A STUDY ON NUMERICAL ANALYSIS OF LATERALLY LOADED PILES

THESIS

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BOĞAZİÇİ UNIVERSITY
Civil Engineering Department

1982

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by
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(B.S. in CE, I.T.U., 1980)

Thesis Submitted to the Faculty of Engineering of
Boğaziçi University
in Partial Fulfillment of
the Requirements for the Degree of

Master of Science
in
Civil Engineering

Boğaziçi University
1982

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176418

ACKNOWLEDGEMENTS

I would like to express may deep appreciation to Doç.Dr.H.Turan Durgunoğlu for his encouraging guidance, advice and invaluable supervision throughout the course of my study.

I also wish to thank to Prof.Dr.Vahit Kumbasar, Doç.Dr. Öktem Vardar and to Dr.Hüsamettin Alper for their invaluable suggestions.

I would also like to express my sincere gratitude to Doç.Dr.Ahmet Sağlamer from Technical University of Istanbul for his kind help and suggestions throughout the study.

Many thanks are due to Mr.Ahmet Çivi Vice Director of Computer Center for his helpful suggestions in the development of the computer program.

A STUDY ON NUMERICAL ANALYSIS OF LATERALLY LOADED PILES

- A B S T R A C T -

The problem of laterally loaded piles is encountered frequently, particularly in the design of waterfront and offshore structures. A procedure for the numerical solution of lateral deflection of piles under various soil conditions are developed. The procedure treats the nonlinear elastoplastic behaviour of the soil, and two particular boundary conditions namely fixed and free headed piles. Criteria are presented for developing soil reaction-pile deflection curves (p-y curves) along the pile. A general computer program is developed which would give numerical solutions of lateral deflection, slope of elastic curve, bending moment and shear force along the pile. A number of computations is made for a hollow steel pile installed into a clay and sand subsoil. Effects of nonlinear soil behaviour are studied by changing the magnitude of lateral load. In addition effects of various soil parameters such as, strain at failure-ε_{50} and lateral soil reaction coefficient-m are numerically evaluated and the results are critically discussed.

YATAY YÜKLÜ KAZIKLARIN ANALİZİ
ÜZERİNE BİR ÇALIŞMA

- Ö Z E T -

Yatay yüklü kazık problemi, özellikle liman yapılarının ve açıkdeniz yapılarının projelendirilmesinde, sık karşılaşılan bir problemdir. Bu çalışmada, değişik zemin şartlarında, kazıkların yatay yerdeğiştirmesinin sayısal çözümü için bir yöntem geliştirilmiştir. Bu yöntem zeminin lineer olmayan elasto-plastik davranışını, serbest ve sabit başlı kazıklar için olmak üzere iki sınır şartını inceler. Kazık boyunca zemin reaksiyonu-kazık yerdeğiştirmesi eğrilerinin (p-y eğrileri) elde edilebilmesi için gerekli kriterler verilmiştir. Kazık boyunca yatay yerdeğiştirme, elastik eğrinin eğimi, eğilme momenti ve kayma kuvvetinin sayısal çözümlerini veren genel bir bilgisayar programı geliştirilmiştir. Kil ve kum zemindeki bir çelik boru kazık için çözümler yapılmıştır. Yatay yükün değeri değiştirilerek, lineer olmayan zemin davranışının etkileri incelenmiştir. Ek olarak kırılma anındaki şekil değiştirme-ε_{50} ve yatay zemin reaksiyonu katsayısı-m gibi değişik zemin parametrelerinin etkileri sayısal olarak değerlendirilmiş ve sonuçlar tartışılmıştır.

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A,B	:	Empirical adjustment factors
A_m, B_m	:	Computational parameters in Gleser Algorithm
b	:	Pile diameter (L)
c	:	Undrained shear strength of undisturbed clay soil sample ($F.L^{-2}$)
D_r	:	Relative density
E_p	:	Elastic modulus of pile ($F.L^{-2}$)
E_s	:	Soil modulus ($F.L^{-2}$)
EI	:	Flexural rigidity (F.L)
E_{50}	:	Secant deformation modulus ($F.L^{-2}$)
E_t	:	Tangent deformation mudulus ($F.L^{-2}$)
H	:	The distance of the application point of lateral load P above the ground surface (L)
I_p	:	Moment of inertia of pile cross section (L^4)
$I_{\rho H}$, $I_{\rho M}$, $I_{\theta H}$, $I_{\rho F}$	:	Influence factors
J	:	Dimensionless constant determined from the type of clay
k	:	Initial soil modulus ($F.L^{-3}$)
k_s, k_c	:	Constants for static and cyclic loading for stiff clays ($F.L^{-3}$)
k_1, k_2, k_h	:	Constants for soil modulus variation
K_A	:	Coefficient of active earth pressure

K_0	: Coefficient of earth pressure at rest
K_p	: Coefficient of passive earth pressure
K_R	: Pile flexibility factor
k_x	: Constant $(F.L^{-3})$
L	: Pile length (L)
m	: Coefficient of lateral soil reaction $(F.L^{-3})$
M	: Moment (F.L)
M_t	: Moment at the depth (x=0) (F.L)
N	: Number of cycles
N_p	: Dimensionless coefficient of ultimate bearing capacity
p	: Soil reaction per unit length of pile $(F.L^{-1})$
P	: Lateral load (F)
P_u	: Ultimate bearing capacity $(F.L^{-1})$
P_u^*, P_m	: Reduced ultimate bearing capacity $(F.L^{-1})$
P_v	: Vertical load (F)
P_w	: The ultimate lateral soil resistance by wedge type failure $(F.L^{-1})$
R_L, R_p, R_s	: Resistance forces on a pile $(F.L^{-2})$
S	: Slope of elastic curve
t	: Number of equal units into which L is divided
V	: Shear force $(F.L^{-2})$
x	: Depth below the ground surface (L)
x_R	: Depth below ground surface to bottom of reduced strength zone (L)

x_T : Critical depth defining the shallow and deep zones (L)

y : Lateral deflection (L)

$y_{50}, y_p, y_u, y_m, y_k$: Some special lateral deflections (L)

$\alpha_{\rho H}, \alpha_{\rho m}, \alpha_{\theta H}, \alpha_{\rho F}$: Interaction Factors

γ' : Effective unit weight of soil ($F.L^{-3}$)

ε_{50} : Strain at failure

ϕ : Angle of internal friction in degrees

$\bar{\rho}$: Mudline deflection of a pile under unit load (L)

ρ_k : Mudline deflection of a pile in a group with a total of m piles (L)

I. INTRODUCTION

New sources are searched in the sea, due to increasing demand on hydrocarbons. The investigation of new hydrocarbon fields in the sea has given a new dimension to offshore engineering. Geotechnical problems of offshore structures become very important because of deep waters and cumbersome environmental conditions. These type of problems had never been solved before 1940's. Every phase of drilling and producing offshore petroleum is affected by soil conditions at the site. Fixed platforms and pipeline construction require accurate soil investigation in order to design footings and piles to be driven. The type of problems which marine geotechnical engineering deals with are shown in Fig. 1.1.

The first petroleum production platform was erected in Maracaibo Lake in Venezuela in 1940's. The depth of the Lake was 30 meters. Offshore oil exploration and production began after the end of World War II. The first steel structure in open water was constructed in 1947, off the coast of Louisiana in 6 meters of water depth.

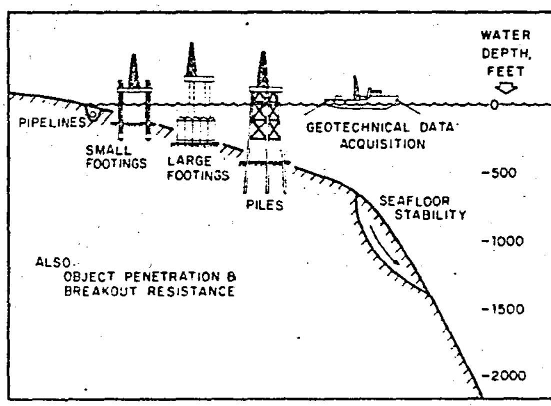


Fig. 1.1.— Marine Geotechnical Engineering Problems (after Focht, 1977)

Almost simultaneously, another steel structure was installed in 15 meters. The pipe pile foundations for these and many other early structures were designed without the benefit of site investigations, and were installed in accordance with the practice at that time drive the piles to practical refusal with the biggest possible hammer. The pile diameters were generally in the range of 0.3 m to 0.6 m and the maximum loads were usually 890-1780 kN, essentially the same as used for land construction. During 1940's little or no attention was given to theoretical geotechnical considerations of bearing capacity, sliding or breakout resistance.

During the 1950's, when the maximum water depth for platform construction increased by about four times to slightly over about sixty meters, offshore activity in Gulf of Mexico blossomed along the Texas and Louisiana coasts. There were significant advances in the applications of geotechnical engineering to the marine environment. As a result of this development a major advancement in geotechnical engineering in the decade of the 1950's was the design and analysis of laterally loaded piles. Early in 1954, Shell Oil Company initiated design efforts for a structure in 23 meters of water in South Pass Block 42 (Focht, 1977). The combination of extremely weak soils at the site and increased concern about hurricane storm loads in deeper water provided a challenge to the geotechnical engineer to analyze and design piles to support cyclic lateral loads of 418 kN per pile.

Shell Oil Company, along with four other oil companies, initiated a research program on laterally-loaded piles that has formed the primary basis for current design procedures for individual piles subject to cyclic lateral loads. The tests which extended over a period of 10 years, were performed on 0.15 m. diameter instrumented piles by Reese, Matlock and Cox at the University of Texas in Austin. The design procedures utilizing the "p/y concepts" was published for piles in soft clay (Matlock, 1970), in sand (Reese, 1974) and in stiff clay (Reese, Cox, Koop, 1975).

Fig. 1.2.- Areas of Offshore Activity (after Mc Kelvey, 1974)

Nineteen percent of World crude oil production is done from offshore petroleum platforms and this percentage will probably increase to 30 percent in a decade.

According to a research done in 1974 (McKelvey, 1974), 34 countries produce petroleum from offshore platforms, 45 countries search for petroleum by drilling platforms and 80 countries use geophysical and other investigation methods in order to find offshore petroleum fields. Areas of offshore activity are shown in Fig.1.2.

In 1976, platforms can be installed at water depths more than 200 meters. Now there are more than three thousand fixed platforms installed in water depths ranging to 150 m . (McKelvey, 1974).

In the same years the United States of America installed new platforms in Cooklet region of Alaska, and along the coastline of California. After exploration of Ekofisk petroleum field in the North Sea in 1960, the problem of proper design of offshore platforms has gained new dimensions. The production area is 320 km to the nearest harbour, with a typical water depth of 70 m and the storm design wave of 20, 30 m.

In general offshore production and exploration platforms could be gathered in three main groups as summarized below:

1- Jacket or template structures
2- Concrete (gravity) structures
3- Hybrid (steel, concrete) structures.

Typical description of these platform types is given in Fig. 1.3.

The jacket or template type of platform is supported by steel tubular piles driven into the sea bottom. These piles must be designed to carry the load imposed by the super-

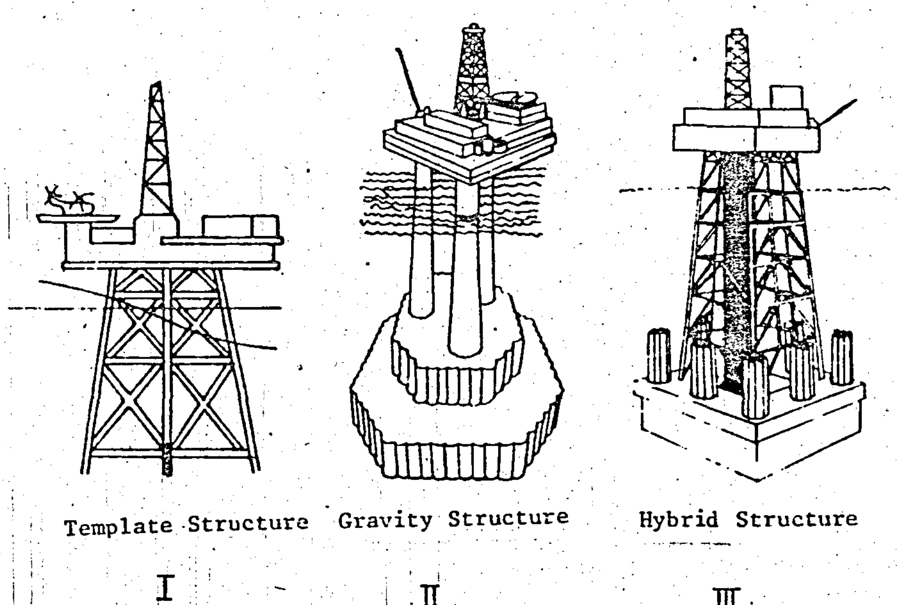


Fig. 1.3.- Offshore Platform Types (after Toğrol, 1981)

structure and environmental forces such as wind forces, wave forces, earthquake forces. Platform jackets are built in shipyards or large plants adjacent to navigable waters.

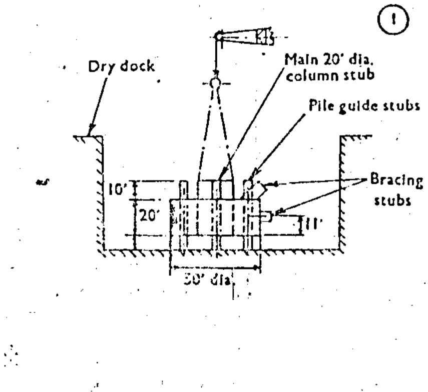
Early steel jackets were moved by barges and positioned with the assistance of derrick barges. Some steel jackets currently use their own structural members for floatation and are towed to their location. The installation procedure is shown (after Hancock, 1975) in Fig. 1.4.

The gravity type structure is more advantagous in the North Sea, because the bulk of the structure affords protection from storms and hostile environment common to the North Sea. Hybrid type of platforms are made of concrete base and a steel jacket. It's weight is only about 25 percent of the gravity type structure and is preferrable at the sites where there is lack of subsoil resistance.

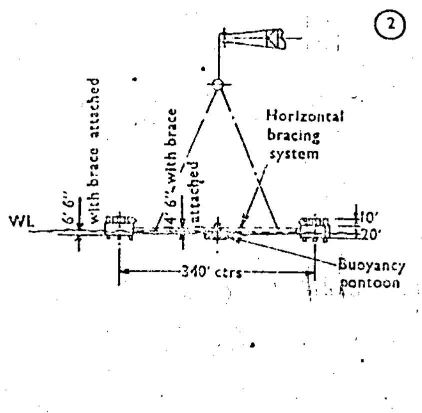
In the late 1940's petroleum production structures were being built for 15 meters of water depth at costs up to 0.4 million US dollars. In late 1970's however, maximum water depth approached to about 300 meters and the costs reached to about 200 million US dollars.

Offshore petroleum production is also very important for our country. Although all eastern neighbouring countries produce petroleum for export, our country could only produce 20 percent of her needs. In 1971, Greece found important hydrocarbon fields in the north of Aegaen Sea. Greece has installed six drilling and production platforms at water depths of 40 meters. Especially in Prinou region from all of the four production platforms crude oil is produced and this shows the petroleum potential of the Aegaen Sea (Petroleum Times, 1976).

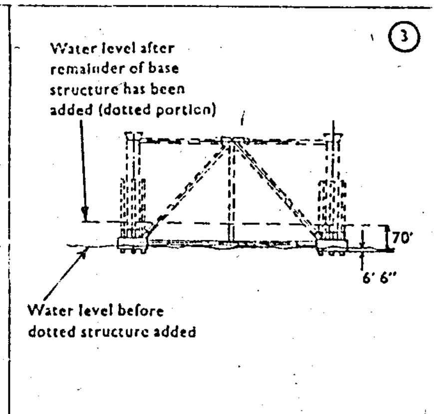
The offshore structures are mostly erected on piled foundations. A typical template structure and its piles are shown in Fig. 1.5. The piled foundations are under great



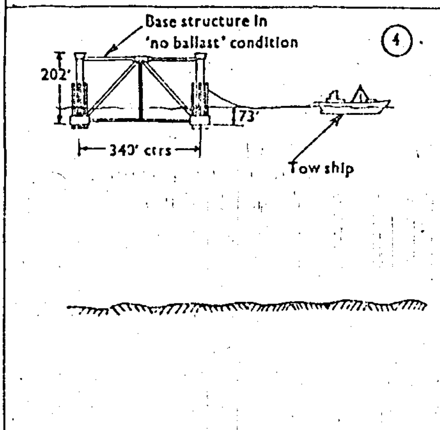
Each of three pontoons with a section of column and pile guides, and bracing stubs, constructed in dry dock, this pontoon structure will draw approx. 6' 3" of water



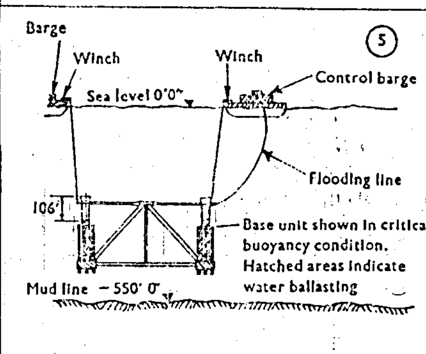
Pontoon structures floated out dry dock, to a site where they will be linked together with lower horizontal bracing



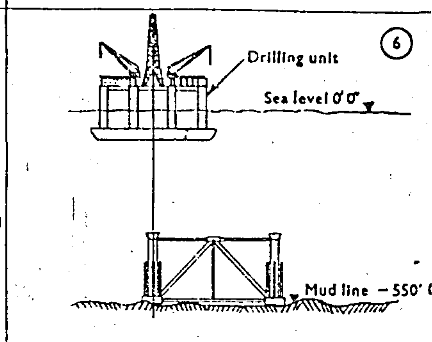
Remainder of base structure is added on and unit is now completed and made ready for tow to site



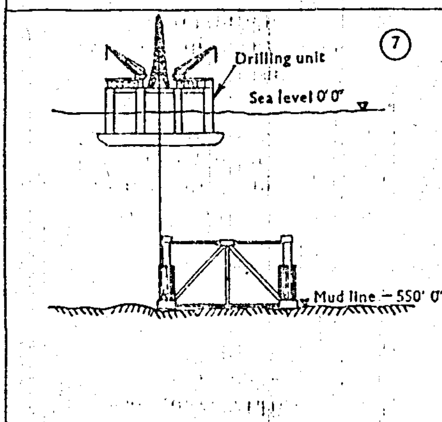
Completed base section towed to site



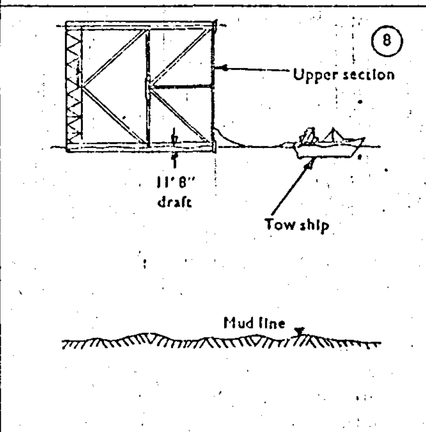
Base section lowered by means of adding water ballast, with sensors placed in each leg to maintain level. As tops of column dip below water level, constant tension winches on barges will control the descent;upon reaching mud,pile guide extension (predetermined length) will penetrate mud



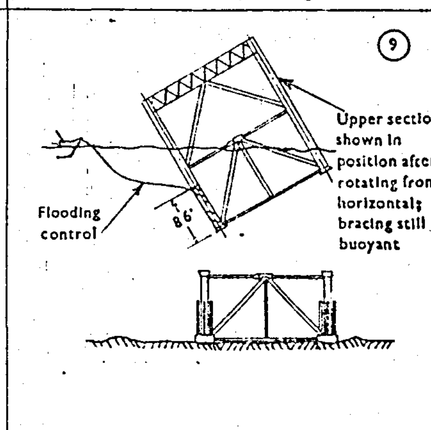
All members in bases are now flooded, but main ballasting in columns is regulated to achieve a level condition in relation to the sea floor. Pile guide extensions will now have penetrated fully, assisted if necessary by water jettin a semi-submersible drilling unit then drills each centre pil pile. When base is level, these piles are grouted in



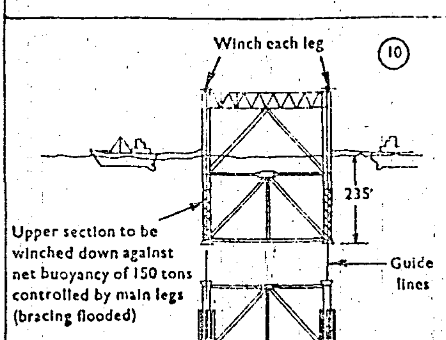
Drilling unit now drills and grouts remaining nine (skirt) piles in each leg



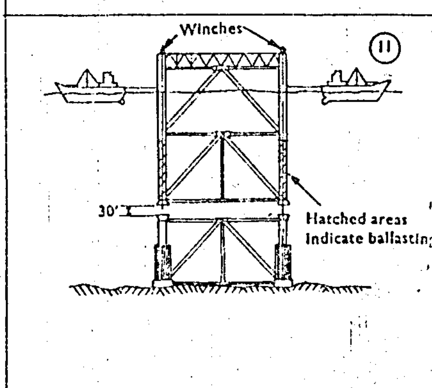
Upper section now towed to site



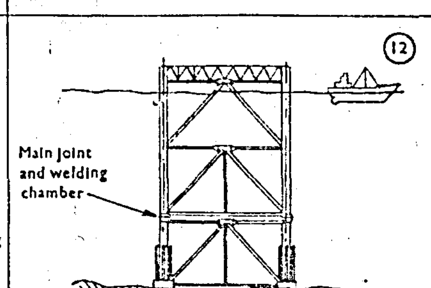
Upper section is upended by means of differential ballasti



Upending now completed by controlled ballasting—upper section has net buoyancy of 150 tons. Diver sent down to attach guide lines to base unit. These are then connected to winches on top of upper section. Lateral position of upper section is maintained from the surface



Upper section is then pulled down by winders against 150 ton buoyancy. Winches are connected to sensing unit, in order to maintain a level condition for the last 30' of descent



Upper section stabbed into base section by winches, and water ballast added to upper section until a pre-determined pressure has been attained between the two surfaces to be welded. Pumps then evacuate each weldin chamber and inspection for water-tightness made and joi pre-heated. Welders then attach temporary lugs at joint and effect main weld. Remaining members then flooded and winches and guide lines removed. Tower is now rea to receive deck modules axial and lateral loads. Especially because of the cyclic behavior of the wave action, the lateral loads have prime importance.

In this thesis a brief explanation of the problem together with soil reaction-pile deflection method is given. The pile behaviour under lateral load is numerically studied considering various subsoil conditions. For this purpose a computer program is developed.

In Chapter 2, the loading conditions, the methods of analysis of piles under lateral loads are given, and pile deflection-soil reaction method, which is widely used is explained in detail. The theory of solution of the differential equation of laterally loaded pile and governing finite difference equations (Gleser, 1953) are given.

In Chapter 3, soil reaction-deflection curves, which are used in the solution of differential equation, are given for soft clays (Matlock,1970), stiff clays (Reese, Cox and Koop, 1975) and sands (Parker and Reese, 1971), (Reese, Cox and Koop, 1974).

In Chapter 4, two methods analyzing the laterally loaded pile groups are summarized. These methods are namely Poulos Method by Poulos (1971) and Combined Elastic/p-y Method by Focht and Koch (1973).

In Chapter 5, the explanation of the developed computer program for the solution of pile deflections and moments according to pile deflection-soil reaction method is given. Beside the main program, the subroutines developing the soil reaction-pile deflection curves for soft clays and sands are documented.

In Chapter 6, problems of a steel pipe pile installed in a soft clay and in a dense sand, are solved using developed computer program. A total of eighty runs are evaluated and

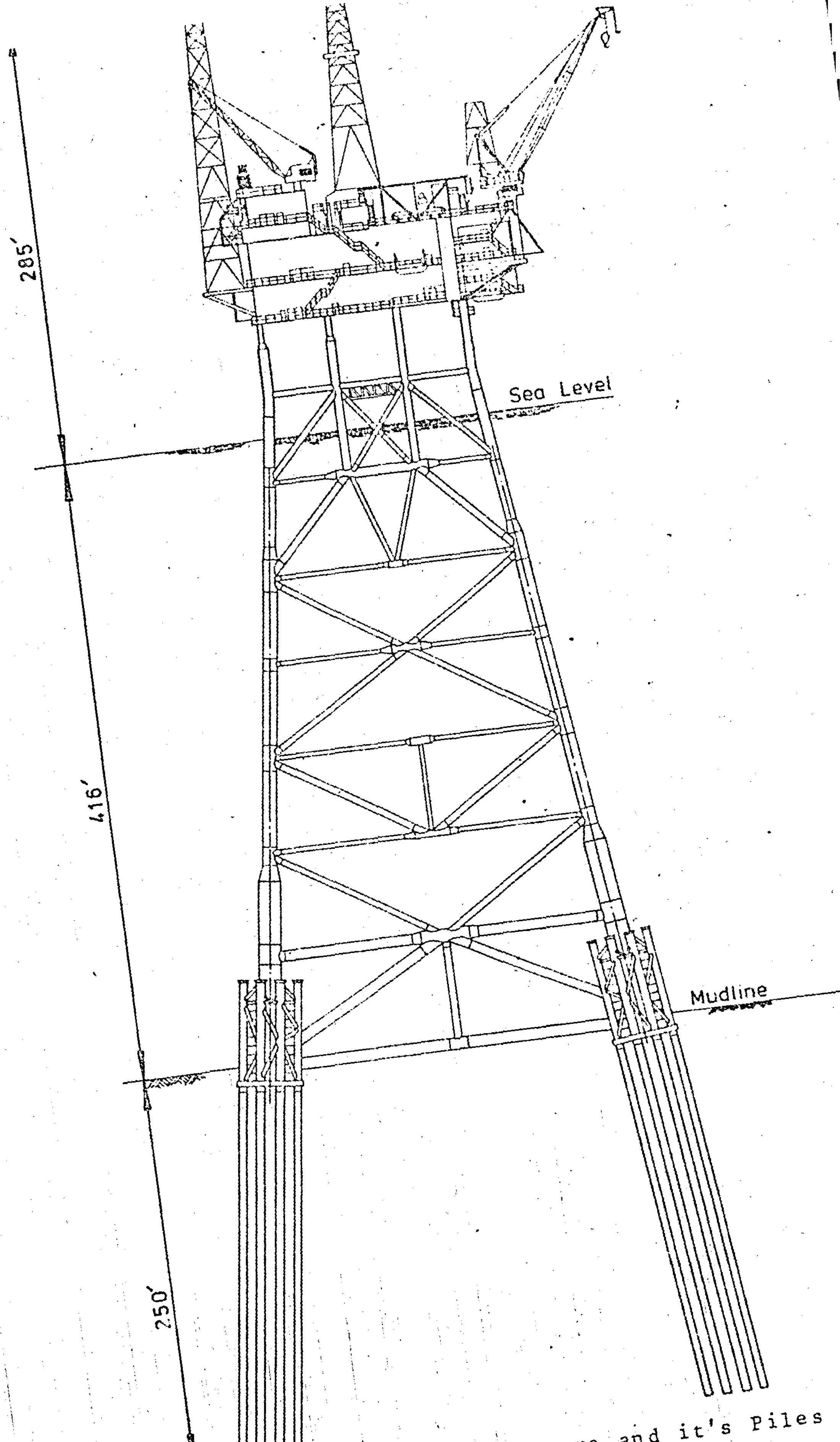


Fig. 1.5.— A Template Structure and it's Piles (after Mc Laughlin, 1975)

the results are presented in graphical forms.

Finally, summary of the thesis together with main conclusions are presented in Chapter 7. The listing of the program developed together with its user's manual are given in the Appendix.

2. ANALYSIS OF LATERALLY LOADED PILES

2.1. INTRODUCTION

The design of piles due to axial loads is done by static and dynamic pile formulae which are currently widely used. In this section the design of piles due to lateral loads will be presented.

The laterally loaded pile occurs as a single structural element, but more frequently it is a part of a complex structure, such as a part of an offshore drilling or production platform as shown in Fig. 2.1.

Piled foundations are under lateral loads due to earthquake, wave, wind forces and lateral soil pressure. Among these earthquake and wave forces are cyclic forces.

