AN ANALYTICAL APPROACH

TO

REINFORCED EARTH SYSTEM

FİKRET EYGÖREN

June 1986

Boğaziçi University

AN ANALYTICAL APPROACH
TO
REINFORCED EARTH SYSTEM

by

FİKRET EYGÖREN

B.Sc. in C.E., Karadeniz Technical University, 1983

Submitted to the Institute for Graduate Studies in
Science and Engineering in Partial Fulfillment of
the Requirements for the Degree of
Master of Science
in
Civil Engineering

Boğaziçi University

June, 1986

AN ANALYTICAL APPROACH

TO

REINFORCED EARTH SYSTEM

APPROVED BY:

Prof.Dr. H.Turan DURGUNOĞLU

(Thesis Supervisor)

Doç.Dr. Erol GÜLER

Doç.Dr. Vural ALTIN

V. Altın

APPROVAL DATE : 16. 6. 1986

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to those who have assisted me during the development of this study, especially to my thesis supervisor Prof. Dr. H. Turan Durgunoğlu for his invaluable suggestions, guidence and encouragement.

I am greatly indebted to the Staff of the Boğaziçi University Civil Engineering Department and Computer Center for their generous help and making the facilities available to me.

I am also grateful to Miss Nükhet Okunur for her patience, care and conscientiousness in typing the manuscript.

İstanbul, June 1986

Fikret EYGÖREN

ABSTRACT

In this study, Reinforced Earth Structures are investigated by using various techniques. Initially the system is handled as a slope stability problem and multi-criterion analysis method is employed to find out the allowable tensile and shear forces for the reinforcing bars. A computer program, which evaluates the forces mobilized in the bars and estimates the safety factor, is prepared.

In the second part of the study, reinforced earth systems are analyzed by the Finite Element Method. For this purpose, the system is considered as a composite material and necessary formulations are derived. In the analysis, the hyperbolic stress-strain parameters are utilized to take into account the real behaviour of the soil.

A finite element computer program, having various capacities is also developed in this study.

ÖZET

Master tezi olarak hazırlanan bu çalışmada donatılı zeminler çeşitli yöntemler kullanılarak incelenmiştir. Sistem önce bir yamaç problemi olarak düşünülmüş ve çok-kriterli analiz metodu kullanılarak donatıların taşıyabileceği maximum çekme ve kesme kuvvetleri hesaplanmıştır. Ayrıca donatılarda oluşan gerilmeleri hesaplayan ve bu kuvvetleri dikkate alarak emniyet katsayısını bulan bir computer programı hazırlanmıştır.

Çalışmanın ikinci bölümünde, donatılı zeminler sonlu elemanlar yöntemi kullanılarak incelenmiş ve bu sistemler hakkında daha detaylı bilgi elde edilmiştir. Bu analizde donatılı zeminler kompozit malzeme olarak düşünülmüş ve bu kabule göre gerekli formüller çıkartılmıştır. Zeminin gerçek davranışınıda dikkate almak amacıyla, hiperbolik gerilme-şekildeğiştirme parametreleri kullanılmıştır.

Bu çalışmada sonlu elemanlar yöntemi için çeşitli özelliklere sahip birde computer programı hazırlanmıştır.

LIST OF FIGURES

LIST OF FIGURES (Cont'd)

LIST OF FIGURES (Cont'd)

LIST OF FIGURES (Cont'd)

LIST OF TABLES

LIST OF SYMBOLS

Symbol	Meaning
σ	Normal stress
τ	Shear Stress
τ_s	Frictional shear resistance
T	Tensile force mobilized in the bar
V	Shear force mobilized in the bar
P	Limit pullout resistance of the reinforcemat
R_T	Rupture strength of the reinforcement
T_f	Tensile force mobilized in the bar at failure
V_f	Shear force mobilized in the bar at failure
R_N	Tensile strength of the bar
R_C	Shearing strength of the bar

LIST OF SYMBOLS - (Cont'd)

Symbol	Meaning
R_T	Tangential component of the strip force
R_R	Radial component of the strip force
$\Delta\tau$	Increase of the overall shear resistance due to the reinforcement
A_{cr}	Cross section area of the failure surface
EI	Bending stiffness of the reinforcement
K_s	Subgrade modulus of the lateral soil reaction
K	Modulus number
n	Modulus exponent
ν	Poisson's ratio
L_o	Transfer length
L_e	Embedment length
ℓ	Length of reinforcement
b	Horizontal spacing of reinforcement
μ	Coefficient of friction between soil and reinforcement
	friction angle between soil and smooth surface

LIST OF SYMBOLS - (Cont'd)

Symbol	Meaning
ϕ	Internal friction angle of soil
μ^*	Apparent friction coefficient
Θ	The angle between the reinforcement and the normal to the failure plane
c	Cohesion
f_{max}	Limit skin friction
u	Pore pressure
F	Factor of safety
$(\sigma_1 - \sigma_3)$	Deviator stress

TABLE OF CONTENTS

Page

CHAPTER-I

AN ANALYTICAL PROCEDURE TO REINFORCED EARTH SYSTEM

INTRODUCTION

Soil is the most abundant and least expensive construction material. Many soils with a suitable water content and density can be strong enough to be structurally sound, especially when loaded only in compression. However, like concrete, soils are very weak in tension. Therefore, in some cases it is not possible to use the soil without any supporting system. As in the case of reinforced concrete, in soils the inclusion of reinforcements which are strong in tension can produce a composite material that combines the best load carrying features of both components. Making use of these aspects of soil and reinforcement, recently a composite material which is named as "Reinforced Earth System" has been introduced.

As a definition, Reinforced Earth is a construction material consisting of a frictional backfill material and linear reinforcing strip. The reinforcements, which are capable of withstanding high tensile forces, have two main effects; namely, reducing the average shear stress carried by the soil and increasing the average normal stress on the failure surface.

Reinforced earth retaining structures have three components:

1. Backfill material
2. Reinforcing Strips
3. Facing, which has only a local role preventing

the backfill material from sloughing away from the wall face. These components are shown on a schematic view of a Reinforced Earth Wall on Figure 1.1.

Reinforced earth structures possess several features that are attractive in many situations requiring retaining structure. In Reinforced Earth structures the in-situ ground is used as one of the main structural elements. The facing elements prevent the collapse of the soil at the face between the strips. The facings, therefore, are relatively thin and inexpensive. The low cost of the elements can provide significant savings in construction materials relative to the conventional solutions. Light construction equipment, adaptability to site condition and easy operation in heterogeneous soils are the other advantages of the reinforced retaining structures. Moreover, these structures are more flexible than classical cast-in-place reinforced concrete retaining structures.

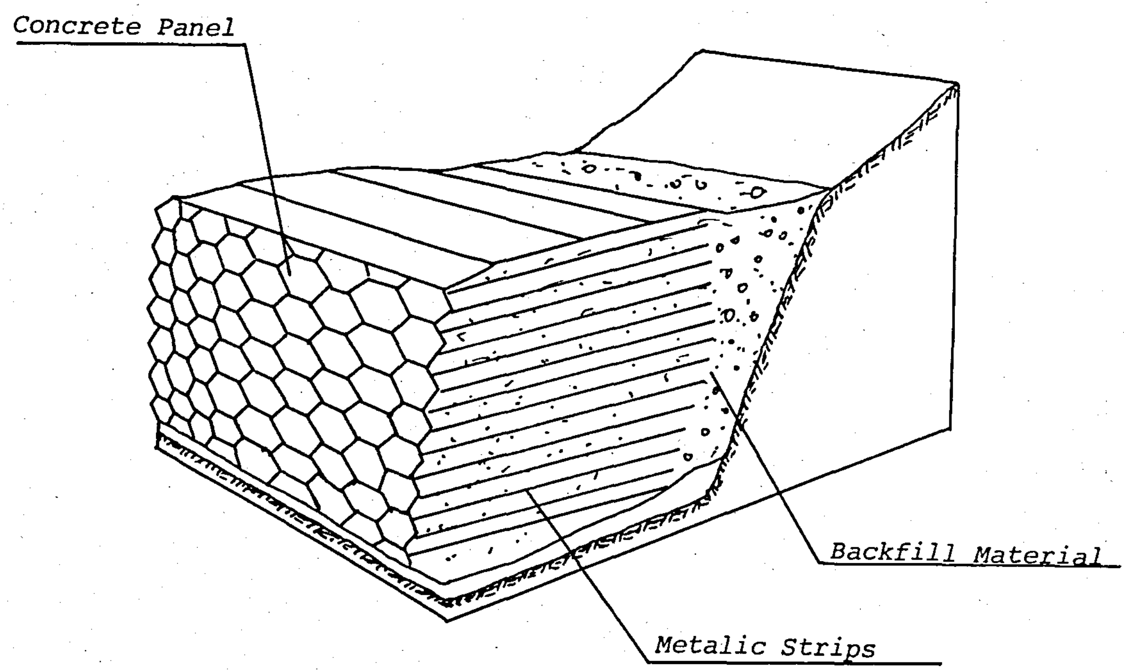


Fig. 1.1 Schematic view of a Reinforced Earth Wall

Consequently, the reinforced earth structures can conform to deformation of surrounding ground and withstand larger total and differential settlements.

Some applications of reinforced erath system in Europe and in U.S.A. are indicated in Fig. 1.2 to 1.3.(Ref.8)

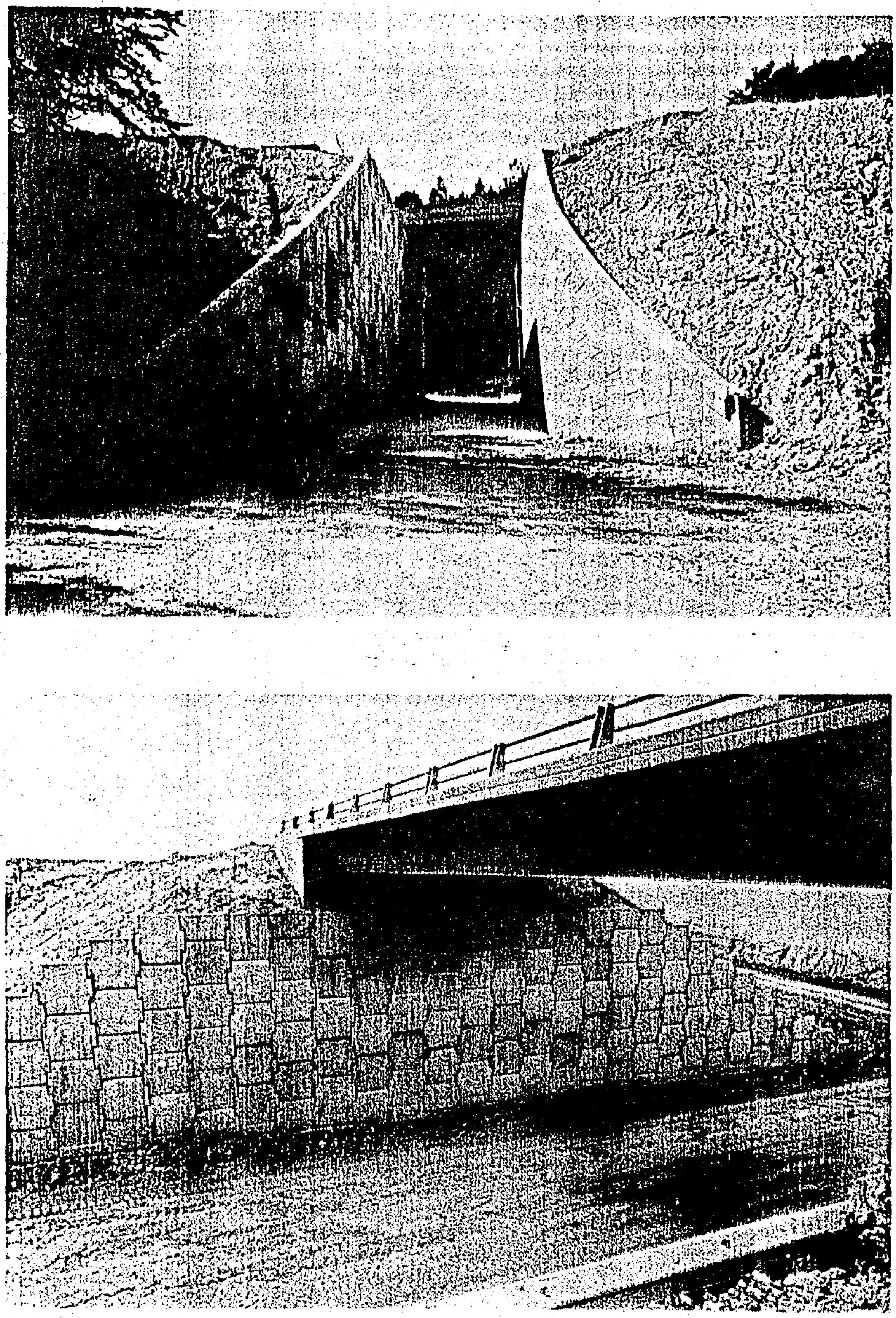


Fig. 1.2. Application of Reinforced Earth System

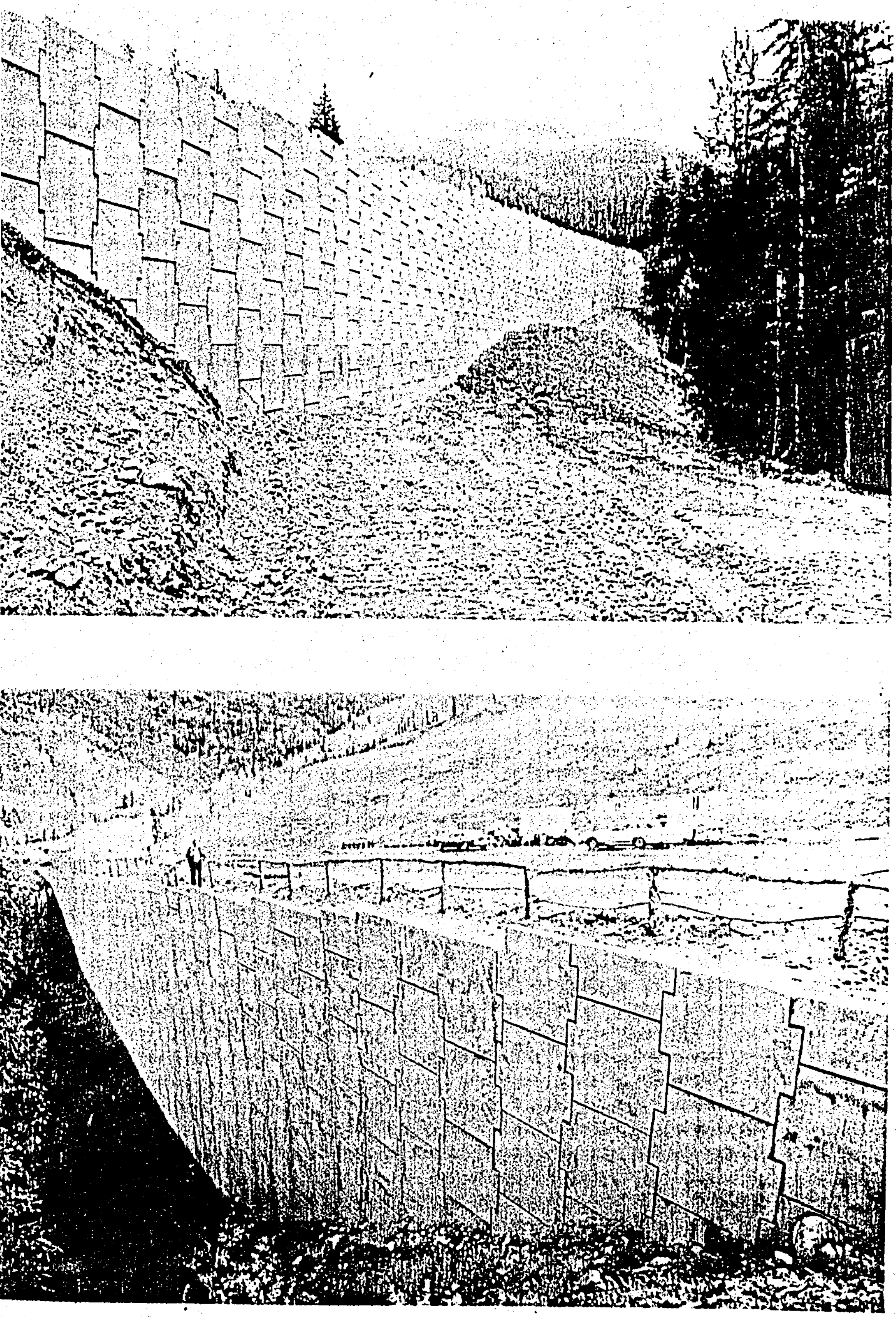


Fig. 1.3. Application of Reinforced Earth Wall

CHAPTER-II

PRINCIPLE OF SOIL-REINFORCEMENT INTERACTION AND DESIGN METHODS

2.1. INTRODUCTION

An essential aspect in the success of any soil reinforcement system is that the two materials sould be compatible in terms of surface characteristics and geometry so that stress can be transferred from one to another.

In Reinforced Earth, the mechanism of soil to reinforcement stress transfer is mainly the friction between the soil and reinforcement surfaces when smooth reinforcement strips are used. When ribbed strips are used, stress transfer is also developed by passive resistance on the ribs. They together determine the bond strength which controls the maximum rate of change of axial force in the reinforcement along its length.

