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SCATTERING OF ACOUSTIC WAVES BY CYLINDERS WITH ARBITRARY CROSS SECTIONS T-MATRIX FORMULATION

bу

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ABSTRACT

In this work, the scattering of steady-state acoustic waves from arbitrarily shaped obstacles in an infinite medium is studied using the T-matrix method. The problem is examined for the two dimensional case where the obstacle is a cylindrical rigid inclusion or a cavity. An acoustic plane wave is considered to be incident on the obstacle.

In the solution of the problem, both incident and scattered wave fields are expanded in series of the circular basis wave functions. The scattered wave field is then evaluated through a transition matrix (T-matrix) which relates the unknown coefficients of the scattered wave series to the coefficients of the incident wave.

Numerical results pertaining to circular, elliptical, rectangular and triangular cross sections are obtained. The results are presented in graphical form and found to be in good agreement compared with the some known exact or approximate solutions available in the literature.

AKUSTİK DALGALARIN RASTGELE KESİTLİ SİLİNDİRLERDEN SAÇILIMI T-MATRİS FORMULASYONU

KISA ŐZET

Bu çalışmada, akustik dalgaların sonsuz bir ortamdaki rastgele şekilli engellerden saçılımı, T-matris metodu kullanılarak ele alınmıştır. Problem, iki boyutta, silindirik rijit cisimler ve boşluklar için incelenmiştir. Engel üzerine gelen dalganın akustik bir düzlem dalga olduğu düşünülmüştür.

Problemin çözümünde, hem gelen hem de saçılan dalga alanları, dairesel temel dalga fonksiyonları cinsinden seri olarak açılmaktadır. Saçılan dalga alanı, saçılan dalga serisinin katsayılarını gelen dalga serisinin katsayılarına bağlayan bir geçiş matrisi (T-matris) vasıtasıyla hesaplanmaktadır.

Dairesel, eliptik, dikdörtgen ve üçgen kesitli silindirik şekiller için nümerik sonuçlar elde edilmiştir. Sonuçlar, grafikler halinde gösterilmiş ve literatürdeki bazı bilinen kesin veya yaklaşık çözümlerle uyuştuğu görülmüştür.

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LIST OF SYMBOLS

A	Constant amplitude factor of the incident wave
a ^o , a _m	Coefficients of the series representing the incident wave field
a ~	Coefficient vector for the incident wave with elements a m
a,b	Half major and minor axes for elliptical and rectangular geometries
В	Bulk modulus of the medium
с	Wave speed
C _s	A constant multiplication factor associated with the Simpson's integration
c ^ơ , c _m	Coefficients of the series representing the scattered wave field
c ~	Coefficient vector for the scattered wave with elements c _m
<u>e</u> 1, <u>e</u> 2	Unit vectors along the x and y directions of the Cartesian coordinate system
$\frac{e}{r}$, $\frac{e}{\theta}$	Unit radial and tangential vectors in polar coor- dinate system
f	Farfield amplitude of the scattered wave field
h .	Height of an isosceles triangle along its symmetry axis
H _n	n-th order cylindrical Hankel function of the first kind
J _n	n-th order cylindrical Bessel function of the first kind
k, <u>k</u>	Wave number and wave normal vector, respectively

n

Unit outward normal vector along the boundary of the scatterer

2/9n

Directional derivative along the unit normal n

N

p

Number of the boundary segments used in the numerical evaluation of the boundary integrals

Ρ

Order of the Simpson's rule used in the numerical

Q ≈

Q ≈

A matrix with elements involving integrals over the boundary of the scatterer

Real part of the Q-matrix

 $Q^{1\,1}\,,\,\,Q^{1\,2}\,,\,\,Q^{2\,1}\,,\,\,Q^{2\,2}$ Submatrices of the Q-matrix

Pressure

integrations

Elements of the Q-matrix

(r,0) Coordinates of a point in circular polar coordinate system

r

Position vector in polar coordinate system

Q_{jm}, Q_{jm}

 r_{iq} , θ_{iq} , q=0,..,P Coordinates of the integration points on the i-th boundary segment

rc S

Corner radius of a round cornered rectangle

Boundary of the cross section of an infinite cylinder

S₁, S₂

Two arbitrary circular curves enclosing S

r_, r_ Radii of the curves S_{+} , S_{m}

 $\Delta S_{i}, i = 1, ..., N$ i-th boundary segment along S

Transition matrix (T-matrix)

≈ ≈

u¹, u^S, u

 T^{11} , T^{12} , T^{21} , T^{22} Submatrices of the T-matrix

Incident, scattered and total wave fields outside the scatterer, respectively

Total wave field at the boundary of the scatterer Velocity field

V.

 u^+

T

w _q , q = 0,,P	Weighing factors associated with the Simpson's formula
(x,y,z)	Coordinates of a point in Cartesian coordinate system
Yn	n-th order cylindrical Bessel function of the second kind
α	Incidence angle of a plane wave
α _m σ, α _m	Coefficients of the series representing the wave field at the surface of the scatterer
a ~	Coefficient vector for the surface wave field with elements $\alpha_{m}^{}$
β	Tip angle for the isosceles triangular cross section
δ _{mn}	Kronecker delta
ε _n	Neumann factor
^{∆θ} i	i-th angular interval associated with S _i
θ _a , θ _b ,	Lower and upper angles of the i-th angular interval
λ	Wavelength
ρ	Density distribution in the medium
ρ _ό	Equilibrium density of the medium
otot	Total scattering cross section
φ	Velocity potential function
ψ	Spatial part of the velocity potential function
$\psi_n^{\sigma}, \psi_n, \hat{\psi}_n^{\sigma}, \hat{\psi}_n$	Basis functions for two-dimensional scalar waves
τ ^σ n	Angular part of the basis functions
ω	Angular frequency

I. INTRODUCTION

When a wave propagating in an unbounded homogeneous medium encounters an obstacle immersed in the medium, its propagation path changes and while a portion of it is reflected back into the medium as a secondary wave emitted by the obstacle the other part of it, if the obstacle is not a cavity, is refracted into the body of the obstacle. The radiation of these secondary waves from the obstacle is called scattering. The obstacle may be a cavity or an inclusion with physical properties differing from those of the surrounding medium.

Scattering and diffraction problems have become increasingly important in the recent years, particularly in the areas of remote sensing, seismic exploration, oil technology, underwater sound detection and especially in non-destructive testing of materials where the scattered wave form is used to identify the shapes and the sizes of the material defects such as voids, cracks or inclusions.

In solving the scattering problems, especially where explicit numerical results are desired, four methods are extensively employed in the literature. These are method of seperation-of-variables, variational method, integral equation method and the transition matrix (Tmatrix) method which has been recently developed [1].

The method of seperation of variables is usually employed in finding exact analytical solutions for only a class of objects bounded by quadric surfaces, [2-5]. Hence, the method restricts the shape of the scatterer to simple geometries, like sphere, circular and elliptic cylinders, such that their boundary geometries can be expressed conveniently in seperable coordinates. The variational method is another method used in the scattering problem solutions and, from the theoretical point of view, it is applicable for arbitrary boundary geometries, [3,6,7]. However, this method especially for general geometries, requires the evaluation of repeated surface or volume integrals with singular kernels. Thus this method is also restricted to relatively simple geometries. The third method, namely the integral equation method, consists of approximating an integral over the surface of the scatterer by a finite sum and then computing the quantities like displacements, velocity potentials at many discrete points by solving the resulting system of equations numerically, [8]. In previous years, several applications of this approach have appeared in the literature [6,9-11].

The latest method developed for the solution of the wave scattering problems is the T-matrix method which was first introduced by Waterman [1] for acoustic waves and reformulated by Pao [12] for elastic waves. The method starts directly with the Helmholtz integral formula, and uses either cylindrical or spherical wave functions for bodies of arbitrary shape. Both incident and scattered waves are represented as a series of the common wave functions, known as the basis functions. The unknown coefficients of the scattered wave series are then related by a transition matrix (T-matrix) to the coefficients of the incident

wave, [1,12]. The elements of the T-matrix are integrals of basis functions and their normal derivatives over the bounding surface of the scatterer, which can be evaluated numerically even for bodies of complex geometry.

The key feature of the method lies in the fact that the T-matrix is fixed for a specific boundary type and geometry and wave number of the incident wave. Hence, once it is created, the scattered field quantities at various regions of the-medium for different incidence angles of the impinging wave can be calculated. The method, comparing with the variational and integral equation techniques, has also a computational advantage because the integrals involved in the formulation are only single surface integrals with no singularities in their integrands. In the recent years, several applications of T-matrix approach to the scattering of acoustic and elastic waves by finite elliptic cylinders [13,14], spheroids [14], finite circular cylinders [15], infinite strips [1,16] have verified the power of the method.

In this work, we have considered the scattering of the plane acoustic waves by infinite cylinders. The near and far field results for cavity and rigid inclusion cases are presented. A brief review of the governing equations for acoustic fields is given in Chapter 2. In Chapter 3, formulation of the transition matrix for the scattering problem is given. The numerical methods employed in the solutions are discussed in Chapter 4 where solutions to specific problems are also given. Results pertaining to circular, elliptical, rectangular and triangular geometries are presented in polar graphical form.

II. THE THEORY OF ACOUSTIC WAVES

In this section, the reduction of the general equations of hydrodynamics to the Helmholtz reduced wave equation describing the motion of the harmonic disturbances in a fluid is discussed briefly. Also, a general solution of the Helmholtz equation is given for polar coordinates.

2.1 EQUATIONS OF HYDRODYNAMICS

To study the wave propagation in a fluid medium, the starting equations are the hydrodynamical equations of motion due to Euler [17]:

$$\rho\left[\frac{\partial v}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right] = -\nabla p \qquad , \qquad (2.1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0 , \qquad (2.2)$$

$$p = p(\rho)$$
 , (2.3)

where ρ is the density, p is the pressure and <u>v</u> is the fluid velocity vector at any point. This set of equations, which are valid for an ideal fluid, is complete and consists of the equation of motion (2.1), the equation of continuity (2.2), and the equation of state (2.3).

Whenever a disturbance is created at any point inside the fluid it will propagate throughout the medium. In order to obtain the governing equations of motion regarding the propagation of this disturbance, one assumes that the relative perturbations from the initial equilibrium state are small, that is,

$$\frac{\rho'}{\rho_0} = \frac{\rho - \rho_0}{\rho_0} \sim \mu , \quad \frac{p'}{p_0} = \frac{p - p_0}{p_0} \sim \mu , \quad (2.4)$$

where ρ_0 and p_0 are the equilibrium density and pressure, respectively, while μ is some small parameter denoting the variations from the equilibrium values. Under these conditions, the fluid velocity \underline{v} with which the fluid particles oscillate is a small quantity of the order of μ relative to the propagation speed of the disturbance. This is the case, actually, in acoustics. Since, acoustical wave lengths are long, the variations in the velocity field, \underline{v} , are very small quantities. Thus, neglecting the term $v \cdot \nabla v$ in Eq. (2.1), we get

$$\rho \frac{\partial v}{\partial t} = -\nabla p \qquad (2.5)$$

Substitution of the expressions $\rho = \rho_0 + \rho'$ and $p = p_0 + p'$ into the Eqs. (2.5), (2.2) and (2.3) yields

$$\rho_{0} \frac{\partial v}{\partial t} = -\nabla p' \qquad , \qquad (2.6)$$

$$\frac{\partial \rho^{i}}{\partial t} + \rho_{0} \nabla \cdot \underline{v} = 0 \qquad , \qquad (2.7)$$

$$p' = \frac{dp}{d\rho} \rho'$$
 (2.8)

where we have neglected the terms involving the products of the perturbed quantities. For the reduction of this set to a single equation, it is convenient to introduce a scalar function ϕ such that

The function ϕ in the above equation is called the velocity potential. Then, Eqs. (2.6) and (2.7) yields

$$p' = -\rho_0 \frac{\partial \phi}{\partial t} , \qquad (2.10)$$

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla^2 \phi = 0$$
 (2.11)

Eliminationg the variables p' and ρ ' from the above equations, we obtain the equation of motion of the disturbance,

$$\frac{\partial^2 \phi}{\partial t^2} - c^2 \nabla^2 \phi = 0 \qquad (2.12)$$

This is a scalar wave equation describing the motion of a disturbance in an acoustic field where the propagation speed of the disturbance is c,

$$c = \sqrt{dp/d\rho}$$
 , (2.13)

or, in general c can also be written in the form

$$c = \sqrt{B/\rho_0} \qquad (2.14)$$

where B is the bulk modulus of the fluid.

<u>v</u> = ⊽¢

2.2 HARMONIC WAVE MOTION IN ACOUSTIC FIELDS

Considering a disturbance which is harmonic in time and has a circular frequency of ω , one can write the velocity potential as

$$\phi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t}$$
, (2.15)

where ψ is the spatial part of the velocity potential function, ϕ . Substituting Eq. (2.15) into Eq. (2.12) and rearranging the terms, we get

$$\nabla^2 \psi + k^2 \psi = 0$$
 (2.16)

where $k = \omega/c$ is the wave number. Eq. (2.16) is known as the Helmholtz reduced wave equation and describes the motion of the acoustic harmonic waves in a fluid medium.

It should also be noted that use of the velocity potential formulation is indeed a convenient way for acoustic wave propagation. However, ϕ is not a measurable quantity like velocity or pressure but it is possible to obtain such quantities from it. For example, substitution of Eq. (2.15) into (2.10) yields

$$p = p_0 + i\omega p_0 \phi$$
 . (2.17)

2.3 SOLUTION OF THE HELMHOLTZ EQUATION

Since throughout this work scattering of the plane waves by infinite cylinders with constant cross-sections will be studied and the wave normals of the incident waves which will be considered are perpendicular to the axes of the cylinders, the dimensionality of the problem reduces to two and it is convenient to obtain the solution of the Helmholtz equation in polar coordinates.

2.3.1 Acoustic Plane Wave

A harmonic plane wave of magnitude A propagating in an acoustic field can be expressed as

$$\phi(\mathbf{r},\theta,\mathbf{t}) = Ae^{\mathbf{i}(\underline{k}\cdot\underline{\mathbf{r}}-\omega\mathbf{t})} \qquad (2.18)$$

Note that we can write the vectors \underline{r} and \underline{k} as

$$\underline{\mathbf{r}} = \mathbf{r}(\cos\theta \ \underline{\mathbf{e}}_1 + \sin\theta \ \underline{\mathbf{e}}_2) \ ; \ \mathbf{r} = |\underline{\mathbf{r}}| \quad , \qquad (2.19)$$

$$\underline{c} = k(\cos\alpha \underline{e}_1 + \sin\alpha \underline{e}_2) ; \quad k = |\underline{k}| = \omega/c , \quad (2.20)$$

respectively. In this case, Eq. (2.18) can be written as

$$\phi = A e^{i k r \cos(\theta - \alpha)} e^{-i\omega t} , \qquad (2.21)$$

where θ and α are the angles that the vectors <u>r</u> and <u>k</u> make with the x-axis and <u>e</u>₁, <u>e</u>₂ are the unit vectors along the x and y directions of the Cartesian coordinate system (Fig. 2.1).



Figure 2.1 - Representation of a plane wave incidence.

Comparing Eq. (2.21) with Eq. (2.15) one can write the spatial part of $_{\varphi}$ as

$$\psi(\mathbf{r},\theta) = A e^{i k \mathbf{r} \cos(\theta - \alpha)} \qquad (2.22)$$

Note that ψ given by Eq. (2.22) does satisfy the Helmholtz equation, Eq. (2.16).

2.3.2 General Solution

In polar coordinates, the Laplacian operator, ∇^2 , is given as

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} , \qquad (2.23)$$

thus, Eq. (2.16) takes the form

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} + k^2 \psi = 0 \qquad (2.24)$$

The most common method used in obtaining the general solution of Eq. (2.24) is the method of separation-of-variables where a solution of the form

$$\psi(\mathbf{r}, \theta) = R(\mathbf{r})T(\theta) \qquad (2.25)$$

is assumed. Upon substitution of the above solution into Eq. (2.24) one can show that the latter equation reduces to two separate equations of the form

$$r^{2} \frac{d^{2}R}{dr^{2}} + r \frac{dR}{dr} + (k^{2}r^{2} - n^{2})R = 0 , \qquad (2.26)$$

$$\frac{d^2T}{d\theta^2} + n^2T = 0 (2.27)$$

Eq. (2.26) is the well-known Bessel's Differential Equation and has its solution either as

$$R = A_n J_n(kr) + B_n Y_n(kr)$$
, (2.28)

or,

$$R = A_n H_n^{(1)}(kr) + B_n H_n^{(2)}(kr) .$$
 (2.29)

 $J_n(kr)$ and $Y_n(kr)$ are known as the n-th order cylindrical Bessel functions of the first and second kind while $H_n^{(1)}(kr)$ and $H_n^{(2)}(kr)$ are the n-th order Hankel functions of the first and second kind, respectively. The unknown constants A_n and B_n are to be determined from the boundary conditions impending on the problem.

The solution of Eq. (2.27) can be written as

$$T = e^{\pm in\theta} = \begin{pmatrix} \cos(n\theta) \\ \sin(n\theta) \end{pmatrix} \qquad (2.30)$$

Hence, combining the solutions (2.28), (2.29), (2.30) and the time factor $exp[-i\omega t]$, one obtains the general solution as

$$\phi(r,\theta,t) = [A_n I_n^{(1)}(kr) + B_n I_n^{(2)}(kr)] (\cos(n\theta)) e^{-i\omega t} , \quad (2.31)$$

where $I_n^{(1)}$ and $I_n^{(2)}$ are the cylindrical Bessel or Hankel functions of the first and second kind, depending on the physics of the problem.

If a wave progressing through a medium encounters an obstacle immersed in the medium, scattering phenomenon takes place and the wave field becomes different from what it would have been in the absence of the obstacle. In general, the solution to the scattering problem of an acoustic wave propagation in an infinite fluid medium, requires the solution of the Helmholtz equation satisfying the boundary conditions prescribed over a discontinuity surface, called scatterer. In order to solve this problem, the usual method of attack is to try to satisfy the boundary conditions impending on the problem so as to obtain an exact solution directly from the general solution (Eq. 2.31) obtained by the method of separation-of-variables. However, this type of approach is successful only if the geometry of the curve S representing the boundary of the scatterer is such that S coincides with an orthogonal curvilinear coordinate system in which the Helmholtz equation seperates. Hence, the analytical solutions are possible only for simple geometries like circular and elliptic cylinders. Exact solutions are available for circular cylinders using circular polar coordinates and Bessel functions, and for elliptic cylinders using elliptic coordinates and Mathieu func-Thus, for general boundary geometries one of the various approxtions. imation techniques must be employed in the solution of the scattering The transition matrix formulation, shortly called T-matrix problems. method, is the latest technique developed for this purpose.

III. T-MATRIX FORMULATION

In the wave scattering problems, as stated in the previous section, the complexity of the boundary geometry of the scatterer necessitates the use of a numerical method to obtain an approximate solution. In the evaluation of the scattered wave field, especially at the far field, the T-matrix method is highly efficient and powerful computational procedure. It uses, depending on the dimensionality of the problem, only cylindrical or spherical wave functions even for bodies of arbitrary shape.

In the formulation, both incident and scattered waves are expanded in series of the common wave functions, known as the basis functions, satisfying the Helmholtz equation (Eq. 2.16). The unknown coefficients of the scattered wave series are then related to the coefficients of the incident wave by a transition matrix, called T-matrix.

3.1 BASIS WAVE FUNCTIONS

A class of solutions for Helmholtz reduced wave equation is the wave functions in circular polar coordinates,

$$\psi_{n}^{\sigma}(r,\theta) = (\varepsilon_{n})^{\frac{1}{2}}H_{n}^{(1)}(kr)\tau_{n}^{\sigma}(\theta) ; \qquad n = 0,1,...,\infty ,(3.1)$$

where $H_n^{(1)}(kr)$ is the cylindrical Hankel function of the first kind, τ_n^{σ} is given by

$$\tau_{n}^{\sigma}(\theta) = \begin{cases} \cos(n\theta) & \text{for } \sigma = 1 \\ \sin(n\theta) & \text{for } \sigma = 2 \end{cases}, \qquad (3.2)$$

and ε_n is the Neumann factor,

$$e_n = \begin{cases} 1 & \text{for } n = 0 \\ 2 & \text{for } n > 0 \end{cases}$$
 (3.3)

Equation (3.1) can be written from the general solution (2.31) by simply setting, $I_n^{(1)}(kr) \rightarrow H_n^{(1)}(kr)$, $B_n \rightarrow 0$.

Assuming a time dependency of the form

$$\phi(\mathbf{r},\theta,t) = \psi_n^{\sigma} e^{-i\omega t} , \qquad (3.4)$$

 ϕ represents an outgoing cylindrical wave with respect to the origin of a coordinate system located as shown in Fig. 3.1. Using the function $H_n^{(2)}(kr)$ in Eq. (3.4) would give the functional form of an incoming wave. However, if $H_n^{(1)}(kr)$ is replaced by $J_n(kr)$, the cylindrical Bessel function of the first kind, we get the expression for a standing wave,

$$\hat{\psi}_{n}^{\sigma}(\mathbf{r},\theta) = (\varepsilon_{n})^{\frac{1}{2}} J_{n}(kr) \tau_{n}^{\sigma}(\theta); \qquad n = 0, 1, \dots, \infty \qquad (3.5)$$

One should also note that $\hat{\psi}_{n}^{\sigma}$ is the regular part of ψ_{n}^{σ} given by Eq. (3.1), and it corresponds to the real part of the latter when kr is real. The functions ψ_{n}^{σ} and $\hat{\psi}_{n}^{\sigma}$ are called the basis wave functions for two dimensional scalar waves.



Figure 3.1 - Geometry of an infinite cylindrical obstacle.

Recalling the orthogonality relations for the trigonometric functions,

$$\int_{0}^{2\pi} \cos(m\theta) \cos(n\theta) d\theta = \frac{2\pi}{(\varepsilon_{m}\varepsilon_{n})^{\frac{1}{2}}} \delta_{mn} , \qquad (3.6)$$

$$\int_{0}^{2\pi} \sin(m\theta) \sin(n\theta) d\theta = \frac{2\pi}{(\varepsilon_m \varepsilon_n)^{\frac{1}{2}}} \delta_{mn} , \qquad (3.7)$$

$$\int_{0}^{2\pi} \cos(m\theta) \sin(n\theta) d\theta = 0 , \qquad (3.8)$$

where $\epsilon_m,~\epsilon_n$ are the Neumann factors and δ_{mn} is the Kronecker delta, one can write orthonogonality relation for $\tau_n^\sigma(\theta)$ over a closed curve as

$$\left(\varepsilon_{m}\varepsilon_{n}\right)^{\frac{1}{2}}\int_{0}^{2\pi}\tau_{m}^{\sigma}\tau_{n}^{\nu}d\theta = 2\pi\delta_{mn} \qquad \text{for} \quad \sigma = \nu \quad . \quad (3.9)$$

Since, only the Hankel functions of the first kind will be utilized in the rest of the work, to simplify the writing the superscript (1) appearing on $H_n(kr)$ will be omitted.

3.2 ORTHOGONALITY OF THE BASIS FUNCTIONS

From the divergence theorem, it can be shown that two arbitrary scalar functions u(x,y,t) and v(x,y,t) satisfy the Green's second identity [18],

$$\int_{C} \left(u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n}\right) dS = \int_{C} \left(u \nabla^2 v - v \nabla^2 u\right) dA \qquad (3.10)$$

The u and v and their first and second derivatives are continuous in a two-dimensional region of area A_s bounded by the closed curve C. The $\partial u/\partial n = \hat{n}$. ∇u is the directional derivative along a unit normal \hat{n} which is pointed outward from the curve C. So far no restriction has been imposed on u and v except the conditions of continuity.

Let u or v represent a scalar wave, satisfying the Helmholtz equation written in the form

$$(\nabla^2 + k_1^2)u = 0$$
, $(\nabla^2 + k_2^2)v = 0$ (3.11)

A time factor $exp(-i\omega t)$ is assumed for the wave. When both u and v have the same wave number, i.e., $k_1 = k_2 = k = \omega/c$, the area integral on the right hand side of Eq. (3.10) vanishes identically and we have

$$\int_{C} \left(u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n} \right) dS = 0 \qquad (3.12)$$

This result is valid for any two functions u and v, so long as they satisfy the same wave equation, and the aforementioned conditions of continuity. For acoustic waves, u and v are the usual velocity potentials.

For the problem of scattering by an infinite cylindrical boundary, consider the geometry shown in Fig. 3.2, where the curve S represents the boundary of the scatterer while S_+ and S_{∞} are somewhat arbitrary circular curves outside the scatterer. By utilizing Eqs. (3.9) and (3.12), the following orthonogonality conditions can be established for the basis wave functions over the circle S_+ :

$$\int_{S_{1}} (\hat{\psi}_{p} - \frac{\partial \hat{\psi}_{q}}{\partial n} - \hat{\psi}_{q} - \frac{\partial \hat{\psi}_{p}}{\partial n}) dS = 0 , \qquad (3.13)$$

$$\int_{S_{1}} \left(\psi_{p} \frac{\partial \psi_{q}}{\partial n} - \psi_{q} \frac{\partial \psi_{p}}{\partial n}\right) dS = 0 , \qquad (3.14)$$

$$\int_{S_{i}} (\psi_{p} \frac{\partial \psi_{q}}{\partial n} - \hat{\psi}_{q} \frac{\partial \psi_{p}}{\partial n}) dS = (-4i)\delta_{pq} \qquad (3.15)$$

For simplicity in writing, only the subscript which indicates the order of the Bessel functions is retained, that is,

$$\psi_p \equiv \psi_p^{\sigma}$$
 , $\psi_q \equiv \psi_q^{v}$, etc. (3.16)

The proof of the first condition is rather simple. Let the region A_s in Eq. (3.10) be bounded externally by the circle S_+ of radius $r = r_+$

as shown in Fig. 3.2. Note that both $\hat{\psi}_p$ and $\hat{\psi}_q$ and their normal derivatives are continuous within S₊, that is, J_{p,q}(kr) have no singularity inside S₊, and furthermore they satisfy the wave equation (3.11) with $k_1 = k_2 = k$. Thus, Eq. (3.13) is a special case of (3.12) with $u = \hat{\psi}_p$ and $v = \hat{\psi}_q$.



Figure 3.2 - Geometry for a constant cross-sectional scatterer.

The same proof, however, cannot be applied to the functions ψ_p and ψ_q because the origin r = 0 is a singularity point for $H_{p,q}(kr)$. For the proof of Eq. (3.14), we consider a region bounded internally by S₊ ($r = r_+$) and externally by S_w($r = r_w$), (Fig. 3.2). Within this region, both $\psi_{p,q}$ and $\partial \psi_{p,q}/\partial n$ are regular, and Eq. (3.12) reduces to

$$-\int_{S_{+}} (\psi_{p} \frac{\partial \psi_{p}}{\partial n} - \psi_{q} \frac{\partial \psi_{p}}{\partial n}) ds + \int_{S_{+}} (\psi_{p} \frac{\partial \psi_{q}}{\partial n} - \psi_{q} \frac{\partial \psi_{p}}{\partial n}) ds = 0, (3.17)$$

where we have replaced u and v by ψ_p and ψ_q , respectively. Note that, while the outer normal along S₊ is in the negative direction of the unit radial vector \underline{e}_r , it is in the positive direction along S_∞. The negative sign in front of the integral along S₊ is introduced because the normal derivative $\partial/\partial n$ is taken to be in the direction of $+\underline{e}_r$ along both curves.

It is seen immediately that both integrals in the above equation vanish identically when p = q. When ψ_p differs from ψ_q , it can be shown that each integral vanishes due to the orthogonality condition given by Eq. (3.9). To show this, note that $\partial/\partial n = \partial/\partial r$ and ds = rd θ along the circular curves S₊ and S_∞, and

$$\frac{\partial \psi_{p}}{\partial r} = (\varepsilon_{p})^{\frac{1}{2}} \tau_{p}^{\sigma}(\theta) \frac{\partial H_{p}(kr)}{\partial r} = (\varepsilon_{p})^{\frac{1}{2}} \tau_{p}^{\sigma} k H_{p}'(kr) , \qquad (3.18)$$

$$\frac{\partial \psi_{q}}{\partial r} = (\varepsilon_{q})^{\frac{1}{2}} \tau_{q}^{\nu}(\theta) \frac{\partial H_{q}(kr)}{\partial r} = (\varepsilon_{q})^{\frac{1}{2}} \tau_{q}^{\nu} k H_{q}^{\prime}(kr) , \qquad (3.19)$$

where primes denote derivatives of the Hankel functions with respect to their argument. Substitution of these into the integral along S_+ in Eq. (3.17) yields

$$\int_{S_{+}} (\dots) ds = \int_{S_{+}} \left[(\varepsilon_{p})^{\frac{1}{2}} \tau_{p}^{\sigma} (\varepsilon_{q})^{\frac{1}{2}} H_{p}(kr) k H_{q}'(kr) - (\varepsilon_{q})^{\frac{1}{2}} \tau_{q}^{\sigma} (\varepsilon_{p})^{\frac{1}{2}} \tau_{p}^{\sigma} H_{q}(kr) k H_{p}'(kr) \right] ds . \qquad (3.20)$$

Since $r \neq r(\theta)$, one can write the above equation in the form

$$\int_{+}^{(\dots)ds} = kr[H_p(kr)H'_q(kr) - H_q(kr)H'_p(kr)][(\epsilon_p\epsilon_q)^{\frac{1}{2}}\int_{0}^{2\pi} \tau_p^{\sigma}\tau_q^{\upsilon}d\theta]. \quad (3.21)$$

One can see that the expression in the second bracket is the same as that given by Eq. (3.9). Thus the integral over S_+ vanishes if $p \neq q$. Going through a very similar procedure, one can show that the integral over S_{∞} also vanishes for $p \neq q$. This completes the proof of Eq. (3.14). From this proof it is seen that the circle S_+ can be replaced by any other circular curve, not necessarily centered at the origin of the coordinate system.

To prove Eq. (3.15), let the region of interest be the same region as in the preceeding case and let $u = \psi_p$ and $v = \hat{\psi}_q$ in Eq. (3.12). Since they are continuous within that region, we can write

$$\int_{+}^{f} (\psi_{p} - \frac{\partial \hat{\psi}_{q}}{\partial n} - \hat{\psi}_{q} - \frac{\partial \psi_{p}}{\partial n}) ds = \int_{S_{\infty}}^{f} (\psi_{p} - \frac{\partial \hat{\psi}_{q}}{\partial n} - \hat{\psi}_{q} - \frac{\partial \psi_{p}}{\partial n}) ds . \quad (3.22)$$

Again, if $p \neq q$, one can show by giving a similar proof as for Eq. (3.17) that the integrals in the above expression vanish. However, they do not, when p = q.

To evaluate the integral on the right-hand side, let S_{∞} recede to infinity. The asymptotical expressions for ψ_p and $\hat{\psi}_q$ as $r \rightarrow \infty$ are [2]

$$\psi_{p} = (\varepsilon_{p})^{\frac{1}{2}} H_{p}(kr) \tau_{p}^{\sigma} \stackrel{\infty}{\rightarrow} (\varepsilon_{p})^{\frac{1}{2}} \tau_{p}^{\sigma} \sqrt{2/\pi kr} e^{i(kr-\pi_{p})} , \qquad (3.23)$$

$$\hat{\psi}_{q} = (\epsilon_{q})^{\frac{1}{2}} J_{q}(kr) \tau_{q}^{\vee} \stackrel{\infty}{\rightarrow} (\epsilon_{q})^{\frac{1}{2}} \tau_{q}^{\vee} \sqrt{2/\pi kr} \cos(kr - \pi_{q}) , \qquad (3.24)$$

where $\pi_p = (2p + 1)\pi/4$. Recalling that, [2],

$$\frac{\partial H_{p}(kr)}{\partial r} = kH'_{p}(kr) = \frac{k}{2} [H_{p-1}(kr) - H_{p+1}(kr)] , \qquad (3.25)$$

$$\frac{\partial J_q(kr)}{\partial r} = k J'_q(kr) = \frac{k}{2} [J_{q-1}(kr) - J_{q+1}(kr)] , \qquad (3.26)$$

and as $r \rightarrow \infty$

$$H_{p\pm 1}(kr) \rightarrow \pm i\sqrt{2/\pi kr} e^{i(kr-\pi_p)} , \qquad (3.27)$$

$$J_{q\pm 1}(kr) \rightarrow \pm \sqrt{2/\pi kr} \sin^{(kr-\pi_q)}$$
, (3.28)

the asymptotic expressions for $\partial\psi_p/\partial r$ and $\partial\hat\psi_q/\partial r$ can be written as

$$\frac{\partial \psi_p}{\partial r} \stackrel{\infty}{\to} i k \sqrt{2/\pi k r} e^{i (k r - \pi_p)} (\varepsilon_p)^{\frac{1}{2}} \tau_p^{\sigma} , \qquad (3.29)$$

$$\frac{\partial \psi_{\mathbf{q}}}{\partial r} \stackrel{\infty}{\rightarrow} -k\sqrt{2/\pi kr} \sin(kr - \pi_{\mathbf{q}}) (\varepsilon_{\mathbf{q}})^{\frac{1}{2}} \tau_{\mathbf{q}}^{\nu} \qquad (3.30)$$

Substituting the expressions (3.23), (3.24), (3.29) and (3.30) into Eq. (3.22), one obtains

$$\int_{S_{+}} (\psi_{p} - \frac{\partial \tilde{\psi}_{q}}{\partial n} - \tilde{\psi}_{q} - \frac{\partial \psi_{p}}{\partial n}) ds = \int_{S_{\infty}} (-\frac{2i}{\pi r}) [(\varepsilon_{p} \varepsilon_{q})^{\frac{1}{2}} \tau_{p}^{\sigma} \tau_{q}^{\nu} e^{i(kr - \pi_{p})}] \\ \times [\cos(kr - \pi_{q}) - isin(kr - \pi_{q})] ds \\ = \int_{T}^{2\pi} (-\frac{2i}{\pi r}) [(\varepsilon_{p} \varepsilon_{q})^{\frac{1}{2}} \tau_{p}^{\sigma} \tau_{q}^{\nu} e^{i(\pi_{q} - \pi_{p})}] r d\theta$$

$$= \left[-\frac{2i}{\pi}\right] \left[\left(\varepsilon_{p}\varepsilon_{q}\right)^{\frac{1}{2}} \int_{0}^{2\pi} \tau_{p}^{\sigma} \tau_{q}^{\nu} d\theta\right] \left[e^{i(\pi_{q}-\pi_{p})}\right]$$
$$= \left[-\frac{2i}{\pi}\right] \left[2\pi\delta_{pq}\right] \left[e^{i(\pi_{q}-\pi_{p})}\right]$$
$$= -4i\delta_{pq} \qquad (3.31)$$

This completes the proof of the third orthogonality condition (3.15).

3.3 WAVE FUNCTION EXPANSIONS

Consider a cylindrical inclusion of cross-sectional area A_c , bounded by a closed curve S, and let a plane wave $u^i(r,\theta)exp(-i\omega t)$ be incident upon this inclusion, as shown in Fig. 3.3. Then, the total wave field $u(r,\theta)$ in the medium will be composed of two parts; the incident wave, u^i , and the scattered wave, u^s , i.e.,

$$u(r,\theta) = u^{\dagger}(r,\theta) + u^{S}(r,\theta)$$
(3.32)

For notational convenience, the time factor $exp(-i\omega t)$ will be supressed in the rest of the work.

Both incident and scattered waves can be expanded into a series of the basis functions in the form

$$u^{i} = A \Sigma a_{m} \hat{\psi}_{m}$$
, $u^{S} = A \Sigma c_{m} \psi_{m}$, (3.33)

where A is the constant amplitude factor of the incident wave, a_m are the incident wave coefficients which can be determined uniquely for a given incident wave type, and c_m are the unknown coefficients of the scattered wave. The coefficients c_m , as will be shown, can be determined



Figure 3.3 - Geometry for a plane wave incidence on a cylindrical scatterer.

by making use of a transition matrix (T-matrix) which relates to $c_{m}^{}$ to $a_{m}^{}$ by

 $c_m = \Sigma T_{mn} a_n$

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(3.34)
3.3.1 The Incident Wave Field

Any known incident wave can be expanded into a series of the basis functions $\hat{\psi}_{\rm m}$ as defined by Eqs. (3.5) and (3.16), [12],

$$u^{i}(r,\theta) = A \Sigma a_{m} \hat{\psi}_{m}(r,\theta) \equiv A \Sigma \Sigma \Delta \sigma_{m=0}^{\infty} \sigma_{=1}^{2} a_{m}^{\sigma} \hat{\psi}_{m}^{\sigma}$$
, $r < r_{\infty}$. (3.35)

Since, $\psi_{\rm m}$ are regular at the origin (r = 0), taken to be inside the surface S, the series converges uniformly within a large circle S_∞ of radius, say, r_∞. For plane incident waves, S_∞ is at the infinity.

