EXPERIMENT 3. Economy of Cost: When the systems are controlled in one-step-ahead manner, the average control energy spent at each step as measured by $(1/50) \sum_{k=0}^{49} u^2(k)$ are respectively given by: 2.392, 1.901 and .426. This is, as one can see, closely related to whether or not and how much the state transition matrix is unstable.

EXPERIMENT 4. Tunability by the Choice of Horizon Length: The choice of horizon length is a variable which supplies the designer the flexibility of being able to determine the strength of control and the rise time of the closed-loop system. Evidently, as the value of horizon length is increased, one obtains a lazier controller which results in a closed-loop system with higher rise time. For the purpose of comparison, the first system is simulated with a four-step-ahead (N=4) controller which resulted in less accurate control with less effort. Another possibility of tuning that remains, is the choice of sampling rate, of course.

4.4. CONCLUSIONS

In this chapter, we have described a stochastic version of the receding horizon controller. As it is sufficient to be a novelty by itself, we have rather done it for the purpose of using it in a stochastic adaptive situation. The choice of the receding horizon control concept in a multitude of many others is not arbitrary but based on its relative simplicity in the choice of penalization matrices and subsequent calculations, flexibility of being able to choose the rise time and strength of control, the favorable feature of securing the asymptotic stability of the closed-loop system, but yet the genuine character in the sense of maintaining the necessary trade-off between perfection and cost.

CHAPTER 5

AN INTRODUCTION TO THE TECHNIQUES OF IDENTIFICATION AND PARAMETER ADAPTIVE CONTROL

5.1. INTRODUCTION

As our ultimate aim is to use our controller in an adaptive setting, it is well worth to try to scan the main themes in the theories of identification and adaptive control. Much has been done in both of these fields for the purposes of (i) to monitor systems to know when a failure occurs; (ii) to sum up what is known about a system into a compact set of knowledge; and (iii) to on-line control systems with minimum possible cost. Since much has been done, there is much to consider, understand and synthesize. Since this is impossible to accomplish in a chapter, we will try to sketch the main trend of researchers in these fields, perhaps in a biased way so as to allow for our immediate use of these results in the following discussion.

The first part is a short introduction to identification and an attempt to gather the seemingly unrelated techniques of parameter estimation in dynamic systems under recursive prediction error identification concept. The second part contains some results on closed-loop identification of systems, which we will make use of in the next chapter. In the third part, the simple z-domain adaptive technique of minimum variance control is mentioned. This constitutes an introduction to adaptive control techniques to which, in the following chapter, we will add the corresponding state space techniques and our novel approach. We will present some remaining parameter-adaptive control approaches and possibilities in the last chapter.

5.2. PREDICTION ERROR IDENTIFICATION

Mathematically speaking, any identification procedure is a transformation of measurable sequences of data to a model that most probably generated these data. Physically, one uses the observed inputs and outputs of a system to uniquely identify the system structure. It is desirable to do this on-line, that is, as the process goes on, since our major aim is adaptive control, which is controlling a system by identifying its structure at the same time. So we will exclusively deal with time domain recursive identification techniques. Recursive because we do not want to load our computer with old and redundant data. The would be user has to choose among different alternatives being in the field of identification much explored. For example, he has to choose:

1) The model set together with its order, between several alternatives such as linear vs non-linear models, input/output vs state space models, etc.
2) The proper design of input to the system which constitutes a favorable condition for proper identification.
3) Which criterion to use for identification. Some criteria naturally correspond to innovation distributions [13].
4) Proper search direction and which gain sequences to use, both affecting the convergence and the convergence rate of the algorithm.
5) Any approximations in the algorithm based on the basic compromise between convergence rate and computational simplicity.
6) Initial conditions to start the algorithm. Actually these do not influence the results very much.

Since there are good references to show how to decide on the above choices, we will not be pursuing this matter any further, but the interested may refer to [14 - 18].

In the following, we will briefly summarize some of the ideas in the theory of prediction error identification. Let us consider the general ARMAX model

$$A(q^{-1})y(k) = B(q^{-1})u(k) + C(q^{-1})v(k) \qquad (5.1)$$

where q^{-1} is the backward shift operator, A, B and C are polynomial operators, such that

$$\begin{aligned} A(q^{-1}) &= 1 + a_1q^{-1} + \ldots + a_nq^{-n} \\ B(q^{-1}) &= b_1q^{-1} + b_2q^{-2} + \ldots + b_nq^{-n} \\ C(q^{-1}) &= 1 + c_1q^{-1} + \ldots + c_nq^{-n} \end{aligned} \qquad (5.2)$$

V(k) is the disturbance term. If we take

$$\Theta^T = (a_1, a_2, \ldots, a_n, b_1, \ldots, b_n) \qquad (5.3)$$

and

$$\Psi^T(k) = (-y(k-1), \ldots, -y(k-n), u(k-1), \ldots, u(k-n)) \qquad (5.4)$$

with $C(q^{-1}) = 1$. Equation 5.1 becomes

$$y(k) = \Theta^T\Psi(k) + v(k) \qquad (5.5)$$

which is called the least squares model. Assuming a least squares criterion

$$\min_{\Theta} \sum_{i=1}^{k} \left[y(i) - \Theta^T \Psi(i)\right]^2 \qquad (5.6)$$

along with the corresponding model, minimization results in

$$\hat{\Theta}(k) = \left[\frac{1}{k} \sum_{i=1}^{k} \Psi(i)\Psi^T(i)\right]^{-1} \frac{1}{k} \sum_{i=1}^{N} \Psi(i)y(i) \qquad (5.7)$$

Let us define

$$R_p(k) = \frac{1}{k} \sum_{i=1}^{k} \Psi(i)\Psi^T(i) \qquad (5.8)$$

which is nothing more than the correlations of the data

$$\hat{\Theta}(k) = R_p^{-1}(k) \frac{1}{k} \sum_{i=1}^{k} \Psi(i)y(i) \qquad (5.9)$$

Writing the equations for $\hat{\Theta}(k)$ and $\hat{\Theta}(k-1)$ and manipulating them along with Equation 5.8, we obtain

$$\hat{\Theta}(k) = \hat{\Theta}(k-1) + \frac{1}{k} R_p^{-1}(k)\Psi(k)\left[y(k) - \hat{\Theta}^T(k-1)\Psi(k)\right] \qquad (5.10)$$

$$R_p(k) = R_p(k-1) + \frac{1}{k}\left[\Psi(k)\,\Psi^T(k) - R_p(k-1)\right] \qquad (5.11)$$

It is also possible to obtain a recursive form by using the matrix inversion lemma for

$$P_*(k) = \frac{1}{k} R_p^{-1}(k) \qquad (5.12)$$

as

$$P_*(k) = P_*(k-1) - \frac{P_*(k-1)\Psi(k)\Psi^T(k)P_*(k-1)}{1 + \Psi^T(k)\ P_*(k-1)\Psi(k)} \tag{5.13}$$

to refrain from taking matrix inverses. The above also constitutes a simple example to obtain a recursive identification algorithm out of an off-line method.

Now introducing the linear finite dimensional predictor model

$$\begin{aligned} \Psi(k+1,\Theta) &= F(\Theta)\Psi(k,\Theta) + G(\Theta)\begin{bmatrix} y(k) \\ u(k) \end{bmatrix} \\ \hat{y}(k|\Theta) &= H(\Theta)\Psi(k,\Theta) \end{aligned} \tag{5.14}$$

defining

$$\psi(k,\Theta) = \frac{d}{d\Theta}\,\hat{y}^T(k,\Theta) = -\frac{d}{d\Theta}\,\varepsilon(k,\Theta)$$

where ε is the prediction error. One can differentiate Ψ and augment to the state equations to get

$$\begin{aligned} \xi(k+1,\Theta) &= A(\Theta)\xi(k,\Theta) + B(\Theta)\,.\begin{bmatrix} y(k) \\ u(k) \end{bmatrix} \\ \begin{bmatrix} \hat{y}(k,\Theta) \\ \mathrm{col}\psi(k,\Theta) \end{bmatrix} &= C(\Theta)\xi(k,\Theta) \end{aligned} \tag{5.15}$$

where col(.) denotes transforming a matrix into a column vector. Let us now minimize the quadratic criterion

$$\frac{1}{2} E \{\varepsilon^2(k,\Theta)\} = V(\Theta) \tag{5.16}$$

in terms of the prediction error by differentiating with respect to Θ. The minimizing Θ can be found by the recursion

$$\hat{\Theta}(k) = \hat{\Theta}(k-1) + \gamma(k)R_p^{-1}\psi[k,\hat{\Theta}(k-1)]\varepsilon[k,\hat{\Theta}(k-1)] \tag{5.17}$$

to calculate R_p as an approximation of the second derivative of the criterion function, we let

$$R_p(k) \cong \frac{1}{k} \sum_{i=1}^{k} \psi(i)\psi^T(i) \tag{5.18}$$

and written recursively as

$$R_p(k) = R_p(k-1) + \gamma(k)[\psi(k)\psi^T(k) - R_p(k-1)] \tag{5.19}$$

this choice is called the stochastic Gauss-Newton algorithm. We could also choose R_p as identity. This would lead to a stochastic gradient algorithm.