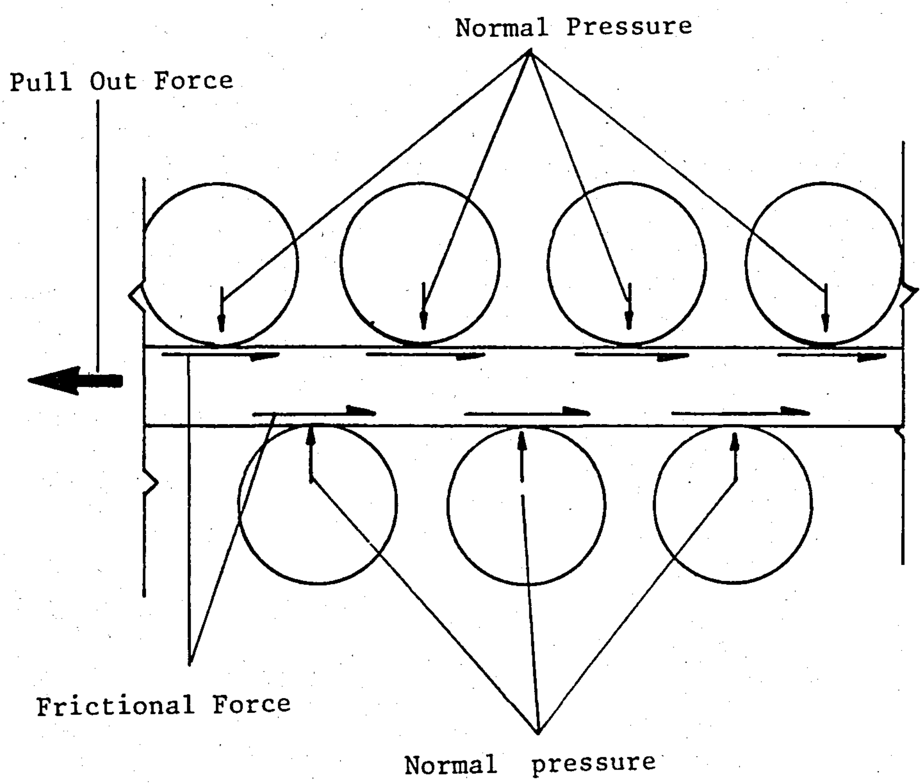


Fig. 2.1 Frictional Load Transfer Between Soil and Reinforcement

2.2. FRICTIONAL LOAD TRANSFER

Frictional stress transfer between soil and reinforcements is illustrated schematically in Fig. 2.1. The load that can be transferred per unit area of reinforcement depends on the interface characteristics of the soil to reinforcing material, and on the normal stress between them. The latter depends on the stress-deformation behavior of the soil, which is itself stress-dependent. Consequently, it is not realiable to estimate the effective friction coefficient by analysis alone. The results of experiments; eg; pullout tests, direct shear tests between soil and reinforcement, instrumented models and full scale structures, are often used as a basis for selection of appropriate values of effective frictional coefficient.

Analysis of the local equilibrium of a section of reinforcement with the soil gives the stress transfer condition, shown in Fig. 2.2. as:

$$dT = T_2 - T_1 = 2 \,.\, b \,.\, \zeta \; (d\ell) \qquad (2.1)$$

where

b = Reinforced width

ℓ = Length along reinforcement

T = Tensile force

ζ = Shear stress along soil-reinforcement interface

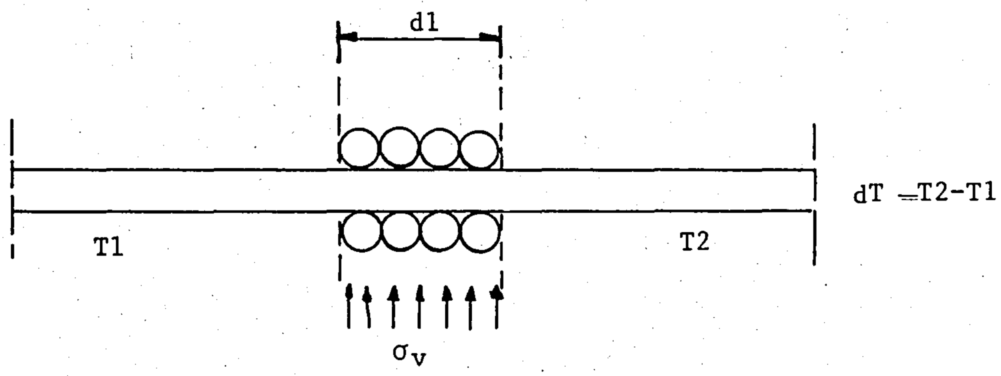


Fig. 2.2 Forces Acting on the Reinforcement

If τ is generated only by interface friction, then

$$\tau = \mu\sigma_v \tag{2.2}$$

where

σ_v = The normal stress exerted on the reinforcement

μ = The coefficient of friction between the soil and reinforcement material.

The interface friction coefficient between different construction material surfaces in direct shear is known to be in the range of about 0.5 to 0.8 times the direct shearing resistance that can be mobilized within the soil. That is

$$\mu = \tan § = (0.5 - 0.8) \tan\phi \tag{2.3}$$

where

§ = Friction angle between soil and smooth surface

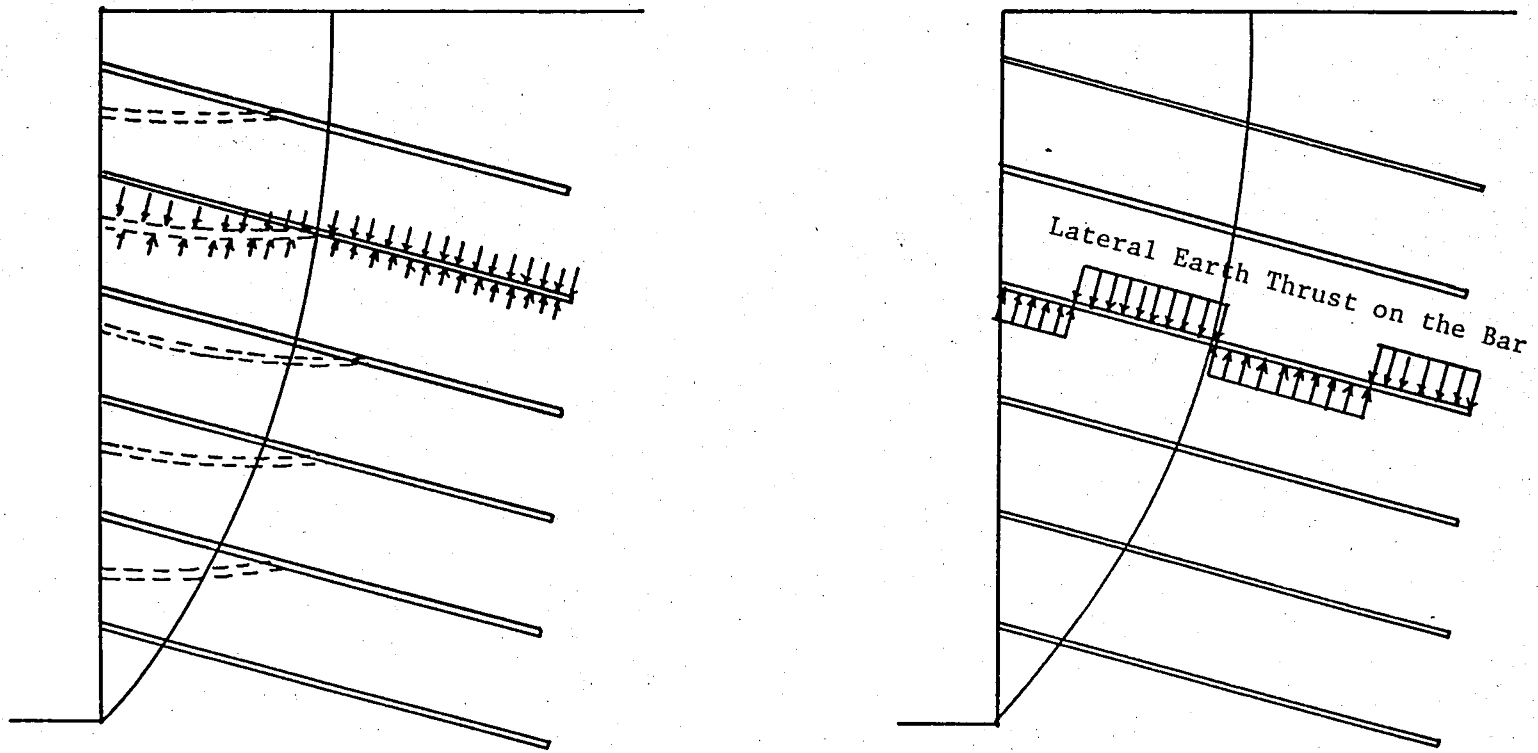
ϕ = Angle of internal friction of the soil.

Thus, if the value of σ is known, it should be a simple mather to calculate the limiting value of reinforcement pullout resistance, in any case. Unfortunately, such a simple calculation can not reliably made, owing to the fact that the effective normal stress is altered by the soil to reinforcement interaction.

Accordingly the most reliable values of friction coefficient are obtained by direct measurement. The value so determined is commonly referred to as the apparent or effective friction coefficient μ^*, and it is usually taken as the average mobilized shear stress along the reinforcement divided by the normal stress as given by the overburden pressure. It is known that the construction methods may influence μ^* in reinforced soil construction. Therefore, it is necessary to evaluate the apparent friction coefficient. μ^* by taking into consideration the specific backfill characteristics and the method of construction.

2.2. PASSIVE EARTH THRUST ON THE REINFORCEMENTS

Although tensile forces constitute the dominant reinforcing mechanism, passive lateral earth resistance can develop against the strip on either side of a potential failure surface, when reinforcing elements are rigid. To illustrate the effect of the rigidity of the reinforcement on the soil-inclusion interaction, the two limiting cases of flexible and completely rigid reinforcements can be considered. As shown in Fig. 2.3 a flexible reinforcement will deform until equilibrium is reached. However, the rigid reinforcement will resist the deformation, consequently passive lateral earth thrust will be mobilized at both sides of the potential sliding surface, and shear stress will be developed on the cross section of the



a) *Flexible Reinforcement* b) *Rigid Reinforcement*

Fig. 2.3. Effect of the Rigidity of the Reinforcement

reinforcement to maintain the equilibrium requirements. Rigit reinforcements, dependent on alignment, may thus have to withstand shearing forces and bending moments as well as tensile forces.

As schematically shown in Fig. 2.4 the overall shear resistance of the reinforced soil can be divided into three components.

1. The apparent cohesion C_o^* due to the shear force, V_o, mobilized in the bars.

$$C_o^* = \Sigma \frac{V_o}{A} \tag{2.4}$$

where A is the area of the shear surface

2. The frictional shear stress, τ_s, mobilized in the soil along the potential failure surface in the absance of the bars

$$\tau_s = \sigma' \tan\phi \tag{2.5}$$

3. The difference in frictional shear stresses between reinforced and non-reinforced soil, $\Delta\tau$, which is caused by the effect of the reinforcing bars on the stresses and displacements of the soil.

Hence the total shearing resistance of the reinforced soil can be written as:

$$\tau = C_{o'} + \sigma' \tan\phi + \Delta\tau \quad (2.6)$$

The finite element analysis results show that the shear forces mobilized in the bars, V_o, and corresponding apparent cohesion, C_o, are practically independent of the applied normal stress σ'. However, the effect of the reinforcing bars on $\Delta\tau$ is high dependent on the applied normal stress, In fact, $\Delta\tau$ varies from possitive to negative as the normal stress increases. Consequently the total apparent cohesino C* is greater than C_o and the apparent internal friction anglc of the reinforced soil is smaller than the internal friction angle of the soil. This can be easily seen in Fig. 2.4.

The mobilization of both the apparent cohesion C^* and the portion of cohesion due to shear forces in the reinforcement bars, C_o, as functions of relative displacement, x, are shown in Fig. 2.5. These results illustrate that the relative soil-inclusion displacement necessary to mobilize this apparent cohesion is generally much greater than that required to mobilize soil-reinforcement friction. This relative displacement is highly dependent on the relative rigidity and diameter of the inclusion. A simplified model of soil-reinforcement interaction has been proposed by Juran to simulate the mobilization of the lateral earth pressure on the inclusion. In this model, the bars are considered as laterally loaded vertical piles supported by a lateral series of elastoplastic springs with spring coefficients which may vary during the loading. In this

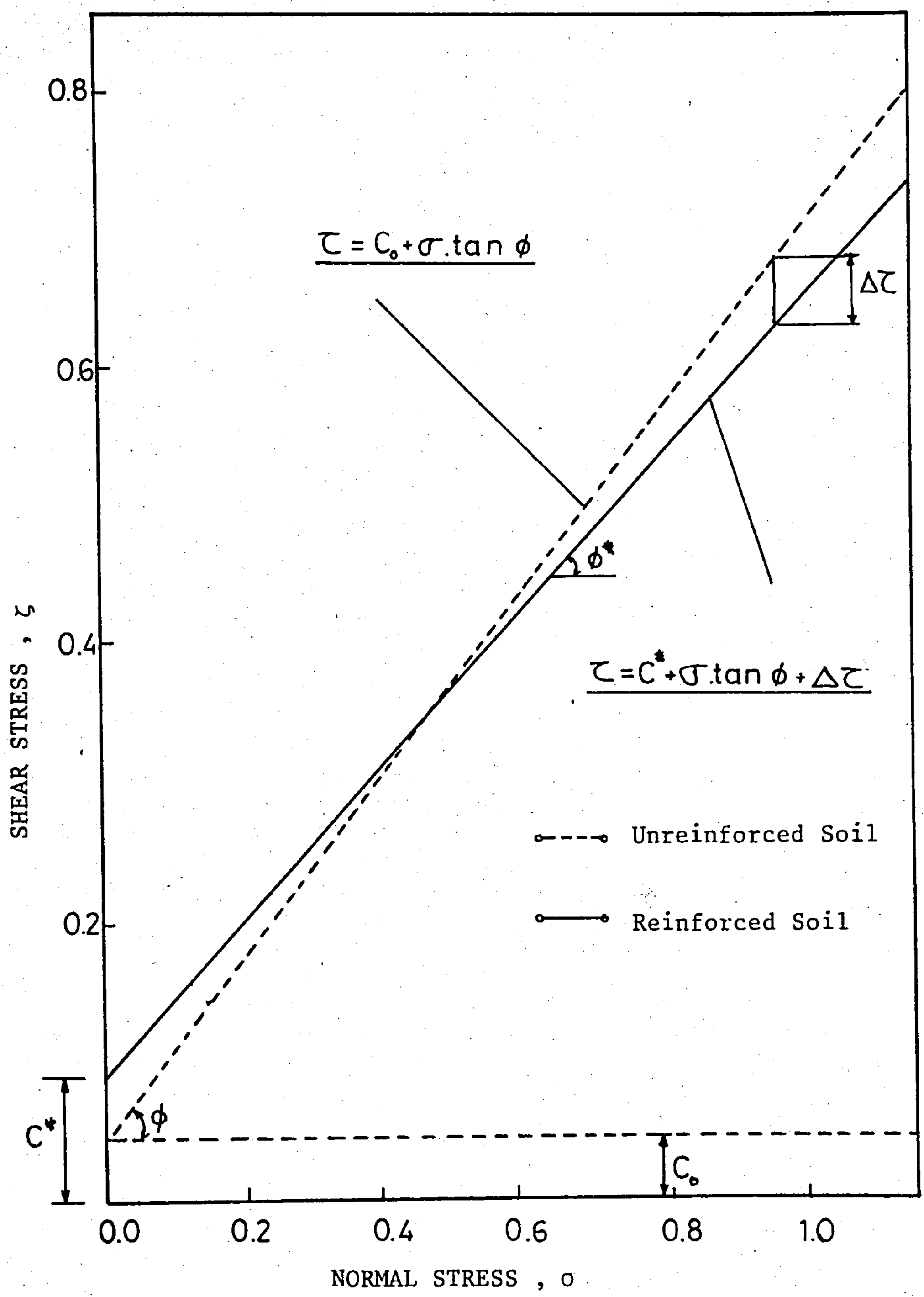


Fig. 2.4. Failure Curves of Reinforced and Unreinforced Soils

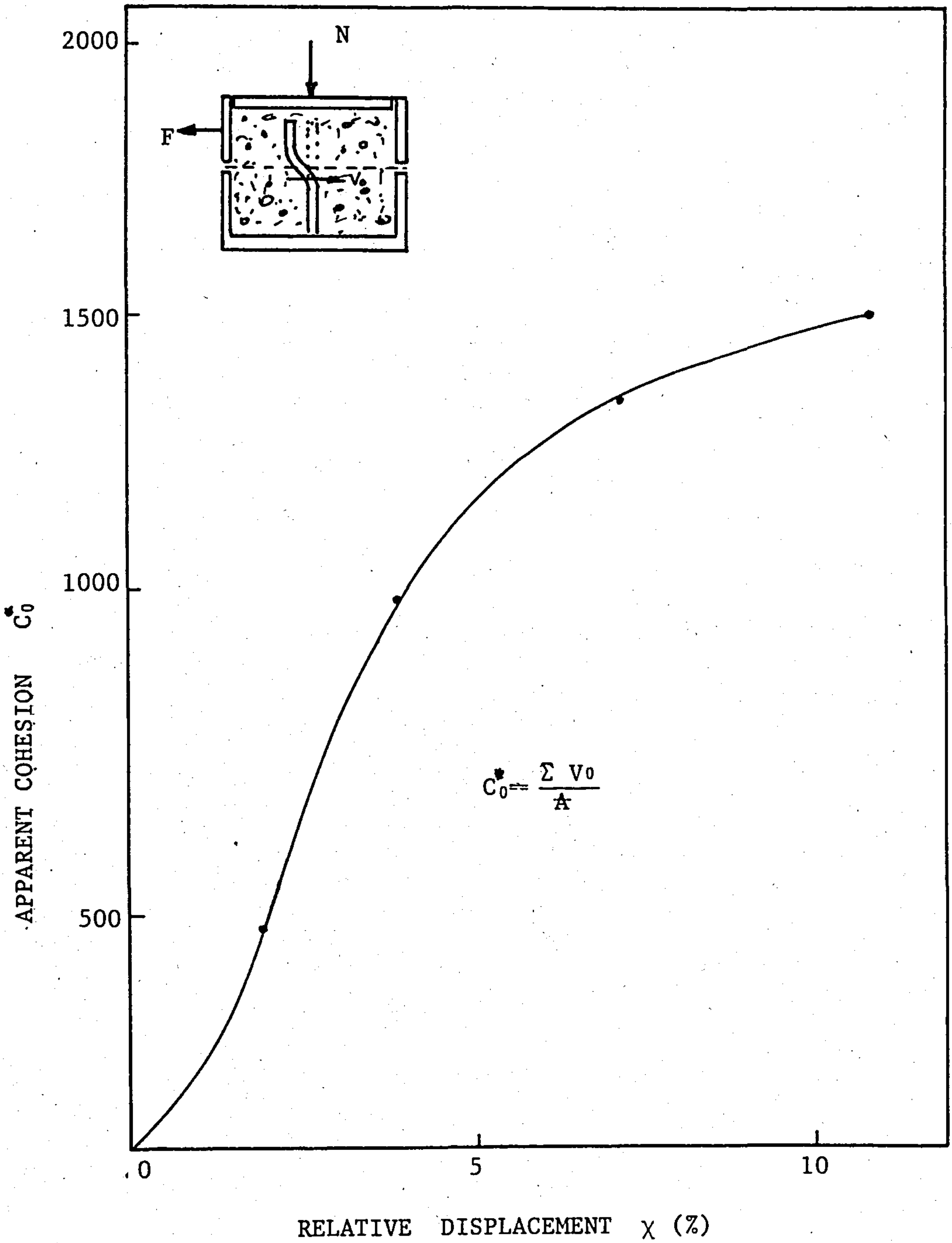


Fig. 2.5. Mobilization of the Apparent Cohesion

model, the relative rigidity of the bar is characterized by its transfer length:

$$l_o = \sqrt[4]{\frac{4EI}{K_s d}} \qquad (2.7)$$

where

EI : bending stiffness of the reinforcement

K_s : subgrade modulus of the lateral soil reaction

d : diameter of the bar

2.4. EFFECT OF THE INCLINATION OF THE REINFORCEMENTS WITH RESPECT TO THE FAILURE SURFACE

The development of tensile forces in the reinforcements during a direct shearing of a reinforced soil mass depends mainly on the orientation of the reinforcements with respect to the failure surface. As shown in Fig. 2.6 the maximum increase of the shear strength of a sand sample reinforced by bars is obtained when the reinforcement is oriented in the same direction as the principal tensile strain increment that would have occured in the unreinforced sand at failure. Orientation of reinforcements in a compressive strain direction can result in a decrease in shear strength of the soil, owing to a reduction of the average normal stress in the soil

