SCATTERING OF ACOUSTIC WAVES BY CYLINDERS WITH ARBITRARY CROSS SECTIONS T-MATRIX FORMULATION

by

BORA GÜNGÖR

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APPROVED BY

Doç. Dr. Ahmet Ceranoğlu
(Thesis Supervisor) ______________

Prof. Dr. Akın Tezel ______________

Prof.Dr. Attila Aşkar ______________

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ABSTRACT

In this work, the scattering of steady-state acoustic waves from arbitrarily shaped obstacles in an infinite medium is studied using the T-matrix method. The problem is examined for the two dimensional case where the obstacle is a cylindrical rigid inclusion or a cavity. An acoustic plane wave is considered to be incident on the obstacle.

In the solution of the problem, both incident and scattered wave fields are expanded in series of the circular basis wave functions. The scattered wave field is then evaluated through a transition matrix (T-matrix) which relates the unknown coefficients of the scattered wave series to the coefficients of the incident wave.

Numerical results pertaining to circular, elliptical, rectangular and triangular cross sections are obtained. The results are presented in graphical form and found to be in good agreement compared with the some known exact or approximate solutions available in the literature.

AKUSTİK DALGALARIN RASTGELE KESİTLİ SİLİNDİRLERDEN SAÇILIMI T-MATRİS FORMULASYONU

KISA ÖZET

Bu çalışmada, akustik dalgaların sonsuz bir ortamdaki rastgele şekilli engellerden saçılımı, T-matris metodu kullanılarak ele alınmıştır. Problem, iki boyutta, silindirik rijit cisimler ve boşluklar için incelenmiştir. Engel üzerine gelen dalganın akustik bir düzlem dalga olduğu düşünülmüştür.

Problemin çözümünde, hem gelen hem de saçılan dalga alanları, dairesel temel dalga fonksiyonları cinsinden seri olarak açılmaktadır. Saçılan dalga alanı, saçılan dalga serisinin katsayılarını gelen dalga serisinin katsayılarına bağlayan bir geçiş matrisi (T-matris) vasıtasıyla hesaplanmaktadır.

Dairesel, eliptik, dikdörtgen ve üçgen kesitli silindirik şekiller için nümerik sonuçlar elde edilmiştir. Sonuçlar, grafikler halinde gösterilmiş ve literatürdeki bazı bilinen kesin veya yaklaşık çözümlerle uyuştuğu görülmüştür.

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LIST OF SYMBOLS

A	Constant amplitude factor of the incident wave
a_m^σ, a_m	Coefficients of the series representing the incident wave field
$\underset{\sim}{a}$	Coefficient vector for the incident wave with elements a_m
a,b	Half major and minor axes for elliptical and rectangular geometries
B	Bulk modulus of the medium
c	Wave speed
C_s	A constant multiplication factor associated with the Simpson's integration
c_m^σ, c_m	Coefficients of the series representing the scattered wave field
$\underset{\sim}{c}$	Coefficient vector for the scattered wave with elements c_m
$\underline{e}_1$, $\underline{e}_2$	Unit vectors along the x and y directions of the Cartesian coordinate system
$\underline{e}_r$, $\underline{e}_\theta$	Unit radial and tangential vectors in polar coordinate system
f	Farfield amplitude of the scattered wave field
h	Height of an isosceles triangle along its symmetry axis
H_n	n-th order cylindrical Hankel function of the first kind
J_n	n-th order cylindrical Bessel function of the first kind
k, $\underline{k}$	Wave number and wave normal vector, respectively

$\hat{n}$	Unit outward normal vector along the boundary of the scatterer
$\partial/\partial n$	Directional derivative along the unit normal $\hat{n}$
N	Number of the boundary segments used in the numerical evaluation of the boundary integrals
p	Pressure
P	Order of the Simpson's rule used in the numerical integrations
$\underset{\approx}{Q}$	A matrix with elements involving integrals over the boundary of the scatterer
$\underset{\approx}{\hat{Q}}$	Real part of the $\underset{\approx}{Q}$-matrix
$\underset{\approx}{Q}^{11}$, $\underset{\approx}{Q}^{12}$, $\underset{\approx}{Q}^{21}$, $\underset{\approx}{Q}^{22}$	Submatrices of the $\underset{\approx}{Q}$-matrix
$Q_{jm}^{\sigma\nu}$, Q_{jm}	Elements of the $\underset{\approx}{Q}$-matrix
(r,θ)	Coordinates of a point in circular polar coordinate system
$\underline{r}$	Position vector in polar coordinate system
r_{iq}, θ_{iq}, $q=0,..,P$	Coordinates of the integration points on the i-th boundary segment
r_c	Corner radius of a round cornered rectangle
S	Boundary of the cross section of an infinite cylinder
S_+, S_∞	Two arbitrary circular curves enclosing S
r_+, r_∞	Radii of the curves S_+, S_∞
ΔS_i, $i = 1,\ldots,N$	i-th boundary segment along S
$\underset{\approx}{T}$	Transition matrix (T-matrix)
$\underset{\approx}{T}^{11}$, $\underset{\approx}{T}^{12}$, $\underset{\approx}{T}^{21}$, $\underset{\approx}{T}^{22}$	Submatrices of the T-matrix
u^i, u^s, u	Incident, scattered and total wave fields outside the scatterer, respectively
u^+	Total wave field at the boundary of the scatterer
$\underline{v}$	Velocity field

w_q, $q = 0,\ldots,P$	Weighing factors associated with the Simpson's formula
(x,y,z)	Coordinates of a point in Cartesian coordinate system
Y_n	n-th order cylindrical Bessel function of the second kind
α	Incidence angle of a plane wave
α_m^σ, α_m	Coefficients of the series representing the wave field at the surface of the scatterer
$\underset{\sim}{\alpha}$	Coefficient vector for the surface wave field with elements α_m
β	Tip angle for the isosceles triangular cross section
δ_{mn}	Kronecker delta
ε_n	Neumann factor
$\Delta\theta_i$	i-th angular interval associated with S_i
θ_{a_i}, θ_{b_i}	Lower and upper angles of the i-th angular interval
λ	Wavelength
ρ	Density distribution in the medium
ρ_o	Equilibrium density of the medium
σ^{tot}	Total scattering cross section
ϕ	Velocity potential function
ψ	Spatial part of the velocity potential function
ψ_n^σ, ψ_n, $\hat{\psi}_n^\sigma$, $\hat{\psi}_n$	Basis functions for two-dimensional scalar waves
τ_n^σ	Angular part of the basis functions
ω	Angular frequency

I. INTRODUCTION

When a wave propagating in an unbounded homogeneous medium encounters an obstacle immersed in the medium, its propagation path changes and while a portion of it is reflected back into the medium as a secondary wave emitted by the obstacle the other part of it, if the obstacle is not a cavity, is refracted into the body of the obstacle. The radiation of these secondary waves from the obstacle is called scattering. The obstacle may be a cavity or an inclusion with physical properties differing from those of the surrounding medium.

Scattering and diffraction problems have become increasingly important in the recent years, particularly in the areas of remote sensing, seismic exploration, oil technology, underwater sound detection and especially in non-destructive testing of materials where the scattered wave form is used to identify the shapes and the sizes of the material defects such as voids, cracks or inclusions.

In solving the scattering problems, especially where explicit numerical results are desired, four methods are extensively employed in the literature. These are method of seperation-of-variables, variational method, integral equation method and the transition matrix (T-matrix) method which has been recently developed [1].

The method of seperation of variables is usually employed in finding exact analytical solutions for only a class of objects bounded by quadric surfaces, [2-5]. Hence, the method restricts the shape of the scatterer to simple geometries, like sphere, circular and elliptic cylinders, such that their boundary geometries can be expressed conveniently in seperable coordinates. The variational method is another method used in the scattering problem solutions and, from the theoretical point of view, it is applicable for arbitrary boundary geometries, [3,6,7]. However, this method especially for general geometries, requires the evaluation of repeated surface or volume integrals with singular kernels. Thus this method is also restricted to relatively simple geometries. The third method, namely the integral equation method, consists of approximating an integral over the surface of the scatterer by a finite sum and then computing the quantities like displacements, velocity potentials at many discrete points by solving the resulting system of equations numerically, [8]. In previous years, several applications of this approach have appeared in the literature [6,9-11].

The latest method developed for the solution of the wave scattering problems is the T-matrix method which was first introduced by Waterman [1] for acoustic waves and reformulated by Pao [12] for elastic waves. The method starts directly with the Helmholtz integral formula, and uses either cylindrical or spherical wave functions for bodies of arbitrary shape. Both incident and scattered waves are represented as a series of the common wave functions, known as the basis functions. The unknown coefficients of the scattered wave series are then related by a transition matrix (T-matrix) to the coefficients of the incident wave, [1,12]. The elements of the T-matrix are integrals of basis functions and their normal derivatives over the bounding surface of the scatterer, which can be evaluated numerically even for bodies of complex geometry.

The key feature of the method lies in the fact that the T-matrix is fixed for a specific boundary type and geometry and wave number of the incident wave. Hence, once it is created, the scattered field quantities at various regions of the medium for different incidence angles of the impinging wave can be calculated. The method, comparing with the variational and integral equation techniques, has also a computational advantage because the integrals involved in the formulation are only single surface integrals with no singularities in their integrands. In the recent years, several applications of T-matrix approach to the scattering of acoustic and elastic waves by finite elliptic cylinders [13,14], spheroids [14], finite circular cylinders [15], infinite strips [1,16] have verified the power of the method.

In this work, we have considered the scattering of the plane acoustic waves by infinite cylinders. The near and far field results for cavity and rigid inclusion cases are presented. A brief review of the governing equations for acoustic fields is given in Chapter 2. In Chapter 3, formulation of the transition matrix for the scattering problem is given. The numerical methods employed in the solutions are discussed in Chapter 4 where solutions to specific problems are also given. Results pertaining to circular, elliptical, rectangular and triangular geometries are presented in polar graphical form.

II. THE THEORY OF ACOUSTIC WAVES

In this section, the reduction of the general equations of hydrodynamics to the Helmholtz reduced wave equation describing the motion of the harmonic disturbances in a fluid is discussed briefly. Also, a general solution of the Helmholtz equation is given for polar coordinates.

2.1 EQUATIONS OF HYDRODYNAMICS

To study the wave propagation in a fluid medium, the starting equations are the hydrodynamical equations of motion due to Euler [17]:

$$\rho\left[\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right] = -\nabla p \quad , \qquad (2.1)$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\underline{v}) = 0 \quad , \qquad (2.2)$$

$$p = p(\rho) \quad , \qquad (2.3)$$

where ρ is the density, p is the pressure and $\underline{v}$ is the fluid velocity vector at any point. This set of equations, which are valid for an ideal fluid, is complete and consists of the equation of motion (2.1), the equation of continuity (2.2), and the equation of state (2.3).

Whenever a disturbance is created at any point inside the fluid it will propagate throughout the medium. In order to obtain the governing equations of motion regarding the propagation of this disturbance, one assumes that the relative perturbations from the initial equilibrium state are small, that is,

$$\frac{\rho'}{\rho_o} = \frac{\rho - \rho_o}{\rho_o} \sim \mu \quad , \quad \frac{p'}{p_o} = \frac{p - p_o}{p_o} \sim \mu \quad , \tag{2.4}$$

where ρ_o and p_o are the equilibrium density and pressure, respectively, while μ is some small parameter denoting the variations from the equilibrium values. Under these conditions, the fluid velocity $\underline{v}$ with which the fluid particles oscillate is a small quantity of the order of μ relative to the propagation speed of the disturbance. This is the case, actually, in acoustics. Since, acoustical wave lengths are long, the variations in the velocity field, $\underline{v}$, are very small quantities. Thus, neglecting the term $\underline{v}\cdot\nabla\underline{v}$ in Eq. (2.1), we get

$$\rho \frac{\partial v}{\partial t} = -\nabla p \quad . \tag{2.5}$$

Substitution of the expressions $\rho = \rho_o + \rho'$ and $p = p_o + p'$ into the Eqs. (2.5), (2.2) and (2.3) yields

$$\rho_o \frac{\partial v}{\partial t} = -\nabla p' \quad , \tag{2.6}$$

$$\frac{\partial \rho'}{\partial t} + \rho_o \nabla\cdot\underline{v} = 0 \quad , \tag{2.7}$$

$$p' = \frac{dp}{d\rho}\rho' \quad . \tag{2.8}$$

where we have neglected the terms involving the products of the perturbed quantities. For the reduction of this set to a single equation, it is convenient to introduce a scalar function ϕ such that

$$\underline{v} = \nabla\phi \quad . \tag{2.9}$$

The function ϕ in the above equation is called the velocity potential. Then, Eqs. (2.6) and (2.7) yields

$$p' = -\rho_o \frac{\partial\phi}{\partial t} \quad , \tag{2.10}$$

$$\frac{\partial\rho'}{\partial t} + \rho_o\nabla^2\phi = 0 \tag{2.11}$$

Eliminationg the variables p' and ρ' from the above equations, we obtain the equation of motion of the disturbance,

$$\frac{\partial^2\phi}{\partial t^2} - c^2\nabla^2\phi = 0 \quad . \tag{2.12}$$

This is a scalar wave equation describing the motion of a disturbance in an acoustic field where the propagation speed of the disturbance is c,

$$c = \sqrt{dp/d\rho} \quad , \tag{2.13}$$

or, in general c can also be written in the form

$$c = \sqrt{B/\rho_o} \quad , \tag{2.14}$$

where B is the bulk modulus of the fluid.

2.2 HARMONIC WAVE MOTION IN ACOUSTIC FIELDS

Considering a disturbance which is harmonic in time and has a circular frequency of ω, one can write the velocity potential as

$$\phi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t} \quad , \tag{2.15}$$

where ψ is the spatial part of the velocity potential function, ϕ. Substituting Eq. (2.15) into Eq. (2.12) and rearranging the terms, we get

$$\nabla^2\psi + k^2\psi = 0 \quad , \tag{2.16}$$

where $k = \omega/c$ is the wave number. Eq. (2.16) is known as the Helmholtz reduced wave equation and describes the motion of the acoustic harmonic waves in a fluid medium.

It should also be noted that use of the velocity potential formulation is indeed a convenient way for acoustic wave propagation. However, ϕ is not a measurable quantity like velocity or pressure but it is possible to obtain such quantities from it. For example, substitution of Eq. (2.15) into (2.10) yields

$$p = p_o + i\omega\rho_o\phi \quad . \tag{2.17}$$

2.3 SOLUTION OF THE HELMHOLTZ EQUATION

Since throughout this work scattering of the plane waves by infinite cylinders with constant cross-sections will be studied and the wave normals of the incident waves which will be considered are perpendicular to the axes of the cylinders, the dimensionality of the problem reduces to two and it is convenient to obtain the solution of the Helmholtz equation in polar coordinates.

2.3.1 Acoustic Plane Wave

A harmonic plane wave of magnitude A propagating in an acoustic field can be expressed as

$$\phi(r,\theta,t) = Ae^{i(\underline{k}\cdot\underline{r}-\omega t)} \quad . \tag{2.18}$$

Note that we can write the vectors $\underline{r}$ and $\underline{k}$ as

$$\underline{r} = r(\cos\theta\ \underline{e}_1 + \sin\theta\ \underline{e}_2)\ ; \quad r = |\underline{r}| \quad , \tag{2.19}$$

$$\underline{k} = k(\cos\alpha\ \underline{e}_1 + \sin\alpha\ \underline{e}_2)\ ; \quad k = |\underline{k}| = \omega/c \quad , \tag{2.20}$$

respectively. In this case, Eq. (2.18) can be written as

$$\phi = Ae^{ikr\cos(\theta-\alpha)}e^{-i\omega t} \quad , \tag{2.21}$$

where θ and α are the angles that the vectors $\underline{r}$ and $\underline{k}$ make with the x-axis and $\underline{e}_1$, $\underline{e}_2$ are the unit vectors along the x and y directions of the Cartesian coordinate system (Fig. 2.1).

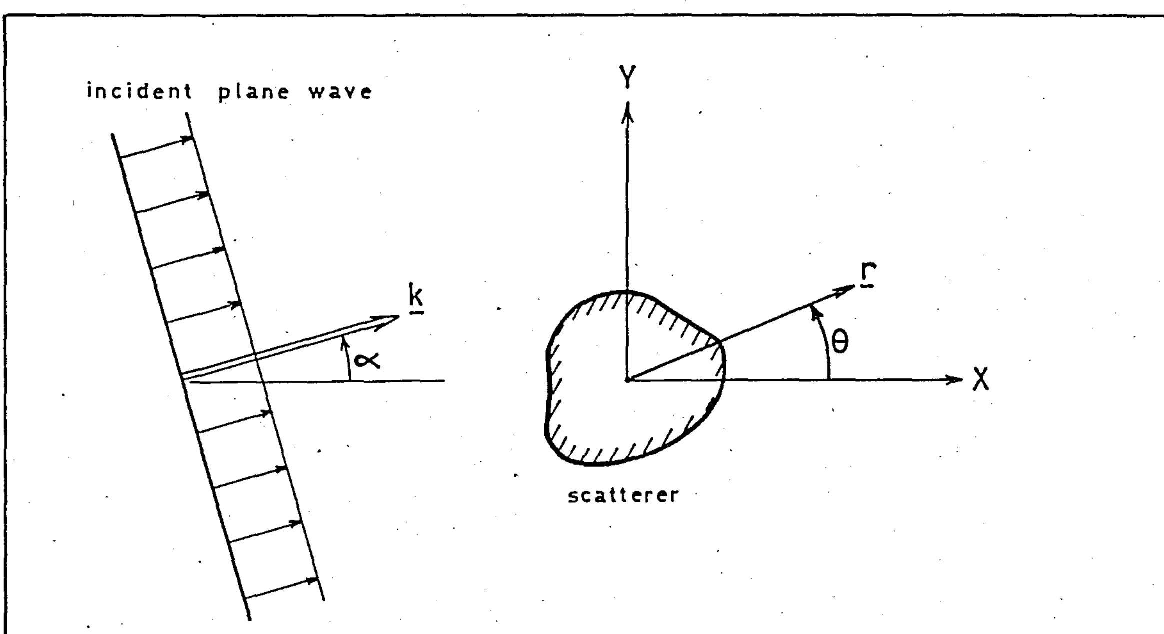


Figure 2.1 - Representation of a plane wave incidence.

Comparing Eq. (2.21) with Eq. (2.15) one can write the spatial part of ϕ as

$$\psi(r,\theta) = Ae^{ikr\cos(\theta-\alpha)} \quad . \tag{2.22}$$

Note that ψ given by Eq. (2.22) does satisfy the Helmholtz equation, Eq. (2.16).

2.3.2 General Solution

In polar coordinates, the Laplacian operator, ∇^2, is given as

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \quad , \tag{2.23}$$

thus, Eq. (2.16) takes the form

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\psi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + k^2\psi = 0 \quad . \tag{2.24}$$

The most common method used in obtaining the general solution of Eq. (2.24) is the method of separation-of-variables where a solution of the form

$$\psi(r,\theta) = R(r)T(\theta) \tag{2.25}$$

is assumed. Upon substitution of the above solution into Eq. (2.24) one can show that the latter equation reduces to two separate equations of the form

$$r^2\frac{d^2R}{dr^2} + r\frac{dR}{dr} + (k^2r^2 - n^2)R = 0 \quad , \tag{2.26}$$

$$\frac{d^2T}{d\theta^2} + n^2T = 0 \quad . \tag{2.27}$$

Eq. (2.26) is the well-known Bessel's Differential Equation and has its solution either as

$$R = A_nJ_n(kr) + B_nY_n(kr) \quad , \tag{2.28}$$

or,

$$R = A_nH_n^{(1)}(kr) + B_nH_n^{(2)}(kr) \ . \tag{2.29}$$

$J_n(kr)$ and $Y_n(kr)$ are known as the n-th order cylindrical Bessel functions of the first and second kind while $H_n^{(1)}(kr)$ and $H_n^{(2)}(kr)$ are the n-th order Hankel functions of the first and second kind, respectively. The unknown constants A_n and B_n are to be determined from the boundary conditions impending on the problem.

The solution of Eq. (2.27) can be written as

$$T = e^{\pm in\theta} = \binom{\cos(n\theta)}{\sin(n\theta)} \quad . \tag{2.30}$$

Hence, combining the solutions (2.28), (2.29), (2.30) and the time factor exp[$-i\omega t$], one obtains the general solution as

$$\phi(r,\theta,t) = [A_nI_n^{(1)}(kr) + B_nI_n^{(2)}(kr)]\binom{\cos(n\theta)}{\sin(n\theta)}e^{-i\omega t} \quad , \tag{2.31}$$

where $I_n^{(1)}$ and $I_n^{(2)}$ are the cylindrical Bessel or Hankel functions of the first and second kind, depending on the physics of the problem.

If a wave progressing through a medium encounters an obstacle immersed in the medium, scattering phenomenon takes place and the wave field becomes different from what it would have been in the absence of the obstacle. In general, the solution to the scattering problem of an acoustic wave propagation in an infinite fluid medium, requires the solution of the Helmholtz equation satisfying the boundary conditions prescribed over a discontinuity surface, called scatterer. In order to solve this problem, the usual method of attack is to try to satisfy the boundary conditions impending on the problem so as to obtain an exact solution directly from the general solution (Eq. 2.31) obtained by the method of separation-of-variables. However, this type of approach is successful only if the geometry of the curve S representing the boundary of the scatterer is such that S coincides with an orthogonal curvilinear coordinate system in which the Helmholtz equation seperates. Hence, the analytical solutions are possible only for simple geometries like circular and elliptic cylinders. Exact solutions are available for circular cylinders using circular polar coordinates and Bessel functions, and for elliptic cylinders using elliptic coordinates and Mathieu functions. Thus, for general boundary geometries one of the various approximation techniques must be employed in the solution of the scattering problems. The transition matrix formulation, shortly called T-matrix method, is the latest technique developed for this purpose.

III. T-MATRIX FORMULATION

In the wave scattering problems, as stated in the previous section, the complexity of the boundary geometry of the scatterer necessitates the use of a numerical method to obtain an approximate solution. In the evaluation of the scattered wave field, especially at the far field, the T-matrix method is highly efficient and powerful computational procedure. It uses, depending on the dimensionality of the problem, only cylindrical or spherical wave functions even for bodies of arbitrary shape.

In the formulation, both incident and scattered waves are expanded in series of the common wave functions, known as the basis functions, satisfying the Helmholtz equation (Eq. 2.16). The unknown coefficients of the scattered wave series are then related to the coefficients of the incident wave by a transition matrix, called T-matrix.

3.1 BASIS WAVE FUNCTIONS

A class of solutions for Helmholtz reduced wave equation is the wave functions in circular polar coordinates,

$$\psi_n^{\sigma}(r,\theta) = (\varepsilon_n)^{\frac{1}{2}} H_n^{(1)}(kr)\tau_n^{\sigma}(\theta) \ ; \qquad n = 0,1,\ldots,\infty \qquad ,(3.1)$$

where $H_n^{(1)}(kr)$ is the cylindrical Hankel function of the first kind, τ_n^σ is given by

$$\tau_n^\sigma(\theta) = \begin{cases} \cos(n\theta) & \text{for } \sigma = 1 \\ \sin(n\theta) & \text{for } \sigma = 2 \end{cases} \quad , \tag{3.2}$$

and ε_n is the Neumann factor,

$$\varepsilon_n = \begin{cases} 1 & \text{for } n = 0 \\ 2 & \text{for } n > 0 \end{cases} \quad . \tag{3.3}$$

Equation (3.1) can be written from the general solution (2.31) by simply setting, $I_n^{(1)}(kr) \to H_n^{(1)}(kr)$, $B_n \to 0$.

Assuming a time dependency of the form

$$\phi(r,\theta,t) = \psi_n^\sigma \, e^{-i\omega t} \quad , \tag{3.4}$$

ϕ represents an outgoing cylindrical wave with respect to the origin of a coordinate system located as shown in Fig. 3.1. Using the function $H_n^{(2)}(kr)$ in Eq. (3.4) would give the functional form of an incoming wave. However, if $H_n^{(1)}(kr)$ is replaced by $J_n(kr)$, the cylindrical Bessel function of the first kind, we get the expression for a standing wave,

$$\hat{\psi}_n^\sigma(r,\theta) = (\varepsilon_n)^{\frac{1}{2}} J_n(kr)\tau_n^\sigma(\theta); \qquad n = 0,1,\ldots,\infty \quad . \tag{3.5}$$

One should also note that $\hat{\psi}_n^\sigma$ is the regular part of ψ_n^σ given by Eq. (3.1), and it corresponds to the real part of the latter when kr is real. The functions ψ_n^σ and $\hat{\psi}_n^\sigma$ are called the basis wave functions for two dimensional scalar waves.

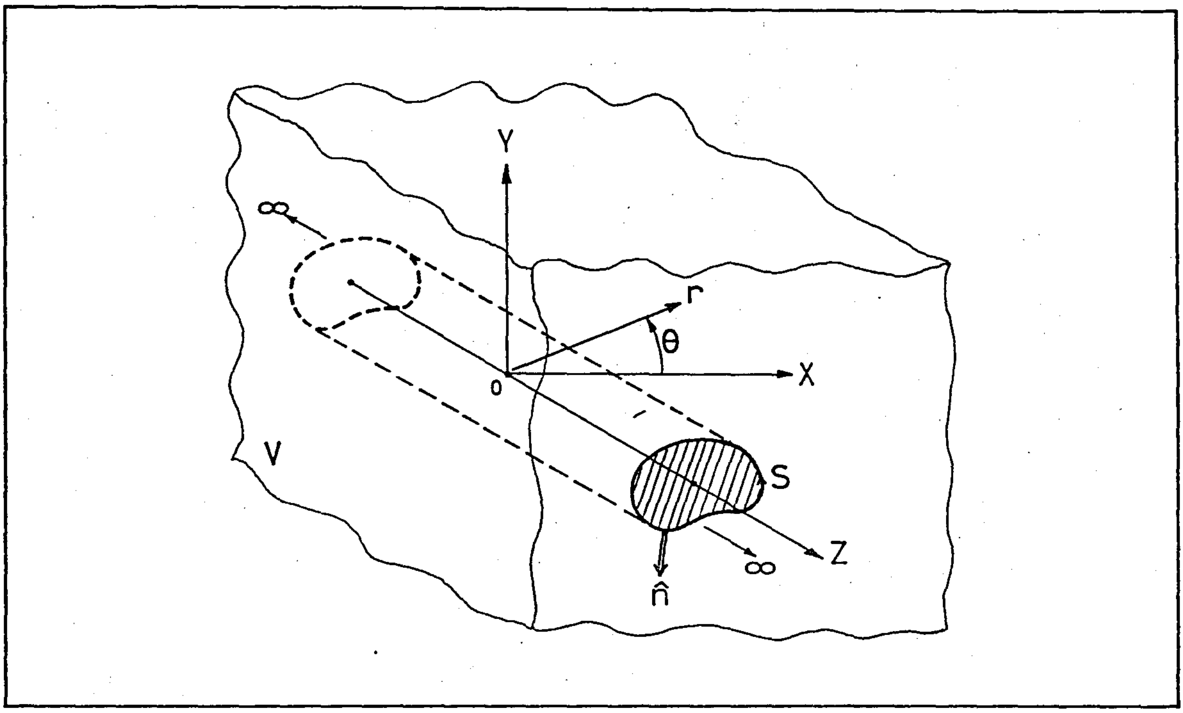


Figure 3.1 - Geometry of an infinite cylindrical obstacle.

Recalling the orthogonality relations for the trigonometric functions,

$$\int_0^{2\pi} \cos(m\theta)\cos(n\theta)d\theta = \frac{2\pi}{(\varepsilon_m\varepsilon_n)^{\frac{1}{2}}}\,\delta_{mn} \quad , \qquad (3.6)$$

$$\int_0^{2\pi} \sin(m\theta)\sin(n\theta)d\theta = \frac{2\pi}{(\varepsilon_m\varepsilon_n)^{\frac{1}{2}}}\,\delta_{mn} \quad , \qquad (3.7)$$

$$\int_0^{2\pi} \cos(m\theta)\sin(n\theta)d\theta = 0 \quad , \qquad (3.8)$$

where ε_m, ε_n are the Neumann factors and δ_{mn} is the Kronecker delta, one can write orthonogonality relation for $\tau_n^\sigma(\theta)$ over a closed curve as

$$(\varepsilon_m \varepsilon_n)^{\frac{1}{2}} \int_0^{2\pi} \tau_m^\sigma \tau_n^\nu \, d\theta = 2\pi\delta_{mn} \qquad \text{for} \quad \sigma = \nu \quad . \tag{3.9}$$

Since, only the Hankel functions of the first kind will be utilized in the rest of the work, to simplify the writing the superscript (1) appearing on $H_n(kr)$ will be omitted.

3.2 ORTHOGONALITY OF THE BASIS FUNCTIONS

From the divergence theorem, it can be shown that two arbitrary scalar functions $u(x,y,t)$ and $v(x,y,t)$ satisfy the Green's second identity [18],

$$\int_C \left(u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n}\right) dS = \iint_{A_s} (u\nabla^2 v - v\nabla^2 u)\, dA \quad . \tag{3.10}$$

The u and v and their first and second derivatives are continuous in a two-dimensional region of area A_s bounded by the closed curve C. The $\partial u/\partial n = \hat{n} \cdot \nabla u$ is the directional derivative along a unit normal $\hat{n}$ which is pointed outward from the curve C. So far no restriction has been imposed on u and v except the conditions of continuity.

Let u or v represent a scalar wave, satisfying the Helmholtz equation written in the form

$$(\nabla^2 + k_1^2)u = 0 \quad , \qquad (\nabla^2 + k_2^2)v = 0 \tag{3.11}$$

A time factor $\exp(-i\omega t)$ is assumed for the wave. When both u and v have the same wave number, i.e., $k_1 = k_2 = k = \omega/c$, the area integral on the right hand side of Eq. (3.10) vanishes identically and we have

$$\int_C (u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n}) dS = 0 \quad . \tag{3.12}$$

This result is valid for any two functions u and v, so long as they satisfy the same wave equation, and the aforementioned conditions of continuity. For acoustic waves, u and v are the usual velocity potentials.

For the problem of scattering by an infinite cylindrical boundary, consider the geometry shown in Fig. 3.2, where the curve S represents the boundary of the scatterer while S_+ and S_∞ are somewhat arbitrary circular curves outside the scatterer. By utilizing Eqs. (3.9) and (3.12), the following orthonogonality conditions can be established for the basis wave functions over the circle S_+:

$$\int_{S_+} (\hat{\psi}_p \frac{\partial \hat{\psi}_q}{\partial n} - \hat{\psi}_q \frac{\partial \hat{\psi}_p}{\partial n}) dS = 0 \quad , \tag{3.13}$$

$$\int_{S_+} (\psi_p \frac{\partial \psi_q}{\partial n} - \psi_q \frac{\partial \psi_p}{\partial n}) dS = 0 \quad , \tag{3.14}$$

$$\int_{S_+} (\psi_p \frac{\partial \hat{\psi}_q}{\partial n} - \hat{\psi}_q \frac{\partial \psi_p}{\partial n}) dS = (-4i)\delta_{pq} \quad . \tag{3.15}$$

For simplicity in writing, only the subscript which indicates the order of the Bessel functions is retained, that is,

$$\psi_p \equiv \psi_p^\sigma \quad , \quad \psi_q \equiv \psi_q^\nu \quad , \quad \text{etc.} \tag{3.16}$$

The proof of the first condition is rather simple. Let the region A_S in Eq. (3.10) be bounded externally by the circle S_+ of radius $r = r_+$

as shown in Fig. 3.2. Note that both $\hat{\psi}_p$ and $\hat{\psi}_q$ and their normal derivatives are continuous within S_+, that is, $J_{p,q}(kr)$ have no singularity inside S_+, and furthermore they satisfy the wave equation (3.11) with $k_1 = k_2 = k$. Thus, Eq. (3.13) is a special case of (3.12) with $u = \hat{\psi}_p$ and $v = \hat{\psi}_q$.

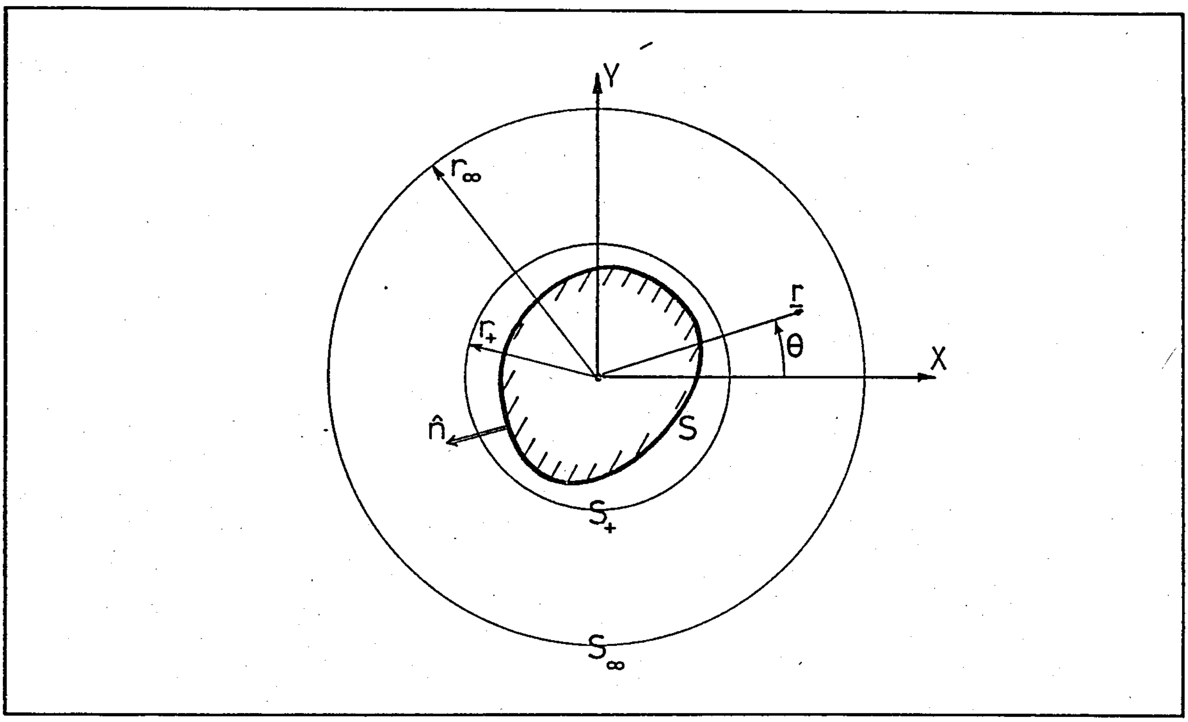


Figure 3.2 - Geometry for a constant cross-sectional scatterer.

The same proof, however, cannot be applied to the functions ψ_p and ψ_q because the origin r = 0 is a singularity point for $H_{p,q}(kr)$. For the proof of Eq. (3.14), we consider a region bounded internally by S_+ $(r = r_+)$ and externally by $S_\infty(r = r_\infty)$, (Fig. 3.2). Within this region, both $\psi_{p,q}$ and $\partial\psi_{p,q}/\partial n$ are regular, and Eq. (3.12) reduces to

$$-\int_{S_+} \left(\psi_p \frac{\partial\psi_p}{\partial n} - \psi_q \frac{\partial\psi_p}{\partial n}\right) ds + \int_{S_\infty} \left(\psi_p \frac{\partial\psi_q}{\partial n} - \psi_q \frac{\partial\psi_p}{\partial n}\right) ds = 0, \quad (3.17)$$

where we have replaced u and v by ψ_p and ψ_q, respectively. Note that, while the outer normal along S_+ is in the negative direction of the unit radial vector $\underline{e}_r$, it is in the positive direction along S_∞. The negative sign in front of the integral along S_+ is introduced because the normal derivative $\partial/\partial n$ is taken to be in the direction of $+\underline{e}_r$ along both curves.

It is seen immediately that both integrals in the above equation vanish identically when p = q. When ψ_p differs from ψ_q, it can be shown that each integral vanishes due to the orthogonality condition given by Eq. (3.9). To show this, note that $\partial/\partial n = \partial/\partial r$ and $ds = r d\theta$ along the circular curves S_+ and S_∞, and

$$\frac{\partial\psi_p}{\partial r} = (\varepsilon_p)^{\frac{1}{2}}\tau_p^\sigma(\theta)\,\frac{\partial H_p(kr)}{\partial r} = (\varepsilon_p)^{\frac{1}{2}}\tau_p^\sigma k H_p'(kr) \quad , \qquad (3.18)$$

$$\frac{\partial\psi_q}{\partial r} = (\varepsilon_q)^{\frac{1}{2}}\tau_q^\nu(\theta)\,\frac{\partial H_q(kr)}{\partial r} = (\varepsilon_q)^{\frac{1}{2}}\tau_q^\nu k H_q'(kr) \quad , \qquad (3.19)$$

where primes denote derivatives of the Hankel functions with respect to their argument. Substitution of these into the integral along S_+ in Eq. (3.17) yields

$$\int_{S_+}(\ldots)ds = \int_{S_+}[(\varepsilon_p)^{\frac{1}{2}}\tau_p^\sigma(\varepsilon_q)^{\frac{1}{2}}H_p(kr)kH_q'(kr)$$

$$- (\varepsilon_q)^{\frac{1}{2}}\tau_q^\nu(\varepsilon_p)^{\frac{1}{2}}\tau_p^\sigma H_q(kr)kH_p'(kr)]ds \; . \qquad (3.20)$$

Since $r \neq r(\theta)$, one can write the above equation in the form

$$\int_{S_+} (\ldots)ds = kr[H_p(kr)H_q'(kr) - H_q(kr)H_p'(kr)][(\varepsilon_p\varepsilon_q)^{\frac{1}{2}} \int_0^{2\pi} \tau_p^\sigma \tau_q^\nu d\theta]. \quad (3.21)$$

One can see that the expression in the second bracket is the same as that given by Eq. (3.9). Thus the integral over S_+ vanishes if $p \neq q$. Going through a very similar procedure, one can show that the integral over S_∞ also vanishes for $p \neq q$. This completes the proof of Eq. (3.14). From this proof it is seen that the circle S_+ can be replaced by any other circular curve, not necessarily centered at the origin of the coordinate system.

To prove Eq. (3.15), let the region of interest be the same region as in the preceeding case and let $u = \psi_p$ and $v = \hat{\psi}_q$ in Eq. (3.12). Since they are continuous within that region, we can write

$$\int_{S_+} (\psi_p \frac{\partial \hat{\psi}_q}{\partial n} - \hat{\psi}_q \frac{\partial \psi_p}{\partial n})ds = \int_{S_\infty} (\psi_p \frac{\partial \hat{\psi}_q}{\partial n} - \hat{\psi}_q \frac{\partial \psi_p}{\partial n})ds . \quad (3.22)$$

Again, if $p \neq q$, one can show by giving a similar proof as for Eq. (3.17) that the integrals in the above expression vanish. However, they do not, when $p = q$.

To evaluate the integral on the right-hand side, let S_∞ recede to infinity. The asymptotical expressions for ψ_p and $\hat{\psi}_q$ as $r \to \infty$ are [2]

$$\psi_p = (\varepsilon_p)^{\frac{1}{2}} H_p(kr)\tau_p^\sigma \overset{\infty}{\to} (\varepsilon_p)^{\frac{1}{2}}\tau_p^\sigma \sqrt{2/\pi kr}\, e^{i(kr-\pi_p)} , \quad (3.23)$$

$$\hat{\psi}_q = (\varepsilon_q)^{\frac{1}{2}} J_q(kr)\tau_q^\nu \overset{\infty}{\to} (\varepsilon_q)^{\frac{1}{2}}\tau_q^\nu \sqrt{2/\pi kr}\, \cos(kr-\pi_q) , \quad (3.24)$$

where $\pi_p = (2p + 1)\pi/4$. Recalling that, [2],

$$\frac{\partial H_p(kr)}{\partial r} = kH_p'(kr) = \frac{k}{2}[H_{p-1}(kr) - H_{p+1}(kr)] \quad , \qquad (3.25)$$

$$\frac{\partial J_q(kr)}{\partial r} = kJ_q'(kr) = \frac{k}{2}[J_{q-1}(kr) - J_{q+1}(kr)] \quad , \qquad (3.26)$$

and as $r \to \infty$,

$$H_{p\pm1}(kr) \to \pm i\sqrt{2/\pi kr}\; e^{i(kr-\pi_p)} \quad , \qquad (3.27)$$

$$J_{q\pm1}(kr) \to \pm\sqrt{2/\pi kr}\; \sin^{(kr-\pi_q)} \quad , \qquad (3.28)$$

the asymptotic expressions for $\partial\psi_p/\partial r$ and $\partial\hat{\psi}_q/\partial r$ can be written as

$$\frac{\partial\psi_p}{\partial r} \overset{\infty}{\to} ik\sqrt{2/\pi kr}\; e^{i(kr-\pi_p)}(\varepsilon_p)^{\frac{1}{2}}\tau_p^{\sigma} \quad ; \qquad (3.29)$$

$$\frac{\partial\psi_q}{\partial r} \overset{\infty}{\to} -k\sqrt{2/\pi kr}\; \sin(kr-\pi_q)(\varepsilon_q)^{\frac{1}{2}}\tau_q^{\nu} \quad . \qquad (3.30)$$

Substituting the expressions (3.23), (3.24), (3.29) and (3.30) into Eq. (3.22), one obtains

$$\int_{S_+} (\psi_p \frac{\partial\hat{\psi}_q}{\partial n} - \hat{\psi}_q \frac{\partial\psi_p}{\partial n})ds = \int_{S_\infty} (-\frac{2i}{\pi r})[(\varepsilon_p\varepsilon_q)^{\frac{1}{2}}\tau_p^{\sigma}\tau_q^{\nu}e^{i(kr-\pi_p)}]$$

$$\times\ [\cos(kr-\pi_q) - i\sin(kr-\pi_q)]ds$$

$$= \int_0^{2\pi} (-\frac{2i}{\pi r})[(\varepsilon_p\varepsilon_q)^{\frac{1}{2}}\tau_p^{\sigma}\tau_q^{\nu}e^{i(\pi_q-\pi_p)}]rd\theta$$

$$= [-\frac{2i}{\pi}][(\varepsilon_p\varepsilon_q)^{\frac{1}{2}}\int_0^{2\pi} \tau_p^{\sigma}\tau_q^{\nu}d\theta][e^{i(\pi_q-\pi_p)}]$$

$$= [-\frac{2i}{\pi}][2\pi\delta_{pq}][e^{i(\pi_q-\pi_p)}]$$

$$= -4i\delta_{pq} \quad . \tag{3.31}$$

This completes the proof of the third orthogonality condition (3.15).

3.3 WAVE FUNCTION EXPANSIONS

Consider a cylindrical inclusion of cross-sectional area A_c, bounded by a closed curve S, and let a plane wave $u^i(r,\theta)\exp(-i\omega t)$ be incident upon this inclusion, as shown in Fig. 3.3. Then, the total wave field $u(r,\theta)$ in the medium will be composed of two parts; the incident wave, u^i, and the scattered wave, u^s, i.e.,

$$u(r,\theta) = u^i(r,\theta) + u^s(r,\theta) \tag{3.32}$$

For notational convenience, the time factor $\exp(-i\omega t)$ will be supressed in the rest of the work.

Both incident and scattered waves can be expanded into a series of the basis functions in the form

$$u^i = A\ \Sigma a_m\hat{\psi}_m \quad , \quad u^s = A\ \Sigma c_m\psi_m \quad , \tag{3.33}$$

where A is the constant amplitude factor of the incident wave, a_m are the incident wave coefficients which can be determined uniquely for a given incident wave type, and c_m are the unknown coefficients of the scattered wave. The coefficients c_m, as will be shown, can be determined

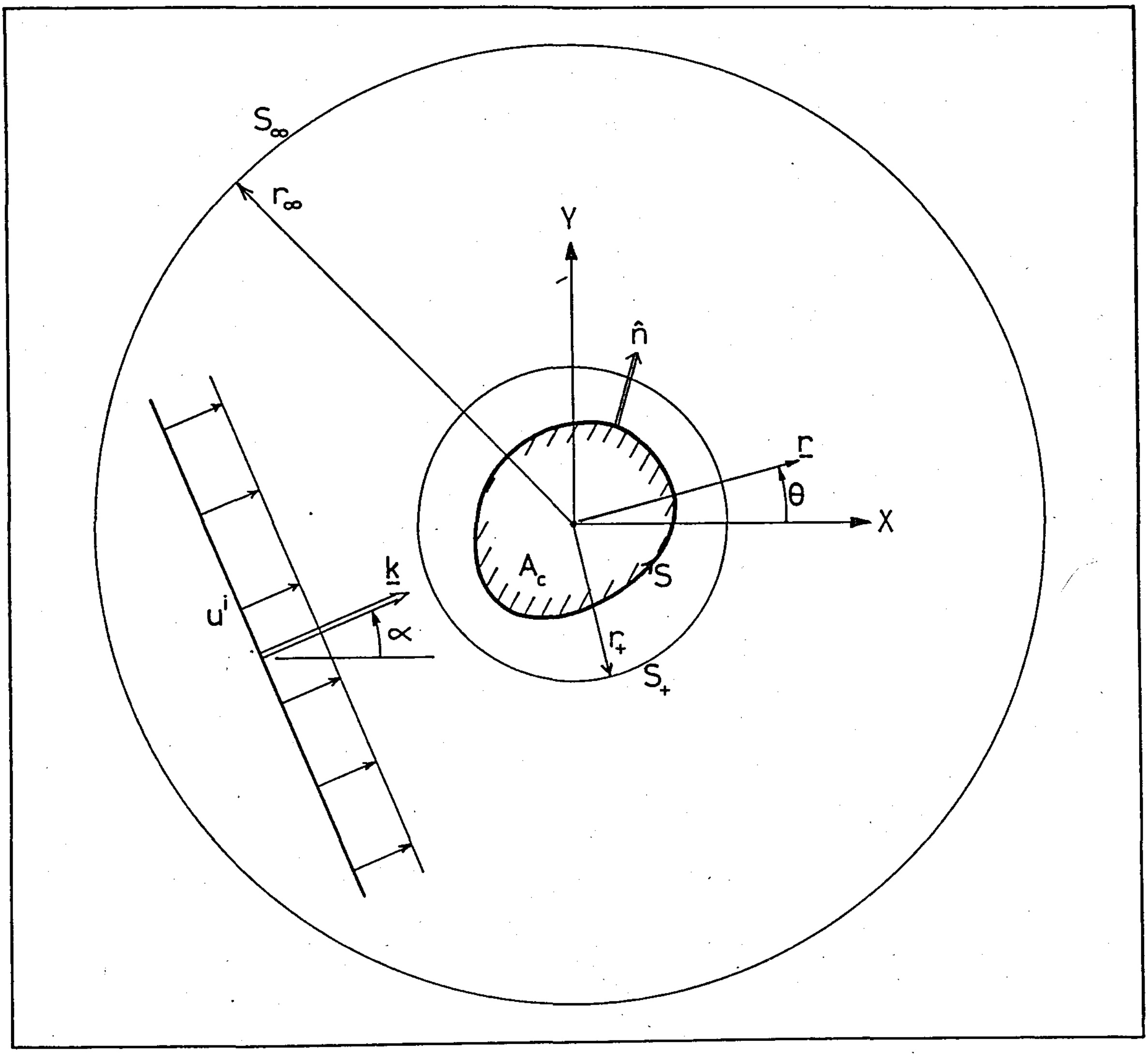


Figure 3.3 - Geometry for a plane wave incidence on a cylindrical scatterer.

by making use of a transition matrix (T-matrix) which relates to c_m to a_m by

$$c_m = \Sigma T_{mn} a_n \tag{3.34}$$

3.3.1 The Incident Wave Field

Any known incident wave can be expanded into a series of the basis functions $\hat{\psi}_m$ as defined by Eqs. (3.5) and (3.16), [12],

$$u^i(r,\theta) = A\, \Sigma a_m \hat{\psi}_m(r,\theta) \equiv A \sum_{m=o}^{\infty} \sum_{\sigma=1}^{2} a_m^{\sigma} \hat{\psi}_m^{\sigma} \quad , \quad r < r_\infty . \quad (3.35)$$

Since, $\hat{\psi}_m$ are regular at the origin ($r = 0$), taken to be inside the surface S, the series converges uniformly within a large circle S_∞ of radius, say, r_∞. For plane incident waves, S_∞ is at the infinity.

In Eq. (3.35) and in the following equations, for the simplicity in writing, the double sum will be represented by a single index and summation. Since the index m is an abbreviation for the two indices m and σ, the symbol Σ means summation on σ from 1 (even) to 2 (odd) and m from 0 to ∞.

Consider a plane incident wave with its wave normal perpendicular to the axis of the scatterer, as shown in Fig. 3.3. This wave can be represented by

$$u^i = Ae^{ikr\cos(\theta-\alpha)} \quad (3.36)$$

as defined in section (2.3.1), where the time factor has been omitted. If the plane wave expression given in Eq. (3.36) is expanded into a series as defined by Eq. (3.35) then the coefficients a_m^{σ} can be determined as explained below.

Trigonometric Fourier series expansion of an even function f(x), in the interval $-L \leq x \leq L$ is given by

$$f(x) = \sum_{n=o}^{\infty} \varepsilon_n \frac{a_n}{2} \cos \frac{n\pi x}{L} \quad , \tag{3.37}$$

where ε_n is the Neumann factor, and

$$a_n = \frac{1}{L} \int_{o}^{2L} f(x) \cos \frac{n\pi x}{L} \, dx \quad , \qquad n = 0,1,\ldots,\infty \tag{3.38}$$

If we now consider the function f(x) to be of the form exp(izcosβ) where we have replaced x by β. If this function is defined in the interval $-\pi \leq \beta \leq \pi$ then its Fourier series expansion is

$$e^{iz\cos\beta} = \sum_{n=o}^{\infty} \varepsilon_n \frac{a_n}{2} \cos(n\beta) \quad , \tag{3.39}$$

where

$$a_n = \frac{1}{\pi} \int_{o}^{2\pi} e^{iz\cos\beta} \cos(n\beta) d\beta \quad . \tag{3.40}$$

Recalling the integral representation of the Bessel functions [19],

$$2\pi i^n J_n(z) = \int_{o}^{2\pi} e^{iz\cos\beta} \cos(n\beta) d\beta \quad , \tag{3.41}$$

equation (3.39) can be written as

$$e^{iz\cos\beta} = \sum_{n=o}^{\infty} \varepsilon_n i^n J_n(z) \cos(n\beta) \quad . \tag{3.42}$$

In order to obtain the Fourier series expansion of the expression given by Eq. (3.36) one then has to replace z by kr and β by (θ-α) in Eq. (3.42). Doing so one gets

$$e^{ikr\cos(\theta-\alpha)} = \sum_{n=0}^{\infty} \varepsilon_n i^n J_n(kr)\cos[n(\theta-\alpha)]$$

$$= \sum_{n=0}^{\infty} \varepsilon_n i^n J_n(kr)[\cos(n\theta)\cos(n\alpha)+\sin(n\theta)\sin(n\alpha)]. \quad (3.43)$$

Rearranging Eq. (3.43) as

$$e^{ikr\cos(\theta-\alpha)} = \sum_{n=0}^{\infty} \sum_{\sigma=1}^{2} [(\varepsilon_n)^{\frac{1}{2}} J_n(kr) {\cos(n\theta) \atop \sin(n\theta)}][(\varepsilon_n)^{\frac{1}{2}} i^n {\cos(n\alpha) \atop \sin(n\alpha)}] \quad (3.44)$$

and then comparing with Eq. (3.35), the coefficients a_m^σ of the incident plane wave are found to be

$$a_m^\sigma = (\varepsilon_m)^{\frac{1}{2}} i^m \begin{matrix} \cos(m\alpha) \; ; \; \sigma = 1 \\ \sin(m\alpha) \; ; \; \sigma = 2 \end{matrix} \quad . \quad (3.45)$$

3.3.2 The Scattered Wave Field

The scattered wave field can be expanded into a series in terms of the functions ψ_m, i.e.,

$$u^s(r,\theta) = A\, \Sigma c_m \psi_m(r,\theta) \equiv A \sum_{m=0}^{\infty} \sum_{\sigma=1}^{2} c_m^\sigma \psi_m^\sigma \quad , \quad r \geq r_+ \; . \quad (3.46)$$

Note that ψ_m are regular outside the region enclosed by the curve S, the boundary of the inclusion. Thus, the above series converges uniformly outside and on a circle S_+, enclosing the inclusion (Fig. 3.3). The radius r_+ of S_+ is yet unspecified, and it can be ascertained when the unknown coefficients c_m are found.

To determine c_m, consider a region which is bounded internally by S, and externally by S_+. Furthermore, to apply Eq. (3.12) for that region, let v be $\hat{\psi}_m$, and u be the total wave field as defined in Eq. (3.32), that is

$$u(r,\theta) = u^i(r,\theta) + u^s(r,\theta) \quad ; \quad (r \text{ inside } S_+, \text{ outside } S). \quad (3.47)$$

At the boundary S one writes

$$u(r,\theta) = u^+ \quad , \frac{\partial u(r,\theta)}{\partial n} = \frac{\partial u^+}{\partial n} \quad ; \quad (r \text{ on } S) \quad . \qquad (3.48)$$

Both u_+ and $\partial u^+/\partial n$ are unknown quantities at the surface S, where (+) indicates that we approach S from the positive direction of $\hat{n}$.

Substituting Eqs. (3.35), (3.46) into (3.47) and evaluating it on S_+ we get

$$u(r,\theta) = A\ \Sigma a_j \hat{\psi}_j(r,\theta) + A\ \Sigma c_j \psi_j(r,\theta); \quad (r \text{ on } S_+) \quad . \qquad (3.49)$$

Since both series are uniformly convergent on the surface (circle) S_+, one can differentiate them term by term to obtain

$$\frac{\partial u(r,\theta)}{\partial n} = A\ \Sigma a_j \frac{\partial \hat{\psi}_j(r,\theta)}{\partial n} + A\ \Sigma c_j \frac{\partial \psi_j}{\partial n} \ ; \quad (r \text{ on } S_+) \quad . \qquad (3.50)$$

Substitution of the expressions (3.48), (3.49) and (3.50) into Eq. (3.12) where the contour C is made out of two curves, S and S_+, gives

$$\int_S (u^+ \frac{\partial \hat{\psi}_m}{\partial n} - \hat{\psi}_m \frac{\partial u^+}{\partial n})ds - \int_{S_+} A[(\sum_j a_j \hat{\psi}_j + \sum_j c_j \psi_j) \frac{\partial \hat{\psi}_m}{\partial n}$$

$$- \hat{\psi}_m(\sum_j a_j \frac{\partial \hat{\psi}_j}{\partial n} + \sum_j c_j \frac{\partial \psi_j}{\partial n})]ds = 0 \quad . \qquad (3.51)$$

Rearranging the above expression, we get

$$\int_S (u^+ \frac{\partial \hat{\psi}_m}{\partial n} - \hat{\psi}_m \frac{\partial u^+}{\partial n}) ds = A \sum_j a_j [\int_{S_+} (\hat{\psi}_j \frac{\partial \hat{\psi}_m}{\partial n} - \hat{\psi}_m \frac{\partial \psi_j}{\partial n}) ds]$$

$$+ A \sum_j c_j [\int_{S_+} (\psi_j \frac{\partial \hat{\psi}_m}{\partial n} - \hat{\psi}_m \frac{\partial \psi_j}{\partial n}) ds]. \qquad (3.52)$$

The integral associated with a_j vanishes according to the orthogonality condition (3.13), and that with c_j is the same as the integral in Eq. (3.15). Hence the right-hand side of the preceding equation reduces to $A\Sigma c_j(-4i\delta_{jm})$, yielding

$$\int_S (u^+ \frac{\partial \hat{\psi}_m}{\partial n} - \hat{\psi}_m \frac{\partial u^+}{\partial n}) ds = -4iAc_m \qquad . \qquad (3.53)$$

Similarly, replacing v in Eq. (3.12) by ψ_m, ad applying the orthogonality conditions (3.14) and (3.15), one obtains

$$\int_S (u^+ \frac{\partial \psi_m}{\partial n} - \psi_m \frac{\partial u^+}{\partial n}) ds = 4iAa_m \qquad . \qquad (3.54)$$

Equation (3.53) states that the unknown coefficients c_m of the series representation for the scattered wave is determined by an integral of the surface sources, u^+ and $\partial u^+/\partial n$, over the boundary of the scatterer. These two sources, u^+ and $\partial u^+/\partial n$, are not independent of each other and are related by Eq. (3.54) where a_m are known.

The unknown quantities, u^+ and $\partial u^+/\partial n$, are to be determined from prescribed boundary conditions.

3.4 BOUNDARY CONDITIONS

In connection with the above formulations, two boundary conditions are of great importance in scattering problems, namely the Neumann and Dirichlet boundary conditions.

3.4.1 Neumann Type Boundary Condition

In the case where the inclusion is a rigid one, the normal component of the velocity field should vanish on the boundary, i.e.,

$$\frac{\partial u^{+}}{\partial n} = 0 \quad , \tag{3.55}$$

or equivalently, from Eq. (3.47),

$$\frac{\partial u^{s}}{\partial n} = -\frac{\partial u^{i}}{\partial n} \quad \text{on } S \quad . \tag{3.56}$$

Such a boundary condition is known as the Neumann type boundary condition.

The other surface quantity u^{+}, which is unspecified, can be represented [1] as a series of the regular wave functions $\hat{\psi}_j$, that is,

$$u^{+} = A\sum_j \alpha_j \hat{\psi}_j \quad , \quad \text{on } S \quad , \tag{3.57}$$

where α_j are the coefficients of the surface field.

Substitution of Eqs. (3.55) and (3.57) into Eqs. (3.53) and (3.54) yields

$$\sum_j \alpha_j [\int_S \hat{\psi}_j \frac{\partial \hat{\psi}_m}{\partial n} ds] = -4ic_m \quad , \tag{3.58}$$

$$\sum_j \alpha_j [\int_S \hat{\psi}_j \frac{\partial \psi_m}{\partial n} ds] = 4ia_m \quad . \tag{3.59}$$

The above pair of equations show that both scattered field coefficients c_m and incident field coefficients a_m are related to the unknown surface field coefficients α_j. If the latter are eliminated from these two equations, the c_m can then be expressed directly in terms of a_m. Before going into the elimination procedure, it will be better obtain the corresponding pair of equations also for Dirichlet boundary condition.

3.4.2 Dirichlet Type Boundary Condition

The Dirichlet boundary condition in acoustic wave scattering corresponds to the case where we have a cavity inside the fluid medium. Thus, the pressure vanishes on the surface of the cavity, i.e.,

$$u^+ = u^i + u^s = 0 \qquad , \qquad \text{on} \quad S \quad . \tag{3.60}$$

The unspecified surface quantity $\partial u^+/\partial n$, in this case, can be represented [1] as a series in terms of the normal gradients of the regular wave functions, that is,

$$\frac{\partial u^+}{\partial n} = A \sum_j \alpha_j \frac{\partial \hat{\psi}_j}{\partial n} \qquad , \qquad \text{on} \quad S \quad . \tag{3.61}$$

Equations (3.60) and (3.61) are then substituted into (3.53) and (3.54) to obtain

$$\sum_j \alpha_j [-\int_S \hat{\psi}_m \frac{\partial \hat{\psi}_j}{\partial n} ds] = -4ic_m \qquad , \tag{3.62}$$

$$\sum_j \alpha_j [-\int_S \psi_m \frac{\partial \hat{\psi}_j}{\partial n} ds] = 4ia_m \quad . \tag{3.63}$$

The above two equations, together with Eqs. (3.58) and (3.59) obtained in the preceding section, form the basis for deriving a transition matrix relating the coefficients c_m to a_m directly.

3.5 THE TRANSITION MATRIX

One can define two matrices with elements Q_{jm} and $\hat{Q}_{jm}$, which are, in the case of Neumann boundary condition, given by

$$Q_{jm} = \frac{1}{4} \int_S \hat{\psi}_j \frac{\partial \psi_m}{\partial n} ds \quad , \tag{3.64}$$

$$\hat{Q}_{jm} = \frac{1}{4} \int_S \hat{\psi}_j \frac{\partial \hat{\psi}_m}{\partial n} ds \quad , \tag{3.65}$$

and in the case of Dirichlet boundary condition, given by

$$Q_{jm} = -\frac{1}{4} \int_S \psi_m \frac{\partial \hat{\psi}_j}{\partial n} ds \quad , \tag{3.66}$$

$$\hat{Q}_{jm} = -\frac{1}{4} \int_S \hat{\psi}_m \frac{\partial \hat{\psi}_j}{\partial n} ds \quad . \tag{3.67}$$

As can be easily seen in both cases, the elements of the $\hat{Q}$ matrix are simply the real part of the elements of the Q matrix, i.e.,

$$\hat{Q}_{jm} = \mathrm{Re}(Q_{jm}) \quad . \tag{3.68}$$

Now, substituting the corresponding Q and $\hat{Q}$ matrices into Eqs. (3.58), (3.59), (3.62) and (3.63), we obtain

$$i\sum_j \alpha_j \hat{Q}_{jm} = c_m \quad , \tag{3.69}$$

$$-i\sum_j \alpha_j Q_{jm} = a_m \quad . \tag{3.70}$$

The equations (3.69) and (3.70) can then be expressed in matrix form,

$$i\hat{\underset{\approx}{Q}}^t \underset{\sim}{\alpha} = \underset{\sim}{c} \quad , \tag{3.71}$$

$$-i\underset{\approx}{Q}^t \underset{\sim}{\alpha} = \underset{\sim}{a} \quad , \tag{3.72}$$

where (t) denotes the transpose matrix. Elimination of the unknown surface field coefficient vector $\underset{\sim}{\alpha}$ from these two equations yields

$$\underset{\sim}{c} = -[\hat{\underset{\approx}{Q}}^t (\underset{\approx}{Q}^t)^{-1}]\underset{\sim}{a} = \underset{\approx}{T}\,\underset{\sim}{a} \quad . \tag{3.73}$$

The $\underset{\approx}{T}$ is called transition matrix which relates the scattered wave field directly to the incident wave field. As shown in Ref. [1,20], the T-matrix is symmetric, i.e., $\underset{\approx}{T}^t = \underset{\approx}{T}$, it can thus be determined from the relation

$$\underset{\approx}{Q}\,\underset{\approx}{T} = -\hat{\underset{\approx}{Q}} \quad , \tag{3.74}$$

or,

$$\underset{\approx}{T} = -\underset{\approx}{Q}^{-1}\hat{\underset{\approx}{Q}} \quad . \tag{3.75}$$

3.6 STRUCTURE OF THE Q-MATRIX

As given in Eqs. (3.63-67), the elements of the Q-matrix are given by integrals involving basis wave functions and their normal gradients. These integrals are evaluated along the boundary of the scatterer. We should note that for a given incident wave field (given wave number) the elements of the Q-matrix are fixed when the geometry and the boundary type of the scatterer are given.

