QUASI-STATIC DESIGN CONSIDERATION OF MICROSTRIP LINES

by

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ABSTRACT

Although microstrip lines have been extensively investigated by a large number of researchers, and their theory and design have been reported in many articles, there is no comprehensive description available at one place. In particular the design information is not readily available.

This study is basically intended for presenting a detailed account of these lines. It describes analysis and design of this transmission medium. The features discussed include quasi-static and fullwave analyses, detailed design considerations including CAD and measurement procedures.

Particularly, the results of six different approaches given for characteristic impedance and effective dielectric constant calculations are compared with each other and among them, equations giving the nearest values to the results of numerical methods and experiments are used to calculate the other microstrip characteristics.

MİKROŞERİTLİ HATLARIN SÖZDE-STATİK YÖNTEMLERLE TASARIMI

ÖZET

Mikroşeritli hatlar birçok araştırmacı tarafından oldukça geniş bir şekilde araştırılmış ve teorisi ile tasarımı birçok makalede sunulmuş olmasına rağmen, bu konuda etraflı bir kaynak elde mevcut değildir.

Bu çalışmanın temel olarak amacı mikroşeritli iletim hatlarının analiz ve tasarımını ayrıntılı bir şekilde ve bir bütün olarak sunmaktır . Üzerinde durulan konular sözde-statik ve tümdalga analizlerini, bilgisayar destekli tasarımı da kapsayan, tasarımda gözönünde bulundurulması gerekli bilgiler ile ölçme işlemlerini içerir.

Özel olarak tasarım sırasında, karakteristik empedans ve etken dielektrik katsayısının hesabı için verilmiş altı değişik yaklaşımın sonuçları birbirleriyle karşılaştırılmış ve bunlar arasında sayısal metodlar ile deneysel çalışmaların sonuçlarına en yakın değerleri veren eşitlikler, mikroşeritli iletim hatlarının diğer karakteristiklerinin hesabında kullanılmıştır.

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APHC	:	Average power handling capability
C	:	Capacitance per unit length
C_a	:	Capacitance with substrate dielectric replaced by air
c	:	Velocity of electromagnetic waves in free space
E	:	Electric field
E_x, E_y, E_z	:	Components of Electric field E
FTD	:	Fourier transform domain
H	:	Magnetic field
H_x, H_y, H_z	:	Components of magnetic field H
h	:	Height of the substrate
K(m), K'(m)	:	Complete elliptic function and its complement
L	:	Inductance per unit length
L_i	:	Incremental inductance

LSE	:	Longitudinal section electric
ln	:	Natural logarithm (to the base e)
log	:	Common logarithm (to the base 10)
MIC	:	Microwave integrated circuit
PPHC	:	Peak power handling capability
Q	:	Charge Quality factor
Q_c, Q_d, Q_r, Q_T, Q_o	:	Various quality factors
q	:	Filling fraction Charge density
R_s	:	Surface resistivity
T, T_{max}, T_{amb}	:	Temperatures
TDR	:	Time domain reflectometer/reflectometry
TEM	:	Transverse electromagnetic
$\tan\delta$	:	Loss tangent
$(\tan\delta)_{eff}$	:	Effective loss tangent
V	:	Voltage
VSWR	:	Voltage standing wave ratio

v_p	:	Phase velocity
w	:	Strip width for microstrip
w_{eff}	:	Effective strip width for microstrip
$w_{eff}(f)$	:	Frequency dependent w_{eff}
Z_o	:	Impedance per unit length for microstrip
Z_o^a	:	Characteristic impedance with substrate replaced by air
α	:	Attenuation constant
α_c	:	α because of conductor loss
α_d	:	α because of dielectric loss
β	:	Phase constant
δ	:	Skin depth
ε	:	Permittivity
ε_o	:	ε for free space
ε_r	:	Relative permittivity
ε_{eff}	:	Effective dielectric constant
$\varepsilon_{eff}(f)$	:	Frequency dependent ε_{eff}
$\varepsilon', \varepsilon''$	:	Real and imaginary parts of ε

ζ	:	Free space impedance (=120Π ohms)
λ	:	Wavelength
λ_o	:	Free space λ
λ_m	:	λ for microstrip
μ	:	Permeability
μ_o	:	Free space permeability
Π	:	A constant (3.1416)
ρ	:	Charge density
σ	:	Conductivity
σ_{eff}	:	Effective conductivity
Ω	:	Ohm
ω	:	Angular frequency (=2Πf)
~	:	Tilde indicates Fourier transform

I. INTRODUCTION

The microstrip line, consisting of a strip conductor on a dielectric substrate backed by a ground plane, has experienced increased popularity in recent years in computer and microwave integrated circuit design with the availability of new low-loss substrate materials and the rapid development of semiconductor devices in the microwave frequency regions. Its simple structure, light weight and ruggedness make it an excellent substitute for coaxial and conventional wave-guides in many applications.

Microstrip lines which are used as a circuit element in microwave integrated circuits (MICs) are called "thin microstrips" in which the ratio of strip conductor thickness, t, to substrate thickness, h, lies between 0.01 and 0.05, whereas "thick microstrips" ($0.05 < t/h < 0.2$) are used as a transmission line for high-speed computers. In this study, there is no special aim to describe one of these versions, especially.

The text is divided into seven chapters. Chapter 2 gives a brief description of planar transmission structures and field configuration of microstrip. A comparison of microstrip with conventional transmission media is also given in this chapter.

Chapter 3 is concerned with the methods of microstrip analysis.

Chapter 4 presents the detailed design considerations including design equations, effects of tolerances and enclosures, losses, etc. and ensures the direct design of microstrip lines without going into details of analysis. Six different sets of equations are given here, enabling the reader to make a comparison.

Chapter 5 outlines the methods for the measurement of microstrip characteristics and for subtrate dielectric constant, briefly.

In chapter 6, it is desired to provide a computer aided design of microstrip. Also, a computer program ensuring to make a comparison of characteristic impedances or effective dielectric constants obtained from different approaches, with each other for desired t/h and w/h ratios by using a special plot algorithm is presented.

II. BASIC CONCEPTS

2.1 PLANAR TRANSMISSION STRUCTURES

A transmission structure should be "planar" in configuration to be used as a circuit element in microwave integrated circuits (MICs). A planar configuration implies that the characteristics of the element can be determined by the dimensions in a single plane. For example, the substrate thickness or the strip conductor width of the microstrip can be adjusted to control its characteristic impedance. When the impedance is controlled by a dimension in a single plane, the fabrication of the circuit can be easily carried out by the photo-etching of thin films and photolithographic techniques. Use of these techniques at microwave frequencies has led to the development of MIC's.

There are several transmission structures having planar configuration. The most common of these are :

(i) Microstrip

(ii) Slotline

(iii) Coplanar strips (CPS)

(iv) Coplanar waveguide (CPW)

The cross-sectional views of these lines are shown in Figure 2.1. Also, a comparison of performance characteristics is given in Table 2.1. Among these transmission structures, microstrip line is the most popular, mainly due to the fact that the mode of propagation on microstrip is almost TEM. This allows an easy approximate analysis.

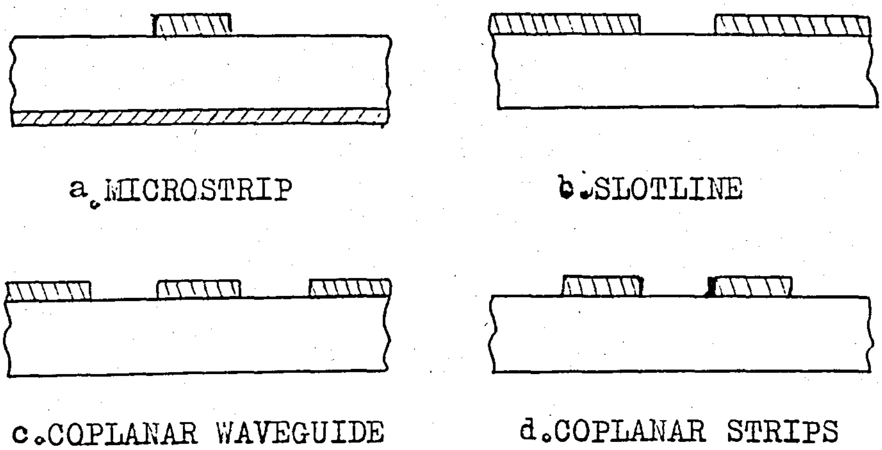


Figure 2.1. Planar Transmission Lines Used in MICs

TABLE 2.1. Comparison of Various MIC Lines

CHARECTERISTIC	MICROSTRIP	SLOTLINE	COPL. WAVEGUIDE	COPL. STRIPS
POWER HANDL. CAP.	HIGH	LOW	MEDIUM	MEDIUM
RADIATION LOSS	LOW	HIGH	MEDIUM	MEDIUM
UNLOADED Q	HIGH	LOW	MEDIUM	LOW
DISPERSION	SMALL	LARGE	MEDIUM	MEDIUM
MOUNTING OF COMP.: IN SHUNT CONF.	DIFF.	EASY	EASY	EASY
IN SERIES CONF.	EASY	DIFF.	EASY	EASY
ENCLOSURE DIM.	SMALL	LARGE	LARGE	LARGE

There are several variations of microstrip configuration that have also been suggested for use in MICs. These include inverted microstrip, suspended microstrip, microstrip with overlay, strip dielectric waveguide and inverted strip dielectric waveguide. Cross-sectional views of these structures are given in Figure 2.2.

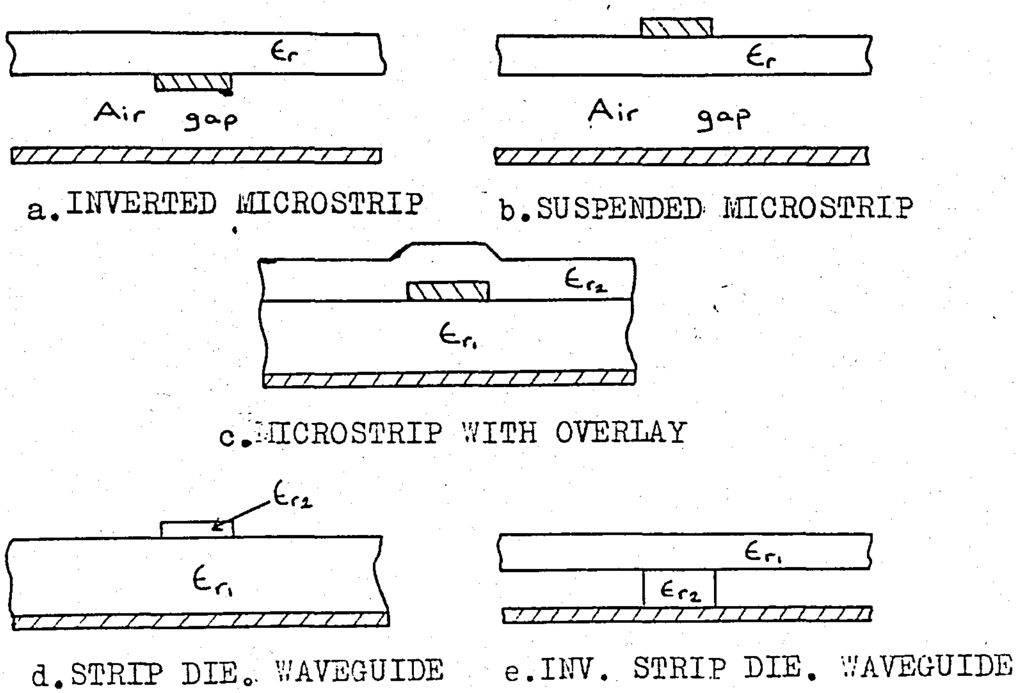


Figure 2.2. Various Transmission Lines Derived from Microstrip

Another very commonly used transmission line that resembles microstrip line is the stripline shown in Figure 2.3. This is also called triplate line. Most of the basic circuit design information available for stripline is also applicable to microstrip line.

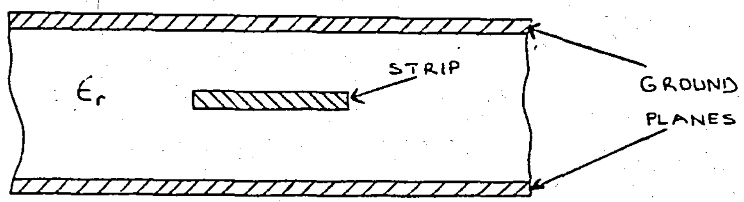


Figure 2.3. Stripline Configuration

2.2. DEFINITION AND FIELD CONFIGURATION

A cross-section of a microstrip is shown in Figure 2.4. It consists of a dielectric substrate with a metal ground-plane on one side and a metal strip on the other.

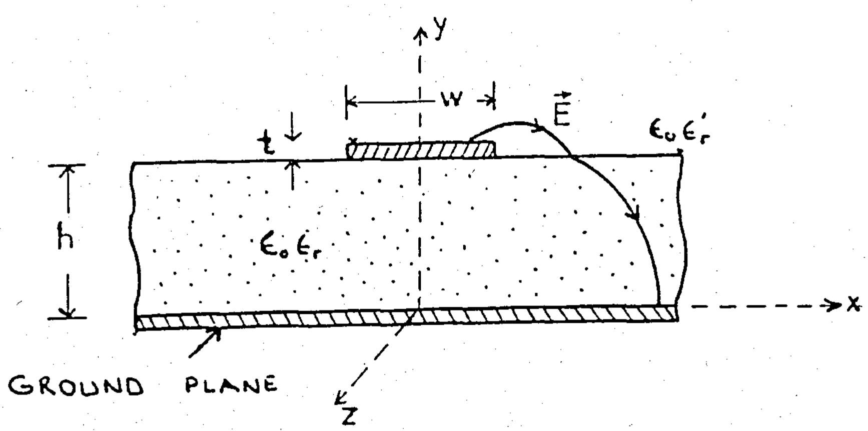


Figure 2.4. Cross-section of a Microstrip

The electrical properties of microstrip will depend on the geometry : strip width w, strip thickness t, and substrate thickness (height) h : and on the dielectric and magnetic properties of the substrate and the medium above, which are considered to have infinite entent. The medium above the substrate is usually air which dielectric constant $\varepsilon_o \varepsilon_r'$ can be assumed to be equal to that of free space $\varepsilon_r' = 1$ and $\varepsilon_o = 8.8542 \times 10^{-12}$ F/m (in the SI system of units). The relative dielectric constant of the substrate is ε_r.

Conceptually, it can be said that microstrip is a two-conductor transmission line which is evolved from a two-wire line as shown in Figure 2.5.

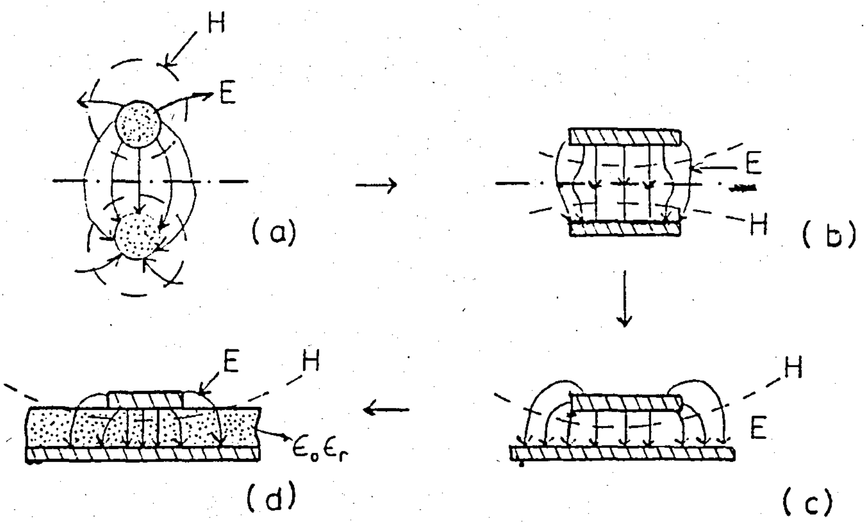


Figure 2.5. Conceptual Evolution of a Microstrip from a Two-wire Line

Transformation from (a) to (b) is essentially a change in the shape of the conductors, whereas that from (b) to (c) involves placing a conducting sheet at the plane of symmetry. The final configuration (d) is obtained by inserting a thin dielectric slab between the two conductors. As a consequence of the last step, the dielectric medium of the transmission line becomes inhomogenous.

Microstrip lines differ considerably from other transmission lines. For example, comparing it with a strip-line one observes that the microstrip structure is open on the top. This open configuration makes microstrip very convenient for use in MICs where discrete lumped devices (active or passive) are to be mounted in the circuit. Also, a slight adjustment or tuning can possibly be incorporated after the circuit has been fabricated. However, along with these advantages, the open structure of microstrip brings in some complications in microstrip analysis and design. This is due to the fact that the presence of dielectric-air interface modifies the mode of propagation in microstrip to a non-TEM hybrid mode.

Simple arguments based on the known quasi-static field distribution of the microstrip and Maxwell's equations can be put forward to show that microstrip structure cannot support a pure TEM wave. Continuity of the tangential component of the electric field along the dielectric-air interface (look at Figure 2.4) gives

$$E_x \Big|_a = E_x \Big|_d \tag{2.1}$$

where subscripts a and d refer to the air and the dielectric side of the interface, respectively. Using Maxwell's equation, we may thus write

$$\varepsilon_r (\vec{\nabla} \times \vec{H})_x \Big|_a = (\vec{\nabla} \times \vec{H})_x \Big|_d \tag{2.2}$$

Expanding Equation (2.2) and using the continuity of normal component of magnetic field, we obtain

$$(\varepsilon_r - 1) \frac{\partial H_y}{\partial z} = \varepsilon_r \frac{\partial H_z}{\partial y} \Big|_a - \frac{\partial H_z}{\partial y} \Big|_d \tag{2.3}$$

As ε_r is not equal to unity and $H_y \neq 0$, Equation (2.3) implies that the expression on its right hand side should be a non-zero quantity, which can be true only if H_z is non-zero. Thus we note that for Maxwell's equations to hold good for the configuration of Figure 2.4, the longitudinal component of H should exist.

Similar arguments can be advanced to show that E_z, the longitudinal component of electric field, is also a non-zero quantity. It may be pointed out that it is only the fringing components E_x and H_x at the air-dielectric interface that lead to the non-TEM nature of the microstrip mode. Since these fringing field components are much smaller than the main field (within the substrate below the strip), the departure from the TEM behavior should be small.

2.3. COMPARISON OF MICROSTRIP WITH CONVENTIONAL TRANSMISSION MEDIA

Microstrip technology is quite mature and offers a superior blend of performance characteristics to the designer of microwave integrated circuits. A comparison of performance characteristics of conventional transmission media is given in Table 2.2.

TABLE 2.2. Comparison of Conventional Trasmission Media

CHARECTERISTIC	MICROSTRIP	STRIPLINE	COAXIAL	WAVEQUIDE
LINE LOSSES	HIGH	HIGH	MEDIUM	LOW
UNLOADED Q	LOW	LOW	MEDIUM	HIGH
POWER CAP.	LOW	LOW	MEDIUM	HIGH
ISOLATION BET. NEIGHBORING CIR.	POOR	FAIR	V.GOOD	V.GOOD
BANDWITH	LARGE	LARGE	LARGE	SMALL
MINIATURIZATION	EXCEL.	V.GOOD	POOR	POOR
VOLUME & WEIGHT	SMALL	MEDIUM	LARGE	LARGE
REALI. OF PAS. CIR.	V.EASY	V.EASY	EASY	EASY
INTEG. WITH CHIP DE.	V.GOOD	FAIR	POOR	POOR
INTEG. WITH FERRITES	GOOD	GOOD	POOR	GOOD
INTEG. WITH LUMP. EL.	V.GOOD	V.GOOD	GOOD	POOR

III. METHODS OF MICROSTRIP ANALYSIS

There are various methods of microstrip analysis. These methods can be divided into three groups as shown in Figure 3.1.

In the first group which comprises quasi-static methods, the nature of the mode of propagation is considered to be pure TEM, and microstrip characteristics are calculated from the electrostatic capacitance of the structure.

In the second group, called dispersion models, the deviation from the TEM nature is accounted for quasi-empirically. Some parameters of the model are determined such that the final expression agrees with the known experimental (or exact theoretical) dispersion behavior of the microstrip.

The methods in the third group take into account the hybrid nature of mode of propagation. Fullwave analysis is carried out for determining the propagation constant instead of the capacitance evaluated in quasi-static analysis.

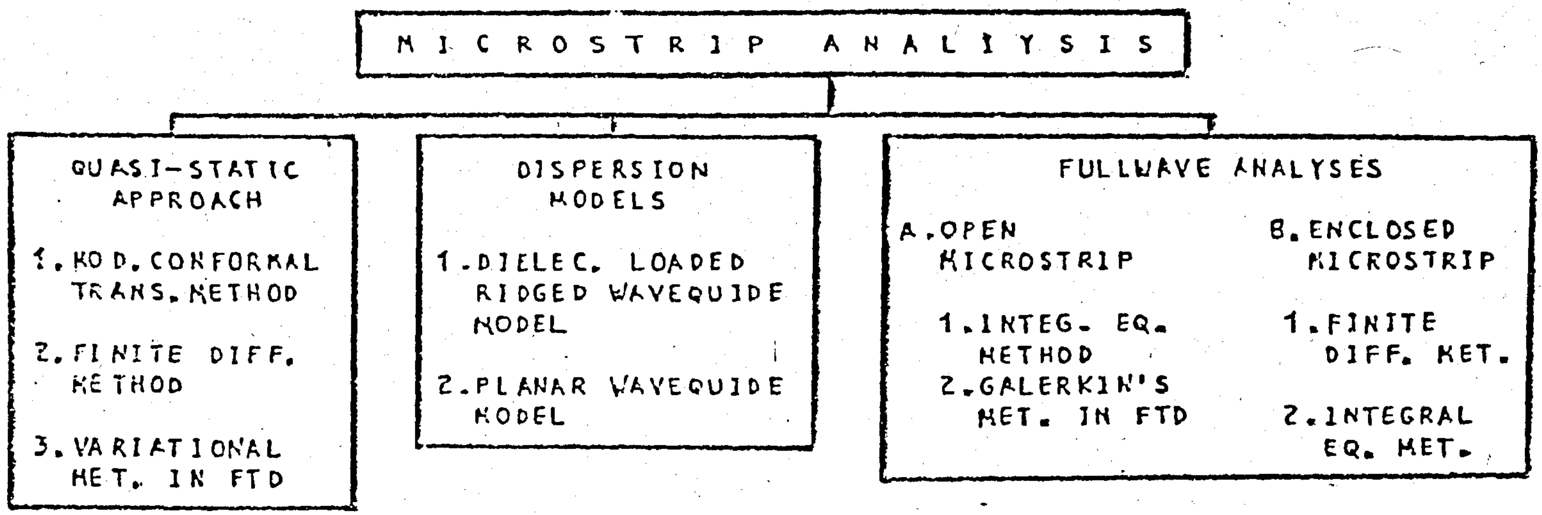


Figure 3.1 : Various Methods of Microstrip Analysis

3.1. QUASI-STATIC ANALYSES OF MICROSTRIP

In quasi-static analyses, the mode of wave propagation in microstrip is assumed to be pure TEM. Transmission characteristics are then calculated from the values of two capacitances :

(i) C_a : Capacitance for a unit length of the microstrip configuration with the dielectric substrate replaced by air

(ii) C : Capacitance for a unit length of the microstrip with the dielectric substrate present.

