ERROR PERFORMANCE OF LINE CODING

by

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ERROR PERFORMANCE OF LINE CODING

ABSTRACT

A tutorial overview of line coding techniques under two broat classifications , neamly linear and nonlinear line codes is presented.A brief discussion of several approaches towards symbol error probability and error performance of line coding is also given.Analysis of a simple model for intersymbol interference due to low-frequency cutoff channel is represented.This model has been developed by Jakubow , Chang and Garcia as it is used to compute symbol error probability for ternary alphabetic line codes and it also gives a design algorithm for the ternary alphabetic line codes that optimizes the symbol error probability.

HAT KODLARINDA HATA BAĞIŞIKLIĞI

ÖZET

Bu tezde , doğrusal ve doğrusal olmayan hat kodları olmak üzere iki ana grup içinde hat kodlama teknikleri anlatılmış ve bu kodların simge hata olasılığı üzerine yapılan araştırmalar kısaca özetlenmiştir. Jakubow, Chang ve Garcia tarafından geliştirilen "alt kesim frekanslı kanal simge girişimine ilişkin modelin" analizi yapılmıştır.Bu model üçlü abecesel hat kodlarının simge hata olasılığını belirlemekte ve bu olasılığı en iyileyen üçlü abecesel kod tasarımında kullanılmaktadır.

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ISI	Intersymbol interference
LF	Low frequency
G(f)	Raised cosine filter transfer function
g(t)	Impulse response of raised cosine filter
B	Roll off factor of raised cosine filter
PRS	Partial response signalling
D	Unit delay
NRZ-L	Non-return to zero level
NRZ-M	Non-return to zero mark
F(D)	PR encoder transfer function (polynomial)
P(D)	Precoder transfer function
h_k	Coefficients of the PR encoder polynomial
AMI	Alternate mark inversion code
WAL-1	Walsh-type-1 code
WAL-2	Walsh-type-2 code
RDS	Running digital sum
DSV	Digital sum variation
S_T or s	Terminal states
S	Allowable states
e(s)	Encoding rule
C(S)	Alphabet of terminal state s.
STPM	State transition probability matrix
Π	STPM
P_i	State transition stationary probability

E	Efficiency
PST	Pair selected ternary
4B3T	Four binary to three ternary alphabetic code
MS43	Four binary to three ternary alphabetic code
FOMOT	For mode ternary
VL43	Variable length code
L742	Four binary to 2-seven level code
3B-2Q	Three binary to 2-four level code
6B-3QI	Six binary to 3-five level code
TPC	Time polarity control
DM	Delay modulation
B6Zs	Bipolar six zeros subsititution code
HDB	High density bipolar
CHDB	Compatible high density bipolar
TIB	Transparent interleaved bipolar
CPF	Crosstalk power factor
P(e)	Probability of symbol error
Z	ISI
n	Noise
σ_n	Variance of Gaussian noise
$A_h(L)$	Moments of ISI
λ_v (a|s)	Encoded word (a) cost in states with Var(s) criteria
λ_v (c(s))	Alphabet cost in states with Var(s)criteria
λ_D(a|s)	Encoded word (a) cost in state s with Var $_D$(s) criteria

λ_D (b(s))	Alphabet cost in state s with Var $_D$(s) criteria
λ_P (a|s)	Encoded word (a) cost in state s with P(e) criteria
P(s,a,t)	Conditional error probability of error in the th symbol, in state s and encoded word a
SNR	Signal to noise ratio

CHAPTER I

INTRODUCTION

The purpose of a communication system is to transfer information from one point to another. This information is represented by an electrical signal in analog or digital form. Analog signals can have continuum values while digital signals can have only finite number of values. In digital communication, the input signal may be data from a computer or may be an analog signal that has been digitized by sampling and quantization. Digital communication is becoming increasingly attractive because of growing demand for data communication and because digital communication systems have several advantages with respect to analog communication systems. The two main advantages of digital transmission are:

1. Regenerative or digital repeaters: The signal enters the transmitter as waveforms which is suitable for transmitter-channel-receiver path. Channel has many different structures such as, wire pairs, coaxial cables, atmosphere (radio), optical fibers. Each of them impose different requirements on the transmitter. But all of them introduce distortion to the transmitted signal and adds noise. In digital communication systems channel is broken into several devices, which the transmitted signals recovered and retransmitted at the boundaries of these devices. These devices are called regenerative or digital repeaters. If the repeater spacings are close enough the regenareted signal will be a copy of the transmitted signal.

2. The second main advantage of digital communication is that the receiver has to decide as to which of finite number of possible transmitted symbols.

The block diagram of a digital communication system is shown in fig 1.1 (25,26)

The upper blocks of the diagram which are line encoder, source encoder, encrypt, channel encoder, multiplex, modulate frequency spread and multiple access which realize the source to transmitter transmission. The lower blocks dictates the receiver back to source transmission. The blocks within the dashed lines intially consisted only of the modulator and demodulator functions named MODEM. But now other signal processings functions are incorporated within the same assembly MODEM. MODEM can be thought of as the brain of the system. The transmitter consists of a frequency up-conversion stage, a high power amplifier and an antenna and the receiver consists of low noise amplifier and down converter stage.

Formatting , Modulation and demodulation are essential steps for digital communications systems, the other processings steps are optional. The signal processing steps (fig 1.2) are

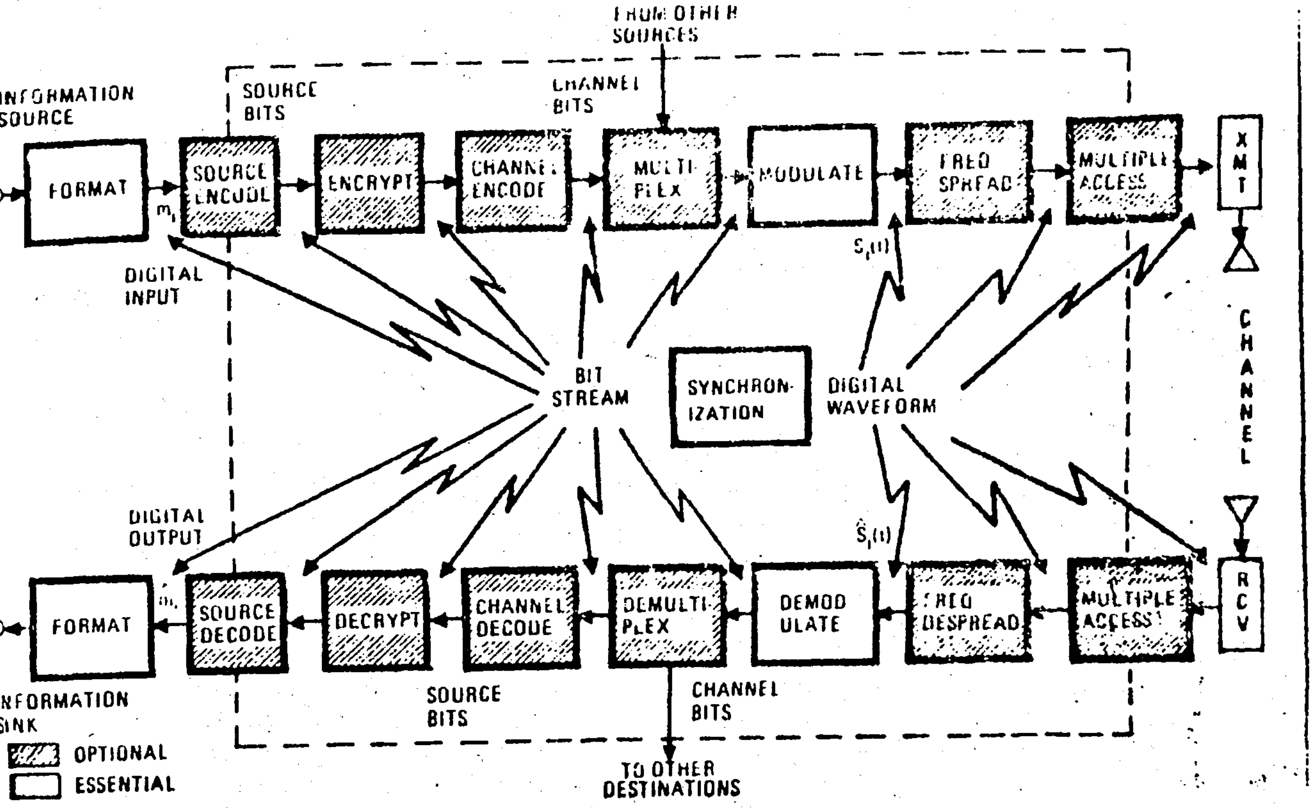


Fig. 1.1 Block diagram of digital communication [25,Fig 1]

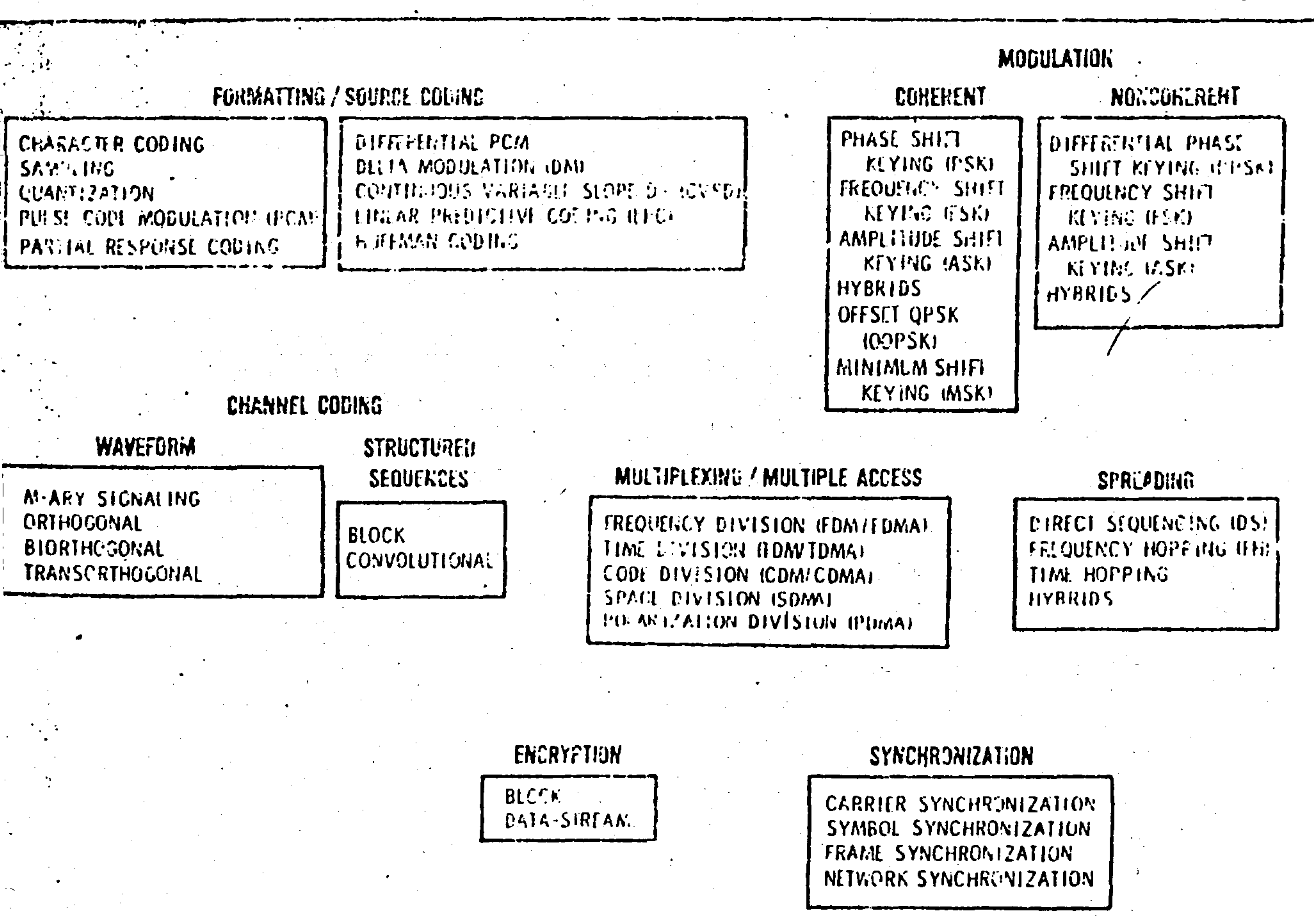


Fig. 1.2 Basic digital communication transformation [25,fig 2]

1) Source Encoding: It removes information redundancy and performs analog to digital (A/D) conversion. Removing the redundancy can be achieved by making data rate reduction. Source encoding can reduce the data rate if the symbols are not equally likely or if they are not statistically independent. Some common source encoding techniques are differential pulse code modulation (DPCM), delta modulation (DM), adaptive versions of these two (ADPCM, ADM), linear predictive coding (LPC) and Huffmancoding.

2) Encryption: It prevents unauthorized persons from extracting information from the channel and from injecting false information to the channel.

3) Channel Coding: This is also known as error coding process. This process trasforms source bits into channel bits. Channe coding is partitioned into two groups, waveform coding and structured sequences. Waveform coding improves the transmission performance in an overall sense because the encoding produces signals with better distance properties than the original signal sets. Structured sequences improves performance by making detection and correction of transmission errors.

4) Frequency Spreading : It is the spread spectrum techniques which allow multiple signals to occupy the same bandwidth transmitted without interfering with one another.

5) Multiplexing and Multiple Access: They both refer to the sharing of a communication resource.

6) Synchronization : It is defined as the alignment of time scales of separated periodic processes, it involves the estimation of time and frequency.

7) Line Coding: Line coding is defined as any operation that transforms data sequence into another sequence. Line coding has 2 main properties, one is compansating for intersymbol interference (ISI) and the other one is error monitoring capability which provides the timing recovery at the receiver. These two main properties make the line codes widely used in digital communication systems. Several line codes exist each of them trying to realize the above properties by increasing the signalling rate, increasing the number of transmitted levels, making spectral shaping, removing long strings zero. These codes can be grouped into two main blocks such as linear and nonlinear line codes.

Linear line codes include partial response (correlative-level) codes, two phase (frequency) modulation codes. In partial response coding, controlled amount of ISI is introduced by correlating the symbols at sampling instants in order to obtain spectral shaping (little amount of low frequency component and nulls at $\frac{1}{T}$ multiples) and to enable transmission at Nyquist rate and even at higher rates. In the two phase modulation coding redundancy is introduced by increasing signalling rate.

In nonlinear line codes, digits of the encoded sequence are grouped in characters of n consecutive digits, encoding being done one character at a time. This group includes alphabetic and nonalphabetic codes In alphabetic codes, m binary bits are mapped into yL level bits by using state transitions and encoding rules. Nonalphabetic codes remove the long strings of zeros by using subsitution sequences instead of consecutive zeros.

One of the goals in the design and the selection of line codes is minimizing the symbol error probability P(e). The evalution of P(e) in the presence of ISI and the noise is the main problem in communication systems. Also there is no direct analytical expression between ISI performance and P(e). There are many methods for determining P(e) due to ISI and additive noise. But none of them allows us to develop code design algorithm that optimize the P(e) performance. Jakubow, Chang and Garcia have developed a ISI model (running digital sum model) which produces a code design algorithm for alphabetic codes which are a branch of nonlinear line codes. These optimize the P(e) performance under their assumptions.

This thesis contains the tutorial review of line coding techniques and analysis of the code design algorithm of Garcia, for 4B3T alphabetic code.

In chapter 2, we present the definition and properties of line coding. Description of baseband system is given in order to explain the compensating property of line coding for ISI. In chapter 3 and chapter 4 linear and nonlinear line codes are described. In chapter 5 some methods of evaluation P(e) due to ISI and additive noise are examined in chronological order. These methods are the method of Shimbo and Celebiler, Glave's method, moment approximation method and running digital sum (RDS) method. In chapter 6 code design algorithm of Garcia depending on therunning digital sum (RDS) model is described and this section includes a computer program in Fortran, ~~this~~ is the design of 4B-3T code with RDS model. In Chapter 7 the new research topics in line coding are tried to discuss.

The purpose of this thesis is to form a basis to serve a collected information and to give a starting point for new research areas about line coding for the future researchers. And also it includes the analysis and programing of Garcia's algorithm for one special code.

CHAPTER II

LINE CODING

Line coding is the encoding procedure which converts data sequence into another sequence to realize certain properties of the baseband transmitted signal.The device that carries out the line coding is call the line coder and the output of the line coder (encoded sequence) is called the line code.The block diagram of a data channel scheme in fig.2.1. indicates the location of the line coder in baseband digital transmission.[27]

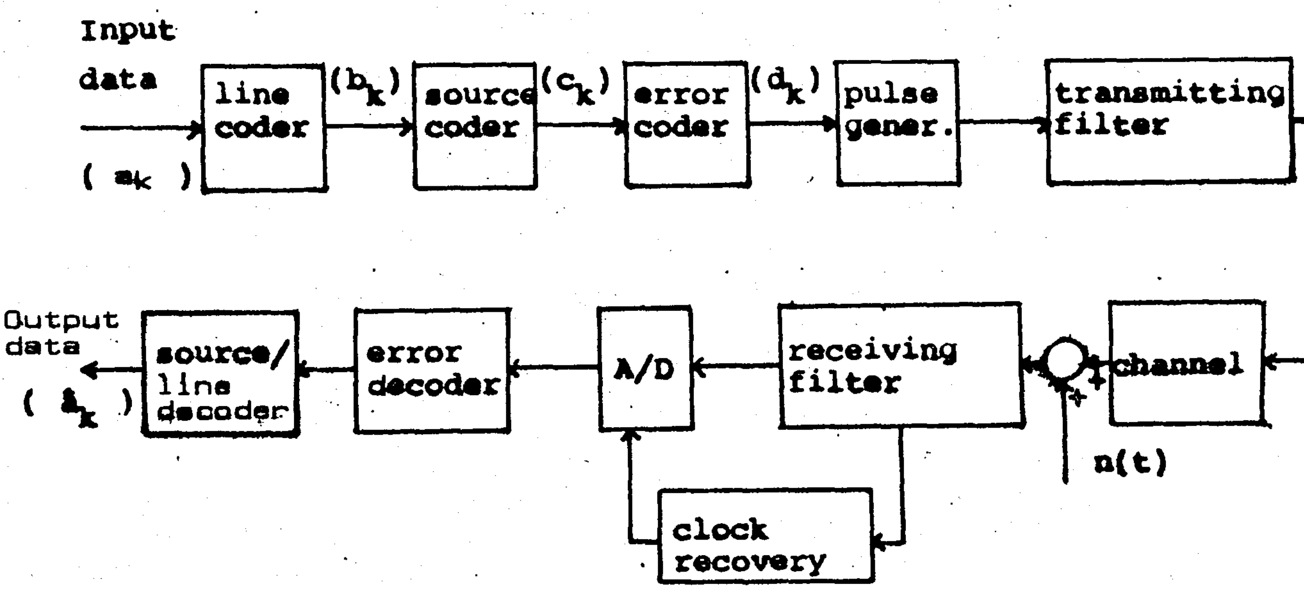


Fig. 2.1. The block diagram of a data channel

The properties of the transmitted signal in baseband digital transmission are:[8],[9],[10].

1. It must have no DC component and must have a small amount of low frequency (LF) component,because of LF cutoff due to the transformer coupling.If DC or LF components are present,they lead to baseline wonder which produces errors in threshold detection.Baseline wonder can be corrected with two methods:One of them uses a pulse pattern and the other uses quantize feedback together with DC restoration.The first method requires that the code must be transparent so that it must be able to transmit long sequence of "zeros" and "ones". In the second method the LF components removed by the coupling network are replaced at the repeater by local generation.

2. It must have timing information so that the clock information can be generated from the received signal or another signal (external clock). The second method is too expensive.

Line codes should have the following properties in addition to the previous properties:[8],[9],[10]

1. They must compansate for intersymbol interference (ISI).
2. They must have in service error monitoring capability.
3. Word or bit synchronization must be provided, if the line code consists of blocks of more than one pulse.
4. It must have the property of lowering line or symbol rate with respect to binary information rate , because the attenuation of coaxial cable is proportional to the square root of the line signal frequency , thus reduction in symbol rate is advantageous. Also this problem can be solved by multilevel signalling.
5. The code must have frequent peak excursion for any binary input data,to provide signal level information to control automatic gain control (AGC) circuits in the repeaters.
6. The code must have simple practical implementation.

The most important properties of the line codes are 1. and 2. The first property of the line coding will be expanded by describing the baseband digital transmission.The second property i.e system monitoring and timing will be examined in section 2.4..

2.1 BASEBAND DIGITAL TRANSMISSION

In baseband system,the digital data is transmitted directly or with some shaping or with using encoded sequences , but does not involve modulation of a sinusoidal carrier signal.The block diagram of the baseband system is in fig. 2.2 (19),(4)

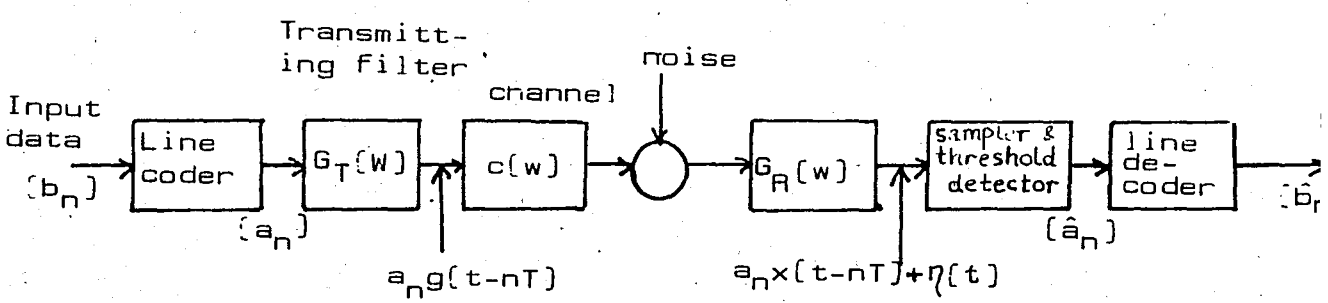


Fig. 2.2 Baseband system

Channel is the coaxial cable. Coaxial cables provide high transmission quality with physical structures. They do not suffer cross talk and impulse interference, and only thermal noise exists.

For describtion of baseband digital transmission, pulse amplitude modulation (PAM) system will be used. Because phase and frequency modulation methods are less efficient as compared to amplitude modulation methods. [19]

In PAM the information is encoded into the amplitude values at T-sec intervals. In fig 2.2 (b_n) is the input sequence consisting of equiprobable binary digits, (a_n) is the encoded sequence which pulse amplitude modulates a train of identically shaped pulse. Input symbols (a_n) are sequence of a narrow impulses whose shape is denoted as g[t-nT]. The number of levels of (a_n) is L. the spacing between levels is uniform as in fig 2.3a Thus the allowed transmitter levels are $\pm$ d, $\pm$ 3d, $\pm$ (L-1)d, where 2d is the distance between adjacent levels. [19]

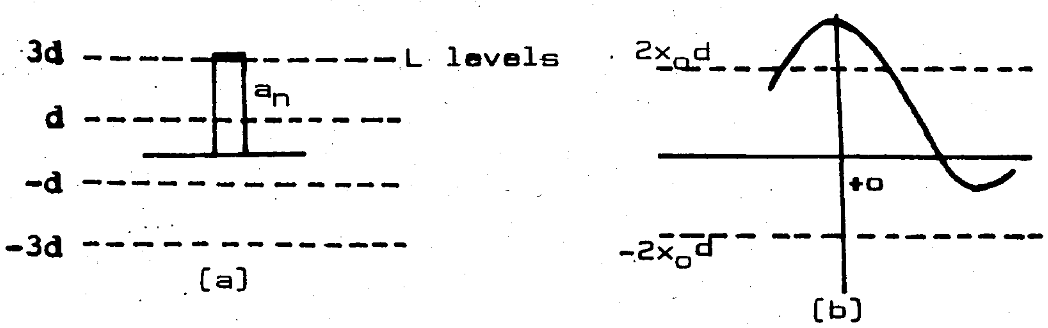


Fig. 2.3 Input symbol a_n at the transmitter (a),at the threshold detector (b).

from fig 2.2 the output of the transmitting filter is;

$$r[\ t\ ]=\sum_{n=-\infty}^{\infty} a_n\ g[t-nT] \qquad (2.1)$$

output of the receiving filter is;

$$y[\ t\ ]=\sum_{n=-\infty}^{\infty} a_n\ x[t-nT] + \eta\ [\ t\ ] \qquad (2.2)$$

where η[t]: additive noise and

$$x[\ t\ ]=\frac{1}{2\pi}\int_{-\infty}^{\infty} G_T\ [W]\ C[W]\ G_R\ [W]\ e^{jwt}\ dw \qquad (2.3)$$

G_T [W] : transfer function of the transmitting filter.

C [W] : coaxial cable characteristics.

G_R [W] : transfer function of the receiving filter.

At the sampler and threshold detector, samples are taken at K'th time interval can be expressed

$$y\,[KT + to] = \sum_{n=-\infty}^{\infty} a_n\, x[KT + t_o - nT] + \eta\,[KT + to] \qquad [2.4]$$

where to is the delay in transmission thru the channel

Let's

$$y_k = \sum_{n=-\infty}^{\infty} a_n\, x_{k-n} + \eta_k \qquad [2.5]$$

$$y_k = x_o\,\left[\, a_k + \frac{1}{x_o} \sum_{\substack{n=-\infty \\ k \neq n}}^{\infty} a_n\, x_{k-n} + \frac{\eta_k}{x_o} \right] \qquad [2.6]$$

x_o factor represents the gain or attenuation of the signal passing thru the system.

The threshold detector can set decesion thresholds at o, $\pm\, 2x_o$ d, $\pm\, 4x_o$ d as in fig 2.3 b. Then error accu..s

when

$$\left| \sum_{n \neq k} a_n\, x_{k-n} + \eta_K \right| > x_o d \qquad [2.7]$$

from equation 2.4 and 2.6

$$y_k = a_k\, x_o + \xi[KT] + \eta\,[KT]\,,\ [KT] = \quad a_n x_{k-n} \qquad [2.8]$$

where ξ[KT] is the intersymbol interference [ISI].
η[KT] is the additive noise.

In the design of a PAM system, the purpose is to minimize the combined effects of ISIand noise in order to achieve the mimimum probability error.

2.2 INTERSYMBOL INTERFERENCE

Intersymbol interference [ISI] is the overlappin tails of other pulses adding to the particular pulse a_k g[t-KT] which is examined at kth sampling time. ISI arises by the imperfections in the amplitude and phase characteristics of the various signal frequency components.Amplitude distortion effects the various signal frequency components while phase distortion effects the transition times.Both types of distortion cause the received signal to be a function of earlier signal values as well as the transmitted values.[2.1]

2.2.1 Ideal Channel Impulse Response

The ideal brick wall channel is as in Fig. 2.4

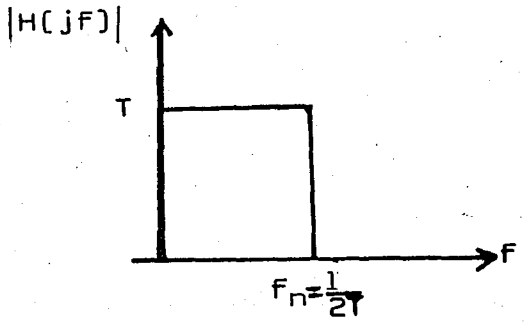


Fig. 2.4 Ideal brick wall channel

$$H(f) = \begin{cases} T & |f| \leqslant \frac{1}{2T} \\ 0 & |f| > \frac{1}{2T} \end{cases} \qquad (2.9)$$

Phase of H(f) = 0

The cutoff frequency $f_n = \frac{1}{2T}$ (Where T is the unit symbol duration) is known as the Nyquist frequency.
The impulse response of the channel (fig 2.6) h (t),

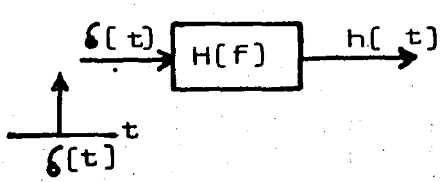


Fig. 2.5

$$h(t) = F^{-1}[H(f)] = \int_{-\infty}^{\infty} H(f)\, e^{j2\pi ft}\, df \qquad (2.10)$$

$$h(t) = \frac{\sin 2\pi f_n t}{2\pi f_n t} = \frac{\sin \pi t/T}{\pi t/T} \qquad (2.11)$$

$$h(nT) = \begin{cases} 1 & n=0 \\ 0 & n=\pm 1, \pm 2, \pm 3 \ldots \end{cases} \qquad (2.12)$$

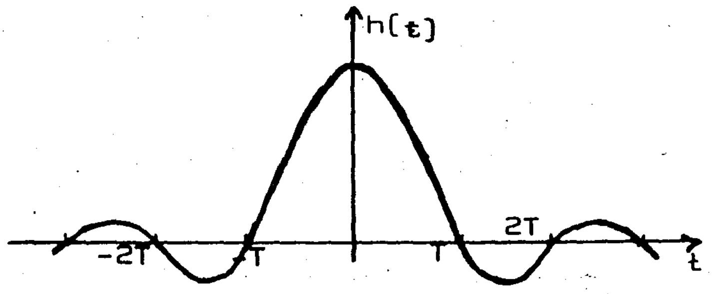


Fig. 2.6 Impulse response of ideal brickwall channel

The channel input $\delta(t)$ (fig 2.5) has an short duration, where as the output has an infinite duration. A desirable property of the described impulse response is that it has a zero crossing for integer multiples of T . Theoretically detection of any of these symbols can be performed without any interference from the preceding or the subsequent impulse patterns as in fig. 2.7 , that can be achieved as ISI-free transmission [11].

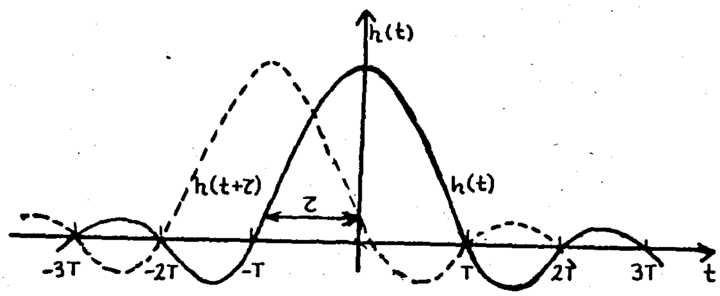


Fig. 2.7 ISI free transmission of bandlimited impulse

2.2.2. ISI and Eye - Diagrams

In pratice it is impossible to realize ISI-free transmission. For an example of the effect of the ISI, the impulse response of the system is shown as in Fig. 2.8 [14].

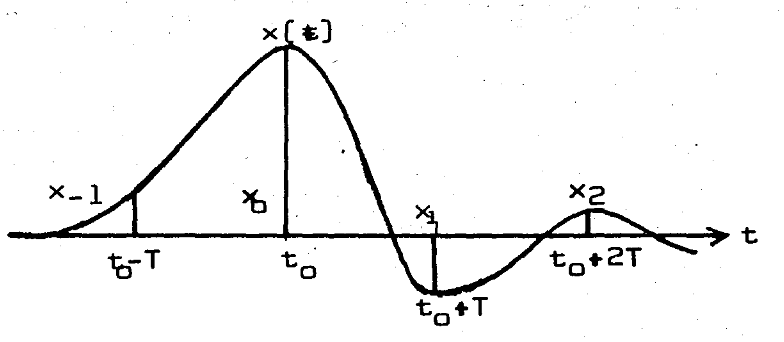


Fig. 2.8 Example of impulse response

The sample x_1 interferes with the succeeding pulse, while the sample x_{-1} interferes with the previous pulse, etc.

From equation 2.6

$$y_k = x_o \left[a_k + \frac{1}{x_o} \sum_{\substack{n=-\infty \\ k\neq n}}^{\infty} a_n \, x_{k-n} + \frac{\eta_k}{x_o} \right]$$

By ignoring the noise samples and letting $x_o = 1$ and assuming that $a_n = \pm 1$ transmitted gives y_o ;

$$y_o = a_o + \sum_n a_n \, x_{-n} \tag{2.13}$$

When $a_o = +1$, the possible values of this quantity as in Fig 2.9 depending on positive or negative a_n .

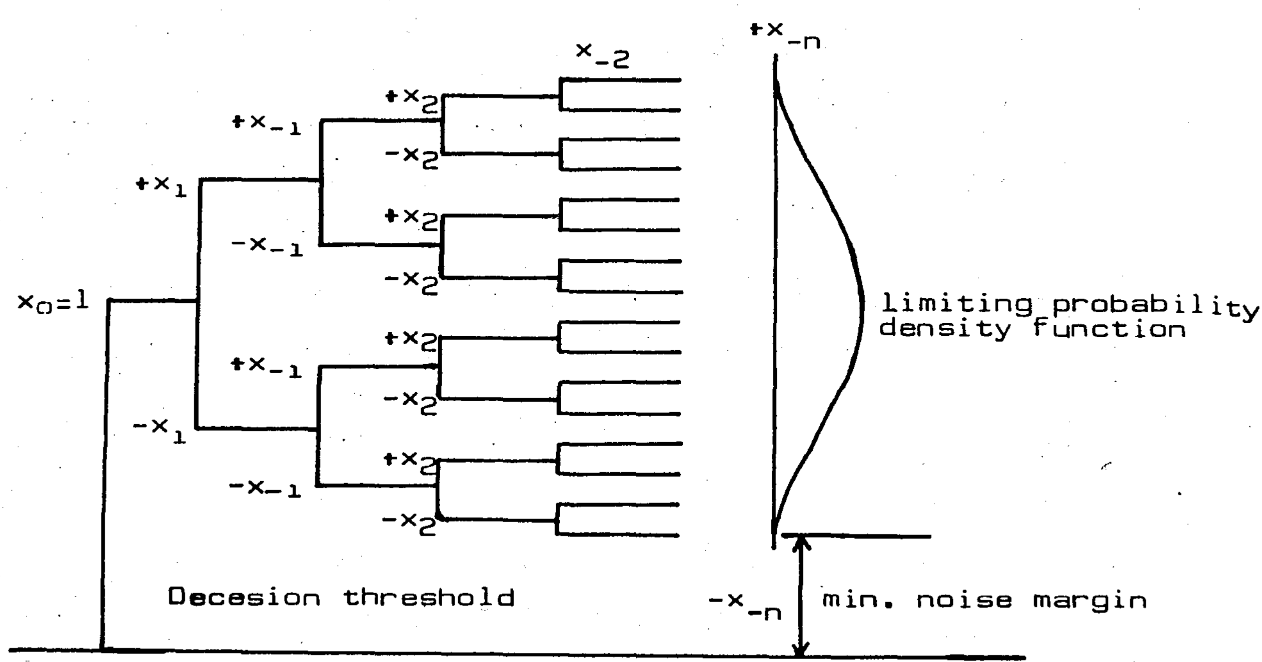


Fig. 2.9 Distiribution of ISI (binary signal)

The limiting distribution of the ISI as an arbitrarily large number of interfering samples are considered could be convolved with the gaussian noise distribution to evaluate the probability error.
The central limit theorem applies to this situation the limit distribution of ISI becomes gaussian, however maximum amount of ISI in this situation is;

$$D = \sum_n |x_n| \tag{2.14}$$

This maximum ISI will be bounded if x(t) decays faster than 1/t, since for any real channel impulse response the asymptotic decay is exponential that the maximum interference in bounded and the ditribution can not be gaussian.

Filling the spaces between branches in fig 2.9, the final distribution becomes smooth enough to possess a probability density. If each successive added interferig sample is half the previous sample, uniform probability density function is obtained. If each successive added interfering sample less than half of the preceding pulse no branch of the tree in fig 2.9 will meet any other branch of the tree.

Let's assume the impulse response is zero for negative time and;

$$x_n = r^n \quad , \quad 0 < r < \frac{1}{2} \tag{2.15}$$

After n branches of the tree the distance between closest branches is $2r^n$. This distance is insufficient to bridge the gap between the branches. In this case there is no limiting probability density function.

Such mathematical abstractions, do not exist in practice, since there must always be noise between branches.

The distribution of ISI can be seen by using oscilloscope in the laboratory. Signal such as $V_o(t)$ as in fig 2.10 is fed to the vertical input of the oscilloscope and the symbol clock is fed to the external trigger of it. The resulting oscilloscope display is known as "eye pattern". The horizantal time base is set approximately equal to the symbol duration.

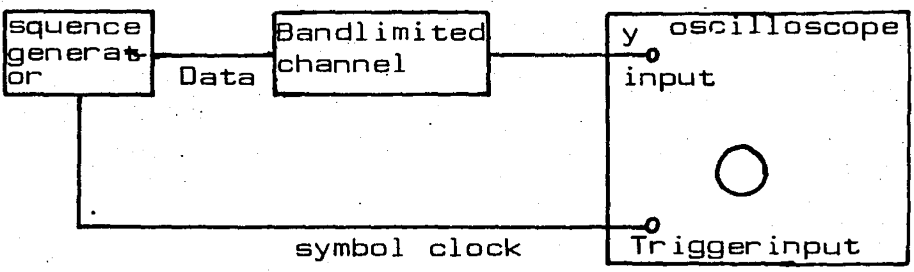


Fig. 2.10 Connection of the oscilloscope for the eye pattern

Let's examine the eye patterns for two binary waveforms. In fig.2.11a undistorted and in fig 2.11b a distorted waveform is shown.