In order to understand the structure of the Q-matrix better, one should restore the full index notation of the basis functions, that is,

$$\psi_j(r,\theta) \equiv \psi_j^\sigma(r,\theta) \quad , \quad \psi_m(r,\theta) \equiv \psi_m^\nu(r,\theta) \quad , \text{ etc.} \tag{3.76}$$

With this notation, Eqs. (3.64) and (3.66) take the form

$$Q_{jm}^{\sigma\nu} = \frac{1}{4} \int_S \hat{\psi}_j^\sigma \frac{\partial \psi_m^\nu}{\partial n} ds \quad , \tag{3.77}$$

$$Q_{jm}^{\sigma\nu} = -\frac{1}{4} \int_S \hat{\psi}_m^\nu \frac{\partial \hat{\psi}_j^\sigma}{\partial n} ds \quad , \tag{3.78}$$

respectively. From these expressions one sees that the Q-matrix actually consists of four submatrices, that is,

$$\underset{\approx}{Q} = \begin{vmatrix} \underset{\approx}{Q}^{11} & \underset{\approx}{Q}^{12} \\ \underset{\approx}{Q}^{21} & \underset{\approx}{Q}^{22} \end{vmatrix} \quad , \tag{3.79}$$

where, in the case of Neumann boundary condition,

$$Q_{jm}^{11} = \frac{1}{4} \int_S \hat{\psi}_j^1 \frac{\partial \psi_m^1}{\partial n} ds \quad , \tag{3.80}$$

$$Q_{jm}^{12} = \frac{1}{4} \int_S \hat{\psi}_j^1 \frac{\partial \psi_m^2}{\partial n} ds \quad , \tag{3.81}$$

$$Q_{jm}^{21} = \frac{1}{4} \int_S \hat{\psi}_j^2 \frac{\partial \psi_m^1}{\partial n} ds \quad , \tag{3.82}$$

$$Q_{jm}^{22} = \frac{1}{4} \int_S \hat{\psi}_j^2 \frac{\partial \psi_m^2}{\partial n} ds \quad , \tag{3.83}$$

and in the case of Dirichlet boundary condition,

$$Q_{jm}^{11} = -\frac{1}{4} \int_S \psi_m^1 \frac{\partial \hat{\psi}_j^1}{\partial n} ds \quad , \tag{3.84}$$

$$Q_{jm}^{12} = -\frac{1}{4} \int_S \psi_m^2 \frac{\partial \hat{\psi}_j^1}{\partial n} ds \quad , \tag{3.85}$$

$$Q_{jm}^{21} = -\frac{1}{4} \int_S \psi_m^1 \frac{\partial \hat{\psi}_j^2}{\partial n} ds \quad , \tag{3.86}$$

$$Q_{jm}^{22} = -\frac{1}{4} \int_{S.} \psi_m^2 \frac{\partial \hat{\psi}_j^2}{\partial n} ds \quad . \tag{3.87}$$

Note that ψ_n^σ and $\hat{\psi}_n^\sigma$ are given by

$$\hat{\psi}_n^1(r,\theta) = (\varepsilon_n)^{\frac{1}{2}} H_n(kr)\cos(n\theta) \quad , \tag{3.88}$$

$$\hat{\psi}_n^2(r,\theta) = (\varepsilon_n)^{\frac{1}{2}} H_n(kr)\sin(n\theta) \quad , \tag{3.89}$$

$$\hat{\psi}_n^1(r,\theta) = (\varepsilon_n)^{\frac{1}{2}} J_n(kr)\cos(n\theta) \quad , \tag{3.90}$$

$$\hat{\psi}_n^2(r,\theta) = (\varepsilon_n)^{\frac{1}{2}} J_n(kr)\sin(n\theta) \quad . \tag{3.91}$$

Depending on the geometry of the scatterer, the Q-matrix has the following properties:

For seperable geometries, i.e., for the circle and ellipse, the Q-matrix is symmetric [1], that is,

$$\underset{\approx}{Q}^t = \underset{\approx}{Q} \quad , \tag{3.92}$$

or in explicit form

$$Q^{11}_{jm} = Q^{11}_{mj} \quad , \qquad Q^{22}_{jm} = Q^{22}_{mj} \quad , \qquad Q^{12}_{jm} = Q^{21}_{mj} \quad . \tag{3.93}$$

Another important property is that, if the cross-sectional geometry of the scatterer has a mirror symmetry with respect to x-axis, i.e., across the plane $y = r\sin\theta = 0$, so that $r(\theta) = r(2\pi-\theta)$, then the integrals involving the mixed products of sines and cosines will vanish and we get

$$Q^{12}_{jm} = Q^{21}_{jm} = 0 \quad , \tag{3.94}$$

and for the nonzero matrices, $\underset{\approx}{Q}^{11}$ and $\underset{\approx}{Q}^{22}$, the integrals are to be evaluated only by considering the half of the boundary.

If the boundary of the scatterer has symmetry with respect to both x and y-axes, then [1],

$$Q_{jm} = 0 \quad \text{if} \quad (j+m) \quad \text{is odd.} \tag{3.95}$$

Also, for such boundaries, real part of the Q-matrix is symmetric [21], i.e.,

$$\hat{\underset{\approx}{Q}}^{t} = \hat{\underset{\approx}{Q}} \quad . \tag{3.96}$$

Note that, for seperable geometries both real and imaginary parts are symmetric.

3.7 PROPERTIES OF THE T-MATRIX

In the light of the notation of Eq. (3.79), a corresponding block notation can be used for the transition matrix in Eq. (3.75),

$$\underset{\approx}{T} = \begin{vmatrix} \underset{\approx}{T}^{11} & \underset{\approx}{T}^{12} \\ \underset{\approx}{T}^{21} & \underset{\approx}{T}^{22} \end{vmatrix} = - \begin{vmatrix} \underset{\approx}{Q}^{11} & \underset{\sim}{Q}^{12} \\ \underset{\approx}{Q}^{21} & \underset{\approx}{Q}^{22} \end{vmatrix}^{-1} \times \begin{vmatrix} \underset{\approx}{\hat{Q}}^{11} & \underset{\approx}{\hat{Q}}^{12} \\ \underset{\approx}{\hat{Q}}^{21} & \underset{\approx}{\hat{Q}}^{22} \end{vmatrix} . \qquad (3.97)$$

For the boundaries having mirror symmetry with respect to x-axis, Eq. (3.97) reduces to the two (single) equations

$$\underset{\approx}{T}^{11} = -(\underset{\approx}{Q}^{11})^{-1}\underset{\approx}{\hat{Q}}^{11} \quad , \quad \underset{\approx}{T}^{22} = -(\underset{\approx}{Q}^{22})^{-1}\underset{\approx}{\hat{Q}}^{22} \quad . \qquad (3.98)$$

Because of the reciprocity principle and energy-conservation requirements, as shown in Refs. [1] and [20], the T-matrix is always symmetric, i.e.,

$$\underset{\approx}{T}^{t} = \underset{\approx}{T} \quad , \qquad (3.99)$$

and

$$\underset{\approx}{T}\,\underset{\approx}{T}^{*} = -\mathrm{Re}(\underset{\approx}{T}) \quad , \qquad (3.100)$$

where the asteriks (*) denotes the complex conjugation and (Re) means the real part. These properties are valid for all geometries and can be used to check the accuracy in the numerical evaluations of the T-matrix.

3.8 PRESENTATION OF THE SCATTERED WAVE FIELD

The scattered field coefficients c_m, which are to be evaluated through the relation (3.73), can be written in a compact form as

$$c_m^{\sigma} = \sum_{n,\nu} T_{mn}^{\sigma\nu} a_n^{\nu} \quad , \qquad (3.101)$$

or, in block matrix notation,

$$\begin{vmatrix} \tilde{c}^1 \\ \\ \tilde{c} \end{vmatrix} = \begin{vmatrix} \underset{\approx}{T}^{11} & \underset{\approx}{T}^{12} \\ \\ \underset{\approx}{T}^{21} & \underset{\approx}{T}^{22} \end{vmatrix} \begin{vmatrix} \tilde{a}^1 \\ \\ \tilde{a}^2 \end{vmatrix} \quad . \tag{3.102}$$

One can then write from Eq. (3.101)

$$c_m^1 = \sum_{n=o}^{\infty} T_{mn}^{11} a_n^1 + \sum_{n=1}^{\infty} T_{mn}^{12} a_n^2 \quad , \qquad m = 0,1,\ldots,\infty \quad , \tag{3.103}$$

$$c_m^2 = \sum_{n=o}^{\infty} T_{mn}^{21} a_n^1 + \sum_{n=1}^{\infty} T_{mn}^{22} a_n^2 \quad , \qquad m = 1,2,\ldots,\infty \quad , \tag{3.104}$$

where

$$a_n^1 = (\varepsilon_n)^{\frac{1}{2}} i^n \cos(n\alpha) \quad , \quad a_n^2 = (\varepsilon_n)^{\frac{1}{2}} i^n \sin(n\alpha) \quad . \tag{3.105}$$

By using the above notation and omitting the time factor, the incident and scattered wave fields can be rewritten as

$$u^i = A \sum_{n=o}^{\infty} (\varepsilon_n)^{\frac{1}{2}} J_n(kr)[a_n^1 \cos(n\theta) + a_n^2 \sin(n\theta)] \quad , \quad r < \infty \tag{3.106}$$

$$u^s = A \sum_{n=o}^{\infty} (\varepsilon_n)^{\frac{1}{2}} H_n(kr)[c_n^1 \cos(n\theta) + c_n^2 \sin(n\theta)] \quad , \quad r \text{ outside } S \tag{3.107}$$

Once the coefficients of the scattered wave field are determined numerically, the scattered field is known. Various field quantities such as the velocity potentials in the near field, surface field potentials, far field amplitudes and scattering cross-sections which are of interest in the acoustic wave scattering problems can then be calculated.

3.8.1 Near-field Solutions

The velocity potentials due to the scattered wave field at finite distances from the scatterer can be obtained from Eq. (3.107). In the case where the near-field solutions are concerned, generally the quantity of interest is the distribution of the velocity potential on the boundary of the scatterer due to scattered wave field. However, this distribution can not be evaluated by the T-matrix formulation directly, because the series representing the scattered wave field is not complete on the surface of the scatterer and hence Eq. (3.107) is not valid on S, [1,12]. The series is complete only on a circular surface, say S_+, which is outside the scatterer as shown in Fig. 3.3. It is possible to evaluate the velocity potential distribution from the intermediate steps of the T-matrix formulation. For this purpose, one can write, from Eq. (3.47) and (3.48),

$$u^+(r,\theta) = u^i(r,\theta) + u^s(r,\theta) \quad , \quad r \text{ on } S \; , \tag{3.108}$$

or

$$u^s = u^+ - u^i \quad , \quad r \text{ on } S \; . \tag{3.109}$$

In the case of Neumann boundary condition, substitution of Eqs. (3.35) and (3.57) into Eq. (3.109) yields

$$u^s = A\,\Sigma\alpha_n\hat{\psi}_n - A\,\Sigma a_n\hat{\psi}_n \quad , \quad r \text{ on } S \; , \tag{3.110}$$

where u^+ is the total wave field at the boundary and α_n are the unknown surface field coefficients. One can immediately see that the unknown coefficients α_n can be uniquely determined through the Q-matrix by solving the matrix equation (3.72). Substituting the expressions for

$\hat{\psi}_n$ in the above equation and rearranging the terms slightly we get

$$u^s = A \sum_{n=o}^{\infty} (\varepsilon_n)^{\frac{1}{2}} J_n(kr)[(\alpha_n^1 - a_n^1)\cos(n\theta) + (\alpha_n^2 - a_n^2)\sin(n\theta)] \;, \quad r \text{ on } S \quad .(3.111)$$

In the case of Dirichlet boundary condition, however, the scattered wave field on the boundary is simply given by

$$u^s = -u^i \quad , \quad \text{on } S \quad , \qquad (3.112)$$

and hence it needs not be evaluated.

It should be noted that, for a circular cylindrical scatterer, the surface field potentials due to scattered wave field can still be determined from Eq. (3.107) directly, because, in this case, the circular surface S_+ can be arbitrarily replaced by S so that the outgoing wave series in Eq. (3.107) is complete also on the boundary of the scatterer.

The near-field results, in fact, have very little practical significance and they are of almost no interest in applications such as non-destructive evaluations, remote sensing etc.

3.8.2 Far Field Solution

From the practical point of view, the far field amplitude is the most important quantity to be determined in the acoustic scattering problems.

The expression for the scattered wave field at distances far from the scatterer is obtained from Eq. (3.107) by using the asymptotical form of the Hankel functions. The asymptotical representation of $H_n(kr)$ as $r \to \infty$ is given by, [2],

$$H_n(kr) \overset{\infty}{=} \sqrt{2/\pi kr}\; e^{i[kr-((2n+1)/4)\pi]} \quad , \tag{3.113}$$

and after simple manipulations, one gets

$$H_n(kr) \overset{\infty}{=} \sqrt{2/i\pi kr}\; e^{ikr}\; i^{-n} \quad . \tag{3.114}$$

Then, substitution of Eq. (3.114) into Eq. (3.107) yields

$$u^s = e^{ikr}\sqrt{2/i\pi kr}\; A \sum_{n=o}^{\infty} (\varepsilon_n)^{\frac{1}{2}} i^{-n}[c_n^1\cos(n\theta) + c_n^2\sin(n\theta)] \;,\quad r \to \infty. \tag{3.115}$$

One can also write Eq. (3.115) as

$$u^s = e^{ikr}\sqrt{2/i\pi kr}\; f(\theta) \quad , \tag{3.116}$$

where $f(\theta)$ is the far field amplitude describing the angular variation of the scattered field at distances far from the scatterer and it is given by

$$f = A \sum_{n=o}^{\infty} (\varepsilon_n)^{\frac{1}{2}} i^{-n}[c_n^1\cos(n\theta) + c_n^2\sin(n\theta)]. \tag{3.117}$$

In the presentation of the scattered wave field results, the field quantities given by Eqs. (3.107), (3.111) and (3.117) are first non-dimensionalized through dividing them by the constant amplitude factor of the incident wave A, and then the angular variations of their norms, i.e., $|u^s/A|$, $|f/A|$ versus θ, are plotted in polar coordinates for the regions of interest.

3.8.3 Total Scattering Cross-Section

In two dimensional wave scattering problems, the total scattering cross-section is defined as the ratio between power generated by the scattered wave over a circle with large radius around the obstacle and the power per unit area generated by the incident wave, and is given by, [13],

$$\sigma^{tot} = \frac{1}{2\pi} \int_{0}^{2\pi} \sigma(\theta) d\theta \quad , \tag{3.118}$$

where $\sigma(\theta)$ is the differential cross-section,

$$\sigma(\theta) = |f(\theta)|^2 \quad . \tag{3.119}$$

The total scattering cross-section can then be written in terms of the scattered wave field coefficients as

$$\sigma^{tot} = \sum_{n=0}^{\infty} (|c_n^1|^2 + |c_n^2|^2) \quad . \tag{3.120}$$

The above quantity is generally used to check the convergency of the scattered wave field results obtained by the T-matrix formulation and employed in the selection of the T-matrix size to be used in the calculations.

IV. NUMERICAL EVALUATIONS

This chapter is devoted to the applications of the transition matrix method described in the previous chapter. The examples presented include cylindrical rigid inclusions and cavities with circular, elliptical, rectangular and triangular cross sections. Some of the results obtained have been compared with the known exact and approximate solutions, [9,22].

In the numerical evaluations, an incident plane wave as given by Eq. (3.36) has been considered and both the "near-field" and, mainly, the "far-field" results have been obtained for various angles of incidence. The results have been presented in polar graphical forms, as discussed in sections (3.8.1) and (3.8.2).

The basic steps in the computation of the scattered wave field using T-matrix method can be outlined as follows:

a) Description of the boundary geometry of the scatterer,

b) Numerical evaluation of the boundary integrals for the generation of the Q-matrix elements,

c) Inversion of the Q-matrix and creation of the T-matrix,

d) Calculation of the scattered field coefficients through T-matrix,

e) Evaluation of the scattered field for various incidence angles

f) Check for the convergency of the numerical results by checking the convergency of the series, Eq. (3.120), representing the total scattering cross section.

Note that, if the convergency of the total scattering cross section is found to be insufficient at the end of the calculations, the steps (b) to (f) should be repeated by using a Q-matrix of the larger size.

4.1 GENERAL PROCEDURE FOR THE EVALUATION OF THE Q-MATRIX ELEMENTS

In order to generate the Q-matrix one should evaluate the integrals given by Eqs. (3.77) and (3.78) either analytically or numerically. Analytical evaluation is possible for only circular geometry, because $r \neq r(\theta)$ for circle and the boundary integrals reduces to simple trigonometric integrals. For general boundary geometries, the boundary integrals are to be evaluated numerically.

In order to illustrate the procedure followed in the computation of the Q-matrix elements, the derivations of the necessary analytical and numerical expressions are presented here for only the $\underset{\approx}{Q}^{11}$ matrix for the rigid inclusion case.

To obtain the elements of the $\underset{\approx}{Q}^{11}$ matrix, first, it is helpful to write Eq. (3.80) with slightly different notation as

$$Q^{11}_{jm} = \frac{1}{4} \int_S [\hat{n}ds \cdot \nabla\psi^1_m(r,\theta)]\hat{\psi}^1_j(r,\theta) \quad , \qquad (4.1)$$

where $\hat{n}$ is the unit normal vector and ∇ is the gradient operator. To put the above expression into a form which is suitable for numerical calculations we need the explicit expressions for the terms inside the integrand.

Consider Fig. 4.1 for the evaluation of the term $\hat{n}ds$. As the incremental quantities Δs, $\Delta\theta$, Δr become infinitesimally small ($\varepsilon \to 0$), we can write

$$\hat{n} = \underline{e}_r\cos\gamma - \underline{e}_\theta\sin\gamma \quad , \qquad (4.2)$$

where $\underline{e}_r$ and $\underline{e}_\theta$ are the radial and tangential unit vectors, respectively, and

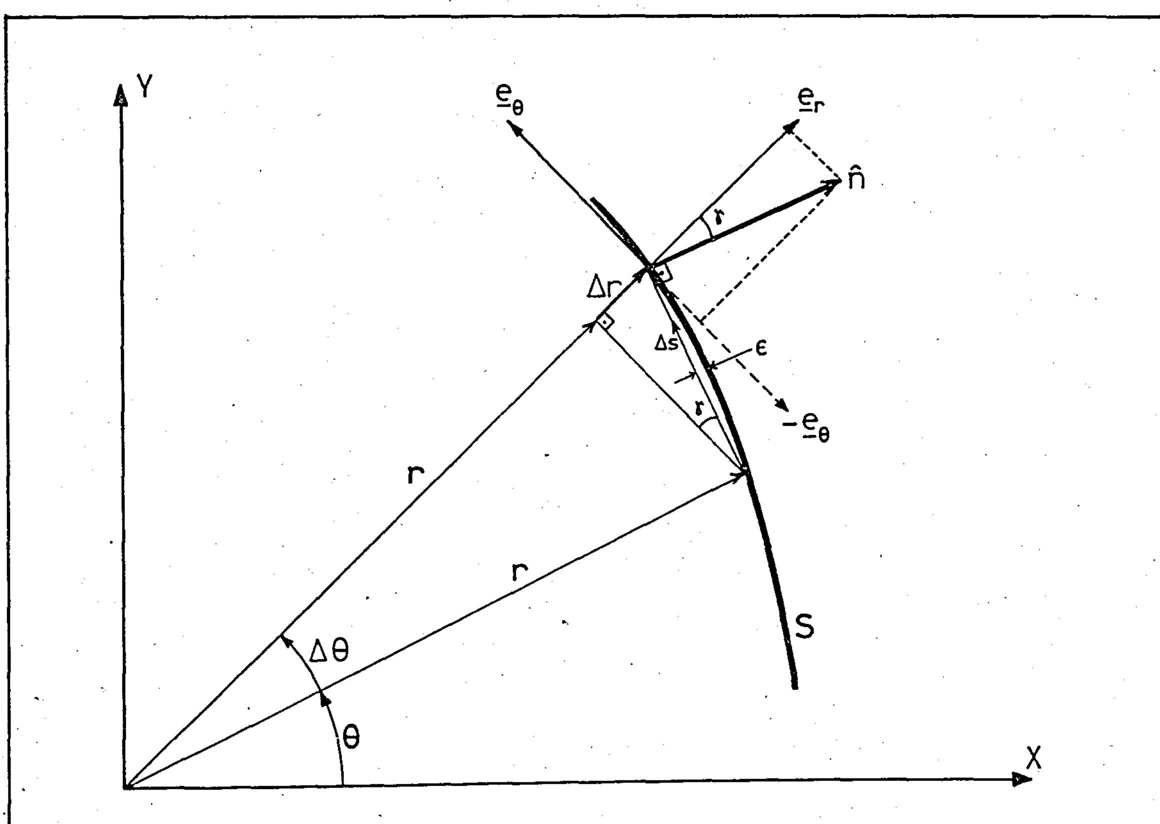


Figure 4.1 - Geometrical representation of the unit normal vector.

$$\cos\gamma = \frac{rd\theta}{ds} \quad , \qquad \sin\gamma = \frac{dr}{ds} \quad . \tag{4.3}$$

Hence, from Eqs. (4.2) and (4.3), one obtains

$$\hat{n}ds = rd\theta[\underline{e}_r - \frac{1}{r}\frac{dr}{d\theta}\underline{e}_\theta] \quad . \tag{4.4}$$

As to the term $\nabla\psi_m^1(r,\theta)$, using the gradient expression in polar coordinates,

$$\nabla = \frac{\partial}{\partial r}\underline{e}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\underline{e}_\theta \quad , \tag{4.5}$$

one can write

$$\nabla\psi_m^1 = (\varepsilon_m)^{\frac{1}{2}}[\underline{e}_r kH_m'(kr)\cos(m\theta) - \underline{e}_\theta \frac{m}{r} H_m(kr)\sin(m\theta)] \ . \tag{4.6}$$

Note that

$$\begin{aligned} H_m'(kr) &= \frac{1}{2}[H_{m-1}(kr) - H_{m+1}(kr)] \\ &= \frac{m}{kr} H_m(kr) - H_{m+1}(kr) \quad , \end{aligned} \tag{4.7}$$

thus, Eq. (4.6) reduces to

$$\begin{aligned} \nabla\psi_m^1 = (\varepsilon_m)^{\frac{1}{2}}\{&\underline{e}_r k[\frac{m}{kr} H_m(kr) - H_{m+1}(kr)]\cos(m\theta) \\ &- \underline{e}_\theta \frac{m}{r} H_m(kr)\sin(m\theta)\} \ . \end{aligned} \tag{4.8}$$

From Eqs. (4.4) and (4.8), one then obtains

$$\hat{n}ds\cdot\nabla\psi_m^1 = (\varepsilon_m)^{\frac{1}{2}} r d\theta \{ k[\frac{m}{kr} H_m(kr) - H_{m+1}(kr)]\cos(m\theta)$$

$$+ \frac{m}{r^2}\frac{dr}{d\theta} H_m(kr)\sin(m\theta)\} \quad . \qquad (4.9)$$

Finally, substituting Eq. (4.9) and corresponding expression for $\hat{\psi}_j^1$, i.e., Eq. (3.90), into Eq. (4.1), and also writing $dr/d\theta$ as $1/k[d(kr)/d\theta]$, one gets

$$Q_{jm}^{11} = \frac{(\varepsilon_j \varepsilon_e)^{\frac{1}{2}}}{4} \int_0^{2\pi} d\theta\{J_j(kr)\cos j\theta\}\{[mH_m(kr) - krH_{m+1}(kr)]\cos(m\theta)$$

$$+ \frac{m}{kr} H_m(kr)\sin(m\theta) \frac{d}{d\theta}(kr)\} \quad . \ (4.10)$$

The quantities r and $dr/d\theta$ both being functions of θ, i.e., $r = r(\theta)$, $d[r(\theta)]/d\theta$, are to be determined from the analytical equations describing the geometry of the boundary of the scatterer of interest. One should also notice that 'r' always occurs as $kr(\theta)$ in the expression for the Q-matrix elements. Therefore, in the numerical evaluations, it is sufficient to specify the wave number in a non-dimensional form, say $k\ell$, where ℓ is a characteristic length in the problem, and the non-dimensional ratios of the geometry parameters such as aspect ratio, corner radius ratio etc.

As stated earlier, in general the angular integrations in Eq. (4.10) which are of the form

$$I = \int_0^{2\pi} F(\theta)d\theta \quad , \qquad \text{on } S \qquad (4.11)$$

can be carried out only numerically. The first step in the numerical evaluation of these integrals is the subdivision of the boundary S into N number of intervals ΔS_i each subtending a central angle of $\Delta\theta_i$

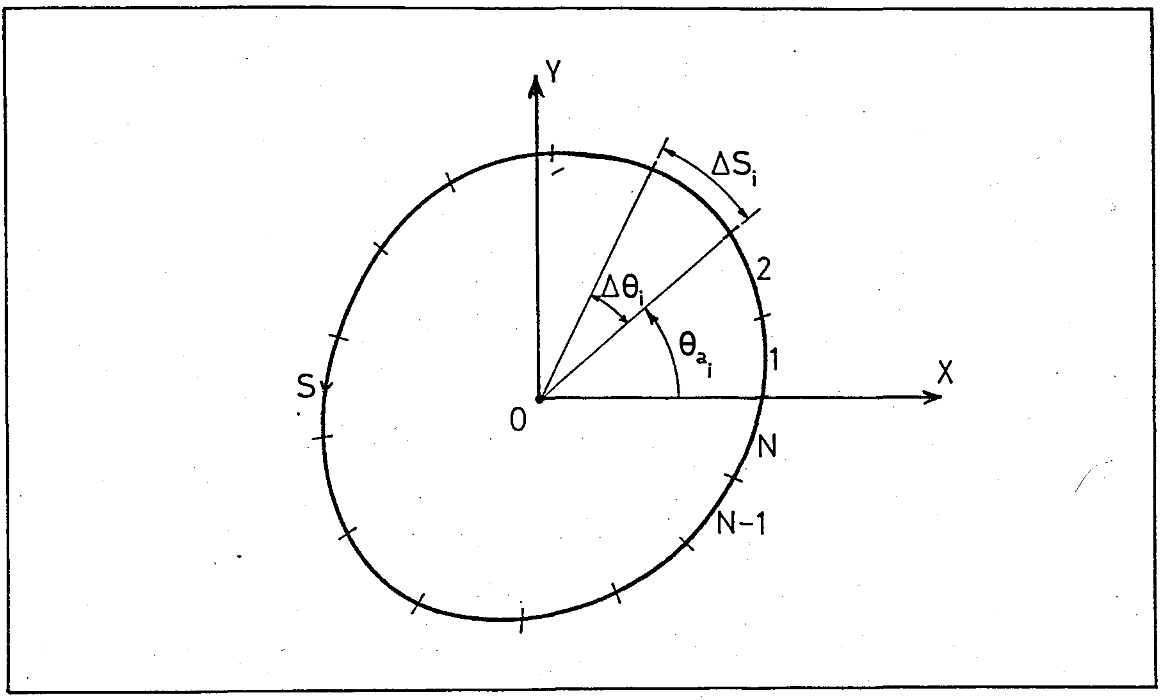


Figure 4.2 - Boundary subdivision.

such that, as shown in Fig. 4.2,

$$\sum_{i=1}^{N} \Delta S_i = S \quad , \quad \sum_{i=1}^{N} \Delta\theta_i = 2\pi \quad . \tag{4.12}$$

The integral given by Eq. (4.11) can then be written as

$$I = \sum_{i=1}^{N} [\int_{\Delta\theta_i} F(\theta)d\theta] \quad , \quad \text{on} \quad S \quad . \tag{4.13}$$

Having divided the boundary into N suitably small segments, one should next approximate the integrals which are to be evaluated separetly for each angular interval $\Delta\theta_i$. This approximation can be made by using the Simpson's rule of any order. Hence, one can approximate the angular integrals in Eq. (4.13) in the form (see Appendix A)

$$\int_{\Delta\theta_i} F(\theta)d\theta \simeq C_s \frac{\Delta\theta_i}{P} \sum_{q=o}^{P} F(\theta_{a_i} + q[\Delta\theta_i/P]) \times w_q \quad , \qquad (4.14)$$

where P is the order of the Simpson's rule used, C_s is a constant multiplication factor and w_q are the weighing factors. Note that, C_s and w_q are fixed numbers depending on the order P. Substituting Eq. (4.14) into (4.13), one gets

$$I \simeq \frac{C_s}{P} \sum_{i=1}^{N} \sum_{q=o}^{P} \Delta\theta_i F(\theta_{a_i} + q\,[\Delta\theta_i/P])w_q \quad , \text{ on } S . \qquad (4.15)$$

Finally, applying the above expression to Eq. (4.10) and using the geometrical definitions shown in Fig. 4.3, one obtains the desired numerical expression for the computation of the $\underset{\approx}{Q}^{11}$-matrix elements,

$$Q^{11}_{jm} \simeq \frac{(\varepsilon_j \varepsilon_m)^{\frac{1}{2}}}{4} \frac{C_s}{P} \sum_{i=1}^{N} \sum_{q=o}^{P} (\Delta\theta_i) w_q \{J_j(kr_{iq})\cos(j\theta_{iq})\}$$

$$\times \{[mH_m(kr_{iq}) - kr_{iq}H_{m+1}(kr_{iq})]\cos(m\theta_{iq})$$

$$+ \frac{m}{kr_{iq}} H_m(kr_{iq})\sin(m\theta_{iq})[\frac{d(kr)}{d\theta}]_{iq}\} \quad , \quad r_{iq} \text{ on } S. \qquad (4.16)$$

The analytical and numerical expressions obtained for the other Q-submatrices, for both rigid inclusion and cavity cases, are given in Appendix B and C.

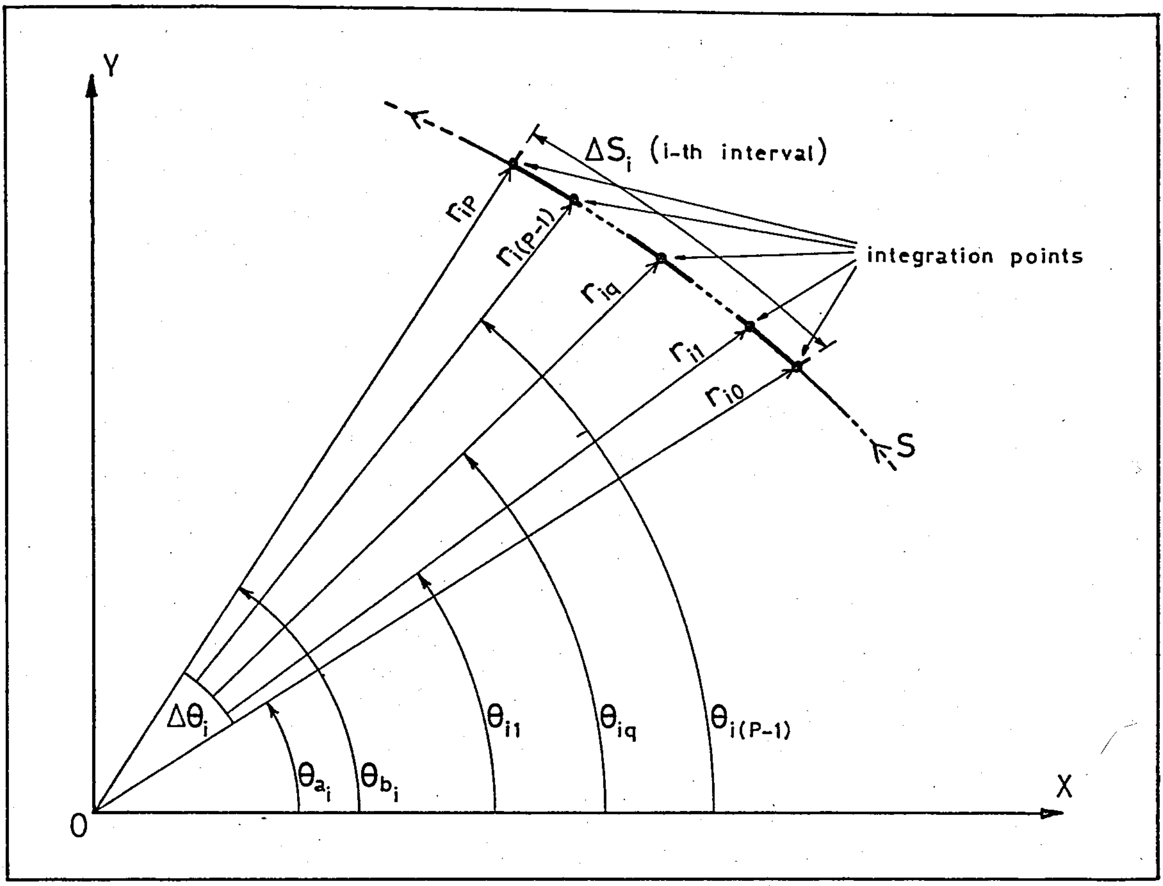


Figure 4.3 - Interval subdivision.

4.2 CREATION OF THE T-MATRIX

After computing the Q-matrix elements according to the procedure outlined in the previous section, one can then create the T-matrix. But, before going into the evaluation of the T-matrix, one should perform the following conditioning on the Q-matrix, [21]. Because of the behaviour of the Hankel functions appearing in the elements of the Q-matrix, the imaginary parts of the elements of the Q-matrix will tend to grow to very large numerical values for the elements above the diagonal. In order to avoid the loss of precision due to the finite precision arithmetic employed by the digital computers, it is convenient at this point to set the imaginary parts of all the mentioned elements to zero, by Gaussian elimination.

The T-matrix can be evaluated, as discussed earlier, using the matrix equation

$$\underset{\approx}{T} = -\underset{\approx}{Q}^{-1} \times \underset{\approx}{\hat{Q}} \tag{4.17}$$

which first requires the inversion of the Q-matrix directly by using a standard matrix inversion technique. However, from the point of view of numerical accuracy, this evaluation can be performed more effectively by first transforming the Q-matrix to a unitary matrix $\underset{\approx}{Q}_{unit}$ and then applying the following matrix equation, [20,21],

$$\underset{\approx}{T} = -[\underset{\approx}{Q}_{unit}]^{t^*} \times [\underset{\approx}{\hat{Q}}_{unit}] \quad , \tag{4.18}$$

where the symbols 't' and '*' denote the matrix transpose and the complex conjugate, respectively. This transformation, i.e., $\underset{\approx}{Q}$ to $\underset{\approx}{Q}_{unit}$, is done by Schmidt orthogonalization [21].

4.3 NUMERICAL EXAMPLES

In this section, the numerical results obtained by the application of the T-matrix method to the cylindrical rigid inclusions and cavities having circular, elliptical, rectangular (round cornered) and triangular (isosceles) cross sectional geometries are presented. The incident wave considered, as stated earlier, is a plane acoustic wave with constant velocity c, angular frequency ω, and wavelength $\lambda = 2\pi/k$.

The scattered wave field is computed for various non-dimensional wave numbers, $k\ell$, and incidence angles, α.

4.3.1 Circular Cylinders

The equation of the boundary of a circular cylindrical scatterer in polar coordinates is simply

$$r = a \quad , \tag{4.19}$$

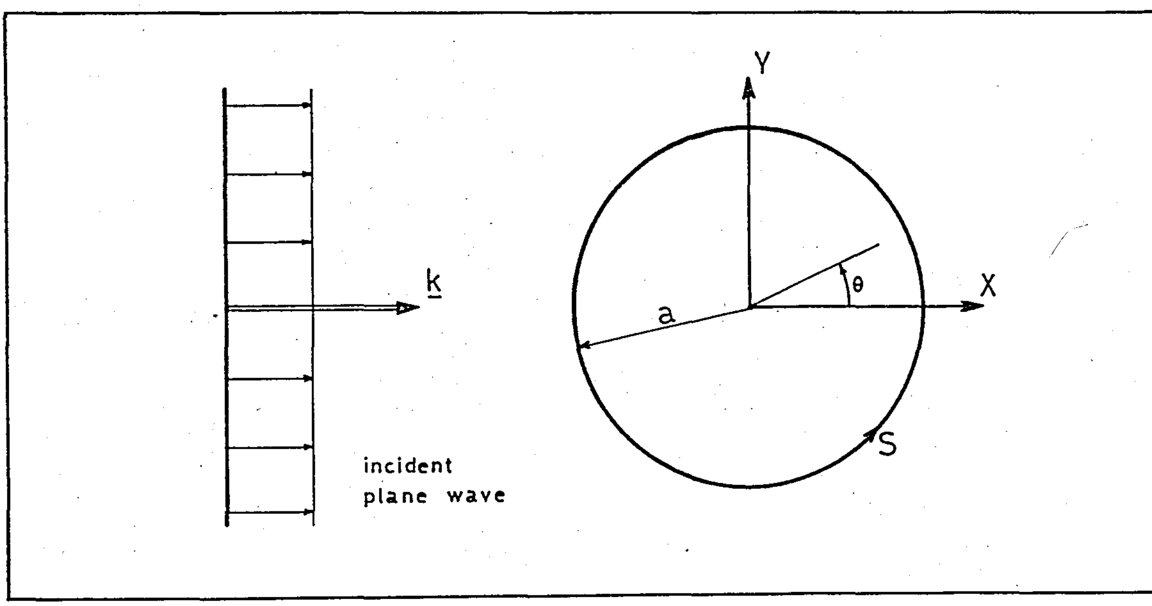


Figure 4.4 - Geometry for the circular cylindrical scatterer.

where a is the radius of the circle, as shown in Fig. 4.4. From Eq. (4.19) one easily sees that $r \neq r(\theta)$, thus,

$$[dr/d\theta] = 0 \quad . \tag{4.20}$$

The polar plots of the scattered field results obtained for the rigid inclusion and cavity cases are presented in Figs. 4.8 to

4.10. Note that, due to the rotational symmetry of the problem, the scattered wave field is to be computed for only zero angle of incidence, i.e., for $\alpha = 0^o$.

4.3.2 Elliptical Cylinders

For the evaluation of the boundary integrals one needs the equation of the ellipse and the angular variation [dr/dθ], which are given by

$$r(\theta) = a[\cos^2\theta + (b/a)^2\sin^2\theta]^{\frac{1}{2}} \quad , \tag{4.21}$$

$$\frac{dr}{d\theta} = a\{(b/a)[(b/a)^2 - 1]\sin\theta\cos\theta[\sin^2\theta + (b/a)^2\cos^2\theta]^{-3/2}\}, \tag{4.22}$$

where 2a and 2b are the major and minor axes, as shown in Fig. 4.5.

The polar plots of the near and far field results obtained for the rigid inclusion and cavity cases for various ka values and aspect ratios (b/a) are presented in Figs. 4.11 to 4.18.

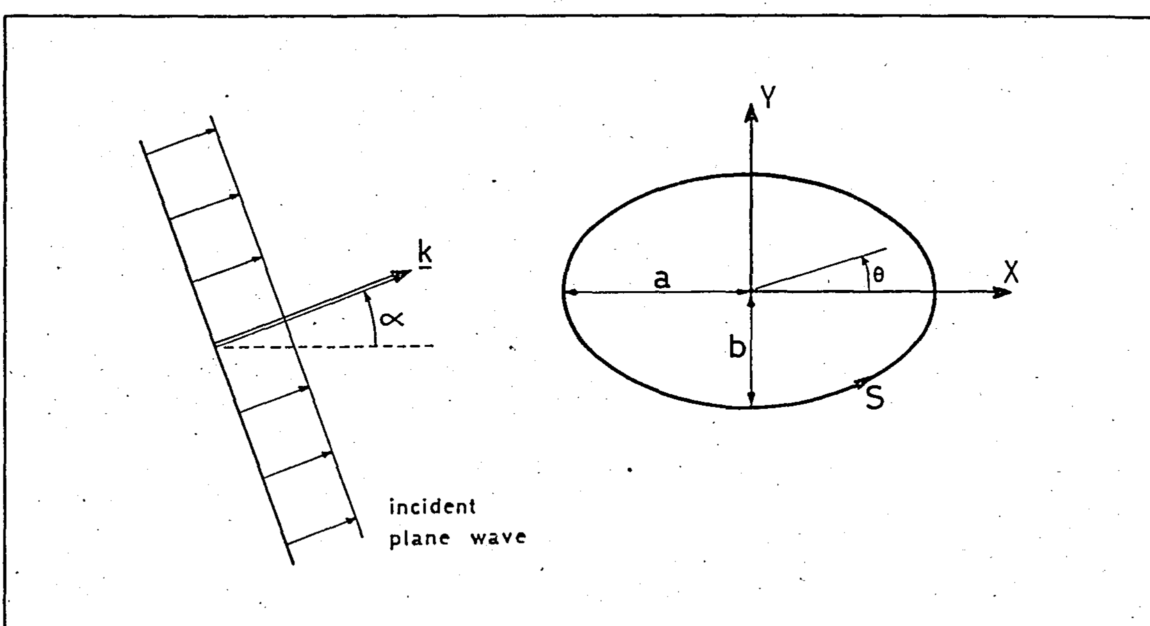


Figure 4.5 - Boundary geometry of the elliptic cylinder.

4.3.3 Rectangular Cylinders

In the application of the T-matrix method to rectangular cylinders, a round cornered type rectangular cross section is considered. To describe the boundary geometry, the following geometrical definitions are made first;

$$\gamma = \arctan[(b/a - r_c/a)/(1 - r_c/a)] \quad , \qquad (4.23)$$

$$(D/a) = (1 - r_c/a)/\cos\gamma \quad , \qquad (4.24)$$

$$\phi_1 = \arctan[b/a - r_c/a] \quad , \qquad (4.25)$$

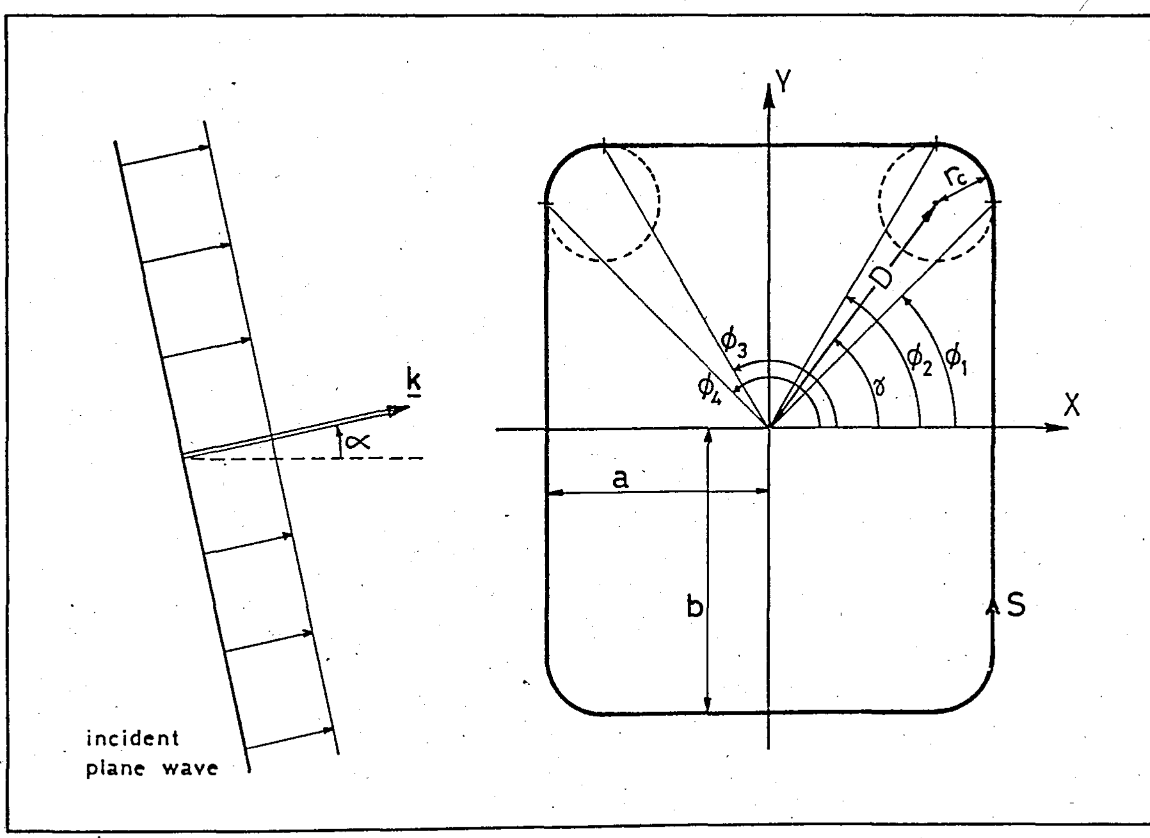


Figure 4.6 - Boundary geometry of the round cornered rectangular cylinder.

$$\phi_2 = \arctan[(b/a)/(1 - r_c/a)] \quad , \qquad (4.26)$$

$$\phi_3 = \pi - \phi_2 \quad , \qquad \phi_4 = \pi - \phi_1 \quad , \qquad (4.27)$$

where 2a and 2b are the width and the length of the rectangle and r_c is the corner radius, as shown in Fig. 4.6. Employing these definitions one can write the analytical equations of the boundary and the $(dr/d\theta)$ values in the range $0 \leq \theta \leq \pi$ as

a) $$r(\theta) = a(1/\cos\theta) \quad ,$$

$$\frac{dr}{d\theta} = a(\sin\theta/\cos^2\theta) \quad \text{for} \quad 0 \leq \theta \leq \phi_1 \quad , \qquad (4.28)$$

b) $$r(\theta) = a\{(D/a)\cos(\theta-\gamma) + [(D/a)^2\cos^2(\theta-\gamma) + (r_c/a)^2 - (D/a)^2]^{\frac{1}{2}}\},$$

$$\frac{dr}{d\theta} = a\{(D/a)\sin(\gamma-\theta) + (D/a)^2\cos(\gamma-\theta)\sin(\gamma-\theta)$$

$$\times [(D/a)^2\cos^2(\gamma-\theta) + (r_c/a)^2 - (D/a)^2]^{-\frac{1}{2}}\}$$

$$\text{for} \quad \phi_1 \leq \theta \leq \phi_2 \quad , \qquad (4.29)$$

c) $$r(\theta) = a[(b/a)/\sin\theta] \quad ,$$

$$\frac{dr}{d\theta} = a[-(b/a)\cos\theta/\sin^2\theta] \quad \text{for } \phi_2 \leq \theta \leq \phi_3, \qquad (4.30)$$

d) $$r(\theta) = a\{(D/a)\cos(\theta+\gamma-\pi) + [(D/a)^2\cos^2(\theta+\gamma-\pi) + (r_c/a)^2 - (D/a)^2]^{\frac{1}{2}}\} \quad ,$$

$$\frac{dr}{d\theta} = a\{(D/a)\sin(\pi-\gamma-\theta) + (D/a)^2\cos(\pi-\gamma-\theta)\sin(\pi-\gamma-\theta)$$

$$\times [(D/a)^2\cos^2(\pi-\gamma-\theta) + (r_c/a)^2 - (D/a)^2]^{-\frac{1}{2}}\}$$

$$\text{for} \quad \phi_3 \leq \theta \leq \phi_4 \quad , \qquad (4.31)$$

e) $r(\theta) = a(-1/\cos\theta)$,

$$\frac{dr}{d\theta} = a(-\sin\theta/\cos^2\theta) \qquad \text{for } \phi_4 \leq \phi < \pi \ . \qquad (4.32)$$

Note that, the values of r and $dr/d\theta$ for the range $\pi \leq \theta \leq 2\pi$ are computed by first setting $\theta \rightarrow \theta - \pi$ and then applying again the formulas given in Eqs. (4.28-32).

The polar plots of the scattered wave field results obtained for various ka values and the ratios (b/a), (r_c/a) are presented in Figs. 4.19 to 4.34.

4.3.4 Triangular Cylinders

We have also considered a scatterer with cross section in the form of an isosceles triangle, as shown in Fig. 4.7. The expressions for the boundary and $dr/d\theta$ in the range $0 \leq \theta < \pi$ are

a) $r(\theta) = h[1/(3\cos\theta)]$,

$$\frac{dr}{d\theta} = h[\sin\theta/(3\cos^2\theta)] \qquad \text{for} \qquad 0 \leq \theta < \phi \ , \qquad (4.33)$$

b) $r(\theta) = h\{(2/3)\tan(\beta/2)/[\sin\theta - \cos\theta\tan(\beta/2)\}$,

$$\frac{dr}{d\theta} = h\{(-2/3)\tan(\beta/2)[\cos\theta + \sin\theta\tan(\beta/2)]/[\sin\theta - \cos\theta\tan(\beta/2)]^2\} \quad \text{for } \phi \leq \theta < \pi \ , \qquad (4.34)$$

where the angle ϕ is given by

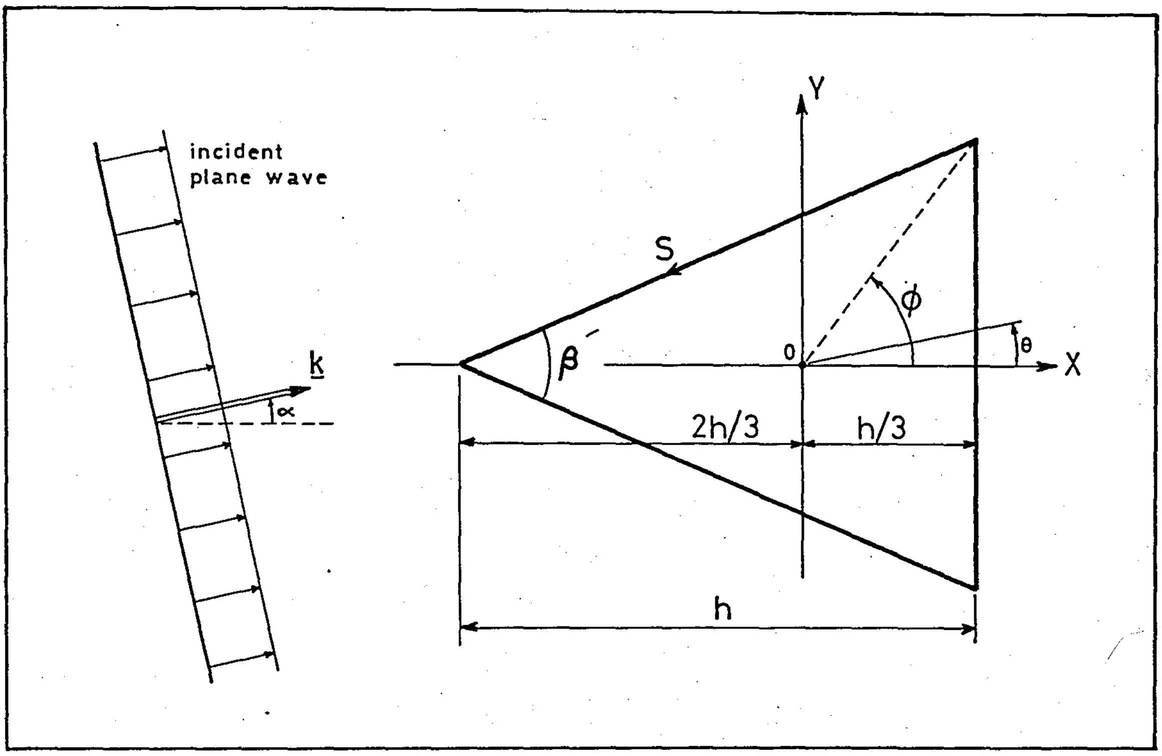


Figure 4.7 - Boundary geometry of the triangular cylinder.

$$\phi = \arctan[3\tan(\beta/2)] \quad . \tag{4.35}$$

Values of $r(\theta)$ and $dr/d\theta$, in the range $\pi \leq \theta < 2\pi$, are obtained by setting $\theta \rightarrow 2\pi-\theta$ in the Eqs. (4.33) and (4.34), and then setting $dr/d\theta$ to $-dr/d\theta$.

The scattered field results obtained for various kh and β values are presented in Figs. 4.35 to 4.41.

V. CONCLUSIONS

In this work, the numerical results for acoustic plane waves scattered by circular, elliptical, rectangular and triangular rigid inclusions and cavities have been obtained utilizing the T-matrix method. The results are presented in polar graphical forms. These results are compared with the exact or approximate solutions [9,22], where available. For the rectangular and triangular cylinders, however, because of the unavailability of any result obtained in other studies, no comparison can be made.

In the evaluations of the elements of the Q-matrix, fourth order Simpson's rule is used to perform the angular integrations along the boundaries of the scatterers. Good convergence is obtained using 36 integration steps in the range 0-π. For triangular geometry, corners may be thought to give rise to difficulties from the analytical point of view. However, since the integrals are evaluated by considering only discrete points taken within each angular interval, no computational problem is encountered. The symmetry and other numerical properties given by Eqs. (3.92), (3.94) and (3.95) is effectively used to check the accuracy of the numerical procedure followed.

The T-matrix is constructed by inverting the Q-matrix using Gauss-Schmidt orthogonalization technique, as discussed in Section 4.2.

The requirements given by Eqs. (3.99) and (3.100) are first used to check the correctness of the computer program then they are, together with Eqs. (3.92), (3.94), (3.95), incorporated into the program reducing computing time significantly.

It is observed that the size of the T-matrix to be used is related to the non-dimensional number kr_{max}, where k is the wave number and r_{max} is the maximum r value which is to be encountered along the boundary of the scatterer. In order to be able to obtain sufficiently convergent results, the size of the T-matrix is generally increased in a direct proportion to the non-dimensional number kr_{max}. The sufficiency of the matrix size, however, is controlled by checking the convergency of the total scattering cross-section, Eq. (3.120). This check is made up to when the total scattering cross section did not differ by more than at least 10^{-5} per cent. A rough estimate for the size requirement of the T-matrix may be given as 6 x 6 for $kr_{max} = 0.1$, 15 x 15 for $kr_{max} = 1.0$ and 40 x 40 for $kr_{max} = 5.0$.

In the case of scattering by circular cylinders, both near (Fig. 4.8) and far field (Figs. 4.9, 3.10) results are found to be in excellent agreement with the exact solutions for all wave numbers, [22]. This is because of the fact that the basis wave functions given by Eqs. (3.1) and (3.5) are exact expressions for the circle and hence the T-matrix formulation yields exact solutions for the circular cross-sections.

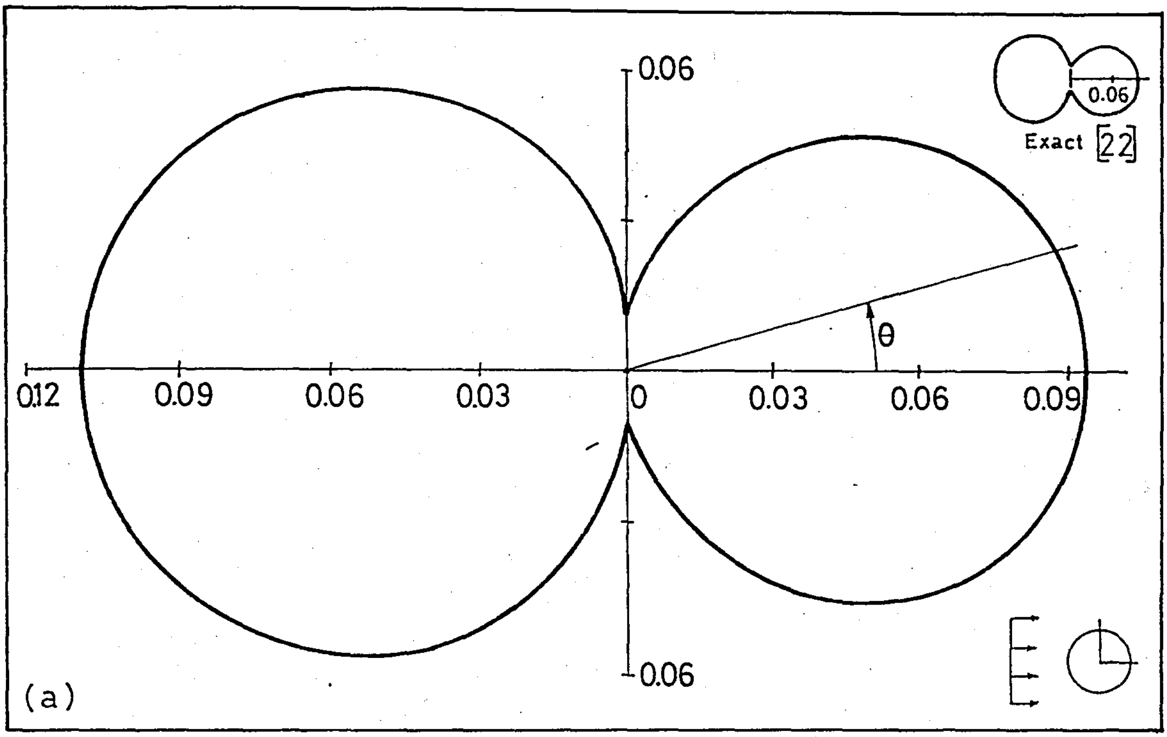
In the case of scattering by elliptical cylinders, the near field results are plotted in Figs. 4.11, 4.12 for the aspect ratios $b/a = 0.5$ and 2.0 and for $\alpha = 0^{0}$. In these figures, we observe that for a given incident wave number the angular spectrum of the scattered field is quite different for two different values of b/a. In the case of scattered far field amplitudes, some of the results were compared with those obtained in Ref. [9] and found to be in good agreement especially for relatively small wave numbers ($ka \leq 1.0$). The far field results have been obtained for the aspect ratios 0.5, 2.0, 5.0 and for different incidence angles and presented in Figs. 4.13-15 for rigid inclusion, and in Figs. 4.16-18 for cavity. In the case of rigid inclusion, the structure of the polar plots varies quite remarkably for different incidence angles and aspect ratios. Especially for the higher aspect ratios, larger angular peaks are observed in the forward direction. However, for a cavity, we observe that, in contrast to the rigid inclusion, the dependency of the structure of the plots on both incidence angle and aspect ratio is not so significant.

The near field results, Figs. 4.19, 4.20, for scatterers with rectangular cross-section show that the surface field potential distributions, comparing with the circular and elliptical geometries, display sharper maxima and minima for all wave numbers. Although, for low frequencies ($ka \leq 1.0$), the shapes of the plots are somewhat similar to the ones obtained in the elliptical case, the picture is entirely different in the high frequency range. For the far field case, we observe that the field scattered by a rectangular cylinder displays more angular structure than that generated by a circular or elliptical cylinder. The far field results obtained for a rigid inclusion (Figs. 4.21-27) indicate that effects of the aspect ratio and the incidence angle on the shape of the polar plots are relatively strong. These effects are more obvious at higher wave numbers ($ka > 1.0$). On the other hand, it is observed that, as shown in Figs. 4.21, 4.22 (rigid inclusion) and in Figs. 4.28, 4.29 (cavity), effect of the corner radius ratio, r_c/a is not significant for a given aspect ratio and the wave number. In this respect, the scattered field results pertaining to the rectangular geometry are presented for $r_c/a = 0.1$ only. In the case of a cavity (Figs. 4.28-34), as for the rigid inclusion, the structure of the far field plots exhibits similar behaviour with respect to the changes in the aspect ratio, incidence angle, and frequency; however, the curves for the cavity are relatively smooth. A comparison of the far field results obtained for a cavity and a rigid inclusion cases reveals that for identical surface geometries no similarity exists for any given wave number in the range $0.1 \leq ka \leq 1.0$. However, at the higher wave numbers the cavity cannot be clearly distinguished from a rigid inclusion.

In the case of cylinders with triangular cross-sections, the near field solution given in Fig. 4.35 are quite different when compared to the previous geometries. This difference may be attributed to the fact that the cross-sectional geometry of the cylinder has only single symmetry axis. The far field results both for cavity and rigid inclusion, Figs. 4.36-41, are given for three different tip angles, $\beta = 60^0$, 30^0 and 0.001^0. For $\beta = 60^0$ and 30^0, we observe that if $\alpha \neq 0^0$ the far field plots are quite similar to the ones obtained for the rectangular cross-sections, while for $\alpha = 0^0$ structure of the plots differs remarkably when compared to the other geometries. Especially at the higher frequencies, for $\alpha = 0^0$ comparably larger angular peaks are observed in the forward scattering than in the backward scattering. For the case $\beta = 0.001^{o}$ where the shape of the scatterer looks like a strip rather than a triangle, as shown in Figs. 4.38 and 4.41 general form of the far field plots is quite different.

In the polar plots of the scattered field results, it is observed that at low frequencies, $kr_{max} \sim 0.1$, the pictures for all boundary geometries are almost the same in appearance and do not change with incidence angle, α, hence, for all cases, the obstacles behave like point scatterers. Thus low frequency results are not useful for practical applications. However, for higher frequencies, quite remarkable differences and angular variations are observed in the structures of the plots for different boundary geometries and incidence angles.

To sum up, the T-matrix method, using only circular or spherical wave functions and removing the geometrical restriction, makes it possible to analyze the scattering of waves from inclusions of any shape. Although the completeness of the series representation of the wave fields, especially in the near field, is still a matter of discussion, the method provides a systematical and powerful computational procedure for a very wide range of wave numbers.



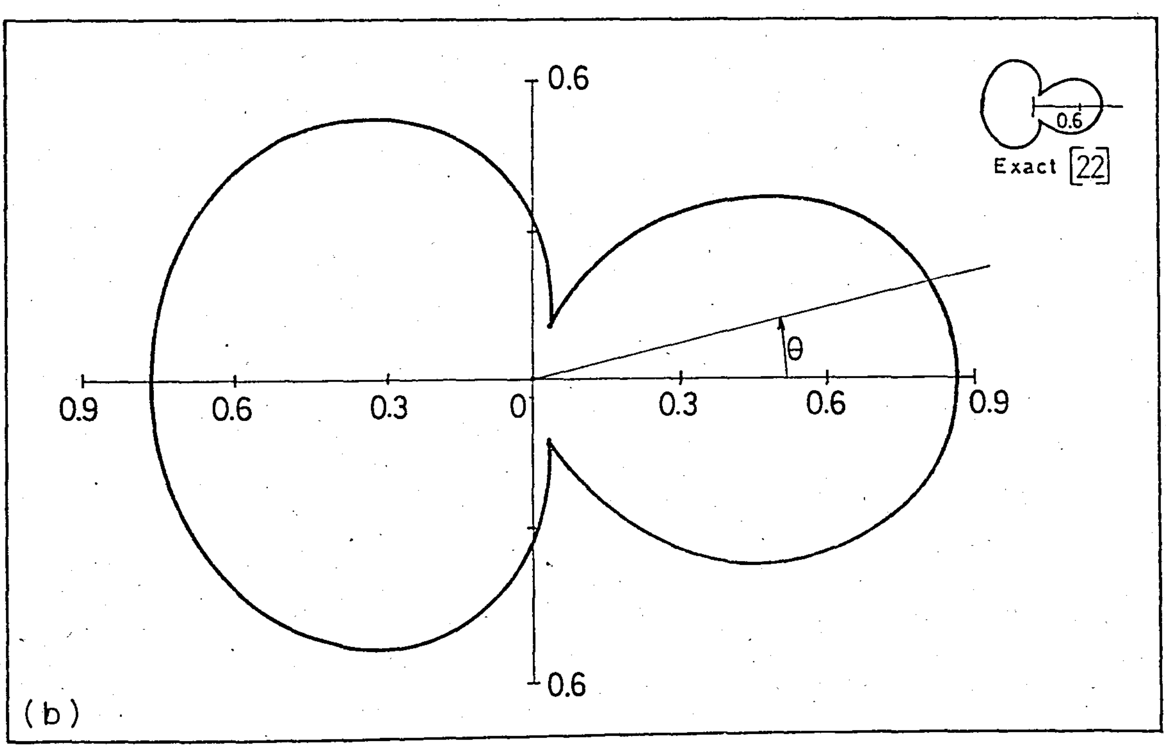


Figure 4.8 - Velocity potential distribution, $|u^S/A|$, at the boundary of a rigid circular inclusion due to the scattered wave field;
(a) ka = 0.1, (b) = 1.0, (c) ka = 5.0

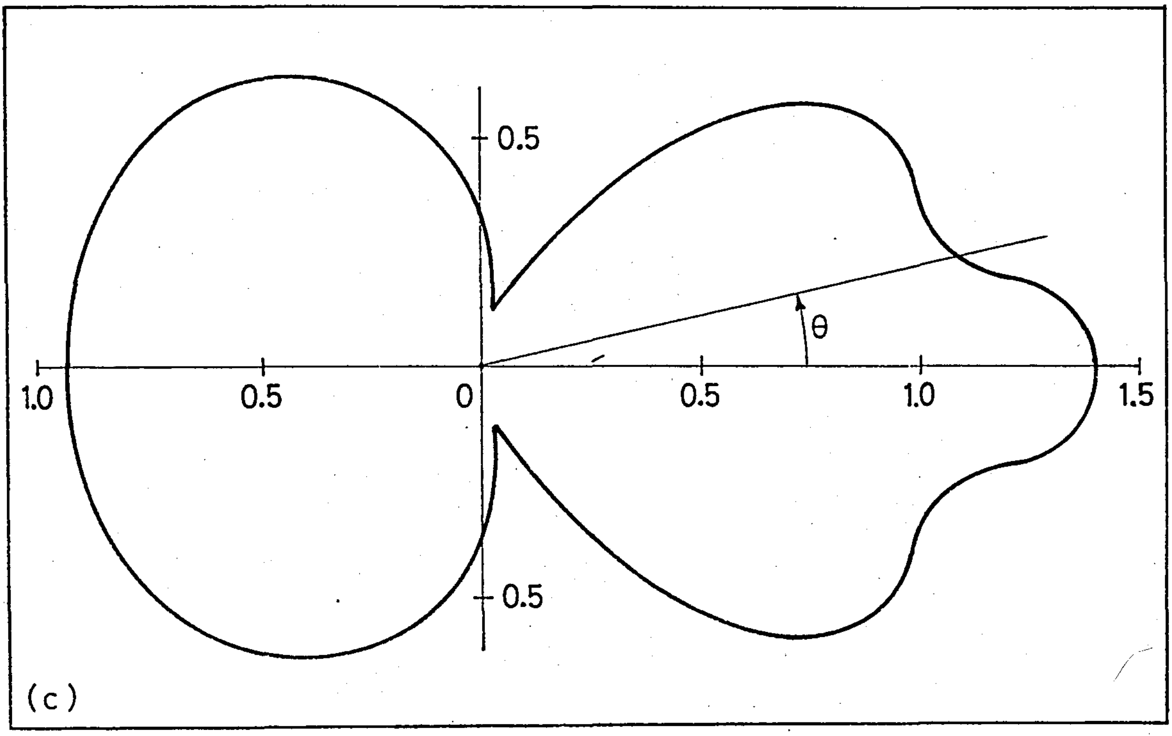
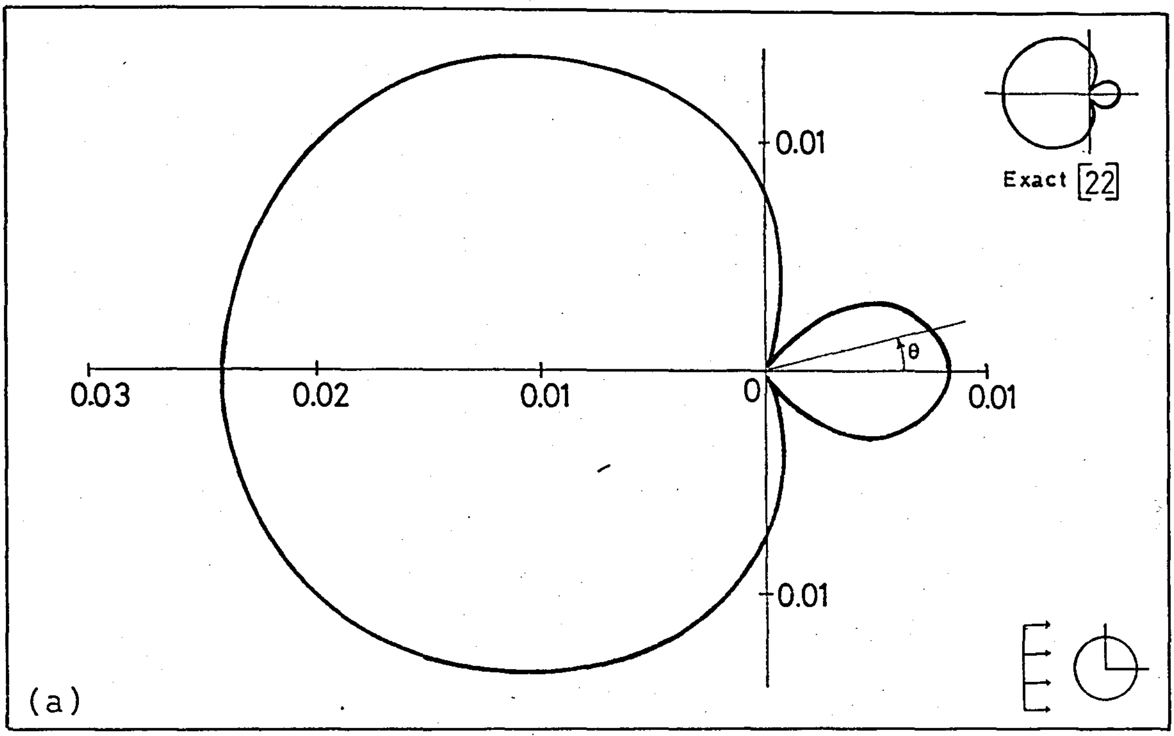


Figure 4.8 (continued).



