COURNOT EQUILIBRIUM WITH FREE ENTRY FOR CAPITALISTIC AND WORKERS' ENTERPRISES

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COURNOT EQUILIBRIUM WITH FREE ENTRY FOR CAPITALISTIC AND WORKERS' ENTERPRISES

ABSTRACT

The purpose of this dissertation is to determine the Cournot Equilibrium with Free Entry for Capitalistic and Workers' Enterprises. An alternative Cournot-like free entry notion to Novshek's CEFE which we call Cournot-Sertel Equilibrium, is used to accomplish our purpose.

Our analysis is carried out in a model such that potential firms all producing a homogeneous good possess identical workers and a common technology where preferences and production functions follow the functional forms of the log-linear model developed previously by Sertel.

Making use of Cournot-Sertel Equilibrium concept; the size of the oligopoly, the production level, the capital and labor inputs of each firm, the number of workers in each firm, total output of the industry and the price of the good are determined at equilibrium for profit-maximizing firms, for utility maximizing workers' enterprises and for the first time for an industry where both coexist.

Furthermore, it is demonstrated that Cournot-Sertel Equilibrium is also Novshek's CEFE for capitalistic and workers' enterprises in a log-linear model.

KAPİTALİST FİRMALAR VE İŞÇİ FİRMALARI İÇİN
SERBEST GİRİŞLİ COURNOT DENGESİ

ÖZET

Bu tezin amacı kârı enbüyükleyen firmalar ile yararı enbüyükleyen işçi firmaları için Serbest Girişli Cournot Dengesi'ni belirlemektir. Bu amaç için, Novshek'in Serbest Girişli Cournot Dengesi kavramına alternatif olan ve Cournot-Sertel Dengesi olarak adlandırdığımız bir kavram kullanılmıştır.

Bu analiz tek tip mal üreten bütün firmalarda çalışan işçilerin ve kullanılan teknolojinin aynı nitelikte olduğu, tercihlerin ve üretim fonksiyonlarının daha önceden Sertel tarafından geliştirilmiş logaritmik doğrusal modele uyduğu bir ana model çerçevesinde yapılmıştır.

Cournot-Sertel denge kavramını kullanarak; denge noktasında, endüstride faaliyet gösteren firma sayısı, her firmanın üretim düzeyi, sermaye ve işçilik girdileri, her firmadaki işçi sayısı, endüstrinin toplam üretimi ve malın fiyatı kârı enbüyükleyen firmalar ve yararı enbüyükleyen işçi firmaları için saptanmıştır. Bu analiz, ayrıca literatürde ilk defa olarak iki tip firmanın da birarada bulunduğu bir endüstri için de yapılmış ve benzer sonuçlar elde edilmiştir.

Bunlara ek olarak, bu modelin varsayımları altında Cournot-Sertel Dengesi'nin aynı zamanda Novshek'in Serbest Girişli Cournot Dengesi olduğu gösterilmektedir.

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INTRODUCTION

The purpose of this dissertation is to determine the Cournot Equilibrium with Free Entry for capitalistic oligopolies and workers' enterprises.

An oligopoly is known as a common form of an economic organization where its members are firms, each producing a homogeneous good and using identical technology. In this study we let the output of anyone of these firms be a function of capital and labor inputs. Moreover, labor input of the firm is considered as the sum of labor inputs of the workers in this firm, and the price of the good in the market is defined to be a function of the total output of the industry, which is in fact the sum of each firm's output.

In the presence of perfect competition and free entry, there exist several questions which deserve more emphasis. Some of them can be stated as follows: Does equilibrium exist in the market? If it exists, what should the number of workers in each firm be, and what should the optimal labor input of each worker be? What should the optimal capital and labor inputs be in determining the firm's optimal production level? What are the total output of the industry and the price of a commodity? Furthermore, what should the number of active firms be in the market at the equilibrium?

In Cournot's Theory of Competition [1], these questions are studied among a given number of firms in a capitalistic (profit maximizing) oligopoly. The important point to be emphasized in this theory is that the number of firms is treated exogenously. Recently, Novshek [2] has proposed the notion Cournot Equilibrium with Free Entry (CEFE) for capitalistic industries to determine the size of the oligopoly by treating the number of firms as an endogenous variable. Laffont and Moreaux [9,10] modified Novshek's CEFE and adapted it to labor-managed (value added per laborer, maximizing type) oligopolies and have shown the nonexistence of such an equilibrium. Later, Sertel [3] has proposed another notion, which he argues to be a Cournot-like notion of free entry equilibrium, as an alternative to Novshek's CEFE providing answers to the above-stated questions. We will refer to this notion throughout this study as the Cournot-Sertel Equilibrium.

In this dissertation, first we apply the Cournot-Sertel Equilibrium notion to a capitalistic oligopoly, secondly to a labor-managed (Sertel's type workers' enterprises) oligopoly, and thirdly to an oligopoly where capitalist and labor-managed firms coexist, and then show the existence and uniqueness of such an equilibrium in all these cases. Finally, we show that Cournot-Sertel Equilibrium for capitalistic and for labor-managed oligopolies is also Novshek's CEFE. We demonstrate all these results in a log-linear model on which our analysis is based.

The organization of this dissertation is as follows:

Chapter II comprises the literature survey of capitalistic and labor-managed oligopolies and the development of the concept of Cournot's Equilibrium with Free Entry. Classical Cournot Equilibrium, Novshek's CEFE and Cournot-Sertel Equilibrium for capitalistic and labor-managed oligopolies are also presented in this chapter.

Chapters III and IV consist of the application of Cournot-Sertel Equilibrium (CSE) concept to capitalistic and workers' enterprises respectively, and reveal the fact that CSE is also Novshek's equilibrium for both cases.

Chapter V shows that capitalistic and workers' enterprises may coexist, they produce at the same level and in fact they are identical even though their objectives are completely different from each other.

Chapter VI concludes this study by providing of a summary of the results, and comparing CSE with Novshek's CEFE.

Our notation will follow the following convention. We will let $\mathbb{N}_+ = \{1,2,\ldots\}$, $\mathbb{R} = (-\infty,\infty)$, and $\mathbb{R}_+ = (0,\infty)$.

II. LITERATURE SURVEY

This chapter is devoted to defining basic classification of oligopolies encountered in literature and then providing a brief literature survey of the studies carried out on the concept of Cournot Equilibrium with Free Entry and its development up to date.

2.1 TYPES OF OLIGOPOLIES

In this dissertation, we deal with two kinds of single good producing oligopolies. An oligopoly means an industry that has countably few number of firms so that each one constitutes a fairly substantial share of the market, and the production level of each firm has an effect on the price of the produced good.

Throughout this dissertation we deal with an industry $M = \{1,2,...,m\}$ with size $m \in \mathbb{N}_+$ whose members ($i \in M$) are all firms each producing quantities $y_i \in \mathbb{R}_+$ of an identical product whose price is given through an inverse demand function $p(Y)$ to be dependent solely on the total quantity $Y = \sum_{i \in M} y_i$ of the output supplied and sold, and where $p'(Y) < 0$. The output is given by $y_i = f[K_i, L_i]$ where $K_i \in \mathbb{R}_+$ and $L_i \in \mathbb{R}_+$ are capital and labor inputs of the i^{th} firm, respectively. Furthermore, the labor input is given by $L_i = \sum_{j=1}^{n_i} x_{ij}$ where $x_{ij} \varepsilon \mathbb{R}_+$ is the labor input of the j^{th} worker in the i^{th} firm and $n_i \varepsilon \mathbb{N}_+$ is the number of workers in the i^{th} firm. Also, C_0 is the fixed cost of production and set up for each $i \varepsilon M$.

2.1.1 Capitalistic Oligopolies

In literature, an oligopoly consisting of only capitalist firms are known as a capitalistic economy, and capitalist firms are known as profit-maximizing firms. Profit maximization problem of the i^{th} firm in the industry can be formulated as:

$$\max \pi_i = \max\{py_i - C(y_i) - C_0\}$$

where π_i is the profit of the i^{th} firm, and $C(y_i)$ is the cost function at production level y_i.

The aim of the i^{th} firm is to determine the output level which yields the maximum profit to it. The first-order conditions for profit maximization are obtained such that marginal revenue should equal marginal cost, i.e.,

$$\frac{\partial \pi_i}{\partial y_i} = p + \frac{\partial p}{\partial y_i} y_i - \frac{\partial C(y_i)}{\partial y_i} = 0 \quad \text{for all } i \varepsilon M .$$

2.1.2 Labor-Managed Oligopolies

A labor-managed oligopoly is a system which consists of only labor-managed firms in which workers come together and form collectives or partnerships to run firms. The members of the firm hire capital and purchase inputs other than capital, and they sell the products of the firm at the best prices they can obtain in the market.

In literature, one encounters frequently two kinds of labor-managed firms. One is explained in Ward [4], Domar [5], and Vanek [6] and is named Wardian L-M firm. The objective of this type of a firm, instead of attempting to maximize profit, is the maximization of net income per laborer or, in other words, maximization of labor's average value added.

The problem underlying in such a firm is to determine the size of labor-force and the amount of capital for an optimum level of output so as to maximize value added per worker. The objective function can be expressed as follows:

$$\max V_i = \frac{py_i - \rho K_i - C_o}{L_i}$$

where V_i is the value-added per laborer in the i^{th} firm, i.e. the residual output per laborer after paying off factors of production other than labor, and ρ is the market rental for capital goods in units of the output.

The first-order conditions of the problem are given by

$$\frac{\partial V_i}{\partial K_i} = \frac{\partial p}{\partial K_i} y_i + \frac{\partial y_i}{\partial K_i} p - \rho = 0$$

$$\frac{\partial V_i}{\partial L_i} = [(\frac{\partial p}{\partial L_i} y_i + \frac{\partial y_i}{\partial L_i} p)L_i - py_i]/L_i^2 = 0$$

for all $i \in M$.

In this type of a labor-managed (L-M) firm if cooperation exists among workers of the firm, each worker contributes a fixed amount of labor, so that $x_{ij} = x$ for some $x \in \mathbb{R}_+$, and then $L_i = n_i x$ for all $i \in M$ and $j \in \mathbb{N}_+$. If x is taken to be unity, then $L_i = n_i$. The problem turns out to be

$$\max V_i = \max \left\{\frac{py_i - \rho K_i - C_o}{n_i}\right\} .$$

The most important deficiency of this type of L-M firm arises from the fact that each worker-partner of the firm contributes a fixed amount of labor. A worker of this firm can be fired when it is better to share among the fewer, or can be hired when it is necessary to put more labor input into the production. In other words, a member of the Wardian L-M firm is not a partner of the firm.

The other type of labor-managed firms is explained in Sertel's [7] workers' enterprises. Partners of a workers' enterprise, instead of maximizing their value added per worker, maximize utility, a function of both income and labor input contribution. Furthermore, utility of a typical worker in the i^{th} firm is defined to be the difference between value added per laborer and disutility of work of a typical member in this firm. Utility maximization problem can be formulated as follows:

$$\max u_{ij} = \max\left\{\frac{py_i - \rho K_i - C_o}{n_i} - bx_{ij}^{\gamma}\right\} \quad , \ \gamma > 1, \ \ b > 0$$

where x_{ij}, u_{ij} and bx_{ij}^{γ} are the labor input, utility level and disutility level of the j^{th} worker in i^{th} firm.

If cooperation exists between workers of the i^{th} firm, i.e. $x_{ij} = x_i$ and $u_{ij} = u_i$ for all $j \in \mathbb{N}_+$, then the problem turns out to be

$$\max u_i = \max\{\frac{py_i - \rho K_i - C_o}{n_i} - bx_i^\gamma\} \quad .$$

In our analysis, we confined ourselves to this representation by assuming that the members of a workers' enterprise cooperate with each other.

It must be emphasized that as explained in Sertel [7], by an enterprise or a firm it is meant a partnership. That is, a firm is a workers' enterprise if and only if its partners are all workers in the firm and its workers are all partners in the firm. Partnership or a partner to a firm is gained through voluntary negotiation between potential and existing partners. The rights and duties of the partners are specified in a contract called "partnership deed". In order to become a partner of the firm, it is necessary to purchase partnership deed, and the retirement of an existing partner is achieved only by purchasing that partner's partnership deed.

THE COMMON SENSE OF THE WORKER-PARTNERSHIP MARKET:

The importance of this type of a market arises from the fact that the worker-partnership (deed) market plays a determinative role in the long-run equilibrium of the market.