In Eq. (3.35) and in the following equations, for the simplicity in writing, the double sum will be represented by a single index and summation. Since the index m is an abbreviation for the two indices m and σ , the symbol Σ means summation on σ from 1 (even) to 2 (odd) and m from 0 to ∞ .

Consider a plane incident wave with its wave normal perpendicular to the axis of the scatterer, as shown in Fig. 3.3. This wave can be represented by

$$u^{i} = Ae^{ikr\cos(\theta - \alpha)}$$
(3.36)

as defined in section (2.3.1), where the time factor has been omitted. If the plane wave expression given in Eq. (3.36) is expanded into a series as defined by Eq. (3.35) then the coefficients a_m^{σ} can be determined as explained below.

Trigonometric Fourier series expansion of an even function f(x), . in the interval -L $\leq x \leq$ L is given by

$$f(x) = \sum_{n=0}^{\infty} \varepsilon_n \frac{a_n}{2} \cos \frac{n \pi x}{L} , \qquad (3.37)$$

where $\boldsymbol{\epsilon}_n$ is the Neumann factor, and

$$a_n = \frac{1}{L} \int_{0}^{2L} f(x) \cos \frac{n \pi x}{L} dx$$
, $n = 0, 1, ..., \infty$ (3.38)

If we now consider the function f(x) to be of the form $exp(izcos\beta)$ where we have replaced x by β . If this function is defined in the interval $-\pi < \beta < \pi$ then its Fourier series expansion is

$$e^{iz\cos\beta} = \sum_{n=0}^{\infty} \epsilon_n - \frac{a_n}{2} \cos(n\beta)$$
, (3.39)

where

$$a_n = \frac{1}{\pi} \int_{0}^{2\pi} e^{iz\cos\beta} \cos(n\beta)d\beta \qquad (3.40)$$

Recalling the integral representation of the Bessel functions [19],

$$2\pi i^{n} J_{n}(z) = \int_{0}^{2\pi} e^{iz\cos\beta} \cos(n\beta) d\beta , \qquad (3.41)$$

equation (3.39) can be written as

$$e^{iz\cos\beta} = \sum_{n=0}^{\infty} \varepsilon_n i^n J_n(z)\cos(n\beta) \qquad (3.42)$$

In order to obtain the Fourier series expansion of the expression given by Eq. (3.36) one then has to replace z by kr and β by (θ - α) in Eq. (3.42). Doing so one gets

$$e^{ikr\cos(\theta - \alpha)} = \sum_{n=0}^{\infty} \varepsilon_{n} i^{n} J_{n}(kr) \cos[n(\theta - \alpha)]$$
$$= \sum_{n=0}^{\infty} \varepsilon_{n} i^{n} J_{n}(kr) [\cos(n\theta) \cos(n\alpha) + \sin(n\theta) \sin(n\alpha)]. \quad (3.43)$$

Rearranging Eq. (3.43) as

$$e^{ikr\cos(\theta-\alpha)} = \sum_{\substack{n=0}}^{\infty} \sum_{\sigma=1}^{2} \left[(\varepsilon_n)^{\frac{1}{2}} J_n(kr) \frac{\cos(n\theta)}{\sin(n\theta)} \right] \left[(\varepsilon_n)^{\frac{1}{2}} i^n \frac{\cos(n\alpha)}{\sin(n\alpha)} \right] \quad (3.44)$$

and then comparing with Eq. (3.35), the coefficients a_m^σ of the incident plane wave are found to be

$$a_{m}^{\sigma} = (\epsilon_{m})^{\frac{1}{2}} i^{m} \qquad \cos(m_{\alpha}); \sigma = 1 \qquad (3.45)$$
$$\sin(m_{\alpha}); \sigma = 2$$

3.3.2 The Scattered Wave Field

The scattered wave field can be expanded into a series in terms of the functions $\psi_{m},$ i.e.,

$$u^{S}(r,\theta) = A \Sigma c_{m} \psi_{m}(r,\theta) \equiv A \sum_{m=0}^{\infty} \sum_{\sigma=1}^{2} c_{m}^{\sigma} \psi_{m}^{\sigma}$$
, $r \ge r_{+}$. (3.46)

Note that $\psi_{\rm m}$ are regular outside the region enclosed by the curve S, the boundary of the inclusion. Thus, the above series converges uniformly outside and on a circle S₊, enclosing the inclusion (Fig. 3.3). The radius r₊ of S₊ is yet unspecified, and it can be ascertained when the unknown coefficients c_m are found.

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To determine c_m , consider a region which is bounded internally by S, and externally by S₊. Furthermore, to apply Eq. (3.12) for that region, let v be $\hat{\psi}_m$, and u be the total wave field as defined in Eq. (3.32), that is

$$u(r,\theta) = u^{i}(r,\theta) + u^{s}(r,\theta)$$
; (r inside S₊, outside S). (3.47)

At the boundary S one writes

$$u(r,\theta) = u^{\dagger}$$
, $\frac{\partial u(r,\theta)}{\partial n} = \frac{\partial u^{\dagger}}{\partial n}$; (r on S). (3.48)

Both u_{+} and $\partial u^{+}/\partial n$ are unknown quantities at the surface S, where (+) indicates that we approach S from the positive direction of \hat{n} .

Substituting Eqs. (3.35), (3.46) into (3.47) and evaluating it on $\rm S_{+}$ we get

$$u(r,\theta) = A \sum_{j} \hat{\psi}_{j}(r,\theta) + A \sum_{j} \psi_{j}(r,\theta); \quad (r \text{ on } S_{+}) . \quad (3.49)$$

Since both series are uniformly convergent on the surface (circle) S_+ , one can differentiate them term by term to obtain

$$\frac{\partial u(r,\theta)}{\partial n} = A \Sigma a_{j} \frac{\partial \hat{\psi}_{j}(r,\theta)}{\partial n} + A \Sigma c_{j} \frac{\partial \psi_{j}}{\partial n}; \quad (r \text{ on } S_{+}) . \quad (3.50)$$

Substitution of the expressions (3.48), (3.49) and (3.50) into Eq. (3.12) where the contour C is made out of two curves, S and S_+ , gives

$$\int_{S} (u^{+} \frac{\partial \hat{\psi}_{m}}{\partial n} - \hat{\psi}_{m} \frac{\partial u^{+}}{\partial n}) ds - \int_{S_{+}} A[(\sum_{j} a_{j} \hat{\psi}_{j} + \sum_{j} c_{j} \psi_{j}) \frac{\partial \hat{\psi}_{m}}{\partial n} - \hat{\psi}_{m}(\sum_{j} a_{j} \frac{\partial \hat{\psi}_{j}}{\partial n} + \sum_{j} c_{j} \frac{\partial \psi_{j}}{\partial n})] ds = 0 \quad . \quad (3.51)$$

Rearranging the above expression, we get

$$\int_{S} (u^{+} \frac{\partial \hat{\psi}_{m}}{\partial n} - \hat{\psi}_{m} \frac{\partial u^{+}}{\partial n}) ds = A \sum_{j} \sum_{j=1}^{S} (\hat{\psi}_{j} \frac{\partial \hat{\psi}_{m}}{\partial n} - \hat{\psi}_{m} \frac{\partial \psi_{j}}{\partial n}) ds] + A \sum_{j=1}^{S} \sum_{j=1}^{S} (\psi_{j} \frac{\partial \hat{\psi}_{m}}{\partial n} - \hat{\psi}_{m} \frac{\partial \psi_{j}}{\partial n}) ds]. \qquad (3.52)$$

The integral associated with a_j vanishes according to the orthogonality condition (3.13), and that with c_j is the same as the integral in Eq. (3.15). Hence the right-hand side of the preceding equation reduces to $A\Sigma c_j(-4i\delta_{jm})$, yielding

$$\int_{S} \left(u^{+} \frac{\partial \tilde{\psi}_{m}}{\partial n} - \hat{\psi}_{m} \frac{\partial u^{+}}{\partial n} \right) ds = -4iAc_{m} \qquad (3.53)$$

Similarly, replacing v in Eq. (3.12) by ψ_m , ad applying the orthogonality conditions (3.14) and (3.15), one obtains

$$\int_{S} (u^{+} \frac{\partial \psi_{m}}{\partial n} - \psi_{m} \frac{\partial u^{+}}{\partial n}) ds = 4iAa_{m} \qquad (3.54)$$

Equation (3.53) states that the unknown coefficients c_m of the series representation for the scattered wave is determined by an integral of the surface sources, u^+ and $\partial u^+/\partial n$, over the boundary of the scatterer. These two sources, u^+ and $\partial u^+/\partial n$, are not independent of each other and are related by Eq. (3.54) where a_m are known.

The unknown quantities, u⁺ and ∂u⁺/∂n, are to be determined from prescribed boundary conditions.

3.4 BOUNDARY CONDITIONS

In connection with the above formulations, two boundary conditions are of great importance in scattering problems, namely the Neumann and Dirichlet boundary conditions.

3.4.1 Neumann Type Boundary Condition

In the case where the inclusion is a rigid one, the normal component of the velocity field should vanish on the boundary, i.e.,

$$\frac{\partial u^+}{\partial n} = 0 \qquad , \qquad (3.55)$$

or equivalently, from Eq. (3.47),

$$\frac{\partial u^{S}}{\partial n} = -\frac{\partial u^{i}}{\partial n} \qquad \text{on } S \qquad (3.56)$$

Such a boundary condition is known as the Neumann type boundary condition.

The other surface quantity u^+ , which is unspecified, can be represented [1] as a series of the regular wave functions $\hat{\psi}_i$, that is,

$$u^{+} = A \Sigma \alpha_{j} \hat{\psi}_{j} , \quad \text{on S} , \qquad (3.57)$$

where α_i are the coefficients of the surface field.

Substitution of Eqs. (3.55) and (3.57) into Eqs. (3.53) and (3.54) yields

$$\sum_{j} \alpha_{j} \left[\int_{S} \hat{\psi}_{j} \frac{\partial \hat{\psi}_{m}}{\partial n} ds \right] = -4ic_{m}, \qquad (3)$$

.58)

$$\sum_{j=1}^{\Sigma \alpha_{j}} \left[\int_{\partial T} \hat{\psi}_{j} - \frac{\partial \Psi_{m}}{\partial n} ds \right] = 4ia_{m} \qquad (3.59)$$

The above pair of equations show that both scattered field coefficients c_m and incident field coefficients a_m are related to the unknown surface field coefficients α_j . If the latter are eliminated from these two equations, the c_m can then be expressed directly in terms of a_m . Before going into the elimination procedure, it will be better obtain the corresponding pair of equations also for Dirichlet boundary condition.

3.4.2 Dirichlet Type Boundary Condition

The Dirichlet boundary condition in acoustic wave scattering corresponds to the case where we have a cavity inside the fluid medium. Thus, the pressure vanishes on the surface of the cavity, i.e.,

$$u^{+} = u^{1} + u^{S} = 0$$
 , on S . (3.60)

The unspecified surface quantity $\partial u^{\dagger}/\partial n$, in this case, can be represented [1] as a series in terms of the normal gradients of the regular wave functions, that is,

$$\frac{\partial u^{+}}{\partial n} = A \Sigma \alpha_{j} \frac{\partial \hat{\psi}_{j}}{\partial n} , \quad \text{on S} . \quad (3.61)$$

Equations (3.60) and (3.61) are then substituted into (3.53) and (3.54) to obtain

$$\sum_{j=1}^{\infty} \hat{\psi}_{m} \frac{\partial \hat{\psi}_{j}}{\partial n} ds] = -4ic_{m} , \qquad (3.62)$$

$$\sum_{j=1}^{\Sigma\alpha_{j}} \left[-\int_{S} \psi_{m} \frac{\partial \psi_{j}}{\partial n} ds \right] = 4ia_{m} \qquad (3.63)$$

The above two equations, together with Eqs. (3.58) and (3.59) obtained in the preceding section, form the basis for deriving a transition matrix relating the coefficients c_m to a_m directly.

3.5 THE TRANSITION MATRIX

One can define two matrices with elements Q_{jm} and \hat{Q}_{jm} , which are, in the case of Neumann boundary condition, given by

$$Q_{jm} = \frac{1}{4} \int_{S} \hat{\psi}_{j} \frac{\partial \psi_{m}}{\partial n} ds , \qquad (3.64)$$
$$\hat{Q}_{jm} = \frac{1}{4} \int_{S} \hat{\psi}_{j} \frac{\partial \hat{\psi}_{m}}{\partial n} ds , \qquad (3.65)$$

and in the case of Dirichlet boundary condition, given by

$$Q_{jm} = -\frac{1}{4} \int_{S} \psi_m \frac{\partial \hat{\psi}_j}{\partial n} ds , \qquad (3.66)$$

$$\hat{Q}_{jm} = -\frac{1}{4} \int_{S} \hat{\psi}_{m} \frac{\partial \hat{\psi}_{j}}{\partial n} ds \qquad (3.67)$$

As can be easily seen in both cases, the elements of the \tilde{Q} matrix are simply the real part of the elements of the Q matrix, i.e.,

 $\hat{Q}_{jm} = Re(Q_{jm})$ (3.68)

Now, substituting the corresponding Q and Q matrices into Eqs. (3.58), (3.59), (3.62) and (3.63), we obtain

$$i\sum_{j} \sum_{j} \hat{Q}_{jm} = c_{m} , \qquad (3.69)$$
$$-i\sum_{j} \alpha_{j} Q_{jm} = a_{m} . \qquad (3.70)$$

The equations (3.69) and (3.70) can then be expressed in matrix form,

$$i\hat{Q}_{\approx}^{\dagger} \hat{\alpha} = \hat{C}$$
, (3.71)

$$-i \underset{\approx}{0} \overset{t}{\alpha} = \underset{\sim}{a} \qquad , \qquad (3.72)$$

where (t) denotes the transpose matrix. Elimination of the unknown surface field coefficient vector α from these two equations yields

$$c_{\omega} = -\left[\hat{Q}^{\dagger} \left(Q^{\dagger} \right)^{-1}\right]_{\omega} = T_{\omega} a_{\omega} \qquad (3.73)$$

The T is called transition matrix which relates the scattered wave field directly to the incident wave field. As shown in Ref. [1,20], the T-matrix is symmetric, i.e., $T_{\approx}^{t} = T_{\approx}$, it can thus be determined from the relation

$$\underset{\approx}{\overset{Q}{\approx}} \underset{\approx}{\overset{T}{\approx}} = - \underset{\approx}{\overset{Q}{\approx}} , \qquad (3.74)$$

or,

$$\mathbf{x} = -\mathbf{Q}^{-1} \hat{\mathbf{Q}} \qquad (3.75)$$

3.6 STRUCTURE OF THE Q-MATRIX

As given in Eqs. (3.63-67), the elements of the Q-matrix are given by integrals involving basis wave functions and their normal gradients. These integrals are evaluated along the boundary of the scatterer. We should note that for a given incident wave field (given wave number) the elements of the Q-matrix are fixed when the geometry and the boundary type of the scatterer are given.

In order to understand the structure of the Q-matrix better, one should restore the full index notation of the basis functions, that is,

$$\psi_{j}(r,\theta) \equiv \psi_{j}^{\sigma}(r,\theta)$$
 , $\psi_{m}(r,\theta) \equiv \psi_{m}^{\nu}(r,\theta)$, etc. (3.76)

With this notation, Eqs. (3.64) and (3.66) take the form

$$Q_{jm}^{\sigma\nu} = \frac{1}{4} \int_{S} \hat{\psi}_{j}^{\sigma} \frac{\partial \psi_{m}^{\nu}}{\partial n} ds , \qquad (3.77)$$

$$Q_{jm}^{\sigma\nu} = -\frac{1}{4} \int_{S} \hat{\psi}_{m}^{\nu} \frac{\partial \hat{\psi}_{j}^{\sigma}}{\partial n} ds , \qquad (3.78)^{\prime}$$

respectively. From these expressions one sees that the Q-matrix actually consists of four submatrices, that is,

$$Q_{\approx} = \begin{vmatrix} Q^{11} & Q^{12} \\ \approx & \approx \\ Q^{21} & Q^{22} \\ \approx & \approx \\ Q^{21} & Q^{22} \end{vmatrix} , \qquad (3.79)$$

where, in the case of Neumann boundary condition,

$$Q_{jm}^{11} = \frac{1}{4} \int_{S} \hat{\psi}_{j}^{1} \frac{\partial \psi_{m}^{1}}{\partial n} ds , \qquad (3.80)$$

$$Q_{jm}^{12} = \frac{1}{4} \int_{S} \hat{\psi}_{j}^{r} \frac{\partial \psi_{m}^{2}}{\partial n} ds , \qquad (3.81)$$

$$Q_{jm}^{21} = \frac{1}{4} \int_{S} \hat{\psi}_{j}^{2} \frac{\partial \psi_{m}^{1}}{\partial n} ds , \qquad (3.82)$$

$$Q_{jm}^{22} = \frac{1}{4} \int_{S} \hat{\psi}_{j}^{2} \frac{\partial \psi_{m}^{2}}{\partial n} ds , \qquad (3.83)$$

and in the case of Dirichlet boundary condition,

$$Q_{jm}^{11} = -\frac{1}{4} \int_{S} \psi_{m}^{1} \frac{\partial \widehat{\psi}_{j}^{1}}{\partial n} ds , \qquad (3.84)$$

$$Q_{jm}^{12} = -\frac{1}{4} \int_{S} \psi_m^2 \frac{\partial \hat{\psi}_j^1}{\partial n} ds , \qquad (3.85)$$

$$Q_{jm}^{21} = -\frac{1}{4} \int_{S} \psi_{m}^{1} \frac{\partial \widehat{\psi}_{j}^{2}}{\partial n} ds , \qquad (3.86)$$

$$Q_{jm}^{22} = -\frac{1}{4} \int_{S_{c}} \psi_{m}^{2} \frac{\partial \widehat{\psi}_{j}^{2}}{\partial n} ds \qquad (3.87)$$

Note that ψ_n^σ and $\widehat{\psi}_n^\sigma$ are given by

$$\hat{\psi}_{n}^{1}(\mathbf{r},\theta) = (\epsilon_{n})^{\frac{1}{2}} H_{n}(k\mathbf{r}) \cos(n\theta) , \qquad (3.88)$$

$$\hat{\psi}_n^2(r,\theta) = (\varepsilon_n)^{\frac{1}{2}} H_n(kr) \sin(n\theta) , \qquad (3.89)$$

$$\hat{\psi}_{n}^{1}(r,\theta) = (\varepsilon_{n})^{\frac{1}{2}} J_{n}(kr) \cos(n\theta) , \qquad (3.90)$$

$$\hat{\psi}_{n}^{2}(r,\theta) = (\varepsilon_{n})^{\frac{1}{2}} J_{n}(kr) \sin(n\theta) \qquad (3.91)$$

Depending on the geometry of the scatterer, the Q-matrix has the following properties:

For seperable geometries, i.e., for the circle and ellipse, the Q-matrix is symmetric [1], that is,

or in explicit form

$$Q_{jm}^{11} = Q_{mj}^{11}$$
, $Q_{jm}^{22} = Q_{mj}^{22}$, $Q_{jm}^{12} = Q_{mj}^{21}$. (3.93)

Another important property is that, if the cross-sectional geometry of the scatterer has a mirror symmetry with respect to x-axis, i.e., across the plane $y = r\sin\theta = 0$, so that $r(\theta) = r(2\pi-\theta)$, then the integrals involving the mixed products of sines and cosines will vanish and we get

$$Q_{jm}^{12} = Q_{jm}^{21} = 0$$
 , (3.94)

and for the nonzero matrices, $Q_{\frac{1}{2}}^{11}$ and $Q_{\frac{2}{2}}^{22}$, the integrals are to be evaluated only by considering the half of the boundary.

If the boundary of the scatterer has symmetry with respect to both x and y-axes, then [1],

$$Q_{jm} = 0$$
 if (j+m) is odd. (3.95)

Also, for such boundaries, real part of the Q-matrix is symmetric [21], i.e.,

$$\hat{\underline{Q}}^{\dagger} = \hat{\underline{Q}} \qquad (3.96)$$

Note that, for seperable geometries both real and imaginary parts are symmetric.

3.7 PROPERTIES OF THE T-MATRIX

In the light of the notation of Eq. (3.79), a corresponding block notation can be used for the transition matrix in Eq. (3.75),

$$\begin{array}{c|cccc} T = \begin{vmatrix} T^{11} & T^{12} \\ \approx & \approx \\ T^{21} & T^{22} \\ \approx & \approx \\ \end{array} \end{vmatrix} = - \begin{vmatrix} Q^{11} & Q^{12} \\ \approx & \sim \\ Q^{21} & Q^{22} \\ \approx & \approx \\ \end{array} \begin{vmatrix} -1 \\ x \\ x \\ \vdots \\ \end{array} \begin{vmatrix} \hat{Q}^{11} & \hat{Q}^{12} \\ \approx & \approx \\ \hat{Q}^{21} & \hat{Q}^{22} \\ \approx & \approx \\ \end{array} \end{vmatrix} .$$
(3.97)

For the boundaries having mirror symmetry with respect to x-axis, Eq. (3.97) reduces to the two (single) equations

$$T_{\approx}^{11} = -(Q_{\approx}^{11})^{-1} \hat{Q}_{\approx}^{11} , \quad T_{\approx}^{22} = -(Q_{\approx}^{22})^{-1} \hat{Q}_{\approx}^{22} . \quad (3.98)$$

Because of the reciprocity principle and energy-conservation requirements, as shown in Refs. [1] and [20], the T-matrix is always symmetric, i.e.,

$$\mathbf{T}_{\approx}^{\mathsf{t}} = \mathbf{T}_{\approx} \qquad , \qquad (3.99)$$

and

$$\underset{\approx}{\mathsf{T}} \underset{\approx}{\mathsf{T}}^{*} = -\operatorname{Re}(\underset{\approx}{\mathsf{T}}) \qquad (3.100)$$

where the asteriks (*) denotes the complex conjugation and (Re) means the real part. These properties are valid for all geometries and can be used to check the accuracy in the numerical evaluations of the T-matrix.

3.8 PRESENTATION OF THE SCATTERED WAVE FIELD

The scattered field coefficients c_m , which are to be evaluated through the relation (3.73), can be written in a compact form as

$$c_{m}^{\sigma} = \sum_{n,\nu} T_{mn}^{\sigma\nu} a_{n}^{\nu} , \qquad (3.101)$$

or, in block matrix notation,

One can then write from Eq. (3.101)

$$c_{m}^{1} = \sum_{n=0}^{\infty} T_{mn}^{11} a_{n}^{1} + \sum_{n=1}^{\infty} T_{mn}^{12} a_{n}^{2} , \qquad m = 0, 1, \dots, \infty , \qquad (3.103)$$

$$c_{m}^{2} = \sum_{n=0}^{\infty} T_{mn}^{21} a_{n}^{1} + \sum_{n=1}^{\infty} T_{mn}^{22} a_{n}^{2} , \qquad m = 1, 2, \dots, \infty , \qquad (3.104)$$

where

$$a_n^1 = (\varepsilon_n)^{\frac{1}{2}} i^n \cos(n\alpha)$$
 , $a_n^2 = (\varepsilon_n)^{\frac{1}{2}} i^n \sin(n\alpha)$. (3.105)

By using the above notation and omitting the time factor, the incident and scattered wave fields can be rewritten as

$$u^{i} = A \sum_{n=0}^{\infty} (\varepsilon_{n})^{\frac{1}{2}} J_{n}(kr) [a_{n}^{1} \cos(n\theta) + a_{n}^{2} \sin(n\theta)], \quad r < \infty, (3.106)$$
$$u^{s} = A \sum_{n=0}^{\infty} (\varepsilon_{n})^{\frac{1}{2}} H_{n}(kr) [c_{n}^{1} \cos(n\theta) + c_{n}^{2} \sin(n\theta)], \quad r \text{ outside S}$$
$$(3.107)$$

Once the coefficients of the scattered wave field are determined numerically, the scattered field is known. Various field quantities such as the velocity potentials in the near field, surface field potentials, far field amplitudes and scattering cross-sections which are of interest in the acoustic wave scattering problems can then be calculated.

3.8.1 <u>Near-field Solutions</u>

The velocity potentials due to the scattered wave field at finite distances from the scatterer can be obtained from Eq. (3.107). In the case where the near-field solutions are concerned, generally the quantity of interest is the distribution of the velocity potential on the boundary of the scatterer due to scattered wave field. However, this distribution can not be evaluated by the T-matrix formulation directly, because the series representing the scattered wave field is not complete on the surface of the scatterer and hence Eq. (3.107) is not valid on S, [1,12]. The series is complete only on a circular surface, say S₊, which is outside the scatterer as shown in Fig. 3.3. It is possible to evaluate the velocity potential distribution from the intermediate steps of the T-matrix formulation. For this purpose, one can write, from Eq. (3.47) and (3.48),

$$u^{+}(r,\theta) = u^{i}(r,\theta) + u^{s}(r,\theta)$$
, r on S, (3.108)

or

$$u^{S} = u^{T} - u^{T}$$
, r on S . (3.109)

In the case of Neumann boundary condition, substitution of Eqs. (3.35) and (3.57) into Eq. (3.109) yields

 $u^{S} = A \Sigma \alpha_{n} \hat{\psi}_{n} - A \Sigma a_{n} \hat{\psi}_{n}$, r on S, (3.110)

where u^+ is the total wave field at the boundary and α_n are the unknown surface field coefficients. One can immediately see that the unknown coefficients α_n can be uniquely determined through the Q-matrix by solving the matrix equation (3.72). Substituting the expressions for $\hat{\psi}_{\mathbf{n}}$ in the above equation and rearranging the terms slightly we get

$$u^{S} = A \sum_{n=0}^{\infty} (\varepsilon_{n})^{\frac{1}{2}} J_{n}(kr) [(\alpha_{n}^{1} - a_{n}^{1})\cos(n\theta) + (\alpha_{n}^{2} - a_{n}^{2})\sin(n\theta)],$$

r on S.(3.111)

In the case of Dirichlet boundary condition, however, the scattered wave field on the boundary is simply given by

$$u^{S} = -u^{i}$$
, on S , (3.112)

and hence it needs not be evaluated.

It should be noted that, for a circular cylindrical scatterer, the surface field potentials due to scattered wave field can still be determined from Eq. (3.107) directly, because, in this case, the circular surface S_+ can be arbitrarily replaced by S so that the outgoing wave series in Eq. (3.107) is complete also on the boundary of the scatterer.

The near-field results, in fact, have very little practical significance and they are of almost no interest in applications such as non-destructive evaluations, remote sensing etc.

3.8.2 Far Field Solution

From the practical point of view, the far field amplitude is the most important quantity to be determined in the acoustic scattering problems.

The expression for the scattered wave field at distances far from the scatterer is obtained from Eq. (3.107) by using the asymptotical form of the Hankel functions. The asymptotical representation of $H_n(kr)$ as $r \rightarrow \infty$ is given by, [2],

$$H_n(kr) \stackrel{\infty}{=} \sqrt{2/\pi kr} e^{i[kr - ((2n+1)/4)\pi]}$$
, (3.113)

and after simple manipulations, one gets

$$H_{n}(kr) \stackrel{\text{e}}{=} \sqrt{2/i\pi kr} e^{ikr} i^{-n} \qquad (3.114)$$

Then, substitution of Eq. (3.114) into Eq. (3.107) yields

$$u^{S} = e^{ikr}\sqrt{2/i\pi kr} A \sum_{n=0}^{\infty} (\varepsilon_{n})^{\frac{1}{2}} i^{-n} [c_{n}^{1} \cos(n\theta) + c_{n}^{2} \sin(n\theta)], \quad r \to \infty.$$
(3.115)

One can also write Eq. (3.115) as

$$u^{s} = e^{ikr} \sqrt{2/i\pi kr} f(\theta)$$
, (3.116)

where $f(\theta)$ is the far field amplitude describing the angular variation of the scattered field at distances far from the scatterer and it is given by

$$f = A \sum_{n=0}^{\infty} (\varepsilon_n)^{\frac{1}{2}} i^{-n} [c_n^1 \cos(n\theta) + c_n^2 \sin(n\theta)]. \qquad (3.117)$$

In the presentation of the scattered wave field results, the field quantities given by Eqs. (3.107), (3.111) and (3.117) are first non-dimensionalized through dividing them by the constant amplitude factor of the incident wave A, and then the angular variations of their norms, i.e., $|u^{S}/A|$, |f/A| versus θ , are plotted in polar coordinates for the regions of interest.

3.8.3 Total Scattering Cross-Section

In two dimensional wave scattering problems, the total scattering cross-section is defined as the ratio between power generated by the scattered wave over a circle with large radius around the obstacle and the power per unit area generated by the incident wave, and is given by, [13],

$$\sigma^{\text{tot}} = \frac{1}{2\pi} \int_{0}^{2\pi} \sigma(\theta) d\theta , \qquad (3.118)$$

where $\sigma(\theta)$ is the differential cross-section,

$$\sigma(\theta) = |f(\theta)|^2 \qquad (3.119)$$

The total scattering cross-section can then be written in terms of the scattered wave field coefficients as

$$\sigma^{\text{tot}} = \sum_{n=0}^{\infty} \left(|c_n^1|^2 + |c_n^2|^2 \right) .$$
 (3.120)

The above quantity is generally used to check the convergency of the scattered wave field results obtained by the T-matrix formulation and employed in the selection of the T-matrix size to be used in the calculations.

IV. NUMERICAL EVALUATIONS

This chapter is devoted to the applications of the transition matrix method described in the previous chapter. The examples presented include cylindrical rigid inclusions and cavities with circular, elliptical, rectangular and triangular cross sections. Some of the results obtained have been compared with the known exact and approximate solutions, [9,22].

In the numerical evaluations, an incident plane wave as given by Eq. (3.36) has been considered and both the "near-field" and, mainly, the "far-field" results have been obtained for various angles of incidence. The results have been presented in polar graphical forms, as discussed in sections (3.8.1) and (3.8.2).

The basic steps in the computation of the scattered wave field using T-matrix method can be outlined as follows:

- a) Description of the boundary geometry of the scatterer,
- b) Numerical evaluation of the boundary integrals for the generation of the Q-matrix elements,
- c) Inversion of the Q-matrix and creation of the T-matrix,
- d) Calculation of the scattered field coefficients through
 T-matrix,

- e) Evaluation of the scattered field for various incidence angles
- f) Check for the convergency of the numerical results by checking the convergency of the series, Eq. (3.120), representing the total scattering cross section.

Note that, if the convergency of the total scattering cross section is found to be insufficient at the end of the calculations, the steps (b) to (f) should be repeated by using a Q-matrix of the larger size.

4.1 GENERAL PROCEDURE FOR THE EVALUATION OF THE Q-MATRIX ELEMENTS

In order to generate the Q-matrix one should evaluate the integrals given by Eqs. (3.77) and (3.78) either analytically or numerically. Analytical evaluation is possible for only circular geometry, because $r \neq r(\theta)$ for circle and the boundary integrals reduces to simple trigonometric integrals. For general boundary geometries, the boundary integrals are to be evaluated numerically.

In order to illustrate the procedure followed in the computation of the Q-matrix elements, the derivations of the necessary analytical and numerical expressions are presented here for only the Q_{\approx}^{11} matrix for the rigid inclusion case.

To obtain the elements of the Q^{11} matrix, first, it is helpful $\propto \sum_{k=1}^{\infty}$ to write Eq. (3.80) with slightly different notation as

$$Q_{jm}^{11} = \frac{1}{4} \int_{S} \left[\hat{n} ds \cdot \nabla \psi_{m}^{1}(r,\theta) \right] \hat{\psi}_{j}^{1}(r,\theta) , \qquad (4.1)$$

where \hat{n} is the unit normal vector and ∇ is the gradient operator. To put the above expression into a form which is suitable for numerical calculations we need the explicit expressions for the terms inside the integrand.

Consider Fig. 4.1 for the evaluation of the term \hat{n} ds. As the incremental quantities Δs , $\Delta \theta$, Δr become infinitesimally small ($\varepsilon \rightarrow 0$), we can write

$$\hat{n} = e_{\rho} \cos \gamma - e_{\rho} \sin \gamma$$
, (4.2)

where \underline{e}_r and \underline{e}_θ are the radial and tangential unit vectors, respectively, and



Figure 4.1 - Geometrical representation of the unit normal vector.

$$\cos \gamma = \frac{r d \theta}{ds}$$
, $\sin \gamma = \frac{d r}{ds}$. (4.3)

Hence, from Eqs. (4.2) and (4.3), one obtains

$$\hat{n}ds = rd\theta \left[\underline{e}_{r} - \frac{1}{r} \frac{dr}{d\theta} \underline{e}_{\theta}\right] \qquad (4.4)$$

As to the term $\nabla \psi_m^1(r,\theta),$ using the gradient expression in polar coordinates,

$$\nabla = \frac{\partial}{\partial r} \frac{\mathbf{e}}{\mathbf{r}} + \frac{1}{r} \frac{\partial}{\partial \theta} \frac{\mathbf{e}}{\theta} , \qquad (4.5)$$

one can write

$$\nabla \psi_{\mathrm{m}}^{1} = (\varepsilon_{\mathrm{m}})^{\frac{1}{2}} [\underline{e}_{\mathrm{r}} \mathrm{kH}_{\mathrm{m}}^{\prime}(\mathrm{kr}) \cos(\mathrm{m}\theta) - \underline{e}_{\mathrm{\theta}} - \frac{\mathrm{m}}{\mathrm{r}} \mathrm{H}_{\mathrm{m}}^{\prime}(\mathrm{kr}) \sin(\mathrm{m}\theta)] . \quad (4.6)$$

Note that

$$H_{m}^{\prime}(kr) = \frac{1}{2} [H_{m-1}(kr) - H_{m+1}(kr)]$$
$$= \frac{m}{kr} H_{m}(kr) - H_{m+1}(kr) , \qquad (4.7)$$

thus, Eq. (4.6) reduces to

$$\nabla \psi_{\mathrm{m}}^{1} = (\varepsilon_{\mathrm{m}})^{\frac{1}{2}} \{ \underline{e}_{\mathrm{r}} k [-\frac{m}{kr} + H_{\mathrm{m}}(kr) - H_{\mathrm{m+l}}(kr)] \cos(m\theta) - \underline{e}_{\theta} - \frac{m}{r} + H_{\mathrm{m}}(kr) \sin(m\theta) \} . \quad (4.8)$$

From Eqs. (4.4) and (4.8), one then obtains

$$\hat{n}ds \cdot \nabla \psi_{m}^{1} = (\varepsilon_{m})^{\frac{1}{2}} r d\theta \{ k [-\frac{m}{kr} H_{m}(kr) - H_{m+1}(kr)] \cos(m\theta) \}$$

$$+ \frac{m}{r^2} \frac{dr}{d\theta} H_{m}(kr) \sin(m\theta) \} \qquad (4.9)$$

Finally, substituting Eq. (4.9) and corresponding expression for $\hat{\psi}_{j}^{1}$, i.e., Eq. (3.90), into Eq. (4.1), and also writing dr/d θ as $1/k[d(kr)/d\theta]$, one gets

$$Q_{jm}^{11} = \frac{\left(\varepsilon_{j}\varepsilon_{e}\right)^{\frac{1}{2}}}{4} \int_{0}^{2\pi} d\theta \{J_{j}(kr)\cos j\theta\} \{[mH_{m}(kr) - krH_{m+1}(kr)]\cos(m\theta)\}$$

$$\frac{d}{kr} H_{m}(kr) \sin(m\theta) \frac{d}{d\theta} (kr) \} \quad . \quad (4.10)$$

The quantities r and dr/d θ both being functions of θ , i.e., $r = r(\theta)$, d[r(θ)]/d θ , are to be determined from the analytical equations describing the geometry of the boundary of the scatterer of interest. One should also notice that 'r' always occurs as kr(θ) in the expression for the Q-matrix elements. Therefore, in the numerical evaluations, it is sufficient to specify the wave number in a non-dimensional form, say k ℓ , where ℓ is a characteristic length in the problem, and the nondimensional ratios of the geometry parameters such as aspect ratio, corner radius ratio etc.

