DETERMINATION OF STAND-BY REQUIREMENTS IN TELECOMMUNICATIONS NETWORKS

by

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We hereby recommend that the thesis entitled "Determination of Stand-by Requirements in Telecommunications Networks" submitted by Dilek Kaptanoğlu be accepted in partial fulfillment of the requirements for the Degree of Master of Science in Industrial Engineering in the Institute for Graduate Studies in Science and Engineering, Boğaziçi University.

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ABSTRACT

To be able to give the required service to telephone customers in case of failures, one of the proposed protection measures is the use of stand-by circuits. Failures are defined as the breakdowns of transmission media. The stand-by protection problem can be solved in two steps.

In the thesis, the first step, which is the determination of stand-by requirements is studied. Methodologies utilized in the calculation of blocking probabilities for obtaining the traffics carried by each route and a simplified heuristic approach called the "equivalent trunk group approach" which is utilized to determine the stand-by requirements are presented. Different mathematical formulations of the problem are given and some special cases are discussed. A solution procedure is developed and applied to a small test network.

ÖZET

Telekomünikasyon şebekesi genellikle santral şebekesi ve iletişim şebekesi olarak ikiye ayrılıp incelenmektedir. Bu ayrım telekomünikasyon şebekelerinin planlanması ve eniyilenmesi problemine büyük kolaylık getirmektedir. Önce santral şebekesinin eniyilenmesi problemi çözülmekte, sonra bu problemin çıktıları girdi olarak kullanılıp iletişim şebekesinin eniyilenmesi problemi çözülmektedir. Bu çalışmada hem santral şebekesinin eniyilenmesi probleminin çıktıları hem de iletişim şebekesinin eniyilenmesinden gelen bazı değerler girdi olarak kullanılmaktadır.

Arıza durumlarında telefon abonelerine istenilen düzeyde servis verebilmek için önerilen koruma önlemlerinden biri "Yedek Bulundurma"dır. Arızalar, iletişim hatlarından herhangi birinin servis veremez duruma gelmesi şeklinde tanımlanmaktadır. "Yedek Bulundurma" problemi iki aşamada çözülebilir. Bu çalışmada, birinci aşama olan "gerekli yedek devre sayısının hesaplanması" problemi ele alınmıştır. İkinci aşama olan yedek optimizasyonu bu çalışmanın kapsamı dışında kalmaktadır.

Her yolda taşınan trafiğin hesaplanmasında, tıkanıklık olasılıklarını elde etmek için kullanılan yöntemler ve gerekli yedek

sayısının hesaplanması için kullanılan basit sezgisel bir 'yöntem olan "eşdeğer hat yaklaşımı" tanıtılmaktadır. Problemin farklı matematiksel formülasyonları verilip, bazı özel durumlar tartışılmaktadır. Geliştirilen çözüm yordamı küçük bir deneme şebekesine uygulanmıştır.

DETERMINATION OF STAND-BY REQUIREMENTS IN TELECOMMUNICATIONS NETWORKS

ABSTRACT

A telecommunications network is a means of interconnecting telephone customers. It is a stochastic service system, where telephone subscribers are the customers and the trunks are the service channels.

The optimization of telecommunication networks is essential for two main purposes. The first purpose is to provide an adequate service level to telephone customers, and the second purpose is to provide this service in a least costly way.

A telecommunications network is generally separated into a switching network and a transmission network. A switching network consists of the switching nodes interconnected by groups of circuits. A transmission network consists of transmission systems interconnecting switching nodes. As a result of this separation and for the sake of simplicity, the telecommunications network optimization problem is treated as two separate problems: The switching network optimization problem and the transmission network optimization problem. The switching network optimization problem and the initial steps of the transmission network optimization problem

which provide necessary input for the problem of determining the stand-by requirements are out of the scope of this study.

To be able to give the required service to telephone customers in case of failures, one of the proposed protection measures is the use of stand-by circuits. Failures are defined as the breakdowns of transmission media. The stand-by proction problem can be solved in two steps. The first step is the determination of stand-by requirements in order to satisfy a predetermined service level. The second step is the stand-by optimization where the optimal routing of these stand-by capacities on the transmission network is obtained.

In the thesis, the first step-determination of stand-by requirements-is studied. Methodologies utilized in the calculation of blocking probabilities for obtaining the traffics carried by each route and a simplified heuristic approach called the "equivalent trunk group approach" which is utilized to determine the stand-by requirements are presented. Different mathematical formulations of the problem are given and some special cases are discussed. A solution procedure is developed and applied to a small test network.

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V : Variance of the global offered traffic.

M : Mean of the global offered traffic.

Z : Peakedness coefficient of the global traffic.

v_i : Variance of the ith offered traffic stream.

m_i : Mean of the ith offered traffic stream.

V_o : Variance of the global overflow traffic.

M_o : Mean of the global overflow traffic.

V_c : Variance of the global carried traffic.

M_c : Mean of the global carried traffic.

v_{io} : Variance of the ith overflowed traffic stream.

m_{io} : Mean of the ith overflowed traffic stream.

v_{ic} : Variance of the ith carried traffic stream.

m_{ic} : Mean of the ith carried traffic stream.

C_k : Amount of traffic carried by route k.

q_{ko} : Number of equivalent circuits for route k in non-failure case.

Q_{ro} : Number of equivalent circuits for relation r in non-failure case.

B_{rf} : End-to-end blocking for relation r when failure f occurs.

$\overline{B}$: Maximum end-to-end blocking allowed in case of failure.

$\overline{Q}_r$: Minimum number of equivalent circuits for relation r.

q_{kf} : Number of equivalent circuits for route k when failure f occurs.

N_{to} : Number of circuits on trunk group t in non-failure case.

N_{tf} : Number of circuits on trunk group t when failure f occurs.

Q_{rf} : Number of equivalent circuits for relation r when failure f occurs.

ΔQ_{rf} : Number of equivalent stand-by circuits required by relation r.

Δq_{kf} : Number of equivalent stand-by circuits required by route k.

ΔN_{tf} : Number of stand-by circuits required by trunk group t.

S_r : Set of routes of relation r.

T_{rf} : Set of trunk groups used by relation r and affected by failure f.

T_k : Set of trunk groups of used by route k.

I. INTRODUCTION

A telecommunications network is a stochastic service system consisting of a number of exchanges, or switching nodes, connected by links which are groups of telephone trunks. In this stochastic service system, telephone subscribers are the customers and the trunks are the service channels.

The optimization of telecommunication networks is essential for two main purposes. The first purpose is to provide an adequate service level to telephone customers, and the second purpose is to provide this service in a least costly way. In other words, satisfactory service should be given to the subscribers while economical use is made of the facilities providing the service.

A telecommunications network is generally separated into a switching network and a transmission network. A switching network consists of the switching nodes interconnected by groups of circuits. A transmission network consists of the transmission systems interconnecting the switching nodes. As a result of this separation and for the sake of simplicity the telecommunications network optimization problem is treated as two separate problems as mentioned in COST 201 project Report, (1980-81), Evranuz, et.al. (1981).

The first problem is the switching network optimization problem (SNOP) in which the optimal trunk group capacities are determined by a dimensioning procedure. Mısırlı (1982) provides an algorithm for SNOP and also introduces the approaches to this problem.

The second problem is the transmission network optimization problem (TNOP) in which the minimum cost facility installation scheme is sought by determining the type of transmission system to be installed on the links. The optimization of the transmission network is carried out in a number of logical steps as mentioned by Nivert and Noort (1983). Baybars and Kortanek (1981), Evranuz (1982), Evranuz and Miraboğlu (1983) have studies on the optimal planning of transmission facilities for telecommunications networks.

The steps of the transmission network optimization problem can be summarized as follows. First a network structure has to be designed, so that a certain degree of structural reliability is achieved and that the routing of circuits will not require major changes in the already existing parts of the network. Secondly the circuit routing is optimized taking into account the diversification requirements.

The network is then analysed in a rather approximate manner to determine whether the service requirements are met in failure conditions. On the basis of this analysis the requirements on a standby protection network can be established, which is to be optimally routed on the remaining capacities in the network.

The switching network optimization problem (SNOP) and the initial steps of the transmission network optimization problem which provide necessary input for the problem of determining the stand-by requirements are out of the scope of this study.

To satisfy the service requirements in failure conditions, three main protection measures are used in the telecommunications network optimization problem as mentioned in COST 201 Project Report (1980-81), Evranuz, et.al., (1981a,b). These measures are:

i. Overdimensioning (overprovisioning)
ii. Multirouting (diversification)
iii. Using stand-by facilities.

The first protection measure can be applied by increasing the trunk group capacities obtained as a result of the switching network optimization problem. The second measure is considered as a part of the circuit routing optimization problem. The third measure which is the use of stand-by facilities must be treated as a separate problem and solved in two steps. The first step which is the determination of stand-by requirements in order to satisfy a predetermined service level is studied in the thesis. The second step which is the optimal routing of these stand-by capacities is not considered in the thesis.

In Section II, basic concepts of teletraffic engineering are introduced to provide a better understanding of the problem definition given in Section III. Section II also provides a literature survey on reliability and availability considerations in telecommunication networks and the methodologies utilized in the solution procedure of the problem.

Section III provides a more concrete description of the telecommunications network optimization problem and the problem studied in the thesis, namely the problem of determining the standby requirements.

Section IV starting with the assumptions of the model describes the steps of the problem together with the approaches utilized and at the end provides a brief summary and flowcharts of the solution procedure applied.

In Section V, the test network is introduced together with the numerical results obtained. The appendices contain the material related to the computer program and the list of the computer program.

II. RELATED TELETRAFFIC ENGINEERING CONCEPTS

This section introduces the basic concepts of teletraffic engineering for the purposes of providing a better understanding of the problem definition. Basic characteristics of telephone traffic are explained and a literature survey is presented concerning the methodologies used in these cases. Some of these methodologies are utilized in the solution procedure. This section also provides a short summary of reliability and availability considerations in telecommunications networks.

2.1 BASIC DEFINITIONS

Telephone traffic, which will be referred to simply as traffic, is defined as the aggregate of telephone calls over a group of circuits (trunks) with regard to the duration of the calls as well as their number (Mina, 1971a). Thus a traffic relation is defined as the total traffic needing to pass from the switching node at the traffic's point of entry to network, to the switching node at the point of exit. An illustration of traffic relation AB is as follows.

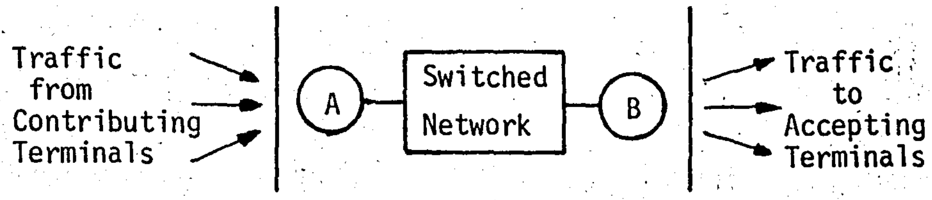


FIGURE 2.1 - Traffic flow for traffic relation AB

One measure of network capacity is the volume of traffic carried over a period of time (Bellamy, 1982). Traffic volume is essentially the sum of all holding times carried during the interval. A more useful measure of traffic is the traffic intensity (also called traffic flow). Traffic intensity is obtained by dividing the traffic volume by the length of time during which it is measured. Thus traffic intensity represents the average activity during a period of time. Although traffic intensity is fundamentally dimensionless (time divided by time), it is usually expressed in units of erlangs, in honor of Danish pioneer traffic theorist A.K. Erlang, or in terms of hundred (century) call seconds per hour (CCS). The relationship between Erlangs and CCS units can be derived by observing that there are 3600 seconds in an hour:

$$1 \text{ erlang} = 36 \text{ CCS}$$

The maximum capacity of a single server (a single trunk in telecommunications terminology) is 1 erlang, which is to say the server is always busy.

Two important parameters used to characterize traffic are the average arrival rate λ and the average holding timet_m. If the traffic intensity A is expressed in erlangs, then

$$A = \lambda t_m \tag{2.1}$$

when λ and t_m are expressed in like units of time (e.g., calls per second and calls per second respectively).

It should be noted that traffic intensity is only a measure of average utilization during a time period and does not reflect the relationship between arrivals and holding times. That is, many short calls can produce the same traffic intensity as a few long ones. For the purposes of this study, as input data traffic intensities are taken which give the traffic offered to each relation. More information related to the mathematical theory of telephone traffic can be found in Bellamy (1982) and Benês (1965).

A switching network is a collection of switching nodes and their interconnecting switching links without regard to the transmission media on which the switching links are carried. A switching node is a switching machine at a specified location and a switching link is the total number of trunks connecting any two specified switching nodes irrespective of their grouping into trunk groups and direction of operation. A trunk group is a set of circuits treated as an entity for dimensioning purposes and is provided to carry a specified amount of traffic between two switching nodes.

The transmission network consists of transmission systems interconnecting switching nodes. It is the collection of transmission nodes and their interconnecting transmission sections (transmission media). A transmission node is a location in the transmission network where a transmission section terminates and which provides multiplexing, demultiplexing or analogue to digital conversion for the interconnection of transmission sections. Some types of transmission media are:

- analogue cable
- analogue radio
- digital cable
- mixed A/D radio
- optical fibre.

2.2 CHARACTERISTICS OF TELEPHONE TRAFFIC

The fundamental relationship coming from the nature of telephone traffic is the following

Offered Traffic = Carried Traffic + Blocked Traffic

where the carried traffic is a measurable quantity and it is the amount of traffic which is actually handled by the system. The offered traffic which is in fact the input traffic to a specific link (trunk group) is usually greater than the carried traffic by the amount of blocked traffic. Blocking or congestion is the situation that a call encounters an "all equipment busy" condition on a given link. So the blocked traffic will either be lost and cleared from the system or overflow to an alternate route, if there exists any.

The traffic distributions which influence the dimensioning procedure applied in the switching network optimization problem (Mısırlı, 1982) are also of great importance for calculating the traffics carried by each route in order to determine the number of equivalent circuits for each route.

There are three types of telephone traffic:smooth, random, and rough (peaky) (Mina, 1971b). To distinguish between those generally the "peakness" concept is used. It is defined as the variance-to-mean ratio, and denoted as,

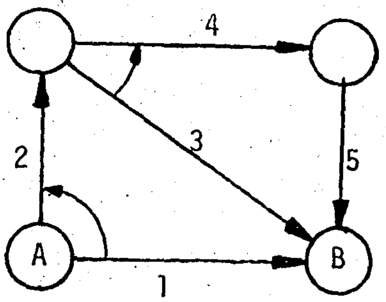
$$\text{v.m.r} = Z = \frac{V}{M} \tag{2.2}$$

Z is usually called the peakedness coefficient or variance-to-mean ratio. When the peakedness coefficient is equal to unity, the traffic is defined as being random and is characterized by Poisson distribution. In the nonrandom case, the traffic is called smooth if the peakedness coefficient is less than unity, and is called peaky or rough if the peakedness coefficient is greater than unity.

The traffic offered to a first choice link is considered as random, because a Poisson distribution of offered traffic is produced by the random arrival of calls. The probability of arrival of a new call in the next instant of time is independent of the number currently present in the system. The overflow traffic is peaky and the carried traffic is smooth.

An alternate routing plan specifies for every origin destination node pair, i.e. for every traffic relation, a first choice and a number of alternate choice routes or paths. Of course, on a switching network it is possible to have a unique path for some traffic relations while having two or more paths for some others.

A simple illustration is the typical organization of a two level hierarchical network up to three choices (Wallström, 1966).



First choice route : {1}
Second choice route: {2,3}
Final route : {2,4,5}

FIGURE 2.2 - Routing plan for origin destination pair AB

2.3 THEORIES AND METHODOLOGIES UTILIZED IN THE CALCULATION OF BLOCKING PROBABILITIES

A telecommunications network is a stochastic service system constructed of a number of exchanges, or switching nodes, connected by links which are groups of telephone trunks. A stream of telephone traffic is a series of events occurring randomly in time, each event being the instant of arrival of a call requesting a trunking connection from an origin exchange to some destination exchange. Like in any stochastic service system some of the traffic offered to a link will be blocked. In case of random (Poissonian) traffic the probability of blocking is given by the Erlang-B formula. But, in cases where the offered traffic is not random, which may be the superposition of several overflow streams or a mixture of random and overflow traffic or some other complex combination, some approximate techniques are used. The use of such techniques arises the need for some other approximate techniques for the calculation of Erlang-B value for non-integral number of trunks.

2.3.1 The Erlang-B Formula

The Erlang Loss formula, which is also known as the Erlang-B formula or the Erlang's first formula given by (2.3)

$$B(N,A) = \frac{A^N}{N!} \Big/ \sum_{j=0}^{N} \frac{A^j}{j!} \qquad (2.3)$$

is fundamental to the study of telephone trunking problems. A.K. Erlang used B(N,A) to express the probability that a call, which is a member of Poisson stream of parameter A, arriving at a group of N telephone trunks will be rejected. Some basic properties of this formula can be found in references; Benês (1965), Cooper (1972), Farmer and Kaufman (1978), Jagerman (1974), Mısırlı (1982), Mina (1971c). Appendix of Cooper (1972) also provides curves for fixed values of N plotted against increasing values of A. The blocking probabilities for $N \leq 80$ and $A \leq 75$ can be determined from these curves.

The numerical computation of B(N,A) as given in (2.3) is awkward when A and N are large since than both numerator and denominator are large. B(N,A) calculated by the recursive relation given in Farmer and Kaufman (1978), Jagerman (1974), Rapp (1964), Szybicki (1964).

$$B(N+1,A) = \frac{A \cdot B(N,A)}{N + 1 + A \cdot B(N,A)}$$

$$B(0,A) = 1 \qquad (2.4)$$

Jagerman (1974), also provides a proof for the derivation of this recursive relation.

2.3.2 Approaches to Calculate Blocking Probabilities for Non-Poissanian Traffic

Starting with Wilkinson's (1956) Equivalent Random Theory many different approximation techniques are developed for the purpose of evaluating the blocking probabilities for trunk groups whose offered traffics are peaky or smooth. Sanders (1981) very briefly summarizes some of those techniques and provides some numerical results for comparative purposes. In this section only Wilkinson's equivalent random theory, Fredericks' and Delbrouck's approximations are introduced. Some other notable studies in this area are by Deschamps (1979). He uses the covariance values between different traffic parcels offered to the same trunk group, whenever possible. So this is a more realistic approach than assuming independency of all traffic streams offered to the same trunk group. Other two studies are by Kuczuro and Bajaj (1977) and by Manfield and Downs (1979). They all make use of the moments of the traffic for their analysis.

2.3.2.1 The Equivalent Random Theory

The equivalent random theory of Wilkinson (1956), is the first technique proposed for the purpose of calculating the blocking probabilities for non-random traffics. It is utilized in the early studies in this area (Rapp, 1964) and recently by Mısırlı (1982).

In case of peaky traffic ($Z > 1$) the main idea behind the equivalent random theory is the following. The offered peaky traffic (M,V) is considered as overflow traffic from an imaginary primary trunk group, having N^* circuits, being offered Poisson Traffic M^*. For each pair (M,V) there corresponds a unique pair (M^*,N^*) M^* and N^* can be found in an iterative way. Rapp (1964), provides very close starting values

$$\begin{aligned} M^* &\simeq V + 3Z(Z - 1) \\ N^* &\simeq (M^*/q) - M - 1 \end{aligned} \qquad (2.5)$$

where

$$Z = V/M \quad \text{and} \quad q = 1 - \{1/(M + Z)\}$$

In a great number of cases these starting values are such that no further iteration steps are needed. But in cases where M is small with high values of peakedness Z, Rapp's iteration scheme may take large number of steps.

After M^* and N^* are obtained, M^* and $N+N^*$ are used instead of M and N respectively in the following calculations, such as obtaining the Erlang-B value and the means and the variances of carried and overflow traffics.

In case of smooth traffic $(Z < 1)$, equivalent random theory can not be applied and generally Poisson traffic is assumed.

2.3.2.2 Fredericks' Approximation

Peakedness was used by Hayward as the basis for an especially simple but surprisingly accurate approximation to the blocking experienced by the overflow traffic on a secondary trunk group (Fredericks, 1980).