To see how this market operates, the demand for worker-partnership deeds and the supply of these are analyzed. Denoting the utility level of a typical member in i^{th} workers' enterprise of size n_i by $\overline{u}_i(n_i)$, and the level of utility elsewhere in the economy by $\underline{u}$, a worker in the sector is willing to pay a demand price up to

$$D_i(n_i + 1) = \overline{u}_i(n_i + 1) - \underline{u}$$

of his income to quit his present job and join the workers' enterprise. On the other hand, his joining the workers' enterprise would cause each current member a loss of $d\bar{u}_i/dn_i$ so the workers' enterprise would admit the marginal applicant to worker-partnership only as long as he is willing to pay at least the supply price

$$S_i(n_i) = -n_i \frac{d\bar{u}_i}{dn_i} \quad .$$

Thus, the workers' enterprise achieves its long-run equilibrium size only when the market for its workers-partnership deeds is equilibrated, i.e. when

$$D_i(n + 1) = S_i(n_i) \quad . \tag{2.1.1}$$

For n_i large enough, $D_i(n_i+1) = \bar{u}_i(n_i+1)-\underline{u}$ is approximated by $D_i(n_i) = \bar{u}_i(n_i)-\underline{u}$. This approximation is employed in the present analysis in order to maintain the continuity and differentiability of (2.1.1). Although for small n_i this approximation diverges from the true situation considerably, as the number of workers increases its approximating effect becomes marginal. Furthermore, long-run equilibrium is achieved when

$$\bar{u}_i(n_i) - \underline{u} = -n_i \frac{d\bar{u}_i}{dn_i} \quad . \tag{2.1.2}$$

The main distinction between Sertel's and Wardian L-M firms is that in Sertel's workers' enterprises the inputs of participants are not fixed. The input level of each participant is chosen so as to maximize his utility by the following first-order conditions:

$$\frac{\partial u_i}{\partial x_i} = (\frac{\partial p}{\partial x_i} y_i + \frac{\partial y_i}{\partial x_i} p)/n_i - b\gamma x_i^{\gamma-1} = 0 \quad ,$$

$$\frac{\partial u_i}{\partial K_i} = (\frac{\partial p}{\partial K_i} y_i + \frac{\partial y_i}{\partial K_i} p - \rho)/n_i = 0 \quad .$$

2.2 THE CONCEPT OF COURNOT EQUILIBRIUM

After defining capitalistic and labor-managed oligopolies, we are ready to provide a literature survey on the concept of Cournot Equilibrium and its development up to date.

2.2.1 Classical Cournot Equilibrium

In this section Classical Cournot Equilibrium is defined briefly for a capitalistic oligopoly where each firm maximizes its profit with respect to quantity as the strategic variable.

Definition (2.2.1): Let M be an industry whose members are each ascribed an identical cost function $C:\mathbb{R}_+ \to \mathbb{R}_+$. A Classical Cournot Equilibrium of M is any point $\bar{y}(m) = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m) > 0$ such that for each $i \in M$ and $y_i \geq 0$

$$P(\sum_{j \in M} \bar{y}_j)\bar{y}_i - C(\bar{y}_i) - C_0 \geq P(\sum_{\substack{j \in M \\ i \neq j}} \bar{y}_j + y_i)y_i - C(y_i) - C_0$$

where y_i is the output level of the i^{th} firm.

Note that, much later, a Classical Cournot Equilibrium came to be known as a Nash Equilibrium in a game where each player maximizes its individual profits.

Cournot Equilibrium as defined above is established among a given number of competitors. In the case of Unlimited Free Competition, Cournot[1] showed that number of firms in the industry grows beyond bound and Dimitriev [8] discussed the consequences of Cournot's theory of competition. The weak point of Cournot's theory is that the equilibrium size i.e. number of firms, is treated exogenously.

2.2.2 Novshek's CEFE

Recently, Novshek [2] has proposed a new equilibrium notion for capitalistic firms, namely "Cournot Equilibrium with Free Entry" (CEFE) by including the number of active firms in the market as an endogenous variable. Then he showed that if firms are small relative to the market, then there exists Cournot Equilibrium with Free Entry, and moreover any CEFE is approximately competitive.

The properties of Novshek's CEFE are:

i. the outputs of active forms yield a Cournot equilibrium (without free entry), i.e. a Nash Equilibrium with quantity as the strategic variable;

ii. all firms make non-negative profit;

iii. there is no profit incentive for additional firms to enter the market. In other words, no potential entrant can earn strictly positive profit by entry, assuming the aggregate output of all other firms to be fixed.

Definition (2.2.2.1): A Novshek's CEFE for an industry M of firms each with an identical cost functions $C:\mathbb{R}_+ \to \mathbb{R}_+$ is a point $\bar{y}(m) = (\bar{y}_1, \bar{y}_2, .., \bar{y}_m) > 0$ such that

i) for each $i \in M$ and $y_i \geq 0$

$$p(\sum_{j\in M} \bar{y}_j)\bar{y}_i - C(\bar{y}_i) - C_o \geq p(\sum_{\substack{j\in M \\ i\neq j}} \bar{y}_j + y_i)y_i - C(y_i) - C_o$$

ii) for any $y > 0$

$$p(\sum_{j\in M} \bar{y}_j + y)y - C(y) - C_o \leq 0 \quad .$$

Although Novshek [2] by (ii) of his above verbal definition stated that all firms make nonnegative profit, he did not formulate this condition mathematically. Condition (ii) of Definition (2.2.2.1) simply states that there is no profit incentive for additional firms to enter the market.

However, a modification of Novshek's CEFE for capitalistic firms which is called Revised Novshek CEFE in [3], is suggested by Laffont and Moreaux [9] who included non-negative profit condition, and can be stated as follows:

Definition (2.2.2.2): Revised Novshek CEFE for an industry M of firms each with an identical cost functions $C:\mathbb{R}_+ \to \mathbb{R}_+$ is a point $\bar{y}(m) = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m) > 0$ such that

i) for each $i \in M$

$$p(\sum_{j\in M} \bar{y}_j)\bar{y}_i - C(\bar{y}_i) - C_o \geq 0$$

ii) for all $i \in M$ and $y_i \geq 0$

$$p(\sum_{j \in M} \bar{y}_j)\bar{y}_i - C(\bar{y}_i) - C_0 \geq p(\sum_{\substack{j \neq i \\ j \in M}} \bar{y}_j + y_i)y_i - C(y_i) - C_0$$

iii) for any $y > 0$

$$p(\sum_{j \in M} \bar{y}_j + y)y - C(y) - C_0 < 0 \quad .$$

In the above formulation, non-negative profit condition is included by (i) and free entry condition is modified by strict inequality as shown in (iii).

In addition to capitalistic firms, Laffont and Moreaux [9] have adapted Novshek's CEFE to the case of competition existing in Wardian labor-managed oligopolies and have shown the nonexistence of such an equilibrium.

Definition (2.2.2.3): Revised Novshek CEFE for a Wardian labor-managed industry M of firms is any pair $(\bar{K},\bar{L})$, where $\bar{K} = (\bar{K}_1,\bar{K}_2,\ldots,\bar{K}_m) \geq 0$, $\bar{L} = (\bar{L}_1,\bar{L}_2,\ldots,\bar{L}_m) > 0$ denoting $\bar{y}_i = f[\bar{K}_i,\bar{L}_i]$ for all $i \in M$ such that

i) for all $i \in M$

$$\frac{p(\sum_{j \in M} \bar{y}_j)\bar{y}_i - \rho\bar{K}_i - C_0}{\bar{L}_i} \geq w$$

where w is per capita added value available elsewhere in the economy;

ii) for all $i \in M$ and for any $y_i \geq 0$, $K_i \geq 0$ and $L_i > 0$;

$$\frac{p(\sum_{j \in M} \bar{y}_j)\bar{y}_i - \rho\bar{K}_i - C_0}{\bar{L}_i} \geq \frac{p(\sum_{\substack{j \neq i \\ j \in M}} \bar{y}_j + y_i)y_i - \rho K_i - C_0}{L_i}$$

iii) for any $y \geq 0$, $K \geq 0$, $L > 0$ such that $y = f[K,L]$

$$\frac{p(\sum_{j\epsilon M} \bar{y}_j + y)y - \rho K - C_o}{L} < w \quad .$$

Not it would be preferable to interpret the conditions imbedded in the above definition in relation with the model under consideration. Condition (i) says that the per capita added value in the L-M firm is at least equal to the per capita added value in the economy. Condition (ii) is the Classical Cournot Equilibrium condition with capital and labor as the strategic variables and finally Condition (iii) is the free entry condition for the potential firms, which is similar to Novshek's free entry condition.

A recent paper by Hill and Waterson [11] analyzes Cournot Equilibrium with Free Entry for labor-managed and capitalistic oligopolies. Their free entry is defined as "firms will enter (leave) an entrepreneurial industry if positive (negative) profits are being earned by the existing firm and firms will enter (leave) a L-M industry if positive (negative) profits per worker are being earned by the existing firms". But we disagree with that kind of free entry notion because only positive (negative) profits or profits per worker of the existing firms are considered, and the profitability of the potential firm is not taken into account before entering.

2.2.3 Cournot-Sertel Equilibrium

Sertel [3] has proposed an alternative free entry notion to Novshek's which helps us in determining the size of the oligopoly. Now, it remains necessary to define this notion for capitalistic oligopolies and for workers' enterprises.

The properties of Cournot-Sertel Equilibrium (CSE) for a capitalistic oligopoly are:

i. all firms make nonnegative profit;

ii. the outputs of the active firms yield a Cournot Equilibrium (without Free Entry), i.e. a Nash Equilibrium with quantity as the strategic variable;

iii. at least one of the potential or the existing firms should earn strictly negative profit by the entrance of the potential firms once a new Cournot Equilibrium is established among potential and existing firms.

These can be mathematically expressed by the following definition.

Definition (2.2.3.1): Cournot-Sertel Equilibrium for a capitalistic industry M of firms, each with an identical cost functions $C:\mathbb{R}_+ \to \mathbb{R}_+$ is any point $\bar{y}(m) = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m) > 0$ such that

i) for all $i \in M$

$$p(\sum_{j \in M} \bar{y}_j)\bar{y}_i - C(\bar{y}_i) - C_o \geq 0;$$

ii) for all $i \in M$ and for any $y_i \geq 0$

$$p(\sum_{j \in M} \bar{y}_j)\bar{y}_i - C(\bar{y}_i) - C_o \geq p(\sum_{\substack{j \neq i \\ j \in M}} \bar{y}_j + y_i)y_i - C(y_i) - C_o$$

iii) for each industry M' such that $M \subsetneq M' \subset N$, the first condition fails whether or not the second condition holds for $i \in M'$.

Cournot-Sertel Equilibrium can also be defined for workers' enterprises in the presence of partnership market.

<u>Definition (2.2.3.2):</u> CSE for industry M of workers' enterprises is any triplet $(\overline{K}, \overline{x}, \overline{n})$ where $\overline{K} = (\overline{K}_1, \overline{K}_2, \ldots, \overline{K}_m) \geq 0$, $\overline{x} = (\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_m) > 0$ and $\overline{n} = (\overline{n}_1, \overline{n}_2, \ldots, \overline{n}_m)$ and for each $i \in M$, $\overline{n}_i \in \mathbb{N}_+$. Furthermore, for each $i \in M$, $\overline{y}_i = f[\overline{K}_i, \overline{L}_i]$ where $\overline{L}_i = \overline{n}_i . \overline{x}_i$ such that

i) for all $i \in M$ and $b > 0$, $\gamma > 1$

$$\frac{p(\sum_{j \in M} \overline{y}_j)\overline{y}_i - \rho\overline{K}_i - C_o}{\overline{n}_i} - b\overline{x}_i \geq \underline{u}$$

where $\underline{u}$ is the "reserve utility" which is the least a worker can count on obtaining elsewhere in the economy.

ii) for all $i \in M$ and for any $y_i \geq 0$, $K_i \geq 0$, $x_i \geq 0$ and $n_i \in \mathbb{N}_+$

$$\frac{p(\sum_{j \in M} \overline{y}_j)\overline{y}_i - \rho\overline{K}_i - C_o}{\overline{n}_i} - b\overline{x}_i^\gamma \geq \frac{p(\sum_{\substack{j \neq i \\ j \in M}} \overline{y}_j + y_i)y_i - \rho K_i - C_o}{n_i} - bx_i^\gamma \quad ;$$

iii) for all $i \in M$

$$\frac{p(\sum_{j \in M} \overline{y}_j)\overline{y}_i - \rho\overline{K}_i - C_o}{\overline{n}_i} - b\overline{x}_i^\gamma - \underline{u} = -\overline{n}_i \frac{d}{dn_i}\left(\frac{p(\sum_{j \in M} \overline{y}_j)\overline{y}_i - \rho\overline{K}_i - C_o}{n_i} - bx_i^\gamma\right)\Big|_{n}$$

iv) for each industry M', $M \subsetneq M' \subset N$, the first condition fails whether or not the second condition holds for some $i \in M'$.

Note that, (iii) is an approximation to (2.1.1).

A close examination of the above definition reveals that the utility of a typical member of the industry M is at least equal to the reserve utility elsewhere in the economy by Condition (i). Condition (ii) is the Classical Cournot Equilibrium condition where the capital of the i^{th} firm and the labor input one of its typical members constitute the strategic variables in the maximization of typical partners utility. Furthermore, Condition (iii) says that there is no utility incentive for a worker to join or leave the workers' enterprise at the equilibrium in the presence of partnership market. Condition (iv) on the other hand implies that for any bigger industry M' utility level of at least one worker in the industry M' will fall below the reserve utility level.

We must pay attention to the fact that the difference between CSE and Novshek's CEFE lies in the Free Entry Condition when a new firm wants to enter the market, according to Novshek's CEFE, the existing firms stick to the same level of production whatever the production level of a new firm is. They do not change their respective production levels even when a new firm tends to enter the market; in fact the potential firm is not taken into account whereas in Cournot-Sertel Equilibrium the existing firms, once they observe that a new firm shows a tendency to enter the market, adjust their output levels as if there is one more firm in the market. In this approach existing firms treat the potential firm as a new competitor and their reactions are adjusted accordingly, because the additional output introduced by the potential firm influence the price of the good. Furthermore, the profit of each one of the existing firms in a capitalistic oligopoly and the utility level of each worker in the labor-managed oligopoly are also affected by the entry of a new firm. From our point of view, to stick to the same production level ignores the newcomer and enforces the existing firms to behave irrationally. Thus, Cournot-Sertel Equilibrium is more appealing and rational in the spirit of Cournot and our analysis will be mainly based on CSE concept.