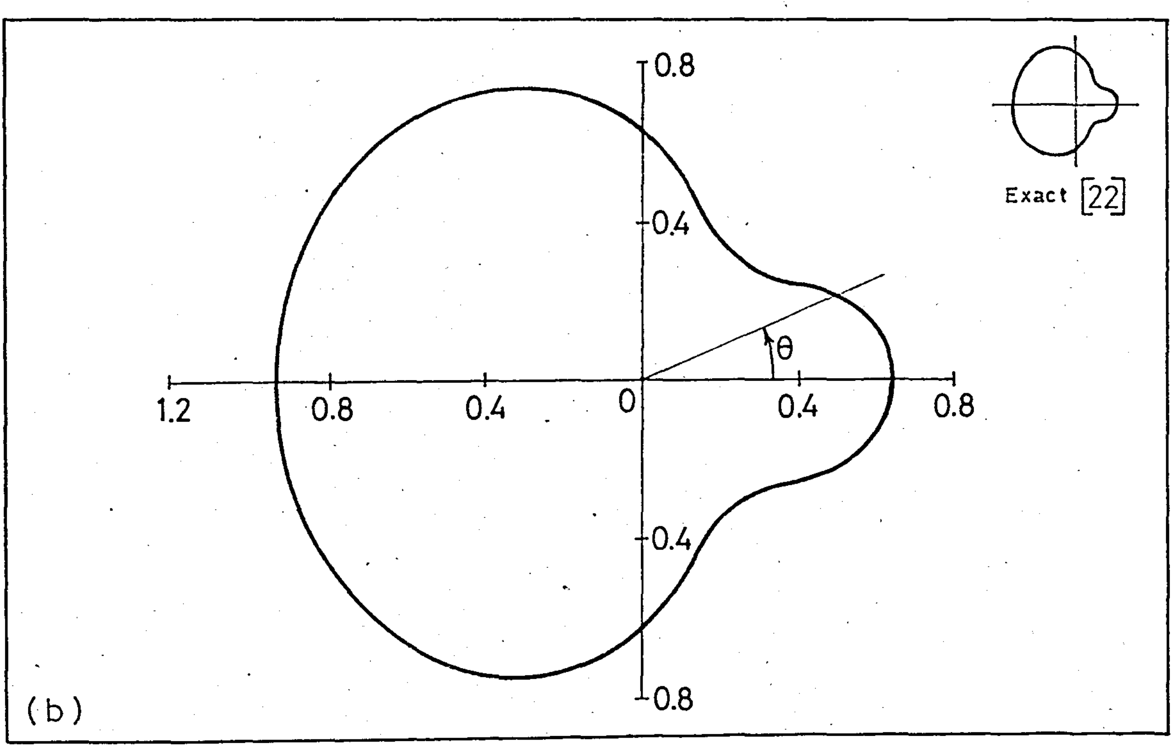


Figure 4.9 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid circular inclusion; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0 .

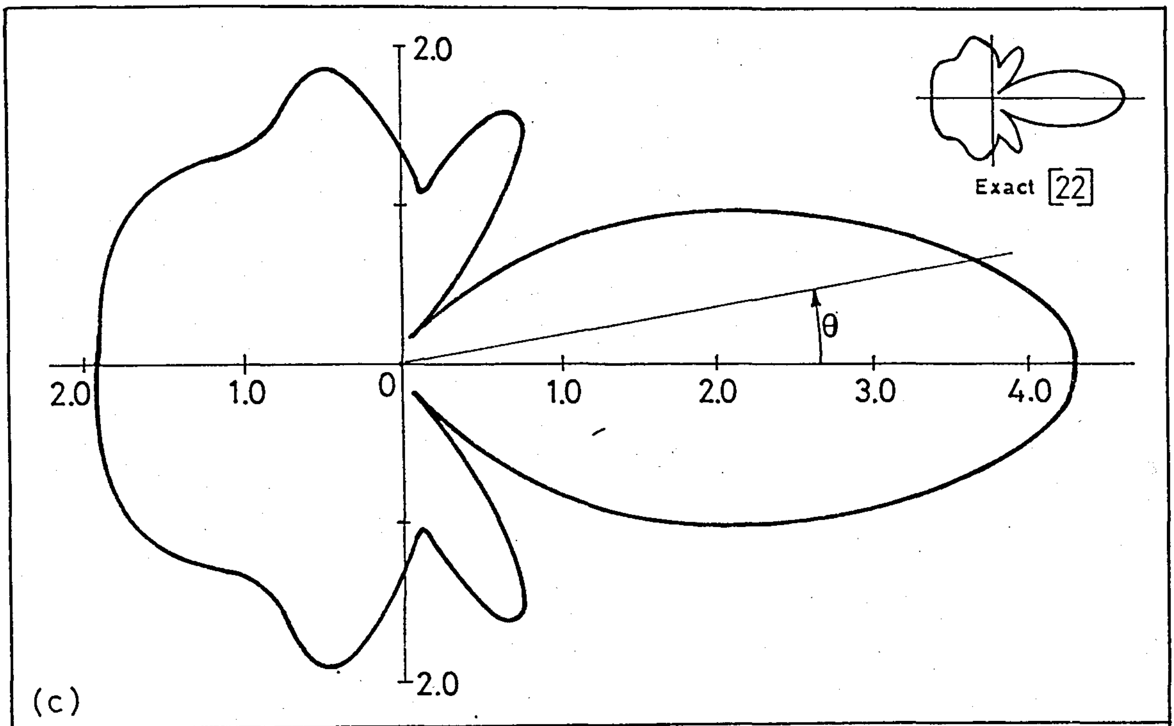
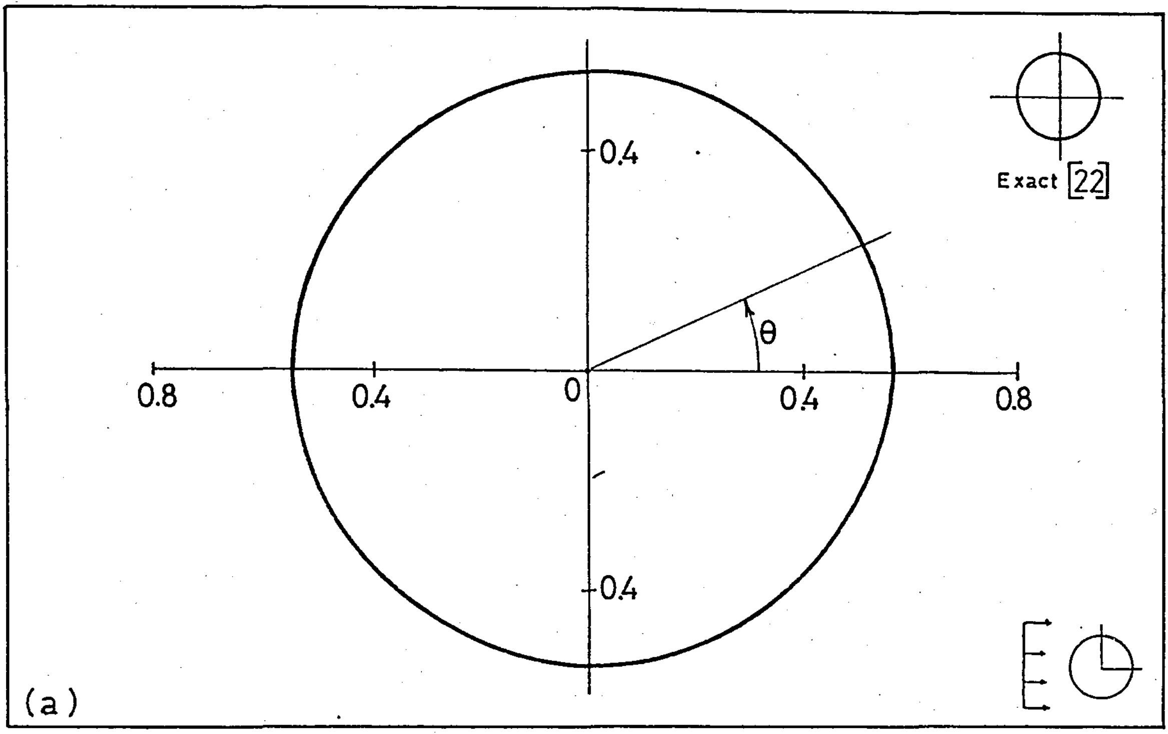


Figure 4.9 (continued).



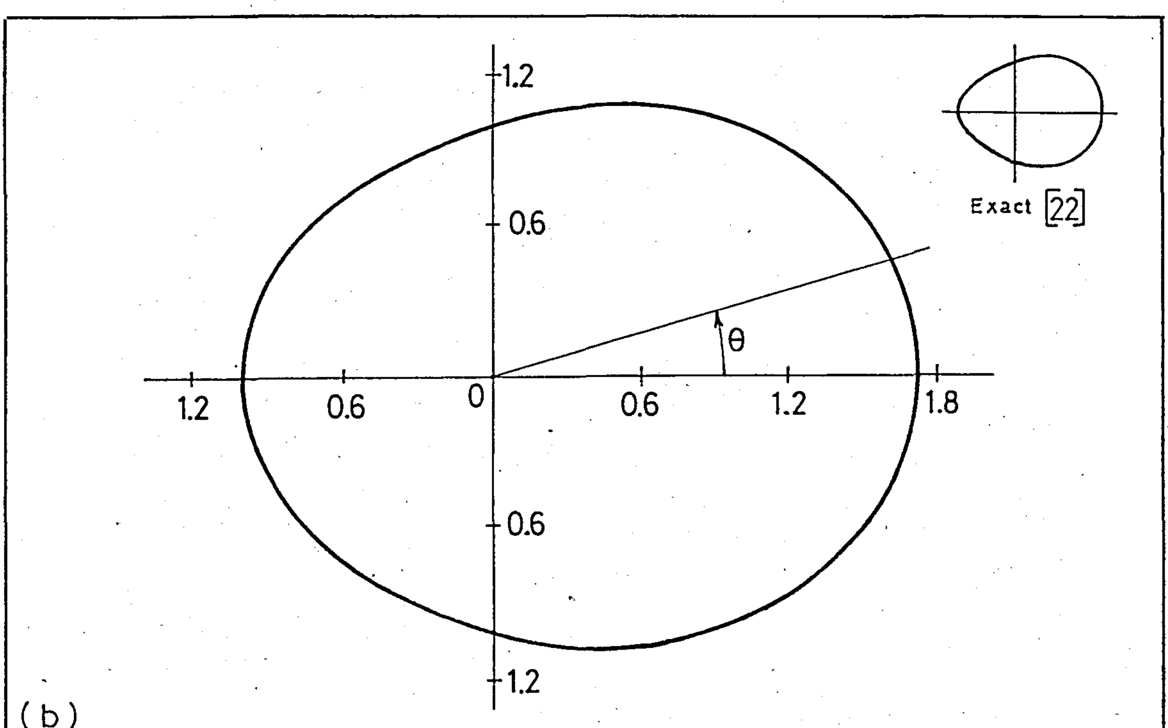


Figure 4.10 - Far field amplitude, $|f/A|$, due to the scattered wave field from a circular cavity; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.

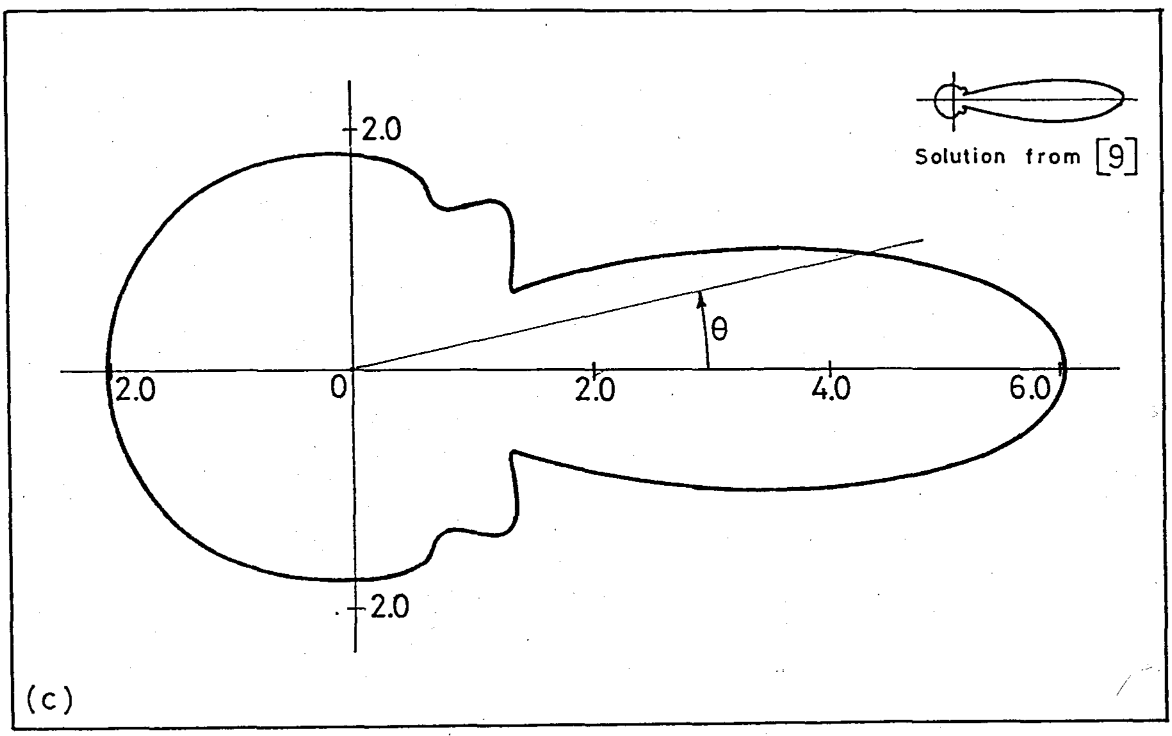


Figure 4.10 (continued).

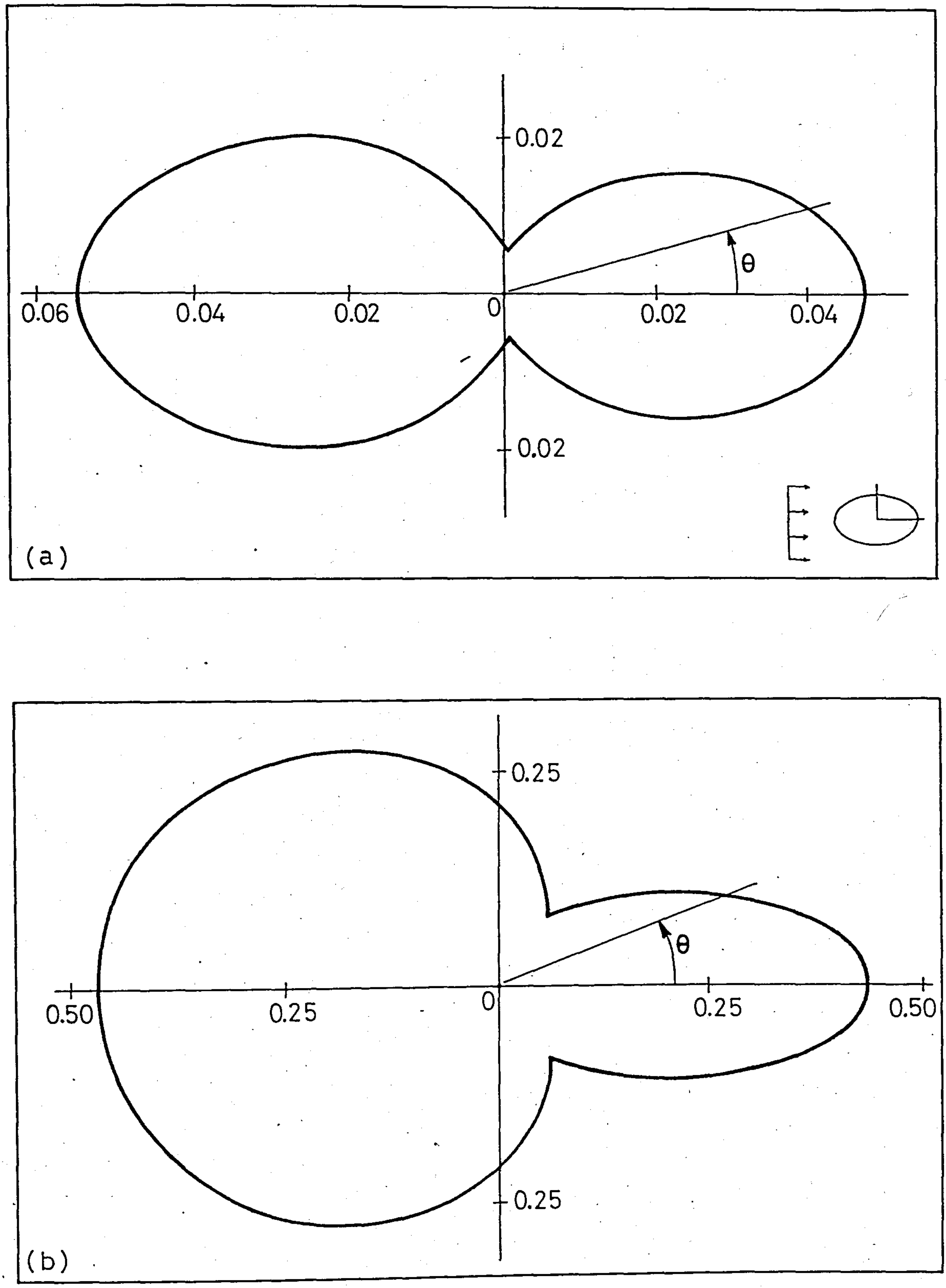


Figure 4.11 - Velocity potential distribution, $|u^s/A|$, at the boundary of a rigid elliptical inclusion due to the scattered wave field for $\alpha = 0^o$ and $b/a = 0.5$; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.

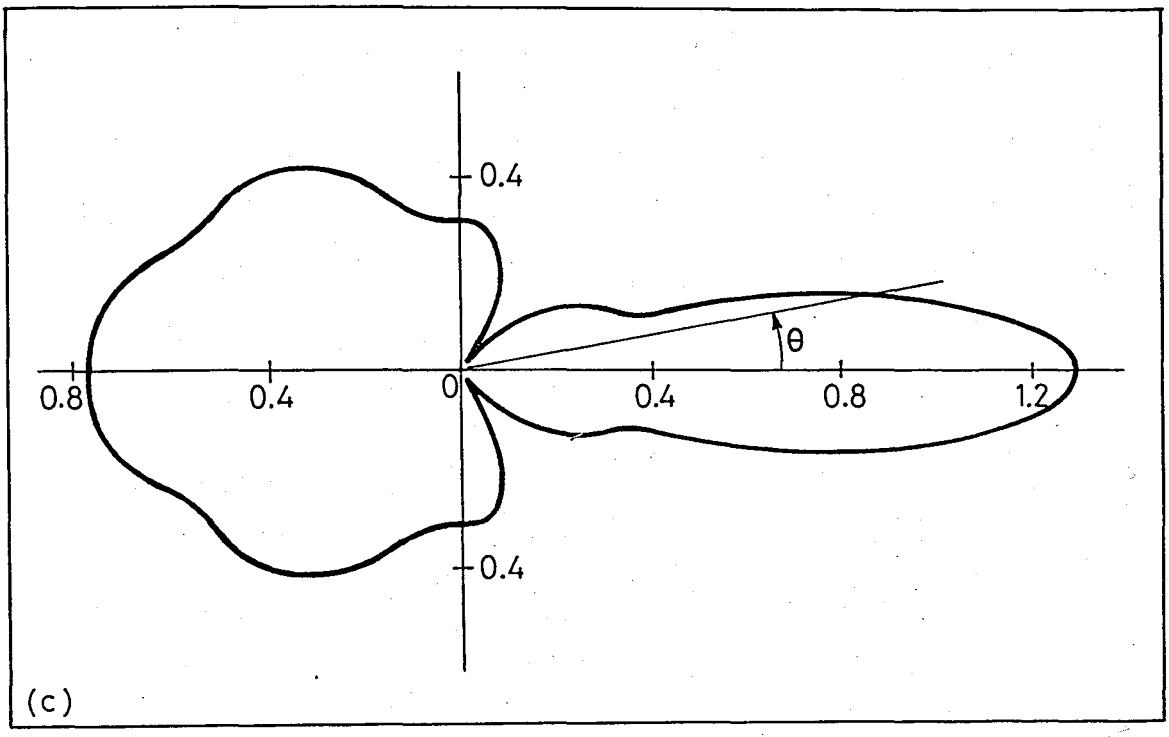
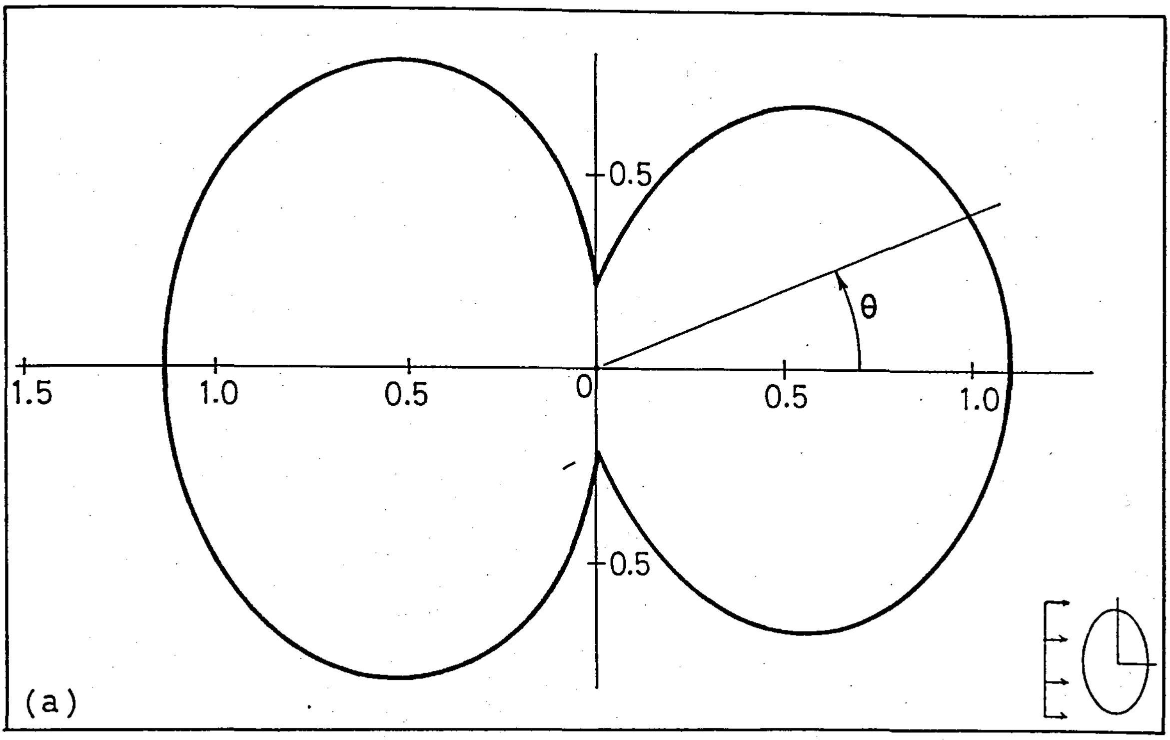


Figure 4.11 (continued).



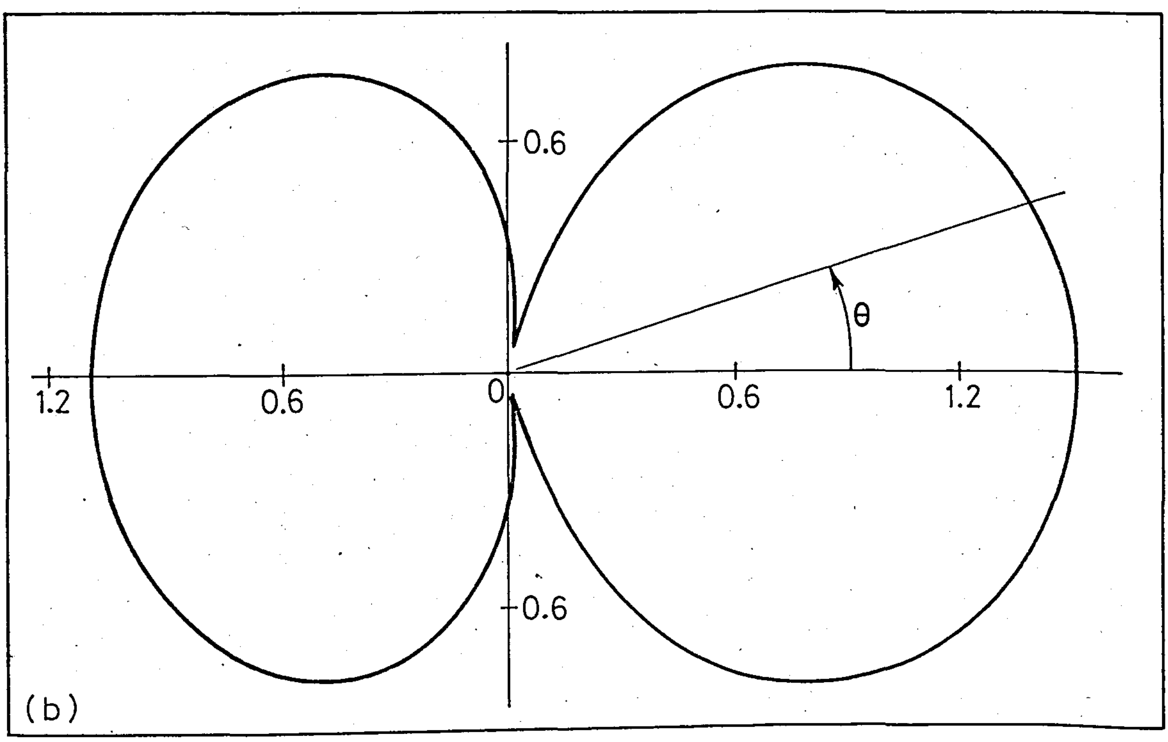


Figure 4.12 - Velocity potential distribution, $|u^s/A|$, at the boundary of a rigid elliptical inclusion due to the scattered wave field for $\alpha = 0^o$ and $b/a = 2.0$; (a) $ka = 0.5$, (b) $ka = 1.0$, (c) $ka = 3.0$

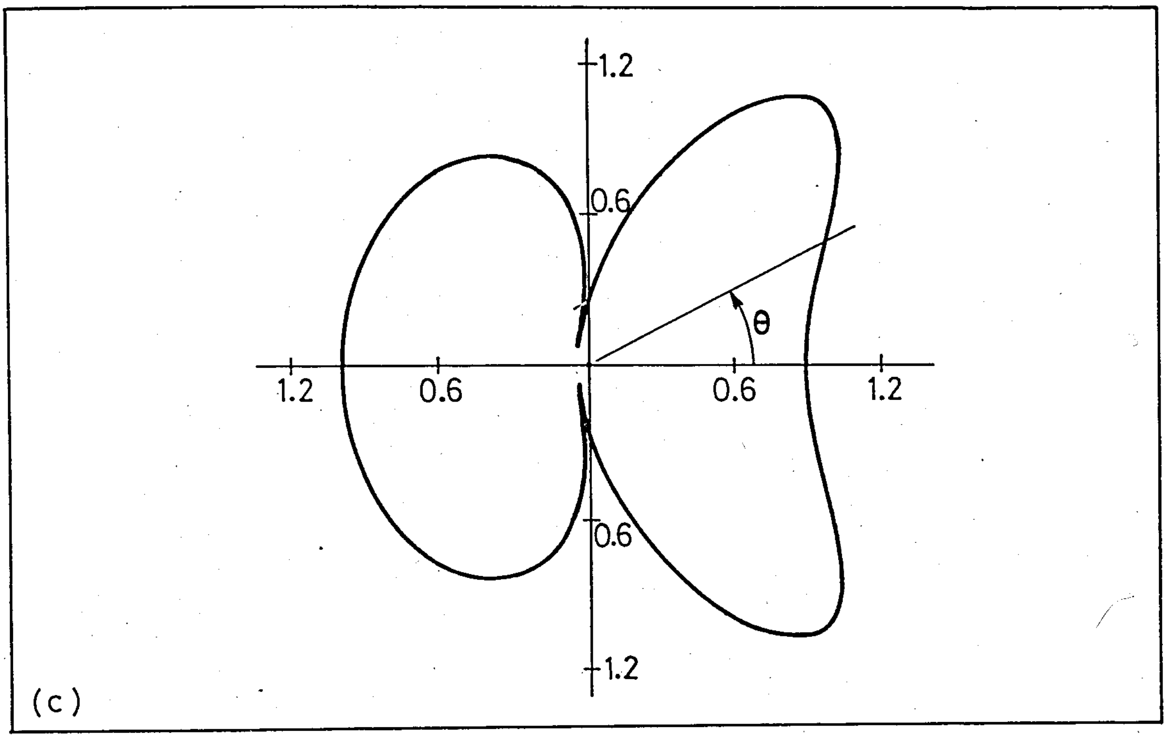
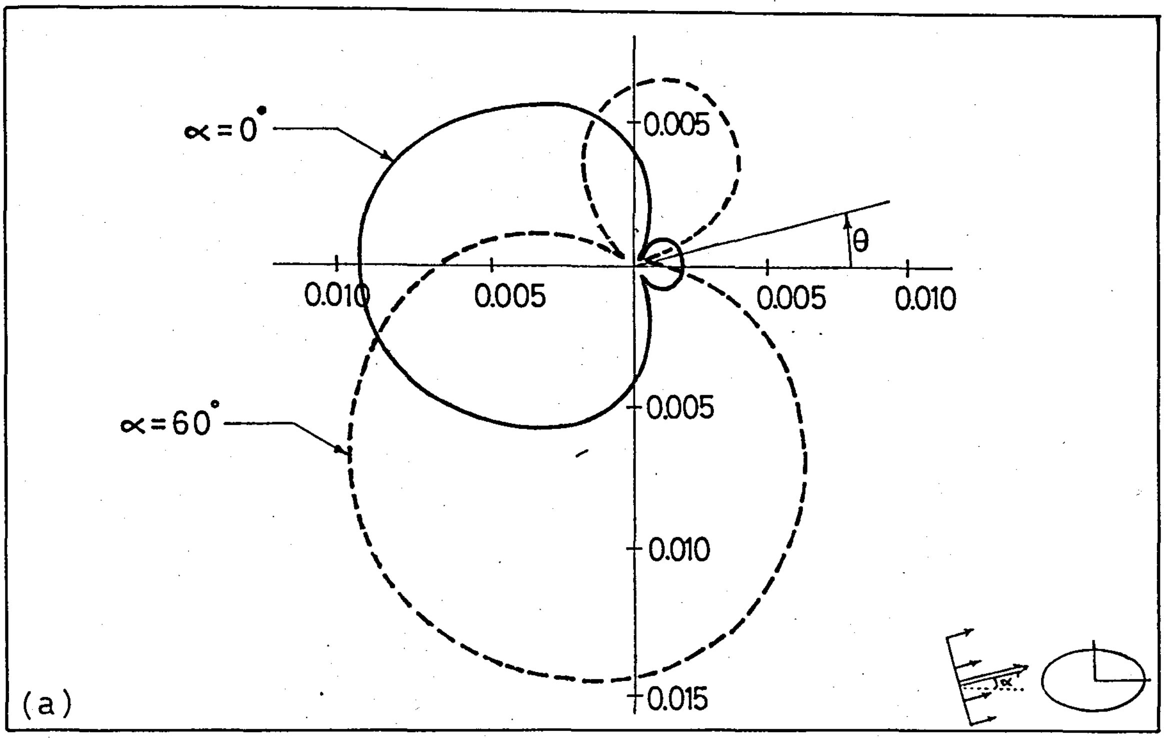


Figure 4.12 (continued).



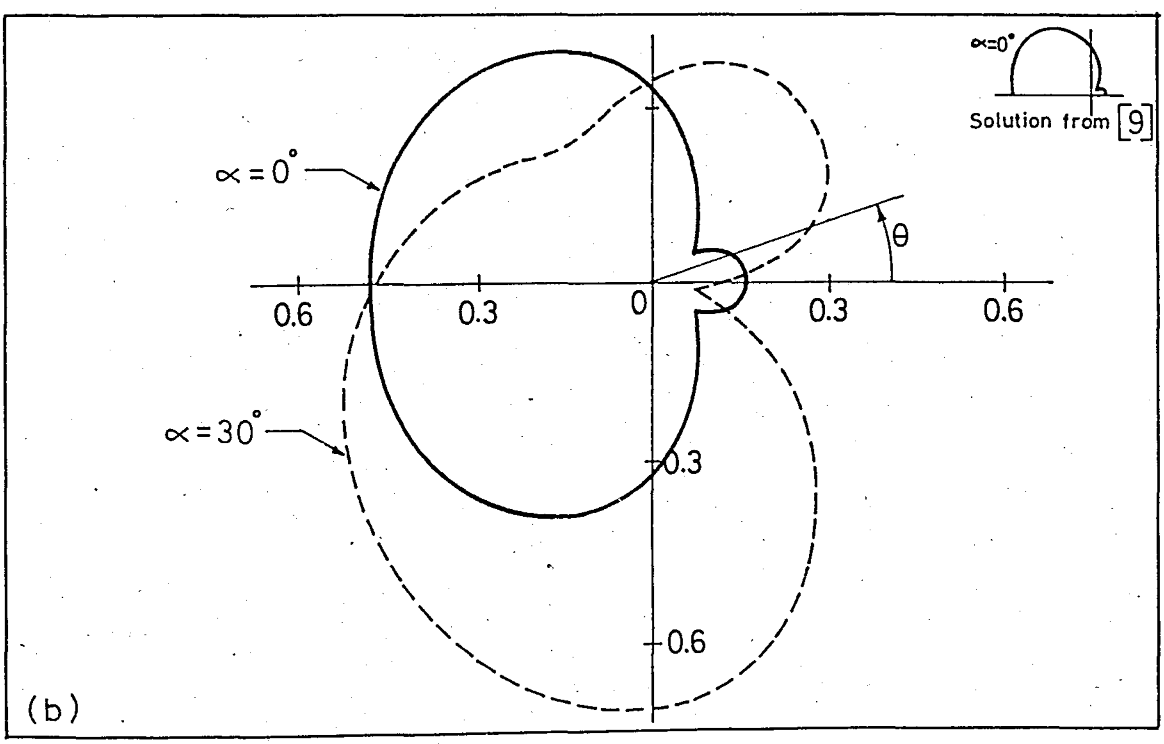


Figure 4.13 - Far field amplitude, $|f/A|$, due to scattered wave field from a rigid elliptical inclusion for $b/a = 0.5$;
(a) $ka = 0.1$, (b) $ka = 1.0$, (c) $ka = 5.0$.

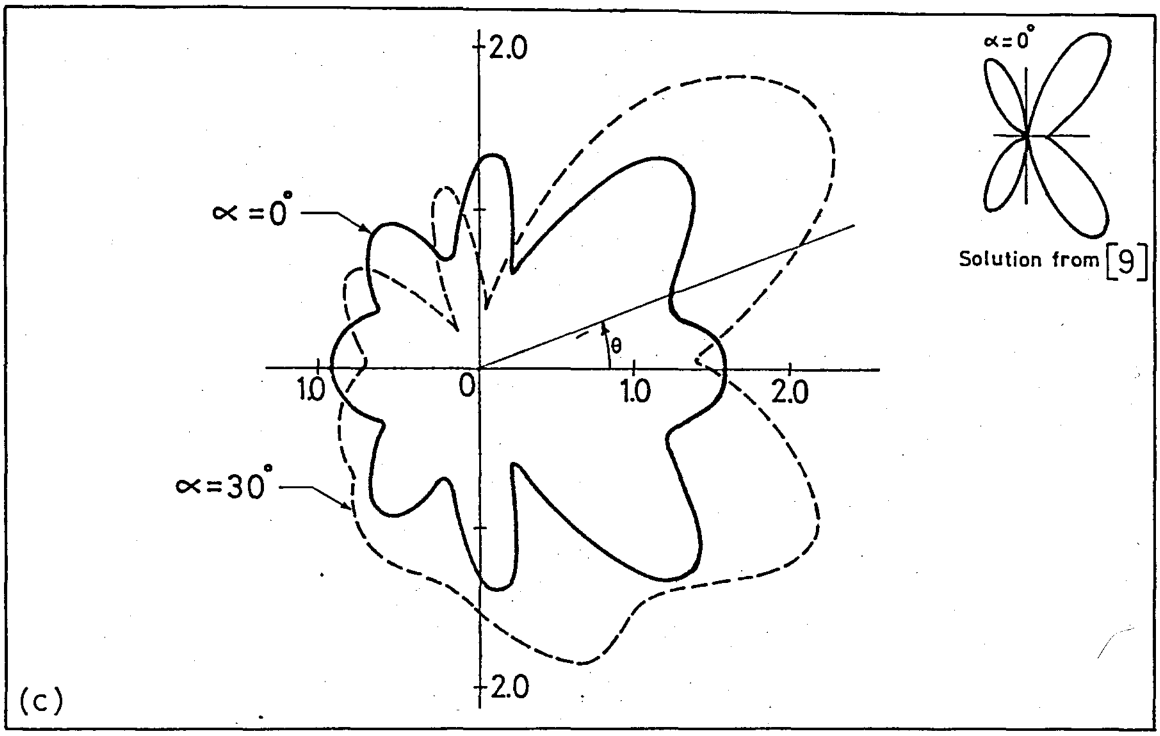


Figure 4.13 (continued).

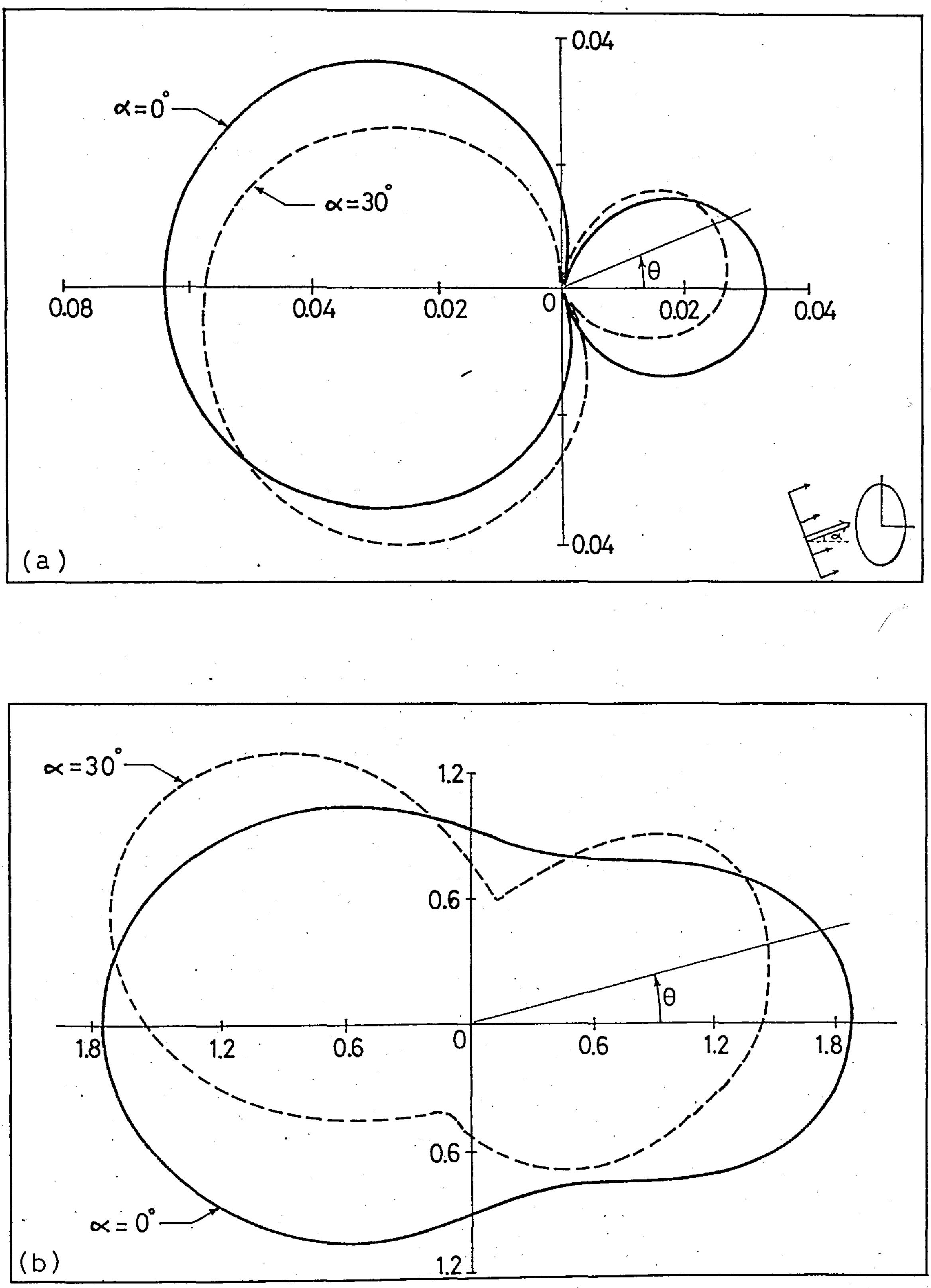


Figure 4.14 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid elliptical inclusion for $b/a = 2.0$;
(a) $ka = 0.1$, (b) $ka = 1.0$, (c) $ka = 3.0$.

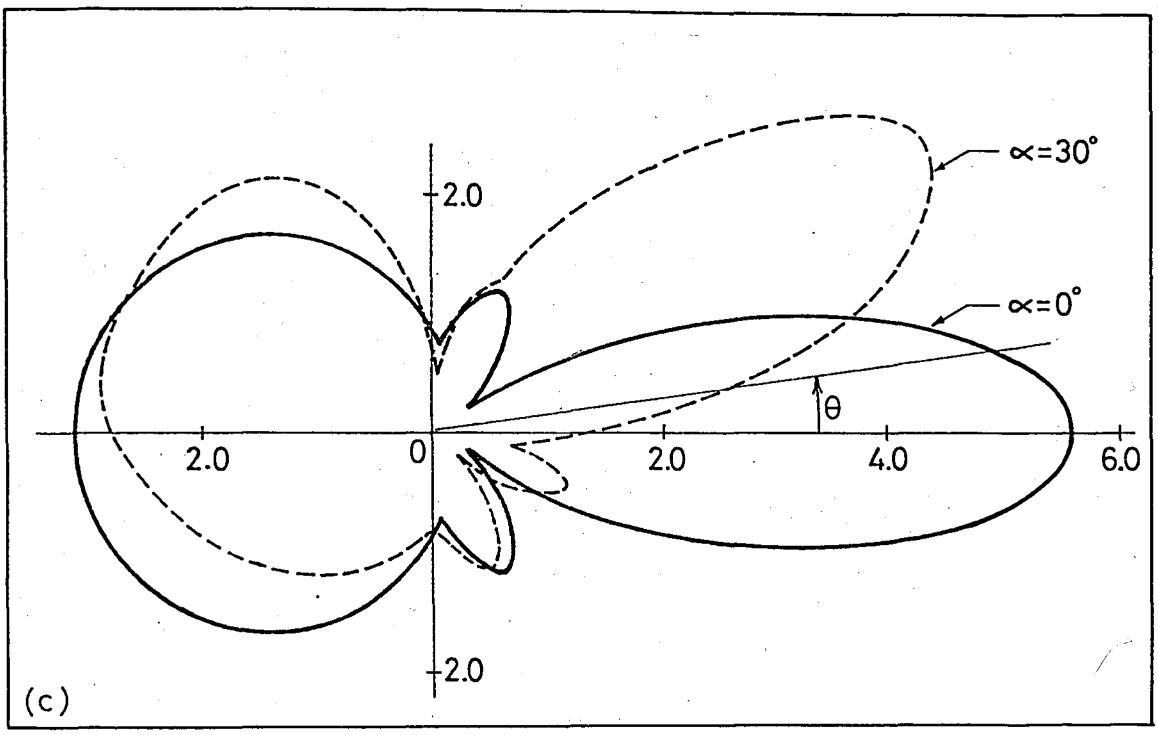
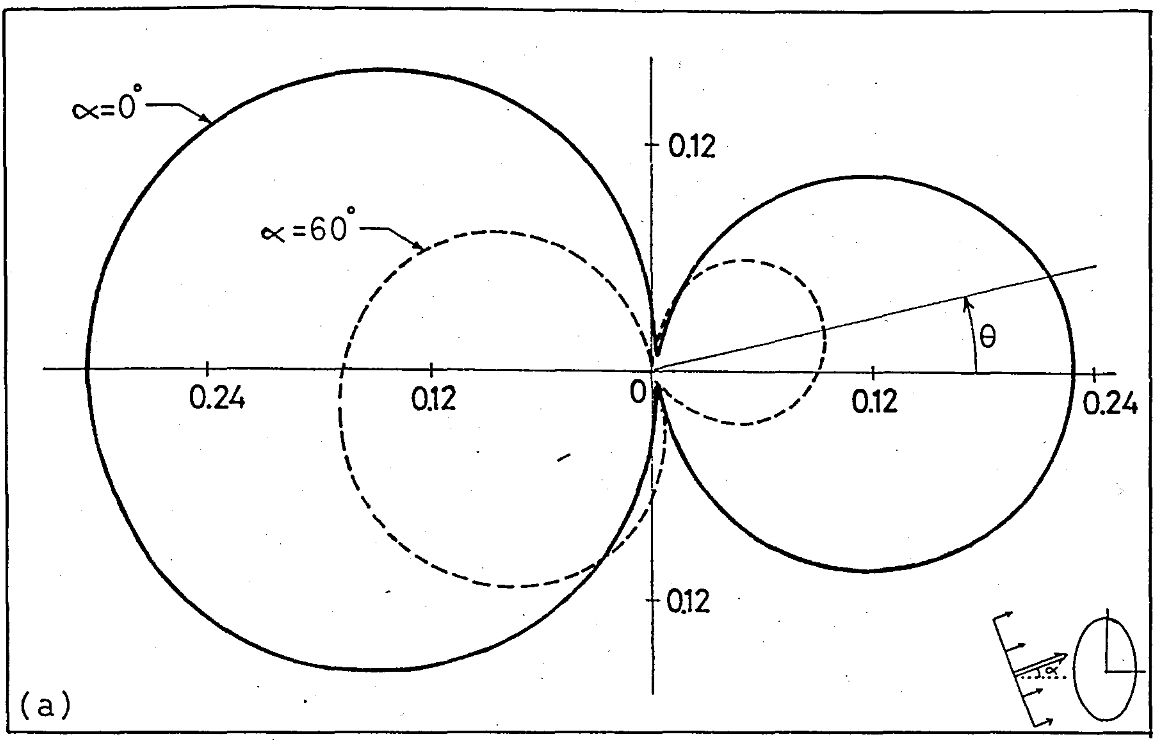


Figure 4.14 (continued).



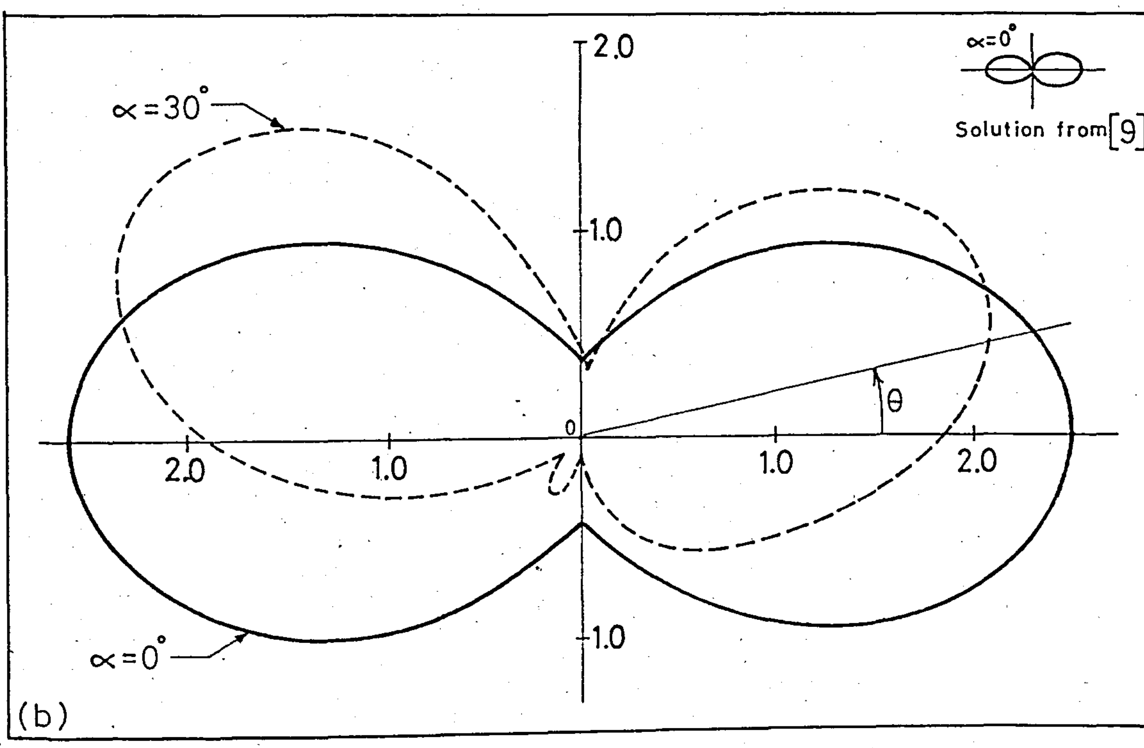


Figure 4.15 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid elliptical inclusion for $b/a = 5.0$;
(a) $ka = 0.1$, (b) $ka = 0.5$, (c) $ka = 1.0$.

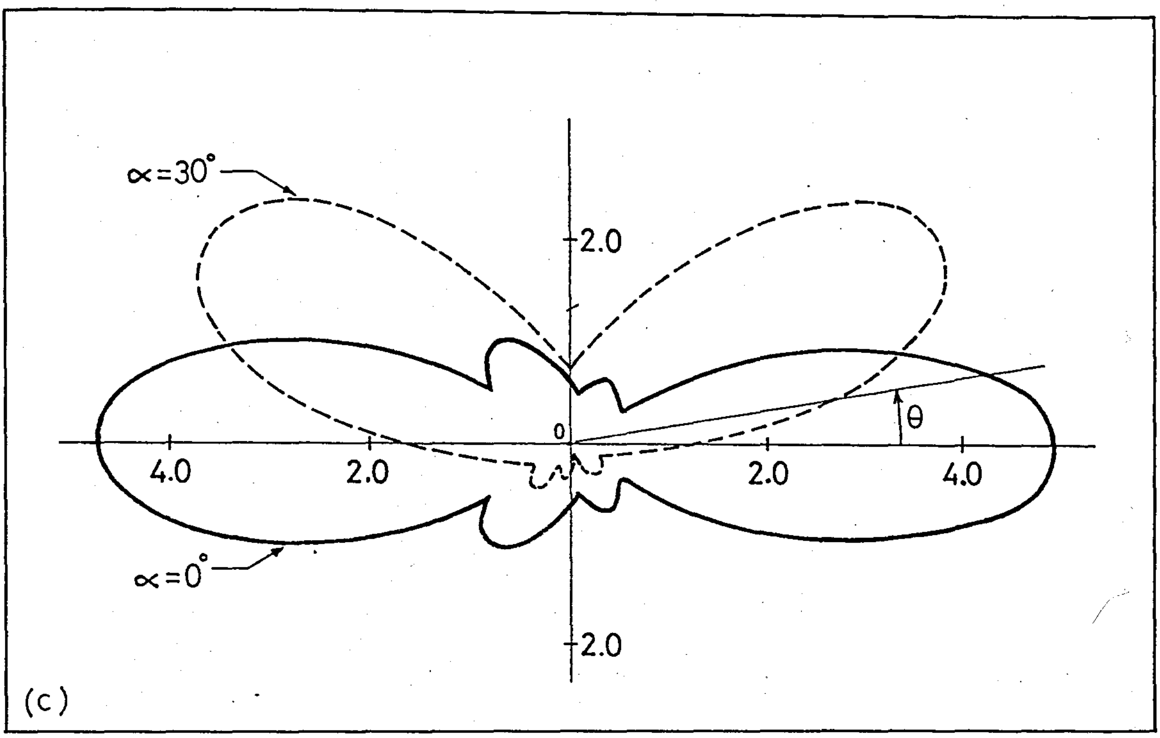


Figure 4.15 (continued).

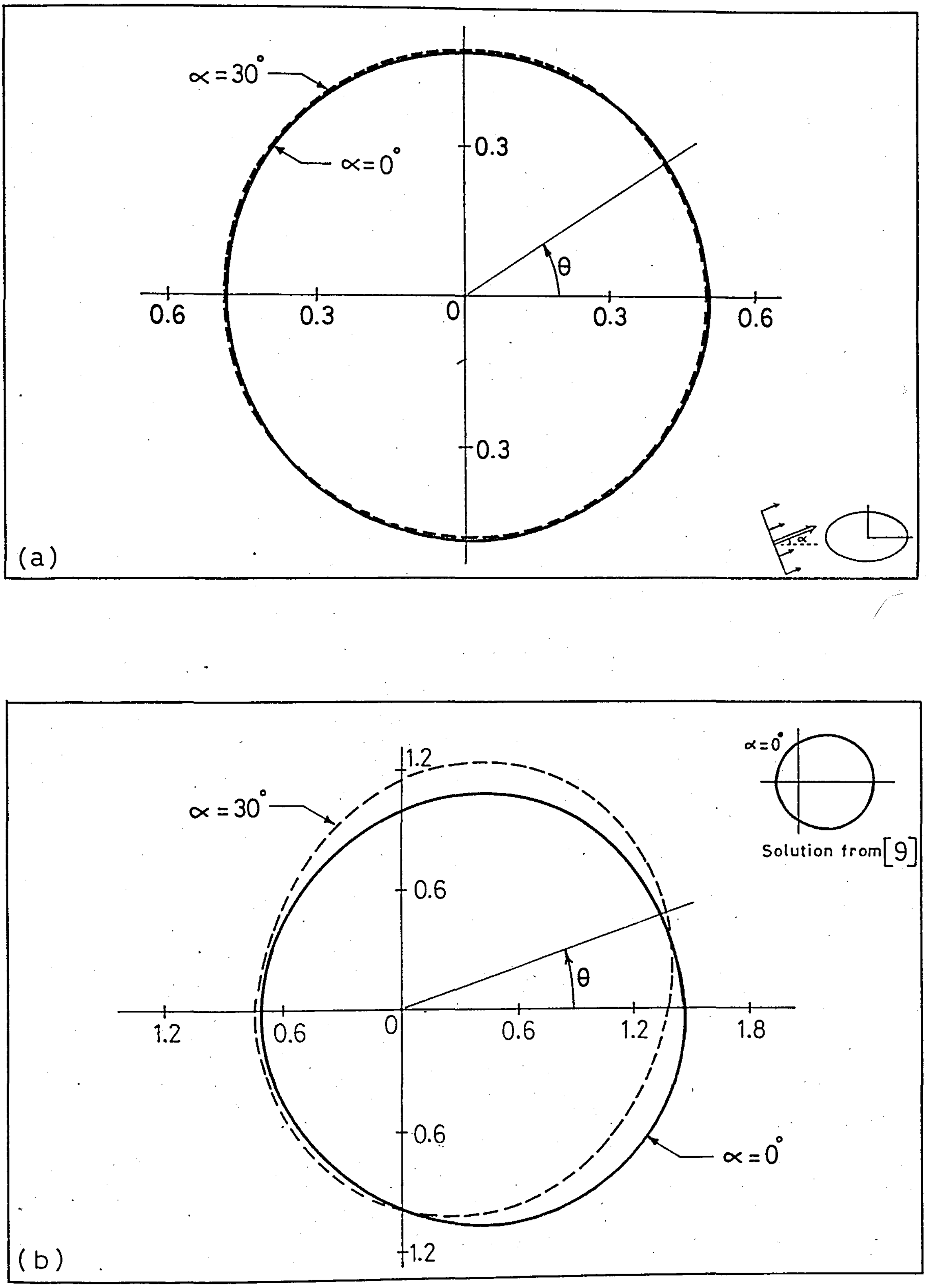


Figure 4.16 - Far field amplitude. |f/A|, due to the scattered wave field from an elliptical cavity for b/a = 0.5; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.

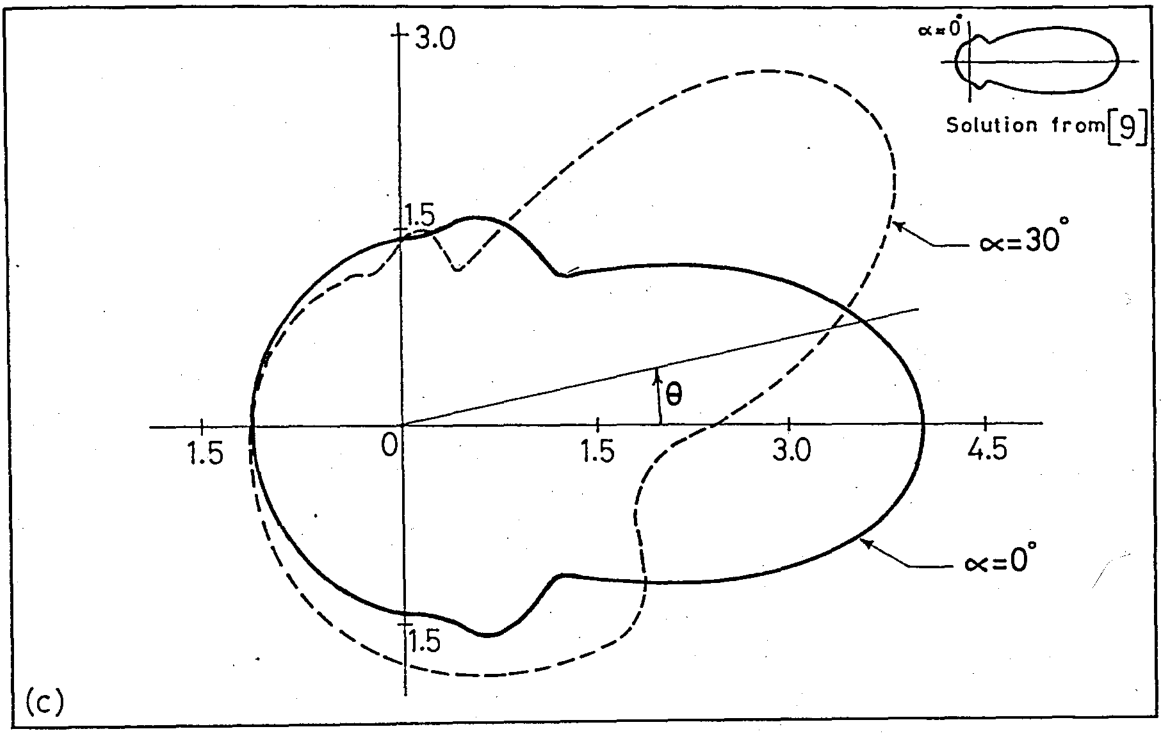


Figure 4.16 (continued).