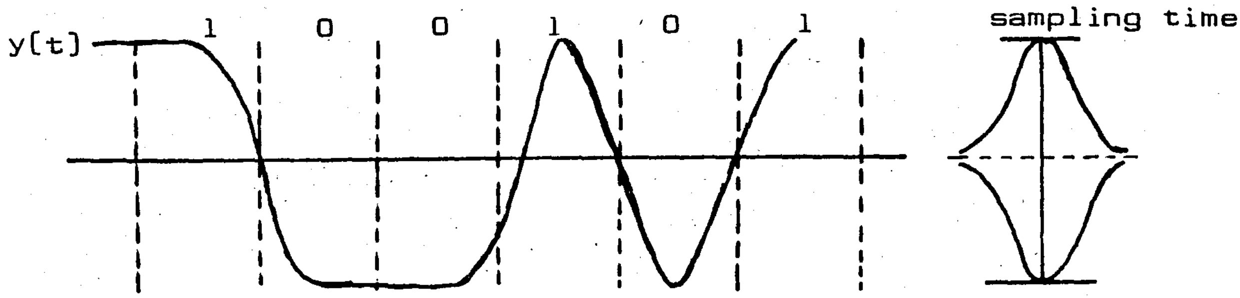


Fig. 2.11 (a)

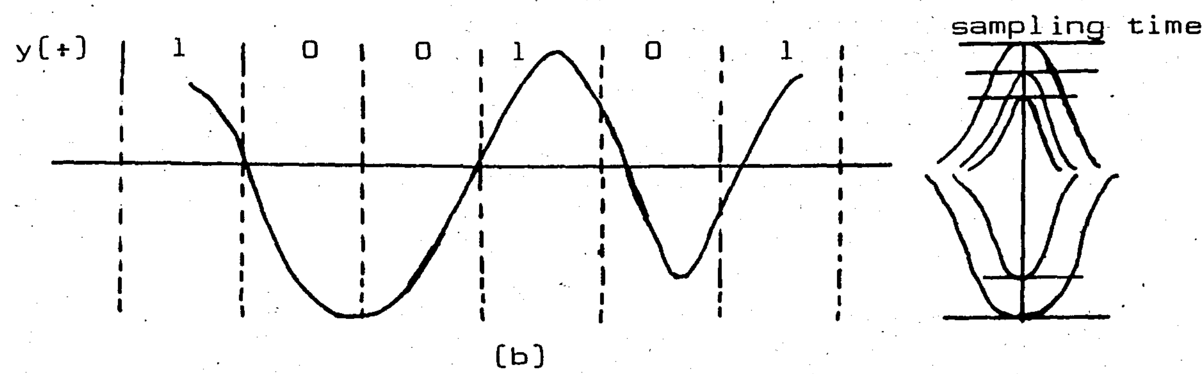


Fig. 2.11 Binary signals and corresponding eye patterns
(a) undistorted (b) distorted

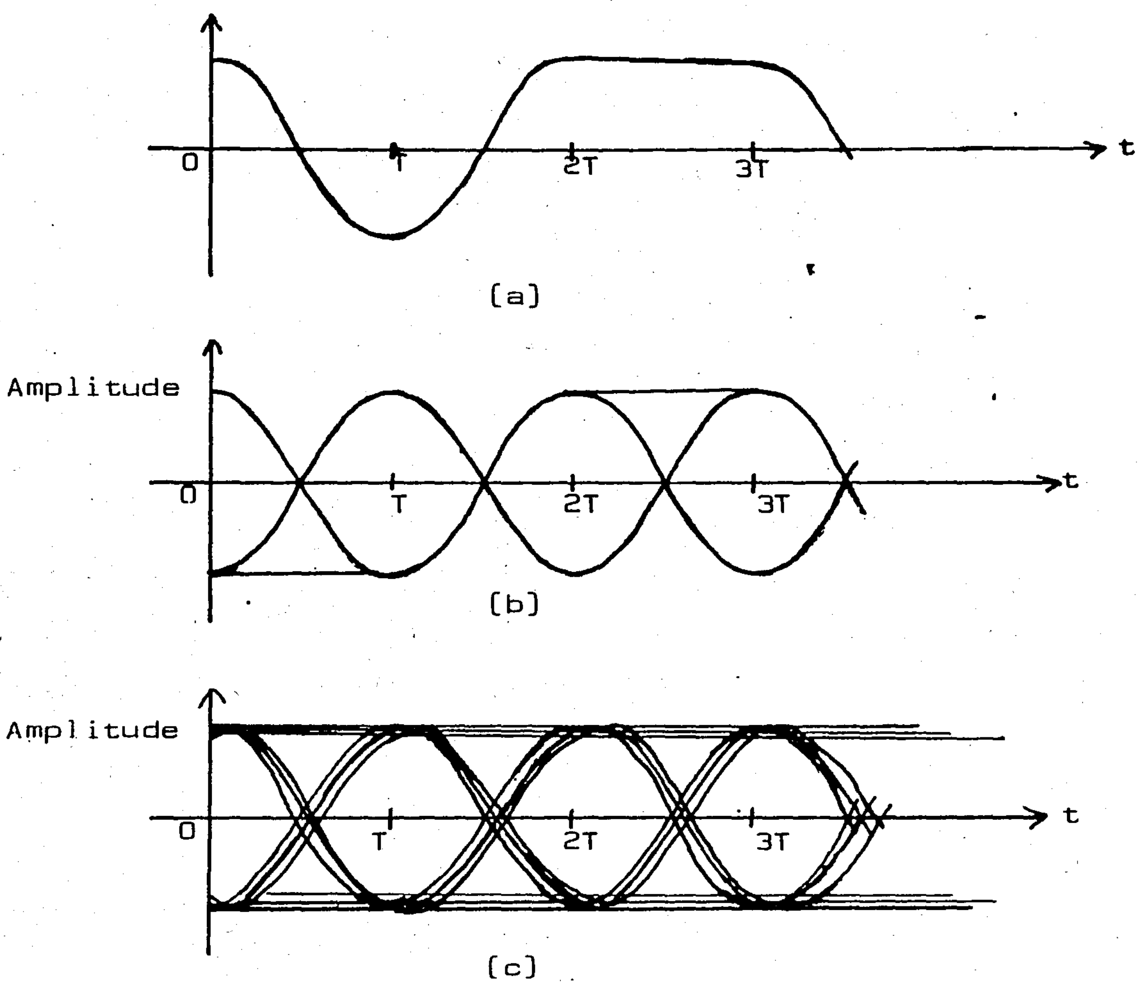


Fig. 2.12 Eye patterns for amplitude modulated systems

(a) Binary system
(b) Overlapping three segments
(c) Overlapping many received waveforms

In undistorded waveform when all T-sec segments of y(t) are superimposed , the "open" eye pattern results as in fig 2.11a Vertical line thru the center shows the superposition of all received sampled values. When the sampling time properly adjusted all sample values are lor las in fig 2.11a an fig 2.11b the waveform is distorted by effects of ISI and noise. The eye pattern for this waveform, y(t) does not pass thru the proper values + 1 and -1 and eye is partially closed. Detection is obviously difficult, the distribution of the received sample values can be observed along the indicated vertical sampling time (14)

Eye patterns provide a great deal of information about the performance characteristics of a data system. It is a well known method of monitoring the quality of the received signal. Fig 2.12 illustrates the formation of an eye pattern as the number of overlapping waveforms and fig 2.13 shows the idealized eye patterns. (21)

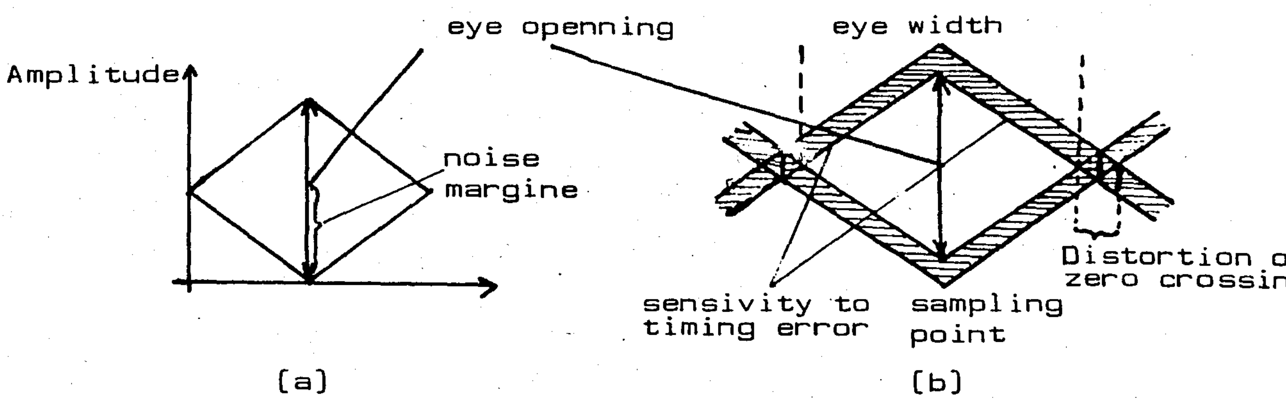


Fig. 2.13 Idealized eye patterns (a) undistorted (b) distorted

From fig 2.12 and 2.13, it is seen that eye pattern is the separation of the received signals into one of several levels which are caused by overlapping, at the sampling instants.
The parameters of the eye pattern are as follows:

1. Eye opening : It is the separation of the nearest received signals which represent different signal levels at the sampling instant. Noise and distortion in the signal cause varying of the signal levels at the sampling instant, the eye opening will decrease as in fig 2.13b.

2. Noise margin : It is the shortest distance from an eye boundary to the midpoint between ideal received signal levels at the sampling instant. It represents the smallest amount of additional noise to a received signal that leads the wrong decesion .

3. Timing jitter : Itis the variation in the zero crossing of the eye signal pattern that is when it does not cross the horizantal zero line at exact integer multiples of the symbol clock. The deviation from the nominal crossing point isknown as peak to peak data transition jitter, J_{pp}.This jitter has an effect on the symbol timing recovery.

4. Sensivity to timing error is revealed by the rate of closure of the eye as the sampling instant is varied.

5. Assymmetries in the eye pattern are caused by the nonlinearities in the transmission channel. (14)

2.3 ISI-FREE TRANSMISSION

ISI-free transmission can be achieved in two ways;

1. With pulse shaping (ex. raised cosine channel)
2. With changing code structure (line coding)

2.3.1 Pulse Shaping for ISI-free Transmission

Let's remember the equation 2.6

$$y_k = a_k x_o + \underbrace{\sum_{\substack{n=-\infty \\ k \neq n}}^{\infty} a_n x_{k-n}}_{ISI} + \eta_k$$

Nyquist Criterion I. (27)

ISI would be nonexistent if the terms for x_{k-n}, $k \neq n$ would be zero. This could be accomplished if the samples of g(t), spaced at T seconds, are all zero except for $x_o = 1$ which is the center of the pulse. The requirement for equally spaced zero crossing can be formulated as:

$$g(kT) = \begin{cases} 1 & k=0 \\ 0 & k \neq 0 \end{cases} \tag{2.16}$$

Hence if g(t) were to be sampled uniformly one would have

$$g(t) \cdot \sum_{k=-\infty}^{\infty} \delta(t-kT) = \delta(t) \tag{2.17}$$

In the frequency domain this expression becomes

$$G[f] * \sum_{k=-\infty}^{\infty} \frac{1}{T} \delta[t-\frac{k}{T}] = 1 \qquad [2.18]$$

that is

$$\frac{1}{T}\sum_{k=-\infty}^{\infty} x[f-\frac{k}{T}] = 1 \qquad [2.19]$$

Many transfer functions that have the odd symmetry condition around $f = \frac{1}{T}$ satisfy the first Nyquist criterion,

for example

$$p[t] = \frac{\sin n\pi/T}{n\pi/T}, \qquad [2.20]$$

Fig 2.14a indicates the waveform satisfying Nyquist criterion I.

Nyquist Criterion II. [27]

The second criterion calls for zero distortion at the pulse edges, at $\pm$ T/2 from center. It can be formulated as;

$$g[\frac{2k-1}{2}T] = \begin{cases} 1/2 & k=0,1 \\ 0 & k \neq 0,1 \end{cases} \qquad [2.21]$$

Expressed as a sampling process one has:

$$g[t] \cdot \sum_{k=-\infty}^{\infty} \delta[t-\frac{2k-1}{2}T] = \frac{1}{2}\delta[t-\frac{T}{2}] + \frac{1}{2}\delta[t+\frac{T}{2}] \qquad [2.22]$$

This means that the half amplitude pulse width exactly equals the pulse to pulse spacing and there are additional zero crossings at $t = \pm\frac{3}{2}T, \pm\frac{5}{2}T, \ldots$ ie pulses will have zero crossings halfway between the pulse centers whenever there is a change in polarity as in Fig 2.14b.

The equation 2.22 in the frequency domain,

$$G[f] * \sum_{k=-\infty}^{\infty} \frac{1}{T}\delta[f-\frac{k}{T}] e^{-jwT/2} = \cos w\frac{T}{2} \qquad [2.23]$$

If both criteria Nyquist I and Nyquist II are to be met then,

$$g[t] \cdot \sum_{k=-\infty}^{\infty} \delta[t-k\frac{T}{2}] = \frac{1}{2}\delta[t-\frac{T}{2}] + \delta[t] + \frac{1}{2}\delta[t+\frac{T}{2}] \qquad [2.24]$$

and in frequency domain;

$$\sum_{k=-\infty}^{\infty} G[f-\frac{2k}{T}] = 1 + \cos\frac{wT}{2} \qquad [2.25]$$

Thus

$$G(f) = 1 + \cos \frac{wT}{2} \qquad (2.26)$$

Then the raised cosine filter has a special importance for the purpose of data communications. In practice a class of raised cosine filters are used,

$$G(f)=\begin{cases} T & |f| \leqslant \frac{1}{2T} - \beta \\ T \cos^2 \frac{\pi}{4\beta}\left(|f| - \frac{1}{2T} + \beta\right) & \frac{1}{2T}-\beta < |f| < \frac{1}{2T} + \beta \qquad (2.27) \\ 0 & |f| \geqslant \frac{1}{2T} + \beta \qquad (2.28) \end{cases}$$

Where T is the bit interval and $0 < \beta < \frac{1}{2T}$ pulse shape in time domain becomes

$$g(t) = \frac{\cos 2\pi\beta t}{1-(4\beta t)^2} \, \frac{\sin \pi t/T}{\pi t/T} \qquad (2.29)$$

Raised cosine filter will be explained later.

Fig 1.14b shows a waveform satisfying Nyquist criterion II

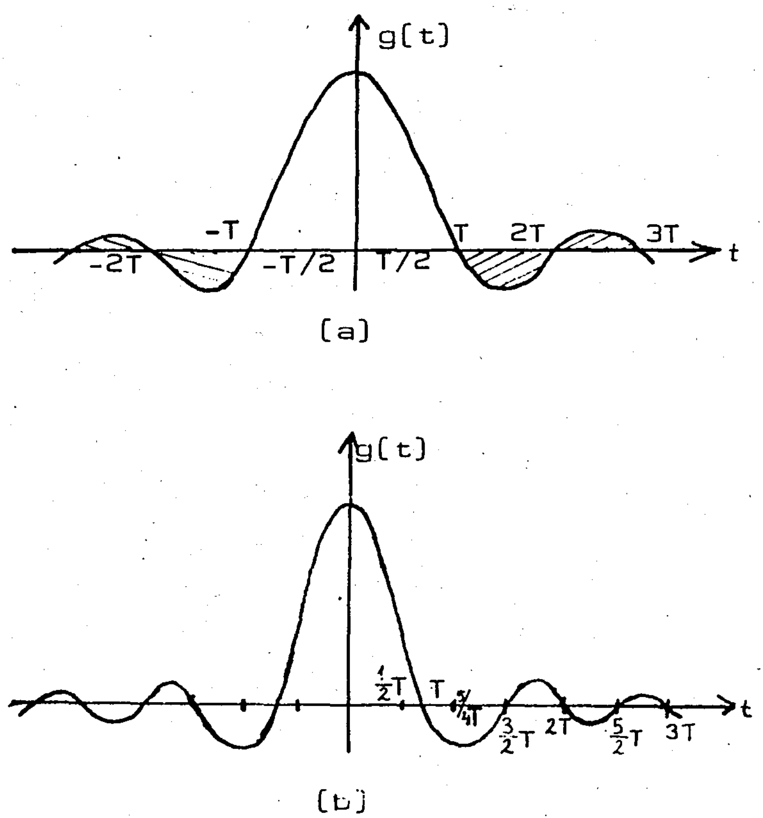


Fig. 2.14 A wave form satisfying Nyquist criterion I (a) and Nyquist criterion II (b)

First criterion serves to open the eye wider in vertical direction the second attempts to open it horizantally.

Nyquist Criterion III. (27)

The third criterion states that for distortionless transmission the area under the system response g(t) during an interval T should be proportional to the signal value. The constraints on the system response g(t) are,

Constraint 1: (28)

$$\int_{\frac{2k-1}{2}T}^{\frac{2k+1}{2}T} g(t)\,dt = \begin{cases} 1 & k=0 \\ 0 & k\neq 0 \end{cases} \qquad (2.29)$$

This implies that the areas under the tail pulses are equal, so that they cancel each other.(in fig.2.14a dashed areas)

Expressing 2.29 in terms of G(f)

$$\frac{T}{2\pi}\int x(w)\; e^{jwkT}\,dw = \begin{cases} 1 & k=0 \\ 0 & k\neq 0 \end{cases} \qquad (2.30)$$

where

$$x(w) = G(w)\,\frac{\sin wT/2}{wT/2} \qquad (2.31)$$

Constraint 2: The sampled system response is

$$Tx(t)\sum_{k=-\infty}^{\infty}\delta(t-kT) = \delta(t) \qquad (2.32)$$

where x(t) is the impulse response of x(w) from the both constraints 2.29 and 2.32 the constraint on x(w) are

$$\sum_{k=-\infty}^{\infty} x\left(w-\frac{2\pi k}{T}\right) = 1 \qquad (2.33)$$

Thus, for G(w) satisfy Nyquist third criterion, x(w) should satisfy Nyquist first criterion, from 2.31 we see that G(w) is given by any x(w) that satisfies 2.33 multiplied by $\frac{W\ T/2}{\mathrm{Sin}\ W\ T/2}$ function. (Such as raised cosine characteristics).

Fig 2.15 shows the spectrum satisfying Nyquist criterion I.and III.

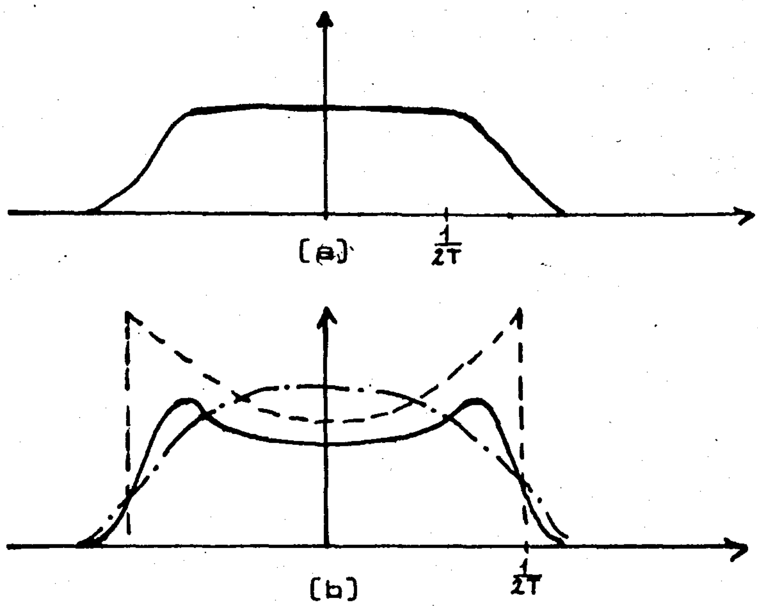


Fig. 2.15 Spectrum satisfying Nyquist I(a) and Nyquist I and II(b)

Raised Cosine Channel : (11)

It is also called Nyquist channel and we know the raised cosine characteristics from eq. 2.27

$$G(f)=\begin{cases} T & |f| \leqslant \frac{1}{2T} - \beta \\ T\cos^2 \frac{\pi}{4\beta}\left(|f| - \frac{1}{2T} + \beta\right) & \frac{1}{2T}-\beta < |f| < \frac{1}{2T} + \beta \\ 0 & |f| \geqslant \frac{1}{2T} + \beta \end{cases}$$

$0<\beta< \frac{1}{2T}$

and from eq.2.28 the impulse response is

$$g(t) = \frac{\cos 2\pi\beta t}{1-(4\beta t)^2} \cdot \frac{\sin \pi t/T}{\pi t/T}$$

The raised cosine filter has a bandwidth up to 100 % larger than the Nyquist bandwidth. Nyquist II conditions are exactly satisfied (zero crossings at $t=\pm\frac{3}{2}T$, $\pm\frac{5}{2}T$) for $\beta=\frac{1}{2T}$, that is 100 % raised cosine filter. For $\beta<\frac{1}{2T}$ the filter bandwidth becomes smaller by foregoing the extra zero crossings at the odd multiples of T/2 . (Fig. 2.16)

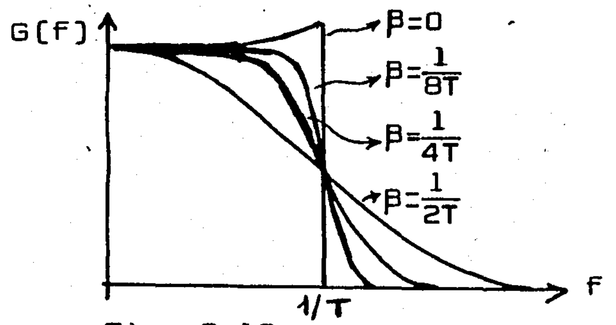


Fig. 2.16 Spectra of raised cosine channel

It is known that if G(f) and its first N-1 derivatives are continous and the N th derivative is discontinous, $|g(t)|$ decays asymptotically as $\frac{1}{|t|^{N+1}}$. for $\beta=0$ it decays slowly and timing errors will occur. For larger value of β, it decays rapidly and does not cause large amount of ISI (for $\beta=\frac{1}{2T}$) (fig 2.17)

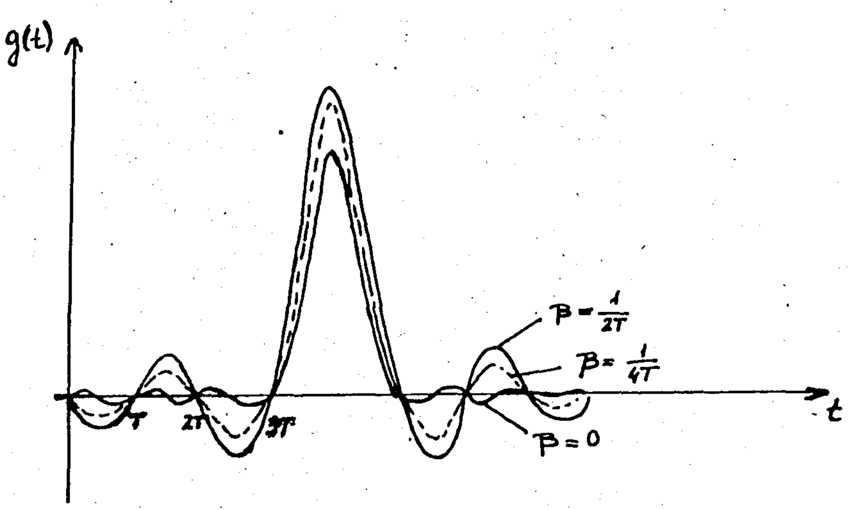
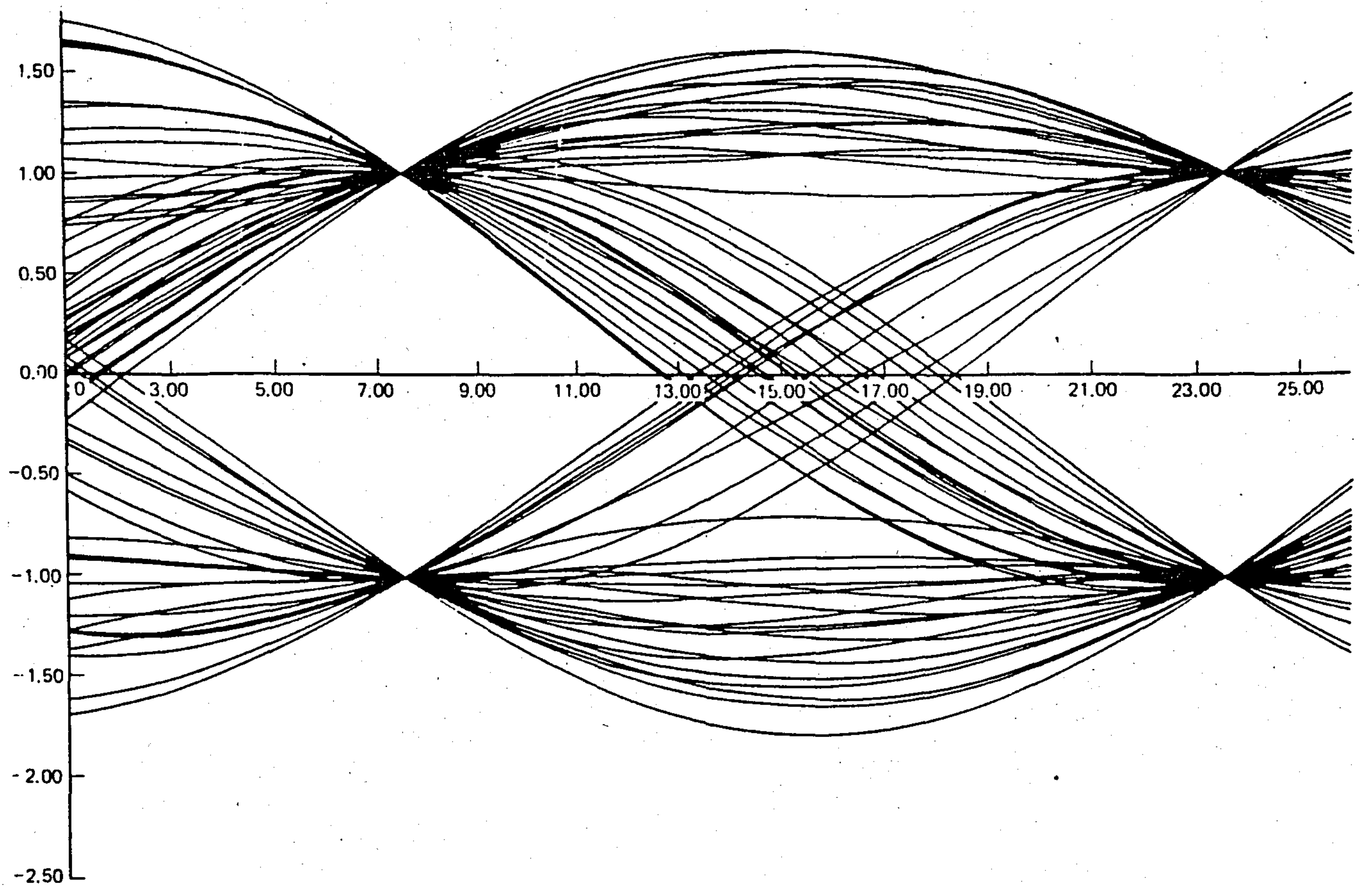


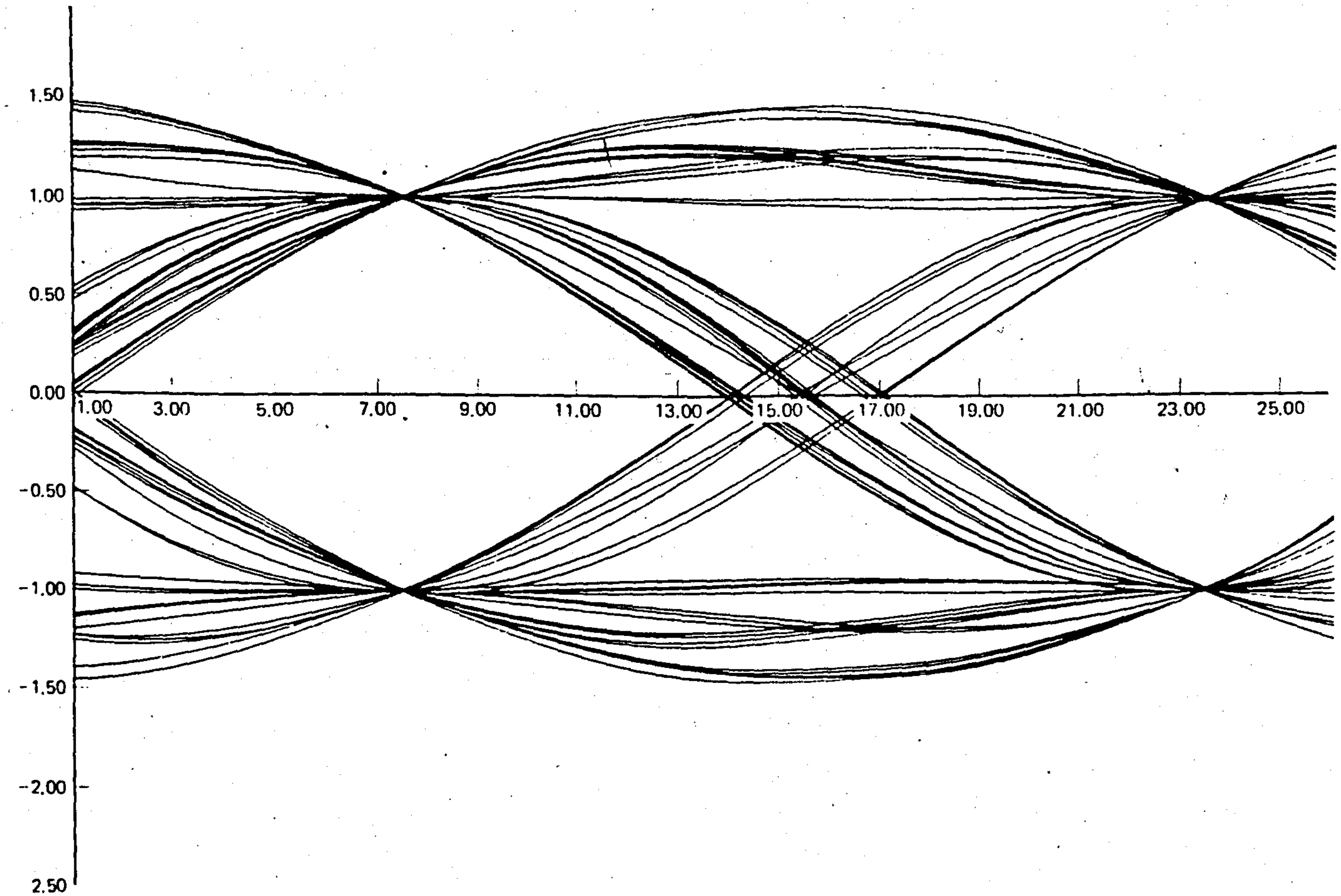
Fig. 2.17 Impulse response of raised cosine channel

Fig 2.18 illustrates the eye diagrams for different volues of β.



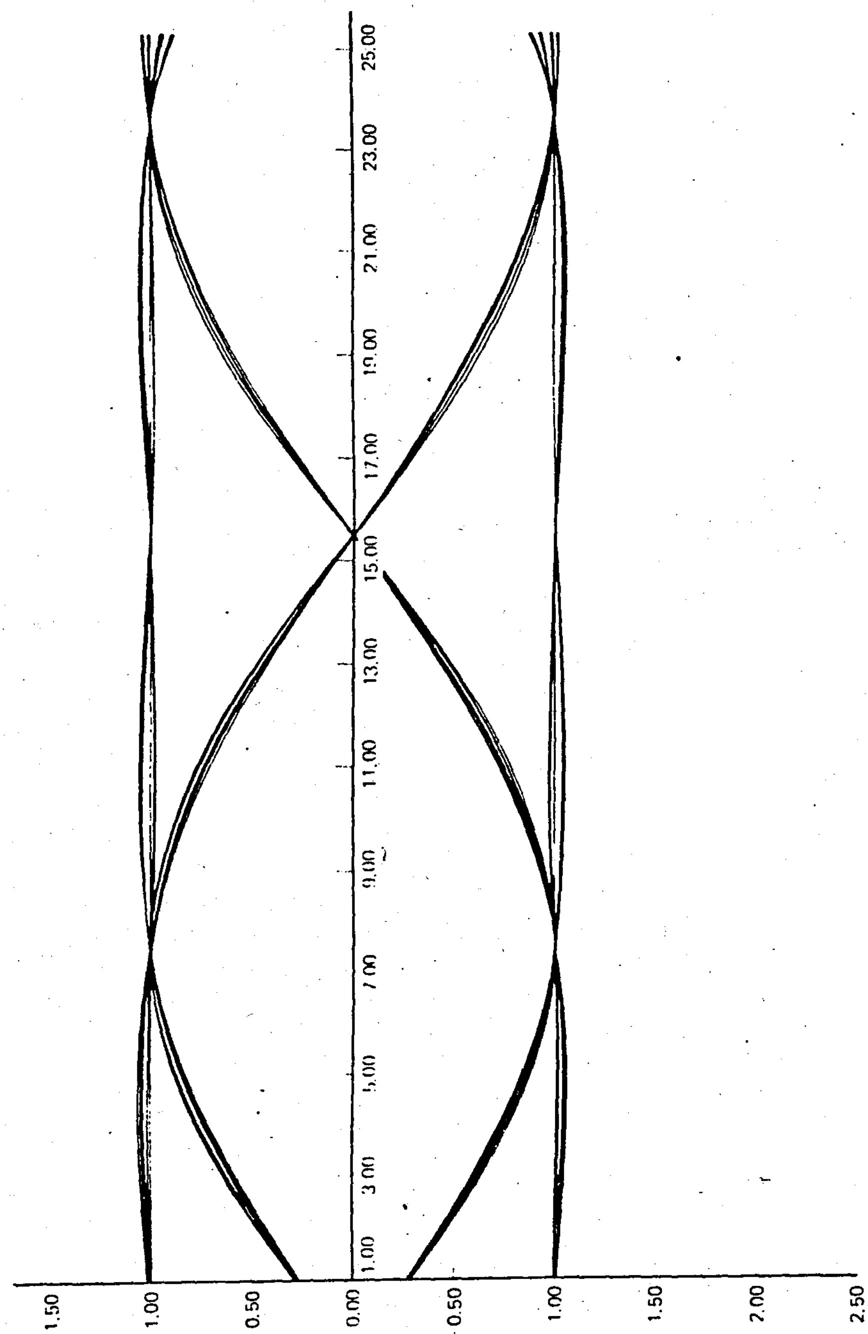
(a)

Fig. 2.18 Eye diagram for different values of β
(a) $\beta=\frac{1}{6T}$ (b) $\beta=\frac{1}{4T}$ (c) $\beta=\frac{1}{2T}$
[11 pp. 105,106,107]


1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
2.50
1.00
3.00
5.00
7.00
9.00
11.00
13.00
15.00
17.00
19.00
21.00
23.00
25.00

(b)

Fig. 2.18



1.50
1.00
0.50
0.00
0.50
1.00
1.50
2.00
2.50
1.00
3.00
5.00
7.00
9.00
11.00
13.00
15.00
17.00
19.00
21.00
23.00
25.00

(c)

Fig. 2.18

2.3.2 Changing Code Structure

Changing code structure process is the line coding. We know that ISI appears as a result of nonlinearities of the amplitude and phase characteristics of the channels. The effect of ISI can be compansated by using coded sequences.This is done in three ways in line coding.

1. By introducing controlled amount of ISI to produce desirable power spectrum (Partial Response codes)
2. By increasing signalling rate (two phase frequency modulation codes alphabetic codes
3. By using some parameters such as running digital sum (RDS) which represents the net accumulation of tails at each symbol instant.

These are will be investigated in detail in chapter 3 and 4

4 ERROR MONITORING CAPABILITY AND TIMING RECOVERY OF LINE CODES

In line codes,timing signal can be derived from the encoded gnal at the receiver end. Timing recovery will be achieved by en-ring sufficient number of transitions to occur in the line signal pecially as long zeros or ones occur in input data. Transitions e caused by correlation between digits in partial response codes. ese transitions may possibly lead to violations and violations are e one of the methods for error detecting and monitoring. For example polar or Alternate-Mark Inversion (AMI) code; data state "zero" is coded and the other state "one"s are encoded as pulses alternate polarity such as + - + - + 0 - + ...(Table 2.1)

For nonalphabetic codes i.e B6ZS and HDB3 code uses the violations to remove the strings of zeros thus improve timing recovery at the receiver end. HDB3 code uses of the sequences 000-, 000+, -00-, +00+ instead of four string zeros with the choice depending on the polarity of both the previous pulse and last pattern violation (Table 2.1). B6ZS code uses the sequence 0VB0BV (where B represents a pulse opposite polarity an V represents a pattern violation) instead of six consecutive zeros. The squence 0VB0VB can be 0+-0+- or 0-+0-+ depending on the polarity of the last pulse.

TABLE 2.1 Violations of AMI, B6ZS, HDB3 code

Binary data	10110000001000001
AMI	+0-+000000-0000+
B6ZS	+0-+0+-0+--0000+ (subsititution sequence)
HDB3	+0-+000+00-000-+ (subsititution sequence)

When alphabetic codes are used, timing information maybe extracted from the encoded signal since the maximum number of successive symbols of the same level is a fixed number (this is the DSV). Error monitoring for alphabetic codes will be possible by tracking the allowable RDS range (where $DSV = RDS_{max} - RDS_{min}$).

These results will be well understood when chapter 3 and Chapter 4 are read.

The Conclusion : Error detecting capability comes from the code structures that are able to timing recovery from the encoded signal at the receiver. So observable errors are detected and monitored during system operation. Additional error detecting bits are not required.

2.5 CLASSIFICATION OF LINE CODES

Line codes can be divided into two classes.

1. Linear Line codes: Encoded seouences are derived from input data by using Linear mathematical relations.

These are

i) Binary signal

ii) Partial response (correlative level) codes

iii) Two phase (frequency) modulation codes

2. Nonlinear line codes: Encoded squence can not be derived from input data by using Linear mathematical relations.

These are

i) Alphabetic codes

ii) Nonalphabetic codes

CHAPTER III

LINEAR LINE CODES

A line code is said to be linear if it can be derived from input data (binary message) by linear operations.Linear codes include:

1. Binary coding

2. Partial response (correlative-level) signalling or coding (PRS)

3. Two phase (frequency) modulation coding

The most well known linear line code is the partial responce (correlative-level) code that is also called linear pseudoternary code which is based on three level pulse amplitude modulation.The linear pseudoternary codes are derived from input data:(6)

$$a_n = \sum_{k=0}^{K} b_{n-k} h_k \qquad (3.1)$$

Where a_n are the encoded bits, b_n are the input (binary) bits and (h_k) are the coefficients that define the code generator.Then the coded time waveform.

$$S(t) = \sum_{k=0}^{K} \sum_{n=-\infty}^{\infty} b_{n-k} h_k \; g(t-nT) \qquad (3.2)$$

Where g(t) is the shape of the pulse.By using,

$m = n-k$

$$S(t) = \sum_{m=-\infty}^{\infty} b_m \sum_{k=0}^{K} h_k \; g(t-(m+k)T) \qquad (3.3)$$

Thus linear pseudoternary code is a particular case of binary code in which the signal element $S_o(t)$ can be replaced by,

$$S_o(t) = g(t) * S_1(t) \qquad (3.4)$$

Where $S_1(t)$ is the sequence of impulses:

$$S_1(t) = \sum_{k=0}^{K} h_k \, \delta(t-nk) \qquad (3.5)$$

It is seen that linear psuedo coding is equivalent to a filtering operation and that the frequency response of the equivalent filter is given by the Fourier transform of $S_1(t)$,

$$S_1(w) = \sum_{k=0}^{K} h_k \, e^{-jwkT} \qquad (3.6)$$

.It is necessary that there be at least two $h_k \neq 0$ and that they should be equal or opposite polarity, if there are only two h_ks.