III. COURNOT EQUILIBRIUM WITH FREE ENTRY FOR A CAPITALISTIC OLIGOPOLY

This chapter is devoted to analyzing a capitalistic industry M, the members of which are profit-maximizing firms. The analysis aims at determining the Cournot-Sertel Equilibrium of the industry under consideration. In the procedure of achieving this, basic data are provided in Section 3.1, and the existence and uniqueness of the optimum production level maximizing the profit function of each firm is proven by showing the strict concavity property of the profit function in Section 3.2. Next the symmetry of the firms in the market is verified under their optimum behaviour in Section 3.3. Then CSE for industry M is determined, so that the equilibrium size of the market is uniquely specified in Section 3.4. Finally CSE obtained in Section 3.4 is proven to satisfy the conditions of Novshek's Equilibrium in Section 3.5.

3.1 BASIC DATA

In this section, we introduce the reader with basic relations inherent in the log-linear model chosen for our purposes mentioned so far. The inverse demand function is taken to be continuous, twice differentiable where its first derivative with respect to total output of industry is strictly negative. Production function of each firm is assumed to be decreasing return to scale with constant capital and labor elasticities which result in a strictly concave function, enabling us to obtain the unique global maximum. The basic advantage of this log-linear model is that it is analytically tractable so that we can solve the problem and tabulate the results explicitly.

Now, we provide the reader with the necessary data concerning the model for a capitalistic oligopoly. Denoting a capitalistic oligopoly by the subscript c, we let

x_c be the amount of labor input of a typical worker,

n_{ic} be the number of workers of the i^{th} firm,

L_{ic} be the labor input of the i^{th} firm given by

$$L_{ic} = n_{ic} . x_{ic} \quad , \tag{3.1.1}$$

K_{ic} be the capital input of the i^{th} firm,

y_{ic} be the output of the i^{th} firm given by

$$y_{ic} = K_{ic}^{\alpha} L_{ic}^{\beta} \quad , \tag{3.1.2}$$

where $0 < \alpha < 1$ and $0 < \beta < 1$ are capital and labor elasticities of output, respectively, and $0 < (\alpha + \beta) \leq 1$,

m_c be the number of firms of the oligopoly,

Y_c be total output of the industry given by

$$Y_c = \sum_{i=1}^{m_c} y_{ic} \quad , \tag{3.1.3}$$

p be the price of the commodity given by

$$p = r\, Y_c^{\theta} \quad , \tag{3.1.4}$$

where $-1 \leq \theta < 0$ is the total output elasticity of price and $r > 0$,

π_{ic} be the profit of the i^{th} firm given by

$$\pi_{ic} = py_{ic} - wL_{ic} - \rho K_{ic} - C_0 \quad , \tag{3.1.5}$$

where w and ρ are factor prices of labor and capital, respectively, and C_0 is the fixed cost of the firm.

The objective of a firm in the industry is to maximize its profit. In our procedure of achieving this, we will first show that the objective function π_{ic} is concave with respect to variables K_{ic} and L_{ic} and then we will derive, the first-order necessary conditions which yield us the unique optimum (global maximum) of the problem due to the concavity property.

3.2 EXISTENCE AND UNIQUENESS OF A GLOBAL MAXIMUM

The aim of this section is to prove that there exists a unique maximum to the objective function of the i^{th} firm. In order to achieve this, the objective function is shown to be concave as stated by the following result.

Corollary (3.2.1): In the log-linear model, π_{ic} is strictly concave with respect to K_{ic} and L_{ic}.

Proof: Letting the model be defined by Equations (3.1.1)-(3.1.5), the problem is to maximize (3.1.5). Then the necessary conditions are obtained as the partial derivatives of the objective function with respect to K_{ic} and L_{ic}, and given by

$$\frac{\partial \pi_{ic}}{\partial K_{ic}} = \alpha \frac{py_{ic}}{K_{ic}} (1 + \theta \frac{y_{ic}}{Y_c}) - \rho = 0 \quad , \tag{3.2.1}$$

and

$$\frac{\partial \pi_{ic}}{\partial L_{ic}} = \beta \frac{py_{ic}}{L_{ic}} (1 + \theta \frac{y_{ic}}{Y_c}) - w = 0 \quad , \tag{3.2.2}$$

To show π_{ic} is concave, it suffices to show the associated Hessian matrix to be negative definite. Thus the Hessian is formed from (3.2.1) and (3.2.2) as

$$H = \begin{bmatrix} \alpha \frac{py_{ic}}{K_{ic}^2} [(\alpha k-1)k + \alpha(k-1)d] & \alpha\beta \frac{py_{ic}}{K_{ic} \cdot L_{ic}} [k^2+(k-1)d] \\ \alpha\beta \frac{py_{ic}}{K_{ic} \cdot L_{ic}} [k^2+(k-1)d] & \beta \frac{py_{ic}}{L_{ic}^2} [(\beta k-1)k + \beta(k-1)d] \end{bmatrix}$$

where $k = 1+(\theta y_{ic}/Y_c) < 1$, $d = 1-(y_{ic}/Y_c) < 1$ and $d < k$. Since $-1 \leq \theta < 0$, $0 < (y_{ic}/Y_c) \leq 1$ and $0 < \alpha, \beta \leq 1$, $(\alpha+\beta) \leq 1$, we get $k^2 > k^2+(k-1)d$ and $(\alpha k-1)k+\alpha(k-1)d < 0$. Thus

$$|H_{11}| = |\alpha \frac{py_{ic}}{K_{ic}^2} [(\alpha k-1)k + \alpha(k-1)d]| < 0$$

which in turn yields

$$|H| = \alpha\beta(\frac{py_{ic}}{K_{ic} \cdot L_{ic}})^2 \{k^2 - (\alpha+\beta)k[k^2 + (k-1)d]\} > 0$$

So the desired result follows immediately.

Now the main result of this section is stated by the following Corollary.

Corollary (3.2.2): There exist unique $(\overline{K}_{ic},\overline{L}_{ic})$ which achieve the maximization of π_{ic}.

Proof: The uniqueness and existence of $(\overline{K}_{ic},\overline{L}_{ic})$ follow from the strictly concave property of π_{ic} as proven by Corollary (3.2.1).

Now, the necessary conditions (3.2.1) and (3.2.2) determine the unique maximum point, so that

$$\alpha p y_{ic}(1 + \theta \frac{y_{ic}}{Y_c}) = \rho K_{ic} \quad , \tag{3.2.3}$$

and

$$\beta p y_{ic}(1 + \theta \frac{y_{ic}}{Y_c}) = w L_{ic} \quad , \tag{3.2.4}$$

In turn, the values of $(\overline{K}_{ic},\overline{L}_{ic})$ satisfying (3.2.3) and (3.2.4) specify the maximum value of π_{ic} given by

$$\overline{\pi}_{ic} = \overline{p}\overline{y}_{ic}[1 - (\alpha+\beta)(1 + \theta \frac{\overline{y}_{ic}}{\overline{Y}_c})] - C_o \quad , \tag{3.2.5}$$

where $\overline{p}$, $\overline{y}_{ic}$ and $\overline{Y}_c$ are optimal with respect to $(\overline{K}_{ic},\overline{L}_{ic})$.

3.3 SYMMETRY OF THE MARKET

Our aim in this section is to show that all firms in the capitalistic oligopoly are identical at their respective optimal reactions, and this property is explained by the symmetric behaviour of the firms and proven by the following theorem.

<u>Theorem (3.3.1):</u> All capitalist firms are symmetric at optimality.

<u>Proof:</u> Letting Equation (3.2.4) be valid for both i^{th} and j^{th} firms, and $L_{ic} = n_{ic}x_c$, Equation (3.2.4) can be rewritten as

$$\beta p y_{ic}(1 + \theta \frac{y_{ic}}{Y_c}) = w n_{ic} x_c \quad ,$$

and

$$\beta p y_{jc}(1 + \theta \frac{y_{jc}}{Y_c}) = w n_{jc} x_c \quad .$$

Dividing the first equation by the second one and substituting $K_{ic}^{\alpha} n_{ic}^{\beta} x_c^{\beta}$ for y_{ic}, we get

$$\frac{K_{ic}^{\alpha} n_{ic}^{\beta}}{K_{jc}^{\alpha} n_{jc}^{\beta}} \cdot \frac{(Y_c + \theta y_{ic})}{(Y_c + \theta y_{jc})} = \frac{n_{ic}}{n_{jc}} \quad . \tag{3.3.1}$$

Furthermore, dividing (3.2.3) by (3.2.4), we obtain

$$\frac{\beta}{\alpha} = \frac{w}{\rho} \frac{n_{ic} x_c}{K_{ic}}$$

which implies

$$K_{ic} = \frac{w\alpha}{\beta\rho} n_{ic} x_c \qquad \text{for} \qquad i = 1, \ldots, m_c \ . \tag{3.3.2}$$

Then substituting (3.3.2) into (3.3.1) for i^{th} and j^{th} firms, we get

$$(\frac{n_{ic}}{n_{jc}})^{1-(\alpha+\beta)} = \frac{Y_c + \theta y_{ic}}{Y_c + \theta y_{jc}} \quad , \tag{3.3.3}$$

Writing (3.3.2) for i^{th} and j^{th} firm and dividing one by the other, we get

$$\frac{K_{ic}}{K_{jc}} = \frac{n_{ic}}{n_{jc}} \equiv a \qquad \text{where} \qquad a \in R_+$$

which implies

$$K_{ic} = aK_{jc} \quad , \tag{3.3.4}$$

$$n_{ic} = an_{jc} \quad . \tag{3.3.5}$$

Since we know that $y_{ic} = K_{ic}^{\alpha}L_{ic}^{\beta} = K_{ic}^{\alpha}n_{ic}^{\beta}x_c^{\beta}$ by (3.1.2), substituting (3.3.4), (3.3.5) and (3.1.2) into (3.3.3), we get

$$\frac{Y_c + \theta x_c^{\beta}K_{jc}^{\alpha}n_{jc}^{\beta}a^{\alpha+\beta}}{Y_c + \theta x_c^{\beta}K_{jc}^{\alpha}n_{jc}^{\beta}} = a^{1-(\alpha+\beta)} \quad . \tag{3.3.6}$$

For the sake of simplicity, let

$$h(a) = \frac{Y_c + \theta x_c^{\beta}K_{jc}^{\alpha}n_{jc}^{\beta}a^{\alpha+\beta}}{Y_c + \theta x_c^{\beta}K_{jc}^{\alpha}n_{jc}^{\beta}}$$

and

$$g(a) = a^{1-(\alpha+\beta)} \quad .$$

Then the first and second derivatives of h(a) and g(a) with respect to a are

$$h'(a) = \frac{\theta x_c^{\beta}K_{jc}^{\alpha}n_{jc}^{\beta}(\alpha + \beta)a^{\alpha+\beta-1}}{Y_c + \theta x_c^{\beta}K_{jc}^{\alpha}n_{jc}^{\beta}} \quad ,$$

$$g(a) = [1 - (\alpha + \beta)]a^{-(\alpha+\beta)} \quad ,$$

and

$$h''(a) = \frac{\theta[(\alpha + \beta) - 1](\alpha + \beta)x_c^{\beta}K_{jc}^{\alpha}n_{jc}^{\beta}a^{\alpha+\beta-2}}{Y_c + \theta x_c^{\beta}K_{jc}^{\alpha}n_{jc}^{\beta}} ,$$

$$g''(a) = -[1 - (\alpha + \beta)](\alpha + \beta)a^{-(\alpha+\beta)-1} .$$

Since $0 < \alpha+\beta \leq 1$, then

$$h'(a) < 0 \quad , \quad g'(a) > 0$$

$$h''(a) > 0 \quad , \quad g'(a) < 0 \quad .$$

So the behaviour of g(.) and h(.) can be schematically depicted as in Fig. (3.3.1) which reveals that the only solution satisfying h(a) = g(a) for $a \in R_+$ is a = 1.

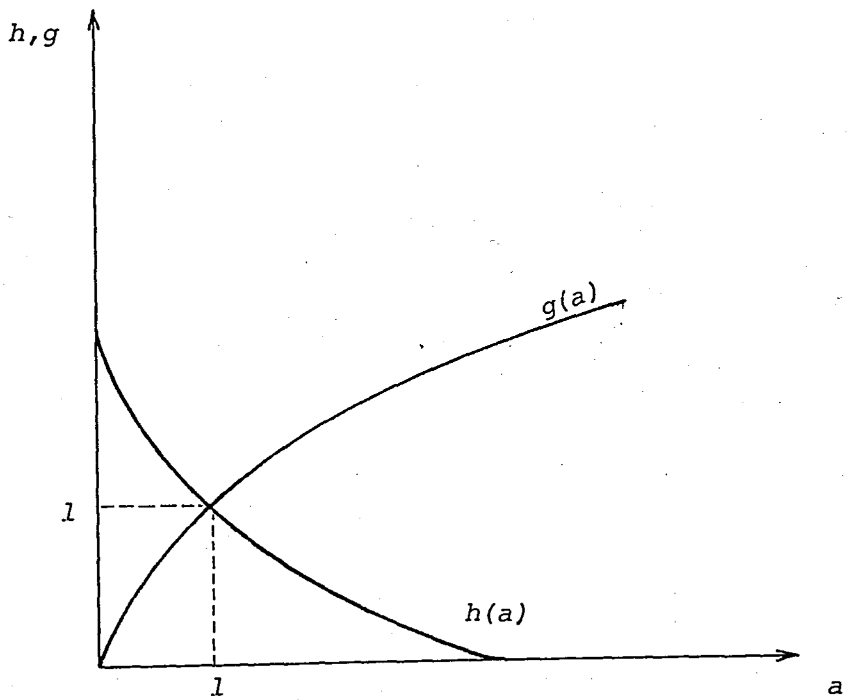

Figure 3.3.1 - The relationship between h(.) and g(.) for a capitalistic oligopoly.

