ULTIMATE STRENGTH AND SERVICEABILITY BASED DESIGN OF PARTIALLY PRESTRESSED CONCRETE BEAMS

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ÖZET

Hem öngerilmeli, hem normal donatıyı haiz kirişler için taşıma gücü gerekleri ve çelik gerilme sınırlarına dayanan basit bir tasarım metodu geliştirilmiştir.

Taşma gücü gerekleri için, öngerilmeli çelikteki gerilme ile, bu çeliğin maximum gerilmesi oranını tahmin için donatım indisi (reinforcing index) kavramı kullanılarak yeni bir formül türetılmiştir.

Bu formül taşıma gücü gereklerini tatmin edecek diğer formullerin türetilmesinde kullanılmıştır.

Kullanışlılık (serviceability) gerekleri yönünde çelik gerilmeleri ile çatlak genişliği ve çekim sınırlamaları arasında bir bağ kurulmıştur.

Bu, taşıma gücü gerekleriyle birleştırılince önerılen tasarım metodunun temelini teşkil etmektedir.

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a	Depth of equivalent stress block
A	Effective tension area of concrete surronding the flexural tension reinforcement having the same centroid as that reinforcement, divided by the number of the directly bonded bars or wires (sq. inches)
A_{ps}	Area of prestressed reinforcement in tension zone
A_s	Area of non-prestressed tension reinforcement
b	width of compression face of member
b_w	web width
c	distance from extrem compression fiber to neutral axis
d	Distance from extrem compression fiber to centroid of tension force in reinforcement
d_c	Thickness of concrete cover measured from extreme tension fiber to bar located closest thereto
d_p	Distance from extreme compresion fiber to centeroid of prestressed reiforcement
d_s	Distance from extreme compression fiber to centroid of non-prestressed reinforcement
f'_c	Specified compressive strength of concrete
f_{ct}	Concrete stress at top fiber
f_p	Calculated stress in prestressed reinforcement at service load
f_{ps}	Calculated stress in prestressed reinforcement at ultimate load
f_{pu}	Ultimate strength of prestressed reinforcement

f_s	Calculated stress in non-prestressed reinforcement at service load
f_{se}	Effective stress in prestressed reinforcement after all losses
f_{ts}	Tensile stress in the non-prestressed tensile reinforcement
f_y	Specified yeild strength of non-prestressed
h_f	thickness of upper flange of flanged member
M	Any moment
M_u	Ultimate moment
M_f	Ultimate moment for flange portion of flanged member
M_w	Ultimate moment for web portion of flanged member
n	Modular ratio
P_e	Effective force in prestressed reinforcement after all losses
T_u	Total tensil force in reinforcement at ultimate load
T_s	Total tensile force in reinforcement at service load
W	Limiting crack width
β	Ratio of distances to neutral axis from extreme tension fiber and from centroid of total tension reinforcement
β_1	A factor defined in section 1o.2.7 of ACI 318-77 code
U_{mc}	Relative required ultimate moment capacity
U	Relative nominal strength for flange portion of flanged member
	Ratio of directly bonded tension reinforcement
w	$=\frac{A_s f_y}{bdf'_c}$ reinforcing index
w_p	$=\frac{A_{ps} f_y}{bdf'_c}$

$$w_{pu} = \frac{A_{ps} f_{pu}}{bdf'_c}$$

CHAPTER I

INTRODUCTION

1.1 General

Today, partial prestressing is accepted world-wide in most engineering practices, because it is a device which permits very high strength steel to be used economically and increase permissble span lengths while keeping the crack width within accepted design limites.

The present method for the design of partially prestressed beams, as suggested by ACI [1] is based on formula 18.3 of ACI 318-77

$$f_{ps}=f_{pu}(1-0.5\frac{f_{pu}}{f'_c}) \qquad (1.1)$$

Where

f_{ps} =calculated stress in prestressed steel atultimate load

f_{pu} =Ultimate strength of prestressing steel

=Ratio of prestressed reinforcement

$=\frac{A_{ps}}{bd}$

f'_c =Specified compressive strength of concrete

This formula was derived from experimental test results on prestressed beams, and suggests that f_{ps} depends solely on the properties of concrete and the prestressing steel.

Equation (1.1) would apply to partially prestressed sections with small amount of non-prestressed steel. however, equation (1.1) is inadequate since it is derived from test results on fully prestressed sections alone ,to accomodate the whole range of partial prestressing. It should be modified for cases of high percentages of partial prestressing.

1.2 Object and Scope

The object of this study was to develope a simple method for the design of partially prestressed concrete beams based on both ultimate strength requirements and steel stress limitations under service loads, taking into consideration the affect of partial prestressing on existing steel stresses.

Equations satisfying both ultimate strength and serviceability criteria are formulated in terms of the required amounts of prestressed and non-prestressed reinforcement and a solution of these two simultaineous equations gives the required amount reinforcement in partially prestressed concrete flexural sections. The formulation is applicable to both T and rectangular sections.

In this study no attemp is made to provide guidelines on how to proportion the cross section of member for a specific design problem. It is assumed that the concrete cross section is given.

CHAPTER II

STRESS IN PRESTRESSING STEEL AT ULTIMATE LOAD

2.1 Introduction

The stress f_{ps} in bonded prestressing steel at ultimate load is predicted by equation (18.3) of ACI code:

$$f_{ps} = f_{pu}(1-o.5 \frac{f_{pu}}{f'_c}) \qquad (2.1)$$

This equation has been derived from experimental test results on prestressed beams. As seen in equation (2.1) f_{ps} depends only on the properties of concrete and prestressing steel. In the case of partial prestressing equation (2.1) apply only to those with small ratios of non-prestressed steel. To accomodate partial prestressing with large ratios of non-prestressed steel equation (2.1) should be modified.

2.2 Proposed Formula for Steel Stress

In order to identify between partially prestressed under -reinforced and over-reinforced sections the ACI code(1) uses a parameter, $\bar{w}$, representing the sum of the partial reinforcing indices:

$$\bar{w} = w_p + w - w' \tag{2.2}$$

Where the partial reinforcing indices w_p , w , w' are defined as:

$$w_p = \frac{A_{ps} f_{ps}}{b\ d\ f'_c} \tag{2.3A}$$

Or

$$w_p = \frac{f_{ps}}{f'_c} \tag{2.3B}$$

$$w = \frac{A_s\ f_y}{b\ d\ f'_c} \tag{2.4A}$$

Or

$$w = \frac{f_y}{f'_c} \tag{2.4B}$$

$$w' = \frac{A'_s\ f'_y}{b\ d\ f'_c} \tag{2.5A}$$

Or

$$w' = \frac{f'_y}{f'_c} \tag{2.5B}$$

Where

d = distance from extreme compression fiber to the centroid of prestressed reinforcement or combined centroid when non-prestressed tension reinforcement is included.
So:

$$d = \frac{A_{ps}\ f_{ps} d_p + A_s\ f_y\ d_{ps}}{A_{ps}\ f_{ps} + A_s\ f_y} \tag{2.6}$$

Equation (2.6) leads $d = d_p$ for purely prestressed section and

$d=d_s$ for purely non-prestressed reinforced section.

Replacing the partial reinforcing indices by their values in equation

(2.2) the combined index $\bar{w}$ becomes:

$$\bar{w} = \frac{A_{ps}\ f_{ps}}{bd\ f'_c} + \frac{A_s\ f_y}{bd\ f'_c} + \frac{A'_s\ f'_y}{bd\ f'_c}$$

Or

$$\bar{w} = \frac{A_{ps}\ f_{ps} + A_s\ f_y + A'_S\ f'_y}{b\ d\ f'_c} \qquad (2.8)$$

Figure 2.1 shows a concrete section reinforced with compressive reinforcement and tensile reinforcement with combination of non-prestressed and prestressed bars. It is also assumed that at ultimate load the non-prestressed tensile and compressive steel yields.

Equilibrium of internal forces in the section shown in figure (2.1) gives:

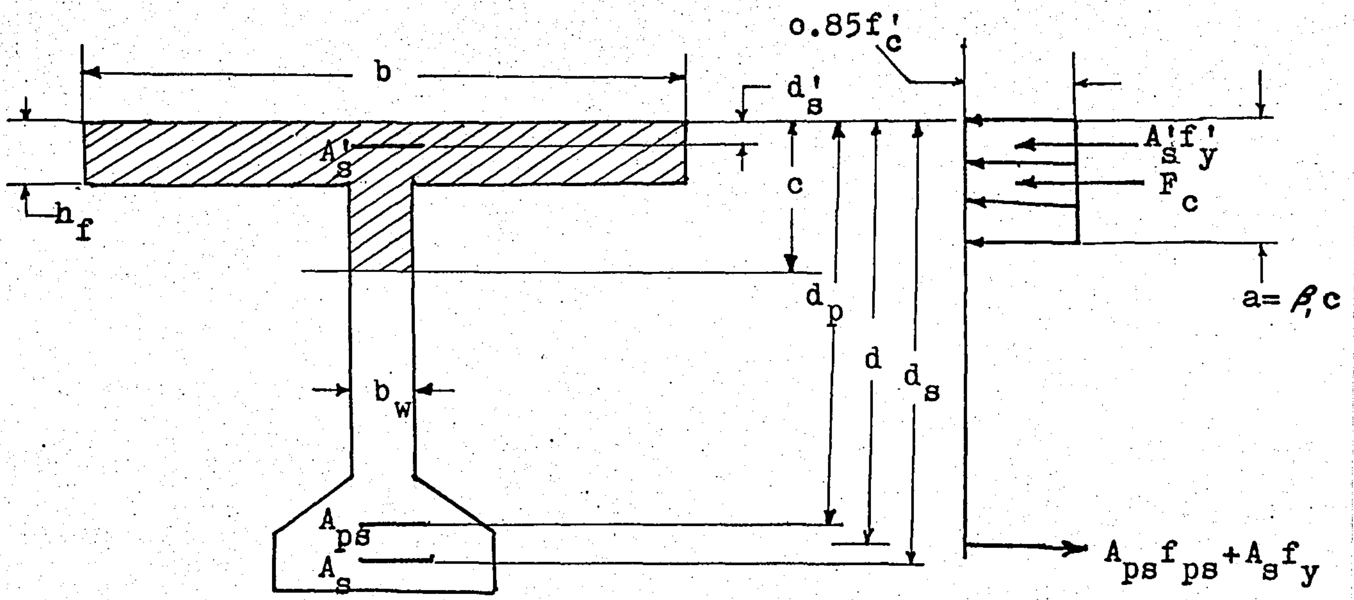


Fig. 2.1 Assumed stresses at ultimate moment capacity.

$$F_c = 0.85 f'_c (b-b_w) h_f + 0.85 f'_c b_w c$$

Where

F_c = force in equivalent compression block in concrete

From equilibrium we have:

$$A_{ps} f_{ps} + A_s f_y = 0.85 f'_c (b-b_w) + 0.85 f'_c b_w \beta_1 c + A'_s f'_s$$

$$a = \beta_1 c$$

Where

a = Depth of equivalent rectangular compression block in concrete.

Substitute a for $\beta_1 c$ and divide both sides by "$b\ d\ f'_c$"

$$\frac{A_{ps} f_{ps} + A_s f_y - A_s f'_y}{b\ d\ f'_c} = \frac{0.85 f'_c (b-b_w) h_f}{b\ d\ f'_c} + \frac{0.85 f'_c b_w}{b\ d\ f'_c} \qquad (2.10)$$

Since the left hand side of equation (2.10) is equal to combined reinforcing index $\bar{w}$ the rigth hand side should also be equal to the $\bar{w}$,

$$\bar{w} = \left[\frac{1}{b\ d\ f'_c} 0.85 f'_c (b-b_w) h_f + 0.85 f'_c b_w a \right] \qquad (2.11)$$

If the neutral axis is within the flange thickness, the section behaves as a rectangular beam, and equation (2.11) reduces to:

$$\bar{w} = 0.85 \frac{a}{d} \qquad (2.12)$$

Equation (2.8) means:

$$\bar{w} = \frac{1}{b\ d\ f'_c} \left[\text{net tensile force in all steel} \right] \qquad (2.13)$$

Similarly equation (2.11) means:

$$\bar{w} = \frac{1}{b\ d\ f'_c} \left[\text{force in concrete compression zone} \right] \qquad (2.14)$$

$\bar{w}$ is proportional to the ultimate resisting force in concrete compressive block regardless of the composition of the tensile reinforcements. Given a tensile force the concrete will balance that force.

