AN ADAPTIVE CONTROL STRUCTURE COMBINING MODEL REFERENCE ADAPTIVE CONTROLLERS AND STOCHASTIC SELF-TUNING REGULATORS

by

Feza Kerestecioğlu

B.S.in E.E.,Boğaziçi University,1983

Submitted to the Institute for Graduate Studies in Science and Engineering in partial fulfillment of the requirements for the degree of

Master of Science

in

Electrical Engineering

Boğaziçi University

1985

AN ADAPTIVE CONTROL STRUCTURE COMBINING MODEL REFERENCE ADAPTIVE CONTROLLERS AND STOCHASTIC SELF-TUNING REGULATORS

APPROVED BY

Doç. Dr. Yorgo ISTEFANOPULOS
(Thesis Supervisor)

Doç. Dr. Okyay KAYNAK

Y. Doç. Dr. İbrahim EKSİN

DATE OF APPROVAL 21.8.1985

ACKNOWLEDGEMENTS

It is my pleasant duty to express my deep gratitude to Doç. Dr. Yorgo ISTEFANOPULOS because of his invaluable guidance, suggestions and helps during the development of this work.

I also appreciate kindly helps of Hakan PEKCAN and Sibel (ÖZTAN) YILANKIRAN in drawing the figures.

Special thanks are to Mine KALKAN not only for her helps in drawings but also for her continous moral support during the whole study.

AN ADAPTIVE CONTROL STRUCTURE COMBINING MODEL REFERENCE ADAPTIVE CONTROLLERS AND STOCHASTIC SELF-TUNING REGULATORS

ABSTRACT

Discrete-time model reference adaptive controllers for single-input single-output minimum-phase plants in deterministic environment are studied. Both explicit and implicit reference models are considered. Similarities and dualities between the model reference adaptive controllers and stochastic self-tuning regulators are indicated and the behaviour of model reference adaptive controllers in stochastic environment is analyzed. Finally, an adaptive control structure combining model reference adaptive controllers and stochastic self-tuning regulators is discussed, which is suitable for regulation and tracking objectives in both deterministic and stochastic environment. Simulations on a digital computer are done to justify theoretical results and investigate various features of the adaptive control structures mentioned.

ÖZETÇE

Gerektirici ortamda tek girdili tek çıktılı enküçük evreli dizgeler için ayrık zamanlı dayanak taslamlı uyarlamalı denetleyiciler incelenmektedir. Hem dolaysız hem de dolaylı dayanak taslamları söz konusu edilmiştir. Dayanak taslamlı uyarlamalı denetleyicilerle stokastik öz-ayarlanan düzengeçler arasındaki benzerlikler ve eşleklik belirtilmekte ve dayanak taslamlı uyarlamalı denetleyicilerin stokastik ortamdaki davranışı incelenmektedir. Son olarak, dayanak taslamlı uyarlamalı denetleyicileri ve stokastik özayarlanan düzengeçleri birleştiren ve hem gerektirici hem de stokastik ortamda düzengeçleme ve izleme amaçları için uygun olan bir uyarlamalı denetim yapısı tartışılmaktadır. Kuramsal sonuçları doğrulamak ve söz konusu uyarlamalı denetleme yapılarını çeşitli yönlerden araştırmak için sayısal bir bilgisayarda benzetimler yapılmıştır.

TABLE OF CONTENTS

LIST OF FIGURES

FIGURE 5.3 Plant output of MRAC with $H_1(q^{-1}) = H_2(q^{-1}) = C_2(q^{-1}) = 1$ and constant adaptation gain ($F = I$; $F = 10 \cdot I$; $F = 100 \cdot I$) for RM3

FIGURE 5.4 Plant output of MRAC with $H_1(q^{-1}) = H_2(q^{-1}) = C_2(q^{-1}) = 1$ and decreasing adaptation gain ($F_o = 100 \cdot I$, $\lambda_2 = 1$; $F_o = 100 \cdot I$, $\lambda_2 = 0.5$; $F_o = 100 \cdot I$ $\lambda_2 = 0.2$) for RM1

FIGURE 5.5 Plant output of MRAC with $H_1(q^{-1}) = H_2(q^{-1}) = C_2(q^{-1}) = 1$ and decreasing adaptation gain ($F_o = 100 \cdot I$, $\lambda_2 = 1$; $F_o = 100 \cdot I$, $\lambda_2 = 0.5$; $F_o=100 \cdot I$ $\lambda_2 = 0.2$) for RM2

FIGURE 5.6 Plant output of MRAC with $H_1(q^{-1}) = H_2(q^{-1}) = C_2(q^{-1}) = 1$ and decreasing adaptation gain ($F_o = 100 \cdot I$, $\lambda_2 = 1$; $F_o = 100 \cdot I$, $\lambda_2 = 0.5$; $F_o = 100 \cdot I$, $\lambda_2 = 0.2$) for RM3

FIGURE 5.7 Plant output of MRAC with $H_1(q^{-1}) = H_2(q^{-1}) = C_2(q^{-1}) = 1$ and constant trace adaptation gain ($F_o = 10 \cdot I$, $\lambda_1(k)/\lambda_2(k) =1$; $F_o = 10 \cdot I$, $\lambda_1(k)/\lambda_2(k) = 4$; $F_o = 10 \cdot I$, $\lambda_1(k)/\lambda_2(k) = 0.25$) for RM1

FIGURE 5.8 Plant output of MRAC with $H_1(q^{-1}) = H_2(q^{-1}) = C_2(q^{-1}) = 1$ and constant trace adaptation gain ($F_o = 10 \cdot I$, $\lambda_1(k)/\lambda_2(k) = 1$; $F_o = 10 \cdot I$, $\lambda_1(k)/\lambda_2(k) = 4$; $F_o = 10 \cdot I$, $\lambda_1(k)/\lambda_2(k) = 0.25$) for RM2.

LIST OF TABLES

LIST OF SYMBOLS

A	Polynomial defining the output dynamics of the plant
B	Polynomial defining the input dynamics of the plant
C	Polynomial defining the disturbance dynamics of the plant
C_1	Polynomial defining the desired dynamics of the plant output
C_2	Polynomial defining the regulation objective
$\hat{C}_2$	Estimated polynomial defining the control objective
D	Polynomial defining the desired dynamics of the reference input
d	Plant delay
F	Adaptation gain matrix
H_1	Numerator polynomial of the a posteriori adaptation error filter
H_2	Denominator polynomial of the a posteriori adaptation error filter

I	Identity matrix
k	Discrete time variable
L	Polynomial defining the filter acting on the plant input and output
p, p_e, p_o, p_{oe}	True parameter vectors
$\hat{p}, \hat{p}_e, \hat{p}_o, \hat{p}_{oe}$	Estimated parameter vectors
q^{-1}	Unit delay operator
t	Time at which parameters change
tr	Trace operator
u	Plant input
u^f	Filtered plant input
u^M	Reference input
v	A posteriori adaptation error
w	Plant disturbance
y	Plant output
y^f	Filtered plant output
y^M	Reference model output
y^{Mf}	Filtered reference model output
z	Complex variable
δ	Kronecker-delta function
ε	Plant-model error

$\hat{\varepsilon}$	Prediction error
$\bar{\varepsilon}$	Auxiliary error
ε^*	A posteriori filtered plant-model error
ε_c	Filtered plant-model error
λ_1, λ_2	Gain updating coefficients
$\phi, \phi_e, \phi_o, \phi_{oe}$	Observation vectors
ϕ^f, ϕ_o^f	Filtered observation vectors

I. INTRODUCTION

In recent years, considerable work has been done to investigate similarities, connections and dualities between the model reference adaptive controllers (MRAC) and stochastic self-tuning regulators (S-STR) [1] – [5].

In an adaptive control system, designed for deterministic environment, the controller parameters are adjusted so that the output of the plant with unknown parameters is able to follow a reference input in a desired dynamics. This dynamics can be modeled explicitly, so the plant model error is used for adaptation (Figure 1.1). Another adaptive control strategie could be to use an adaptive predictor to obtain a control such that the output of the plant becomes the desired output(Figure 1.2). If the output of the predictor is identical to that of the explicit reference model, two systems can be equivalent [1], [2], [6]. In that case, the controller and the predictor form an implicit reference model (shown with dashed lines in Figure 1.2), and therefore this type adaptive controllers is referred as MRAC with implicit reference model.

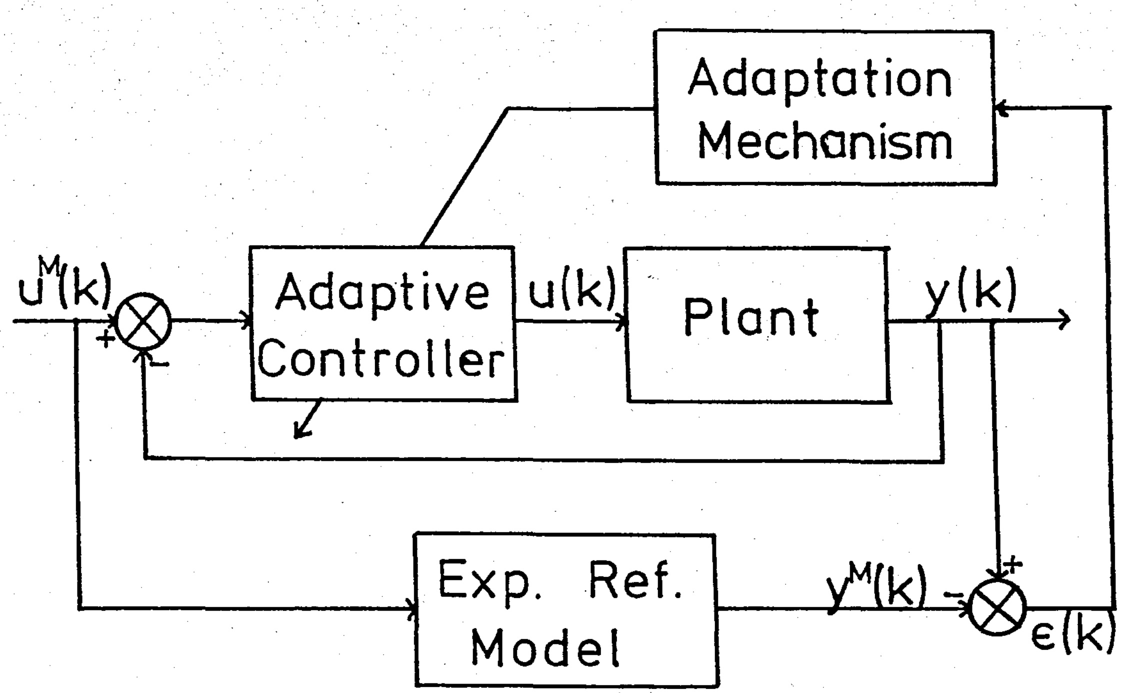


Figure 1.1 Explicit Model Reference Adaptive Control System

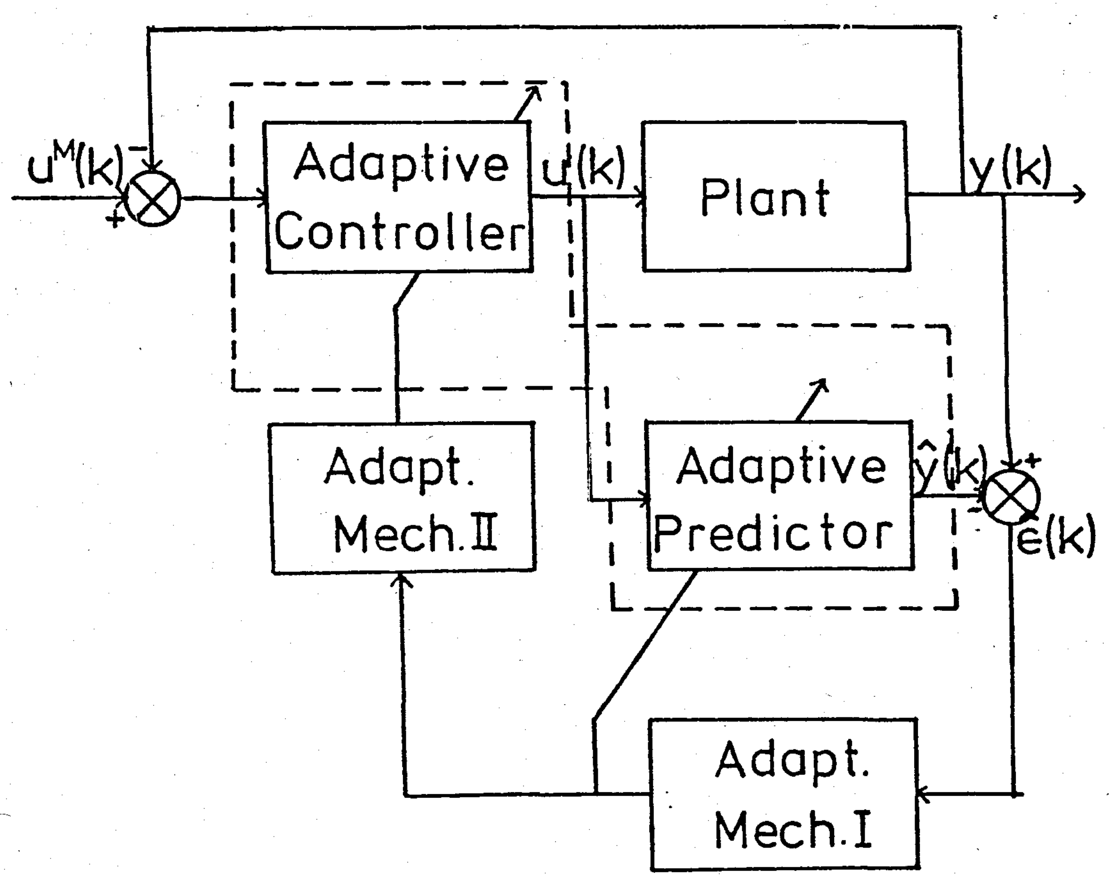


Figure 1.2 Implicit Model Reference Adaptive Control System

On the other hand, a structure similar to that of the MRAC with implicit reference model appears in case of stochastic self-tuning regulators for which the control objectives are given with ARMA models [7]. Starting from the equivalencies between explicit and implicit model reference controllers, stochastic self-tuning regulators with explicit reference models can be defined [2]. These are equivalent to the stochastic self-tuning regulators where adaptive predictors are used, which from the class of stochastic self-tuning regulators mostly considered in literature.

As shown in [1] and [2], similarities can be found between model reference adaptive control systems and stochastic self-tuning regulators. Both of them use the same control law and parameter adaptation algorithms which have a similar structure. On the other hand, the positive realness conditions which appear as a result of the stability analysis of a MRAC and the convergence analysis of a S-STR are of the same type [2], [3], [5].

Furthermore, it is shown in [2] and [5] that model reference adaptive controllers and stochastic self-tuning regulators can be combined to built an adaptive control structure which operates as a desired MRAC in deterministic environment and as a desired S-STR in stochastic environment.

This thesis aims, after analyzing model reference adaptive controllers and stochastic self-tuning regulators with explicit and implicit reference models and exhibiting their similarities and dualities, to discuss the adaptive control structure combining them. Simulations are done on a digital computer to investigate various properties of these control structures and their adaptation algorithms.

In chapter II, the tracking and regulation of a plant with known parameters are considered. Chapter III analyzes the adaptive control problem in deterministic environment. In chapter IV, the problem is extended to cover the stochastic context. The similarities and duality between MRAC and S-STR, the behaviour of MRAC in stochastic environment and the combined MRAC-S-STR design are discussed. Chapter V interprets the simulation results. The sixth and last chapter consist of conclusions and areas of further research.

II. MODEL REFERENCE CONTROL FOR TRACKING AND REGULATION

Before analyzing the MRAC for a discrete-time single -input single-output (SISO) minimum phase-plant with unknown parameters (see Chapter III), we will consider the model reference control for the same type of plant with known parameters in both deterministic and stochastic environment.

II.1 DETERMINISTIC ENVIRONMENT

Consider the discrete linear time invariant SISO system described by the following ARMA equation

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) \qquad d>0 \qquad y(0)\neq(0) \qquad (2.1)$$

where

$$A(q^{-1}) = 1 + a_1q^{-1} + \dots + a_{n_a}q^{-n_a} \qquad (2.2)$$

$$B(q^{-1}) = b_o + b_1q^{-1} + \dots + b_{n_b}q^{-n_b} \qquad b_o\neq 0 \qquad (2.3)$$

q^{-1} is the unit delay operator and $u(k)$ and $y(k)$ are the plant input and output, respectively. It is assumed that the zeroes of $B(z^{-1})$ are inside the unit circle $(|z|<1)$ so that they can be cancelled without leading to an unbounded control input.

The control objectives in tracking and regulation are defined as follows [2], [6], [8]:

i) In tracking, the output of the plant is wanted to satisfy the equation

$$C_1(q^{-1})y(k) = q^{-d}D(q^{-1})u^M(k) \tag{2.4}$$

where

$$C_1(q^{-1}) = 1 + C_{11}q^{-1} + \dots + C_{1n_{c1}}q^{-n_{c1}} \tag{2.5}$$

$$D(q^{-1}) = d_o + d_1q^{-1} + \dots + d_{n_d}q^{-n_d} \tag{2.6}$$

and $u^M(k)$ is a bounded reference input.

ii) In regulation $(u^M(k) \equiv 0)$ an initial disturbance $y(0) \neq 0$ is wanted to be eliminated with the dynamics defined as

$$C_2(q^{-1})y(k+d) = 0 \qquad k \geqslant 0 \tag{2.7}$$

where

$$C_2(q^{-1}) = 1 + C_{21}q^{-1} + \dots + C_{2n_{c_2}}\, q^{-n_{c2}} \tag{2.8}$$

is an asymptotically stable polynomial.

Two methods can be considered to obtain a solution to such a control problem :

i) Explicit Reference Model :

We define the reference model as

$$C_1(q^{-1})y^M(k) = q^{-d}D(q^{-1})u^M(k) \tag{2.9}$$

and the plant-model error as

$$\varepsilon(k) = y(k) - y^M(k) \tag{2.10}$$

Then, if the equation

$$C_2(q^{-1})\,\varepsilon\,(k+d) = 0 \qquad\qquad k > 0 \tag{2.11}$$

holds, both control objectives are accomplished.

To compute the control input $u(k)$, the following polynomial identity is used

$$C_2(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}R(q^{-1}) \tag{2.12}$$

where

$$S(q^{-1}) = 1 + s_1q^{-1} + \ldots + s_{n_s}\, q^{-n_s} \tag{2.13}$$

$$R(q^{-1}) = r_o + r_1\, q^{-1} + \ldots + r_{n_r}q^{-n_r} \tag{2.14}$$

Using (2.10),(2.12) and (2.1)

$$C_2(q^{-1})\ \varepsilon(k+d) = C_2(q^{-1})\ \left[y(k+d) - y^M(k+d)\right]$$

$$= \left[A(q^{-1})S(q^{-1}) + q^{-d}R(q^{-d})\right]y(k+d)$$

$$- C_2(q^{-1})y^M(k+d)$$

$$= B(q^{-1})S(q^{-1})u(k) + R(q^{-1})y(k) - C_2(q^{-1})y^M(k+d) \quad (2.15)$$

To achieve (2.11),

$$B(q^{-1})S(q^{-1})u(k) = C_2(q^{-1})y^M(k+d) - R(q^{-1})y(k) \quad (2.16)$$

$$u(k) = \frac{C_2(q^{-1})y^M(k+d) - R(q^{-1})y(k) - B_S(q^{-1})u(k)}{b_o} \quad (2.16a)$$

where

$$B_S(q^{-1}) = B(q^{-1})S(q^{-1}) - b_o \quad (2.17)$$

It is shown in [6] and [9], if one chooses

$$n_s = d-1 \quad \text{and} \quad n_r = \max(n_A-1,\ n_{c_2}-d) \quad (2.18)$$

(2.12) has a unique solution for $S(q^{-1})$ and $R(q^{-1})$ and the total number of coefficients to be computed (n_s+n_r) is minimum.

For such a case (2.16a) can be written as

$$u(k) = \frac{C_2(q^{-1})y^M(k+d) - P_o^T \emptyset_o(k)}{b_o} \tag{2.19}$$

where

$$\left.\begin{aligned} P_o^T &= \left[b_o s_1 + b_1,\ b_o s_2 + b_1 s_1 + b_2 \ldots, b_{n_b} s_{d-1},\ r_o,\ \ldots, r_{n_r} \right] \\ \emptyset_o^T(k) &= \left[u(k-1),\ \ldots,\ u(k-d-n_b+1), y(k), \ldots, y(k-n_r) \right] \end{aligned}\right\} \tag{2.20}$$

(2.19) can also be expressed as

$$C_2(q^{-1})y^M(k+d) = p^T \emptyset(k) \tag{2.21}$$

where

$$\left.\begin{aligned} p^T &= \left[b_o,\ p_o^T \right] \\ \emptyset^T(k) &= \left[u(k),\ \emptyset_o^T(k) \right] \end{aligned}\right\} \tag{2.22}$$

The block diagram of this control scheme is given in Figure 2.1. The control objective is shown with dashed lines.

Note that the closed-loop poles are defined by $C_2(q^{-1})$ polynomial. In the adaptive case, it will have considerable importance on the control performances (see Chapter V). In case of known parameters, if $C_2(q^{-1})=1$ all the closed loop poles are at the origin, which means the model output is reached by the plant output after d steps.

ii) Implicit Reference Model :

This method is an application of the separation theorem. One first designs a predictor for the plant output, then a control will be computed such that the predictor output becomes equal to the desired output.

Defining the prediction error as

$$\hat{\varepsilon}(k) = y(k) - \hat{y}(k) \tag{2.23}$$

we design a predictor such that

$$C_2(q^{-1})\,\hat{\varepsilon}(k+d) = 0 \qquad k > 0 \tag{2.24}$$

where $C_2(q^{-1})$ is given by (2.8)

Using (2.23), (2.12) and (2.1),

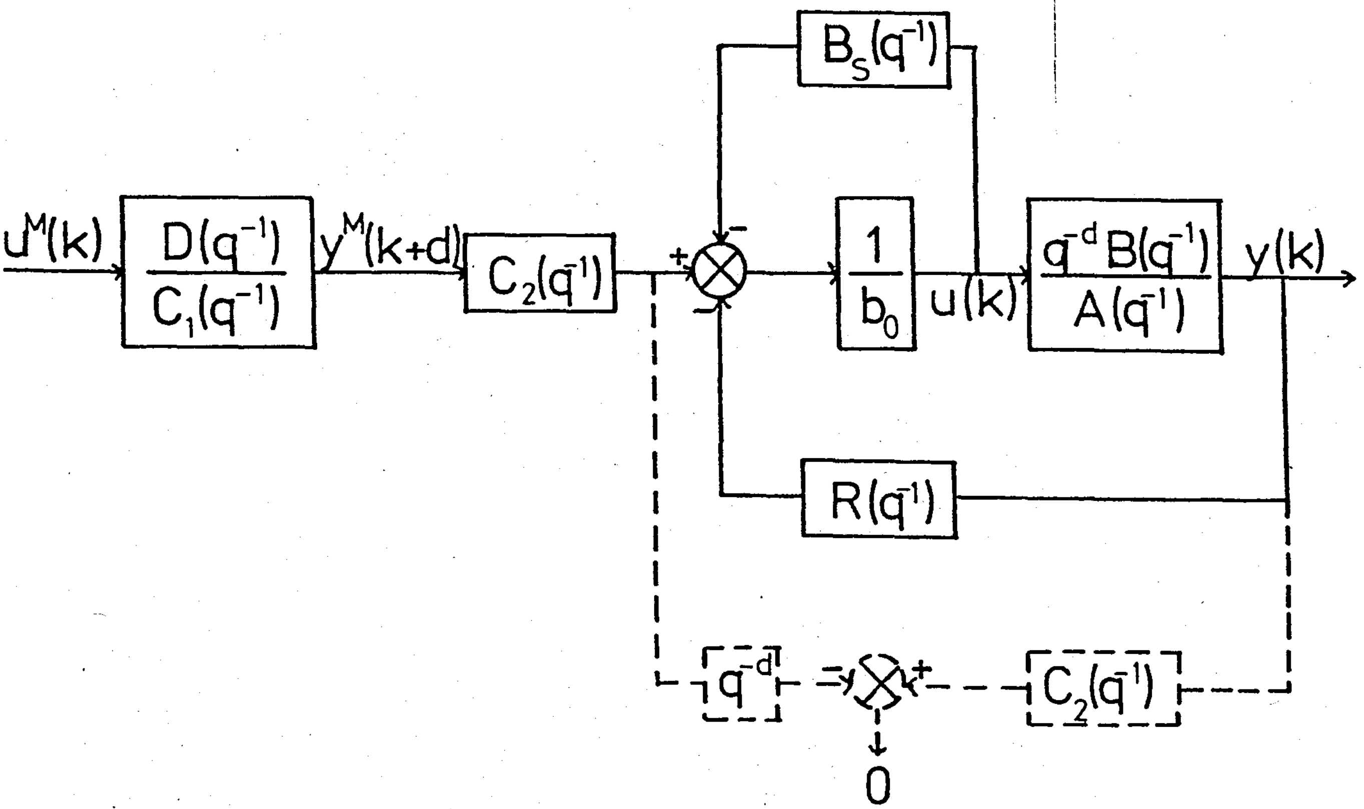


Figure 2.1 Model Reference Control System (known parameters)

$$C_2(q^{-1})\,\hat{\varepsilon}(k+d) = C_2(q^{-1})\left[y(k+d) - \hat{y}(k+d)\right]$$

$$= \left[A(q^{-1})S(q^{-1}) + q^{-d}R(q^{-1})\right]y(k+d) - C_2(q^{-1})\hat{y}(k+d)$$

$$= B(q^{-1})S(q^{-1})u(k) + R(q^{-1})y(k) - C_2(q^{-1})\hat{y}(k+d) \quad (2.25)$$

Hence, to achieve (2.24) the following predictor can be used

$$C_2(q^{-1})\hat{y}(k+d) = B(q^{-1})S(q^{-1})u(k) + R(q^{-1})y(k) \quad (2.26)$$

If (2.18) is satisfied, (2.26) can be written as

$$C_2(q^{-1})\hat{y}(k+d) = p^T\,\emptyset(k) \quad (2.27)$$

where p and $\emptyset(k)$ are given by (2.22).

Now, a control can be computed such that

$$\hat{y}(k+d) = y^M(k+d)$$

which gives as again the control expressed in (2.19).

II.2 STOCHASTIC ENVIRONMENT

In this section, the behaviour of the control structure shown in Figure 2.1 in stochastic environment is analyzed.

The plant in stochastic environment is defined as

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + C(q^{-1})w(k) \qquad (2.28)$$

where

$$C(q^{-1}) = 1 + c_1q^{-1} + \dots + c_{n_c}q^{-n_c} \qquad (2.29)$$

is an asymptotically stable polynomial, $w(k)$ is a sequence of independently and identically distributed normal zero-mean random variables and $A(q^{-1})$ and $B(q^{-1})$ are given by (2.2) and (2.3), respectively.

If the control law given in (2.16a) is used one has

$$C_2(q^{-1})y^M(k+d) = B(q^{-1})S(q^{-1})u(k) + R(q^{-1})y(k) \qquad (2.30)$$

Using (2.28), (2.12)

$$\begin{aligned} C_2(q^{-1})y^M(k+d) &= A(q^{-1})S(q^{-1})y(k+d) + q^{-d}R(q^{-1})y(k+d) \\ &\quad - C(q^{-1})S(q^{-1})w(k+d) \\ &= C_2(q^{-1})y(k+d) - C(q^{-1})S(q^{-1})w(k+d) \qquad (2.31) \end{aligned}$$

So, in tracking the plant-model error becomes

$$C_2(q^{-1})\,\varepsilon\,(k+d) = C(q^{-1})S(q^{-1})w(k+d) \qquad (2.32)$$

In other words, for the case of stochastic environment, the control given with (2.16a) achieves the control objectives defined by the equation

$$y(k) = y^{M}(k) + \frac{C(q^{-1})S(q^{-1})}{C_2(q^{-1})} w(k) \tag{2.33}$$

(2.33) becomes in regulation

$$y(k) = \frac{C(q^{-1})S(q^{-1})}{C_2(q^{-1})} w(k) \tag{2.34}$$

Furthermore if one chooses $C_2(q^{-1}) = C(q^{-1})$, then the plant output becomes

$$y(k) = S(q^{-1})w(k) \tag{2.35}$$

which corresponds to the minimum variance regulator [10] .

III. MODEL REFERENCE ADAPTIVE CONTROL : DETERMINISTIC ENVIRONMENT

In this chapter the design of a MRAC for a discrete -time SISO minimum-phase plant with unknown parameters is given.

III.1 A STABILITY THEOREM USEDFOR THE DESIGN OF THE MRAC

In adaptive case, the aim is to augment the linear control strategie given in the previous chapter with a parameter adaptation algorithm so that the objectives defined are achieved asymptotically. It is also desired, that the input and output of the plant remain bounded, so the design of the MRAC must be done from a stability point of view.

Below, a stability theorem [6] , [8] , [11] is given, which will be used in the next section to justify the MRAC design.

Theorem 3.1. Assume that the following adaptation algorithm for updating the parameter vector $\hat{p}(k)$ is used

$$\hat{p}(k) = \hat{p}(k-1) + F_k \emptyset(k-d) v_k \qquad (3.1)$$

where

$$F_{k+1}^{-1} = \lambda_1(k) F_k^{-1} + \lambda_2(k) \emptyset(k-d) \emptyset^T(k-d); \quad F_o > 0 \qquad (3.2)$$

with

$$0 < \lambda_1(k) \leq 1 \qquad 0 \leq \lambda_2(k) < 2 \qquad \forall k \qquad (3.3)$$

Assume that the relation between $\emptyset(k-d)$ and v_k is given by

$$v_k = H(q^{-1}) \left[p - \hat{p}(k)\right]^T \emptyset(k-d) \qquad (3.4)$$

where $\emptyset(k-d)$ is a bounded or unbounded vector sequence, $H(z^{-1})$ is a rational discrete transfer function normalized under the form

$$H(z^{-1}) = \frac{1 + h_1 z^{-1} + \ldots + h_\alpha z^{-\alpha}}{1 + h_1' z^{-1} + \ldots + h_\beta' z^{-\beta}} \qquad (3.5)$$

and p is a constant parameter vector. Then if the transfer function

$$H'(z^{-1}) = H(z^{-1}) - \frac{\lambda}{2} \qquad (3.6)$$

is strictly positive real where

$$2 > \lambda \geqslant \max_{0 < k < \infty} \lambda_2(k) \tag{3.7}$$

one has for v_o and $\hat{p}(0)$ bounded

$$\text{i)} \quad \lim_{k \to \infty} v_k = 0 \tag{3.8}$$

$$\text{ii)} \quad \lim_{k \to \infty} x_k = 0 \tag{3.9}$$

(where x_k is the state vector of any of the state realizations of $H(z^{-1})$)

$$\text{iii)} \quad \lim_{k \to \infty} \emptyset^T(k-d)\{\left[\hat{p}(k+1) - p\right] + F_k \emptyset(k-d)\, v_k\} = 0 \tag{3.10}$$

$$\text{iv)} \quad \lim_{k \to \infty} \left[1-\lambda_1(k)\right] \left\| \left[\hat{p}(k-1)-p\right] + F_k \emptyset(k-d) v_k \right\|^2_{F_k^{-1}} = 0 \tag{3.11}$$

$$\text{v)} \quad \lim_{k \to \infty} \| \emptyset(k-d) v_k \|^2_{F_k} = \lim_{k \to \infty} \Delta \hat{p}(k)^T F_k^{-1} \Delta \hat{p}(k) = 0 \tag{3.12}$$

(where $\Delta\hat{p}(k) = \hat{p}(k) - \hat{p}(k-1)$)

$$\text{vi)} \quad \left[\hat{p}(k-1)-p\right]^T F_k^{-1} \left[\hat{p}(k-1)-p\right] < M_1 < \infty \quad k > 0 \tag{3.13}$$

$$\text{vii)} \quad \lim_{k \to \infty} \left[\hat{p}(k-1)-p\right]^T F_k^{-1} \left[\hat{p}(k-1)-p\right] = \text{const.} \tag{3.14}$$

If in addition $F_k^{-1} > \varepsilon F_o^{-1}$; $F_o > 0$; $\varepsilon > 0, k > 0$ and F_k^{-1} is nondecreasing for $k \geq k_o$ (3.15)

$$\text{viii)} \lim_{k \to \infty} \Delta\hat{p}(k) = \lim_{k \to \infty} F_k \emptyset(k-d) v_k = 0 \tag{3.16}$$

$$\text{ix)} \quad \| \hat{p}(k) \| \leq M_2 < \infty \qquad k \geq 0 \tag{3.17}$$

The proof of this theorem can be found in [11]. It is done by making use of the equivalent feedback representation (EFR); i.e. the MRAC to be designed is represented by an equivalent feedback system defined by (3.1) and (3.4) with a linear time invariant block (defined by $H(z^{-1})$) and a time varying non-linear block. Then an appropriate adaptation mechanism is chosen such that the global asymptotic stability of the equivalent feedback system is assured.

