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ON THE CLASSICAL SOLUTIONS

OF FIELD THEORETICAL MODELS

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Submitted to the Institute for Graduate Studies in Science and Engineering in partial fulfillment of the requirements for the degree of

Doctor

of

Philosophy

Boğaziçi University

1984

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DATE OF APPROVAL: APRIL 12, 1984

ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to my thesis supervisor Mahmut Hortaçsu not only for his guidance in this work but also for his invaluable contributions to our other joint research. I also wish to express my special thanks to my friend Gediz Akdeniz for his continous encouragement and his contributions to this thesis.

I wish to thank Doç. Dr. Alpar Sevgen and all the members of the Physics Department for creating a stimulating and friendly atmosphere throughout my graduate work. Special thanks to Daniel Alfandari for his assistance in the computer work and graphs.

I would like to use this opportunity to thank my high school physics teacher Vahan Acemyan who guided me into physics and always gave me the moral support I needed.

And finally I wish to express my endless gratitude to my mother and my sister Janet for their understanding and sacrifice throughout my educational carreer.

ON THE CLASSICAL SOLUTIONS

OF FIELD THEORETICAL MODELS

In this thesis we study two important aspects of classical solutions of two specific field theoretical models.The importance of classical solutions lies in the fact that, they constitute the first step in understanding the formal aspects of the underlying quantum theory and sometimes lead to results which are not obtainable by perturbation theory. Here, two kinds of classical solutions, monopole and meron, are under consideration.

In the first part we quantize a purely fermionic model with non-polynomial conformal invariant Lagrangian. Then we present a static solution to the classical field equations when we restrict the internal symmetry group to SU(2). The quantized version of the model contains composite vector and axial-vector gluon fields. The classical solution for the vector field is precisely that of the Wu-Yang monopole.

In the second part the stability properties of merons

-classical solutions in four dimensional Euclidean space which are singular at origin and infinity with divergent energy- are investigated by taking the DeAFF model as a theoretical laboratory. We find that in gravitational models with Yang-Mills fields, merons are unstable. Two special cases, conformally flat and flat space are taken under consideration. At the end we outline a suitable ansatz about the fields which in Minkowski domain gives an expression that can be interpreted as the potential of the model. Some graphs of the potential are added for various values of λ^2/e^2.

ALAN KURAMSAL MODELLERİN KLASİK ÇÖZÜMLERİ ÜZERİNE

Bu tezde, alan kuramlarının klasik çözümlerinin iki önemli niteliği iki ayrı alan kuramı modeli ele alınarak incelenmiştir. Klasik çözümlerin önemi, kuantum alan kuramlarının formel yönlerinin anlaşılmasında bir ilk adım olmasında ve bazen tedirgeme yöntemleriyle elde edilemeyecek sonuçlar vermesindedir. Burada sözkonusu edilen iki tür klasik çözüm, monopol ve meron çözümleridir.

Birinci kısımda konform invaryant ve polinom olmayan, yalnızca fermiyonlar içeren bir Lagrange fonksiyonunun kuantalaştırılması sunulmuştur. Ardından iç simetri grubu SU(2) ile sınırlanarak klasik hareket denklemlerinin statik çözümleri bulunmuştur. Modelin kuantalaşmış biçimi bileşik vektör ve eksensel-vektör gluon alanları içermektedir. Vektör alanın klasik çözümü bir Wu-Yang monopolü olarak alınmıştır.

İkinci kısımda ise kütleçekimsel modellerde meronların -dört boyutlu Öklitsel uzayda, orijinde ve sonsuzda singüler olan ıraksak enerjili klasik çözümler- kararlılık problemi DeAFF modeli ele alınarak incelenmiştir. Sonuçta, Yang-Mills alanlarını içeren kütleçekimsel modellerde meronların kararsız olduğu bulunmuştur. İki özel durum, konform düz ve düz uzay çözümleri inceleme konusu edilmişlerdir. Son bölümde alanlar için uygun bir ansatz kullanılarak, Minkowski uzayında modelin potansiyeli olarak yorumlanabilecek bir ifade elde edilmiştir. λ^2/e^2'nin bazı değerlerine karşılık gelen potansiyellerin grafikleri bölümün sonuna eklenmiştir.

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LIST OF SYMBOLS

A^a_μ : Axial-vector field

E_i, B_i : Electric and magnetic fields

e : Colour charge

$F_{\mu\nu}$: Field strength tensor

g : Copling constant, determinant of $g_{\mu\nu}$

$g_{\mu\nu}$: Metric coefficients

I : Action

$\mathcal{L}$: Lagrangian density

R : Scalar curvature

r : Length of a vector in three dimensional Euclidean space

Tr : Trace

t : Ordinary time

V : Potential

y : Scalar product of γ_μ and V_μ in Minkowski space

V^a_μ : Vector field

$\vec{x}$: Vector in three dimensional Euclidean space

x : Length of a vector in four dimensional Euclidean space

γ_μ : Dirac matrices

δ : Variation symbol

∂_μ : Four-divergence

δ_{ij} : Kronecker symbol

θ : Polar angle in three dimensional Euclidean space, angle between isovector and isoscalar

$\theta_{\mu\nu}$: Energy-momentum tensor

ε_{ijk} : Levi-Civita tensor

$\varepsilon_{\mu\nu\rho\sigma}$: Four dimensional Levi-Civita tensor

Λ : Momentum cut-off

λ : Cosmological constant

σ_i : Pauli spin matrices or SU(2) generators

τ : Cosmological or proper time

τ_a : SU(n) group generators

φ : Scalar field

ψ : Spinor field

$\cdot$: Scalar product

$[\,,\,]$: Commutator

$\{\,,\,\}$: Anti-commutator

$\Box$: D'Alembertian

Greek indices takes values 0,1,2,3 whereas Latin indices from 1 to N, where N is the number of group generators.

The representation for γ matrices is,

$$\gamma_0 = \begin{bmatrix} 0 & 1\!\!1 \\ 1\!\!1 & 0 \end{bmatrix} \qquad \gamma^i = \begin{bmatrix} 0 & -\sigma_i \\ \sigma_i & 0 \end{bmatrix} \qquad \gamma^5 = \begin{bmatrix} -1\!\!1 & 0 \\ 0 & 1\!\!1 \end{bmatrix}$$

where σ_i's are given by,

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

PART I

MONOPOLES IN PURELY FERMIONIC MODELS

I.1. INTRODUCTION

It has been over fifty years since the foundations of the theory of magnetic monopoles were laid down by Dirac[1]. After the discovery of non-abelian gauge theory[2] and subsequently of its classical magnetic monopole solutions[3] it was realized that the mechanism of spontaneous symmetry breaking can be utilized to make these solutions have finite energy[4]. These solutions have received a great deal of attention concerning their implication for Grand Unified Theories[5]. In this work we present the counterpart of the Wu-Yang monopole[3] in a class of recently proposed purely fermionic models[6,7]. These theories, when quantized, have composite vector fields which behave[7] more or less like the vector bosons of pure non-abelian gauge theories. These composite vector fields essentially arise from introducing auxiliary fields that are necessary in putting the conformally invariant non-polynomial fermion self-interaction into polynomial form. The auxiliary fields have to be chosen such that when the functional integral over them is performed, the original non-polynomial purely fermionic Lagrangian is obtained. If, instead of integrating over the auxiliary fields first, one integrates over the fermion fields, one obtains an effective Lagrangian in which a propagating vector field appears. One can devise an algorithm in which such a model is regularizable.

Encouraged by the similarities of this model with non-abelian gauge theories, we searched for solutions to the classical equations of motion such that the expression for the composite vector field is of the Wu-Yang monopole type. We considered a model with SU(2) internal symmetry and found that we were not able to satisfy the equations of motion. Was it possible, then, to modify the Lagrangian such that one can find monopole type solutions? It turns out that such a solution exists provided that the model contains an axial vector interaction as well as a vector one.Before going into the details we find it more appropriate to give some historical information about monopoles and make clear what is meant by a Wu-Yang monopole in the next section.In this section we also elaborate on the importance of these "classical" monopole solutions,the quantization of the electric charge, and the elimination of the disturbing Dirac's string by the Wu-Yang ansatz.In section I.3 presentation of the model, its quantization and regularization algorithm are given. The propagators of vector field and of auxiliary field are calculated and some information about them is extracted. After this preparation the monopole solution is presented in section I.4. Since a SU(2) internal symmetry is also required, some explanation about the notation is given and the ansatz for ψ is written explicitly.Unknown parameters are determined such that the equation of motion is satisfied. For this purpose four-potential of the monopole field is calculated. At the final stage a discussion on the solution is given and its explicit form is written. The conclusions are given in section I.5.

I.2. A SUMMARY ON DIRAC[1,11] AND WU-YANG MONOPOLES[3,11]

If one permits the presence of magnetic charges in the Maxwell's theory, one ends up with the following equations of motion,

$$\partial^{\mu} F_{\mu\nu} = j_{\nu}$$
$$\partial^{\mu} \tilde{F}_{\mu\nu} = -g_{\nu} \qquad (I.2.1)$$

Here j_{ν} and g_{ν} are electric and magnetic charge currents respectively and the field strength tensors $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are given by,

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} = \begin{bmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & -E_3 & E_2 \\ -B_2 & E_3 & 0 & -E_1 \\ -B_3 & -E_2 & E_1 & 0 \end{bmatrix} \qquad (I.2.2)$$

$\varepsilon^{\mu\nu\alpha\beta}$ is the totally anti-symmetric tensor with $\varepsilon^{0123} = +1$.