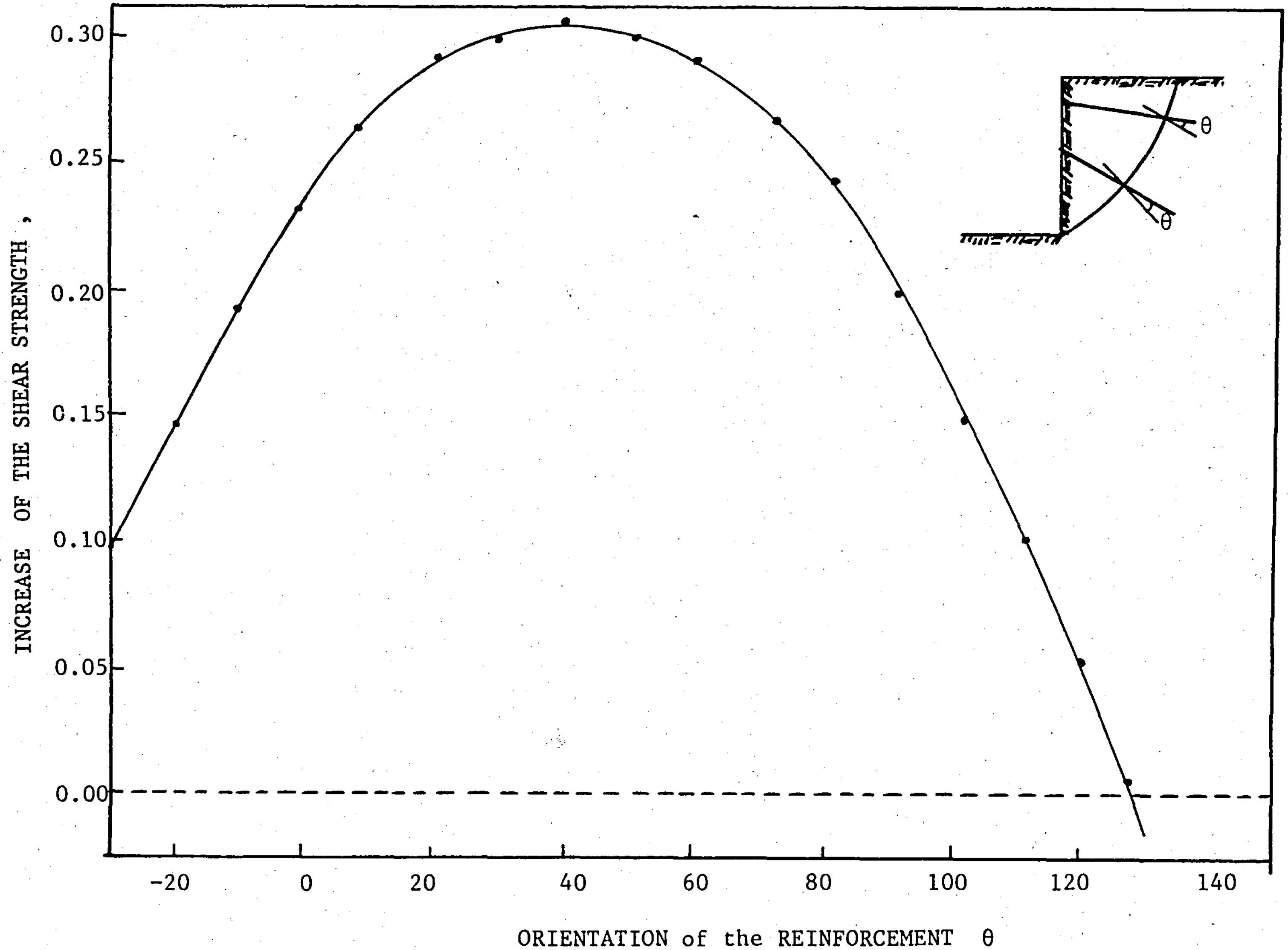


Fig. 2.6. Effect of Orientation of Reinforcements

on the failure surface. If the potential foilure surface in the reinforced soil mass is considered to be a zero extension plane, these results suggest that inclining the reinforcement vertically in an unstable slope or excavation can significantly reduce the efficiency of the reinforcing system.

The apparent friction angle of the reinforced soil mass along the failure plane can be written as:

$$\tan\phi^* = \tan\phi' + \frac{T_{(max)}}{\sigma A_{cr}} (\cos\theta \tan\phi' + \sin\theta)$$

$$= \tan\phi' + \frac{\Delta\tau}{\sigma} \qquad (2.8)$$

where T(max) is the lesser of P or R_T, respectively the limit pullout resistance and rupture strength of the reinforcement.

σ and $\Delta\tau$ are, respectively, the applied normal stress and the increase of the overall shear resistance due to the reinforcement

A_{cr} = the cross section area of the failure surface

ϕ' = the internal friction angle of the unreinforced sond

θ = the angle between the reinforcement and the normal to the failure plane

Although it seems easy to optimize the effectiveness of reinforcements by changing the orientation, it is complicated by the fact that, in slopes and embankments and within earth walls, the principal stress directions and the failure plane orientation are not the same at all points. Thus, the optimal reinforcement orientation would complicate the construction process. Therefore, the optimization of reinforoeement inclination would have its greatest applicability to soil nailing and root pile system.

2.5. DESIGN METHOD

2.5.1 Introduction

Reinforced Earth Structures must be designed so that they are stable both internally and externally. In order to be internally stable the reinforced soil structure must be coherent and self-supporting under the action of its own weight and any externally applied forces. This is accomplished through stress transfer from the soil to the reinforcement.

The reinforcements must be sized and spaced so that they are neither ruptured by the stress that they are required to carry nor taken out of the soil mass.

Reinforced soil structures should satisfy the same external design criteria as conventional retaining walls, independent of reinforcing system chosen. The wall must be stable against sliding due to the lateral pressure of the soil retained by the wall and resist overturning about its toe. It should be safe against foundation failure, and there must be overall slope stability.

Classical methods of soil mechanics have been found satisfactory for analysis of the external stability of reinforced soil structures. Therefore, this study is focused on internal stability analysis.

2.5.2. Design Parameters

The main design parameters of a Reinforced Earth System concern the mechanical properties of the soil and reinforcements, as well as the parameters characterizing the different mechanisms of soil-reinforcement interaction. These parameters can be classified in five main groups.

1. Mechanical proparties of the backfill material, particulary internal friction angle and density

2. Mechanical properties of the reinforcements including allowable tensile and shearing resistance and bending stiffness.

3. Parameters related to lateral earth thrust on the reinforcement, particularly the limit passive pressure of the soil and modulus of lateral soil reaction.

4. Parameters related to the soil-reinforcements interaction, particularly the apparent friction coefficient.

5. The geometric properties of the reinforcements area, length, horizontal and vertical spacing between the reinforcements.

2.5.3. Internal Stability Analysis

In order to evaluate the internal stability of reinforced soil system, two analytical approaches have been developed. The first considers local internal stability of the active zone in the structure with respect to two failure modes namely: (1) Failure by pullout of the reinforcements and (2) failure by rupture of the reinforcements. The second approach, on the other hand, considers the general stability of the structure and surroundings. In the proceeding section, Bishop's Method of Slices, which is one of the classical slope stability anlysis methods, has been adapted to evaluate the factor of safety with respect to failure along circular potential sliding surfaces, taking into account the available tensile and pullout resistance of the reinforcements crossing the potential sliding surface.

2.5.4. Calculation of the Forces in Reinforcement

In order to calculate the load that can be transferred safely by the reinforcements, four failure criteria have been considered.

1. Shear Resistance of the Soil

The classical Mohr-Coulomb's Failure Criterion is used

$$\tau = c + \sigma \tan \phi \qquad (2.9)$$

where

c is cohesion

ϕ is internal friction angle of the soil

2. Soil-Reinforcement Friction

The mobilized tensile fonce T must be balanced by the effective friction along the soil to reinforcement interface in the resistant zone behind the failure surface.

For a circular inclusion with diameter D, assuming that the limit skin friction f_{max} is constant all along the embedment length L_e, the mobilized pullout resistance T_m, can be evaluated as

$$T_m < \pi D L_e . f_{max} = T_{pl} \qquad (2.10)$$

where

T_{pl} is the pullout force

Although the limit unit skin friction, f_{max}, is considered to be constant, pullout tests on actual reinforcements are necessary to determine a reliable value of f_{max} to be used for design.

3. Normal Interaction Between the Soil and the Reinforcements

The normal interaction between the soil and a relatively rigid reinforcement results in a progressive mobilization of the passive lateral earth thrust on the reinforcement. This lateral earth pressure must be less than the maximum passive resistance that can be mobilized in the soil.

The shear forces mobilized in the inclusion are calculated considering the equation of elastic bending of the inclusion and assuming that the soil can be represented by a series of elostoplastic springs. The response of the soil to loading is thus characterized by a lateral reaction modulus R_s and relative rigidity of the inclusion and soil termed "the transfer length", as previously defined.

The maximum shear force, V_o, mobilized at the point of intersection with the failure surface is:

$$V_o = p \frac{D}{2} L_o \qquad (2.11)$$

where

P = the passive pressure on the bar

L_o = transfer length

4. Strength of the Inclusion

When the inclusion has to withstand both tensile and shearing forces, denoted respectively by T and V, the design criterion is derived from an analysis of the Mohr's Circle for the stresses in the inclusion considering that the metalic reinforcing element follows Tresca's failure criterion:

$$\frac{T^2}{R_N^2} + \frac{V^2}{R_C^2} < 1 \qquad (2.12)$$

$$R_C = R_N/2 \qquad (2.13)$$

where

R_N = Tensile strength of the bar

R_C = Shearing strength of the bar

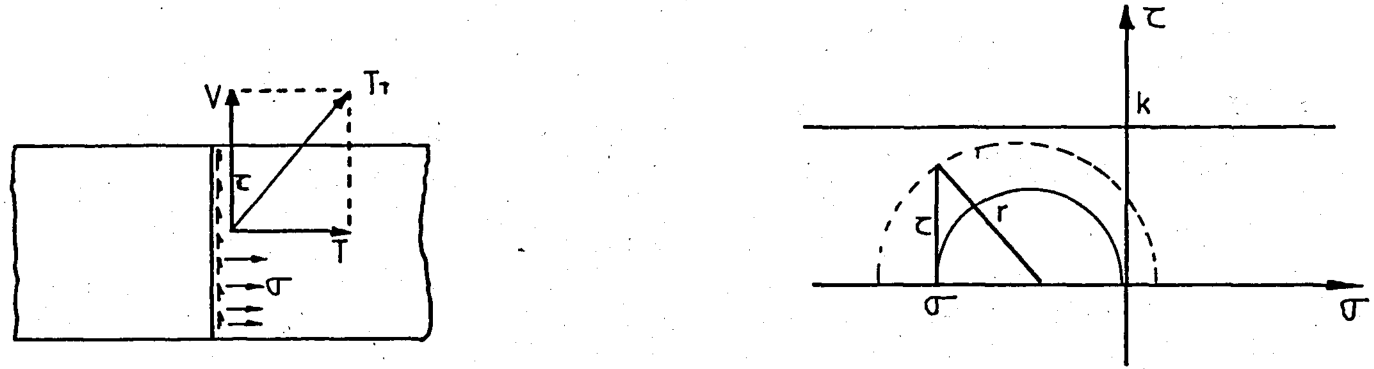
Figure 2.7.a shows the mohr's circle for the state of stresses in the inclusion. The tensile and shear forces, T_f and V_f, respectively, mobilized in the bar at failure depend on the inclination of the reinforcement with respect to the tangent of the failure surface. The failure criterion and the actual total force T mobilized in the inclusion and displacement vector $\vec{\delta}$ can be represented on the same axes as shown in Fig. 2.7.b.

The principle of maximum plastic work implies that at the point $\vec{T}_T(T_1V)$ corresponding to the failure state of stresses in the bar, the tangent to the ellipse, representing the failure surface, must be orthogonal to the direction of the displacement vector $\vec{\delta}$. From this principle and the Tresca failure criterion it can be shown that the tensile and shear forces at failure of a bar can be computed as a function of the angle between displacement vector and the bar, as.

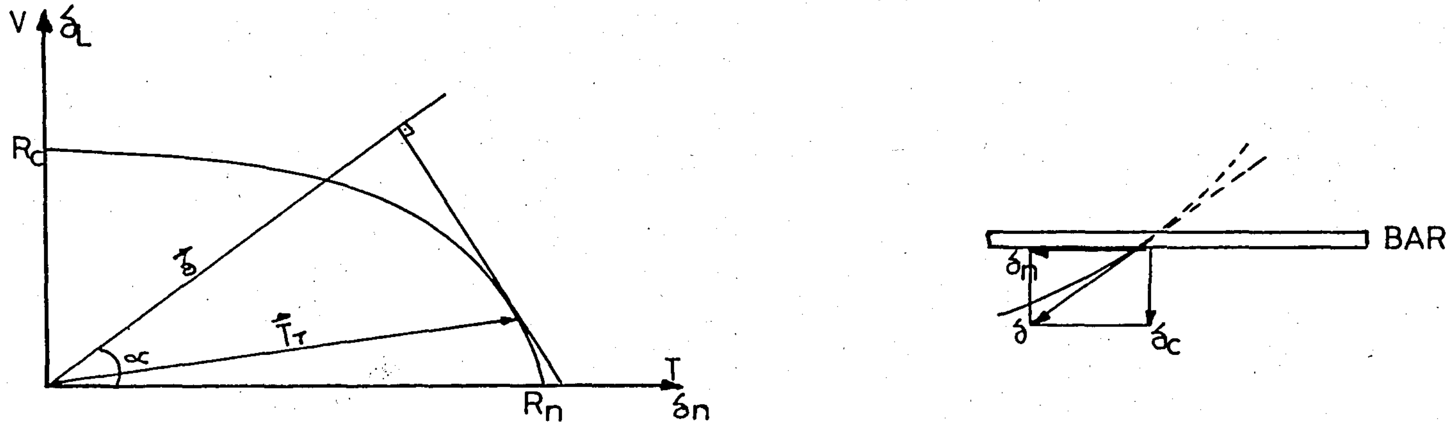
$$V_f = \frac{R_C}{\sqrt{1+4\tan^2(\frac{\pi}{2} - \alpha)}} \tag{2.14}$$

$$T_f = 4\, V_f \tan(\frac{\pi}{2} - \alpha) \tag{2.15}$$

It should be noted that for $\alpha=0$ only tensile force develops, while for $\alpha = \frac{\pi}{2}$ only shear force is mobilized as shown in Fig. 2.8.



