DECENTRALIZED STABILIZATION WITH CONTROLLER CONSTRAINTS: STRONG AND RELIABLE STABILIZATION

by

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ABSTRACT

In this thesis we study two problems in decentralized stabilization. The first is the strong decentralized stabilization problem, which can be stated as follows. Given a plant Z, does there exist a block-diagonal stable compensator C that internally stabilizes Z ? The second is the reliable decentralized stabilization problem. Given a plant Z, does there exist a block-diagonal internally stabilizing compensator C that maintains its stabilizing property in case of interconnection failures in the plant? We show that for two-channel systems the two problems are equivalent in the following sense. The problem of reliable decentralized stabilization for a given plant is solvable if and only if the problem of strong decentralized stabilization for another plant (defined explicitly in terms of the original plant) is solvable.

Using this main result, we show that:

i) For a two-input-two-output plant with all of its zeros stable, the strong decentralized stabilization problem is solvable.

ii) For a two-input-two-output plant which has a tansfer matrix with the diagonal elements stable and the off-diagonal elements minimum phase, the reliable decentralized stabilization problem is solvable.

KISITLI DENETİMCİ İLE AYRIŞIK KARARLILAŞTIRMA: KUVVETLİ VE GÜVENİLİR KARARLILAŞTIRMA

ÖZET

Bu tezde ayrışık kararlılaştırmada iki problem incelenmektedir. Birincisi, verilen bir Z dizgesini iç kararlılaştıracak öbek-köşegen ve kendisi kararlı olan bir denetimci C bulunmasıdır. Buna kuvvetli ayrışık kararlılaştırma diyoruz. İkincisi ise verilen bir Z dizgesini iç kararlılaştıracak öbek-köşegen ve Z'nin ara bağlantılarındaki kopukluklarda kararlılaştırma özelliğini yitirmeyen bir denetimci C bulunması diye tanımlanan güvenilir ayrışık kararlılaştırmadır. Burada iki kanallı dizgelerde bu iki problemin birbirleri ile sıkı sıkıya ilişkili oldukları gösterilmektedir. Yani, verilen bir dizgeyi güvenilir ayrışık kararlılaştırma problemini çözmek için bu dizgenin parametreleri ile tanımlanan başka bir dizge için kuvvetli ayrışık kararlılaştırma problemini çözmek gerekli ve yeterlidir.

Bu ana sonuçtan yararlanılarak gösterilebilir ki:
i) Sıfırları kararlı olan iki-girdili-iki-çıktılı bir dizge için kuvvetli ayrışık kararlılaştırma problemi her zaman çözülebilir.
ii) Köşegen üzerindeki dönüşüm işlevlerinin kutupları kararlı ve köşegen dışı dönüşüm işlevlerinin sıfırları kararlı olan bir iki-girdili-iki-çıktılı dizge her zaman güvenilir ayrışık kararlılaştırılabilir.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

C	Transfer matrix of a stabilizing compensator
C_i	Elements of $C \in R(s)^{m \times n}$ in block form
c_i	Elements of $C \in R(s)^{2 \times 2}$
N	Numerator matrix of Z
n_i	Elements of numerator matrix N
$R(s)$	Set of rational functions in s
R_{sp}	Set of proper rational functions with all of its poles stable
$R(s)^{n \times m}$	Set of nxm matrices whose elements all belong to $R(s)$
$R_{sp}^{m \times n}$	Set of mxn matrices whose elements all belong to R_{sp}
Z	Transfer matrix of a linear time-invariant multivariable plant
Z_{ij}	Elements of $Z \in R(s)^{n \times m}$ in block form
z_{ij}	Elements of $Z \in R(s)^{2 \times 2}$
μ	Characteristic polynomial of $Z \in R(s)^{2 \times 2}$
σ	A stable polynomial
$=:$	.... defines ....
$:=$	.... is defined as ....

I. INTRODUCTION

Since the satisfactory resolution of pole assignment and internal stabilization problems in linear control theory via a dynamic output feedback scheme of Figure 2.1, (see e.g., ROSENBROCK [1]), the more difficult problems where the feedback compensator satisfies certain extra requirements are being considered. One of these special internal stabilization problems is the decentralized stabilization problem, where the stabilizing compensator has a block-diagonal structure. The first satisfactory solution to decentralized stabilization problem is due to WANG and DAVISON [2], in which the concept of decentralized fixed modes has been shown to be central to the existence of a decentralized compensator. The synthesis procedure of WANG and DAVISON [2], however, does not provide an explicit expression for the compensator transfer matrix. This is a major obstacle in imposing further engineering constraints on the stabilizing decentral compensator, such as reliability, compensator stability, minimality, etc.. Another novel approach to solve decentralized stabilization problem is that of CORFMAT and MORSE [3], where the concept of strong connectedness as well as decentralized fixed modes has been basic to their synthesis procedure. The main idea of CORFMAT and MORSE [3] is to use constant output feedbacks in all but one input-output channels of a strongly connected system to make the system reachable and observable from the remaining channel. They use a dynamic output feedback compensator in the remaining channel to achieve overall internal stability. The synthesis procedure of CORFMAT and MORSE [3] suffers from the same drawback of the original procedure of WANG and DAVISON [2] in that the procedure does not yield explicit expressions for the decentral compensator; although one can draw certain conclusions of immense practical value from the work of CORFMAT and MORSE [3] such as **almost all strongly connected systems can be decentrally stabilized.**

In certain special cases, decentralized control procedures which yield explicit expressions for the feedback compensators do exist. One such synthesis procedure is due to GUCLU and OZGULER [4] in the special case of diagonal stabilization problem. In this work it is shown that given an N-input-N-output plant, an internally stabilizing diagonal compensator can be determined by solving a nonlinear polynomial equation which can in turn easily be solved via Smith Canonical Forms (see Section III).

Another special stabilization problem is the strong stabilization problem of YOULA, BONGIORNO, and LU [5], where the compensator itself is required to be stable in addition to its internal stability property. The practical motivation for strong stabilization is that such closed loop systems exhibit superior sensitivity properties compared to plants which are internally stabilized by an unstable compensator.

This classical paper of YOULA, BONGIORNO, and LU [5] yields some conditions for the solvability of the problem purely in terms of the zeros and the poles of the plant to be internally stabilized. The result of YOULA, BONGIORNO, and LU [5] is for a general m-input-p-output plant. The central concept that emerges is the **parity interlacing property**. Later through the works of VIDYASAGAR and VISWANADHAM [6] and GHOSH [7], it has been realized that strong stabilization is also a central subproblem in **simultaneous stabilization problems**. In the context of decentralized stabilization problems, one can easily consider **strong decentralized stabilization problem**, where the compensator is block-diagonal, stable, and internally stabilizes a given plant. There has not been any noteworthy progress in this direction mainly due to the fact that most of the existing decentralized stabilization procedures do not yield explicit expressions for the compensator.

Finally, still another special stabilization problem is **decentralized reliable stabilization problem** which can roughly be described as determining a block-diagonal, internally stabilizing compensator which remains functioning in case of interconnection failures in the plant. Decentralized reliable stabilization has been the main concern of the book by SILJAK [8] in which decentralized stabilization of a system by (usually nonlinear) state-feedback has been considered. The conclusion SILJAK draws through his works (SILJAK [9,10]) and the work of DAVISON [11] is that for a large class of systems reliable decentralized stabilization is possible and does not constitute a serious constraint on the set of decentrally stabilizing compensators. In the case of decentralized schemes via dynamic output feedback, however, the decentral linear compensator might exhibit bad reliability properties with respect to interconnection failures (see the example of Section V). It thus remains a challenging question whether one can synthesize a decentral compensator that internally stabilizes a given plant and that remains reliable (i.e., maintains its stabilizing feature) in the case of interconnection failures. Another motivation for decentralized reliable stabilization is that a reliable stabilization scheme is also sub-reliable with respect to failures in the feedback loop. This point is further elaborated in Section IV of the thesis.

A sound conceptual framework in solving any special internal stabilization problem such as the ones described in the preceding paragraphs is the following : (i) Characterize the set of all compensators that solve the main problem (internal stabilization problem) in terms of a parameter set and (ii) Choose particular elements in the parameter set to obtain corresponding compensators with desired additional features. Such a scheme has in fact been the starting point of ZAMES [12], YOULA, BONGIORNO, and JABR [13], DESOER, LIU, MURRAY, and SAEKS [14], SAEKS and MURRAY [15] in a variety of problems ranging from sensitivity minimization, quadratic optimal control to output regulation and tracking. The success of such an approach is mainly due to the fact that it is relatively easy to characterize the set of all linear compensators that internally stabilize a given linear plant (see Section II). The question one can ask at this point is whether a similar characterization is possible for the set of all decentral compensators that stabilize a given plant in terms of a simple parameter set.

In this thesis, we exploit the main result of GUCLU and OZGULER [4] in obtaining the set of all diagonal stabilizing compensators in the simplest case of a two-input- two-output plant (Theorem 3.1). Although the result applies to a very restricted decentralized stabilization problem, it is the first of its kind and the same line of reasoning as in Theorem 3.1 yields the set of all solutions to the completeness equation (Theorem 3.2), which is tightly connected to the decentralized fixed modes in the multivariable case (see OZGULER [16]). We then rigorously define and study decentralized strong stabilization and decentralized reliable stabilization problems, again in the simplest cases of two-input-two-output and two-channel systems in the spirit of the conceptual framework of the preceding paragraph. The main outcome of this study is that strong stabilization is an integral part of reliable stabilization problems. In fact, in the special cases examined in this thesis the reliable stabilization problem for a given plant can be shown to be equivalent to a strong stabilization problem defined for a new plant. See Theorems 4.1 and 4.2. We also show in the same theorems that both problems are eventually reducible to solving equations of the type

$$a + bx + cy + dxy = u,$$

$$A + BXC + DYE = U,$$

where the unknowns; u is a unit in the ring of stable rational functions and x,y are elements in the ring; U is a unimodular stable rational matrix and X, Y are stable proper rational matrices. We also state some sufficient conditions for the solvability of these equations in Section III and IV.