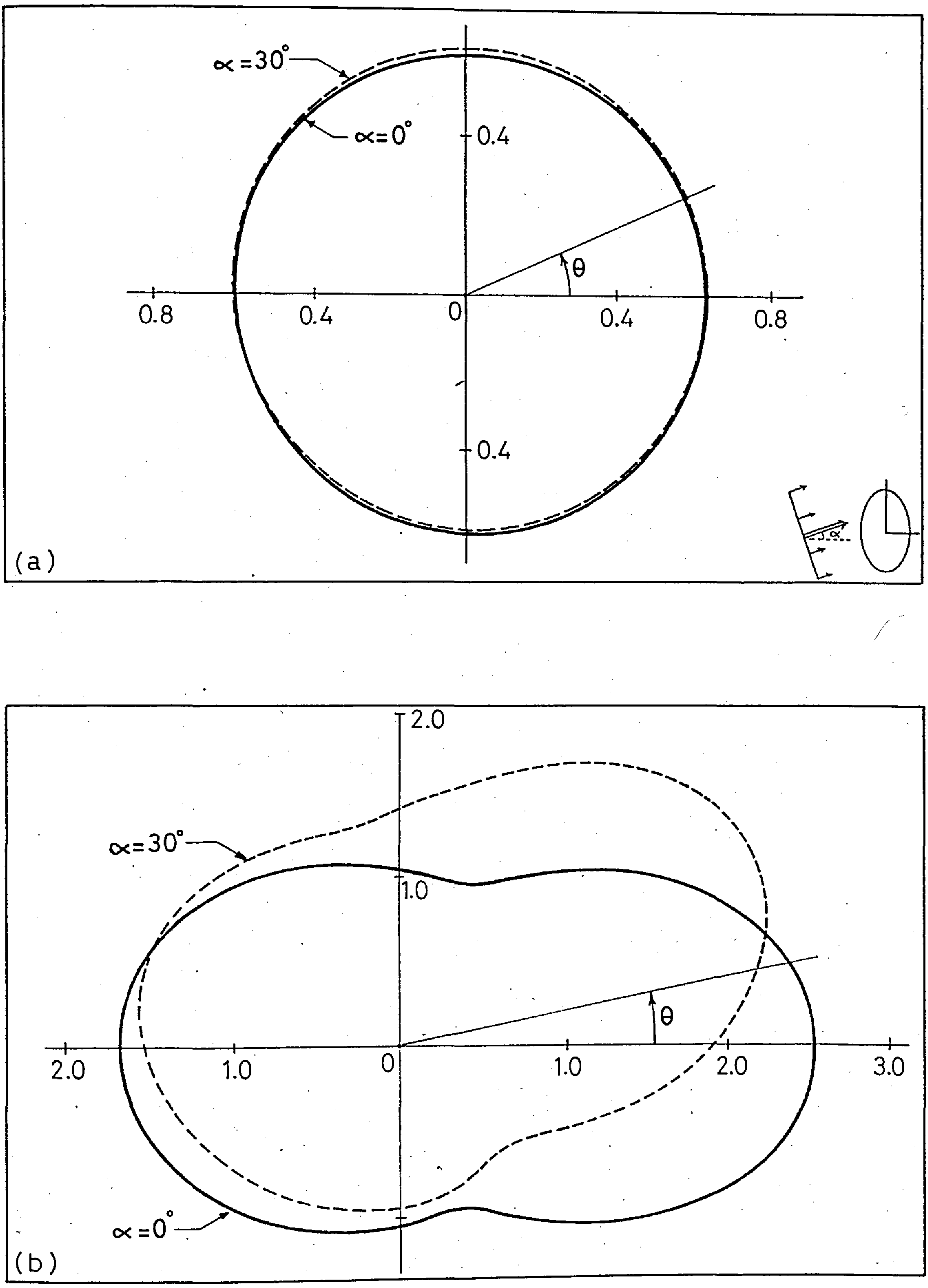


Figure 4.17 - Far field amplitude, $|f/A|$, due to the scattered wave field from an elliptical cavity for $b/a = 2.0$; (a) $ka = 0.1$, (b) $ka = 1.0$, (c) $ka = 3.0$.

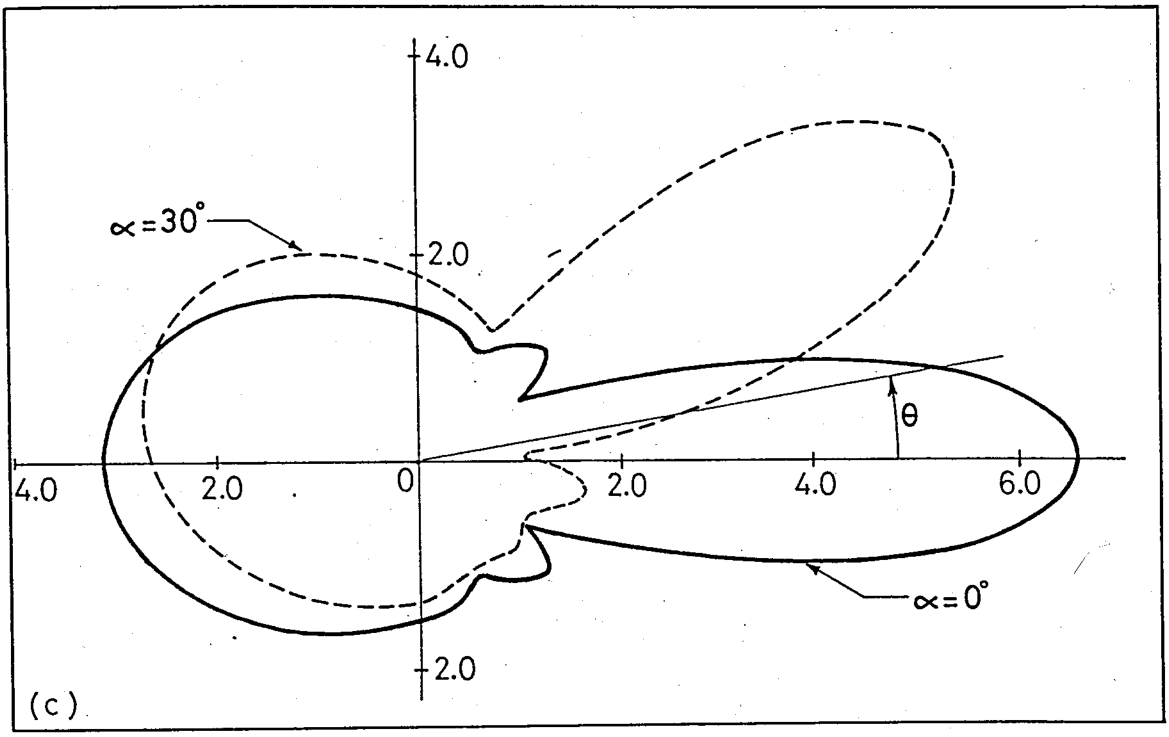
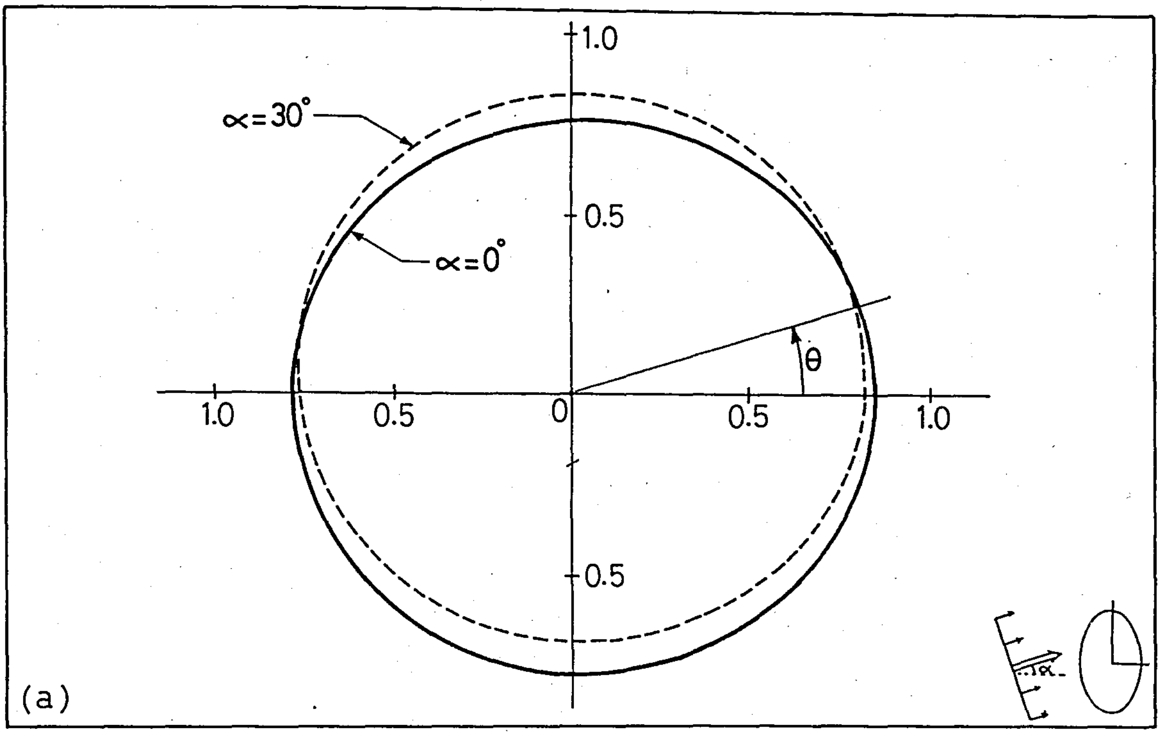


Figure 4.17 (continued).



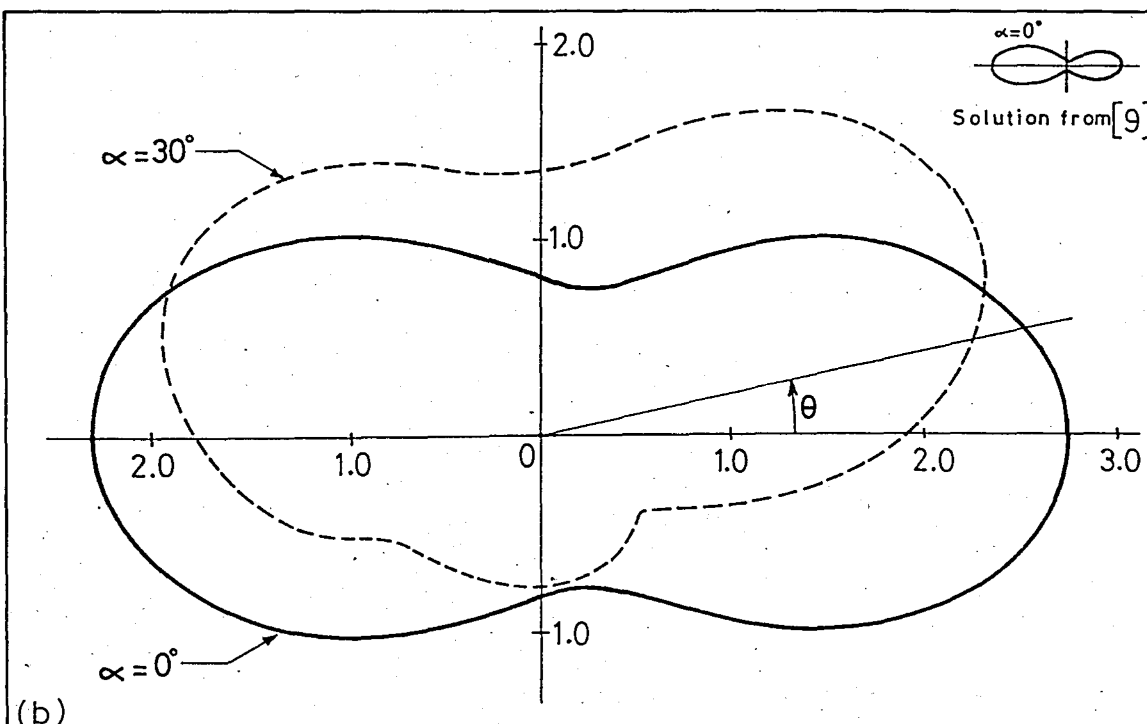


Figure 4.18 - Far field amplitude, $|f/A|$, due to the scattered wave field from an elliptical cavity for b/a = 5.0; (a) ka = 0.1, (b) ka = 0.5, (c) ka = 1.0.

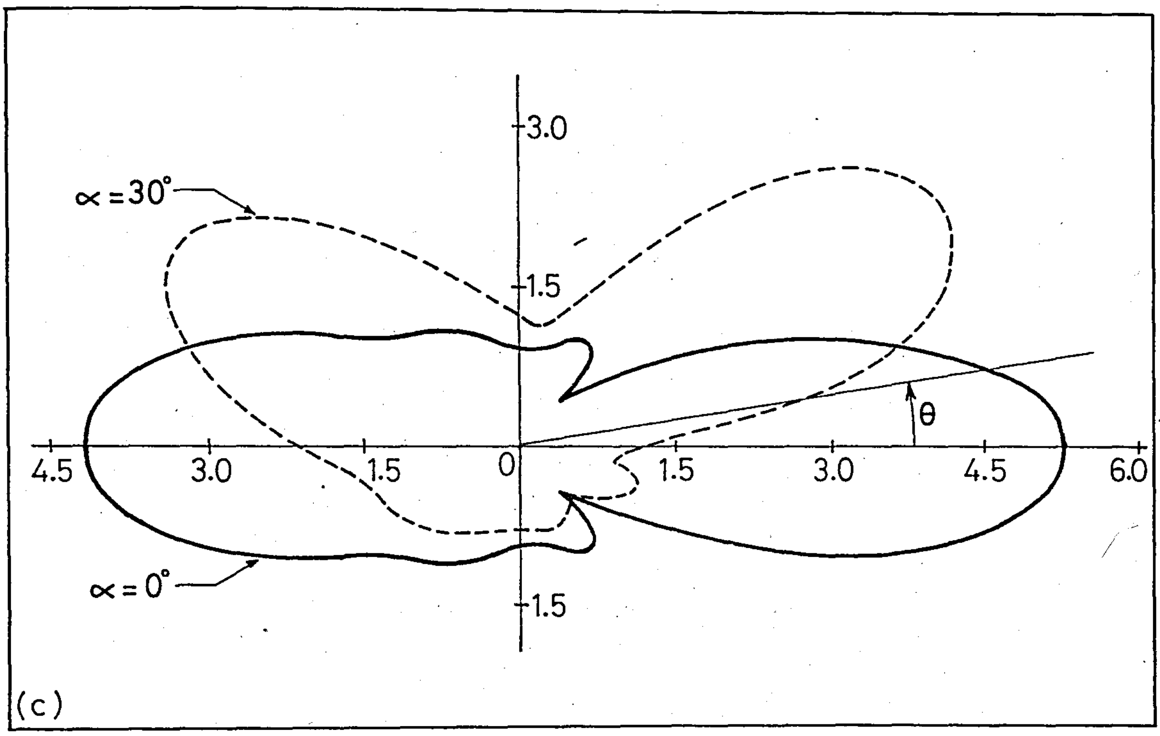


Figure 4.18 (continued).

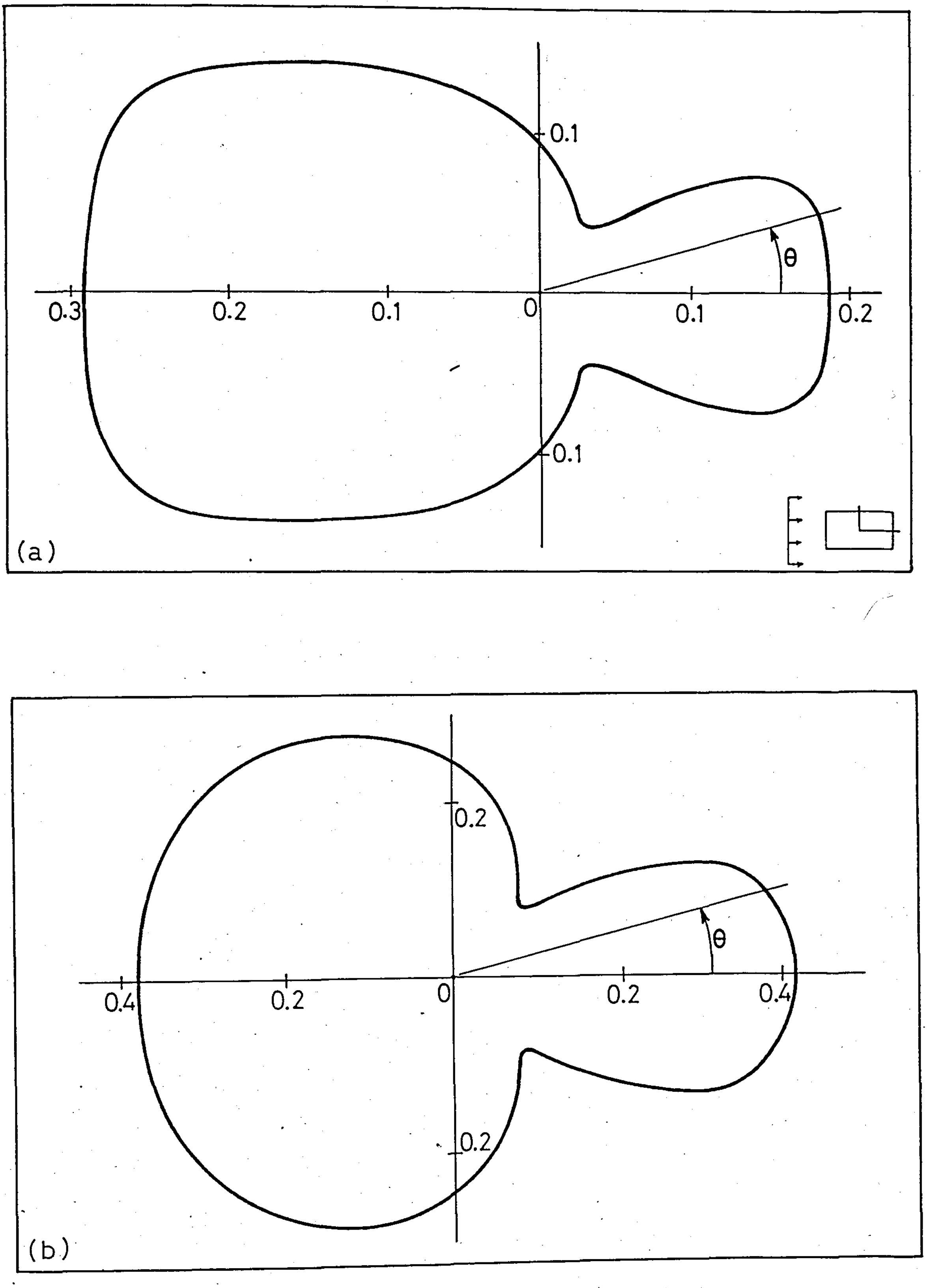


Figure 4.19 - Velocity potential distribution, $|u^s/A|$, at the boundary of a rigid rectangular inclusion due to scattered wave field for $\alpha = 0^o$, $r_c/a = 0.1$ and $b/a = 0.5$;
(a) ka = 0.5, (b) ka = 1.0, (c) ka = 3.0

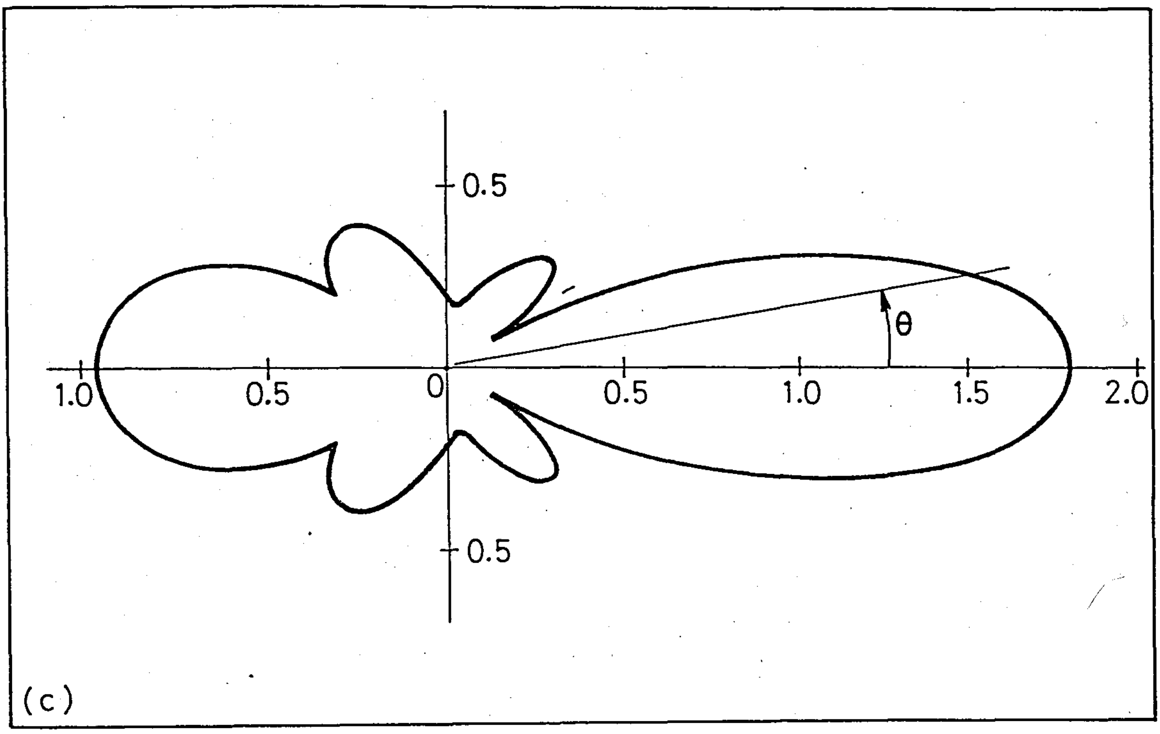


Figure 4.19 (continued).

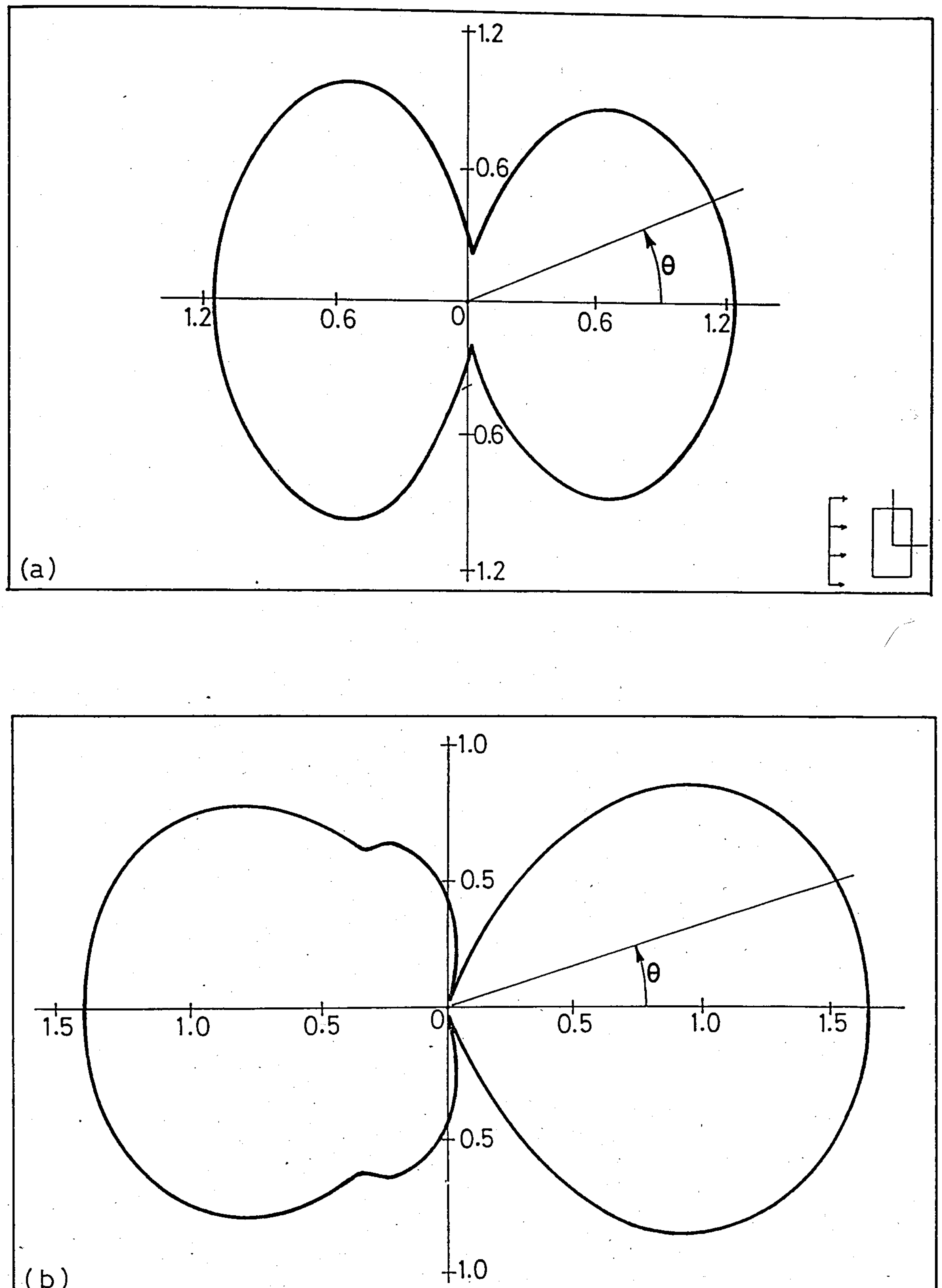


Figure 4.20 - Velocity potential distribution, $|u^S/A|$, at the boundary of a rigid rectangular inclusion due to the scattered wave field for $\alpha = 0^o$. $r_c/a = 0.1$ and $b/a = 2.0$;
(a) ka = 0.5, (b) ka = 1.0, (c) ka = 3.0.

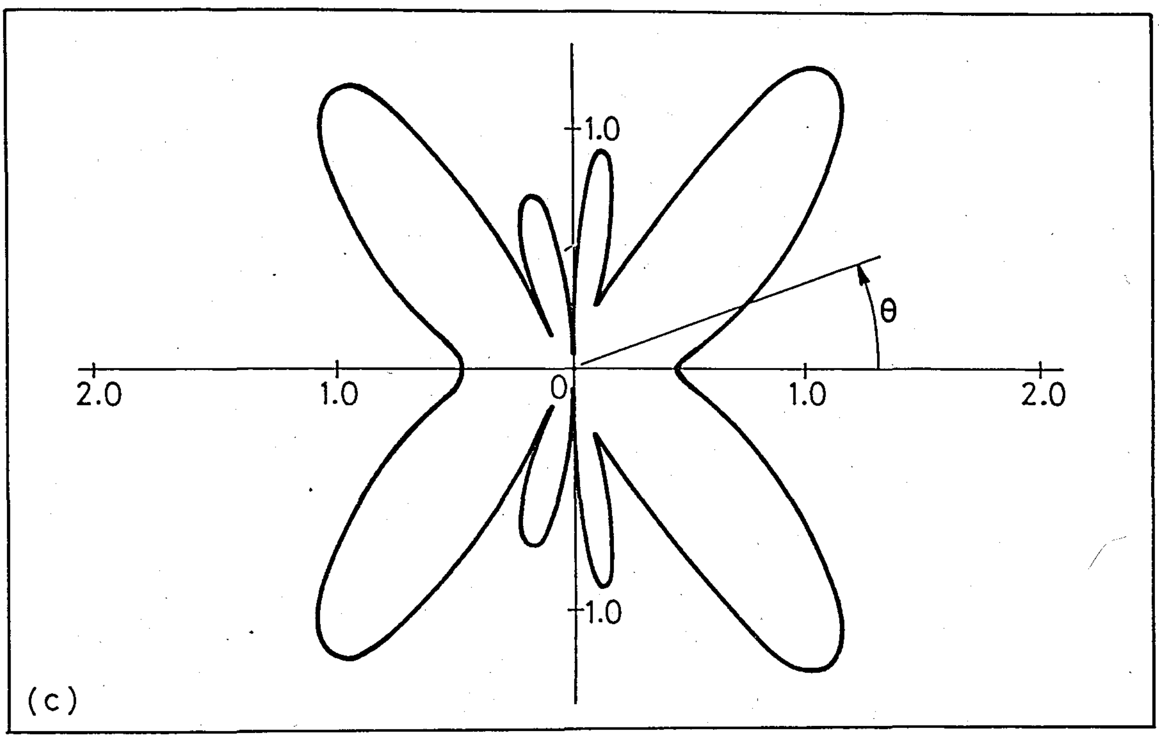


Figure 4.20 (continued).

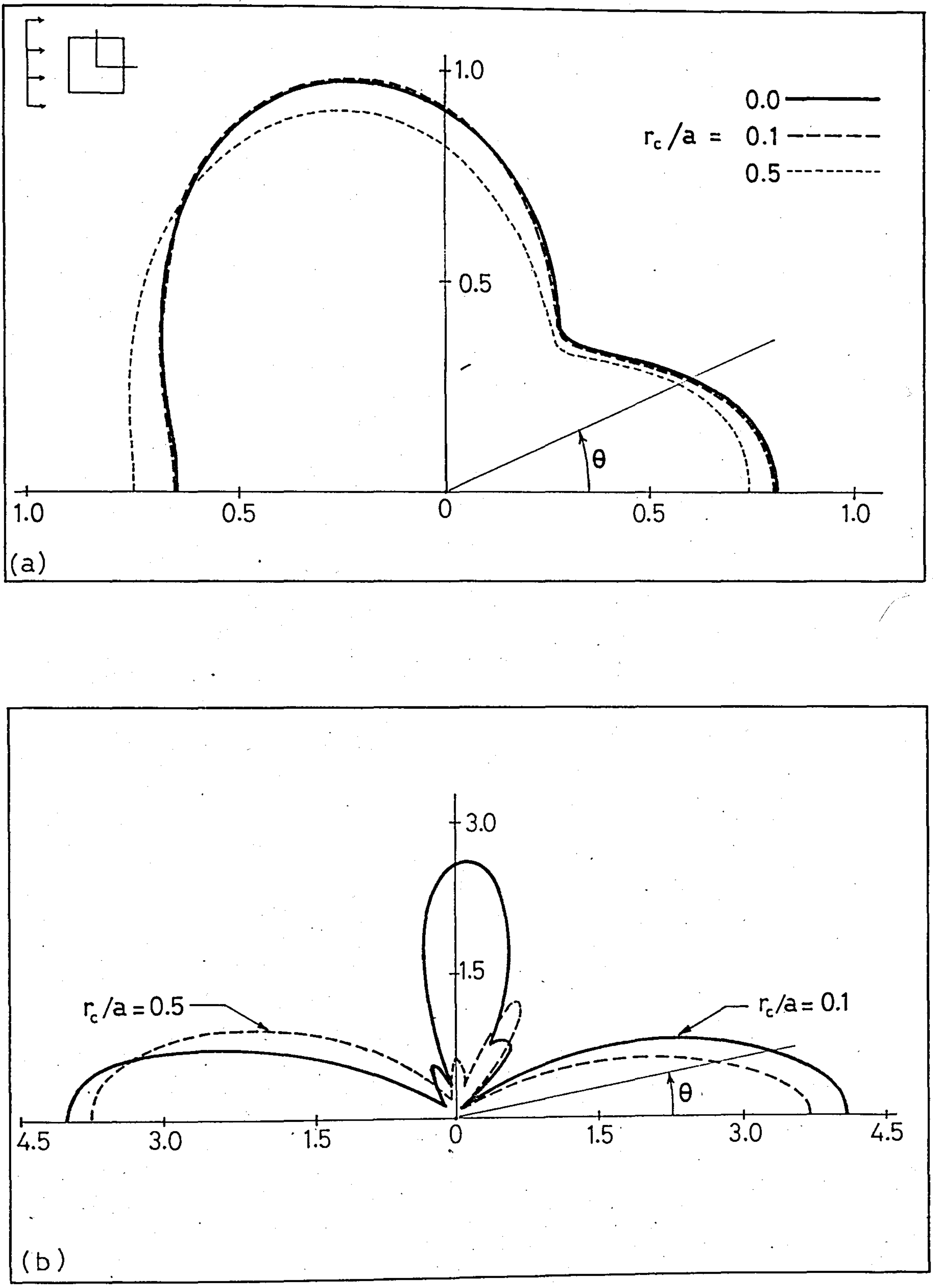


Figure 4.21 - Effect of the corner radius on the far field amplitude, $|f/A|$, due to the scattered wave field from a rigid rectangular inclusion for $\alpha = 0^o$ and $b/a = 1.0$; (a) $ka = 1.0$, (b) $ka = 5.0$.

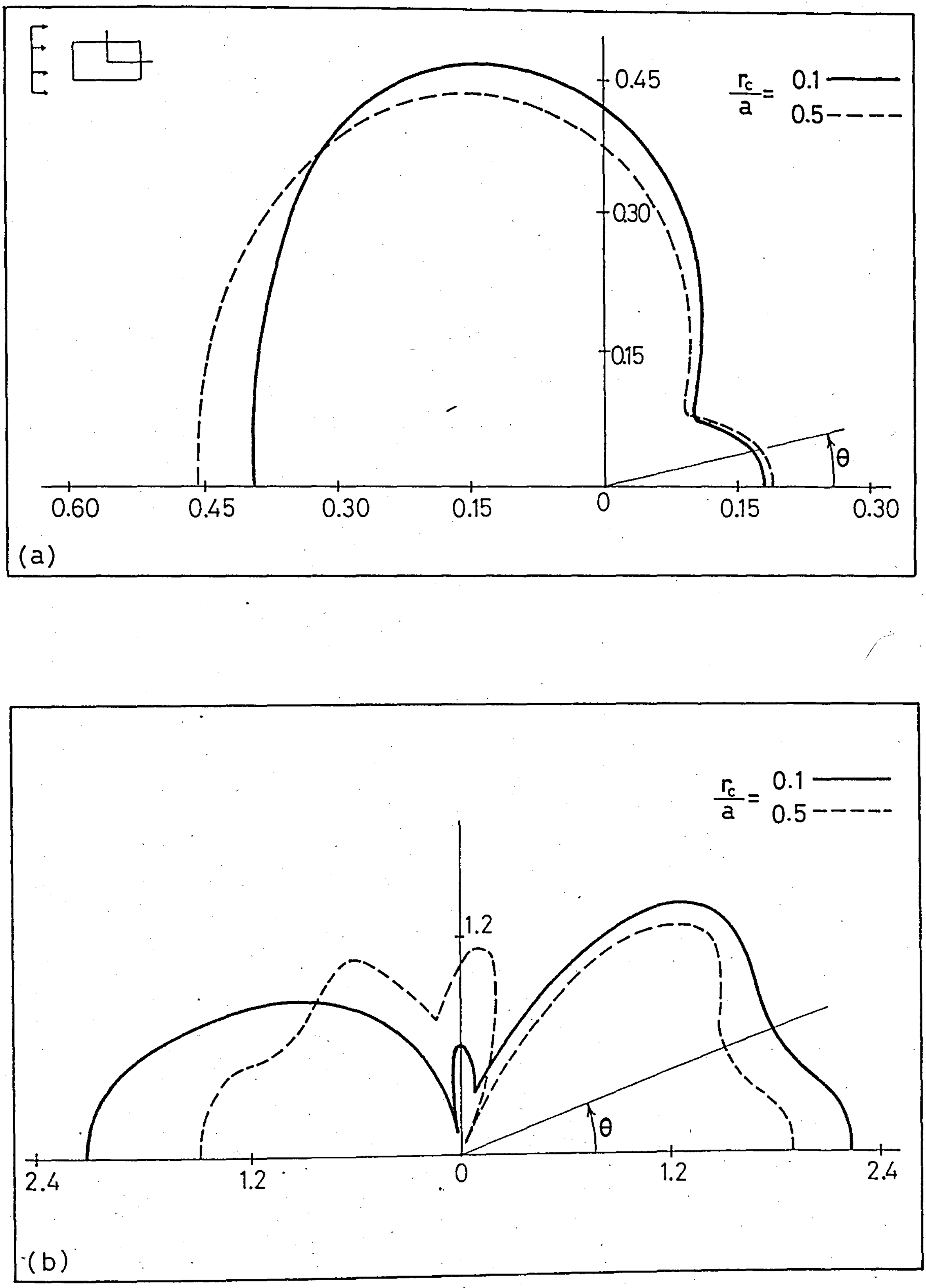
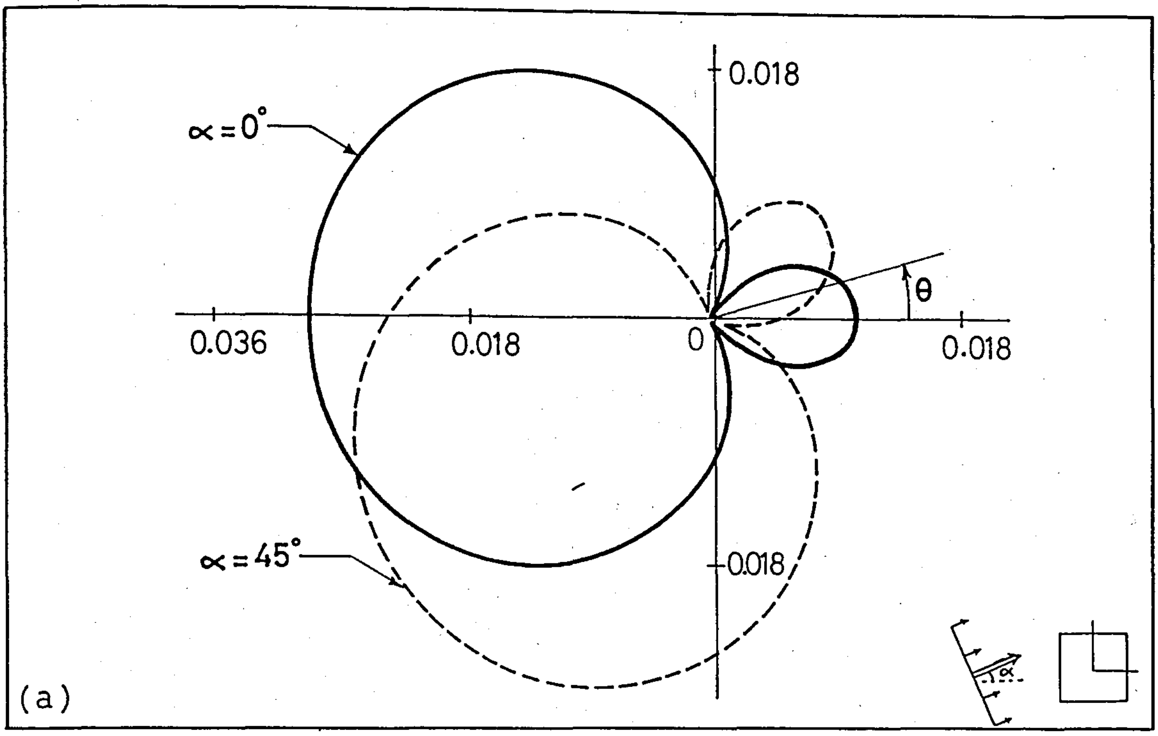


Figure 4.22 - Effect of the corner radius on the far field amplitude, $|f/A|$, due to the scattered wave field from a rigid rectangular inclusion for $\alpha = 0^0$ and $b/a = 0.5$; (a) ka = 1.0, (b) ka = 5.0.



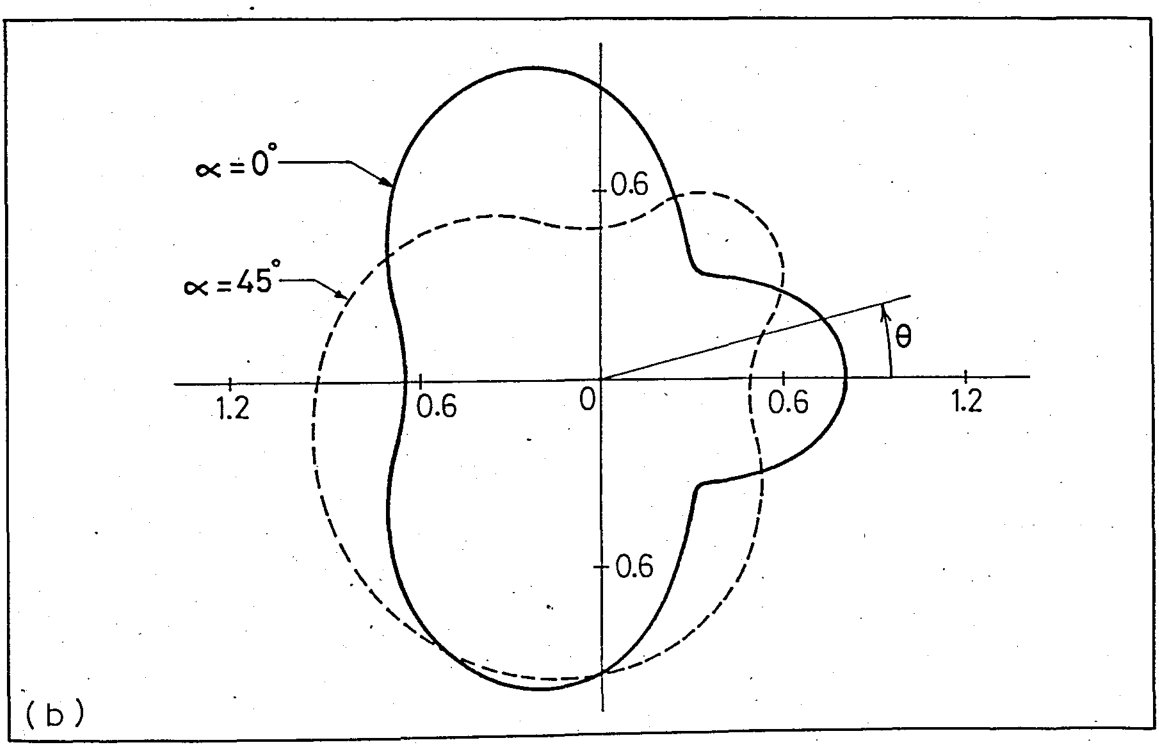


Figure 4.23 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid rectangular inclusion for $r_c/a = 0.1$ and $b/a = 1.0$; (a) $ka = 0.1$, (b) $ka = 1.0$, (c) $ka = 5.0$.

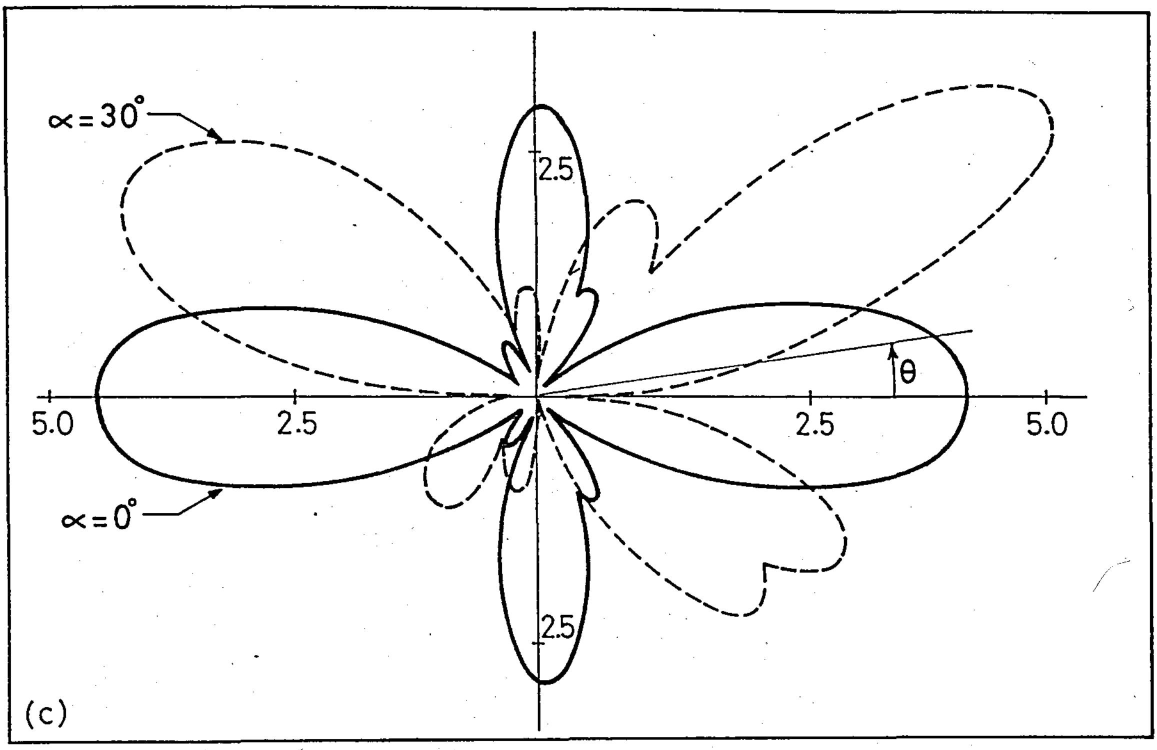
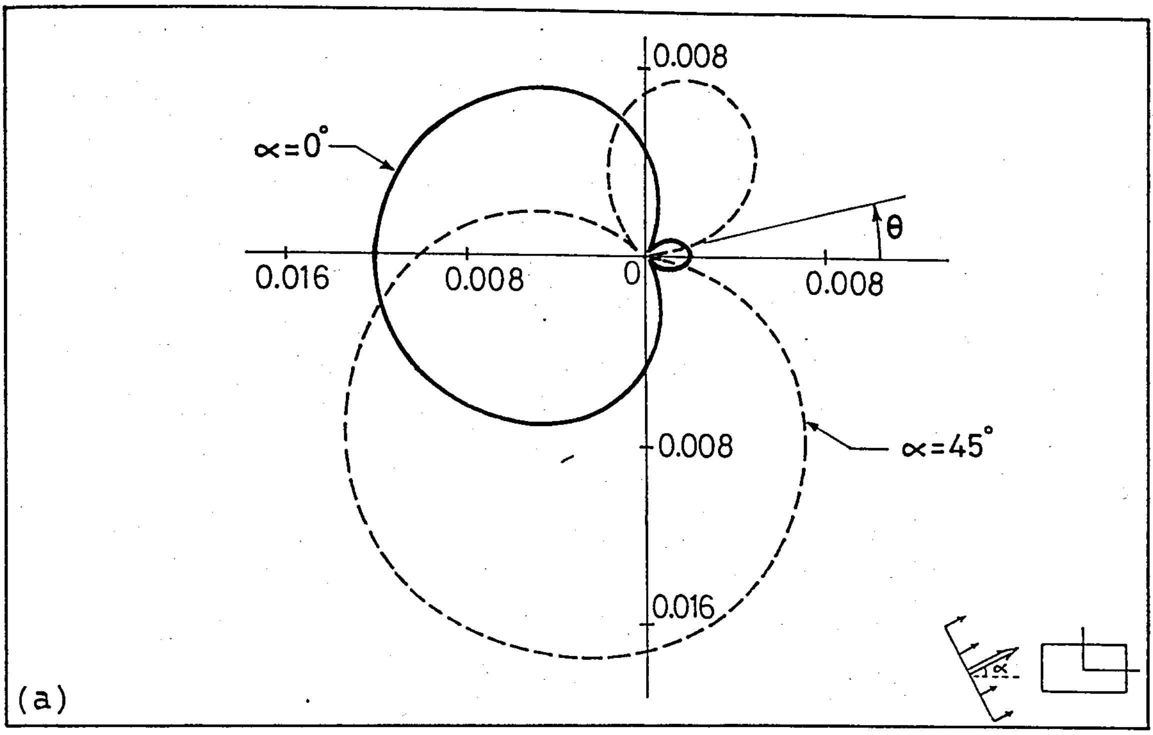


Figure 4.23 (continued).



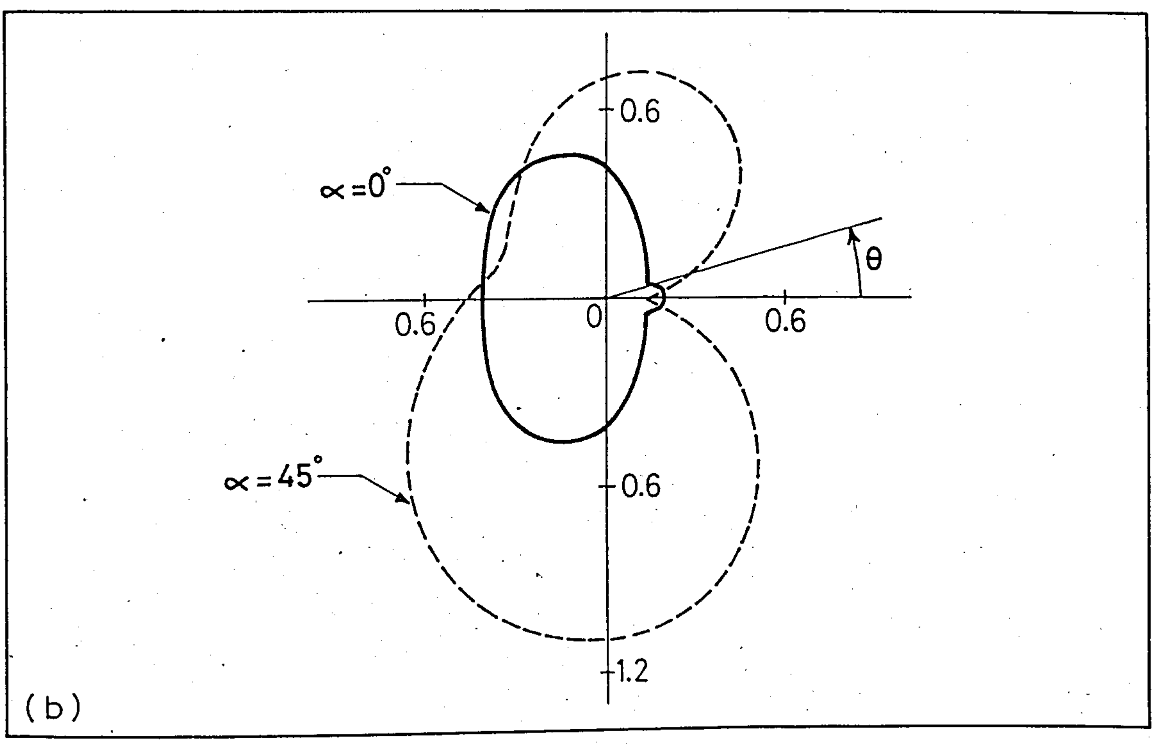


Figure 4.24 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid rectangular inclusion for $r_c/a = 0.1$ and $b/a = 0.5$; (a) ka = 0.1, (b) ka = 1.0, (c) ka = 5.0.

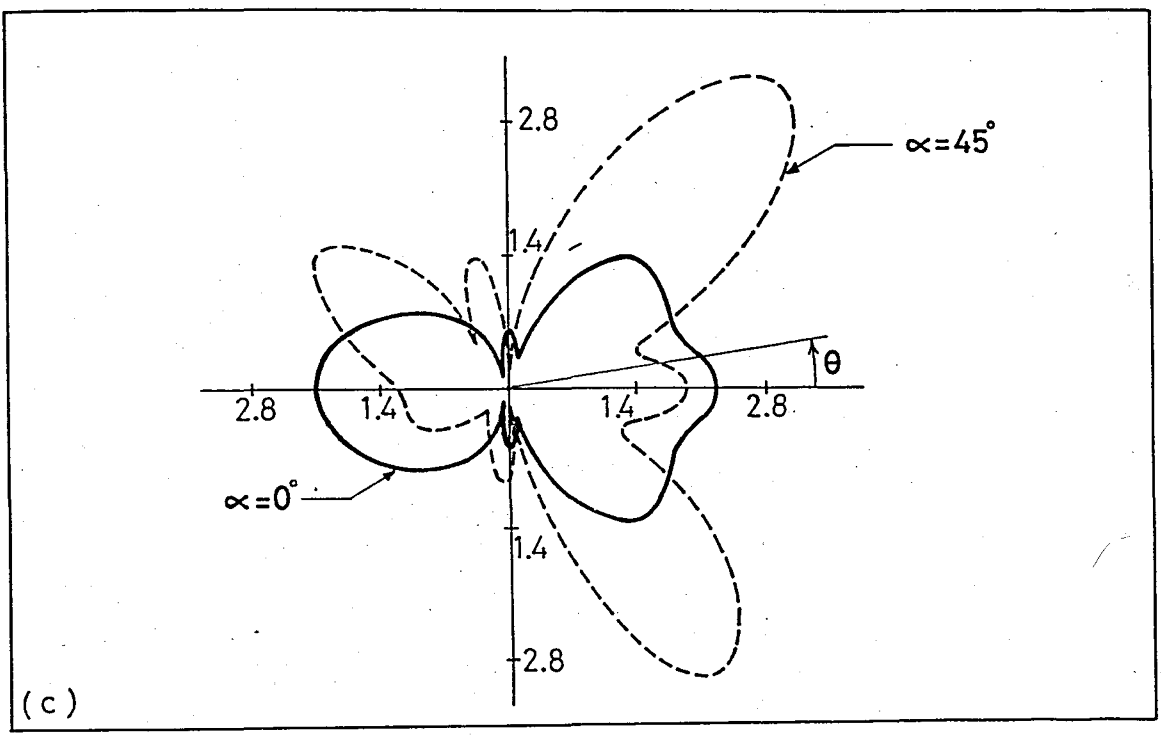


Figure 4.24 (continued).

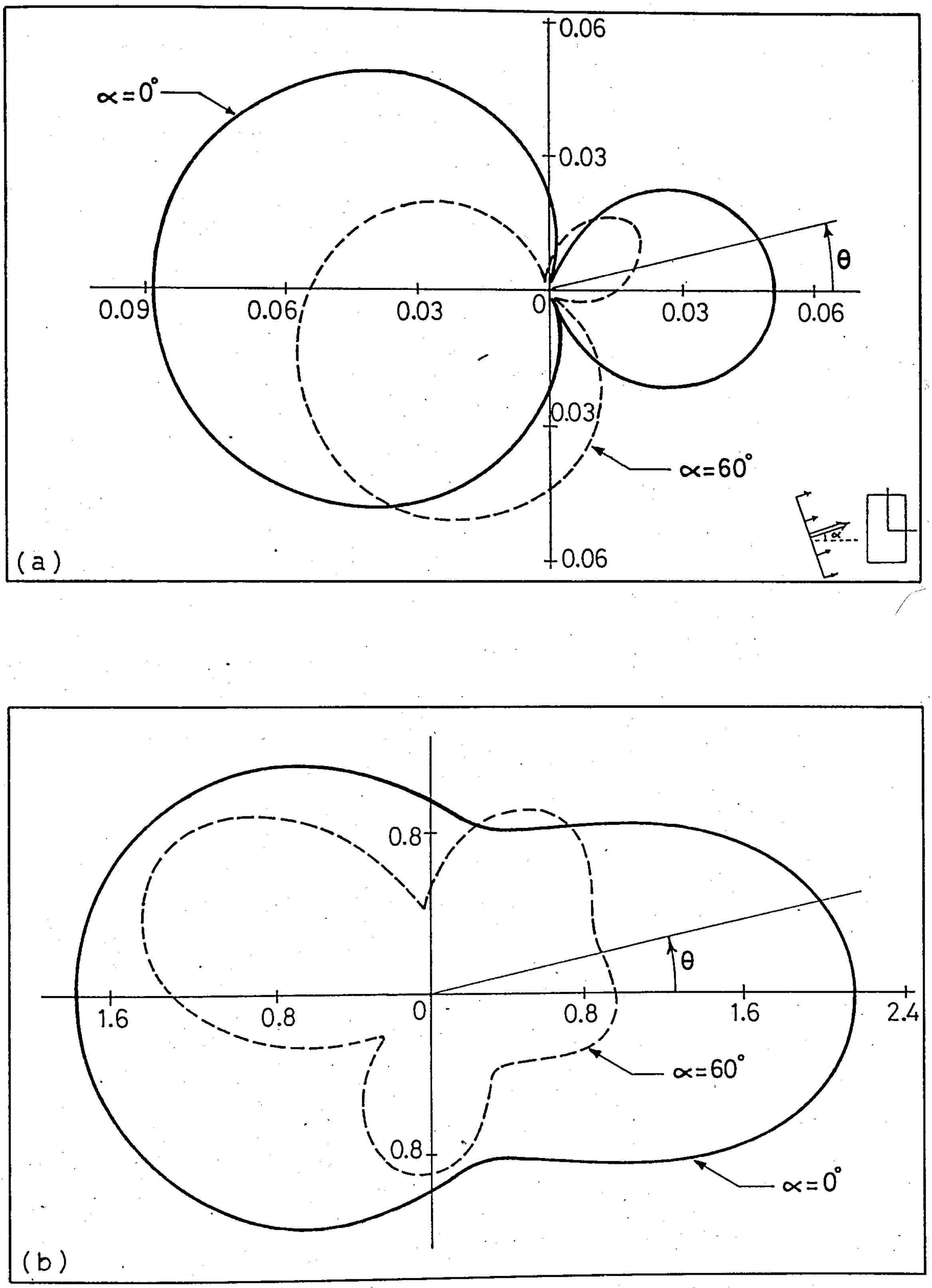


Figure 4.25 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid rectangular inclusion for $r_c/a = 0.1$ and $b/a = 2.0$; (a) $ka = 0.1$, (b) $ka = 1.0$, (c) $ka = 3.0$.

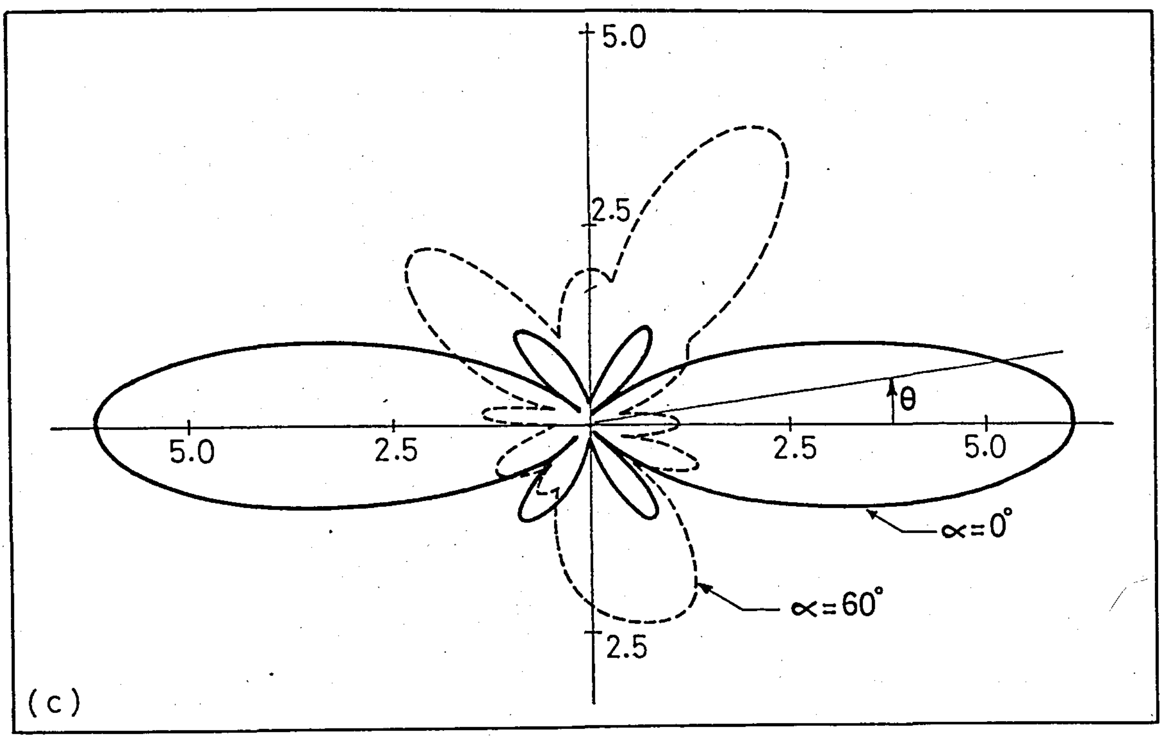


Figure 4.25 (continued).

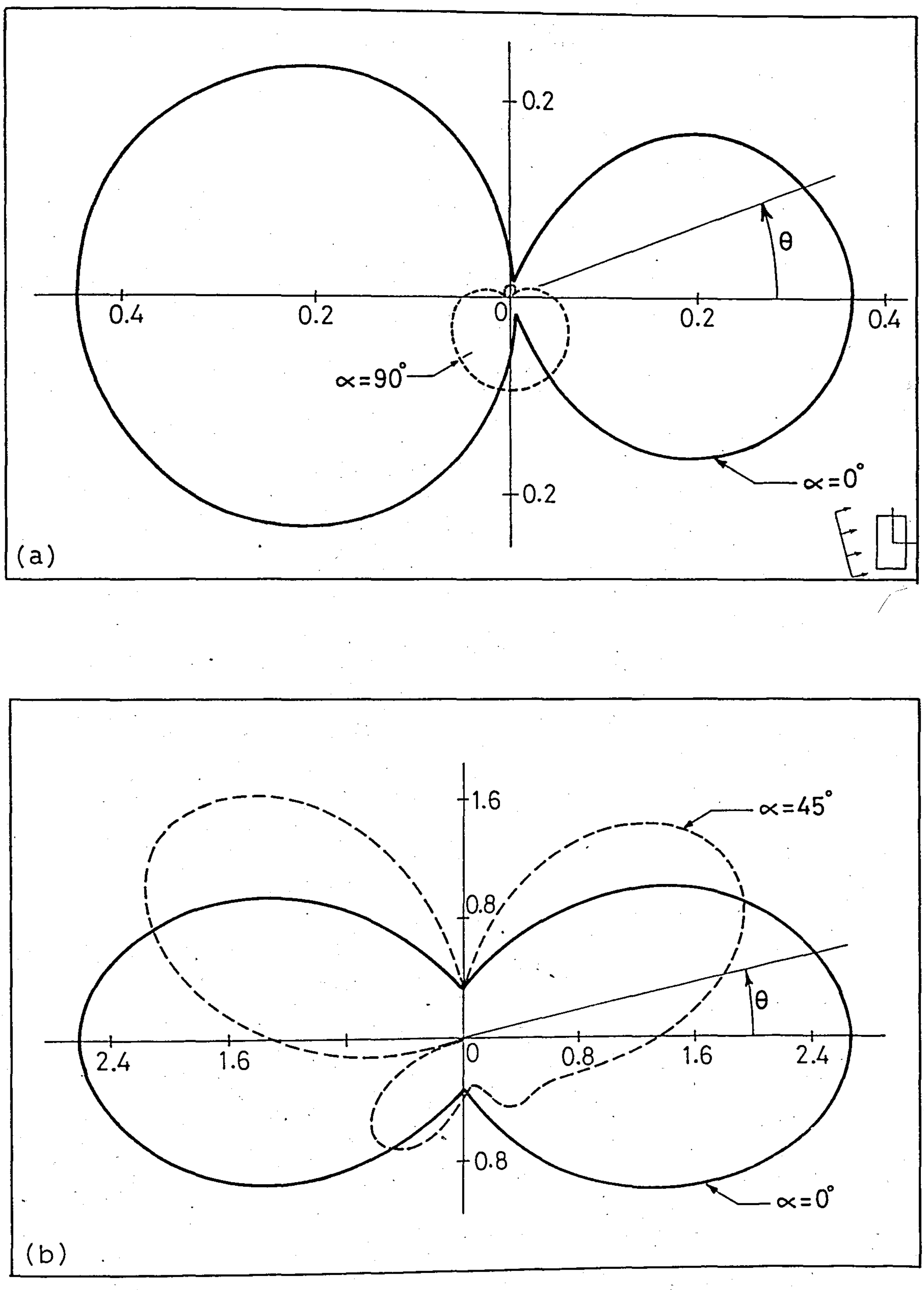


Figure 4.26 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid rectangular inclusion for $r_c/a = 0.1$ and $b/a = 5.0$; (a) ka = 0.1, (b) ka = 0.5, (c) ka = 1.0.