In case of $Z > 1$ the system of N servers (trunks) is split up in m subsystems each with N/m servers and offered traffic M/m, Where M is the mean offered traffic in erlangs. The subsystems are stochastically equivalent. The blocking values are equal and equal to the blocking of the original system. Fredericks computes the peakedness $\hat{Z}$ of the traffic offered to each of these groups. Since m is an independ variable one may manipulate $\hat{Z}$ such that $\hat{Z} = 1$. If the correlation between the subgroups is taken to be one, Fredericks then shows that $m = Z$. Making the (amittedly wrong) assumption that $Z = 1$ implies Poisson traffic to each of the subsystems, the Erlang loss function $B(N/m, M/m) = B(N/Z, M/Z)$ describes the blocking values.

In case of smooth traffic, $Z < 1$, the above development can be inverted. The original system is now thought of as being a subsystem of a greater system with mN servers and mM erlang offered traffic. Fredericks show that $m = 1/Z$, which leads again to the blocking $B(mN, mM) = B(N/Z, M/Z)$.

2.3.2.3 Delbrouck's Approximation

In Mısırlı (1982) it is mentioned that peaky traffic has the negative binomial distribution while smooth traffic has the binomial distribution. Because of certain parametric similarities between these two distributions and their limiting relationships to Poisson distributions, Delbrouck (1981) mentions that it is possible to implement a unified approximating procedure to estimate the main congestion functions associated with lost-calls-cleared trunk groups offered peaky or smooth traffics.

M, V, and Z are the mean variance and the peakedness of the offered traffic. m and v are the mean and variance of the carried traffic. x denoting the number of simultaneously occupied trunks, was assumed to have pascal distribution. By using the known values M, V, Z and N (capacity of the trunk group), m, v and B(N,M) are obtained in the following manner. Let,

$$p = \frac{M}{V} \quad , \quad n = \frac{Mp}{1 - p} \quad ,$$
$$q = 1 - p \quad , \quad nq = \frac{M}{Z} \qquad (2.6)$$

using Eq. (2.6)

$$p(x) = p^n(x^{-n})(-q)^x$$
$$p(0) = p^n \qquad (2.7)$$
$$p(x) = q\left(\frac{n + x - 1}{x}\right)p(x - 1) \quad , \quad x = 1,2,\ldots$$

The third equation in (2.7) provides a recursive relation. The distribution is truncated at x = N.

$$q(x) = \frac{p(x)}{\sum_{n=0}^{N} p(n)} \quad , \quad x = 0,1,2,\ldots,N \qquad (2.8)$$

using equations (2.6) to (2.8), m and v are obtained as follows.

$$m = \frac{nq - (n + N)q \cdot q(N)}{1 - q} \qquad (2.9)$$
$$v = \frac{nq + m(n + 1)q - (N + 1)(n + N)q \cdot q(N)}{1 - q} - m^2$$

where q(N) is calculated from (2.7) which is also calculated by using (2.5) and (2.6). Blocking probability is given by

$$B(N,M) = q(N)(1 + \frac{N}{M}(Z - 1)) \tag{2.10}$$

According to Delbrouck the above approximation can be extended to the case of smooth traffic when the parameters n and q are negative. He also provides some approximations for cases where several traffic parcels with different Z-factors are offered to the same trunk group.

2.3.3 Approaches to Calculate Erlang-B Value for Nonintegral Number of Trunks

As seen in Sections 2.3.2.1 and 2.3.2.2, both the Wilkinson's ERT and Fredericks' approximation for the calculation of blocking probabilities for nonpoissonian traffics leads to the calculation of Erlang-B value for nonintegral number of trunks. In fact, in such cases the easiest way is to take the nearest integer value as the parameter of the Erlang-B formula. To be more precise some interpolation techniques are given in literature for the purpose of calculating the Erlang-B value for nonintegral number of trunks. Here four of such techniques are presented and in Appendix A some numerical results are given for three of those techniques.

2.3.3.1 Rapp's Approximation

Recalling the recursive relation of the Erlang-B formula given by equation set (2.4), Rapp (1964) states that the recursion formula can be started by a value δ where, $0 < \delta < 1$ and he provides an approximate value for $B(\delta,M)$. So if X is the nonintegral value for number of trunks coming from the approximations of Wilkinson or Fredericks, $X = N + \delta$ where N is the integer part of X. Rapp approximates $B(\delta,M)$ as follows.

$$B(\delta,M) \simeq C_0 + C_1\delta + C_2\delta^2 \tag{2.11}$$

where,

$$C_0 = 1$$

$$C_1 = \frac{M + 2}{(1 + M)^2 + M}$$

$$C_2 = \frac{1}{(1 + M)[(1 + M)^2 + M]}$$

After obtaining $B(\delta,M)$ by (2.11) $B(X,M)$ can be easily obtained by using the recursive relation.

2.3.3.2 Szybicki's Approximation

Szybicki (1964) like Rapp also uses the recursive relation and provides an approximation for $B(\delta,M)$ which is different than Rapp's. He takes,

$$B(\delta,M) \simeq \frac{(2 - \delta) \cdot M + M^2}{\delta + 2M + M^2} \tag{2.12}$$

Szybicki also mentions Rapp's approximation and he concludes that both of these interpolations give quite accurate results.

2.3.3.3 Jagerman's Approximation

Jagerman (1974) proposes a different approximation technique as an extension of a theorem which is stated in his paper. As in the previous techniques let X denote the nonintegral value for the number of trunks coming from the approximations of Wilkinson or Fredericks, $X = N + \delta$ where N is the integral part of X. Jagerman approximates B(X,M) as follows.

$$B(X,M) \simeq \frac{B^{1-\delta} \cdot B_1^{\delta}}{1 - \frac{1}{2}\delta(1-\delta)\left(\frac{B_1^2}{BB_2} - 1\right)} \qquad (2.13)$$

where,

$$B = B(N,M), \quad B_1 = B(N+1,M), \quad B_2 = (N+2,M)$$

2.3.3.4 Approximation Through a Continued Fraction

Lévy Soussan proposes a technique for the numerical evaluation of the Erlang function through a continued fraction algorithm. The basic characteristic of this algorithm is that it can be simply applied either when the number of trunks is a nonintegral value or an integral one. But this technique can not be applied for all values of N and M. Farmer and Kaufman (1978) also states this algorithm and they specify the cases when this algorithm is applicable.

To apply the continuous fraction algorithm first the number of terms in the continued fraction is calculated by

$$k = \frac{5\sqrt{N^2 + 500N}}{4(M - N + \sqrt{N})} + 2 \qquad (2.14)$$

Absolute value of k rounded to the nearest integer value will yield B(X,M) accurate to the 6 decimal points in the cases where this algorithm is applicable. B(X,M) is approximated by the following continued fraction.

$$B(X,M) = \frac{M + \cfrac{-X}{1 + \cfrac{1}{M + \cfrac{-X+1}{1 + \cfrac{2}{M + \cfrac{-X+2}{1 + \cdots\cdots}}}}}}{M} \qquad (2.15)$$

This algorithm can be used for values M < X when N is a noninteger smaller than 15 and also for all values of M and X satisfying M > X.

2.4 RELIABILITY AND AVAILABILITY CONSIDERATIONS

Before discussing the meaning of reliability and availability concepts in telecommunications networks it is essential to state some general definitions given by Barlow and Proschan (1965).

"Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered".

"Pointwise availability is the probability that the system will be able to operate within the tolerances at a given instant of time".

"Interval availability is the expected fraction of a given interval of time that the system will be able to operate within the tolerances".

2.4.1 Approaches to Determine Availability in Telecommunications Networks

The usual simple reliability concept which is entirely based on the component failure is not adequate for a telephone network. In a telecommunications network both equipment failure and traffic congestion affect the service given to users. A path is said to be failed if it cannot be used for connecting any pair of telephone customers.

Lee (1970) defines the "network unserviceable probability" as the combined effect of equipment failure and traffic congestion. A connecting path of a network is unavailable if (i) it is failed or if (ii) it is not failed but busy. When all paths are unavailable the network is said to be unserviceable. The network unserviceable probability at any time t is given by Lee (1970) as follows.

$$P(t) = \prod_{i=1}^{M} F_i(t) + \sum_{n=1}^{M-1} B_{M-n}(t)\Big(\sum_{k=1}^{\binom{M}{n}} \prod_{i \varepsilon G_n(k)} F_i(t) \cdot \prod_{i \notin G_n(k)} (1 - F_i(t))\Big) + B_M(t) \prod_{i=1}^{M} (1 - F_i(t) \qquad (2.16)$$

where M is the number of parallel paths. $0 < n < M$, $F_i(t)$ is the failure probability distribution of path i, $B_{M-n}(t)$ is the conditional probability that M-n paths are busy in serving telephone calls given that the other n paths are failed, P(t) is the network unsercriceable probability at any time t and G_n is the set containing combinations $\binom{M}{n}$ where $G_n(k)$ denotes one of these combinations. In equation (2.16) the first term is the probability that exactly M paths has failed, second term is the probability that n paths has failed and the remaining (M-n) paths are busy, and the last term is the probability that all the M paths are busy and none of them has failed. Lee (1971) also discusses the use of computer aided methods for calculating the unserviceable probability of a class of telecommunications networks.

Lajtha (1975) of the Research Institute of the Hungarian Post Office has stated that there was need for empirical data to measure reliability and availability of the telecommunications network. To characterize reliability they tried to determine the number of interruptions, n, for a year for different units and circuits. Down Time Ratio (DTR) is mentioned as a widely used availability characteristic

$$DTR = \frac{\text{down time}}{\text{up time} + \text{down time}} .$$

If A is the availability defined as the ratio between the time period during which the element tested can be used for the requisite purpose and the test interval, we have

$$DTR = 1 - A \tag{2.17}$$

Letting L_i denote the down time (in hours/year/unit) for the i^{th} fault and having n interruptions during the year. We can write

DTR as follows.

$$DTR = \sum_{i}^{n} L_i \qquad \text{or} \qquad DTR = n.\overline{L_i} \qquad (2.18)$$

Since the telecommunications service is characterized by the values n, L_i and DTR Lajtha proposes survey methods to collect statistics on these values. He concluded that "availability in a switched network can be characterized only by the changes in the probability of establishing the connection."

According to COST 201 Study Group (1980-81) the terminal availability of two operating nodes is the probability of successful communication between the two. They define the global network availability as the probability that an operating path exists between any two terminal nodes.

Chan (1980) defines the availability of a link as having at least one idle trunk. He does not consider failure of circuits.

2.4.2 Grade of Service and Service Quality Concepts

Wright (1970) mentions three different aspects of grade of service. These are:

- nominal grade of service for a group of circuits
- nominal end-to-end grade of service
- nominal network grade of service.

Nominal grade of service for a group of circuits is usually taken as being equal to the congestion function that is used to calculate how many switches are required for some specific traffic.

Nominal end-to-end grade of service corresponds to the average probability that a caller will fail to establish a connection because of the limited number of circuits.

Nominal network grade of service is the average of the nominal end-to-end grades of service over the whole network when weighted according to the traffic in the different relations.

Grade of service can be referred to any practicable means of measuring congestions Datrois (1977) gives the following definitions for grade of service and service quality. "Exchange grade of service is a component of the exchange service quality, qualifying the normal exchange reaction to traffic variations in the ideal situation in which the exchange is completely fault and trouble free".

"Exchange service quality is a measure of exchange's contribution to the overall service quality in a specified environment. It covers all possible disturbances to the call handling process caused by the exchanges's reaction to traffic variations and by its reaction to faults, failures, and troublesome situations within the exchange".

"Overall service quality is a measure of the call handling properties of a telephone network as observed by the users for stated traffic conditions. The overall service quality depends not only upon individual exchangee service qualities but also on the network performance characteristics which are associated with subscriber and trunk network behaviour".

For our purposes, grade of service is the probability that a call arriving in the network will not receive service. In teletraffic terminology this probability is usually referred to as point-to-point congestion or end-to-end blocking.

2.4.3 Increasing the Reliability and Use of Stand-by Facilities

To increase the reliability of telecommunications networks COST 201 Study Group (1979) points out two important facts which are explained according to the interaction between the switching network and the transmission network.

i. No part of a trunk group carrying alternatively routed traffic should have a transmission path sharing a transmission section with any trunk group from which it receives overflow traffic (its "contributing groups") or, if it does, those sections common to the trunk group and its contributing groups should have a high reliability, for example, be protected by an adequate service protection network.

ii. No traffic route should have more than a specified percentage of its capacity carried on any one transmission path, probably depending upon the size of the route and subject to the avoidance of excessively circuitous transmission paths or excessive costs.

To achieve these objectives a rough assessment of reliability might be made by the switching network optimization and a fine adjustment by the transmission optimization.

Some specific factors to increase reliability are the protection measures, which are overdimensioning, multirouting and use of stand-by facilities.

Overdimensioning is simply providing more circuits than the optimum ones. Multirouting or diversification is considered in the circuit routing optimization problem. To increase reliability in failure conditions the most effective way is to use stand-by facilities. It is obvious that in any system of components, use of stand-by facilities increase the reliability.

In the thesis, rather than reliability, availability concept will be used and it will be related to the predetermined value of the grade of service. The problem is to ensure that for every origin-destination exchange pair, i.e. for every traffic relation, the grade of service falls within a given acceptable level for failure conditions. If the required grade of service value is not achieved, stand-by circuits are required. Determination of those stand-by requirements is the subject of this study and the solution procedure will be explained in Section IV.

III. DEFINITION OF THE PROBLEM

This section has two main purposes. The first purpose is to describe briefly the inter-relationship between the switching and transmission models in the context of the overall optimization model. The second purpose is to state a more concrete definition of the problem of determining the stand-by requirements as a part of the global optimization problem.

3.1 AN OVERVIEW OF THE TELECOMMUNICATIONS NETWORK OPTIMIZATION PROBLEM

A telecommunications network is optimized in two main steps. The first step is the switching network optimization. Main inputs to this problem are the forcasted values of traffic flow between each origin destination pair, mean circuit costs and the required service level. As a result of the switching network optimization problem optimal trunk group capacities and the routing pattern for each traffic relation are obtained.

The second step of the overall optimization model is the transmission network optimization. Inputs to the transmission network optimization problem are a maximal graph of transmission nodes and media optimal trunk group capacities obtained as a result of the switching network optimization problem, and also the cost values and service requirements in failure situations. Nivert and Noort (1983) and Lindberg, et.al. (1983) who are members of the COST 201 Project Study Group describe the steps of the transmission network optimization problem by the following flowchart shown in Figure 3.1.

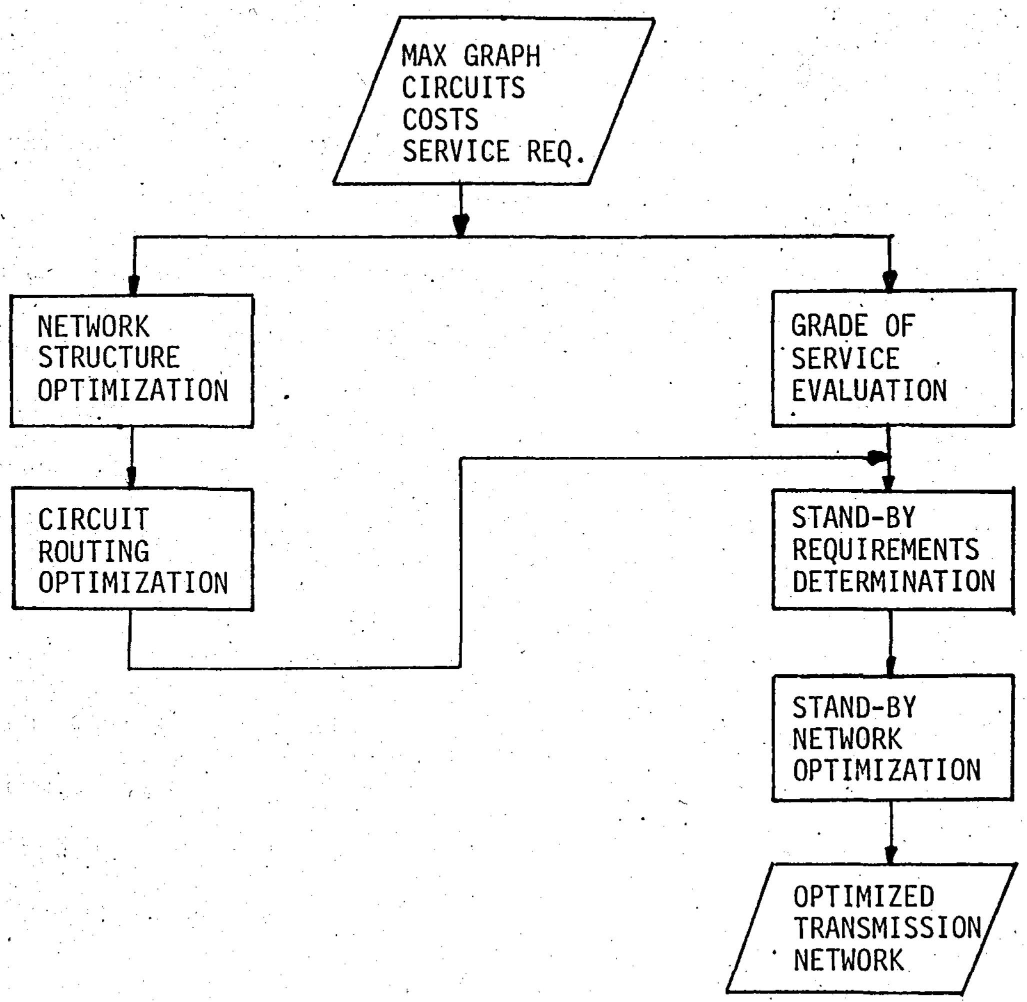


FIGURE 3.1 - Flowchart of the Tranmission Network Optimization Problem

By Network Structure Optimization, basic structure of the transmission network is determined from a maximal set of existing or proposed media. As a result some of the edges of the maximal media graph are deleted (Evranuz, 1981).

By Circuit Routing Optimization optimal routing of trunk groups in the transmission network is determined. The trunk groups are multirouted either if this is required or if this can be done at a relatively low cost. Different transmission techniques such analogue, digital or their combination are considered.

Grade of Service Evaluation and Stand-by Requirements Determination which are the subject of the thesis have the purpose of calculating the end-to-end blockings for each traffic relation and determining the number of stand-by circuits to be added to each trunk group routed on the failed transmission media. Grade of Service Evaluation and Determination of Stand-by requirements will be explained in more detail in the next section.

While determining the stand-by requirements no cost values are considered, the only objective was to satisfy the required service level. In the next step which is the stand-by optimization, the stand-by requirements are routed in the transmission network at a minimum cost.

3.2 THE PROBLEM OF DETERMINING THE STAND-BY REQUIREMENTS

The subproblems of the transmission network optimization problem such as network structure optimization and circuit routing optimization are solved on the transmission network but the problem of determining the stand-by requirements is solved on the switching network while making use of the results of the circuit routing optimization problem.

The inter-relation between the switching and the transmission networks can be made clear by the following example. COST 201 Project (1980).

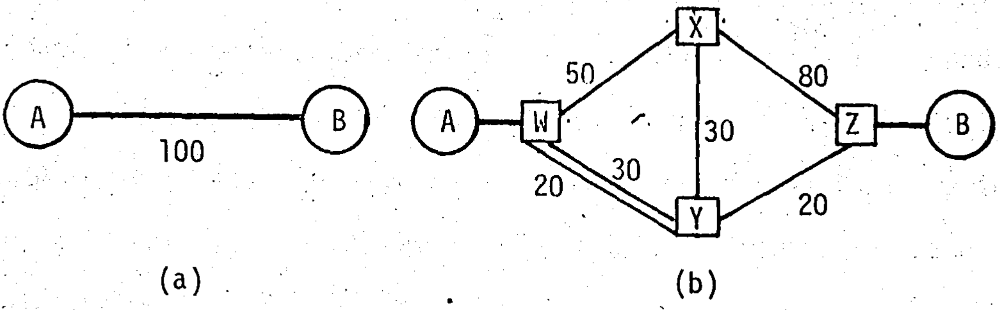


FIGURE 3.2 - Routing of trunk group AB on the Transmission Network

Figure 3.2(a) shows a trunk group of capacity 100 between the switching nodes A and B, Figure 3.2(b) shows the routing of these circuits in the transmission network where X, Y, Z, and W are transmission nodes. The allocation of 100 circuits to three transmission paths is as follows.