This thesis is organized as follows. In Section II, we give some necessary definitions and notation we use in this thesis. We characterize all two-dimensional diagonal compensators that stabilize a given plant in Section III, and we show that they are given in terms of the compensators of another but a stable plant. We also give a comment on how to solve decentralized strong stabilization problems in view of this characterization. Multivariable version of this characterization, yielding the set of all solutions to completeness equation, is also studied in that section. In Section IV, we show that for two-channel multivariable systems and for two-input, two-output systems, reliable decentralized stabilization problem is equivalent to strong decentralized stabilization problem in the sense that the problem of reliable decentralized stabilization for a given plant can be reduced to the problem of stabilizing a new plant using a stable decentral compensator. In Section V we give some consequences of the main results of Section IV and we give a large class of transfer matrices for which the reliable decentralized stabilization problem is solvable. Finally, we give an example to show that a decentralized stabilizing compensator for a given plant does not necessarily maintain its stabilizing feature in case of interconnection failures in the plant, and an example illustrating the synthesis procedure for the reliable decentralized stabilization problem using the results of Section IV.

II. BACKGROUND AND NOTATION

In this section we set up the notation and state some preliminary results that will be frequently used in the subsequent sections. For the details of notation and terminology and results given without proof the reader is referred to KHARGONEKAR and OZGULER [17].

Throughout the thesis we let R(s) denote the set of rational functions in s with real coefficients and we let R_{sp} denote the subset of R(s) consisting of proper rational functions whose poles lie in the open left-half plane. The set R_{sp} is a ring; thus if two functions f_1 and f_2 belong to R_{sp} so do their difference and product. The ring R_{sp} is clearly commutative ($f_1.f_2=f_2.f_1$) and is an integral domain ($f_1.f_2=0$ implies $f_1=0$ or $f_2=0$). The set R(s) is the quotient field generated by R_{mp} ; i.e. every $g \in R(s)$ can be written as $g = f_1/f_2$ such that $f_1, f_2 \in R_{mp}$ and $f_2 \neq 0$ and conversely every ratio f_1/f_2 where $f_1, f_2 \in R_{mp}$, $f_2 \neq 0$ belongs to R(s). If we define the degree of an element f in R_{mp} as its relative degree (i.e. the degree of the denominator polynomial minus the degree of the numerator polynomial) plus the number of its finite zeros in the right-half plane, then R_{mp} can be seen to be an Euclidean domain, i.e., given any two elements f and $g \neq 0$ in R_{mp}, there exist an h, and an l such that $f = gh + l$, where the degree of l is less than the degree of g. In other words, a division algorithm can be performed in R_{mp}. Note that the degree of an element in R_{mp} is precisely the number of its unstable zeros counting the multiplicities and the zeros at infinity. A most useful property of an Euclidean domain is that a matrix with elements from an Euclidean domain can be brought to Smith Canonical Form under unimodular equivalence (MacDUFFEE [18]).

A function f in R_{mp} is called a unit if its reciprocal belongs to R_{mp}. Clearly the units in R_{mp} are those functions with relative degree zero and with stable zeros.

Given any rational function h, we can find two functions f and g in R_{mp} such that $h = f/g$, and such that f and g are relatively prime (i.e. one is a greatest common divisor of f and g). In other words there exist a and b in R_{mp} such that $a.f + b.g = 1$. Such a pair (f,g) is called a coprime factorization of h.

It is essential to recognize that we are expressing a given rational function h as a ratio of proper stable transfer functions with no common factors, rather than as a ratio of polynomials with no common zeros.

We let $R_{wp}^{n \times m}$ denote the set of nxm matrices whose elements all belong to R_{wp}. Thus $R_{wp}^{n \times m}$ is the set of transfer functions of stable linear time-invariant systems with m inputs and n outputs. A matrix $F \in R_{wp}^{n \times n}$ is unimodular if its inverse exists and belongs to $R_{wp}^{n \times n}$. Clearly, F is unimodular if and only if det(F) is a unit.

Given any $Z \in R(s)^{n \times m}$ (which means Z is an nxm matrix whose elements are rational functions of s). We can find matrices $N_R \in R_{wp}^{n \times m}$ and $D_R \in R_{wp}^{m \times m}$ such that $Z=N_R D_R^{-1}$ and the matrices N_R, D_R are right-coprime, i.e. there exist $P \in R_{wp}^{m \times n}$ and $Q \in R_{wp}^{m \times m}$ such that

$$PN_R + QD_R = I_m \qquad \text{for all s.}$$

Similarly, we can find $N_L \in R_{wp}^{n \times m}$, $D_L \in R_{wp}^{n \times n}$, $P_1 \in R_{wp}^{m \times n}$ and $Q_1 \in R_{wp}^{n \times n}$ such that $Z=D_L^{-1}N_L$ and

$$D_L Q_1 + N_L P_1 = I_n \qquad \text{for all s.}$$

We refer to (N_R, D_R) as a right-coprime factorization (r.c.f.) of Z and to (D_L, N_L) as a left-coprime factorization (l.c.f.) of Z.

If (N_R, D_R) is a right-coprime factorization of Z so is $(N_R U, D_R U)$ whenever U is an mxm unimodular matrix. Conversely, if (N_R, D_R), (N_{R1}, D_{R1}) are two r.c.f. of Z, then $N_R = N_{R1} U$, $D_R = D_{R1} U$ for some unimodular U.

If (D_L, N_L) is a left-coprime factorization of Z, so is (UD_L, UN_L) whenever U is an nxn unimodular matrix. Conversely, if (D_L, N_L), (D_{L1}, N_{L1}) are two l.c.f. of Z, then $D_L = UD_{L1}$, $N_L = UN_{L1}$ for some unimodular U.

Now, we briefly summarize some results on feedback stability. Consider the feedback system shown below in Figure 2.1, where Z and C are rational matrices of order nxm and mxn respectively, and assume that $\det(I_n+ZC)\neq 0$ (otherwise the system is not well-defined).

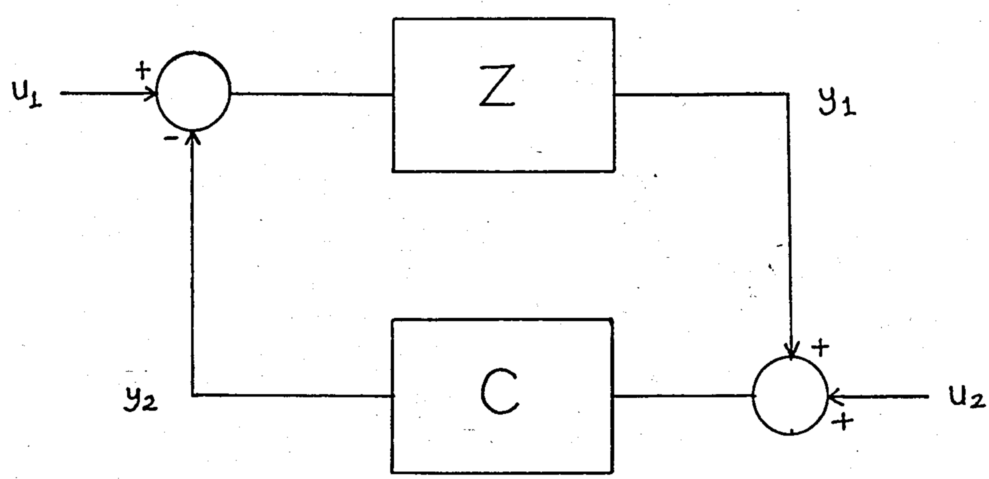

Figure 2.1 A feedback system with dynamic compensator C

Then it is easy to verify that

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} (I_m+CZ)^{-1}C & -(I_m+CZ)^{-1}CZ \\ (I_n+ZC)^{-1}ZC & (I_n+ZC)^{-1}Z \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

or more concisely

$$y = H\,u \quad .$$

We will say that the pair (Z,C) is internally stable if and only if $H \in R_{sp}^{(n+m)\times(n+m)}$ (VIDYASAGAR, SCHNEIDER, and FRANCIS [19]). We say that C internally stabilizes Z if (Z,C) is internally stable.

Next, we state without proof a necessary and sufficient condition for a pair (Z,C) to be stable. The proof is essentially contained in DESOER and CHAN [20].

LEMMA 2.1 Let $Z \in R(s)^{n\times m}$ be represented as $Z=PQ^{-1}R$, where P, Q, R belong to $R_{sp}^{n\times k}$, $R_{sp}^{k\times k}$, and $R_{sp}^{k\times m}$, respectively and (P,Q) is right-coprime, (Q,R) is left-coprime. Let $C \in R(s)^{m\times n}$ be represented as $C=ED^{-1}F$, where E, D, F belong to $R_{sp}^{m\times s}$, $R_{sp}^{s\times s}$, $R_{sp}^{s\times n}$ respectively and (E,D) is right-coprime, (D,F) is left-coprime. Then the following statements are equivalent:

i) The pair (Z,C) is internally stable.

ii) The matrix $\Phi := \begin{bmatrix} Q & RE \\ -FP & D \end{bmatrix}$ is unimodular.