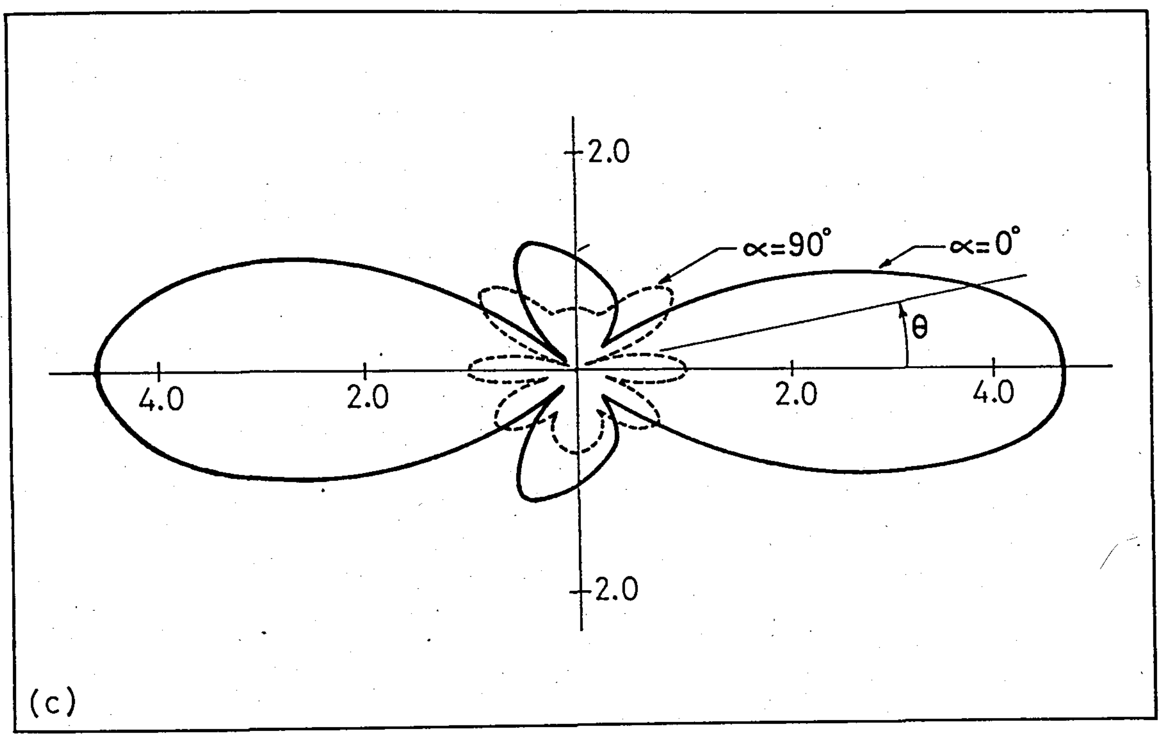
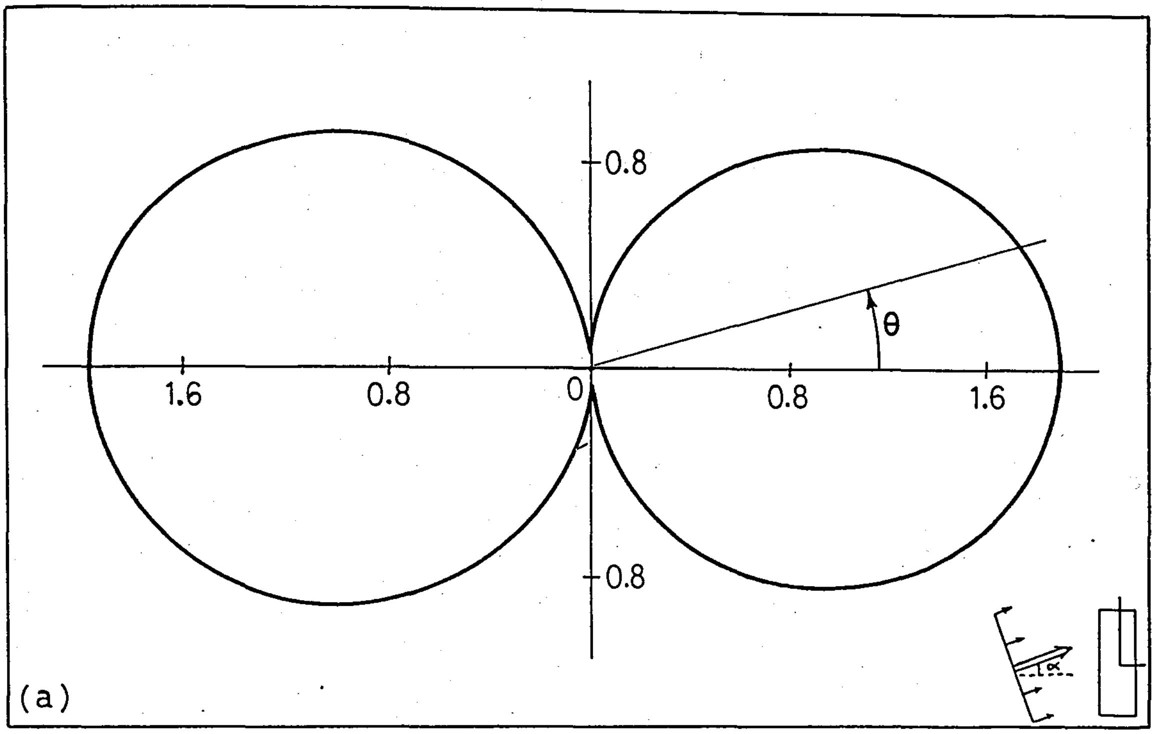


Figure 4.26 (continued).



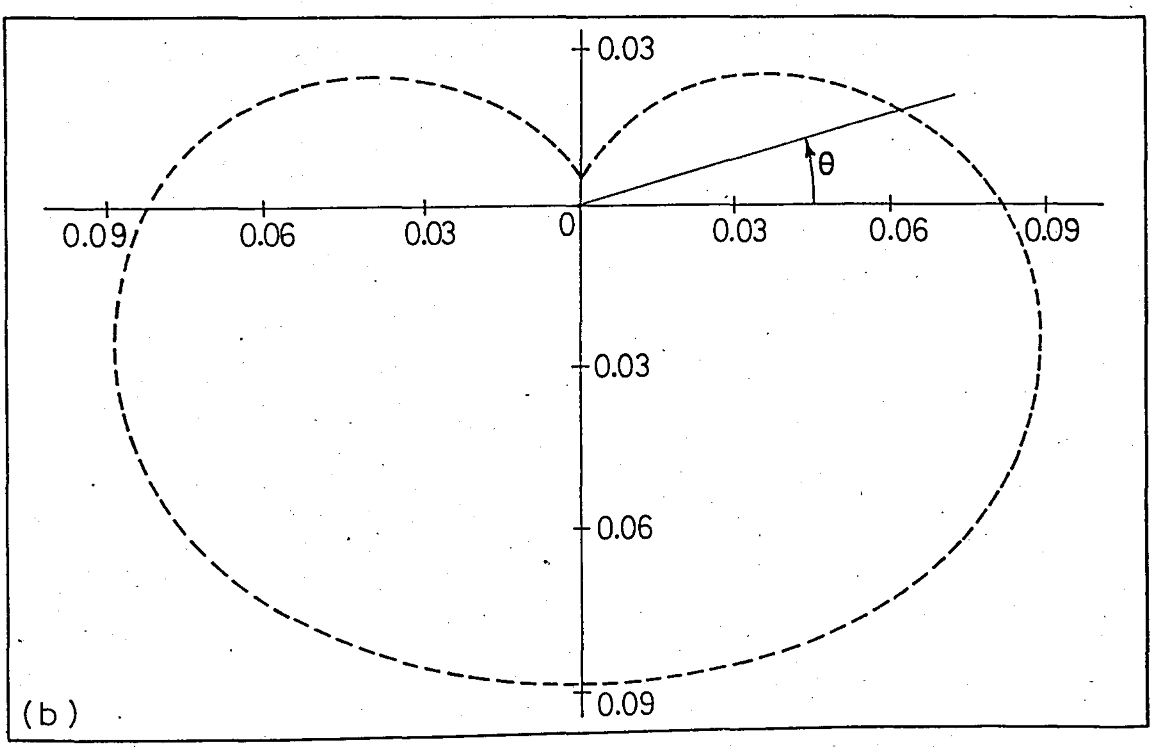
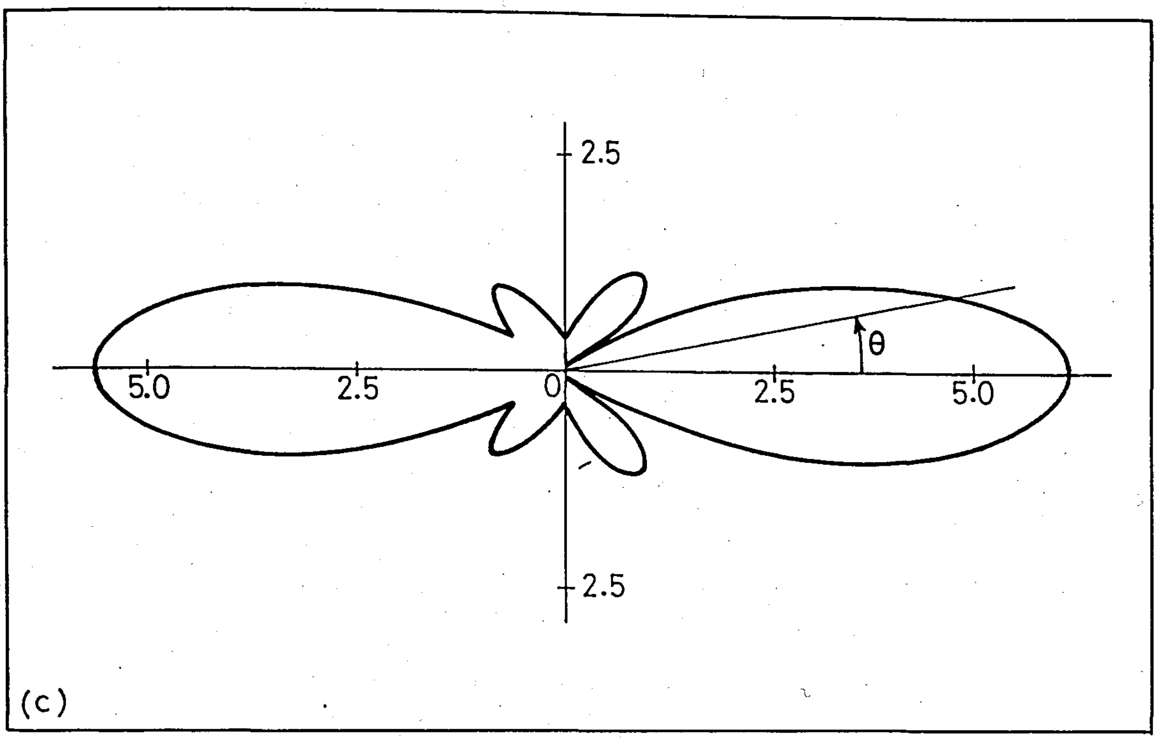


Figure 4.27 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid rectangular inclusion for $r_c/a = 0.1$ and $b/a = 10.0$;
(a) $ka = 0.1$, $\alpha = 0^o$, (b) $ka = 0.1$, $\alpha = 90^o$,
(c) $ka = 0.5$, $\alpha = 0^o$, (d) $ka = 0.5$, $\alpha = 90^o$.



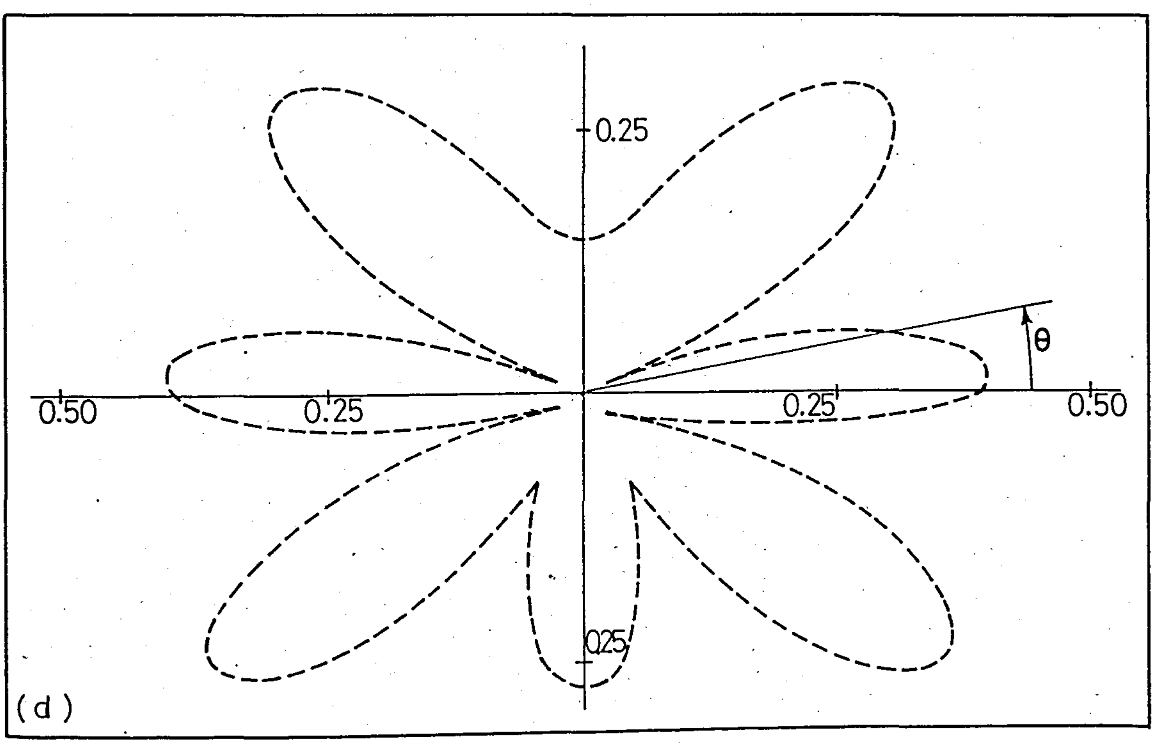


Figure 4.27 (continued).

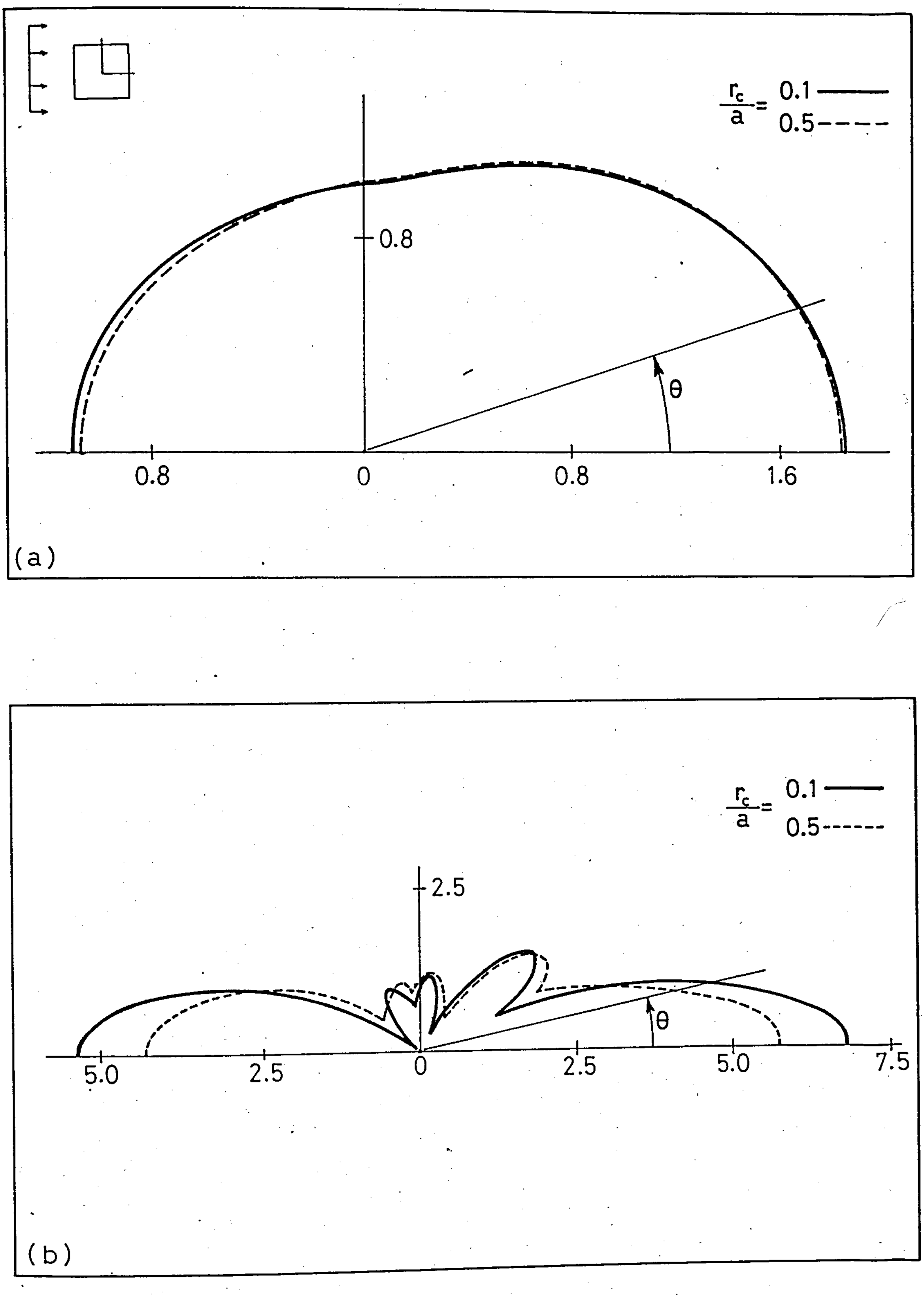


Figure 4.28 - Effect of the corner radius on the far field amplitude, $|f/A|$, due to the scattered wave field from a rectangular cavity for $\alpha = 0^o$ and $b/a = 1.0$; (a) ka = 1.0, (b) ka = 5.0

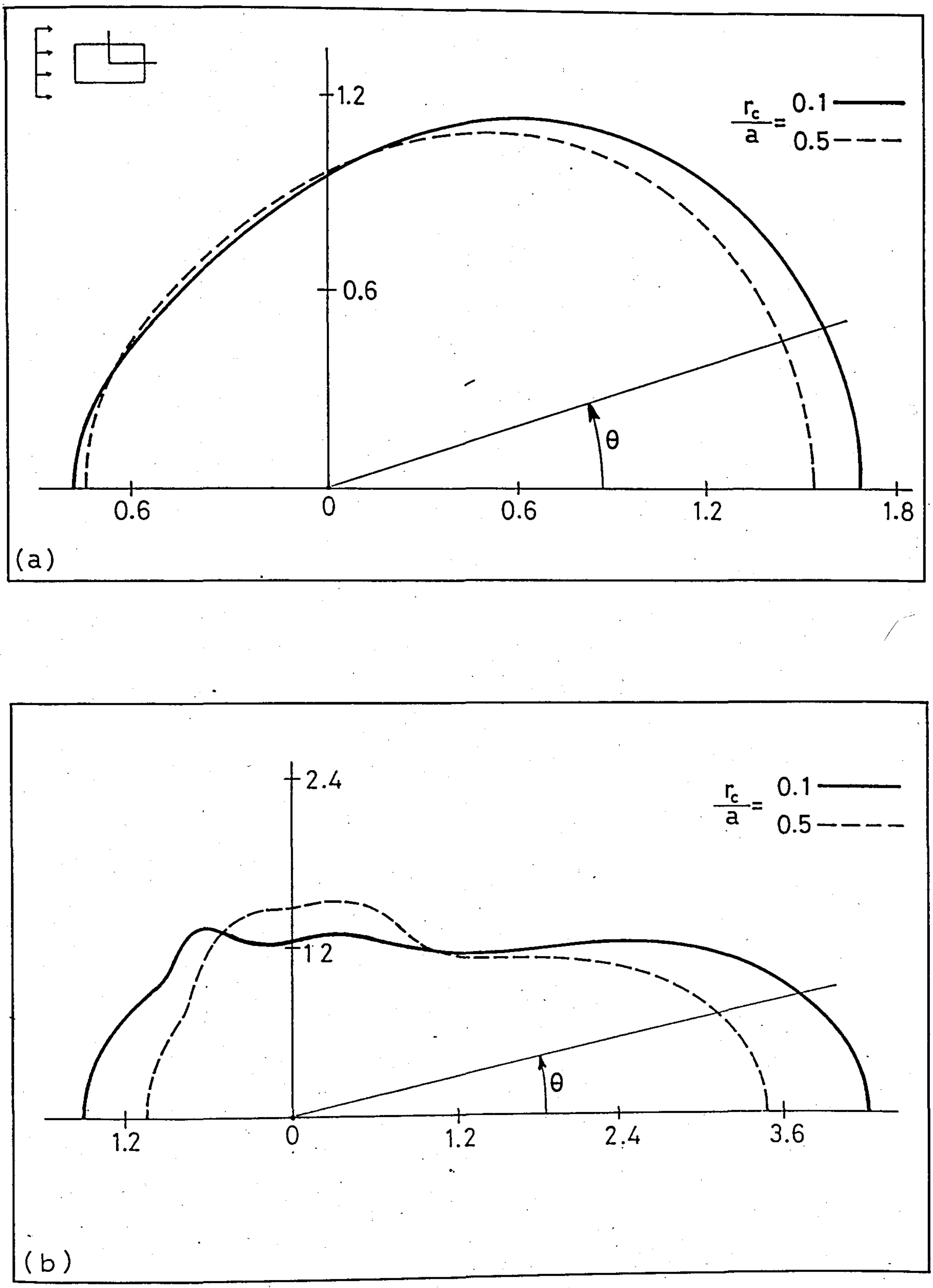
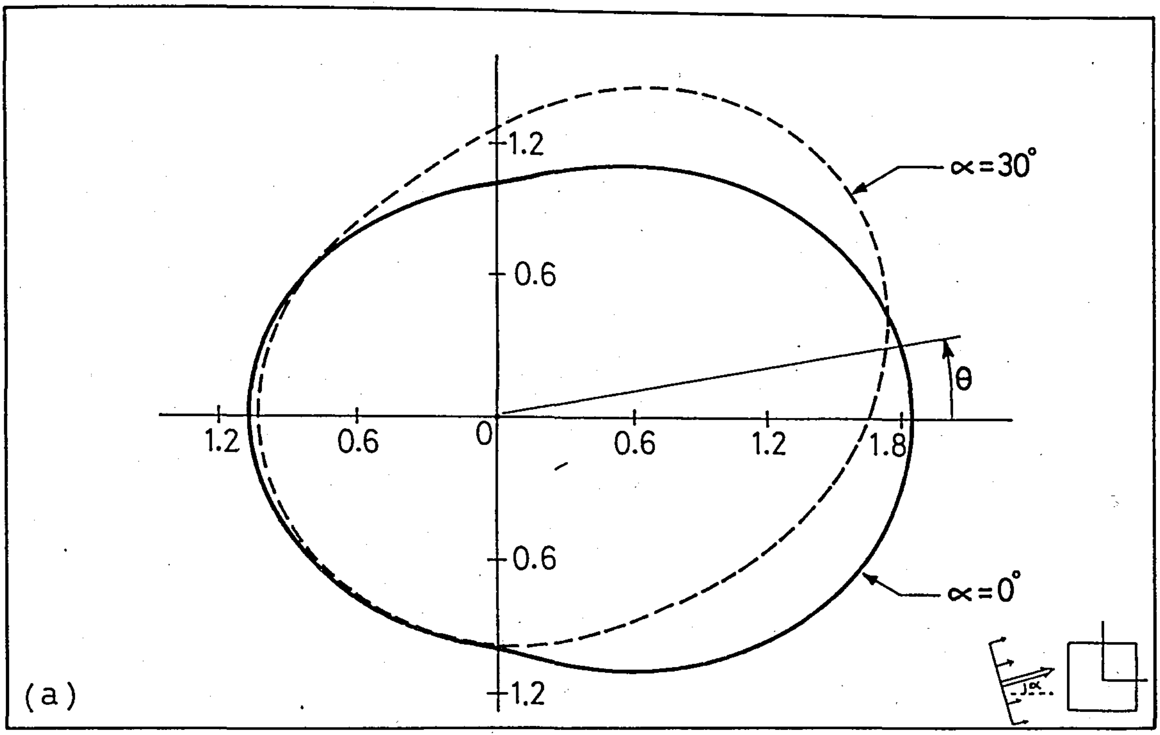


Figure 4.29 - Effect of the corner radius on the far field amplitude, $|f/A|$, due to the scattered wave field from a rectangular cavity for $\alpha = 0^o$ and $b/a = 0.5$; (a) $ka = 1.0$, (b) $ka = 5.0$.



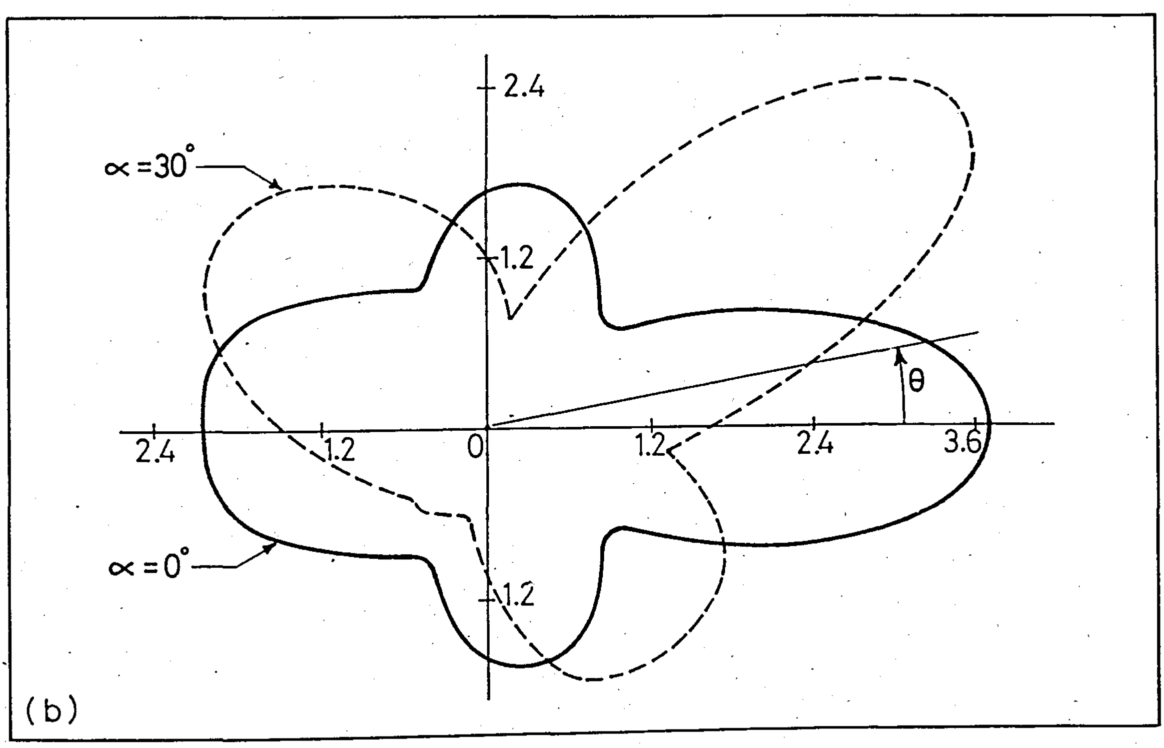


Figure 4.30 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rectangular cavity for $r_c/a = 0.1$ and $b/a = 1.0$;
(a) $ka = 1.0$, (b) $ka = 3.0$, (c) $ka = 5.0$.

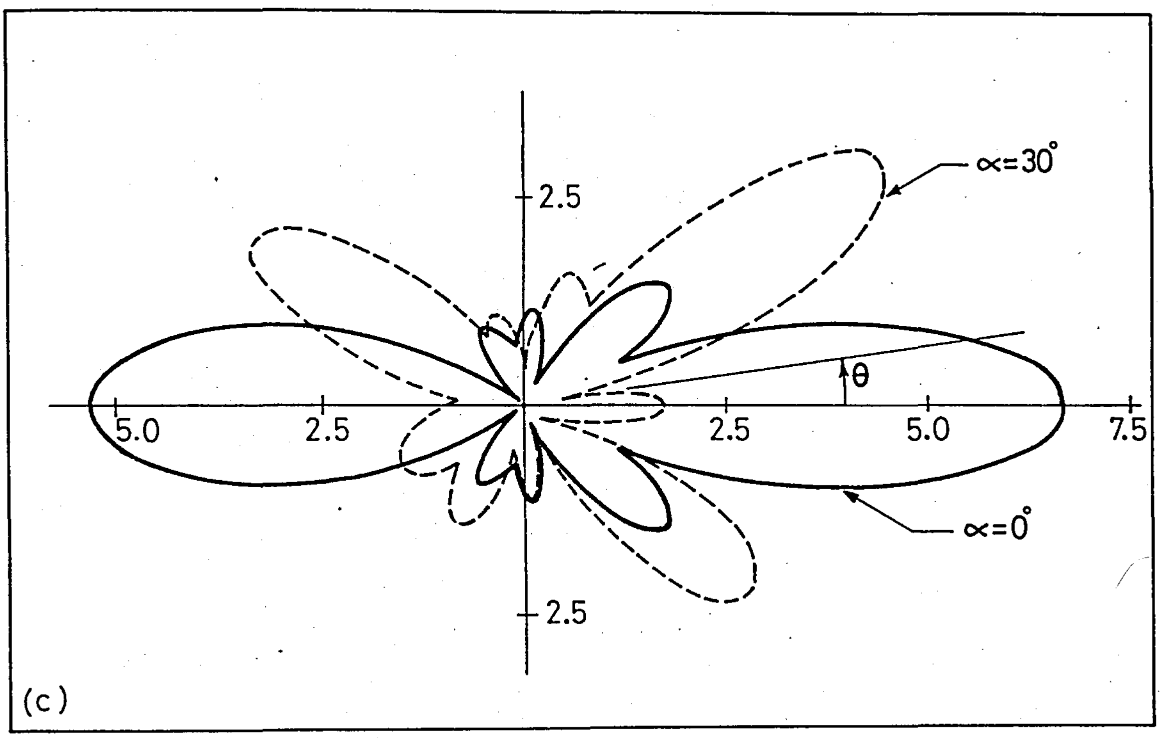
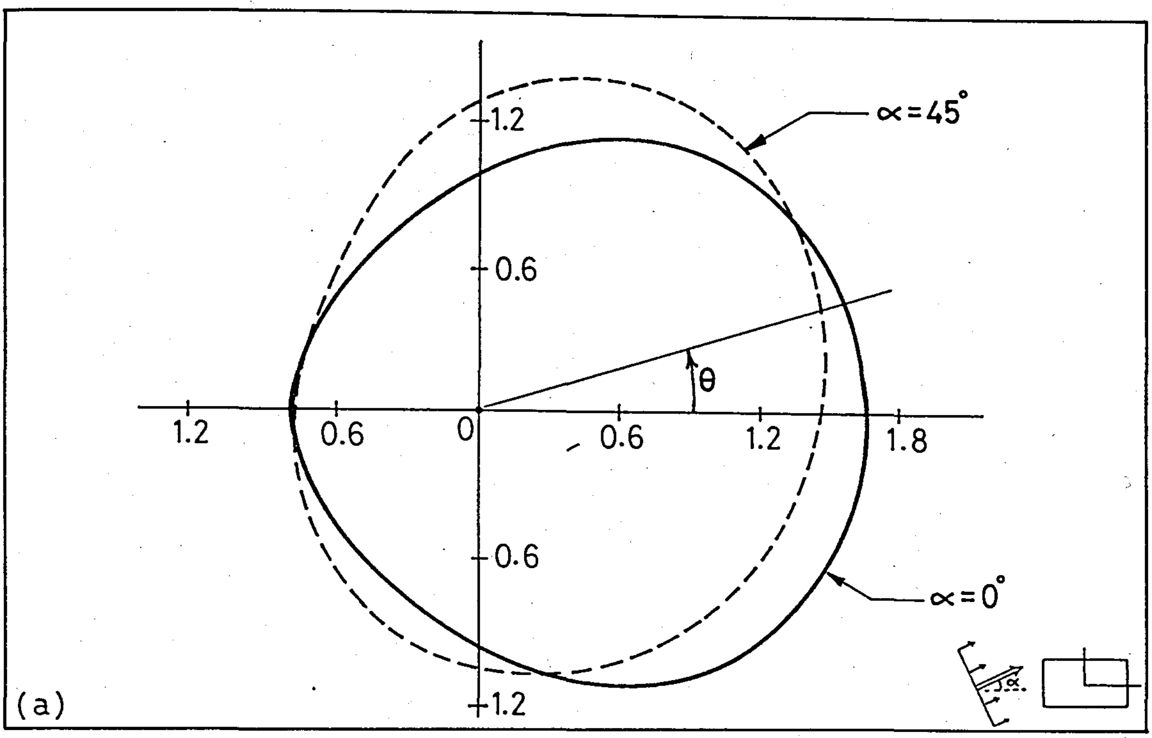


Figure 4.30 (continued).



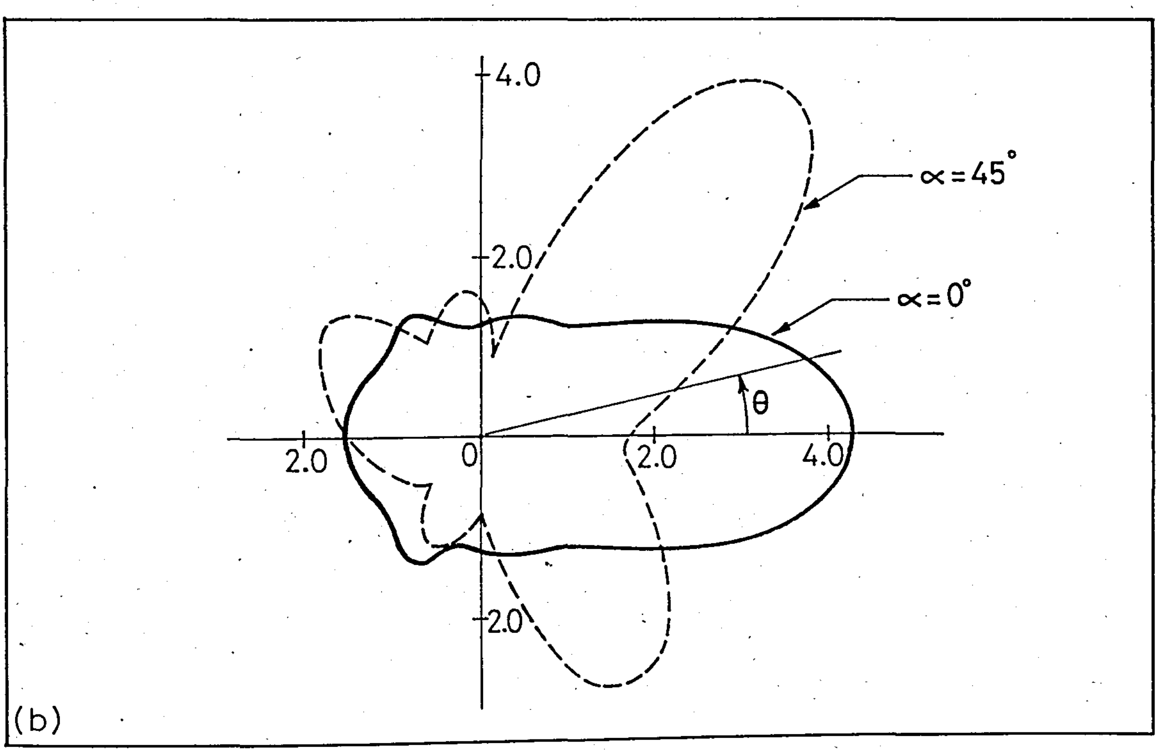
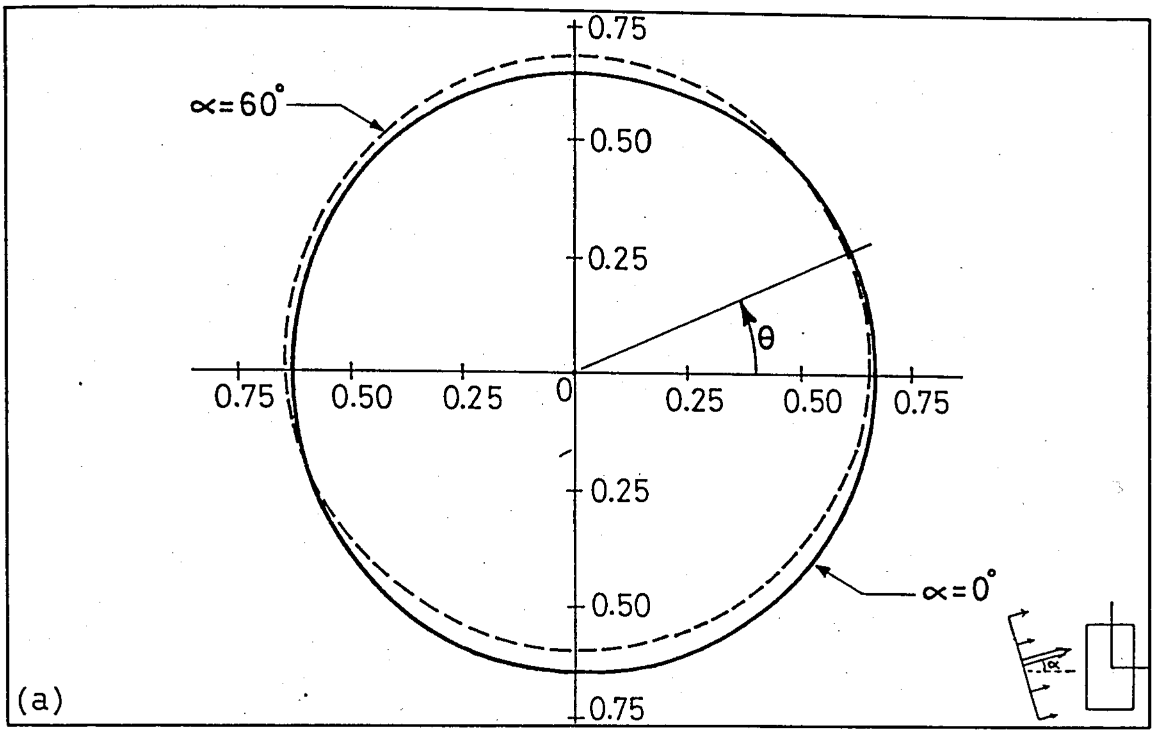


Figure 4.31 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rectangular cavity for $r_c/a = 0.1$ and $b/a = 0.5$;
(a) $ka = 1.0$, (b) $ka = 5.0$.



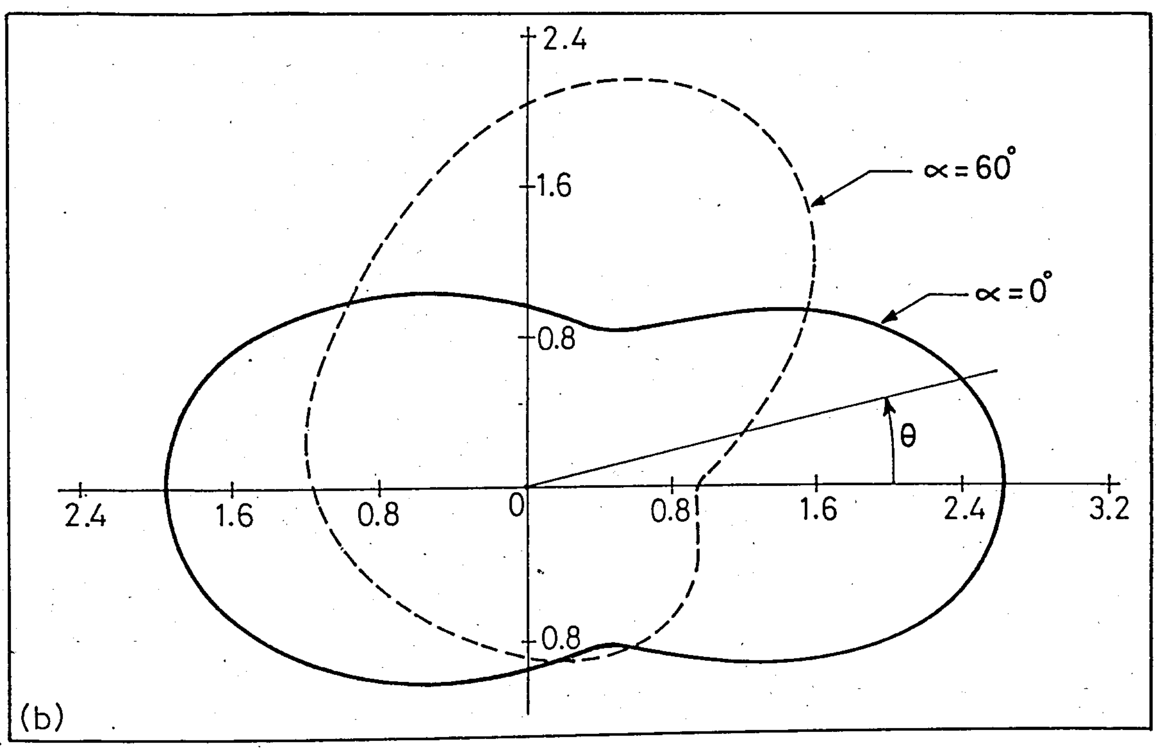


Figure 4.32 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rectangular cavity for $r_c/a = 0.1$ and $b/a = 2.0$;
(a) $ka = 0.1$, (b) $ka = 1.0$, (c) $ka = 3.0$.

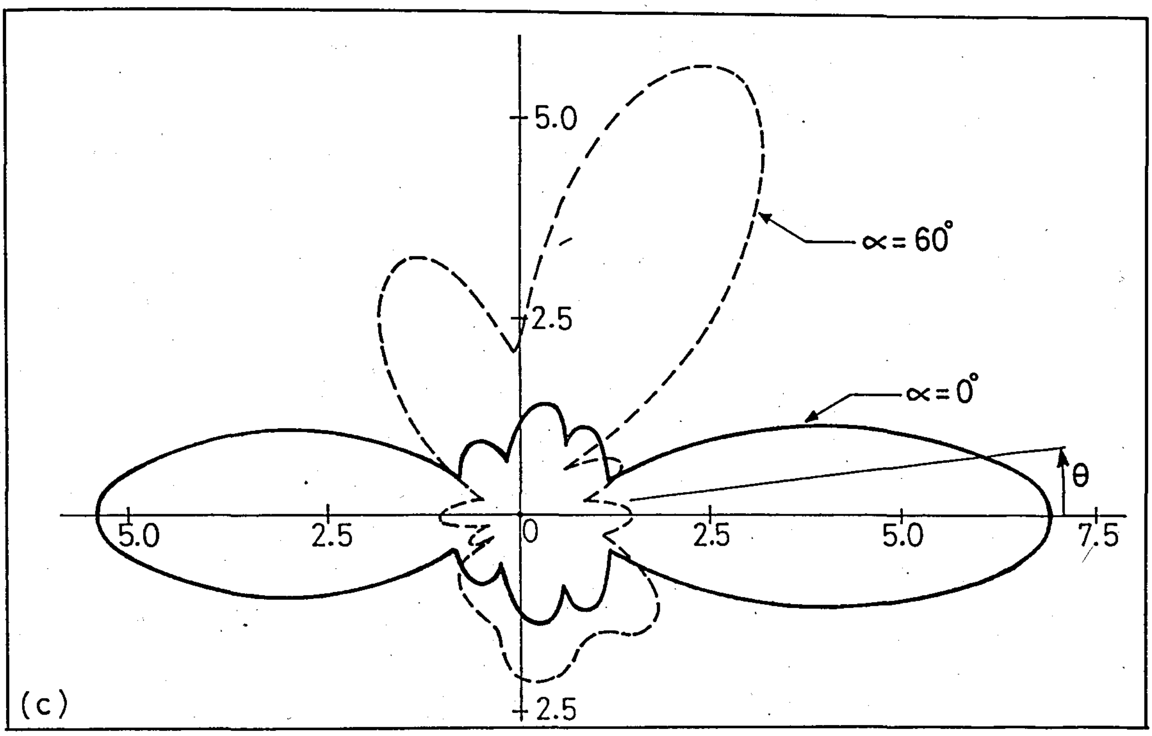
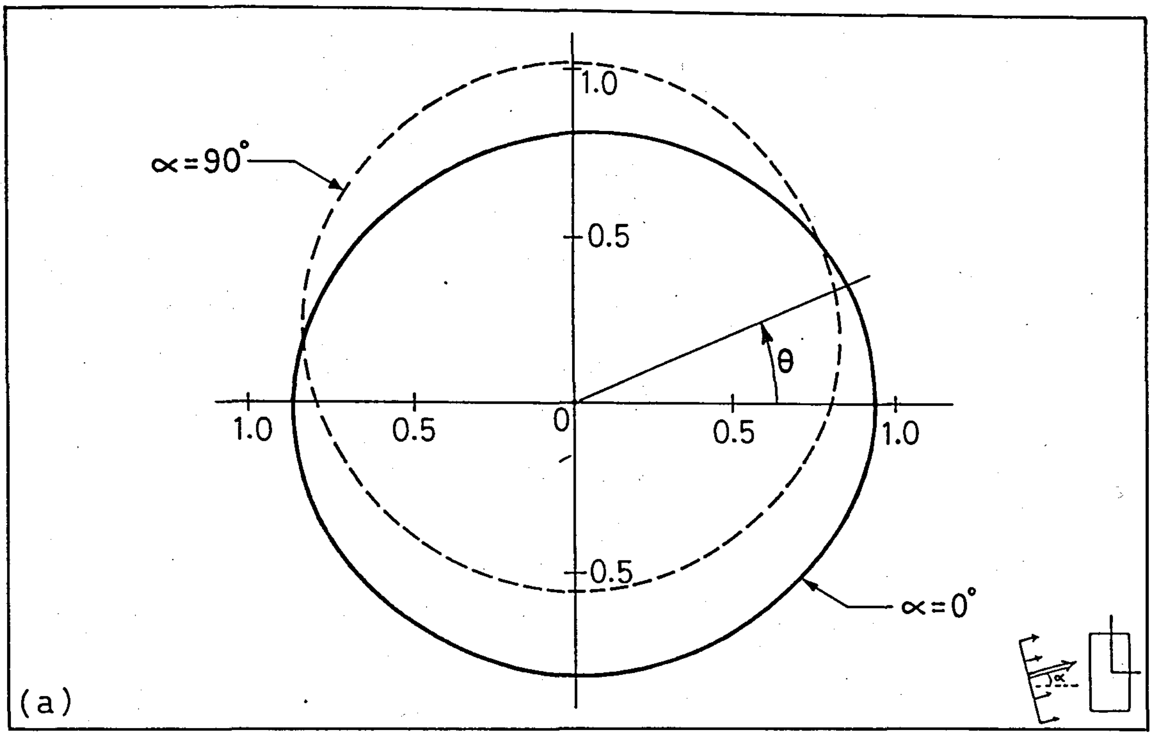


Figure 4.32 (continued).



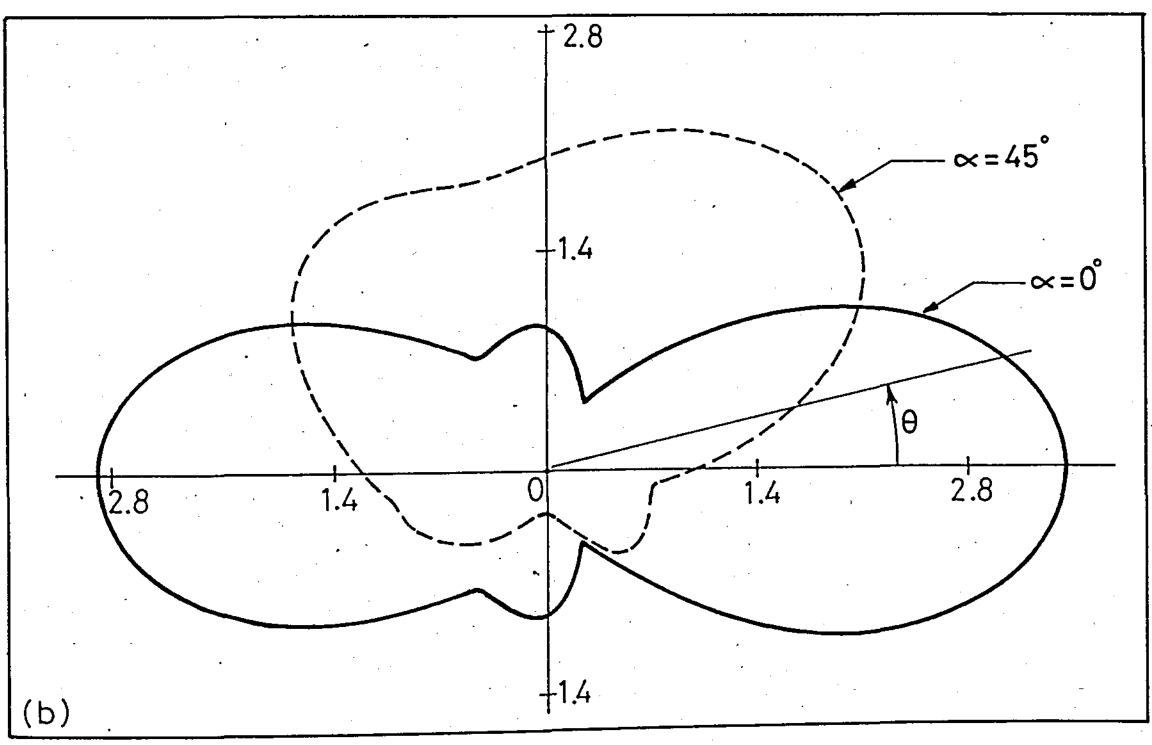


Figure 4.33 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rectangular cavity for $r_c/a = 0.1$ and $b/a = 5.0$;
(a) $ka = 0.1$, (b) $ka = 0.5$, (c) $ka = 1.0$.

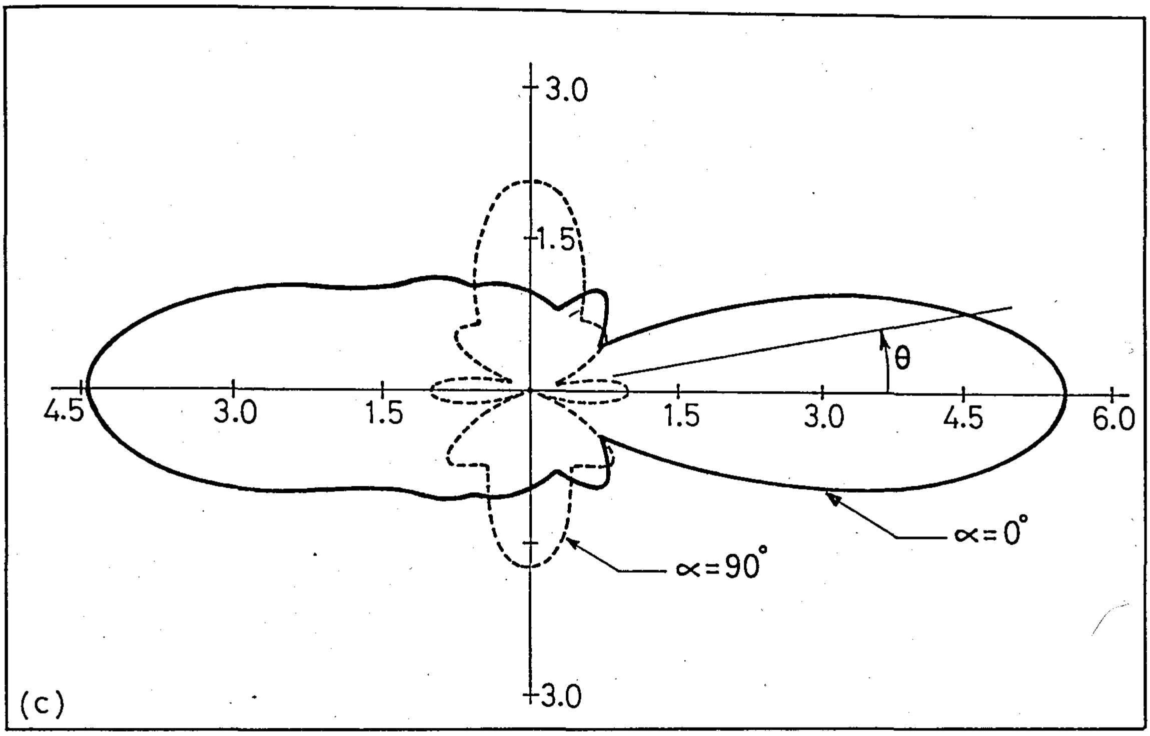
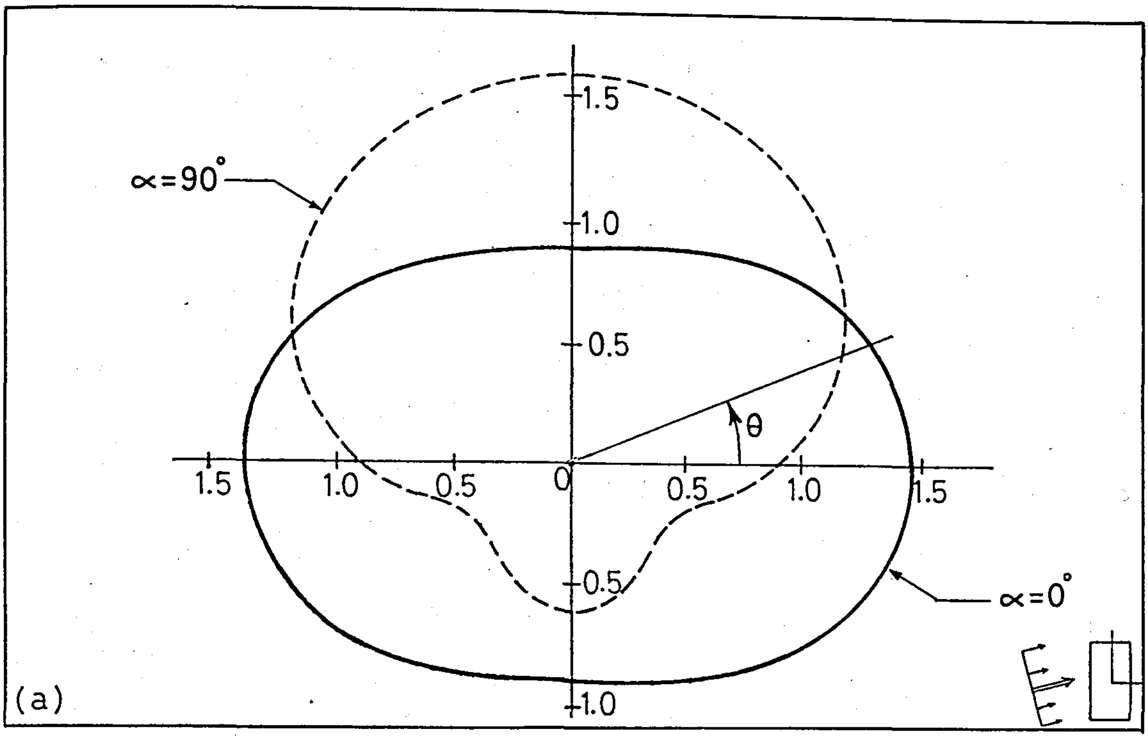


Figure 4.33 (continued).



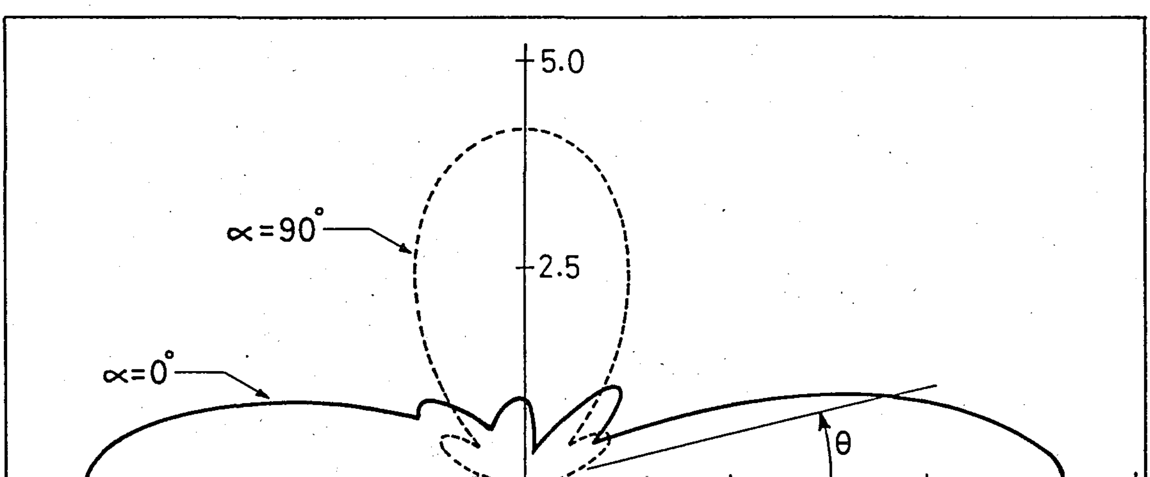
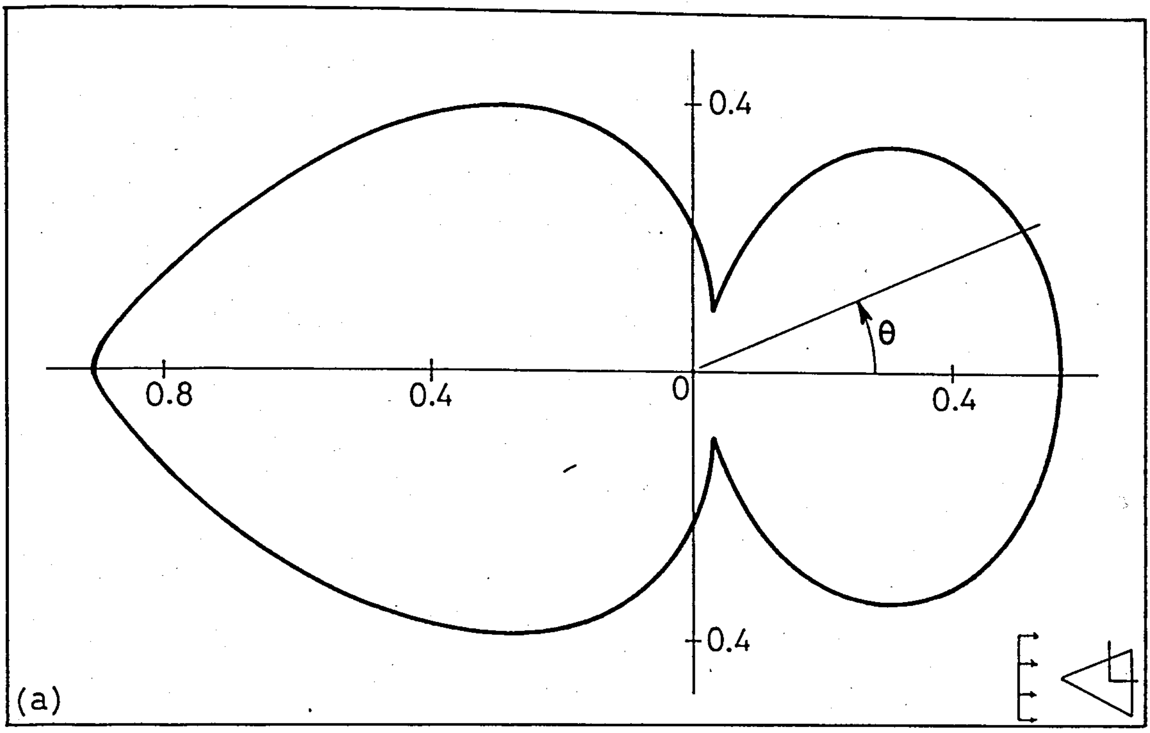


Figure 4.34 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rectangular cavity for $r_c/a = 0.1$ and $b/a = 10.0$.
(a) $ka = 0.1$, (b) $ka = 0.5$.



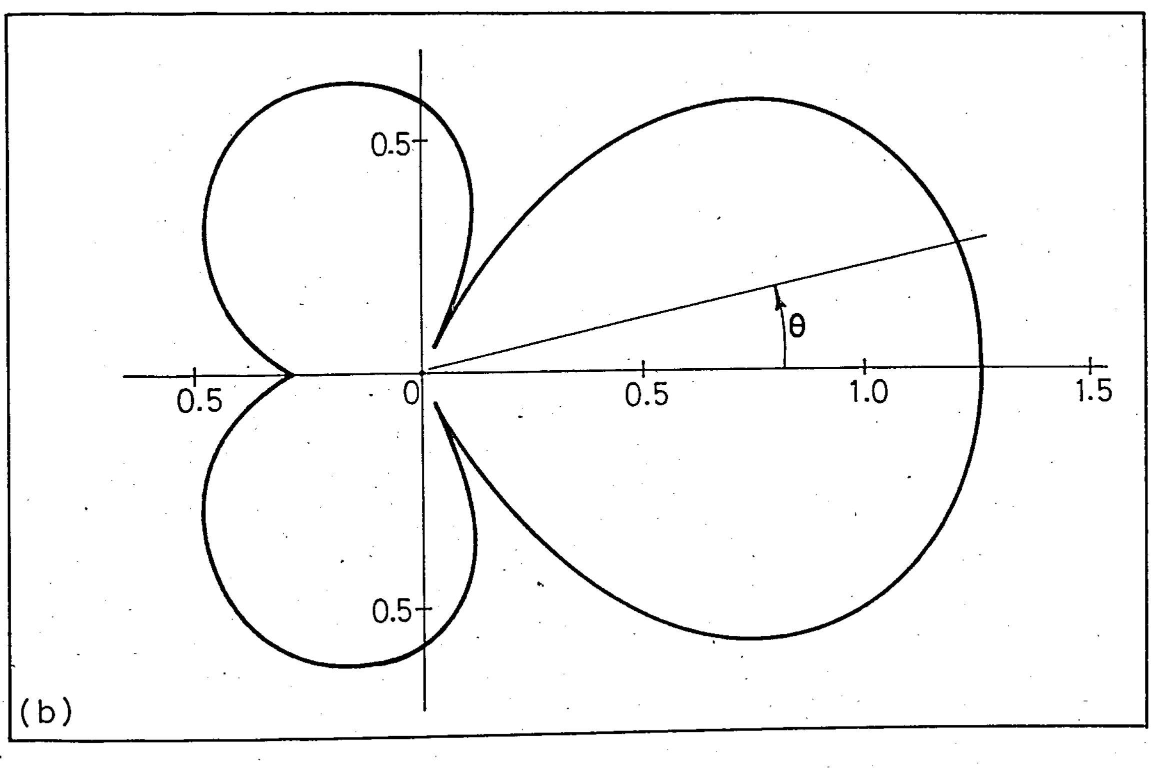


Figure 4.35 - Velocity potential distribution, $|u^s/A|$, at the boundary of a rigid triangular inclusion due to the scattered wave field for $\alpha = 0^0$ and $\beta = 60^0$; (a) kh = 1.5, (b) kh = 3.0, (c) kh = 9.0.