The water depth at a site where the platform is installed may be as high as 150 m - 200 m. The piles are driven to the depths where sufficient axial and lateral bearing capacities are reached. The forces acting on a laterally loaded pile are

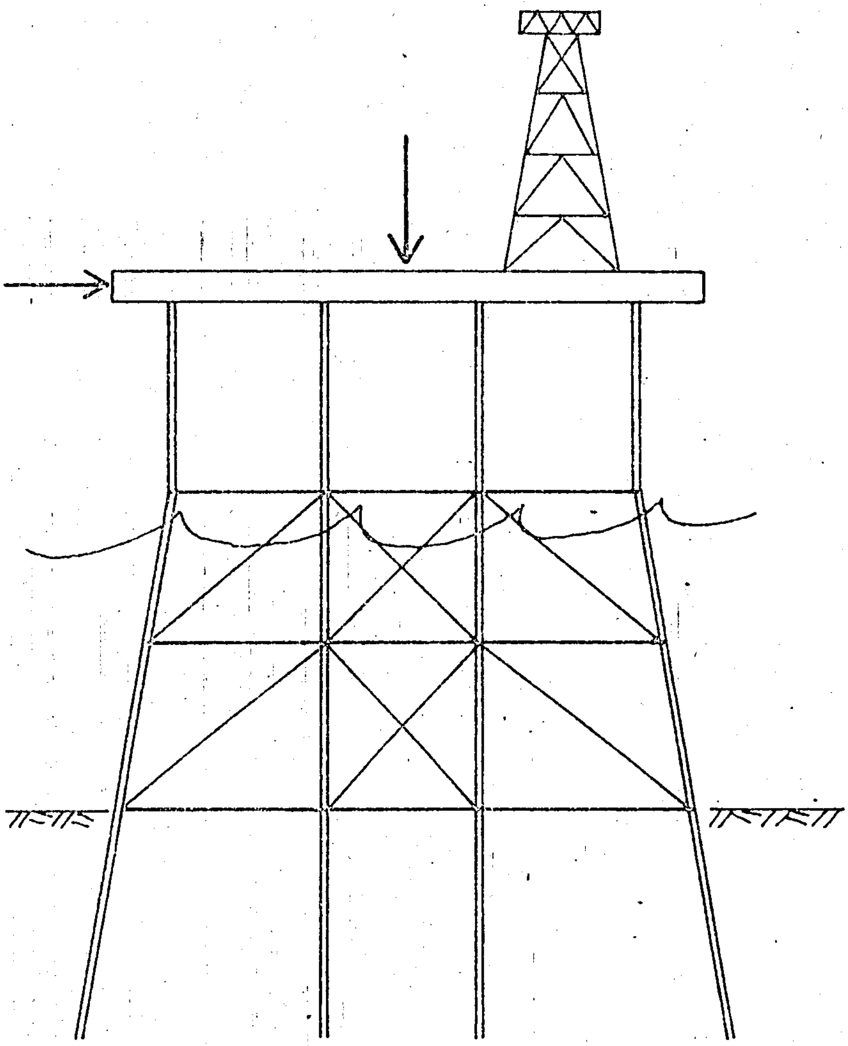
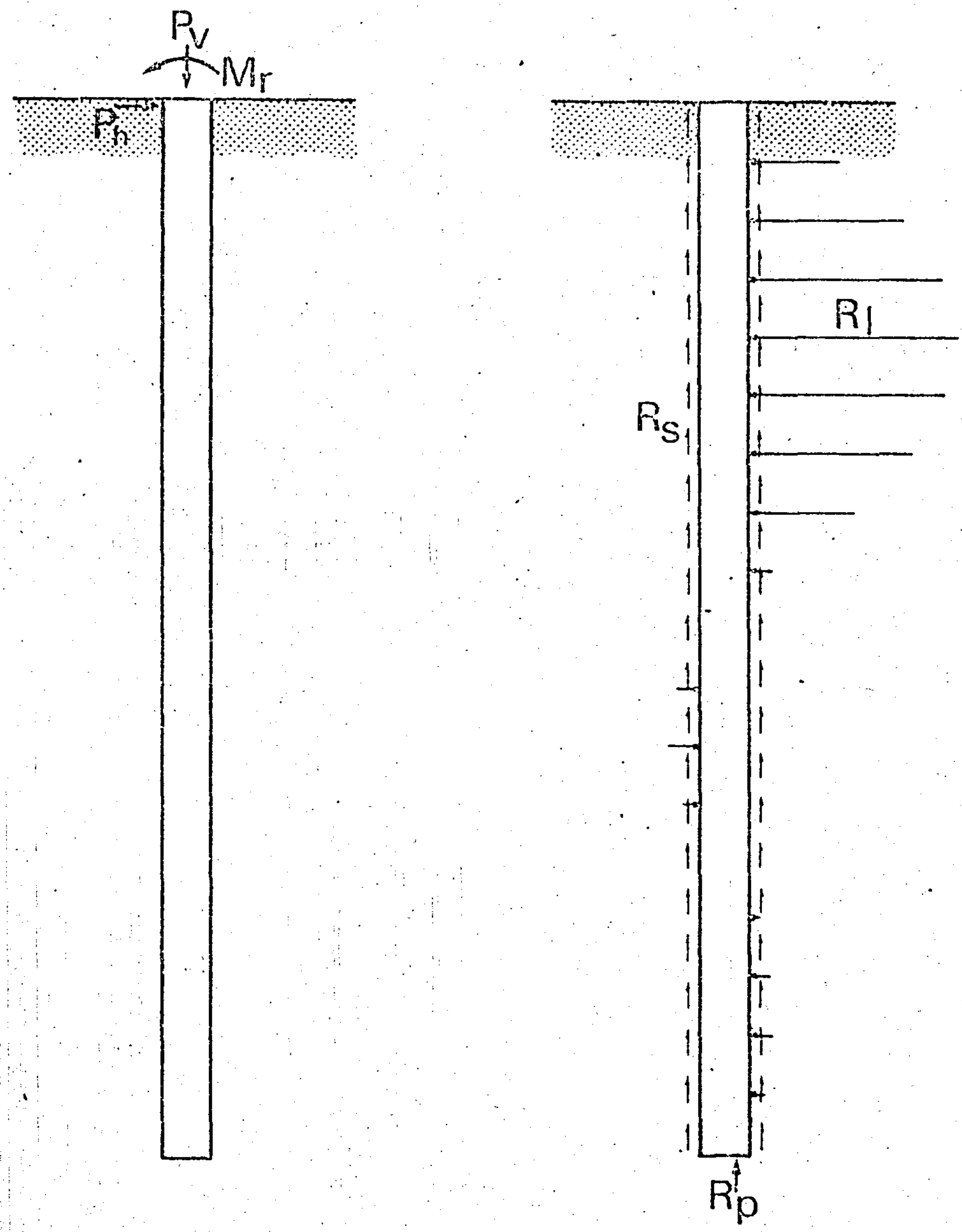


Fig. 2.1.– Forces Acting on an Offshore Drilling Platform (after Sağlamer, 1977)



P_v = 15000-30000 kN (compression)

P_v = 10000 kN (tension)

P_h = 1000-2500 kN

M_r = 5000-10000 kN-meters

Fig. 2.2.- The Forces and the Soil Reaction Acting on an Offshore Structure's Pile (after Sağlamer, 1977)

shown in Fig.2.2.

In the design of offshore structures due to lateral loading, three types of loads are considered.

1- Short-term static loading
2- Cyclic loading where equilibrium is reached
3- Static or cyclic loading after an application of cyclic load.

The short term static loading is the collision of a ship to the offshore structure. During this instant although there is some dynamic effect between the ship and the structure, with the assumption of sufficient touching time, visco-elastic effects on the soil are not considered. The problem is solved by using static stress-strain characteristics of the soil.

In the geotechnical design of offshore structures the most important force is cyclic loading due to wave forces which takes its maximum value during a storm. When the phase angle of the wave is $3\pi/2$, lateral wave force and the wind force act in the same direction as shown in Fig.2.3. As an example, at 150 m water depth design wave height is H=30 m, wave period is T=15-17 sec and wave length is L=350-430 m in the North Sea.

The reloading after cyclic loading, is the case of, loading the pile having already a plastic deformation by the effect of former cyclic loading.

2.2. METHOD OF ANALYSIS FOR LATERAL LOADS

In the design of laterally loaded piles there are basically three approaches:

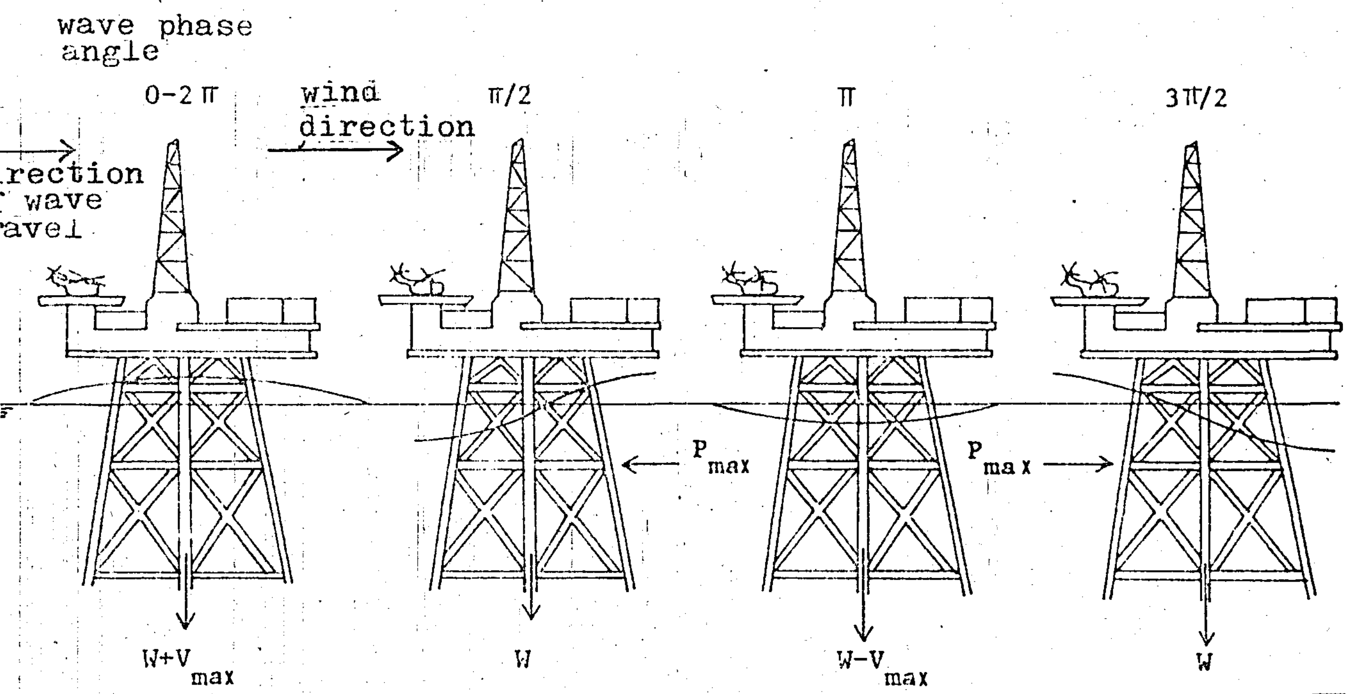


Fig. 2.3.- Principal Forces Acting on a Template Structure (after Sağlamer, 1977)

A. Methods of limit analysis
B. Elastic method
C. Soil reaction-pile deflection method

A. Methods of Limit Analysis

Limit analysis methods assume that the limiting or maximum soil resistance is acting against the pile when the ultimate load is placed against the pile. The limit analysis method assumes (1) a soil of constant strength with depth, and (2) that the pile deflects sufficiently to develop the full soil resistance all along the length considered. The second assumption is obviously untrue where there are small deflections. The force diagram for limit analysis is shown in Fig.2.4.

B. Elastic Methods

If the reaction-pressure relations of a soil are known, it would seem preferrable to use an elastic method.

Figure 2.5. shows graphically some of the possible complexities in the behaviour of the soil. Forms of the springs indicate that the stiffness of the soil will vary with depth and deflection. The friction blocks indicate that there is some limit to the soil reaction which can be developed. The spaces near the top between the springs and the pile indicate that there is some possibility of a pile under going some deflection with no resistance being developed in the soil. The dashpots suggest that the resistance of the soil will vary with time after the load has been applied and with rate of loading. Also it is shown in Fig.2.5(d) the reaction deflection relationships of a given spring or of a given part of the soil will vary with cyclic loading due to molding away or softening of the soil (Matlock, 1956).

With elastic methods, it is commonly assumed that a soil behaves in a series of separate elements. It is believed that the assumption could be applied to the problem of the laterally loaded pile without a large error being caused.

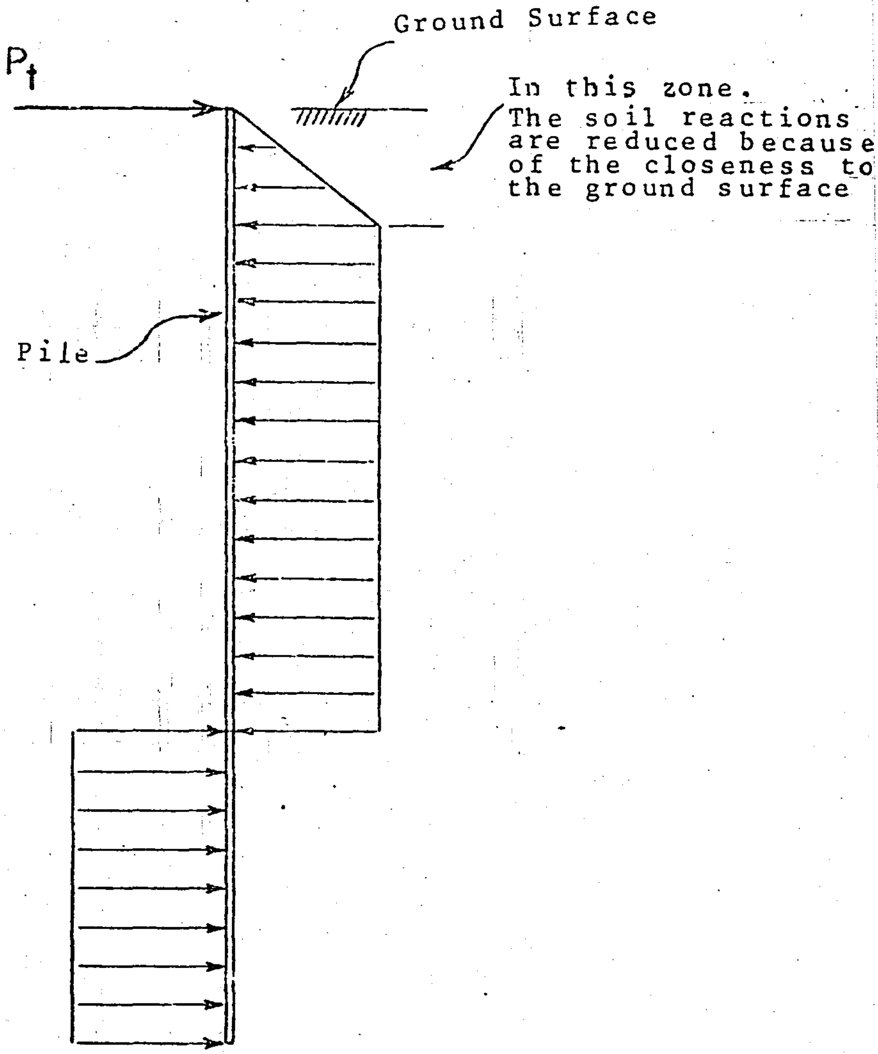


Fig. 2.4.— Example of a Force Diagram for Limit Analysis (after Matlock, 1956)

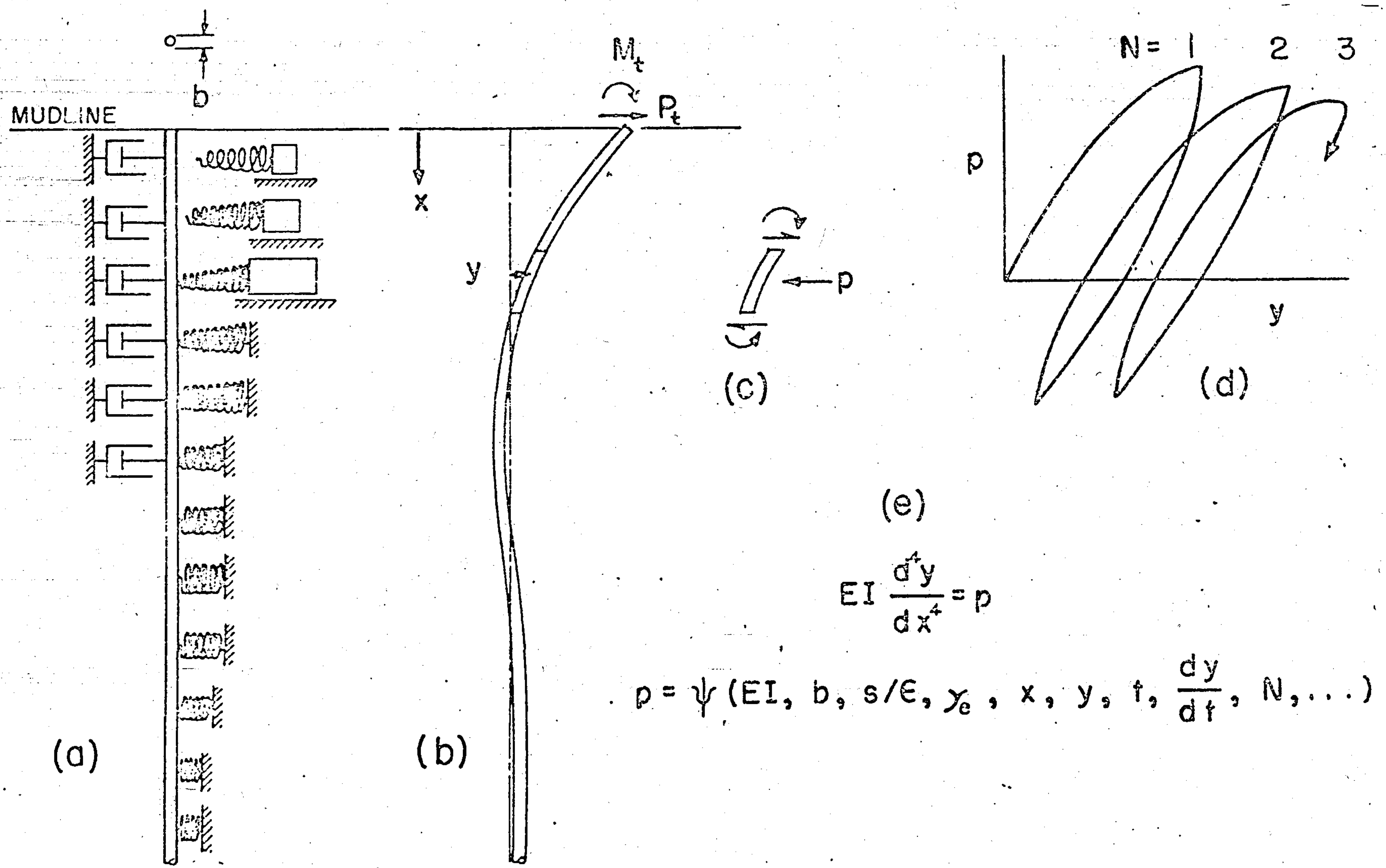


Fig. 2.5.— A Representation of the General Problem in Elastic Method (after Matlock, 1956)

Ordinary beam theory can be used to develop the differential equation for the laterally loaded pile. The solution of the equation depends upon the development of a mathematically convenient function for soil reaction p. The soil reaction may be a function of the pile properties, the stress-strain relationships of the soil, the effective unit weight of the soil, the depth of the overburden at the point considered, the deflection of the pile, the rate of loadings, the number of cycles of loading time consolidation of the soil and perhaps other parameters.

There are two steps to obtain satisfying results in the analysis of laterally loaded pile by elastic method.

1- Complete information describing the behaviour of the soil must be obtained.

2- The differential equation must be solved.

Assuming that the basic soil information can be expressed mathematically and the resulting differential equation can be solved, the results of a typical case might be shown as in Fig.2.6.

If the relationship between soil reaction p and pile deflection y is linear and defined by the soil modulus E_s as shown in Fig. 2.7., the soil resistance per unit length of pile is equal to the modulus multiplied by the deflection.

There are three possible variations of soil modulus with depth. Soil modulus may be constant with depth, or may have a linear variation with depth or may have any variation with depth.

Case A. Constant Modulus with Depth

If the soil modulus is constant with depth as shown by Curve 1 in Fig. 2.7 and if the pile can be considered to be of infinite length, a differential equation could be solved rat-

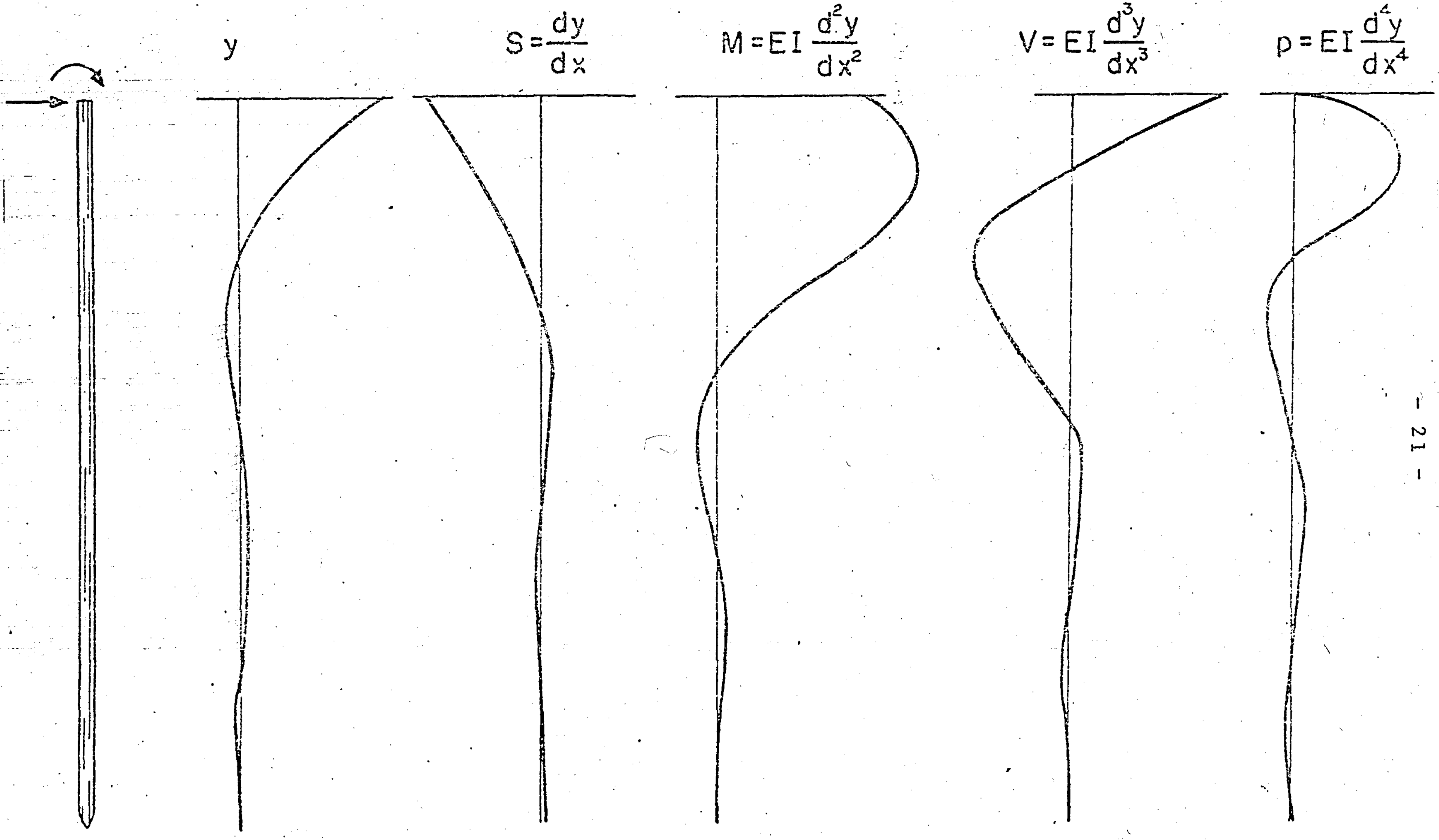


Fig. 2.6.– Form of the Results Obtained from a Complete Solution (after Matlock, 1956)

her easily.

Soil resistance function is given by:

$$p = -E_s \cdot y \tag{2.1}$$

and the typical form of solution is as follows:

$$y = e^{\beta x}(A\cos\beta x + B\sin\beta x) + e^{-\beta x}(C\cos\beta x + D\sin\beta x) \tag{2.2}$$

The coefficients, A,B,C and D can be evaluated by the boundary conditions at the top of the pile.

Case B. Linear Modulus Variation with Depth

If the soil modulus has a linear variation with depth x as shown by Curve 2 in Fig.2.7, the soil resistance p is equal to $(k_1 + k_2x)$ multiplied by the deflection y. Solution of the differential equation can be made by substituting for the deflection a series written in terms of a depth parameter Z.

Soil resistance function is given by:

$$p = -(k_1 + k_2x)y \tag{2.3}$$

and the typical form of the solution is as follows:

$$y = C_1y_1 + C_2y_2 + C_3y_3 + C_4y_4 \tag{2.4}$$

$$\text{e.g. } y_3 = \frac{Z^2}{2!} - \frac{3\alpha}{7!} Z^7 + \frac{3.8\ \alpha^2}{12!} Z^{12} \ \ldots \text{ etc.} \tag{2.5}$$

The undetermined coefficients must be evaluated from the boundary conditions at the top of the pile.

Case C. Random Modulus Variation with Depth

If the soil modulus has a random variation with depth as

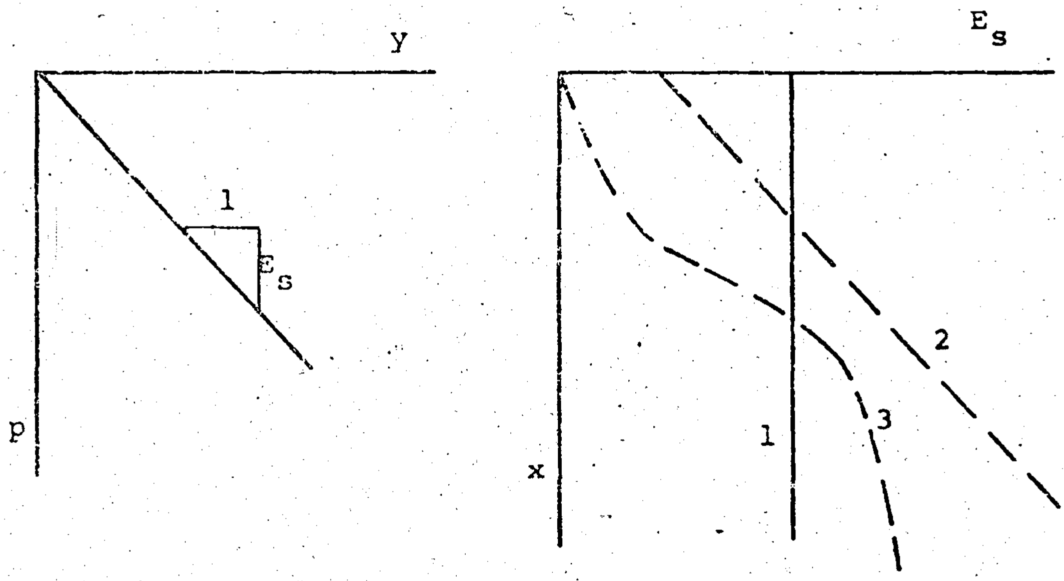


Fig. 2.7.— Methods of Solution where Resistance is Proportional to Deflection (after Matlock, 1956)

shown by curve 3 in Fig.2.7, the soil resistance is equal to some function of x multiplied by the deflection. The solution of the differential equation can be made by writing the differential equation in difference form as shown in Fig.2.7.

Soil resistance function is given

$$P = -\phi(x).y \tag{2.6}$$

and the typical form of solution is as follows:

$$y_m = \frac{EI}{E_{sm}}\left(\frac{t}{L}\right)^4 (y_{m+2} - 4y_{m+1} + 6y_m - 4y_{m-1} + y_{m-2}) \tag{2.7}$$

where,

EI = Flexural rigidity of the pile

E_{sm} = Soil modulus at point m

t = Number of pile divisions in finite difference formulation

L = Pile length

y_m = Pile deflection at point m.

A number of simultaneous algebraic equations are thus obtained. A convenient method for solving these simultaneous equations has been suggested by Gleser (1953), which will be explained in detail later.

If the soil resistance-pile deflection relationship is not a straight line but has some other configuration like in Fig.2.8. then in the solution the elasto-plastic behaviour of the soil must be considered. In this case, either the inelastic behaviour of the soil is simulated by the iterative applications of one of the elastic methods, or the behavior of the laterally loaded pile is examined by considering the physical model of the pile-soil system.

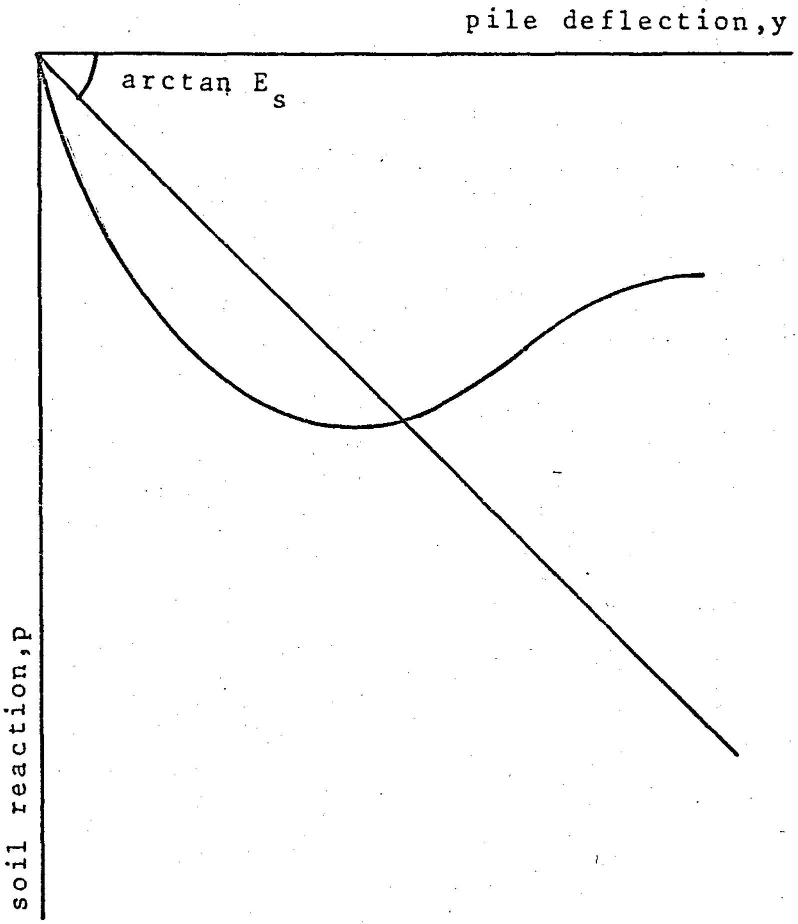


Fig. 2.8.– The Nonlinear Relationship Between Pile Deflection and Soil Reaction (after Sağlamer, 1977)

C. Soil Reaction-Pile Deflection Method

Offshore structures involve unusually large ratios of lateral to vertical loading and flexural stresses may become a major factor in determining pile sizes. Thus bending moments in piles must be reliably predicted.

The differential equation of the laterally loaded pile is obtained by cutting a segment through the pile and applying equilibrium equations as seen in Fig.2.9. (Hetenyi, 1946).