A → WYZ → B	20 circuits
A → WYXZ → B	30 circuits
A → WXZ → B	50 circuits

As it can be seen from Figure 3.2(b) there may be more than one transmission section between two transmission nodes. This example also makes clear that usually failure of a transmission media does not affect all the circuits of a trunk group.

In this study failures are defined as the breakdowns of transmission media. Only single failures that is one transmission media at a time are considered.

Grade of service evaluation has the purposes of calculating the end-to-end blocking for each relation and determining the minimum number of equivalent circuits for each relation, in failure conditions.

Determination of stand-by requirements is the calculation of stand-by circuits required for each transmission medium failure in order to achieve minimum number of equivalent circuits for each relation.

IV. FORMULATION AND THE SOLUTION PROCEDURE

This section states the main assumptions of the model used and provides a detailed description of the solution procedure which will be separated into two steps, namely the grade of service evaluation and the determination of stand-by requirements. Three mathematical formulations of the model are stated and the special cases that may occur as a result of a failure are discussed with their related solution procedures.

4.1 ASSUMPTIONS OF THE MODEL

The main assumptions of the model can be summarized as follows.

i. Traffic originates from an infinite number of traffic sources, implying Poisson traffic input.

ii. Lost calls are cleared from the system with zero holding time.

iii. Poissonian traffic is first offered to the first choice route of the traffic relation to which it belongs to.

iv. The global traffic offered to any trunk group is obtained by combining the indivudally offered streams under the assumption of independence.

v. Any trunk group may be carrying a mixture of random, peaky, and smooth traffics. Mean trunk group blockings are calculated by using the global traffic.

vi. Traffic carried or overflowed by the preceding trunk group, disregarding the influence of the following ones, is used for computing the stream offered to a trunk group by a particular relation (Cavellero and Tonietti, 1981).

vii. Mean trunk group blocking is assigned to each traffic stream using that trunk group.

viii. Each traffic stream overflowed or carried by a trunk group is assumed to have the same peakedness with the global overflowed and carried traffics (Butto, et.al., 1976).

ix. Failures are defined as the breakdowns of transmission media. Only single failures, one transmission medium at a time, are considered. When a transmission medium is in failure all the circuits routed on it are down (Cost Project 201, 1980-81, Lindberg, et.al., 1983).

x. The stand-by requirements for each trunk group are calculated as real numbers and rounded off to yield integral values.

A few other assumptions coming from the equivalent trunk group approach and the simplified approach will be stated in the related sections.

4.2 GRADE OF SERVICE EVALUATION

As mentioned in the previous section, grade of service evaluation is done for two purposes (Cavellero and Tonietti, 1981; Nivert and Noort, 1983; Lindberg, et.al., 1983).

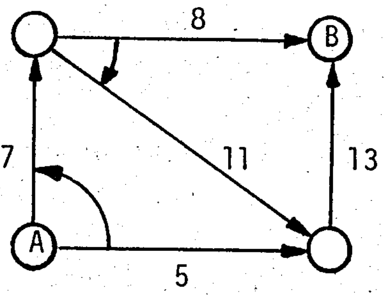
The first purpose is to calculate the end-to-end blockings for each traffic relation, taking overprovision into account. Prior to end-to-end blockings computation the traffic carried by each route has to be calculated. The second purpose is to calculate the number of circuits of equivalent trunk groups for each route, and the minimum number of equivalent circuits for each traffic relation which has to meet the required service level in failure conditions.

4.2.1 Calculation of Traffic Carried by Each Route

Optimal trunk group capacities which are the output of the switching network optimization problem are changed as a result of overprovisioning. Using these new values of trunk group capacities, the routing scheme for each traffic relation, and the means of the traffic offered to each relation, the traffic carried by each route is calculated.

The traffic offered to each traffic relation was assumed to be Poissonian and offered to the first choice route of this relation.

The traffic blocked overflows to the next route from the congestioned trunk group and continues in this manner. If a call arrives to a congestioned trunk group and there exists no alternative route to overflow, then the call does not receive service and cleared from the system.



First Choice Route {5,13}
Second Choice Route {7,8}
Final Route {7,11,13}

FIGURE 4.1 - Routing pattern for traffic relation AB

In Figure 4.1, a specific traffic relation taken from the test network that will be introduced in Section V is given as an illustration (Traffic relation 10). The numbers of the trunk groups are also the ones used in the test network.

The traffic flow from A to B is first offered to trunk group 5 which is the initial trunk group of the first choice route. Some of this traffic is carried by trunk group 5 and due to congestion the remaining is blocked. The blocked amount is offered to trunk group 7 as overflow traffic which is no more random but peaky and the carried amount is offered to trunk group 13 which is the next and the last trunk group of the first choice route. This traffic is also no more random but smooth. The traffic carried by trunk group 7 is offered to trunk group 8 which is again smooth, but the traffic blocked at trunk group 7 is lost since there is no alternative route to overflow from trunk group 7. The traffic blocked at trunk group 8 overflows to trunk group 11, the blocked amount at 11 is also lost, and the carried amount is offered to trunk group 13. According to the given routing pattern; trunk group 13, which is the last trunk group of both the first choice and the final route, carries two traffic streams belonging to the same relation.

In fact, each trunk group may carry two or more traffic streams either belonging to the same traffic relation or not. By assumption (iv), the mean and the variance of the global traffic offered to a trunk group equals to the sum of the means and the sum of the variances of all the traffic streams offered to that trunk group respectively. Thus the peakedness coefficient Z is calculated as:

$$Z = \frac{V}{M} = \frac{\Sigma v_i}{\Sigma m_i} \qquad (4.1)$$

where, V and M are the variance and the mean of the global traffic respectively. v_i and m_i denote the variance and the mean of the i'th traffic stream respectively. According to its peakedness value we can call the global traffic as random, smooth or peaky as mentioned in Section 2.2.

For the computation purposes, the trunk groups of the switching network are numbered in such a way that, if a trunk group is the initial trunk group of any first choice route, traffic stream offered to that trunk group originates from the traffic data, on the other hand, traffic stream offered to trunk groups other than trunk group 1 either flows from the preceding trunk groups (as overflowed or carried) or originates from the traffic data. For example, to trunk group 5 traffic flow only from trunk groups 1, 2, 3, and 4.

The blocking probabilities for each trunk group are calculated by using N, the mean M of the global offered traffic. The peakedness coefficient Z is used to determine the type of the traffic. If Z equals to unity then the global traffic is random and the blocking probability for the trunk group having N trunks is given by the Erlang-B formula.

$$B(N,M) = \frac{M^N}{N!} \Big/ \sum_{j=0}^{N} \frac{M^j}{j!} \tag{4.2}$$

If Z is greater than unity then the global traffic is peaky and the blocking probability is calculated by using Frederick's (1980) approximation, which is introduced in section 2.3.2.2, as the parameters of the Erlang-B formula N/Z and M/Z are taken instead of N and M respectively. N/Z may not be an integer so another approximation is needed for calculating the Erlang-B value for nonintegral number of trunks and Rapp's (1964) approximation of Section 2.3.3.1 is utilized for this purpose in the thesis. If Z is less than unity then the global traffic is smooth and the blocking probability is calculated in the same way it was done for peaky traffic.

The means and the variances of the overflow traffics are calculated by the following formulae (Mina 1971b; Mısırlı, 1982; Rapp, 1964; Szybicki, 1964). In case of smooth or peaky offered traffics, the parameters of B(•,•) are changed according to Fredericks' approximation.

$$M_o = M.B(N,M) \qquad (4.3)$$

$$V_o = M\left(1 - M_o + \frac{M}{N + 1 + M_o - M}\right) \qquad (4.4)$$

where M_o and V_o are the mean and the variance of the global overflow traffic respectively.

The means and the variances of the carried traffic are given by the following formulae..

$$M_c = M.1 - B(N,M) \qquad (4.5)$$

$$V_c = M_c\{1 - M[B(N-1,M) - B(N,M)]\} \qquad (4.6)$$

where M_c and V_c are the mean and the variance of the global carried traffic respectively.

Starting with the first trunk group, the global traffic offered to this trunk group is determined, mean blocking is determined, means and variances of the global carried and overflowed (lost) traffics are calculated by using the set of equations (4.3 - 4.6), then the means and the variances of each traffic stream. overflowed from or carried by that trunk group are calculated. The means of overflowed and carried streams are calculated by simple ratio as follows.

$$\frac{m_i}{M} = \frac{m_{ic}}{M_c} = \frac{m_{io}}{M_o} \qquad \forall\, i \qquad i = 1,\ldots,k \qquad (4.7)$$

where k is the number of different traffic streams offered to that trunk group. The variances of overflowed and carried traffic streams are obtained by using assumption (iii), i.e., by equating the peakedness of the streams to the peakedness of the global overflowed and carried traffics respectively as proposed by Butto, et.al. (1976)

$$\frac{v_{ic}}{m_{ic}} = \frac{V_c}{M_c} \qquad \forall\, i \quad , i = 1,\ldots,k \tag{4.8}$$

$$\frac{v_{io}}{m_{io}} = \frac{V_o}{M_o} \qquad \forall\, i \quad , i = 1,\ldots,k \tag{4.9}$$

Each overflowed and carried traffic stream is offered to its following trunk group if there exists such a trunk group. For the traffic streams offered to a trunk group the following information is needed.

i. The preceding trunk group
ii. The related traffic route
iii. The related traffic relation.

The traffic carried on a route equals to the traffic carried on the last trunk group of that route. Hence, if a traffic stream carried on a trunk group belongs to a route whose last trunk group is that trunk group traffic carried by this route is obtained and there is no "following trunk group" for this stream. The overflow traffic of a traffic stream is lost if there exists no alternative route to overflow from that trunk group.

At the end of the procedure explained above the blocking probabilities of each trunk group and the traffics carried by each route are obtained.

4.2.2 Calculation of End-to-End Blockings

The probability that a call at any orinating node does not reach to its destination was defined as the end-to-end blocking. There are many approaches in literature for the calculation of end-to-end blockings in switching networks. Some of those are given by Butto, et.al. (1976), Chan (1980), Gaudreau (1980), and Horn (1979). But in this study, since the traffics carried by each route are calculated, end-to-end blockings are calculated simply by utilizing Berry's (1970) formula.

$$\sum_{k \varepsilon S_r} C_k = M_r(1 - B_r) \tag{4.10}$$

where C_k is the traffic carried by route k, S_r is the set of the routes of relation r, M_r is the mean of the traffic offered to relation r, and B_r is the end-to-end blocking for relation r. Knowing C_k and M_r one can easily calculate B_r by (4.10).

4.2.3 Equivalent Trunk Group Approach

The Equivalent Trunk Group Approach which is proposed by Cavellero and Tonietti (1981) of COST 201 Study Group is used for handling failures. In large networks it would require very long computer times to calculate end-to-end blockings for every traffic relation in each failure state. Therefore the computations are performed in a simplified heuristic way based on substituting each traffic route by an equivalent trunk group (Lindberg, et.al., 1983).

The main assumption of this approach is the independence of traffic relations.

4.2.3.1 Calculation of Equivalent Circuits for Each Route

According to the Equivalent Trunk Group Approach the end-to-end blocking of a traffic relation is defined as the loss probability of an equivalent trunk group whose capacity is the sum of the capacities of trunk groups equivalent to the routes used by the relations.

If we think of a traffic relation with three alternate routes, as given in Figure 4.1, the mean traffic M is first offered to q_1 circuits, where q_1 is the number of equivalent circuits of the first route and C_1 Erlangs are carried. The overflow is then offered to q_2 circuits of the equivalent trunk group of the second route and etc. Now using the Erlang-B formula and since C_k's are known, the quantities q_k can be calculated from the following set of equations.

$$C_1 = M[1 - B(q_1,M)] \tag{4.11a}$$

$$C_1 + C_2 = M[1 - B(q_1+q_2,M)] \tag{4.11b}$$

$$C_1 + C_2 + C_3 = M[1 - B(q_1+q_2+q_3,M)] \tag{4.11c}$$

From (4.11a) q_1, the number of equivalent circuits for the first route, is determined by using the recursive relation of the Erlang-B formula given by equation (2.3). From (4.11a) we obtain

$$B(q_1,M) = \frac{M - C_1}{M} = B_1 \qquad (4.11a')$$

So the problem is to determine q_1, when M and B_1 are known. Having M fixed Erlang-B value decreases as the number of circuits increases. Hence by the recursive relation the integer part of q_1 is determined. For obtaining more percise values for q_1 Jagerman's approximation is used to determine the fraction part.

Similarly, from (4.11b) q_1+q_2 is determined since q_1 is previously calculated q_2 is obtained by simply subtracting. From (4.11c) $q_1+q_2+q_3$ is determined which is in fact, the number of equivalent circuits for relation AB, gives q_3, which is the number of equivalent circuits for the 3rd route.

4.2.3.2 Calculation of Minimum Equivalent Circuits for Each Relation

Equivalent circuits for traffic relation r, in non-failure conditions is defined as the sum of the equivalent circuits of its routes again calculated for non-failure cases.

$$Q_{ro} = \sum_{k\varepsilon S_r} q_{ko} \qquad (4.12)$$

where Q_r denotes the number of equivalent circuits of relation r and the second subscript o denotes the non-failure situation.

In failure cases each relation must have an end-to-end blocking value not greater than a predetermined end-to-end blocking value allowed for failure cases (EEB_f), that is,

$$B_{rf} \leq \overline{B} \tag{4.13}$$

where B_{rf} denotes the end-to-end blocking for relation r in failure f, $\overline{B}$ denotes the maximum end-to-end blocking allowed in case of failure. From now on the second subscript f will denote the failure and correspond to the number of the transmission medium failed.

Minimum number of equivalent circuits for a relation, which is denoted by $\overline{Q}_r$ is calculated by using the maximum end-to-end blocking allowed ($\overline{B}$) and the mean of the offered traffic (M) and utilizing the Erlang-B formula.

$$\overline{B} = B(\overline{Q}_r, M) \tag{4.14}$$

Again the recursive relation and the Jagerman's approximation is utilized to calculate $\overline{Q}_r$.

4.3 DETERMINATION OF STAND-BY REQUIREMENTS

Stand-by requirements are calculated by utilizing the equivalent trunk group approach (Cavellero and Tonietti, 1981) when failure f occurs some trunk groups have their capacities reduced. If trunk group t whose capacity is reduced by the failure is used in route k, the reduction in equivalent circuits of route k is assumed to be in the same proportion as the reduction of actual circuits of trunk group t. So, the number of equivalent circuits of route k in failure f is:

$$q_{kf} = q_{ko} \cdot \frac{N_{tf}}{N_{to}} \tag{4.15}$$

where N_{to} is the capacity of trunk-group t in non-failure condition and N_{tf} the capacity of the same trunk group when failure f occurs.

If in route k more than one trunk group fail, it is assumed that the number of equivalent circuits is reduced according to the "worst" (i.e. the one affected the most from the failure) trunk-group. Thus the number of equivalent circuits of route k in failure f is given by

$$q_{kf} = q_{ko} \cdot \min_{t \varepsilon T_k} \left(\frac{N_{tf}}{N_{to}}\right) \tag{4.16}$$

where T_k is the set of trunk groups of route k. The number of equivalent circuits of relation r in case of failure is given by

$$Q_{rf} = \sum_{k \varepsilon S_r} q_{kf} \tag{4.17}$$

If inequality (4.18) is satisfied

$$Q_{rf} \geq \overline{Q}_r \tag{4.18}$$

no stand-by circuits are required for relation r, otherwise it is necessary to increase the number of equivalent circuits of relation r by an amount:

$$\Delta Q_{rf} = \overline{Q}_r - Q_{rf} \tag{4.19}$$

4.3.1 Mathematical Formulation of the Problem

In order to increase the capacity of a route in terms of number of equivalent circuits, it is necessary to increase the number of operating circuits of failed trunk groups by adding stand-by circuits. The amount of stand-by circuits that has to be added to each route is obtained by equation (4.20) which is derived from equation (4.16)

$$q_{kf} = q_{ko} \cdot \min_{t \varepsilon T_k} \{ \frac{N_{tf} + \Delta N_{tf}}{N_{to}} \} - q_{kf} \qquad (4.20)$$

where ΔN_{tf} is the number of stand-by circuit that has to be added to trunk group t when failure f occurs. So the mathematical formulation of the problem is as follows (Cavellero and Tonietti, 1981),

$$\min \sum_{t \varepsilon T_{rf}} \Delta N_{tf} \qquad (P1)$$

$$\text{s.t.} \quad \sum_{k \varepsilon S_r} [q_{ko} \cdot \min_{t \varepsilon T_k} \{ \frac{N_{tf} + \Delta N_{tf}}{N_{to}} \} - q_{kf}] = \Delta Q_{rf}$$

$$\Delta N_{tf} \geq 0$$

$$\Delta N_{tf} \leq N_{to} - N_{tf}$$

where T_{rf} is the set of trunk groups used by relation r and affected by failure f. Problem (P1) is simply the minimization of stand-by requirements. The equality constraint is the quality of service constraint. Last two constraints provides bounds on the variables ΔN_{tf}. With the stand-by circuits the trunk group can have capacity at most equal to its capacity in non-failure case. This provides an upper bound for each variable and the nonnegativity constraint provides a lower bound. Since the equivalent trunk group approach requires the independence of relations, problem (P1) is formulated just for a single traffic relation.

4.3.2 Reformulation as a Linear Programming (LP) Problem

Problem (P1) is reformulated as an LP problem by eliminating the minimization term in the equality constraint. A set of new variables are defined for the minimization terms and a set of constraints are added in the following manner to obtain (P2)

$$\min \sum_{t \in T_{rf}} \Delta N_{tf} \tag{P2}$$

$$\text{s.t.} \quad \sum_{k \in S_r} [q_{ko} \cdot y_k - q_{kf}] = \Delta Q_{rf}$$

$$N_{to} y_k \leq N_{tf} + \Delta N_{tf} \qquad \forall t \in T_k \; ; \; \forall k \in S_r$$

$$\Delta N_{tf} \geq 0$$

$$\Delta N_{tf} \leq N_{to} - N_{tf}$$

A numerical example for the LP formulation is given in Appendix B.

4.3.3 A Simplified Approach

The simplified approach proposed by Cavellero and Tonietti (1981) assumes the independence of the routes and uses the number of stand-by circuits added to each route as the variables. Then the stand-by circuits for the trunk groups is calculated by using equation (4.21)

$$\Delta N_{tf} = \frac{N_{to}}{q_{ko}} (\Delta q_{kf} + q_{kf}) - N_{tf} \qquad t \varepsilon T_{kf} \qquad (4.21)$$

where T_{kf} is the set of trunk groups of route k affected by failure f. In equation (4.21) let

$$\alpha_{kt} = \frac{N_{to}}{q_{ko}} \qquad t \varepsilon T_{kf} \qquad (4.22)$$

and

$$\beta_{kt} = \frac{N_{to}}{q_{ko}} q_{kf} - N_{tf} \qquad t \varepsilon T_{kf} \qquad (4.23)$$

So (4.21) becomes,

$$\Delta N_{tf} = \alpha_{kt} \cdot \Delta q_{kf} + \beta_{kt} \qquad t \varepsilon T_{kf} \qquad (4.24)$$

Equation (4.24) gives the stand-by capacity ΔN_{tf} of trunk group t in failure f, necessary to guarantee an increase of equivalent capacity Δq_{kf} of route k.

The formulation according to the simplified approach is as follows.

$$\min \sum_{k \varepsilon S_r} \alpha_k \cdot \Delta q_{kf} \qquad (P3)$$

$$\text{s.t.} \sum_{k \varepsilon S_r} \Delta q_{kf} = \Delta Q_{rf}$$

$$\Delta q_{kf} \geq 0$$

$$\Delta q_{kf} \leq q_{ko} - q_{kf}$$

where α_k is defined as,

$$\alpha_k = \sum_{t \varepsilon T_{kf}} \alpha_{kt} = \frac{1}{q_{ko}} \sum_{t \varepsilon T_{kf}} N_{to} \tag{4.25}$$

(P3) can be easily solved by the following algorithm.