Several types of algorithms can be classified as recursive prediction error methods, such as maximum likelihood [20,21], generalized least squares [22], instrumental variables [23], modified extended Kalman filter [24], and pseudo linear regression methods. We will be dealing with modified extended Kalman filter in Chapter 7. In the following, we will deal a little bit with pseudo linear regression methods as they represent approximations to the recursive prediction error methods, rather than direct applications as in other algorithms. We start with linear prediction model

$$\Psi(k+1,\Theta) = F(\Theta)\Psi(k,\Theta) + G(\Theta)\cdot\begin{bmatrix} y(k) \\ u(k) \end{bmatrix}$$
$$\hat{y}(k|\Theta) = \Theta^T\Psi(k,\Theta) \quad (5.20)$$

Let us assume for a moment as if Ψ were not dependent upon Θ, and so not consider Θ dependancy of Ψ when taking derivatives. Therefore,

$$y(k) = \Theta^T\Psi(k) + e(k) \quad (5.21)$$

and the estimator becomes

$$\hat{\Theta}(k) = \hat{\Theta}(k-1) + \gamma(k)R_p^{-1}(k)\Psi(k)\varepsilon(k)$$
$$\varepsilon(k) = y(k) - \hat{\Theta}^T(k-1)\Psi(k)$$
$$\Psi(l+1) = F[\hat{\Theta}(k)]\Psi(k) + G[\hat{\Theta}(k)]\begin{bmatrix} y(k) \\ u(k) \end{bmatrix} \quad (5.22)$$

This class of parameter estimation schemes, which includes extended least squares [25,26], extended matrix method [27], modified reference method of Landau [28] can be seen as approximations to the prediction error identification methods.

Analysis of recursive prediction error methods leads to certain interesting results, one of which is that these algorithms converge with probability one to a local minimum of the expected value of the chosen criterion. That means their convergence properties are the same as off-line prediction error methods. Another point is that the asymptotic distribution of these algorithms are the same as the asymptotic distribution of off-line methods. A third point is that the convergence of pseudo-linear regression methods requires the positive realness of certain transfer functions related to the unknown system. So the convergence cannot be ascertained beforehand [13].

5.3. CLOSED-LOOP IDENTIFIABILITY

In some cases, identification of systems cannot be done open-loop due either to security reasons as in industrial processes or that the system is inherently closed-loop as in biological or economic systems. To extend the identifiability concept to closed-loop systems, we will present the following discussion which appears at length in [29]. We will use these closed-loop identifiability results in the next chapter in proving the convergence of our algorithm.

Let us assume that the true system is given by

$$y(k) = G_s(q^{-1})u(k) + H_s(q^{-1})e(k) \tag{5.23}$$

$\{e(k)\}$ is a sequence of independent random vectors with zero means and covariance Σ. Let us also assume that, without loss of generality, $e(k)$ has the same dimension as $y(k)$. Let $H_s(o) = I$ and $\det[H_s(z)]$ has zeros outside the unit circle. This assumption is for securing $H_s^{-1}(q^{-1})$ to be a well-defined stable filter. To ensure that this puts no restriction, see the representation theorems in Chapter 3. Consider,

$$u(k) = F_i(q^{-1})y(k) + L_i(q^{-1})v(k) \tag{5.24}$$

$$1 \leq i \leq h$$

Notice that the feedback law and the term consisting of the outside disturbances and set point effects are allowed to change between h different cases.

A model for a certain value of Θ is

$$y(k) = G_m(q^{-1})u(k) + H_m(q^{-1})\varepsilon(k) \qquad (5.25)$$

$\{\varepsilon(k)\}$ is a sequence of independent random vectors with zero mean values and covariances $\hat{\Sigma}$. We assume that we use a prediction error identification method. In direct identification, inputs and outputs are processed as if they were obtained from an open loop system, whereas in indirect identification, in the first step, the closed-loop system is identified, then we solve for the open-loop system using knowledge of F_i and L_i. Let us define

$$D_T(S,M) = \{\Theta | G_M(z) = G_S(z) \text{ and } H_M(z) = H_S(z) \text{ a.e.} z\}$$

which is the set of parameter values which result in the models having the same system and noise transfer matrices as the true system. Let us present two handy definitions of identifiability:

DEFINITION 1:

The system S is said to be system identifiable if $\hat{\Theta}_N \to D_T(S,M)$ with probability one as $N \to \infty$.

DEFINITION 2:

The system S is said to be strongly system identifiable if it is system identifiable and $D_T(S,M)$ is non-empty.

The main theorem for identifiability analysis is the following:

THEOREM 5.1:

Consider the system in Equation 5.23, with the condition in Equation 5.24, and identification with the model in Equation 5.25 with either the direct or indirect method. Other assumptions are:

1) There is at least one delay in the system and/or in the feedback law.
2) The closed-loop system is asymptotically stable.
3) $D_T(S,M)$ is non-empty, which means that the system is included in the considered class of models.
4) The possible correlation of v(k) and e(k) is described by

$$v(k) = K_i(q^{-1})e(k) + \tilde{v}(k) \tag{5.27}$$

   where K_i is a causal asymptotically stable filter, $\tilde{v}(k)$ is independent of e(k) and persistently exciting of any finite order [30].
5) There is a delay from e(k) to $G_S(q^{-1})u(k)$.

Then the necessary and sufficient condition for strong system identifiability is that

$$\text{rank } R_h = \text{dimy} + \text{dimu} \qquad (5.28)$$

where

$$R_h(z) = \begin{bmatrix} I & \cdots & I & 0 & \cdots & 0 \\ F_1(z) & \cdots & F_n(z) & L_1(z) & \cdots & L_n(z) \end{bmatrix} \qquad (5.29)$$

The proof is given in [29].

SPECIAL CASE:

In the case of pure linear feedback law with $L_i \equiv 0$ R_h reduces to

$$R_h(z) = \begin{bmatrix} I & \cdots & I \\ F_1 & \cdots & F_h \end{bmatrix} \qquad (5.30)$$

a necessary condition for strong system identifiability is that $h \geq h_o$ = smallest integer $\geq 1 + \text{dimu}|\text{dimy}$. Proof is given in Appendix C. In the next chapter, we will utilize this special case.

5.4. AN INTRODUCTION TO PARAMETER ADAPTIVE CONTROL

In this last section of this chapter, we make a brief introduction to the simplest type of adaptive controllers which make use of parameter estimation, namely self-tuning algorithms. In the next chapter, we will mention alternative design techniques in state space.

It is customary to classify the adaptive control techniques as passive (non-dual or feedback) and active (dual or closed-loop). What makes the difference is the information amount available to the controlling mechanism and this, in turn, depends on the structure of the performance index. If the performance index is one-step ahead as in non-dual controllers, the controller then takes into account only the previous measurements and assume no further information will be available. Minimization of a loss function of several steps ahead, however, as in the case of dual controllers, means that the loop will remain closed in the future and will lead to a dependency on the future observations, but we do not mean the violation of causality. The dual controllers, in general, ensure better compromise between control and estimation but are more complex in structure. The non-dual controllers can also be classified as certainty-equivalent and cautious controllers. The certainty equivalent controllers do not take into consideration that estimated parameters are not always equal to the true ones, but use these estimates wherever parameters are needed to form the control law. Cautious controllers are designed according to separation principle which allows the use of the parameter estimate in the control law with its associated uncertainty, so these controllers are more "cautious" as the name implies. More on these classifications and relative merits of each type of controller can be found in [31] and [32] among others. We shall deal in this and the next chapter with passive controllers and in the last chapter with active controllers which entail extra computational requirements. Let us, for the moment, restrict ourselves to the self-tuning algorithm of Åström [33].