Writing the equilibrium equation for moment in the element shown:

$$M_{m-1} - M_m - V_m\, dx = 0$$

$$dM - V_m\, dx = 0$$

$$\frac{dM}{dx} - V_m = 0$$

Differantiating with respect to x,

$$\frac{d^2M}{dx^2} - \frac{dV_m}{dx} = 0$$

$$\frac{d^2M}{dx^2} = EI\,\frac{d^4y}{dx^4} \quad \text{and} \quad \frac{dV}{dx} = p$$

$$EI\,\frac{d^4y}{dx^4} = p \tag{2.8}$$

where

y = deflection of the pile
x = depth from the ground surface
p = soil reaction.

The value of p depends upon the interaction of pile and soil, and the method most commonly adopted to deal with this

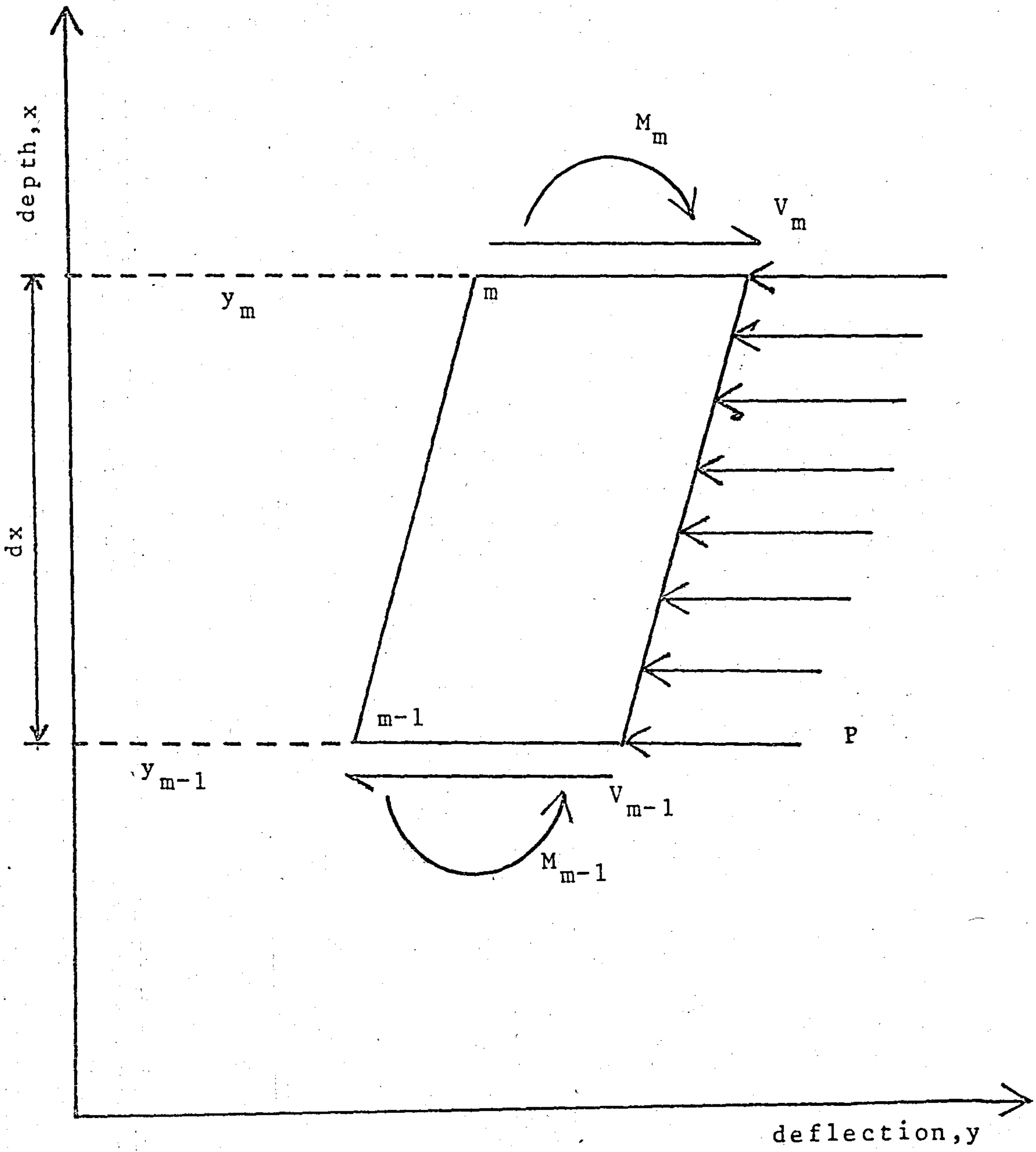


Fig. 2.9.– Segment of a Pile

is the 'p-y' method in which a secant modulus E_s is defined.

$$E_s = -p/y \qquad (2.9)$$

where

E_s depends on;

a) stress-strain properties of the soil and will vary with depth;

b) pile width;

c) pile deflection.

The concept of p-y curves can be explained by Fig.2.10 according to Reese and Cox (1969). In Fig. 2.10 (a) a section is shown through a pile which has been driven into soil, with the soil stratum to be examined indicated at the depth below the ground surface as x_1. The earth pressure distribution around the pile after driving and prior to lateral loading is shown in Fig.2.10 (b). If the pile is deflected a distance y_1, soil pressures would be developed which might have a form as shown in the Fig. 2.10(c). Integration of the soil pressures around the pile would yield an unbalanced force p per unit length of pile. y_1 is the deflection at depth x_1, when the pile is loaded by a particular lateral load and p_1 is the corresponding soil reaction. It is important to point out that for a different lateral load at depth x_1, pile deflection and soil reaction will be different from y_1 and p_1. In general as the depth increases, the soil reaction also increases. A set of p-y curves representing soil behavior along a laterally loaded pile is shown in Fig.2.11. It is assumed that these curves are independent of the consolidation effects of clay subsoils under working loads.

In the problem analyzed, if the soil reaction pile deflection curves are obtained along the pile length, the diffe-

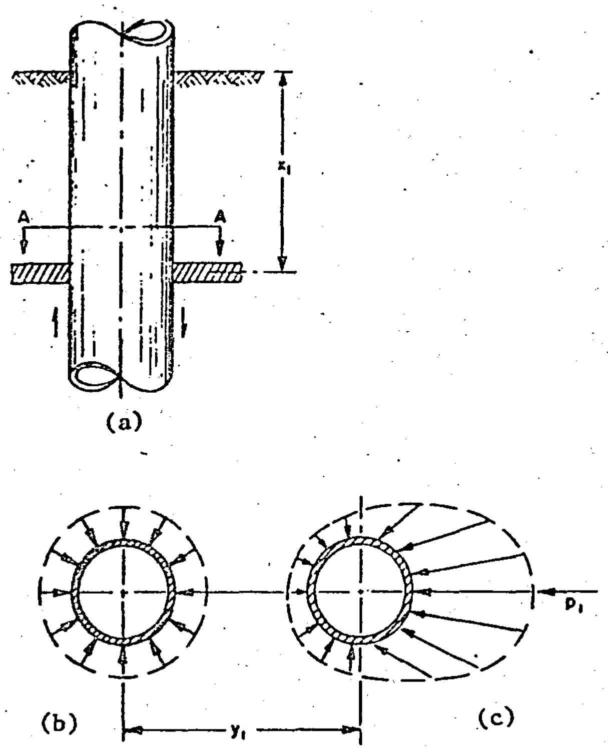


Fig. 2.10.- Graphical Definition of p and y (after Reese and Cox, 1969)

rential equation could be solved considering the nonlinear behavior of the soil.

As p-y curves are nonlinear and function of depth as shown in Fig.2.11 in order to solve the laterally loaded pile problem, several iterations must be made.

The solution of this problem may be done by using finite differences method and the advantage of this method is that it allows the values of E_s and I to vary along the pile.

The basic equation as written previously is:

$$EI\,\frac{d^4y}{dx^4} = -E_s y \tag{2.10}$$

and in difference form it becomes

$$\frac{EI}{(L/t)^2}\left[y_{m+2} - 4y_{m+1} + 6y_m - 4y_{m-1} + y_{m-2}\right] = -E_{s_m}\, y_m \tag{2.11}$$

where

L = length of pile

t = total number of pile sections.

Similar expressions could be found for slope (S_m), moment (M_m), Shear (V_m) and soil reaction (P_m) as:

$$S_m = \frac{1}{2(L/t)}\,(-y_{m+1} + y_{m-1}) \tag{2.12}$$

$$M_m = \frac{EI}{(L/t)^2}\,(y_{m+1} - 2y_m + y_{m-1}) \tag{2.13}$$

$$V_m = \frac{EI}{2(L/t)^3}\,(-y_{m\,2} + 2y_{m+1} - 2y_{m-1} + y_{m-2}) \tag{2.14}$$

A total of t+1 expressions of the form of equation can be written and also four expressions for the boundary conditions at the top and base of the pile.

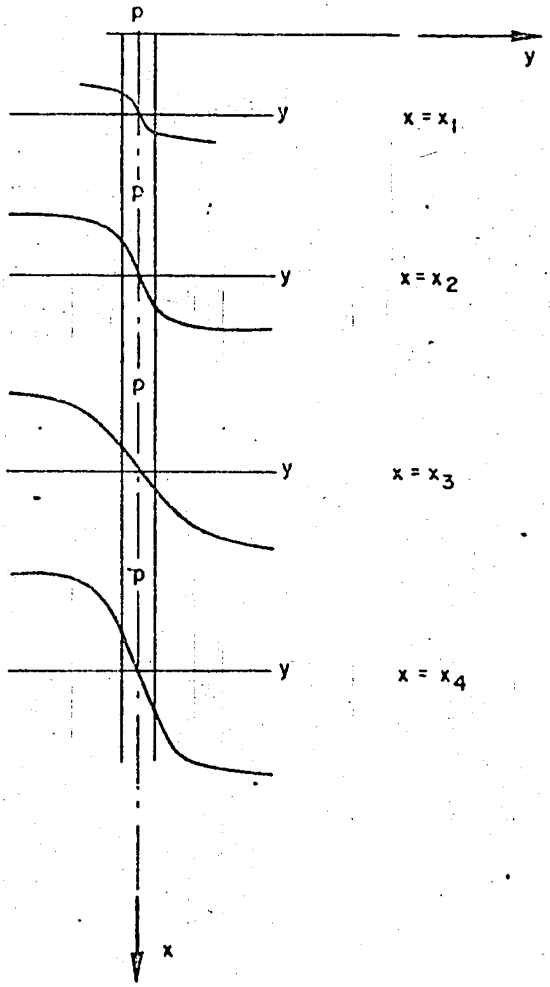


Fig. 2.11.– Set of p-y Curves (after Reese, 1971)

at base of pile; $M_o = V_o = 0$

at top of pile; P_t and M_t are known (Free head)

at top of pile; P_t is known, $S_t = 0$ (Fixed head)

Then the t+5 equations can be solved by matrix methods or by the method of Gleser (1953) based on successive application of boundary conditions.

Generalized Equations for Free Head Pile

The equations (Gleser, 1953) are written in finite difference form for corressponding points shown in Fig.2.12 are documented below:

Point 1 $Y_1 - 2Y_o + Y_{-1} = 0$ (2.15)

Point 2 $-Y_2 + 2Y_1 + Y_{-2} = 0$ (2.16)

Point 3 $Y_2 - 4Y_1 + 6Y_o - 4Y_{-1} + Y_{-2} = -A_oY_o$ (2.17)

Point (m+3) $Y_{m+2} - 4Y_{m+1} + 6Y_m - 4Y_{m-1} + Y_{m-2} = -A_mY_m$

$$Y_{t+2} - 4Y_{t+1} + 6Y_t - 4Y_{t-1} + Y_{t-2} = -A_tY_t$$

$$-Y_{t+2} + 2Y_{t+1} - 2Y_{t-1} + Y_{t-2} = -2L^3P/t^3EI$$

(2.18÷2.20)

$$Y_{t+1} - 2Y_t + Y_{t-1} = -\frac{L^2.H.P}{t^2.\ EI} \quad (2.21)$$

Definitions of terms in Equations are given at the end of this section.

The solution for deflections could be obtained as follows:

From equations (2.15), (2.16) and (2.17);

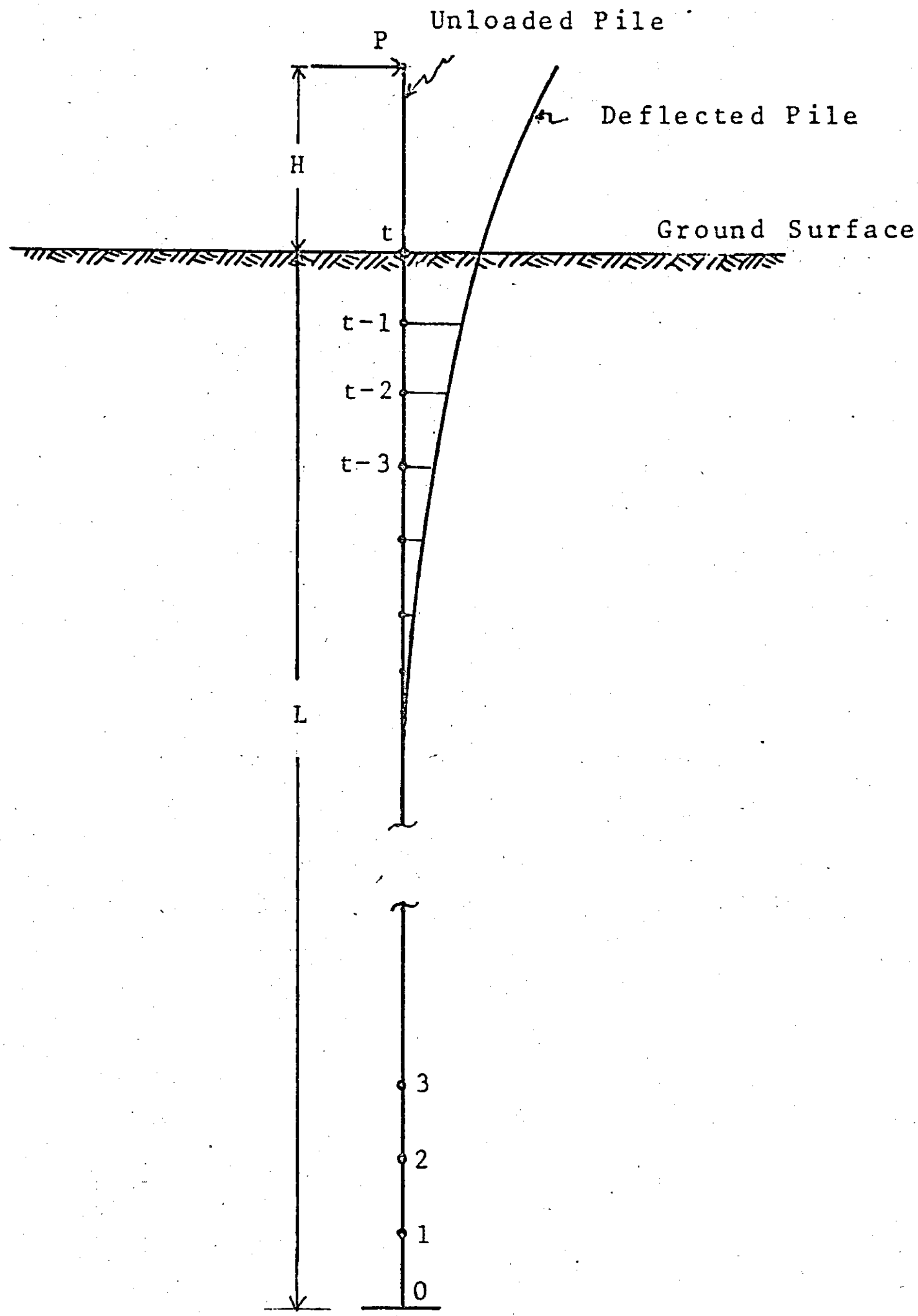


Fig. 2.12.- Representation of Deflected Pile

$$Y_o = \frac{-2Y_2 + 4Y_1}{2 + A_o} = -B_1Y_2 + 2B_1Y_1 \qquad (2.22)$$

From equations (2.18), and (2.22):

$$Y_1 = \frac{-Y_2 + Y_2(4-2B_1)}{5 + A_1 - 4B_1} = -B_1Y_3 + B_3Y_2 \qquad (2.23)$$

From equations (2.23), (2.22) and (2.19):

$$Y_2 = \frac{-Y_4 + Y_3\left[4 - B_2(4-2B_1)\right]}{6+A_2 - B_1 - B_3(4 - 2B_1)} = -B_4Y_4 + B_5Y_3 \qquad (2.24)$$

In general, thereafter

$$Y_n = \frac{-Y_{n+2} + Y_{n-1}\left[4-B_{2m-2}(4-B_{2m-3})\right]}{6+A_m - B_{2m-4} - B_{2m-1}(4-B_{2m-3})} = -B_{2m}Y_{m+2} + B_{2m+1}Y_{m+1} \qquad (2.25)$$

until a solution is obtained for Y_t. Substituting this and the solution for Y_{t-1} into (2.21):

$$Y_{t+2} = \frac{1}{B_{2t}(2-B_{2t-1})}\left\{\frac{-L^2HP}{t^2\ EI} + \left[B_{2t-2} - 1 + B_{2t-1}(2-B_{2t-1})\right]Y_{t+1}\right\} =$$

$$- B_{2t+2} + B_{2t+3}Y_{t+1} \qquad (2.26)$$

Substituting into equation (2.20):

$$Y_{t+1}\left[-B_{2t-3} + 2 + \{(2-B_{2t-3})B_{2t-1} + B_{2t-4}\}\{B_{2t}B_{2t-3} - B_{2t+1}\}\right.$$

$$+\{2-B_{2t-3}\}B_{2t-2}\Big] = \frac{-2PL^3}{t^3EI} - B_{2t+2}\ B_{2t+2}B_{2t}(2B_{2t-1}$$

$$+ B_{2t-4} - B_{2t-3}B_{2t-1}) \tag{2.27}$$

To obtain deflections, substitute value for Y_{t+1} from equation (2.27) into (2.26) to obtain Y_{t-2}. Substitute these values into equation (2.25) to obtain Y_t, etc. until Y_o, Y_{-1} and Y_{-2} are obtained.

The constants of the above equations are as follows:

$$B_1 = \frac{2}{2+A_o} \tag{2.28}$$

$$B_2 = \frac{1}{5+A_1-4B_1} \tag{2.29}$$

$$B_3 = B_2(4 - 2B_1) \tag{2.30}$$

$$B_4 = \frac{1}{6+A_2-B_1-B_3(4-2B_2)} \tag{2.31}$$

$$B_5 = B_4(4-B_3) \tag{2.32}$$

For all even constants B_6 through B_{2t} inclusive:

$$B_{2m} = \frac{1}{6 + A_m - B_{2m-4} - B_{2m-1}(4-B_{2m-3})} \tag{2.33}$$

For all odd constants B_7 through B_{2t+1} inclusive:

$$B_{2m+1} = B_{2m}(4 - B_{2m-1}) \tag{2.34}$$

$$B_{2t+2} = \frac{-L^2 \ HP}{t^2 EIB_{2t}(2-B_{2t-1})} \qquad (2.35)$$

$$B_{2t+3} = \frac{B_{2t-2} - 1 + B_{2t+1}(2 - B_{2t-1})}{B_{2t} \ (2 - B_{2t-1})} \qquad (2.36)$$

Generalized Equations for Fixed Head Pile

Equations (2.15) through (2.20) for the fixed head pile are identical with those for the fixed head pile as are the solutions in equations (2.22) through (2.25) inclusive

$$Y'_{t-1} - Y'_{t+1} = 2 \frac{L}{t} \text{ (slope of pile at ground = 0)}$$

or

$$Y'_{t-1} = Y'_{t+1} \qquad (2.37)$$

In this case solution could be obtained as follows:

Substituting the value of (2.37) and the results of equations (2.22) through (2.25) into equation (2.20):

$$Y'_{t+2} = \frac{-2L^3 P(1+B'_{2t-2} - B'_{2t-1}B'_{2t+1})}{t^3 EI(-1+B'_{2t}B'_{2t-4} + B'_{2t}B'_{2t-2}B'_{2t-4} - B'_{2t}B'_{2t-1}B'_{2t-3} - B'_{2t-2} + B'_{2t+1}B'_{2t-1}} \qquad (2.38)$$

$$Y'_{t+1} = Y'_{t-1} = \frac{-B'_{2t-1} \ B'_{2t}}{1 + B'_{2t-2} - B'_{2t+1}B'_{2t-1}} Y'_{t+2} \qquad (2.39)$$

$$Y'_t = B_2 + Y_{t+2} + B_{2t+1}Y_{t+1} \qquad (2.40)$$

To obtain remaining deflections, substitute values of Y'_{t+2}, Y'_{t+1} and Y'_t into equation (2.25) for Y'_{t-1}, etc. until value for Y'_3 is obtained, after which use equations (2.24), (2.23), (2.22), (2.17), (2.16), (2.15) to obtain Y'_2, Y'_1, Y'_o Y'_{-1} and Y'_{-2}.

For fixed head pile, all constants are the same as for free head pile to and including B'_{2t} and B'_{2t+1}. Thereafter use equations (2.38) and (2.39).

Nomenclature

In the above equations:

Y_m is the deflection at point m
t is the number of equal units into which L is divided
P is the applied load

H is the distance of P above the ground surface

E is Young's Modulus for the pile material
I is the moment of inertia of the pile
A_m is the soil resistance at point m multiplied by $\frac{L^4 b}{t^4 EI}$
b is the width of pile.

Method of Solution

1- Divide embedded length of pile into t elements with additional elements t+1, t+2, at the top of the pile and elements t-1, t-2 below the pile tip.

2- A series of E_s values are assumed along the pile length according to: $E_s = k_h . x$ (2.41)

where; k_h = a constant of soil modulus variation
x = depth measured along the pile length.

3- For every node, using finite difference equations, y_n displacements are found according to Gleser (1953).

4- These y_n values are plotted in the referring p-y curves and p soil reactions are found.

5- A new set of soil modulus is obtained using:

$$E_s = -p/y \qquad (2.42)$$

6- Using the new set of soil modulus values, y_n displacements are refound.

7- This iterative procedure continues until the new set of soil modulus values are equal to the previous set of soil modulus values.

8- With the equations given previously S_m (slope), V_m (shear), M_m (moment) distribution along the pile is computed.

2.3. SUMMARY

In this chapter loading conditions of the laterally loaded piles are given. Three types of methods of analysis, which are the limit analysis methods, the elastic methods and the soil reaction-pile deflection method, are described.

The limit analysis methods assume that the limiting or maximum soil resistance is acting against the pile when the ultimate load is placed on the pile.

The elastic methods assume that, soil reaction is linearly proportional to depth. It is preferrable to use the elastic methods, if the reaction-pressure relationships of the soil are known.

The pile deflection-soil reaction method is the iterative application of the elastic method, considering the nonlinear relationship between pile deflection and soil reaction.

The derivation of the fourth order differential equation of the laterally loaded pile, and the solution using finite difference equations are summarized. In addition an iterative procedure for pile deflections soil reaction method is described.

3. SOIL CRITERIA FOR LATERALLY LOADED PILES

3.1. INTRODUCTION

In this chapter soil reaction-deflection (p-y) curves which are recommended by various researchers are summarized. The p-y curves for soft clays (Matlock, 1970), for stiff clays (Reese, Cox, Koop, 1975) are summarized. In the case of sands, the p-y curves recommended by Reese and Parker (1971) and by Reese, Cox, Koop (1974) are summarized.

Each of the procedures described in this chapter is based on experimental studies using full-sized, instrumented piles. In each case, p-y curves were derived from experimental results and were employed in developing the recommended procedures.

The recommended criteria relating to foundation design are devoted primarily to pile foundations and more specifically to steel cylindrical pile foundations.

The pile foundation shall be designed to carry lateral loads, whether static or cyclic. Generally under lateral loading soil reaction-deflection relationships of soils are nonlinear.

3.2. SOIL REACTION-DEFLECTION CURVES FOR SOFT CLAYS

A. Ultimate Resistance

Based on field tests as well as on laboratory tests, Matlock (1970) developed soil criteria for constructing p-y curves for static and cyclic loading in soft clays. Matlock assumed that the ultimate resistance per unit length of pile is expressed as:

$$P_u = N_p . c . b \tag{3.1}$$

where

c = undrained shear strength of undisturbed clay soil sample in N/cm^2

b = pile diameter in cm, and

N_p = dimensionless coefficient of ultimate bearing capacity.

The value of the coefficient N_p for the depth where plastic failure occurs is given as:

$$N_p = 9 \tag{3.2}$$

Near the ground surface where the overburden pressure is not high enough to prevent the formation of the upward wedge, the coefficient N_p is given by

$$N_p = 3 + \frac{\gamma' x}{c} + J \frac{x}{b} \tag{3.3}$$

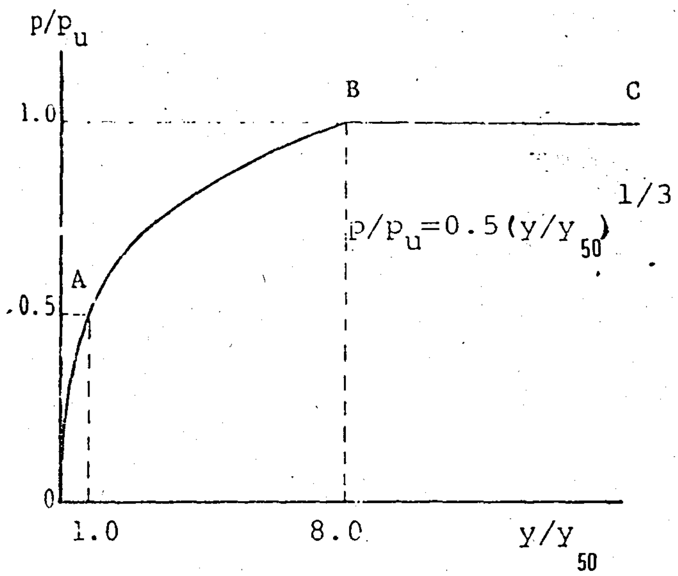


Fig. 3.1.– p-y Curves for Soft Clays Under Static Loading (after Matlock, 1970)

where

γ' = effective unit weight of clay in N/cm^3,
x = depth in cm,
c = cohesion of a clay in N/cm^2
b = pile diameter in cm, and
J = dimensionless constant determined from the type of clay.

Matlock's experimental value of J ranged from 0.25 to 0.50. Matlock (1970) points out that the magnitude of J is relatively insignificant in the more important upper layers. He proposes a value of 0.5 for use in connection with offshore clays in the Gulf of Mexico.

Near the ground surface the ultimate resistance per unit legnth of pile is determined by the lower value of p_u computed.

B. p-y Curve For Static Loading

The p-y curve for the static loading as seen in Fig. 3.1, is constructed by the following procedure (Matlock, 1970):

1- Choose ε_{50}, the strain which occurs at one-half the maximum stress on laboratory undrained compression test of undisturbed soil samples. Typical values given by Skempton (1951) for ε_{50}, in Table 1. Could be used if a stress strain curve is unavailable.

2- Calculate the pile deflection y_{50} which corresponds to the strain ε_{50} on the stress-strain curve, by a formula

$$y_{50} = 2.5\ \varepsilon_{50}\ b \qquad (3.4)$$

where

b = diameter or width of a pile in cm.

3- Draw a nondimensional cubic curve between points 0 and B in Fig. 3.1 given by:

$$\frac{p}{P_u} = 0.5 \left(\frac{Y}{Y_{50}}\right)^{1/3} \text{ for } 0 \leq p / p_u \leq 1 \tag{3.5}$$

4- The curve between points B and C in Fig. 3.1 is a horizontal line given by:

$$\frac{p}{P_u} = 1 \tag{3.6}$$

5- Calculate the values of p and y from the nondimensional p-y curve by multiplying the abcissa with y_{50}.

TABLE 3.1- Strain at Failure of Clays in Undrained Triaxial Test (after Matlock, 1970)

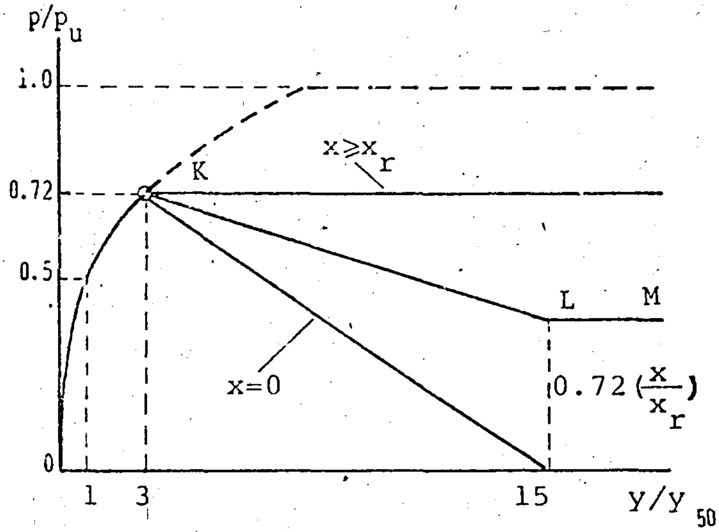
ε_{50}	clay
0.005	brittle or stiff clay
0.02	soft clays
0.01	other clays

The coordinates of the normalized p-y curve are as seen in Fig.3.1:

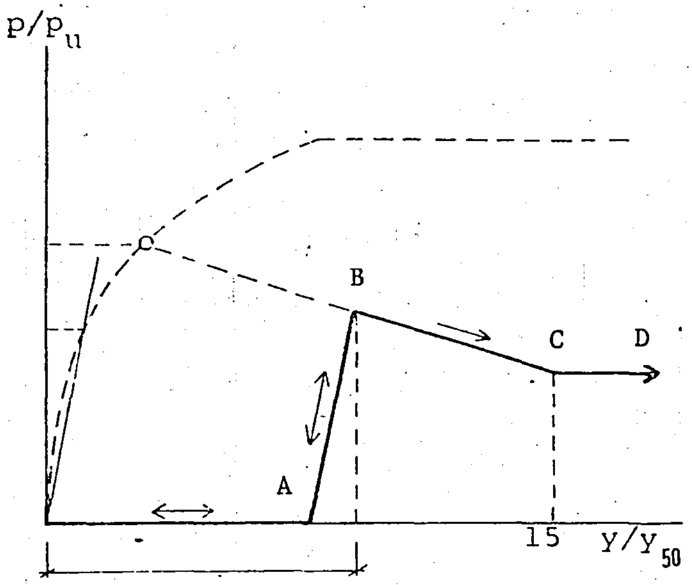
y/y_{50}	p/P_u
0	0
1.0	0.5
8.0	1.0
∞	1.0

C. Effect of Cyclic Loading on p-y Curve

Cyclic loads cause decrease in lateral bearing capacity below that for static loads. In this case normalized p-y curve shown in Fig.3.2 recommended by Matlock (1970) could be utilized: In this case equation of the curve between points 0 and K is given as in Equation 3.5:



Cyclic Loading



Reloading after Cyclic Loading

Fig. 3.2.— p-y Curves for Soft Clays Under Cyclic Loading and Reloading (after Matlock, 1970)

$$\frac{p}{P_u} = 0.5 \left(\frac{Y}{Y_{50}}\right)^{1/3} \text{ for } 0 \leq p/P_u < 1 \qquad (3.7)$$

It should be pointed out that Equation 3.7 is applied only for $0 \leq p/P_u \leq 0.72$. A maximum value of 0.72 for p/P_u is adopted to allow for the decrease in lateral bearing capacity under cyclic loading.

