THREE DIMENSIONAL ANALYSIS OF POINT-BEARING PILE GROUPS

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-ABSTRACT-

Piles in the foundation of structures like bridge piers, quays, retaining walls and the like are usually installed in groups resulting in a group reaction.This brings about the problem of determining the distribution of this reaction among individual piles in the group. Especially, if a group of piles consists of both vertical and battered piles, if there are both horizontal and vertical loads or if the loads on the group are eccentric with respect to the pile group, the question of what load is carried by each pile in the group naturally arises. This study attempts to develop a computer program providing the three dimensional analysis of such systems. Here it is assumed that external loads from the superstructure are transferred to the piles through an infinitely rigid pile cap, and piles reach the bearing strata. Each pile is idealized as a beam having a uniformly distributed linearly elastic spring reaction in the lateral direction. Thus; shears, axial forces and moments acting on the piles within the group are determined. The displacements and rotations of each single pile are also calculated. The results of the analysis are checked against the solutions of some particular examples given in the literature.

SAĞLAM ZEMİNDE OTURAN KAZIK GRUPLARININ ÜÇ BOYUTLU ANALİZİ

- ÖZET -

Köprü tabliyeleri, rıhtım ve istinat duvarları ve benzeri yapıların temellerinde, kazıklar genellikle gruplar halinde kullanılmakta bu da grup reaksiyonunun doğmasına neden olmaktadır. Ayrıca ortaya çıkan grup reaksiyonunun kazıklar arasındaki dağılımının belirlenmesi problemi de ortaya çıkmaktadır. Özellikle, eğer kazık grubu düşey ve eğik kazıklardan meydana gelmişse ve bu grub yatay ve düşey yükler etkisi altındaysa, veya bu yükler eksantrik olarak etkiyorsa; her kazığın ne kadar yük taşıdığı sorusu daha da önem kazanmaktadır. Çalışmada bu tip sistemlerin üç boyutlu analizi ele alınmıştır. Üstyapıdan etkiyen dış yüklerin sonsuz rijit bir tabliye tarafından kazıklara aktarıldığı ve kazıkların sağlam zemine ulaştıkları varsayılmıştır. Her kazık yatay yönde eşit sayılı lineer elastik yay reaksiyonlarıyla desteklenmiş bir kiriş olarak idealize edilmiştir. Böylelikle gruptaki her bir kazığa etkiyen kesme ve eksenel kuvvetlerle momentler bulunmuştur. Ayrıca yükler altında ortaya çıkan deplasman ve dönmeler de hesaplanmıştır. Literatürden verilen bazı örnekler geliştirilen programdan elde edilen neticelerle karşılaştırılmıştır.

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LIST OF SYMBOLS

A	:	Cross sectional area (L^2)
A_{ij}	:	Elements of the stiffness matrice
$\|A\|_i$	:	Transformation matrice
$\|a\|_i$	:	Pile head location matrice
α	:	Torsional factor (L^4)
b_{ij}	:	Coefficients of the second side (elements of the loading matrice)
BG	:	Width of the band matrice
c	:	Coefficient of subgrade reaction
c_1, c_2, c_3 and c_4	:	Integration constants
C(I)	:	Sequence of soil modulus (Coefficient of subgrade reaction)
CO(I)	:	Sequence of direction cosines
$\|d\|_o$	:	Pile head displacements matrice in the global axis
$\|d\|_i$	:	Pile head displacements matrice in pile axis
E	:	Modulus of elasticity of the pile material ($F.L^{-2}$)

EL(I) : Sequence for lengths of pile elements (L)

$|ER|_i$: Element stiffness matrice

EX : Stiffness against moment in x direction for each pile element

EY : Stiffness against moment in y direction for each pile element

G : Shear modulus of elasticity of the pile material ($F.L^{-2}$)

γ : Pile stiffness factor = $(\frac{cb}{4EI})$** 0.25

GJ : Stiffness against torsion for each pile element

GL(I) : Sequence for Gamma (L) for each element

H : Horizontal force (F)

I : Moment of inertia of the section of the pile (L^4)

$|I|$: Identity matrice

$|IDKR|_i$: Gaussian-eliminated and re-arranged pile head stiffness matrice

$|IDKRT|$: Equivalent pile group stiffness matrice

$|IKR|_i$: Re-arranged pile stiffness matrice

ITS : Number of columns on the second side of the equation (number of loadings)

i : Row number in the element stiffness matrice

j	:	Column number in the element stiffness matrice
KS (i,j)	:	Function that transfers two dimensional matrice into one dimensional one.
l	:	Length of the pile element (L)
M	:	Moment (F.L)
ND	:	Total number of node number in one pile element
NI	:	Node number
$\lvert NIKR \rvert_i$	:	New-order pile stiffness matrice
NLOAD	:	Total number of loadings
NPILE	:	Total number of piles in a group
p	:	Lateral subgrade reaction at any point $(F.L^{-1})$
P_1, P_2, P_3, P_4, P_5, and P_6	:	Elements of the loading matrice
$\lvert POC \rvert_i$	:	Pile head reactions matrice
Q	:	Axial force (F)
$\lvert q \rvert_o$	:	External pile head loading matrice in the global axis
$\lvert q \rvert_i$	:	Pile head reactions matrices in the pile axis
R	:	Resultant force (F)
$\lvert T_1 \rvert_i$	:	Translation matrice

$\|T_2\|_i$	:	Rotation matrice
V	:	Vertical force (F)
w	:	Lateral deflection of pile (L)
X(I)	:	Sequence of x, y, z coordinates
YBG	:	Width of the half-band matrice

I. INTRODUCTION

Design and analysis of pile foundations has become an important area of research and several approaches have been developed to tackle them. Its importance arises not only because pile foundations constitute a large proportion of substructures but a better understanding of their behaviour helps the analysis of superstructures as well.

Due to the complex nature of the problem, with variation of subgrade reaction from one site to another, and along the length of the pile, and also diversity of pile group arrangements a definitive general solution is not possible. Modelling the problem mathematically is difficult. Rather than developing a mathematical model for a direct solution experimental studies have been carried out for single piles with different soil properties. As a result of these, charts have been developed giving the individual pile stresses for different soil properties. After Broms, many charts have been developed determining subgrade reaction and resultant pile stresses for single piles under vertical and horizontal forces and bending moments (4).

Although engineering codes and specifications specify capacities for single piles, no assistance is provided for determining the reaction capacity of different piles located in the same group (1). Yet pile configuration in a group influences the magnitude and distribution of indivudual pile reactions.

With the help of microcomputers representative models for pile groups can be analized to determine individual pile stresses within the group (5). A recent study including the previous works of three-dimensional analysis has been done by Bowles and a computer program has been developed. A successive approximation analysis may be performed by this method if the soil data are sufficiently reliable. This method of pile group analysis makes the following assumptions.

1. The load carried by any pile is proportional to the displacement of the pile head. The displacement consists of an axial, transverse and rotational component,

2. The footing (pile cap) in infinitly rigid,

3. The footing undergoes only small displacements,

4. The pile heads are pinned to the pile cap.

Although these assumptions do not strictly represent the true situation, they do not introduce serious errors. The program analyzes the group under vertical and a horizontal loads and for any pile the axial and transverse forces are computed. However it is not possible to compute either settlements of the pile groups or the bending and torsional moments exerted in each single pile. Inorder to overcome all these difficulties, another approach is adopted in this study by modelling the piles as an elastically supported beam in the lateral direction. This implies that the objective of this study is to present the three dimensional analysis of pile groups giving the forces and displacements developed in individual piles in each of the six-degrees of freedom in space as shown in figure 1.1. A program is developed considering point-bearing piles only. The skin friction acting on the piles is ignored. It is assumed that load effects from the superstructure are transferred to the individual piles by a rigid pile cap. In practice piles groups of this type constitute a significant proportion of total number of piled foundations. It is also assumed that subgrade reaction is proportional to the lateral displacement at any point along the pile based on "Winkler hypothesis" (7). Any other model than the Winkler's can easily be used in the program if desired.

Inorder to serve the needs of engineering design offices a simple data input is formulated , and effort is made to use a minimum number of memory locations. In the program, piles may individually be defined by the user as vertical or inclined. The subgrade reaction may be defined along the pile length to simulate actual soil properties encountered at the site of the foundation. The program analyzes the group and determines displacements and rotations as well as axial forces, shears and moments generated in each pile.

Once the analysis is complete, it will be necessary to establish the acceptability of the individual pile stresses and displacements both from structural and geotechnical view point. The overall group reaction capacity should also be investigated.

Finally, solutions of some particular examples given in the literature are compared to the ones obtained from the proposed method and computer outputs of these study.

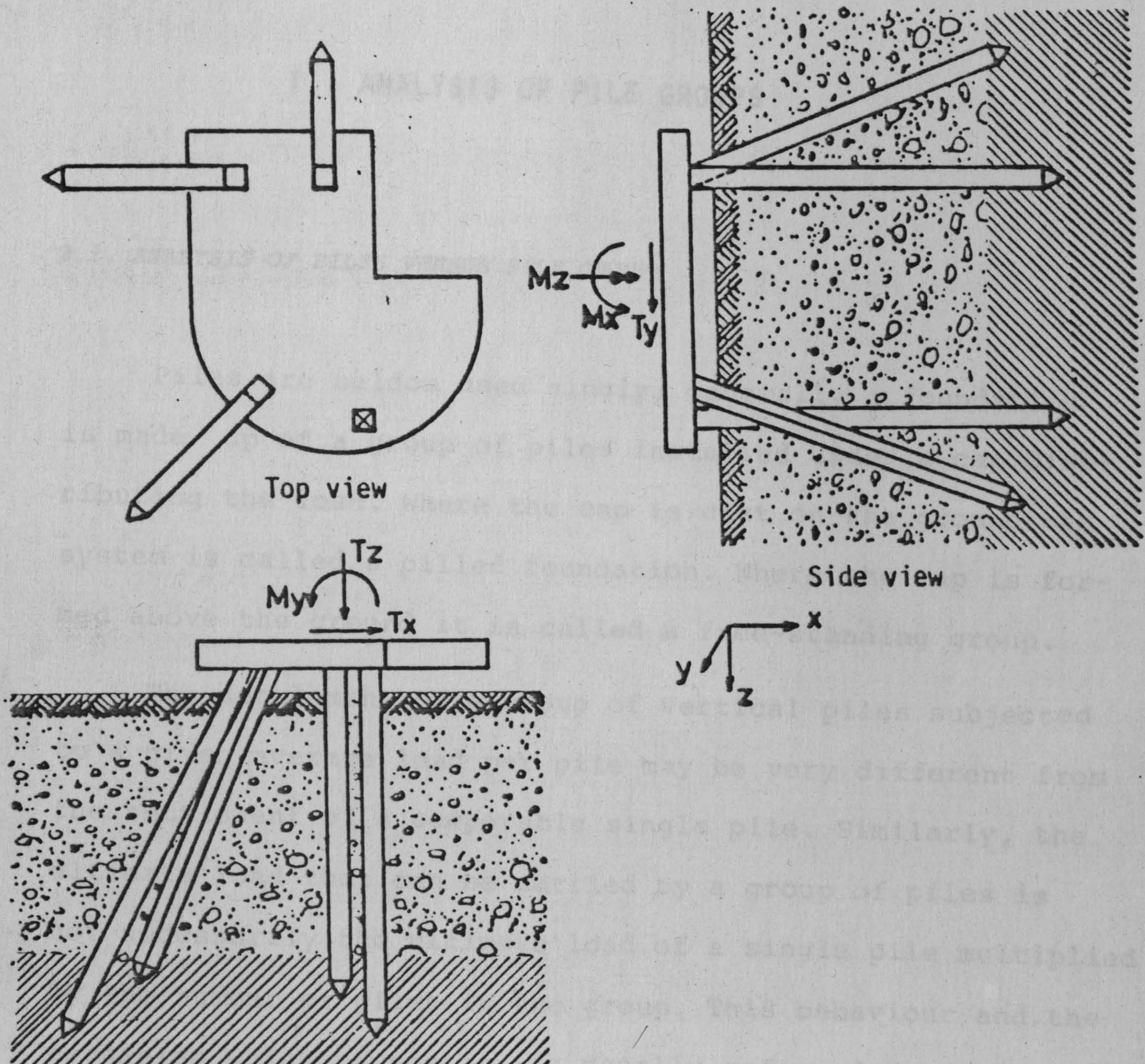

FIGURE 1.1 - Three dimensional modelling of point-bearing pile groups and the external forces and moments applied from the superstructure.

II. ANALYSIS OF PILE GROUPS

2.1. ANALYSIS OF PILES VERSUS PILE GROUPS

Piles are seldom used singly; generally a foundation is made up of a group of piles installed under a cap distributing the load. Where the cap is cast on the ground the system is called a pilled foundation. Where the cap is formed above the ground it is called a free-standing group.

The settlement of a group of vertical piles subjected to a given average load per pile may be very different from the settlement of a comparable single pile. Similarly, the ultimate load that can be carried by a group of piles is not necessarily the ultimate load of a single pile multiplied by the number of piles in the group. This behaviour and the mechanism which causes it, is usually referred to as "group action". It is important in the case of friction piles in clay, not quite so important with point-bearing piles in dense or gravel and generally unimportant where piles are driven to rock. Pile groups of this type are object of this study. The pile ends are assumed to reach the hard bearing strata and interference among adjacent piles in neglected.

In such a case the bearing capacity should be calculated as the load per pile multiplied by the number of piles in the group. The settlement of the foundation is usually little more than the elastic shortening of the pile under load.

Another aspect of the study for piled foundaitons is the resistance of such foundations against horizontal forces. In many applications these forces are small enough to be neglected, but with large buildings and bridges to resist to wind forces, and in earthquake areas the resistance to horizontal forces caused by shocks are of considerable importance. In the case of a bridge the forces due to traffic acceleration, breaking and turning may also be important. For the design of retaining walls, quays and dolphins horizontal forces form a major part of the loading system.

When a vertical pile is deflected from its initial position by a horizontal force applied to the pile head, the deflected form of the pile depends on the head conditions, the pile length, and the stiffness of both pile and the soil. The differential equation for the flexure of a uniform pile embedded in the soil is

$$EI \frac{d^4W}{dx^4} + p = 0 \qquad (2.1)$$

where,

W = The deflection of the pile at any point

X = The depth of that point from the soil surface

P = Lateral subgrade reaction per unit length at any point.

E = The modulus of elasticity of the pile material

I = The moment of inertia of the section of the pile

In uniform clay it is often assumed that c, the coefficient of lateral reaction is constant, so that p=cw. For granular soils c is usually considered to vary linearly with depth, so that $c=n_h x$, and therefore $p=n_h xw$ where n_h is the constant of horizontal subgrade reaction, as defined by Terzaghi (4). Palmer and Brown (5) examined the case where the value of c varies according to the equation $c=c_1(x/L)^n$, where c_1 is the value of c at the depth L, L being the pile length. They found that values of the parameter n in the range $0 < n < 1$ agreed best with test results.

For the case where p=cw and for a pile with no head restraint, and with a horizontal force P applied at the ground level.

$$W = \frac{H}{2\,EI\beta^3}\, e^{-\beta x} \cos \beta x \qquad (2.2)$$

$$M = -\frac{H}{\beta}\, e^{-\beta x} \sin \beta x \qquad (2.3)$$

where M = the moment on the pile at depth x

$$\beta = (c/4EI)^{1/4}$$

$$W = \frac{H}{4EI\,\beta^3} e^{-\beta x} (\cos\beta x + \sin\beta x) \tag{2.4}$$

$$M = \frac{H}{2\beta} e^{-\beta x} (\sin\beta x - \cos\beta x) \tag{2.5}$$

Although there are many approaches of calculation for laterally loaded single piles, the application of the theoretical solutions to practical design is handicapped by the difficulty of obtaining the value of c. However the value assigned to the parameter c is considerable importance. It is known to vary with the type of soil, the confining pressures, the width of the face, the amount of deflection and the duration of loading.

2.2 GROUPS OF VERTICAL PILES

The following approximate methods are commonly used for groups of identical piles subjected forces and moments for practical design purposes. The pile cap is assumed to be rigid and the reaction of any pile is assumed to be proportional to the displacement of the pile head.

If the vertical load V is applied at the centre of gravity of the pile group, the displacement of the head of each pile will be the same and load distribution is therefore assumed to be equal. Thus V = nQ where Q is the load per pile and n the number of piles :

$$Q_1/x_1 = Q_2/x_2 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = Q_n / x_n \quad (2.6)$$

so that

$$Q_1 = Q_1\ x_1/x_1,\ Q_2 = Q_1\ x_2/x_1, \ldots\ldots\ldots = Q_n = Q_1\ x_n/x_1 \quad (2.7)$$

It is obvious that

$$M = Q_1 x_1 + Q_2 x_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + Q_n x_n \quad (2.8)$$

Thus,

$$M = Q_1 x_1^2/x_1 + Q_1\ x_2/x_1 + \ldots\ldots\ldots\ldots + Q_n x_n^2/x_1 \quad (2.9)$$

Therefore

$$Q_1 = Mx_1 / \sum_{i=1}^{n} x^2 \quad (2.10a)$$

Similarly

$$Q_2 = Mx_2 / \sum_{i=1}^{n} x_i^2;\ \ldots\ldots\ Q_n = Mx_n / \sum_{i=1}^{n} x^2 \quad (2.10b)$$

Thus the total load Q on pile 1 due to a vertical force and a moment applied at the center of gravity is.

$$Q_1 = V/n \pm M\ x_1 / \sum_{i=1}^{n} x^2 \quad (2.11)$$

If a rectangular group of piles is subjected to moments about both axes xx and yy through the centre of gravity of the group as well as a vertical force acting at the centre of gravity, then

$$Q_1 = V/n \pm Myy\ x_1 \ / \sum_{i=1}^{n} x^2 \pm Mxx\ Y \ / \sum_{i=1}^{n} y^2 \qquad (2.12)$$

The sign of the second term will be positive for piles to the left of yy and the third term will be positive for piles above xx for the moment directions in right-handed coordinate system.

2.3 GROUPS WITH VERTICAL AND INCLINED PILES

When a piled foundation is subjected to a horizontal force or a moment as well as a vertical force, it is usual for some of the piles to be inclined in order that the re-sultant of the external forces will be appliced approxima-tely axially to some of the piles. The calculation of the forces and moments transmitted to the each pile in the group is an extremely complicated problem for which no true solution exists. The usual approach to the subject has been made from the direction of structural engineering, in which the piles are treated as members of a frame, the cap is assumed to be rigid. There is a high order of in-determinancy and various simplifications are introduced to make a solution possible.

Mostly piles are regarded as hignes at their upper ands and carry axial loads to hignes on rigid bearings

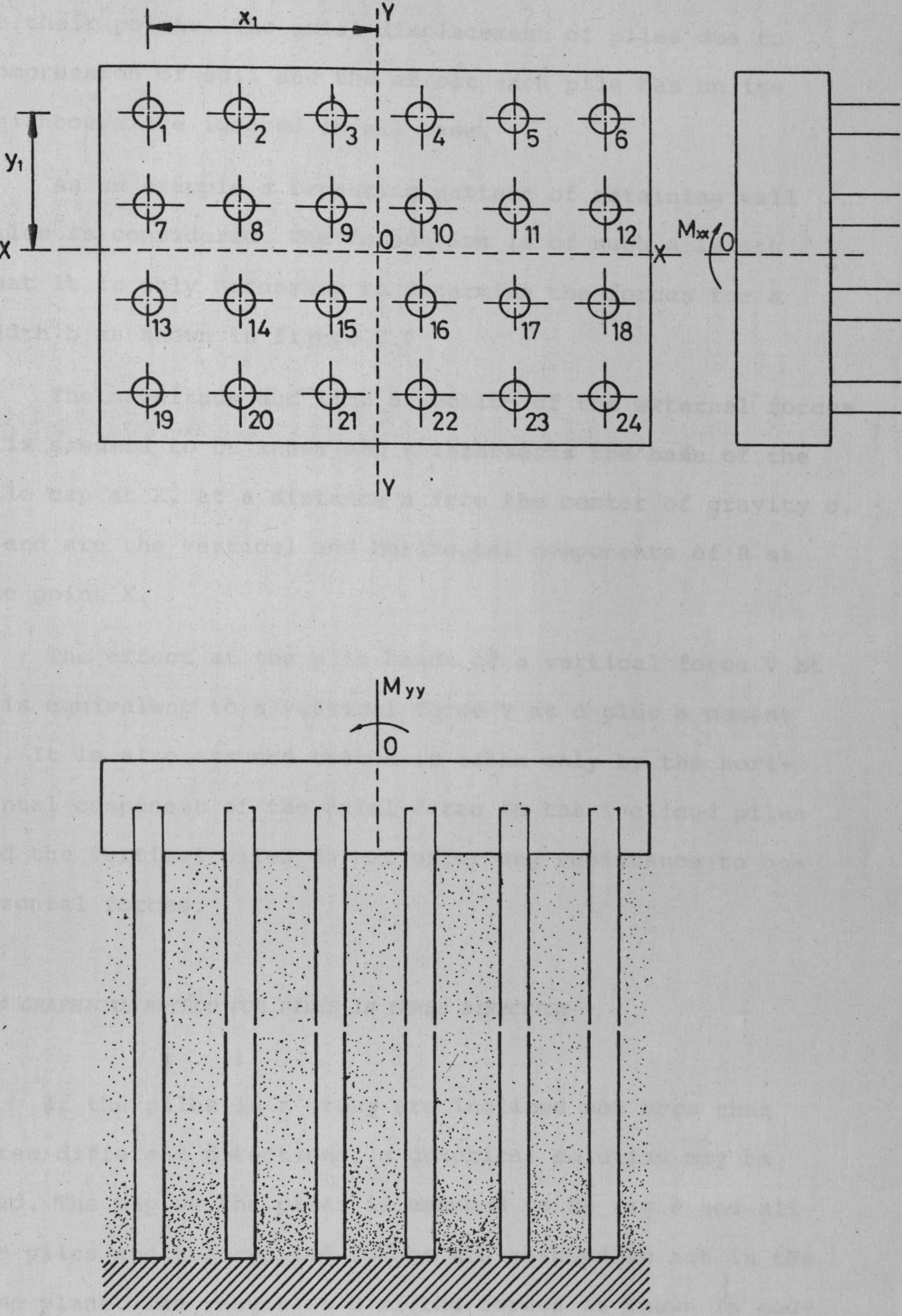

FIGURE 2.1. A pile group subjected to bending moments in two direction.

at their points. The axial displacement of piles due to compression of soil and the effect each pile has on its neighbours are ignored in all case.