Equations in (I.2.1) are symmetric under the interchange of electric and magnetic quantities $E \leftrightarrow B$, $j_{\mu} \leftrightarrow g_{\mu}$. However the usual Maxwell's equations do not have this symmetry. Therefore, the inclusion of magnetic charges or monopoles into the theory seems to enjoy some aesthetic advantage (the nature does not seem to exhibit this symmetry, since till now nobody has been succeeded to detect a magnetic monopole experimentally). The price for this advantage is a singularity line or a string. This can be seen as follows. The defining equation for the monopole four-vector potential,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{I.2.3}$$

when used in Eq.(I.2.1) leads to the result,

$$g_\nu = \varepsilon_{\nu\mu\alpha\beta}\partial^\mu \partial^\alpha A^\beta \tag{I.2.4}$$

If monopoles exists, then $g_\nu \neq 0$ and this implies that,

$$\partial_\mu \partial_\alpha A_\beta \neq \partial_\alpha \partial_\mu A_\beta \tag{I.2.5}$$

Eq.(I.2.5) is the mathematical statement that A_β is singular On any simply connected surface surrounding the monopole, A_β need only be singular at one point on the surface. If one imagines an outward succession of such surfaces, one is led to visualize a continous line of points extending from the monopole to infinity, along which the four-potential is singular. A simple example of such a potential is,

$$A_0 = 0 \qquad \vec{A} = -\hat{\phi} g \frac{\sin\theta}{r(1-\cos\theta)} \tag{I.2.6}$$

This expression describes a monopole at rest at the origin, since

$$\vec{\nabla} \times \vec{A} = g \frac{\vec{r}}{r^3} \tag{I.2.7}$$

It is singular along the positive z-axis because when $\theta = 0$ $\vec{A}$ blows up.

It was Dirac[1)] who first treated the magnetic monopoles in the context of Maxwell's theory fifty three years ago, in 1931, and since then the string is found to be a disturbing feature of the monopole theory. Elimination of the Dirac string has been one of the main goals of the researches. It is clear that new potentials have to be introduced to accomplish this (Wu-Yang construction[3)]). But,the presence of the string is the source of an important result if one tolerates its existence as a mathematical necessity and assumes that it is unphysical. It can be visualized as an infinitely long, thin solenoid. Magnetic flux lines emanate in all directions from the monopole and return from infinity through the string. Clearly the position of the string and any motion it may experience must be unphysical and undetectable.In other words, an electron or any other particle should not exhibit unusual behaviour in the vicinity of a string. In particular, The phase of the particles' wave function ψ should change by at most some integral multiple of 2π when a small closed loop is traversed around the string.If the monopole is assumed far away and no other forces act on the particle, the ψ satisfies the wave equation for a free particle. ψ can be written in the form $\psi = \phi e^{i\beta}$, ϕ being a function with a definite phase at every point. Then ϕ satisfies the wave equation for a particle in an electromagnetic potential $e\vec{A}=\vec{\nabla}\beta$. This is entirely trivial if β is an integrable function, for there is no electromagnetic field. However, β will be non-integrable near a string. For a small loop enclosing the string, the change in β is,

$$\oint d\beta = e\int d\vec{S}\cdot\vec{\nabla}\times\vec{A} = e(Flux) = 4\pi e g = 2\pi n \qquad \text{(I.2.8)}$$

The flux in this equation is the total magnetic flux within the string, which is equal to the total magnetic flux $\Phi = 4\pi g$ of the monopole. Dirac's famous quantization condition then follows immediately,

$$eg = \frac{n}{2} \tag{I.2.9}$$

The existence of monopoles explains why the electric charge is quantized.

So, Eq.(I.2.9) is the condition to be obeyed if the string is required to be an unobservable object. A straightforward way to demonstrate this is by showing that there is a gauge transformation that moves the string from one place to any other desired location by transforming the potential $\vec{A}$. But there is an unpleasant feature to this argument: the gauge transformation is necessarily singular at both the old and new locations of the string.

Wu and Yang gave a refinement of this argument that avoids this difficulty. In the Wu-Yang construction[3)] one does not have to deal with singular gauge transformations, nor with singular potentials(except at the origin). The price paid for this is the necessity of using different vector potentials in different regions of space. But we will not go into the details of this construction. We merely sketch the SU(2) monopole solution of Wu and Yang, in order to present the form of the vector potential which is used when we search for monopole solution of the model given in the next section.

The solution that is found by Wu and Yang is a pointlike monopole without a string. The ansatz they used as a solution to pure SU(2) Yang-Mills equation of motion is,

$$A^a_{\ 0} = i r_a\, g(r)/r^2 \qquad A^a_{\ i} = \varepsilon_{ain} r_n \left(1 - h(r)\right)/r^2 \tag{I.2.10}$$

which reduces the equation of motion to the following coupled equations,

$$r^2 g'' = 2 g h^2$$

$$r^2 h'' = h\left(h^2 - 1 + g^2\right) \tag{I.2.11}$$

which has nontrivial constant solution

$$h = 0 \qquad g = \text{constant} = C \tag{I.2.12}$$

Constant g and h evidently imply unbroken local SU(2) gauge invariance, because $A^a_{\ \mu}$ is a pointlike long-range potential in this case. Substituting Eq.(I.2.12) into Eq.(I.2.10) gives

$$A^a_{\ 0} = C = \text{constant} \qquad A^a_{\ i} = \varepsilon_{ain} r_n / r^2 \tag{I.2.13}$$

It is this form which we used with C=0. To see that this can be transformed into the vector potential of a monopole with Dirac string, consider the gauge transformation,

$$\omega = \begin{bmatrix} \cos\theta/2 & e^{-i\phi}\sin\theta/2 \\ -e^{i\phi}\sin\theta/2 & \cos\theta/2 \end{bmatrix} \tag{I.2.14}$$

which rotates the $\hat{r} = (\theta, \phi)$ direction in group space to the z-axis z=(0,0) and is discontinous along the negative z-axis. This gauge transformation when applied to the ansatz (I.2.13) gives,

$$A^{a}_{0} = \delta_{a3}\, iC/r \qquad A^{1}_{i} = A^{2}_{i} = 0$$

$$A^{3}_{i} = -\frac{1}{2r}\tan\theta/2\,\hat{\phi}_{i} \tag{I.2.15}$$

With C=0 this is same with Eq.(I.2.6) hence proving that Eq.(I.2.13) is the vector potential of a monopole.

I.3. THE MODEL

Our starting point is the classical Lagrangian given by

$$\mathcal{L} = \bar{\psi}\, i\not{\partial}\psi + g_1\left[(\bar{\psi}\gamma_\mu\tau_a\psi)(\bar{\psi}\gamma^\mu\tau_a\psi)\right]^{2/3}$$
$$+ g_2\left[(\bar{\psi}\gamma_\mu\gamma_5\tau_a\psi)(\bar{\psi}\gamma^\mu\gamma_5\tau_a\psi)\right]^{2/3} \tag{I.3.1}$$

where τ_a are the group generators in the represantation to which the fermions belong. Note that ψ carry a spinor index as well as a group index on which the matrices τ_a act. It was shown in Ref.(6) that this model can be quantized when $g_2=0$. We mimick the same procedure for this model and introduce two vector and two axial vector auxiliary fields to put the Lagrangian into a polynomial form. The final expression reads,

$$\mathcal{L} = \bar{\psi}\, i\not{\partial}\psi + \frac{1}{C_2(T)}\left\{\mathrm{Tr}\left[\lambda_\mu\left(J^\mu - \frac{1}{C_2(T)}\mathrm{Tr}(V^2)V^\mu\right)\right]\right\}$$

$$+\frac{1}{C_2(T)}\,\mathrm{Tr}(J_\mu V^\mu)$$
$$-\frac{1}{C_2(T)}\left\{\mathrm{Tr}\left[\rho_\mu\left(J_5^\mu-\frac{1}{C_2(T)}\mathrm{Tr}(A^2)A^\mu\right)\right]\right\}$$
$$-\frac{1}{C_2(T)}\,\mathrm{Tr}(J_5^\mu A_\mu) \tag{I.3.2}$$

where

$$\lambda_\mu=\lambda_\mu^a T^a \tag{I.3.3}$$

$$V_\mu=V_\mu^a T^a \tag{I.3.4}$$

$$V^2=V_\mu V^\mu \tag{I.3.5}$$

$$\rho_\mu=\rho^a_\mu T^a \tag{I.3.6}$$

$$A_\mu=A_\mu^a T^a \tag{I.3.7}$$

$$A^2=A_\mu A^\mu \tag{I.3.8}$$

$$J_a^\mu=g_1\bar\Psi\gamma_\mu\tau_a\Psi \tag{I.3.9}$$

$$J_5^{a\mu}=-g_2\bar\Psi\gamma_\mu\gamma_5\tau_a\Psi \tag{I.3.10}$$

$C_2(T)$ is the second order Casimir operator of the group. Note that all the auxiliary fields belong to the adjoint representation of the group G.

To quantize the Lagrangian given by Eq.(I.3.2) we calculate the Fadeev-Popov determinant of the model. The functional integral reads,

$$Z = \int D\bar{\Psi}\, D\Psi\, DA_\mu\, DV_\mu\, D\lambda_\mu\, D\rho_\mu\, Dc^*_\mu\, Dc_\mu\, De^*_\mu\, De_\mu$$

$$\cdot \exp i \int d^4x\, \mathcal{L}_{eff} \qquad \text{(I.3.11)}$$

where

$$\mathcal{L}_{eff} = \bar{\Psi}\left(i\partial\!\!\!/ + g_1 V\!\!\!/ + g_1 \lambda\!\!\!/ + g_2 A\!\!\!/ + g_2 \rho\!\!\!/\right)\Psi$$
$$- \frac{1}{(C_2(T))^2}\left[\mathrm{Tr}(V^2)\,\mathrm{Tr}\,\lambda.V - \mathrm{Tr}(A^2)\,\mathrm{Tr}\,\rho.A\right] \qquad \text{(I.3.12)}$$
$$+ \mathcal{L}_{ghost}$$

Here

$$\mathcal{L}_{ghost} = \frac{1}{(C_2(T))^2}\Big[2\,\mathrm{Tr}(c^*_\mu V^\mu)\,\mathrm{Tr}(c_\mu V^\mu) + \mathrm{Tr}(V^2)\,\mathrm{Tr}(c^*_\mu c^\mu)$$

$$+ 2\,\mathrm{Tr}\,e^*_\mu A^\mu\,\mathrm{Tr}\,e_\mu A^\mu + \mathrm{Tr}\,A^2\,\mathrm{Tr}(e^*_\mu e^\mu)\Big] \qquad \text{(I.3.13)}$$

$C_\mu = C^\alpha_\mu T^\alpha$ are the anticommuting fields in the adjoint representation.