Between points K and L in Fig. 3.2 there is a straight line reducing p/P_u to $0.72\ x/x_r$ at $y/y_{50} = 15$. x_R is the depth below soil surface to bottom of reduced strength zone. For soft clay x_R may be approximated as (Matlock, 1970):

$$x_R = \frac{6b}{\frac{\gamma' b}{c} J} \qquad (3.8)$$

where

b = pile diameter

γ' = effective unit weight of soil

c = undrained shear strength of clay

J = is an empirical adjustment factor, which is taken as 0.5.

The curve between L and M in Fig. 3.2 is given as a horizontal line corresponding to a value of

$$\frac{p}{P_u} = 0.72 \frac{x}{x_R} \qquad (3.9)$$

It must be noted that at $x = 0$, $p/P_u = 0$ for the values of $y/y_{50} \geq 15$. The value of $p/P_u = 0.72$ from K to M in Fig. 3.2, for the depths $x > x_R$.

The coordinates of the p-y curve mentioned above may be summarized as (Fig. 3.2):

$x>x_R$	
Y/Y_{50}	p/P_u
0	0
1.0	0.5
3.0	0.72
∞	0.72

$x<x_R$	
Y/Y_{50}	p/P_u
0	0
1.0	0.5
3.0	0.72
15.0	$0.72x/x_R$
∞	$0.72x/x_R$

D. Reloading After Cyclic Loading Condition

For reloading after cyclic loading an initial straight line is assumed with slope

$$E_{50} = 0.5 \frac{(p/P_u)}{(Y/Y_{50})} \tag{3.10}$$

up to the post failure line obtained in cyclic loading as shown in Fig. 3.2.(b). In this case, for deflections less than the value B shown in Fig.3.2.(b), it is assumed that the soil reaction is zero. The AB line in the same figure is parallel to OC.

3.3. SOIL REACTION-DEFLECTION CURVES FOR STIFF CLAYS

These p-y criteria are derived from the pile tests done in 1967-68 (Reese, Cox, Koop, 1968). The tests were made by deriving, 18 meters long, 0.61 m diameter pipe piles in sea origined overconsolidated clay near Austin in Texas. The p-y curves (Fig.3.3) for overconsolidated clays are given below:

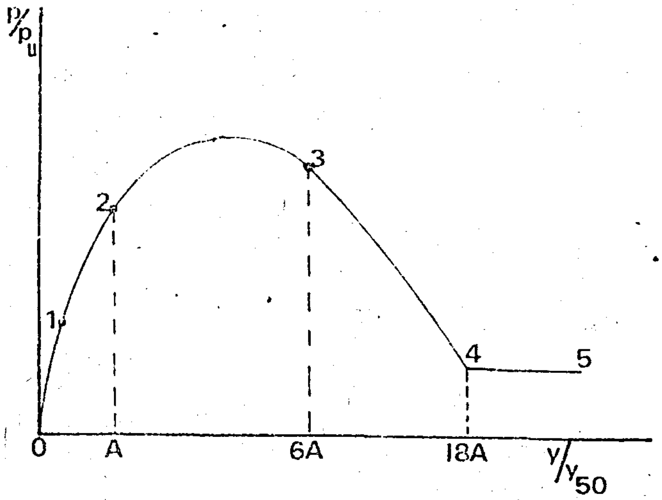
A. Ultimate Resistance

At shallow depths (Reese, Cox, Koop, 1975)

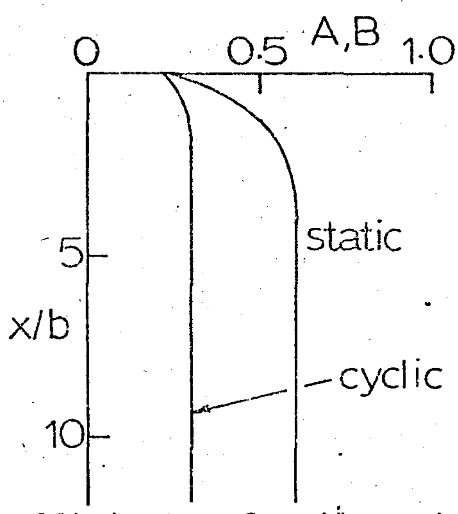
$$P_u = 2\ c\ b + \gamma' bx + 2.83\ c\ x \tag{3.11}$$

At depths well below ground surface

$$P_u = 11\ c\ b \tag{3.12}$$



Stiff Clay p-y curves (static loading)



Coefficients of soil resistance
A and B against depth.

Fig. 3.3.- p-y Curves for Stiff Clays Under Static Loading
(after Reese, Cox and Koop, 1975)

B. Value of Y_{50}

The reference value (Reese, Cox, Koop, 1975) Y_{50} is given by

$$y_{50} = \varepsilon_{50}.b \qquad (3.13)$$

where ε_{50} is given in Table 3.2.

C. Standart Curve for Static Loading (Fig. 3.3)

As shown in Fig. 3.3, from 0 to 1 there is a straight line of $p/y = k_s x$ (3.14)

where k_s and k_c (for cycling loading) are given in Table 3.3. The section 0 to 1 can also be written in dimensionless from:

$$\frac{(p/P_u)}{(y/y_{50})} = k_s.x.(\frac{y_{50}}{P_u}) \qquad (3.15)$$

From 1 to 2 the curve is a parabola:

$$p/P_u = 0.5\ (y/y_{50})^{1/2} \qquad (3.16)$$

and from 2 to 3

$$p/P_u = 0.5(y/y_{50})^{1/2} - 0.055\ (\frac{y-Ay_{50}}{A.y_{50}})^{1.25} \qquad (3.17)$$

for $Ay_{50} < y_{50} \leq 6Ay_{50}$ where A is given in Fig. 3.3

from 3 to 4

$$p/P_u = 0.5(6A)^{1/2} - 0.411 - \frac{0.0625}{y_{50}}(y-6Ay_{50}) \qquad (3.18)$$

for $6Ay_{50} < y < 18\ Ay_{50}$.

Finally from 4 to 5 a straight line portion:

$$p/P_u = 0,5\ (6A)^{1/2} - 0.411 - 0.75.P_u.A \qquad (3.19)$$

for $y > 18Ay_{50}$.

TABLE 3.2- Strain at Failure Values for Stiff Clays (after Reese, Cox and Koop, 1975)

Undrained Cohesion, C (kN/m^2)	ε_{50}
50-100	0.007
100-200	0.005
200-400	0.004

TABLE 3.3- K_s and k_c Values for Stiff Clays (after Reese, Cox, Koop, 1975).

k_s and k_c x 10^6 N/m_3	Undrained Cohesion, C kN/m^2		
	50-100	100-200	200-400
k_s (static loading)	135	270	540
k_c (cyclic loading)	54	108	216

D. Standart curve for Cyclic Loading (Fig.3.4)

A reference deflection $y_p = 4.1\ A.y_{50}$ is defined. Then the standart curve consists of a straight line 0 to 1:

$$\frac{p/P_u}{y/y_{50}} = k_c \times \left(\frac{y_{50}}{P_u}\right) \qquad (3.20)$$

a parabola 1 to 2:

$$p/P_u = B \left[1 - \left|\frac{y-0.45y_p}{0.45\ y_p}\right|^{2.5}\right] \qquad (3.21)$$

which is fitted to form a cap between the straight lines 0 to 1 and 2 to 3 with the peak at $y = 0.45\ y_p$. Values of A and B are given in Fig. 3.3

Section 2 to 3 is a straight line:

$$p/P_u = 0.936B - \frac{0.085}{y_{50}}\ (y-0.6y_p) \qquad (3.22)$$

for $0.6\ y_p < y < 1.8\ y_p$,

and finally, section 3 to 4 is a straight line:

$$p/P_u = 0.936B - \frac{0.102}{y50} \cdot y_p \qquad (3.23)$$

for $y > 1.8\ y_p$

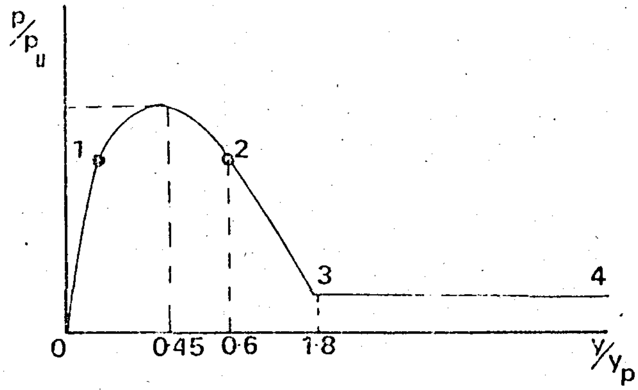


Fig. 3.4.— p-y Curves for Stiff Clays Under Cyclic Loading (after Reese, Cox and Koop, 1975)

3.4. SOIL REACTION-DEFLECTION CURVES FOR SAND

A. Parker and Reese Procedure

The criteria proposed by Parker and Reese (1971) for obtaining a set of p-y curves for sand are based on formulas for the ultimate lateral soil resistance per unit length of pile and on recommendations by Terzaghi (1955) for the shape of the early part of the p-y curve. The detailed development of the formulas for the ultimate lateral soil resistance is given by Parker and Reese (1971).

The ultimate lateral soil resistance is given by the equation 3.24. of plastic failure of sand. The equation is derived by considering the successive failure of the square block soil elements as in the case of clay (Reese, 1958).

$$P_u = \gamma' b\ x\ \{K_p^3 + 2\ K_o \tan\Phi\ (K_p^2 + 1) - K_A\} \qquad (3.24)$$

where

P_u = ultimate soil resistance per unit length of pile in N/cm,

γ' = effective unit weight of soil in N/cm^3,

x = depth in cm,

b = pile width in cm,

K_A = $\tan^2(45^o - \phi/2)$ coefficient of active earth pressure,

K_p = $\tan^2(45^o + \phi/2)$ coefficient of passive earth pressure,

K_o = coefficient of earth pressure at rest which is assumed to be 0.5, and

Φ = angle of internal friction of a sand in degrees.

The examination of Equation 3.24 for P_u reveals that the first term within the paranthesis K_p^3 is by far the major contributing factor in determining the ultimate lateral soil resistance per unit length of pile. Therefore, the ultimate lateral soil resistance P_u is proportional to the cube of the coefficient of the passive earth pressure.

Near the ground surface the ultimate soil resistance on a pile is obtained by computing the force exerted from a soil wedge moving upward. The ultimate lateral soil resistance by wedge type failure P_w is obtained by differentiating the total force exerted from the soil wedge on to the pile with respect to depth, x (Parker and Reese, 1971).

$$P_w = \gamma' \; x \; \{b(K_p - K_A) + x \tan\beta \; [K_p \tan\alpha + K_o \; (\tan\Phi - \tan\alpha)]\} \tag{3.25}$$

where

$\beta = 45^o + \Phi/2$, and

α = angle to define the shape of wedge, and is assumed to be equal to one half of Φ.

The early portion of the p-y curve is constructed from Terzaghi's (1955) recommendation. Terzaghi used the theory of elasticity to derive the relationship between the horizontal deflection of a vertical pile and the lateral soil resistance as:

$$p/y = k = mx \tag{3.26}$$

where

p = lateral soil resistance in N/cm,
y = lateral pile deflection in cm,
k = soil modulus in N/cm^3,
m = coefficient of lateral soil reaction in N/cm^3,
x = depth from ground surface in cm.

The values recommended by Terzaghi (1955) for coefficient m are summarized in Table 3.4.

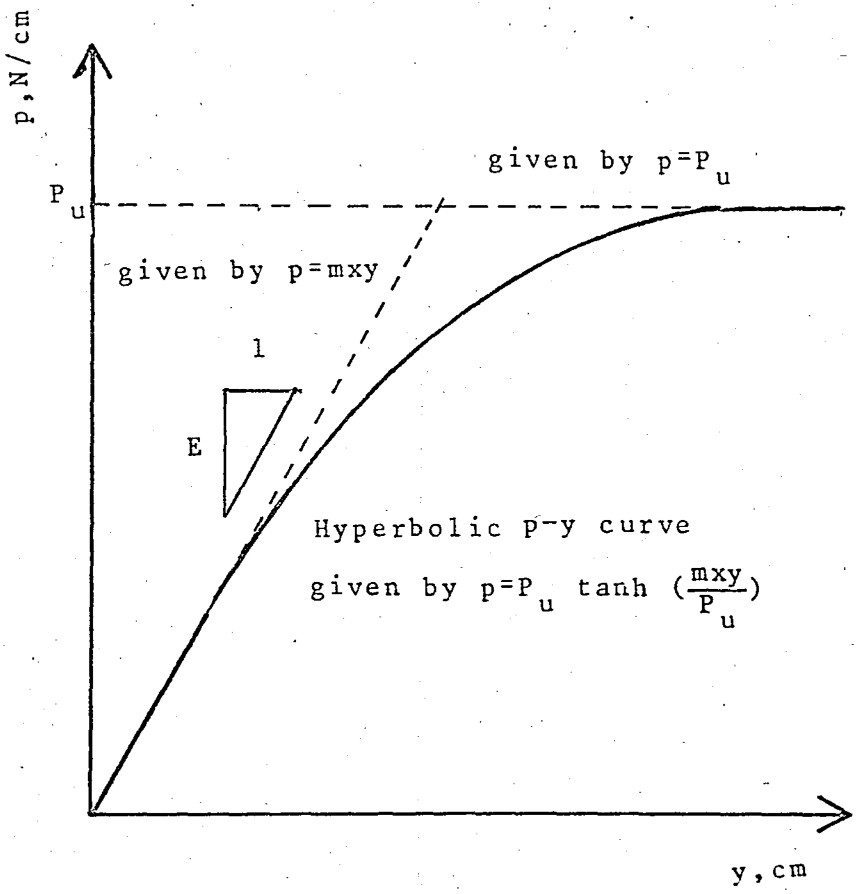


Fig. 3.5.– Form of p-y Curves for Sands (after Reese and Parker, 1971)

TABLE 3.4- Values of Coefficient of Lateral Soil Reaction-m, for Sands (after Reese, 1971) N/cm^3

Relative Density	Dry or Moist Sand	Submerged Sand
Loose	0.96 - 2.84	0.57 - 1.75
Medium	3.49 - 10.95	2.18 - 7.29
Dense	13.87 - 27.74	8.76 - 17.50

The procedures described to this point define two straight lines, one from the origin at a slope given by Table 3.2. and the other a horizontal line given either by P_u or P_w. The transition between these two straight lines is given by the following equation.

$$p = P_u \tanh \left(\frac{mxy}{P_u}\right) \tag{3.27}$$

where

p = lateral soil resistance in N/cm,
P_u= ultimate lateral soil resistance in N/cm,
m = coefficient of lateral soil resistance in N/cm^3, and
y = lateral deflection of pile in cm.

The hyperbolic p-y curve generated by Equation 3.27 for p is tangent to the straight line defining the early portion of the p-y curve given by Equation 3.26 and is asymptotic to the ultimate lateral resistance given by Equations 3.24 and 3.25 as seen in Fig. 3.5.

The procedures for constructing the p-y curves are summarized as follows.

1- Compute the two types of ultimate lateral soil resistance per unit length of pile by Equations 3.24 and 3.25 for P_u and P_w along the pile. Typical distributions of P_u and P_w could be seen in Fig. 3.6.

2- Take the smaller value as the governing ultimate

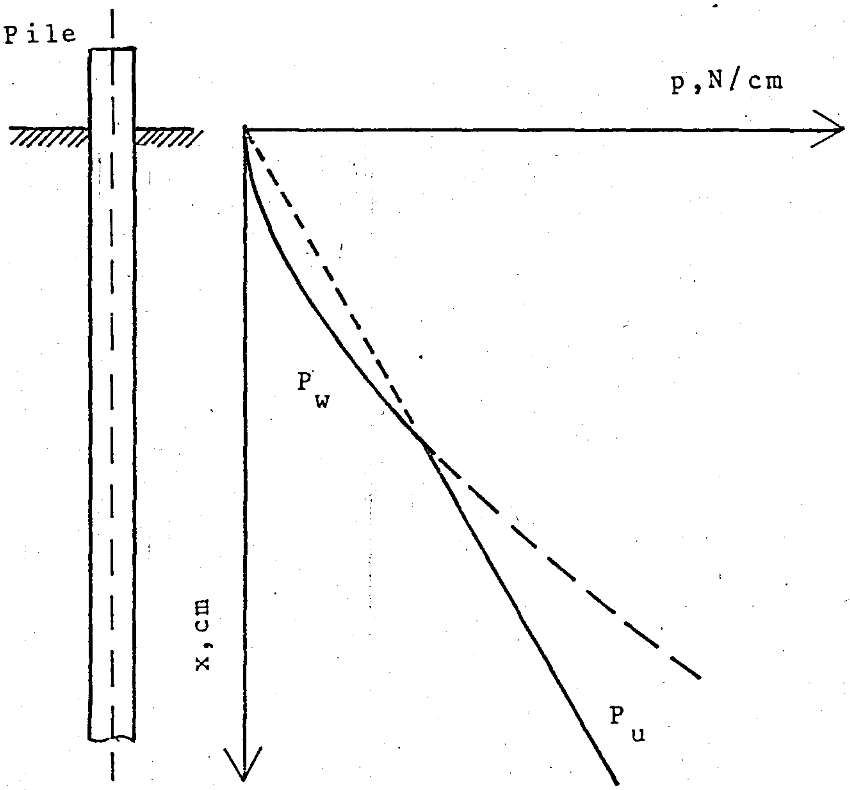


Fig. 3.6.– Ultimate Lateral Soil Resistance for Sand (after Reese and Parker, 1971)

value.

3- Choose the appropriate value of k, depending on the state of the sand using Table 3.2.

4- Construct a p-y curve as it is shown in Fig 3.5.

5- Repeat steps 3 and 4 for various depths to obtain a set of p-y curres.

It should be maintained that the recommandations by Parker and Reese (1971) do not include the consideration of cyclic loading.

B. Reese, Cox and Koop's (1974) Procedure

The p-y curves are based on formulae for the ultimate lateral soil resistance and on an initial linear relation with a coefficient of soil reaction.

The following relationships could be used in calculation of ultimate resistance.

a) Near the surface, $x < x_T$,

$$p_u = \gamma' x \left| \frac{K_o \, x \tan\Phi \, \sin\beta}{\tan(\beta-\Phi)\cos\alpha} + \frac{\tan\beta}{\tan(\beta-\Phi)} (b + x \tan\beta \, \tan\alpha) + K_o \, x \tan\beta \, (\tan\Phi \, \sin\beta - \tan\alpha) - K_A b \right| \qquad (3.28)$$

b) Well below the ground surface, $x > x_T$

$$p_u = K_A b\gamma' \, x \, (\tan^8\beta - 1) + K_o b\gamma' \, x \tan\Phi \, \tan^4\beta \qquad (3.29)$$

In equations 3.28 and 3.29; x_t is the intersection point of curves defined by equations 3.28 and 3.29 as shown in Fig. 3.7.

$$\alpha = \Phi/2; \; \beta = 45 + \Phi/2; \; K_o = 0.4; \; K_A = \tan^2(45 - \Phi/2)$$

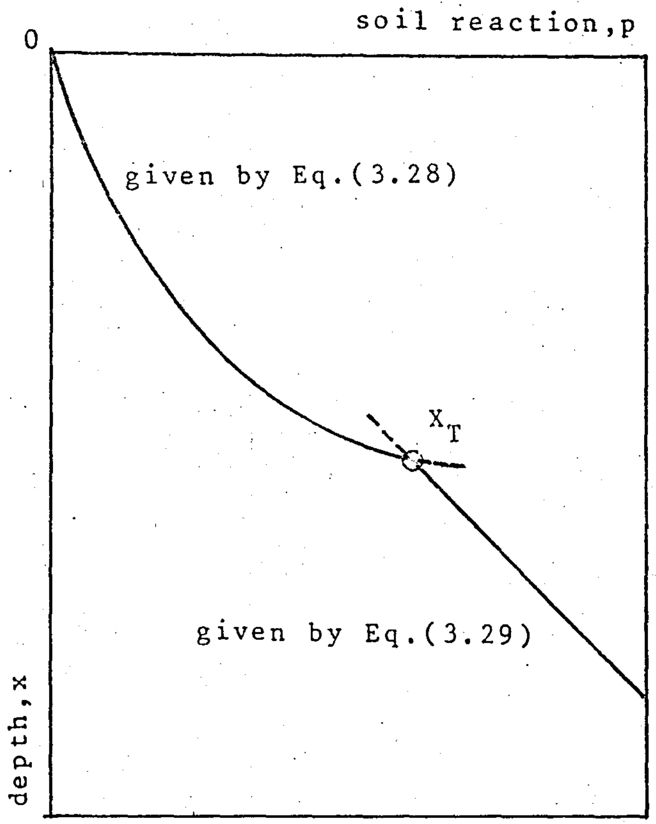


Fig. 3.7.- Variation of Ultimate Soil Reaction with Depth (after Reese, Cox and Koop, 1974)

where Φ = angle of shearing resistance

Standart Curve for Sand is given in Fig 3.8.

The first section, K to L, is a straight line given by:

$$p/y = kx \tag{3.30}$$

where k has the values given below for submerged sand for both static and cyclic loading.

Relative Density, Dr	Loose	Medium	Dense
$k(N/cm^3)$	5.43	16.29	33.94

The third section between points M and N is a straight line with a slope of m. To complete the curve the following quantities are calculated:

$$y_u = \frac{3b}{80} \tag{3.30}$$

$$y_m = \frac{b}{60} \tag{3.31}$$

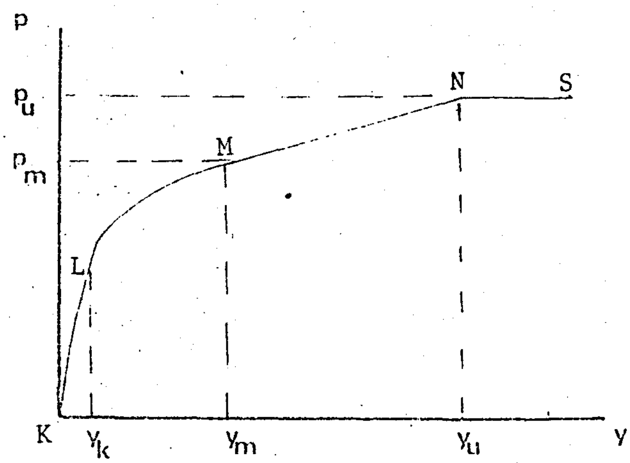
$$P_u^* = A.P_u \tag{3.32}$$

$$P_m = B.P_u$$

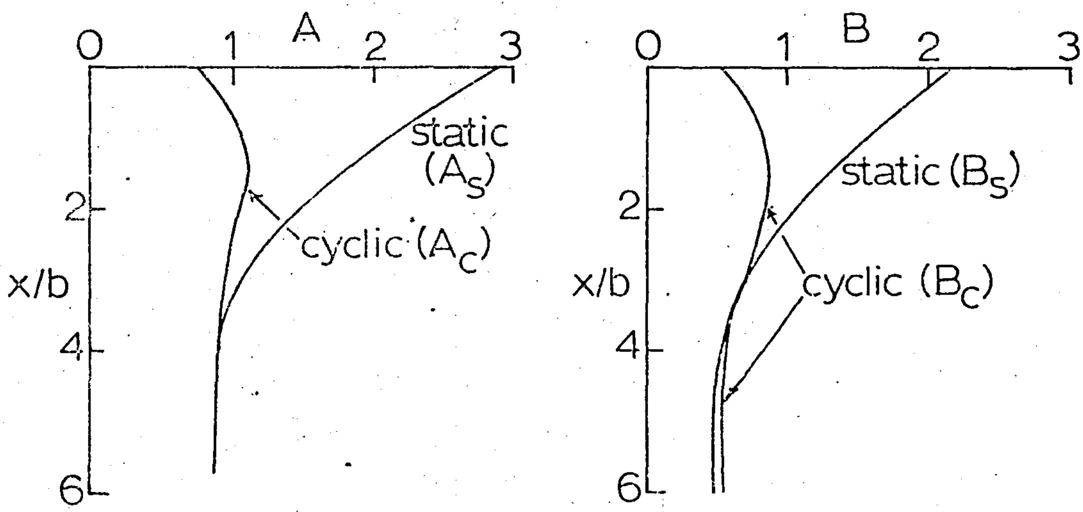
where A and B are modifying factors used in the light of experience from full-scale observations as given in Fig. (3.8). Then the slope of the line MN is calculated by:

$$m = \frac{p_u^* - p_m}{y_u - y_m} \tag{3.34}$$

The second section between L and M is a curve linking the two straight lines. To locate point L Y_k is calculated from:



Sand p-y curves



Coefficients of soil resistance A and B against depth

Fig. 3.8.– p-y Curves for Sands (after Reese, Cox and Koop, 1974)

$$Y_k = \left[\frac{p_m}{k_x \; y_m^{1/n}}\right]^{\frac{n}{n-1}} \qquad (3.35)$$

where

$$n = \frac{p_m}{m \cdot y_m}$$

A parabola given by:

$$p = \left(\frac{p_m}{y_m^{1/n}}\right) y^{1/n} \qquad (3.36)$$

is used between the points L and M.

The fourth section, N to S, is again a straight line given by:

$$p = P_u^* = A \cdot P_u \qquad (3.37)$$

3.5. SUMMARY

In this chapter, soil reaction-deflection (p-y) curves for soft clays, stiff clays and clean sands are summarized.

The p-y curves for soft clays recommended by Matlock (1970) consider three types of loading conditions, namely, static loading, cyclic loading and reloading after cyclic loading conditions.

The p-y curves for stiff clays which are recommended by Reese, Cox and Koop (1975) consider static and cyclic loading conditions.

Finally for sands, two different procedures are summarized Parker and Reese (1971) procedure considers one set of curves for both static and cyclic loading conditions. Reese, Cox and Koop's (1974) procedure considers both cyclic and static loading cases.

The drawing instructions and the equations of the p-y curves including the necessary tables for some soil parameters are summarized in this chapter.

4. LATERALLY LOADED PILE GROUPS

4.1. INTRODUCTION

In practice piles are mostly found in groups. The neighbouring piles effect the deflections of the single pile. In the Poulos Method (1971), the increment in mudline deflection, which is the deflection at the ground surface, of a single pile according to group effect is calculated.

4.2. METHOD PROPOSED BY POULOS

Poulos (1971) has considered this problem for piles embedded in a homogenous elastic mass. Consider initially two piles each under lateral load P as shown in Fig.4.1. Then if the mudline deflection of a single pile under unit load is $\bar{\rho}$ the increased deflection of each pile as a consequence of the group effect is

$$\rho = P\ \bar{\rho}\ (1+\alpha) \qquad (4.1)$$

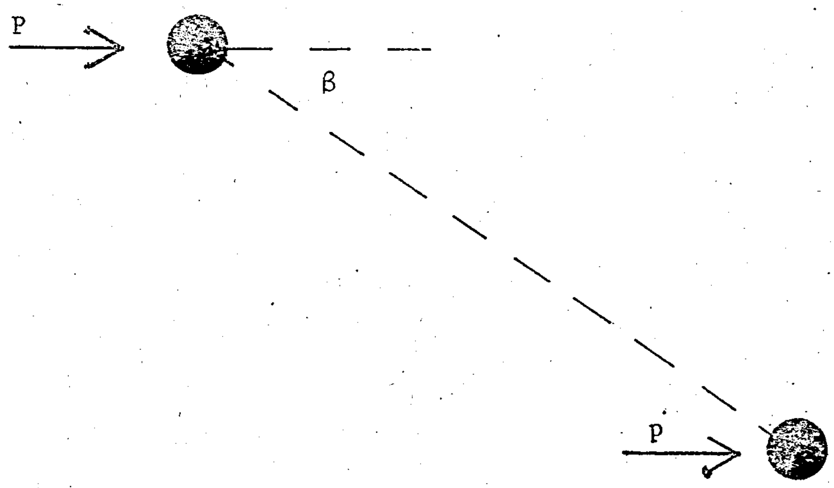


Fig. 4.1.– Group Effect (after Poulos, 1971)

where α depends upon the pile spacing, length, stiffness, the departure angle β as given in Fig. 4.1, and pile head fixity. Extending this to a group of piles, and assuming that superposition holds,

$$\rho_k = \bar{\rho} \left[\sum_{\substack{j=1 \\ j \neq k}}^{m} P_j \alpha_{kj} + P_k \right] \tag{4.2}$$

where

ρ_k = mudline deflection of pile k in a group with a total of m piles,

P_j, P_k = loads on piles j, k respectively,

α_{kj} = interaction coefficient between piles k and j.

This form of the equation is general and could be used for both fixed-headed or free-headed piles and for lateral load or moment applied to pile head.