The first result, (3.8) is used for the design. The other results, (3.9)-(3.17) are used to prove the boundedness of the input and output of the plant.

III.2 A MRAC DESIGN

Having introduced the stability theorem in the previous section, we are now ready to discuss a MRAC design for minimum-phase plants where the time delay and the upperbounds of the degrees of polynomials $A(q^{-1})$ and $B(q^{-1})$ are known.

To deal with unknown plant parameters, a natural way is to replace the parameter vector by an estimated one and use an adaptation algorithm to update it. Therefore the control in adaptive case will be computed by

$$u(k) = \frac{C_2(q^{-1})y^M(k+d) - \hat{p}_o^T(k)\emptyset_o(k)}{\hat{b}_o(k)} \tag{3.18}$$

or equivalently,

$$C_2(q^{-1})y^M(k+d) = \hat{p}^T(k)\emptyset(k) \tag{3.19}$$

where

$$\hat{p}^T(k) = \left[\hat{b}_o(k),\ \hat{p}_o^T(k)\right] \tag{3.20}$$

with $\hat{b}_o(k)$ and $\hat{p}_o^T(k)$ being the estimated values of b_o and p_o.

The design objective is to achieve the objective defined for the case of known parameters (expressed in (2.11)) asymptotically; i.e.

$$\lim_{k \to \infty} C_2(q^{-1})\, \varepsilon(k) = 0 \qquad \varepsilon(0) \neq 0 \tag{3.21}$$

or

$$\lim_{k \to \infty} \varepsilon_c(k) = 0 \tag{3.22}$$

where $\varepsilon_c(k)$ is the filtered plant-model error defined as

$$\varepsilon_c(k) = C_2(q^{-1})\ \varepsilon(k) \tag{3.23}$$

and the plant input and output remain bounded, which can be expressed as

$$\|\emptyset(k)\| \leq M < \infty \qquad \forall k \tag{3.24}$$

Since, the use of (2.12),(2.1),(2.20) and (2.22) yields

$$\begin{aligned} C_2(q^{-1})y(k) &= \left[A(q^{-1})S(q^{-1}) + q^{-d}R(q^{-1})\right]y(k) \\ &= B(q^{-1})S(q^{-1})u(k-d) + R(q^{-1})y(k-d) \\ &= p\ \emptyset(k-d) \end{aligned} \tag{3.25}$$

(3.23) can be written as (using (2.10) and (3.19))

$$\begin{aligned} \varepsilon_c(k) &= C_2(q^{-1})\left[y(k) - y^M(k)\right] \\ &= \left[p - \hat{p}(k-d)\right]^T \emptyset(k-d) \end{aligned} \tag{3.26}$$

A comparison of (3.26) with (3.4) shows that Theorem 3.1 is not directly applicable to solve the problem. To apply the theorem, one can define an a posteriori filtered plant-model error (called also augmented error

$$\varepsilon^*(k) = \left[p-\hat{p}(k)\right]^T \emptyset(k-d) \tag{3.27}$$

Note that $\varepsilon^*(k)$ can also be expressed as

$$\varepsilon^*(k) = \varepsilon_c(k) + \bar{\varepsilon}(k) \tag{3.28}$$

where $\bar{\varepsilon}(k)$ is the auxiliary error and given by

$$\bar{\varepsilon}(k) = \left[\hat{p}(k-d) - \hat{p}(k)\right]^T \emptyset(k-d) \tag{3.29}$$

Now, defining the a posteriori adaptation error as

$$v(k) = \frac{H_1(q^{-1})}{H_2(q^{-1})} \varepsilon^*(k) \tag{3.30}$$

where $H_1(q^{-1})$ and $H_2(q^{-1})$ are monic polynomials of q^{-1}, one can straightforwardly apply Theorem 3.1 to obtain the following result :

If one uses the adaptation algorithm given by

$$\hat{p}(k) = \hat{p}(k-1) + F_k \emptyset(k-d) v(k) \tag{3.31}$$

with

$$F_{k+1} = \frac{1}{\lambda_1(k)} \left[F_k - \frac{F_k \emptyset(k-d) \emptyset^T(k-d) F_k}{\lambda_1(k)/\lambda_2(k) + \emptyset^T(k-d) F_k \emptyset(k-d)} \right] \tag{3.32}$$

where

$$0 < \lambda_1(k) \leqslant 1; \quad 0 \leqslant \lambda_2(k) < 2 \qquad F_o > 0 \tag{3.33}$$

one obtaines

$$\lim_{k \to \infty} v(k) = 0 \tag{3.34}$$

if

$$\frac{H_1(z^{-1})}{H_2(z^{-1})} - \frac{\lambda}{2} \tag{3.35}$$

is strictly positive real with λ given in (3.7),

Note that (3.32) in obtained from (3.2) applying the matrix inversion lemma [12] .

With further analysis, it can be shown that the objective expressed in (3.22) is achieved too. This proof, together with that of the boundedness of the plant input and output is given in Appendix A.

To make the algorithm implementable, an expression for $v(k)$ depending on parameters estimated up to the (k-1)st step should be found. This can be obtained using (3.30) (3.27),(3.31) and (3.25)

$$v(k) = \frac{H_1(q^{-1})}{H_2(q^{-1})} \left[p-\hat{p}(k) \right]^T \emptyset(k-d)$$

$$= \frac{H_1(q^{-1})}{H_2(q^{-1})} \left[p^T\emptyset(k-d) - \hat{p}^T(k-1)\emptyset(k-d) \right.$$

$$\left. - \emptyset^T(k-d)F_k\emptyset(k-d)v(k) \right.$$

$$=\{H_1(q^{-1})\left[C_2(q^{-1})y(k) - \hat{p}^T(k-1)\emptyset(k-d)\right] + \left[1-H_2(q^{-1})\right]v(k)$$
$$+\left[1-H_1(q^{-1})\right]\emptyset^T(k-d)F_k\emptyset(k-d)v(k)\}$$
$$x\frac{1}{1+\emptyset^T(k-d)F_k\emptyset(k-d)} \qquad (3.36)$$

The block diagram of this adaptive control structure is given in Figure 3.1.

III.3 SOME PARTICULAR CASES

As shown in the previous section, once the theorem 3.1 is introduced, the design of a MRAC becomes rather straightforward. Another benefit of this theorem is that it constitutes a unifying framework among the discrete-time adaptive control mechanisms so that the resulting design comprises many different types of adaptation algorithms.

i) Particular Adaptation Gains:

(3.32) gives us different possibilities for the adaptation gain updating

a) If $\lambda_1(k) = 1$, $\lambda_2(k) = 0$, one obtains the simplest case, namely the constant adaptation gain ($F_{k+1}=F_k$), used for example in [13].

b) If $\lambda_1(k) = 1$, $\lambda_2(k) = \lambda_2$ $0 < \lambda_2 < 2$, the time decreasing adaptation gain is obtained [14].

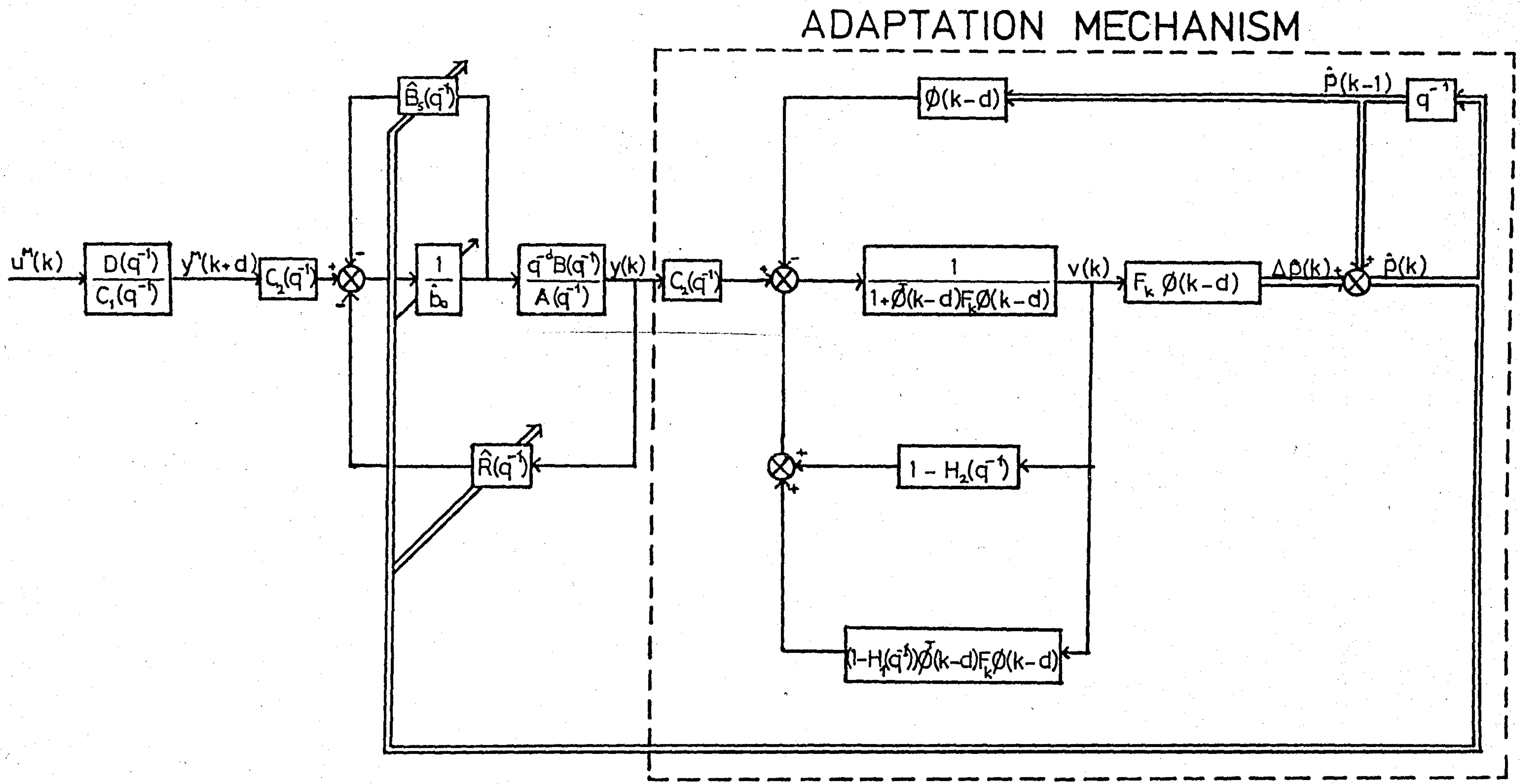


Figure 3.1 Model Reference Adaptive Control System (deterministic environment)

c) $\lambda_1(k) = \lambda_1$; $\lambda_2(k) = \lambda_2$ (usually $0.95<\lambda_1<0.99$) corresponds to time varying adaptation gain (also called adaptation gain with forgetting factor), which is useful for the case of slowly time-varying plants [15].

d) If $\lambda_1(k)$ and $\lambda_2(k)$ are chosen such that trace F_k is constant, a real time adaptation algorithm for tracking time varying plants (called constant trace adaptation gain algorithm) is obtained [6], [8].

Simulations comparing all of these algorithms can be found in Chapter V.

ii) If $H_1(q^{-1}) = 1$; $H_2(q^{-1}) = C_2(q^{-1})$ and $C_2(q^{-1})$ is such that

$$\frac{1}{C_2(z^{-1})} - \frac{\lambda}{2} \tag{3.37}$$

is strictly positive real, with λ as in (3.7), the a posteriori adaptation error can be calculated as

$$v(k) = \frac{C_2(q^{-1})y(k) - \hat{p}^T(k-1)\emptyset(k-d) + \left[1-C_2(q^{-1})\right]v(k)}{1 + \emptyset^T(k-d)F_k\emptyset(k-d)} \tag{3.38}$$

Using (3.25), (3.30), (3.28), (3.29), (3.26) one obtains

$$v(k) = \frac{\varepsilon(k) + \left[1-C_2(q^{-1})\right]\bar{\varepsilon}(k)}{1+\emptyset^T(k-d)F_k\emptyset(k-d)} \tag{3.39}$$

Furthermore, if the poles in tracking and regulation are same, i.e. $C_1(q^{-1}) = C_2(q^{-1})$, the algorithm is called parallel MRAC algorithm.

Note that (3.37) drastically limits the region allowed for the roots of $C_2(z^{-1})$, if stability is wanted to be assured.

iii) If $H_1(q^{-1}) = H_2(q^{-1}) = 1$, the a posteriori adaptation error can be given as

$$v(k) = \frac{C_2(q^{-1})y(k) - \hat{p}^T(k-1)\emptyset(k-d)}{1 + \emptyset^T(k-d)F_k\emptyset(k-d)}$$

$$= \frac{C_2(q^{-1})\,\varepsilon(k)}{1 + \emptyset^T(k-d)F_k\emptyset(k-d)} \tag{3.40}$$

with no positive realness conditions to be fulfilled. Comparing (3.40) with (3.39), note that filtering the plant-model error has the effect of removing the positive realness condition.

Furthermore, if one chooses $C_2(q^{-1}) = C_1(q^{-1})$, (3.40) can be written as [8] :

$$v(k) = \frac{y(k) - y_s^M(k) + \left[\hat{p}(k-d) - \hat{p}(k-1)\right]^T\emptyset(k-d)}{1 + \emptyset^T(k-d)F_k\emptyset(k-d)} \tag{3.41}$$

where $y_s^M(k)$ is the output of a series-parallel reference model given as

$$y_s^M(k) = \left[1-C_1(q^{-1})\right] y(k) + q^{-d} D(q^{-1}) u^M(k) \tag{3.42}$$

This algorithm is called series-parallel MRAC algorithm.

III.4 GENERALIZED MRAC DESIGN

As shown in the previous section the positive realness conditions can be removed by using an appropriate filter for the plant-model error. An alternative way could be to introduce a filter which acts on the plant input and output. Such a filter will allow us to obtain more flexibility in the design, since the degree of freedom in choosing the appropriate polynomials is increased. On the other hand, as shown in Chapter IV, it may also be useful in fulfilling the convergence conditions in stochastic environment. In this section, this generalized design of MRAC is covered although it is not considered in simulation studies.

Let us define the following filtered variables

$$\left.\begin{aligned} L(q^{-1})y^f(k) &= y(k) \\ L(q^{-1})u^f(k) &= u(k) \\ L(q^{-1})y^{Mf}(k) &= y^M(k) \\ L(q^{-1})\phi_o^f(k) &= \phi_o(k) \end{aligned}\right\} \tag{3.43}$$

where

$$L(q^{-1}) = 1 + \ell_1 q^{-1} + \ldots + \ell_{n_\ell} q^{-n} \tag{3.44}$$

is an asymptotically stable polynomial.

Using filtered variables, (2.15) can be written as

$$C_2(q^{-1})\varepsilon(k+d) = L(q^{-1})\Big[E(q^{-1})S(q^{-1})u^f(k)+R(q^{-1})y^f(k) - C_2(q^{-1})y^{Mf}(k+d)\Big] \tag{3.45}$$

Hence if the parameters are known, to achieve the objective defined in (2.11), the control input

$$u(k) = L(q^{-1})\ u^f(k) \tag{3.46}$$

with

$$u^f(k) = \frac{C_2(q^{-1})y^{Mf}(k+d) - p_o^T \phi_o^f(k)}{b_o} \tag{3.47}$$

has to be used.

In case of unknown parameters, again, the parameters must be replaced by their estimated values, i.e.

$$u^f(k) = \frac{C_2(q^{-1})y^{Mf}(k+d) - \hat{p}_o^T(k)\phi_o^f(k)}{\hat{b}_o(k)} \tag{3.48}$$

or

$$C_2(q^{-1})y^{Mf}(k+d) = \hat{p}^T(k)\phi^f(k) \tag{3.49}$$

with

$$\phi^{fT}(k) = \left[u^f(k), \phi_o^{fT}(k)\right] \tag{3.50}$$

and $\hat{p}^T(k)$ as in (3.20).

Using (3.49) and (3.43), the a posteriori adaptation error defined in (3.30) can now be given as

$$v(k) = \frac{H_1(q^{-1})L(q^{-1})}{H_2(q^{-1})}\left[p-\hat{p}(k)\right]^T\phi^f(k-d) \tag{3.51}$$

A straightforward application of Theorem3.1 gives that if the adaptation algorithm

$$\hat{p}(k) = \hat{p}(k-1) + F_k\phi^f(k-d)v(k) \tag{3.52}$$

$$F_{k+1} = \frac{1}{\lambda_1(k)}\left[F_k - \frac{F_k\phi^f(k-d)\,\phi^{fT}(k-d)F_k}{\lambda_1(k)/\lambda_2(k) + \phi^{fT}(k-d)F_k\phi^f(k-d)}\right] \tag{3.53}$$

with (3.33) is used, one has

$$\lim_{k\to\infty} v(k) = 0 \tag{3.54}$$

if

$$\frac{H_1(z^{-1})L(z^{-1})}{H_2(z^{-1})} - \frac{\lambda}{2} \tag{3.55}$$

is strictly positive real, where λ is given in (3.7).

It is shown in [8], that with this adaptation algorithm the convergence of the plant-model error and the boundedness of plant input and output is assured.

In general the a posteriori adaptation error can be implemented as [8],

$$\begin{aligned} v(k) = \{ & H_1(q^{-1})L(q^{-1})\left[C_2(q^{-1})y^f(k) - \hat{p}^T(k-1)\phi^f(k-d)\right] \\ & + \left[1-H_2(q^{-1})\right]v(k) + \left[1-H_1(q^{-1})L(q^{-1})\right] \phi^{fT}(k-d)F_k\phi^f(k-d)v(k)\} \\ & x \frac{1}{1+\phi^{fT}(k-d)F_k\phi^f(k-d)} \end{aligned} \tag{3.56}$$

Note that (3.56) reduces to (3.36) for $L(q^{-1}) = 1$.

IV. MODEL REFERENCE ADAPTIVE CONTROL : STOCHASTIC ENVIRONMENT

In this chapter, the adaptive control problem for a discrete time SISO minimum-phase plant in stochastic environment is discussed.

After introducing a convergence theorem which will enable us to do the convergence analyses in stochastic environment, two examples of S-STR are analyzed in order to establish connections, similarities with corresponding MRAC schemes. Then the behaviour of MRAC in a stochastic environment is discussed. Finally, an adaptive control scheme combining model reference adaptive controllers and stochastic self-tuning regulators is considered, which is able to accomplish both tracking and regulation objectives in deterministic and stochastic environment.

IV.1. A THEOREM FOR CONVERGENCE ANALYSIS IN STOCHASTIC ENVIRONMENT

In the convergence analyses in stochastic environment we will use the following theorem [5] :

Theorem 4.1. Consider the adaptation algorithm (3.31)-(3.32) where

$$\lambda_1(k) = 1, \quad \lambda_2(k) = \lambda; \quad 0 \leqslant \lambda < 2 \qquad \forall k \tag{4.1}$$

Assume that the stationary processes $\bar{\phi}(k,\hat{p})$ and $\bar{v}(k,\hat{p})$ can be defined for $\hat{p}(k) = \hat{p}$ and $\hat{p}(k)$ belongs infinitely often to the domain for which these stationary processes can be defined. Assume that for $\hat{p}(k) = \hat{p}$

$$\bar{v}(k,\hat{p}) = H^*(q^{-1})\bar{\phi}^T(k-d\ ,\hat{p})\left[p^*-\hat{p}\right] + w^*(k) \tag{4.2}$$

where $w^*(k)$ is the image of the stochastic disturbance in the equation governing the a posteriori adaptation error and is a white random sequence (or incorrelated to $\bar{\phi}(k-d),\hat{p})$).

Then, if

$$H^*(z^{-1}) - \frac{\lambda}{2} \tag{4.3}$$

is strictly positive real, then

$$\text{Prob}\left\{\lim_{k \to \infty} \hat{p}(k) \in D\right\} = 1 \tag{4.4}$$

where

$$D : \{p | [p^* - p]^T \emptyset(k) = 0\} \tag{4.5}$$

and

$$\text{Prob}\{\lim_{k \to \infty} v(k) = w^*(k)\} = 1 \tag{4.6}$$

If in addition, the input is sufficiently rich and the controller is of adequate order, it follows

$$\text{Prob}\{\lim_{k \to \infty} \hat{p}(k) = p^*\} = 1 \tag{4.7}$$

The proof of this theorem canbe found in [16] or [17] . It is made by making use of the ordinary differential equation (ODE) method [19].

IV.2 STOCHASTIC SELF-TUNING REGULATORS:TWO EXAMPLES

In stochastic environment, for which the plant is described by (2.28), when the plant parameters together with the disturbance dynamics are unknown, it is natural to replace the parameters in the control law with their estimated values to achieve the control objectives asymptotically.

Considering (2.33), the objective in the case of known parameters, a rather general objective for this case can be stated as

$$\text{Prob}\ \{\lim_{k\to\infty} \frac{C_2(q^{-1})}{C(q^{-1})S(q^{-1})}\varepsilon(k) = w(k)\} = 1 \tag{4.8}$$

where $S(q^{-1})$ is uniqely defined with (2.12) and (2.18).

To show the similarities and connections of the stochastic self-tuning regulators with model reference adaptive controllers we will give to examples.

i) Example 1 :

The control objective is chosen as

$$\text{Prob}\ \{\lim_{k\to\infty} \varepsilon(k) = S(q^{-1})w(k)\} = 1 \tag{4.9}$$

which corresponds in (4.8) to the particular case

$$C_2(q^{-1}) = C(q^{-1}) \tag{4.10}$$

Together with the control law expressed in (3.18), the adaptation algorithm given in (3.31)-(3.32) with (4.1) is used. Below we will show that if the a posteriori adaptation error is calculated as

$$v(k) = \frac{\varepsilon(k)}{1 + \emptyset^T(k-d)F_k\emptyset(k-d)} \tag{4.11}$$

and if

$$\frac{1}{C(z^{-1})} - \frac{\lambda}{2} \tag{4.12}$$

is strictly positive real the objective defined in (4.9) will be achieved.

To show this we rewrite (4.11) as

$$v(k) + \emptyset^T(k-d)F_k\emptyset(k-d)v(k) = \varepsilon(k) \tag{4.13}$$

and use (3.31) to obtain

$$v(k) + \left[\hat{p}(k) - \hat{p}(k-1)\right]^T\emptyset(k-d) = \varepsilon(k) \tag{4.14}$$

Since

$$\varepsilon(k) = \frac{\varepsilon_c(k)}{C_2(q^{-1})} = \frac{1}{C_2(q^{-1})}\{\left[p-\hat{p}(k-d)\right]^T\emptyset(k-d) + S(q^{-1})C(q^{-1})w(k)\} \tag{4.15}$$

(4.14) becomes

$$v(k)+\left[\hat{p}(k) + \hat{p}(k-1)\right]^T\emptyset(k-d) = \frac{1}{C_2(q^{-1})}\left[p-\hat{p}(k-d)\right]^T\emptyset(k-d) + S(q^{-1})C(q^{-1})w(k) \tag{4.16}$$

Now, introducing the stationary sequences $\bar{\emptyset}(k,\hat{p})$ $\bar{v}(k,\hat{p})$

and $\bar{\varepsilon}(k,\hat{p})$ for $\hat{p}(k) = \hat{p}$ and considering (4.10)

$$\bar{v}(k,\hat{p}) = \frac{1}{C(q^{-1})} \bar{\emptyset}^T(k-d,\hat{p}) \left[p-\hat{p}\right] + S(q^{-1})w(k) \tag{4.17}$$

On the other hand, from (4.14), it follows that

$$\bar{v}(k,\hat{p}) = \bar{\varepsilon}(k,\hat{p}) \tag{4.18}$$

Now, Theorem 4.1 can be applied to (4.17) with (4.18) to obtain the control objective defined in (4.9) if the positive realness condition in (4.12) is satisfied.

ii) Example 2 :

For this case, the following objective is wanted to be achieved.

$$\text{Prob}\{ C_2(q^{-1})\,\varepsilon(k) = S(q^{-1})w(k) \} = 1;\; C(q^{-1}) = 1 \tag{4.19}$$

Consider the following a posteriori adaptation error with the control law (3.18) and the adaptation algorithm (3.31),(3.32),(4.1)

$$v(k) = \frac{C_2(q^{-1})\,\varepsilon(k)}{1 + \emptyset^T(k-d)F_k\emptyset(k-d)} \tag{4.20}$$

or

$$v(k) + \emptyset^{T}(k-d)F_k\emptyset(k-d)v(k) = C_2(q^{-1})\varepsilon(k) \quad (4.21)$$

$$v(k) + \left[\hat{p}(k) - \hat{p}(k-1)\right]^T\emptyset(k-d) = \left[p-\hat{p}(k-d)\right]^T\emptyset(k-d)$$

$$+ S(q^{-1})C(q^{-1})w(k) \quad (4.22)$$

Using stationary sequences $\bar{\emptyset}(k,\hat{p}), \bar{v}(k,\hat{p})$, for $\hat{p}(k) = \hat{p}$ and the fact that $C(q^{-1}) = 1$,

$$\bar{v}(k,\hat{p}) = \bar{\emptyset}^{T}(k-d,\hat{p})\left[p-\hat{p}\right] + S(q^{-1})w(k) \quad (4.23)$$

It follows from (4.21),

$$\bar{v}(k,\hat{p}) = C_2(q^{-1})\bar{\varepsilon}(k,\hat{p}) \quad (4.24)$$

So, (4.23) can be rewritten as

$$C_2(q^{-1})\bar{\varepsilon}(k,\hat{p}) = \emptyset^{T}(k,\hat{p})\left[p-\hat{p}\right] + S(q^{-1})w(k) \quad (4.25)$$

Now applying Theorem 4.1 to (4.25) directly gives the objective expressed in (4.19) without any positive realness condition to be satisfied.

IV.3 SIMILARITIES AND DUALITY BETWEEN MRAC AND S-STR

The two examples introduced in the previous section make us possible to analyze the structural similarities between MRAC designed for deterministic environment and S-STR designed for stochastic environment.

The first similarity between them arises in their control laws. As it was the case for a plant with known parameters, in the case of unknown parameters the same control law provides a control input to achieve the objectives in stochastic environment as well as in deterministic environment. Not only in deterministic but also in stochastic case the same control law can be obtained using explicit or implicit reference models. But in literature, usually, the model reference adaptive controllers are considered with explicit reference models whereas the stochastic self-tuning regulators with implicit reference models.

On the other hand, similar adaptation algorithms can be used for both the MRAC and S-STR (Note that, only a certain class among all possible adaptation algorithms suitable for MRAC is allowed to be used for S-STR if convergence is wanted to be assured; namely the class defined by (4.1)). For example, the adaptation algorithm of a MRAC given in III.3. iii ($H_1(q^{-1}) = H_2(q^{-1}) = 1$) considered with (4.1) is identical with the algorithm used in the S-STR introduced as the second example in IV.3.ii. On the other side, the adaptation algorithm which corresponds to the MRAC given in III.3.ii differs from that of the S-STR in the first example (See. IV.3.i) only in the a posteriori adaptation error during transients (compare (3.39) and (4.11) and consider that $\bar{\varepsilon}(k) \rightarrow 0$ as $k \rightarrow \infty$). Morever, if S-STR in Example 1 is operated in deterministic environment $(w(k)=0 \; \forall k, \; c_i=0 \; i=1,\ldots,c_{n_c})$ the MRAC corresponding to the a posteriori adaptation error given by (3.39) for $C_2(q^{-1}) = 1$ is obtained.

Another similarity between the model reference adaptive controllers and stochastic self-tuning regulators shows itself up in the positive realness conditions to ensure stability in deterministic environment and convergence in stochastic environment.

However in spite of these similarities MRAC and S-STR accomplish different tasks in different environments, so the introduction of the following definition is justified [2

Definition 4.1. (Asymptotic) Duality between MRAC and S-STR. A MRAC (implicit or explicit) designed for a deterministic environment is (asymptotically) dual with respect to a S-STR (implicit or explicit) designed for a stochastic environment if and only if

i) The adjustable parameter vectors are updated by (asymptotically) identical adaptation algorithms [same structure, same observation vector ($\emptyset$), same a posteriori adaptation error (as $k \rightarrow \infty$)].

ii) The positive realness conditions for global asymptotic stability of the MRAC and for w.p.1 convergence of the S-STR are the same.

iii) The control laws are (asymptotically) the same. (If both control laws and a posteriori adaptation errors are identical for any k, they will be called dual).

After having introduced this definition, two questions come into the picture :

i) What is the behaviour of MRAC designed for a deterministic environment in a stochastic environment?

ii) How can an adaptive control structure be built which behaves as a desired MRAC in a deterministic environment and as a desired S-STR in a stochastic environment?

This questions are tried to be answered in the next two sections of this chapter.

IV.4 BEHAVIOUR OF MRAC IN STOCHASTIC ENVIRONMENT

In this section we will analyze the behaviour of the MRAC, designed in Chapter III for a deterministic environment, in a stochastic environment.

Since the plant is described in stochastic environment with (2.28), the filtered plant-model error becomes

$$
\begin{aligned}
\varepsilon_c(k) &= C_2(q^{-1})\left[y(k) - y^M(k)\right] \\
&= A(q^{-1})S(q^{-1})y(k) + R(q^{-1})y(k-d) - C_2(q^{-1})y^M(k) \\
&= B(q^{-1})S(q^{-1})u(k-d) + R(q^{-1})y(k-d) - C_2(q^{-1})y^M(k) \\
&\qquad + S(q^{-1})C(q^{-1})w(k) \\
&= \left[p-\hat{p}(k-d)\right]^T\emptyset(k-d) + S(q^{-1})C(q^{-1})w(k) \qquad (4.26)
\end{aligned}
$$

Now, for the case of regulation ($y^M(k) \equiv 0$; hence, $\varepsilon(k) = y(k)$), one can write (4.26) as

$$
\begin{aligned}
C_2(q^{-1})\,\varepsilon(k) - \left[C_2(q^{-1}) - C(q^{-1})\right]\varepsilon(k) &= \left[p-\hat{p}(k-d)\right]^T\emptyset(k-d) \\
&\quad - \left[C_2(q^{-1}) - C(q^{-1})\right] y(k) \\
&\quad + S(q^{-1})C(q^{-1})w(k) \qquad (4.27)
\end{aligned}
$$

or

$$C(q^{-1})\,\varepsilon(k) = \left[p'-\hat{p}(k-d)\right]^T \emptyset(k-d) + S'(q^{-1})C(q^{-1})w(k) \qquad (4.28)$$

where $S'(q^{-1})$ is given by the polynomial identity

$$C(q^{-1}) = A(q^{-1})S'(q^{-1}) + q^{-d}R'(q^{-1}) \qquad (4.29)$$

and

$$p' = \left[b_o, b_o s_1' + b_1 \ldots, b_{nb} s_{d-1}', r_o', \ldots, r_{n_r}'\right] \qquad (4.30)$$

with s_i', $i = 1,\ldots,d-1$ and r_j', $j = 1,\ldots,n_r$ being the coefficients of $S'(q^{-1})$, $R'(q^{-1})$ respectively (i.e. p' contains the parameters corresponding to the case when the closed loop poles are determined by the disturbance dynamics).

(4.28) gives the following relation between the corresponding stationary processes for $\hat{p}(k) = \hat{p}$

$$\bar{\varepsilon}(k,\hat{p}) = \frac{1}{C(q^{-1})}\bar{\emptyset}^T(k,\hat{p})\left[p'-\hat{p}\right] + S'(q^{-1})w(k) \qquad (4.31)$$

Here follows

$$\bar{\varepsilon}_c(k,\hat{p}) = \frac{C_2(q^{-1})}{C(q^{-1})}\bar{\emptyset}^T(k,\hat{p})\left[p'-\hat{p}\right] + C_2(q^{-1})S'(q^{-1})w(k) \qquad (4.32)$$

So,

$$\bar{v}(k,\hat{p}) = \frac{H_1(q^{-1})}{H_2(q^{-1})}\bar{\varepsilon}_c(k,\hat{p})$$

$$= \frac{H_1(q^{-1})}{H_2(q^{-1})} \frac{C_2(q^{-1})}{C(q^{-1})} \bar{\emptyset}^T(k,\hat{p})\left[p'-\hat{p}\right]$$

$$+ \frac{H_1(q^{-1})}{H_2(q^{-1})} C_2(q^{-1})S'(q^{-1})w(k) \tag{4.33}$$

The application of Theorem 4.1 at this point yields that if

$$\frac{H_1(z^{-1})}{H_2(z^{-1})} \frac{C_2(z^{-1})}{C(z^{-1})} - \frac{\lambda}{2} \tag{4.34}$$

is strictly positive real then

$$\text{Prob}\left\{ \lim_{k \to \infty} v(k) = \frac{H_1(q^{-1})}{H_2(q^{-1})} C_2(q^{-1})S'(q^{-1})w(k) \right\} = 1 \tag{4.35}$$

and

$$\text{Prob}\left\{ \lim_{k \to \infty} \hat{p}(k) \in D_1 \right\} = 1 \tag{4.36}$$

with

$$D_1 = \{ p \,|\, [p'-p]^T \emptyset(k) \} = 0 \tag{4.37}$$

On the other hand, if one considers the filtered variables introduced in the generalized MRAC design in III.4, a similar analysis method as through (4.26)-(4.37) will yield the result that if

$$\frac{H_1(z^{-1})}{H_2(z^{-1})} \frac{C_2(z^{-1})}{C(z^{-1})} L(z^{-1}) - \frac{\lambda}{2} \qquad (4.38)$$

is strictly positive real then (4.35) and (4.36) with (4.37) holds.

