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**COMPUTER MODELS FOR ESTIMATING
THE PERFORMANCE OF A GIVEN AXIAL-FLOW GAS TURBINE STAGE
AND FOR OPTIMIZING THE EFFICIENCY**

by

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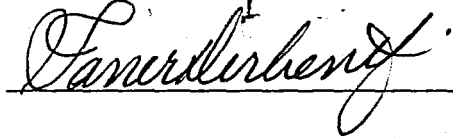
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ABSTRACT

The present thesis involves the study of the aerodynamic performance of an axial-flow gas turbine stage. Existing studies have been reviewed and based on the most developed performance evaluation and efficiency optimization techniques, two computer programs have been developed. Given the geometry of a turbine stage, the inlet gas conditions and the rotational speed of the moving blades, the first program calculates the total-to-total isentropic efficiency of the stage and determines the gas conditions at the inlet and outlet of the blade rows. The second program, on the other hand, solves the inverse problem: For specified inlet gas conditions and blade rotational speed, the program determines the geometry of the turbine stage for which the isentropic efficiency is an optimum.

Both programs are applied to turbine stages operating at various inlet gas conditions and blade rotational speeds. The isentropic stage efficiency is observed to depend primarily on the overall pressure ratio, the initial mass flow parameter and the dimensionless speed of the stage. The results which are summarized in tabular and graphical forms are in conformity with those reported by the original authors. Finally, recommendations are given for improving the precision of the performance prediction method used.

Ö Z E T

Bu tezde eksenel akışlı bir gaz türbini kademesinin aerodinamik performansı incelenmiştir. Konuyla ilgili çalışmalar araştırılmış, en gelişmiş performans değerlendirme ve verim eniyileştirme teknikleri esas alınarak iki bilgisayar programı hazırlanmıştır. Birinci program, boyutları, akışkan giriş şartları ve dönme hızı verilen bir türbin kademesinin tersinir adiabatik verimini ve kanat dizinlerine giriş ve çıkış noktalarındaki gaz özelliklerini hesaplamaktadır. İkinci program ise ters problemi çözmekte yani, akışkan giriş şartları ve dönme hızı verilen bir türbin kademesinin boyutlarını tersinir adiabatik verimin eniyi olmasını sağlayabilecek şekilde hesaplamaktadır.

Her iki programın da değişik akışkan giriş şartları ve dönme hızları ile işleyen türbin kademelerine uygulaması yapılmıştır. Sonuçta tersinir adiabatik verimin özellikle giriş ve çıkıştaki basınçların oranı, girişteki gaz akımı parametresi, ve basamağın hız parametresine bağımlı olarak değişiklikler gösterdiği gözlenmiştir. Sonuçlar, çizelge ve grafik biçiminde toplanmış ve bunların daha önce yayınlanmış metodlarda görünen sonuçlarla uyduğu saptanmıştır. Son olarak da performans tahmininin hassasiyetini arttıracak bazı değişiklikler tavsiye edilmiştir.

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LIST OF SYMBOLS

A	Gas flow area, measured normal to flow direction
A_n	Annulus area
A_t	Blade passage throat area
B	Constant defined by Eq. (13) and Eq. (14)
C_L	Tangential lift coefficient
C_p	Specific heat of the working fluid at constant pressure
c	Blade chord
c_a	Axial chord
D	Mean (reference) diameter of the turbine annulus
D_e	Equivalent diffusion ratio
D_p	Diffusion factor for the pressure surface
D_s	Diffusion factor for the suction surface
D_t	Total diffusion factor for a blade passage
e	Mean radius of curvature of upper blade surface between the throat and the trailing edge
\vec{G}	Vector formed by the constraint equations of the optimization problem
H	Total enthalpy
$H(R_{op})$	Objective function of the one-dimensional maximization problem
h	Annulus height
ID	Inner diameter of the turbine annulus
i	Gas incidence
i_s	Stalling gas incidence

J_{in}	Jacobian matrix
k	Radial tip clearance of the rotor blades
L	Blade camber length
M	Mach number
M_{ijk}	Three dimensional matrix used in the differentiation of the penalty function
\dot{m}	Turbine inlet mass flow rate
N	Rotational blade speed
n	Number of blades in a row
OD	Tip (outer) diameter of the turbine annulus
o	Blade opening or throat
P	Total gas pressure
P_c	Total critical gas pressure
$P(\vec{x}, r_k)$	Interior penalty function
R	Degree of reaction of the stage
R_a	Gas constant
Re	Reynolds number
Re_h	Reynolds number based on throat mean hydraulic diameter
R_{op}	Optimal step size of the one-dimensional optimization of $H(R_{op})$
r_k	Non-negative resequencing factor defined by Eq. (101)
s	Blade pitch or spacing
T	Total gas temperature
ΔT	Total temperature drop across the stage
t	Maximum blade thickness
t_e	Blade trailing edge thickness, measured normal to the camber line at the trailing edge

U	Linear velocity of the rotor blades measured at the reference diameter
V	Total gas velocity
V_a	Axial component of the absolute gas velocity
$V_{p-\min}$	Minimum gas velocity over the pressure surface
$V_{s-\max}$	Maximum gas velocity over the suction surface
V_t	Tangential component of the absolute gas velocity
ΔW	Specific work output of the stage
\vec{X}	Vector formed by the design variables of the optimization problem
x	Distance along the camber line to the leading edge of the blade
Y_e	Endwall loss coefficient
Y_k	Clearance loss coefficient
Y_p	Profile loss coefficient
$Y_{p(\beta_i=0)}$	Profile loss coefficient of a nozzle blade ($\beta_i=0$) of same gas inlet angle and of same pitch-per-chord ratio as the actual blade
$Y_{p(\beta_i=-\alpha_0)}$	Profile loss coefficient of a zero-reaction blade of same gas inlet angle and of same pitch-per-chord ratio as the actual blade
$Y_{p(i=0)}$	Profile loss at zero incidence
Y_s	Secondary loss coefficient
Y_t	Overall mean stagnation pressure loss coefficient
Y_{te}	Trailing edge loss coefficient
Z	Blade loading parameter

Greek Symbols

α	Gas flow angle, measured relative to the axial direction
α_c	Included angle of divergence of the turbine annulus walls
β	Blade (metal) angle, measured relative to the axial direction
$\vec{\nabla}$	Gradient vector operator
γ	Ratio of the specific heats of the working fluid
δ_m	Momentum thickness
δ_u	Blade loading factor
ϵ	Gas deflection
ξ	Blade stagger angle
η	Overall isentropic efficiency of the stage
λ	Empirical function correlating the secondary losses to annulus geometry
λ_N	Stator total loss coefficient based on temperature drop
λ_R	Rotor total loss coefficient based on temperature drop
μ	Dynamic viscosity of the working fluid
μ_a	Acceleration ratio
ν	Kinematic viscosity of the working fluid
ϕ	Flow coefficient
ρ	Density of the working fluid
ψ	Temperature drop coefficient
$\bar{\omega}$	Mean loss of the total pressure through a blade row due to friction and flow separation

Subscripts

i	Inlet to a stage
is	Isentropic
m	Mean
N	Nozzle
o	Outlet from a stage
R	Rotor
s	Static
1	Inlet to the stator row
2	Outlet from the stator row
3	Inlet to the rotor row
4	Outlet from the rotor row

I. INTRODUCTION

Since the early days of the development of gas turbine engines and their progressive use in industrial, vehicular, railroad, marine, aviation, nuclear and spatial applications, the necessity for high efficiency required for economic operation has become more and more dominant. In recent years, turbines of considerable size and capacity have been produced with many stages. Along with the demand for larger gas turbines, factors like efficiency and component weight have become major design considerations.

Compared to that of the turbine used in aerospace applications, the design of the turbine for power plants does not necessitate the accomplishment of the lightest weight condition, which is, in most of the cases, incompatible with the attainment of the best efficiency. The premium placed on performance almost invariably result in the employment of a multistage construction. In land applications, turbine efficiency is thereby considerably improved over the level which can be obtained with light-weight, single-stage turbines ordinarily used in aircraft units. In return, for the case of stationary power turbines, a low axial velocity is desirable in order to avoid high leaving losses which are proportional to the residual kinetic energy contained in the

gas leaving the turbine at the last stage.

On contemporary aero-engines, every effort is made to reduce the number of stages and turbine weight to a minimum. Within the non-aeronautical field, R @ D on axial flow gas turbines has two principal aims. The first is to explore the gas flow processes within a turbine stage, which would lead to a better understanding of the aerodynamics and to the determination of the associated aerodynamic efficiencies. The second is the achievement of reliable operation at progressively higher turbine inlet gas temperature closely related to metallurgical problems, leading to improvements in specific power output and specific fuel consumption.

Not considering the metallurgical aspect, if a turbine designer is to select the best design for his purpose, he must be able to predict the design point performance of a wide range of possible turbines, as well as the performance of his designs over a wide range of operation conditions. Moreover, he would like to be able to design a turbine stage with the objective of minimizing aerodynamic losses.

However, tests conducted on turbines have often revealed efficiencies considerably inferior to those predicted during the preliminary design stages. For this reason, aerodynamic research into the performance of turbines has been much accelerated.

Prediction of the aerodynamic efficiency of gas turbines requires the accurate calculation of aerodynamic losses. These losses are due to gas friction, vorticities, gas leakages boundary layer development on blade passages and adverse pressure gradients encountered through the blade passages of the turbine, and they

constitute the main cause of the drop in total efficiency.

Since the flow of gas within the turbine at high pressure and temperature is quite a complicated process, it is nearly impossible to bring about a wholly analytical solution. Results of experimental work conducted by companies, research centers and universities, as well as theoretical and analytical considerations of aerodynamic concepts are therefore used together for the formation of generalized loss-geometry correlations.

With appropriate design input information such as rotative speed, mass flow rate, inlet state conditions of the working fluid, pressure ratio or power desired and with specified values for the significant dimensions of the blade profiles such as axial width, maximum thickness, pitch and chord lengths, blade inlet, exit and stagger angles, the use of aerodynamic equilibrium conditions in conjunction with the available loss correlations can lead to an accurate prediction of the design point efficiency of a turbine.

In the present thesis, two principal aims are considered: The first comprises the estimation of performance of an axial flow gas turbine stage of predetermined geometry, under different conditions of pressure ratio, inlet fluid conditions and machine rotative speed. The second is the determination of the geometry of a gas turbine stage for which the aerodynamic losses would be a minimum. Two separate computer programs are developed for each of the purposes cited above. The first program adopts the loss system of Ainley and Mathieson [1] with the suggested modification of Dunham and Came [2] as its basis. This loss system is chosen because it is one of the best known and

and most completely documented of the gas turbine loss evaluation systems so far described in the literature. The second computer program is based on the optimization method developed by Rao and Gupta [3], the aerodynamic losses to be minimized being mostly those predicted in [1] and [2], although the evaluation of the stage efficiency is carried out in a slightly different manner.

II. LITERATURE SURVEY

In the past thirty years, a number of loss evaluation systems as well as concrete surveys on design practices for gas turbines power elements have been described in the literature. Results of experimental works together with theoretical and analytical considerations have been published and served as the greatest contributors to the improvements achieved in gas turbine efficiency, dependability and low first and operating costs. Without doubt, the analytical and numerical procedures used in designing such turbomachines have now undergone a major revision. The empirical relations used in design process are being replaced by more sophisticated methods of analysis.

The purpose of the literature survey presented here is to review the present day basis of turbine performance calculations and turbine stage design, indicating where improvements have been made in recent years and where further work is needed.

Expressions, correlations and schematic representation of concepts related to the works presented in the following paragraphs can be found in the Appendix.

2.1 LITERATURE ON PERFORMANCE PREDICTION

According to Horlock [4], there are three principal sources of turbine cascade data:

- i) The results of Zweifel [5] and miscellaneous turbine cascade results which have been brought together by Soderberg [6] in a simple correlation.
- ii) British cascade data (the results of Todd [7], Reeman [8], and Dunsby [9]) which have been brought together in correlations by Ainley and Mathieson [1].
- iii) Early data from steam turbine cascades, principally that of the Steam Nozzle Research Committee reported in reference [10].

The first two sets of data are reviewed and to some extent compared here. In addition, some more recent studies are reported.

An efficient method for the calculation of pressure loss is to represent the pressure gradient in flow direction by choosing the adequate cascade parameters. An early approach to this method was presented by Zweifel [5], in 1945. From a series of cascade tests, Zweifel suggested that the lift coefficient based on the tangential loading of the blade (aerodynamic blade loading coefficient, ψ_z , can be expressed as a function of pitch-to-chord ratio and of the gas inlet and outlet angles. But, to arrive to this conclusion, Zweifel had made the highly simplified assumption that the actual velocity distribution on the suction and pressure sides of the blade could be approximated to a distribution independent of the distance from the

leading edge, along the camber line. Zweifel concluded that the interrelation between cascade total loss coefficient, Y_t , and aerodynamic blade loading coefficient, ψ_z , showed that the losses decreased to a minimum for the value of $\psi_z = .8$. Therefore, for given gas inlet and outlet angles (deflection), an optimum pitch-to-chord ratio could be obtained.

This suggestion, although based on an oversimplified flow model, has been confirmed to a large degree, by later investigations.

Soderberg [6] in 1949, correlated the losses on a basis of pitch-to-chord ratio, Reynolds number, aspect ratio (based on axial chord), thickness-to-chord ratio and blading geometry. Soderberg used the work of Zweifel [5] to obtain the optimum pitch-to-chord ratio for a given change of direction through a cascade. Then, based on this optimum, he formulated the "nominal" loss coefficient, for turbine rows operating at a Reynolds number of 10^5 and with an aspect ratio of 3:1 as well as the "total" loss coefficient for aspect ratios other than 3:1 and for Reynolds numbers (based on throat hydraulic mean diameter) other than 10^5 . Soderberg included a correction on the efficiency for clearance losses simply by multiplying the final calculated stage efficiency by the ratio of blade area to total area including leakage space.

Soderberg's correlation implies that the effect of profile shape is limited. Although the correlation is affected by thickness-to-chord ratio, no effect of trailing edge thickness is included. Further, it is implied that degree of reaction (or stagger) is unimportant, so long as the optimum pitch-to-chord ratio is chosen. The

neglect of these parameters renders Soderberg's correlation open to criticism. Further, the aspect ratio alone cannot be expected to be the only important parameter in the correction for secondary loss, since the entry boundary layer and the blade geometry must be vital parameters. However, it is understood that the correlation gives turbine efficiencies to within 3 percent over a wide range of Reynolds number and aspect ratio, and it is useful for obtaining quick estimates of turbine performance.

Stewart, Whitney and Wong [11], in 1960, using experimental data of Dunavant and Erwin [12], offered a relationship between the boundary layer momentum thickness and the total diffusion factor. The boundary layer momentum thickness is an important parameter in calculating the growth of the momentum loss due to boundary layer formation on blade surface. The total diffusion factor, on the other hand, is related to the blade surface velocity distribution. As a result, therefore, the work of [11] correlated the losses in a turbine cascade to the type of velocity distribution over the blade surface. The relationship showed an increasing momentum thickness for increasing total diffusion ratio, indicating also that for diffusion ratios greater than .6, flow separation would be imminent. Stewart, Whitney and Wong adopted a surface pressure distribution similar to that proposed by Zweifel [5], in order to interrelate surface velocities with cascade geometry. They assumed that the maximum suction surface velocity was equal to the cascade exit velocity and that the minimum pressure surface velocity had a value which produced an actual tangential force corresponding to the aerodynamic blade loading coefficient of [5].

These assumptions then led, through the use of aerodynamic blade loading coefficient and the assumed surface velocities, to an inter-relation between the total diffusion factor and the pitch-to-chord ratio, for given gas inlet and exit angles. The momentum loss and hence the energy loss may then be obtained as a function of pitch-to-chord ratio for the given gas angles.

This is a useful correlation but it can only be a guide to the choice of velocity distribution on the blade surface. The blade shape to give a required velocity distribution must be separately determined.

In 1968, Balje and Binsley [13] reported that the correlations of Stewart, Whitney and Wong [11] did not represent a description of the real situation because of the fact that for nozzle blades operating at small gas exit angles it would be possible to get negative values for the total diffusion factor. Moreover, for an aerodynamic blade loading coefficient chosen to be greater than 1, the total diffusion factor became imaginary. According to Balje and Binsley, the reason for the failure of this flow model was related to the inability to obtain a satisfactory expression for the pressure distribution in terms of cascade geometry.

The model that Balje and Binsley used in [13] as an alternative was based on the assumption that the surface velocity varied linearly with camber line, from the leading edge to the trailing edge. Using this linear variation of the free stream velocity in Truckenbradt [14] expression for the momentum thickness at a point in a turbulent boundary layer with pressure gradient, and integrating over the total camber length, they obtained a relation for the momentum thickness in terms

of cascade inlet and exit angles. This relation led to the calculation of the profile loss coefficient through the use of the correlation of Stewart, Whitney and Wong [11].

In another paper published in 1968, Balje [15] correlated the pressure losses (except for secondary losses) of a cascade to three key parameters. These parameters were the aerodynamic load coefficient of Zweifel, the accelerating rate formed by taking the ratio of gas exit and inlet velocities and the dimensionless boundary layer momentum thickness formed by dividing boundary layer momentum thickness to the camber length of the blade.

In a report presented to NGTE in 1951, Ainley and Mathieson [1] offered a method for estimating the performance of an axial flow gas turbine. Ainley and Mathieson calculated the profile loss at zero incidence first and then corrected the resulting loss coefficient according to the actual incidence on the blade, through a number of experimentally driven correlations. The clearance and secondary losses were correlated to the square of the lift coefficient, based on the work by Carter [16]. A method of calculation of absolute gas conditions depending on the total loss was also shown and the expression of the total-to-total aerodynamic efficiency was derived. The method was illustrated by a worked example.

This method constitutes one of the most complete and accurate surveys among the loss evaluation systems so far published.

The most notable refinement on the Ainley and Mathieson system was the one due to Dunham and Came [2], which was published in 1970. In this paper, the additional profile losses, incurred when a blade row chokes due to the development of shock waves have been accounted

for by the introduction of an arbitrary correction factor incorporating a numerical constant chosen to fit available efficiency data, the correction being applied only for Mach numbers greater than unity. Also, instead of correcting the final efficiency by a Reynolds number correction factor as was done by Ainley and Mathieson [1], an optional correction has been applied directly to the profile and secondary losses using the Reynolds number appropriate to the particular blade row.

Concerning the secondary losses, the blade loading parameter of Ainley and Mathieson [1] was adopted since it represented the effect of blade loading satisfactory, but the empirical constant of Ainley and Mathieson [1] used in the derivation of secondary losses has been replaced by a more realistic expression including the chord-to-blade height ratio and the ratio of gas outlet angle to cascade inlet angle. The final expression for tip clearance loss coefficient, on the other hand, contained the .78th power of clearance-to-chord ratio in the revised system [2].

As already stated, the loss model of Ainley and Mathieson [1] together with the corrections proposed by Dunham and Came [2] constitutes the basis of the first part of the present thesis. For this reason, both methods have been explained in detail in the following chapter.

2.2 LITERATURE ON TURBINE DESIGN

Research on conventional aerodynamic design of gas turbines and considerations on material limitations have a considerably short history which dates back to the early 1940's. At that time, the

design of gas turbines was much influenced by previous experiences on steam turbines for which, neither the problem of high component efficiencies for reasonable economic operation nor that of new material which would withstand high stresses at much higher temperatures had ever been encountered. The necessity of more efficient operation at higher temperatures and with lighter stage weight, however, has soon led gas turbine engineers to review the design considerations both from an aerodynamic and a mechanical standpoint.

The first person to conceive that the blade design should allow the gas stream to preserve a constant angular momentum, instead of completely neglecting the centrifugal forces upon the swirling gas, was the Air-Commodore Sir Frank Whittle. Sir Whittle explained his theory in 1945 [17] and designed the turbines in his jet, propulsion engines accordingly. This "free vortex" assumption of Whittle was followed by a number of other assumption of flow model, since in theory, the flow can take, in a blade passage an infinite number of radial forms as long as the centrifugal forces tending to throw the gas outwards are always equally balanced by an opposing radial static pressure gradient. As a widely used example to these proposed flow models, the one in which the gas mass flow per unit area of annulus is maintained constant over the flow area at any point may be cited. Resultingly, the old distinction between reaction and impulse turbines, as used by the steam engineers lost a great deal of its meaning, because the calculated reaction of a turbine stage began to vary along the blade height, the point of least reaction occurring at the root of the blade.

In 1948, Ainley wrote a paper on the performance of axial flow turbines [18]. In the paper Ainley discussed the various flow models proposed up to his time and made the comparison between the impulse steam-turbine blading with constant sections at all radii and the typical gas turbine stage providing impulse (least reaction) conditions at the root of the blades. Ainley discussed also in this paper a technique developed by the NGTE on turbine testing, giving regard to power output, rotational speed, gas flow (aerodynamic losses), inlet and outlet gas pressures (both total and static), inlet and outlet total head temperatures, instrumentation and safety precautions. The paper may be considered as one of the key works which constituted a basis to the system [1] he developed later with Mathieson.

Again in 1948, Reeman [19] outlined the mathematical approach to the design of gas turbines for jet engines and surveyed very comprehensively the mechanical problems that are involved, drawing attention to the balance that must be sought between aerodynamic efficiency and material limitations. He stated out that low axial velocity and high total-to-static efficiency as required for the turbocharger turbine was not a necessary requirement in the aircraft gas turbine, for the leaving velocity was used in the final nozzle. His conclusions were as follows: "Larger axial velocities lead to smaller annulus areas and to lower centrifugal stresses the latter constituting the most important factor in fixing the nozzle angle. With nozzle angle fixed, a compromise has to be made between reaction, swirl, disc stresses and weight. Increasing the reaction lowers the Mach number relative to the blade but raises the disc stresses and blade root temperature.

Increasing the swirl on the other hand, decreases the disc stresses but increases the Mach number relative to blade also increasing the blade root temperature". Reeman finally suggested an optimum in which there was little root reaction and an $10-16^\circ$ outlet swirl. The preliminary design given in Reeman's discussion was relevant to single stage turbines in which the blade loading coefficient [5] was of order 2 at the root where the reaction approached zero.

Emmert [20] in 1950, supported the flow model in which the mass of flow through the stage is to be evenly distributed over the stage annulus and he stated that a value of .6 or greater was required for maximum blading efficiency for the reaction type of blading.

Miser and Stewart [21] presented in 1961, full details of the design of two stage air-cooled turbine for a jet engine. Their report covered the choice of velocity triangles and the blade design. Velocity distributions, both radial and blade-to-blade were also calculated.

In 1970, Horlock [4] outlined an example to the preliminary design study of an axial flow turbocharger. Choosing the pressure ratio, the total entry temperature, the design mass flow and the target total-to-total efficiency from the start, he calculated the reaction of the stage and the flow coefficient, defined by the ratio of axial velocity through the stage to the blade speed at reference diameter. From the velocity triangles, he made the choice of the required blade speed and he calculated the outlet gas angle and Mach numbers. Following the change of the blade height and annulus area having regard to allowable blade stress, root reaction, Mach number and nozzle and rotor deflections, Horlock calculated

finally the pitch-to-chord ratio through the use of Zweifel's blade loading criterion [5].

In general, the problem under discussion comprises mostly the calculation techniques using vector diagrams which allow the fluid dynamics requirements to be satisfied throughout the stage. The basic premise of the correlations adopted in most of the works in the literature is that the shape of the velocity triangles at the mean radius is the principal factor which determines the efficiency of the turbine stage. The determination of the velocity triangles will provide the designer the flow outlet angle from the nozzle, flow inlet and outlet angle to and from the rotor and related relative and absolute velocities. With given velocity triangles, the efficiency will, of course, vary with blade geometry and, in particular, with incidence. The aerodynamic design problem becomes therefore, that of selecting a suitable blade profile and blade number.

The blade profile design methods are discussed in detail by Ainley [18]. Ainley presents the results of tests he conducted at NGTE, by comparing aerofoil and conventional blade profiles. He concludes, for most of the cases, that the aerofoil blading gives a less value for especially the profile loss coefficient. In accordance, Rao [3] proposes to superimpose a standard aerofoil section, such as A_3K_7 , after a camber line is predicted from aerodynamic and mechanical considerations. The number of blades in a row, or blade pitch, would be determined either from the results of cascade experiments or directly from the use of Zweifel's loading coefficient [5].

Very little work has been reported so far, concerning the

application of optimization techniques in the design of turbines. In 1966, George [22] reported about the optimization of a rocket engine turbine using a differential calculus approach. Balje and Binsley [13], in 1968 applied optimization techniques in the design of axial turbines using Wood's Direct Search Technique [23]. The optimizing variables were those geometric ratios which may be selected independently in order to achieve a maximum or optimum efficiency. These were chosen as the nozzle outlet angle, rotor inlet angle, blade height-to-mean diameter ratio, nozzle and rotor chord and pitch, mean diameter ratio and the degree of admission which varied from values just above 0 to values just below 1 in order to avoid indeterminate forms in the equations.

At the end of this work, Balje and Binsley concluded that the attainable efficiency might be significantly affected by the compressibility effects. It was noted also that the optimum turbine performance at low specific speeds was much more sensitive to the Reynolds number than performance at high specific speeds. In any case the optimum performance was very sensitive to chord length, i.e., blade number, especially at lower Reynolds numbers where large chords, hence, low blade numbers are recommended for optimum performance.

In 1980, Rao and Gupta [3] optimized the design of an axial gas turbine stage using the penalty function method of Fiacco and Mc Cormick [24]. They formulated the problem as a nonlinear mathematical programming problem with the objective of minimizing aerodynamic losses and mass of the stage. The aerodynamic losses were calculated according to the loss system of Ainley and Mathieson [1] as corrected by Dunham and Came [2]. The analysis of stress,

deflection and natural frequencies of vibration of doubly tapered and twisted cantilever beams (blades) was performed by finite element methods. At the end of the paper, discussion of optimum variables is made and a sensitivity analysis is presented.

The method of Rao and Gupta (3) constitutes the second part of this thesis. It will therefore be treated in detail in the following chapter.

III. ANALYSIS

The thesis is formed of two parts. In the first part, the performance evaluation of a gas turbine stage is realized, using the method developed by Ainley and Mathieson [1] with the suggested refinements by Dunham and Came [2]. In the second part, some important parameters of a gas turbine stage are optimized in order to get maximum aerodynamic efficiency from the stage. The loss correlations as given by Ainley and Mathieson [1], the performance evaluation technique and constraints of the optimization problem as given by Rao and Gupta [3] and the sequential unconstrained minimization technique as given by Fiacco and Mc Cormick [24] are adopted as the basis. The analysis is accordingly subdivided into two parts each of which concerning the related part of the thesis.

3.1 GAS TURBINE STAGE PERFORMANCE EVALUATION

The importance and durability of the Ainley and Mathieson loss system [1] results from the fact that it provides a relatively high accuracy compared to works that followed during the last 30 years, and that it achieves a remarkable level of simplicity, reducing the number

of variables appearing in the correlations to a minimum. The system has become a foundation worthy of subsequent refinement, the most notable of which was published in 1970 by Dunham and Came [2].

The procedure of the method of Ainley and Mathieson is formed of two parts. The first is the determination of pressure loss and gas efflux angle for each blade in the turbine under a wide range of inlet conditions. The second is to fix the gas inlet conditions at a given turbine speed and to follow the course of a fixed mass of gas from one blade row to the next, calculating the new gas conditions at each step. Finally, stage and overall pressure ratios, stage and overall efficiencies can be drawn out from the general picture.

Throughout the method, the following assumptions are made:

- i) The flow path through each stage is considered at one diameter only - this diameter being termed the "reference diameter".
- ii) The pressure loss coefficients in each blade row are not influenced by the gas Mach numbers.
- iii) The gas outlet angles from a blade are not influenced by the incidence angle.

It is to be noted, throughout the following analysis, that gas conditions written without the subscript, s , refer to total conditions and that primed quantities imply absolute inlet conditions to the next stage.

3.1.1 Determination of Gas Outlet Angle from a Blade Row

It is assumed [1] that gas outlet angle, α_o , from any row remains constant over the gas outlet Mach number, M_o , range $0. < M_o < .5$ and given over this range by

$$\alpha_o = [-1.17\cos^{-1}(o/s) + 12.58] - 4. * (s/e) \quad (1)$$

where

o = blade opening or throat

s = blade pitch

e = mean radius of curvature of upper blade surface
between throat and trailing edge.

The parameters used in Eq. (1) are also illustrated in Fig. 3.1.

At a gas outlet Mach number unity, it is assumed that the outlet angle is given by [1]

$$\alpha_o = -\cos^{-1}[(A_t(1 - k/h) + \pi \cdot OD \cdot k)/A_{n,o}] \quad (2)$$

where, as also shown in Fig. 3.2,

A_t = passage throat area

$A_{n,o}$ = annulus area downstream of the row

k = radial tip clearance

h = annulus height

OD = tip (outer) diameter of the turbine.

The throat area, A_t and the annulus area $A_{n,o}$ are calculated as follows:

$$A_t = o \times n \times (OD - ID)/2 , \quad (2a)$$

and

$$A_{n,o} = (OD^2 - ID^2) \times \pi/4 , \quad (2b)$$

where,

ID = inner diameter of the turbine,

and

n = blade number of the row.

For the Mach number range $.5 < M_o < 1.$, a linear variation of α_o may be assumed [1] with reasonable accuracy.

At low Mach numbers, α_o of Eq. (1) may be corrected as a function of radial clearance as [1]

$$\alpha_{o,corr} = \tan^{-1}\{[1 - 1.35(k/h)(\cos\beta_i/\cos\alpha_o)] * \tan\alpha_o + 1.35(k/h)(\cos\beta_i/\cos\alpha_o) * \tan\alpha_o\} \quad (3)$$

where

β_i = blade angle at inlet, as shown in Fig. 3.1.

3.1.2 Determination of Pressure Losses in a Blade Row

For a gas flow through a blade passage, the overall mean stagnation pressure loss coefficient, Y_t , can be expressed as:

$$Y_t = (P_i - P_o)/(P_o - P_{o,s}) \quad (4)$$

where, as illustrated in Fig. 3.3,

P_i = inlet total pressure to the stage

P_o = outlet total pressure from the stage

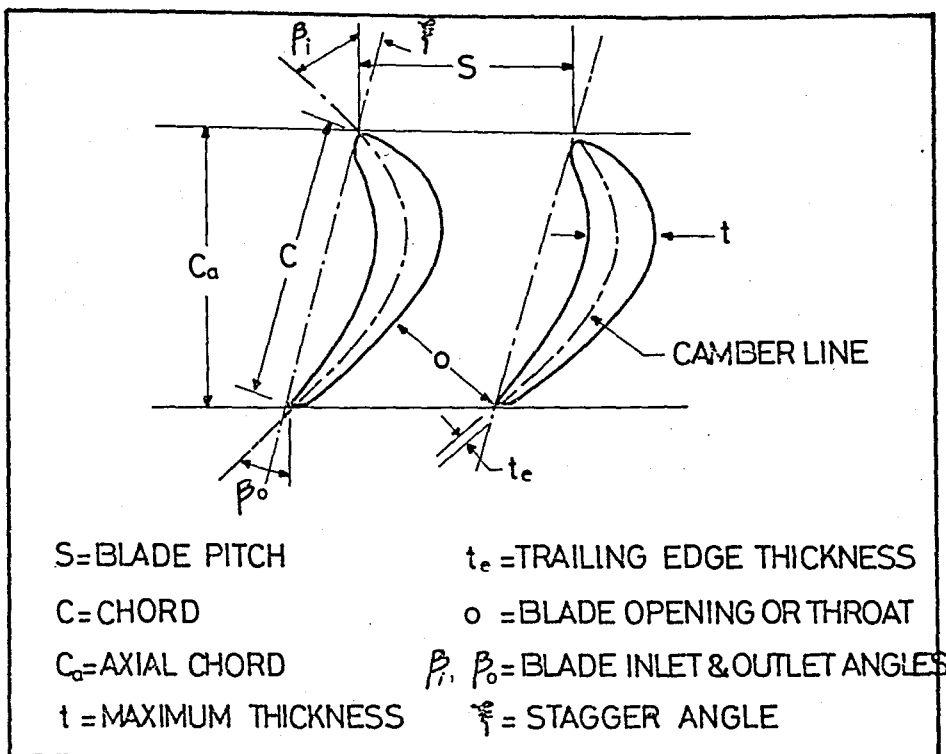


FIGURE 3.1 - Turbine blade nomenclature.

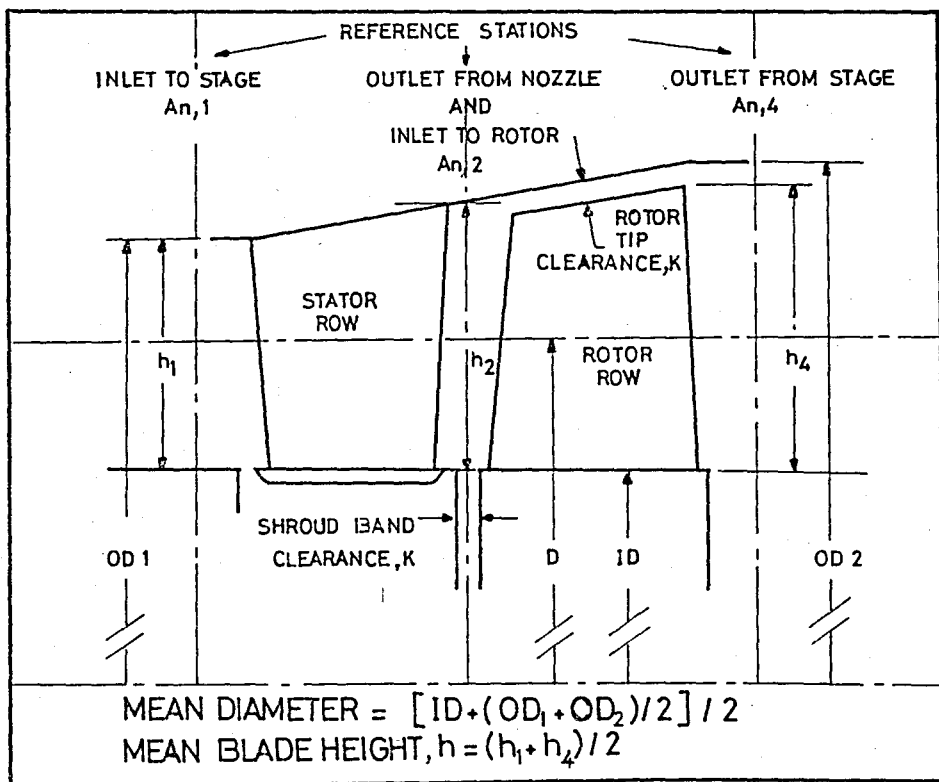


FIGURE 3.2 - Typical turbine stage showing choice of reference stations and reference heights.

and

$P_{0,s}$ = outlet static pressure from the stage

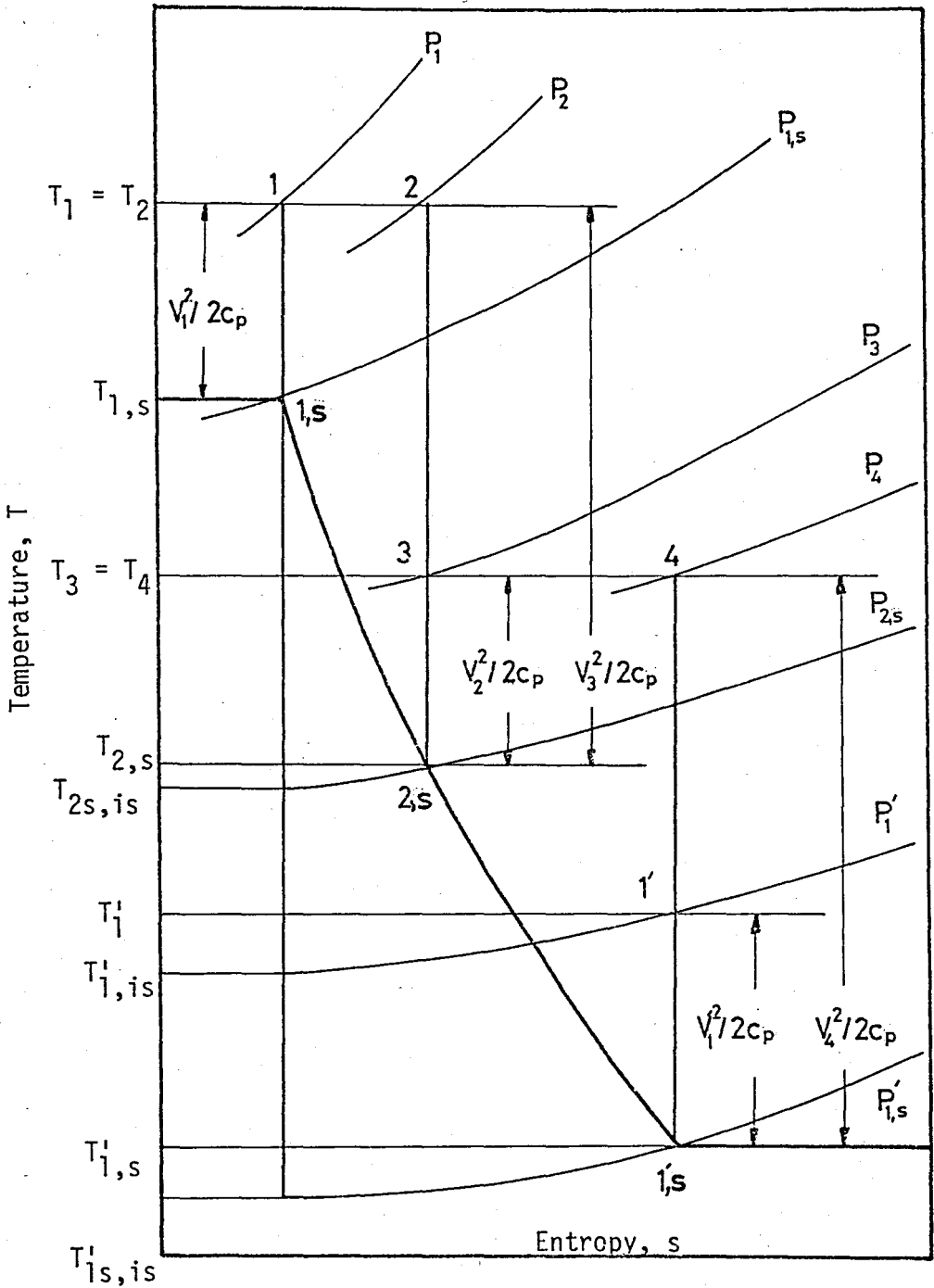


FIGURE 3,3 - Temperature-entropy diagram for a reaction stage.

The overall mean stagnation pressure loss coefficient, Y_t , may be split down into

- i) The profile loss coefficient, Y_p , associated with boundary layer growth over the blade profile,
- ii) The secondary loss coefficient, Y_s , associated with the turning of the boundary layer on the end walls of the blade and the related separation of flow
- iii) The loss coefficient associated with blade tip clearance, Y_k ,
- and iv) The trailing edge loss coefficient, Y_{te} , caused by the trailing edge thickness of the blade.

3.1.2.1 Determination of Profile Loss Coefficient

In a turbine blade passage, friction losses are caused by the interaction of the flowing medium and the blade walls. A thin sheet of decelerated fluid, the boundary layer, is interposed between the blade surface and the flowing medium and its thickness increases with flow path length. Thus, the average flow in the cascade is slightly accelerated due to the boundary layer displacement thickness and a momentum loss results. The growth of the displacement thickness and the resulting momentum loss is strongly affected by the adverse pressure gradients in direction of flow. Strong adverse pressure gradients are typical for the suction surface of decelerating cascades. Actually a critical pressure gradient exists which causes boundary layer separation. As long as the flow is not separated, it is possible

to calculate the boundary layer displacement thickness and hence the momentum (profile) loss, either from known pressure gradient or experimentally, from cascade data.

In the Ainley and Mathieson loss system [1], the profile loss coefficient, Y_p , is given as a function of the incidence, i , of the blade, expressed as:

$$i = -(\beta_i - \alpha_i) \quad (5)$$

where

α_i = gas inlet angle to blade, as shown in Fig. 3.4,
also, it is a function of the gas outlet angle, α_o , from the blade.

First, the profile loss is determined at zero incidence, then, the stalling incidence, i_s , of the blade row is determined; stalling incidence being defined as the incidence at which the profile loss is twice the loss at zero incidence.

Profile loss at zero incidence, $Y_{p(i=0)}$, is given by, [1],

$$Y_{p(i=0)} = \{Y_{p(\beta_i=0)} + (\beta_i/\alpha_o)^2 [Y_{p(\beta_i=-\alpha_o)} - Y_{p(\beta_i=0)}]\} \\ * ((t_e/c)/.2)^{-\beta_i/\alpha_o} \quad (6)$$

where

$Y_{p(\beta_i=0)}$ = profile loss coefficient of a nozzle blade
($\beta_i=0$) of same α_o and pitch-per-chord ratio,
(s/c), as the actual blade (Fig. 3.5)

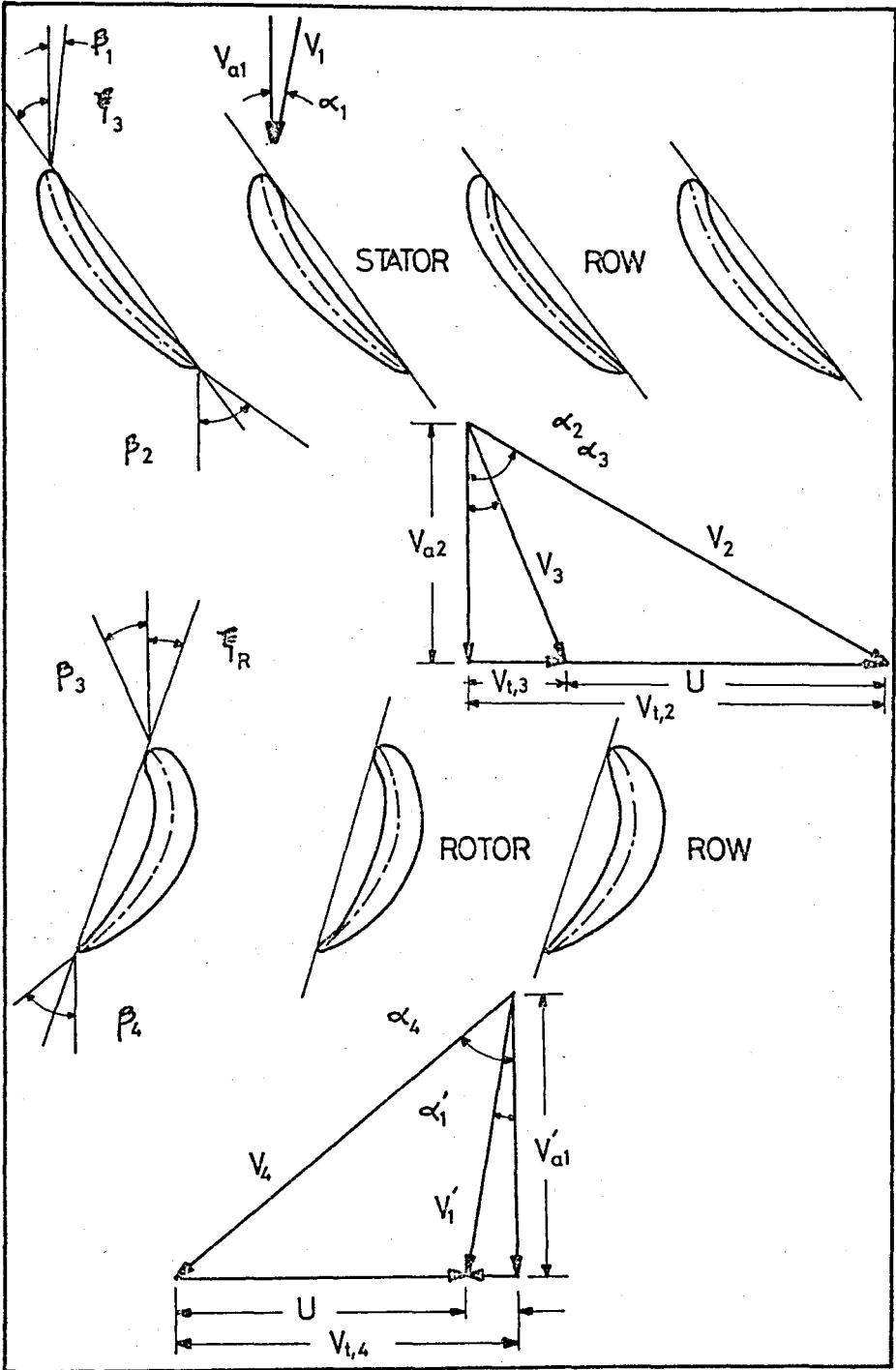


FIGURE 3.4 - Velocity triangles for an axial-flow turbine stage.

$Y_{p(\beta_i = -\alpha_0)}$ = profile loss coefficient of a zero-reaction blade (impulse blade) of same α_0 and s/c as the actual blade (Fig. 3.6)

c = blade chord.

Once the loss coefficient at zero incidence is determined, the value of the stalling incidence, i_s , is needed in order to calculate the actual value of the profile loss coefficient. It is assumed in [1] that the ratio of profile loss at any incidence to profile loss at zero incidence, $Y_p/Y_{p(i=0)}$, is a function of the ratio of actual incidence to stalling incidence, i/i_s . This relationship is given in Fig. 3.7.

The stalling incidence can be determined as follows, [1]

- i) First, the outlet angle for a blade having $s/c = .75$ is determined. Knowing the actual outlet angle, α_0 , and the actual s/c , [1] gives

$$\alpha_{0(s/c=.75)} = \alpha_0 / [-0.35(s/c) + 1.25] \quad (7)$$

- ii) The stalling incidence for a blade of $s/c = .75$, $i_{s(s/c=.75)}$, is determined. This is a function of α_0 and $\beta_i/\alpha_{0(s/c=.75)}$, as shown in Fig. 3.8 [1].
- iii) To calculate the actual stalling incidence of the blade, Δi_s is needed. This is a function of s/c and α_0 [1], and is given in Fig. 3.9.

The actual stalling incidence can be derived from

$$i_s = i_{s(s/c = 0.75)} + \Delta i_s \quad (8)$$

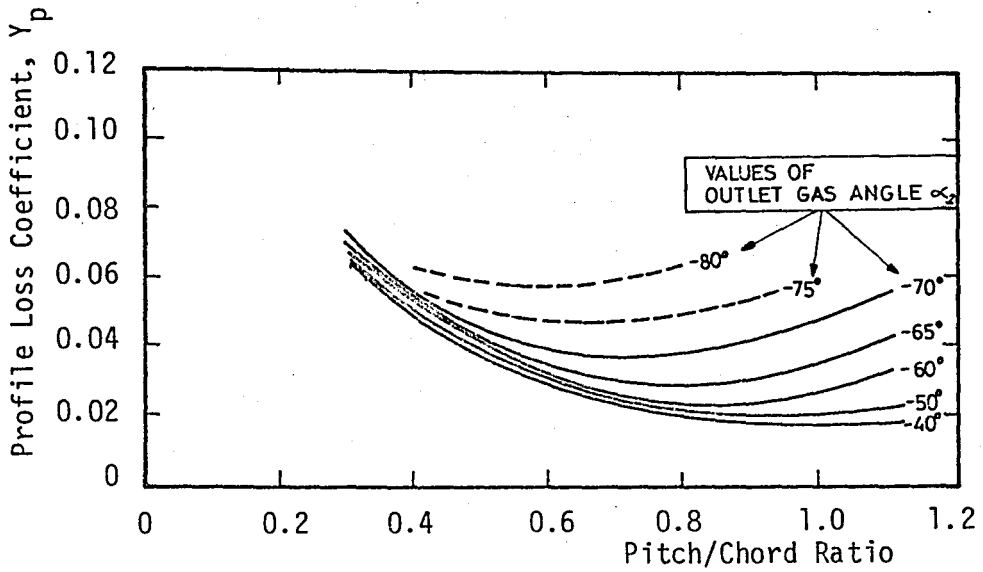


FIGURE 3.5 - Profile loss coefficient for nozzle blades ($\beta_i = 0$).