Changes in pile head fixity and type of loading will, of course, change $\bar{\rho}$ and α_{kj}.

For single piles, Poulos (1971) gives

$$\bar{\rho} = \frac{I\rho}{LE_s} \tag{4.3}$$

where $I\rho$ has values:

$I\rho_H$ for a laterally loaded, free-ended pile;
$I\rho_M$ for moment on a free-ended pile;
$I\rho_F$ for a fixed-head pile.

and these parameters are plotted in Figs.4.2. through 4.4. respectively against pile flexibility factor K_R for different values of L/d where

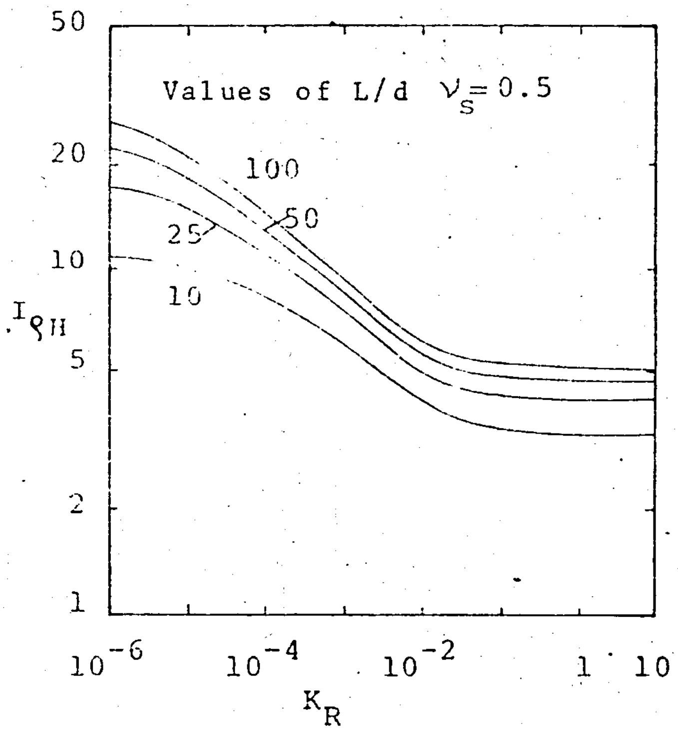


Fig. 4.2.— Influence Factors $I_{\rho H}$ for Free-Head Piles (after Poulos, 1971)

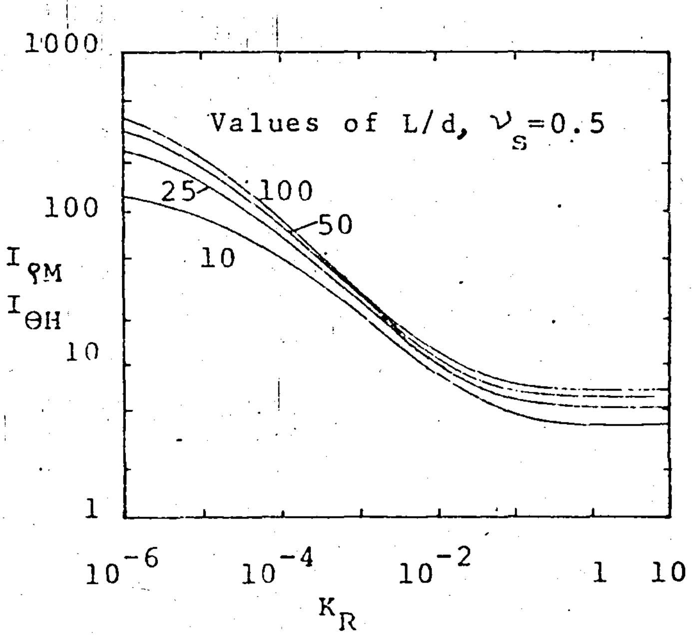


Fig. 4.3.– Influence Factors $I_{\rho M}$ and $I_{\theta H}$ for Free-Head Piles (after Poulos, 1971)

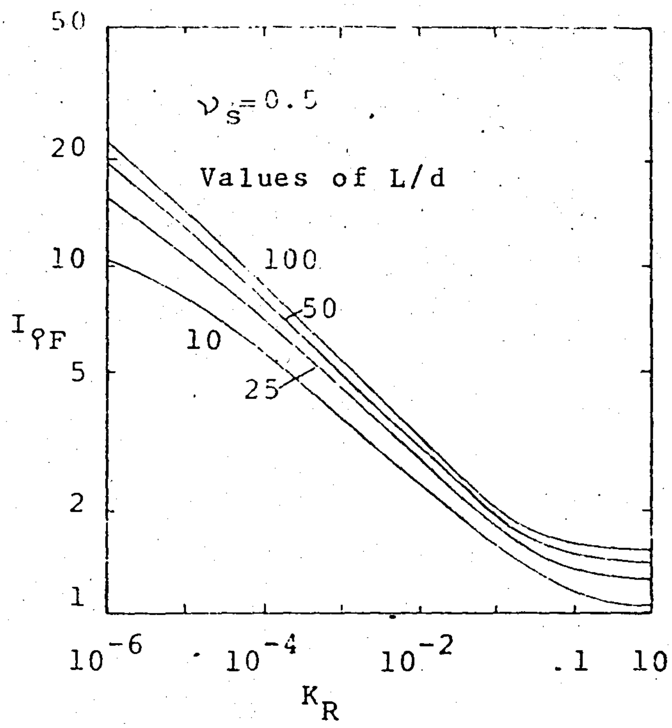


Fig. 4.4.— Influence Factors $I_{\rho F}$ for Fixed-Head Piles (after Poulos, 1971)

$$K_R = \frac{E_p I_p}{E_s L^4} \tag{4.4}$$

Where:

E_p = elastic modulus of pile,
I_p = moment of inertia of pile cross section,
E_s = soil modulus,
L = pile length.

Values of the interaction factor α are given in Figs. 4.5. through 4.7, here

$\alpha\rho_H$ is for laterally loaded, free-headed piles
$\alpha\rho_M$ is for moment on free-headed piles
$\alpha\rho_F$ is for fixed-headed piles.

Equation 4.2. could be used either

a) to calculate mudline deflections of all piles when the loads on the piles are all the same; or

b) to calculate pile loads when deflections of all piles are constrained to be the same.

4.3. COMBINED ELASTIC/p-y METHOD (Focht and Koch, 1973)

An obvious deficiency in the Poulos method is the calculation of single pile deflection by an elastic technique. A combined method using the p-y method for the single pile but then using the elastic group effect has been suggested by Focht and Koch (1973).

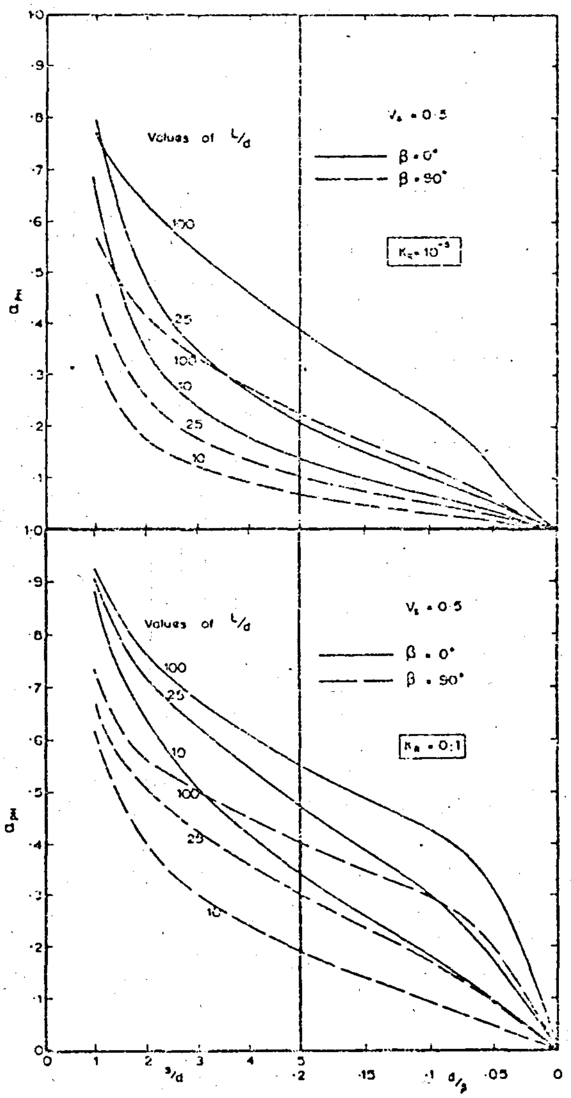


Fig. 4.5.– Interaction Factors $\alpha_{\rho H}$ for Free-Head Piles for Horizontal Loads (after Poulos, 1971)

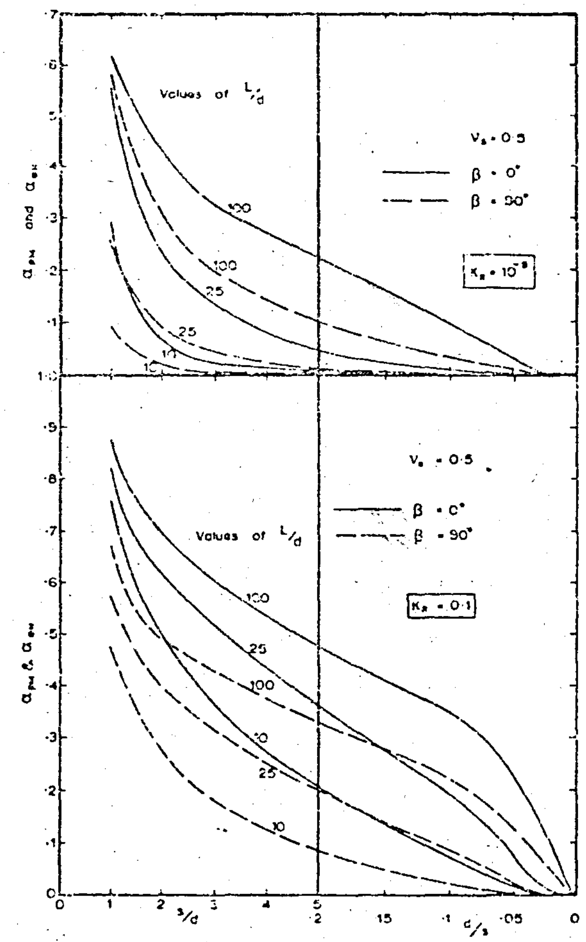


Fig. 4.6.– Interaction Factors $\alpha_{\rho M}$ and $\alpha_{\theta H}$ for Free-Head Piles for Moments (after Poulos, 1971)

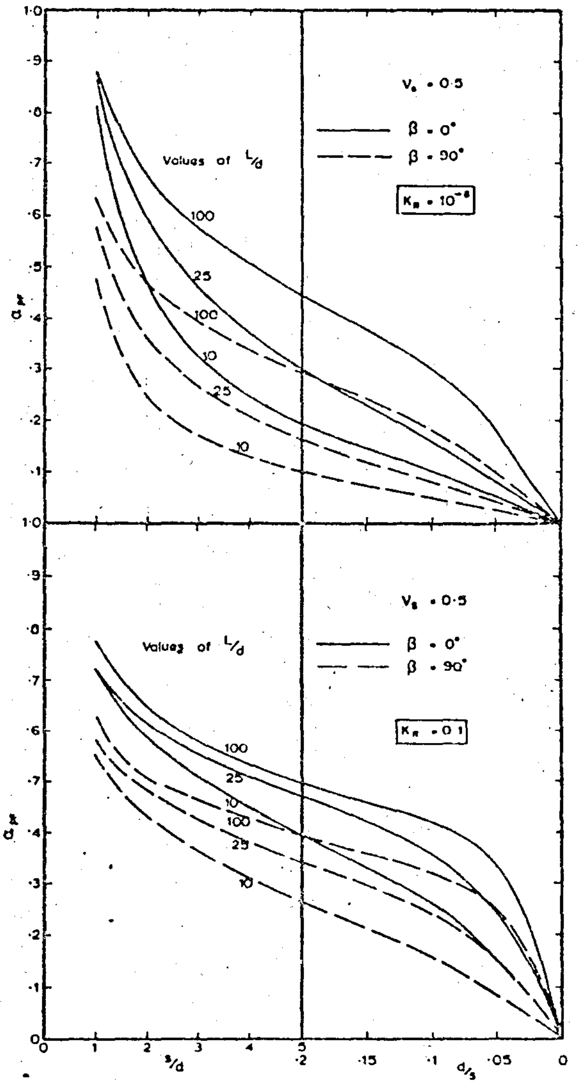


Fig. 4.7.– Interaction Factors $\alpha_{\rho F}$ for Fixed-Head Piles (after Poulos, 1971)

Then:

$$\rho_k = \bar{\rho} \sum_{\substack{j=1 \\ j \neq k}}^{m} P_j \alpha_{kj} + y_t \tag{4.5}$$

where

$\bar{\rho}$ = unit elastic deflection at mudline

y_t = deflection obtained by p-y method for single pile at mudline

Thus the group effect increases the p-y deflection y_t at the mudline by the amount in the elastic portion of the Equation 4.5.

It is suggested that the elastic modulus for the soil, E_s used in this part of the calculation, should be at least as high as E_{50}, or, if there is an initially linear portion to the stress-strain curve the initial tangent modulus, E_t can be used (see Fig.4.8).

The Procedure in Modifying p-y Curves

The Poulos technique can only be used to give deflections at the mudline, whereas pile design requires moments and shear forces-and hence deflections- to be found along the length of the pile. This can be dealt with empirically by modifying the p-y curves, by multiplying the y abscissae by constant factors to produce a new set of p-y curves for each constant as shown in Fig.4.9.

The procedure is to find the modification factor that gives a mudline deflection for a single pile equal to that calculated for the pile in the group. This set of modified p-y curves is then used to compute the deflections, shears and moments along the pile.

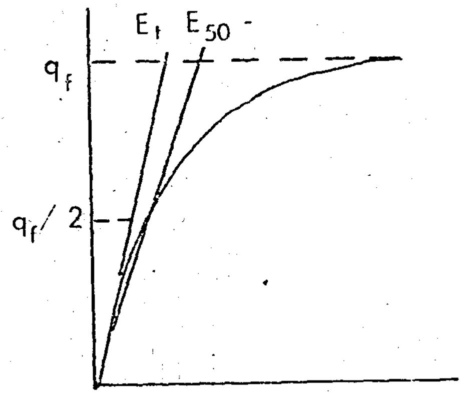


Fig. 4.8.— Tangent and Secant Deformation Modulus (after Focht and Koch, 1973)

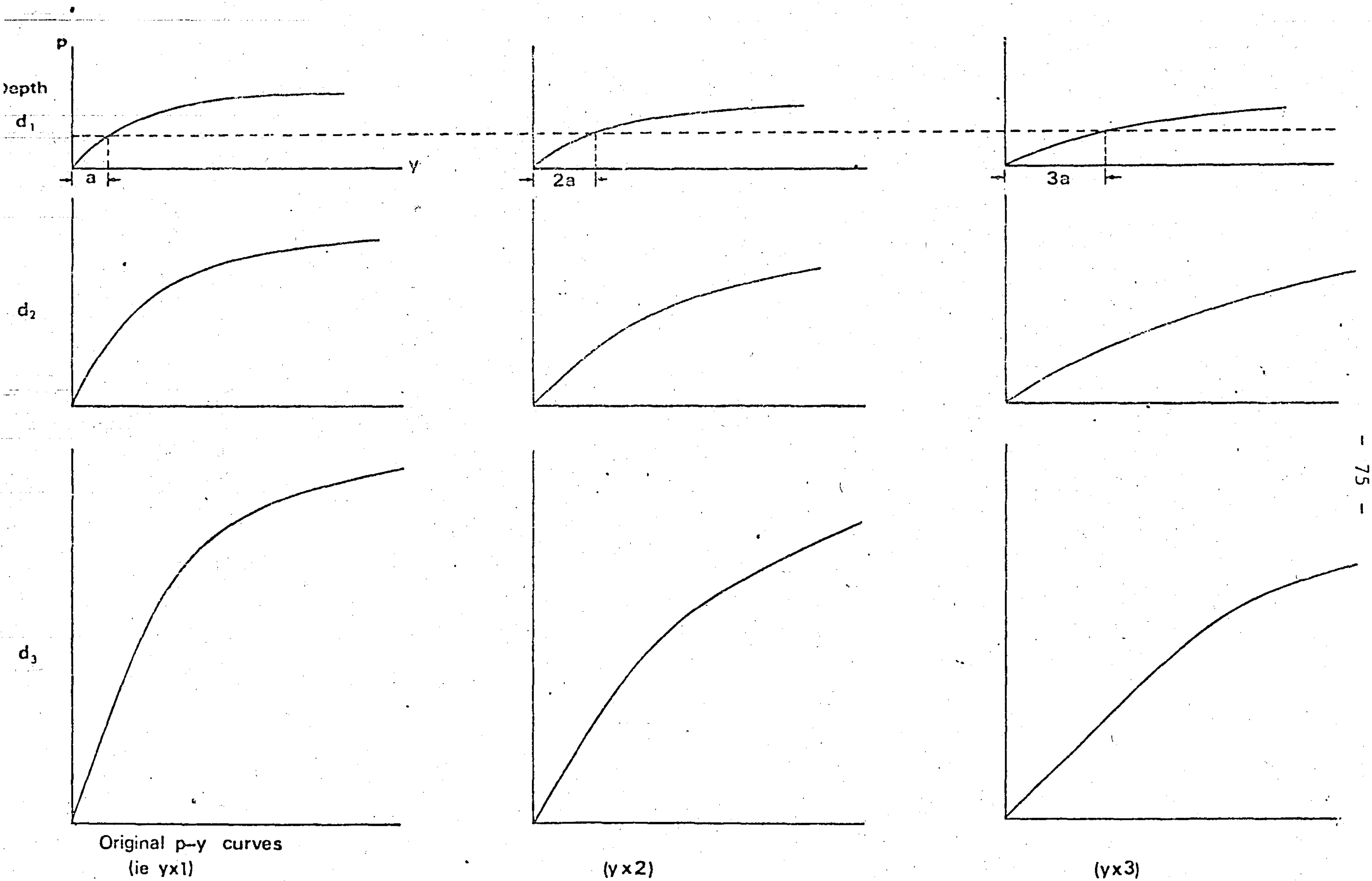


Fig. 4.9.– Modified p-y Curves (after Focht and Koch, 1973)

4.4. SUMMARY

In the pile groups, the neighbouring piles increase the deflections of the single pile. In this chapter two methods are summarized in the design of laterally loaded pile groups. Both of the methods give the mudline deflections of the pile. The Poulos method (1971) uses an elastic technique in the computation of mudline deflection. The second method is the combined elastic/p-y method which is suggested by Focht and Koch (1973). This method uses the p-y method in computation of mudline deflection, then uses the elastic group effect. In order to calculate pile deflections all along the pile length, the p-y curves must be modified by multiplying the soil reaction axes by constant factors.

5. NUMERICAL TECHNIQUE FOR THE ANALYSIS OF LATERALLY LOADED PILES

5.1. INTRODUCTION

In this chapter, a computer program for the solution of laterally loaded piles, which is developed, is presented. The method used in programming is recommended by Matlock and Reese (1966). The details of pile deflection soil reaction method which is used in programming, is given in Chapter 2. In the computer program, Gleser Algoritm (1953) is used. In programming the p-y curves, for soft clay, recommendation of Matlock (1970) in used. Static and cyclic loading cases are considered. The second subroutine which creates the p-y curves for sand is programmed according to the recommendation of Parker and Reese (1971).

A general computer program is developed for the determination of deflections and bending moments of a pile under lateral loading as a function of depth. Various boundary conditions could be considered at the top of the pile. Soil properties are defined by a set of curves which give soil reaction as a function of pile deflection. The detailed information about these curves are given in Chapter 3.

5.2. PROCEDURE

A summary of procedure used in programming could be written as follows:

1- Assumption for soil modulus along the pile length is made. 2. Using this assumption, the fourth order differential equation (2.8) of laterally loaded pile is solved. 3. With the deflections found, the subroutine developing the pile deflection-soil reaction curves are called and corressponding soil reactions are computed for every depth. 4. The soil reaction values obtained from the curves are divided by pile deflections, and these values are taken as the new soil modulus values. 5. Using this new soil modulus values the differential equation (2.8) is resolved. Pile deflections are refound. 6. The procedure has continued until the new soil modulus values found are equal to former values. 7. At this stage by using the last pile deflection values, from equations (2.12), (2.13), (2.14) slope of elastic curve, bending moment and shear forces are computed and printed along the pile. The flow chart of the computer program is given in Fig. 5.1.

As seen above there are basically five different parts in the program.

These are as follows:

1- Input
2- Assumption of soil modulus
3- The Iteration Procedure

A. Solution of the Differential Equation

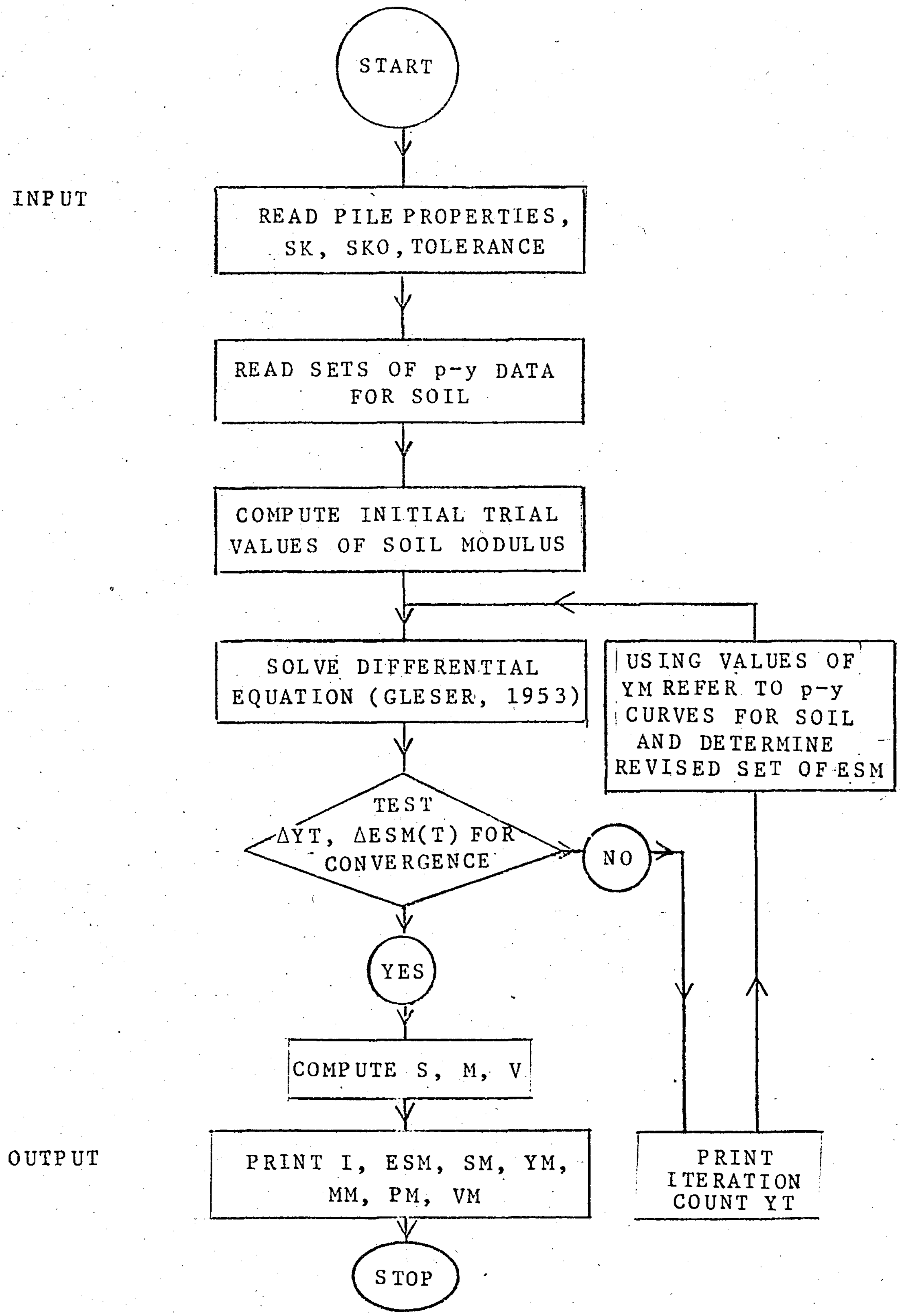


Fig. 5.1.– Flowing Chart of Computer Program for Laterally Loaded Pile

B. Calculation of Soil Reactions

C. Computation of Soil Modulus Values

D. Evaluation Criteria

4- Computation of bending moments, slope of elastic curve and shear force

5- Output

5.3. INPUT

In this section pile properties, soil properties and supplemental instructions are read.

Pile properties are:

EPI = flexural rigidity of the pile, EI

PL = pile length, L

DD = pile diameter, b

P = lateral load, P

H = height of application point of lateral load from ground surface, H

Supplemental instructions are:

T = number of pile sections

TOL = tolerance is such a value that when the convergence criteria is less than this value the iteration procedure ends

SKO = soil modulus at the ground surface taken as the first assumption

SK = slope of soil modulus linearly increasing with depth

As two problems for two different types of soils are considered, there are two different sets of input parameters, for the development of pile deflection soil reaction curves. One set is for clay and other set is for clean sand.

Case A. Clayey Soils

IZ = key for cyclic or static loading, IZ=1 for cyclic loading, and IZ=2 for static loading.

GAMA = effective unit weight of the soil, γ'

CC = undrained shear strength of clay, C

EPSO = strain at failure ε_{50} for which the typical values to be used are given in table.

Case B. Sandy Soils

DM = coefficient of lateral soil reaction m in N/cm^2, for which Terzaghi (1955) gave the values as shown in Table 3.4.

GAMA = effective unit weight of soil, γ'

FI = angle of internal friction of sand in radians, ϕ.

5.4. ASSUMPTION OF SOIL MODULUS

SK and SKO values define the assumption of soil modulus. There are two possible variations of soil modulus with depth. Soil modulus may be constant with depth, or may have a linear variation with depth.

The values of SKO and SK may be summarized as follows:

Case A. Constant Modulus with Depth

$$SKO \neq 0 \quad , \text{ and}$$
$$SK = 0$$

Case B. Linear Modulus Variation with Depth

$SKO = 0$, and

$SK \neq 0$

or

$SKO \neq 0$

$SK \neq 0$

5.5. THE ITERATION PROCEDURE

A. Solution of the Differential Equation

Using soil modulus values, in the first iteration from the first assumption and in the preceeding iterations from the values computed from curves as will be defined in part D, the fourth order differential equation (2.8) is solved. In the solution the equations from 2.15 to 2.40 are used (Gleser, 1953). After the solution, pile deflections along the pile are found.

B. Calculation of Soil Reactions

The subroutine is called to find soil reactions. Two different subroutines may be called according to soil type. In the case of soft clay subroutine SOFCL is called, and in the case of clean sand subroutine PSAND is called. Both of the subroutines develop soil reaction-pile deflection curves for every depth. With the pile deflections found in the above section, soil reactions are computed from the subroutines. The detailed information about subroutines will be given in the end of this section.

C. Computation of Soil Modulus Values

By dividing the soil reaction values computed in the previous section B, by the pile deflections found in section A, a new set of soil modulus values are obtained.

D. Evaluation Criteria

In the first iteration the first assumption of soil modulus values and the values found in step C are compared. If they are not equal, using the new soil modulus values found in step C, the procedures in steps A,B,C and D are repeated until the new soil modulus values are equal to the formerly calculated values, or the difference between them is less than a tolerance specified in the program.

5.6. COMPUTATION OF BENDING MOMENTS, SLOPE OF ELASTIC CURVE AND SHEAR FORCE

After the convergence is satisfied in section D, the slope of the elastic curve, the bending moment and the shear forces are computed from equations (2.12), (2.13) and (2.14).

5.7. OUTPUT

For every node, node number, soil modulus, pile deflection, slope of elastic curve, moment, shear force, soil reaction values are printed.

5.8. SUBROUTINE SOFCL

This subroutine develops pile deflection-soil reaction curves for soft clay which is recommended by Matlock (1970). The explanation and equations for these curves are given in Chapter 3. In the subroutine both static and cyclic loading cases are considered. The procedure is as follows:

1- The x_R value, which is the depth below soil surface to bottom of reduced strength zone is computed from equation (3.8).