Note that

$$g(0) = 0 \quad , \quad g(\infty) = \infty$$

$$h(0) = \frac{Y_c}{Y_c + \theta x_c^{\beta} K_{jc}^{\alpha} n_{jc}^{\beta}} \quad , \quad h(\infty) = -\infty$$

Therefore, the unique solution a = 1 is the unique solution which in turn yields for all $i \varepsilon M$

$$\overline{K}_{ic} = \overline{K}_c \quad ,$$

$$n_{ic} = n_c \quad ,$$

$$\overline{L}_{ic} = \overline{L}_c \quad ,$$

$$\overline{y}_{ic} = \overline{y}_c \quad ,$$

$$\overline{\pi}_{ic} = \overline{\pi}_c \quad .$$

Thus, all the firms in a capitalistic oligopoly are symmetric. As the above relation show, number of workers in each firm, the capital and labor inputs, the output and consequently the profit of each firm turn out to be equal at their respective optimal behaviour.

3.4 COURNOT-SERTEL EQUILIBRIUM

Now, we are at a point to determine CSE for a capitalistic oligopoly.

By the symmetry of the firms at their optimal reactions as proven by Theorem (3.3.1), Equations (3.1.1)-(3.1.5) can be rewritten as

$$\overline{L}_c = n_c . x_c \quad , \qquad (3.4.1)$$

$$\overline{y}_c = \overline{K}_c^{\alpha} \, \overline{L}_c^{\beta} \quad , \qquad (3.4.2)$$

$$\overline{Y}_c = m_c . \overline{K}_c^{\alpha} \, \overline{L}_c^{\beta} \quad , \qquad (3.4.3)$$

$$\bar{p} = r.Y_c^{\theta} \quad , \tag{3.4.4}$$

$$\bar{\pi}_c = \bar{p}\,\bar{y}_c - w\bar{L}_c - \rho\bar{K}_c - C_o \quad , \tag{3.4.5}$$

and the first-order conditions (3.2.3) and (3.2.4) can also be transformed into

$$\alpha\bar{p}\,\bar{y}_c(1 + \theta\frac{\bar{y}_c}{\bar{Y}_c}) = \rho\bar{K}_c \quad , \tag{3.4.6}$$

and

$$\beta\bar{p}\,\bar{y}_c(1 + \theta\,\frac{\bar{y}_c}{\bar{Y}_c}) = w\bar{L}_c \quad . \tag{3.4.7}$$

Dividing (3.4.6) by (3.4.7), we get

$$\bar{K}_c = \frac{w\alpha}{\beta\rho}\,\bar{L}_c \quad , \tag{3.4.8}$$

Substituting (3.4.1)-(3.4.4), and (3.4.8) into (3.4.6), we obtain

$$\bar{L}_c = n_c . x_c = [(\frac{\beta r}{w})(\frac{w\alpha}{\beta\rho})^{\alpha(1+\theta)}\; m_c^{\theta}(1 + \frac{\theta}{m_c})]^{1/\varepsilon_c} \quad , \tag{3.4.9}$$

where

$$\varepsilon_c = 1 - (\alpha+\beta)(1+\theta) \quad . \tag{3.4.10}$$

Then $\bar{L}_c$, $\bar{K}_c$, $\bar{y}_c$, $\bar{Y}_c$, $\bar{p}$ and $\bar{\pi}_c$ can be calculated as functions of m_c by using the Equations (3.4.1)-(3.4.5) and (3.4.8). Now it suffices to determine the size of the market by employing the zero profit condition which states that optimal profit of each firm is to fall to the level zero. In other words, the size of the market should be determined by

$$\bar{\pi}_c = \bar{p}\,\bar{y}_c[1 - (\alpha + \beta)(1 + \theta\,\frac{\bar{y}_c}{\bar{Y}_c})] - C_o = 0 \quad . \tag{3.4.11}$$

Expressing (3.4.11) as a function of m_c only, we get

$$r[(\frac{w\alpha}{\beta\rho})^{\alpha}(\frac{\beta r}{w})^{\alpha+\beta}]^{(1+\theta)/\varepsilon_c}[m_c^{\theta}(1+\frac{\theta}{m_c})^{(\alpha+\beta)(1+\theta)}]^{1/\varepsilon_c}$$

$$\cdot [1-(\alpha+\beta)(1+\frac{\theta}{m_c})] = C_o \quad (3.4.12)$$

Letting

$$C_o' = \frac{C_o}{r[(\frac{w\alpha}{\beta\rho})^{\alpha}(\frac{\beta r}{w})^{\alpha+\beta}]^{(1+\theta)/\varepsilon_c}} \quad (3.4.13)$$

(3.4.12) is reduced to

$$[m_c^{\theta}(1+\frac{\theta}{m_c})^{(\alpha+\beta)(1+\theta)}]^{1/\varepsilon_c}[1-(\alpha+\beta)(1+\frac{\theta}{m_c})] = C_o'. \quad (3.4.14)$$

Then $[m_c]$ satisfying (3.4.14) determines the number of capitalistic firms in the market at the equilibrium.

According to Cournot-Sertel Equilibrium, a new firm which tends to enter the market should be symmetric with respect to the already existing firms since a new equilibrium point is established among the (m+1) firms. On the other hand, since the solution found by (3.4.14) is the maximum number of symmetric firms that can exist in the market, a potential firm gains no positive profit upon its entrance into the market, so $[m_c]$ is the CSE.

Corollary (3.4.1): There exists a unique solution to (3.4.14).

Proof: Letting

$$f(m_c) = [m_c^{\theta}(1 + \frac{\theta}{m_c})^{(\alpha+\beta)(1+\theta)}]^{1/\varepsilon_c} [1 - (\alpha+\beta)(1 + \frac{\theta}{m_c})]$$

and by first-order differentiation with respect to m_c, we obtain

$$f'(m_c) = \frac{\theta}{m_c} m_c^{\theta/\varepsilon_c} (1 + \frac{\theta}{m_c})^{((\alpha+\beta)(1+\theta))/\varepsilon_c}$$

$$\cdot \{[1 - (\alpha+\beta)(1 + \frac{\theta}{m_c})]\frac{1}{\varepsilon_c} [1 - \frac{(\alpha+\beta)(1+\theta)}{m_c + \theta}] + \frac{\alpha + \beta}{m_c}\}$$

Since $f(0) = \infty$, $f(\infty) = 0$, and $f'(m_c) < 0$, then $f(m_c)$ should be equal to C_0' at a single finite point by the fact that $f(.)$ is strictly decreasing.

In order to illustrate the basic points in the above argument, we present a numerical example below.

Example (3.4.1): We let

$\alpha = 1/3$, $\beta = 2/3$, $\theta = -1/2$, $w = 1/10$, $\rho = 1/10$, $r = 1$ and $C_0 = 1/100$. By (3.4.10) and (3.4.13), $\varepsilon_c = 1/2$ and $C_0' = 0.00189$. Substituting the above values into (3.4.14), we get

$$\frac{1}{2m_c^2} (1 - \frac{1}{2m_c}) = C_0' \quad .$$

Then the greatest integer solution of the above equation is $[m_c] = 16$ which means that the equilibrium size of the market is at most 16 firms and the values of L_c, K_c, y_c, Y_c, p and π_c at optimality are found to be

$\overline{L}_c = 2.069$

$\overline{K}_c = 1.03454$

$\overline{y}_c = 1.64223$

$\overline{Y}_c = 26.2757$

$\overline{p} = 0.19508$

$\overline{\pi}_c = 0.00001168 \sim 0$.

by using the Equations (3.1.1)-(3.1.5).

3.5 RELATIONSIHP BETWEEN CSE and NOVSHEK'S CEFE

In this section we prove an important result in the sense that Cournot-Sertel Equilibrium for capitalistic oligopoly is also Novshek's CEFE.

Suppose that the equilibrium size of the market is determined by (3.4.14), which is the CSE. Thus $\overline{y}_c$, $\overline{K}_c$ and $\overline{L}_c$ are related through Equations (3.4.1)-(3.4.5) and calculated by the procedure of Section (3.4) for $[m_c]$ existing firms. Furthermore assume that there exists a potential firm which wants to enter the market with production level $y = K^{\alpha}. L^{\beta}$ according to Novshek's CEFE concept where K and L are its capital and labor inputs.

Now we show that although the $[m_c]$ existing firms in the market insist upon their current production levels, the potential firm cannot obtain any positive gain in the market, thus won't be able to enter it.

<u>Corollary (3.5.1):</u> $[m_c]$ as determined by (3.4.14) is also the Novshek's CEFE.

Proof: The first-order conditions for the potential firm are:

$$\alpha(m_c\bar{y}_c + y)^{\theta}\ y(1 + \frac{\theta y}{m_c\bar{y}_c + y}) = \rho K \quad , \tag{3.6.1}$$

and

$$\beta(m_c\bar{y}_c + y)^{\theta}\ y(1 + \frac{\theta y}{m_c\bar{y}_c + y}) = wL \quad . \tag{3.6.2}$$

Note that $\bar{y}_c$, $\bar{K}_c$ and $\bar{L}_c$ are treated as fixed numbers in Novshek's CEFE concept.

The values of K and L satisfying (3.6.1) and (3.6.2) will determine the optimum production level of the potential firm. Then by (3.6.1) and (3.6.2) we get

$$L = \frac{w\alpha}{\beta\rho} K \ . \tag{3.6.3}$$

Letting

$$L = a\ \bar{L}_c \quad , \qquad a \in R_+ \tag{3.6.4}$$

by (3.6.3) and (3.4.8), we obtain

$$K = a\ \bar{K}_c \quad , \tag{3.6.5}$$

$$y = a^{\alpha+\beta}\bar{y}_c \quad . \tag{3.6.6}$$

Substituting (3.6.4)-(3.6.6) into (3.6.2), we get

$$\beta(m_c + a^{\alpha+\beta})^{\theta}\bar{y}_c^{\ 1+\theta}\ (1 + \frac{\theta a^{\alpha+\beta}}{m_c + a^{\alpha+\beta}}) = w\ \bar{L}_c . a^{1-(\alpha+\beta)} \tag{3.6.7}$$

Now, let

$$h(a) = \beta(m_c + a^{\alpha+\beta})^{\theta}\bar{y}_c^{\ 1+\theta}\ (1 + \frac{\theta a^{\alpha+\beta}}{m_c + a^{\alpha+\beta}})$$

and

$$g(a) = w\bar{L}_c . a^{1-(\alpha+\beta)} \quad .$$

By some straight forward calculations, we can show that

$$h'(a) < 0 \quad , \quad g'(a) > 0 \quad ,$$

$$h''(a) > 0 \quad , \quad g''(a) < 0 \quad .$$

Thus functions h(.) and g(.) can be represented as in Fig. (3.5.1) which shows that functions under consideration intersect at a single point a_o.

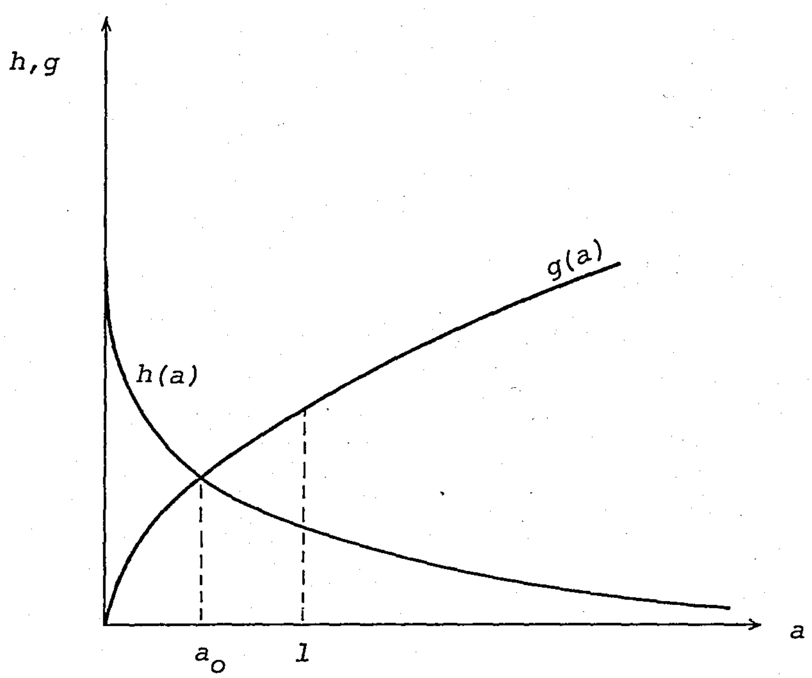

Figure 3.5.1 - The relationship between h(.) and g(.) for a capitalistic oligopoly using Novshek's CEFE.