So if the disturbance dynamics is known, $L(q^{-1})$ makes us possible to assure stability without loosing the freedom in choosing the control objective (defined by $C_2(q^{-1})$) and the filter used in calculating the a posteriori adaptation error.

Note that, in stochastic environment the controller parameters will depend on the disturbance dynamics, in general. But in the control law expressed by (3.18), the closed loop poles are defined by $C_2(q^{-1})$, so the parameters are incompatible with them.

In regulation, this drift in the controller parameters is not a handicap in eliminating the plant-model error (Note that $y^M(k+d) \equiv 0$ in (3.18)), but the regulation objective is dictated by the disturbance dynamics.

In tracking, if the disturbance dynamics is known the parameters can be matched by choosing the disturbance dynamics as the regulation objective (i.e $C_2(q^{-1})=C(q^{-1})$) For the case of unknown disturbance dynamics the control law must be extended in a way such that the drift in parameters is compensated. This will lead us to an adaptive control scheme which is suitable for tracking and regulation in both deterministic and stochastic environment, in the next section.

IV.5 AN ADAPTIVE CONTROL STRUCTURE COMBINING MODEL REFERENCE ADAPTIVE CONTROLLERS AND STOCHASTIC SELF-TUNING REGULATORS

In the previous section, it is concluded that tracking in a stochastic environment with known disturbance dynamics can be done by choosing the closed loop poles appropriately ($C_2(q^{-1}) = C(q^{-1})$). Therefore, if the disturbance dynamics is unknown, it is reasonable to estimate the control objective besides the parameter vector p. That is, the control law is modified as

$$u(k) = \frac{\hat{C}_2(q^{-1},k)y^M(k+d) - \hat{p}_o^T(k)\phi_o(k)}{\hat{b}_o(k)} \tag{4.39}$$

or equivalently,

$$\hat{C}_2(q^{-1},k)y^M(k+d) = \hat{p}^T(k)\phi(k) \tag{4.40}$$

where

$$\hat{C}_2(q^{-1},k) = 1 + \hat{c}_{21}(k)q^{-1} + \dots \hat{c}_{2n_{\hat{c}}}(k)q^{-n_{\hat{c}}} \tag{4.41}$$

is a polynomial in q^{-1} with estimated coefficients.(4.40) can also be written as

$$C_2(q^{-1})y^M(k+d) - \left[C_2(q^{-1}) - \hat{C}_2(q^{-1},k)\right] y^M(k+d)$$

$$= \hat{p}^T(k)\ \emptyset(k) \tag{4.42}$$

or

$$C_2(q^{-1})y^M(k+d) = \hat{p}_e^T(k)\emptyset_e(k) \tag{4.43}$$

where

$$\left.\begin{aligned} \hat{p}_e^T(k) &= [\hat{p}(k), \hat{c}(k)] \\ \emptyset_e(k) &= [\emptyset(k), y^M(k+d-1), \ldots, y^M(k+d-n_{\hat{c}})] \end{aligned}\right\} \tag{4.44}$$

with $\hat{c}(k)$ containing the coefficients of $\left[C_2(q^{-1})-\hat{C}_2(q^{-1},k)\right]$. So, the control input can be calculated as

$$u(k) = \frac{C_2(q^{-1})y^M(k+d) - \hat{p}_{oe}(k)\emptyset_{oe}(k)}{\hat{b}_o(k)} \tag{4.45}$$

where $\hat{p}_{oe}(k)$ and $\emptyset_{oe}(k)$ are obtained by extending $\hat{p}_o(k)$ and $\emptyset_o(k)$ in the same way as $\hat{p}_e(k)$ and $\emptyset_e(k)$ are obtained from $\hat{p}(k)$ and $\emptyset(k)$.

The adaptation algorithm will be asfollows

$$\hat{p}_e(k) = \hat{p}_e(k-1) + F_k\emptyset_e(k-d)v(k) \tag{4.46}$$

$$\left.\begin{aligned} F_{k+1} &= F_k - \frac{F_k \emptyset_e(k-d)\emptyset_e^T(k-d)F_k}{1/\lambda_2 + \emptyset_e^T(k-d)F_k\emptyset_e(k-d)} \\ 0 &\leqslant \lambda_2 < 2 \qquad\qquad F_o > 0 \end{aligned}\right\} \quad (4.47)$$

$$v(k) = \frac{H_1(q^{-1})}{H_2(q^{-1})}\varepsilon^*(k) = \frac{H_1(q^{-1})}{H_2(q^{-1})}\left[p_e - \hat{p}_e(k)\right]^T \emptyset_e(k-d) \quad (4.48)$$

with

$$p_e = \left[p, 0, \ldots .0\right] \quad (4.49)$$

(Note that this scheme is reduced to MRAC in case of regulation $y^M(k) \equiv 0$.)

The block diagram of this scheme is given in Figure 4.1, where the adaptation algorithm is analogous to that in Figure 3.1. The analysis in deterministic and stochastic environments can be made as follows.

i) Deterministic Environment :

In deterministic environment, the analysis can be done by applying Theorem 3.1 and following the same way as in the MRAC design in III.2. (Only replace $p, \hat{p}(k)$ and $\emptyset(k)$ by $p_e, \hat{p}_e(k)$ and $\emptyset_e(k)$, respectively.) Therefore it is omitted here.

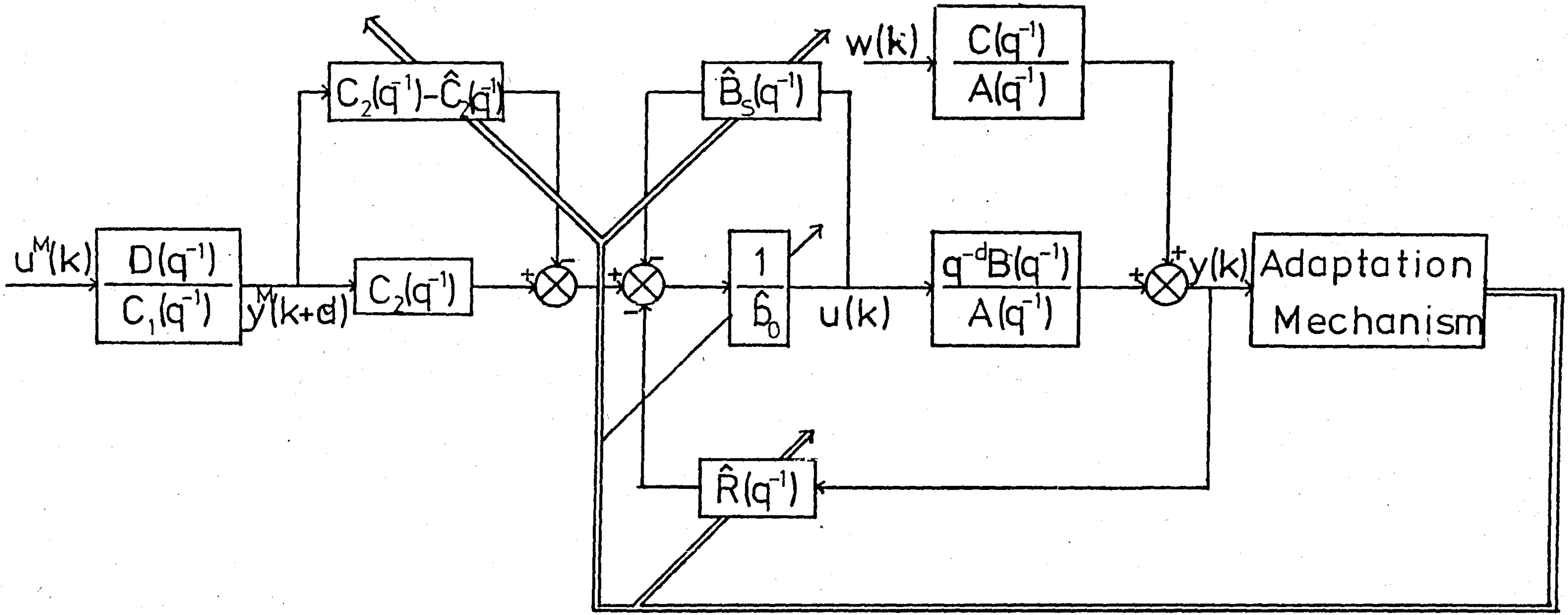


Figure 4.1 Combined MRAC-S-STR Control Structure (stochastic environment)

ii) Stochastic Environment :

The analysis in stochastic environment can be made using Theorem 4.1. The filtered plant-model error, in this case is given as

$$\varepsilon_c(k) = \left[p_e - \hat{p}_e(k-d)\right]^T \phi_e(k-d) + S(q^{-1})C(q^{-1})w(k) \quad (4.50)$$

(obtained in a similar way as done in (4.26))

Hence,

$$C_2(q^{-1})\varepsilon(k) - \left[C_2(q^{-1}) - C(q^{-1})\right]\varepsilon(k)$$

$$= \left[p_e - \hat{p}_e(k-d)\right]^T \phi_e(k-d) - \left[C_2(q^{-1}) - C(q^{-1})\right]\left[y(k) - y^M(k)\right]$$

$$+ S(q^{-1})C(q^{-1})w(k) \quad (4.51)$$

or

$$C(q^{-1})\varepsilon(k) = \left[p_e' - \hat{p}_e(k-d)\right]^T \phi_e(k-d) + S'(q^{-1})C(q^{-1})w(k) \quad (4.52)$$

where

$$p_e' = \left[p', c_{21} - c_1, \ldots, c_{2n_c} - c_{n_c}\right] \quad (4.53)$$

and

$$\hat{p}_e(k) = \left[\hat{p}(k), c_1 - \hat{c}_1(k), \ldots, c_{n_c} - \hat{c}_{n_c}(k)\right] \quad (4.54)$$

In terms of the stationary processes for $\hat{p}(k) = \hat{p}$

$$\bar{\varepsilon}(k,\hat{p}) = \frac{1}{C(q^{-1})}\bar{\phi}_e^T(k-d,\hat{p})\left[p_e'-\hat{p}_e\right] + S'(q^{-1})w(k) \quad (4.55)$$

Here follows

$$\bar{v}(k,\hat{p}) = \frac{H_1(q^{-1})}{H_2(q^{-1})}\bar{\varepsilon}_c(k,\hat{p})$$

$$= \frac{H_1(q^{-1})}{H_2(q^{-1})}\frac{C_2(q^{-1})}{C(q^{-1})}\bar{\phi}_e^T(k,\hat{p})\left[p_e'-\hat{p}_e\right]$$

$$+ \frac{H_1(q^{-1})}{H_2(q^{-1})}C_2(q^{-1})S'(q^{-1})w(k) \quad (4.56)$$

If theorem 4.1 is applied, it is concluded that if

$$\frac{H_1(z^{-1})}{H_2(z^{-1})}\frac{C_2(z^{-1})}{C(z^{-1})} - \frac{\lambda}{2} \quad (4.57)$$

is strictly positive real then

$$\mathrm{Prob}\left\{\lim_{k\to\infty} v(k) = \frac{H_1(q^{-1})}{H_2(q^{-1})}C_2(q^{-1})S'(q^{-1})w(k)\right\} = 1 \quad (4.58)$$

and

$$\text{Prob}\{\lim_{k \to \infty} \hat{p}_e(k) \in D_2\} = 1 \qquad (4.59)$$

where

$$D_2 = \{ p_e \,|\, [p_e' - p_e]^T \, \emptyset_e(k) = 0 \} \qquad (4.60)$$

Furthermore; since

$$\bar{v}(k,\hat{p}) = \frac{H_1(q^{-1})}{H_2(q^{-1})} C_2(q^{-1}) \bar{\varepsilon}(k,\hat{p}) \qquad (4.61)$$

the plant model error will converge such that

$$\varepsilon(k) = S'(q^{-1}) \; w(k) \qquad (4.62)$$

so the plant output becomes asymptotically

$$y(k) = y^M(k) + S'(q^{-1}) w(k) \qquad (4.63)$$

Comparing (4.63) with (2.33), it can be seen that in stochastic environment the control objective accomplished takes asympotically the form dictated by the disturbance dynamics ($C_2(q^{-1}) = C(q^{-1})$). But, in a deterministic environment ($c_i = 0$, $i = 1, \ldots, n_c$), contrary to some S-STR schemes, which behave as having $C_2(q^{-1}) = 1$ (See IV.2.i and IV.3), the combined MRAC-S-STR scheme can accomplish any control objective defined by an asymptotically stable polynomial $C_2(q^{-1})$. This means, it behaves as a desired MRAC in deterministic environment and as a desired S-STR in stochastic environment.

V. SIMULATION STUDIES

Simulations are done an a CDC 170/815 CYBER to examine the performances of particular MRAC and combined MRAC-S-STR schemes in both deterministic and stochastic environments (The computer program used in simulations is given in Appendix B.).

The plant used in simulation studies is given in deterministic environment before parameters change by the discrete transfer function.

$$\frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-1}(1 + 0.4\ z^{-1})}{(1-0.5z^{-1})\left[1-(0.8+0.3\ j)z^{-1}\right]\left[1-(0.8-0.3j)z^{-1}\right]} \quad (5.1)$$

A change of parameters is made at k=t so the plant transfer function becomes

$$\frac{z^{-d}B'(z^{-1})}{A'(z^{-1})} = \frac{z^{-1}(0.9 + 0.5z^{-1})}{(1-0.5z^{-1})\left[1-(0.9+0.5j)z^{-1}\right]\left[1-(0.9-0.5j)z^{-1}\right]} \quad (5.2)$$

(5.1) and (5.2), which differ from the plant transfer functions used in [8] for simulations only in their time delays (d=2 in [8]), correspond in time domain to

$$y(k+1) = u(k) + 0.4u(k-1) + 2.1y(k) - 1.53y(k-1) + 0.365y(k-2) \qquad k < t, \quad (5.3)$$

$$y(k+1) = 0.9\,u(k) + 0.5u(k-1) + 2.3y(k) - 1.96\,y(k-1) + 0.53\,y(k-2) \qquad k \geqslant t \quad (5.4)$$

respectively.

On the other hand, in stochastic environment the plant is characterized by

$$y(k+1)=u(k) + 0.4u(k-1) + 2.1y(k) - 1.53y(k-1) + 0.365y(k-2) + w(k) - 0.25w(k-1) \quad (5.5)$$

before parameters change and by

$$y(k+1) = 0.9u(k) + 0.5u(k-1) + 2.3y(k) - 1.96y(k-1) + 0.53y(k-2) + w(k) - 0.1w(k-1) \quad (5.6)$$

after parameters change, where w(k) is a sequence of independent and identically distrubuted zero-mean normal random variables with variance 0.25.

Three different reference models are used in simulations for tracking :

i) RM1 :

$$\frac{z^{-d}D(z^{-1})}{C_1(z^{-1})} = \frac{z^{-1}(0.28 + 0.22z^{-1})}{(1-0.5z^{-1})\left[1-(0.7+0.2j)z^{-1}\right]\left[1-(0.7-0.2j)z^{-1}\right]} \tag{5.7}$$

or in time domain :

$$y^M(k+1) = 0.28u^M(k) + 0.22\, u^M(k-1) + 1.9y^M(k) - 1.23\, y^M(k-1) + 0.265\, y^M(k-2) \tag{5.8}$$

which, again, differs from the reference model used in [8] only in time delay. The reference input applied in the case of this reference model is given as

$$u^M(k) = \begin{cases} 1 & 0 < k < 50 \quad \text{or} \quad 100 \leqslant k < 150 \\ 0 & \text{elsewhere} \end{cases} \tag{5.9}$$

The output of this reference model is such that the system operates partly in regulation ($y^M(k) = 0$) and partly in tracking.

ii) RM2 :

$$\frac{z^{-d}D(z^{-1})}{C_1(z^{-1})} = \frac{z^{-1}}{1-e^{-0.1}z^{-1}} \tag{5.10}$$

or equivalently,

$$y^M(k+1) = u^M(k) + e^{-0.1}y^M(k) \tag{5.11}$$

The reference input in this case in chosen as a train of impulses

$$u^M(k) = 7 \sum_{n=0}^{3} \delta(k-50n) \tag{5.12}$$

Note that the first differences in the reference model output is relatively high at some instants; a fact which will cause problems in tracking as we will see in simulation results.

iii) RM3 :

$$\frac{z^{-1}D(z^{-1})}{C_1(z^{-1})} = \frac{z^{-1} \sin(\pi/60)}{1-2\cos(\pi/60)z^{-1}+z^{-2}} \tag{5.13}$$

or

$$y^M(k+1) = \sin(\pi/60)u^M(k) + 2\cos(\pi/60)y^M(k) - y^M(k-1) \tag{5.14}$$

The reference input is taken to be a single impulse at k=0,i.e.

$$u^M(k) = \delta(k) \tag{5.15}$$

so that the model let the system operate continously in tracking.

On the other hand, in regulation an initial output level ($y(k) = 2, k \leqslant 2$) is wanted to be reduced to zero with a dynamics specified by $C_2(q^{-1})$.

In simulations, where RM1 or RM3 is used, parameters are changed at k=25. In the case of RM2 and regulation the parameter changes occur at k=15 and k=0 respectively.

In Figures 5.1-5.43 the reference model outputs are shown with thick lines.

Figures 5.1-5.16 exhibit how the plant output is affected by different types of adaptation gains used in a MRAC.

Figures 5.1-5.3 and 5.13. indicate that the magnitude of the adaptation gain, if constant adaptation gain ($\lambda_1(k)=1, \lambda_2(k)=0$) is used, has not any effect on the plant output in tracking or in regulation.

But as seen in Figures 5.4-5.9 and 5.14-5.15, in case of decreasing adaptation gain ($\lambda_1(k)=1$, $\lambda_2(k)=\lambda$) and constant trace adaptation gain ($\lambda_1(k), \lambda_2(k)$ such that $tr[F_{k+1}] = tr[F_k]$) the choice of λ's are more or less important to improve the performance of the system in tracking, whereas this is not the case in regulation. For finding $\lambda_1(k)$ and $\lambda_2(k)$ in case of constant trace adaptation gain the ratio $\lambda_1(k)/\lambda_2(k)$ is fixed to a certain value and a $\lambda_1(k)$ is chosen at each step such that $tr\left[F_{k+1}\right] = tr\left[F_k\right]$. The best performance is obtained in the case of $\lambda_2(k)=0.2$ among all simulations done with the decreasing adaptation gain algorithm; i.e. a relatively slowly decreasing gain is more successful in tracking the model output than a rapidly decreasing gain. On the other hand, the constant trace adaptation gain with $\lambda_1(k)/\lambda_2(k) = 0.25$ shows the best performance, which means one have to weight the measurement vector more than the gain value at time k, in calculating the gain for time k+1.

In Figures 5.10-5.12, and 5.16 the performances of three types of adaptation gains can be compared. Obviously, in regulation (see Figure 5.16) the decreasing and constant trace adaptation gains are equally more successfull than constant adaptation gain. This is true also for tracking (provided) that $\lambda_2(k)$ or $\lambda_1(k)/\lambda_2(k)$ are chosen properly). The success of the constant trace and decreasing adaptation gains is observed especially in Figure 5.11. We see that the plant-model error increases after each impulse input (i.e. when the first differences of the model output are relatively high) in case of constant adaptation gain algorithm But if decreasing or constant trace adaptation gains are used although the transients after the first impulse input after parameters change are large in magnitude than that in case of constant gain, after the later impulse inputs the plant-model error decreases to zero. That exhibits the "learning" character of the decreasing and constant trace adaptation gain algorithms.

In Figure 5.17 a disadvantage of decreasing adaptation gain algorithm is seen. There, after parameters change at k=25, they take their previous values back at k=125. One observes that the convergence time of decreasing adaptation gain is relatively increased. The reason for this is that the gain continous to decrease even when no adaptation is needed any more ($50 < k < 125$) and one way to overcome this effect may be reinitialize the adaptation gain periodically (or before each adaptation process).

Although it is considered in literature usually for slowly time-varying plants, simulations using adaptation gain with forgetting factor are done, too (only for RM1, see Figure 5.18). It is obvious, how important is to make an appropriate choice for λ_1 and λ_2 to obtain a rapid convergence and reasonable transients. The best results are obtained with $\lambda_1=0.95$ and $\lambda_2=0.2$. That is, for a time-invariant plant it is better to let the adaptation gain "forget" the past measurements slowly than rapidly.

The effects of $C_2(q^{-1})$ polynomial and $H_1(q^{-1})/H_2(q^{-1})$ filter can be observed in Figures 5.19-5.28. In these simulations, constant trace adaptation gain with

$\lambda_1(k)/\lambda_2(k)=1$ and $F_o=10\cdot I$ is used.

In Figures 5.19-5.21 and 5.25 the plant outputs of model reference adaptive systems with different control objectives (defined by $C_2(q^{-1})$ polynomial) and $H_1(q^{-1}) = H_2(q^{-1})=1$ are shown. The $C_2(q^{-1})$ polynomials used in these simulations are

i) $C_2(q^{-1})=1$ (5.16a)

ii) $$C_2(q^{-1}) = \left[1-(0.3+0.2j)q^{-1}\right]\left[1-(0.3-0.2j)q^{-1}\right]$$
$$= 1-0.6q^{-1} + 0.13q^{-2} \tag{5.16b}$$

iii) $$C_2(q^{-1}) = \left[1-(0.9+0.2j)q^{-1}\right]\left[1-(0.9-0.2j)q^{-1}\right]$$
$$= 1-1.8q^{-1} + 0.85q^{-2} \tag{5.16c}$$

It is clearly seen that as the roots of the $C_2(q^{-1})$ polynomial (the closed loop poles) approaches the unit circle, the adaptation process is smoothed and abrupt changes in plant output are avoided. In regulation (Figure 5.25), a proper choice of the regulation objective((5.16b) in this case) highly improves the performance. On the other hand, in tracking, as seen in Figures 5.19-5.21, the best choice for $C_2(q^{-1})$ polynomial seems to be (5.16a), since as the roots are taken away from the origin both the transient plant-model error and the convergence time increase. But two examples are presented in Figures 5.27-5.28 showing the usefullness of choosing the roots of $C_2(q^{-1})$ polynomial away from the origin. In these simulations the plant delay is chosen to be d=2, which is a rather "harder" case for an adaptive controller; and the control objectives are defined by (5.16a) and

$$C_2(q^{-1}) = (1-0.4q^{-1})^3 \tag{5.17}$$

It is seen clearly, how important may the placement of the closed-loop poles be in avoiding catastrophic transients.

In Figures 5.22-5.24 and 5.26 the simulation results of model reference adaptive controllers with $H_1(q^{-1})=1$ and $H_2(q^{-1})=C_2(q^{-1})$, where $C_2(q^{-1})$ is given by (5.16b), are shown. It can be concluded from these Figures that the transient errors are increased in tracking, whereas the regulation performance is not affected at all.

From all simulations for model reference adaptive controllers done in deterministic environment it can be deduced that the convergence of the controller parameters to the true values is not guaranteed always. In Table 5.1, the values to which the controller parameters converge (p_∞) are tabulated for different initial parameter values. These are the results obtained from regulation simulations with $H_1(q^{-1}) = H_2(q^{-1}) = C_2(q^{-1}) = 1$ and constant trace adaptation gain $\lambda_1(k)/\lambda_2(k) = 1$, $F_0=10 \cdot I$). The true parameter vector corresponding to this case is $p^T = [0.9 \quad 0.5 \quad 2.3 \quad -1.96 \quad 0.53]$. It is seen that p_∞ is closer to the true parameter vector if the initial estimates are not near to it. This is a reasonable result, since if the differences between the initial estimates and true values of the parameters are high the input is expected to be richer. (Large values and variations at the input are observed in such cases).

TABLE 5.1

$\hat{p}(0)$					p_∞				
0.80	0.40	2.20	-1.86	0.43	0.851	0.473	2.173	-1.859	0.511
1.00	0.60	2.00	-1.86	0.63	0.911	0.505	2.329	-1.982	0.539
-1.00	-1.00	0.00	1.00	-1.00	0.899	0.499	2.297	-1.958	0.529
0.10	0.10	0.10	0.10	0.10	0.900	0.500	2.299	-1.959	0.530
0.50	0.50	0.50	0.50	0.50	0.900	0.500	2.300	-1.960	0.530
1.00	1.00	1.00	1.00	1.00	0.900	0.500	2.299	-1.959	0.530

The results of simulations for stochastic environment and combined MRAC-S-STR scheme are presented in Figures 5.29-5.43. In this part of simulation studies $H_1(q^{-1})$ and $H_2(q^{-1})$ polynomials are chosen equal to unity.

The performances of MRAC with constant adaptation gain and $C_2(q^{-1})=1$ in stochastic environment for RM1, RM3 and regulation are shown in Figures 5.29-5.31. The simulation result for RM2 is omitted since in that case the plant output turned out to be practically unstable.

On the other hand, the plant outputs of the combined MRAC-S-STR scheme in stochastic environment are shown in Figures 5.36-5.39. As in the case of MRAC, constant adaptation gain algorithm and $C_2(q^{-1})=1$ is used in these simulations, too. Comparing Figures 5.29-5.31 with 5.36-5.39, one can conclude that the combined control scheme improves the performance in stochastic environment in case of tracking. But the performances of MRAC and combined MRAC-S-STR scheme are equal in regulation, as expected. This facts are made more clear in Table 5.2 where sum of the plant-model error squares after parameters change are tabulated.

TABLE 5.2 $\sum_{k=t}^{n} \varepsilon^2(k)$

	MRAC	MRAC-S-STR
RM1	325	198
RM2	36,305	248
RM3	143	103
REG	37	37

n = 180 for tracking, n=50 for regulation

It should be noted that, tracking the reference model RM2, where the first differences of the model output is relatively high at some instants, is practically impossible for a MRAC in stochastic environment; whereas the combined MRAC-S-STR is at least not unstable in that case although the plant-model errors are high after each impulse input.

On the other hand, the performances of combined MRAC-S-STR in deterministic environment are figured in Figures 5.32-5.35. The smoothing effect of $C_2(q^{-1})$ polynomial $(C_2(q^{-1})=1-0.6q^{-1})$ can be observed in these simulation results.

Although, Theorem 4.1 allows only the use of constant or decreasing adaptation gains to assure convergence in stochastic environment; simulations are done with constant trace adaptation gain, too. Figures 5.40-5.43 show the results of simulations done with combined MRAC-S-STR scheme using constant trace adaptation gain ($F_o=10\cdot I$, $\lambda_1(k)/\lambda_2(k)=1$). A comparison between the combined schemes which use constant trace and constant adaptation gains is made in Table 5.3.

Although no convergence proof is given for this case, the simulations with constant trace adaptation gain yield quite satisfactory results; except in tracking RM2 the sum of the plant-model error squares increases because of the large transients after each impulse input; which is a behaviour contrary to that in deterministic environment.

TABLE 5.3

$$\sum_{k=t}^{n} \varepsilon^2(k)$$

	Const. Adap. Gain	Const. Trace Adap. Gain
RM1	198	165
RM2	248	832
RM3	103	138
REG	37	24

n= 180 for tracking, n=50 for regulation

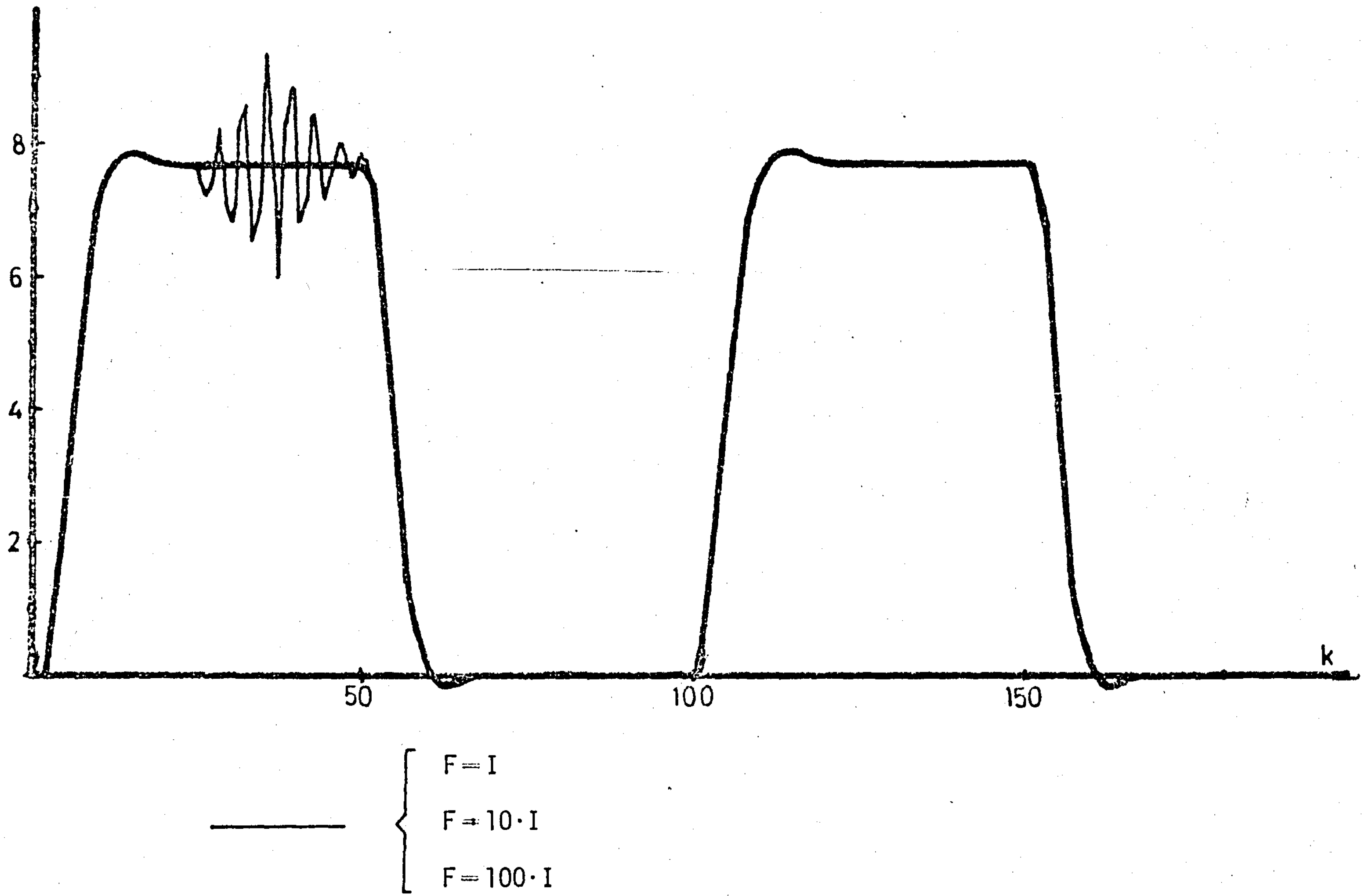


Figure 5.1 Plant output of MRAC $(H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. ad. gain) for RM1

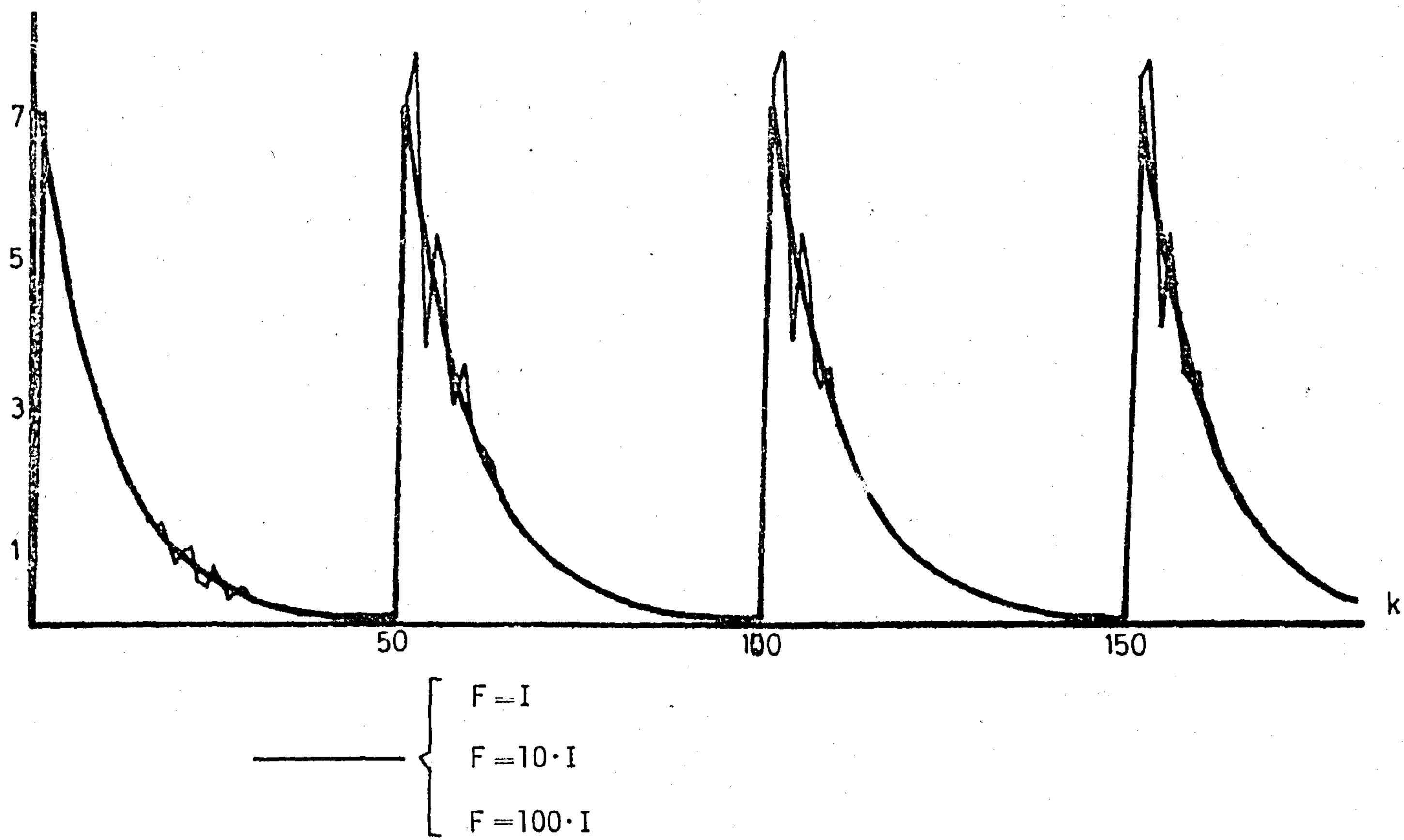


Figure 5.2 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. ad. gain) for RM2