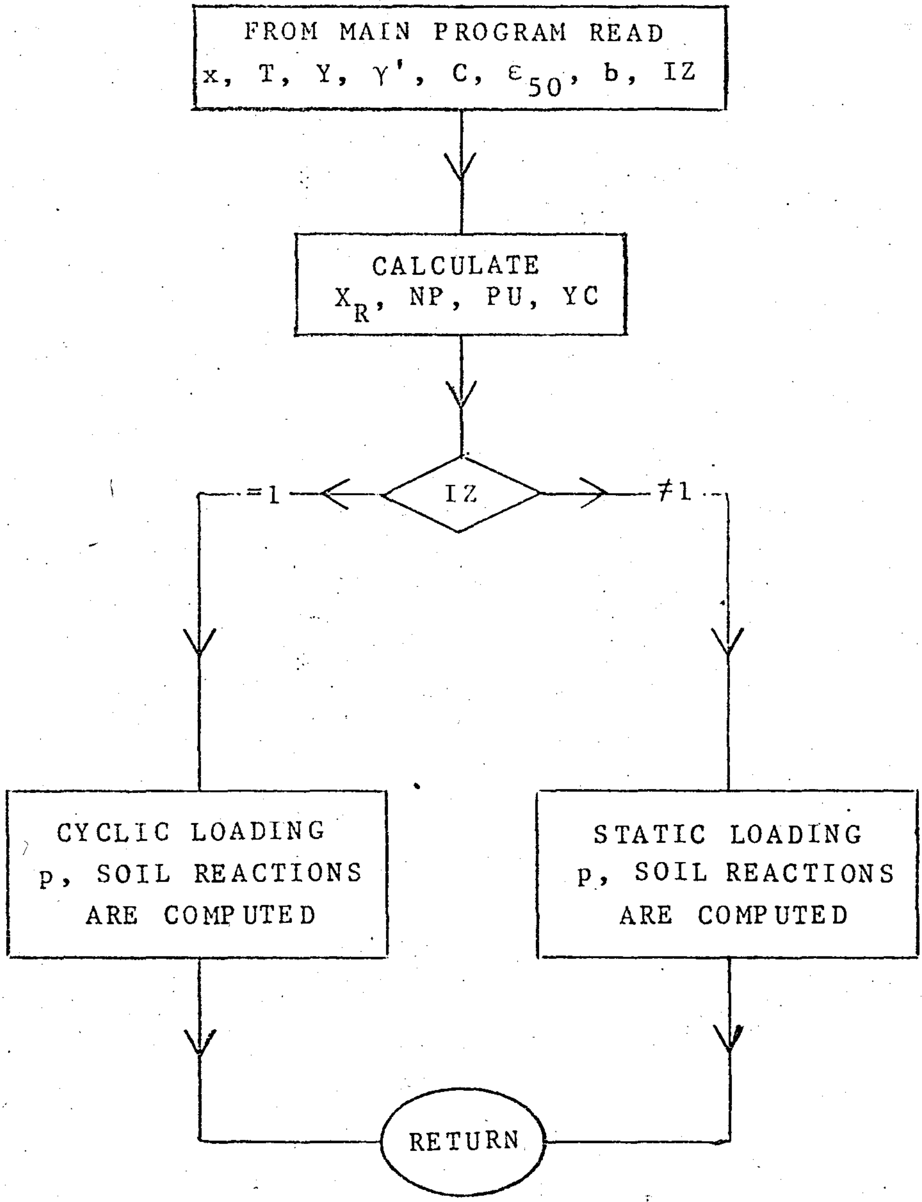


Fig. 5.2.— Flowing Chart of Subroutine SOFCL

2- Dimensionless coefficient of ultimate bearing capacity ENP is computed from equations (3.2) and (3.3) along the pile length.

3- Ultimate bearing resistance PU per unit length of pile is computed from equation (3.1).

4- Y_{50} value is computed from equation (3.4).

5- Soil reaction values are computed. According to IZ key either the equations for static loading case or cyclic-loading case are used.

In the case of static loading, with the pile deflections computed in the main program, from equations (3.5) and (3.6) soil reactions are computed. In the case of cyclic loading, from equations (3.7) and (3.9) soil reactions are computed along the pile length.

The flowing chart of subroutine is shown in Fig.5.2.

5.9. SUBROUTINE PSAND

This subroutine develops pile deflection soil reaction curves for every depth along the pile for clean sands (Reese and Parker, 1971). Detailed explanation of these curves are given in Chapter 3. FI, GAMA, DM values which are read in the main program and the computed pile deflections in the main program are attented to the subroutine. The procedure is as follows:

1- Ultimate bearing capacities from equations (3.24) and (3.25) are computed for every depth and the smaller value is chosen.

2- From equation (3.27) using the pile deflections found in the main program, soil reaction values are computed. The

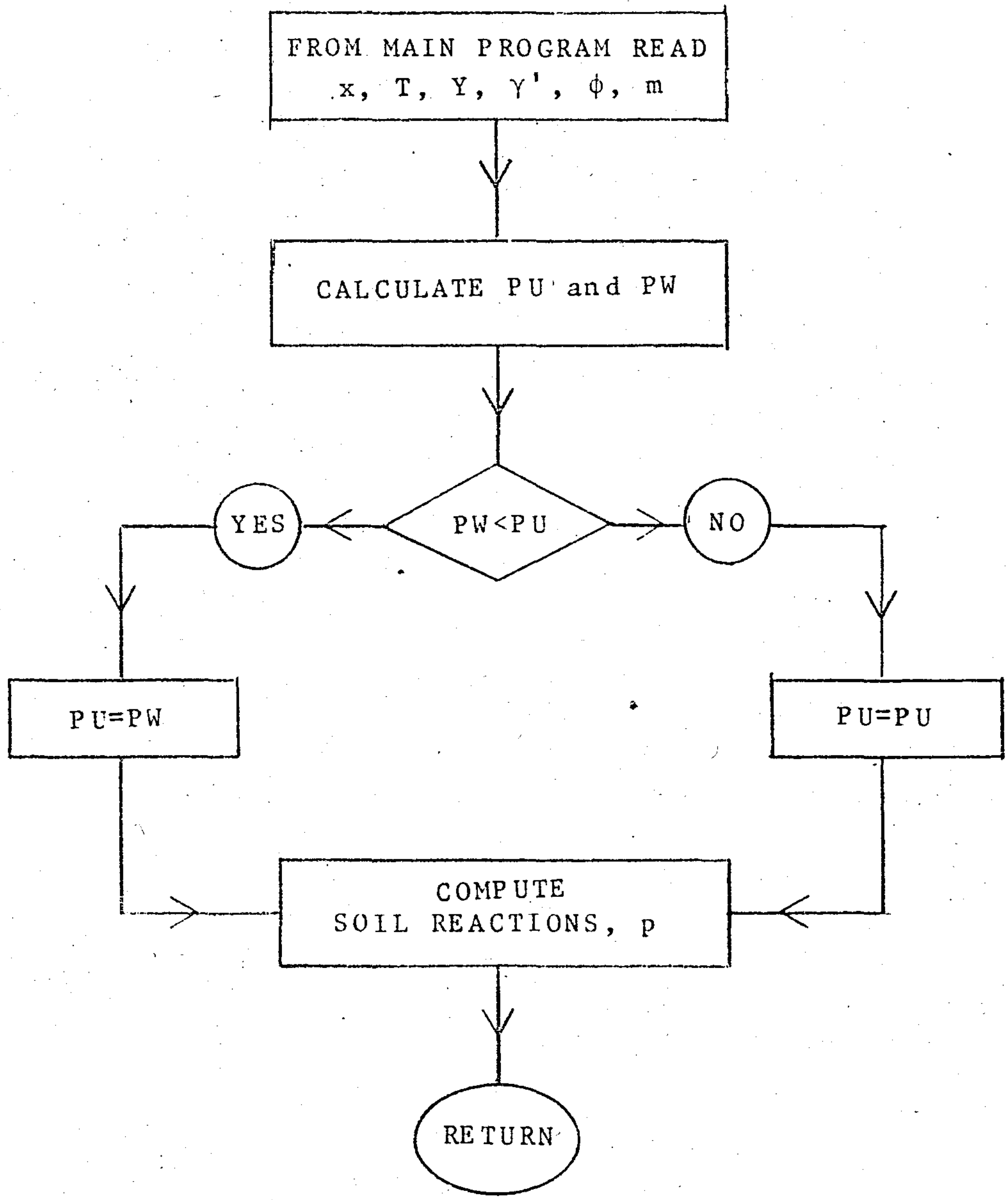


Fig. 5.3.– Flowing Chart of Subroutine PSAND

value of soil reactions are then attended to the main program. The flow chart of this subroutine is given in Fig.5.3.

5.8. SUMMARY

In this chapter, the explanation of the developed computer program for the solution of pile deflections and moments according to pile deflection - soil reaction method is given. Beside the main program, the subroutines developing the soil reaction - pile deflection curves for soft clays and sands are documented. The flowing charts of the main program and the two subroutines are given. The p-y curves for stiff clay are not programmed. The p-y curves for stiff clays may be programmed using the same technique for programming the p-y curves for soft clays.

6. NUMERICAL COMPUTATIONS

6.1. INTRODUCTION

Using the computer program developed in Chapter 5, four sets of problems are solved, two for soft clay and the other two for dense sand.

Effects of nonlinear soil behavior are studied by changing the magnitude of lateral load. In addition effects of various soil parameters such as strain at failure-ε_{50}, and lateral soil reaction coefficient-m are numerically evaluated.

In the problems a steel pipe-pile of 18.3 meters in length and 0.406 m in diameter is selected. Flexural rigidity of the pile is 9.076×10^{11} N-cm^2 and is constant along the pile. The properties of the pile and the soil are given in Fig. 6.1.

In problems 1 and 2, the pile is installed in a soft clay subsoil and in problems 3 and 4, the pile is installed in a dense sand subsoil. In problem 1, the strain at failure-ε_{50} is changed in order to study the effect of strain at failure. In problems 2 and 3, the magnitude of lateral load is changed and the effect of nonlinear soil behavior are studied. In

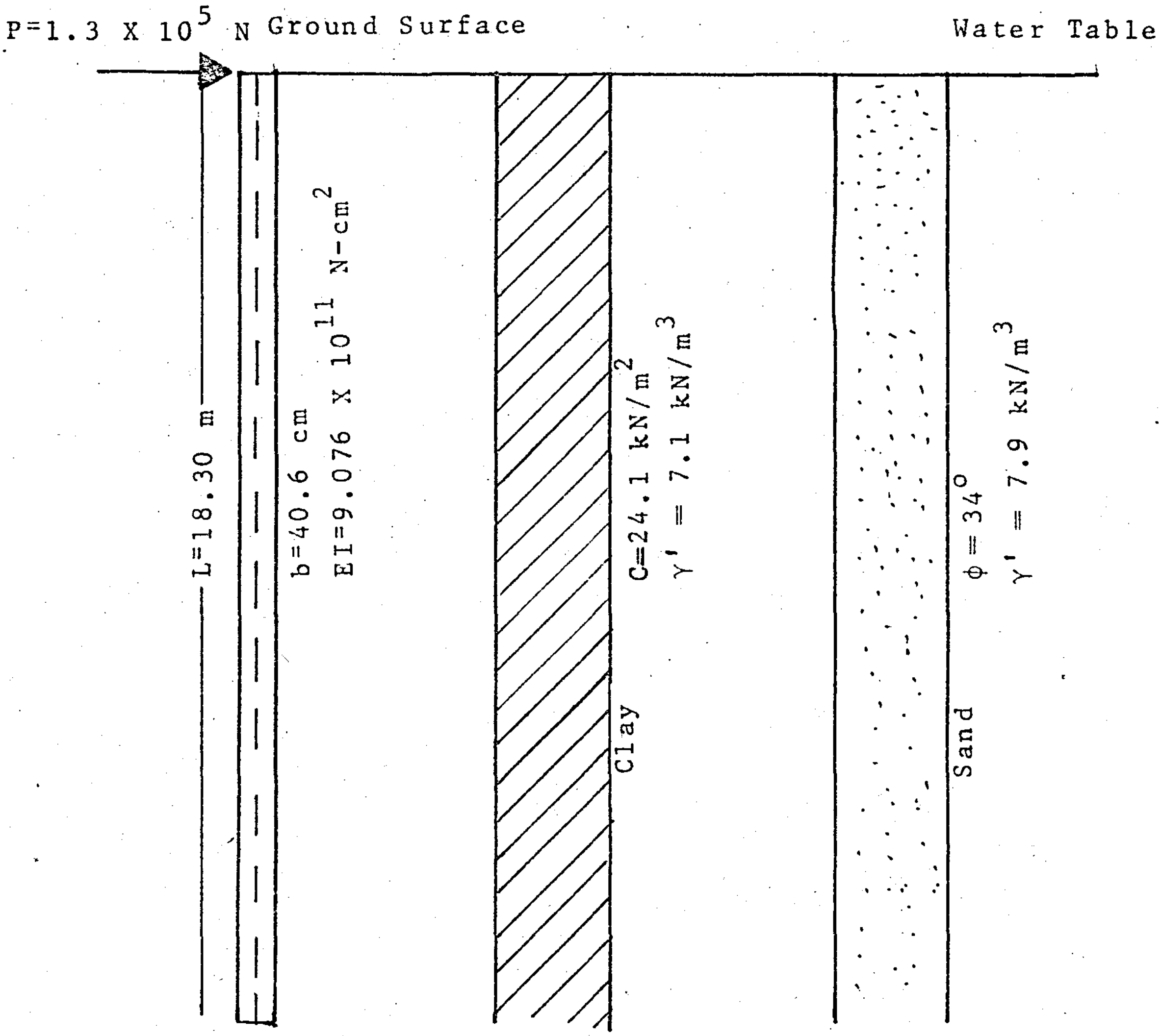


Fig. 6.1.– Pile Problem for Numerical Computations

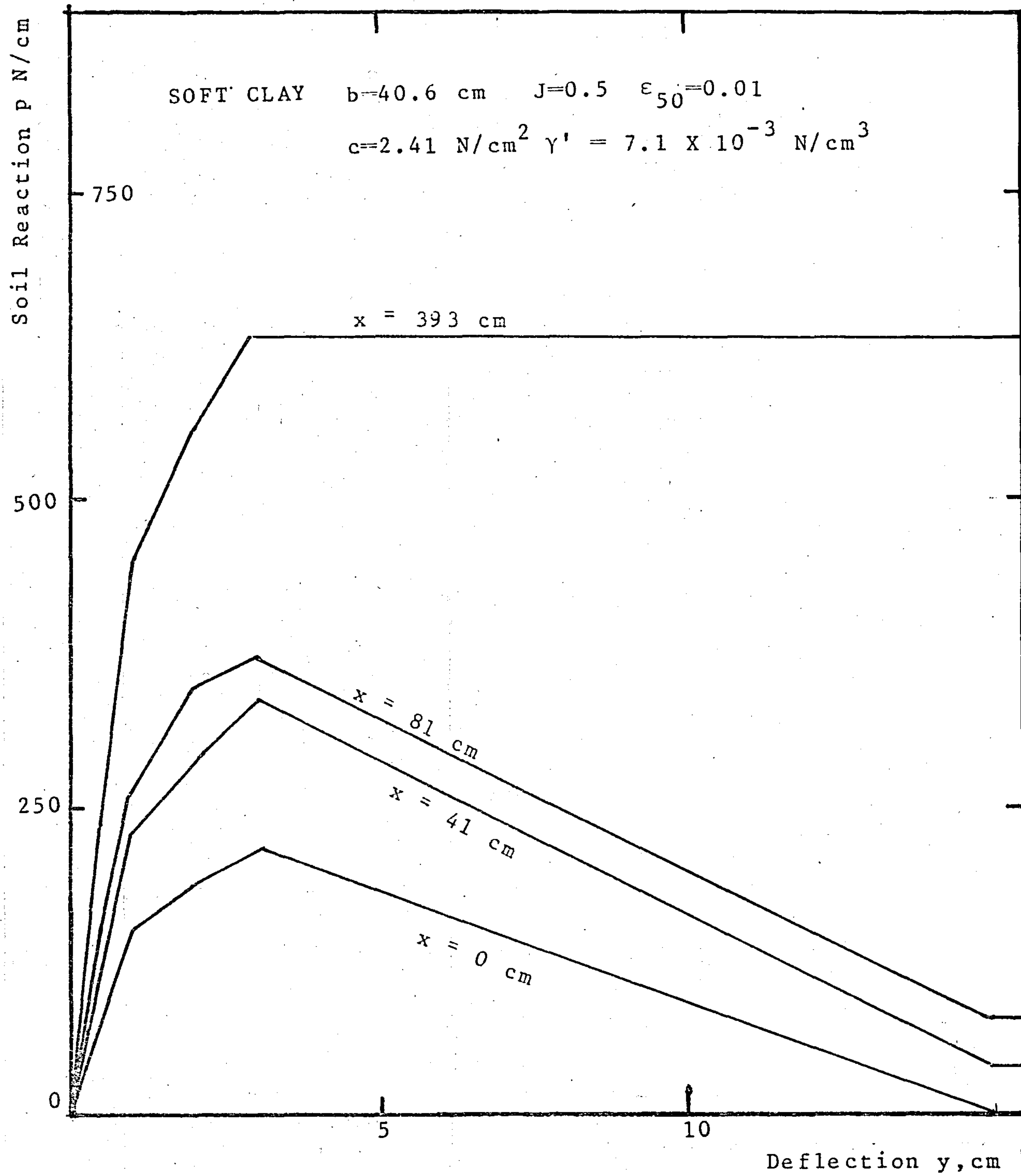


Fig. 6.2.– p-y Curves for 40.6 cm Diameter Pile in Soft Clay

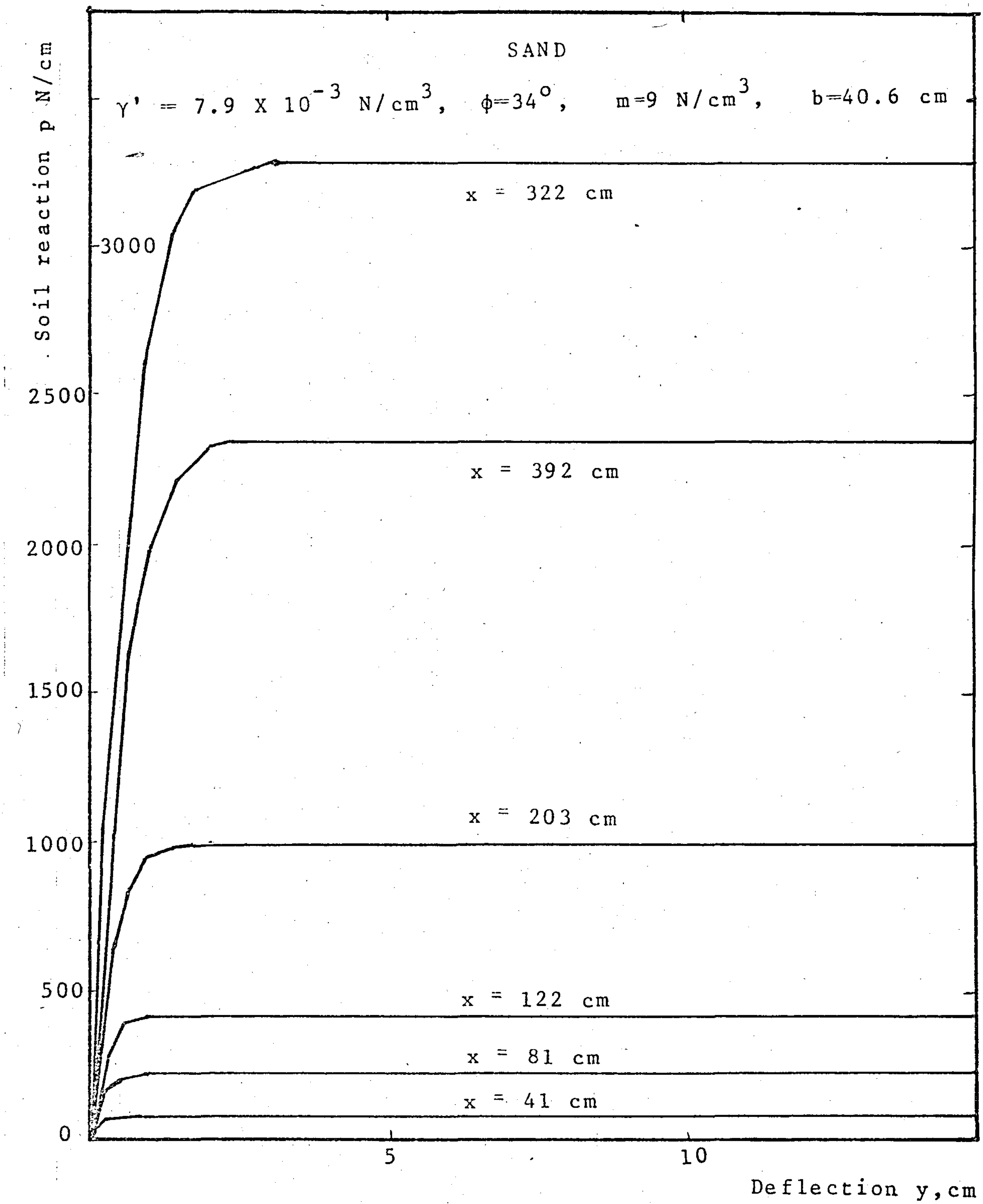


Fig. 6.3.— p-y Curves for 40.6 cm Diameter Pile in Dense Sand

problem 4, the effect of lateral soil reaction coefficient m is studied.

The soil reaction-deflection curves for soft clay and dense sand are shown in Fig. 6.2. and Fig. 6.3. respectively. In the figures 6.2 and 6.3, p-y curves for some depths are shown. The two subroutines developed in Chapter 5. compute p-y curves all along the pile length.

80 runs are made to solve four sets of problems. Computed pile top deflections and maximum bending moments for fixed and free headed piles, for every set of problems are plotted in Figures 6.4 through 6.11. The computer time for each run is 30 seconds.

The mathematical compatibility conditions of the solutions are explained in the end of this chapter.

6.2. PROBLEM NUMBER 1

The properties of clay subsoil in which the pile is installed are given as follows:

$C = 24.1\ kN/m^2$

$\gamma' = 7.1\ kN/m^3$

where,

C is the undrained shear strength of clay, and
γ' is the effective unit weight of clay.

C and γ' remained constant in every problem and only the strain at failure, ε_{50} values are changed in order to study the effect of strain at failure.

The range of change in ε_{50} is from 0.005 to 0.02 in the

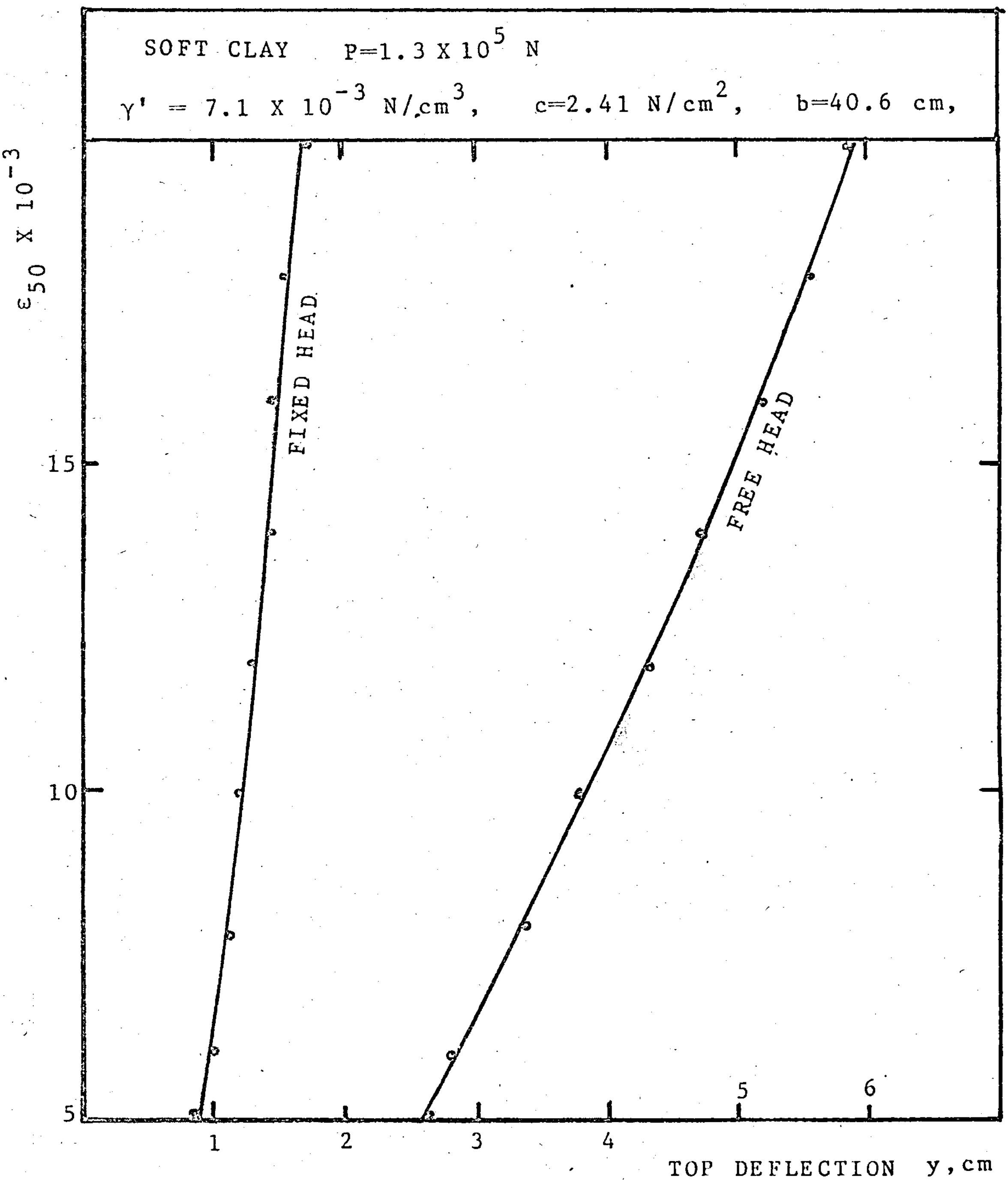


Fig. 6.4.– Lateral Top Deflections vs. Strain at Failure in Soft Clay

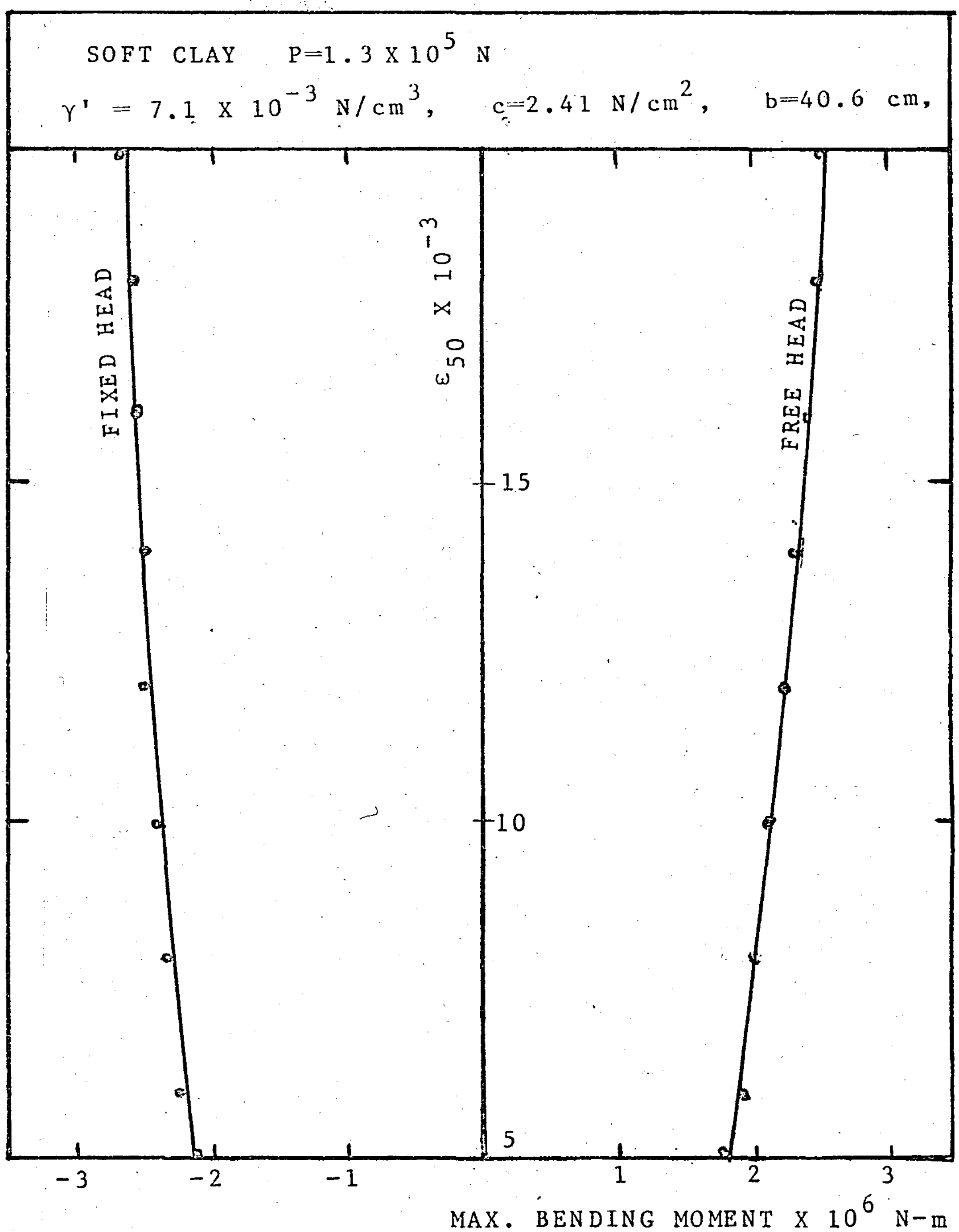


Fig. 6.5.— Maximum Bending Moments vs. Strain at Failure in Soft Clay

case of soft clay, and the corresponding deflections and maximum moments are computed for each case and plotted in Fig. 6.4 and Fig. 6.5. Both of the curves are nonlinear. As it is expected, when the strain at failure - ε_{50} increases, the top deflections and maximum bending moments increase.

6.3. PROBLEM NUMBER 2

In this problem soil parameters used in problem 1 remained constant and ε_{50} is taken as 0.01. On the other hand magnitude of lateral load is changed to investigate the non-linear soil behavior.

In figures 6.6 and 6.7 top deflections and maximum moments are plotted as functions of lateral load, P. For the free head pile case a remarkable result is obtained. After the lateral load is increased to such a value, the curves for deflection and moments flatten, and this indicates the rapid deterioration of the clay due to cyclic loading. The figure shows that, in the design of pile to sustain lateral loading, it is necessary to compute the pile response for a full range of loads. For the fixed head case the curves don't flatten like in the free head case. The values of maximum moments for fixed head case are greater than the values for free head case for the load range from 0.25×10^5 N to 2×10^5 N. This values are for this particular problem. But for every type of pile behavior, soil behavior, and loading condition, there will be a critical value. For the load range from 2.0×10^5 N to 4.0×10^5 N the maximum moments in free head case, have larger values than the maximum moments in fixed head case.

6.3. PROBLEM NUMBER 3

The pile installed in a sand is analyzed and the results are presented in Figs 6.8 through 6.11.