At this point we would like to note that our starting point to the quantization procedure is the expression given by Eq.(I.3.2). However, the classical field equations of both of the Lagrangians, given by Eq.(I.3.1) and Eq.(I.3.2) are equivalent. To correctly quantize the expression given by Eq.(I.3.2) a constrained system, one has to add the Fadeev-Popov determinant to the Lagrangian. If one integrates over the auxiliary and the ghost fields of the latter Lagrangian, Eq.(I.3.13), one obtains the purely fermionic Lagrangian of Eq.(I.3.1). Although the interpretation of the fractional powers in Eq.(I.3.1) is problematic, it is in this sense that one can give a meaning to the purely fermionic Lagrangian.

We redefine the fields as,

$$
\begin{aligned}
V'_\mu &= V_\mu + \lambda_\mu \\
G_\mu &= V_\mu \\
A'_\mu &= A_\mu + \rho_\mu \\
H_\mu &= A_\mu
\end{aligned}
\tag{I.3.14}
$$

Substitute into the Lagrangian in terms of primed fields and drop the primes for notational convenience. After integrating over the fermion fields we obtain the effective Lagrangian given by,

$$
\begin{aligned}
\mathcal{L}_{eff} = \mathrm{Tr}\ln\left(i\not{\partial} + g_1 \not{V} + g_2 \not{A}\right) \\
- \frac{1}{C_2(T)}\Big[\mathrm{Tr}\left((G_\mu)^2\right)\mathrm{Tr}\left(G_\mu(V^\mu - G^\mu)\right) \\
+ \mathrm{Tr}\left((H_\mu)^2\right)\mathrm{Tr}\left(H_\mu(A^\mu - H^\mu)\right)\Big] + \mathcal{L}_{ghost}
\end{aligned}
\tag{I.3.15}
$$

The saddle point conditions are,

$$
\begin{aligned}
V_\mu = 0 \qquad & A_\mu = 0 \qquad & G_\mu = 0 \\
H_\mu = 0 \qquad & C_\mu = 0 \qquad & e_\mu = 0
\end{aligned}
\tag{I.3.16}
$$

The tadpole condition,

$$
\left.\frac{\partial S_{eff}}{\partial V_\mu}\right|_{vac} = 0 = \frac{g_1}{(2\pi)^4}\mathrm{Tr}\int d^4p\, \frac{\gamma_\mu \not{p}}{p^2}
\tag{I.3.17}
$$

and

$$\left.\frac{\partial S_{eff}}{\partial A_\mu}\right|_{vac} = 0 = \frac{g_2}{(2\pi)^4}\,\mathrm{Tr}\int d^4p\;\frac{\gamma_\mu\gamma_5 \not{p}}{p^2} \tag{I.3.18}$$

are satisfied trivially. This is also true for the other fields and ghosts. The propagator for the V_μ field is,

$$\begin{aligned}\frac{\partial^2 S_{eff}}{\partial V_\mu^a\,\partial V_\nu^b} &= -\delta^{ab} C_2(T)\frac{g_1^2}{(2\pi)^4}\,\mathrm{Tr}\int d^4p\;\frac{\gamma_\mu \not{p}\gamma_\nu(\not{p}+\not{q})}{p^2(p+q)^2}\\ &= -\delta^{ab} C_2(T)\frac{g_1^2}{6\pi^2}\left(q_\mu q_\nu - g_{\mu\nu}q^2\right)\\ &\quad\cdot\left(\ln\Lambda + \text{finite part}\right)\end{aligned} \tag{I.3.19}$$

Here Λ is the cut-off.

We see that if we take

$$\frac{g_1^2}{6\pi^2}\ln\Lambda = 1 \tag{I.3.20}$$

we obtain the propagator of a massless vector field. Here since we get the tadpole condition free, we could fix our coupling constant at this stage. This is in contrast to the situation in other models[8,9], where the tadpole condition already fixes the coupling constant. Fixing g_1 as in Eq.(I.3.20) makes this model regularizable. Since the bare coupling goes to zero as the cut-off Λ goes to infinity, the interaction is asymptotically free as in the CP^{n-1} model[8] and as in QCD.

Note that the regular algorithm for renormalization, i.e. of introducing wave function renormalization instead of fixing the coupling constant at this stage, does not work for this model. Introducing such a term would mean, via Eqn's. (I.3.14) a quadratic term in our original λ_μ and ρ_μ fields. However, such a counterterm would destroy our constraints,

$$\bar{\psi}\gamma_\mu\psi - V^2V_\mu = 0$$

$$\bar{\psi}\gamma_\mu\gamma_5\psi - A^2A_\mu = 0 \qquad \text{(I.3.21)}$$

, and render the model non-renormalizable in this scheme. To give a meaning to the model we had to devise the new algorithm which is applied also by other authors[10).]

One calculates the propagator of the axial vector field in the same manner. Its propagator is given by,

$$\begin{aligned}\frac{\partial^2 S_{eff}}{\partial A^a_\mu \partial A^b_\nu} &= -\delta^{ab} C_2(T)\frac{g_2^2}{(2\pi)^4}\, \mathrm{Tr}\int d^4p\, \frac{\gamma_\mu\gamma_5 \not{p}\gamma_\nu\gamma_5(\not{p}+\not{q})}{p^2(p+q)^2} \\ &= -\delta^{ab}C_2(T)\frac{g_2^2}{6\pi^2}\left(q_\mu q_\nu - g_{\mu\nu}q^2\right) \\ &\quad \cdot \left(\ln\Lambda + \text{finite part}\right)\end{aligned} \qquad \text{(I.3.22)}$$

This expression is finite and describes the propagator of a massless axial vector field if and only if g_2 obeys the same relation as g_1, Eq.(I.3.20). So, in order to get a regularized model, we have to fix g_1 equal to g_2. If the fermion field had a mass in our Lagrangian, we see that the composite axial vector boson would be massive, although the composite vector field is still massless[7)], since,

$$\begin{aligned} &-\delta^{ab}C_2(T)\frac{g_2^2}{(2\pi)^4}\,\mathrm{Tr}\int d^4p\,\frac{\gamma_\mu\gamma_5(\not{p}+m)\gamma_\nu\gamma_5(\not{p}+\not{q}+m)}{(p^2-m^2)\left[(p+q)^2-m^2\right]} \\ &= -\delta^{ab}C_2(T)\frac{g_2^2}{6\pi^2}\left[q_\mu q_\nu - g_{\mu\nu}\left(q^2-12m^2\right)\right]\left(\ln\Lambda + \text{finite part}\right)\end{aligned} \qquad \text{(I.3.23)}$$

Here we stick to the model with massless fermions since we found the monopole solution only in this case.

One sees easily that with the saddle point conditions Eqn's.(I.3.16), all the other fields and all the ghosts do not propagate.We get,

$$\frac{\partial^2 S_{eff}}{\partial X_\mu \partial Y_\nu} = 0 \tag{I.3.24}$$

where X_μ and Y_μ are generic fields not equal to V_μ and A_μ. Also the mixed terms where X_μ and Y_ν are different terms, including A_μ and V_μ are zero. As far as perturbation theory is involved all the terms with zero propagators decouple and we are left with an effective Lagrangian which reads,

$$\mathcal{L}_{eff} = \mathrm{Tr}\,\ln\left(i\not{\partial} + g\not{V} + g\not{A}\right) \tag{I.3.25}$$

I.4. THE MONOPOLE SOLUTION[12)]

In this section we present the classical magnetic monopole solution of the model given by the Lagrangian in Eq.(I.31) We consider the symmetry group to be SU(2). The composite vector and axial vector fields are given in terms of the spinor fields by,

$$\bar{\psi}\gamma_\mu \tau_a \psi = V^a{}_\mu V^2$$

$$\bar{\psi}\gamma_\mu \gamma_5 \tau_a \psi = A^a{}_\mu A^2 \tag{I.4.1}$$

As it was stated in the previous section ψ carries a group index as well as a spinor index. Therefore the right-hand sides of the Eqn's.(I.4.1) must be understood as,

$$\bar{\psi}\gamma^\mu\tau_a\psi = \bar{\psi}_{i\alpha}\gamma^\mu_{\kappa\beta}\tau^a_{ij}\psi_{\beta j}$$

$$= \mathrm{Tr}\,\bar{\psi}\gamma^\mu\psi\tau_a^T \qquad (I.4.2)$$

If $\psi\to\psi\epsilon$ where $\epsilon = i\sigma_2$, the above expression takes the form

$$\bar{\psi}\gamma^\mu\tau_a\psi = -\mathrm{Tr}\,\bar{\psi}\gamma^\mu\psi\tau_a \qquad (I.4.3)$$

Similarly for axial-vector field,

$$\bar{\psi}\gamma^\mu\gamma_5\tau_a\psi = -\mathrm{Tr}\,\bar{\psi}\gamma^\mu\gamma_5\psi\tau_a \qquad (I.4.4)$$

For the classical fields these relations can be also written as,

$$V^a_\mu = \frac{\bar{\psi}\gamma_\mu\tau_a\psi}{(\bar{\psi}\gamma^\lambda\tau_a\psi\,\bar{\psi}\gamma_\lambda\tau_a\psi)^{1/3}}$$

$$A^a_\mu = \frac{\bar{\psi}\gamma_\mu\gamma_5\tau_a\psi}{(\bar{\psi}\gamma^\lambda\gamma_5\tau_a\psi\,\bar{\psi}\gamma_\lambda\gamma_5\tau_a\psi)^{1/3}} \qquad (I.4.5)$$

Strictly speaking one can not divide by fermion operators. However, one obtains the same set of equations of motion by starting from the Lagrangian as given in Eq.(I.3.1) and then eliminating the Lagrange multipliers from the equations of motion obtained after varying the spinor, vector, axial-vector fields as well as Lagrange multipliers.

The ansatz for the spinor field is taken as,

$$\psi = r^{-5/2} \begin{bmatrix} (ia\vec{x}\cdot\vec{\sigma} + br)\epsilon \\ \\ (ic\vec{x}\cdot\vec{\sigma} + dr)\epsilon \end{bmatrix} \qquad (I.4.6)$$

where $r = |\vec{x}|$. Here a,b,c,d are real numbers. However for a ge-ral treatment they will be considered as functions of r.