a) State of the Stresses in the Bar



b) Application of the Principle of Maximum Work

Fig. 2.7. Determination of the Maximum Force in the Bar

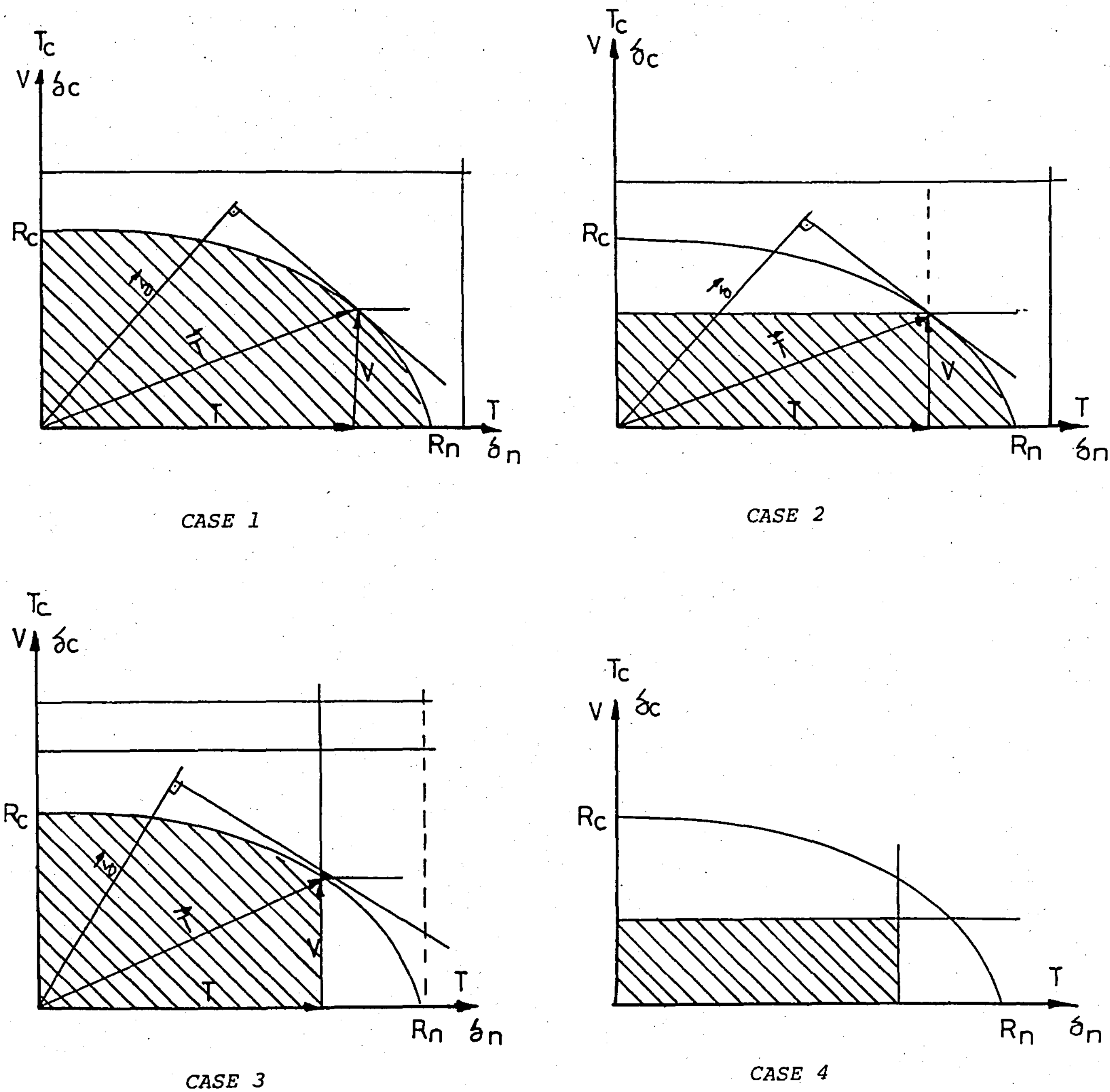


Fig. 2.8. Example of the Multi-Criteria Analysis

2.5.5. General Slope Stability

Bishop's Method of Slices has been utilized to evaluate the safety factor with respect to failure along potential sliding surface, taking into account the forces in the bars, calculated in previous section.

This method considers the potential failure surface as a circular arc, and defines the factor of safety, F, as the ratio of the available shear strength of the soil to that required to maintain the equilibrium.

Fig. 2.9 shows the assumed potential failure surface and the free body diagram of the forces.

The shear strength mobilized is

$$S = C' + (\sigma_n - u) \tan\phi' \tag{2.16}$$

where

C' is cohesion in terms of effective stress

ϕ' is angle of shearing resistance

σ_n is total normal stress

and u denotes pore pressure.

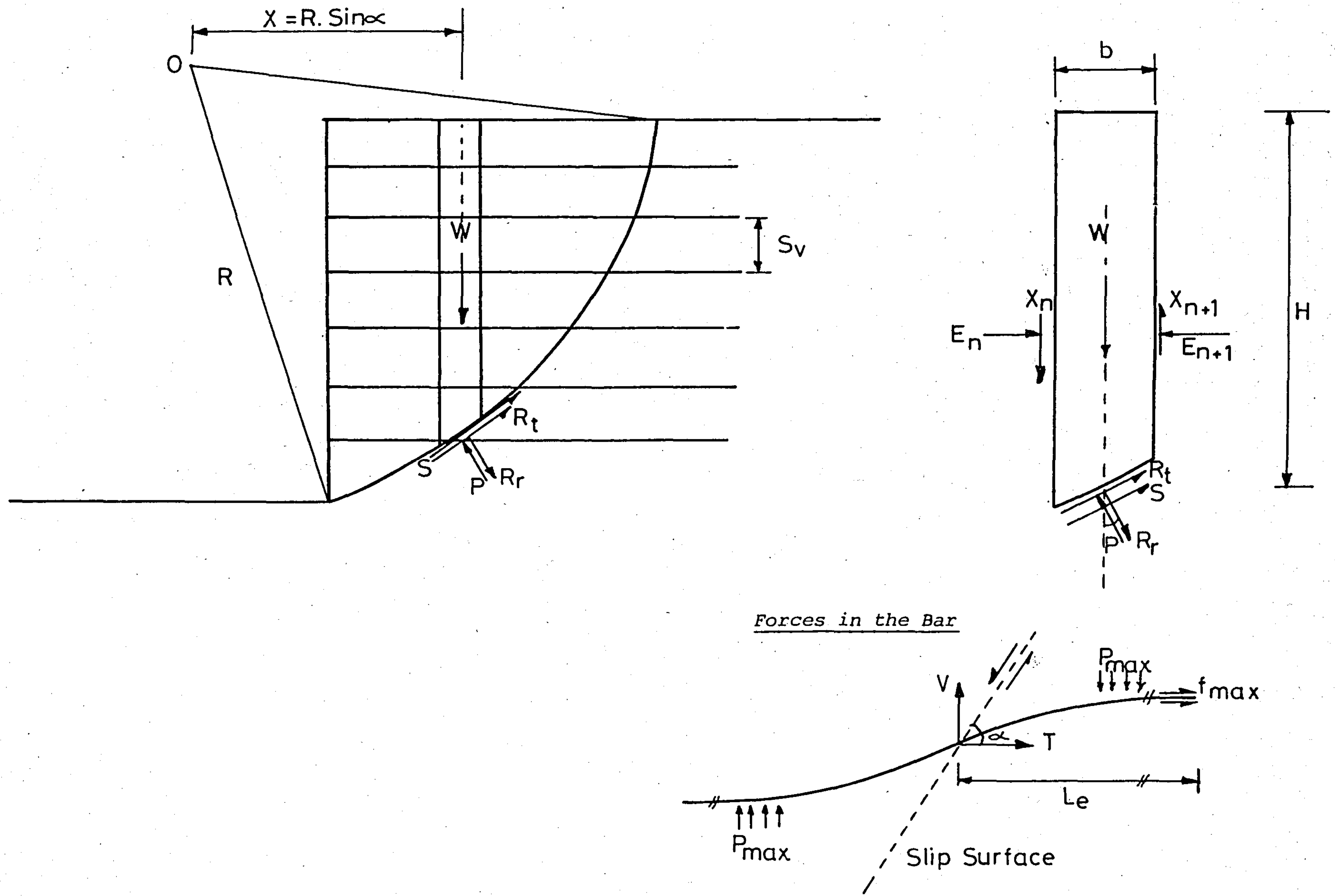


Fig. 2.9. Free Body Diagram of the Forces

In order to examine the equilibrium, it is necessary to know the value of normal stress, and pore pressure at each point on the slip surface, as well as the contribution of the reinforcements to whole equilibrium.

If the sum of the moment around the center of the failure surface is taken, one gets;

$$R\Sigma W \sin\alpha = R\Sigma(S + R_t) \tag{2.17}$$

Shearing force can be written as:

$$S = \frac{s\ell}{F} \tag{2.18}$$

By substituting equation (2.18) into (2.17) factor of safety can be expressed as;

$$F = \frac{\Sigma \dfrac{S \cos\alpha}{b}}{\Sigma W\sin\alpha - \Sigma R_t} \tag{2.19}$$

Writing the sum of the forces in Y direction on free body diagram yields the following equation;

$$W = (P-R_r)\cos\alpha + (S+R_t)\sin\alpha \tag{2.20}$$

$$W = (P-R_r)\cos\alpha + \left(\frac{s\ell}{F} + R_t\right)\sin\alpha \tag{2.21}$$

$$P\cos\alpha = W + R_r\cos\alpha - \frac{s\ell}{F}\sin\alpha - R_t\sin\alpha \tag{2.22}$$

The normal stress on the slip surface is

$$\sigma = \frac{P}{\ell} = \frac{P\cos\alpha}{b} \tag{2.23}$$

By substituting Equation 2.22 into Equation 2.23 the normal stress may be written as follows:

$$\sigma = \frac{W}{b} + \frac{R_r\cos\alpha}{b} - \frac{s}{F}\frac{b}{b}\frac{\sin\alpha}{\cos\alpha} - \frac{R_t\sin\alpha}{b} \tag{2.24}$$

Substituting equation 2.24 into equation 2.16, while considering equation 2.18 the shear strength of the soil may be written in the following form:

$$S = C + \left(\left(\frac{W}{b} + \frac{R_r\cos\alpha}{b} - \frac{S}{F}\tan\alpha - \frac{R_t\sin\alpha}{b} - u\ell\right)\tan\phi\right.$$

$$S\left(1 + \frac{\tan\phi\tan\alpha}{F}\right) = C + \left(\left(\frac{W}{b} + \frac{R_r\cos\alpha}{b} - \frac{R_t\sin\alpha}{b}\right) - u\right)\tan\phi$$

$$S = \frac{C + \left(\frac{W}{b} + \frac{R_r \text{Cos}\alpha}{b} - \frac{R_t \text{Sin}\alpha}{b} - u\ell\right) \tan\phi}{1 + \frac{\tan\phi\tan\alpha}{F}} \qquad (2.25)$$

If equation 2.25 is substituted into equation 4 one gets

$$F = \frac{\Sigma \dfrac{C + \left(\left(\frac{W}{b} + \frac{R_r \text{Cos}\alpha - R_t \text{Sin}\alpha}{b}\right) - u\ell\right) \tan\phi}{1 + \frac{\tan\phi\tan\alpha}{F}} \; \dfrac{b}{\text{Cos}\alpha}}{\Sigma W \text{Sin}\alpha - \Sigma R_t} \qquad (2.26)$$

Finally the equation for determining the factor of safety is determined ~~fut~~ as:

$$F = \frac{\Sigma \dfrac{\left(C + \frac{W}{b} \tan\phi - u\ell\tan\phi\right) \frac{b}{\text{Cos}\alpha} + \left(R_r \text{Cos}\alpha - R_t \text{Sin}\alpha\right) \frac{\tan\phi}{\text{Cos}\alpha}}{1 \quad \frac{\tan\phi\tan\alpha}{F}}}{\Sigma W \text{Sin}\alpha - \Sigma R_t}$$

(2.27)

2.6. DEVELOPMENT OF THE COMPUTER PROGRAM FOR INTERNAL STABILITY ANALYSIS

A computer program (SLOPER) has been developed to estimate the factor of safety in slopes with or without reinforcing strips, using Bishop's Method of Slices. In the case of reinforcement, program also gives the strip forces, calculated according to multi-failure criteria which has been explained in detail in previous sections.

The program listing and user's manual and typical program output are given in Appendix-C.

2.7. SUMMARY

The principle of soil reinforcement interaction and design methods have been covered in this chapter. The equations for determining the frictional load transfer from soil to reinforcement were derived and the factors that effect on soil-reinforcement interaction have been examined. Tensile and, if it is required, shear forces were found out using multicriteria analysis method, for which an example is given in Fig. 2.8. For internal stability analysis of reinforced earth system, Bishop's Method of Slices was utilized and equation for evaluating the factor of safety has been derived. Finally a computer program which has capability of calculating the forces mobilized in reinforcing strips was developed.

CHAPTER-III

SELECTION OF COMPONENTS AND CONSTRUCTION

3.1. INTRODUCTION

In general, placement of succesive layers of backfill material, reinforcements, and facing elements doesn't require specialized contractors, skilled labor, or specialized equipment. Many of the components of the available earth reinforcement systems are prefabricated, thus providing ease of forming and handling and allowing relatively quick construction.

Generally, only minimal working space is required in front of the earth structure, which is specially advantageous when working along existing highways or in restricted areas.

A fairly wide range of backfill materials has been used for reinforced soil structures. Suitable quality backfill material can frequently be found near the construction site. Therefore it is not necessary to import the backfill material for construction purposes.

3.2. REINFORCING STRIPS

Ideally reinforcements should have the following characteristics.

* High tensile strength
* High apparent friction coefficient with the backfill material
* High durability
* Low deformability under working loads
* Flexibility
* Low cost

Recently ordinary mild galvanized steel is the most frequently used. The steel bars have a thickness of 3 mm, and a width of 50, 60 or 90 mm, and a yield strength of 3500 kg/cm^2.

These steel elements can be either driven into the ground or placed in prebored holes and filled with a suitable grout. After the boreholes have been drilled, the bars are placed and sealed with cement grout. Generally the boreholes are inclined slightly downwards from facing to enable gravity filling. In same cases, grouting is performed under small pressure using a packer, placed close to the facing.

3.3. THE FACING

The main role of facing is to prevent the backfill material sloughing away from the wall face. This purpose can be achieved in many ways.

There are three kinds of facing commonly used in practice:

* Welded wire mesh
* Prefabricated panels
* shotcrete facing

Typical facing elements are illustrated on Fig 3.1 and Fig. 3.2.(Ref.8) Welded wire mesh is used with fragmented rock or intermediate soils, such as chalk marl or shales, to prevent block falls. Prefabricated panels are being developed for permanent structures. The third type of facing method, shoterete, is a concrete applied to soil using special equipment. The maximum aggregate size for shotcrete is usually limited to 10 to 15 mm.

There are two methods for placing the shotcrete; dry and wet. In the dry method, water is introduced at the end of transportation of the dry cement and granular material by compressed air. In the wet method, on the other hand, the wetted mixture is trasported under pessure by means of a concrete pump. It is generally necessary to reinforce the shotcrete. Typically, reinforcement consists of welded wire mesh with wire diameter varying from 5 to