Algorithm Simple

```
Step 1.   Let $\Delta = 0$.

Step 2.   For every $k \varepsilon S_r$ calculate $\alpha_k$ by using (4.25).  If
          $T_{kf} = \phi$ let $\alpha_k = \infty$.

Step 3.   If $\Delta \geq \Delta Q_{rf}$. Stop.  Otherwise go to step 4.

Step 4.   Choose route $\tilde{k}$, satisfying $\alpha_{\tilde{k}} = \min_k \alpha_k$.  Let

          $q = \min\{\Delta Q_{rf} - \Delta, q_{\tilde{k}} - q_{\tilde{k}f}\}$

          $\Delta q_{\tilde{k}f} = q$

          $\Delta = \Delta + q$

          $\alpha_{\tilde{k}} = \infty$

          and go to step 3.
```

At the end of the algorithm the quantities Δq_{kf} are known and the stand-by requirements are obtained by equation (4.21).

4.3.4 Special Cases

As a result of a failure one of the following four cases describes the situation of a relation which is affected by the failure. Those cases are illustrated on the small network given in Figure 4.2.

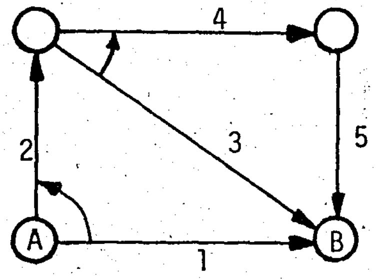


FIGURE 4.2 - Example

TABLE 4.1 - Routing Pattern and the Equivalent Circuits for the Example

Route No.	Trunk Groups	Equivalent Circuits
1	1	q_1
2	2, 3	q_2
3	2,4,5	q_3

The routing pattern and the number of equivalent circuits are given in Table 4.1.

Case (i): Only one route fails. Since no problem of distributing the equivalent circuits between the failed routes exists, this is the simplest case. Let $\tilde{k}$ be the route affected by the failure then

$$\Delta q_{\tilde{k}f} = \Delta Q_{rf} \tag{4.26}$$

and

$$\Delta N_{tf} = \frac{N_{to}}{q_{ko}} \cdot \Delta q_{kf} \tag{4.27}$$

In the example of Figure 4.2, if trunk group 5 fails only route 3 is affected by the failure f. Thus the remaining equivalent circuits equals to:

$$Q_{ABf} = q_{10} + q_{20} + q_{3f}$$

Having

$$Q_{ABf} < \overline{Q}_{AB}$$

implies that the equivalent circuits of route 3 must be increased by the amount,

$$\Delta q_{3f} = \overline{Q}_{AB} - Q_{ABf}$$

Thus the stand-by requirement of trunk group 5 is obtained as

$$\Delta N_{5f} = \frac{N_{50}}{q_{30}} \cdot \Delta q_{3f}$$

<u>Case (ii)</u>: More than one route is in failure and no failed trunk group is common to more than one route. Problems in this case are solved by Algorithm Simple.

In the example of Figure (4.2) if trunk groups 3 and 4 fails, the second and the third routes are affected. The remaining equivalent circuits equals to:

$$Q_{ABf} = q_{10} + q_{2f} + q_{3f}$$

and

$$\Delta Q_{ABf} = \overline{Q}_{AB} - Q_{ABf}$$

α_k values given by equation (4.25) are calculated as,

$$\alpha_2 = \frac{N_{30}}{q_{20}} \quad , \qquad \alpha_3 = \frac{N_{40}}{q_{30}}$$

Algorithm Simple provides the Δq_{2f} and Δq_{3f} values. Then using equation (4.21) ΔN_{3f} and ΔN_{4f} are obtained.

Case (iii): More than one route fails, the same trunk group is common to all the failed routes and now other trunk group is common to all the failed routes and no other trunk group fail.

Algorithm Simple cannot be used in this case because of the dependence between the routes; but, since every route in failure uses the same failed trunk group $\tilde{t}$ they are aggregated together to constitute a fictitious route $\tilde{k}$ with equivalent number of circuits,

$$q_{\tilde{k}o} = \sum_{k \varepsilon S_{r\tilde{t}}} q_{ko} \tag{4.28}$$

where $S_{r\tilde{t}}$ is the set of routes of relation r used by trunk group $\tilde{t}$ in failure. Then the fictitious route is treated as a real one and we have the situation in case (i).

In the example of Figure 4.2 if trunk group 2 fails only, the second and the third routes fail. So the routes 2 and 3 can be replaced by the fictitious route $\tilde{2}$ whose equivalent circuits are given by

$$q_{\tilde{2}0} = q_{20} + q_{30}$$

and

$$q_{\tilde{2}f} = \frac{N_{2f}}{N_{20}} \cdot q_{\tilde{2}0}$$

and ΔN_{2f} is obtained as in case (i).

Case (iv): More than one route is in failure, some trunk groups are common to the failed routes, some others not.

This is the most general case which can be handled neither by Algorithm Simple nor by using fictitious routes.

In the example of Figure 4.2 such a case occurs if trunk groups 2 and 3 fail. 2 is common to the second and the third routes while 3 is used only by the third route.

For this case what is proposed by Cavellero and Tonietti (1981) is to use the Algorithm Simple neglecting the fact that some of the trunk groups are used in more than one route. They call such trunk groups as "anomalous trunk groups". Since the stand-by capacities of anomalous trunk groups are different in different routes using that trunk group; the maximum of those numbers is accepted as the stand-by capacity for that trunk group. On the other hand, using the LP formulation given by (P2) gives more accurate results in such cases since in that formulation the independence of the routes is not assumed. A numerical example is given for this case in Appendix B.

4.4 SOLUTION PROCEDURE

This section provides a brief summary of the solution procedure applied in the thesis. The flowchart illustrated in Figure 4.3 shows the major steps of the solution. The operations, denoted by 1 through 4 in Figure 4.3, constitute the grade of service evaluation step, which is explained thoroughly in Section 4.2 and a detailed flowchart of this step is given by Figure 4.4. The remaining operations, denoted by 5 through 7 in Figure 4.4 constitute the determination of stand-by requirements step which is explained thoroughly in Section 4.3 and a detailed flowchart of this step is given by Figure 4.5.

The computer program for the solution procedure, whose main steps have been presented through the flowcharts, is written in FORTRAN IV. The computer program was run on a UNIVAC 1106 System for a test network. In Appendices C and D the description of the input data together with the routing description is given and the explanation of the computer program is presented in Appendix E. The computer program is listed in Appendix F.

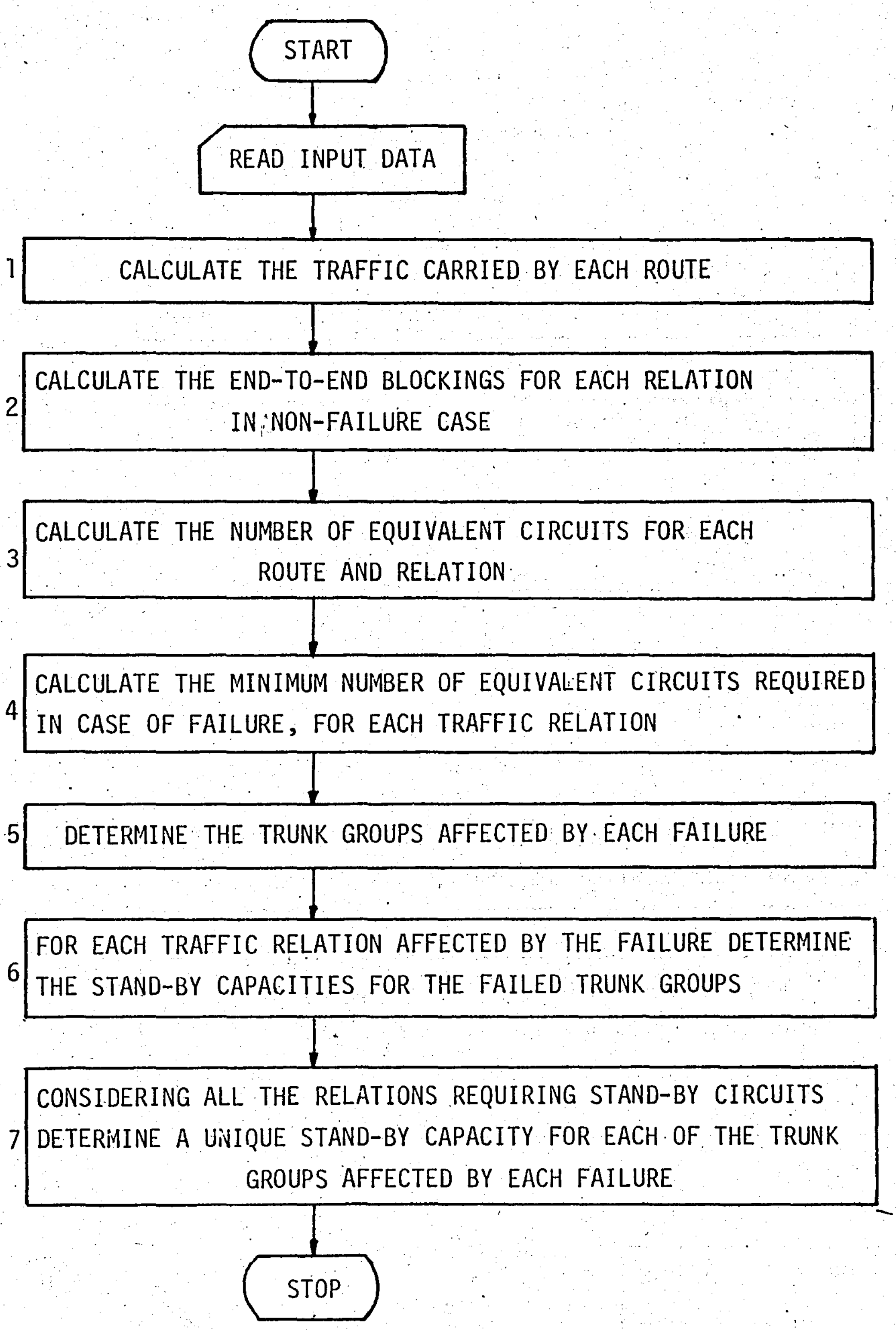
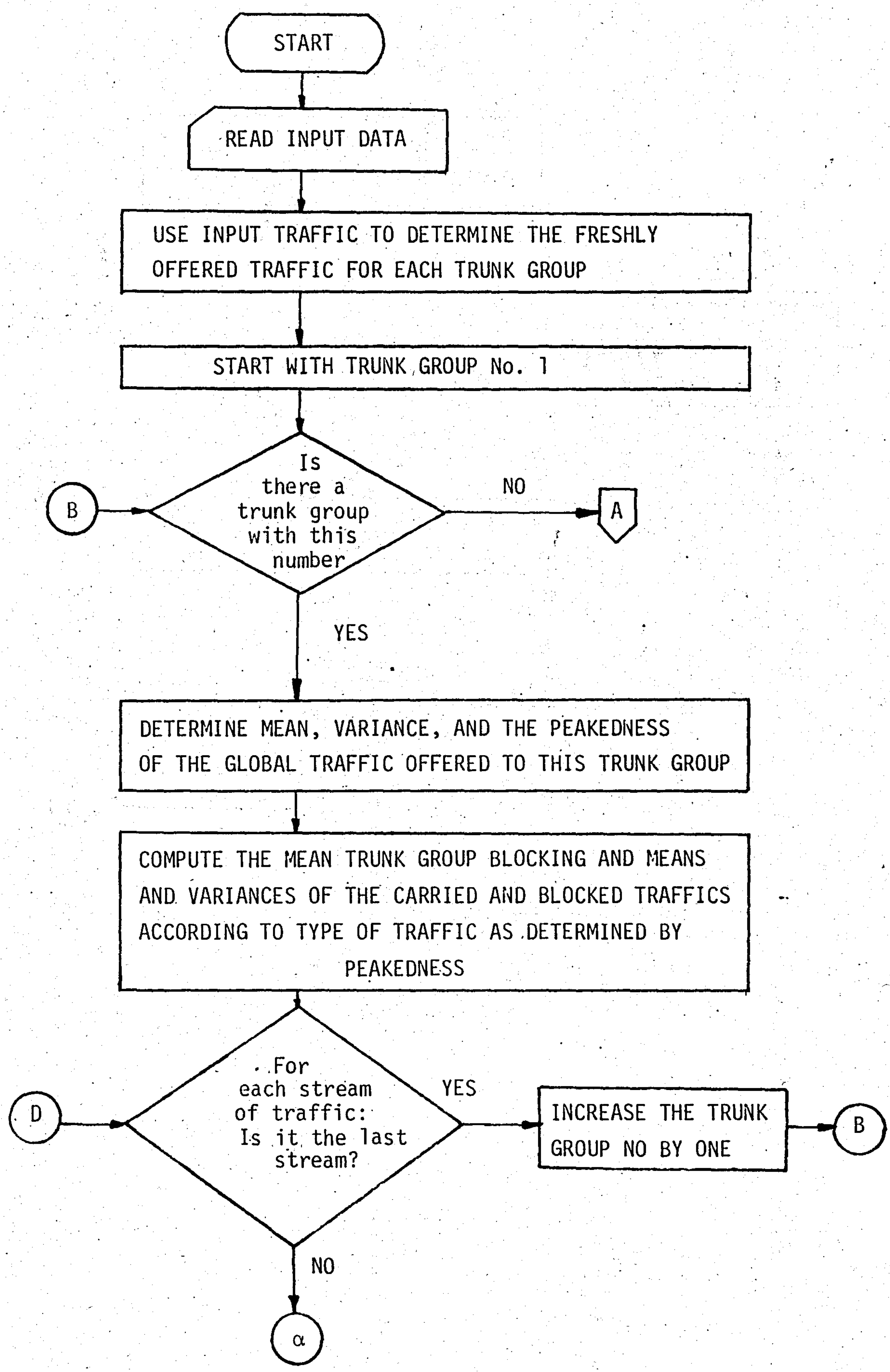
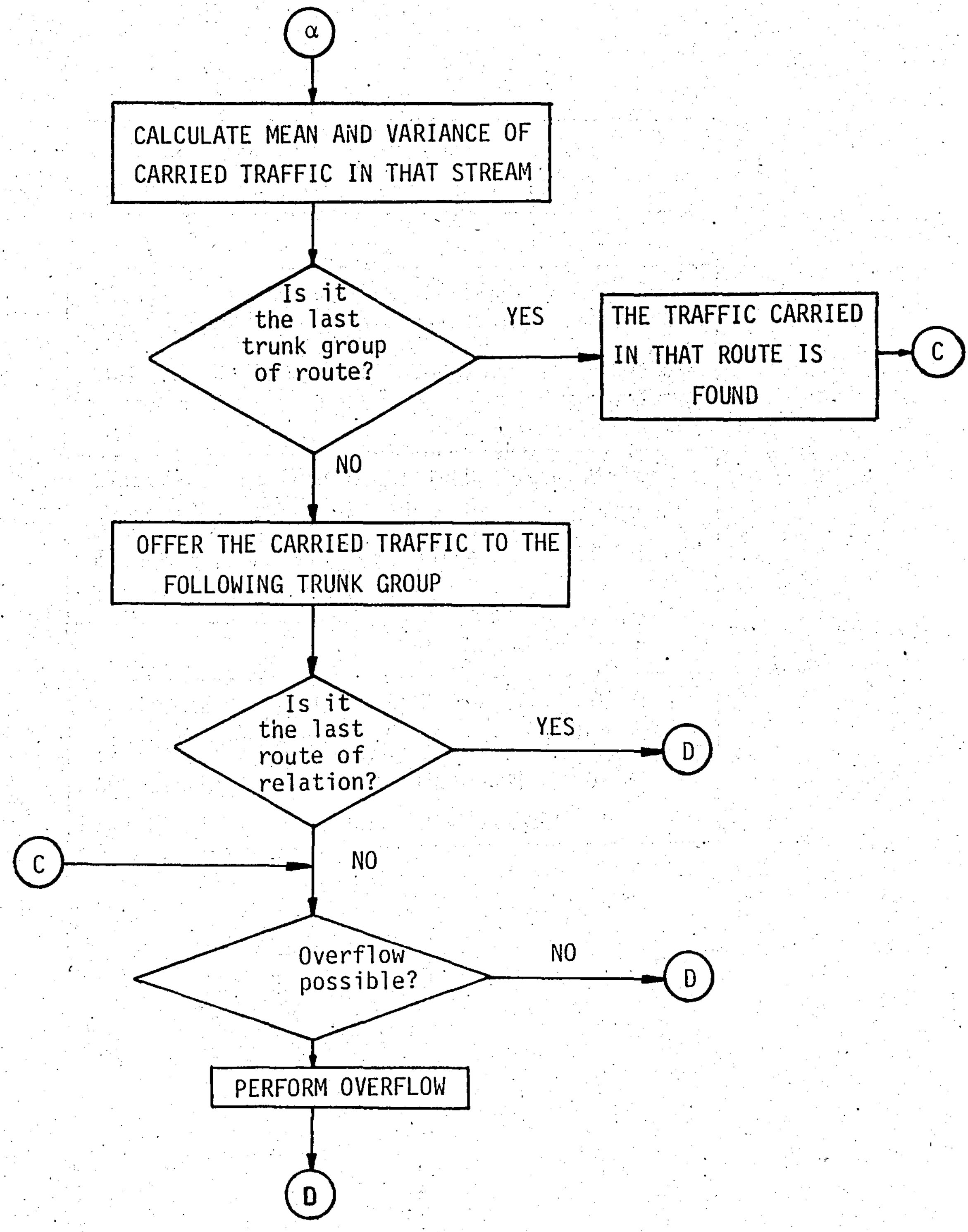


FIGURE 4.3 - Flowchart of the solution procedure


START
READ INPUT DATA
USE INPUT TRAFFIC TO DETERMINE THE FRESHLY OFFERED TRAFFIC FOR EACH TRUNK GROUP
START WITH TRUNK GROUP No. 1
B
Is there a trunk group with this number
NO
A
YES
DETERMINE MEAN, VARIANCE, AND THE PEAKEDNESS OF THE GLOBAL TRAFFIC OFFERED TO THIS TRUNK GROUP
COMPUTE THE MEAN TRUNK GROUP BLOCKING AND MEANS AND VARIANCES OF THE CARRIED AND BLOCKED TRAFFICS ACCORDING TO TYPE OF TRAFFIC AS DETERMINED BY PEAKEDNESS
D
For each stream of traffic: Is it the last stream?
YES
INCREASE THE TRUNK GROUP NO BY ONE
B
NO
α


α
CALCULATE MEAN AND VARIANCE OF CARRIED TRAFFIC IN THAT STREAM
Is it the last trunk group of route?
YES
THE TRAFFIC CARRIED IN THAT ROUTE IS FOUND
C
NO
OFFER THE CARRIED TRAFFIC TO THE FOLLOWING TRUNK GROUP
Is it the last route of relation?
YES
D
C
NO
Overflow possible?
NO
D
PERFORM OVERFLOW
D