■

Let (N_R, D_R) be any r.c.f. of $Z \in R(s)^{n\times m}$ and let (D_{CL}, N_{CL}) be any l.c.f. of $C \in R(s)^{m\times n}$. Then letting $P=N_R$, $Q=D_R$, $R=I$ and $E=I$, $D=D_{CL}$, $F=N_{CL}$, where I is the identity matrix, the above lemma simplifies.

Corollary 2.1.1 The following statements are equivalent:

i) The pair (Z,C) is internally stable.

ii) The matrix $(D_{CL}D_R + N_{CL}N_R)$ is unimodular.

■

Similarly, let (D_L, N_L) be any l.c.f. of $Z \in R(s)^{n\times m}$ and let (N_{CR}, D_{CR}) be any r.c.f. of $C \in R(s)^{m\times n}$. Then letting $P=I$, $Q=D_L$, $R=N_L$ and $E=N_{CR}$, $D=D_{CR}$, $F=I$, where I is the identity matrix, we have a dual result to Corollary 2.1.1.

Corollary 2.1.2 The following statements are equivalent.

i) The pair (Z,C) is internally stable.

ii) The matrix $(D_L D_{CR} + N_L N_{CR})$ is unimodular.

■

Note that if $C \in R_{sp}^{m\times n}$, i.e., if the transfer matrix of the compensator is a stable proper rational matrix then $N_R=C$, $D_R=I$ and $N_L=C$, $D_L=I$ yield right and left-coprime factorization for C, respectively. This easily yields the following result.

Corollary 2.1.3 Let (N_R, D_R), (D_L, N_L) be any r.c.f. and l.c.f. of $Z \in R(s)^{n\times m}$ and suppose $C \in R_{sp}^{m\times n}$. Then the following conditions are equivalent.

i) The pair (Z,C) is internally stable.

ii) The matrix D_R+CN_R is unimodular.

iii) The matrix $D_L+N_L C$ is unimodular.

Throughout the thesis we often encounter the problem of characterizing all solutions to the equation

$$PN_R + QD_R = I \quad .$$

Solution to this problem is related to the characterization of all compensators of a plant. Before we characterize all compensators that stabilize a given strictly proper plant, we give general solution to this equation.

LEMMA 2.2 Let $Z \in R(s)^{n \times m}$ and let (N_R, D_R), (D_L, N_L) be any r.c.f. and l.c.f. of Z. General solution to the equation $PN_R+QD_R=I$ in the unknowns $P \in R_{sp}^{m \times n}$, $Q \in R_{sp}^{m \times m}$ is given by

$$P = P^o + R\, D_L$$

$$Q = Q^o - R\, N_L$$

where (P^o, Q^o) is a particular solution of the equation $PN_R+QD_R=I$ and $R \in R_{sp}^{m \times n}$.

General solution to the equation $D_LQ+N_LP=I$ is given by, for arbitrary S in $R_{sp}^{m \times n}$

$$P = P_1^o + D_R S$$
$$Q = Q_1^o - N_R S$$

where (P_1^o, Q_1^o) is a particular solution of the equation $D_LQ+N_LP=I$. Various procedures exist to obtain a particular solution to these equations (see e.g. KHARGONEKAR and OZGULER [17], PERNEBO [21]).

The next result characterizes all compensators that stabilize a given strictly proper plant. (see VIDYASAGAR, SCHNEIDER, and FRANCIS [19]).

LEMMA 2.3 Let $Z \in R(s)^{n \times m}$ be strictly proper and let (N_R, D_R), (D_L, N_L) be any r.c.f. and l.c.f. of Z. Select matrices P, Q, P_1, Q_1 such that

$$PN_R + QD_R = I_m$$

$$N_L P_1 + D_L Q_1 = I_n$$

Then the set of all compensators that internally stabilize Z is given by

$$C = (Q-RN_L)^{-1}(P+RD_L), \qquad R \in R_{sp}^{m\times n}$$

or

$$C = (P_1+D_RS)(Q_1-N_RS)^{-1}, \qquad S \in R_{sp}^{m\times n}$$

REMARK 2.1 : The matrices $Q-RN_L$ and Q_1-N_RS are nonsingular for any choice of matrices $R \in R_{sp}^{m\times n}$ and $S \in R_{sp}^{m\times n}$. To see this note that by $PN_R+QD_R=I_m$, we have $D_R^{-1}=PZ+Q$ implying that D_R is biproper, i.e. D_R^{-1} is also proper. Consequently, $N_R=ZD_R$ is strictly proper. This in turn implies that $Q=(I_m-PN_R)D_R^{-1}$ is biproper. Similarly, it follows that N_L is strictly proper and that Q_1 is biproper.

Now, $Q-RN_L=Q(I-Q^{-1}RN_L)$ where Q is biproper and $I-Q^{-1}RN_L$ is also biproper for any $R \in R_{sp}^{m\times n}$. Therefore, $Q-RN_L$ is nonsingular for any $R \in R_{sp}^{m\times n}$. Similarly, it follows that Q_1-N_RS is nonsingular for any $S \in R_{sp}^{m\times n}$.

III. CHARACTERIZATION

In this section we give a characterization of all diagonal compensators that internally stabilize a given two-input-two-output plant.

Consider the strictly proper transfer matrix

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix},$$

where z_{11}, z_{12}, z_{21}, and z_{22} are strictly proper rational functions. Let μ be a least common denominator of all minors of Z, i.e. a least degree polynomial which is divisible by the denominator polynomials of z_{11}, z_{12}, z_{21}, z_{22}, and $(z_{11}z_{22}-z_{12}z_{21})$. Then, μZ is easily seen to be a polynomial matrix; denoted as

$$\mu Z =: \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}. \tag{3.1}$$

Let σ be any polynomial having all its zeros stable and with degree equal to the degree of μ. It follows that

$$m := \mu/\sigma$$

is biproper and is in R_{sp}. Further, let $n_1:=v_{11}/\sigma$, $n_2:=v_{12}/\sigma$, $n_3:=v_{21}/\sigma$, and $n_4:=v_{22}/\sigma$, which are in R_{sp} so that

$$\mu/\sigma\ Z = 1/\sigma \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} = 1/m \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix} .$$

We claim that m divides $n_1n_4-n_2n_3$, i.e. for some d in R_{sp}, we have

$$md = n_1n_4-n_2n_3 \quad .$$

To see this note that on taking the determinants in (3.1), we have $\mu^2(z_{11}z_{22}-z_{12}z_{21})=v_{11}v_{22}-v_{12}v_{21}$. By the choice of μ, $\mu(z_{11}z_{22}-z_{12}z_{21})=:\delta$ is a polynomial. Hence, $\mu\delta=v_{11}v_{22}-v_{12}v_{21}$ which on division by σ^2 yields $md=n_1n_4-n_2n_3$, where $d:=\delta/\sigma$. Consequently we have a representation

$$Z = 1/m \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix} \tag{3.2}$$

of Z which has the property that

i) m divides $n_1n_4-n_2n_3$,

ii) $m = \mu/\sigma$, where σ is a stable polynomial and μ is the characteristic polynomial of Z.

Let

$$C = \begin{bmatrix} \alpha_1/\beta_1 & 0 \\ 0 & \alpha_2/\beta_2 \end{bmatrix}$$

be the fractional representation over R_{mp} of unknown compensator transfer matrix C. Where $\alpha_1, \beta_1, \alpha_2, \beta_2$ are in R_{mp} and (α_i, β_i) i=1,2 are coprime pairs.

Then we can state the following lemma.

LEMMA 3.1 C internally stabilizes Z if and only if

$$u := m\beta_1\beta_2 + n_1\alpha_1\beta_2 + n_4\beta_1\alpha_2 + d\alpha_1\alpha_2 \tag{3.3}$$

is a unit in R_{mp}.

Proof: C internally stabilizes Z if and only if d.det(I+ZC) is a unit, GUCLU and OZGULER [4]. An easy calculation yields that

$$d.\det(I+ZC) = m\beta_1\beta_2 + n_1\alpha_1\beta_2 + n_4\beta_1\alpha_2 + d\alpha_1\alpha_2 \quad .$$

■

By this lemma, the problem of finding all C's that internally stabilize a given plant Z turns out to be a question of characterizing all α_1, β_1, α_2, β_2 that satisfy equation (3.3). We will give an answer to this question below.

Let (a,b,c,d) be in R_{mp}^4 such that the greatest common factor of (a,b,c,d) is a unit. Define two sets A and M as

$$A=\{(\alpha_1,\alpha_2,\beta_1,\beta_2)\in R_{sp}^4 : a\beta_1\beta_2+b\beta_1\alpha_2+c\alpha_1\beta_2+d\alpha_1\alpha_2=1\},$$

and

$$M=\{(m_1,m_2,m_3,m_4)\in R_{sp}^4 : m_1m_4+\Omega m_2m_3=1\},$$

where Ω := ad-bc.

Let

$$U^o := \begin{bmatrix} \beta_1^o & \alpha_1^o \\ \delta_1^o & \tau_1^o \end{bmatrix} \qquad V^o := \begin{bmatrix} \beta_2^o & \delta_2^o \\ \alpha_2^o & \tau_2^o \end{bmatrix}$$

be unimodular matrices with $\det U^o=\det V^o=1$ satisfying

$$U^o\begin{bmatrix} a & b \\ c & d \end{bmatrix}V^o = \begin{bmatrix} 1 & 0 \\ 0 & \Omega \end{bmatrix} =: S .$$

Here S is the Smith Canonical Form of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in $R_{sp}^{2\times 2}$ and such unimodular matrices exist by the fact that R_{sp} is an Euclidean domain. Then we can state the following theorem.