Let us consider the system

$$y(k) = \frac{B(z)}{A(z)} u(k) + \frac{D(z)}{C(z)} \varepsilon(k) \qquad (5.31)$$

where

$$B(z) = b_d a^{-d} + \ldots + b_n z^{-n}$$
$$A(z) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

Let us, for the moment, assume that $B^*(z) = z^n B(z)$ without any roots outside the unit circle. Then the control sequence which minimizes

$$J = E\{y^2(k+d) | Y_k , U_k\} \qquad (5.32)$$

where

$$Y_k = \{y(k), y(k-1), \ldots\}, \quad U_k = \{u(k), u(k-1), \ldots\}$$

is given by

$$u(k) = -\left[A(z)G(z)\,|\,z^{d}B(z)C(z)F(z)\right]y(k) \qquad (5.33)$$

where F and G are found by

$$D(z)\,|\,C(z) = F(z) + z^{-d}G(z)\,|\,C(z) \qquad (5.34)$$

$$F(z) = 1 + f_1 z^{-1} + \ldots + f_{d-1}z^{-(d-1)} \qquad (5.35)$$

PROOF:

Substitute Equation 5.34 into Equation 5.31

$$y(k+d) = \frac{z^{d}B(z)}{A(z)}\,u(k) + F(z)\varepsilon(k+d) + \frac{G(z)}{C(z)}\,\varepsilon(k)$$

$$= \frac{z^{d}B(z)}{A(z)}\,u(k) + F(z)\varepsilon(k+d)$$

$$+ \frac{G(z)}{C(z)}\left\{\frac{C(z)}{D(z)}\left(y(k) - \frac{B(z)}{A(z)}\,u(k)\right)\right\} \qquad (5.37)$$

So

$$E\{\ {}^{2}(k+d)\ Y_k, U_k\} = \left[\frac{B(z)}{A(z)}\left(z^{d} - \frac{G(z)}{D(z)}\right)u(k) + \frac{G(z)}{D(z)}\,y(k)\right]^2$$

$$+ (1 + f_1^{\,2} + \ldots f_{d-1}^{\,2})\ \Sigma$$

$E\{y^2(k+d)\}$ is minimized when $u(k)$ is chosen to satisfy

$$\frac{B(z)}{A(z)}\left[z^{d} - \frac{G(z)}{D(z)}\right]u(k) + \frac{G(z)}{D(z)}\,y(k) = 0 \qquad (5.38)$$

Manipulation gives Equation 5.33.

For models in least squares structure

$$A(z)y(k) = B(z)u(k) + \varepsilon(k) \quad (5.39)$$

$$A(z) = 1 + a_1 z^{-1} + \dots + a_n z^{-n}$$

$$B(z) = b_1 z^{-1} + \dots + b_n z^{-n} \quad (5.40)$$

Let

$$\beta^T = (b_1, \Theta^T) \quad (5.41)$$

where

$$\Theta^T = (b_2, b_3, \dots, b_n, a_1, a_2, \dots, a_n) \quad (5.42)$$

then

$$y(k) = \xi^T(k)\Theta + b_1 u(k-1) + \varepsilon(k) \quad (5.43)$$

$$\xi^T(k) = [u(k-2), \dots, u(k-n), -y(k-1), \dots, -y(k-n)] \quad (5.44)$$

The minimum variance controller is derived using the fact that the particular u(k) which minimizes $E\{y^2(k+1)\}$ is

$$u(k) = \xi^T(k+1)\Theta | b_1 \quad (5.45)$$

To be able to use the minimum variance controller, one needs to know the parameter values. Since the true values are not always known, then they are replaced by their current estimates which results in a certainty equivalent non-dual controller. A common parameter estimation scheme used with minimum variance controller is the recursive least squares identification method. The resulting self-tuning regulator has the unexpected property that the scheme may converge to the correct controller even though the original system is not in least squares class [33].

5.5. CONCLUDING REMARKS

In this chapter, we have introduced the problems of open-loop and closed-loop identification of systems and adaptive control. The subject matters we touched upon were completely determined from a utilitarian viewpoint, since we will be using and extending the concepts involved in the following chapters. We will be using the recursive prediction error identification method presented in this chapter in a certainty equivalent control structure and the results in the section on closed-loop identifiability will be used in proving the convergence of the overall resulting scheme.

CHAPTER 6

STOCHASTIC ADAPTIVE RECEDING HORIZON CONTROLLERS

6.1. INTRODUCTION

A suboptimal adaptive control algorithm for stochastic systems with unknown parameters is proposed in this chapter. The linear control law is certainty equivalent in the sense that it is linear in the estimates of the states and that the feedback gain matrix is calculated using the estimates of the unknown parameters. In the sequel, the control scheme is separated into an adaptive estimator which simultaneously estimates the states and identifies the parameters of the system, and a certainty-equivalent controller which makes use of the state and parameter estimates as if they were the true values. For the estimation part, the adaptive estimator of Ljung [34] is employed and for the control stage, the receding horizon concept is made use of. We allow some of the parameters in the system and measurement equations to be unknown. The system dynamics and measurement equations are given by

$$x(k+1) = A(\Theta)x(k) + Bu(k) + Ky(k) \tag{6.1}$$

$$Y(k) = \Theta C\ x(k) + v(k) \tag{6.2}$$

where x(k) is the nxl state vector at the k^{th} time instant, u(k) is the mxl deterministic input vector, y(k) is the corresponding output vector of dimensions pxl and v(k) is the noise sequence whose statistics are known. The noise sequence is assumed to be zero mean white Gaussian with covariance $E\{v(k)v^T(j)\} = Q_v\delta(k,j)$. In this formulation, the pxs matrix Θ contains all the unknown parameters in the model. Therefore, the system matrix $A(\Theta)$ and the output matrix ΘC are completely determined if the parameter matrix Θ is known. Furthermore, in this formulation, a particular parameterization suggested by Ljung [34] is adopted and the system matrix is assumed to be in the form $A(\Theta) = A + G\Theta C$. This formulation is general enough to contain stochastic difference equations with random parameter [34].

The problem is to obtain the control sequence u(k) for k=0,1,...,N-1, which minimizes

$$E\{\sum_{k=0}^{N-1} u^T(k)Ru(k)\} \tag{6.3}$$

subject to the system dynamics of Equation 6.1 and also to the constraint

$$E\ \{x(N)\} = 0 \qquad (6.4)$$

where R is a positive definite matrix and N is the predetermined horizon length. For the overall controller structure to be implementable, we require the control at the k^{th} instant to be a function of the information state $\{Y_k, U_{k-1}\}$ where $Y_k = \{y(o), y(1), \ldots, y(k)\}$, and $U_{k-1} = \{u(o), u(1), \ldots, u(k-1)\}$. If the parameter matrix is known, the results are the extension of Thomas' receding horizon controller to the stochastic case as demonstrated in Chapter 4. But with unknown parameters, the controller structure must be improved to include adaptation to the parameter identification process. The configuration of the controller to be used is shown in Figure 1.

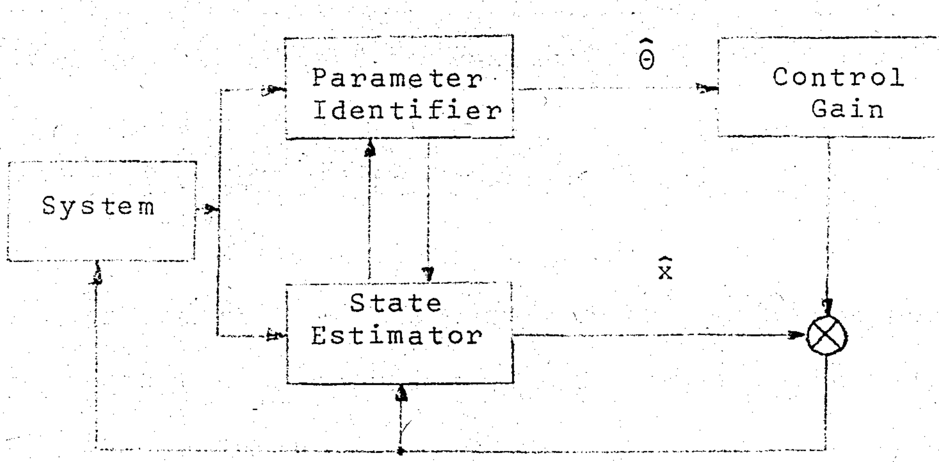

FIGURE 1. Stochastic Adaptive Controller Structure.

As can be noticed from the figure, the controller does not take into account the uncertainty associated with the identification of the parameters, but accepts the parameter estimates as if they were the true values of the parameters; that is, in the terms defined by Wittenmark [32], the controller is not "continuous" but simply "certainty equivalent".

Similar controller configurations in state space have been implemented so far with different realizations for the constituent subsystems. The parameter-adaptive self organizing controller of Saridis [35], for example, is realized with a first order stochastic approximation algorithm for parameter identification, a Kalman filter for state estimation, whereas the control gains are computed by either the steady state dynamic programming equations or by "one-step-ahead" approximations (called per-interval control). Alag and Kaufman [36] have designed a compensator which is composed of an on-line weighted least squares parameter identifier, a Kalman state estimator and a model-following control law making use of a single-step performance index. Kreisselmeier [37] suggests a controller configuration where the feedback laws are computed based on the current estimates of the parameters and states. Cao [8] has used the "per-interval" controller of Saridis in conjunction with a first order stochastic approximation type parameter estimator and a steady state Kalman filter for state estimation. But the bias in the parameter estimates led to control inconsistencies.