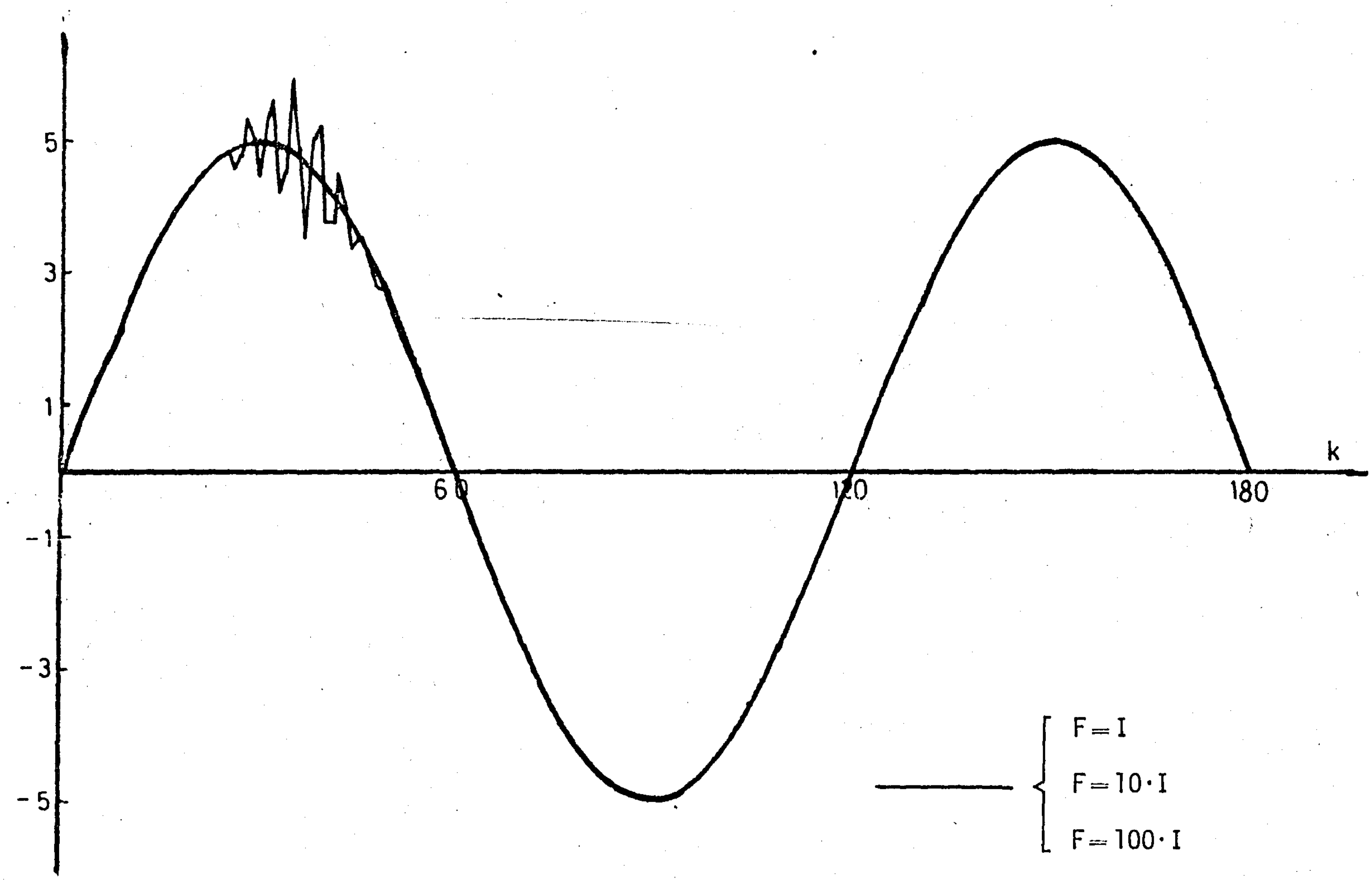


Figure 5.3 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$; const. ad. gain) for RM3.

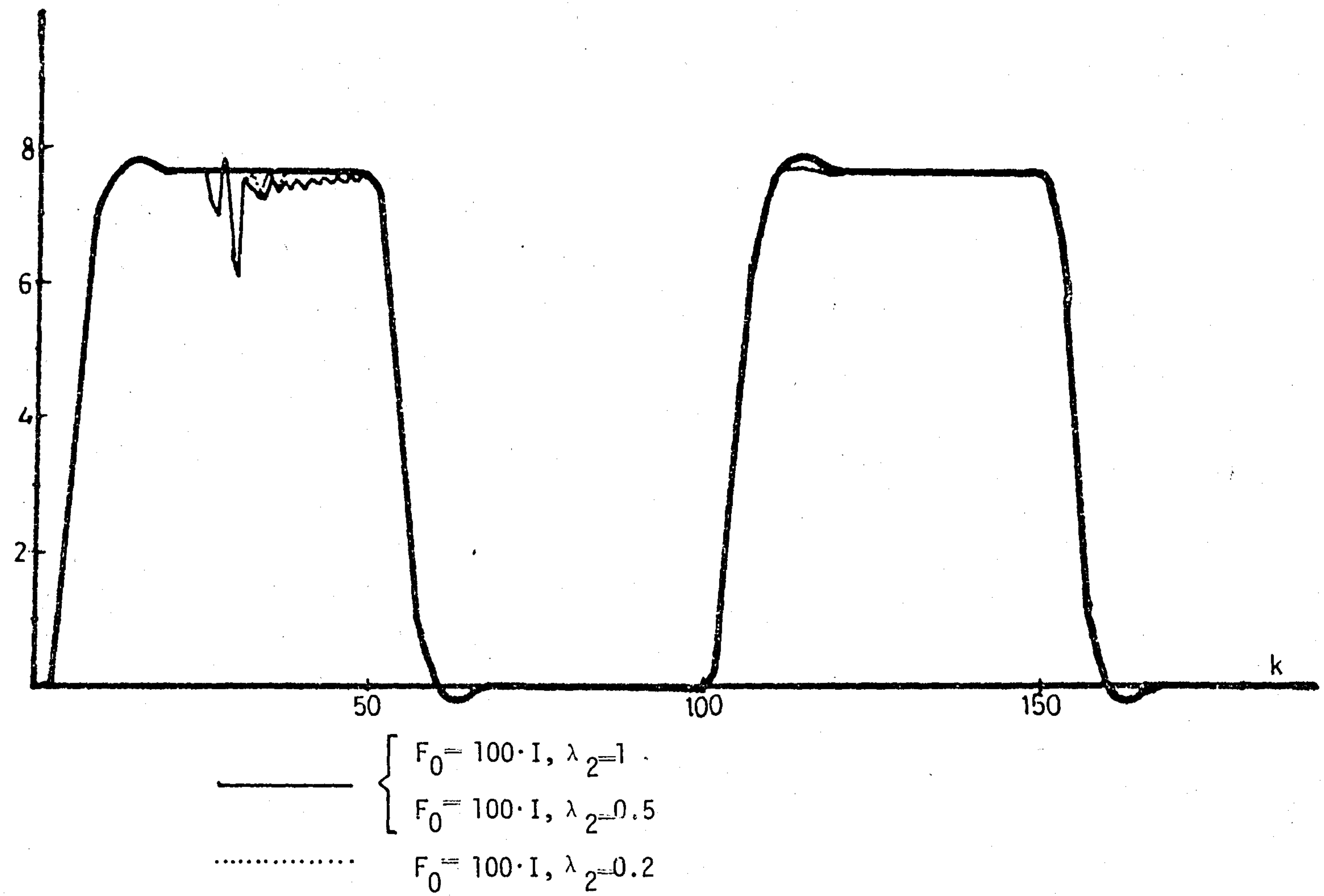


Figure 5.4 Plant output of MRAC($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, dec. ad. gain) for RM1

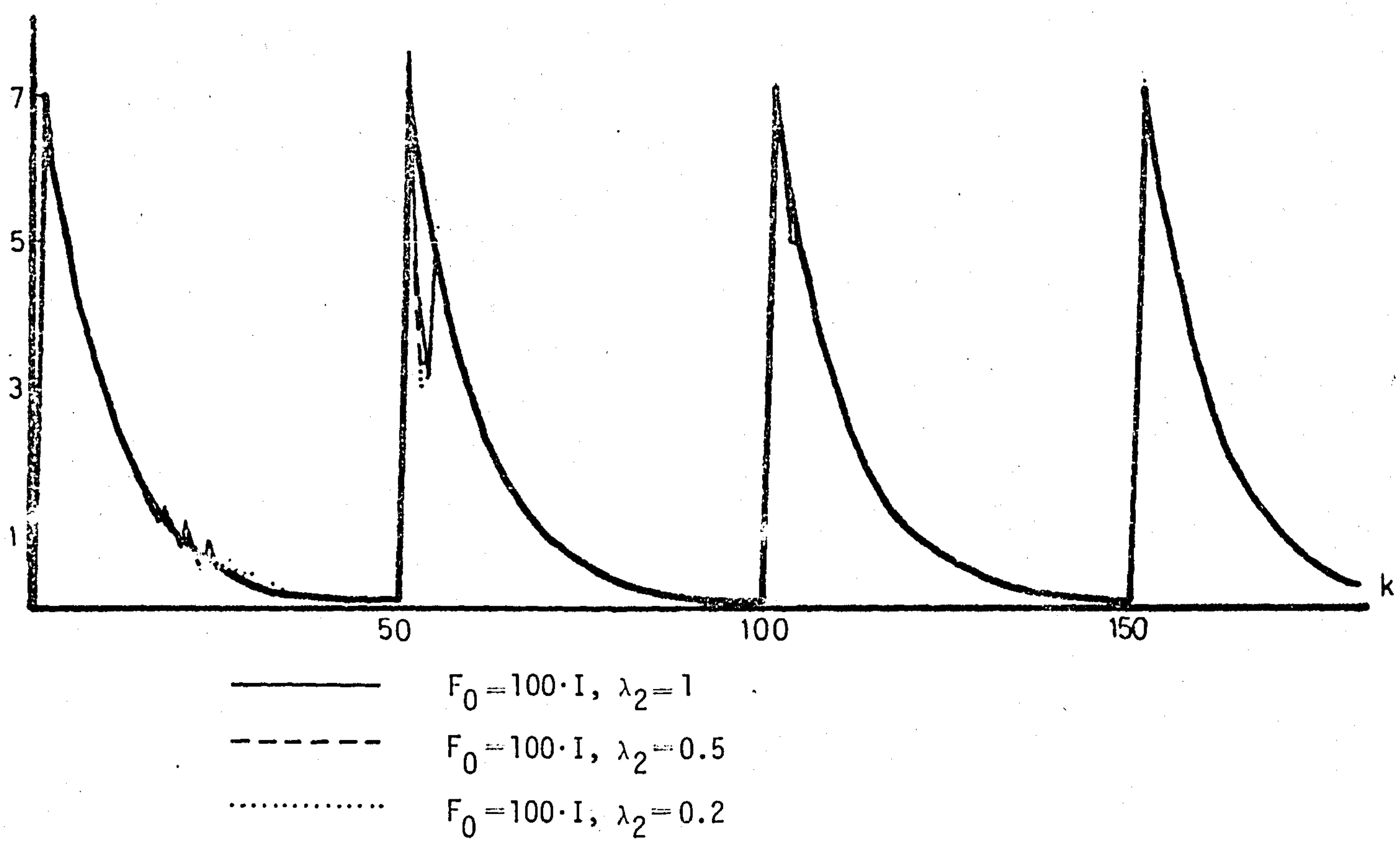


Figure 5.5 Plant output of MRAC ($H_1(q^{-1}) = H_2(q^{-1}) = C_2(q^{-1}) = 1$, dec.ad.gain) for RM2

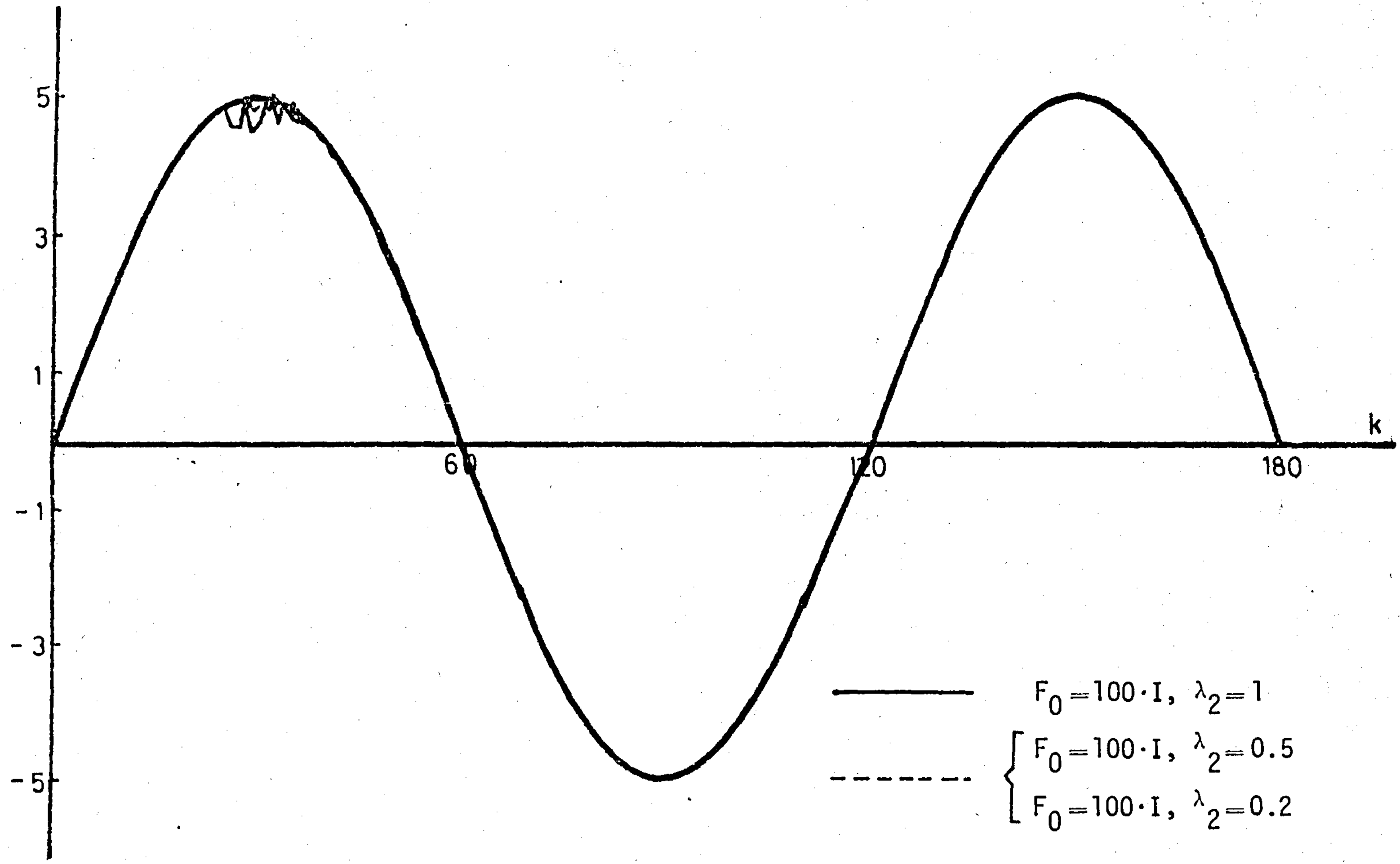
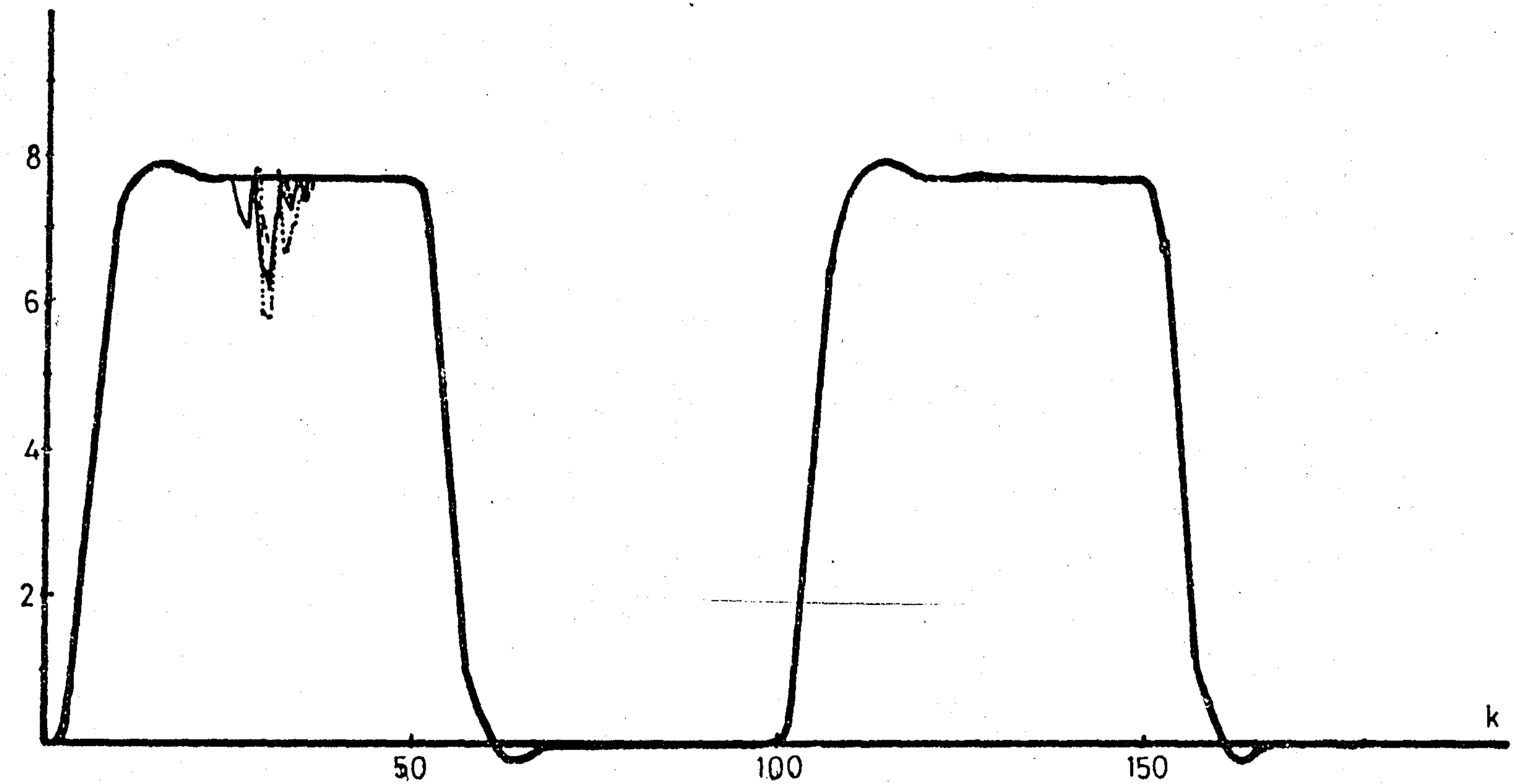


Figure 5.6 Plant output of MRAC ($H_1(q^{-1}) = H_2(q^{-1}) = C_2(q^{-1}) = 1$, dec. ad. gain) for RM3



$F_0 = 10 \cdot I$, $\lambda_1(k)/\lambda_2(k) = 1$

$F_0 = 10 \cdot I$, $\lambda_1(k)/\lambda_2(k) = 4$

$F_0 = 10 \cdot I$, $\lambda_1(k)/\lambda_2(k) = 0.25$

Figure 5.7 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. trace ad. gain) for RM1

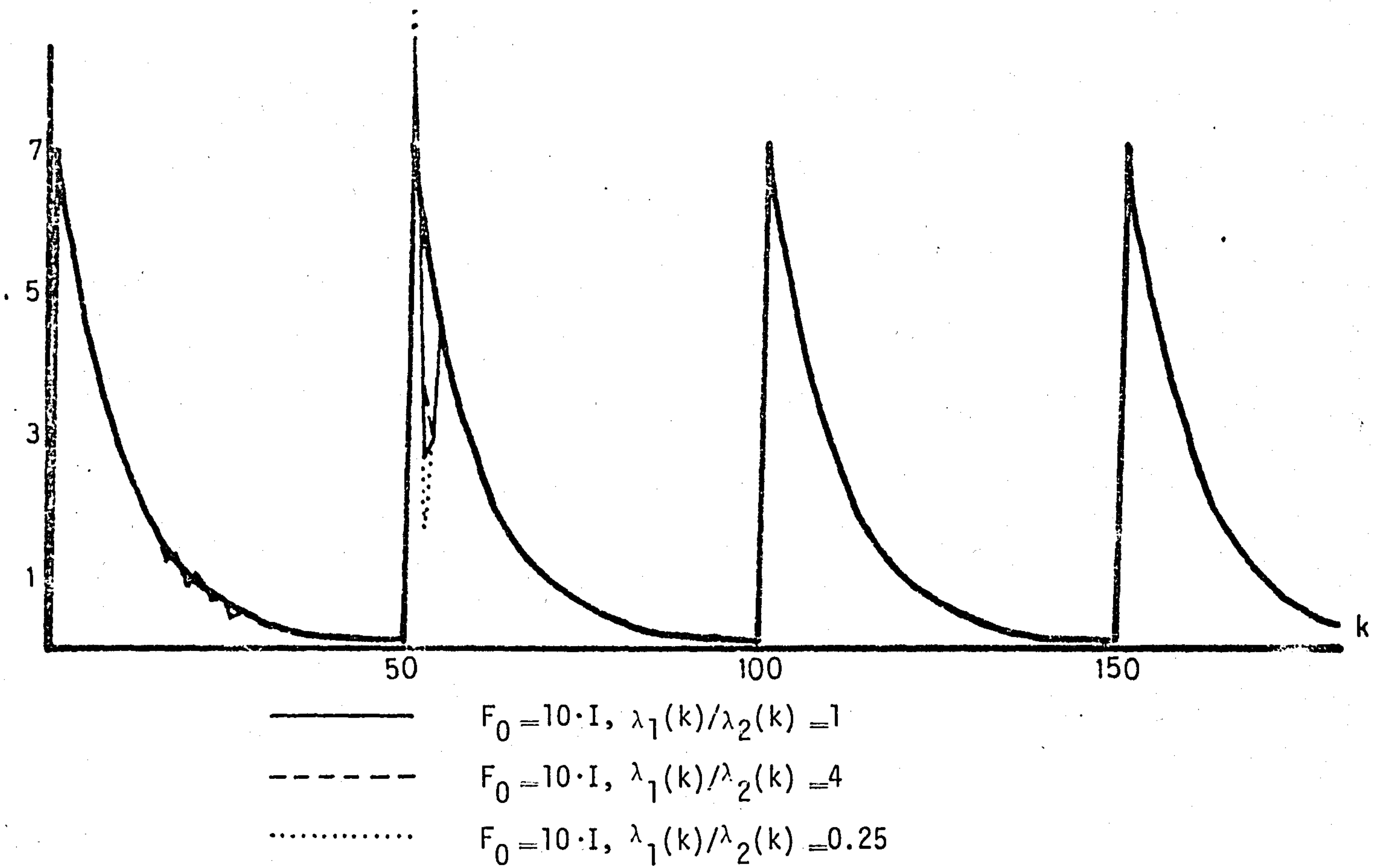


Figure 5.8 Plant output of MRAC ($H_1(q^{-1}) = H_2(q^{-1}) = C_2(q^{-1}) = 1$, const. trace ad. gain) for RM2

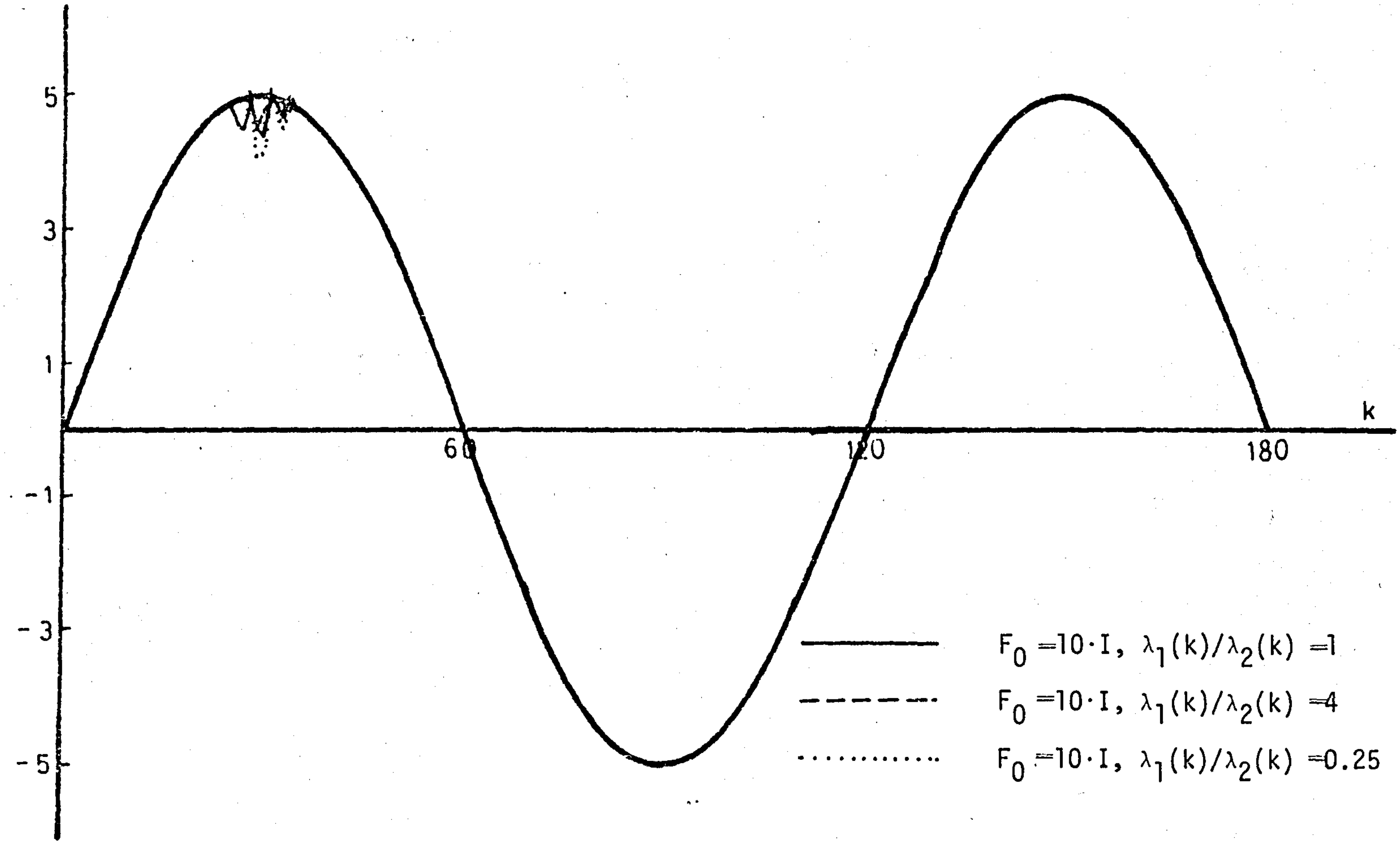


Figure 5.9 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. trace ad. gain) for RM3

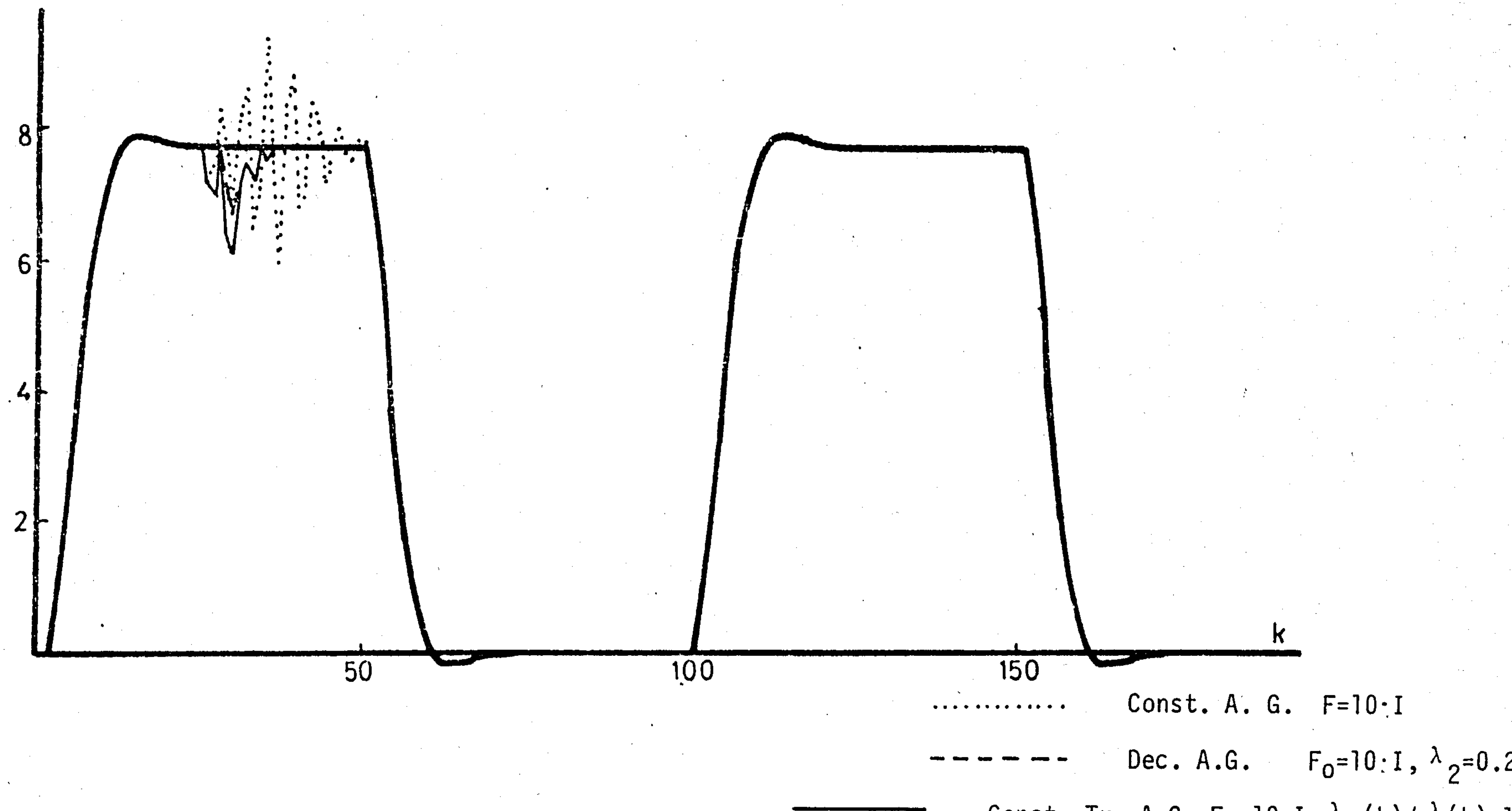
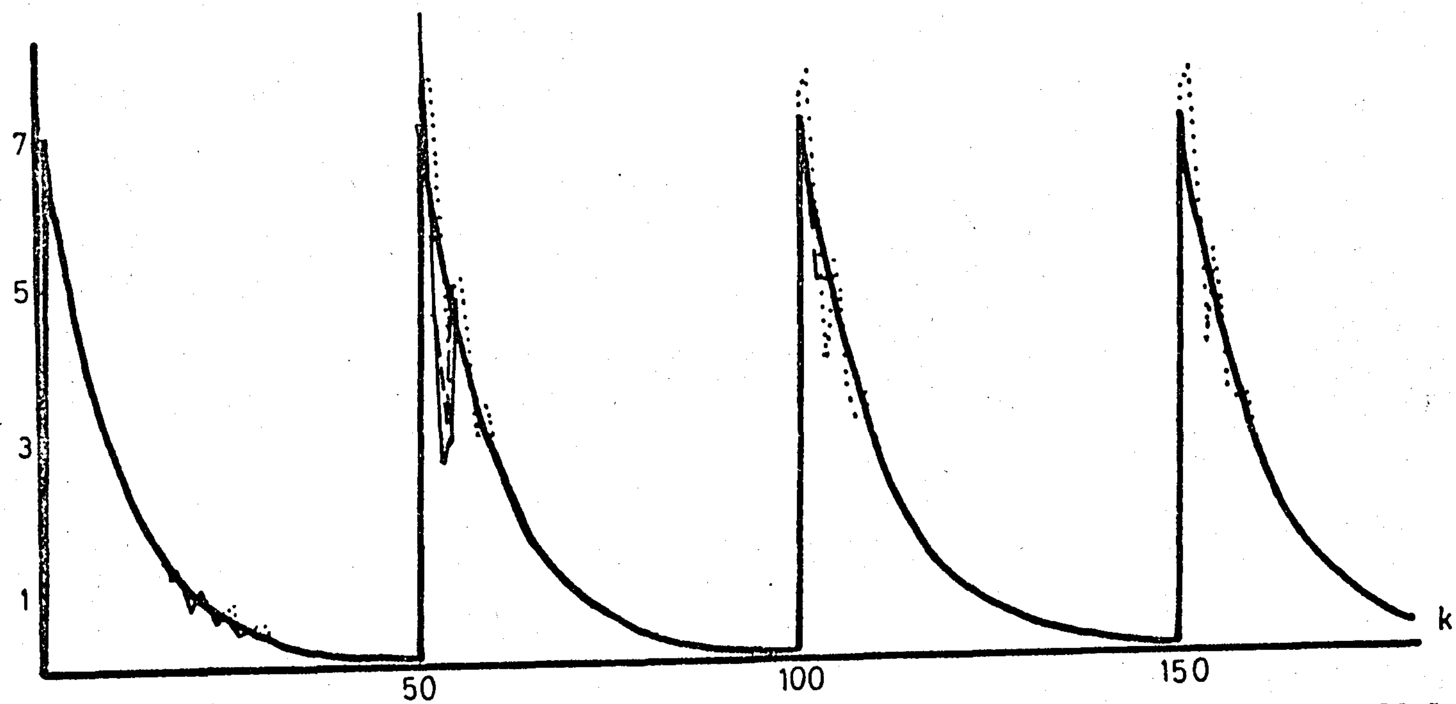