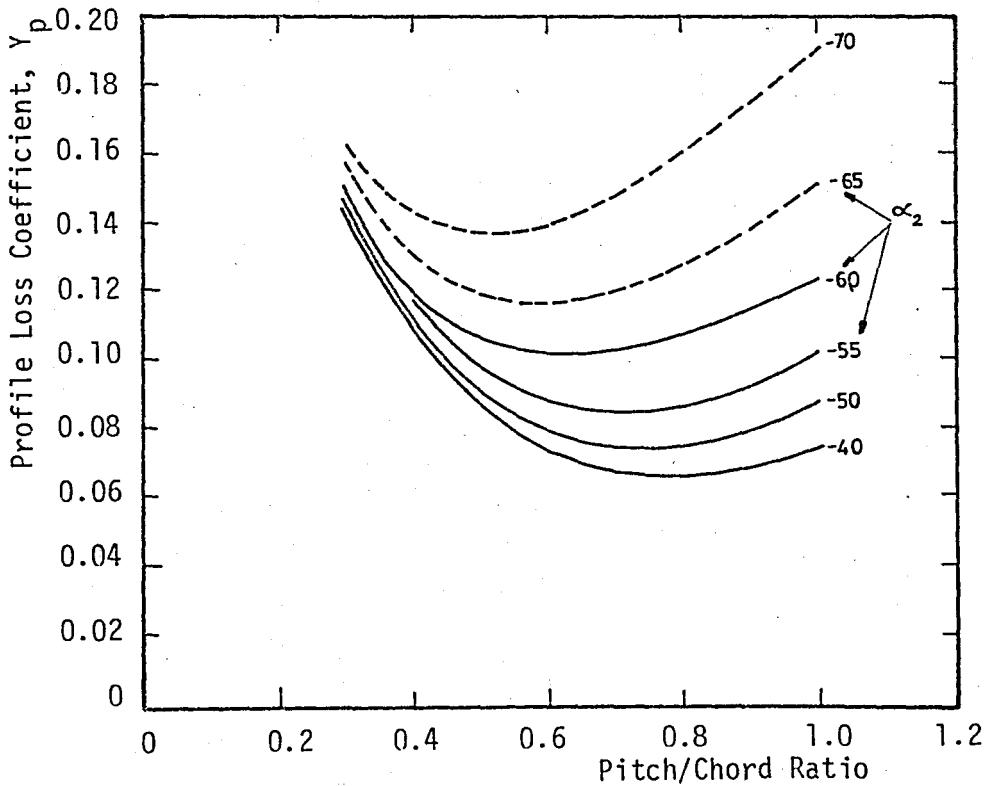


FIGURE 3.6 - Profile loss coefficients for impulse blades ($\beta_i = -\alpha_0$).

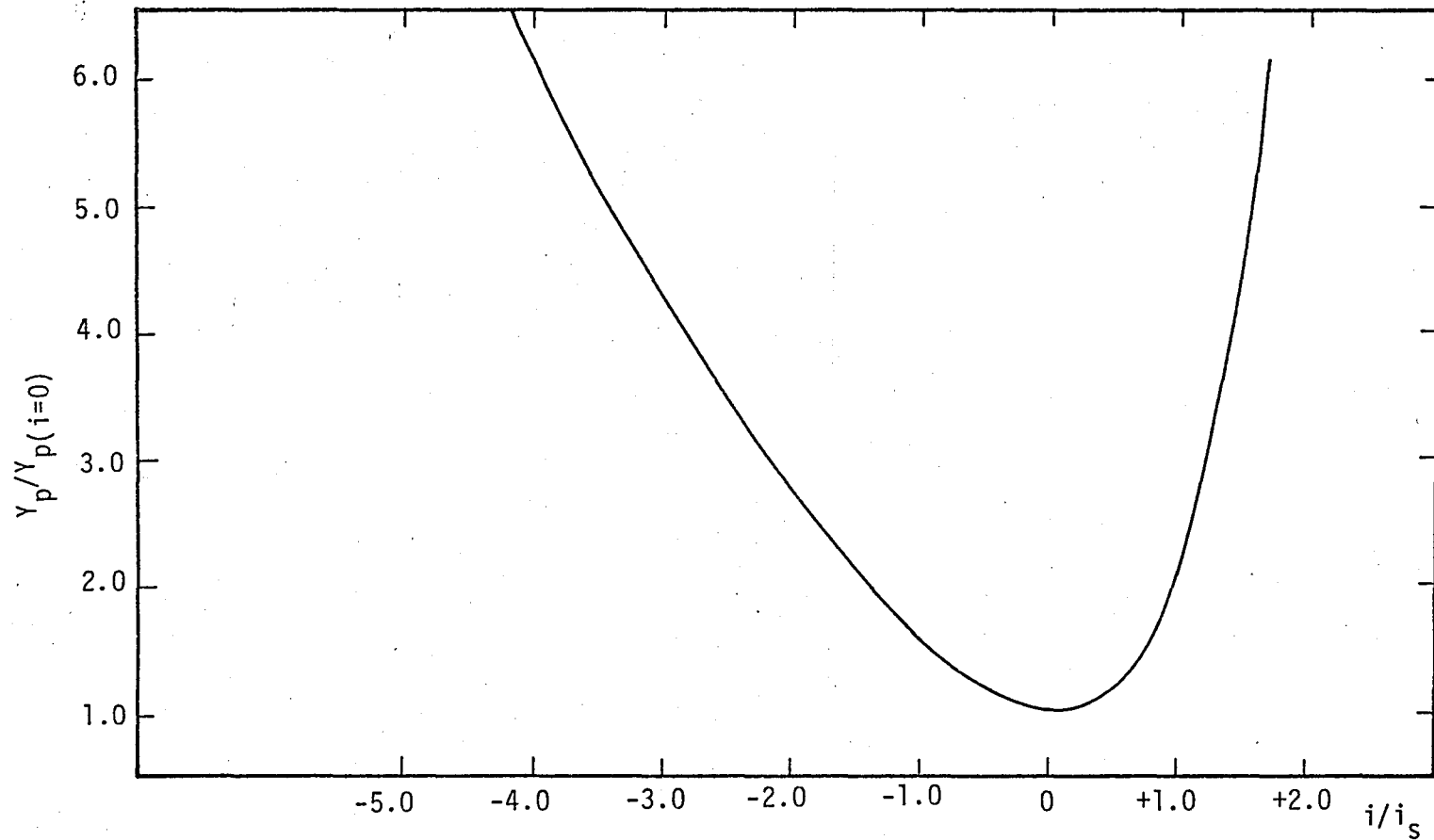


FIGURE 3.7 - Variation of the profile loss with incidence.

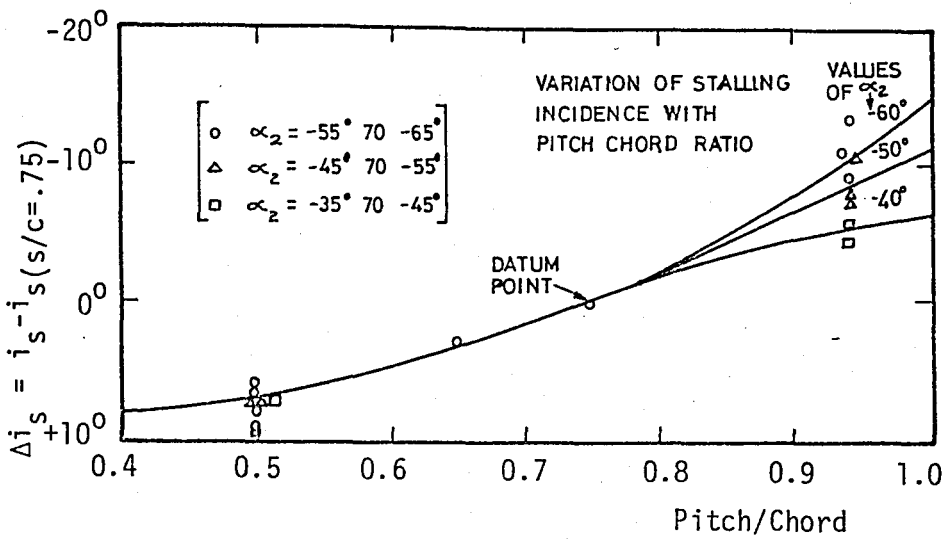


FIGURE 3.8 - Variation of the stalling incidence with pitch-per-chord ratio.

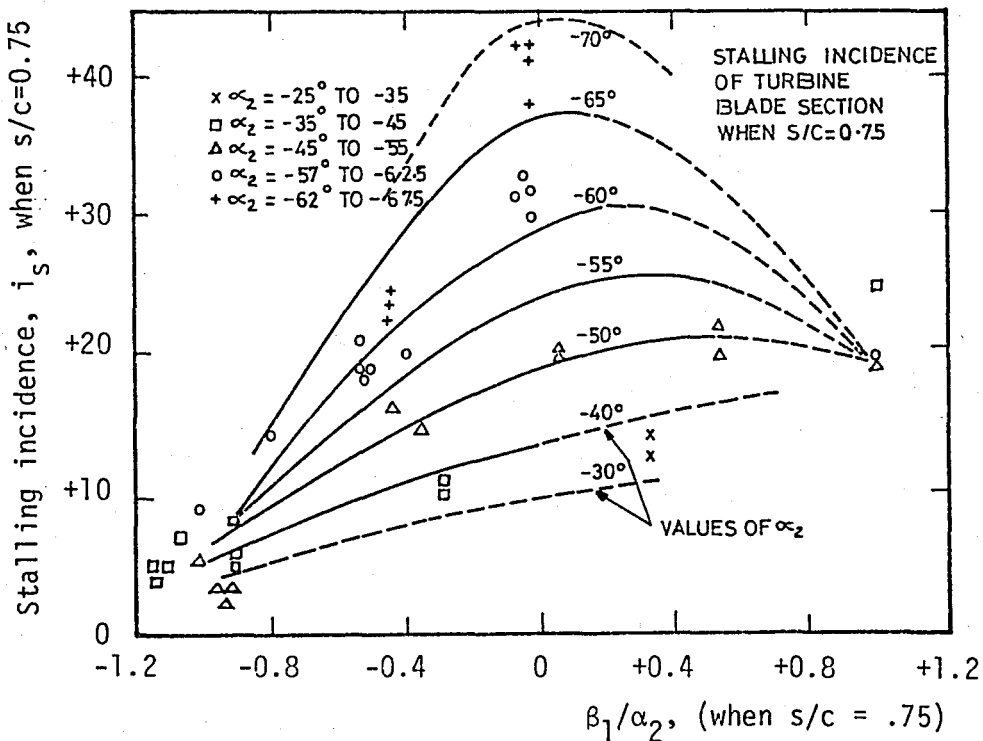


FIGURE 3.9 - Stalling incidence of turbine blade sections when pitch-per-chord ratio equals .75.

- iv) Having now determined i_s and $Y_{p(i=0)}$, the profile loss coefficients over a wide range of incidence can be deduced from Fig. 3.7 [1].

3.1.2.2 Determination of Secondary and Tip Clearance Loss Coefficients

The secondary losses arise from the interaction between the 2-dimensional flow through the blade passage and the endwall boundary layer. The boundary layer at the annulus walls is deflected through a blade row and a secondary vortex is superimposed on the two-dimensional flow through the passage. This secondary flow is directed from the pressure side to the suction side of the blade near the top and bottom walls and in the opposite direction in the middle of the channel. The secondary losses, therefore depend on the blade loading, the blade shape (aspect ratio) and the endwall boundary layer.

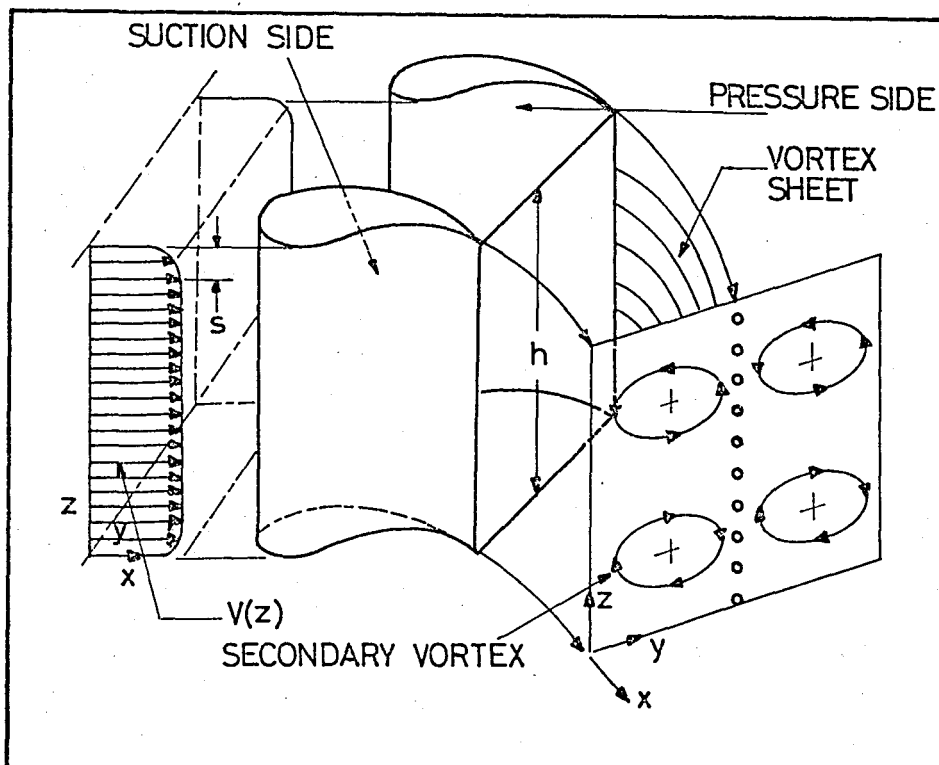


FIGURE 3.10 - Formation of secondary flow in a turbine blade passage.

Several empirical formulas have been suggested for the effect of blade loading (Sec. 2), one that fits best to experimental data being that due to Ainley and Mathieson [1].

The blade loading parameter of Ainley and Mathieson [1], Z , is in the form:

$$Z = \left(\frac{C_L}{s/c}\right)^2 (\cos^2 \alpha_o / \cos^3 \alpha_m) \quad (9)$$

where

$$C_L/(s/c) = 2(\tan \alpha_i - \tan \alpha_o) \cos \alpha_m,$$

and

$$\alpha_m = \tan^{-1} \{(\tan \alpha_i + \tan \alpha_o)/2\}.$$

Ainley and Mathieson relate the secondary loss coefficient, Y_s , to the above defined blade loading parameter, Z , as [1]

$$Y_s = \lambda Z, \quad (10)$$

where λ is an empirical function given as [1]

$$\lambda = f\left\{\left(\frac{A_{n,o} \cos \alpha_o}{A_{n,i} \cos \beta_i}\right)^2 / (1 + ID/OD)\right\}, \quad (11)$$

where

$A_{n,i}$ = annulus area upstream of row, as shown in Fig. 3.2.

The functional relationship of λ , expressed in Eq. (11) is shown in Fig. 3.11.

Equation (11) has been corrected later by Dunham and Came [2]. Their correlation represents a significant improvement over that of Ainley and Mathieson [1], particularly for blades of low aspect ratio

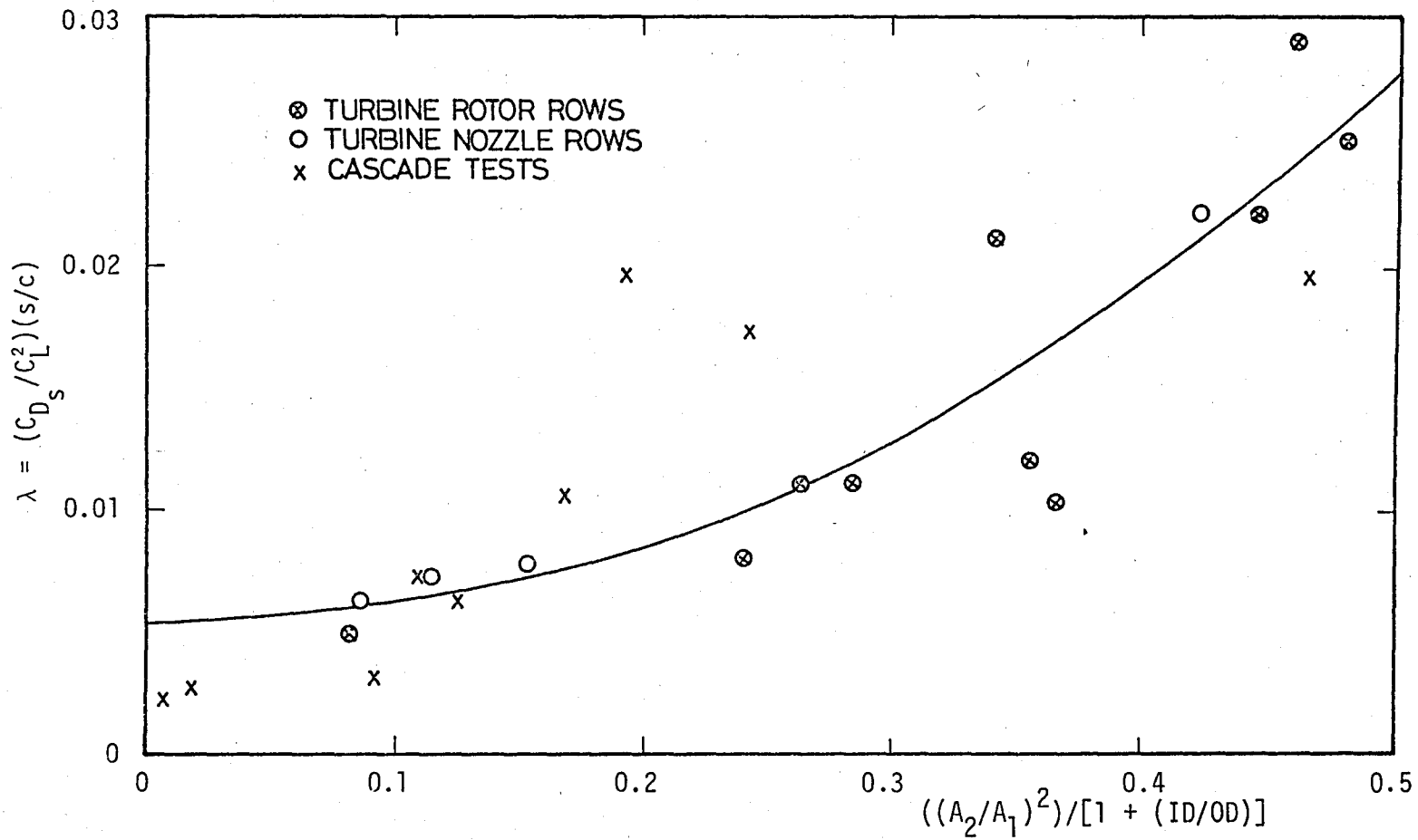


FIGURE 3.11 - Secondary losses in turbine blade rows.

(c/h) , as encountered in small turbines. Although for more advanced methods of performance prediction, it would be necessary to calculate the wall boundary layer development, Dunham and Came [2] avoided this complication by the adoption of a single constant, in place of λ ,

$$Y_s = 0.0334(c/h)(\cos\alpha_0/\cos\beta_i)Z \quad (12)$$

where, as shown in Fig. 3.1 and Fig. 3.2,

c/h = aspect ratio.

If there is a clearance between the tip of the blade and the side wall, there occurs a flow through the clearance directed from the pressure side towards the suction side. The intensity of this leakage flow through the clearance is proportional to the pressure difference between both sides of the blade, and thus proportional to the deflection [25]. The tip clearance loss coefficient, Y_k , depends consequently on the blade loading [1] and the size and nature of the clearance. Examination of both cascade and turbine data confirmed that Ainley and Mathieson's [1] blade loading parameter, Z , represents the effect of blade loading on clearance losses, satisfactorily.

In the method of Ainley and Mathieson, Y_k is expressed as [1],

$$Y_k = B(k/h)Z \quad (13)$$

where the constant, B , accounts for the type of the clearance and

$B = 0.5$ for plain tip clearance

$B = 0.25$ for shrouded tips.

This correlation has been corrected by Dunham and Came [2], according to Hubert's cascade data [26], in the following way

$$Y_k = B(c/h)(k/c)^{.78} Z \quad (14)$$

where $B = 0.47$ for plain tip clearance

$B = 0.37$ for shrouded tips.

3.1.2.3 Determination of Total Loss Coefficient and the Effect of Trailing Edge Thickness

In the Ainley and Mathieson Loss System [1], the profile, secondary and clearance loss coefficients are simply added to yield the total pressure loss coefficient, Y_t

$$Y_t = Y_p + Y_s + Y_k \quad (15)$$

This procedure applies to "conventional" blades [18], having a trailing edge thickness, t_e , roughly equal to 2% of the blade pitch, s . If the ratio of trailing edge thickness to blade pitch, t_e/s , differs from 0.02, then the total loss coefficient should be corrected by a factor, Y_{te} , [1]

$$Y_t = [Y_p + Y_s + Y_k] * Y_{te} \quad (16)$$

where, the trailing edge loss coefficient, Y_{te} , is defined as

$$Y_{te} = 21.6(t_e/s)^2 + 3.86(t_e/s) + 0.91 \quad (17)$$

and

t_e = blade trailing edge thickness, as shown in Fig. 3.1.

3.1.3 Determination of Gas Conditions

Once the total loss coefficient related to a particular blade is determined, the performance of the stage at particular entry flow conditions and at particular blade speed is determined. The calculations are then repeated for different values of these parameters and the overall performance map is built up. The method explained in this section is completely based on the system of Ainley and Mathieson [1]. It is assumed that the turbine is single stage and that the nozzle operates at zero incidence. The method can easily be adopted for use in a multistage turbine, the incidence onto the nozzle of any stage being calculated from the flow at exit from the previous stage.

3.1.3.1 Determination of Gas Flow Conditions at Outlet from a Nozzle Row

The procedure for determining the outlet gas conditions from the known inlet conditions and the calculated loss coefficient, Y_t , is outlined as follows.

- i) Since the Mach number at outlet, M_2 , is initially unknown, a preliminary guess for α_2 is obtained from Eq. (1) on the basis of geometric parameters.
- ii) The non-dimensional inlet mass flow, $\dot{m} \sqrt{T_1} / A_2 P_1$, is calculated from inlet conditions, where
 - \dot{m} = turbine inlet mass flow
 - T_1 = absolute total gas temperature at inlet to first nozzle row

P_1 = absolute total gas pressure at inlet to first nozzle row

$$A_2 = A_{n,2} \cdot \cos \alpha_2$$

- iii) It is assumed that the maximum non-dimensional mass flow that can be passed through a specified blade passage, $(\dot{m}\sqrt{T_i}/A_0 P_i)_{\max}$, is a function of the total loss coefficient, Y_t . This relationship is plotted in Fig. 3.12.

The mean pressure drop within the blade passage is expressed in terms of a non-dimensional quantity, $\bar{\omega}/P_i$, where

$\bar{\omega}$ = mean loss of total pressure through a blade row due to friction and flow separation.

The value of $\bar{\omega}/P_i$ at the point where the non-dimensional mass flow reaches its maximum value, is termed the critical pressure drop ratio, $(\bar{\omega}/P_i)_{\text{crit}}$, and is a function of the total loss coefficient, Y_t . This relation is plotted in Fig. 3.13.

Therefore, the values of $(\dot{m}\sqrt{T_i}/A_2 P_i)_{\max}$ and $(\bar{\omega}/P_i)_{\text{crit}}$ for given Y_t is found by referring to Fig. 3.12 and Fig. 3.13, respectively.

- iv) Over the range of outlet Mach number $0 < M_o < 1.2$ and total loss coefficient $0 \leq Y_t < 1$, the actual ratio of pressure loss to inlet pressure, $\bar{\omega}/P_i$, can be related to the actual non-dimensional mass flow, $\dot{m}\sqrt{T_i}/A_0 P_i$, by a single curve, plotting $(\bar{\omega}/P_i)/(\bar{\omega}/P_i)_{\text{crit}}$ against $(\dot{m}\sqrt{T_i}/A_0 P_i)/(\dot{m}\sqrt{T_i}/A_0 P_i)_{\max}$. This relationship is given in Fig. 3.14.

Thus, from the above determined values of $(\bar{\omega}/P_1)_{\text{crit}}$, $(\dot{m}\sqrt{T_1}/A_2P_1)_{\text{max}}$ and $\dot{m}\sqrt{T_1}/A_2P_1$, one can find the corresponding value of actual $\bar{\omega}/P_1$ and from this, the absolute total pressure, P_2 , of gas at outlet of the stator can be found from:

$$P_2 = P_1 - \bar{\omega} \quad . \quad (18)$$

Then, the non-dimensional mass flow at outlet of stator, $\dot{m}\sqrt{T_2}/A_2P_2$, can be calculated, knowing that for an uncooled nozzle, $T_2 = T_1$, where T_2 is the total absolute gas outlet temperature from nozzle (Fig. 3.3).

- v) From the value of $\dot{m}\sqrt{T_2}/A_2P_2$, the value of M_2 and $V_2/\sqrt{T_2}$, where V_2 is the absolute gas outlet velocity from stator, can be deduced. The curves represented on Fig. 3.15 and Fig. 3.16, respectively, should be used for this purpose.
- vi) It is stated in Section 3.1.1 that the gas outlet angle from a blade row, α_0 , is a known function of outlet Mach number, M_0 . Therefore, one can compare the value of α_2 corresponding to M_2 found in (v) and the initial value of α_2 guessed in (i). If the two values differ from each other by more than a given tolerance, the new value of α_2 is taken as the improved guess and the procedure is repeated.

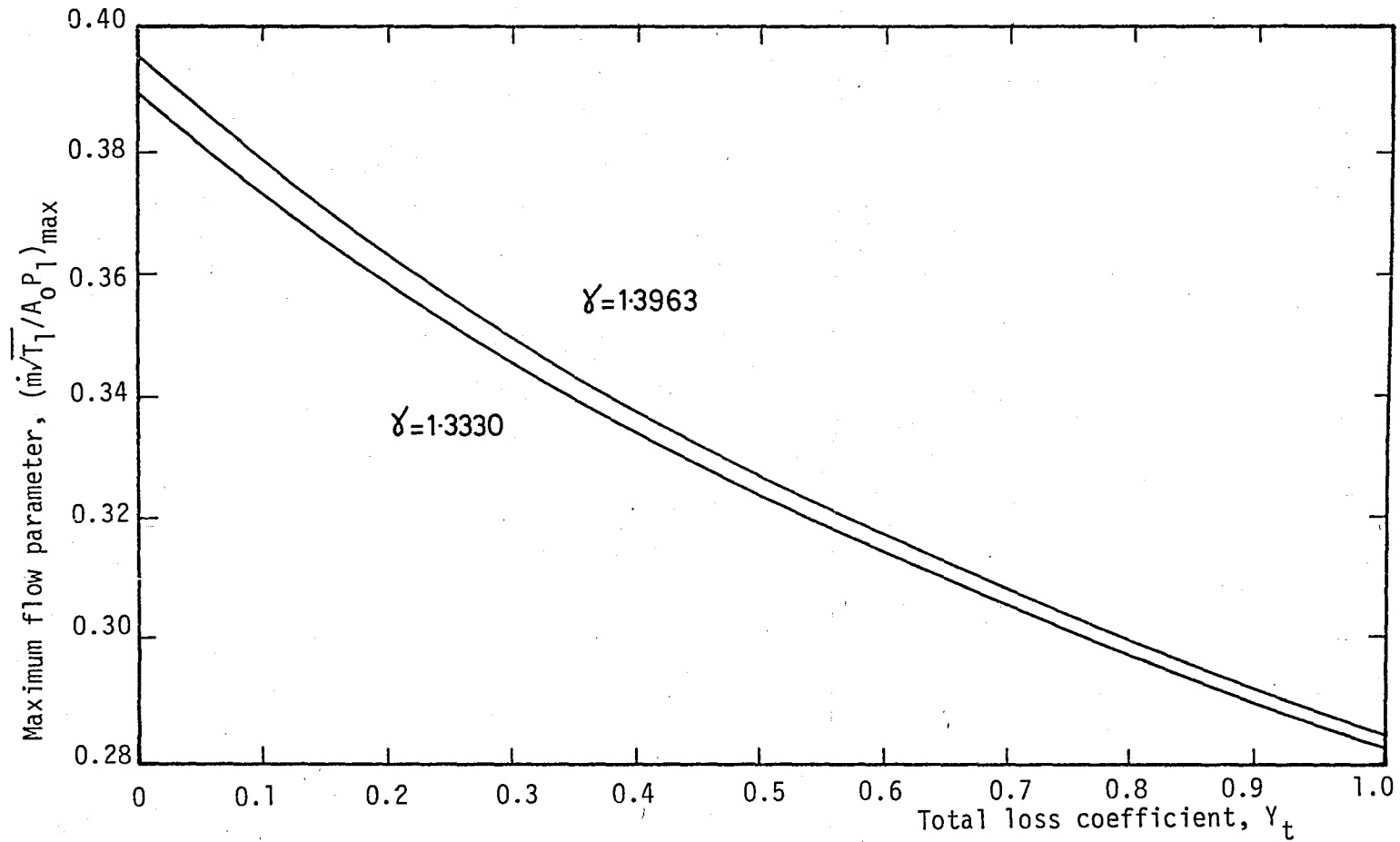


FIGURE 3.12 - Fundamental relation for compressible adiabatic flow.

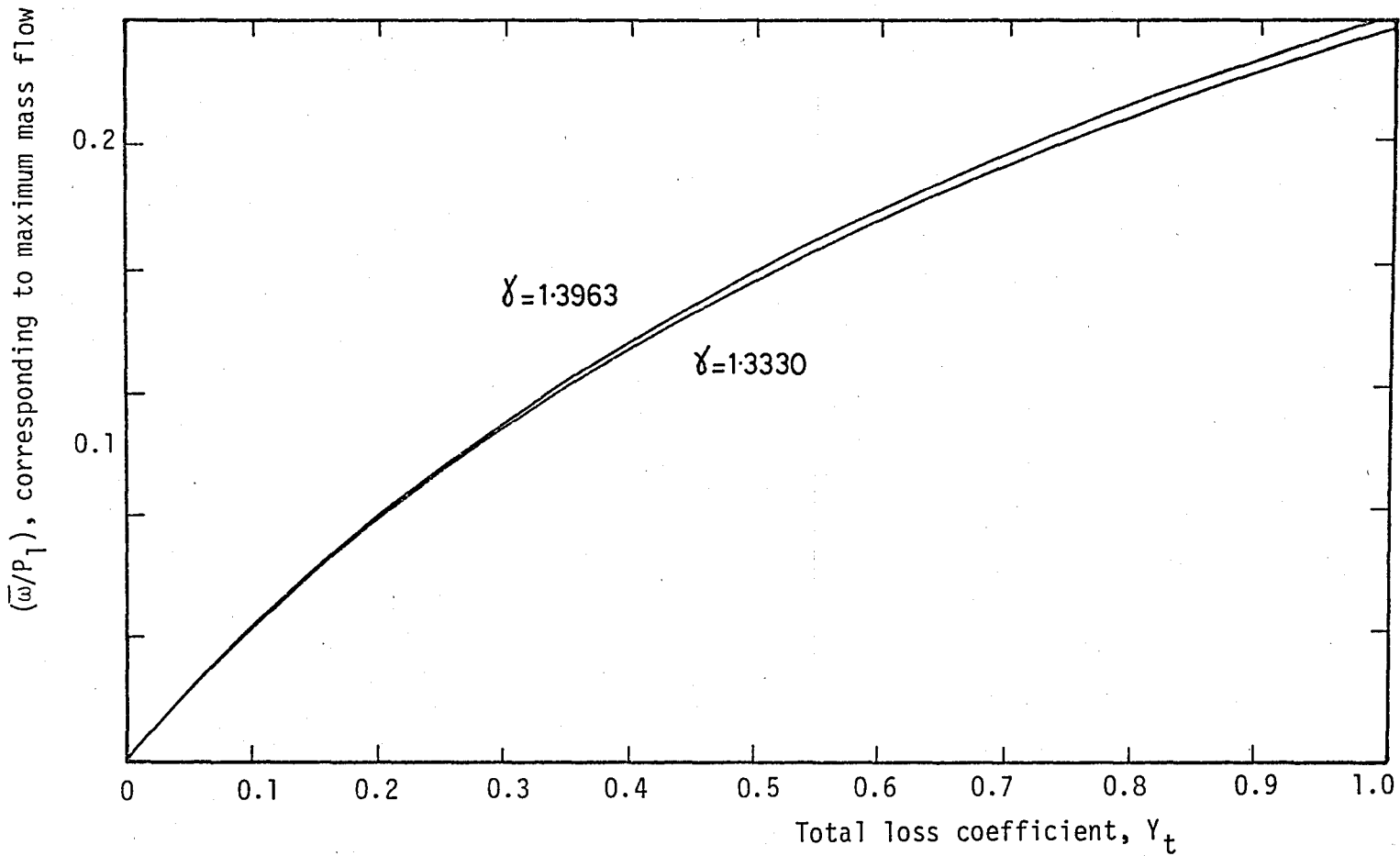


FIGURE 3.13 - Fundamental relation for compressible adiabatic flow.

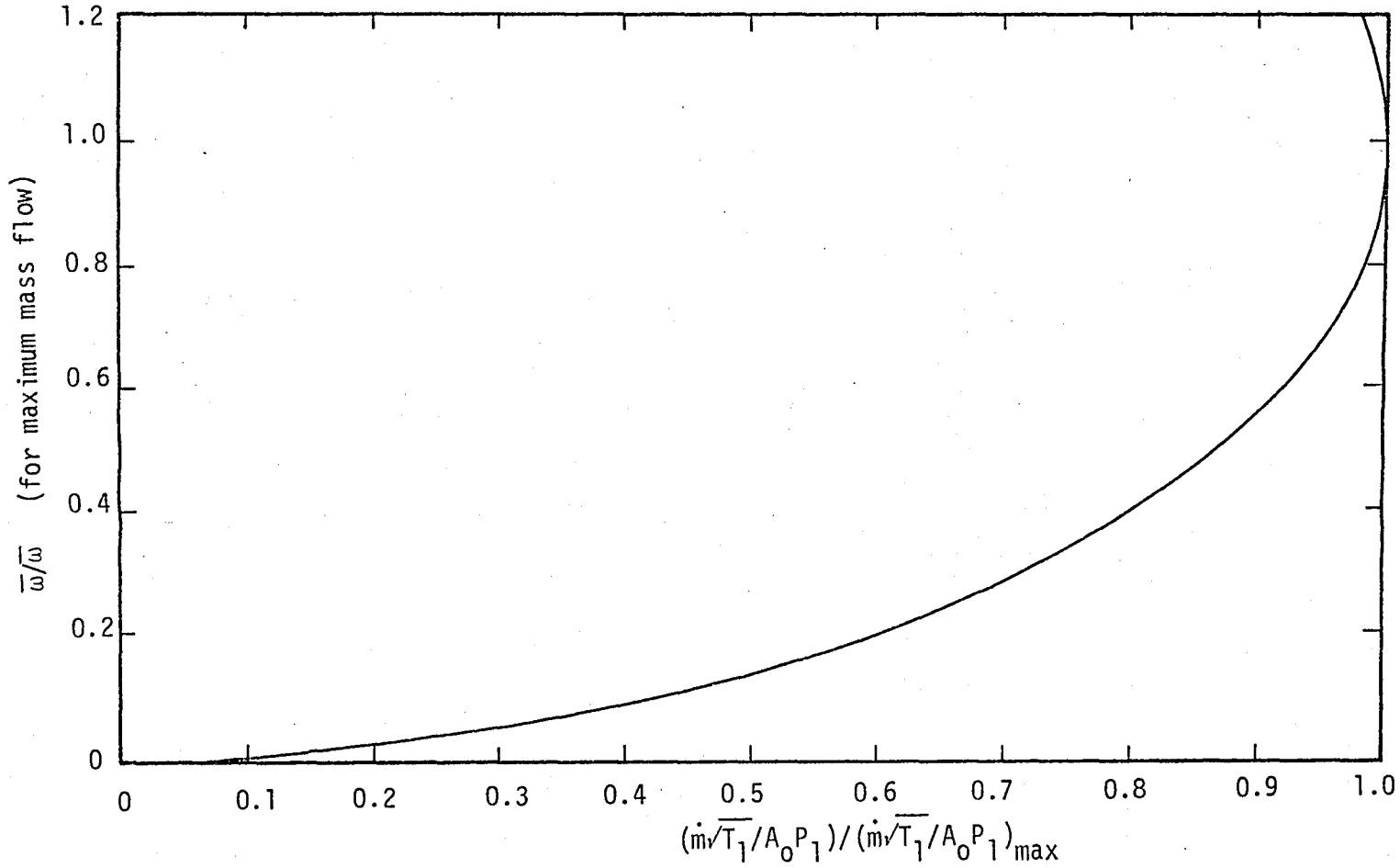


FIGURE 3.14 - Fundamental relation for compressible adiabatic flow.

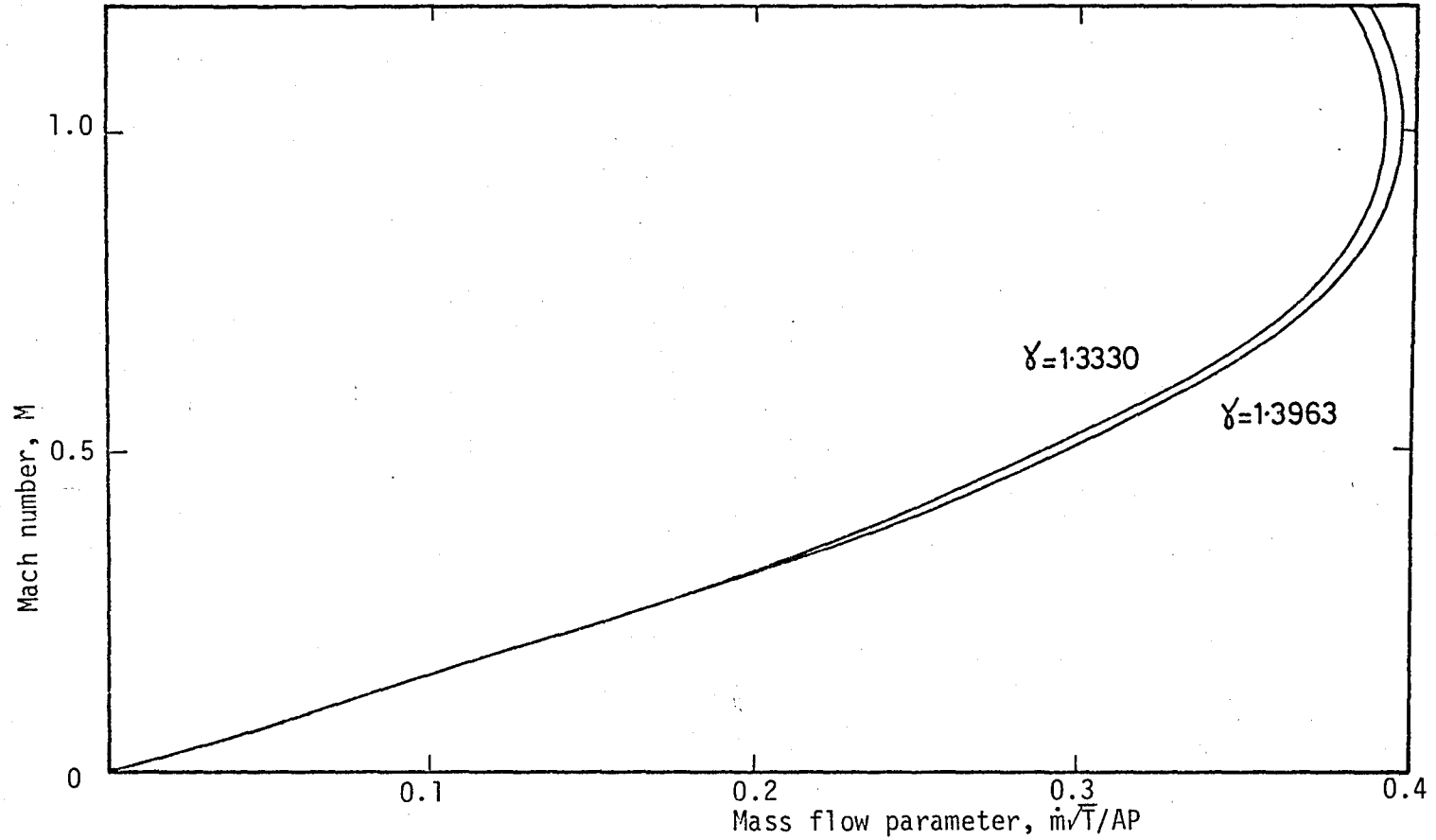


FIGURE 3.15 - Fundamental relation for compressible isentropic flow.

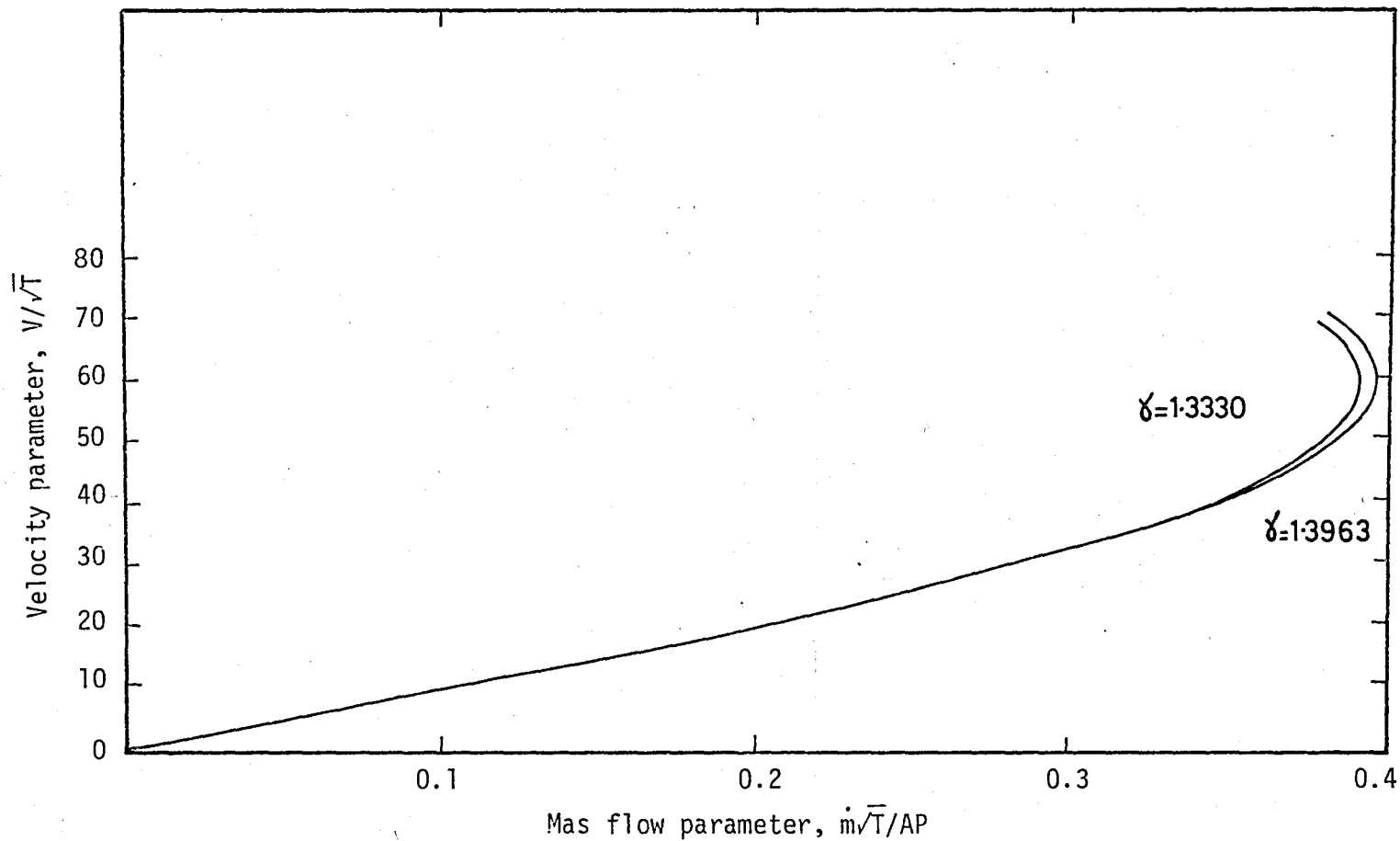


FIGURE 3.16 - Fundamental relation for compressible isentropic flow.

3.1.3.2 Determination of Gas Flow Conditions Relative to Rotor at Inlet

Knowing the gas velocity, flow angle from the nozzle row and the blade speed at the reference diameter, the gas conditions relative to the inlet of the rotor can be determined from the velocity triangles (Fig. 3.4)

$$V_{a,3} = V_2 \cos\alpha_2 \quad , \quad (19)$$

and

$$\alpha_3 = \tan^{-1}[(U/V_{a,3}) - \tan\alpha_2] \quad , \quad (20)$$

where

$V_{a,3}$ = axial component of gas velocity at rotor inlet,

U = rotor blade speed at reference diameter,

α_3 = relative gas inlet angle to rotor.

Also from velocity triangles,

$$V_3 \cos\alpha_3 = V_2 \cos\alpha_2 \quad . \quad (21)$$

Thus,

$$V_3/V_2 = \cos\alpha_2/\cos\alpha_3 \quad , \quad (21.a)$$

where

V_3 = relative gas inlet velocity to rotor.

The energy equation gives:

$$T = T_s + V^2/2C_p \quad (22)$$

where

C_p = specific heat of gas at constant pressure.

From Eq. (22) and from the fact that the static temperature at exit from the nozzle is equal to the static temperature relative to the rotor entry (i.e. $T_{s,2} = T_{s,3}$) the total head temperature, T_3 , relative to the rotor inlet can be found:

$$T_2 = T_{s,2} + V_2^2/2C_p \quad (23)$$

$$T_3 = T_{s,3} + V_3^2/2C_p \quad (24)$$

$$T_3 = T_{s,2} + V_3^2/2C_p \quad (25)$$

From Eqs. (23) and (25)

$$T_2 - T_3 = (V_2^2 - V_3^2)/2C_p \quad (26)$$

Combining Eqs. (26) and (21.a) and rearranging,

$$T_3 = T_2 + V_2^2[1 - (\cos^2\alpha_2/\cos^2\alpha_3)]/2C_p \quad (27)$$

Having found T_3 , total head pressure relative to rotor at inlet, P_3 , may be deduced from:

$$P_3 = P_2 \left[1 - \left(\frac{T_1 - T_2}{T_1} \right) \right]^{\gamma/\gamma-1}, \quad (28)$$

where

γ = ratio of specific heats for the gas.

3.1.3.3 Determination of Gas Flow Conditions Relative to Rotor at Outlet

The flow is calculated along an axial line at constant diameter, and no heat is transferred to or from the blades. Therefore,

there must be no changes of total gas temperature relative to the rotor, i.e. $T_4 = T_3$, where T_4 is the total head temperature relative to rotor at outlet.