As an example a repeating pattern of retaining wall piles is considered. The foundation is of such a length that it is only necessary to determine the forces for a width b as shown in figure 2.2

The magnitude and line of action of the external forces R is assumed to be known and R intersects the base of the pile cap at X, at a distance a from the center of gravity o. V and are the vertical and horizontal components of R at the point X.

The effect at the pile heads of a vertical force V at X is equivalent to a vertical force V at o plus a moment Va. It is also assumed that H is taken only by the horizontal component of the axial force in the inclined piles and the vertical piles do not offer any resistance to horizontal forces.

2.4 GRAPHICAL METHOD FOR PILES IN THREE DIRECTION

If the piles in a group are inclined not more than three different directions, a graphical solution may be used. The cap on the piles is assumed to be rigid and all the piles and the applied forces are assumed to act in the same plane. The resultant R of the forces is known in magnitude, direction and position. Lines P, Q and S are drawn representing the lines of action of the rows of piles inclined in the same direction through the centers of gravity of their respective rows.

The direction of R meets line P in X. Lines Q and S meet in Y. Using Bow's notation as shown in figure 2.3 the forces on the lines XY and P are first determined from these the forces on lines Q and S are calculated. It is also assumed that the force in each of the three directions is equally shared among the piles inclined in that direction.

2.5 METHODS BASED ON ELASTIC THEORY

The method proposed by Vetter (1939) provides a means of estimating pile loads when there are more than three rows or when piles are fixed headed. It is confined to two dimensional systems and the following assumptions are made(11).

1. The pile cap is rigid
2. The piles are elastic
3. The whole load is carried by the piles
4. The resistance of a pile is concentrated at its base in the case of an end-bearing pile and at one-third of the length of the pile up from the base in friction piles.
5. The soil provides rigid axial bearing but gives no other support.

The methods proposed by Asplund and Francis introduced the lateral resistance of the soil to the calculations to approximate to reality in a better way although they require the knowledge of pile soil interaction.

However all the methods put forward ignore the displacement of the supporting soil, the influence of individual pile on its neighbours and the effect of pile cap when it is flexible or exerts a vertical pressure on the ground. It is assumed that all the piles act independently in all the methods, but if they are closely spaced they do influence each other. Jampel (1949) and Francis (1964) suggest to reduce the value of c for groups of closely spaced piles and group action can be as important in any given instance for horizontal as well as for vertical forces (11).

In the design of retaining walls, bridge abutments or quays, the use of complex design method does not seem to be easily applicable because of the uncertanties involved regarding c coefficient.

III. PROGRAMMING

Inorder to analyze pile groups each pile is assumed to act as a beam elastically supported in the lateral direction. The related stiffness matrix is produced out of the mathematical model obtained using "Winkler hypothesis" (7).

3.1 FORMATION OF THE ELEMENT STIFFNESS MATRICE :

It is assumed that the reaction force for a unit length is directly proportional to the displacements for each element. Thus, the horizontal reaction p, is equal to a constant times horizontal displacement.

$$p(x) = cbw(x), \quad c > 0 \qquad (3.1)$$

where, b is the width of the element.

The degrees of freedom and resultant reactions for an element is shown in figure 3.1

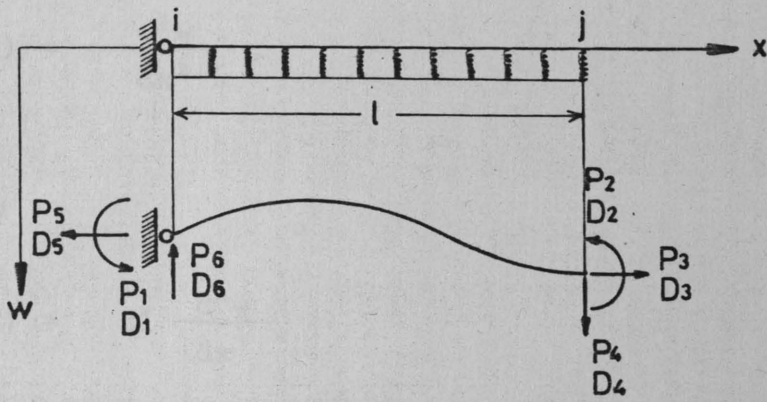

FIGURE 3.1. A schematic representation for forces and displacement of a pile element.

The elastic curve for a span of ℓ is governed by the differential equation

$$\frac{d^4 w}{dx^4} = - \frac{P(x)}{EI} = - \frac{cb}{EI} \; w(x) \qquad (3.2)$$

For a homogenous solution;

$$W = C_1 \cosh \gamma x . \cos \gamma x + C_2 \sinh \gamma x \cos \gamma x + C_3 \cosh \gamma x \sin \gamma x + C_4 \sinh \gamma x . \sin \gamma x$$

Where C_1, C_2, C_3 and C_4 are integration constrants and

$$\gamma = \sqrt[4]{\frac{Cb}{4EI}}$$

Moments and shear forces can be written out of the solution for displacements as

$$M(x) = - \frac{d^2 w}{dx^2} \qquad (3.4)$$

and

$$T(x) = - EI \frac{d^3 w}{dx^3} \qquad (3.5)$$

Therefore the reactive forces will be

$$P_1 = M(0) = EI \frac{d^2 w(0)}{dx^2} \qquad (3.6)$$

$$P_2 = M(\ell) = - EI \frac{d^3 w(\ell)}{dx^2} \qquad (3.7)$$

$$P_4 = T(\ell) = - EI \frac{d^3 w(\ell)}{dx^3} \qquad (3.8)$$

$$P_6 = T\,(0) = -EI \frac{d^3 w(0)}{dx^3} \qquad (3.9)$$

and the related displacements are

$$D_1 = - \frac{dw\ (0)}{dx} \qquad (3.10)$$

$$D_2 = \frac{dw\ (\ell)}{dx} \qquad (3.11)$$

$$D_4 = w(\ell) \tag{3.12}$$

$$D_6 = -w(0) \tag{3.13}$$

Betti's reciprocal theorem is also applicable for this case therefore the, flexibility matrice should be symmetrical to its diagonal. Thus some elements of the flexibility matrice will be,

$$f_{11} = f_{22},\ f_{66} = f_{44},\ f_{24} = f_{16},\ f_{26} = f_{62} = f_{14} \tag{3.14}$$

Furthermore, considering that the pile is a straight prismatic beam, it can be stated that

$$f_{13} = f_{24} = f_{34} = f_{36} = 0, \quad f_{33} = \frac{\ell}{EA} \tag{3.15}$$

where

ℓ is the length of the element

A is the cross-sectional area

Inorder to determine the other elements of the flexibility matrice two different loadings are made on the beam.

For $P_1 = P_3 = P_4 = P_6 = 0$ and $P_2 = 1$ integration constants C_1, C_2, C_3 and C_4 are determined using equations 3.6 through 3.9 By definition,

$D_1 = f_{12}$, $D_2 = f_{22} = f_{11}$, $D_4 = f_{42} = f_{16}$, $D_6 = f_{62} = f_{14}$ which leads to

$$f_{11} = \frac{\ell}{EI} \cdot \frac{1}{\gamma\ell} \cdot \frac{\sinh\gamma\ell \,.\cosh\gamma\ell + \sin\gamma\ell \,.\, \cos\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.17)$$

$$f_{12} = \frac{\ell}{EI} \cdot \frac{1}{\gamma\ell} \cdot \frac{\sinh\gamma\ell \,.\, \cos\gamma\ell + \cosh\gamma\ell \,.\, \sin\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.18)$$

$$f_{14} = \frac{\ell^2}{EI} \cdot \frac{1}{(\gamma\ell)^2} \frac{\sinh\gamma\ell \,.\, \sin\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.19)$$

$$f_{16} = \frac{\ell^2}{EI} \cdot \frac{1}{2(\gamma\ell)^2} \frac{\sinh^2\gamma\ell - \sin^2\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.20)$$

In a second loading for P_1, = P_2, = P_3, = P_6, and P_4 = 1 the integration constants are determined using equations 3.6 through 3.9 Consequently, $D_4 = f_{44}$ and $D_6 = f_{46}$ are obtained.

$$f_{44} = \frac{\ell^3}{EI} \cdot \frac{1}{2(\gamma\ell)^3} \frac{\sinh\gamma\ell \,.\, \cosh\gamma\ell - \sin\gamma\ell \,.\, \cos\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.21)$$

$$f_{46} = \frac{3}{EI} \frac{1}{2(\gamma\ell)^3} \frac{\sinh\gamma\ell \,.\, \cos\gamma\ell - \cosh\gamma\ell \,.\, \sin\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.22)$$

By definition stiffness matrix is the inverse of the flexibility matrice so the elements of the stiffness matrice are the reciprocals of the flexibilty elements. Thus,

$$k_{11} = \frac{EI}{\ell} \cdot 2\gamma\ell \cdot \frac{\sin\gamma\ell.\cosh\gamma\ell - \sin\gamma\ell \cosh\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.23)$$

$$k_{12} = \frac{EI}{\ell} \cdot 2\gamma\ell \cdot \frac{\sin\gamma\ell \,.\, \sin\gamma\ell - \sin\gamma\ell \cos\gamma\ell}{\sinh^2\gamma\ell - \sin\gamma\ell} \quad (3.24)$$

$$k_{14} = \frac{EI}{\ell^2} 4(\gamma\ell)^2 \frac{\sinh\gamma\ell \sin\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.25)$$

$$k_{16} = \frac{EI}{\ell^2} 2(\gamma\ell)^2 \frac{\sinh^2\gamma\ell + \sin^2\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.26)$$

$$k_{44} = \frac{EI}{\ell^2} 4(\gamma\ell)^3 \frac{\sinh\gamma\ell \cosh\gamma\ell + \sin\gamma\ell \cos\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.27)$$

$$k_{46} = \frac{EI}{\ell^3} 4(\gamma\ell)^3 \frac{\cosh\gamma\ell \sin\gamma\ell + \sinh\gamma\ell \cosh\gamma\ell}{\sinh^2\gamma\ell - \sin^2\gamma\ell} \quad (3.28)$$

and the resulting stiffness matrix will be as shown in figure 3.2

[K] =

	1	2	3	4	5=3	6	
	K 11	K 12	0	K 14	0	K 16	1
	K 21	K 11	0	K 16	0	K 14	2
	0	0	EA/l	0	EA/l	0	3
	K 14	K 16	0	K 44	0	K 46	4
	0	0	EA/l	0	EA/l	0	5=3
	K 16	K 14	0	K 46	0	K 44	6

FIGURE 3.2 Stiffness matrice of a pile element

A subroutine is introduced to the program for the calculation of element stiffness matrix element. The subroutine is executed twice for each element. One for the reactions in the plane of the beam and the other for the reactions perpendicular to the plane of the beam. In the second execution the torsional stiffness $G\alpha$ is substituted for the axial stiffness EA, where

G is Shear modulus of elasticity

α is the torsional factor

Some of the magnitudes are calculated at the beginning for the reasons of simplicity. By introducing some intermediate values stiffness matrice elements can easily be obtained. The calculation procedure is,

GL = L* (C*B/(4*E*I)) **0.25

The magnitudes that are to be calculated once,

CH = $\cosh\gamma\ell$

Cϕ = $\cos\gamma\ell$

SH = $\sinh\gamma\ell$

SI = $\sin\gamma\ell$

SHCH = $\sin\gamma\ell * \cosh\gamma\ell$

CHSI = $\cosh\gamma\ell * \sin\gamma\ell$

SHCϕ = $\sinh\gamma\ell * \sin\gamma\ell$

SICϕ = $\sin\gamma\ell * \cos\gamma\ell$

PYD = $\sinh^2\gamma\ell - \sin^2\gamma\ell$

PYDT = 1/PYD

The coefficients calculated according to this magnitudes are

L11 = (CHCH - SICϕ) * PYDT

L44 = (SHCH - SICϕ) * PYDT

L12 = (CHSI - SHCϕ) * PYDT

L46 = (SHSI + SHCϕ) * PYDT

L14 = SH * SI * PYDT

L16 = (SH * SH + SI * SI) * PYDT

EI = E*I

GJ = G*α

CK1 = (EI * GL * 2)/L

CK2 = CK1 * GL * 2/L

CK3 = CK2 * GL/L

Hence the elements of the stiffness matrice are,

K11 = CK1 * L11

K12 = CK1 * L12

K14 = CK2 * L14

K16 = CK2 * 0.5 * L16

K44 = CK3 * L44

K46 = CK3 * L44

It is also possible to re-arrange matrice inorder to put the degrees of freedom of one node on the side and the second one on the other. Thus the new-order of the element stiffness matrice will be as given in figure 3.3

[K] =

1	5	6	2	3	4	
K11	0	K16	K12	0	K14	1
0	EA/l	0	0	EA/l	0	5
K16	0	K44	K14	0	K46	6
K12	0	K14	K11	0	K16	2
0	EA/l	0	0	EA/l	0	3
K14	0	K46	K16	0	K44	4

FIGURE 3.3 Re-arranged stiffness matrice of a pile element

A further attempt is made to form the two dimensional stiffness matrice in a dimensional one. Using the property of symmetry the unnecessary elements are eliminated. A function transfers the row and column number into the related array number of the one-dimensional sequence as shown in figure 3.4.

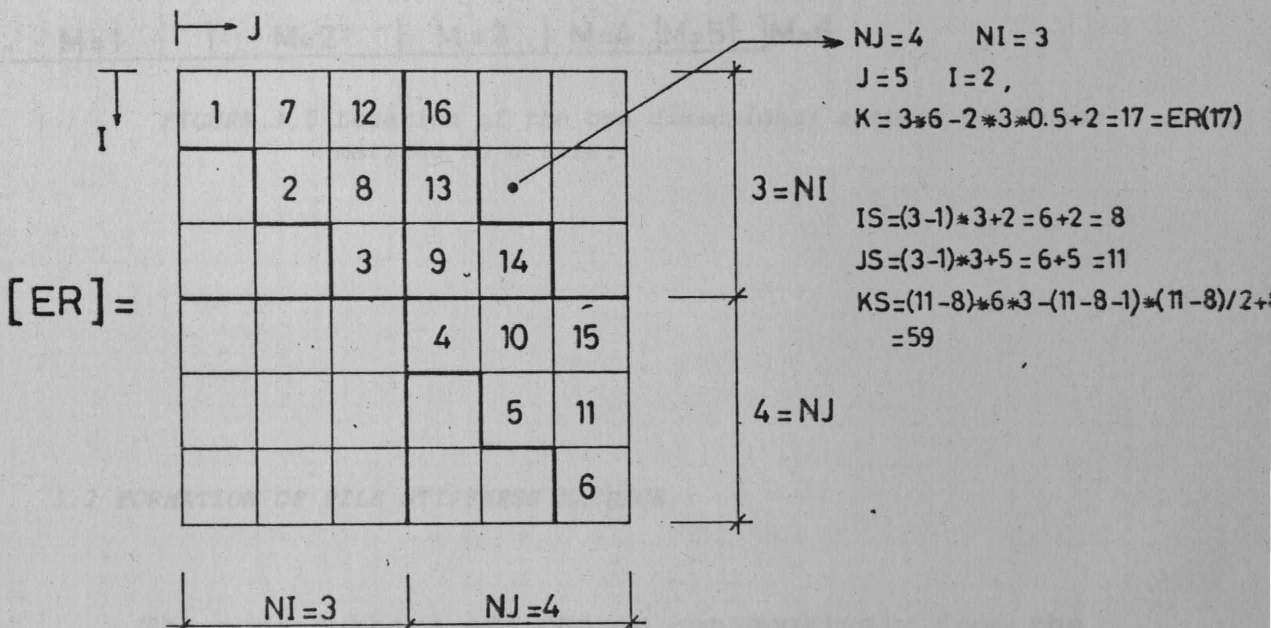

FIGURE 3.4 One dimensional order of element stiffness matrice

The one dimensional |ER| sequence is formed for each element and kept in a file. The same procedure is repeated for the degrees of freedom perpendicular to the plane of the element. A record is reserved for each element and twenty-one elements are kept in each record as shown in figure 3.5

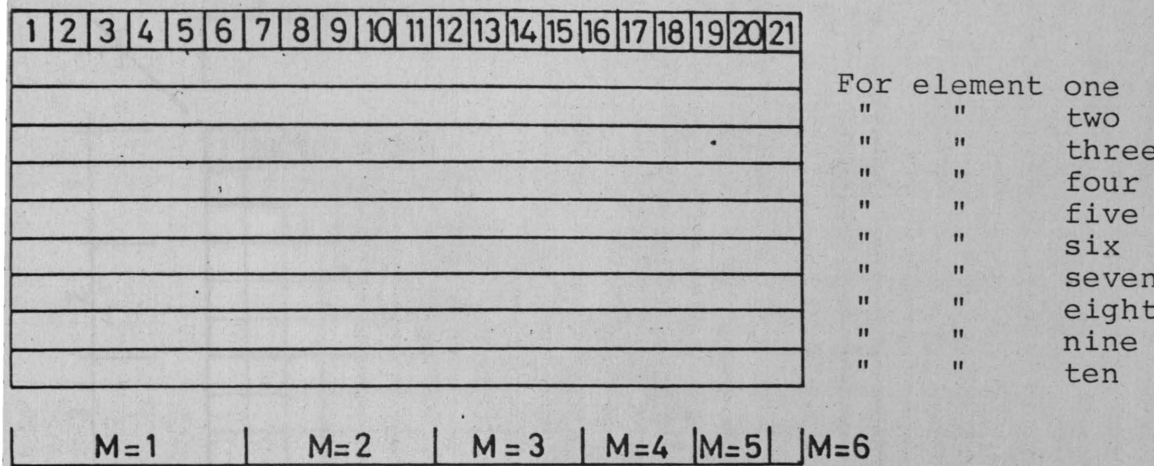

FIGURE 3.5 Location of the one dimensional element stiffness matrice in a file.

3.2 FORMATION OF PILE STIFFNESS MATRICE :

The node numbers are chosen consecutively from the bearing strata to the pile cap for each pile. Since the node numbers are consecutive; the pile stiffness matrix can be easily formed by the superposition of element stif-nesses. Pile stiffness matrix is also formed in one dimen-sional sequence and kept in a file. For a pile having five elements and six node numbers the one-dimensional pile-stiffness matrice is shown in figure 3.6.

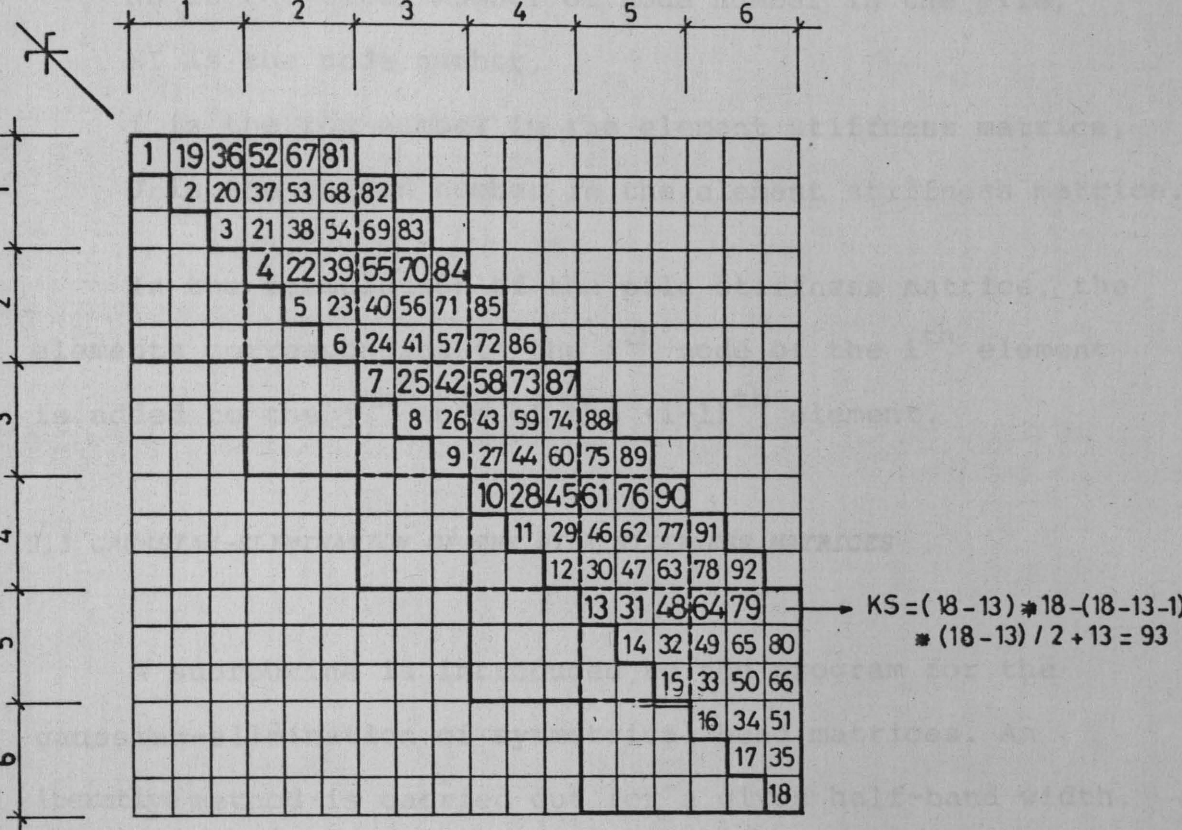

FIGURE 3.6 One dimensional order of pile stiffness matrice

The corresponding row number of the one-dimensional pile stiffnes matrice is calculated by a function, once the row and column numbers are determined. The function is

KS(IS,JS) = (JS-IS) * ND*3 - (JS-IS-1)*(XS-IS)/2+IS (3.28)

and

IS = (NI-1) * 3+I (3.29)

JS = (NI-1) * 3+J (3.30)

where

ND is the total number of node number in the pile,

NI is the node number,

I is the row number in the element stiffness matrice,

J is the column number in the element stiffness matrice.