If the section is fully prestressed one can rewrite equation (2.1) as a function of the reinforcing index,

$$w = w_p$$

$$\frac{f_{ps}}{f_{pu}} = 1 - 0.5 \rho_p \frac{f_{pu}}{f'_c} \qquad (2.15)$$

From equation (2.3.B) we have $\rho_p = w_p \dfrac{f'_c}{f_{ps}}$ substitute into equation (2.15).

$$\frac{f_{ps}}{f_{pu}} = 1 - 0.5\ w_p \frac{f'_c}{f_{ps}} \frac{f_{pu}}{f'_c}$$

$$\frac{f_{ps}}{f_{pu}} = 1 - 0.5\ w_p\ \frac{f_{pu}}{f_{ps}} \qquad (2.16)$$

It was shown earlier that reinforcing index is proportional to the resulting force in the steel or in the concrete.

These forces are equal and opposite, and from concrete's point of view it does not matter weather the force comes from the prestressing steel, non-prestressing steel or their combination. Thus equation (2.16) is valid for fully prestressed and partially prestressed section.

so assume that the section is partially prestressed with some tensile reinforcement only:

$$\bar{w} = w_p + w - w'$$

where

$$w' = 0$$

$$\frac{f_{ps}}{f_{pu}} = 1 - 0.5\ (w_p + w)\ \frac{f_{pu}}{f_{ps}} \qquad (2.17)$$

We know that

$$w_{pu} = \rho_p\ \frac{f_{pu}}{f'_c} \qquad (2.18)$$

Multiply equation (2.3.B) by $\frac{f_{pu}}{f_{pu}}$

$$w_p = \rho_p\ \frac{f_{ps}}{f'_c}\ \times\ \frac{f_{pu}}{f_{pu}}$$

$$w_p = w_{pu}\ \frac{f_{ps}}{f_{pu}} \qquad (2.19)$$

Substitute equation (2.19) into eq. (2.17)

$$\frac{f_{ps}}{f_{pu}} = 1 - 0.5 \left(w_{pu} \frac{f_{ps}}{f_{pu}} + w\right) \frac{f_{pu}}{f_{ps}}$$

Which is a quadratic equation in $\frac{f_{ps}}{f_{pu}}$

$$\left(\frac{f_{ps}}{}\right)^2 - (1 - 0.5w_{pu}) \frac{f_{ps}}{f_{pu}} + 0.5w = 0 \qquad (2.20)$$

$$2\left(\frac{f_{ps}}{f_{pu}}\right)^2 - (2 - w_{pu}) \frac{f_{ps}}{f_{pu}} + w = 0$$

$$\frac{f_{ps}}{f_{pu}} = \frac{(2 - w_{pu}) + \sqrt{(2 - w_{pu})^2 - 8w}}{4} \leqslant 1 \qquad (2.21)$$

Based on the results obtained in reference 2, we could assume $w_{pu} = w_p$, and follow the same reasoning as before to introduce $\bar{w}$ in the equation (2.I).

$$\frac{f_{ps}}{f_{pu}} = 1 - 0.5 \rho_p \frac{f_{pu}}{f'_c}$$

From equation (2.I8) we have :

$$w_{pu} = \rho_p \frac{f_{pu}}{f'_c}$$

Substitute the above equation into eq. (2.I).

$$\frac{f_{ps}}{f_{pu}} = 1 - 0.5\, w_{pu}$$

Above we have assumed $w_p = w_{pu}$ so:

$$\frac{f_{ps}}{f_{pu}} = 1 - 0.5\ w_p$$

By the same reasoning as before:

$$\frac{f_{ps}}{f_{pu}} = 1 - 0.5\ (w_{pu}\ \frac{f_{ps}}{f_{pu}} + w)$$

Solve the above equation for $\frac{f_{ps}}{f_{pu}}$

$$\frac{f_{ps}}{f_{pu}} = \frac{1 - 0.5\ w}{1 + 0.5\ w_{pu}} \leqslant 1 \qquad (2.23)$$

The equation (2.23) is simpler than equation (2.21) and it provides a reasonably good prediction of f_{ps} .

In order to see how the equation (2.23) compares with more exact equation (2.21) they are ploted in figure (2.2) versus the prestressed reinforcement ratio. It can be seen that equation (2.23) provides almost as good as an approxiation as the equation (2.21), within 2 percent if   is less than 0.5 percent.

Also in figure (2.3) both of the equation (2.21) and (2.23) are ploted with equation (18.3) of ACI code. From figure (2.3) it is obvious that equation (18.3) of ACI code is in unsafe side. The similar result is obtained by stress, strain compatibility analysis in reference (2).

The equation (2.23) is simpler and will be used in design to replace the equation (18.3) of the ACI code for partially prestressed beams.

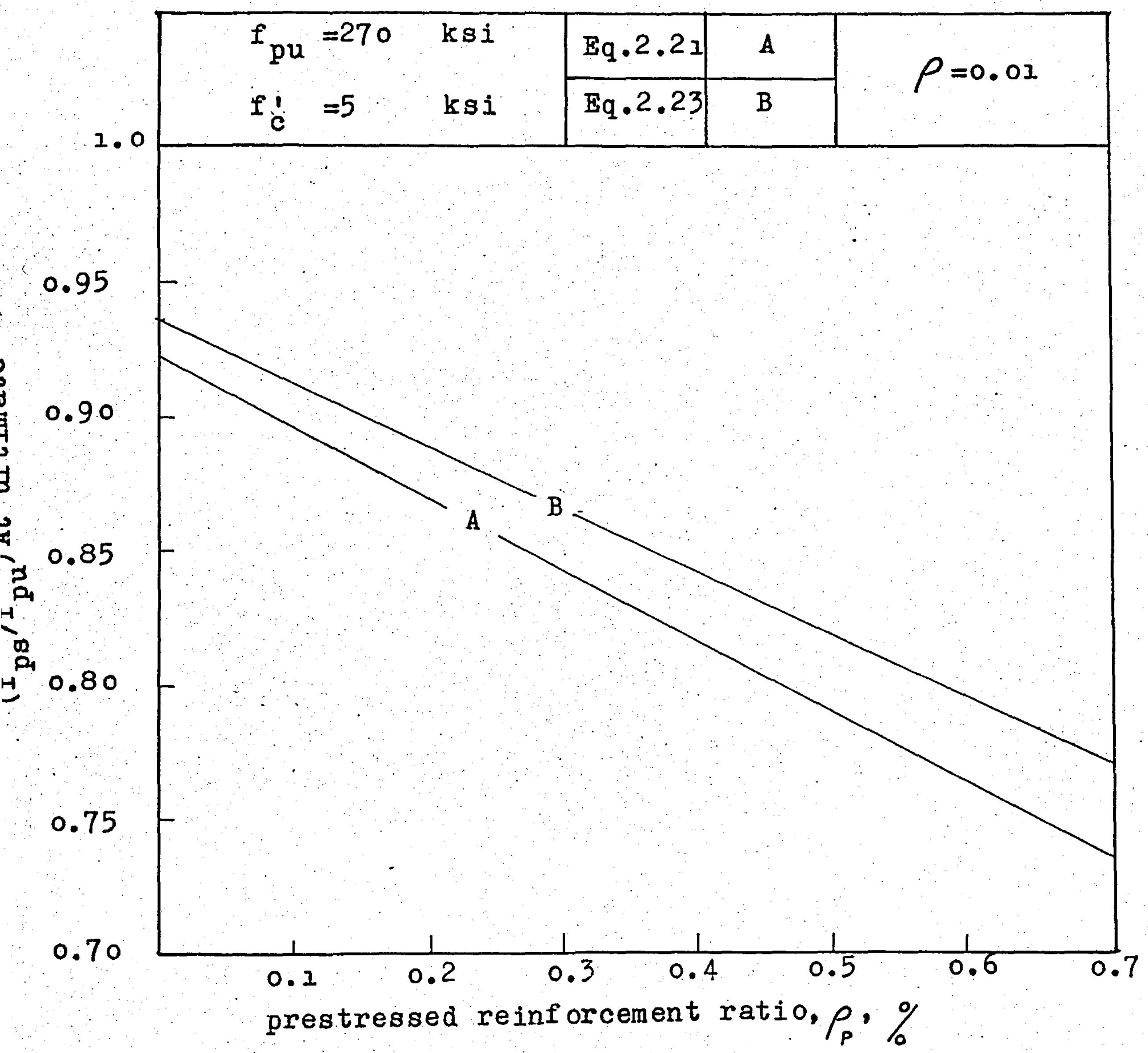


Fig.2.2 Comparison of the two prediction equation of f_{ps} at ultimate for partially prestressed beams.

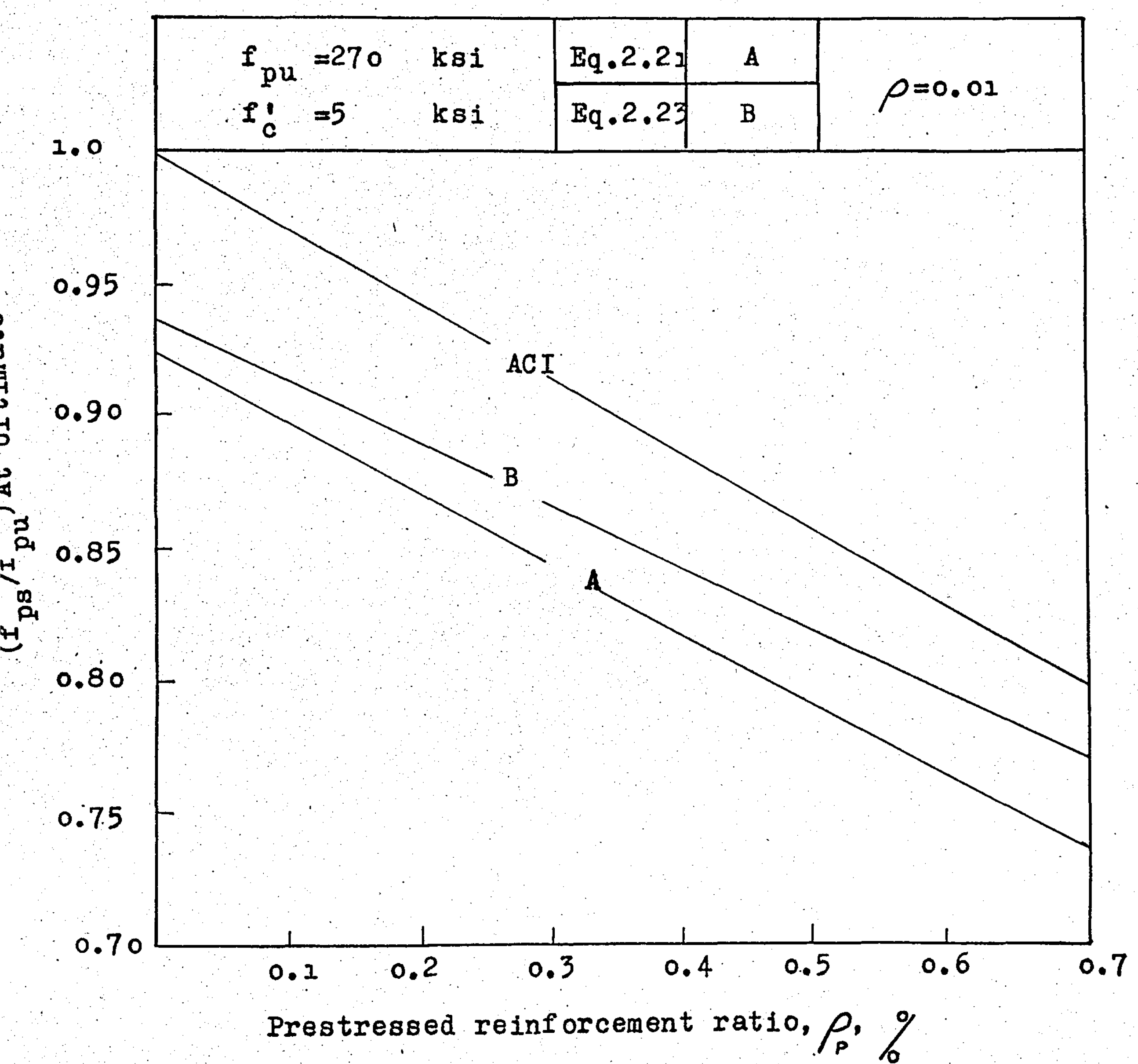


Fig.2.3 Comparison of the eq. (2.21) and (2.23) with eq.(18.3) of ACI code.