The procedure to be followed is given as:

- i) From known rotor inlet gas angle, α_3 (Eq. (20)), and inlet blade angle, β_3 , the value of the incidence on rotor can be determined from Eq. (5). Using the incidence, rotor total loss coefficient, $Y_{t, \text{rotor}}$, can be found as explained in Section 3.1.2.
- ii) The value of $\dot{m}\sqrt{T_3}/A_4 P_3$ is calculated. This is the non-dimensional mass flow at inlet to rotor, where

$$A_4 = A_{n,4} \cos \alpha_4 \quad (29)$$

where

$A_{n,4}$ = Annulus area downstream the rotor row

α_4 = relative gas outlet angle from rotor.

Since the value of rotor outlet Mach number, M_4 , is initially unknown, a preliminary guess for rotor outlet gas angle, α_4 , is obtained from Eq. (1), on the basis of the geometric parameters.

- iii) From the value of $Y_{t, \text{rotor}}$, values of $\dot{m}\sqrt{T_3}/A_4 P_3)_{\text{max}}$ and $(\bar{\omega}/P_3)_{\text{crit}}$ may be found from Fig. 3.12 and Fig. 3.13, respectively.

- iv) The ratio $(\dot{m}\sqrt{T_3}/A_4P_3)/(\dot{m}\sqrt{T_3}/A_4P_3)_{\max}$ is calculated, the corresponding value for $(\bar{\omega}/P_3)/(\bar{\omega}/P_3)_{\text{crit}}$ is found from Fig. 3.14 and finally from the known $(\bar{\omega}/P_3)_{\text{crit}}$, the actual $\bar{\omega}/P_3$ is deduced.

Since P_3 is known, the total gas pressure relative to rotor at outlet, P_4 , can be derived from:

$$P_4 = P_3 - \bar{\omega} \quad (30)$$

The magnitude of rotor outlet non-dimensional mass flow, $\dot{m}\sqrt{T_4}/A_4P_4$, can now be calculated, and from Fig. 3.16 and Fig. 3.15, the corresponding values of $V_4/\sqrt{T_4}$ and M_4 may be found, where,

V_4 = gas velocity relative to rotor at outlet,

and M_4 = outlet Mach number from rotor.

- v) As before, if the value of α_4 corresponding to the value of M_4 found in (iv), differs from the value approximated in (i), the process is repeated using the latest value of α_4 .

3.1.3.4 Determination of Absolute Gas Flow Conditions at Outlet from Rotor

From the velocity triangles shown in Fig. 3.4

$$V_{a,4} = V_4 \cos \alpha_4 \quad (31)$$

$$\alpha_1' = \tan^{-1} [(U/V_{a,4}) - \tan \alpha_4] \quad (32)$$

where

$V_{a,4}$ = axial component of gas velocity relative to rotor
at outlet,

and α_1' = absolute gas outlet angle from rotor, or
absolute gas inlet angle to the next stator.

From the energy equation (Eq. (22)),

$$T_1' = T_4 - V_4^2 [1 - (\cos^2 \alpha_4 / \cos^2 \alpha_1')] / 2C_p \quad (33)$$

and

$$P_1' = P_4 \left[1 - \left(\frac{T_3 - T_4}{T_3} \right) \right]^{\gamma / \gamma - 1} \quad (34)$$

where, as shown in Figure 3.3,

T_1' = absolute total gas temperature at rotor outlet, or
absolute total gas temperature at inlet to next stator

P_1' = absolute total gas pressure at rotor outlet, or
absolute total gas pressure at inlet to next stator.

3.1.4 Determination of Turbine Overall Characteristics

The overall stage pressure ratio, P_1'/P_1 and the temperature drop ratio, $(T_1 - T_1')/T_1$, corresponding to the initially selected value of non-dimensional mass flow at inlet, $\dot{m} \sqrt{T_1}/P_1$, may now be determined.

The overall stage isentropic efficiency, η , is then given by

$$\eta = [(T_1 - T_1')/T_1] / [1 - (P_1'/P_1)^{\gamma - 1 / \gamma}] \quad (35)$$

The magnitudes of pressure and temperature drop ratio and the isentropic efficiency for different values of non-dimensional mass flow can be calculated to construct a performance characteristic.

The process may be extended by choosing several blade speeds for the turbine and by constructing a constant speed characteristic curve for each speed chosen.

3.1.5 Performance of a Choked or Nearly Choked Stage

Choking is assumed to occur in a turbine row when the non-dimensional mass flow, $\dot{m}\sqrt{T_1}/P_0 A_0$, in nozzle or rotor, attains its maximum value for the given total loss coefficient, Y_t , and blade outlet angle, α_0 . The flow upstream of the choked row then remains unchanged as the overall pressure ratio is increased, the flow downstream becoming supersonic and the gas direction changing slightly to preserve continuity. The flow may choke again downstream as the pressure ratio is further increased.

Ainley and Mathieson [1] suggest the following procedure for calculating the flow through a choked row, assuming that the flow quantity, $\dot{m}\sqrt{T_0}/P_0$, and the loss coefficient, Y_t , remain constant as the pressure ratio is reduced below that required to choke the blade passage.

- i) The value of outlet Mach number, M_0 , is arbitrarily selected.
- ii) Knowing M_0 , the values of non-dimensional flow parameter at blade outlet, $\dot{m}\sqrt{T_0}/A_0 P_0$, and the total to static pressure ratio, $P_{0,s}/P_0$, can be found referring to Fig.3.15 and Fig. 3.17 respectively.

iii) Now, from the definition of total loss coefficient (Eq. (4)), from known total to static pressure ratio, $P_{0,s}/P_0$, and from total loss coefficient, Y_t , which is unaltered, one gets

$$P_i - P_0 = Y_t [P_0 - P_{0,s}] \quad (36)$$

Dividing throughout by P_0 , and using Eq. (18), one gets,

$$\bar{\omega}/P_0 = Y_t [1 - P_{0,s}/P_0] \quad (37)$$

and

$$\bar{\omega}/P_i = (\bar{\omega}/P_0) / (1 + \bar{\omega}/P_0) \quad (38)$$

so that, finally,

$$P_0 = P_i - \bar{\omega}.$$

Knowing that $T_0 = T_i$, one can find from $\dot{m}\sqrt{T_0}/A_0 P_0$, the necessary value of A_0 . Since

$$A_0 = A_{n,0} \cos \alpha_0 \quad ,$$

the required value of blade outlet gas angle, α_0 , can be determined. Having thus determined the new outlet conditions relative to the row, the absolute conditions at the outlet can be calculated as before. This procedure is repeated until the outlet Mach number is equal to unity. Only those results for which the mass flow attains its maximum value and the outlet Mach number reaches unity are taken into consideration. Although the intermediate steps are necessary for predicting the exact value of the pressure ratio for which the turbine fully chokes (when $M_0 = 1$) and although they will appear on the output of the computer program, the data points corresponding to those intermediate

steps will not be shown on the final performance curves.

3.1.6 Effect of Reynolds Number

In reference [1], it is stated that the magnitude of the efficiency and turbine performance calculated using the data presented, would correspond to a mean operating Reynolds number of approximately 2×10^5 , the Reynolds number being defined by blade chord, outlet gas velocity (relative to the row), outlet gas density and outlet gas viscosity.

Thus, it is recommended in [1] that an approximate correction be applied to the overall characteristics by assuming that the turbine overall efficiency is affected by Reynolds number variations as

$$1 - \eta_{\text{corrected}} = (1 - \eta) + \left(\frac{\text{Re}_{,m}}{2 \times 10^5} \right)^{-0.2} \quad (39)$$

where

$\text{Re}_{,m}$ = Mean Reynolds number of stator and nozzle.

In the revised method of Dunham and Came [2], however, an optional correction is applied directly to the profile and secondary losses, using the Reynolds number appropriate to the particular blade row:

$$(Y_p + Y_s)_{\text{corrected}} = (Y_p + Y_s) \left(\frac{\text{Re}}{2 \times 10^5} \right)^{-0.2} \quad (40)$$

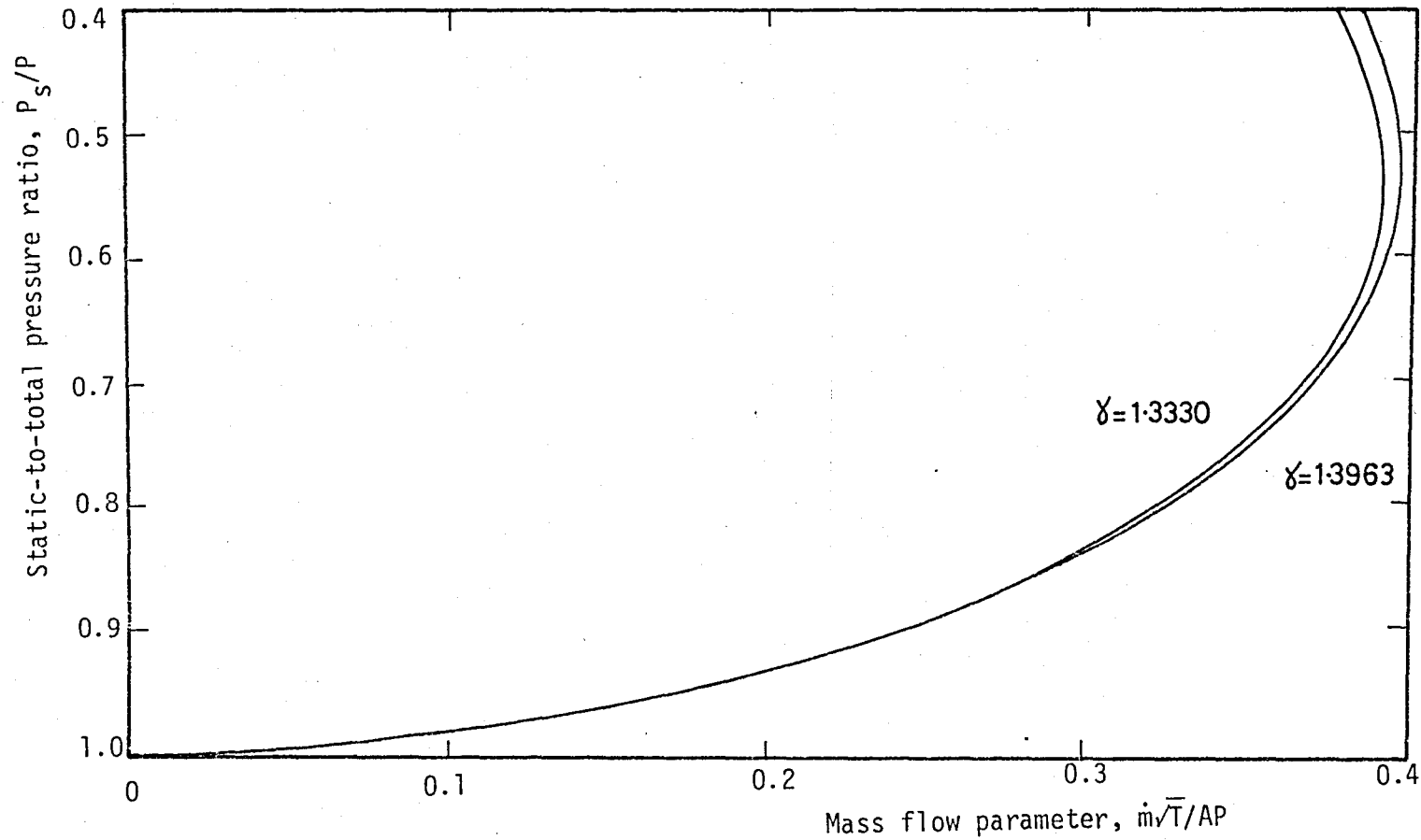


FIGURE 3.17 - Fundamental relation for compressible isentropic flow.

3.2 OPTIMUM DESIGN OF A GAS TURBINE STAGE

The second part of the thesis involves the design of a gas turbine stage in which the aerodynamic losses are minimized (or, the aerodynamic total-to-total efficiency is maximized). The analysis may be split down into two parts: the calculation of the efficiency and the optimization of the efficiency.

3.2.1 Calculation of the Efficiency of the Stage

The efficiency calculation is performed according to the method given by Rao and Gupta [3]. The method consists in relating the efficiency to the most relevant turbine parameters, namely, the turbine mean diameter, the chord and pitch lengths of the stator and rotor blades, the rotor blade inlet and exit angles and the axial velocity through the stage. The remaining parameters of the problem are expressed in terms of these 8 design variables. Initial values are given to the design variables at beginning of the analysis and these are modified through the iterative process explained in Section 3.2.2.

In the present design method, it is assumed that the total pressure and temperature at the inlet to the stage, the mass flow rate across the stage and the speed of the rotor are known parameters. Further, it is assumed that the properties of air such as the specific heat at constant temperature, the gas constant, the viscosity and the ratio of the specific heats are also known.

Before discussing the procedure, it must be noted that the effects of incidence and deviation of the gas stream onto and away

the rotor blades are completely neglected in the present method, i.e., the gas angles at the inlet and outlet of the rotor blades are assumed to be equal to the respective blade angles. Consequently, the independent choice of the rotor inlet and outlet gas angles constitutes the main difference between the procedures of the first and the second parts of the thesis. In contrast to the method of Ainley and Mathieson [1] in which the outlet gas angles from the blade rows are determined through a trial and error process after having obtained the respective total losses of the rows, in the procedure of Rao and Gupta [3], the trial and error process is imposed directly on the total loss of the stator and on the total-to-total efficiency.

The performance estimation method of [3] may be outlined as follows:

First, the static gas conditions at the stator inlet are determined using the known inlet conditions. From the energy equation (Eq. (22)) it is deduced that:

$$T_{1s} = T_1 - V_a^2 / 2C_p \quad (41)$$

where the inlet gas velocity to the (first) stator row is assumed axial (and of an assigned magnitude since V_a is a design variable).

For an isentropic expansion as shown in Fig. 2.3, the static pressure can be found from the thermodynamic relation,

$$P_{1s} = P_1 (T_{1s} / T_1)^{\gamma / (\gamma - 1)} \quad (42)$$

and from the perfect gas law, the inlet gas density, ρ_1 , is determined as:

$$\rho_1 = P_{1s} / (R_a T_{1s}) \quad (43)$$

where R_a is the gas constant.

The geometry of the turbine at inlet can now be determined from the consideration of the continuity equation:

$$A_{n,1} = \dot{m} / (\rho_1 V_a) \quad , \quad (44)$$

where, as also shown in Fig. 2.2,

$A_{n,1}$ = annulus area at inlet to stator.

The annulus height at turbine inlet, h_1 , is given by

$$h_1 = A_{n,1} / \pi \cdot D \quad , \quad (45)$$

where D , the mean diameter, is a design variable of preassigned value.

Since the rotor inlet gas angle, α_3 , is also a design variable with an assigned magnitude, the velocity triangles shown in Fig. 3.4 can be used to determine the gas angle at the outlet of the stator.

Thus,

$$\alpha_2 = \tan^{-1}(\tan \alpha_3 + V_a / U) \quad (46)$$

where the blade speed, U , at reference diameter, D , is determined from the known rotational blade speed, N , as

$$U = \pi \cdot D \cdot N \quad (47)$$

The gas outlet velocity from the stator is also determined from the velocity triangles as

$$V_2 = V_a / \cos \alpha_2 \quad (48)$$

Now, the static pressure, P_{2s} , and the temperature, T_{2s} , at the stator outlet must be determined. The energy equation (Eq. 22) gives T_{2s} easily. Hence,

$$T_{2s} = T_1 - \frac{V_2^2}{2c_p} \quad (49)$$

However, the evaluation of P_{2s} is not straight forward and requires the use of the stator total loss coefficient, λ_N , based on the temperature drop (or, the enthalpy loss). Horlock [4] defines λ_N as

$$\lambda_N = \frac{T_{2s} - T_{2s,is}}{T_1 - T_{2s}} \quad (50)$$

where $T_{2s,is}$ is the absolute isentropic static temperature at the stator outlet, as shown in Fig. 2.3.

It is noted that Eq. (50) contains two unknowns, namely, λ_N and $T_{2s,is}$. Therefore, this step requires a trial and error process to be carried out. Accordingly, a first trial value is assigned to λ_N , and then, combining Eq. (49) and Eq. (50), the value of $T_{2s,is}$ is obtained:

$$T_{2s,is} = T_1 - \frac{V_2}{2c_p} (1 - \lambda_N) \quad (51)$$

The absolute static pressure at the stator outlet, P_{2s} , can now be determined from the thermodynamic equation for isentropic expansion as follows:

$$P_{2s} = P_1 / (T_1 / T_{2s,is})^{\gamma / (\gamma - 1)} \quad (52)$$

The gas density at the rotor inlet is given from the ideal gas law:

$$\rho_2 = P_{2s} / (R_a T_{2s}) \quad (53)$$

and as before the flow area at the rotor inlet is determined from the continuity equation:

$$A_{n,2} = \dot{m} / (\rho_2 V_a) \quad (54)$$

The annulus height at rotor inlet, h_2 , is given by:

$$h_2 = A_{n,2} / (\pi \cdot D) \quad (55)$$

In order to determine the gas conditions at the rotor outlet, an important aerodynamic parameter is used. This is the stage total temperature drop, ΔT , which is defined as:

$$\Delta T = T_1 - T_1' \quad (56)$$

where

T_1' = total absolute gas temperature leaving the stage.

However, another expression for ΔT can be obtained through the following analysis. For adiabatic flow in a turbine stage, the specific work output in the rotor blades, ΔW , is equal to the change in specific total enthalpy, i.e.,

$$\Delta W = H_1 - H_1' \quad (57)$$

where H_1 and H_1' are the total enthalpies of the gas entering and leaving the stage, respectively. Eq. (57) can also be expressed in terms of the total temperatures at the entry and exit of the stage. Therefore,

$$\begin{aligned}\Delta W &= C_p(T_1 - T_1') \\ &= C_p \Delta T\end{aligned}\quad (58)$$

On the other hand, considering the momentum equation applied to a control surface enclosing the rotor, the specific work output, ΔW , can be expressed as the total change of momentum of gas per unit flow, i.e.,

$$\Delta W = U(V_{t_3} + V_{t_4}) \quad (59)$$

where, as shown in Fig. 3.4.

V_{t_3} , V_{t_4} = tangential components of absolute gas velocities at rotor inlet and outlet, respectively.

Again considering the velocity triangles in Fig. 3.4, Eq. (59) can be modified as follows

$$\Delta W = UV_a(\tan\alpha_3 + \tan\alpha_4) \quad (60)$$

Finally, combining Eq. (58) and Eq. (60), a second expression for the total temperature drop across the stage is obtained:

$$\Delta T = UV_a(\tan\alpha_3 + \tan\alpha_4)/C_p \quad (61)$$

and the absolute total gas temperature at the rotor outlet follows directly from Eq. (56)

$$T_1' = T_1 - \Delta T \quad (62)$$

The determination of the total gas pressure at the outlet of the rotor, P_1' , is not as easy as in the case of the stator, because

it involves the expression of the total-to-total stage efficiency, η_s . Horlock [4] defines the stage efficiency as follows:

$$\eta = \frac{T_1 - T_1'}{T_1 - T_{1, is}'} \quad (63)$$

where $T_{1, is}'$ is the total absolute isentropic temperature at the stage outlet. On the other hand, the thermodynamic equation for isentropic expansion gives,

$$P_1' = P_1 \left(\frac{T_{1, is}'}{T_1} \right)^{\gamma/(\gamma-1)} \quad (64)$$

Combining Eqs. (62), (63) and (64), one gets

$$P_1' = P_1 \left(1 - \frac{\Delta T}{\eta T_1} \right)^{\gamma/(\gamma-1)} \quad (65)$$

It is noted that, again, there are two unknowns in Eq. (65) namely, P_1' and η . Therefore, the procedure must involve here the assumption of a trial value for the efficiency, η . This guessed value is replaced by the more accurate value of η at the end of the first iteration, and the iterative process is continued until the values of the stage efficiency in two consecutive iterations are sufficiently close to each other.

From the velocity triangles at the rotor outlet (Fig. 3.4) and using the assigned value of the design variable, α_4 , the absolute gas outlet angle from the rotor (or, the absolute gas angle at the inlet to the next stator row), α_1' , can be determined as

$$\alpha_1^i = \tan^{-1}(\tan\alpha_4 - V_a/U) \quad , \quad (66)$$

and the absolute gas velocity at the outlet of the rotor row is given by

$$V_1^i = V_a / \cos\alpha_1^i \quad . \quad (67)$$

The absolute static temperature at the rotor outlet is determined from the energy equation (Eq. (22)):

$$T_{1,s}^i = T_1^i - (V_1^{i2} / 2c_p) \quad , \quad (68)$$

whereas the absolute static pressure at the same station is determined from the thermodynamic equation for isentropic expansion:

$$P_{1,s}^i = P_1^i (T_{1,s}^i / T_1^i)^{\gamma / (\gamma - 1)} \quad (69)$$

The density of air leaving the rotor is found from the ideal gas law. The annulus area $A_{4,n}$ and the annulus height, h_4 , are determined from the continuity equation as before:

$$\rho_4 = P_{4s} / (R_a T_{4s}) \quad (70)$$

$$A_{4,n} = \dot{m} / (\rho_4 V_a) \quad (71)$$

$$h_4 = A_{4,n} / (\pi \cdot D) \quad (72)$$

After the gas conditions at each station are evaluated, the pressure loss coefficients of the stator and rotor, based on guessed blade outlet angles are calculated according to the procedure of Ainley and Mathieson [1] as corrected by Dunham and Came [2] (Section 3.1.2). The pressure loss coefficients for stator (Y_N) and for rotor

(Y_R) lead to the improved value of λ_N and λ_R and thereafter to the improved value of η . The correlations of pressure loss coefficients, Y , of Ainley and Mathieson [1] and enthalpy drop coefficients, λ , of Horlock [4] are given by Rao and Gupta [3] as follows:

$$\lambda_{N, \text{improved}} = Y_N \frac{T_{2s, is}}{T_1} \quad (73)$$

and

$$\lambda_{R, \text{improved}} = Y_R \frac{T_{4s, is}}{T_4} \quad (74)$$

where, $T_{4s, is}$ is the absolute static isentropic temperature at the rotor outlet and can be determined from the thermodynamic relation for isentropic expansion as,

$$T_{4s, is} = T_{2s} / \left(\frac{P_{2s}}{P_{4s}} \right)^{(\gamma-1)/\gamma} \quad (75)$$

and

$$T_4 = T_{4s} + V_1^2 / 2c_p \quad (76)$$

The expression for η , as given by Horlock [4] is as follows:

$$\eta = \frac{H_1 - H_1^t}{H_1 - H_{1, is}^t} \quad (77)$$

where

$$H_{1, is}^t = \text{absolute total isentropic enthalpy at rotor outlet.}$$

Rearranging and noting that the total enthalpy drop is equal to the static enthalpy drop for the rotor blades, Eq. (77) may be put in the conventional form which is,

$$\eta = \frac{1}{1 + (\lambda_R (V_1^2 / 2c_p) + (T_{3s} / T_{2s}) \lambda_{N, \text{improved}} (V_2^2 / 2c_p)) / (T_1 - T_1')} \quad (78)$$

The final step of the procedure is the evaluation of the Reynolds number, Re , of the stage:

$$Re = (\rho_2 V_2 C_N + \rho_4 V_1' C_R) / (2\mu) \quad (79)$$

where μ is the viscosity of the fluid.

The correction for Re applied to η is realized as suggested in [1]:

$$\eta_{,Re} = 1 - (1 - \eta) \left(\frac{2 \times 10^5}{Re} \right)^{0.2} \quad (80)$$

If $\eta_{,Re}$ differs from η assumed in Eq. (65), then the value of $\eta_{,Re}$ and $\lambda_{N, \text{improved}}$ are taken as the improved guesses to be used in Eq. (65) and Eq. (51) respectively, and the procedure is repeated until convergence.

3.2.2 Optimization of the Efficiency of the Stage

Inspection of the efficiency equations together with the loss relations presented earlier shows that analytical optimization is not possible due to the high non-linearity of the expressions (Sec. 3.2.1). It is therefore, necessary to resort to a numerical optimization technique.

It is stated in Section 3.2.1 that the isentropic efficiency, η , of the stage is considered to be a function of the eight design variables, including the mean diameter, rotor & stator chord & pitch lengths, rotor inlet and outlet blade angles and the axial velocity. The initial values of the design variables assigned at the beginning of the analysis are modified during the iterative optimization process. The remaining data of the problem are either given at the start or can be expressed in terms of the design variables during the analysis.

Grouping the design variables as a vector quantity, \vec{X} , the optimization problem in hand can be stated in the standard form:

$$\text{Find } \vec{X} = \{D, C_R, C_N, S_N, S_R, \alpha_3, \alpha_4, V_a\} \quad \text{which} \\ \text{maximizes } \eta(\vec{X}) \quad (81)$$

However, a practical turbine design, in most cases, is the result of a multitude of compromises. The maximum efficiency to be obtained would turn out to correspond to a geometry which poses either fabrication problems such as excessive rotative speeds, huge diameters, etc., or violates the aerodynamic requirements such as excessive Mach numbers, choking, etc. In order to avoid such undesired results, it is therefore necessary to meet the requirements rising from aerodynamic, vibrational and strength considerations. In the present thesis, only those requirements which are related to aerodynamic considerations are considered. The constraints of the numerical problem are as visualized in Ref. [3] and they are stated in a non-dimensional form as follows:

1. The rotational velocity of the rotor should be within some upper and lower bounds. In the present study it will be required that

$$\frac{50}{U} - 1.0 \leq 0 \quad (82)$$

$$\frac{U}{400} - 1.0 \leq 0$$

2. The aspect ratio (height/chord) of rotor and nozzle blades should be within some specified upper and lower bounds.

The bounds used in the present study are as follows:

$$(h_R/8D) - (C_R/D) \leq 0$$

$$(C_R/D) - (h_R/2D) \leq 0 \quad (83)$$

$$(h_N/10D) - (C_N/D) \leq 0$$

$$(C_N/D) - (h_N/2D) \leq 0$$

where h_N and h_R correspond to the average nozzle and rotor annulus heights, respectively and are expressed as follows:

$$h_N = (h_1 + h_2)/2 \quad (84)$$

$$h_R = (h_2 + h_4)/2.$$

3. The pitch-to-chord ratio of the rotor and nozzle blades should lie within certain limits. The present analysis will require that

$$0.5 < S_N/C_N < 1.0 \quad (85)$$

$$0.5 < S_R/C_R < 1.0$$

4. The relative gas velocity angles at inlet and outlet of the rotor blades should be within some specified bounds. Here, it will be assumed that

$$\begin{aligned} 0.01 < \alpha_3 < 1 \\ 40/57 < \alpha_4 < 1.4 \end{aligned} \quad (86)$$

5. The axial velocity of flow should lie within some upper and lower bounds which will be given in the present analysis as

$$\begin{aligned} (50/V_a) - 1 &\leq 0 \\ (V_a/400) - 1 &\leq 0 \end{aligned} \quad (87)$$

6. The actual pressure ratio across the nozzle blades should be below the critical pressure ratio:

$$(P_1/P_{2s}) - (P_1/P_c) \leq 0 \quad (88)$$

where the critical pressure, P_c , across the nozzle is found from frictionless, adiabatic flow approximation [27] applied to the throat section for critical conditions, and is given as follows:

$$P_c = P_1 \left(\frac{2}{\gamma + 1} \right)^{\gamma/(\gamma+1)} \quad (89)$$

7. The Mach number at the exit from the stage should be less than a specified maximum value. Here, the maximum value will be assumed to be unity.

$$M_4 - 1 \leq 0 \quad (90)$$

where, M_4 is given in [3] as

$$M_4 = \frac{V_1'}{\sqrt{\gamma R_a T_{3s}}} \quad (91)$$

8. The included angle of divergence, α_c , of the turbine annulus walls should not exceed some specified upper limit taken in the present study as

$$(12\alpha_c/\pi) - 1.5 \leq 0 \quad (92)$$

where, α_c is defined as follows [3]

$$\alpha_c = \tan^{-1} \frac{h_4 - h_1}{2(C_N + 1.25C_R)} \quad (93)$$

9. The flow coefficient, ϕ , as defined by the ratio of axial velocity to blade speed (V_a/U) and the stage temperature drop coefficient, ψ , defined as:

$$\psi = 2c_p \Delta T / U^2 \quad (94)$$

should lie within certain limits. The present study will assume that these limits are given by

$$0.25 < \phi < 2 \quad (95)$$

$$0.5 < \psi < 6 \quad (96)$$

10. The degree of reaction at mean radius must be within some specified upper and lower bounds. Here,

$$0.3 < R < 0.7 \quad (97)$$

where the reaction is defined as [3]

$$R = \frac{\phi}{2}(\tan\alpha_4 - \tan\alpha_3) \quad (98)$$

Grouping the constraints given by Eq. (82) through Eq. (98) as a vector quantity, \vec{G} , and arranging so that

$$\vec{G} \leq 0 \quad (99)$$

the nonlinear programming model of Eq. (81) can be modified as

$$\begin{aligned} &\text{Find } \vec{X} \\ &\text{which maximizes } \eta(\vec{X}) \\ &\text{subject to } \vec{G}(\vec{X}) \end{aligned} \quad (100)$$

However, no analytical or numerical solution to the constrained case of a nonlinear programming problem has so far been offered in the literature. Therefore, it is necessary to convert the problem of Eq. (100) into an unconstrained optimization problem. The transformed problem can then be solved using the Steepest Ascent Method [29].

The transformation is realized through the widely used nonlinear programming method of Fiacco and Mc Cormick [24,28,29] known as the Sequential Unconstrained Maximization Technique (SUMT). The procedure is more or less similar to the use of the Lagrange multipliers method [29,30] and involves the transformation of the objective function into an interior penalty function, P , augmented by a penalty term consisting of the constraints.

This concept, applied to Eq. (100) results in the new function:

$$P(\vec{X}, r_k) = \eta(\vec{X}) + r_k \sum_{i=1}^{10} \frac{1}{G_i(\vec{X})} \quad (101)$$

where r_k is a non-negative parameter, called the resequencing factor.

It is assumed that the optimization of Eq. (101) is equivalent to the optimization of Eq. (100). To clarify the idea, the following reasoning is given.

First, the constraint equations, G_i , as given in Eqs. (82) through (98), are convex [29]. It follows that $1/G_i$ is concave as η also is. Consequently, P is concave and thus possesses a unique maximum.

Furthermore, the optimization of P is performed for a decreasing sequence of r_k , i.e.,

$$0 < r_{k+1} < r_k \quad (102)$$

This means that, in the limit, when r_k is sufficiently small,

$$\lim_{r_k \rightarrow 0} P(\vec{X}, r_k) = \eta(\vec{X}) \quad (103)$$

Therefore, at the end of the procedure, the contribution of the constraints to the penalty function will be minimized and the resulting optimum of $P(\vec{X}, r_k)$ will correspond, with approximately no error, to an optimum efficiency, $\eta(\vec{X})$.

The algorithm is initiated by arbitrarily selecting an initial non-negative value for r_k . In the present work, this initial r_k is

chosen, as suggested in Ref. [3], to be

$$r_1 = 4 \times 10^{-4} \quad (104)$$

and the decreasing sequence imposed on r_k is realized through:

$$r_{k+1} = r_k/10 \quad (105)$$

An initial point, \vec{X}_0 , is selected as the first trial solution. Since the method of [24] is essentially an "Interior Point Unconstrained Maximization", \vec{X}_0 must be an interior point, that is it must not lie on or beyond the boundaries of the solution space. The value of \vec{X}_0 given in [3] as calculated according to the constraint equations, is adopted in the present thesis.

Given the value of r_1 and \vec{X}_0 , the Steepest Ascent Method (SAM) [29] is used to determine the corresponding optimal solution of $P(\vec{X}, r_k)$. The object of the S.A.M. is to generate \vec{X}_{n+1} once \vec{X}_n is given. Logically, the next trial point, \vec{X}_{n+1} , should be selected in the direction of the fastest rate of improvement in the objective value. This direction is determined by using the information provided by the gradient of the objective function, $\vec{\nabla}P(\vec{X}_n, r_n)$, at the current trial point, \vec{X}_n . The relation between the next and the actual trial points is given as:

$$\vec{X}_{n+1} = \vec{X}_n + R_{op,n} \vec{\nabla}P(\vec{X}_n, r_n) \quad (106)$$

where R_{op} is known as the optimal step size.

Following Eq. (106), in order to determine the next trial point, it is therefore necessary to determine both a feasible direction, $\vec{\nabla}P$, and the optimal step size in this direction, R_{op} . In the

present thesis, the determination of $\vec{\nabla}P$ and R_{op} at any point is performed in the following way:

Referring to Eq. (101), the gradient, $\vec{\nabla}P$, of the penalty function can be given as:

$$\vec{\nabla}P(\vec{X}, r_k) = \vec{\nabla}\eta(\vec{X}) + \vec{\nabla}\left[r_k \sum_{i=1}^{10} \frac{1}{G_i(\vec{X})}\right] \quad (107)$$

or

$$\vec{\nabla}P(\vec{X}, r_k) = \vec{\nabla}\eta(\vec{X}) - r_k \vec{\nabla}\left[\sum_{i=1}^{10} \frac{1}{G_i^2(\vec{X})}\right] \quad (108)$$

As presented in Eqs. (41) through (80), the efficiency equations expressed in terms of the design variables, \vec{X} , are implicit. Therefore a straight analytic differentiation would be quite cumbersome. The same is also valid for the determination of $\vec{\nabla}(1/G_i^2(\vec{X}))$. Consequently, the general chain rule using the Jacobian Matrix evaluation is applied to this problem. The general chain rule states that [31], the Jacobian Matrix, J_{in} , for a multivariable, continuously differentiable function, u , can be expressed as:

$$\begin{aligned} J_{in} &= \frac{\partial u}{\partial z_m} \\ &= \sum \frac{\partial u}{\partial v_i} \frac{\partial v_i}{\partial \omega_j} \dots \frac{\partial x_k}{\partial y_\ell} \frac{\partial y_\ell}{\partial z_m} \end{aligned} \quad (109)$$

where

$$\begin{aligned} u &= u(v_i) & i &= 1, n_1 \\ v_i &= v_i(\omega_j) & j &= 1, n_2 \\ \vdots & & \vdots & \end{aligned}$$

$$\begin{aligned} \vdots \\ x_k &= x_k(y_\ell) & k = 1, n_3 & ; \ell = 1, n_4 \\ y_\ell &= y_\ell(z_m) & m = 1, n_5 \end{aligned}$$

The differentiation of $n(\vec{X})$ and $1/G_i^2(\vec{X})$ is thus performed by the use of equation (109). R_{op} in Eq. (106) can be determined as follows. Since the objective is to determine \vec{X}_{n+1} which gives the largest improvement in the value of $P(\vec{X}, r_k)$, R_{op} is determined as the optimal value for which

$$H(R_{op}) = P(\vec{X}_n + R_{op} \vec{\nabla} P(\vec{X}_n, r_k)) \quad (110)$$

is maximized over R_{op} .

Noting that, in Eq. (110) both \vec{X}_n and $P(\vec{X}_n, r_k)$ are known and fixed for the moment, H is a function of R_{op} only. Therefore, the value of R_{op} which maximizes $H(R_{op})$ can be determined by a numerical trial and error process. This process is started from $R_{op} = 0$ and continued until, for some X_i ,

$$X_{n+1,i} > \text{Upper bound of } X_i$$

or

$$X_{n+1,i} < \text{Lower bound of } X_i.$$

(111)

Once the optimal step size, R_{op} of Eq. (106) is determined from Eq. (110), this optimal step size is fed into Eq. (106) and the new starting trial point \vec{X}_{n+1} is found. Then, the gradient of P at this new point is determined and the procedure is repeated until the values of P corresponding to two consecutive iterations are approximately

the same. This process is called the "one-dimensional maximization" of P.

Note that at the point where one-dimensional maximization is terminated, the optimum design vector, \vec{X} , and the corresponding optimum efficiency, η , are expressed for the relevant value of r_k . Therefore, once an optimum η is found for some r_k , r_k is updated according to Eq. (105) and the whole procedure is repeated, until for two successive values of r_k , the corresponding optimum efficiencies are approximately the same.

Numerical applications of the above procedure have indicated that for $r_k < 4 \times 10^{-6}$ further iterations result in negligible improvements in η . For computer time saving purposes, therefore, the calculations are stopped beyond this value of r_k . Consequently, for each run with new input conditions, three sequential maximization steps are performed in the range,

$$4 \times 10^{-6} < r_k < 4 \times 10^{-4}.$$

IV. RESULTS AND DISCUSSIONS

Based on the analysis given in Chapter 3, two computer programs have been developed. Input and output descriptions together with the program listings are presented in Appendices B and C. The application of the programs to turbine stages with various inlet gas conditions and blade rotational speeds are discussed in this chapter and the numerical results are presented in graphical and tabular form.

4.1 RESULTS ON PERFORMANCE EVALUATION

As presented in Chapter 3, the program developed is based on the study of Ainley and Mathieson [1] with the suggested modification of Dunham and Came [2]

The variation of the blade outlet gas angles with the outlet Mach number is plotted in Fig. 4.1. The curve obtained for the case of the stator is in agreement with the results of Ref. [1], but a deviation ($\sim 1.5^\circ$) in the outlet angle is observed for the case of the rotor, especially for Mach numbers exceeding 0.5. It should be remembered that the calculations of Ref. [1] are hand-made. Therefore, the computerized calculations of the present work can be considered as more precise.

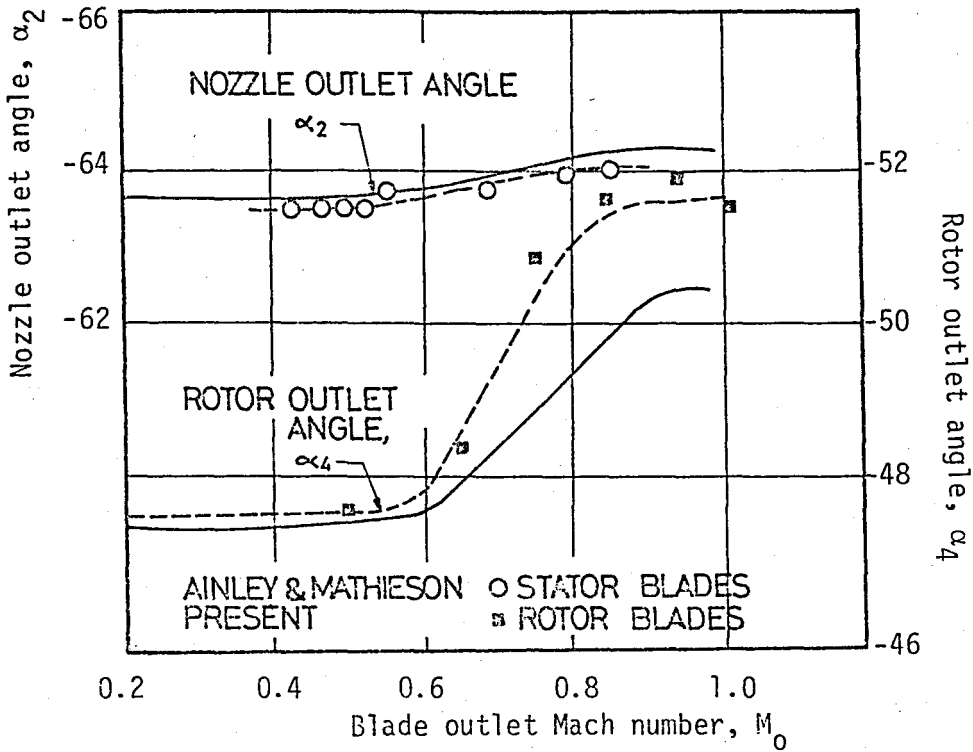


FIGURE 4.1 - Variation of outlet gas angle with outlet Mach number.

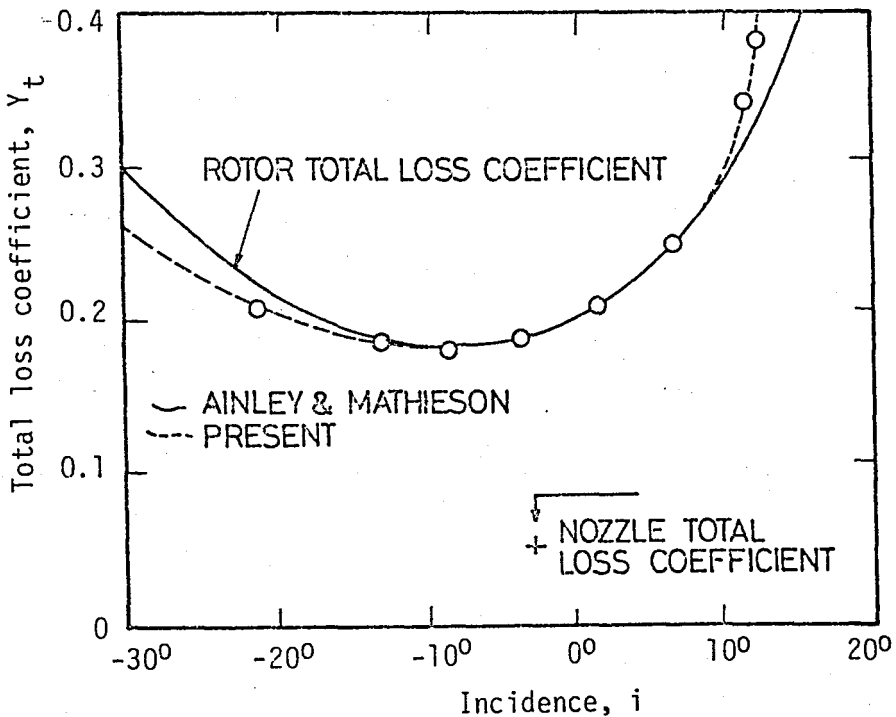


FIGURE 4.2 - Variation of total loss coefficient with incidence.

The variation of the rotor loss coefficient with respect to the incidence is given in Fig. 4.2. The deviation of the results from those of the original method [1], especially for incidences lower than -15° and higher than $+10^{\circ}$ are due to the fact that secondary and tip clearance losses are calculated in this thesis, according to the suggested modifications of Dunham and Came to the original Ainley and Mathieson method. Since Dunham and Came's correlations have proven, over the last three decades, to give more adequate results, especially for the case of the turbines with low aspect ratio, the outputs of the present thesis may be found more reliable than those reported in Ref. [1].

The total-to-total aerodynamic efficiency, η , of the stage is plotted in Fig. 4.3, against the overall pressure ratio. A plot of the initial mass flow parameter is also given on the same figure. Three curves, corresponding each to a different value of the speed parameter, N/\sqrt{T} , are shown on the figure. It is observed, in general, that the efficiency decreases slightly for decreasing values of the speed parameter. The same is also valid for the mass flow parameter, which decreases from 10.268 (corresponding to $N/\sqrt{T} = 435$, as given by Ref. [1]) first to 10.159 (for $N/\sqrt{T} = 424$) and then to 10.113 (for $N/\sqrt{T} = 393$) at the point where the rotor is choked. For both cases, the pressure ratio for which the stage completely chokes (i.e. Mach number equals unity) is correctly predicted as 0.51.

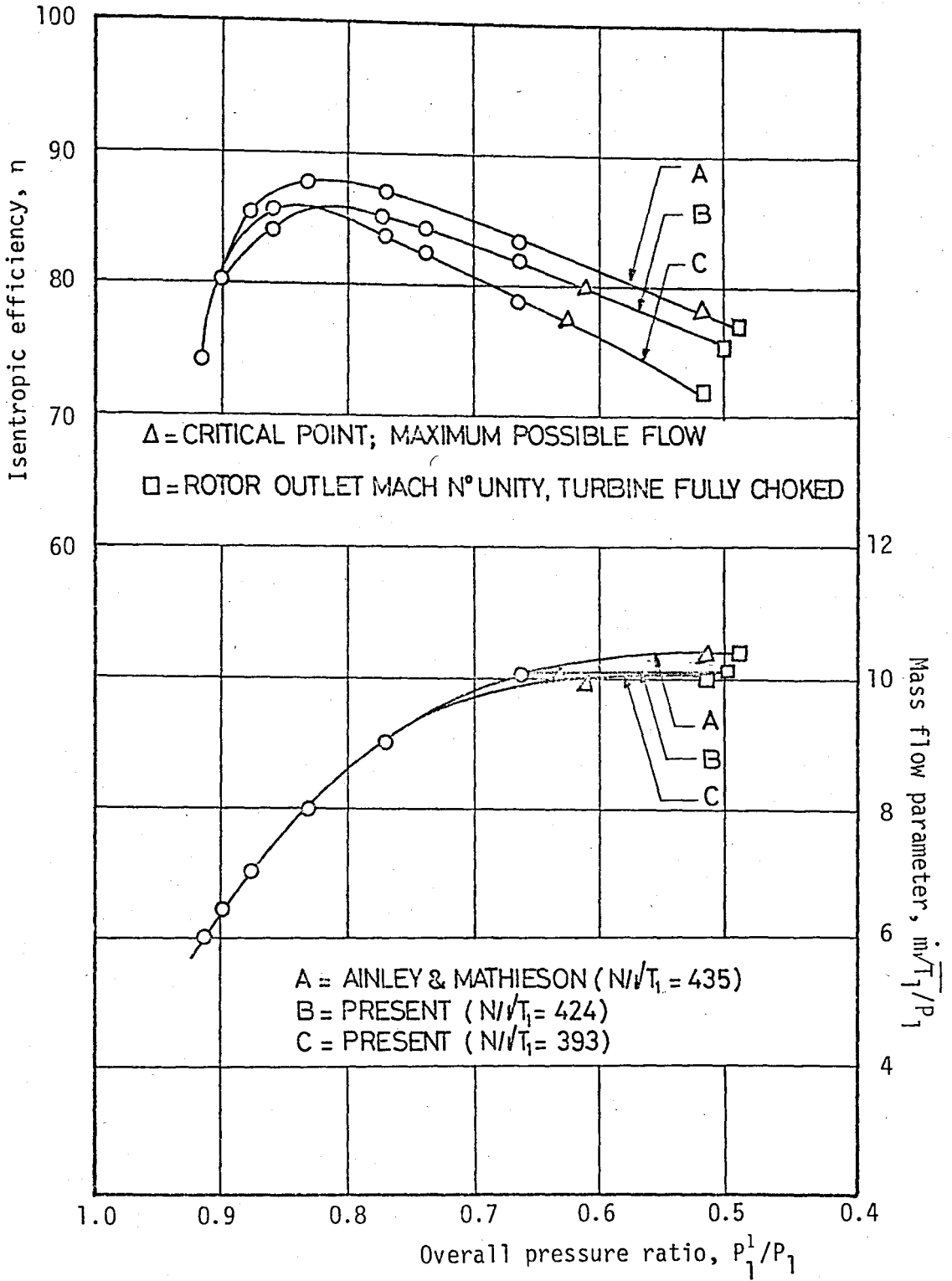


FIGURE 4.3 - Variation of the isentropic efficiency and the non-dimensional mass flow with overall pressure ratio.

4.2 RESULTS ON OPTIMUM STAGE DESIGN

A computer program is developed, based on the turbine stage performance evaluation model suggested by Rao and Gupta [3] and on the optimization method of Fiacco and Mc Cormick [24]. Table 4.1 summarizes the results obtained by the application of the program to various inlet gas conditions and to various blade speeds. The results are also presented in graphical form within Fig. 4.4 through Fig. 4.9.