Figure 5.10 Plant output of MRAC $(H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1)$ for RM1



.... Const. A.G. $F=10 \cdot I$

— — — — — Dec. A.G. $F_0=100 \cdot I$, $\lambda_2=0.2$

———————— Const. Tr. A.G. $F_0=10 \cdot I, \lambda_1(k)/\lambda_2(k)=1$

Figure 5.11 Plant output of MRAC $(H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1)$ for RM2

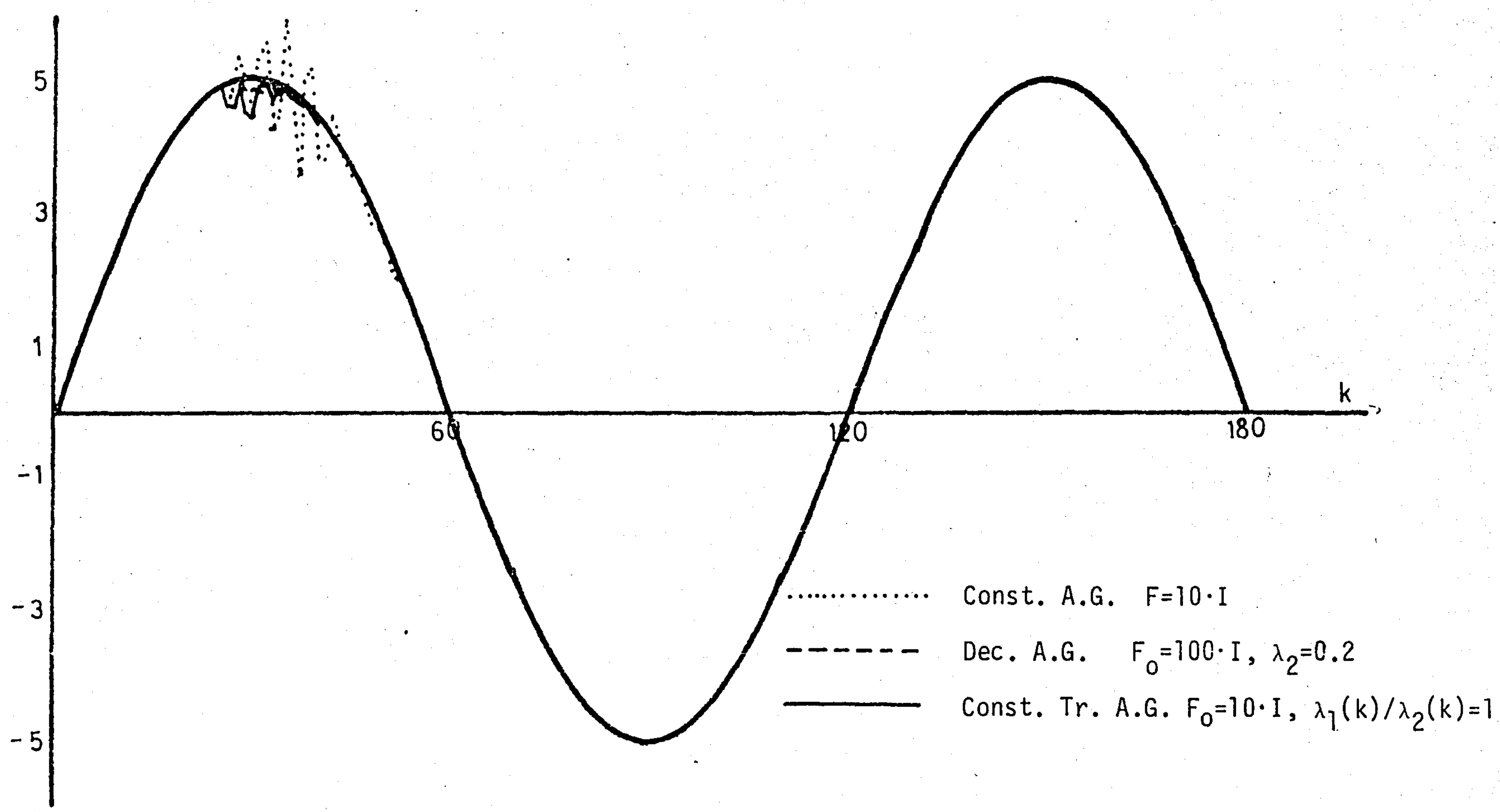


Figure 5.12 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$) for RM3

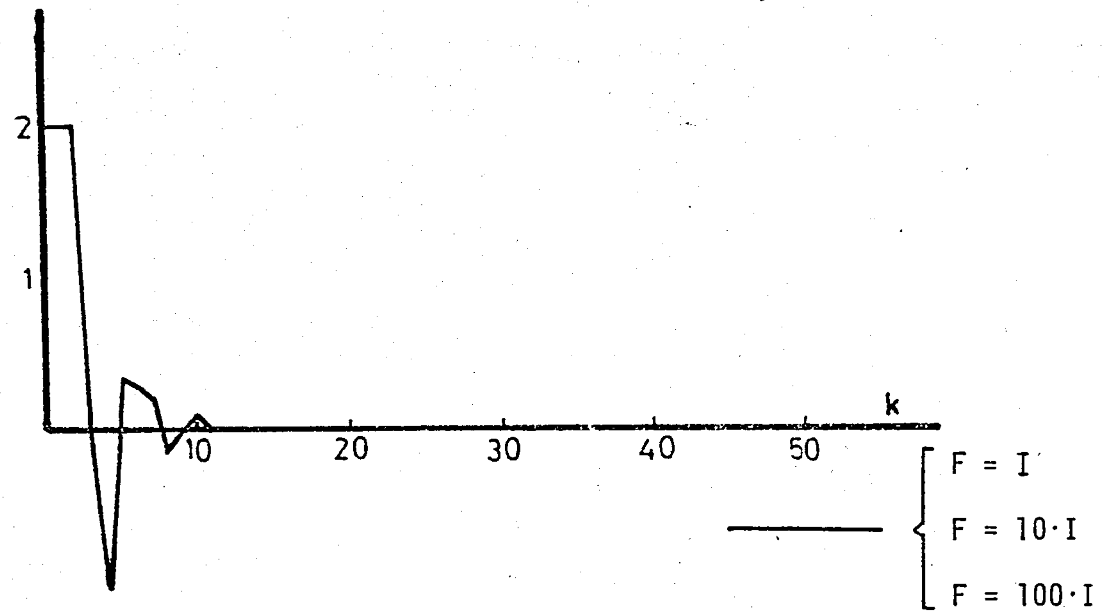


Figure 5.13 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})$ $=C_2(q^{-1})=1$, const. ad. gain) for regulation

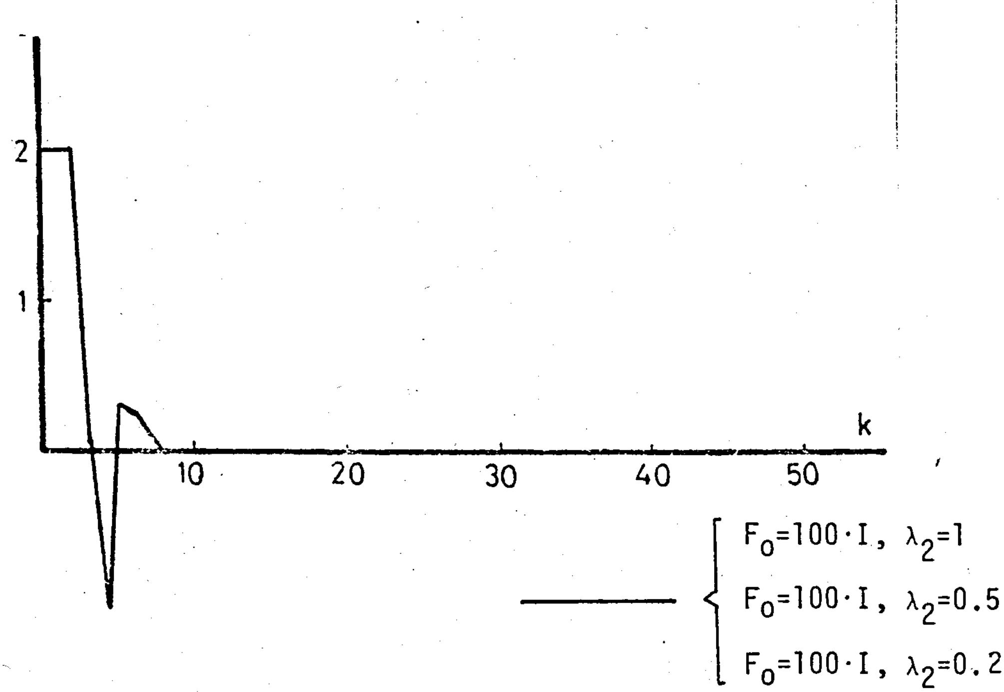


Figure 5.14 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, dec. ad. gain) for regulation

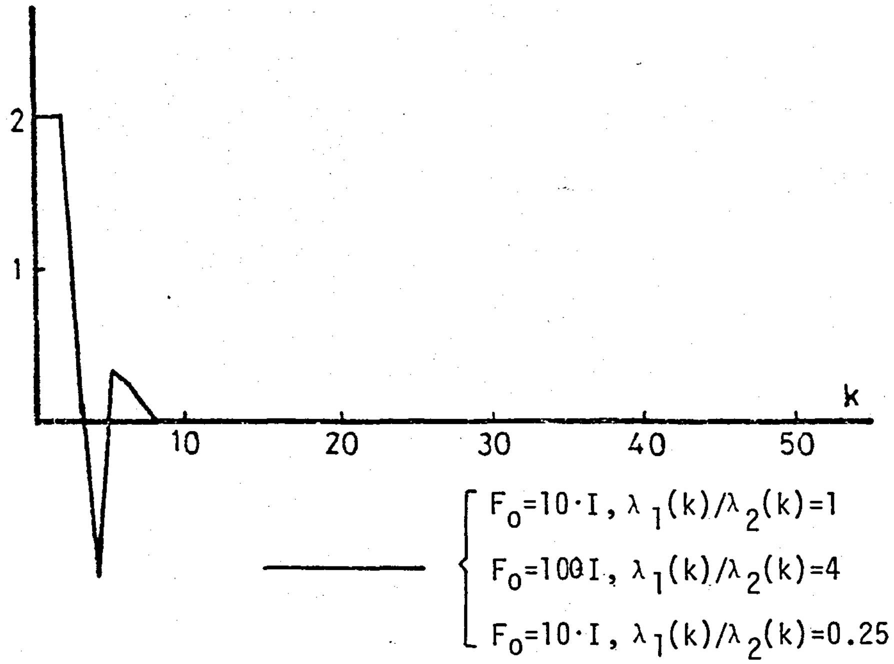


Figure 5.15 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. trace ad. gain) for regulation

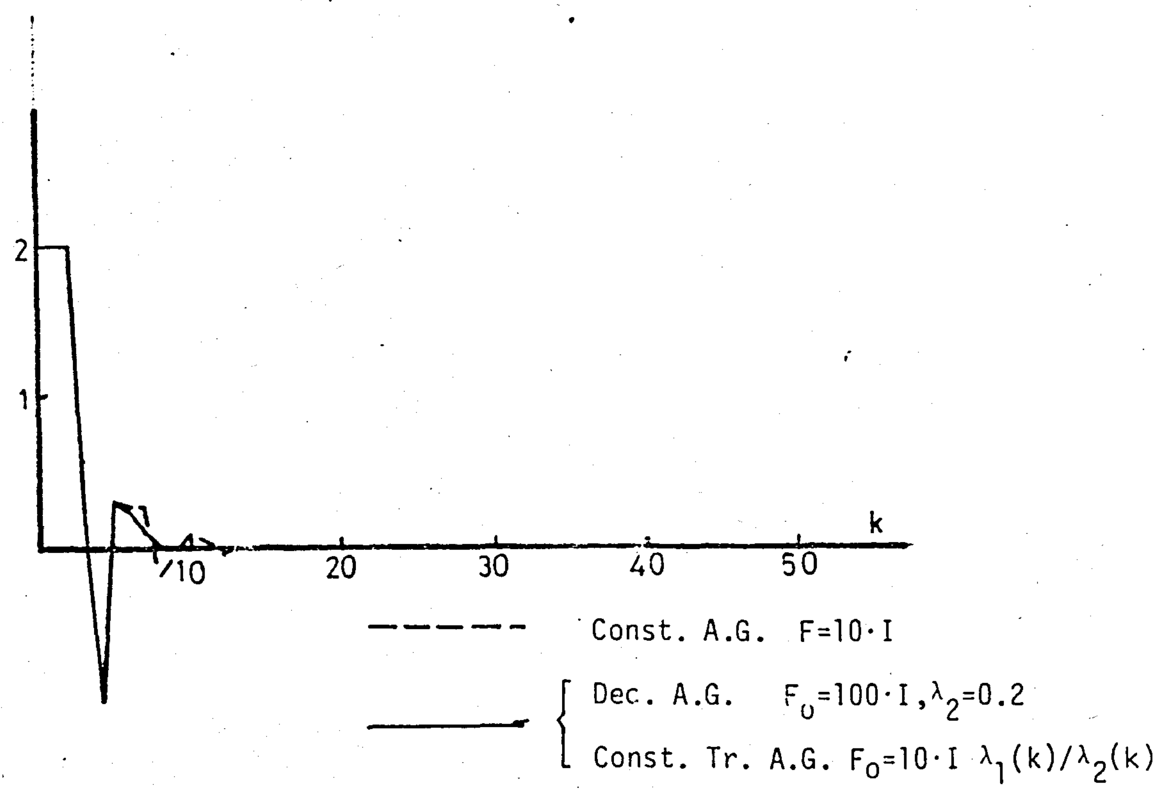
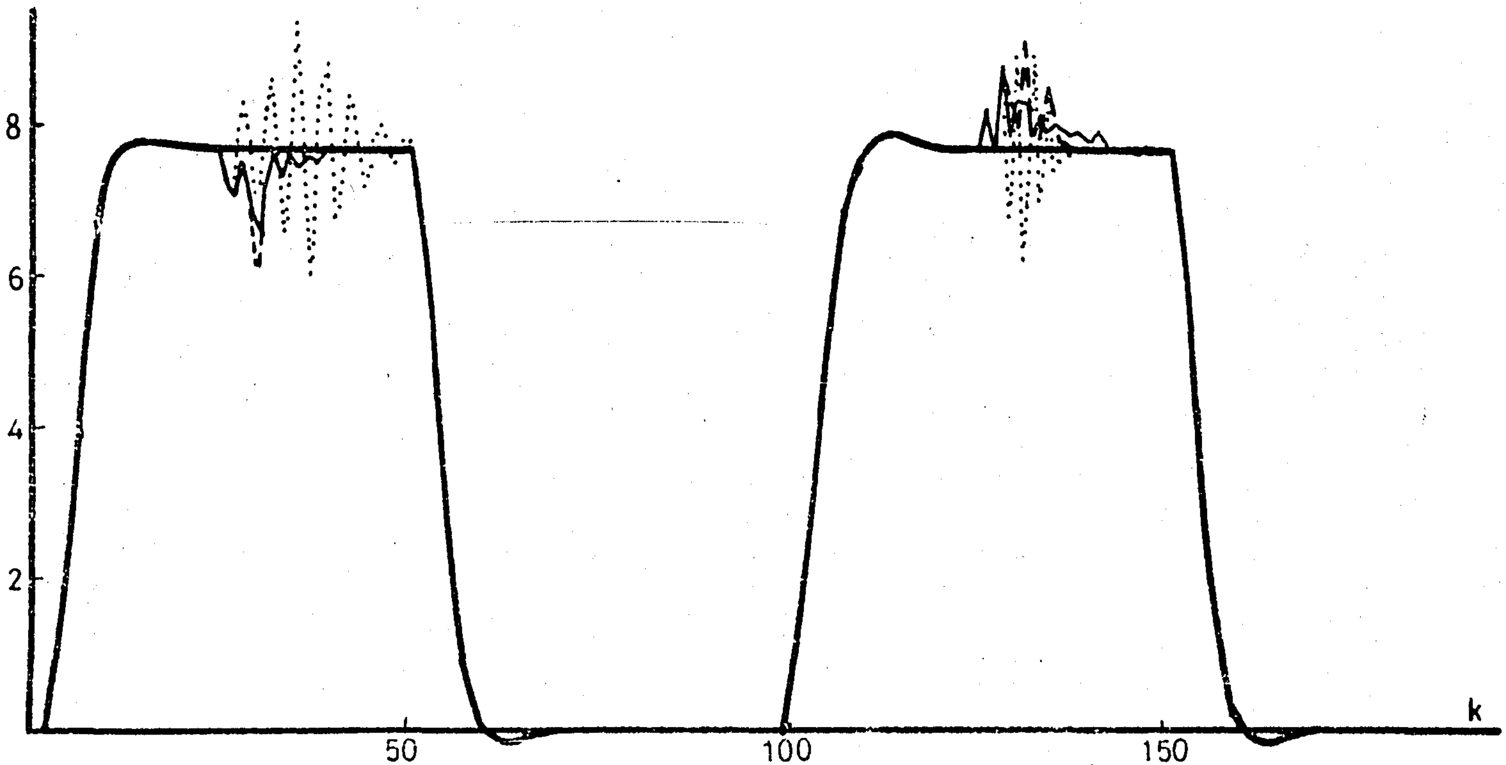


Figure 5.16 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$) for regulation



.... Const. A.G. $F=10\cdot I$

———— Dec.A.G. $F_0=100\cdot I$, $\lambda_2=0.2$

– – – – – – Const.Tr.A.G. $F_0=10\cdot I$, $\lambda_1(k)/\lambda_2(k)=1$

Figure 5.17 Plant output of MRAC $(H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1)$ for RM1

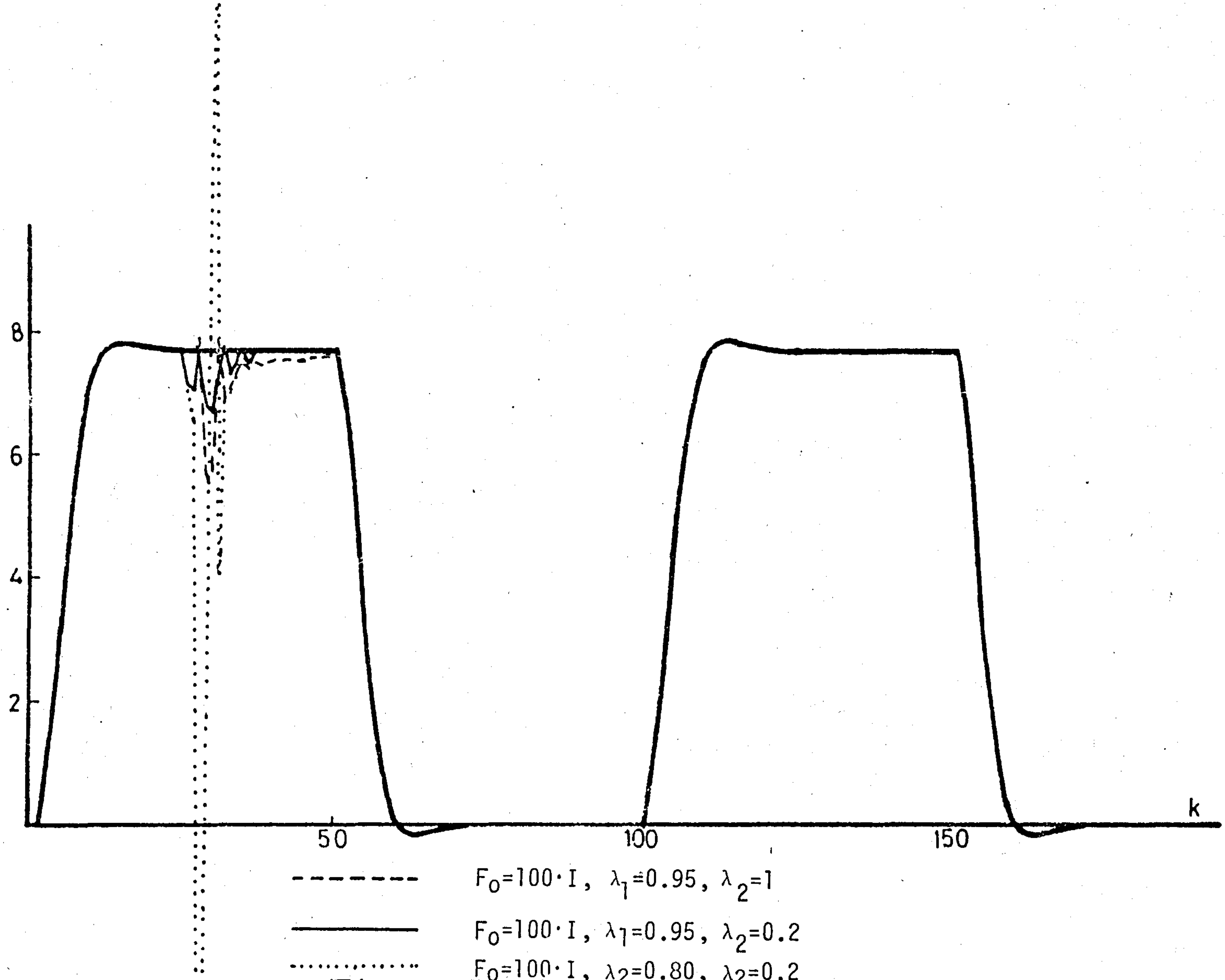
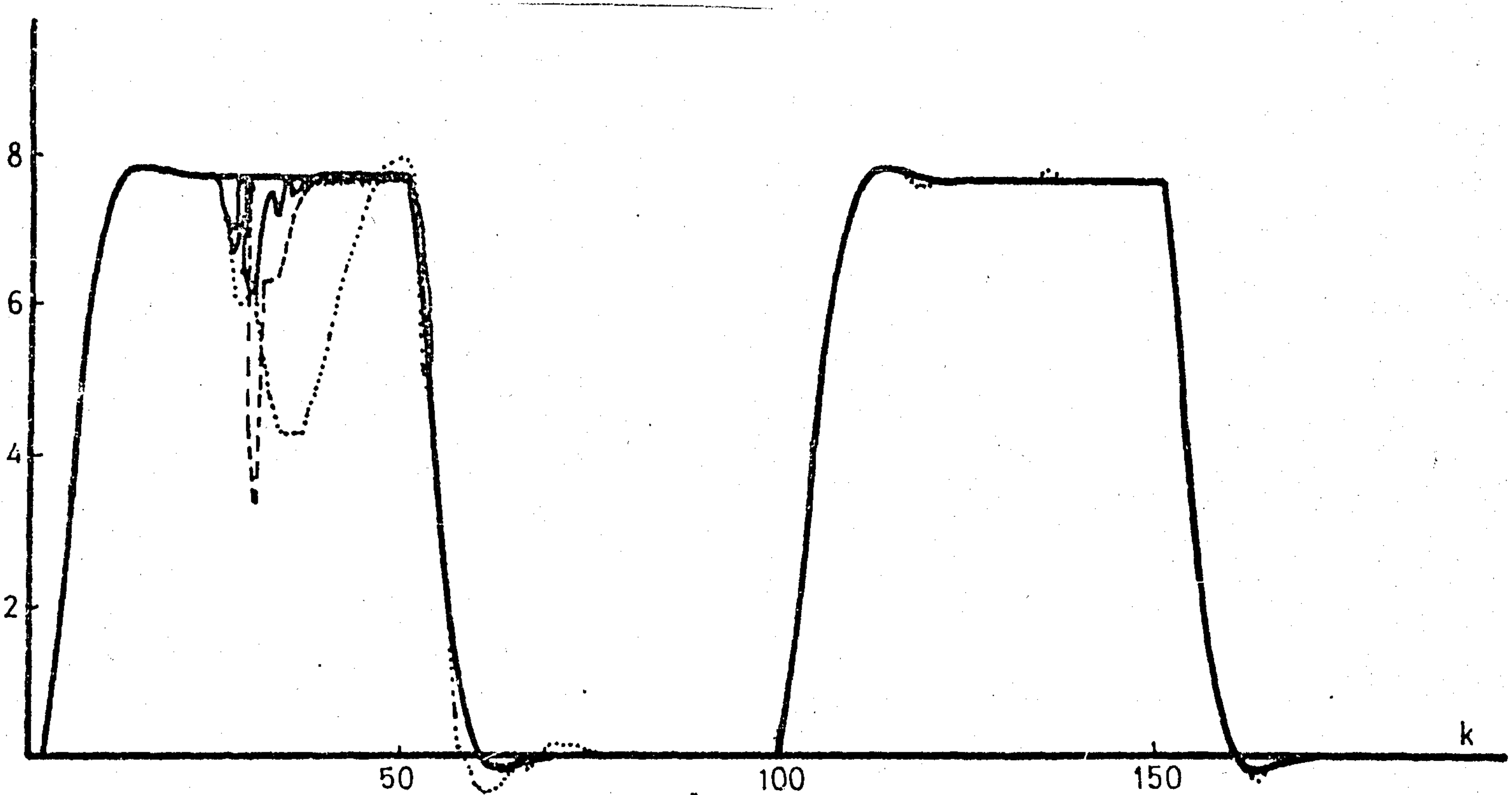


Figure 5.18 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, ad. gain with forg. factor) for RM1



$C_2(q^{-1})=1$

$C_2(q^{-1})=1-0.6\,q^{-1}+0.13\,q^{-2}$

$C_2(q^{-1})=1-1.8\,q^{-1}+0.85\,q^{-2}$

Figure 5.19 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=1$, const. trace ad. gain) for RM1

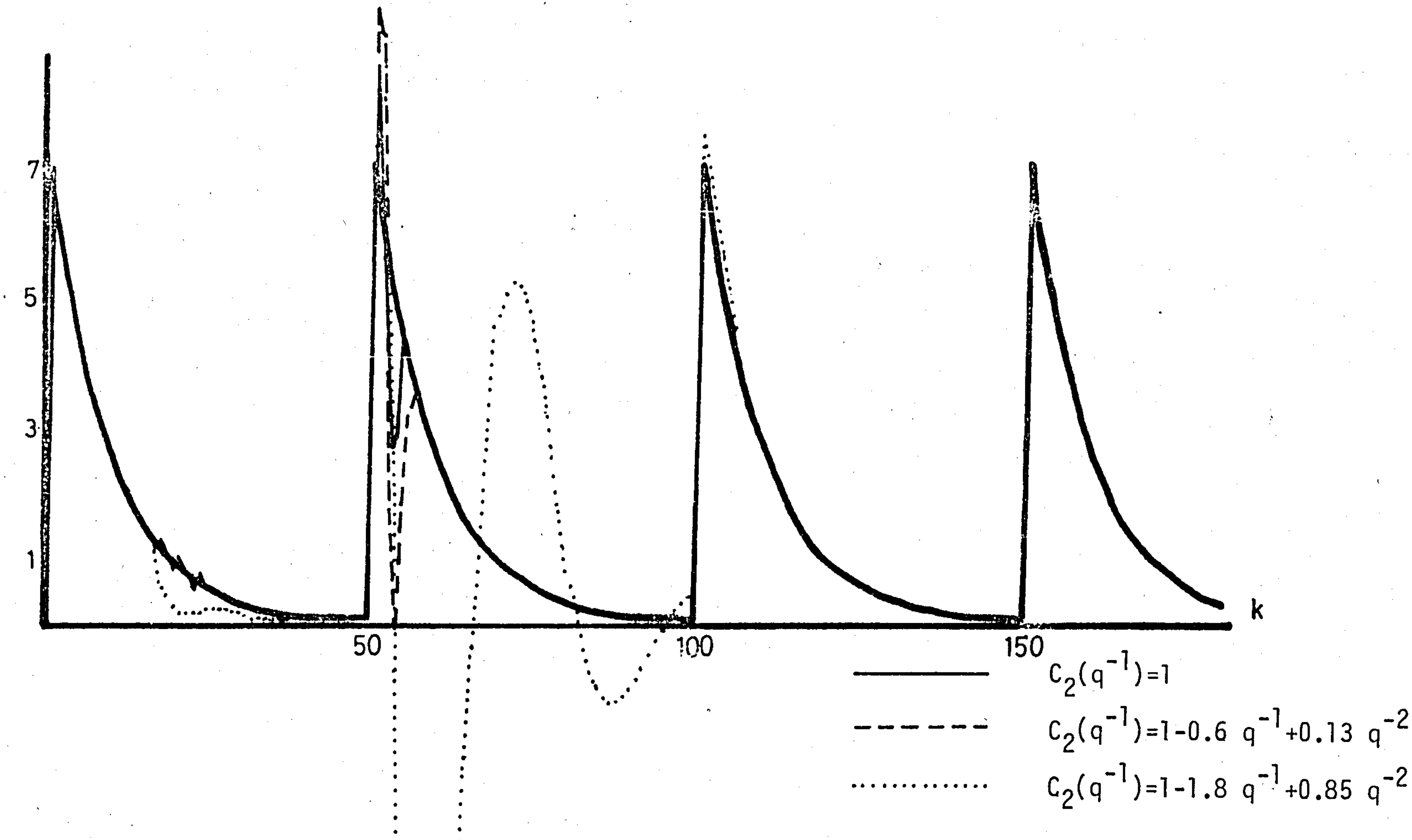


Figure 5.20 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=1$, cons
trace ad. gain) for RM2