As stated earlier, in general the angular integrations in Eq. (4.10) which are of the form

$$I = \int_{\Omega}^{2\pi} F(\theta) d\theta ,$$

on S

(4.11)

can be carried out only numerically. The first step in the numerical evaluation of these integrals is the subdivision of the boundary S into N number of intervals ΔS_i each subtending a central angle of $\Delta \theta_i$



Figure 4.2 - Boundary subdivision.

such that, as shown in Fig. 4.2,

$$\sum_{i=1}^{N} \Delta S_{i} = S , \qquad \sum_{i=1}^{N} \Delta \theta_{i} = 2\pi . \qquad (4.12)$$

The integral given by Eq. (4.11) can then be written as

$$I = \sum_{i=1}^{N} [f F(\theta)d\theta], \text{ on } S. \qquad (4.13)$$

Having divided the boundary into N suitably small segments, one should next approximate the integrals which are to be evaluated separetly for each angular interval $\Delta \theta_i$. This approximation can be made by using the Simpson's rule of any order. Hence, one can approximate the angular integrals in Eq. (4.13) in the form (see Appendix A)

$$\int_{\Delta \theta_{i}} F(\theta) d\theta \approx C_{s} \frac{\Delta \theta_{i}}{P} \sum_{q=0}^{P} F(\theta_{a_{i}} + q[\Delta \theta_{i}/P]) \times w_{q} , \quad (4.14)$$

where P is the order of the Simpson's rule used, C_s is a constant multiplication factor and w_q are the weighing factors. Note that, C_s and w_q are fixed numbers depending on the order P. Substituting Eq. (4.14) into (4.13), one gets

$$I \simeq \frac{C_{s}}{P} \sum_{i=1}^{N} \sum_{q=0}^{P} \Delta \theta_{i} F(\theta_{a_{i}} + q [\Delta \theta_{i}/P]) w_{q}, \text{ on } S. \quad (4.15)$$

Finally, applying the above expression to Eq. (4.10) and using the geometrical definitions shown in Fig. 4.3, one obtains the desired numerical expression for the computation of the Q^{11} -matrix elements,

$$Q_{jm}^{l_{1}} \simeq \frac{\left(\varepsilon_{j}\varepsilon_{m}\right)^{\frac{1}{2}}}{4} \frac{C_{s}}{P} \frac{\sum_{i=1}^{N} \sum_{q=0}^{P} \left(\Delta\theta_{i}\right)w_{q}\left\{J_{j}\left(kr_{iq}\right)\cos\left(j\theta_{iq}\right)\right\}}{x \left\{\left[mH_{m}\left(kr_{iq}\right) - kr_{iq}H_{m+1}\left(kr_{iq}\right)\right]\cos\left(m\theta_{iq}\right)\right\} + \frac{m}{kr_{iq}}H_{m}\left(kr_{iq}\right)\sin\left(m\theta_{iq}\right)\left[\frac{d(kr)}{d\theta}\right]_{iq}\right\}, r_{iq} \text{ on S. (4.16)}$$

The analytical and numerical expressions obtained for the other Q-submatrices, for both rigid inclusion and cavity cases, are given in Appendix B and C.



Figure 4.3 - Interval subdivision.

4.2 CREATION OF THE T-MATRIX

After computing the Q-matrix elements according to the procedure outlined in the previous section, one can then create the T-matrix. But, before going into the evaluation of the T-matrix, one should perform the following conditioning on the Q-matrix, [21]. Because of the behaviour of the Hankel functions appearing in the elements of the Q-matrix, the imaginary parts of the elements of the Q-matrix will tend to grow to very large numerical values for the elements above the diagonal. In order to avoid the loss of precision due to the finite precision arithmetic employed by the digital computers, it is convenient at this point to set the imaginary parts of all the mentioned elements to zero, by Gaussian elimination.

The T-matrix can be evaluated, as discussed earlier, using the matrix equation

$$\mathbf{T}_{\approx} = -\mathbf{Q}^{-1} \times \hat{\mathbf{Q}}$$
(4.17)

which first requires the inversion of the Q-matrix directly by using a standard matrix inversion technique. However, from the point of view of numerical accuracy, this evaluation can be performed more effectively by first transforming the Q-matrix to a unitary matrix Q_{unit} and then applying the following matrix equation, [20,21],

$$\sum_{k=0}^{T} = -\left[\underset{\text{sunit}}{\mathbb{Q}} \right]^{t} \times \left[\underset{\text{sunit}}{\mathbb{Q}} \right] , \qquad (4.18)$$

where the symbols 't' and '*' denote the matrix transpose and the complex conjugate, respectively. This transformation, i.e., Q to Q_{unit} , is done by Schmidt orthogonalization [21].

4.3 NUMERICAL EXAMPLES

In this section, the numerical results obtained by the application of the T-matrix method to the cylindrical rigid inclusions and cavities having circular, elliptical, rectangular (round cornered) and triangular (isosceles) cross sectional geometries are presented. The incident wave considered, as stated earlier, is a plane acoustic wave with constant velocity c, angular frequency ω , and wavelength $\lambda = 2\pi/k$.

The scattered wave field is computed for various non-dimensional wave numbers, k^{ℓ} , and incidence angles, α .

4.3.1 Circular Cylinders

The equation of the boundary of a circular cylindrical scatterer in polar coordinates is simply





Figure 4.4 - Geometry for the circular cylindrical scatterer.

where a is the radius of the circle, as shown in Fig. 4.4. From Eq. (4.19) one easily sees that $r \neq r(\theta)$, thus,

$$[dr/d\theta] = 0 \qquad (4.20)$$

The polar plots of the scattered field results obtained for the rigid inclusion and cavity cases are presented in Figs. 4.8 to 4.10. Note that, due to the rotational symmetry of the problem, the scattered wave field is to be computed for only zero angle of incidence, i.e., for $\alpha = 0^{\circ}$.

4.3.2 Elliptical Cylinders

For the evaluation of the boundary integrals one needs the equation of the ellipse and the angular variation [dr/d θ], which are given by

$$r(\theta) = a[\cos^2\theta + (b/a)^2 \sin^2\theta]^{\frac{1}{2}} , \qquad (4.21)$$

$$\frac{dr}{d\theta} = a\{(b/a)[(b/a)^2 - 1]sin\thetacos\theta[sin^2\theta + (b/a)^2cos^2\theta]^{-3/2}\},$$
(4.22)

where 2a and 2b are the major and minor axes, as shown in Fig. 4.5.

The polar plots of the near and far field results obtained for the rigid inclusion and cavity cases for various ka values and aspect ratios (b/a) are presented in Figs. 4.11 to 4.18.



Figure 4.5 - Boundary geometry of the elliptic cylinder.

4.3.3 Rectangular Cylinders

In the application of the T-matrix method to rectangular cylinders, a round cornered type rectangular cross section is considered. To describe the boundary geometry, the following geometrical definitions are made first;

$$\gamma = \arctan[(b/a - r_c/a)/(1 - r_c/a)]$$
, (4.23)

$$(D/a) = (1 - r_c/a)/\cos\gamma$$
, (4.24)

$$\phi_1 = \arctan[b/a - r_c/a]$$
, (4.25)



Figure 4.6 - Boundary geometry of the round cornered rectangular cylinder.

$$\Phi_{2} = \arctan[(b/a)/(1 - r_{c}/a)] , \qquad (4.26)$$

$$\Phi_{3} = \pi - \Phi_{0} , \qquad \Phi_{1} = \pi - \Phi_{1} , \qquad (4.27)$$

where 2a and 2b are the width and the length of the rectangle and r_c is the corner radius, as shown in Fig. 4.6. Employing these definitions one can write the analytical equations of the boundary and the $(dr/d\theta)$ values in the range $0 \le \theta \le \pi$ as

a)
$$r(\theta) = a(1/\cos\theta)$$
,

 $\frac{dr}{d\theta} = a(\sin\theta/\cos^2\theta) \quad \text{for} \quad 0 \le \theta \le \phi_1 \quad , \quad (4.28)$

b)
$$r(\theta) = a\{(D/a)\cos(\theta - \gamma) + [(D/a)^2\cos^2(\theta - \gamma) + (r_c/a)^2 - (D/a)^2]^{\frac{1}{2}}\},$$

 $\frac{dr}{d\theta} = a\{(D/a)\sin(\gamma - \theta) + (D/a)^2\cos(\gamma - \theta)\sin(\gamma - \theta)$
 $x [(D/a)^2\cos^2(\gamma - \theta) + (r_c/a)^2 - (D/a)^2]^{-\frac{1}{2}}\}$

for
$$\phi_1 \leq \theta \leq \phi_2$$
 , (4.29)

$$r(\theta) = a[(b/a)/\sin\theta],$$

$$\frac{dr}{d\theta} = a[-(b/a)\cos\theta/\sin^2\theta] \quad \text{for } \phi_2 \le \theta \le \phi_3, \quad (4.30)$$

d)
$$r(\theta) = a\{(D/a)\cos(\theta+\gamma-\pi) + [(D/a)^2\cos^2(\theta+\gamma-\pi) + (r_c/a)^2 - (D/a)^2]^{\frac{1}{2}}\}$$
,

$$\frac{dr}{d\theta} = a\{(D/a)\sin(\pi-\gamma-\theta) + (D/a)^2\cos(\pi-\gamma-\theta)\sin(\pi-\gamma-\theta)\}$$

$$x [(D/a)^{2}\cos^{2}(\pi - \gamma - \theta) + (r_{C}/a)^{2} - (D/a)^{2}]^{-\frac{1}{2}}$$

for $\phi_{3} \leq \theta \leq \phi_{4}$, (4.31)

e)
$$r(\theta) = a(-1/\cos\theta)$$

$$\frac{dr}{d\theta} = a(-\sin\theta/\cos^2\theta) \quad \text{for } \phi_4 \le \phi < \pi \quad . \quad (4.32)$$

Note that, the values of r and dr/d θ for the range $\pi \leq \theta \leq 2\pi$ are computed by first setting $\theta \rightarrow \theta - \pi$ and then applying again the formulas given in Eqs. (4.28-32).

The polar plots of the scattered wave field results obtained for various ka values and the ratios (b/a), (r_c/a) are presented in Figs. 4.19 to 4.34.

4.3.4 Triangular Cylinders

We have also considered a scatterer with cross section in the form of an isosceles triangle, as shown in Fig. 4.7. The expressions for the boundary and dr/d θ in the range 0 $\leq \theta < \pi$ are

a)
$$r(\theta) = h[1/(3\cos\theta)]$$

 $\frac{dr}{d\theta} = h[\sin\theta/(3\cos^2\theta)] \quad \text{for} \quad 0 \le \theta \le \phi , \quad (4.33)$

$$r(\theta) = h\{(2/3)\tan(\beta/2)/[\sin\theta - \cos\theta\tan(\beta/2)\}$$

$$\frac{d\mathbf{r}}{d\mathbf{\rho}} = h\{(-2/3)\tan(\beta/2)[\cos\theta + \sin\theta\tan(\beta/2)]/[\sin\theta]$$

- $\cos\theta \tan(\beta/2)$]²} for $\phi \leq \theta < \pi$, (4.34)

where the angle ϕ is given by



Figure 4.7 - Boundary geometry of the triangular cylinder.

 $\phi = \arctan[3\tan(\beta/2)]$

(4.35)

Values of $r(\theta)$ and $dr/d\theta$, in the range $\pi \le \theta < 2\pi$, are obtained by setting $\theta \rightarrow 2\pi - \theta$ in the Eqs. (4.33) and (4.34), and then setting $dr/d\theta$ to $-dr/d\theta$.

The scattered field results obtained for various kh and β values are presented in Figs. 4.35 to 4.41.

V. CONCLUSIONS

In this work, the numerical results for acoustic plane waves scattered by circular, elliptical, rectangular and triangular rigid inclusions and cavities have been obtained utilizing the T-matrix method. The results are presented in polar graphical forms. These results are compared with the exact or approximate solutions [9,22], where available. For the rectangular and triangular cylinders, however, because of the unavailability of any result obtained in other studies, no comparison can be made.

In the evaluations of the elements of the Q-matrix, fourth order Simpson's rule is used to perform the angular integrations along the boundaries of the scatterers. Good convergence is obtained using 36 integration steps in the range $0-\pi$. For triangular geometry, corners may be thought to give rise to difficulties from the analytical point of view. However, since the integrals are evaluated by considering only discrete points taken within each angular interval, no computational problem is encountered. The symmetry and other numerical properties given by Eqs. (3.92), (3.94) and (3.95) is effectively used to check the accuracy of the numerical procedure followed.

The T-matrix is constructed by inverting the Q-matrix using Gauss-Schmidt orthogonalization technique, as discussed in Section 4.2.

The requirements given by Eqs. (3.99) and (3.100) are first used to check the correctness of the computer program then they are, together with Eqs. (3.92), (3.94), (3.95), incorporated into the program reducing computing time significantly.

It is observed that the size of the T-matrix to be used is related to the non-dimensional number kr_{max} , where k is the wave number and r_{max} is the maximum r value which is to be encountered along the boundary of the scatterer. In order to be able to obtain sufficiently convergent results, the size of the T-matrix is generally increased in a direct proportion to the non-dimensional number kr_{max} . The sufficiency of the matrix size, however, is controlled by checking the convergency of the total scattering cross-section, Eq. (3.120). This check is made up to when the total scattering cross section did not differ by more than at least 10^{-5} per cent. A rough estimate for the size requirement of the T-matrix may be given as 6 x 6 for $kr_{max} = 0.1$, 15 x 15 for $kr_{max} = 1.0$ and 40 x 40 for $kr_{max} = 5.0$.

In the case of scattering by circular cylinders, both near (Fig. 4.8) and far field (Figs. 4.9, 3.10) results are found to be in excellent agreement with the exact solutions for all wave numbers, [22]. This is because of the fact that the basis wave functions given by Eqs. (3.1) and (3.5) are exact expressions for the circle and hence the T-matrix formulation yields exact solutions for the circular cross-sections.

In the case of scattering by elliptical cylinders, the near field results are plotted in Figs. 4.11, 4.12 for the aspect ratios b/a = 0.5 and 2.0 and for $\alpha = 0^{\circ}$. In these figures, we observe that for a given incident wave number the angular spectrum of the scattered

field is quite different for two different values of b/a. In the case of scattered far field amplitudes, some of the results were compared with those obtained in Ref. [9] and found to be in good agreement especially for relatively small wave numbers ($ka \le 1.0$). The far field results have been obtained for the aspect ratios 0.5, 2.0, 5.0 and for different incidence angles and presented in Figs. 4.13-15 for rigid inclusion, and in Figs. 4.16-18 for cavity. In the case of rigid inclusion, the structure of the polar plots varies quite remarkably for different incidence angles and aspect ratios. Especially for the higher aspect ratios, larger angular peaks are observed in the forward direction. However, for a cavity, we observe that, in contrast to the rigid inclusion, the dependency of the structure of the plots on both incidence angle and aspect ratio is not so significant.

The near field results, Figs. 4.19, 4.20, for scatterers with rectangular cross-section show that the surface field potential distributions, comparing with the circular and elliptical geometries, display sharper maxima and minima for all wave numbers. Although, for low frequencies (ka \leq 1.0), the shapes of the plots are somewhat similar to the ones obtained in the elliptical case, the picture is entirely different in the high frequency range. For the far field case, we observe that the field scattered by a rectangular cylinder displays more angular structure than that generated by a circular or elliptical cylinder. The far field results obtained for a rigid inclusion (Figs. 4.21-27) indicate that effects of the aspect ratio and the incidence angle on the shape of the polar plots are relatively strong. These effects are more obvious at higher wave numbers (ka > 1.0). On the
other hand, it is observed that, as shown in Figs. 4.21, 4.22 (rigid inclusion) and in Figs. 4.28, 4.29 (cavity), effect of the corner radius ratio, r_c/a is not significant for a given aspect ratio and the wave number. In this respect, the scattered field results pertaining to the rectangular geometry are presented for $r_c/a = 0.1$ only. In the case of a cavity (Figs. 4.28-34), as for the rigid inclusion, the structure of the far field plots exhibits similar behaviour with respect to the changes in the aspect ratio, incidence angle, and frequency; however, the curves for the cavity are relatively smooth. A comparison of the far field results obtained for a cavity and a rigid inclusion cases reveals that for identical surface geometries no similarity exists for any given wave number in the range $0.1 \le ka \le 1.0$. However, at the higher wave numbers the cavity cannot be clearly distinguished from a rigid inclusion.

In the case of cylinders with triangular cross-sections, the near field solution given in Fig. 4.35 are quite different when compared to the previous geometries. This difference may be attributed to the fact that the cross-sectional geometry of the cylinder has only single symmetry axis. The far field results both for cavity and rigid inclusion, Figs. 4.36-41, are given for three different tip angles, $\beta = 60^{\circ}$, 30° and 0.001° . For $\beta = 60^{\circ}$ and 30° , we observe that if $\alpha \neq 0^{\circ}$ the far field plots are quite similar to the ones obtained for the rectangular cross-sections, while for $\alpha = 0^{\circ}$ structure of the plots differs remarkably when compared to the other geometries. Especially at the higher frequencies, for $\alpha = 0^{\circ}$ comparably larger angular peaks are observed in the forward scattering than in the

backward scattering. For the case $\beta = 0.001^{\circ}$ where the shape of the scatterer looks like a strip rather than a triangle, as shown in Figs. 4.38 and 4.41 general form of the far field plots is quite different.

In the polar plots of the scattered field results, it is observed that at low frequencies, $kr_{max} \sim 0.1$, the pictures for all boundary geometries are almost the same in appearance and do not change with incidence angle, α , hence, for all cases, the obstacles behave like point scatterers. Thus low frequency results are not useful for practical applications. However, for higher frequencies, quite remarkable differences and angular variations are observed in the structures of the plots for different boundary geometries and incidence angles.

To sum up, the T-matrix method, using only circular or spherical wave functions and removing the geometrical restriction, makes it possible to analyze the scattering of waves from inclusions of any shape. Although the completeness of the series representation of the wave fields, especially in the near field, is still a matter of discussion, the method provides a systematical and powerful computational procedure for a very wide range of wave numbers.











Figure 4.9 - Far field amplitude, |f/A|, due to the scattered wave field from a rigid circular inclusion; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.



Figure 4.9 (continued).



Figure 4.10 - Far field amplitude, |f/A|, due to the scattered wave field from a circular cavity; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.





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Figure 4.11 - Velocity potential distribution, $|u^S/A|$, at the boundary of a rigid elliptical inclusion due to the scattered wave field for $\alpha = 0^0$ and b/a = 0.5; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.



Figure 4.11 (continued).



Figure 4.12 - Velocity potential distribution, $|u^S/A|$, at the boundary of a rigid elliptical inclusion due to the scattered wave field for $\alpha = 0^{\circ}$ and b/a = 2.0; (a) ka = 0.5, (b) ka = 1.0, (c) ka = 3.0



Figure 4.12 (continued).





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Figure 4.13 - Far field amplitude, |f/A|, due to scattered wave field from a rigid elliptical inclusion for b/a = 0.5; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.



Figure 4.13 (continued).

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Figure 4.14 - Far field amplitude, |f/A|, due to the scattered wave field from a rigid elliptical inclusion for b/a = 2.0; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 3.0.



Figure 4.14 (continued).





Figure 4.15 - Far field amplitude, |f/A|, due to the scattered wave field from a rigid elliptical inclusion for b/a = 5.0; (a) ka = 0.1, (b) ka = 0.5, (c) ka = 1.0.

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Figure 4.16 - Far field amplitude. |f/A|, due to the scattered wave field from an elliptical cavity for b/a = 0.5; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.



Figure 4.16 (continued).



Figure 4.17 - Far field amplitude, |f/A|, due to the scattered wave field from an elliptical cavity for b/a = 2.0; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 3.0.



Figure 4.17 (continued).

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Figure 4.18 - Far field amplitude, |f/A|, due to the scattered wave field from an elliptical cavity for b/a = 5.0; (a) ka = 0.1, (b) ka = 0.5, (c) ka = 1.0.

81.1



Figure 4.18 (continued).



Figure 4.19 - Velocity potential distribution, $|u^S/A|$, at the boundary of a rigid rectangular inclusion due to scattered wave field for $\alpha = 0^{\circ}$, $r_c/a = 0.1$ and b/a = 0.5;

(a) ka = 0.5, (b) ka = 1.0, (c) ka = 3.0



Figure 4.19 (continued).



Figure 4.20 - Velocity potential distribution, $|u^{S}/A|$, at the boundary of a rigid rectangular inclusion due to the scattered wave field for $\alpha = 0^{\circ}$. $r_{c}/a = 0.1$ and b/a = 2.0; (a) ka = 0.5, (b) ka = 1.0, (c) ka = 3.0.



Figure 4.20 (continued).

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Figure 4.21 - Effect of the corner radius on the far field amplitude, |f/A|, due to the scattered wave field from a rigid rectangular inclusion for $\alpha = 0^{\circ}$ and b/a = 1.0; (a) ka = 1.0, (b) ka = 5.0.





Figure 4.22 - Effect of the corner radius on the far field amplitude, |f/A|, due to the scattered wave field from a rigid rectangular inclusion for $\alpha = 0^{\circ}$ and b/a = 0.5; (a) ka = 1.0, (b) ka = 5.0.



Figure 4.23 - Far field amplitude, |f/A|, due to the scattered wave field from a rigid rectangular inclusion for $r_c/a = 0.1$ and b/a = 1.0; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.



Figure 4.23 (continued).



Figure 4.24 - Far field amplitude, |f/A|, due to the scattered wave field from a rigid rectangular inclusion for $r_c/a = 0.1$ and b/a = 0.5; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.



Figure 4.24 (continued).

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Figure 4.25 - Far field amplitude, |f/A|, due to the scattered wave field from a rigid rectangular inclusion for $r_c/a = 0.1$ and b/a = 2.0; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 3.0.



Figure 4.25 (continued).




Figure 4.26 - Far field amplitude, |f/A|, due to the scattered wave field from a rigid rectangular inclusion for $r_c/a = 0.1$ and b/a = 5.0; (a) ka = 0.1, (b) ka = 0.5, (c) ka = 1.0.



Figure 4.26 (continued).





Figure 4.27 - Far field amplitude, [f/A], due to the scattered wave field from a rigid rectangular inclusion for $r_c/a = 0.1$ and b/a = 10.0; (a) ka = 0.1, $\alpha = 0^{\circ}$, (b) ka = 0.1, $\alpha = 90^{\circ}$, (c) ka = 0.5, $\alpha = 0^{\circ}$, (d) ka = 0.5, $\alpha = 90^{\circ}$.





Figure 4.27 (continued).







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Figure 4.29 - Effect of the corner radius on the far field amplitude, |f/A|, due to the scattered wave field from a rectangular cavity for $\alpha = 0^{\circ}$ and b/a = 0.5; (a) ka = 1.0, (b) ka = 5.0.





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Figure 4.30 - Far field amplitude, |f/A|, due to the scattered wave field from a rectangular cavity for $r_c/a = 0.1$ and b/a = 1.0; (a) ka = 1.0, (b) ka = 3.0, (c) ka = 5.0.



Figure 4.30 (continued).





Figure 4.31 - Far field amplitude, |f/A|, due to the scattered wave field from a rectangular cavity for $r_c/a = 0.1$ and b/a = 0.5; (a) ka = 1.0, (b) ka = 5.0.





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Figure 4.32 - Far field amplitude, |f/A|, due to the scattered wave field from a rectangular cavity for $r_c/a = 0.1$ and b/a = 2.0; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 3.0.



Figure 4.32 (continued).



Figure 4.33 - Far field amplitude, |f/A|, due to the scattered wave field from a rectangular cavity for $r_c/a = 0.1$ and b/a = 5.0; (a) ka = 0.1, (b) ka = 0.5, (c) ka = 1.0.



Figure 4.33 (continued).





Figure 4.34 - Far field amplitude, $\lceil f/A \rceil$, due to the scattered wave field from a rectangular cavity for $r_c/a = 0.1$ and b/a = 10.0. (a) ka = 0.1 , (b) ka = 0.5.



Figure 4.35 - Velocity potential distribution, $|u^S/A|$, at the boundary of a rigid triangular inclusion due to the scattered wave field for $\alpha = 0^{\circ}$ and $\beta = 60^{\circ}$; (a) kh = 1.5, (b) kh = 3.0, (c) kh = 9.0.



Figure 4.35 (continued).





Figure 4.36 - Far field amplitude, |f/A|, due to the scattered wave field from a rigid triangular inclusion for $\beta = 60^{\circ}$; (a) kh = 0.3, (b) kh = 3.0, (c) kh = 9.0.



Figure 4.36 (continued).



Figure 4.37 - Far field amplitude, |f/A|, due to the scattered wave field from a rigid triangular inclusion for $\beta = 30^{\circ}$; (a) kh = 0.3, (b) kh = 3.0, (c) kh = 9.0.



Figure 4.37 (continued).





Figure 4.38 - Far field amplitude, |f/A|, due to the scattered wave field from a rigid triangular inclusion for $\beta = 0.001^{\circ}$; (a) kh = 0.3, (b) kh = 3.0, (c) kh = 9.0.



Figure 4.38 (continued).



Figure 4.39 - Far field amplitude, |f/A|, due to the scattered wave field from a triangular cavity for $\beta = 60^{\circ}$; (a) kh = 0.3, (b) kh = 30, (c) kh = 9.0.



Figure 4.39 (continued).



Figure 4.40 - Far field amplitude, |f/A|, due to the scattered wave field from a triangular cavity for $\beta = 30^{\circ}$; (a) kh = 0.3, (b) kh = 3.0, (c) kh = 9.0.



Figure 4.40 (continued).



Figure 4.41 - Far field amplitude, |f/A|, due to the scattered wave field a triangular cavity for $\beta = 0.001^{\circ}$; (a) kh = 0.3 , (b) kh = 3.0 , (c) kh = 9.0.

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Figure 4.41 (continued).

APPENDICES

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APPENDIX A

SIMPSON'S APPROXIMATE INTEGRATON FORMULA

Consider an integral in the form

$$I = \int_{a}^{b} f(x) dx$$

The above integral can be evaluated approximately by using Simpson's rule which can be written as

$$\int_{a}^{D} f(x) dx \simeq C_{s} h[w_{0}f(a) + w_{1}f(a+h) + w_{2}f(a+2h) + ... + w_{p}f(b)]$$
(A.2)

where C_s is a constant factor, w_i 's are the weight factors, P is the order of the Simpson's rule used for the approximation and h is given by

$$h = (b - a)/P$$
 (A.3)

The expression given by Eq. (A.2) may also be written in a compact form as

$$\int_{a}^{b} f(x) dx \simeq C_{s} h \sum_{i=0}^{p} w_{i} f(a + i x h) . \qquad (A.4)$$

(A.1)

As an example, for a fourth order Simpson's rule (Bode's formula) [19], the values of C_s and w_i are

$$C_s = 2/45$$
, $w_0 = w_4 = 7$, $w_1 = w_3 = 32$, $w_2 = 12$.
(A.5)

Thus Eq. (A.2) takes the form

$$\int_{a}^{D} f(x)dx \simeq (2/45)h[7f(a) + 32f(a + h) + 12f(a + 2h) + 32f(a + 3h) + 7f(b)], \quad (A.6)$$

where

$$h = (b - a)/4$$

(A.7)

APPENDIX B ANALYTICAL EXPRESSIONS FOR THE ELEMENTS OF THE Q-MATRIX

a. <u>Rigid Inclusion</u>:

b.

$$\begin{aligned} Q_{jm}^{12} &= \frac{\left(\epsilon_{j}\epsilon_{m}\right)^{\frac{1}{2}}}{4} \int_{0}^{2\pi} d\theta \{J_{j}(kr)\cos(j\theta)\}\{[mH_{m}(kr) - krH_{m+1}(kr)]\sin(m\theta) \\ &- \frac{m}{kr} H_{m}(kr)\cos(m\theta) - \frac{d}{d\theta}(kr)\}. \quad (B.1) \end{aligned}$$

$$\begin{aligned} Q_{jm}^{21} &= \frac{\left(\epsilon_{j}\epsilon_{m}\right)^{\frac{1}{2}}}{4} \int_{0}^{2\pi} d\theta \{J_{j}(kr)\sin(j\theta)\}\{[mH_{m}(kr) - krH_{m+1}(kr)]\cos(m\theta) \\ &+ \frac{m}{kr} H_{m}(kr)\sin(m\theta) - \frac{d}{d\theta}(kr)\}. \quad (B.2) \end{aligned}$$

$$\begin{aligned} Q_{jm}^{22} &= \frac{\left(\epsilon_{j}\epsilon_{m}\right)^{\frac{1}{2}}}{4} \int_{0}^{2\pi} d\theta \{J_{j}(kr)\sin(j\theta)\}\{[mH_{m}(kr) - krH_{m+1}(kr)]\sin(m\theta) \\ &- \frac{m}{kr} H_{m}(kr)\cos(m\theta) - \frac{d}{d\theta}(kr)\}. \quad (B.3) \end{aligned}$$

$$\begin{aligned} \frac{Cavity:}{4} &= -\frac{\left(\epsilon_{j}\epsilon_{m}\right)^{\frac{1}{2}}}{4} \int_{0}^{2\pi} d\theta \{H_{m}(kr)\cos(m\theta)\}\{[jJ_{j}(kr) - krJ_{j+1}(kr)]\cos(j\theta)\} \end{aligned}$$

+ $\frac{j}{kr} J_j(kr) sin(j\theta) \frac{d}{d\theta}(kr)$ (B.4)

$$Q_{jm}^{12} = - \frac{(\varepsilon_j \varepsilon_m)^{\frac{1}{2}}}{4} \int_{0}^{2\pi} d\theta \{H_m(kr) \sin(m\theta)\} \{[jJ_j(kr) - krJ_{j+1}(kr)]\cos(j\theta)\}$$

+
$$\frac{j}{kr} J_j(kr)sin(j\theta) \frac{d}{d\theta}(kr)$$
}. (B.5)

$$Q_{jm}^{21} = - \frac{\left(\varepsilon_{j}\varepsilon_{m}\right)^{\frac{1}{2}}}{4} \int_{0}^{2\pi} d\theta \{H_{m}(kr)\cos(m\theta)\}\{[jJ_{j}(kr) - krJ_{j+1}(kr)]\sin(j\theta)\}$$

$$-\frac{j}{kr_{j}} J_{j}(kr)\cos(j\theta) - \frac{d}{d\theta}(kr)\}. \qquad (B.6)$$

$$Q_{jm}^{22} = - \frac{\left(\varepsilon_{j}\varepsilon_{m}\right)^{\frac{1}{2}}}{4} \int_{0}^{2\pi} d\theta \{H_{m}(kr)\sin(m\theta)\}\{[jJ_{j}(kr) - krJ_{j+1}(kr)]\sin(j\theta)\}$$

$$-\frac{j}{kr} J_{j}(kr)\cos(j\theta) \frac{d}{d\theta}(kr)\}. \qquad (B.7)$$

APPENDIX C NUMERICAL EXPRESSIONS FOR THE ELEMENTS OF THE Q-MATRIX

Rigid Inclusion:

a.

$$Q_{jm}^{12} \simeq \frac{\left(\varepsilon_{j}\varepsilon_{m}\right)^{\frac{1}{2}}}{4} \frac{C_{s}}{P} \frac{\sum_{i=1}^{N} \sum_{q=0}^{P} \left(\Delta\theta_{i}\right)w_{q}\left\{J_{j}\left(kr_{iq}\right)\cos\left(j\theta_{iq}\right)\right\}}{x\left\{\left[mH_{m}\left(kr_{iq}\right) - kr_{iq}H_{m+1}\left(kr_{iq}\right)\right]\sin\left(m\theta_{iq}\right)\right.\right.}$$
$$\left. - \frac{m}{kr_{iq}} H_{m}\left(kr_{iq}\right)\cos\left(m\theta_{iq}\right)\left[\frac{d(kr)}{d\theta}\right]_{\theta_{iq}}\right\} . \tag{C.1}$$
$$Q_{jm}^{21} \simeq \frac{\left(\varepsilon_{j}\varepsilon_{m}\right)^{\frac{1}{2}}}{4} \frac{C_{s}}{P} \sum_{i=1}^{N} \sum_{q=0}^{P} \left(\Delta\theta_{i}\right)w_{q}\left\{J_{j}\left(kr_{iq}\right)\sin\left(j\theta_{iq}\right)\right\}$$

$$\times \{ [mH_{m}(kr_{iq}) - kr_{iq}H_{m+1}(kr_{iq})] cos(m\theta_{iq})$$

$$+ \frac{m}{kr_{iq}} H_{m}(kr_{iq}) sin(m\theta_{iq}) [\frac{d(kr)}{d\theta}]_{\theta_{iq}} \} .$$
(C.2)

$$Q_{jm}^{22} \simeq \frac{\left(\varepsilon_{j}\varepsilon_{m}\right)^{\frac{1}{2}}}{4} \frac{C_{s}}{P} \sum_{i=1}^{N} \sum_{q=0}^{P} \left(\Delta\theta_{i}\right) w_{q} \{J_{j}(kr_{iq})sin(j\theta_{iq})\}$$

$$\times \{[mH_{m}(kr_{iq}) - kr_{iq}H_{m+1}(kr_{iq})]sin(m\theta_{iq})$$

$$- \frac{m}{kr_{iq}} H_{m}(kr_{iq})cos(m\theta_{iq})[\frac{d(kr)}{d\theta}]_{\theta_{iq}}\}. \quad (C.3)$$

$$\mathbf{D}_{\bullet} = \frac{\mathbf{C}_{\bullet} \mathbf{V}_{\bullet} \mathbf{C}_{\bullet}}{\mathbf{C}_{\bullet} \mathbf{V}_{\bullet}}$$

$$Q_{jm}^{11} \simeq - \frac{(\varepsilon_{j}\varepsilon_{m})^{\frac{1}{2}}}{4} - \frac{C_{s}}{P} + \sum_{i=1}^{N} \sum_{q=0}^{P} (\Delta\theta_{i})w_{q}\{H_{m}(kr_{iq})\cos(m\theta_{iq})\}$$

$$\times \{[jJ_{j}(kr_{iq}) - kr_{iq}J_{j+1}(kr_{iq})]\cos(j\theta_{iq})$$

$$+ \frac{j}{kr_{iq}} J_{j}(kr_{iq})\sin(j\theta_{iq})[\frac{d(kr)}{d\theta}]_{\theta_{iq}}\}. \quad (C.4)$$

$$Q_{jm}^{12} \simeq - \frac{(\varepsilon_{j}\varepsilon_{m})^{\frac{1}{2}}}{4} - \frac{C_{s}}{P} + \sum_{i=1}^{N} \sum_{q=0}^{P} (\Delta\theta_{i})w_{q}\{H_{m}(kr_{iq})\sin(m\theta_{iq})\}$$

$$\times \{[jJ_{j}(kr_{iq}) - kr_{iq}J_{j+1}(kr_{iq})]\cos(j\theta_{iq})$$

$$+ \frac{j}{kr_{iq}} J_{j}(kr_{iq})\sin(j\theta_{iq})[\frac{d(kr)}{d\theta}]_{\theta_{iq}}\}. \quad (C.5)$$

$$Q_{jm}^{21} \simeq - \frac{\left(\varepsilon_{j}\varepsilon_{m}\right)^{\frac{1}{2}}}{4} \frac{C_{s}}{P} \frac{\sum_{i=1}^{N} \sum_{q=0}^{P} \left(\Delta\theta_{i}\right)w_{q}\left\{H_{m}\left(kr_{iq}\right)\cos\left(m\theta_{iq}\right)\right\}}{x \left\{\left[jJ_{j}\left(kr_{iq}\right) - kr_{iq}J_{j+1}\left(kr_{iq}\right)\right]\sin\left(j\theta_{iq}\right)\right.\right.} \\ \left. - \frac{j}{kr_{iq}} J_{j}\left(kr_{iq}\right)\cos\left(j\theta_{iq}\right)\left[\frac{d(kr)}{d\theta}\right]_{\theta_{iq}}\right\}. \quad (C.6)$$

$$Q_{jm}^{22} \simeq - \frac{\left(\varepsilon_{j}\varepsilon_{m}\right)^{\frac{1}{2}}}{4} \frac{C_{s}}{P} \sum_{i=1}^{N} \sum_{q=0}^{P} \left(\Delta\theta_{i}\right)w_{q}\left\{H_{m}\left(kr_{iq}\right)\sin\left(m\theta_{iq}\right)\right\} \\ x \left\{\left[jJ_{j}\left(kr_{iq}\right) - kr_{iq}J_{j+1}\left(kr_{iq}\right)\right]\sin\left(j\theta_{iq}\right)\right. \\ \left. - \frac{j}{kr_{iq}} J_{j}\left(kr_{iq}\right)\cos\left(j\theta_{iq}\right)\left[\frac{d(kr)}{d\theta}\right]_{\theta_{iq}}\right\}. \quad (C.7)$$
APPENDIX D