CHAPTER III

Ultimate Strength Requirements

3.I Assumptions

1-Plane sections are assumed to remain plane. This assumption has been found to be reasonably true.

2-The steel and concrete are completely bonded.

3-The stress diagram of the concrete at equilibrium is such that the average concrete stress is $0.85f'_c$ and has a rectangular shape.

3.2 Formulations

The equilibrum of the internal for the cross section and material properties in figure (3.1) can be written, assuming T-section behavior.

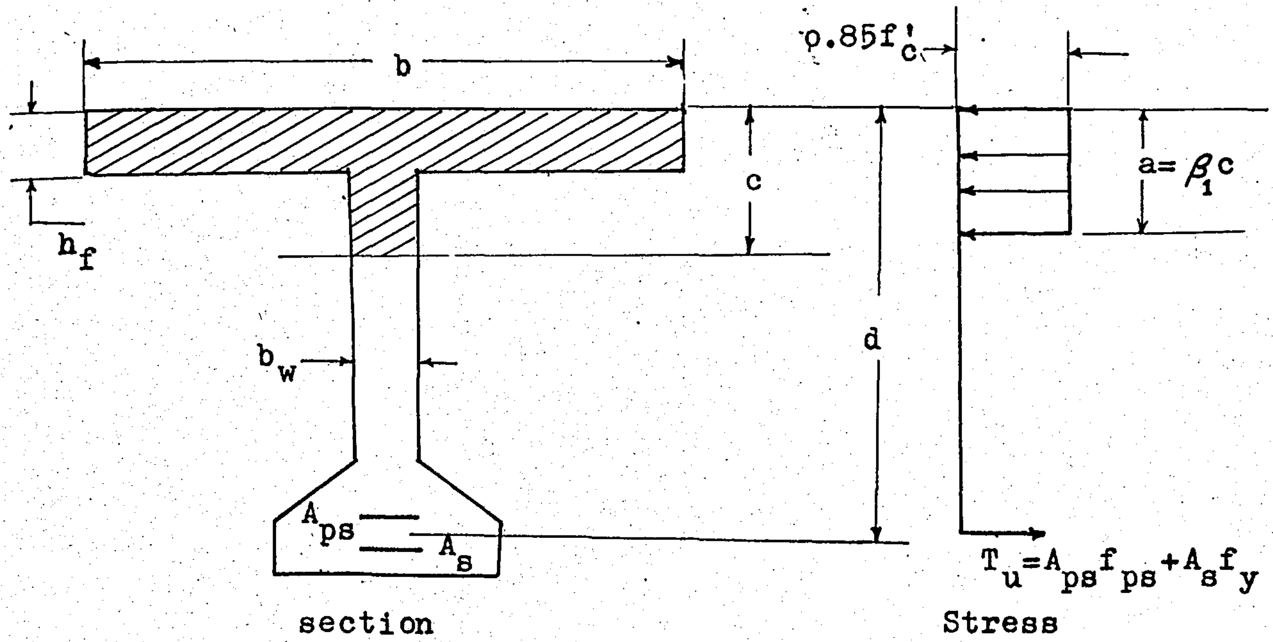


Fig.3.1 stress at ultimate load.

$$0.85 \left[f'_c \; b_w \, a + (b-b_w) \, h_f \right] = A_{ps} \, f_{ps} + A_s \, f_{ts} \qquad (3.1.A)$$

It is possible to substitute f_{ts}, tensile stress in non-prestressed steel as f_y, by exact analysis it had been proven(2) that generally f_{ts} equals to f_y at ultimate. So equation (3.1.A) could be written as:

$$0.85 \, f'_c \left[b_w \, a + (b-b_w) \, h_f \right] = A_{ps} \, f_{ps} + A_s \, f_y \qquad (3.1.B)$$

Divide both side of equation (3.1.B) by "b d f'_c "

$$\frac{0.85 \, f'_c}{b \, d \, f'_c} \left[b_w \, a + (b-b_w) \, h_f \right] = \frac{A_{ps} \, f_{ps}}{b \, d \, f'_c} + \frac{A_s \, f_y}{b \, d \, f'_c}$$

Or

$$0.85 \left[\frac{b_w}{b} \left(\frac{a}{d}\right) + \left(1 - \frac{b_w}{b}\right) \frac{h_f}{d} \right] = w_p + w \qquad (3.2)$$

Where from definition of reinforcing index:

$$w_p = \frac{A_{ps} \ f_{ps}}{b \ d \ f'_c} \qquad (3.3)$$

$$w = \frac{A_s \ f_y}{b \ d \ f'_c} \qquad (3.4)$$

Solve equation (3.2) for $(\frac{a}{d})$

$$\frac{a}{d} = \frac{1}{0.85} \left(\frac{b}{b_w}\right) (w_p + w) - \left(\frac{b}{b_w} - 1 \right) \left(\frac{h_f}{d}\right) \qquad (3.5)$$

In case of T-section behavior the ultimate moment, M_u is partly carried by the flange and partly by the web.

$$M_u = M_f + M_w$$

Take moment about the centroid of area of reinforcing bars.

$$M_u = \phi \left[0.85 \ f'_c \ (b - b_w) \ h_f \ (d - 0.5 \ h_f) + 0.85 \ f'_c \ b_w \ a \ (d - 0.5 \ a) \right]$$

Divide both side by " $b \ d^2 f'_c$ "

Where

ϕ = 0.9 given by ACI, strength reduction factor for flexure.

$$U_{mc} = \frac{M_u}{b \ d^2 f'_c} \qquad (3.8)$$

Where

U_{mc} = Ultimate moment capacity of the section

$$U = 0.85\left[\left(1 - \frac{b_w}{b}\right)\left(\frac{h_f}{d}\right)\left(1 - 0.5\frac{h_f}{d}\right)\right] \qquad (3.9)$$

Substitute equation (3.8) and (3.9) into (3.7)

$$U_{mc} = \phi U + 0.85\left[\frac{b_w}{b}\left(\frac{a}{d}\right) - 0.5\left(\frac{b_w}{b}\right)\left(\frac{a}{d}\right)^2\right]$$

Solve the above equation for " $\frac{a}{d}$ "

$$\left(\frac{a}{d}\right)^2 - 2\left(\frac{a}{d}\right) + \frac{2}{0.85\phi}\left(\frac{b}{b_w}\right)(U_{mc} - \phi U) = 0$$

$$\frac{a}{d} = 1 \mp \sqrt{1 - 2.614\left(\frac{b}{b_w}\right)(U_{mc} - \phi U)}$$

Take (-) sign, because " $\frac{a}{d}$" should always be smaller than one.

$$\frac{a}{d} = 1 - \sqrt{1 - 2.614\left(\frac{b}{b_w}\right)(U_{mc} - \phi U)} \qquad (3.10)$$

For T-beam behavior it is necessary to have:

$$a \geqslant h_f$$

Or

$$\frac{a}{d} \geqslant \frac{h_f}{d}$$

$$1 - \sqrt{1 - 2.614\left(\frac{b}{b_w}\right)(U_{mc} - 0.9\,U)} \geqslant \frac{h_f}{d}$$

$$-\sqrt{1 - 2.614 \left(\frac{b}{b_w} \right)\left(U_{mc} - 0.9\ U \right)} \geqslant \frac{h_f}{d}$$

Square both side

$$1 - 2.614 \left(\frac{b}{b_w} \right)\left(U_{mc} - 0.9\ U \right) \leqslant \left[1 + \left(\frac{h_f}{d} \right)^2 - 2\ \frac{h_f}{d} \right]$$

$$- U_{mc} + 0.9\ U \leqslant 0.382 \left(\frac{b_w}{b} \right)\left[\left(\frac{h_f}{d} \right)^2 - 2\ \frac{h_f}{d} \right]$$

Substitute u for equation (3.9)

$$- U_{mc} + 0.765 \left(\frac{h_f}{d} \right) - 0.765 \left(\frac{h_f}{d} \right)\left(0.5 \right) \left(\frac{h_f}{d} \right) +$$
$$0.765 \left(\frac{b_w}{b} \right)\left(\frac{h_f}{d} \right)\left(0.5 \right)\left(\frac{h_f}{d} \right) \leqslant$$
$$0.382 \left(\frac{b_w}{b} \right)\left(\frac{h_f}{d} \right)^2 - 0.765 \left(\frac{b_w}{b} \right)\left(\frac{h_f}{d} \right).$$

$$-U_{mc} + 0.765 \left(\frac{h_f}{d} \right) - 0.765 \left(\frac{h_f}{d} \right)\left(0.5 \right)\left(\frac{h_f}{d} \right) \leqslant 0$$

$$U_{mc} \geqslant 0.765 \left(\frac{h_f}{d} \right)\left(1 - 0.5\ \frac{h_f}{d} \right) \qquad (3.11)$$

The equation (3.11) is necessery condition for a section to behave as T-beam.

Sum of reinforcement in equation (3.2) can be written as:

A) T-beam behavior :

$$U_{mc} \geqslant 0.765 \left(\frac{h_f}{d} \right)\left(1 - 0.5\ \frac{h_f}{d} \right)$$

$$w_p + w = 0.85 \left(\frac{b_w}{b} \right) \left[1 - \sqrt{1 - 2.614 \left(\frac{b}{b_w} \right) \left(U_{mc} - 0.9\, U \right)} \right] +$$

$$0.85 \left(1 - \frac{b_w}{b} \right) \left(\frac{h_f}{d} \right) \qquad (3.12)$$

B) Rectangular section

$$U_{mc} < 0.765 \left(\frac{h_f}{d} \right) \left(1 - 0.5 \frac{h_f}{d} \right)$$

Equation (3.10) for rectangular behavior will become:

$$\frac{a}{d} = 1 - \sqrt{1 - 2.614\, U_{mc}} \qquad (3.13)$$

Equation (3.2) becomes :

$$w_p + w = 0.85 \left[1 - \sqrt{1 - 2.614\, U_{mc}} \right] \qquad (3.14)$$

The stress in the bonded prestressed steel " f_{ps} " is predicted from equation (18.3) of ACI code. Then substitute into the equation (3.12).

$$\frac{f_{ps}}{f_{pu}} = \frac{1 - 0.5\, w}{1 + 0.5\, w_{pu}} \qquad (3.15)$$

Where

$$w_{pu} = \frac{A_{ps}\, f_{pu}}{b\, d\, f'_c} \qquad (3.16)$$

$$w_p = \frac{A_{ps}\, f_{ps}}{b\, d\, f'_c}$$

$$w_p = \frac{A_{ps}\, f_{pu}}{b\, d\, f'_c} \times \frac{f_{ps}}{f_{pu}}$$

$$w_p = w_{pu} \frac{f_{ps}}{f_{pu}}$$

In the above equation substitute the value of " $\frac{f_{ps}}{f_{pu}}$ " from equation (3.I5).