Fig. 4.4 is a plot of the overall efficiency to the overall pressure ratio. The data points are grouped under two different sets: regions corresponding to three ranges of the mass flow parameter based on annulus diameter, $\dot{m}\sqrt{T_1}/P_1 D^2$, and various lines corresponding to constant values of the turbine speed parameter, $ND/\sqrt{T_1}$. Inspection of Fig. 4.4 indicates that along constant $ND/\sqrt{T_1}$ lines, the efficiency tends to increase for increasing values of the pressure ratio. Compared to the corresponding pressure ratio range of Fig. 4.3, this efficiency variation is in accordance with the results of Ainley and Mathieson [1]. It is also observed from Fig. 4.4 that the mass flow parameter being held constant, the efficiency tends to decrease as the pressure ratio is increased.

The relation between the mass flow parameter, the pressure ratio and the efficiency is also plotted on Fig. 4.5 in an altered form. The mass flow parameter is shown on the ordinate axis versus the pressure ratio and the data points are grouped for different ranges of the efficiency. The general trend of the figure indicates that higher efficiencies tend to be associated with lower pressure

TABLE 4.1 - Aerodynamic Variables Corresponding to the Optimum Efficiency, as calculated by the computer program RAO.

Variable Point #	$\frac{\dot{m}_v \bar{T}_1}{P} \times 10^5$	$\frac{\dot{m}_v \bar{T}_1}{P_1 D^2} \times 10^2$	$ND/V\bar{T}_1$	P'_1/P_1	$Re_x 10^{-3}$	$\frac{\Delta T}{\bar{T}_1} \times 10$	η
1	166	1.77	2.77	0.562	268	1.12	0.837
2	193	2.07	2.37	0.631	199	0.082	0.840
3	232	1.97	1.77	0.729	246	0.061	0.806
4	237	1.53	1.86	0.654	422	0.079	0.784
5	237	1.69	2.36	0.596	429	0.100	0.826
6	248	3.94	2.65	0.599	275	0.105	0.874
7	263	2.23	2.18	0.619	256	0.093	0.821
8	290	2.18	2.35	0.672	253	0.081	0.853
9	322	2.75	1.77	0.734	156	0.061	0.818
10	387	3.06	2.29	0.684	240	0.079	0.868
11	387	2.89	1.89	0.730	308	0.065	0.854

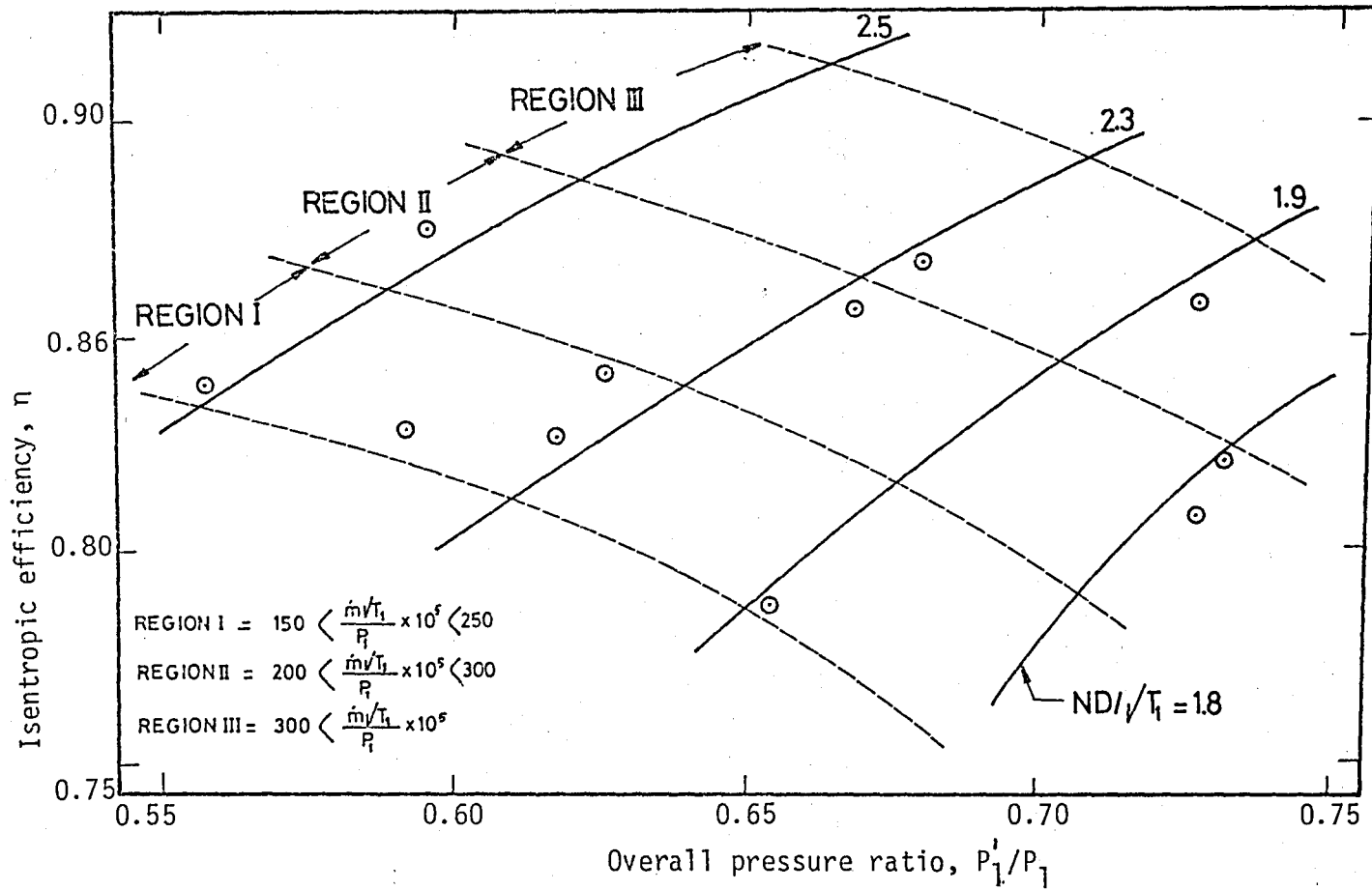


FIGURE 4.4 - Variation of the isentropic efficiency with overall pressure ratio.

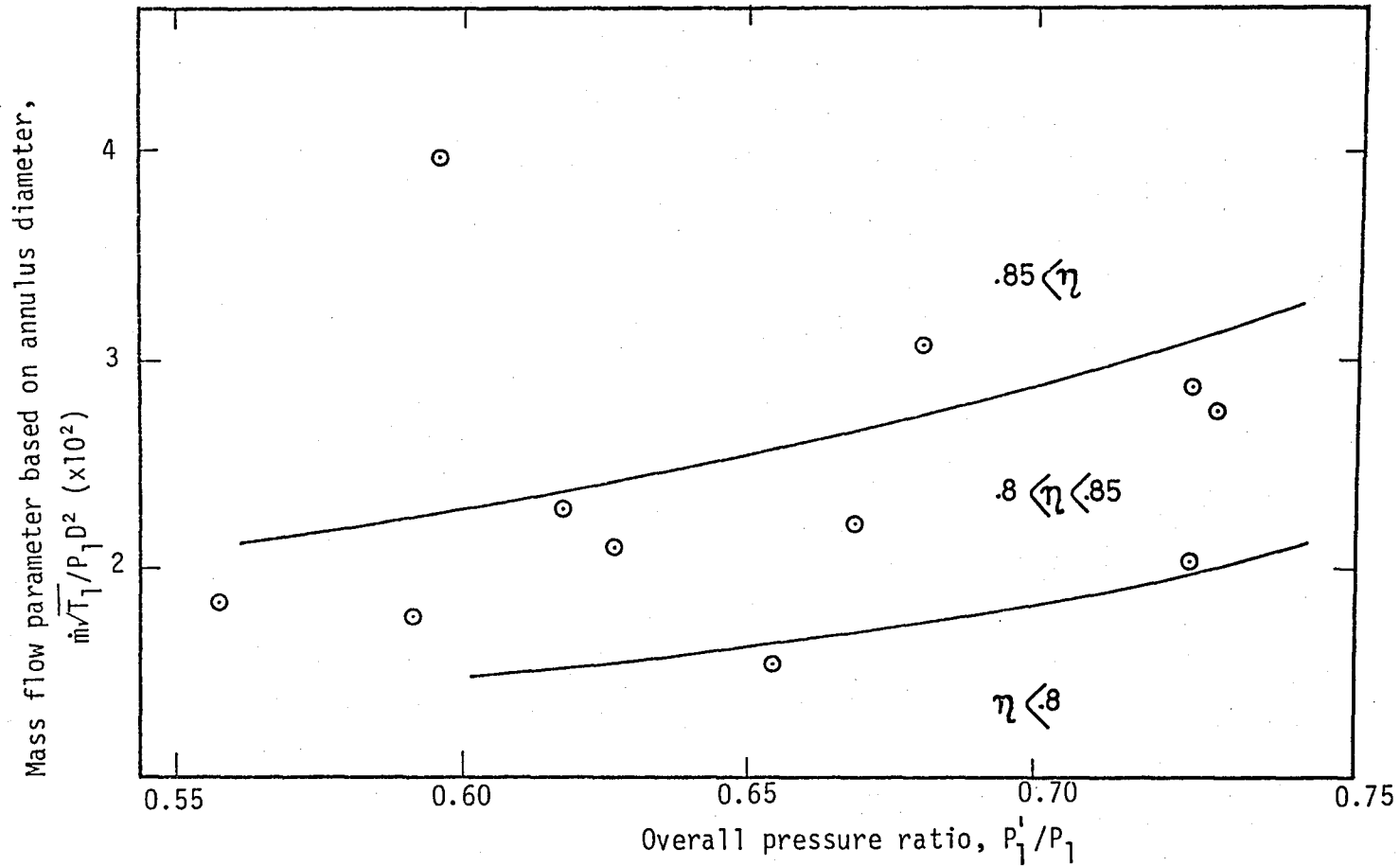


FIGURE 4.5 - Variation of the non-dimensional flow parameter with the overall pressure ratio.

ratios, since the mass flow parameter required is smaller.

The isentropic efficiency is plotted in Fig. 4.6 as a function of the speed parameter. It is observed from the figure that along (approximately) constant speed parameter lines, and for increasing mass flow rates, the efficiency also tends to increase. However, the rate of increase in the efficiency is lowest at higher pressure ratios. This trend is in conformity with the results appearing in Fig. 4.3.

Finally, the temperature drop coefficient, $\Delta T/T_1$ is plotted against the overall pressure ratio and the speed parameter in Fig. 4.7 and 4.8, respectively. It is interesting to note from Fig. 4.7 that the curve goes through a minimum, corresponding to a range of pressure ratio such as, $0.6 < P_1'/P_1 < 0.7$. Recalling that the temperature drop, ΔT , is directly related to the work output, ΔW , of the stage through Eq. (58), it can be deduced that an optimization of the specific work output can be realized by controlling the back pressure of the turbine. Fig. 4.8, on the other hand, points out that the specific work output of the stage tends to increase with increasing blade rotative speed.

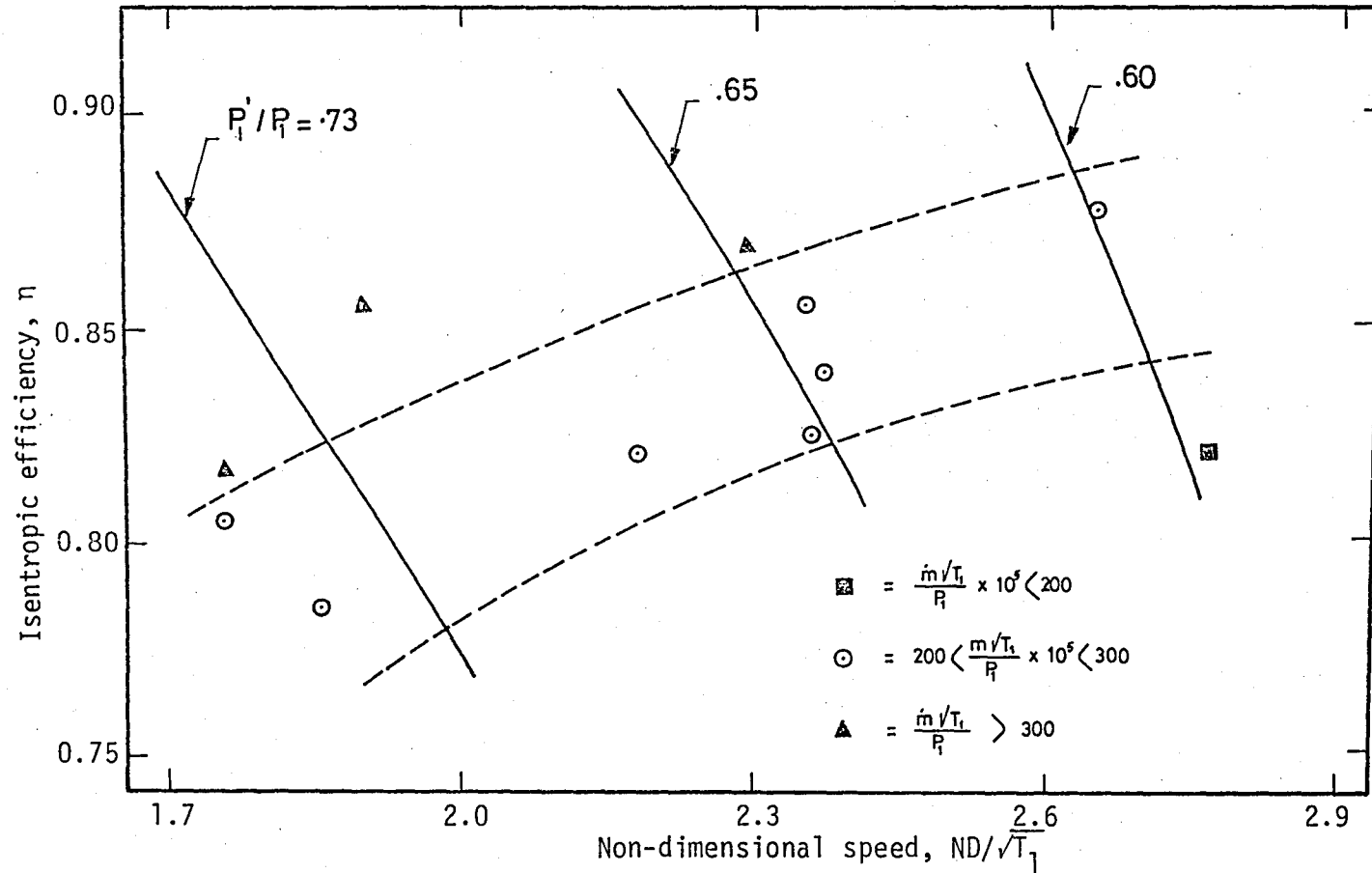


FIGURE 4.6 - Variation of the isentropic efficiency with the non-dimensional speed parameter.

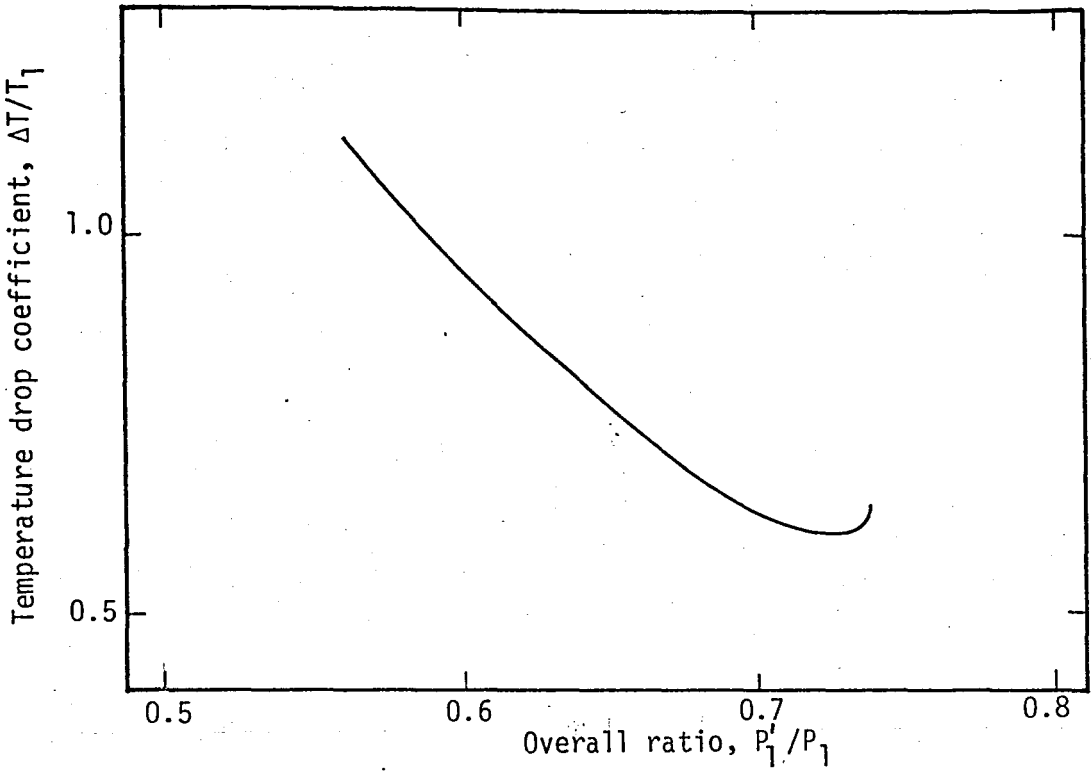


FIGURE 4.7 - Variation of the temperature drop coefficient with the overall pressure ratio.

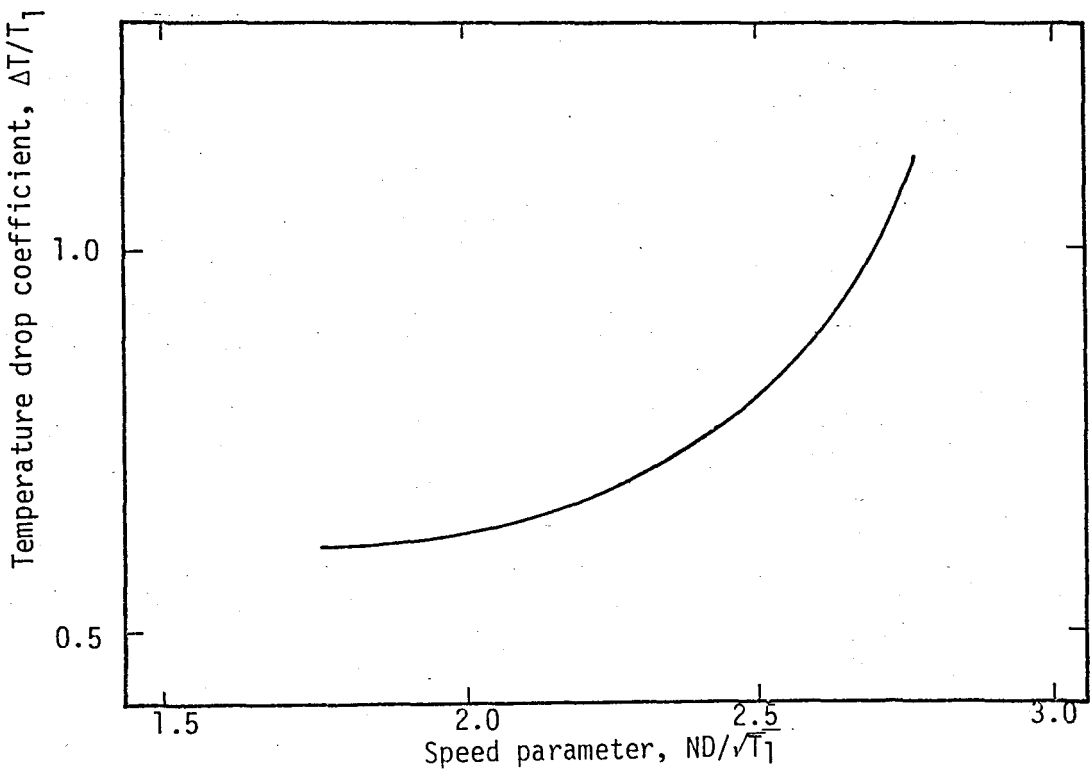


FIGURE 4.8 - Variation of the temperature drop coefficient with the non-dimensional speed parameter.

V. CONCLUSIONS AND RECOMMENDATIONS

1. Two computer programs have been developed to calculate the performance of an axial flow gas turbine stage and to predict the geometry of a turbine stage which would develop the maximum efficiency.
2. The first program is based on the loss evaluation and performance prediction method of Ainley and Mathieson [1], as modified by Dunham and Came [2] and the second program, on the efficiency evaluation method of Rao and Gupta [3] and on the optimization technique of Fiacco and Mc Cormick [24]. It might be understood that the contribution of the present thesis is not in working out a new method of its own, but rather, in combining the existing ones and computerizing them in order to offer a useful tool for the researchers as well as for the designers working on gas turbine aerodynamics.

Indeed in the method presented in Ref. [1], it took about six to eight man-hours to calculate a single performance map (curves A, for example, of Fig. (4.3)) of a given turbine. However in the related computer program the performance map

containing three different curves (curves A, B and C of Fig. 4.3) together with the gas conditions at each stage, are obtained in not more than three seconds. The second computer program, on the other hand, provides an efficient means for the user, especially in the preliminary design stage of the turbine (The computer program of Rao and Gupta's method [3] was not available). The original method of Ref. [3] is slightly altered to incorporate the Sequential Unconstrained Maximization Technique of Fiacco and Mc Cormick [24]. The use of this technique allowed a considerable computer time saving and the 61 minutes of computer time on an IBM 7044 computer in the original method were reduced down to 320 CDC seconds (cpu time).

3. The performance characteristics are studied by making use of the dimensionless turbine parameters, and this non-dimensional analysis enlarged the scope of the observations, allowing the prediction of the behaviour of any aerodynamic variables when the others remain fixed.
4. The following conclusions have been obtained from the application of the programs to various gas turbine data.
 - a) The isentropic efficiency is a function of the inlet mass flow parameter (which can be expressed by various groups such as $\dot{m}\sqrt{T_1}/P_1$, $\dot{m}\sqrt{T_1}/P_1 A_1$ or $\dot{m}\sqrt{T_1}/P_1 D^2$). This dependence is in conformity with the results given by

Ainley and Mathieson [1]. The effect of the rotative speed parameter, $N/\sqrt{T_1}$, is also observed and it is found that the efficiency increases with increasing rotative speeds.

- b) The functioning of the programs are seen to be largely restricted by the interpolation subroutines in that for very small and for very large mass flows, some parameters fall out of the interpolation domain, making further calculations impossible.
 - c) The results of the optimization program are in good agreement with those of the performance prediction program. The values of the design variables corresponding to optimum efficiency lie within acceptable bounds. It is noted that the efficiency depends least on the axial velocity, C_a , since the final value of this variable shows very small difference from the initial value. This consequence is due to the fact that the axial velocity is related to the physical variables as given by Eq. (46) and Eq. (47).
5. The mathematical model, the interpolation technique and the optimization method need to be improved in the following areas.
- a) The present gas flow model is 2-D and does not describe the flow adequately, especially in stages having small hub-to-tip radius ratios. In such cases a 3-D solution is required.

- b) The interpolation methods used are quite restrictive and do not allow calculations beyond the limits of the interpolation domain. To overcome this difficulty the capacity of the related subroutine may be extended to allow extrapolation, or the interpolation domain may be slightly enlarged.
- c) The computer program AINLEY may be modified such that the correlations of the blade outlet angles and of the stage losses as well as the presentations of the final performance maps can automatically be obtained in a graphical form. Also the velocity triangles corresponding to a fixed value of the inlet gas conditions and of the rotational speed can be displayed together with the final results, in order to provide a considerable help for the design engineer, especially in obtaining quick estimates of the turbine performance.
- d) The variation of the stage efficiency with respect to small perturbations of the design variables can be obtained by introducing a sensitivity analyses to the present optimization technique. Such an improvement would give information about the relative importance of the stage parameters during the preliminary design of the turbine. The sensitivity analyses can be carried out by means of a control variable which decides whether the program will optimize or will directly give the efficiency related to the input design vector.

APPENDICES

APPENDIX A

TURBINE AERODYNAMIC LOSS ANALYSES

A.1 CORRELATION OF ZWEIFEL

The lift coefficient, C_L , based on tangential blade loading is given by Zweifel [5] as:

$$C_L = 2(s/c_a)(\tan\alpha_o + \tan\alpha_i)\cos^2\alpha_o \quad (\text{A.1})$$

where

c_a = axial chord.

A.2 CORRELATION OF SODERBERG

The nominal stage aerodynamic loss coefficient, Y_t , corresponding to an aspect ratio $h/c_a = 3$ and to a Reynolds number, $Re = 10^5$, is given by Soderberg [6] as a function of the gas deflection, ϵ , that may be expressed as

$$\epsilon = \alpha_o + \alpha_i \quad (\text{A.2})$$

The functional relationship of Y_t and ϵ is given in Fig. A.1.

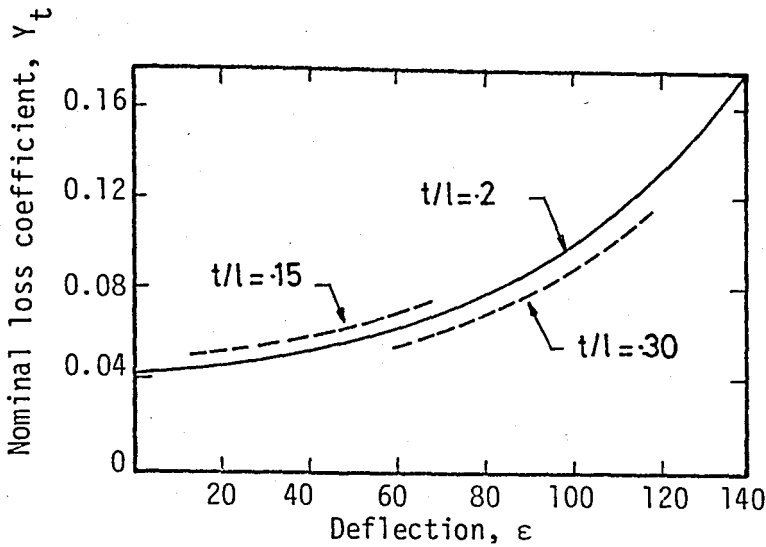


FIGURE A.1 - Soderberg's correlation of the total loss coefficient.

In Fig. A.1, the variable t , is the maximum blade thickness (Fig. 2.1).

If the "corrected" total loss coefficient, $Y_{t,corr}$, of a blade row operating with an aspect ratio other than 3 and at a Reynolds number other than 10^5 , is desired, the following correlation suggested by Soderberg can be used:

$$Y_{t,corr} = \left(\frac{10^5}{Re_h}\right)^{1/4} [(1 + Y_t)(0.975 + 0.075(C_a/h))] \quad (A.3)$$

where

Re_h = Reynolds number based on throat mean hydraulic diameter,

and Y_t = Stage total aerodynamic loss coefficient as given in

Fig. A.1.

A.3 CORRELATION OF STEWART, WHITNEY AND WONG

According to Stewart et.al [11] the diffusion factor for the suction surface, D_s , is given as:

$$D_s = 1 - (V_o/V_{s,max}) \quad , \quad (A.4)$$

where $V_{s,max}$ = maximum gas velocity over the suction surface, and the diffusion factor for the pressure surface, D_p , can be expressed as:

$$D_p = 1 - (V_{p,min}/V_i) \quad (A.5)$$

where $V_{p,min}$ = minimum gas velocity over the pressure surface.

A total diffusion factor, D_t , for a blade passage can therefore be defined using Eq. (A.4) and Eq. (A.5) as follows:

$$D_t = D_s + D_p \quad (A.6)$$

or

$$D_t = (1 - (V_o/V_{s-max})) + (1 - (V_{p-min}/V_i)) \quad (A.7)$$

The total diffusion factor, D_t , can be related to a dimensionless momentum thickness parameters, δ_m/ℓ , as [11],

$$\delta_m/\ell = 0.003/(1 - 1.4D_t) \quad (A.8)$$

where

δ_m = momentum thickness

ℓ = camber length of the blade.

This relationship is also given in Fig. A.2.

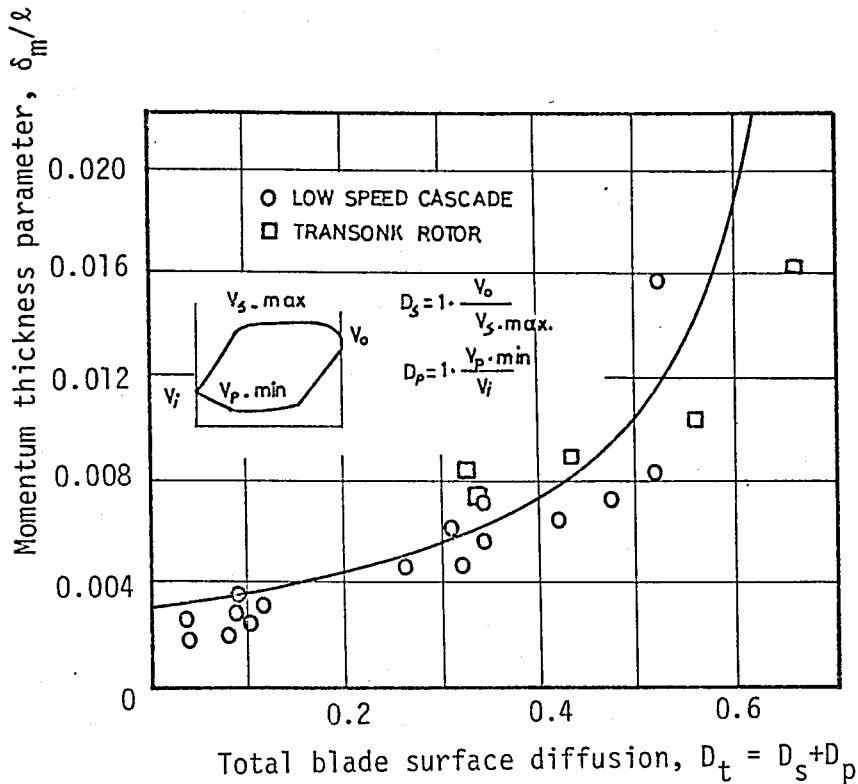


FIGURE A.2 - Correlation of the momentum thickness with the total blade surface diffusion factor.

The total diffusion factor, D_t , can also be related to the tangential lift coefficient, C_L , by the following analysis [12]:

By definition, C_L is given by:

$$C_L = \frac{[(P_o - \frac{1}{2} \rho V_{p-min}^2) - (P_o - \frac{1}{2} \rho V_{s-max}^2)]}{(1/2) \rho V_o^2} \quad (A.9)$$

But, assuming a constant suction surface gas velocity through the blade passage, i.e., assuming

$$V_{s-max} = V_o, \quad (A.10)$$

one can rearrange Eq. (A.9) as follows:

$$C_L = 1 - V_{p-\min}^2/V_0^2 \quad (\text{A.11})$$

On the other hand, combining Eq. (A.7) and Eq. (A.10), D_t can be expressed as:

$$D_t = 1 - \frac{V_{p-\min}}{V_i} \quad (\text{A.12})$$

Considering the velocity triangles as shown in Fig. 3.4, it is seen that

$$(V_i/V_0)^2 = (\cos\alpha_0/\cos\alpha_i)^2 \quad (\text{A.13})$$

and finally knowing that

$$\left(\frac{V_{p-\min}}{V_0}\right)^2 = \left(\frac{V_{p-\min}}{V_i}\right)^2 (V_i/V_0)^2, \quad (\text{A.14})$$

one can obtain, by the combination of Eqs. (A.11), (A.12), (A.13) and (A.14), a correlation of C_L and D_t , in terms of blade inlet and outlet gas angles as:

$$C_L = 1 - (\cos\alpha_0/\cos\alpha_i)^2 (1 - D_t)^2 \quad (\text{A.15})$$

A.4 CORRELATION OF BALJE and BINSLEY

Balje and Binsley [13] assumed a velocity profile of the form:

$$V = V_0 \left[\frac{\sin\beta_0}{\sin\beta_i} + \frac{x}{\ell} \left(1 - \frac{\sin\beta_0}{\sin\beta_i} \right) \right], \quad (\text{A.16})$$

where

x = distance along the camber line

β_0 = blade outlet angle.

This velocity profile is inserted in the Truckenbrodt expression [14], which is given as:

$$\delta_m = \left(\frac{V_0 \delta_{m,0}}{v_0} \right)^{1/n} = V_0^{-3-(2/n)} a \int_{x=0}^{x=l} v^{3+(2/n)} dx \quad (\text{A.17})$$

where

$\delta_{m,0}$ = momentum thickness at blade outlet,

v = kinematic viscosity of the gas,

and a, n = constants.

Resultingly, a relation for the momentum thickness is obtained:

$$\frac{\delta_m}{l} = 0.0021 \left[\frac{1 - \mu_a^{4.5}}{1 - \mu_a} \right]^{0.8}, \quad (\text{A.18})$$

where, the acceleration ratio, μ_a , is defined as:

$$\mu_a = \sin \beta_0 / \sin \beta_i \quad (\text{A.19})$$

A.5 CORRELATION OF BALJE

Balje, in Ref. [15], defines the blade loading, δu , as:

$$\delta u = \cot \alpha_0 - \cot \alpha_i, \quad (\text{A.20})$$

and the equivalent diffusion ratio, D_e , as

$$D_e = [1 + 0.9(t/c) + \frac{0.3 C_L \mu_a^2}{\sqrt{1 + \cot^2 \alpha_m}} \frac{\sin \xi}{\sin \alpha_m}] \frac{1.025}{\mu_a} \quad (\text{A.21})$$

where, also shown in Fig. 3.1,

ξ = blade stagger angle.

In Ref. [15], the relation between the momentum thickness and the equivalent diffusion ratio is given as follows:

$$\frac{\delta_m}{\ell} = \frac{0.013}{2.62 - D_e} - 0.004 \quad (\text{A.22})$$

Using Eqs. (A.20), (A.21) and (A.22), Balje gives the profile clearance and endwall loss coefficients as follows

$$Y_p = \frac{1.93(c/s)(\delta_m/\ell)(\ell/c)(1 + W^2)^{1.5}}{1 - 1.08(c/s)(\delta_m/\ell)(\ell/c)(1 + W^2)^{1.5}} \quad (\text{A.23})$$

where

$$W = \cot\alpha_m + (\delta u/2),$$

$$Y_k = 0.19(k/h)^{1/2}(c/h)^{1/2}\delta u(1 + W\delta/\sqrt{W}) \quad (\text{A.24})$$

and

$$Y_e = Y_p(c/h)C_L \frac{1 + W^2}{2\delta u W} \quad (\text{A.25})$$

where

$$Y_e = \text{endwall loss coefficient.}$$

APPENDIX B

COMPUTER PROGRAM AINLEY

B.1 GENERAL DESCRIPTION

The computer program AINLEY calculates total aerodynamic losses for nozzle and rotor separately, combines the results to determine the total stage loss and also prints the gas flow conditions at each station (inlet and outlet of blade rows), following a fixed mass of gas throughout its trajectory.

The data fed into the program may be divided into two parts:

- i) the data representing the geometrical variables of the turbine stage in hand and the properties of the gas (usually air) which passes through. These data are grouped under the data file TURBO1 and must be fed separately by the user,
- ii) the data corresponding to the figures and graphs presented in Chapter 3, which are used to carry out the necessary interpolations by means of the library subroutines ICS1VE, ICS2VE and ICS1VU. These data form the datafile TUBRO2.

The program AINLEY first calculates gas outlet angle values corresponding to a range of Mach numbers at blade outlet from $M = 0$ to $M = 1$. This is done conformly to the analysis given in Section 3.1.1 and by calling the subroutine SUB1. If the value of the control variable, G , which appears in the argument list of SUB1 is set equal to 1, SUB1 performs the calculations for the stator row, but if G equals 2, then rotor outlet gas angles vs rotor outlet Mach numbers are evaluated. Another correlation is established in the subroutine SUB2, between the stator outlet gas angles and the corresponding stator total loss coefficient, following the theory explained in Section 3.1.2.

Following the choice of an initial guess for the stator outlet gas angle, the stator total loss coefficient and the corresponding value of the non-dimensional mass flow parameter are determined. Then the gas conditions at the outlet of the stator are calculated (Section 3.1.3.1), a check being imposed upon the actual inlet and exit non-dimensional mass flows, for the case in which they would exceed the maximum allowable flow for the stator (choking of the row). If such a situation occurs, the subroutine CHOKe is called, in order to perform the procedure explained in Section 3.1.2. Subroutine CHOKe returns the results to the calling program which uses them as absolute gas conditions at the inlet to the next rotor row.

Once the absolute outlet gas conditions from the stator are determined and the guessed outlet angle checked, an initial value for the rotor outlet gas angle must be chosen and the related rotor total loss coefficient calculated (as for the stator case, subroutine SUB31 calculates the value of the rotor total loss coefficient corresponding to an outlet gas angle). The maximum non-dimensional mass

flow corresponding to this loss coefficient is easily obtained by interpolation. Similarly to the case of stator, the non-dimensional mass flows at inlet and outlet of the rotor are calculated (Section 3.1.3.3) and checked against the maximum flow for any possible choking of the row: if the gas has overexpanded in the row, the subroutine CHOKE is called again.

According to the gas flow conditions at the rotor outlet, the absolute gas conditions at the inlet to the next stage and the total-to-total stage aerodynamic efficiency are calculated. A final correction is applied for the stage mean Reynolds number.

At the final part of the first iteration, the magnitudes of gas properties at each station, the stage efficiency and the efficiency corrected for Reynolds number are printed, the non-dimensional flow quantity is augmented by an increment and a new set of calculations is started for the new value of the mass flow. This procedure is repeated until choking occurs in one of the blade rows.

Once the rotor (or stator) chokes, the subroutine CHOKE takes into account the reasoning given in Section 3.1.5: When choking occurs in a turbine blade passage, no greater quantity of gas can pass through the passage throat. Therefore, the mass flow upstream must remain constant, and the non-dimensional mass flow at turbine inlet is no more incremented in the next loop. In return, the subroutine CHOKE starts to increase the Mach number at the outlet of the choked row until, the outlet Mach number reaches unity. This point represents the fully choked state of the blade passage and the calculations are stopped.

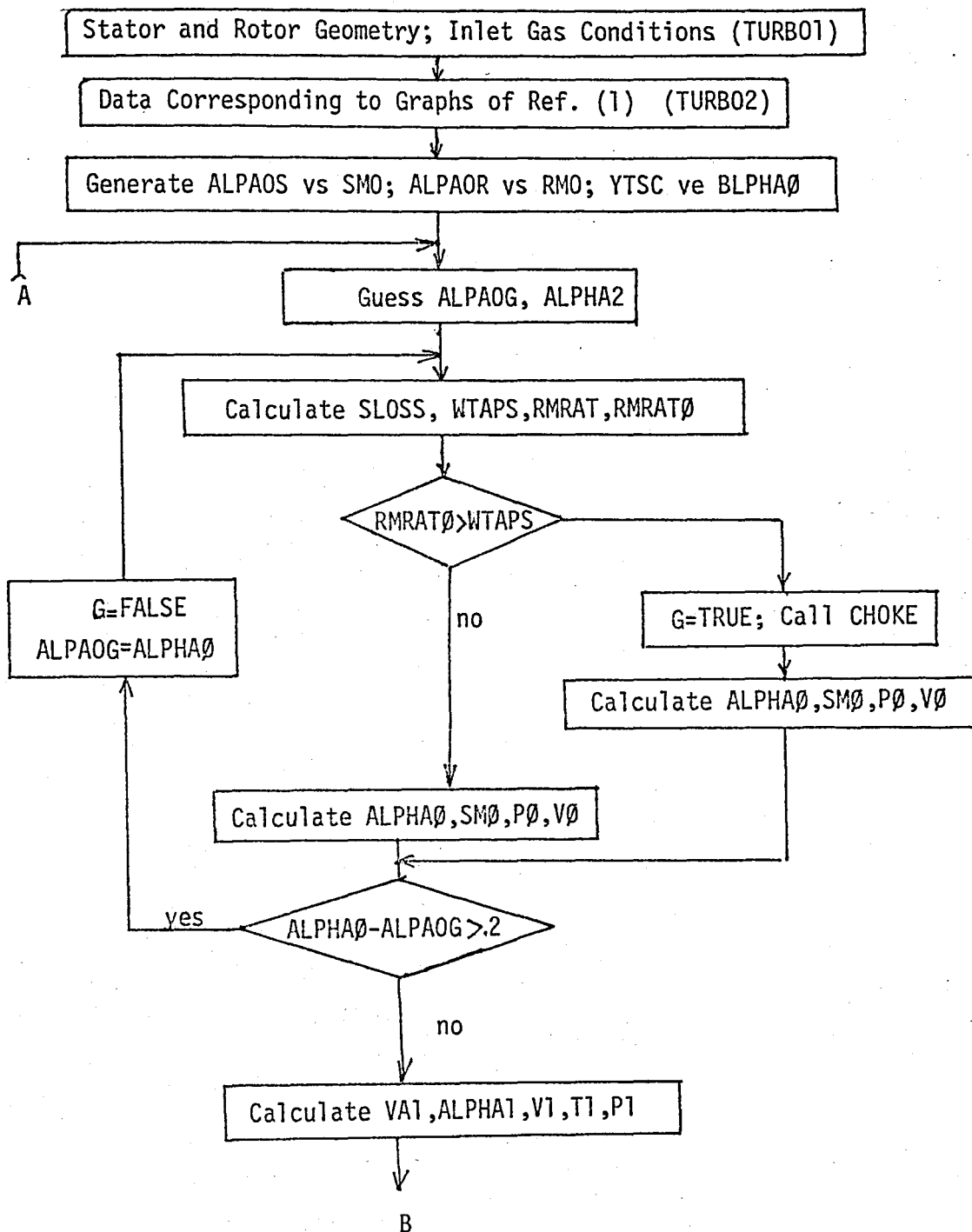


FIGURE B.1 - Flow chart of the computer program "AINLEY"

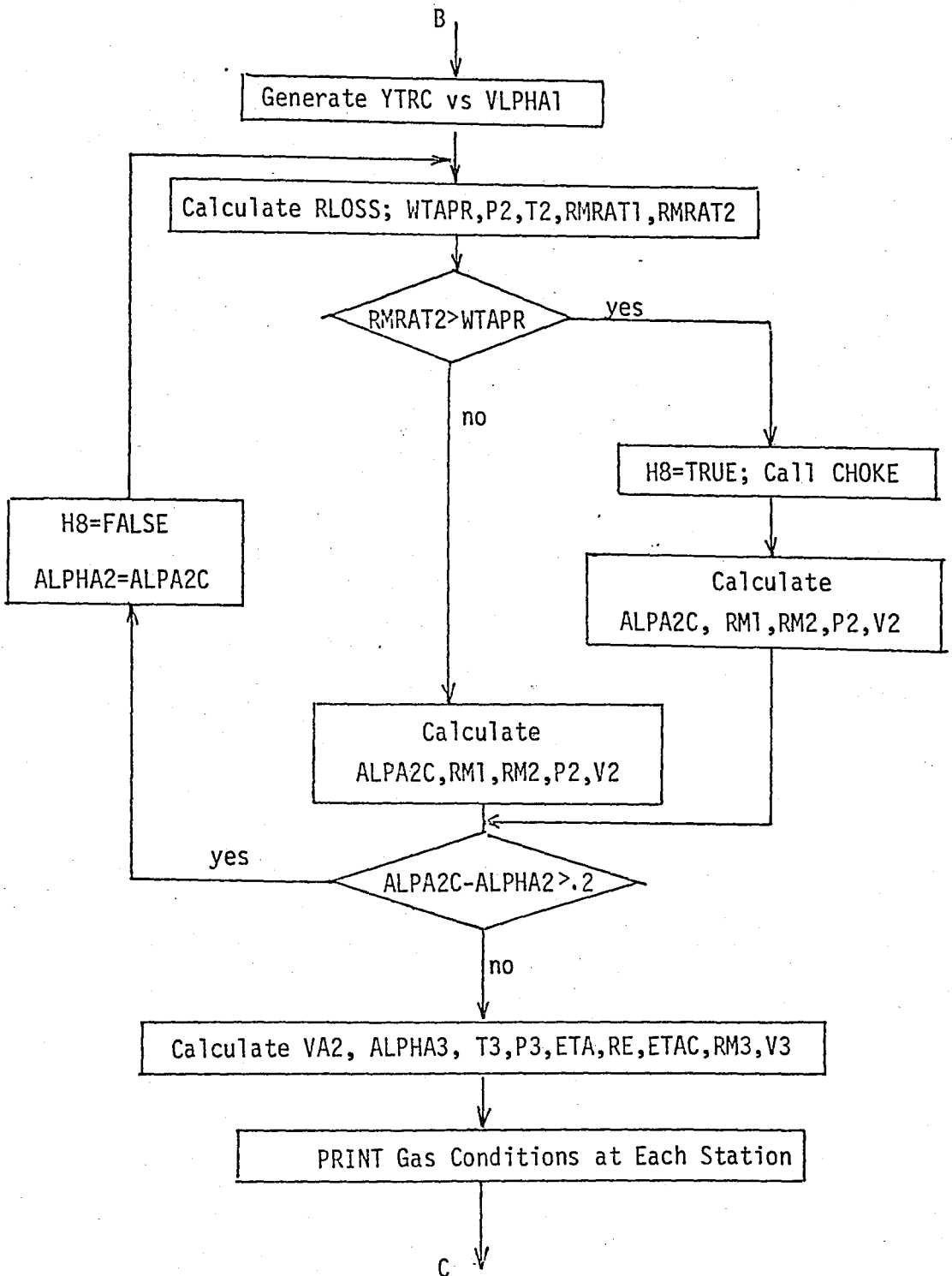


FIGURE B.1 (continued).

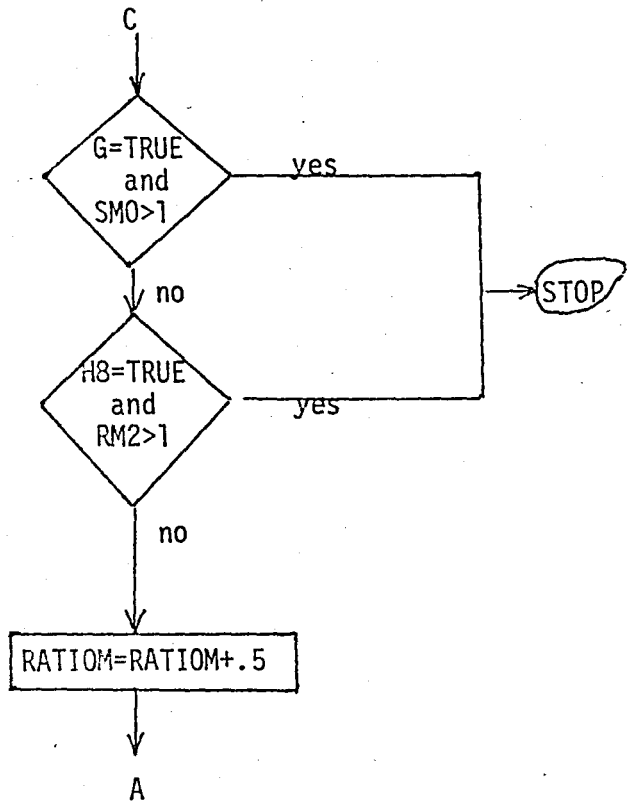


FIGURE B.1 (continued).

In the output, the value of the non-dimensional mass flow for which a row chokes is designated. Also, the overall pressure ratio corresponding to a Mach number of unity at the outlet of the choked row is printed as the limit pressure ratio of the turbine.