COMPUTER PROGRAM LISTING

Ċ ź č ***** . 3 C 4 ± _ SCATTERING OF ACOUSTIC WAVES Ċ - 22 5 С - 6 С BY THE CYLINDERS OF ARBITRARY CROSS-SECTION * 7 С ÷ 8 (((T-MATRIX FORMULATION))) * C g C 10 ************** Ć 11 С 12 C * * ***** P R O G R A M-(A) *** 13 C 14 C THIS PROGRAM CREATES THE T-MATRIX FOR A GIVEN BOUNDARY GEOMETRY AND WAVE NUMBER, THEN, STORES THE ELEMENTS OF 15 С 16 С THE T-MATRIX INTO A DATA FILE(10) TO BE USED BY THE PROGRAM-(B) FOR EVALUATION OF THE SCATTERED FIELD. 17 С 18 С 19 С 20 С --FOLLOWING CROSS-SECTIONS FOR THE INFINITE CYLINDERS 21 С 22 CAN BE HANDLED: С 1-) CIRCULAR 23 С 2-) ELLIPTICAL 3-) RECTANGULAR (ROUND CORNERED) 4-) TRIANGULAR (ISOSCELES) 24 C 25 С 26 C 27 С 28 Č 29 С ** THE PROCEDURES AND GENERAL STEPS FOLLOWED BY THE ** 30 C ** PROGRAM ARE AS FOLLOWS : ** 31 С 32 C 33 С 34 ---ELEMENTS OF THE Q11,Q12,Q21,Q22-MATRICES(SUBMATRICES) C OF THE Q-MATRIX) ARE EVALUATED BY PERFURMING THE ANGULAR INTEGRATIONS ALONG THE BOUNDARY OF THE SCATTERER. 35 C 36 de Verd С 37 C --ANGULAR INTEGRATIONS ARE EVALUATED NUMERICALLY BY THE SIMPSON'S RULE OF VARIOUS ORDER. (RECOMMENDED ORDER IS 4) Č 38 39 C 40 C 41 C 42 C____ --THEN, THE Q-MATRIX IS INVERTED AND THE T-MATRIX WHICH 43 С 44 C IS GIVEN BY: -1 45 C 46 C 47 C IS CONSTRUCTED, BY THE GAUSS-SCHMIDT OR THOGONALIZATION 48 C TECHNIQUE. 49 С C --FINALLY, THE T-MATRIX AND SOME PARAMETERS DESCRIBING C --FINALLY, THE T-MATRIX AND SOME PARAMETERS DESCRIBING THE BOUNDARY ARE STORED INTO A FILE (REFERRED AS UNIT. 10). 50 C 51 52 53 С 54 C * NOTE: * 55 С 56 ___ C ¹ Q , ORTHUGONALIZED Q AND T-MATRICES ARE STORED ALSO INTO TWO DATA FILES(REFERRED AS UNIT 11 & 22) TO BE USED BY TWO ADDITIONAL PROGRAMS TO MAKE THE ********* 57 С . 58 С 59 C 60 C FOLLOWING CHECKS :--1 61 С 1-) 0 * <u>0 =</u> I 62 C 63 С .64 C 2-) $T \neq CONJ(T) = -RE(T)$ 65 С 66 C 67 68 C INPUT DATA 69 С 69 C 70 C 71 С CARD 1 : 72 C 2======= 73 C COLUMN 1-2: NI (READING UNIT OR FILE ND.) 74 C COLUMN 3-4: NO (WRITING UNIT OR FILE NO.) 75 C 76 C CARD 2 : 77 C ====== 78 C COLUMN 1-2: NO (HALF SIZE OF THE T-MATRIX.MAX(NO)=20) 79 C COLUMN 3-4: NOSF (ORDER OF THE SIMPSON'S FURMULA.) 80 C

81	C
82	C CARD 3:
84	COLUMN 1-2: BF1 (BF1 EBE2 ARE THE TWO MULTIPLICATION
85	C COLUMN 3-8: BF2 FACTORS INVOLVED IN THE SIMPSON'S
87	C INT(A TO B)F(X)+DX=2./45.+H+(7+F0+32+F1+12+F2+32+F3+7+F4)
88	C WHERE: FO=F(A) , F1=E(A+H) , F2=E(A+2H) ,, F4=F(B)
89	C THEN, BF1=2. & BF2=45.)
91	C
92	C CARD 4 :
94	COLUMN 1-4: BCON (BOUNDARY CONDITION, ENTER:
95	C 1-) 'NEUM' FOR NEUMANN TYPE B.C.
96	C - C - C - C - C - C - C - C - C - C -
98	C CARD 5 :
99 100	C ====================================
101	C
102	C CARD 6 :
105	C VER COLUMN 1-6: SHAPE (CROSS-SECTIONAL GEONETRY OF THE AND A CONTRACTOR AND A
105	C SCATTERER. ENTER:
107	C 2-) 'ELLIPS' FOR ELLIPTICAL BOUNDARY
108	C 3-) ! RECTAN! EUR RECTANGULAR BOUNDARY
109	C 4-) 'IRIANG' FUR IRIANGULAR BUUNDARY Co. Adamstrationer all block hader of the chronical distribution of the second states of the distribution of the
111	C CARD 7 :
112	C COLUMN 1-4 - 5-8 - 9-10 :
114	C FOR CIRCLE: RAD (RADIUS)
115	C FOR ELLIPSE: AA, BB (HALF MAJOR & MINOR AXES)
117	C CORRAD (CORNER RADIUS)
118	C FOR TRIANGLE: H (HEIGHT IN X-DIRECTION), BET (ANGLE
119	C DEINEEN IND CUDAL SIDES.VIN DEOREES//
121	C CARD 3 :
122	C COLUMN 1-4: BGEN (TYPE OF THE ANGULAR DIVISION TO BE USED
124	C IN THE NUMERICAL EVALUATION OF THE
125	C BOUNDARY INTEGRALS. ENTER: C INTERVALS
127	C (DIVISIONS ARE MADE AUTOMATICALLY)
128	C (DIVISIONS MUST BE GIVEN IN CARD SET(11))
130	C COLUMN 5-6: NSYM (NSYM=2 FOR THE BOUNDARIES HAVING MIRROR
131	C SYMMETRY W.R.T. X-AXIS ,THEN INTEGRATIONS
133	C BOUNDARY (BETWEEN U-180 DEGREES)
134	C. AND THE PORT OF
135	C 2-) FOR HSYM=2,012(I,J) & 021(I,J)=0.
137	C. T12(I,J) $\&$ T21(I,J)=0.
139	C
140	C CARD 9 :
141	C COLUMN 1-6: ERR (ERROR FACTOR FOR THE BESSEL FUNCTIONS.
143	C TAKE ERR=1.D-10 - 1.D-15)
144	C ELEMENTS. TAKE TRUNC=1.D-20' - 1.D-25)
146-	C. AND A CARL THAN A CARD A
147	L LAKU IO : (IF BGEN= MAN ')UNII (MIS LAKU) Com ==========
149	C CULUMN 1-3: NINTV (NUMBER OF ANGULAR INTERVALS.FOR A
150	C CALLER AND CONTRACTION CONVERGENCY INCLUE NUMERICALE INTEG → THE CONTRACTION TAKE NINTV=72-90)
152	C
153	C CAPD SET 11 . (15 BEENEVAUTULE INHITETHIS CARD SET)
155	C ====================================
156	C +EACH CARD CONTAINS: *
157	CULUMN 4-7: CIN (CORRESPONDING ANGLE FOR THE
159	C INTERVALS (IN DEGREES))
-56 160 -5	ACCENTER ON THE CONTRACTOR OF A CONTRACTOR AND A CONTRACTOR OF A CONTRACTOR AND AND A CONTRACTOR AND A CONTRACTOR AND A CONTRACTOR AND AND A CONTRACTOR AND A CONTRACTOR AND A CONTRACTOR AND A CONTRACTOR AND A CONTRACTOR AND A CONTRACTOR AND A CONTRACTOR AND A CONTRACTOR AND A CONTRACTOR AND A CONTRACTOR AND A CONT

SUM(NCIN * CIN)=360 DEG,FUR NSYM=1 I == 180 DEG,FOR NSYM=2 161 C 162 C 163 С 164 C CARD SET 12 : 165 С *EACH CARD CONTAINS:* COLUMN 1-6: W(I) (WEIGHING -166 C COEFFICIENTS IN SIMPSON'S FORMULA. 167 C EXAMPLE: W(1)=7. (CARD 12-1) 168 C ÷ 169 W(2)=32. (CARD 12-2) C W(3)=12. (CARD 12-3) 170 - C . . 171 W(4)=32. (CARD 12-4) 172 Ċ W(5)=7. (CARD 12-5) 173 C 174 -----C 175 C ****** 176 C SAMPLE DATA: **** 177 C 178 C A-1 EQUAL ANGULAR INTERVALS B-1 NON-EQUAL ANG. INTERVALS 179 C COLUMN 180 C COLUMN 181 _____ 182 C CARD 1123456789012 CARD 1123456789012 183 C ----------1 5 8 184 in 1 1 5 8 С 1 --1417 110 4 185 5 4 2 I 2 186 С 3 12. 45 3 12. 45. 187 С 4 INFUM 4 INEUM 188 5 3 5 11.0 С 10.5 189 С 6 ICIRCLE 6 **IELLIPS** 190 11.0 11.0 2.0 С 7 7 IAUTO 2 191 С 8 8 IMAN Z 192 9 I1.D-121.D-25 9 11.D-151.D-24 С 193 C 10 1 72 10 I 20 4. 17. 194 C -**11** 1 10 6. 11 195 1 32. 12 Ŧ 10 C 12 4. 196 С 13 I 12. 13 I 7. ·197 c 14 I 32. 14 I 32. 198 C 15 I 7. 15 I 12. 32. 199 16 C T 7. 200 C 17 I 201 C C 202 203 204 C 205 С 206 IMPLICIT REAL*8(A-H,O-Z) 207 С 208 REAL #8 RQ(40,40),W(10),R(10),OTDR(10),TETL(10),XI(10),CINTV(145), DREAL, DIMAG, DABS, CDABS, DFLOAT, DSQRT, DCOS, DSIN, DATAN 209 8 210 C COMPLEX #16 011(20,20),012(20,20),021(20,20),022(20,20), 211 212 3 TM(40,40),0(40,40),BES1(22),BES2(22),HANK(22), DUM1, DUM2, X1, X2, P, RR, C, D, DZ, DN, D1, D2, DCMPLX 213 ٤. 214 C CHARACTER*4.BCON,BGEN,WRIT(8) CHARACTER*6 SHAPE 215 . . . 216 Âρ CHARACTER*1 LINE(132) 217 CHARACTER H1+12,H2+25,H3+18,H4+33,H5+8,H6+10,H7+29,H8+37, 218 H9*9,H10*13,H11*28,H12*23,H13*27,MATR*8,H14*10 219 ٤ 2 2 0 C ---CONMON/INVO/Q 221 - 222 CUMMON/REO/RO 223 COMMON/TMAT/TM COMMON/GRAD/RAD 224 225 COMMON/GAX/AA,BB 226 COMMON/REC/A, B, CORRAD, TT1, TT2, TT3, TT4, ALPH, DIS 227 COMMON/TRIAN/H,BETA, TETA1 228 C EQUIVALENCE (RO(1),W(1)),(RO(11),R(1)),(RO(21),DTDR(1)), 229 (RO(31), TETE(1)), (RO(41), XI(1)), (RO(51), CINTV(1)) 230 231 C 3 232 G(V)=V*36Q./(2.*P1) C 233 DATA 011,012,021,022/1600#(0.D0,0.D0)/ 234 DATA LINE/132*'='/ 235 DATA WRIT/10-11', '0-12', '0-21'; '0-22'; 'T-11'; T-12'; 'T-21'; 'T-22'/ 236 DATA H1,H2,H3/'B.C. TYPE : ', 'NEUMANN (RIGID INCLUSION)', 'DIRICHLE 237 ET (CAVITY).'/ 238 DATA H4/'CROSS-SECTION OF THE SCATTERER : '/ 239 240 DATA H5, H6, H7/ CIRCULARI FELLIPTICALI, RECTANGULAR (ROUND CORNERE

2 4 1		n izm	
271 ME242	Sala n	л. т.л 1 1 л. т. л 1	R / ISYMMETRY, CONDITION COMPONENTIATY AVIS 1. THE PARTY AND A COMPANY
243	n D	ATA 1	9.410//SYMMETRIC'. 'NON-SYMMETRIC'/
244	alatar a D	ATA	11/*TYPE OF BOUNDARY #DIVISION: : :: //: mastered to the first water and it
245	D	ATA I	12,H13/'EQUAL ANGULAR INTERVALS','NON-EQUAL ANGULAR INTERVAL
246	٤ <u>۶</u>	517	a da ante en la companya de la companya da ante en la companya da ante en la companya da companya da companya d
247	D	ATA	ATR/' MATRIX'/
248	D	ATA	14/'TRIANGULAR'/
249 (
250			
251	R	READC	,777) NI,NO
HE 252 HE	<u> </u>	UKMA	
293		10 2 M A	(212/E2.0.E6.0)
255	N N	IG P=11	<pre>\$E+1</pre>
256	N. N	102=N	*2
257	N	10 T = N(+2
258	P	1 = () A 1	AN(1.D0) *4.D0
259	W	RITE	N0,613)
260	613 F	ORMAT	(1H1)
261	W	IKITE	
262	100 F	URHA	(///5X,4/('*')/5X,'*',45X,'*'/5X,'*',8X,'SCATTERING UF ACUUS
263	ן ג ר ז יייי	IC W/	YES' 99X9'*'/DX9'*' 94DX9'*'/DX9'* BT THE UTLINDERS OF ARBITRA
265	ι	ст ско стањ	111.0V.141/5V.141.65V.141/5V.67(141)////5V.144 DD0CDAM-(/):
266	്ന്റ്റ്	GENE	ATION OF THE THATRIX SEVENENTS: **!/5X.54(!=!)////)
267	R	EAD()	I,1) BCON, ANO, SHAPE
268	1 F	ORMA	(A4/F4.0/A6)
269	• I	FISH	PE.EQ. 'CIRCLE') READ(NI,81) RAD
270	defetae I	F(SH	PE.EQ. 'ELLIPS') READ(NI,81) AA,BB
271	I	F(SH	PE.EQ. 'RECTAN') READ(NI,81) A,B,CORRAD
272	I	F-(SH)	PE.EQ. 'TRIANG') THEN
273	matrix (1.200)	REA	D(NI,81) H,BET
274	1996 - BU		$A = BE I + Z \cdot + P I / 3 b U \cdot B = D + A + B + B + B + B + B + B + B + B + B$
213	ital e te 🖻		A 1-DATAN(J.TUTAN(DETA/2.))
277	é,≣≕iie F R	END 11	1.821 RCEN.NSYM.ERR.TRUNC
278	82 F	ORMA	(A4, 12/2D6, 0)
279	81 F	URMA	(2F4.0,F10.0)
280	I	(F (& G	N.EQ. 'AUTO') THEN
281		RE.	D(NI,30) NINTV
282	30	FO	MAT(13)
283 (.**** A	NUTUM.	TIC ANGULAR INTERVAL GENERATION ####
20284		DU C T	31 I=1, NINIV/NOTA
200	21		(V(I)-FI*C)/UFLUAI(NINIV)
287 (**** M		ANGIN AR INTERVAL GENERATION ****
288		NI	TV=0
289	14. Tr. 1	кк	0
290		. TO	=0.
291		DO	32 I=1,150/NSYM
292		RE	D(NI,33) NCIN, CIN
293	33	FO	NAILIS, F4.0)
294	2011.a.s.		34 JEINUIN <u>Constant and an and an and an an an an an an an an an an an an an </u>
273	(1) 2 A	ו≒א זי 1	TV(K)=2.*PT/360.*([N
297	a .aasta a 1,273	лт. ПТ	= TOT+NCIN*CIN
298		Т NI	TV=NINTV+NSYM*NCTN
299		IF	TOT.EQ.(360./DFLOAT(NSYM))) GO TO 35
300	32	KK	KK+NCIN
301	35 E	END I	
302	C .	06 18	I=1,NGP
303	. 18 H	READ()	1,19) W(1)
304		JURNA	
306	90 F	TIRMA	(63(1=1)/)
307	1	IF (BC	IN.EQ. 'NEUN') WRITE(NO,47) H1,H2
308	47 F	FORMA	(1X, A12, A25/)
309	1	IF (BC	N.EQ.'DRIC') WRITE(ND,47) H1,H3
、310	•	IRITE	N0,421 ANO
311	42 F	FORMA	(1X, 'WAVE NUMBER : ', F4.1)
312	i la la la la la la la la la la la la la	IF(SH	PE.EQ. CIRCLETTENRILLEINU, 431.H49H79KAU ER HERREN AND AND AND AND AND AND AND AND AND AN
313	43 F	FURNA	(//LX,AJJ,AO//LX, KAUIUJ · ' 'FD.J/
		11124	1//14. A33. A10//1X. '4-AXIS : '. F6.3.4X. '8-AXIS : '. F6.3)
315 	44 ł	LETZH IETZH	PE-FQ. RECTAN') WRITE (ND, 45) H4, H7, A, B, CORRAD
	45 5	FORMA	(//1X, A33, A29//1X, 'A-AXIS:', F6.3, 'B-AXIS:', F6.3, 4X,
318	3		'CORNER RAD.: ', F13.101
319	· · · · ·	IF(SH	PE.EQ. 'TRIANG') WRITE(ND,998) H4,H14,H,BET
320	998.1	FORMA	(//1X,A33,A10//1X; H:',E6;3, BETA:',F7.3)

IF(NSYM.EQ.1) WRITE(N0,46) H8,H10 321 322 46 FORMAT(/1X,A37,A13) 323 IF (NSYM.EQ.2) WRITE(NO,46) H8,H9 WRITE(NO,48) NINTV 324 48 FORMAT(//1X, 'NUMBER OF BOUNDARY SEGMENTS : ',13) IF(BGEN.EQ.'AUTO') THEN 325 326 327 WRITE(N0,49) H11,H12 328 ELSE 329 WRITE(N0,49) H11,H13 330 - 49 FORMAT(/1X, A28, A27) 331 END IF WRITE(N0,50) NOSF 50 FORMAT(/1X,'ORDER OF THE SIMPSON',1H','S RULE USED IN NUMERICAL IN &TEGRATION : ',12) 332 333 334 WRITE(NO,51) NQ2,NO2,TRUNC,ERR 335 336 51 FORMAT(//1X,'TOTAL DIMENSION OF THE T-MATRIX : ',13,' X',13// 337 & 1X,'TRUNCATION FACTOR FOR THE Q-MATRIX ELEMENTS : ',09.3// 338 & £1X,'ERROR FACTOR FOR THE BESSEL FUNCTIONS : ',D9.3//63('=')/1H1) IF (BGEN.EQ.'AUTO') GO TO 53 339 TREEPENSATION AND AND AND 340 WRITE(N0,54) 54 FORMAT(10X, BOUNDARY INTERVAL 1/13X, 'NO ANGULAR '/10X, 'SEGMENT 341 (DEGREE)'/10X,19('=')) 342 3 DO 55 I=1,NINTV/NSYM 343 344 55 WRITE(N0,56) 1,G(CINTV(1)) > 56 FURMAT(12X,13,7X,F4.1/10X,19('=')) 345 WRITE(NO,57) 346 57 FORMAT(1H1) 347 348 53 TEB=CINTV(1) 349 TE A=0.D0 350 IF (NO.GE.13. AND.NO.LE.18) NOT=22 351 IF (SHAPE.EQ. 'RECTAN') CALL RECT 352 C**** START FOR BOUNDARY INTEGRATION AND GENERATION OF **** 353 C**+ THE Q-MATRIX ELEMENTS * * * * DD 2 K=1,NINTV/NSYM IF(SHAPE.EO.'ELLIPS') CALL ELLIP(TEA,TEB,R,TETL,NGP) 354 355 IF (SHAPE.EQ. 'CIRCLE') THEN 356 357 TETL(1) = TEADO 999 KKK=1,NGP 358 IF(KKK.GT.1) TETL(KKK)=TETL(KKK-1)+(TEB-TEA)/DFLOAT(NGP-1) 359 360 999 R(KKK)=RAD 361 END - IE IF (SHAPE.EO. TRIANG') CALL TRIAL(TEA, TEB, R, TETL, NGP, DTDR) IF (SHAPE.EQ. 'RECTAN') CALL RECTAL(TEA, TEB, R, TETL, NGP, DTDR) IF (SHAPE.EO. 'CIRCLE'.DR. SHAPE.EO. 'ELLIPS') 362 363 364 CALL ANGDER (SHAPE, TETL, DTDR, NGP) 3 365 366 DO 3 II=1,NGP 367 ARG=AN()*R(II) 368 DZ = DCMPLX (AR G, 0. DO) DN=DCMPLX(0.D0,0.D0) 369 _____ 370 DO 4 I=1,2 CALL DBESS(1,DZ,ON,DUM2,BES2(I),ERR) 371 DN=DN+DCMPLX(1.D0,0.00) 372 4 373 DO 5 I=3,NQT 374 BES2(I)=2.*(I-2)/DZ*BES2(I-1)-BES2(I-2) DN=DCMPLX(((NGT-1)*1.DO),0.DO) 375 DU 40 I=1,2376 CALL DBESS(0,DZ,DN,BES1(NQT+1-I),DUM1,ERR) 377 DN=DN-DCMPLX(1.D0,0.D0) 378 40 379 DO 41 1=3.NOT 380 41 BES1(NOT+1-I)=2.*(NOT+1-I)/DZ*BES1(NOT-I+2)-BES1(NOT-I+3) 381 DU 6 1=1.NQT HANK(I)=DCMPLX(DREAL(BES1(1)),DREAL(BES2(1))) . 382 6 00 8 J=1,NQ+1 383 384 DO 8 M=1,NQ+1 SINCE, Q-MATRIX IS SYMMETRIC FUR SEPERABLE GEOMETRIES, ONLY **** THE LOWER TRIANGULAR PART IS TO BE EVALUATED FOR CIRCLE AND *** 385 C * * * * 386 C**** * * * * ELLIPSE. C * * * * ั่ง 87 IF ((SHAPE.EQ. 'CIRCLE'.OR.SHAPE.EQ. 'ELLIPSE') .AND. = 388 M.GT.J) GO TO 8 389 £ 390 C**** FOR 'CIRCLE' , (0) IS A DIAGONAL MATRIX. **** 391 C**** THAT IS: 0IJ(J,M)=0.0 FOR (J) NOT EQ. TO (M)**** * * * * 392 C**** AND NEED NOT BE EVALUATED . IF (SHAPE.EQ. 'CIRCLE'.AND.J.NE.M) GO TU 8 393 C**** IF CROSS-SECTIONAL GEOMETRY OF THE SCATTERER HAS **** C**** A SYMMETRY W.R.T. Y-AXIS,THEN, OIJ(J,M)=0.0 FUR **** 394 395 396 C**** (J+N) IS ODD, AND NEED NOT BE EVALUATED. *** C**** (*NOTE: TRIANGLE , LOCATED ON THE COORDINATE **** C**** SYSTEM SUCH THAT ITS SYMMETRY AXIS COINCIDES **** 397 398 C++++ WITH THE X-AXIS, HAS NO SYMMETRY W.R.T. Y-AXIS) **** · 399 C * * * *

401 EJEM=1./2. 402 IF((J.EQ.1.AND.M.NE.1).OR.(M.EQ.F.AND.J.NE.1)) EJEM=DSQRT(2.)/4.DO 403 IF (J.EQ.1. AND. M.EQ.1) EJEM=.25D0 404 P=BF1*(TEB-TEA)/DFLOAT(NGP-1)/BF2*NSYM 405 RR = P406 IF (BCON.EQ. 'NEUM') THEN $P = P \neq BES1(J) \neq DCUS((J-1) \neq TETL(II))$ 407 408 RR=RR*BES1(J)*DSIN((J-1)*TETE(II)) 409 D1=HANK(M+1) 410 D2=HANK(M) 411 MM = M412 ELSE $P = P \neq HANK(M) \neq DCOS((M-1) \neq TETL(II))$ 413 414 RR=RR+HANK(M)*DSIN((M-1)*TETL(II)) 415 01=BES1(J+1) 416 D2=8E51(J) 417 MN = J418 EJEM=-1.DO*EJEM END IF 419 420 C**** (X1)E(X2) ARE FUNCTION TYPE SUBPROGRAMS USED FOR **** 421 C**** CALCULATION OF THE Q-MATRIX ELEMENTS **** C=X1(R(II),D1,TETL(II),DTDR(II),D2,W(II),MM,ANG,EJEM) 422 D=X2(R(II),D1,TETL(II),DTDR(II),D2,W(II),MM,ANO,EJEM) 423 IF (M.LE.NO.AND.J.LE.NO) 011(J,M)=011(J,M)+C*P 424 425 IF (M.GT.1.AND.J.GT.1) Q22(J-1,M-1)=Q22(J-1,M-1)+D*RR 426 C++++ IF THE BOUNDARY HAS A MIRROR SYMMETRY W.R.T. X-AXIS, **** 4 4 # # 427 C * * * * THEN, ELEMENTS OF THE 012 & 021 MATRICES ARE ALL 428 C**** ZERO AND NEED NOT BE EVALUATED. **** IF(2-NSYM) 52,8,52 52 IF(BCON.EQ.'NEUM') THEN 429 430 IF(M.GT.1.AND.J.LE.NQ) 012(J,M-1)=012(J,N-1)+D*P 431 432 IF(J.GT.1.AND.M.LE.NQ)=021(J-1,M)=021(J-1,M)+C*RR ELSE 433 -IF(M.GT.1.AND.J.LE.NO) 012(J,M-1)=012(J,M-1)+C*RR IF(J;GT.1.AND.M.LE.NO) 021(J-1,M)=021(J-1,M)+D*P 434 435 END IF 436 8 CONTINUE 437 438 3 CONTINUE IF (K-NINTV/NSYM) 58,2,2 439 440 58 TEA=TEA+CINTV(K) TEB=TEB+CINTV(K+1) 441 C**** DISPLAY OF THE COMPLETED INTERVALS (HELPFUL IN 442 C**** INTERACTIVE EXECUTION OF THE PROGRAM IN TERMINAL)**** 443 444 2 WRITE(6,61) K 61 FORMAT (12X, ' ** ', 12, ' **') 445 446 C**** TRUNCATION DF THE O-MATRIX ELEMENTS TO INCREASE **** 447 C**** THE ACCURACY OF THE INVERSION, AND CONSTRUCTION DF**** 448 C**** THE O-MATRIX FROM QIJ-SUBMATRICES *** CALL CTRUNC(011, NO2, 0, 0, TRUNC, NO) 449 450 CALL CTRUNC(012, NO2, 0, NO, TRUNC, NO) CALL CTRUNC(Q21,NQ2,NQ,0,TRUNC,NQ) 451 CALL CTRUNC(Q22, NQ2, NQ, HQ, TRUNC, NO) 452 IF (SHAPE.EQ. 'RECTAN'.OR. SHAPE.EQ. 'TRIANG') GO TO 627 453 454 DD 881 I=1,NO2 455 DD 881 J=1,NQ2 456 IF(J,GT,I) Q(I,J)=O(J,I)457 881 CUNTINUE 458 627 WR ITE(11,777) NO,NO2 459 NN0=N0*13 NT =0 460 IF (NO.GT.10) NNG=130 461 462 LL =0 NPRINT=2*NSYM-1 463 464 73 D0 71 L=1,4,NPRINT 465 NTEST=0 MM = 1 466 NN = 110467 IF(L.GT.2) THEN 468 469 MM = NO + 1NN=N02 470 471 END IF 472 IF (L.EQ.2. OR . L.EQ. 4) THEN K = NQ + 1473 N=NQ2' 474 475 ELSE 1.000 47.6 K=1 N=NQ 477 478 END IF 479 IF (NQ.GT.10) THEN 480 N=10

ł

481 IF(L.EQ.2.0R.1.EQ.4) N=NO+10
Att482 END IF second in the second second second second second second second second second second second
483 WRITE(N0,65) (LINE(I),I=1,NN0)
484 65 FORMAT(1X, 130A1)
405 ARTICINU,065 ARTICIVLI, MATK
487 70 WRITE(N0,65) (LINE(I),I=1,NN0)
488 DO 68 I=MM,NN
$489 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad$
491 67 FGRMAT(/1X,10013.5)
492 69 FORMAT(2X,10D13.5)
493 IF(NTEST.EO.1) GO TO 71
495 NTEST=1
496 K = N+1
497 N=NQ
498 IF(L.EQ.2.OR.L.EQ.4) N=NQ2
501 71 CONTINUE
502 IF(LL.GT.O) GO TO 74
503 IF(NT.EQ.1) GO TO 444
505 D0 76 M=1+N02
506 76 R0(J,M)=DREAL(Q(J,M))+(-1:00)
507 WRITE(6,63)
508 ABA ABA AS FORMAT(///1X,'** INVERSION STARTED ***) BARABANA AS THE AAAA
510 C**** T-MATRIX BY GAUSS-SCHMIDT ORTHOGONALIZATION
511 CALL PRCSSM(NO2)
512 WRITE(6,64)
513 64 FUKMA((/1X,'**** CUMPLE(ED ****)) #x514 (Pales - DH 122]=1.NQ2
515 DO 122'J=1,NQ2
516 122 WRITE(11,80) Q(I,J)
517 DO 121 J=1,NO2
519 121 W8 ITE(11.80) DCOUJG(0(I.J))
520 NT=NT+1
521 WRJTE(N0,673)
523 GD TD 73
524 444 D0 72 1=1, NO 2
525 D0 72 J=1,NQ2
526 72 9(1,J)=1M(1,J) 527 11=4
526 TO 73
529 C**** STORAGE OF THE RESULTS INTO THE FILES TO BE USED ****
530 C++++ BY THE PROGRAM-(B) AND OTHER CHECKING PROGRAMS ++++
531 74 WRITELIU,75) BOUN, AND, SHAPE 532 75 EDRMAT(1X, A4.) (B. C.) PTX.E4TT.E (WAVE ND) //1X, A6. (SCATTERER))
533 IF (SHAPE.EQ.'CIRCLE') WRITE(10,93) RAD
534 IF (SHAPE.EQ. 'ELLIPS') WRITE (10,77) AA, BB
535 IF (SHAPE.EQ. 'RECTAN') WRITE (10,991) A, B, CURRAD
537 997 FORMAT(1X,2F6.3,' (H & BETA) ')
538 91 FORMAT(1X,2F6.3,F13.10,'(A & B AXES , CORNER RADIUS)')
539 93 FORMAT(1X,F6.3,' (RADIUS)')
541 WOITE(10.78) NSYN.NO
542 78 FORMAT(1X,12,' (SYM. COND.)'/1X,13,' (NO)')
543 WRITE(22,777) NQ,NQ2
5.44
545 00 79 J=1;102 WRITE(22,80) IN(1,J)
547 79 WRITE(10,80) TM(1,J)
548 80 FORMAT(1X,2D30.23)
549 STOP
551 C
2552 C
553 C
555 C
556 SUBROUTINE ELLIP (T1, T2:R, TET; NGP)
557 C
559 C SUBPROGRAM FOR CALCULATION OF R & TETA VALUES IN POLAR
560 C COORDINATES FOR EACH ANGULAR INTERVAL ALONG THE

561	C ELLIPTICAL BOUNDARY.
562	LE CARACTER TI : LOWER ANGLE OF THE INTERVALENCE AND A DATA AND A DATA AND A DATA AND A DATA AND A DATA AND A D
562	G 12 : UPPER ANGLE OF THE INTERVAL
565	C C C C C C C C C C C C C C C C C C C
566	
567	IMPLICIT REAL*8(A-H,O-Z)
569	OIMENSION R(10), IEI(10)
570	F(T) = A + B/(DSQRT(A + A + (DSIN(T)) + 2 + B + B + (DCOS(T)) + 2))
571	T3=T2-T1
573	E[1] = [1]
574	DD 1 1 = 2, NGP
575	TET(I)=TET(1-1)+T3/DFLOAT(NGP-1)
576	1 = R(I) = F(TET(I))
578	END
579	C
580) is <mark>C</mark> entre lie bein met in helicity operations with the birth of the statistic states of the states of the state
582	
583	
584	SUBROUT INE ANGDER (SHAPE, TETL DTDR, NGP)
586)
587	C SUBPROGRAM FOR CALCULATION OF THE ANGULAR DERIVATIVES
588	Construction (D(R)/D(TETA))) FOR EACH INTERVAL ALONG THE
589	/ C CIRCULAR & ELLIPTICAL BOUNDARIES.) 泰 CIRCARDETO IETL (I) : TETA VALUESEDED THE與INTERVALUE ACCESSION () / ACCESSION () / ACCESSION () / ACCESSION
591	C (I=1,NGP)
592	C DTDR(I) : CORRESPONDING ANGULAR DERIVATIVES
593	
595	IMPLICIT REAL+8(A-H, D-Z)
596	DIMENSION TETL(10), DTDR(10)
597	CHARACTER*6 SHAPE
599	COMMON/GAX/A,B
600	F(T)=(A+B+(B+B-A+A)+DSIN(T)+DCOS(T))/DSORT((A+A+DSIN(T)+DSIN(T)+B+
601	& B # DCUS (T) # DCUS (T)) # # 3) TE (SHAPE: EA ' (T) C (E)) # CONTACT A SHAPE (A S
603	GO TO 2
604	1 DO 3 I=1,NGP
605	3 DTDR(I)=0.DO
607	2 IF(SHAPE.EQ.'ELLIPS') GO TO 4
608	GO TO 5
609	4 D0 6 I=1,NGP
611	5 RETURN
612	END
613	o C 1916 <mark>- Namana sono a sua del control de control de la control de la control de la control de la control de control de la contro</mark>
615	C
616	Ç
617	с С составляется с с с с с с с с с с с с с с с с с с
619	ins internet source in the second state of the second state internet in the second state of the second state of the second second second state second second state second second state second s
620	C+++++++++++++++++++++++++++++++++++++
621	C SUBPROGRAM FOR CALCULATION OF SOME REQUIRED PARAMETERS
623	C THROUGHOUT THE MAIN PROGRAM
624	C INPUT DATA: (A , B , CORRAD
625	
627	
628	IMPLICIT REAL+8(A-H,O-Z)
629	COMNON/REC/A, B, CORKAD, T1, T2, T3, T4, ALPH, D15
631	T1=DATAN((B-CURRAD)/A)
632	IF (A.EO.CORRAD) THEN
633	T2 = P I/2.
634	ALrD - rL/2 DIS= (B-CORRAD)
636	ELSE
637	$T_2 = DATAN(B/(A-CORAD))$
638	L THE ALPHEDATAN (IB-CURRAD)/IA-CURRAD)/TA-CURRAD)
640	END IF

641 T3 = PI - T2642 T4 = PI - T1643 RETURN 644 END 645 С . 646 С 647 С 648 £ 649 С 650 SUBROUTINE RECTALITEA, TEB, R, TETX, NGP, DTDR) 651 С 652 653 SUBPROGRAM FOR CALCULATION OF R & TETA VALUES AND ALSO С 654 ANGULAR DERIVATIVES (D(R)/D(TETA)) FOR EACH ANGULAR C INTERVAL ALONG THE RECTANGULAR BOUNDARY 655 C. 656 С TEA : LOWER ANGLE OF THE INTERVAL 657 TFR : UPPER ANGLE OF THE INTERVAL С : NUMBER OF ANGULAR DIVISION 658 NGP С FOR THAT INTERVAL 659 С 661 С 662 INPLICIT REAL*8(A-H, 0-Z) COMMON/REC/A, B, CORRAD, T1, T2, T3, T4, ALPH, DIS 663 DIMENSION TETL(10),R(10),DJDR(10),TETX(10) 664 F(T,X)=DIS+DCOS(T-X)+DSQRT(DIS++2+(DCOS(T-X))++2+CORRAD++2-DIS++2) 665 G(TE)=DIS+DSIN(TE)+DIS++2+DCUS(TE)+DSIN(TE)/DSORT(DIS++2+DCUS(TE)+ 666 £*2+CORRAD**2-DIS**2) 667 668 PI=DATAN(1.D0)#4.00 TEC=TEB-TEA 669 670 TETX(1) = TEA00 1 I=2,NGP 671 672 1 TETX(I)=TETX(I-1)+TEC/DFLOAT(NGP-1) 673 DU 3 I=1,NGP TETL(I)=TETX(I) 674 IF(TETX(I).GT.PI) TETL(I)=TETX(I)-PI 675 676 3 CONTINUE 677 DO 2 I=1:NGP 678 IF (TETL(I).LE.TI.OR.TETL(I).GE.T4) THEN R(I)=DABS(A/DCOS(TETL(I))) 679 DTDR(I)=A*DSIN(TETL(I))/DCOS(TETL(I))**2 680 IF(TETL(I).GE.T4) DTDR(I)=-DTDR(I) 681 682 GO TO 2 END IF 683 IF (TETL(I).GT.TI.AND.TETL(I).LT.T2) THEN 684 685 R(I) = F(TETL(I), ALPH)TET=TETL(I)-ALPH 686 687 DTDR(I) = -G(TET)688 GO TO 2 689 END IF 690 IF (TETL(I).GE.TZ.AND.TETL(I).LE.T3) THEN 691 R(I) = B/DSIN(TETL(I))DTDR(I)=-B*DCOS(TETL(I))/DSIN(TETL(I))**2 692 693 GU TO 2 694 ENDIF IF(TETL(I).GT.T3.AND.TETL(I).LT.T4) 695 THEN R(I) = F(TETL(I), PI-ALPH) 696 TET=PI-TETL(1)-ALPH 697 698 DTDR(I) = G(TET)699 END IF 700 CONTINUE 2 701 RETURN 702 END 703 Ċ __704 C 705 C 706 C 707 Ć SUBROUTINE TRIAL(TEA, TEB, R, TETX, NGP, DTDR) 708 709 °C. SUBPROGRAM FOR CALCULATION OF R & TETA VALUES AND ALSO 711 С ANGULAR DERIVATIVES (D(R)/D(TETA)) FOR EACH ANGULAR 712 C INTERVAL ALONG THE TRIANGULAR BOUNDARY 713 С TEA : LOWER ANGLE OF THE INTERVAL TEB : UPPER ANGLE OF THE INTERVAL 714 : C ÷÷. UPPER ANGLE TEB 715 : NUMBER OF ANGULAR DIVISION 716 C N GP 717 718 719 C 720 IMPLICIT REAL*8(A-H,0-Z)