In the formulation of the pile stiffness matrice, the elements corresponding to the i^{th} node of the i^{th} element is added to the j^{th} note of the $(i-1)^{th}$ element.

3.3 GAUSSIAN-ELIMINATION OF THE PILE STIFFNESS MATRICES

A subroutine is introduced to the program for the gaussian-elimination of symmetrical band matrices. An iterative method is carried out for a given half-band width. The iteration taken are shown illustratively on an example having N numbers of unknowns and ITS number of columns on the second side of the equation. It is obvious that the property of symmetry is preserved in each step. A graphical representation is shown in figure 3.7

where,

BG = Width of the band matrice

YBG = Width of the half band matrice = BG/2+1

ITS = Number of columns on the second side of the equation.

and BG,YBG and ITS being integers

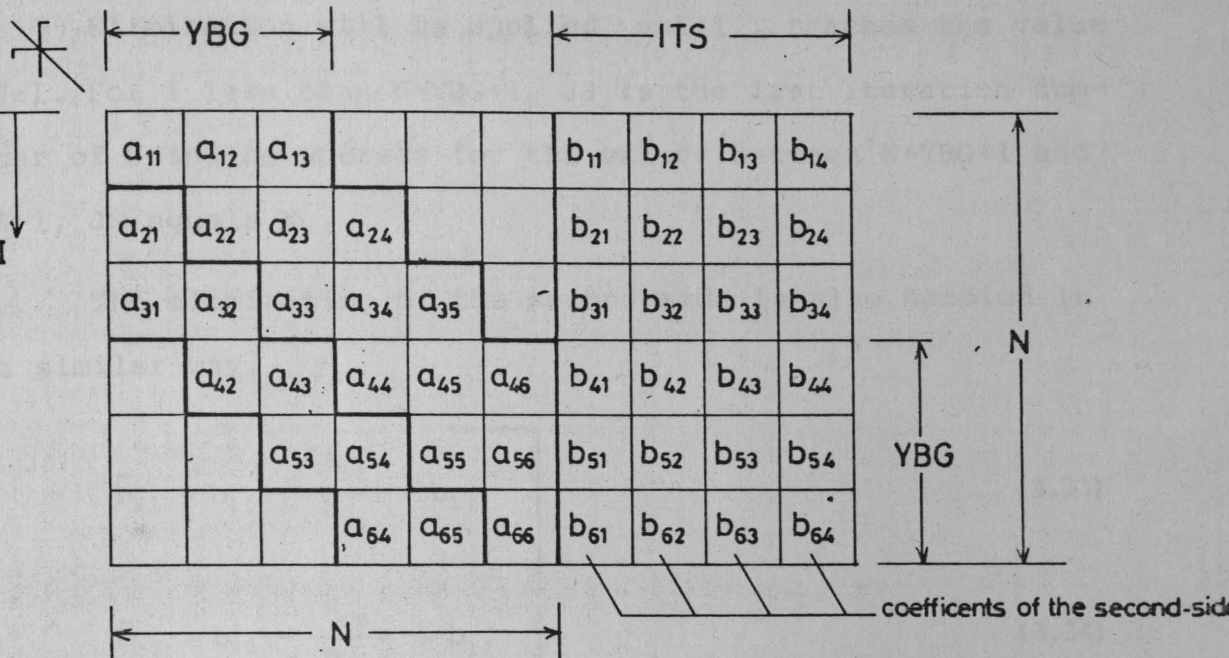

FIGURE 3.7 Gaussian-elimination of symmetrical matrices

I=1, N-YBG+1

J=1,JS   JS=1, I+YBG-1

$$\bar{a}_{22} = a_{22} - \frac{a_{12}}{a_{11}} * a_{21} = a_{22} - \frac{a_{21}}{a_{11}} * a_{12} = a_{22} - \frac{a_{12}}{a_{11}} * a_{12} \qquad (3.31)$$

$$\bar{a}_{23} = a_{23} - \frac{a_{13}}{a_{11}} * a_{21} = a_{23} - \frac{a_{21}}{a_{11}} * a_{13} = a_{23} - \frac{a_{12}}{a_{11}} * a_{13} \qquad (3.32)$$

.
.
.

The gaussian-eliminated elements of the element stiffness matrice will be calculated as shown in equations 3.31 and 3.32 and the iteration will proceeduntil the row number I reaches to the value N-YBG+1

Elimination will be applied until I reaches the value N-1. For I less than N-YBG+1, JS is the last iteration number of J indice whereas for the values between N-YBG+1 and N-1, JS equals N.

The elimination of the second side is also handled in a similar way.

$$\bar{b}_{21} = b_{21} - \frac{a_{12}}{a_{11}} * b_{11} \tag{3.33}$$

$$\bar{b}_{22} = b_{22} - \frac{a_{12}}{a_{11}} * b_{12} \tag{3.34}$$

$$\vdots \qquad J = 1,ITS$$

It is easily proved that the eliminated band matrice will be symmetrical as well. As an example,

$$\bar{a}_{32} = a_{32} - \frac{a_{31}}{a_{11}} * a_{12} = a_{23} - \frac{a_{12}}{a_{11}} * a_{13} = \bar{a}_{23}$$

3.4 CALCULATION OF DIRECTION COSINES:

A right-handed global coordinate system is choosen for each group which has an origin at the force of application with z-axis showing the vertical direction. The nodal coordinates of each pile is given to the program as input values. The axial direction of the pile is taken to be the axis of the pile coordinate system and the related direction cosines are calculated. Also direction cosines of the local y direction have to be given as an input value to the program. The direction cosines of the local z axis is found out from the other pre-determined direction cosines by the formula

$$C\phi\ (7) = C\phi\ (2) * C\phi\ (6) - C\phi\ (3) * C\phi\ (5) \qquad (3.35)$$

$$C\phi\ (8) = C\phi\ (3) * C\phi\ (4) - C\phi\ (1) * C\phi\ (6) \qquad (3.36)$$

$$C\phi\ (9) = C\phi\ (1) * C\phi\ (5) - C\phi\ (2) * C\phi\ (4) \qquad (3.37)$$

where

$$x = C\phi\ (1)\ \underline{i} + C\phi\ (2)\ \underline{j} + C\phi\ (3)\ \underline{k} \qquad (3.38)$$

$$y = C\phi\ (4)\ \underline{i} + C\phi\ (5)\ \underline{j} + C\phi\ (6)\ \underline{k} \qquad (3.39)$$

$$z = C\phi\ (7)\ \underline{i} + C\phi\ (8)\ \underline{j} + C\phi\ (9)\ \underline{k} \qquad (3.40)$$

and $C\phi$ is the sequence of direction cosines. Direction cosines of a vertical pile and an inclined pile are given in figures 3.8 and 3.9 respectively.

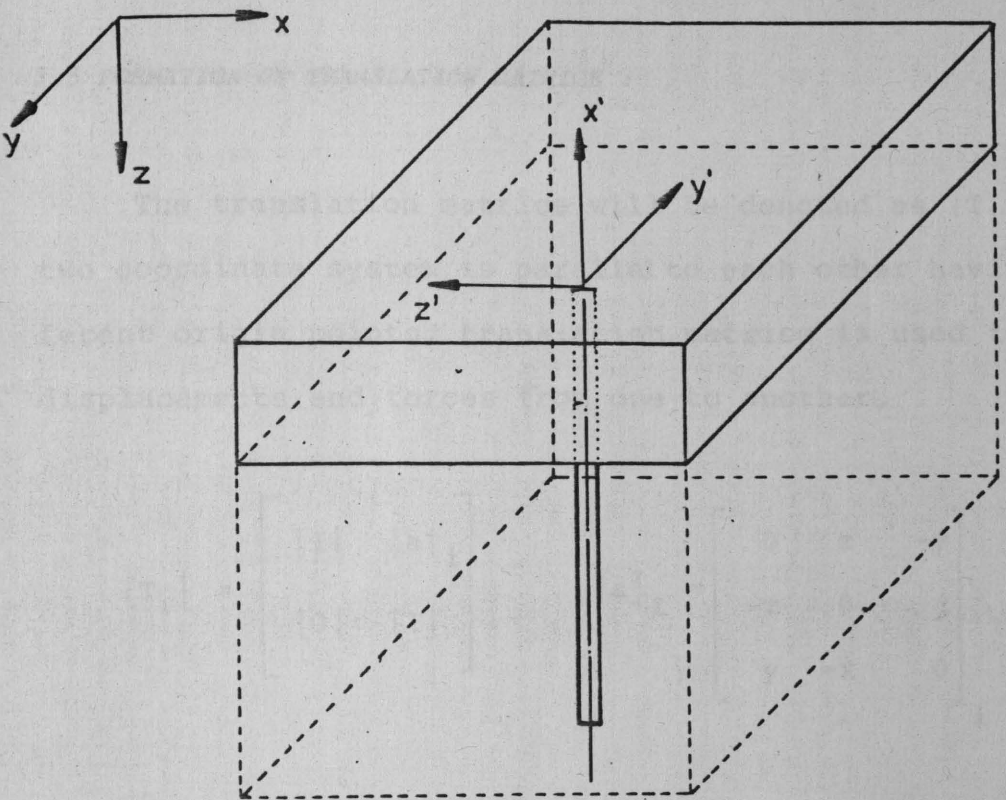

FIGURE 3.8 Direction cosines of a vertical pile

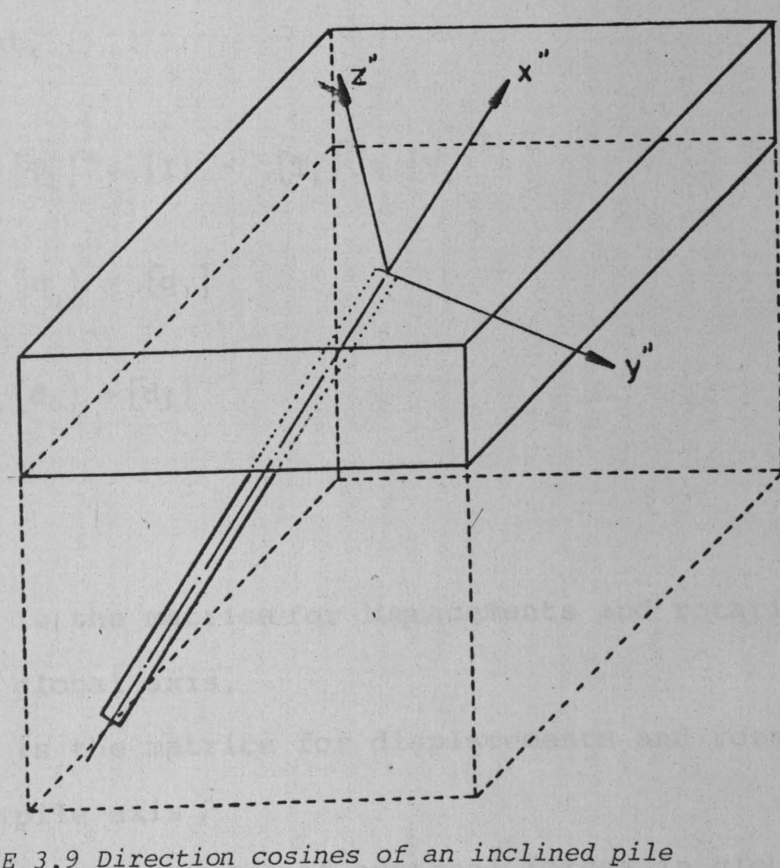

FIGURE 3.9 Direction cosines of an inclined pile

3.5 FORMATION OF TRANSLATION MATRICE :

The translation matrice will be denoted as $|T_1|$. If two coordinate system is parallel to each other having different origin points, translation matrice is used transfer displacements and forces from one to another,

$$[T_1] = \begin{bmatrix} |I| & |a|_i \\ |0| & |I| \end{bmatrix} \qquad [a]_i = \begin{bmatrix} 0 & z & -y \\ -z & 0 & j \\ y & -x & 0 \end{bmatrix}_i$$

where x,y and z are the coordinates of the each pile head according to the global axis. By definition it can be stated that,

$$[T_1].[T_1]^{-1} = [I| \rightarrow [T_1]^{-1} = [T_1|^T \qquad (3.41)$$

$$[T_1|^T.[q_o] = [q_1] \qquad (3.42)$$

$$[T_1].[d_o] = [d_i] \qquad (3.43)$$

where

$[d]_o$ is the matrice for displacements and rotations in global axis,

$[d]_i$ is the matrice for displacements and rotations in pile axis ,

$[q]_o$ is the matrice of external forces in global axis,

$[q|_i$ is the matrice of external forces in pile axis .

To write it in explicit from,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 1 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Ux \\ Uy \\ Uz \\ Wx \\ Wy \\ Wz \end{bmatrix}_0 = \begin{bmatrix} Ux \\ Uy \\ Uz \\ Wx \\ Wy \\ Wz \end{bmatrix}_i$$

thus,

$$(Ux)i = (Ux)_o + Z\ (Wy)_o - y\ (Wz)_o$$

$$(Uy)i = (Uy)_o + -z\ (Wx)_o + x\ (Wz)_o$$

$$(Uz)i = (Uz)_o + y\ (Wx)_o - x\ (Wy)_o$$

$$(Wx)i = (Wx)_o$$

$$(Wy)i = (Wy)_o$$

$$(Wz)i = (Wz)_o$$

3.6 FORMATION OF THE ROTATION MATRICE :

The rotation matrice will be denoted as $[T_2]$. If two different coordinate axis having the same origin are considered, rotation matrice transfers reactions and displacements from one coordinate axis to the other.

$$|T_2| = \left[\begin{array}{c|c} |\lambda_i| & |0| \\ \hline |0| & |\lambda_i| \end{array}\right]_{6x6} \rightarrow |T_2|^T = \left[\begin{array}{c|c} |\lambda_i|^T & |0| \\ \hline |0| & |\lambda_i|^T \end{array}\right]$$

where λ_i is

$$\lambda_i = \left[\begin{array}{c|c|c} C\phi(1) & C\phi(2) & C\phi(3) \\ \hline C\phi(4) & C\phi(5) & C\phi(6) \\ \hline C\phi(7) & C\phi(8) & C\phi(9) \end{array}\right]$$

and

$C\phi(1)$, $C\phi(2)$, $C\phi(3)$ are the direction cosines of the x axis

$C\phi(4)$, $C\phi(5)$, $C\phi(6)$ are the direction cosines of the y axis

$C\phi(7)$, $C\phi(8)$, $C\phi(9)$ are the direction cosines of the z axis

It can be easily established that

$$[T_2] \cdot [T_2]^{-1} = [I] \rightarrow [T_2]^{-1} = [T_2]^T \qquad (3.44)$$

$$[T_2]^T \cdot [d_o] = [d_1] \qquad (3.45)$$

$$[T_2] \cdot [q_o] = [q_1] \qquad (3.46)$$

In the explicit form,

$$\left[\begin{array}{ccc|ccc} C\phi(1) & C\phi(2) & C\phi(3) & 0 & 0 & 0 \\ C\phi(4) & C\phi(5) & C\phi(6) & 0 & 0 & 0 \\ C\phi(7) & C\phi(8) & C\phi(9) & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & C\phi(1) & C\phi(2) & C\phi(3) \\ 0 & 0 & 0 & C\phi(4) & C\phi(5) & C\phi(6) \\ 0 & 0 & 0 & C\phi(7) & C\phi(8) & C\phi(9) \end{array}\right] \begin{bmatrix} Ux \\ Uy \\ Uz \\ Wx \\ Wy \\ Wz \end{bmatrix}_0 = \begin{bmatrix} Ux \\ Uy \\ Uz \\ Wx \\ Wy \\ Wz \end{bmatrix}_i$$

thus,

$$(Ux)i = C\phi(1) * Ux_o + C\phi(4) * Uy_o + C\phi(7) * Uz_o$$

$$(Uy)i = C\phi(2) * Ux_o + C\phi(5) * Uy_o + C\phi(8) * Uz_o$$

$$(Uz)i = C\phi(3) * Ux_o + C\phi(6) * Uy_o + C\phi(9) * Uz_o$$

$$(Wx)i = C\phi(1) * Wx_o + C\phi(4) * Wy_o + C\phi(7) * Wz_o$$

$$(Wy)i = C\phi(2) * Wx_o + C\phi(5) * Wy_o + C\phi(8) * Wz_o$$

$$(Wz)i = C\phi(3) * Wx_o + C\phi(6) * Wy_o + C\phi(9) * Wz_o$$

3.7 FORMATION OF THE TRANSFORMATION MATRICE

Transformation matrice will be denoted as |A|. Its a combination of rotation and translation matrices and obtained by multiplication of them.

It enables the transformation of displacements and reactions from one coordinate system to another having different direction cosines and origin points. Further, it can be stated that,

$$|A| = |T_2|^T . |T_1| \quad (3.47)$$

$$|A|^T . |q_o| = |q_i| \quad (3.48)$$

$$|A| . |d_o| = |d_i| \quad (3.49)$$

3.8 FORMATION OF THE GAUSSIAN-ELIMINATED PILE HEAD STIFFNESS MATRICE

A new subrouting "KURIKR" is introduced to the program, and elements corresponding to the head node of the gaussian -eliminated pile stiffness matrice are obtained inorder to form the pile head stiffness matrice. The execution is carried twice. One for the degrees of freedom in the plane of the beam and one for the degrees of freedom perpendicular to the plane of the beam.

Furthermore, the pile head stiffness matrices are re-arranged to make transformations to the force of application possible. Both matrices are sketched in figure 3.10 and 3.11.

[IKR] =

	6	1	2	5	4	3
6	(16) k(1,1)	(34) k(1,2)	(51) k(1,3)	0	0	0
1	(34) k(2,1)	(17) k(2,2)	(35) k(2,3)	0	0	0
2	(51) k(3,1)	(35) k(3,2)	(18) k(3,3)	0	0	0
5	0	0	0	(16) k(4,4)	(34) k(4,5)	(51) k(4,6)
4	0	0	0	(34) k(5,4)	(17) k(5,5)	(35) k(5,6)
3	0	0	0	(51) k(6,4)	(35) k(6,5)	(18) k(6,6)

Influences in the plane of the beam

First execution

Influences perpendicular to the plane of the beam

Second execution

FIGURE 3.10 Gaussian-eliminated pile head stiffness matrice

[NIKR]=

	1	2	3	4	5	6
1	k(2,2)	k(2,3)	0	0	0	k(2,1)
2	k(3,2)	k(3,3)	0	0	0	k(3.1)
3	0	0	k(6,6)	k(6,5)	k(6.4)	0
4	0	0	k(5,6)	k(5,5)	k(5.4)	0
5	0	0	k(4,6)	k(4,5)	k(4,4)	0
6	k(1,2)	k(1,3)	0	0	0	k(1,1)

FIGURE 3.11 Re-arrangement of pile head stiffness matrice.

3.9 FORMULATION OF THE EQUIVALENT PILE-GROUP STIFFNESS MATRICE :

The Gaussian-eliminated and re-arranged pile head stiffness matrices are rotated and translated to the point of application of forces from the superstructure. It will be denoted as $|IDKR|_i$. By definition,

$$|IDKR|_i = |A|_i \; |NIKR|_i \; |A|_i^T \tag{3.50}$$

$$= |T_1|_i^T \; |T_2|_i \; |NIKR|_i \; |T_2|_i^T \; |T_1|_i \tag{3.51}$$

$$= \left[\begin{array}{c:c} [I] & [0] \\ \hdashline [a_i]^T & [I] \end{array}\right]_i \left[\begin{array}{c:c} [\lambda_i] & [0] \\ \hdashline [0] & [\lambda_i] \end{array}\right]_i \left[\begin{array}{ccc:ccc} k(2,2) & k(2,3) & 0 & 0 & 0 & k(2,1) \\ k(3,2) & k(3,3) & 0 & 0 & 0 & k(3,1) \\ 0 & 0 & k(6,6) & k(6,5) & k(6,4) & 0 \\ \hdashline 0 & 0 & k(5,6) & k(5,5) & k(5,4) & 0 \\ 0 & 0 & k(4,6) & k(4,5) & k(4,4) & 0 \\ k(1,2) & k(1,3) & 0 & 0 & 0 & k(1,1) \end{array}\right]_i \left[\begin{array}{c:c} [\lambda_i] & [0] \\ \hdashline [0] & [\lambda_i] \end{array}\right]_i \left[\begin{array}{c:c} [I] & [a_i] \\ \hdashline [0] & [I] \end{array}\right]_i$$

The equivalent pile group stiffness matrice is determined by the summation of each individual transformed and gaussian-eliminated pile head stiffness matrices.

$$|IDKRT| = \sum_{i=1}^{n\ pile} |IDKR|_i \tag{3.52}$$

3.10 SOLUTION FOR THE DISPLACEMENTS OF THE EQUIVALENT PILE

Once the equivalent pile stiffness matrice is formed, the relation between the external force and related displacements is established by,

$$\left(\sum_{i=1}^{i=npile} |T_1|_i^T \, |T_2|_i^T \, |NIKR|_i \, |T_2|_i \, |T_1|_i \right) |d_o| = |q_o| \qquad (3.53)$$

where the loading matrice, and the resulting displacements can be explicity stated as

$$q_o = \begin{bmatrix} Qx_o \\ Qy_o \\ Qz_o \\ Mx_o \\ My_o \\ Mz_o \end{bmatrix} \qquad d_o = \begin{bmatrix} Ux_o \\ Uy_o \\ Uz_o \\ Wx_o \\ Wy_o \\ Wz_o \end{bmatrix}$$

Six equations for six unknowns are solved for each loading and the displacement and rotations at the force of application of the infinitely rigid pile cap are obtained. For each loading six different loads can be applied, three forces and three moments in space. The loads and corresponding displacements are kept in a file.