B.2 LIST OF THE IMPORTANT VARIABLES OF THE PROGRAM AINLEY

RATIOM	:	Mass flow parameter, $\dot{m}\sqrt{T_1}/P_1$
ALPAOS	:	Vector formed by the stator outlet gas angle values, calculated in SUB1
SMO	:	Vector formed by the stator outlet Mach number values, ranging from zero to unity
RMO	:	Vector formed by the rotor outlet Mach number values, ranging from zero to unity
ALPAOR	:	Vector formed by the rotor outlet gas angle values, calculated in SUB1
YTSC	:	Vector formed by the stator total loss coefficient values, calculated in SUB2
BLPHAØ	:	Vector formed by the stator outlet gas angle values, created in SUB2
ALPAOG	:	Initial guess for α_2
ALPHA2	:	Initial guess for α_4
SLOSS	:	Stator total loss coefficient, Y_N
WTAPS	:	Maximum value of the mass flow parameter, $(\dot{m}\sqrt{T}/AP)_{\max}$, that can be passed through the stator
RMRAT	:	Mass flow parameter, $\dot{m}\sqrt{T_1}/P_1A_1$, at inlet to the stator

- RMRATØ : Mass flow parameter, $\dot{m}/\sqrt{T_2}/P_2A_2$, at outlet from the stator
- G : Logical control variable
TRUE, when stator choked
FALSE, when stator not choked.
- ALPHAØ : Stator outlet gas angle, α_2
- SMØ : Stator outlet Mach number, M_2
- PØ : Total gas pressure, P_2 , at outlet of the stator
- VØ : Absolute gas velocity, V_2 , at outlet of the stator
- VA1 : Axial component of V_2
- ALPHA1 : Gas angle, α_3 , relative to the rotor at inlet
- V1 : Gas velocity, V_3 , relative to the rotor at inlet
- T1 : Total gas temperature, T_3 , at inlet to the rotor
- P1 : Total gas pressure, P_3 , at inlet to the rotor
- YTRC : Vector formed by the rotor total loss coefficient values, calculated in SUB31.
- VLPHA1 : Vector formed by the rotor inlet gas angle values, created in SUB31
- RLOSS : Rotor total loss coefficient, Y_R
- WTAPR : Maximum value of the mass flow parameter, $(\dot{m}/\sqrt{T}/PA)_{\max}$, that can be passed by the rotor
- T2 : Total gas temperature, T_4 , at outlet from the rotor
- P2 : Total gas pressure, P_4 , at outlet from the rotor
- RMRAT1 : Mass flow parameter, $\dot{m}/\sqrt{T_3}/P_3A_3$, at inlet to the rotor

- RMRAT2 : Mass flow parameter, $\dot{m}\sqrt{T_4}/P_4 A_4$, at outlet from the rotor
- H8 : Logical control variable
TRUE , when rotor choked
FALSE, when rotor not choked
- ALPA2C : Gas angle, α_4 , relative to the rotor at outlet
- V2 : Gas velocity, V_4 , relative to the rotor at outlet
- RM1 : Mach number, M_3 , relative to the rotor at inlet
- RM2 : Mach number, M_4 , relative to the rotor at outlet
- VA2 : Axial component of V_4
- T3 : Total temperature, T_1' , at inlet to the next stage
- P3 : Total pressure, P_1' , at inlet to the next stage
- V3 : Absolute velocity, V_1' , at inlet to the next stage
- RM3 : Mach number, M_1' , at inlet to the next stage
- ETA : Total-to-total isentropic efficiency, η , of the stage
- RE : Mean Reynolds number, Re , of the stage
- ETAC : Total-to-total isentropic efficiency corrected for the Reynolds number.

B.3 SUBROUTINE DESCRIPTION

The functional relationship among the subroutines of the program AINLEY is given below:

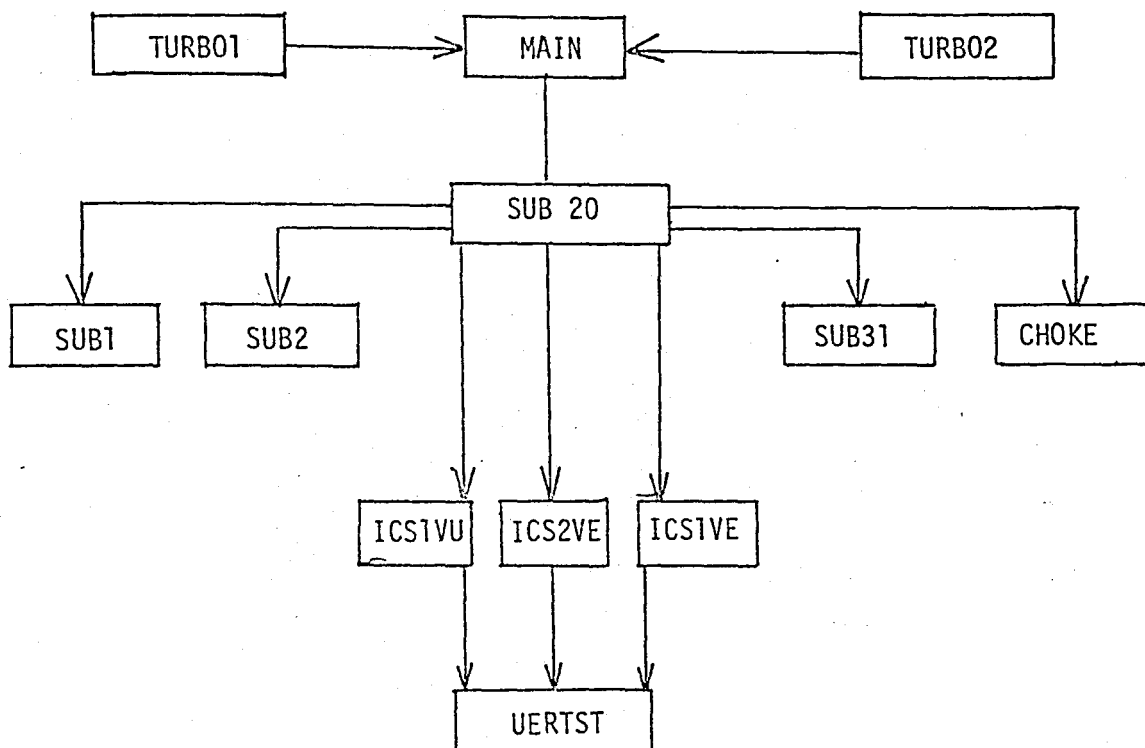


FIGURE B.2 - Functional relationship among the subroutines of the program "AINLEY".

A summary describing the function of each subroutine used in AINLEY is given as follows:

- MAIN : The main program reads the datafiles TURB01 and TURB02 and calculates the blade speed, annulus areas down and upstream of the blade rows, throat areas of the blade passages and works out the non-dimensional geometrical variables of the stage such as the pitch-to-chord ratio, etc. It then transmits the data to SUB20 by means of the "common" declarations.
- SUB20 : Subroutine SUB20 has no argument list, the interconnection with the main program is realized by means of "common declarations". It is SUB20 that perform the procedure described in Section B.1. This subroutine acts as the principal execution control consol.
- SUB1 : Subroutine SUB1 generates the two vectors SMO and ALPAOS which represent the vector of the outlet Mach numbers from a blade and the vector of the related outlet gas angles, respectively. The value of the control variable, G, which appears on the argument list decides whether the blade is a rotor or stator blade. For $G = 1$, SUB1 performs the calculations for a stator blade and returns the vectors ALPAOS and SMO to SUB20. But if $G = 2$, rotor values are calculated and vectors ALPAOR and RMO are returned to the calling program.
- The calculations performed in SUB1 are as explained in Section 3.1.1.

SUB2 : Subroutine SUB2 calculates the stator total loss coefficient as a function of the blade outlet angle, and returns two vectorial quantities to SUB20:BLPHAØ, the set of outlet angles ranging from -70° to -45° and YTSC, the set of total loss coefficients, each corresponding to one element of BLPHAØ.

In SUB2, the calculations are performed according to the theory explained in Section 3.1.2 and the results are returned to SUB20 by means of a common declaration.

SUB31 : The rotor total loss coefficient, Y_R , is calculated in the subroutine SUB31. The important input quantity is the rotor outlet gas angle as guessed in SUB20. The subroutine SUB31 generates rotor inlet gas angles stored in the vector VLPHA1 and calculates the corresponding total loss coefficients stored in the vector YTRC. Then both vectors are returned to SUB20 by means of a common declaration. The calculation of the rotor total loss coefficient is realized through the analysis outlined in Section 3.1.2. Accordingly, if the rotor outlet angle guessed at the beginning differs from that calculated at the end of the calling program, SUB20 uses this latest value of the outlet angle and, by interpolation out of the vector YTRC, finds the corresponding total loss coefficient.

CHOKE : The subroutine CHOKE performs the calculations which are valid only for a choked blade row. Within the subprogram SUB20, if it is found that for any blade row the non-dimensional flow quantity equals or exceeds the maximum allowable non-dimensional flow, then CHOKE is called. The input quantities in the argument list are the total loss coefficient of the row, the inlet pressure and temperature and the non-dimensional mass flow. According to the analysis explained in Section 3.1.5, the performance of the choked row is calculated and the values of the corrected gas outlet angle, Mach number, the outlet gas pressure and velocity related to the blade are returned to SUB20.

ICS1VE, ICS1VU, ICS2VE: These subroutines are available in the Boğaziçi University Computer Center library and they perform the interpolations (one or two dimensional, equally or unequally distributed data) from the figures of Ref. [1] or from the vector quantities generated in SUB1, SUB2 and SUB31. Their usage, function and parameter listings are explained in detail in Section B.5.

B.4 INPUT DESCRIPTION

An explanation of the usage of the program AINLEY is given. The five cards composing the data file TURB01 must be fed in the following sequence:

Card 1 : RA, GAMA, CP

RA : Gas constant (ft. pdls/lb/°C)

GAMA : Ratio of the specific heats

CP : Specific heat at constant pressure (ft.pdls/lb/°C)

Card 2 : TID, TOD, RK

TID : Turbine inner diameter (in)

TOD : Turbine outer diameter (in)

RK : Clearance of the rotor blades (in)

Card 3 : OPS, PITS, RCS, TMAXS, CHOS, TETS, BETTAI, BNS

OPS : Opening, or throat, of the stator blades (in)

PITS : Pitch length of the stator blades (in)

RCS : Mean radius of curvature of the upper stator
blade surface between throat and trailing edge (in)

TMAXS : Maximum thickness of the stator blades (in)

CHOS : Chord length of the stator blades (in)

TETS : Trailing edge thickness of the stator blades (in)

BETTAI: Inlet angle of the stator blades (deg)

BNS : Number of the stator blades

Card 4 : OPR, PITR, RCR, TMAXR, CHOR, TETR, BETTAI, BNR, HEIR, BR

OPR : Opening or throat of the rotor blades (in)

PITR : Pitch length of the rotor blades (in)

RCR : Mean radius of curvature of the upper rotor blade
surface between throat and trailing edge (in)

TMAXR : Maximum thickness of the rotor blades (in)

CHOR : Chord length of the rotor blades (in)
 TETR : Trailing edge thickness of the rotor blades (in)
 BETTA1: Inlet angle of the rotor blades (deg)
 BNR : Number of the rotor blades
 HEIR : Annulus height at reference station (in)
 BR : Constant of Eq. (14)

Card 5 : ALPHA1, TI, PI, RPM, EMRAH

ALPHA1: Gas inlet angle to the stator (deg)
 TI : Absolute total gas inlet temperature to the stator ($^{\circ}\text{K}$)
 PI : Absolute total gas inlet pressure to the stator
 (lbf/in 2)
 RPM : Blade rotative speed (rev.per min)
 EMRAH : Inlet non-dimensional mass flow-initial value
 ($\dot{m}\sqrt{T_1}/P_1$)

The five cards are all in free format. A sample data for forming TURBO1 is given below:

Card 1 : 3073., 1.333, 12300.
 Card 2 : 9.5, 13.0, .030
 Card 3 : .429, .9817, 3.516, .266, 1.33, .0196, 0., 36.
 Card 4 : .447, .7113, 2.000, .143, 0.95, .0071, 36., 50.,
 1.75, .5
 Card 5 : 0., 1100., 40., 14000., 7.

B.5 OUTPUT DESCRIPTION

The description of the output is automatically printed by the computer. The output consists of the results corresponding to each value of the non-dimensional flow parameter, $\dot{m}\sqrt{T_i}/P_i$. The program stops when that value of the pressure ratio for which the stage completely chokes is obtained.

A sample output of the program AINLEY, operating with the data given above is as follows:

*** ROTOR 1 EFFICIENCY = 21.19162305433

 STATOR TOTAL LOSS COEFFICIENT, L(S) = .05762254

 FROTH TOTAL LOSS COEFFICIENT, L(F) = .21572176

 * ABSOLUTE STATOR OUTLET GAS CONDITIONS *

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-83.519246	39.721	1100.0000	884.22546	.422697

 * INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
14.806377	36.226	1074.9781	467.81255	.286595

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575504	35.629	1074.9781	604.99123	.290666

 * ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
-30.423724	31.058	1071.2249	480.45010	.223690

** UNCONNECTED EFFICIENCY = .0690991028409

 * THE EFFICIENCY OF THE STAGE AS GIVEN BY AER 15 ETAT(STAGE) = .34033576 *

NON-DIRECTIONAL MASS FLOW W/SKTIT11/PI = 7.000010
 OVERALL PRESSURE RATIO P3/PI = .476688

STATOR TOTAL LOSS COEFFICIENT, L(51) = .05702254

ROTOR TOTAL LOSS COEFFICIENT, L(10) = .19140632

 * ABSOLUTE STATOR OUTLET GAS CONDITIONS *

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET PACH NUMBER
-63.514295	39.682	1100.0700	963.55292	.462466

 * INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET PACH NUMBER
22.149011	35.659	1071.0109	464.00828	.314104

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET PACH NUMBER
-47.575304	35.233	1071.0109	621.25117	.311402

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET PACH NUMBER
-47.575304	35.233	1071.0109	621.25119	.311402

 * ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET PACH NUMBER
-21.940541	54.178	1162.9126	604.06706	.732497

95 UNCORRECTED EFFICIENCY = .8750214164211

 * THE EFFICIENCY OF THE STAGE AS GIVEN BY AEG IS STAG14GL = .85474936 *

NON-DIMENSIONAL MASS FLOW W/SORT111/PI = 7.5000000
 OVERALL PRESSURE RATIO P7/PI = .8544550

*** ROTOR INCIDENCE = -7.576978571095

 STATOR TOTAL LOSS COEFFICIENT, L(S) = .05702254

 ROTOR TOTAL LOSS COEFFICIENT, L(R) = .18241619

 * ABSOLUTE STATOR OUTLET GAS CONDITIONS *

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-53.525449	39.622	1190.0000	1051.92513	.507072

 * INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
28.423021	35.026	1066.6334	532.77063	.344092

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575334	34.534	1066.6334	724.75945	.397427

 * ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
-17.294170	33.169	1055.9407	506.38671	.244103

*** UNCORRECTED EFFICIENCY = .8764146575703

 * THE EFFICIENCY OF THE STAGE AS GIVEN BY AEM IS LTA(STAGE) = .45695356 *

NON-DIMENSIONAL MASS FLOW W/SORT(T11/P1) = 8.0000000
 OVERALL PRESSURE RATIO P3/P1 = .5292301

*** ROTOR INCIDENCE = -2.32519d263076

STATOR TOTAL LOSS COEFFICIENT, L(S) = .05702254

ROTOR TOTAL LOSS COEFFICIENT, L(R) = .16967657

 * ABSOLUTE STATOR OUTLET GAS CONDITIONS *

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-03.520743	39.456	1100.0000	1147.07506	.552944

 * INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
33.674902	34.333	1061.8024	613.28575	.377738

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-7.575304	53.745	1061.8024	754.37820	.385899

 * ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
-10.555783	32.052	1048.2384	542.10456	.259054

** UNCORRECTED EFFICIENCY = .8741267615209

 * THE EFFICIENCY OF THE STAGE AS GIVEN BY AEN IS ETA(STAGE) = .60245349 *

NON-DIMENSIONAL MASS FLOW $W/S0C(T01)/PI = 8.5000000$
 OVERALL PRESSURE RATIO $P3/P1 = .9013103$

*** ROTOR INCIDENCE=2.411946932934

STATOR TOTAL LOSS COEFFICIENT, L(S)= .0571393

ROTOR TOTAL LOSS COEFFICIENT, L(R)= .21055816

 * ABSOLUTE STATOR OUTLET GAS CONDITIONS *

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-63.645719	39.467	1100.0000	1262.97977	.613612

 * INLET & OUFLET GAS CONDITIONS (RELATIVE TO ROTOR) *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
31.411947	33.514	1055.9657	715.45332	.41535

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575304	32.723	1055.9657	877.34418	.428393

 * ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
-3.627610	30.666	1031.9799	556.93296	.264773

*** CALCULATED EFFICIENCY= .3637467401073

 * THE EFFICIENCY OF THE STAGE AS GIVEN BY AIN IS ETA(STAGE)= .59035416 *

NON-DIMENSIONAL MASS FLOW W*ORTITII/PI = 9.000000
 OVERALL PRESSURE RATIO P3/PI = .7666563

*** ROTOR INCIDENCE = 0.300000020488

 STATOR TOTAL LOSS COEFFICIENT, L(S) = .05718393

 ROTOR TOTAL LOSS COEFFICIENT, L(R) = .26443306

 * ABSOLUTE STATOR OUTLET GAS CONDITIONS *

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-43.763704	37.347	1100.0000	1391.58431	.679431

 * INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
47.300069	32.630	1049.4818	832.92254	.459163

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575304	31.547	1049.4818	976.09657	.450175

 * ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
2.844077	29.069	1028.4233	625.08876	.380051

** UNCORRECTED EFFICIENCY = .6507111441153

 * THE EFFICIENCY OF THE STAGE AS GIVEN BY ZEN IS LTA(1STAGE) = .839-2204 *

ADI-DIMENSIONAL MASS FLOW W*SQRT(T11)/P1 = 9.5000000
 OVERALL PRESSURE RATIO P3/P1 = .7272169

*** ROTOR INCIDENCE=10.55017639105

 STATOR TOTAL LOSS COEFFICIENT, L(S)= .01744417

 ROTOR TOTAL LOSS COEFFICIENT, L(R)= .31536691

 * ABSOLUTE STATOR OUTLET GAS CONDITIONS *

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-23.845019	39.214	1100.0300	1589.75521	.765517

 * INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
46.550178	31.264	1039.4707	1018.97027	.523644

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.920973	29.413	1039.4707	1154.40119	.574113

 * ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
12.307042	20.297	1010.7942	727.35719	.389255

** UNCORRECTED EFFICIENCY= .815262230223

 * THE EFFICIENCY OF THE STAGE AS GIVEN BY AER IS ETA(STAGE)= .8071551 *

NON-DIMENSIONAL MASS FLOW *50RTETI/P1 =10.007000
 OVERALL PRESSURE RATIO P3/P1 = .6574291

STATOR TOTAL LOSS COEFFICIENT, L(S)= .05744417

ROTOR TOTAL LOSS COEFFICIENT, L(R)= .35820546

 * ABSOLUTE STATOR OUTLET GAS CONDITIONS *

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-53.911016	30.169	1100.0000	1695.51261	.943548

 * INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
46.254422	30.541	1034.1188	1119.35688	.560687

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-48.265683	28.217	1034.1188	1272.24139	.638693

 * ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
17.200630	24.698	1000.2725	850.92153	.440742

NO UNCORRECTED EFFICIENCY= .7989506260921

 * THE EFFICIENCY OF THE STAGE AS GIVEN BY /EM IS ETA(STAGE)= .79348611 *

NON-DIMENSIONAL MASS FLOW $\sqrt{P3/PI} = 10.1851964$
 OVERALL PRESSURE RATIO $P3/PI = .6174556$

THE MAXIMUM NON-DIMENSIONAL FLOW WHICH PASSES THROUGH THIS STATOR IS EQUAL TO 10.2114399

THE MAXIMUM NON-DIMENSIONAL MASS FLOW WHICH PASSES THROUGH THIS ROTOR IS EQUAL TO 10.1351964

*** CORRECTED BLADE OUTLET ANGLE =38.56165143591
 *** ROTOR INCIDENCE=12.2942291448

STATOR TOTAL LOSS COEFFICIENT, L(S) = .05744417

ROTOR TOTAL LOSS COEFFICIENT, L(R) = .35829546

 * ABSOLUTE STATOR OUTLET GAS CONDITIONS *

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-33.911618	39.106	1100.6800	1695.51251	.743846

 * INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
48.254422	30.541	1034.1168	1119.85968	.560687

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-31.420062	26.219	1034.1168	1905.24477	1.000000

 * ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
34.039794	20.308	971.0354	1322.66779	.722066

96 UNCORRECTED EFFICIENCY = .7577615025438

 * THE EFFICIENCY OF THE STAGE AS GIVEN BY ACP IS ETASTAGE2 = .75949621 *

1D-6-DIMENSIONAL MASS FLOW P/SCRT(T11/P1) = 16.1551964
 OVERALL PRESSURE RATIO P2/P1 = .5074976

ROTOR MACH # EXCEEDS UNITY BEYOND PRESSURE RATIO = .5074976

B.6 PROGRAM LISTING OF "AINLEY"

```

1      PROGRAM AINLEY(INPUT,OUTPUT,TURBO1,TAPE8=TURBO1,
2      #                                     TURBO2,TAPE5=TURBO2,
3      #                                     RESULT,TAPE6=RESULT)
4      COMMON/D55/VWOP(51,2),VWTAP(51,2),RRN(80,2),VWON(100),VVOT(100)
5      COMMON/S11/RA,RINCR,VO
6      COMMON/S12/TI,IF
7      COMMON/S13/DOSS,SOES
8      COMMON/S14/AND,AN2,ATS,ATR
9      COMMON/S17/B
10     COMMON/S18/DOSR,SOER
11     COMMON/S19/RK,RKOH
12     COMMON/S99/TID,TOD
13     COMMON/S21/YP1(17,8),YP2(15,6),STAIN(21,9),DELSIS(31,4),
14     #      YP9YPS(100),SLAN(100)
15     COMMON/S22/BETTA1,ALPHA1
16     COMMON/S23/TOCS,SOCS,TOSS,SR,CS
17     COMMON/S24/BLPHA1(100),YTSC(100),SAS,M,PF
18     COMMON/S15/ALPAOS(500),SMO(500)
19     COMMON/S31/SOCR,TOCR,TOSR,BR,RR,CR
20     COMMON/S32/VLPHA1(500),YTRC(500),BETTA1
21     COMMON/S33/K,KF
22     COMMON/S34/ALPHA2,A2,BE
23     COMMON/S55/PIN
24     COMMON/D56/RPH,U,ALPAOG,SLOSS,PI,CP
25     COMMON/S77/GAMA
26     COMMON/S88/VWU(100),VRO(100)
27     COMMON/C11/CKTAP(500),DPOP(500),OVOT(500),URMN(500),BWTAP(500)
28     COMMON/C12/RK(100),H(500)
29     COMMON/E13/ENRAH
30     COMMON/F14/SINCR
31     COMMON/P1/CHOS,HEIR,CHOR
32     READ(8,*)FA,GAMA,CP
33     READ(8,*)TID,TOD,RK
34     READ(8,*)OPS,PITS,RCS,TMAXS,CHOS,TETS,BETTA1,BNS
35     READ(8,*)OPR,PITR,RCK,TMAXR,CHOR,TETR,BETTA1,BNR,HEIR,BR
36     READ(8,*)ALPHA1,TI,PI,RPH,ENRAH
37     READ(5,*)M,P,K,SAS,RINCR,VO
38     READ(5,1)((YP1(I,J),I=1,17),J=1,8)
39     1 FORMAT(8(16F5.3,/,),8F5.3)
40     READ(5,2)((YP2(I,J),I=1,15),J=1,6)
41     2 FORMAT(5(16F5.3,/,),10F5.3)
42     READ(5,3)((STAIN(I,J),I=1,21),J=9,1,-1)
43     3 FORMAT(9(20F4.1,/,),9F4.1)
44     READ(5,4)(YP9YPS(I),I=1,30)
45     4 FORMAT(20F4.2,/,10F4.2)
46     READ(5,5)((DELSIS(I,J),I=1,31),J=1,4)
47     5 FORMAT(18F4.2,/,13F6.2,/,16F5.2,/,5(13F6.2,/,),12F6.2)
48     READ(5,*)(SLAN(I),I=1,41)
49     READ(5,*)(RRN(I,J),I=1,80),J=1,2)
50     READ(5,*)(VWTAP(I,J),I=1,51),J=1,2)
51     READ(5,*)(VWOP(I,J),I=1,51),J=1,2)
52     READ(5,*)(VVOT(I),I=1,80)
53     READ(5,*)(VWON(I),I=1,51)
54     READ(5,*)(VWU(I),I=1,34)
55     READ(5,*)(VRO(I),I=1,34)
56     READ(5,*)(UNTAP(I),I=1,60)
57     READ(5,*)(DPOP(I),I=1,60)
58     READ(5,*)(OVOT(I),I=1,60)
59     READ(5,*)(URMN(I),I=1,60)
60     READ(5,*)(BWTAP(I),I=1,50)
61     PIN=ATAH(1.)*4.
62     DOSS=OPS/PITS
63     SOES=PITS/RCS
64     TOCS=TMAXS/CHOS
65     SOCS=PITS/CHOS
66     TOSS=TETS/PITS
67     DOSS=OPR/PITR
68     SOER=PITR/RCK
69     TOCR=TMAXR/CHOR
70     SOCR=PITR/CHOR
71     TOSR=TETR/PITR
72     SR=TID/TOD
73     RR=TID/TOD
74     RKOH=RK/HEIR
75     ALPAOG=-1.17*ACOS(DOSS)*180./PIN+12.58-4.*SOES
76     CS=CHOS/12.
77     CR=CHOR/12.
78     B=BETTA1*PIN/180.
79     AND=(TOD*2.-TID*2.)*PIN/4.
80     AN2=AND

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81      U=-1.*PIN*(TID+TOD)*RPH/1440.
82      ATS=12.*DOSS*SOCS+CS+BNS*(TOD-TID)/2.
83      ATR=12.*DOSSR*SUCR+CR+BNR*(TOD-TID)/2.
84      CALL SUB20
85      STOP
86      END
87
88      C
89      C
90      C
91      C
92      SUBROUTINE SUB1(G,ALPA0S,SMO)
93      DIMENSION ALPA0S(500),SMO(500)
94      COMMON/S11/RA,RINCRC,VOO
95      COMMON/S12/T1,NF
96      COMMON/S13/DOSS,SOES
97      COMMON/S14/ANO,AN2,ATS,ATR
98      COMMON/S17/B
99      COMMON/S18/DOSSR,SOER
100     COMMON/S19/RK,RKOH
101     COMMON/S99/T10,T00
102     COMMON/S55/PIN
103     COMMON/S77/GAMA
104     ALPHA(A1,A2)=-1.17*ACOS(A1)*180./PIN+12.58-4.*A2
105     CO=(GAMA+A2*TI)*(.5)
106     IF(CO.GT.1.) THEN
107     E=-COS(CO-15./G)
108     ALPA02=E*180./PIN
109     ALPA01=ALPHA(DOSS,SIN 5)
110     END IF
111     IF(CO.EC.2.) THEN
112     D=ALPHA(DOSSR,SOER)*PIN/180.
113     F=ATAN((1.-1.35*AKOH)+COS(B)/COS(D))+ATAN((1.-1.35*AKOH)+COS(B)/COS(D))
114     # +TAN(B))
115     ALPA01=F*180./PIN
116     ATR=ATR*(1.-RKOH)+PIN*TOD*RK
117     ALPA02=-/COS(ATR/AN2)*180./PIN
118     END IF
119     SMO(1)=0.
120     DO 22 I=1,500
121     1 IF(SMO(I).GT.1.)GOTO 8
122     IF(SMO(I).GT..5)GOTO 3
123     ALPA0S(I)=ALPA01
124     GO TO 7
125     3 IF(SMO(I).EQ.1.)GOTO 4
126     ALPA0S(I)=2.*(ALPA02-ALPA01)+SMO(I)+2.*ALPA01-ALPA02
127     GO TO 7
128     4 ALPA0S(I)=ALPA02
129     7 SMO(I+1)=SMO(I)+.01
130     22 CONTINUE
131     8 NF=I-1
132     9 RETURN
133     END
134
135     C
136     C
137     C
138     C
139     SUBROUTINE SUB2
140     COMMON/S21/YP1(17,8),YP2(15,6),STAIN(21,9),DELSIS(31,4),
141     # YP3YPS(100),SLAR(100)
142     COMMON/S22/BETTAI,ALPHAI
143     COMMON/S23/TUCS,SOCS,TOSS,SE,CS
144     COMMON/S14/ANO,AN2,ATS,ATR
145     COMMON/S24/BLPHAG(100),YTSC(100),SAS,M,NF
146     COMMON/S55/PIN
147     COMMON/S17/B
148     COMMON/P1/CHOS,HEIR,CHOR
149     DIMENSION WK(100),YPBIO(200),YPIA(200),ALPASC(100),STSCS(100),DEL
150     #TIS(90),STINS(90),YPS(90),G(90),YPSIO(90),AMS(100),ADS(100),YSS(10
151     #0),AD(100),SF(100),YTS(100),Y(100)
152     TFFAC=21.6*TOSS**2.+3.86*TOSS+.91
153     AIS=ALPHAI*PIN/180.
154     AT=ANO*COS(B)
155     ALPHAO(1)=SAS
156     GO TO 10
157     9 BLPHAO(1)=SAS+1.
158     10 XG=SOCS
159     DO 11 I=1,K
160     IF(BLPHAO(1).GT.-45.)GOTO 14

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```

161 Y(I)=BLPHAO(I)
162 CALL ICS2VE(YP1,X0,.30,1.10,-80.,-45.,17,8,1,17,WK,Y(I),IER)
163 YPB10(I)=Y(I)
164 Y(I)=BLPHAO(I)
165 CALL ICS2VE(YP2,X0,.30,1.00,-70.,-45.,15,6,1,15,WK,Y(I),IER)
166 YPB1A(I)=Y(I)
167 YPS10(I)=(YPB10(I)+((BETTA1/(BLPHAO(I))*2)*(YPB1A(I)-YPB10(I)))*
168 * (TUCS/.2)*(-BETTA1/BLPHAO(I))))
169 ALPASC(I)=(-.35*SOCS+1.25)*BLPHAO(I)
170 Y(I)=BLPHAO(I)
171 X0=BETTA1/ALPASC(I)
172 CALL ICS2VE(STAIN,X0,-1.0,1.0,-70.,-30.,21,9,1,21,WK,Y(I),IER)
173 S1SCS(I)=Y(I)
174 X0=SOCS
175 Y(I)=BLPHAO(I)
176 CALL ICS2VL(DELSTIS,X0,.40,1.0,-70.,-40.,31,4,1,31,WK,Y(I),IER)
177 DELTIS(I)=Y(I)
178 STINS(I)=S1SCS(I)+DELTIS(I)
179 G(I)=(ALPHA1-BETTA1)/STINS(I)
180 CALL ICS1VE(YPGYPS,-4.2,1.6,0.,0.,0.,0.,30,1,G(I),PSCOND,IER)
181 YPS(I)=G(I)*YPS10(I)
182 AOS(I)=BLPHAO(I)*PIN/180.
183 AMS(I)=ATAN((TAN(AIS)+TAN(AOS(I)))/2.)
184 AO(I)=AM0*COS(AOS(I))
185 SF(I)=(AO(I)/A1)**2./(1.+SK)
186 IF(SF(I).GT.0.5)GOTO 9
187 IF(SF(I).LT.0.0)GOTO 12
188 CALL ICS1VE(SLAN,0.0,0.5,0.,0.,0.,0.,41,1,SF(I),PSCOND,IER)
189 DCF=.0534*(CHGS/HEIR)*(COS(AOS(I))/COS(B))
190 YSS(I)=DCF*(2.*(TAN(AIS)-TAN(AOS(I)))*COS(AMS(I))**2.+COS(AOS(I))
191 * **2./COS(AMS(I))**3.
192 YTS(I)=YPS(I)+YSS(I)
193 IF(TUSS.EQ.0.02) GOTO 8
194 YTSC(I)=YTS(I)*TEFAC
195 GO TO 11
196 8 YTS(I)=YTS(I)
197 11 BLPHAO(I+1)=BLPHAO(I)+1.
198 GO TO 14
199 12 WRITE(6,13)
200 13 FORMAT(//,20X,'ERROR IN FORMULATION OF SF(I)')
201 14 MF=I-1
202 RETURN
203 END