721	COMMON/TRIAN/H,BETA,TETA1
722	DIMENSION TETL(10), R(10); DTUR(10); TETX(10)
723	PI=DATAN(1.)*4.
724	TEC=TEB-TEA -
725	TETX(1)=TEA
-/26	DU 1 I=2,NGP
728	$1 \in I \times (I) = I \in I \times (I-I) + I \in C/DF \cup UAT(NGP-1)$
720	
710	
731 3	CONTINUE
732	DO = 1 + NOP
733	IF (TETL (I).LE.TETA1) THEN
734	# R(I)=H/3.*(1./DCOS(TETU(<u>I)))</u>
735	DTDk(1)=H/3.*DSIN(TETL(1))/(DCOS(TETL(1)))**2
736	IF(TETX(I).GT.PI) DTDR(I)=DTDR(I)
737	GO TO 2
730	
739	1F(1)=1=(1)+0=(1)+1=(A1)+1=(A) R(1)=2+(A)(3+(A)(A)(A)(A)(2)(A)(A)(A)(A)(A)(A)(A)(A)(A)(A)(A)(A)(A)
741	DTAN(BETA/2.))
742	$DTDR(1) = -2 \cdot + H/3 \cdot + DTAN(BETA/2 \cdot) + (DCOS(TETL(1)) + DSIN(TETL(1)) + D$
743	<pre>E DTAN(BETA/2.))/(DSIN(TETL(I))-DCOS(TETL(I))*</pre>
-744 ·····	δ DTAN(BETA/2.)) * *2 2000 2000 2000 2000 2000 2000 20
745	$IF(TETX(I) \cdot GT \cdot PI) DTDR(I) = -DTDR(I)$
746	END IF a second second second second second second second second second second second second second second second
747 2	CONTINUE
748	
749	END A second second second second second second second second second second second second second second second sec
751 C	ara uni manaisme o ni kanainin <mark>a kanainin kana</mark> kana kana kanainin kana kana kanainin uni kanainin kana kana kana
752 0	ar a daga a sana ang ang panganan <mark>ang ang ang ang ang ang ang ang ang ang </mark>
753 C	ann an an tarra an ann airseachadh a nn an an ann an ann an ann an ann an ann an a
754 C	
755	COMPLEX FUNCTION X1+16(XRL, XBESP, XTET, XDT, XBES, XW, IND, ANO, EJEM)
756 C	k se en en de se a se a se a se a se a se a se a s
757 C****	****************************
758 C	SUBPROGRAM USED IN CALCULATION OF THE
759 C	Q-MATRIX ELEMENIS
700	an se an an an an an an an an an an an an an
761 C	
761 C 762 763	COMPLEX*16 XBESP,XBES REAL *8 XRL,XTET,XDT,XW,DCOS,DSIN,ANO,EJEM,DFLOAT
761 C 762 763 764	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XH, DCOS, DSIN, ANO, EJEM, DFLOAT X1={((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN
761 C 762 763 764 765	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XH, DCOS, DS IN, ANO, E JEM, DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBES)*DCOS((IND-1)*XTET)+(IN &D-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*E JEM
761 C 762 763 764 765 766	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XH, DCOS, DSIN, ANO, EJEM, DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBES)*DCOS((IND-1)*XTET)+(IN &D-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN
761 C 762 763 764 765 766 767	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XH, DCOS, DSIN, ANO, EJEM, DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END
761 C 762 763 764 765 766 767 768 C	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XH, DCOS, DSIN, ANO, EJEM, DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END
761 C 762 763 764 765 766 767 767 768 769 769 769 769 769 769 769 769 769 769	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XH, DCOS, DSIN, ANO, EJEM, DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBES)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END
761 C 762 763 764 765 766 767 768 769 769 769 770 770 770 770 770 770 770 770 770 77	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XH, DCOS, DSIN, AND, EJEM, DFLOAT X1={((IND-1)*XBES-AND*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN CD-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END
761 C 762 763 764 765 766 767 766 767 768 769 769 770 770 771 C 771 C	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XW, DCOS, DSIN, AND, EJEM, DFLOAT X1={((IND-1)*XBES-AND*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END
761 C 762 763 764 765 766 766 767 768 C 769 C 770 C 771 C 771 C 773	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XW, DCOS, DS IN, ANO, E JEM, DF LOAT X1 = (((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DS IN((IND-1)*XTET)*XDT*XBES/XRL)*XW*E JEM RETURN END COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XW2, IND2, ANO,
761 C 762 763 764 765 766 767 768 C 769 C 770 C 771 C 771 C 773 773 774	COMPLEX #16 XBESP, XBES REAL #8 XRL, XTET, XDT, XW, DCOS, DS IN, ANO, E JEM, DF LOAT X1 = (((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*E JEM RETURN END COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XW2, IND2, AND, E JEM)
761 C 763 763 764 765 766 767 768 769 767 768 769 770 770 770 771 772 773 774 775 775 775	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XW,DCOS,DSIN,ANO,EJEM,DFLDAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, &EJEM)
761 C 763 763 764 765 766 767 768 C 769 C 770 C 771 C 773 C 773 773 774 775 C 776 C*****	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XW,DCOS,DSIN,ANO,EJEM,DFLDAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, &EJEM)
761 C 763 763 764 765 766 767 768 C 779 C 770 C 771 C 772 C 773 773 774 775 C 776 C 777 C	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XW,DCOS,DSIN,ANO,EJEM,DFLDAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, &EJEM) ************************************
761 C 763 763 764 765 766 767 768 C 769 C 770 C 771 C 772 C 773 773 773 774 775 C 776 C 777 C 778 C	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XW,DCOS,DSIN,ANO,EJEM,DFLDAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, GEJEM) ************************************
760 C 761 C 763 764 765 766 767 768 C 768 C 770 C 770 C 771 C 773 774 775 C 776 C***** 777 C 778 C 778 C 779 C***** 779 C*****	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XW, DCOS, DSIN, ANO, EJEM, DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XW2, IND2, ANO, EJEM) ************************************
760 C 761 C 763 764 765 766 767 768 767 769 767 767 768 C 770 C 771 C 772 C 773 774 775 C 776 C***** 777 C 778 C 7780 C	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XH, DCOS, DS IN, ANO, E JEM, DF LOAT X1 = (((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN CD-1)*DS IN((IND-1)*XTET)*XDT*XBES/XRL)*XW*E JEM RETURN END COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XH2, IND2, ANO, EE JEM) ************************************
760 C 761 C 762 763 764 765 766 767 768 C 779 C 770 C 771 C 772 C 773 773 774 775 C 776 C 777 C 778 C 779 C**** 780 C 781	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XN, DCOS, DSIN, AND, EJEM, DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN CD-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XW2, IND2, AND, GEJEM) ************************************
761 C 761 C 763 763 764 765 766 766 767 768 767 768 767 767 768 C 770 C 771 C 773 774 775 C 776 C***** 777 C 778 C 780 C 781 782 783 783	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XW, DCOS, DS IN, ANO, E JEM, DFLOAT X1 = (((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DS IN((IND-1)*XTET)*XDT*XBES/XRL)*XW*E JEM RETURN END COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XW2, IND2, ANO, 6E JEM) ************************************
761 C 761 C 763 763 764 765 766 767 768 C 770 C 771 C 773 774 775 C 776 C***** 777 C 778 C 780 C 781 783 784 784	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XW, DCOS, DSIN, ANO, EJEM, DFLOAT X1={((IND-1)*XBES-ANO*XRL*XBESP; DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XW2, IND2, ANO, 6EJEM) ************************************
761 C 761 C 763 763 764 765 766 767 767 768 769 C 770 C 771 C 773 774 775 C 776 C***** 777 C 778 C 780 C 781 783 784 785	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XW,DCOS,DSIN,ANO,EJEM,DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN CD-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, 6EJEM) ************************************
760 C 761 C 763 764 765 766 767 768 769 C 770 C 771 C 772 C 773 774 775 C 776 C***** 777 C 778 C 780 C 781 782 783 784 785 786	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XW, DCOS, DS IN, ANO, E JEM, DF LOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN CD-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*E JEM RETURN END COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XW2, IND2, ANO, CEJEM) ************************************
761 C 763 763 764 765 766 766 767 768 769 C 770 C 771 C 772 C 773 774 775 C 776 C***** 777 C 778 C 780 C 781 782 783 784 785 786 786 787	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XUT,XW,DCOS,DSIN,ANO,EJEM,DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, EEJEM) ************************************
761 C 761 C 763 764 765 766 767 768 769 C 770 C 771 C 772 C 773 773 774 775 776 C***** 777 C 778 C 780 C 781 782 783 784 785 786 787 C 788 C	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XUT,XW,DCOS,DSIN,ANO,EJEM,DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)#DCOS((IND-1)*XTET)+(IN ED-1)*DSIN(((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, EEJEM) ************************************
761 C 761 C 763 763 765 766 767 768 769 C 770 C 771 C 772 C 773 773 774 775 776 C***** 777 C 778 C 780 C 781 782 783 784 785 786 787 C 788 C 788 C 789 C 789 C 789 C	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XH,OCOS,DSIN,ANO,EJEM,DFLOAT X1=(((IND-1)*XEE)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, EEJEM) ************************************
760 C 761 C 763 764 765 766 767 768 767 768 767 767 768 C 770 C 771 C 772 C 773 774 775 C 776 C**** 777 C 778 C 780 C 781 782 783 784 785 786 787 C 788 C 789 C 790	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XH,DCOS,DSIN,ANO,EJEM,DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, EEJEM) ************************************
760 C 761 C 763 764 765 766 767 768 767 768 767 767 768 C 770 C 771 C 772 C 773 774 775 C 776 C**** 777 C 778 C 779 C**** 780 C 781 782 783 784 785 786 787 C 788 C 789 C 789 C 789 C 789 C 790 C 791 C	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XW,OCOS,DSIN,ANO,EJEM,OFLOAT X1=(((IND-1)*XBES_AND*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, 6EJEM) ************************************
760 C 761 C 763 764 765 766 767 768 767 768 767 767 768 C 770 C 771 C 772 C 773 774 775 C 776 C**** 777 C 778 C 779 C**** 780 C 781 782 783 784 785 786 787 C 788 C 789 C 790 C 791 C 792 C	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XW,DCOS,DSIN,AND,EJEM,DFLOAT X1=(((IND-1)*XBES_AND*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,AND, EEJEM) ************************************
761 C 763 763 763 764 765 766 767 768 767 767 768 C 770 C 771 C 772 C 773 774 775 C 776 C**** 777 C 778 C 780 C 781 782 786 C 786 C 787 C 788 C 789 C 790 C 791 C 793 C	COMPLEX*16 XBESP, xBES REAL*8 XRL, XTET, XDT, XW, DCOS, DSIN, ANO, EJEM, DFLOAT X1=(((IND-1)*XDES_ANO*XRL*XBES)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XW2, IND2, ANO, GEJEM) ************************************
761 C 763 763 763 764 765 766 767 768 767 767 768 C 770 C 771 C 772 C 773 774 775 C 776 C**** 777 C 778 C 779 C**** 780 C 781 782 783 784 785 786 788 C 789 C 790 C 793 C 793 C 794 C*****	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XDT, XW, DCOS, DSIN, ANO, EJEM, DFLDAT XI=(((IND-1)*XXET)*XDT*XBES/XRL)*XBESP2((IND-1)*XTET)+(IN COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XW2, IND2, ANO, GEJEM) ************************************
761 C 763 763 763 764 765 766 767 768 767 768 767 767 768 C 770 C 771 C 773 771 774 775 775 C 776 C**** 777 C 778 C 778 C 780 C 781 782 783 784 786 C 789 C 789 C 790 C 791 C 793 C 795 C 795 C	COMPLEX*16 XBESP, XBES REAL*8 XRL, XTET, XD, YN, DCOS, DSIN, ANO, E JEM, DFLDAT X1=(((IND-1)*XBES_AND*XRL*XBESP!+DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*E JEM RETURN END COMPLEX FUNCTION X2*16(XRL2, XBESP2, XTET2, XDT2, XBES2, XW2, IND2, ANO, 6EJEM) ************************************
761 C 763 763 763 764 765 766 767 768 767 768 767 767 768 C 770 C 771 C 773 774 775 C 776 C**** 777 C 778 C 778 C 778 C 780 C 781 783 784 785 786 C 787 C 788 C 789 C 790 C 793 C 795 C 795 C 795 C 796 C 795 C	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XH,DCOS,DSIN,ANO,EJEM,DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN 6D-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO, 6EJEM) ************************************
760 C 761 C 763 764 765 766 767 768 767 768 767 767 768 C 770 C 771 C 772 C 773 774 775 C 776 C**** 777 C 778 C 778 C 780 C 781 782 783 784 785 786 788 C 790 C 791 C 792 793 793 C 795 C 795 C 797 C**** 798 C	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XW,DCOS,DSIN,ANO,EJEM,DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN ED-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2;XBESP2;XTET2,XDT2;XBES2;XW2;IND2;ANO; 6EJEM) ************************************
760 C 761 C 763 764 765 766 767 768 767 768 767 767 768 C 770 C 771 C 773 774 775 C 776 C**** 777 C 778 C 778 C 781 782 783 784 785 786 788 C 790 C 791 C 792 793 793 C 795 C 795 C 795 C 796 C 797 C**** 798 C 797 C****	COMPLEX*16 XBESP,XBES REAL*8 XRL,XTET,XDT,XW,DCOS,DSIN,AND,EJEM,DFLOAT X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN 6D-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM RETURN END COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,AND, 6EJEM) ************************************

801 REAL*8 CDABS, TRUNC 802 DO 1 1=1,NGT/2 DU 1 J=1,NOT/2 803 804 IF (CDABS(GIJ(I,J)).LT.TRUNC) GIJ(I,J)=DCMPLX(0.D0,0.D0) 805 Q(I+M,J+N) = QIJ(I,J)1 RETURN 806 807 END 808 C 809 С :: C : 810 С 811 812 С **С**-813 SUBROUT INE PRCSSM(NBGR) 814 C 815 816 C SUBPROGRAM FOR GAUSS-SCHMIDT ORTHOGONALIZATION AND CREATION OF THE T-MATRIX 817 C 819 C 820 IMPLICIT REAL#8(A-H, 0-Z) COMPLEX #16 RI(40,40), TM(40,40), DUM 821 822 REAL*8 RR1(40,40), DREAL, DIMAG, TMMX(40,40), RI1(40,40) COMMON/INVO/R1 823 824 COMMON/G/RI1 825 COMMON/TMAT/TM 826 COMMON/REQ/RR1 827 00 1 I=1,NBGR 828 DO 1 J=1,NBGR 1 RI1(I, J) = DIMAG(KI(I, J))829 830 _C++++ CONDITIONING OF THE Q−MATRIXEBEEORE OR THOG ONALIZATION ++++ 831 CALL CNDTHO(NBGR) 832 C++++ NURHALIZE THE N'TH ROW OF AN (N) BY (N) MATRIX ++++ SUM1=0.00 833 DO 20 K=1,NBGR 834 SUM1=RR'1(NBGR,K) ** 2+ RI1(NBGR,K) ** 2+ SUM1 835 di di s 836 20 CONTINUE SUM1=DSQRT(SUM1) 837 838 DO 28 K=1, NBGR RR1(NBGR,K)=RR1(NBGR,K)/SUM1 839 840 RI1(NBGR,K)=RI1(NBGR,K)/SUM1 28 CONTINUE 841 SET UP A LOOP FOR THE N-1 REMAINING ROWS. 842 -C**** * * * * 843 NMI=NBGR-1 844 NROW=NBGR DO 100 I=1, NMI 845 846 NROW=NROW-1 847 MR OW = NR OW 848 DO 36 K=1,NBGR TMMX(1,K)=RR1(NRUW,K) 849 IMMX(2,K 36 CONTINUE DD #0 850 TMMX(2,K)=RI1(NROW,K) 851 852 DD 80 J=NROW,NMI 853 Sk1=0.D0 854 SI1=0.D0 855 MROW=MROW+1 856 DU 40 K=1,NBGR SR1=SR1+RR1(MROW,K)*RR1(NROW,K)+RI1(MROW,K)*RI1(NROW,K) 857 SI1=SI1+RR1(MROw,K) #RI1(NROW,K)-RI1(MROw,K) #RR1(NROW,K) 858 40 CONTINUE 859 DO 48 K=1,NBGR 860 TMMX(1,K)=TMMX(1,K)-SK1*RR1(MROW,K)+SI1*RI1(MROW,K) 861 TMMX(2,K)=TMMX(2,K)-SR1*R11(MROW,K)-SI1*RR1(MROW,K) 862 863 **48 CUNTINUE** 864 80 CONTINUE SUM1=0.D0 865 DO 84 K=1,NBGR 866 -SUM1=SUM1+TMMX(1,K)**2+TMMX(2,K)**2 867 868 - 84 CONTINUE-869 SUM1=DSQRT(SUM1) DO 88 K=1, NB GR 870 RR1(NROW,K)=TNMX(1,K)/SUM1 871 RI1(NROW,K)=TMMX(2,K)/SUH1 872 88 CONTINUE 873 100 CONTINUE 874. DU 2 I=1,NBGR 875 DO 2 J=1,NBGR 876 2 RI(1, J) = DCMPLX(RR1(1, J), RI1(1, J)) 877 878 DD 3 I=1,NBGR 3 J=1,NBGR 879 DO - provide the A state and A when the A state and A 880 DUM=RI(I,J)

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8.8	3 0	* * * *	PEREORM OFTRANSPOSE & DEALION TO CET THATDIV ANALAS
88	4	e E nete	DO 160 I=1,NBGR () - CONTRACTOR AND AND A CONTRACTOR AND
88	5		DO 152 J=1,NBGR
88	6	107	THMX(1,J)=0.D0
88	1	152	
88	9		DO 180 I=1.NBGR
89	0	8. <u>-</u> - 1	DU 176 J=1,NBGR
89	1		DO 172 K=1,NBGR
. 89	2		TNMX(I,J)=TMMX(I,J)-R11(K,I) *RR1(K,J)
89	3	172	CONTINUE
89	5	180	CONTINUE
89	6		DU 196 I=1,NBGR
89	7		DO 192 J=1,NBGR
89	8		TM(1, J)=DCMPLX(0.D0, TMMX(1, J))
89	9	192	CONTINUE
90	0	140	
3090	z	and th	DO = 204 J=1 • NBGR
90	3	· · · · · · · · · · · ·	TMMX(I,J)=0.D0 ,
90	4	204	CONTINUE
90	5	208	CONTINUE
	0		DU 220 I=1,NBGR
00	8	t i e	D0.212 K=1 NBGR - CONTRACTOR
90	9		TMMX(1,J)=TMMX(1,J)+RR1(K,1)*RR1(K,J)
91	0	212	CONTINUE
91	1	216	CONTINUE
<u>91</u>	2	220	CONTINUE
91 12:01	3	214	DD 230 I=LINBGR
91	5	a de la composición de la composición de la composición de la composición de la composición de la composición d	TM(I,J) = TM(I,J) - DCMPLX(TMMX(I,J),0,D0)
91	6	232	CONTINUE
91	7	236	CONTINUE
	8	÷	RETURN
0.1			END
91	9 0-5 °C	titee ee	END
91 92 92	9 0 C 1 C		END
91 92 92 -92	9 0 C 1 C 2 C		END
91 92 92 92 92	9 0 C 1 C 2 C 3 C		END
91 92 92 92 92 92 92	9 0 C 1 C 2 C 3 C 4 C		END
91 92 92 92 92 92 92	0 C 1 C 2 C 3 C 4 C 5		END SUBROUTINE CNDTNQ(NBGR)
91 92 92 92 92 92 92 92 92 92	9 0 C 2 C 3 C 4 C 5 6 C 7 C		END SUBROUTINE CNDTNQ(NBGR)
91 92 92 92 92 92 92 92 92 92 92 92	9 0 C 2 C 3 C 4 C 5 6 C 7 C 8 C	* * * * * \$ UBP	END SUBROUTINE CNDTNO(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	0 C 1 C 2 C 3 C 4 C 5 6 C 7 C 8 C 9 C	* * * * * SUBP THE	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 C 1 C C 2 C C 3 4 C 5 C C 6 C C 9 0 C	* * * * * SUBP THE * * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 C 1 C C 2 C C 3 4 C 5 C C 7 C C 8 C C 9 C C 10 C C 11 C C 12 C C 13 C C 14 C C 15 C C 16 C C 17 C C	**** SUBP THE ****	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 C 1 C C 2 C C 3 C C 5 C C 7 C C 0 C C 1 C C 2 3 C 2 2 C 3 C C 3 C C 3 C C 3 C C	**** SUBP THE ****	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 C 1 C C 2 C C 3 C C 5 C C 6 7 C 9 0 C 1 C C 2 C C 2 C C 3 C C 2 C C 3 C C 2 C C 3 C C 2 C C 3 C C 2 C C 3 C C 3 C C 3 C C 3 C C 3 C C 3 C C 3 C C 3 C C 3 C C 3 C C 3 C C	**** SUBP THE ****	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 93 93 93 93 93 93 93 93	9 0 1 2 3 4 5 5 6 7 C 2 3 4 5 6 7 C 2 8 9 C C 2 3 4 5 5 7 C 2 3 7 C 2 3 7 C 2 3 5 5 5 7 C 2 3 C 2 3 C 2 5 5 5 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	**** \$UBP THE ****	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 93 93 93 93 93 93 93 93	9 0 C 1 C C 2 C C 3 C C 4 C C 5 C C 1 C C 2 C C 3 C C	* * * * * SUBP THE * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 C 1 C C 2 C C 3 C C 4 C C 5 6 C 2 C C 3 C C 4 C C 5 C C	* * * * * SUBP THE * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 93 93 93 93 93 93 93 93 93 93 93 93	9 0 1 C 2 C 3 C 5 C 67 C 7 C 3 4 5 C 6 C 7 C 3 4 5 C 6 C 7 C 9 C	**** \$U\$P THE ****	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 C 3 C 5 C 67 C 7 C 3 4 5 C 6 7 6 7 7 C 9 0	**** \$U\$P THE ****	END SUBROUTINE CNDTNQ(NBGR) ************************************
911 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 C 3 C 5 C 6 C 7 C 3 C 7 C 3 C 7 C 9 C 1 C 2 C 1 C 2 C 1 C 2 C 1 C	* * * * * \$ U \$ P THE * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 C 3 C 5 C 6 C 7 C 9 0 1 C 23 C 6 C 7 C 9 0 1 C 2 C 1 C 2 C 1 C 2 C 2 C 2 C	**** * & * * * THE * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 C 3 C 5 C 6 C 7 C 9 0 1 C 23 C 3 C 3 C 3 C	* * * * * * * * * * * * * * *	END SUBROUTINE CNDTNO(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 3 4 5 6 7 6 7 6 7 6 7 6 7 6 7 7 6 7 7 6 7 7	* * * * THE * * * * * * * * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 3 4 5 6 7 6 7 7 6 7 7 6 7 6 7 7 6 7 7 6 7 6 7	* * * * * THE * * * * * * * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 3 4 5 6 7 6 7 7 8 9 0 1 2 3 4 5 6 7 2 3 4 5 6 7 5	* * * * * THE * * * * * * * * * *	END SUBROUTINE CNDTNG(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 3 4 5 6 7 6 7 7 8 9 0 1 2 3 4 5 6 7 8 7 8 7 8 7 8 7 8	* * * * * THE * * * * * * * * * *	END SUBROUTINE CNDTNG(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 3 4 5 6 7 6 7 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 1 2 3 6 7 8 9	* * * * * THE * * * * * * * * * *	END SUBROUTINE CNDTNG(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 3 4 5 6 7 6 7 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 1	* * * * * THE * * * * * * * * * *	END SUBROUTINE CNDTNO(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 92 92	9 0 1 C 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2	* * * * * THE * * * * * * * * * * 8 * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 93 93 93 93 93 93 93 93 93 93 93 93 93	9 C C 1 C C 2 3 C 5 C C 7 C C 3 C C 7 C C 3 C C 7 C C 7 C C 7 C C 7 S C 7 S C 1 C C 2 C C 1 C C 2 C C	* * * * * THE * * * * * * * * * * * * * * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91 92 92 92 92 92 92 92 92 92 92 92 93 93 93 93 93 93 93 93 93 93 93 93 93	9 0 1 C 2 3 4 5 6 7 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6	* *	END SUBROUTINE CNDING(NBGR) ************************************
91292 92992992 9299929999 9929999999 99399999999	9 C 1 C 2 C 3 4 5 C 6 C 7 C 2 C 2 C 2 C 3 C 7 C 9 C 1 C 3 C 7 S 8 9 1 C 3 C 7 S 8 9 1 C 2 C 2 C 2 C 2 C 2 C 2 C 2 C 2 C 2 C 2 C 2 C 2 C 2 C 2 C 2 C C C C	* 8 * * * * 16 20 * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91299299299999999999999999999999999999	9 C	* 8 * * * * 16 20 * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************
91299299299299999999999999999999999999	9 0 1 2 2 1	* 8 * * * * 8 * * * * 16 20 * * * *	END SUBROUTINE CNDTNG(NBGR) ************************************
91299299299999999999999999999999999999	9 C	* 8 * * * * 8 * * * * 16 20 * * * *	END SUBROUTINE CNDTNQ(NBGR) ************************************

72 CONTINUE 961 962 80 CONTINUE 963 RETURN 964 END 965 C 966 С 967 С 968 C 969 C 970 SUBROUTINE DBESS(MO, DZ, DN, DB1, DB2, E) 971 C 972 С DOCUMENTATION ADDED AT CORNELL UNIVERSITY 8/5/74 FOR THE SUBROUTINE D 973 C DZ IS THE VALUE OF WHICH WE ARE TAKING THE BESSEL FUNCTION. 974 C DN IS THE ORDER OF THE BESSEL FUNCTION. THE VALUE OF THE BESSEL FUNCTION IS STORED IN DB1 IF THE BESSEL FUNCTION WAS OF THE FIRST KIND, IE. A J-BESSEL FUNCTION. 975 С 976 С 977 С THE VALUE OF THE BESSEL FUNCTION IS STORED IN DB2 IF THE BESSEL FUNCTION WAS OF THE SECOND KIND, IE. A Y-BESSEL FUNCTION. 978 C 979 C 980 С 981 C IMPLICIT REAL*8(A-H, 0-Z) 982 IMPLICIT REAL*8(A-H,O-Z) CCALCULATES BESSEL FUNCTION (COMPLEX ORDER AND COMPLEX ARGUMENT) OF THE 983 - 984 CFIRST KIND IF MO=0, AND ALSO OF THE SECOND KIND (NEUMANN FUNCTION) IF 985 CMO = 1986 CCALCULATES BESSEL FUNCTION OF CONPLEX ORDER AND COMPLEX ARGUMENT USING CPOWER SERIES FOR ABS(Z) LESS THAN ZOAND ASYMPTUTIC SERIES FOR ABS(Z) CGREATER THAN ZU. CHOOSES ZO=10 IF GIVEN ZO LESS THAN 1 987 988 C E1 DETERMINES ROUNDOFF OF EN TO INTEGER. IF E1 LE O., SETS T 1002 FORMAT(45H NEITHER SERIES FOR BESSEL FUNCTION CONVERGES) 989 SETS TO .001 990 991 1022 FORMAT(46H NEITHER SERIES FOR NEUMANN FUNCTION CONVERGES) 1011 FORMAT(50H THIS IS A SINGULAR POINT OF THE NEUMANN FUNCTION 992 14H ZX=,1PE14.5,3X,4H ZY=,E14.5) DIMENSIUN C(10),C1(100),C2(100),C3(100),C4(100),C5(100),T3(101) 993 994 995 DIMENSION C6(101) 996 COMPLEX #16 Z, EN, B, ARG, G, T1, EX, S1, CF, SF, T3 COMPLEX #16 FNS,S,T,U,V,S2,T2,SOZ,ZLG,A,A1,B1,B2,B1T,ZH,ZHS,C1 COMPLEX #16 X,DZ,DN,DB1,DB2,CDUM 997 998 999 REAL*8 DATAN2, DREAL, DIMAG, DLOG, DFLOAT, DSIGN, DABS, CDABS 1000 COMPLEX*16 CDEXP, CDSQRT, CDCOS, CDSIN, DCMPLX INTEGER*4 IDINT 1001 DATA IFLAG /0/ 1002 ER0=.1D-2 1003 Z=DZ 1004 1005 X = 71006 ZX=DREAL(DZ) 1007 ZY = DIMAG(DZ)EN=DN 1008 ENX=DREAL(DN) 1009 ENY=DIMAG(DN) 1010 M = M(1 + 1)1011 ABSZ=CDABS(Z) 1012 DB1=DCMPLX(0.D0,0.D0) 1013 1014 DB2=DCMPLX(0.D0,0.D0) 1015 NFLAG=2 1016 NF = 0 1017 KFLAG=1 1018 CKFLAG DENUTES QUADRANT OF Z IF (ZX.LT.O.DO.AND.ZY.GE.O.DO)KFLAG=2 1019 IF(ZX.LT.0.DO.AND.ZY.LT.0.D0)KFLAG=3 1020 IF(ZX.GE.O.DO.AND.ZY.LT.O.DO)KFLAG=4 1021 IF (KFLAG.E0. 2. UR . KFLAG.E0. 3) X=-Z 1022 CROUTINE MOVES Z FROM LEFT-HALF PLANE TO RIGHT-HALF PLANE IF ASYMPTOTIC 1023 1024 CSERIES TO BE USED IF(IFLAG.GT.0)G0 T0 2 1025 아린카 1026 C(1)=3.141592653589793D0 = C(1) /2.D0 1027 C(2) C(3) = C(2)/2.001028 =1.DO/DSQRT(C(2)) 1029 C(4) 1030 C(5) = DLOG(2.D0)1031 C(6)=8.D0 C(7)=64.D0: 1032 C(9)=2.D0*C(1) 1033 1034 C(10)=1.D0/C(1) EUL=.5772156649015338D0-C(5) 1035 1036 CI=DCMPLX(0.90,2.00) A1=DCMPLX(0.D0,C(1)) 1037 C6(1)=1.D0 1038 DD 100 I=1,100 1039 -1040 EYE=I