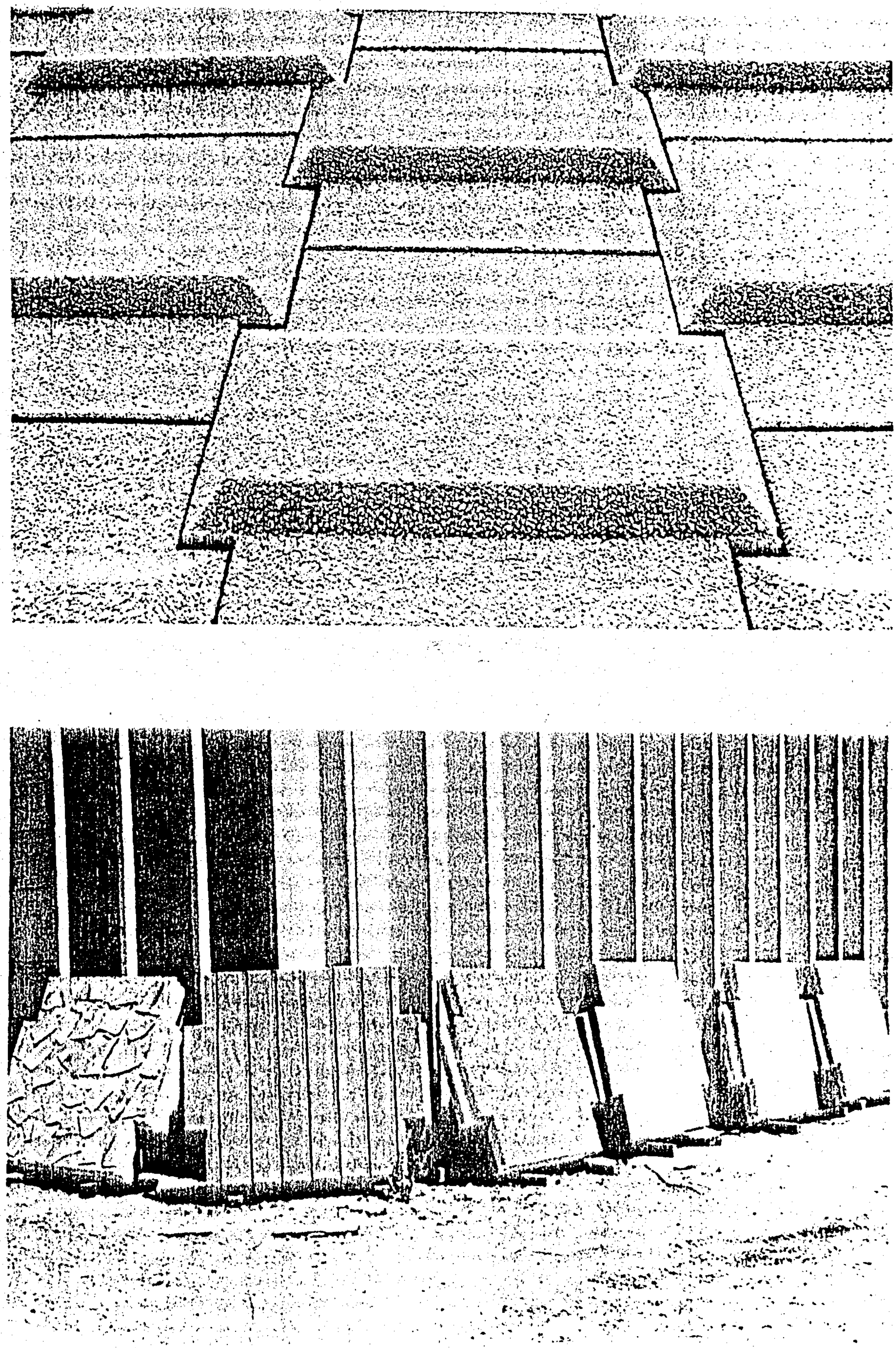


Fig. 3.1. Typical Facing Elements

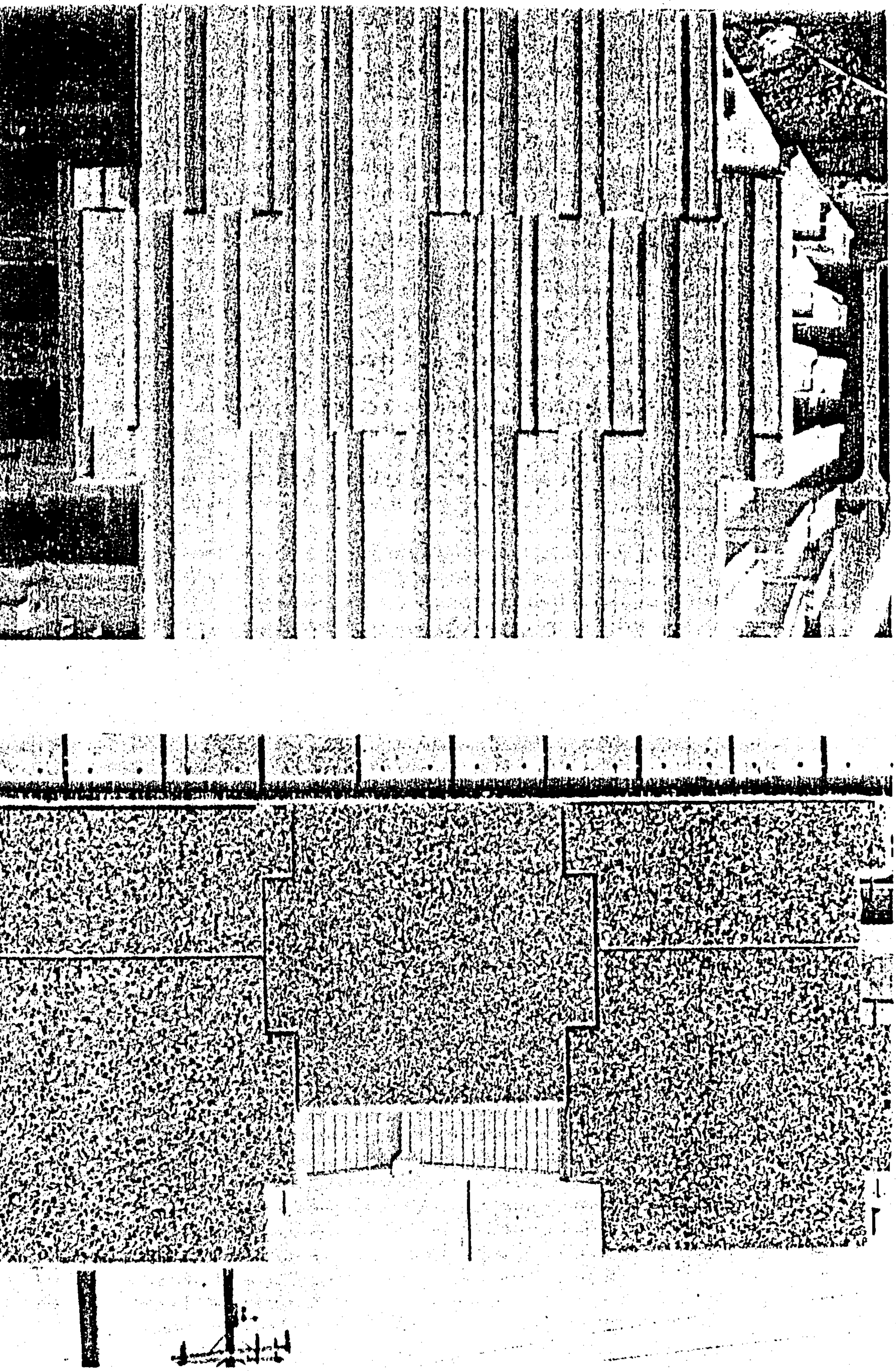


Fig. 3.2. Typical Facing Elements

10 mm. The attachment of grouted reinforcements to the facing is generally made by bolting the bars to a square steel plate 15 to 20 mm thick and 300 to 400 mm wide.

For large spacings between the reinforcements, the facing must be designed taking into account the maximum mobilized bending moment or tensile stress.

3.4. BACKFILL

Backfill material for a Reinforced Earth Structure is selected to satisfy the following requirements:

* Internal friction should be high enough to insure the necessary soil-reinforcement interaction.
* Moisture content may have to be limited to avoid difficulties during compaction.
* Should not cause excessive corrosion in rein forcing elements.

In addition, all backfill material should be free from organic and other compressible materials.

Since a reinforced earth wall is constructed without external support framework, it must be inherently stable at all stages of construction. Thus there must be an immediate transfer of effective normal stress between the backfill soil and the reinforcing strips with every added layer of soil. This requires the backfill material to be properly selected. The selection of the backfill material for all internally reinforred soil systems should be prepared based on the following criteria:

A. Gradation

Sieve Size	Percent Passing
6"	100
3"	100-75
300	15-0

B. Plasticity

Plasticity Index I_p shall not exceed 6.

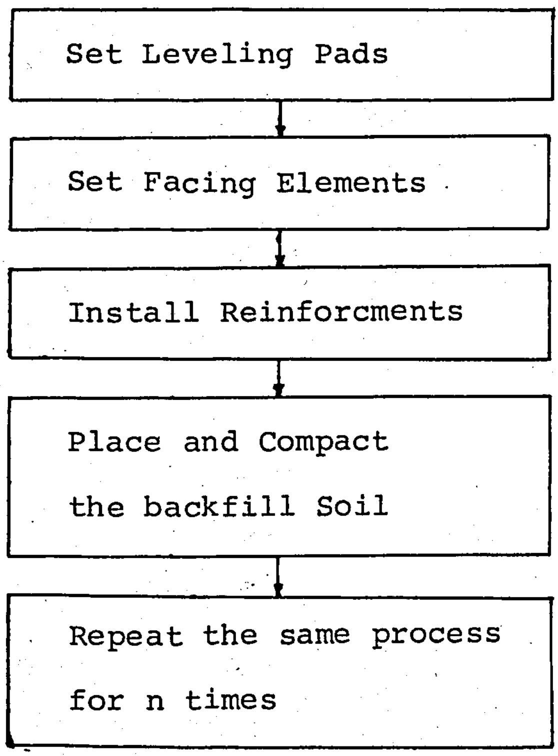
Results of the laboratory and field pullout tests have indicated that all mateirals having up to 25 precent passing the No.200 steve will provide adequate pallout and frictional resistance. However, some materials having 15 to 25 percent passing the No. 200 sieve may produce problems related to frost susceptibility, compaction and drainage. Backfill requirements therefore, should be determined on an individual project basis by taking into consideration of the specific backfill characteristics of the anticipated barrow source.

3.5. CONSTRUCTION

In this section typical construction phases are briefly described in the cases of reinforced earth wall and soil nailing during excavation.

3.5.1. Reinforced Earth Wall

The phases of construction in reinforced soil retaining structures are illustrated on following diagram and are illustrated on Fig.'s 3.3, 3.4; 3.5 and 3.6 (Ref.8).



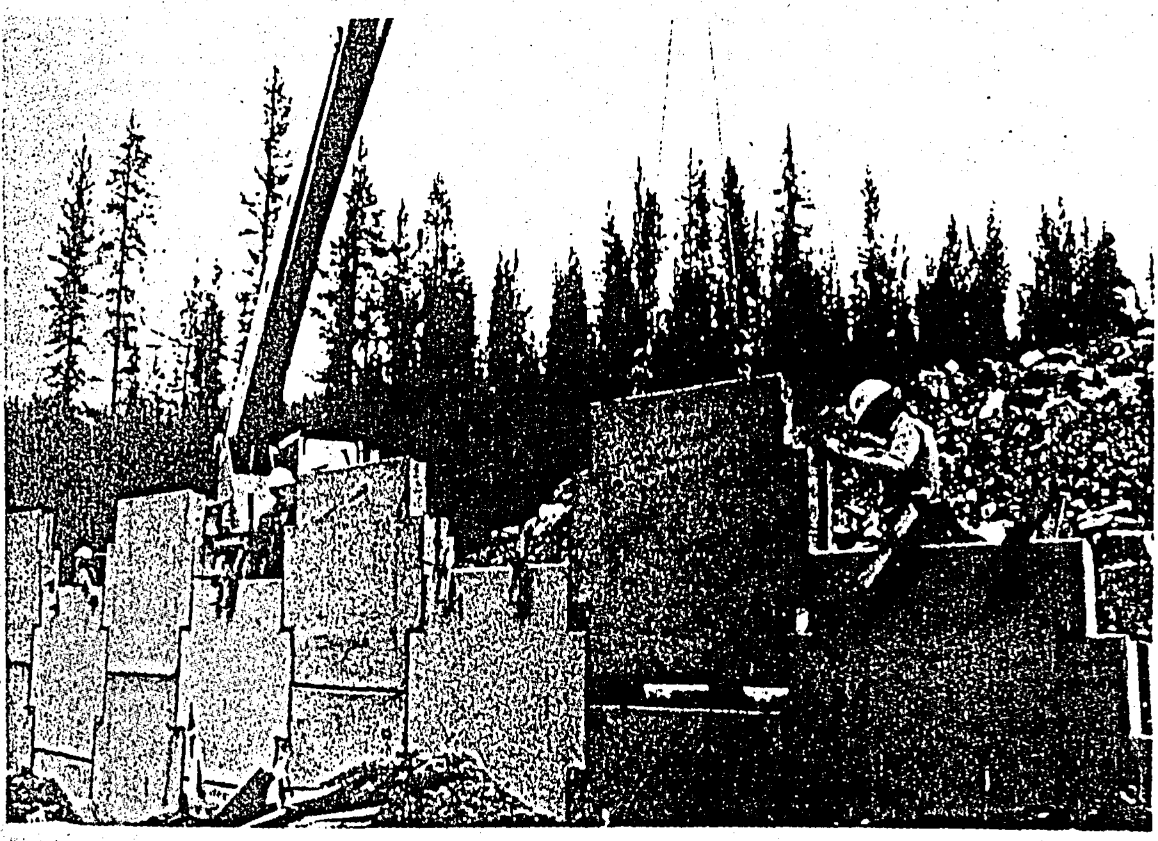
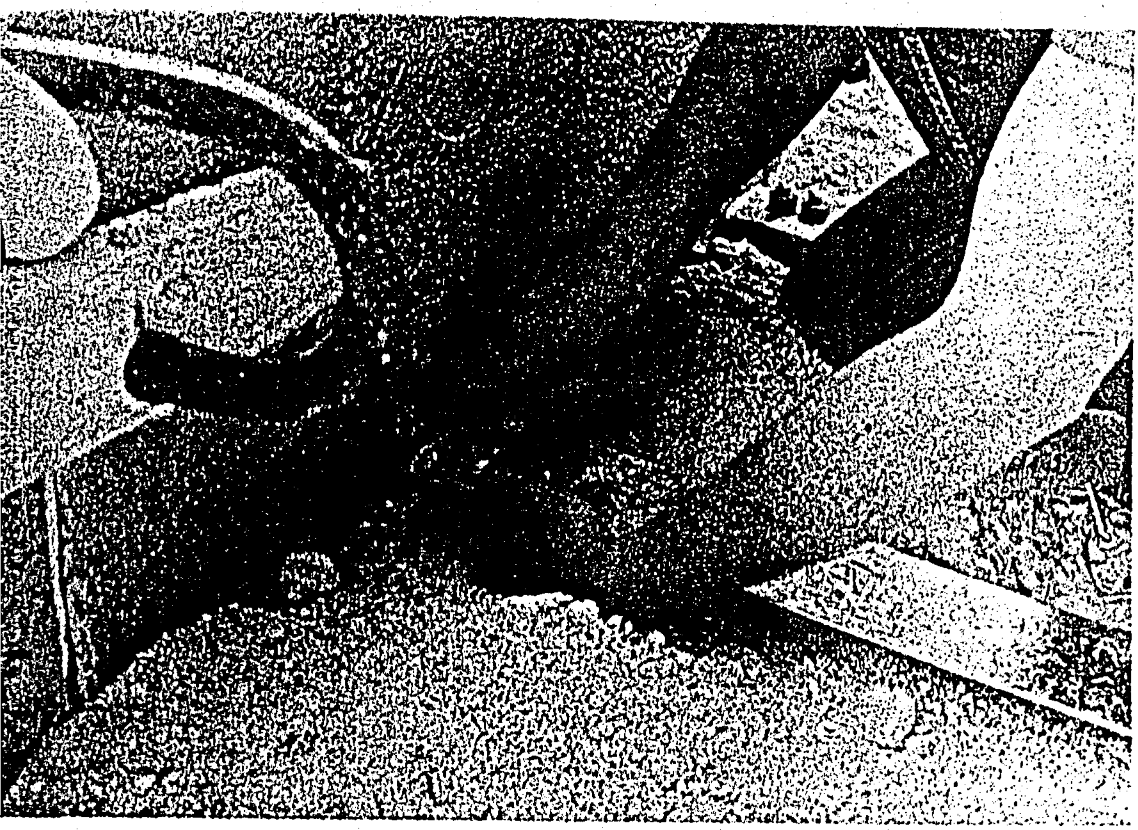


Fig. 3.3. Setting the Facings



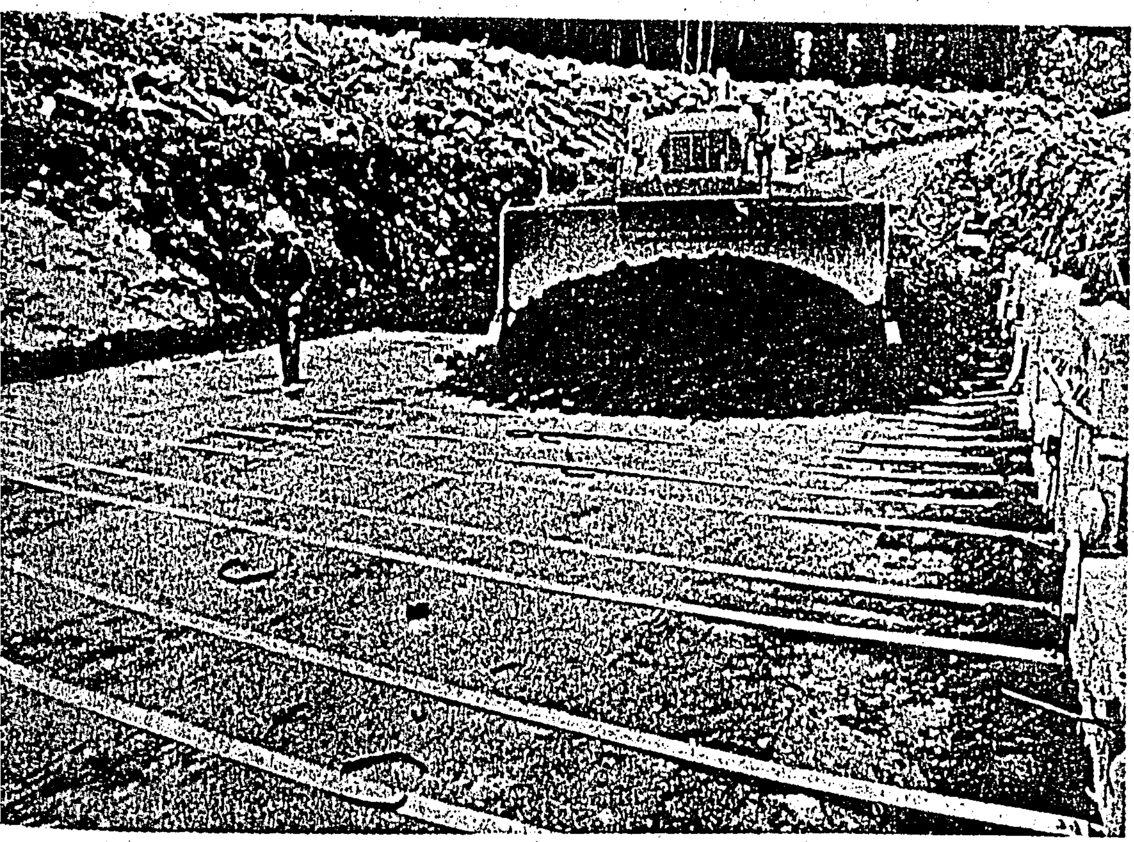


Fig. 3.4. Installing the Reinforcements

A. Setting the Leveling Pads

The first step of reinforced earth wall is to locate the footing beneath the facing panels. It must be correctly levelled in order to ensure an appropriate alignment for the first row of panels and to facilitate the setting up to the whole facing.

B. Setting the Facing Elements

The stability of the facing during the backfilling operation is ensured for the first row of panels by temporary struts placed on the external side of the wall, and for the succesive levels by temporarily securing facing panels by wooden wedges and screw clamps.

Concrete facing panels are joined as shown in Fig. 3.5 and vertical and horizontal joints are sealed by a geotextile called "Filter Fabric".

C. Placement of Reinforcements

The reinforcements should be laid flat on the compacted embankment and fixed to the tie-strips protruding from the panels, Fig. 3.4. Before backfilling, all of the reinforcements must be bolted to the tie-strips and corrosive protection if necessary, should be applied.