```

C
C
C
C
C
C

```

204
205
206
207
208
209 SUBROUTINE SUB31
210 COMMON/S21/YP1(17,8),YP2(15,6),STAIN(21,9),DELSTIS(31,4),
211 * YPYPS(100),SLAN(100)
212 COMMON/S31/SOCR,TOCR,TOSR,BR,RR
213 COMMON/S14/AM0,AM2,ATS,ATR
214 COMMON/S32/VLPHAL(500),YTRC(500),BETTA1
215 COMMON/S53/W,K,FE
216 COMMON/S34/ALPHOR,A2,FE
217 COMMON/S55/PIN
218 COMMON/S18/DOSR,SOLR
219 COMMON/S19/PK,PKOH
220 COMMON/P1/CHGS,HEIR,CHOR
221 DIMENSION RINS(500),E(500),YPR(500),F(500),AMR(500),YSKR(500),
222 * YTR(500),KK(500)
223 TEFAC=21.6*TOSR**2.+3.86*TOSR+.91
224 ALPSCR=(-.35*SOCS+1.25)*ALPHOR
225 B=BETTA1*PIN/180.
226 A1=AM0*COS(B)
227 D=(A2/A1)**2./(1.+RR)
228 A01=ALPHOR
229 A02=ALPHOR
230 A03=ALPHOR
231 A05=ALPHOR
232 A04=BETTA1/ALPSCR
233 CALL ICS2VL(YP1,SOCR,.30,1.10,-80.,-45.,17,8,1,17,WK,A01,IER)
234 CALL ICS2VL(YP2,SOCR,.30,1.00,-70.,-45.,15,6,1,15,WK,A02,IER)
235 CALL ICS2VL(STAIN,A04,-1.0,1.0,-70.,-30.,21,9,1,21,WK,A03,IER)
236 CALL ICS2VL(DELSTIS,SOCR,.40,1.0,-70.,-40.,31,4,1,31,WK,A05,IER)
237 CALL ICS1VE(SLAN,0.0,0.5,0.,0.,0.,0.,41,1,D,PSCOND,IER)
238 YPB10=(A01+(BETTA1/ALPHOR)**2.*(A02-A01))*(TUCR/.2)**(-BETTA1/
239 * ALPHOR)
240 STINR=703+AG5

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241      VLPHA1(1)=BETTA1-4.2*STINP
242      GO TO 2
243      1 VLPHA1(1)=VLPHA1(1)+.2
244      2 DO 10,I=1,K
245          RINS(I)=- (BETTA1-VLPHA1(I))
246          F(I)=RINS(I)/STINR
247          IF(F(I).LT.-4.2)GOTO 1
248          IF(F(I).GT.1.6)GO TO 11
249          CALL ICSIVF(YPOYPS,-4.2,1.6,0.,0.,0.,0.,30,1,F(I),PSCOND,IER)
250          ASAF=F(I)
251          YPR(I)=F(I)*YPRIO
252          F(I)=VLPHA1(I)*PIR/180.
253          ANR(I)=ATAN((TAN(F(I))+TAN(EE))/2.)
254          DCFK=.0334*(CHKR/HEIR)*(COS(EE)/COS(8))
255          *NR*(CHOR/HEIR)*(KK/CHOR)**.76
256          YSKR(I)=DCFK*(2.*(TAN(F(I))-TAN(EE))*COS(ANR(I))**.2.*(COS(EE))
257          *2./COS(ANR(I))**.3.
258          YTK(I)=YPR(I)+YSKR(I)
259          IF(TUSE.F0..02)GOTO 7
260          YTRC(I)=YTK(I)*TEFAC
261          GOTO 10
262      7 YTRC(I)=YTK(I)
263      10 VLPHA1(I+1)=VLPHA1(I)+1.
264      11 *F=I-1
265          RETURN
266      END
267      C
268      C
269      C
270      C
271      C
272      SUBROUTINE SUB20
273      DIMENSION ALPABS(500),SMD(500),ALPADR(500),RMN(500)
274      COMMON/055/VWDP(51,2),VWTAP(51,2),RMN(60,2),VWDR(100),VVGT(100)
275      COMMON/S11/EA,F INCR,VO
276      COMMON/S12/I1,WF
277      COMMON/S13/OUSS,SOES
278      COMMON/S14/AN0,AN2,ATS,ATR
279      COMMON/S17/B
280      COMMON/S18/OSR,SDER
281      COMMON/S19/RK,KKOH
282      COMMON/S99/TID,TOD
283      COMMON/S21/YP1(17,8),YP2(15,6),STAIN(21,9),DELSIS(31,4),
284      * YPOYPS(100),SLAN(100)
285      COMMON/S22/BETTA1,ALPHA1
286      COMMON/S23/TGCS,SOCS,TGSS,SR,CS
287      COMMON/S24/BLPHAO(100),YTSC(100),SAS,N,KF
288      COMMON/S31/SOCR,TOCR,TUSK,BR,RR,CR
289      COMMON/S32/VLPHA1(500),YTPC(500),BETTA1
290      COMMON/S33/K,KF
291      COMMON/S34/ALPHA2,A2,EE
292      COMMON/S55/PIN
293      COMMON/D56/KPM,U,ALPAGG,SLOSS,PI,CP
294      COMMON/S77/GAMA
295      COMMON/S88/VBU(100),VRG(100)
296      COMMON/C11/GATAP(500),OPOP(500),OVUT(500),ORMN(500),BHTAP(500)
297      COMMON/C12/WK(100),H(5000)
298      COMMON/C13/G,H1,H2,PUS,H3,H4,H8
299      COMMON/C25/TRDAT
300      COMMON/E13/EMRAH
301      COMMON/E14/SINCRE
302      LOGICAL G,H1,H2,PUS,H3,H4,H6,H7,H8,H9,H10,H11
303      II=0
304      JJ=0
305      KK=0
306      LL=0
307      PUS=.FALSE.
308      G=.FALSE.
309      H1=G
310      H6=G
311      H7=G
312      H9=G
313      H10=G
314      H11=G
315      RATIO4=EMRAH
316      C
317      C
318      C      **** GENERATE BLADE OUTLET ANGLES VS. MACH NUMBERS ****
319      C
320      C

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```

321          CALL SUB1(1.,ALPAOS,SMO)
322          CALL SUB1(2.,ALPAOR,RMO)
323      C
324      C
325      C      **** GENERATE STATOR LOSS VS. OUTLET ANGLES ****
326      C
327      C
328          CALL SUB2
329      C
330      C
331      C      **** GUESS STATOR OUTLET ANGLE FROM THE START ****
332      C
333      C
334          AA=ACOS(OOSR)
335          ALPHA2=-1.17*AA*180./PIN+12.49-4.*SOER
336      C
337      C
338          ALPAOG=-1.17*ACOS(OOSS)*180./PIN+12.58-4.*SOES
339      C      6 SLOSS=ALPAOG
340          CALL ICS1VU(YTSC,PLPHAO,RF,1,SLOSS,H,IER)
341          STALUS=SLOSS
342          WTAPS=GAMA
343      C
344      C
345      C      **** FIND MAX. NON-DIM. FLOW THROUGH STATOR ****
346      C
347          CALL ICS2VE(VHTAP,SLOSS,0.,1.,1.333,1.3963,51,2,1,51,WK,WTAPS,IER)
348          AXFL=WTAPS
349      C
350      C
351          BB=ALPAOG*PIN/180.
352          AO=AND*CGS(BB)
353          GO TO 290
354      C
355      C      275 RATION=RATION-.05
356          GO TO 290
357      C
358      C
359      C      280 RATION=RATION-SINCR
360      C      290 TMDAT=RATION*PI/SQRT(TI)
361          RMRAT=TMDAT*SQRT(TI)/(AO*PI)
362          WDS=RMRAT/WTAPS
363      C
364      C
365      C      **** CHECK INLET MASS FLOW TO STATOR ****
366      C
367      C
368          IF(WDS.GT.1.) THEN
369              II=II+1
370              GO TO 275
371          END IF
372      C
373      C
374      C
375      C
376          IF(G) GO TO 325
377      C      300 WOPCS=GAMA
378          CALL ICS2VE(VWOP,SLOSS,0.,1.,1.333,1.3963,51,2,1,51,WK,WOPCS,IER)
379          CALL ICS1VE(VWGW,0.,1.,0.,0.,0.,0.,51,1,WOWS,PSCOND,IER)
380          WBAR5=WOPCS*WOWS*PI
381          PO=PI-WBAR5
382          RMRATO=TMDAT*SQRT(TI)/(AO*PO)
383      C
384      C
385      C      **** CHECK CHOKE ON STATOR ****
386      C
387      C
388          W1=ABS(RMRATO/WTAPS-1.)
389          IF(W1.GT..01)GO TO 330
390          G=.TRUE.
391          IF(LL.NE.0) GO TO 324
392          SMAXFL=RATION
393      C      324 GO TO 331
394      C      325 H8=.FALSE.
395          CALL CHOKE(SLOSS,PI,TI,AO,AND,ALPHAO,SMO,RATION,PO,VO)
396          IF(PUS.OR.H2) STOP
397          AGAH1=SPG
398          AGAH=ALPHAO
399          GO TO 326
400      C

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```

401 C
402 C **** CHECK OUTLET MASS FLOW FROM STATOR ****
403 C
404 C
405 330 IF(RMRATO.GT.WTAPS) THEN
406 JJ=JJ+1
407 DIFFER=1.-(WTAPS/RMRATO)
408 RATION=RATION-DIFFER
409 GO TO 290
410 END IF
411 C
412 C
413 331 SMG=RMRATO
414 CALL ICS1VU(GRMN,BWTAP,50,1,SMO,H,IER)
415 VOTO=SMO
416 CALL ICS1VE(VVOT,.02,1.2,0.,0.,0.,0.,60,1,VOTO,PSCOND,IER)
417 VELDCI=VOTO
418 ALPHAO=SMO
419 CALL ICS1VU(ALPAOS,SMO,NF,1,ALPHAO,H,IER)
420 C
421 C
422 DIFF1=ABS(ALPAOG-ALPHAO)
423 IF(DIFF1.LT..1)GOTO 7
424 ALPAOG=ALPHAO
425 G=.FALSE.
426 GO TO 6
427 7 VO=VOTO*SQRT(TI)
428 C
429 C
430 C **** START CALCULATIONS ON ROTOR GAS CONDITIONS ****
431 C
432 C
433 326 CC=ALPHAO*PIN/180.
434 VA1=VO*COS(CC)
435 DD=ATAN(U/VA1)-TAN(CC)
436 ALPHA1=DD*180./PIN
437 V1=VA1/COS(DD)
438 T1=T1-(VO**2.*(1.-(COS(CC)/COS(DD))**2.))/(2.*CP)
439 T2=T1
440 P1=PG*(T1/T1)**(GAMA/(GAMA-1.))
441 IF(N7) GO TO 825
442 1 EF=ALPHA2*PIN/180.
443 A2=AN2*COS(EF)
444 C
445 C
446 C **** GENERATE ROTOR LOSS VS. INCIDENCE ANGLE ****
447 C
448 C
449 CALL SUB31
450 2 FLUSS=ALPHA1
451 CALL ICS1VU(YTAG,VLPHAL,KF,1,RLOSS,H,IER)
452 C
453 C
454 IF(IER.NE.129) GO TO 2600
455 RATION=RATION+.05
456 GO TO 290
457 C
458 C
459 2600 ROTLUS=RLOSS
460 WTAPR=GAMA
461 C
462 C
463 C **** FIND MAX. NON-DIM. FLOW THROUGH ROTOR ****
464 C ****AS A FUNCTION OF INCIDENCE****
465 C
466 C
467 CALL ICS2VE(VWTAP,RLOSS,0.,1.,1.333,1.3963,51,2,1,51,WK,WTAPR,IER)
468 BOXEL=WTAPR
469 IF(RL) GO TO 825
470 C
471 C
472 C **** CHECK INLET MASS FLOW TO ROTOR ****
473 C
474 C
475 NONR=(TKDAT*SQRT(T1)/(A2*P1))/WTAPR
476 IF(NONR.GT.1.)THEN
477 KK=KK+1
478 GO TO 275
479 END IF
480 C

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481      C
482      800 CALL ICS1VE(VWDR,0.,1.,0.,0.,0.,0.,51,1,WDR,PSCOND,IER)
483          WOPCR=GAMA
484      CALL ICS2VE(VWGP,RLOSS,0.,1.,1.333,1.3963,51,2,1,51,WK,WOPCR,IER)
485      WBARK=WDR*WOPCR*P1
486      P2=P1-WBARK
487      RMRAT1=TRDAT*SQRT(T1)/(A2*P1)
488      RMRAT2=TRDAT*SQRT(T2)/(A2*P2)
489
490      C
491      C      **** CHECK CHOKE ON ROTOR ****
492      C
493      C
494          W2=ABS(RMRAT2/WTAPR-1.)
495          IF(W2.GT,.01) GO TO 830
496          H1=.TRUE.
497          GO TO 831
498      825 H8=.TRUE.
499          CALL CHOKE(RLOSS,P1,T2,A2,AN2,ALPHA2,RH2,RATION,P2,V2)
500          IF(PUS.OF.H2) STOP
501          GO TO 826
502
503      C
504      C
505      C
506      C      **** CHECK OUTLET MASS FLOW FROM ROTOR ****
507      C
508      830 IF(RMRAT2.GT.WTAPR) THEN
509          LL=LL+1
510          DIFFES=1.-(WTAPR/RMRAT2)
511          RATION=RATION-DIFFES
512          G=.FALSE.
513          GO TO 290
514          END IF
515
516      C
517      C
518      831 RM1=RMRAT1
519          RM2=RMRAT2
520          CALL ICS1VU(ORRN,BWTAP,50,1,RM1,H,IER)
521          CALL ICS1VU(ORRN,BWTAP,50,1,RM2,H,IER)
522          ALPA2C=RM2
523          CALL ICS1VU(ALPA2C,RHO,NF,1,ALPA2C,H,IER)
524
525      C
526          DIFF2=ABS(ALPHA2-ALPA2C)
527          IF(DIFF2.LT..2) GO TO 12
528          ALPHA2=ALPA2C
529          H1=.FALSE.
530          GO TO 1
531      12 VOT2=RH2
532          CALL ICS1VE(UVOT,.02,1.2,0.,0.,0.,0.,60,1,VOT2,PSCOND,IER)
533          V2=VOT2*SQRT(T2)
534      826 GG=ALPHA2*PIN/LBO.
535          VA2=V2*COS(GG)
536          HH=ATAN((U/VA2)-TAN(GG))
537          ALPHA3=HH*130./PIN
538          T3=T2-V2**2.*(1.-(COS(GG)/COS(HH))**2.)/(2.*CP)
539          P3=P2*(T3/T2)**(GAMA/(GAMA-1.))
540          ETA=((T1-T3)/T1)/(1.-(P3/PI)**((GAMA-1.)/GAMA))
541
542      C
543      C      **** CALCULATE STAGE MEAN REYNOLDS NUMBER ****
544      C
545          STMU=TI
546          RORU=T2
547          ROST=TI
548          RORO=T2
549          CALL ICS1VE(VRU,100.,2500.,0.,0.,0.,0.,34,1,STMU,PSCOND,IER)
550          CALL ICS1VE(VRU,100.,2500.,0.,0.,0.,0.,34,1,RORU,PSCOND,IER)
551          CALL ICS1VE(VRU,100.,2500.,0.,0.,0.,0.,34,1,ROST,PSCOND,IER)
552          CALL ICS1VE(VRU,100.,2500.,0.,0.,0.,0.,34,1,RORO,PSCOND,IER)
553          RENST=CS*VU*ROST*10000./STMU
554          RENU=CR*V2*RORO*10000./RORU
555          RENM=(RENST+RENU)/2.
556
557      C
558      C
559          ETAC=1.-(1.-ETA)/((RENM/20000.)**(.2))
560          A3=AN2*COS(HH)

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561      VOT3=TIMEAT*SQRT(T3)/(A3*P3)
562      RM3=GAMA
563      CALL ICS2VL (RMN,VOT3,0.,.395,1.333,1.3963,80,2,1,80,WK,RM3,IER)
564      IF (IER.EQ.129) GO TO 1001
565      CALL ICS1VL (VVT,0.,.4,0.,0.,0.,0.,80,1,VOT3,PSCOND,IFR)
566      IF (IER.EQ.129) GO TO 1001
567      V3=VOT3*SQRT(T3)
568
569      C
570      C
571      C
572      1001 RIN=(BETA1-ALPHA1)
573      WRITE(6,*) ' *** ROTOR INCIDENCE=',PIN
574
575      C
576      *PIII(0,4)STALOS,ROTLOS
577      4 FORMAT(6(/),20X,'STATOR TOTAL LOSS COEFFICIENT, L(S)=' ,F10.8,/,15
578      #X,57('*'),///,20X,'ROTOR TOTAL LOSS COEFFICIENT, L(R)=' ,F10.8,/,15
579      #X,57('*'),8(/))
580      WRITE(6,5)
581      5 FORMAT(///,50X,47('*'),/,50X,'*',4X,'ABSOLUTE STATOR OUTLET GAS CO
582      #NDITIONS',4X,'*',/,50X,47('*'),/)
583      WRITE(6,8)
584      8 FORMAT(///,5X,'OUTLET GAS ANGLE',4X,'OUTLET GAS PRESSURE',6X,'OUT
585      #LET GAS TEMPERATURE',7X,'OUTLET GAS VELOCITY',4X,'OUTLET MACH NUMB
586      #ER',/)
587      WRITE(6,9)ALPHA0,PO,TI,VO,SHO
588      9 FORMAT(/,7X,F10.6,16X,F7.3,17X,F9.4,16X,F10.5,11X,F10.6,/)
589      WRITE(6,13)
590      13 FORMAT(/,45X,60('*'),/,45X,'*',4X,'INLET & OUTLET GAS CONDITIONS (
591      #RELATIVE TO ROTOR)',5X,'*',/,45X,60('*'),5(/),5X,'INLET GAS ANGLE'
592      #,7X,'INLET GAS PRESSURE',7X,'INLET GAS TEMPERATURE',7X,'INLET VELO
593      #CITY',10X,'INLET MACH NUMBER',/)
594      WRITE(6,14) ALPHA1,P1,T1,V1,RM1
595      14 FORMAT(/,7X,F10.6,16X,F7.3,17X,F9.4,16X,F10.5,11X,F10.6)
596      WRITE(6,40)
597      40 FORMAT(///,5X,'OUTLET GAS ANGLE',4X,'OUTLET GAS PRESSURE',6X,'OUT
598      #LET GAS TEMPERATURE',7X,'OUTLET GAS VELOCITY',4X,'OUTLET MACH NUMB
599      #ER',/)
600      WRITE(6,16) ALPHA2,P2,T2,V2,RM2
601      16 FORMAT(/,7X,F10.6,16X,F7.3,17X,F9.4,16X,F10.5,11X,F10.6,/)
602      WRITE(6,18)
603      18 FORMAT(///,45X,60('*'),/,45X,'*',4X,'ABSOLUTE GAS CONDITIONS AT IN
604      #LET TO THE NEXT STAGE',4X,'*',/,45X,60('*'),/)
605      WRITE(6,20)
606      20 FORMAT(/,5X,'INLET GAS ANGLE',7X,'INLET GAS PRESSURE',7X,'INLET GA
607      #S TEMPERATURE',7X,'INLET VELOCITY',10X,'INLET MACH NUMBER',/)
608      WRITE(6,19)ALPHA3,P3,T3,V3,RM3
609      19 FORMAT(/,7X,F10.6,16X,F7.3,17X,F9.4,16X,F10.5,11X,F10.6,/)
610      WRITE(6,*) '** UNCORRECTED EFFICIENCY= ',ETA
611      WRITE(6,17) ETAC
612      17 FORMAT(///,39X,81('*'),/,39X,'*',4X,'THE EFFICIENCY OF THE STAGE A
613      #S GIVEN BY A&A IS ETA(STAGE)=' ,F10.6,4X,'*',/,39X,81('*'),/)
614      PRATIO=P3/P1
615      WRITE(6,2000) RATION,PFATIO
616      2000 FORMAT(5(/),10X,'NON-DIMENSIONAL MASS FLOW',5X,'W*SQRT(T1)/P1',2X,
617      #,=' ,F10.7,/,10X,'OVERALL PRESSURE RATIO',12X,'P3/P1',2X,'=' ,F10.7
618      #,/)
619
620      C
621      C
622      C
623      IF(M3.AND.H4) GO TO 5000
624      IF(H6) GO TO 6001
625
626      C
627      IF(G) THEN
628      WRITE(6,320) SMAXFL
629      320 FORMAT(/,10X,'THE MAXIMUM NON-DIMENSIONAL FLOW WHICH PASSES THROU
630      #GH THIS STATOR IS EQUAL TO',5X,F10.7,/)
631      H6=.TRUE.
632      END IF
633
634      C
635      C
636      C
637      C
638      IF(H1) THEN
639      WRITE(6,820) PATION
640      820 FORMAT(/,10X,'THE MAXIMUM NON-DIMENSIONAL MASS FLOW WHICH PASSES

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641      #THROUGH THIS ROTOR IS EQUAL TO',5X,F10.7,/)
642      H7=.TRUE.
643      END IF
644      C
645      C
646      7001 IF(H9)GO TO 8001
647      C
648      C
649      IF(H3) THEN
650      WRITE(6,7002) PRATIO
651      7002 FORMAT(//,10X,'STATOR MACH # EXCEEDS UNITY AT PRESSURE RATIO=
652      #',2X,F10.7,/)
653      H9=.TRUE.
654      END IF
655      C
656      C
657      8001 IF(H10) GO TO 9001
658      C
659      C
660      IF(H4) THEN
661      WRITE(6,8002)PRATIO
662      8002 FORMAT(//,10X,'ROTOR MACH # EXCEEDS UNITY BEYOND PRESSURE RATIO=
663      #',2X,F10.7,/)
664      H10=.TRUE.
665      END IF
666      C
667      C
668      9001 IF(H10.OR.H9) GO TO 5000
669      C
670      C
671      IF((H6.AND.H7).AND.(SMAXFL.GT.RATIO)) THEN
672      G=.FALSE.
673      GO TO 6
674      END IF
675      C
676      C
677      IF(H7) GO TO 6
678      C
679      C
680      IF(G) GO TO 325
681      C
682      C
683      IF((II.NE.0).OR.(KK.NE.0)) THEN
684      RATIO=R/TION+.02
685      GO TO 290
686      END IF
687      C
688      C
689      RATIO=RATIO+.5
690      GO TO 6
691      5000 RETURN
692      END
693      C
694      C
695      C
696      C
697      C
698      SUBROUTINE CHCKE (LOSS,PM,TH,AN,AIN,ALPANC,MACH,RATIO,PH,VN)
699      COMMON/C11/ORTAP(500),GDP(500),OVOT(500),GRN(500),BWTAP(500)
700      COMMON/C12/HZ(100),H(500)
701      COMMON/C13/G,H1,H2,PUS,H3,H4,H8
702      COMMON/S77/GAMA
703      COMMON/S55/PIN
704      COMMON/C25/TKDAT
705      REAL LOSS,MACH,NWTAP,IVOTN
706      LOGICAL G,H1,H2,H3,H4,PUS,H8
707      MACH=MACH+.1
708      C
709      C
710      IF(MACH.GT.1.2) THEN
711      PUS=.TRUE.
712      GO TO 9
713      END IF
714      C
715      C
716      IF(H3.OR.H4)GO TO 1
717      IF(MACH.GT.1.) GO TO 5
718      1 NWTAP=MACH
719      CALL JCS1VE(ORTAP,.02,1.20,0.,0.,0.,0.,60,1,NWTAP,PSCOND,IER)
720      PRSDPH=MACH

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721 CALL ICSIVE(DPOP,.02,1.20,0.,0.,0.,0.,.60,1,PNSOPN,PSCOND,IER)
722 NVOTN=NACH
723 CALL ICSIVE(OVGT,.02,1.20,0.,0.,0.,0.,.60,1,NVOTN,PSCOND,IER)
724 WBUPN=L0SS*(1.-PNSOPN)
725 WBOPN=WBOPN/(1.+WBUPN)
726 PN=PI-WBOPN*PI
727 ANC=(1./NVOTN)*TNDAT*SQRT(TM)/PI
728 WRITE(6,*) ' *** CORRECTED BLADE OUTLET ANGLE =',ANC
729 FF=ACOS(ANC/ANN)
730 ALPANC=-(FF*180./PI)
731 VN=NVOTN*SQRT(TM)
732 GO TO 9
733 C
734 C
735 9 IF(.NOT.H3) THEN
736 H3=.TRUE.
737 NACH=1.
738 GO TO 1
739 ELSE
740 H4=.TRUE.
741 NACH=1.
742 GO TO 1
743 END IF
744 C
745 C
746 9 RETURN
747 END
748 C
749 C
750 C
751 C
752 C
753 C SUBROUTINE ICSIVE (F,A,B,U1,D1,UN,DN,N,M,G,PSCOND,IER)
754 C
755 C-----S-----LIBRARY 2-----
756 C
757 C FUNCTION - CUBIC SPLINE ONE-DIMENSIONAL INTERPOLATION -
758 C EQUALLY SPACED DATA
759 C USAGE - CALL ICSIVE(F,A,B,U1,D1,
760 C UN,DN,N,M,G,PSCOND,IER)
761 C PARAMETERS F - VECTOR OF N EQUALLY SPACED
762 C FUNCTIONAL VALUES F(1),F(2),...,
763 C F(N), WHERE F(1) IS THE
764 C FUNCTIONAL VALUE AT
765 C  $X=A+(I-1)*H$  AND  $H=(B-A)/(N-1)$ 
766 C A - VALUE OF THE ABSCISSA FOR F(1)
767 C B - VALUE OF THE ABSCISSA FOR F(N)
768 C U1 - BOUNDARY RELATED PARAMETER
769 C (SEE ELEMENT DOCUMENTATION)
770 C D1 - BOUNDARY RELATED PARAMETER
771 C (SEE ELEMENT DOCUMENTATION)
772 C UN - BOUNDARY RELATED PARAMETER
773 C (SEE ELEMENT DOCUMENTATION)
774 C DN - BOUNDARY RELATED PARAMETER
775 C (SEE ELEMENT DOCUMENTATION)
776 C N - NUMBER OF DATA POINTS
777 C M - NUMBER OF ANSWERS SOUGHT
778 C G - VECTOR G(1),G(2),...,G(M) OF ANSWERS
779 C WHERE G(I) IS THE I-TH ABSCISSA
780 C UPON ENTRY AND THE I-TH
781 C ORDINATE UPON COMPLETION.
782 C PSCOND - PSEUDO CONDITION NUMBER FOR BOUNDARY
783 C PARAMETERS. PSCOND.GT.1.E5
784 C INDICATES VALUES RETURNED IN G(I)
785 C MAY BE UNRELIABLE.
786 C IER - ERROR PARAMETER
787 C TERMINAL ERROR = 128*N
788 C N = 1 INDICATES G(I) NOT IN INTERVAL (A,B)
789 C FOR SOME I=1,2,...,M
790 C N = 2 INDICATES UNIQUE SOLUTION NOT POSSIBLE
791 C FOR GIVEN BOUNDARY VALUES
792 C PRECISION - SINGLE
793 C REQ'D INSL ROUTINES - UFTST
794 C LANGUAGE - FORTRAN
795 C-----
796 C LATEST REVISION - FEBRUARY 3,1971
797 C
798 C SUBROUTINE ICSIVE (F,A,B,U1,D1,UN,DN,N,M,G,PSCOND,IER)
799 C
800 C DIMENSION F(1),G(1)

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801      EQUIVALENCE      (IDET,DET)
802      REAL             LAM,LAMN
803      DATA            S3,LAM/1.7320508,-.26794919/
804      IER=0
805      H=(B-A)/(N-1)
806      LAMN=LAM**2*(N-1)
807      C                  CALCULATE LAM AND TAU
808      GAM=F(1)
809      TAU=F(N)
810      NI=N+1
811      DO 5 I=2,N
812      J=NI-1
813      GAM=GAM*LAM+F(I)
814      TAU=TAU*LAM+F(J)
815      5 CONTINUE
816      GAM=3.*GAM
817      TAU=3.*TAU
818      C1=1.-U1*LAM
819      C2=1.-U1/LAM
820      C3=C1*LAMN
821      C4=U1*H/(2.*S3)*(CT*TAU-S3*((1.+6.*U1)*F(1)-U1*F(2)))/H
822      C5=1.-UN/LAM
823      C6=C1*LAMN
824      C7=1.-UN*LAM
825      C8=UN*H/(2.*S3)*(CT*GAM-S3*((1.+6.*UN)*F(N)-UN*F(N-1)))/H
826      DET=C1*C6-C2*C7
827      IF (IDET.EQ.0) GO TO 55
828      C                  CALCULATE BOUNDARY PARAMETERS B1,BN
829      B1=(C2*C6-C3*C5)/DET
830      BN=(C1*C6-C4*C3)/DET
831      C                  CALCULATE PSCOND
832      C1=ABS(C1)
833      C2=ABS(C2)
834      C4=ABS(C4)
835      C5=ABS(C5)
836      PSCOND=AM/X1(C1,C2,C4,C5)/ABS(DET)
837      KP=0
838      N2=N-1
839      C                  CALCULATE G(I)'S
840      DO 50 I=1,M
841      XI=A
842      IF(G(I).LT.A) GO TO 15
843      DO 10 J=1,N2
844      XI=XI+H
845      IF(G(I).GT.XI) GO TO 10
846      K=J
847      GO TO 20
848      10 CONTINUE
849      IF(G(I).GT.B) GO TO 15
850      K=N2
851      GO TO 20
852      15 IER=129
853      GO TO 9000
854      20 X=(G(I)-(A+(K-1)*H))/H
855      IF(K.EQ.KP) GO TO 45
856      GAMK=F(1)
857      TAUk=F(N)
858      IF(K.LE.1) GO TO 30
859      DO 25 J=2,K
860      GAMK=GAMK*LAM+F(J)
861      25 CONTINUE
862      30 K1=N-K
863      IF(K1.LE.1) GO TO 40
864      DO 35 J=2,K1
865      JJ=N1-J
866      TAUk=TAUk*LAM+F(JJ)
867      35 CONTINUE
868      40 GAMK=3.*GAMK
869      TAUk=3.*TAUk
870      KP=K
871      45 T=(1.-LAM**2)*X+(LAM+1.)*X*X)*X
872      Y=1.-X
873      TY=(1.-LAM**2)*Y+(LAM+1.)*Y*Y)*Y
874      G(I)=GAMK*TX+TAUk*TY+F(K)*Y**3+F(K+1)*X**3+B1*H*TX*LAM**2*(K-1)
875      1 -EN*H*TY*LAM**2*(N-K-1)
876      50 CONTINUE
877      GO TO 9005
878      55 IER=130
879      9000 CONTINUE
880      CALL UERTST( IER, 'ICSIVE ' )

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881      9005 RETURN
882      END
883      C
884      C
885      C
886      C
887      C
888      C      SUBROUTINE ICS2VE (F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
889      C
890      C-----S-----LIBRARY 2-----
891      C
892      C      FUNCTION          - BICUBIC SPLINE TWO-DIMENSIONAL INTERPOLATOR -
893      C                        - EQUALLY SPACED DATA
894      C      USAGE          - CALL ICS2VE(F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
895      C      PARAMETERKS    F      - MATRIX OF I*J FUNCTIONAL VALUES F(1,1),F(2,1),
896      C                        F(3,1),...,F(1,2),...,F(I,J). THE VALUE
897      C                        F(I1,J1) IS THE FUNCTIONAL VALUE AT X(I1),
898      C                        Y(J1) WHERE X(I1)=A+(I1-1)*HX,Y(J1)=C+(J1-1)
899      C                        *HY, HX=(B-A)/(I-1), AND HY=(D-C)/(J-1)
900      C                        XO      - X-DIRECTION ABSCISSA FOR WHICH INTERPOLATES
901      C                        ARE SOUGHT
902      C                        A      - LOWER LIMIT OF THE INTERVAL (A,B)
903      C                        B      - UPPER LIMIT OF THE INTERVAL (A,B)
904      C                        C      - LOWER LIMIT OF THE INTERVAL (C,D)
905      C                        D      - UPPER LIMIT OF THE INTERVAL (C,D)
906      C                        I      - NUMBER OF MESH POINTS IN THE FIRST COORDINANT,
907      C                        X, DIRECTION
908      C                        J      - NUMBER OF MESH POINTS IN THE SECOND COORDINANT
909      C                        Y, DIRECTION
910      C                        N      - NUMBER OF INTERPOLATED VALUES DESIRED
911      C                        IF     - FIRST DIMENSION OF F IN CALLING PROGRAM
912      C                        WK     - WORK AREA OF DIMENSION J
913      C                        Y      - VECTOR Y(1),Y(2),...,Y(N) WHERE Y(J1) IS THE
914      C                        J1-TH ABSCISSA UPON ENTRY AND THE INTERPO-
915      C                        LATED FUNCTIONAL VALUE AT COORDINANT
916      C                        (X0,Y(J1)) UPON COMPLETION
917      C                        IER    - ERROR PARAMETER
918      C                        TERMINAL ERROR = 128+N
919      C                        N = 1 INDICATES X0 NOT IN INTERVAL (A,B)
920      C                        N = 2 INDICATES Y(J1) NOT IN INTERVAL (C,D)
921      C                        FOR SOME J1=1,2,....,N
922      C      PRECISION      - SINGLE
923      C      REQ'D IMSL ROUTINES - ICS1VE,UERTST
924      C      LANGUAGE      - FORTRAN
925      C-----S-----LIBRARY 2-----
926      C      LATEST REVISION - JANUARY 18, 1971
927      C
928      C      SUBROUTINE ICS2VE (F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
929      C
930      C      DIMENSION      F(IF,1),Y(1),WK(1)
931      C                        INTERPOLATE IN Y DIRECTION AT X
932      C      DO 5 I1=1,J
933      C      WK(I1)=X0
934      C      CALL ICS1VE(F(1,I1),A,B,C,D,I,1,WK(I1),PSCOND,IER)
935      C      IF(IER.NE.0) GO TO 9000
936      C      5 CONTINUE
937      C      CALL ICS1VE (WK,C,D,C,0,0,0,0,J,N,Y,PSCOND,IER)
938      C      IF(IER.NE.0) GO TO 10
939      C      GO TO 9005
940      C      10 IER=130
941      C      9000 CONTINUE
942      C      CALL UERTST(IER,'ICS2VE')
943      C      9005 RETURN
944      C      END
945      C
946      C
947      C
948      C
949      C
950      C      SUBROUTINE ICS1VU (F,X,N,M,G,H,IER)
951      C
952      C-----S-----LIBRARY 2-----
953      C
954      C      FUNCTION          - CUBIC SPLINE ONE-DIMENSIONAL INTERPOLATION -
955      C                        - UNEQUALLY SPACED DATA
956      C      USAGE          - CALL ICS1VU(F,X,N,M,G,H,IER)
957      C      PARAMETERS    F      - VECTOR OF N UNEQUALLY SPACED
958      C                        FUNCTIONAL VALUES F(1),F(2),...,
959      C                        F(N), WHERE F(I) IS THE FUNCTIONAL
960      C                        VALUE AT X(I)

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961 C X - VECTOR OF N ABSCISSA X(1),X(2),...,X(N)
962 C N - NUMBER OF DATA POINTS
963 C M - NUMBER OF ANSWERS SOUGHT
964 C G - VECTOR G(1),G(2),...,G(M) OF ANSWERS
965 C WHERE G(I) IS THE I-TH ABSCISSA
966 C UPON ENTRY AND THE I-TH ORDINATE
967 C UPON COMPLETION.
968 C H - WORK AREA OF DIMENSION 8*N+M
969 C IER - ERROR PARAMETER
970 C TERMINAL ERROR = 128+M
971 C N = 1 INDICATES G(I) NOT IN INTERVAL
972 C (X(1),X(N)) FOR SOME I=1,2,...,M
973 C N = 2 INDICATES THAT CONVERGENCE WAS
974 C NOT OBTAINED IN 5*M ITERATIONS
975 C PRECISION - SINGLE
976 C KFO'D IMSL ROUTINES - UFRIST
977 C LANGUAGE - FORTRAN
978 C -----
979 C LATEST REVISION - MARCH 5, 1971
980 C
981 C SUBROUTINE ICSIVU(F,X,N,M,G,H,IER)
982 C
983 C DIMENSION F(1),X(1),G(1),H(1)
984 C DATA EPSLN,OMEGA/1.E-6,1.0717968/
985 C SET UP WORK AREAS
986 C 12=N
987 C 13=N+M
988 C 14=13+M
989 C 15=14+M
990 C 16=15+M
991 C 17=16+M
992 C 18=17+M
993 C 19=18+M
994 C NT=16
995 C IER = 0
996 C H1 = M-1
997 C
998 C DERIVATIVES,H(J6), USING CENTRAL
999 C DIFFERENCES
1000 C DO 5 I=1,M1
1001 C J2=12+I
1002 C H(I)=X(I+1)-X(I)
1003 C H(J2)=(F(I+1)-F(I))/H(I)
1004 C 5 CONTINUE
1005 C DO 10 I=2,M1
1006 C J2=12+I
1007 C J3=13+I
1008 C J4=14+I
1009 C J5=15+I
1010 C J6=16+I
1011 C J7=17+I
1012 C H(J3)=H(I-1)+H(I)
1013 C H(J4)=.5*H(I-1)/H(J3)
1014 C H(J5)=(H(J2)-H(J2-1))/H(J3)
1015 C H(J6)=H(J5)+H(J5)
1016 C H(J7)=H(J6)+H(J5)
1017 C 10 CONTINUE
1018 C H(I6+1)=0.
1019 C J6=16+M
1020 C H(J6)=0.
1021 C BEGIN ITERATION ON SECOND DERIVATIVES
1022 C KCOUNT=0
1023 C 15 ETA=0.
1024 C KCOUNT=KCOUNT+1
1025 C DO 25 I=2,M1
1026 C J4=14+I
1027 C J6=16+I
1028 C J7=17+I
1029 C W=(H(J7)-H(J4)*H(J6-1)-(.5-H(J4))*H(J6+1)-H(J6))*OMEGA
1030 C IF (ABS(W).LE.ETA) GO TO 20
1031 C ETA=ABS(W)
1032 C 20 H(J6)=H(J6)+W
1033 C 25 CONTINUE
1034 C IF(KCOUNT.GT.NT) GO TO 75
1035 C IF (LTA.GE.EPSLN) GO TO 15
1036 C CONVERGENCE OBTAINED
1037 C DO 30 I=1,M1
1038 C J6=16+I
1039 C J8=18+I
1040 C H(J8)=(H(J6+1)-H(J6))/H(I)
1041 C 30 CONTINUE

```



```
1121      20 IER2=IER2-IBIT(IEP1)
1122      C
1123      PRINT ERROR MESSAGE
1124      WRITE (PRINTR,25) (ITYP(I,IER1),I=1,3),NAME,IER2,IER
1125      25 FORMAT(' *** I P S L(UERTST) *** ',3A6,2X,A6,2X,I2,
1126      1 ' (IER = ',I3,')')
1126      RETURN
1127      END
```

APPENDIX C

COMPUTER PROGRAM RAO

C.1 GENERAL DESCRIPTION

The computer program RAO calculates the geometry and aerodynamic properties of a gas turbine stage for which the aerodynamic efficiency is an optimum.

The input data of RAO is grouped under two data files:

- i) As for the case of the program AINLEY (App. B) data are necessary to evaluate the interpolations encountered on loss calculations. These data are fixed and automatically fed into the program under the data file TURB04.
- ii) The variable data of the program is fed by the user. These constitute the datafile TURB03. The content of TURB03 is explained in detail in Section C.4.

The program RAO calculates first the efficiency, η_{ASRE} , of the stage according to the given input trial vector, \vec{X}_0 . The loss calculations required for the efficiency evaluation are performed with the help of the interpolation subroutine ICS2VE, making use of

the data provided by TURB04. The calculation of the sum, CONST, of the inverse of the constraint equations allows then, the evaluation of the first value of the penalty function, PENFUN.

The next step is the evaluation of the gradients of ETA and CONST. A three dimensional matrix, M, is created. Each slice of M corresponds to a two-dimensional matrix. The first slice, M_1 , for example, is formed of the partial derivatives of Eq. (78) combined with Eq. (80), simply with respect to the present variables, i.e.,

$$\begin{aligned}
 M(1,1,1) &= \partial\eta/\partial\lambda_R \\
 M(1,1,2) &= \partial\eta/\partial\lambda_N \text{ improved} \\
 M(1,1,3) &= \partial\eta/\partial T_{2s} \\
 &\vdots \\
 M(1,1,8) &= \partial\eta/\partial Re
 \end{aligned}
 \tag{C.1}$$

The remaining elements of $M(1,i,j)$ are zero.

The second slice, $M(2,i,j)$ is formed of the partial derivatives of λ_R , $\lambda_{N,i}$, T_{2s} , ..., Re , with respect to their own components. That is,

$$\begin{aligned}
 M(2,1,1) &= \partial\lambda_R/\partial Y_R \\
 M(2,1,5) &= \partial\lambda_R/\partial T_4
 \end{aligned}
 \tag{C.2}$$

or

$$M(2,2,2) = \partial\lambda_{N,i}/\partial Y_N$$

Finally, the dependent variables of the problem are eliminated and the partial derivative of the efficiency with respect to each one of the design variables can be obtained. This procedure can be explained as follows:

The first slice, M_1 , of the M vector, is multiplied with the next slice, M_2 , such that $M_{1,2}$ is obtained.

$$M_{1,2} = \left[\frac{\partial n}{\partial \lambda_R}, \frac{\partial n}{\partial \lambda_N}, \dots, \frac{\partial n}{\partial Re} \right] \times \begin{vmatrix} \frac{\partial \lambda_R}{\partial Y_R} & \frac{\partial \lambda_R}{\partial Y_N} & \dots & \frac{\partial \lambda_R}{\partial C_R} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \frac{\partial Re}{\partial Y_R} & \frac{\partial Re}{\partial Y_N} & \dots & \frac{\partial Re}{\partial C_R} \end{vmatrix}$$

$$M_{1,2} = \left[\frac{\partial n}{\partial Y_R}, \dots, \frac{\partial n}{\partial C_R} \right] \quad (C.3)$$

Then $M_{1,2}$ is multiplied with the next slice, to obtain $M_{1,3}$, i.e.

$$M_{1,3} = \left[\frac{\partial n}{\partial Y_R}, \dots, \frac{\partial n}{\partial C_R} \right] \times \begin{vmatrix} \frac{\partial Y_R}{\partial Y_{p,r}} & \dots & \frac{\partial Y_R}{\partial C_R} \\ \frac{\partial C_R}{\partial Y_{p,r}} & \dots & \frac{\partial C_R}{\partial C_R} \end{vmatrix}$$

$$M_{1,3} = \left[\frac{\partial n}{\partial Y_{p,r}}, \dots, \frac{\partial n}{\partial C_R} \right] \quad (C.4)$$

such that, when the resulting vector is multiplied with the last, nth slice, one is left with:

$$M_{1,n} = \left[\frac{\partial n}{\partial d} \frac{\partial n}{\partial C_R} \frac{\partial n}{\partial C_N} \dots \frac{\partial n}{\partial V_a} \right] \quad (C.5)$$

which is recognized as $\nabla \eta$, or DETA in the program. The multiplication of each slice of M with the following slice is performed by means of the subroutine MULTIP. The gradient of the constraint equation,

DCONST, is performed in the same way. When DETA and DCONST are evaluated, the gradient, DPEN, of the penalty function is evaluated according to Eq. (108).

The next step, is the determination of the maximizing step length, R_{op} . The idea is explained in Eq. (111). For each variable, the upper bound, $XUP(i)$ and the lower bound $XLOW(i)$ are initially given in the datafile TURBO3. Therefore, the next value to be assigned to that variable cannot exceed these limits. Thus, replacing $XUP(i)$ by $X_{n+1}(i)$ in Eq. (106) one gets

$$R_{op,n}(i) = \frac{XUP(i) - X(i)}{\nabla P(i)} \quad i = 1,8 \quad (C.6)$$

Note that $XUP(i)$ must be replaced by $XLOW(i)$ in Eq. (C.6) for the case where $\nabla P(i)$ is negative.

Therefore, eight values of R_{op} for which each design variable would violate its bounds is determined. The lowest value of R_{op} is chosen among those eight possible R_{op} 's and, this value is divided by 50 to find the final value of R_{op} to be used in Eq. (110). Consequently, the search for the maximum of $H(R_{op})$ can be carried out safely for 50 increments of R_{op} .

The maximum value of $H(R_{op})$ is called PENMAX in the program. Once PENMAX is determined, the corresponding values of the design vector $X(i)$, ETASRE, pressure ratio, PRATIO, Mach number, MC3, and Reynolds number, RE, are assigned to XMAX(i), ETAMAX, PRMAX, MAKMAX and REMAX respectively. If PENMAX turns out to be the absolute maximum of the objective function, PENFUN, these maxima are printed in the output.

In the opposite case, they are changed with new values corresponding to the latest values of PENMAX.

After PENMAX is obtained, the program calculates a new DPEN, using the values of XMAX(i) and comes up with a new PENMAX. If the last value is greater, the procedure is repeated. If not, a counter, PCOUNT is increased by 1, and the procedure is repeated. When, at any point, PCOUNT is greater than 2, the procedure is stopped, for the renewal of the resequencing factor, RF. Then the second sequential maximization step starts and a new PENMAX is found corresponding to the new value of RF. RF is updated three times, and as indicated in Eq. (114), the procedure is terminated for

$$RF < 4 \times 10^{-6} .$$

The absolute maxima of each design variable and those of ETASRE, PRATIO, MC3, RE, PENFUN and the other aerodynamic variables are printed in the output.

C.2 LIST OF THE IMPORTANT VARIABLES OF THE PROGRAM RAO

X(I)	:	Vector formed by the design variables
FPENF	:	Final value (corresponding to the optimum) of the penalty function
L01	:	Logical control variable
L02	:	Logical control variable
ETASRE	:	Total-to-total isentropic efficiency, η , of the stage (corrected for Reynolds number)

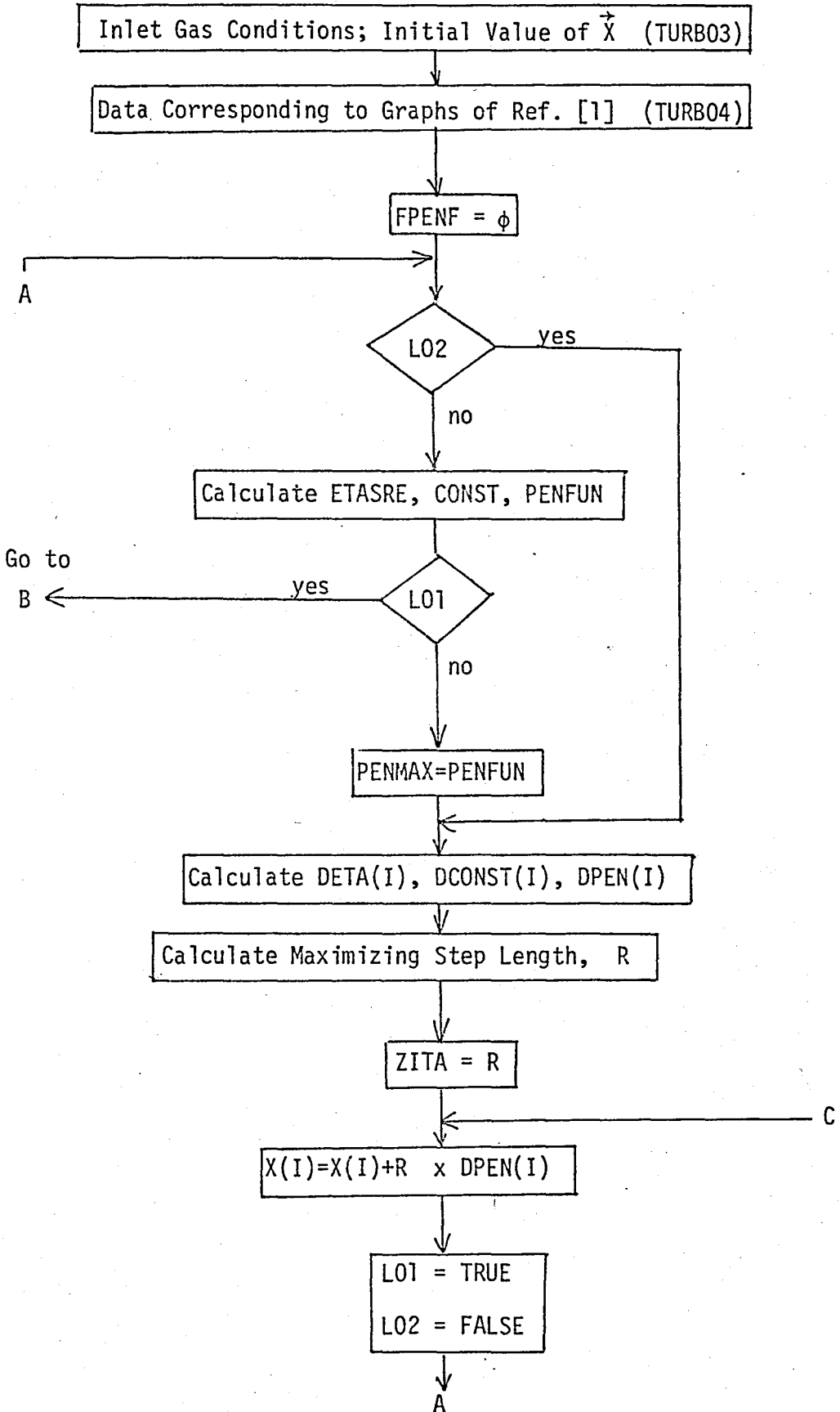


FIGURE C.1 - Flow chart of the computer program "RAO".

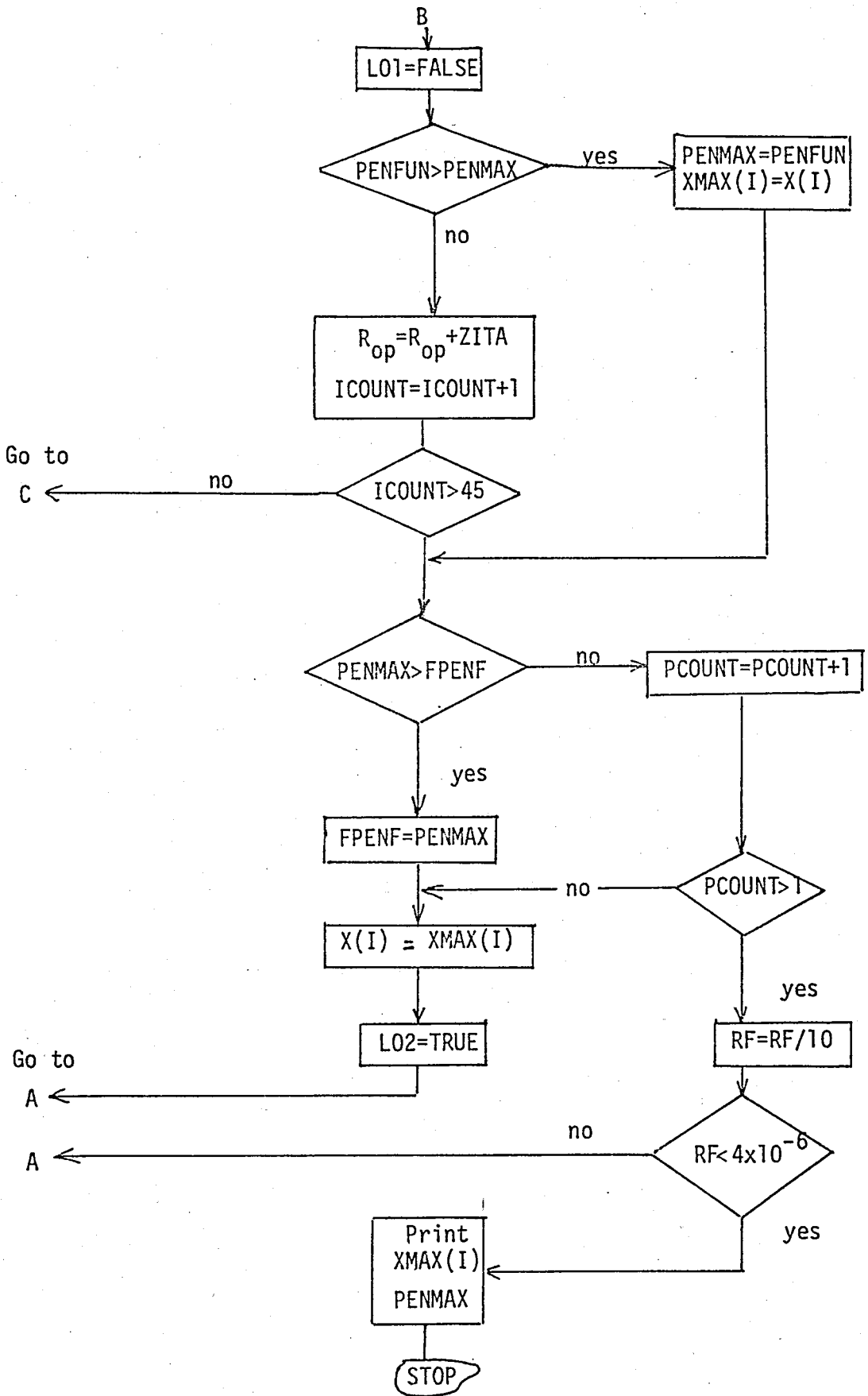


FIGURE C.1 (continued).

CONST : Sum of the constraint equations
 PENFUN : Penalty function
 PENMAX : Local maximum of the penalty function
 DELTA(I) : Gradient of the efficiency
 DCONST(I) : Gradient of the constant equations
 DPEN(I) : Gradient of the penalty function
 R : Maximizing step length
 ZITA : Increment
 XMAX(I) : Absolute maxima of the design variables
 ICOUNT : Counter
 PCOUNT : Counter
 RF : Resequencing factor

C.3 SUBROUTINE DESCRIPTION

The functional relationship between the subroutines of the program RAO is given below.

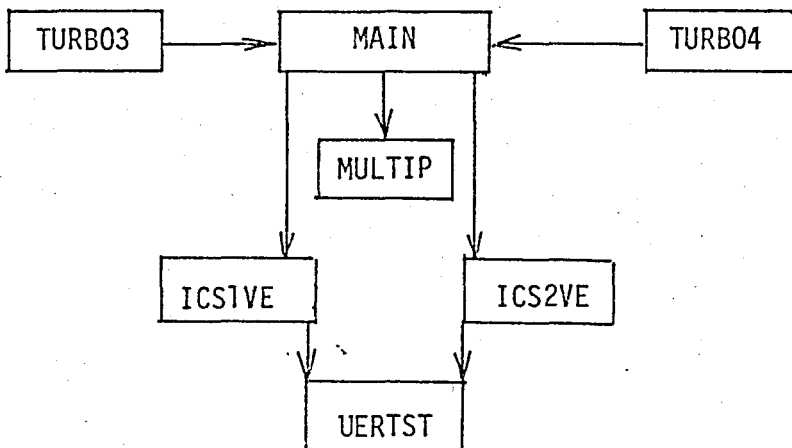


FIGURE C.2 - Functional relationship among the subroutines of the program "RAO".

The functions of the main program and of the subroutines are as follows:

MAIN : The main program reads the datafiles TURB03 and TURB04 and executes the job explained in Section C.1

MULTIP : The subroutine MULTIP performs the matrix multiplications. The argument list comprises the names of the two matrices to be multiplied and their dimensions. This subroutine is called by the statement,

```
CALL MULTIP(A,B,M,N)
```

The first matrix A is an $l \times M$ matrix and the second matrix B is an $M \times N$ matrix. The resulting vector is thus an $l \times N$ matrix and returned to the calling program under the name A.

ICS1VE, ICS2VE, UERTST: These subroutines are explained in Section B.2.1.

C.4 INPUT DESCRIPTION

The datafile TURB04 is automatically fed to RAO. The datafile TURB03, to be fed by the user, must be prepared as follows:

Card 1 : XI(I), I = 1,8

XI : Vector of the first trial values of the design variables, where

XI(1) : Mean diameter of the turbine (m)

XI(2) : Chord length of the rotor blades (m)

XI(3) : Chord length of the stator blades (m)
 XI(4) : Pitch length of the stator blades (m)
 XI(5) : Pitch length of the rotor blades (m)
 XI(6) : Gas inlet angle to the rotor (rad)
 XI(7) : Gas outlet angle from the rotor (rad)
 XI(8) : Axial gas velocity, constant throughout the
 stage (m/s)

Card 2 : RPM, MDAT, P01, T01

RPM : Blade rotational speed (rev/s)
 MDAT : Gas mass flow rate at inlet to the stage (kg/s)
 P01 : Absolute total pressure at inlet to the stage (N/m^2)
 T01 : Absolute total temperature at inlet to the stage ($^{\circ}K$)

Card 3 : AL1, B, CLEAR

AL1 : Inlet gas angle to the stage (deg)
 B : Constant of Eq. (14)
 CLEAR : Tip clearance of the rotor blades (m)

Card 4 : ILAMN, IETAS

ILAMN : Initial guess for λ_N , to be used in Eq. (51)
 IETAS : Initial guess for η , to be used in Eq. (65)

Card 5 : RESFAC, MSL, EPS

RESFAC: First value of the resequencing factor of Eq. (101)
 MSL : First value of the maximizing step length of Eq. (106)
 EPS : Tolerance to be used in the iterative efficiency
 estimation.

Card 6 : CP, GAMA, RA

CP : Specific heat at constant pressure (N.m/kg⁰K)

GAMA : Ratio of the specific heats

RA : Gas constant (N.m/kg⁰K)

Card 7 : G

G = 0 , optimize

G ≠ 0 , calculate the efficiency, only.

The seven input cards must be given in free format. A sample data set for TURBO3 is given below:

Card 1 : .430, .024, .017, .015, .018, .358, .953, 270.

Card 2 : 250., 20., 400 000., 1100.

Card 3 : 0., .5, .000154

Card 4 : .0570, .80

Card 5 : 4×10^{-4} , 0., .001

Card 6 : 1160., 1.333, 287.

Card 7 : 0.

C.5 OUTPUT DESCRIPTION

On the output of the computer program RAO, appears first the initial trial of the design vector, \vec{X}_0 . Then, the chosen upper and lower limits of the design variables are shown and the initial gas conditions and the selected rotative speed are printed.

The results of the program, then, are given: The values of the design variables corresponding to the calculated optimum of the

efficiency are printed; also, various aerodynamic parameters calculated at optimum conditions are shown, in the following order:

- i) Efficiency, η , of the stage
- ii) The relevant value of the penalty function, P.
- iii) The inlet non-dimensional mass flow, $\dot{m}\sqrt{T_1}/P_1$.
- iv) The pressure ratio, P_1^*/P_1 .
- v) Rotor outlet Mach number, M_4
- vi) Stage mean Reynolds number, Re
- vii) The inlet mass flow parameter based on the optimum annulus diameter, $\dot{m}\sqrt{T_1}/P_1 D^2$ (DQMMAX)
- viii) The overall temperature drop ratio, $\Delta T/T_1$ (DQTMAX)
- ix) The speed parameter, ND/\sqrt{T} (DQMAMX)
- x) The work output parameter of the stage, $C_p \Delta T/N^2 D^2$ (DQM2MX)
- xi) The flow coefficient, ϕ (FIMAX)
- xii) The temperature drop coefficient, ψ (PSIMAX)

A sample output of the program RAO is given below:

*** THE FIRST TRIAL OF THE DESIGN VECTOR IS **

.432
 .024
 .0173
 .0151
 .0151
 .358
 .953
 272.

*** THE CHOSEN UPPER & LOWER BOUNDS ARE AS FOLLOWS **

.064 < X(1) < .512
 .00128 < X(2) < .04096
 .00064 < X(3) < .04096
 .000512 < X(4) < .04096
 .000256 < X(5) < .04096
 .0105 < X(6) < .9948
 .698 < X(7) < 1.396
 50. < X(8) < 400.