It is obvious that

$$h(0) = \beta(m_c)^\theta \, \bar{y}_c^{1+\theta} \quad , \qquad g(0) = 0$$

$$h(\infty) = 0 \quad , \qquad g(\infty) = \infty \; .$$

Now the next step is to show that $a_o < 1$. Note that

$$h(1) = \beta(m_c + 1)^\theta \, \bar{y}_c^{\,1+\theta} \, (1 + \frac{\theta}{m_c + 1})$$

and

$$g(1) = w \, \bar{L}_c$$

We know that (3.4.6) is valid for the existing firms, so

$$m_c^\theta \, \bar{y}_c^{\,1+\theta}(1 + \frac{\theta}{m_c}) = w\bar{L}_c \quad . \tag{3.6.8}$$

Let

$$f(m_c) = m_c^\theta \, \bar{y}_c^{\,1+\theta}(1 + \frac{\theta}{m_c}) \quad .$$

Since $f'(m_c) < 0$,

$$h(1) < f(m_c) = g(1) \tag{3.6.9}$$

Therefore the intersection point a_o of $h(a)$ and $g(a)$ is $0 < a_o < 1$. Then the optimal profit for the potential firm is given by

$$\bar{\pi} = (m_c + a^{\alpha+\beta})^\theta \, \bar{y}_c^{\,1+\theta}[1 - (\alpha + \beta)(1 + \frac{\theta \, a^{\alpha+\beta}}{m_c + a^{\alpha+\beta}})] - C_o \; ,$$

and the optimal profit $\bar{\pi}_c$ for the existing firms is given by

$$\bar{\pi}_c = m_c^\theta \, \bar{y}_c^{\,1+\theta}[1 - (\alpha + \beta)(1 + \frac{\theta}{m_c})] - C_o \simeq 0.$$

For $a < 1$, we get

$$0 = \overline{\pi}_c > \pi \tag{3.6.10}$$

which means that there is no profit incentive for the potential firm. Thus, Cournot-Sertel Equilibrium is also Novshek's CEFE.

Note that, in this section, the constant r which appears in the inverse demand function is taken to be unity.

IV. COURNOT EQUILIBRIUM WITH FREE ENTRY FOR WORKERS' ENTERPRISES

An industry M consisting of only workers' enterprises the partners of which are utility maximizing are considered in this chapter. The analysis aims at determining the Cournot-Sertel Equilibrium of the industry under consideration. To achieve this, basic data are given in Section 4.1. and the existence and uniqueness of the optimum production level maximizing the utility function of each individual is proven by showing the concavity property of the utility function in Section 4.2. Then, in Section 4.3 the symmmetry of the firms is shown under their optimum behaviour. Then CSE for this industry M is determined, so that the equilibrium size of the market is uniquely specified in Section 4.4. Finally, CSE obtained in Section 4.4 is proven to be Novshek's Equilibrium in Section 4.5.

4.1 BASIC DATA

As far as the log-linear model is concerned we again assume the inverse demand function and the production function of each firm to satisfy the conditions we have imposed in Section 3.1 in our analysis of capitalistic oligopoly.

Data relevant with the model of workers' enterprises are described below in order to shed light upon the theoretical framework to be developed. Denoting workers' enterprises by the subscript w, we let

x_{iw} be the amount of labor input of a typical worker in the i^{th} firm,

n_{iw} be the number of workers of the i^{th} firm,

L_{iw} be the labor input of the i^{th} firm given by

$$L_{iw} = n_{iw} . x_{iw} \quad , \qquad (4.1.1)$$

K_{iw} be the capital input of the i^{th} firm,

y_{iw} be the output of the i^{th} firm given by

$$y_{iw} = K_{iw}^{\alpha} L_{iw}^{\beta} \quad , \qquad (4.1.2)$$

where $0 < \alpha < 1$ and $0 < \beta < 1$ are capital and labor elasticities of output, respectively, and $0 < \alpha+\beta \leq 1$,

m_w be the number of firms,

y_w be total output of the industry given by

$$y_w = \sum_{i=1}^{m_w} y_{iw} \quad , \qquad (4.1.3)$$

p be the price of the commodity given by

$$p = r \, Y_w^{\theta} \quad , \qquad (4.1.4)$$

where $-1 \leq \theta < 0$ is the total output elasticity of price and $r > 0$,

bx_{iw}^{γ} be the disutility of work of a typical worker in the i^{th} firm, and $b > 0$, $\gamma > 1$,

u_{iw} be the utility of a typical worker in the i^{th} firm given by

$$u_{iw} = \frac{py_{iw} - \rho K_{iw} - C_o}{n_{iw}} - bx_{iw}^{\gamma} \qquad (4.1.5)$$

where ρ is the market rental for capital goods in units of the output and C_o is the fixed cost of the firm.

The objective of a worker in the industry is to maximize his utility given by (4.1.5). As a first step in our effort to achieve the maximization of utility of a typical worker, we show that the objective function u_{iw} is concave with respect to variables K_{iw} and x_{iw}, and then obtain the first-order necessary conditions which will yield the unique optimum (global maximum) of the problem by the strictly concave property of the objective function.

4.2 EXISTENCE AND UNIQUENESS OF A GLOBAL MAXIMUM

In this section our aim is to show that there exists a unique global maximum of the utility function of a typical worker in a workers' enterprise. We employ the argument of Chapter III and verify the concavity of the utility function in showing the existence and uniqueness result.

Corollary (4.2.1): In the log-linear model, u_{iw} is strictly concave with respect to K_{iw} and x_{iw}.

Proof: Letting the model be defined by Equations (4.1.1)-(4.1.5), the aim is reduced to maximizing (4.1.5). The necessary conditions are obtained by the partial differentiation of the utility function with respect to K_{iw} and x_{iw}, so

$$\frac{\partial u_{iw}}{\partial K_{iw}} = \frac{1}{n_{iw}} [\alpha \frac{py_{iw}}{K_{iw}} (1 + \theta \frac{y_{iw}}{Y_w}) - \rho] = 0 \qquad (4.2.1)$$

and

$$\frac{\partial u_{iw}}{\partial x_{iw}} = \frac{\beta p y_{iw}}{n_{iw} x_{iw}} \left(1 + \theta \frac{y_{iw}}{Y_w}\right) - b\gamma x_{iw}^{\gamma-1} = 0 \qquad (4.2.2)$$

To show u_{iw} is strictly concave, it suffices to show that the Hessian matrix is negative definite. So it is constructed from (4.2.1) and (4.2.2) to be

$$H = \begin{bmatrix} \frac{\alpha p y_{iw}}{n_{iw} K_{iw}^2}[(\alpha k-1)k+\alpha(k-1)d] & \frac{\alpha\beta p y_{iw}}{n_{iw} x_{iw} K_{iw}}[k^2+(k-1)d] \\ \frac{\alpha\beta p y_{iw}}{n_{iw} x_{iw} K_{iw}}[k^2+(k-1)d] & \frac{\beta p y_{iw}}{n_{iw} x_{iw}^2}[(\beta k-1)k+\beta(k-1)d]-b\gamma(\gamma-1)x_{iw}^{\gamma-2} \end{bmatrix}$$

where $k = 1 + \theta(y_{iw}/Y_w) < 1$, $d = 1 - (y_{iw}/Y_w) < 1$ and $d < k$. Since $-1 \leq \theta < 0$, $0 < (y_{iw}/Y_w) \leq 1$ and $0 < \alpha, \beta \leq 1$, $(\alpha+\beta) \leq 1$, $\gamma > 1$, we get $k^2 > k^2+(k-1)d$ and $(\alpha k-1)k+\alpha(k-1)d < 0$. Thus

$$|H_{11}| = \alpha \frac{p y_{ic}}{K_{iw}^2}[(\alpha k - 1)k + \alpha(k - 1)d] < 0$$

and

$$|H| = \alpha\beta\left(\frac{p y_{iw}}{n_{iw} x_{iw} K_{iw}}\right)^2 \{k^2 - (\alpha + \beta)k[k^2 + (k - 1)d]\}$$

$$- \left(\frac{\alpha p y_{iw}}{n_{iw} K_{iw}^2}\right)[(\alpha k - 1)k + \alpha(k - 1)d]b\gamma(\gamma - 1)x_{iw}^{\gamma-2} > 0 .$$

Thus, the Hessian is negative definite.
So the concavity follows immediately.

A result closely related with the concavity property is stated in terms of existence and uniqueness.

Corollary (4.2.2): There exist unique $(\overline{K}_{iw},\overline{x}_{iw})$ which achieve the maximization of u_{iw}.

Proof: The uniqueness and existence of $(\overline{K}_{iw},\overline{x}_{iw})$ follow from the strictly concave property of u_{iw} as proven by Corollary (4.2.1).

Since each worker maximizes his utility, first-order (necessary) conditions (4.2.1) and (4.2.2) can be written as

$$\alpha p y_{iw}(1 + \theta \frac{y_{iw}}{Y_w}) = \rho K_{iw} \tag{4.2.3}$$

and

$$\frac{\beta}{\gamma} \frac{p y_{iw}}{n_{iw}} (1+ \theta \frac{y_{iw}}{Y_w}) = b x_{iw}^{\gamma} \ . \tag{4.2.4}$$

Then the values of $(\overline{K}_{iw},\overline{x}_{iw})$ are obtained by solving (4.2.3) and (4.2.4). Furthermore, substituting (4.2.3) and (4.2.4) into (4.1.5) we obtain the optimal value of u_{iw} as

$$\overline{u}_{iw} = \frac{\overline{p}\overline{y}_{iw}}{n_{iw}} [1 - (\alpha + \frac{\beta}{\gamma})(1 + \theta \frac{\overline{y}_{iw}}{\overline{Y}_w})] - \frac{\dot{C}}{n_{iw}}$$

where $\overline{p}$, $\bar{y}_{iw}$ and $\bar{Y}_w$ are optimal values of price, output of the i^{th} firm and total output with respect to $(\bar{K}_{iw},\bar{x}_{iw})$.

The important distinction in workers' enterprises arises from the need to determine the number of partners of each firm. In the presence of worker-partnership market, $\bar{u}_{iw}$ is the optimal utility level of a typical member in the i^{th} firm and $-n_{iw}(d\bar{u}_{iw}/dn_{iw})$ is the least amount that a typical worker in the sector should pay in order to join the i^{th} firm. Now, defining η_{iw} to be the net utility gain of a typical worker when he joins the firm, it is given by

$$\eta_{iw} = \bar{u}_{iw} - (-n_{iw} \frac{d\bar{u}_{iw}}{dn_{iw}}) = \bar{u}_{iw} + n_{iw} \frac{d\bar{u}_{iw}}{dn_{iw}} \qquad (4.2.6)$$

As (4.2.6) reveals, η_{iw} is the utility of a typical worker gains by becoming a partner of i^{th} firm diminished by the amount he has to pay for joining it. Now it is necessary to obtain a more explicit representation of η_{iw}.

Differentiating (4.2.5) with respect to n_{iw}, we get

$$\frac{d\bar{u}_{iw}}{dn_{iw}} = \frac{\bar{p}\bar{y}_{iw}}{n_{iw}^2} [(\alpha + \beta)(1 + \theta \frac{\bar{y}_{iw}}{\bar{Y}_{iw}}) - 1] + \frac{C_o}{n_{iw}^2} \quad .$$

Note that $\bar{x}_{iw}$ and $\bar{K}_{iw}$ are functions of n_{iw}. Then, (4.2.6) can be expressed as

$$\eta_{iw} = \frac{\bar{p}\bar{y}_{iw}}{n_{iw}} \frac{\beta}{\gamma}(\gamma - 1)(1 + \theta \frac{\bar{y}_{iw}}{\bar{Y}_{iw}}) \quad , \qquad (4.2.7)$$

which in turn can be simplified to

$$\eta_{iw} = (\gamma - 1)b\bar{x}_{iw}^{\gamma} \quad . \qquad (4.2.8)$$

Since $(d\eta_{iw}/dn_{iw}) < 0$, as the number of partners of the i^{th} firm increases, the net utility gain of the potential entrant decreases down to $\underline{u}$ which is the reserve utility level elsewhere in the economy as it is stated in Condition 3 of CSE for workers' enterprises. Thus, at long-run equilibrium achieved by (2.1.2),

$$\eta_{iw} = \underline{u} \quad , \qquad \text{for each} \qquad i \in M \quad . \qquad (4.2.9)$$

which says that no more entrance will occur from this point on.

4.3 SYMMETRY OF THE MARKET

In this section, the workers' enterprises in a labor-managed oligopoly are shown to be symmetric in the sense that they all behave identically at the optimum.

Theorem (4.3.1): In the log-linear model, all workers' enterprises are symmetric at optimality.