THEOREM 3.1 A quadruple $(\alpha_1,\alpha_2,\beta_1,\beta_2)$ belongs to A if and only if there exists a quadruple (m_1,m_2,m_3,m_4) in M such that

$$(\beta_1\ \ \alpha_1) = (m_1\ \ m_2)\begin{bmatrix} \beta_1^o & \alpha_1^o \\ \delta_1^o & \tau_1^o \end{bmatrix}, \qquad \begin{bmatrix} \beta_2 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \beta_2^o & \delta_2^o \\ \alpha_2^o & \tau_2^o \end{bmatrix}\begin{bmatrix} m_4 \\ m_3 \end{bmatrix}$$

Proof: If $(\alpha_1, \alpha_2, \beta_1, \beta_2)$ belongs to A then,

$$(\beta_1 \quad \alpha_1) \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \beta_2 \\ \alpha_2 \end{bmatrix} = 1$$

$$(\beta_1 \quad \alpha_1)\; U^{o-1}\; S\; V^{o-1} \begin{bmatrix} \beta_2 \\ \alpha_2 \end{bmatrix} = 1 \quad .$$

Let $(m_1 \;\; m_2) = (\beta_1 \;\; \alpha_1)\, U^{o-1}$ and,

$$\begin{bmatrix} m_4 \\ m_3 \end{bmatrix} = V^{o-1} \begin{bmatrix} \beta_2 \\ \alpha_2 \end{bmatrix}$$

then obviously we have,

$$(m_1 \quad m_2) \begin{bmatrix} 1 & 0 \\ 0 & \Omega \end{bmatrix} \begin{bmatrix} m_4 \\ m_3 \end{bmatrix} = 1$$

namely, $m_1 m_4 + \Omega m_2 m_3 = 1$. Thus (m_1, m_2, m_3, m_4) belongs to M.

Conversely, if (m_1, m_2, m_3, m_4) belongs to M then,

$$(m_1 \quad m_2) \begin{bmatrix} 1 & 0 \\ 0 & \Omega \end{bmatrix} \begin{bmatrix} m_4 \\ m_3 \end{bmatrix} = 1 , \qquad (m_1 \quad m_2)\; U^{o} \begin{bmatrix} a & b \\ c & d \end{bmatrix} V^{o} \begin{bmatrix} m_4 \\ m_3 \end{bmatrix} = 1 .$$

Then $(\beta_1 \quad \alpha_1) \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \beta_2 \\ \alpha_2 \end{bmatrix} = 1.$ Therefore $(\alpha_1, \alpha_2, \beta_1, \beta_2)$ belongs to A.

■

Letting

$a := m, \quad b := n_4, \quad c := n_1,$ and $d := d,$

in Theorem 3.1, we obtain the set of all solutions $(\alpha_1, \alpha_2, \beta_1, \beta_2)$ of equation (3.3). This in turn yields a characterization of the set of all decentral (diagonal) stabilizing compensators

$$C = \begin{bmatrix} \alpha_1/\beta_1 & 0 \\ 0 & \alpha_2/\beta_2 \end{bmatrix}$$

of the given plant Z. It is actually possible to state this characterization in a more system-theoretic setting. To do this note that

$$\Omega = ad-bc = md-n_1n_4 = -n_2n_3,$$

and consider a subsidiary stable transfer matrix

$$Z_1 = \begin{bmatrix} 0 & n_2 \\ n_3 & 0 \end{bmatrix} .$$

This transfer matrix consists of the off-diagonal entries of the numerator matrix of original plant Z. It follows by Lemma 3.1 applied to Z_1 that diagonal compensator

$$C_1 = \begin{bmatrix} m_2/m_1 & 0 \\ 0 & m_3/m_4 \end{bmatrix}$$

is such that (Z_1, C_1) is internally stable if and only if (m_1, m_2, m_3, m_4) is in M. It follows that the set of all diagonal stabilizing compensators of Z is described by the parameter set M.

REMARK 3.1 Note that in view of this characterization, one procedure to solve decentralized strong stabilization problem for the plant of (3.2) is to search for an element (m_1, m_2, m_3, m_4) in M such that

$$\beta_1 = m_1\beta_1^o + m_2\delta_1^o, \qquad \beta_2 = m_4\beta_2^o + m_3\delta_2^o$$

are units in R_{mp}.

Multivariable version of theorem 3.1 can be proved by a similar reasoning and its use will be in determination of all K,L,M,N such that completeness equation (see OZGULER [16])

$$(K\ L)\begin{bmatrix} Q & R \\ P & 0 \end{bmatrix}\begin{bmatrix} M \\ N \end{bmatrix} = I$$

is satisfied. We state the following result without proof as it is only loosly connected with the rest of the material in this thesis.

THEOREM 3.2 The set

$$\Sigma = \{ (K,L,M,N) : (K\ L)\begin{bmatrix} Q & R \\ P & 0 \end{bmatrix}\begin{bmatrix} M \\ N \end{bmatrix} = I \}$$

is given by

$$\Sigma = \{ (K,L,M,N) : (K\ L) = (U_1\ U_2)\begin{bmatrix} K^o & L^o \\ K_1^o & L_1^o \end{bmatrix}, \begin{bmatrix} M \\ N \end{bmatrix} = \begin{bmatrix} M^o & M_1^o \\ N^o & N_1^o \end{bmatrix}\begin{bmatrix} U_4 \\ U_3 \end{bmatrix},$$

$$U_1U_4 + U_2SU_3 = I \}$$

where

$$\begin{bmatrix} K^o & L^o \\ K_1^o & L_1^o \end{bmatrix}\begin{bmatrix} Q & R \\ P & 0 \end{bmatrix}\begin{bmatrix} M^o & M_1^o \\ N^o & N_1^o \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & S \end{bmatrix}.$$

IV. STRONG AND RELIABLE DECENTRALIZED STABILIZATION PROBLEMS

One of the most important aspects of large-scale system control is decentralization. This implies that various controllers in the system are only allowed to measure certain outputs of the system and control certain inputs. The decentralized information structure often appears in practice in large-scale systems where it may be impractical, unreliable, and costly to utilize all inputs and measurements.

4.1 TWO-CHANNEL MULTIVARIABLE SYSTEMS

In this section we consider the strong and reliable decentralized stabilization problems for two-input-channel and two-output-channel systems. Two-input-channel and two-output-channel systems are those systems that provide two groups of outputs to measure and two groups of inputs to control.

4.1.1 STRONG DECENTRALIZED STABILIZATION PROBLEM

The problem of stabilizing a given plant using a stable compensator is called strong stabilization problem, YOULA, BONGIORNO, and LU [5]. If the stabilizing compensator is required to be block-diagonal, then it is called a strong decentralized stabilization problem, which can formally be defined as follows.

Given a linear time-invariant multivariable system transfer matrix

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

where Z_{11}, Z_{12}, Z_{21}, Z_{22} are elements of $R(s)^{p\times r}$, $R(s)^{p\times q}$, $R(s)^{m\times r}$, $R(s)^{m\times q}$ respectively. Determine a block diagonal feedback compensator

$$C = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix},$$

such that

i) C is stable rational. i.e. C_1, C_2 are elements of $R_{sp}^{r\times p}$, $R_{sp}^{q\times m}$ respectively, and

ii) C internally stabilizes Z.

Let Z be represented in coprime matrix fractional representation as

$$Z = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} Q^{-1} \quad (R_1 \quad R_2) \tag{4.1}$$

where P_1, P_2, Q, R_1, R_2 belong to $R_{sp}^{p\times k}$, $R_{sp}^{m\times k}$, $R_{sp}^{k\times k}$, $R_{sp}^{k\times r}$, $R_{sp}^{k\times q}$ respectively, with (P_1,P_2,Q) right-coprime, (Q,R_1,R_2) left-coprime. It follows by LEMMA 2.1 that C internally stabilizes Z if and only if

$$\Phi = \begin{bmatrix} Q & R_1C_1 & R_2C_2 \\ -P_1 & I & 0 \\ -P_2 & 0 & I \end{bmatrix}$$

is unimodular in $R_{sp}^{(k+p+m)\times(k+p+m)}$.

Multiplying the second and the third columns by P_1 and P_2 from the right, respectively, and adding to the first column we have

$$\Phi_1 = \begin{bmatrix} Q+R_1C_1P_1+R_2C_2P_2 & R_1 & R_2 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} .$$

Clearly, Φ_1, therefore Φ, is unimodular if and only if $Q+R_1C_1P_1+R_2C_2P_2=U$ is unimodular in $R_{sp}^{k\times k}$.

This proves the following statement:

Proposition 4.1.1 Strong decentralized stabilization problem is solvable if and only if there exist C_1 in $R_{sp}^{r\times p}$ and C_2 in $R_{sp}^{q\times m}$ such that

$$Q + R_1C_1P_1 + R_2C_2P_2 =: U \quad .$$

is a unimodular matrix; in which case, $C:=\text{diag}(C_1,C_2)$ is a solution to the problem.

■

By the result of this proposition one can concentrate on the equation $Q+R_1C_1P_1+R_2C_2P_2=U$, where the unknown U is unimodular and unknowns C_1, C_2 are stable proper rational matrices.

4.1.2 RELIABLE DECENTRALIZED STABILIZATION PROBLEM

In this section we pose further requirements on the feedback compensator. These requirements improve the reliability of the system. Here by reliability we mean that in the case of complete break-down of any one of the interconnections the subsystems remain stable. It is possible, however, to have an unstable system due to a disconnection of a controller. But in that case the remaining compensator makes the system sub-reliable (i.e. not worse than the original system with no compensators).