*** INPUT CONDITIONS ARE AS FOLLOWS **

** REV. PER SECOND = 250.
** GAS TEMPERATURE = 1100.
** GAS PRESSURE = 400000.
** GAS MASS FLOW = 20.

** VALUES OF DESIGN VARIABLES CORRESPONDING TO OPTIMUM EFFICIENCY

** X(1) = .2517366221649
** X(2) = .03189277987009
** X(3) = .02273274830175
** X(4) = .01700489146619
** X(5) = .009887143687339
** X(6) = .3579559169991
** X(7) = .9690816785905
** X(8) = 272.0000124397

** VARIOUS OPTIMUM PARAMETERS ARE **

** EFFICIENCY = .7980291526246 **
** PENALTY FUNCTION = .7963460573962 **
** INLET NON-DIM MASS FLOW = .001658312395178 **
** PRESSURE RATIO = .663191088576 **
** ROTOR MACH NUMBER = .5540689163215 **
** REYNOLDS NUMBER = 305084.2288297 **

** DQM MAX = .02616818165689
** DQT MAX = .07781462381428
** DQM AMX = 1.89753618571
** DQW MAX = 24.85300627392
** DQM 2MX = 3.95790509403
** FIMAX = 1.375728207634
** PSIMAX = 5.036272025487

C.6 PROGRAM LISTING OF "RAO"

```

1      PROGRAM RAO(INPUT,OUTPUT,TURBO3,TAPF7=TURBO3,
2      "                               TURBO4,TAPF5=TURBO4)
3      C
4      C
5      C
6      DIMENSION M(17,25,25),G(13,11,11),H(13,9,9),O(10,10,10),
7      #       SIZE1(17,2),SIZE2(13,2),SIZE3(13,2),SIZE4(10,2),
8      #       DETA(30),RD2(30,30),GG(31),DCONST(50),
9      #       C(31,8),D3(30),D4(30),D5(30),D6(30),D19(30),
10     #       X(8),DPER(8),XMAX(8),ABSX(8),DIFF(8),DIFFX(8),
11     #       YP1(17,8),YP2(15,6),VMU(34),WKL(500),
12     #       XLW(8),XUP(8),XI(8),RMAX(8),RFI(10)
13     C
14     C
15     INTEGER SIZE1,SIZE2,SIZE3,SIZE4
16     REAL    M,RD2,LAMB,L/MS,LANCN,LANCR,LAMB1,LAMB2,MDAT,HUAVG,MSL,
17     #       ILAMB,LETAS,RC3,MAKMAX
18     LOGICAL LO1,LO2,LO3,LO4,LO5,LL1,LL2
19     C
20     C
21     DATA ((SIZE1(I,J),J=1,2),I=1,17) /1,8,8,17,17,20,20,24,
22     #       24,23,23,19,19,16,16,17,
23     #       17,20,20,19,19,17,17,14,
24     #       14,11,11,10,10,9,9,8,8,8/
25     C
26     DATA ((SIZE2(I,J),J=1,2),I=1,13) /1,3,3,4,4,5,5,5,5,7,
27     #       7,9,9,10,10,11,11,8,
28     #       6,7,7,9,9,8,8,8/
29     C
30     DATA ((SIZE3(I,J),J=1,2),I=1,13) /1,3,3,4,4,5,5,5,5,7,
31     #       7,6,6,6,6,7,7,7,7,6,
32     #       6,9,9,8,8,8/
33     C
34     DATA ((SIZE4(I,J),J=1,2),I=1,10) /1,4,4,5,5,6,6,8,8,9,
35     #       9,8,8,10,10,6,6,7,7,8/
36     C
37     READ(5,*) (VMU(I),I=1,34)
38     READ(5,*) (XLW(I),I=1,8)
39     READ(5,*) (XUP(I),I=1,8)
40     READ(5,1) ((YP1(I,J),I=1,17),J=1,8)
41     1 FORMAT(8(10F5.3,/),8F5.3)
42     READ(5,2) ((YP2(I,J),I=1,15),J=1,6)
43     2 FORMAT(5(10F5.3,/),10F5.3)
44     C
45     C
46     READ(7,*) (XI(I),I=1,8)
47     READ(7,*) RPM,MDAT,POL,TC1
48     READ(7,*) AL1,8,CLEAR
49     READ(7,*) ILAMB,LETAS
50     READ(7,*) RESFAC,MSL
51     READ(7,*) EPS
52     READ(7,*) CP,GAMA,KA
53     READ(7,*) CONTY
54     C
55     C
56     PRINT*, ' *** THE FIRST TRIAL OF THE DESIGN VECTOR IS *** '
57     PRINT*, ' '
58     DO 3 I=1,8
59     3 PRINT*, '          XI(I)
60     PRINT*, ' '
61     PRINT*, ' *** THE CHOSEN UPPER & LOWER BOUNDS ARE AS FOLLOWS *** '
62     DO 4 I=1,8
63     4 PRINT*, ' XLW(I), ' < X('I, ' < ',XUP(I)
64     PRINT*, ' '
65     PRINT*, ' *** INPUT CONDITIONS ARE AS FOLLOWS *** '
66     PRINT*, ' '
67     PRINT*, ' ** REV. PER SECOND = ',RPM
68     PRINT*, ' ** GAS TEMPERATURE = ',TC1
69     PRINT*, ' ** GAS PRESSURE = ',POL
70     PRINT*, ' ** GAS MASS FLOW = ',MDAT
71     PRINT*, ' '
72     C
73     ODM=1.
74     SRS=1.
75     II=1
76     SFI(1)=RESFAC
77     FPERF=0.
78     ICDUNT=0
79     PIN=ALAN(1)*4.
80     C

```

```

81          LU4=.FALSE.
82      C
83      C      *** SET THE RESPNSL FACTOR FOR SEQUENTIAL MINIMIZATION ***
84      C
85      C
86          RF=RLSFAC
87      C
88      C
89      C
90      5          LU1=.FALSE.
91                LU2=.FALSE.
92                LU3=.FALSE.
93                LU5=.FALSE.
94                LL1=.FALSE.
95                LL2=.FALSE.
96                L=0
97      C
98      7          DO 6 I=1,8
99      6          X(I)=X1(I)
100
101      C
102      C      *** GIVE INITIAL SET OF VARIABLES ,X(I) ***
103      C
104      C
105      10          D=      X(1)
106                  CR=    X(2)
107                  CN=    X(3)
108                  SS=    X(4)
109                  SK=    X(5)
110                  BET2=  X(6)
111                  BET3=  X(7)
112                  CA=    X(8)
113      C
114      C
115                  LLL=0
116                  TR=.2*CR
117                  TN=.2*CN
118      C
119      C
120      C
121      C
122      C
123      C      **** CALCULATE EFFICIENCY WITH GIVEN SET OF VARIABLES ,X(I) ****
124      C
125      C
126          RRAT=MDAT*SQRT(TG1)/P01
127          ETAS=1ETAS
128          LAHN=1LAHN
129          U=P1#*RP#*D
130          FI=CA/U
131          SIAC=FI*(TAN(BET3)-TAN(BET2))/2.
132          DTOS=U*CA*(TAN(BET2)+TAN(BET3))/CP
133          PSI=2.*CP*DTOS/U**2.
134          AL2=ATAN(TAN(BET2)+1./FI)
135          IF(AL2.LT..768) THEN
136              GO TO 2000
137          END IF
138          AL3=ATAN(TAN(BET3)-1./FI)
139          CC2=CA/COS(AL2)
140          CC3=CA/COS(AL3)
141          V2=CA/COS(BET2)
142          V3=CA/COS(BET3)
143          CW2=CC2*SIN(AL2)
144          CW3=CC3*SIN(AL3)
145          T2=T01-CC2**2./(2.*CP)
146      C
147      C
148      C
149      C
150      20          T2PR=T2-CC2**2.*LAHN/(2.*CP)
151                  P2=P01/(TG1/T2PR)**(GAMA/(GAMA-1.))
152                  IF(P2.GT.10000000.) GO TO 2000
153                  PC=P01/((GAMA+1.)/2.)**(GAMA/(GAMA-1.))
154                  R02=P2/(RA*T2)
155                  A2=MDAT/(R02*CA)
156                  A2N=MDAT/(R02*CC2)
157                  H2=A2*RP./U
158                  CC1=CA
159                  T1=T01-(CC1**2./(2.*CP))
160                  P1=P01*(T1/T01)**(GAMA/(GAMA-1.))

```

```

161 R01=P1/(RA*T1)
162 A1=MDAT/(K01*CA)
163 H1=A1*RPM/U
164 T03=T01-0105
165 T3=TC3-(CC3**2./(2.*CP))
166 P03=P01*(1.-DT05/(ETAS*T01))***(GAMA/(GAMA-1.))
167 PRATI0=P03/P01
168 P3=P03*(T3/T03)***(GAMA/(GAMA-1.))
169 R03=P3/(RA*T1)
170 A3=MDAT/(R03*CA)
171 H3=A3*RPM/U
172 HR=(H2+H3)/2.
173 HN=(H2+H1)/2.
174 ALG=ATAN((H3-H1)/(2.*(CN+1.25*CR)))
175
176 C
177 C
178 %C3=CC3/(GAMA*RA*T3)**.5
179 T3DPR=T2/(P2/P3)**((GAMA-1.)/GAMA)
180 LAMK=2.*CP*(T3-T3DPR)/V3**2.
181 S0CS=SS/CN
181 S0CR=SR/CR
182 YPB20=-1.*BET3*180./PIN
183 YPB2B3=YPB20
184 YPA10=-1.*AL2*180./PIN
185 YPA1A2=YPA10
186 CALL ICS2VE(YP1,S0CR,.3,1,1,-80.,-45.,17,8,1,17,WK,YPB20,IEF)
187 CALL ICS2VE(YP1,S0CS,.3,1,1,-80.,-45.,17,8,1,17,WK,YPA10,IEK)
188 CALL ICS2VE(YP2,S0CR,.3,1,,-70.,-45.,15,6,1,15,WK,YPB2B3,IFK)
189 CALL ICS2VE(YP2,S0CS,.3,1,,-70.,-45.,15,6,1,15,WK,YPA1A2,IFL)
190 YPR=(YPB20+(BET2/BET3)**2.*(YPB2B3-YPB20))
191 *(TR/(.2*CR))***(BET2/BET3)
192 YPH=(YPA10+(AL1/AL2)**2.*(YPA1A2-YPA10))
193 *(TN/(.2*CN))***(AL1/AL2)
194 ALH=ATAN((TAN(AL2)-TAN(AL1))/2.)
195 BETH=ATAN((TAN(BET3)-TAN(BET2))/2.)
196 COSKS=COS(AL2)**2./COS(ALH)**43.
197 COSKR=COS(BET3)**2./COS(BETH)**3.
198 CLR=2.*SR*(TAN(BET3)+TAN(BET2))*COS(BETH)/CR
199 CLN=2.*SS*(TAN(AL2)+TAN(AL1))*COS(ALH)/CN
200 LAMCN=.0334*CN*COS(AL2)/(HN*COS(AL1))
201 LAMCR=.0334*CR*COS(BET3)/(HR*COS(BET2))
202 CLR=CR
203 YSYKR=(LAMCR+B*CR*(CLEAR/CR))**.78/HR)
204 *CR*CLR**2.*COSKR/SR
205 YSYKN=LAMCN*CN*CLN**2.*COSRS/SS
206 YN=YPN+YSYKN
207 YR=YPR+YSYKR
208 LAMN1=YN*T2PR/T01
209 T03RFL=T3+V3**2./(2.*CP)
210 LAMR1=YR*T3DPR/T03REL
211 DENOM=(1.+(LAMR1*V3**2./(2.*CP)+T3*LAMN1*CC2**2.
212 /(.2.*T2*CP)))/(T01-T03)**2.
213 ETAS=1./SORT(DENOM)
214 BLADEN=PIN*D/SS
215 BLADER=PIN*D/SR
216 STMU=T01
217 ROMU=T2
218 CALL ICS1VE(VMU,100.,2500.,0.,0.,0.,0.,34,1,STMU,PSCOND,IEF)
219 CALL ICS1VE(VMU,100.,2500.,0.,0.,0.,0.,34,1,ROMU,PSCOND,ILF)
220 MUAVG=(STMU+ROMU)/200000.
221 RE=(RU2*CC2*CN+R03*V3*CR)/(2.*MUAVG)
222 PEFAC=(2.*100000./RE)**.2
223 ETASRE=1.-(1.-ETAS)*PEFAC
224 DGM=MDAT*SOR1(T01)/(P01*D**2.)
225 DGT=DGT0S/T01
226 DGM=RPM*D/SORT(T01)
227 DGN=CP*DGT0S/(RPM*D)**2.
228 DGM2=MDAT*KA*T01/(P01*RPM*D**3.)
229
230 C
231 C
232 DIFFET=ABS(ETASRE-ETAS)
233 IF(DIFFET.GT.EPS) THEN
234 ETAS=ETASRE
235 LAMN=LAMN1
236 LLL=LLL+1
237 IF(LLL.GT.5) GO TO 37
238 GO TO 20
239 END IF
240
241 C
242 C

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241 C      **** CALCULATE SUM OF INVERSE CONSTRAINT EQUATIONS ****
242 C
243 C
244     37      IF (CONTY.NE.0.) GO TO 4444
245             CONST=0.
246             GG(1)=1./(50./U-1.)
247             GG(2)=1./(U/400.-1.)
248             GG(3)=1./(HR/(B.*D)-CR/D)
249             GG(4)=1./(CR/C-HR/(2.*D))
250             GG(5)=1./(HR/(10.*D)-CH/D)
251             GG(6)=1./(CH/D-HR/(2.*D))
252             GG(7)=1./(.5-SS/CN)
253             GG(8)=1./(SS/CN-1.)
254             GG(9)=1./(.3-SR/CR)
255             GG(10)=1./(SR/CR-1.)
256             GG(11)=1./(.01-BET2)
257             GG(12)=1./(BET2-1.)
258             GG(13)=1./(40./57.-BET3)
259             GG(14)=1./(BET3-1.4)
260             GG(15)=1./(50./CA-1.)
261             GG(16)=1./(CA/400.-1.)
262             GG(17)=1./(P01/P2-PC1/PC)
263             GG(18)=1./(MC3-1.)
264             GG(19)=1./(12.*ALC/PIN-1.5)
265             GG(20)=1./(.25-FI)
266             GG(21)=1./(FI-2.)
267             GG(22)=1./(.5-PSI)
268             GG(23)=1./(.3-REAC)
269             GG(24)=1./(.3-REAC)
270             GG(25)=1./(REAC-.7)
271             GG(26)=1./(40./57.-AL2)
272             GG(27)=-1./D
273             GG(28)=-1./CR
274             GG(29)=-1./CN
275             GG(30)=-1./SS
276             GG(31)=-1./SR
277 C
278             GO 51 I=1,31
279     51      CONST=CONST+GG(I)
280 C
281 C
282     IF (CONST.GT.0.) THEN
283     GO TO 1140
284     END IF
285 C
286     **** CALCULATE PENALTY FUNCTION ****
287 C
288 C
289     PENFUN=ETASKE+RF*CONST
290 C
291 C
292     IF (PENFUN.LT.0) GO TO 1140
293 C
294 C
295     IF (L01) GO TO 5000
296 C
297 C
298 C
299     PENMAX=PENFUN
300     4444     ETAMAX=ETASKE
301             CONMAX=CONST
302             FIMAX=FI
303             PSIMAX=PSI
304             REMAX=RE
305             PRMAX=PRATIO
306             MAXMAX=MC3
307 C
308             PSIMAX=PSI
309             FIMAX=FI
310             DCMAX=DCM
311             DUTMAX=DUT
312             DCMAX=DCM
313             DCMAX=DCM
314             DCM2MAX=DCM2
315     DO 53 I=1,8
316     53      XMAX(I)=X(I)
317             IF (CONTY.NE.0.) GO TO 4445
318 C      *** CALCULATE DERIVATIVE OF PENALTY FUNCTION ***
319 C
320 C

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321 52 DENOM=(1.+(LAMR1*V3**2./(2.*CP)+T3*LAMN1*CC2**2.
322 # /((2.*T2*CP))/(T01-T03))**2.
323 CLE=CR
324 REFAC=(2.*107000./RE)**.2
325 COSRR=(COS(BET3))**2./(COS(BETN))**3.
326 COSRS=COS(AL2)**2./COS(ALN)**3.
327 M(1,1,1)=(-1.)*REFAC*V3**2./(2.*CP*DENOM*(T01-T03))
328 M(1,1,2)=(-1.)*REFAC*T3*CC2**2./(T2**2.*CP*DENOM*(T01-T03))
329 M(1,1,3)=REFAC*T3*CC2**2.*LAMN1/(T2**2.*2.*CP*DENOM
330 *(T01-T03))
331 M(1,1,4)=(-1.)*REFAC*CC2**2.*LAMN1/(T2**2.*CP*DENOM*(T01-T03))
332 M(1,1,5)=(-1.)*REFAC*(LAMR1*V3**2./(2.*CP)+LAMN1*T3*CC2**2.
333 # /((2.*CP))/(T01-T03)**2.*DENOM)
334 M(1,1,6)=(-1.)*REFAC*T3*CC2**2.*LAMN1/(T2**2.*CP*DENOM*(T01-T03))
335 M(1,1,7)=(-1.)*REFAC*V3**2.*LAMR1/(2.*CP*DENOM*(T01-T03))
336 M(1,1,8)=2.*ZG0000.)***2*(1.-1./SQRT(DENOM))/F1**1.2
337 R(2,1,1)=T3DPR/T03RLL
338 R(2,1,5)=(-1.)*YR*T3DPR/T03REL**2.
339 R(2,1,6)=YR/T03REL
340 R(2,2,2)=T2PR/T01
341 R(2,2,7)=YH/T01
342 M(2,3,9)=(-1.)*CC2/CP
343 M(2,4,4)=1.
344 M(2,4,10)=(-1.)*CC3/CP
345 M(2,5,3)=-1.
346 M(2,5,4)=1.
347 M(2,6,8)=1./COS(AL2)
348 M(2,6,9)=1.
349 M(2,6,12)=CA*SIN(AL2)/COS(AL2)**2.
350 M(2,7,8)=1./COS(BET3)
351 M(2,7,13)=CA*SIN(BET3)/COS(BET3)**2.
352 M(2,7,11)=1.
353 M(2,8,9)=R02*CN/(2.*MUAVG)
354 M(2,8,11)=R03*CR/(2.*MUAVG)
355 M(2,8,14)=CC2*CN/(2.*MUAVG)
356 M(2,8,15)=V3*CR/(2.*MUAVG)
357 M(2,8,16)=R02*CC2/(2.*MUAVG)
358 M(2,8,17)=R03*V3/(2.*MUAVG)
359 M(3,1,1)=1.
360 M(3,1,2)=1.
361 M(3,2,3)=1.
362 M(3,2,4)=1.
363 M(3,3,5)=CA*(TAN(BET2)+TAN(BET3))/CP
364 M(3,3,6)=U*(TAN(BET2)+TAN(BET3))/CP
365 M(3,3,7)=U*CA/(CP*(COS(BET2))**2.)
366 M(3,3,8)=U*CA/(CP*(COS(BET3))**2.)
367 M(3,3,9)=1.
368 M(3,4,9)=-1.
369 M(3,5,10)=1.
370 M(3,5,11)=V3/CP
371 M(3,6,12)=(P3/P2)**((GAMA-1.)/GAMA)
372 M(3,6,13)=(GAMA-1.)*T2/(GAMA*(P2**((GAMA-1.)/GAMA)))*(P3**((1./GAMA
373 # )))
374 M(3,6,14)=(1.-GAMA)*T2*(P3**((GAMA-1.)/GAMA))*(P2**((1.-2.*GAMA)/
375 # GAMA))/GAMA
376 M(3,7,12)=1.
377 M(3,7,15)=-1.*LAMN*CC2/CP
378 M(3,8,6)=1.
379 M(3,9,15)=1.
380 M(3,9,6)=1./COS(AL2)
381 M(3,9,16)=CA*SIN(AL2)/(COS(AL2)**2.)
382 M(3,10,6)=1./COS(AL3)
383 M(3,10,17)=CA*SIN(AL3)/(COS(AL3))**2.
384 M(3,11,6)=1./COS(BET3)
385 M(3,11,11)=1.
386 M(3,11,8)=CA*SIN(BET3)/(COS(BET3))**2.
387 M(3,12,7)=1./((1.+(TAN(BET2)+1./F1)**2.)*COS(BET2)**2.)
388 M(3,12,18)=-1./((1.+(TAN(BET2)+1./F1)**2.)*F1**2.)
389 M(3,12,16)=1.
390 M(3,13,8)=1.
391 M(3,14,14)=1./(RA*T2)
392 M(3,14,12)=-1.*P2/(RA*T2**2.)
393 M(3,15,10)=-1.*P3/(RA*T3**2.)
394 M(3,15,13)=1./(RA*T3)
395 M(3,16,19)=1.
396 M(3,17,20)=1.
397 M(4,1,1)=2.*BET2*(YPB2B3-YPB20)/BET3**2.
398 M(4,1,2)=-2.*BET2**2.*(YPB2B3-YPB20)/BET3**3.
399 M(4,2,2)=(LARCX+B*CR*(CLEAR/CR))**2.76/HR)*CLR**2.*CR*(-2.)
400 # *COS(BET3)*SIN(BET3)/(SR*COS(BETN))**3.)

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401      H(4,2,3)=(LAMCR*CLR**2.*(COS(BET3))**2./(SR*(COS(BETN))**3.))+
402      # 1.22*B*CLEAR**2.78*CLR**2.*CR**2.22*(COS(BET3))**2./(HR*SR
403      # *COS(BETN))**3.)
404      H(4,2,4)=CLR**2.*(COS(BET3))**2.*CR/(SR*(COS(BETN))**3.)
405      H(4,2,5)=-1.*B*(CLEAR/CR)**2.78*CLR**2.*(COS(BET3))**2.*CR/
406      # (HR**2.*SR*(COS(BETN))**3.)
407      H(4,2,6)=(LAMCR+B*CR*(CLEAR/CR)**2.78/HR)**2.*CLR*CR*CGSRR/SR
408      H(4,2,7)=(LAMCR+B*CR*(CLEAR/CR)**2.78/HR)*(-1.)*(CLR/SR)**2.
409      # *CR*CGSRR
410      H(4,2,8)=(LAMCR+B*CR*(CLEAR/CR)**2.78/HR)*CLR**2.*CR*CGSRR
411      # *SIN(BETN)**3./(SR*CGS(BETN))
412      H(4,3,9)=-2.*AL1**2.*(YPA1A2-YPA10)/AL2**3.
413      H(4,4,10)=CLN**2.*CN*CGSRS/SS
414      H(4,4,11)=2.*LAMCN*CLN*CN*CGSRS/SS
415      H(4,4,12)=LAMCN*CLN**2.*CGSRS/SS
416      H(4,4,13)=-1.*LAMCN*CLN**2.*CGSRS*CN/SS**2.
417      H(4,4,9)=-2.*LAMCN*CLN**2.*COS(AL2)*CN*SIN(AL2)/(SS*(COS(ALN))**
418      #3.)
419      H(4,4,14)=3.*LAMCN*CLN**2.*CN*CGSRS*SIN(ALN)/(SS+CGS(ALN))
420      H(4,5,17)=1.
421      H(4,5,15)=PI*HR*PRM
422      H(4,6,16)=1.
423      H(4,7,1)=1.
424      H(4,8,2)=1.
425      H(4,9,17)=H(3,3,5)
426      H(4,9,16)=H(3,3,6)
427      H(4,9,1)=H(3,3,7)
428      H(4,9,2)=H(3,3,8)
429      H(4,10,18)=1.
430      H(4,10,19)=H(2,4,16)
431      H(4,10,22)=1.
432      H(4,11,16)=H(2,7,8)
433      H(4,11,2)=H(2,7,13)
434      H(4,12,24)=H(2,3,9)
435      H(4,13,21)=(T3/T03)**(GAMA/(GAMA-1.))
436      C
437      H(4,13,22)=(GAMA/(GAMA-1.))*P03*T3**((1./(GAMA-1.))/T03)**(GAMA/
438      # (GAMA-1.))
439      H(4,13,15)=(GAMA/(1.-GAMA))*P03*T3**((GAMA/(GAMA-1.))*T03**((2.*
440      # GAMA-1.)/(1.-GAMA))
441      H(4,14,23)=(GAMA/(GAMA-1.))*P01*T2PR**((1./(GAMA-1.))*T01**((GAMA/
442      # (1.-GAMA))
443      H(4,15,20)=1.
444      H(4,15,16)=H(2,6,9)
445      H(4,15,9)=H(2,6,12)
446      H(4,16,1)=H(3,12,7)
447      H(4,16,24)=H(3,12,18)
448      H(4,16,9)=1.
449      H(4,17,2)=1./((1.+(TAN(BET3)-1./FI)**2.)*COS(BET3)**2.)
450      H(4,17,24)=1./((1.+(TAN(BET3)-1./FI)**2.)*FI**2.)
451      H(4,18,16)=1./U
452      H(4,18,17)=-1.*CA/U**2.
453      H(4,18,24)=1.
454      H(4,19,12)=1.
455      H(4,20,3)=1.
456      H(5,1,1)=1.
457      H(5,2,2)=1.
458      H(5,3,3)=1.
459      H(5,4,5)=-.0334*COS(BET3)/(HR*COS(BET2))
460      H(5,4,4)=-.0334*CGS(BET3)*CR/(HR**2.*COS(BET2))
461      H(5,4,2)=-.0334*SIN(BET3)*CR*COS(BET3)/(HR*COS(BET2))
462      H(5,4,1)=-.0334*CR*SIN(BET2)*COS(BET3)/(HR*(COS(BET2)**2.))
463      H(5,5,8)=-.5
464      H(5,5,9)=-.5
465      H(5,6,1)=2.*SR*COS(BETN)/(CR*COS(BET2)**2.)
466      H(5,6,2)=2.*SR*COS(BETN)/(CR*COS(BET3)**2.)
467      H(5,6,3)=-2.*SR*(TAN(BET2)+TAN(BET3))*COS(BETN)/CR**2.
468      H(5,6,10)=2.*(TAN(BET2)+TAN(BET3))*COS(BETN)/CR
469      H(5,6,11)=-2.*(TAN(BET2)+TAN(BET3))*SR*SIN(BETN)/CR
470      H(5,7,10)=1.
471      H(5,8,1)=-2./((4.+(TAN(BET2)-TAN(BET3))**2.)*COS(BET2)**2.)
472      H(5,8,2)=2./((4.+(TAN(BET3)-TAN(BET2))**2.)*COS(BET3)**2.)
473      H(5,8,11)=1.
474      H(5,9,6)=1.
475      H(5,9,1)=H(3,12,7)
476      H(5,9,12)=H(3,12,18)
477      H(5,10,5)=-.0334*CGS(AL2)/(HR*COS(AL1))
478      H(5,10,6)=-.0334*SIN(AL2)*CN/(HR*COS(AL1))
479      H(5,10,7)=-.0334*CGS(AL2)*CN/(HR**2.*COS(AL1))
480      H(5,11,13)=2.*(TAN(AL1)+TAN(AL2))*COS(ALN)/CN

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481 $M(5, 11, 14) = -2. * (\text{TAN}(\text{AL1}) + \text{TAN}(\text{AL2})) * \text{SS} * \text{SIN}(\text{ALM}) / \text{CN}$
 482 $M(5, 11, 5) = -2. * \text{SS} * (\text{TAN}(\text{AL1}) + \text{TAN}(\text{AL2})) * \text{COS}(\text{ALM}) / \text{CN} * * 2.$
 483 $M(5, 11, 6) = 2. * \text{SS} * \text{COS}(\text{ALM}) / (\text{CN} * \text{COS}(\text{AL2})) * * 2.$
 484 $M(5, 12, 5) = 1.$
 485 $M(5, 13, 13) = 1.$
 486 $M(5, 14, 14) = 1.$
 487 $M(5, 14, 6) = 2. / ((4. + (\text{TAN}(\text{AL2}) - \text{TAN}(\text{AL1})) * * 2.) * \text{COS}(\text{AL2})) * * 2.)$
 488 $M(5, 15, 15) = 1.$
 489 $M(5, 16, 16) = 1.$
 490 $M(5, 17, 23) = 1.$
 491 $M(5, 17, 15) = \text{PIN} * \text{RPH}$
 492 $M(5, 18, 17) = -1.$
 493 $M(5, 18, 19) = 1.$
 494 $M(5, 19, 16) = M(3, 10, 6)$
 495 $M(5, 19, 18) = M(3, 10, 17)$
 496 $M(5, 19, 20) = 1.$
 497 $M(5, 20, 16) = M(3, 9, 6)$
 498 $M(5, 20, 6) = M(3, 9, 16)$
 499 $M(5, 20, 22) = 1.$
 500 $M(5, 21, 17) = \text{PO1} * \text{GAMA} * (1. - \text{DTOS} / (\text{ETAS} * \text{T01})) * * (1. / (\text{GAMA} - 1.)) / ((1. - \text{GAMA}$
 501 $* \text{A}) * \text{ETAS} * \text{T01})$
 502 $M(5, 22, 19) = 1.$
 503 $M(5, 22, 20) = -1. * \text{CC3} / \text{CP}$
 504 $M(5, 23, 21) = 1.$
 505 $M(5, 23, 22) = \text{LANN} * \text{CC2} * (-1.) / \text{CP}$
 506 $M(5, 24, 12) = 1.$
 507 $M(5, 24, 16) = 1. / \text{U}$
 508 $M(5, 24, 23) = M(4, 18, 17)$
 509 $M(6, 1, 1) = 1.$
 510 $M(6, 2, 2) = 1.$
 511 $M(6, 3, 3) = 1.$
 512 $M(6, 4, 4) = .5$
 513 $M(6, 4, 5) = .5$
 514 $M(6, 5, 6) = 1.$
 515 $M(6, 6, 12) = 1.$
 516 $M(6, 6, 1) = M(3, 12, 7)$
 517 $M(6, 6, 7) = M(3, 12, 18)$
 518 $M(6, 7, 8) = .5$
 519 $M(6, 7, 4) = .5$
 520 $M(6, 8, 9) = \text{RPH} / \text{U}$
 521 $M(6, 8, 10) = -1. * \text{RPH} * \text{A2} / \text{U} * * 2.$
 522 $M(6, 8, 4) = 1.$
 523 $M(6, 9, 10) = -1. * \text{RPH} * \text{A3} / \text{U} * * 2.$
 524 $M(6, 9, 11) = \text{RPH} / \text{U}$
 525 $M(6, 9, 5) = 1.$
 526 $M(6, 10, 13) = 1.$
 527 $M(6, 11, 1) = M(5, 8, 1)$
 528 $M(6, 11, 2) = M(5, 8, 2)$
 529 $M(6, 12, 14) = 1. / \text{U}$
 530 $M(6, 12, 10) = M(4, 18, 17)$
 531 $M(6, 12, 7) = 1.$
 532 $M(6, 13, 15) = 1.$
 533 $M(6, 14, 12) = M(5, 14, 6)$
 534 $M(6, 15, 16) = 1.$
 535 $M(6, 16, 14) = 1.$
 536 $M(6, 17, 17) = 1.$
 537 $M(6, 17, 10) = M(3, 3, 5)$
 538 $M(6, 17, 14) = M(3, 3, 6)$
 539 $M(6, 17, 1) = M(3, 3, 7)$
 540 $M(6, 17, 2) = M(3, 3, 8)$
 541 $M(6, 18, 2) = M(4, 17, 2)$
 542 $M(6, 18, 7) = M(4, 17, 24)$
 543 $M(6, 18, 18) = 1.$
 544 $M(6, 19, 17) = -1.$
 545 $M(6, 20, 14) = M(3, 10, 6)$
 546 $M(6, 20, 18) = M(3, 10, 17)$
 547 $M(6, 21, 19) = M(2, 3, 9)$
 548 $M(6, 22, 14) = M(3, 9, 6)$
 549 $M(6, 22, 12) = M(3, 9, 16)$
 550 $M(6, 22, 19) = 1.$
 551 $M(6, 23, 16) = M(5, 17, 15)$
 552 $M(6, 23, 10) = 1.$
 553 $M(7, 1, 1) = 1.$
 554 $M(7, 2, 12) = 1.$
 555 $M(7, 3, 3) = 1.$
 556 $M(7, 4, 4) = M(6, 8, 9)$
 557 $M(7, 4, 5) = M(6, 8, 10)$
 558 $M(7, 5, 6) = M(6, 9, 11)$
 559 $M(7, 5, 5) = M(6, 9, 10)$
 560 $M(7, 6, 7) = 1.$

561 $R(7,7,8)=1./U$
562 $R(7,7,5)=R(4,18,17)$
563 $R(7,7,12)=1.$
564 $R(7,8,9)=RPH/U$
565 $R(7,8,5)=-1.*RPH*A1/U**2.$
566 $R(7,9,10)=-1.*MDAT/(R02**2.*CA)$
567 $R(7,9,8)=-1.*MDAT/(R02*CA**2.)$
568 $R(7,9,4)=1.$
569 $R(7,10,5)=1.$
570 $R(7,10,15)=PIN*RPH$
571 $R(7,11,6)=1.$
572 $R(7,11,6)=-1.*MDAT/(R03*CA**2.)$
573 $R(7,11,11)=-1.*MDAT/(R03**2.*CA)$
574 $R(7,12,1)=R(3,12,7)$
575 $R(7,12,12)=R(3,12,13)$
576 $R(7,12,16)=1.$
577 $R(7,13,13)=1.$
578 $R(7,14,8)=1.$
579 $R(7,15,14)=1.$
580 $R(7,16,15)=1.$
581 $R(7,17,5)=R(3,3,5)$
582 $R(7,17,6)=R(3,3,6)$
583 $R(7,17,1)=R(3,3,7)$
584 $R(7,17,2)=R(3,3,8)$
585 $R(7,18,2)=R(4,17,2)$
586 $R(7,18,12)=R(4,17,24)$
587 $R(7,19,8)=R(3,9,6)$
588 $R(7,19,16)=R(3,9,16)$
589 $R(8,1,1)=1.$
590 $R(8,2,2)=1.$
591 $R(8,3,3)=1.$
592 $R(8,4,4)=R(7,9,10)$
593 $R(8,4,5)=R(7,9,8)$
594 $R(8,5,6)=PIN*RPH$
595 $R(8,5,14)=1.$
596 $R(8,6,7)=R(7,11,11)$
597 $R(8,6,5)=R(7,11,8)$
598 $R(8,7,8)=1.$
599 $R(8,8,5)=1.$
600 $R(8,9,9)=-1.*MDAT/(CA*R01**2.)$
601 $R(8,9,5)=-1.*MDAT/(R01*CA**2.)$
602 $R(8,10,10)=1./(RA*T2)$
603 $R(8,10,11)=-1.*P2/(RA*T2**2.)$
604 $R(8,10,4)=1.$
605 $R(8,11,12)=1./(RA*T3)$
606 $R(8,11,13)=-1.*P3/(RA*T3**2.)$
607 $R(8,11,7)=1.$
608 $R(8,12,5)=1./U$
609 $R(8,12,14)=R(4,18,17)$
610 $R(8,12,17)=1.$
611 $R(8,13,15)=1.$
612 $R(8,14,16)=1.$
613 $R(8,15,6)=1.$
614 $R(8,16,1)=R(3,12,7)$
615 $R(8,16,17)=R(3,12,13)$
616 $R(9,1,1)=1.$
617 $R(9,2,2)=1.$
618 $R(9,3,3)=1.$
619 $R(9,4,4)=R(8,10,10)$
620 $R(9,4,5)=R(9,10,11)$
621 $R(9,5,6)=1.$
622 $R(9,6,7)=1.$
623 $R(9,7,8)=R(8,11,12)$
624 $R(9,7,9)=R(8,11,13)$
625 $R(9,8,10)=1.$
626 $R(9,9,11)=1./(RA*T1)$
627 $R(9,9,12)=-1.*P1/(RA*T1**2.)$
628 $R(9,10,13)=R(4,14,23)$
629 $R(9,10,4)=1.$
630 $R(9,11,14)=-1.*CC2/CP$
631 $R(9,11,5)=1.$
632 $R(9,12,15)=R(4,13,21)$
633 $R(9,12,9)=R(4,13,22)$
634 $R(9,12,16)=R(4,13,18)$
635 $R(9,12,8)=1.$
636 $R(9,13,9)=1.$
637 $R(9,13,17)=-1.*CC3/CP$
638 $R(9,13,16)=1.$
639 $R(9,14,7)=PIN*RPH$
640 $R(9,14,20)=1.$

```

641      H(9,15,18)=1.
642      H(9,16,19)=1.
643      H(9,17,6)=1./O
644      H(9,17,20)=H(4,18,17)
645      H(10,1,1)=1.
646      H(10,2,2)=1.
647      H(10,3,3)=1.
648      H(10,4,4)=H(4,14,23)
649      H(10,5,5)=H(9,11,14)
650      H(10,5,14)=1.
651      H(10,6,6)=1.
652      H(10,7,7)=1.
653      H(10,8,8)=H(4,13,21)
654      H(10,8,9)=H(4,13,22)
655      H(10,8,10)=H(4,13,18)
656      H(10,9,9)=1.
657      H(10,9,10)=1.
658      H(10,9,11)=H(9,13,17)
659      H(10,10,12)=1.
660      H(10,11,13)=(GAMA/(GAMA-1.))*PO1*TO1**((GAMA/(1.-GAMA))*T1**((1.
661      / (GAMA-1.))
662      H(10,12,6)=-1.*CA/CP
663      H(10,12,13)=1.
664      H(10,13,14)=1.
665      H(10,13,4)=1.
666      H(10,13,5)=H(5,23,22)
667      H(10,14,6)=H(3,9,6)
668      H(10,14,15)=H(3,9,16)
669      H(10,14,5)=1.
670      H(10,15,16)=H(5,21,17)
671      H(10,15,8)=1.
672      H(10,16,10)=1.
673      H(10,16,16)=-1.
674      H(10,17,6)=H(3,10,6)
675      H(10,17,17)=H(3,10,17)
676      H(10,17,11)=1.
677      H(10,18,16)=1.
678      H(10,19,19)=1.
679      H(10,20,7)=PIN*RPB
680      H(11,1,1)=1.
681      H(11,2,2)=1.
682      H(11,3,3)=1.
683      H(11,4,4)=1.
684      H(11,4,5)=H(5,23,22)
685      H(11,5,5)=1.
686      H(11,5,6)=H(3,9,6)
687      H(11,5,7)=H(3,9,16)
688      H(11,6,6)=1.
689      H(11,7,8)=1.
690      H(11,8,9)=H(5,21,17)
691      H(11,9,10)=1.
692      H(11,9,11)=H(5,22,20)
693      H(11,10,10)=1.
694      H(11,10,9)=-1.
695      H(11,11,6)=H(3,10,6)
696      H(11,11,12)=H(3,10,17)
697      H(11,11,11)=1.
698      H(11,12,13)=1.
699      H(11,13,6)=H(10,12,6)
700      H(11,14,5)=H(9,11,14)
701      H(11,14,4)=1.
702      H(11,15,1)=H(3,12,7)
703      H(11,15,14)=H(3,12,18)
704      H(11,15,7)=1.
705      H(11,16,15)=H(3,3,5)
706      H(11,16,6)=H(3,3,6)
707      H(11,16,1)=H(3,3,7)
708      H(11,16,2)=H(3,5,8)
709      H(11,16,9)=1.
710      H(11,17,2)=H(4,17,2)
711      H(11,17,12)=1.
712      H(11,17,14)=H(4,17,24)
713      H(11,18,16)=1.
714      H(11,19,17)=1.
715      H(12,1,1)=1.
716      H(12,2,2)=1.
717      H(12,3,3)=1.
718      H(12,4,4)=H(9,11,14)
719      H(12,5,4)=1.
720      H(12,5,5)=H(3,9,6)

```

721	$M(12,5,6)=M(3,9,16)$
722	$M(12,6,5)=1.$
723	$M(12,7,6)=1.$
724	$M(12,7,1)=M(3,12,7)$
725	$M(12,7,7)=M(3,12,18)$
726	$M(12,8,8)=1.$
727	$M(12,9,9)=M(3,3,5)$
728	$M(12,9,5)=M(3,3,6)$
729	$M(12,9,1)=M(3,3,7)$
730	$M(12,9,2)=M(3,3,8)$
731	$M(12,9,10)=1.$
732	$M(12,10,10)=-1.$
733	$M(12,11,5)=M(3,10,6)$
734	$M(12,11,11)=M(3,10,17)$
735	$M(12,12,11)=1.$
736	$M(12,12,2)=M(4,17,2)$
737	$M(12,12,7)=M(4,17,24)$
738	$M(12,13,12)=1.$
739	$M(12,14,7)=1.$
740	$M(12,14,5)=1./U$
741	$M(12,14,9)=M(4,18,17)$
742	$M(12,15,9)=1.$
743	$M(12,15,8)=M(10,20,7)$
744	$M(12,16,13)=1.$
745	$M(12,17,14)=1.$
746	$M(13,1,1)=1.$
747	$M(13,2,2)=1.$
748	$M(13,3,3)=1.$
749	$M(13,4,4)=M(3,9,6)$
750	$M(13,4,5)=M(3,9,16)$
751	$M(13,5,4)=1.$
752	$M(13,6,5)=1.$
753	$M(13,6,1)=M(3,12,7)$
754	$M(13,6,6)=M(3,12,18)$
755	$M(13,7,6)=1.$
756	$M(13,7,4)=1./U$
757	$M(13,7,7)=M(4,18,17)$
758	$M(13,8,8)=1.$
759	$M(13,9,7)=1.$
760	$M(13,9,8)=M(10,20,7)$
761	$M(13,10,7)=M(3,3,5)$
762	$M(13,10,4)=M(3,3,6)$
763	$M(13,10,1)=M(3,3,7)$
764	$M(13,10,2)=M(3,3,8)$
765	$M(13,11,2)=M(4,17,2)$
766	$M(13,11,6)=M(4,17,24)$
767	$M(13,12,9)=1.$
768	$M(13,13,10)=1.$
769	$M(13,14,11)=1.$
770	$M(14,1,1)=1.$
771	$M(14,2,2)=1.$
772	$M(14,3,3)=1.$
773	$M(14,4,4)=1.$
774	$M(14,5,1)=M(3,12,7)$
775	$M(14,5,5)=M(3,12,18)$
776	$M(14,6,5)=1.$
777	$M(14,6,4)=1./U$
778	$M(14,6,6)=M(4,18,17)$
779	$M(14,7,6)=1.$
780	$M(14,7,7)=M(10,20,7)$
781	$M(14,8,7)=1.$
782	$M(14,9,8)=1.$
783	$M(14,10,9)=1.$
784	$M(14,11,10)=1.$
785	$M(15,1,1)=1.$
786	$M(15,2,2)=1.$
787	$M(15,3,3)=1.$
788	$M(15,4,4)=1.$
789	$M(15,5,4)=1./U$
790	$M(15,5,5)=M(4,18,17)$
791	$M(15,6,5)=1.$
792	$M(15,6,6)=M(10,20,7)$
793	$M(15,7,6)=1.$
794	$M(15,8,7)=1.$
795	$M(15,9,8)=1.$
796	$M(15,10,9)=1.$
797	$M(16,1,1)=1.$
798	$M(16,2,2)=1.$
799	$M(16,3,3)=1.$
800	$M(16,4,4)=1.$

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801      M(16,5,5)=M(10,20,7)
802      M(16,6,5)=1.
803      M(16,7,6)=1.
804      M(16,8,7)=1.
805      M(16,9,8)=1.
806      M(17,1,6)=1.
807      M(17,2,7)=1.
808      M(17,3,2)=1.
809      M(17,4,3)=1.
810      M(17,5,1)=1.
811      M(17,6,3)=1.
812      M(17,7,5)=1.
813      M(17,8,4)=1.
814      C
815      C
816      C      ***** OBTAIN VECTOR DETA(I) *****
817      C
818      C
819      C
820      DO 91 J=1,SIZE1(1,2)
821      DETA(J)=M(1,1,J)
822      C
823      DO 93 K=2,17
824      DO 92 KK=1,SIZE1(K,1)
825      DO 92 KKK=1,SIZE1(K,2)
826      MD2(KK,KKK)=M(K,KK,KKK)
827      C
828      C
829      C
830      C
831      CALL MULTIP(DETA,MD2,SIZE1(K,1),SIZE1(K,2))
832      C
833      C
834      C
835      C
836      C
837      C      93      CONTINUE
838      C
839      C
840      C
841      G(2,1,1)=1.
842      G(2,2,2)=1.
843      G(2,3,3)=.5
844      G(2,3,4)=.5
845      G(3,1,1)=1.
846      G(3,2,2)=1.
847      G(3,3,3)=M(6,8,9)
848      G(3,3,4)=M(6,8,10)
849      G(3,4,5)=M(6,9,11)
850      G(3,4,6)=M(6,9,10)
851      G(4,1,1)=1.
852      G(4,2,2)=1.
853      G(4,3,3)=M(7,9,10)
854      G(4,3,4)=M(7,9,8)
855      G(4,4,1)=M(7,10,15)
856      G(4,5,5)=M(7,11,11)
857      G(4,5,4)=M(7,11,8)
858      G(5,1,1)=1.
859      G(5,2,2)=1.
860      G(5,3,4)=M(8,10,10)
861      G(5,3,5)=M(8,10,11)
862      G(5,4,3)=1.
863      G(5,5,6)=M(8,11,12)
864      G(5,5,7)=M(8,11,13)
865      G(6,1,1)=1.
866      G(6,2,2)=1.
867      G(6,3,3)=1.
868      G(6,4,4)=M(9,10,13)
869      G(6,5,5)=M(9,11,14)
870      G(6,6,6)=M(4,13,18)
871      G(6,6,7)=M(4,13,21)
872      G(6,6,8)=M(4,13,22)
873      G(6,7,6)=1.
874      G(6,7,9)=M(9,13,17)
875      G(6,7,8)=1.
876      G(7,1,1)=1.
877      G(7,2,2)=1.
878      G(7,3,3)=1.
879      G(7,4,4)=M(5,23,22)
880      G(7,4,5)=1.