1042	61711	= F Y F ·	
	C2 (1)	=2.00 #EYE	
1043	C3(1)	=2.D0*EYE-1	
1044	C4(I)=	(4.D0*EYE-1.0	DDO) **2
1045	C6(I+1)=C6(I)+1.D0	/DFLOAT(I+1)
1046 100	C5(I)=	(4.DO*EYE-3.	D0)**2
1047	IFLAG=	1	
1048 2	ERR = .1	D-07	
1049	IF(E+G	1.0.00)ERR=E	
1050	1=0 77-5 D	n - 1990 - 1990 - 1990 - 1990 - 1990 - 1990 - 1990 - 1990 - 1990 - 1990 - 1990 - 1990 - 1990 - 1990 - 1990 - 19	
1051	22-0+0	U Stennyn her rom	
1053	T	TEMP-ENVADET	J. AND. DABSTIDINITENX J-ENXJ.LE. ERUJNFLAGE-1
-1054		TEMP=ITEMP	SN(1)DOJENA)
1055	IF (NFL	AG.EQ1) EN:	=DCMPLX(TEMP,0.DO)
1056	ENX1=D	REAL(EN)	
1057	IF (NFL	AG.EQ1.AND.	ENX1.EQ.O.DO)NFLAG=0
1058	IF (NFL)	AG.EQ1.AND	ENX1.GT.O.DOJNFLAG=1
1059 CNFLA	G=-1,0,	+1,+2 MEANS	(ENX, ENY) A NEGATIVE INTEGER, ZERO, A POSITIVE INT
ELOGO CEGER	AND A	NON-INTEGER	RESPECTIVEL Y HERE AND AND AND AND AND AND AND AND AND AND
1061	IF(NFL)	AG.EQ.Z.AND.	
1062 CFUK I	1=3 • EX.	PRESSES NEUN	ANN FUNCTIONEINEIERMS OF BESSEL FUNCTIONS
1064	CONTIN	AU•EU• - IJEN=-	−E Ν - Σταταδιτά μ <mark>εταιρικά παι παι παι π</mark> αταιριτας μεταξή τους θεταλογός, με μετά τους ματα τους βαταλουργου.
1065		0	n de la companya de la companya de la companya de la companya de la companya de la companya de la companya de s
1066	A=CDEX	P (EN\$A1)	and and an and an an and the state of the
1067	IF (ABS	Z.GE.ZZ) GO 1	na en la contracta de la contracta de la contracta de la contracta de la contracta de la contracta de la contra FO-6
1068	IFIENY	. EQ. 0. DO . AND	DABS(IDINT(ENX)+0.5D0-ENX).LE.ERD) GO TO 6
1069	IF (ABS)	Z.NE.0.D0)GU	TO 8
1070	IF(M.E)	0.2) WRITE(6)	1011) DZ
1071	IF (NFL)	AG.NE.0)GO TO) 16
1072	DB1=DC	MPLX(1.00,0.0	
1073 16	CONTIN	ae en en en entre	
1075	T=1	·	
10765min 1	Z = C	PLX OLOGIABS	7) • DATAN2(DIMAG(7) • DREAL(7))
1077 CCH005	SES PRI	NCIPAL VALUE	OF, Z IN CALCULATING CLUG(Z)
1078	ARG=EN	+C1(1)	
1079	CALL D	GAMM (ARG, G, CE	DUM, ERR, 0)
1080	ZH≍EN≉	(ZLG-C(5))	
1081	ZHS=CD	EXP(ZH)	
1082	-1311)=	2H3/6	
1084	EX=00E	1) Y 9 C 2 (1) # (7) (
-1085 11	I=I+1	~	
1085 11	I=I+1 T3(I)=	-T3(I-1)*EX/(<pre>>=C(21/1/1/2/2/10/2/2/10/10/10/2/2/2/2/2/2/2</pre>
1085 11 1086 1087	I = I + 1 T3(I) = S1 = S1 +	-T3(I-1)*EX/(T3(I)	→TC())///////////////////////////////////
1085 11 1086 1087 1088	I = I + 1 T3(I) = S1 = S1 + S1 S = CD	-T3(I-1)*EX/(T3(I) ABS(S1)	
1085 11 1086 1087 1088 1089	I = I + 1 T3 (I) = S1 = S1 + S1 S = CD T1 S = CD	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(T3(I))	
1085 11 1086 1087 1088 1089 1090	I = I + 1 T3(I) = S1 = S1 + S1 S = CD T1 S = CD IF(T1S	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(T3(I)) .LE.ERR*SIS)	G0 T0 9
1085 11 1086 1087 1088 1089 1090 1091	I = I + 1 T3(I) = S1 = S1 + S1 S = CD T1 S = CD IF(T1S IF(I.L	-T3(I-1)*EX/(T3(I) ABS(S1) .LE.ERR*S1S) .LE.ERR*S1S) T.101)GO TO 1	G0 T0 9
1085 11 1086 1087 1088 1089 1090 1091 1092	I=I+1 T3(I)= S1=S1+ S1S=CD T1S=CD IF(T1S IF(I)L IF(JFL JELAG=	-T3(I-1)*EX/(T3(I) ABS(S1) .LE.ERR*S1S) T.101)G0 T0 J AG.GT.0)G0 T0	((EN+C1(1+1))+C1(1-1)) 60 T0 9 11 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093	I = I+1 T3(I) = S1=S1+ S1S=CD T1S=CD IF(T1S IF(I-L IF(JFL JFLAG= GD_T0	-T3(I-1)*EX/(T3(I) ABS(S1) .LE.ERR*S1S) T.101)G0 T0 1 AG.GT.0)G0 TC 1 6	
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 14	I = I + 1 T3 (I) =- S1 = S1 + S1 S = CD IF (T1S IF (I - L IF (J FL JF LAG = GO T0 WR ITE (-T3(I-1)*EX/(T3(I) ABS(S1) .LE.ERR*S1S) T.101)G0 T0 J AG.GT.0)G0 TC 1 6 6	((EN+C1(1+1))*C1(1-1)) GO TO 9 11 14
1085 11 1086 1087 1088 1099 1090 1091 1092 1093 1094 1095 14	I = I + 1 T3 (I) = S1 = S1 + S1 S = CD IF (T1S IF (I + L IF (J + L JF LAG = GO T0 WR ITE (I STOP	-T3(I-1)*EX/(T3(I) ABS(S1) .LE.ERR*S1S) .LE.ERR*S1S) T.101)G0 T0 1 AG.GT.0)G0 TC 1 6 6	((EN+C1(1-1))+C1(1-1)) GO TO 9 11 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 14 1095 14	I = I + 1 T3 (I) = S1 = S1 + S1 = S1 + S1 = CD IF (T1S IF (I + L IF (JFL JF LAG= GU T0 WR ITE (STOP CONTINU	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(T3(I)) .E.ERR*S1S) T.101)GO TO J AG.GT.0)GU TC 1 6 6,1002)	((EN+C1(1-1))*C1(1-1)) 60 T0 9 11 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 14 1095 14 1096 1097 81	I = I + 1 T 3 (I) = S1 = S1 + S1 = S1 + S1 = CD I F (T1S IF (T1S IF (I + L IF (JFL JFLAG= GU T0 WR ITE (STOP CONTINU JFLAG=	-T3(I-1)*EX/(T3(I) ABS(S1) .LE.ERR*S1S) T.101)G0 TO J AG.GT.0)G0 TC 1 6 6,1002) UE	<pre></pre>
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1095 14 1096 1097 1097 81 1098 1099	I = I + 1 T 3 (I) = S1 = S1 + S1 = S1 + S1 = CD I F (T1S IF (I - L IF (JFL JFLAG= GO TO WR ITE(STOP CONTINU JFLAG= GO TO	-T3(I-1)*EX/(T3(I) ABS(S1) .LE.ERR*S1S) T.101)60 T0 J AG.GT.0)60 TC 1 6,1002) UE 1	GO TO 9 11 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1095 14 1096 1097 1097 81 1098 1099 1091 82	I = I + 1 T = I + 1 T = I + 1 T = S = CD T = S = CD I = C = S = CD I = C = S = CD I = C = C = C = C = C I = C = C = C = C = C I = C = C = C = C = C I = C = C = C = C = C I = C = C = C = C = C I = C = C = C = C = C I = C = C = C = C = C I = C = C = C = C = C I = C = C = C = C = C = C = C = C = C =	-T3(I-1)*EX/(T3(I) ABS(S1) .LE.ERR*S1S) JG.GT.O)GO TO AG.GT.O)GO TO 6 6,1002) UE 1 6 6 (6,1022)	GO TO 9 1 1 1 1 1 1 1 1 1 1 1 1 1
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1099 1099 1098 1099 1100 1099 1102	I = I + 1 T = I + 1 T = I = S S = S S = S I =	-T3(I-1)*EX/(T3(I) ABS(S1) .LE.ERR*SIS) .LE.ERR*SIS) T.101)G0 T0 J AG.GT.0)GU TC 1 6 5,1002) UE 1 6 6,1022)	((EN+C1(1-1))*C1(I-1)) GO TO 9 11 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1095 14 1096 1097 1097 81 1098 1099 1100 82 1101 1102 1103 9	I = I + 1 T = I + 1 T = I = S = CD T = S = CD I = C = S = CD I = C = S = CD I = C = S = S = C I = C = S = S = S = S = S = S = S = S = S	-T3(I-1)*EX/(T3(I) ABS(S1) .LE.ERR*SIS) .LE.ERR*SIS) T.101106 T0 T0 AG.GT.0)GU T0 6 6,1002) UE 1 6 (6,1022) E.2)GU T0 55	((EN+C1(1-1))*C1(I-1)) GO TO 9 11 1 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1095 14 1096 1097 1098 1098 1099 1100 1102 9 1103 1104	I = I + 1 T = I + 1 T = I + 1 T = C = S = C = S = C = S = S = S = S = S	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(T3(I)) .LE.ERR*SIS) T.101)GO TO 1 AG.GT.0)GO TC 1 6 6,1002) UE 1 6 6,1002) UE 1 6 6,1022) E.2)GO TO 55 (ENX1)	((EN+C1(1-1))*C1(I-1)) GO TO 9 11) 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1095 14 1096 1097 1097 81 1098 1099 1100 82 101 1102 1103 1104 1105 1105	I = I + 1 T = I + 1 T = I + 1 T = S = CD I = S = CD I = S = CD I = C = S = CD I = C = S = S = S = S = S = S = S = S = S	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(T3(I)) .LE.ERR*SIS) T.101)GO TO J AG.GT.0)GO TC 1 6 6,1002) UE 1 6 (6,1022) E.2)GO TO 55 (ENX1) * (ZLG+EUL)	((EN+C1(1-1))*C1(I-1)) CO TO 9 11) 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1099 1100 82 101 1102 1103 1104 1105 1106	I = I + 1 T = I + 1 T = I + 1 T = S = CD I = S = CD I = S = CD I = C = S = CD I = C = S = S = S = S = S = S = S = S = S	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(T3(I)) .LE.ERR*SIS) T.101)GO TO J AG.GT.0)GO TC 1 6 6,1002) UE 1 6 (6,1022) E.2)GO TO 55 (ENX1) *(ZLG+EUL)	((EN+C1(1-1))*C1(I-1)) CO TO 9 11 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1099 1100 82 101 1102 1103 1104 1105 1106 1107 1107	I = I + 1 T = I + 1 T = I + 1 T = S = CD I = S = CD I = S = CD I = C = S = CD I = C = S = S = S = S = S = S = S = S = S	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(T3(I)) .LE.ERR*SIS) T.101)GO TO J AG.GT.0)GO TC 1 6 6,1002) UE 1 6 (6,1022) E.2)GO TO 55 E.2)GO TO 55 (ENX1) * (ZLG+EUL)	((EN+C1(1-1))*C1(I-1)) CO TO 9 11 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1099 1100 82 101 1102 1103 1104 1105 1106 1107 1108	I = I + 1 T3 (I) = S1 = S1 + S1 = S1 + S1 = CD T1 S = CD IF (T1S IF (I • L IF (JFL JFLAG= G0 T0 WR ITE (STOP CONTINU JFLAG= G0 T0 WR ITE (STOP B = S1 IF (M.N N= DABS U= 2 · D0 S2 = B*U IZ Z = 1 D0 75	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(T3(I)) .LE.ERR*SIS) T.101)GO TO J AG.GT.0)GO TO J AG.GT.0)GO TO J 4 6 6 6 (6,1022) UE 1 6 (6,1022) E.2)GO TO 55 (ENX1) * (ZLG+EUL) J=[ZZZ;1]	((EN+C1(1-1))*C1(I-1)) GO TO 9 11) 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1099 1100 82 101 1102 1103 1104 1105 1106 1107 1108 1109 77 1109 110	I = I + 1 T = I + 1 T = I + 1 T = S = S = S = S = S = S = S = S = S =	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(T3(I)) .E.ERR*S1S) T.101)GO TO J AG.GT.0)GO TO AG.GT.0)GO TO 1 6 6 (6,1022) UE 1 6 (6,1022) E.2)GO TO 55 (ENX1) *(ZLG+EUL) J=[ZZZ;1]	((EN+C1(1-1))*C1(I-1)) GO TO 9 11) 14
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1099 1101 102 1097 81 1098 1099 1100 82 1101 1102 1105 1106 1107 1108 1109 1110	I = I + 1 I = I + 1 T3 (I) = S1 = S1 + S1 = S1 + S1 = CD IF (T1S IF (I + L IF (JFL JF LAG= G0 T0 WR ITE (STOP G0 T0 WR ITE STOP B = S1 IF (M • N N = DABS U = 2 · D0 S2 = B * U IZ ZZ = 1 D0 75 J1 = J - 1 J1 N = N + TE	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(T3(I))	<pre></pre>
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1097 1099 1093 1097 81 1098 1097 1009 82 1101 1102 1103 1104 1105 1106 1107 1108 1109 1110 1111 1112	I = I + 1 T = I + 1 T = I + 1 T = S = S = S = S = S = S = S = S = S =	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(S1) .LE.ERR*S1S) T.101)G0 TO J AG.GT.0)G0 TC 1 6 6,1002) UE 1 6 (6,1022) E.2)G0 TO 55 (ENX1) \$ (ZLG+EUL) J=IZZZ;I J-1 (J1.LE.0) G0 (J1.GT.101) (<pre></pre>
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1094 1097 81 1098 1097 1099 1100 1101 1102 1103 1104 1105 1106 1107 1108 1109 1111 1112 1113	I = I + 1 T = I + 1 T = I + 1 T = S = S = S = S = S = S = S = S = S =	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(S1) LE.ERR*S1S) T.101)G0 TO J AG.GT.0)G0 TC 1 6 6,1002) UE 1 6 (6,1022) E.2)G0 TO 55 (ENX1) 4 (ZLG+EUL) J=IZZZ,1 J-1 (J1.LE.0) G0 (J1.GT.101) (P1=C6(J1)	<pre>G0 T0 9 G0 T0 9 I1 I I I I I I I I I I I I I I I I I I</pre>
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1094 1097 81 1098 1097 1099 1100 1102 9 1103 1104 1105 1106 1107 1108 1107 1108 1101 112 113 1114	I = I + 1 T = I + 1 T = I + 1 T = S = CD T = S = CD T = S = CD I = C = S = CD I = C = C = C I = C = C = C I = C = C = C I = C = C = C = C I = C = C = C = C = C = C = C = C = C =	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(S1) .LE.ERR*S1S) T.101)G0 TO J AG.GT.0)G0 TC 1 6 6,1002) UE 1 6 (6,1022) E.2)G0 TO 55 (ENX1) V(ZLG+EUL) J=IZZZ;1 J-1 (J1.LE.0) G0 (J1.GT.101) (P 1=C6(J1) 203	GO TO 9 11 14 10 14 10 10 10 10 10 10 10 10 10 10
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1097 1097 81 1098 1097 1109 82 101 102 1102 9 1103 1104 1105 1106 1107 1108 1109 1110 1111 1112 113 1114 115 200	I = I + 1 T = I + 1 T = I + 1 T = S = CD T = S = CD I = C = S = CD I = C = S = CD I = C = S = CD I = C = S = CD I = C = S = C = C = C = C = C = C = C = C	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(S1) LE.ERR*S1S) T.101)G0 TO J AG.GT.0)G0 TO 1 6 6,1002) UE 1 6 (6,1022) E.2)G0 TO 55 (ENX1) * (ZLG+EUL) J=[ZZZ;1] J-1 (J1.LE.0) G0 (J1.GT.101) (C P1=C6(J1) 203 0.D0	GO TO 9 11 14 TO 200 50 TO 201
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 81 1098 1099 1097 81 1098 1099 1100 82 101 1102 1103 1104 1105 1106 1107 1108 1109 111 112 113 114 115 115 200 1116 200	I = I + 1 I = I + 1 T3 (I) = S1 = S1 + S1 S = CD. IF (T1S IF (I • L IF (JFL JF LAG= GO TO WR ITE (STOP CONTINU JF LAG= GO TO WR ITE STOP B = S1 IF (M•N N=DABS U=2 · DO S2 = B *U IZZ = 1 DO 75 J1 = J-1 J1 N=N+ IF TEM GO TO TEMP1= GO TO	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(S1) LE.ERR*S1S) T.101)G0 TO J AG.GT.0)G0 TC 1 6 6,1002) UE 1 6 (6,1022) E.2)G0 TO 55 (ENX1) * (ZLG+EUL) J=[ZZZ;1] J-1 (J1.LE.0) G0 (J1.GT.101) (P 1=C6(J1) 203 0.D0 203	((EN+C1(1-1))*C1(I-1)) G0 T0 9 11 14 T0 200 C0 T0 201
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 14 1096 1097 1097 81 1098 1099 1100 82 1101 1102 1103 1104 1105 1106 1107 1108 1109 111 112 113 114 115 115 200 1116 117 1117 201	I = I + 1 I = I + 1 T3 (I) = S1 = S1 + S1 S = CD. IF (T1S IF (I • L IF (JFL JFLAG= GO TO WR ITE (STOP CONTINU JFLAG= GO TO WR ITE STOP B = S1 IF (M • N N=DABS U=2 • DO S2 = B * U IZ Z = 1 DO 75 J1 = J - 1 J1 N=N+ IF TEM GO TO TEMP1= GO TO TEMP1= GO TO	-T3(I-1)*EX/(T3(I) ABS(S1) ABS(S1) LE.ERR*S1S) T.101)G0 TO J AG.GT.0)G0 TO 1 6 6,1002) UE 1 6 (6,1022) E.2)G0 TO 55 (ENX1) *(ZLG+EUL) J=[ZZZ;1] J-1 (J1.LE.0) G0 (J1.GT.101) (P 1=C6(J1) 203 0.00 203 C6(101)	<pre>((EN+C1(1-1))*C1(I-1)) (0 T0 9 11 1 1 14 T0 200 50 T0 201</pre>
1085 11 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1097 14 1096 1097 1097 81 1098 1099 1100 82 1101 1102 1103 1104 1105 1106 1107 1108 1109 1110 1111 1112 1113 1114 1115 200 1118 1118	I = I + 1 I = I + 1 T3 (I) = S1 = S1 + S1 S = CD IF (T1S IF (I • L IF (JFL JF LAG= GO TO WR ITE (STOP GO TO WR ITE GO TO WR ITE STOP B = S1 IF (M • N N = DABS U = 2 • DO S2 = B * U = Z = 1 J1 = J - 1 J1 N = N + IF IF GO TO TE MP 1 = GO TO TE MP 1 = DO 2022	-T3(I-1)*EX/1 T3(I) ABS(S1) ABS(S1) LE.ERR*S1S) T.101)G0 T0 J AG.GT.0)G0 T0 1 6 6,1002) UE 1 6 (6,1022) E.2)G0 T0 55 (ENX1) *(ZLG+EUL) J=IZZZ;1 J-1 (J1.LE.0) G0 (J1.GT.101) (P 1=C6(J1) 203 0.00 203 C6(101) JJ=102,J1 P1=T6(J1) P1	<pre>C(C),,,, ((EN+C1(I-1))*C1(I-1)) GO TO 9 I1 D 14 TO 200 CO TO 201 CO CO CO CO CO CO CO CO CO CO CO CO CO C</pre>

1121 203 CONTINUE 1122 1F (J1N.LE.0) GO TO 205 1123 IF(J1N.GT.101) GO TO 206 1124 TEMP2=C6(J1N) 1125 GU TO 208 1126 205 TEMP 2=0.00 1127 GO TO 208 1128 206 TEMP2=C6(101) DO 207 JJ=102, J1N 1129 TEMP2=TEMP2+1.DO/DFLOAT(JJ) 1130 1131 207 CONTINUE 1132 208 CONTINUE T2 = T3(J) * (TEMP 1+ TEMP 2) 1133 1134 T2R=DABS(DREAL(T2)) 1135 T2I=DABS(DIMAG(T2)) 1136 75 S2=S2-T2 1137 S2R=DABS(DREAL(S2)) S2I=DABS(DIMAG(S2)) 1138 IF(T2R.GT.ERR*S2R) GO TO 78 IF(T2I.LE.ERR*S2I) GO TO 76 1139 1140 1141 78 I=I+1 1142 IF(1.GT.101.AND.JFLAG.E0.0)G0 T0 81 IF(1.GT.101.AND.JFLAG.NE.0)G0 T0 82 1143 T3(I)=-T3(I-1)*EX/((EN+C1(I-1))*C1(I-1)) 1144 1145 1222=1 1146 GO TO 77 76 B2=S2*C(10) 1147 IF (N.EQ.0) GO TO 55 1148 1149 S1=DCMPLX(0.D0,0.D0) 1150 T1=-C(10)/ZHS LUP=N-11151 IF(LUP.EQ.0)GU TO 72 1152 1153 D0 70 LL=1,LUP **三1154**篇 T1=T1+DFLOAT(LL) 70 1155 S1 = S1 + F100 71 LL=1,LUP 1156 T1=T1*EX/(DFLOAT(LL)*DFLOAT(LUP-LL+1)) 1157 S1=S1+T1 1158 -7----1159 GO TO 73 1160 72 S1=S1+T1 -11. 73 B2=B2+S1 1161 1162 GO TO 55 IF (NFLAG.LT.O) B= A*B 1163 55 1164 BX=DREAL(8) BY = DIMAG(B)1165 1166 BXA=DABS(BX)1167 BYA=DABS(BY) IF(ZX.EQ.0.D0.0R.ZY.EQ.0.D0)NF=1 1168 IF (NFLAG.NE. 2. AND. BXA.LT. BYA.AND.NF.EQ.1) BX=0.DO 1169 IF(NFLAG.NE.2.AND.BYA.LT.BXA.AND.NF.E0.1)BY=0.D0 G0 T0(56,57,58,59),M 1170 1171 1172 ARG=X-EN*C(2)-C(3) 6 CF=CDCOS(ARG) 1173 SF=C(6)*CDSIN(ARG) ±1174 FNS=C1(4)*EN*EN 1175 1176 I=0 S1=DCMPLX(1.D0,0.D0) 1177 S2=DCMPLX(0.D0,0.D0) 1178 U=DCMPLX(1.00,0.D0) 1179 1180 T1S=1.D0 . . . S=CF 1181 I = I + 1 1182 18 V=-(FNS-C5(I))/(C(7)*X*C3(I))*U 1183 U=V*(FNS-C4(I))/(C2(I)*X) 1184 US=CDABS(U) 1185 IF (US.GT.TIS) GO TO 20 1186 1187 12 CUNTINUE T=U*CF+V*SF 1188 TR=DABS (DREAL(T)) 1189 TI=DABS(DIMAG(T)) 1190 1191 S = S + T1192 SR=DABS(DREAL(S)) SI=DABS(DIMAG(S)) 1193 IF (TT.LE.ERR*SI) GO TO 24 IF (TT.LE.ERR*SI) GO TO 26 T1=U 1194 1195 1196 24 1197 T2=V i e e e 1198 S1=S1+T1 1199 S2 = S2 + T21200 T1S=US

17 IF(I-100)18,25,25 1201 17 IF(1-100/10,25,25 20 IF(1.EQ.1) GO TO 12 1202 1203 IF(JFLAG.GT.O) GD TO 23 1204 JFLAG=1 1205 GO TO B 23 WRITE(6,1002) -1206 1207 29 CONT INUE 1208 STOP 26 1209 CONTINUE SQZ=CDSQRT(X) 1210 IF (DREAL(SQZ).LT.0.DO) SQZ=-SQZ 1211 1212 CCHOOSES PROPER BRANCH FOR SQUARE ROOT 1213 B=C(4)/SQZ#S 1214 IF(KFLAG.EQ.2) $B=A \neq B$ 1215 IF(KFLAG.EQ.3)B=B/A IF (M.E0.3) M=2 -1216 IF(M.NE.2)G0 TO 55 1217 B2=C(4)/SOZ*(SF*(S1+U)/CT6)=CF*(S2+V)*C(6)) IF(KFLAG.EO.2)B2=(B2+CDCOS(C(1)*EN)*CI*B)/A 1218 1219 1220 IF(KFLAG.EQ.3)B2=(B2-COCOS(C(1)*EN)*C1*B)*A 1221 GO TO 55 IF(JFLAG.GT.0)GD TO 28 25 1222 .JFLAG=1 1223 GO TO 8 1224 1225 28 WRITE(6,1002) GG TO 29 1226 58 B1=DCMPLX(BX,BY) 1227 -1228 M=4 1229 FN=-FN GO TO 3 1230 59 BIT=DCMPLX(BX,BY) 1231 EN=-EN 1232 1233 ARG=C(1) #EN B2=(CDCUS(ARG)*B1-B1T)/CDSIN(ARG) 1234 1235 -DB1 = B11236 DB2=B2 sa utada RETURN 1237 1238 57 IF(NFLAG.EQ.-1)82=A*B2 1239 DB2=B21240 56 DB1=DCMPLX(BX, BY) 1241 RETURN 1242 END SUBROUTINE DGAMM(DZ,DGM,DPS,ERR,JJ) 1243 1244 CIF JJ=0, CALCULATES ONLY GAMMA FUNCTION, IF JJ=1, CALCULATES ONLY PSI CFUNCTION, IF JJ=2, CALCULATES BOTH 1245 12.46 IMPLICIT REAL*8(A-H, 0-Z) COMPLEX*16 GAM,Z,DZ,DGM,DPS,DPSI 1247 CUMPLEX *16 TERM1, ZT1, TERM, SUM, ZLG, ZTGAM, ZT 1248 COMPLEX#16 COLOG, DCMPLX, CDEXP, CDSIN 1249 1250 REAL*6 DREAL, DIMAG, DLOG, DABS 1251 INTEGER #4 IDINT DIMENSION B(10) 1252 DIMENSION C(100) 1253 1254 DATA IFLAG /0/ 1255 1001 FORMAT(1H ,///,24H SERIES DID NOT CONVERGE) 1256 1010 FORMAT(1H ,///,47H THIS IS & SINGULAR POINT OF THE GAMMA FUNCTION 1257 X,/,5X,6HARG R=,E12.5,3X,6HARG I=,E12.5) IF(JJ.E0.0)G0 T0 60 1258 1259 DPS=DPSI(DZ,ERR) 1260 IF(JJ.EQ.1)RETURN 1261 60 E=ERR If (E.LE.0.D0) E=. 10-07 1262 ZX=DREAL(DZ) 1263 126.4 ZY = DIMAG(DZ)1265 Z=DZ IF(ZX.LT.0.D0)Z=-Z 1266 NFLAG=2 1267 1268 J=0 CK=NUMBER OF TERMS IN SERIES 1269 1270 IF(ZY.EQ.0.DO.AND.(IDINT(ZX)-ZX).EQ.0.DO) NFLAG=1 IF (NFLAG.EQ. 1. AND. ZX.LE. 0. DO)NFLAG=0 1271 1272 CNFLAG=0 MEANS Z=0 OR Z A NEGATIVE INTEGER NELAG=1MEANS Z A POSITIVE IF (NFLAG.NE. 0) GU TO 51 1273 WRITE(6,1010) DZ 1274 DGM=DCMPLX(0.D0,0.D0) 1275 127.6 RETURN 51 IF (NFLAG.EQ. 2)GD TO 42 1277 IF (ZX.GT.2.0D0) GD TD 55 1278 DGM=DCMPLX(1.D0,0.D0) .1279 1280 RETURN

· .			
1	289	IF(IFLAG.NE.O)GU TU 20	
4	290	DU 100 I=1,100	3
1	291 100		
÷1	292	P1=3.141592653589793D0	Ξ.
_ 1	293	P12=DL0G(2.D0*P1)/2.D0	
<u>-</u> 1	294	B(1)=1.0D0/12.0D0	
_1	295	B(2) = -1.000/360.000	
1	296	B(3)=1.0D0/1260.0D0	÷
1	297	B(4) = -1.000/1680.000	
21	298	8(5)=1.0D0/1188.0D0	÷.
1	299	B(6) = -691.000/360360.000	
₿ 1	300	B(7)=1.0D0/156.0D0	
1	301	B(8) = -3617.000/122400.000	
-1	302	B(9)=43867.000/244188.000	2
ົ1	303	B(10) = -174611.000/125400.000	2
ાં	304 CB(I)	ARE THE BERNOULL I COFFEICLENTS TINESTIREINGS FORMULA	-i-;
៊ី	305	IF I AG =)	
៍រិ	306 20	7 T = 7	4
1	307		1
21	300		
꼴북	200		2
្នុ	309	$\mathbf{I} + \mathbf{I} + $	
14	310	에 부분 수 있 는 것이 있는 것 같은 것이 있는 것이 있는 것이 것이 있다. 이 것이 있는 것이 있다. 것이 있는 것이 있는 것이 있는 것이 있는 것이 있는 것이 있는 것이 있는 것이 있는 것이 없다. 것이 있는 것이 있는 것이 있는 것이 있는 것이 있는 것이 있는 것이 있는 것이 있 같이 있는 것이 있다. 것이 있는 것이 있는 것이 있는 것이 있는 것이 있다. 것이 있는 것이 있는 것이 있는 것이 있는 것이 있다. 것이 있	È.
្ពុ	311		
ę ł	312	WRITE (6,1001) and a second second second second second second second second second second second second second	1
1	313	STUP	<u>.</u> .
7 1	314		
្រា	315	ZT=ZT+1.00	
21	316	CO TO 5	2
1	317 3	FIF=I	~
- 1	318	<pre>> ZLG=CDLOG(ZT)</pre>	
_1	319	SUM=(ZT5 D0)*ZLG -ZT+PI2	
1	320	TERM1=SUM	-
1	321	ATER1R=DABS(DREAL(TERM1))	
21	322	ATER1I=DABS(DIMAG(TERM1))	-
1	323	J=0	-,
÷.1	324	(s l=J+1) skalate tot (state tot (state tot (state tot)) skalate tot (state tot)) state tot (state tot) state	7
_			
ົ້າ	325	$T \in RM = B(J) * CD \in XP(-(2, D0 * C(J) - 1, D0) * ZLG)$	
〕 三1 三1	325	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM))	
1	.325 .326 .327	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM))	il.
1	325 326 327 328	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM	141 H 11
	325 326 327 328	TERM=B(J)*CDEXP(-(2.DO*C(J)-1.DO)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMPEDABS(DREAL(SUM))	Lord Rock
	325 326 327 328 329	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATER=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMEDABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM))	the better the
	325 326 327 328 329 330	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATER=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMR=DABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM)) E(ATERPOOR ATERNIGO TO 24	tern burd Bage
	325 326 327 328 329 330 331	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMR=DABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM)) IF(ATERR.GT.ATERIR)GO TO 24 IF(ATERR.GT.ATERIR)GO TO 24	212 4440 2440 1924
	325 326 327 328 329 330 331 332	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMR=DABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM)) IF(ATERR.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERII)GO TO 24	
	325 326 327 328 329 330 331 332 333	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUM=DABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM)) IF(ATERR.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERII)GO TO 24 IF(ASUMR.EQ.0.D0)GO TO 9 IE(ATERDEASUMP CT.E)GD TO 27	ter 2010 April 2010 Day
	325 326 327 328 329 330 331 332 333 333 334	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATER=DABS(DREAL(TERM)) ATER I=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMEDABS(DREAL(SUM)) ASUM I=DABS(DIMAG(SUM)) IF(ATERR.GT.ATER1R)GD TO 24 IF(ATERRIGT.ATER1I)GD TO 24 IF(ATERRIGT.ATER1I)GD TO 24 IF(ATERR/ASUMR.GT.E)GD TO 27 CONTINUES	
	325 326 327 328 329 330 331 332 333 334 335	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATER=DABS(DREAL(TERM)) ATER = DABS(DIMAG(TERM)) SUM=SUM+TERM ASUM=DABS(DREAL(SUM)) ASUM = DABS(DIMAG(SUM)) IF(ATER.GT.ATERIN)GD TO 24 IF(ATERI.GT.ATERIN)GD TO 24 IF(ATERI.GT.ATERIN)GD TO 24 IF(ATERI.GT.ATERIN)GD TO 24 IF(ATERR/ASUMK.GT.E)GD TO 27 CUNTINUE IF(ASUMI = 0.0.D0) CO TO 5	
	325 326 327 328 329 330 331 332 333 334 335 336	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATER=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMR=DABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM)) IF(ATER.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERII)GO TO 24 IF(ATERI.GT.ATERI.FICI.GT.ATERI)GO TO 24 IF(ATERI.GT.ATERI.FICI.GT.ATERI IF(ATERI.GT.ATERI.FICI.GT.ATERI.FICI.GT.ATERI IF(ATERI.GT.ATERI.FICI.GT.ATERI.FICI.GT.ATERI IF(ATERI.GT.ATERI.FICI.GT.ATERI.FICI.GT.ATERI IF(ATERI.GT.ATERI.FICI.GT.ATERI.FICI.GT.ATERI IF(ATERI.FICI.GT.ATERI.FICI.GT.ATERI.FICI.GT.ATERI IF(ATERI.FICI.GT.ATERI.FICI.GT.ATERI.FICI.GT.ATERI.FICI.GT.ATERI.FICI.GT.ATERI.FICI.GT.ATERI.FICI.FICI.FICI.FICI.FICI.FICI.FICI.FI	
	325 326 327 328 329 330 331 332 333 333 334 335 336 337	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATER=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMF=DABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM)) IF(ATERR.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERIR)GO TO 24 IF(ASUMR.EQ.0.D0)GO TO 9 IF(ATERR/ASUMR.GT.E)GO TO 27 CONTINUE IF(ASUMI.EQ.0.D0) GO TO 6 IF(ATERI/ASUMI.LE.E)GO TO 6	
	325 326 327 328 329 330 331 332 333 335 335 335 336 337 338 22	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMEDABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM)) IF(ATERR.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERII)GO TO 24 IF(ATERI.GT.ATERII)GO TO 9 IF(ATERR/ASUMK.GT.E)GO TO 27 OCUNTINUE IF(ASUMI.EQ.O.DO) GO TO 6 IF(ATERI/ASUMI.LE.E)GO TO 6 CONTINUE	
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	325 326 327 328 329 330 331 332 333 334 335 336 337 338 2 339 340 341 342 24 343 344 345 346 347 348 200	TERM=B(J)*CDEXP(-(2.DO*C(J)-1.DO)*ZLG) ATER=DABS(DREAL(TERM)) ATER = DABS(DIMAG(TERM)) SUM=SUM+TERM ASUM=DABS(DREAL(SUM)) ASUM = DABS(DIMAG(SUM)) IF(ATERR.GT.ATERIR)GO TO 24 IF(ATERR.GT.ATERIR)GO TO 24 IF(ATERR.GT.ATERII)GO TO 24 IF(ATERR/ASUMR.EQ.O.DO)GO TO 9 IF(ATERR/ASUMR.GT.E)GO TO 27 CONTINUE IF(ASUMI.E0.O.DU) GO TO 6 CONTINUE IF(ASUMI.E0.O.DU) GO TO 6 CONTINUE ATERIFATERR ATERIFATERR ATERIFATERR ATERIFATERR ATERIFATERR ATERIFATERR ATERIFATERR ATERIFATERR ATERIFATERR IF(J.LT.10)GO TO 8 GO TO 4 ZTGAM=SUM IF(IF.EQ.O)GO TO 31 DO 200 K=1', IF	
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	325 326 327 328 329 330 331 332 333 334 335 336 336 337 338 2 339 340 341 342 22 343 344 345 345 346 347 348 200 347 348 200 349 350	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMEDABS(DREAL(SUM)) IF(ATERR.GT.ATERIR)GD TO 24 IF(ATERR.GT.ATERIR)GO TO 24 IF(ATERR.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERIR)GO TO 24 IF(ATERI.GT.ATERIR)GO TO 27 OUNTINUE IF(ASUMI.E0.0.D0)GO TO 9 IF(ATER/ASUMK.GT.E)GO TO 27 OUNTINUE IF(ASUMI.E0.0.D0)GO TO 6 CONTINUE IF(ASUMI.E0.0.D0)GO TO 6 CONTINUE ATERIR=ATERR ATERII=ATERR ATERII=ATERR ATERII=ATERI GO TO 7 CONTINUE IF(J.LT.I0)GO TO 8 GO TO 4 2TGAM=SUM IF(IF.EQ.0)GO TO 31 DO 200 K=1,IF ZTGAM=ZTGAM-CDLOG(ZT1+C(K)) GAM=CDEXP(ZTGAM) IF(ZX.LT.0.D0)GAM=-P1/(GAM*CDSIN(P1*Z)*Z) DCM=GAM	
	325 326 327 328 329 330 331 332 333 335 336 335 336 337 338 2' 339 340 341 342 24 343 344 344 345 346 347 346 347 348 20 350 351 352	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMR=DABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM)) IF(ATERR.GT.ATERIN)GO TO 24 IF(ATERR.GT.ATERIN)GO TO 24 IF(ATERI.GT.ATERIN)GO TO 24 IF(ATERI.GT.ATERIN)GO TO 24 IF(ATERI.ASUMR.GT.E)GO TO 27 CONTINUE IF(ASUMI.EQ.O.DO) GO TO 6 CONTINUE IF(ASUMI.EQ.O.DO) GO TO 6 CONTINUE ATERIR=ATERR ATERII=ATERI GO TO 7 CONTINUE IF(J.LT.IO)GO TO 8 GO TO 4 ZTGAM=SUM IF(IF.EQ.O)GO TO 31 DO 200 K=1,IF ZTGAM=ZTGAM-CDLOG(ZT1+C(K)) GAM=CDEXP(ZTGAM) IF(ZX.LT.O.DO)GAM=-P1/(GAM*CDSIN(PI*Z)*Z) DGM=GAM RETURN	
	325 326 327 328 329 330 331 332 333 334 335 336 337 338 22 339 340 341 342 24 343 344 345 346 347 344 345 346 347 348 200 349 351 350	TERM=B(J)*CDEXP(-(2.DO*C(J)-1.DO)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DREAL(TERM)) SUM=SUM+TERM ASUMR=DABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM)) IF(ATERR,GT.ATERIN)GO TO 24 IF(ATERR,GT.ATERIN)GO TO 24 IF(ATERR,GT.ATERIN)GO TO 24 IF(ATERR/ASUMK.GT.E)GO TO 27 CUNTINUE IF(ASUMI.EC.O.DO)GO TO 6 CONTINUE IF(ATERI/ASUMI.LE.E)GO TO 6 CONTINUE ATERIA=ATERR ATERII=ATERI GO TO 7 CONTINUE IF(J.LT.IO)GO TO 8 GO TO 4 ZTGAM=SUM IF(IF.EQ.O)GO TO 31 DO 200 K=1/IF ZTGAM=ZTGAM-CDLOG(ZTI+C(K)) GAM=CDEXP(ZTGAM) IF(ZX.LT.O.DO)GAM=-P17(GAM*CDSIN(PI*Z)*Z) DGM=GAM RETURN FND	
	325 326 327 328 329 330 331 332 333 334 335 336 336 337 338 27 339 340 341 342 24 343 344 344 345 346 344 345 346 347 348 20 348 20 351 351 353 354	TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) SUM=SUM+TERM ASUMR=DABS(DREAL(SUM)) ASUMI=DABS(DIMAG(SUM)) IF(ATERR,GT.ATERINIGO TO 24 IF(ATERR,GT.ATERINIGO TO 24 IF(ATERR,GT.ATERINIGO TO 24 IF(ATERR/ASUMR.GT.E)GO TO 27 CUNTINUE IF(ASUMI.E0.0.D0) GO TO 6 IF(ATERI/ASUMI.LE.E)GO TO 6 CONTINUE ATERIFATERR ATERIFATERR ATERIFATERR ATERIFATERR ATERIFATERR ATERIFATERR IF(J.LT.10)GO TO 8 GO TO 4 JTGAM=SUM IF(IF.E0.0)CO TO 31 DO 200 K=1,IF ZTGAM=ZTGAM-CDLOG(ZTI+C(K)) GAM=CDEXP(ZTGAM) IF(Z.LT.0.D0)GAM=-P1/(GAM*CDSIN(P1*Z)*Z) DCM=GAM RETURN END COMPLEX FUNCTION DPSI*16(Z,E)	
	325 326 327 328 329 330 331 332 333 334 335 336 337 338 2 339 340 341 342 2 343 344 345 346 344 345 346 347 348 20 350 351 352 353 354	TERM=B(J)*CDEXP(-(2.DO*C(J)-1.DO)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIAG(TERM)) SUM=SUM+TERM ASUMR=DABS(DIAG(TERM)) ASUMI=DABS(DIAG(SUM)) IF (ATERR.GT.ATERIN)GO TO 24 IF (ATERR.GT.ATERIN)GO TO 24 IF (ATERI.GT.ATERIN)GO TO 27 CONTINUE IF (ASUMI.E0.O.DO) GO TO 6 IF (ATERI/ASUMI.LE.E) GO TO 6 CONTINUE ATERIR=ATERR ATERII=ATERI GO TO 7 CONTINUE IF (J.LT.10)GO TO 31 IO 20 K=1',IF ZTGAM=ZTGAM-CDLOG(ZTI+C(K)) GAM=CDEXP(ZTGAM) IF (ZX.LT.O.DO) GAM=-P1/(GAM*CDSIN(PI*Z)*Z) DGMEGM RETURN END COMPLEX FUNCTION DPSI*16(Z.E) IMPLICIT COMPLEX*16 (A-H,D-Z)	
	325 326 327 328 329 330 331 332 333 335 336 336 337 338 2 339 340 341 342 2 343 344 344 344 344 344 344 345 345 345	TERM=B(J)*CDEXP(-(2.DO*C(J)-1.DO)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMT=DABS(DIMAG(SUM)) ASUMI=DABS(DIMAG(SUM)) IF(ATERR.GT.ATERINGO TO 24 IF(ATERR.GT.ATERNI)GO TO 24 IF(ATERR.GT.ATERNI)GO TO 24 IF(ATERR/ASUMK.GT.E)GO TO 27 OUNTINUE IF(ASUMR.E0.0.DO) GO TO 6 CONTINUE IF(ASUMI.E0.0.DO) GO TO 6 CONTINUE ATERIX=ATERR ATERII=ATERR ATERII=ATERR ATERII=ATERI GO TO 7 CONTINUE IF(J.LT.10)GO TO 8 GO TO 4 ZIGAM=SUM IF(IF.E0.0)GO TO 31 DO 200 K=1,IF ZIGAM=ZTGAM-CDLOG(ZTI+C(K)) GAM=CDEXP(ZTGAM) IF(ZX.LT.0.DO)GAM=-P1/IGAM*CDSIN(PI*Z)*Z) DGM=GAM RETURN END COMPLEX FUNCTION DPSI*16(Z;E) IMPLICIT COMPLEX*16 (A-H,O-Z) COMPLEX	
	325 326 327 328 329 330 331 332 333 335 336 337 338 2 339 340 341 342 2 343 344 344 344 344 344 345 346 347 348 20 350 351 352 353 355 355 356	TERN=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMR=DABS(DIMAG(SUM)) IF(ATERR.GT.ATERLI)GO TO 24 IF(ATERR.GT.ATERLI)GO TO 24 IF(ATERI.GT.ATERLI)GO TO 24 IF(ATERI.GT.ATERLI)GO TO 24 IF(ATERR/ASUMR.GT.E)GO TO 27 OUNTINUE IF(ASUMI.E0.0.D0) GO TO 6 IF(ATERI/ASUMI.LE.E)GO TO 6 CONTINUE ATERLR=ATERR ATERLI=ATERR ATERLI=ATERR ATERLI=ATERR GO TO 4 ZIGAM=ZUGM-CDLOG(ZTI+C(K)) GAM=CDEXP(ZIGAM) IF(ZX_LT.0.D0)GAM=-PI/(GAM*CDSIN(PI*Z)*Z)) DGM=GAM RETURN END COMPLEX FUNCTION DPSI*16(Z,E) IMPLICIT COMPLEX*16 (A-H;0-Z) COMPLEX*16 CDEXP;0DL0G,CDC05;CDSIN,DCMPLX REAL*0 DATAN2,DFLOAT,DABS,CDABS	计计算机 计数字 医胃 医结节 医结节 医结节 计段 医结节 化化二化物 计错误 医结节 化化二化物 化化二化物
	325 326 327 328 329 330 331 332 333 334 335 336 337 338 2 339 340 341 342 24 343 344 344 345 340 341 342 24 343 344 345 347 348 20 351 350 351 355 353 355 356 357 358	TERN=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DREAL(TERM)) ATERI=DABS(DIMAG(TERM)) SUM=SUM+TERM ASUMR=DABS(DIMAG(SUM)) ASUMT=DABS(DIMAG(SUM))	정말 이 회장에 정말한 정말한 방법에 들었다. 정말 이 것은 것을 것을 것을 것을 것을 것을 것을 것을 것을 것 같다. 것은 것은 것은 것은 것은 것을 것 같은 것을 것 같다. 것은 것 같은 것 같은 것 같은 것 같은 것 같은 것 같은 것 같은
	325 326 327 328 329 330 331 332 333 334 335 336 336 337 338 27 339 340 341 342 24 343 344 342 24 343 344 345 346 347 348 20 348 20 351 352 353 354 355 356 357 358	TERR=DABS(DEXP(-(2.D0*C(J)-1.D0)*ZLG) ATERR=DABS(DEAL(TERM)) ATERI=DABS(DEAL(TERM)) SUM=SUM+TERM ASUMR=DABS(DEAL(SUM)) ASUMI=DABS(DEAL(SUM)) ASUMI=DABS(DIMAG(SUM)) TF(ATER,GT.ATERIR)GO TO 24 IF(ATER,GT.ATERIR)GO TO 24 IF(ATER,GT.ATERIR)GO TO 24 IF(ATER,GT.ATERIR)GO TO 27 CONTINUE IF(ATER/ASUMK.GT.E)GO TO 27 CONTINUE IF(ATERI/ASUMK.GT.E)GO TO 6 CONTINUE ATERIRAJER ATERIAJER ATERIAJER ATERIAJER ATERIAJER TF(ATERI/ASUMI.LE.E)GO TO 6 CONTINUE IF(J.LT.IO)GO TO 8 GO TO 4 ZIGAM=SUM IF(IF.EQ.O)GO TO 31 DO 200 K=1',IF ZIGAM=CDEXP(ZTAM=CDLOG(ZT1+C(K))) GAM=CDEXP(ZTAM=CDLOG(ZT1+C(K))) IF(ZX.LT.0,D)GAM=P1/(GAM*CDSIN(P1*Z)*Z) DGM=GAM RETURN END COMPLEX FUNCTION DPSI*16(Z.E) IMPLICIT COMPLEX*16 (A-H;0-Z) COMPLEX*16 (DEXP;CDLOG;CDCSTCSTN;DCMPLX REAL*86 E;ERR,B;ABTER;ABTERI;ABSUMI;PI,ZX;ZY,ZXI;EN;EUL;DREAL;DIMAG INTEGRE*4 IDINT	· 2011년 1월 2011년 1월 2011년 1월 21일 - 1월 21일 - 1월 21일 - 1월 21일 - 1월 21일 - 1월 21일 - 1월 21일 - 1월 21일 - 1월 21일 - 1월 2