Our aim is to find a,b,c,d such that ψ satisfies the equation of motion of Lagrangian Eq.(I.3.1),

$$i\partial\!\!\!/\psi + \frac{4g_1}{3} V^{a\mu}\gamma_\mu \tau_a \psi + \frac{4g_2}{3} A^{a\mu}\gamma_\mu \gamma_5 \tau_a \psi = 0 \qquad (I.4.7)$$

provided that V^a_μ is a Wu-Yang monopole, that is,

$$V^a_0 = 0 \qquad V^a_i = \varepsilon_{aij}\, x_j / r^2 \qquad (I.4.8)$$

Under the above assumptions, various terms when calculated gives,

$$\bar{\psi}\gamma^0 \tau_a \psi = 0$$

$$\bar{\psi}\gamma^i \tau_a \psi = -r^{-5}\left[4(ab-cd) r x_j \varepsilon_{jia} + 4(a^2-c^2) x_i x_a + 2(c^2-a^2+b^2-d^2) r^2 \delta_{ia}\right] \qquad (I.4.9)$$

Eq. (I.4.9) immediately gives a restriction on a,b,c,d because of Eqn's. (I.4.8). They are,

$$a^2=c^2 \quad \text{and} \quad b^2=d^2 \tag{I.4.10}$$

A convenient choice is,

$$a=c \quad \text{and} \quad b=-d \tag{I.4.11}$$

which simplifies the Eq. (I.4.9) and gives,

$$\bar{\psi}\gamma^i\tau_a\psi=-8ab\epsilon_{jia}x_j/r^4 \tag{I.4.12}$$

By using the Eq. (I.4.1) one obtains,

$$V^a_0=0 \qquad V^a_i=2^{1/3}(ab)^{1/3}\epsilon_{jia}x_j/r^2$$

$$V^2=-128a^2b^2/r^6 \tag{I.4.13}$$

and for axial-vector part

$$A^a_0=0$$

$$A^a_i=\frac{4a^2x_ix_a+2(b^2-a^2)r^2\delta_{ia}}{2^{1/3}r^3(2a^2b^2-3a^4-3b^4)^{1/3}} \tag{I.4.14}$$

Substitutions of the above expressions into the Eq. (I.4.7), gives the following coupled equations,

$$a'r+\frac{a}{2}-2^{5/3}G_1(ab)^{1/3}a-\frac{2G_2(a^2-3b^2)b}{2^{1/3}(2a^2b^2-3a^4-3b^4)^{1/3}}=0$$

$$b'r-\frac{3}{2}b+2^{5/3}G_1(ab)^{1/3}b-\frac{2G_2(3a^2-b^2)a}{2^{1/3}(2a^2b^2-3a^4-3b^4)^{1/3}}=0 \tag{I.4.15}$$

where $G_1=4g_1/3$ and $G_2=4g_2/3$, prime denotes differentiation with respect to r. The importance of the following cases is obvious.

i) One can hope that without axial-vector part, the monopole solution may exist. This can be seen in Eq.(I.4.15) by putting $G_2=0$. If a,b are considered as constants, the only solution is a=b=0 which is not desirable. If a,b are treated as functions of r, there exists a solution,

$$a=(r/r_o)^{-1/2}\exp\left[3\lambda(r/r_o)^{1/3}\right]$$

$$b=(r/r_o)^{3/2}\exp\left[-3\lambda(r/r_o)^{1/3}\right] \qquad \text{(I.4.16)}$$

where r_o is an integration constant and $\lambda=2^{5/3}G_1$. However, this can not be considered as a monopole solution, because it blows up not only at r=0 but also at $r=\infty$. Therefore, for this case we can safely conclude that monopole solution do not exist.

ii) Assuming $g_2\neq 0$ and a,b,c,d=constant, we end up with the following equations,

$$\frac{a}{2}-2^{5/3}G_1(ab)^{1/3}a-\frac{2G_2(a^2-3b^2)b}{2^{1/3}(2a^2b^2-3a^4-3b^4)^{1/3}}=0$$

$$-\frac{3b}{2}+2^{5/3}G_1(ab)^{1/3}b-\frac{2G_2(3a^2-b^2)a}{2^{1/3}(2a^2b^2-3a^4-3b^4)^{1/3}}=0 \qquad \text{(I.4.17)}$$

If a=kb then an equation for k,

$$4k^{4/3}(3k^4-2k^2+3)^{1/3}=3k^4+2k^2-9 \qquad \text{(I.4.18)}$$

can be found provided that $g_1=g_2$. This last condition is the regularizability condition of the model as it is explained in the previous section, therefore it is not an additional assumption. The reason to treat them as different couplings is obvious from (i). Eq.(I.4.18) has a root $k^2=3$ all the others being complex. So, the solution is,

$$a=c=\mp 3^{7/4}/2^7 g^{3/2}$$

$$b=-d=\mp 3^{5/4}/2^7 g^{3/2} \tag{I.4.19}$$

and

$$\psi=\mp\left(3^{5/4} r^{-3/2}/2^6 g^{3/2}\right)\begin{bmatrix}\exp\left(i\frac{\pi}{3}\,\frac{\vec{x}\cdot\vec{\sigma}}{r}\right)\\ -\exp\left(-i\frac{\pi}{3}\,\frac{\vec{x}\cdot\vec{\sigma}}{r}\right)\end{bmatrix} \tag{I.4.20}$$

The interesting feature of this solution is that, there is an angle between isovector and isoscalar, which is a consequence of the regularizability of the model.

Our attempts to find a solution with a parity violating interaction, given by the Lagrangian,

$$\mathcal{L}=\bar{\psi}i\partial\psi+g_1\left(\bar{\psi}\gamma^\mu\tau_a\psi\,\bar{\psi}\gamma_\mu\tau_a\psi\right)^{2/3}$$

$$+g_2\left(\bar{\psi}\gamma^\mu\frac{1-\alpha\gamma_5}{\sqrt{1+\alpha^2}}\tau_a\psi\,\bar{\psi}\gamma_\mu\frac{1-\alpha\gamma_5}{\sqrt{1+\alpha^2}}\tau_a\psi\right)^{2/3} \tag{I.4.21}$$

failed to give a solution unless α goes to infinity in which case this expression reduces to the Lagrangian in Eq.(I.3.1).

Another interesting question is whether it is possible to choose the parameters in the ansatz (I.4.6) so that for the Lagrangian given in Eq.(I.3.1) one obtains a solution where the axial-vector field rather than the vector field is of monopole type. We have found that the answer to this question is also in negative.

I.5. CONCLUSION

In previous publications these types of models were considered for composite scalar and vector fields separately. This work shows that composite vector and axial-vector fields can be considered together provided that their couplings are equal. This equality is a consequence of the regularizability of the model. In general a similar construction can be given for models containing several kinds of bosons with consequent equalities for their couplings provided that the fermions are massless[6].

As far as the existence of the monopole solution is concerned we have discovered two important facts. An axial-vector term is necessary, and the regularizability of the new interaction requires that the angle found in Eq.(I.4.20) is 60°. In general the condition for the vector field to behave as a monopole is very hard to satisfy together with the spinor field equation. It is this behaviour of these type of purely spinorial models which makes our solution unique. In fact requiring that the model give a monopole solution together with the requirement of regularizability uniquely determines the form of the Lagrangian with the consequent parity doubling for the gluons.

PART II

SEMI-CLASSICAL APPROACH TO THE STABILITY OF MERONS IN A GRAVITATIONAL MODEL AND THE POTENTIAL OF THE MODEL

II.1. INTRODUCTION

The study of the classical solutions to field equations has represented an interesting ground of investigation both for the physical insights that such configurations can offer and also for a deeper understanding of the formal aspects of the theory. Much attention has also been devoted to the stability properties of the classical solutions[1]. Furthermore, semiclassical stability[2,3,4,5], i.e. small perturbations around Euclidean vacuum "bounce" solutions has been considered as a new approach to stability of gravity. For example, instability of flat space at finite temperature, stability of gravity with cosmological constant in the deSitter background and instability of Kaluza-Klein vacuum have been investigated, respectively by Gross and Perry[3], Abbott and Deser[4] and Witten[5].

The aim of this work is to present a discussion of meron solutions and the potential in the case of gravitation where one might find a good interpretation for merons. As is well known, merons are the classical solutions of conformal invariant field theories with singularity at the origin as well as at infinity[6] and they are unstable in pure Yang-Mills theories[7] even in the presence of fermions[8] and in CP^2 models[9,10], but they are stable in pure spinor models[11].

In order to perform our study we shall take as a theoretical laboratory a model which has been examined by De Alfaro, Fubini and Furlan (henceforth DeAFF)[12]. In particular DeAFF considered a model of gravitation coupled to matter fields, which is just the effective part of N=4 Lagrangian for supergra vity with SU(2)XSU(2) local invariance[13]-by effective we mean that the odd parity and spinorial fields are not taken into account, having the corresponding classical configurations vanishing-, where supersymmetry fixes uniquely the ratio between the cosmological constant and the color charge. The DeAFF model is also complete from the cosmological point of view[14], and a class of meronic solutions of this model has recently been found[15] (more details are given in the next section).

Since merons are not bounce solutions in Euclidean space namely they are vacuum solutions with divergent energy in Euclidean space, one has to work in the Minkowski domain where the energy of merons turns out to be real and finite. In the succeeding sections we shall define the stability conditions which will help us to discuss, at least in a particular case, the stability of merons for the considered model. In particular, the stability properties will be investigated for both the flat and conformally flat space backgrounds in which merons exist.