Binary and two phase (frequency) modulation codes are derived from input binary data by direct mapping.

In some linear line codes precoding gives a certain decoding advantage for error monitoring. These techniques and concept redundancy of the code will appear several times in the sequel so that it is convenient to discuss them at the begining of the chapter.

Precoding: Precoding is transforming the basic data sequence into another sequence with the same number of levels. Precoding does not alter the statistics of the data sequence so that after precoding the encoded symbols are still equally likely and statistically independent. Let's examine the precoding operation by using the example of simple binary precoder in Fig. 3.1

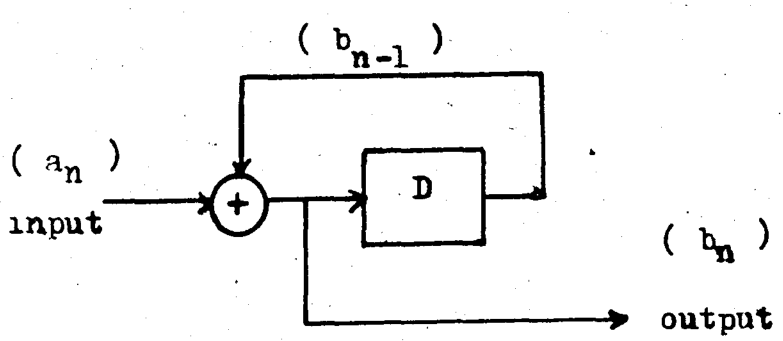


Fig 3.1 Simple binary precoder

D : Unit delay
O : Modulo 2 adder (EX-OR) for binary system
Modulo m adder (EX-OR) m-ary system

The precoded sequence $[b_n]$ is obtained by modulo 2 addition of input data sequence and the sequence $[b_n]$ which is delayed by a single period .

$$b_n = a_n + b_{n-1}$$

The above techniques is in fact the differential encoding method, given a binary data sequence consists of transtions corresponding to the ones and no transtions for zero. Non-Return-Zero-Level (NRZ-L) or Non-Return-Zero-Inverse (NRZ-I) waveforms are known as binary waveforms because 1 represents mark and 0 represents space. Their precoded version is indentical to the Non-Return-to Zero-Mark (NRZ-M) while 1 represents change in amplitude, 0 represents no change in amplitude. Binary data and precoded data are illustrated in fig. 3.2. [10]

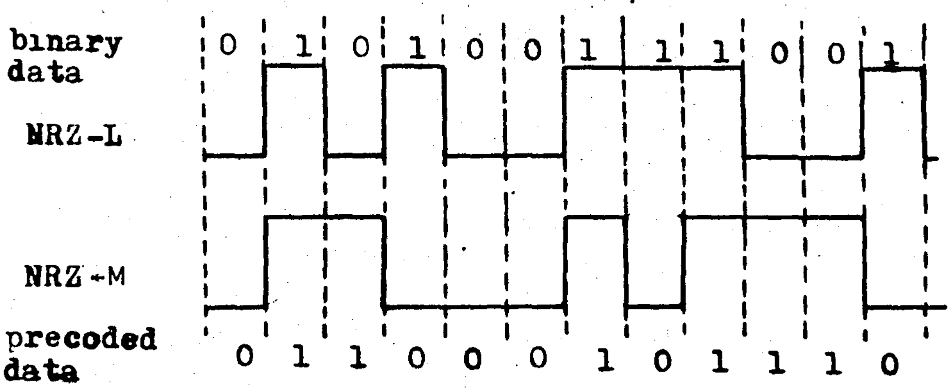


Fig. 3.2 NRZ-L, NRZ-M representation of binary data

<u>Redundancy in line codes:</u> Consider the baseband digital system as in fig 3.3.

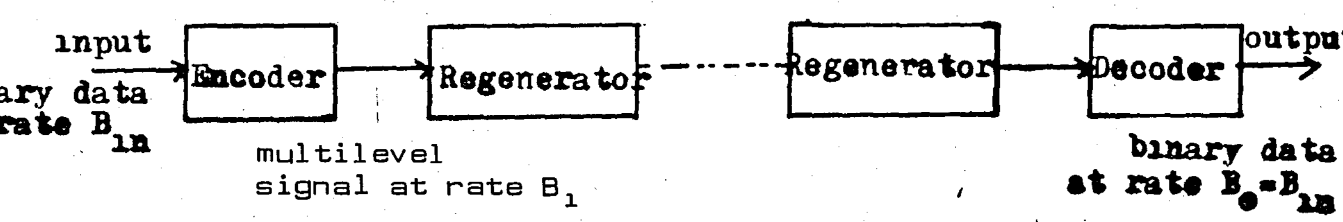


Fig. 3.3 Block diagram of baseband digital transmission

input data is in binary form and the signal to be transmitted over the line has L_1 levels which is greater than 2. Bin is the rate at which binary data is fed to the encoder, B_1 is the rate at which the multilevel (L Levels) information is transmitted. The redundancy in information rate is given by, [10]

$$\text{Redundancy in information rate} = B_1 \log_2 L - B_{in} \log_2 2 \qquad (3.8)$$

and the redundacy percantage with respect to the binary input is:

$$r= \frac{B_1 \log_2 L-B_{in}}{B_{in}} .100 \qquad [3.9]$$

3.1 BINARY CODING

The simplest type of the line codes. Input binary signal is encoded in binary form again. There is no redundancy. But the following two requirements must be satisfied. [10]

1. The binary signal must ensure the necessary transitions such that timing information can be recovered from received signal.

2. The DC or low frequency [LF] component must be filtered or spectrally shaped before the transmission.

3.2 PARTIAL RESPONSE (CORRELATIVE-LEVEL) CODING

In the Nyquist theorems, ISI can be eliminated at the sampling instants. [See Chapter 2,2.3.1] Thus Nyquist designed the impulse response g[t] that eliminates ISI at the sampling instants. He showed that the minimum bandwidth without ISI is R/2 HZ. In order to transmit R symbols/second. The equivalent channel [raised cosine channel] and corresponding pulse shape as in fig 3.4

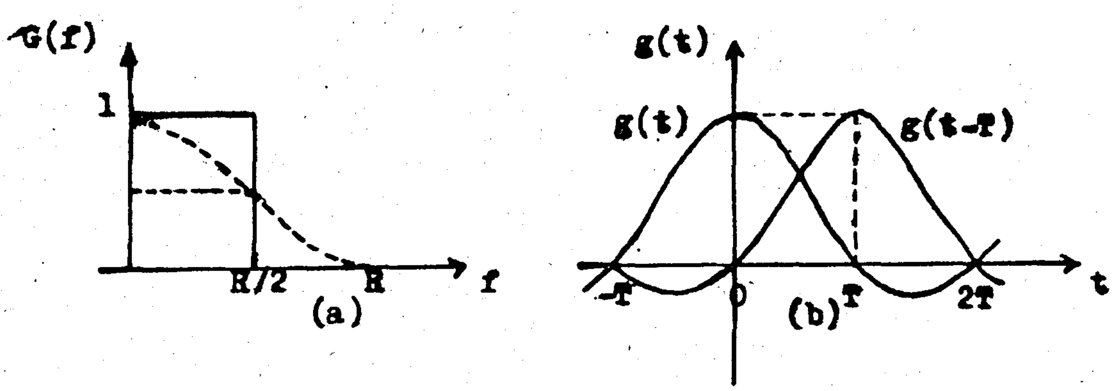


Fig. 3.4 Raised cosine channel [a] and corresponding impulse response [b]

It is seen that g[t] has a long tail which passes thru zero at other sampling points. When the transmitted symbols are independent and the noise uncorrelated at the sampling instant, then each symbol can be recovered without overlapping with the previous symbol of the waveform. Such sysytems are called zero-memory systems.Nyquist found that if G(f) has smoother transitions at the spectral band edge,there will be no ISI.For this purpose,the pratical filters require more bandwidth that the Nyquist bandwidth of $R/2$ HZ.One common filter shape is the raised cosine filter with 100 percent excess bandwidth with twice the Nyquist bandwidth (as in fig 3.4 a dotted line).Is given by,(22)

$$G(f) = \begin{cases} \cos^2 \left(\frac{\pi f}{2R}\right) & |f| \leq R \\ 0 & |f| > R \end{cases} \qquad (3.10)$$

And the theoretical maximum value of the symbol rate is 2 symbols/S/HZ but 1 symbol/S/HZ ~~is achieved~~ with Nyquist II.

In summary the key assumption in the Nyquist criteria is that the transmitted amplitudes be selected independently by designing ideal raised cosine filter (excess bandwidth range from 15 to 100 percent).But it is impossible to have a precise relationship between cut off frequency of the ideal filter. Thus , the Nyquist rate with Nyquist type zero memory system can not be achieved in practice.During early 1960's Dr. A.Lender discovered the correlative transmission method.He achieved the symbol rate 2W Symbols/second with bandwidth W H_z by correlating the transmitted amplitudes and changing the detection procedure.Then Kretzmer achived the teoretical maximum value of the symbol rate 2 symbols/sec./HZ by using perturbation-tolerant filters.

In correlative coded systems a controlled amount of ISI is introduced in order to simplify the filter desing and to enable transmission at Nyquist rate and even higher rates.The transmission channel does not respond fully within one symbol interval at the sampling instants,but only responds partially.That is each sampled impulse response extends over more than one interval thus causing the amplitude levels to be correlated in the transmitted signal.The properties (correlative level) coding (PRS) are listed as the following: (17)

1. For a given number of input data levels,the number of transmitted signal levels are increased,this leads to higher required SNR for a given error rate.

2. It provides the shaping of the power spectral density into a convenient format for transmission,for instance by placing nulls in the frequency response.Spectral shaping can make the system less sensitive to timing errors.This allows practical PRS systems to transmit at Nyquist rate and even higher rates.In addition PRS spectrum might be chosen to complement nonideal charactriestics in order to reduce the residual undesired ISI.

3. PRS format has the advantage for error monitoring with the violations in the code. Owing to correlation between digits, correlative pulse trains have distinctive patterns. [22]

A block diagram of PRS transmitter is illustrated in fig 3.5 [9]

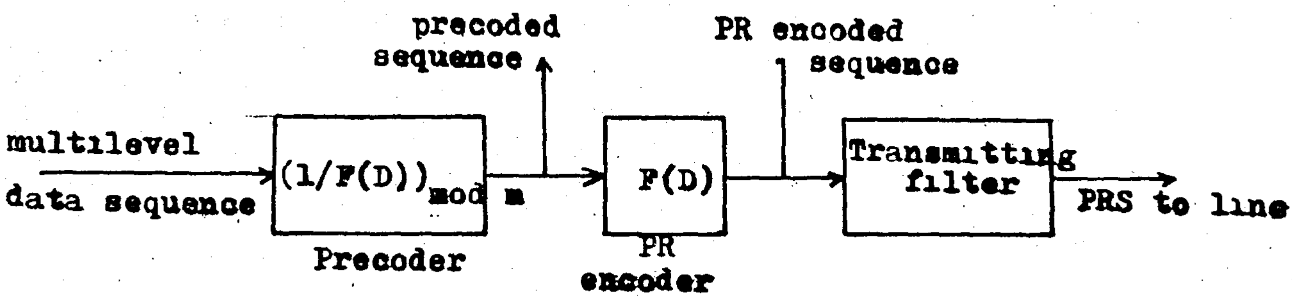


Fig. 3.5 PRS transmitter

Where F [D] Is the PR encoder transfer function [as in equation 3.6]

$$F[D] = \sum_{k=0}^{N-1} h_k \, D^k \qquad [3.11]$$

Where the coefficients h_k are integers which may be positive and negative, D is the unit delay function. By choosing an appropriate function F[D] the spectral shaping is achieved.

Since the levels in the transmitted PRS are correlated a decoding error at the receiver can yield additional errors by propagation. To avoid this error propagation, the input data sequence is converted into another sequence by the precoder whose transfer function

$$P[D] = \left[\frac{1}{F[D]}\right]_{\text{mod } m} \qquad [3.12]$$

Which is the inverse over mod m of PRS encoder transfer function F[D].

Some commonly used PRS line codes are duobinary, modified duobinary, bipolar or Alternate-Mark-Inversion [AMI] Code, biternary, polibinary, twinned binary, multileved bipolar, dicode, higher order bipolar.

3.2.1 DUOBINARY CODE

Binary data is transformed into a three level signal, each of the three resulting levels is associated with the existing binary digit and precoding bits. The three signalling levels of a duobinary signal are numbered as [-2,0,2].

Consider a binary input train consisting of "one"'s and "zero"s represented by $\delta[t]$ and $-\delta[t]$ as in fig 3.6 at A. The cascade of a simple one-tap transversal filter $H_1[f]$ with Nyquist rectangular filter $G[f]$. [12]

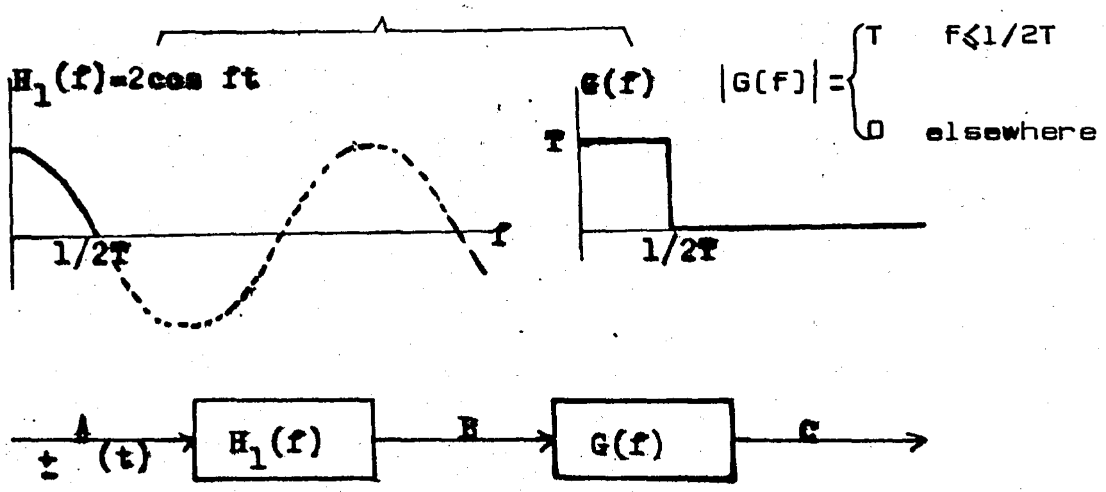


Fig 3.6 Block diagram of duobinary system

The output at B is

$$h_1[t] = \delta[t] + \delta[t-T] \qquad [3.13]$$

with the appropriate sign. $H_1[f]$ is the fourier transform of the $h_1[t]$;

$$H_1[f] = F[h_1[t]] = \int_{-\infty}^{\infty} [\delta[t] + \delta[t-T]]\, e^{-j2\pi f t} dt \qquad [3.14]$$

$$H_1[f] = 1 + e^{-j2\pi fT} \quad \text{or} \qquad [3.15]$$

$$|H_1[f]| = 2\cos\pi fT \qquad [3.16]$$

The overall transfer function in fig. 3.6 is H[f];

$$H[f] = H_1[f]\ G[f]$$

$$H[f] = T[1 + e^{-j2\pi fT}] \qquad f < \frac{1}{2T} \qquad [3.17]$$

$$|H[f]| = |H_1[f]\ G[f]|$$

$$|H[f]| = \begin{cases} 2T\cos\pi fT & f \leqslant \frac{1}{2T} \\ 0 & f > \frac{1}{2T} \end{cases} \qquad [3.18]$$

The shape of H(f) as in fig 3.7

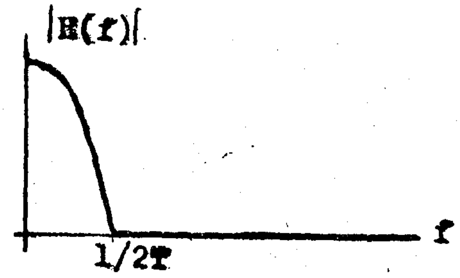


Fig 3.7 Overall transfer function of duobinary

The impulse response of the H(f) is h(t);

$$h(t) = F^{-1}\left[H(f)\right] = \int_0^{\frac{1}{2T}} T(1 + e^{-j2\pi fT})\, e^{-j2\pi ft}\, df$$

$$h(t) = \frac{\sin \pi t/T}{\pi t/T} + \frac{\sin \pi(t-T)/T}{\pi(t-T)/T} \qquad (3.19)$$

Hence for every input at A, the output at C is h(t) with an appropriate polarity (fig 3.6) since the bit interval is T seconds, there will be overlap or ISI between digits. Then at the sampling instant there will be three distinct amplitude levels (-2,0,+2) as shown in fig 3.8. It is assumed that the input at A consists of two successive "one"s represented by delta function $\delta(t)$ followed by $\delta(t-T)$. The output at C by assuming no delay in the system is h(t) followed by h(t-T) as in fig 3.8 clearly both h(t) and h(t-T) consists of two sinc pulses numbered 1 and 2 for h(t) and 3 and 4 for h(t-T). Their sum at sampling point T is +2. For the binary input 00 the waveform will be similar to fig 3.8 but with negative polarity and their sum a+T will be -2. For binary input 01 and 10 their sum at T will be zero. Thus the duobinary waveform has one of the following three amplitudes at sampling instants (-2,0,+2). However the bandwidth is still the Nyquist bandwidth.

Now Let's consider the case of a duobinary pulse used in conjuction with binary PAM. The received signal at the sampling instant in the absence of noise is, Fig 3.9 (23)

$$B_n = A_n + A_{n-1} \qquad n=1,2,\ldots \qquad (3.20)$$

Where (A_n) is the set of amplitude values of the transmitted pulses with each A_n taking the values of 1, -1 B_n has the three possible values. If A_{n-1} is the decoded symbol from the (n-1)st sampling interval, its effect on B_n, the received signal in the nth sampling interval can be eliminated by subtraction thus allowing A_n to be decoded. This process can be repeated sequentially.

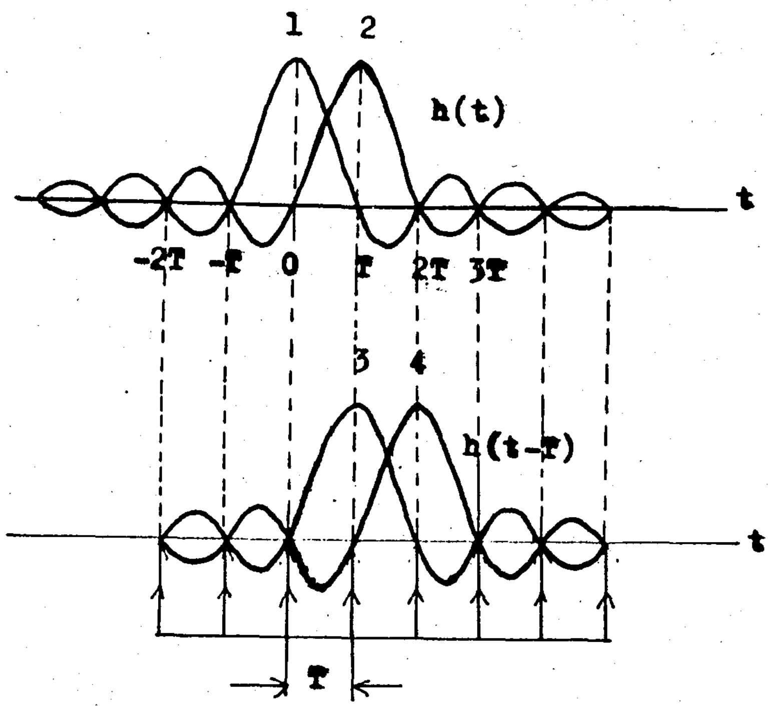


Fig. 3.8 Impulse patterns in duobinary system

Error propagation can be avoided by precoding the data at the transmitter instead of eliminating the ISI by subtraction at the receiver. The precoding operation is as follows: Let D_n be the data sequence which consists of zeros and ones. From data sequence (D_n) precoded sequence (P_n) is generated:

$$P_n = D_n \ominus P_{n-1} \qquad n = 1,2,\ldots\ldots \tag{3.21}$$

Where the symbol $\ominus$ is modulo-2 subtraction, for multilevel input it will be mod.m subtraction. The transmitted pulse $x(t)$ in n th sampling interval;

$$-x(t) \quad \text{if} \quad P_n = 0$$

$$x(t) \quad \text{if} \quad P_n = 1$$

The transmitted signal amplitude (A_n) that modulates duobinary pulse $x(t)$ is obtained from the sequence(p_n) which converts binary $(0,1)$ signals to bipolar (± 1) signals.

$$A_n = 2\,P_n - 1 \tag{3.22}$$

Then the received signal (B_n) from equation 3.20;

$$B_n = 2(P_n + P_{n-1} - 1) \tag{3.23}$$

since

$$D_n = P_n \oplus P_{n-1} \quad \text{mod } 2 \tag{3.24}$$

Then

$$D_n = \frac{B_n}{2} + 1 \quad \text{mod } 2 \tag{3.25}$$

The operation described above is given in table 3.1
In the presence of the noise, the received signal having the form of $B_n + V_n$ is compared with two thresholds -1 and +1. The data squence D_n is obtained according to the rule:

$$D_n = \begin{cases} 1 & -1 < B_n + V_n < 1 \\ 0 & |B_n + V_n| \geq 1 \end{cases}$$

TABLE 3.1 Binary signals in duobinary coding scheme

Data Sequence	D_n		1	1	1	0	1	0	0	1	0	0	0
Precoded Sequence	P_n	0	1	0	1	1	0	0	0	1	1	1	1
Transmitted Sequence	A_n	-1	1	-1	1	1	-1	-1	-1	1	1	1	1
Received Sequence	B_n		0	0	0	2	0	-2	-2	0	2	2	2
Decoded Sequence	D_n		1	1	1	0	1	0	0	1	0	0	0

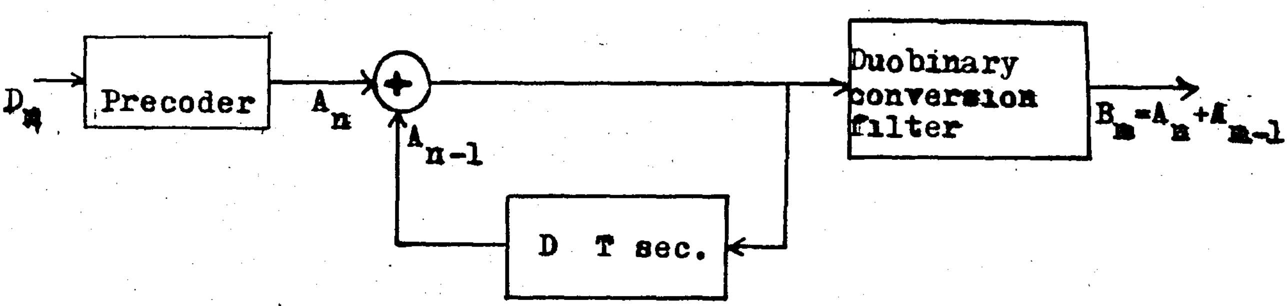


Fig. 3.9 Duobinary encoder

Duobinary encoder transfer function F(D) can be written as from equations 3.11 and 3.20;

$$F(D) = 1 + D \rightarrow H(f) = T(1 + e^{-j2\pi fT}) \qquad (3.26)$$

And precoder transfer function P(D) from 3.12 and 3.21:

$$P(D) = \left(\frac{1}{1 + D} \right)_{mod2} \qquad (3.27)$$

F(D) expression yields nulls at odd multiples of Nyquist frequency in the spectral density characteristics of the encoded squence. Let's find the spectral density of duobinary system.

$$G_h(f) = H(f)\,H^*(f) = |H(f)|^2 \tag{3.28}$$

From equation 3.26

$$|H(f)|^2 = T(1+e^{-j2\pi fT})\;T(1+e^{-j2\pi fT})^*$$

$$|H(f)|^2 = 2T^2\cos^2\pi fT \tag{3.29}$$

Nulls at f=1/2T , 3/2T , ... as in fig. 3.10

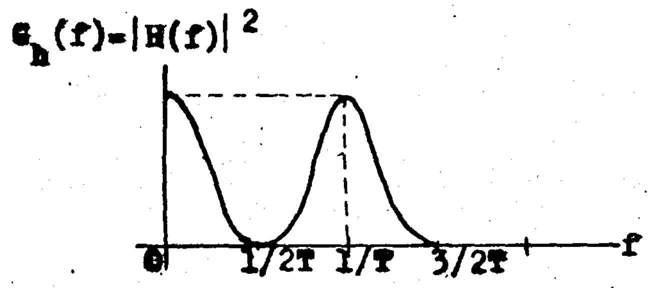


Fig 3.10 Normalized spectral density of duobinary

The application of duabinary coding is more or less restricted by high values of the signal spectrum at low frequencies although it is used in frequency modulated systems.

3.2.2 Modified Duobinary Code (Partial Response Class 4 Code)

In this case;

$$H_1(f) = 1-e^{-j4\pi fT} \tag{3.30}$$

$$h_1(t) = \delta(t)-\delta(t-2T) \tag{3.31}$$

$$|H_1(f)| = 2\sin 2\pi ft \tag{3.32}$$

$$G(f) = T \qquad \text{for} \qquad f \leqslant \frac{1}{2T} \tag{3.33}$$

$$|H(f)| = |H_1(f)\,G(f)| = \begin{cases} 2T\sin 2\pi ft & \text{for } f \leqslant \frac{1}{2T} \\ 0 & \text{elsewhere} \end{cases} \tag{3.34}$$

And the impulse response of modified duobinary system is

$$h(t) = \frac{\sin \pi t/T}{\pi t/T} - \frac{\sin \pi(t-2T)/T}{\pi(t-2T)/T} \tag{3.35}$$

And (Fig 3.11)

$$B_n = A_n - A_{n-2} \qquad n= 2,3,\ldots\ldots \tag{3.36}$$

$$P_n = D_n \oplus P_{n-2} \quad \text{mod } 2 \quad n=2,3,\ldots\ldots \tag{3.37}$$

$$A_n = 2P_{n-1} \tag{3.38}$$

$$B_n = 2 \, [P_{n-} \; P_{n-2}] \tag{3.39}$$

Since

$$D_n = P_n \ominus P_{n-2} \tag{3.40}$$

$$D_n = \frac{Bn}{2} \quad \text{mod } 2 \tag{3.41}$$

The operation described above is given in table 3.2 when additive noise is present, the received signal at the sampling instant can be expressed as $B_n + V_n$. For this case the decesion variable $B_n + V_n$ is compared with the thresholds -1 and +1 and data squence is obtained according to the rule.

$$D_n = \begin{cases} 0 & \text{if} \quad -1 < B_n + V_n < 1 \\ 1 & \text{if} \quad |B_n + V_n| \geqslant 1 \end{cases}$$

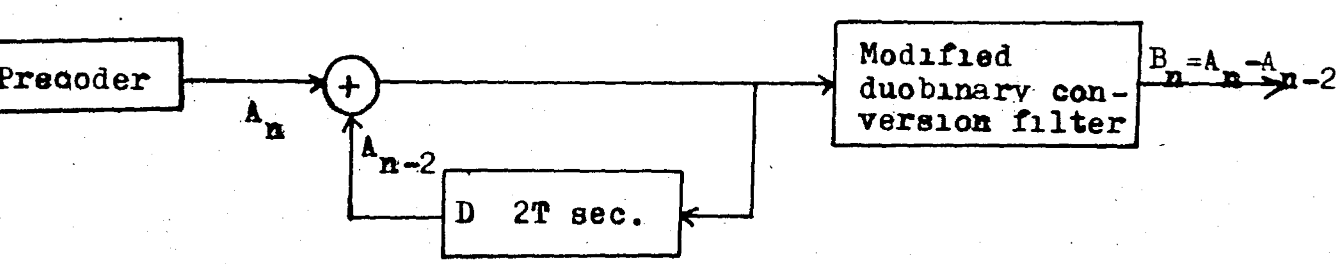


Fig. 3.11 Modified duobinary encoder

TABLE 3.2 Binary signals in modified duabinary scheme

Data Sequence	D_n			1	1	1	0	1	0	0	1	0	0	0
Precoded Sequence	P_n	0	0	1	1	0	1	1	1	1	0	1	0	1
Transmitted Sequence	A_n	-1	-1	1	1	-1	1	1	1	1	-1	1	-1	1
Received Sequence	B_n			2	2	-2	0	2	0	0	-2	0	0	0
Decoded Sequence	D_n			1	1	+1	0	1	0	0	+1	0	0	0

For modified duobinary case the transfer function of the modified duobinary encoder is,

$$F(D) = 1-D^2 = (1+D)(1-D) \qquad H(f)=T(1-e^{-j4\pi ft}) \qquad (3.42)$$

and the precoder is,

$$P(D)=\left(\frac{1}{1-D^2}\right)_{mod2} \qquad (3.43)$$

$F(D)$ is zero at $D=\mp 1$ $(D=e^{-j2\pi fT})$ i.e at $f=0,1/2T,1/T,3/2T,\ldots$ $F(D)$ implies that nulls exist at zero frequency and integral multiples of Nyquist frequency. The spectral density is

$$G_n(f) = 2T^2\sin^2 2\pi fT \qquad (3.44)$$

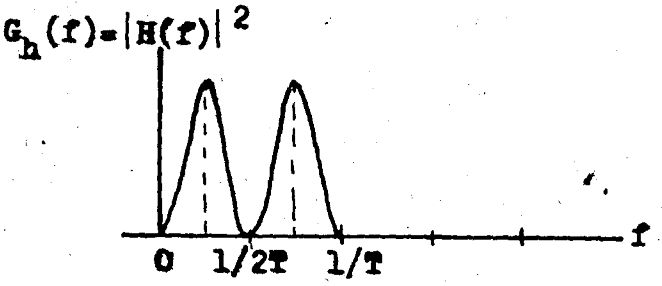


Fig 3.12 spectral density of modified duobinary system

3.2.3 Polibinary Code

This code is the multilevel transmission with duobinary or modified duobinary code. The transmission of one of M equally spaced levels ($M=2^k$, k number of digits) at the Nyquist rate gives 2M-1 equally spaced levels at each sampling instant. The (2M -1) level signal is mapped by the receiver back into M- level signal. The input data sequence D_n consists of elements from the alphabet of the M levels 0,1,2,.... M-1. (23)

At this case for multilevel duobinary code

$$P_n= D_n \ominus P_{n-1} \quad \text{mod } M \qquad (3.45)$$

$$A_n= 2P_n - (M-1) \qquad (3.46)$$

$$B_n= 2\,(P_n + P_{n-1} - (M-1)\,) \qquad (3.47)$$

$$D_n= \frac{B_n}{2} + (M-1) \quad \text{mod } M \qquad (3.48)$$

In the presence of noise the received signal noise is quantized to the nearest of the acceptable signal levels and the rule given above is used on the quantized values to obtain the data sequence (D_n).

For modified duobinary case,

$$B_n = 2\,(P_{n-}\ P_{n-2}) \qquad (3.49)$$

$$D_n = P_n - P_{n-2} \qquad \text{(Mod M)} \qquad (3.50)$$

$$D_n = \frac{B_n}{2} \qquad \text{(Mod M)} \qquad (3.51)$$

İf the source output is coded using Gray code, that an error in an adjacent level of the received signal results in only a single binary digit error. Table 3.3 and 3.4 shows two examples of four level signalling operation.

TABLE 3.3 Four level signal transmission with duobinary

Data Sequence	D_n		0	0	1	3	1	2	0	3	3
Precoded Sequence	P_n	0	0	0	1	2	3	3	1	2	1
Transmitted Sequence	A_n	-3	-3	-3	-1	1	3	3	-1	1	-1
Received Sequence	B_n		-6	-6	-4	0	4	6	2	0	0
Decoded Sequence	D_n		0	0	1	3	1	2	0	3	3

TABLE 3.4 Fourlevel signal transmission with modified duobinary

Data Sequence	D_n			0	0	1	3	1	2	0	3	3
Precoded Sequence	P_n	0	0	0	1	3	2	1	2	0	1	2
Transmitted Sequence	A_n	-3	-3	-3	-1	3	1	-1	1	-3	-1	1
Received Sequence	B_n			0	2	6	2	-4	0	-2	-2	4
Decoded Sequence	D_n			0	0	1	3	1	2	0	3	3

3.2.4 Bipolar or Alternate Mark İnvesion (AMI) Code

This is a widely used method of line coding which can be described as follows: (10)

1. Binary input zero is uncoded and is represented by a space or absence of a pulse

2. Binary input one is represented by a mark which alternate in polarity.

In this code the transfer fuction of the encoder is,

$$F(D) = 1-D \qquad H(f) = T(1-e^{-j2\pi ft}) \qquad (3.52)$$

F(D) is zero at f=0,1/T,...
and the precoder transfer function is,

$$P(D) = \left(\frac{1}{1-D} \right)_{mod2} \qquad (3.53)$$

The block diagram of the system as in fig. 3.13 and the operation is as in table 3.5

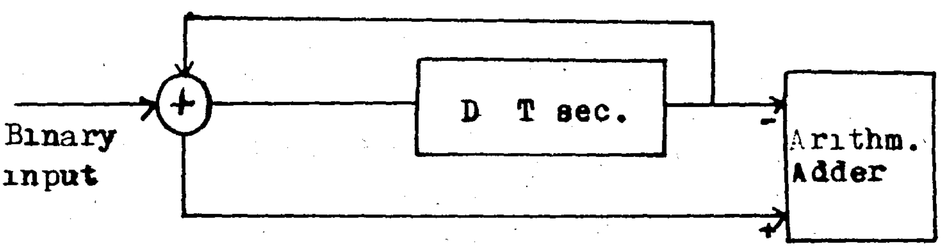


Fig. 3.13 Block diagram of AMI encoder

TABLE 3.5 Binary signal with bipolar or AMI code

Binary Input	1	0	0	1	1	1	0	1	1	0	0
Bipolar Output	1	0	0	-1	1	-1	0	1	-1	0	0

The spectral density of this code is (fig 3.14)

$$G_h(f) = |H(f)|^2 = 4T^2\sin^2\pi fT \qquad (3.54)$$

The nulls exist at zero frequency and integral multiple of Nyquist frequency

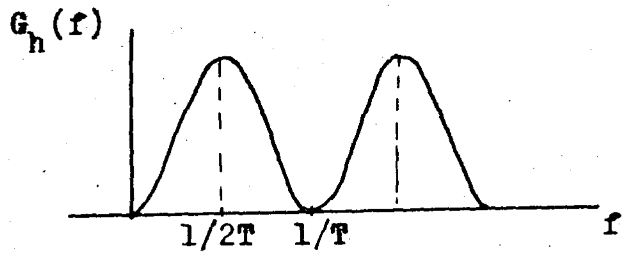


Fig. 3.14 Spectral density of AMI code

It possesses several characteristics that are desirable in baseband transmission. It has no DC component and contains only small amount of LF components. Timing information can be easily recovered from the received signal by simple full-wave rectification followed by a narrow bandpass filter or a phase locked loop. Errors in the received signal can also be detected and monitored without reference to the transmitted information using the pulse polarity alternation (violation) rules. Non-precoded version of this code is twinned binary and dicode. (9)

3.2.5 Twinned Binary Code

The block diagram of the system as in fig 3.15 and operation as in table 3.6 (9.8)

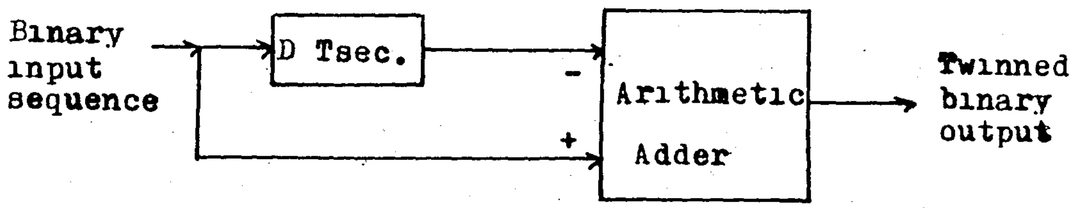


Fig. 3.15 Block diagram of twinned binary encoder

TABLE 3.6 Binary signals with twinned binary signals

Binary input	1	0	0	1	1	1	0	1	1	0
Twinned binary output	1	-1	0	1	0	0	-1	1	0	-1

It is seen that from table 2.5 positive (negative) pulse is generated at every positive (negative) going transition in binary data. Twinned binary can be therefore thought of as a digitally differentiated version of binary input. Thus to decode it, on simply requires a digital integrator.

The transfer function of the twinned binary encoder is

$$F(D) = 1-D \qquad H(f) = T(1-e^{-j2\pi fT}) \qquad (3.55)$$

Then the spectral density is the same with bipolar see fig 3.14. This code has no DC component and clock recovery can be easily derived. But it suffers error propagation.

3.2.6 Dicode

Pulses indicate transitions in digital formation, the pulses alternate in polarity for sucsessive transitions. Signal has no DC component and the spectrum is identical to bipolar encoding. The transfer function of the encoder as in equation 3.55. Table 3.7 illustrates the operation of this system. (9.8)

TABLE 3.7 Binary signals with dicode signals

Binary input data	0	0	1	1	0	1	1	0	1
dicode	1	0	-1	0	1	-1	1	0	-1

3.2.7 Biternary Code

Biternary transmission technique is a method where by two separate NRZ pulse trains, one of which is delayed by one half a bit period, are added to form the bitermary signal; the results being a doubling of the bit rate with an increase in the required transmission bandwidth. The waveforms are shown in fig 3.16. The resulting wave is a three level signal. (9.8)

Detection entails sampling the biternary signal at half the bit -period intervals (every T/2 seconds). Individual samples of the biternary signal related to the two original signal, $F_1(t)$, $F_1(t-T/2)$ follow;

F_1	F_t	F_1 (t-T/2)
0	-1	0
0	0	1
1	0	0
1	+1	1

The obvious resulting from detection of a zero in the biternary signal can be resolved by using information that is avaible from a previous sample.