$$w_p = w_{pu} \frac{1 - 0.5w}{1 + 05w_{pu}} \leqslant w_{pu} \qquad (3.I7)$$

Substitute equation (3.I3) into (3.I2).

$$w_p + w = 0.85\left(\frac{b_w}{b}\right)\left[1 - \sqrt{2.614\left(\frac{b}{b_w}\right)(U_{mc} - 0.9U)}\right] + 0.85\left(1 - \frac{b_w}{b}\right)\left(\frac{h_f}{d}\right).$$

$$w_{pu} \frac{1 - 0.5\ w}{1 + 0.5\ w_{pu}} + w =$$

$$0.85\left(\frac{b_w}{b}\right)\left[1 - \sqrt{2.614\left(\frac{b}{b_w}\right)(U_{mc} - 0.9\ U)}\right] + 0.85\left(1 - \frac{b_w}{b}\right)\frac{h_f}{d}$$

The left side of the above equation would be:

$$\frac{w_{pu} - 0.5\ w_{pu} + w + 0.5\ w_{pu}\ (w)}{1 + 0.5\ w_{pu}}$$

Or

$$\frac{w_{pu} + w}{1 + 0.5w_{pu}}$$

The whole equation would be :

T-beam behavior

$$A_{PS}\ f_{ps}\left\{1-0.425\left\{\left(\frac{b_w}{b}\right)\left[1-\sqrt{1-2.614\left(\frac{b}{b_w}\right)(U_{mc}-0.9U)}\right]+\left(1-\frac{b_w}{b}\right)\left(\frac{h_f}{d}\right)\right\}\right\}+A_s\ f_y=$$

$$0.85\ f_c'\ b\ d\left\{\frac{b_w}{b}\left[1-\sqrt{1-2.614\left(\frac{b}{b_w}\right)(U_{mc}-0.9U)}\right]+\left(1-\frac{b_w}{b}\right)\left(\frac{h_f}{d}\right)\right\} \qquad (3.18)$$

For rectangular behavior substitute equation (3.17) into eq. (3.14).

$$\frac{w_{pu}+w}{1+0.5w_{pu}}=0.85\left(1-\sqrt{2.614U_{mc}}\right)$$

Substitute the values for "w_{pu}" and "w".

$$A_{ps}\ f_{ps}\left\{1-0.425\left[1-\ 1-2.614U_{mc}\right]\right\}+A_s\ f_s=$$

$$0.85\ b\ d\ f_c'\left(1-\sqrt{1-2.614U_{mc}}\right) \qquad (3.19)$$

The equation (3.18) or (3.19) would be solved simultaneously with the equation (4.6) satisfying steel stress limitations under service load.

CHAPTER IV

Serviceability Based Design of

Partially Prestressed Beams

4.1 Literature Survey

The change in stress in the steel (prestressed and non-prestressed) in a cracked section at service load is one of the most important parameters in predicting the serviceability of partially prestressed concrete beams.

Based on observations in references 5 and 6, all the major cracks usually developed at a net steel stress of 15 to 25 ksi. At higher stresses the existing cracks widened and new cracks of narrow width formed between major cracks.

For beams with non-prestressing steel the number of flexural cracks was almost twice as many as those with no mild steel. These cracks were more evenly distributed with considerably less spacing and finer widths.

This behavior can be attributed to the fact that bond of mild steel to the surronding concrete played a pronouced role in crack control.

Also it is suggested that would be advantageous to locate the non-prestressing steel below the prestressed tendons. This is due to the fact that mild steel has larger diameters than the prestressing reinforcement, hence a larger bond area of contact with the surrounding concrete. Also by replacing the mild steel close to the tensile concrete face, cracks will be more evenly distributed, hence crack spacing and consequently crack width would be smaller. The effect of spacing of stirrups on the crack spacing had been found that the final crack spacing and crack pattern did not necessarily follow the vertical shear reinforcement. even-though the first few cracks usually started at the stirrups, the vertical legs of the stirrups served only as initial weak areas of stress concentration. In most cases the stabilized mean crack spacing were smaller than spacings of stirrups.

The suggested steel limit for given crack widths:

Limiting Crack Width	Upper limit of Steel stress
W	f_s
0.04 in	15 ksi
0.08 in	25 ksi

Table 4.1 Crack width and upper limit of steel stress

Conclusions

1. Initial flexural cracks randomly form at irregular spacings at low net steel stresses of 3 to 8 ksi.

2. Visible crack spacing stabilizes at 50 to 70 percent of the ultimate load.

3. The presence of non-prestressed steel in the prestressed members has a significant effect on crack control. Such that the crack becomes more evenly distributed, spacing and width becomes smaller.

4. An increase in the percentage of reinforcement decreases substantially the crack spacings and width in the partially prestressed beams.

5. For loads above the first cracking load, the deflections are smaller for prestressed concrete beams containing non-prestressing reinforcement than for a similar beam without non-prestressed reinforcement.

4.2 Steel Stress Limitation Under Service Loading

The analysis is based on the following assumptions:

1. The cross section is symmetrical about the plane of bending.
2. Plane sections remains plane.
3. Stress strain relations for concrete and reinforcement are linear within the range of service load.
4. Tensile strength of concrete is neglected.

Formulations:

Figure (4.1) illustrates the stress distribution across the section and applied external loads.

The applied normal force "p_e" at level of prestressing tendon should be replaced by the value of the tendon force.

$$p_e = A_{ps} f_{se} \tag{4.1}$$

Assume $\epsilon_c = \epsilon_s$

$$\frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$f_c = \frac{f_s}{n}$$

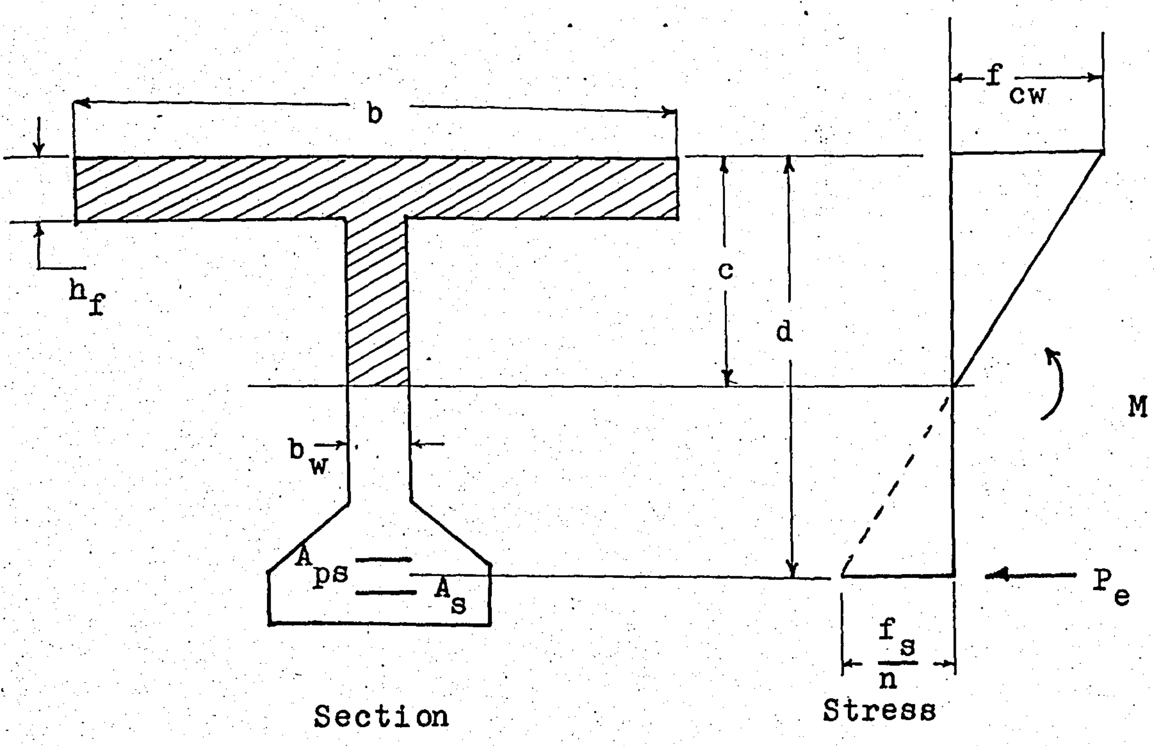


Fig. 4.1 Stress at Service Load.

From similarity of stress triangles in figure 4.1

$$\frac{f_s/n}{d - c} = \frac{f_{ct}}{c}$$

$$f_{ct} = \frac{f_s}{n} \frac{c}{d - c} \tag{4.2}$$

$$\frac{f_{ct}}{c} = \frac{f_{cw}}{c - h_f}$$

$$f_{cw} = \frac{f_{ct}(c-h_f)}{c}$$

$$f_{cw} = \frac{f_s}{n} \frac{c}{d-c} \frac{c-h_f}{c}$$

$$f_{cw} = \frac{f_s}{n} \frac{c-h_f}{d-c} \tag{4.3}$$

From equilibrium of forces:

$$f_{ct}\frac{c}{d-c}\frac{c}{2}(b)-f_{cw}\frac{c-h_f}{2}(b-b_w)-(A_{ps}+A_s)f_s=p_e$$

Substitute values of "f_{ct}" and "f_{cw}" from equation (4.2) and (4.3).

$$\frac{f_s}{n}\frac{c}{d-c}\frac{c}{2}(b)-\frac{f_s}{n}\frac{c-h_f}{2}(b-b_w)-(A_{ps}+A_s)f_s=p_e$$

$$\frac{f_s}{n}\frac{1}{d-c}\frac{1}{2}\left[bc^2-(b-b_w)(c-h_f)^2\right]-(A_{ps}+A_s)f_s=p_e \quad (4.4)$$

Equilibruim of moment:

$$\frac{f_s}{n}\frac{1}{d-c}\frac{1}{2}\left[bc^2-(b-b_w)(c-h_f)^2\right]\left[(d-c)+\frac{2}{3}\frac{(bc^2)c-(b-b_w)(c-h_f)^2(c-h_f)}{bc^2-(b-b_w)(c-h_f)^2}\right]=M \quad (4.5$$

Note that, the difference of moment of tensile forces in prestressed non-prestressed reinforcement is small and will be ignored. Or simply it is assumed that both normal prestressed reinforcement are located at the same level. Equation (4.4) could be written as follow:

$$p_e = A_{ps}\, f_{se}$$

$$A_{ps}(f_{se}+f_s)+A_s\, f_s = \frac{1}{2}\frac{f_s}{n}\left[\frac{bc^2-(b-b_w)(c-h_f)}{d-c}\right]$$

In the rigth hand side factor out "b" and multiply by "$\frac{d^2}{d^2}$"

$$A_{ps}(f_s + f_{se})+A_s\ f_s= \frac{f_s}{n}\ \frac{b\ d}{2}\left[\frac{(\frac{c}{d})^2 - (1-\frac{b_w}{b})(\frac{c}{d} - \frac{h_f}{d})^2}{1-\frac{c}{d}}\right] \qquad (4.6)$$

The equation (4.6) would be solved along with equation (3.18)oor (3.19) inorder to get steel areas "A_{ps}" and"A_s".