Proof: By (4.2.9),

$$\eta_{iw} = \eta_{jw} = \underline{u} \qquad \text{for all} \quad i,j \in M .$$

Furthermore by (4.2.8), we obtain

$$\bar{x}_{iw} = \bar{x}_{jw} = \bar{x}_w \qquad \text{for all} \qquad i,j \in M . \tag{4.3.1}$$

Thus, the labor inputs of the workers in the industry are equal. Dividing (4.2.3) by (4.2.4) and using (4.3.1), we obtain

$$\frac{\bar{K}_{iw}}{\bar{K}_{jw}} = \frac{n_{iw}}{n_{jw}} \quad , \qquad \text{for all} \quad i,j \in M . \tag{4.3.2}$$

Writing (4.2.4) for the i^{th} and j^{th} firms and dividing one by the other, we get

$$\frac{\bar{y}_{iw}}{\bar{y}_{jw}} \cdot \frac{n_{jw}}{n_{iw}} \cdot \frac{\bar{Y}_w + \theta\bar{y}_{iw}}{\bar{Y}_w + \theta\bar{y}_{jw}} = 1 \quad . \tag{4.3.3}$$

Substituting (4.1.1), (4.1.2) and (4.3.2) into (4.3.3), we obtain

$$\left(\frac{n_{iw}}{n_{jw}}\right)^{\alpha+\beta-1} \cdot \frac{\bar{Y}_w + \theta\bar{x}_w^{\beta}\,\bar{K}_{iw}^{\alpha}\,n_{iw}^{\beta}}{\bar{Y}_w + \theta\bar{x}_w^{\beta}\,\bar{K}_{jw}^{\alpha}\,n_{jw}^{\beta}} = 1 \quad . \tag{4.3.4}$$

Letting

$$\frac{\bar{K}_i}{\bar{K}_j} = \frac{n_{iw}}{n_{jw}} \equiv a \qquad \text{where} \qquad a \in R_+ \tag{4.3.5}$$

and substituting (4.3.5) into (4.3.4), we obtain

$$\frac{\bar{Y}_w + \theta\bar{x}_w^{\beta}\,\bar{K}_{jw}^{\alpha}\,n_{jw}^{\beta}\,a^{\alpha+\beta}}{\bar{Y}_w + \theta\bar{x}_w^{\beta}\,\bar{K}_{jw}^{\alpha}\,n_{jw}^{\beta}} = a^{1-(\alpha+\beta)} \quad . \tag{4.3.6}$$

In order to simplify our analysis of (4.3.6), we let

$$h(a) = \frac{\bar{Y}_w + \theta\bar{x}_w^{\beta}\,\bar{K}_{jw}^{\alpha}\,n_{jw}^{\beta}\,a^{\alpha+\beta}}{\bar{Y}_w + \theta\bar{x}_w^{\beta}\,\bar{K}_{jw}^{\alpha}\,n_{jw}^{\beta}} \quad ,$$

and

$$g(a) = a^{1-(\alpha+\beta)} \quad .$$

Then,

$$h'(a) = \frac{\theta\bar{x}_w^{\beta}\,\bar{K}_{jw}^{\alpha}\,n_{jw}^{\beta}\,(\alpha+\beta)a^{\alpha+\beta-1}}{\bar{Y}_w + \theta\bar{x}_w^{\beta}\,\bar{K}_{jw}^{\alpha}\,n_{jw}^{\beta}} \quad ,$$

$$h''(a) = \frac{\theta(\alpha+\beta)[(\alpha+\beta)-1]\bar{x}_w^{\beta}\,\bar{K}_{jw}^{\alpha}\,n_{j}^{\beta}\,a^{\alpha+\beta-2}}{\bar{Y}_w + \theta\bar{x}_w^{\beta}\,\bar{K}_{jw}^{\alpha}\,n_{jw}^{\beta}} \quad ,$$

and

$$g'(a) = [1-(\alpha+\beta)]a^{-(\alpha+\beta)} \quad ,$$

$$g''(a) = -[1 - (\alpha + \beta)](\alpha + \beta)a^{-(\alpha+\beta)-1} \quad .$$

Now, it remains necessary to identify a solution to (4.3.6) if it exists at all.

Since $(\alpha + \beta) \leq 1$,

$$h'(a) < 0 \quad ; \quad g'(a) > 0$$

$$h''(a) > 0 \quad ; \quad g''(a) < 0 \quad ,$$

and

$$h(0) = \frac{\bar{Y}_w}{\bar{Y}_w + \theta\bar{y}_{jw}} > 1 \quad ; \quad g(0) = 0$$

$$h(\infty) = -\infty \quad ; \quad g(\infty) = \infty \ .$$

These observations are summarized in Fig. (4.3.1), which shows that the only solution satisfying (4.3.6) is $a_0 = 1$.

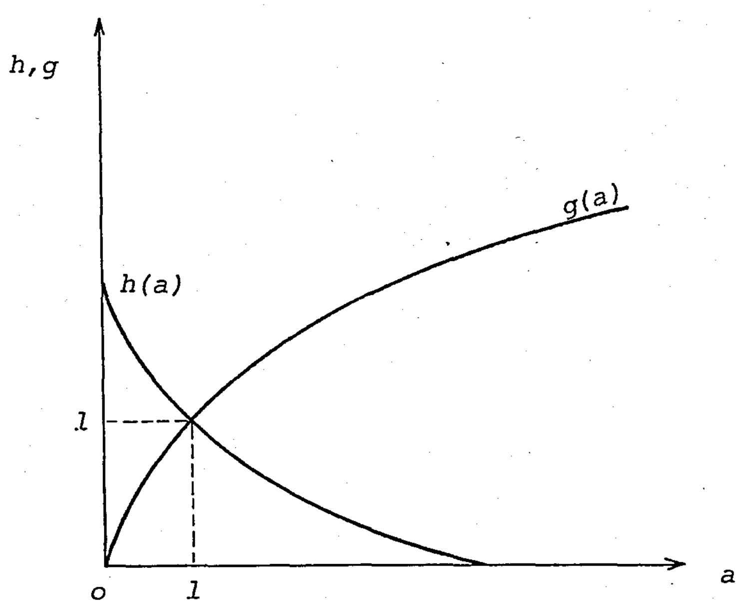

Figure 4.3.1 - The relationship between h(.) and g(.) for workers' enterprises.

The fact that a = 1 implies for all $i \in M$,

$$\bar{K}_{iw} = \bar{K}_w$$
$$\bar{x}_{iw} = \bar{x}_w$$
$$n_{iw} = n_w$$
$$\bar{L}_{iw} = \bar{L}_w$$
$$\bar{y}_{iw} = \bar{y}_w$$
$$\bar{u}_{iw} = \bar{u}_w \ .$$

Thus all workers' enterprises are symmetric.

4.4 COURNOT-SERTEL EQUILIBRIUM

Now we are at a point to determine the Cournot-Sertel Equilibrium for workers' enterprises.

Because of the symmetry of the market at optimality, Equations (4.1.1)-(4.1.5) and (4.2.7)-(4.2.8) can be rewritten as

$$\bar{L}_w = n_w \bar{x}_w \quad , \tag{4.4.1}$$

$$\bar{y}_w = \bar{K}_w^{\alpha} \bar{L}_w^{\beta} \quad , \tag{4.4.2}$$

$$\bar{Y}_w = m_w \bar{K}_w^{\alpha} \bar{L}_w^{\beta} \ , \tag{4.4.3}$$

$$\bar{p} = r \bar{Y}_w^{\theta} \tag{4.4.4}$$

$$\bar{u}_w = \frac{\bar{p}\bar{y}_w - \rho\bar{K}_w - C_o}{n_w} - b\bar{x}_w = \frac{\bar{p}\bar{y}_w^{\gamma}}{n_w}[1 - (\alpha + \frac{\beta}{\gamma})(1 + \frac{\theta}{m_w})] - \frac{C_o}{n_w} \ , \tag{4.4.5}$$

$$\eta_w = \frac{\bar{p}\bar{y}_w}{n_w} \cdot \frac{\beta}{\gamma}(\gamma - 1)(1 + \theta \frac{\bar{y}_w}{\bar{Y}_w}) = (\gamma - 1)b\bar{x}_w^{\gamma} \quad . \qquad (4.4.6)$$

The first-order necessary conditions stated by (4.2.3) and (4.2.4) can also be transformed into

$$\alpha\bar{p}\bar{y}_w(1 + \theta \frac{\bar{y}_w}{\bar{Y}_w}) = \rho\bar{K}_w \quad , \qquad (4.4.7)$$

and

$$\frac{\beta}{\gamma} \frac{\bar{p}\bar{y}_w}{n_w}(1 + \theta \frac{\bar{y}_w}{\bar{Y}_w}) = b\bar{x}_w^{\gamma} \quad . \qquad (4.4.8)$$

Substituting (4.4.1)-(4.4.4) into (4.4.7) and dividing (4.4.7) by (4.4.8), we get

$$\bar{K}_w = [\frac{\alpha r}{\rho} m_w^{\theta} n_w^{\beta(1+\theta)} \bar{x}_w^{\beta(1+\theta)} (1 + \frac{\theta}{m_w})]^{1/(1-\alpha(1+\theta))} \qquad (4.4.9)$$

and

$$\frac{\beta}{\gamma} \frac{\rho}{\alpha} \frac{\bar{K}_w}{n_w} = b\bar{x}_w^{\gamma} \quad . \qquad (4.4.10)$$

which in turn yields

$$\bar{x}_w = [(\frac{b\gamma\alpha}{\beta\rho})^{\alpha(1+\theta)} (\frac{\beta r}{b\gamma})m_w^{\theta} n_w^{(\alpha+\beta)(1+\theta)-1}(1 + \frac{\theta}{m_w})]^{1/\varepsilon_w} , \qquad (4.4.11)$$

where $\varepsilon_w = \gamma - (\gamma\alpha+\beta)(1+\theta)$.

Then the values $\bar{L}_w$, $\bar{K}_w$, $\bar{y}_w$, $\bar{Y}_w$, $\bar{p}$, $\bar{u}_w$, η_w can be calculated by Equations (4.4.1)-(4.4.6) and (4.4.9) as functions of n_w and m_w.

The equilibrium size of the market is established when the net utility gain of a newcoming worker, the utility of a typical partner in an existing firm and the reserve utility level elsewhere in the economy become equal. This is mathematically expressed by the following equality

$$\eta_w = \bar{u}_w = \underline{u} \quad . \tag{4.4.12}$$

The facts that $\eta_w = \bar{u}_w$ and $\eta_w = \underline{u}$ imply

$$\bar{p}\bar{y}_w[1 - (\alpha + \beta)(1 + \frac{\theta}{m_w})] = C_o \quad , \tag{4.4.13}$$

and

$$\frac{\bar{p}\bar{y}_w}{n_w}(\gamma - 1)\frac{\beta}{\gamma}(1 + \frac{\theta}{m_w}) = \underline{u} \quad . \tag{4.4.14}$$

Furthermore by $\bar{u}_w = \underline{u}$, we have

$$\frac{\bar{p}\bar{y}_w}{n_w}[1 - (\alpha + \frac{\beta}{\gamma})(1 + \frac{\theta}{m_w})] = \bar{u} + \frac{C_o}{n_w} \quad . \tag{4.4.15}$$

Dividing (4.4.14) by (4.4.13), we get

$$n_w = \frac{\frac{\beta}{\gamma}(\gamma - 1)(1 + \frac{\theta}{m_w})C_o}{[1 - (\alpha + \beta)(1 + \frac{\theta}{m_w})]\underline{u}} \quad . \tag{4.4.16}$$

The way η_w is defined by (4.2.8) and the fact that $\eta_w = \underline{u}$ together imply

$$(\gamma - 1)b\bar{x}_w^{\gamma} = \underline{u}$$

which can be rewritten by (4.4.11) as

$$(\gamma - 1)b[(\frac{b\gamma\alpha}{\beta\rho})^{\alpha(1+\theta)}(\frac{\beta r}{b\gamma})m_w^{\theta}\, n_w^{(\alpha+\beta)(1+\theta)-1}(1 + \frac{\theta}{m_w})]^{1/\varepsilon_w} = \underline{u} \quad , \tag{4.4.17}$$

where n_w is given by (4.4.16).

Then $[m_w]$ satisfying (4.4.17) determines the size of the market at the equilibrium point.

If a particular value of $\underline{u}$ is not readily given in the market, the value of $\bar{u}$ can be determined theoretically. We let

x be the labor input of a typical worker elsewhere in the economy,

u be the utility level of a typical worker elsewhere in the economy given by

$$u = wx - bx^{\gamma} \quad , \quad b > 0 \ , \quad \gamma > 1$$

where w is the wage of labor and bx^{γ} is the disutility of a worker.

The optimal utility level $\underline{u}$ should be chosen so as to maximize u with respect to x and can be obtained by solving the first-order necessary condition as given by

$$\frac{du}{dx} = w - \gamma b x^{\gamma-1} = 0$$

Thus the optimal labor input is found to be

$$\underline{x} = \left(\frac{w}{\gamma b}\right)^{1/(\gamma-1)} ,$$

and the optimal utility is found to be

$$\underline{u} = b(\gamma - 1)\left(\frac{w}{\gamma b}\right)^{\gamma/(\gamma-1)} . \qquad (4.4.18)$$

By (4.4.18) and (4.4.16), we obtain

$$n_w = \frac{\frac{\beta}{\gamma}\left(1 + \frac{\theta}{m_w}\right)C_o}{\left[1 - (\alpha + \beta)\left(1 + \frac{\theta}{m_w}\right)\right]\left(\frac{w}{\gamma b}\right)^{\gamma/(\gamma-1)} . b} . \qquad (4.4.19)$$

Then substituting (4.4.18) and (4.4.19) into (4.4.17), we obtain

$$(\frac{\beta}{w})^{\beta(1+\theta)}(\frac{\alpha}{\rho})^{\alpha(1+\theta)} r.m_w^{\theta}(1 + \frac{\theta}{m_w})^{(\alpha+\beta)(1+\theta)}[\frac{C_o}{1-(\alpha+\beta)(1+\frac{\theta}{m_w})}]^{(\alpha+\beta)(1+\theta)-1} = 1 \quad , \qquad (4.4.20)$$

which is equivalent to (3.4.14).

The greatest integers $[m_w]$ and $[n_w]$ satisfying (4.4.19) and (4.4.20) determine the number of firms and the number of workers in each firm at the equilibrium of the market, respectively.

By the reasoning employed in Chapter III, it can be shown that $[m_w]$ is the CSE. Furthermore by Corollary (3.4.1), $[m_w]$ is the unique solution to (4.4.20).

Example (4.4.1): Let's illustrate the argument of this section by using the model of Example (3.4.1).

Let $\alpha = 1/3$, $\beta = 2/3$, $\gamma = 2$, $\theta = -1/2$, $w = 1/10$, $\rho = 1/10$, $r = 1$, $b = 2$ and $C_o = 1/100$.

From the above values $\varepsilon_w = 4/3$ and Equation (4.4.20) turns out to be

$$\frac{2m_w - 1}{m_w^3} = 0.00756 \quad . \qquad (4.5.1)$$

The greatest integer solution $[m_w]$ of (4.5.1) is 16. Substituting this value into (4.4.19), we get $[n_w] = 82$. Thus

$$\bar{x}_w = 0.02509$$
$$\bar{L}_w = 2.05714$$

$\bar{L}_w = 1.03215$

$\bar{y}_w = 1.63464$

$\bar{Y}_w = 26.15425$

$\bar{p} = 0.19554$

$\bar{u}_w = 0.0012585$

$\eta_w = 0.0012587$

$\underline{u} = 0.00125$

by using the Equations (4.4.1)-(4.4.6) and (4.4.1).