```

881 $G(7,5,3)=H(3,9,6)$
 882 $G(7,5,6)=H(3,9,16)$
 883 $G(7,5,4)=1.$
 884 $G(7,6,7)=-1.$
 885 $G(7,6,8)=1.$
 886 $G(7,7,7)=H(5,21,17)$
 887 $G(7,8,8)=1.$
 888 $G(7,8,9)=H(9,13,17)$
 889 $G(7,9,3)=H(4,10,6)$
 890 $G(7,9,10)=H(3,10,17)$
 891 $G(7,9,9)=1.$
 892 $G(8,1,1)=1.$
 893 $G(8,2,2)=1.$
 894 $G(8,3,3)=1.$
 895 $G(8,4,3)=H(3,9,6)$
 896 $G(8,4,5)=H(3,9,16)$
 897 $G(8,4,4)=1.$
 898 $G(8,5,4)=H(9,11,14)$
 899 $G(8,6,5)=1.$
 900 $G(8,6,6)=H(3,12,7)$
 901 $G(8,6,7)=H(3,12,18)$
 902 $G(8,7,8)=H(3,3,5)$
 903 $G(8,7,3)=H(3,3,6)$
 904 $G(8,7,6)=H(3,3,7)$
 905 $G(8,7,9)=H(3,3,8)$
 906 $G(8,7,10)=1.$
 907 $G(8,8,10)=-1.$
 908 $G(8,9,3)=H(3,10,6)$
 909 $G(8,9,11)=H(3,10,17)$
 910 $G(8,10,7)=H(4,17,24)$
 911 $G(8,10,9)=H(4,17,2)$
 912 $G(8,10,11)=1.$
 913 $G(9,1,1)=1.$
 914 $G(9,2,2)=1.$
 915 $G(9,3,3)=1.$
 916 $G(9,4,3)=H(3,9,6)$
 917 $G(9,4,6)=H(3,9,16)$
 918 $G(9,5,6)=1.$
 919 $G(9,5,4)=H(3,12,7)$
 920 $G(9,5,7)=H(3,12,18)$
 921 $G(9,6,4)=1.$
 922 $G(9,7,7)=1.$
 923 $G(9,7,3)=1./U$
 924 $G(9,7,8)=H(4,18,17)$
 925 $G(9,9,5)=1.$
 926 $G(9,10,3)=H(3,3,5)$
 927 $G(9,10,3)=H(3,3,6)$
 928 $G(9,10,4)=H(3,3,7)$
 929 $G(9,10,5)=H(3,3,8)$
 930 $G(9,11,5)=H(4,17,2)$
 931 $G(9,11,7)=H(4,17,24)$
 932 $C(9,8,8)=1.$
 933 $G(9,8,1)=H(12,15,8)$
 934 $G(10,1,1)=1.$
 935 $G(10,2,2)=1.$
 936 $G(10,3,3)=1.$
 937 $G(10,4,4)=1.$
 938 $G(10,5,5)=1.$
 939 $G(10,6,4)=H(3,12,7)$
 940 $G(10,6,6)=H(3,12,18)$
 941 $G(10,7,6)=1.$
 942 $G(10,7,3)=1./U$
 943 $G(10,7,7)=H(4,18,17)$
 944 $G(10,8,7)=1.$
 945 $G(10,8,1)=H(12,15,8)$
 946 $G(11,1,1)=1.$
 947 $G(11,2,2)=1.$
 948 $G(11,3,8)=1.$
 949 $G(11,4,8)=1.$
 950 $G(11,5,7)=1.$
 951 $G(11,6,8)=1./U$
 952 $G(11,6,9)=H(4,18,17)$
 953 $G(11,7,9)=1.$
 954 $G(11,7,11)=H(12,15,8)$
 955 $G(12,1,1)=1.$
 956 $G(12,2,2)=1.$
 957 $G(12,3,3)=1.$
 958 $G(12,4,4)=1.$
 959 $G(12,5,5)=1.$
 960 $G(12,6,6)=1.$

961 $G(12,7,7)=1.$
 962 $G(12,8,8)=1.$
 963 $G(12,9,11)=K(12,15,8)$
 964 $G(13,1,1)=1.$
 965 $G(13,2,2)=1.$
 966 $G(13,3,3)=1.$
 967 $G(13,4,4)=1.$
 968 $G(13,5,5)=1.$
 969 $G(13,6,6)=1.$
 970 $G(13,6,6)=1.$
 971 $G(13,7,7)=1.$
 972 $G(13,8,8)=1.$
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$H(2,1,1)=1.$
 $H(2,2,2)=1.$
 $H(2,3,3)=.5$
 $H(2,3,4)=.5$
 $H(3,1,1)=1.$
 $H(3,2,2)=1.$
 $H(3,3,3)=RPR/U$
 $H(3,3,4)=H(7,8,5)$
 $H(3,3,5)=H(7,4,4)$
 $H(3,3,6)=H(7,4,5)$
 $H(4,1,1)=1.$
 $H(4,2,2)=1.$
 $H(4,3,4)=H(8,9,9)$
 $H(4,3,3)=H(8,9,5)$
 $H(4,4,1)=H(8,5,6)$
 $H(4,5,3)=H(7,9,2)$
 $H(4,5,5)=H(7,9,10)$
 $H(5,2,2)=1.$
 $H(5,1,1)=1.$
 $H(5,3,3)=1.$
 $H(5,4,4)=H(9,9,11)$
 $H(5,4,5)=H(9,9,12)$
 $H(5,5,6)=H(8,10,10)$
 $H(5,5,7)=H(8,10,11)$
 $H(6,1,1)=1.$
 $H(6,2,2)=1.$
 $H(6,3,3)=1.$
 $H(6,4,4)=H(10,11,13)$
 $H(6,5,3)=H(10,12,6)$
 $H(6,5,4)=1.$
 $H(6,6,5)=H(4,14,23)$
 $H(6,7,6)=H(9,11,14)$
 $H(7,1,1)=1.$
 $H(7,2,2)=1.$
 $H(7,3,3)=1.$
 $H(7,4,3)=H(10,12,6)$
 $H(7,5,4)=1.$
 $H(7,5,5)=H(5,23,22)$
 $H(7,6,5)=1.$
 $H(7,6,5)=H(3,9,6)$
 $H(7,6,6)=H(3,9,16)$
 $H(8,1,1)=1.$
 $H(8,2,2)=1.$
 $H(8,3,3)=1.$
 $H(8,4,4)=H(9,11,14)$
 $H(8,5,4)=1.$
 $H(8,5,3)=H(3,9,6)$
 $H(8,5,5)=H(3,9,16)$
 $H(8,6,5)=1.$
 $H(8,6,6)=H(3,12,7)$
 $H(8,6,7)=H(3,12,18)$
 $H(9,1,1)=1.$
 $H(9,2,2)=1.$
 $H(9,3,4)=1.$
 $H(9,4,4)=H(3,9,6)$
 $H(9,4,5)=H(3,9,16)$
 $H(9,5,5)=1.$
 $H(9,5,3)=H(3,12,7)$
 $H(9,5,6)=H(3,12,13)$
 $H(9,6,3)=1.$
 $H(9,7,6)=1.$
 $H(9,7,4)=1./U$
 $H(9,7,7)=H(4,19,17)$
 $H(10,1,1)=1.$
 $H(10,2,2)=1.$

1041	$H(10,3,3)=1.$
1042	$H(10,4,4)=1.$
1043	$H(10,5,3)=H(3,12,7)$
1044	$H(10,5,5)=H(3,12,18)$
1045	$H(10,6,5)=1.$
1046	$H(10,6,4)=1./0$
1047	$H(10,6,6)=H(4,18,17)$
1048	$H(10,7,6)=1.$
1049	$H(10,7,1)=H(8,5,6)$
1050	$H(11,1,1)=1.$
1051	$H(11,2,3)=1.$
1052	$H(11,3,6)=1.$
1053	$H(11,4,8)=1.$
1054	$H(11,5,8)=1./0$
1055	$H(11,5,9)=H(4,18,17)$
1056	$H(11,6,9)=1.$
1057	$H(11,6,11)=H(8,5,6)$
1058	$H(12,1,1)=1.$
1059	$H(12,2,2)=1.$
1060	$H(12,3,3)=1.$
1061	$H(12,4,4)=1.$
1062	$H(12,5,5)=1.$
1063	$H(12,6,6)=1.$
1064	$H(12,7,7)=1.$
1065	$H(12,8,8)=1.$
1066	$H(12,9,1)=H(8,5,6)$
1067	$H(13,1,1)=1.$
1068	$H(13,2,2)=1.$
1069	$H(13,3,3)=1.$
1070	$H(13,4,4)=1.$
1071	$H(13,5,5)=1.$
1072	$H(13,6,6)=1.$
1073	$H(13,7,7)=1.$
1074	$H(13,8,8)=1.$
1075	
1076	
1077	
1078	$0(2,1,1)=1.$
1079	$0(2,2,2)=1.$
1080	$0(2,3,3)=H(7,8,9)$
1081	$0(2,3,4)=H(7,8,5)$
1082	$0(2,4,5)=H(8,9,11)$
1083	$0(2,4,4)=H(8,9,10)$
1084	$0(3,1,1)=1.$
1085	$0(3,2,2)=1.$
1086	$0(3,3,3)=H(8,9,5)$
1087	$0(3,3,5)=H(8,9,9)$
1088	$0(3,4,4)=H(8,5,6)$
1089	$0(3,5,3)=H(8,6,5)$
1090	$0(3,5,6)=H(8,6,7)$
1091	$0(4,1,3)=1.$
1092	$0(4,2,2)=1.$
1093	$0(4,3,4)=1.$
1094	$0(4,4,1)=1.$
1095	$0(4,5,5)=H(9,9,11)$
1096	$0(4,5,6)=H(9,9,12)$
1097	$0(4,6,7)=H(9,7,8)$
1098	$0(4,6,8)=H(9,7,9)$
1099	$0(5,1,1)=1.$
1100	$0(5,2,2)=1.$
1101	$0(5,3,3)=1.$
1102	$0(5,4,4)=1.$
1103	$0(5,5,5)=H(10,11,13)$
1104	$0(5,6,5)=1.$
1105	$0(5,6,4)=H(10,12,6)$
1106	$0(5,7,6)=H(4,13,21)$
1107	$0(5,7,7)=H(4,13,22)$
1108	$0(5,7,8)=H(4,13,18)$
1109	$0(5,8,7)=1.$
1110	$0(5,8,8)=1.$
1111	$0(5,8,9)=H(9,13,17)$
1112	$0(6,1,1)=1.$
1113	$0(6,2,2)=1.$
1114	$0(6,3,3)=1.$
1115	$0(6,4,4)=1.$
1116	$0(6,5,4)=H(10,12,6)$
1117	$0(6,6,5)=H(5,21,17)$
1118	$0(6,7,6)=1.$
1119	$0(6,7,7)=H(9,13,17)$
1120	$0(6,8,6)=1.$

C
C
C

```

1121          O(6,8,5)=-1.
1122          O(6,9,4)=M(3,16,6)
1123          O(6,9,8)=M(3,16,17)
1124          O(6,9,7)=1.
1125          U(7,1,1)=1.
1126          O(7,2,2)=1.
1127          O(7,3,3)=1.
1128          U(7,4,4)=1.
1129          O(7,5,7)=M(3,3,5)
1130          O(7,5,4)=M(3,3,6)
1131          O(7,5,5)=M(3,3,7)
1132          O(7,5,6)=M(3,3,8)
1133          O(7,5,8)=1.
1134          O(7,6,8)=-1.
1135          O(7,7,4)=M(3,16,6)
1136          O(7,7,9)=M(3,16,17)
1137          O(7,8,9)=1.
1138          O(7,8,6)=M(4,17,2)
1139          O(7,8,10)=M(4,17,24)
1140          O(8,1,1)=1.
1141          O(8,2,2)=1.
1142          O(8,3,3)=1.
1143          O(8,4,6)=1.
1144          O(8,5,4)=1.
1145          O(8,6,5)=1.
1146          O(8,7,7)=1.
1147          O(8,7,1)=M(10,20,7)
1148          O(8,8,7)=M(3,3,5)
1149          O(8,8,6)=M(3,3,6)
1150          O(8,8,4)=M(3,3,7)
1151          O(8,8,5)=M(3,3,8)
1152          O(8,9,5)=M(4,17,2)
1153          O(8,9,8)=M(4,17,24)
1154          O(8,10,8)=1.
1155          O(8,10,6)=1./U
1156          O(8,10,7)=M(4,18,17)
1157          U(9,1,1)=1.
1158          O(9,2,2)=1.
1159          U(9,3,3)=1.
1160          O(9,4,4)=1.
1161          O(9,5,5)=1.
1162          O(9,6,6)=1.
1163          U(9,7,7)=1.
1164          O(9,8,7)=M(4,18,17)
1165          O(9,7,1)=M(10,20,7)
1166          U(9,8,6)=1./U
1167          O(10,1,1)=1.
1168          O(10,2,2)=1.
1169          O(10,3,3)=1.
1170          O(10,4,6)=1.
1171          O(10,5,7)=1.
1172          O(10,6,8)=1.
1173          O(10,7,1)=M(10,20,7)
1174          C
1175          C      **** OBTAIN PARTIAL DERIVATIVE(S) OF INVERSE CONSTRAINT EQ'NS
1176          C
1177          C
1178          D#3DT3=-.5*CC3/((GAMA*RA)**.5*T3**1.5)
1179          D#3DC3=1./(GAMA*RA*T3)**.5
1180          PDC19=1./(1.+((H2-H1)/(2.*(CN+1.25*CR)))**2.)
1181          DC22DT=2.*CP*(U/(.5*U**2.-2.*CP*DTOS))**2.
1182          DC22DU=-1.*DC22DT*DTOS/U**2.
1183          DC23DT=-2.*CP*(U/(2.*CP*DTOS-6.*U**2.))**2.
1184          DC23DU=-2.*DC23DT*DTOS/U**2.
1185          C
1186          C17FAC=(PC/(PC-P2))**2./P01
1187          C17FAD=(1.+LAMN)*CA**2.*SIN(AL2)/(2.*CP*COS(L2)**3.)
1188          C18FAC=-1./(MC3-1.)**2.
1189          C18FAD=M(3,10,6)+M(3,10,17)*M(4,16,24)*M(4,18,16)
1190          C18FAF=M(3,10,17)*M(4,17,24)*M(4,18,17)*PIN*RPH
1191          C19FAC=-12.*PIN/(12.*ALC-1.5*PIN)**2.
1192          C24FAC=1./(C3-R)**2.
1193          C25FAC=-1./(K-.7)**2.
1194          C26FAC=(57./(40.-57.*AL2))**2.
1195          C
1196          C
1197          C(1,1)=50.*PIN*RPH/(50.-PIN*RPH*D)**2.
1198          C
1199          C(2,1)=-400.*PIN*RPH/(PIN*RPH*D-400.)**2.
1200          C

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```

1201      G(1,1,1)=8./(HR-B.*CR)
1202      G(1,1,2)=64.*D/(HR-B.*CR)**2.
1203      G(1,1,3)=-8.*D/(HR-B.*CR)**2.
1204      C
1205      C
1206      C      ****OBTAIN VECTOR C(3) ****
1207      C
1208      DO 201 J=1,SIZE2(1,2)
1209      201      D3(J)=G(1,1,J)
1210      C
1211      DO 401 K=2,13
1212      DO 301 KK=1,SIZE2(K,1)
1213      DO 301 KKK=1,SIZE2(K,2)
1214      301      MD2(KK,KKK)=G(K,KK,KKK)
1215      CALL MULTIP(D3,MD2,SIZE2(K,1),SIZE2(K,2))
1216      401      CONTINUE
1217      C
1218      DO 402 I=1,8
1219      402      C(3,I)=D3(I)
1220      C
1221      C
1222      G(1,1,1)=2./(2.*CR-HR)
1223      G(1,1,2)=-4.*D/(2.*CR-HR)**2.
1224      G(1,1,3)=2.*D/(2.*CR-HR)**2.
1225      C
1226      C
1227      C      ****OBTAIN VECTOR C(4) ****
1228      C
1229      DO 501 J=1,SIZE2(1,2)
1230      501      D4(J)=G(1,1,J)
1231      C
1232      DO 701 K=2,13
1233      DO 601 KK=1,SIZE2(K,1)
1234      DO 601 KKK=1,SIZE2(K,2)
1235      601      MD2(KK,KKK)=G(K,KK,KKK)
1236      CALL MULTIP(D4,MD2,SIZE2(K,1),SIZE2(K,2))
1237      701      CONTINUE
1238      C
1239      C
1240      DO 702 I=1,8
1241      702      C(4,I)=D4(I)
1242      C
1243      C
1244      C
1245      H(1,1,1)=10./(HN-10.*CN)**2.
1246      H(1,1,3)=-10.*D/(HN-10.*CN)**2.
1247      H(1,1,2)=100.*D/(HN-10.*CN)**2.
1248      C
1249      C      ****OBTAIN VECTOR C(5) ****
1250      C
1251      DO 801 J=1,SIZE3(1,2)
1252      801      D5(J)=H(1,1,J)
1253      C
1254      DO 1001 K=2,13
1255      DO 901 KK=1,SIZE3(K,1)
1256      DO 901 KKK=1,SIZE3(K,2)
1257      901      MD2(KK,KKK)=H(K,KK,KKK)
1258      CALL MULTIP(D5,MD2,SIZE3(K,1),SIZE3(K,2))
1259      1001      CONTINUE
1260      C
1261      C
1262      DO 1011 I=1,8
1263      1011      C(5,I)=D5(I)
1264      C
1265      H(1,1,1)=2./(2.*CN-HN)
1266      H(1,1,2)=-4.*D/(2.*CN-HN)**2.
1267      H(1,1,3)=2.*D/(2.*CN-HN)**2.
1268      C
1269      C      ****OBTAIN VECTOR C(6) ****
1270      C
1271      DO 1002 J=1,SIZE3(1,2)
1272      1002      D6(J)=H(1,1,J)
1273      C
1274      DO 1004 K=2,13
1275      DO 1003 KK=1,SIZE3(K,1)
1276      DO 1003 KKK=1,SIZE3(K,2)
1277      1003      MD2(KK,KKK)=H(K,KK,KKK)
1278      CALL MULTIP(D6,MD2,SIZE3(K,1),SIZE3(K,2))
1279      1004      CONTINUE
1280      C

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1281      C
1282      DO 1077 I=1,8
1283      C(6,I)=D6(I)
1284      C
1285      C(7,3)=-1.*SS/(.5*CN-SS)**2.
1286      C(7,4)=CR/(.5*CN-SS)**2.
1287      C
1288      C(8,3)=SS/(SS-CN)**2.
1289      C(8,4)=-1.*CR/(SS-CN)**2.
1290      C
1291      C(9,2)=-1.*SR/(.3*CR-SR)**2.
1292      C(9,5)=CR/(.3*CR-SR)**2.
1293      C
1294      C(10,2)=SR/(SR-CR)**2.
1295      C(10,5)=-1.*CR/(SR-CR)**2.
1296      C
1297      C(11,6)=1./(1.01-BET2)**2.
1298      C(12,6)=-1./(BET2-1.0)**2.
1299      C
1300      C(13,7)=1./(40./57.-BET3)**2.
1301      C(14,7)=-1./(BET3-1.4)**2.
1302      C
1303      C(15,8)=50./(50.-CA)**2.
1304      C(16,8)=-400./(CA-400.0)**2.
1305      C
1306      C(17,1)=C17FAC*N(4,14,23)*C17FAD/(FI*0*(1.+TAN(AL2)**2.))
1307      C(17,6)=C17FAC*N(4,14,23)*C17FAD/(COS(BET2)**2.*(1.+TAN
1308      # (AL2)**2.))
1309      C(17,6)=C17FAC*N(4,14,23)*(1.+LAMBDA)*(U*SIN(2.*AL2)-2.*CA)
1310      # /(2.*CP*COS(AL2)**2.)
1311      C
1312      C(18,1)=C18FAC*(DH3DC3*C18FAE+DH3DT3*(H(2,4,10)*C18FAE
1313      # -H(3,3,5)*PIN*RPH))
1314      C(18,6)=-1.*C18FAC*DH3DT3*N(3,3,7)
1315      C(18,7)=C18FAC*(DH3DC3*N(3,10,17)*H(4,17,2)+DH3DT3*(H(2,4,10)
1316      # *H(3,10,17)*H(4,17,2)-H(3,3,8)))
1317      C(18,8)=C18FAC*(C18FAD*(DH3DC3+H(2,4,10)+DH3DT3)-DH3DT3
1318      # *(3,3,6))
1319      C
1320      C
1321      C
1322      Q(1,1,1)=-2.*C19FAC*PDC19*(H3-H1)/(2.*CN+2.5*CR)**2.
1323      Q(1,1,2)=Q(1,1,1)*2.5/2.
1324      Q(1,1,3)=-1.*C19FAC*PDC19/(2.*CN+2.5*CP)
1325      Q(1,1,4)=-1.*Q(1,1,3)
1326      C
1327      C      ****OBTAIN VECTOR C(19) ****
1328      C
1329      DO 1005 J=1,SIZE4(1,2)
1330      Q19(J)=Q(1,1,J)
1331      C
1332      DO 1007 K=2,10
1333      DO 1006 KK=1,SIZE4(K,1)
1334      DO 1006 KKK=1,SIZE4(K,2)
1335      H52(KK,KKK)=Q(K,KK,KKK)
1336      CALL MULTIP(B19,BD2,SIZE4(K,1),SIZE4(K,2))
1337      1007      CONTINUE
1338      C
1339      C
1340      DO 1088 I=1,6
1341      C(19,I)=Q19(I)
1342      C
1343      C(20,1)=-1.*PIN*RPH*CA/(.25*U-CA)**2.
1344      C(20,8)=PIN*RPH*D/(.25*U-CA)**2.
1345      C(21,1)=PIN*RPH*CA/(CA-2.*U)**2.
1346      C(21,8)=-1.*PIN*D*RPH/(CA-2.*U)**2.
1347      C
1348      C(22,1)=(DC22DU+DC22DT*N(3,3,5))*PIN*RPH
1349      C(22,6)=DC22DT*U*CA/(CP*COS(BET2)**2.)
1350      C(22,7)=DC22DT*U*CA/(CP*COS(BET3)**2.)
1351      C(22,8)=DC22DT*U*(TAN(BET2)+TAN(BET3))/CP
1352      C
1353      C(23,1)=(DC23DU+DC23DT*N(3,3,5))*PIN*RPH
1354      C(23,6)=DC23DT*U*CA/(CP*COS(BET2)**2.)
1355      C(23,7)=DC23DT*U*CA/(CP*COS(BET3)**2.)
1356      C(23,8)=DC23DT*U*(TAN(BET2)+TAN(BET3))/CP
1357      C
1358      C(24,1)=C24FAC*(TAN(BET3)-TAN(BET2))*K(4,18,17)*PIN
1359      # *RPH/2.
1360      C(24,8)=(TAN(BET3)-TAN(BET2))*C24FAC/(2.*U)

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```

1361          C(24,6)=-1.*FI*C24FAC/(2.*COS(BET2)**2.)
1362          C(24,7)=FI*C24FAC/(2.*COS(BET3)**2.)
1363      C
1364          C(25,1)=C(24,1)*C25FAC/C24FAC
1365          C(25,8)=C(24,8)*C25FAC/C24FAC
1366          C(25,6)=C(24,6)*C25FAC/C24FAC
1367          C(25,7)=C(24,7)*C25FAC/C24FAC
1368      C
1369          C(26,1)=C26FAC*K(3,12,18)*K(4,18,17)*M(10,20,7)
1370          C(26,6)=C26FAC*M(3,12,7)
1371          C(26,8)=C26FAC*K(3,12,18)/U
1372      C
1373      C
1374          C(27,1)=1./D**2.
1375          C(28,2)=1./CR**2.
1376          C(29,3)=1./CN**2.
1377          C(30,4)=1./SS**2.
1378          C(31,5)=1./SR**2.
1379      C
1380      C
1381      C
1382      C   *** CALCULATE SUM OF DERIVATIVE OF CONSTRAINT EQ'NS ***
1383      C
1384      C
1385          DO 1030 I=1,8
1386      1030      DCONST(I)=0.
1387          DO 1100 MMM=1,8
1388          DO 1090 NNN=1,31
1389      1090      DCONST(MMM)=DCONST(MMM)+C(NNN,MMM)
1390      1100      CONTINUE
1391      C
1392      C
1393      C   **** OBTAIN FINAL FORM OF DERIVATIVE OF PENALTY FUNCTION ****
1394      C
1395      C
1396      C
1397          DO 1110 I=1,8
1398          DPER(I)=DETA(I)+RF*DCONST(I)
1399      1110      CONTINUE
1400      C
1401      C
1402      C
1403      C   *** SET MAXIMIZING STEP LENGTH, R ***
1404      C
1405      C
1406      9999      RTOT=45L
1407          XL0W(5)=-.31*XI(2)
1408          XL0W(4)=-.31*XI(3)
1409          DO 1027 I=1,8
1410          IF(DPEN(I).EQ.0.) GO TO 1027
1411          IF(DPEN(I).GT.0.) THEN
1412              RMAX(I)=(XUP(I)-XI(I))/DPEN(I)
1413          ELSE
1414              RMAX(I)=(XL0W(I)-XI(I))/DPEN(I)
1415          END IF
1416      C
1417          IF(RMAX(I).LT.RTOT) THEN
1418              RTOT=RMAX(I)
1419          END IF
1420      1027      CONTINUE
1421          R=RTOT/51.
1422          ZITA=R
1423      C
1424          IF(DPEN(5).LT.0.) THEN
1425              X2FIN=XI(2)+45.*R*DPEN(2)
1426              XL0X(5)=-.31*X2FIN
1427              RMAX(5)=(XL0X(5)-XI(5))/DPEN(5)
1428              IF(RMAX(5).LT.RTOT) THEN
1429                  RTOT=RMAX(5)
1430              R=RTOT/51.
1431              ZITA=R
1432              END IF
1433              GO TO 1120
1434              END IF
1435      C
1436      C
1437      1120      ICOUNT=ICOUNT+1
1438          IF(ICOUNT.GT.45) GO TO 2000
1439      C
1440      C   *** CALCULATE NEW SET OF VARIABLES ***

```

```

1441 C
1442 C
1443 DO 1121 I=1,8
1444 X(I)=X1(I)+R*OPEN(I)
1445 IF(X(I).LT.XLOW(I)) THEN
1446 LU2=.TRUE.
1447 END IF
1448 1121 CONTINUE
1449 C
1450 IF(LU2) GO TO 2000
1451 C
1452 LU1=.TRUE.
1453 LU2=.FALSE.
1454 GO TO 10
1455 C
1456 C
1457 C
1458 5000 LU1=.FALSE.
1459 C
1460 IF(LL1) THEN
1461 LL1=.FALSE.
1462 GO TO 1024
1463 END IF
1464 C
1465 IF(LL2) THEN
1466 LL2=.FALSE.
1467 L=L+1
1468 PENLES=PENFUN
1469 LL1=.FALSE.
1470 GO TO 1140
1471 END IF
1472 C
1473 C
1474 1022 IF(PENFUN.GT.PENMAX) THEN
1475 IF(PENFUN.GT.1.) THEN
1476 BLK=0
1477 GO TO 1140
1478 END IF
1479 PENMAX=PENFUN
1480 ETAMAX=ETASRE
1481 CONMAX=CONST
1482 REMAX=RE
1483 PRMAX=PRATIO
1484 MAXMAX=MC3
1485 PSIMAX=PSI
1486 FIRAX=F1
1487 DORMAX=DOR
1488 DSTMAX=DST
1489 DORMAX=DORA
1490 DORMAX=DOR
1491 DORMAX=DOR2
1492 DO 1130 I=1,8
1493 1130 XMAX(I)=X(I)
1494 GO TO 1140
1495 ELSE
1496 L=L+1
1497 PENLES=PENFUN
1498 LL1=.TRUE.
1499 GO TO 1140
1500 END IF
1501 C
1502 C
1503 C
1504 1024 IF(PENLES.GT.PENFUN) THEN
1505 C
1506 IF(BLK.EQ.0) THEN
1507 PENLES=PENFUN
1508 L=L+1
1509 IF(L.GT.100) GO TO 2000
1510 LL1=.TRUE.
1511 GO TO 1140
1512 ELSE
1513 BLK=0
1514 L=L+1
1515 PENLES=PENFUN
1516 LL1=.TRUE.
1517 GO TO 1140
1518 END IF
1519 C
1520 ELSE

```

```

1521         IF (MLK.EQ.0) THEN
1522             MLK=MLK+1
1523             L=0
1524             KLM=KLM+1
1525             IF (KLM.GT.50) GO TO 2000
1526             GO TO 1025
1527         ELSE
1528             GO TO 1025
1529         END IF
1530     C
1531     C
1532     1025     IF (PENFUN.GT.PENMAX) THEN
1533             IF (PENFUN.GT.1.) THEN
1534                 MLK=0
1535                 GO TO 1140
1536             END IF
1537             PENMAX=PENFUN
1538             MLK=0
1539             LTAMAX=ETASRE
1540             CONMAX=CONST
1541             REIMAX=RE
1542             PENMAX=PERATIO
1543             HAKMAX=RC3
1544             PSIMAX=PSI
1545             FINAX=FI
1546             DGMMAX=DOM
1547             DDTMAX=DDT
1548             DGMHAX=DGMA
1549             DGMHAX=DDM
1550             DGM2HAX=DDM2
1551             GO 1026 I=1,8
1552     1026     XMAX(I)=X(I)
1553             GO TO 1140
1554         ELSE
1555             PERLES=PENFUN
1556             LL1=.TRUE.
1557             GO TO 1140
1558         END IF
1559     C
1560     C
1561     1140     R=R+ZITA
1562             GO TO 1120
1563     C
1564     C
1565     2000     GO 2001 I=1,8
1566     2001     XI(I)=XMAX(I)
1567             LO7=.FALSE.
1568             LO1=.FALSE.
1569             DIFPEN=PENMAX-FPENF
1570             FPENF=PENMAX
1571             IF (DIFPEN.LT..0001) THEN
1572                 PCOUNT=PCOUNT+1.
1573                 IF (PCOUNT.GT.1.) THEN
1574                     KK=II+1
1575                     RF1(KK)=RF1(II)/10.
1576                     PRINT*, ' '
1577                     PRINT*, ' '
1578                     PCOUNT=0.
1579                     LO5=.TRUE.
1580                     ODMN=1.
1581                     GO TO 2006
1582                 END IF
1583             END IF
1584             ODMN=ODMN+1.
1585     2006     LL1=.FALSE.
1586             LL2=.FALSE.
1587             KLM=0
1588             MLK=0
1589             L=0
1590             RTOT=MSL
1591             ICOUNT=0
1592             IF (LO5) GO TO 2100
1593             GO TO 7
1594     C
1595     C
1596     C
1597     2100     LO5=.FALSE.
1598             RF=RF/10.
1599     4445     IF (CNTY.NE.0.) RF=.0000004
1600             IF (RF.LT..000004) THEN

```



```

1601 PRINT*, ' ** VALUES OF DESIGN VARIABLES CORRESPONDING TO OPTIMUM
1602 # EFFICIENCY ARE AS FOLLOWS **'
1603 PRINT*, ' '
1604 DO 2004 I=1,8
1605 2004 PRINT*, ' ** X(' ,I, ') = ',XI(I)
1606 PRINT*, ' '
1607 PRINT*, ' ** VARIOUS OPTIMUM PARAMLTERS ARE **'
1608 PRINT*, ' '
1609 PRINT*, ' ** EFFICIENCY = ',ETAMAX, ' **'
1610 PRINT*, ' '
1611 PRINT*, ' ** PENALTY FN.= ',PENMAX, ' **'
1612 PRINT*, ' '
1613 PRINT*, ' *** INLET NON-DIM MASS FLOW = ',RHRAT, ' **'
1614 PRINT*, ' '
1615 PRINT*, ' ** PRESSURE RATIO = ',PRMAX, ' **'
1616 PRINT*, ' ** ROTOR MACH # = ',MAKMAX, ' **'
1617 PRINT*, ' '
1618 PRINT*, ' ** REYNOLDS # = ',REMAX, ' **'
1619 PRINT*, ' '
1620 PRINT*, ' ** DOMMAX= ',DOMMAX
1621 PRINT*, ' ** DGTMAX= ',DGTMAX
1622 PRINT*, ' ** DQMAX= ',DQMAX
1623 PRINT*, ' ** DQWMAX= ',DQWMAX
1624 PRINT*, ' ** DQZMAX= ',DQZMAX
1625 PRINT*, ' ** FIMAX= ',FIMAX
1626 PRINT*, ' ** PSIMAX= ',PSIMAX
1627 GO TO 2200
1628 FND IF
1629 SMS=SMS+1.
1630 GO TO 5
1631 2200 STOP
1632 FND
1633 C
1634 C
1635 C
1636 C
1637 SUBROUTINE MULTIP(A,B,M,N)
1638 DIMENSION A(30),B(30,30),C(30)
1639 C
1640 C
1641 C
1642 DO 1 I=1,N
1643 T=0.
1644 DO 2 J=1,M
1645 C
1646 C
1647 2 T=T+B(J,I)*A(J)
1648 1 C(I)=T
1649 C
1650 C
1651 DO 12 I=1,N
1652 12 A(I)=C(I)
1653 C
1654 RETURN
1655 END
1656 C
1657 C
1658 SUBROUTINE ICSIVE (F,A,B,U1,D1,UN,DN,N,H,G,PSCOND,IER)
1659 C
1660 C-----S-----LIBRARY 2-----
1661 C
1662 C FUNCTION - CUBIC SPLINE ONE-DIMENSIONAL INTERPOLATION -
1663 C EQUALLY SPACED DATA
1664 C USAGE - CALL ICSIVE(F,A,B,U1,D1,
1665 C UN,DN,N,H,G,PSCOND,IER)
1666 C PARAMETERS F - VECTOR OF N EQUALLY SPACED
1667 C FUNCTIONAL VALUES F(1),F(2),...,
1668 C F(N), WHERE F(I) IS THE
1669 C FUNCTIONAL VALUE AT
1670 C X=A+(I-1)*H AND H=(B-A)/(N-1)
1671 C A - VALUE OF THE ABSCISSA FOR F(1)
1672 C B - VALUE OF THE ABSCISSA FOR F(N)
1673 C U1 - BOUNDARY RELATED PARAMETER
1674 C (SEE ELEMENT DOCUMENTATION)
1675 C D1 - BOUNDARY RELATED PARAMETER
1676 C (SEE ELEMENT DOCUMENTATION)
1677 C UN - BOUNDARY RELATED PARAMETER
1678 C (SEE ELEMENT DOCUMENTATION)
1679 C DN - BOUNDARY RELATED PARAMETER
1680 C (SEE ELEMENT DOCUMENTATION)

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1681 C N - NUMBER OF DATA POINTS
1682 C M - NUMBER OF ANSWERS SOUGHT
1683 C G - VECTOR G(1),G(2),...,G(N) OF ANSWERS
1684 C WHERE G(I) IS THE I-TH ABCISSA
1685 C UPON ENTRY AND THE I-TH
1686 C ORDINATE UPON COMPLETION.
1687 C PSCOND - PSEUDO CONDITION NUMBER FOR BOUNDARY
1688 C PARAMETERS. PSCOND.GT.1.E5
1689 C INDICATES VALUES RETURNED IN G(I)
1690 C MAY BE UNRELIABLE.
1691 C IER - ERROR PARAMETER
1692 C TERMINAL ERROR = 128+N
1693 C N = 1 INDICATES G(I) NOT IN INTERVAL (A,B)
1694 C FOR SOME I=1,2,...,N
1695 C N = 2 INDICATES UNIQUE SOLUTION NOT POSSIBLE
1696 C FOR GIVEN BOUNDARY VALUES
1697 C PRECISION - SINGLE
1698 C REQ'D IMSL ROUTINES - UERTST
1699 C LANGUAGE - FORTRAN
1700 C -----
1701 C LATEST REVISION - FEBRUARY 3, 1971
1702 C
1703 C SUBROUTINE ICS1VE (F,A,B,U1,D1,UN,DN,N,M,G,PSCOND,IER)
1704 C
1705 C DIMENSION F(34),G(1)
1706 C EQUIVALENCE (IDET,DET)
1707 C REAL LAM,LAMN
1708 C DATA 53,LAM/1.7320508,-.26794919/
1709 C IER=0
1710 C H=(B-A)/(N-1)
1711 C LAMN=LAM**N
1712 C CALCULATE LAM AND TAU
1713 C GAM=F(1)
1714 C TAU=F(N)
1715 C N1=N+1
1716 C DO 5 I=2,N
1717 C J=N1-I
1718 C GAM=GAM*LAM+F(I)
1719 C TAU=TAU*LAM+F(J)
1720 C 5 CONTINUE
1721 C GAM=3.*GAM
1722 C TAU=3.*TAU
1723 C C1=1.-U1*LAM
1724 C CT=1.-U1/LAM
1725 C C2=CT*LAMN
1726 C C3=D1/H/(2.*S3)+(CT*TAU-S3*((1.+6.*U1)*F(1)-U1*F(2)))/H
1727 C CT=1.-UN/LAM
1728 C C4=CT*LAMN
1729 C C5=1.-UN*LAM
1730 C C6=DN/H/(2.*S3)+(CT*GAM-S3*((1.+6.*UN)*F(N)-UN*F(N-1)))/H
1731 C DET=C1*C5-C2*C4
1732 C IF (IDET.EQ.0) GO TO 55
1733 C CALCULATE BOUNDARY PARAMETERS B1,BN
1734 C B1=(C2*C6-C3*C5)/DET
1735 C BN=(C1*C6-C4*C3)/DET
1736 C CALCULATE PSCOND
1737 C C1=ABS(C1)
1738 C C2=ABS(C2)
1739 C C4=ABS(C4)
1740 C C5=ABS(C5)
1741 C PSCOND=AMAX1(C1,C2,C4,C5)/ABS(DET)
1742 C KP=0
1743 C N2=N-1
1744 C CALCULATE G(I)'S
1745 C DO 50 I=1,M
1746 C XI=A
1747 C IF(G(I).LT.A) GO TO 15
1748 C DO 10 J=1,N2
1749 C XI=XI+H
1750 C IF(G(I).GT.XI) GO TO 10
1751 C K=J
1752 C GO TO 20
1753 C 10 CONTINUE
1754 C IF(G(I).GT.B) GO TO 15
1755 C K=N2
1756 C GO TO 20
1757 C 15 IER=129
1758 C GO TO 9000
1759 C 20 X=(G(I)-(A+(K-1)*H))/H
1760 C IF(K.EQ.KP) GO TO 45

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1761      GAMK=F(1)
1762      TAUk=F(N)
1763      IF(K.LE.1) GO TO 30
1764      DO 25 J=2,K
1765      GAMK=GAMK*LAN+F(J)
1766 25      CONTINUE
1767 30      K1=N-K
1768      IF(K1.LE.1) GO TO 40
1769      DO 35 J=2,K1
1770      JJ=N1-J
1771      TAUk=TAUK*LAN+F(JJ)
1772 35      CONTINUE
1773 40      GAMK=3.*GAMK
1774      TAUk=3.*TAUK
1775      KP=K
1776 45      TX=(1.-(LAN+2.)*X+(LAN+1.)*X*X)*X
1777      Y=1.-X
1778      TY=(1.-(LAN+2.)*Y+(LAN+1.)*Y*Y)*Y
1779      G(I)=GAMK*TX+TAUK*TY+F(K)*Y**3+F(K+1)*X**3+B1*N*TX*LAN***(K-1)
1780      -BN*N*TY*LAN***(N-K-1)
1781 50      CONTINUE
1782      GO TO 9005
1783 55      IER=130
1784 9000 CONTINUE
1785      CALL ULRTST( IER, 'ICS1VE' )
1786 9005 RETURN
1787      END

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1788 C
1789 C
1790 C
1791 C
1792 C      SUBROUTINE ICS2VE (F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
1793 C

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C-ICS2VE-----S-----LIBRARY 2-----

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1794 C
1795 C
1796 C      FUNCTION          - BICUBIC SPLINE TWO-DIMENSIONAL INTERPOLATOR -
1797 C                      - EQUALLY SPACED DATA
1798 C      USAGE            - CALL ICS2VE(F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
1799 C      PARAMETERS      F - MATRIX OF 1*J FUNCTIONAL VALUES F(1,1),F(2,1),
1800 C                      F(3,1),...,F(1,2),...,F(1,J). THE VALUE
1801 C                      F(I1,J1) IS THE FUNCTIONAL VALUE AT X(I1),
1802 C                      Y(J1) WHERE X(I1)=A+(I1-1)*HX,Y(J1)=C+(J1-1)
1803 C                      *HY, HX=(B-A)/(I-1), AND HY=(D-C)/(J-1)
1804 C                      X0 - X-DIRECTION ABSCISSA FOR WHICH INTERPOLATES
1805 C                      ARE SOUGHT
1806 C                      A - LOWER LIMIT OF THE INTERVAL (A,B)
1807 C                      B - UPPER LIMIT OF THE INTERVAL (A,B)
1808 C                      C - LOWER LIMIT OF THE INTERVAL (C,D)
1809 C                      D - UPPER LIMIT OF THE INTERVAL (C,D)
1810 C                      I - NUMBER OF MESH POINTS IN THE FIRST COORDINANT,
1811 C                      X, DIRECTION
1812 C                      J - NUMBER OF MESH POINTS IN THE SECOND COORDINANT
1813 C                      Y, DIRECTION
1814 C                      N - NUMBER OF INTERPOLATED VALUES DESIRED
1815 C                      IF - FIRST DIMENSION OF F IN CALLING PROGRAM
1816 C                      WK - WORK AREA OF DIMENSION J
1817 C                      Y - VECTOR Y(1),Y(2),...,Y(N) WHERE Y(J1) IS THE
1818 C                      J1-TH ABSCISSA UPON ENTRY AND THE INTERPOLATED
1819 C                      FUNCTIONAL VALUE AT COORDINANT
1820 C                      (X0,Y(J1)) UPON COMPLETION
1821 C                      IER - ERROR PARAMETER
1822 C                      TERMINAL ERROR = 128+N
1823 C                      N = 1 INDICATES X0 NOT IN INTERVAL (A,B)
1824 C                      N = 2 INDICATES Y(J1) NOT IN INTERVAL (C,D)
1825 C                      FOR SOME J1=1,2,...,N
1826 C      PRECISION        - SINGLE
1827 C      REQ'D IMSL ROUTINES - ICS1VE,UERTST
1828 C      LANGUAGE         - FORTRAN

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C-----

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1829 C
1830 C      LATEST REVISION  - JANUARY 18, 1971
1831 C
1832 C      SUBROUTINE ICS2VE (F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
1833 C
1834 C      DIMENSION        F(IF,1),Y(1),WK(1)
1835 C                      INTERPOLATE IN Y DIRECTION AT X
1836 C                      DO 5 I1=1,J
1837 C                      WK(I1)=X0
1838 C                      CALL ICS1VF(F(1,I1),A,B,0.,0.,0.,0.,I,1,WK(I1),PSCOND,IER)
1839 C                      IF( IER.NE.0 ) GO TO 9000
1840 5      CONTINUE

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1841          CALL ICS1VE (IK,C,D,0.,0.,0.,0.,J,N,Y,PSCOND,IER)
1842          IF (IER.NE.0) GO TO 10
1843          GO TO 9005
1844          10 IER=130
1845          9000 CONTINUE
1846          CALL UERTST(IER,'ICS2VE')
1847          9005 RETURN
1848          END
1849      C
1850      C
1851      C
1852      C
1853      C
1854      C          SUBROUTINE ICS1VU (F,X,N,M,G,H,IER)
1855      C
1856      C-----S-----LIBRARY 2-----
1857      C
1858      C          FUNCTION          - CUBIC SPLINE ONE-DIMENSIONAL INTERPOLATION -
1859      C                          UNEQUALLY SPACED DATA
1860      C          USAGE          - CALL ICS1VU(F,X,N,M,G,H,IER)
1861      C          PARAMETERS      F - VECTOR OF N UNEQUALLY SPACED
1862      C                          FUNCTIONAL VALUES F(1),F(2),...,
1863      C                          F(N), WHERE F(I) IS THE FUNCTIONAL
1864      C                          VALUE AT X(I)
1865      C                          X - VECTOR OF N ABSCISSA X(1),X(2),...,X(N)
1866      C                          N - NUMBER OF DATA POINTS
1867      C                          M - NUMBER OF ANSWERS SOUGHT
1868      C                          G - VECTOR G(1),G(2),...,G(M) OF ANSWERS
1869      C                          WHERE G(I) IS THE I-TH ABSCISSA
1870      C                          UPON ENTRY AND THE I-TH ORDINATE
1871      C                          UPON COMPLETION.
1872      C                          H - WORK AREA OF DIMENSION 8*N+M
1873      C                          IER - ERROR PARAMETER
1874      C                          TERMINAL ERROR = 128*N
1875      C                          N = 1 INDICATES G(I) NOT IN INTERVAL
1876      C                          (X(1),X(N)) FOR SOME I=1,2,...,M
1877      C                          N = 2 INDICATES THAT CONVERGENCE WAS
1878      C                          NOT OBTAINED IN 5*N ITERATIONS
1879      C          PRECISION          - SINGLE
1880      C          REG'D INSL ROUTINES - UERTST
1881      C          LANGUAGE          - FORTRAN
1882      C-----
1883      C          LATEST REVISION    - MARCH 5, 1971
1884      C
1885      C          SUBROUTINE ICS1VU(F,X,N,M,G,H,IER)
1886      C
1887      C          DIMENSION          F(1),X(1),G(1),H(1)
1888      C          DATA              EPSLN,OMEGA/1.E-6,1.0717968/
1889      C                          SET UP WORK AREAS
1890      C          12=N
1891      C          13=N+N
1892      C          14=13+N
1893      C          15=14+N
1894      C          16=15+N
1895      C          17=16+N
1896      C          18=17+N
1897      C          19=18+N
1898      C          NT=16
1899      C          IER = 0
1900      C          N1 = N-1
1901      C
1902      C          DERIVATIVES,H(J6), USING CENTRAL
1903      C          DIFFERENCES
1904      C          DO 5 I=1,N1
1905      C              J2=12+I
1906      C              H(I)=X(I+1)-X(I)
1907      C              H(J2)=(F(I+1)-F(I))/H(I)
1908      C          5 CONTINUE
1909      C          DO 10 I=2,N1
1910      C              J2=12+I
1911      C              J3=13+I
1912      C              J4=14+I
1913      C              J5=15+I
1914      C              J6=16+I
1915      C              J7=17+I
1916      C              H(J3)=H(I-1)+H(I)
1917      C              H(J4)=.5*H(I-1)/H(J3)
1918      C              H(J5)=(H(J2)-H(J2-1))/H(J3)
1919      C              H(J6)=H(J5)+H(J5)
1920      C              H(J7)=H(J6)+H(J5)
1921      C          10 CONTINUE

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1921      H(16+J)=0.
1922      J6=16+J
1923      H(J6)=0.
1924      C
1925      KCOUNT=0
1926      15  ETA=0.
1927      KCOUNT=KCOUNT+1
1928      DO 25  I=2,N1
1929          J4=14+I
1930          J6=16+I
1931          J7=17+I
1932          W=(H(J7)-H(J4)+H(J6-1)-(.5-H(J4))*H(J6+1)-H(J6))*OMEGA
1933          IF (ABS(W).LE.ETA) GO TO 20
1934          ETA=ABS(W)
1935      20  H(J6)=H(J6)+W
1936      25  CONTINUE
1937      IF (KCOUNT.GT.NT) GO TO 75
1938      IF (.C.TA.GE.FPSLN) GO TO 15
1939      C
1940      GO 30  I=1,N1
1941      J6=16+I
1942      J8=18+I
1943      H(J8)=(H(J6+1)-H(J6))/H(I)
1944      30  CONTINUE
1945      DO 65  J=1,N
1946          I=1
1947          J1=19+J
1948          IF (G(J)-X(I)) 70,60,35
1949      35  IF (G(J)-X(N)) 45,50,70
1950      40  IF (G(J)-X(I)) 55,60,45
1951      45  I=I+1
1952          GO TO 40
1953      50  I=4
1954      55  I=I-1
1955      C
1956      60  J6=16+I
1957          J2=12+I
1958          J8=18+I
1959          HT1=G(J)-X(I)
1960          HT2=G(J)-X(I+1)
1961          PRDD=HT1+HT2
1962          H(J9)=(H(J6)+HT1)*H(J8)
1963          DELSQS=(H(J6)+H(J6+1)+H(J9))/6.
1964          G(J)=F(I)+HT1* H(J2)+PRDD*DELSQS
1965      65  CONTINUE
1966          GO TO 9005
1967      70  IER=129
1968          GO TO 9000
1969      75  IER=130
1970      9000 CONTINUE
1971          CALL UERTST(IER,'ICSLVU')
1972      9005 RETURN
1973      END
1974      C
1975      C
1976      C
1977      C
1978      C      SUBROUTINE UERTST (IER,NAME)
1979      C
1980      C-----LIBRARY 2-----
1981      C
1982      C      FUNCTION          - ERROR MESSAGE GENERATION
1983      C      USAGE           - CALL UERTST(IER,NAME)
1984      C      PARAMETERS      IER      - ERROR PARAMETER. TYPE + N WHERE
1985      C                          TYPE= 128 IMPLIES TERMINAL ERROR
1986      C                          64 IMPLIES WARNING WITH FIX
1987      C                          32 IMPLIES WARNING
1988      C                          N    = ERROR CODE RELEVANT TO CALLING ROUTINE
1989      C                          NAME  - INPUT SCALAR (DOUBLE PRECISION ON DEC)
1990      C                          CONTAINING THE NAME OF THE CALLING ROUTINE
1991      C                          AS A 6-CHARACTER LITERAL STRING.
1992      C      LANGUAGE        - FORTRAN
1993      C-----
1994      C      LATEST REVISION  - OCTOBER 1,1975
1995      C
1996      C      SUBROUTINE UERTST(IER,NAME)
1997      C
1998      C      DIMENSION          IITP(3,4),IBIT(4)
1999      C      CHARACTER*6        IITP
2000      C      INTEGER           IARD,IARF,TERN,PRINTR

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2001      EQUIVALENCE      (IBIT(1),WARN),(IBIT(2),WARF),(IBIT(3),TERM)
2002      DATA            ITYP/'WARNIN','G',,,'
2003      *                'WARNIN','G(WITH',' FIX) ',,
2004      *                'TERMIN','AL',,,'
2005      *                'NON-DE','FINED',,,'
2006      *                IBIT/ 32,64,128,C/
2007      DATA            PRINTR/ 6/
2008      IER2=IER
2009      IF (IER2 .GE. WARN) GO TO 5
2010
2011      C                  NON-DEFINED
2012      IER1=4
2013      GO TO 20
2014      5 IF (IER2 .LT. TERM) GO TO 10
2015      C                  TERMINAL
2016      IER1=3
2017      GO TO 20
2018      10 IF (IER2 .LT. WARF) GO TO 15
2019      C                  WARNING(WITH FIX)
2020      IER1=2
2021      GO TO 20
2022      C                  WARNING
2023      15 IER1=1
2024      C                  EXTRACT 'N'
2025      20 IER2=IER2-IBIT(IER1)
2026      C                  PRINT FRROR MESSAGE
2027      WRITE (PRINTR,25) (ITYP(I,IER1),I=1,3),NAME,IER2,IER
2028      25 FORMAT(' *** I M S L(UEPST) *** ',3A6,2X,A6,2X,I2,
2029      1 ' (IER = ',I3,')')
2030      RETURN
2031      END

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