55 IF (ZX.GT.20.D0)G0 T0 42 IF = 1DINT(ZX)-1 IJ=1

DO 300 N=2,IF 300 IJ=IJ*N

DGM=IJ

RETURN 42 CONTINUE

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1284 1285

1286

1287 1288 ŀ

1361 DATA IFLAG /0/ 1362 FRR=FIF (ERR . LE. 0. D0) ERR = . 10-5 1363 1364 78=7 1365 ZT = Z**1366** ZX=DREAL(ZP) 1367 ZY = DIMAG(ZP)1368 NI = IDINT(ZX)1369 $Z \times I = DFL DAT(NI) - 7 \times I$ 1370 IF (ZY.NE.O.DO.OR.ZX.GT.O.DO.OR.ZXI.NE.O.DO)GO TU 1 1371 DPSI=DCMPLX(0.D0,0.D0) WRITE (6,1010) Z 1372 1373 RETURN 1374 CCALCULATE PSI HERE IF Z IS A POSITIVE INTEGER 1375 1 IF(ZY.NE.O.DO.OR.ZXI.NE.O.DO)GO TO 2 EUL=-.5772156649 1376 1377 DPSI=EUL 1378 IF (NI.EQ.1)RETURN 1379 NE = NI - 100 100 N=1.NF 1380 1381 100 DPSI=DPSI+1.DO/DFLOAT(N) 1382 RETURN 1383 ISIGN=0 1384 $IF(ZX \cdot LT \cdot O \cdot DO) IS IGN = 1$ IF(ISIGN.EQ.1)ZT=1.00-ZT 1385 1386 CREFLECTS Z INTO 1-Z IF Z IS IN LEFT-HALF PLANE 1387 ZS=ZT 1388 NR = 01389 IF(IFLAG.NE.O)GU TO 3 1390 PI=DATAN2(0.D0,-1.D0) 1391 B(1)=1.00/(2.00*6.00)1392 B(2)=-1.D0/(4.D0*30.D0) 1393 $B(3) = 1 \cdot D0/(6 \cdot D0 \neq 42 \cdot D0)$ 1394 B(4) =-1.D0/(8.D0*30.D0) 1395 $B(5) = 5.00/(10.00 \div 66.00)$ 1396 B(6) =-691.D0/(12.D0*2730.D0) B(7)=7.D0/(14.00*6.D0) 1397 1398 B(8)=-3617.D0/(16.D0*510.D0) 1399 B(9)=43867.D0/(18.D0*798.D0) 1400 B(10)=-174611.D0/(20.D0*330.D0) IFLAG=1 1401 IF ((NR+NI).GT.10)GD TO 4 1402 1403 NR = NR + 11404 GO TO 3 CINCREASES REAL PART OF Z UNTIL GREATER THAN 10 1405 4 ZT=ZT+DFLOAT(NR) 1406 $7L = CDL \Omega G(2T)$ 1407 TER1=.500/ZT 1408 1409 SUM=ZL-TER1 1410 ABTER1=CDABS(TER1) N=0 1411 N=N+1 1412 8 EN=DFLOAT(N) 1413 1414 TER = -B(N) * CDEXP(-2.DO * EN * ZL)ABTER=CDABS(TER) 1415 IF (ABTER.LT. ABTER1)GO TO 5 1416 1417 NR = NR + 1ZT=ZS ≣1418≊ IF (NR.LT.100) GO TU 4 1419 WRITE (6,1011) 1420 1421 RETURN 1422 SUM=SUM+TER 5 ABSUMI=DABS(DIMAG(SUM)) 1423 IF (ABSUMI.NE.O.DO) GO TO 6 1424 7 IF (ABTER/CDABS(SUM).LE.ERR)GO TO 1425 1426 GO TO 9 6 IF (DABS (DREAL (TER))/ DABS (DREAL (SUM)) .LE.ERR. AND. DABS (DIMAG (TER))/ 1427 1ABSUMI.LE.ERR)GO TO 7 1428 9 ABTER1 = ABTER 1429 7 DPSI=SUM 1430 1431 IF(NR.EQ.0) GO TO 10 1432 DO 200 N=1.NR 1433 200 DPSI=DPSI-1.DO/(ZT-DFLDAT(N)) 1434 10 IF(ISIGN.EQ.0)RETURN 1435 1436 $ARG = PI \neq ZP$ DPS1=DPSI-PI*CDCOS(ARG)/CDSIN(ARG) 1437 RETURN 1438 THE PSI 1010 FURMAT(1H ,//,45H THIS IS & SINGULAR POINT OF FUNCTION ./. 1439 130X,6HARG R=,1PE12.5,3X,6HARG [=,E12.5] 1440 1011 FORMAT(1H ,//,24H SERIES DID NOT CONVERGE) 1441

1442

END-

********** SAMPLE RUN WITH THE SAMPLE DATA-(A) GIVEN IN THE PROGRAM-(A) **\$** SCATTERING OF ACOUSTIC WAVES ***** * BY THE CYLINDERS OF ARBITRARY CROSS-SECTION * (((T-MATRIX FORMULATION))) * **** ** PROGRAM-(A): GENERATION OF THE T-MATRIX ELEMENTS \$ \$ B.C. TYPE : NEUMANN (RIGID INCLUSION) WAVE NUMBER -: .5 CROSS-SECTION OF THE SCATTERER : CIRCULAR RADIUS : 1.000 SYMMETRY CONDITION (W.R.T. X-AXIS) NUMBER OF BOUNDARY SEGMENTS : 72 EQUAL ANGULAR INTERVALS TYPE OF BOUNDARY DIVISION : 4 ORDER OF THE SIMPSON'S RULE USED IN NUMERICAL INTEGRATION : -TOTAL DIMENSION OF THE T-MATRIX : 10 X 10 TRUNCATION FACTOR FOR THE 0-MATRIX ELEMENTS : . 100-025 ERROR FACTOR FOR THE BESSEL FUNCTIONS 1. . 100-013 -----

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2 С ****** 3 C 4 С **1** SCATTERING OF ACOUSTIC WAVES 5 С 6 BY THE CYLINDERS OF ARBITRARY CROSS-SECTION * С С 8 С (((T-MATRIX FORMULATION))) # g С **10** Ċ 11 C *** P R O G R A M-(B) *** 12 С EVALUATION OF THE SCATTERED WAVE FIELD AT VARIOUS DISTANCES 13 С 14 FROM THE SCATTERER AND FOR VARIOUS INCIDENCE ANGLES OF THE ACOUSTIC PLANE WAVE , BY USING THE T-MATRIX CREATED BY THE ः C 15 C 16 C PROGRAM-(A). ·17 С 18 C *** THE GENERAL STEPS FOLLOWED BY THE PROGRAM ARE AS FOLLOWS *** 19 С 20 C 21 -- READING OF THE PARAMETERS DESCRIBING THE BOUNDARY OF THE C 22 С SCATTERER AND ELEMENTS OF THE T-MATRIX FROM THE FILE(10) 23 C 24 Č CREATION OF THE INCIDENT WAVE FIELD COEFFICIENTS 25 С 26 <u></u> C A1(I) & A2(I) FUR A DESIRED INCIDENCE ANGLE (AINC) 27 £ 28 C -- CALCULATION OF THE SCATTERED WAVE FIELD COEFFICIENTS 29 C 30 C C1(1) & C2(1) . WHERE: 31 С 32 C = T11 + A1 + T12 + A2 = C2 = T21 + A1 + T22 + A2 33 С 34 C (C1 & C2 ARE COLUMN VECTORS & TIJ'S ARE (NO X NO) MATRICES) 35 С 36 C EVALUATION AND PRESENTATION OF THE FOLLOWING RESULTS BOTH IN TABULAR AND POLAR GRAPHICAL FURMS: 37 38 C A-) NEAR FILLD SOLUTIONS (+ONLY FOR CIRCULAR BOUNDARY) 39 AT DESIRED REGIONS OR, ON THE BOUNDARY OF THE C SCATTERER. 40 C 41 Ċ 1-) VELOCITY POTENTIALS DUE TO SCATTERED WAVE FIELD 42 C 2-) WAVE VELOCITIES DUE TO SCATTERED WAVE FIELD B-) FAR-FIELD SCATTERED WAVE FIELD AMPLITUDES 43 С 44 C C-) TOTAL SCATTERING CROSS-SECTION (ALSO CHECK FOR 45 С CONVERGENCY) 46 Č ***** IMPORTANT NOTE ***** 47 48 Č CONVERGENCY OF THE TUTAL SCATTERING CRUSS-SECTION MUST BE CHECKED UP TO AT LEAST 1.E-3 %.IF (%) DIFFERENCE IS GREATER THAN 1.E-3,THEN THE SIZE OF THE T-MATRIX USED MUST BE INCREASED.IN SUCH A CASE, A NEW T-MATRIX OF THE LARGER SIZE MUST BE RECREATED BY THE PROGRAM-(A). 49 C. 50 C 51 С <u>5 2</u> Č 53 С 54 C 55 C 56 C 57 C 58 C 59 C CARD 1 : 60 C COLUMN 1-2: NI (READING UNIT OR FILE NO.) 61 C 62 C COLUMN 3-4: NO (WRITING UNIT OR FILE NO.) 63 C 64 С CARD 2 : 65 С COLUMN 1-3: NYAZ (ANGULAR INCREMENT IN PRINTING THE 66 TABULAR RESULTS.EXAMPLE: IF NYAZ=10, 67 С THEN, SCATTERED WAVE FIELD RESULTS 68 C VERSUS POLAR ANGLE ARE PRINTED FOR EACH 69 С 10 DEGREE INCREMENT. MIN(NYAZ)=2. 70 C (*NOTE*: NYAZ SHOULD BE AN EVEN NUMBER) 71 С COLUMN 4-6 : XAX (XAX & YAX ARE THE SCALE PARAMETERS 72 C REQUIRED FOR 'SUBROUTINE GRAPH4'. 73 COLUMN 7-9 : YAX C TAKE XAX & YAX=6.0-11.0) 74 Č (*NOTE:* 'GRAPH4' IS A LIBRARY PRUGRAM IN UNIVAC-1106 SYSTEM) 75 С 76 č 77 C CARD 3 :

81 C	CAPD SET A
83 C	THE CARD SET 17 . AND AN AN AN AN AN AN AN AN AN AN AN AN AN
84 C	*EACH CARD CONTAINS*
86 0	COLOWN 1-4: DUN4 (INCIDENCE ANGLE OF THE WAVE.(IN DEGREES)) (*NOTE:* NO OF CARDS IN SET(4) MUST
88 0	BE EQUAL IU 'NINC'.)
89 C	CARD 5 : The second second second second second second second second second second second second second second second s
91 C	COLUMN 1-2: NPOT (IF VELOCITY POTENTIALS ARE DESIRED TO
92 0	BEEVALUATED NPOT=1 ,OTHERWISE NPOT=0)
94 C	OTHERWISE NVEL=0
95 C	COLUMN 5-6: NFAR (IF FAR-FIELD SOLUTION IS DESIRED
97 C	COLUMN 7-8:NCROS (NCROS=1 IF TOTAL SCATTERING CROSS-SECTION
98 C	COLUMN 9-10: NDEC (NUMPER OF DECIONS AT UNIC) THE ACAD FEED
-100 C	SOLUTIONS ARE DESIRED TU BE EVALUATED.
101 C	(*NOTE:* IF BOTH NPOT & NVEL=U,TAKE NREG=O)
103 C	CARD SET 6 : (*IF NREG=0 ; UMIT THIS CARD SET)
104 C	#FACH CAUD CONTAINS#
E 106 C	COLUMN 1-4: REG(I) (REGION(DISTANCE FROM THE ORIGIN OF THE
107 C	POLAR COORDINATE SYSTEM))
109 C	SOLUTIONS ARE DESIRED TO BE
110 C	EVALUATED ON THE BOUNDARY OF
1112 Č	2=)I=1,NREG
113 C	
115 C	
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121 CC 122 C 123 C 124 C 125 C 126 C 127 C 128 C 129 C 130 C 131 C 132 C 133 C 134 C 135 C 136 C 137 C 136 C 137 C 138 C 137 C 138 C 139 C 136 C 137 C 138 C 139 C 136 C 137 C 138 C 139 C 136 C 137 C 138 C 139 C 136 C 137 C 138 C 139 C 136 C 137 C 138 C 139 C 136 C 137 C 138 C 139 C 139 C 139 C 139 C 130 C 131 C 132 C 133 C 134 C 135 C 136 C 137 C 138 C 139 C 136 C 137 C 138 C 139 C 136 C 137 C 138 C 139 C 136 C 137 C 138 C 139 C 140 C 141 142 C 149 C 146 C 149 C 150 C 155 C 155 C	COLUMN CARD [1234567890 CARD N/REC/A; B
$\begin{array}{c} 121 & C \\ 122 & C \\ 123 & C \\ 124 & C \\ 125 & C \\ 126 & C \\ 127 & C \\ 128 & C \\ 130 & C \\ 130 & C \\ 131 & C \\ 133 & C \\ 133 & C \\ 133 & C \\ 134 & C \\ 135 & C \\ 136 & C \\ 137 & C \\ 136 & C \\ 137 & C \\ 138 & C \\ 137 & C \\ 138 & C \\ 139 & C \\ 131 & C \\ 134 & C \\ 141 & 142 \\ 143 & 144 \\ 142 & 143 \\ 144 & C \\ 146 & 147 \\ 148 & C \\ 146 & 147 \\ 148 & C \\ 149 & C \\ 151 & 152 & C \\ 151 & 152 & C \\ 155 & 156 & C \\ 1$	COLUMN CARD 11234567890 CARD 1234567890 CARD 11234567890 CARD EX 16 T11(20,20),T12(20,20),T22(20,20),T22(20,20), COMPLEX 16 T11(20,20),T12(20,20),T22(20,20),T22(20,20), COMPLEX 16 T11(20,20),T12(20,20),T12(20,20),T22(20,20), COMPLEX 16 T11(20,20),T12(20,20),T12(20,20),T22(20,20), COMPLEX 16 T11(20,20),T12(20,20),T12(20,20),T22(20,20),T22(20,20), COMPLEX 16 T11(20,20),T12(20,20),T12(20,20),T22(20,20),T22(20,20), COMPLEX 16 T11(20,20),T12(20,20),T12(20,20),T22(20
121 CC 122 C 123 C 124 C 125 C 126 C 127 C 128 C 129 C 131 C 132 C 131 C 132 C 133 C 134 C 135 C 135 C 136 C 137 138 C 137 138 C 137 138 C 137 138 C 137 138 C 137 138 C 137 138 C 137 138 C 137 138 C 137 138 C 137 140 C 141 142 143 144 145 C 146 147 148 C 149 150 C 154 155 156 C 157	COLUMN CARD [1234567890 1 [5 6 2 [10 5. 5. 2 [10 9. 7. 3 1 1 4 [0. 5 [1 0 1 1 2 6 [0 0 1 1 0 7 [5. COMPLEX+16 T11(20,20),T12(20,20),T21(20,20),T22(20,20), 6 [BES1(22),BES2(22),HANK(22),OUH,DUM2,D2,DN,CPART, 6 [Al(20),A2(20),C1(20),C2(20),D35F(20), 6 [BES1(22),BES2(22),HANK(22),OUH,DUM2,D2,DN,CPART, 6 [Al(20),A2(20),C1(20),C2(20),BASF(20), 6 [BES1(22),BES2(22),HANK(22),OUH,DUM2,D2,DN,CPART, 6 [Al(20),A2(20),C1(20),C2(20),BASF(20), 7 [S. COMMON/REC/A,B,CORRAD,T1,T2,T35I(4,ALPH,DIS COMMON/REC/A,BA,CORRAD,T1,T2,T35I(4,ALPH,DIS COMMON/RIAN/H,BETA,TETA1 REAL*4 X(190),Y(190),XAX,YAX CHARACTER BCON*4,SHAPE*6,H1*12,H2*25,H3*18,H4*33,H8*37, 8 [H5*8,H6*10,H7*34,H9*9,H10*13,H11*10] G(V)=V*360./(2,*PI) F(U)=U*2.*PI/360. DATA H1,H2;H3/'B,C. TYPE : ','NEUMANN (RIGID INCLUSION)','DIRICHLE
121 CC 122 C 123 C 124 C 125 C 126 C 127 C 128 C 129 C 130 C 131 C 132 C 133 C 134 C 135 C 136 C 137 C 136 C 137 C 138 C 137 C 138 C 137 C 138 C 137 C 138 C 137 C 138 C 137 C 136 C 137 C 136 C 137 C 136 C 137 C 136 C 137 C 136 C 137 C 138 C 141 C 141 C 149 C 151 C 153 C 155 C 156 C 157 C 158 C 157 C	CARD [1234567890 CARD [1234567890 CARD [1234567890 CARD [1234567890 CARD [1234567890 CARD [1234567890 CARD [1234567890 CARD [1234567890 CARD [1234567890 CARD [1234567890 CARD [1234567890 CARD [1234567890 CARD [1234567890 CINC [10 9.7. 3 1 1 3 1 2 4 10. 5 [1 0 1 1 2 5 [45. 6 10. 7 15. COMPLEX+16 T11(20,20),T12(20,20),T21(20,20),T22(20,20), 6 BES1(22),BES2(22),HANK (22),OUH,JOUM2,D2,D0,CPART; 6 A1(20),A2(20),C1(20),C2(20),BASF(20), 6 USPOT(190),USVEL(190),USFAR(190) COMMON/REC/A,B,CORRAD,T1;T2,T3;14;ALPH,JIS COMMON/REC/A,B,CORRAD,T1;T2,T3;14;ALPH,JIS COMMON/REC/A,B,CORRAD,T1;T2,T3;14;ALPH,JIS COMMON/REA,A,HAPE *6,H1*12,H2*25,H3*18,H4*33,H8*37, 6 H5*8,H6*10,H7*34,H9*9,H10*13,H11*10 G(V)=V*360./(2,*P1) F(U)=U*2,*P1/360. DATA H1,H2;H3/'B.C. TYPE : ','NEUMANN (RIGID INCLUSION)','DIRICHLE 5T (CAVITY)'/ DATA H4/CROSS-SECTION OF THE SCATTERER : '/

1	61		DATA H5,H6,H7/'CIRCULAR', 'ELLIPTICAL', 'RECTANGULAR (WITH ROUNDED C
1	.63		DATA H9,H10/'SYMMETRIC', 'NON-SYMMETRIC'/
	64	ſ	DATA H11/'TRIANGULAR'/
្រា	.66		READ(5,10) NI,NO,NYAZ,XAX,YAX
1	.67 68	10	FORMAT(212/13,2F3.0) WRITE(N().613)
1	.69	613	FORMAT(1H1)
	.70 71	100	WRITE(NU, 100) EDRMAT(///5X, $47(1+1)/5X$, $141/6X$, $141/6X$, $141/6X$, $141/6X$, $160/6X$, $160/6X$
1	72	ંટર	TIC WAVES',9X, '*'/5X, '*',45X, '*'/5X, '* BY THE CYLINDERS OF ARBITRA
। जन्म	.73 .74)) 11 رائی چین (RY CROSS-SECTION *'/5X,'*',45X,'*'/5X,'*',8X,'(((T-MATRIX FORMULA TION)))',9X,'*'/5X,'*',45X,'*',5X,47('*')////5X,'** PROCEAM-(A);
	75		EVALUATION OF THE NEAR AND FAR FIELD SOLUTIONS **'/5x,65('=')///)
231 1	.76 .77	C***	READING OF THE DATA, CREATED BY THE PROGRAM-(A) AND ****
	78	C + + + +	STORED INTO THE FILE(10) ++++
1	.79 .80	1	FORMAT(1X,A4/1X,F4.1/1X,A6)
1	81		IF (SHAPE.EO. 'CIRCLE') READ(10,2) RAD
1	. 8 2 . 8 3	fet sin foi F	IF (SHAPE.EQ. 'RECTAN') READ(10,2) AA, BB
	84	Strand	IF (SHAPE.EQ. 'TRIANG') THEN A SAME AND A SA
1	.02 .86	en en en en en en en en en en en en en e	BETA=F(BET)
] 	87	, and the second second second second second second second second second second second second second second se	TETA1=DATAN(3.*DTAN(BETA/2.))
1	.89	2	FORMAT(1X,2F6.3,F13.10)
្នា	.90		READ(10,4) NSYM, NO
<u></u> 1	92	7	NQ2=NQ*2
1	93		NQT=NQ+2 IF(NQ,GE,13,AND,NO,LE,18)=NOT=22
1	95	un e n en la L	00 5 I=1,NQ
· 1 1	.96 .97	6	$DC_{6} J=1,NQ$ READ(10,7) T11(1,J)
<u>_</u>	98		DO 5 J=1,NO
1 1 2 2	.99 100	5	READ(10,7) [12(1,J) D0 8. I=1,NQ
2	01		D0 9 J=1,N0
2	02	-11.5-5-11 -7 - 1	$\frac{\text{READ}(10,7)}{\text{D}} = \frac{1}{2} \frac{1}{1} 1$
223	04	8	READ(10,7) T22(1,J)
ź	06	n na seo seo seo seo seo seo seo seo seo seo	WRITE(N0,11)
2	07	11	FORNAT(63('=')/) TE(BCON.EO.'NEUR') WRITE(NOC12) H):H2
2	09	12	FURMAT(1X,A12,A25)
원 물2	10		IF(BCON.EQ.'DRIC') WRITE(NO,12) H1,H3 TF(SHAPF.FQ.'CIRCLF') WRITE(NO,13) H4,H5,RAD
2	12	13	FORMAT(/1X,A33,A8//1X, 'RADIUS : ',F6.3)
2	13	14	IF(SHAPE.EQ.'ELLIPS') WRITE(NU,14) H4,H6,AA,BB FORMAT(/1X,A33,A10//1X,'A-AXIS::'',F6.3,4X,'B-AXIS : ',F6.3)
2	15		IF (SHAPE.EQ. 'RECTAN') WRITE (NO, 15) H4, H7, A, B, CORRAD
2 2	17	10 المراجع	CORNER RAD.: ', F13.10)
<u></u> 2	18		IF(SHAPE.EC. 'TRIANG') WRITE(ND;998) H4;H11;H;BET
ź	20		IF(NSYM.EQ.1) WRITE(NO,16) H8,H10
2	21	16	FORMAT(/1X,A37,A13) TE(NSYM.F0.2) WRITE(ND.16) H8;H9
2 2 2 2	23		WRITE(NO,17) ANO,NO2,NO2
====2 2	24	17 ۶	FURNAL(//1X, 'WAVE ITPE TAGUUSIIC PLANE WAVE //1X, WAVE NU : ', F4.1//1X, 'TOT.DIM. OF THE T-MATRIX USED : ',13,'X',13//63('='))
2	26		READ(NI,18) NINC
2	:27 :28		READ(NI,20) DUH4
	29	19	AINC(I)=F(DUM4)
2		20	FORMAT(F4+0)
=2	32	71	READ(NI,21) NPOT, NVEL, NFAR, NCROS, NREG
2	. 3 3 2 3 4	<u>۲۱</u>	IF (SHAPE.EQ. 'RECTAN') CALL RECT
2	35	Maria de la composición de la composición de la composición de la composición de la composición de la composición	IF(NREG.E0.0) GO TO 23 DO 22 I=1.NREG
2	37	22	READ(NI,20) REG(1)
	38	23	UU 24 II=1,NINU WRITE(NO,46) II,G(AINC(II))
1.25	40	46	FORMAT(///6X+12,'- ANGLE OF INCIDENCE : ', F4.0, ' (DEGREES) / 45('='

241 ((3 242 C**** CREATION OF THE INCIDENTEWAVE FIELD COEFFICIENTS **** 243 A1(1) = (1.00, 0.00)244 DO 25 I=1,NO-245 IF(NO-1) 26,27,26 Z46 A1(I+1)=DSQRT(2.D0)*(0.D0,1.D0)**I*DCQS(I*AINC(II)) 26 247 27 A2(I)=DSQRT(2.D0)*(0.D0,1.D0)**I*DSIN(I*AINC(II)) 248 25 CONTINUE CALCULATION OF THE SCATTERED WAVE FIELD COEFFICIENTS **** 249 C * * * * 250 CALL MULVEC(T11,A1,C1,N0) IF (NSYM-2) 28,29,28 251 CALL MULVEC(T12,A2,C2,NQ) D0 30 I=1,NQ 252 28 253 254 30 C1(I) = C1(I) + C2(I)255 29 CALL MULVEC(T22, A2, C2, NO) 256 IF(NSYM-2) 31,32,31 257 31 CALL MULVEC(T21,A1,BASF,NO) 258 DO 33 I=1,NQ C2(I) = C2(I) + BASF(I)259 33 260 32 TEB=PI *4.D0/360. I = 1 261 262 IF (NREG) 64,65,64 263 64 DO 34 I=1,NREG 264 65 -----TEA=0.DO C**** 265 CALCULATION OF THE NEAR & FAR-FIELD SOLUTIONS **** 266 00 35 J=1,181 IF(NREG.EQ.0) GD TO 36 REGION=REG(I) 267 268 269 DTDR = 0.00IF (REG(I).EQ.O.) THEN IF (SHAPE.EQ.'CIRCLE') REGION=RAD IF (SHAPE.EQ.'ELLIPS') THEN 270 271 272 REGION=AA*BB/DSQRT(AA*AA*(DSIN(TEA))**2+BB*BB*(DCOS(TEA))* 273 274 E #2) 275 DTDR=(AA*BB*(BB*BB-AA*AA)*DSIN(TEA)*DCOS(TEA))/DSQRT((AA*AA*DSIN & (TEA) * * 2 + B B * B B * D C O S (TEA) * * 2) * * 3) 276 277 END IF 278 IF (SHAPE.EQ. TRECTAN) CALL RECTAZ (TEA, REGION, DTDR) IF(SHAPE.EQ. 'TRIANG') CALL TRIAZ(TEA, REGION, DTDR) 279 280 END IF DZ=DCMPLX((ANO*REGION),0.DO) 281 282 DN = (0.D0, 0.D0)DO 37 K=1,2 283 284 CALL DBESS(1,DZ,DN,DUM2,BES2(K),1.0-15) 285 37 DN = DN + (1.00, 0.00)286 D0 38 K=3,N0+1 BES2(K)=2.*(K-2)/DZ*BES2(K-1)-BES2(K-2) 287 38 288 DN=DCMPLX(((NQT-1)*1.D0),0.D0) DO 70 K=1,2 289 CALL DBESS(0,DZ,DN,BES1(NOT+1-K),DUM1,1.D 290 291 70 DN=DN-(1.D0,0.D0) 292 DU 71 K=3,NQT BES1(NQT+1-K)=2.*(NQT+1-K)/DZ*BES1(NQT-K+2)-BES1(NQT-K+3) 293 71 294-DO 39 K=1,NO+1 295 HANK(K)=DCMPLX(DREAL(BES1(K)),DREAL(BES2(K))) 39 USPDT(J)=(0.00,0.00) 296 297 USVEL(J) = (0.00, 0.00)298 36 USFAR(J)=(0.D0,0.D0) 299 00 40 N=1,NQ 300 EJEM=1.00 IF(N.GT.1) EJEM=DSQRT(2.DO) 301 IF(N.EQ.1) THEN ------302 CPART=C1(N) *DCOS((N-1)*TEA) 303 304 GO TO 414 END IF 305 CPART=C1(N)+DCOS((N-1)+TEA)+C2(N-1)+DSIN((N-1)+TEA) 306 414 IF (NPOT) 41,42,41 307 USPOT(J)=USPOT(J)+CPART+HANK(N)+EJEM 308 41 309 42 IF(NVEL) 43,44,43 USVEL(J)=USVEL(J)+EJEM+(((N-1)+HANK(N)-DZ+HANK(N+1))+CPART+(N-1)/ 310 43 ®ION+HANK(N)+(C1(N)+DSIN((N-1)+TEA)-C2(N)+DCOS(N+TEA))+DTDR) 311 312 44 IF(I.GT.1).GO TO 40 IF(NFAR) 45,40,45 313 USFAR(J)=USFAR(J)+CPART#EJEM#(0.D0,1.D0)##(1-N) 45 314 315 40 CONTINUE TEA=TEA+TEB 316 35 IF(I.GT.1) GO TO 49 317 IF(NFAR) 91,50,91 318 91 WRITE(N0,47) 319 320 47 FORMAT(//1X,63('=')/6X, *** FAREFIELD SCATTERED FIELD AMPLITUDE ***

$321 \{ (1X, 63(1=1)) \}$	
323 48 FORMAT(/18X.'RFAL'.7X.'IMAGINARY '.6X.'P	EAK VALUE!/3X. !ANGLE!. 10X.
324 E'PART',9X, 'PART',13X, '(NORM)'/1X,63('=')	<pre>} last the state of the st</pre>
325 CALL GRAFIK(ND,USFAR, PI, NSYM, NYAZ, XAX, YA	X, AINC(II))
326 50 IF (NCROS) 51,490,51	te de la participa de la tractica de la compañía de la compañía de la compañía de la compañía de la compañía d
328 52 FURMAT(//1X,36('=')/,2X,'CHECK FUR THE C	ONVERGENCY OF THE /3X TOT
329 EAL SCATTERING CRUSS-SECTION'/37('='),/,1	X, URDER TOT.S.C.SECTION (
330 6%)DIFFERENCE'/37('='))	
331 USC=0.00 332 C4444 CALCULATION OF THE TOTALES CATTERING CROSS	ST SECTIONS #### ST TO SECTIONS
333 00 53 N=1.NO	
334 PR C=US C	
335 USC=USC+CDABS(C1(N))**2+CDABS(C2(N))**2	· · · · · · · · · · · · · · · · · · ·
336 IF (N.EQ.1) THEN	ar de la ser antigen de la companya de la companya de la companya de la companya de la companya de la companya
338 - GU TO 53	, and a state of the state of
339 ELSE	
340 PERDIF=DABS(USC-PRC)/DABS(PRC)*100.D0	
341 WRITE(NU,54) N,USC,PERDIF	in a second second second
343 END IF	
344 53 CONTINUE	
345 54 FORMAT(1X,14,D16.7,D14.7)	
347 55 EDDMAT(141)	filiais fil Fifther a contain a thairte
348 490 IF(NREG.EQ.O) GO TO 24	en in tel Vice the Although Constants
349 49 IF(REG(I).EQ.O.) THEN	
350 WRITE(ND, 56)	
351 ELSE 352 URITE(NO. 57) REC(1)	na 1999 - Maria Maria, Maria Maria da Angela da Angela (m. 1990) 1999 - Angela Maria, Maria Mar
353 END IF	i terre anticipation de la construction de la construction de la construction de la construction de la constru La construction de la construction de la construction de la construction de la construction de la construction d
354 56 FORMAT(//6X, 'REGION : *BOUNDARY*'/6X,19(! = '))))))))))))))))))
355 57 FORMAT(//6X, 'REGION : ', F4.1/6X, 10('='))	
357 58 WRITE(N0.60)	
358 60 FORMAT (//1X, 63('=')/8X, '** VELOCITY POTE	NTIALS DUE TO SCATTERED FI-Z
359 & ELD **'/1X,63('='))	· · · · · · · · · · · · · · · · · · ·
360 WRITE(NO,48)	
362 59 IF (NVFL) 61.34.61	A JAING (III)) Storigenski stranski stranski skladat (Statisticka)
363 61 WRITE(N0,63)	
364 63 FORMAT(//1X,63('=')/8X,'** WAVE VELOCITI	ES DUE TO SCATTERED FIELD
365 + 447 / 13 + 65 (= 1) 366 WRITE(N0+48)	Million and the first state of the training state of the state of the state of the state of the state of the st
367 CALL GRAFIK(NO,USVEL, PI, NSYM, NYAZ, XAX, YA	X;AINC(II))
368 34 CONTINUE	
369 24 CONTINUE	er en en del de la companya de la companya de la companya de la companya de la companya de la companya de la c
371 END	ninin, saletta il suo suo suo si suo suo sentetta si sentetta il suo si si si si si si si si si si si si si
372 C	
373 C	ł <u>konstantowa za statu za s</u>
375 SUBROUTINE MULVEC(A,B,C,N)	al landa ini ini ini la sukan sa kata na kata na kata ini kata i
376 C	
377 C***********************************	¢φφφ αναιτική προγραφική το το το το το το το το το το το το το
378 C SUBPROGRAM FUR MAIRIX & VECTUR MULTIPLICATIO	UN → \to
380	
381 COMPLEX #16 A(20,20),B(20),C(20),T	
382 DO 1 I=1,N	ana kata mba kata kata kata kata kata kata kata ka
383 1= (0.00,0.00)	e de la critecia con el Male de la Male de la Male de la Male de la Male de la Male de la Male de la Male de la
385 2 T=T+A(I,J) +B(J)	n na sense sense sense sense sense sense sense na sense na sense na sense sense sense sense sense sense sense Na sense sense sense sense sense sense sense sense sense sense sense sense sense sense sense sense sense sense s
386 1 C(I)=T	
387 RETURN	na en la comercia de la como de comercia de la esta de la esta de la comercia de la esta de la esta de la esta
389 C	to a state de la constante en la transférica. A
390 C	
. 391 C	
392 C	
394 SUBROUTINE RECT	
395 C	. <u>1997 - Mariana Managaran</u> (managara) ang kanalar dalah kan
396 C++++++++++++++++++++++++++++++++++++	
397 C SUBPROGRAM FOR CALCULATION OF SOME REQUIRED	
399 C THROUGHOUT THE MAIN PROGRAM	1. A second sec second second
400 C INPUT DATA: (A , B , CORRAD]	e na stra se se que s ua sego da 1977 -