In the sequel, a new suboptimal control algorithm is suggested for the same controller configuration, which we think has the advantages of simplicity of implementation, generality of application and good performance qualities.

6.2. THE ALGORITHM

Let us first consider the case where the parameter matrix is known. The results of this certainty-about-parameters (CAP) control problem are given in Chapter 4. The optimal control vector u^* is obtained by

$$u^*(k) = F\hat{x}(k) \qquad k=0,1,... \tag{6.5}$$

with the controller gain matrix F, as in the deterministic case being calculated as

$$F = -R^{-1}B^T[W(o)A^T]^{-1} \tag{6.6}$$

where W(o) is the zeroth index solution of the backward iteration

$$W(i) = A^{-1}[W(i+1) + BR^{-1}B^{T}]A^{-T}, \quad W(N) = 0 \tag{6.7}$$

provided that the system is controllable and $N \geq n-m+1$. The state vector estimate is obtained by

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + Ky(k) \tag{6.8}$$

or equivalently, upon substitution of Equation 6.5 into Equation 6.8 by

$$\hat{x}(k+1) = (A + BF)\hat{x}(k) + Ky(k) \tag{6.9}$$

when there exist some unknown parameters either in the system equation or in the measurement equation, the state vector can be augmented to include the unknown parameters and the augmented vector can be estimated. However, this procedure would lead to estimation and subsequenct control of systems with nonlinear dynamics which we shall treat in the next chapter. To avoid this nonlinear control problem, a certainty- equivalence is imposed both with respect to state estimates as well as the parameter estimates ensuring ease of implementation and realizability of the control scheme for high order systems.

As a result of this enforced certainty-equivalence the stochastic adaptive control algorithm becomes

$$u(k) = \hat{F}(k)\hat{x}(k) = -R^{-1}B^T[\hat{W}(k,o)\hat{A}^T(k)]^{-1}\hat{x}(k) \quad (6.10)$$

where $\hat{A}(k)=A+G\hat{\Theta}(k)C$ and $\hat{W}(k,o)$ is the zeroth index solution of the backward iteration

$$\hat{W}(k,i) = \hat{A}(k)^{-1}[\hat{W}(k,i+1) + BR^{-1}B^T]A^{-T}(k),\ \hat{W}(k,N) = 0 \quad (6.11)$$

If the chosen horizon length N is large, then doubling algorithms may be employed as demonstrated in Chapter 2, thus avoiding matrix inversion at every step of this backward iteration.

Notice that the controller gain Equation 6.10 depends on $\hat{A}$ which changes as the parameter estimates are changed at every stage along with parameter identification.

The estimates for the states and the parameters are computed by

$$\hat{x}(k+1) = [\hat{A}(k) + B\hat{F}(k)]\hat{x}(k) + Ky(k) \quad (6.12)$$

$$\hat{\Theta}^T(k) = \hat{\Theta}^T(k-1) + [\gamma(k)|r(k)]C\hat{x}(k)[y(k) - \hat{\Theta}(k-1)C\hat{x}(k)] \quad (6.13)$$

$$r(k) = r(k-1) + \gamma(k)[||C\hat{x}(k)||^2 - r(k-1) + \delta] \quad (6.14)$$

In this algorithm, $\gamma(k)$ is an arbitrary scalar gain sequence which satisfies Dvoretzky's conditions [39]. The arbitrary constant γ is a small positive term used to prevent r(k) from taking on the null value. It can also be noticed that the parameter identifier is of a stochastic approximation type where r(k) is the trace of the uncertainty matrix associated with the parameter identification.

The algorithm is started with an arbitrary $\hat{\Theta}(o)$ for any given $\hat{x}(o)$.

6.3. CONVERGENCE CONDITIONS

The convergence of the proposed algorithm is closely related to the conditions of identifiability for systems operating under feedback. These conditions have been established by Söderström et.at. [29] and in the previous chapter, for multi-variable systems in a feedback loop. Here, we will demonstrate that the proposed algorithm meets the conditions for closed-loop identifiability.

Figure 2 depicts the configuration of the adaptive stochastic controller together with the system whose closed-loop identifiability will be examined. Substituting Equation 6.2 into Equation 6.1, we get

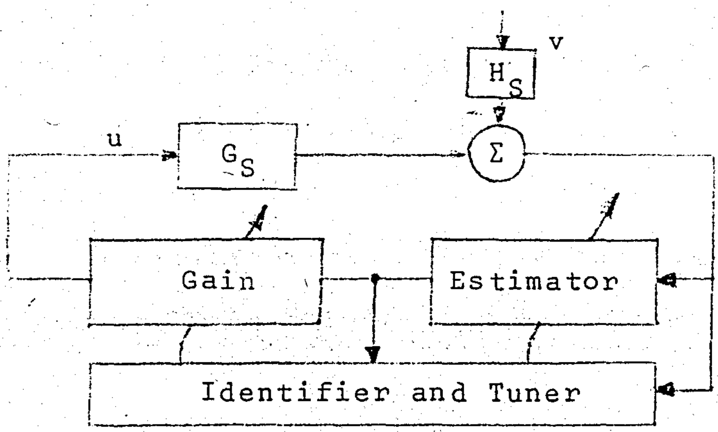

FIGURE 2. System Operating Under Feedback.

$$x(k+1) = [A + (G+K)\Theta C]x(k) + Bu(k) + Kv(k)$$

$$= \mathbb{A}(\Theta)x(k) + Bu(k) + Kv(k) \qquad (6.15)$$

the transfer matrix H_S from v to y is found to be

$$H_S(z) = \Theta C(zI - \mathbb{A})^{-1}K + I \qquad (6.16)$$

Considering the estimator of Equation 6.12, the transfer matrix from y to $\hat{x}$ is obtained as $[zI - (\hat{A}+B\hat{F})]^{-1}K$ and thus the transfer matrix from y to u becomes

$$F_i = \hat{F}[zI - (\hat{A}+B\hat{F})]^{-1}K \qquad (6.17)$$

Finally using Equation 6.15 again, we find the transfer matrix from u to x to be

$$G_S(z) = \Theta C(zI - \mathbb{A})^{-1}B \qquad (6.18)$$

The subscript i in the feedback matrix of Equation 6.17 denotes the different values of F(z) due to tuning by the parameter identifier. That is, the feedback law shifts between h different cases, where each case is to be used in a non-neglible part of the total control period.

Examination of the transfer matrices given above will show compliance with the identifiability conditions of Chapter 5. One can observe from Equation 6.17 that the feedback loop contains the necessary time delay for closed-loop identifiability. Also, it is assumed that the parameter estimates do not change drastically, and that they constitute a set of pseudo-stationary points of operation each taking sufficiently long duration to ensure an h-shift in feedback laws, where h is the smallest integer such that $h \geq 1 + \dim u | \dim y$. This assumption is strongly backed up by simulations. If these weak conditions are satisfied then the only remaining condition for identifiability is the theorem in Chapter 5. The asymptotic stability is, in turn, secured by the identifiability. That is, in the limit, if the true values of the parameters are known, then the proposed stochastic receding horizon controller is sufficient to render the system asymptotically stable (Chapter 4). This means that, in this case, asymptotic stability implies and is, in turn, implied by the identifiability of the closed-loop system.

6.4. SIMULATION RESULTS

Several systems have been simulated and the proposed algorithm converged in all cases. Some of the simulated systems and the achieved results are reported below:

SYSTEM 1) $x(k+1) = (.5 + 1.5\Theta)x(k) + u(k) + 1.5v(k)$

$y(k) = \Theta x(k) + v(k)$ with $v(k):N(0,.25)$, $\Theta = 1$

Clearly, this system is unstable but controllable.

SYSTEM 2) $x(k+1) = (-.7 + 1.5\Theta)x(k) + u(k) + 1.5v(k)$

$y(k) = \Theta x(k) + v(k)$ with $v(k):N(0,.25)$, $\Theta = 1$

This system is the first standard example reported in Söderström [40].

SYSTEM 3) $x(k+1) = (.83 + .15\Theta)x(k) + .1u(k) + .15v(k)$

$y(k) = \Theta x(k) + v(k)$ with $v(k):N(0,.25)$, $\Theta = 1$

This system is obtained from the same continuous time system that System 2 was obtained, but with one tenth the sampling period.