Values of characteristic impedance Z_o and the phase constant β can be written in terms of these capacitances as follows :

$$Z_o = Z_o^a \; (C_a/C)^{1/2} \qquad (3.1)$$

and

$$\beta = \beta_o (C/C_a)^{1/2} \qquad (3.2)$$

where $Z_o^a = 1/(c.C_a)$ and $\beta_o = \omega/c$; c being the velocity of electromagnetic waves in free space. There are various methods available for calculation of electrostatic capacitances C_a and C. Three of these are listed in Figure 3.1 and only the first one will be discussed in detail in the following sub-section.

3.1.1. MODIFIED CONFORMAL TRANSFORMATION METHOD

An exact conformal transformation for the impedance of a zero thickness, homogeneous dielectric microstrip has been given by Schneider at 1969[1].

The transformation from the microstrip (Z_1-plane) to a parallel plate capacitor (Z-plane) is expressed in terms of the derivative of the logarithm of the theta function Q_4 and its parameter $\kappa = K'/K$ as follows :

$$Z_1 = Z_1(Z) = -\frac{2hK}{\Pi}\frac{\partial}{\partial Z}\ln\left[Q_4(Z,\kappa)\right] \qquad (3.3)$$

where $K = K(m)$ and $K' = K'(m)$ are complete elliptic integrals of first kind with modulus m.

The characteristic impedance Z_o^a of the microstrip of width w and height h (and $t = 0$) is obtained by solving the following equations :

$$\frac{w}{h} = \frac{2}{\Pi}\frac{\partial}{\partial \S}\ln\left[Q_4(\S,\kappa)\right] \qquad (3.4)$$

$$dn^2(2K\S) = E/K \qquad (3.5)$$

$$Z_o^a = 1/2(\mu_o/\varepsilon_o)^{1/2}\,K'/K \qquad (3.6)$$

where μ_o and ε_o are the free space permeability and permittivity respectively, and Z_o^a is the characteristic impedance of microstrip with dielectric substrate replaced by air. $E = E(m)$ is the complete elliptic integral of second kind and dn is the Jacobian elliptic function. The derivative of the logarithm of theta function is given by the following (rapidly converging) series expansion

$$\frac{\partial}{\partial \S} \ln\left[Q_4(\S, \kappa)\right] = 4\Pi \sum_{n=1}^{\infty} \frac{\mathrm{Sin}(2n\pi\S)}{\exp(n\pi\kappa)-\exp(-n\pi\kappa)} \quad (3.7)$$

Equations from (3.4) to (3.6) can be used for the design of microstrip lines. The exact computation for one intermediate case by means of the series expansion for the derivative of the logarithm of the theta function is treated in APPENDIX B step by step.

The practical solution to this set of equations must be done with the help of a computer or with numerical tables. Because of this limitation the above method is not used very frequently.

The most widely used technique for microstrip analysis was introduced by Wheeler (2,3) in 1964-65. The method involves use of a conformal transformation for evaluation of C_a and introduces a concept of effective dielectric constant for evaluation of C. The conformal transformation selected is such that the resulting expressions are explicit and can be written in terms of simple functions. The transformation used for the wide strip (w/h>2) is

$$Z_1 = J\Pi + d\tanh(Z/2) - Z \qquad (3.8)$$

The variable Z_1 refers to the microstrip plane (see Figure 3.2) and Z is the plane where in the microstrip configuration transforms into a parallel plate capacitor configuration. The parameter d is approximately equal to g' of Figure 3.2 (b). The dielectric-air boundary of the microstrip substrate is transformed into an elliptical-looking curve ba' as shown in Figure 3.2(b). For evaluating the capacitance C, it is necessary to introduce approximations to modify the dielectric-air boundary. The curved dielectric-air boundary of Figure 3.2(b) is approximated by a rectangular boundary as shown in Figure 3.2(c). The area ($\Pi S'$) over the curve is written in terms of a "parallel area" $\Pi S''$ and a "series area" $\Pi(S'-S'')$. These series and parallel areas can be written in terms of an equivalent parallel area s given by (Figure 3.2(d))

$$S = S'' + (S' - S'')/\varepsilon_r \qquad (3.9)$$

The effective filling factor can then be written as

$$q = (g' - a' + s)/g' \qquad (3.10)$$

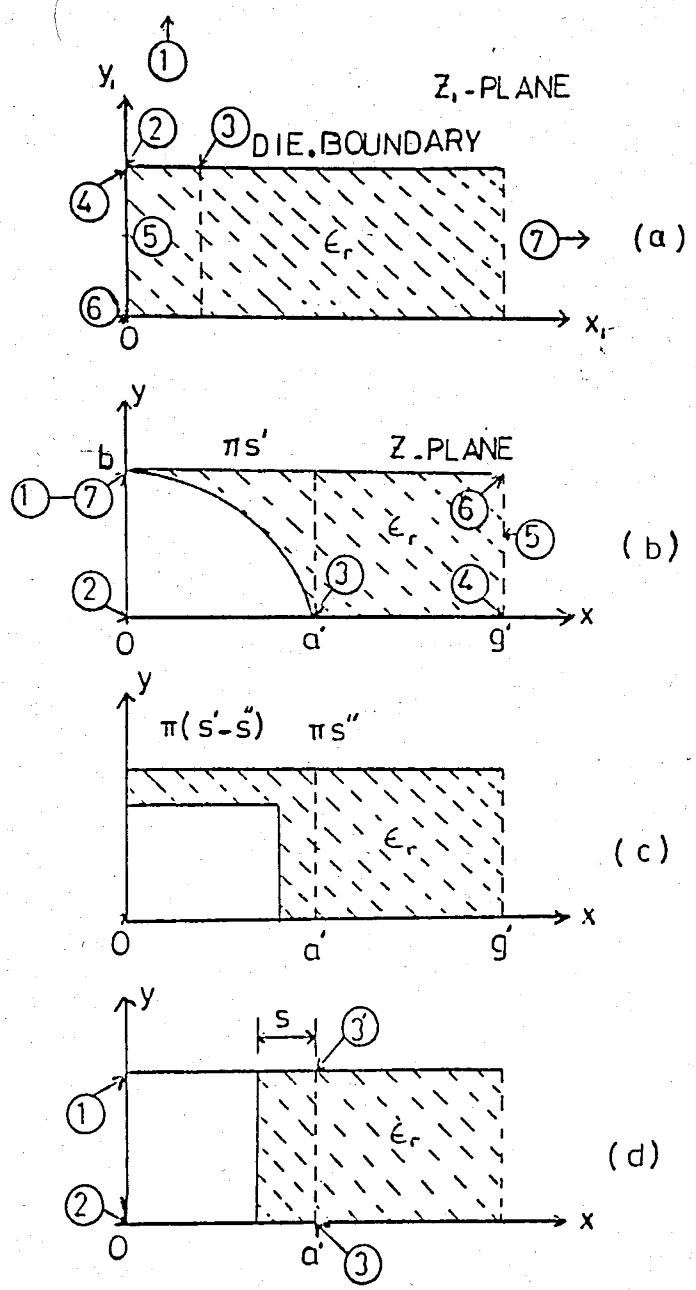


Figure 3.2. Conformal Transformation and Evaluation of Effective Dielectric Constant (from[3])

The effective dielectric constant is related to effective filling factor as

$$\varepsilon_{eff} = (1-q) + q\varepsilon_r \tag{3.11}$$

Different expressions for ε_{eff} are derived for wide microstrip ($w/h > 2$) and for narrow microstrip ($w/h < 2$) because of the different approximations used in two cases. For wide strips (4)

$$q = 1 - \frac{1}{d} \ln \frac{d+c}{d-c} + \frac{0.732}{d\varepsilon_r} \left[\ln \frac{d+c}{d-c} - \text{Cosh}^{-1}(0.358d + 0.595) \right] + \frac{\varepsilon_r - 1}{d\varepsilon_r} \left[0.386 - \frac{1}{2(d-1)} \right] \tag{3.12}$$

where $d = 1 + \sqrt{1 + c^2}$, and c is found implicitly from

$$\frac{\Pi}{2} \frac{w}{h} = c - \text{Sinh}^{-1} c \tag{3.13}$$

and for narrow strips

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \, \frac{\ln \frac{\Pi}{2} + \frac{1}{\varepsilon_r} \ln \frac{4}{\Pi}}{\ln \frac{8h}{w}} \tag{3.14}$$

Formulas for the impedance of microstrip lines can be derived from these results. It is an advantage of the transformation selected by Wheeler that the impedance formulas can be written explicitly both for analysis (Z_o in terms of w/h and ε_r) and for synthesis (w/h in terms of Z_o and ε_r). These relations are listed below. For wide strips (w/h < 2)

$$Z_o = \frac{377}{(\varepsilon_r)^{1/2}} \left[\frac{w}{h} + 0.883 + \frac{\varepsilon_r + 1}{\Pi_r} \left\{ \ln\left(\frac{w}{2h} + 0.94\right) + 1.451 \right\} + 0.165 \frac{\varepsilon_r - 1}{\varepsilon_r^2} \right]^{-1} \qquad (3.15)$$

and for narrow strips (w/h < 2)

$$Z_o = \frac{377}{2\Pi \left(\frac{\varepsilon_r + 1}{2}\right)^{1/2}} \left[\ln\left(\frac{8h}{w}\right) + \frac{1}{8}\left(\frac{w}{2h}\right)^2 - \frac{1}{2} \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \left\{ \ln \frac{\Pi}{2} + \frac{1}{\varepsilon_r} \ln \frac{4}{\Pi} \right\} \right] \qquad (3.16)$$

Expressions giving strip width for a desired impedance may be written as :

For wide strips (w/h < 2)

$$\frac{w}{2h} \Pi = \frac{377\Pi}{2(\varepsilon_r)^{1/2} Z_o} - 1 - \ln \left\{ \frac{377\Pi}{(\varepsilon_r)^{1/2} Z_o} - 1 \right\}$$

$$+ \frac{\varepsilon_r - 1}{2\varepsilon_r} \left[\ln \left\{ \frac{377\Pi}{2(\varepsilon_r)^{1/2} Z_o} - 1 \right\} + 0.293 - \frac{0.517}{\varepsilon_r} \right] \quad (3.17)$$

and for narrow strips ($w/h < 2$)

$$\frac{2h}{w} = \frac{1}{4} e^{h'} - \frac{1}{2} e^{-h'} \quad (3.18)$$

where

$$h' = \left(\frac{\varepsilon_r + 1}{2} \right)^{1/2} \frac{Z_o}{60} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \left(0.226 + \frac{0.120}{\varepsilon_r} \right) \quad (3.19)$$

Values of the characteristic impedance and effective dielectric constant of microstrip, based on the method discussed above, may be obtained from Figure 3.3. In this figure the value of $\sqrt{\varepsilon_{eff}}$ is plotted as a function of w/h for various values of the subtrate dielectric constant. ε_r. The variation of characteristic impedance for air microstrip (Z_o^a for $\varepsilon_r = 1$) is also shown by the dotted curve. Impedance for any value of ε_r can be obtained by dividing Z_o^a by the corresponding value of $\sqrt{\varepsilon_{eff}}$. It may be seen from Figure 3.3 that the impedance value decreases when the strip width to substrate height ratio (w/h) is increased because an increase in w (or decrease in h) increases the line capacitance.

It may be pointed out that in the modified conformal transformation method discussed in this section, the thickness of the microstrip conductor is ignored and the analysis is restricted to an open microstrip without any enclosure.

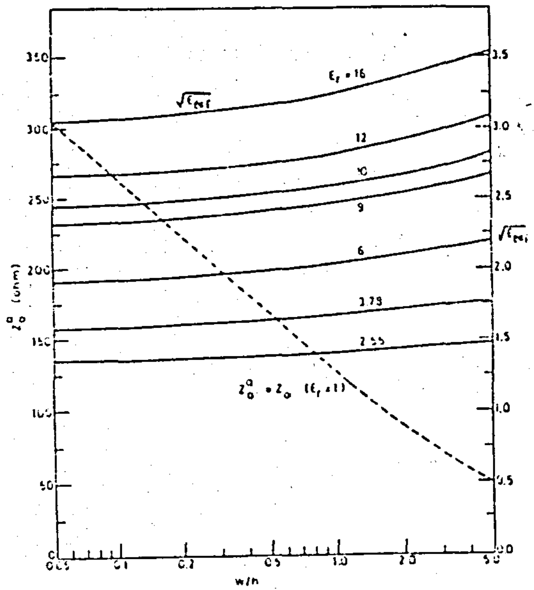


Figure 3.3. Characteristic Impedance and Effective Dielectric Constant of Microstrip Lines Calculated Using Wheeler's Method.

3.1.2 FINITE DIFFERENCE METHOD

Another method for quasi-static analysis of microstrip lines is based on the numerical solution of Laplace's equation in finite difference form[5]. The solution of Laplace's equation yields information about the potential distribution in the microstrip cross-section. Field distribution and the charge on the strip can be calculated therefrom.

Since this method is more suitable for enclosed microstrip, it is not explained in detail here.

3.1.3. VARIATIONAL METHOD IN FOURIER TRANSFORM DOMAIN

The search for microstrip analysis techniques which are computationally more efficient has led to the "Variational method in Fourier Transform Domain (FTD)". There are two significant features of this method [6]. First, a variational method for calculating the capacitance C from the charge density ρ is used. This avoids the need for knowing the charge density distribution accurately. Secondly, the major portion of analysis is carried out in Fourier Transform Domain with the result that the integral equation for the potential is replaced by an ordinary product of an approximate $\tilde{\rho}$ and a factor $\tilde{g}$ derived in FTD. (The superscript $\tilde{}$ indicates a transformed function and is obtained from $\tilde{f}(\alpha) = \int_{-\infty}^{\infty} f(x)\, e^{j\alpha x} dx$).

Results obtained by this method agree well with those of the modified conformal transformation method discussed earlier.

This method can also be used to take into account the effect of finite strip thickness and enclosure. It can be easily extended for microstrip on composite substrates or where a dielectric overlay exists over the microstrip (7).

3.2. MICROSTRIP DISPERSION MODELS

The quasi-static methods of microstrip analysis peresented here do not take into account the non-TEM nature of the microstrip mode. The non-TEM behavior causes the effective dielectric constant (ε_{eff}) and impedance Z_o of the microstrip to be functions of frequency. The frequency dependence of the effective dielectric constant describes the influence of dispersion on the phase velocity, whereas the frequency dependence of the effective width describes the influence of the dispersion on the characteristic impedance.

Fortunately, changes in effective dielectric constant and characteristic impedance with frequency are very small. However the frequency below which dispersion effects may be neglected is given by the relation[8]

$$f_o(\mathrm{GHz}) = 0.3 \sqrt{\frac{Z_o}{h(\varepsilon_r - 1)^{1/2}}} \quad (h \text{ in cm}) \qquad (3.20)$$

Equation (3.20) shows that f_o is higher for high impedance lines on thin substrates.

The numerical analysis for dispersion in open and shielded microstrip transmission lines has been treated extensively. These analyses, however, require extensive computations and fail to provide insight into the dominant physical phenomenon at work. For these reasons, the numerical approach is not convenient for microstrip circuit design and therefore, not discussed in this study.

However, there are several semi-empirical techniques available that lead to a closed form solution for the dependence of effective dielectric constant and characteristic impedance on frequency. These dispersion models are described in the following sub-sections.

3.2.1. DIELECTRIC-LOADED RIDGED WAVEGUIDE MODEL

This model for microstrip dispersion is based on the study of another structure which resembles microstrip as far as inhomogeneity of dielectric medium is concerned, but has a shape that can be analysed mathematically.[9] This structure is shown in Figure 3.4.

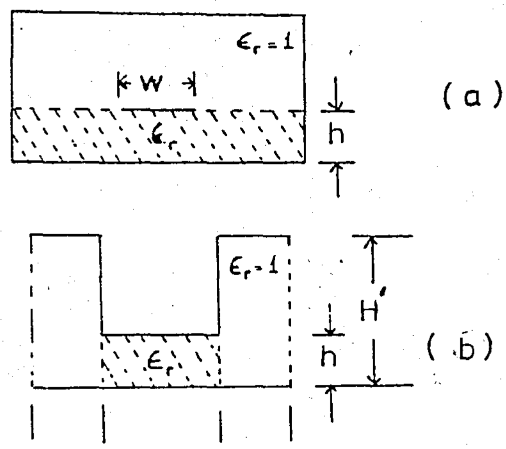


Figure 3.4. Ridged Waveguide Model for Microstrip Line

The configuration shown in Figure 3.4(b) corresponds to the microstrip cross-section shown in Figure 3.4(a). The dimensions of the structure are chosen such that it has the same electrical characteristics at zero frequency as the microstrip.

The structure is analysed for dispersion and the results are compared with the measured microstrip dispersion values to determine unknown parameter H'/h. The comparison shows that H'/h and related parameters are nearly constant or vary linearly with characteristic impedance. This feature is used to derive a simple formula that can be used to predict the dispersion of a microstrip transmission line. The resulting dispersion formula may be written as

$$\varepsilon_{eff}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff}(0)}{1 + \left(\frac{f}{f_p}\right)^2 G} \tag{3.21}$$

where f_p, being a parameter of the dispersion function, equals to

$$f_p(\text{GHz}) = \frac{0.398\ Z_o}{h(\text{in mm})} \tag{3.22}$$

and G, being an emprical parameter used to simplify the microstrip dispersion function, equals to

$$G = 0.6 + 0.009\ Z_o \tag{3.23}$$

and $\varepsilon_{eff}(0)$ is the zero frequency value of ε_{eff}.

From equation (3.21), it can be seen that for $f_p \gg f$

$$\varepsilon_{eff}(f) = \varepsilon_{eff}(0)$$

In other words, high-impedance lines on thin substrates are less dispersive.

3.2.2 PLANAR WAVEGUIDE MODEL

It has been shown (10) that the dynamic properties of microstrip can be approximated by a planar waveguide model. In this model, microstrip is represented by a parallel plate waveguide of width w_{eff} and height h as shown in Figure 3.5.

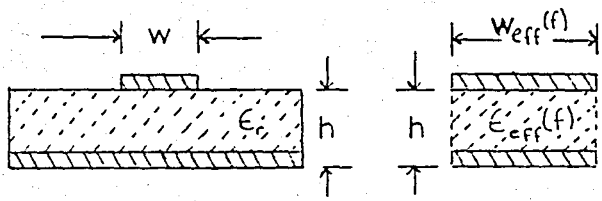


Figure 3.5. Planar Waveguide Model for Microstrip Line

The top and bottom plates are of infinite conductivity, and there are magnetic walls at the sides. It is filled with a medium of dielectric constant ε_{eff}. The value of ε_{eff} at zero frequency is determined from quasi-static analysis. The effective width w_{eff} and the effective dielectric constant ε_{eff} are frequency dependent. The frequency dependence of the dielectric constant describes the influence of the dispersion on the phase velocity, whereas the frequency dependence of the effective width describes the influence of the dispersion on the characteristic impedance. The phase velocity of the waves in microstrip line decreases with increasing frequency, hence effective dielectric constant increases with frequency. Also, the characteristic impedance of a microstrip increases with frequency. This increase in impedance can be explained only by a hypothetical decrease in the effective strip width caused by the concentration of electric field lines below the strip at higher frequency. The decrease of the effective strip width with frequency is described by the following empirical relation [10]

$$w_{eff}(f) = w + \frac{w_{eff}(0) - w}{1 + f/f_g} \quad (3.24)$$

where

$$f_g = c/(2w\sqrt{\varepsilon_r})$$

$w_{eff}(0)$ = effective width calculated from quasi-static analysis

$$= 120\pi h/(Z_o\sqrt{\varepsilon_{eff}(0)})$$

And the frequency dependent impedance is given by

$$Z_o(f) = 120\pi h/(w_{eff}(f)\sqrt{\varepsilon_{eff}(f)}) \quad (3.25)$$

3.3. FULLWAVE ANALYSES

As pointed out in Chapter 2, the microstrip configuration is not capable of supporting a pure TEM mode, and longitudinal components of both the electric and magnetic fields are present. The hybrid modes supported by the microstrip cannot be fully described in terms of static capacitances and inductances. Therefore, one has to introduce time varying electric and magnetic fields and solve the wave equation. Moreover, the charge density used in the electrostatic analyses becomes time varying and is replaced by the electric current density for fullwave analyses. Fullwave analysis is carried out for determining the propagation constant instead of the capacitance evaluated in quasi-static analysis.

Methods of studying wave propagation on microstrip without making any quasi-static assumption may be divided into two groups. (See Figure 3.1). In one group, the microstrip is considered with a rectangular enclosure, and the other group deals with open microstrip lines. This grouping becomes convenient since different types of mathematical tools are needed for handling closed and open geometries. For example a Fourier series representation can be used to express the fields in a closed rectangular structure whereas a Fourier integral representation is suited for open structure.