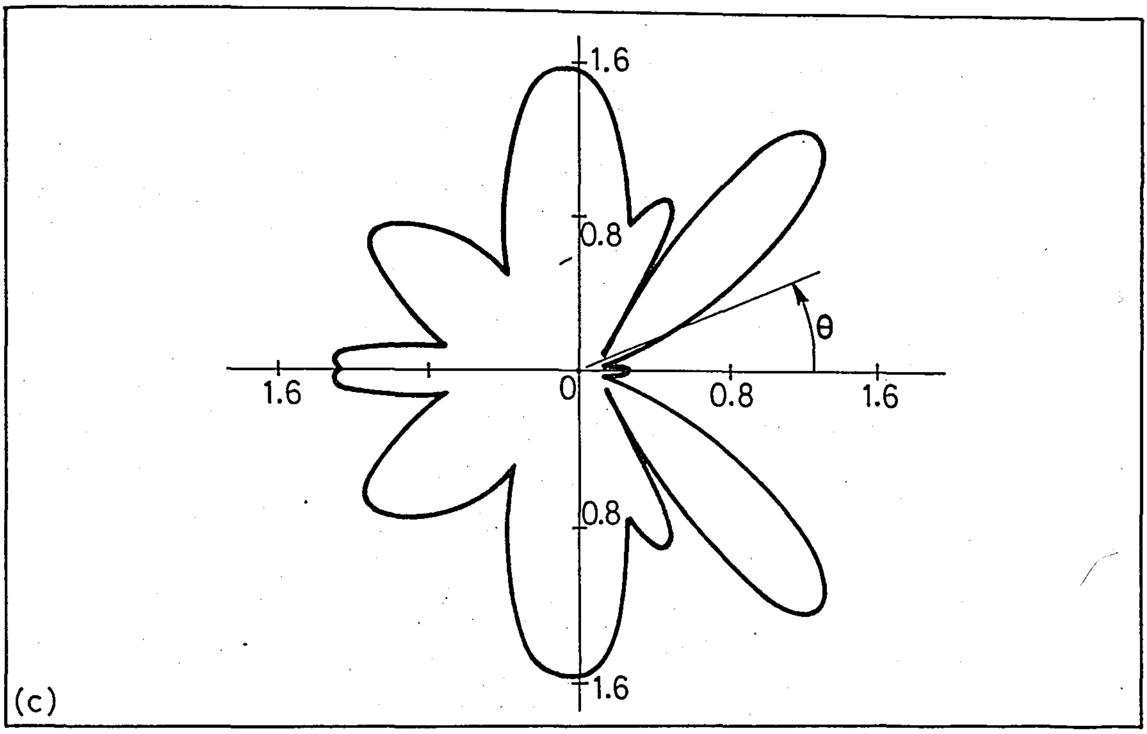
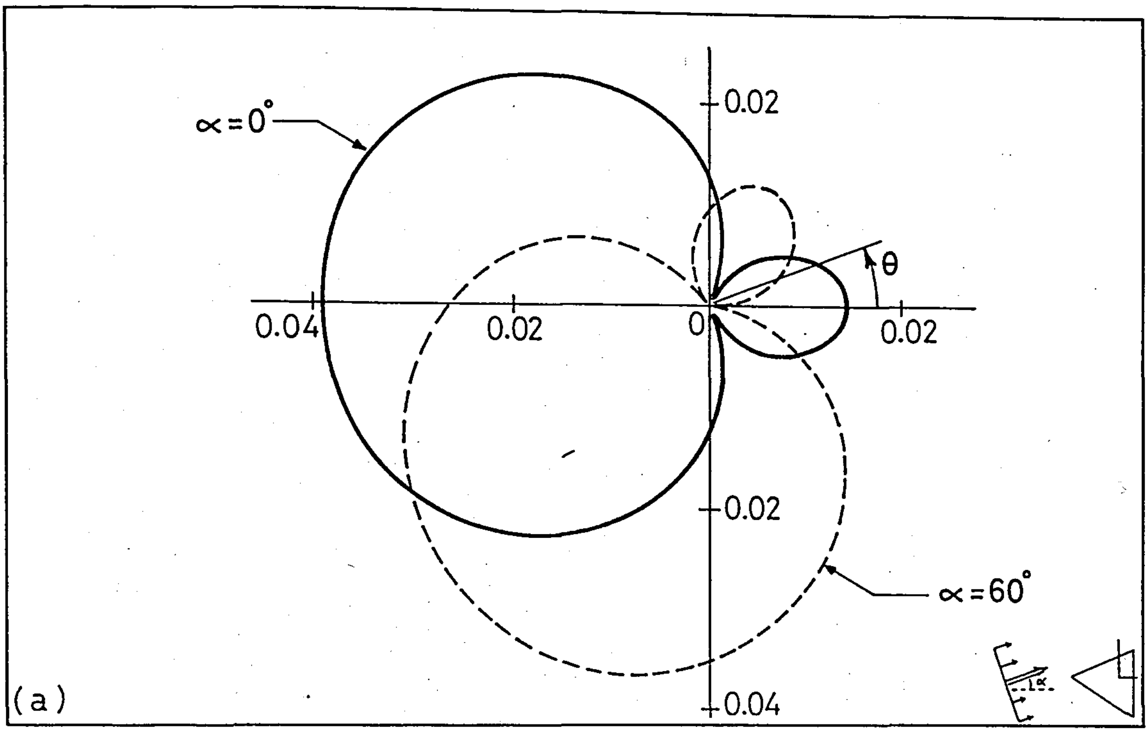


Figure 4.35 (continued).



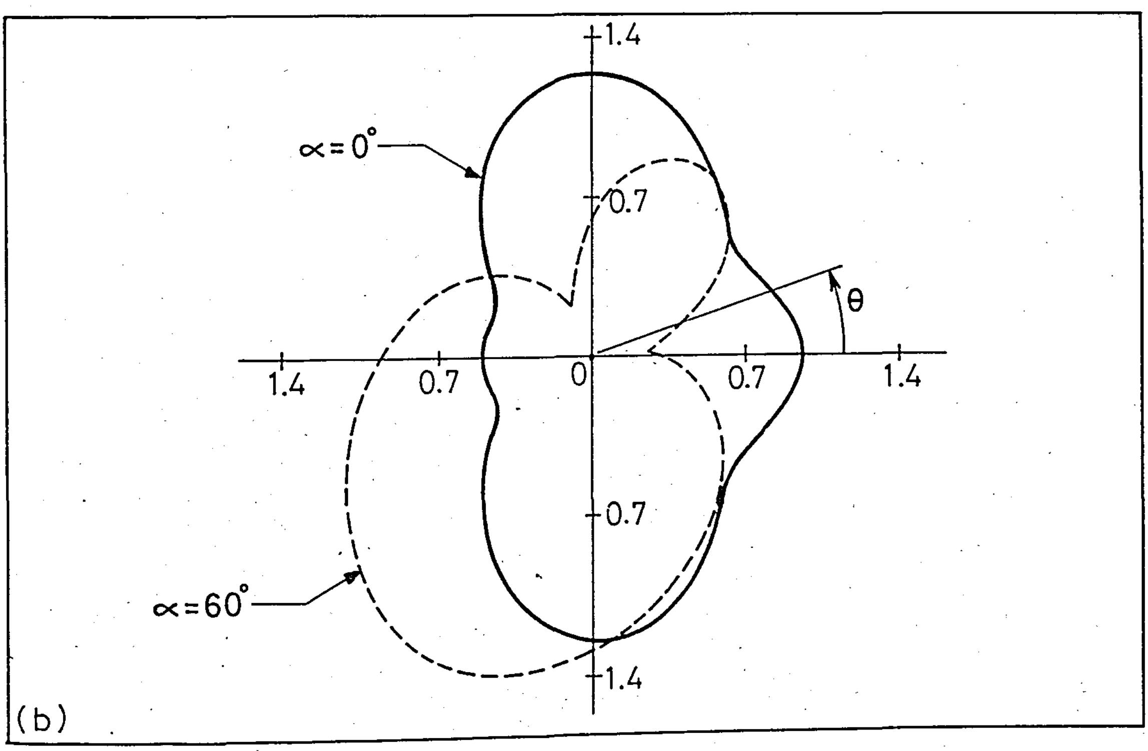


Figure 4.36 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid triangular inclusion for $\beta = 60^0$;
(a) $kh = 0.3$, (b) $kh = 3.0$, (c) $kh = 9.0$.

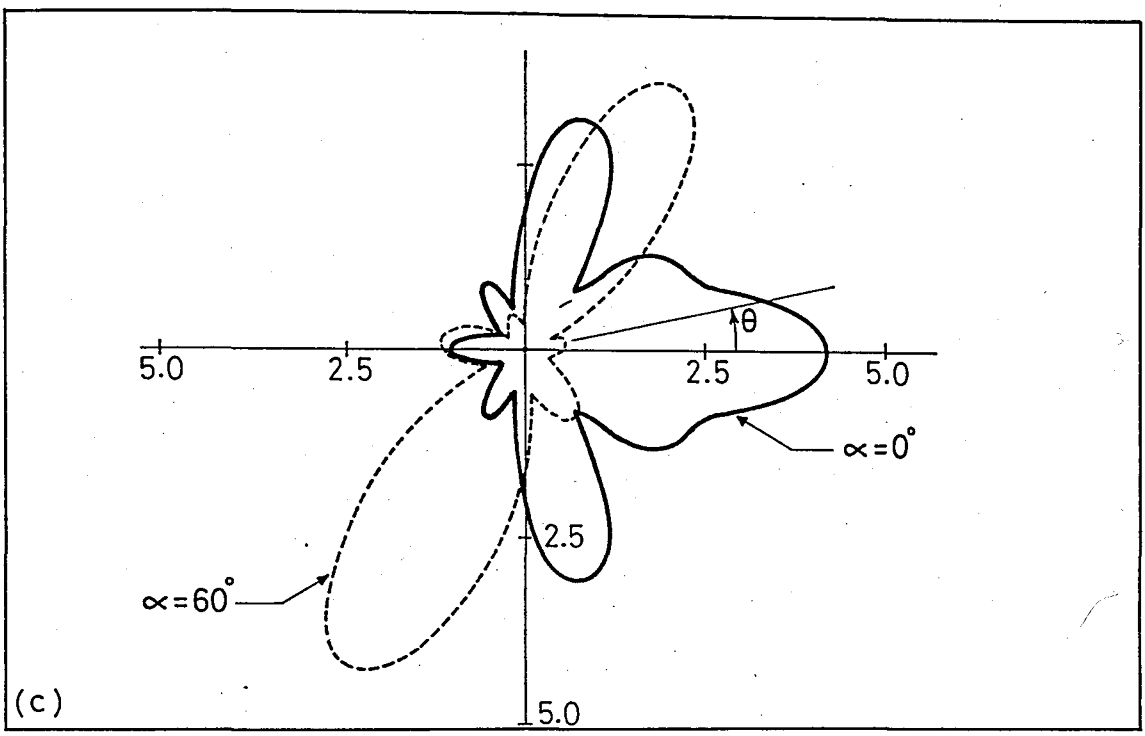
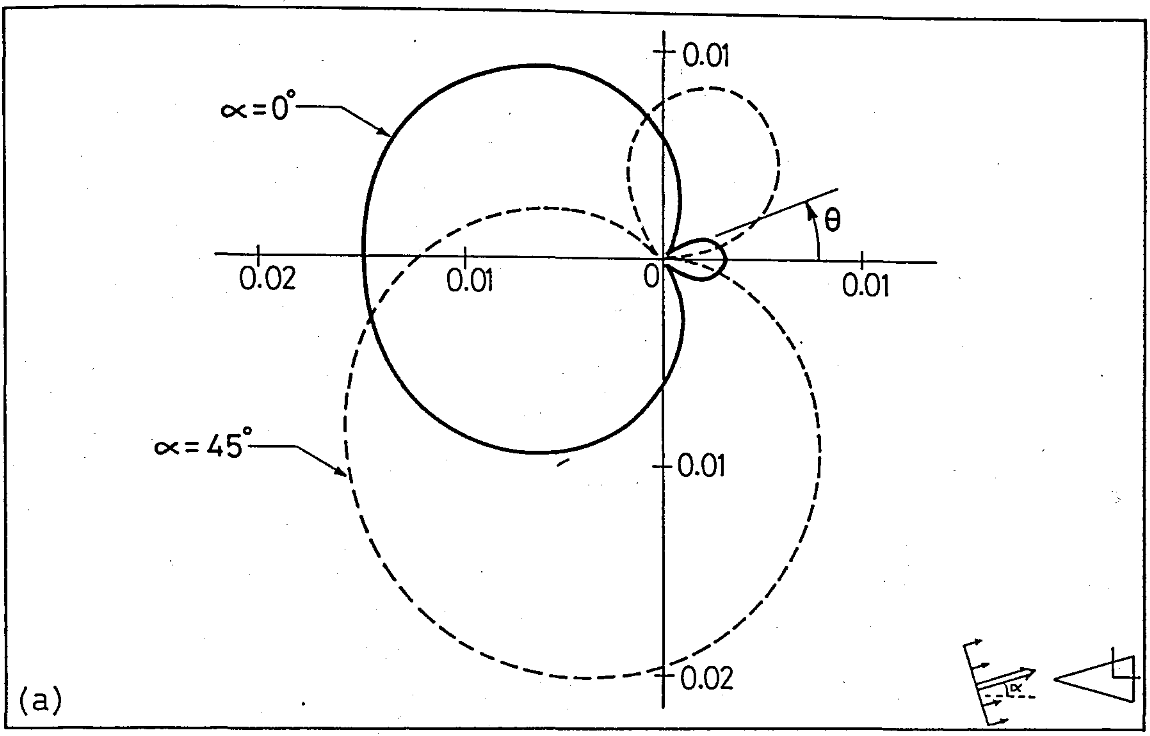


Figure 4.36 (continued).



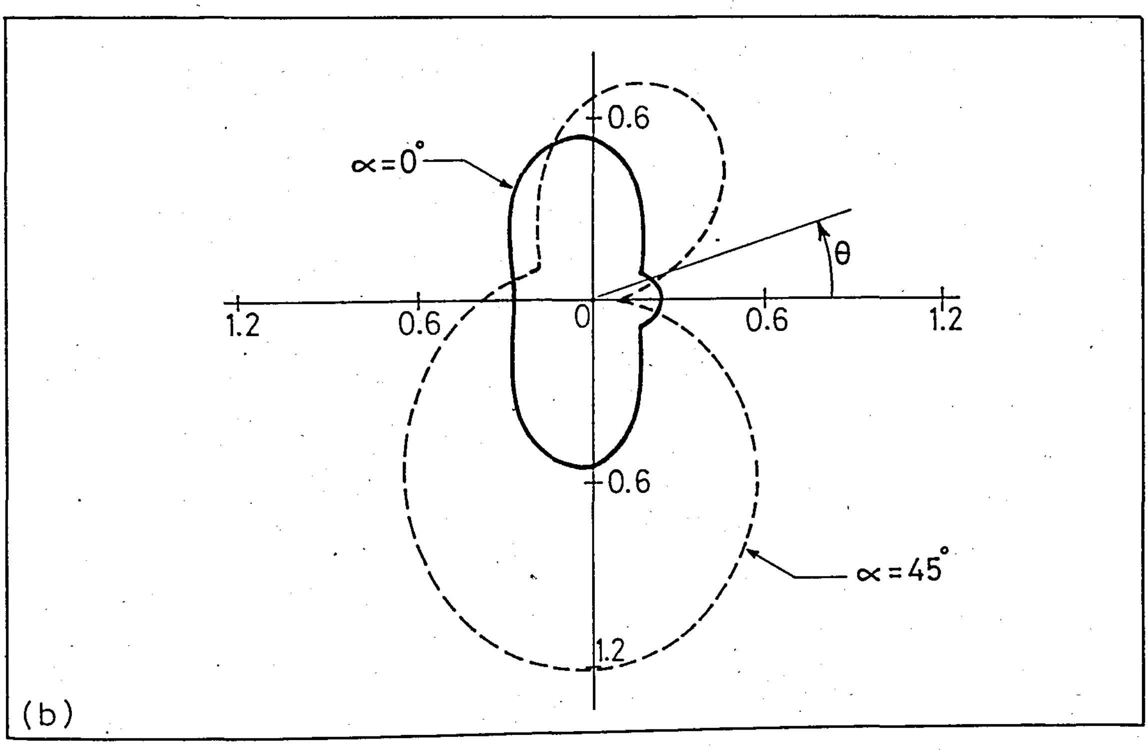


Figure 4.37 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid triangular inclusion for $\beta = 30^0$;
(a) kh = 0.3, (b) kh = 3.0, (c) kh = 9.0.

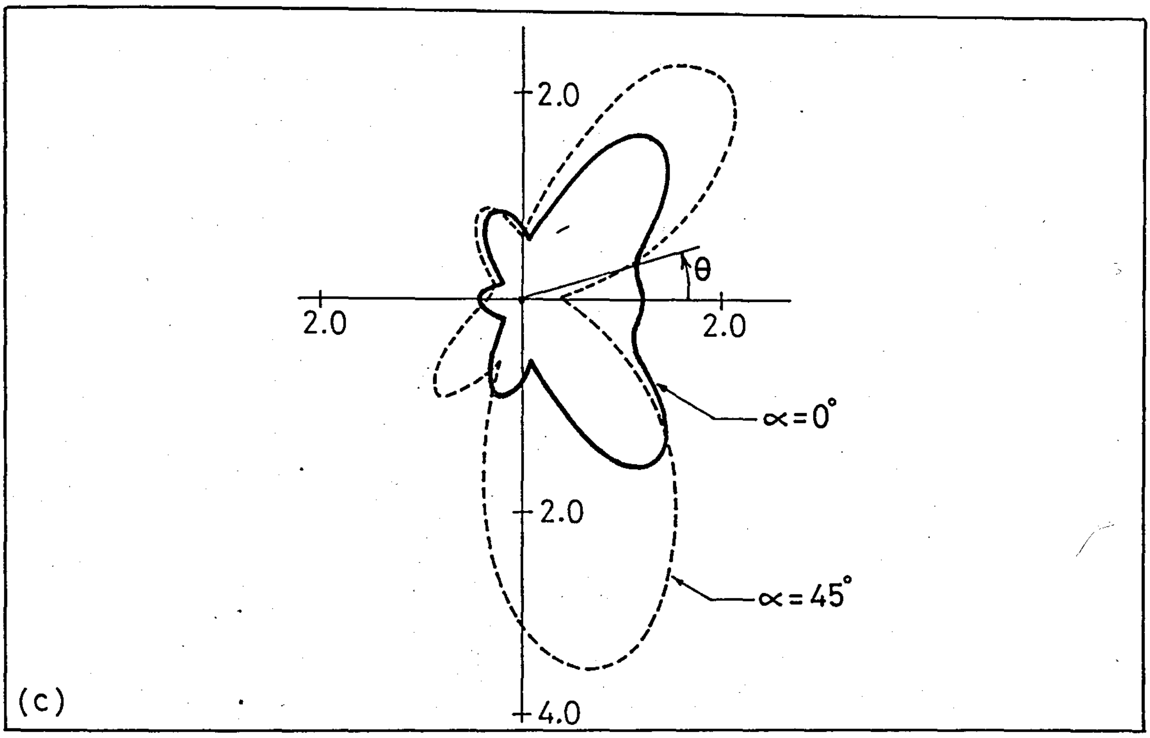


Figure 4.37 (continued).

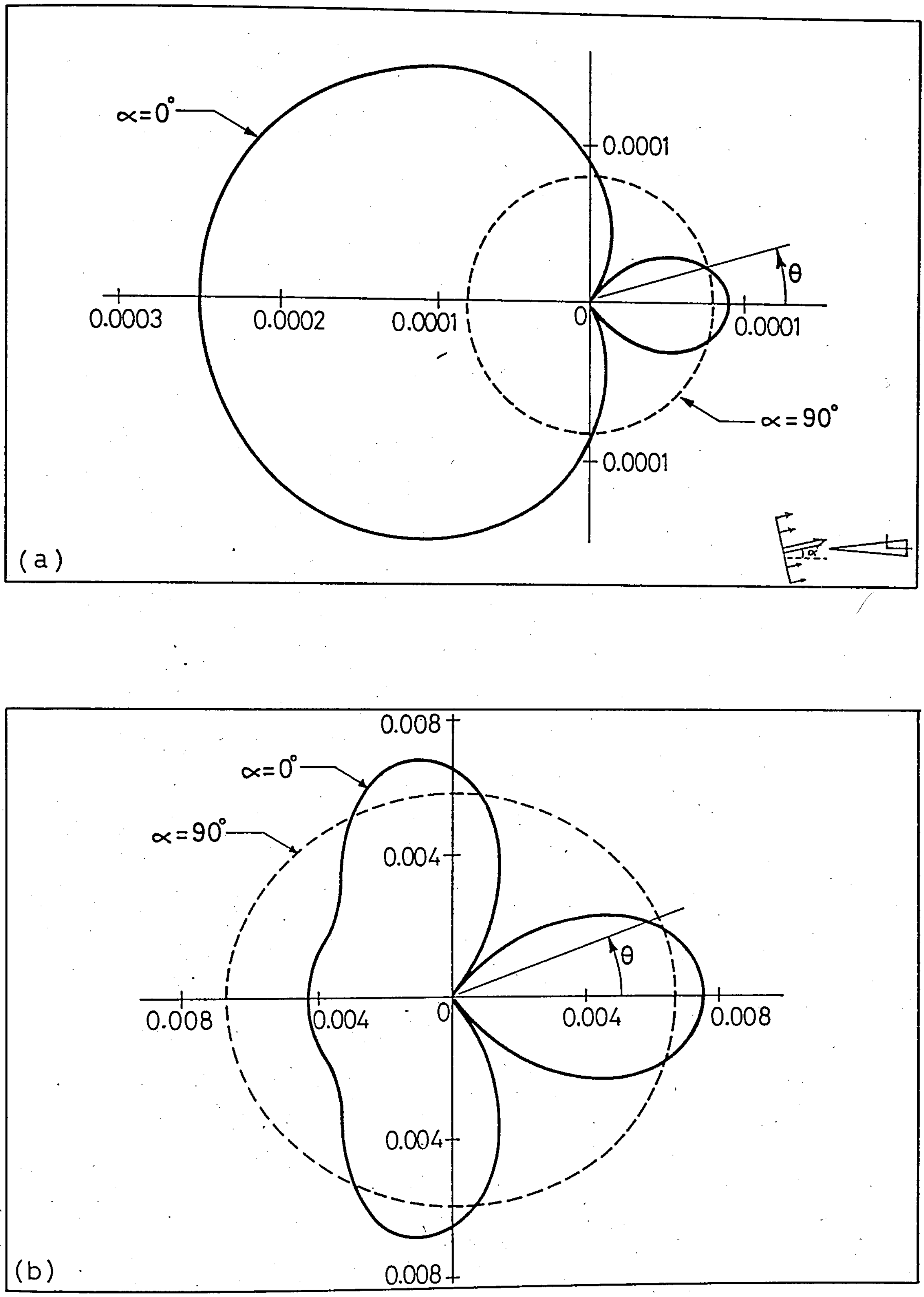


Figure 4.38 - Far field amplitude, $|f/A|$, due to the scattered wave field from a rigid triangular inclusion for $\beta = 0.001^{0}$;
(a) kh = 0.3, (b) kh = 3.0, (c) kh = 9.0.

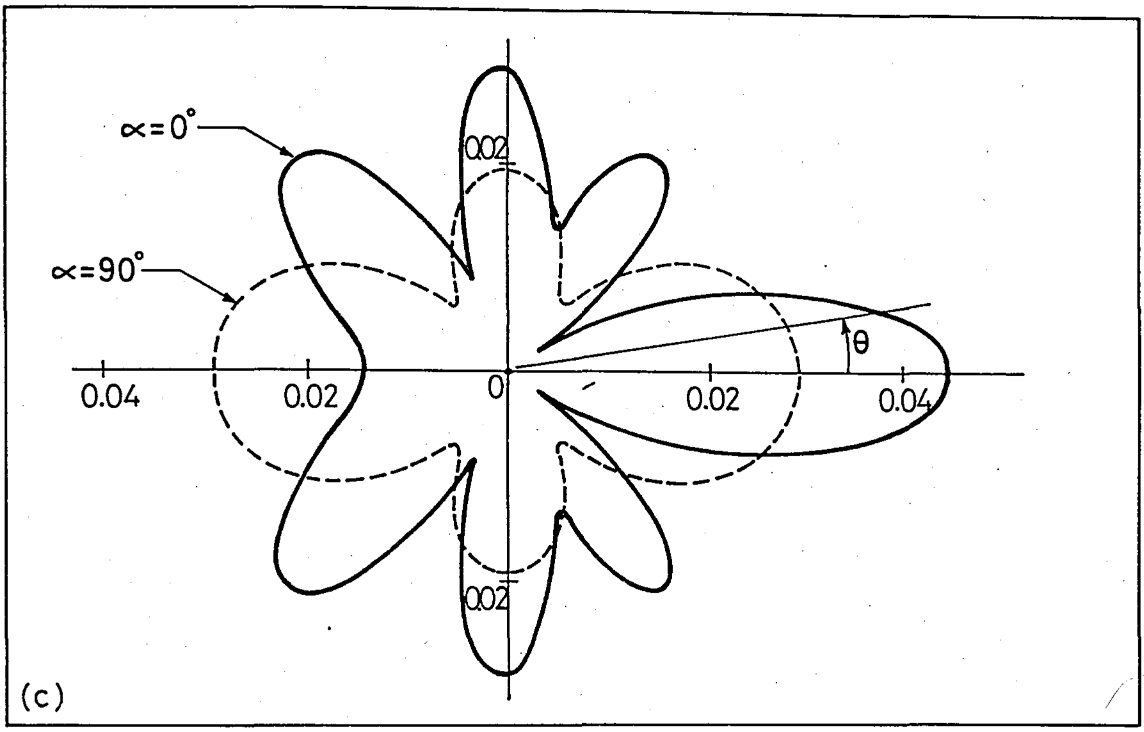
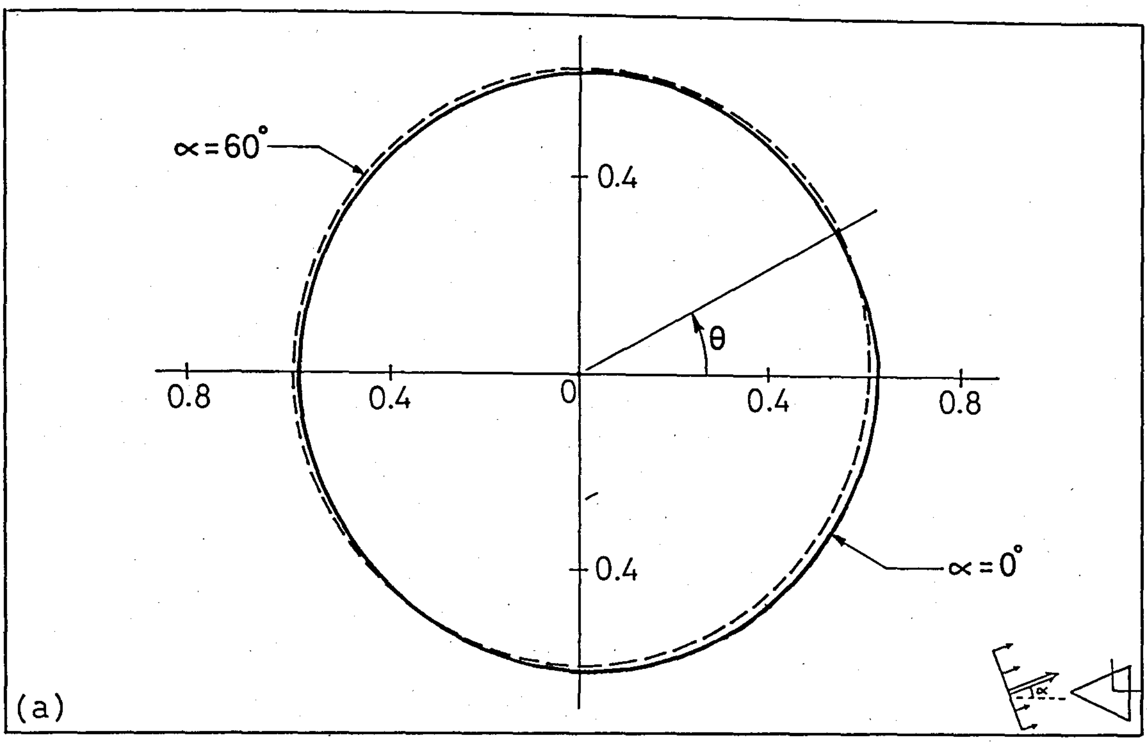


Figure 4.38 (continued).



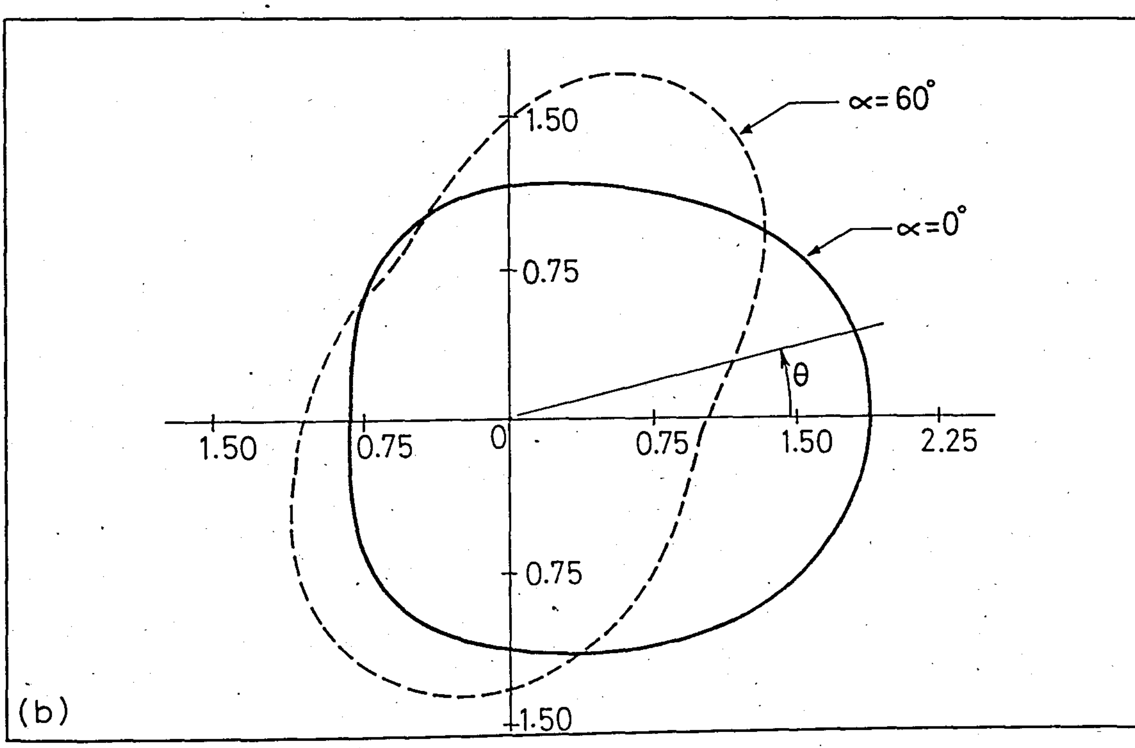


Figure 4.39 - Far field amplitude, $|f/A|$, due to the scattered wave field from a triangular cavity for $\beta = 60^0$; (a) kh = 0.3, (b) kh = 3.0, (c) kh = 9.0.

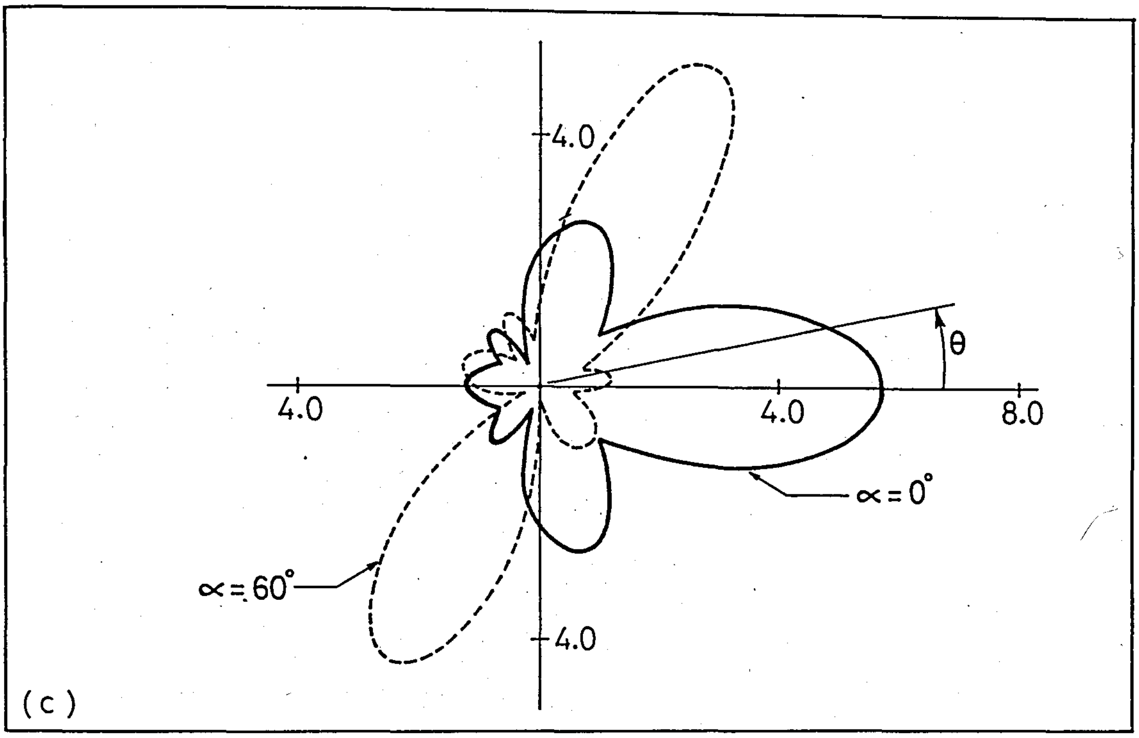


Figure 4.39 (continued).

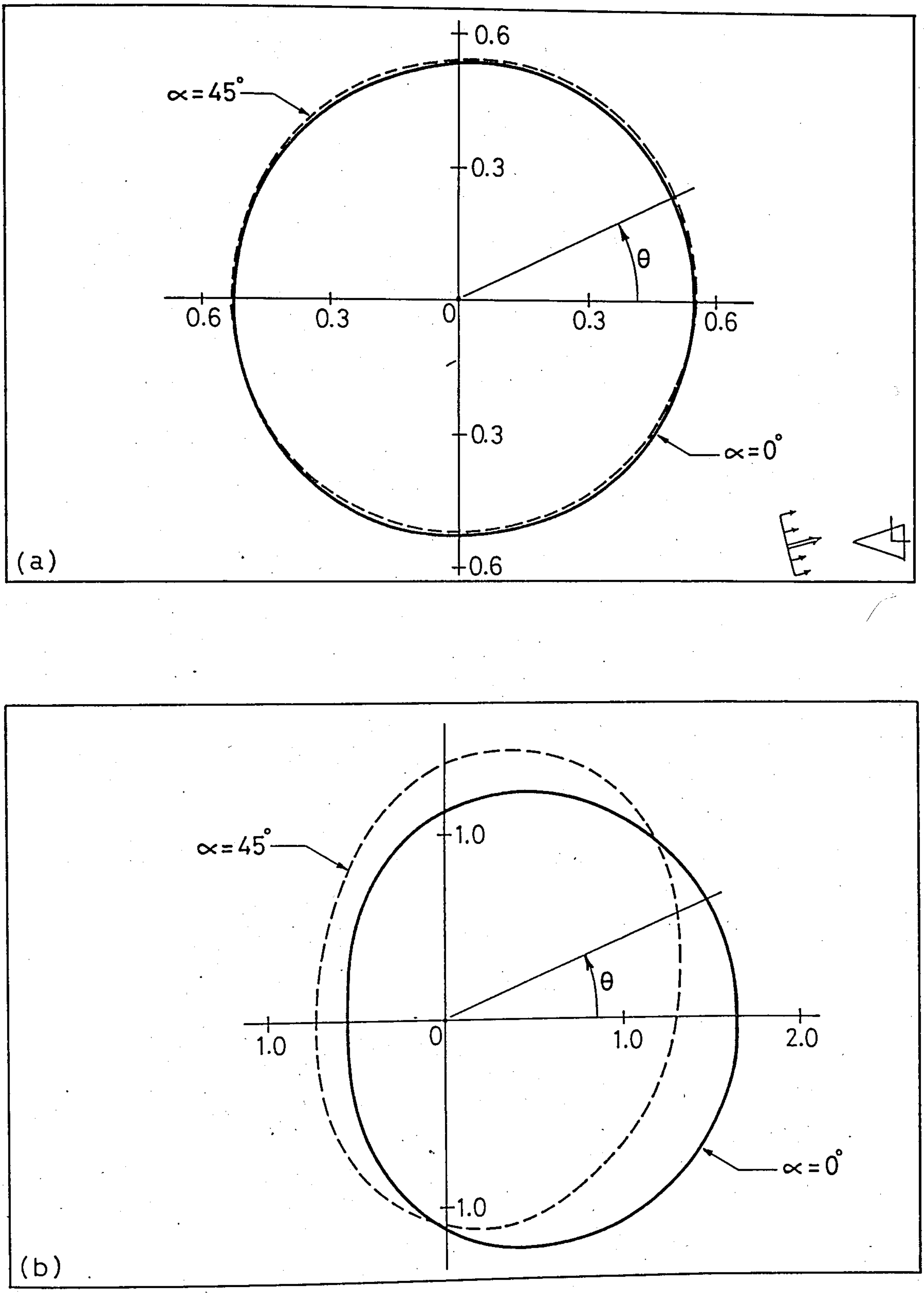


Figure 4.40 - Far field amplitude, $|f/A|$, due to the scattered wave field from a triangular cavity for $\beta = 30^o$; (a) kh = 0.3, (b) kh = 3.0, (c) kh = 9.0.

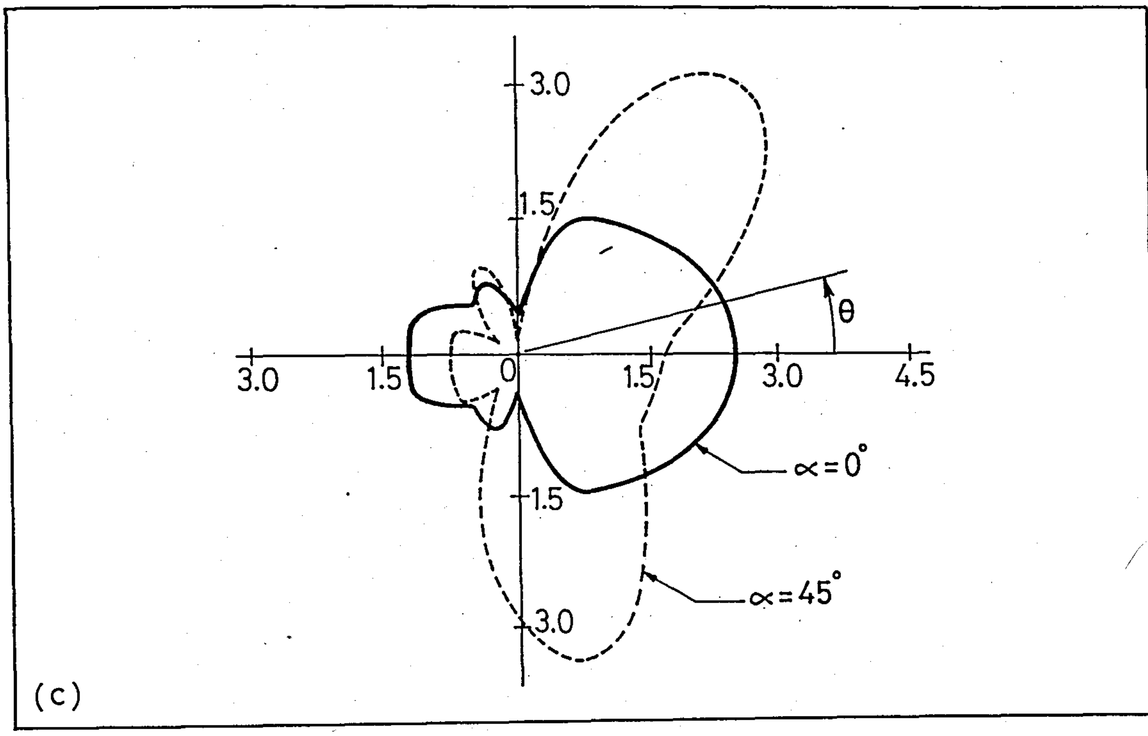
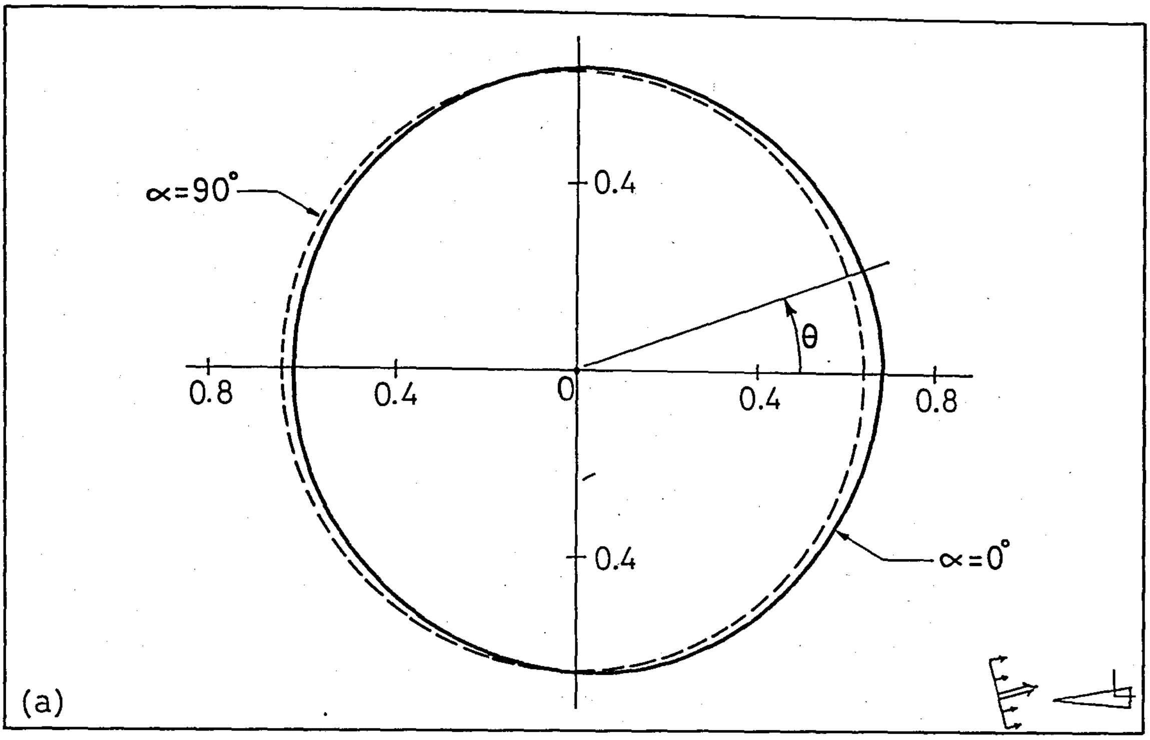


Figure 4.40 (continued).



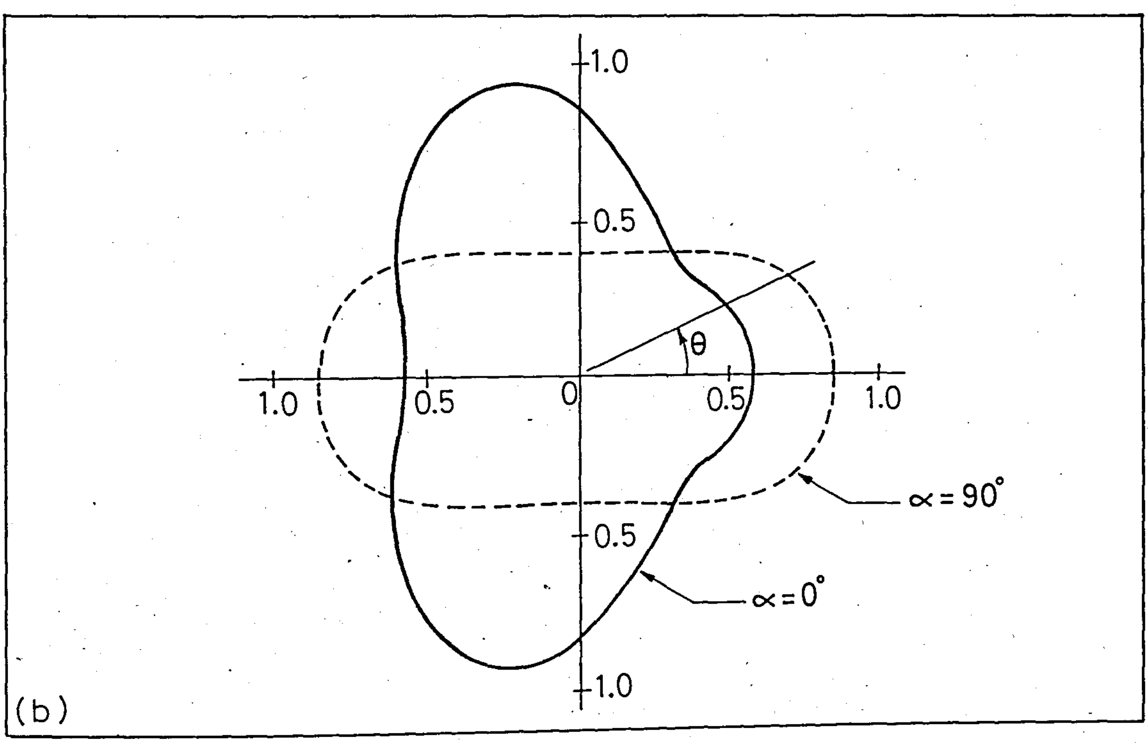


Figure 4.41 - Far field amplitude, $|f/A|$, due to the scattered wave field a triangular cavity for $\beta = 0.001^{\circ}$; (a) kh = 0.3 , (b) kh = 3.0 , (c) kh = 9.0.

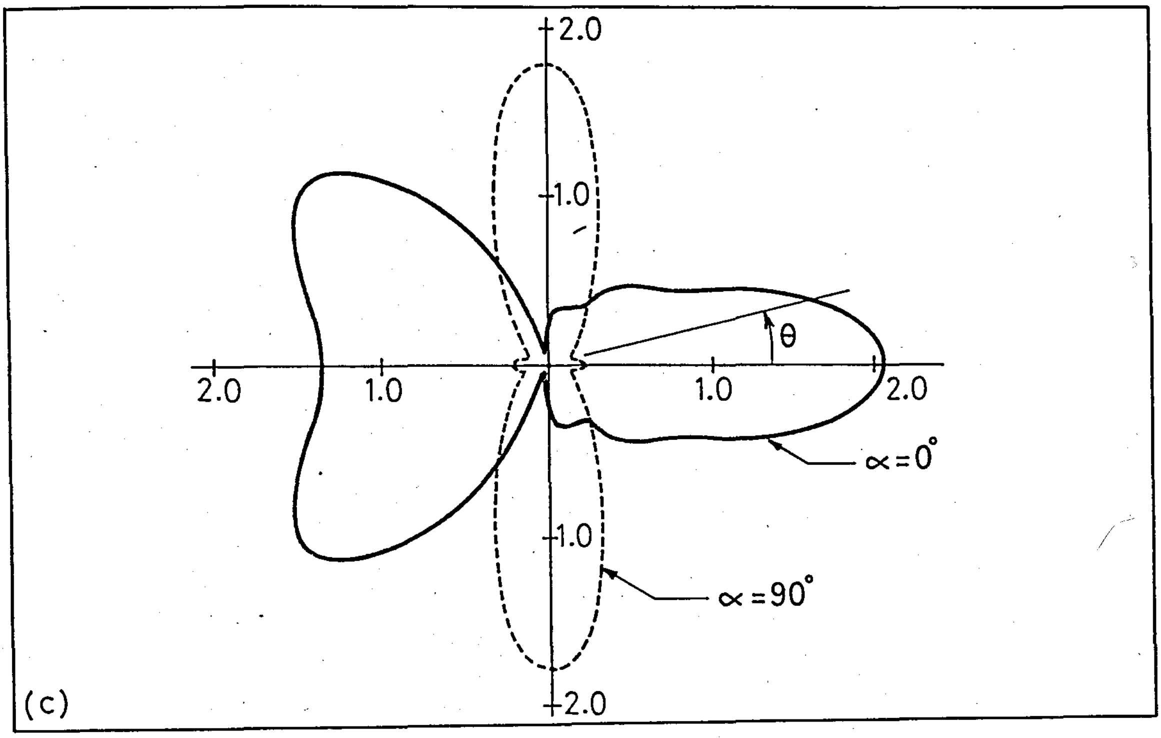


Figure 4.41 (continued).

APPENDICES

APPENDIX A

SIMPSON'S APPROXIMATE INTEGRATON FORMULA

Consider an integral in the form

$$I = \int_a^b f(x)dx \qquad \text{(A.1)}$$

The above integral can be evaluated approximately by using Simpson's rule which can be written as

$$\int_a^b f(x)dx \simeq C_s h[w_o f(a) + w_1 f(a+h) + w_2 f(a+2h) + \ldots + w_P f(b)] \,, \qquad \text{(A.2)}$$

where C_s is a constant factor, w_i's are the weight factors, P is the order of the Simpson's rule used for the approximation and h is given by

$$h = (b - a)/P \qquad \text{(A.3)}$$

The expression given by Eq. (A.2) may also be written in a compact form as

$$\int_a^b f(x)dx \simeq C_s h \sum_{i=o}^{P} w_i f(a + i \times h) \ . \qquad \text{(A.4)}$$

As an example, for a fourth order Simpson's rule (Bode's formula) [19], the values of C_s and w_i are

$$C_s = 2/45 \quad , \quad w_o = w_4 = 7 \; , \qquad w_1 = w_3 = 32 \; , \qquad w_2 = 12. \tag{A.5}$$

Thus Eq. (A.2) takes the form

$$\int_a^b f(x)dx \simeq (2/45)h[7f(a) + 32f(a + h) + 12f(a + 2h) + 32f(a + 3h) + 7f(b)] \; , \tag{A.6}$$

where

$$h = (b - a)/4 \; . \tag{A.7}$$

APPENDIX B

ANALYTICAL EXPRESSIONS FOR THE ELEMENTS OF THE Q-MATRIX

a. Rigid Inclusion:

$$Q^{12}_{jm} = \frac{(\varepsilon_j\varepsilon_m)^{\frac{1}{2}}}{4} \int_0^{2\pi} d\theta\{J_j(kr)\cos(j\theta)\}\{[mH_m(kr) - krH_{m+1}(kr)]\sin(m\theta)$$

$$- \frac{m}{kr} H_m(kr)\cos(m\theta) \frac{d}{d\theta}(kr)\}. \quad (B.1)$$

$$Q^{21}_{jm} = \frac{(\varepsilon_j\varepsilon_m)^{\frac{1}{2}}}{4} \int_0^{2\pi} d\theta\{J_j(kr)\sin(j\theta)\}\{[mH_m(kr) - krH_{m+1}(kr)]\cos(m\theta)$$

$$+ \frac{m}{kr} H_m(kr)\sin(m\theta) \frac{d}{d\theta}(kr)\} . \quad (B.2)$$

$$Q^{22}_{jm} = \frac{(\varepsilon_j\varepsilon_m)^{\frac{1}{2}}}{4} \int_0^{2\pi} d\theta\{J_j(kr)\sin(j\theta)\}\{[mH_m(kr) - krH_{m+1}(kr)]\sin(m\theta)$$

$$- \frac{m}{kr} H_m(kr)\cos(m\theta) \frac{d}{d\theta}(kr)\}. \quad (B.3)$$

b. Cavity:

$$Q^{11}_{jm} = - \frac{(\varepsilon_j\varepsilon_m)^{\frac{1}{2}}}{4} \int_0^{2\pi} d\theta\{H_m(kr)\cos(m\theta)\}\{[jJ_j(kr) - krJ_{j+1}(kr)]\cos(j\theta)$$

$$+ \frac{j}{kr} J_j(kr)\sin(j\theta) \frac{d}{d\theta}(kr)\} . \quad (B.4)$$

$$Q^{12}_{jm} = -\frac{(\varepsilon_j\varepsilon_m)^{\frac{1}{2}}}{4}\int_0^{2\pi} d\theta\{H_m(kr)\sin(m\theta)\}\{[jJ_j(kr) - krJ_{j+1}(kr)]\cos(j\theta)$$

$$+ \frac{j}{kr} J_j(kr)\sin(j\theta) \frac{d}{d\theta}(kr)\}. \qquad (B.5)$$

$$Q^{21}_{jm} = -\frac{(\varepsilon_j\varepsilon_m)^{\frac{1}{2}}}{4}\int_0^{2\pi} d\theta\{H_m(kr)\cos(m\theta)\}\{[jJ_j(kr) - krJ_{j+1}(kr)]\sin(j\theta)$$

$$- \frac{j}{kr} J_j(kr)\cos(j\theta) \frac{d}{d\theta}(kr)\}. \qquad (B.6)$$

$$Q^{22}_{jm} = -\frac{(\varepsilon_j\varepsilon_m)^{\frac{1}{2}}}{4}\int_0^{2\pi} d\theta\{H_m(kr)\sin(m\theta)\}\{[jJ_j(kr) - krJ_{j+1}(kr)]\sin(j\theta)$$

$$- \frac{j}{kr} J_j(kr)\cos(j\theta) \frac{d}{d\theta}(kr)\}. \qquad (B.7)$$

APPENDIX C

NUMERICAL EXPRESSIONS FOR THE ELEMENTS OF THE Q-MATRIX

a. <u>Rigid Inclusion:</u>

$$Q^{12}_{jm} \simeq \frac{(\varepsilon_j \varepsilon_m)^{\frac{1}{2}}}{4} \frac{C_s}{P} \sum_{i=1}^{N} \sum_{q=o}^{P} (\Delta\theta_i) w_q \{J_j(kr_{iq}) \cos(j\theta_{iq})\}$$

$$x\{[mH_m(kr_{iq}) - kr_{iq}H_{m+1}(kr_{iq})]\sin(m\theta_{iq})$$

$$- \frac{m}{kr_{iq}} H_m(kr_{iq}) \cos(m\theta_{iq}) [\frac{d(kr)}{d\theta}]_{\theta_{iq}}\} \; . \qquad (C.1)$$

$$Q^{21}_{jm} \simeq \frac{(\varepsilon_j \varepsilon_m)^{\frac{1}{2}}}{4} \frac{C_s}{P} \sum_{i=1}^{N} \sum_{q=o}^{P} (\Delta\theta_i) w_q \{J_j(kr_{iq}) \sin(j\theta_{iq})\}$$

$$x \{[mH_m(kr_{iq}) - kr_{iq}H_{m+1}(kr_{iq})]\cos(m\theta_{iq})$$

$$+ \frac{m}{kr_{iq}} H_m(kr_{iq}) \sin(m\theta_{iq}) [\frac{d(kr)}{d\theta}]_{\theta_{iq}}\} \; . \qquad (C.2)$$

$$Q_{jm}^{22} \simeq \frac{(\varepsilon_j\varepsilon_m)^{\frac{1}{2}}}{4} \frac{C_s}{P} \sum_{i=1}^{N} \sum_{q=o}^{P} (\Delta\theta_i) w_q \{J_j(kr_{iq})\sin(j\theta_{iq})\}$$

$$\times \{[mH_m(kr_{iq}) - kr_{iq}H_{m+1}(kr_{iq})]\sin(m\theta_{iq})$$

$$- \frac{m}{kr_{iq}} H_m(kr_{iq})\cos(m\theta_{iq})[\frac{d(kr)}{d\theta}]_{\theta_{iq}}\} . \qquad (C.3)$$

b. <u>Cavity</u>:

$$Q_{jm}^{11} \simeq - \frac{(\varepsilon_j\varepsilon_m)^{\frac{1}{2}}}{4} \frac{C_s}{P} \sum_{i=1}^{N} \sum_{q=o}^{P} (\Delta\theta_i) w_q \{H_m(kr_{iq})\cos(m\theta_{iq})\}$$

$$\times \{[jJ_j(kr_{iq}) - kr_{iq}J_{j+1}(kr_{iq})]\cos(j\theta_{iq})$$

$$+ \frac{j}{kr_{iq}} J_j(kr_{iq})\sin(j\theta_{iq})[\frac{d(kr)}{d\theta}]_{\theta_{iq}}\}. \qquad (C.4)$$

$$Q_{jm}^{12} \simeq - \frac{(\varepsilon_j\varepsilon_m)^{\frac{1}{2}}}{4} \frac{C_s}{P} \sum_{i=1}^{N} \sum_{q=o}^{P} (\Delta\theta_i) w_q \{H_m(kr_{iq})\sin(m\theta_{iq})\}$$

$$\times \{[jJ_j(kr_{iq}) - kr_{iq}J_{j+1}(kr_{iq})]\cos(j\theta_{iq})$$

$$+ \frac{j}{kr_{iq}} J_j(kr_{iq})\sin(j\theta_{iq})[\frac{d(kr)}{d\theta}]_{\theta_{iq}}\}. \qquad (C.5)$$

$$Q_{jm}^{21} \simeq - \frac{(\varepsilon_j \varepsilon_m)^{\frac{1}{2}}}{4} \frac{C_s}{P} \sum_{i=1}^{N} \sum_{q=o}^{P} (\Delta\theta_i) w_q \{H_m(kr_{iq}) \cos(m\theta_{iq})\}$$

$$\times \{[jJ_j(kr_{iq}) - kr_{iq} J_{j+1}(kr_{iq})] \sin(j\theta_{iq})$$

$$- \frac{j}{kr_{iq}} J_j(kr_{iq}) \cos(j\theta_{iq}) [\frac{d(kr)}{d\theta}]_{\theta_{iq}}\}. \qquad (C.6)$$

$$Q_{jm}^{22} \simeq - \frac{(\varepsilon_j \varepsilon_m)^{\frac{1}{2}}}{4} \frac{C_s}{P} \sum_{i=1}^{N} \sum_{q=o}^{P} (\Delta\theta_i) w_q \{H_m(kr_{iq}) \sin(m\theta_{iq})\}$$

$$\times \{[jJ_j(kr_{iq}) - kr_{iq} J_{j+1}(kr_{iq})] \sin(j\theta_{iq})$$

$$- \frac{j}{kr_{iq}} J_j(kr_{iq}) \cos(j\theta_{iq}) [\frac{d(kr)}{d\theta}]_{\theta_{iq}}\}. \qquad (C.7)$$