3.11 DETERMINATION OF PILE HEAD REACTIONS AND DISPLACEMENTS :

The previously calculated $|A|_i^T$ and $|NIKR|_i$ matrices are taken back to the memory from the file and by transforming the original displacements pile head displacements and reactions are determined. Therefore it can be stated that ;

$$|d|_i = |T_2|_i^T \, |T_1|_i \, |d|_o \qquad (3.54)$$

$$= |A|^T \, |d|_o \qquad (3.55)$$

$$|POC|_i = |NIKR|_i \, |d|_i \qquad (3.56)$$

where

$|d|_o$ = Pile cap displacements matrice at the force of application

$|d|_i$ = Pile head displacements matrice

$|A|^T$ = Transpose of the translation matrice

$|POC|_i$ = Pile head reactions matrice

3.12 DETERMINATION OF NODAL DISPLACEMENTS AND REACTIONS :

The pile element stiffness matrices which have been kept in a file are taken back to memory for superposition and displacements and reactions are found out for each element in an iterative solution. Once the displacements of the j^{th} node the top-most pile element is calculated; the reactions caused in the i^{th} node due to these displacements can easily be obtained. Then the displacements exerted in the i^{th} node due to the reactions in i^{th} node of the some pile element are determined which are also the reactions of the j^{th} node of $(ND-1)^{th}$ pile element. In an iterative solution; from nodal displacements the nodal reactions and from nodal reaction the nodal displacements are successively obtained until the i^{th} node of the first pile element. In figure 3.12 the relation between element stiffness matrices and nodal points are schematically shown.

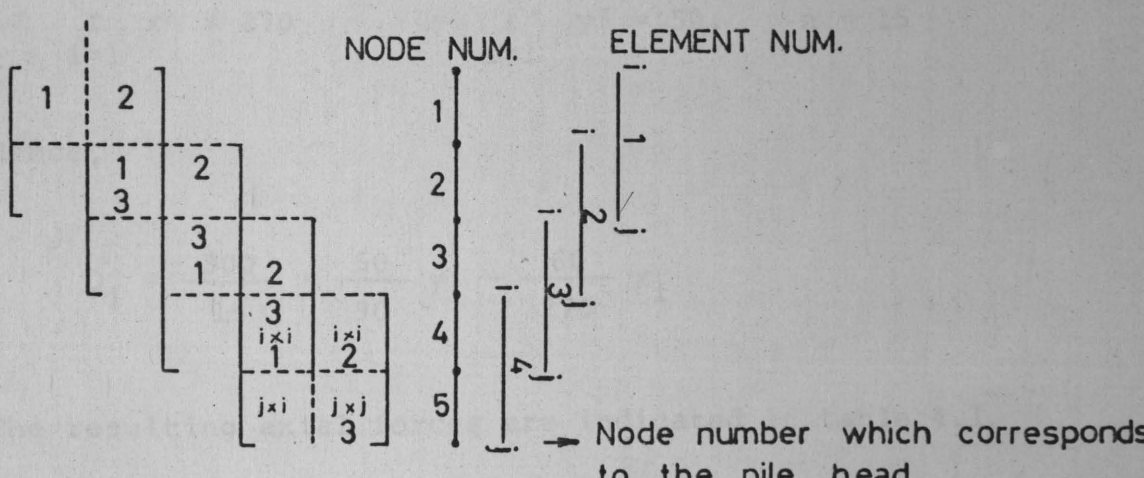

FIGURE 3.12 A schematical representation of the relation between element stiffness matrices and node numbers.

IV. COMPARISON BY OTHER CALCULATION METHODS

IV.a Example from "Grundbau Taschenbuch"

A pile group of fifteen piles is subjected to a vertical load of 300 t as well as bending moments of 50 tm and 60 tm in x and y directions respectively as shown in figure 4.1. The pile material properties are assumed to be equal. All the piles are assumed to reach the hard-bearing strata. The axial reactions arised due to these external loads are given by the formula 2.12

$$\sum_{i=1}^{15} x^2 = 270 \qquad \sum_{i=1}^{15} y^2 = 50, \qquad n = 15$$

Hence,

$$Q_i = \frac{300}{15} + \frac{50}{90} y_i - \frac{60}{270} x_i$$

The resulting axial forces are indicated in table 4.1

Pile No	1	2	3	4	5	6	7	8	9	10	11
x (m)	-6	-3	0	+3	+6	-6	-3	0	+3	+6	-6
y (m)	+3	+3	+3	+3	+3	0	0	0	0	0	-3
Q_i (t)	23.00	22.33	21.67	21.00	20.33	21.33	20.67	20.00	19.33	18.67	19.67

Pile No	12	13	14	15
x (m)	-3	0	+3	+6
y (m)	-3	-3	-3	-3
Q_i (t)	19.00	18.33	17.67	17.00

TABLE 4.1 The axial reactions of the piles for the example from "Grundbau Taschenbuch"

The same pile group is also solved by the developed programme and the resulting pile head axial reactions as well as shears, bending moments, torsional moments, displacement and rotations are obtained for each pile. The outputs are reasonably close to the results obtained by the rough calculation method as shown in table 4.2.

Pile Number	Axial forces computed by the rough calculation method	Axial forces computed by the developed programme
1	23.00	22.960
2	22.33	22.297
3	21.67	21.635
4	21.00	20.973
5	20.33	20.310
6	21.33	21.325
7	20.67	20.662
8	20.00	20.000
9	19.33	19.338
10	18.67	18.675
11	19.67	19.690
12	19.00	19.027
13	18.33	18.365
14	17.67	17.703
15	17.00	17.040

TABLE 4.2 The axial and lateral reactions of the piles given in the example by Bowles (Foundation Analysis and Design) for three different interference ratios.

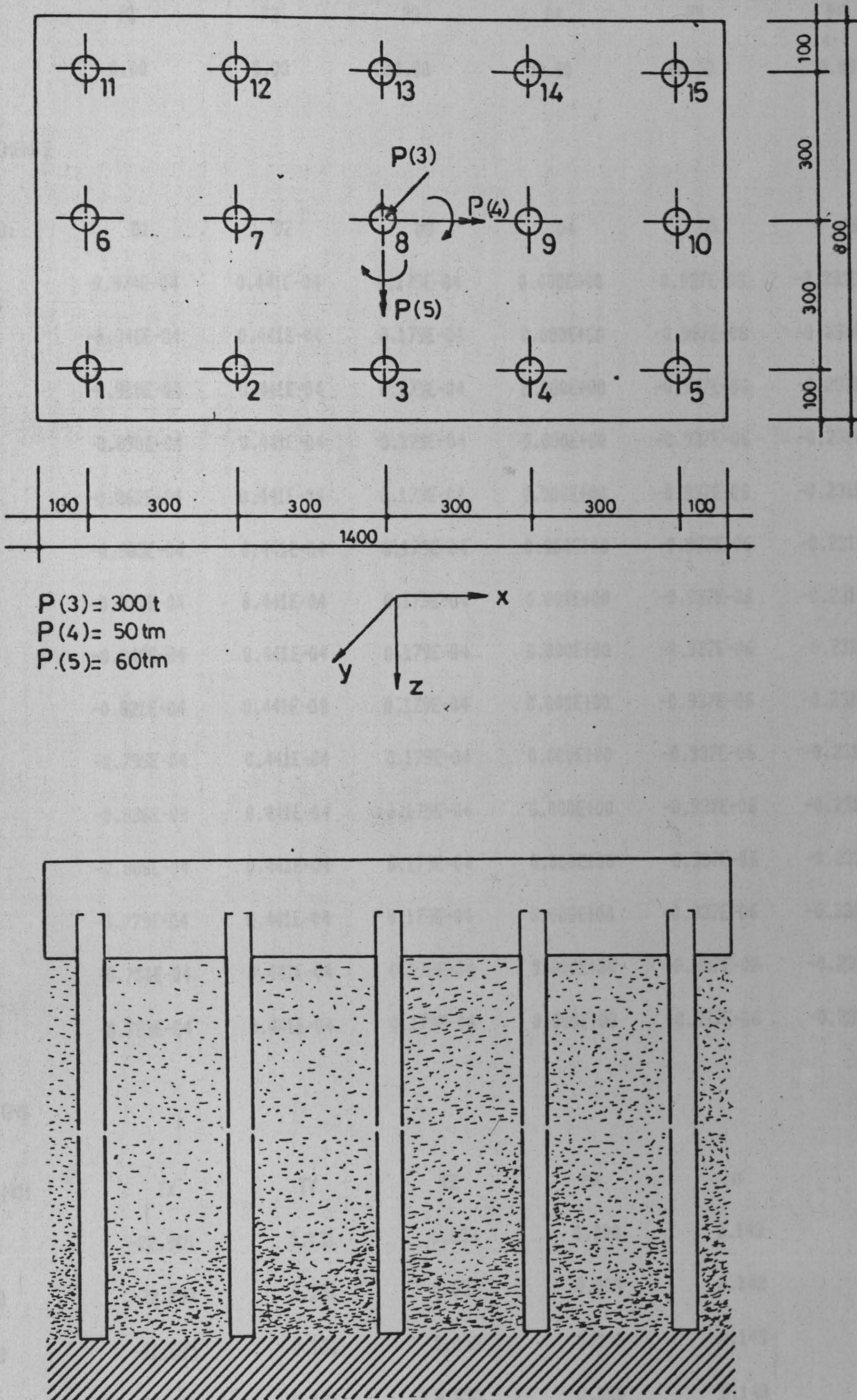

FIGURE 4.1 Example from "Grundbau Taschenbuch"

AMPLE FROM "GRUNDBAU TASCHENBUCH"(C=500t/m3)

DAL LOADS

AD NO:	P1	P2	P3	P4	P5	P6
1	0.00	0.00	300.00	50.00	60.00	0.00

ILE HEAD DISPLACEMENTS

OAD NO:	PILE NO:	D1	D2	D3	D4	D5	.D6
1	1	-0.974E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	2	-0.946E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	3	-0.918E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	4	-0.890E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	5	-0.862E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	6	-0.905E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	7	-0.877E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	8	-0.849E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	9	-0.821E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	10	-0.793E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	11	-0.836E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	12	-0.808E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	13	-0.779E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	14	-0.751E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05
1	15	-0.723E-04	0.441E-04	0.179E-04	0.000E+00	-0.937E-06	-0.231E-05

PILE HEAD REACTIONS

LOAD NO:	PILE NO:	TX	TY	TZ	MX	MY	MZ
1	1	-22.960	0.131	0.053	0.000	0.143	0.353
1	2	-22.297	0.131	0.053	0.000	0.143	0.353
1	3	-21.635	0.131	0.053	0.000	0.143	0.353
1	4	-20.973	0.131	0.053	0.000	0.143	0.353
1	5	-20.310	0.131	0.053	0.000	0.143	0.353
1	6	-21.325	0.131	0.053	0.000	0.143	0.353

1	7	-20.662	0.131	0.053	0.000	0.143	0.353
1	8	-20.000	0.131	0.053	0.000	0.143	0.353
1	9	-19.338	0.131	0.053	0.000	0.143	0.353
1	10	-18.675	0.131	0.053	0.000	0.143	0.353
1	11	-19.690	0.131	0.053	0.000	0.143	0.353
1	12	-19.027	0.131	0.053	0.000	0.143	0.353
1	13	-18.365	0.131	0.053	0.000	0.143	0.353
1	14	-17.703	0.131	0.053	0.000	0.143	0.353
1	15	-17.040	0.131	0.053	0.000	0.143	0.353

IV.b Example From "Foundation Analysis an Design, Bowles"

As a second example another pile group is solved by the developed programme. The group consists of thirteen vertical and batter piles and is subjected to a vertical force of 1000 kips as well as a horizontal load of 150 kips as shown in figure 4.2.

According to the results given by Bowles, three different solutions are possible depending on the interference ratio, r. The stiffness matrix elements and resultant axial and lateral forces are indicated in table 4.3. Although the reactions change in a big range according to the interference ratio, they are compatible to the ones obtained from the developed programme.

Matrix						Solution (-indicates compression)		
For r ratio = 0.22616						Pile No	TX (Kips) (axial force)	TZ (Kips) (lateral for
-3.249	0.000	0.000	0.000	7.003	0.000	1	-64.81	17.65
0.000	-3.559	0.000	-11.956	0.000	0.000	2	-74.09	13.70
0.000	0.000	-12.071	0.000	0.000	0.000	3	-86.12	10.84
0.000	-11.956	0.000	-388.569	0.000	0.000	4	-89.41	10.84
7.003	0.000	0.000	0.000	-408.379	0.000	5	-82.84	10.84
0.000	0.000	0.000	0.000	0.000	-202.641	6	-79.55	10.84
						7	-83.86	7.71
						8	-76.26	10.84
						9	-79.55	10.84
						10	-83.36	7.71
						11	-74.09	13.70
						12	-86.12	10.84
						13	-64.81	17.65

For r ratio = 0.00560

						Pile No	TX (Kips) (axial force)	TZ (Kips) (lateral force)
-0.470	0.000	0.000	0.000	9.000	0.000	1	10.88	3.32
0.000	-0.868	0.000	-15.364	0.000	0.000	2	-75.75	3.36
0.000	0.000	-11.806	0.000	0.000	0.000	3	-135.69	3.15
0.000	-15.364	0.000	-371.630	0.000	0.000	4	-186.68	3.15
9.000	0.000	0.000	0.000	-397.087	0.000	5	-84.69	3.15
0.000	0.000	0.000	0.000	0.000	-5.026	6	-33.70	3.15
						7	-162.40	3.02
						8	17.29	3.15
						9	-33.70	3.15
						10	-162.40	3.02
						11	-75.75	3.16
						12	-135.69	3.15
						13	10.88	3.32

For r ratio = 0.000

						Pile No	TX(Kips) (axial force)	TZ(Kips) (lateral force)
-0.400	0.000	0.000	0.000	9.050	0.000	1	82.82	0.0
0.000	-8.800	0.000	-15.450	0.000	0.000	2	-21.87	0.0
0.000	0.000	-11.800	0.000	0.000	0.000	3	-197.15	0.0
0.000	-15.450	0.000	-371.200	0.000	0.000	4	-274.67	0.0
9.050	0.000	0.000	0.000	-396.800	0.000	5	-133.25	0.0
0.000	0.000	0.000	0.000	0.000	0.000	6	-55.73	0.0
						7	-106.12	0.0
						8	8.16	0.0
						9	-69.35	0.0
						10	-128.34	0.0
						11	-58.64	0.0
						12	-210.77	0.0
						13	157.04	0.0

TABLE 4.3 The axial and lateral forces computed by the programme developed by Bowles

Pile No	TX (Kips (Axial Force)	TZ (Kips) (Lateral force)
1	132.838	21.927
2	- 3.554	21.295
3	- 36.627	20.924
4	- 69.699	20.618
5	-113.922	20.618
6	- 80.850	20.924
7	-212.960	22.330
8	-158.146	20.618
9	-191.218	20.312
10	- 3.965	20.006
11	224.270	20.633
12	-146.995	20.312
13	218.623	21.105

TABLE 4.4 The axial and lateral forces computed by the developed programme.

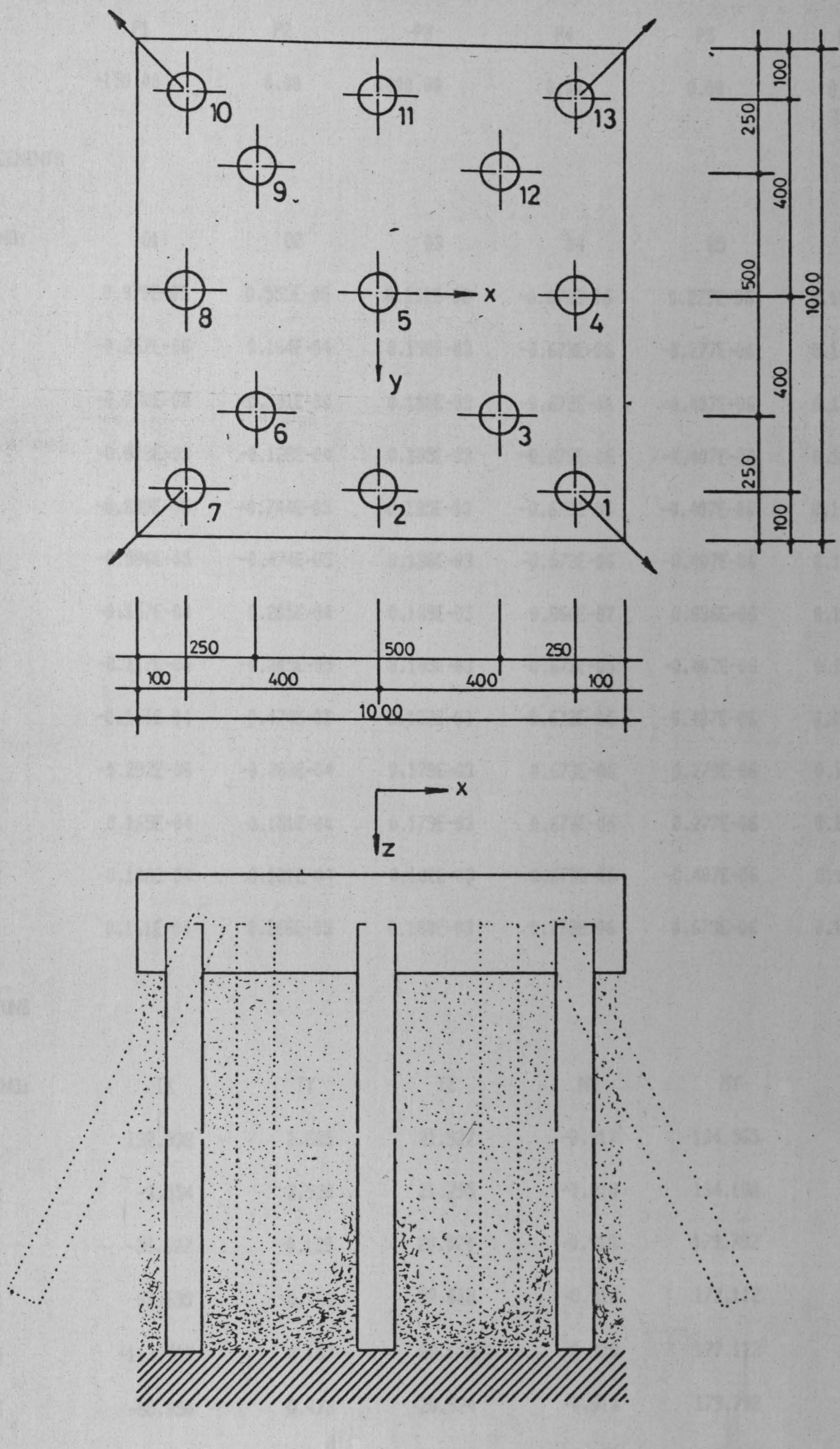

FIGURE 4.2 Example from "Foundation Analysis and Design"(Bowles)

AMPLE FROM "FOUNDATION ANALYSIS AND DESIGN,BOWLES"(C=4320lb/ft3)

OAL LOADS

AD NO:	P1	P2	P3	P4	P5	P6
1	-150.00	0.00	1000.00	0.00	0.00	0.00

ILE HEAD DISPLACEMENTS

OAD NO:	PILE NO:	D1	D2	D3	D4	D5	D6
1	1	0.979E-05	0.555E-05	0.191E-03	-0.673E-06	0.279E-06	0.106E-05
1	2	-0.262E-06	0.164E-04	0.190E-03	-0.673E-06	-0.277E-06	0.106E-05
1	3	-0.270E-05	-0.101E-04	0.188E-03	-0.673E-06	-0.407E-06	0.102E-05
1	4	-0.513E-05	-0.128E-04	0.185E-03	-0.673E-06	-0.407E-06	0.102E-05
1	5	-0.839E-05	-0.744E-05	0.185E-03	-0.673E-06	-0.407E-06	0.102E-05
1	6	-0.596E-05	-0.474E-05	0.188E-03	-0.673E-06	-0.407E-06	0.102E-05
1	7	-0.157E-04	0.205E-04	0.189E-03	0.866E-07	0.836E-06	0.106E-05
1	8	-0.117E-04	-0.205E-05	0.185E-03	-0.673E-06	-0.407E-06	0.102E-05
1	9	-0.141E-04	-0.474E-05	0.182E-03	-0.673E-06	-0.407E-06	0.102E-05
1	10	-0.292E-06	-0.283E-04	0.179E-03	0.673E-06	-0.279E-06	0.106E-05
1	11	0.165E-04	-0.151E-04	0.179E-03	0.673E-06	0.277E-06	0.106E-05
1	12	-0.108E-04	-0.101E-04	0.182E-03	-0.673E-06	-0.407E-06	0.102E-05
1	13	0.161E-04	0.266E-05	0.180E-03	0.279E-06	0.673E-06	0.106E-05

PILE HEAD REACTIONS

LOAD NO:	PILE NO:	TX	TY	TZ	MX	MY	MZ
1	1	132.838	1.685	21.927	-0.917	194.565	23.994
1	2	-3.554	2.920	21.295	-0.918	184.182	34.819
1	3	-36.627	-0.139	20.924	-0.918	179.792	7.643
1	4	-69.699	-0.445	20.618	-0.918	177.112	4.963
1	5	-113.922	0.167	20.618	-0.918	177.112	10.323
1	6	-80.850	0.473	20.924	-0.918	179.792	13.003
1	7	-212.960	3.385	22.330	0.118	202.947	38.892
1	8	-158.146	0.779	20.618	-0.918	177.112	15.683

1	9	-191.218	0.473	20.312	-0.918	174.432	13.003
1	10	-3.965	-2.157	20.006	0.917	172.863	-9.673
1	11	224.270	-0.659	20.633	0.918	183.209	3.457
1	12	-146.995	-0.139	20.312	-0.918	174.432	7.643
1	13	218.623	1.356	21.105	0.381	190.791	21.113

V. CONCLUSION AND REMARKS

Calculation of the compressive and tensile forces on a group of raking piles or combined vertical and raking piles under inclined loading is very complex, and there is no established procedure with any sound theoretical basis, since a piled foundation is a three-dimensional structure with a high degree of indeterminancy. Inorder to tackle the problem some analytical methods are proposed in the literature. However an exact solution is not available. This study is adopted elastic beam model for piles as a better approach for the group analysis of point-bearing piles. Comparison of some numerical examples given in the literature by the proposed model with other calculation methods has proved that the results obtained are in reasonable limits.