IV. MICROSTRIP DESIGN CONSIDERATIONS

For design purposes it is necessary to know how the characteristic impedance, phase velocity and attenuation constant depend on geometrical factors, on the electronic properties of the substrate and conductors and on the frequency. Also, power handling capability, effect of tolerances and dielectric anisotropy are the some important design parameters.

In this section, these design parameters of microstrip lines are discussed. And various expressions, useful for microstrip design, are summarized in the last subsection called "DESIGN EQUATIONS".

4.1. MICROSTRIP LOSSES

Attenuation constant, α, is one of the most important characteristics of any transmission line. There are two sources of dissipative losses in a microstrip circuit : CONDUCTOR LOSS and SUBSTRATE DIELECTRIC LOSS. Since we consider only nonmagnetic dielectric substrates, no magnetic loss component will be present. And also since the radiation loss is a small quantity near conductor and dielectric loss, it is neglected and not considered here.

4.1.1 CONDUCTOR LOSS

Conductor loss in microstrip structure is described by Pucel (11) and Schneider (1) comprehensively. Both of these analyses are based on the "incremental inductance rule" of wheeler (12). (This method is based on the fact that inductance and conductor attenuation are related since the inductance is the normalized magnetic field energy of the circuit element and attenuation is proportional to the magnetic field energy stored in the metal conductor). In this method the series surface resistance R_s per unit length is expressed in terms of that part of the total inductance per unit length which is attributable to the skin effect, i.e., the inductance L_i produced by the magnetic field within the conductors.

It is well known that for a conductor the surface impedance, $Z_s(= R_s + JX_s)$, has a real part R_s(surface resistance per unit length) which (10) is equal to the imaginary part X_s. That is

$$R_s = X_s = \omega L_i : \tag{4.1}$$

According to wheeler, L_i can be found from the external inductance L per unit length. L_i is obtained as the incrementa increase in L caused by an incremental recession of all metallic walls due to skin effect. This situation is shown in Figure 4.1. The amount of recession is equal to half the skin depth $\delta = (2/\omega \mu \sigma_c)^{1/2}$.

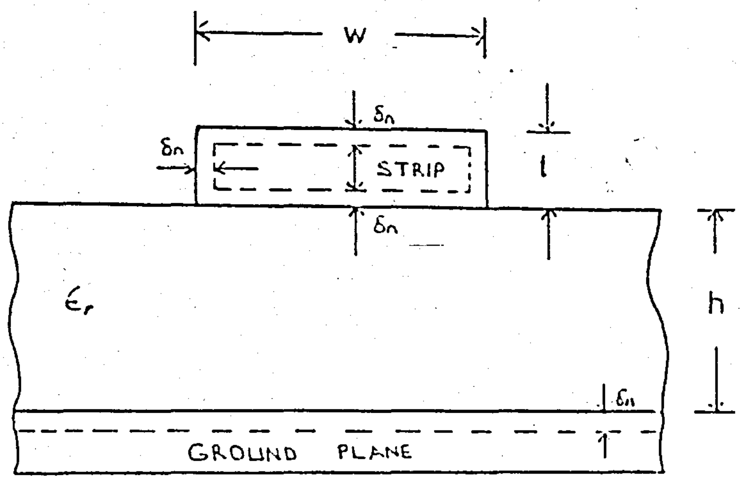


Figure 4.1. Recession of Conducting Walls of Microstrip for Loss Calculation Using "Incremental Inductance Rule"

An assumption underlying this rule is that the radius of curvature and the thickness of the conductors exposed to the electromagnetic fields be greater than skin depth. According to Wheeler (12), we have

$$L_i = \sum_m \frac{\mu_m}{\mu_o} \frac{\partial L}{\partial n_m} \frac{\delta_m}{2} \qquad (4.2)$$

$$R_s = \sum_m \frac{R_{sm}}{\mu_o} \frac{\partial L}{\partial n_m} \tag{4.3}$$

where $\partial L/\partial n_m$ denotes the derivative of L with respect to incremental recession of wall m, n_m is the normal direction to this wall, and $R_{sm} = \omega\mu_m\delta_m/2$ the surface resistance of the wall m.

The attenuation constant because of conductor (ohmic) loss is defined as

$$\alpha_c \cong \frac{P_c}{2P(Z)} = \frac{\text{Power loss in conductors}}{2\ (\text{Power transmitted})}$$

$$\text{(nepers/unit length)} \tag{4.4}$$

In terms of R_s and Z_o (characteristic impedance), α_c may be written as

$$\alpha_c = \frac{|I|^2 R_s}{2|I|^2 Z_o} = \frac{1}{2\mu_o Z_o} \sum_m R_{sm} \frac{\partial L}{\partial n_m} \tag{4.5}$$

Inductance L of the microstrip structure can be expressed in terms of the characteristic impedance for the microstrip with the substrate replaced by air (Z_o^a) and is given as

$$L = \frac{Z_o^a}{c} \tag{4.6}$$

where c is the velocity of electromagnetic waves in free space.

Expression for attenuation constant based on Equation (4.5) is presented later in subsection 4.5.2 where all other design information is also summarized.

4.1.2. DIELECTRIC LOSS

For a uniformly field transmission line the dielectric loss α_{du} is independent of the geometry of the line and can be written as :

$$\alpha_{du} = \frac{g Z_o}{2} = \frac{\omega}{2} \sqrt{\frac{\mu}{\varepsilon'}} \, \varepsilon'' = \frac{\omega}{2} \sqrt{\mu \varepsilon'} \tan \delta \qquad (4.7)$$

where g is the shunt conductance per unit length of the line, ε' is the real part and ε'' is the imaginary part of the permittivity of the dielectric.

However, when the dielectric is not uniform over the cross-section of the line, the equation (4.7) does not apply. Calculation of loss for the mixed dielectric case has been considered by Schneider (13). This loss can be calculated if one knows the loss tangent of the dielectric substrate.

When the upper dielectric (air) is assumed to be lossless, the following expression for α_d, attenuation constant considering mixed dielectrics, is obtained

$$\alpha_d = \frac{\omega}{2}\sqrt{\mu\varepsilon_{reff}}\,(\tan\delta)_{eff}$$

$$= \frac{\omega}{2}\sqrt{\mu\varepsilon_{reff}}\,\frac{\sigma_{eff}}{\omega\varepsilon_{reff}} \qquad (4.8)$$

If σ is the conductivity of the dielectric and σ_o the conductivity of air, we may write

$$\sigma_{eff} = q\sigma + (1-q)\sigma_o \cong q\sigma \quad (\text{Since } \sigma_o \ll \sigma\,)$$

Also,

$$\varepsilon_o\varepsilon_{eff} = q\varepsilon + (1-q)\varepsilon_o \quad \text{with} \quad \varepsilon = \varepsilon_o\varepsilon_r$$

There, the Equation (4.8) becomes

$$\alpha_d = q\left\{\frac{\varepsilon}{q\varepsilon + (1-q\varepsilon_o)}\right\}^{1/2} \cdot \alpha_{du} = \left(\frac{\varepsilon_r}{\varepsilon_{eff}}\right)^{1/2} q\alpha_{du} \qquad (4.9)$$

where q is the dielectric filling fraction, and α_{du} is the attenuation constant for a line uniformly filled with the dielectric ε_r, given by Equation (4.7). Since,

$$q = \frac{\partial\varepsilon_{eff}}{\partial\varepsilon_r} = \frac{\varepsilon_{eff}-1}{\varepsilon_r-1}$$

the attenuation constant α_d can be written as

$$\alpha_d = 27.3 \frac{\varepsilon_r}{\sqrt{\varepsilon_{eff}}} \frac{\varepsilon_{eff}-1}{\varepsilon_r-1} \frac{\tan\delta}{\lambda_o} \text{ dB/Unit length} \tag{4.10}$$

4.2. POWER HANDLING CAPABILITY

The power handling capability of a microstrip is limited by heating caused because of ohmic and dielectric losses and by dielectric breakdown. Increase in temperature due to conductor and dielectric losses limits the average power of the microstrip line, while the breakdown between the strip conductor and ground plane limits the peak power.

4.2.1. AVERAGE POWER HANDLING CAPABILITY

The average power handling capability of microstrip is determined by the temperature rise of the strip conductor and the supporting substrate. The parameters which play major roles in the calculation of average power capability are :

(i) transmission line losses
(ii) thermal conductivity of the substrate material
(iii) surface area of the strip conductor and
(iv) ambient temperature.

Therefore, dielectric substrates with low loss tangent and large thermal conductivity will increase the average power capability of microstrip lines.

The maximum average power for a given line may be calculated from

$$P_{av} = (T_{max} - T_{amb})/\Delta T \tag{4.13}$$

where ΔT denotes rise in temperature perwatt and given by

$$\Delta T = \frac{0.2303h}{K}\left\{\frac{\alpha_c}{w_{eff}} + \frac{\alpha_d}{2w_{eff}(f)}\right\} \quad (^{o}C/watt)$$

where K is the thermal conductivity of the substrate.

The maximum operating temperature T_{max} of microstrip circuits can be limited due to

(i) change of substrate properties with temperature

(ii) change of physical dimensions with temperature and

(iii) connectors.

For $T_{max} = 100^{o}C$, $T_{amb} = 25^{o}C$ and $Z_o = 50$ ohm, values of Average power handling capability (APHC) for various substrates at 2 GHZ, 10 GHZ and 20 GHZ are calculated by computer and given in Table 4.1. Among the dielectrics considered, APHC is lowest for poly styrene and it is maximum for BeO. At lower frequencies GaAs microstrip lines have better APHC than Si microstrip lines have better APHC than Si microstrip lines, but at higher frequencies APHC is better in the case of Si.

SUBSTRATE	MAX. AVERAGE POWER (KW)		
	2 GHZ	10 GHZ	20 GHZ
POLYSTYRENE	0.321	0.124	0.075
QUARTZ	1.200	0.523	0.357
SILICON(SI)	3.190	2.230	1.640
GAAS	3.550	1.470	0.930
SAPPHIRE	11.650	5.100	3.460
ALUMINA	12.120	5.170	3.400
BEO	174.500	75.700	51.500

Table 4.1. Comparison of Average Power Handling Capability (APHC) for Various Substrates

4.2.2. PEAK POWER HANDLING CAPABILITY

The peak voltage which can be applied without causing dielectric breakdown determines the peak power handling capability of the microstrip. If Z_o is the characteristic impedance of the microstrip and V_o is the maximum voltage the line can withstand, the maximum peak power is given by

$$P_p = \frac{V_o^2}{2Z_o} \quad (4.14)$$

Thick substrates can support higher voltages than thin substrates. Therefore, low impedance lines and lines on thick substrates have higher peak power handling capability.

The sharp edges of the strip conductor serve as field concentrators. The electric fieldtends to a large value at the sharp edges of the conductor if it is a flat strip and decreases as the edge of the conductor is rounded off more and more. Therefore, thick and rounded strip conductors will increase breakdown voltage.

The dielectric strength of the substrate material as well as of the air plays an important role. The breakdown strength of dry air is approximately 30 kv/cm. Thus the maximum electric field (tangential) near the strip edge should be less than 30 kv/cm. In order to avoid air breakdown near the strip edge, the edge of the strip conductor is painted with a dielectric paint which has the same dielectric constant as that of the substrate and is lossless.

4.3. EFFECT OF TOLERANCES

Characteristics of microstrip lines (namely Z_o and ε_{eff}) are primarily functions of strip width w and substrate parameters ε_r and h. These are also influenced by factors such as strip thickness, frequency of operation (dispersion), size of enclosure etc. Any changes in the values of w, ε_r or h give rise to corresponding changes in Z_o and ε_{eff}.

The substrate properties, like surface finish, metallization thickness, the fabrication process and so on, determine the accuracy of fabrication of strip width. In addition to the error in fabrication of strip width, the thickness and the dielectric constant of the substrate have some manufacturing tolerances. All these factors contribute to variations in Z_o and ε_{eff} of the microstrip. Since in microwave integrated circuits (MIC's) it is very difficult to incorporate arrangements for post fabrication adjustments, it is necessary to take into account the effect of tolerances at the design stage itself. However, unlike the effects of deterministic parameters like dispersion, strip thickness, etc., the effect of tolerances cannot be incorporated exactly because of the uncertainty in Z_o and ε_{eff} arising from tolerances.

4.4 EFFECT OF DIELECTRIC ANISOTROPY

Some of the dielectric substrates used for microstrip circuits exhibit anisotropy in permittivity. The most common examples are sapphire and Epsilom-10 (trade name for a ceramic loaded resin). In both cases, the substrates are manufactured such that one of the principal axis of the permittivity tensor is perpendicular to the dielectric interface y = h. If there were no fringing of electric fields in microstrip configuration all the field lines will coincide with this axis, and the capacitance will be determined by the value of permittivity in this direction. In such a case there is no effect of anisotropy.

4.5. DESIGN EQUATIONS

The numerical methods to characterize the performance of microstrip line discussed so farinvolve extensive computations. Closed form expressions are necessary for optimization and computer-aided-design of microstrip circuits. A complete set of design equations for microstrip are presented in this section. These include closed form expressions for the characteristic impedance and effective dielectric constant, and their variation with metal strip thickness, enclosure size and dispersion. Expressions for microstrip loss and quality factor Q are also described.

4.5.1. CHARACTERISTIC IMPEDANCE AND EFFECTIVE DIELECTRIC CONSTANT

The advent of high-speed digital circuitry has given emphasis to the need for a ready method of determining the characteristics of interconnection between individual components. This is particularly important for integrated circuits where the close proximity of the interconnections may induce adverse effects on the circuit performance ; for example overshoot, ringing and crosstalk. For these effects to be minimised, it is necessary to have a knowledge of the characteristic impedance of the microstriplines.

There are a number of closed form approximate expressions available for the characteristic impedance and effective dielectric constant. In this section, first a method based on line-capacitance computation (a) is described and two modifications (b,c) are given. Next, a set of equations in terms of series expansions obtained by generalization of Schneider's standard equations are given (d), and a modification to equivalent relative-permittivity, expression which takes account of the finite strip thickness is presented (e). Finally, a set of closed form expressions being more accurate and based on the works of wheeler (3) and Schneider (1) are given (f).

Since a number of closed form approximate expressions are available for the characteristic impedance and effective dielectric constant, it is worthwhile to compare these results with each other and available experimental and numerical results by using tables. These tables are presented in each sub-section.

a. Line-Capacitance Computation Method

This method was pioneered by Schwarzmann (16) and based on the subdivision of the capacitance associated with a single microstripline into three components : the parallel plate capacitance, C_{pp}, between the lower surface of the conductor and the ground plane ; the fringing capacitance, C_F, at the edges of the conductor ; and the capacitance between the conductor's upper surface and the ground plane, C_{ppu} (See Figure 4.2),

These capacitances C_{pp}, C_F and C_{ppu} are given as follows (16) :

$$C_{pp} = \frac{\varepsilon_r}{c\zeta} \quad \frac{w}{h} \tag{4.15}$$

$$C_{ppu} = \frac{2}{3}\left(\frac{\varepsilon_r}{c\zeta} \quad \frac{w}{h}\right) \tag{4.16}$$

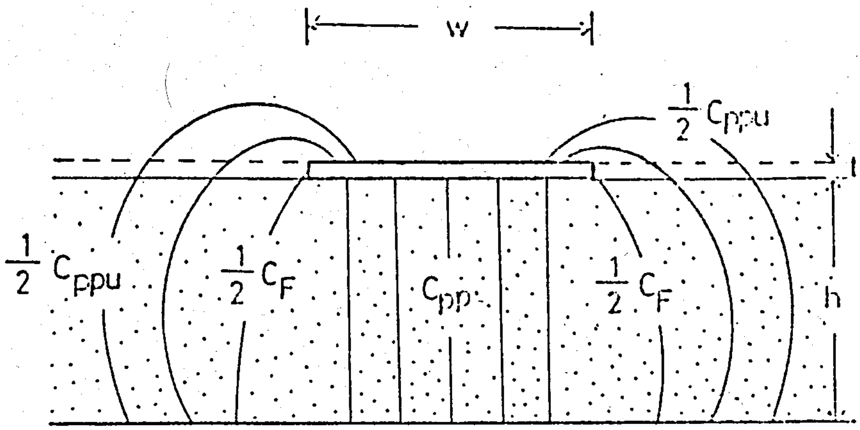


Figure 4.2. Microstripline with Associated Capacitances

and

$$C_F = \frac{\varepsilon_r}{c\zeta} \quad \frac{2.7}{\log\left(\frac{4h}{t}\right)} \tag{4.17}$$

where ζ is the free space impedance and c is the speed of light.

As we know, for TEM mode the characteristic impedance of an elementary transmission line can be related to the velocity of propagation down the line and the capacitance per unit length by

$$Z_o = 1/\nu C \tag{4.18}$$

where the velocity of propagation ν is given by the relationship

$$\nu = c/\sqrt{\varepsilon_r} \tag{4.19}$$

and

$$C = C_{pp} + C_{ppu} + C_F$$

Substituting equations (4.15), (4.16), (4.17) in Eq. (4.18) yields

$$Z_o = \frac{\zeta}{\sqrt{\varepsilon_r}} \left[\frac{w}{h} + \frac{A_1}{\log\left(\frac{4h}{t}\right)} + \left(\frac{A_2}{\sqrt{\varepsilon_r}} \quad \frac{w}{h} \right) \right]^{-1} \tag{4.20}$$

where $A_1 = 2.7$ and $A_2 = 2/3$

b. Modification to the Method Given in a :

Since one major source of error in the line capacitance computation method arises from the use of the relative dielectric permittivity, John and Arlett (17) have considered it in the straight-line approximation neglecting the dependence on microstrip geometry as follows :

$$\varepsilon_{eff} = 0.4475\ \varepsilon_r + 0.6965 \tag{4.21}$$

They have also changed the constants A_2 in C_{ppu} and A_1 in C_F with the following emprical formulae :

$$A_1 = -0.6895\ \{h/(\varepsilon_{eff} w)\} + 1.2163 \tag{4.22}$$

$$A_2 = 0.0515(h/t) + 2.183 \tag{4.23}$$

c) Modification to the Method Given in b :

In the method given in b, the expressions for A_1, A_2 and ε_{eff} have been obtained by supposing that A_1, associated with the fringing capacitance, is a function of the strip thickness t and the dielectric height h. Similarly, A_2 associated with the upper-plate capacitance is a function of the dielectric height h, the strip conductor width w and the effective dielectric permittivity ε_{eff}.

But there is no physical significance to believe that the constants A_1 and A_2 depend on microstrip parameters while these parameters have already been accounted for in the capacitance calculations. Also since the microwave energy propagates both in the dielectric substrate below the strip and in the air region above, the effective dielectric permittivity should be considered as a function of microstrip geometry as follows (18) :

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1 + \frac{w}{10t}\right)^{-1} \tag{4.24}$$

Also, the capacitance C_{ppu} between upper surface of microstrip and ground plane can be neglected since the metallic strip is thin and good conducting (18).