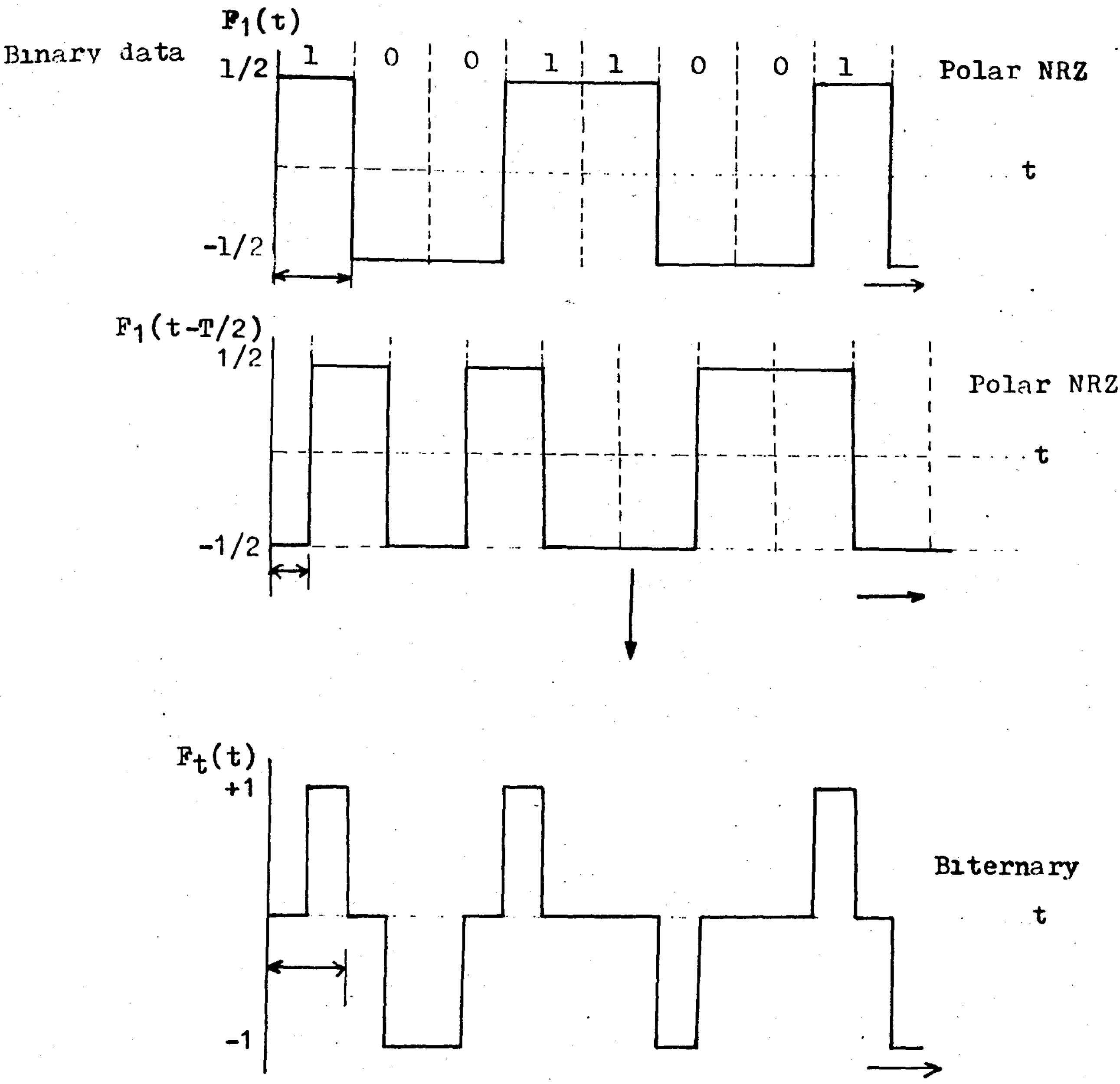


Fig. 3.16 Biternary waveform

3.2.8 Interleaved or High-order Bipolar Code

Block diagram of the system as in fig 3.17. As it is seen seperating the even and odd bits of binary data and encoding these squences seperately into bipolar format and the output is summation of two bipolar codes. The main advantage of this interleaving process is the creation of a spectral zero at the Nyquist frequency, same as modified duobinary. But the distortion of the interleaved signal due to coupling networks is twice as much as that of the non-interleaved sequence.

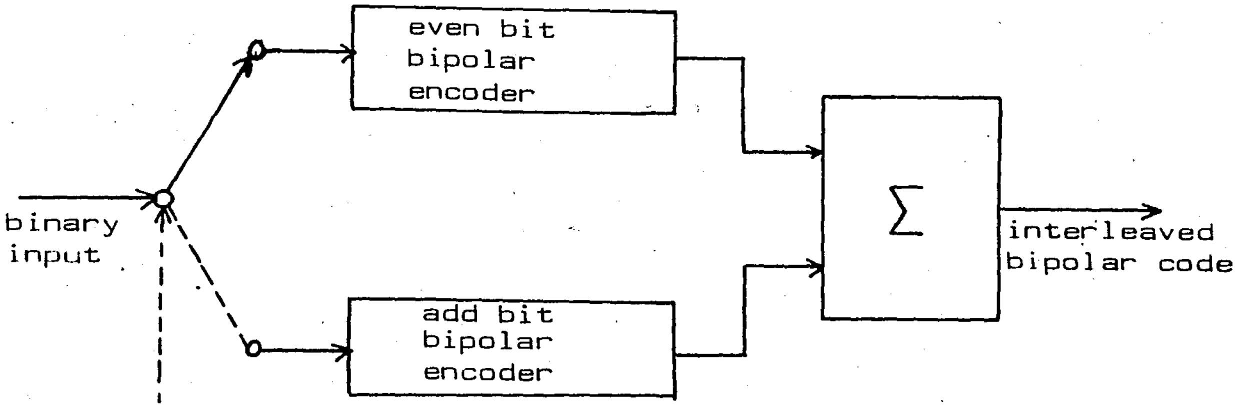


Fig. 3.17 Block diagram of interleaved bipolar encoder

TABLE 3.8 Binary signals with interleaved bipolar signals

Binary Input	1	0	0	1	1	1	0	1	1	0
Interleaved bipolar output	1	0	0	1	-1	-1	0	1	1	0

3.2.9 Multilevel Bipolar Code

If an m-level data sequence is fed into the encoder in fig 3.13 with modulo m adder, m-ary bipolar signals will result. Again the multilevel pulses alternate in polarity. Generalized version of these codes, is the multilevel feedback balanced codes. [10]

3.2.10 Multilevel Feedback Balanced Code

In multilevel bipolar codes, the pulses alternate in polarity and the number of levels is unchanged.
In the feedback balanced codes, the number of levels L of the output stream need not be the the same as that of the m-ary input data. In addition the polarity of the output pulses is controlled by a negative feedback of the code running digital sum. This sum is in effect kept within certain bounds and the code is said to be balanced. An example of a quaternary five-level encoder is illustrated in fig 3.18. [10]

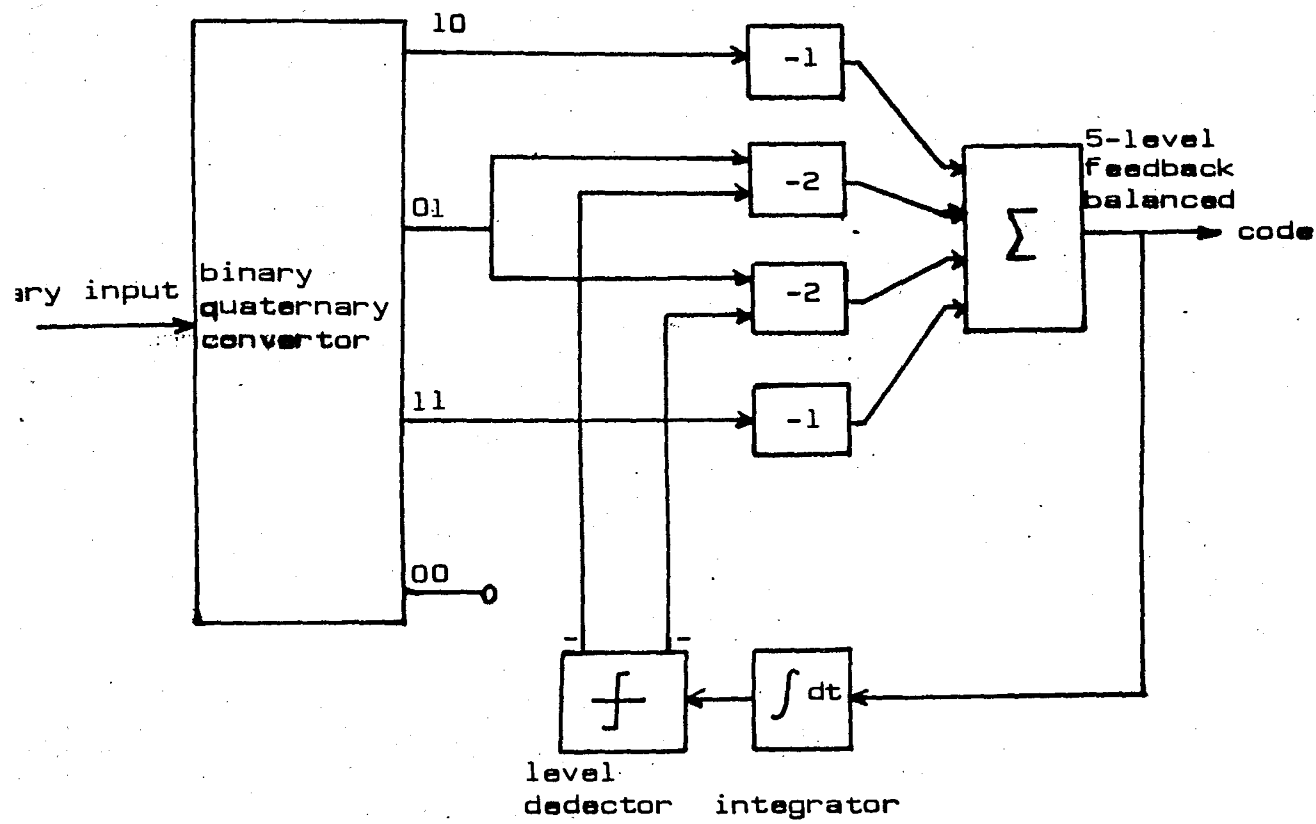


Fig. 3.18 Quaternary five-level encoder

3.2.11 Generalized PRS System

[17]We know that from equation 3.11 PRS system polynomial is

$$F(D) = \sum_{k=0}^{N-1} h_k D^k$$

Where N is the smallest number of contigous samples that span all the nonzero samples. For a given input symbol X_K, the output squence y_K is given by

$$y(D) = x(D)\ F(D) \tag{3.56}$$

Where

$$x(D) = \sum_{k=0}^{\infty} x_k D^k \quad \text{and} \quad y(D) = \sum_{k=0}^{\infty} y_k D^k \tag{3.57}$$

The $\{X_K\}$ Will be assumed to be independent m-ary symbols taking on the equally likely values -(m-1),-(m-3),(m-3), (m-1).

Fig 3.19 shows the method of generating PRS system function H(W). The system consists of a tapped delay line with coefficients $\{h_k\}$ in cascade with the frequency response G(W) Where

$$F(w) = F(D)\Big|_{D=\exp(-jwT)} \tag{3.58}$$

or

$$F(w) = \sum_{k=0}^{N-1} h_k e^{-jwkT} \qquad (3.59)$$

Most of the desirable properties of PRS systems can be stated in terms of the impulse response h(t), many are the best illuminated by frequency domain. h(t), has the sample values (h_k) if and only if G(W) satisfies Nyquist's first criterion (see chapter 2.3.1) that is

$$\sum_{k=-\infty}^{\infty} G\left(w-\frac{2\pi k}{T}\right) = T \qquad (3.60)$$

$$g(kT) = \begin{cases} 0 & k \neq 0 \\ 1 & k=0 \end{cases} \qquad (3.61)$$

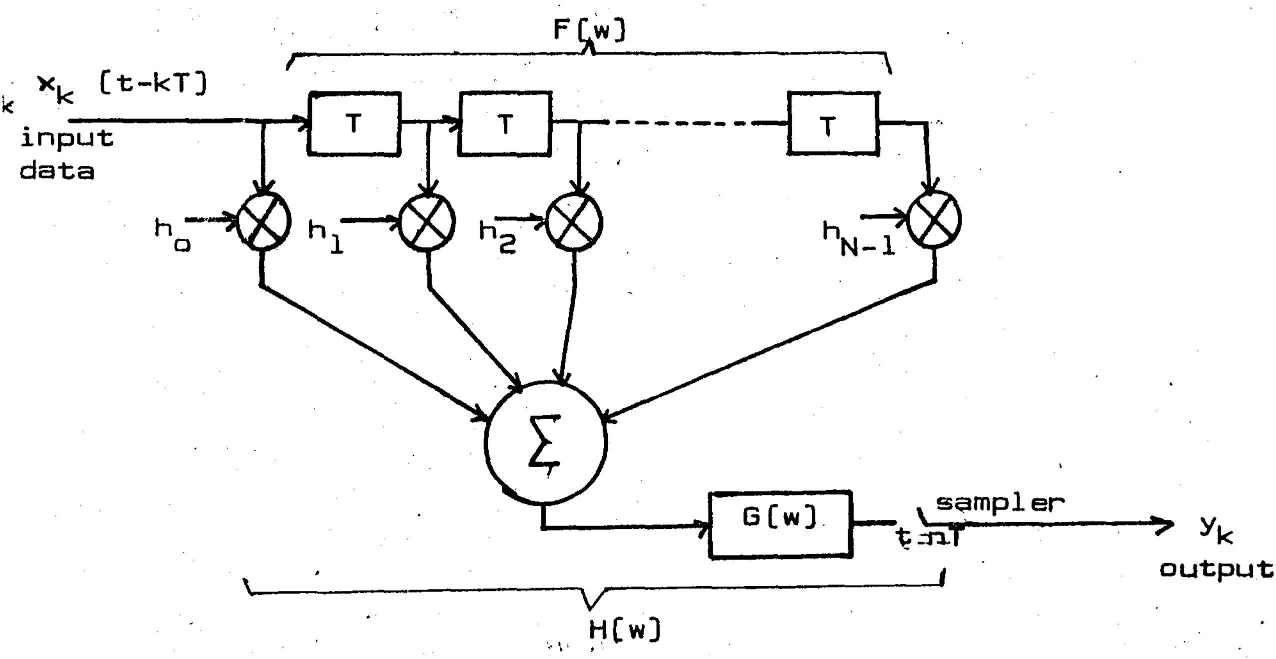


Fig. 3.19 General partial-response system model

The design of PRS is equivalent to the problem of selecting the coefficients in discrete time filter in Fig 3.19 to achieve a desirable spectral shaping. The spectral shaping can be achieved in two ways, one is designing the pulse shape the other is correlation properties of the output sequence (y_k). Since

$$y_k = \sum_{k=0}^{N-1} h_k x_{n-k} \qquad (3.62)$$

The autocorrelation function for (y_k);

$$\emptyset(m)=E\,(y_k\, y_{k+m}) \qquad (3.63)$$

$$\emptyset(m)=\sum_{l=0}^{N-1}\sum_{n=0}^{N-1} h_n h_e E(x_{k-n}x_{k+m-l}) \qquad (3.64)$$

When the input sequence is zero mean and white

$$E(\, x_{k-n}\, x_{k+m-l}) = \delta_{m+n-l} \qquad (3.65)$$

Where we have used the normalization

$$E\,(\, x_n^2\,) = 1 \qquad (3.66)$$

Then

$$\emptyset(m)=\sum_{n=0}^{N-1-|m|} h_n\, h_{n+|m|} \qquad m=0,\pm1,\ldots \pm(N-1) \qquad (3.67)$$

The corresponding power spectrum density,

$$\emptyset(f) = \sum_{m=-(N-1)}^{N-1} \emptyset(m)\, e^{-j2\pi fmt} \qquad (3.68)$$

$$\emptyset(f) = \left|\sum_{m=0}^{N-1} h_m\, e^{-j2\pi fmt}\right|^2 \qquad (3.69)$$

PR System Polynomial:

A. System Bandwidth: (17)

For maximizing the data rate in the available bandwidth, PRS are designed to occupy the minimum bandwidth, without undesired ISI. Such that $H(w)=0$ for $|w| > \pi/T$.
For minimum bandwidth system

$$G(w) = \begin{cases} T & |w|\leqslant \pi/T \\ 0 & \text{elsewhere} \end{cases} \qquad (3.70)$$

Corresponding system impulse response is

$$h(t) = \sum_{k=0}^{N-1} h_k \frac{\sin\frac{\pi}{T}(t-kT)}{\frac{\pi}{T}(t-kT)} \qquad (3.71)$$

For an example using the system polynomial $(1+D)$ as in duo-binary code which are insuitable for minimum bandwidth systems

B. Spectral null at $w=\pi/T$.(17)

The total signal energy in the tails of h(t) can be reduced for minimum bandwidth systems by making H(W) continous. For this F(W) must have a zero at π/T (Where G(w) has a discontinuity) for H(W) to be continous. The condition for the continuity of H(W) and its derivatives are as follows

Preposition 1: The first (K-1) derivatives of a minimum bandwidth F(W) are continous iff F(D) has $(1+D)^K$ as a factor.

If H(D) has more than one zero at D=-1, the roll-off near $W=\pi/T$ becomes less sharp,then the design of a filter will be easier. If F(D) has a large multiplicity of (1+D) factors, the error performence tends to be degraded due to the increose in the number of output levels. The sensitivity to timing offsets also suffers because more controlled amount of ISI terms are introduced.

C. Spectral null at w=o (17)

(1-D) factor of the H(D) is achieved the spectral null at W=0. Multiple zeros at D=1 causes more gradual roll-off the frequency components just above dc which may be desirable.

D.The number of output Levels: (17)

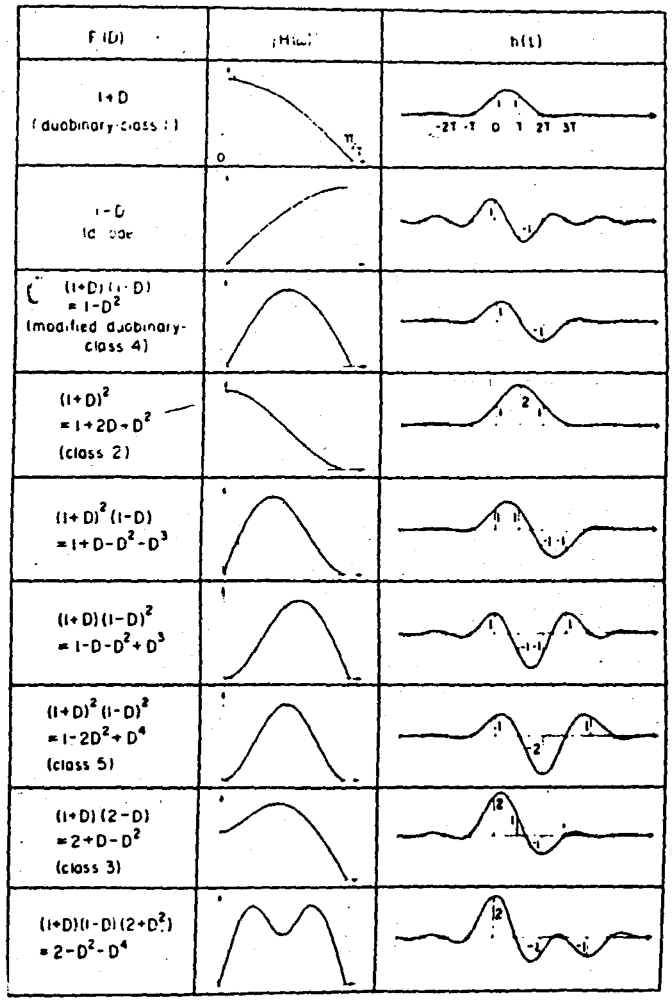
For m-ary input with M nonzero pulse samples will lead the m^M output levels in PRS system. The number of output levels L lies in the range

$$M(m-1)+1 \leqslant L \leqslant m^M \qquad (3.72)$$

With the factors $(1\pm D)$, the number of output levels is reduced.

Several PRS systems are shown in Table 3.9

TABLE 3.9 Several PRS systems [17]



SYSTEM POLYNOMIAL F(D)	FREQUENCY RESPONSE $H(\omega)$ for $\lvert\omega\rvert \le \pi/T$	IMPULSE RESPONSE $h(t)$	NO. OF OUTPUT LEVELS L
$1+D$	$2T\cos\frac{\omega}{2}T$	$\frac{4T^2}{\pi}\frac{\cos(\pi t/T)}{T^2-4t^2}$	$2m-1$
$1-D$	$j2T\sin\frac{\omega}{2}T$	$\frac{8Tt}{\pi}\frac{\cos(\pi t/T)}{4t^2-T^2}$	$2m-1$
$1-D^2$	$j2T\sin\omega T$	$\frac{2T^2}{\pi}\frac{\sin(\pi t/T)}{t^2-T^2}$	$2m-1$
$1+2D+D^2$	$4T\cos^2\frac{\omega}{2}T$	$\frac{2T^3}{\pi t}\frac{\sin(\pi t/T)}{T^2-t^2}$	$4m-3$
$1+D-D^2-D^3$	$j4T\cos\frac{\omega T}{2}\sin\omega T$	$-\frac{64T^3t}{\pi}\frac{\cos(\pi t/T)}{(4t^2-9T^2)(4t^2-T^2)}$	$4m-3$
$1-D-D^2+D^3$	$-4T\sin\frac{\omega T}{2}\sin\omega T$	$\frac{16T^2}{\pi}\frac{\cos(\pi t/T)(4t^2-3T^2)}{(4t^2-9T^2)(4t^2-T^2)}$	$4m-3$
$1-2D^2+D^4$	$-4T\sin^2\omega T$	$\frac{8T^3}{\pi t}\frac{\sin(\pi t/T)}{t^2-4T^2}$	$4m-3$
$2+D-D^2$	$T+T\cos\omega T+j3T\sin\omega T$	$\frac{T^2}{\pi t}\sin(\pi t/T)\left(\frac{3t-T}{t^2-T^2}\right)$	$4m-3$
$2-D^2-D^4$	$-T+T\cos 2\omega T+j3T\sin 2\omega T$	$\frac{2T^2}{\pi t}\sin(\pi t/T)\left(\frac{2T-3t}{t^2-4T^2}\right)$	$4m-3$

3.3 TWO PHASE (or FREQUENCY) MODULATION CODING

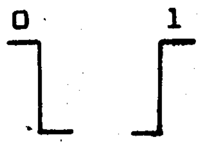
In this class of line coding, transmission redundancy is introduced by using a signalling rate equal to twice the input binary data rate. The line signal is DC free and contains a large number of transitions from which timing information can be recovered. [9]

Biphase, diphase, pulse doublet, two level AMI classl, WAL1, WALZ, 90 carrier diphase, pulse position modulation (PPM), Manchester code are some of this class codes.

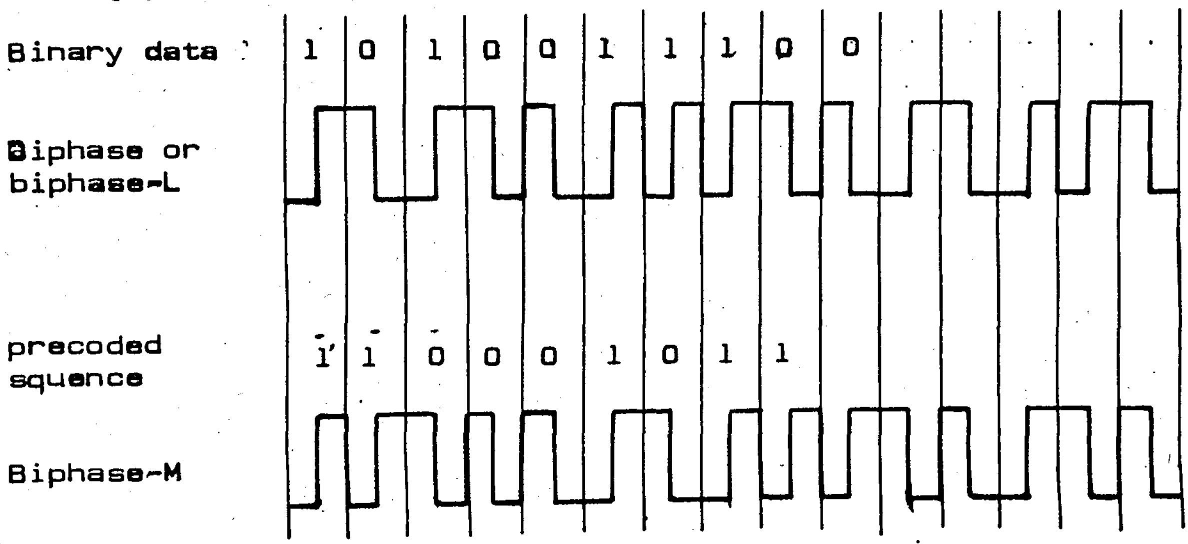
3.3.1 Diphase Code

It is derived by allocating respectively two complementary phases. To represent the two states in binary data. to be transmitted. The two basic elements are in fig 3.20 a and the resulting signal which is known diphase or biphase-l in fig 3.20 b.

If the binary data is first precoded and then transmitted as above, precoded diphase is obtained asin fig. 3.20b. Thelatter signal is also known as biphase-M or frequency doubling coding or conditioned diphase or two level AMI-class II coding. A common alternative description of precoded diphase is that there is always a signal transition in the middle of each symbol period and additional transition at the beginning where there is binary zero in the original data sequence



(a)



(b)

Fig. 3.20 Basic elements of diphase (a) and waveform of diphase and biphase-M codes (b)

Both diphase and precoded diphase, sometimes collectively known as Manchester codes, and also can be interpreted as Walsh-Type, (WAL-1) modulations of a NRZ binary sequence and of precoded version. Power spectral density is, (fig 3,22)

$$G(f) = A^2T\left[\frac{\sin \pi fT/2}{\pi fT/2}\right]^2 \sin^2 \pi f\ T/2 \qquad (3.73)$$

Where A is one half of the peak to peak amplitude.

In summary these type of codes can be described in terms of transitions or as a form of modulation either phase reversal modulation or binary squence shift modulation or amplitude modulation with carrier frequency equal to the data rate.

3.3.2 90° Carrier Diphase

It shifts the transitions in the original diphase by 90°. The resultant line signal can be interpreted as Walsh-Type-2 (WAL-2) modulation of a NRZ binary Squence. The waveforms are shown in fig 3.21 The power spectral density is (Fig.3.)

$$G(f) = 4A^2T\left[\frac{\sin \pi f\ T/2}{\pi f\ T/2}\right]^2 \sin^4 \pi f\ T/4 \qquad (3.74)$$

This code has a self equalizing property in cable transmission when coherent detection is used.

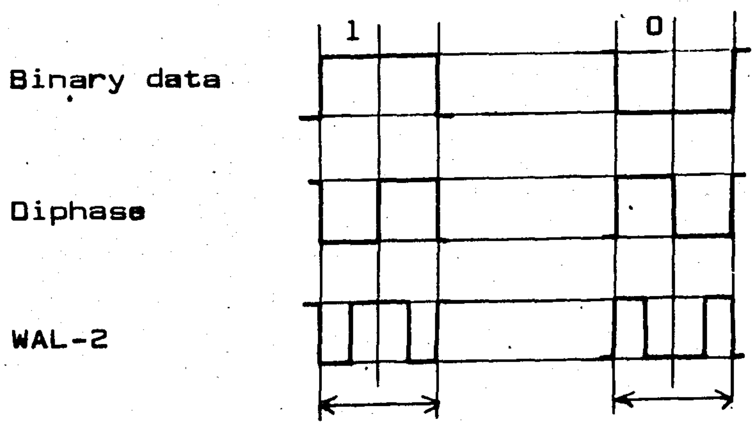


Fig. 3.21 Waveforms of diphase and WAL-2

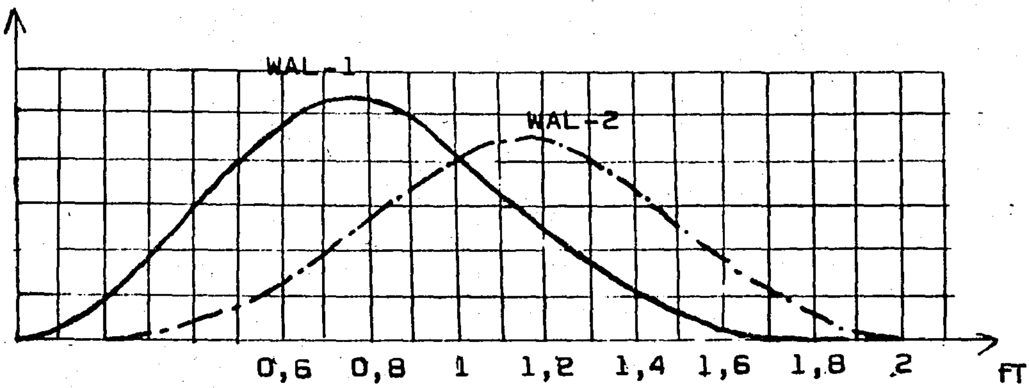


Fig. 3.22 Power spectral densities of WAL-1 and WAL-2

3.3.3 Modified Two Phase Modulation Code

They are linear combinations of diphase and PRS and a timing component and they may or may not involve an increase in transmitted levels. Two level AMI class-I coding, pulse doublet coding, pulse position coding are some of these codes.

3.3.4 Two Level AMI Class-I Code

A linear combination of a bipolar or AMI full-width signal, a diphase signal and a timing component at binary data rate. This signal is used for digital fibre optic communication systems. (Fig.3.23)

3.3.5 Pulse Doublet Code

Encoding procedure consists of transmitting a pair of contigous half-width pulses of opposite polarities for every binary one in the data squence while binary zeros are not encoded. For modified pulse doublet coding binary one is encoded into pulse doublets and zero is uncoded. The polarity of doublets is controlled by both the altrnate-mark-inversion rule and the even-odd time slot rule. A simpler explanation of this code is as follows: the encoded signal can be regarded as the result of a doubleside band modulation with suppressed carrier of an AMI signal. The carrier frequency is half the bit rate and its phase is shifted by a symbol period relative to AMI signal. (fig.3.23)

3.3.6 Intertran Pulse Position Code

It is combination of a 90° carrier diphase and a timing component at twice the binary data rate. (fig.3.23)

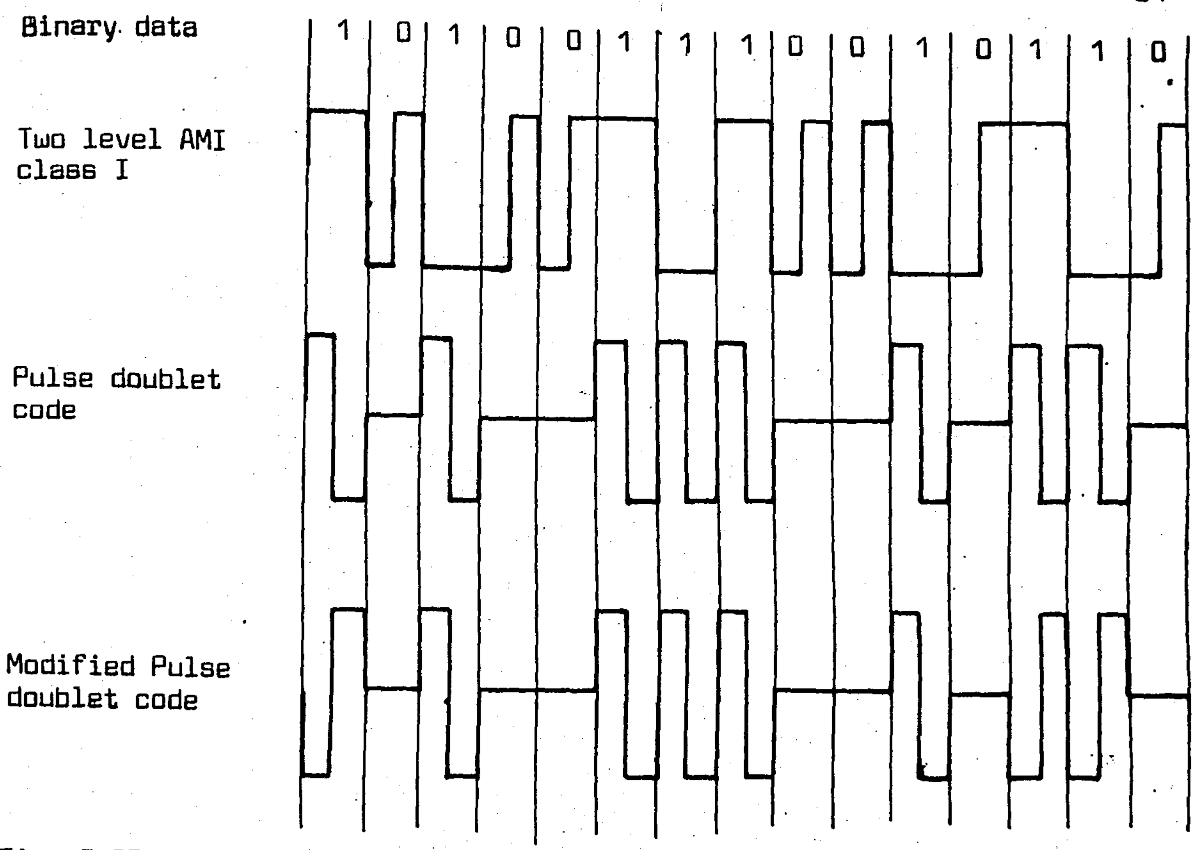


Fig. 3.23 Waveforms of Modified two-phase Modulation schemes

3.4 SUMMARY

Linear line codes are divided into three basic groups.

1. Binary coding: Binary input is transmitted in binary form.

2. Partial-Response (correlativel-level) signalling (PRS) or coding: It is based on spectral shaping by introducing controlled amount of ISI.

3. Two phase (or frequency) modulation coding: it is based on the increase in signalling rate.

In some of the linear line codes precoding operation is used. The purpose of the precoding is taking the advantage on error monitoring by reducing the decision threshold levels. The block diagram of the linear line codes is in fig 3.24. And the waveforms of several linear line codes given a binary in put data is shown in fig 3.25 and table 3.10.

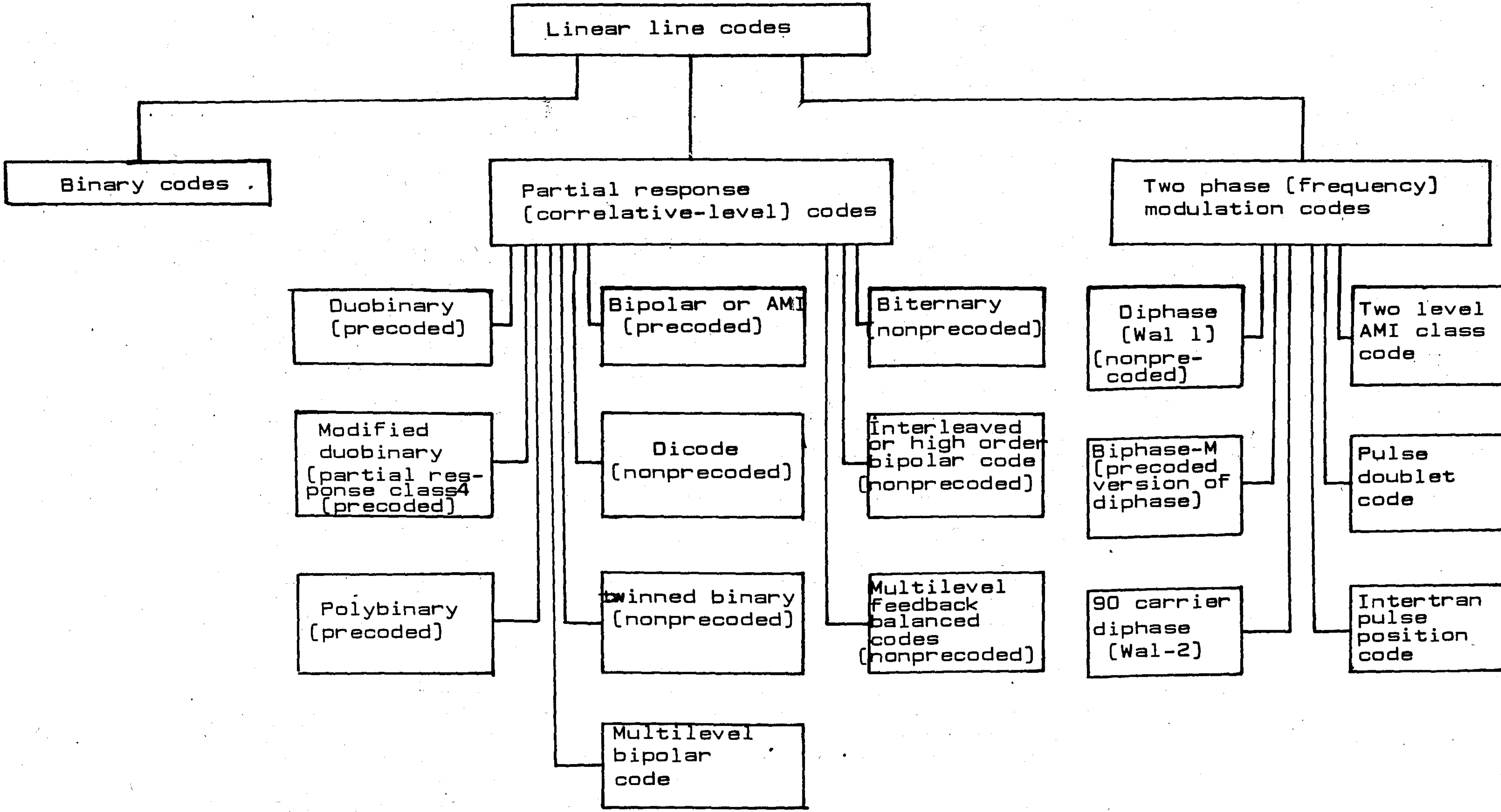


Fig. 3.24. Block diagram of linear line codes

TABLE 3.10 Some PRS codes

Binary data	1	1	1	0	1	0	0	1	0	0	0
Duobinary	0	0	0	2	0	2	2	0	2	2	2
Modified duobinary	2	2	-2	0	2	0	0	-2	0	0	0
Bipolar	1	-1	1	0	-1	0	0	1	0	0	0
twinned binary	1	0	0	-1	1	-1	0	1	-1	0	0
Dicode	1	0	0	-1	1	-1	0	1	-1	0	0
Interleaved bipolar	1	1	-1	0	1	0	0	-1	0	0	0

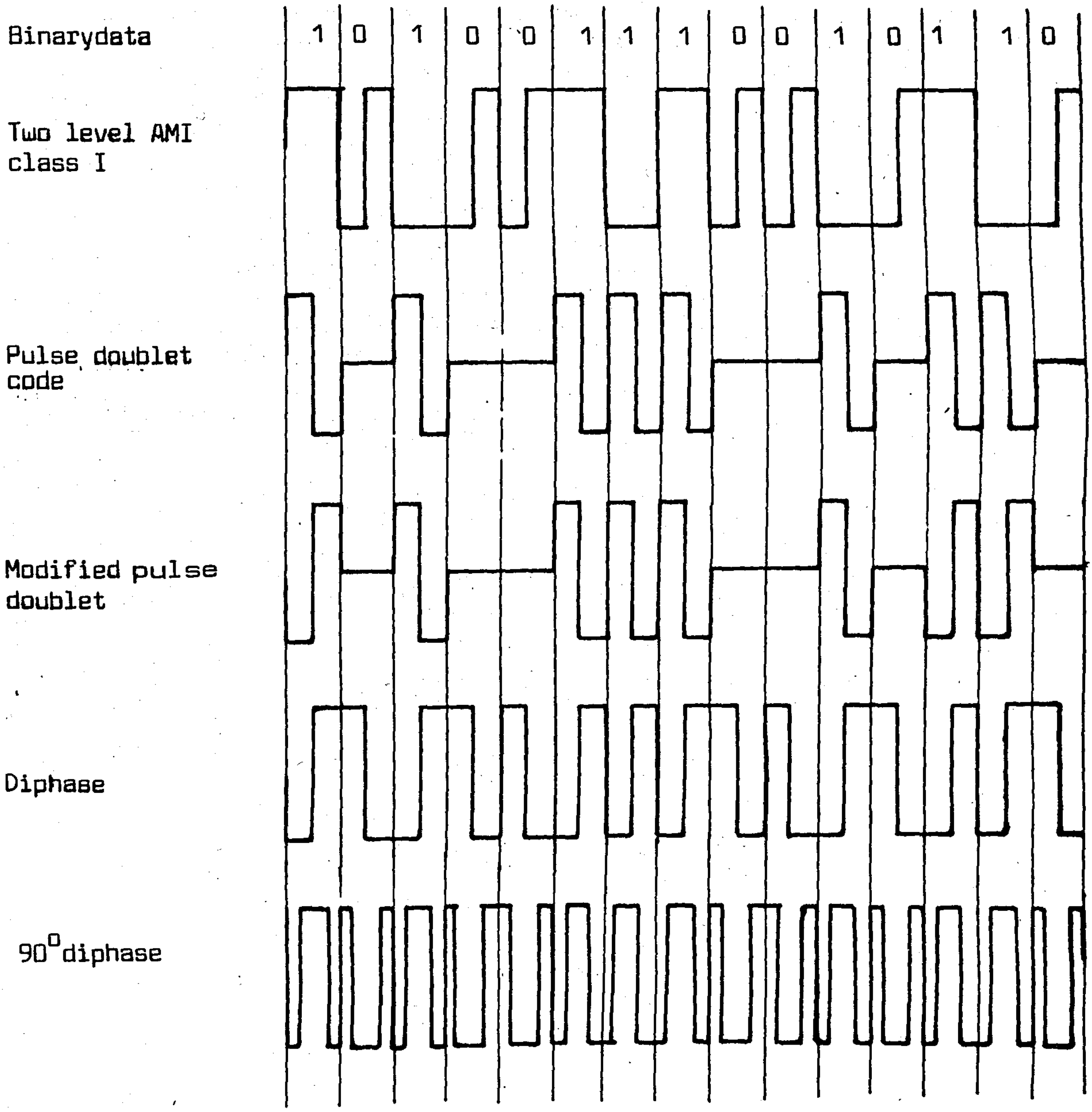


Fig. 3.25 Waveforms of two phase Modulation codes

CHAPTER IV

NONLINEAR LINE CODES

The encoded sequences produced by nonlinear codes can not be completely derived from the input data sequence by means of a linear relationship. Digits of the encoded sequence are grouped in "characters" of n consecutive digits, encoding being done one character at a time. For example in ternary codes if a ternary character is n digis long, there are 3^n possiblecharacter combinations. For coding procedure binary data is divided into frames of m bits each; to each binary character frame there corresponds one or several possible ternary character.combinations.