The equation (4.5) is rearranged to obtain a cubic equation in ratio of,distance from extreme compression fiber to neutral axis, to ,distance from extreme compression fiber to centroid of tensile reinforcement area.

$$\left[b\ c^2-(b - b_w)(c - h_f)^2\right](d - c)+\left[\frac{2}{3}\ b\ c^3-(b - b_w)(c - h_f)^3\right]=$$

$$\frac{2\ n\ M}{f_s}(d - c)$$

$$\left[b\ c^2-(b - b_w)(c^2- h_f^2 - 2\ c\ h_f)\right](d - c)+$$

$$\frac{2}{3}\left[b\ c^3-(b - b_w)(c^3- h_f^3 - 3c\ h_f)\right]- \frac{2\ n\ m}{f_s}d+ \frac{2\ n\ M'}{f_s}c = 0$$

$$-b_wc^3+3b_wdc^2+(6b\ d\ h_f-6b_wd\ h_f-3b\ h_f^2 +3b_wh_f^2 + \frac{6\ n\ M}{f_s})c-$$

$$3b\ d\ h_f^2 +3b_wd\ h_f^2 +2b\ h_f^3 -2b_wh_f^2 - \frac{6\ n\ M}{f_s}d = 0$$

Divide both side by"$-b_wd^3$"

$$(\frac{c}{d})^3-3(\frac{c}{d})^2-6\left[\frac{b}{b_w}\frac{h_f}{d} - \frac{h_f}{d} - \frac{1}{2}\frac{b}{b_w}(\frac{h_f}{d})^2- \frac{1}{2}(\frac{h_f}{d})^2+ \frac{n\ M}{f_sb_wd}\right]\frac{c}{d} +$$

$$6\left[\frac{n\ M}{f_sb_wd^2} + \frac{1}{2}\frac{b}{b_w}(\frac{h_f}{d})^2- \frac{1}{2}(\frac{h_f}{d})^2- \frac{1}{3}\frac{b}{b_w}(\frac{h_f}{d})^3+ \frac{1}{3}(\frac{h_f}{d})^3\right] =0$$

$$\left(\frac{c}{d}\right)^3-3\left(\frac{c}{d}\right)^2-6\left[\frac{n\ M}{f_s b_w d^2}+\frac{h_f}{d}\left(\frac{b}{b_w}-1\right)-\frac{1}{2}\frac{h_f^2}{d^2}\left(\frac{b}{b_w}-1\right)\right]\frac{c}{d}+$$

$$6\left[\frac{n\ M}{f_s b_w d^2}+\frac{1}{2}\left(\frac{h_f}{d}\right)^2\left(\frac{b}{b_w}-1\right)-\frac{1}{3}\left(\frac{h_f}{d}\right)^3\left(\frac{b}{b_w}-1\right)+\frac{1}{3}\left(\frac{h_f}{d}\right)\right]=0$$

Or

$$\left(\frac{c}{d}\right)^3-3\left(\frac{c}{d}\right)^2-6\left[\frac{n\ M}{f_s b_w d^2}+\left(\frac{b}{b_w}-1\right)\frac{h_f}{d}\left(1-\frac{1}{2}\frac{h_f}{d}\right)\right]\left(\frac{c}{d}\right)+$$

$$6\left[\frac{n\ M}{f_s b_w d^2}+\left(\frac{b}{b_w}-1\right)\left(\frac{h_f}{d}\right)^2\left(\frac{1}{2}-\frac{1}{3}\frac{h_f}{d}\right)\right]=0 \qquad (4.7)$$

From equation (4.7), $\frac{c}{d}$ ratio is obtained, then substituted in the equation (4.6) to give an equation with two unknowns A_{ps} and A_s. Which can satisfy steel stress limitations under working moment M.

However the maximum compressive stress at top fiber of concrete must not exceed it's allowable stress "f_{ct}" which is limited by the ACI code to $0.45f'_c$.

$$f_{ct} \geqslant \frac{f_s}{n}\,\frac{c}{d-c}$$

$$n\, f_{ct}(d-c) \geqslant f_s\, c$$

Where

f_{ct} = concrete stress at fiber

Divide both side by "d".

$$f_s \frac{c}{d} \leqslant n\ f_{ct}(1 - \frac{c}{d})$$

$$f_s \frac{c}{d} + n\ f_{ct} \frac{c}{d} \leqslant n\ f_{ct}$$

$$\frac{c}{d}(f_s + n\ f_{ct}) \leqslant n\ f_{ct}$$

$$\frac{c}{d} \leqslant \frac{n\ f_{ct}}{f_s + n\ f_{ct}} \qquad (4.8)$$

$$\frac{c}{d} \leqslant \frac{0.45n\ f'_c}{f_s + 0.45n\ f'_c} \qquad (4.9)$$

If the above computed "$\frac{c}{d}$" can not satisfy the above condition, hence it is necessary to reduce "f_s" or increase "d" and revise the design.

4.3 Determination Of Steel Area and Detailing of Reinforcement

The solution of simultaneous equations (3.18) or (3.19) and (4.6) can give one set of positive values of A_{ps} and A_s. It can satisfy both design criteria.
The influnce of steel stress, f_s on the area of prestressed A_{ps} and non-prestressed A_s reinforcement is illustrated in figure (4.2) for the case of rectangular section.
A solid line corresponds to equation (3.19), the ultimate flexural strength requirement, and two dotted lines represent equation (4.6) for two different values of f_s at service load. The figure 4.2 is plotted based on the following datum.

M_D =44.3 ft-kip

M_L =51.7 ft-kip

M_u =1.4M +1.7M_L =149.91 ft-kip

f'_c =5 ksi

f_y =60 ksi

f_{pu}=270 ksi

f_{se}=155 ksi

n =7

d =10 in

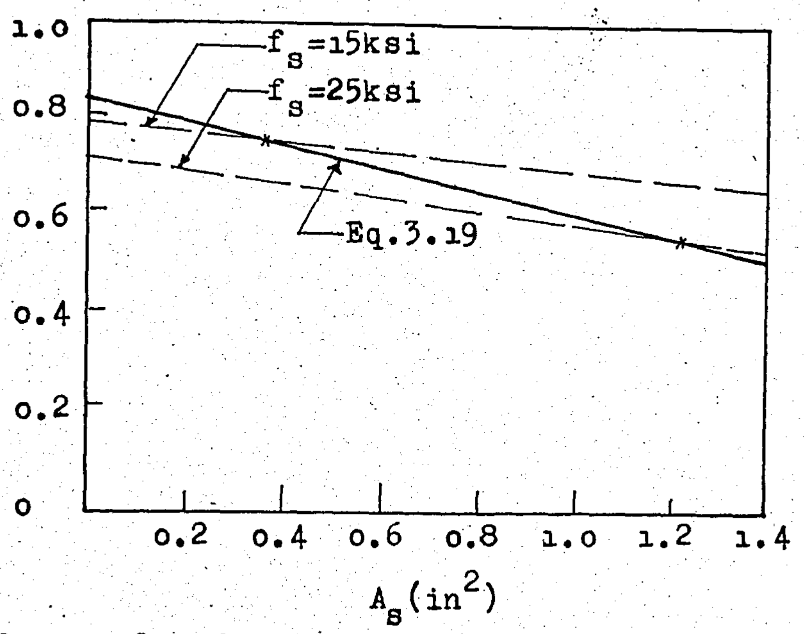


Fig. 4.2 Influence of steel stress on area of reinforcement.

As expected the quantity of non-prestressed steel decreases with decrease in the steel stress, f_s and the quantity of prestressed steel increases accordingly.

In the formulas the distance "d" had been taken as the distance between extreme concrete compression fiber to the centroid of tensile force. In partially prestressed members because the non-prestressed steel is usually closer to the surface than prestressed reinforcement. In order to obtain the centroid of total tensile force at the same location namely distance "d" from top concrete compression fiber. We have the following condition, which can be observed from figure (4.3).

1. At Ultimate strength

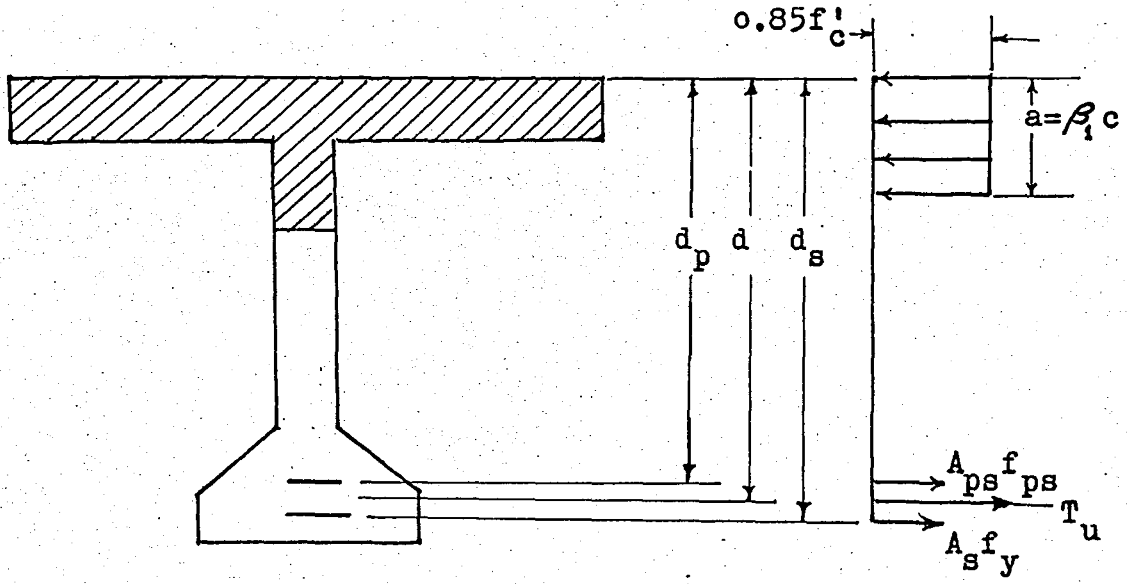


Fig.4.3 Ultimate load condition.

$$T_u = A_{ps}\, f_{ps} + A_s\, f_y \qquad (4.10)$$

To have T_u at the distance "d" the following condition should satisfy:

$$A_s\, f_y(d_s - d_p) = T_u(d - d_p)$$

$$A_s\, f_y(d_s - d_p) = (A_{ps}\, f_{ps} + A_s\, f_y)(d - d_p)$$

$$(d_s - d_p) = (d - d_p)\left(\frac{A_{ps}\, f_{ps}}{A_s\, f_y} + \frac{A_s\, f_y}{A_s\, f_y} \right)$$

$$(d_s - d_p) = (d - d_p)\left(1 + \frac{A_{ps}\, f_{ps}}{A_s\, f_y}\right)$$

$$(d_s - d_p) = (d - d_p)\left(1 + \frac{A_{ps}\, f_{pu}}{A_s\, f_y}\right) \qquad (4.11.A)$$

Or

$$A_{ps}\ f_{ps}(d_s - d_p) = (A_{ps}\ f_{ps} + A_s\ f_y)(d_s - d)$$

$$d_s - d_p = (1 + \frac{A_s\ f_y}{A_{ps}\ f_{ps}})(d_s - d)$$

$$d_s - d_p = (d_s - d)(1 + \frac{A_s\ f_y}{A_{ps}\ f_{pu}}) \quad (4.11.B)$$

At Service Load:

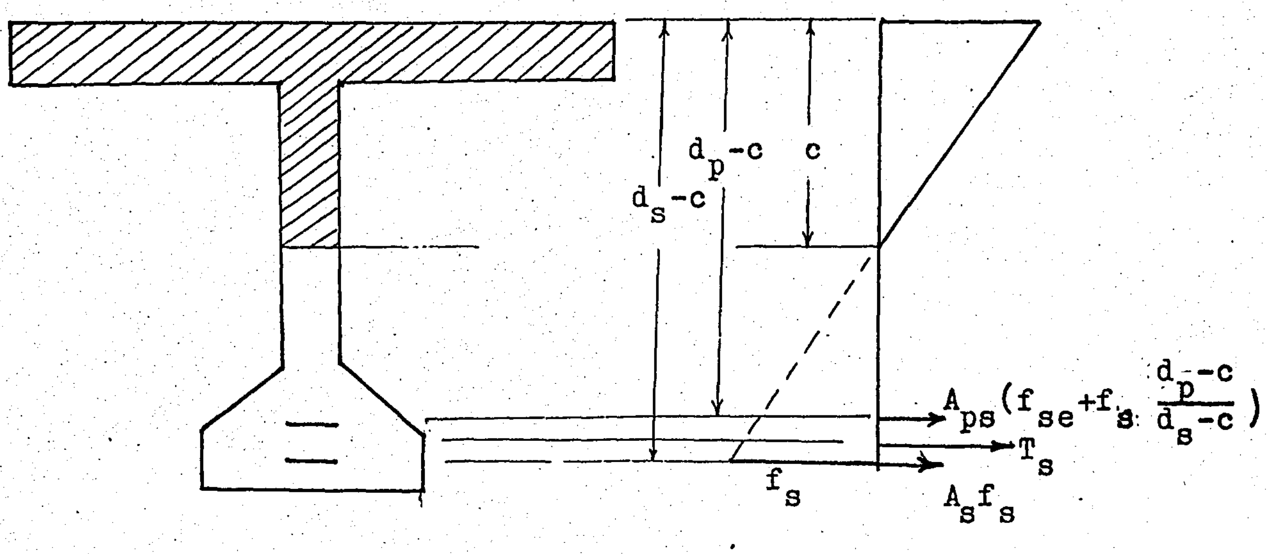


Fig. 4.4 Service Load Condition.

$$\frac{f_s}{d_s - c} = \frac{f_p}{d_p - c}$$

$$f_p = f_{se} + f_s \frac{d_p - c}{d_s - c} \quad (4.12)$$

$$f_{tp}=f_{se}+f_s\left(\frac{d_p-c}{d_s-c}\right) \qquad (4.13)$$

Where

f_{tp}=Total calculated stress in prestressing tendon at service load.