II.2. THE MODEL

The Lagrangian of the model is the following conformally invariant Lagrangian[12],

$$\mathcal{L} = -\frac{\sqrt{g}}{4}\left\{R + \frac{3}{2}\lambda^2\varphi^2 + \frac{1}{e^2\varphi^2}\sum_\alpha F^\alpha_{\mu\nu}F^\alpha_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma} + \frac{2E}{\varphi^2}g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi\right\} \tag{II.2.1}$$

which describes the interaction among the gravitational field, the SU(2) gauge vector fields and a neutral scalar field with a dimensionless cosmological constant λ. The equation of motion which follow from the Lagrangian are,

$$\partial_\mu\left(\sqrt{g}\, g^{\mu\mu'} F_{\mu'\nu}\, \varphi^{-2}\right) = \sqrt{g}\,\varphi^{-2}\left[F_{\mu'\nu}, A_\mu\right] g^{\mu'\mu} \tag{II.2.2a}$$

$$\xi\,\partial_\mu\left(\sqrt{g}\, g^{\mu\nu} \varphi^{-2}\partial_\nu\varphi\right) = \frac{\sqrt{g}}{4}\Big(3\lambda^2\varphi - \frac{4\xi}{\varphi^3} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{2}{e^2\varphi^3}\sum_\alpha F^\alpha_{\mu\nu} F^\alpha_{\lambda\rho}\, g^{\mu\lambda} g^{\nu\rho}\Big) \tag{II.2.2b}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -2\,\theta_{\mu\nu} \tag{II.2.2c}$$

with the energy momentum tensor,

$$\theta_{\mu\nu} = \frac{1}{e^2\varphi^2}\Big(\sum_\alpha F^\alpha_{\mu\lambda} F^\alpha_{\nu\rho}\, g^{\lambda\rho} - \frac{1}{4} g_{\mu\nu}\sum_\alpha F^\alpha_{\lambda\rho} F^\alpha_{\lambda'\rho'}\, g^{\lambda\lambda'} g^{\rho\rho'}\Big) + \frac{\xi}{\varphi^2}\Big(\partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2} g_{\mu\nu}\partial_\lambda\varphi\,\partial_\rho\varphi\, g^{\lambda\rho}\Big) - \frac{3\lambda^2\varphi^2}{8} g_{\mu\nu} \tag{II.2.3}$$

The Lagrangian in Eq.(II.2.1) exhibits a simple covariance property $\mathcal{L} \to u^2 \mathcal{L}$ under the rescaling of the fields,

$$g_{\mu\nu} \to u^{-2} g_{\mu\nu} \qquad A_\mu \to A_\mu \qquad \varphi \to u\varphi \tag{II.2.4}$$

which leads one to the following class of meron solutions[15],

$$\begin{aligned} g_{\mu\nu} &= \frac{1}{c^2}\,\delta_{\mu\nu}\,\frac{x^{2\gamma}}{x^2} \\ A_\mu &= -i\,\sigma_{\mu\nu}\frac{x_\nu}{x^2} \\ \varphi &= \sqrt{\frac{2}{e^2}}\, c\, a\, x^{-\gamma} \end{aligned} \tag{II.2.5}$$

where "a" is a normalization constant which is fixed by the theory, while c remains an arbitrary constant.

Inserting the solutions in Eq.(II.2.5) into the equations of motion (II.2.2a,b,c) we obtain the following algebraic constraints,

$$3\lambda^2/e^2 = \left[\xi_5\gamma^2 + 3(\gamma^2-1)\right]\left[(\gamma^2-1) - \xi_5\gamma^2\right]$$
$$1/a^2 = \xi_5\gamma^2 - (\gamma^2-1)$$
$$4\xi_5\gamma^2 = (3/a^2)\left(1-\lambda^2/e^2\right) \qquad \text{(II.2.6)}$$

Here, one should make some remarks about the reality of solutions (II.2.5) which are important for the stability criterion. Meron solutions Eqn's.(II.2.5) take a more convenient form if the singularities from zero and infinity are displayed in $\pm b_\mu$, $b_\mu = (0,0,0,1)$ by a suitable conformal transformation followed by a Wick rotation to Minkowski space[16] $x_4 = it$,

$$x \rightarrow \left\{\frac{(x-b)^2}{(x+b)^2}\right\}^{1/2} \rightarrow \frac{(it-1)+r^2}{(it+1)+r^2} \equiv \exp(-i\tau) \qquad \text{(II.2.7)}$$

where

$$\tau = \arctan t_+ + \arctan t_- \quad ; \quad t_\mp = t \mp r \qquad \text{(II.2.8)}$$

Consequently, taking into account the transformation properties of fields one gets the following complete Minkowski solutions,

$$g_{\mu\nu} = \frac{\delta_{\mu\nu}}{c^2}\,\frac{1}{(1+t_+^2)(1+t_-^2)}\exp(-2i\gamma\tau) \qquad \text{(II.2.9a)}$$

$$A_\mu = -i\,\sigma_{\mu\nu}\,s_\nu \qquad \text{(II.2.9b)}$$

where

$$s_\nu = \frac{t_+}{1+t_+^2} y_\nu^+ + \frac{t_-}{1+t_-^2} y_\nu^- \quad ; \quad y_\nu^\pm \equiv \left(1, \pm \frac{\vec{x}}{|\vec{x}|}\right) \tag{II.2.9c}$$

and

$$\varphi = c\alpha \exp(i\gamma\tau) \tag{II.2.9d}$$

For the meron solutions Eqn's.(II.2.5) the energy momentum tensor is,

$$\Theta_{\mu\nu} = \frac{1}{(x^2)^2}\left\{\left(\frac{1}{4a^2} - \frac{\xi\gamma^2}{2} - \frac{3a^2}{4e^2}\right)\delta_{\mu\nu} + \left(\xi\gamma^2 - \frac{1}{a^2}\right)\frac{x_\mu x_\nu}{x^2}\right\} \tag{II.2.10}$$

which is conserved in the covariant sense,

$$\left(\Theta^\nu{}_\mu\right)_{;\nu} = 0 \tag{II.2.11}$$

The energy momentum tensor in Eq.(II.2.10) can be improved by means of the conformal transformation which gives finite energy in Minkowski space,

$$E \sim (\text{positive constant})\left[\frac{\gamma^2\xi}{2} - \frac{3}{4}\left(\frac{1}{a^2} - \frac{a^2}{e^2}\right)\right] \tag{II.2.12}$$

II.3. DEFINITION OF STABILITY AND STABILITY OF MERONS [18)]

Before starting to investigate the stability of the meron solutions Eqn's.(II.2.5) in the gravitational DeAFF model, we shall recall the definition of stability for merons in the Minkowski domain. By this we mean that the quantities (co-ordinates, scalar, vectorial and tensorial fields) are transformed from Euclidean space to Minkowski space by using a combined conformal transformation, translation-inversion-translation

(TIT), followed by a Wick rotation, i.e. improved quantities in Minkowski space[16]. As is well known, meron solutions have finite improved energy and action, and they are invariant under the compact O(4)XO(2) subgroup of the O(4,2) Minkowski conformal group in Minkowski domain. These improved meron properties allow us to study the stability of merons in the gravitational field theories.

Now let us define the stability for merons: by making the ansatz $\exp(-ik\tau)$ with the proper time τ in Eq.(II.2.8) for the small perturbations around the meron solutions (II.2.9a,b) in the Minkowski domain, stability or unstability will be determined by k being real or complex. On the other hand, the small fluctuations in the Euclidean space are also corresponding to the small fluctuations in Minkowski domain by conformal transformation (TIT), so for the small fluctuations around Euclidean solutions Eqn's.(II.2.5) we can take the ansatz x^k which turns out to be, as given in Eq.(II.2.7),

$$x^k \longrightarrow \exp(-ik\tau) \qquad \text{(II.3.1)}$$

in Minkowski domain which leads one to work in Euclidean space.

If one examines the stability of merons in CP^2 and pure Yang-Mills models by using the above instability definitions in Minkowski domain, the results are same, they are unstable, as in Refs. 7 and 9[10].

Now let us investigate the stability of merons in the DeAFF model. For this study we should like to investigate two special cases of the solutions (II.2.5). The first one is the conformally flat space, which by substituting $\gamma=0$ leads to the constraint $\lambda^2=e^2$. Consequences of this constraint have been discussed by Cervero[14]. The second one is the flat space (which corresponds to $\gamma=1$ in the solutions (II.2.5)) with the constraint $\lambda^2=-\varepsilon_1^2 e^2/3$ which coincides with the prediction of extended N=4 supergravity on the cosmological constant λ[17].

i) Conformally flat space:

Substituting $\gamma=0$, the solutions (II.2.5) take the form,

$$g_{\mu\nu}=\frac{1}{c^2}h^2(x)\delta_{\mu\nu} \qquad A_\mu=i\sigma_{\mu\nu}\partial_\nu \ln h \qquad \varphi=\sqrt{\frac{2}{e^2}}\,ca \tag{II.3.2}$$

where $h(x)=1/x$ and the solution (II.3.2) has finite improved energy in Minkowski space which is positive for $a^4 > e^2$.

Let us now take small fluctuations around the solution (II.3.2).

$$g_{\mu\nu}\rightarrow g_{\mu\nu}+\delta g_{\mu\nu} \qquad A_\mu\rightarrow A_\mu+\delta A_\mu \qquad \varphi\rightarrow\varphi+\delta\varphi \tag{II.3.3}$$

Owing to the mathematical difficulties of a general treatment we shall limit ourselves for a very preliminary indication for the particular simple case where the fluctuations in $g_{\mu\nu}$ and A_μ are assumed to be generated as a result of a variation in $h(x)=1/x$ of the form,

$$h\rightarrow h+\delta h \tag{II.3.4}$$

Namely, our fluctuations are still in the flat or conformally flat space. Then, according to the above assumption, we find,

$$\delta g_{\mu\nu}=2h\,\delta h\,\delta_{\mu\nu} \tag{II.3.5}$$

$$\delta F_{\mu\nu}=i\left\{2h(x.\partial)\delta h\,\sigma_{\mu\nu}-B_{\lambda\nu}\sigma_{\mu\lambda}+B_{\lambda\mu}\sigma_{\nu\lambda}\right\} \tag{II.3.6}$$

where,

$$\sigma_{\mu\nu}=\frac{1}{4i}\left(s_\mu \bar{s}_\nu - s_\nu \bar{s}_\mu\right) \tag{II.3.7}$$

with

$$s_\mu=(1, i\vec{\sigma}) \qquad \bar{s}_\mu=(1,-i\vec{\sigma}) \tag{II.3.8}$$

and

$$B_{\lambda\nu}=x_\lambda x_\nu h^3 \delta h + 2x_\lambda h \partial_\nu \delta h + 2x_\nu h \partial_\lambda \delta h + \frac{1}{h}\partial_\nu \partial_\lambda \delta h \tag{II.3.9}$$

Here $F_{\mu\nu}$ is given by,

$$F_{\mu\nu}=\partial_\mu A_\nu - \partial_\nu A_\mu + e\left[A_\mu, A_\nu\right] \tag{II.3.10}$$

where e is the colour charge. We shall also use the result,

$$\left\{F_{\mu\nu}, \delta F_{\mu\nu}\right\}=2h\Box\delta h - 2h^3(x.\partial)^2\delta h - 4h^3(x.\partial)\delta h \tag{II.3.11}$$