Consider the decentralized control system below

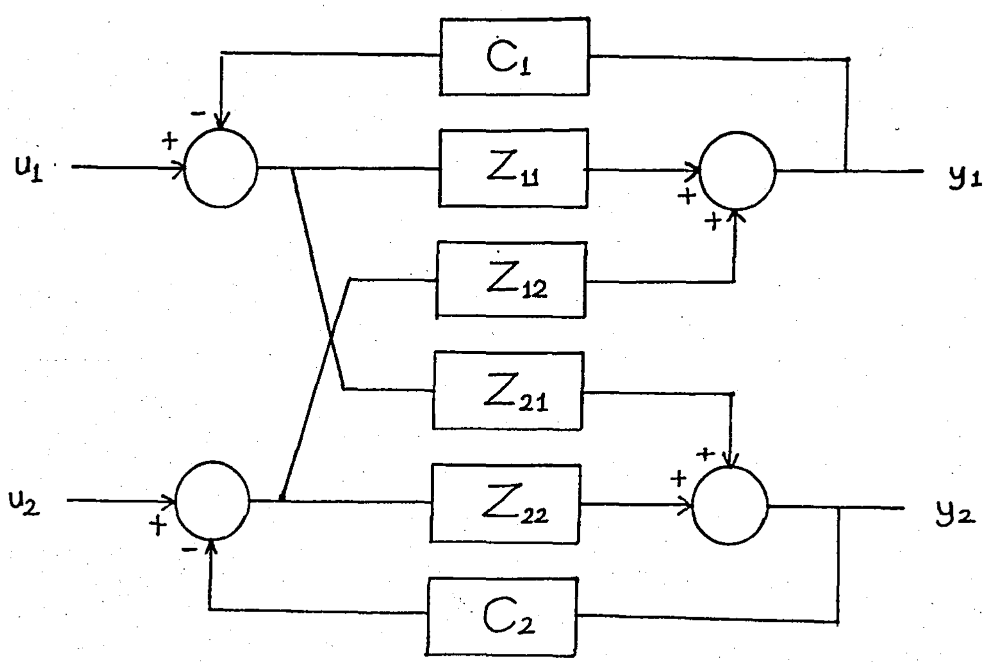

Figure 4.1.1 Decentralized control system

where the compensator $C=\text{diag}(C_1,C_2)$ internally stabilizes the plant $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$.

Here we call Z_{11}, Z_{22} as subplant transfer matrices, and Z_{12}, Z_{21} as interconnection transfer matrices.

Now, suppose that any one of the interconnections, namely Z_{12} or Z_{21}, breaks down completely. In such a situation, if the controllers are chosen such that C_1 internally stabilizes Z_{11} and C_2 internally stabilizes Z_{22}, then the subsystems, namely (Z_{11},C_1) and (Z_{22},C_2), remain stable. Clearly, if both of the interconnections fail, then the system again remains stable. Such a system is called reliable.

In case of controller failure, namely $C_1=0$ or $C_2=0$, however, the system may become unstable. But in that case the remaining compensator makes the system not worse than the original unstable system. We call such a system as sub-reliable.

On the other hand, if C_1 and C_2 do not have reliability property, then the overall system may become unstable in case of interconnection failures.

Consider a linear time-invariant system represented by a transfer matrix

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}.$$

where Z_{11}, Z_{12}, Z_{21}, Z_{22} belong to $R(s)^{p\times r}$, $R(s)^{p\times q}$, $R(s)^{m\times r}$, $R(s)^{m\times q}$, respectively. The reliable decentralized stabilization problem is formally defined as follows:

Determine a decentralized feedback compensator

$$C = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$$

such that

i) C_1 internally stabilizes Z_{11},
ii) C_2 internally stabilizes Z_{22},
iii) C internally stabilizes Z.

Let Z be represented in coprime matrix fractional representation as $Z = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} Q^{-1} (R_1 \quad R_2)$.

Also let $Z_{11}=P_1Q^{-1}R_1$ and $Z_{22}=P_2Q^{-1}R_2$ be represented by coprime matrix fractional representation as

$$Z_{11} = Q_{11}^{-1}R_{11} = P_{1o}Q_{1o}^{-1}$$

and

$$Z_{22} = Q_{22}^{-1}R_{22} = P_{2o}Q_{2o}^{-1}$$

where Q_{11}, R_{11}, Q_{22}, R_{22}, P_{1o}, Q_{1o}, P_{2o}, Q_{2o} belong to $R_{mp}^{p\times p}$, $R_{mp}^{p\times r}$, $R_{mp}^{m\times m}$, $R_{mp}^{m\times q}$, $R_{mp}^{p\times r}$, $R_{mp}^{r\times r}$, $R_{mp}^{m\times q}$, and $R_{mp}^{q\times q}$ respectively, and (Q_{11},R_{11}), (Q_{22},R_{22}) are left-coprime, (P_{1o},Q_{1o}), (P_{2o},Q_{2o}) are right-coprime.

By Lemma 2.1, a compensator

$$C = \begin{bmatrix} E_1 D_1^{-1} & 0 \\ 0 & E_2 D_2^{-1} \end{bmatrix}, \qquad (4.1.1)$$

where E_1, E_2, D_1, D_2 are in $R_{mp}^{r\times p}$, $R_{mp}^{q\times m}$, $R_{mp}^{p\times p}$, $R_{mp}^{m\times m}$, with (E_1,D_1), (E_2,D_2) right-coprime, internally stabilizes Z if and only if

$$U := \begin{bmatrix} Q & R_1E_1 & R_2E_2 \\ -P_1 & D_1 & 0 \\ -P_2 & 0 & D_2 \end{bmatrix}$$

is unimodular. For the compensator C to satisfy the additional constraints (i) and (ii) it is also necessary by Corollary 2.1.2 that

$$U_1 := Q_{11}D_1 + R_{11}E_1,$$
$$U_2 := Q_{22}D_2 + R_{22}E_2$$

are also unimodular. Conversely, if there exist E_1, D_1, E_2, D_2 in $R_{mp}^{r\times p}$, $R_{mp}^{p\times p}$, $R_{mp}^{q\times m}$, $R_{mp}^{m\times m}$, respectively such that the stable rational matrices U_1, U_2, U are all unimodular, then the compensator defined by (4.1.1) satisfies (i), (ii), and (iii) above, i.e., it is a solution to reliable decentralized stabilization problem. Consequently, in the light of the above discussion we can state the following preliminary result.

LEMMA 4.1.2 The reliable stabilization problem for Z is solvable if and only if there exist right-coprime pairs (E_1, D_1) and (E_2, D_2) such that U_1, U_2 and U, defined above, are all unimodular matrices. In this case, the compensator C of (4.1.1) is a solution to the problem.

■

Now we can give the main result of this section. Let E_i, D_i, i=1,2 be particular solutions to the equations

$$Q_{ii}D_i + R_{ii}E_i = I \qquad i=1,2$$

(such particular solutions exist by the fact that (Q_{ii}, R_{ii}) are left-coprime pairs).

Define

$$Q^c := \begin{bmatrix} Q & R_1E_{11}^c & R_2E_{22}^c \\ -P_1 & D_{11}^c & 0 \\ -P_2 & 0 & D_{22}^c \end{bmatrix},$$

$$R_1^c := \begin{bmatrix} -R_1Q_{1c} \\ P_{1c} \\ 0 \end{bmatrix}, \quad R_2^c := \begin{bmatrix} -R_2Q_{2c} \\ 0 \\ P_{2c} \end{bmatrix}, \quad \begin{matrix} P_1^c := (0 \quad I \quad 0), \\ P_2^c := (0 \quad 0 \quad I). \end{matrix}$$

Since Z is strictly proper, by similar reasoning in REMARK 2.1 $\det Q^c \neq 0$, so that Q^{-1} is well-defined. It is easy to show also that Q^c, R_1^c, R_2^c are left-coprime and P_1^c, P_2^c, Q^c are right-coprime.

THEOREM 4.1 The following statements are equivalent:

i) The reliable decentralized stabilization problem for Z is solvable.

ii) There exist X and Y in $R_{mp}^{r\times p}$ and $R_{mp}^{q\times m}$ respectively, such that

$$Q^p + R_1^p X P_1^p + R_2^p Y P_2^p =: U^p$$

is unimodular.

iii) The strong decentralized stabilization problem for

$$Z^p = \begin{bmatrix} P_1^p \\ P_2^p \end{bmatrix} Q^{p-1} \ (R_1^p \quad R_2^p)$$

is solvable.

Proof: [(i)<=>(ii)] If the reliable decentralized stabilization problem is solvable, then there exist D_1', E_1', D_2', E_2' such that

$$Q_{11}D_1'+R_{11}E_1'=:U_1, \quad (4.1.2)$$

$$Q_{22}D_2'+R_{22}E_2'=:U_2 \quad (4.1.3)$$

and

$$\begin{bmatrix} Q & R_1E_1' & R_2E_2' \\ -P_1 & D_1' & 0 \\ -P_2 & 0 & D_2' \end{bmatrix} =: U_3$$

are all unimodular matrices.