The total tensile force in reinforcement:

$$T_s=A_{ps}\left(f_{se}+f_s\,\frac{d_p-c}{d_s-c}\right)+A_s\,f_s \qquad (4.14)$$

To have the same centroid for the total tensile force at distance "d" from concrete extreme compression fiber the following condition should satisfy.

$$A_s\,f_s(d_s-d_p)=\left[A_{ps}\left(f_{se}+f_s\,\frac{d_p-c}{d_s-c}\right)+A_s\,f_s\right](d-d_p)$$

Or

$$\left[A_{ps}\left(f_{se}+f_s\,\frac{d_p-c}{d_s-c}\right)\right](d_s-d_p)=$$

$$\left[A_{ps}(f_{se}+f_s\,\frac{d_p-c}{d_s-c}+A_s\,f_y\right](d_s-d)$$

Or

$$(d-d\ )=1\left[+\frac{A_s}{A_{ps}}\,\frac{f_s}{A_{ps}\,f_{se}+f_s\,\frac{d_p-c}{d_s-c}}\right](d_s-d) \qquad (4.15)$$

Also it is possible to calculate A_s, non-prestressed reinforcement steel area from equation (4.14) since T_s could be found from right hand side of equation (4.6).

$$A_s = \frac{T_s}{f_s} - A_{ps}\left(\frac{f_{se}}{f_s} + \frac{d_p - c}{d_s - c} \right) \qquad (4.16)$$

4.4 Complete Analysis

After distribution of the reinforcement in the section member will be analyzed more completely by taking into consideration of "d_s", distance between centroid of area of normal reinforcing steel and top concrete compression fiber and "d_p", distance between centroid of prestressing tendon and top concrete compression fiber. Also the effect of tensile forces, prestressed and non-prestressed reinforcing steel is considered in the equilibrium equation of forces and moments.

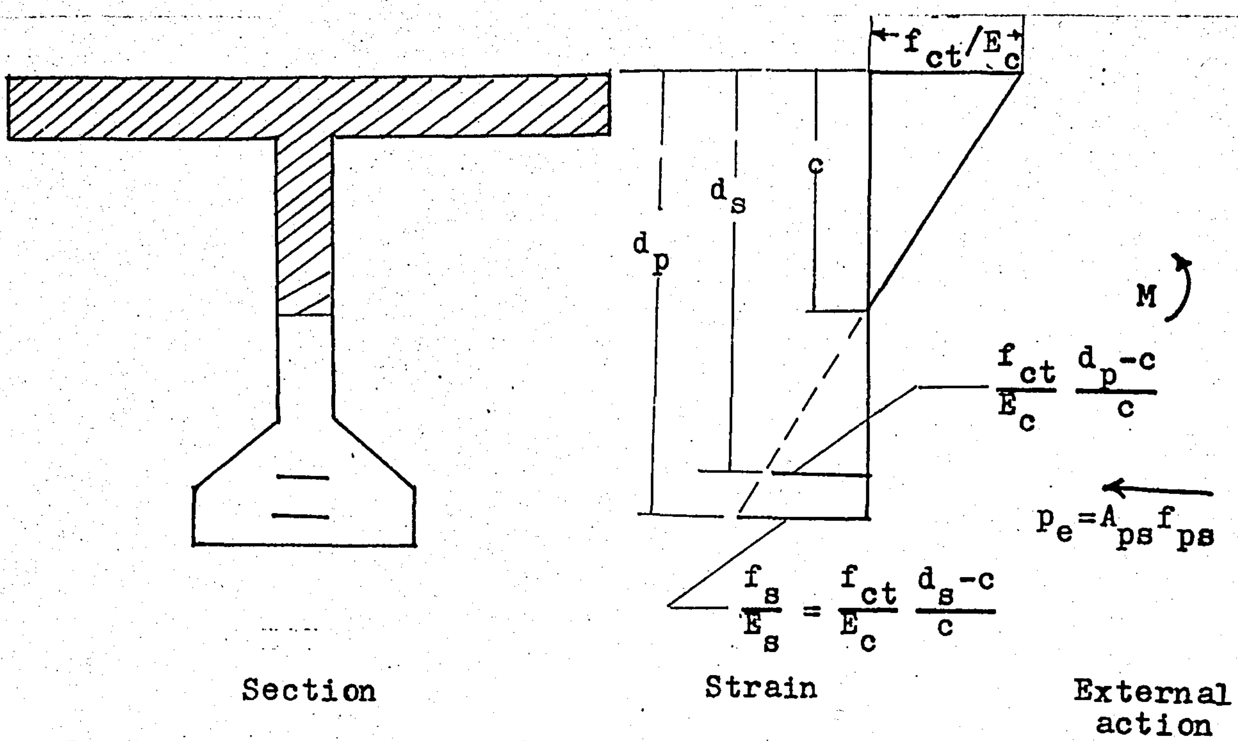


Fig. 4.5 Equilibrium of forces

From figure (4.5)

$$\frac{f_{ct}/E_c}{c} = \frac{f_s/E_s}{d_s - c}$$

$$f_s = \frac{n\, f_{ct}}{c}\,(d_s - c)$$

Force in prestressed tendon $= n\,\frac{f_{ct}}{c}\,(d_p - c)A_{ps}$

Force in non-prestressed tendon $= f_s\, A_s$

$$= n\,\frac{f_{ct}}{c}\,(d_s - c)A_s$$

Force in concrete compression block=

$$\frac{f_{ct}}{2c}\left[b\, c^2 - (b - b_w)(c - h_f)^2\right]$$

Equilibrium of forces:

$$\frac{f_{ct}}{2c}\left[b\, c^2 - (b - b_w)(c - h_f)^2\right] - n\,\frac{f_{ct}}{c}\left[A_{ps}(d_p - c) + A_s(d_s - c)\right] = p_e \tag{4.17}$$

Equilibrium of moments:

$$\frac{f_{ct}}{2c}\left[b\, c^2 - (b - b_w)(c - h_f)^2\right]\left[(d_p - c) + \frac{2}{3}\,\frac{bc^3 - (b-b_w)(c-h_f)^3}{bc^2 - (b-b_w)(c-h_f)^2}\right] +$$

$$n\,\frac{f_{ct}}{c}\,A_s(d_s - c)(d_s - d_p) = M \tag{4.18}$$

The equation (5.8) is solved for "f_{ct}" and then substituted in the equation (5.9). The resulted equation is rearranged to obtain the following equation:

$$\left(\frac{c}{d_p}\right)^2+3\left(\frac{M}{P_e\,d_p}-1\right)\left(\frac{c}{d_p}\right)^2+6\left[\left(\frac{b}{b_w}-1\right)\left(\frac{h_f}{d_p}\right)\left(\frac{M}{P_e d_p}-1+\frac{1}{2}\frac{h_f}{d}\right)+\right.$$

$$n\frac{A_{ps}+A_s}{b_w\,d_p}\frac{M}{P_e\,d_p}+n\frac{A_s}{b_w d_p}\left(\frac{d_s}{d_p}-1\right)\Big]\left(\frac{c}{d_p}\right)-3\Big[\left(\frac{b}{b_w}-1\right)\left(\frac{h_f}{d_p}\right)^2\Big($$

$$\frac{M}{P_e\,d_p}-1+\frac{2}{3}\frac{h_f}{d}\Big)+2n\left(\frac{A_{ps}}{b_w\,d_p}+\frac{A_s}{b_w\,d_p}\frac{d_s}{d_p}\right)\frac{M}{P_e\,d_p}+$$

$$\left.2n\frac{A_s}{b_w\,d_p}\frac{d_s}{d_p}\left(\frac{d_s}{d_p}-1\right)\right]=0 \qquad (4.19)$$

From figure 4.5

$$\frac{f_s}{d_s-c}=n\,\frac{f_{ct}}{c}$$

$$f_s=n\,f_{ct}\left(\frac{d_s}{c}-1\right) \qquad (4.20)$$

From figure 4.4

$$f_p=f_{se}+f_s\,\frac{d_p-c}{d_s-c}$$

substitute for "f_s" from equation (4.20)

$$f\;=f_{se}+nf_{ct}\left(\frac{d_p}{c}-1\right) \qquad (4.21)$$

solve equation (4.17) for "f_{ct}"

$$f_{ct}=\frac{2p_e\ c}{bc^2-(b-b_w)(c-h_f)^2-2n\left[A_{ps}(d_p-c)+A_s(d_s-c)\right]} \tag{4.22}$$

The stresses in the prestressed steel, non-prestressed reinforcing steel and concrete compression fiber are derived above. The stress should be checked in both steel and concrete assuming a cracked section under full service load.

4.5 Crack Control

Although the steel stress f_s is the most important variable in predicting crack widths in flexural members, the thickness of concrete cover the area of concrete in the zone of maximum tension surrounding each individual reinforcing bar or strand and the effect of loading condition are also significant varibles. It woud be preferable to follow the same crack control method, crack width computation as the ACI code,but the ACI formula is not directly applicable to partially prestressed sections. It could be assumed in evaluating the area of tensile concrete zone per bar that reinforcing bars and prestressed strands have the same effect as long as the prestressed strands are bonded directly. The effective tension area per bar, "A" can be written as:

$$A = (\text{Area largest bar or strand}) \times \frac{\text{tension area of concrete having the same centroid of reinforcem}}{\text{total area of reinforcement directly bonded}}$$

$$A = \frac{1}{\rho^*} \times (\text{Area of largest bar or strand})$$

Where ρ^* is:

$\rho^* = \dfrac{\text{total area of reinforcement directly bonded}}{\text{tension area of concrete having the same centroid of reinforce}}$

the crack width prediction formula proposed by Gergly and and Lutz can be written as:

$$W = 0.076x10^{-3} \beta f_s \sqrt{\frac{d_c}{\rho^*}(\text{Area of largest bar or strand})} \qquad (4.23$$

Where

W, is t e limiting crack width in inches.

Alternatively area of largest bar or strand should satisfy the following equation:

$$\text{Area of largest bar or strand} \leqslant \left(\frac{13.2x10^3 W}{\beta f_s}\right) \frac{\rho^*}{d_c} \qquad (4.24)$$

β, Is ratio of distances to neutral axis from the extreme tension fiber and from the centroid of reinforcement.

$$\beta = \frac{d_{total} - c}{d - d' - c}$$

d_c, Is thickness of concrete cover measured from extreme tension fiber to the bar located closest thereto.

d', Is distance from extreme tensile face of member to the centroid of total reinforcement in the tensile zone.