Substituting the results above in the equation of motion in Eq.(II.2.2b) with $\xi=1, c=1, a=(2/e^2)^{1/2}$ (this value of "a" is a consequence of having meron solutions[15]) and taking the variation of the resulting equations with respect to h and φ we get

$$\begin{aligned} h^2\Box\delta\varphi - 2h^4(x.\partial)\delta\varphi = {} & 6h^3\left(\frac{\lambda^2}{e^2}-1\right)\delta h + \frac{3}{2}h^4\left(\frac{\lambda^2}{e^2}+3\right)\delta\varphi \\ & +2h\Box\delta h - 2h^3(x.\partial)^2\delta h \\ & -4h^3(x.\partial)\delta h + 6h^3\delta h \end{aligned} \tag{II.3.12}$$

Similarly, Eq.(II.2.2c), becomes,

$$\Box\delta h + 3h^2\delta h + 4\frac{\lambda^2}{e^2}h^3\delta\varphi = 0 \tag{II.3.13}$$

According to the stability definition of merons, as given at the beginning of this section, we can take the following ansatz for the fluctuation part,

$$\delta h = x^k Y_\ell \tag{II.3.14}$$

where Y_1 depends on the three polar angles in four dimensions. By using the identities,

$$\frac{1}{h^2} \Box \delta h = \left[(k+1)^2 - (\ell+1)^2\right] \delta h \tag{II.3.15}$$

and

$$(x.\partial)\delta h = k\,\delta h \tag{II.3.16}$$

Eq.(II.3.13) becomes,

$$4\frac{\lambda^2}{e^2}\delta\varphi = \left[-(k+1)^2 + (\ell+1)^2 - 3\right] x^{k+1} Y_\ell \tag{II.3.17}$$

Substituting this and $\lambda^2/e^2=1$ (this is because of constraint Eqn's.(II.2.6)), we get for the l=0 ground state the following quartic equation,

$$k^4 + 11k^3 + 22k^2 + 25k + 18 = 0 \tag{II.3.18}$$

This equation has two real and two complex roots. To ensure stability, all four roots for k must be real. Thus there are no absolutely stable solutions.

Before going into the investigation of stability properties of flat space, we would like to check another ansatz, which will be used in the following section in order to find the potential of the model. This is again a conformally flat ansatz, which differs from the former in the A_μ 's form, that is,

$$g_{\mu\nu} = h^2(x)\delta_{\mu\nu} \qquad A_\mu = -i\,\sigma_{\mu\nu} h\,\partial_\nu x \qquad \varphi = \sqrt{\frac{2}{e^2}} \tag{II.3.19}$$

Following the same steps as above one finds,

$$(k+1)^4 - 4(k+1)^2 + 9 = 0 \tag{II.3.20}$$

All the four roots are complex, so no stability for this ansatz too.

ii) Flat space

Substituting $\gamma=1$, the solutions (II.2.5) take the form,

$$g_{\mu\nu} = \frac{1}{c^2}\delta_{\mu\nu} \qquad A_\mu = i\,\sigma_{\mu\nu}\partial_\nu \ln h \qquad \varphi = \sqrt{\frac{2}{e^2}}\, c a h \tag{II.3.21}$$

where again $h(x)=1/x$. This solution has also finite improved energy in Minkowski space which is positive for $\frac{2}{3} > \frac{e^2-a^4}{a^2e^2}$
Let us again take a small fluctuation around the solution Eq.(II.3.21).

$$g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu} \qquad A_\mu \to A_\mu + \delta A_\mu \qquad \varphi \to \varphi + \delta\varphi \tag{II.3.22}$$

where the fluctuations in A_μ and φ are also assumed to be generated as a variation (II.3.11) and $\delta g_{\mu\nu} = \delta g\,\delta_{\mu\nu}$. Eq.(II.3.6), Eq.(II.3.9) and Eq.(II.3.11) again hold in this case too. Only the variation of φ differs,

$$\delta\varphi = \sqrt{\frac{2}{e^2}}\,\delta h \tag{II.3.23}$$

Following the same steps as done in the conformally flat case, we arrive to the coupled equations given below,

$$3h(x\cdot\partial)\delta\varphi - h\delta\varphi + 15\delta h - 2(x\cdot\partial)\delta h - 2(x\cdot\partial)^2\delta h + \frac{1}{h^2}\Box\delta h = 0 \qquad \text{(II.3.24)}$$

$$\frac{6}{h^3}\Box\delta h + \frac{18}{h}\delta h + 10h^2\delta\varphi - 8h\delta h - 4h(x\cdot\partial)\delta h = 0 \qquad \text{(II.3.25)}$$

Taking the same ansatz as in Eq.(II.3.14) and using the identities Eq.(II.3.15) and Eq.(II.3.16) one obtains,

$$\delta\varphi = \frac{1}{5}\left[(2k+4)x^{k+1}Y_\ell - 3\{(k+1)^2 - (\ell+1)^2\}x^{k+3}Y_\ell\right] \qquad \text{(II.3.26)}$$

Substituting this into Eq.(II.3.24) one finds, with l=0,

$$\begin{aligned}&\left[\frac{3}{5}(2k+4)(k+1) - \frac{2k+4}{5} + 14 - 2k - 2k^2 + (k+1)^2\right]x^k \\ &+ \left[-\frac{9}{5}(k+1)^2(k+3) - \frac{9}{5}(k+3) + \frac{3}{5}(k+1)^2 + \frac{9}{5}\right]x^{k+2} = 0\end{aligned} \qquad \text{(II.3.27)}$$

The coefficients of x^k and x^{k+2} must then be equal to zero,

$$\begin{aligned}&k^2 + 16k + 83 = 0 \\ &(k^2 + 2k + 3)(3k + 8) = 0\end{aligned} \qquad \text{(II.3.28)}$$

This again gives solutions with complex k, hence the solutions are not stable for this case as well.

II.4. THE POTENTIAL OF THE MODEL[19)]

As it is known, in models with gravity it is not possible to express the potential separately. In such cases, the conventional method to obtain the potential is tie the different fields by a new field with a redefinition of the kinetic term.

We will use the above approach and investigate the potential of the Lagrangian in Eq.(II.2.1) in the Minkowski domain which will also provide us with a way of defining a new kinetic term via a new variable and field. For this purpose we wish to use a suitable ansatz which is already given in the last section in Eq.(II.3.19). Inserting this ansatz in the Lagrangian in Eq.(II.2.1) we get the action in the spherical coordinates,

$$I = 3\pi^2 \int d(\ln x)\Big[\Big(1-\frac{1}{E^2}\Big)\Big(\frac{d}{d(\ln x)}hx\Big)^2 - \Big(\frac{\Lambda^2}{4}+\frac{1}{E^2}\Big)(hx)^4 + \frac{4}{E^2}(hx)^3 + \Big(1-\frac{4}{E^2}\Big)(hx)^2\Big] \tag{II.4.1}$$

where $\Lambda^2 = a^2\lambda^2$, $E^2=a^2e^2$ and φ=constant=a. From section two and from Eq.(II.2.7) we can change the variable x to τ which may be called as cosmological time in the model[14] by,

$$\ln x \rightarrow -i\tau \equiv T \tag{II.4.2}$$

Since the invariant quantity is the action its invariance is achieved by the following change of the h(x) as,

$$h(x) \rightarrow \frac{1}{x} f(x) \tag{II.4.3}$$

Then, the action in Eq.(II.4.1) becomes,

$$I = 3\pi^2 \int dT \Big[\Big(1-\frac{1}{E^2}\Big)\Big(\frac{df}{dT}\Big)^2 - \Big(\frac{\Lambda^2}{4}+\frac{1}{E^2}\Big)f^4 + \frac{4}{E^2}f^2 + \Big(1-\frac{4}{E^2}\Big)f^2\Big] \tag{II.4.4}$$

Now, we can interpret f(T) as the position of a particle and T as a proper time and Eq.(II.4.4) as the mechanical equ ation for a particle in a potential,

$$V(f) = 3\pi^2\Big[\Big(\frac{\Lambda^2}{4}+\frac{1}{E^2}\Big)f^4 - \frac{4}{E^2}f^3 - \Big(1-\frac{4}{E^2}\Big)f^2\Big] \tag{II.4.5}$$

In order to get a correct kinetic term one should fix the coeff-icient of kinetic term as,

$$1-\frac{1}{E^2}=\frac{1}{2} \tag{II.4.6}$$

or

$$E^2=2 \qquad a=\sqrt{\frac{2}{e^2}} \tag{II.4.7}$$

This is also a very convenient choice in finding the place of meron on the potential which we shall discuss in the following paragraph.

For various values of λ^2/e^2, some graphs of the potential may be found at the end of this section.

The potential is unbinding when $\lambda^2/e^2<-1$, and for $\lambda^2/e^2>5/2$ it has only one minimum at f=0, which is not well defined physically. The only physical interval therefore is $-1<\lambda^2/e^2<5/2$ where there exist another ground state for $f\neq 0$ which may allow transition between two ground states.

The place of meron in the potential is the point where $\lambda^2/e^2=1$, f=1 and V=0.

In figure 2 the intersection with $\lambda^2/e^2=1$ plane is presented and it is seen that the meron is on the minimum. Therefore it can be interpreted as a vacuum solution. This solution can decay only when λ^2/e^2 is allowed to change even if the change is very small.

II.5. CONCLUSION

In the sections (II.1), (II.2) and (II.3) we have investigated the stability of the improved meron solutions in the conformally flat space and flat space. Our results indicate that particular forms of this solutions are unstable and thus cannot be interpreted as a possible candidates for a vacuum state. Since the fluctuations for metrics were still in the flat space, i.e. $g_{\mu\nu} = \delta_{\mu\nu}(h + \delta h)$ our results are very particular.

In the last section (II.4) we presented a suitable ansatz which provides us an expression which can be interpreted as the potential of the model in the Minkowski domain. We have shown that the model is physically meaningful when λ^2/e^2 is limited to the interval $-1 < \lambda^2/e^2 < 5/2$.