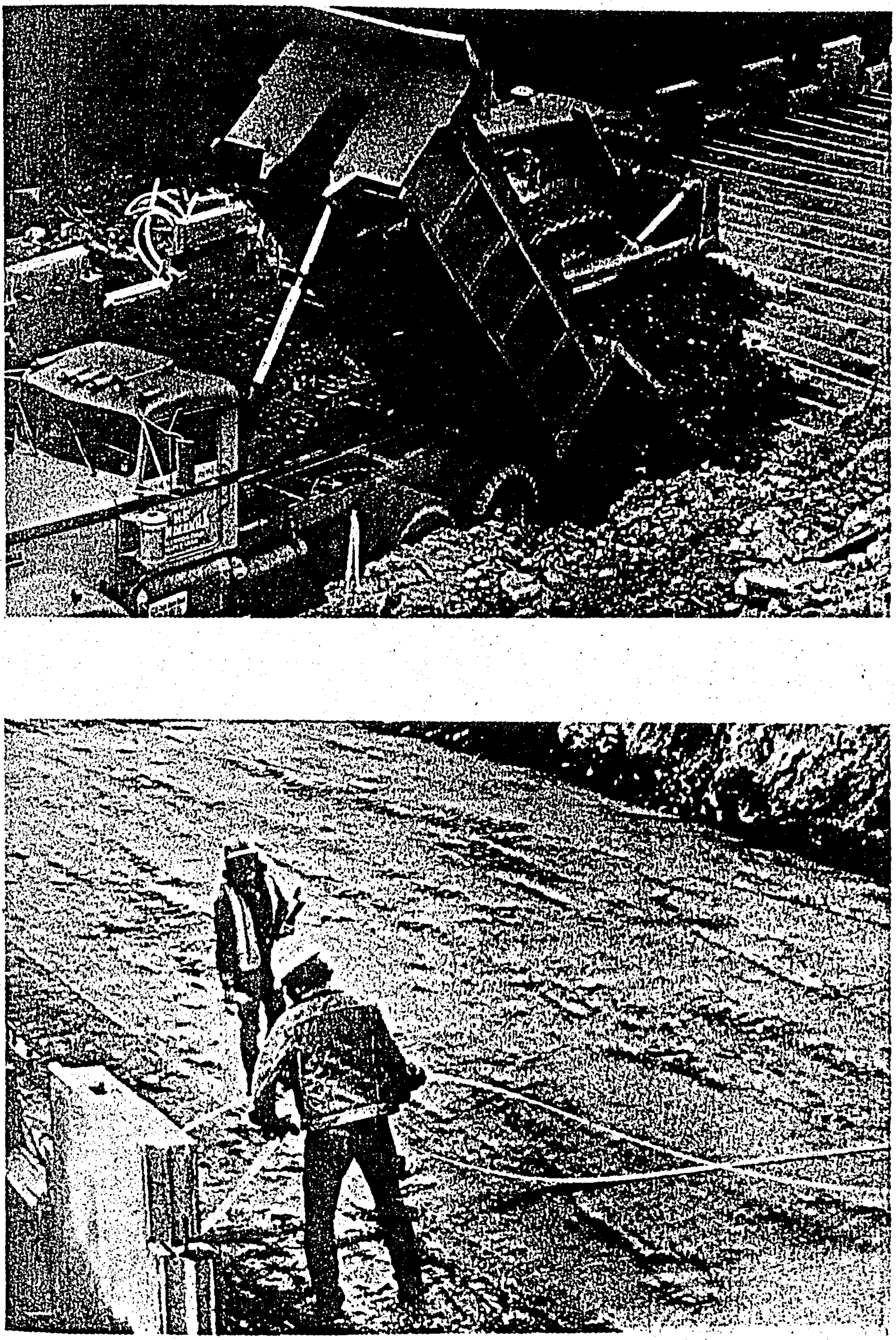
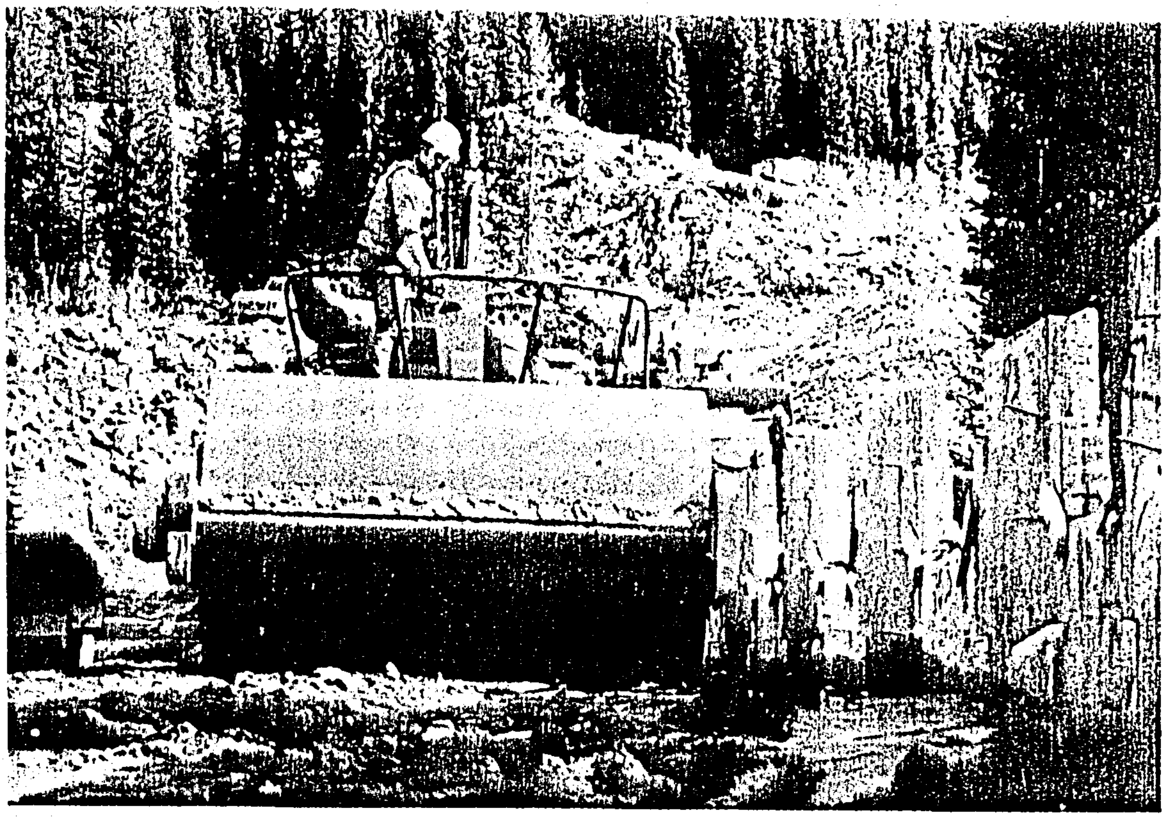


Fig. 3.5. Backfilling



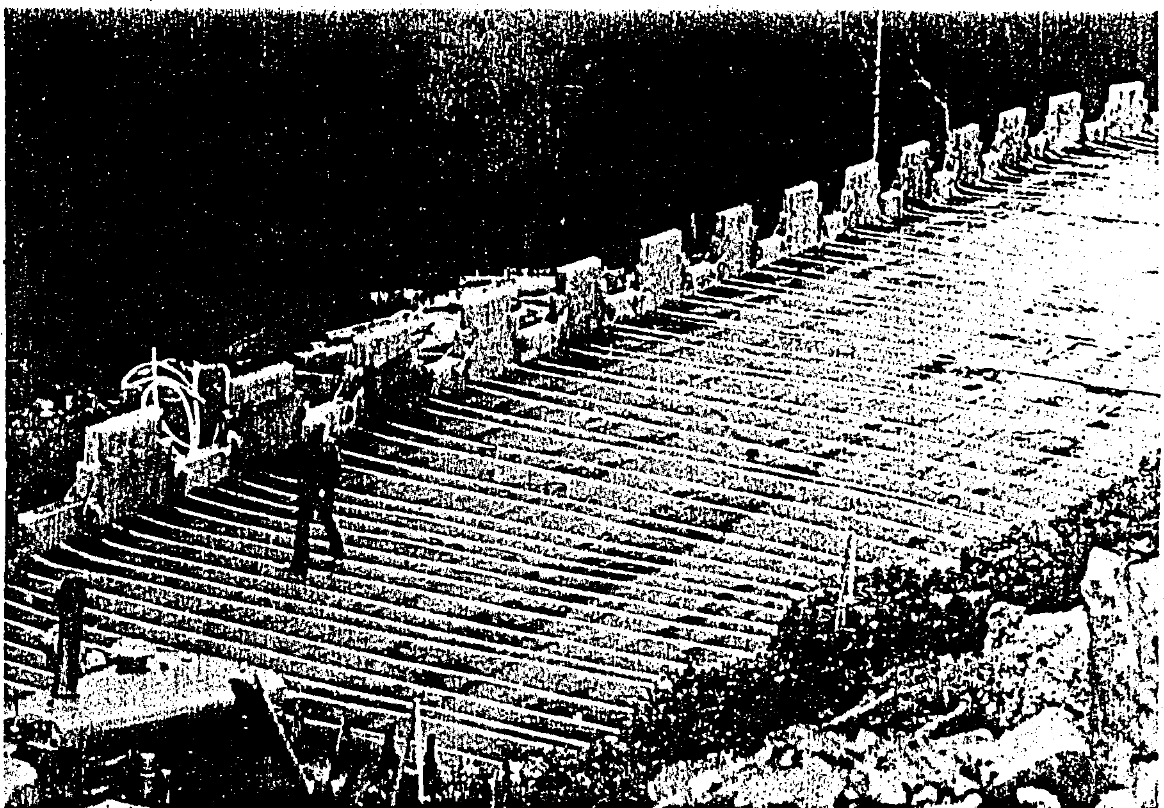


Fig. 3.6. Compacting the Backfill

D. Placement and Compaction of Backfill Soil

The placement of the backfill material on a layer of reinforcement should begin at the center of the first reinforcement reached by the equipment. Equipment should not run over exposed reinforcements. Care should be taken to insure that the reinforcing strips are properly aligned after dumping the backfill. Thickness of the backfill layer should be 30 cm in average for the case of concrete facing.

Proper compaction of the backfill soil is required to minimize subsequent settlements and to insure good soil-reinforcement stress transfer. Each fill layer must be levelled after compaction to ensure that all the reinforcements are in contact with the soil over their entire bottom surface. This may require some manual filling and tamping, particularly near the connection of reinforcements to the facing and in zones of difficult access.

3.5.2. Excavation

Excavation and installation of the reinforcing members is done in sequential steps, like in the case of reinforced earth wall. These steps are shown in the following diagram and illustrated in Fig. 3.7. Excavation process is carried out using small conventional earth work equipment, starting at the top and processing in incremental steps towards the bottom. Generally the short-term cohesion

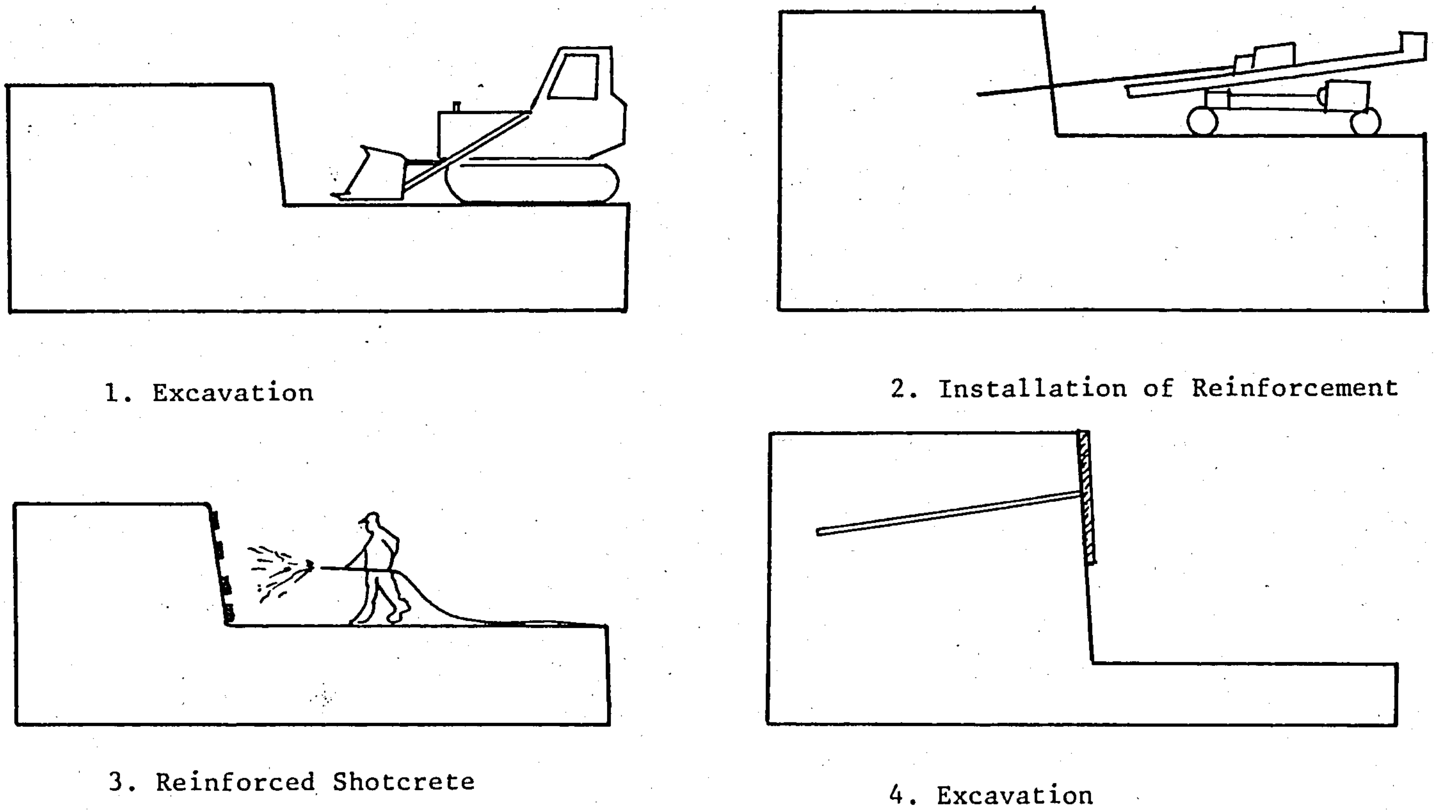


Fig. 3.7 Construction Phases for Soil Nailing

of the soil is sufficient to ensure local stability of each excavated step, which is commonly limited to 150 cm.

Excavate First Layer of Soil

↓

Install First layer of Reinforcements and Panel Facing

↓

Excavate Second layer of Soil

↓

Install Second layer of Reinforcements and Panel Facing

↓

Excavate last Layer of Soil

↓

Install last layer of Reinforcements and Panel Facing

3.6. SUMMARY

In this chapter, selection of components of reinforced earth structures have been briefly explained and construction squences for the cases of reinforced earth wall and excavation have been shortly described Aditionally, specifications for a proper beckfill material are provided.

CHAPTER-IV

THE FINITE ELEMENT METHOD

4.1. INTRODUCTION

The Finite Element Method (F.E.M) is a technique used for the analysis of the stresses, strains and displacements in a continuous bodies. In an elastic halfspace the number of interconnection points is infinite, but in finite element idelization the real continuum is divided into finite number of smaller units. Instead of solving the problem for the entire body in one operation, the solutions are formulated for each unit or element and later combined to obtain the solution for whole body. The primary unknowns to be determined are generally the displacements at the nodal points. Then a set of functions called "Shape Functions" is chosen to define the state of displacement and strain within each element in terms of its nodal displacements. These strains with any initial strains and constitutive relationships define the state of the stress throughout the element.

4.2. PRINCIPLE OF MINIMUM POTENTIAL ENERGY

In order to make stress or strain analysis of continuous media by Finite Element method, a set of equilibrium equations for each element should be obtained to build up the equilibrium equations for the whole system. The equilibrium equations are obtained by utilizing the principle of minimum potential energy.

The potential energy of a loaded elactic body is represented by the sum of internal strain energy stored as a result of deformations and the potenial energy of external loads, and can be characterized by the following equation

$$\Pi_{system} = U_{int} + U_{ext} \tag{4.1}$$

where

Π_{system} = Total potential energy of the system

U_{int} = Internal strain energy

U_{ext} = Potential energy of applied external loads,

The potential energy of the whole system should be equal to the sum of the potential energies of the elements assembling the whole body, that is

$$\Pi_{system} = \sum_{e=1}^{N} \Pi e \tag{4.2}$$

where e = Element number and N = the total number of elements used to represent the body. Potential energy of external load is:

$$U_{ext} = -W \tag{4.3}$$

Where W denotes work done by external forces. Due to the fact that "Total potential energy of the system must be minimum in order the system to be in equilibrium state", the potential energy of each element must be mimimum. Minimization of the potential energy of each element will result in the minimization of the potential energy of the whole system.

The necessary formulas for a single element can be derived as follows

$$U_{in} = \frac{1}{2} \int_{\nu} \{\varepsilon\}^T \{\sigma\} \, d\nu \tag{4.4}$$

where $\{\varepsilon\}$ and $\{\sigma\}$ = the stress and strain vectors, respectively, v = the volume of the element.

U_{nex} = Work donebybody forces + work done by distributed edge loads + work done by inertiaforces + Work done by concentrated loads applied at the nodes

$$U_{ext} = \int_v \{U\}^T \{x\}dv + \int_s \{U\}^T \{P\}ds - \int \rho \{U\}^T \{\ddot{U}\}dv \quad \{d\}^T\{P\} \tag{4.5}$$

where

$\{d\}$ is displacement vector at the nodes

S is the boundary line of the element

$\{U\}$ is the displacement vector at any point on the element which is related to the nodal displacement by displacement functions as shown Fig. 4.1.