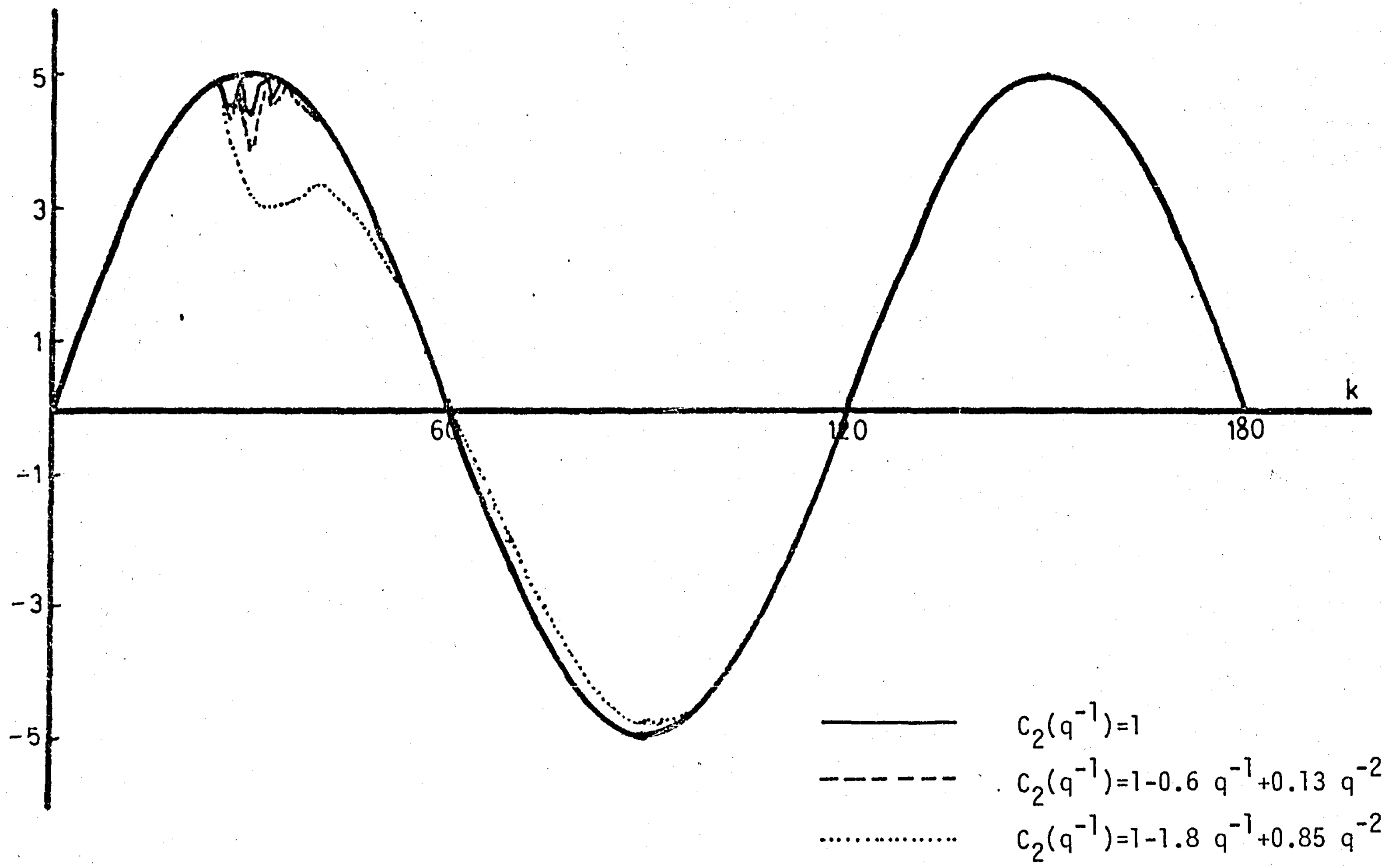


Figure 5.21 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=1$, const. trace ad. gain) for RM3

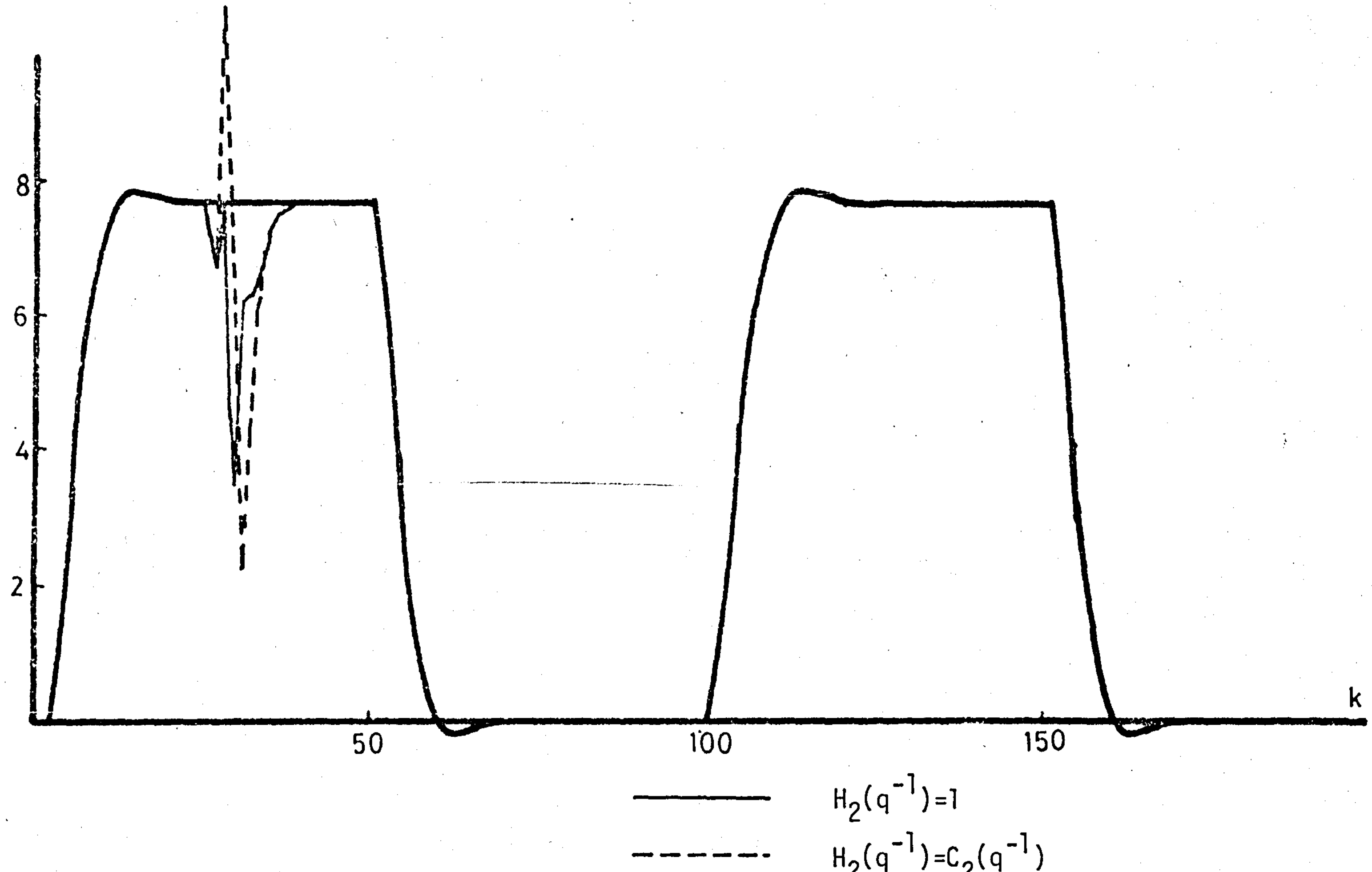


Figure 5.22 Plant output of MRAC ($H_1(q^{-1})=1$, $C_2(q^{-1})=1-0.6\ q^{-1}+0.13\ q^{-2}$, const. trace ad. gain)

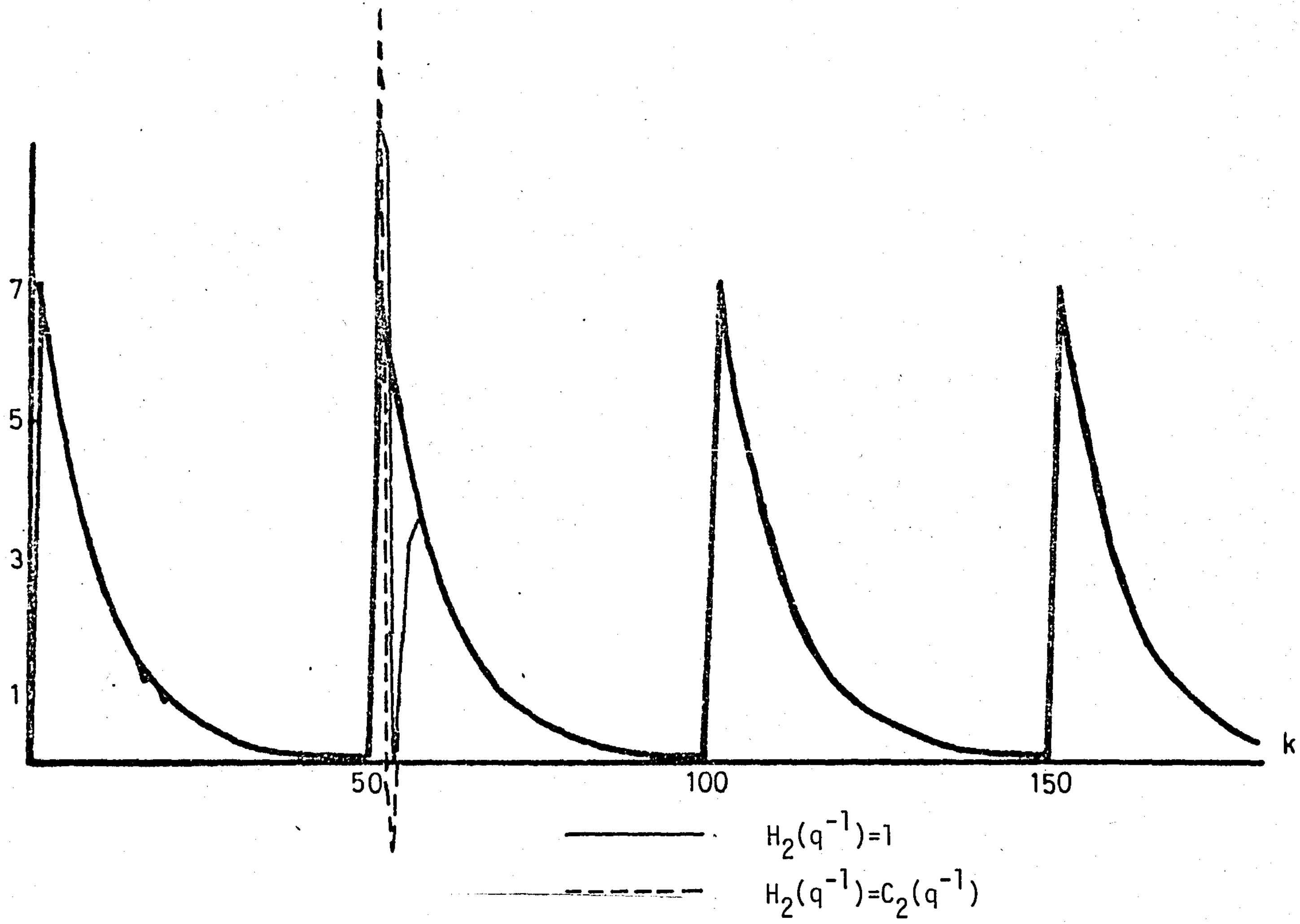


Figure 5.23 Plant output of MRAC ($H_1(q^{-1})=1$, $C_2(q^{-1})=1-0.6\ q^{-1}+0.13\ q^{-2}$, const trace ad. gain)

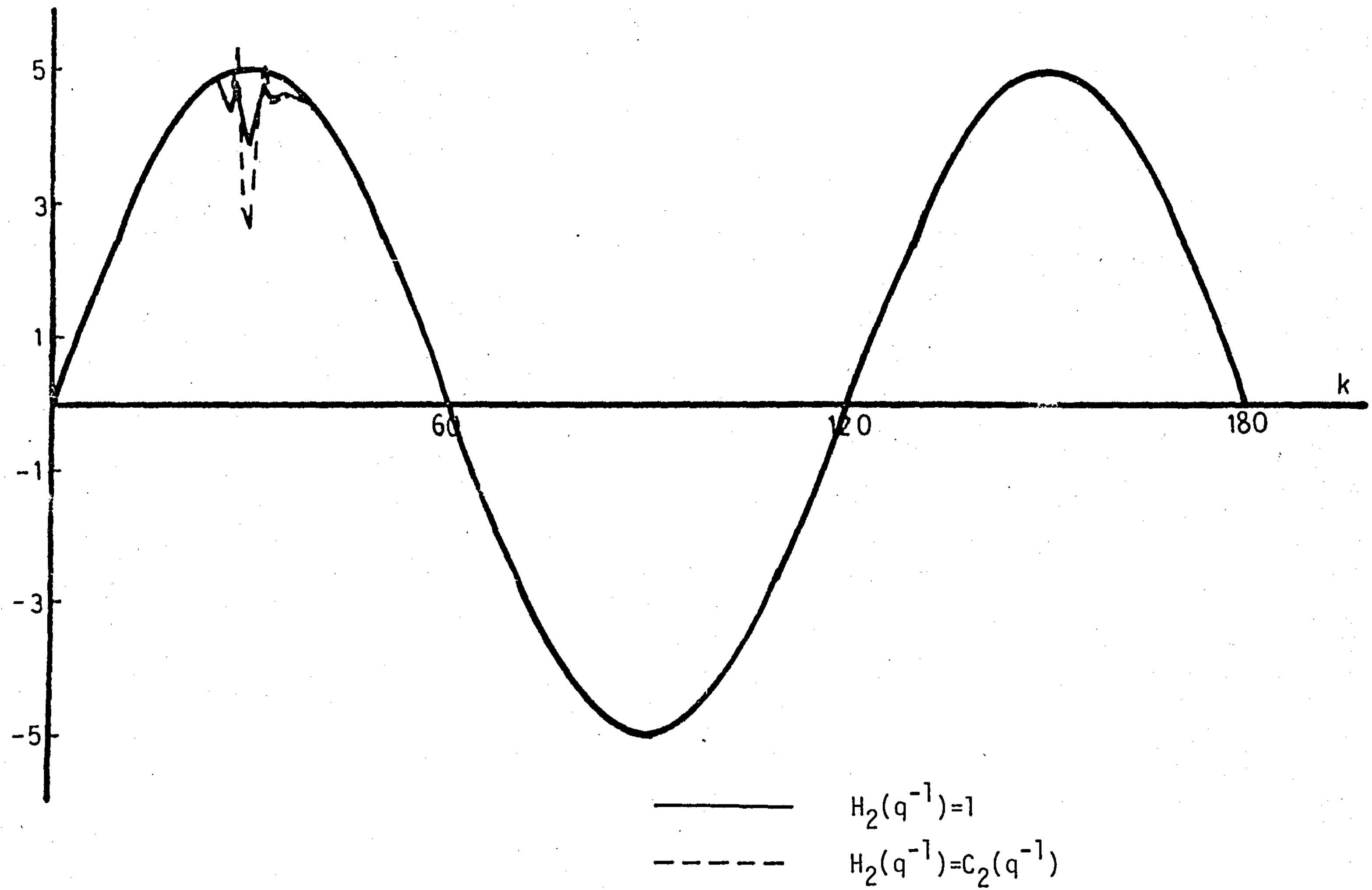


Figure 5.24 Plant output of MRAC ($H_1(q^{-1})=1$, $C_2(q^{-1})=1-0.6\ q^{-1}+0.13\ q^{-2}$, const. trace ad. gain)

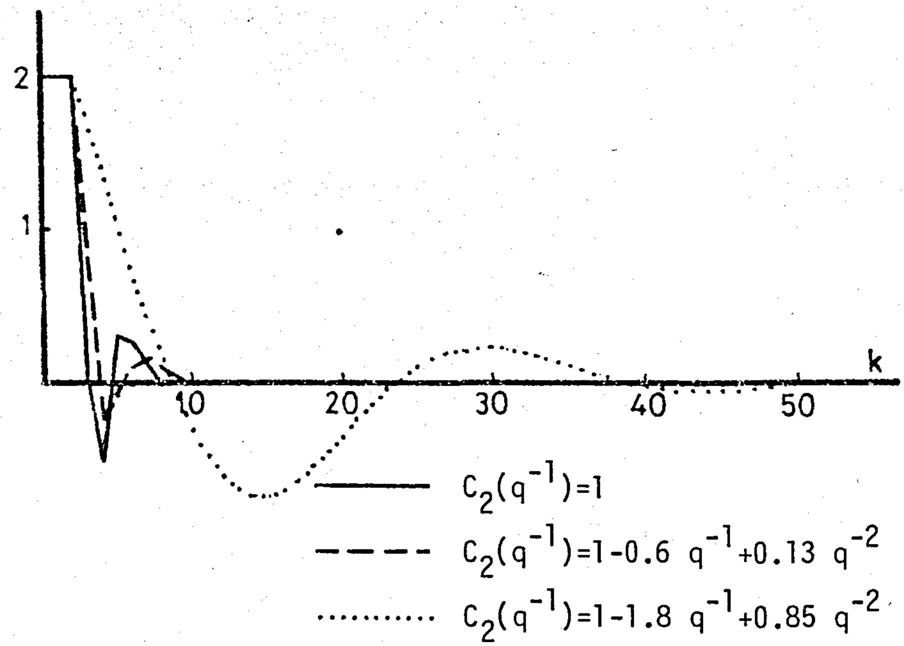


Figure 5.25 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-2})=1$, const. trace ad. gain) for regulation

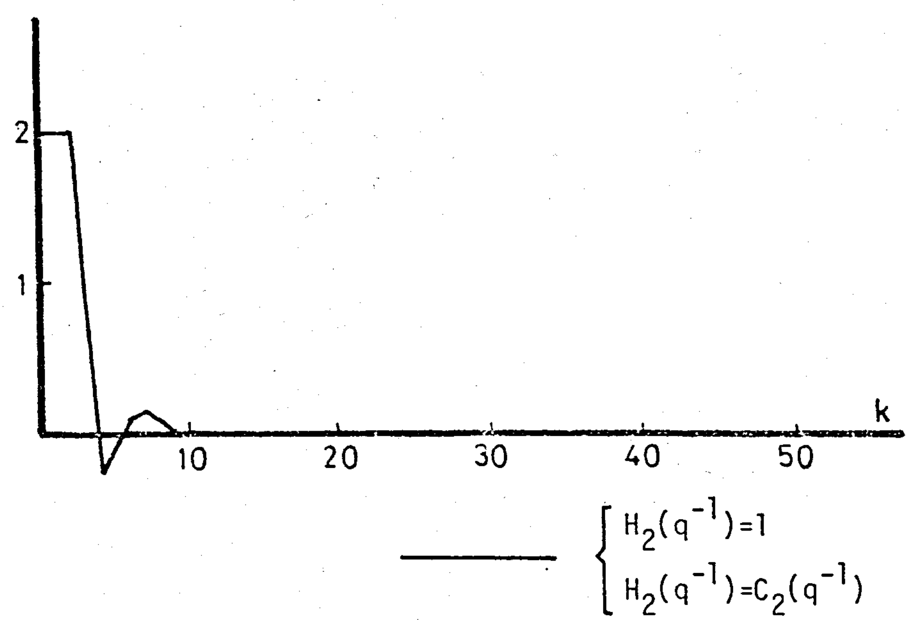


Figure 5.26 Plant output of MRAC ($H_1(q^{-1})=1$, $C_2(q^{-1})$ $=1-0.6\ q^{-1}+0.13\ q^{-2}$, const. trace ad. gain) for regulation

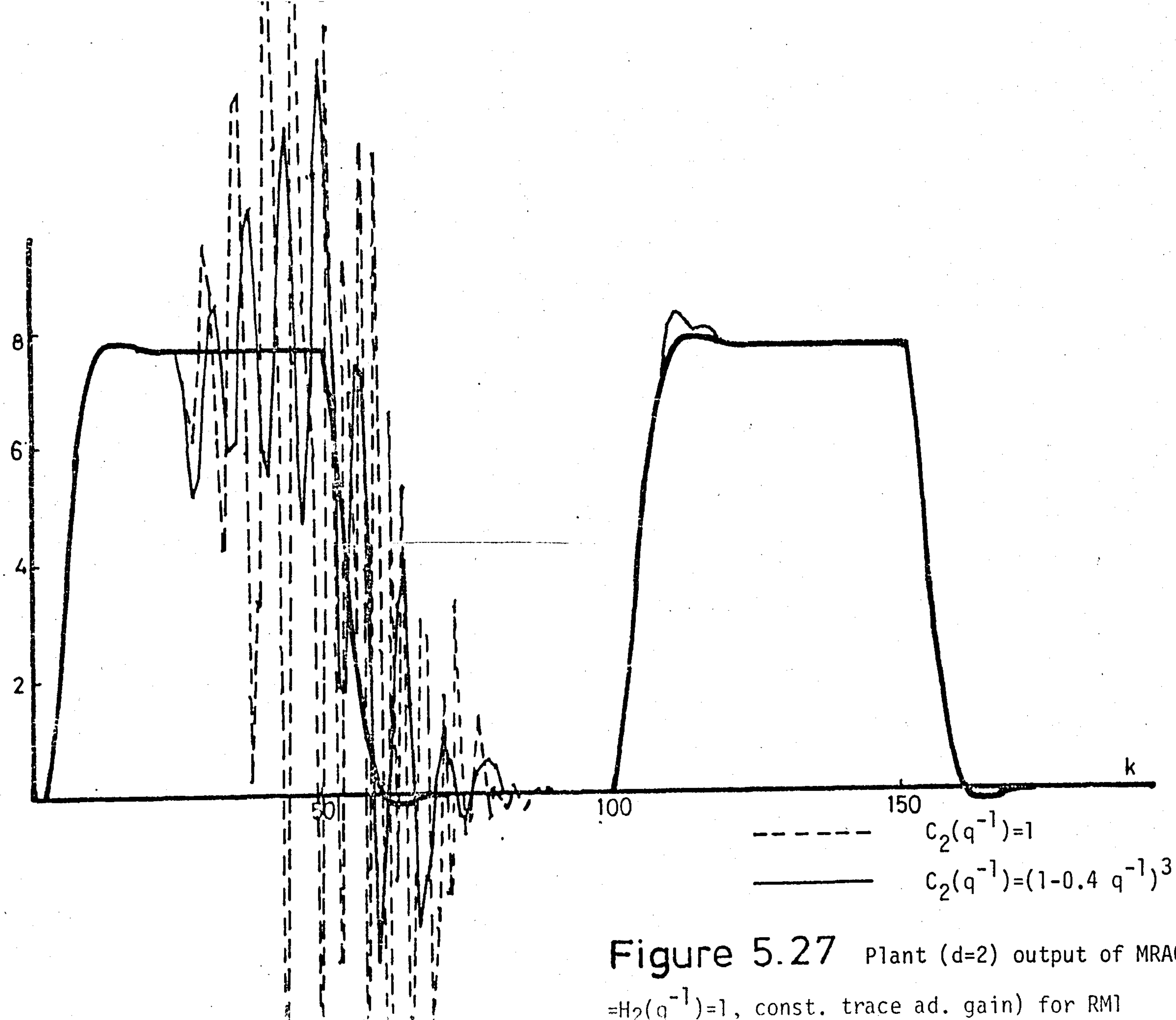
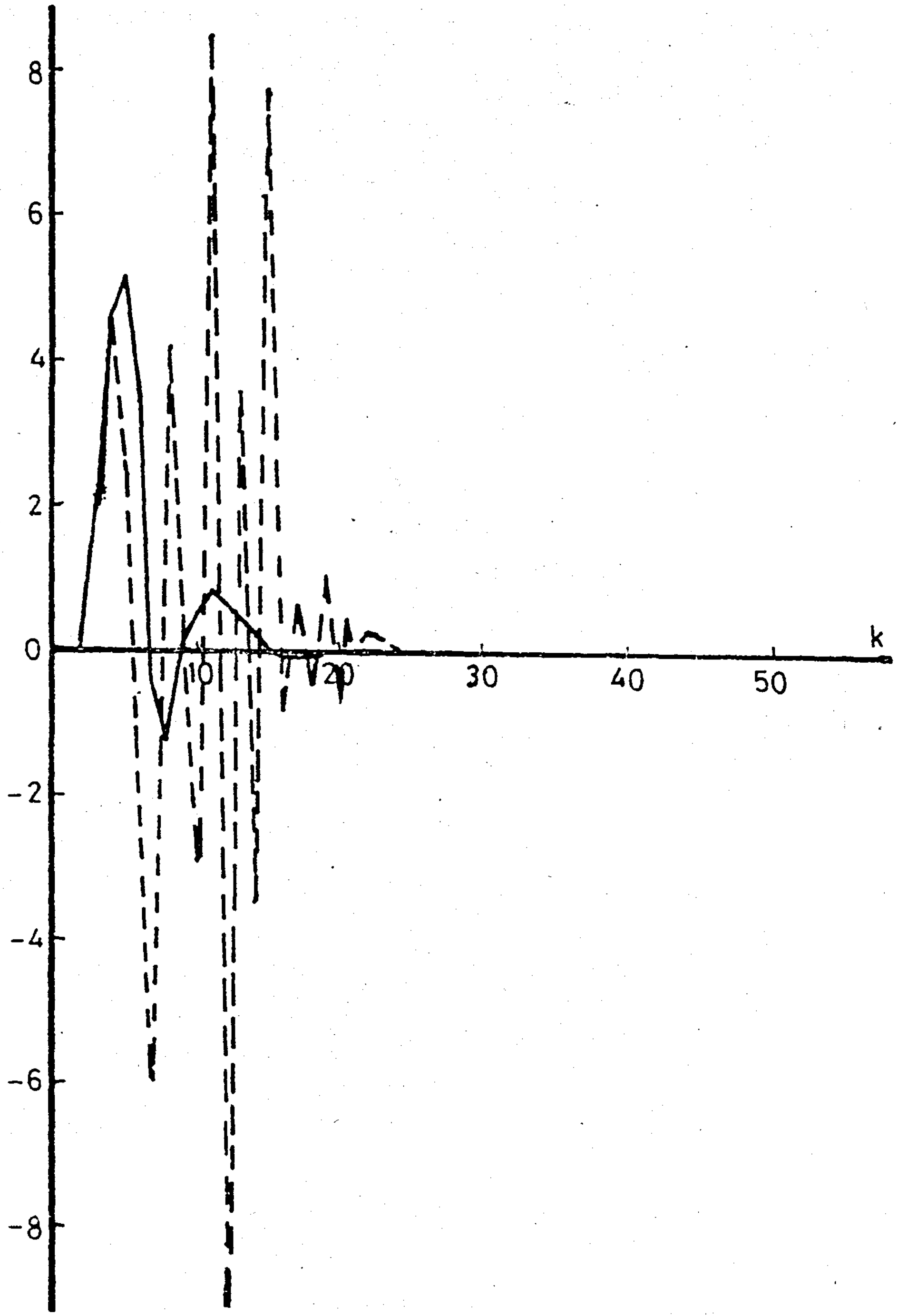


Figure 5.27 Plant (d=2) output of MRAC ($H_1(q^{-1})$ $=H_2(q^{-1})=1$, const. trace ad. gain) for RM1



- - - - - - $C_2(q^{-1})=1$

——— $C_2(q^{-1})=(1-0.4\ q^{-1})^3$

Figure 5.28 Plant (d=2) output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=1$, const. trace ad. gain) for regulation

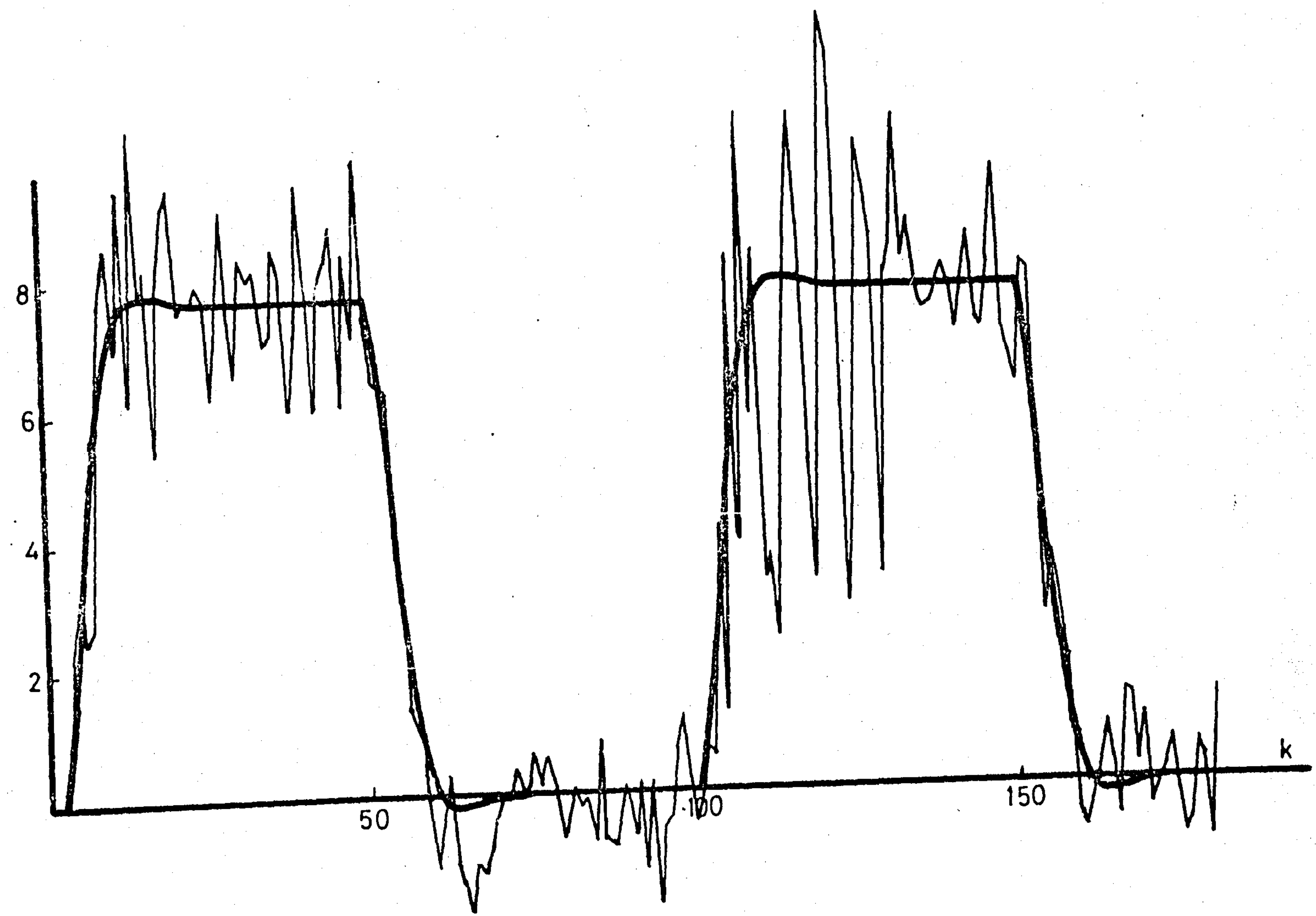


Figure 5.29 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. ad. gain) for RM1 in

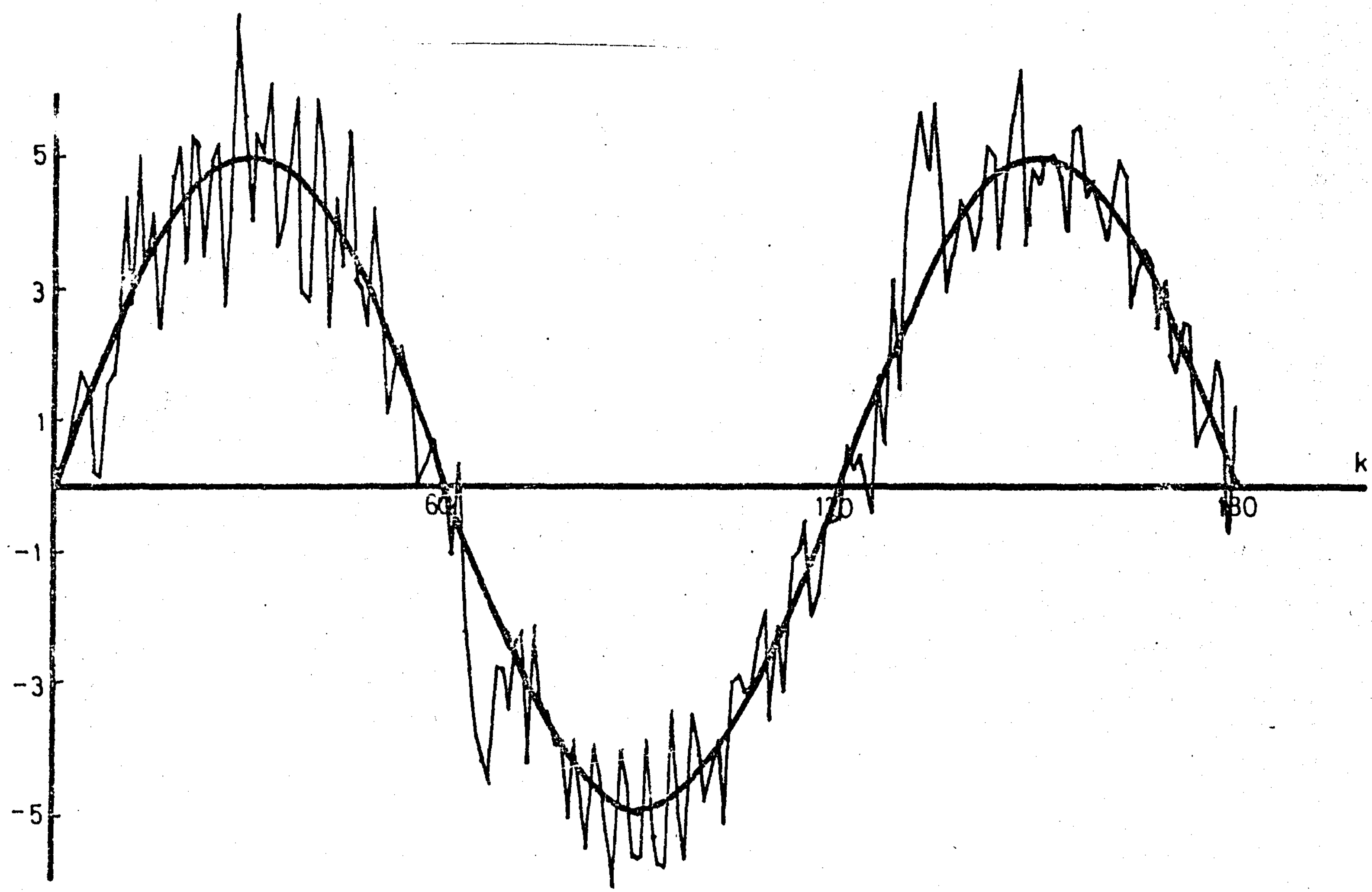


Figure 5.30 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. ad. gain) for RM3 in stochastic environment

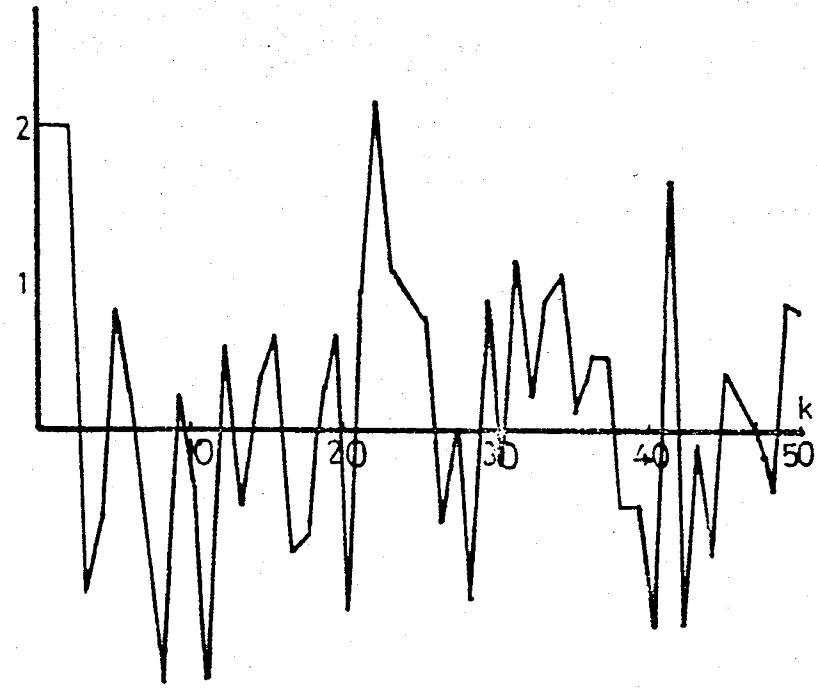


Figure 5.31 Plant output of MRAC ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$ const. ad. gain) for regulation in stochastic environment.