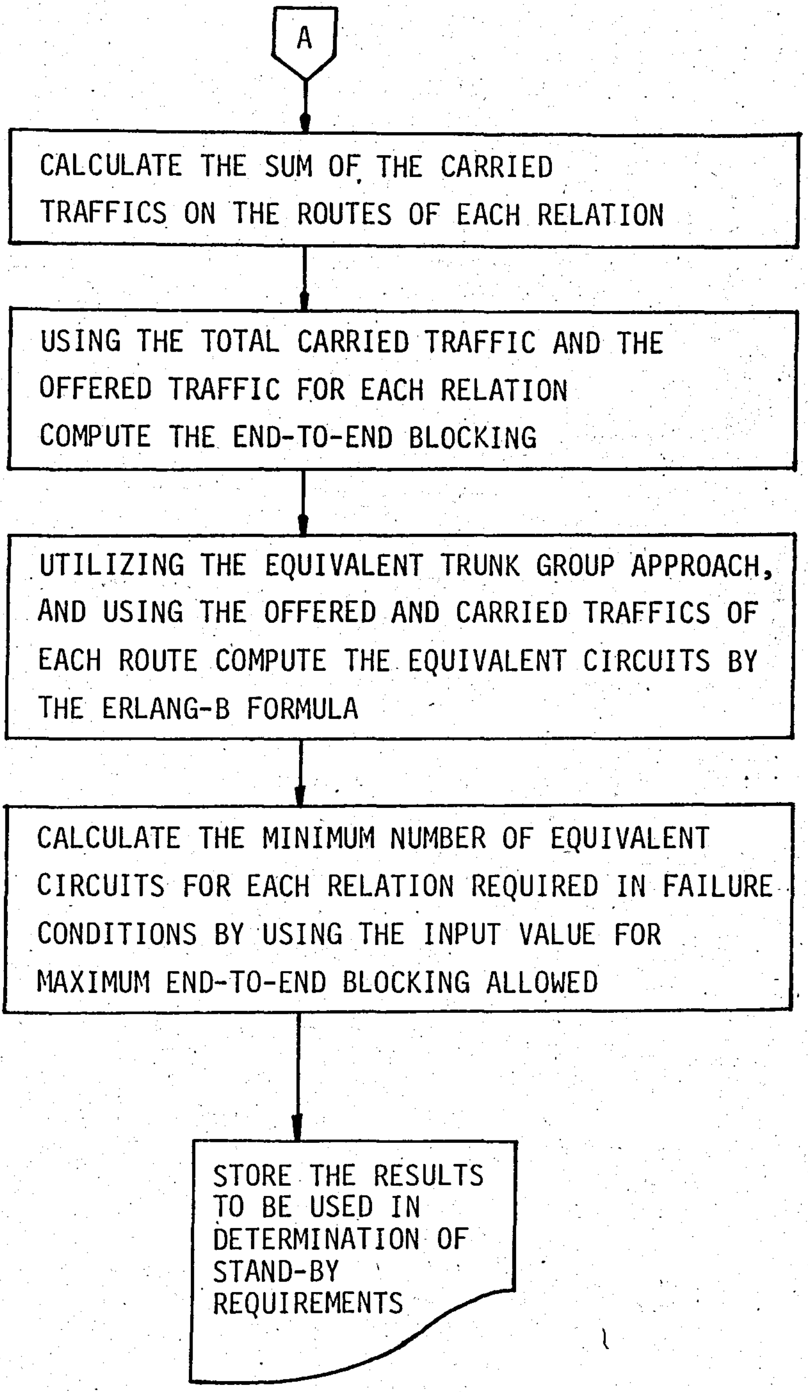
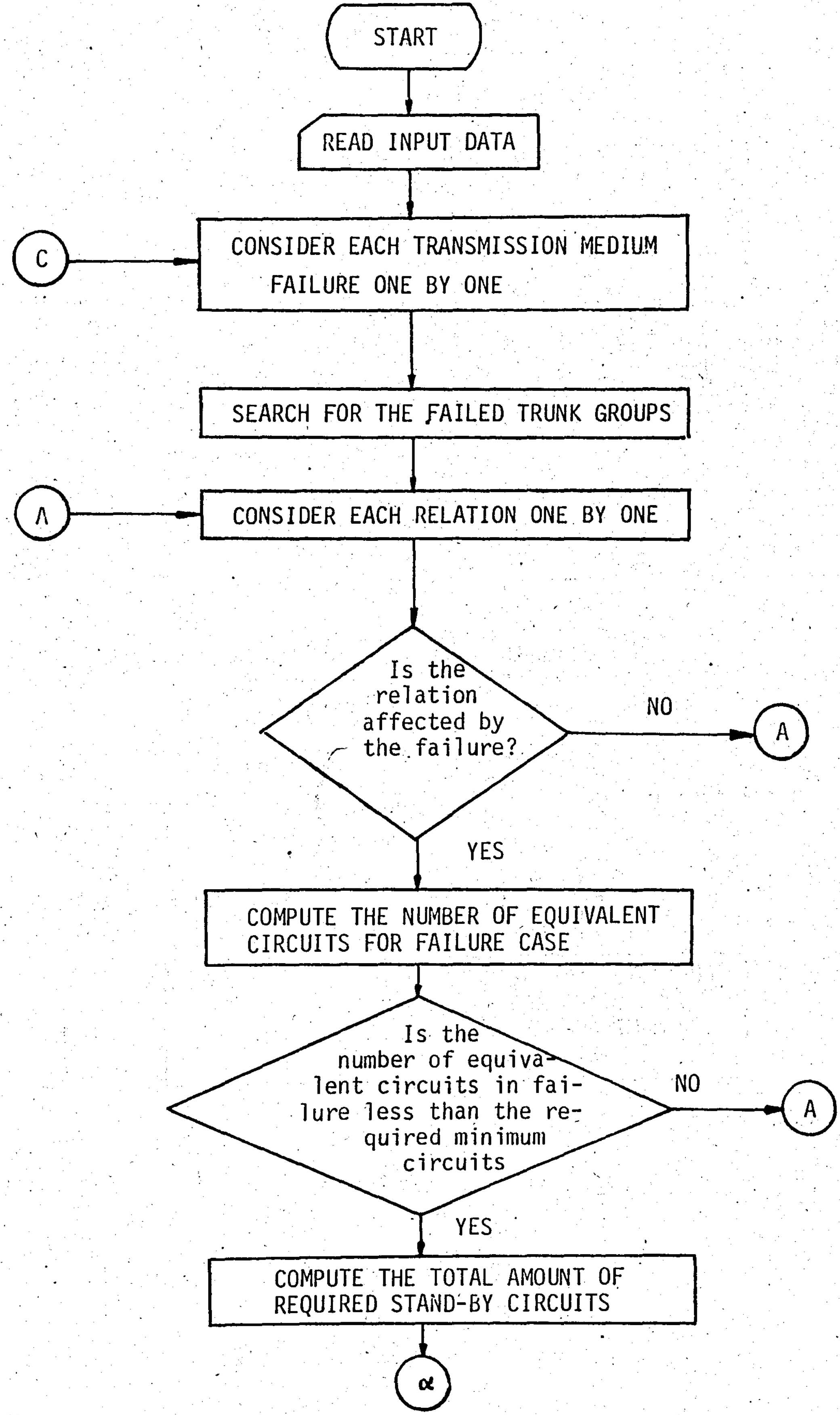
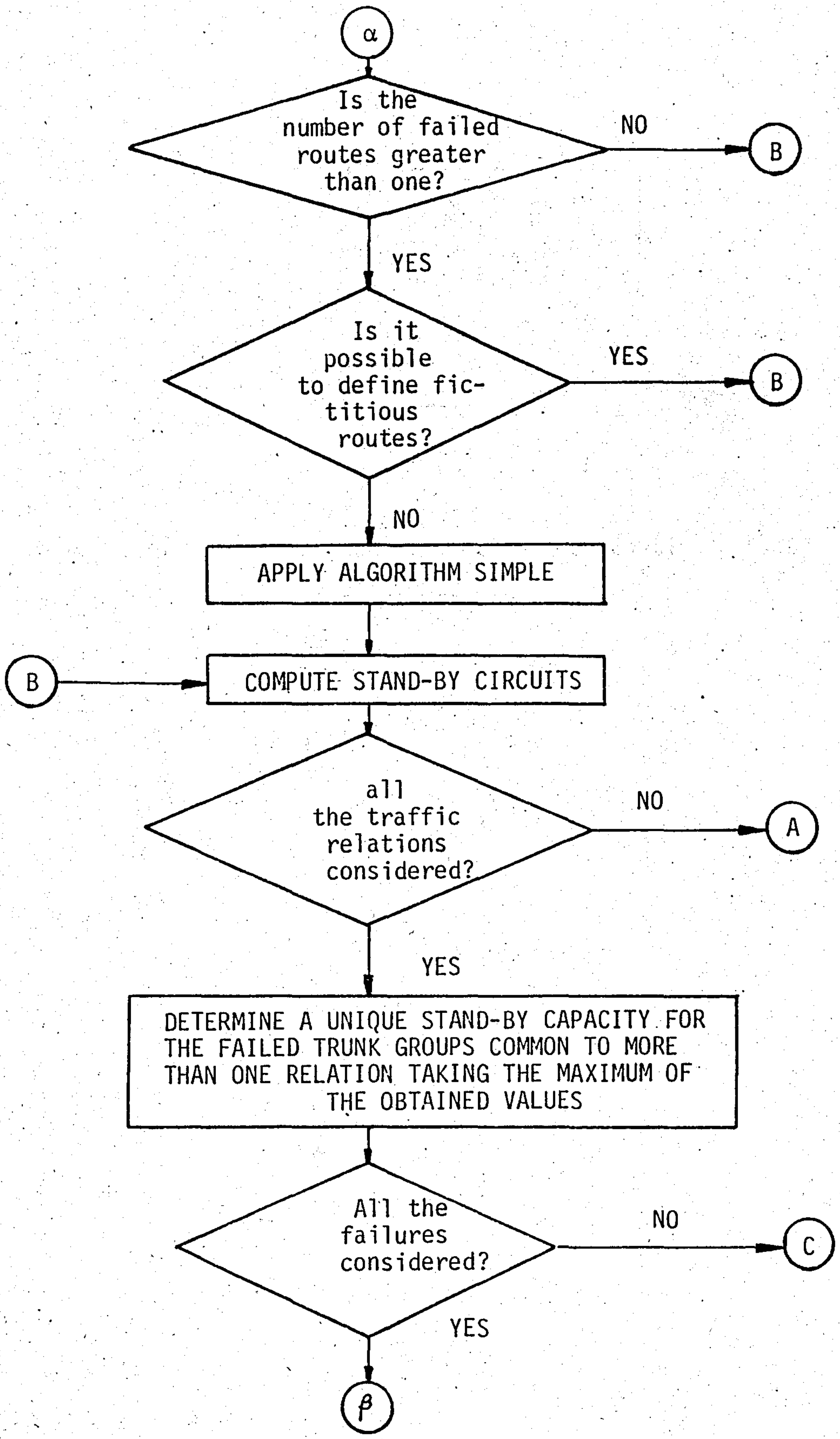


FIGURE 4.4 - Flowchart of the grade of service evaluation step


START
READ INPUT DATA
C
CONSIDER EACH TRANSMISSION MEDIUM FAILURE ONE BY ONE
SEARCH FOR THE FAILED TRUNK GROUPS
A
CONSIDER EACH RELATION ONE BY ONE
Is the relation affected by the failure?
NO
A
YES
COMPUTE THE NUMBER OF EQUIVALENT CIRCUITS FOR FAILURE CASE
Is the number of equivalent circuits in failure less than the required minimum circuits
NO
A
YES
COMPUTE THE TOTAL AMOUNT OF REQUIRED STAND-BY CIRCUITS
α


α
Is the number of failed routes greater than one?
NO
B
YES
Is it possible to define fictitious routes?
YES
B
NO
APPLY ALGORITHM SIMPLE
B
COMPUTE STAND-BY CIRCUITS
all the traffic relations considered?
NO
A
YES
DETERMINE A UNIQUE STAND-BY CAPACITY FOR THE FAILED TRUNK GROUPS COMMON TO MORE THAN ONE RELATION TAKING THE MAXIMUM OF THE OBTAINED VALUES
All the failures considered?
NO
C
YES
β

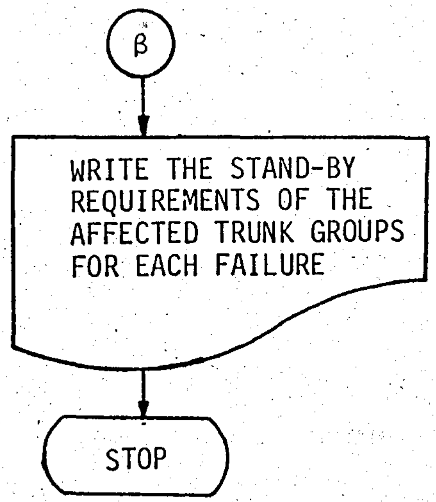


FIGURE 4.5 - Flowchart of the determination of stand-by requirements step

V. NUMERICAL RESULTS

The solution procedure presented in Chapter IV is applied to a test network which is commonly used by eleven European Countries in the context of COST (European Cooperation in Scientific and Technical Research) Project 201: "Methods for Planning and Optimisation of Telecommunications Networks".

In this chapter, first the test network is introduced and the input data is tabulated. Then the results of grade of service evaluation such as the traffic carried by each route, end-to-end blockings for each traffic relation and the equivalent circuits for each route and relation are listed. Minimum number of equivalent circuits for each relation are also listed for different values of end-to-end blockings allowed, in case of failure.

Equivalent circuits for each route, equivalent circuits for each relation, and the minimum equivalent circuits for each relation are used to calculate the stand-by requirements. Stand-by requirements for each trunk group and for each transmission medium failure are listed for given values of end-to-end blocking, in case of failure.

A sensitivity analysis has been carried out with respect to value of allowed end-to-end blocking, in case of failure. Those results are given as the analysis of the numerical results.

5.1 TEST NETWORK

The test network used in this study is a small network with 6 switching nodes and 15 trunk groups. For each pair of switching nodes two traffic relations are defined with each direction defining a different relation. So, 30 traffic relations are defined on this switching network. The transmission network consists of 10 transmission nodes and 21 transmission media.

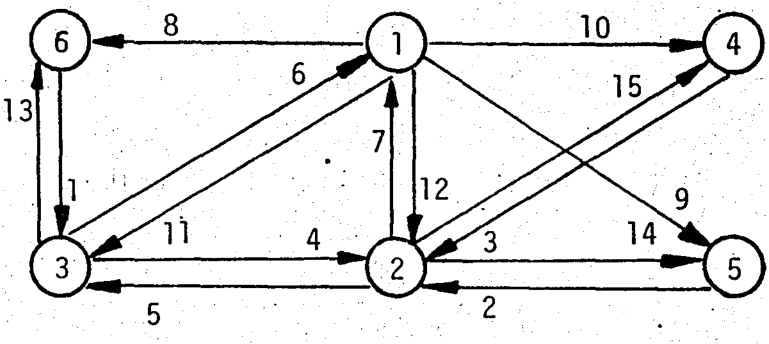


FIGURE 5.1 - Switching network of the test network

TABLE 5.1 - Originating and Terminating Switching Nodes for each Trunk Group and the Number of Circuits

Trunk Group No.	Origin Sw. Node	Terminating SW Node	Number of Circuits
1	6	3	180
2	5	2	180
3	4	2	240
4	3	2	60
5	2	3	60
6	3	1	300
7	2	1	600
8	1	6	60
9	1	5	90
10	1	4	120
11	1	3	180
12	1	2	300
13	3	6	60
14	2	5	60
15	2	4	60

TABLE 5.2 - Traffic Data and the Routing Pattern

Traffic Relation No.	Origin-Destination Node Pair	Mean Offered Traffic	First Route	Second Route	Third Route
1	(1,2)	253.99	12		
2	(1,3)	148.25	11		
3	(1,4)	113.48	10	12,15	
4	(1,5)	96.05	9	12,14	
5	(1,6)	85.03	8	11,13	
6	(2,1)	326.22	7		
7	(2,3)	29.43	5	7,11	
8	(2,4)	16.91	15		
9	(2,5)	7.64	14		
10	(2,6)	4.14	5,13	7,8	7,11,13
11	(3,1)	193.71	6		
12	(3,2)	29.89	4	6,12	
13	(3,4)	6.85	4,15	6,10	6,12,15
14	(3,5)	5.92	4,14	6,9	6,12,14
15	(3,6)	3.52	13		
16	(4,1)	144.89	3,7		
17	(4,2)	16.71	3		
18	(4,3)	6.66	3,5	3,7,11	
19	(4,5)	2.27	3,14		
20	(4,6)	1.68	3,5,13	3,7,8	3,7,11,13
21	(5,1)	119.99	2,7		
22	(5,2)	7.44	2		
23	(5,3)	5.19	2,5	2,7,11	
24	(5,4)	2.23	2,15		
25	(5,6)	1.97	2,5,13	2,7,8	2,7,11,13
26	(6,1)	106.11	1,6		
27	(6,2)	4.02	1,4	1,6,12	
28	(6,3)	3.35	1		
29	(6,4)	1.65	1,4,15	1,6,10	1,6,12,15
30	(6,5)	1.96	1,4,14	1,6,9	1,6,12,14

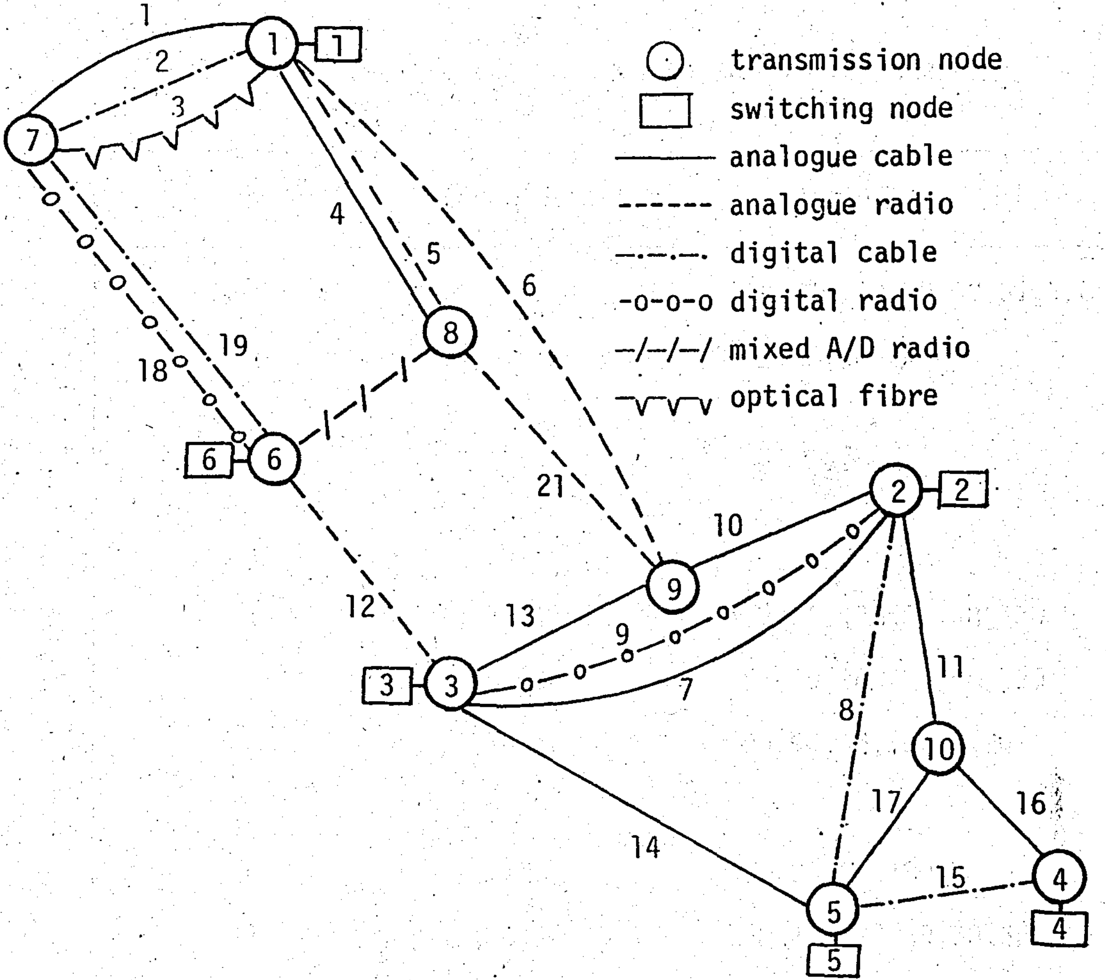


FIGURE 5.2 - Transmission network of the test network

TABLE 5.3 - List of Trunk Groups to Media with the Corresponding Number of Circuits

Transmission Media	Trunk Groups	Circuits
1	6,7,8,9,10,11,12	113,240,30,30,60,67,120
2	6,8,9,11	75,30,30,45
3	-	-
4	-	-
5	-	-
6	6,7,9,10,11,12	112,360,30,60,68,180
7	4,5,7,9,10,12	60,60,240,30,60,120
8	2,3,9,10,14,15	90,96,30,60,30,24
9	-	-
10	7,9,10,12	360,30,60,180
11	2,3,9,10,14,15	90,144,30,60,30,36
12	1,6,9,11,13	180,188,30,112,60
13	6,7,9,10,11,12	112,240,30,60,68,120
14	9	30
15	2,3,9,10,14,15	90,96,30,60,30,24
16	2,3,9,10,14,15	90,144,30,60,30,36
17	6,7,8,9,10,11,12	113,240,30,30,60,67,120
18	6,8,9,11	75,30,30,45
19	7,9,10,12	240,30,60,120
20	-	-
21	7,9,10,12	240,30,60,120

5.2 RESULTS OF THE TEST NETWORK

The results obtained by using the solution procedure given in Section IV and the data given in the previous section can be tabulated in two main groups. The results of grade of service evaluation constitutes the first group where, all the traffic streams offered to each trunk group,means and variances of the global traffics offered to each trunk group, mean blockings for each trunk group, traffics carried by each route, equivalent trunks for each route and relation, and the minimum equivalent trunks for each relation are listed. The results of grade of service evaluation are then used to obtain the results given in the second group which are the stand-by requirements. Stand-by requirements are obtained by the simplified approach.