In conclusion, it can be stated that a suitable programme has been developed for the design of end-bearing pile groups which will turn tedious calculation into a practical and efficient aid to the needs of engineering design offices.

The computed axial, shear forces as well as bending and torsional moments for each single pile is also useful in the structural design of the piles. The cross-section of piles should be chosen in accordance with the reactive forces to which it is subjected to.

Another importance of the study is that the settlements of the piled foundation under the forces from superstructure can also be obtained, which gives the designer reliable information to make sure that the movements are within acceptable limits provided the pile is founded on a hard yielding strata.

The numerical computations with the developed programme has shown that the subgrade coefficient, c, does not effect the reactive forces and moments if the group consists of only vertical piles. However if the group includes or consists of battered piles the computed reactive forces and moments vary considerably due to the change in subgrade coefficient. Therefore if can be concluded that reliable subsoil properties are necessary for the three-dimensional analysis of pile groups including raking piles, but not so adequate if the group consists of only vertical piles.

The analysis giving the maximum pile head reactions and moments is also useful for the design of pile cap.

Another important aspect of the study is the calculation of settlements and movements if detailed subsoil data is available. Since the study is confined to point-bearing piles only, it is found out that the vertical settlements are little more than the elastic shortening of piles. The computation of movements approaching to exactness will enable the designer, to check the previously calculated support reactions which had been done assuming vertical and horizontal movements to be zero. If the movements are not within the acceptable limits, a modification of support reactions can be achieved by modelling fixed support as an elastic support undergoing the calculated amount of displacements.

The dynamic analysis of the pile groups or introduction of friction piles can be accepted as a continuation of research subjects to improve this programme for a complete analysis. The matrix methods used for this study is also very efficient in these research areas of pile group analysis.

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VII. APPENDIX

APPENDIX 1 - LIST OF THE PROGRAM

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PILFON.FTN;1             /F77/OP/TR:BLOCKS/WR

        PROGRAM PILFON
C************************************************************************
C       NPILE  :NUMBER OF PILES                                         *
C       NLOAD  :NUMBER OF LOADINGS                                      *
C       X(I)   :SEQUENCE OF X,Y,Z COORDINATES OF A SINGLE PILE          *
C       C(I)   :SEQUENCE OF SOIL MODULUS FOR VARIOUS DEPTHS FOR EACH PILE*
C       E      :ELASTICITY MODULUS FOR EACH GIVEN PILE                  *
C       MU     :POISSION'S RATIO  FOR EACH PILE                         *
C       G      :ELASTICITY MODULUS FOR TORSION: G=E/(1+2*EMU)           *
C       EL(I)  :SEQUENCE FOR THE LENGTHS OF EACH ELEMENT FOR EACH PILE  *
C       GL(I)  :SEQUENCE FOR GAMMA(L) FOR EACH PILE                     *
C       EX     :STIFFNESS AGANIST MOMENT IN X DIRECTION FOR EACH PILE   *
C       EY     :STIFFNESS AGANIST MOMENT IN Y DIRECTION FOR EACH PILE   *
C       GJ     :STIFFNESS AGANIST TORSION FOR EACH PILE                 *
C       P(I,J) :SEQUENCE OF LOADING MATRICE                             *
C************************************************************************
        COMMON/BLOCK1/SR(200),IBB,ISS,ND,IKR(6,6)
        COMMON/BLOCK2/ER(21),RN
        COMMON/BLOCK3/ NPILE,NLOAD
        COMMON/BLOCK4/ID,IREC,COF
        DIMENSION ATRANS(6,6),DEP(6,10),P(6,10),DEPOR(6,10),POC(6,10)
        REAL*4 IKR,NIKR(6,6),IDKRM(6,6),IDKRT(6,6)
        REAL*4 COF(6,6),DIS(6,10),POF(6,10)
        INTEGER RN,ROW(6)
        CHARACTER*4  BAS(17)
        DATA ROW/6,1,2,5,4,3/
        OPEN(UNIT= 1,FILE='ELEMNT',ACCESS='DIRECT',RECL=21,TYPE='SCRATCH')
        OPEN(UNIT= 2,FILE='PILE'  ,ACCESS='DIRECT',RECL=36,TYPE='SCRATCH')
        OPEN(UNIT= 3,FILE='ATRANS',ACCESS='DIRECT',RECL=36,TYPE='SCRATCH')
        OPEN(UNIT= 7,FILE='NIKR'  ,ACCESS='DIRECT',RECL=36,TYPE='SCRATCH')
        OPEN(UNIT= 9,FILE='DEPOR' ,ACCESS='DIRECT',RECL=06,TYPE='SCRATCH')
        OPEN(UNIT=10,FILE='FORCES',ACCESS='DIRECT',RECL=06,TYPE='SCRATCH')
        OPEN(UNIT=11,FILE='DISP'  ,ACCESS='DIRECT',RECL=30,TYPE='SCRATCH')
        OPEN(UNIT=12,FILE='PREAC' ,ACCESS='DIRECT',RECL=30,TYPE='SCRATCH')
C
C       A PROGRAM FOR THE CALCULATION OF GROUP CAPACITY OF PILES
C
        NTL=1
    750 READ(4,'(A1,17A4)',END=650) TOP,BAS
        IF(TOP.NE.1H*) GO TO 750
        IF(NTL.NE.1) WRITE(6,'(1H1)')
        WRITE(6,'(10X,17A4)') BAS
        READ(4,'(5I5)') NPILE,NLOAD,ND,ITR,NTR
        DO 10 J=1,NLOAD
     10 READ(4,'(6F10.0)')(P(I,J),I=1,6)
        RN=1
        DO 11 IPIL=1,NPILE
        CALL PILRI(IPIL)
     11 CONTINUE
        DO 12 I=1,6
        DO 12 J=1,6
     12 IDKRT(I,J)=0
        DO 13 IPIL=1,NPILE
        READ(2,REC=IPIL)((IDKRM(I,J),J=1,6),I=1,6)
        DO 13 I=1,6
```

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```
      DO 13 J=1,6
   13 IDKRT(I,J)=IDKRT(I,J)+IDKRM(I,J)
      CALL SOLVE(6,NLOAD,IDKRT,P,DEP)
      IF(NTR.EQ.0) GO TO 640
      WRITE(6,'(//,10X,36HSUMMATION OF PILE STIFFNESS MATRICES:)')
      WRITE(6,'(6(/10X,6E15.5))')((IDKRT(I,J),I=1,6),J=1,6)
      WRITE(6,'(//,10X,13HDISPLACEMENTS)')
      WRITE(6,'(6(/10X,6E15.5))')((DEP(I,J),I=1,6),J=1,NLOAD)
  640 JREC=1
      NREC=1
      DO 20 IPIL=1,NPILE
      READ(3,REC=IPIL)((ATRANS(I,J),J=1,6),I=1,6)
      READ(7,REC=IPIL)  ((NIKR(I,J),J=1,6),I=1,6)
      CALL MATCAR(6,NLOAD,ATRANS,DEP,DEPOR)
      CALL MATCAR(6,NLOAD,NIKR,DEPOR,POC)
      DO 21 JLOAD=1,NLOAD
      WRITE( 9,REC=JREC)(DEPOR(I,JLOAD),I=1,6)
      WRITE(10,REC=JREC)  (POC(I,JLOAD),I=1,6)
   21 JREC=JREC+1
      DO 20 IK=1,2
      IREC=2*IPIL*(ND-1)+IK-2
      DO 211 I=1,3
      DO 211 J=1,NLOAD
  211 DIS(I,J)=DEPOR(ROW(I+3*(IK-1)),J)
      CALL READER(3,ND)
      CALL MATCAR(3,NLOAD,COF,DIS,POF)
      WRITE(11,REC=NREC)((DIS(I,J),I=1,3),J=1,NLOAD)
      WRITE(12,REC=NREC)((POF(I,J),I=1,3),J=1,NLOAD)
      NREC=NREC+1
      DO 212 ID=1,ND-1
      CALL READER(2,ND)
      CALL MATCAR(3,NLOAD,COF,DIS,POF)
      CALL READER(1,ND)
      CALL SOLVE(3,NLOAD,COF,POF,DIS)
      WRITE(11,REC=NREC)((DIS(I,J),I=1,3),J=1,NLOAD)
      WRITE(12,REC=NREC)((POF(I,J),I=1,3),J=1,NLOAD)
      NREC=NREC+1
  212 IREC=IREC-2
   20 CONTINUE
      CALL OUTPUT(P,ND,ITR)
      NTL=NTL+1
      GO TO 750
  650 CLOSE(UNIT=8,DISP='DELETE')
      STOP
      END
```

PROGRAM SECTIONS

Name	Size		Attributes
$CODE1	003562	953	RW,I,CON,LCL
$PDATA	000760	248	RW,D,CON,LCL
$IDATA	000010	4	RW,D,CON,LCL
$VARS	004110	1060	RW,D,CON,LCL

```
$TEMPS  000010     4         RW,D,CON,LCL
BLOCK1  001666   475         RW,D,OVR,GBL
BLOCK2  000126    43         RW,D,OVR,GBL
BLOCK3  000004     2         RW,D,OVR,GBL
BLOCK4  000224    74         RW,D,OVR,GBL

Total Space Allocated = 013136  2863
```

```
      SUBROUTINE PILRI(IPIL)
      DIMENSION X(300),C(100),EL(100),GL(100),CO(12)
     1          ,NIKR(6,6),IDKR(6,6),IDKRM(6,6),A(6,6),ATRANS(6,6)
     2          ,ROTA(6,6),TRNSLT(6,6)
      COMMON/BLOCK1/SR(200),IBB,ISS,ND,IKR(6,6)
      COMMON/BLOCK2/ER(21),RN
      INTEGER RN,ROW(6)
      REAL MU
      REAL*4 IKR,NIKR,IDKR,IDKRM
      CHARACTER*5 KP,KPILE
      DOUBLE PRECISION PI,ALFA,ALFAY
      DATA KPILE,PI/'PILE:',3.141592654/
      DATA ROW/6,1,2,5,4,3/
      KS(I,J)=(J-I)*ND*3-(J-I-1)*(J-I)*0.5+I
      READ(4,'(I2,3X,A5)') IROW,KP
      IF(IROW.NE.IPIL.OR.KP.NE.KPILE) THEN
      WRITE(6,'(5X,I2,3X,23HPILE HEAD CARD IS WRONG)') IPIL
      GO TO 75
      ENDIF
      READ(4,61)D,E,MU,C(1),KST,KTY,(CO(I),I=4,6)
      READ(4,'(8F10.0)')(X(I),I=1,ND*3,3)
      READ(4,'(8F10.0)')(X(I),I=2,ND*3,3)
      READ(4,'(8F10.0)')(X(I),I=3,ND*3,3)
      IF(KTY.EQ.0) THEN
      DO 12 I=2,ND-1
   12 C(I)=C(1)
      ELSEIF(KTY.EQ.1) THEN
      READ(4,'(8E10.0)')(C(I),I=1,ND-1)
      ENDIF
      G=E/(2*(1+2*MU))
      IF(KST.EQ.0) THEN
      EF=E*PI*D**2/4
      EIX=E*PI*D**4/64
      EIY=EIX
      GJ=G*PI*D**4/32
      ELSEIF(KST.EQ.1) THEN
      EF=E*D**2
      EIX=E*D**4/12
      EIY=EIX
      GJ=G*D**4/6
      ENDIF
      J=1
      DO 20 I=1,3*(ND-1),3
      EL(J)=SQRT((X(I+3)-X(I))**2
     1          +(X(I+4)-X(I+1))**2
     2          +(X(I+5)-X(I+2))**2)
      GL(J)=EL(J)*SQRT(SQRT(C(J)*D*1000/(4*EIX)))
   20 J=J+1
      IL=3*ND-5
      SS=KS(IL,3*ND)
      IBB=1
      ISS=3
      DO 8.K=1,2
      DO 30 I=1,200
```

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PILFON.FTN;1               /F77/OP/TR:BLOCKS/WR

      30 SR(I)=0
         DO 9 NI=1,ND-1
         IF(K.EQ.1) THEN
         CALL ELRI(EF,EL(NI),EIX,GL(NI))
         ELSE
         CALL ELRI(GJ,EL(NI),EIY,GL(NI))
         ENDIF
         IS=(NI-1)*3+1
         JS=IS
         LS=6
         LM=1
         DO 10 L=1,6
         KL=KS(IS,JS)
         DO 11 M=1,LS
         SR(KL+M-1)=SR(KL+M-1)+ER(LM)
      11 LM=LM+1
         LS=LS-1
      10 JS=JS+1
       9 CONTINUE
         CALL ELE
         CALL KURIKR
         IBB=4
         ISS=6
       8 CONTINUE
C
C        NEW ORDER OF PILE STIFFNESS MATRICE
C
         DO 21 I=1,6
         DO 21 J=1,6
      21 NIKR(I,J)=0
         DO 22 I=1,6
         DO 22 J=1,6
      22 NIKR(ROW(I),ROW(J))=IKR(I,J)
         WRITE(7,REC=IPIL)((NIKR(I,J),J=1,6),I=1,6)
C
C        CALCULATION OF DIRECTION COSINES
C
         CO(1)=X(1)
         CO(2)=X(2)
         CO(3)=X(3)
         MS=ND*3-2
         CO(10)=X(MS)
         CO(11)=X(MS+1)
         CO(12)=X(MS+2)
         DO 31 I=1,3
      31 CO(I)=CO(9+I)-CO(I)
         DELTA=1/(SQRT(CO(1)*CO(1)+CO(2)*CO(2)+CO(3)*CO(3)))
         DO 32 I=1,3
      32 CO(I)=CO(I)*DELTA
         IF(CO(3).EQ.-1) THEN
         CO(4) =0.
         CO(5)=-1.
         CO(6) =0.
         ENDIF
         CO(7)=CO(2)*CO(6)-CO(3)*CO(5)
```

```
PDP-11 FORTRAN-77 V5.0-0        11:02:05    30-Nov-85             Page 6
PILFON.FTN;1              /F77/OP/TR:BLOCKS/WR

              CO(8)=CO(3)*CO(4)-CO(1)*CO(6)
              CO(9)=CO(1)*CO(5)-CO(2)*CO(4)
              REWIND 08
              WRITE(8,'(12F10.4)')(CO(I),I=1,12)
              REWIND 08
              READ (8,'(12F10.4)')(CO(I),I=1,12)
        C     WRITE(6,'(3F15.4)')(CO(I),I=1,12)
        C
        C     FORMATION OF THE ROTATION MATRICE
        C
              DO 41 I=1,6
              DO 41 J=1,6
           41 ROTA(I,J)=0
              DO 42 K=1,4,3
              DO 42 I=K,K+2
              DO 42 J=K,K+2
           42 ROTA(I,J)=CO(3*(I-K)+(J-K+1))
        C
        C     FORMATION OF THE TRANSPOSE OF THE TRANSLATION MATRICE
        C
              DO 51 I=1,6
              DO 51 J=1,6
              IF(I.NE.J) THEN
              TRNSLT(I,J)=0
              ELSE
              TRNSLT(I,J)=1.
              ENDIF
           51 CONTINUE
              TRNSLT(4,3)= CO(11)
              TRNSLT(5,3)=-CO(10)
              TRNSLT(6,1)=-CO(11)
              TRNSLT(6,2)= CO(10)
        C
        C     FORMATION OF THE TRANSFORMATION MATRICE
        C
              CALL MATCAR(6,6,TRNSLT,ROTA,A)
              DO 52 I=1,6
              DO 52 J=1,6
           52 ATRANS(I,J)=A(J,I)
              WRITE(3,REC=IPIL)((ATRANS(I,J),J=1,6),I=1,6)
        C
        C     ROTATION AND TRANSLATION OF GAUSSIAN ELIMINATED PILE STIFFNESS MATRICE
        C
              CALL MATCAR(6,6,NIKR,ATRANS,IDKR)
              CALL MATCAR(6,6,A,IDKR,IDKRM)
              WRITE(2,REC=IPIL)((IDKRM(I,J),J=1,6),I=1,6)
           61 FORMAT(4E10.0,2I5,3E10.0)
           75 RETURN
              END


PROGRAM SECTIONS

 Name        Size                  Attributes
```

```
PDP-11 FORTRAN-77 V5.0-0        11:02:05 . 30-Nov-85          Page 7
PILFON.FTN;1              /F77/OP/TR:BLOCKS/WR

$CODE1  005166  1339             RW,I,CON,LCL
$PDATA  000210    68             RW,D,CON,LCL
$IDATA  000032    13             RW,D,CON,LCL
$VARS   006766  1787             RW,D,CON,LCL
$TEMPS  000014     6             RW,D,CON,LCL
BLOCK1  001666   475             RW,D,OVR,GBL
BLOCK2  000126    43             RW,D,OVR,GBL

Total Space Allocated = 016446  3731
```

PDP-11 FORTRAN-77 V5.0-0 11:02:49 30-Nov-85 Page 8
PILFON.FTN;1 /F77/OP/TR:BLOCKS/WR

```
      SUBROUTINE ELRI(EF,EL,EI,GL)
      COMMON/BLOCK2/ER(21),RN
      INTEGER RN
C
C     FORMATION OF THE ELEMENT STIFFNESS MATRICE ON ELASTIC FOUNDATION
C
      CH=COSH(GL)
      CO=COS(GL)
      SH=SINH(GL)
      SI=SIN(GL)
      SHCH=SH*CH
      CHSI=CH*SI
      SHCO=SH*CO
      SICO=SI*CO
      PYDT=1./(SH*SH-SI*SI)
C
C     MEMBERS OF THE STIFFNESS MATRICE
C
      CK1=2*EI*GL/EL
      CK2=2*CK1*GL/EL
      CK3=CK2*GL/EL
C
C     FORMATION OF ELEMENT STIFFNESS MATRICE IN ONE DIMENSIONAL [ER] SEQUENCE
C
      ER(1) =CK1*(SHCH-SICO)*PYDT
      ER(2) =EF/EL
      ER(3) =CK3*(SHCH+SICO)*PYDT
      ER(4) =ER(1)
      ER(5) =ER(2)
      ER(6) =ER(3)
      ER(9) =CK2*(SH*SI*PYDT)
      ER(12)=CK2*0.5*(SH*SH+SI*SI)*PYDT
      ER(15)=ER(12)
      ER(16)=CK1*(CHSI-SHCO)*PYDT
      ER(17)=ER(2)
      ER(18)=CK3*(CHSI+SHCO)*PYDT
      ER(21)=ER(9)
      WRITE(1,REC=RN)ER
      RN=RN+1
      RETURN
      END
```

PROGRAM SECTIONS

Name	Size		Attributes
$CODE1	000650	212	RW,I,CON,LCL
$IDATA	000014	6	RW,D,CON,LCL
$VARS	000060	24	RW,D,CON,LCL
BLOCK2	000126	43	RW,D,OVR,GBL

Total Space Allocated = 001072 285

```
PDP-11 FORTRAN-77 V5.0-0          11:02:56    30-Nov-85          Page 9
PILFON.FTN;1              /F77/OP/TR:BLOCKS/WR

      SUBROUTINE KURIKR
      COMMON/BLOCK1/SR(200),IBB,ISS,ND,IKR(6,6)
      REAL*4 IKR
      M=3*ND-2
      ME=M
      K=1
      DO 10 I=IBB,ISS
      ART=ND*3
      DO 20 J=I,ISS
      IKR(I,J)=SR(ME)
      IF(I.NE.J) IKR(J,I)=IKR(I,J)
      ME=ME+ART
   20 ART=ART-1
      ME=M+K
   10 K=K+1
      RETURN
      END

PROGRAM SECTIONS

 Name     Size               Attributes

$CODE1  000314   102         RW,I,CON,LCL
$VARS   000016     7         RW,D,CON,LCL
BLOCK1  001666   475         RW,D,OVR,GBL

Total Space Allocated = 002220   584
```

```
PDP-11 FORTRAN-77 V5.0-0          11:03:00    30-Nov-85          Page 10
PILFON.FTN;1             /F77/OP/TR:BLOCKS/WR