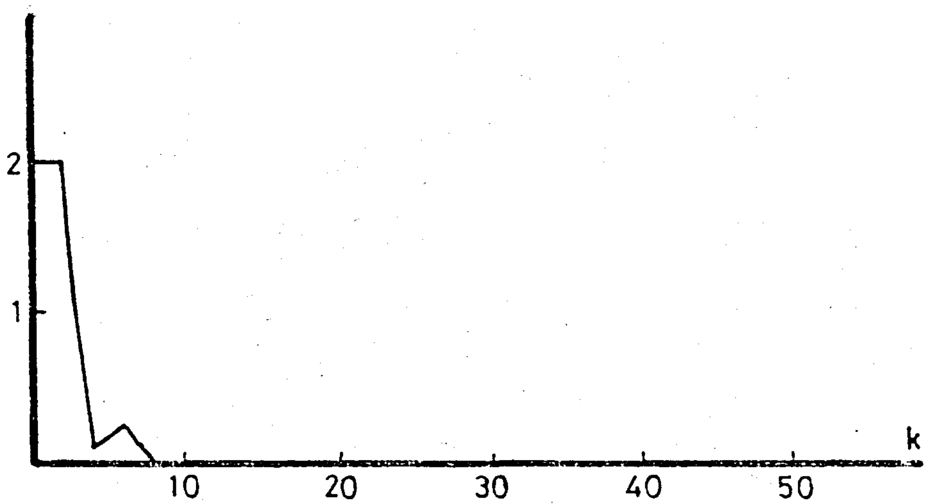


Figure 5.32 Plant output of combined MRAC-S-STR ($H_1(q^{-1})$ $=H_2(q^{-1})=1$, $C_2(q^{-1})=1-0.6\ q^{-1}$, const. ad. gain) for regulation in deterministic environment

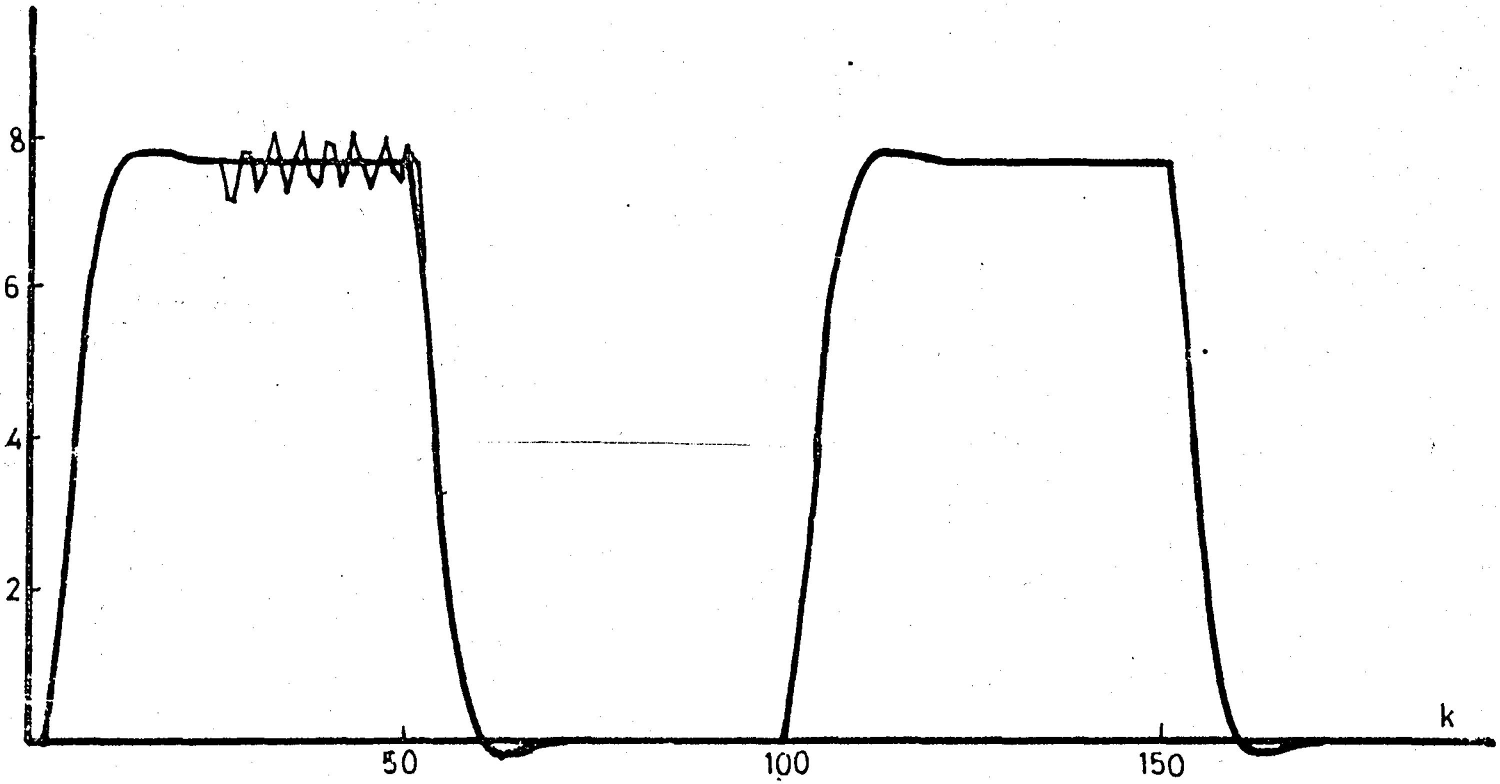


Figure 5.33 Plant output of combined MRAC-S-STR ($H_1(q^{-1})=H_2(q^{-1})=1$, $C_2(q^{-1})=1-0.6\,q^{-1}$, const. ad. g
for RM1 in deterministic environment

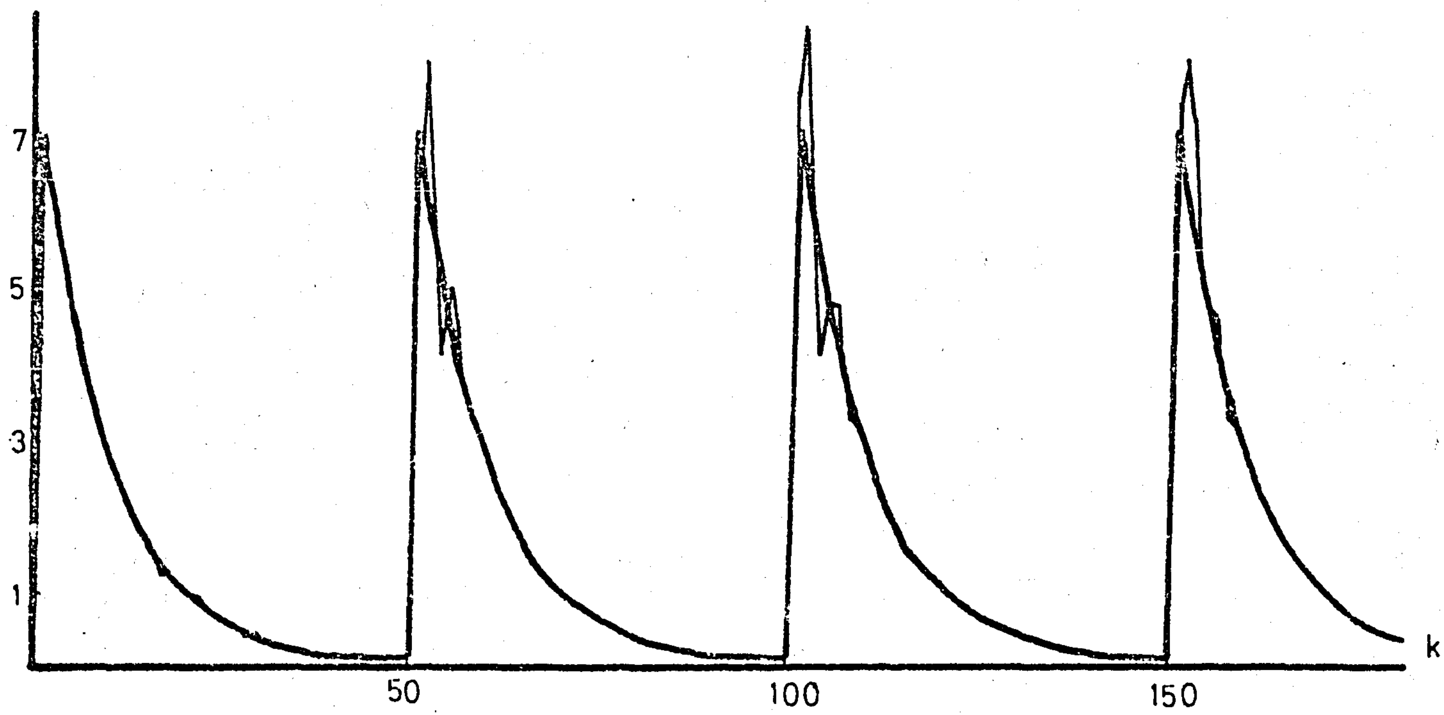


Figure 5.34 Plant output of combined MRAC-S-STR $(H_1(q^{-1})=H_2(q^{-1})=1,\ C_2(q^{-1})=1-0.6\ q^{-1}$, const. ad. gain) for RM2 in deterministic environment

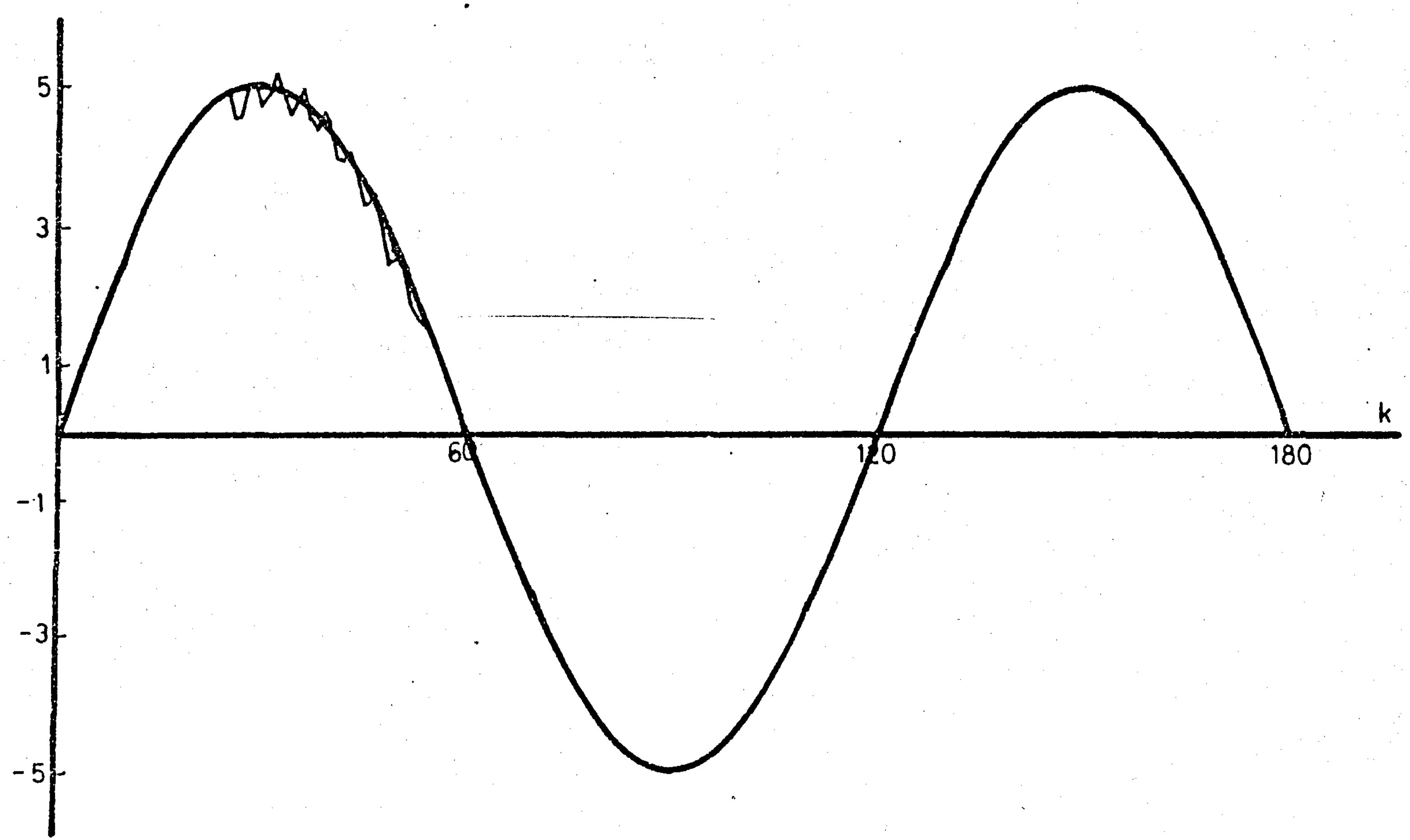


Figure 5.35 Plant output of combined MRAC-S-STR ($H_1(q^{-1})=H_2(q^{-1})=1$, $C_2(q^{-1})=1-0.6\ q^{-1}$, const. ad. ga

for RM3 in deterministic environment

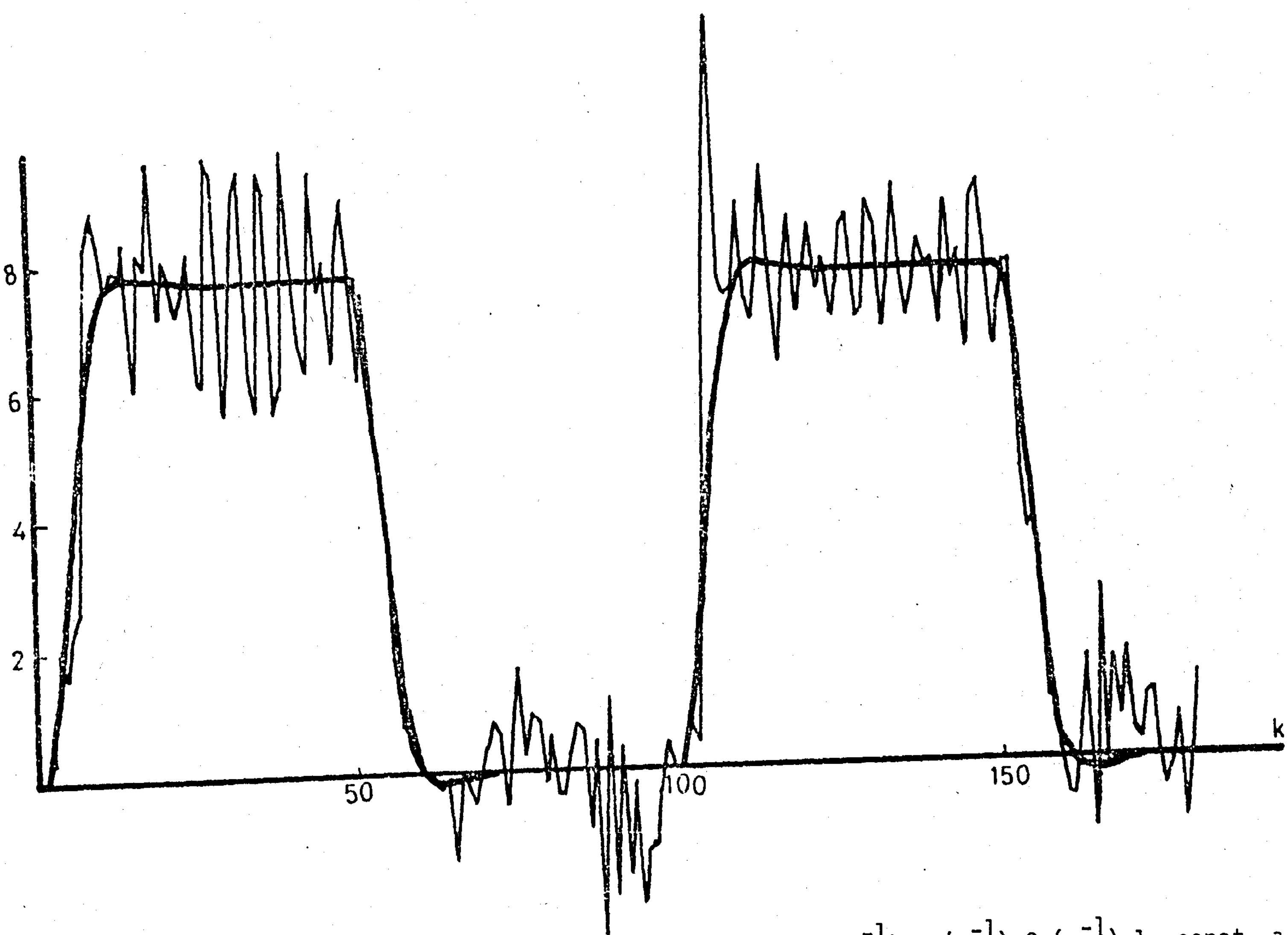


Figure 5.36 Plant output of combined MRAC-S-STR ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. ad. gain) for RM in stochastic environment

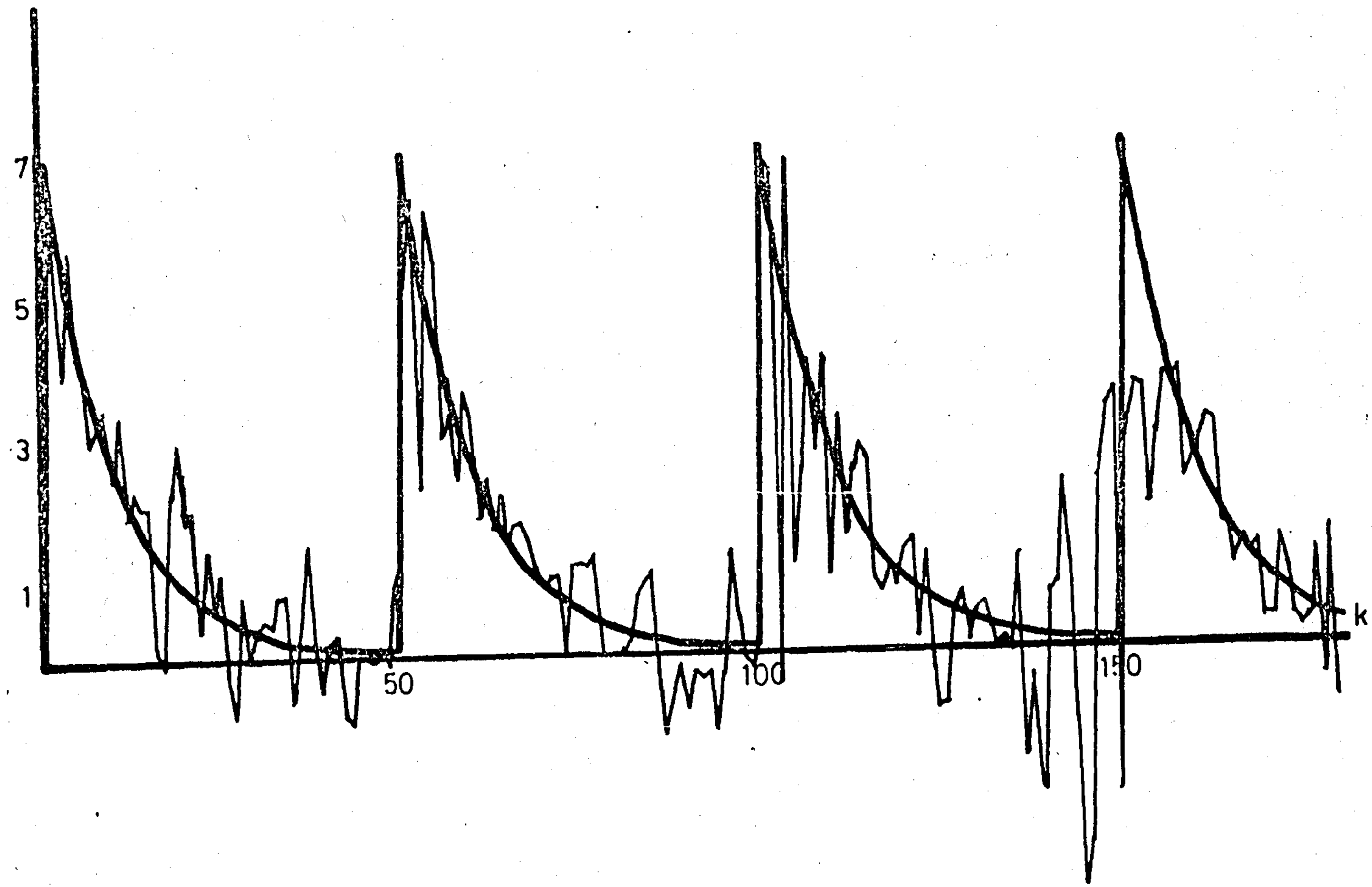


Figure 5.37 Plant output of combined MRAC-S-STR ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. ad. gain) for PM2 in stochastic environment.

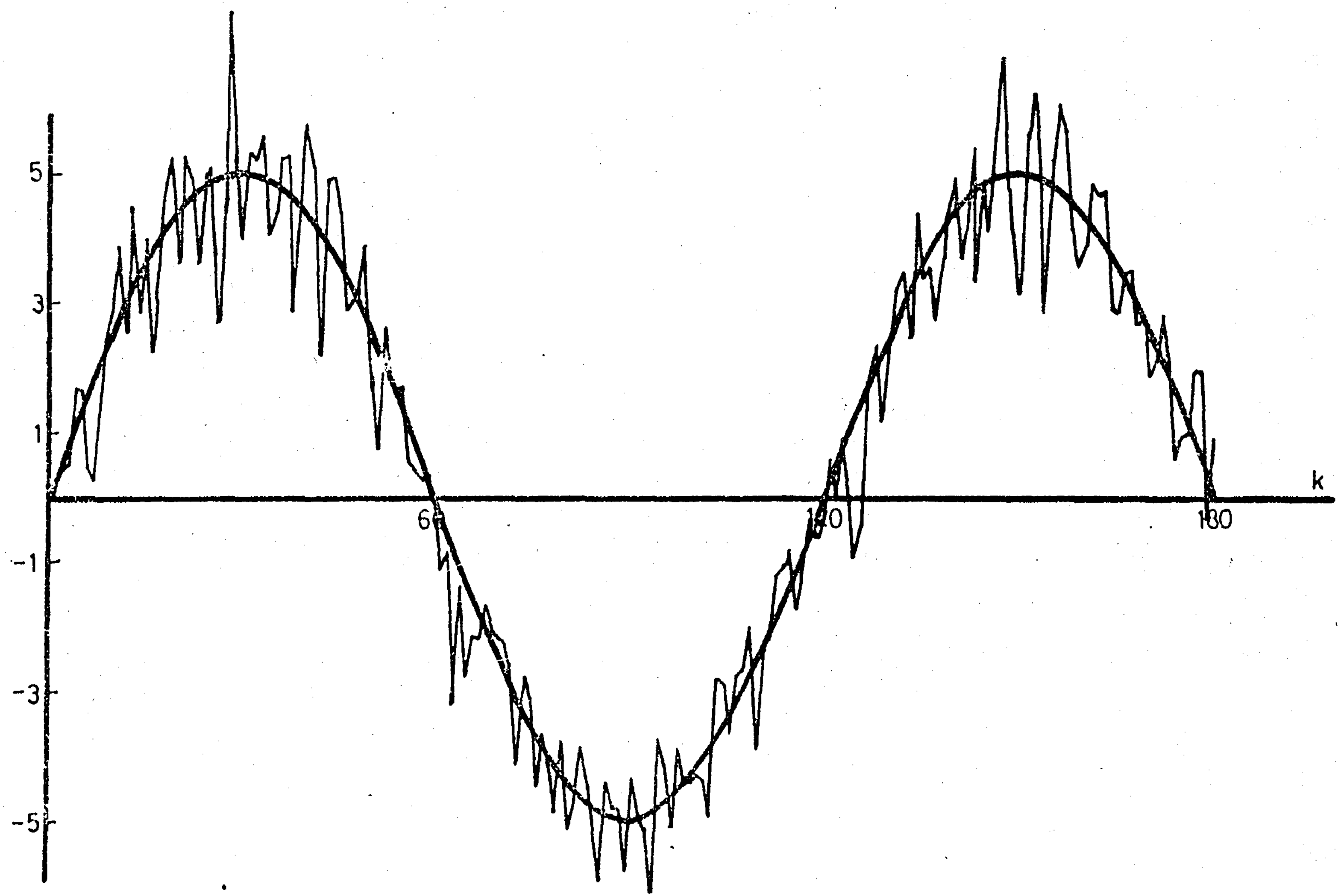


Figure 5.38 Plant output of combined MRAC-S-STR ($H_1(q^{-1})=H_2(q^{-1})=1$, const. ad. gain) for RM3 in stochastic environment

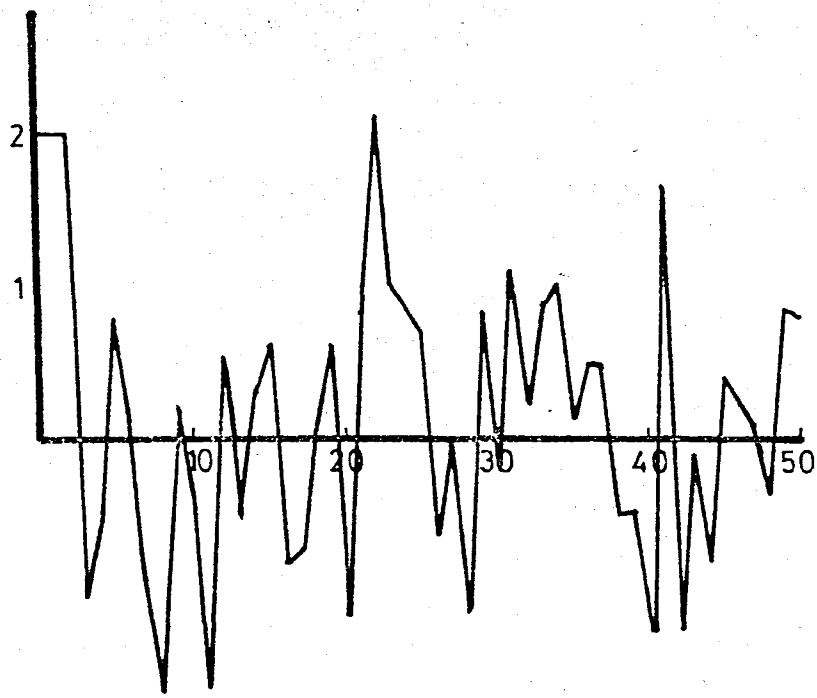


Figure 5.39 Plant output of combined MRAC-S-STR ($H_1(q^{-1})$ $=H_2(q^{-1})C_2(q^{-1})=1$, const. ad. gain) for regulation in stochastic environment

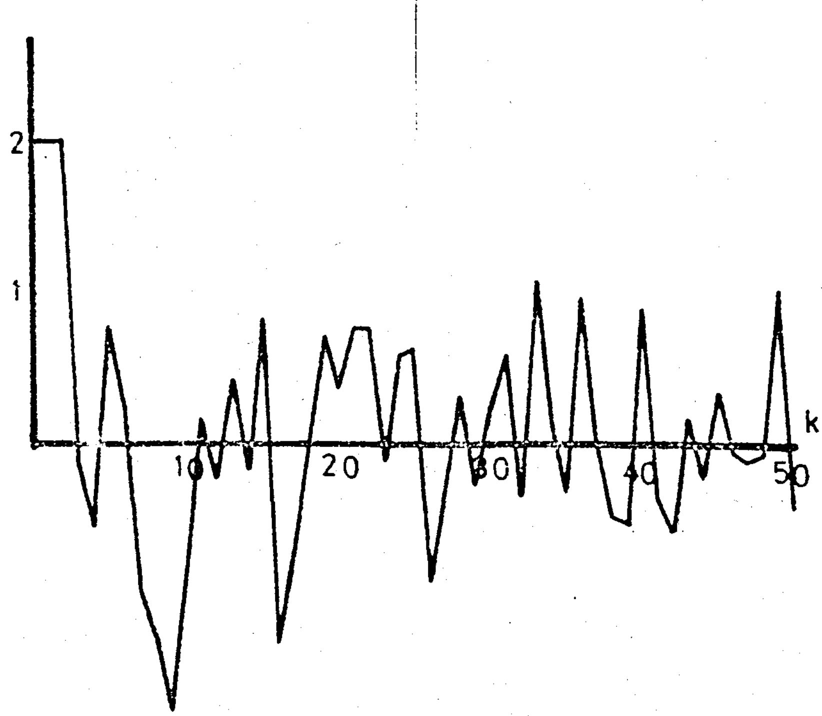


Figure 5.40 Plant output of combined MRAC-S-STR ($H_1(q^{-1})$ $=H_2(q^{-1})=C_2(q^{-1})$ $=1$ const. trace ad. gain) for regulation in stochastic environment.