Let $E_1'':=E_1'U_1^{-1}$, $D_1'':=D_1'U_1^{-1}$, $E_2'':=E_2'U_2^{-1}$, $D_2'':=D_2'U_2^{-1}$, note that by unimodularity of U_1, U_2, U_3 the matrices E_1'', D_1'', E_2'', D_2'' are stable proper rational matrices, and substituting into (4.1.2),(4.1.3) we obtain

$$Q_{11}D_1''+R_{11}E_1''=I,$$
$$Q_{22}D_2''+R_{22}E_2''=I.$$

By Lemma 2.2, there exist X in $R_{mp}^{r\times p}$ and Y in $R_{mp}^{m\times q}$ such that

$$E_1'' = E_1^o - Q_{1o}X, \quad E_2'' = E_2^o - Q_{2o}Y \qquad (4.1.4)$$
$$D_1'' = D_1^o + P_{1o}X, \quad D_2'' = D_2^o + P_{2o}Y \qquad (4.1.5)$$

Note that

$$U_3':=\begin{bmatrix} Q & R_1E_1'' & R_2E_2'' \\ -P_1 & D_1'' & 0 \\ -P_2 & 0 & D_2'' \end{bmatrix} = \begin{bmatrix} Q & R_1E_1' & R_2E_2' \\ -P_1 & D_1' & 0 \\ -P_2 & 0 & D_2' \end{bmatrix}\begin{bmatrix} I & 0 & 0 \\ 0 & U_1^{-1} & 0 \\ 0 & 0 & U_2^{-1} \end{bmatrix}$$

is unimodular as U_1, U_2, U_3 are unimodular.

Substituting expressions (4.1.4), (4.1.5) into the expression for U_3', we obtain

$$U_3' = \begin{bmatrix} Q & R_1(E_1^o-Q_{1o}X) & R_2(E_2^o-Q_{2o}Y) \\ -P_1 & D_1^o+P_{1o}X & 0 \\ -P_2 & 0 & D_2^o+P_{2o}Y \end{bmatrix}.$$

It is straightforward to verify that

$$U_3' = Q^o + R_1^oXP_1^o + R_2^oYP_2^o \quad .$$

Conversely, if for some X in $R_{up}^{r\times p}$ and Y in $R_{up}^{m\times q}$, U^{c} is unimodular, then letting

$$E_1 := E_1^{c} - Q_{1c}X, \qquad E_2 := E_2^{c} - Q_{2c}Y,$$
$$D_1 := D_1^{c} + P_{1c}X, \qquad D_2 := D_2^{c} + P_{2c}Y$$

we obtain

$$Q_{11}D_1 + R_{11}E_1 = I, \qquad Q_{22}D_2 + R_{22}E_2 = I,$$

and

$$U^{c} = \begin{bmatrix} Q & R_1E_1 & R_2E_2 \\ -P_1 & D_1 & 0 \\ -P_2 & 0 & D_2 \end{bmatrix}.$$

As U^{c}, by hypothesis, is unimodular, the local compensators $E_1D_1^{-1}$ and $E_2D_2^{-1}$ solve the reliable decentralized stabilization problem for Z.

Equivalence of (ii) and (iii) is a direct consequence of Proposition 4.1.1.

■

4.2 TWO-INPUT-TWO-OUTPUT SYSTEMS

In the special case where the plant has two inputs and two outputs, the representation

$$Z = 1/m \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix} \tag{3.2}$$

is more convenient for a closer examination of the relation between strong decentralized stabilization and reliable decentralized stabilization problems. For the purpose of obtaining simpler equations for the solutions of these problems, we now use representation (3.2) in proving a counterpart of Theorem 4.1.

We consider the problems of strong decentralized stabilization and reliable decentralized stabilization in terms of the more convenient representation (3.2) of Z rather than the representation $Z=PQ^{-1}R$, where P, Q, R are in $R_{mp}^{2\times 2}$ with (P,Q) right-coprime and (Q,R) left-coprime (see Remark 4.1).

Let

$$C = \begin{bmatrix} \alpha_1/\beta_1 & 0 \\ 0 & \alpha_2/\beta_2 \end{bmatrix}.$$

where α_1, α_2, β_1, β_2 are in R_{sp} and (α_1, β_1), (α_2, β_2) are coprime pairs, be a candidate compensator for Z.

Let

$$n_{11}/m_{11} := n_1/m, \qquad n_{22}/m_{22} := n_2/m$$

where n_{11}, m_{11}, n_{22}, m_{22} are in R_{sp} and (n_{11}, m_{11}), (n_{22}, m_{22}) are coprime pairs.

Proposition 4.2 Consider the transfer matrix Z of (3.2).

a) The strong decentralized stabilization problem is solvable if and only if there exist x and y in R_{sp} such that

$$m + n_1 x + n_4 y + dxy =: u$$

is a unit in R_{sp}; in which case C=diag(x,y) is a solution to the problem.

b) The reliable decentralized stabilization problem is solvable if and only if there exist α_1, α_2, β_1, and β_2 in R_{sp} such that

i) $m_{11}\beta_1 + n_{11}\alpha_1 =: u_1$

ii) $m_{22}\beta_2 + n_{22}\alpha_2 =: u_2$

iii) $m\beta_1\beta_2 + n_1\alpha_1\beta_2 + n_4\beta_1\alpha_2 + d\alpha_1\alpha_2 =: u_3$

are all units in R_{mp}; in which case $C=\mathrm{diag}(\alpha_1/\beta_1, \alpha_2/\beta_2)$ is a solution to the problem.

Proof: a) By the definition of strong decentralized stabilization problem

$$C = \begin{bmatrix} \alpha_1/\beta_1 & 0 \\ 0 & \alpha_2/\beta_2 \end{bmatrix} \in R_{mp} .$$

Since (α_1,β_1) and (α_2,β_2) are coprime, and since β_1, β_2 are units in R_{mp}, By Lemma 3.1 (Z,C) is internally stable if and only if

$$m\beta_1\beta_2 + n_1\alpha_1\beta_2 + n_4\beta_1\alpha_2 + d\alpha_1\alpha_2 =: v$$

is a unit. Multiplying both sides with $\beta_1^{-1}\beta_2^{-1}$, we have

$$m + n_1\alpha_1\beta_1^{-1} + n_4\alpha_2\beta_2^{-1} + d\alpha_1\beta_1^{-1}\alpha_2\beta_2^{-1} = v\beta_1^{-1}\beta_2^{-1}$$

which implies with $x:=\alpha_1\beta_1^{-1}$ and $y:=\alpha_2\beta_2^{-1}$ that

$$m + n_1x + n_4y + dxy =: u,$$

where $u:=v\beta_1^{-1}\beta_2^{-1}$ is a unit.

Conversely, if u is a unit, then by the choice of $\beta_1=1$, $\beta_2=1$ and $\alpha_1=x$, $\alpha_2=y$, C solves strong decentralized stabilization problem.

b) By definition, the reliable decentralized stabilization problem is solvable if and only if (Z,C), (z_{11},c_1), and (z_{22},c_2) are internally stable. Since $z_{11}=n_{11}/m_{11}$, $z_{22}=n_{22}/m_{22}$, $c_1=\alpha_1/\beta_1$, and $c_2=\alpha_2/\beta_2$ are coprime fractional representations

i) (z_{11}, c_1) is internally stable if and only if $m_{11}\beta_1+n_{11}\alpha_1$ is a unit.

ii) (z_{22}, c_2) is internally stable if and only if $m_{22}\beta_2+n_{22}\alpha_2$ is a unit.

iii) (Z,C) is internally stable, by Lemma 3.1, if and only if $m\beta_1\beta_2+n_1\alpha_1\beta_2+n_4\beta_1\alpha_2+d\alpha_1\alpha_2$ is a unit.

■

Let

$$m_{11}\beta_1^o + n_{11}\alpha_1^o = 1, \qquad (4.2.1)$$

$$m_{22}\beta_2^o + n_{22}\alpha_2^o = 1 \qquad (4.2.2)$$

for some β_1^o, α_1^o, β_2^o, α_2^o in R_{mp}. Since (m_{11}, n_{11}) and (m_{22}, n_{22}) are coprime pairs, such elements exist.

Define

$$m^o := m\beta_1^o\beta_2^o + n_1\alpha_1^o\beta_2^o + n_4\beta_1^o\alpha_2^o + d\alpha_1^o\alpha_2^o \quad .$$

Note that m^o is in R_{mp} (not necessarily a unit).

THEOREM 4.2 The following statements are equivalent :

i) The reliable decentralized stabilization problem for Z of (3.2) is solvable.

ii) There exist x and y in R_{mp} such that

$$m^o+(dm_{11}-n_4n_{11})\alpha_2^o x+(dm_{22}-n_1n_{22})\alpha_1^o y+(dm_{11}-n_4n_{11})m_{22}xy =: u^o$$

is a unit.

iii) The strong decentralized stabilization problem for

$$Z^{o} := 1/m^{o}\begin{bmatrix} (dm_{11}-n_4n_{11})\alpha_2^{o} & n_3 \\ n_3 & (dm_{22}-n_1n_{22})\alpha_1^{o} \end{bmatrix}$$

is solvable.

Proof: [(i)<=>(ii)] By Proposition 4.2, if reliable decentralized stabilization problem is solvable, then there exist α_1', α_2', β_1', β_2' in R_{wp} such that $u_1:=m_{11}\beta_1'+n_{11}\alpha_1'$, $u_2:=m_{22}\beta_2'+n_{22}\alpha_2'$, and $u_3:=m\beta_1'\beta_2'+n_1\alpha_1'\beta_2'+n_4\beta_1'\alpha_2'+d\alpha_1'\alpha_2'$ are units in R_{wp}.