Redundancy in the line coded signal is in the form of either an increase in the number of transmitted levels or an increase in the signalling rate or bandwidth combination of these two, just like in the case of linear codes. Nonlinear line codes can be divided into two main catagories:

1. Alphabetic codes
2. Nonalphabetic codes

Before describing the above two classes, it will be convenient to give a number of definitions about nonlinear line codes. We will illustrate these definitions by means of several examples.

Definition 1 : Running digital sum (RDS):
Running digital sum at time n is defined as

$$RDS_n = RDS(0)+\sum a_i \qquad (4.1)$$

Where a is the encoded sequence and RDS (0) is the initial terminal state of the encoder.

Example 1: Let's assume the encoded sequence at terminal state $S_T = 1$ for three bit ternary code is

+0- $(a_1=+1, a_2=0, a_3=-1)$ then,

$$RDS_1=S_T+a_1 = 1+1=2$$

$$RDS_2=S_T+\sum_{i=1}^{2} a_i = 1+1+0=2$$

$$RDS_3=S_T+\sum_{i=1}^{3} a_i = 1+1+0-1=1$$

<u>Definition 2</u> : Allowable states (S): are the set of values (States) that RDS can assume.

<u>Example 2</u>: For terminal states $S_T=1$ and $S_T=0$ the ternary encoded squences are

for $S_T=1$	for $S_T=0$
+0-	---
$RDS_1=2$	$RDS_1=-1$
$RDS_2=2$	$RDS_2=-2$
$RDS_3=1$	$RDS_3=-3$

Allowable states are (-1, -2, -3, 1, 2)

<u>Definition 3</u> : Terminal state (S_T): the state which the encoded sequence occupies at the end of an encoded codeword. It is a subset of allowable states.

<u>Example 3</u>: From example 2 for the first codeword $S_T= RDS_3=1$, for the second codeword $S_T= RDS_3=-3$

<u>Definition 4</u> : Alphabet C(S): S_T has an alphabet C(S) which is consisting of 2^m encoded codewords. m is the number of binary digits.

<u>Example 4</u>: Let's assume the case m=2, and encoded sequence is 2 ternary bits word.

input binary data	Encoded data for $S_T=1$ alphabet C(1)	Encoded data for $S_T=-2$ alphabet C(-2)
0 0	0 +	0 +
0 1	- 0	+ 0
1 0	0 -	0 +
1 1	+ +	- -

Alphabet C(1) and C(-2) consist of $2^2= 4$ ternary code words such as for C(1),0+,-0,0-,++.

Definition 5 : Encoding rule e[S]: Encoding is a state dependent conversion from input binary sequences to encoded sequences and it is excuted by a set of encoding rules that depend on the state of the transmitted sequence at the time of encoding. [4],[3].

Example 5:

Input binary data	Encoded for $S_T=1$	data for $S_T=-1$
0 0 0 0	0 - -	0 + -
0 0 0 1	+ - 0	0 + +

If the first input data is 0000 then 0001 and the initial terminal state is $S_T=1$ then the encoded sequences will be

input data	0 0 0 0	0 0 0 1	0 0 0 0
encoded squence	0 - -	0 + +	0 - +
	$RDS_3=S_T=-1$	$RDS_3=S_T=1$	

At state $S_T=1$, 0000 binary word corresponds 0-- ternary word. At the end of this ternary word, $RDS_3=-1$, then the second binary word is encoded to ternary word at state $S_T=-1$ which is 0++. At the end of this ternary word $RDS_3=1$, then the third binary word will be encoded to corresponding ternary word at state $S_T=1$.

Definition 6 : Digital sum variation [DSV]: If the set of allowable states [S] of a line coder is finite, the line coder produces the sequences which have a finite digital sum variation [DSV]. The DSV is equal to the difference between maximum value of allowable states [S] and minimum value of allowable states [S] i.e,

$$DSV = S_{max} - S_{min} \text{ or}$$
$$DSV = RDS_{max} - RDS_{min} \quad [4.2]$$

Also maximum number of consecutive equal polarity pulses is equal to the DSV.

Example 6: The DSV of the code in example 2 is $DSV= RDS_{max} - RDS_{min}= 2-[-1]=3$. So the maximum number of consecutive like polarity pulses must be two such as

input data	0 0 0 0	0 0 0 1	1 0 0 0
encoded squence	0 + +	0 + -	0 - 0
	2 positive pulses	2 negative pulses	

The physical meaning of the DSV is proportional to peak to peak value of low frequency (LF) distortion (2). For example if the DSV is 2, there are 2 consecutive positive or negative pulses. So the frequency spectrum will be as a DC like behaviour.

Definition 7 : Balanced codes : If a codeword "a" is in the alphabet corresponding to some state (S) it is possible to use "-a" for the state $(S_{max}-(S-S_{min}))$

Example 7:

a: 0++ in S=1 and $S_{max}=2$

-a: 0-- in S=2 $S_{min}=1$

Balanced codes generate sequences a power spectrum containing a null at dc an also integer multiples of 1/T. The balanced property simplifies the decoding procedure, since the code is completely specified once half of the alphabets have been selected. (4)

Definition 8 : Line coder operation for alpabetic codes: In the terminal s, the coder accepts M binary digits and encodes them into codewords consistings of N digits by using the alphabet c(s) and encoding rule e(s). It takes the RDS thru allowable states and finally ending after N digits at the terminal state s', $s' = s + \sum_{i=1}^{N} a_i$. For the next block of M binary digits the whole encoding process will be repeated. (4)

Definition 9 : Terminal state stationary probabilities : These values can be found by modelling the encoding procedure as a Markov process (4), (3), that is,

$$\underline{P} = \underline{\underline{\Pi}}\, \underline{P} \qquad (4.3)$$

where pi's are the terminal state stationary probabilities,

$$\underline{P} = (P(S_1), P(S_2), \ldots, P(Sn)) \quad \text{or} \qquad (4.4)$$

$$\underline{P} = (P_1, P_2, \ldots\ldots\ldots, Pn)$$

where S_1, S_2, ...S_n are the terminal states.

$\underline{\underline{\Pi}}$ is the transition matrix of the terminal state process, it is called state transition probability matrix (STPM).It is defined as,

$$\underline{\underline{\Pi}} = \|\Pi_{ij}\| \qquad (4.5)$$

(4.5)

The STPM can be easily found by

$$\pi_{ij} = 2^{-M} n_{ij} \qquad (4.6)$$

where M is the number of the binary input digits, nij is the number of inputwords which cause the encoder to change from terminal state i to j. It will be illustrated latter.

Definition 10: Efficiency (E) : Efficiency of a code is defined to be the ratio of the average bits per symbol of the coded signal stream to that of the random (uncoded) signal stream. It is formulated as,

$$E = \frac{M}{N} \mathrm{Log}_L K \qquad (4.7)$$

where M = number of digits of binary input
N = number of digits of encoded sequence
K = number of levels of encoded sequence
L = number of levels of input sequence (for binary input L=2)

Definition 11: State independent decodable (SID): Encoding procedure depends on the states of the sequence occupie at the end of a codeword (from the previous definitions) In decoding procedure, additional circuits are necessary to track the sequence state for state dependent decoding. Thus, state independent decoding may often be necessary. Decoding independently of the state is possible if and only if one binary word "b" corresponds to each codeword "a_r" in c(s).That is the state dependent mapping of (b) into a must have a unique inverse. Given a group of terminal states (S_{Ti}), i=1,...n and their associated alphabets c(s_i), it is desirable to determine whether it is possible to code so that decoding independent of i. The following two necessary and sufficient conditions must be satisfied, for the above:

Condition 1 : A sufficient but not necessary, condition for the existence of an assignment of binary words to the members of c(si), i=1,2,...n so that decoding is independent of is that any integers u,v such that $1 \leq u \leq v \leq n$

$$a_r \in c(s_u) \cap c(s_v) \rightarrow a_r \in c(s_{u+1})$$

That a binary word "b" has been assigned to "a_r" in the alphabet c[s_u]. Then it is possible to give "a_r" the same binary assignment in c[s_{u+1}].....c[s_v]. (13)

Condition 2 : A necessary and sufficient condition for the possibility of assigning binary words to the members of c[s_i] i=1,2,...n so that decoding is independent of i is that for $1 \leq u \leq v \leq x \leq n$ and for each $a_r \in c[s_u] \cap c[s_x]$ if $a_r \notin c[s_v]$ then some other word $a_p \in c[s_v]$ must be assigned to a_r in u^{th} alphabet. But if $a_p, a_r \in c[s_m]$ where $1 < m < n$.

State dependent decoding is precluded. [13]

Example : Let the binary words be b_1, b_2, b_3, b_4 and states are 1,2,3,4 and the alphabets c[1], c[2], c[3], c[4] and the set of alphabets as in following.

	c[1]	c[2]	c[3]	c[4]
b_1	a_1	a_2	a_1	a_2
b_2	a_3	a_4	a_4	a_4
b_3	a_5	a_7	a_8	a_5
b_4	a_6	a_6	a_9	a_{10}

This is the state independent decodable code structure. Because binary word b_1 corresponds words a_1 and a_2.

① $b_1 \rightarrow a_1 \in c[1] \cap c[3]$, $b_1 \rightarrow a_2 \in c[2] \cap c[4]$

$b_1 \rightarrow a_1 \notin c[2]$ $\quad$ $b_1 \rightarrow a_2 \notin c[1]$

$b_1 \rightarrow a_2 \in c[2]$ $\quad$ $b_1 \rightarrow a_1 \in c[1]$

$a_1, a_2 \notin c[4]$ $\quad$ $a_2 a_1 \notin c[4]$

② $b_2 \rightarrow a_3 \in c[1]$

$b_2 \rightarrow a_4 \in c[2] \cap c[4]$

$b_2 \rightarrow a_4 \in c[3]$

$a_4, a_3 \notin c[1]$

$a_4, a_3 \notin c[2]$

$a_4, a_3 \notin c[3]$

4.1 ALPHABETIC CODES

An alphabetic code is defined as one in which XK-level digits are converted into yL level digits using alphabet c[s] and encoding rule e[s]. Such a code is denoted as an XK-yL code, although this notation is not always adopted.

For the conversion to be possible, the following relationship must be satisfied.

$$K^x \leq L^y \quad \text{or}$$

$$x \leq y \log_k L \qquad [4.8]$$

In most cases, the input data is binary, i.e K=2 then

$$x \leq y \log_2 L \qquad [4.9]$$

And the ratio of symbol [line] rate to the input rate is [Fig 3.]

$$\frac{B_1}{B_{in}} = \frac{y}{x} \qquad [4.10]$$

Conversion of input data into individual blocks or words need to be framed or synchronized at the receiving terminal before the information can be correctly recovered. Synchronization is achieved by monitoring the various built-in properties of the code.such as RDS, state transitions, violations etc.

We can summarize the alphabetic codes into two blocks.

1. Binary alphabetic codes
2. Multilevel alphabetic codes.
   a) without rate increase.
   b) with rate increase.

4.1.1 Binary Alphabetic Codes

In binary alphabetic codes binary data is converted into another binary squence. Zero-disparity, unitdisparity, Neu, Harvey and 2B-3B codes are examples of the binary alphabetic codes.

i) Zero disparity code

Codewords are 2n bit words which must contain n "ones" and n "zeros". For keeping redundancy to a low level, n must be large.

ii) Unit disparity code

Codewords are digit groups in which number of "ones" differs by only one from the number of "zeros".

iii) Neu code

Binary is first converted into a ternary stream using an appropriate conversion procedure (ex. 3 binary to 2 ternary, 3B-2T) the resultant squence is then translated back into binary using one of the rules given in table 4.1.

TABLE 4.1 Neu code translation

Ternary symbol	A-code	B-code
+ 1	01	10
0	00	00 and 11 alternating
- 1	10	01

The B code is balanced code, number of zeros equal to the number of ones. Error monitoring can also be carried out using the alternation rule in the translation. Intermediate binary to ternary conversion yields increase in signalling rate. (For 3B-2T, signalling rate is 4/3)

iv) Harvey code

The generation of this code is similar to Neu codes Binary-ternary-binary translation is used by using rules give in table 4.2.

TABLE 4.2 Harvey code translation

Ternary symbol	Binaryword
+ 1	1000
0	1100
- 1	1110

Each codeword can be identified by the location of the one to zero transition. Large amount of redundancy is introduced but signalling rate is high. For example for the intermediate 3B-2T conversion signalling rate is 8/3.

v) 2B-3B code
This class of coding is where 2 binary digits are converted into 3 binary digits by using a set of rules.

4.1.2 Multilevel Alphabetic Codes

Binary input data is converted into multilevel sequences.

i) Multilevel alphabetic codes without rate increase
One such code is Pair-Selected-Ternary (PST). The PST code translation is shown in table 4.3. It consists of two alphabets (c (s))as mode A and mode B which alternate each time 01 or 10 is detected in the binary input stream. ++, --, 00 ternary pairs are not used and they can provide framing information and alternation property gives the error monitoring capability. (8) (Signalling rate is 2/2 = 1, thus no rate increase)

TABLE 4.3 PST code translation

Binary word	Ternary words Mode A	Mode B
00	- +	- +
01	0 +	0 -
10	+ 0	- 0
11	+ -	+ -

Example 8 : Binary input 00 10 10 11 01
Ternary output -+ +0 -0 +- 0+
ModeA ModeB ModeA →Mode B

When the probabilities of occurence of "ones" and "zeros" differ a modified PST is prefered in Table 4.4.

TABLE 4.4 Modified PST code translation

Binary word	Ternary word Mode A	Mode B
00	0+	-0
01	-+	-+
10	+-	+-
11	+0	0-

Modes alternate after each 00 or 01.

2] Multilevel alphabetic codes with rate increase

There exist unlimited number of codes which belong to this lass for a given number of levels L in the required coded signal, such code constructed provided that the condition in 4.8 is satisfied o the sufficient redundancy is introduced. Ternary alphabetic codes, 742, VL43, 3B-2Q, 6B-3QI codes are some of them.

i] Ternary alphabetic codes

Ternary alphabetic line codes are translated M binary input igits into N ternary channel symbols by using encoding rules that lepend on the state of the encoder. These codes are described as B-NT codes. Number of levels of ternary digits are three as +1, 0, -1. hen the information capacity of a ternary channel is $\log_2 3 = 1.58$ its /symbol, so these codes can potentially increase the data rate y up to 5 percent over straight binary signalling.

MS43, 4B-3T, Fomot type of ternary alphabetic codes will e explained in the following pragraphs.

MS43 Code

Four binary digits are translated into three ternary digits. he translation is shown in table 4.5 [2]

TABLE 4.5 MS43 Code

Binary word	Ternary words S_T=1	S_T=2 or 3	S_T=4
0000	+++	-+-	-+-
0001	++0	00-	00-
0010	+0+	0-0	0-0
0011	0++	-00	-00
0100	+-+	+-+	---
0101	0-+	0-:	0-+
0110	-0+	-+0	-0+
0111	00+	00+	--0
1000	0+0	0+0	-0-
1001	+00	+00	0--
1010	-+0	-+0	-+0
1011	+-0	+-0	+-0
1100	+0-	+0-	+0-
1101	0+-	0+-	0+-
1110	-++	-++	--+
1111	++-	+--	+--

MS43 ternary words are grouped into three alphabets [c(s)] corresponding to S_T=1,2 or3,4. The procedure for finding the allowable states (s) is indicated in Table 4.6 as in example 2.One can easily see that at the end of a ternary word, the RDS is equal to the one of the terminal states.

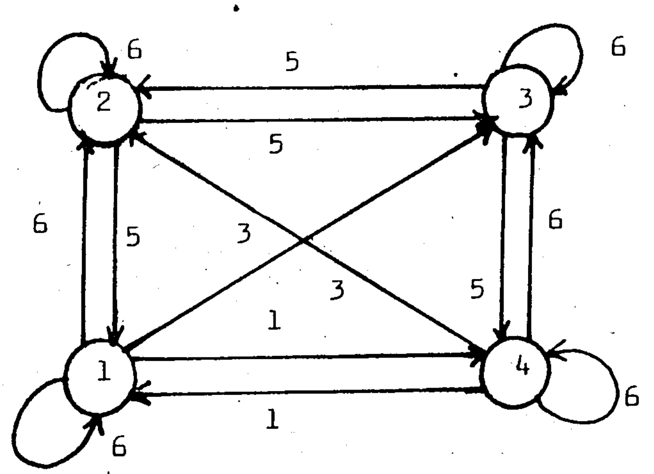
TABLE 4.6 Allowable states of MS43 code

Binary word	Ternary word $S_T=1$ RDS$_1$	RDS$_2$	RDS$_3$	$S_T=2$or3 RDS$_1$	RDS$_2$	RDS$_3$		$S_T=4$ RDS$_1$	RDS$_2$	RDS$_3$
0000	2	3	4	1	2	1	232	3	4	3
0001	2	3	3	2	2	1	332	4	4	3
0010	2	2	3	2	1	1	322	4	3	3
0011	1	2	3	1	1	1	222	3	3	3
0100	2	1	2	3	2	3	434	3	2	1
0101	1	0	1	2	1	2	323	4	3	4
0110	0	0	1	1	2	2	233	3	3	4
0111	1	1	2	2	2	3	334	3	2	2
1000	1	2	2	2	3	3	344	3	3	2
1001	2	2	2	3	3	3	444	4	3	2
1010	0	1	1	1	2	2	233	3	4	4
1011	2	1	1	3	2	2	433	5	4	4
1100	2	2	1	3	3	2	443	5	5	4
1101	1	2	1	2	3	2	343	4	5	4
1110	0	1	2	1	2	3	234	3	2	3
1111	2	3	2	3	2	1	432	5	4	3

The allowable states (S) that the RDS can assume is $S=\{0,1,2,3,4,5\}$ for this code. And The DSV is

$DSV = S_{max}-S_{min} = 5-0=5$

Terminal state diagram of MS43 code is in Fig 4.1 Let's show the computation of this diagram by looking the alphabet for $S_T=1$, from table 4.6. There are six words that end with $RDS_3=1$ then the number of transitions that do not lead the change in state is $n_{11}=6$. There are six words that end with $RDS_3=2$, then the number of transitions from $S_T=1$ to $S_T=2$ are 6 ($n_{12}=6$). There are three words that end with RDS_3:3, then the number of transitions from $S_T=1$ to $S_T=3$ are 3 ($n_{13}=3$). There is one word that ends with $RDS_3=4$, then number of transitions from $S_T=1$ to $S_T=4$ is 1 ($n_{14}=1$).



Finding

Fig.4.1 State transition diagram of MS43 code the terminal state tronsition probabilities will be easier by using diagram in fig 4.1. Thus state transition probability matrix(STPM) is

$$\Pi = \frac{1}{16} \cdot \begin{bmatrix} 6 & 6 & 3 & 1 \\ 5 & 6 & 5 & 0 \\ 0 & 5 & 6 & 5 \\ 1 & 3 & 6 & 6 \end{bmatrix} \quad (4.11)$$

Where $\Pi_{ij} = \frac{n_{ij}}{2^M} = \frac{n_{ij}}{16}$ (ex. $\Pi_{11} = \frac{n_{11}}{16} = \frac{6}{16}$, $\Pi_{41} = \frac{n_{41}}{16} = \frac{1}{16}$

Then the terminal state stationary probabilities can be found by using equation 4.3

$$\left.\begin{array}{l} \underline{P}\,\underline{\Pi} = \underline{P} \\ \sum_{i=1}^{4} P_i = 1 \end{array}\right\} \begin{array}{l} P_1 = P_4 = 5/28 \\ P_2 = P_3 = 9/28 \end{array} \quad (4.12)$$

4B-3T Code

In this code four binary digits are converted into three ternary digits as shown in table 4.7. Six binary words are converted directly into balanced or zero disparity ternary words. The remaining ten are translated into ternary words of +1,+2.+3 digital sums. These ternary words are grouped into two alphabets according to the polarity of the digital sums. Mode A is used for RDS:-1,-2,-3 and mode B is used for RDS=0,1,2. Mode alternation can provide error monitoring and synchronizing the received sequence.

TABLE 4.7 4B-3T code

Binary words	Ternary words S_T=-1,-2,-3	S_T=0,1,2
0000	0- +	0-+
0001	-+ 0	-+0
0010	-0 +	-0+
0011	+- +	-+-
0100	0+ +	0--
0101	0+ 0	0-0
0110	00 +	00-
0111	-+ +	+--
1000	0+ -	0+-
1001	+- 0	+-0
1010	+0 -	+0-
1011	+0 0	-00
1100	+0 +	-0-
1101	++ 0	--0
1110	++ -	--+
1111	++ +	---

From the same procedure as in MS43

Allowable states : s= [-4,-3,-2,-1,0,1,2,3]

DSV: $S_{max} - S_{min}$: 3-[-4]: 7

There must be 7 consecutive like polarity pulses if the initial state s=-3 and the encoded ternary word is -0+, again in the same state and let's assume the corresponding ternary word is 0+ +, now it is at state s=-1, and let's assume encoded ternary word is + + +, so it is at state S=2, again Let's assume encoded ternary word is +-- then -0+ 0++ +++ +--

7 positive pulses

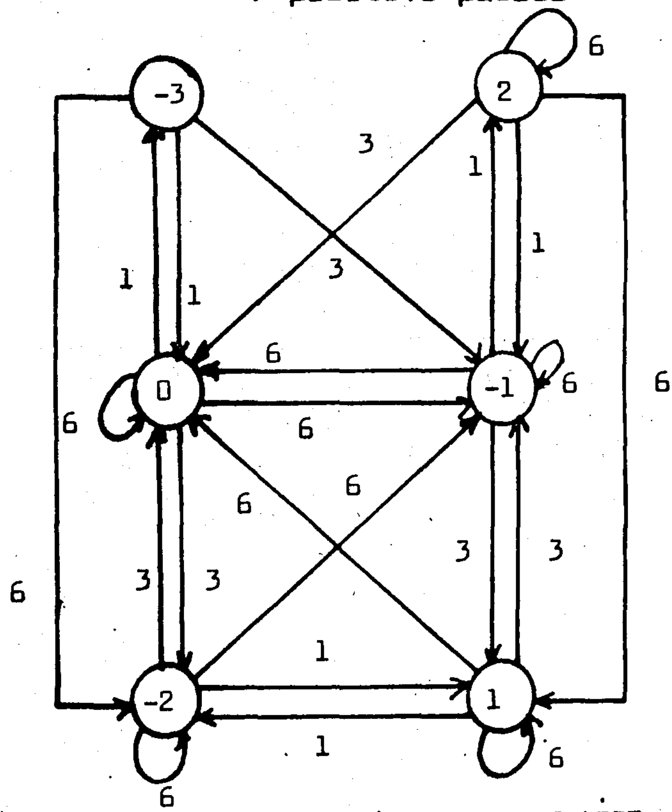


Fig.4.2 State transition diagram of 4B3T code

and STPM and terminal state stationary probabilities are

$$\pi = \frac{1}{16}\begin{bmatrix} 6 & 6 & 3 & 1 & 0 & 0 \\ 0 & 6 & 6 & 3 & 1 & 0 \\ 0 & 0 & 6 & 6 & 3 & 1 \\ 1 & 3 & 6 & 6 & 0 & 0 \\ 0 & 1 & 3 & 6 & 6 & 0 \\ 0 & 0 & 1 & 3 & 6 & 6 \end{bmatrix}$$

$P_{-3} = P_2 = 1/30$

$P_{-2} = P_1 = 4/30$

$P_{-1} = P_0 = 10/30$

Another 4B-3T code is "Alternate-Mode 4B-3T" code. In this code mode is changed when nonzero digital sums, otherwise it remains unaltered. The DSV of this code is infinity. Infinitely many consecutive same polarity pulsses will be possible.

Fomot code

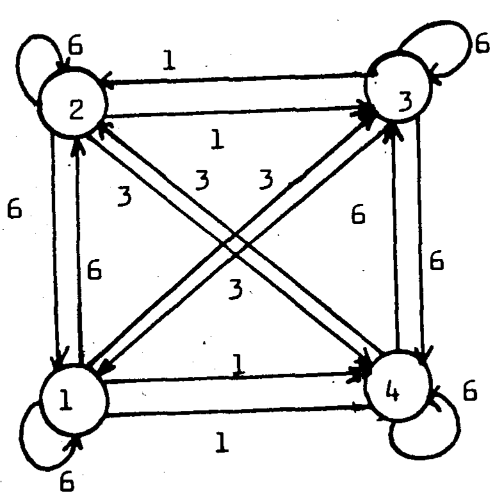
Fomot (Four mode ternary)code is the short word version of 4B-3T code. The structure of the code is shown in table 4.8.For RDS= 1,2,3,4 there are four alphabets.

TABLE 4.8 Fomot code

binary word	Ternary words $S_T=1$	$S_T=2$	$S_T=3$	$S_T=4$
0000	- ++	-00	-++	-00
0001	- +0	-+0	-+0	-+0
0010	+ -0	+-0	+-0	+-0
0011	+ 00	+--	+00	+--
0100	- 0+	-0+	-0+	-0+
0101	+ ++	-+-	-+-	-+-
0110	+ 0+	+0+	-0-	-0-
0111	+ 0-	+0-	+0-	+0-
1000	0 ++	0++	--0	--0
1001	0 +0	0-0	0+0	0-0
1010	+ -+	+-+	+-+	---
1011	+ +0	++0	0--	0--
1100	0 0+	--+	00+	--+
1101	0 +-	0+-	0+-	0+-
1110	0 -+	0-+	0-+	0-+
1111	+ +-	00-	++-	00-

Allowable s ates : s:(0,1,2,3,4,5)

$DSV = S_{max} - S_{min}$: 5-0=5



Fgg. 4.3 State transition diagram FOMOT code

The STPM and terminal state stationary probabilities are

$$\Pi = \frac{1}{16} \begin{bmatrix} 6 & 6 & 3 & 1 \\ 6 & 6 & 1 & 3 \\ 3 & 1 & 6 & 6 \\ 1 & 3 & 6 & 6 \end{bmatrix} \qquad P_1 = P_2 = P_3 = P_4 = \frac{1}{4}$$

Other ternary alphabetic codes such as 6B-4T, 10B-7T can be produced.

ii) L742 code

In this transmission scheme, block of four binary digits are converted into words of 2-seven level symbols grouped into three alphabets as in table 4.9 [10]

TABLE 4.9 L742 code

Binary word	Encoded wors		
	S_T=1 or 2	S_T=3 or 4	S_T=5 or 6
0000	3 -3	-1 1	-1 1
0001	3 0	-2 1	-2 1
0010	0 1	0 1	-1 3
0011	2 0	2 0	-3 1
0100	0 2	0 2	-2 -2
0101	3 1	-2 2	-2 2
0110	1 1	1 1	0 -3
0111	1 0	1 0	-3 2
1000	3 -2	-1 0	-1 0
1001	3 -1	-2 0	-2 0
1010	2 -2	2 -2	-3 1
1011	2 -1	2 -1	-3 0
1100	0 3	-1 -1	-1 -1
1101	2 2	0 -1	0 -1
1110	1 3	0 -2	0 -2
1111	1 -1	1 -1	-3 3

Allowable state : S= (1,2,3,4,5,6,7)

$DSV = S_{max} - S_{min} = 7-1=6$

iii) VL43 Code (Variable Length Code)

This is the variable length alphabetic code. It attempts to increase fixed length state-dependent code efficiency by increasing the word length. In this code there are three alphabets with S_T=RDS= S_1, S_2, S_3. The alphabet $c(S_2)$ contains only words of length three. $c(S_1)$ and $c(S_3)$ each contains 16 words of length six (each carriying 8 bits) and 15 words of length three (Table 4.10),(13) Thus first 4 binary digits are converted into 3 ternary digits and second 8 binary digits are converted into 6 ternary digits. The algorithm for VL43 code can be summarized as follows.

1) Compute RDS
2) Examine the next 4 bits
3) Look in the table for 3.digit ternary codeword corresponding to these 4 bits and the digital sum.
4) If no codeword is found increase the number of bits examined to 8 and look for a suitable 6 bits ternary word.
5) Record the selected word and go back to 1 for next bits.

TABLE 4.10 VL43 Code Book (13)

Binary Equivalent	Alphabet for Coder State C(S1)	C(S2)	C(S3)
0000	+-+	+ +	
0001		[illegible]	-+-
0010	0+0	0+0	-0-
0100	+00	+--	+--
1000	00+	00+	[illegible]
0011	0+-	0+-	0+-
0101	+0-	+0-	+0-
1001	++0	00-	00-
1010	-0+	0-0	0-0
1100	0--	--+	---
0110	--0	--0	+-0
1110	-+0	-+0	--0
1101	-0+	-0+	-0-
1011	0-+	0-+	0--
0111	++-	++-	0--
1111	-++	-++	-00

Binary Equivalent	C(S1) Alphabet	Binary Equivalent	C(S2) Alphabet
00010000	-+-+-+	00000000	--+-+-
00010001	+-+-+-	00000001	+-+---
00010010	---0+0	00000010	+-+-0-
00010100	---+00	00000100	000+--
00011000	-+-00+	00001000	+-+--0
00010011	+-+-0-	00000011	---+0+
00010101	+++--0	00000101	---+-0
00011001	-+-++0	00001001	+-+00-
00011010	-+-+0+	00001010	+-+0-0
00011100	-+-0++	00001100	+-+--+
00010110	000-+-	00000110	000+-0
00011110	000-+0	00001110	000---
00011101	-++--+	00001101	---+--
00011011	-++-00	00001011	---+00
00010111	-+-++-	00000111	+-+0--
00011111	000-++	00001111	+-+-00

For example if 110000010000 is to be coded when state is s_2 the first 4 bits are encoded as --+, after which the state is s_1. The next 8 bits are encoded into a 6 bits word, - + - + -+ .

VL43 codes combine the advantages of short and long word lengths. It uses minimum number of possible words instead of many number of words in fixed length codes.

iv) 3B-2Q Code

In this code three binary bits are converted into words of two four-level symbols as in table 4.11 [10] There are two alphabets and the modes alternate when the first two input data bits are both zeros or both ones.

TABLE 4.11 3B-2Q Code

Binary word	Encoded words Mode A	Mode B
000	-3, -1	3, 1
001	-1, -3	1, 3
010	-1, 1	-1, 1
011	-3, 3	-3, 3
100	1, -1	1, -1
101	3, -3	3, -3
110	-3, 1	3, -1
111	1, -3	-1, 3

The DSV = 9 is quite large and the LF distortion is expected to be high.

v) 6B-3QI Code

In this code blocks of six binary digits are translated into words of three five level symbols grouped into two alphabets. Mode alternation is controlled by the running digital sum at the end of the quinary (5 level) word.

4.2 NONALPHABETIC CODES

These codes derived from basic bipolar encoding algorithm. We know that in the bipolar code, zero binary is transmitted uncoded. Long sequences of zeros are not desirable as no timing information can be derived. One such approach is using PST or modified pair selected ternary for removing long strings of zeros. The other methods are similar in that they substitute for sequences of consecutive zeros some chosen non-zero pattern. These methods are realized by the nonalphabetic codes. Nonalphabetic codes can be explained into two groups as in alphabetic codes

1. Binary nonalphabetic codes

2. Multilevel nonalphabetic codes

4.2.1 Binary Nonalphabetic Codes

Polarity pulse coding, Miller code are the codes in this group.

i) Polarity Pulse Code

One takes n-bit data block and transmits it either unchanged or with the bits inverted, as an (n+1) bit word so as to reduce the running digital sum since the beginning of the transmission. The extra digit, that is polarity digit, indicates whether or not the original data has been inverted. The othe name of this code is Carter code.

ii) Miller code or Delay modulation (DM)

This code is a variation of the diphase or biphase-L code. A transition from one level to the other is placed at the midpoint of the bit period when the binary data contains a "one". No transition is used for a "zero" unless it is followed by another "zero", in which case the transition is placed at the end of the bit period of the first "zero". Fig 4.4 (6) indicates the relation between diphase and delay modulation.

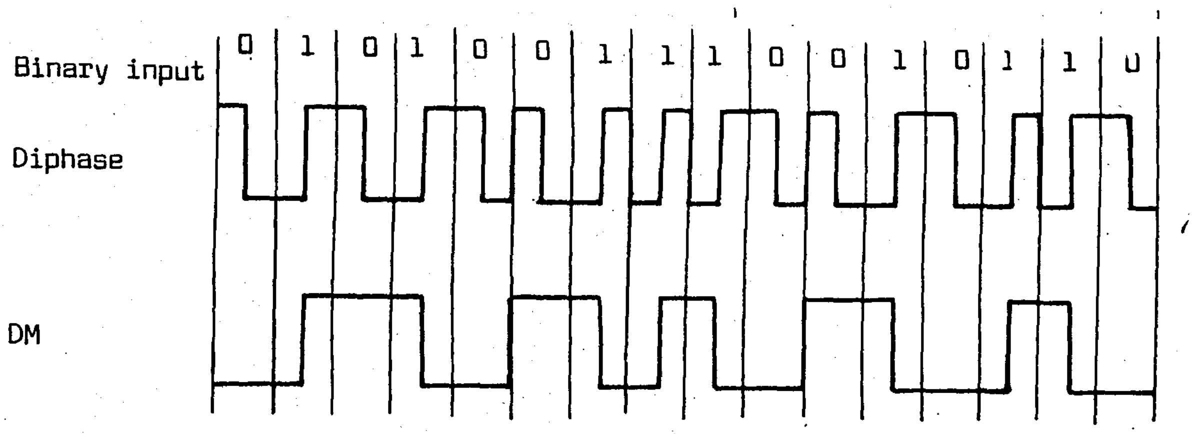


Fig.4.4 Diphase and delay modulation

4.2.2 Multilevel Nonalphabetic Codes

Time polarity control, filled bipolar or AMI, feedback balanced codes are some of these codes.