The equation (4.24) will provide a distribution of reinforcement that will reasonably control crackig.

CHAPTER V

Design Procedure And Examples

5.1 Design Procedure

1-Compute U_{mc}, the ultimate moment capacity of the section by using equation (3.11) and substitute into equation (3.11) to check weather the section behaves as rectangular or T beam.

2-Compute one set of values of A_{ps} and A_s, solving simultaneous equations (3.18) or (3.19) and calculate T_u and T_s.

3-Determine the integer number of prestressing strands wires which can give the nearest steel area A_{ps} to the value computed in step 1.

4-Calculate the non-prestressed steel area A_s from equation (3.18) or (3.19) and equation (4.16), for given A_{ps}. In equation (4.16) the ratio $(d_p-c)/(d_s-c)$ is not yet known, however this value may be taken as 0.90 to 0.98 .

The largest amount of two computed A_s values gives the required steel area. Determine the integer number of reinforcing bars considering the fact that several small diameter bars at moderate spacing are more effective in controlling cracking than small number large bars of equivalent area.

5-Non-prestressed bars should be distributed in the tensile zone of concrete. In doing so compute the distance d_s from the extreme compression fiber to the centroid of A_s.

6-Calculate the distance from the centroid of A_s to the centroid of A_{ps} ,using equation (4.11.B) and (4.5) and the lesser of two values gives a solution. prestressed strands or wires are arranged in the tensile zone of concrete in such a way that the distance from the extreme compression fiber to its centroid must be equal to:

$$d_s-(d_p- d_s)$$

After distributing the reinforcement in the section, the member should be analyzed more completely and its adequacy checked for the service load criteria. The following steps can be used:

1-Check the stress in the steel and the concrete assuming a cracked section under full service load. The neutral axis depth, c can be obtained for more complete analysis using equation (4.19).

2-If a check for fatique is necessary, calculate the stress changes in the steel and in the concrete for combination of repetitive live load moment and dead load moment and compare these values with the admissible fatique stress range for each material.

3-Check the adequacy of reinforcing detailes corresponding to limiting crack width. For relatively deep beams the reinforcement should also be placed near the vertical faces in the tension zone.

5.2 Design Example

Consider a prestressed concrete T beam with dimensions given in figure 5.2 , and design to carry :

Dead load moment = M_D =2353 ft-kips

Live load moment = M_L =966 ft-kips

Limiting crack width=W=0.06in.(under full service load)

Given material properties

Compressive strength of concrete = f'_c =5 ksi

Yield strength of non-prestressed reinforcement = f_y =60 ksi

Ultimate strength of prestressed reinforcement = f_{pu} =270 ksi

Effective stress in prestressed reinforcement

after all losses = 155 ksi

Ultimate moment = M_u

M_u= 1.4 M_D + M_L

M_u=1.4(2353)+1.7(966)

M_u=4936.4 ft-kips

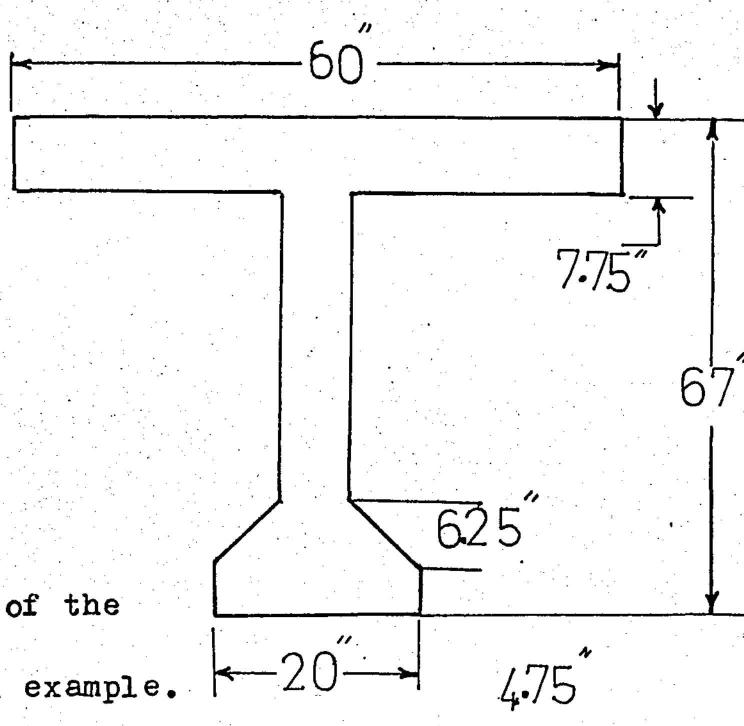


Fig. 5.1 Cross section of the beam in design example.

1-Ultimate Strength Requirements

$$U_{mc}=\frac{M_u}{f'_c\ b\ d^2}$$

$$U_{mc}=\frac{4936.4x12}{5x60x62.4^2}=0.05071$$

Check to see if the section behaves as T-beam, from eq.(3.11)

$$U_{mc}\geqslant 0.765\ \frac{h_f}{d}(1-0.5\ \frac{h_f}{d})$$

$$0.765\ \frac{7.75}{62.4}(1-0.5\ \frac{7.75}{62.4})=0.08911$$

The equation (3.11) is not satisfied so the beam behaves as rectangular beam. Then use equation (3.19) to obtain a relationship between A_s and A_{ps}.

$$A_{ps}\ f_{pu}\left[1-0.425(1-\sqrt{1-2.614U_{mc}})\right]+A_s\ f_y=$$

$$0.85f'_c\ b\ d\left[1-\sqrt{1-2.614U_{mc}}\right]$$

$$A\ \ (270)\left[1-0.425(1-\sqrt{1-2.614(0.0507)}\right]+A_s(60)=$$

$$0.85(5)(60)(62.4)\left[1-\sqrt{1-2.614(0.0507)}\right].$$

$$262.1A_{ps}+60A_s=1092\quad \text{kips}\qquad (5.1)$$

2. Serviceability Requirements

Limiting crack width, W=0.06 in. then from table (4.1) it is found that calculated stress in non-prestressed reinforcement at service load , f_s=20 ksi.

$$\text{Modular ratio} = n = \frac{E_s}{E_c} = \frac{E_p}{E_c} = 7$$

Using equation (4.7):

$$(\tfrac{c}{d})^3-3(\tfrac{c}{d})^2-6\left[\frac{n\ M}{f_s b_w d^2}+(\frac{b}{b_w}-1)(\frac{h_f}{d})(1-0.5\frac{h_f}{d})\right](\tfrac{c}{d})+$$

$$6\left[\frac{n\ M}{f_s b_w d^2}+(\frac{b}{b_w}-1)(\frac{h_f}{d})^2(\frac{1}{2}-\frac{1}{3}\frac{h_f}{d})\right]=0$$

$$M = M_D + M_L = 3319 \qquad \text{ft-kips}$$

$$(\tfrac{c}{d})^3-3(\tfrac{c}{d})^2-6\left[\frac{7x3319x12}{20x7x62.4^2}+(\frac{60}{7}-1)(\frac{7.75}{62.4})(1-\frac{1}{2}\frac{7.75}{62.4})\right](\tfrac{c}{d})+$$

$$6\left[\frac{7x3319x12}{20x7x62.4^2}+(\frac{60}{7}-1)(\frac{h_f}{d})^2(\frac{1}{2}-\frac{1}{3}\frac{h_f}{d})\right]=0$$

$$(\tfrac{c}{d})^3-3(\tfrac{c}{d})^2-8.3604(\tfrac{c}{d})+3.3899=0$$

$$\frac{c}{d}=0.3637$$

To check $\quad \frac{c}{d} \leqslant \frac{n\ f_{ct}}{f_s + n\ f_{ct}}$

The AcI code limites $f_{ct} = 0.45\ f'_c$

$$f_{ct} = 0.45(5) = 2.25 \qquad \text{ksi}$$

$$\frac{c}{d} \leqslant \frac{7x2.25}{20+7x2.25} = 0.4406$$

The value of $\frac{c}{d}$ =0.3637 is O.K.

Use the equation (4.6).

$$A_{ps}(f_{se}+f_s)+A_s\ f_s = \frac{f_s}{n}\ \frac{b\ d}{2}\left[\frac{(\frac{c}{d})^2-(1-\frac{b_w}{d})(\frac{c}{d}-\frac{h_f}{d})^2}{1-\frac{c}{d}}\right]$$

$$A_{ps}(155+20)+A_s(20) = \frac{20}{7}\ \frac{60x62.4}{2}\left[\frac{(0.3637)^2-(1-\frac{7}{60})(0.3637-\frac{7.75}{62.4})^2}{1-0.3637}\right]$$

$175A_{ps} + 20A_s = 686$ kips (5.2)

Solve equation (5.1) and (5.2) simultanously

$262.1A_{ps} + 60A_s = 1092$

$175A_{ps} + 20A_s = 686$

$A_{ps} = 3.675$ sq. in.

$As = 2.146$ sq. in.

T_u and T_s (tensile force at ultimate and at service load respectively) could be calculated from right side of eq. (3.11) and eq. (4.6) which has been calculated as:

$$T_u=0.85f'_c\ b\ d\left[1-\sqrt{1-2.614U_{mc}}\right]=1092 \quad \text{kips}$$

$$T_s = \frac{b}{2}\frac{d}{n}\left[\frac{(\frac{c}{d})^2-(1-\frac{b_w}{b})(\frac{c}{d}-\frac{h_f}{d})^2}{1-\frac{c}{d}}\right] f_s=686 \quad \text{kips}$$

For prestressing provide 24 strand in two groups of 12 strands of 0.5 in. diameter each:

A_{ps}=24x0.153=3.672 in.

For non-prestressed reinforcing:

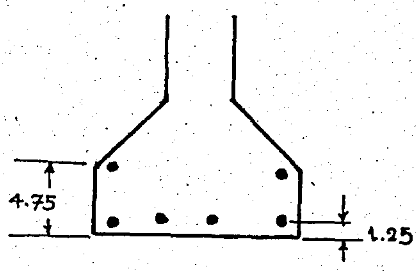
Four No. 6 bars and two No. 5 bars

A_s=4xo.44 + 2xo.31 =2.380 in.^{2}

Distribution of non-prestressed reinforcement.

$$d_s = \frac{(4x0.44x65.25+2x0.31x63)}{2.38}$$

d_s=64.66 in.