Now, let $\beta_1'':=\beta_1'u_1^{-1}$, $\alpha_1'':=\alpha_1'u_1^{-1}$, $\beta_2'':=\beta_2'u_2^{-1}$, and $\alpha_2'':=\alpha_2'u_2^{-1}$. These rational functions ,clearly, are in R_{wp}. Then, we can write

$$m_{11}\beta_1'' + n_{11}\alpha_1'' = 1,$$
$$m_{22}\beta_2'' + n_{22}\alpha_2'' = 1 .$$

Using expressions (4.2.1) and (4.2.2), and Lemma 2.2, it follows that for some x, y in R_{wp} $\beta_1''=\beta_1^{o}+n_{11}x$, $\alpha_1''=\alpha_1^{o}-m_{11}x$, $\beta_2''=\beta_2^{o}+n_{22}y$, and $\alpha_2''=\alpha_2^{o}-m_{22}y$. Consequently,

$$u_3u_1u_2 = m\beta_1''\beta_2'' + n_1\alpha_1''\beta_2'' + n_4\beta_1''\alpha_2'' + d\alpha_1''\alpha_2'' \qquad (4.2.3)$$

is also a unit in R_{wp}. If we substitute α_1'', α_2'', β_1'', β_2'' into the equation (4.2.3), then we obtain by a straightforward calculation that $u_3u_1u_2 = u^{o}$.

Conversely, if for some x, y in R_{mp}, u^o is a unit in R_{mp}, then letting $\beta_1 := \beta_1^o + n_{11}x$, $\alpha_1 := \alpha_1^o - m_{11}x$, $\beta_2 := \beta_2^o + n_{22}y$, and $\alpha_2 := \alpha_2^o - m_{22}y$, we obtain $m_{11}\beta_1 + n_{11}\alpha_2 = 1$, $m_{22}\beta_2 + n_{22}\alpha_2 = 1$, and $m\beta_1\beta_2 + n_1\alpha_1\beta_2 + n_4\beta_1\alpha_2 + d\alpha_1\alpha_2 = u^o$.

Consequently, α_1/β_1 and α_2/β_2 solve reliable decentralized stabilization problem for Z.

[(ii)<=>(iii)] Let us first compute d^o associated with Z^o. Let the numerator matrix of Z^o be N^o, namely $N^o = m^o Z^o$. Then, the determinant of N^o is

$$\det N^o = (dm_{11} - n_4 n_{11})(dm_{22} - n_1 n_{22})\alpha_1^o \alpha_2^o - n_2 n_3 .$$

Since $n_{11}/m_{11} = n_1/m$, and $n_{22}/m_{22} = n_4/m$, for some g_1, g_2 in R_{mp}, we have $m = m_{11}g_1 = m_{22}g_2$, $n_1 = n_{11}g_1$, and $n_4 = n_{22}g_2$. Using these equalities and $md = n_1 n_4 - n_2 n_3$, we obtain

$$\det N^o = (dm_{11} - n_4 n_{11})[(dm_{22} - n_4 n_{11})\alpha_1^o \alpha_2^o + g_1] .$$

And using (4.2.1), (4.2.2) we can write

$$g_1 g_2 = (m\beta_1^o + n_1\alpha_1^o)(m\beta_2^o + n_4\alpha_2^o) .$$

Substituting this into the term in the square brackets, we get

$$\det N^o = m^o (dm_{11} - n_4 n_{11}) m_{22} .$$

Therefore m^o divides $\det N^o$ and the quotient is $(dm_{11} - n_4 n_{11}) m_{22} =: d^o$. By straightforward manipulations it can further be shown that $m^o = \mu^o \sigma_1/\sigma_2$, where μ^o is the characteristic polynomial of Z^o and σ_1, σ_2 are stable polynomials. Hence the representation of Z^o is of the form (3.2). Thus, by Proposition 4.2, (iii) is equivalent to (ii).

■

By the result of Theorem 4.2, $a + bx + cy + dxy = u$, where u is a unit, is a central problem for both strong and reliable decentralized stabilization problems. Given a fixed x in R_{mp}, a necessary and sufficient condition for the existence of y in R_{mp} such that $a+bx+cy+dxy$ is a unit, is well-known (VIDYASAGAR, and VISWANADHAM [6]). By a straightforward examination of this equation, some sufficient conditions for the solvability can be obtained.

Corollary 4.2.1 If d is a unit in

$$a + bx + cy + dxy = u,$$

then there exist x, y and a unit u in R_{mp} satisfying the above equation.

Proof: Let d be a unit in R_{mp}, then we can find an x in R_{mp} such that $(c+dx)$ is a unit. In fact, $x=d^{-1}(u_1-c)$, where u_1 is a unit. Then we have $a+bx+u_1y=u$. Since u_1 is a unit, similarly we can find y in R_{mp} such that $a+bx+cy+dxy$ is a unit.

■

REMARK 4.1 The representations (3.2) and (4.1) are closely related. In fact

$$Z = 1/m \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix} = PQ^{-1}R$$

where $m = u.\det Q$ and $d = u.\det P.\det R$ for some u in R_{mp}. Theorem 4.2 is, of course, a special case of Theorem 4.1. In fact, it is possible to give an alternative proof of Theorem 4.2 using Theorem 4.1 and the relation between the two representations. In the notation of Theorem 4.1, an alternative expression for Z^{∞} turns out to be

$$Z^{\infty} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ & & & \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 & q_2 & r_1\alpha_1^{\infty} & r_2\alpha_2^{\infty} \\ q_3 & q_4 & r_3\alpha_1^{\infty} & r_4\alpha_2^{\infty} \\ -p_1 & -p_2 & \beta_1^{\infty} & 0 \\ -p_3 & -p_4 & 0 & \beta_2^{\infty} \end{bmatrix}^{-1} \begin{bmatrix} -r_1 m_{11} & -r_2 m_{22} \\ -r_3 m_{11} & -r_4 m_{22} \\ n_{11} & 0 \\ 0 & n_{22} \end{bmatrix}$$

where $\begin{bmatrix} p_1 & p_2 \\ p_3 & p_4 \end{bmatrix} = P$, $\begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} = Q$, and $\begin{bmatrix} r_1 & r_2 \\ r_3 & r_4 \end{bmatrix} = R$

(n_{11}, m_{11}), (n_{22}, m_{22}) are coprime and $n_{11}/m_{11} = z_{11}$, $n_{22}/m_{22} = z_{22}$.

V. CONSEQUENCES OF THE MAIN RESULTS AND EXAMPLES

In this last section, we investigate certain implications of the main theorems of the previous section to give some interesting sufficient conditions for the solvability of strong and reliable decentralized stabilizations problems. We also give examples indicating the significance of reliable stabilization problem and illustrating the synthesis of strong and reliable diagonal compensators.

It is well known (YOULA, BONGIORNO, and LU [5]) that a minimum-phase multivariable plant can always be (centrally) strong stabilized. We show below that a similar result holds in the case of two-input-two-output diagonal stabilization. Consider

$$Z = 1/m \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix} = Q^{-1}R, \qquad (5.1)$$

where m, n_1, n_2, n_3, n_4 are in R_{sp}, Q and R are in $R_{sp}^{2\times2}$ and (Q,R) is left-coprime. As we have shown in Remark 4.1 that m may differ from detQ by a factor of u, where u is a unit in R_{sp}. But Q and R can be chosen such that m=detQ and d=detR.

Also let s denote the smallest invariant factor of Z, i.e., the greatest common factor of all entries of R. Then it can also be shown that $s = g.c.f.(n_1, n_4, d)$.

THEOREM 5.1 i) If d is minimum phase (i.e., its zeros are stable), then the plant Z of (5.1) is strong decentralized stabilizable.

ii) If $d \equiv 0$, then Z is strong decentralized stabilizable if and only if Z is strong (centralized) stabilizable, i.e., if and only if there exists w in R_{mp} such that m+sw is a unit.

Proof: i) If d is minimum phase, then the greatest common factor of (n_4, d) is also minimum phase (due to the fact that zeros of d are stable, zeros of any factor of d are also stable). Let $g := g.c.f.(n_4, d)$ so that g is minimum phase and

$$n_4 = g n_4', \qquad d = g d'.$$

Clearly (n_4', d') is coprime. By the theorem of YOULA, BONGIORNO, and LU [5], since d′ has no unstable zeros there exists an x′ in R_{mp} such that $n_4' + d'x' =: u'$, where u′ is a unit in R_{mp}. Let $a' := m + n_1 x'$ and note that a′ is biproper due to the fact that m is biproper and n_1 is strictly proper. Since gu′ is minimum phase and (a′,gu′) is coprime, there exists y′ in R_{mp} such that $a' + gu'y'$ is a unit. Letting x=x′and y=y′, x and y satisfy the equation

$$(m + n_1 x) + (n_4 + dx) y = u ,$$

where u is a unit in R_{mp}. Thus the plant Z of (5.1) is strong decentralized stabilizable.

ii) If $d \equiv 0$, then s is the greatest common factor of (n_1, n_4). We can write $n_1 = sn_1'$, $n_4 = sn_4'$. Clearly (n_1', n_4') is coprime and there exist x' and y' in R_{wp} such that $n_1'x' + n_4'y' = 1$. If there exists w in R_{wp} such that $m+sw$ is a unit then letting $x = x'w$ and $y = y'w$, we have $w = n_1'x + n_4'y$. Since by hypothesis, $m+sw$ is a unit, a straightforward manipulation yields that $m + n_1x + n_4y + dxy$ is a unit. Therefore $diag(x,y)$ solves the strong decentralized stabilization problem for Z.

Conversely, if there exist x and y in R_{wp} such that $diag(x,y)$ solves the strong decentralized stabilization problem for Z, then $m + n_1x + n_4y + dxy$ is a unit in R_{wp}. Letting $w = n_1'x + n_4'y$ it follows that $m+sw$ is a unit.