5.2.1 Results of Grade of Service Evaluation

Table 5.4 tabulates the information about the traffic streams offered to each trunk group. Each traffic stream offered to each trunk group are numbered so that the number of streams offered to any trunk group can be easily seen from Table 5.4. Types of the traffics are denoted by capital letters R, C, and O, meaning random traffic (freshly offered Poissonian traffic), carried traffic (smooth) and overflow traffic (peaky) respectively. C-R means carried traffic coming from a trunk group whose offered traffic is random and has zero blocking. As mentioned in Section IV, different types of traffics can be offered to the same trunk group. This situation can be easily detected from Table 5.4.

TABLE 5.4 - Traffic Streams Offered to each Trunk Group

Trunk Group	Stream No.	Mean	Variance	Type	From which trunk group	Belongs to which route	Belongs to which relation
1	1	106.11	106.11	R	-	43	26
	2	4.02	4.02	R	-	44	27
	3	3.35	3.35	R	-	46	28
	4	1.65	1.65	R	-	47	29
	5	1.96	1.96	R	-	50	30
2	1	119.99	119.99	R	-	35	21
	2	7.44	7.44	R	-	36	22
	3	5.19	5.19	R	-	37	23
	4	2.23	2.23	R	-	39	24
	5	1.97	1.97	R	-	40	25
3	1	144.84	144.84	R	-	27	16
	2	16.71	16.71	R	-	28	17
	3	6.66	6.66	R	-	29	18
	4	2.27	2.27	R	-	31	19
	5	1.68	1.68	R	-	32	20
4	1	29.89	29.89	R	-	18	12
	2	6.85	6.85	R	-	20	13
	3	5.42	5.42	R	-	23	14
	4	4.02	4.02	C-R	1	44	27
	5	1.65	1.65	C-R	1	47	29
	6	1.96	1.96	C-R	1	50	30
5	1	29.43	29.43	R	-	10	7
	2	4.14	4.14	R	-	14	10
	3	5.19	5.17	C	2	37	23
	4	1.97	1.96	C	2	40	25
	5	6.660	6.659	C	3	29	18
	6	1.68	1.679	C	3	32	20
6	1	193.71	193.71	R	-	17	11
	2	106.11	106.109	C	1	43	26
	3	0.618	2.496	0	4	19	12
	4	0.142	0.572	0	4	21	13
	5	0.112	0.453	0	4	24	14
	6	0.083	0.336	0	4	45	27
	7	0.034	0.138	0	4	48	29
	8	0.041	0.164	0	4	51	30

Table 5.4 continued...

Trunk Group	Stream No.	Mean	Variance	Type	From which trunk group	Belongs to which route	Belongs to which relation
7	1	326.22	326.22	R	-	9	6
	2	119.98	119.67	C	2	35	21
	3	144.840	144.838	C	3	27	16
	4	0.514	2.043	0	5	11	7
	5	0.072	0.287	0	5	15	10
	6	0.091	0.360	0	5	38	23
	7	0.034	0.137	0	5	41	25
	8	0.116	0.462	0	5	30	18
	9	0.029	0.117	0	5	33	20
8	1	85.03	85.03	R	-	7	5
	2	0.071	0.032	C	7	15	10
	3	0.034	0.015	C	7	41	25
	4	0.029	0.013	C	7	33	20
9	1	96.05	96.05	R	-	5	4
	2	0.107	0.038	C	6	24	14
	3	0.039	0.014	C	6	51	30
10	1	113.48	113.48	R	-	3	3
	2	0.135	0.048	C	6	21	13
	3	0.033	0.012	C	6	48	29
11	1	148.25	148.25	R	-	2	2
	2	0.502	0.224	C	7	11	7
	3	0.088	0.040	C	7	38	23
	4	0.114	0.051	C	7	30	18
	5	27.077	59.321	0	8	8	5
	6	0.022	0.049	0	8	16	10
	7	0.011	0.023	0	8	42	25
	8	0.009	0.020	0	8	34	20
12	1	253.99	253.99	R	-	1	1
	2	0.589	0.210	C	6	19	12
	3	0.079	0.028	C	6	45	27
	4	11.562	49.525	0	9	6	4
	5	0.013	0.055	0	9	25	14
	6	0.005	0.020	0	9	52	30
	7	4.667	26.542	0	10	4	3
	8	0.006	0.032	0	10	22	13
	9	0.001	0.008	0	10	49	29

Table 5.4 continued...

Trunk Group	Stream No.	Mean	Variance	Type	From which trunk group	Belongs to which route	Belongs to which relation
13	1	3.52	3.52	R	-	26	15
	2	4.068	3.216	C	5	14	10
	3	1.935	1.530	C	5	40	25
	4	1.651	1.305	C	5	32	20
	5	25.734	8.728	C	11	8	5
	6	0.021	0.007	C	11	16	10
	7	0.010	0.003	C	11	42	25
	8	0.009	0.003	C	11	34	20
14	1	7.64	7.64	R	-	13	9
	2	2.27	2.269	C	3	31	19
	3	5.308	4.048	C	4	23	14
	4	1.919	1.464	C	4	50	30
	5	11.472	8.660	C	12	6	4
	6	0.013	0.010	C	12	25	14
	7	0.005	0.003	C	12	52	30
15	1	16.91	16.91	R	-	12	8
	2	2.230	2.224	C	2	39	24
	3	6.708	5.117	C	4	20	13
	4	1.616	1.232	C	4	47	29
	5	4.631	3.496	C	12	4	3
	6	0.006	0.004	C	12	22	13
	7	0.0013	0.0010	C	12	49	29

TABLE 5.5 - Means, Variances and Mean Blockings of the Global Traffics Offered to Each Trunk Group

Trunk Group No.	Mean	Variance	Blocking
1	117.09	117.0900	0.00000
2	136.82	136.8200	0.00006
3	172.16	172.1600	0.00000
4	49.79	49.7899	0.02067
5	49.070	49.0507	0.01747
6	300.850	303.9771	0.04661
7	591.900	594.13162	0.02412
8	85.163	85.0890	0.31844
9	96.195	96.1018	0.12038
10	113.648	113.5396	0.04113
11	176.073	207.9781	0.04960
12	270.912	330.4095	0.00779
13	36.948	28.3114	0.00000
14	28.627	24.0960	0.00000
15	32.102	28.9840	0.00000

TABLE 5.6 - Traffic Carried by each Route and the Equivalent Circuits of each Route

Route No.	Traffic Carried	Equivalent Circuits
1	252.0102	277.18
2	140.8961	150.09
3	108.8130	118.17
4	4.6306	27.99
5	84.4877	88.12
6	11.4722	34.19
7	57.9531	58.08
8	25.7338	39.04
9	318.3507	336.05
10	28.9159	37.37
11	0.4768	7.07
12	16.9200	38.36
13	7.6400	27.05
14	4.0677	7.80
15	0.0481	1.29
16	0.0214	2.05
17	184.6809	195.07
18	29.2721	37.32
19	0.5846	8.00
20	6.7084	11.15
21	0.1295	3.25
22	0.0055	0.76
23	5.3080	9.27
24	0.0940	2.27
25	0.0128	1.42
26	3.52	15.88
27	141.346	155.08
28	16.71	40.23
29	6.5437	11.17

Table 5.6 continued....

Route No.	Traffic Carried	Equivalent Circuits
30	0.1080	3.33
31	2.27	12.75
32	1.6507	4.24
33	0.0195	0.87
34	0.0087	1.40
35	117.088	130.060
36	7.4396	19.05
37	5.099	9.22
38	0.084	3.00
39	2.2297	9.25
40	1.9355	4.66
41	0.0229	0.89
42	0.0102	1.55
43	101.1640	110.06
44	3.9369	7.39
45	0.0786	3.06
46	3.3500	16.64
47	1.6159	4.10
48	0.0312	1.78
49	0.0013	0.37
50	1.9195	4.49
51	0.0340	1.56
52	0.0046	0.85

TABLE 5.7 - Equivalent Circuits, Minimum Equivalent Circuits, End-to-End Blockings for each Relation

Relation No.	Equivalent Circuits	EEB (in non-failure Case)	Minimum Equivalent Circuits in Case of Failure According to given EEB_f Values				
			0.01	0.05	0.1	0.2	0.3
1	277.18	0.00779	275.06	252.04	234.06	205.04	179.01
2	150.09	0.04960	150.09	150.07	139.02	121.01	104.05
3	146.16	0.00032	130.02	116.14	107.07	93.02	80.04
4	122.31	0.00094	111.23	99.16	91.09	79.03	68.03
5	97.12	0.01580	97.12	89.01	81.09	70.04	60.05
6	336.05	0.02412	336.05	321.05	300.01	263.03	229.02
7	44.44	0.00127	39.23	33.30	30.07	25.09	21.10
8	38.36	0.00000	24.73	20.40	18.12	15.03	12.16
9	27.05	0.00000	13.25	10.41	9.07	7.10	5.43
10	11.14	0.00069	8.41	6.36	5.28	4.09	3.13
11	195.07	0.04661	195.07	194.05	180.03	157.03	136.04
12	45.32	0.00112	40.01	34.08	30.21	25.18	21.16
13	15.16	0.00097	12.21	9.51	8.17	6.27	5.16
14	12.96	0.00098	10.25	8.04	6.58	5.15	4.09
15	15.88	0.00000	7.50	5.60	4.59	3.39	2.56
16	155.08	0.02412	155.08	147.03	136.01	118.03	102.03
17	40.23	0.00000	24.44	20.26	18.05	14.34	12.12
18	14.50	0.00127	12.05	9.32	8.06	6.18	5.05
19	12.75	0.00000	5.61	4.13	3.29	2.36	1.83
20	6.51	0.00069	4.66	3.32	2.64	2.02	1.39
21	130.06	0.02418	130.06	123.02	113.06	98.04	85.01
22	19.05	0.00006	13.07	10.24	8.66	7.03	5.33
23	12.22	0.00133	10.04	7.60	3.36	5.05	4.02
24	9.25	0.00006	5.54	4.10	3.25	2.33	1.78
25	7.20	0.00075	5.17	3.69	3.06	2.17	1.57
26	110.06	0.04661	110.06	109.11	100.11	87.03	75.03
27	10.45	0.00112	8.27	6.25	5.19	4.03	3.09
28	16.64	0.00000	7.28	5.40	4.41	3.28	2.44
29	6.25	0.00097	4.61	3.29	2.60	1.99	1.38
30	6.90	0.00098	5.16	3.68	3.05	2.17	1.56

As EEB_f (End-to-End Blocking allowed in failure case) decreases minimum equivalent circuits for each relation increases. In Table 5.7 equivalent circuits for five different values of EEB_f are listed.

5.2.2 Stand-by Requirements Obtained by the Simplified Approach

In this sub-section the stand-by requirements calculated by the simplified approach are listed. Stand-by requirements are calculated for five different values of EEB_f by using the minimum equivalent circuits for those values listed in Table 5.7.

TABLE 5.8 - Stand-by Requirements

Transmission Media	Trunk Groups	Stand-by Requirement for Different EEB_f Values				
		0.01	0.05	0.1	0.2	0.3
1	6	113	111	90	54	22
	7	240	213	176	110	49
	8	30	30	30	22	12
	9	30	20	12	0	0
	10	58	44	35	20	7
	11	67	67	54	32	12
	12	118	93	73	42	14
2	6	75	73	52	16	0
	8	30	30	30	22	12
	9	19	6	0	0	0
	11	45	53	32	10	0
6	6	112	110	89	53	21
	7	360	333	296	230	169
	9	30	27	19	7	0
	10	60	47	37	23	10
	11	68	68	55	33	13
	12	178	153	133	102	74
7	4	60	55	49	40	34
	5	60	53	48	40	34
	7	344	213	176	110	49
	9	39	20	12	0	0
	10	58	44	35	20	7
	12	118	93	73	42	14

Table 5.8 continued...

Transmission Media	Trunk Groups	Stand-by Requirement for Different EEB_f Values				
		0.01	0.05	0.1	0.2	0.3
8	2	90	80	66	46	28
	3	96	84	66	39	14
	9	30	24	16	3	0
	10	58	44	35	20	7
	14	24	10	1	0	0
	15	19	4	0	0	0
10	7	360	333	296	230	169
	9	30	27	19	7	0
	10	60	47	37	23	10
	12	178	153	133	102	74
11	2	90	80	66	46	28
	3	144	132	114	87	62
	9	30	24	16	3	0
	10	60	47	37	23	10
	14	24	10	1	0	0
	15	33	19	11	1	0
12	1	180	178	164	142	123
	6	188	186	165	129	97
	9	19	6	0	0	0
	11	112	112	99	77	57
	13	60	48	35	22	14
13	6	112	110	89	53	21
	7	240	213	176	110	49
	9	30	20	12	0	0
	10	58	44	35	20	7
	11	68	68	55	33	13
	12	118	93	73	42	14
14	9	19	6	0	0	0
15	2	90	80	66	46	28
	8	96	84	66	39	14
	9	30	24	16	3	0
	10	58	44	35	20	7
	14	24	10	1	0	0
	15	19	4	0	0	0
16	2	90	80	66	46	28
	3	144	132	114	87	62
	9	30	24	16	3	0
	10	60	47	37	23	10
	14	24	10	1	0	0
	15	33	19	11	1	0

Table 5.8 continued....

Transmission Media	Trunk Groups	Stand-by Requirement for Different EEB_f Values				
		0.01	0.05	0.1	0.2	0.3
18	6	113	111	96	54	22
	7	240	213	176	110	49
	8	30	30	30	22	12
	9	30	20	12	0	0
	10	58	44	35	20	7
	11	67	67	54	32	12
	12	118	93	73	42	14
19	6	75	73	52	16	0
	8	30	30	30	22	12
	9	19	6	0	0	0
	11	45	53	32	10	0
20	7	240	213	176	110	49
	9	30	20	12	0	0
	10	58	44	35	20	7
	12	118	93	73	42	14
21	7	240	213	176	110	49
	9	30	20	12	0	0
	10	58	44	35	20	7
	12	118	93	73	42	14

As it can easily be detected from Table 5.8 the number of required stand-by circuits decrease as the end-to-end blocking allowed in case of failure increases. Similarly number of relations requiring stand-by facilities decrease as the the end-to-end blocking allowed in case of failure increases. The number of relations requiring stand-by facilities with respect to five different end-to-end blockings allowed in case of failure are listed together with the total number of relations affected by the failure of any transmission medium.

TABLE 5.9 - Number of Relations Requiring Stand-by

Transmission Media	Number of Relations Affected by the Failure	Number of Relations Requiring Stand-by with respect to EEB_f				
		0.01	0.05	0.1	0.2	0.3
1	22	10	10	10	9	9
2	17	5	5	4	4	1
6	22	10	10	9	9	8
7	18	18	18	18	17	17
8	18	16	11	6	4	3
10	15	6	6	6	6	5
11	18	17	16	12	6	3
12	19	13	13	12	12	12
13	22	10	10	9	8	8
14	3	1	1	-	-	-
15	18	16	11	6	4	3
16	18	17	16	12	6	3
18	22	10	10	10	9	9
19	17	5	5	4	4	1
20	15	6	6	6	5	5
21	15	6	6	6	5	5
TOTAL	279	166	154	130	108	94

5.3 ANALYSIS OF THE RESULTS

TABLE 5.10 - Behaviour of Some Global Measures

EEB_f	Number of Unfeasible Relations	Percentage of Unfeasible Relations	Percentage of Total Stand-by Requirements	Percentage of Affected Relation Requiring Stand-b
0	30	1	1	1
0.01	7	0.23	0.9996	0.5950
0.05	1	0.03	0.8534	0.5520
0.1	0	0	0.6926	0.4659
0.2	0	0	0.4444	0.3871
0.3	0	0	0.2434	0.3369
1	0	0	0	0

Table 5.10 provides the behaviour of some global measures again according to different values of EEB_f (End-to-end blocking allowed in case of failure). In the second column number of unfeasible relations are listed. Unfeasible relations are defined as the relations whose end-to-end blockings in non-failure conditions are greater than or equal to the end-to-end blockings allowed in case of failure (i.e. $EEB_0 \geq EEB_f$). Third column gives the percentage values for the unfeasible relations.

Fourth column gives the percentages of total stand-by requirements. As end-to-end blocking allowed in case of failures increases the percentage of stand-by requirements decreases.

Some of the relations affected by the failure do not require stand-by circuits. This case occurs if $Q_{rf} \geq \overline{Q}_r$ as explained in Section 4.3. The relations whose number of equivalent circuits in failure cases is greater than or equal to the minimum number of equivalent circuits do not require stand-by circuits. The percentages of affected relations requiring stand-by also decrease as EEB_f increases.

The change in the percentage of total stand-by requirements is plotted for different values of EEB_f. Figure 5.10 shows this relation.

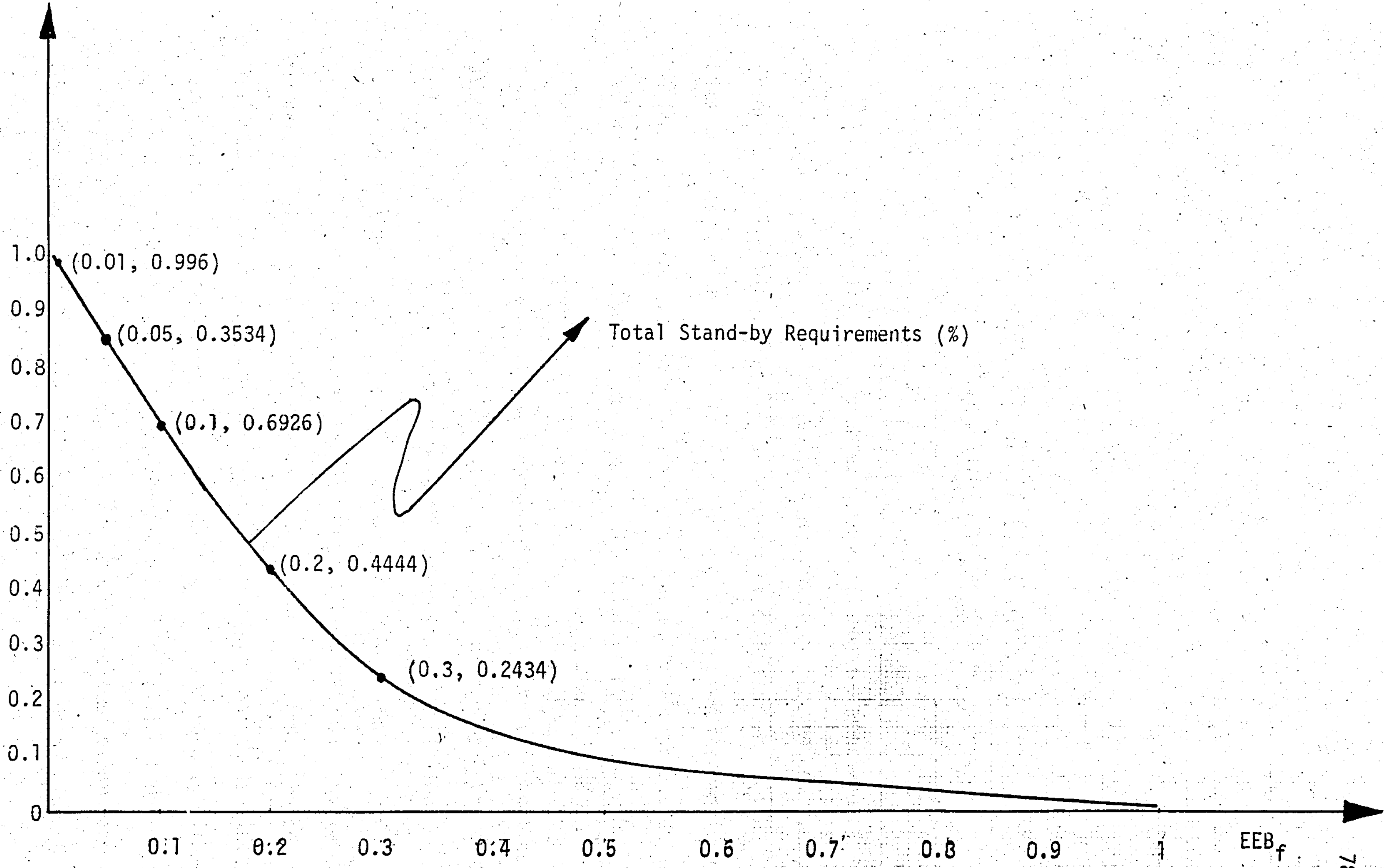


FIGURE 5.3 - The change in the percentage of total stand-by requirements

VI. CONCLUSION AND EXTENSIONS FOR FUTURE WORK

In this study, the problem of determining the required stand-by capacities in telecommunications networks is studied. Prior to this problem, grade of service evaluation problem, which aims to determine the minimum number of equivalent circuits required in failure cases is studied. A literature survey on the proposed methodologies and approximations for calculation of blocking probabilities is presented.