SYSTEM 4)

$$x(k+1) = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v(k)$$

$$y(k) = \begin{bmatrix} a & b \end{bmatrix} x(k) + v(k)$$

with $v(k):N(0,1)$, $a = .95$, $b = -.05$

This system represents the dynamics of the pharmacodynamical application reported by Koivo [44], who studied the control of infusion rate of a drug for blood pressure regulation. In his paper, Koivo used a minimum variance regulator not penalizing the cost of input energy to comply with microprocessor requirements. The proposed algorithm penalizes the input energy meaning that it restricts the infusion rate of the drug while regulating the blood pressure.

All systems considered above are obtained from an ARMAX model of the type

$$y(k) = ay(k-1) + bu(k-1) + v(k) + cv(k-1)$$

with the following constants:

System 1) $a = 2$, $b = 1$, $c = -.5$

System 2) $a = .8$, $b = 1$, $b = .7$

System 3) $a = .98$, $b = .1$, $c = -.83$

System 4) $a = .95$, $b = -.05$, $c = 0$

The simulation of these systems exhibited the following general properties of the proposed algorithm:

1. Effective regulation to Zero: System 1 and System 2 were controlled by the proposed adaptive receding horizon scheme. In 50 iterations, the following results were obtained for the average value of the estimate $(1/50) \sum_{k=1}^{50} \hat{x}(k)$:

System 1 with 1-step ahead controller: .0056

System 1 with 4-step ahead controller: .0109

System 2 with 1-step ahead controller: .0316

2. Regulation to Non-zero Set Points: For System 4, we applied the concept of regulation to non-zero set points mentioned in Chapter 2. The desired final state for x_1 was 50 mm Hg below that of the operating one. In a few iterations, we had 1.64 percent error in control and zero percent error in state-estimation. As can be seen, the non-singularity of the transition matrix is well taken care of by the method proposed in Chapter 2.

3. Success of Adaptation: Jacobs et.al. [42] suggests that the incremental costs after many stages of operation can be used as a measure to evaluate the asymptotic properties of controllers. Small incremental costs as compared with the incremental costs of the certainty about parameters case would suggest successful adaptation. The performance for System 1 of the 1-step ahead adaptive controller was compared with that of the 1-step ahead CAP controller using the incremental cost $\hat{x}(1000) + u^2(999)$. These costs differ from each other by 2×10^{-4} which means very good adaptation.

4. Tunability by the Choice of Horizon Length: Better performance was observed with fewer-step-ahead controller. The table below gives a comparison of the performance of 1-step ahead and 4-step ahead controllers for System 1 at k=3000.

	$r(k)$	$\lvert\hat{\theta}-\theta\rvert$	$\lvert\hat{x}-x\rvert$	$\frac{1}{k}\sum_{i=1}^{k}\hat{x}(i)$	$\frac{1}{k}\sum_{i=1}^{k-1}u^2(i)$
1-step ahead	.5571	.0080	.0000	.0207	2.718
4-step ahead	.7446	.0028	.0000	.0415	2.564

This behavior is attributed to the lazy character of the 4-step ahead controller since it is given the information that it has 4-steps before it to achieve the control.

5. Tunability by the Choice of Sampling Period: System 2 and System 3 are obtained from the same continuous time system. However, System 3 has a sampling period equal to one tenth of that used for System 2. The accuracy of control as measured by $1/k \sum_{i=1}^{k} \hat{x}(i)$ is .023 for System 2 and .0065 for System 3 with k=3000, which indicates that, in general, shortening the sampling interval results in a better controller. The original continuous time system was

$$\mathring{x}(t) = -.2x(t) + .8u(t) + 1.5v(t)$$

$$y(t) = x(t) + v(t)$$

and a first-order approximation has been used for discretization.

6.5. DISCUSSION

As can be noticed from the above presentation, the algorithm has several distinct features to be emphasized.

1. Due to the inherent property of receding horizon controllers, the designer does not have to choose any state penalization matrix whose choice of relative magnitude

with respect to the input penalization matrix is more or less a trial and error procedure as mentioned in Chapter 1. For single input systems, one does not even have to choose the input penalization constant, because it eventually cancels in the calculations. However, the necessary trade-off between the input energy and the system behavior quality is always conserved.

2. The choice of horizon length adds a flexibility to design. Stronger control is associated with less number of steps before the controller. If the choice of N is large, one can use the doubling algorithms of Chapter 2.

3. Even though this exposition includes multidimensional systems with multi-parameter uncertainty, no use is made of matrix update equations for state and parameter estimation uncertainties whose presence constitute most of the computational burden of other methods.

4. The chosen parametrization is general enough to contain the ARMAX model

$$y(k) + A_1 y(k-1) + \ldots + A_n y(k-n_a) = B_1 u(k-1) + \ldots + B_m u(k-n_b) + v(k) + C_1 v(k-1) + \ldots + C_p v(k-n_c)$$

with

$$x(k) = [y^T(k-1) \ldots y^T(k-n_a)u^T(k-1) \ldots$$
$$u^T(k-n_b)v^T(k-1) \ldots v^T(k-n_c)]^T$$

and proper choices of coefficient matrices and vectors [34].

5. All the above simulations and the convergence analysis is done without resort to an external perturbation signal. Presence of such a sufficiently exciting signal is considered to be a must by many authors such as [34, 35, 37, etc]. Our simulations verified that the identifier sufficiently tunes the parameters of the controller to provide the necessary shifts in control law, needed for closed loop identifiability.

6.6. CONCLUSIONS

An adaptive controller for linear stochastic systems with parameter uncertainty has been introduced. The controller is certainty equivalent in both the parameters and the states in the sense that it used the parameter estimates which depend on the state estimates instead of the true parameters in the controller gain calculation and makes use of the state estimates depending on the parameter estimates instead of the true states in the feedback law. Another way to pose the situation is that the identifier tunes the parameters of both the constant deterministic feedback gain and state estimator.

The proposed control algorithm represents an improvement over the self-tuning regulators which do not penalize the energy spent in control, thus achieving their aim by using in some cases unacceptably high energy. The proposed controller is also an extention of the popular controllers using linear quadratic cost criteria, but which consider only one-step-ahead effects. This algorithm gives the designer the possibility of penalizing the amount of energy spent and the flexibility of tuning with different horizon lengths. This controller with the enforced certainty equivalence with respect to both the state estimates and the parameter estimates is by far simpler to implement as compared with the control law using the on-line solution of the matrix Riccati equation.

CHAPTER 7

SOME OTHER POSSIBILITIES OF DESIGNING STOCHASTIC ADAPTIVE CONTROLLERS

7.1. INTRODUCTION

Up to now, we have been using a very simple but effective identification scheme to find the unknown constant parameters in the signal model. Actually, neither this scheme nor the other identification schemes exhaust the possibilities to search for the unknown parameters. For example, other methods such as time-series analysis exists to be used with ready test data calculating off-line the signal model output covariance and the problem then is to match a proper signal model to this output covariance. If one is supposed to find the parameters on-line, as in the case of an adaptive control situation, an obvious possibility is to use non-linear filters to accommodate for augmentation of the parameters to the state equation so as to estimate them as states. Another possible approach is a parallel processing scheme in which for all possible values of parameters, one uses Kalman filters operating in parallel, each designed using a discrete value out of that possible parameter set.

Evidently, these methods would greatly increase the computational requirements and therefore not resorted to in this work whose main purpose is to design easily implementable schemes. However, we shall briefly mention them here, first to indicate the possibilities they present in stochastic adaptive control and, second, to excite further research work in adaptive estimation, for adaptive estimation by itself stands as a vast area for further research. It is demanded from the research workers to try to reduce the computational complexity of the adaptive methods, so that they can be used more effectively in a real situation, perhaps with small-size digital computing facilities.

Another extension in stochastic adaptive control might be to estimate the characteristics of the random disturbances acting on the system. Actually one idealizes the situation a bit saying that the characteristics of the noise acting on the process is known exactly from the start. It may be that the disturbances are too obscure to yield to easy formulas and one may be obliged to estimate the characteristics as the process goes on.

In the first section we deal with extended Kalman filtering for simultaneous on-line state and parameter estimation. Recent convergence results of Ljung will also be included. In the next section, parallel processing methods that make use of parameter detection are introduced. Lastly, some highlights from among the techniques of adaptive noise estimation or of determining the filter gain without knowing the noise covariances are included.