The fringing capacitance C_F has been evaluated by considering the round edge of the metallic strip in the method given in a. Hence, it can be calculated by taking a conducting wire parallel to the ground plane as follows (18) :

$$C_F = \frac{\varepsilon_r}{c\zeta} \quad \frac{2\Pi}{\ln\left(\frac{4h}{t}\right)} \tag{4.25}$$

Finally, the expression for the characteristic impedance comes out to be simply :

$$Z_o = \frac{\zeta}{\sqrt{\varepsilon_{eff}}}\left[\frac{w}{h} + \frac{2\Pi}{\ln\left(\frac{4h}{t}\right)}\right]^{-1} \tag{4.26}$$

Results of calculations by using the methods given in a, b and c are compared with experimental values of Kauppp (19) where w/h varies from 0.227 to 2.500 and t/h varies from 0.046 to 0.350 in Table 4.2. The results of calculations are also compare with the numerical method of Yamashita and

Mettra (6) in Table 4.3. (The relative errors in percent are given in parenthesis). As it is clear from these tables, the accuracy of the line-capacitance method given in a and its derivatives is not good. Also, for effective dielectric constant, although the numerical methods predict a decrease with increase in strip thickness, the method given in c show it otherwise i.e. ε_{eff} increases with t/h. This situation can be seen from Table 4.10. This is of course not acceptable since the fraction of total energy propagating in air increases with strip thickness, there by decreasing the effective dielectric constant.

d. Generalization of Schneider's Standard Equation Obtained by Exact Conformal Mapping

The rigorous solution for computing Z_o from equations (3.4), (3.5), (3.6) is not recommended for most engineering applications. Useful expressions for the characteristic impedance of the microstrip in terms of rational functions or series expansions have been obtained by generalization of equations obtained by exact conformal mapping as follows (1) :

For $w \leqslant h$

$$Z_o = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left[\sum_{n=1}^{\infty} a_n \left(\frac{h}{w}\right)^n\right] \qquad (4.27)$$

W/H	T/H	MEAS. ZO(OHM)-(19)	CALCULATED ZO(OHM)		
			METHOD A	METHOD B	METHOD C
0.227	0.063	111.2	96.856 (-14.80)	97.763 (-13.70)	105.577 (-5.30)
0.250	0.063	111.0	101.105 (-9.90)	94.492 (-17.60)	113.028 (1.60)
0.340	0.063	99.2	89.489 (-10.80)	91.921 (-7.90)	101.002 (1.60)
0.483	0.090	90.5	76.569 (-18.10)	85.138 (-6.2)	87.505 (-3.4)
0.526	0.147	82.0	67.643 (-21.2)	79.796 (-2.7)	75.626 (-8.40)
0.645	0.090	81.0	70.0353 (-15.6)	80.795 (-0.6)	82.6405 (1.90)
0.681	0.127	76.5	64.556 (-18.5)	75.729 (-1.0)	74.796 (-2.20)
0.714	0.200	73.0	57.769 (-26.3)	70.189 (-4.0)	65.304 (-11.70)
0.909	0.127	67.05	81.229 (-15.2)	68.8869 (2.7)	70.404 (3.70)
1.157	0.147	65.0	51.205 (-26.9)	61.872 (-5.0)	62.501 (-3.90)
1.070	0.200	61.0	50.030 (-21.9)	61.307 (+0.5)	59.082 (-3.20)
1.000	0.280	59.4	47.682 (-19.7)	59.201 (-0.3)	54.597 (-8.70)
1.250	0.350	53.0	41.520 (-27.6)	52.056 (-1.8)	47.946 (-10.50)
1.875	0.350	42.5	34.740 (-22.3)	43.848 (+3.0)	42.016 (-1.10)
2.500	0.350	36.4	29.803 (-20.5)	37.876 (+4.9)	37.429 (+3.00)

Table 4.2. Comparison of Calculated and Measured Characteristic Impedances for Methods a, b, and c (ε_r = 4.7)

EPSR	W / H	T / H	NUM. METH. (6)	CALCULATED ZO(OHM)		
				METHOD A	METHOD B	METHOD C
1.0	0.10	0.010	253.0	312.895 (-14.80)	162.374 (-13.70)	328.083 (-5.30)
1.0	0.10	0.050	230.0	237.677 (+3.20)	317.815 (+27.60)	245.699 (6.30)
1.0	0.10	0.200	194.0	168.073 (-15.40)	274.413 (29.30)	171.508 (-13.10)
1.0	1.00	0.010	125.0	139.368 (+10.30)	117.165 (-6.6)	183.940 (32.0)
1.0	1.00	0.050	122.0	122.149 (0+2)	180.668 (32.47)	154.833 (13.40)
1.0	1.00	0.200	112.0	100.714 (-11.2)	165.764 (32.4)	121.676 (7.90)
1.0	2.00	0.01	88.0	86.231 (-2.0)	89.386 (+1.5)	123.602 (28.80)
1.0	2.00	0.05	86.0	79.314 (-8.4)	112.116 (24.5)	109.740 (+21.60)
1.0	2.00	0.20	82.0	67.684 (-17.6)	115.120 (28.7)	91.970 (10.80)
11.7	0.10	0.01	96.5	95.201 (-1.3)	38.194 (-152.0)	109.209 (11.60)
11.7	0.10	0.05	89.0	71.614 (-24.2)	62.792 (-41.7)	74.735 (-19.00)
11.7	0.10	0.20	82.0	50.192 (-63.3)	56.724 (-44.5)	50.696 (-61.70)
11.7	1.00	0.01	45.0	49.347 (8.8)	29.110 (-54.5)	70.350 (+36.00)
11.7	1.00	0.05	43.7	42.151 (-3.6)	41.501 (-5.2)	54.291 (19.50)
11.7	1.00	0.20	41.5	33.688 (-23.1)	38.763 (-7.0)	38.636 (-7.40)
11.7	2.00	0.01	31.0	32.144 (3.5)	23.026 (-34.6)	48.035 (35.50)
11.7	2.00	0.05	30.4	28.927 (-5.0)	30.1403 (-0.8)	40.2876 (24.50)
11.7	2.00	0.20	29.5	24.673 (-19.5)	28.673 (-2.8)	30.614 (+3.60)

Table 4.3. Comparison of Characteristic Impedances Calculated from Methods a, b and c with Characteristic Imp. obtained by Numerical Methods (6)

and for $\omega > h$

$$Z_o = \frac{120\pi}{\sqrt{\varepsilon_{eff}}} \left[\sum_{n=1}^{-\infty} b_n \left(\frac{w}{h} \right)^n \right]^{-1} \tag{4.28}$$

The number of terms after which the series is terminated determines the accuracy of the approximations. The following formulas obtained by rational function approximation give an accuracy of ±0.80 percent for $0 \leqslant w/h \leqslant 10$ which is the range of importance for most engineering applications :

For $w/h \leqslant 1$

$$Z_o = \frac{60}{(\varepsilon_{eff})^{1/2}} \ln\left(\frac{8h}{w} + \frac{w}{4h} \right) \tag{4.29}$$

and for $w/h \geqslant 1$

$$Z_o = \frac{120\Pi}{(\varepsilon_{eff})^{1/2} \left(\frac{w}{h} + 2.42 - 0.44 \frac{h}{w} + \left(1 - \frac{h}{w} \right)^6 \right)} \tag{4.30}$$

The effective dielectric constant can be found by using Wheeler's method by starting from an approximate conformal mapping transformation for the transformed parallel plate capacitance (3). The square root of the effective dielectric constant $(\varepsilon_{eff})^{1/2}$ obtained by this method is shown in Fig. 4.3 as a function of w/h an ε_r.

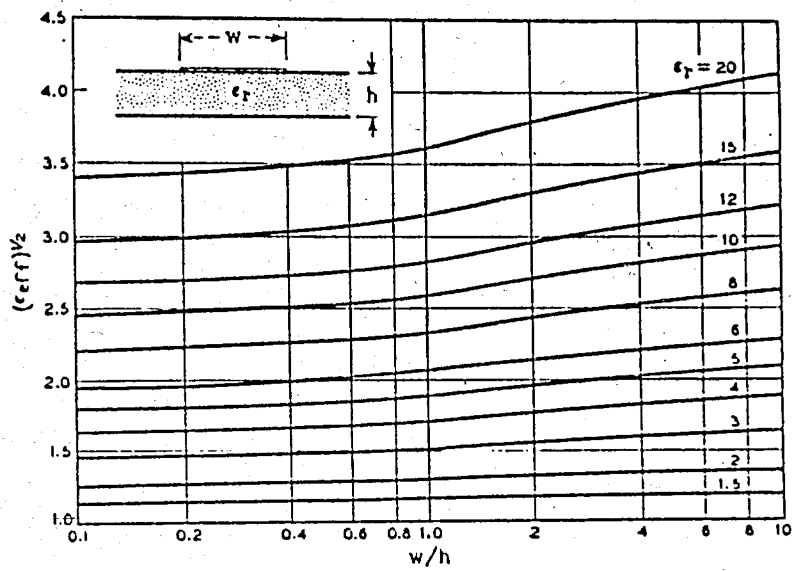


Fig. 4.3. Square Root of the Effective Dielectric Constant for the Microstrip.

In order to find a function which approximates the set of curves of Fig. 4.3 over the total range $0 < w/h < \infty$ and $1 < \varepsilon_r < \infty$ Schneider has defined a function $F(\varepsilon_r, \frac{w}{h})$ by

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} F(\varepsilon_r, \frac{w}{h}) \tag{4.31}$$

where

$$F(\varepsilon_r, \frac{w}{h}) = (1 + 10 \frac{h}{w})^{-\frac{1}{2}} \qquad (4.32)$$

By using Eq. (4.31) in equations (4.29) and (4.30) characteristic impedance can be found.

e. Modification to Schneider's Effective Dielectric Constant Equation given in d :

The original work of Schneider (1) has considered only strips of zero thickness, a condition never achieved in practice. Yamashita and Mittra (6) have shown that the effect of strip thickness is appreciable for $w/h < 1$, the effect being a lowering of the characteristic impedance as found by Stinehelfer (5) by experiment. A modificaiton to Schneider's effective dielectric constant equation has been given by Ross and Howes (20) approximately agreeing with the work of Yamashita and Mittra (6), Bryant and Weiss (21), Stinehelfer (5) and Wheeler (3). This expression is

For $\frac{w}{h} \leqslant 1$

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2} (1 + \frac{10h}{w})^{-\frac{1}{2}} + 0.468\left(\frac{\varepsilon_r+0.5}{1.5}\right)\sqrt{\frac{t}{w}} \qquad (4.33)$$

and for $\frac{w}{h} \geq 1$

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1+\frac{10h}{w}\right)^{-\frac{1}{2}} \qquad (4.34)$$

This ε_{eff} expression is identical with strips, where the term in t is swamped by the other two w and h. It is assumed that $\frac{t}{w}$ becomes small as $\frac{w}{h}$ increases, which is likely to be true in the practical case, since it implies $t/h < 0.2$.

The expressions for characteristic impedance are the same as equations (4.29) and (4.30) given in d.

Results of calculations by using the methods given in d and e are compared with experimental values of Kaupp (19) where w/h varies from 0.227 to 2.500 and t/h varies from 0.046 to 0.350 and given in Table 4.4. The results of calculations are also compared with the numerical method of Yamashita and Mittra (6) in Table 4.5. (The relative errors in percent are given in parenthesis). As it is clear from these tables, the accuracy is not good for these methods, also. And just like in the method c, for the set of equations given in e, the value of ε_{eff} increases with t/h and for some set of parameters becomes even greater than ε_r. (Look at Table 4.10).

W / H	T / H	MEAS. Z0(OHM)-(19)	CALCULATED Z0(OHM)	
			METHOD D	METHOD E
0.227	0.063	111.2	120.949 (+8.00)	107.180 (-3.70)
0.250	0.063	111.0	117.436 (+5.30)	106.247 (-4.60)
0.340	0.063	99.2	106.294 (6.60)	96.264 (-3.00)
0.483	0.090	50.5	93.712 (3.40)	84.994 (-6.4)
0.526	0.147	82.0	90.688 (9.5)	80.701 (-1.6)
0.645	0.090	81.0	83.524 (1.6)	76.781 (5.46)
0.681	0.127	76.5	81.635 (6.2)	74.176 (-3.1)
0.714	0.200	73.0	79.997 (8.7)	71.322 (-2.3)
0.909	0.127	67.0	71.767 (6.6)	66.089 (-1.3)
1.157	0.147	65.0	63.499 (-2.3)	58.759 (-10.6)
1.070	0.200	61.0	66.129 (7.7)	60.247 (-1.2)
1.000	0.280	59.4	68.587 (13.3)	61.299 (3.0)
1.250	0.350	53.0	60.992 (13.10)	54.604 (2.90)
1.875	0.350	42.5	48.887 (13.0)	44.711 (4.9)
2.500	0.350	36.0	41.016 (12.2)	41.016 (12.2)

Table 4.4. Comparison of Calculated and Measured Characteristic Impedances for Methods d and e ($\varepsilon_r = 4.7$)

EPSR	W / H	T / H	NUM. METH. (6)	CALCULATED ZO(OHM)	
				METHOD D	METHOD E
1.0	0.10	0.010	253.0	262.90 (3.70)	245.40 (-3.0)
1.0	0.10	0.050	230.0	262.90 (12.50)	227.90 (-0.90)
1.0	0.10	0.200	194.0	262.903 (26.20)	204.00 (4.90)
1.0	1.00	0.010	125.0	126.40 (1.10)	123.50 (1.26)
1.0	1.00	0.050	122.0	126.40 (3.4)	120.30 (-1.4)
1.0	1.00	0.20	112.0	126.40 (11.3)	115.10 (2.6)
1.0	2.00	0.01	88.0	89.332 (1.5)	87.90 (-0.1)
1.0	2.00	0.05	86.0	89.382 (3.7)	86.20 (0.2)
1.0	2.00	0.20	82.0	89.382 (8.2)	83.40 (1.6)
11.7	0.10	0.01	96.5	100.228 (3.7)	92.50 (-4.3)
11.7	0.10	0.05	89.0	100.228 (11.22)	85.00 (-4.77)
11.7	0.10	0.20	82.0	100.228 (18.10)	75.10 (-9.1)
11.7	1.00	0.01	45.0	44.808 (-0.4)	43.70 (-2.9)
11.7	1.00	0.05	43.7	44.808 (2.4)	42.60 (-2.5)
11.7	1.00	0.20	41.5	44.808 (7.3)	40.70 (-1.9)
11.7	2.00	0.01	31.0	30.596 (-1.3)	30.60 (-1.3)
11.7	2.00	0.05	30.4	30.596 (0.6)	29.60 (-2.7)
11.7	2.00	0.20	29.5	30.596 (3.5)	28.70 (-2.7)

Table 4.5. Comparison of Characteristic Impedances Calculated from Methods d and e with Characteristic Impedance Obtained by Numerical Methods (6).

f. New Equations

New equations with accuracy fulfilling the 1 percent goal have been obtained by modifying those of Wheeler (3) and Schneider (1). Schneider's equations for effective dielectric constant ε_{eff} are given in Eq. (4.31). Making $F(\varepsilon_r, \frac{w}{h})$ independent of ε_r is an approximation, but for $\varepsilon_r < 16$, it is good enough and the following equation gives relative errors in ε_{eff} not larger than 1 (See Table 4.7) percent for a practical range of microstrip lines ($0.05 < w/h < 20$ and $\varepsilon_r < 16$) :

For $w/h \leqslant 1$

$$F(w/h) = \left(1 + \frac{12h}{w}\right)^{-\frac{1}{2}} + 0.04\left(1 - \frac{w}{h}\right)^2 \qquad (4.35)$$

and for $\frac{w}{h} \geqslant 1$,

$$F(w/h) = \left(1 + \frac{12h}{w}\right)^{-\frac{1}{2}} \qquad (4.36)$$

To obtain a better approximation above it would be necessary to let F vary with ε_r. The range covered and the accuracy obtained with the approximation above should, however, be sufficient for most practical needs. (The dependence of F on ε_r decreases for increasing ε_r, thus it is belived that the largest error in the above approximation will not be above 2 % for any value of $\frac{w}{h}$ and ε_r with greatest error for w/h < 0.05).

Impedance equations for analysis are best constructed with those for single-dielectric together with the ε_{eff} equation. The error in Schneider's equation for w/h > 1 is approximately three times that claimed and efforts to improve on this showed that the best result (Look at Table 4.6) is obtained by Hammerstad (22) by modifying Wheeler's equations as follows :

For $w/h \leqslant 1$

$$Z_o = \frac{\zeta}{2\Pi \sqrt{\varepsilon_{eff}}} \ln\left(\frac{8h}{w} + 0.25 \frac{w}{h}\right) \tag{4.37}$$

For $w/h \geqslant 1$

$$Z_o = \frac{\zeta}{\sqrt{\varepsilon_{eff}}} \left[\frac{w}{h} \quad 1.393 + 0.667 \ln\left(\frac{w}{h} + 1.444\right)\right]^{-1} \tag{4.38}$$

where $\zeta = 120\Pi$ ohm is the free-space wave impedance.

The same accuracy has been obtained by modifying the constants of Wheeler's synthesis ($w/h = h(Z_o)$) equations as follows :

For $w/h \leq 2$

$$\frac{w}{h} = \frac{8}{\left[\exp(A) - 2\exp(-A)\right]} \tag{4.39}$$

For $w/h \geq 2$

$$\frac{w}{h} = \frac{2}{\Pi}\left\{B-1-\ln(2B-1)+\frac{\varepsilon_r-1}{2\varepsilon_r}\left[\ln(B-1)+0.39-\frac{0.61}{\varepsilon_r}\right]\right\} \tag{4.40}$$

where

$$A = \frac{Z_o}{60}\left(\frac{\varepsilon_r+1}{2}\right)^{1/2} + \frac{\varepsilon_r-1}{\varepsilon_r+1}\left(0.23+\frac{0.11}{\varepsilon_r}\right) \tag{4.41}$$

and

$$B = \frac{60\ \Pi^2}{Z_o\sqrt{\varepsilon_r}} \tag{4.42}$$

Effect of Strip Thickness

The equations given above assume a two-dimensional strip conductor. But in practice, the strip is three-dimensional ___ its thickness, t, must be considered. The zero-thickness ($t/h = 0$) formulas can be modified to consider the thickness of the strip when the strip width, w, is replaced by an effective strip width, w_{eff}.

Simple and accurate formulas for Z_o and ε_{eff} with finite strip thickness are (23) :

For $w/h \leqslant 1$

$$Z_o = \frac{\zeta}{2\Pi\sqrt{\varepsilon_{eff}}} \ln\left(\frac{8h}{w_{eff}} + 0.25\,\frac{w_{eff}}{h}\right) \qquad (4.43)$$

For $\frac{w}{h} \geqslant 1$

$$Z_o = \frac{\zeta}{\sqrt{\varepsilon_{eff}}}\left[\frac{w_{eff}}{h} + 1.393 + 0.667\ln\left(\frac{w_{eff}}{h} + 1.444\right)\right]^{-1} \qquad (4.44)$$

where for $\frac{w}{h} \leqslant \frac{1}{2\Pi}$

$$\frac{w_{eff}}{h} = \frac{w}{h} + \frac{1.25}{\Pi}\,\frac{t}{h}\left(1 + \ln\frac{4\Pi w}{t}\right) \qquad (4.45)$$

for $\frac{w}{h} \geqslant \frac{1}{2}$

$$\frac{w_{eff}}{h} = \frac{w}{h} + \frac{1.25}{\Pi}\,\frac{t}{h}\left(1 + \ln\frac{2h}{t}\right) \qquad (4.46)$$

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}F\left(\frac{w}{h}\right) - \frac{\varepsilon_r-1}{4.6}\,\frac{t/h}{\sqrt{\frac{w}{h}}} \qquad (4.47)$$

in which $F\left(\frac{w}{h}\right)$ is given by equations (4.35) and (4.36).

A summary of typical results obtained by using above equations is given in Tables 4.6, 4.7, 4.8, 4.9 and 4.10. To determine the percentage error inherent in using present equations, the available numerical results are also presented.

In table 4.6 characteristic impedance for zero thickness (t/h = 0) microstrip and in Table 4.7 effective dielectric constant for zero thickness microstrip calculated by the method given in f are compared with numerical results of Green (24), Wheeler (3) and the method given in d (another zero thickness case). As can be seen from these tables, the results for Z_o and ε_{eff} are correct to within 1 percent (The maximum relative error in Z_o and ε_{eff} is less than ±0.6 percent and 0.8 percent respectively) for the following sets of parameters $0.1 \leqslant w/h \leqslant 10$ and $\varepsilon_r \leqslant 11.7$ as claimed.

But in practice, the strip is three-dimensional and zero thickness case is a condition never achieved. (It may be approximated when $t/h \leqslant 0.005$ and $0.1 \leqslant w/h \leqslant 5$). So, the zero thickness (t/h = 0) formulas given here at the beginning are modified to consider the thickness of the strip. The results obtained for microstrip including strip thickness are compared with available experimental and nemerical results in Tables 4.8, 4.9 and 4.10. It is found that the results for Z_o and ε_{eff} obtained by this method are correct to within 2 percent for the following sets of parameters $0 < t/h \leq 0.350$, $0.1 \leq w/h \leq 10$ and $\varepsilon_r \leq 11.7$.

4.5.2. EFFECT OF ENCLOSURE

Most of microstrip circuit applications require a metallic enclosure for hermetic sealing, mechanical strength, electromagnetic shielding, mounting connectors and ease of handling. Both the top cover and side walls tend to lower impedance and effective dielectric constant. This is because the fringing flux lines are prematurely terminated on the enclosure walls. This increases the electric flux in air.

The closed from equations for a microstrip with top cover (without side walls) are obtained as follows (26) :

For $\frac{w}{h} \leq 1$

$$Z_o = \frac{\zeta}{2\Pi\sqrt{\varepsilon_{eff}}}\left(\frac{8h}{w} + 0.25\,\frac{w}{h}\right) - P \qquad (4.48)$$

For $\frac{w}{h} \geq 1$

W/H	EPSR	NUM.METH.(24)	WHEELER(2)	METHOD F WITH ZERO THICK.	METHOD D
0.1	1	262.38	262.392 (0.02)	262.940 (0.2)	262.940 (0.2)
0.2	1	221.22	221.270 (0.03)	221.408 (0.08)	221.408 (0.08)
0.5	1	166.67	166.697 (0.02)	166.822 (0.09)	166.822 (0.09)
1.0	1	126.40	126.520 (0.1)	126.060 (-0.2)	126.613 (0.03)
2.0	1	89.0	90.080 (1.2)	89.335 (0.4)	89.382 (0.5)
5.0	1	49.34	49.380 (0.1)	49.347 (0.01)	49.617 (0.5)
10.0	1	28.99	29.13 (0.5)	28.943 (-0.1)	29.192 (0.7)
0.1	2.62	189.68	189.960 (0.1)	190.263 (0.3)	191.010 (0.7)
0.2	2.62	158.82	158.970 (0.1)	159.190 (0.2)	159.610 (0.5)
0.5	2.62	118.28	118.510 (0.2)	118.552 (0.2)	118.310 (0.03)
1.0	2.62	88.45	88.620 (0.2)	88.763 (0.3)	88.270 (-0.2)
2.0	2.62	61.05	61.910 (1.4)	61.411 (0.5)	61.090 (0.03)
5.0	2.62	32.78	32.450 (-1.0)	32.903 (0.3)	32.870 (0.3)
10.0	2.62	18.93	18.960 (0.2)	18.856 (-0.3)	18.912 (-0.1)
0.1	3.78	163.93	164.050 (0.1)	164.293 (0.2)	165.250 (0.8)
0.2	3.78	137.66	137.330 (0.2)	137.817 (0.1)	137.610 (0.4)
0.5	3.78	101.78	101.960 (0.2)	101.867 (0.08)	101.570 (-0.2)
1.0	3.78	75.84	76.060 (0.3)	75.999 (0.2)	75.460 (-0.5)
2.0	3.78	52.08	52.810 (1.4)	52.321 (0.4)	51.970 (-0.2)
5.0	3.78	27.73	27.400 (-1.2)	27.831 (0.3)	27.750 (+0.1)
10.0	3.78	15.83	15.860 (0.2)	15.867 (0.2)	15.890 (0.4)
0.1	9.6	108.92	109.130 (0.2)	108.897 (-0.02)	109.865 (0.8)
0.2	9.6	90.87	91.050 (0.2)	90.687 (-0.2)	91.136 (0.2)
0.5	9.6	67.17	67.370 (0.3)	66.981 (-0.2)	66.791 (-0.6)

Table 4.6. Comparison of Characteristic Impedance for Zero Thickness Microstrip Calculated by Different Methods.