The problem of determining the required stand-by capacities is solved by using a simple heuristic approach proposed by Cavellero and Tonietti (1981) and it is completely based on the equivalent trunk group idea.

A linear programming formulation of the problem is developed which will give more accurate results than the simplified approach utilized in the thesis. In fact, in the first three of the special cases discussed, the simplified approach give the same results with linear programming. In the case, where some of the failed trunk groups are common to the routes of a relation while some others are not, the simplified approach overestimates the number of required stand-by circuits because of the independence assumption for the routes, which not true always. This situation is illustrated on a numerical example where results are obtained both by the simplified approach and by linear programming for a specific traffic relation of the test network.

The change in the number of total stand-by circuits is examined with respect to different values of maximum end-to-end blocking allowed in failure cases. As expected, the number of required stand-by circuits decreases as the maximum end-to-end blocking allowed in failure cases increases.

The first natural extension of this study is performing the stand-by optimization process. Lindberg (1980) suggests some solution procedures to this problem, and also in Lindberg, et.al, (1983) a brief description of the stand-by optimization problem is given.

In the solution procedure, for all cases the simplified approach is utilized, but as explained above, it would be more realistic to switch to linear programming whenever situations as "having more than one route in failure, with some trunk groups common and some others are not" occur. In all the remaining cases the simplified approach can be used.

While determining the global traffic offered to a trunk group the independence of the traffic streams were assumed which is not true in reality. To be more realistic, a covariance term similar to the one given by Deschamps (1979) may be introduced or a procedure for the decomposition of traffic in loss systems similar to the one proposed by Manfield and Downs (1979) may be utilized.

A failure probability or an availability value may be introduced for the transmission media which will probably lead to examination of the transmission media only having considerably high failure probability.

Finally, a quite different approach to this problem may be the consideration of cost factor at the beginning which may lead to solving a unique problem to obtain the optimum stand-by network.

APPENDICES

APPENDIX A

NUMERICAL COMPARISON OF THE METHODS FOR CALCULATING THE ERLANG-B VALUE FOR NONINTEGRAL NUMBER OF TRUNKS

Here a numerical comparison of Jagerman's Approximation, Approximation Through a Continued Fraction and Rapp's Approximation are is given. For a set of 10 values of X and M, where X is either integer or any real number, the computer results are given for B(X,M) by those three methods in Table A.1. Where X is the number of trunks and M is the mean of the offered traffic (Table A.2). B(N,M) values are tabulated where N are the nearest integer values of the noninteger values considered in Table A.1. Table A.2 is provided to show the possible error that is made by simply taking the nearest integer value rather than using one of the proposed approaches for this purpose. N is taken as the nearest integer to X.

Looking at Table A.1, one can conclude that all of those methods give almost the same results except for the continued fraction for X = 80 and M = 60. But in Section II, it was mentioned that the continued fraction technique is not applicable in cases where $15 < M < X$.

TABLE A.1 - Numerical Comparison

X	M	Jager	Continued Fraction	Rapp
120.70	255.20	0.530460	0.530460	0.530460
10.50	6.75	0.053409	0.053384	0.053384
10.82	9.50	0.15015	0.15012	0.15012
3.50	0.90	0.024345	0.024313	0.024311
20.00	40.00	0.521310	0.521310	0.521310
60.00	45.00	0.0054342	0.0054603	0.0054342
80.00	60.00	0.0021987	-0.44181	0.0021986
10.50	13.00	0.022790	0.022785	0.022786
4.00	9.00	0.61381	0.61381	0.61381
15.50	50.00	0.69833	0.69833	0.69833

TABLE A.2 - B(N,M) Values

N	M	B(N,M)
121.00	255.20	0.52930
11.00	6.75	0.040618
4.00	0.9	0.011141
20.00	13.00	0.018110
16.00	50.00	0.68870

Looking at Tables A.1 and A.2 simultaneously, it can be seen that especially for small values of M rounding off X to the nearest integer leads to quite notable errors. For example,

$$B(3.50,0.9) = 0.0243$$

But,

$$B(4.00,0.9) = 0.0111.$$

APPENDIX B

A NUMERICAL EXAMPLE FOR THE LP FORMULATION

As mentioned in case (iv) of the special cases stated in Section 4.3.4, the Simplified Approach utilized in the thesis overestimates the required stand-by capacities because of the assumption of independence between the routes.

Consider the failure of transmission medium 12. This failure affects the trunk groups; 1, 6, 9, 11, and 13. Considering specifically relation 30, which has three routes, all of those routes are affected by the failure. The list of the trunk groups in the routes of relation 30 are:

1st Route :	1, 4, 14
2nd Route :	1, 6, 9
3rd Route :	1, 6, 12, 14

Trunk group 1 is common to all the routes, 6 is common to the 2nd and 3rd routes and 9 is used only by the 2nd route. So this is typically the situation defined in case (iv) of section 4.3.4. Using the notation used in Section IV the LP formulation for this case is as follows.

$$\min \Delta N_{1,12} + \Delta N_{6,12} + \Delta N_{9,12}$$

$$\text{s.t.} \quad q_{1,0} \frac{N_{1,12} + \Delta N_{1,12}}{N_{1,0}} - q_{1,12} + q_{2,0}y_2 - q_{2,12} + q_{3,0}y_3 - q_{3,12} = \Delta Q_{30,12}$$

$$N_{1,0}\, y_2 \leq N_{1,12} + \Delta N_{1,12}$$
$$N_{6,0}\, y_2 \leq N_{6,12} + \Delta N_{6,12}$$
$$N_{9,0}\, y_2 \leq N_{9,12} + \Delta N_{9,12}$$
$$N_{1,0}\, y_3 \leq N_{1,12} + \Delta N_{1,12}$$
$$N_{6,0}\, y_3 \leq N_{6,12} + \Delta N_{6,12}$$
$$\Delta N_{1,12} \leq N_{1,0} - N_{1,12}$$
$$\Delta N_{6,12} \leq N_{6,0} - N_{6,12}$$
$$\Delta N_{9,12} \leq N_{9,0} - N_{9,12}$$

$$\Delta N_{1,12}, \Delta N_{6,12}, \Delta N_{9,12} \geq 0$$
$$y_2, y_3 \geq 0$$

Now, if we plug in the values and order the terms, we obtain,

$$\min \Delta N_{1,12} + \Delta N_{6,12} + \Delta N_{9,12}$$

$$\text{s.t.} \quad 4.49\Delta N_{1,12} + 280.8y_2 + 153y_3 = 549$$
$$180\, y_2 - \Delta N_{1,12} \leq 0$$
$$300\, y_2 - \Delta N_{6,12} \leq 112$$
$$90\, y_2 - \Delta N_{9,12} \leq 60$$
$$180\, y_3 - \Delta N_{1,12} \leq 0$$
$$300\, y_3 - \Delta N_{6,12} \leq 112$$

$$\Delta N_{1,12} \leq 180$$
$$\Delta N_{6,12} \leq 188$$
$$\Delta N_{9,12} \leq 30$$

$$\Delta N_{1,12},\ \Delta N_{6,12},\ \Delta N_{9,12} \geq 0$$
$$y_2, y_3 \geq 0$$

Solving the above LP yields

$$\Delta N_{1,12} = 79.56 \simeq 80$$
$$\Delta N_{6,12} = 0$$
$$\Delta N_{9,12} = 0$$

But,solving the same problem by the simplified approach, we obtain,

$$\alpha_1 = \frac{1}{4.49}(180) = 40.09$$

$$\alpha_2 = \frac{1}{1.56}(180 + 300 + 90) = 365.38$$

$$\alpha_3 = \frac{1}{0.85}(180 + 300) = 564.706$$

$$\min\{\alpha_1, \alpha_2, \alpha_3\} = \alpha_1$$

Thus,

$$q = \min\{3.05,\ 4.49\} = 3.05$$
$$\Delta q_{1,12} = 3.05$$
$$\Delta N_{1,12} = (N_{1,0}/q_{1,0})(\Delta q_{1,12})$$
$$\Delta N_{1,12} = (180/4.49)(3.05) = 122.27 \simeq 122$$

As it is shown above LP and the simplified approach give quite different results in such cases.

APPENDIX C

DESCRIPTION OF THE INPUT DATA

The description of the input data together with the notation used in the computer program is as follows:

NR : Number of Traffic Relations

KN : Number of Routes

NT : Number of Trunk Groups

RA(IR) : Traffic offered to Relation IR. IR = 1,...,NR.

NTCM(IT) : Capacity of Trunk Group IT. IT = 1,...,NT.

NRPK(IR) : First Route Pointer for Relation IR. IR = 1,...,NR.

KPT(IK) : First Trunk Group Pointer for Route IK. IK = 1,...,KN.

LKT(IL) : List of Trunk Groups in Each Route. IL = 1,...,LKTN.

LKTN : Number of Elements of List LKT.

RBF : End-to-End Blocking Allowed in Case of Failure.

MVN : Number of Transmission Media.

MPT(IM) : Media Pointer to Trunk Groups. IM = 1,...,MVN.

LMTN : Number of Elements of Lists LMT and LCM.

LMT(IL) : List of Trunk Groups to Media.

LCM(IL) : List of Circuits of Media for each Trunk Group.

MF(IM) : Failure Condition of Media.

In the above list of the input data every parameter and array is understood easily other than NRPK(IR), KPT(IK), LKT(IL) which define the routing pattern. List of trunk groups is given by list LKT and the relations and routes are separated by the use of pointers NRPK and KPT. Routing description and the use of those pointers are explained on the test network in Appendix D.

MPT(IM) is also a pointer which helps to find the list of the trunk groups routed on medium IM from the list LMT(IL). Taking IL = MPT(IM), LMT(IL) gives the first trunk group routed on this medium and LCM(IL) gives the number of circuits of that trunk group routed on IM.

MF(IM) can be either zero or one, which shows a non-failing or a failing medium respectively. In the test network studied in this thesis MF(IM) = 1, meaning a failing transmission medium, for all media.

APPENDIX D

ROUTING DESCRIPTION

Routing description is given by the following three arrays.

NRPK(IR) gives the first route used by relation IR.

KPT(IK) gives the place of the first trunk group used by route IK, on list LKT.

LKT(IL) gives one after another all trunk groups used by each route.

In the test network of Figure 5.1, studied in Section V,

NR = 30, NT = 15 , KN = 52

NRPK = 1,2,3,5,7,9,10,12,13,14,17,18,20,23,26,27,28,29,
31,32,35,36,37,39,40,43,44,46,47,50,53.

KPT = 1,2,3,4,6,7,9,10,12,13,14,16,17,18,20,22,25,26,
27,29,31,33,36,38,40,43,44,46,47,49,52,54,57,60,
64,66,67,69,72,74,77,80,84,86,88,91,92,95,98,102,
105,108,112.

LKT = 12//11//10//12,15//9/12,14//8/11,13//7//5/7,11//
15//14//5,13/7,8/7,11,13//6//4/6,12//4,15/6,10/
6,12,15//4,14/6,9/6,12,14//13//3,7//3//3,5/3,7,11//

LKT continued...

3,14//3,5,13/3,7,8/3,7,11,13//2,7//2//2,5/2,7,11//
2,15//2,5,13/2,7,8/2,7,11,13//1,6//1,4/1,6,12//1//
1,4,15/1,6,10/1,6,12,15//1,4,14/1,6,9/1,6,12,14//

In list LKT double slashes separate the relations and single slashes separate the routes and they are determined from the pointers NRPK and KPT.

For example if we want to read the trunk groups on the routes of relation 18. NRPK(18) = 29 and NRPK(19) = 31. Meaning that the first route of relation 18 is the route 29 and this relation has two routes (31 - 29 = 2) KPT(29) = 47 and KPT(31) = 52. Meaning that from LKT(47) to LKT(51) we have the list of the trunk groups of relation 18.

LKT(47) - LKT(51) is:
3,5/3,7,15.

APPENDIX E

EXPLANATION OF THE COMPUTER PROGRAM

The variables utilized in the computer program to designate the necessary input parameters and data were explained in Appendix C. The variables denoting the basic outputs of the program and some additional variables used within the computer program are as follows.

TMA(.) : Mean of each traffic stream offered to a trunk group.

TMV(.) : Variance of each traffic stream offered to a trunk group.

MT(.) : The number of previous trunk group for each traffic stream.

MK(.) : The number of the route to which the traffic stream belongs to.

MR(.) : The number of the relation to which the traffic stream belongs to.

INF(.) : Maximum number of streams that can be passing through each trunk group.

NPT(.) : Pointer used to determine the place of information related to each trunk group in the arrays TMA, TMV, MT, MK and MR.

TB(.) : The realized blocking probability of each trunk group.

TA(.) : Mean of traffic offered to each trunk group.

TV(.) : Variance of traffic offered to each trunk group.

REQV(.) : Number of equivalent circuits for each traffic relation.

REQMN(.) : Minimum number of equivalent circuits for each traffic relation.

EEB(.) : The realized end-to-end blocking probability for each traffic relation.

CR(.) : Traffic carried by each route.

MFT(.,.) : The matrix storing the information related to each failure.

In MFT, the first column except for the last row, is used to store numbers of the trunk groups affected by the failure. This column is updated for each failure. The second, third and the fourth columns again except for the last row are used to show whether the failed trunk group is used by the first, second and third routes of a relation simply by substituting zero or one. For example, if MFT(4,3) = 1, it means that the failed trunk group whose number given by MFT(4,1) is used by the second route of that relation. The fifth column simply stores the rowwise sum of the values in the second, third, and the fourth columns, in order to check the anomalous trunk groups. The last column is used to store the number of stand-by circuits. The last row keeps the total number of trunk groups failed in each route of a specific relation. All the columns other than the first one are updated for each relation. Number of columns of this matrix is obtained by adding three to the maximum number of alternate routes that a traffic relation may have (which is given as three in the test network so MFT has six columns). Number of rows is obtained by adding one to the maximum number of trunk groups that may be affected by a failure (which is given as seven in the test network).

ETG(.) : Number of equivalent circuits for each route.

ETGF(.) : Number of equivalent circuits for each route as a result of a failure.

QRF(.) : Number of equivalent circuits for each relation as a result of a failure.

NF(.) : Capacity of each trunk group as a result of a failure.

ITGFR(.) : Pointer to the failures described in matrix MFT.

NALFA(.) : Failed route numbers for a relation.

ALFA(.) : α values for the routes given by NALFA.

DELQK(.) : Δq_{kf} values for the failed routes.

MSBR(.) : Total number of required stand-by circuits for each transmission medium failure.

NFTG : Maximum number of trunk groups that may be affected by a failure.

The objectives of the subroutines with their input requirements and issuing outputs are given in the following paragraphs.

SUBROUTINE OVERFL

The mean and variance of the overflowing traffic distributions are calculated. The necessary inputs are the blocking value, the amount of traffic and the number of circuits which are determined according to Fredericks' approximation in case of nonrandom traffic.

SUBROUTINE CARRY

The mean and the variance of the carried traffic distributions are calculated. The necessary inputs are the amount of traffic blocking values for the actual circuits and for one less than that value.

SUBROUTINE RECUR

The Erlang-B value is calculated by its recursive relation. The inputs are the initial value obtained by Rapp's approximation in case of noninteger number of circuits, number of circuits, amount of traffic.

SUBROUTINE EQVCIR

Determines the number of equivalent circuits by making use of the recursive relation of the Erlang-B formula and Jagerman's approximation. The inputs are the amount of traffic and the Erlang-B value.

SUBROUTINE ALFAC

Determines the alfa values for each route as defined by the simplified approach the inputs are the information related to the failure such as matrix MFT, arrays ITGFR, N1CM, ETG, and the number of the initial route of the interested relation.

SUBROUTINE SIMPLE

Performs the procedure given by Algorithm Simple. That is; determines the equivalent stand-by circuits for each route. The necessary inputs are the total stand-by requirements for the interested relation, alfa values obtained by SUBROUTINE ALFAC and information related to the equivalent circuits of the routes given by ETG and ETGF.

SUBROUTINE SBRC

Determines the stand-by requirements for the trunk groups affected by the failure and used by a specific relation. The necessary inputs are given by ITGFR, DELQK, NTLM, NALFA, ETG, ETGF, NF, which are explained previously, and the number of elements stored in array ITGFR.