      SUBROUTINE ELE
      INTEGER YBG,BG
      COMMON/BLOCK1/A(200),IBB,ISS,ND,IKR(6,6)
      K(I,J)=(J-I)*N-(J-I-1)*(J-I)*0.5+I
      N=3*ND
      NS=N-3
      YBG=6
      IL=N-YBG+1
      KS=K(IL,N)
C
C     ELEMINATION
C
      DO 10 I=2,NS
      IF(I.GT.IL) GO TO 19
      JS=I+YBG-2
      GO TO 20
   19 JS=N
   20 ORAN=A(K(I-1,I))/A(K(I-1,I-1))
      DO 11 J=I,JS
   11 A(K(I,J))=A(K(I,J))-ORAN*A(K(I-1,J))
   10 CONTINUE
      RETURN
      END
```

PROGRAM SECTIONS

Name	Size		Attributes
$CODE1	000622	201	RW,I,CON,LCL
$PDATA	000014	6	RW,D,CON,LCL
$IDATA	000022	9	RW,D,CON,LCL
$VARS	000026	11	RW,D,CON,LCL
$TEMPS	000004	2	RW,D,CON,LCL
BLOCK1	001556	439	RW,D,OVR,GBL

Total Space Allocated = 002470   668

```
      SUBROUTINE OUTPUT(P,ND,ITR)
      DIMENSION P(6,10),DISP(6,10),PREC(6,10)
      COMMON/BLOCK3/ NPILE,NLOAD
      INTEGER ROW(6)
      CHARACTER*2  SEQ(30),HDEP(6),HFOR(6),HLOAD(6)
      CHARACTER*10 ISM1,ISM2,ISM3
      DATA ROW/6,1,2,5,4,3/
      DATA ISM1,ISM2,ISM3/' LOAD NO: ','  PILE NO:','  NODE NO:'/
      DATA HDEP /'D1','D2','D3','D4','D5','D6'/
      DATA HFOR /'TX','TY','TZ','MX','MY','MZ'/
      DATA HLOAD/'P1','P2','P3','P4','P5','P6'/
      DATA SEQ(1),SEQ(2),SEQ(3),SEQ(4)/'ST','ND','RD','TH'/
      DATA SEQ(21),SEQ(22),SEQ(23)/'ST','ND','RD'/
      DO 110 I=4,30
      IF(I.LE.20.OR.I.GE.24) SEQ(I)=SEQ(4)
  110 CONTINUE
      WRITE(6,'(//,11X,11HNODAL LOADS)')
      WRITE(6,101)ISM1,HLOAD
      DO 10 JLOAD=1,NLOAD
      WRITE(6,102)JLOAD,(P(I,JLOAD),I=1,6)
   10 CONTINUE
      WRITE(6,'(//,11X,23HPILE HEAD DISPLACEMENTS)')
      WRITE(6,103)ISM1,ISM2,HDEP
      DO 20 JLOAD=1,NLOAD
      DO 21 IPIL=1,NPILE
      IREC=(IPIL-1)*NLOAD+JLOAD
      READ(9,REC=IREC) (DISP(I,JLOAD),I=1,6)
   21 WRITE(6,105)JLOAD,IPIL,(DISP(I,JLOAD),I=1,6)
   20 CONTINUE
      WRITE(6,'(//,11X,19HPILE HEAD REACTIONS)')
      WRITE(6,104)ISM1,ISM2,HFOR
      DO 30 JLOAD=1,NLOAD
      DO 31 IPIL=1,NPILE
      IREC=(IPIL-1)*NLOAD+JLOAD
      READ(10,REC=IREC)(PREC(I,JLOAD),I=1,6)
   31 WRITE(6,106)JLOAD,IPIL,(PREC(I,JLOAD),I=1,6)
   30 CONTINUE
      IF(ITR.EQ.0) RETURN
      WRITE(6,'(1H1,//,11X,26HPILE ELEMENT DISPLACEMENTS)')
      DO 40 IPIL=1,NPILE
      WRITE(6,'(//,10X,I2,A2,2X,5HPILE:)') IPIL,SEQ(IPIL)
      WRITE(6,103)ISM1,ISM3,HDEP
      DO 40 ID=1,ND
      NREC=2*ND*(IPIL-1)+ND-ID+1
      READ(11,REC=NREC)((DISP(ROW(I),J),I=1,3),J=1,NLOAD)
      NREC=NREC+ND
      READ(11,REC=NREC)((DISP(ROW(I),J),I=4,6),J=1,NLOAD)
      WRITE(6,105)(J,ID,(DISP(I,J),I=1,6),J=1,NLOAD)
   40 CONTINUE
      WRITE(6,'(1H1,//,11X,22HPILE ELEMENT REACTIONS)')
      DO 50 IPIL=1,NPILE
      WRITE(6,'(//,10X,I2,A2,2X,5HPILE:)') IPIL,SEQ(IPIL)
      WRITE(6,104)ISM1,ISM3,HFOR
      DO 50 ID=1,ND
```

```
      NREC=2*ND*(IPIL-1)+ND-ID+1
      READ(12,REC=NREC)((PREC(ROW(I),J),I=1,3),J=1,NLOAD)
      NREC=NREC+ND
      READ(12,REC=NREC)((PREC(ROW(I),J),I=4,6),J=1,NLOAD)
      WRITE(6,106)(J,ID,(PREC(I,J),I=1,6),J=1,NLOAD)
   50 CONTINUE
  101 FORMAT(//10X,A10,21X,6(A2,13X))
  102 FORMAT(/15X,I2,12X,6F15.2)
  103 FORMAT(//10X,2A10,12X,6(A2,13X))
  104 FORMAT(//10X,2A10,13X,6(A2,13X))
  105 FORMAT(/15X,I2,8X,I2,5X,6E15.3)
  106 FORMAT(/15X,I2,8X,I2,5X,6F15.3)
      RETURN
      END
```

PROGRAM SECTIONS

Name	Size		Attributes
$CODE1	004050	1044	RW,I,CON,LCL
$PDATA	000424	138	RW,D,CON,LCL
$IDATA	000042	17	RW,D,CON,LCL
$VARS	001170	316	RW,D,CON,LCL
$TEMPS	000006	3	RW,D,CON,LCL
BLOCK3	000004	2	RW,D,OVR,GBL

Total Space Allocated = 005740 1520

No FPP Instructions Generated

```
      SUBROUTINE SOLVE(N,MD,COEFMA,LOAD,X)
C
C     SOLUTION FOR LINEAR EQUATIONS BY MEANS OF GAUSSIAN ELIMINATION
C
      DIMENSION D(6,16),X(6,10)
      REAL*4 COEFMA(6,6),LOAD(6,10)
      DO 100 I=1,N
      DO 100 J=1,N+MD
      IF(J.LE.N) THEN
      D(I,J)=COEFMA(I,J)
      ELSE
      D(I,J)=LOAD(I,J-N)
      ENDIF
  100 CONTINUE
C
C     ELEMINATION
C
      DO 10 K=1,N-1
      DO 10 I=K+1,N
      OR=D(I,K)/D(K,K)
      DO 10 J=1,N+MD
   10 D(I,J)=D(I,J)-OR*D(K,J)
C
C     SUBSTITUTION
C
      DO 80 I=1,N
      DO 40 L=1,MD
      X(N,L)=D(N,N+L)/D(N,N)
      DO 40 M=N-1,1,-1
      TOP=0
      DO 50 J=M+1,N
   50 TOP=TOP+D(M,J)*X(J,L)
      D(M,N+L)=D(M,N+L)-TOP
   40 X(M,L)=D(M,N+L)/D(M,M)
   80 CONTINUE
      RETURN
      END
```

PROGRAM SECTIONS

Name	Size		Attributes
$CODE1	001464	410	RW,I,CON,LCL
$IDATA	000036	15	RW,D,CON,LCL
$VARS	000622	201	RW,D,CON,LCL
$TEMPS	000014	6	RW,D,CON,LCL

Total Space Allocated = 002360   632

```
      SUBROUTINE MATCAR(IM,IL,Q,R,S)
      REAL*4 Q(6,6),R(6,10),S(6,10)
      DO 10 I=1,IM
      DO 10 J=1,IL
      S(I,J)=0
      DO 10 K=1,IM
   10 S(I,J)=S(I,J)+Q(I,K)*R(K,J)
      RETURN
      END
```

PROGRAM SECTIONS

Name	Size		Attributes
$CODE1	000366	123	RW,I,CON,LCL
$IDATA	000036	15	RW,D,CON,LCL
$VARS	000006	3	RW,D,CON,LCL
$TEMPS	000006	3	RW,D,CON,LCL

Total Space Allocated = 000440 144