1. Time polarity control code (TPC)

This code is not derived from the bipolar code. In this transmission, time slots are labelled alternatively positive and negative. Pulses occuring in the negatively labelled slots are transmitted with a negative polarity or viceversa. Zeros are sent out unaltered. LF and DC components are produced. Generation of TPC and waveform in Fig 4.5 a and b.(10)

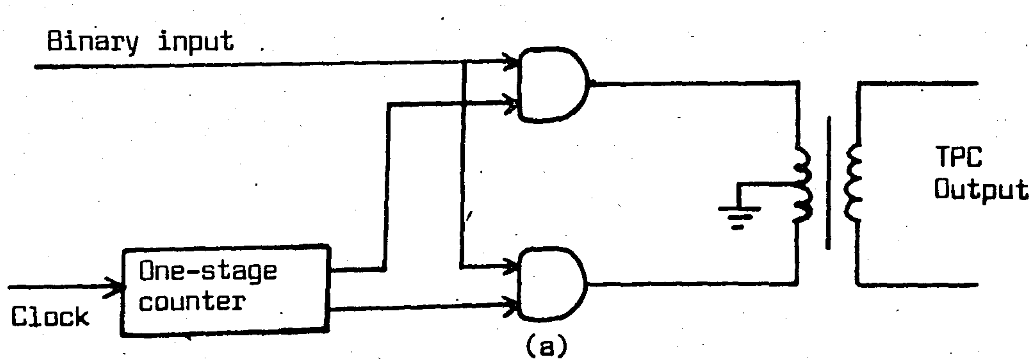


Fig 4.5

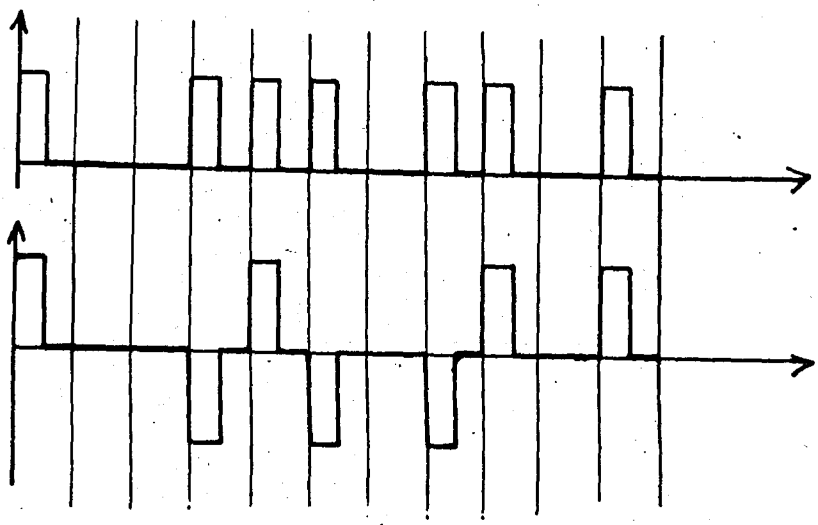


Fig.4.5 Generation of TPC(a) and TPC waveform (b)

2. Filled bipolar or AMI code

Spaces are denoted as "zero", positive and negative pulses generated using the bipolar encoding rules are denoted as "B" pulses. On the other hand pulses (whether inserted or inverted from the original sequence that violate, are denoted as "V" pulses. A "V" pulse has the same polarity of the preceding pulse. B6ZS, high density bipolar (HDB), compatible high density bipolar (CHDB), uniformed bipolar code, transparent interleaved bipolar (T1B) code are the codes of filled bipolar codes group.

i) B6ZS code

With this code the encoded sequence is obtained from the original bipolar stream upon substituting following pattern for any strings of six consecutive zeros. (10) (9)

B0V B0V

Alternatively 0VB0VB can also be used depending of the polarity of the previous pulse and last pattern violation. Removing the consecutive zeros increases the DSV from 1 to 3. (DSV of the bipolar code is one)

Other bipolar with n zero substitution codes can also be derived, in fig 4.6 block diogram of a BnZS encoder is illustrated. Bipolar stream is unchanged, except where the number of consecutive zeros exceed (n-1). At the decoder substituting sequences are removed. B6ZS code is shown in fig 4.8.

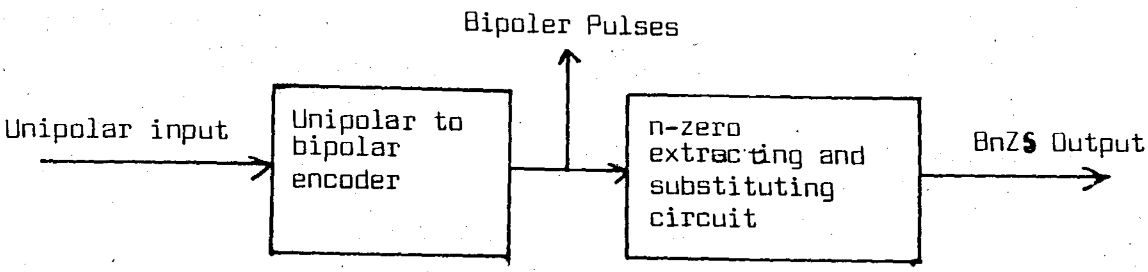


Fig.4.6 Block diagram of BnZS code

ii] Uniformed bipolar code

In this code, sequences of consecutive bipolar pulses are also substituted, in addition to consecutive zeros. The BOVOBOOV sequences are used for eight consecutive zeros and BOOVBOOV sequences are used for eight consecutive bipolar pulses. If timing content of bipolar stream is low, the fillins format tends to increase it, viceversa. The pulses are therefore uniformly distributed.

iii] High density bipolar (HDB) and Compatible high density bipolar (CHDB) code

These codes have all common characteristics;their digital sum variations are identical and equal to two, irrespective of the length of the substituted zero squence.

A HDB code of order n (n≥2] denoted as HDBn, is one in which (n + 1) consecutive zeros are replaced by one of the following seauences depending on keeping of the number of B pulses between the consecutive V pulses odd. (10] (9]

BOOO V
or
0000 V
n+ 1 digits

The remaining pulses are in the original unipolar stream. If the filling sequences are

OO.....OBOV
or 0000 ..... OOV
n + 1 digits

the codes are called compatible HDBn (CHDBn). The block diagram of the encoders of these two codes are in fig 4.7 and waveforms in fig 4.8

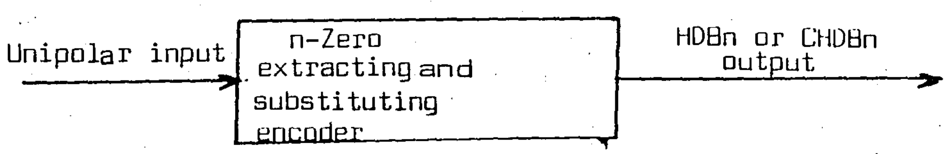


Fig.4.7 Block diagram of HDBn or CHDBn encoder

iv) Transparent interleaved bipolar (TIB) code

A TIB code of order n is derived from two ternary subsequences : (10)

1. Each subsequence normally follows bipolar encoding rule.

2. When (n+1) consecutive zeros are found in the resulting squence, they are filled by the following pattern

$$\underbrace{00\ldots00XXVV}_{n+1 \text{ digits}}$$

x denotes either a zero or a bipolar pulse chosen such that the number of B pulses between successive V pulses is add.

3. The two V pulses in the same filling pattern have opposite polarity.

The waveform of TIB code is in fig 4.8

3. Feedback balanced codes

There are two types of feedback balanced codes, one involving redundancy in the number of symbols or digits and the other in the number of amplitude levels.

The first scheme is the multilevel version of the polarity pulse coding The second cheme involves an increase in the number of possible levels for each digit or symbol. This redundancy is then used to minimize the LF components by controlling the polarity of some of the output pulses by negative feedback.

4.3 SUMMARY OF NONLINEAR LINE CODES

Nonlinear line codes are derived from input data sequence that the digitsof the encoded squence are grouped in characters of n consecutive digits, encoding being done one character at a time. Thus there is not a linear relationship between input and encoded sequences. These codes are divided into two main parts.

1. Alhabetic codes : M binary input digits are converted into the multilevel symbols, such as N ternary. Encoding procedure is state-dependent and states are determined by the RDS. Synchronization and timing recovery is achieved by the property of the state transitions, RDS, DSV and violations.

2. Nonalphabetic codes: These codes are derived for the purpose of removing the long string zeros. They used some sequence patterns with violation rule instead of consecutive zeros, so the timing recovery and error detecting will be easier.

The block diagram of the nonlinear line codes is shown in fig 4.9

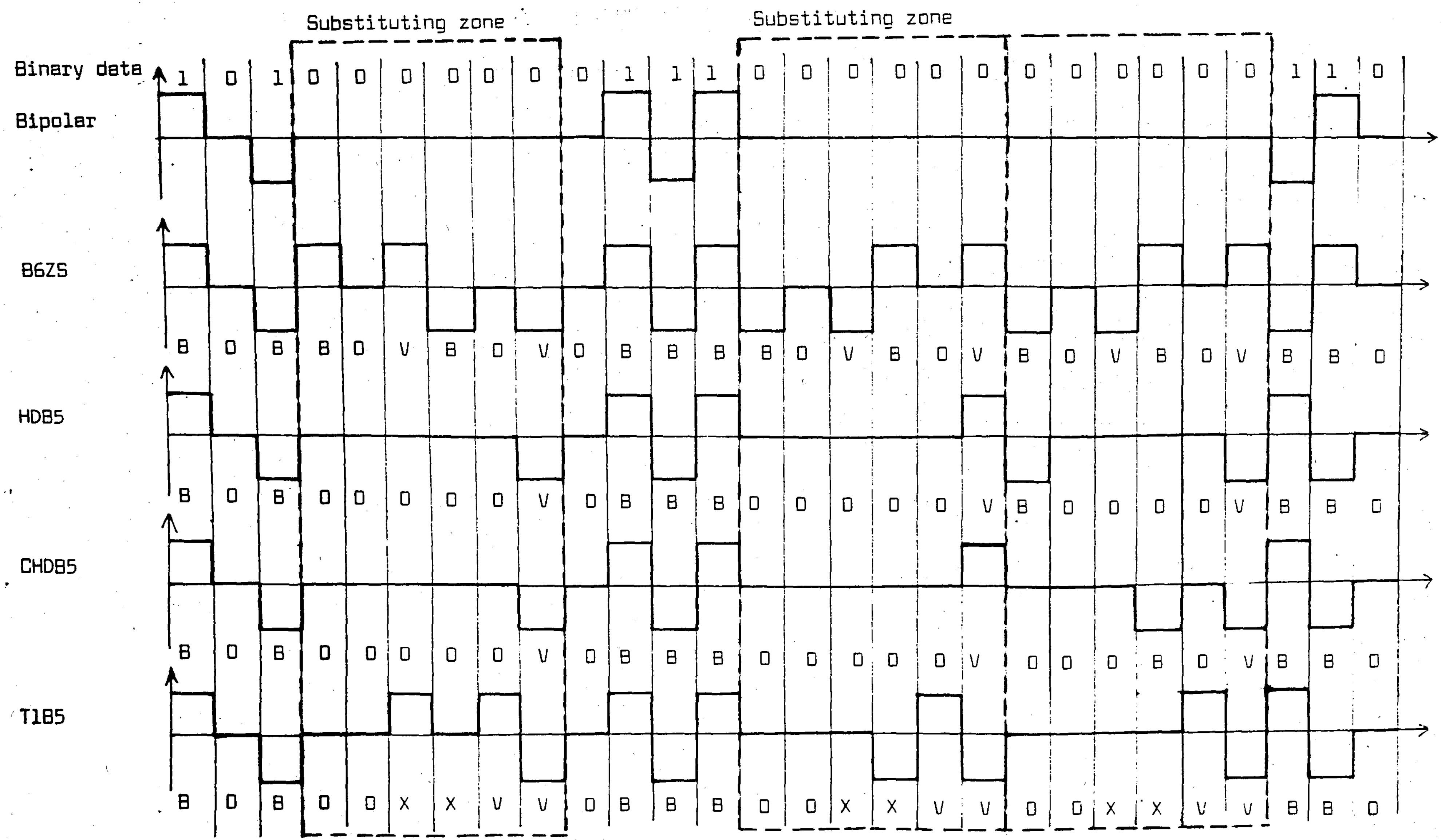


Fig. 4.8 Bipolar, B6ZS, HDB5, CHDB5, T1B5 waveforms

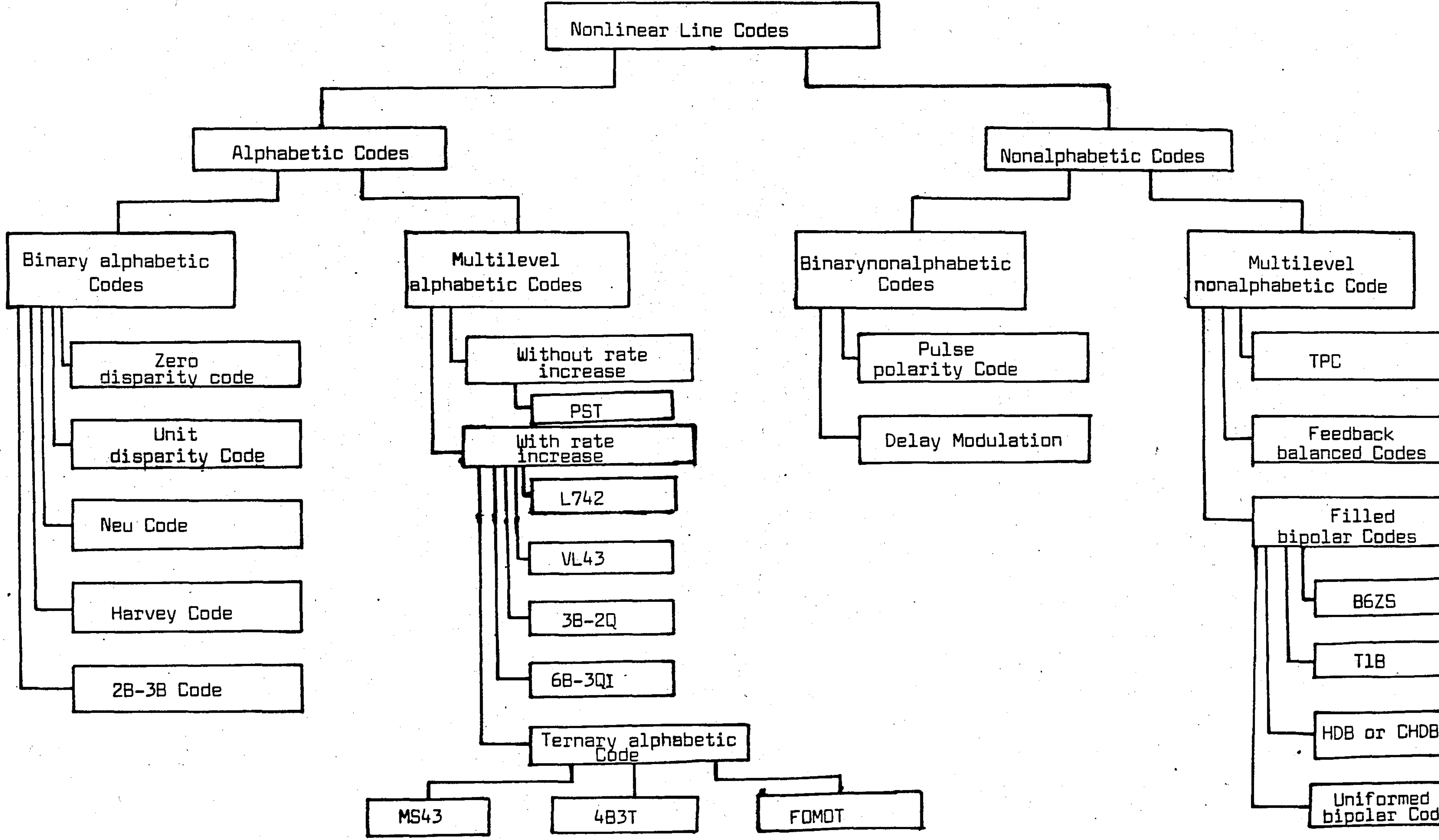


Fig 4.9 Block diagram of nonlinear line codes

4.4 COMPARISON OF VARIOUS CODES

4.4.1 Comparison of various codes without rate increase

The results of this comparison are outlined in table 4.12 [10] All of the codes in the table have three levels and information redundancy is $log_2 3$:1.58 [58%]

TABLE 4.12 Comparison of various codes without rate increase

	Bipolar	PST	HDB2,CHDB2 B3Zs	HDB3	B6Zs	CHDB5
Max.no of level	3	3	3	3	3	3
Redundancy%	58	58	58	58	58	58
DSV	1	3	2	2	3	2
Normalized timing content	1	1.25	1.57	1.02	1.28	0.98
Normalized average power	1	1.50	1.21	1.10	1.06	1.02
Max.no of consecutive zeros	∞	2	2	3	5	5

Normalized timing content is the magnitude of the spectral density at half line rate. Normalized average power of a code or its pulse density is the mean value of the spectral density that is evaluated over the whole range of the continous spectrum.

Bipolar code is the most well known code. It has lowest possible value of DSV=1, but it has disadvantage is that zeros are uncoded therefore timing information can not be derived when long strings of zeros are present.

The PST codes introduces the following problems.

1. Binary input is encoded into words of two symbols, these words.

2. More power is needed asin fig 4.12 [10].The PST code uses 50 % more power than the bipolar code so this leads increase in ISI for some transmission channels. But PST timing content is greater than bipolar timing content by 0.25 factor.

The extraction of zeros is much better achieved by adopting the B6Zs code. On the other hand HDB2, CHDB2, B3Zs are favourable for both timing and distortion indices.

LF distortion: DSV is used as a measure for the peak to peak value of the LF distortion. All LF distortion leads to a degradation of the noise margin. The average power of the destructive LF-distortion of the some codes is calculated as in table 4.13 [Fig 4.10] [2]

TABLE 4.13 Average power of destructive LF distortion of some codes.

Code	LF distortion
AMI	$\frac{1}{4} q$
HDB3	$\frac{P^2 q(1+2q+3q^2)}{(2-q^4)(1-q^4)}$
B6Zs	$\frac{q(1-q^5-2Pq^5)}{4(1-q^6)}$

Where $q = 1-p$ probability of binary zeros

p probability of binary ones

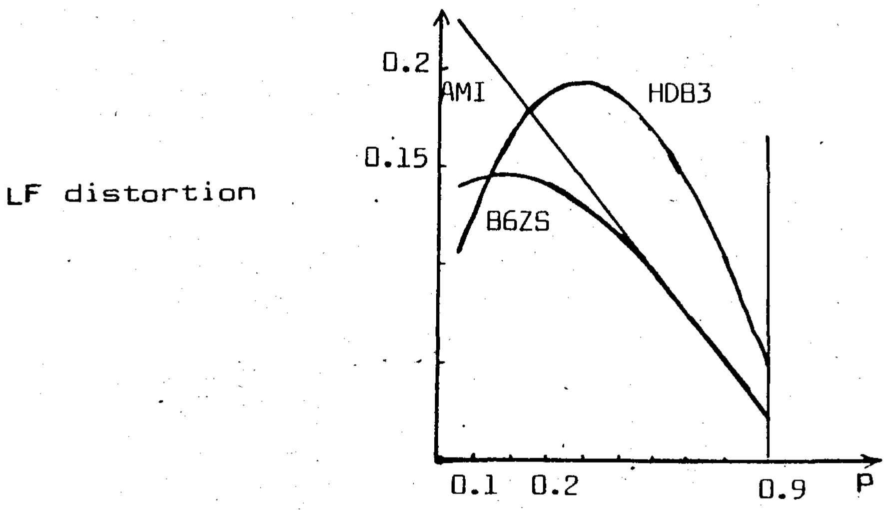


Fig.4.10 LF distortion

ISI power factor (cross talk factor) is defined for n-level digital signal as (2)

$$CPF = \sum_{i=1}^{n} \left(\frac{n-i}{n-1}\right)^2 P_{n-i} \qquad (4.17)$$

where Pn-i is the probability of transition between pulses which differ (n-i) levels in amplitude. In ternary transmission, the worst transitions are + -, - +. The transition +0, 0+, -0 and 0- contribute only one fourth as much ISI power. Thus the ISI power becomes

$CPF = \frac{1}{4} P_1 + P_2$ from equation 4.17. Table 4.14 gives the CPF for some codes. (Fig 4.11)

TABLE 4.14 crosstalk power factor of some codes

Code	CPF
AMI	$\frac{1}{2}\ p\ (1+p)$
HDB3	$\frac{p(2+2p+q^5)}{2(2-q^4)(1-q^4)}$
B6Zs	$\frac{p(1+p\ +5q^6)}{2\ (1-q^6)}$

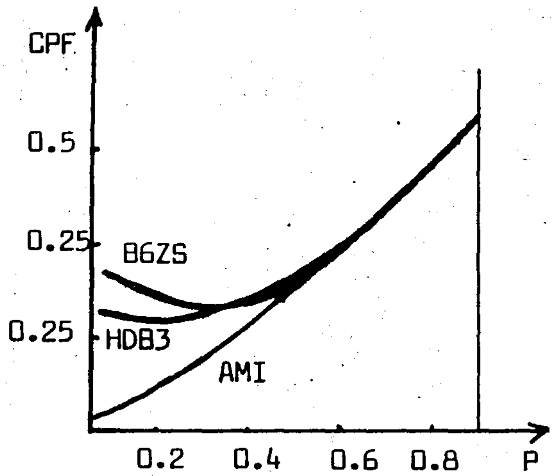


Fig.4.11 ISI factor (CPF)

4.4.2 Comparison of codes with rate increase

The most interesting codes with rate increase are the alphabetic codes with exception of PST. Various parameters of these codes can be seen from table 4.15 (10) and power spectrums is in fig 4.13 (10)

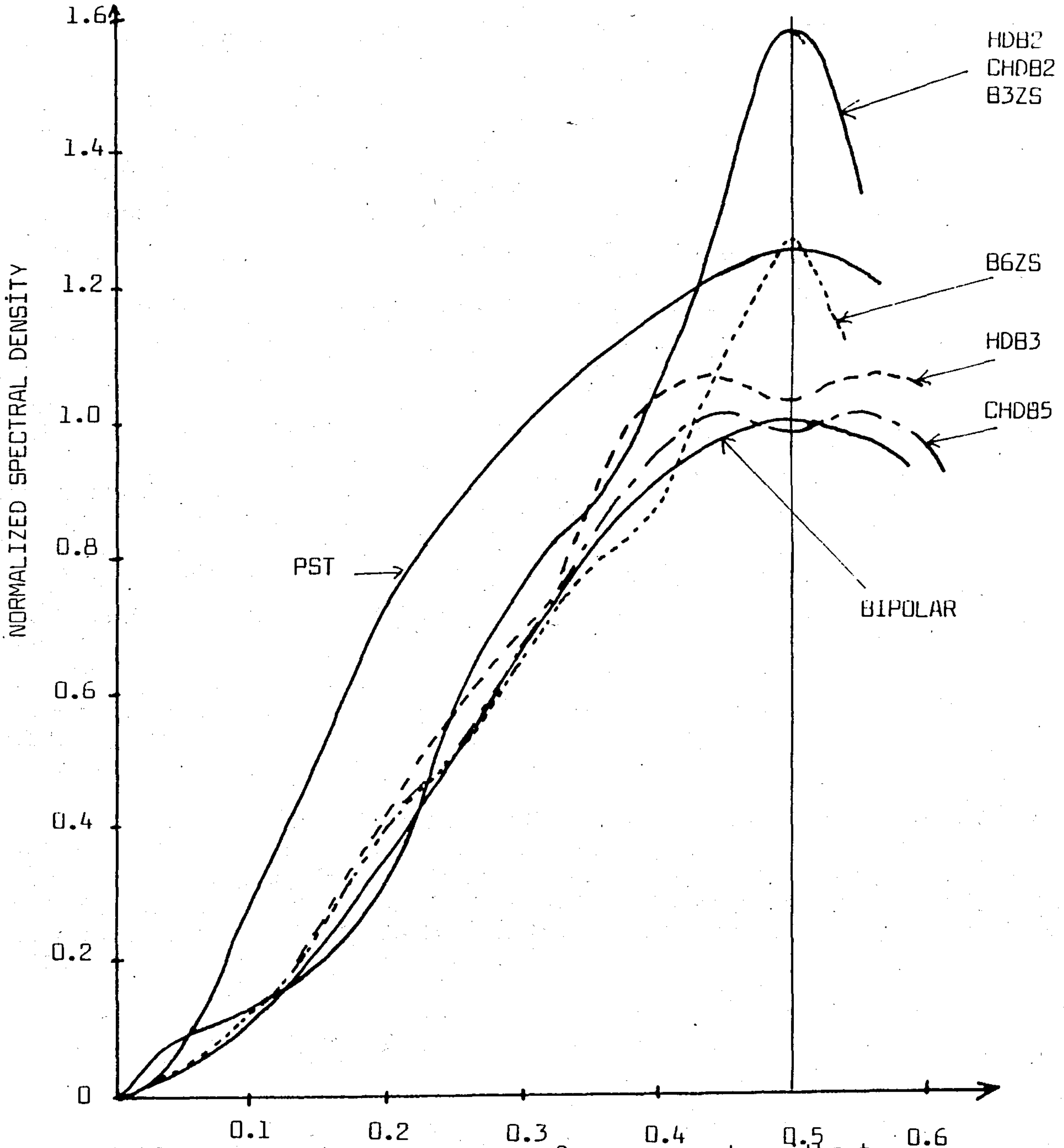


Fig.4.12 Power spectrum comparison of various codes without rate increase for symmetric random binary data

TABLE 4.15 Comparison of various alphabetic codes

	PST	4B3T (Alternate mode)	4B-3T	MS43	VL43	3B-2Q	L742
No of levels	3	3	3	3	3	3	3
Bits/word	2	4	4	4	4 or 8	3	4
Symbol/word	2	3	3	3	3 or 6	2	2
Inputrate/ linerate	1	1.33	1.33	1.33	1.33	1.5	2
Redundancy %	58	18.8	18.8	18.8	18.8	33.3	40.5
DSV	3	∞	7	5	4	9	6
Normalized timing content	1.25	0.80	0.76	0.84	-	-	0.94
Normalized average power	1.5	1.31	1.36	1.26	-	-	1.20
No.of alphabets	2	2	2	3	5	2	3
Max.no.of consecutive zeros	2	4	4	4	5	0	2
Max.no of cons. pulses of same pol.	2	5	6	6	-	2	2

There exists little difference between the above codes. The MS43 code has better performance indices than 4B-3T. It utilizes less average power for a given symmetric r ndom binary data(Fig 4.14) (10). It has lower DSV, so it has lower distortion (Fig 4.15)(10). On the other hand 4B-3T code has only two alphabets as against three for MS43.

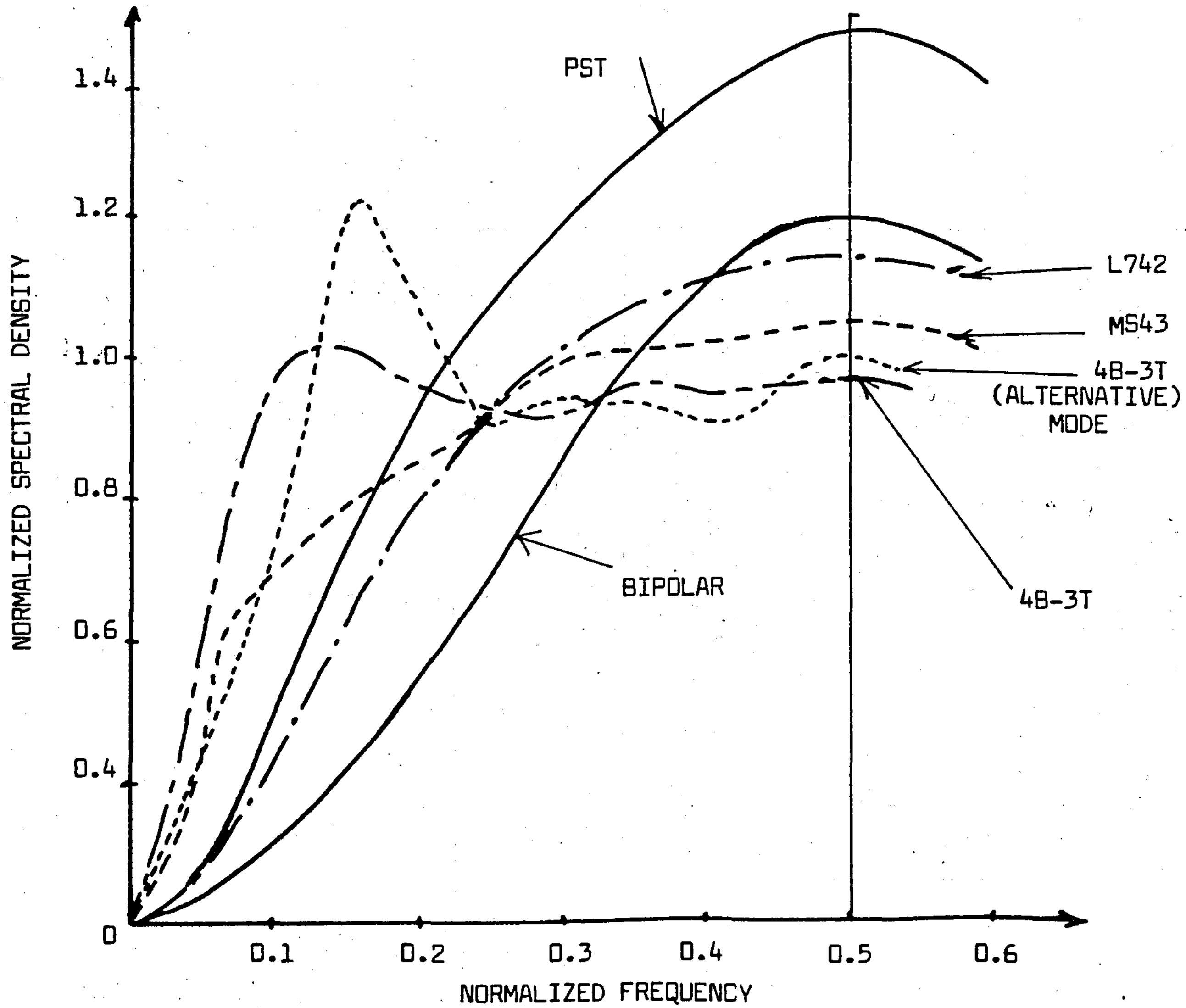


Fig.4.13 Power spectrum comparison of bipolar code and various alphabetic codes for symmetric random binary data

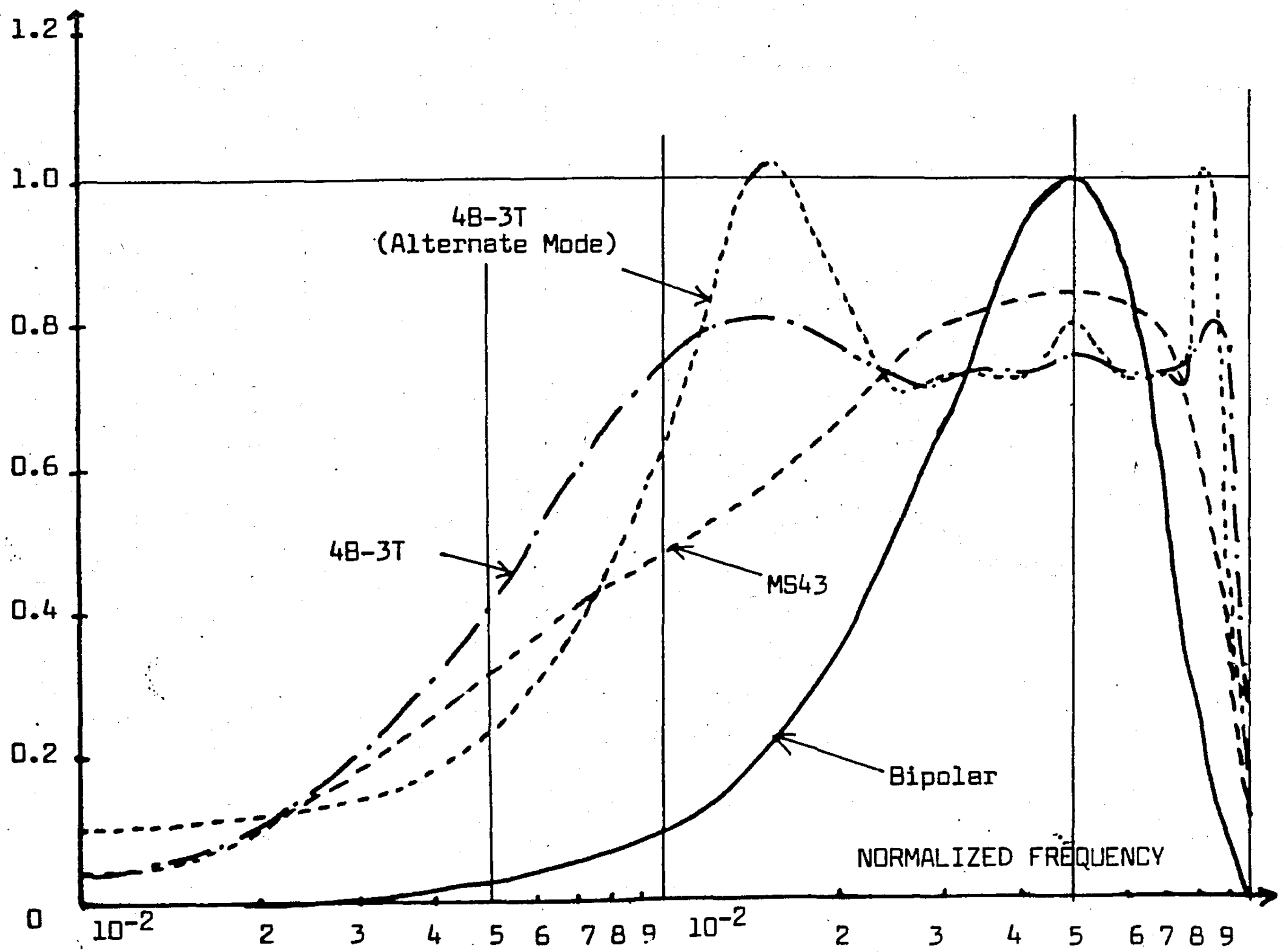


Fig.4.14 Power spectral densities of 4B-3T and MS 43 codes as compared with bipolar code

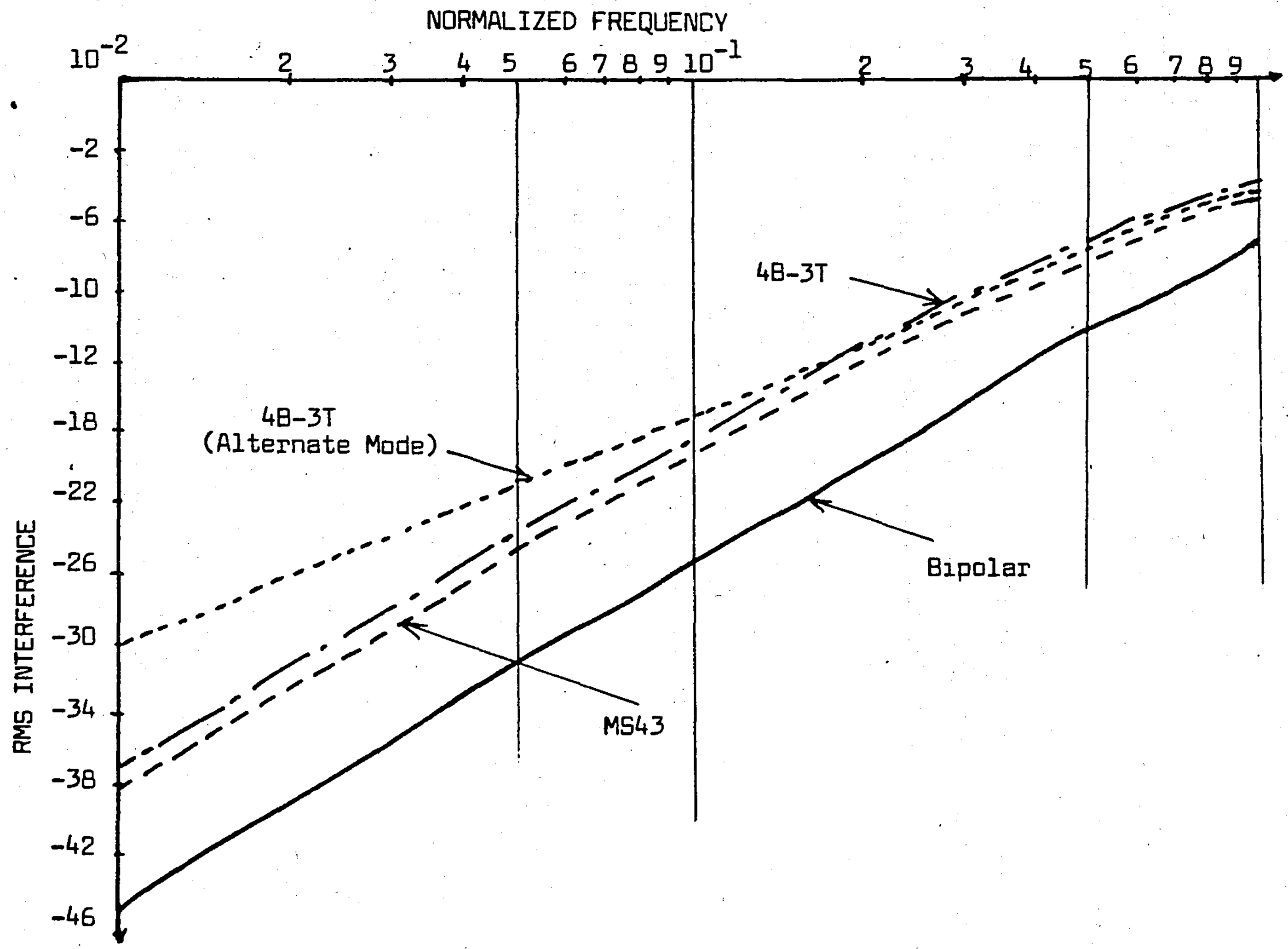


Fig.4.15 Comparison of RMS intersymbol interference due to transmission low-frequency cutoff

CHAPTER V

EVALUATION OF PROBABILITY OF ERROR IN THE PRESENCE OF ISI AND ADDITIVE NOISE

The main purpose, for designing and selecting the line codes is to minimize the symbol probability of error, P(e). This purpose presents the determination of error probability (P(e)) in the presence of ISI and additive noise that is a major problem in digital communication systems. Several methods have been proposed to evaluate the probability of error due to ISI and noise.We will try to explain inshortly form some of these methods in chronological order, detailed derivation of this methods in references.