APPENDIX F

LISTING OF THE COMPUTER PROGRAM

```
3R.MAIN
/83-11:46(,0)
      PARAMETER LMR=112
      PARAMETER NT=15
      PARAMETER NFT=16
      PARAMETER NR=30
      PARAMETER NRPO=31
      PARAMETER KNPO=53
      PARAMETER LKTN=111
      PARAMETER KN=52
      PARAMETER MRL=4
      PARAMETER MVN=21
      PARAMETER NFTG=7
      PARAMETER LMTN=82
      PARAMETER MXF=6
      PARAMETER MVNPO=22
      DIMENSION TMA(LMR),MT(LMR),MK(LMR),MR(LMR),TMV(LMR),INF(NT),NTCM(N
     *T),TB(NT),TA(NT),TV(NT),NPT(NFT),RA(NR),REQV(NR),NRPK(NRPO),KPT(KN
     *PO),LKT(LKTN),CR(KN),NF(NT),ETG(KN),REQMN(NR),FEB(NR),MF(MVN),MFT(
     *8,6),LMT(LMTN),LCM(LMTN),ETGF(KN),QRF(NR),ITGFR(MXF),NALFA(3),ALFA
     *(3),DELQK(3),MSHR(MVN),MPT(MVNPO)
      RBF=0.05
      MW=6
      READ(5,1)(RA(IR),IR=1,NR)
    1 FORMAT(9F8.4,8X)
      READ(5,2)(NTCM(IT),IT=1,NT)
    2 FORMAT(16I4)
      READ(5,3)(NRPK(IR),IR=1,NRPO)
      READ(5,3)(KPT(IK),IK=1,KNPO)
      READ(5,3)(LKT(IL),IL=1,LKTN)
      READ(5,3)(LMT(IL),IL=1,LMTN)
      READ(5,3)(LCM(IL),IL=1,LMTN)
    3 FORMAT(24I3,8X)
      READ(5,3223)(MF(IM),IM=1,MVN)
      READ(5,3223)(MPT(I),I=1,MVNPO)
 3223 FORMAT(25I2)
      DO 921 I=1,NT
      TA(I)=0.
      TB(I)=0.
      INF(I)=0.
      TV(I)=0.
  921 CONTINUE
      DO 922 I=1,NFT
      NPT(I)=0
  922 CONTINUE
C     DETERMINE THE STORAGE CAPACITIES FOR EACH TRUNK GROUP
      DO 51 IL=1,LKTN
      IT=LKT(IL)
      INF(IT)=INF(IT)+1
   51 CONTINUE
      NPT(1)=1
      DO 88 IT=2,NFT
      NPT(IT)=NPT(IT-1)+INF(IT-1)
   88 CONTINUE
      DO 920 I=1,LMR
      TMA(I)=0
      TMV(I)=0
      MT(I)=0
      MK(I)=0
      MR(I)=0
  920 CONTINUE
      DO 923 I=1,KN
      CR(I)=0
      ETG(I)=0.
      ETGF(I)=0.
```

```
  923 CONTINUE
      DO 224 I=1,NR
      REQV(I)=0.
      REQMN(I)=0.
      QRF(I)=0.
      EEB(I)=0.
  224 CONTINUE
C     STORING THE FRESH TRAFFIC
      DO 111 IR=1,NR
      IK=NRPK(IR)
      IL=KPT(IK)
      IT=LKT(IL)
      A=RA(IR)
      JO=NPT(IT)
      JD=NPT(IT+1)-1
      DO 41 J=JO,JD
      IF(MR(J).NE.0)GO TO 41
      TMA(J)=A
      TMV(J)=A
      MT(J)=0
      MK(J)=IK
      MR(J)=IR
      GO TO 111
   41 CONTINUE
  111 CONTINUE
      DO 555 IT=1,NT
      JO=NPT(IT)
      JD=NPT(IT+1)-1
      DO 5 J=JO,JD
      TA(IT)=TA(IT)+TMA(J)
      TV(IT)=TV(IT)+TMV(J)
    5 CONTINUE
      Z=TV(IT)/TA(IT)
      N=NTCM(IT)
      IF(Z.EQ.1.)GOTO 48
C     NONPOISSON TRAFFIC
      XN=FLOAT(N)/Z
      NP=INT(XN)
      DIF=XN-NP
      A=TA(IT)/Z
      TETA=1.-((A+2.)*DIF)/((1.+A)**2+A)+DIF**2/((1.+A)*((1.+A)**2+A))
      CALL RECUR(TETA,NP,A,EF,DIF)
      TB(IT)=EF
      XNMO=XN-1.
      NPM=INT(XNMO)
      CALL RECUR(TETA,NPM,A,EFF,DIF)
      A=TA(IT)
   50 CALL OVERFL(A,XN,EF,OVM,OVV)
      CALL CARRY(A,EF,EFF,CRM,CRV)
      ZOV=OVV/OVM
      ZCR=CRV/CRM
C     HANDLE EACH TRAFFIC OFFERED TO THIS TRUNK GROUP ONE BY ONE
      DO 4 J=JO,JD
      IF(MR(J).EQ.0)GO TO 555
      AJ=TMA(J)
      CRMJ=CRM*TMA(J)/TA(IT)
      CRVJ=CRMJ*ZCR
      IK=MK(J)
      IR=MR(J)
      IL=KPT(IK)
      ITT=LKT(IL)
C     IS IT THE FIRST TRUNK GROUP OF THIS ROUTE?
      IF(IT.NE.ITT)GO TO 118
      ILS=KPT(IK+1)-1
      ITG=LKT(ILS)
C     IS IT THE LAST TRUNK GROUP OF THIS ROUTE
      IF(IT.EQ.ITG)GO TO 39
      ITK=LKT(IL+1)
      JH=NPT(ITK)
      JE=NPT(ITK+1)-1
      DO 99 KD=JH,JE
      IF(MR(KD).NE.0)GO TO 99
      TMA(KD)=CRMJ
      TMV(KD)=CRVJ
      MT(KD)=IT
      MK(KD)=IK
      MR(KD)=IR
      GO TO 19
   99 CONTINUE
   19 IF(TB(IT).EQ.0)GO TO 4
      KI=NRPK(IR+1)-1
C     IS IT THE LAST ROUTE OF THIS RELATION?
```

```
      IF(KI.EQ.IK)GO TO 4
      KP=IK+1
C     SEARCH FOR THE OVERFLOW POSSIBILITIES
      DO 77 KL=KP,KI
      IL=KPT(KL)
      ITG=LKT(IL)
      IF(ITG.EQ.IT)GO TO 77
      OVMJ=OVM*TMA(J)/TA(IT)
      OVVJ=OVMJ*ZOV
      JS=NPT(ITG)
      JF=NPT(ITG+1)-1
      DO 72 K=JS,JF
      IF(MR(K).NE.0)GO TO 72
      TMA(K)=OVMJ
      TMV(K)=OVVJ
      MT(K)=IT
      MK(K)=KL
      MR(K)=IR
      GO TO 4
   72 CONTINUE
   77 CONTINUE
      GO TO 4
   39 CR(IK)=CRMJ
      GO TO 19
  118 ILS=KPT(IK+1)-1
      ITG=LKT(ILS)
C     IS IT THE LAST TRUNK GROUP OF THIS ROUTE?
      IF(IT.EQ.ITG)GO TO 399
      ILS=KPT(IK+1)-1
      DO 11 I=IL,ILS
      ITS=LKT(I)
      IF(ITS.NE.IT)GO TO 11
      IS=I-IL
      ITSS=LKT(I+1)
      JB=NPT(ITSS)
      JE=NPT(ITSS+1)-1
      DO 24 KD=JB,JE
      IF(MR(KD).NE.0)GO TO 24
      TMA(KD)=CRMJ
      TMV(KD)=CRVJ
      MT(KD)=IT
      MK(KD)=IK
      MR(KD)=IR
      GO TO 18
   24 CONTINUE
   11 CONTINUE
   18 IF(TB(IT).EQ.0)GO TO 4
      KI=NRPK(IR+1)-1
C     IS IT THE LAST ROUTE OF THIS RELATION?
      IF(KI.EQ.IK)GO TO 4
      KP=IK+1
C     SEARCH FOR THE OVERFLOW POSSIBILITIES
      DO 71 KL=KP,KI
      ILP=KPT(KL)+IS
      IF(LKT(ILP).EQ.IT)GO TO 71
      OVMJ=OVM*TMA(J)/TA(IT)
      OVVJ=OVMJ*ZOV
      ITP=LKT(ILP)
      MB=NPT(ITP)
      ME=NPT(ITP+1)-1
      DO 25 MM=MB,ME
      IF(MR(MM).NE.0)GO TO 25
      TMA(MM)=OVMJ
      TMV(MM)=OVVJ
      MR(MM)=IR
      MT(MM)=IT
      MK(MM)=KL
      GO TO 4
   25 CONTINUE
   71 CONTINUE
      GO TO 4
  399 CR(IK)=CRMJ
      IS=KPT(IK+1)-KPT(IK)-1
      GO TO 18
    4 CONTINUE
      GO TO 555
C     POISSON TRAFFIC
   48 A=TA(IT)
      XN=FLOAT(N)
      TETA=1.
      CALL RECUR(TETA,N,A,EF,0.)
      TB(IT)=EF
```

```
      NPO=N-1
      CALL RECUR(TETA,NPO,A,FFF,0.)
      GO TO 50
  555 CONTINUE
      WRITE(MW,1000)
 1000 FORMAT(///,12X,,TMA,,20X,,TMV,,20X,,MT,,12X,,MK,,12X,,MR,)
      DO 1001 I=1,LMR
      WRITE(MW,1002)I,TMA(I),TMV(I),MT(I),MK(I),MR(I)
 1002 FORMAT(3X,I3,4X,F10.5,10X,F10.5,10X,3(I5,10X))
 1001 CONTINUE
      WRITE(MW,1003)
 1003 FORMAT(///,30X,,TB,,20X,,TA,,20X,,TV,)
      DO 1010 I=1,NT
      WRITE(MW,1011)I,TB(I),TA(I),TV(I)
 1011 FORMAT(10X,I2,15X,3(F10.5,10X))
 1010 CONTINUE
C     NUMBER OF EQUIVALENT CIRCUITS FOR EACH ROUTE
      DO 29 IR=1,NR
      IR1=NRPK(IR)
      IR2=NRPK(IR+1)-1
      Y=0.
      Q=0.
      DO 92 IK=IR1,IR2
      Y=Y+CR(IK)
      PR=(RA(IR)-Y)/RA(IR)
      OCR=RA(IR)
      CALL EQVCIR(PR,OCR,EQCIR)
      ETG(IK)=EQCIR-Q
      Q=Q+ETG(IK)
   92 CONTINUE
      EEB(IR)=PR
   29 CONTINUE
C     NUMBER OF EQUIVALENT CIRCUITS FOR EACH RELATION
      DO 612 IR=1,NR
      IB=NRPK(IR)
      IE=NRPK(IR+1)-1
      DO 66 IK=IB,IE
      REQV(IR)=REQV(IR)+ETG(IK)
   66 CONTINUE
  612 CONTINUE
C     MINIMUM NUMBER OF EQUIVALENT CIRCUITS IN FAILURE
      DO 69 IR=1,NR
      OREL=RA(IR)
      CALL EQVCIR(RBE,OREL,EQCMIN)
      IF(EQCMIN.LT.REQV(IR))GO TO 96
      REQMN(IR)=REQV(IR)
      GO TO 69
   96 REQMN(IR)=EQCMIN
   69 CONTINUE
      WRITE(MW,1012)
 1012 FORMAT(///,25X,,CR,,20X,,ETG,)
      DO 1016 I=1,KN
      WRITE(MW,1014)I,CR(I),ETG(I)
 1014 FORMAT(12X,I3,5X,F10.5,15X,F10.5)
 1016 CONTINUE
      WRITE(MW,1015)
 1015 FORMAT(20X,,REQMN,,15X,,REQV,,15X,,EEB,)
      DO 1019 I=1,NR
      WRITE(MW,1017)REQMN(I),REQV(I),EEB(I)
 1017 FORMAT(15X,F10.5,8X,F10.5,8X,F8.5)
 1019 CONTINUE
      DO 712 IM=1,MVN
      IF(MF(IM).EQ.0) GO TO 712
      DO 113 K=1,NFTG
      MFT(K,1)=0
      MFT(K,6)=0
  113 CONTINUE
      IS=0
      I1=MPT(IM)
      IF(I1.EQ.0) GO TO 712
  464 IS=IS+1
      I2=MPT(IM+IS)
      IF(I2.EQ.0) GO TO 464
      I2=I2-1
      K=1
      DO 735 I=I1,I2
      IT=LMT(I)
      MFT(K,1)=IT
      NF(IT)=NTCM(IT)-LCM(I)
      K=K+1
  735 CONTINUE
      DO 818 IR=1,NR
```

```
      DO 332 I=1,NFTG+1
      DO 334 J=2,5
      MFT(I,J)=0
  334 CONTINUE
  332 CONTINUE
      DO 543 I=1,3
      NALFA(I)=0
      ALFA(I)=0.
      DELQK(I)=0.
  543 CONTINUE
      DO 615 I=1,NXF
      ITGFR(I)=0
  615 CONTINUE
      IK1=NRPK(IR)
      IK2=NRPK(IR+1)-1
      DO 219 K=1,NFTG
      ITF=MFT(K,1)
      DO 919 IK=IK1,IK2
      IL1=KPT(IK)
      IL2=KPT(IK+1)-1
      DO 317 IL=IL1,IL2
      ITG=LKT(IL)
      IF(ITG.NE.ITF)GO TO 317
      KR=IK-IK1+2
      MFT(K,KR)=1
      GO TO 919
  317 CONTINUE
  919 CONTINUE
      ISUM=0
      DO 524 KM=2,4
      ISUM=ISUM+MFT(K,KM)
  524 CONTINUE
      MFT(K,5)=ISUM
  219 CONTINUE
      M=1
      NFR=0
      DO 32 J=2,4
      DO 33 I=1,NFTG
      IF(MFT(I,J).EQ.0)GO TO 33
      ITGFR(M)=I
      M=M+1
      MFT(NFTG+1,J)=MFT(NFTG+1,J)+MFT(I,J)
   33 CONTINUE
      IF(MFT(NFTG+1,J).EQ.0)GO TO 32
      NFR=NFR+1
   32 CONTINUE
      NIT=M-1
      WRITE(MW,1113)NIT
 1113 FORMAT(30X,,NIT=,,I3)
      IF(NFR.EQ.0)GO TO 818
      IB=1
      ORAN=1.
      JK=2
      NRK=0
      QRF(IR)=REQV(IR)
   44 NRK=NRK+MFT(NFTG+1,JK)
      IF(MFT(NFTG+1,JK).NE.0)GO TO 4141
      JK=JK+1
      IF(JK.EQ.5)GO TO 94
      GO TO 44
 4141 IK=IK1+JK-2
      WRITE(MW,103)IB,NRK,IK
  103 FORMAT(30X,,IB=,,I3,5X,,NRK=,I3,5X,,IK=,,I3)
      DO 693 M=IB,NRK
      K=ITGFR(M)
      IT=MFT(K,1)
      ORANM=FLOAT(NF(IT))/FLOAT(NTCM(IT))
      IF(ORANM.GE.ORAN)GO TO 693
      ORAN=ORANM
  693 CONTINUE
      IB=NRK+1
      ETGF(IK)=ETG(IK)*ORAN
      QRF(IR)=QRF(IR)+ETGF(IK)-ETG(IK)
      JK=JK+1
      IF(JK-1.GT.NFR)GO TO 94
      GO TO 44
   94 IF(QRF(IR).GE.REQMN(IR))GO TO 818
      DELQR=REQMN(IR)-QRF(IR)
      WRITE(MW,222)DELQR,ORAN
  222 FORMAT(30X,,DELQR=,,F8.4,5X,,ORAN=,,F8.4)
      IF(NFR.EQ.1)GO TO 913
      NANO=0
```

```
      DO 43 K=1,NIT
      I=ITGFR(K)
      IF(MFT(I,5).EQ.1)GO TO 43
      NANO=NANO+1
   43 CONTINUE
      IF(NANO.EQ.NIT)GO TO 914
      CALL ALFAC(MFT,ITGFR,NTCM,NALFA,ALFA,ETG,IK1)
      CALL SIMPLE(DELQR,ALFA,NALFA,ETG,ETGF,DELQK)
      CALL SBRC(ITGFR,DELQK,NTCM,MFT,NALFA,ETG,ETGF,NF,NIT)
      GO TO 823
  914 K=ITGFR(1)
      IT=MFT(K,1)
      SBR=FLOAT(NTCM(IT))/REQV(IR)*DELQR
      NSBR=INT(SBR+0.5)
      IF(MFT(K,6).GE.NSBR)GO TO 823
      MFT(K,6)=NSBR
  823 WRITE(MW,122)IM,IR
  122 FORMAT(20X,,IM=,,I3,20X,,IR=,,I3)
      WRITE(MW,503)
  503 FORMAT(30X,,MFT,)
      DO 505 I=1,NFTG
      WRITE(MW,504)MFT(I,1),MFT(I,5),MFT(I,6)
  504 FORMAT(25X,3I4)
  505 CONTINUE
      GO TO 818
  913 JS=IK-IK1+1
      NALFA(JS)=IK
      DELQK(JS)=DELQR
      CALL SBRC(ITGFR,DELQK,NTCM,MFT,NALFA,ETG,ETGF,NF,NIT)
      GO TO 823
  818 CONTINUE
      IMSBR=0
      DO 542 K=1,NFTG
      IMSBR=IMSBR+MFT(K,6)
  542 CONTINUE
      MSBR(IM)=IMSBR
      WRITE(MW,1456)
 1456 FORMAT(///,30X,,NF,)
      DO 1056 IT=1,NT
      WRITE(MW,6501)NF(IT)
 6501 FORMAT(28X,I4)
 1056 CONTINUE
      WRITE(MW,1457)
 1457 FORMAT(///,30X,,ETGF,)
      DO 1057 IK=1,KN
      WRITE(MW,7501)ETGF(IK)
 7501 FORMAT(26X,F10.5)
 1057 CONTINUE
  712 CONTINUE
      ISUMCR=0
      ISUMSB=0
      DO 7 IM=1,MVN
      ISUMSB=ISUMSB+MSBR(IM)
    7 CONTINUE
      DO 9 IC=1,LMTN
      ISUMCR=ISUMCR+LCM(IC)
    9 CONTINUE
      WRITE(MW,7877) ISUMCR,ISUMSB
 7877 FORMAT(///,30X,,ISUMCR=,,I8,30X,,ISUMSB=,,I8)
      STOP
      END
```

```
K 2085 DBANK
VERFL
3-11:48(,0)
      SUBROUTINE OVERFL(AF,XNJ,EF,OVM,OVV)
      OVM=AF*EF
      EOF=AF/(XNJ+1.+OVM-AF)
      OVV=OVM*(1.-OVM+EOF)
      RETURN
      END

8 DBANK
ARRY
3-11:48(,0)
      SUBROUTINE CARRY(AC,EF,EFF,CRM,CRV)
      CRM=AC*(1.-EF)
      CLC=AC*(EFF-EF)
      CRV=CRM*(1.-CLC)
      RETURN
      END
```

```
 DBANK
CUR
-11:48(,0)
    SUBROUTINE RECUR(TETA,N,A,APROX3,DIF)
    APROX3=(A*TETA)/(DIF+1.+A*TETA)
    DO 4 I=1,N-1
    APROX3=(A*APROX3)/(I+DIF+1.+A*APROX3)
  4 CONTINUE
    RETURN
    END

1 DBANK
VCTR
-11:48(,0)
    SUBROUTINE EQVCTR(PO,ACR,EQC)
    N=0
    P=1.0
418 PP=P
    N=N+1
    P=ACR*P/(N+1+ACR*P)
    IF(P.GT.PO)GO TO 418
    K=N+1
    PK=(ACR*P)/(K+1+ACR*P)
    DO 88 I=1,99
    DELTA=FLOAT(I)/100.0
    PAY=(PP**(1.-DELTA))*(P**DELTA)
    PAYDA=1.-0.5*DELTA*(1.-DELTA)*((P**2)/(PP**PK)-1.)
    PX=PAY/PAYDA
    IF(PX.GT.PO)GO TO 88
    GO TO 811
 88 CONTINUE
811 EQC=N-1.+DELTA
    RETURN
    END
```

```
NK 35 DBANK
.ALFAC
/83-11:49(,0)
      SUBROUTINE ALFAC(MFT,ITGFR,NTCM,NALFA,ALFA,ETG,IK1)
      DIMENSION MFT(8,6),ITGFR(6),NTCM(15),NALFA(3),ALFA(3),ETG(52)
      ITL=1
      SIGM=0.
      NRL=0
      DO 105 J=2,4
      NRL=NRL+MFT(8,J)
      IF(MFT(8,J).EQ.0)GO TO 105
      DO 103 K=ITL,NRL
      I=ITGFR(K)
      ITR=MFT(I,1)
      SIGM=SIGM+NTCM(ITR)
  103 CONTINUE
      ITL=NRL+1
      IK=IK1+J-2
      NALFA(J-1)=IK
      ALFA(J-1)=SIGM/ETG(IK)
  105 CONTINUE
      RETURN
      END
NK 32 DBANK
.SIMPLE
/83-11:49(,0)
      SUBROUTINE SIMPLE(DELQR,ALFA,NALFA,ETG,ETGF,DELQK)
      DIMENSION ALFA(3),NALFA(3),ETG(52),ETGF(52),DELQK(3)
      DEL=0.
  118 IF(DEL.GE.DELQR)GO TO 538
      ALF=100000.
      DO 719 J=1,3
      IF(ALFA(J).EQ.0)GO TO 719
      IF(ALF.LT.ALFA(J))GO TO 719
      ALF=ALFA(J)
      K=J
 719  CONTINUE
      IK=NALFA(K)
      DELQK(K)=MIN(DELQR-DEL,ETG(IK)-ETGF(IK))
      DEL=DEL+DELQK(K)
      ALFA(K)=100000.
      GO TO 118
  538 RETURN
      END
K 24 DBANK
```

```
SBRC
83-11:49(,0)
      SUBROUTINE SBRC(ITGFR,DELQK,NTCM,MFT,NALFA,ETG,ETGF,NF,NIT)
      DIMENSION ITGFR(6),DELQK(3),NTCM(15),MFT(8,6),NALFA(3),ETG(52),ETG
     *F(52),NF(15)
      MFTP=0
      J=1
      DO 967 I=1,NIT
      K=ITGFR(I)
      IT=MFT(K,1)
   91 IF(NALFA(J).NE.0)GO TO 451
      J=J+1
      GO TO 91
  451 IK=NALFA(J)
      IF(MFT(8,J+1).GT.1)GO TO 911
      SBR=FLOAT(NTCM(IT))/ETG(IK)*DELQK(J)
      NSBR=INT(SBR+0.5)
      GO TO 511
  911 SBR=FLOAT(NTCM(IT))/ETG(IK)*(DELQK(J)+ETGF(IK))-NF(IT)
      NSBR=INT(SBR+0.5)
  511 IF(MFT(K,6).GE.NSBR)GO TO 119
      MFT(K,6)=NSBR
  119 MFTP=MFTP+MFT(8,J+1)
      IF(I.LT.MFTP)GO TO 967
      J=J+1
  967 CONTINUE
      RETURN
      END
```

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