METHOD	NUM. METH. (24)			WHEELER (3)			METHOD D			METHOD F		
EPSR / W/H	2.62	3.78	9.6	2.62	3.78	9.6	2.62	3.78	9.6	2.62	3.78	9.60
0.1	1.92	2.57	5.82	1.91 (-0.2)	2.56 (-0.3)	5.80 (-0.3)	1.89 (-1.5)	2.52 (-1.6)	5.72 (-1.6)	1.91 (-0.5)	2.56 (-0.3)	5.83 (0.2)
0.2	1.94	2.61	5.93	1.93 (-0.2)	2.60 (-0.3)	5.90 (-0.4)	1.92 (-0.9)	2.58 (-0.8)	5.90 (-0.4)	1.93 (-0.3)	2.60 (-0.1)	5.96 (0.6)
0.5	1.98	2.68	6.15	1.97 (-0.4)	2.67 (-0.5)	6.14 (-0.4)	1.98 (+0.1)	2.69 (+0.4)	6.23 (+1.3)	1.98 (-0.3)	2.68 (0.0)	6.20 (0.8)
1,0	2,04	2.77	6.45	2.03 (-0.2)	2.76 (-0.3)	6.42 (-0.4)	2.05 (+0.6)	2.81 (+1.2)	6.60 (+2.3)	2.03 (-0.4)	2.77 (-0.1)	6.51 (0.7)
2.0	2.12	2.92	6.89	2.07 (-2.2)	2.99 (+2.6)	7.12 (+3.2)	2.14 (+0.8)	2.95 (+1.2)	7.05 (+2.3)	2.11 (-0.4)	2.91 (-0.2)	6.92 (0.5)
5.0	2.26	3.16	7.65	2.29 (+1.2)	3.20 (+1.4)	7.79 (+1.8)	2.27 (+0.5)	3.19 (+0.9)	7.78 (+1.6)	2.25 (-0.7)	3.14 (-0.7)	7.67 (0.3)
10.0	2.37	3.34	8.23	2.38 (+0.5)	3.36 (+0.6)	8.30 (+0.8)	2.38 (+0.5)	3.37 (+0.8)	8.34 (+1.3)	2.35 (-0.6)	3.32 (-0.6)	8.26 (0.4)

Table 4.7. Comparison of Effective Dielectric Constant, ε_{eff}, for Zero Thickness Microstrip Calculated by Different Methods.

W / H	T / H	MEAS. ZO(OHM)-(19)	CALCULATED ZO(OHM)	
			METHOD F	INCL.THICK.
0.227	0.063	111.2	120.551 (7.7)	108.935 (-2.0)
0.250	0.063	111.0	117.119 (+5.00)	108.850 (-1.90)
0.340	0.063	99.2	106.229 (6.60)	98.121 (-1.00)
0.483	0.090	90.5	93.906 (3.60)	88.824 (-1.8)
0.526	0.147	82.0	90.936 (9.8)	82.057 (0.03)
0.645	0.090	81.0	83.883 (3.4)	79.684 (-1.6)
0.681	0.127	76.5	82.018 (6.7)	75.616 (-1.1)
0.714	0.200	73.0	80.398 (9.2)	72.616 (-0.5)
0.909	0.127	67.0	72.222 (7.2)	67.466 (0.6)
1.157	0.147	65.0	64.121 (-1.3)	65.211 (0.3)
1.070	0.200	61.0	66.595 (8.4)	61.596 (0.9)
1.000	0.280	59.4	69.041 (13.9)	60.420 (1.7)
1.250	0.350	53.0	61.683 (14.0)	53.974 (1.8)
1.875	0.350	42.5	49.297 (13.8)	43.32 (1.9)
2.500	0.350	36.0	41.198 (12.6)	36.73 (2.0)

Table 4.8. Comparison of Calculated and Measured Characteristic Impedances for Method f with Zero Thickness and Including Strip Thickness (ε_r = 4.7)

EPSR	W / H	T / H	NUM. METH.(6)	CALCULATED Z0(OHM)	
				INCL.THICK.	ZERO THICK.(F)
1.0	0.10	0.010	253.0	250.427 (-1.0)	262.94 (+3.7)
1.0	0.10	0.050	230.0	226.389 (-1.5)	262.94 (12.50)
1.0	0.10	0.200	194.0	192.251 (-1.0)	262.94 (26.20)
1.0	1.00	0.010	125.0	125.220 (1.70)	126.613 (1.20)
1.0	1.00	0.050	122.0	121.262 (-0.6)	126.613 (3.6)
1.0	1.00	0.20	112.0	113.684 (1.4)	126.613 (11.5)
1.0	2.00	0.01	88.0	88.706 (0.7)	89.335 (1.4)
1.0	2.00	0.05	86.0	87.042 (1.1)	89.335 (3.7)
1.0	2.00	0.20	82.0	83.184 (1.2)	89.335 (8.2)
11.7	0.10	0.01	96.5	95.087 (-1.4)	99.313 (2.8)
11.7	0.10	0.05	89.0	87.843 (-1.3)	99.313 (10.3)
11.7	0.10	0.20	82.0	81.690 (-0.30)	99.313 (17.4)
11.7	1.00	0.01	45.0	44.630 (-0.8)	45.039 (0.0)
11.7	1.00	0.05	43.7	43.650 (-0.1)	45.039 (2.9)
11.7	1.00	0.20	41.5	41.880 (0.9)	45.039 (7.8)
11.7	2.00	0.01	31.0	30.688 (-1.0)	30.875 (-0.4)
11.7	2.00	0.05	30.4	30.231 (-0.56)	30.875 (1.5)
11.7	2.00	0.20	29.5	29.331 (-0.5)	30.875 (4.4)

Table 4.9. Comparison of Characteristic Impedances Calculated by New Equations (Method f) with Numerical Methods (6).

EPSR	W/H	T/H	EFFECTIVE DILECTRIC CONS.			
			NUM MET. (25)	MET. C	MET. E	MET. F (WITH T)
3.78	0.1	0.01	2.52	3.09	2.95	2.54
"	"	0.10	2.36	3.65	3.86 *	2.37
"	"	0.20	-	3.70	4.42 *	2.18
"	1.0	0.01	2.77	2.52	2.94	2.77
"	"	0.10	2.70	3.09	3.23	2.72
"	"	0.20	-	3.32	3.41	2.66
"	2.0	0.01	2.92	2.46	3.05	2.91
"	"	0.10	2.87	2.85	3.26	2.87
"	"	0.20	-	3.09	3.38	2.83
9.60	0.1	0.01	5.67	7.45	6.72	5.77
"	"	0.10	5.11	9.21	8.88	5.24
"	"	0.20	-	9.40	10.18 *	4.65
"	1.0	0.01	6.42	5.69	6.91	6.47
"	"	0.10	6.19	7.45	7.53	6.31
"	"	0.20	-	8.17	8.01	6.12
"	2.0	0.01	6.89	5.51	7.28	6.91
"	"	0.10	6.73	6.73	7.76	6.79
"	"	0.20	-	7.45	8.05	6.66

Table 4.10. Comparison of Effective Dielectric Constant, ε_{eff}, for Microstrip Including Strip Thickness Calculated by Different Methods

$$Z_o = \frac{\zeta}{\sqrt{\varepsilon_{eff}}} \left[\frac{w}{h} + 1.393 + 0.667 \ln\left(\frac{w}{h} + 1.444\right)\right]^{-1}$$

$$- \left[1 - \tanh \left(1 + \frac{0.48\sqrt{\frac{w}{h} - 1}}{\left(1 + \frac{h'}{h}\right)^2} \right)\right] \cdot P \qquad (4.49)$$

where

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2} F\left(\frac{w}{h}\right)\right) \tanh \left[0.18 + 0.235 \frac{h'}{h} - \frac{0.415}{\left(\frac{h'}{h}\right)^2} \right] \qquad (4.50)$$

and

$$P = 270 \left[1 - \tanh(0.28 + 1.2 \sqrt{\frac{h'}{h}}) \right] \qquad (4.51)$$

The factor $F(\frac{w}{h})$ has been defined in the equations (4.35) and (4.36) ; h' is the spacing between the strip and the top cover and $\frac{h'}{h} \geqslant 1$.

4.5.3. EFFECT OF DISPERSION

The effect of frequency (dispersion) on ε_{eff} is described accurately by the dispersion model given by Getsinger (9) and modified by Edwards and Owens (27), as discussed in Section 3.2. The effect of frequency on the characteristic impedance of the microstrip has been described by several investigators (28, 29, 30, 31). Among these, Bianco et. al. (30) has approximate results closer to numerical values.

The results of Bianco et. al. for $Z_o(f)$, and of Edwards and Owen for $\varepsilon_{eff}(f)$ may be stated as follows :

$$Z_o(f) = Z_T - \frac{Z_T - Z_o}{1+G\left(\frac{f}{f_p}\right)^2} \tag{4.52}$$

$$\varepsilon_{eff}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff}}{1+G\left(\frac{f}{f_p}\right)^2} \tag{4.53}$$

where

$$G = \left(\frac{Z_o - 5}{60}\right)^{\frac{1}{2}} + 0.004\, Z_o \tag{4.54}$$

$$f_p(GHZ) = \frac{6.16\text{x}10^3 \cdot Z_o}{h}$$

Here h is in cm, Z_o is in ohms and Z_T is twice the characteristic impedance of a stripline of width w and heihgt 2h. Z_o and ε_{eff} are quasi-static values obtained earlier.

4.5.4 LOSSES

A closed form expression for total loss can be derived using Equations (4.43-4.44) as follows :

$$\alpha_T = \alpha_c + \alpha_d \tag{4.56}$$

The two components α_c and α_d are given by for $\frac{w}{h} \leq 1$

$$\alpha_c = 1.38\ A\ \frac{R_s}{hZ_o}\ \frac{32 - (w_{eff}h)^2}{32 + \left(\frac{w_{eff}}{h}\right)^2}\ \text{dB/cm} \tag{4.57}$$

For $\frac{w}{h} \geq 1$

$$\alpha_c = 6.1\text{x}10^{-5}A \cdot \frac{R_s Z_o \varepsilon_{eff}}{h}\left(\frac{w_{eff}}{h} + \frac{0.667\ \frac{w_{eff}}{h}}{\frac{w_{eff}}{h} + 1.444}\right)\ \text{dB/cm} \tag{4.58}$$

and

$$\alpha_d = \begin{cases} 4.34\zeta\sigma\ \dfrac{\varepsilon_{eff}-1}{\sqrt{\varepsilon_{eff}}(\varepsilon_r-1)}\ \text{dB/cm}\left(\dfrac{w}{h} \leq 1\right) \\ 27.3\ \dfrac{\varepsilon_r}{\varepsilon_r-1}\ \dfrac{\varepsilon_{eff}-1}{\sqrt{\varepsilon_{eff}}}\ \dfrac{\tan\delta}{\lambda_o}\ \text{dB/cm}\left(\dfrac{w}{h} \geq 1\right) \end{cases} \tag{4.59}$$

where

$$A = 1 + \frac{h}{w_{eff}} \left(1 + \frac{1.25}{\Pi} \ln \frac{2B}{t} \right)$$

$$R_s = \frac{\Pi f \mu_o}{\rho_c} \; ; \; \rho_c = \text{resistivity of the strip conductor}$$

$$\sigma = \omega \varepsilon_o \varepsilon_r \tan \delta = \text{conductivity of the dielectric substrate}$$

and

$$B = \begin{cases} h & \left(\frac{w}{h} \geqslant \frac{1}{2\Pi} \right) \\ 2\,\Pi\, w & \left(\frac{w}{2} \leqslant \frac{1}{2\Pi} \right) \end{cases}$$

The dielectric loss is normally very small compared with the conductor loss for dielectric substrates. The dielectric loss in silicon substrate (used for monolithic MICs), however, is usually of the same order, or even larger than the conductor loss, this is because of lower resistivity available in silicon wafers. However, higher resistivity can be obtained in GaAs, and therefore the dielectric loss is lower for this material.

Values of conductor and dilectric losses per unit length for 50 ohm microstrip lines on various substrates (dielectric as well as semiconductor) are plotted in Figure 4.4 as functions of frequency. At a given frequency the total loss can be obtained by adding the two values.

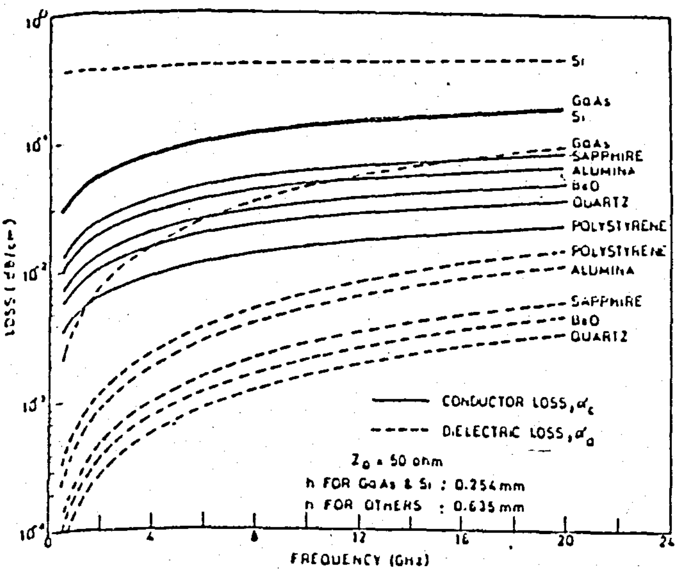


Figure 4.4. Conductor and Dielectric Losses as Functions of Frequency for Microstrip Lines on Various Substrates.

4.5.5. QUALITY FACTOR

The quality factor, Q, of a microstrip can be related to the total loss in the line by (32)

$$Q_T = \frac{\beta}{2\,\alpha_T} \tag{4.60}$$

where Q_T is the equivalent Q of the resonator (quarter wavelength), α_T is the total loss in the resonator, and $\beta = 2\Pi/\lambda_m$. When losses in a resonant line are considered, another loss factor, α_R, due to radiation at the open end discontinuities must also be taken into account. The corresponding radiation Q-factor is given by (32)

$$Q_R = \frac{Z_o}{480\Pi.\left(\frac{h}{\lambda_o}\right)^2.R} \tag{4.61}$$

where

$$R = \frac{\varepsilon_{eff}(f)+1}{\varepsilon_{eff}(f)} - \frac{\left[\varepsilon_{eff}(f)-1\right]^2}{2\left[\varepsilon_{eff}(f)\right]^{3/2}} \ln\left[\frac{\sqrt{\varepsilon_{eff}(f)}+1}{\sqrt{\varepsilon_{eff}(f)}-1}\right] \tag{4.62}$$

in which $\varepsilon_{eff}(f)$ is given by the Eq. (4.53). The total Q of the resonator can be expressed by

$$\frac{1}{Q_T} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_R} \tag{4.63}$$

Here Q_c, Q_d and Q_R are the quality factors corresponding to conductor, dielectric and radiation losses, respectively. Finally, the circuit quality factor Q_o, is defined as

$$\frac{1}{Q_o} = \frac{1}{Q_c} + \frac{1}{Q_d} = \frac{\lambda_o(\alpha_c + \alpha_d)}{\Pi\sqrt{\varepsilon_{eff}(f)}} \tag{4.64}$$

The variation with frequency of Q_O, Q_R and Q_T for a quarter wave resonator on different substrates is shown in Figure 4.5.

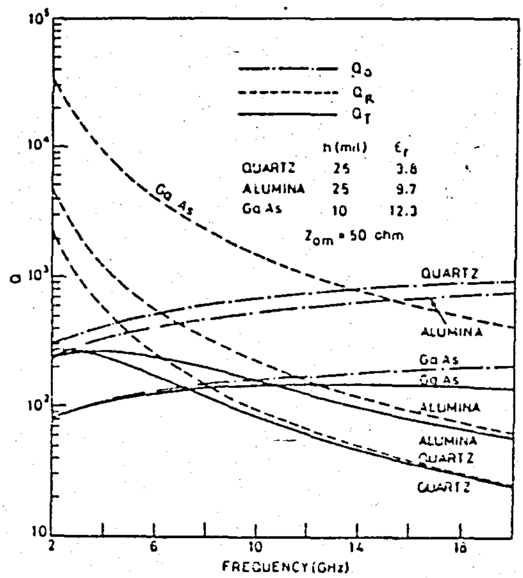


Figure 4.5. Variation of Q-factors with Frequency for Quarterwave Microstrip Resonators.

V. MICROSTRIP MEASUREMENTS

Just as in case of any other transmission structure the important characteristics that need be measured for a microstrip are :

(i) characteristic impedance
(ii) phase velocity and
(iii) attenuation constant

Also, in several cases, the dielectric constant of the substrate material is not known accurately. Since this information is required for microstrip circuit design, several methods have been devised for quick determination of the dielectric constant of metallized substrates.

In this chapter, the methods for the measurement of microstrip characteristics and for substrate dielectric constant are briefly outlined.

5.1. SUBSTRATE DIELECTRIC CONSTANT

In the normal course of fabricatingdielectric substrates for MICs, a dielectric filled resonator cavity is automatically constructed. (The substrate is metallized on each side, hence forming a tiny resonant cavity). The rectangular substrate with its top-and bottom-surface metallization (sides not metallized) becomes, in effect, a parallel plate dielectric-loaded-waveguide resonator. This structure is shown in Figure 5.1(a). For high dielectric constant substrates the fringing field is very small, and the sides of the resonator are good approximations to open circuits. The boundary conditions, therefore, consist of an open circuit at the side walls of the resonator.

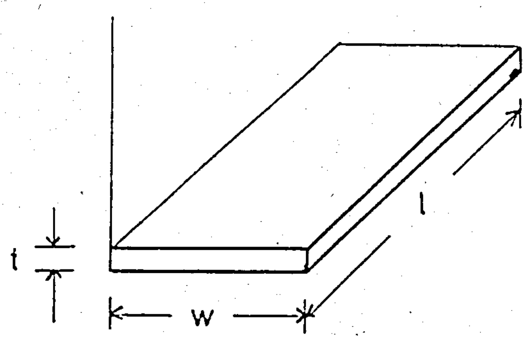


Figure 5.1(a) Metallized Substrate as a Parallel Plate Resonator

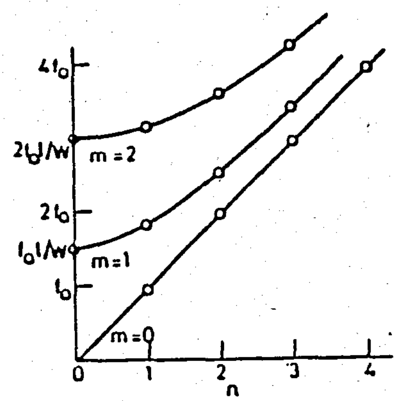


Figure 5.1(b) Resonance Frequencies for Various Modes of a Metallized Substrate Resonator (from (33)).

The resonance frequency f_{mn} for a wavequide with open-circuit side walls are given by

$$f_{mn}^2 = \frac{c^2}{\varepsilon_r}\left\{\left(\frac{m}{2w}\right)^2 + \left(\frac{n}{2\ell}\right)^2\right\} \tag{5.1}$$

where m and n represent the mode of resonance and correspond to the number of half cycle variations along w and ℓ, respectively. Values of f_{mn} are shown in Figure 5.1(b) as dots superimposed on the dispersion curves.

Excitation of the resonator at the substrate corner and detection of resonance are accomplished as shown in

Figure 5.2 (33). Transmission measurements may be carried out by using a network analyser or any other suitable system (sweep generator and crystal detector etc.). The modes are identified by referring to Figure 5.1(b).

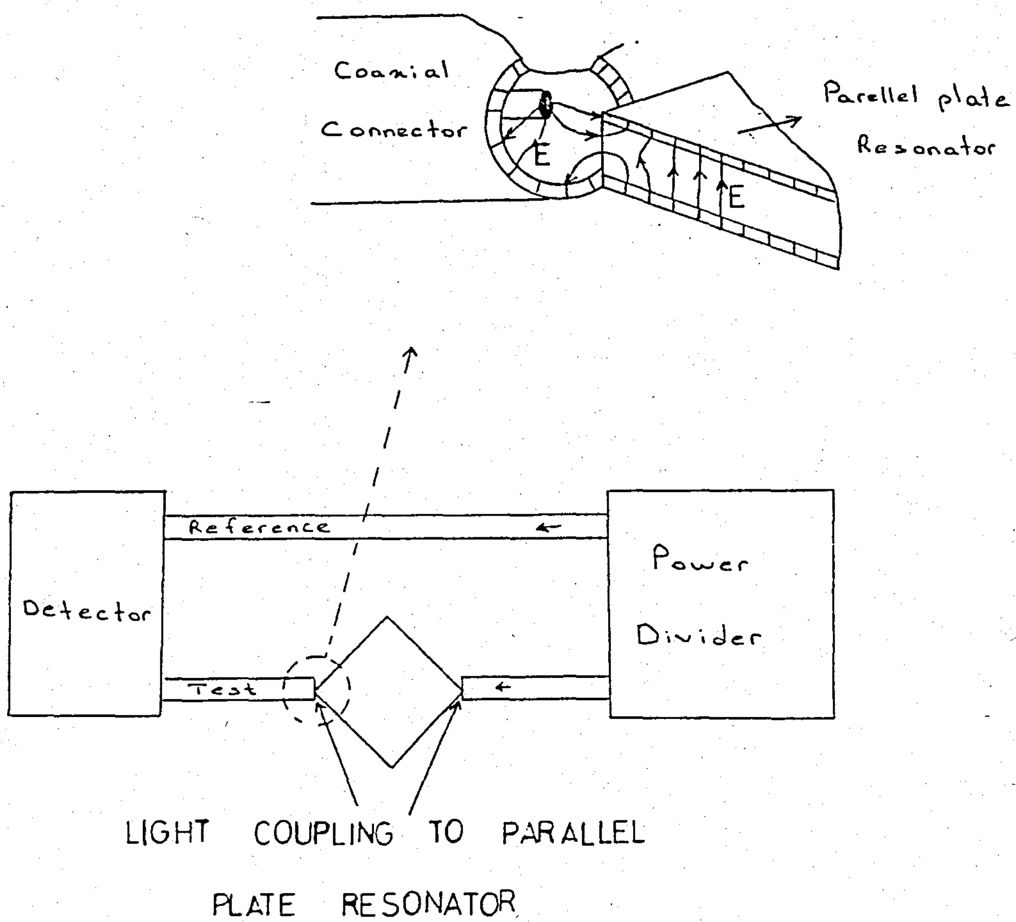


Figure 5.2. Test Configuration for Resonance Measurements.