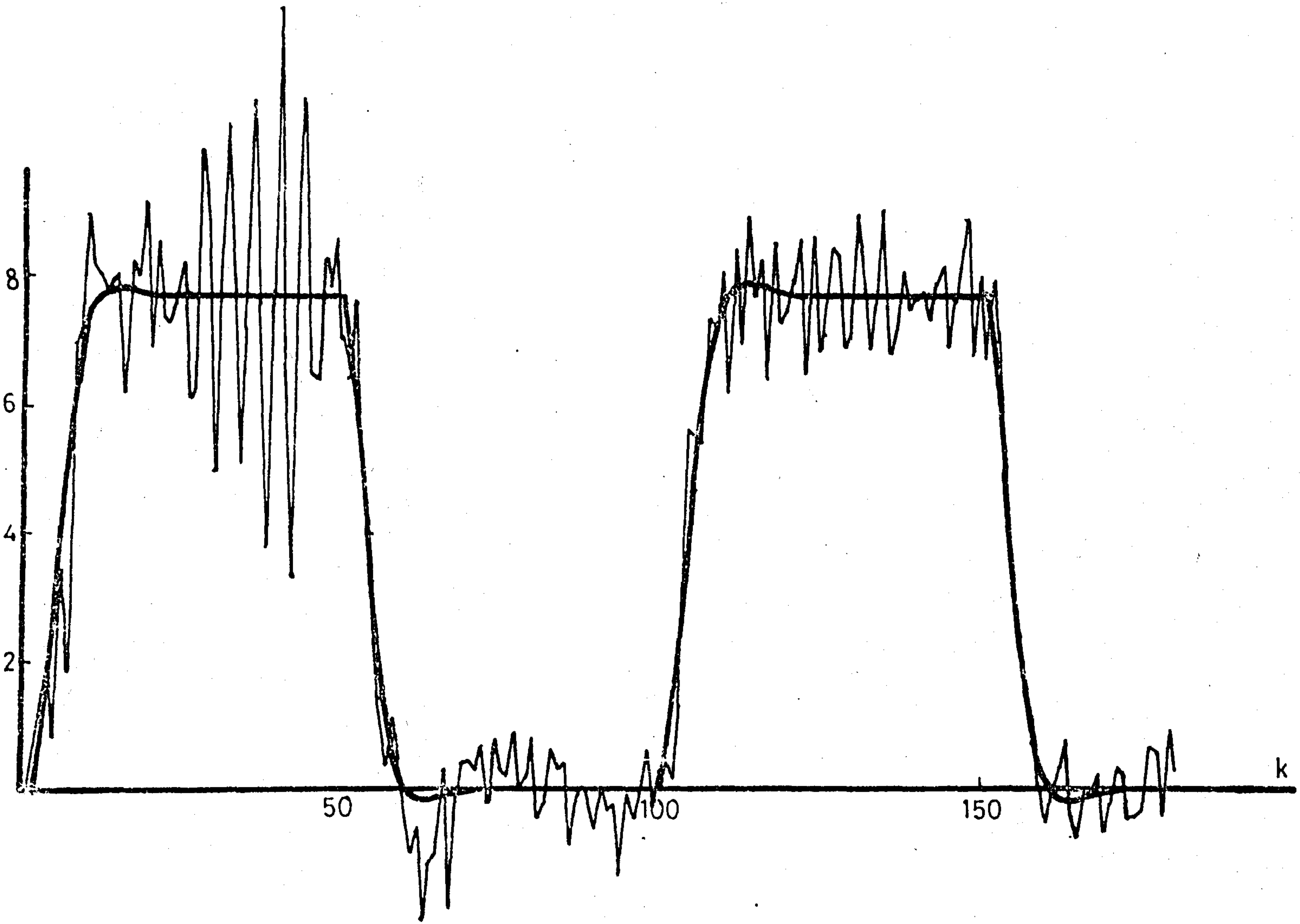


Figure 5.41 Plant output of combined MRAC-S-STR ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. trace ad. gain) for RM1 in stochastic environment

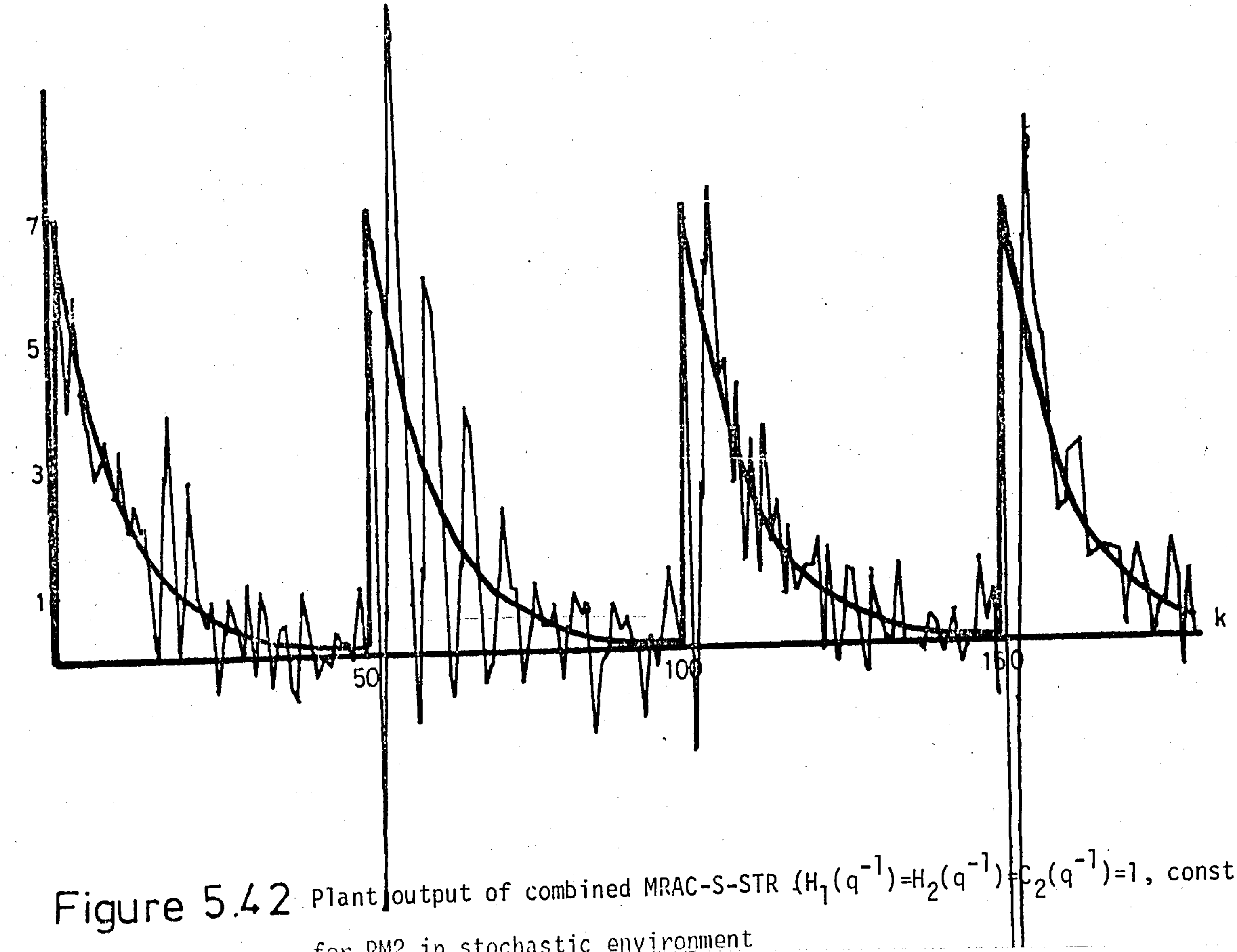


Figure 5.42 Plant output of combined MRAC-S-STR ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. trace ad. gain) for PM2 in stochastic environment

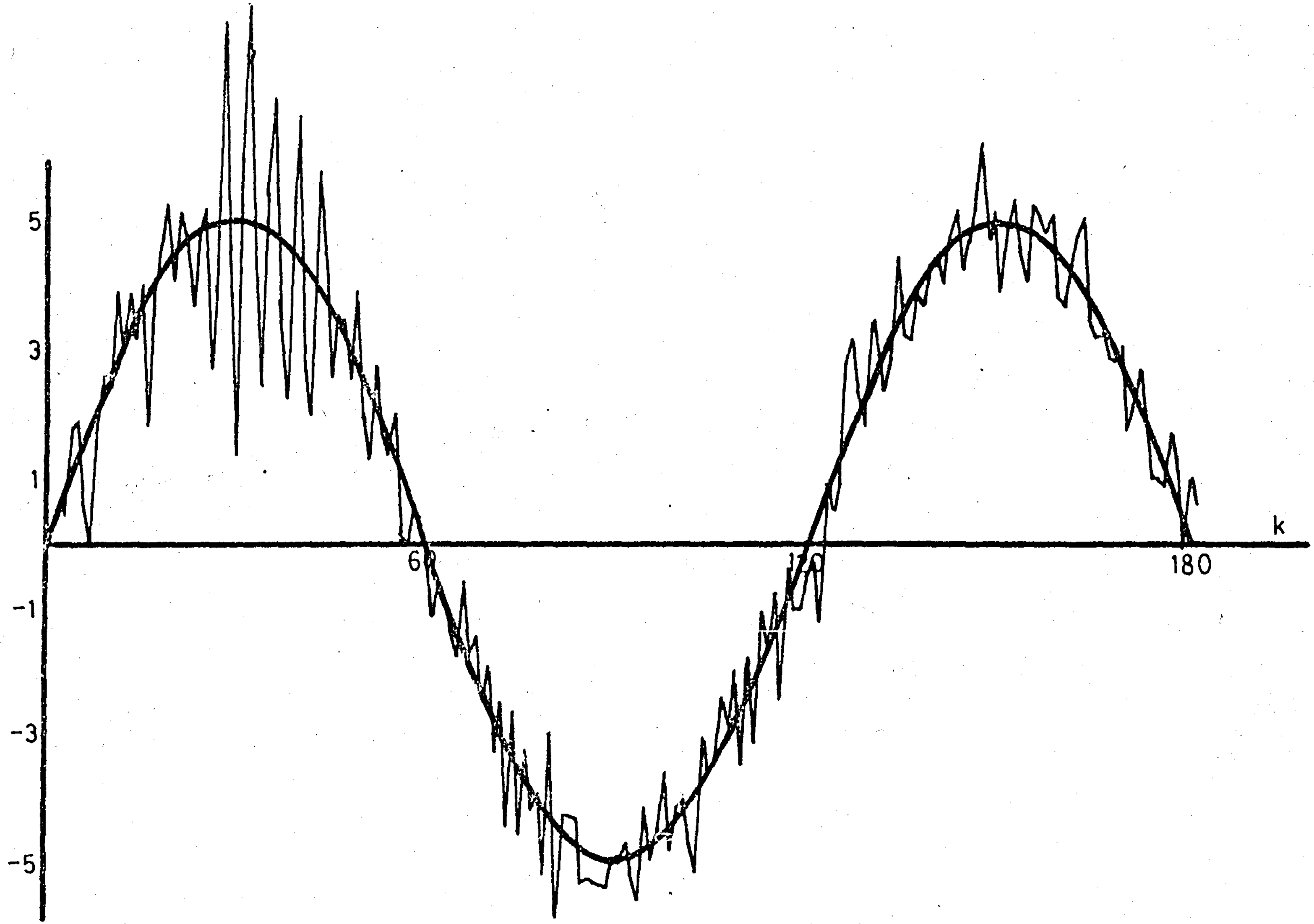


Figure 5.43 Plant output of combined MRAC-S-STR ($H_1(q^{-1})=H_2(q^{-1})=C_2(q^{-1})=1$, const. trace ad. gain) for RM3 in stochastic environment

VI. CONCLUSIONS AND AREAS OF FURTHER RESEARCH

In this thesis, a model reference adaptive control structure, which is applicable to minimum-phase single-input single-output discrete-time plants, is discussed. It is assumed that the plant delay and an upper bound for the degrees of polynomials $A(q^{-1})$ and $B(q^{-1})$ are known. Two equivalent strategies (explicit and implicit reference models) are presented which yield the same control structure.

Furthermore, the similarities between the model reference adaptive controllers and stochastic self-tuning regulators, where the control objectives are defined by ARMA models, are indicated and a definition of duality between then is given.

Starting from this duality an extension of model reference adaptive controllers is considered, which is capable of doing regulation and tracking in both deterministic and stochastic environment.

Various features of deterministic and stochastic model reference adaptive controllers are examined in simulations. It is shown that, the control performances are considerably affected by the adaptation gains used and polynomials chosen to define the control objectives. The simulations yield also the result that the performance of the combined MRAC-S-STR scheme is better than that of the MRAC, in a stochastic environment.

Although the simulations yield satisfactory results for MRAC in deterministic environment and for combined MRAC-S-STR in both deterministic and stochastic environments, still there are important problems to be solved, which present new subjects of research in this area :

i) To guarantee that the zeroes of the plant are cancelled with a bounded control input, the plant is assumed to be minimum-phase in all adaptive schemes considered in this thesis. The adaptive control of non-minimum-phase plants is still an open question.

ii) Another subject to be investigated is the extension of discrete-time model reference adaptive control techniques to the multi-input multi-output case.

iii) RM2 and RM3 used in simulations are reference models of the order less than that of the plant. Although the simulations yield satisfactory results in many cases, the theoretical analysis of MRAC using reduced order models is still missing.

iv) The model reference adaptive control of time-varying plants is another question to be answered.

v) On the other hand, another area of further research may be the adaptive control of (at least certain classes) of nonlinear systems.

vi) In simulations, it is seen that even in cases where the plant-model error is within reasonable limits, large values for the control input may be obtained. Therefore, also the problem of adaptive control with constraints on the control input and its variation must be considered in future.

vii) In Chapter IV., the disturbance in stochastic environment is assumed to be normally distrubuted with zero-mean. It is still an open question whether the combined MRAC-S-STR scheme given can deal with other types of disturbances.

viii) Theorem 4.1 limit ourselves to constant or decreasing adaptation gain algorithms, if convergence is to be assured. But simulations have shown, better results may be obtained with a constant trace adaptation gain. So the convergence analysis in stochastic environment must be extended to constant trace adaptation gain (and other types of gains) case in order to obtain convergence conditions in those cases.

ix) Lastly, a major problem in this area is the analysis of the effects of unmodeled plant dynamics to the performance of MRAC and combined MRAC-S-STR schemes. Such an analysis may allow a robust redesign of descrete time adaptive control schemes.

It is clearly seen, although growing effort has been made in this area in recent years, there are still question to be answered. As these questions are solved, which actually arise since the cases where adaptive control schemes are applied increase, the extension of the use of adaptive control will continue.

APPENDIX A

PROOF OF THE BOUNDEDNESS OF $\emptyset(k)$ AND CONVERGENCE TO ZERO OF $\varepsilon_c(k)$

For this proof, which is given in [8], one uses Theorem 3.1 and the following lemmas.

Lemma A.1. [18]. The vector $\emptyset(k)$ given in (2.22) containing the inputs and outputs of the plant in (2.1), which is minimum-phase, verifies

$$\| \emptyset(k-d) \| < C_1 + C_2 \max_{0 \leqslant \ell \leqslant k} | y(\ell) | \tag{A.1}$$

$$0 \leqslant C_1 < \infty , \quad 0 < C_2 < \infty$$

Lemma A.2. [18]. If

$$\lim_{k \to \infty} \frac{\varepsilon_c(k)}{\left[1+\emptyset^T(k-d) F_k \emptyset(k-d)\right]^{1/2}} = 0 \tag{A.2}$$

where $\{\varepsilon_c(k)\}$ is a real sequence, $\{\emptyset(k-d)\}$ is a real vector sequence and F_k is a sequence of positive definite real matrices, then subject to

i) $F_k^{-1} > \varepsilon\ F_o^{-1}$, $\varepsilon > 0$ $\quad F_o > 0 \qquad \forall k > 1$ (A.3)

ii) $$\|\emptyset(k-d)\| < C_3 + C_4 \max_{0 \leq \ell \leq k} |\varepsilon_c(k)| \tag{A.4}$$

$$0 \leq C_3 < \infty \quad , \quad 0 < C_4 < \infty$$

It follows that

$$\lim_{k \to \infty} \varepsilon_c(k) = 0 \tag{A.5}$$

and

$$\|\emptyset(k-d)\| < C_5,\ 0 < C_5 < \infty \qquad \forall k > 0 \tag{A.6}$$

From Theorem 3.1, one has

$$\lim_{k \to \infty} v(k) = 0 \tag{A.7}$$

which implies that

$$\lim_{k \to \infty} \varepsilon^*(k) = 0 \tag{A.8}$$

given that $H_1(q^{-1})$ and $H_2(q^{-1})$ are asymptotically stable polynomials (See (3.30)). Then we can write

$$\lim_{k \to \infty} \frac{\varepsilon^*(k)}{\left[1+\emptyset^T(k-d)\ F_k\emptyset(k-d)\right]^{1/2}} = 0 \tag{A.9}$$

because

$$\frac{1}{\left[1+\emptyset^T(k-d)F_k\emptyset(k-d)\right]^{1/2}} \leqslant 1 \tag{A.10}$$

Consider now

$$\Omega(k) = \frac{\bar{\varepsilon}(k)}{\left[1+\emptyset^T(k-d)F_k\emptyset(k-d)\right]^{1/2}}$$

$$= \frac{\left[\hat{p}(k-d) - \hat{p}(k)\right]^T\emptyset(k-d)}{\left[1+\emptyset^T(k-d)F_k\emptyset(k-d)\right]^{1/2}} \tag{A.11}$$

Using (3.31)

$$\Omega(k) = \frac{-\emptyset(k-d) \sum_{i=0}^{d-1} F_{k-i}\emptyset(k-d-i).v(k-i)}{\left[1+\emptyset^T(k-d)F_k\emptyset(k-d)\right]^{1/2}} \tag{A.12}$$

From Theorem 3.1 (with the additional requirement that $\lambda_1(k)$ and $\lambda_2(k)$ are such that $\lim F_k > \delta F_o$ $\delta > 0$) it results that $\hat{p}(k)$ is bounded, and from (2.1) and (3.18) are concludes that $u(k)$ and $y(k)$ cannot become unbounded for finite k. (Division in 3.18 can be avoided by choosing $\lambda_1(k)$ and $\lambda_2(k)$ properly.) Therefore,

$\emptyset(k-d)$ can eventually become unbounded only asymptotically. Therefore, the following relation is always true

$$\frac{\|\emptyset(k-d)\|}{\left[1+\emptyset^T(k-d)F_k\emptyset(k-d)\right]^{1/2}} \leqslant \frac{\|\emptyset(k-d)\|}{\left[1+\lambda\min(F_k)\|\emptyset(k-d)\|\right]^{1/2}} \leqslant C_6 < \infty \qquad (A.13)$$

and one concludes from equations (A.12) and (3.16)

$$\lim_{k \to \infty} \Omega(k) = 0 \qquad (A.14)$$

From (A.9), (A.11), (A.14) and (3.29), one obtains (A.2) of the lemma A.2, (A.4) is obtained as follows from (2.10), we have

$$|\varepsilon(k)| \geqslant |y(k)| - |y^M(k)| \geqslant |y(k)| - C_7; \quad 0 < C_7 < \infty \qquad (A.15)$$

since $y^M(k)$ is bounded.

From (3.23), and knowing that $C_2(q^{-1})$ is an asymptotically stable polynomial, we have

$$|\varepsilon(k)| \leqslant C_8 + C_9 \max_{0 \leqslant \ell \leqslant k} |\varepsilon_c(\ell)| \qquad (A.16)$$

$$0 \leqslant C_8 < \infty \quad , \quad 0 < C_9 < \infty$$

On the other hand,

$$|y(k)| > C_{10} + C_{11} \max_{0 \leq \ell \leq k} |y(\ell)| \tag{A.17}$$

$$0 \leq C_{10} < \infty; \quad 0 < C_{11} \leq \infty$$

Using (A.15)-(A.17) and (A.1) we obtain (2.11). Thus from lemma A.2, one concludes that lim $\varepsilon_c(k)=0$ and that $\emptyset(k-d)$ is bounded. And given that C_2-polynomial is asymptotically stable, we can also say that

$$\lim_{k \longrightarrow \infty} \varepsilon(k) = 0 \tag{A.18}$$

APPENDIX B
COMPUTER PROGRAM

```
********************************************************
*  THIS PROGRAM SIMULATES A CONTROL SYSTEM WHERE A MRAC  *
*  OR A COMBINED MRAC-S-STR SCHEME IS USED                *
********************************************************
C
C     DIMENSION OR TYPE DECLARATIONS
C
      DIMENSION UM(0:180),YM(0:182),U(0:180),Y(0:182)
      DIMENSION P(7),PO(6),FI(7),FIO(5)
      DIMENSION F(7,7),FIFI(7,7),FFIFI(7,7),FFIFIF(7,7),FFI(7)
      CHARACTER BOS,YIL,CIZ,ART,NOK,C(-90:115)
      INTEGER D
C
C     INPUT: PLANT DELAY(D), CONTROL STRUCTURE IDENTIFIER(ISTR),
C            MODEL IDENTIFIER(IM), ENVIRONMENT IDENTIFIER(IENV),
C            ADAPTATION GAIN IDENTIFIER(IG), A POSTERIORI ADAPTATION
C            ERROR IDENTIFIER(IH), TIME AT WHICH PARAMETERS CHANGE(KT)
C            SIMULATION ENDING TIME(IST)
C
      READ(6,*)D,ISTR,IM,IENV,IG,IH,KT,IST
C
C     INPUT: COEFFICIENTS OF C2-POLYNOMIAL
C
      READ(6,*)C21,C22,C23
C
C     INITIALIZE PLANT AND MODEL
C
      READ(6,*)(U(I),I=2-2*D,1),(Y(I),I=-D,D+1)
      READ(6,*)(UM(I),I=1,D+1),(YM(I),I=D-1,D+1)
C
C     INPUT: INITIAL GAIN MATRIX(F), GAIN UPDATING COEFFICIENTS
C            (AL1,AL2,AL), INITIAL PARAMETERS(P)
C
      L=D+4
      IF(ISTR.EQ.1) GO TO 10
      L=L+1
   10 DO 20 I=1,L
   20 READ(6,*)(F(I,J),J=1,L)
      READ(6,*)AL1,AL2,AL
      READ(6,*)(P(I),I=1,L)
C
C     DATA
C
      DATA BOS,YIL,CIZ,ART/' ','*','I','+'/
      DATA V,V1,V2,V3/0.,0.,0.,0./
      DATA ISEED,W,W1/1000000000,0.,0./
      DATA YMIN,YMAX/10.,0./
C
      L1=L-1
      WRITE(6,30)
   30 FORMAT(3X,'K',7X,'YM(K+D)',7X,'Y(K+D)',7X,'U(K)',7X,'V(K)',6X,'
     $+D)',28X,'P',/,2X,3('-'),5X,9('-'),5X,8('-'),5X,6('-'),5X,8('-'
     $X,7('-'),8X,43('-'))
C
      DO 320 K=2,IST
C
C     FORM FI(K-D) VECTOR
C
      DO 40 I=1,D+1
   40 FI(I)=U(K-D+1-I)
      DO 50 I=1,3
   50 FI(I+D+1)=Y(K-D+1-I)
      IF(ISTR.EQ.1) GO TO 60
```

```
C     CALCULATE THE A POSTERIORI ADAPTATION ERROR, V(K)
C
   60 DO 70 N=1,L
      FFI(N)=0.
      DO 70 I=1,L
   70 FFI(N)=FFI(N)+F(N,I)*FI(I)
      CALL VECMUL(L,FI,FFI,Q)
      CALL VECMUL(L,P,FI,VD)
      YC=Y(K)+C21*Y(K-1)+C22*Y(K-2)+C23*Y(K-3)
      VH=0.
      IF(IH.EQ.1) GO TO 80
      V3=V2
      V2=V1
      V1=V
      VH=-(C21*V1+C22*V2+C23*V3)
   80 V=(YC-VD+VH)/(1+Q)
C
C     PARAMETER ADAPTATION
C
      DO 90 I=1,L
   90 P(I)=P(I)+FFI(I)*V
C
C     FORM THE PO(K) AND FIO(K) VECTORS
C
      DO 100 I=1,L1
  100 PO(I)=P(I+1)
      DO 110 I=1,D
  110 FIO(I)=U(K-I)
      DO 120 I=1,3
  120 FIO(I+D)=Y(K+1-I)
      IF(ISTR.EQ.1) GO TO 130
      FIO(D+4)=YM(K+D-1)
C
C     THE MODEL OUTPUT FOR THE (K+1)ST STEP
C
  130 IF(IM-2)150,160,140
  140 IF(IM-3)160,170,180
C
C     REFERENCE MODEL 1
C
  150 UM(K)=1.
      IF(K.GE.50.AND.K.LT.100) UM(K)=0.
      IF(K.GE.150) UM(K)=0.
      YM(K+D)=.28*UM(K)+.22*UM(K-1)+1.9*YM(K+D-1)-1.23*YM(K+D-2)+.265*
     $(K+D-3)
      GO TO 190
C
C     REFERENCE MODEL 2
C
  160 IF((K+D-1).EQ.50.OR.(K+D-1).EQ.100.OR.(K+D-1).EQ.150) UM(K+D-1)=
      YM(K+D)=UM(K+D-1)+EXP(-0.1)*YM(K+D-1)
      GO TO 190
C
C     REFERENCE MODEL 3
C
  170 YM(K+D)=2*COS(3.14159/60)*YM(K+D-1)-YM(K+D-2)
      GO TO 190
C
C     REFERENCE MODEL 4 (REGULATION)
C
  180 YM(K+D)=0.
C
  190 YMC=YM(K+D)+C21*YM(K+D-1)+C22*YM(K+D-2)+C23*YM(K+D-3)
C
```

```
      CALL VECMUL(L1,PO,FIO,PFI)
      U(K)=(YMC-PFI)/P(1)
C
C     APPLY THE CONTROL TO THE PLANT
C     LET THE PLANT PARAMETERS CHANGE AT K=KT
C
      IF(K.GE.KT) GO TO 200
      Y(K+D)=U(K)+0.4*U(K-1)+2.1*Y(K+D-1)-1.53*Y(K+D-2)+0.365*Y(K+D-3
      GO TO 210
  200 Y(K+D)=.9*U(K)+.5*U(K-1)+2.3*Y(K+D-1)-1.96*Y(K+D-2)+.53*Y(K+D-3
C
C     IN CASE OF STOCHASTIC ENVIRONMENT LET THE DISTURBANCE ACT
C     ON THE PLANT OUTPUT
C
  210 IF(IENV.EQ.1) GO TO 240
      W=0.
      DO 220 I=1,12
      CALL RANSET(ISEED)
      RV=RANF()
      RV=RV/2
      ISEED=10*RV*ISEED
  220 W=W+RV
      W=W-3.
      IF(K.GE.KT) GO TO 230
      Y(K+D)=Y(K+D)+W-0.25*W1
      GO TO 240
  230 Y(K+D)=Y(K+D)+W-0.1*W1
  240 W1=W
C
C     UPDATE GAIN
C
      IF(IG.EQ.1) GO TO 300
      DO 250 I=1,L
      DO 250 J=1,L
  250 FIFI(I,J)=FI(I)*FI(J)
      CALL MATMUL(L,F,FIFI,FFIFI)
      CALL MATMUL(L,FFIFI,F,FFIFIF)
      IF(IG.EQ.3) GO TO 260
      F(I,J)=(F(I,J)-FFIFIF(I,J)/(AL1/AL2+Q))/AL1
      GO TO 300
  260 DO 270 I=1,L
      DO 270 J=1,L
  270 F(I,J)=F(I,J)-FFIFIF(I,J)/(AL+Q)
      AL1=0.
      DO 280 I=1,L
  280 AL1=AL1+F(I,I)/(10*L)
      DO 290 I=1,L
      DO 290 J=1,L
  290 F(I,J)=F(I,J)/AL1
C
C     OUTPUT
C
  300 WRITE(6,310)K,YM(K+D),Y(K+D),U(K),V,W,(P(I),I=1,L)
  310 FORMAT(2X,I3,5X,F7.4,6X,F8.4,3X,F8.3,5X,E3.2,5X,F7.3,7X,6(F6.3,
     $)
C
  320 CONTINUE
C
C     PLOT
C
      WRITE(6,330)
  330 FORMAT(10(/))
      DO 340 K=0,IST
      IF(Y(K).GT.YMAX) YMAX=Y(K)
```

```
      IE=INT(YMAX*SC)
      DO 370 K=0,IST
      IP=INT(Y(K)*SC)
      IS=INT(YM(K)*SC)
      DO 350 I=IB,IE
  350 C(I)=BOS
      C(0)=CIZ
      C(IS)=ART
      C(IP)=YIL
      WRITE(6,360)K,(C(I),I=IB,IE)
  360 FORMAT(1X,I3,123(A1))
  370 CONTINUE
C
      STOP
      END
C
C
C
      SUBROUTINE VECMUL(N,A,B,S)
C
C     THIS SUBROUTINE IS USED FOR VECTOR MULTIPLICATIONS
C
      DIMENSION A(N),B(N)
      S=0.
      DO 600 I=1,N
  600 S=S+A(I)*B(I)
      RETURN
      END
C
C
C
      SUBROUTINE MATMUL(N,A,B,C)
C
C     THIS SUBROUTINE IS USED FOR MATRIX MULTIPLICATIONS
C
      DIMENSION A(N,N),B(N,N),C(N,N)
      DO 800 I=1,N
      DO 800 J=1,N
      C(I,J)=0.
      DO 800 K=1,N
  800 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
14.42.21.UCLP, AA, P04      ,        0.277KLNS.
```

BIBLIYOGRAPHY

[1] Landau, I.D., Adaptive Controllers with Explicit and Implicit Reference Models and Stochastic Self-Tuning Regulators - Equivalence . and Duality Aspects," Proc.17th IEEE-CDC Conference, San Diego, U.S.A., 1979.

[2] Landau, I.D., "Combining Model Reference Adaptive Controllers and Stochastic Self-Tuning Regulators," Automatica, Vol. 18,77,1982.

[3] Ljung· L. and I.D. Landau, "Model Reference Adaptive Systems and Self-Tuning Regulators - Some. Connections," Proc. 7th IFAC Congress , Vol 3, 1973, 1978.

[4] Landau, I.D., "Model Reference Adaptive Control and Stochastic Self-Tuning Regulators - Towards Cross Fertilization," Proc. of AFOSR Workshop : Adaptive Control, Champaign, U.S.A. 1979.

[5] Dugard, L. and I.D. Landau, "Stochastic Model Reference Adaptive Controllers," Proc 19th IEEE-CDC Conference, Alburquerque, U.S.A. 1980.

[6] Lozano, L.R. and I.D. Landau, "Redesign of Explicit and Implicit Discrete Time Model Reference Adaptive Control Schemes," Int. Jour. Control, Vol 33, 247,1981.

[7] Gawthrop, P.J., "Same Interpretations of the Self-tuning Controllers," Proc.IEE, Vol 124,889,1977.

[8] Landau, I.D. and Lozano R. "Unification of Discrete Time Explicit Model Reference Adaptive Control Designs," Automatica, Vol 17,593,1981.

[9] Kucera, V., "Discrete Linear Control: The Polynomial Equation Approach," Wiley, New York, 1979.

[10] Aström, K.J. and B. Wittenmark, "On Self-tuning Regulators," Automatica, Vol 9,185,1973.

[11] Landau, I.D. and H.M. Silveria, "A Stability Theorem with Application to Adaptive Control," IEEE Trans Aut. Control, AC-24, 305, 1974.

[12] Goodwin, G.C. and K.S. Sin, Adaptive Filtering Prediction and Control, Prentice-Hall, New Jersey, 1984.

[13] Ionescu T. and R. Monopoly, "Discrete Model Reference Adaptive Control with an Augmented Error Signal," Automatica, Vol 13,507,1977.

[14] Landau, I.D., "Unbiased Recursive Identification Using Model Reference Adaptive Techniques," IEEE Trans. Aut. Control, AC-21,200,1976.

[15] Lozano L.R., "Independent Tracking and Regulation Adaptive Control with Forgetting Factor," Automatica, Vol 18,455,1982.

[16] Dugard, L. and I.D. Landau, "Recursive Output Error Identification Algorithms Theory and Evaluation," Automatica, Vol 16,443,1980.

[17] Dugard, L., I.D. Landau and H.M. Silveira," Adaptive State Estimation Using M.R.A.S. Techniques Convergence Analysis and Evaluation," IEEE Trans. Aut. Control, AC-25,1169,1980.

[18] Goodwin, G.C., P. Ramadge, P.Caines, "Discrete Time Multivariable Adaptive Control,"IEEE. Trans. Aut. Control AC-25, 449, 1980.

[19] Ljung L., "On Positive Real Transfer Functions and the Convergence of Some Recursive Schemes," IEEE Trans. Aut. Control, AC-22, 539, 1977.

REFERENCES NOT CITED

Aström, K.J., V. Borisson, L. Ljung and B. Wittenmark, "Theory and Applications of Self-tuning Regulators", Automatica, Vol 13, 457, 1977.

Aström, K.J. and B. Wittenmark, "Self-tuning Controllers Based on Pole-Zero Placement,"Proc IEE, Vol 127, 120, 1980.

Egardt, B., "Unification of Some Continous-Time Adaptive Control Schemes," IEEE Trans Aut. Control, AC-24, 588,1979.

Egardt, B., "Unification of Some Discrete-Time Adaptive Control Schemes," IEEE Trans. Aut. Control AC-25, 693,1980.

Egardt, B. "Stability Analysis of Discrete-Time Adaptive Control Schemes", IEEE Trans. Aut. Control, AC-25, 710,1980.

Johnson, C.R., "Input Matching, Error Augmentation Self-tuning and Output Error Identification:

Algorithm in Discrete Adaptive Model Following," Proc. 18th IEEE-CDC Conference, Fort Lauderdale, U.S.A.

Landau, I.D., " A Survey of Model Reference Adaptive Techniques - Theory and Applications," Automatica, Vol 10, 353, 1974.

Landau, I.D., "An Extension of a Stability Theorem Applicable to Adaptive Control," IEEE Trans, Aut. Control, AC-25, 1980

Landau, I.D., "Deterministic and Stochastic Adaptive Pole-Zero Placement for Minimum-Phase Systems," Proc. ISIS Workshop on Adaptive Control, Florence, Italy, 1982.

Landau, I.D., Adaptive Control: The Model Reference Approach, Dekker, New York, 1979.

Wellstead, P.E. and P. Zanker, "Servo Self-Tuners", Int. J. Control, Vol. 3C,27,1979.