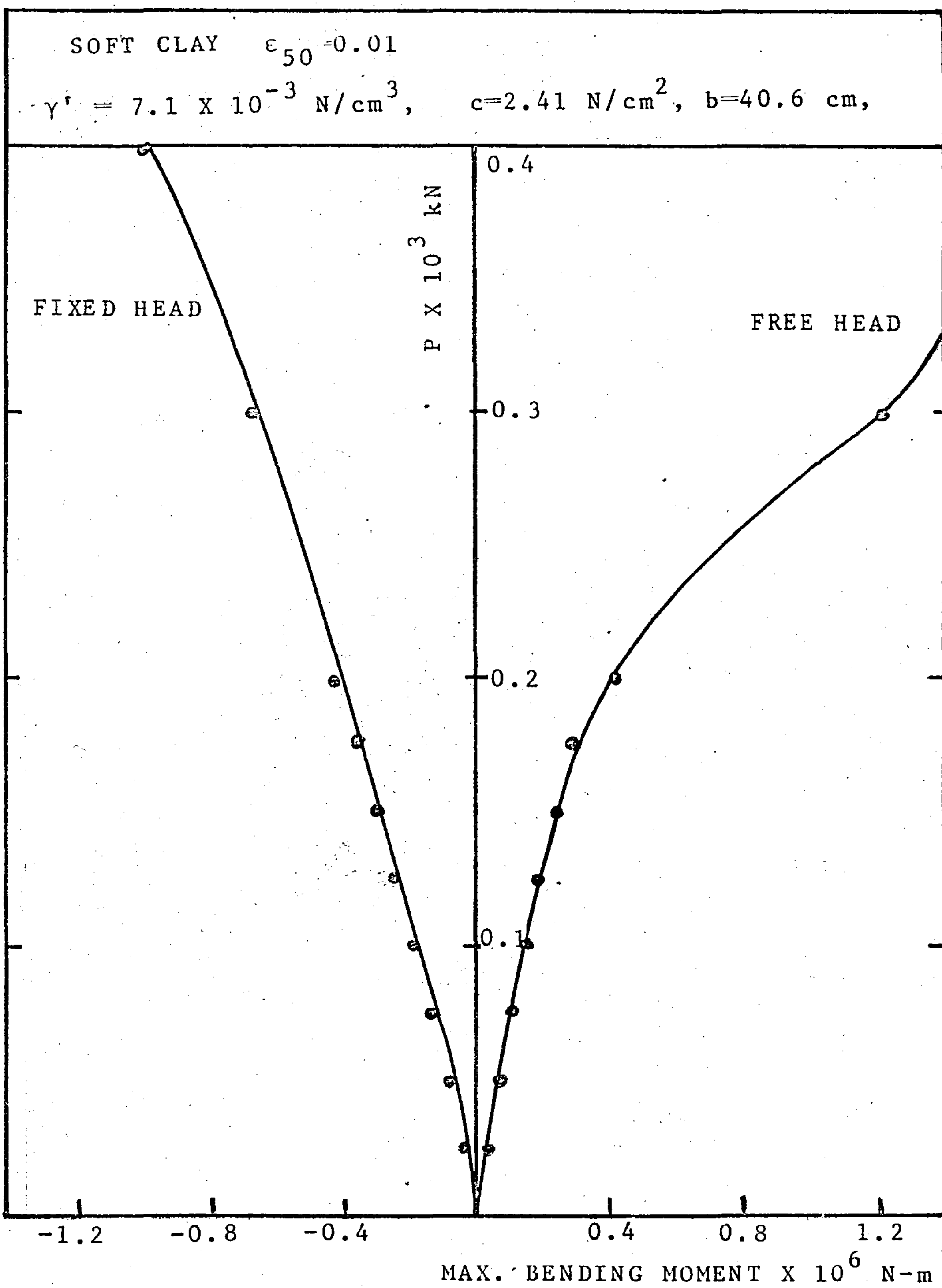


Fig. 6.6.— Maximum Bending Moments vs. Lateral Load in Soft Clay

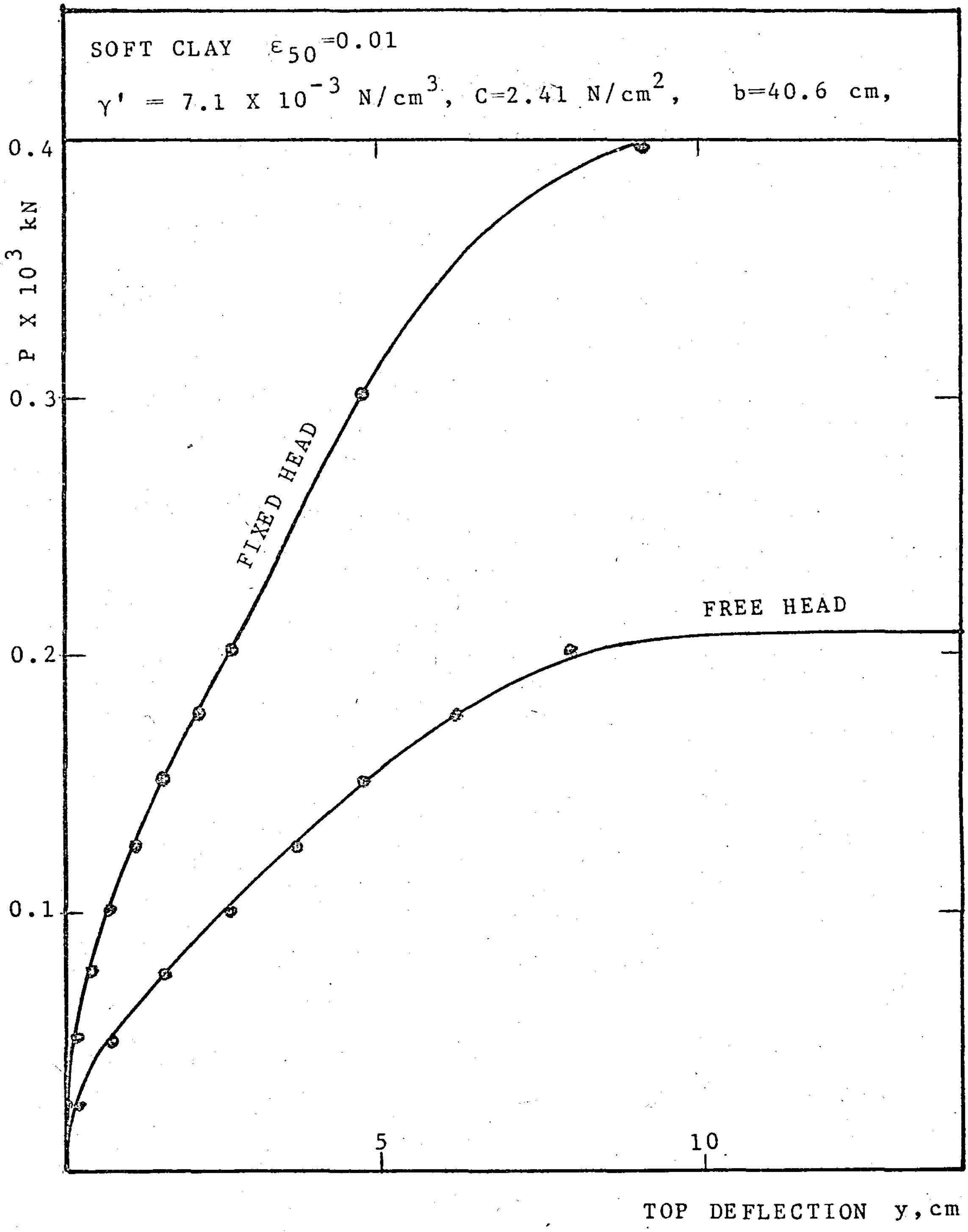


Fig. 6.7.- Lateral Top Deflections vs. Lateral Load in Soft Clay

The properties of sand subsoil in which the pile is installed are given as follows:

$\emptyset = 34^o$

$\gamma' = 7.9\ kN/m^3$

where,

$\emptyset$ is the angle of internal friction of sand, and

γ' is the effective unit weight of soil.

Coefficient of lateral soil reaction, m is taken as 9 N/cm^3. The water table is assumed at the ground level. In Figs. 6.8, 6.9 in the ordinate axes the lateral load is increased from 0.25×10^5 N to 4.0×10^5 N and corresponding deflections and maximum moments are plotted. The both curves are nonlinear. As the soil is dense sand, at first glance the curves seem to be linearly increasing but it would not be satisfactory to employ a design method which predicts a linear response under lateral load. Sand behaves in a more-favorable way than the clay. There is not any flattening in the curves due to increasing values of lateral load. In this case the maximum moment values for fixed piles are slightly more than maximum moments for free head piles for a full range of loads.

6.4. PROBLEM NUMBER 4

The effect of lateral soil reaction coefficient, m is studied in Figs 6.11 and 6.12, in the ordinate axes m values are plotted and the corresponding computed values of maximum moments and deflections are given. All the curves are nonlinear. For various sand densities three different m values are taken. In the dense sand range where m changes from 8.76 to 17.50 N/cm^3, the increase in deflection is very small and the curve is moderately a straight line having a very large slope. In this range the slope of the curves in Fig. 6.10 for free head case and fixed head case are more or less equal.

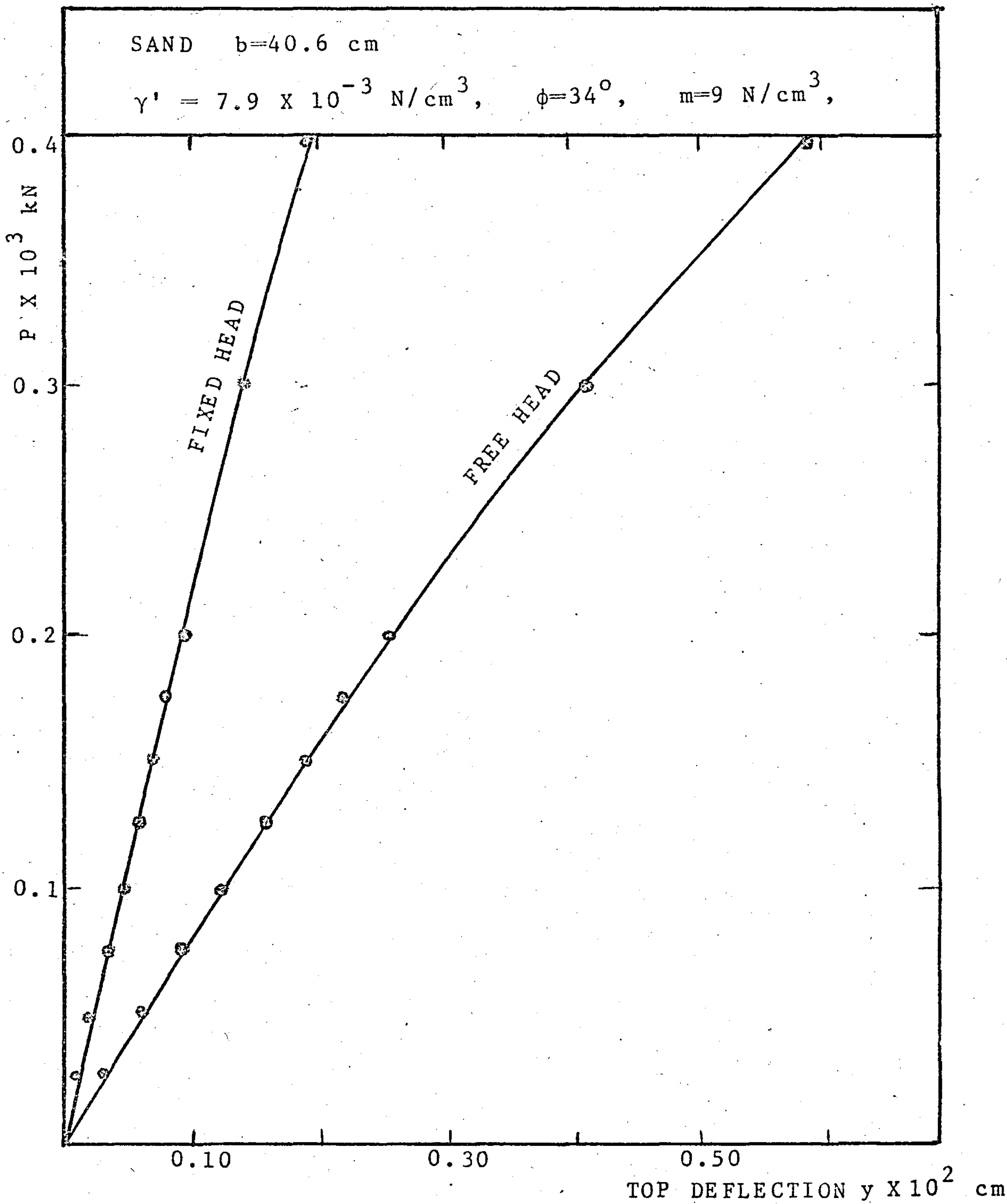


Fig. 6.8.– Lateral Top Deflections vs. Lateral Load in Dense Sand

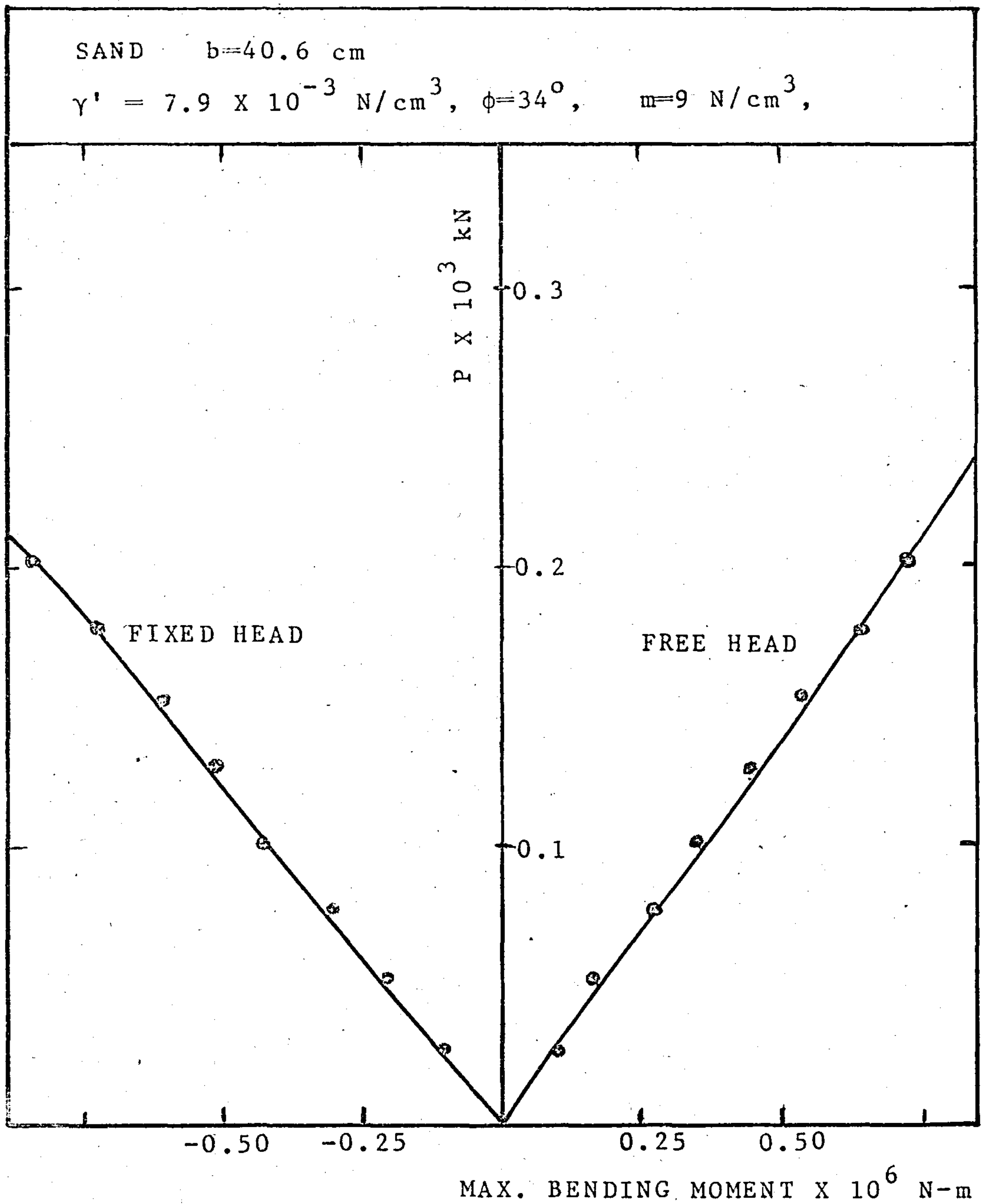


Fig. 6.9.— Maximum Bending Moments vs. Lateral Load in Dense Sand

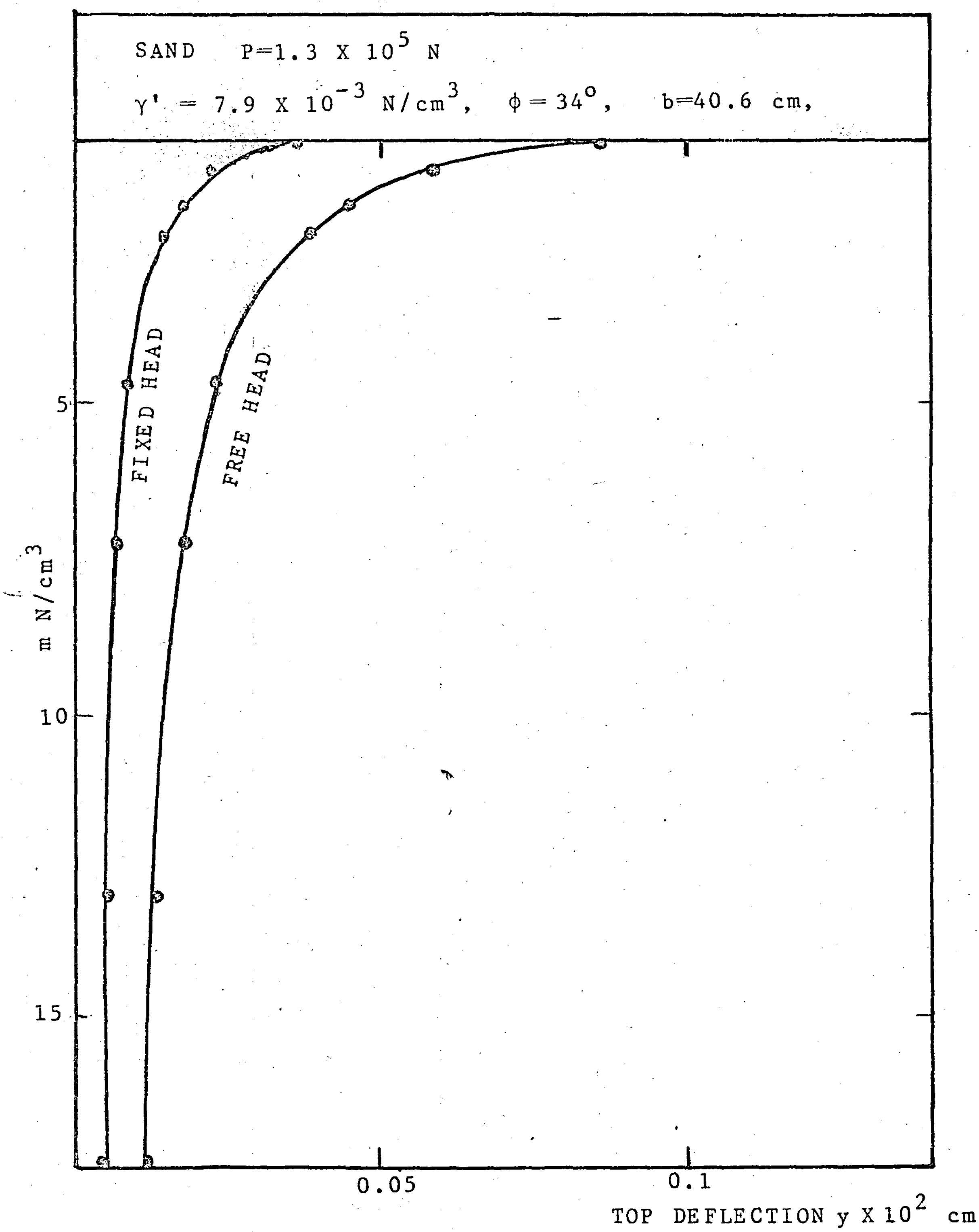


Fig. 6.10.– Lateral Top Deflections vs. Lateral Soil Reaction Coefficient in Dense Sand

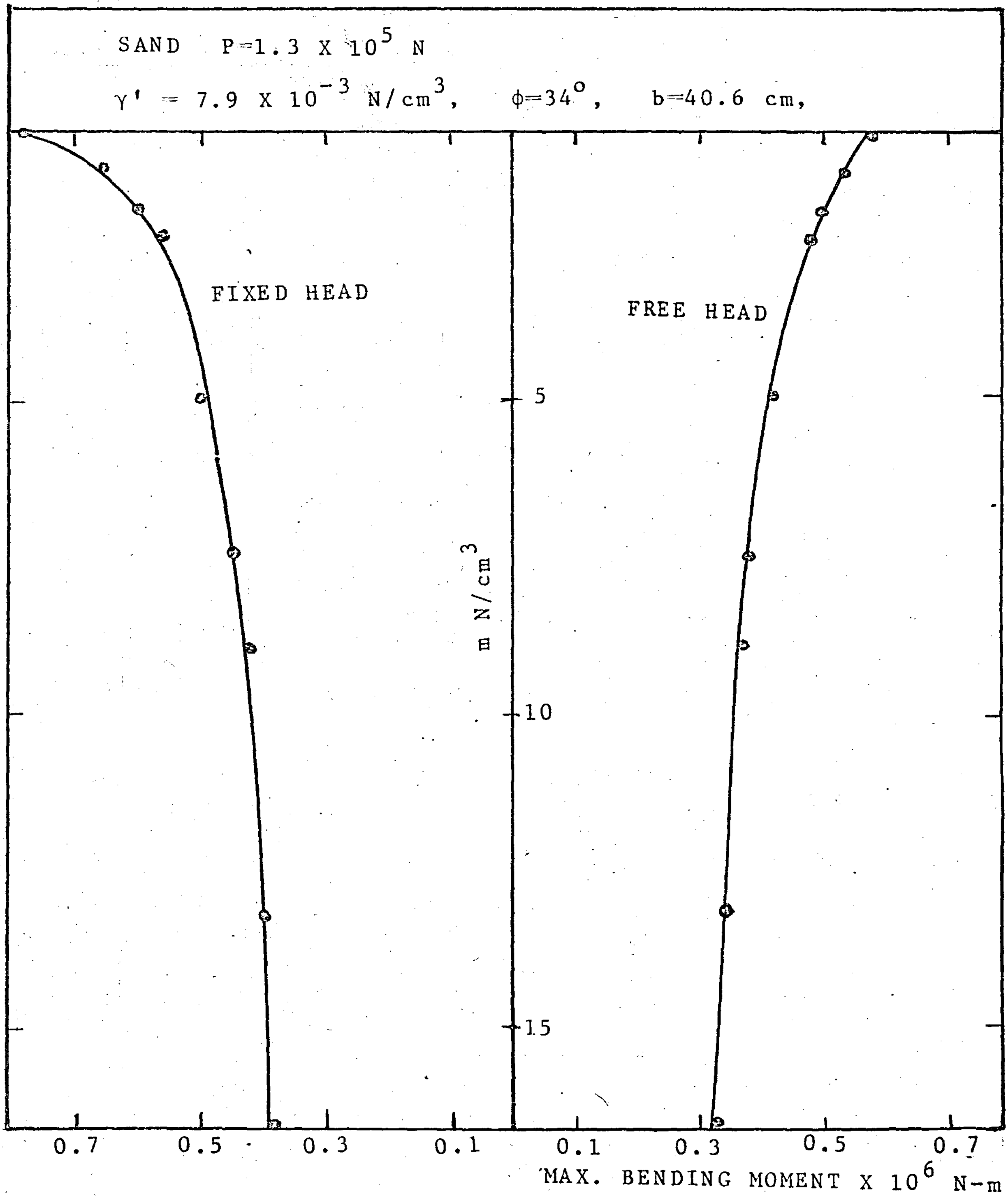


Fig. 6.11.- Maximum Bending Moments vs. Lateral Soil Reaction Coefficient in Dense Sand

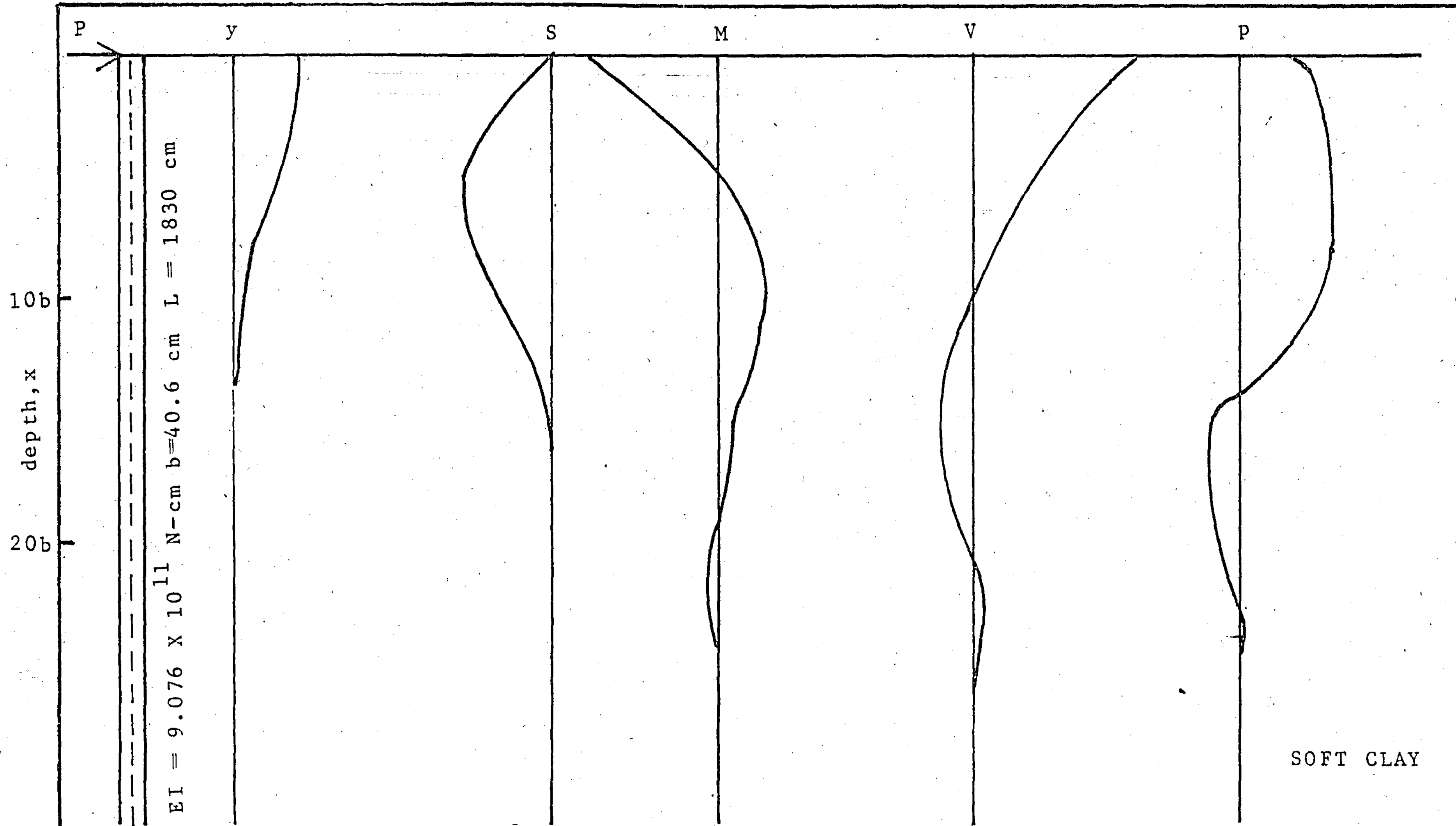
In the range of medium dense sand where lateral soil reaction coefficient, m is from 2.18 to 7.29 N/cm^3, the curves are nonlinear and they are flattened as the values of m decreases from 7.29 to 2.18 N/cm^3. The amount of increase in deflection is not very rapid for this range. The maximum deflection is obviously obtained for loose sand. The curves for free head case and fixed head case are nonlinear. The deflections increase very rapidly as the sand becomes looser. In full range of m values from 0.57 to 17.50 N/cm^3, the ratio of the top deflections computed in the case of free read pile, to the top deflection in the case of fixed head pile is between 2.61 to 2.76.

The maximum moments corresponding to different types of sands are shown in Fig. 6.11. The moment curves are nonlinear. For m values of 0.57 to 17.50 N/cm^3, the maximum moment values for fixed head pile are larger than maximum moment values for free head case. In loose sand the values of maximum moments increase more rapidly than in the medium dense sand. As the sand becomes looser the importance of m values increase so the exact value for lateral soil reaction coefficient must be chosen for every different problem.