The accuracy of this method is limited because of the fringing field and radiation due to the open sides of the resonator. A modification has been suggested (34) wherein the substrate is metallized on all the sides so that a small microwave cavity is formed. Equation (5.1) for various resonant frequencies is still valid. Resonant frequencies are measured by clearing metallization from the corners of the cavity and positioning the center conductor in coaxial connectors as shown in Figure 5.2.

5.2. CHARACTERISTIC IMPEDANCE

Impedance is a fundamental concept in microwave circuit design because the impedances of circuit elements and their interconnections determine the distribution of power within a circuit. The ability of a microwave engineer to predict circuit performance will partially depend on the accuracy of the knowledge of impedances for available circuit elements.

The characteristic impedance can be measured by terminating the microstrip with a precision coaxial load and measuring maximum and minimum VSWR as a function of frequency. With Z_{OR} as the reference line impedance (50 ohms) the characteristic microstrip line impedance Z_o is determined by

$$\left(\frac{Z_o}{Z_{OR}}\right)^{\pm 1} = \sqrt{VSWR_{max}\ VSWR_{min}} \qquad (5.2)$$

where the exponent +1 goes with high impedance lines, -1 with low impedance lines. Since the general behavior of Z_o/Z_{OR} as a function of geometry is known from Time Domain Reflectometry (TDR) measurements as well as from previous theoretical and experimental results, the question whether to use +1 or -1 never arises.

5.3. PHASE VELOCITY OR EFFECTIVE DIELECTRIC CONSTANT

Phase velocity or effective dielectric constant can be determined by using Reflection Cancellation Method (35) which is based upon the fact that a section of mismatched transmission line of length ℓ will appear to be a perfect match at frequencies such that $n\lambda/2 = \ell$. At other frequencies the line is mismatched. If the source frequency is adjusted the line will appear to be matched at frequencies such that $\ell = \lambda_1/2$, λ_2, $3\lambda_3/2$, etc., thus allowing λ_1, λ_2, λ_3, etc. to be obtained. The phase velocity v_p is given by

$$v_p = f\lambda = c/\sqrt{\varepsilon_{eff}(f)} \qquad (5.3)$$

where c is the velocity of light in free space. Thus, at the matching frequencies

$$\varepsilon_{eff}(f) = \left(\frac{c}{f\lambda}\right)^2 \quad (5.4)$$

which allows $\varepsilon_{eff}(f)$ to be found. When dispersion effects are negligible, Equation (5.4) gives the value of ε_{eff}. In the case of significant dispersion the value of $\varepsilon_{eff}(f)$ obtained at the lowest frequency is used as a first approximation to ε_{eff}.

In this method, The frequencies at which the reflection coefficient at the input to the mismatched section is zero are determined using a network analyzer and the length ℓ of the microstrip line is measured. This method is quite simple to apply and is not wasteful of substrate material. It can also determine the permittivity associated with a specific section of microstrip.

5.4. ATTENUATION CONSTANT

The most satisfactory method of measuring the attenuation constant for low-loss substrates is from the Q-factor of a resonant section of line. The attenuation constant and the unloaded quality factor Q_o are related by the following expression

$$\alpha = \frac{27.3}{Q_o \lambda_m} \quad (\text{dB/cm}) \quad (5.5)$$

An approximate but very simple method to measure the attenuation constant is based on the comparison technique. In this method two microstrip lines with identical electrical characteristics but different lengths are taken. Their insertion losses are measured. The difference between the two values of insertion loss is used for evaluating the attenuation constant. This procedure avoids the systematic errors caused by radiation, coaxial to microstrip transition, and so forth.

VI COMPUTER AIDED DESIGN OF MICROSTRIP LINES

6.1. PROGRAM FOR MICROSTRIP DESIGN

A complete set of design algorithm for microstrip is presented in this program. This includes closed form expressions for the characteristic impedance, Z_o, and effective dielectric constant, ε_{eff}, and their variation with metal strip thickness, enclosure size and dispersion. Expressions for microstrip loss and quality factor Q are also given.

Programming language is FORTRAN and the equations for Z_o and ε_{eff} given in this are correct to within 2 percent for the following sets of parameters : $0 \leq t/h \leq 0.350$, $0.1 \leq w/h \leq 10$ and $\varepsilon_r \leq 11.7$

6.2. COMPARISON BY A COMPUTER PROGRAM USING GRAPHS

A computer program ensuring to make a comparison of characteristic impedance or effective dielectric constant obtained from different approaches with each other for desired t/h and w/h ratios and for different ε_r's by using a special plot algorithm is given here.

```
**************************************
*                                    *
*    PROGRAM  FOR  MICROSTRIP  DESIGN      *
*                                    *
**************************************
PROGRAM MSTRP(GIRDI,VERI,TAPE5=GIRDI,TAPE6=VERI)
DATA EPSR,W,H,HE,T,FREQ,RESOSC,CODS/X1,X2,X3,X4,X5,X6,X7,X8
W       : STRIP CONDUCTOR WIDTH (IN CM)
H       : SUBSTRATE THICKNESS (IN CM)
HE      : SPACING BETWEEN STRIP AND TOP COVER
T       : STRIP CONDUCTOR THICKNESS (IN CM)
C       : VELOCITY OF LIGHT IN FREE SPACE
FREQ    : OPERATING FREQUENCY (IN HZ)
EPSR    : RELATIVE PERMITTIVITY
EPSO    : PERMITTIVITY FOR FREE SPACE
FSP     : FREE SPACE PERMEABILITY
RESOSC  : RESISTIVITY OF STRIP CONDUCTOR
CODS    : CONDUCTIVITY OF DIELECTRIC SUBTRAT
C=3E10
EPSO=8.854E-14
FSP=(4E-9)*3.1415
RATWH=W/H
RATWHE=0
RATTH=T/H
*****************************************************************
*                                                               *
*  CHARECTERISTIC IMPEDANCE AND EFFECTIVE DIELECTRIC            *
*                                                               *
*         CONSTANT FOR ZERO THICKNESS MICROSTRIP                *
*                                                               *
*****************************************************************
EPSEF : EFFECTIVE DIELEKTRIC COSTANT FOR METHOD F
ZOF   :CHARECTERISTIC IMPEDANCE FOR METHOD F
IF(RATWH.LE.1) EPSEF=(((1./SQRT(1+(12/RATWH)))+(0.04*((1-
&RATWH)**2)))*((EPSR-1)/2))+((EPSR+1)/2.)
IF(RATWH.GT.1) EPSEF=(((EPSR-1)/2.)*(1./SQRT(1+(12./
&RATWH))))+((EPSR+1)/2)
IF(RATWH.LE.1) ZOF=(ALOG((8./RATWH)+(0.25*RATWH)))*
&(60./SQRT(EPSEF))
IF(RATWH.GT.1) ZOF=(1./(RATWH+1.393+0.667*(ALOG(1.444+
&RATWH))))*(376.8/SQRT(EPSEF))
**************************************
*                                    *
*    EFFECT  OF  STRIP  THICKNESS    *
*                                    *
**************************************
EPSEFT : EFF. DIEL. CONST. INCLUDING STRIP THICKNESS
RATWHE : RATIO OF W TO HE (RATWHE=W/HE)
ZOST   : ZO FOR MICROSTRIP INCLUDING STRIP THICKNESS
A=1./(2*3(22/7))
IF(RATWH.LE.1) EPSEFT=((((1./SQRT(1+(12./RATWH)))+
&(0.04*((1-RATWH)**2)))*((EPSR-1)/2.))+((EPSR+1)/2.))
&-(((EPSR-1)/4.6)*(RATTH/SQRT(RATWH)))
IF(RATWH.GT.1) EPSEFT=((((EPSR-1)/2.)*(1./SQRT(1+
&(12./RATWH))))+((EPSR+1)/2.))-(((EPSR-1)/4.6)*(RATTH
&/SQRT(RATWH)))
IF(RATWH.LE.A) RATWHE=((1+(ALOG(12.564*(RATWH/RATH
&))))*(RATTH*0.39788))+RATWH
IF(RATWH.GT.A) RATWHE=RATWH+((1+ALOG(2./RATT))*(
&0.39788*RATTH))
IF(RATWH.LE.1) ZOST=(ALOG((8./RATWHE)+(0.25*RATWHE)))
&*(60/SQRT(EPSEFT)))
IF(RATWH.GT.1) ZOST=(1./(RATWH)+1.393+(0.667*(ALOG
&(1.444+RATWHE)))))*(376.8/SQRT(EPSEFT))
```

```
 ********************************
 *                              *
 *     EFFECT  OF  ENCLOSURE    *
 *                              *
 ********************************
 EPSEFF : EFF. DILEC. CONSTANT FOR AN ENCLOSED MICROSTRIP
 RATHEH : RATIO OF HE TO H $(HE/H)>10
 ZOE    : CHARACTERISTIC IMP. FOR AN ENCLOSED MICROSTRIP
 RATHEH=HE/H
 IF(RATWH.LE.1) EPSEFE=((((1/SQRT(1+(12/RATWH)))+(0.04*((1-
&RATWH)**2)))*((EPSR-1)/2))TANH(0.18+(0.235*RATHEH)-(0.415/
&((RATHEH)**2))))))+((EPSR+)/2.)
 IF(RWH.GT.1) EPSEFE=((1/SQRT(1+(12/RATWH)))*(TANH(0.13+(
&0.235*RATHEH)-(0.415/((RATHEH)**2))))))+((EPSR+1)/2)
 IF(RATWH.LE.1) ZOE=(((8/RATWH)+(0.25*RATWH))*(60/SQRT(EPSEFF
&)))-270*1-(TANH(0.28+(1.2*SQRT(RATHEH)))))
 IF(RATWH.GT.1) ZOE=((1/(RATWH+1.393+(0.667*(ALOG(RATWH+1.444
&))))))*(376.8/SQRT(EPSEFF)))-(270*(1-(TANH(0.28+(1.2*SQRT(
 RATHEH))))))*(1-(TANH(1+((0.48*(SQRT(RATWH-1)))/((1+RATHEH)
&**2))))))
 ****************************************
 *                                      *
 *     EFFECT  OF  DISPERSION (FREQUENCY)    *
 *                                      *
 ****************************************
 ZOD    : EFF. DILEC. CNSTANT INCL. EFFECT OF DISPERSION
 ZOD    : CHARECTERISTIC IMP. INCL.EFFECT OF DISPERSION
 ZO     : TWICE THE CHARECTERISTIC IMP. OF A STRIPLINE OF
          WIDTH W AND HEIGHT 2H
 FREK=FREQ/1E9
 RATWHD=(W/(2*H))
 IF(RATWH.LE.1) ZOT=2*((ALOG((8./RATWH)+(0.25*RATWH)))*(60.
&/SQRT(EPSEF)))
 IF(RATWH.GT.1) ZOT=2*(1./(RATWH+1.393+0.667*(ALOG(1.444+
RATWH))))*(376.8/SQRT(EPSEF)))
 EPSEFD=EPSR-((EPSR-EPSEF)/(((((FREK/((15.66*ZOF)/H))**2)*(
&SQRT((ZOF-5)/60)+(0.004*ZOF)))+1))
 ZOD=ZOT-((ZOT-ZOF)/((((FRK/((15.66*ZOF)/H))**2)*(SQRT((
&ZOF-5)/60+(0.004*ZF)))+1))
 ********************************
 *                              *
 *        L  O  S  S  E  S      *
 *                              *
 ********************************
 CLOSS : CONDUCTOR LOSS
 DLOSS : DILECTRIC LOSS
 TLOSS : TOTAL LOSS
 IF(RATWH.GT.A) EQU=1+((1/RATWHE)*(1+(0.398*(ALOG(?/RATTH)))))
 IF(RATWH.LE4) EQU=1+((1/RATWHE)*(1+(0.398*(ALOG(12.56*(RATWH
&/RATTH))))))
 IF(RATWH.LE.1) CLOSS=(1.38*EQU)*(SQRT((3.14*FREQ*FSP)/RESOSC)
&/(H*ZOST))*((32-((RATWHE)**2))/(32+((RATWHE)**2)))
 IF(RATWH.GT.1) CLOSS=(RATWHE+((0.667*RATWHE)/(RATWHE+1.444)))*
&(0.00061*EQU)*(((SQRT((3.14*FREQ*FSP)/RESOSC))*ZOST*EPSEFT)/H)
 IF(RATWH.LE.1) DLOSS=1636.14*CODS*((ESEFT-1)/(SQRT(EPSEFT)*(
&EPSR-1)))
 IF(RATWH.GT.1) DLOSS=27.3*(EPSR/(EPSR-1))*((EPSEFT-1)/SQRT
&(EPSEFT))*((CCDC/(6.28*FREQ*EPSO*EPSR))/(C*FREQ))
 TLOSS=CLOSS+DLOSS
```

```
******************************************
*                                        *
*      Q U A L I T Y   F A C T O R       *
*                                        *
******************************************
QT : EQUIVALENT Q OF THE RESONATOR
QC : QUALITY FACTOR CORRESPONDIG TO CONDUCTOR LOSS
QD : QUALITY FACTOR CORRESPONDING TO DIELECTRIC LOSS
QR : QUALITY FACTOR CORRESPONDING TO RADIATION LOSS
QO : CIRCUIT QUALITY FACTOR
QT=(6.28*FREQ*(SQRT(EPSEFFT)))/C
RAD=((EPSEFD+1)/EPSEFD)-((((EPSEFD-1)**2)/(2*SQRT(EPSEFD**3)))
$*(ALOG((SQRT(EPSEFD)+1)/(SQRT(EPSEFD)-1))))
QR=(Z0F/(480*3.14*((H/(C*FREQ))**2)*RAD)
QO=(3.14*SQRT(EPSEFD))/((C*FREQ)*(CLOSS+DLOSS))
STOP
END
```

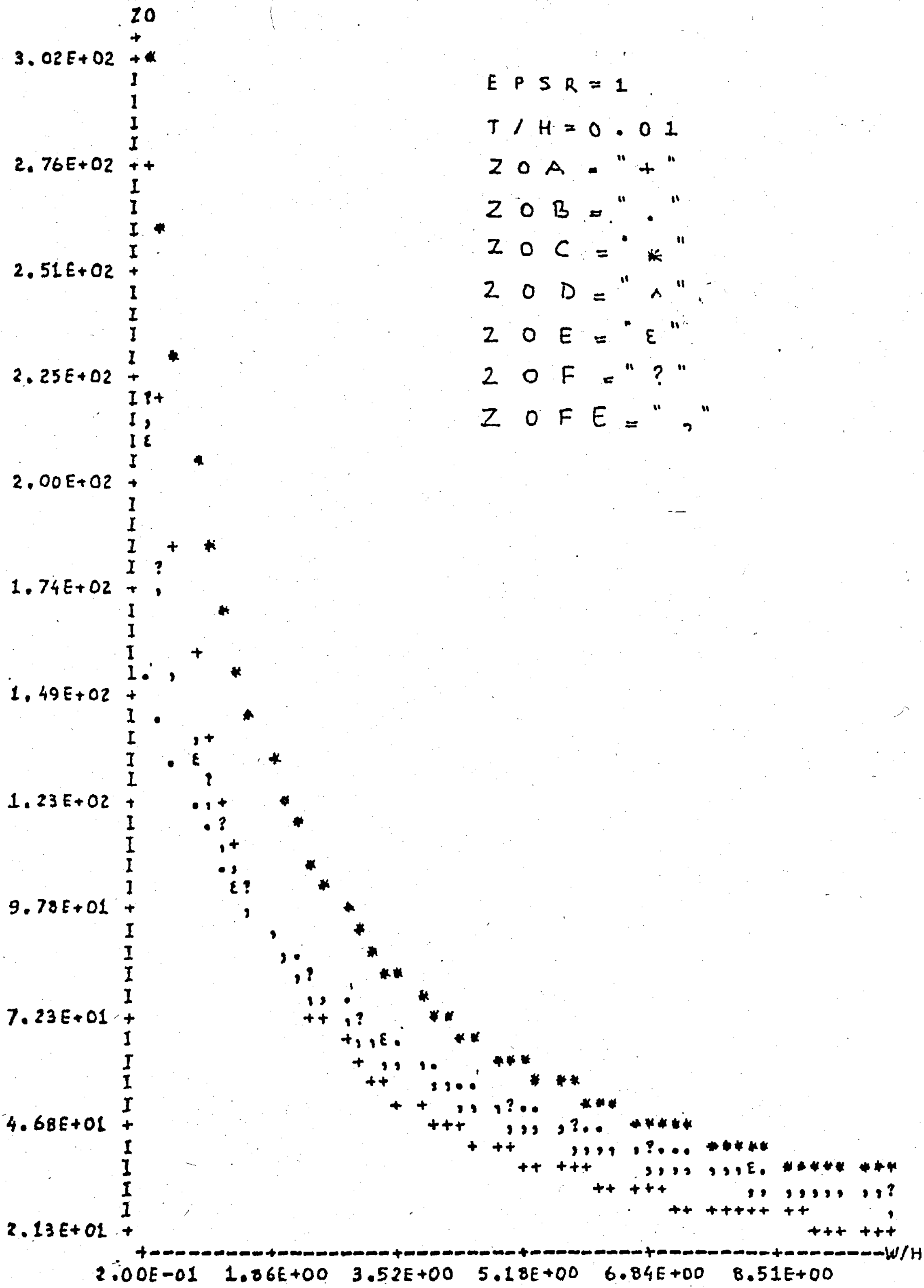
```
C***********************************************************************
C*                                                                     *
C*    P R O G R A M   F O R   M A K I N G   C O M P A R I S O N       *
C*                                                                     *
C*                  B Y   U S I N G   G R A P H S                      *
C*                                                                     *
C***********************************************************************
      PROGRAM FS1(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION ZOA(60),ZOB(60),ZOC(60),ZOD(60),ZOE(60),ZOF(60)
      DIMENSION ZOFE(60),IN(7),X(7,60),Y(7,60)
C     ZOA  :CHARACTERISTIC IMP. FOR METHOD A
C     ZOB  :CHARACTERISTIC IMP. FOR METHOD B
C     ZOC  :CHARACTERISTIC IMP. FOR METHOD C
C     ZOD  :CHARACTERISTIC IMP. FOR METHOD D
C     ZOE  :CHARACTERISTIC IMP. FOR METHOD E
C     ZOF  :CHARACTERISTIC IMP. FOR METHOD F
C     ZOFE :CHA. IMP. FOR MET. F INCLUDING THICKNESS
      EPSR=11.7
      RATWHE=0
      RATWH=0
      RATTH=0.01
      DO 20 I=1,50
      RATWH=RATWH+0.2
      EPSEFA=EPSR
      ZOA(I)=(376.8/SQRT(EPSR))*(1./(RATWH+((0.606/
     $SQRT(EPSR))*RATWH)+(2.7/ALOG10(4./RATTH))))
      EPSEFB=(0.4475*EPSR)+0.6965
      A1=(0.0315*(1/RATTH))+2.183
      A2=(-0.0695*(1/(EPSEFB*RATWH)))+1.2165
      ZOB(I)=(376.8/SQRT(EPSEFB))*(1./(RATWH+(A1/(ALOG10(4/
     $RATTH)))+((A2*RATWH)/SQRT(EPSEFB))))
      EPSEFC=((1./((RATWH/(10.*RATTH))+1.))*
     $((EPSR-1)/2.))+((EPSR+1.)/2.)
      ZOC(I)=(376.8/SQRT(EPSEFC))*(1./(RATWH+(0.282/(ALOG
     $(4./RATTH)))))
      EPSEFD=0.5*((EPSR+1)+((EPSR-1)*(1./SQRT(1+(10./RATWH)))))
      IF(RATWH.LE.1) ZOD(I)=ALOG((8./RATWH)+(RATWH/4.))*
     $(60/(SQRT(EPSEFD)))
      IF(RATWH.GT.1) ZOD(I)=376.8/((((1-(1/RATWH))**6)-(0.44/
     $RATWH)+2.42+RATWH)*(SQRT(EPSEFD)))
      IF(RATWH.LE.2) EPSEFE=((EPSR+1)/2)+(((EPSR-1)/2)*(1./
     $SQRT(1+(10/RATWH))))+(0.466*((EPSR+0.5)/1.5)*(SQRT
     $(RATTH/RATWH)))
      IF(RATWH.GT.2) EPSEFE=((EPSR+1)/2.)+((EPSR-1)/2.)*(1./
     $(SQRT(1+(10./RATWH))))
      IF(RATWH.LE.1) ZOE(I)=ALOG((8./RATWH)+(RATWH/
     $4.))*(60./(SQRT(EPSEFE)))
      IF(RATWH.GT.1) ZOE(I)=376.8/((((1-(1./RATWH))**6)
     $-(0.44/RATWH)+2.42+RATWH)*(SQRT(EPSEFE)))
      IF(RATWH.LE.1) EPSEFF=(((1./SQRT(1+(12/RATWH)))+
     $(0.04*((1-RATWH)**2)))*((EPSR-1)/2))+((EPSR+1)/2.)
      IF(RATWH.GT.1) EPSEFF=(((EPSR-1)/2.)*(1./SQRT(1+(12./
     $RATWH))))+((EPSR+1)/2)
      IF(RATWH.LE.1) ZOF(I)=(ALOG((8./RATWH)+(0.25*RATWH
     $)))*(60./SQRT(EPSEFF))
```

```
      IF(RATWH.GT.1) ZOF(I)=(1./(RATWH+1.393+0.667*(ALOG(
     &1.444+RATWH))))*(376.8/SQRT(EPSEFF))
      A=0.15915494
      IF(RATWH.LE.1) EPSLFG=((((1./SQRT(1+(12./RATWH)))+
     &(0.04*((1-RATWH)**2)))*((EPSR-1)/2.))+((EPSR+1)/2.))-(((
     &EPSR-1)/4.6)*(RATTH/SQRT(RATWH)))
      IF(RATWH.GT.1) EPSLFG=((((EPSR-1)/2.)*(1./SQRT(1+
     &(12./RATWH))))+((EPSR+1)/2.))-(((EPSR-1)/4.6)*(RATTH
     &/SQRT(RATWH)))
      IF(RATWH.LE.A) RATWHE=((1+(ALOG(12.564*(RATWH/
     &RATTH))))*(RATTH*0.39788))+RATWH
      IF(RATWH.GT.A) RATWHE=RATWH+((1+ALOG(2./RATTH
     &))*((0.39788*RATTH))
      IF(RATWH.LE.1) ZOFE(I)=(ALOG((8./RATWHE)+(0.25*
     &RATWHE)))*(60/SQRT(EPSEFG))
      IF(RATWH.GT.1) ZOFE(I)=(1./(RATWHE+1.393+(0.667*
     &(ALOG(1.444+RATWHE)))))*(376.8/SQRT(EPSEFG))
      X(1,I)=RATWH
      X(2,I)=RATWH
      X(3,I)=RATWH
      X(4,I)=RATWH
      X(5,I)=RATWH
      X(6,I)=RATWH
      X(7,I)=RATWH
      Y(1,I)=ZOA(I)
      Y(2,I)=ZOB(I)
      Y(3,I)=ZOC(I)
      Y(4,I)=ZOD(I)
      Y(5,I)=ZOE(I)
      Y(6,I)=ZOF(I)
      Y(7,I)=ZOFE(I)
20    CONTINUE
      IN(1)=50
      IN(2)=50
      IN(3)=50
      IN(4)=50
      IN(5)=50
      IN(6)=50
      IN(7)=50
      CALL GRAFIK (60,56,X,Y,IN,0,7)
      WRITE(6,109)
109   FORMAT(////,15X,'ZO VS. W/H',//)
      STOP
      END
      SUBROUTINE GRAFIK(IX,IY,X,Y,IN,NL,IDIM)
*************************************************************
*    IX    :LENGTH OF X-AXIS
*    IY    :LENGTH OF Y-AXIS
*    X     :INDEPENDENT VARIABLE
*    Y     :DEPENDENT VARIABLE
*    IN    :NUMBER OF POINTS
*    NL    :LOGARITHMIC INDICATOR
*    IDIM  :NUMBER OF GRAPHS
**************************************************************
      DIMENSION X(7,60),Y(7,60),U(111),V(56),D(12),E(12),IN(7)
      CHARACTER *1 A(56,111),F(111)*1
      REAL MIN,MIN1,MAX,MAX1,INCX,INCY
      DO 5 I=1,56
      DO 5 J=1,111
      A(I,J)=' '
```