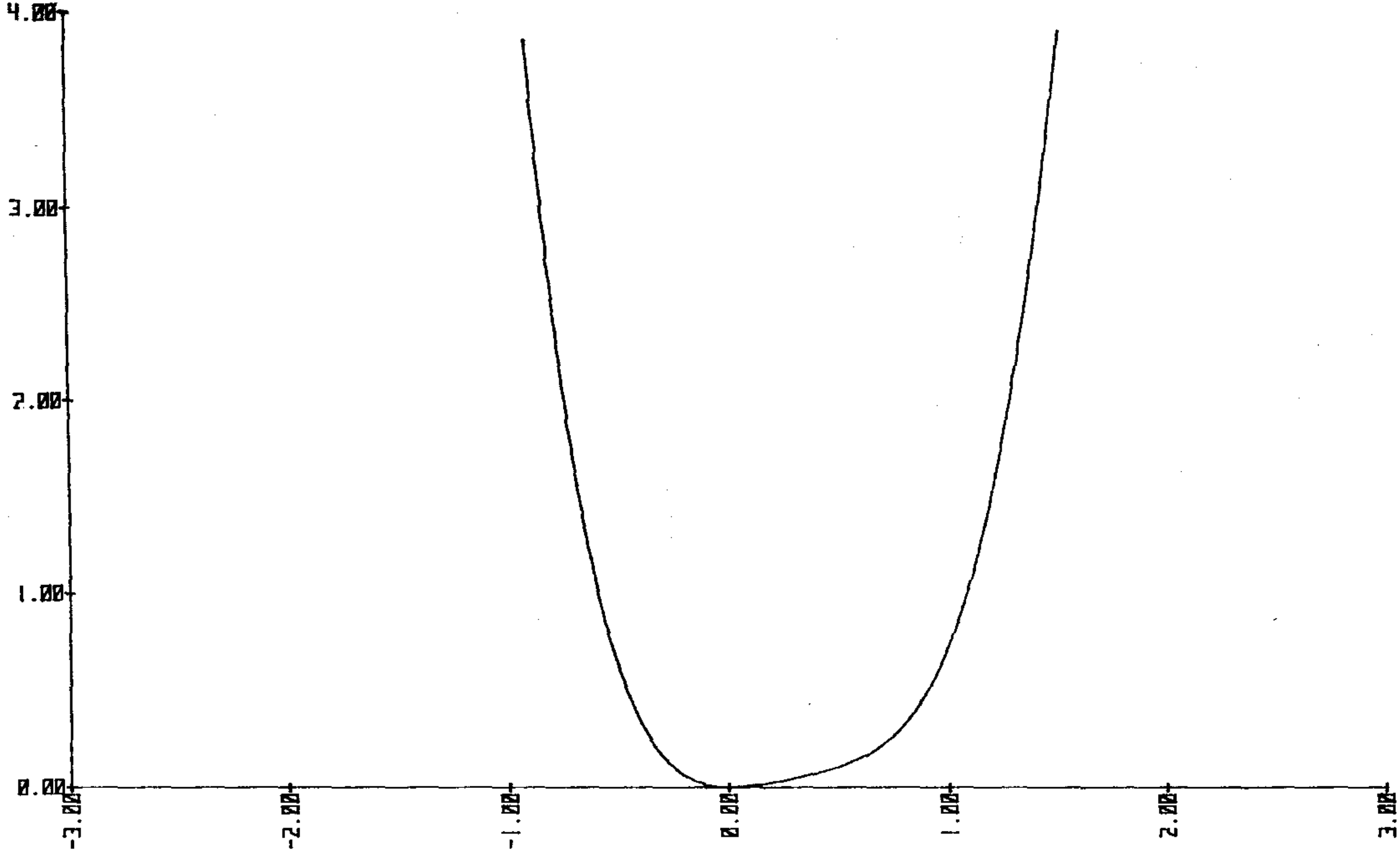

Figure II.4.1

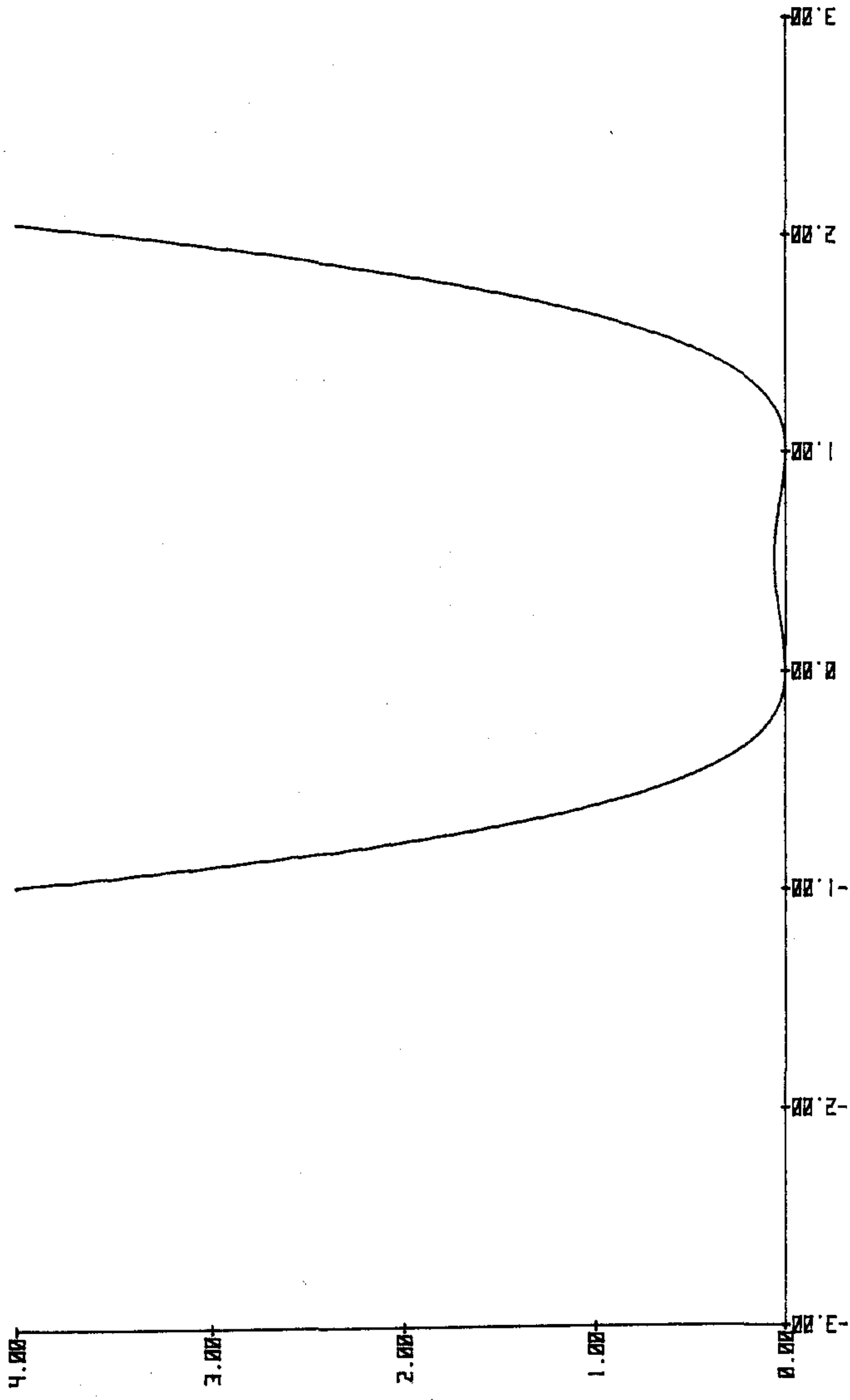

Figure II.4.2

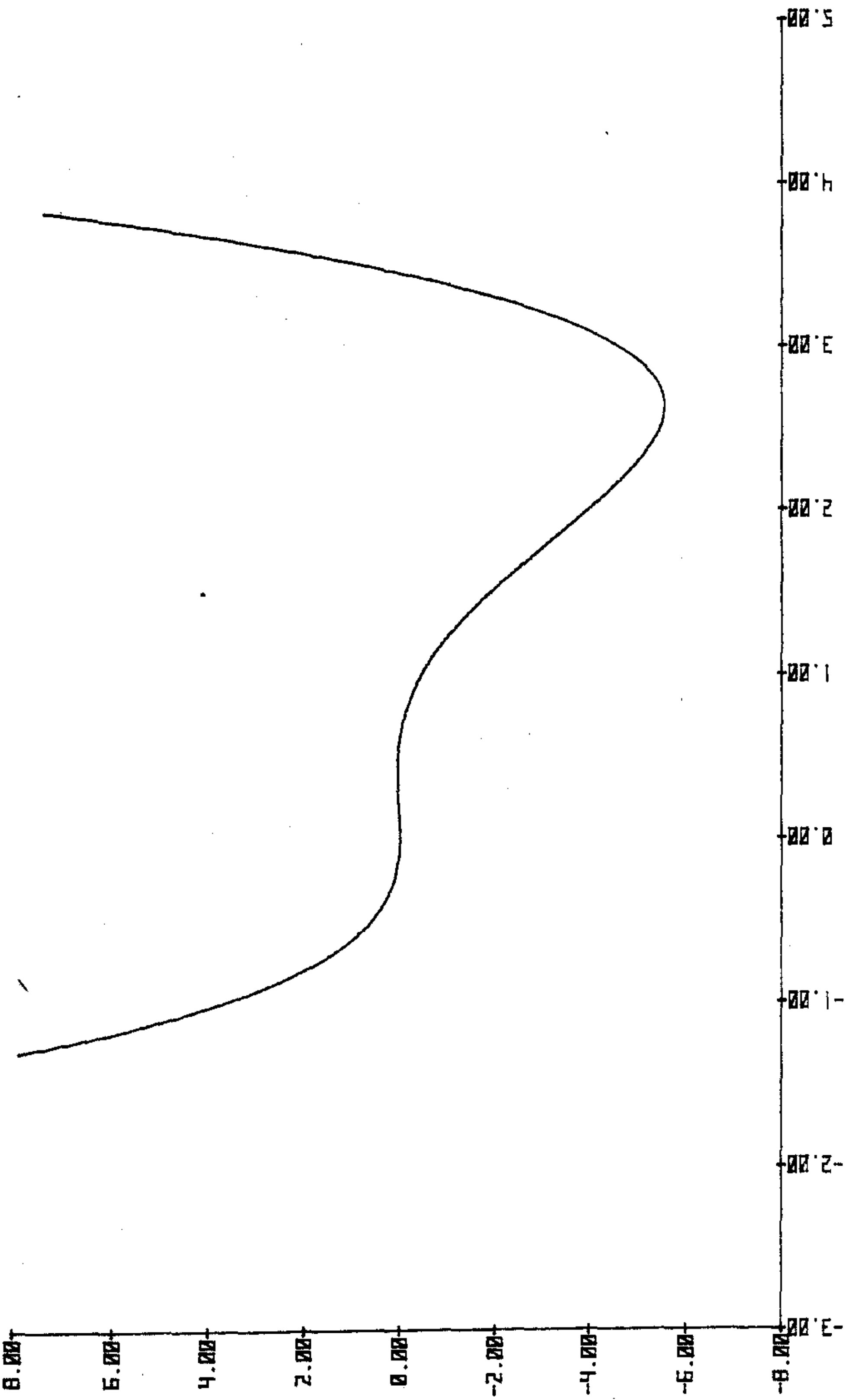

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Figure II.4.3

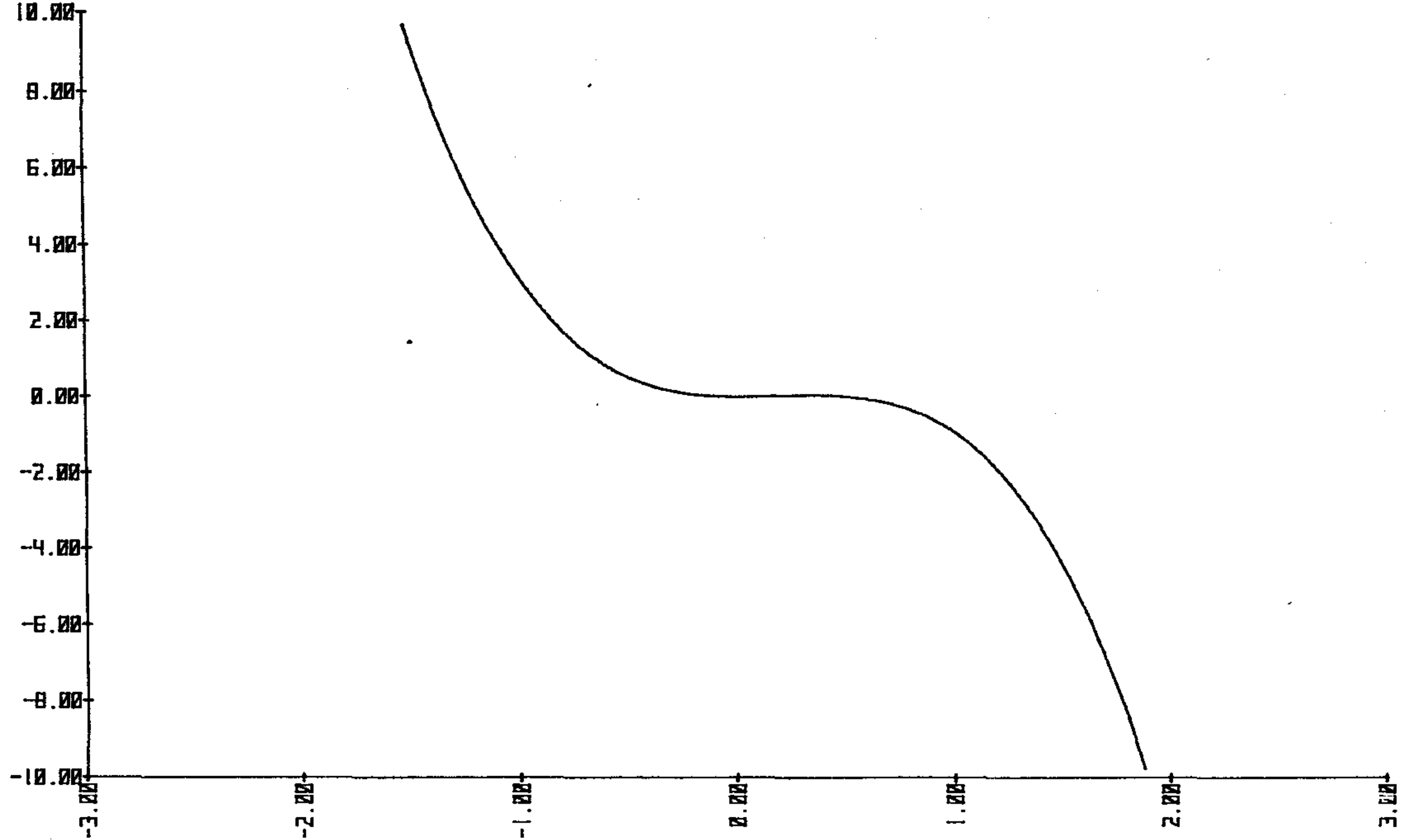
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Figure II.4.4

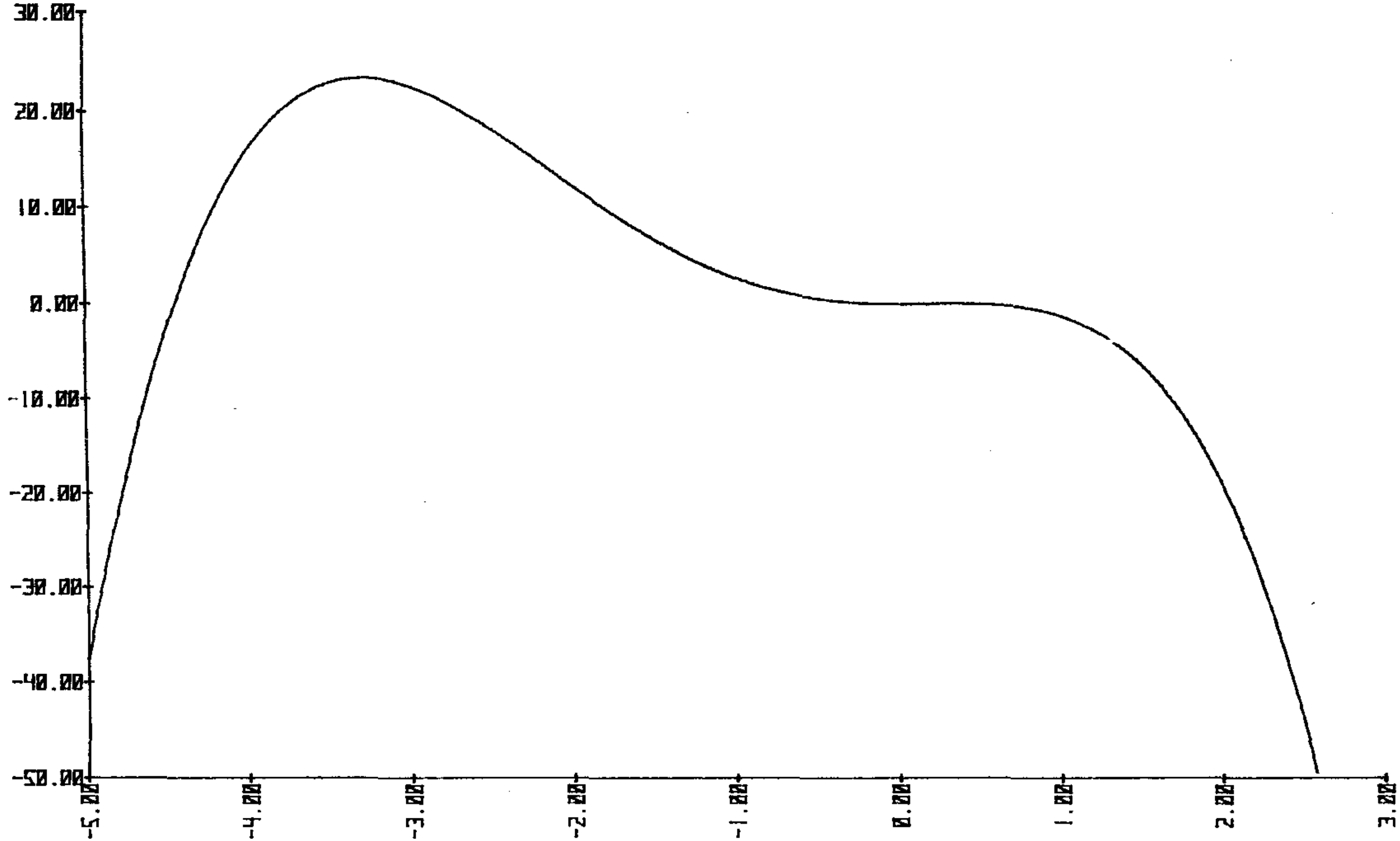
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Figure II.4.5

SUMMARY

In the first part we are concerned with monopole solutions in a model which contains massless composite vector and axial-vector gluons. We first present the quantization and regularization of the model and look for its monopole solutions. Axial-vector interaction is necessary if one requires to satisfy the equation of motion when vector part is restricted to be a Wu-Yang monopole. We also imposed the condition that the internal symmetry to be SU(2). It is found that there exists an angle $\pi/3$ between vector and isovector parts which stems from the regularizability of the model. We also observed that vector part itself has a solution, though it is not interpreted as a monopole. This solution may have a topological interpretation, since for negative coupling constant, the angle between the isovector and the isoscalar parts change from zero to $\pi/2$ as r goes from zero to infinity. When an axial-vector interaction is added to the Lagrangian this angle becomes constant and the solution takes the form of a Wu-Yang monopole.

In the second part we investigated stability properties of a field theoretic model which contains gravitational,Yang-Mills and scalar interactions. For this purpose we used the semi-classical approach, that is, we perturbed the equations of motion of the model around a field which is taken to be common to the original fields. Two special cases are taken under consideration, conformally flat and flat spaces. We found that meron solutions are not stable in both cases. After this we found an ansatz which when used in the action of

the model gave an expression which is very similar to a classical action with its kinetic and potential terms. We observed that this is possible in Minkowski space where the variable is no longer four-dimensional Euclidean radial distance but a proper time which is also interpreted as cosmological time by other authors.

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