To compute d_p use equation (4.11) and (4.15) and take lesser of $d_s - d_p$.

$$d_s - d_p=(1+\frac{A_s}{A_{ps}}\frac{f_y}{f_{pu}})(d_s-d) \qquad (4.11.B)$$

$$d_s - d_p=\left[1+\frac{A_s}{A_{ps}}\frac{f_s}{f_{se}+f_s\frac{d_p-c}{d_s-c}}\right](d_s-d) \qquad (4.15)$$

The ratio of $\frac{d_p-c}{d_s-c}$ is varying usually between 0.9 and 0.98 , take as 0.95 . d_s-d_p=2.428 in. (4.15)

d_p=d_s-2.428=64.66-2.428= 62.232 in. (4.15)

d_s-d_p=2.586 in. (4.11.B)

To check the design

$$p_e = A_{ps}\ f_{se}$$

$$p_e = 3.672(155) = 569.16 \qquad \text{kips}$$

Compute the stresses in the concrete and steel under full service load assuming cracked section using equation (4.19).

$$\left(\frac{c}{d_p}\right)^3 + 3\left(\frac{M}{p_e d_p} - 1\right)\left(\frac{c}{d_p}\right)^2 + 6\left[\left(\frac{b}{b_w} - 1\right)\left(\frac{h_f}{d_p}\right)\left(\frac{M}{p_e d_p} - 1 + \frac{1}{2}\frac{h_f}{d}\right) + n\frac{A_{ps} + A_s}{b_w d_p}\ \frac{M}{p_e d_p} + n\frac{A_s}{b_w d_p}\left(\frac{d_s}{d_p} - 1\right)\right]\left(\frac{c}{d_p}\right) - 3\left[\left(\frac{b}{b_w} - 1\right)\left(\frac{h_f}{d_p}\right)^2\left(\frac{M}{p_e d_p} - 1 + \frac{2}{3}\frac{h_f}{d}\right) + 2n\left(\frac{A_{ps}}{b_w d_p} + \frac{A_s}{b_w d_p}\ \frac{d_s}{d_p}\right)\frac{M}{p_e d_p} + 2n\frac{A_s}{b_w d_p}\ \frac{d_s}{d_p}\left(\frac{d_s}{d_p} - 1\right)\right] = 0$$

$$\left(\frac{c}{d_p}\right)^3 + 3\left(\frac{3319\text{x}12}{569.16\text{x}62.232} - 1\right)\left(\frac{c}{d_p}\right)^2 + 6\left[\left(\frac{60}{7}\right)1\right)\left(\frac{7.75}{62.232}\right)\left(\frac{3319\text{x}12}{569.16\text{x}62.232} - 1 + \frac{1}{2}\ \frac{7.75}{62.4}\right) + 7\ \frac{3.672+2.382}{7\text{x}62.232}\ \frac{3319\text{x}12}{569.16\text{x}62.232} + 7\frac{2.382}{7\text{x}62.232}\left(\frac{64.66}{62{,}232} - 1\right)\right]\left(\frac{c}{d_p}\right) - 3\left[\left(\frac{60}{7} - 1\right)\left(\frac{7.75}{62.4}\right)^2\left(\frac{3319\text{x}12}{569.16\text{x}62.232} - 1 + \frac{2}{3}\left(\frac{7.75}{62.4}\right) + 2\text{x}7\left(\frac{3.672}{7\text{x}62.232} + \frac{2.382}{7\text{x}62.232}\ \frac{64.66}{62.232}\right)\left(\frac{3319\text{x}12}{569.16\text{x}62.232}\right) + 2\text{x}7\ \frac{2.382}{7\text{x}62.232}\ \text{x}\ \frac{64.66}{62.232} - 1\right)\right] = 0$$

$$(\frac{c}{d_p})^3 + 0.3726(\frac{c}{d_p})^2 + 1.7191(\frac{c}{d_p}) - 0.7479 = 0$$

$$\frac{c}{d_p} = 0.3742$$

$c = 23.29$ in.

$$f_{ct} = \frac{2p_e c}{bc^2 - (b-b_w)(c-h_f)^2 - 2n\left[A_{ps}(d_p - c) + A_s(d_s - c)\right]}$$

$$f_{ct} = \frac{2 \times 569.16 \times 23.29}{60(23.29)^2 - (60-7)(23.29-7.75)^2 - 2 \times 7\left[3.672(62.232-23.29) + 2.380(64.66-23.29)\right]} = 1.62 \text{ ksi}$$

$f_{ct} = 1.62 < 0.45 f'_c$ O.K.

$$f_s = nf_{ct}\left(\frac{d_s}{c} - 1\right)$$

$$f_s = 7(1.62)\left(\frac{64.66}{23.29} - 1\right) = 20.14 \text{ ksi}$$

$$f_p = f_{se} + nf_{ct}\left(\frac{d_p - c}{c}\right)$$

$$f_p = 155 + 7(1.62)\left(\frac{62.232}{23.29} - 1\right) = 173.96 \text{ ksi}$$

To check the adequacy of reinforcement details using equation (4.24) the steel area of grouted tendons is excluded from computation of the ratio of total area of reinforcement to tension area of concrete.

$$d' = \frac{A_s(d_{total} - d_s) + A_{ps}(d_{total} - d_p)}{A_s + A_{ps}}$$

$$d' = \frac{2.38(67.0-64.66)+3.672(67-62.25)}{2.38+3.672}$$

$d' = 3.80$ in.

The computation of the tension area of concrete having it's centroid at 3.80 in. above the bottom fiber is too complicated because of the non symmetery of the shape about the axis passing through the centroid. So the tension area of concrete is assumed as:

$$(20 \times 3.80) \times 2 = 152 \text{ sq.in.}$$

The ratio of area of directly bonded non-prestressed reinforcement to the tension area of concrete.

$$\rho^* = \frac{2.38}{152} = 0.0157$$

$$\beta = \frac{d_{total} - c}{d - d' - c}$$

$$\beta = \frac{67-23.29}{67-3.80-23.29} = 1.095$$

Fictitious steel stress at the centroid of area of total tension reinforcement.

$$f_s = nf_{ct}\left(\frac{d_s}{c} - 1\right)$$

$$f_s = 7(1.62)\left(\frac{67-3.8}{23.29} - 1\right) = 19.43 \quad \text{ksi}$$

Rigth side of equation (4.24) for a limiting crack width of o.oo6 inches.

$$\left(\frac{13.2x10^3W}{\beta f_s} \right) \frac{\rho^*}{d_c}$$

$$\left(\frac{13.2x10^3x0.006}{1.095(19.43)} \right)^3 \frac{0.0157}{1.75} = 0.462 \quad \text{sq. in.}$$

Which is greater than the largest area of reinforcing bar that is o.44 sq. in.

So adequacy of reinforcing detail is varified.

Solution By The Equation 18.3 of The ACI Code:

Ultimate strength requirements

$$0.85f'_c\ b\ a = A_{ps}\ f_{ps} + A_s f_y$$

From ACI equation 18.3

$$f_{ps} = f_{pu}\left(1 - 0.5\ \frac{f_{pu}}{f'_c}\right)$$

$$f_{ps} = f_{pu}\left(1 - 0.5\ \frac{A_{ps}}{b\ d}\ \frac{f_{pu}}{f'_c}\right)$$

$$f_{ps} = 270\left(1 - 0.5\ \frac{A_{ps}}{7x62.4}\ \frac{270}{5}\right)$$

$$f_{ps} = 270 - 16.69A_{ps}$$

$$\frac{a}{d} = 1 - \sqrt{1 - 2.614U_{mc}}$$

$$U_{mc} = 0.0507$$

$$\frac{a}{d} = 0.0686 \quad , \quad a = 4.283 \text{ in.}$$

substitute the above values in the ultimate strength requirement equation.

$$A_{ps}^2 + 15.27A_{ps} - 57.83 = 0$$

$$A_{ps} = 3.20 \quad \text{sq. in.}$$

CHAPTER VI

Discussion and Conclusions

The design procedure described here is based on steel stress limitations at service load by which the serviceability of members may be assured and ultimate strength requirements. The procedure can lead to the direct computation of the necessary quantities of prestressed and non-prestressed reinforcements that can fullfill simultaneously the above two requirements.

Applying equations (2.21) and (2.23) for steel stress will result in a more exact and realistic approximation than equation (18.3) of ACI code, which takes into consideration only properties of concrete and prestressing steel. Applying equations (4.6) and (4.7) for a given limiting crack width, the value of f_s can not be straightforwardly chosen in computations. Thus, in design the table (4.1) is suggested until more reliable and simple methods for crack width prediction are developed.

To compare the result obtained in this study with the result based on equation (18.3) of the ACI code, area of prestressed tendon is found. As expected the area is smaller than what have been found in design example. The main reason is as observed in charter 2 the ACI formula is on unsafe side. It predictes a higher value for ratio of $\frac{f_{ps}}{f_{pu}}$. In figure (6.1) the ratio of $\frac{f_{ps}}{f_{pu}}$ is ploted vs. prestressed reinforcement ratio. It is shown in figure (6.1) that equation 18.3 of the ACI code predicts the ratio of $\frac{f_{ps}}{f_{pu}}$, three to five percent higher than both equation (2.21) and (2.23).

Similar result is obtained in exact non-linear analysis in reference 2.

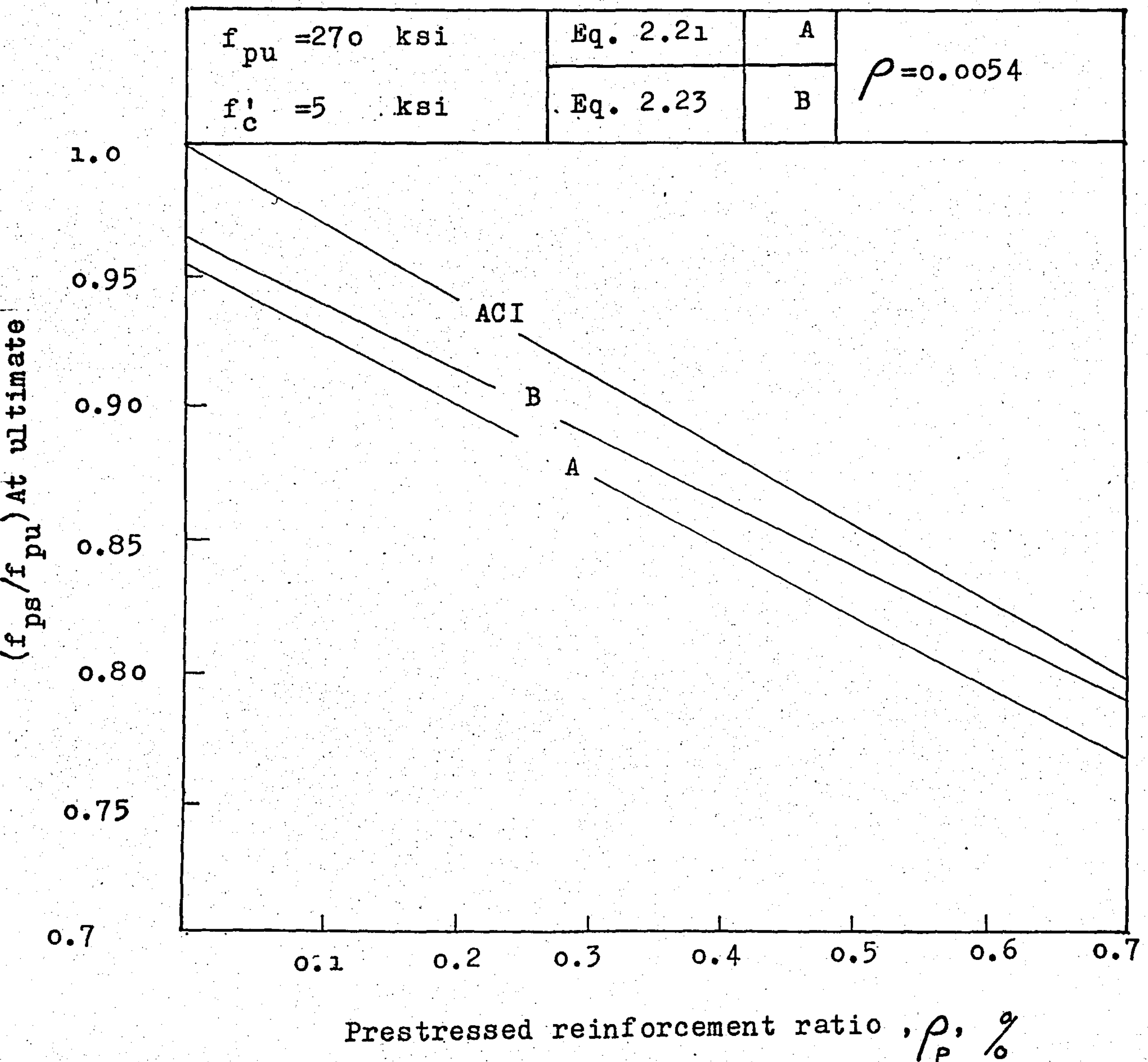


Fig. 5.2 Comparison of results.

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