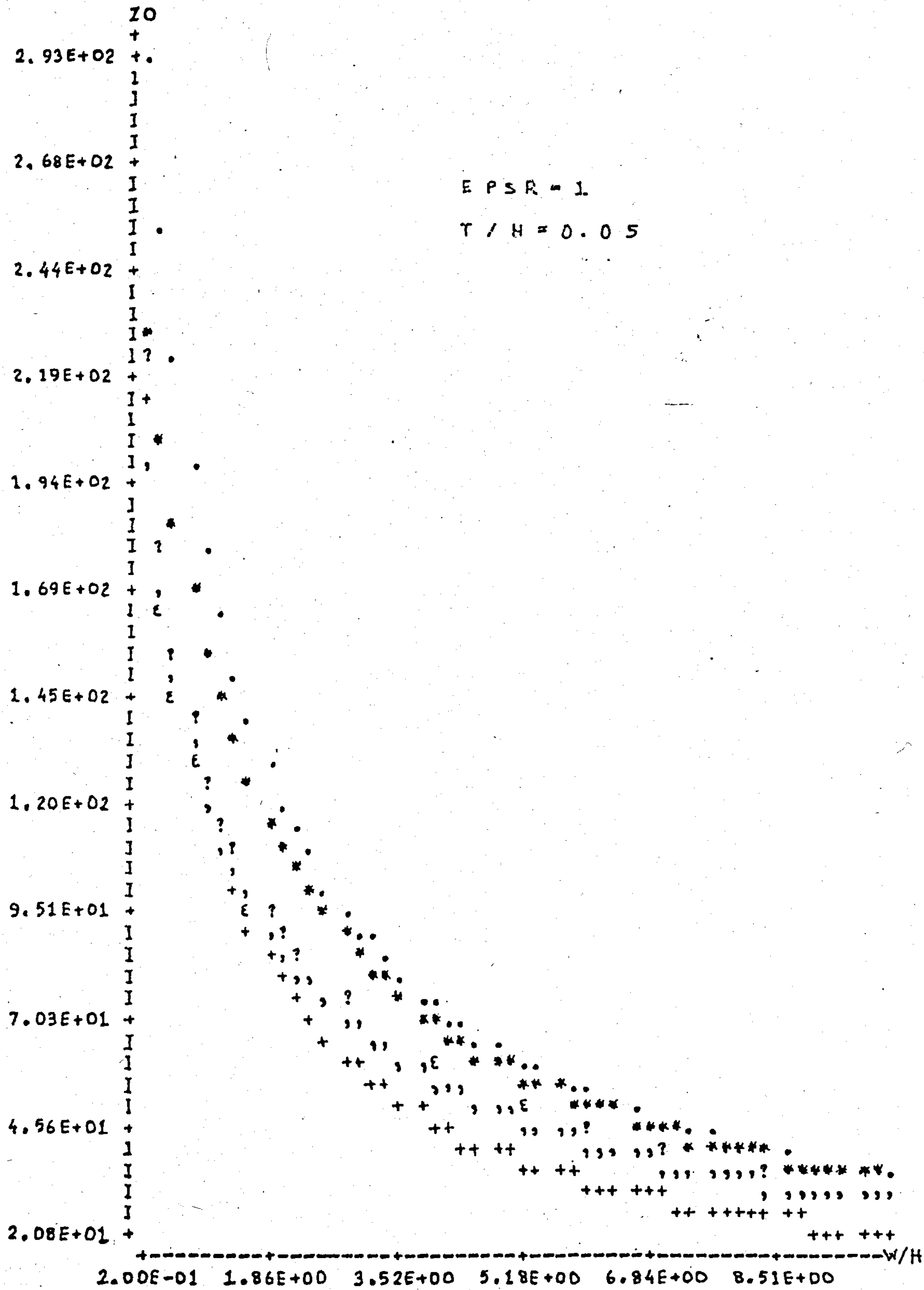
```
5   CONTINUE
    IF(IX.GT.111.OR.IX.LT.2) IX=111
    IF(IY.GT.50.OR.IY.LT.2) IY=50
    IF(NL.EQ.1.OR.NL.EQ.2) THEN
    DO 10 M=1,IDIM
    DO 10 I=1,IN(M)
    IF(X(M,I).NE.0) THEN
    X(M,I)=ALOG10(X(M,I))
    ENDIF
10  CONTINUE
    END IF
    IF(NL.EQ.1.OR.NL.EQ.3) THEN
    DO 15 M=1,IDIM
    DO 15 I=1,IN(M)
    IF(Y(M,I).NE.0) THEN
    Y(M,I)=ALOG10(Y(M,I))
    ENDIF
15  CONTINUE
    END IF
    DO 20 I=1,IX
    F(I)='-'
20  CONTINUE
    DO 25 I=1,IX,10
    F(I)='+'
25  CONTINUE
    MIN=X(1,1)
    MAX=X(1,1)
    MIN1=Y(1,1)
    MAX1=Y(1,1)
    DO 35 M=1,IDIM
    DO 30 I=1,IN(M)
    IF(X(M,I).GT.MAX) MAX=X(M,I)
    IF(X(M,I).LT.MIN) MIN=X(M,I)
    IF(Y(M,I).GT.MAX1) MAX1=Y(M,I)
    IF(Y(M,I).LT.MIN1) MIN1=Y(M,I)
30  CONTINUE
35  CONTINUE
    INCX=(MAX-MIN)/(IX-1)
    INCY=(MAX1-MIN1)/(IY-1)
    U(1)=MIN
    V(1)=MIN1
    DO 40 I=1,IX-1
    U(I+1)=U(I)+INCX
40  CONTINUE
    DO 45 I=1,IY-1
    V(I+1)=V(I)+INCY
45  CONTINUE
    DO 65 M=1,IDIM
    DO 60 I=1,IN(M)
    M1=1
    DIFF=ABS(X(M,I)-U(1))
    DO 50 J=1,IX-1
    IF(ABS(X(M,I)-U(J+1)).LT.DIFF) THEN
    DIFF=ABS(X(M,I)-U(J+1)).
    M1=J+1
    END IF
50  CONTINUE
    L=1
```

```
      DIFF1=ABS(Y(M,I)-V(1))
      DO 55 J=1,IY-1
      IF(ABS(Y(M,I)-V(J+1)).LT.DIFF1) THEN
      DIFF1=ABS(Y(M,I)-V(J+1))
      L=J+1
      END IF
   55 CONTINUE
      IF(M.EQ.1) A(L,M1)='+'
      IF(M.EQ.2) A(L,M1)='.'
      IF(M.EQ.3) A(L,M1)='*'
      IF(M.EQ.4) A(L,M1)='L'
      IF(M.EQ.5) A(L,M1)='&'
      IF(M.EQ.6) A(L,M1)='?'
      IF(M.EQ.7) A(L,M1)=','
   60 CONTINUE
   65 CONTINUE
      L2=0
      L3=0
      DO 70 I=1,IY,5
      L2=L2+1
      IF(NL.EQ.1.OR.NL.EQ.3) D(L2)=10**V(I)
      IF(NL.NE.1.AND.NL.NE.3) D(L2)=V(I)
   70 CONTINUE
      DO 75 I=1,IX,10
      L3=L3+1
      IF(NL.EQ.1.OR.NL.EQ.2) E(L3)=10**U(I)
      IF(NL.NE.1.AND.NL.NE.2) E(L3)=U(I)
   75 CONTINUE
      IF(NL.EQ.1.OR.NL.EQ.2) THEN
      DO 80 M=1,IDIM
      DO 80 I=1,IN(M)
      X(M,I)=10**X(M,I)
   80 CONTINUE
      END IF
      IF(NL.EQ.1.OR.NL.EQ.3) THEN
      DO 85 M=1,IDIM
      DO 85 I=1,IN(M)
      Y(M,I)=10**Y(M,I)
   85 CONTINUE
      END IF
      K7=((IY-1)/5)*5+1
      IP=(IX-1)/10+1
      PRINT 95
      DO 90 I=IY,1,-1
      IF(I.EQ.K7) THEN
      K7=K7-5
      L=I/5+1
      PRINT 100, D(L),(A(I,J),J=1,IX)
      GO TO 90
      END IF
      PRINT 105,(A(I,J),J=1,IX)
   90 CONTINUE
      PRINT 110,(F(I),I=1,IX)
      PRINT 115,(E(I),I=1,IP)
      RETURN
   95 FORMAT(1H1,/)
  100 FORMAT(3X,1PE10.2,1X,'+',111A1)
  105 FORMAT(14X,'I',111A1)
  110 FORMAT(15X,111A1,/)
  115 FORMAT(11X,11(1PE9.2,1X),1PE9.2,/)
      END
```

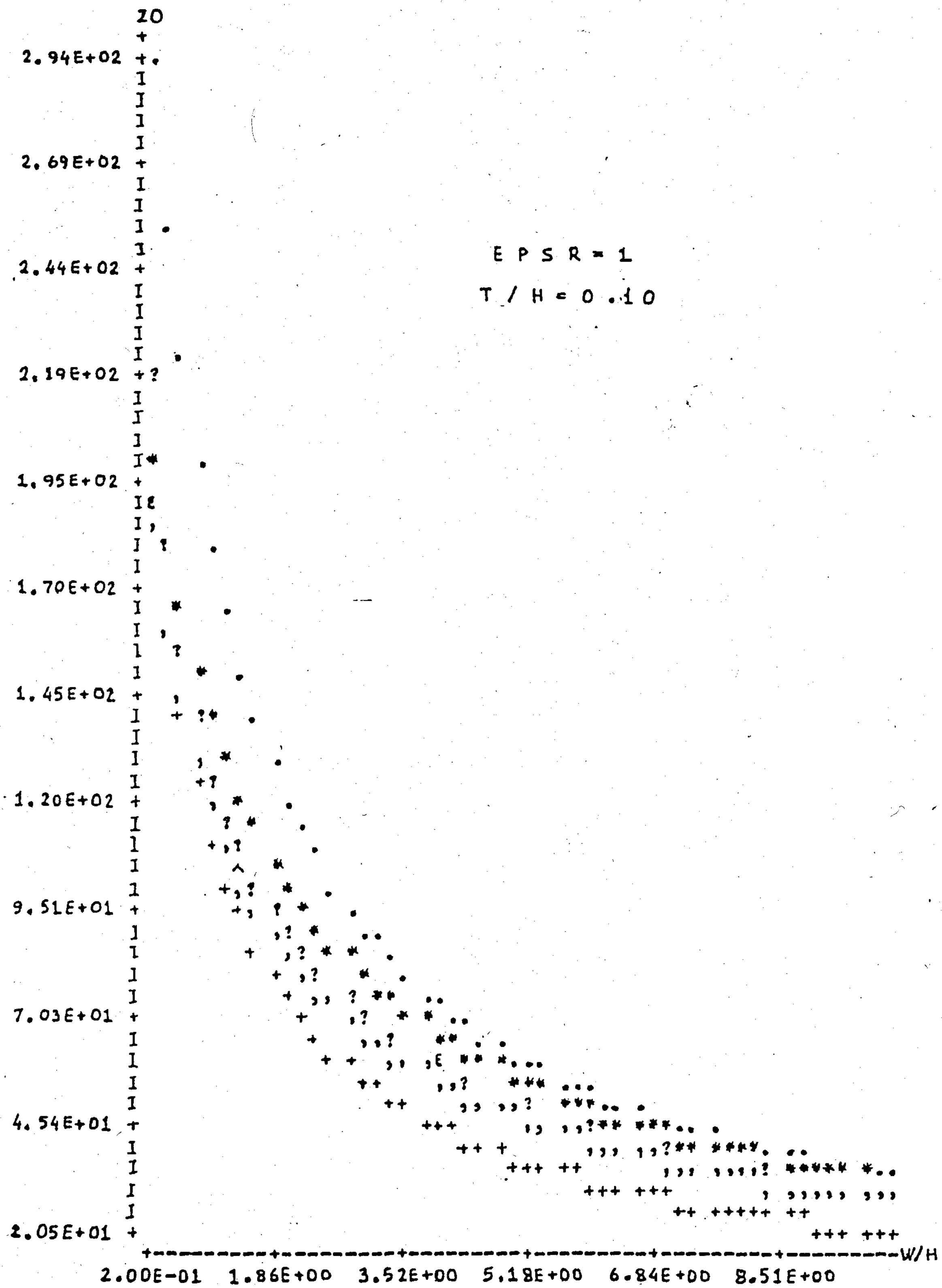
OUTPUTS FOR 6.2



ZO vs. W/H

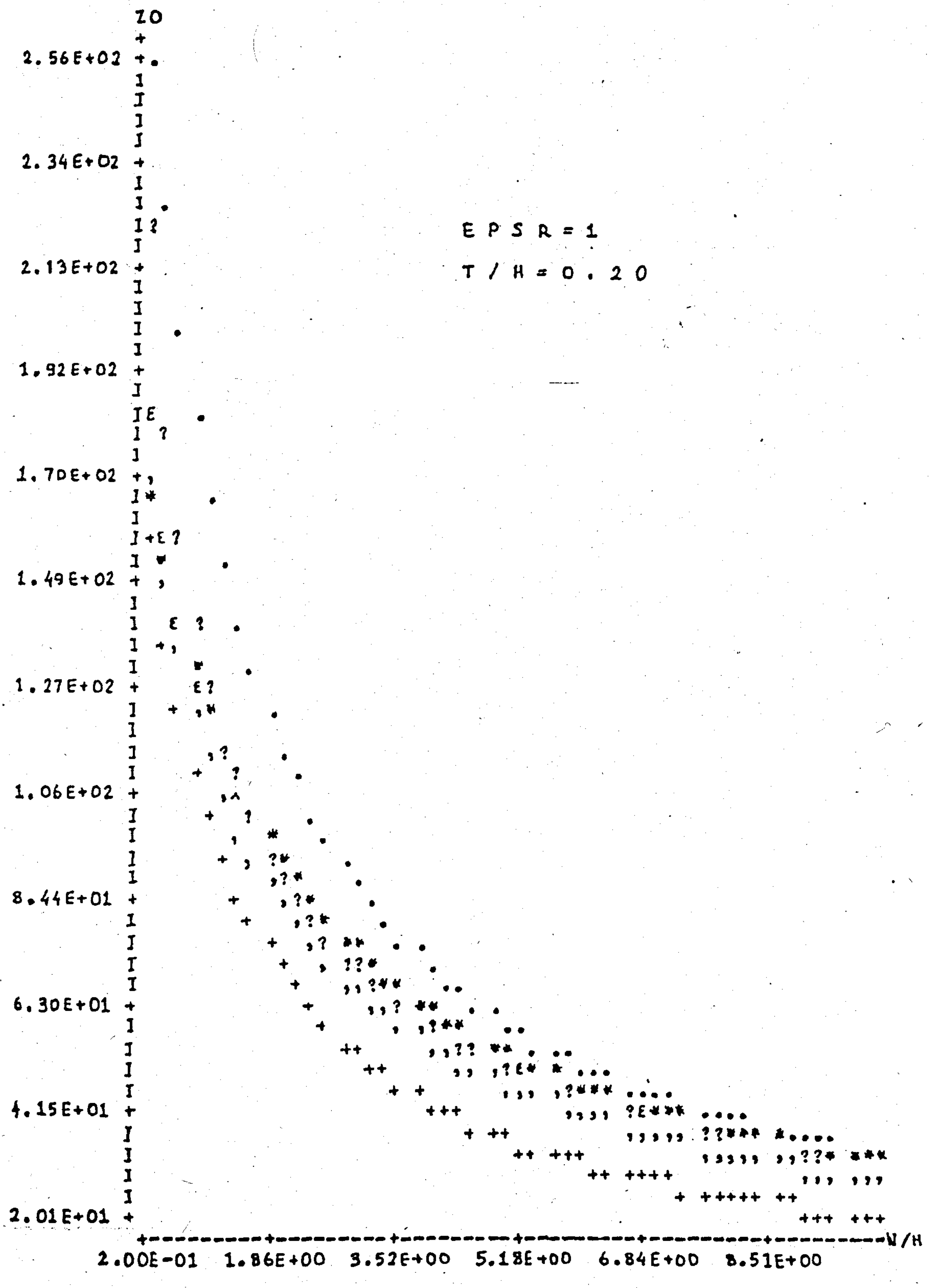

ZO
2.93E+02
2.68E+02
2.44E+02
2.19E+02
1.94E+02
1.69E+02
1.45E+02
1.20E+02
9.51E+01
7.03E+01
4.56E+01
2.08E+01
EPSR = 1
T / H = 0.05
2.00E-01
1.86E+00
3.52E+00
5.18E+00
6.84E+00
8.51E+00
W/H