APPENDIX D

COMPUTER PROGRAM LISTING

```
C
C                  ****************************************
C                  *                                      *
C                  *       SCATTERING OF ACOUSTIC WAVES   *
C                  *                                      *
C                  * BY THE CYLINDERS OF ARBITRARY CROSS-SECTION *
C                  *                                      *
C                  *     ((( T-MATRIX FORMULATION )))     *
C                  *                                      *
C                  ****************************************
C
C
C                         ***  P R O G R A M-(A)  ***
C                         ===========================
C        THIS PROGRAM CREATES THE T-MATRIX FOR A GIVEN BOUNDARY
C        GEOMETRY AND WAVE NUMBER,THEN,STORES THE ELEMENTS OF
C        THE T-MATRIX INTO A DATA FILE(10) TO BE USED BY THE
C        PROGRAM-(B) FOR EVALUATION OF THE SCATTERED FIELD.
C
C
C     --FOLLOWING CROSS-SECTIONS FOR THE INFINITE CYLINDERS
C       CAN BE HANDLED:
C             1-) CIRCULAR
C             2-) ELLIPTICAL
C             3-) RECTANGULAR (ROUND CORNERED)
C             4-) TRIANGULAR (ISOSCELES)
C
C
C
C     ** THE PROCEDURES AND GENERAL STEPS FOLLOWED BY THE **
C     ** PROGRAM ARE AS FOLLOWS :                         **
C
C
C     --ELEMENTS OF THE Q11,Q12,Q21,Q22-MATRICES(SUBMATRICES
C       OF THE Q-MATRIX) ARE EVALUATED BY PERFORMING THE
C       ANGULAR INTEGRATIONS ALONG THE BOUNDARY OF THE SCATTERER.
C
C
C     --ANGULAR INTEGRATIONS ARE EVALUATED NUMERICALLY BY THE
C       SIMPSON'S RULE OF VARIOUS ORDER.(RECOMMENDED ORDER IS 4)
C
C
C     --THEN, THE Q-MATRIX IS INVERTED AND THE T-MATRIX WHICH
C       IS GIVEN BY:
C                                     -1
C                         T=-Q   * RE(Q)
C       IS CONSTRUCTED, BY THE GAUSS-SCHMIDT ORTHOGONALIZATION
C       TECHNIQUE.
C
C
C     --FINALLY, THE T-MATRIX AND SOME PARAMETERS DESCRIBING
C       THE BOUNDARY ARE STORED INTO A FILE(REFERRED AS UNIT 10).
C
C
C            * NOTE: *
C            =========
C               Q , ORTHOGONALIZED Q AND T-MATRICES ARE STORED
C            ALSO INTO TWO DATA FILES(REFERRED AS UNIT 11 & 22)
C            TO BE USED BY TWO ADDITIONAL PROGRAMS TO MAKE THE
C            FOLLOWING CHECKS :
C                                     -1
C                      1-)  Q * Q   = I
C
C                                  T
C                      2-)  T * CONJ(T) = -RE(T)
C
C
C                      ********************
C                       I N P U T   D A T A
C                      ********************
C     CARD 1 :
C     ========
C         COLUMN 1-2: NI (READING UNIT OR FILE NO.)
C         COLUMN 3-4: NO (WRITING UNIT OR FILE NO.)
C
C     CARD 2 :
C     ========
C         COLUMN 1-2: NQ (HALF SIZE OF THE T-MATRIX.MAX(NQ)=20)
C         COLUMN 3-4: NOSF (ORDER OF THE SIMPSON'S FORMULA.)
C
```

```
C
C   CARD 3 :
C   ========
C       COLUMN 1-2: BF1 (BF1 & BF2 ARE THE TWO MULTIPLICATION
C       COLUMN 3-8: BF2  FACTORS INVOLVED IN THE SIMPSON'S
C                         FORMULA. EX: 4'TH ORDER S.F. IS GIVEN BY:
C         INT(A TO B)F(X)*DX=2./45.*H*(7*F0+32*F1+12*F2+32*F3+7*F4)
C            WHERE: F0=F(A) , F1=F(A+H) , F2=F(A+2H) ,..., F4=F(B)
C                      THEN, BF1=2.  &  BF2=45. )
C              (*SEE CARD SET(12) FOR THE REMAINING COEFFICIENTS)
C
C   CARD 4 :
C   ========
C       COLUMN 1-4: BCON (BOUNDARY CONDITION. ENTER:
C                          1-) 'NEUM' FOR NEUMANN TYPE B.C.
C                          2-) 'DRIC' FOR DIRICHLET TYPE B.C. )
C
C   CARD 5 :
C   ========
C       COLUMN 1-4: ANO (WAVE NUMBER)
C
C   CARD 6 :
C   ========
C       COLUMN 1-6: SHAPE (CROSS-SECTIONAL GEOMETRY OF THE
C                           SCATTERER. ENTER:
C                          1-) 'CIRCLE' FOR CIRCULAR BOUNDARY
C                          2-) 'ELLIPS' FOR ELLIPTICAL BOUNDARY
C                          3-) 'RECTAN' FOR RECTANGULAR BOUNDARY
C                          4-) 'TRIANG' FOR TRIANGULAR BOUNDARY
C
C   CARD 7 :
C   ========
C       COLUMN 1-4 , 5-8 , 9-19 :
C          FOR CIRCLE: RAD (RADIUS)
C          FOR ELLIPSE: AA,BB (HALF MAJOR & MINOR AXES)
C          FOR RECTANGLE: A,B (HALF MAJOR & MINOR AXES),
C                          CORRAD (CORNER RADIUS)
C          FOR TRIANGLE: H (HEIGHT IN X-DIRECTION),BET (ANGLE
C                          BETWEEN TWO EQUAL SIDES.(IN DEGREES))
C
C   CARD 8 :
C   ========
C       COLUMN 1-4: BGEN (TYPE OF THE  ANGULAR DIVISION TO BE USED
C                          IN THE NUMERICAL EVALUATION OF THE
C                          BOUNDARY INTEGRALS.    ENTER:
C                     1-) 'AUTO' FOR EQUAL ANGULAR INTERVALS
C                          (DIVISIONS ARE MADE AUTOMATICALLY)
C                     2-) 'MAN ' FOR NON-EQUAL ANGULAR INTERVALS
C                          (DIVISIONS MUST BE GIVEN IN CARD SET(11))
C       COLUMN 5-6: NSYM (NSYM=2 FOR THE BOUNDARIES HAVING MIRROR
C                          SYMMETRY W.R.T. X-AXIS ,THEN INTEGRATIONS
C                          ARE PERFORMED ONLY FOR HALF OF THE
C                          BOUNDARY (BETWEEN 0-180 DEGREES)
C                          NSYM=1 FOR NON-SYMMETRIC ONES)
C                          (*NOTE:* 1-) FOR THIS PROGRAM TAKE NSYM=2
C                                   2-) FOR NSYM=2,Q12(I,J) & Q21(I,J)=0.
C                                                  T12(I,J) & T21(I,J)=0.
C                                       AND NOT TO BE PRINTED)
C
C   CARD 9 :
C   ========
C       COLUMN 1-6: ERR (ERROR FACTOR FOR THE BESSEL FUNCTIONS.
C                          TAKE ERR=1.D-10 - 1.D-15)
C       COLUMN 7-12: TRUNC (TRUNCATION FACTOR FOR THE Q-MATRIX
C                            ELEMENTS. TAKE TRUNC=1.D-20 - 1.D-25)
C
C   CARD 10 :  (IF BGEN='MAN ' ,OMIT THIS CARD)
C   =========
C       COLUMN 1-3: NINTV (NUMBER OF ANGULAR INTERVALS.FOR A
C                     GOOD CONVERGENCY IN THE NUMERICAL INTEG-
C                     RATION, TAKE NINTV=72-90)
C                     (*NOTE:* NINTV SHOULD BE AN EVEN NUMBER)
C
C   CARD SET 11 : (IF BGEN='AUTO' ,OMIT THIS CARD SET)
C   =============
C         *EACH CARD CONTAINS:*
C       COLUMN 1-3: NCIN (NO OF EQUAL SUCCESSIVE INTERVALS REPEATED)
C       COLUMN 4-7: CIN (CORRESPONDING ANGLE FOR THE
C                         INTERVALS (IN DEGREES))
C         *NOTE.* CARDS MUST CONTINUE UP TO WHEN :
```

```
C                         SUM( NCIN * CIN  )=360 DEG,FOR NSYM=1
C                                  I     I  =180 DEG,FOR NSYM=2
C
C  CARD SET 12 :
C  ==============
C     *EACH CARD CONTAINS:*
C      COLUMN 1-6: W(I) (WEIGHING COEFFICIENTS IN SIMPSON'S FORMULA.
C                          EXAMPLE: W(1)=7.  (CARD 12-1)
C                                   W(2)=32. (CARD 12-2)
C                                   W(3)=12. (CARD 12-3)
C                                   W(4)=32. (CARD 12-4)
C                                   W(5)=7.  (CARD 12-5)
C
C
C                            ****************
C                             SAMPLE DATA:
C                            ****************
C  A-) EQUAL ANGULAR INTERVALS         B-) NON-EQUAL ANG.INTERVALS
C
C                COLUMN                             COLUMN
C      ==================                  ==================
C      CARD I123456789012                  CARD I123456789012
C      =====I============                  =====I============
C        1  I 5 8                             1  I 5 8
C        2  I 5 4                             2  I10 4
C        3  I2. 45.                           3  I2. 45.
C        4  INEUM                             4  INEUM
C        5  I0.5                              5  I1.0
C        6  ICIRCLE                           6  IELLIPS
C        7  I1.0                              7  I1.0 2.0
C        8  IAUTO 2                           8  IMAN  2
C        9  I1.D-121.D-25                     9  I1.D-151.D-24
C       10  I 72                             10  I 20 4.
C       11  I 7.                             11  I 10 6.
C       12  I 32.                            12  I 10 4.
C       13  I 12.                            13  I 7.
C       14  I 32.                            14  I 32.
C       15  I 7.                             15  I 12.
C                                            16  I 32.
C                                            17  I 7.
C
C
C*********************************************************************
C
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      REAL*8 RQ(40,40),W(10),R(10),DTDR(10),TETL(10),XI(10),CINTV(145),
     &       DREAL,DIMAG,DABS,CDABS,DFLOAT,DSQRT,DCOS,DSIN,DATAN
C
      COMPLEX*16 Q11(20,20),Q12(20,20),Q21(20,20),Q22(20,20),
     &           TM(40,40),Q(40,40),BES1(22),BES2(22),HANK(22),
     &           DUM1,DUM2,X1,X2,P,RR,C,D,DZ,DN,D1,D2,DCMPLX
C
      CHARACTER*4 BCON,BGEN,WRIT(8)
      CHARACTER*6 SHAPE
      CHARACTER*1 LINE(132)
      CHARACTER   H1*12,H2*25,H3*18,H4*33,H5*8,H6*10,H7*29,H8*37,
     &            H9*9,H10*13,H11*28,H12*23,H13*27,MATR*8,H14*10
C
      COMMON/INVO/Q
      COMMON/REQ/RQ
      COMMON/TMAT/TM
      COMMON/GRAD/RAD
      COMMON/GAX/AA,BB
      COMMON/REC/A,B,CORRAD,TT1,TT2,TT3,TT4,ALPH,DIS
      COMMON/TRIAN/H,BETA,TETA1
C
      EQUIVALENCE (RQ(1),W(1)),(RQ(11),R(1)),(RQ(21),DTDR(1)),
     &            (RQ(31),TETL(1)),(RQ(41),XI(1)),(RQ(51),CINTV(1))
C
      G(V)=V*360./(2.*PI)
C
      DATA Q11,Q12,Q21,Q22/1600*(0.D0,0.D0)/
      DATA LINE/132*'='/
      DATA WRIT/'Q-11','Q-12','Q-21','Q-22','T-11','T-12','T-21','T-22'/
      DATA H1,H2,H3/'B.C. TYPE : ','NEUMANN (RIGID INCLUSION)','DIRICHLE
     &T (CAVITY)'/
      DATA H4/'CROSS-SECTION OF THE SCATTERER : '/
      DATA H5,H6,H7/'CIRCULAR','ELLIPTICAL','RECTANGULAR (ROUND  CORNERE
```

```
     &D)'/
      DATA H8/'SYMMETRY CONDITION (W.R.T. X-AXIS) : '/
      DATA H9,H10/'SYMMETRIC','NON-SYMMETRIC'/
      DATA H11/'TYPE OF BOUNDARY DIVISION : '/
      DATA H12,H13/'EQUAL ANGULAR INTERVALS','NON-EQUAL ANGULAR INTERVAL
     &S'/
      DATA MATR/'  MATRIX'/
      DATA H14/'TRIANGULAR'/
C
C
      READ(5,777) NI,NO
  777 FORMAT(2I2)
      READ(NI,7) NQ,NOSF,BF1,BF2
    7 FORMAT(2I2/F2.0,F6.0)
      NGP=NOSF+1
      NQ2=NQ*2
      NQT=NQ+2
      PI=DATAN(1.D0)*4.D0
      WRITE(NO,613)
  613 FORMAT(1H1)
      WRITE(NO,100)
  100 FORMAT(///5X,47('*')/5X,'*',45X,'*'/5X,'*',8X,'SCATTERING OF ACOUS
     &TIC WAVES',9X,'*'/5X,'*',45X,'*'/5X,'* BY THE CYLINDERS OF ARBITRA
     &RY CROSS-SECTION *'/5X,'*',45X,'*'/5X,'*',8X,'((( T-MATRIX FORMULA
     &TION )))',9X,'*'/5X,'*',45X,'*'/5X,47('*')////5X,'** PROGRAM-(A):
     & GENERATION OF THE T-MATRIX ELEMENTS **'/5X,54('=')////)
      READ(NI,1) BCON,ANO,SHAPE
    1 FORMAT(A4/F4.0/A6)
      IF(SHAPE.EQ.'CIRCLE') READ(NI,81) RAD
      IF(SHAPE.EQ.'ELLIPS') READ(NI,81) AA,BB
      IF(SHAPE.EQ.'RECTAN') READ(NI,81) A,B,CORRAD
      IF(SHAPE.EQ.'TRIANG') THEN
         READ(NI,81) H,BET
         BETA=BET*2.*PI/360.
         TETA1=DATAN(3.*DTAN(BETA/2.))
      END IF
      READ(NI,82) BGEN,NSYM,ERR,TRUNC
   82 FORMAT(A4,I2/2D6.0)
   81 FORMAT(2F4.0,F10.0)
      IF(BGEN.EQ.'AUTO') THEN
         READ(NI,30) NINTV
   30    FORMAT(I3)
C**** AUTUMATIC ANGULAR INTERVAL GENERATION ****
         DO 31 I=1,NINTV/NSYM
   31    CINTV(I)=PI*2./DFLOAT(NINTV)
      ELSE
C**** MANUAL ANGULAR INTERVAL GENERATION ****
         NINTV=0
         KK=0
         TOT=0.
         DO 32 I=1,150/NSYM
         READ(NI,33) NCIN,CIN
   33    FORMAT(I3,F4.0)
         DO 34  J=1,NCIN
         K=KK+J
   34    CINTV(K)=2.*PI/360.*CIN
         TOT=TOT+NCIN*CIN
         NINTV=NINTV+NSYM*NCIN
         IF(TOT.EQ.(360./DFLOAT(NSYM))) GO TO 35
   32    KK=KK+NCIN
   35 END IF
      DO 18 I=1,NGP
   18 READ(NI,19) W(I)
   19 FORMAT(F6.0)
      WRITE(NO,90)
   90 FORMAT(63('=')/)
      IF(BCON.EQ.'NEUM') WRITE(NO,47) H1,H2
   47 FORMAT(1X,A12,A25/)
      IF(BCON.EQ.'DRIC') WRITE(NO,47) H1,H3
      WRITE(NO,42) ANO
   42 FORMAT(1X,'WAVE NUMBER : ',F4.1)
      IF(SHAPE.EQ.'CIRCLE') WRITE(NO,43) H4,H5,RAD
   43 FORMAT(//1X,A33,A8//1X,'RADIUS : ',F6.3)
      IF(SHAPE.EQ.'ELLIPS') WRITE(NO,44) H4,H6,AA,BB
   44 FORMAT(//1X,A33,A10//1X,'A-AXIS : ',F6.3,4X,'B-AXIS : ',F6.3)
      IF(SHAPE.EQ.'RECTAN') WRITE(NO,45) H4,H7,A,B,CORRAD
   45 FORMAT(//1X,A33,A29//1X,'A-AXIS:',F6.3,'B-AXIS:',F6.3,4X,
     &       'CORNER RAD.:',F13.10)
       IF(SHAPE.EQ.'TRIANG') WRITE(NO,998) H4,H14,H,BET
  998 FORMAT(//1X,A33,A10//1X,'H:',F6.3,'  BETA:',F7.3)
```

```
      IF(NSYM.EQ.1) WRITE(NO,46) H8,H10
   46 FORMAT(/1X,A37,A13)
      IF(NSYM.EQ.2) WRITE(NO,46) H8,H9
      WRITE(NO,48) NINTV
   48 FORMAT(//1X,'NUMBER OF BOUNDARY SEGMENTS : ',I3)
      IF(BGEN.EQ.'AUTO') THEN
         WRITE(NO,49) H11,H12
      ELSE
         WRITE(NO,49) H11,H13
   49    FORMAT(/1X,A28,A27)
      END IF
      WRITE(NO,50) NOSF
   50 FORMAT(/1X,'ORDER OF THE SIMPSON',1H','S RULE USED IN NUMERICAL IN
     &TEGRATION : ',I2)
      WRITE(NO,51) NQ2,NQ2,TRUNC,ERR
   51 FORMAT(//1X,'TOTAL DIMENSION OF THE T-MATRIX : ',I3,' X',I3//
     &        1X,'TRUNCATION FACTOR FOR THE Q-MATRIX ELEMENTS : ',D9.3//
     &1X,'ERROR FACTOR FOR THE BESSEL FUNCTIONS : ',D9.3//63('=')/1H1)
      IF(BGEN.EQ.'AUTO') GO TO 53
      WRITE(NO,54)
   54 FORMAT(10X,'BOUNDARY   ANGULAR'/10X,'SEGMENT     INTERVAL'/13X,'NO
     &      (DEGREE)'/10X,19('='))
      DO 55 I=1,NINTV/NSYM
   55 WRITE(NO,56) I,G(CINTV(I))
   56 FORMAT(12X,I3,7X,F4.1/10X,19('='))
      WRITE(NO,57)
   57 FORMAT(1H1)
   53 TEB=CINTV(1)
      TEA=0.D0
      IF(NQ.GE.13.AND.NQ.LE.18) NQT=22
      IF(SHAPE.EQ.'RECTAN') CALL RECT
C**** START FOR BOUNDARY INTEGRATION AND GENERATION OF ****
C**** THE Q-MATRIX ELEMENTS                            ****
      DO 2 K=1,NINTV/NSYM
      IF(SHAPE.EQ.'ELLIPS') CALL ELLIP(TEA,TEB,R,TETL,NGP)
      IF(SHAPE.EQ.'CIRCLE') THEN
      TETL(1)=TEA
      DO 999 KKK=1,NGP
      IF(KKK.GT.1) TETL(KKK)=TETL(KKK-1)+(TEB-TEA)/DFLOAT(NGP-1)
  999 R(KKK)=RAD
      END IF
      IF(SHAPE.EQ.'TRIANG') CALL TRIA1(TEA,TEB,R,TETL,NGP,DTDR)
      IF(SHAPE.EQ.'RECTAN') CALL RECTA1(TEA,TEB,R,TETL,NGP,DTDR)
      IF(SHAPE.EQ.'CIRCLE'.OR.SHAPE.EQ.'ELLIPS')
     &                                CALL ANGDER(SHAPE,TETL,DTDR,NGP)
      DO 3 II=1,NGP
      ARG=AN0*R(II)
      DZ=DCMPLX(ARG,0.D0)
      DN=DCMPLX(0.D0,0.D0)
      DO 4 I=1,2
      CALL DBESS(1,DZ,DN,DUM2,BES2(I),ERR)
    4 DN=DN+DCMPLX(1.D0,0.D0)
      DO 5 I=3,NQT
    5 BES2(I)=2.*(I-2)/DZ*BES2(I-1)-BES2(I-2)
      DN=DCMPLX(((NQT-1)*1.D0),0.D0)
      DO 40 I=1,2
      CALL DBESS(0,DZ,DN,BES1(NQT+1-I),DUM1,ERR)
   40 DN=DN-DCMPLX(1.D0,0.D0)
      DO 41 I=3,NQT
   41 BES1(NQT+1-I)=2.*(NQT+1-I)/DZ*BES1(NQT-I+2)-BES1(NQT-I+3)
      DO 6 I=1,NQT
    6 HANK(I)=DCMPLX(DREAL(BES1(I)),DREAL(BES2(I)))
      DO 8 J=1,NQ+1
      DO 8 M=1,NQ+1
C**** SINCE, Q-MATRIX IS SYMMETRIC FOR SEPERABLE GEOMETRIES,ONLY ****
C**** THE LOWER TRIANGULAR PART IS TO BE EVALUATED FOR CIRCLE AND ***
C**** ELLIPSE.                                                   ****
      IF((SHAPE.EQ.'CIRCLE'.OR.SHAPE.EQ.'ELLIPSE').AND.
     &                              M.GT.J) GO TO 8
C**** FOR 'CIRCLE' , ( Q ) IS A DIAGONAL MATRIX.  ****
C**** THAT IS: QIJ(J,M)=0.0 FOR (J) NOT EQ. TO (M)****
C**** AND NEED NOT BE EVALUATED.                  ****
      IF(SHAPE.EQ.'CIRCLE'.AND.J.NE.M) GO TO 8
C**** IF CROSS-SECTIONAL GEOMETRY OF THE SCATTERER HAS ****
C**** A SYMMETRY W.R.T. Y-AXIS,THEN, QIJ(J,M)=0.0 FOR  ****
C**** (J+M) IS ODD,AND NEED NOT BE EVALUATED.        ****
C**** (*NOTE: TRIANGLE , LOCATED ON THE COORDINATE     ****
C****  SYSTEM SUCH THAT ITS SYMMETRY AXIS COINCIDES    ****
C****  WITH THE X-AXIS, HAS NO SYMMETRY W.R.T. Y-AXIS) ****
      IF(SHAPE.NE.'TRIANG'.AND.((J+M)/2)*2.NE.J+M)   GO TO 8
```

```
      EJEM=1./2.
      IF((J.EQ.1.AND.M.NE.1).OR.(M.EQ.1.AND.J.NE.1)) EJEM=DSQRT(2.)/4.D0
      IF(J.EQ.1.AND.M.EQ.1) EJEM=.25D0
       P=BF1*(TEB-TEA)/DFLOAT(NGP-1)/BF2*NSYM
      RR=P
      IF(BCON.EQ.'NEUM') THEN
         P=P*BES1(J)*DCOS((J-1)*TETL(II))
         RR=RR*BES1(J)*DSIN((J-1)*TETL(II))
         D1=HANK(M+1)
         D2=HANK(M)
         MM=M
      ELSE
         P=P*HANK(M)*DCOS((M-1)*TETL(II))
         RR=RR*HANK(M)*DSIN((M-1)*TETL(II))
         D1=BES1(J+1)
         D2=BES1(J)
         MM=J
         EJEM=-1.D0*EJEM
      END IF
C**** (X1)&(X2) ARE FUNCTION TYPE SUBPROGRAMS USED FOR ****
C**** CALCULATION OF THE Q-MATRIX ELEMENTS              ****
      C=X1(R(II),D1,TETL(II),DTDR(II),D2,W(II),MM,ANG,EJEM)
      D=X2(R(II),D1,TETL(II),DTDR(II),D2,W(II),MM,ANG,EJEM)
      IF(M.LE.NQ.AND.J.LE.NQ) Q11(J,M)=Q11(J,M)+C*P
      IF(M.GT.1.AND.J.GT.1) Q22(J-1,M-1)=Q22(J-1,M-1)+D*RR
C**** IF THE BOUNDARY HAS A MIRROR SYMMETRY W.R.T. X-AXIS, ****
C**** THEN, ELEMENTS OF THE  Q12 & Q21  MATRICES ARE ALL   ****
C**** ZERO AND NEED NOT BE EVALUATED.                      ****
      IF(2-NSYM) 52,8,52
   52 IF(BCON.EQ.'NEUM') THEN
         IF(M.GT.1.AND.J.LE.NQ) Q12(J,M-1)=Q12(J,M-1)+D*P
         IF(J.GT.1.AND.M.LE.NQ) Q21(J-1,M)=Q21(J-1,M)+C*RR
      ELSE
         IF(M.GT.1.AND.J.LE.NQ) Q12(J,M-1)=Q12(J,M-1)+C*RR
         IF(J.GT.1.AND.M.LE.NQ) Q21(J-1,M)=Q21(J-1,M)+D*P
      END IF
    8 CONTINUE
    3 CONTINUE
      IF(K-NINTV/NSYM) 58,2,2
   58 TEA=TEA+CINTV(K)
      TEB=TEB+CINTV(K+1)
C**** DISPLAY OF THE COMPLETED INTERVALS.(HELPFUL IN    ****
C**** INTERACTIVE EXECUTION OF THE PROGRAM IN TERMINAL)****
    2 WRITE(6,61) K
   61 FORMAT(12X,'** ',I2,' **')
C**** TRUNCATION OF THE Q-MATRIX ELEMENTS TO INCREASE   ****
C**** THE ACCURACY OF THE INVERSION,AND CONSTRUCTION OF****
C**** THE Q-MATRIX FROM QIJ-SUBMATRICES                 ****
      CALL CTRUNC(Q11,NQ2,0,0,TRUNC,NQ)
      CALL CTRUNC(Q12,NQ2,0,NQ,TRUNC,NQ)
      CALL CTRUNC(Q21,NQ2,NQ,0,TRUNC,NQ)
      CALL CTRUNC(Q22,NQ2,NQ,NQ,TRUNC,NQ)
      IF(SHAPE.EQ.'RECTAN'.OR.SHAPE.EQ.'TRIANG') GO TO 627
      DO 881 I=1,NQ2
      DO 881 J=1,NQ2
      IF(J.GT.I) Q(I,J)=Q(J,I)
  881 CONTINUE
  627 WRITE(11,777) NQ,NQ2
      NNQ=NQ*13
      NT=0
      IF(NQ.GT.10) NNQ=130
      LL=0
      NPRINT=2*NSYM-1
   73 DO 71 L=1,4,NPRINT
      NTEST=0
      MM=1
      NN=NQ
      IF(L.GT.2) THEN
         MM=NQ+1
         NN=NQ2
      END IF
      IF(L.EQ.2.OR.L.EQ.4) THEN
         K=NQ+1
         N=NQ2
      ELSE
         K=1
         N=NQ
      END IF
      IF(NQ.GT.10) THEN
         N=10
```

```
            IF(L.EQ.2.OR.L.EQ.4) N=NQ+10
         END IF
         WRITE(NO,65) (LINE(I),I=1,NNQ)
      65 FORMAT(1X,130A1)
         WRITE(NO,66) WRIT(L+LL),MATR
      66 FORMAT(20X,A4,A8)
   70    WRITE(NO,65) (LINE(I),I=1,NNQ)
         DO 68 I=MM,NN
         WRITE(NO,67) (DREAL(Q(I,J)),J=K,N)
      68 WRITE(NO,69) (DIMAG(Q(I,J)),J=K,N)
      67 FORMAT(/1X,10D13.5)
      69 FORMAT(2X,10D13.5)
         IF(NTEST.EQ.1) GO TO 71
         IF(NQ.GT.10) THEN
         NTEST=1
            K=N+1
            N=NQ
            IF(L.EQ.2.OR.L.EQ.4) N=NQ2
            GO TO 70
         END IF
      71 CONTINUE
         IF(LL.GT.0) GO TO 74
         IF(NT.EQ.1)  GO TO 444
         DO 76 J=1,NQ2
         DO 76 M=1,NQ2
      76 RQ(J,M)=DREAL(Q(J,M))*(-1.D0)
         WRITE(6,63)
      63 FORMAT(///1X,'** INVERSION STARTED **')
C**** INVERSION OF THE Q-MATRIX AND CREATION OF THE      ****
C**** T-MATRIX BY GAUSS-SCHMIDT ORTHOGONALIZATION        ****
         CALL PRCSSM(NQ2)
         WRITE(6,64)
      64 FORMAT(/1X,'***  COMPLETED  ***')
         DO 122 I=1,NQ2
         DO 122 J=1,NQ2
     122 WRITE(11,80) Q(I,J)
         DO 121 J=1,NQ2
         DO 121 I=1,NQ2
     121 WRITE(11,80)  DCONJG(Q(I,J))
         NT=NT+1
         WRITE(NO,673)
     673 FORMAT(///5X,'** ORTHOGONALIZED Q-MATRICES **'/)
         GO TO 73
     444 DO 72 I=1,NQ2
         DO 72 J=1,NQ2
      72 Q(I,J)=TM(I,J)
         LL=4
         GO TO 73
C**** STORAGE OF THE RESULTS INTO THE FILES TO BE USED ****
C**** BY THE PROGRAM-(B) AND OTHER CHECKING PROGRAMS   ****
      74 WRITE(10,75) BCON,ANO,SHAPE
      75 FORMAT(1X,A4,' (B.C.)'/1X,F4.1,' (WAVE NO)'/1X,A6,' (SCATTERER)')
         IF(SHAPE.EQ.'CIRCLE') WRITE(10,93) RAD
         IF(SHAPE.EQ.'ELLIPS') WRITE(10,77) AA,BB
         IF(SHAPE.EQ.'RECTAN') WRITE(10,91) A,B,CORRAD
         IF(SHAPE.EQ.'TRIANG') WRITE(10,997) H,BET
     997 FORMAT(1X,2F6.3,' ( H & BETA ) ')
      91 FORMAT(1X,2F6.3,F13.10,'(A & B  AXES , CORNER RADIUS)')
      93 FORMAT(1X,F6.3,' (RADIUS)')
      77 FORMAT(1X,2F6.3,' (A & B -AXES)')
         WRITE(10,78) NSYM,NQ
      78 FORMAT(1X,I2,' (SYM. COND.)'/1X,I3,' (NQ)')
         WRITE(22,777) NQ,NQ2
         DO 79 I=1,NQ2
         DO 79 J=1,NQ2
         WRITE(22,80) TM(I,J)
      79 WRITE(10,80) TM(I,J)
      80 FORMAT(1X,2D30.23)
         STOP
         END
C
C
C
C
C
         SUBROUTINE ELLIP(T1,T2,R,TET,NGP)
C
C***************************************************************
C   SUBPROGRAM FOR CALCULATION OF  R & TETA  VALUES IN POLAR
C   COORDINATES FOR EACH ANGULAR INTERVAL ALONG THE
```

```
C    ELLIPTICAL BOUNDARY.
C          T1 : LOWER ANGLE OF THE INTERVAL
C          T2 : UPPER ANGLE OF THE INTERVAL
C         NGP : NUMBER OF ANGULAR DIVISION FOR THAT INTERVAL
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION R(10),TET(10)
      COMMON/GAX/A,B
      F(T)=A*B/(DSQRT(A*A*(DSIN(T))**2+B*B*(DCOS(T))**2))
      T3=T2-T1
      TET(1)=T1
      R(1)=F(T1)
      DO 1 I=2,NGP
      TET(I)=TET(I-1)+T3/DFLOAT(NGP-1)
    1 R(I)=F(TET(I))
      RETURN
      END
C
C
C
C
C
      SUBROUTINE ANGDER(SHAPE,TETL,DTDR,NGP)
C
C************************************************************
C    SUBPROGRAM FOR CALCULATION OF THE ANGULAR DERIVATIVES
C    ( D(R)/D(TETA) ) FOR EACH INTERVAL ALONG THE
C    CIRCULAR & ELLIPTICAL BOUNDARIES.
C         TETL(I) : TETA VALUES OF THE INTERVAL
C                     ( I=1,NGP )
C         DTDR(I) : CORRESPONDING ANGULAR DERIVATIVES
C************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION TETL(10),DTDR(10)
      CHARACTER*6 SHAPE
      COMMON/GRAD/RAD
      COMMON/GAX/A,B
      F(T)=(A*B*(B*B-A*A)*DSIN(T)*DCOS(T))/DSQRT((A*A*DSIN(T)*DSIN(T)+B*
     &B*DCOS(T)*DCOS(T))**3)
      IF(SHAPE.EQ.'CIRCLE') GO TO 1
      GO TO 2
    1 DO 3 I=1,NGP
    3 DTDR(I)=0.D0
      RETURN
    2 IF(SHAPE.EQ.'ELLIPS') GO TO 4
      GO TO 5
    4 DO 6 I=1,NGP
    6 DTDR(I)=F(TETL(I))
    5 RETURN
      END
C
C
C
C
C
      SUBROUTINE RECT
C
C************************************************************
C    SUBPROGRAM FOR CALCULATION OF SOME REQUIRED PARAMETERS
C    DESCRIBING THE RECTANGULAR BOUNDARY TO BE USED
C    THROUGHOUT THE MAIN PROGRAM
C         INPUT DATA:( A , B , CORRAD )
C         RETURN PARAMETERS:(T1,T2,T3,T4,ALPH,DIS)
C************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/REC/A,B,CORRAD,T1,T2,T3,T4,ALPH,DIS
      PI=DATAN(1.D0)*4.D0
      T1=DATAN((B-CORRAD)/A)
      IF(A.EQ.CORRAD) THEN
        T2=PI/2.
        ALPH=PI/2.
        DIS=(B-CORRAD)
      ELSE
      T2=DATAN(B/(A-CORRAD))
      ALPH=DATAN((B-CORRAD)/(A-CORRAD))
      DIS=(A-CORRAD)/DCOS(ALPH)
      END IF
```

```
      T3=PI-T2
      T4=PI-T1
      RETURN
      END
C
C
C
C
C
      SUBROUTINE RECTA1(TEA,TEB,R,TETX,NGP,DTDR)
C
C*****************************************************
C  SUBPROGRAM FOR CALCULATION OF  R & TETA  VALUES AND ALSO
C  ANGULAR DERIVATIVES ( D(R)/D(TETA) ) FOR EACH ANGULAR
C  INTERVAL ALONG THE RECTANGULAR BOUNDARY
C           TEA : LOWER ANGLE OF THE INTERVAL
C           TEB : UPPER ANGLE OF THE INTERVAL
C           NGP : NUMBER OF ANGULAR DIVISION
C                     FOR THAT INTERVAL
C*****************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/REC/A,B,CORRAD,T1,T2,T3,T4,ALPH,DIS
      DIMENSION TETL(10),R(10),DTDR(10),TETX(10)
      F(T,X)=DIS*DCOS(T-X)+DSQRT(DIS**2*(DCOS(T-X))**2+CORRAD**2-DIS**2)
      G(TE)=DIS*DSIN(TE)+DIS**2*DCOS(TE)*DSIN(TE)/DSQRT(DIS**2*DCOS(TE)*
     &*2+CORRAD**2-DIS**2)
      PI=DATAN(1.D0)*4.D0
      TEC=TEB-TEA
      TETX(1)=TEA
      DO 1 I=2,NGP
    1 TETX(I)=TETX(I-1)+TEC/DFLOAT(NGP-1)
      DO 3 I=1,NGP
      TETL(I)=TETX(I)
      IF(TETX(I).GT.PI) TETL(I)=TETX(I)-PI
    3 CONTINUE
      DO 2 I=1,NGP
      IF(TETL(I).LE.T1.OR.TETL(I).GE.T4) THEN
        R(I)=DABS(A/DCOS(TETL(I)))
        DTDR(I)=A*DSIN(TETL(I))/DCOS(TETL(I))**2
        IF(TETL(I).GE.T4) DTDR(I)=-DTDR(I)
        GO TO 2
      END IF
      IF(TETL(I).GT.T1.AND.TETL(I).LT.T2) THEN
        R(I)=F(TETL(I),ALPH)
        TET=TETL(I)-ALPH
        DTDR(I)=-G(TET)
        GO TO 2
      END IF
      IF(TETL(I).GE.T2.AND.TETL(I).LE.T3) THEN
        R(I)=B/DSIN(TETL(I))
        DTDR(I)=-B*DCOS(TETL(I))/DSIN(TETL(I))**2
        GO TO 2
      END IF
      IF(TETL(I).GT.T3.AND.TETL(I).LT.T4) THEN
        R(I)=F(TETL(I),PI-ALPH)
        TET=PI-TETL(I)-ALPH
        DTDR(I)=G(TET)
      END IF
    2 CONTINUE
      RETURN
      END
C
C
C
C
C
      SUBROUTINE TRIA1(TEA,TEB,R,TETX,NGP,DTDR)
C
C*****************************************************
C  SUBPROGRAM FOR CALCULATION OF  R & TETA  VALUES AND ALSO
C  ANGULAR DERIVATIVES ( D(R)/D(TETA) ) FOR EACH ANGULAR
C  INTERVAL ALONG THE TRIANGULAR BOUNDARY
C           TEA : LOWER ANGLE OF THE INTERVAL
C           TEB : UPPER ANGLE OF THE INTERVAL
C           NGP : NUMBER OF ANGULAR DIVISION
C                     FOR THAT INTERVAL
C*****************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
```

```
      COMMON/TRIAN/H,BETA,TETA1
      DIMENSION TETL(10),R(10),DTDR(10),TETX(10)
      PI=DATAN(1.)*4.
      TEC=TEB-TEA
      TETX(1)=TEA
      DO 1 I=2,NGP
    1 TETX(I)=TETX(I-1)+TEC/DFLOAT(NGP-1)
      DO 3 I=1,NGP
      TETL(I)=TETX(I)
      IF(TETX(I).GT.PI)   TETL(I)=2.*PI-TETX(I)
    3 CONTINUE
      DO 2 I=1,NGP
      IF(TETL(I).LE.TETA1) THEN
      R(I)=H/3.*(1./DCOS(TETL(I)))
      DTDR(I)=H/3.*DSIN(TETL(I))/(DCOS(TETL(I)))**2
      IF(TETX(I).GT.PI) DTDR(I)=-DTDR(I)
      GO TO 2
      END IF
      IF(TETL(I).GT.TETA1) THEN
      R(I)=2.*H/3.*DTAN(BETA/2.)/(DSIN(TETL(I))-DCOS(TETL(I))*
     &     DTAN(BETA/2.))
      DTDR(I)=-2.*H/3.*DTAN(BETA/2.)*(DCOS(TETL(I))+DSIN(TETL(I))*
     &     DTAN(BETA/2.))/(DSIN(TETL(I))-DCOS(TETL(I))*
     &     DTAN(BETA/2.))**2
      IF(TETX(I).GT.PI) DTDR(I)=-DTDR(I)
      END IF
    2 CONTINUE
      RETURN
      END
C
C
C
C
C
      COMPLEX FUNCTION X1*16(XRL,XBESP,XTET,XDT,XBES,XW,IND,ANO,EJEM)
C
C*******************************************
C     SUBPROGRAM  USED IN CALCULATION OF THE
C                 Q-MATRIX ELEMENTS
C*******************************************
C
      COMPLEX*16 XBESP,XBES
      REAL*8 XRL,XTET,XDT,XW,DCOS,DSIN,ANO,EJEM,DFLOAT
                   X1=(((IND-1)*XBES-ANO*XRL*XBESP)*DCOS((IND-1)*XTET)+(IN
     &D-1)*DSIN((IND-1)*XTET)*XDT*XBES/XRL)*XW*EJEM
      RETURN
      END
C
C
C
C
C
      COMPLEX FUNCTION X2*16(XRL2,XBESP2,XTET2,XDT2,XBES2,XW2,IND2,ANO,
     &EJEM)
C
C*******************************************
C     SUBPROGRAM  USED IN CALCULATION OF THE
C                 Q-MATRIX ELEMENTS
C*******************************************
C
      COMPLEX*16 XBESP2,XBES2
      REAL*8 XRL2,XTET2,XDT2,XW2,DCOS,DSIN,EJEM,ANO
                   X2=(((IND2-1)*XBES2-ANO*XRL2*XBESP2)*DSIN((IND2-1)*XTET
     &2)-(IND2-1)*DCOS((IND2-1)*XTET2)*XDT2*XBES2/XRL2)*XW2*EJEM
      RETURN
      END
C
C
C
C
C
      SUBROUTINE CTRUNC(QIJ,NQT,M,N,TRUNC,NQ)
C
C******************************************
C    THIS SUBPROGRAM IS USED IN TRUNCATION &
C    CONSTRUCTION OF THE Q-MATRIX
C******************************************
C
      COMPLEX*16 QIJ(20,20),Q(40,40),DCMPLX
      COMMON/INVQ/Q
```

```
      REAL*8 CDABS,TRUNC
      DO 1 I=1,NOT/2
      DO 1 J=1,NOT/2
      IF(CDABS(QIJ(I,J)).LT.TRUNC) QIJ(I,J)=DCMPLX(0.D0,0.D0)
    1 Q(I+M,J+N)=QIJ(I,J)
      RETURN
      END
C
C
C
C
C
      SUBROUTINE   PRCSSM(NBGR)
C
C**********************************************************
C   SUBPROGRAM FOR GAUSS-SCHMIDT ORTHOGONALIZATION
C   AND CREATION OF THE T-MATRIX
C**********************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMPLEX*16 RI(40,40),TM(40,40),DUM
      REAL*8 RR1(40,40),DREAL,DIMAG,TMMX(40,40),RI1(40,40)
      COMMON/INVO/RI
      COMMON/Q/RI1
      COMMON/TMAT/TM
      COMMON/REQ/RR1
      DO 1 I=1,NBGR
      DO 1 J=1,NBGR
    1 RI1(I,J)=DIMAG(RI(I,J))
C**** CONDITIONING OF THE Q-MATRIX BEFORE ORTHOGONALIZATION ****
      CALL CNDTNO(NBGR)
C**** NORMALIZE THE N'TH ROW OF AN (N) BY (N) MATRIX    ****
      SUM1=0.D0
      DO 20 K=1,NBGR
      SUM1=RR1(NBGR,K)**2+RI1(NBGR,K)**2+SUM1
   20 CONTINUE
      SUM1=DSQRT(SUM1)
      DO 28 K=1,NBGR
      RR1(NBGR,K)=RR1(NBGR,K)/SUM1
      RI1(NBGR,K)=RI1(NBGR,K)/SUM1
   28 CONTINUE
C****  SET UP A LOOP FOR THE N-1 REMAINING ROWS. ****
      NMI=NBGR-1
      NROW=NBGR
      DO  100 I=1,NMI
      NROW=NROW-1
      MROW=NROW
      DO 36 K=1,NBGR
      TMMX(1,K)=RR1(NROW,K)
      TMMX(2,K)=RI1(NROW,K)
   36 CONTINUE
      DO 80 J=NROW,NMI
      SR1=0.D0
      SI1=0.D0
      MROW=MROW+1
      DO 40 K=1,NBGR
      SR1=SR1+RR1(MROW,K)*RR1(NROW,K)+RI1(MROW,K)*RI1(NROW,K)
      SI1=SI1+RR1(MROW,K)*RI1(NROW,K)-RI1(MROW,K)*RR1(NROW,K)
   40 CONTINUE
      DO 48 K=1,NBGR
      TMMX(1,K)=TMMX(1,K)-SR1*RR1(MROW,K)+SI1*RI1(MROW,K)
      TMMX(2,K)=TMMX(2,K)-SR1*RI1(MROW,K)-SI1*RR1(MROW,K)
   48 CONTINUE
   80 CONTINUE
      SUM1=0.D0
      DO 84 K=1,NBGR
      SUM1=SUM1+TMMX(1,K)**2+TMMX(2,K)**2
   84 CONTINUE
      SUM1=DSQRT(SUM1)
      DO 88 K=1,NBGR
      RR1(NROW,K)=TMMX(1,K)/SUM1
      RI1(NROW,K)=TMMX(2,K)/SUM1
   88 CONTINUE
  100 CONTINUE
      DO 2 I=1,NBGR
      DO 2 J=1,NBGR
    2 RI(I,J)=DCMPLX(RR1(I,J),RI1(I,J))
      DO 3 I=1,NBGR
      DO 3 J=1,NBGR
      DUM=RI(I,J)
```

```
      RI(I,J)=RI(J,I)
    3 RI(J,I)=DUM
C**** PERFORM Q-TRANSPOSE * REAL(Q) TO GET T-MATRIX . *****
      DO 160 I=1,NBGR
      DO 152 J=1,NBGR
      TMMX(I,J)=0.DO
  152 CONTINUE
  160 CONTINUE
      DO 180 I=1,NBGR
      DO 176 J=1,NBGR
      DO 172 K=1,NBGR
      TMMX(I,J)=TMMX(I,J)-RI1(K,I)*RR1(K,J)
  172 CONTINUE
  176 CONTINUE
  180 CONTINUE
      DO 196 I=1,NBGR
      DO 192 J=1,NBGR
      TM(I,J)=DCMPLX(0.DO,TMMX(I,J))
  192 CONTINUE
  196 CONTINUE
      DO 208 I=1,NBGR
      DO 204 J=1,NBGR
      TMMX(I,J)=0.DO
  204 CONTINUE
  208 CONTINUE
      DO 220 I=1,NBGR
      DO 216 J=1,NBGR
      DO 212 K=1,NBGR
      TMMX(I,J)=TMMX(I,J)+RR1(K,I)*RR1(K,J)
  212 CONTINUE
  216 CONTINUE
  220 CONTINUE
      DO 236 I=1,NBGR
      DO 232 J=1,NBGR
      TM(I,J)=TM(I,J)-DCMPLX(TMMX(I,J),0.DO)
  232 CONTINUE
  236 CONTINUE
      RETURN
      END
C
C
C
C
C
      SUBROUTINE    CNDTNQ(NBGR)
C
C*************************************************************
C SUBPROGRAM FOR CONDITIONING OF THE Q-MATRIX TO INCREASE
C THE ACCURACY IN THE ORTHOGONALIZATION
C*************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8      RR1(40,40),RI1(40,40)
      COMMON/Q/RI1
      COMMON/REQ/RR1
      NROW=NBGR
      DO 60 KR=2,NBGR
C**** RESCALE THE CURRENT ROW *****
      SCLE1=1.DO/RI1(NROW,NROW)
      DO 8 LC=1,NBGR
      RR1(NROW,LC)=SCLE1*RR1(NROW,LC)
      RI1(NROW,LC)=SCLE1*RI1(NROW,LC)
    8 CONTINUE
C**** RESCALE ALL THE ROWS UP TO THE CURRENT ROW. *****
      MROW=NROW-1
      DO 20 MR=1,MROW
      RSCL1=RI1(MR,NROW)
      DO 16 MC=1,NBGR
      RR1(MR,MC)=RR1(MR,MC)-RSCL1*RR1(NROW,MC)
      RI1(MR,MC)=RI1(MR,MC)-RSCL1*RI1(NROW,MC)
   16 CONTINUE
   20 CONTINUE
      NROW=NROW-1
   60 CONTINUE
C**** SET THE IMAGINARY ELEMENTS ABOVE THE MAIN DIAGONAL=0. ****
      NROW=NBGR-1
      DO 80 I=1,NROW
      IB=I+I
      DO 72 J=IB,NBGR
      RI1(I,J)=0.DO
```

```
   72 CONTINUE
   80 CONTINUE
      RETURN
      END
C
C
C
C
C
      SUBROUTINE DBESS(MO,DZ,DN,DB1,DB2,E)
C
C
C  DOCUMENTATION ADDED AT CORNELL UNIVERSITY 8/5/74 FOR THE SUBROUTINE D
C  DZ IS THE VALUE OF WHICH WE ARE TAKING THE BESSEL FUNCTION.
C  DN IS THE ORDER OF THE BESSEL FUNCTION.
C  THE VALUE OF THE BESSEL FUNCTION IS STORED IN DB1 IF THE
C  BESSEL FUNCTION WAS OF THE FIRST KIND, IE. A J-BESSEL FUNCTION.
C  THE VALUE OF THE BESSEL FUNCTION IS STORED IN DB2 IF THE
C  BESSEL FUNCTION WAS OF THE SECOND KIND, IE. A Y-BESSEL FUNCTION.
C
C
      IMPLICIT REAL*8(A-H,O-Z)
CCALCULATES BESSEL FUNCTION (COMPLEX ORDER AND COMPLEX ARGUMENT) OF THE
CFIRST KIND IF MO=0,AND ALSO OF THE SECOND KIND (NEUMANN FUNCTION) IF
CMO=1
CCALCULATES BESSEL FUNCTION OF COMPLEX ORDER AND COMPLEX ARGUMENT USING
CPOWER SERIES FOR ABS(Z) LESS THAN ZOAND ASYMPTOTIC SERIES FOR ABS(Z)
CGREATER THAN ZO. CHOOSES ZO=10 IF GIVEN ZO LESS THAN 1
C E1 DETERMINES ROUNDOFF OF EN TO INTEGER. IF E1 LE 0., SETS TO .001
 1002 FORMAT(45H NEITHER SERIES FOR BESSEL FUNCTION CONVERGES)
 1022 FORMAT(46H NEITHER SERIES FOR NEUMANN FUNCTION CONVERGES)
 1011 FORMAT(50H THIS IS A SINGULAR POINT OF THE NEUMANN FUNCTION    ,
     14H ZX=,1PE14.5,3X,4H ZY=,E14.5)
      DIMENSION C(10),C1(100),C2(100),C3(100),C4(100),C5(100),T3(101)
      DIMENSION C6(101)
      COMPLEX*16 Z,EN,B,ARG,G,T1,EX,S1,CF,SF,T3
      COMPLEX *16 FNS,S,T,U,V,S2,T2,SOZ,ZLG,A,A1,B1,B2,B1T,ZH,ZHS,CI
      COMPLEX *16 X,DZ,DN,DB1,DB2,CDUM
      REAL*8 DATAN2,DREAL,DIMAG,DLOG,DFLOAT,DSIGN,DABS,CDABS
      COMPLEX*16 CDEXP,CDSQRT,CDCOS,CDSIN,DCMPLX
      INTEGER*4 IDINT
      DATA IFLAG /0/
      ERO=.1D-2
      Z=DZ
      X=Z
      ZX=DREAL(DZ)
      ZY=DIMAG(DZ)
      EN=DN
      ENX=DREAL(DN)
      ENY=DIMAG(DN)
      M=MO+1
      ABSZ=CDABS(Z)
      DB1=DCMPLX(0.D0,0.D0)
      DB2=DCMPLX(0.D0,0.D0)
      NFLAG=2
      NF=0
      KFLAG=1
CKFLAG DENOTES QUADRANT OF Z
      IF(ZX.LT.0.D0.AND.ZY.GE.0.D0)KFLAG=2
      IF(ZX.LT.0.D0.AND.ZY.LT.0.D0)KFLAG=3
      IF(ZX.GE.0.D0.AND.ZY.LT.0.D0)KFLAG=4
      IF(KFLAG.EQ.2.OR.KFLAG.EQ.3) X=-Z
CROUTINE MOVES Z FROM LEFT-HALF PLANE TO RIGHT-HALF PLANE IF ASYMPTOTIC
CSERIES TO BE USED
      IF(IFLAG.GT.0)GO TO 2
      C(1)=3.14159265358979300
      C(2)  =C(1)  /2.D0
      C(3)=C(2)/2.D0
      C(4)  =1.D0/DSQRT(C(2))
      C(5)=DLOG(2.D0)
      C(6)=8.D0
      C(7)=64.D0
      C(9)=2.D0*C(1)
      C(10)=1.D0/C(1)
      EUL=.57721566490153380D0-C(5)
      CI=DCMPLX(0.D0,2.D0)
      A1=DCMPLX(0.D0,C(1))
      C6(1)=1.D0
      DO 100 I=1,100
      EYE=I
```

```
      C1(I)   =EYE
      C2(I)   =2.D0*EYE
      C3(I)   =2.D0*EYE-1.D0
      C4(I)=(4.D0*EYE-1.0D0)**2
      C6(I+1)=C6(I)+1.D0/DFLOAT(I+1)
  100 C5(I)=(4.D0*EYE-3.D0)**2
      IFLAG=1
    2 ERR=.1D-07
      IF(E.GT.0.D0)ERR=E
      I=0
      ZZ=5.D0
      IF(DABS(ENY).LE.ERO.AND.DABS(IDINT(ENX)-ENX).LE.ERO)NFLAG=-1
         ITEMP=ENX+DSIGN(.5D0,ENX)
          TEMP=ITEMP
      IF(NFLAG.EQ.-1) EN=DCMPLX(TEMP,0.D0)
      ENX1=DREAL(EN)
      IF(NFLAG.EQ.-1.AND.ENX1.EQ.0.D0)NFLAG=0
      IF(NFLAG.EQ.-1.AND.ENX1.GT.0.D0)NFLAG=1
CNFLAG=-1,0,+1,+2 MEANS (ENX,ENY) A NEGATIVE INTEGER,ZERO,A POSITIVE INT
CEGER, AND A NON-INTEGER,RESPECTIVELY
      IF(NFLAG.EQ.2.AND.M.EQ.2)M=3
CFOR M=3, EXPRESSES NEUMANN FUNCTION IN TERMS OF BESSEL FUNCTIONS
      IF(NFLAG.EQ.-1)EN=-EN
    3 CONTINUE
      JFLAG=0
      A=CDEXP(EN*A1)
      IF(ABSZ.GE.ZZ) GO TO 6
      IF(ENY.EQ.0.D0.AND.DABS(IDINT(ENX)+0.5D0-ENX).LE.ERO) GO TO 6
      IF(ABSZ.NE.0.D0)GO TO 8
      IF(M.EQ.2) WRITE(6,1011) DZ
      IF(NFLAG.NE.0)GO TO 16
      DB1=DCMPLX(1.D0,0.D0)
   16 RETURN
    8 CONTINUE
      I=1
      ZLG=DCMPLX(DLOG(ABSZ),DATAN2(DIMAG(Z),DREAL(Z)))
CCHOOSES PRINCIPAL VALUE OF Z IN CALCULATING CLOG(Z)
      ARG=EN+C1(1)
      CALL DGAMM(ARG,G,CDUM,ERR,0)
      ZH=EN*(ZLG-C(5))
      ZHS=CDEXP(ZH)
      T3(1)=ZHS/G
      S1=T3(1)
      EX=CDEXP(C2(1)*(ZLG-C(5)))
   11 I=I+1
      T3(I)=-T3(I-1)*EX/((EN+C1(I-1))*C1(I-1))
      S1=S1+T3(I)
      S1S=CDABS(S1)
      T1S=CDABS(T3(I))
      IF(T1S.LE.ERR*S1S) GO TO 9
      IF(I.LT.101)GO TO 11
      IF(JFLAG.GT.0)GO TO 14
      JFLAG=1
      GO TO 6
   14 WRITE(6,1002)
      STOP
   81 CONTINUE
      JFLAG=1
      GO TO 6
   82 WRITE (6,1022)
      STOP
    9 B=S1
      IF(M.NE.2)GO TO 55
      N=DABS(ENX1)
      U=2.D0*(ZLG+EUL)
      S2=B*U
      IZZZ=1
   77 DO 75 J=IZZZ,I
      J1=J-1
      J1N=N+J-1
         IF (J1.LE.0) GO TO 200
         IF (J1.GT.101) GO TO 201
         TEMP1=C6(J1)
      GO TO 203
  200 TEMP1=0.D0
      GO TO 203
  201 TEMP1=C6(101)
      DO 202 JJ=102,J1
         TEMP1=TEMP1+1.D0/DFLOAT(JJ)
  202 CONTINUE
```

```
  203 CONTINUE
         IF (J1N.LE.0) GO TO 205
      IF(J1N.GT.101) GO TO 206
      TEMP2=C6(J1N)
      GO TO 208
  205 TEMP2=0.D0
      GO TO 208
  206 TEMP2=C6(101)
         DO 207 JJ=102,J1N
         TEMP2=TEMP2+1.D0/DFLOAT(JJ)
  207 CONTINUE
  208 CONTINUE
      T2=T3(J)*(TEMP1+TEMP2)
      T2R=DABS(DREAL(T2))
      T2I=DABS(DIMAG(T2))
   75 S2=S2-T2
      S2R=DABS(DREAL(S2))
      S2I=DABS(DIMAG(S2))
      IF(T2R.GT.ERR*S2R) GO TO 78
      IF(T2I.LE.ERR*S2I) GO TO 76
   78 I=I+1
      IF(I.GT.101.AND.JFLAG.EQ.0)GO TO 81
      IF(I.GT.101.AND.JFLAG.NE.0)GO TO 82
      T3(I)=-T3(I-1)*EX/((EN+C1(I-1))*C1(I-1))
      IZZZ=I
      GO TO 77
   76 B2=S2*C(10)
      IF(N.EQ.0) GO TO 55
      S1=DCMPLX(0.D0,0.D0)
      T1=-C(10)/ZHS
      LUP=N-1
      IF(LUP.EQ.0)GO TO 72
      DO 70 LL=1,LUP
   70 T1=T1*DFLOAT(LL)
      S1=S1+T1
      DO 71 LL=1,LUP
      T1=T1*EX/(DFLOAT(LL)*DFLOAT(LUP-LL+1))
   71 S1=S1+T1
      GO TO 73
   72 S1=S1+T1
   73 B2=B2+S1
      GO TO 55
   55 IF(NFLAG.LT.0)B=A*B
      BX=DREAL(B)
      BY=DIMAG(B)
      BXA=DABS(BX)
      BYA=DABS(BY)
      IF(ZX.EQ.0.D0.OR.ZY.EQ.0.D0)NF=1
      IF(NFLAG.NE.2.AND.BXA.LT.BYA.AND.NF.EQ.1) BX=0.D0
      IF(NFLAG.NE.2.AND.BYA.LT.BXA.AND.NF.EQ.1)BY=0.D0
      GO TO(56,57,58,59),M
    6 ARG=X-EN*C(2)-C(3)
      CF=CDCOS(ARG)
      SF=C(6)*CDSIN(ARG)
      FNS=C1(4)*EN*EN
      I=0
      S1=DCMPLX(1.D0,0.D0)
      S2=DCMPLX(0.D0,0.D0)
      U=DCMPLX(1.D0,0.D0)
      T1S=1.D0
      S=CF
   18 I=I+1
      V=-(FNS-C5(I))/(C(7)*X*C3(I))*U
      U=V*(FNS-C4(I))/(C2(I)*X)
      US=CDABS(U)
      IF(US.GT.T1S) GO TO 20
   12 CONTINUE
      T=U*CF+V*SF
      TR=DABS(DREAL(T))
      TI=DABS(DIMAG(T))
      S=S+T
      SR=DABS(DREAL(S))
      SI=DABS(DIMAG(S))
      IF(TR.GT.ERR*SR) GO TO 24
      IF(TI.LE.ERR*SI) GO TO 26
   24 T1=U
      T2=V
      S1=S1+T1
      S2=S2+T2
      T1S=US
```

```
      17 IF(I-100)18,25,25
      20 IF(I.EQ.1) GO TO 12
         IF(JFLAG.GT.0) GO TO 23
         JFLAG=1
         GO TO 8
      23 WRITE(6,1002)
     29  CONTINUE
         STOP
      26 CONTINUE
         SQZ=CDSQRT(X)
         IF(DREAL(SQZ).LT.0.D0)SQZ=-SQZ
CCHOOSES PROPER BRANCH FOR SQUARE ROOT
         B=C(4)/SQZ*S
         IF(KFLAG.EQ.2)B=A*B
         IF(KFLAG.EQ.3)B=B/A
         IF(M.EQ.3)M=2
         IF(M.NE.2)GO TO 55
         B2=C(4)/SQZ*(SF*(S1+U)/C(6)-CF*(S2+V)*C(6))
         IF(KFLAG.EQ.2)B2=(B2+CDCOS(C(1)*EN)*CI*B)/A
         IF(KFLAG.EQ.3)B2=(B2-CDCOS(C(1)*EN)*CI*B)*A
         GO TO 55
      25 IF(JFLAG.GT.0)GO TO 28
         JFLAG=1
         GO TO 8
      28 WRITE(6,1002)
         GO TO 29
      58 B1=DCMPLX(BX,BY)
         M=4
         EN=-EN
         GO TO 3
      59 B1T=DCMPLX(BX,BY)
         EN=-EN
         ARG=C(1)*EN
         B2=(CDCOS(ARG)*B1-B1T)/CDSIN(ARG)
         DB1=B1
         DB2=B2
         RETURN
      57 IF(NFLAG.EQ.-1)B2=A*B2
         DB2=B2
      56 DB1=DCMPLX(BX,BY)
         RETURN
         END
         SUBROUTINE DGAMM(DZ,DGM,DPS,ERR,JJ)
CIF JJ=0,CALCULATES ONLY GAMMA FUNCTION, IF JJ=1, CALCULATES ONLY PSI
CFUNCTION, IF JJ=2, CALCULATES BOTH
         IMPLICIT REAL*8(A-H,O-Z)
         COMPLEX*16 GAM,Z,DZ,DGM,DPS,DPSI
         COMPLEX *16 TERM1,ZT1,TERM,SUM,ZLG,ZTGAM,ZT
         COMPLEX*16 CDLOG,DCMPLX,CDEXP,CDSIN
         REAL*8 DREAL,DIMAG,DLOG,DABS
         INTEGER*4 IDINT
         DIMENSION B(10)
         DIMENSION C(100)
         DATA IFLAG /0/
    1001 FORMAT(1H ,///,24H SERIES DID NOT CONVERGE)
    1010 FORMAT(1H ,///,47H THIS IS A SINGULAR POINT OF THE GAMMA FUNCTION
        X,/,5X,6HARG R=,E12.5,3X,6HARG I=,E12.5)
         IF(JJ.EQ.0)GO TO 60
         DPS=DPSI(DZ,ERR)
         IF(JJ.EQ.1)RETURN
      60 E=ERR
         IF(E.LE.0.D0)E=.1D-07
         ZX=DREAL(DZ)
         ZY=DIMAG(DZ)
         Z=DZ
         IF(ZX.LT.0.D0)Z=-Z
         NFLAG=2
         J=0
CK=NUMBER OF TERMS IN SERIES
         IF(ZY.EQ.0.D0.AND.(IDINT(ZX)-ZX).EQ.0.D0) NFLAG=1
         IF(NFLAG.EQ.1.AND.ZX.LE.0.D0)NFLAG=0
CNFLAG=0 MEANS Z=0 OR Z A NEGATIVE INTEGER   NFLAG=1MEANS Z A POSITIVE
         IF(NFLAG.NE.0)GO TO 51
         WRITE(6,1010) DZ
         DGM=DCMPLX(0.D0,0.D0)
         RETURN
      51 IF(NFLAG.EQ.2)GO TO 42
         IF(ZX.GT.2.0D0) GO TO 55
         DGM=DCMPLX(1.D0,0.D0)
         RETURN
```

```
   55 IF(ZX.GT.20.D0)GO TO 42
      IF=IDINT(ZX)-1
      IJ=1
      DO 300 N=2,IF
  300 IJ=IJ*N
      DGM=IJ
      RETURN
   42 CONTINUE
      IF(IFLAG.NE.0)GO TO 20
      DO 100 I=1,100
  100 C(I)=I
      PI=3.14159265358979 3D0
      PI2=DLOG(2.D0*PI)/2.D0
      B(1)=1.0D0/12.0D0
      B(2)=-1.0D0/360.0D0
      B(3)=1.0D0/1260.0D0
      B(4)=-1.0D0/1680.0D0
      B(5)=1.0D0/1188.0D0
      B(6)=-691.0D0/360360.0D0
      B(7)=1.0D0/156.0D0
      B(8)=-3617.0D0/122400.0D0
      B(9)=43867.0D0/244188.0D0
      B(10)=-174611.0D0/125400.0D0
CB(I) ARE THE BERNOULLI COEFFICIENTS IN STIRLINGS FORMULA
      IFLAG=1
   20 ZT=Z
      ZT1=ZT-1.D0
      I=0
    5 IF(DREAL(ZT).GT.10.D0) GO TO 3
    4 I=I+1
      IF(I.LE.100)GO TO 30
      WRITE (6,1001)
      STOP
   30 CONTINUE
      ZT=ZT+1.D0
      GO TO 5
    3 IF=I
      ZLG=CDLOG(ZT)
      SUM=(ZT-.5          D0)*ZLG      -ZT+PI2
      TERM1=SUM
      ATER1R=DABS(DREAL(TERM1))
      ATER1I=DABS(DIMAG(TERM1))
      J=0
    8 J=J+1
      TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG)
      ATERR=DABS(DREAL(TERM))
      ATERI=DABS(DIMAG(TERM))
      SUM=SUM+TERM
      ASUMR=DABS(DREAL(SUM))
      ASUMI=DABS(DIMAG(SUM))
      IF(ATERR.GT.ATER1R)GO TO 24
      IF(ATERI.GT.ATER1I)GO TO 24
      IF(ASUMR.EQ.0.D0)GO TO 9
      IF(ATERR/ASUMR.GT.E)GO TO 27
    9 CONTINUE
      IF(ASUMI.EQ.0.D0) GO TO 6
      IF(ATERI/ASUMI.LE.E)GO TO 6
   27 CONTINUE
      ATER1R=ATERR
      ATER1I=ATERI
      GO TO 7
   24 CONTINUE
    7 IF(J.LT.10)GO TO 8
      GO TO 4
    6 ZTGAM=SUM
      IF(IF.EQ.0)GO TO 31
      DO 200 K=1,IF
  200 ZTGAM=ZTGAM-CDLOG(ZT1+C(K))
   31 GAM=CDEXP(ZTGAM)
      IF(ZX.LT.0.D0)GAM=-PI/(GAM*CDSIN(PI*Z)*Z)
      DGM=GAM
      RETURN
      END
      COMPLEX FUNCTION DPSI*16(Z,E)
      IMPLICIT COMPLEX*16 (A-H,O-Z)
      COMPLEX*16 CDEXP,CDLOG,CDCOS,CDSIN,DCMPLX
      REAL*8 DATAN2,DFLOAT,DABS,CDABS
      REAL*8 E,ERR,B,ABTER,ABTER1,ABSUMI,PI,ZX,ZY,ZXI,EN,EUL,DREAL,DIMAG
      INTEGER*4 IDINT
      DIMENSION B(10)
```

```
      DATA IFLAG /0/
      ERR=E
      IF(ERR.LE.0.D0)ERR=.1D-5
      ZP=Z
      ZT=Z
      ZX=DREAL(ZP)
      ZY=DIMAG(ZP)
      NI=IDINT(ZX)
      ZXI=DFLOAT(NI)-ZX
      IF(ZY.NE.0.D0.OR.ZX.GT.0.D0.OR.ZXI.NE.0.D0)GO TO 1
      DPSI=DCMPLX(0.D0,0.D0)
      WRITE (6,1010) Z
      RETURN
CCALCULATE PSI HERE IF Z IS A POSITIVE INTEGER
    1 IF(ZY.NE.0.D0.OR.ZXI.NE.0.D0)GO TO 2
      EUL=-.5772156649
      DPSI=EUL
      IF(NI.EQ.1)RETURN
      NF=NI-1
      DO 100 N=1,NF
  100 DPSI=DPSI+1.D0/DFLOAT(N)
      RETURN
    2 ISIGN=0
      IF(ZX.LT.0.D0)ISIGN=1
      IF(ISIGN.EQ.1)ZT=1.D0-ZT
CREFLECTS Z INTO 1-Z IF Z IS IN LEFT-HALF PLANE
      ZS=ZT
      NR=0
      IF(IFLAG.NE.0)GO TO 3
      PI=DATAN2(0.D0,-1.D0)
      B(1)=1.D0/(2.D0*6.D0)
      B(2)=-1.D0/(4.D0*30.D0)
      B(3)=1.D0/(6.D0*42.D0)
      B(4)=-1.D0/(8.D0*30.D0)
      B(5)=5.D0/(10.D0*66.D0)
      B(6)=-691.D0/(12.D0*2730.D0)
      B(7)=7.D0/(14.D0*6.D0)
      B(8)=-3617.D0/(16.D0*510.D0)
      B(9)=43867.D0/(18.D0*798.D0)
      B(10)=-174611.D0/(20.D0*330.D0)
      IFLAG=1
    3 IF((NR+NI).GT.10)GO TO 4
      NR=NR+1
      GO TO 3
CINCREASES REAL PART OF Z UNTIL GREATER THAN 10
    4 ZT=ZT+DFLOAT(NR)
      ZL=CDLOG(ZT)
      TER1=.5D0/ZT
      SUM=ZL-TER1
      ABTER1=CDABS(TER1)
      N=0
    8 N=N+1
      EN=DFLOAT(N)
      TER=-B(N)*CDEXP(-2.D0*EN*ZL)
      ABTER=CDABS(TER)
      IF(ABTER.LT.ABTER1)GO TO 5
      NR=NR+1
      ZT=ZS
      IF(NR.LT.100)GO TO 4
      WRITE (6,1011)
      RETURN
    5 SUM=SUM+TER
      ABSUMI=DABS(DIMAG(SUM))
      IF(ABSUMI.NE.0.D0)GO TO 6
      IF(ABTER/CDABS(SUM).LE.ERR)GO TO 7
      GO TO 9
    6 IF(DABS(DREAL(TER))/DABS(DREAL(SUM)).LE.ERR.AND.DABS(DIMAG(TER))/
     1ABSUMI.LE.ERR)GO TO 7
    9 ABTER1=ABTER
      GO TO 8
    7 DPSI=SUM
      IF(NR.EQ.0) GO TO 10
      DO 200 N=1,NR
  200 DPSI=DPSI-1.D0/(ZT-DFLOAT(N))
   10 IF(ISIGN.EQ.0)RETURN
      ARG=PI*ZP
      DPSI=DPSI-PI*CDCOS(ARG)/CDSIN(ARG)
      RETURN
 1010 FORMAT(1H ,//,45H THIS IS A SINGULAR POINT OF THE PSI FUNCTION,/,
     130X,6HARG R=,1PE12.5,3X,6HARG I=,E12.5)
 1011 FORMAT(1H ,//,24H SERIES DID NOT CONVERGE)
      END
```

```
*****************************************************************
SAMPLE RUN WITH THE SAMPLE DATA-(A) GIVEN IN THE PROGRAM-(A)
*****************************************************************

***************************************************
*                                                 *
*          SCATTERING OF ACOUSTIC WAVES           *
*                                                 *
*  BY THE CYLINDERS OF ARBITRARY CROSS-SECTION    *
*                                                 *
*        ((( T-MATRIX FORMULATION )))             *
*                                                 *
***************************************************

** PROGRAM-(A): GENERATION OF THE T-MATRIX ELEMENTS **
=====================================================

=============================================================

B.C. TYPE : NEUMANN (RIGID INCLUSION)

WAVE NUMBER :    .5

CROSS-SECTION OF THE SCATTERER : CIRCULAR

RADIUS :  1.000

SYMMETRY CONDITION (W.R.T. X-AXIS) :       SYMMETRIC

NUMBER OF BOUNDARY SEGMENTS :  72

TYPE OF BOUNDARY DIVISION :      EQUAL ANGULAR INTERVALS

ORDER OF THE SIMPSON'S RULE USED IN NUMERICAL INTEGRATION :  4

TOTAL DIMENSION OF THE T-MATRIX :  10 X 10

TRUNCATION FACTOR FOR THE Q-MATRIX ELEMENTS : .100-025

ERROR FACTOR FOR THE BESSEL FUNCTIONS : .100-013

=============================================================
```

```
=================================================================
                    Q-11   MATRIX
=================================================================

   -.17857+000    .00000         .00000         .00000         .00000
    .10846+001    .00000         .00000         .00000         .00000

    .00000        .86373-001     .00000         .00000         .00000
    .00000        .47539+000     .00000         .00000         .00000

    .00000        .00000         .28808-002     .00000         .00000
    .00000        .00000         .48779+000     .00000         .00000

    .00000        .00000         .00000         .30650-004     .00000
    .00000        .00000         .00000         .49718+000     .00000

    .00000        .00000         .00000         .00000         .16132-006
    .00000        .00000         .00000         .00000         .49892+000
=================================================================
                    Q-22   MATRIX
=================================================================

    .86373-001    .00000         .00000         .00000         .00000
    .47539+000    .00000         .00000         .00000         .00000

    .00000        .28808-002     .00000         .00000         .00000
    .00000        .48779+000     .00000         .00000         .00000

    .00000        .00000         .30650-004     .00000         .00000
    .00000        .00000         .49718+000     .00000         .00000

    .00000        .00000         .00000         .16132-006     .00000
    .00000        .00000         .00000         .49892+000     .00000

    .00000        .00000         .00000         .00000         .50729-009
    .00000        .00000         .00000         .00000         .49947+000

       ** ORTHOGONALIZED Q-MATRICES **

=================================================================
                    Q-11   MATRIX
=================================================================

    .16246+000    .00000         .00000         .00000         .00000
    .98672+000    .00000         .00000         .00000         .00000

    .00000       -.17876+000     .00000         .00000         .00000
    .00000        .98389+000     .00000         .00000         .00000

    .00000        .00000        -.59057-002     .00000         .00000
    .00000        .00000         .99998+000     .00000         .00000

    .00000        .00000         .00000        -.61647-004     .00000
    .00000        .00000         .00000         .10000+001     .00000

    .00000        .00000         .00000         .00000        -.32333-006
    .00000        .00000         .00000         .00000         .10000+001
=================================================================
                    Q-22   MATRIX
=================================================================

   -.17876+000    .00000         .00000         .00000         .00000
    .98389+000    .00000         .00000         .00000         .00000

    .00000       -.59057-002     .00000         .00000         .00000
    .00000        .99998+000     .00000         .00000         .00000

    .00000        .00000        -.61647-004     .00000         .00000
    .00000        .00000         .10000+001     .00000         .00000

    .00000        .00000         .00000        -.32333-006     .00000
    .00000        .00000         .00000         .10000+001     .00000

    .00000        .00000         .00000         .00000        -.10157-008
    .00000        .00000         .00000         .00000         .10000+001
```