6.5. MATHEMATICAL COMPATIBILITY OF THE SOLUTIONS

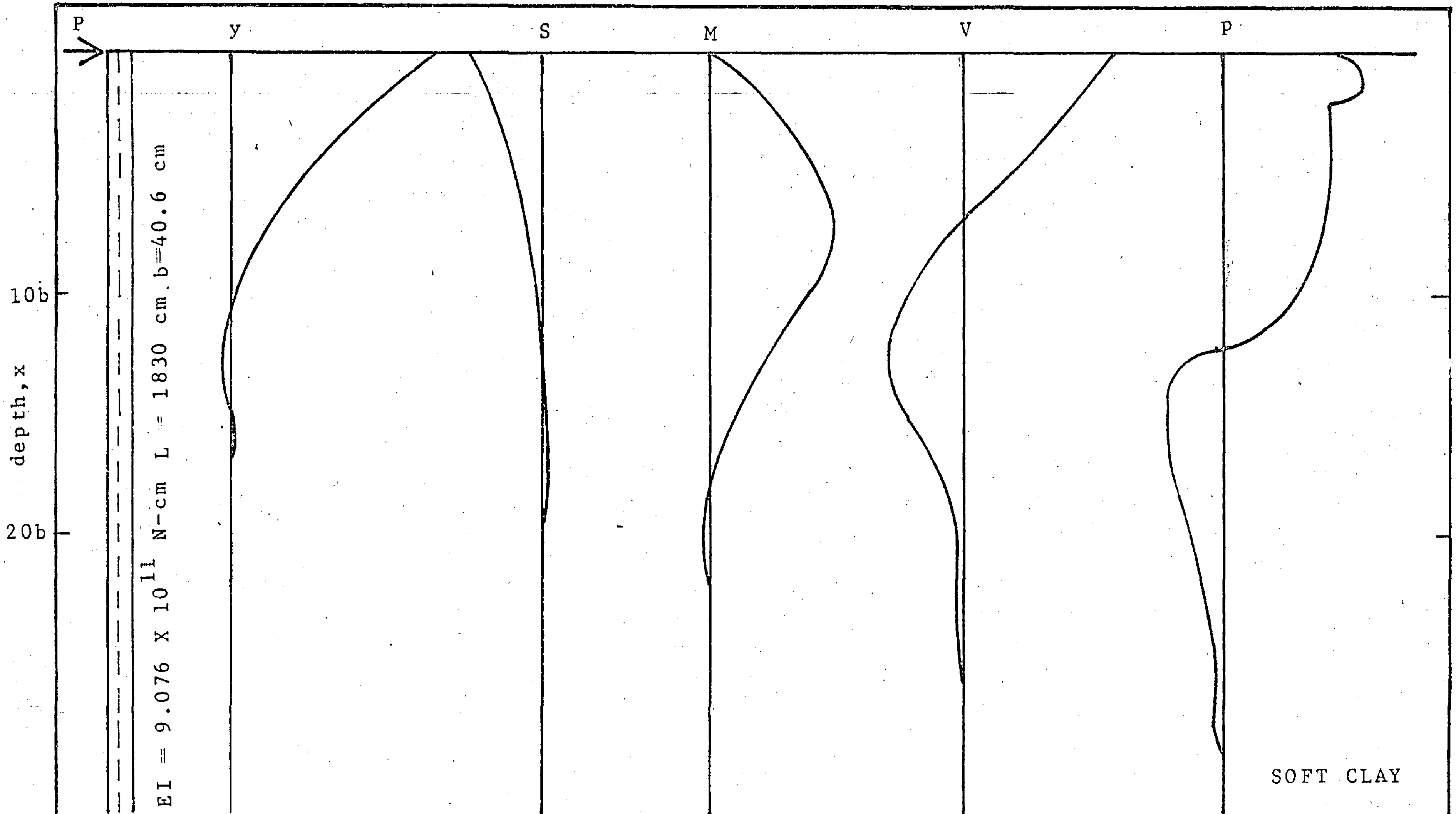
Lateral deflection, slope of elastic curve, moment distribution and shear force distribution are shown in Figs 6.12 and 6.13, along a free head and a fixed head pile respectively. In both of the cases the pile is subjected to a 1.3×10^5 N lateral load. The load is applied just at the top of the pile at the ground surface so the moment according to lateral load at the top of the pile is zero. As it is seen in both of the figures, all mathematical compatibility conditions are satisfied and the boundary conditions used for the solution of the differential equation are obviously met.

Moment and shear force at the bottom of the pile is zero



$P = 1.3 \times 10^5$ N, $\gamma' = 7.1 \times 10^{-3}$ N/cm^3, c=2.41 N/cm^2, ε_{50}=0.01,

Fig. 6.12.– Complete Solution for a Fixed Head Pile in Soft Clay



$P = 1.3 \times 10^5$ N $\gamma' = 7.1 \times 10^{-3}$ N/cm^3, $c = 2.41$ N/cm^2, $\varepsilon_{50} = 0.01$,

Fig. 6.13.– Complete Solution for a Free Head Pile in Soft Clay

and shear force is equal to lateral force at the top of the pile for both cases. For the free head case the fourth boundary condition is the value of moment known at the top of the pile and in this special case it is taken as zero. In the fixed head pile shown in Fig 6.13, the value of the slope of elastic curve is zero at the top of the pile. The difference in the upper part of the deflection curves in both cases are also compatible with the slope of elastic curves. In fixed head case as the slope of elastic curve is zero the top of the deflection curve is parallel to the pile axes. Also in both free and fixed head cases, maximum moments are found to be at the depth where shear force is zero. In free head case, maximum moments are computed at the depths equal to 7 to 8 pile diameters below the ground surface. It is observed that the upper 10 pile diameter zone of the soil is important in designing piles against lateral loads. Therefore improvement of soil conditions for depth of 10 pile diameters below the ground surface may be a reasonable and economical solution in the case of very weak surface soil conditions.

6.6. SUMMARY

In this chapter, problems of a steel pipe pile installed in a soft clay and in a dense sand are solved using developed computer program. Effects of nonlinear soil behavior are studied by changing the magnitude of lateral load. In addition effects of various soil parameters such as, strain at failure-ε_{50} and lateral soil reaction coefficient-m are numerically evaluated and the results are critically discussed.

Four sets of problems are solved using totally 80 runs, and computed pile top deflections and maximum bending moments for fixed and free headed piles, for every set of problems, are plotted in Figs 6.4 through 6.11.

In problem number 1, in the case of soft clay, ε_{50} which is the strain at failure is changed in order to study the effect of strain at failure.

In problem number 2, the pile is installed in a soft clay subsoil. The magnitude of lateral load is changed. After the lateral load is increased to a critical value, the curves for top deflections flatten, and this indicates the rapid deterioration of the clay due to cyclic loading.

In problem number 3, the pile is installed in a dense sand subsoil. The magnitude of the lateral load is changed. Sand behaves in a more favorable way than the clay.

In problem number 4, in the case of sand, the effect of lateral soil reaction coefficient, m is studied.

All the curves obtained are nonlinear because of the nonlinear soil behavior.

All mathematical compatibility conditions are satisfied and the boundary conditions used for the solution of the differential equation are met.

In free headed piles, maximum moments are computed at the depths equal to 7 to 8 pile diameters below the ground surface. It is observed that the upper 10 pile diameter zone is important in designing piles.

7. SUMMARY AND CONCLUSIONS

A rational approach to the analysis of a laterally loaded pile is presented. The method is very useful because detailed response of a pile under lateral loading could be predicted for a small cost. The results of the computations allow the dimensions of the pile to be selected so as to satisfy design requirements and to achieve maximum economy.

The advantages of using the method are as follows:

1. Changes in pile properties with depth, changes in diameter of the pile and changes in pile stiffness may be considered while analyzing the pile by the presented method.

2. Various boundary conditions may be treated. In this thesis only the free head and the fixed head cases are analyzed. However, the theory may also be applied to restrained-head case.

3. Various soil conditions may be considered. Through the use of p-y curves, it is possible to take into account changes in soil resistance as a function of pile deflection and as a function of depth.

The possible weakness of the method lies in uncertainties in regard to the prediction of p-y curves. Criteria are presented for soft to medium clay and for clean sands. While these criteria are thought to reflect the behavior of these soils under lateral loading, they are based on the analyses of a relatively small number of experiments exist in the literature. Criteria are not yet available for other types of soils.

In this thesis, a general computer program is developed which would give numerical solutions of lateral deflection, slope of elastic curve, bending moment and shear force along the laterally loaded pile. Soil properties are defined by a set of curves which give soil reaction as a function of pile deflection. Two subroutines are developed for soft clays (Matlock, 1970) and for clean sands (Parker and Reese, 1971). Both of the subroutines developed soil reaction-deflection curves for every depth along the pile.

A summary of procedure used in programming could be written as follows:

1. Assumption for soil modulus along the pile length is made.

2. Using this assumption, the fourth order differential equation (2.8) of laterally loaded pile is solved.

3. With the deflections found, the subroutine developing the soil reaction-deflection curves are called and corresponding soil reactions are computed for every depth.

4. The soil reaction values obtained from the curves are divided by pile deflections, and these values are taken as the new soil modulus values.

5. Using these new soil modulus values, the differential equation (2.8) is resolved. Pile deflections are refound.

6. The procedure has continued until the new soil modulus values are equal to former values.

7. At this stage by using the last pile deflection values, from equations (2.12), (2.13), (2.14) slope of elastic curve, bending moment and shear forces are computed and printed along the pile.

Problems of a steel pipe pile installed in a soft clay and in a dense sand are solved using developed computer program. Effects of non-linear soil behavior are studied by changing the lateral load. In addition effects of various soil parameters such as, strain at failure-ε_{50} and lateral soil reaction-m are numerically evaluated and the results are critically discussed.

Four sets of problems are solved using totally 80 runs. Computed pile top deflections and maximum bending moments, for every set of problems for fixed and free headed piles, are plotted in Figures 6.4 through 6.11.

In problem number 1, the pile is installed in a soft clay subsoil, strain at failure-ε_{50} is changed in order to study the effect of strain at failure. When the strain at failure-ε_{50} increases, the top deflections and maximum bending moments increase.

In problem number 2, the pile is installed in a soft clay subsoil. The magnitude of lateral load is changed and the effects of non-linear soil behavior are studied. After the lateral load is increased to a critical value, the curves for top deflections flatten, and this indicates the rapid deterioration of the clay due to cyclic loading.

In problem number 3, the pile is installed in a soft clay subsoil. The magnitude of lateral load is changed. Sand behaves in a more favorable way than the clay.

In problem number 4, in the case of sand, the effect of lateral soil reaction coefficient-m is studied.

All the curves obtained for maximum bending moments and pile top deflections are non-linear because of the non-linear soil behavior.

All mathematical compatibility conditions are satisfied and the boundary conditions used for solution of the differential equation (2.8) are met.

For free headed piles, maximum bending moments are computed at the depths equal to 7 to 8 pile diameters below the ground surface. It is observed that the upper 10 pile diameter zone is important in designing piles against lateral loads. Therefore improvement of soil conditions for depth of 10 pile diameters below the ground surface may be a reasonable and economical solution in the case of very weak surface soil conditions.

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APPENDIX

A.1. USER'S MANUAL

The computer program for the numerical analysis of laterally loaded piles is given in this chapter. SI units are used for the parameters used in the program. Force unit is Newton and Length unit is cm.

<u>INPUT</u>

1. Identification of problem
   1st card: TITLE (20A4)
2. Pile and Loading properties
   2nd card: PT,H,DD,PL,T,EPI (5F10.5,E20.8)
3. Soil and pile type, evaluation criteria
   3rd card: TOL,ISOIL,KODE (2I2,F10.5)
4. Assumption of soil modulus values
   4th card: SKO,SK (2F10.5)
5. Soil data
   For soft clays,

5th card: GAMA,CC,EPSO,IZ (3F10.5,I2)

For sands,

5th card: GAMA,FI,DM (3F10.5)

OUTPUT

1. TITLE
2. PL,P,H,T,SK,SKO,EPI
3. ISOIL,KODE,TOL
4. For soft clays

   GAMA,CC,EPSO,IZ

   For sands

   GAMA,FI,DM
5. NITE,YT
6. I,ESM,S,Y,TM,V,SR

SYMBOLS USED IN THE COMPUTER PROGRAM

TITLE : Title of the Problem

PT : Lateral Load, P

H : Distance of Application Point of P above the Ground Surface, H

DD : Pile Diameter, b

PL : Pile Length, L

T : Number of Equal Units into which L is divided, t

EPI : Flexural Rigidity, EI

ISOIL : Key specifying the type of soil (SOFT CLAY = 1, SAND = 2)

KODE : Boundary Condition Key (FREE HEAD = 1, FIXED HEAD = 2)

TOL : Tolerance (taken as 0.0001 for this problem)

SKO : Assumed soil modulus value at the ground surface

SK : Slope of linear Soil Modulus Variation

GAMA : Effective Unit Weight of Soil, γ'

CC : Undrained Shear Strength, c

EPSO : Strain at Failure, ε_{50}

IZ : Loading Key, CYCLIC = 2, STATIC = 1

FI : Angle of Internal Friction in Radians

DM : Coefficient of Lateral Soil Reaction, m

YT : Top Deflection of the Pile

NITE : Number of Iterations

ESM : Soil Modulus, E_s

S : Slope of Elastic Curve, S

Y : Pile Deflection, y

TM : Bending Moment, M

V : Shear Force, V

SR : Soil Reaction, p

A.2. COMPUTER PROGRAM

CE*LPILE(1).MAIN

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      C      THIS PROGRAM IS DEVELOPED BY GOKHAN BAYKAL AS MASTER THESIS.
      C      FEBRUARY,1982
      C      SOLVES PILE DEFLECTIONS,SLOPE OF ELASTIC CURVE,SHEAR FORCE,
      C      BENDING MOMENT AND SOIL REACTIONS ALONG LATERALLY LOADED PILES.
             DIMENSION ESM(150),A(150),B(300),Y(150),S(150),TM(150),
            1V(150),SR(150),YESM(150),PR(150)
             CHARACTER*80  TITLE
       900   READ(5,1,END=999)TITLE
             READ(5,2) P,H,DD,PL,T,EPI
             READ(5,3) ISOIL,KODE,TOL
             READ(5,4) SKO,SK
       1     FORMAT(A80)
       2     FORMAT(5F10.5,E12.5)
       3     FORMAT(2I2,F10.5)
       4     FORMAT(2F10.5)
             WRITE(6,15) TITLE
       15    FORMAT(1H1,///,20X,A80)
             WRITE(6,7)PL,P,H,T,SK,SKO,EPI
       7     FORMAT(15X,'PL=',F10.2,5X,'P=',F10.2,5X,'H=',F8.2,5X,'T=',F8.2,
            15X,'SK=',F8.6,5X,'SKO=',F8.6,5X,'EPI=',E12.5)
             IF(ISOIL.EQ.2) GO TO 101
             WRITE(6,8)ISOIL,KODE,TOL
       8     FORMAT(15X,'SOIL TYPE=SOFT CLAY=',I2,5X,'KODE(FREE HEAD=1,FIXED
            1HEAD =2)=',I2,5X,'TOLERANCE=',F10.5)
             READ(5,5) GAMA,CC,EPS0,IZ
       5     FORMAT(3F10.5,I2)
             WRITE(6,10)GAMA,CC,EPS0,IZ
       10    FORMAT(15X,'GAMA=',F10.6,5X,'CC=',F10.6,5X,'EPS0=',F8.6,5X,
            1,'IZ=',I2)
             GO TO 102
       101   WRITE(6,9) ISOIL,KODE,TOL
       9     FORMAT(15X,'SOIL TYPE=SAND=',I2,5X,'KODE(FREE HEAD=1,FIXED HEAD=
            12)=',I2,5X,'TOLERANCE=',F10.5)
             READ(5,6) GAMA,FI,DM
       6     FORMAT(3F10.5)
             WRITE(6,11) GAMA,FI,DM
       11    FORMAT(15X,'GAMA=',F10.6,5X,'FI=',F8.4,5X,'DM=',F8.4)
       102   CONTINUE
             IT=T+1
             IN=IT+1
             IM=IN+1
             X=PL/T
             DO 50 I=1,IT
             Z=I-1
             J=IN-I
             IF(I.EQ.1) GOTO41
             GO TO 42
       41    YESM(J)=SKO
             GO TO 50
       42    YESM(J)=(Z*X*SK)+SKO
       50    CONTINUE
             NITE=0
       56    CONTINUE
             NITE=NITE+1
             DO 13 I=1,IT
       13    ESM(I)=YESM(I)
      C      COMPUTE A VALUES
             DO 100 I=1,IT
             A(I)=ESM(I)*((PL/T)**4)/EPI
       100   CONTINUE
      C      COMPUTE B VALUES
             B(1)=2./(2.+A(1))
             B(2)=1./(5.+A(2)-4.*B(1))
             B(3)=B(2)*(4.-2.*B(1))
             B(4)=1./(6.+A(3)-B(1)-B(3)*(4.-2.*B(1)))
             B(5)=B(4)*(4.-B(3))
             NL=2.*T+1.
             MN=1
             DO 200 MM=6,NL
             GOTO (20,30) ,MN
       20    MO=MM/2+1
             B(MM)=1./(6.+A(MO)-B(MM-4)-B(MM-1)*(4.-B(MM-3)))
             MN=2
             GO TO 200
       30    B(MM)=B(MM-1)*(4.-B(MM-2))
             MN=1
       200   CONTINUE
             KL=NL+1
             ML=KL+1
             B(KL)=(PL**2.)*H*P/(T**2.*EPI*B(NL-1)*(2.-B(NL-2)))
             B(ML)=(B(NL-3)-1.+B(NL)*(2.-B(NL-2)))/(B(NL-1)*(2.-B(NL-2)))
             IF(KODE.EQ.2)GO TO 33
      C   ** F R E E   H E A D   C A S E **
      C      LATERAL LOAD P AND MOMENT M AT THE TOP OF THE PILE
      C      ARE KNOWN
             Y(IN)=((-2.*P*PL**3.)/(EPI*T**3.)-B(KL)*B(KL)*B(KL-2)*(2.*B
            1(KL-3)*B(KL-6)-B(KL-5)*B(KL-3)))/(-B(ML)+2.+((2.-B(KL-5)).
            2B(KL-3)+B(KL-6))*(B(KL-2)*B(ML)-B(NL))*(2.*B(KL-5))*B(KL-4))
             Y(IM)=-B(KL)+B(ML)*Y(IN)
             GO TO 43
       33    CONTINUE
      C      ** F I X E D   H E A D   C A S E **
      C      LATERAL LOAD AND SLOPE OF ELASTIC CURVE AT THE TOP OF THE PILE
      C      ARE KNOWN
             Y(IM)=(-2.*PL**3.*P*(1.+B(KL-4)-B(KL-3)*B(KL-1)))/(T**3.*
            1EPI*(-1.+B(KL-2)*B(KL-6)+B(KL-2)*B(KL-4)*B(KL-6)-B(KL-2)*
            2B(KL-3)*B(KL-5)-B(KL-4)+B(KL-1)*B(KL-3)))
             Y(IN)=(-B(KL-3)*B(KL-2)/(1.+B(KL-4)-B(KL-1)*B(KL-3)))*Y(IM)
       43    DO 300 I=1,IN
             IL=IN-I
             ILO=2.*IL
             IF(IL.EQ.1) GOTO 301
             Y(IL)=-B(ILO-2)*Y(IL+2)+B(ILO-1)*Y(IL+1)
       300   CONTINUE
       301   Y(1)=-B(1)*Y(3)+2.*B(1)*Y(2)
             YM1=-Y(2)+2.*Y(1)
             YM2=Y(3)-2.*Y(2)+2.*YM1
             NT=T
             IF(ISOIL.EQ.2) GO TO 103
             CALL SOFCL(X,NT,Y,PR,GAMA,CC,EPS0,DD,IZ)
             GO TO 104
       103   CALL PSAND(X,NT,Y,PR,GAMA,FI,DD,DM)
       104   CONTINUE
             T=NT
             DO 63 I=1,IT
       63    YESM(I)=PR(I)/Y(I)
```

```
 66   FORMAT(15X,I3,' ITERATION',5X,'TOPDEFLECTION=',F10.5)
      KT=T
      DR=TESM(KT)-ESM(KT)
      T=KT
      IF(ABS(DR).LE.TOL) GO TO 55
      GO TO 56
 55   CONTINUE
C     COMPUTE S AND M VALUES
      S(1)=-(T/(2.*PL))*(YM1-Y(2))
      TM(1)=-((EPI*T**2.)/(PL**2.))*(Y(2)-2.*Y(1)+YM1)
      DO 400 I=2,IT
      S(I)=-(T/(2.*PL))*(Y(I-1)-Y(I+1))
      TM(I)=-((EPI*T**2.)/(PL**2.))*(Y(I+1)-2.*Y(I)+Y(I-1))
 400  CONTINUE
C     COMPUTE V VALUES
      V(1)=-((EPI*T**3.)/(2.*PL**3.))*(-Y(3)+2.*Y(2)-2.*YM1+YM2)
      V(2)=-((EPI*T**3.)/(2.*PL**3.))*(-Y(4)+2.*Y(3)-2.*Y(1)+YM1)
      DO 500 I=3,IT
      V(I)=-((EPI*T**3.)/(2.*PL**3.))*(-Y(I+2)+2.*Y(I+1)-2.*Y(I-1)
     1+Y(I-2))
 500  CONTINUE
C     COMPUTE SOIL RESISTANCE VALUES
      DO 600 I=1,IT
      SR(I)=-ESM(I)*Y(I)
 600  CONTINUE
      WRITE(6,12)
 12   FORMAT(1H1,'NODE',9X,'ESM',21X,'S',17X,'Y',15X,'TM',
     117X,'V',20X,'SR')
      WRITE(6,23)(I,ESM(I),Y(I),S(I),TM(I),V(I),SR(I),I=1,IT)
 23   FORMAT(/I4,6F19.7)
      GO TO 900
 999  STOP
      END
```

CE*LPILE(1).SOFCL

```
      SUBROUTINE SOFCL(H,N,YY,P,GAMA,C,EPS0,D,IZ)
      DIMENSION P(100),X(100),PU(100),ENP(100)
      DIMENSION YY(100)
      XR=6.*D/(GAMA*D/C+.50)
      NPI=N+1
      DO 27 I=1,NPI
      X(I)=(N-I+1)*H
      IF(X(I).LE.D*5) GO TO 2
      IF(X(I).GT.XR) GO TO 3
      ENP(I)=3+GAMA*XR/C+0.5*X(I)/D
      GO TO 24
 2    ENP(I)=3.
      GO TO 24
 3    ENP(I)=9.
 24   PU(I)=ENP(I)*C*D
 27   CONTINUE
      YC=2.5*EPS0*D
      IF(IZ.EQ.1)GO TO 29
C     SOFT CLAY SHORT TERM STATIC LOADING CASE
      DO 100 I=1,NPI
      Y=YY(I)
      IF(ABS(Y).LT.8*YC)GO TO 12
      GO TO 13
 12   P(I)=(0.5*PU(I)*ABS((Y/YC))**0.3333)*Y/ABS(Y)
      GO TO 100
 13   P(I)=PU(I)*Y/ABS(Y)
 100  CONTINUE
      RETURN
 29   CONTINUE
C     SOFT CLAY UNDER CYCLIC LOADING WHERE EQUILUBRIUM HAS BEEN REACHED
      DO 200 I=1,NPI
      Y=YY(I)
      IF(ABS(Y).LT.3*YC) GO TO 21
      IF(X(I).LE.0.1) GO TO 25
      IF(X(I).GT.XR) GO TO 22
      IF(ABS(Y).GT.15*YC)GO TO 23
      P(I)=(.06*(1.-X(I)/XR)*(3.-Y/YC)+.72)*PU(I)*Y/ABS(Y)
      GO TO 200
 20   P(I)=.5*PU(I)/YC*Y
      GO TO 200
 21   P(I)=(0.5*PU(I)*(ABS(Y)/YC)**0.333)*Y/ABS(Y)
      GO TO 200
 22   P(I)=.72*PU(I)*Y/ABS(Y)
      GO TO 200
 25   P(I)=0.06*PU(I)*(15.-Y/YC)*Y/ABS(Y)
      GO TO 200
 23   P(I)=.72*X(I)/XR*PU(I)*Y/ABS(Y)
 200  CONTINUE
      RETURN
      END
```

CE*LPILE(1).PSAND

```
      SUBROUTINE PSAND(H,N,Y,P,GAMA,FI,D,DM)
      DIMENSION X(101)
      DIMENSION P(101),Y(101)
      AKO=0.5
      ALFA=0.5*FI
      BETA=0.7855+ALFA
      ZETA=0.7855-ALFA
      TF=TAN(FI)
      TA=TAN(ALFA)
      TB=TAN(BETA)
      TZ=TAN(ZETA)
      AKA=TZ*TZ
      AKP=TB*TB
      NPI=N+1
      DO 28 I=1,NPI
      X(I)=(N-I+1)*H
      WA=AKP**2+1.
      WB=AKP**3+2.*AKO*TF*WA-AKA
      PF=GAMA*D*X(I)*WB
      WC=AKP*TA+AKO*(TF-TA)
      WD=D*(AKP-AKA)+X(I)*TB*WC
      PW=GAMA*X(I)*WD
      IF(PW.LT.PF) GO TO 15
      PU=PF
      GO TO 16
 15   PU=PW
 16   CONTINUE
      Z=((DM*X(I)*Y(I))/PU)*3.14159/180.
      P(I)=PU*TANH(Z)
 28   CONTINUE
      RETURN
      END
```

```
      LATERALLY LOADED PILE PROBLEM
PL=   1850.00       P= 130000.00      H=        .00      T=    50.00      SK= .200000     SKO= .000000      EPI=  .90760+012
SOIL  TYPE=SAND= 2       KODE(FREE HEAD=1,FIXED HEAD=  2)= 1     TOLERANCE=       .00001
GAMA=     .007900      FI=      .5900      DM= 15.0000
  1.ITERATION        TOPDEFLECTION= -13.71940
  2.ITERATION        TOPDEFLECTION= -12.44295
  3.ITERATION        TOPDEFLECTION= -12.32176
  4.ITERATION        TOPDEFLECTION= -12.31359
  5.ITERATION        TOPDEFLECTION= -12.31204
  6.ITERATION        TOPDEFLECTION= -12.31204
```

NODE	ESM	S	Y	TM	V	SP
1	479.0921097	-.0934886	.0005458	.0000000	.0000000	44.7896495
2	469.5103951	-.0735126	.0005464	-29999.2145996	1451.2413483	34.5149307
3	459.9286537	-.0534923	.0005491	-106230.8652344	2533.0519714	24.6026556
4	450.3468628	-.0333153	.0005556	-215417.9863281	3257.8106384	15.0034271
5	440.7650528	-.0128203	.0005669	-344702.5976562	3635.7840271	5.6507249
6	431.1832085	.0081835	.0005836	-481557.2148437	3674.6356201	-3.5285869
7	421.6013298	.0298980	.0006057	-613685.9218750	3379.4190674	-12.6050404
8	412.0194206	.0525183	.0006327	-728930.6875000	2752.7828979	-21.6385520
9	402.4374619	.0762144	.0006639	-815189.6328125	1795.5439453	-30.6715338
10	392.8554602	.1011137	.0006977	-860364.5000000	507.3207054	-39.7230735
11	383.2734489	.1272828	.0007322	-852325.5078125	-1112.4298706	-48.7871353
12	373.6913757	.1547100	.0007651	-778934.6328125	-3063.1302185	-57.8137794
13	364.1092529	.1832867	.0007935	-628104.3750000	-5342.2977295	-66.7363977
14	354.5270882	.2127906	.0008139	-387878.4492187	-7943.9671631	-75.4400167
15	344.9448891	.2428669	.0008227	-46605.9902344	-10857.4837646	-83.7756824
16	335.3626518	.2730120	.0008154	406889.3515625	-14066.0554199	-91.5580120
17	325.7804108	.3025565	.0007874	983029.2500000	-17545.3381348	-98.5669804
18	316.1981621	.3306502	.0007335	1691208.0937500	-21262.4372559	-104.5509720
19	306.6159592	.3562477	.0006482	2539439.6250000	-25174.6650391	-109.2312298
20	297.0338173	.3780972	.0005257	3533993.5625000	-29228.4035645	-112.3076515
21	287.4517746	.3947307	.0003601	4678958.7500000	-33359.3793945	-113.4660492
22	277.8698692	.4044584	.0001453	5975900.0625000	-37492.2172852	-112.3868093
23	268.2881355	.4053661	-.0001249	7423389.0000000	-41538.9907227	-108.7549067
24	258.7065887	.3953173	-.0004564	9016554.1250000	-45400.5312500	-102.2711802
25	249.1252327	.3719606	-.0008548	10746707.8750000	-48967.5336914	-92.6647692
26	239.5440369	.3327425	-.0013256	12600977.6250000	-52121.6596680	-79.7064743
27	229.9629421	.2749261	-.0018733	14562010.7500000	-54737.0527344	-63.2228165
28	220.3818321	.1956171	-.0025018	16607727.2500000	-56682.9257812	-43.1104636
29	210.8005199	.0917962	-.0032139	18711199.5000000	-57825.9404297	-19.3506913
30	201.2187996	-.0396412	-.0040114	20840586.0000000	-58034.0166016	7.9765624
31	191.6361427	-.2018381	-.0048945	22959289.5000000	-57180.2124023	38.6794715
32	182.0517216	-.3979214	-.0058621	25026177.5000000	-55146.9643555	72.4422722
33	172.4643040	-.6309417	-.0069110	26996047.2500000	-51830.2944336	108.8149261
34	162.8721027	-.9038065	-.0080364	28820155.0000000	-47145.0688477	147.2048702
35	153.2725983	-1.2192081	-.0092314	30447066.2500000	-41031.5664062	186.8711872
36	143.6622963	-1.5795475	-.0104870	31823665.7500000	-33457.9951172	226.9214230
37	134.0364513	-1.9868567	-.0117920	32896191.5000000	-24429.7351074	266.3112183
38	124.3886919	-2.4427186	-.0131330	33611922.0000000	-13993.2657471	303.8465652
39	114.7105827	-2.9481895	-.0144946	33920498.5000000	-2244.3070679	338.1885338
40	104.9910717	-3.5037249	-.0158596	33776205.5000000	10674.8028564	367.8598328
41	95.2158699	-4.1091119	-.0172088	33139103.0000000	24566.0043945	391.2526627
42	85.3667946	-4.7634101	-.0185218	31977974.0000000	39167.2412109	406.6370506
43	75.4213247	-5.4649057	-.0197769	30272061.0000000	54149.1494141	412.1704292
44	65.3530188	-6.2110810	-.0209522	28014256.2500000	69113.4033203	405.9128914
45	55.1346326	-6.9986036	-.0220254	25212959.7500000	83593.2666016	385.8654366
46	44.7488050	-7.8233389	-.0229752	21895229.2500000	97052.4892578	350.0850639
47	34.2192526	-8.6803901	-.0237818	18108717.7500000	108894.1328125	297.0364609
48	23.6940241	-9.5641687	-.0244277	13924178.8750000	118472.7119141	226.6136417
49	13.6414249	-10.4684985	-.0248987	9436515.3750000	125226.5976562	142.8052349
50	5.1724999	-11.3867559	-.0251849	4757592.0000000	128914.1464844	58.8979936
51	.0000000	-12.3120353	-.0252809	.0000000	129997.6816406	.0000000