ρ is mass density

4.3. INITIAL STRESSES AND STRAINS

Stress-strain relationship with initial stresses and strains may be written as

$$\{\sigma\} = [D]\,(\{\varepsilon\} - \{\varepsilon\}_0) + \{\sigma\}_0 - \alpha\Delta T\{D_T\} \tag{4.6}$$

where

$\{\varepsilon\}_0$: initial strains

$[D]$: material properties matrix

$\{\sigma\}_0$: initial streses

α : coefficient of thermal expansion

$\{D\}_T$: temperature-material matrix

ΔT : change in temperature

Hence, equation (4.4) takes the following form,

$$U_{in} = \frac{1}{2} \int_v \{\varepsilon\}^T [D] \{\varepsilon\} dv - \frac{1}{2} \int_v \{\varepsilon\}^T [D] \{\varepsilon_0\} dv$$

$$+ \frac{1}{2} \int_v \{\varepsilon\}^T \{\sigma_0\} dv - \frac{1}{2} \int_v \{\varepsilon\}^T \alpha \Delta T \{D_T\} dv \qquad (4.7)$$

4.4. FINITE ELEMENT FORMULATION

The strain-displacement relations can be written in the matrix form as

$$\{\varepsilon\} = [\Delta] \{U\} \qquad (4.8)$$

in which

$[\Delta]$ = a matrix operator which relates strains to derivatives of displacements

The displacement vector, $\{U\}$, can be written as

$$\{U\} = [N]\{d\} \qquad (4.9)$$

where

$[N]$ is the shape matrix, relating displacements at any point on the element to the nodal displacements.

Substituting equation 4.9 into equation 4.8 the expression for strain becomes

$$\{\varepsilon\} = [\Delta]\ [N]\ \{d\} \tag{4.10}$$

$$[G] = [\Delta]\ [N]$$

$$\{\varepsilon\} = [G]\ \{d\} \tag{4.11}$$

Total potential energy given by equation 4.1 for a Unique finite element takes the following form

$$\begin{aligned}\Pi_e = {} & \frac{1}{2}\int \{d\}^T\ [G]^T[D]\ [G]\ \{d\}dV - \frac{1}{2}\int_V \{d\}^T[G]^T\ [D]\ \{\varepsilon_o\}dV \\ & + \frac{1}{2}\int_V\{d\}^T\ [G]^T\{\sigma_o\}dV - \frac{1}{2}\alpha\Delta T\int_V\{d\}^T[G]^T\{D_T\}dV \\ & - \int_V\{d\}^T[N]^T\{x\}dV + \int_S d^T\ [N]^T\{p\}\,ds \\ & + \rho\int_V\{d\}^T[N]^T[N]\{\ddot{d}\}\ dV - \{d\}^T\{P\}\end{aligned} \tag{4.12}$$

The potential energy has to be minimized for the condition of equilibrium to be satisfied.

$$\frac{\partial\Pi e}{\partial d_i} = 0 \quad (I = 1, 2, \ldots n)$$

where

i = row number in the displacement vector

n = Degree of freedom in one element

When |G| and |N| matrices are appropriately substituted, and derivation prodcedure is completed, the following expression is obtained.

$$[k]\{d\} + [m]\{\ddot{d}\} = \{P\} - \Sigma\{f\} \qquad (4.13)$$

where

$$\{f\} = \{f\}_{\varepsilon_0} + \{f\}_x + \{f\}_p + \{P\} + \{f\}_{\sigma_0} + \{f\}_T$$

Stiffness Matrix $[k] = \int_V [G]^T[D][G]\,dV$

Mass Matrix $[m] = \int_V [N]^T[N]\,dV$

Initial strains $\{f\}_{\varepsilon_0} = -\frac{1}{2}\left(\int[G]^T[D]\,dv\right)\{\varepsilon\}_0$

Initial stresses $\{f\}_{\varepsilon_0} = \frac{1}{2}\left(\int[G]^T dv\right)\{\sigma\}_0$

Temperature forces $\{f\}_T = -\frac{1}{2}\alpha\Delta T\left(\int[G]^T dv\right)\{D\}_T$

Body forces $\{f\}_x = -\int [N]^T\{x\}\,dv$

Edge forces $\{f\}_s = - \int_s [N]^T \{P\}_s \, ds.$

4.5. PROCEDURE OF THE FINITE ELEMENT ANALYSIS

The Equation 13 derived for the most general case takes the following form for steady state condition

$$[K]\{d\} = \{P\} \tag{4.14}$$

In this equation, the element stiffness matrix, $|K|$, varies according to the type of element selected to represent the media. The content of this matrix, therefore, can be determined using the geometrical properties of chosen element.

Evaluation of the right hand side of Equation 5 gives a simultaneous linear equation which may be solved by various techniques, eg. Gauss-Jourdan Method. Solution of these equations give the magnitude of displacements at nodal points. By utilizing these values, stresses and strains for each element can be determined from the following equations.

$$\{\varepsilon\} = [G]\ \{d\} \tag{4.15}$$

$$\{\sigma\} = [D]\ [G]\ \{d\} \tag{4.16}$$

$|D|$ and $|G|$ contain physical properties of material under consideration and geometrical pecularities of the element chosen to represent the structure. It means, by altering the content of these two matrices, various materials and different types of elements can be employed in the finite element analysis.

For the case of plane stress condition, ie. there is no stress in one of the three axes, the content of material property matrix, D , is as follows.

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{4.17}$$

where

E is elasticity modulus

and

ν is poisson's ratio

In soil structures, e.g. earth dams, the system may be loaded in all three directions. But it is the case that, in these structures, the extention of the system in a particular direction is restrained. In other words, it is assumed that there is no strain, say, in axial direction of an earth dam. This is called plane strain condition and at this condition soil problem can possibly be converted from three dimensions into two dimensions by setting the amount of movement in this particular direction to zero.

The material propety matrix for plane strain condition will be different than it was for plane stress case. If the neccessary derivation is accomplished, |D| matrix for plane strain condition will be as follows.

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1}{2} - \nu \end{bmatrix} \qquad (4.18)$$

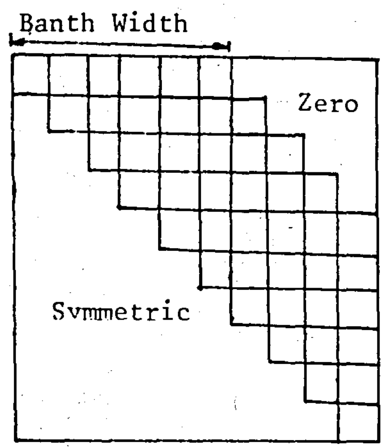
A similiar procedure will be employed to derive appropriate material property matrix for reinforced earth structure, which will be considered as a composite material.

4.6. SOLUTION OF SYSTEMS EQUATIONS

After computing the element stiffness matrix for each element, the system stiffness matrix is assembled by superimposing the element stiffness matrices, using "code number technigue". Calculating the load vector, {P}, gives simultaneous linear equation as follows.

$$[K]\{D\} = \{P\}$$

Since the element stiffness matrices are symmetric only one half of the matrix needs to be generated. Moreover, all of the non-zero coefficients in the system are confined within a band in stiffness matrix as shown below.



The band width depends on the largest difference between the code numbers for a single element. In the computer program, only the storage of the elements within the upper half of the band is sufficient. This assures significant amount of memory to be saved.

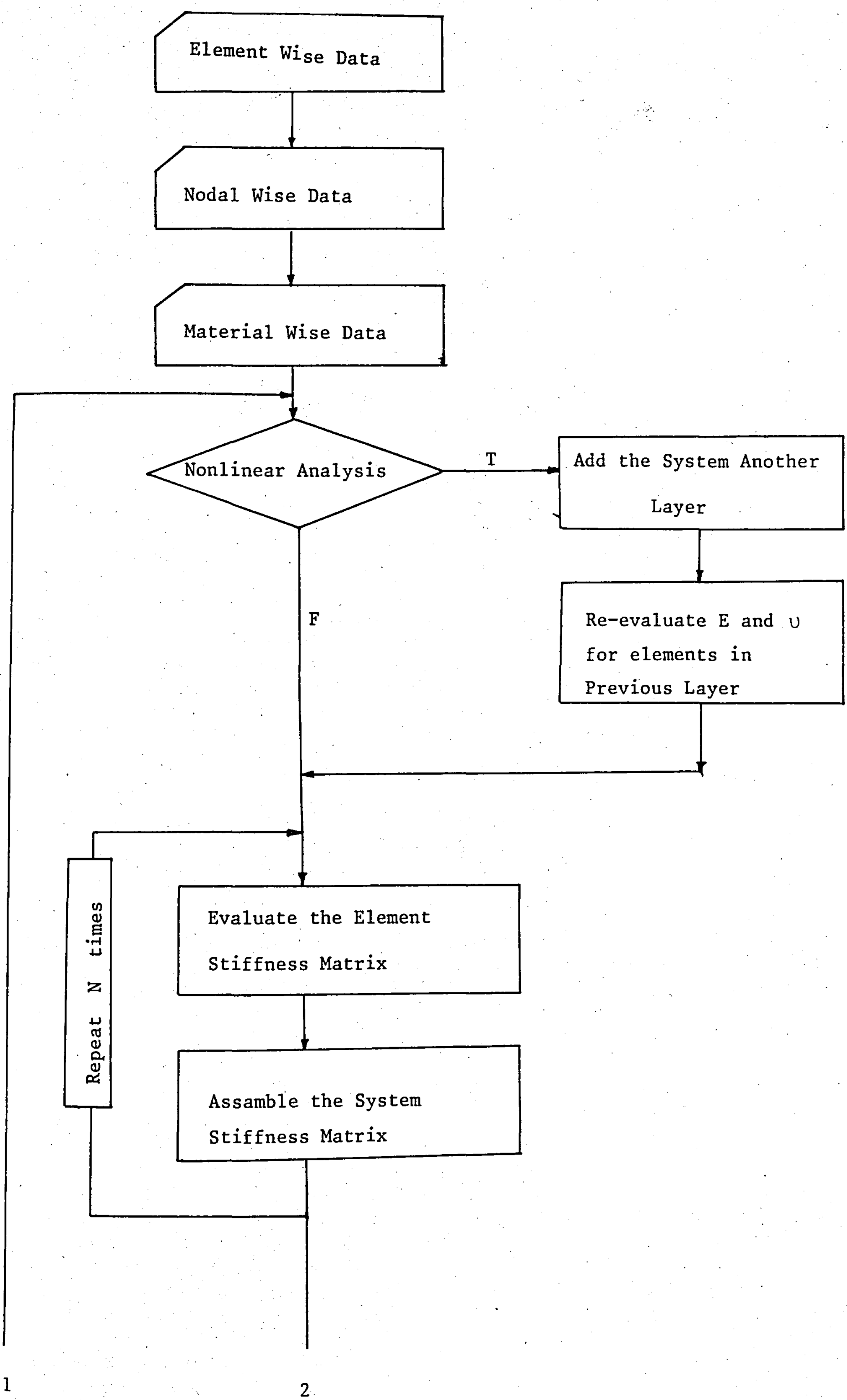
4.7. FLOW-CHART OF THE FINITE ELEMENTCOMPUTER PROGRAM

The finite element technique, infact, is a technique developed for analyzing a media by subdividing it into finite number of smaller units, which yields hundreds of unknowns to be determined. This technique, therefore, definitely requires computers to be employed.

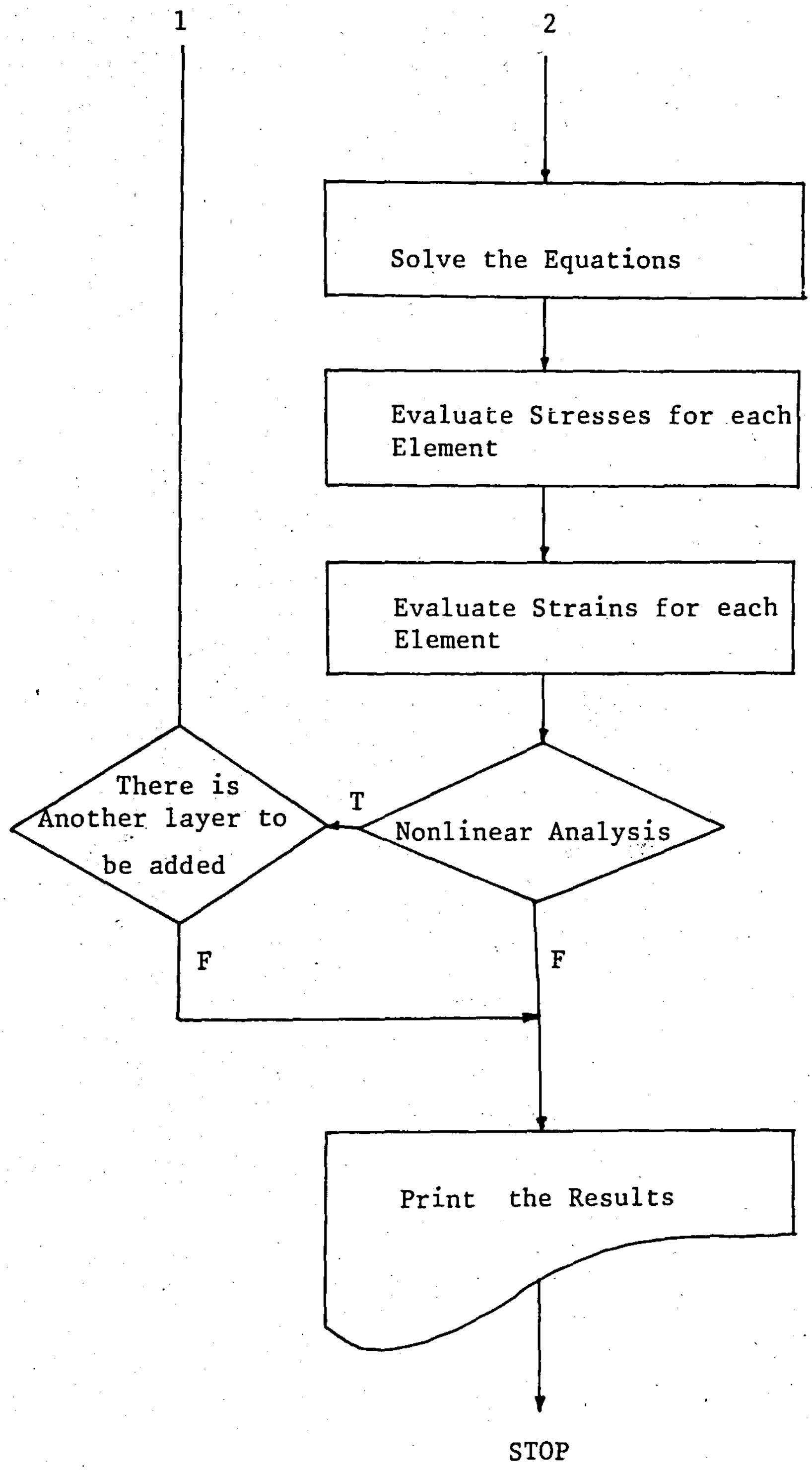
In reality, the memory of a micro-computer may not be enough to solve a medium-size finite element problem requiring more than 100 elements. Special techniques for appropriate usage of computer memory have to be utilized. One of the most efficient method of this purpose is to convert the system stiffness matrix into banded form, as previously described, and to solve the equation using "Frontal Technique".

In a typical finite element computer program, first of all, input data should be entered to describe the problem and the material under consideration. Input data may possibly be handled in three parts; element-data, nodal-point data and data for material properties. After reading these inputs, the program branches to a subroutine to evaluate the element stiffness matrix, then returns to the main program and again goes to another subroutine to add the effect of this element for system stiffness, by utilizing "Code Number Technique". It repeats the same steps as many times as number of elements. The program, then, determines the load vector which may contain concentrated or distributed edge load, gravity force, initial stresses or strains, and/or forces due to thermal expansion.

After evaluating the load vector, program ramifies to dissolve the set of equation and estimate the magnitude of displacements at nodal points, then it determines the stresses and strains for each element by back substituting process.



Element Wise Data
Nodal Wise Data
Material Wise Data
Nonlinear Analysis
T
F
Add the System Another Layer
Re-evaluate E and υ for elements in Previous Layer
Repeat N times
Evaluate the Element Stiffness Matrix
Assamble the System Stiffness Matrix
1
2


1
2
Solve the Equations
Evaluate Stresses for each Element
Evaluate Strains for each Element
There is Another layer to be added
T
Nonlinear Analysis
F
F
Print the Results
STOP

The flow chart for a typical finite element program may be written as shown on previous pages.

4.8. THE PROCEDURE FOR NONLINEAR STRESS ANALYSES

Nonlinear, stress-dependent stress-strain behavior may be approximated in finite element analyses by assigning different modulus values to each of the elements into which the soil is subdivided for purposes of analysis. The modulus values assigned to each element is selected on the basis of the stresses or strains in each element since the modulus values depend on the stresses and the stresses inturn depend on the modulus values, it is necessary to make repeated analyses to insure that the modulus values and the stress conditions correspond for each element in the system.

Two techniques for approximate nonlinear stress analyses are illustrated in Fig. 4.1. By the iterative procedure, the same change in external loading is analyzed repeatedly. After each analysis the values of stress and strain within each element are examined to determine if they satisfy the appropriate nonlinear relationship between stress and strain. If the values of stress and strain don't correspond, a new value of modulus is selected for that element for the next analysis.