RETURN PARAMETERS: (T1, T2, T3, T4, ALPH, DIS) 401 С 402 403 C. 404 IMPLICIT REAL*8(A-H, 0-Z) 405 COMMON/REC/A, B, CORRAD, T1, T2, T3, T4, ALPH, DIS 406 PI=DATAN(1.DO)*4.DO 407 T1 = DATAN((B - CORRAD)/A)408 IF(A.EQ.CORRAD) THEN 409 T2=PI/2. 410 ALPH=PI/2. 411 DIS = (B-CORRAD)412 ELSE 413 T2 = DATAN(B/(A - CORRAD))414 ALPH=DATAN((B-CORRAD)/(A-CORRAD)) 415 DIS=(A-CORRAD)/DCOS(ALPH) 416 END IF elektore setteme T3 = PI - T2417 418 T4=PI-T1 Ä19 RETURN 420 END 421 C 422 C 423 C 424 C 425 SUBROUTINE RECTAZ(TEX, R, DTDR) 428 C*********************************** 429 С SUBPROGRAM FOR CALCULATION OF R & D(R)/D(TETA) 430 C VALUES FOR A GIVEN ANGLE (TETA) ALONG THE 431 RECTANGULAR BOUNDARY C 432 C*********************** 433 C 434 IMPLICIT REAL#8(A-H, 0-Z) 435 COMMON/REC/A, B, CORRAD, T1, T2, T3, T4, ALPH, DIS 436 F(T,X)=DIS+DCOS(T-X)+DSORT(DIS++2+(DCOS(T-X))++2+CORRAD++2-DIS++2) 437 G(TE)=DIS+DSIN(TE)+DIS++2+DCOS(TE)+DSIN(TE)/DSORT(DIS++2+DCOS(TE)+ 438 € #2 +CORRAD##2 -D1S##2) PI=DATAN(1.D0)*4.D0 439 440 TET=TEX 441 TETL=TET 442 IF (TET.GT.PI) TETL=TET-PI IF (TETL.LE.T1.OR.TETL.GE.T4) THEN 443 444 R = DABS(A/DCOS(TETL))445 DTDR = A*DSIN(TETL)/DCOS(TETL)**2 IF(TETL.GE.T4) DTDR=-DTDR 446 447 RETURN 448 5TI IT TOL TUEN ENDIF IF (TETL.GT.T1.AND.TETL.LT.T2) 449 THEN 450 R=F(TETL;ALPH) TET=TETL-ALPH 451 452 DTDR =-G(TET) 453 RETURN 454 END 1F IF (TETL.GE.T2.AND.TETL.LE.T3) 455 THEN -456 R=B/DSIN(TETL) DTDR=-B*DCOS(TETL)/DSIN(TETL)**2 457 458 RETURN 459 END IF 460 IF (TETL.GT.T3.AND.TETL.LT.T4) THEN R=F(TETL,PI-ALPH) 461 462 TET=PI-TETL-ALPH 463 DTDR=G(TET) 464 END IF 465 RETURN • A set of a probability of an experimental set of a set of a balance of the set of a set 466 3.7 END 467 C 468 C 469 С 470 C 승분영 471 С 472 SUBROUTINE TRIA2(TEX,R,DTDR) 473 C SUBPROGRAM FOR CALCULATION OF R & D(R)/D(TETA) 475 С 476 C VALUES FOR A GIVEN ANGLE (TETA) ALONG THE TRIANGULAR B HUND AR Y 477 479 C 480 IMPLICIT REAL*8(A-H,O-Z)

481 COMMON/TRIAN/H,BETA, TETA1 482 PI=DATAN(1.)*4. 483 TET=TEX 484 TETL=TET IF(TET.GT.PI) TETL=2.*PI-TET 485 IF (TETL.LE.TETAL) THEN 486 R=H/3.*(1./DCOS(TETL)) 487 488 DTDR=H/3.*DSIN(TETL)/(DCOS(TETL))**2 489 IF(TEX.GT.PI) DTDR=-DTDR 490 RETURN 491 END IF IF(TETL.GT.TETA1) THEN 492 493 R=2.*H/3.*DTAN(BETA/2.)/(DSIN(TETL)-DCOS(TETL)* == 494 3 DTAN(BETA/2.)) Contestas 495 DT DR = - 2 . * H/3 . * DT AN (BETA/2.)* (DCOS(TETL)+DS IN (TETL) * 496 DTAN(BETA/2.))/(DSIN(TETL)-DCOS(TETL)* 3 497 3 DTAN(BETA/2.))**2 498 IF(TEX.GT.PI) DTDR=-DTDR 499 END IF RETURN 500 501 END 502 C 503 С 504 С 505 C 506 C 507 SUBROUTINE GRAFIK(NO,US, PI, NSYM, NYAZ, XAX, YAX, AINC) 508 C 509 510 C SUBPROGRAM FOR PRESENTATION OF THE RESULTS IN TABULAR AND POLAR GRAPHICAL FORMS 511 С 512 C*********************** 513 C 514 COMPLEX#16 US(190),DCMPEX 515 REAL*8 DIMAG, DREAL, CDABS, PI, TG, T 516 REAL*4 X(190), Y(190), XAX, YAX 517 T=0. 518 K=NYAZ/2 519 00 1 I=1,181 IF(K.E0.(NYAZ/2+1)) K=1 520 521 TG=T/360.D0*2.D0*P1 522 C**** DATA TO BE PRESENTED IN GRAPHS, ARE STORED INTO X(1) & Y(1) ARRAYS 523 C**** (*NOTE: FIRST, POLAR TO RECTANGULAR CONVERSION IS NECESSARY 524 C**** FOR 'GRAPH4') 525 X(I) = CDABS(US(I)) + DCOS(TG)526 $Y(I) = CDABS(US(I)) \neq DSIN(TG)$ IF (NSYM.EQ.2.AND.I.GT.91.AND.AINC.EQ.O.) GO TO 1 527 528 IF(K.EQ.NYAZ/2) WRITE(N0,2) T, DREAL(US(I)), DINAG(US(T)), CDABS(US(I 529 ((3 530 K = K + 1FORMAT(4X,F4.0,8X,F8.5,4X,F8.5,9X,F8.5) 531 2 532 T=T+2.D0 1 533 CALL GRAPH4(XAX, YAX, 181, X, Y) 534 C*** 'GRAPH4' IS A' LIBRARY PROGRAM IN UNIVAC-1106 SYSTEM **** 535 WRITE(N0,3) 536 3 FORMAT(1H1) 537 RETURN 538 END 539 C 540 С С 541 542 C 543 С 544 SUBROUTINE DBESS(MO, DZ, DN, DB1, DB2, E) 545 C 546 C DOCUMENTATION ADDED AT CORNELL UNIVERSITY 8/5/74 FOR THE SUBROUTINE D 547 C DZ IS THE VALUE OF WHICH WE ARE TAKING THE BESSEL FUNCTION. DN IS THE ORDER OF THE BESSEL FUNCTION. 548 C 549 Ć THE VALUE OF THE BESSEL FUNCTION IS STORED IN DB1 IF THE BESSEL FUNCTION WAS DF THE FIRST KIND, IE. A J-BESSEL FUNCTION. 550 C 551 C THE VALUE OF THE BESSEL FUNCTION IS STORED IN DB2 IF THE 552 C BESSEL FUNCTION WAS UF THE SECOND KIND, IE. A Y-BESSEL FUNCTION. 553 С 554 · C 555 С IMPLICIT REAL*8(A-H,0-Z) 55.6 CCALCULATES BESSEL FUNCTION (COMPLEX ORDER AND COMPLEX ARGUMENT) OF THE 557 CFIRST KIND IF MO=0, AND ALSO OF THE SECOND KIND (NEUMANN FUNCTION) IF 559 CMO = 1560 CCALCULATES BESSEL FUNCTION OF COMPLEX ORDER AND COMPLEX ARGUMENT USING

<u>.</u>				<u> </u>		
561	CPOWER SERIE	ES FOR ABS(Z)	LESS THAN	ZOAND ASYMP	TOTIC SERIES FO	JR ABS(Z)
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263	L EL DELEKAL	INES RUUNDUFF	UF EN IU 1	NIEGER. IF	EF LE U., SEIS	
565	1022 FURMAI	11458 NEITHER 11468 NEITHER	SERIES FUR	BESSEL FUN	NETION CONVERGES	5)
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567	14H 7X=	= 19614.5.3X	A SINGULAR	SPUINT OF 1	HE NEUHANN FUN	
568		STON C(10).C1		0).03(100).	C4(100).C5(100	•та(101) С база
569	DIMENS	SION C6(101)	(1007)02(10		0112007,051200	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
570	Спирія	EX#16 7.EN.B.	ARG.G.TI.FX	STACE SEAT	311 - 2010 - T. M. W. W. S.	
571	COMPLE	EX #16 ENS.S.	T.U.V.S2.T2	• S07 • ZLG•A•	A1.81.82.81T.Z	H, ZHS, CI
572	COMPLE	EX *16 X,DZ,D	N, D81, D82, C	DUM		
573	REAL*	8 DATAN2, DREA	L, DIMAG, DLC	G,DFLOAT,DS	IGN, DABS, CDABS	
574	COMPLE	EX*16 CDEXP,C	DS OR T, CD CO S	CDSIN, DCMP	LX	
575	INTEG	ER#4 IDINT				
576	DATA	IFLAG /0/				e de la companya de la companya de la companya de la companya de la companya de la companya de la companya de l La companya de la companya de la companya de la companya de la companya de la companya de la companya de la comp
577	ERO=•1	10-2				
578	Z=DZ	generation data di Tella				
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580		EAL(UZ)			Gerref Alfender († 1995)	
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583	ENY=DA	REAL (DN)			erio de la Companya de la Companya de la Companya de la Companya de la Companya de la Companya de la Companya d	entral de la servició de la servició de la servició de la servició de la servició de la servició de la servició
	ENY=D	IMAG(DN)			us here e ekon	eachar 🔥 coileanach
585	M=M0+	1	······································			n de la constructivitation. Na
	ABSZ=0	CDABS(Z)		and a second second second second second second second second second second second second second second second	mette tradice -	a philip 2444.
587	DB1=D0	CMPLX(0.00,0.	D0)			
588	DB2=D	CMPLX (0.D0,0.	DO)		n a grannaga.	
589	NFLAG	= 2				
590	NF = 0	an an tha a g				
591	KFLAG	= 1				
592	CKFLAG DEND	TES QUADRANT	OF Z	e de la le sui	• • • • • • • • • • • •	
593	IF(ZX	.LT.0.DO.AND.	ZY.GE.0.001	IKFLAG=2	a construction and the second second	. Sa contrat an ago ata
594	IF(ZX	LT.O.DO.AND.	ZY .L I . 0. DO	KFLAG=3	nen en både stille	n i kan sen senten i
595	1212	• GE• U• UU• ANU•	21.LI.U.UUI	/KFLAG=4 -V-27		. talan aggi
	CDOUTINE HO	LAG+EV+Z+UK+K VEC 7 EDOM IE	ГЦА0,0 С.9.8.37 Ст_на с DI A	NE TO RIGHT	-HALE PLANE TE	ASYMPTOTIC
291	CREDIES TO	VES Z EKUN LE DE HSEN	F1-11AL1 1.64		the first and	
500	CSERIES IU	1 AG. GT. 0160 T	n 2			i shi kata kata ka Air
600	≦ani i C(1)=	3.14159265358	979300			
601	C(2)	=C(1) /2.DO			•	
602	C(3)=	C(2)/2.DO				
603	C(4)	=1.D0/DSQRT(C(2))			
604	C(5)=	DL0G(2.D0)				an an an 100 ann abhraí 1
605	C(6)=	8.DO				in intervention
606	C(7)=	64.D0				t shahiye matag
607	C(9)=	2.D0+C(1)	-		n e stationet de	ander tot ere eren t
608	C(10)	=1.00/0(1)	22000 C/EV		Alle të Huk Pretsori -	
609	EUL=.	5//2156649015	33800-0151		e da su sega d'actor	이 나는 아이는 바람을 했다.
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011	AI-UU AI-UU	-1.00			e para ang	相关 计可以指示 法律师
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614	EYE=I				المحمول معالم محمول المراجع المراجع المحمول المحمول المحمول المحمول المحمول المحمول المحمول المحمول المحمول ال المحمول المحمول	
615	C1(I)	=EYE				
616	C2(1)	=2.DO*EYE				
617	C3(1)	=2.D0*EYE-1	.DO		and the second second	أهمه بشمطريوج الوارا
618	- C4(1)	= (4.D0*EYE-1.	0D0) **2			
619	C6(1+	1 = C6(1) + 1.00	/DFLOAT(I+)	1)	4. st. stratistication	11111111111111111111111111111111111111
620	100 C5(I)	$= (4.00 \pm EYE - 3.$	001**2		on the subset Tetra	
	IFLAG	= 1			والمعقلة أنعا يتعاصف موجدا الم	
622	2 ERR=•	10-07			Totel in the state of the	i Chingtonia anazari na ma Wana.
623	1+(L.	GI.U.DUJERK=E	- eruszaitelek		e l'estre encle	
624	1=0	nn				And the second
= 625		BS (ENY) I F. FR	D. AND. DABS	(IDINT(ENX)-	ENX) .LE.ERO)NF	LAG=-1
627	LEVEL BUILT VUA	ITEMP=ENX+DSI	GN (. 500, EN	X)		en en en en en en en en en en en en en e
628		TEMP=ITEMP			en og het det er die boerden op Style in det en die besken	
629	IF (NF	LAG.EQ1) EN	=DCMPLX(TE	MP .0.D01	الم المعامة مورد رايون الد	n al contra contra contra en la contra en la contra en la contra en la contra en la contra en la contra en la c
630	EN X1 =	DREAL(EN)				
631	IF (NF	LAG.EQ1.AND	ENX1.EQ.0	DOINFLAG=0	 usulgations of 	n da ser la constante de la constante de la constante de la constante de la constante de la constante de la con La constante de la constante de la constante de la constante de la constante de la constante de la constante de
632	IF(NF	LAG.EO1.ANL	ENXI.GI.O	A NECATIVE T	NTEGER.7EDO.A	POSITIVE INT
633	CNFLAG =-1,0),+1,+2 MEANS	LENX (ENT)	A NEGALIVE I	INT LOLN 72 LNO 94	· • • • • • • • • • • • • • • • • • • •
634	CEGER, AND	A NUN-INTEGER	M ED 21M-2	te te tij generalite	in the second second second	
635	LI- (NF	-LAG.EV.Z.ANU - Monceese NEIN	TANN FUNCTI	ONTINTERMS	OF BESSEL FUNC	TIONS
636	: LFUK M=39 b ⊺c/∺r		EN	eseren i Filliñ.		
160	1 C L N T	LAG.LW. 1701			waka shiftin ka	
639	JFIA(G=0				a ta a popta car popta.
640	A= CDE	EXP(EN#A1)				
	Make an arrest the second seco		(a) A set of the se			

641	IF (ABSZ-GE-77) GU TO 6
642	IF (ENY . EQ. 0. DO. AND . DABS(IDINT(ENX)+0.5D0-ENX), IE. FR() GO TO 6
643	IF (ABSZ-NE.O.DO) GO TO B
644	IF(M.EQ.2) KRITE(6,1011) DZ
645	$IF (NFLAG.NE.0) GO TO 16$ $DB1=DCMP1 \times (1 OO.0 DO)$
647 16	RETURN
648 8	CONTINUE
649	I=1
650	ZLG=DCHPLX(DLOG(ABSZ), DATAN2(DIMAG(Z), DREAL(Z)))
651 CCHUUS	APC=CN+C1/1)
653	CALL DGAMM(ARG,G,CDUM,ERR,O)
654	ZH=EN*(ZLG-C(5))
655	ZHS=CDEXP(ZH)
656	T3(1)≠ZHS/G
658	S1= 3(1) FX=C0FXP(C2(1)*(7) G=C(5)))
659 11	I=I+1
660	T3(I)=-T3(I-1)*EX/((EN+C1(I-1))*C1(I-1))
661	S1=S1+T3(I)
662	SIS=CDABS(SI)
664	
665	IF(1.LT.101)G0 TO 11
666	IF(JFLAG.GT.O)GO TO 14
667	
660 14	VU. IV. Distance in the second s
670	STOP
671 81	CONTINUE
672	JFLAG=1 Benefit of the state of
674 87	UDITE (6.1022)
675	
676 9	B=S1
677	IF(M.NE.2)GO TO 55
678	N=DABS(ENX1)
D79	U#2.DU#(2LG#COL) (\$2=B#11-second) - UTTO - State - State St
681	1 / 2 / 2 = 1
682 77	DO 75 J=I222,I
683	
684	JIN=N+J-I
686	IF (J1.6T.101) 60 TO 201
687	TEMP1=C6(J1)
688	CO' TO 203
689 200	
690 201	
692	D0 202 JJ=102, J1
693	TEMP1=TEMP1+1.DO/DFLDAT(JJ)
694 202	CONTINUE de la contraction de la contraction de la contraction de la contraction de la contraction de la contra
695 203	
697	IF(J1N.GT.101) GO TO 206
698	TEMP2=C6(J1N)
699	
700 205	
702 206	TEMP2=C6(101)
703	DO 207 JJ=102, J1N
704	TEMP2=TEMP2+1.DO/DFLUAI(JJ)
705 207	CONTINUE
707	T2=T3(J)+(TEMP1+TEMP2)
708	T2R=DABS(DREAL(T2))
709	T2I=DABS(DIMAG(T2))
710 75	SZ=SZ=1Z SZERZENER (SZ)
111 112355510 -	S2I=DABS(DIMAG(S2))
713	IF(T2K.GT.ERR#S2R) GO TO 78
714	IF (T2I.LE.ERR*S21) GO TO 76
715 78	I=I+1 TE (I CT 101 AND 1ELAC E0.0) GO TO 81 1000 AND 1000 AND 1000
717	IF(1.6T.101.AND.JFLAG.NE.0)GO TO 82
718	$T3(I) = -T3(I-1) \neq EX/((EN+C1(I-1)) \neq C1(I-1))$
719	IZZZ=I
720	GO TO 77 a de la completa de managementa de la completa de la completa de la completa de la completa de la comp

721 76	82=S2*C(10)
722	IF(N.EQ.O) GO TU 55 https://www.europer
723	S1=DCMPLX(0.D0,0.D0)
724	$T_1 = -C(10)/Z_HS$
725	
725	
121	DU /U LL=1,LUP
770	11-11*DFLUAT(LL): STATE STAT
727	
731	$D = 11 \pm E \times (1 D = 1 \cap 1 (1 + 1 \to 1 \cap 1 \cap 1 \cap 1 (1 + 1 + 1 + 1 \cap 1 \to 1 \cap 1 \cap 1 \cap 1 \cap 1 (1 + 1 + 1 \cap 1 \to 1 \cap 1 \cap 1 \cap 1 \cap 1 \cap 1 \cap 1 \cap 1 \cap$
732 71	1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 -
733	
734 72	SI=SI+TI
735 73	B2=B2+S1
736	GO TO 55 STATE AND A STATE AND
737 55	IF (NFLAG.LT.O) $B = A \div B$
738	BX=DREAL(B)
739	BY=DIMAG(B)
740	BXA=DABS(BX) and a set of the set
(41	
7.7	I = (ZA + EQ + 0.5 U + 0.5 U + 2.1 + EQ + 0.5 U + 0.
744	If the Lagence 2, and, by a cliptic and one Eq. () by -0.00
745	(0 TO (56.57.58.59) M
746 6	AR G=X-EN*C(2)-C(3)
747	CF=CDCOS(ARG)
748	SF=C(6)*CDSIN(ARG)
749	FNS=C1(4)*EN*EN
750	I = O
751	S1=DCMPLX(1.D0,0.D0)
152	
() 3	U=DCMPLX(1.D0,0.D0)
755	
756 18	
757	V = -(FNS - C5(1))/(C(7) * x * C3(1)) * U
758	U=V*(FNS-C4(1))/(C2(1)*X)
759	US = CDABS(U)
760	IF (US.GT.TIS) GO TO 20
761 12	CONTINUE
762	1 単口菜(トキマ菜)というに、こことは小豆類類類的な実施が完全な実施になった。こことの「「「「「「」」」の含い、「「」」では、「」」「「」」」「「」」」「「」」」」「」」」」
765	T = DAS(DTREAL(T))
765	TI-DADS (DIMAGNING STORAGENER) AND AND AND AND AND AND AND AND AND AND
766	SR=DABS(DREAL(S))
767	SI=DABS(DIMAG(S))
768	IF(TR.GT.ERR*SR) GO TO 24
769	IF(TI.LE.ERR*SI) GO TO 26
770 24	
771	
772	31=31+11-4.574-000-00000000000000000000000000000000
	TISELS AND AND AND AND AND AND AND AND AND AND
775 17	F(1-100) 18,25,25
776 20	IF(1.E0.1) GO TO 12
777	IF(JFLAG.GT.O) GD TU 23
778	JFLAG=1Rectained and a second second second second second second second second second second second second
779	GO TO 8
780 23	WRITE(6,1002) Contraction Research and the second states of the second s
781 29	
782	
783 26	
785	16 (DFA1 (S07) -1 T.O.DO) SQZ=-SQZ
786 CCH005	ES PROPER BRANCH FUR SQUARE ROOT
787	B=C(4)/SQZ#S
788	IF(KFLAG.EQ.2)B=A*B
789	IF (KFLAG.EO. 3)B=B/A
790	IF(N.E0.3)M=2
. 791	IF (M.NE.2) GO TO 55
792	BZ = C(4)/SQZ + (5F + (5L + U)/U(0) + U(5A + V) + C(0)/U(0)/U(0) + U(0)/U(0) + U(0)/U(0) + U(0)/U(0)/U(0) + U(0)/U(0)/U(0)/U(0)/U(0)/U(0)/U(0)/U(0)/
(75	TE (VELACIED 3) R2=(R2-CDCOS(C(1)*EN)*CI*B)*A
705	AR TA 55
796 25	IF (JFLAG.GT.0)GO TO 28
797	JFLAG=1
798	CO TO 8
• 799 28	WRITE(6,1002)
800	GO TO 29

801 58 B1=DCMPLX(BX,BY) 802 M = 4EN=-EN 803 804 со то з 805 59 BIT=DCMPLX(BX,BY) EN=-EN 806 807 $ARG=C(1)\neq EN$ B2=(CDCOS(ARG)*B1-B1T)/CDSIN(ARG) 808 809 081 = 81810 DB2=B2811 RETURN 812 57 IF (NFLAG.EQ. -1) B2=A*B2 813 DB2=B2DB1=DCMPLX(BX,BY) 814 56 815 RETURN 816 END 817 SUBROUTINE DGAMM(DZ, DGM, DPS, ERR, JJ) 818 CIF JJ=0, CALCULATES ONLY GAMMA FUNCTION, IF JJ=1, CALCULATES ONLY PSI CFUNCTION, IF JJ=2, CALCULATES BOTH IMPLICIT REAL+8(A-H,O-Z) 819 820 821 CUMPLEX #16 GAM, Z, DZ, DGM, DPS, DPSI 822 COMPLEX #16 TERM1, ZT1, TERM, SUM, ZLG, ZTGAM, ZT COMPLEX#16 CDLOG, DCMPLX, CDEXP, CDSIN 823 REAL*8 DREAL, DIMAG, DLOG, DABS 824 825 INTEGER #4 ID INT 826 DIMENSION B(10) 827 DIMENSION C(100) 828 DATA IFLAG /0/ 1001 FORMAT(1H ,///,24H SERIES DID NOT CONVERGE) 1010 FORMAT(1H ,///,47H THIS IS A SINGULAR POINT OF THE GAMMA FUNCTION 829 830 X,/,5X,6HARG R=,E12.5,3X,6HARG I=,E12.5) IF(JJ.EQ.0)GO TU 60 831 - 832 833 UPS=DPSI(DZ, ERR) 834 冊 IF(JJ.EQ.1)RETURN 835 60 E=ERR 836 1F(E.LE.0.D0)E=.10-07 837 ZX=DREAL(DZ) 838 ZY = DIMAG(DZ)839 Z = 0 Z 840 IF (ZX.LT.0.D0)Z=-Z 841 NFLAG=2 842 1.12.04 J = 0CK=NUMBER OF TERMS IN SERIES 843 IF (ZY.EQ.0.DO.AND. (IDINT (ZX)-ZX).EQ.0.DO) NFLAG=1 844 845 IF (NFLAG.EO. 1. AND. ZX.LE.O.DO)NFLAG=0 846 CNFLAG=0 MEANS Z=0 OR Z A NEGATIVE INTEGER NFLAG=1MEANS Z A POSITIVE IF(NFLAG.NE.0)G0 TO 51 847 848 WRITE(6,1010) DZ DGM=DCMPLX(0.D0,0.D0) 849 850 RETURN IF (NFLAG.EQ. 2) GO TO 42 51 851 IF(ZX.GT.2.0D0) GU TO 55 852 DGM=DCMPLX(1.D0,0.D0) 853 854 RETURN 855 55 IF (ZX.GT.20.D0)GU TO 42 856 IF = IOINT(ZX) - 1857 IJ=1 DO 300 N=2, IF -858 859 300 IJ=IJ*N 860 DGM=IJ 861 RETURN 862 42 CONTINUE IF(IFLAG.NE.0)GU TO 20 863 DO 100 I=1,100 864 100 C(I)=I 865 PI=3.141592653589793D0 866 867 PI2=DL0G(2.D0*PI)/2.D0 868 B(1)=1.0D0/12.0D0 B(2) = -1.000/360.000869 B(3)=1.0D0/1260.0D0 870 871 B(4) = -1.000/1680.000B(5)=1.000/1188.0D0 = 872 B(6)=-691.0D0/360360.0D0 873 B(7)=1.0D0/156.0D0 874 B(8)=-3617.0D0/122400.0D0 875 B(9)=43867.0D0/244188.0D0 876 B(10)=-174611.0D0/125400.0D0 877 ARE THE BERNOULLI COEFFICIENTS IN STIRLINGS FORMULA CB(I) 878 879 IFLAG=1880 20 ZT = Z

881 ZT1 = ZT - 1.00882 I = 05 883 IF (DREAL(ZT) .GT. 10.DO) GO TO 3 884 I = I + 14 885 IF(I.LE.100)G0 TU 30 886 WRITE (6,1001) 887 STOP 888 30 CONTINUE 889 2T = ZT + 1.00890 GO TO 5 891 3 I = I892 ZLG=CDLOG(ZT) 893 SUM= (7T-.5 DO) #ZLG -ZT+P12 894 TERM1=SUM: 895 ATER1R=DABS(DREAL(TERM1)) 896 ATER1I=DABS(DIMAG(TERM1)) 897 J = 0 898 J=J+1 8 TERM=B(J)+CDEXP(-(2.DO+C(J)-1.DO)+ZLG) 899 900 ATERR=DABS(DREAL(TERM)) 1.1211-0.211 901 ATER I=DABS (DIMAG(TERM)) 902 SUM=SUM+TERM 903 ASUMR=DABS (DREAL (SUM)) 904 ASUMI=DABS(DIMAG(SUM)) IF (ATERR.GT.ATER1R)GO TO 24 905 IF (ATER I.GT. ATER 11) GO TO 24 906 907 IF (ASUMR.EQ.0.D0)G0 TO 9 908 IF (ATERR/ASUMR.GT.E) GU TO 27 9 CONTINUE 909 IF(ASUMI.EQ.O.DO) GO TO 6 IF(ATERI/ASUMI.LE.E)GO TO 6 910 911 912 CONTINUE 27 913 ATER1R = ATERR 914 ATER1I=ATERI 915 GO TO 7 916 24 CONTINUE 917 IF(J.LT.10)G0 T0 7 8 918 GO 10 4 919 6 ZTGAM=SUM IF(IF.EQ.0)G0 T0 31 -920 921 D0.200 K=1,IF 92Ż 200 ZTGAM=ZTGAM-CDL0G(ZT1+C(K)) GAM=CDEXP(ZTGAM) 923 31 924 IF(ZX.LT.0.D0)GAM=-PI/(GAM*CDSIN(PI*Z)*Z) 925 DGM=GAM 926 RETURN 927 END COMPLEX FUNCTION DPSI*16(Z + E) 928 IMPLICIT COMPLEX#16 (A-H,0-Z) 929 COMPLEX#16 CDEXP, CDLOG, CDCOS, CDSIN, DCMPLX 930 REAL*8 DATAN2, DFLOAT, DABS, CDABS 931 932 REAL #8 E, ERR, B, ABTER, ABTER, LABSUMI, PI, ZX, ZY, ZXI, EN, EUL, DREAL, DIMAG 933 INTEGER*4 .IDINT 934 DIMENSION B(10) 935 DATA IFLAG /0/ 936 ERR=E IF (ERR.LE.O.DO)ERR=.10-5 937 938 ZP = Z939 Z T = Z 940 ZX=DREAL(ZP) 941 7Y = DIMAG(2P)942 NI = IDINT(ZX)943 ZXI=DFLOAT(NI)-ZX 944 IF (ZY.NE.O.DO.OR.ZX.GT.O.DO.OR.ZXI.NE.O.DO)GO TO 1 945 DPSI=DCMPLX(0.D0,0.D0) 946 WRITE (6,1010) Z 947 RETURN 948 CCALCULATE PSI HERE IF Z IS A POSITIVE INTEGER 1 IF(ZY.NE.O.DO.OR.ZXI.NE.O.DO)GO TO 2 949 EUL=-.5772156649 950 951 DPSI=EUL 952 IF(NI.EQ.1)RETURN 953 NF=NI-1 C. 3. 11. s 954 DO 100 N=1,NF 100 DPSI=DPSI+1.DO/DFLOAT(N) 955 956 RETURN 957 ISIGN=02 IF(ZX.LT.0.D0) IS IGN=1 958 IF(15IGN.EQ.1)ZT=1.D0-ZT 959 960 CREFLECTS Z INTO 1-Z IF Z IS IN LEFT-HALF PLANE 1. **.**

961	ZS=ZT
962	NR=0 states to pay the provide the MANDIA STREAM STRE
963	IF (IFLAG, NE, O)GO TO 3
964	PI=DATAN2(0.D0,-1.D0)
965	B(1)=1.D0/(2.D0*6.D0)
966	$B(2) = -1 \cdot DO/(4 \cdot DO + 30 \cdot DO)$
967	$B(3) = 1 \cdot D0/(6 \cdot D0 + 42 \cdot D0)$
960	$B(4) = 1 \cdot BU/(10 \cdot B(4) + 30 \cdot D0)$
970	B(6) = -691, 00/(12, 0.0+2730, 0.0)
971	$B(7) = 7 \cdot D0/(14 \cdot D0 + 6 \cdot D0)$
972	B(8)=-3617.D0/(16.D0*510.D0)
973	B(9)=43867.D0/(18.D0*798.D0)
974	B(10) = -174611.00/(20.00+330.00)
975	IFLAG=1
077	
978	
979 CINCRE	ASES REAL PART OF 7 UNTIL GREATER THAN 10
980 4	ZT=ZT+DFLOAT(NR)
981	ZL=CDLOG(ZT)
982	TER1=.5D0/2T
983	SUM=ZL-TER1
- 984	ABTER1=CDABS(TER1)
900 199086 8	N = 0 . $N = N + 1$ $N = N + 1$
987	EN=DELOAT(N)
988-	$TER = -B(N) \neq CDEXP(-2.DO \neq EN \neq ZL)$
989	ABTER=CDABS(TER)
990	IF (ABTER.LT. ABTER1) GO TO 5
991	NR=NR+1
992	
993	
995	RETURN ·
996 5	SUM=SUM+TER.
997	AbSUMI=DABS(DIMAG(SUM))
998	IF (ADSUMI.NE.O.DO) GO TO 6
999	IF (ABTER/CDABS(SUM).LE.ERR)GO TO 7
1000	
1001 6	IF (DABS(DREAL(TER))/DABS(DREAL(SUN)).LE.ERK.AND.DABS(DIMAG(TER))/
1002 9	ABDUMI.LE.EKK/GU IU. <u>A anterinterinterinterinterinterinterinteri</u>
1004 Land L	
1005 7	DPSI=SUM
1006	IF (NR.EQ.0) GO TO 10
1007	00 200 N=1,NR
1008 200	DPSI=DPSI=1.00/(ZT-DFLOAT(N))
1009 10	IF (ISIGN.EQ. 0) RETURN
81010 (Black of C	AKU+rittr DDSI-DI&CDCAS(ARC)/CDSIN(ARC)
1012	RETURN
1013 1010	FORMAT(1H .//,45H THIS IS A SINGULAR POINT OF THE PSI FUNCTION,/,
1014	30X,6HARG R=,1PE12,5,3X,6HARG I=,E12,5)
1015 1011	FORMAT(111 ,//,24H SERIES DID NOT CONVERGE)
1016	FND STATES AND AND AND AND AND AND AND AND AND AND

******* SAMPLE RUN WITH THE SAMPLE DATA-(A) GIVEN IN THE PROGRAM-(B) ******** A. M. W. Miller and Super-induction of the second system o Production and the second secon ****** * SCATTERING UF ACOUSTIC WAVES * * BY THE CYLINDERS OF ARBITRARY CROSS-SECTION * (((T-MATRIX FORMULATION))) < +)) .. + + ***** ** PROGRAM-(B): EVALUATION OF THE NEAR AND FAR FIELD SOLUTIONS ** Manan Andreas (1997) and a state of the _____ B.C. TYPE : NEUMANN (RIGID INCLUSION) CROSS-SECTION OF THE SCATTERER : CIRCULAR RADIUS : 1.000 SYMMETRY CONDITION (W.R.T. X-AXIS) : SYMMETRIC WAVE TYPE : ACOUSTIC PLANE WAVE WAVE NO : . . . TOT.DIM. OF THE T-NATRIX USED : 10 X 10
••• FAR-FIELD SCATTERED FIELD ARVLITUDE •• ••• FAR-FIELD SCATTERE FIELD ARVLITUDE •• ••• FAR-FIELD SCATTERED FIELD ARVLITUDE ••	1- A	NGLE OF INCIDE	NCE : 0. (DE	GREES)		
** FAA-FIELD SCATTERED FIELD ANULTUDE ** ANGLE REAL PART IMAGINARY FEAK VALUE (MORN) 0 09370 2030 2050 10 09370 2030 2050 20 -08560 17730 19913 10 08570 1106 19913 20 -08560 17736 19913 30 -08178 1102 1106 40 -07330 11116 13430 90 -09327 20322 12010 60 -09323 -17219 17411 100 -00430 228953 22922 110 -00440 -22832 23926 100 -02323 -12832 23930 100 -03440 -38837 38031 100 -03445 -43972 49707 100 -03445 -43972 49707 100 -03445 -43972 49707 -210000 -44972 -44972		an Alama an Ala				
*** FAX-FIELD SCATTERED FIELD AAPLITUDE ** ANGLE REAL INGGNARYPEAN VALUE 0. 09037 .20340 .22257 10. 08650 .1733 .1063 20. 06550 .1733 .1663 20. 06736 .11116 .1330 20. 06736 .00355 .05707 20. 06736 .00355 .05707 20. 06736 .11116 .1330 20. 06736 .00417 .06633 20. 06420 .26955 .05707 20. .00414 .26955 .23958 110. 00440 .26955 .23950 130. .01470 .38805 .49701 130. .02252 .427037 .49707 100. .03361 .249972 .49707 130. .0275 .3003 .9017 .424000 .484000 .244000 .49707 .424000 .484000<				==================		na internetien.
REAL IMACI NARY PEAK VALUE (NORM) 0. 09037 20340	++ FA =========::::::::::::::::::::::::::::	R-FIELD SCATTE	RED FIELD AMPL	ITUDE		
ANCLE PART PART ON THE UNDER OF THE CONTROL OF THE		REAL Sec.	IMAGINARY	PEAK VAL	UENIA	
009937 20340 22257 1009340 19733 21663 2008560 17936 19913 3008766 06263 09275 6008766 1.00955 09275 6008766 1.00955 0505 6008767 1116 13430 6008767 1116 1443 10008768 1.10955 12272 10008768 1.1166 13620 10008768 1.1166 13620 10008768 1.1166 13620 10008768 1.1210 17211 1.17411 1000944028955 2225 12009560 1.24196 13620 12009560 9972 1.45000 1.46800 10001775 1.5007 1007 10001775 1.5007 1007 10001775 1.5007 1007 100214000 1.24400	ANGLE	PART	PART	(NORM)	en o sogniogo Da el iterat
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	** VE	LOCITY POTENT	IALS DUE TO SC	ATTERED FIELD **	

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