7.2. SIMULTANEOUS STATE AND PARAMETER ESTIMATION OF EXTENDED KALMAN FILTERING

Let us suppose that the model of the system whose states and parameters are to be estimated, is as follows:

$$x(k+1) = A(\Theta)x(k) + B(\Theta)u(k) + v(k)$$
$$y(k) = C(\Theta)x(k) + w(k) \qquad (7.1)$$

where

$$E\left[v(k)v^T(\ell)\right] = Q_v\delta(k,\ell), \quad E\left[w(k)w^T(\ell)\right] = Q_w\delta(k,\ell)$$
$$E\left[v(k)w^T(\ell)\right] = Q_c\delta(k,\ell), \; E\left[x(o)\right] = 0,$$
$$E\left[x(o)x^T(o)\right] = X_o(\Theta) \qquad (7.2)$$

We assume differentiability with respect to Θ of the parameter dependent matrices. Notice that in the above equations, we suppose that the noise covariances to be independent of the unknown parameters. In the last section of this chapter, we will take up the converse approach that the only unknowns are the noise covariances. To simultaneously estimate the states and parameters of the system, we augment the parameter vector to the state vector, considering the parameters as constant states. Therefore,

$$z(k) = \begin{bmatrix} x(k) \\ \Theta(k) \end{bmatrix} \qquad (7.3)$$

and

$$z(k+1) = f\left[z(k),u(k)\right] + \begin{bmatrix} v(k) \\ 0 \end{bmatrix}$$

$$y(k) = h(z(k)) + w(k) \qquad (7.4)$$

where

$$f\left[z(k),u(k)\right] = \begin{bmatrix} A(\Theta)x(k) + B(\Theta)u(k) \\ \Theta \end{bmatrix} \qquad (7.5)$$

and

$$h(z(k)) = C(\Theta)x(k)$$

So now the problem is to estimate the states of a nonlinear system. Applying the well-known extended Kalman filter equations, we obtain,

$$\hat{z}(k+1) = f[\hat{z}(k),u(k)] + N(k)[y(k) - h(\hat{z}(k))], \quad \hat{z}(o) = \hat{z}_o \tag{7.6}$$

$$N(k) = \{F[\hat{z}(k),u(k)]\bar{P}(k)H^T(\hat{z}(k)) + \bar{Q}_c\}$$
$$\{H(\hat{z}(k))\bar{P}(k)H^T(\hat{z}(k)) + Q_w\}^{-1}$$

$$\bar{P}(k+1) = F[\hat{z}(k),u(k)]\bar{P}(k)F^T[\hat{z}(k),u(k)] + \bar{Q}_v$$
$$- N(k)[H(\hat{z}(k))\bar{P}(k)H^T(\hat{z}(k)) + Q_w].N^T(k), \quad \bar{P}(o) = \bar{P}_o$$

$$F[\hat{z}(k),u(k)] = \frac{\partial}{\partial z} f(z,u)\Big|_{z=\hat{z}(k)}$$

$$= \begin{bmatrix} A(\hat{\Theta}(k)) & \frac{\partial}{\partial \Theta} \quad A(\Theta)\hat{x} + B(\Theta)u \\ 0 & I \end{bmatrix} \tag{7.7}$$

$$H|\hat{z}(k)| = \frac{\partial}{\partial z} h(z)\Big|_{z=\hat{z}(k)} = [C(\hat{\Theta}(k)) \quad \frac{\partial}{\partial \Theta} C(\Theta)\hat{x}\Big|_{\Theta=\hat{\Theta}}]$$

$$\bar{Q}_v = \begin{bmatrix} Q_v & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{Q}_c = \begin{bmatrix} Q_c \\ 0 \end{bmatrix}, \quad \hat{z}_o = \begin{bmatrix} \hat{x}_o \\ \hat{\Theta}_o \end{bmatrix}, \quad \bar{P}_o = \begin{bmatrix} x(\hat{\Theta}_o) & 0 \\ 0 & \Sigma_o \end{bmatrix}$$

Σ_o is an arbitrarily assumed value for the initial parameter uncertainty.

Although it appears that the problem is posed in a good setting and the solution can be found in a straight forward manner, in some cases and most often when the residuals are large (which means that the uncertainty in the parameter estimate is large) and/or the input to the system is small, the filter may diverge [24]. This is attributed to the independence of the filter gain of the uncertainty in the parameter estimates. For deterministic models, where the steady state filter gain does not involve the unknown parameter in a natural fashion, good convergence is obtained. The convergence analysis of Ljung [24] resulted in both an understanding of the convergence properties of the filter and a remedial modification of the algorithm which involved the inclusion of a term in the algorithm to obtain global convergence. Extended Kalman filter is used in conjunction with dynamic programming in control situations, for example, by Bar Shalom [31], and presents a more exact answer than other suboptimal controllers, but it is so computationally complex that it can but be used with very simple examples.

7.3. ADAPTIVE ESTIMATION BY COMBINED DETECTION/ ESTIMATION APPROACH

In this approach, the unknown parameter vector Θ is assumed known to belong to a discrete set Θ_i, which is attached to an arbitrary probability distribution at the outset. Next, Kalman filters are built to estimate the states of the system with parameters Θ_i. They operate in parallel, each producing outputs $\hat{x}_{k|\Theta}$, which are the conditional state estimates based on their knowledge of the system parameters. The mean estimate of the state is found either as a weighted sum of the conditional state estimates, the weights being the a-posteriori probabilities of the parameters. Or the estimate can be found as one that maximizes the a-posteriori probabilities. The update of parameter probabilities is based on a likelihood ratio approach utilizing conditional Kalman filter innovations and their associated covariances. The a-posteriori probabilities $p(\Theta_i|Y_k)$, after some simple manipulations, can be written in terms of likelihood functions $p(Y_k|\Theta_i)$ recursively as [5]:

$$p(\Theta_i|Y_k) = \frac{p(y(k)|Y_{k-1},\Theta_i)p(\Theta_i|Y_{k-1})}{\sum_{i=1}^{m} p(y(k)|Y_{k-1},\Theta_i)p(\Theta_i|Y_{k-1})} \qquad (7.8)$$

where the choice of a-priori probabilities is immaterial and the denominator of the expression is just a normalization constant. We assume, there are M different parameters in the set. For Gaussian signal models, using conditional Kalman filter innovations $\tilde{y}_i(k)$ and covariances $E[\tilde{y}_i(k)\tilde{y}_i^T(k)] = \Omega_i(k)$

$$p(\Theta_i|Y_k) = c|\Omega_i^{-T}(k)|^{\frac{1}{2}}\exp[-\frac{1}{2}\tilde{y}_i^T(k)\Omega_i^{-1}(k)\tilde{y}_i(k)]p(\Theta_i|Y_{k-1}) \tag{7.9}$$

Therefore, Kalman filters constructed for all Θ_i, driven by y(k), produce at each stage, the innovations sequences $\tilde{y}_i(k)$ which are not true ones if $\Theta_i=\Theta$ that correspond to the true signal model. So if $\Theta_i \neq \Theta$, the sequence is not white. Notice that the covariances $\Omega_i(k)$ can be computed off-line. We will also mention a condition for distinguishing the true one among others: For different parameters, either the innovations are not equal or their covariances are not equal or both as time processes. This provides a uniqueness condition of the true parameter value as that which, provides the smallest covariance. If y(k) are Gaussian, the convergence is almost sure but the method is not constrained either by Gaussianity or ergodicity assumption. It is possible to include non-Gaussian, asymptotically stationary or some non-stationary situations.

We assumed at the outset that the unknown parameter is a member of a finite set. If the reality is that the unknown parameter is just a point in a compact region, then some representative points Θ_i in that region are selected built on the compromise between finer approximation of the region and the complexity of the resulting calculation. The following theorem will illustrate the convergence properties of the scheme:

THEOREM 7.1:

Suppose that the innovations are asymptotically ergodic in the autocorrelation function, $\Omega_i(k) \rightarrow \Omega_i > 0$ as $k \rightarrow \infty$ and show that the limiting covariance of the filter innovations Σ_i such that

$$\Sigma_i = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=k}^{k+n-1} \tilde{y}_i(j)\tilde{y}_i(j) \tag{7.10}$$

Assume a-priori pseudo-probabilities for the parameters, and realize the update of these probabilities by Equation 7.8. When one has for some β

$$\beta < \beta_j \tag{7.11}$$

for

$$\beta_i = \ln|\Omega_i| + tr(\Omega_i^{-1}\Sigma_i) \tag{7.12}$$

then the probability for the parameter which is closest, in the sense that it minimizes Kullback information measure to the true one, will approach 1 and the probabilities of other parameters will approach zero asymptotically. For an infinite measurement sequence, we have asymptotic per sample Kullback information function as

$$\bar{J}(\Theta_s,\Theta_r) = \lim_{k\to\infty} k^{-1} \; E\{\ln \frac{p(Y_k|\Theta_s)}{p(Y_k|\Theta_r)} \Big| \Theta_s\} \qquad (7.13)$$

Convergence of the scheme is exponentially fast. Such a scheme can also be useful for time-varying parameters as well. Deshpande et al. [43] used this scheme together with dynamic programming in control situations. But both "the curse of dimensionality" of dynamic programming and also of the method renders the overall scheme quite involved from a computational standpoint.