It is interesting to note that the results we have obtained both in capitalistic and workers' enterprises are identical.

4.5 RELATIONSHIP BETWEEN CSE AND NOVSHEK's CEFE

In this section, we will show that CSE for workers' enterprises is also Novshek's CEFE. Suppose that the equilibrium size of the market is determined by (4.4.20) using CSE concept. Now at the equilibrium point, $\bar{y}_w$, $\bar{K}_w$ and $\bar{L}_w$ are related by Equations (4.4.1)-(4.4.6) and computed accordingly by the reasoning of Section (4.4) for $[m_w]$ existing firms and $[n_w]$ workers in each one of the existing firms.

Suppose that a firm wants to enter the market with production level $y = K^{\alpha}.L^{\beta} = K^{\alpha}.n^{\beta}.x^{\beta}$ according to Novshek's CEFE concept where K is the capital input of the potential firm, x is the labor input of a typical worker of the potential firm and n is the number of partners of the potential firm.

Now we prove that even if the $[m_w]$ existing firms in the market insist upon their current levels of production, typical worker of the potential firm cannot attain a net utility gain and thus does not show any tendency to enter the market.

Corollary (4.5.1): $[m_w]$ and $[n_w]$ as found by (4.4.20) and (4.4.19), respectively, are also Novshek's CEFE.

Proof: The first-order necessary conditions for the potential firm are given by

$$\alpha(m_w\bar{y}_w + y)^{\theta}\, y(1 + \frac{\theta y}{m_w\bar{y}_w + y}) = \rho K \quad , \tag{4.5.1}$$

and

$$\frac{\beta}{\gamma}\frac{(m_w\bar{y}_w + y)^{\theta}}{n}\, y(1 + \frac{\theta y}{m_w\bar{y}_w + y}) = bx^{\gamma} \quad . \tag{4.5.2}$$

Note that $\bar{y}_w$, $\bar{K}_w$, m_w, $\bar{x}_w$ are treated as fixed numbers in Novshek's CEFE concept, and the values of K, n, x, satisfying (4.5.1) and (4.5.2) will determine the optimum level of production of the potential firm. At equilibrium, since

$$\eta = (\gamma - 1)bx^{\gamma} = \underline{u}$$

for a potential firm, in the presence of worker-partnership market, the labor input of a worker in anyone of the existing firms should be equal to that of a worker of the potential firm. In other words,

$$x = \bar{x}_w \quad . \tag{4.5.3}$$

From (4.5.1) and (4.5.2), we get

$$\frac{\beta}{\gamma} \cdot \frac{p}{\alpha} \cdot \frac{K}{n} = bx^{\gamma} \quad . \tag{4.5.4}$$

Letting

$$K = a\bar{K}_w \quad , \tag{4.5.5}$$

and by (4.5.3) and (4.5.4), we obtain

$$n = an_w \tag{4.5.6}$$

$$y = a^{\alpha+\beta}\,\bar{y}_w \tag{4.5.7}$$

Substituting (4.5.5)-(4.5.7) into (4.5.1), we get

$$\alpha(m_w + a^{\alpha+\beta})\bar{y}_w^{\,1+\theta}\left(1 + \frac{\theta a^{\alpha+\beta}}{m_w + a^{\alpha+\beta}}\right) = \rho\bar{K}_w a^{1-(\alpha+\beta)} \quad . \tag{4.5.8}$$

The solution a_o to (4.5.8) can be shown to satisfy

$$0 < a_o < 1$$

by a similar analysis previously carried out in Section (3.5).

Using the relations expressed by (4.5.5)-(4.5.7), the optimal utility for a worker of the potential firm is given by

$$\bar{u} = \frac{(m_w + a^{\alpha+\beta})^{\theta} a^{\alpha+\beta}\bar{y}_w^{\,1+\theta}\left[1 - \left(\alpha + \frac{\beta}{\gamma}\right)\left(1 + \frac{\theta a^{\alpha+\beta}}{m_w + a^{\alpha+\beta}}\right)\right] - C_o}{a\, n_w} \quad . \tag{4.5.9}$$

Substituting (4.4.13) into (4.5.9) and differentiating $\bar{u}$ with respect to a, we get

$$\frac{d\bar{u}}{da} = \frac{\bar{y}_w^{\,1+\theta}}{a^2 n_w}\{(m_w+a^{\alpha+\beta})^{\theta}a^{\alpha+\beta}[1 - (\alpha + \frac{\beta}{\gamma})(1 + \frac{\theta a^{\alpha+\beta}}{m_w+a^{\alpha+\beta}})]$$

$$\cdot [(\alpha + \beta)(1 + \frac{\theta a^{\alpha+\beta}}{m_w + a^{\alpha+\beta}})-1] + m_w^{\theta}[1 - (\alpha+\beta)(1 + \frac{\theta}{m_w})]$$

$$- \theta(\alpha + \frac{\beta}{\gamma})\frac{m_w}{(m_w + a^{\alpha+\beta})^2}(\alpha+\beta)a^{\alpha+\beta}.a^{\alpha+\beta}(m_w + a^{\alpha+\beta})^{\theta}\}$$

For $0 < a < 1$, we see that

$$d\bar{u}/da > 0 \quad . \tag{4.5.10}$$

Furthermore, evaluating $\bar{u}$ as given by (4.5.9) at unity, we obtain

$$\bar{u}(1) = \frac{(m_w + 1)^{\theta}\bar{y}_w^{\,1+\theta}[1 - (\alpha + \frac{\beta}{\gamma})(1 + \frac{\theta}{m_w + 1})] - C_o}{n_w}$$

On the other hand, the utility level of a typical worker in the existing firm is given to be

$$\bar{u}_w = \frac{m_w^{\theta}\,\bar{y}_w^{\,1+\theta}[1 - (\alpha + \frac{\beta}{\gamma})(1 + \frac{\theta}{m_w})] - C_o}{n_w} = \underline{u}$$

So

$$\bar{u}(1) < \bar{u}_w = \underline{u} \tag{4.5.11}$$

and

$$\bar{u}(0) = -\infty \quad .$$

Then (4.5.10) and (4.5.11) are summarized in Fig. (4.5.1). Note that $\bar{u}$ does not intersect with $\underline{u}$ for $a < 1$, so the utility of a worker in a potential firm cannot attain the reserve utility level elsewhere in the economy.

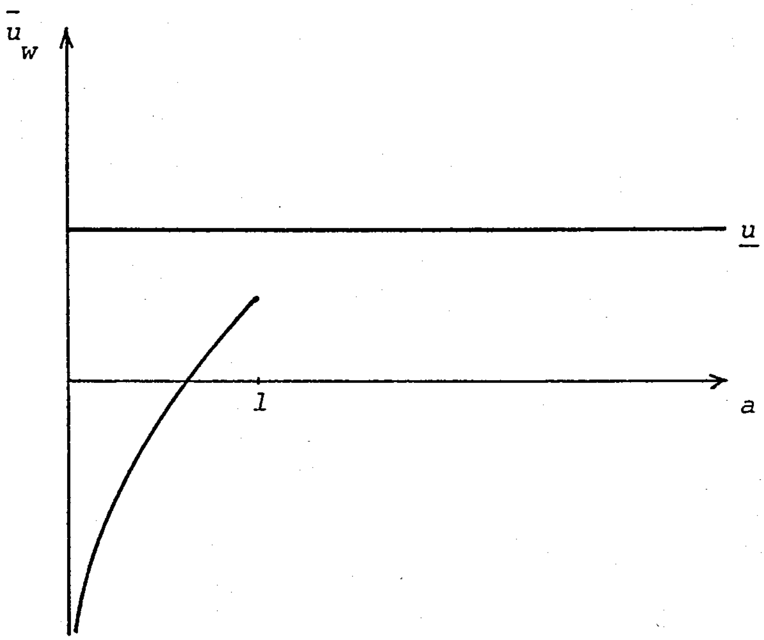

Figure 4.5.1 - The relationship between $\underline{u}$ and $\bar{u}_w$ using Novshek's CEFE.

Thus it is obvious that there is no utility incentive for a worker of the potential firm to enter the market since $a_o < 1$. Consequently CSE is also Novshek's CEFE.

Note that, as in Section (3.5) r is taken to be unity.

V. COURNOT EQUILIBRIUM WITH FREE ENTRY FOR A MIXED ECONOMY

In this chapter we will consider a mixed economy where workers' enterprises and capitalistic firms co-exist. Relevant Data are provided in Section 5.1 to clarify the variables inherent in this problem. In Section 5.2 it is shown that all the firms in industry M are symmetrical regardless of whether they are capitalistic firms or workers' enterprises. Then CSE is established for a mixed economy in Section 5.3 in a manner similar to the one employed in previous chapters.

5.1 BASIC DATA

In a mixed economy, we let

Y be the total output of the industry given by

$$Y = Y_W + Y_C \quad , \tag{5.1.1}$$

p be the price of the commodity given by

$$p = r\, Y^{\theta} \quad , \tag{5.1.2}$$

and the other variables and relations be defined as in Sections 3.1 and 4.1.

By the use of the above data, the necessary conditions (3.2.3) and (3.2.4) in capitalistic economy turn out to be

$$\alpha p y_{ic}(1 + \frac{\theta\, y_{ic}}{Y}) = \rho K_{ic} \quad , \tag{5.1.3}$$

$$\beta p y_{ic}(1 + \frac{\theta\, y_{ic}}{Y}) = w L_{ic} \quad , \tag{5.1.4}$$

and the necessary conditions (4.2.3) and (4.2.4) in a labor-managed economy turn out to be

$$\alpha p y_{iw}(1 + \theta \frac{y_{iw}}{Y}) = \rho K_{iw} \quad , \tag{5.1.5}$$

$$\frac{\beta}{\gamma} \frac{p y_{iw}}{n_{iw}} (1 + \theta \frac{y_{iw}}{Y}) = b x_{iw}^{\gamma} \ . \tag{5.1.6}$$

The reasoning we have employed in Sections 3.2 and 4.2 can immediately be applied to this case, and we can similarly verify that the values $(\bar{K}_{ic}, \bar{L}_{ic}, \bar{K}_{iw}, \bar{x}_{iw})$ satisfying (5.1.3)-(5.1.6) are the solution of the problem.

5.2 SYMMETRY OF THE MARKET

This section is basically concerned with showing the symmetry of the market. All the workers' enterprises and the capitalistic firms in a mixed economy are identical since they all have the same production level of the output at their optimum behaviour. This result is proven by the following theorem.

Theorem (5.2.1): All the firms of a mixed economy are symmetric at their optimum behaviour.

Proof: Since Equations (5.1.3)-(5.1.6) possess the form of Equations (3.2.3), (3.2.4), (4.2.3) and (4.2.4) the results of Sections 3.3 and 4.3 can be obtained in the same manner.

Thus, all the capitalistic and workers' enterprises are symmetric among themselves.

Recall that the condition of CSE for capitalistic firms as obtained in Section 3.4 was

$$\bar{p}\bar{y}_c[1 - (\alpha + \beta)(1 + \theta \frac{\bar{y}_c}{\bar{Y}})] = C_0 \quad , \tag{5.2.1}$$

and the condition of CSE for workers' enterprises as obtained in Section 4.4 was

$$\bar{p}\bar{y}_w[1 - (\alpha + \beta)(1 + \theta \frac{\bar{y}_w}{\bar{Y}}] = C_0 \quad . \tag{5.2.2}$$

Dividing (5.2.1) by (5.2.2) we obtain

$$\frac{1 - (\alpha + \beta)(1 + \theta \frac{\bar{y}_w}{\bar{Y}})}{1 - (\alpha + \beta)(1 + \theta \frac{\bar{y}_c}{\bar{Y}})} = \frac{\bar{y}_c}{\bar{y}_w} \quad . \tag{5.2.3}$$

Now if we let $\bar{y}_w/\bar{y}_c \equiv a$ for some $a \in R_+$, (5.2.4)

and substitute (5.2.4) into (5.2.3), we get

$$\frac{\bar{Y} - (\alpha + \beta)(\bar{Y} + \theta a\bar{y}_c)}{\bar{Y} - (\alpha + \beta)(\bar{Y} + \theta\bar{y}_c)} = \frac{1}{a} \quad . \tag{5.2.5}$$

Let

$$h(a) = \frac{\bar{Y} - (\alpha + \beta)(\bar{Y} + \theta a\bar{y}_c)}{\bar{Y} - (\alpha + \beta)(\bar{Y} + \theta\bar{y}_c)}$$

and

$$g(a) = 1/a \quad .$$

Then,

$$h'(a) = \frac{-\theta(\alpha + \beta)\bar{y}_c}{\bar{Y} - (\alpha + \beta)(\bar{Y} + \theta\bar{y}_c)} > 0$$

$$h''(a) = 0$$

and

$$g'(a) = -\frac{1}{a^2} < 0$$

$$g''(a) = \frac{2}{a^3} > 0 \quad .$$

Since

$$h(0) = \frac{\bar{Y}[1 - (\alpha + \beta)]}{\bar{Y} - (\alpha + \beta)(\bar{Y} + \theta\bar{y}_c)} \quad ; \qquad g(0) = \infty$$

$$h(\infty) = \infty \quad ; \qquad g(\infty) = 0 \quad ,$$

the above results enable us to depict h(.) and g(.) as in Fig. (5.2.1).