T-11 MATRIX

-.26392-001	.00000	.00000	.00000	.00000
-.16030+000	.00000	.00000	.00000	.00000
.00000	-.31956-001	.00000	.00000	.00000
.00000	.17588+000	.00000	.00000	.00000
.00000	.00000	-.34877-004	.00000	.00000
.00000	.00000	.59056-002	.00000	.00000
.00000	.00000	.00000	-.38004-008	.00000
.00000	.00000	.00000	.61647-004	.00000
.00000	.00000	.00000	.00000	-.10454-012
.00000	.00000	.00000	.00000	.32333-006

T-22 MATRIX

-.31956-001	.00000	.00000	.00000	.00000
.17588+000	.00000	.00000	.00000	.00000
.00000	-.34877-004	.00000	.00000	.00000
.00000	.59056-002	.00000	.00000	.00000
.00000	.00000	-.38004-008	.00000	.00000
.00000	.00000	.61647-004	.00000	.00000
.00000	.00000	.00000	-.10454-012	.00000
.00000	.00000	.00000	.32333-006	.00000
.00000	.00000	.00000	.00000	-.10316-017
.00000	.00000	.00000	.00000	.10157-008

```
C
C           **********************************************************
C           *                                                        *
C           *          SCATTERING OF ACOUSTIC WAVES                  *
C           *                                                        *
C           * BY THE CYLINDERS OF ARBITRARY CROSS-SECTION            *
C           *                                                        *
C           *         ((( T-MATRIX FORMULATION )))                   *
C           **********************************************************
C
C                          *** P R O G R A M-(B) ***
C                          =========================
C        EVALUATION OF THE SCATTERED WAVE FIELD AT VARIOUS DISTANCES
C        FROM THE SCATTERER AND FOR VARIOUS INCIDENCE ANGLES OF THE
C        ACOUSTIC PLANE WAVE ,BY USING THE T-MATRIX CREATED BY THE
C        PROGRAM-(A).
C
C
C    *** THE GENERAL STEPS FOLLOWED BY THE PROGRAM ARE AS FOLLOWS ***
C
C     -- READING OF THE PARAMETERS DESCRIBING THE BOUNDARY OF THE
C        SCATTERER AND ELEMENTS OF THE T-MATRIX FROM THE FILE(10)
C
C
C     -- CREATION OF THE INCIDENT WAVE FIELD COEFFICIENTS
C        A1(I) & A2(I) FOR A DESIRED INCIDENCE ANGLE (AINC)
C
C
C     -- CALCULATION OF THE SCATTERED WAVE FIELD COEFFICIENTS
C        C1(I) & C2(I) , WHERE:
C
C          C1 = T11*A1 + T12*A2      &     C2 = T21*A1 + T22*A2
C
C        (C1 & C2 ARE COLUMN VECTORS & TIJ'S ARE (NO X NO) MATRICES)
C
C     -- EVALUATION AND PRESENTATION OF THE FOLLOWING RESULTS
C        BOTH IN TABULAR AND POLAR GRAPHICAL FORMS:
C          A-) NEAR FIELD SOLUTIONS (*ONLY FOR CIRCULAR BOUNDARY)
C              AT DESIRED REGIONS OR,ON THE BOUNDARY OF THE
C              SCATTERER.
C              1-) VELOCITY POTENTIALS DUE TO SCATTERED WAVE FIELD
C              2-) WAVE VELOCITIES DUE TO SCATTERED WAVE FIELD
C          B-) FAR-FIELD SCATTERED WAVE FIELD AMPLITUDES
C          C-) TOTAL SCATTERING CROSS-SECTION (ALSO CHECK FOR
C                                                  CONVERGENCY)
C
C                    ***** IMPORTANT NOTE *****
C                    ==========================
C         CONVERGENCY OF THE TOTAL SCATTERING CROSS-SECTION
C         MUST BE CHECKED UP TO AT LEAST 1.E-3 %.IF (%) DIFFERENCE
C         IS GREATER THAN 1.E-3,THEN THE SIZE OF THE T-MATRIX
C         USED MUST BE INCREASED.IN SUCH A CASE, A NEW T-MATRIX
C         OF THE LARGER SIZE MUST BE RECREATED BY THE PROGRAM-(A).
C
C
C                  *********************
C                  I N P U T   D A T A
C                  *********************
C    CARD 1 :
C    ========
C       COLUMN 1-2: NI (READING UNIT OR FILE NO.)
C       COLUMN 3-4: NO (WRITING UNIT OR FILE NO.)
C
C    CARD 2 :
C    ========
C       COLUMN 1-3: NYAZ (ANGULAR INCREMENT IN PRINTING THE
C                         TABULAR RESULTS.EXAMPLE: IF NYAZ=10,
C                         THEN, SCATTERED WAVE FIELD RESULTS
C                         VERSUS POLAR ANGLE ARE PRINTED FOR EACH
C                         10 DEGREE INCREMENT. MIN(NYAZ)=2.
C                         (*NOTE*: NYAZ SHOULD BE AN EVEN NUMBER)
C       COLUMN 4-6 : XAX ( XAX & YAX ARE THE SCALE PARAMETERS
C       COLUMN 7-9 : YAX  REQUIRED FOR 'SUBROUTINE GRAPH4'.
C                         TAKE XAX & YAX=6.0-11.0)
C         (*NOTE:* 'GRAPH4' IS A LIBRARY PROGRAM IN UNIVAC-1106 SYSTEM)
C
C    CARD 3 :
C    ========
C       COLUMN 1-2: NINC (NUMBER OF INCIDENCE ANGLES FOR WHICH
C                         THE SCATTERED FIELD SOLUTIONS ARE DESIRED)
```

```
C
C   CARD SET 4 :
C   ============
C      *EACH CARD CONTAINS*
C     COLUMN 1-4: DUM4 (INCIDENCE ANGLE OF THE WAVE.(IN DEGREES))
C                       (*NOTE:*  NO OF CARDS IN SET(4) MUST
C                                  BE EQUAL TO 'NINC'. )
C
C   CARD 5 :
C   ========
C     COLUMN 1-2: NPOT (IF VELOCITY POTENTIALS ARE DESIRED TO
C                       BE EVALUATED NPOT=1 ,OTHERWISE NPOT=0)
C     COLUMN 3-4: NVEL (IF WAVE VELOCITIES ARE DESIRED NVEL=1,
C                       OTHERWISE NVEL=0)
C     COLUMN 5-6: NFAR (IF FAR-FIELD SOLUTION IS DESIRED
C                       NFAR=1 ,OTHERWISE NFAR=0 )
C     COLUMN 7-8:NCROS (NCROS=1 IF TOTAL SCATTERING CROSS-SECTION
C                       IS DESIRED)
C     COLUMN 9-10: NREG (NUMBER OF REGIONS AT WHICH THE NEAR FIELD
C                        SOLUTIONS ARE DESIRED TO BE EVALUATED.
C                   (*NOTE:* IF BOTH NPOT & NVEL=0,TAKE NREG=0 )
C
C   CARD SET 6 : (*IF  NREG=0 ,OMIT THIS CARD SET)
C   ============
C        *EACH CARD CONTAINS*
C     COLUMN 1-4: REG(I) (REGION(DISTANCE FROM THE ORIGIN OF THE
C                         POLAR COORDINATE SYSTEM))
C                 (*NOTE:* 1-) TAKE REG(I)=0. ,IF THE NEAR FIELD
C                              SOLUTIONS ARE DESIRED TO BE
C                              EVALUATED ON THE BOUNDARY OF
C                              THE SCATTERER.
C                          2-) I=1,NREG      )
C
C
C
C                         *************
C                          SAMPLE  DATA
C                         *************
C                 ( A )                          ( B )
C
C                   COLUMN                        COLUMN
C           ===============               ===============
C           CARD I1234567890              CARD I1234567890
C           =====I==========              =====I==========
C              1 I 5 6                       1 I 5 6
C              2 I 10 5. 5.                  2 I 10 9. 7.
C              3 I 1                         3 I 2
C              4 I0.                         4 I30.
C              5 I 1 0 1 1 2                 5 I45.
C              6 I0.                         6 I 0 0 1 1 0
C              7 I5.
C
C
C*********************************************************************
C
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      DIMENSION AINC(10),REG(10)
C
      COMPLEX*16 T11(20,20),T12(20,20),T21(20,20),T22(20,20),
     &           BES1(22),BES2(22),HANK(22),DUM1,DUM2,DZ,DN,CPART,
     &           A1(20),A2(20),C1(20),C2(20),BASF(20),
     &           USPOT(190),USVEL(190),USFAR(190)
C
      COMMON/REC/A,B,CORRAD,T1,T2,T3,T4,ALPH,DIS
      COMMON/TRIAN/H,BETA,TETA1
C
      REAL*4 X(190),Y(190),XAX,YAX
C
      CHARACTER BCON*4,SHAPE*6,H1*12,H2*25,H3*18,H4*33,H8*37,
     &          H5*8,H6*10,H7*34,H9*9,H10*13,H11*10
C
      G(V)=V*360./(2.*PI)
      F(U)=U*2.*PI/360.
C
      DATA H1,H2,H3/'B.C. TYPE : ','NEUMANN (RIGID INCLUSION)','DIRICHLE
     &T (CAVITY)'/
      DATA H4/'CROSS-SECTION OF THE SCATTERER : '/
      DATA H8/'SYMMETRY CONDITION (W.R.T. X-AXIS) : '/
```

```
      DATA H5,H6,H7/'CIRCULAR','ELLIPTICAL','RECTANGULAR (WITH ROUNDED C
     &ORNERS)'/
      DATA H9,H10/'SYMMETRIC','NON-SYMMETRIC'/
      DATA H11/'TRIANGULAR'/
C
      READ(5,10) NI,NO,NYAZ,XAX,YAX
   10 FORMAT(2I2/I3,2F3.0)
      WRITE(NO,613)
  613 FORMAT(1H1)
      WRITE(NO,100)
  100 FORMAT(///5X,47('*')/5X,'*',45X,'*'/5X,'*',8X,'SCATTERING OF ACOUS
     &TIC WAVES',9X,'*'/5X,'*',45X,'*'/5X,'* BY THE CYLINDERS OF ARBITRA
     &RY CROSS-SECTION *'/5X,'*',45X,'*'/5X,'*',8X,'((( T-MATRIX FORMULA
     &TION )))',9X,'*'/5X,'*',45X,'*'/5X,47('*')/////5X,'** PROGRAM-(B):
     & EVALUATION OF THE NEAR AND FAR FIELD SOLUTIONS **'/5X,65('=')///)
      PI=DATAN(1.D0)*4.D0
C**** READING OF THE DATA,CREATED BY THE PROGRAM-(A) AND ****
C**** STORED INTO THE FILE(10)                           ****
      READ(10,1) BCON,ANO,SHAPE
    1 FORMAT(1X,A4/1X,F4.1/1X,A6)
      IF(SHAPE.EQ.'CIRCLE') READ(10,2) RAD
      IF(SHAPE.EQ.'ELLIPS') READ(10,2) AA,BB
      IF(SHAPE.EQ.'RECTAN') READ(10,2) A,B,CORRAD
      IF(SHAPE.EQ.'TRIANG') THEN
        READ(10,2) H,BET
        BETA=F(BET)
        TETA1=DATAN(3.*DTAN(BETA/2.))
      END IF
    2 FORMAT(1X,2F6.3,F13.10)
      READ(10,4) NSYM,NQ
    4 FORMAT(1X,I2/1X,I3)
      NQ2=NQ*2
      NQT=NQ+2
      IF(NQ.GE.13.AND.NQ.LE.18) NQT=22
      DO 5 I=1,NQ
      DO 6 J=1,NQ
    6 READ(10,7) T11(I,J)
      DO 5 J=1,NQ
    5 READ(10,7) T12(I,J)
      DO 8 I=1,NQ
      DO 9 J=1,NQ
    9 READ(10,7) T21(I,J)
      DO 8 J=1,NQ
    8 READ(10,7) T22(I,J)
    7 FORMAT(1X,2D30.23)
      WRITE(NO,11)
   11 FORMAT(63('=')/)
      IF(BCON.EQ.'NEUM') WRITE(NO,12) H1,H2
   12 FORMAT(1X,A12,A25)
      IF(BCON.EQ.'DRIC') WRITE(NO,12) H1,H3
      IF(SHAPE.EQ.'CIRCLE') WRITE(NO,13) H4,H5,RAD
   13 FORMAT(/1X,A33,A8//1X,'RADIUS : ',F6.3)
      IF(SHAPE.EQ.'ELLIPS') WRITE(NO,14) H4,H6,AA,BB
   14 FORMAT(/1X,A33,A10//1X,'A-AXIS : ',F6.3,4X,'B-AXIS : ',F6.3)
      IF(SHAPE.EQ.'RECTAN')  WRITE(NO,15) H4,H7,A,B,CORRAD
   15 FORMAT(/1X,A33,A34//1X,'A-AXIS:',F6.3,4X,'B-AXIS:',F6.3,4X,
     &      'CORNER RAD.:',F13.10)
      IF(SHAPE.EQ.'TRIANG') WRITE(NO,998) H4,H11,H,BET
  998 FORMAT(/1X,A33,A10//1X,'H:',F6.3,'   BETA:',F7.3)
      IF(NSYM.EQ.1) WRITE(NO,16) H8,H10
   16 FORMAT(/1X,A37,A13)
      IF(NSYM.EQ.2) WRITE(NO,16) H8,H9
      WRITE(NO,17) ANO,NQ2,NQ2
   17 FORMAT(//1X,'WAVE TYPE : ACOUSTIC PLANE WAVE'//1X,'WAVE NO : ',
     &F4.1//1X,'TOT.DIM. OF THE T-MATRIX USED : ',I3,' X',I3//63('='))
      READ(NI,18) NINC
      DO 19 I=1,NINC
      READ(NI,20)  DUM4
   19 AINC(I)=F(DUM4)
   18 FORMAT(I2)
   20 FORMAT(F4.0)
      READ(NI,21) NPOT,NVEL,NFAR,NCROS,NREG
   21 FORMAT(5I2)
      IF(SHAPE.EQ.'RECTAN') CALL RECT
      IF(NREG.EQ.0) GO TO 23
      DO 22 I=1,NREG
   22 READ(NI,20) REG(I)
   23 DO 24  II=1,NINC
      WRITE(NO,46) II,G(AINC(II))
   46 FORMAT(///6X,I2,'- ANGLE OF INCIDENCE : ',F4.0,' (DEGREES)'/45('='
```

```
     &))
C**** CREATION OF THE INCIDENT WAVE FIELD COEFFICIENTS ****
      A1(1)=(1.D0,0.D0)
      DO 25 I=1,NQ
      IF(NQ-I) 26,27,26
   26 A1(I+1)=DSQRT(2.D0)*(0.D0,1.D0)**I*DCOS(I*AINC(II))
   27   A2(I)=DSQRT(2.D0)*(0.D0,1.D0)**I*DSIN(I*AINC(II))
   25 CONTINUE
C**** CALCULATION OF THE SCATTERED WAVE FIELD COEFFICIENTS ****
      CALL MULVEC(T11,A1,C1,NQ)
      IF(NSYM-2) 28,29,28
   28 CALL MULVEC(T12,A2,C2,NQ)
      DO 30 I=1,NQ
   30 C1(I)=C1(I)+C2(I)
   29 CALL MULVEC(T22,A2,C2,NQ)
      IF(NSYM-2) 31,32,31
   31 CALL MULVEC(T21,A1,BASF,NQ)
      DO 33 I=1,NQ
   33 C2(I)=C2(I)+BASF(I)
   32 TEB=PI*4.D0/360.
      I=1
      IF(NREG) 64,65,64
   64 DO 34 I=1,NREG
   65 TEA=0.D0
C**** CALCULATION OF THE NEAR & FAR-FIELD SOLUTIONS ****
      DO 35 J=1,181
      IF(NREG.EQ.0) GO TO 36
      REGION=REG(I)
      DTDR=0.D0
      IF(REG(I).EQ.0.) THEN
          IF(SHAPE.EQ.'CIRCLE') REGION=RAD
          IF(SHAPE.EQ.'ELLIPS') THEN
                 REGION=AA*BB/DSQRT(AA*AA*(DSIN(TEA))**2+BB*BB*(DCOS(TEA))*
     &*2)
         DTDR=(AA*BB*(BB*BB-AA*AA)*DSIN(TEA)*DCOS(TEA))/DSQRT((AA*AA*DSIN
     &(TEA)**2+BB*BB*DCOS(TEA)**2)**3)
          END IF
          IF(SHAPE.EQ.'RECTAN') CALL RECTA2(TEA,REGION,DTDR)
          IF(SHAPE.EQ.'TRIANG') CALL TRIA2(TEA,REGION,DTDR)
      END IF
      DZ=DCMPLX((ANO*REGION),0.D0)
      DN=(0.D0,0.D0)
      DO 37 K=1,2
      CALL DBESS(1,DZ,DN,DUM2,BES2(K),1.D-15)
   37 DN=DN+(1.D0,0.D0)
      DO 38 K=3,NQ+1
   38 BES2(K)=2.*(K-2)/DZ*BES2(K-1)-BES2(K-2)
      DN=DCMPLX(((NQT-1)*1.D0),0.D0)
      DO 70 K=1,2
      CALL DBESS(0,DZ,DN,BES1(NQT+1-K),DUM1,1.D-15)
   70 DN=DN-(1.D0,0.D0)
      DO 71 K=3,NQT
   71 BES1(NQT+1-K)=2.*(NQT+1-K)/DZ*BES1(NQT-K+2)-BES1(NQT-K+3)
      DO 39 K=1,NQ+1
   39 HANK(K)=DCMPLX(DREAL(BES1(K)),DREAL(BES2(K)))
      USPOT(J)=(0.D0,0.D0)
      USVEL(J)=(0.D0,0.D0)
   36 USFAR(J)=(0.D0,0.D0)
      DO 40 N=1,NQ
      EJEM=1.D0
      IF(N.GT.1) EJEM=DSQRT(2.D0)
      IF(N.EQ.1) THEN
          CPART=C1(N)*DCOS((N-1)*TEA)
          GO TO 414
      END IF
      CPART=C1(N)*DCOS((N-1)*TEA)+C2(N-1)*DSIN((N-1)*TEA)
  414 IF(NPOT) 41,42,41
   41 USPOT(J)=USPOT(J)+CPART*HANK(N)*EJEM
   42 IF(NVEL) 43,44,43
   43 USVEL(J)=USVEL(J)+EJEM*(((N-1)*HANK(N)-DZ*HANK(N+1))*CPART+(N-1)/
     &REGION*HANK(N)*(C1(N)*DSIN((N-1)*TEA)-C2(N)*DCOS(N*TEA))*DTDR)
   44 IF(I.GT.1) GO TO 40
      IF(NFAR) 45,40,45
   45 USFAR(J)=USFAR(J)+CPART*EJEM*(0.D0,1.D0)**(1-N)
   40 CONTINUE
   35 TEA=TEA+TEB
      IF(I.GT.1) GO TO 49
      IF(NFAR) 91,50,91
   91 WRITE(NO,47)
   47 FORMAT(//1X,63('=')/6X,'** FAR-FIELD SCATTERED FIELD AMPLITUDE **'
```

```
     &/1X,63('='))
      WRITE(NO,48)
   48 FORMAT(/18X,'REAL',7X,'IMAGINARY ',6X,'PEAK VALUE'/3X,'ANGLE',10X,
     &'PART',9X,'PART',13X,'(NORM)'/1X,63('='))
      CALL GRAFIK(NO,USFAR,PI,NSYM,NYAZ,XAX,YAX,AINC(II))
   50 IF(NCROS) 51,490,51
   51 WRITE(NO,52)
   52 FORMAT(//1X,36('=')/,2X,'CHECK FOR THE CONVERGENCY OF THE'/3X,'TOT
     &AL SCATTERING CROSS-SECTION'/37('='),/,1X,'ORDER TOT.S.C.SECTION (
     &%)DIFFERENCE'/37('='))
      USC=0.D0
C**** CALCULATION OF THE TOTAL SCATTERING CROSS-SECTION ****
      DO 53 N=1,NO
      PRC=USC
      USC=USC+CDABS(C1(N))**2+CDABS(C2(N))**2
      IF(N.EQ.1) THEN
      WRITE(NO,54) N,USC
      GO TO 53
      ELSE
      PERDIF=DABS(USC-PRC)/DABS(PRC)*100.D0
      WRITE(NO,54) N,USC,PERDIF
C
      END IF
   53 CONTINUE
   54 FORMAT(1X,I4,D16.7,D14.7)
      WRITE(NO,55)
   55 FORMAT(1H1)
  490 IF(NREG.EQ.0) GO TO 24
   49 IF(REG(I).EQ.0.) THEN
         WRITE(NO,56)
         ELSE
         WRITE(NO,57) REG(I)
      END IF
   56 FORMAT(//6X,'REGION : *BOUNDARY*'/6X,19('='))
   57 FORMAT(//6X,'REGION : ',F4.1/6X,10('='))
      IF(NPOT) 58,59,58
   58 WRITE(NO,60)
   60 FORMAT(//1X,63('=')/8X,'** VELOCITY POTENTIALS DUE TO SCATTERED FI
     &ELD **'/1X,63('='))
      WRITE(NO,48)
      CALL GRAFIK(NO,USPOT,PI,NSYM,NYAZ,XAX,YAX,AINC(II))
   59 IF(NVEL) 61,34,61
   61 WRITE(NO,63)
   63 FORMAT(//1X,63('=')/8X,'** WAVE VELOCITIES DUE TO SCATTERED FIELD
     &**'/1X,63('='))
      WRITE(NO,48)
      CALL GRAFIK(NO,USVEL,PI,NSYM,NYAZ,XAX,YAX,AINC(II))
   34 CONTINUE
   24 CONTINUE
      STOP
      END
C
C
C
      SUBROUTINE MULVEC(A,B,C,N)
C
C*********************************************
C  SUBPROGRAM FOR MATRIX & VECTOR MULTIPLICATION
C*********************************************
C
      COMPLEX*16 A(20,20),B(20),C(20),T
      DO 1 I=1,N
      T=(0.D0,0.D0)
      DO 2 J=1,N
    2 T=T+A(I,J)*B(J)
    1 C(I)=T
      RETURN
      END
C
C
C
C
C
      SUBROUTINE RECT
C
C*************************************************
C   SUBPROGRAM FOR CALCULATION OF SOME REQUIRED PARAMETERS
C   DESCRIBING THE RECTANGULAR BOUNDARY TO BE USED
C   THROUGHOUT THE MAIN PROGRAM
C       INPUT DATA:( A , B , CORRAD )
```

```
C        RETURN PARAMETERS:(T1,T2,T3,T4,ALPH,DIS)
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/REC/A,B,CORRAD,T1,T2,T3,T4,ALPH,DIS
      PI=DATAN(1.D0)*4.D0
      T1=DATAN((B-CORRAD)/A)
      IF(A.EQ.CORRAD) THEN
        T2=PI/2.
        ALPH=PI/2.
        DIS=(B-CORRAD)
      ELSE
      T2=DATAN(B/(A-CORRAD))
      ALPH=DATAN((B-CORRAD)/(A-CORRAD))
      DIS=(A-CORRAD)/DCOS(ALPH)
      END IF
      T3=PI-T2
      T4=PI-T1
      RETURN
      END
C
C
C
C
C
      SUBROUTINE RECTA2(TEX,R,DTDR)
C
C*****************************************************
C    SUBPROGRAM FOR CALCULATION OF R & D(R)/D(TETA)
C    VALUES FOR A GIVEN ANGLE(TETA) ALONG THE
C             RECTANGULAR BOUNDARY
C*****************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/REC/A,B,CORRAD,T1,T2,T3,T4,ALPH,DIS
      F(T,X)=DIS*DCOS(T-X)+DSQRT(DIS**2*(DCOS(T-X))**2+CORRAD**2-DIS**2)
      G(TE)=DIS*DSIN(TE)+DIS**2*DCOS(TE)*DSIN(TE)/DSQRT(DIS**2*DCOS(TE)*
     &*2+CORRAD**2-DIS**2)
      PI=DATAN(1.D0)*4.D0
      TET=TEX
      TETL=TET
      IF(TET.GT.PI) TETL=TET-PI
      IF(TETL.LE.T1.OR.TETL.GE.T4) THEN
        R=DABS(A/DCOS(TETL))
        DTDR=A*DSIN(TETL)/DCOS(TETL)**2
        IF(TETL.GE.T4) DTDR=-DTDR
      RETURN
      END IF
      IF(TETL.GT.T1.AND.TETL.LT.T2) THEN
        R=F(TETL,ALPH)
        TET=TETL-ALPH
        DTDR=-G(TET)
      RETURN
      END IF
      IF(TETL.GE.T2.AND.TETL.LE.T3) THEN
        R=B/DSIN(TETL)
        DTDR=-B*DCOS(TETL)/DSIN(TETL)**2
      RETURN
      END IF
      IF(TETL.GT.T3.AND.TETL.LT.T4) THEN
        R=F(TETL,PI-ALPH)
        TET=PI-TETL-ALPH
        DTDR=G(TET)
      END IF
      RETURN
      END
C
C
C
C
C
      SUBROUTINE TRIA2(TEX,R,DTDR)
C
C*****************************************************
C    SUBPROGRAM FOR CALCULATION OF R & D(R)/D(TETA)
C    VALUES FOR A GIVEN ANGLE(TETA) ALONG THE
C             TRIANGULAR  BOUNDARY
C*****************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
```

```
      COMMON/TRIAN/H,BETA,TETA1
      PI=DATAN(1.)*4.
      TET=TEX
      TETL=TET
      IF(TET.GT.PI) TETL=2.*PI-TET
      IF(TETL.LE.TETA1) THEN
      R=H/3.*(1./DCOS(TETL))
      DTDR=H/3.*DSIN(TETL)/(DCOS(TETL))**2
      IF(TEX.GT.PI) DTDR=-DTDR
      RETURN
      END IF
      IF(TETL.GT.TETA1) THEN
      R=2.*H/3.*DTAN(BETA/2.)/(DSIN(TETL)-DCOS(TETL)*
     &     DTAN(BETA/2.))
      DTDR=-2.*H/3.*DTAN(BETA/2.)*(DCOS(TETL)+DSIN(TETL)*
     &     DTAN(BETA/2.))/(DSIN(TETL)-DCOS(TETL)*
     &     DTAN(BETA/2.))**2
      IF(TEX.GT.PI) DTDR=-DTDR
      END IF
      RETURN
      END
C
C
C
C
C
      SUBROUTINE GRAFIK(NO,US,PI,NSYM,NYAZ,XAX,YAX,AINC)
C
C*********************************************
C    SUBPROGRAM FOR PRESENTATION OF THE RESULTS
C    IN TABULAR AND POLAR GRAPHICAL FORMS
C*********************************************
C
      COMPLEX*16 US(190),DCMPLX
      REAL*8 DIMAG,DREAL,CDABS,PI,TG,T
      REAL*4 X(190),Y(190),XAX,YAX
      T=0.
      K=NYAZ/2
      DO 1 I=1,181
      IF(K.EQ.(NYAZ/2+1)) K=1
      TG=T/360.D0*2.D0*PI
C**** DATA TO BE PRESENTED IN GRAPHS,ARE STORED INTO X(I) & Y(I) ARRAYS
C**** (*NOTE: FIRST, POLAR TO RECTANGULAR CONVERSION IS NECESSARY
C****             FOR  'GRAPH4' )
      X(I)=CDABS(US(I))*DCOS(TG)
      Y(I)=CDABS(US(I))*DSIN(TG)
      IF(NSYM.EQ.2.AND.I.GT.91.AND.AINC.EQ.0.) GO TO 1
      IF(K.EQ.NYAZ/2) WRITE(NO,2) T,DREAL(US(I)),DIMAG(US(I)),CDABS(US(I
     &))
      K=K+1
    2 FORMAT(4X,F4.0,8X,F8.5,4X,F8.5,9X,F8.5)
    1 T=T+2.D0
      CALL GRAPH4(XAX,YAX,181,X,Y)
C**** 'GRAPH4' IS A LIBRARY PROGRAM IN UNIVAC-1106 SYSTEM ****
      WRITE(NO,3)
    3 FORMAT(1H1)
      RETURN
      END
C
C
C
C
C
      SUBROUTINE DBESS(MO,DZ,DN,DB1,DB2,E)
C
C
C  DOCUMENTATION ADDED AT CORNELL UNIVERSITY 8/5/74 FOR THE SUBROUTINE D
C  DZ IS THE VALUE OF WHICH WE ARE TAKING THE BESSEL FUNCTION.
C  DN IS THE ORDER OF THE BESSEL FUNCTION.
C  THE VALUE OF THE BESSEL FUNCTION IS STORED IN DB1 IF THE
C  BESSEL FUNCTION WAS OF THE FIRST KIND, IE. A J-BESSEL FUNCTION.
C  THE VALUE OF THE BESSEL FUNCTION IS STORED IN DB2 IF THE
C  BESSEL FUNCTION WAS OF THE SECOND KIND, IE. A Y-BESSEL FUNCTION.
C
C
      IMPLICIT REAL*8(A-H,O-Z)
CCALCULATES BESSEL FUNCTION (COMPLEX ORDER AND COMPLEX ARGUMENT) OF THE
CFIRST KIND IF MO=0,AND ALSO OF THE SECOND KIND (NEUMANN FUNCTION) IF
CMO=1
CCALCULATES BESSEL FUNCTION OF COMPLEX ORDER AND COMPLEX ARGUMENT USING
```

```
CPOWER SERIES FOR ABS(Z) LESS THAN ZOAND ASYMPTOTIC SERIES FOR ABS(Z)
CGREATER THAN ZO. CHOOSES ZO=10 IF GIVEN ZO LESS THAN 1
C E1 DETERMINES ROUNDOFF OF EN TO INTEGER. IF E1 LE 0., SETS TO .001
 1002 FORMAT(45H NEITHER SERIES FOR BESSEL FUNCTION CONVERGES)
 1022 FORMAT(46H NEITHER SERIES FOR NEUMANN FUNCTION CONVERGES)
 1011 FORMAT(50H THIS IS A SINGULAR POINT OF THE NEUMANN FUNCTION    ,
     14H ZX=,1PE14.5,3X,4H ZY=,E14.5)
      DIMENSION C(10),C1(100),C2(100),C3(100),C4(100),C5(100),T3(101)
      DIMENSION C6(101)
      COMPLEX*16 Z,EN,B,ARG,G,T1,EX,S1,CF,SF,T3
      COMPLEX *16 FNS,S,T,U,V,S2,T2,SQZ,ZLG,A,A1,B1,B2,B1T,ZH,ZHS,CI
      COMPLEX *16 X,DZ,DN,DB1,DB2,CDUM
      REAL*8 DATAN2,DREAL,DIMAG,DLOG,DFLOAT,DSIGN,DABS,CDABS
      COMPLEX*16 CDEXP,CDSQRT,CDCOS,CDSIN,DCMPLX
      INTEGER*4 IDINT
      DATA IFLAG /0/
      ERO=.1D-2
      Z=DZ
      X=Z
      ZX=DREAL(DZ)
      ZY=DIMAG(DZ)
      EN=DN
      ENX=DREAL(DN)
      ENY=DIMAG(DN)
      M=MO+1
      ABSZ=CDABS(Z)
      DB1=DCMPLX(0.D0,0.D0)
      DB2=DCMPLX(0.D0,0.D0)
      NFLAG=2
      NF=0
      KFLAG=1
CKFLAG DENOTES QUADRANT OF Z
      IF(ZX.LT.0.D0.AND.ZY.GE.0.D0)KFLAG=2
      IF(ZX.LT.0.D0.AND.ZY.LT.0.D0)KFLAG=3
      IF(ZX.GE.0.D0.AND.ZY.LT.0.D0)KFLAG=4
      IF(KFLAG.EQ.2.OR.KFLAG.EQ.3) X=-Z
CROUTINE MOVES Z FROM LEFT-HALF PLANE TO RIGHT-HALF PLANE IF ASYMPTOTIC
CSERIES TO BE USED
      IF(IFLAG.GT.0)GO TO 2
      C(1)=3.14159265358979300
      C(2)  =C(1)  /2.D0
      C(3)=C(2)/2.D0
      C(4)  =1.D0/DSQRT(C(2))
      C(5)=DLOG(2.D0)
      C(6)=8.D0
      C(7)=64.D0
      C(9)=2.D0*C(1)
      C(10)=1.D0/C(1)
      EUL=.5772156649015338D0-C(5)
      CI=DCMPLX(0.D0,2.D0)
      A1=DCMPLX(0.D0,C(1))
      C6(1)=1.D0
      DO 100 I=1,100
      EYE=I
      C1(I)  =EYE
      C2(I)  =2.D0*EYE
      C3(I)  =2.D0*EYE-1.D0
      C4(I)=(4.D0*EYE-1.0D0)**2
      C6(I+1)=C6(I)+1.D0/DFLOAT(I+1)
  100 C5(I)=(4.D0*EYE-3.D0)**2
      IFLAG=1
    2 ERR=.1D-07
      IF(E.GT.0.D0)ERR=E
      I=0
      ZZ=5.D0
      IF(DABS(ENY).LE.ERO.AND.DABS(IDINT(ENX)-ENX).LE.ERO)NFLAG=-1
            ITEMP=ENX+DSIGN(.5D0,ENX)
             TEMP=ITEMP
      IF(NFLAG.EQ.-1) EN=DCMPLX(TEMP,0.D0)
      ENX1=DREAL(EN)
      IF(NFLAG.EQ.-1.AND.ENX1.EQ.0.D0)NFLAG=0
      IF(NFLAG.EQ.-1.AND.ENX1.GT.0.D0)NFLAG=1
CNFLAG=-1,0,+1,+2 MEANS (ENX,ENY) A NEGATIVE INTEGER,ZERO,A POSITIVE INT
CEGER, AND A NON-INTEGER,RESPECTIVELY
      IF(NFLAG.EQ.2.AND.M.EQ.2)M=3
CFOR M=3, EXPRESSES NEUMANN FUNCTION IN TERMS OF BESSEL FUNCTIONS
      IF(NFLAG.EQ.-1)EN=-EN
    3 CONTINUE
      JFLAG=0
      A=CDEXP(EN*A1)
```

```
      IF(ABSZ.GE.ZZ) GO TO 6
      IF(ENY.EQ.0.D0.AND.DABS(IDINT(ENX)+0.5D0-ENX).LE.ERO) GO TO 6
      IF(ABSZ.NE.0.D0)GO TO 8
      IF(M.EQ.2) WRITE(6,1011) DZ
      IF(NFLAG.NE.0)GO TO 16
      DB1=DCMPLX(1.D0,0.D0)
   16 RETURN
    8 CONTINUE
      I=1
      ZLG=DCMPLX(DLOG(ABSZ),DATAN2(DIMAG(Z),DREAL(Z)))
CCHOOSES PRINCIPAL VALUE OF Z IN CALCULATING CLOG(Z)
      ARG=EN+C1(1)
      CALL DGAMM(ARG,G,CDUM,ERR,0)
      ZH=EN*(ZLG-C(5))
      ZHS=CDEXP(ZH)
      T3(1)=ZHS/G
      S1=T3(1)
      EX=CDEXP(C2(1)*(ZLG-C(5)))
   11 I=I+1
      T3(I)=-T3(I-1)*EX/((EN+C1(I-1))*C1(I-1))
      S1=S1+T3(I)
      S1S=CDABS(S1)
      T1S=CDABS(T3(I))
      IF(T1S.LE.ERR*S1S) GO TO 9
      IF(1.LT.101)GO TO 11
      IF(JFLAG.GT.0)GO TO 14
      JFLAG=1
      GO TO 6
   14 WRITE(6,1002)
      STOP
   81 CONTINUE
      JFLAG=1
      GO TO 6
   82 WRITE (6,1022)
      STOP
    9 B=S1
      IF(M.NE.2)GO TO 55
      N=DABS(ENX1)
      U=2.D0*(ZLG+EUL)
      S2=B*U
      IZZZ=1
   77 DO 75 J=IZZZ,I
      J1=J-1
      J1N=N+J-1
          IF (J1.LE.0) GO TO 200
          IF (J1.GT.101) GO TO 201
          TEMP1=C6(J1)
      GO TO 203
  200 TEMP1=0.D0
      GO TO 203
  201 TEMP1=C6(101)
      DO 202 JJ=102,J1
          TEMP1=TEMP1+1.D0/DFLOAT(JJ)
  202 CONTINUE
  203 CONTINUE
          IF (J1N.LE.0) GO TO 205
      IF(J1N.GT.101) GO TO 206
      TEMP2=C6(J1N)
      GO TO 208
  205 TEMP2=0.D0
      GO TO 208
  206 TEMP2=C6(101)
          DO 207 JJ=102,J1N
          TEMP2=TEMP2+1.D0/DFLOAT(JJ)
  207 CONTINUE
  208 CONTINUE
      T2=T3(J)*(TEMP1+TEMP2)
      T2R=DABS(DREAL(T2))
      T2I=DABS(DIMAG(T2))
   75 S2=S2-T2
      S2R=DABS(DREAL(S2))
      S2I=DABS(DIMAG(S2))
      IF(T2R.GT.ERR*S2R) GO TO 78
      IF(T2I.LE.ERR*S2I) GO TO 76
   78 I=I+1
      IF(I.GT.101.AND.JFLAG.EQ.0)GO TO 81
      IF(I.GT.101.AND.JFLAG.NE.0)GO TO 82
      T3(I)=-T3(I-1)*EX/((EN+C1(I-1))*C1(I-1))
      IZZZ=I
      GO TO 77
```

```
   76 B2=S2*C(10)
      IF(N.EQ.0) GO TO 55
      S1=DCMPLX(0.D0,0.D0)
      T1=-C(10)/ZHS
      LUP=N-1
      IF(LUP.EQ.0)GO TO 72
      DO 70 LL=1,LUP
   70 T1=T1*DFLOAT(LL)
      S1=S1+T1
      DO 71 LL=1,LUP
      T1=T1*EX/(DFLOAT(LL)*DFLOAT(LUP-LL+1))
   71 S1=S1+T1
      GO TO 73
   72 S1=S1+T1
   73 B2=B2+S1
      GO TO 55
   55 IF(NFLAG.LT.0)B=A*B
      BX=DREAL(B)
      BY=DIMAG(B)
      BXA=DABS(BX)
      BYA=DABS(BY)
      IF(ZX.EQ.0.D0.OR.ZY.EQ.0.D0)NF=1
      IF(NFLAG.NE.2.AND.BXA.LT.BYA.AND.NF.EQ.1) BX=0.D0
      IF(NFLAG.NE.2.AND.BYA.LT.BXA.AND.NF.EQ.1)BY=0.D0
      GO TO(56,57,58,59),M
    6 ARG=X-EN*C(2)-C(3)
      CF=CDCOS(ARG)
      SF=C(6)*CDSIN(ARG)
      FNS=C1(4)*EN*EN
      I=0
      S1=DCMPLX(1.D0,0.D0)
      S2=DCMPLX(0.D0,0.D0)
      U=DCMPLX(1.D0,0.D0)
      T1S=1.D0
      S=CF
   18 I=I+1
      V=-(FNS-C5(I))/(C(7)*X*C3(I))*U
      U=V*(FNS-C4(I))/(C2(I)*X)
      US=CDABS(U)
      IF(US.GT.T1S) GO TO 20
   12 CONTINUE
      T=U*CF+V*SF
      TR=DABS(DREAL(T))
      TI=DABS(DIMAG(T))
      S=S+T
      SR=DABS(DREAL(S))
      SI=DABS(DIMAG(S))
      IF(TR.GT.ERR*SR) GO TO 24
      IF(TI.LE.ERR*SI) GO TO 26
   24 T1=U
      T2=V
      S1=S1+T1
      S2=S2+T2
      T1S=US
   17 IF(I-100)18,25,25
   20 IF(I.EQ.1) GO TO 12
      IF(JFLAG.GT.0) GO TO 23
      JFLAG=1
      GO TO 8
   23 WRITE(6,1002)
   29 CONTINUE
      STOP
   26 CONTINUE
      SQZ=CDSQRT(X)
      IF(DREAL(SQZ).LT.0.D0)SQZ=-SQZ
CCHOOSES PROPER BRANCH FOR SQUARE-ROOT
      B=C(4)/SQZ*S
      IF(KFLAG.EQ.2)B=A*B
      IF(KFLAG.EQ.3)B=B/A
      IF(M.EQ.3)M=2
      IF(M.NE.2)GO TO 55
      B2=C(4)/SQZ*(SF*(S1+U)/C(6)-CF*(S2+V)*C(6))
      IF(KFLAG.EQ.2)B2=(B2+CDCOS(C(1)*EN)*CI*B)/A
      IF(KFLAG.EQ.3)B2=(B2-CDCOS(C(1)*EN)*CI*B)*A
      GO TO 55
   25 IF(JFLAG.GT.0)GO TO 28
      JFLAG=1
      GO TO 8
   28 WRITE(6,1002)
      GO TO 29
```

```
   58 B1=DCMPLX(BX,BY)
      M=4
      EN=-EN
      GO TO 3
   59 B1T=DCMPLX(BX,BY)
      EN=-EN
      ARG=C(1)*EN
      B2=(CDCOS(ARG)*B1-B1T)/CDSIN(ARG)
      DB1=B1
      DB2=B2
      RETURN
   57 IF(NFLAG.EQ.-1)B2=A*B2
      DB2=B2
   56 DB1=DCMPLX(BX,BY)
      RETURN
      END
      SUBROUTINE DGAMM(DZ,DGM,DPS,ERR,JJ)
CIF JJ=0,CALCULATES ONLY GAMMA FUNCTION, IF JJ=1, CALCULATES ONLY PSI
CFUNCTION, IF JJ=2, CALCULATES BOTH
      IMPLICIT REAL*8(A-H,O-Z)
      COMPLEX*16 GAM,Z,DZ,DGM,DPS,DPSI
      COMPLEX *16 TERM1,ZT1,TERM,SUM,ZLG,ZTGAM,ZT
      COMPLEX*16 CDLOG,DCMPLX,CDEXP,CDSIN
      REAL*8 DREAL,DIMAG,DLOG,DABS
      INTEGER*4 IDINT
      DIMENSION B(10)
      DIMENSION C(100)
      DATA IFLAG /0/
 1001 FORMAT(1H ,///,24H SERIES DID NOT CONVERGE)
 1010 FORMAT(1H ,///,47H THIS IS A SINGULAR POINT OF THE GAMMA FUNCTION
     X,/,5X,6HARG R=,E12.5,3X,6HARG I=,E12.5)
      IF(JJ.EQ.0)GO TO 60
      DPS=DPSI(DZ,ERR)
      IF(JJ.EQ.1)RETURN
   60 E=ERR
      IF(E.LE.0.D0)E=.1D-07
      ZX=DREAL(DZ)
      ZY=DIMAG(DZ)
      Z=DZ
      IF(ZX.LT.0.D0)Z=-Z
      NFLAG=2
      J=0
CK=NUMBER OF TERMS IN SERIES
      IF(ZY.EQ.0.D0.AND.(IDINT(ZX)-ZX).EQ.0.D0) NFLAG=1
      IF(NFLAG.EQ.1.AND.ZX.LE.0.D0)NFLAG=0
CNFLAG=0 MEANS Z=0 OR Z A NEGATIVE INTEGER   NFLAG=1MEANS Z A POSITIVE
      IF(NFLAG.NE.0)GO TO 51
      WRITE(6,1010) DZ
      DGM=DCMPLX(0.D0,0.D0)
      RETURN
   51 IF(NFLAG.EQ.2)GO TO 42
      IF(ZX.GT.2.0D0) GO TO 55
      DGM=DCMPLX(1.D0,0.D0)
      RETURN
   55 IF(ZX.GT.20.D0)GO TO 42
      IF=IDINT(ZX)-1
      IJ=1
      DO 300 N=2,IF
  300 IJ=IJ*N
      DGM=IJ
      RETURN
   42 CONTINUE
      IF(IFLAG.NE.0)GO TO 20
      DO 100 I=1,100
  100 C(I)=I
      PI=3.141592653589793D0
      PI2=DLOG(2.D0*PI)/2.D0
      B(1)=1.0D0/12.0D0
      B(2)=-1.0D0/360.0D0
      B(3)=1.0D0/1260.0D0
      B(4)=-1.0D0/1680.0D0
      B(5)=1.0D0/1188.0D0
      B(6)=-691.0D0/360360.0D0
      B(7)=1.0D0/156.0D0
      B(8)=-3617.0D0/122400.0D0
      B(9)=43867.0D0/244188.0D0
      B(10)=-174611.0D0/125400.0D0
CB(I) ARE THE BERNOULLI COEFFICIENTS IN STIRLINGS FORMULA
      IFLAG=1
   20 ZT=Z
```

```
      ZT1=ZT-1.D0
      I=0
    5 IF(DREAL(ZT).GT.10.D0) GO TO 3
    4 I=I+1
      IF(I.LE.100)GO TO 30
      WRITE (6,1001)
      STOP
   30 CONTINUE
      ZT=ZT+1.D0
      GO TO 5
    3 IF=I
      ZLG=CDLOG(ZT)
      SUM=(ZT-.5        D0)*ZLG       -ZT+PI2
      TERM1=SUM
      ATER1R=DABS(DREAL(TERM1))
      ATER1I=DABS(DIMAG(TERM1))
      J=0
    8 J=J+1
      TERM=B(J)*CDEXP(-(2.D0*C(J)-1.D0)*ZLG)
      ATERR=DABS(DREAL(TERM))
      ATERI=DABS(DIMAG(TERM))
      SUM=SUM+TERM
      ASUMR=DABS(DREAL(SUM))
      ASUMI=DABS(DIMAG(SUM))
      IF(ATERR.GT.ATER1R)GO TO 24
      IF(ATERI.GT.ATER1I)GO TO 24
      IF(ASUMR.EQ.0.D0)GO TO 9
      IF(ATERR/ASUMR.GT.E)GO TO 27
    9 CONTINUE
      IF(ASUMI.EQ.0.D0) GO TO 6
      IF(ATERI/ASUMI.LE.E)GO TO 6
   27 CONTINUE
      ATER1R=ATERR
      ATER1I=ATERI
      GO TO 7
   24 CONTINUE
    7 IF(J.LT.10)GO TO 8
      GO TO 4
    6 ZTGAM=SUM
      IF(IF.EQ.0)GO TO 31
      DO 200 K=1,IF
  200 ZTGAM=ZTGAM-CDLOG(ZT1+C(K))
   31 GAM=CDEXP(ZTGAM)
      IF(ZX.LT.0.D0)GAM=-PI/(GAM*CDSIN(PI*Z)*Z)
      DGM=GAM
      RETURN
      END
      COMPLEX FUNCTION DPSI*16(Z,E)
      IMPLICIT COMPLEX*16 (A-H,O-Z)
      COMPLEX*16 CDEXP,CDLOG,CDCOS,CDSIN,DCMPLX
      REAL*8 DATAN2,DFLOAT,DABS,CDABS
      REAL*8 E,ERR,B,ABTER,ABTER1,ABSUMI,PI,ZX,ZY,ZXI,EN,EUL,DREAL,DIMAG
      INTEGER*4 IDINT
      DIMENSION B(10)
      DATA IFLAG /0/
      ERR=E
      IF(ERR.LE.0.D0)ERR=.1D-5
      ZP=Z
      ZT=Z
      ZX=DREAL(ZP)
      ZY=DIMAG(ZP)
      NI=IDINT(ZX)
      ZXI=DFLOAT(NI)-ZX
      IF(ZY.NE.0.D0.OR.ZX.GT.0.D0.OR.ZXI.NE.0.D0)GO TO 1
      DPSI=DCMPLX(0.D0,0.D0)
      WRITE (6,1010) Z
      RETURN
CCALCULATE PSI HERE IF Z IS A POSITIVE INTEGER
    1 IF(ZY.NE.0.D0.OR.ZXI.NE.0.D0)GO TO 2
      EUL=-.5772156649
      DPSI=EUL
      IF(NI.EQ.1)RETURN
      NF=NI-1
      DO 100 N=1,NF
  100 DPSI=DPSI+1.D0/DFLOAT(N)
      RETURN
    2 ISIGN=0
      IF(ZX.LT.0.D0)ISIGN=1
      IF(ISIGN.EQ.1)ZT=1.D0-ZT
CREFLECTS Z INTO 1-Z IF Z IS IN LEFT-HALF PLANE
```

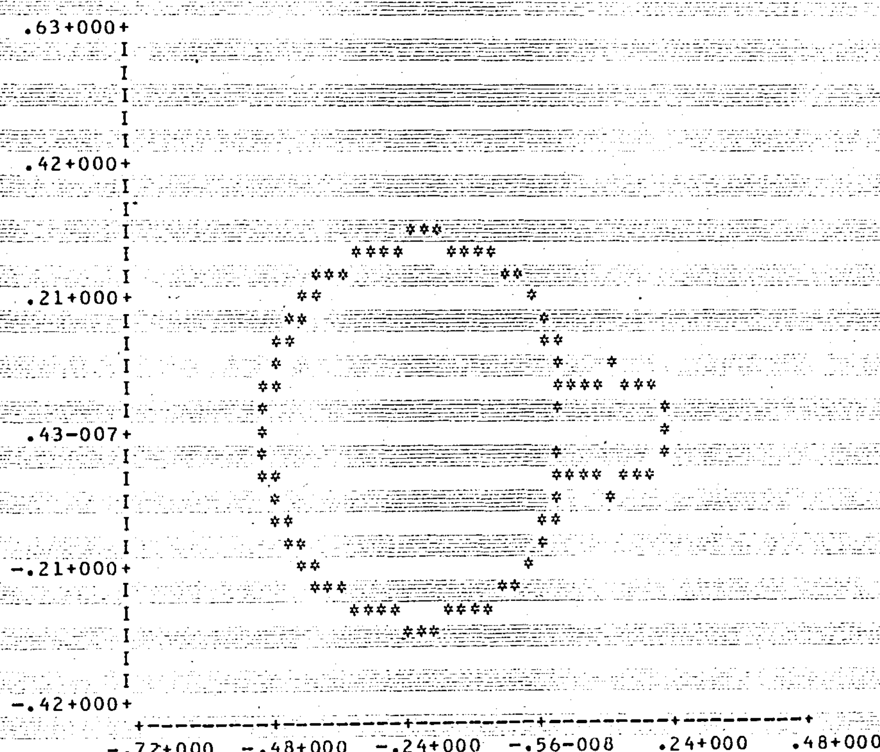
```
      ZS=ZT
      NR=0
      IF(IFLAG.NE.0)GO TO 3
      PI=DATAN2(0.D0,-1.D0)
      B(1)=1.D0/(2.D0*6.D0)
      B(2)=-1.D0/(4.D0*30.D0)
      B(3)=1.D0/(6.D0*42.D0)
      B(4)=-1.D0/(8.D0*30.D0)
      B(5)=5.D0/(10.D0*66.D0)
      B(6)=-691.D0/(12.D0*2730.D0)
      B(7)=7.D0/(14.D0*6.D0)
      B(8)=-3617.D0/(16.D0*510.D0)
      B(9)=43867.D0/(18.D0*798.D0)
      B(10)=-174611.D0/(20.D0*330.D0)
      IFLAG=1
    3 IF((NR+NI).GT.10)GO TO 4
      NR=NR+1
      GO TO 3
CINCREASES REAL PART OF Z UNTIL GREATER THAN 10
    4 ZT=ZT+DFLOAT(NR)
      ZL=CDLOG(ZT)
      TER1=.5D0/ZT
      SUM=ZL-TER1
      ABTER1=CDABS(TER1)
      N=0
    8 N=N+1
      EN=DFLOAT(N)
      TER=-B(N)*CDEXP(-2.D0*EN*ZL)
      ABTER=CDABS(TER)
      IF(ABTER.LT.ABTER1)GO TO 5
      NR=NR+1
      ZT=ZS
      IF(NR.LT.100)GO TO 4
      WRITE (6,1011)
      RETURN
    5 SUM=SUM+TER
      ABSUMI=DABS(DIMAG(SUM))
      IF(ABSUMI.NE.0.D0)GO TO 6
      IF(ABTER/CDABS(SUM).LE.ERR)GO TO 7
      GO TO 9
    6 IF(DABS(DREAL(TER))/DABS(DREAL(SUM)).LE.ERR.AND.DABS(DIMAG(TER))/
     1ABSUMI.LE.ERR)GO TO 7
    9 ABTER1=ABTER
      GO TO 8
    7 DPSI=SUM
      IF(NR.EQ.0) GO TO 10
      DO 200 N=1,NR
  200 DPSI=DPSI-1.D0/(ZT-DFLOAT(N))
   10 IF(ISIGN.EQ.0)RETURN
      ARG=PI*ZP
      DPSI=DPSI-PI*CDCOS(ARG)/CDSIN(ARG)
      RETURN
 1010 FORMAT(1H ,//,45H THIS IS A SINGULAR POINT OF THE PSI FUNCTION,/,
     130X,6HARG R=,1PE12.5,3X,6HARG I=,E12.5)
 1011 FORMAT(1H ,//,24H SERIES DID NOT CONVERGE)
      END
```

```
    ****************************************************************
    SAMPLE RUN WITH THE SAMPLE DATA-(A) GIVEN IN THE PROGRAM-(B)
    ****************************************************************

    ************************************************
    *                                              *
    *        SCATTERING OF ACOUSTIC WAVES          *
    *                                              *
    * BY THE CYLINDERS OF ARBITRARY CROSS-SECTION  *
    *                                              *
    *        ((( T-MATRIX FORMULATION )))          *
    *                                              *
    ************************************************

    ** PROGRAM-(B): EVALUATION OF THE NEAR AND FAR FIELD SOLUTIONS **
    =================================================================

 ===============================================================

 B.C. TYPE : NEUMANN (RIGID INCLUSION)

 CROSS-SECTION OF THE SCATTERER : CIRCULAR

 RADIUS :  1.000

 SYMMETRY CONDITION (W.R.T. X-AXIS) :     SYMMETRIC

 WAVE TYPE :  ACOUSTIC PLANE WAVE

 WAVE NO :   .5

 TOT.DIM. OF THE T-MATRIX USED :  10 X 10

 ===============================================================
```

1- ANGLE OF INCIDENCE : 0. (DEGREES)

** FAR-FIELD SCATTERED FIELD AMPLITUDE **

ANGLE	REAL PART	IMAGINARY PART	PEAK VALUE (NORM)
0.	-.09037	.20340	.22257
10.	-.08940	.19733	.21663
20.	-.08650	.17936	.19913
30.	-.08178	.15024	.17106
40.	-.07536	.11116	.13430
50.	-.06746	.06365	.09275
60.	-.05831	.00955	.05909
70.	-.04820	-.04914	.06883
80.	-.03742	-.11038	.11655
90.	-.02632	-.17211	.17411
100.	-.01523	-.23242	.23292
110.	-.00448	-.28955	.28958
120.	.00560	-.34196	.34201
130.	.01470	-.38835	.38863
140.	.02255	-.42765	.42825
150.	.02892	-.45903	.45994
160.	.03361	-.48186	.48303
170.	.03648	-.49572	.49707
180.	.03745	-.50037	.50177



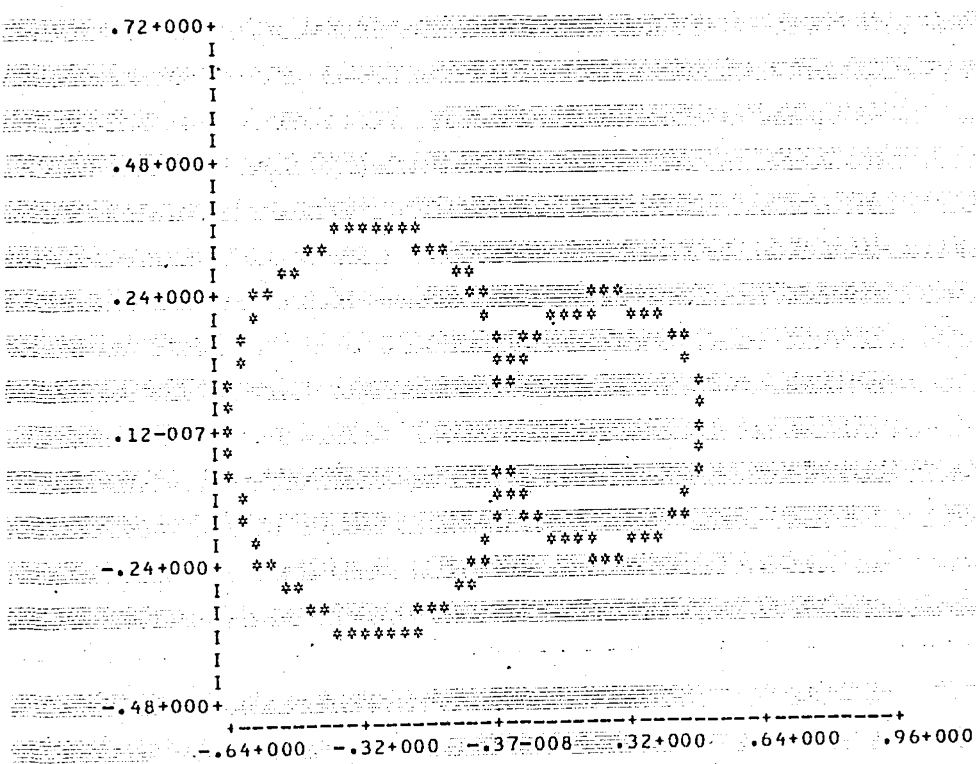
CHECK FOR THE CONVERGENCY OF THE TOTAL SCATTERING CROSS-SECTION

ORDER	TOT.S.C.SECTION	(%)DIFFERENCE
1	.2639208-001	
2	.9030308-001	.2421598+003
3	.9037284-001	.7724488-001
4	.9037284-001	.8410429-005
5	.9037284-001	.2313573-009

REGION : *BOUNDARY*

** VELOCITY POTENTIALS DUE TO SCATTERED FIELD **

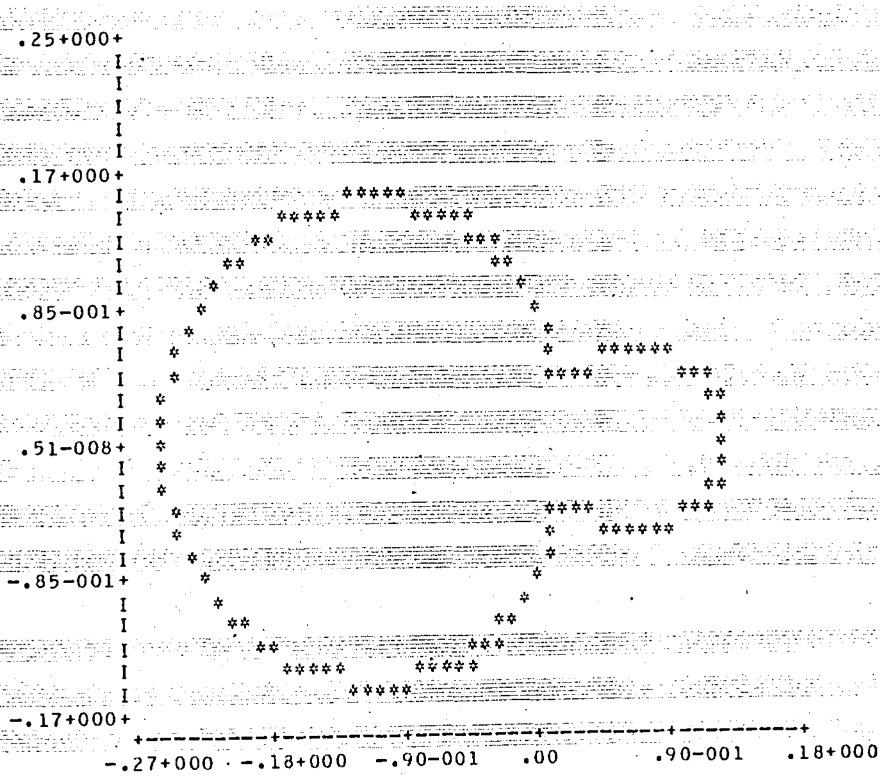
ANGLE	REAL PART	IMAGINARY PART	PEAK VALUE (NORM)
0.	-.33923	.35750	.49283
10.	-.33271	.35061	.48334
20.	-.31365	.32998	.45526
30.	-.28357	.29578	.40975
40.	-.24481	.24841	.34877
50.	-.20040	.18868	.27524
60.	-.15368	.11792	.19371
70.	-.10805	.03809	.11457
80.	-.06651	-.04822	.08215
90.	-.03143	-.13796	.14150
100.	-.00426	-.22779	.22783
110.	.01458	-.31436	.31470
120.	.02558	-.39458	.39541
130.	.03006	-.46583	.46680
140.	.02984	-.52608	.52692
150.	.02693	-.57393	.57456
160.	.02325	-.60852	.60897
170.	.02037	-.62941	.62974
180.	.01930	-.63638	.63668



REGION : 5.0

** VELOCITY POTENTIALS DUE TO SCATTERED FIELD **

ANGLE	REAL PART	IMAGINARY PART	PEAK VALUE (NORM)
0.	-.08886	-.09388	.12926
10.	-.08608	-.09228	.12620
20.	-.07785	-.08758	.11718
30.	-.06448	-.08000	.10275
40.	-.04648	-.06992	.08396
50.	-.02453	-.05780	.06279
60.	.00056	-.04420	.04420
70.	.02790	-.02967	.04073
80.	.05656	-.01480	.05847
90.	.08559	-.00009	.08559
100.	.11408	.01397	.11493
110.	.14118	.02701	.14374
120.	.16615	.03871	.17060
130.	.18832	.04886	.19455
140.	.20716	.05730	.21494
150.	.22224	.06393	.23125
160.	.23323	.06869	.24313
170.	.23991	.07155	.25035
180.	.24215	.07251	.25277



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