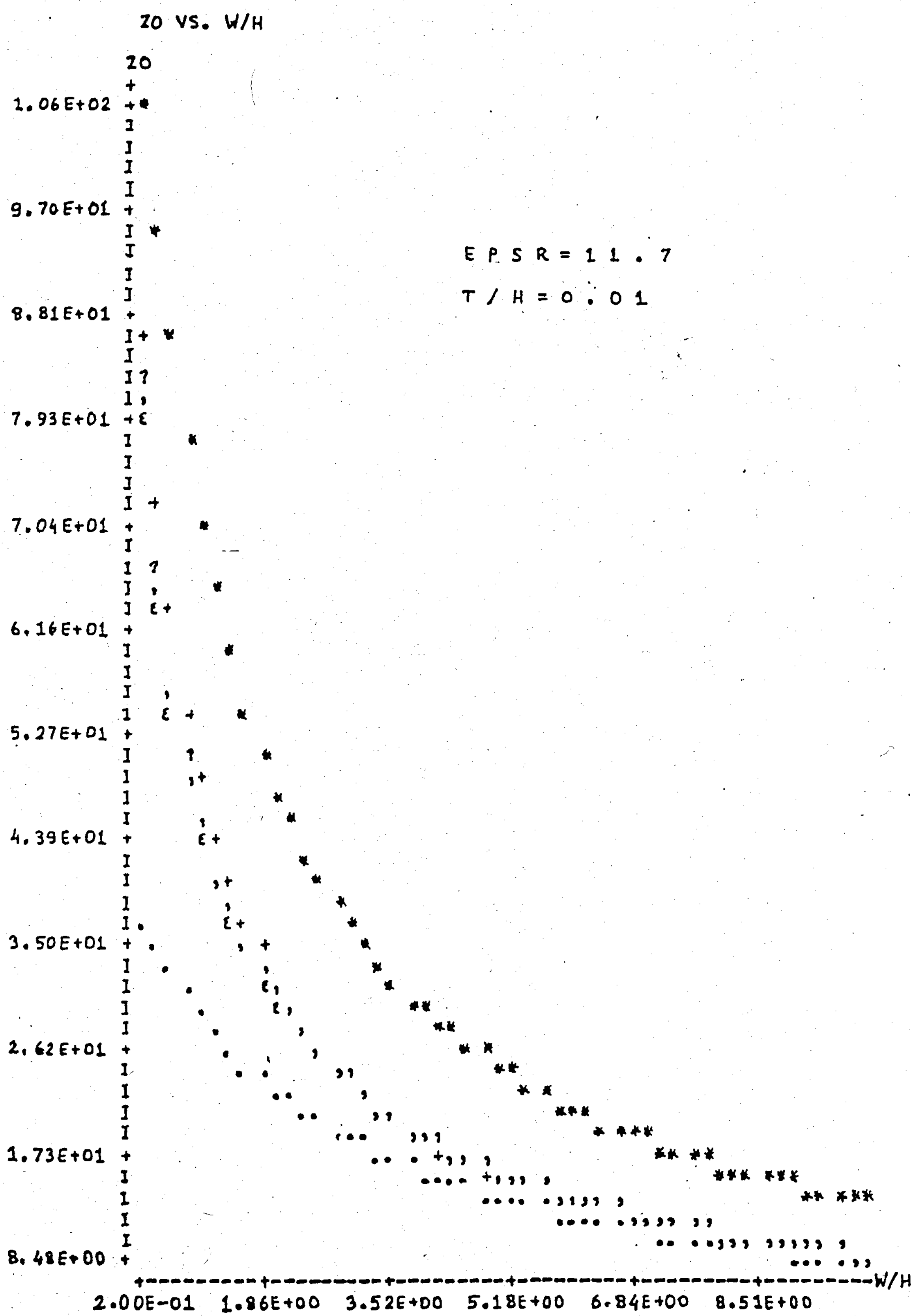
ZO VS. W/H

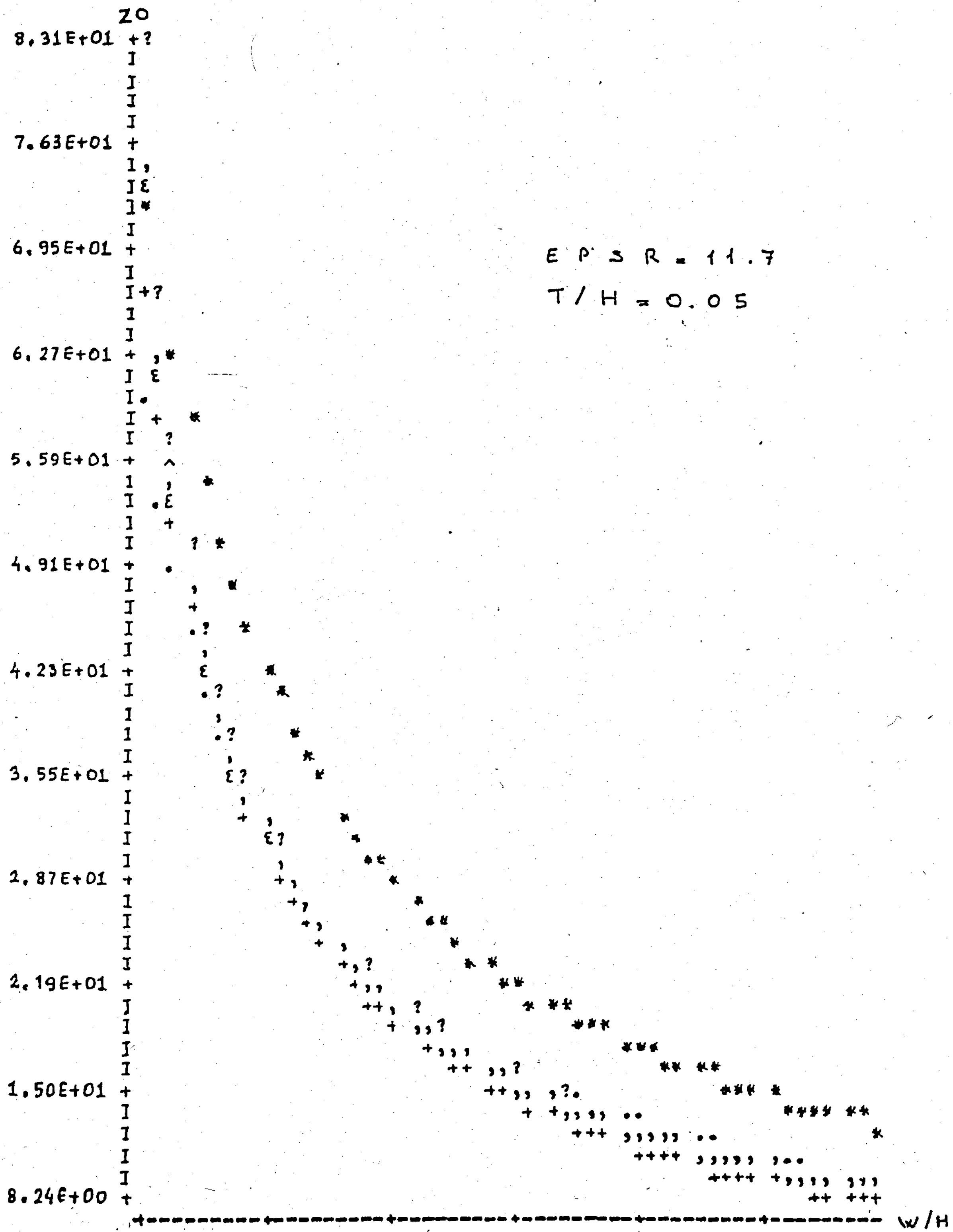

ZO
2.94E+02
2.69E+02
2.44E+02
2.19E+02
1.95E+02
1.70E+02
1.45E+02
1.20E+02
9.51E+01
7.03E+01
4.54E+01
2.05E+01
EPSR = 1
T/H = 0.10
W/H
2.00E-01
1.86E+00
3.52E+00
5.18E+00
6.84E+00
8.51E+00

ZO VS. W/H

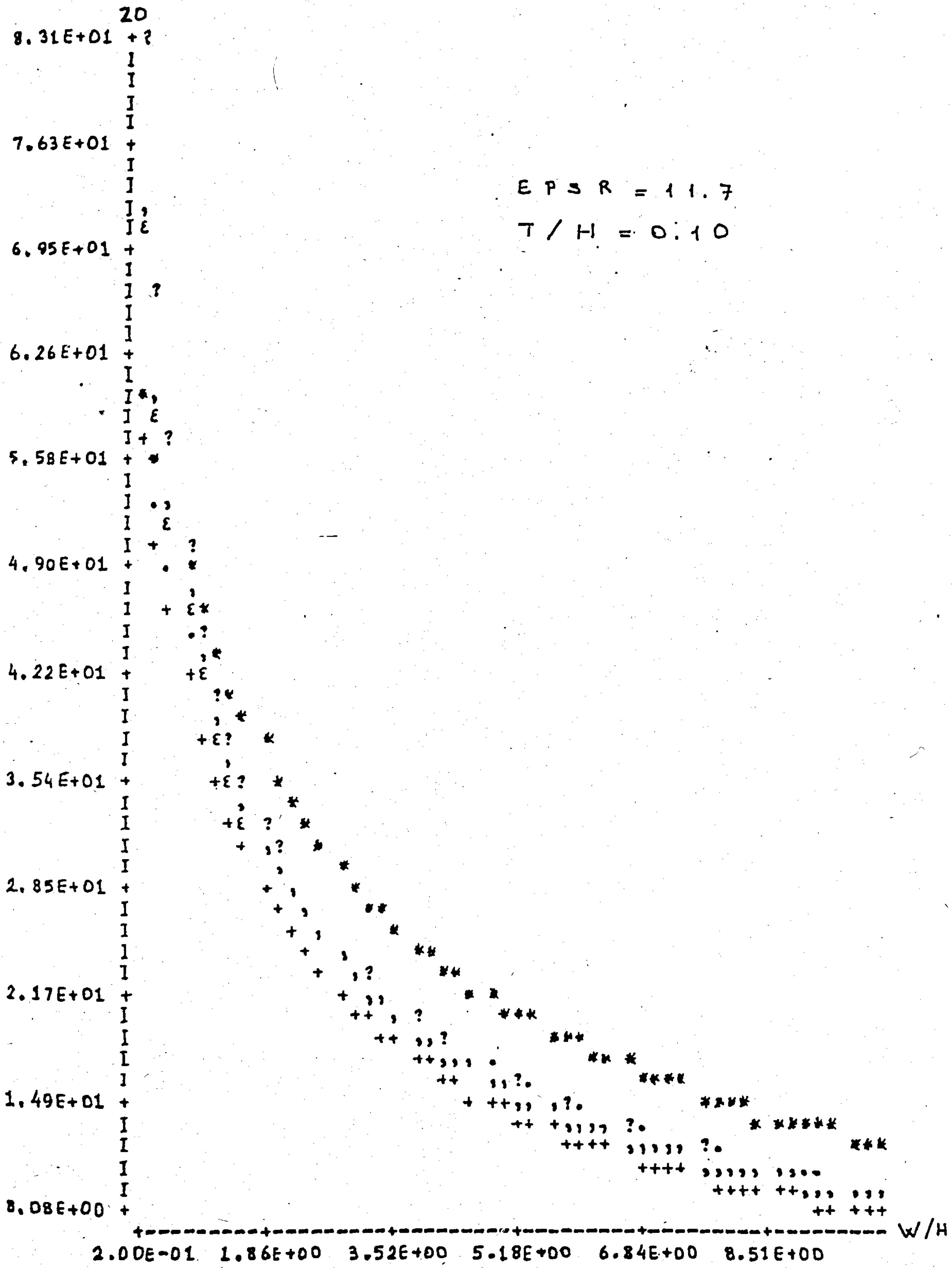
ZO VS. W/H


ZO
2.56E+02
2.34E+02
2.13E+02
1.92E+02
1.70E+02
1.49E+02
1.27E+02
1.06E+02
8.44E+01
6.30E+01
4.15E+01
2.01E+01
EPSR=1
T/H=0.20
2.00E-01
1.86E+00
3.52E+00
5.18E+00
6.84E+00
8.51E+00
W/H


ZO VS. W/H
ZO
1.06E+02
9.70E+01
8.81E+01
7.93E+01
7.04E+01
6.16E+01
5.27E+01
4.39E+01
3.50E+01
2.62E+01
1.73E+01
8.48E+00
EPSR = 11.7
T/H = 0.01
W/H
2.00E-01
1.86E+00
3.52E+00
5.18E+00
6.84E+00
8.51E+00

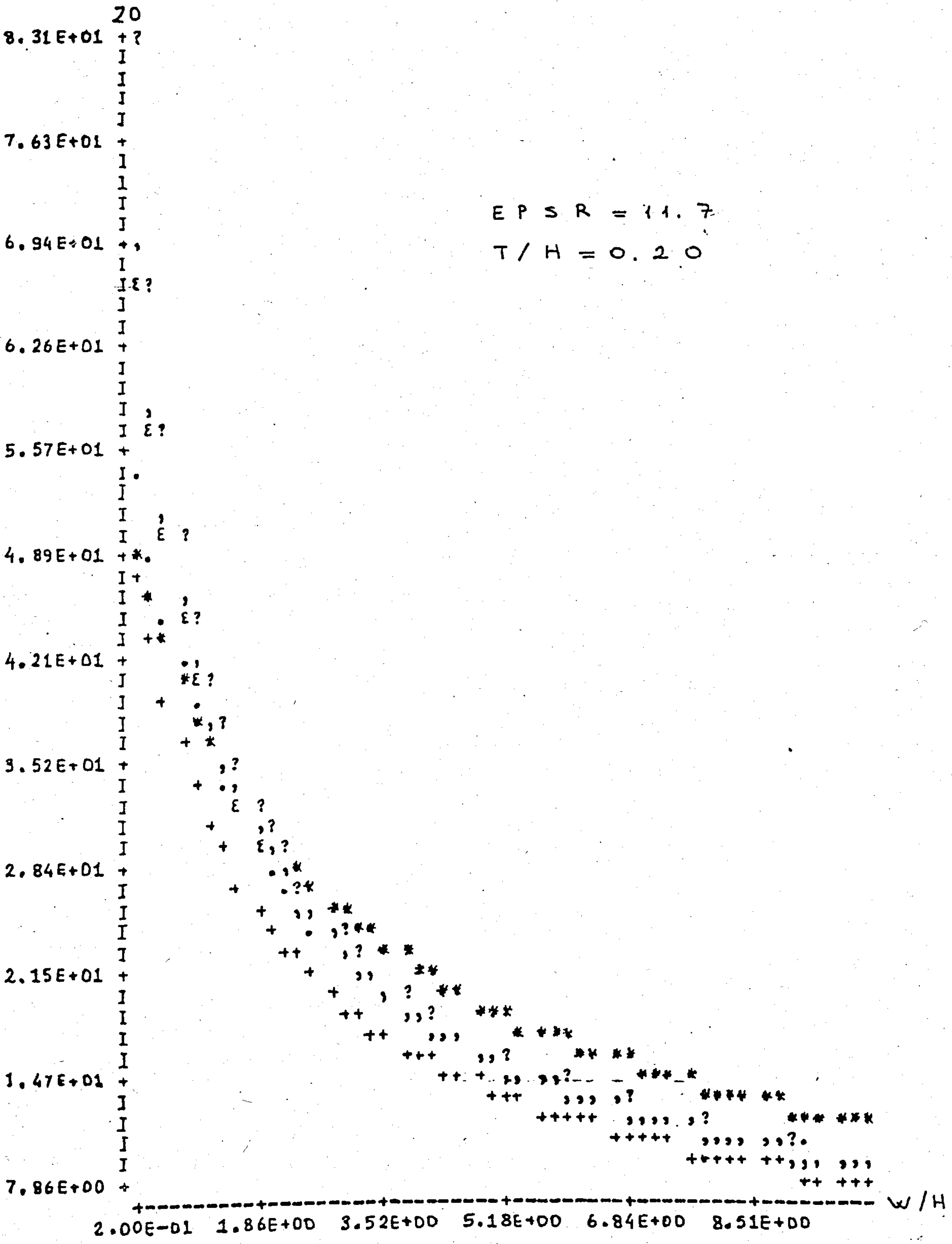

ZO
8.31E+01
7.63E+01
6.95E+01
6.27E+01
5.59E+01
4.91E+01
4.23E+01
3.55E+01
2.87E+01
2.19E+01
1.50E+01
8.24E+00
2.00E-01
1.86E+00
3.52E+00
5.18E+00
6.84E+00
8.51E+00
W/H
EPSR = 11.7
T/H = 0.05

ZO VS. W/H


ZO
8.31E+01
7.63E+01
6.95E+01
6.26E+01
5.58E+01
4.90E+01
4.22E+01
3.54E+01
2.85E+01
2.17E+01
1.49E+01
8.08E+00
EPSR = 11.7
T/H = 0.10
W/H
2.00E-01
1.86E+00
3.52E+00
5.18E+00
6.84E+00
8.51E+00

ZO VS. W/H

ZO VS. W/H


ZO
8.31E+01
7.63E+01
6.94E+01
6.26E+01
5.57E+01
4.89E+01
4.21E+01
3.52E+01
2.84E+01
2.15E+01
1.47E+01
7.86E+00
EPSR = 11.7
T/H = 0.20
W/H
2.00E-01
1.86E+00
3.52E+00
5.18E+00
6.84E+00
8.51E+00

VII. CONCLUSION

Microstrip transmission line has in the last few years become very popular in computer and microwave integrated circuit design as can be judged from the large number of papers published on microstrip and microstrip networks. But there is no comprehensive description available at one place. In particular the design information is not readily available. In this study, a detailed account of this line is presented. Section 4.5 on design considerations can be used directly for the design of microstrip line without going into details of analysis. A computer program is also presented in Chapter VI for CAD.

The accuracy needed in equations for microstrip impedance and effective dielectric constant should be set in accordance with typical variations in substrate parameters and manufacturing tolerances, and 1 % as a maximum relative error seems to be a reasonable goal for both. Most of the equations given in section 4.5.1 (except the equations given in f) do not have that accuracy as can be seen from the tables prepared to compare the typical results with the available numerical and experimental results.

From these tables, it is seen that considerable error can arise in the direct application of Eq. 4.20 given in the line-capacitance computation method. Here, one major source of error arises from the use of the relative dielectric permittivity. By considering this point, the method given in b has taken the relative dielectric permittivity in the straight-line approximation neglecting the dependence on microstrip geometry. It has also obtained the constant A_1, associated with the fringing capacitance as a function of the strip thickness t and the dielectric height h. Similarly A_2, associated with the upper-plate capacitance has been obtained as a function of the dielectric height h, the strip conductor width w and the effective dielectric permittivity, ε_{eff}. But there is no physical significance to believe that the constants A_1 and A_2 depend on microstrip parameters while these parameters have already been accounted for in the capacitance calculations. Also, since the microwave energy propagates both in the dielectric substrate below the strip and in the air region above, the effective dielectric permittivity should be considered as a function of microstrip geometry.

The numerical methods predict a decrease with increase in strip thickness, but the above method (given in c) shows it otherwise i.e. ε_{eff} increases with t/h. (See Table 4.10).

This is ofcourse not acceptable since the fraction of total energy propagating in air increases with strip thickness, there by decreasing the effective dielectric constant. It can be said here that the large amount of error in the results of methods given in b and c appear to be due to the questionable basis for curve fitting. They have tried to fit the experimental results of Kaupp (19) using the equations given in line-capacitance computation method. But these equations have a relative error as large as ±27.6 percent.

Just like the above situation occurred in the methods given in b and c, the formulas for Z_o and ε_{eff} are obtained by curve fitting the numerical results of Yamashita and Mittra (6) in the method given in e. But these formulas have also an error as large as 12 percent.

In f, new equations intending to have 1 % maximum relative error for both characteristic impedance and effective dielectric permittivity are given first for zero thickness microstrip. Next, since the microstrip is three dimensional and so the strip conductor thickness must be consididered, these equations are modified to include strip thickness. A summary of typical results is given in Tables 4.8, 4.9 and 4.10. A comparison of these results with available experimental and numerical results has shown that the results for Z_o and ε_{eff} given here are correct to within 2 percent for the following set, of parameters : $0 \leq t/h \leq 0.350$; $0.1 \leq w/h \leq 10$ and $\varepsilon_r \leq 11.7$.

These simple and most accurate closed form expressions are employed to calculate microstrip characteristics. The results find wide applications in thick microstrips ($0.05 \leq t/h \leq 0.2$) used in computers and in thin microstrips ($0.01 \leq t/h \leq 0.05$) used in MIC's.

APPENDIX A

DATA ON COMMONLY USED SUBSTRATES FOR MICROSTRIP (24)

TYPE	EPSR	TEMP. RANGE (°C)	LOSS FAC. (X-BAND)	DIE. STR. (KV/mm)	THER. CON. K(W/M°C)	RELATIVE PRICE
REXOLENE S	2.32	(-60)-(+70)	0.0005	19.7	0.40	1
REXOLITE 1422	2.53	(-60)-(+100)	0.0005	19.7	0.15	3
REXOLITE 2200	2.62	(-75)-(+190)	0.0010	19.7	0.20	2
POLYPHENYL OXIDE	2.55	(-60)-(+260)	0.0016	19.7	0.20	4
3M,K6098 CUCLD	2.50	(-60)-(+260)	0.0015	31.5	0.10	5
RI/DUROID 5880	2.35	(-60)-(+260)	0.0010	11.8	0.25	10
FUSED SILICA	3.82	NO PRACTICAL LIMITATION	0.0001	15.7	1.40	3
BERYLLIA	6.80	"	0.0003	15.7	230.	8
SAPPHIRE	9.3-11.7	"	0.0004	19.7	40.0	6
ALUMINA	9.6-9.9	"	0.0004	19.7	35.0	1

APPENDIX A (CONTINUED)

TYPE	ADVANTAGES / DISADVANTAGES
REXOLENE S	LOW LOSSES & INEXPENSIVE
	VERY POOR MECHANICAL QUALITIES
REXOLITE 1422	VERY GOOD ELECTRICAL QUALITIES
	POOR MECHANICAL QUALITIES
REXOLITE 2200	FAIR ELECTRICAL & MECHANICAL QUALITIES
	MODERATE PEEL STRENGTH
POLYPNENY OXIDE	FAIR ELECTRICAL QUALITIES
	POOR MECHANICAL QUALITIES AT THIS PRICE
3M,K6098	VERY GOOD MECHANICAL QUALITIES
	TEMPERATURE DEPENDENT ELECTRICAL QUALITIES
RI/DUROID	EXCELLENT ELECTRICAL & MECHANICAL QUALITIES
	EXPENSIVE
FUSED SILICA	LOW EPSR (ADVENTAGEOUS AT HIGH FREQ.,X-BAND & ABOVE)
	-
BERYLLIA	HIGH THERMAL CONDUCTIVITY
	EXPENSIVE. BERYLLIA DUST IS DANGEROUS, ESPECIALLY IN MOIST ATMOSPHERES AT HIGH TEMPERATURES
SAPPHIRE	VERY GOOD SURFACE FINISH
	EXPENSIVE & ANISOTROPIC
ALUMINA	INEXPENSIVE
	VERY POOR MACHINABILITY

APPENDIX - B

DESIGN OF MICROSTRIP LINES BY USING THETA FUNCTIONS

STEP 1 : For a given characteristic impedance Z_o^a, $\frac{K'}{K}$ is calculated from Equation (3.6).

Assume Z_o^a = 126.95 ohm. Then by using $Z_o^a = 60\pi\kappa$

$\kappa = \frac{K'}{K} = 0.673532.$

STEP 2 : Modulus m for elliptic functions is found by looking up tables for $\frac{K'}{K}$.

Modulus m is found as = 0.86 from tables (36).

STEP 3 : E, K and K' are found using this value of m.

E = 1.136

K = 2.42093

K'= 1.63058

STEP 4 : The solution of equation $dn^2(2K\S) = \frac{E}{K}$ gives the value of §.

$$dn^2(2K\S) = \frac{E}{K} = \frac{1.136}{2.42093} = 0.46924$$

$$2K\S = \text{arc } dn\left(\frac{E}{K}\right)^{1/2} = 1.02806$$

$$\S = \frac{1.02806}{2 \times 2.42093} = 0.212328$$

STEP 5 : For known and $\kappa = \frac{K'}{K} = 0.673532$, $\frac{w}{h}$ for the microstrip is calculated from equations (3.4) and (3.7).

$$\frac{\partial}{\partial \S} \ln\left[Q_4(\S,\kappa)\right] = 4\Pi \sum_{n=1}^{\infty} \frac{\exp(-n\Pi\kappa)}{1-\exp(-2n\Pi\kappa)} \text{Sin}(2n\Pi\S)$$

For the first 10 terms, this equation equals to

$$\frac{\partial}{\partial \S} \ln\left[Q_4(\S,\kappa)\right] = 4\Pi \times (0.124095) = 1.55942$$

From equation (3.4)

$$\frac{w}{h} = \frac{2}{\Pi} \frac{\partial}{\partial \S} \ln\left[Q_4(\S,\kappa)\right] = \frac{2}{\Pi} \times (1.55942)$$

$$\frac{w}{h} = 0.99276$$

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