7.4. ADAPTIVE NOISE COVARIANCE AND FILTER GAIN DETERMINATION

In some practical situations, where the statistics of the noises acting on the system are not known beforehand, the use of Kalman filters for optimal state estimation does not give good results, since Kalman filters need exact a-priori knowledge of the noise statistics. Also it is possible to formulate the problem so that the errors associated with the modelling of the parameters occuring in the various system matrices are considered as unknown additive disturbances. Then one has to resort to some special techniques which help to find the unknown statistics. In the following brief sketch, we will only consider, whenever possible, techniques to find the unknown gain matrix of the filter without explicitly finding the unknown noise covariances.

1. Maximum Likelihood Approach [44 - 47]:

The idea is to estimate the unknown parameters in such a way as to maximize either (a) the joint density of the states and the parameters conditional on the previous measurements or (b) the marginal density of the parameteres, or (c) the marginal density of the state, which results in the techniques of Section 7.3. (a) and (b) result in similar equations which we shall present below. Direct maximization of the densities leads to equations non-linear in the estimate of the unknown parameters and Newton-Raphson iterations can be used but the computation of the derivatives presents a major problem. For systems with:

i) time-invariant state transition and input matrices,

ii) controllability and observability conditions guaranteed,

iii) the steady state condition is reached so that the filter gain and the covariances associated with the uncertainty in the state estimate are constant,

iv) no a-priori information on the parameters is available it can be shown that [48], the sub-optimal maximum likelihood adaptive filter has the describing equations:

$$\hat{x}(k+1) = A\hat{x}(k) + A\hat{K}(k)e(k), \quad \hat{x}(o) \text{ chosen}$$

$$e(k) = y(k) - C\hat{x}(k) \qquad (7.14)$$

$$\frac{\partial \hat{x}(k+1)}{\partial K^{jm}} = A(I - \hat{K}(k)C) \frac{\partial \hat{x}(k)}{\partial K^{jm}} + I_{jm} v(k)$$

$$\frac{\partial \hat{x}(o)}{\partial K^{jm}} = 0 \qquad (7.15)$$

where $j=1,\ldots,n$, $n=1,\ldots,r$, and $\hat{K}(k)$ is updated according to

$$\Lambda(k+1) = \Lambda(k) + \text{tr}\left[C \frac{\partial \hat{x}(k+1)}{\partial k^{jm}} \frac{\partial \hat{x}(k+1)}{\partial k^{jm}} C^T\right] \qquad (7.16)$$

$$g(k+1) = g(k) + \text{tr}\left[e(k+1) \frac{\partial \hat{x}(k+1)}{\partial k^{jm}} C^T\right] \qquad (7.17)$$

$$\hat{K}^{jm}(k+1) = \hat{K}^{jm}(k) + \Lambda^{-1}(k+1)g(k+1) \qquad (7.18)$$

$\hat{K}^{jm}(k+1)$ is the unique estimate of the filter gain based on the measurements up to time k. Under steady state filtering conditions, the input and output noise covariance can be found easily. The details of relevant optimization and subsequent approximation can be found in [48].

2. Correlation Methods

In the correlation methods, we utilize equations that relate the unknown parameters and the autocorrelation function of observations. The unknown parameters are solved in terms of autocorrelations. Either the autocorrelations of the output or the innovations can be used. We assume the system is completely controllable and observable.

a) Output Autocorrelation Method:

This method is only applicable in the cases where the output is a stationary process and the state transition matrix is stable. Assume that $\tilde{C}(i)$ be the d'th lag autocorrelation of the output $y(k)$:

$$\tilde{C}(d) = E\{y(k)y^T(k-d)\} \tag{7.19}$$

Since the output is stationary, autocorrelation is only a function of the lag d. Estimate of $\hat{C}^N(d)$ is obtained recursively from

$$\hat{C}^N(d) = \hat{C}^{N-1}(d) + \frac{1}{N}\left[y(N)y^T(N-d) - \hat{C}^{N-1}(k)\right] \tag{7.20}$$

where N is the sample size. Defining Σ as

$$\Sigma = E\{x(k)x^T(k)\} \tag{7.21}$$

which can be shown to satisfy

$$\Sigma C^T = (\phi^T\phi)^{-1}\phi^T \begin{bmatrix} \tilde{C}_1 \\ \vdots \\ \tilde{C}_n \end{bmatrix} \tag{7.22}$$

where

$$\phi = [A^T C^T, \ldots, (A^T)^n C^T] \tag{7.23}$$

Using Equation 7.22, one can solve

$$\Pi = A[\Pi + (\Sigma-\Pi)C^T(\tilde{C}(o)-C\Pi C^T)^{-1}C(\Sigma-\Pi)]A^T \tag{7.24}$$

and substitute in

$$K = (\Sigma-\Pi)C^T(C(o) - C\Pi C^T)^{-1} \tag{7.25}$$

to find an estimate of the filter gain. Complete derivations are given in [48].

b) Innovation Correlation Method:

Optimal Kalman filtering requires that the innovations sequence be a zero mean Gaussian white noise sequence. But for a suboptimal filter, this is not so, and

Mehra in [52] developed a scheme which makes use of this property. Namely, innovations are tested for zero correlation. Using M_1C^T from

$$M_1C^T = (\phi^T\phi)^{-1}\phi^T \begin{bmatrix} \Gamma(1)+CAK_o\Gamma(o) \\ \cdots \\ \Gamma(n)+CAK_o\Gamma(n-1)+ \ldots CAK_o\Gamma(o) \end{bmatrix}$$

where $\Gamma(d) = E\{v(k)v^T(k-d)\}$

One solves for δM in

$$\delta M = A\big[\delta M - (M_1C^T + \delta MC^T)(\tilde{C}(o) + C\delta MC^T)^{-1}(CM_1 + C\delta M) + K_oCM_1 + M_1C^TK_o{}^T - K_o\Gamma(o)K_o{}^T\big]A^T$$

and then substitutes in the following equation to find K

$$K = (M_1C^T + \delta MC^T)(\tilde{C}(o) + C\delta MC^T)^{-1}$$

As can be noticed, one has to start the procedure either by choosing a K_o or use output correlation method for starting.

Ohab and Stubberud [53] developed a method also based on uncorrelating the innovations. They first measure the correlation in the innovations. If the innovations are correlated, then the gain matrix will be so adjusted that the innovations are less correlated. This goes on until the innovations are uncorrelated. The method of steepest descent is used to find the filter gain that secures the innovations being uncorrelated.

3. Covariance Matching Techniques [54,55]:

In this classification are the techniques which equate the measured and theoretical (as obtained from the Kalman filter) covariances of the innovations. As an example, one may have the measured sample covariance be larger than that calculated by the filter, then increasing the input covariance which increases the uncertainty in the state estimate results in an increase in innovations covariance.

One case in which success has been achieved is when the input noise covariance is known but the output noise covariance is not known. It can then be estimated by

$$Q_v = \frac{1}{N} \sum_{j=1}^{N} v(k-j)v^T(k-j) - CPC^T$$

where N is the sample size and P(k) is obtained from the Kalman filter.

7.5. CONCLUDING REMARKS

In this chapter, we have presented other ways of obtaining stochastic adaptive controllers which are based on adaptive estimation and deterministic controller blocks used in series configuration. These techniques are not effectively used because of the computational load associated with them. So our proposed technique is evidently superior to them. In the last section of this chapter, we have mentioned the work on adaptively estimating the noise statistics and filter gains.

APPENDIX A

CONTINUOUS RECEDING HORIZON CONTROLLER

For the linear time-invariant continuous system

$$\overset{\circ}{x} = Ax + Bu, \qquad x(o) = x_o \tag{A.1}$$

the performance criterion

$$J(x_o,u(,)) = \frac{1}{2} \int_o^{T_1} u^T R u d\tau \ , \quad R > 0 \tag{A.2}$$

and the equality constraint

$$x(T_1) = 0 \tag{A.3}$$

if the system (A,B) is controllable.