```
      SUBROUTINE READER(NR,ND)
      COMMON/BLOCK4/ID,IREC,COF
      DIMENSION COEFFI(6,6),COF(6,6),ER(21),INTI(3),INTJ(3)
      DATA INTI/0,0,1/
      DATA INTJ/0,1,1/
      KS(I,J)=(J-I)*6-(J-I-1)*(J-I)*0.5+I
      READ(1,REC=IREC)(ER(I),I=1,21)
      DO 10 I=1,6
      DO 10 J=I,6
      COEFFI(I,J)=ER(KS(I,J))
   10 COEFFI(J,I)=COEFFI(I,J)
      DO 11 I=1,3
      DO 11 J=1,3
   11 COF(I,J)=COEFFI((3*INTI(NR)+I),(3*INTJ(NR)+J))
      IF(NR.EQ.1.AND.ND.GT.2.AND.ID.LT.(ND-1)) THEN
      READ(1,REC=IREC-2)(ER(I),I=1,21)
      NR=3
      DO 20 I=1,6
      DO 20 J=I,6
   20 COEFFI(I,J)=ER(KS(I,J))
      DO 21 I=1,3
      DO 21 J=I,3
   21 COF(I,J)=COF(I,J)+COEFFI((3*INTI(NR)+I),(3*INTJ(NR)+J))
      ENDIF
      RETURN
      END
```

PROGRAM SECTIONS

Name	Size		Attributes
$CODE1	001360	376	RW,I,CON,LCL
$PDATA	000006	3	RW,D,CON,LCL
$VARS	000364	122	RW,D,CON,LCL
$TEMPS	000002	1	RW,D,CON,LCL
BLOCK4	000224	74	RW,D,OVR,GBL

Total Space Allocated = 002200   576

APPENDIX 2 - INPUT DATA FORMAT

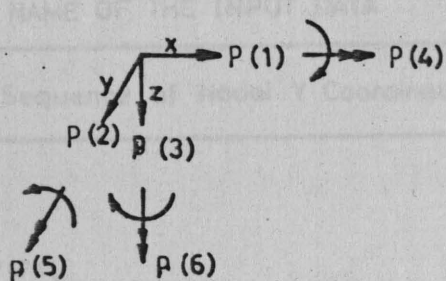
INPUT DATA FORMAT

TITLE

ROW NO	SYMBOL	COLUMN NO	NAME OF THE INPUT DATA	FORMAT
1	TOP	1	'*'	A1
2	BAS	2_77	Heading	19A4

SYSTEM PROPERTIES

ROW NO	SYMBOL	COLUMN NO	NAME OF THE INPUT DATA	FORMAT
1	NPILE	1_5	Number of Piles	I5
2	NLOAD	6_10	Number of Loadings	I5
3	ND	11_15	Number of Node Numbers	I5
4	ITR	16_20	Key for Printing Element Displacements and Reactions	I5
5	NTR	21_24	Key for Printing Equivalent Pile Stiffness Matrix and Displacements	I5



LOADINGS

ROW NO	SYMBOL	COLUMN NO	NAME OF THE INPUT DATA	FORMAT
1	P (1)	1_10	Force in X direction	F10.0
2	P (2)	11_20	Force in Y direction	F10.0
3	P (3)	21_30	Force in Z direction	F10.0
4	P (4)	31_40	Moment in X direction	F10.0
5	P (5)	41_50	Moment in Y direction	F10.0
6	P (6)	51_60	Moment in Z direction	F10.0

PILE PROPERTIES

ROW NO	SYMBOL	COLUMN NO	NAME OF THE INPUT DATA	FORMAT
1	D	1_10	Pile Diameter	E10.0
2	E	11_20	Elasticity Modulus	E10.0
3	MU	21_30	Poisson's Ratio	E10.0
4	C	31_40	Soil Modulus (If Constant)	E10.0
5	KST	41_45	0_ for Circle 1_ for Square	I5
6	KTY	46_50	0_If Soil Modulus is Constant 1_ If Soil Modulus is Variable	I5

NODAL X COORDINATES

ROW NO	SYMBOL	COLUMN NO	NAME OF THE INPUT DATA	FORMAT
1_2_3_4 5_6_7_8	X(I), I=1,4,7,10...	1_80	Sequence of Nodal X Coordinates	8F10.0

NODAL Y COORDINATES

ROW NO	SYMBOL	COLUMN NO	NAME OF THE INPUT DATA	FORMAT
1_2_3_4 5_6_7_8	X(I), I=2,5,8,11...	1_80	Sequence of Nodal Y Coordinates	8F10.0

NODAL Z COORDINATES

ROW NO	SYMBOL	COLUMN NO	NAME OF THE INPUT DATA	FORMAT
1_2_3_4 5_6_7_8	X(I) I=3,6,9,12...	1_80	Sequence of Nodal Z Coordinates	8F10.0

SEQUENCE OF SOIL MODULUS

ROW NO	SYMBOL	COLUMN NO	NAME OF THE INPUT DATA	FORMAT
1_2_3_4 5_6_7_8	C(I), I=1,2,3...	1_80	Sequence of Soil Modulus if KTY=1	8F10.0

APPENDIX 3 - NUMERICAL EXAMPLES

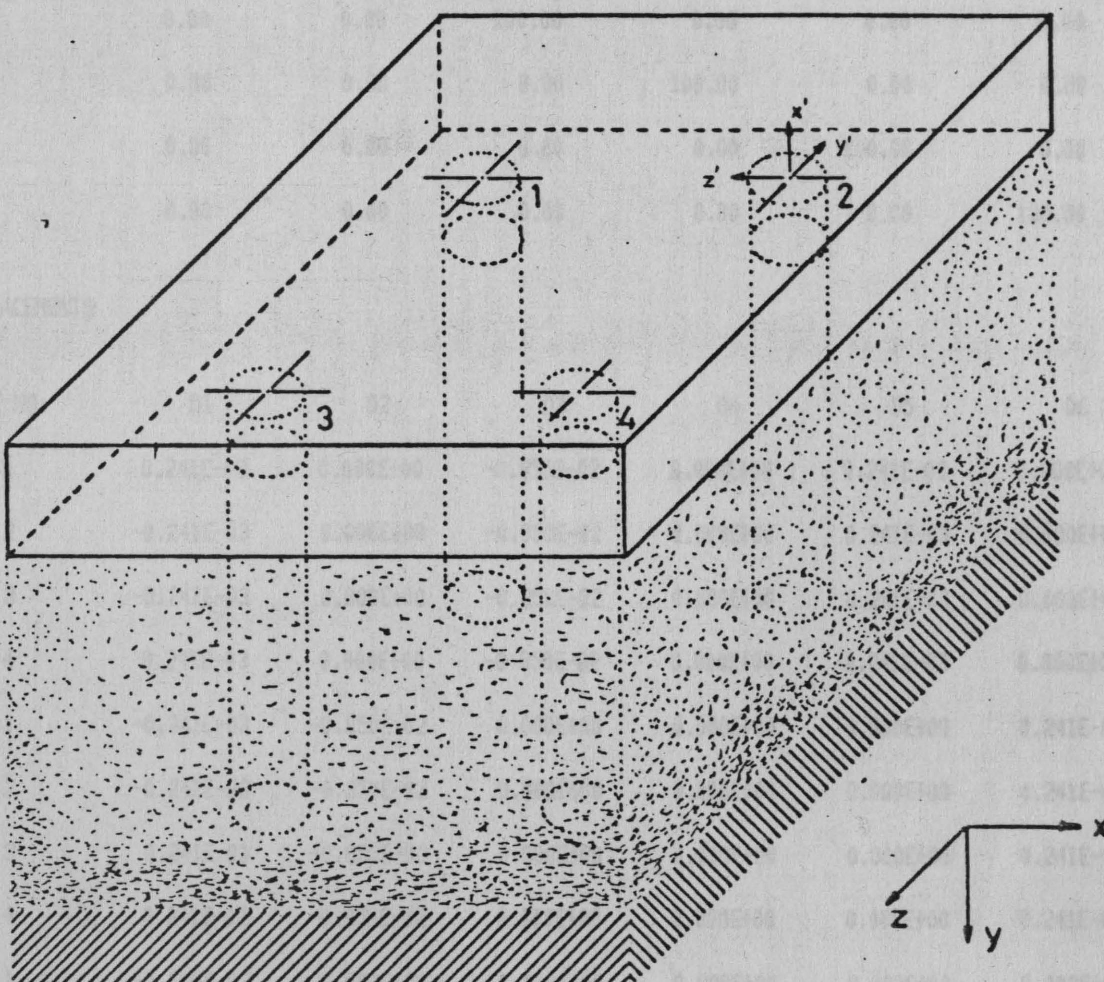


FIGURE A-1 Example number one

EXAMPLE NUMBER ONE(C=1000t/m3)

NODAL LOADS

LOAD NO:	P1	P2	P3	P4	P5	P6
1	100.00	0.00	0.00	0.00	0.00	0.00
2	0.00	100.00	0.00	0.00	0.00	0.00
3	0.00	0.00	100.00	0.00	0.00	0.00
4	0.00	0.00	0.00	100.00	0.00	0.00
5	0.00	0.00	0.00	0.00	100.00	0.00
6	0.00	0.00	0.00	0.00	0.00	100.00

PILE HEAD DISPLACEMENTS

LOAD NO:	PILE NO:	D1	D2	D3	D4	D5	D6
1	1	0.241E-03	0.000E+00	-0.850E-02	0.000E+00	0.241E-03	0.000E+00
1	2	-0.241E-03	0.000E+00	-0.850E-02	0.000E+00	0.241E-03	0.000E+00
1	3	-0.241E-03	0.000E+00	-0.850E-02	0.000E+00	0.241E-03	0.000E+00
1	4	0.241E-03	0.000E+00	-0.850E-02	0.000E+00	0.241E-03	0.000E+00
2	1	-0.241E-03	-0.850E-02	0.000E+00	0.000E+00	0.000E+00	0.241E-03
2	2	-0.241E-03	-0.850E-02	0.000E+00	0.000E+00	0.000E+00	0.241E-03
2	3	0.241E-03	-0.850E-02	0.000E+00	0.000E+00	0.000E+00	0.241E-03
2	4	0.241E-03	-0.850E-02	0.000E+00	0.000E+00	0.000E+00	0.241E-03
3	1	-0.106E-03	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	2	-0.106E-03	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	3	-0.106E-03	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	4	-0.106E-03	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4	1	0.933E-04	0.145E-02	0.000E+00	0.000E+00	0.000E+00	-0.933E-04
4	2	0.933E-04	0.145E-02	0.000E+00	0.000E+00	0.000E+00	-0.933E-04
4	3	-0.933E-04	0.145E-02	0.000E+00	0.000E+00	0.000E+00	-0.933E-04
4	4	-0.933E-04	0.145E-02	0.000E+00	0.000E+00	0.000E+00	-0.933E-04
5	1	-0.933E-04	0.000E+00	0.145E-02	0.000E+00	-0.933E-04	0.000E+00
5	2	0.933E-04	0.000E+00	0.145E-02	0.000E+00	-0.933E-04	0.000E+00
5	3	0.933E-04	0.000E+00	0.145E-02	0.000E+00	-0.933E-04	0.000E+00

5	4	-0.933E-04	0.000E+00	0.145E-02	0.000E+00	-0.933E-04	0.000E+00
6	1	0.000E+00	0.119E-02	-0.119E-02	-0.119E-02	0.000E+00	0.000E+00
6	2	0.000E+00	-0.119E-02	-0.119E-02	-0.119E-02	0.000E+00	0.000E+00
6	3	0.000E+00	-0.119E-02	0.119E-02	-0.119E-02	0.000E+00	0.000E+00
6	4	0.000E+00	0.119E-02	0.119E-02	-0.119E-02	0.000E+00	0.000E+00

ILE HEAD REACTIONS

OAD NO:	PILE NO:	TX	TY	TZ	MX	MY	MZ
1	1	56.899	0.000	-41.438	0.000	-99.430	0.000
1	2	-56.899	0.000	-41.438	0.000	-99.430	0.000
1	3	-56.899	0.000	-41.438	0.000	-99.430	0.000
1	4	56.899	0.000	-41.438	0.000	-99.430	0.000
2	1	-56.899	-41.438	0.000	0.000	0.000	-99.430
2	2	-56.899	-41.438	0.000	0.000	0.000	-99.430
2	3	56.899	-41.438	0.000	0.000	0.000	-99.430
2	4	56.899	-41.438	0.000	0.000	0.000	-99.430
3	1	-25.000	0.000	0.000	0.000	0.000	0.000
3	2	-25.000	0.000	0.000	0.000	0.000	0.000
3	3	-25.000	0.000	0.000	0.000	0.000	0.000
3	4	-25.000	0.000	0.000	0.000	0.000	0.000
4	1	21.992	6.353	0.000	0.000	0.000	13.430
4	2	21.992	6.353	0.000	0.000	0.000	13.430
4	3	-21.992	6.353	0.000	0.000	0.000	13.430
4	4	-21.992	6.353	0.000	0.000	0.000	13.430
5	1	-21.992	0.000	6.353	0.000	13.430	0.000
5	2	21.992	0.000	6.353	0.000	13.430	0.000
5	3	21.992	0.000	6.353	0.000	13.430	0.000
5	4	-21.992	0.000	6.353	0.000	13.430	0.000
6	1	0.000	6.251	-6.251	-12.497	-16.174	16.174
6	2	0.000	-6.251	-6.251	-12.497	-16.174	-16.174
6	3	0.000	-6.251	6.251	-12.497	16.174	-16.174
6	4	0.000	6.251	6.251	-12.497	16.174	16.174

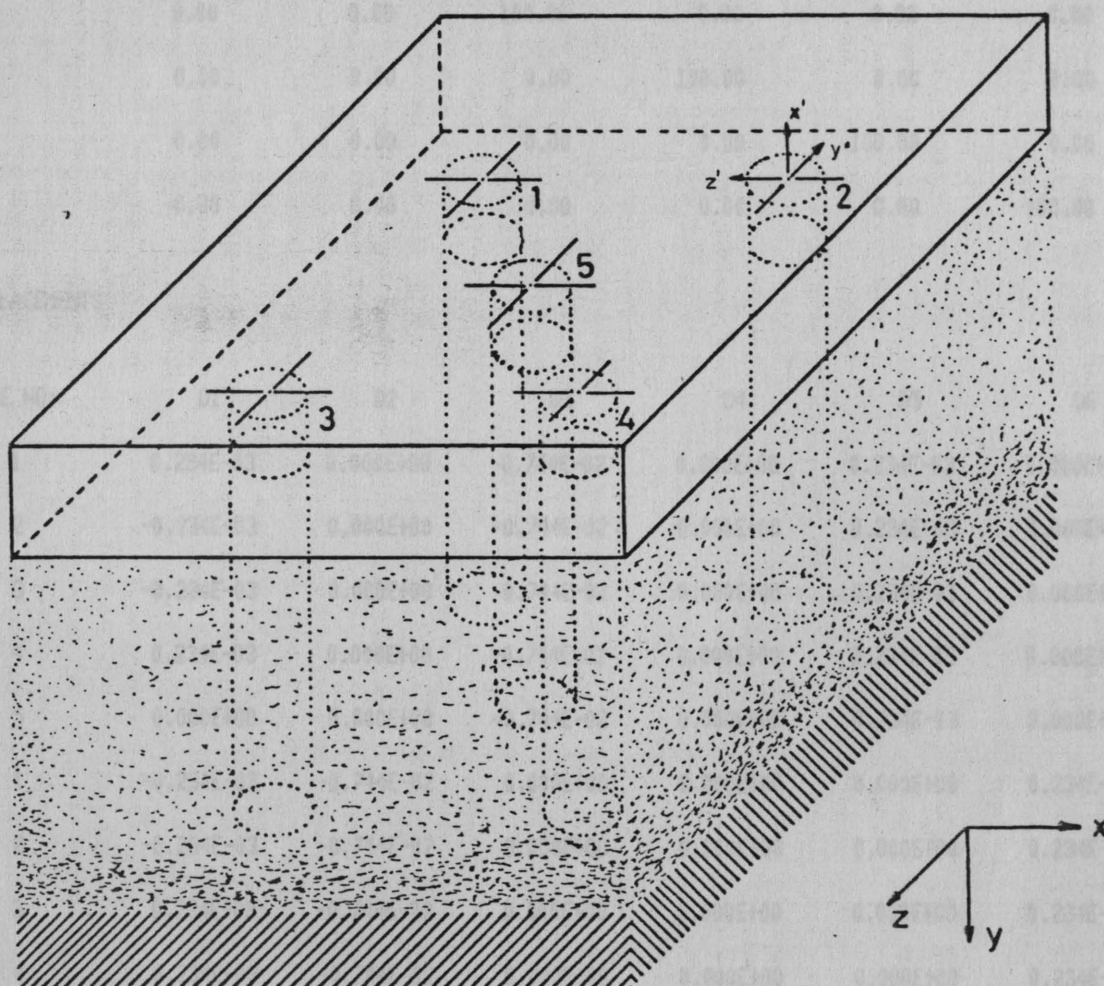


FIGURE A-2 Example number two

EXAMPLE NUMBER TWO(C=1000t/m3)

NODAL LOADS

LOAD NO:	P1	P2	P3	P4	P5	P6
1	100.00	0.00	0.00	0.00	0.00	0.00
2	0.00	100.00	0.00	0.00	0.00	0.00
3	0.00	0.00	100.00	0.00	0.00	0.00
4	0.00	0.00	0.00	100.00	0.00	0.00
5	0.00	0.00	0.00	0.00	100.00	0.00
6	0.00	0.00	0.00	0.00	0.00	100.00

PILE HEAD DISPLACEMENTS

LOAD NO:	PILE NO:	D1	D2	D3	D4	D5	D6
1	1	0.234E-03	0.000E+00	-0.744E-02	0.000E+00	0.234E-03	0.000E+00
1	2	-0.234E-03	0.000E+00	-0.744E-02	0.000E+00	0.234E-03	0.000E+00
1	3	-0.234E-03	0.000E+00	-0.744E-02	0.000E+00	0.234E-03	0.000E+00
1	4	0.234E-03	0.000E+00	-0.744E-02	0.000E+00	0.234E-03	0.000E+00
1	5	0.000E+00	0.000E+00	-0.744E-02	0.000E+00	0.234E-03	0.000E+00
2	1	-0.234E-03	-0.744E-02	0.000E+00	0.000E+00	0.000E+00	0.234E-03
2	2	-0.234E-03	-0.744E-02	0.000E+00	0.000E+00	0.000E+00	0.234E-03
2	3	0.234E-03	-0.744E-02	0.000E+00	0.000E+00	0.000E+00	0.234E-03
2	4	0.234E-03	-0.744E-02	0.000E+00	0.000E+00	0.000E+00	0.234E-03
2	5	0.000E+00	-0.744E-02	0.000E+00	0.000E+00	0.000E+00	0.234E-03
3	1	-0.849E-04	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	2	-0.849E-04	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	3	-0.849E-04	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	4	-0.849E-04	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	5	-0.849E-04	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4	1	0.906E-04	0.141E-02	0.000E+00	0.000E+00	0.000E+00	-0.906E-04
4	2	0.906E-04	0.141E-02	0.000E+00	0.000E+00	0.000E+00	-0.906E-04
4	3	-0.906E-04	0.141E-02	0.000E+00	0.000E+00	0.000E+00	-0.906E-04
4	4	-0.906E-04	0.141E-02	0.000E+00	0.000E+00	0.000E+00	-0.906E-04

4	5	0.000E+00	0.141E-02	0.000E+00	0.000E+00	0.000E+00	-0.906E-04
5	1	-0.906E-04	0.000E+00	0.141E-02	0.000E+00	-0.906E-04	0.000E+00
5	2	0.906E-04	0.000E+00	0.141E-02	0.000E+00	-0.906E-04	0.000E+00
5	3	0.906E-04	0.000E+00	0.141E-02	0.000E+00	-0.906E-04	0.000E+00
5	4	-0.906E-04	0.000E+00	0.141E-02	0.000E+00	-0.906E-04	0.000E+00
5	5	0.000E+00	0.000E+00	0.141E-02	0.000E+00	-0.906E-04	0.000E+00
6	1	0.000E+00	0.106E-02	-0.106E-02	-0.106E-02	0.000E+00	0.000E+00
6	2	0.000E+00	-0.106E-02	-0.106E-02	-0.106E-02	0.000E+00	0.000E+00
6	3	0.000E+00	-0.106E-02	0.106E-02	-0.106E-02	0.000E+00	0.000E+00
6	4	0.000E+00	0.106E-02	0.106E-02	-0.106E-02	0.000E+00	0.000E+00
6	5	0.000E+00	0.000E+00	0.000E+00	-0.106E-02	0.000E+00	0.000E+00

PILE HEAD REACTIONS

LOAD NO:	PILE NO:	TX	TY	TZ	MX	MY	MZ
1	1	55.238	0.000	-35.958	0.000	-85.479	0.000
1	2	-55.238	0.000	-35.958	0.000	-85.479	0.000
1	3	-55.238	0.000	-35.958	0.000	-85.479	0.000
1	4	55.238	0.000	-35.958	0.000	-85.479	0.000
1	5	0.000	0.000	-35.958	0.000	-85.479	0.000
2	1	-55.238	-35.958	0.000	0.000	0.000	-85.479
2	2	-55.238	-35.958	0.000	0.000	0.000	-85.479
2	3	55.238	-35.958	0.000	0.000	0.000	-85.479
2	4	55.238	-35.958	0.000	0.000	0.000	-85.479
2	5	0.000	-35.958	0.000	0.000	0.000	-85.479
3	1	-20.000	0.000	0.000	0.000	0.000	0.000
3	2	-20.000	0.000	0.000	0.000	0.000	0.000
3	3	-20.000	0.000	0.000	0.000	0.000	0.000
3	4	-20.000	0.000	0.000	0.000	0.000	0.000
3	5	-20.000	0.000	0.000	0.000	0.000	0.000
4	1	21.349	6.168	0.000	0.000	0.000	13.038
4	2	21.349	6.168	0.000	0.000	0.000	13.038
4	3	-21.349	6.168	0.000	0.000	0.000	13.038

4	5	0.000	6.168	0.000	0.000	0.000	13.038
5	1	-21.349	0.000	6.168	0.000	13.038	0.000
5	2	21.349	0.000	6.168	0.000	13.038	0.000
5	3	21.349	0.000	6.168	0.000	13.038	0.000
5	4	-21.349	0.000	6.168	0.000	13.038	0.000
5	5	0.000	0.000	6.168	0.000	13.038	0.000
6	1	0.000	5.557	-5.557	-11.109	-14.378	14.378
6	2	0.000	-5.557	-5.557	-11.109	-14.378	-14.378
6	3	0.000	-5.557	5.557	-11.109	14.378	-14.378
6	4	0.000	5.557	5.557	-11.109	14.378	14.378
6	5	0.000	0.000	0.000	-11.109	0.000	0.000

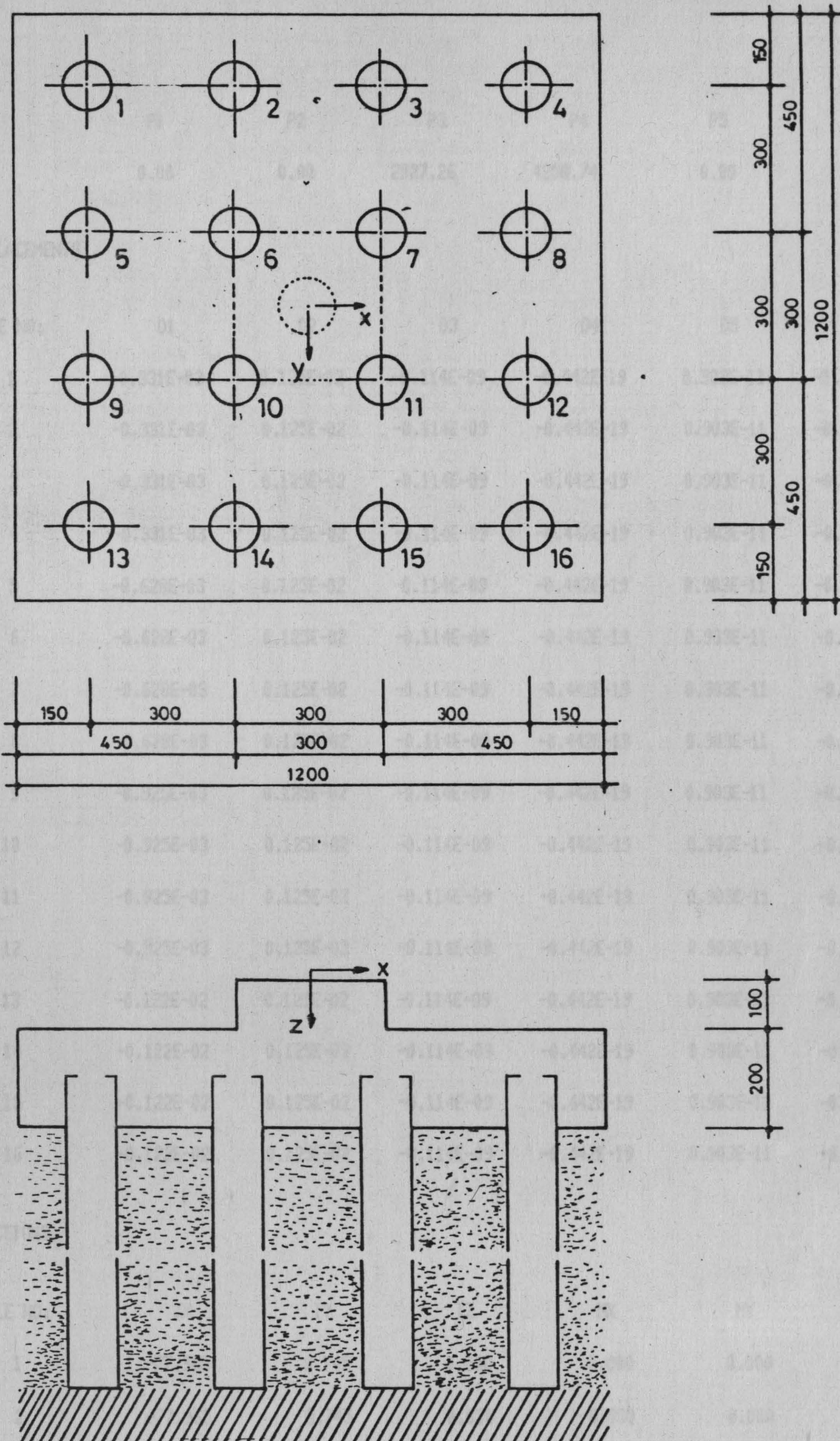


FIGURE A-3 Example number three

XAMPLE NUMBER THREE(C=2000t/m3)

ODAL LOADS

OAD NO:	P1	P2	P3	P4	P5	P6
1	0.00	0.00	2927.26	4258.74	0.00	0.00

ILE HEAD DISPLACEMENTS

OAD NO:	PILE NO:	D1	D2	D3	D4	D5	D6
1	1	-0.331E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	2	-0.331E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	3	-0.331E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	4	-0.331E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	5	-0.628E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	6	-0.628E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	7	-0.628E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	8	-0.628E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	9	-0.925E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	10	-0.925E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	11	-0.925E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	12	-0.925E-03	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	13	-0.122E-02	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	14	-0.122E-02	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	15	-0.122E-02	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04
1	16	-0.122E-02	0.125E-02	-0.114E-09	-0.442E-19	0.903E-11	-0.990E-04

ILE HEAD REACTIONS

OAD NO:	PILE NO:	TX	TY	TZ	MX	MY	MZ
1	1	-77.965	8.745	0.000	0.000	0.000	14.676
1	2	-77.965	8.745	0.000	0.000	0.000	14.676
1	3	-77.965	8.745	0.000	0.000	0.000	14.676
1	4	-77.965	8.745	0.000	0.000	0.000	14.676
1	5	-147.957	8.745	0.000	0.000	0.000	14.676