As Fig. (5.2.1) reveals, the solution of (5.2.5) is $a = 1$, which says that the outputs of capitalistic and workers' enterprises are equal, i.e.

$$\bar{y}_c = \bar{y}_w \quad . \tag{5.2.6}$$

Using (5.2.6), we can equate (5.1.3) to (5.1.5) and obtain the result

$$\bar{K}_c = \bar{K}_w = \bar{K} \tag{5.2.7}$$

which furthermore implies

$$\bar{L}_c = \bar{L}_w = \bar{L} \tag{5.2.8}$$

since $\bar{K}_c^{\alpha}\,\bar{L}_c^{\beta} = \bar{K}_w^{\alpha}\,\bar{L}_w^{\beta}$ by the fact that $\bar{y}_c = \bar{y}_w$.

Thus, all the firms in the industry are symmetric, so that each firm acquires the same amount of capital and labor inputs to yield the same output level.

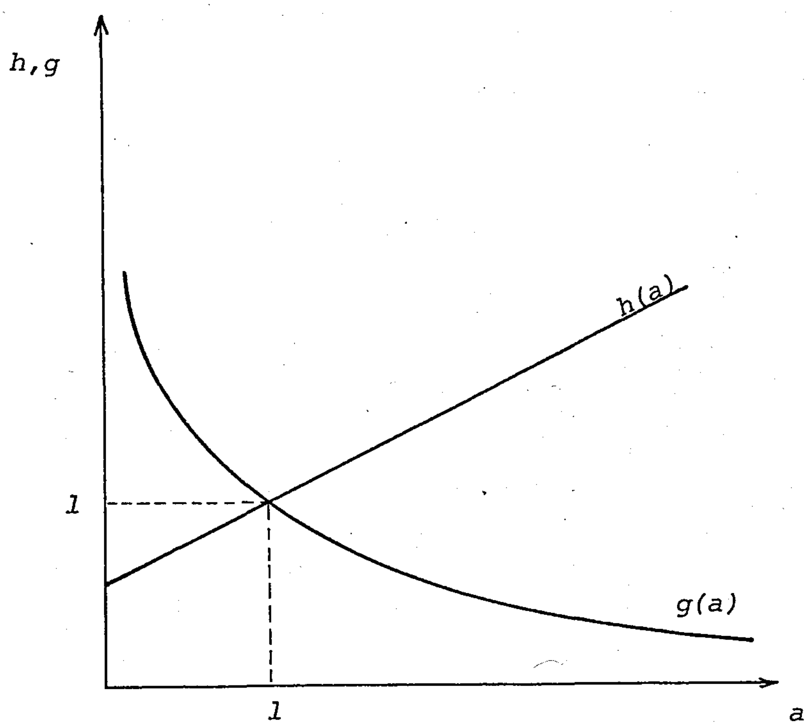

Figure 5.2.1 - The relationship between h(.) and g(.) for a mixed economy.

5.3 COURNOT-SERTEL EQUILIBRIUM

By the symmetry of the market, at optimality the equations of the model can be simplified into

$$\bar{p} = r\bar{Y}^{\theta} \quad , \tag{5.3.1}$$

$$\bar{Y} = (m_c + m_w)\bar{y} \quad , \tag{5.3.2}$$

$$\bar{y} = \bar{K}^{\alpha}\,\bar{L}^{\beta} \quad , \tag{5.3.3}$$

$$\bar{K} = (w\alpha/\beta\rho)\bar{L} \quad , \tag{5.3.4}$$

$$\bar{L} = n_w\,\bar{x}_w \quad , \tag{5.3.5}$$

$$\bar{\pi} = \bar{p}\bar{y}[1 - (\alpha + \beta)(1 + \theta\frac{\bar{y}}{\bar{Y}}] - C_o \quad , \tag{5.3.6}$$

$$\bar{u}_w = \frac{\bar{p}\bar{y}}{n_w}[1 - (\alpha + \frac{\beta}{\gamma})(1 + \theta\frac{\bar{y}}{\bar{Y}})] - \frac{C_o}{n_w} \quad , \tag{5.3.7}$$

$$\eta_w = (\gamma - 1)b\bar{x}_w^{\gamma} = \underline{u} \quad . \tag{5.3.8}$$

Furthermore, the first-order necessary conditions (5.1.3)-(5.1.6) and the equilibrium conditions (5.2.1) and (5.2.2) can be rewritten as

$$\alpha\bar{p}\bar{y}(1 + \theta\frac{\bar{y}}{\bar{Y}}) = \rho\bar{K} \quad , \tag{5.3.9}$$

$$\beta\bar{p}\bar{y}(1 + \theta\frac{\bar{y}}{\bar{Y}}) = w\bar{L} \quad , \tag{5.3.10}$$

$$\frac{\beta}{\gamma}\,\frac{\bar{p}\bar{y}}{n_w}(1 + \theta\frac{\bar{y}}{\bar{Y}}) = b\bar{x}_w^{\gamma} \, , \tag{5.3.11}$$

and

$$\bar{p}\bar{y}[1 - (\alpha + \beta)(1 + \theta\frac{\bar{y}}{\bar{Y}})] = C_o \, . \tag{5.3.12}$$

Substituting (5.3.1)-(5.3.4) into (5.3.10) and (5.3.12), we obtain

$$[\frac{\beta r}{w}(m_c + m_w)^{\theta}(\frac{w\alpha}{\beta\rho})^{\alpha(1+\theta)}\;(1 + \frac{\theta}{m_c + m_w})]^{1/\varepsilon_c} = \bar{L} \quad , \tag{5.3.13}$$

where $\varepsilon_c = 1 - (\alpha + \beta)(1 + \theta)$. Then

$$r(m_c + m_w)^{\theta}(\frac{w\alpha}{\beta\rho})^{\alpha(1+\theta)} L^{(\alpha+\beta)(1+\theta)}[1 - (\alpha + \beta)(1 + \frac{\theta}{m_c + m_w})] = C_o \cdot \tag{5.3.14}$$

By (5.3.13), (5.3.14) can be restated as

$$[(m_c + m_w)^{\theta}(1 + \frac{\theta}{m_c + m_w})^{(\alpha+\beta)(1+\theta)}]^{1/\varepsilon_c} [1 - (\alpha + \beta)(1 + \frac{\theta}{m_c + m_w})] = C_o' \tag{5.3.15}$$

where C_o' is given by Equation (3.4.13).

Note that (5.3.14) possesses the form of (3.4.14) while the only difference arises from the fact that m_c is now replaced by m_c+m_w.

Similarly the greatest integer $[m_c+m_w]$ satisfying (5.3.15) determines the equilibrium size of the market. The uniqueness and existence of $[m_c+m_w]$ at the equilibrium can be established as in Corollary (3.4.1). Knowing the optimum sum of workers' enterprises and capitalistic firm in the mixed economy all the other unknowns can then be determined by solving Equations (5.3.1)-(5.3.8).

As this argument reveals, the important result of this chapter lies in the fact that the total number of firms in the market bears great importance and in fact it is a fixed number at the equilibrium; furthermore the number of capitalistic firms and the number of workers' enterprises are not important as long as their sum is fixed since they are all identical.

Example (5.3.1): Consider Examples (3.4.1) and (4.5.1) and recall that

$\alpha = 1/3$, $\beta = 2/3$, $\gamma = 2$, $\theta = -1/2$, $w = 1/10$, $\rho = 1/10$,

$r = 1$, $b = 2$ and $C_o = 1/100$.

Then, the solution $[m_c+m_w]$ to (5.3.14) is found to be 16 which coincides with the solutions we have obtained in Examples (3.4.1) and (4.4.1). So

$\underline{u} = 0.00125$

$\bar{L} = 2.069$

$\bar{K} = 1.03454$

$\bar{y} = 1.64223$

$\bar{Y} = 26.2757$

$\bar{p} = 0.19508$

$\bar{\pi}_c = 0.00001168 \simeq 0$

$\bar{u}_w = 0.0012585$

$\eta_w = 0.0012587$

$[n_w] = 82$

$\bar{x}_w = 0.025087$

Note that in all the examples considered so far, the number of firms that the industry M can absorb remains the same regardless of the nature of the industry.

The result that the labor inputs of capitalist firms and workers' enterprises are equal leads us to conclude that the labor input of a typical worker in a capitalist firm is equal to the labor input of a worker-partner in a workers' enterprise since the worker in a capitalist firm is expected to determine his labor input by maximizing his own utility as the worker in a workers' enterprise does.

This argument shows us that the number of workers in a capitalist firm coincides with the number of worker-partners in a workers' enterprise. So in Example (5.3.1) a capitalist firm in a mixed economy will also employ 82 workers.

VI. SUMMARY AND RESULTS

This dissertation is concerned with equilibrium problem of oligopolies where perfect competition and free entry exist in the industry. A new Cournot Equilibrium with Free Entry notion suggested by Sertel is applied to capitalistic and workers' enterprises. The existence and uniqueness of such equilibrium are proven in a log-linear model where each firm produces a homogeneous good and uses an identical technology.

Once the outputs of the firms are determined to obtain Cournot Equilibrium with quantity as the strategic variable, the symmetry of the market is proven for both capitalistic and workers' enterprises, and the equilibrium is obtained by (3.4.11) for capitalistic oligopolies and by (4.4.12) for workers' enterprises. The values of the variables at equilibrium obtained as functions of m and n can be tabulated as follows.

For A Capitalistic Oligopoly:

$$\bar{L}_c = [(\frac{\beta r}{w})(\frac{w\alpha}{\beta\rho})^{\alpha(1+\theta)}\,\bar{m}_c(1 + \frac{\theta}{\bar{m}_c})]^{1/\varepsilon_c}$$

where $\varepsilon_c = 1 - (\alpha + \beta)(1 + \theta)$,

$$\bar{K}_c = \frac{w\alpha}{\beta\rho} \bar{L}_c \quad ,$$

$$\bar{y}_c = \bar{K}_c^{\alpha} \bar{L}_c^{\beta} \quad ,$$

$$\bar{Y}_c = \bar{m}_c \bar{y}_c \quad ,$$

$$\bar{p} = r \bar{Y}_c^{\theta} \quad ,$$

$$\bar{\pi}_c = \bar{p}\bar{y}_c[1 - (\alpha + \beta)(1 + \frac{\theta}{\bar{m}_c})] - C_o$$

where $\bar{m}_c$ is the greatest integer solution of (3.4.14)

For Workers' Enterprises:

$$\bar{x}_w = [(\frac{b\gamma\alpha}{\beta\rho})^{\alpha(1+\theta)} (\frac{\beta r}{b\gamma}) \bar{m}_w^{\theta} \bar{n}_w^{(\alpha+\beta)(1+\theta)-1} (1 + \frac{\theta}{\bar{m}_w})]^{1/\varepsilon_w}$$

where $\varepsilon_w = \gamma - (\gamma\alpha + \beta)(1 + \theta)$,

$$\bar{L}_w = \bar{n}_w \bar{x}_w \quad ,$$

$$\bar{K}_w = \frac{\gamma\alpha b}{\beta\rho} \bar{n}_w \bar{x}_w^{\gamma} \quad ,$$

$$\bar{y}_w = \bar{K}_w^{\alpha} \bar{L}_w^{\beta} \quad ,$$

$$\bar{Y}_w = \bar{m}_w \bar{y}_w \quad ,$$

$$\bar{p} = r \bar{Y}_w^{\theta} \quad ,$$

$$\bar{u}_w = \frac{\bar{p}\bar{y}_w}{\bar{n}_w} [1 - (\alpha + \frac{\beta}{\gamma})(1 + \frac{\theta\bar{y}_w}{\bar{Y}_w})] - C_o$$

where $\bar{m}_w$ is the greatest integer solution of (4.4.20) and $\bar{n}_w$ is given by (4.4.19).

Since the equilibrium size of the industry for both capitalistic and workers' enterprises is the solution of the same equation, each industry can absorb the same number of firms. Furthermore, the capital and labor inputs, and accordingly, the outputs of the firms turn out to be equal regardless of the type of industry being considered. In other words, under this model the firms are identical at equilibrium.

A more interesting conclusion is made in Chapter V; the capitalistic firms and workers' enterprise can co-exist and maintain the Cournot-Sertel Equilibrium preserving the symmetry of the industry. Furthermore the total number of firms in the market at equilibrium turns out to be the same as in capitalistic and labor-managed oligopolies but the number of firms of each type does not play an important role since they are symmetric. The number of workers in a capitalist firm and a workers' enterprise also turn out to be equal to each other.

In addition to the above results, in this model we proved that CSE also satisfies the conditions of Novshek's Equilibrium for both capitalistic and workers' enterprises.

In order to clarify the novelty of this dissertation, it suffices to bear in mind the distinction of Cournot-Sertel Equilibrium notion resulting in a more Cournot-like argument. If labor-managed firms are designed properly as the workers' enterprises which is proposed by Sertel and employed in this study, and if this more Cournot-like Equilibrium notion is applied; CEFE can be determined in spite of the nonexistence of CEFE result obtained by Laffont and Moreaux for a labor-managed oligopoly.

The studies carried out so far on this theory have dealt with determining CEFE for either a capitalist oligopoly or a labor-managed oligopoly separately. As far as we know, this study is the first one aiming at finding CEFE in a situation where capitalist and labor-managed firms co-exist and achieving to obtain the CEFE for the so-mentioned situation.

Furthermore the fact that labor-managed and capitalist firm which all possess the basic data on parameters involved in the model are identical at their respective optimal behaviour in a mixed economy alleviates the controversy going on between systems where capitalist firms or labor-managed firms constitute the majority.

In spite of the nice results obtained in this dissertation, we should confess that the log-linear model which our analysis is carried upon sacrifices greater generality but permits a detailed tabulation and complete comparison of equilibrium values among workers' enterprises and profit-maximizing firms.

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