The optimal control minimizing Equation A.2 and satisfying Equation A.1 and Equation A.3 is given by

$$u^*(x(t)) = -R^{-1}B^T\bar{M}^{-1}x(t) \tag{A.4}$$

where $\bar{M}$ is the solution at instant 0 of the equation

$$\overset{\circ}{M} = AM + MA^T - BR^{-1}B^T, \quad M(T_1) = 0 \qquad (A.5)$$

or in an explicit form

$$\bar{M} = \int_o^{T_1} e^{-A\tau}BR^{-1}B^T e^{-A^T\tau} d\tau \qquad (A.6)$$

And the application of this feedback results in a system which is asymptotically stable. To prove what we have stated, let us apply variational calculus to form the canonical equations:

$$\overset{\circ}{x} = Ax + Bu, \quad x(o) = x_o$$

$$\overset{\circ}{\lambda} = -A^T\lambda \;, \quad x(T_1) = 0$$

$$u = -R^{-1}B^T\lambda \qquad (A.5)$$

where λ and x are related by

$$\lambda(\tau) = K(t)x(t) \qquad (A.6)$$

with K being the solution of

$$-\overset{\circ}{K} = KA + A^TK - KBR^{-1}B^TK \qquad (A.7)$$

x(k) has the solution

$$x(t) = e^{At}x_o - \int_o^t e^{A(t-\tau)} BR^{-1}B^Te^{-A^T\tau}\lambda_o d\tau \quad (A.8)$$

with $x(T_1) = 0$, it gives

$$x_o = (\int_o^{T_1} e^{-A\tau} BR^{-1}B^T e^{-A^T\tau} d\tau)\lambda_o = M_o\lambda_o \qquad (A.9)$$

If M_o is invertible, and using Equation A.6

$$K(o) = (\int_o^{T_1} e^{-A\tau}BR^{-1}B^T e^{-A^T\tau}d\tau)^{-1} \qquad (A.10)$$

so

$$\bar{M} = K^{-1}(o) \qquad (A.11)$$

Next we prove that $\bar{M}$ is invertible. The term $e^{-A\tau}BR^{-1}B^T e^{-A^T\tau}$ is at least positive semi-definite due to positive definiteness of R. So let us establish strict positiveness of $\bar{M}$. If it were semi-definite there exists a constant vector $x \neq 0$ such that $x^T e^{-At}BR^{-1}B^T e^{-A^T t}x=0$ for $t\varepsilon[0,T_1]$. This is impossible since $x^T e^{-At}B=0$ for $t\varepsilon[0,T_1]$, which contradicts the hypothesis of the controllability of the pair (A,B). So $\bar{M} > 0$.

Let us now prove that the resulting closed loop system is asymptotically stable. That is,

$$\mathring{x} = (A - BR^{-1}B^T\bar{M}^{-1})x = \mathbb{A}\, x \qquad (A.12)$$

is asymptotically stable. It is the same as proving the asymptotic stability of the adjoint system

$$\overset{\circ}{x} = A^T x \tag{A.13}$$

we first establish that

$$V(x) = x^T \bar{M} x \tag{A.14}$$

is a Lyapunov function for the system in Equation A.13.

i) $V(x) > 0$ since $\bar{M} > 0$

ii) $\overset{\circ}{V}(x) = x^T(A\bar{M} + \bar{M}A^T - 2BB^T)x$ (A.15)

after Equations A.10 and A.11

$$\bar{M}A^T + A\bar{M} = \int_o^{T_1} \frac{d}{d\tau}(e^{-A\tau}BB^T e^{-A^T\tau})d\tau =$$

$$= -e^{AT_1}BB^T e^{-A^T\tau_1} + BB^T \tag{A.16}$$

and

$$\overset{\circ}{V}(x) = -x^T\left[e^{AT_1}BB^T e^{-A^T T_1} + BB^T\right]x \leq 0 \tag{A.17}$$

So Equation A.14 describes a Lyapunov function for the system in Equation A.13. For asymptotic stability, it is sufficient to have $\overset{\circ}{V}(x(t)) \not\equiv 0$ for $x(t_o) \not\equiv 0$, Equation A.13 gives

$$x(t) = e^{\mathbb{A}^T(t-t_o)} x(t_o) \tag{A.18}$$

and

$$\mathring{V} = -x^T(t_o) e^{\mathbb{A}(t-t_o)} (e^{AT_1} BB^T e^{-A^T T_1} + BB^T) e^{\mathbb{A}^T(t-t_o)} x(t_o) \tag{A.19}$$

By a similar reasoning to the one we have given for positive definiteness of $\bar{M}$, to have $\mathring{V} \equiv 0$ would contradict the hypothesis of contrallability, so $\mathring{V} \not\equiv 0$ and the system is asymptotically stable.

APPENDIX B

SPECTRAL FACTORIZATION OF Φ_{η}

We briefly indicate the main steps leaving aside the intermediate calculations. One has to first establish

$$
\begin{aligned}
H(z)\Sigma H^{T}(z^{-1}) = {} & C(zI-A)^{-1}APC^{T}(CPC^{T}\,{}^{T}Q_{v})^{-1}CPA^{T}(z^{-1}I-A^{T})^{-1}C^{T} \\
& + C(zI-A)^{-1}APC^{T} + CPA^{T}(zI-A^{T})^{-1}C^{T} \\
& + CPC^{T} + Q_{v} \qquad (B.1)
\end{aligned}
$$

Matrix Riccati equation can be used to show

$$
\begin{aligned}
H(z)\Sigma H^{T}(z^{-1}) = {} & C(zI-A)^{-1}\{APA^{T} - P + Q_{W}\}(z^{-1}I-A^{T})^{-1}C^{T} \\
& + C(zI-A)^{-1}APC^{T} + CPA^{T}(zI-A^{T})^{-1}C^{T} \\
& + CPC^{T} + Q_{v} \qquad (B.2)
\end{aligned}
$$

The first three terms are collected to give

$$H(z)\Sigma^{-1}H^T(z^{-1}) = C(zI-A)^{-1}\{-P + Q_w + z^{-1}AP + zPA^T - APA^T\}$$
$$x(z^{-1}I-A^T)^{-1}C^T + CPC^T + Q_v \qquad (B.3)$$

Factorizing the first term gives

$$H(z)\Sigma^{-1}H^T(z^{-1}) = C(zI-A)^{-1}\{Q_w - (zI-A)P(z^{-1}I-A^T)\}$$
$$\cdot (z^{-1}I-A^T)^{-1}C^T + CPC^T + Q_v$$
$$= \Phi_\eta(z) \qquad (B.4)$$

APPENDIX C

PROOF OF CLOSED-LOOP IDENTIFIABILITY RESULT

Let us neglect the initial value effects, so

$$\hat{y}(k|k-1;M) = (I - H_M^{-1})y(k) + H_M^{-1}G_M u(k) \quad (C.1)$$

Denoting

$$G = G_S(q^{-1}), \quad \hat{G} = \hat{G}_M|q^{-1}), \text{ etc} \quad (C.2)$$

$$L_i = (I - H^{-1}) + H^{-1}GF_i$$

$$\hat{L}_i = (I - \hat{H}^{-1}) + \hat{H}^{-1}\hat{G}F_i \quad (C.3)$$

Then since

$$\hat{y}(k|k-1,S) = [I - H^{-1}]y(k) + H^{-1}Gu(k)$$

$$= [I - H^{-1} + H^{-1}GF_i]y(k) = L_i y(k) \quad (C.4)$$

the set when using direct identification with a prediction error method is

$$D_I(S,M) = \{\Theta | \sum_{i=1}^{h} \gamma_i E|(L_i - \hat{L}_i y(k)|^2 = 0\}$$

$$= \{\Theta | L_i = \hat{L}_i, \ i=1, \ldots, h\} \qquad (C.5)$$

Since y is filtered white noise, collecting Equations C.3 and C.5,

$$[\hat{H}^{-1} - H^{-1}; H^{-1}G - \hat{H}^{-1}\hat{G}]R_h = [0 \ldots 0] \qquad (C.6)$$

where R_h is

$$R_h = \begin{bmatrix} I & \ldots & I \\ F_1 & \ldots & F_h \end{bmatrix} \qquad (C.7)$$

rank R_h = dim y + dim u.

So

$$\hat{H}^{-1} - H^{-1} = 0, \ H^{-1}G - \hat{H}^{-1}\hat{G} = 0 \qquad (C.8)$$

i.e.,

$D_I(S,M) = D_T(S,M)$ and system identifiability is satisfied. Notice that it is only a condition on regulators which implies that, it is a sufficient condition for strong system identifiability. A necessary condition is

$$h \dim y \geq \dim y + \dim u \rightarrow h \geq \frac{\dim y + \dim u}{\dim y} \qquad (C.9)$$

if

$$h_o \geq 1 + \frac{\dim u}{\dim y}, \ h \geq h_o$$

so we can choose h_o to have strong system identifiability. If dim u = dim y, you choose two regulators (proportional ones will suffice) such that

$$\det[F_1(z) - F_2(z)] \not\equiv 0. \qquad (C.10)$$

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