5.1 THE METHOD OF THE SHIMBO AND CELEBILER

In recent years Shimbo and Celebiler have developed a method to compute the P(e) due to ISI for binary signals. They have considered the binary baseband system. Their main assumptions are that the symbols in data stream are mutually independent and the additive noise is gaussian noise with zero mean and variance σ_n indpendent of the (a_k). we know that the output of the sampler in binary baseband system is asin equation 2.2

$$y[\tau] = a_0 x[\tau] + \sum_{\substack{-\infty \\ k\neq 0}}^{\infty} a_k x[\tau - nT] \; n[\tau] \qquad [5.1]$$

They assumed that

$$a_k = \begin{cases} +1 & \text{with probability } 1/2 \\ -1 & \text{with probability } 1/2 \end{cases} \qquad [5.2]$$

The middle term in (5.1) is ISI and the last term in (5.1) is the additive gaussian noise. The receiver decides that

$a_0 = 1$ if $y(\tau)$ is positive

$a_0 = 1$ if $y(\tau)$ is negative

A decision error occurs if $y(\tau)$ is positive while $a_0 = -1$ or vice versa.

if the following notations are used

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} = \sum{}'$$

$$ISI = z(\tau) = \sum{}' a_n x(\tau - nT) \qquad (5.3)$$

and

$$ISI + noise = z(\tau) + n(\tau) \qquad (5.4)$$

Then the P(e) in decision can be written as

$$P(e)= \frac{1}{2}\left[\Pr\left(Z(\tau)+n(\tau)>x(\tau)\Big|_{a_0=-1}\right] +\Pr\left[Z(\tau)+n(\tau)<-x(\tau)\Big|_{a_0=1}\right]\right] \quad (5.5)$$

and

$$P(e)= \frac{1}{2}\left[1- \Pr\left[-x(\tau)\leq Z(\tau) + n(\tau)\leq x(\tau)\right]\right] \quad (5.6)$$

if

$$Q(e) = \left[\Pr\left[-x(\tau)\leq Z(\tau)+n(\tau) \leq x(\tau)\right]\right] \quad (5.7)$$

Then

$$P(e)= \frac{1}{2}\left[1-Q(e)\right] \quad (5.8)$$

AISO

$$P(e)\hat{=} \Pr\left[|Z(\tau)+n(\tau)| > X(\tau)\right] \quad (5.9)$$

$$Q(e)\hat{=} \Pr\left[|Z(\tau)+n(\tau)| < X(\tau)\right] \quad (5.10)$$

They obtained the P(e) given by (5.9) by using Gram-charlier expansion (24). The result is that the P(e) dueto additive gaussian noise of power $\sigma^2 = \sigma_n^2 + \sum_1^{\infty} x_n^2$ [where $x_k = |x(\tau - kT)|$] plus the ISI power that is not gaussian. If ISI power $\sum_1^{\infty} x_n^2$ is smaller than the noise power σ_n^2 then the sum term issmall and the series converges very fast or viceversa.

They examined their methods by using finite number of interfering pulses. First they took 10 interfering pulses (five on each side of the main pulse), they had seen that the results were the same as the exact values of P(e), then they increased the interfering pulses such as 49 pulses, they observed no change in P(e).

Ho and yeh derived this same results for multilevel signals with the same assumption that the symbols in data stream are mutually independent. For most baseband data transmission systems in which line coding is used that produces a correlated data stream. For such a data signal the evaluation of P(e) becomes even more difficult.

Glave tried to derive a general upper bound on the P(e) due to ISI for correlated data signals. (14)

5.2 THE METHOD OF GLAVE

Glave considered the system as in fig 5.1

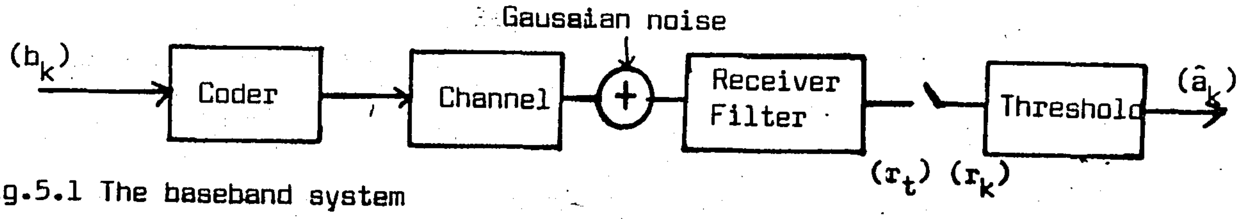


Fig.5.1 The baseband system

We know that again from 2.2

$y(\tau)= a_o x(\tau) + a_k x(\tau-kT) + n(\tau)$

if a sequence (a_k) represents an in dependent binary sequence ($a_k = \mp 1$) then the probability of error will be as in Shimbo and Celebiler

$$P(e)= \frac{1}{2} Pr\left[| Z(\tau)+n(\tau)| > x(\tau)\right] \tag{5.11}$$

Let's consider a code with corelated symbols such as bipolar and PST codes. For the bipolar code, the probability distribution for each a_k is

$$\begin{aligned} Pr\{a_k=1\} &= Pr\{a_k=-1\} = \frac{1}{4} \\ Pr\{a_k=o\} &= \frac{1}{2} \end{aligned} \tag{5.12}$$

For a fixed k the preceding symbol and succeeding symbol are each correlated with the symbol a_k is

$$R(n)= E\{a_k \; a_{k\mp n}\} = \begin{cases} -\frac{1}{4} & n=1 \\ 0 & n>1 \end{cases} \tag{5.13}$$

In the PST code asin table 4.3

$$\begin{aligned} Pr\{a_k=1\} &= Pr\{a_k\}= -1 = \frac{3}{8} \\ Pr\{a_k=o\} &= \frac{1}{4} \end{aligned} \tag{5.14}$$

and

$$R(n) = E\{a_k \; a_{k\mp n}\} = \begin{cases} -3/8 & n=1 \\ 0 & n>1 \end{cases}$$

For either of these ternary codes P(e) can be expressed as

$P(e)= Pr\left[z(\tau)+n(\tau) > x(\tau)/_2 \mid a_o=-1\right] Pr\left[a_o=-1\right] +$

$$+ \Pr\left[|Z(\tau)+n(\tau)| > x(\tau)/2 \mid a_o=0\right] \Pr\left[a_o=0\right] + \quad (5.16)$$

$$+ \Pr\left[Z(\tau)+n(\tau) < -x(\tau)/2 \mid a_o=-1\right]\left(\Pr a_o=-1\right]$$

An upper bound on this probability can be obtained as

$$P(e) \leqslant \Pr\left[|Z(\tau)+n(\tau)| \geqslant x(\tau)/2\right] \quad (5.17)$$

The peak value of ISI can be bounded by some constant A. For the independent binary code

$$A = \sum_{-\infty}^{\infty} |x(\tau-kT)| \quad (5.18)$$

For the correlated codes the above value A can be used or a bound on the peak value of ISI is derived by using knowledge of the structure of the code and the pulse response x(t). With this knowledge, there appears the following question; what is the distribution for random variable Z that maximizes the $\Pr\left[|Z+n| \geqslant K\right]$ (5.19)

Subject to the constraints

i) $|Z| \leqslant A$

ii) $E\{Z^2\} < \sigma_o^2$

iii) $n \sim N(0, \sigma_n^2)$

if this distribution can be found the resulting value of $\Pr\left[|Z+n| \geqslant K\right]$ will provide an upper bound on the P(e) due to ISI. (14) Glave found that the bound on $\Pr\left[|Z+n| \geqslant K\right]$ is; with the condition

$$(K-A) \geqslant 3\sqrt{\sigma_n} \quad (5.20)$$

$$\Pr\left[|Z+n| \geqslant K\right] \frac{\sigma_o^2}{A}\left[Q\left(\frac{K-A}{\sigma_n}\right)+Q\left(\frac{K+A}{\sigma_n}\right)\right]+2\left(1-\frac{\sigma_o^2}{\sigma_n}\right)Q\left[\frac{K}{\sigma_n}\right] \quad (5.21)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2}\, dy \quad (5.22)$$

(K-A) is the eye openning, that the condition in (5.20) requires the effective eye openning be almost two standart deviations.

These bounds have a general validity but they often turn out to be quite loose. Cariolaro and Rupolin developed the moment approximation method for correlated digital signals depending on the assumption that correlated symbols are produced by a general finite sequential machine.

5.3 THE MOMENT APPROXIMATION METHOD

Generation of correlated signals are show in Fig 5.2

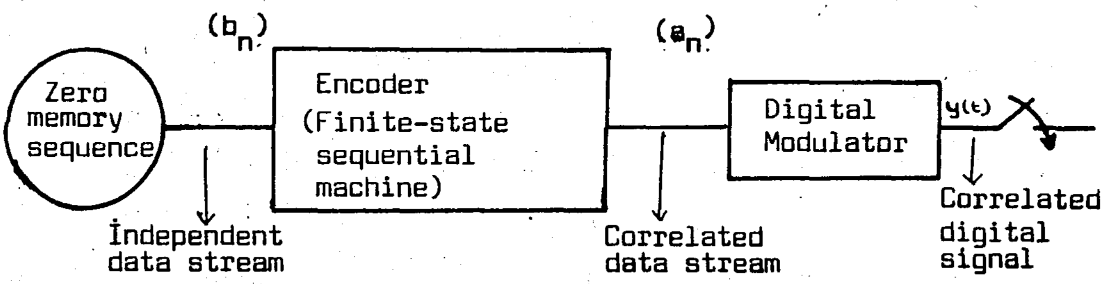


Fig.5.2 Generation of correlated digital signals

The output of the encoder (M,N encoder) is correlated data stream which is correlated in dependence on the sequential-machine laws and the source probabilities. The correlated data stream is converted into a digital signal by a PAM modulator.

Representation of correlated digital signals:

1. A_i and A_o be respectively $\Lambda_i$ary and $\Lambda_o$ary alphabets
2. Sequential machine which is the (M, N) encoder is described as:

$$M= (\phi ,O, \varphi, g,h) \tag{5.23}$$

Where

$\phi= A_i^M$ (in put set of source words)

$O= A_o^N$ (output set of codewords)

$\varphi = (S_1.........S_I)$ (state set)

g: transition function $O x \varphi \rightarrow \varphi$

h: output function $\varphi x \phi \rightarrow O$

This definitions are valid for alphabetic line codes. They found P(e) by using the conditional moments of ISI where the contidition is $(a_o^{(v)}=1)$=(symbol 1 has been transmitted) 1 EA_o. ($a_o^{(v)}$ means the wth digits of a_o codeword).

From previous two methods, the P(e) is

$$P(e) \triangleq Pr\left[|z+n| > K\right]$$

Now

$$P(e) = Pr\left[|z+n| > K \mid a_o^{(v)}=1\right] \qquad (5.24)$$

and

$$F_{Z+n}(K|1) = \int_{\text{range } z} Fn(K-\xi)\, dF_z(\xi|1) \qquad (5.25)$$

Where $F_{Z+n}(K/1)$ is the conditional distribution of (z+n). (density function of (z+n))

And this distribution can be described in terms of conditional ISI moments $R_h(1)$.

$$F_{z+n}(K/1) = \sum_{h=o}^{\infty} (-1)^h\, Fn^{(h)}(k) \frac{Rh(1)}{h!} \qquad (5.26)$$

where

$$R_h(1) = \int \xi^h\, dF_z(\xi/1) \qquad (5.27)$$

If we summarize the above methods. All of the three methods are used the same following assumption Additive noise is gaussian noise with zero mean and variance σ_n^2 , and are connected at the same point that finding the P(e) = $Pr(|z+N| > K)$ and the distribution function (density function) of the (z+n) $F_{z+n}(.)$. Shimbo and celebiler tried to derive these for binary signals with assumption that the symbols of the in put data is not correlated. Glave developed a methot for correlated signals but not complicated. Moment approximation method is used the correlated digital signals which are produced by a general finite state sequential machine and it determines the P(e) in terms of conditional moments of ISI with condition $(a_o^{(v)}=1)$.

Celebiler's and Glave's methods consider an L_t interfering sample approximation to the real system impulse response. Large value of L_t is needed to represent the real systems. When L_t is large, the calculations will be difficult and complex. In moment approximation method Large number of interferes are used while avoiding the computational complexity. In this method, it is assumed that ISI goes thru a finite length of L consecutive codewords, thus including L_t=LN-1 interfering terms (Where N is the encoded word length.)

In 1982 Jakubow, Chang and Garcia developed a model for finding P(e) due to ISI and additive noise. This model is also called RDS model. (4)

5.4 RDS METHOD

We know that P(e) written as for ternary codes,

$$\begin{aligned} P(e) = \; & Pr((Z+n)<-1/2\,|+)\;P(+)+ \\ & Pr((Z+n)>1/2\,|-)\;P(-)+ \\ & Pr(|Z+n|>\tfrac{1}{2}\,|0)\;\;P(0) \end{aligned} \qquad (5.28)$$

From the balanced property implies that P(+)=P(-)
and

$$\begin{aligned} P((Z+n)>1/2\,|-) &= P((Z+n)<-1/2\,|+) \\ P((Z+n)>1/2\;\;0) &= P((Z+n)<-1/2\,|0) \end{aligned} \qquad (5.29)$$

Then

$$P(e)=2\;P((Z+n)<-1/2\,|+)\;P(+)+2\;P((Z+n)<-1/2\,|0)\;P(o) \qquad (5.30)$$

and

$$\begin{aligned} P(e)= \; & P(+)\int_{\text{range}Z} Q\left[\frac{\frac{1}{2}+I}{\sigma_n}\right] F_Z\,(I\,|+)dI\;+ \\ & P(o)\int_{\text{range}Z} Q\left[\frac{\frac{1}{2}+I}{\sigma_n}\right] F_Z\,(I\,|\,o)\;dI \end{aligned} \qquad (5.31)$$

This method is based on the assumption that the overall channel consists of a raised cosine characteristics $x(f)$ in tandem with two single pole transformers. The transfer function of the transformer network is

$$x'(f)= \left(\frac{if}{if+fc}\right)^2 \qquad (5.32)$$

Then the overall channel transfer function is

$$G'(f) = x(f)x'(f) \qquad [5.33]$$

where $x[f] = G_R[f]\ C[f]\ G_T[f]$ [see section 1.1]

The pulse sample can be found by DFT, $g'(KT)$ for values of $0<\beta<\frac{1}{2T}$ and $Fc = (\frac{fc}{f})$ $0.1 < Fc < 1$ (where $f = \frac{1}{T}$). In all cases the pulse shapes have similar features: ISI sample prior to the main sample is very small and taking the shape of very sbwly decaying tail. P(e) can be found very accurate for all values of channel parameters and for all codes.

The raised cosine channel takes the shape of long decaying tail that follows the main sample. As the tail length increases, the RDS can be used as an estimator of the distortion because it represents the net accumulation of tails at each symbol instant. The model is that ISI induced by a symbol consisting of a infinite duration constant amplitude tail of polarity opposite of the symbol. (Fig 5.3)

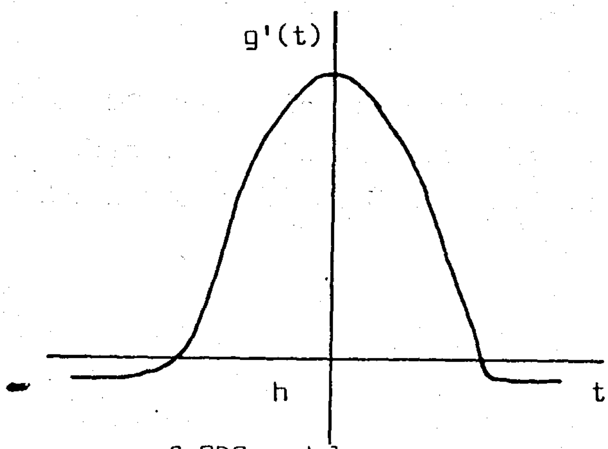


Fig.5.3 Impulse response of RDS model

DSV of the code is finite and take only the discrete values. Distribution of ISI will be symmetric about zero from the balanced property of the code. Then the set of allowable states (S) must be

$$S = (0, \pm, \ldots.\pm DSV/2) \text{ when DVS is even}$$
$$S = (\pm 1/2, \ldots.+, DSV/2) \text{ when DSV is odd} \qquad [5.34]$$

Depending on this model, the actual value of ISI associated with states is,

$$Z(s) = -s.h \qquad [5.35]$$

where h is the amplitude of the tail.
Then the density function of the ISI is

$$F_{\mathcal{Z}}(\xi/\imath) = \sum_{s \in s_T} P(\mathcal{Z}(s)/\imath)\ \delta(\xi - \mathcal{Z}(s)) \qquad \imath \ (-1,0,+1) \qquad [5.36]$$

and

$$P(e) = \sum_{s \in s_T} Q\left(\frac{\frac{1}{2} + \mathcal{Z}(s)}{\epsilon_n}\right)\left(P(+)\ P(s/+) + P(o)P(s/o)\right) \qquad [5.37]$$

5.4.1 Comparisons of P(e) calculations dependingon moment-approximation and RDS method

Bipolar, MS43 and 8B6T codes are used to test the methods . For design of Fc $\leq$0,5 percent RDS model deviates from the moment approximation model by less than 0,2 dB at $P(e)=10^{-10}$, Fig 5.4 shows the curves for RDS model at Fc=0,5. The deviation from the moment method increases with the increase in cutoff frequency Fc. Because increment in Fc causes the rate of decay of the tail to increase. This making the RDS model less valid. In Fig 4.5 the difference between RDS and moment method are shown at Fc=0.7, Itis seen that deviation is worst at high SNR.

In general RDS model is an accurate methods and useful for calculating P(e) when Fc $\leq$0.5 percent (by using 5.37) And the moment approximation method for values of Fc>0.5 percent.

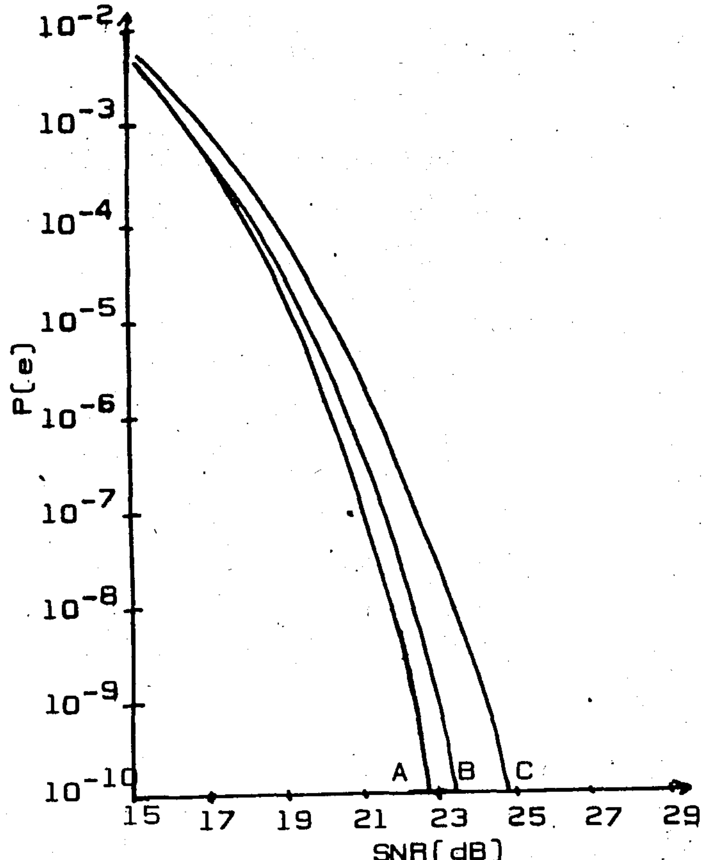


Fig 5.4 Performance of codes in RDS model
A: Bipolar B:8B6T C:MS43 (4)

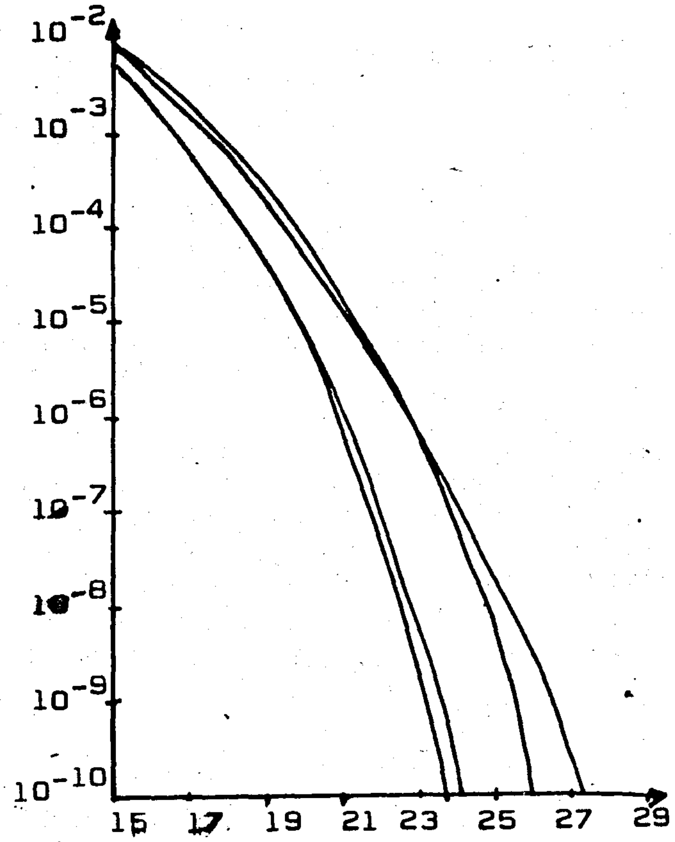


Fig 5.5 Performance of codes
A: 8B6T(RDS) B:8B6T (moment)
C: MS43 (RDS) B:MS43 (moment) (4)

CHAPTER VI

CODE DESIGN ALGORITHM BASED ON RDS MODEL

RDS model allows us to develope new code design algorithms that optimize the P[e] performance.

6.1 CODE DESIGN CRITERIA

P[e] performance depends on the ISI performance we must set up the criteria that measure the ISI performance.DSV can be related to the ISI performance i.e. ISI can be controlled by making DSV as small as possible, but it fails as an indicator of P[e] performance because bipolar code has the DSV= 1 that is less than the DSV of PST code which is three.

Power spectral density of the transmitted sequence is another indicator of the ISI performance, because rms ISI is related analyti cally to the power spectral density , in particular low frequency [LF] component.The power spectral density is difficult to compute since it involves computing autocorrelation function of transmitted sequence:

In RDS model ISI is modelled as [4]

$$Z = -s.h \tag{6.1}$$

where s is the state,h is the tail amplitude.

Then the variance of the RDS will be proportional to the mean ISI power.

6.1.1 Var[S] [4]

From [6.1]

$$E\left\{Z^2\right\} = h^2 var[S] \tag{6.2}$$

where

$$var[S] = \frac{1}{N}\sum_{j=1}^{N} E\left\{S_j^2\right\} = E\left\{\frac{1}{N}\sum_{j=1}^{N} S_j^2\right\} \tag{6.3}$$

because $E\left\{S_j\right\} = 0 \quad j=1,\ldots.N$ from the balanced property.

Where N is the encoded word lenght and S_j is the RDS at time j.

By assuming on the initial terminal state S_o and then each code word traces a specific trajectory of states we obtain,

$$Var[S] = \sum_{S_o \in S_T} P_o[s_o]\, E\left\{\frac{1}{N}\sum_{j=1}^{N} S_j^2 \quad S_o = s_o\right\}$$

$$= \sum_{s_o \in S_T} P_o[s_o] \sum_{a \in c(s_o)} Pr[a/s_o] \left[\frac{1}{N}\sum_{j=1}^{N}[s_o + \sum_{j=1}^{t} a_j]^2\right]$$

for an independent equiprobable binary information source $Pr[a/s_0]=2^{-M}$ yielding

$$Var[S]=\sum_{s_0 \in S_T} P_0[s_0] \sum_{a \in c[s_0]} 2^{-M} \left[\frac{1}{N} \sum_{j=1}^{N} [s_0 + \sum_{J=1} a_j]^2 \right] \quad [6.4]$$

where $P_0[s]$ terminal state stationary probabilities, M is the number of input data bits. $a_j=[a_j[1]\ldots a_j[N]]$ is the encoded jth code word in C[S]. If we use the following substitutions

$$\lambda_v(a/s)=\frac{1}{N}\sum_{t=1}^{N}(s+\sum_{j=1} a_j)^2 \quad [6.5]$$

we can also write the encoded word [a] cost $\lambda_v[a/s]$ as

$$\lambda_v(a/s)=\frac{1}{N}\sum_{t=1}^{N} RDS_t^2 \quad [6.6]$$

and the alphabet cost [C[s]] is

$$\lambda_v(C(s))=\sum_{a \in C(s)} 2^{-M}\lambda_v(a/s) \quad [6.7]$$

then

$$Var(s) = \sum_{s \in S_T} P_0(s)\ \lambda_v(C(s)) \quad [6.8]$$

The second approach is adding the level information of the transmitted sequence. This occurs when a nonzero transmitted signal encounters the ISI of the polarity.

6.1.2 $Var_D[s]$ [4]

It is the same as [6.8]

$$var_D[s]=\sum_{s \in S_T} P_0[s]\ [C[s]] \quad [6.9]$$

where

$$\lambda_D[C[s]]=\sum_{a \in C[s]} 2^{-M}\lambda_D[a/s] \quad [6.10]$$

where

$$\lambda_D[a/s]= \frac{1}{N}\sum_{t=1}^{N} RDS_{t-1}^2 F[a_t, RDS_{t-1}] \quad [6.11]$$

where

$$RDS_{t-1}=s_0+\sum_{j=0}^{t-1} a_j \quad [6.12]$$

where $a_0=0$ and s_0 is the state of the encoder and

$$F(a_t, RDS_{t-1}) = \begin{cases} \delta_{a_{t,0}} + \delta_{a_{t,-}} & RDS_{t-1} \geq 0 \\ \delta_{a_{t,0}} + \delta_{a_{t,+}} & S_{t-1} < 0 \end{cases} \qquad [6.13]$$

where $\delta_{m,n}$ is kronecker delta function.

where RDS_{t-1} $[S_{t-1}]$ be the state at time $(t-1)$ and the symbol a_t is transmitted at time t.Then the ISI will add destructively to a_t if a_t is zero or if a_t is opposite polarity than RDS_{t-1}.$F(a_t, RDS_{t-1})$ is an indicator function for destructive ISI.
Example: Finding procedure of var(S) and $var_D(S)$ of 4B-3T code which is given in table 6.1 .

a) Var(S):From table 4.3
N=3,M=4

$P_o(-3)=P_o(2)=\frac{1}{30}$

$P_O(-2)=P_o(1)=\frac{4}{30}$

$P_o(-1)=P_o(0)=\frac{10}{30}$

and from equation 6.6

$$\lambda_\sigma(a/s)=\frac{1}{3}\sum_{t=1}^{3} RDS_t^2$$

TABLE 6.1 ternary word cost $[\lambda_\sigma(a/s)]$ of 4B-3T code

input data/ST	-3	-2	-1	0	1	2
0000	34/3	17/3	6/3	1/3	2/3	9/3
0001	34/3	17/3	6/3	1/3	2/3	9/3
0010	41/3	22/3	9/3	2/3	1/3	6/3
0011	17/3	6/3	1/3	2/3	1/3	6/3
0100	14/3	5/3	2/3	5/3	2/3	5/3
0101	17/3	6/3	1/3	2/3	1/3	6/3
0110	22/3	9/3	2/3	1/3	2/3	9/3
0111	29/3	14/3	5/3	2/3	5/3	14/3
1000	22/3	9/3	2/3	1/3	6/3	17/3
1001	22/3	9/3	2/3	1/3	6/3	17/3
1010	17/3	6/3	1/3	2/3	9/3	22/3
1011	12/3	3/3	0/3	3/3	0/3	3/3
1100	9/3	2/3	1/3	6/3	1/3	2/3
1101	6/3	1/3	2/3	9/3	2/3	1/3
1110	9/3	2/3	1/3	6/3	1/3	2/3
1111	5/3	2/3	5/3	14/3	5/3	2/3

and

$$\lambda_v\ (C(s)) = 2^{-M} \sum_{a\ C(s)} \lambda_v\ (a/s)$$

$$\lambda_v\ (C(-3)) = \frac{310}{3} \cdot \frac{1}{16} = 6.4583$$

$$\lambda_v\ (C(-2)) = \frac{130}{3} \cdot \frac{1}{16} = 2.7083$$

$$\lambda_v\ (C(-1)) = \frac{46}{3} \cdot \frac{1}{16} = 0.9583$$

$$\lambda_v\ (C(0)) = \frac{58}{3} \cdot \frac{1}{16} = 1.2083$$

$$\lambda_v\ (C(1)) = \frac{46}{3} \cdot \frac{1}{16} = 0.9583$$

$$\lambda_v\ (C(2)) = \frac{130}{3} \cdot \frac{1}{16} = 2.7083$$

and

$$var(s) = \sum_{\epsilon} P_0(s)\ \lambda_v\ (C(s))$$

$$= \frac{1}{30} \cdot 6.4583 + \frac{4}{30} \cdot 2.7083 + \frac{10}{30} \cdot 0.9583 + \frac{10}{30}\ 1.2083 +$$
$$+ \frac{4}{30}\ 0.9583 + \frac{1}{30}\ 2.7083$$

$$var(s) = 1.5196$$

b) $Var_D(s)$

$$\lambda_D\ (a/s) = \frac{1}{3} \sum_{t=1}^{3} RDS^2_{t-1} F(a_t, s_{t-1})$$

$$\lambda_D(C(s)) = 2^{-16} \sum_{a \in C(s)} \lambda_D(a/s)$$

$$Var(s) = \sum_{s \in s_T} P_o(s)\, \lambda_D(C(s))$$

TABLE 6.2 ternary word cost (λ_D (a/s))of 4B-3T code

input data/S_T	-3	-2	-1	0	1	2
0000	25/3	13/3	5/3	0	1/3	2/3
0001	25/3	13/3	5/3	0	2/3	8/3
0010	32/3	18/3	8/3	1/3	1/3	5/3
0011	18/3	8/3	2/3	0	2/3	8/3
0100	22/3	9/3	2/3	1/3	2/3	9/3
0101	22/3	9/3	2/3	1/3	2/3	9/3
0110	27/3	12/3	3/3	0	3/3	12/3
0111	25/3	13/3	5/3	1/3	5/3	13/3
1000	18/3	8/3	2/3	1/3	5/3	13/3
1001	22/3	8/3	2/3	1/3	5/3	13/3
1010	13/3	5/3	1/3	2/3	8/3	18/3
1011	17/3	6/3	1/3	2/3	1/3	6/3
1100	17/3	6/3	1/3	2/3	1/3	6/3
1101	14/3	5/3	2/3	5/3	2/3	5/3
1110	13/3	5/3	1/3	1/3	1/3	5/3
1111	13/3	5/3	2/3	5/3	2/3	5/3

$$\lambda_D(C(-3)) = \frac{323}{3} \cdot \frac{1}{16}$$

$$\lambda_D(C(-2)) = \frac{143}{3} \cdot \frac{1}{16}$$

$$\lambda_D(C(-1)) = \frac{44}{3} \cdot \frac{1}{16}$$

$$\lambda_D(C(0)) = \frac{23}{3} \cdot \frac{1}{16}$$

$$\lambda_D(C(1)) = \frac{44}{3} \cdot \frac{1}{16}$$

$$\lambda_D(C(2)) = \frac{143}{3} \cdot \frac{1}{16}$$

$var_D(s) = 1.308$

The above example indicates that $var(s) < var_D(s)$. This result is obvious because destructive ISI power and the P(e) tended to be greater for Var(s) criterion.

Another measure of the line codes ISI performance is the symbol error probability.

6.1.3 Symbol error probability (4)

It is possible to compute the conditional probability of error for a given symbol and given RDS.

$$P(e) = \sum_{s \in S_{T}} P_o(s) \quad \lambda_p(C[s]) \tag{6.14}$$

where

$$\lambda_p(C(s)) = \sum_{a \in C(s)} 2^{-M} \lambda_p(a/s) \tag{6.15}$$

where

$$\lambda_p(a/s) = \frac{1}{N} \sum_{t=1}^{N} P(s,a,t) \tag{6.16}$$

where P(s,a,t) is the conditional probability of error in the t th symbol in a given an initial values of RDS.

$$P(s,a,t) = \frac{1}{2} \, Q\left[\frac{1/2 + RDS_{t-1} \cdot h}{6n}\right] \left(\delta_{a_{t,+}} + \delta_{a_{t,0}} \right) + \frac{1}{2} \, Q\left[\frac{1/2 + RDS_{t-1} \cdot h}{6n}\right] \left(\delta_{a_{t,-}} + \delta_{a_{t,0}} \right) \tag{6.17}$$

The design criterion requires the tail amplitude h and SNR be specified.

6.2 THE CODE DESIGN ALGORITHM

This algorithm involves the spesification of code words or alphabet C[s] for each of the terminal states s of the code which is a given class MBNT code, so that, the cost is minimized. Cost is var[S] as in (6.8)

$$Cost=var[S]=\sum_{S\in s_T} P_o[s]\,\lambda_v[C[s]]$$

where from (6.7)

$$\lambda_v[C[s]]=2^{-M}\sum_{a\in C[s]}\lambda_v[a/s]$$

Algorithm is computed into two steps. These steps are:

Step 1: For each terminal state s, order the set of codeword allowed for use in state s, than choose lovest cast 2^M codewords, call this alphabet $C^{[1]}[s]$ and find the set of terminal state stationary probabilities $P_o^{[1]}[s]$.

Step 2: Total average cost (6.8) can be reduced by increasing the terminal state stationary probabilities. Increase these probabilities then test the reduction in average cost. This is done as follows: For each terminal state s, examine the allowable codewords that are not in $C^{[1]}[s]$. If a codeword leads to a state which has a low $\lambda_v[a/s]$, it is then exchanged with a codeword in $C^{[1]}[s]$ has higher $\lambda_v[a/s]$. Then the resulting code measure is computed if the measure is reduced , choose the new code. This procedure is repeated untill all possible exchanges have been tested.

6.2.1 Results of Algorithms

Table 6.3 shows the performance ofthe best code (which is found by the defined algorithm) found for each three criteria (var[S], $var_D[S]$, P[e]). For the P[e] criterion three values of tail amplitude h were tested. The measurements are taken at a value of SNR=23dB. The results indicated that :

1. Codes designed according to $var_D[S]$ and P[e] criteria yielded codes with very similar performance in destructive ISI power, probability of error and power spectral density.
2. Codes designed according to var[S] tended to differ from those using $var_D[S]$ and P[e] criteria; the destructive ISI power and the probability of error tended to be greater for var[S] criterion. However var[S] yielded the codes with the lowest power content at low frequencies.

For a DSV less than 3 there was no difference in the spectra; for DSV greater than 3 the spectra of the var[S] codes were lower. However good P[e] performance implies little destructive and high constructive ISI.

TABLE 6.3. Performance of optimized codes [4,TableI]

CODE CLASS	EFFICIENCY	DSV	VAR(s)	$VAR_D(s)$	P(e) Fc=0.1% α=0.5%	P(e) Fc=0.5% α=0.5%	P(e) Fc=1.0% α=0.5%	# TERMINAL STATES (# CODEBOOKS)
6B5T	75.7%	2	.4523	.1398	$1.16x10^{-12}$	$8.33x10^{-11}$	$1.30x10^{-8}$	3(3)
5B4T	78.9%	3	.5929	.1875	$1.24x10^{-12}$	$2.47x10^{-10}$	$2.81x10^{-7}$	4(2)
8B6T	84.1%	4	.6691	.2203	$1.29x10^{-12}$	$9.92x10^{-11}$	$2.05x10^{-6}$	3(3)
4B3T	84.1%	5	.9481	.3462	$1.58x10^{-12}$	$8.12x10^{-9}$	$4.80x10^{-6}$	4(4)
7B5T	88.3%	5	1.0050	.3954	$1.72x10^{-12}$	$6.08x10^{-9}$	$3.38x10^{-6}$	4(4)
10B7T	90.1%	6	1.1096	.4578	$1.88x10^{-12}$	$9.44x10^{-9}$	$5.45x10^{-5}$	4(4)

3. Chien [5] has found, the efficiency [E] is related to the number of allowable states or allowable RDS states. This relation is

$$E = \frac{Log_2 \lambda_{max}}{Log_2 K} \qquad [6.18]$$

where λ_{max} is the maximum eigen value of the state transition matrix and K is the level of the code. The coding requirement is represented by the constraint put upon the RDS of the coded signal stream. Thus the efficiency is related to the limits of RDS [fig.6.1].

Since efficiency determines the required transmission bandwith [see 6.21]. The selected code classes in table 6.1 depends on the relation between efficiency and allowable states.

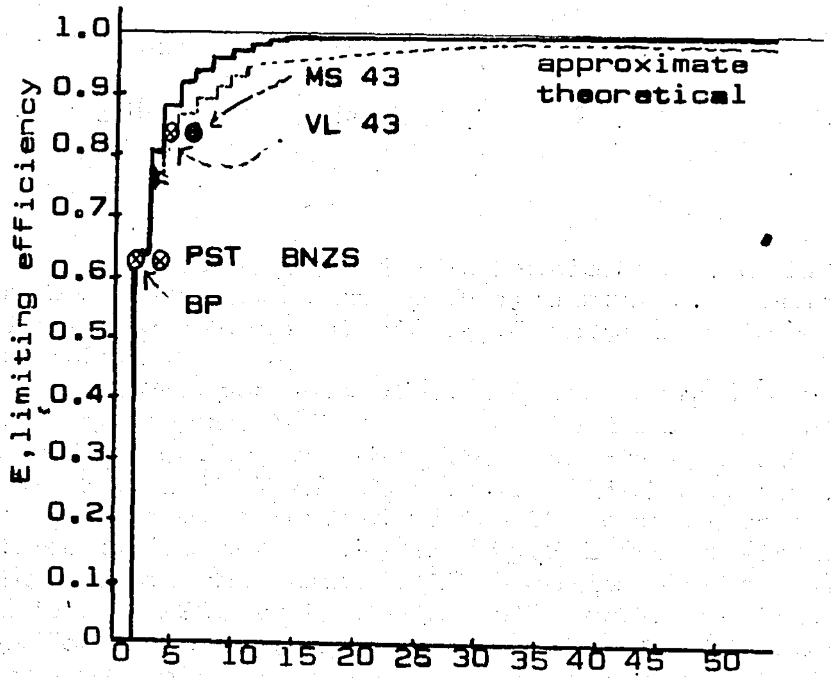


Fig. 6.1. Limiting efficiency vs allowable states ternary alphabet [$n=M^{+}-M^{-}+1$, M^{+} and M^{-} are the upper bound of allowable states (RDS)].

Another comparison was made by using three criteria, the SNR required to achieve an error of $P(e)=10^{-10}$ and $\beta=1/4T$, $F_c=0,5$ percent channel was computed. It was found that codes differ verl little SNR, these results indicated that :

1. All three criteria tended to choose the same codewords.

2. DSV may be a good indicator of P(e) performance, because the results showed that all the DSV 5 codes had very similar SNR values.

In order to test this conjecture they modified the design algorithm that the highest cost 2^{-M} codewords were selected for each alphabet. The results are in table 6.4 it can be seen that SNR values for best codes differs only slightly from the SNR values for bad codes. Then the question appears , when is DSV a good indicator of P(e) performance. They conjecture that When a code class is close to DSV-efficiency curve (fig 6.1). DSV is a good indicator of P(e) performance because high efficiency implies that there are fever choices in the selection of good codewords.