1	6	-147.957	8.745	0.000	0.000	0.000	14.676
1	7	-147.957	8.745	0.000	0.000	0.000	14.676
1	8	-147.957	8.745	0.000	0.000	0.000	14.676
1	9	-217.950	8.745	0.000	0.000	0.000	14.676
1	10	-217.950	8.745	0.000	0.000	0.000	14.676
1	11	-217.950	8.745	0.000	0.000	0.000	14.676
1	12	-217.950	8.745	0.000	0.000	0.000	14.676
1	13	-287.943	8.745	0.000	0.000	0.000	14.676
1	14	-287.943	8.745	0.000	0.000	0.000	14.676
1	15	-287.943	8.745	0.000	0.000	0.000	14.676
1	16	-287.943	8.745	0.000	0.000	0.000	14.676

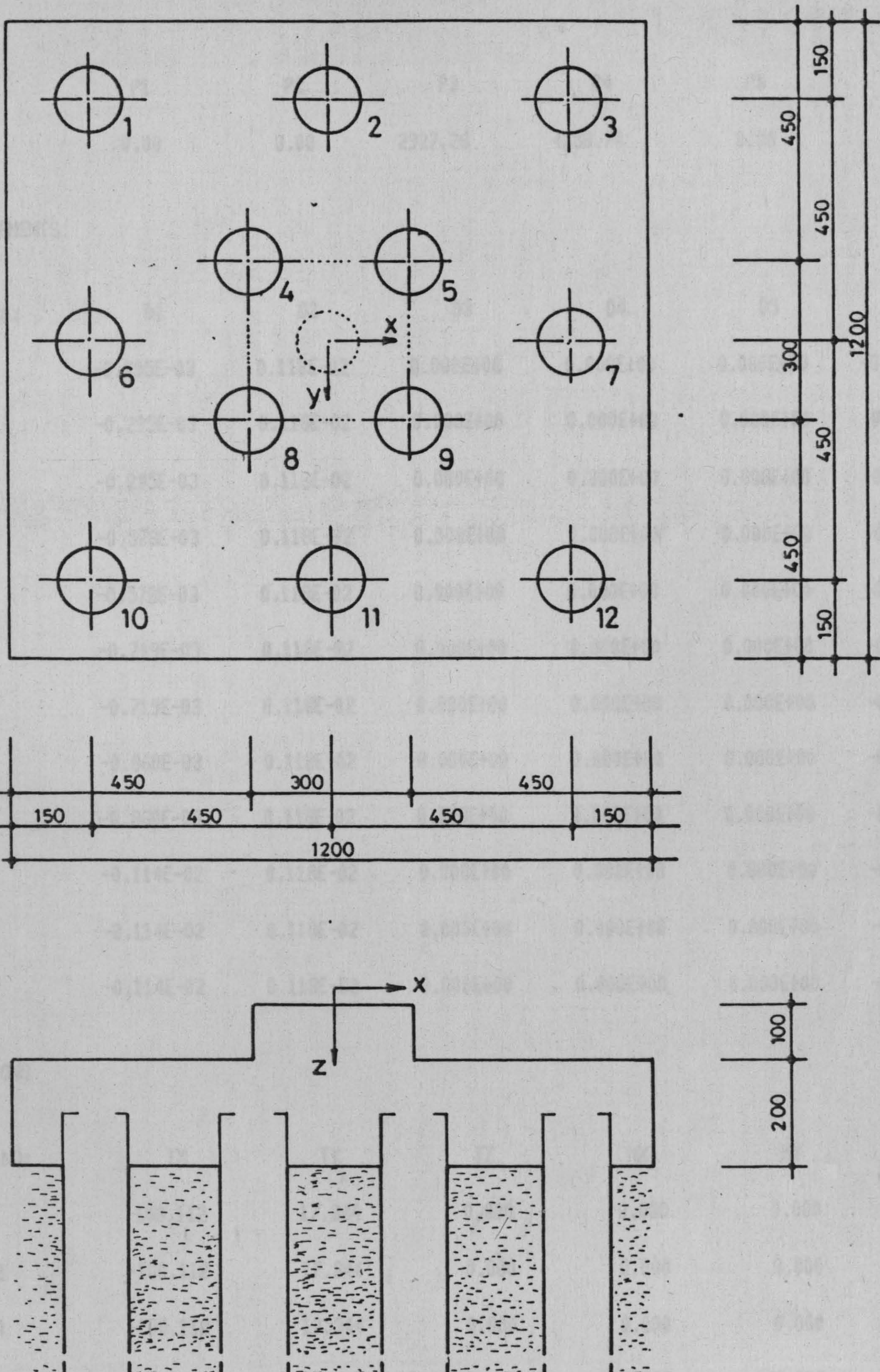


FIGURE A-4 Example number four

EXAMPLE NUMBER FOUR(C=3500t/m3)

NODAL LOADS

LOAD NO:	P1	P2	P3	P4	P5	P6
1	0.00	0.00	2927.26	4258.74	0.00	0.00

PILE HEAD DISPLACEMENTS

LOAD NO:	PILE NO:	D1	D2	D3	D4	D5	D6
1	1	-0.295E-03	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	2	-0.295E-03	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	3	-0.295E-03	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	4	-0.578E-03	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	5	-0.578E-03	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	6	-0.719E-03	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	7	-0.719E-03	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	8	-0.860E-03	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	9	-0.860E-03	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	10	-0.114E-02	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	11	-0.114E-02	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04
1	12	-0.114E-02	0.118E-02	0.000E+00	0.000E+00	0.000E+00	-0.942E-04

PILE HEAD REACTIONS

LOAD NO:	PILE NO:	TX	TY	TZ	MX	MY	MZ
1	1	-100.112	17.344	0.000	0.000	0.000	28.996
1	2	-100.112	17.344	0.000	0.000	0.000	28.996
1	3	-100.112	17.344	0.000	0.000	0.000	28.996
1	4	-195.996	17.344	0.000	0.000	0.000	28.996
1	5	-195.996	17.344	0.000	0.000	0.000	28.996
1	6	-243.938	17.344	0.000	0.000	0.000	28.996
1	7	-243.938	17.344	0.000	0.000	0.000	28.996
1	8	-291.880	17.344	0.000	0.000	0.000	28.996
1	9	-291.880	17.344	0.000	0.000	0.000	28.996

1	10	-387.765	17.344	0.000	0.000	0.000	28.996
1	11	-387.765	17.344	0.000	0.000	0.000	28.996
1	12	-387.765	17.344	0.000	0.000	0.000	28.996

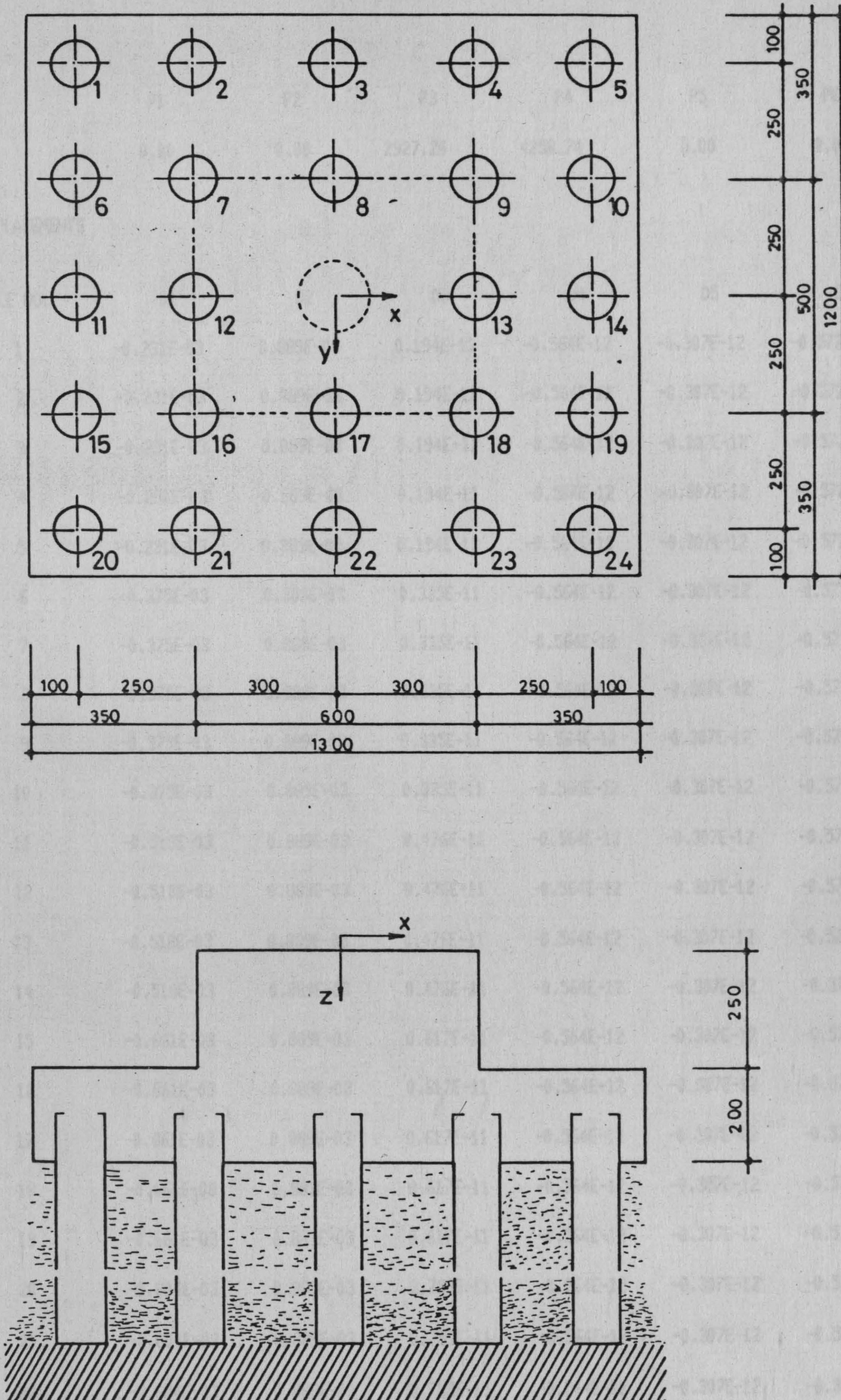


FIGURE A-5 Example number five

EXAMPLE NUMBER FIVE(C=1000t/m3)

NODAL LOADS

LOAD NO:	P1	P2	P3	P4	P5	P6
1	0.00	0.00	2927.26	4258.74	0.00	0.00

PILE HEAD DISPLACEMENTS

LOAD NO:	PILE NO:	D1	D2	D3	D4	D5	D6
1	1	-0.231E-03	0.889E-03	0.194E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	2	-0.231E-03	0.889E-03	0.194E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	3	-0.231E-03	0.889E-03	0.194E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	4	-0.231E-03	0.889E-03	0.194E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	5	-0.231E-03	0.889E-03	0.194E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	6	-0.375E-03	0.889E-03	0.335E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	7	-0.375E-03	0.889E-03	0.335E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	8	-0.375E-03	0.889E-03	0.335E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	9	-0.375E-03	0.889E-03	0.335E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	10	-0.375E-03	0.889E-03	0.335E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	11	-0.518E-03	0.889E-03	0.476E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	12	-0.518E-03	0.889E-03	0.476E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	13	-0.518E-03	0.889E-03	0.476E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	14	-0.518E-03	0.889E-03	0.476E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	15	-0.661E-03	0.889E-03	0.617E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	16	-0.661E-03	0.889E-03	0.617E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	17	-0.661E-03	0.889E-03	0.617E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	18	-0.661E-03	0.889E-03	0.617E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	19	-0.661E-03	0.889E-03	0.617E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	20	-0.804E-03	0.889E-03	0.759E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	21	-0.804E-03	0.889E-03	0.759E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	22	-0.804E-03	0.889E-03	0.759E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	23	-0.804E-03	0.889E-03	0.759E-11	-0.564E-12	-0.307E-12	-0.572E-04
1	24	-0.804E-03	0.889E-03	0.759E-11	-0.564E-12	-0.307E-12	-0.572E-04

LE HEAD REACTIONS

AD NO:	PILE NO:	TX	TY	TZ	MX	MY	MZ
1	1	-54.538	3.896	0.000	0.000	0.000	8.236
1	2	-54.538	3.896	0.000	0.000	0.000	8.236
1	3	-54.538	3.896	0.000	0.000	0.000	8.236
1	4	-54.538	3.896	0.000	0.000	0.000	8.236
1	5	-54.538	3.896	0.000	0.000	0.000	8.236
1	6	-88.253	3.896	0.000	0.000	0.000	8.236
1	7	-88.253	3.896	0.000	0.000	0.000	8.236
1	8	-88.253	3.896	0.000	0.000	0.000	8.236
1	9	-88.253	3.896	0.000	0.000	0.000	8.236
1	10	-88.253	3.896	0.000	0.000	0.000	8.236
1	11	-121.969	3.896	0.000	0.000	0.000	8.236
1	12	-121.969	3.896	0.000	0.000	0.000	8.236
1	13	-121.969	3.896	0.000	0.000	0.000	8.236
1	14	-121.969	3.896	0.000	0.000	0.000	8.236
1	15	-155.685	3.896	0.000	0.000	0.000	8.236
1	16	-155.685	3.896	0.000	0.000	0.000	8.236
1	17	-155.685	3.896	0.000	0.000	0.000	8.236
1	18	-155.685	3.896	0.000	0.000	0.000	8.236
1	19	-155.685	3.896	0.000	0.000	0.000	8.236
1	20	-189.401	3.896	0.000	0.000	0.000	8.236
1	21	-189.401	3.896	0.000	0.000	0.000	8.236
1	22	-189.401	3.896	0.000	0.000	0.000	8.236
1	23	-189.401	3.896	0.000	0.000	0.000	8.236
1	24	-189.401	3.896	0.000	0.000	0.000	8.236

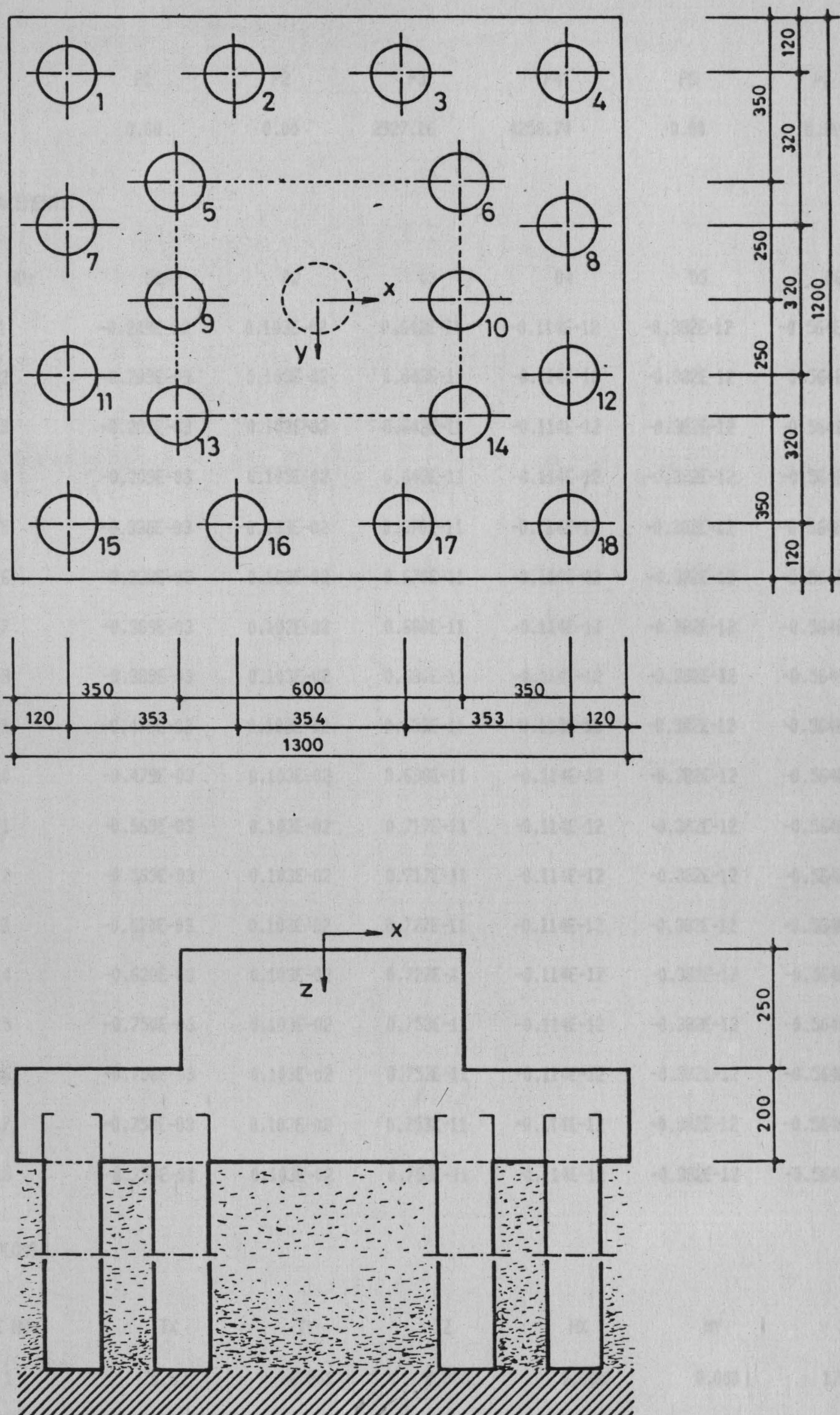


FIGURE A-6 Example number six

EXAMPLE NUMBER SIX(C=1000t/m3)

NODAL LOADS

LOAD NO:	P1	P2	P3	P4	P5	P6
1	0.00	0.00	2927.26	4258.74	0.00	0.00

PILE HEAD DISPLACEMENTS

LOAD NO:	PILE NO:	D1	D2	D3	D4	D5	D6
1	1	-0.209E-03	0.103E-02	0.643E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	2	-0.209E-03	0.103E-02	0.643E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	3	-0.209E-03	0.103E-02	0.643E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	4	-0.209E-03	0.103E-02	0.643E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	5	-0.338E-03	0.103E-02	0.670E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	6	-0.338E-03	0.103E-02	0.670E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	7	-0.389E-03	0.103E-02	0.680E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	8	-0.389E-03	0.103E-02	0.680E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	9	-0.479E-03	0.103E-02	0.698E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	10	-0.479E-03	0.103E-02	0.698E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	11	-0.569E-03	0.103E-02	0.717E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	12	-0.569E-03	0.103E-02	0.717E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	13	-0.620E-03	0.103E-02	0.727E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	14	-0.620E-03	0.103E-02	0.727E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	15	-0.750E-03	0.103E-02	0.753E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	16	-0.750E-03	0.103E-02	0.753E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	17	-0.750E-03	0.103E-02	0.753E-11	-0.114E-12	-0.382E-12	-0.564E-04
1	18	-0.750E-03	0.103E-02	0.753E-11	-0.114E-12	-0.382E-12	-0.564E-04

PILE HEAD REACTIONS

LOAD NO:	PILE NO:	TX	TY	TZ	MX	MY	MZ
1	1	-70.827	6.841	0.000	0.000	0.000	17.537
1	2	-70.827	6.841	0.000	0.000	0.000	17.537
1	3	-70.827	6.841	0.000	0.000	0.000	17.537

1	4	-70.827	6.841	0.000	0.000	0.000	17.537
1	5	-114.814	6.841	0.000	0.000	0.000	17.537
1	6	-114.814	6.841	0.000	0.000	0.000	17.537
1	7	-132.026	6.841	0.000	0.000	0.000	17.537
1	8	-132.026	6.841	0.000	0.000	0.000	17.537
1	9	-162.626	6.841	0.000	0.000	0.000	17.537
1	10	-162.626	6.841	0.000	0.000	0.000	17.537
1	11	-193.225	6.841	0.000	0.000	0.000	17.537
1	12	-193.225	6.841	0.000	0.000	0.000	17.537
1	13	-210.437	6.841	0.000	0.000	0.000	17.537
1	14	-210.437	6.841	0.000	0.000	0.000	17.537
1	15	-254.424	6.841	0.000	0.000	0.000	17.537
1	16	-254.424	6.841	0.000	0.000	0.000	17.537
1	17	-254.424	6.841	0.000	0.000	0.000	17.537
1	18	-254.424	6.841	0.000	0.000	0.000	17.537

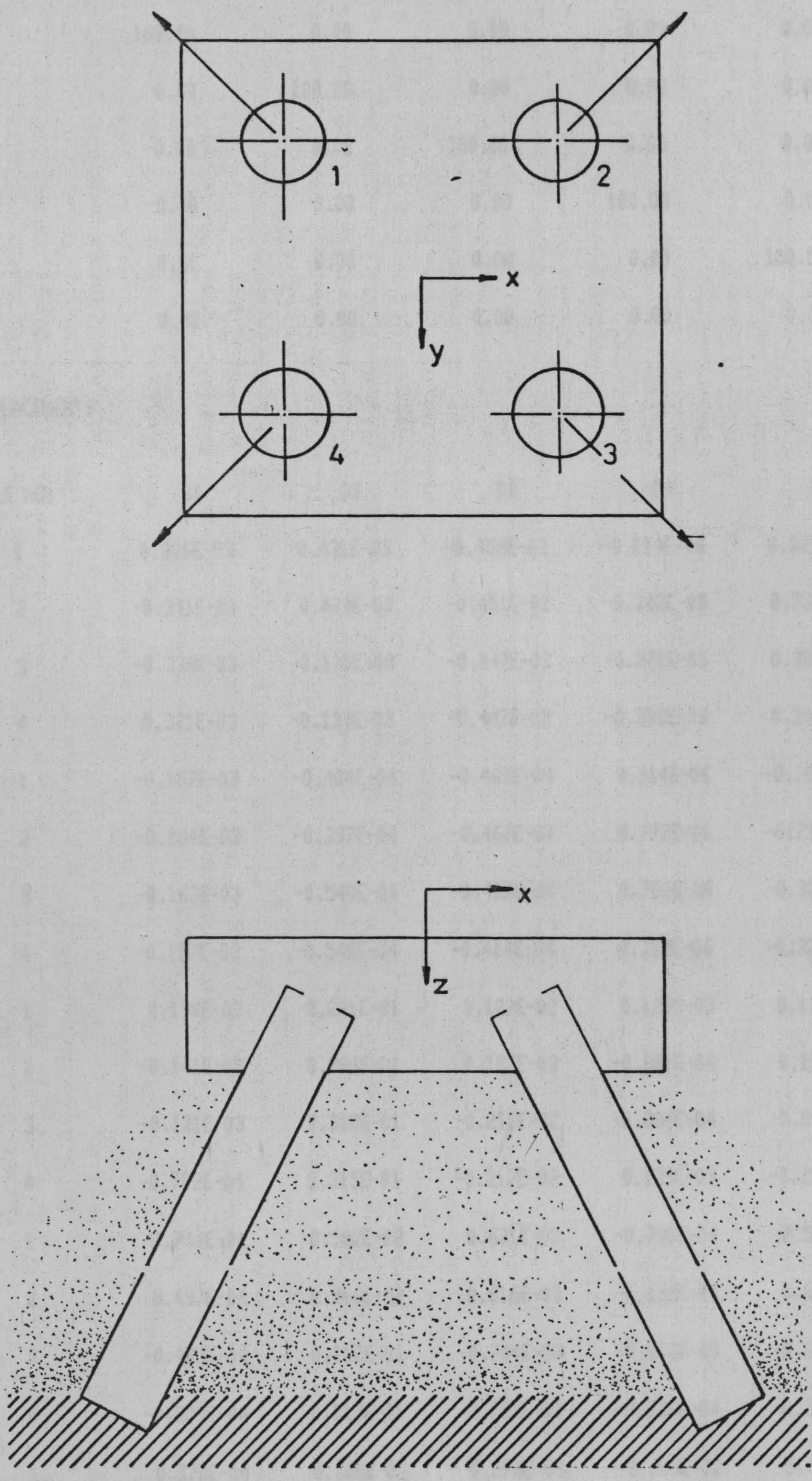


FIGURE A-7 Example number seven

XAMPLE NUMBER SEVEN(C=1000t/m3)

ODAL LOADS

OAD NO:	P1	P2	P3	P4	P5	P6
1	100.00	0.00	0.00	0.00	0.00	0.00
2	0.00	100.00	0.00	0.00	0.00	0.00
3	0.00	0.00	100.00	0.00	0.00	0.00
4	0.00	0.00	0.00	100.00	0.00	0.00
5	0.00	0.00	0.00	0.00	100.00	0.00
6	0.00	0.00	0.00	0.00	0.00	100.00

PILE HEAD DISPLACEMENTS

OAD NO:	PILE NO:	D1	D2	D3	D4	D5	D6
1	1	0.326E-03	0.431E-03	-0.452E-02	-0.114E-06	0.329E-04	-0.152E-04
1	2	-0.341E-03	0.429E-03	-0.451E-02	-0.243E-05	0.332E-04	-0.152E-04
1	3	-0.338E-03	-0.135E-03	-0.447E-02	-0.271E-05	0.309E-04	-0.194E-04
1	4	0.321E-03	-0.134E-03	-0.447E-02	-0.396E-06	0.306E-04	-0.194E-04
2	1	-0.157E-03	-0.404E-04	-0.467E-04	0.314E-06	-0.786E-06	0.340E-05
2	2	-0.164E-03	-0.397E-04	-0.467E-04	0.777E-06	-0.791E-06	0.340E-05
2	3	-0.163E-03	-0.540E-04	-0.416E-04	0.782E-06	-0.328E-06	0.347E-05
2	4	-0.157E-03	-0.546E-04	-0.414E-04	0.319E-06	-0.323E-06	0.347E-05
3	1	0.198E-03	0.281E-01	0.182E-02	0.139E-03	0.138E-03	-0.171E-02
3	2	0.141E-03	0.284E-01	0.182E-02	-0.881E-04	0.139E-03	-0.171E-02
3	3	-0.121E-03	0.318E-01	-0.212E-02	-0.884E-04	-0.889E-04	-0.171E-02
3	4	-0.702E-04	0.315E-01	-0.212E-02	0.139E-03	-0.892E-04	-0.171E-02
4	1	0.744E-04	0.982E-02	0.631E-03	-0.730E-04	0.599E-04	-0.738E-03
4	2	0.497E-04	0.966E-02	0.616E-03	-0.173E-03	0.600E-04	-0.738E-03
4	3	-0.407E-04	0.112E-01	-0.744E-03	-0.173E-03	-0.396E-04	-0.739E-03
4	4	-0.209E-04	0.113E-01	-0.762E-03	-0.731E-04	-0.396E-04	-0.739E-03
5	1	0.370E-04	0.563E-02	0.376E-03	-0.205E-02	0.497E-04	-0.593E-03
5	2	0.166E-04	0.148E-02	0.970E-04	-0.215E-02	0.498E-04	-0.593E-03
5	3	-0.279E-05	0.295E-02	-0.196E-03	-0.215E-02	-0.481E-04	-0.593E-03

5	4	-0.198E-04	0.709E-02	-0.474E-03	-0.205E-02	-0.481E-04	-0.593E-03
6	1	0.140E-03	0.484E-03	-0.281E-03	-0.454E-06	0.116E-03	-0.541E-04
6	2	-0.128E-03	0.478E-03	-0.267E-03	-0.872E-05	0.117E-03	-0.541E-04
6	3	-0.117E-03	0.577E-03	-0.110E-03	-0.972E-05	0.109E-03	-0.692E-04
6	4	0.120E-03	0.582E-03	-0.125E-03	-0.145E-05	0.108E-03	-0.692E-04

PILE HEAD REACTIONS

LOAD NO:	PILE NO:	TX	TY	TZ	MX	MY	MZ
1	1	51.000	1.947	-21.936	-0.001	-52.840	4.318
1	2	-53.390	1.938	-21.912	-0.017	-52.773	4.298
1	3	-52.923	-0.904	-21.719	-0.019	-52.370	-2.807
1	4	50.129	-0.897	-21.744	-0.003	-52.441	-2.790
2	1	-24.620	-0.159	-0.240	0.002	-0.614	-0.286
2	2	-25.581	-0.155	-0.240	0.005	-0.614	-0.278
2	3	-25.528	-0.225	-0.210	0.005	-0.525	-0.447
2	4	-24.543	-0.228	-0.209	0.002	-0.522	-0.454
3	1	30.970	118.199	10.664	0.973	30.365	238.413
3	2	22.038	119.687	10.700	-0.615	30.460	242.065
3	3	-18.954	136.528	-11.563	-0.617	-31.076	283.300
3	4	-10.984	134.891	-11.566	0.971	-31.088	279.282
4	1	11.639	39.592	3.848	-0.510	11.261	74.818
4	2	7.770	38.809	3.775	-1.205	11.085	72.895
4	3	-6.368	46.183	-4.159	-1.206	-11.408	90.951
4	4	-3.264	46.901	-4.248	-0.511	-11.627	92.714
5	1	5.780	20.657	2.461	-14.323	7.548	32.720
5	2	2.604	0.131	1.083	-15.006	4.167	-17.659
5	3	-0.437	7.398	-1.550	-15.006	-5.261	0.173
5	4	-3.090	27.860	-2.929	-14.323	-8.646	50.395
6	1	21.844	1.736	0.023	-0.003	3.586	2.621
6	2	-19.948	1.706	0.107	-0.061	3.821	2.548
6	3	-18.307	2.012	0.783	-0.068	5.231	2.843