■

In case of reliable stabilization, a consequence of Theorem 4.2 is the following.

Let a two-input-two-output plant Z be such that the elements on the main diagonal is stable and the other elements are minimum phase. Then it can be represented as

$$Z = \begin{bmatrix} n_{11}/m_{11} & n_{12}/m_{12} \\ n_{21}/m_{21} & n_{22}/m_{22} \end{bmatrix} = 1/m \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix} \tag{5.2}$$

where m_{11}, m_{22} are units in R_{wp}, (m_{12}, m_{21}) is coprime, and n_{12}, n_{21} have stable zeros. Then we can state the following theorem.

THEOREM 5.2 A plant with transfer matrix Z of (5.2) always admits a diagonal reliable stabilizing compensator.

Proof: If we represent Z in the form (3.2), then we have

$$m = m_{12}m_{21}, \quad n_1 = n_{11}m_{12}m_{21}m_{11}^{-1}, \quad n_2 = n_{12}m_{21}, \quad n_3 = n_{21}m_{12}$$

and $n_4 = n_{22}m_{12}m_{21}m_{22}^{-1}$. Then

$$d = (n_1n_4 - n_2n_3)/m = n_{11}n_{22}m_{12}m_{21}m_{11}^{-1}m_{22}^{-1} - n_{12}n_{21} \ .$$

If we calculate d^o associated with Z^o of Theorem 4.2, then we obtain $d^o = -m_{22}n_{12}n_{21}$, which has stable zeros. Therefore by Theorem 5.1 Z^o is strong decentralized stabilizable, and by Theorem 4.2, Z admits a diagonal reliable stabilizing compensator.

■

Example 1

In this example we will find a diagonal stabilizing compensator for Z below and show that it does not stabilize the subplants.

Let unknown compensator be $C=\mathrm{diag}(c_1, c_2)$ and let

$$Z = \begin{bmatrix} \dfrac{2s-3}{(s-1)(s-2)} & \dfrac{1}{(s-2)} \\ \\ \dfrac{1}{(s-2)} & \dfrac{1}{(s-2)} \end{bmatrix}$$

If we represent Z in the form (3.2), then we have

$m = (s-1)(s-2)/(s+1)^2$, $n_1 = (2s-3)/(s+1)^2$

$n_4 = n_2 = n_3 = (s-1)/(s+1)^2$, and $d = 1/(s+1)^2$.

Let $n_1/m = n_{11}/m_{11}$, where $n_{11} = (2s-3)/(s+1)^2$, $m_{11} = (s-1)(s-2)/(s+1)^2$. Note that (n_{11}, m_{11}) is coprime.

Let $n_4/m = n_{22}/m_{22}$, where $n_{22} = 1/(s+1)$, $m_{22} = (s-2)/(s+1)$. Note that (n_{22}, m_{22}) is coprime.

Let $\beta_2 = (s-5)/(s+1)$ and $\alpha_2 = (s-10)/(s+1)$, then

$$c_2 = \alpha_2/\beta_2 = (s-10)/(s-5) .$$

Substituting β_2 and α_2 into the equation

$$u = m\beta_1\beta_2 + n_1\alpha_1\beta_2 + n_4\beta_1\alpha_2 + d\alpha_1\alpha_2 ,$$

where u is a unit, we obtain

$$u = \frac{s(s-1)(s-6)}{(s+1)^2}\beta_1 + \frac{2s^2-12s+5}{(s+1)^3}\alpha_1 \qquad (5.3)$$

Since the coefficients of β_1 and α_1 are coprime we can find β_1 and α_1 satisfying (5.3).

In fact,

$$\alpha_1 = \frac{2834s^2-2999s+5}{(s+1)^2}$$

$$\beta_1 = \frac{25s^2-5368s+2530}{(s+1)^2}$$

gives u=25, which is a unit in $R_{\infty p}$.

Hence

$C = \mathrm{diag}\{\alpha_1/\beta_1, \alpha_2/\beta_2\}$ solves decentralized stabilization problem for Z.

But

$$\sigma_1 = m_{11}\beta_1 + n_{11}\alpha_1$$

$$= \frac{25s^4+225s^3+4184s^2-9319s+5045}{(s+1)^4}$$

which is not unit in R_{sp}, thus (z_{11}, c_1) is unstable, and

$$\sigma_2 = m_{22}\beta_2 + n_{22}\alpha_2$$

$$= \frac{s(s-6)}{(s+1)^2}$$

which is not unit in R_{sp}, thus (z_{22}, c_2) is unstable.

Example 2

In this example we solve a diagonal reliable stabilization problem. Consider a 2x2 transfer matrix:

$$Z = \begin{bmatrix} \dfrac{(s-1)}{(s+1)^2} & \dfrac{(s+2)}{(s-3)^2} \\ \\ \dfrac{(s+3)}{(s-4)^2} & \dfrac{(s-1)}{(s+2)^2} \end{bmatrix} = \begin{bmatrix} \dfrac{n_{11}}{m_{11}} & \dfrac{n_{12}}{m_{12}} \\ \\ \dfrac{n_{21}}{m_{21}} & \dfrac{n_{22}}{m_{22}} \end{bmatrix}$$

Let

$$n_{11} = \frac{(s-1)}{(s+1)^2}, \quad m_{11} = 1, \quad n_{12} = \frac{(s+2)}{(s+1)^2}, \quad m_{12} = \frac{(s-3)^2}{(s+1)^2},$$

$$n_{21} = \frac{(s+3)}{(s+1)^2}, \quad m_{21} = \frac{(s-4)^2}{(s+1)^2}, \quad n_{22} = \frac{(s-1)}{(s+2)^2}, \quad m_{22} = 1 .$$

Note that m_{11}, m_{22} are units in R_{mp}, (m_{12}, m_{21}) is coprime and n_{12}, n_{21} are minimum phase. If we represent Z in the form (3.2), then we have

$$m = \frac{(s-3)^2(s-4)^2}{(s+1)^4}, \quad n_1 = \frac{(s-1)(s-3)^2(s-4)^2}{(s+1)^6},$$

$$n_2 = \frac{(s+2)(s-4)^2}{(s+1)^4}, \quad n_3 = \frac{(s+3)(s-3)^2}{(s+1)^4},$$

$$n_4 = \frac{(s-1)(s-3)^2(s-4)^2}{(s+1)^4(s+2)^2}, \quad d = \frac{-27s^5+53s^4-441s^3+411s^2-548s+120}{(s+1)^6(s+2)^2} .$$

By Theorem 5.2, Z admits a diagonal reliable stabilizing compensator.

In fact, $C=\mathrm{diag}(\alpha_1/\beta_1, \alpha_2/\beta_2)$ where

$$\alpha_1 = -1$$

$$\alpha_2 = \frac{203s^3-1209s^2+3053s-2735}{(s+2)(s+3)(s+19)}$$

$$\beta_1 = \frac{s(s+3)}{(s+1)^2}$$

$$\beta_2 = \frac{s^5-175s^4+1613s^3-3648s^2+6648s-2279}{(s+2)^3(s+3)(s+19)}$$

solves the reliable stabilization problem for Z. To see this, note that

$$m_{11}\beta_1+n_{11}\alpha_1 = 1,$$

thus (z_{11},c_1) is internally stable,

$$m_{22}\beta_2+n_{22}\alpha_2 = 1,$$

thus (z_{22},c_2) is internally stable, and

$$m\beta_1\beta_2+n_1\alpha_1\beta_2+n_4\beta_1\alpha_2+d\alpha_1\alpha_2 = \frac{s+1}{s+19}$$

which is a unit in R_{sp}, thus (Z,C) is internally stable.

Note also that $C=\mathrm{diag}(x,y)$ where $x = 1$,

$$y = \frac{-203s^3+1209s^2-3053s+2735}{(s+2)(s+3)(s+19)}$$

solves the diagonal strong stabilization problem for

$$Z^{D} = \begin{bmatrix} 0 & \dfrac{(s+2)}{(s-3)^2} \\ \dfrac{(s+3)}{(s-4)^2} & 0 \end{bmatrix}.$$

VI. CONCLUSIONS

In this thesis, we have studied the reliable decentralized stabilization and strong decentralized stabilization problems for two-channel systems (scalar or multivariable). We have shown that the reliable decentralized stabilization problem for a given plant is equivalent to a strong decentralized stabilization problem for a new plant defined in terms of the original plant (Theorem 4.1 and Theorem 4.2). Both problems are reducible to solving equations of the type

$$a + bx + cy + dxy = u,$$
$$A + BXC + DYE = U$$

where the unknowns; u is a unit in R_{mp}, x, y are in R_{mp}, U is a unimodular matrix in $R_{mp}^{m \times m}$ and X, Y are stable rational matrices. We have given some sufficient conditions to solve these equations for the scalar case and a large class of transfer matrices for which the reliable stabilization problem is solvable (Theorem 5.1 and Theorem 5.2).

We have also given a set of all diagonal stabilizing compensators in the simplest case of a two-input-two-output plant. Although the result applies to a very restricted decentralized control problem, it is the first of its kind and by similar reasoning the set of all solutions to the completeness equation can be found.

Using the main results, we have shown that :

i) For a two-input-two-output plant with all of its zeros stable, the strong decentralized stabilization problem is solvable .

ii) For a two-input-two-output plant which has a transfer matrix with diagonal elements stable and the off-diagonal elements minimum phase, the reliable decentralized stabilization problem is solvable.

Finally, we have given some examples using the technique outlined in this thesis and we have shown that a decentralized stabilizing compensator does not have built-in reliability properties. It has to satisfy further requirements.

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