TABLE 6.4 SNR range for code class

$\beta=\frac{1}{4T}$ $F_c=0.5$ Percent, $P(e)=10^{-10}$

Code class	SNR best code	SNR bad code	Difference
6B5T	22.89	23.04	0.15
5B4T	23.21	23.32	0.11
8B6T	23	24.06	1.06
4B3T	24.35	24.50	0.15
7B5T	24.26	24.67	0.41
10B7T	24.39	24.73	0.34

The design and a selection of a code involves other criteria in addition to P(e) performance. The transmitted signal must contain timing framing and in-service monitoring functions.

An important class of codes that has been achieved in high capacity PCM systems in Europe is 4B3T class. And standart codes are the MS43 and the jessop waters (BR4B3T) code. Since 8B6T has the same efficiency as the 4B3T code so it can be compared with 4B3T code. A comparison between these codes are shown in fig.6.2 for desired $P(e)=10^{-10}$. The curves show that 4B3T code is better than the BR 4B3T code and MS43 code. The BR 4B3T code has DSV=7 than the P(e) performance is poorer than MS43 and 4B3T codes. The 8B6T code has a smaller DSV and much larger of choices for selecting codewords.

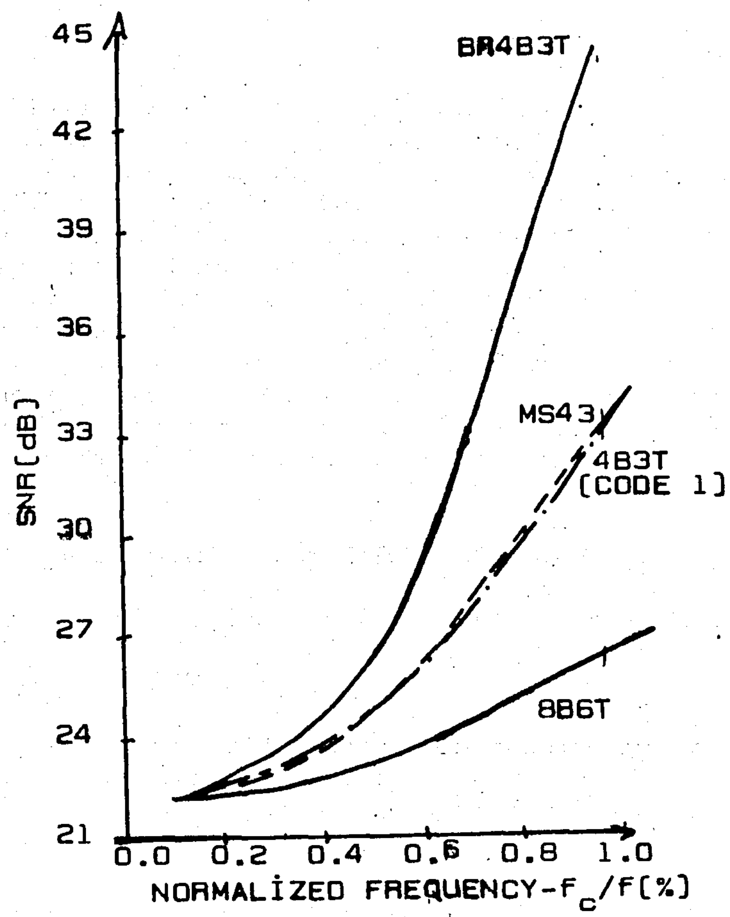


Fig. 6.2 4B3T code comparison
$P(e)=10^{-10}$, $B=1/4T$

6.3 CODE CLASS COMPARISON

In the previous section, codes were choosen according to the P(e) performance. There is no clear way which code class is optimum for a desired information since the transmission bandwith will differ according to the efficiency of the code. Consider on MBNT code on a raised cosine channel with parameter β.

The bandwith is

$$Bw=(1+\beta)\frac{R_b}{2} \tag{6.19}$$

where R_b is the baud rate and

$$R_b = \frac{N}{M} R_i \tag{6.20}$$

where R_i is the information rate in bit/sec since for ternary codes efficiency is

$$E = \frac{M}{N} \log_2 3$$

Then

$$Bw = \frac{(1+\beta)\ R_i}{2\ \log_2 3}\ \frac{1}{E} \tag{6.21}$$

Thus for a given information rate , the bandwith for transmission is invesly proportional to the efficiency of code.The different symbol rates and bandwith requirements imply different levels of noise at the decision threshold. So making comparisons between codes at the same SNR invalid. The design objective is to find the necessary signal level at the transmitter to achieve a desired error rate at the receiver given the constraints of the physical channel.Fig 6.2 and fig.6.3 shows the comparisons between above codes depending on signal level gain and symbol error probability P(e) for T4 link and European link.Signal level gain is defined as the difference in SNR between code 1 and code 2. Such as

$$\text{Signal level gain}=SNR_{1,2}=SNR_1-SNR_2 \qquad (6.22)$$

The additive noise is white.The power spectral density is

$$P_N(f) = 2KT\,Z_o \qquad (6.23)$$

where K and T are Boltzmann's constant and the absolute temperature.The noise is shaped and amplified by the receiver filter $G_R(f)$.The noise power at the decision node is given by

$$G_n^2 = 2KT_oQ\int_{-Bw}^{Bw} G_R(f)^2 df \qquad (6.24)$$

where Q is the noise figure of equalizing amplifier and Bw is the badwith of $G_R(f)$.
The equation 6.22 becomes

$$SNR_{1,2}= 20\log_{10}\left(\frac{6_2}{6_1}\right)=10\log_{10}\left(\frac{\int_0^{Bw_2}\left|G_{R2}(f)\right|^2 df}{\int_0^{Bw_1}\left|G_{R1}(f)\right|^2 df}\right) \qquad (6.25)$$

The codes should be compared to a common reference point.This is an arbitrary choice ;in this comparison the bipolar code is selected as reference where $R_i = R_b$ on channel B = 0 and $Bw_2=R_i/2$ for these links for $F_c<0.5$ and 0.6 percent the 10B7T code is optimum and efficiency consideration dominate.For $F_c > 0.5$ and 0.6 less efficient 8B6T code is optimum.The more efficient codes (4B3T,7B5T, 10B7T) do poorly at controlling dc wonder at high cut off frequencies where as the less efficient codes (PST,6B5T, 5B4T) do much better in all cases the bipolar and PST codes out performe.

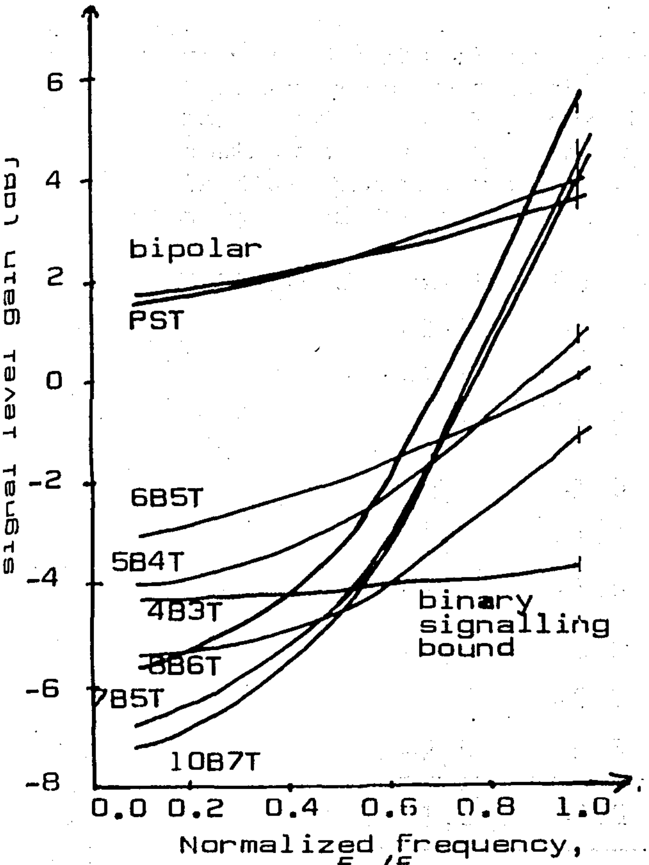


Fig 6.3 Code class comparison T4 link, $P(e)=10^{-10}$, $B=1/4T$

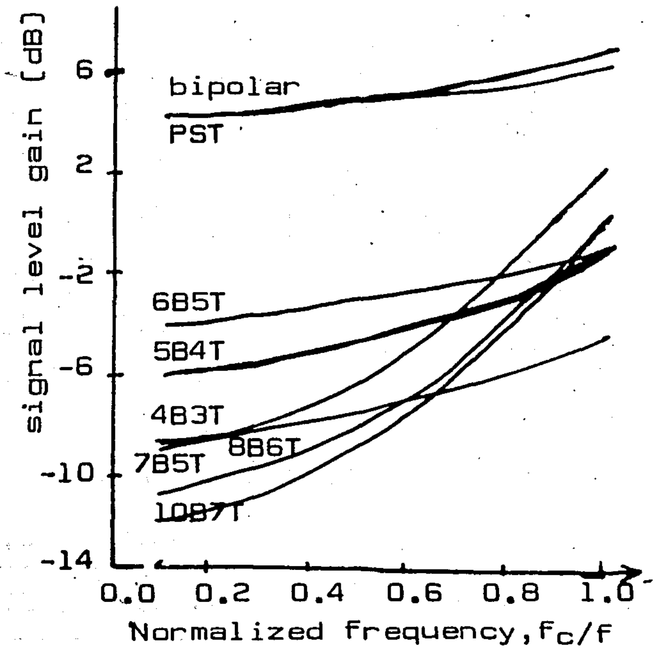


Fig. 6.4 Code class comparison: European link $P(e)=10^{-10}$ $B=1/4T$

6.4 FORTRAN PROGRAM

This program involves the optimization of 4B3T alphabetic code with the defined algorithm of RDS model in var(S) criteria.The result of this program will be compared with values given in table 6.3.From table 6.3,

Givens: number of terminal states=4
number of code books (alphabets)=4
DSV=5

from 5.33 if DSV is odd , the allowable states (s) are $S=\pm\frac{1}{2}$,....$\pm\frac{DSV}{2}$. Then

$$S=\pm\frac{1}{2}\ldots\ldots\pm\frac{3}{2}\ \pm\frac{5}{2} \qquad (6.26)$$

Terminal states will be four of the allowable states. Then the combinations of terminal states can be as in table 6.5 . The marked combination is possible,because it is impossible to find 16 three lenght ternary words that causes the state transition.(In table 6.6 you can see the combinations of three lenght ternary words) Let's take the combination;

-2,5 -1,5 -0,5 0,5

from state -2,5 to -1,5 need word sum= +1
from state -2,5 to -0,5 need word sum= +2

from state -2,5 to 0,5 need word sum= +3

for word sum= +1,there are 6 combinations of 3 lenght ternary word,for word sum= +2 there are 3 combinations, for word sum- +3 there is 1 combination.Total are 10 combinations.But there are 16 words and the constraint is that there is one to one correspondence between binary word to ternary word.

TABLE 6.5 Combinations of terminal states for 4B3T

-2,5	-1,5	-0,5	0,5
-2,5	-1,5	-0,5	1,5
-2,5	-1,5	-0,5	2,5
-2,5	-1,5	-0,5	1,5
-2,5	-1,5	-0,5	2,5
-2,5	-1,5	1,5	2,5
-2,5	-0,5	0,5	1,5
-2,5	-0,5	0,5	2,5
-2,5	0,5	1,5	2,5
-1,5	-0,5	0,5	1,5
-1,5	-0,5	0,5	2,5
-1,5	-0,5	1,5	2,5
-1,5	0,5	1,5	2,5
-0,5	0,5	1,5	2,5

Ternary bits are (0 ,+1 , -1).There are 26 combinations except (000) word for three lenght ternary words.

TABLE 6.6 Possible three-lenght ternary words

ternary word	digital sum
0+-	
0-+	
+0-	0
-0+	
-+0	
+-0	
00+	
0+0	
+00	1
++-	
-++	
+-+	
0++	
+0+	2
++0	
+++	3
00-	
0-0	
-00	

ternary word	digital sum
-+- --+ +--	-1
0-- -0- --0	-2
---	-3

The possible codewords for terminal states -1.5,-0.5 , 0.5,1.5 are as in table 6.7 .

TABLE 6.7 Possible codewords terminal states

(-1.5,-0.5,0.5,1.5)

Input data / S_T^-	-1.5	-0.5	0,5	1.5
	0+-	0+-	0+-	0+-
	0-+	0-+	0-+	0-+
	+0-	+-0	+-0	+-0
	-0+	-0+	-0+	-0+
	-+0	-+0	-+0	-+0
	+-0	+-0	+-0	+-0
	00+	00-	00+	00-
	+00	-00	+00	-00
	0+0	0-0	0+0	0-0
	++-	--+	++-	--+
	+-+	-+-	+-+	-+-
	-++	+--	-++	+--
	++0	00+	00-	--0
	0++	0+0	0-0	0--
	+0+	+00	-00	-0-
	+++	++-	--+	---
		+-+	-+-	
		-++	+--	
		0++	0--	
		++0	--0	
		+0+	-0-	

Operation is not necessary for the alphabet c[-1.5] and c[1.5] because 16 possible codewords exist.For the alphabets c[-0.5] and [0.5] 16 codewords will be selected out of 21 codewords.

<u>Data:</u> 1. Terminal States

S[1] = -1.5

S[2] = -0.5

S[3] = 0.5

S[4] = 1.5

2. Codewords A[I,J,K], I=1,4 J=1,16 or 21 K=1,3

3. M = 4
N = 3

<u>Steps of the program:</u>

Step 1: Finding codeword costs $\lambda_v[a/s]$, which is

$$\lambda_v[a/s] = \frac{1}{N}\sum_{i=1}^{3} RDS_i^2$$ where

$$RDS_i = s + \sum_{t=1}^{i} a_t$$ where s is the terminal state of the codeword

<u>Step 2:</u> For alphabet c[-0.5], c[0.5],the codeword costs are ordered from smaller value to larger value. And accept the first 16 of them

<u>Step 3:</u> Find the alphabet cost which is

$$\lambda[c[s]] = \sum_{a\in c(s)} 2^{-M} \lambda_v[a/s]$$

<u>Step 4:</u> Find the state transition probability matrix $[\pi]$ which parameters of it as

$$\pi_{ij} = \frac{1}{16} n_{ij}$$ where

n_{ij} is the number of transitions from state i to j. This matrix is symmetric.

<u>Step 5:</u> Evaluate the terminal state stationary probabilities from

$$\underline{P}\,\pi_{ij} = \underline{P}$$

$$\sum_{i=1}^{4} P_i = 1$$

The procedure for evaluation of probabilities:

Let's

$$\Pi = \begin{bmatrix} n_{11} & n_{12} & n_{13} & n_{14} \\ n_{21} & n_{22} & n_{23} & n_{24} \\ n_{24} & n_{23} & n_{22} & n_{21} \\ n_{14} & n_{13} & n_{12} & n_{11} \end{bmatrix}$$

and

$$[P_1 \; P_2 \; P_3 \; P_4] \begin{bmatrix} \Pi \end{bmatrix} = [P_1 \; P_2 \; P_3 \; P_4]$$

Then

$$P_1 n_{11} + P_2 n_{21} + P_3 n_{24} + P_4 n_{14} = P_1$$

$$P_1 n_{12} + P_2 n_{22} + P_3 n_{23} + P_4 n_{13} = P_2$$

$$P_1 n_{13} + P_2 n_{23} + P_3 n_{22} + P_4 n_{12} = P_3$$

$$P_1 n_{14} + P_2 n_{24} + P_3 n_{21} + P_4 n_{11} = P_4$$

and

$$P_1 + P_2 + P_3 + P_4 = 1$$ that is

$$P_1 = 1 - P_2 - P_3 - P_4$$

by putting P_1 into first equation then

$$(n_{11}-1)(1-P_2-P_3-P_4) + P_2 n_{21} + P_3 n_{24} + P_4 n_{14} = 0$$

$$P_2(n_{21}-n_{11}\ 1) + P_3(n_{24}-n_{11}\ 1) + P_4(n_{14}-n_{11}\ 1) = (1-n_{11})$$

$$P_1 n_{12} + P_2 (n_{22}-1) + P_3 n_{23} + P_4 n_{13} = 0$$

$$P_1 n_{13} + P_2 n_{23} + P_3 (n_{22}-1) + P_4 n_{12} = 0$$

$$P_1 n_{14} + P_2 n_{24} + P_3 n_{21} + P_4 (n_{11}-1) = 0$$

Then

$$\begin{bmatrix} 0 & (n_{21}-n_{11} \quad 1) & (n_{24}-n_{11} \quad 1) & (n_{14}-n_{11} \quad 1) \\ n_{12} & n_{22}-1 & n_{23} & n_{13} \\ n_{13} & n_{23} & n_{22}-1 & n_{12} \\ n_{14} & n_{24} & n_{21} & (n_{11}-1) \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 1-n_{11} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

it can be write as

$$\underline{A}\ \underline{P} = \underline{M}$$

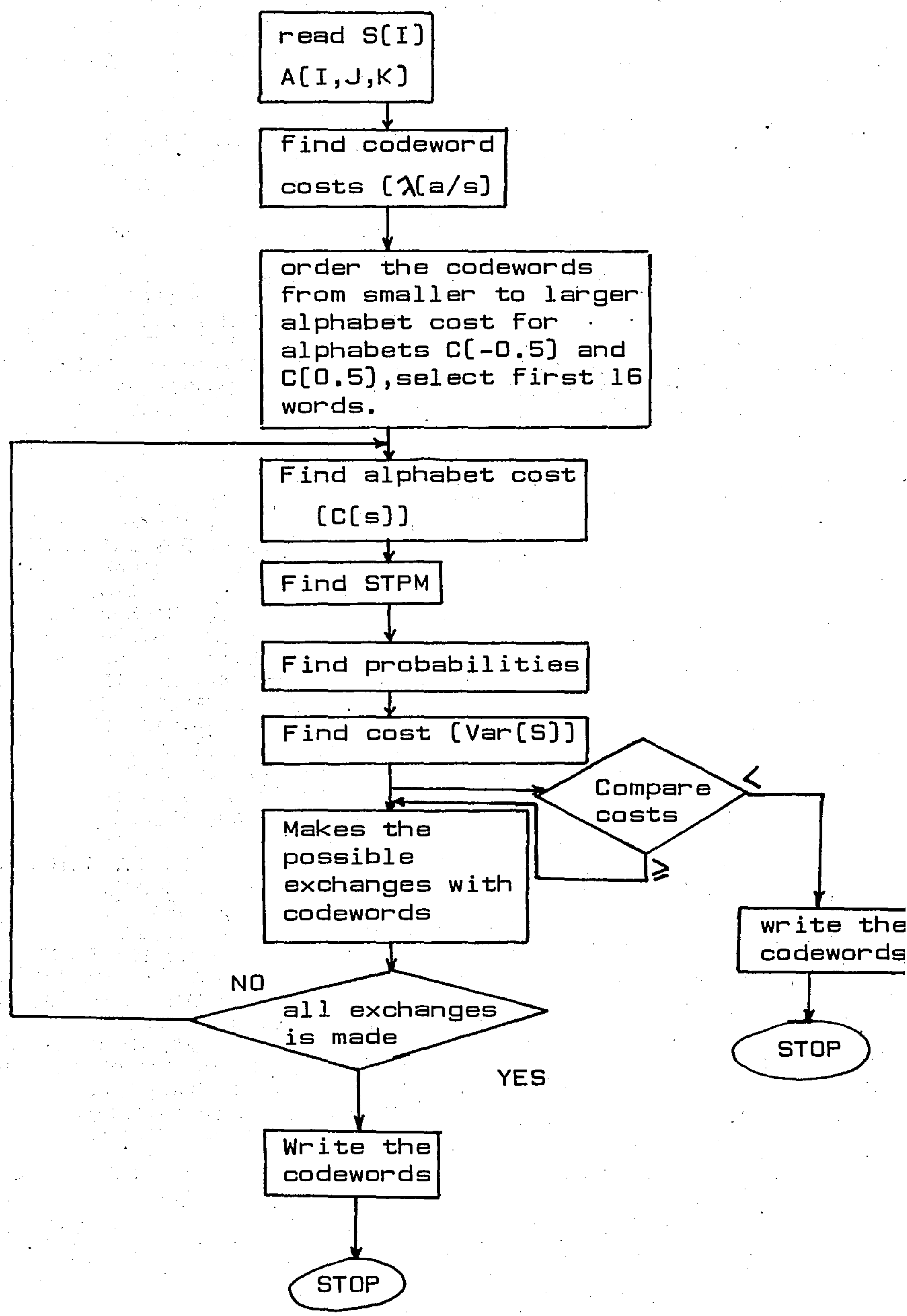
$$\underline{P} = \underline{A}^{-1}\ \underline{M} \qquad (6.27)$$

Step 6: Find the cost var(s) Which is

$$\mathrm{Var}(s) = \sum_{S \in s_T} P[s]\,\lambda[C[s]]$$

Step 7: For alphabets c(-0.5) and C(0.5) choose the 17 th codeword for 16 th codeword, and repeat the setps 1-6 and compare th costs, if the second cost is less than first one, select this codeword (17 tH) lf it is not choose the 18 th codeword for 16th codeword and repeat this process.

FLOW CHART OF PROGRAM



```
      DIMENSION A(4,21,3),S(4),SUM(10),B(10,25,5),MA(10,10),
     $ E(10,25,5),APCOST(10,25),CBCOST(10),P(10,10),
     $ DEL(10),PROB(10),VAR(50),NA(2,21,3),Y(25,5),Z(25)
      DATA NA/0,0,0,0,1,1,-1,-1,-1,-1,1,1,0,0,1,-1,0,0,1,-1,
     $ 1,-1,-1,1,1,0,0,0,1,1,1,1,0,1,0,-1,0,0,0,1,0,1,
     $ 1,1,-1,-1,0,0,0,0,1,1,-1,-1,0,0,0,0,1,-1,1,-1,-1,1,
     $ 1,-1,1,0,1,1,0,0,1,1,0,-1,0,1,0,1,0,1,0,0,-1,-1,
     $ 1,1,-1,-1,1,1,0,0,0,0,1,-1,0,0,0,0,-1,1,1,-1,
     $ 1,-1,0,1,1,0,1,0,1,-1,0,1,0,1,0,1,0,0,0,1/
      DATA S,M,N/-1.5,-0.5,0.5,1.5,4,3/
      I=1
      DO 62 J=1,16
      DO 63 K=1,3
   63 A(I,J,K)=FLOAT(NA(I,J,K))
   62 CONTINUE
      I=2
      DO 65 J=1,21
      DO 66 K=1,3
   66 A(I,J,K)=FLOAT(NA(I,J,K))
   65 CONTINUE
      BM=FLOAT(M)
      DN=FLOAT(N)
C FINDING THE VAR(S) BY CHOOSING FIRST 16 MIN.COST CODEWORDS FOR S=-0.5
      CALL ECCOST (SUM,B,S,A,E,APCOST,CBCOST,BM,DN)
      CALL HCCOST (SUM,B,S,A,E,APCOST,CBCOST,BM,DN)
      CALL PROBE (MA,B,S,P,PROB,BM)
      CALL VARIAN (VR,PROB,CBCOST,VAR,16)
      WRITE(2,75)(((A(I,J,K),K=1,3),I=1,2),J=1,16),(CBCOST(I)
     $ ,I=1,4),(PROB(I),I=1,4),VAR(16)
   75 FORMAT(/,5X,16(3F3.0,3X,3F3.0,/,5X),9E16.5)
C REPLACING THE 16 MIN. COST CODEWORDS WITH THE REMAINING 5CODEWORDS
      KA=16
      N=1
   81 L=KA+N
      DO 76 J=L,21
      DO 430 K=1,3
  430 Z(K)=A(2,KA,K)
      XD=APCOST(2,KA)
      XE=B(2,KA,3)
      A(2,KA,1)=A(2,J,1)
      A(2,KA,2)=A(2,J,2)
      A(2,KA,3)=A(2,J,3)
      APCOST(2,KA)=APCOST(2,J)
      B(2,KA,3)=B(2,J,3)
CFINDING THE NEW VAR(S)
      CALL NCCOST(SUM,APCOST,CBCOST,BM)
      CALL PROBE (MA,B,S,P,PROB,BM)
      CALL VARIAN (VR,PROB,CBCOST,VAR,L)
C MAKING THE COMPARISON BETWEEN FIRST AND SECOND VAR(S) IF THE
C  SECOND ONE IS LESS CHOOSE THIS EXCHANGED CODEWORD AND GO ON
C EXCHANGING THE CODEWORDS
      IF(VAR(L).LT.VAR(KA)) GO TO 80
   76 CONTINUE
      DO 410 K=1,3
  410 A(2,KA,K)=Z(K)
      APCOST(2,KA)=XD
      B(2,KA,3)=XE
      KA=KA-1
      N=N+1
      IF(KA.EQ.1) GO TO 50
```

```
          GO TO 81
    80    WRITE(2,75)(((A(I,J,K),K=1,3),I=1,2),J=1,16),(CBCOST(I),I=1,4)
         $ ,(PROB(I),I=1,4),VAR(L)
          VAR(L)=VAR(KA)
          KA=KA-1
          N=N+1
          GO TO 81
    50    STOP
          END
          SUBROUTINE ECCOST (SUM,B,S,A,E,APCOST,CBCOST,BM,DN)
          DIMENSION SUM(10),B(10,25,5),S(4),A(4,21,3),E(10,25,5),
       $ APCOST(10,25),CBCOST(10)
C FINDING THE CODEWORD COSTS AND ALPHABET COST FOR S=-1.5
CWHICH ARE THE SAME FOR S=1.5
          I=1
          SUM(I)=0.
          DO 1 J=1,16
          DO 2 K=2,3
          B(I,J,1)=S(I)+A(I,J,1)
          E(I,J,1)=B(I,J,1)**2
          LA=K-1
          B(I,J,K)=B(I,J,LA)+A(I,J,K)
    2     E(I,J,K)=E(I,J,LA)+(B(I,J,K)**2)
          APCOST(I,J)=E(I,J,K)/DN
    1     SUM(I)=SUM(I)+APCOST(I,J)
          CBCOST(I)=SUM(I)/(2**BM)
          CBCOST(4)=CBCOST(1)
          RETURN
          END
          SUBROUTINE HCCOST (SUM,B,S,A,E,APCOST,CBCOST,BM,DN)
          DIMENSION SUM(10),B(10,25,5),S(4),A(4,21,3),E(10,25,5)
       $   ,APCOST(10,25),CBCOST(25),Y(25,5)
C FINDING CODEWORD COSTS AND ALPHABET COST FOR S=-0.5 WHICH ARE
C THE SAME FOR S=0.5
          I=2
          SUM(I)=0.
          DO 3 J=1,21
          DO 4 K=2,3
          B(I,J,1)=S(I)+A(I,J,1)
          E(I,J,1)=B(I,J,1)**2
          LA=K-1
          B(I,J,K)=B(I,J,LA)+A(I,J,K)
    4     E(I,J,K)=E(I,J,LA)+(B(I,J,K)**2)
    3     APCOST(I,J)=E(I,J,K)/DN
C ORDERING THE  21 CODEWORDS FROM MIN. TO MAX. CODEWORD COST
          DO 6 J=1,21
          LN=J+1
          DO 7 KS=LN,21
          IF(APCOST(I,J).LE.APCOST(I,KS)) GO TO 7
          X=APCOST(I,J)
          APCOST(I,J)=APCOST(I,KS)
          APCOST(I,KS)=X
          DO 107 K=1,3
          Y(J,K)=A(I,J,K)
          A(I,J,K)=A(I,KS,K)
          A(I,KS,K)=Y(J,K)
          XA=B(I,J,3)
          B(I,J,3)=B(I,KS,3)
          B(I,KS,3)=XA
   107    CONTINUE
```

```
 7      CONTINUE
 6      CONTINUE
        DO 8 J=1,16
        SUM(I)=SUM(I)+APCOST(I,J)
 8      CONTINUE
        CBCOST(I)=SUM(I)/(2**BM)
        CBCOST(3)=CBCOST(2)
        RETURN
        END
        SUBROUTINE PROBE (MA,B,S,P,PROB,BM)
        DIMENSION MA(10,10),B(10,25,5),S(4),P(10,10),DEL(10),PROB(10)
C DEFINING THE STPM WHICH IS SYMMETRIC  AND FINDING TERMINAL
C STATE STATIONARY PROBABILITIES
        DO 10 I=1,2
        DO 11 L=1,4
 11     MA(I,L)=0
 10     CONTINUE
        K=3
        DO 12 I=1,2
        DO 13 L=1,4
        DO 14 J=1,16
        IF(B(I,J,K).NE.S(L)) GO TO 14
        MA(I,L)=MA(I,L)+1
 14     CONTINUE
 13     CONTINUE
 12     CONTINUE
        DO 16 I=1,2
        DO 17 L=1,4
 17     P(I,L)=FLOAT(MA(I,L))/(2**BM)
 16     CONTINUE
        F=1.-P(1,1)
        FA=P(2,2)-1.
        FB=(P(2,3)**2-FA**2)
        FC=P(1,2)**2-P(1,3)**2
        FD=P(2,1)*P(1,3)+P(2,4)*P(1,2)
        FE=P(2,4)*P(1,3)+P(2,1)*P(1,2)
        FG=F*P(1,2)+P(1,3)*P(1,4)
        FH=P(1,4)*P(1,2)+F*P(1,3)
        DEL(1)=(F*FB)-(FA*FE)+(FD*P(2,3))
        DEL(2)=(P(2,1)*FC)+(FA*FG)-(FH*P(2,3))
        DEL(3)=(FA*FH)-(FC*P(2,4))-(FG*P(2,3))
        DEL(4)=(FB*P(1,4))+(FA*FD)-(FE*P(2,3))
        DET=(F+P(2,1))*DEL(2)+(F+P(2,4))*DEL(3)+(F+P(1,4))*DEL(4)
        DO 20 I=1,4
 20     PROB(I)=(DEL(I)*F)/DET
        GO TO 102
 102    RETURN
        END
        SUBROUTINE VARIAN (VR,PROB,CBCOST,VAR,K)
        DIMENSION PROB(10),CBCOST(10),VAR(K)
C FINDING THE VAR(S)
        VR=0.
        DO 21 I=1,4
 21     VR=VR+(PROB(I)*CBCOST(I))
        VAR(K)=VR
        RETURN
        END
        SUBROUTINE NCCOST (SUM,APCOST,CBCOST,BM)
        DIMENSION SUM(10),APCOST(10,25),CBCOST(10)
        I=2
```

```
          SUM(I)=0.
          DO 300 J=1,16
    300   SUM(I)=SUM(I)+APCOST(I,J)
          CBCOST(I)=SUM(I)/(2**BM)
          CBCOST(3)=CBCOST(2)
          RETURN
          END
```

```
          C[-1.5]     C[-0.5]
A(I,J,K)
①        0. 1.-1.     0.-1.-1.
          0.-1. 1.     1. 0.-1.
          1. 0.-1.     1.-1. 0.
         -1. 0. 1.     0. 0. 1.
         -1. 1. 0.     0. 1. 0.
          1.-1. 0.     1. 0. 0.
          0. 0. 1.     1.-1. 1.
          1. 0. 0.    -1. 1. 0.
          0. 1. 0.     0.-1. 1.
          1. 1.-1.     0. 0.-1.
          1.-1. 1.     1. 1.-1.
         -1. 1. 1.     1.-1.-1.
          1. 1. 0.    -1. 1. 1.
          0. 1. 1.     0. 1. 1.
          1. 0. 1.     1. 0. 1.
          1. 1. 1.    -1. 0. 1.
   CBCOST(1)=.15833E+01 CBCOST(2)=.66667E+00 CBCOST(3).66667E+00 CBCOST(4)=.15833E+01   P(1)=.16434
 E+01  P(2)=.19549E+00  P(3)=.38144E+00  P(4)=.27972E+00
 Var(S)=.34295E+01
②
          0. 1.-1.     0.-1.-1.
          0.-1. 1.     1. 0.-1.
          1. 0.-1.     1.-1. 0.
         -1. 0. 1.     0. 0. 1.
         -1. 1. 0.     0. 1. 0.
          1.-1. 0.     1. 0. 0.
          0. 0. 1.     1.-1. 1.
          1. 0. 0.    -1. 1. 0.
          0. 1. 0.     0.-1. 1.
          1. 1.-1.     0. 0.-1.
          1.-1. 1.     1. 1.-1.
         -1. 1. 1.     1.-1.-1.
          1. 1. 0.    -1. 1. 1.
          0. 1. 1.     0. 1. 1.
          1. 0. 1.     1. 0. 1.
          1. 1. 1.     0.-1. 0.
              .15833E+01      .66667E+00      .66667E+00      .15833E+01      .15213
 E+01         .18924E+00      .35622E+00      .29685E+00
             .32424E+01
```

(3)

```
 0.  1.-1.     0.-1.-1.
 0.-1.  1.     1.  0.-1.
 1.  0.-1.     1.-1.  0.
-1.  0.  1.    0.  0.  1.
-1.  1.  0.    0.  1.  0.
 1.-1.  0.     1.  0.  0.
 0.  0.  1.    1.-1.  1.
 1.  0.  0.   -1.  1.  0.
 0.  1.  0.    0.-1.  1.
 1.  1.-1.     0.  0.-1.
 1.-1.  1.     1.  1.-1.

-1.  1.  1.    1.-1.-1.
 1.  1.  0.   -1.  1.  1.
 0.  1.  1.    0.  1.  1.
 1.  0.  1.    1.-1.-1.
 1.  1.  1.    0.-1.  0.
          .15833E+01      -.15226E+30      -.15226E+30      .15833E+01      .16269
E+01      .19774E+00       .37919E+00       .29622E+00
    -.87843E+29
```

(4)

```
 0.  1.-1.     0.-1.-1.
 0.-1.  1.     1.  0.-1.
 1.  0.-1.     1.-1.  0.
-1.  0.  1.    0.  0.  1.
-1.  1.  0.    0.  1.  0.
 1.-1.  0.     1.  0.  0.
 0.  0.  1.    1.-1.  1.
 1.  0.  0.   -1.  1.  0.
 0.  1.  0.    0.-1.  1.
 1.  1.-1.     0.  0.-1.
 1.-1.  1.     1.  1.-1.
-1.  1.  1.    1.-1.-1.
 1.  1.  0.   -1.  1.  1.
 0.  1.  1.    0.-1.  0.
 1.  0.  1.    1.-1.-1.
 1.  1.  1.    0.-1.  0.
          .15833E+01      -.15226E+30      -.15226E+30      .15833E+01      .16111
E+01      .19988E+00       .37704E+00       .31193E+00
    -.87843E+29
```

```
(5)        0. 1.-1.    0.-1.-1.
           0.-1. 1.    1. 0.-1.
           1. 0.-1.    1.-1. 0.
          -1. 0. 1.    0. 0. 1.
          -1. 1. 0.    0. 1. 0.
           1.-1. 0.    1. 0. 0.
           0. 0. 1.    1.-1. 1.
           1. 0. 0.   -1. 1. 0.
           0. 1. 0.    0.-1. 1.
           1. 1.-1.    0. 0.-1.
           1.-1. 1.    1. 1.-1.
          -1. 1. 1.    1.-1.-1.
           1. 1. 0.    0.-1. 0.
           0. 1. 1.    0.-1. 0.
           1. 0. 1.    1.-1.-1.
           1. 1. 1.    0.-1. 0.
              .15833E+01     -.15226E+30     -.15226E+30     .15833E+01     .16224
     E+01     .17639E+00      .35932E+00      .34190E+00
           -.81569E+29

(6)        0. 1.-1.    0.-1.-1.
           0.-1. 1.    1. 0.-1.
           1. 0.-1.    1.-1. 0.
          -1. 0. 1.    0. 0. 1.
          -1. 1. 0.    0. 1. 0.
           1.-1. 0.    1. 0. 0.
           0. 0. 1.    1.-1. 1.
           1. 0. 0.   -1. 1. 0.
           0. 1. 0.    0.-1. 1.
           1. 1.-1.    0. 0.-1.
           1.-1. 1.    1. 1.-1.
          -1. 1. 1.    0.-1. 0.
           1. 1. 0.    0.-1. 0.
           0. 1. 1.    0.-1. 0.
           1. 0. 1.    1.-1.-1.
           1. 1. 1.    0.-1. 0.
              .15833E+01     -.15226E+30     -.15226E+30     .15833E+01     .16224
     E+01     .17639E+00      .35932E+00      .34190E+00
           -.81569E+29
```

CHAPTER VII

COCLUSION

We have summarized all the line coding techniques such as partial response codes, two phase frequency modulation codes , alphabetic codes and nonalphabetic codes.

These line coding schemes are primarily developed for pulse code modulation (PCM) systems , high density magnetic recording and high data transmission systems in order to reduce dc wandering (baseline wander),to suppress ISI , maintain the self clocking capability and allow for effective error monitoring.Line codes are different from error detection and correction codes which are intended to combat against the random or burst noise effects.These techniques are used to compansate for undesired characteristics of a digital communication channel.

Also various methods to compute the symbol error probability P(e) due to the ISI and the noise have been discussed.One of these methods developed by Jakubow , Garcia and Chang is the running digital sum method which is based on the assumption that the raised cosine channel is cascaded with two single pole transformer.The impulse response of the overall system consists of a waveform with an infinite duration constent amplitude tail of polarity opposite to that of the symbol.This model allows us to set a direct relation between the ISI and the P(e) and also develop new code design algorithms for ternary alphabetic codes that optimize the P(e) performance.But the experiments indicated that this model is usefull only for some values of f_c/f (f_c is the transformer frequency , $f=1/T$) such as $f_c/f<0.5$ percent.

For future investigation the following items can be recommended.

As a result of code conversion , the encoded sequence includes some redundancy which should be exploited as much as possible to improve the reliability against random noise. Kobayashi has found practical solutions to a class of correlative level codes i.e maximum likelihod decoding (MLD) and ambiguity zone decoding (AZD). (30) An extension of similar approaches to other type of codes (ex. alphabetic codes) can be investigated.

Franaszek developed a method of constructing synchronous variable length codes.(31) He reduced the problem of finding an optimal variable code to that of selecting a set of terminal states (principal states).This notion is equivalent to finding an optimal terminal states set in the method of constructing a fixed length code.A dynamic programming algorithm has been applied to a systematic search of principal states and optimal variable length codes for various constraints have been found.(32) This algorithm and its results can be future developed.

The communication model is assumed to be baseband channel with pulse amplitude modulation.The results are extendable for other modulation systems.Many authors report applications of correlative coding technique to FM (33) , phase-shift keyed (PSK),quadrature amplitude modulation (QAM) (34),vestigal sideband (VSB) or single sideband (SSB) systems.These applications can be adopted for other line codes.

The code design algorithm of RDS model has been used for ternary alphabetic codes.The application of this algorithm to the other fixed length alphabetic codes can be examined.

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