By the incremental procedure the change in loading is analyzed in a series of steps, or increments. At the begining of each

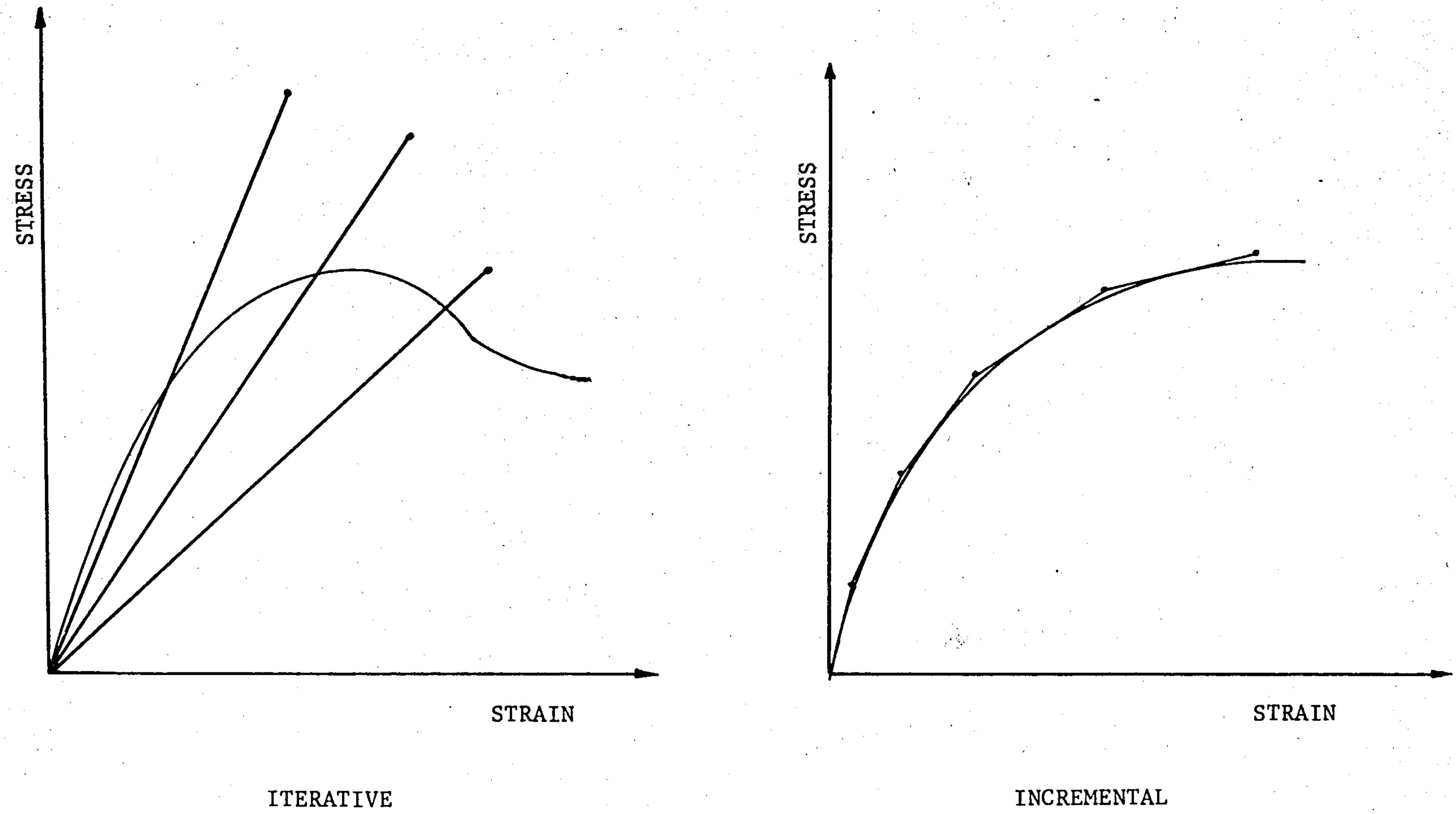


Fig. 4.1. Non-linear Analysis Of the Soil

new increment of loading an appropriate modulus value is selected for each element on the basis of the values of stress or strain in that element. Thus the nonlinear stress-strain relationship is approximated by a series of straight lines.

Both of these methods has advantages and shortcomings. The principal advantage of the iterative procedure is the fact that it is possible, by means of this procedure, to represent stress-strain relationships in which the stress decreases with increasing strain after reaching a peak value. This capability may be very important because the occurrence of progressive failure of soils is believed to be associated with this type of stress-strain behavior. The shortcoming of the iterative procedure is that it is very difficult to take into account nonzero initial stresses, which has an important rôle in many soil problems.

The principal advantages of the incremental procedure is that initial stresses maybe readily accounted for. It also has the advantage that, in the process of analyzing the effects of a given loading, stresses and strains are calculated for smaller loads as well. For example, if the application of a 50 ton load to a footing was analyzed using 10 steps, or increments, the settlement of the footing, and the stresses and strains in the soil, would be calculated for footing loads in increments of 5 tons up to 50 tons the shortcoming of the incremental procedure is that it is not possible to simulate by this technique a stress-strain relationship in which the stress decreases beyond the peak. In order to do so, the use of a negative value of modulus would be required, which is not possible in the finite element method. The accuracy of the incremental procedure may be improved if each load increment is analyzed more than once.

4.9. SUMMARY

At the begining ofthis chapter, the principle of minimum potential energy was explained, then by using this concept, Finite element formulations for the most general case have been derived. Aditionally these formulations were adapted for steady state condition so as to use them in soil problems. More over, plane strain and plane stress conditions were revealed. Additionally, some techniques, employed in computer programs to decrease the amount of memory needed in an operation, have been described. Furthermore, flow chart for a finite element program, capable of making linear and nonlinear analysis, have also been explained. Finally, the techniques for making nonlinear stress analysis and their advantageous and shortcomings were described in this chapter.

CHAPTER-V

HYBERBOLIC STRESS STRAIN PARAMETERS*

5.1. INTRODUCTION

The Finite Element Method provides a powerful technique for analysis of stresses and movements in earth masses, and it has already been applied to a number of practical porblems including embankment dams, open excavation, braced excavation, and a variety of soil structure interaction problems including reinforced earth soils.

Due to the availability of high-speed computers and these powerful numerical analytical techniques, it is possible to approximate nonlinear, inelastic soil behavior in stress analyses. However, in order to perform nonlinear stress analyses of soils, it is necessary to be able to describe the stress-strain behavior of the soil in quantative terms, and to develop techniques for incorporating this behavior in the analyses. This is difficult, because the stress-strain characteristics of soils are extremely

---

*Ref.2.9

complex, and the behavior of soil is highly dependent on the magnitudes of the streses in the soil.

The hyperbolic stress-strain relationships described give ~~reference~~ in this chapter have been developed by to provide a simple framework encompassing the most important characteristics of soil stress-strain behavior, using the data available from conventional laboratory tests. These relationships have been used in Finite Element Analyses of a number of different types of static soil mechanics problems and values of the hyperbolic parameters have been determined for over hundred different soils, which are summarized in Appendix A.

5.2. HYPERBOLIC STRESS-STRAIN RELATIONSHIPS

The hyperbolic stress-strain relationships are developed for use in nonlinear incremental analyses of soil deformations. In each increment of such analyses the stress-strain behavior of the soil is treated as being linear and the relationship between stress and strain is assumed to be governed by the generalized Hooke's Law of elastic deformations, which may be expressed as follows for conditions of plane strain

$$\begin{bmatrix} \Delta\sigma x \\ \Delta\sigma y \\ \Delta\tau xy \end{bmatrix} = \frac{E_t}{(1+\nu_t)(1-2\nu_t)} \begin{bmatrix} 1-\nu_t & \nu_t & 0 \\ \nu_t & (1-\nu_t) & 0 \\ 0 & 0 & \frac{1-2\nu_t}{2} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon x \\ \Delta\varepsilon y \\ \Delta\gamma_{xy} \end{bmatrix}$$

where

$\Delta\sigma x$, $\Delta\sigma y$, $\Delta\tau xy$ denote increments of stress during a step of analysis.

E_t denotes tangent value of deformation modulus

ν_t denotes tangent value of poisson's ratio

By reevaluating the Young's Modulus and Poisson's Ratio in each element corresponding to the computed strees values in that element, it is possible to model three important characteristics of the stress-strain behavior of soils, namely, nonlinearity, stress dependency, and inelasticity. The procedures used to account for these characteristics are described in the following sections.

5.2.1. Nonlinear Stress-Strain Curves Represented By Hyperbola

It has been shown that the stress-strain curves for a number of soils could be approximated by reasonable accuracy by hyperbolas like the one shown in Fig. 5.1. This hyperbola can be represented

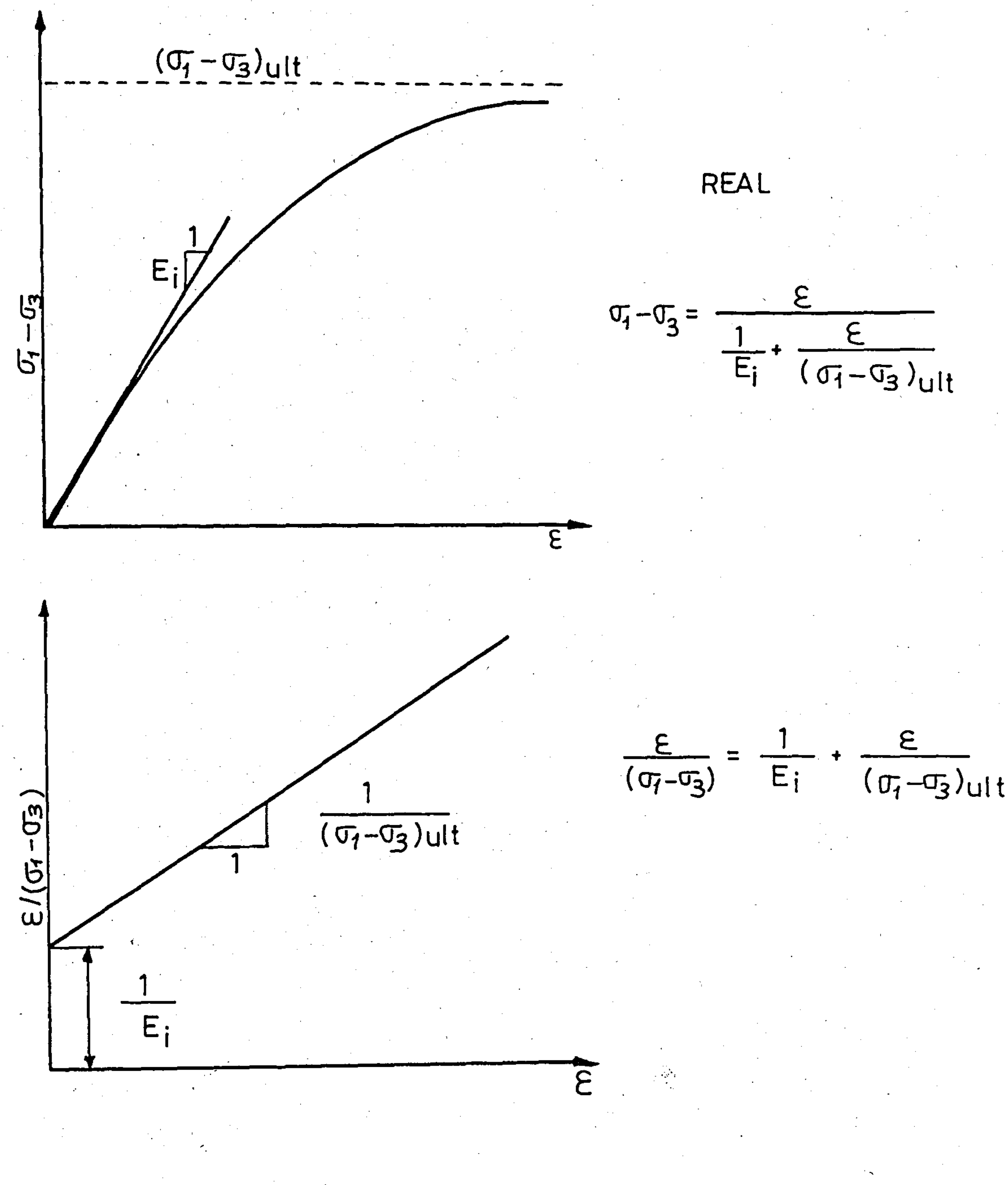


Fig. 5.1. Hyperbolic Representation of Stress-Strain Curve (Konder,1963)

by an equation of the form.

$$\sigma_1 - \sigma_3 = \frac{\varepsilon}{\frac{1}{E_1} \quad \frac{\varepsilon}{(\sigma_1-\sigma_3)_{ult}}} \tag{5.1}$$

in which

σ_1, σ_3 are major and minor principal streses

ε is axial strain

$(\sigma_1-\sigma_3)_{ult}$ is ultimate deviator stress

E_i is initial tangent modulus

when data from actual tests are plotted on the transformed plot, the points frequenly are found to deviate from the ideal linear relationship. Experience indicates that a good match is usually achieved by selecting the straight line so that it passes through the points where 70% and 95% of the strength are mobilized

5.2.2. Effect of Confining Pressure on $(\sigma_1-\sigma_3)_{ult}$ and E_i

For all soils except fully saturated tested under unconsolidated undrained conditions, an increase in confining pressure will result in a steeper stress-strain curve. It shows that the values of E_i and $(\sigma_1-\sigma_3)_{ult}$ increase with increasing confining pressure.

This stress-dependency is taken into account by using empirical equations suggested by Janbu

$$E_i = K\,P_a\left(\frac{\sigma_3}{P_a}\right)^n \tag{5.2}$$

The variation of E_i with σ_3 corresponding to this equation is shown in Fig. 5.2.

The parameter K in equation (5.2) is the modulus number, and n is the modulus exponent. P_a is atmospheric pressure, introduced to equation to make conversion from one system of units to another. Both K and n are dimensionless while the units of E_i are the same as the units of P_a.

The variation of $(\sigma_1-\sigma_3)_{ult}$ with σ_3 is accounted for as shown in Fig. 5.3 by relating $(\sigma_1-\sigma_3)_{ult}$ to the compressive strength or stress difference at failure, $(\sigma_1-\sigma_3)_f$, and then using the Mohr-Coulomb strength equation to relate $(\sigma_1-\sigma_3)_f$ to σ_3.

The values of $(\sigma_1-\sigma_3)_{ult}$ and $(\sigma_1-\sigma_3)_f$ are related by

$$(\sigma_1-\sigma_3)_f = R_f(\sigma_1-\sigma_3)_{ult} \tag{5.3}$$

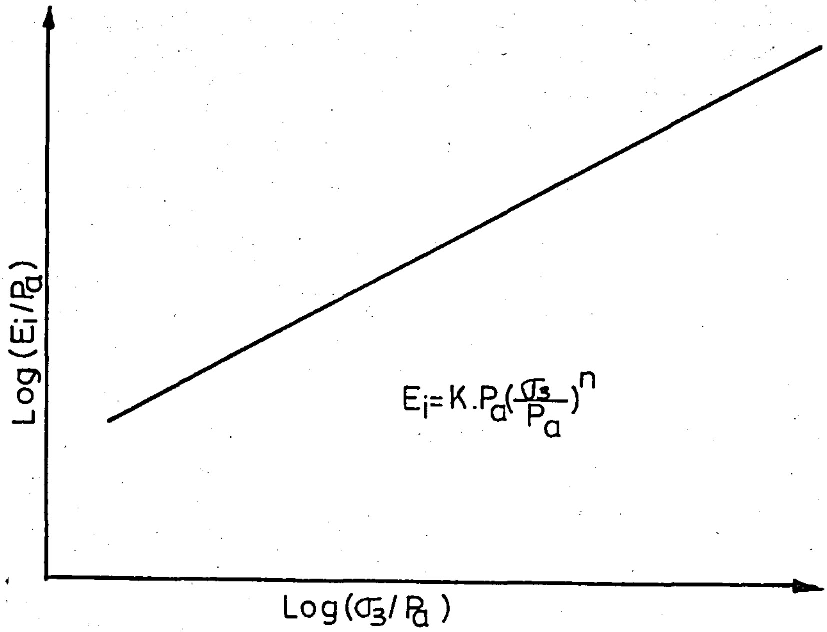


Fig. 5.2. Variation of Initial Tangent Modulus with Confining Pressure (Duncan, Wong, 1974)

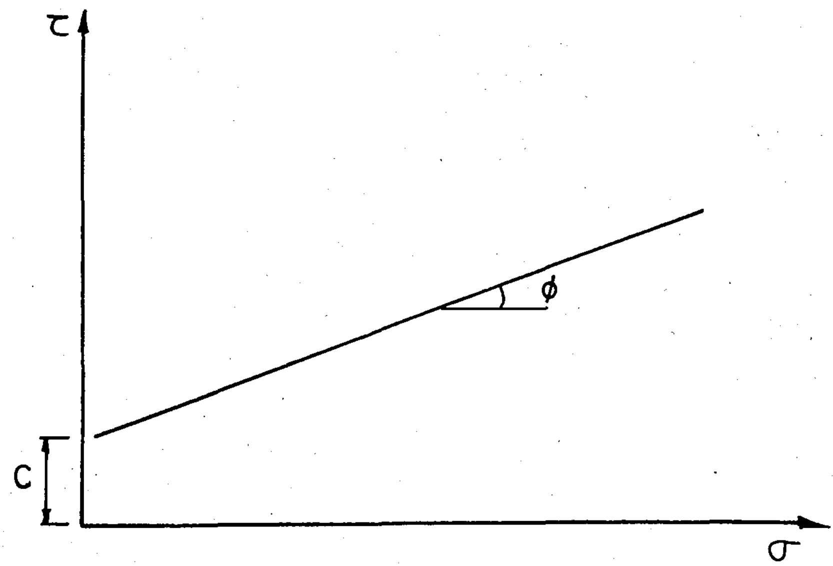


Fig. 5.3. Variation of Strength with Confining Pressure (Duncan, Wong, 1974)

in which R_f is the failure ratio. Since $(\sigma_1-\sigma_3)_f$ is always smaller than $(\sigma_1-\sigma_3)_{ult}$ the value of R_f is always smaller than unity, and varies from 0.5 to 0.9 for most soils.

The variation of $(\sigma_1-\sigma_3)_f$ with σ_3 is represented by the familiar Mohr-Coulomb Strength relationship, which can be expressed as follows.

$$(\sigma_1-\sigma_3)_f = \frac{2\ C\ Cos\phi + 2\sigma_3 Sin\phi}{1 - Sin\phi} \tag{5.4}$$

in which C and ϕ are the cohesion intercept and friction angle respectively.

5.2.3. Relationsip Between E_t and Stresses

The tangent deformation modulus E_t can be defined as the slope of the stress-strain curve at any point. By differentiating equation Fig. 5.1 with respect to ε and substituting the expression of equation Fig. 5.2, through 5.4 into the resulting expression for E_t, the following equation can be derived

$$E_t = \left[1 - \frac{R_f(1-Sin\phi)(\sigma_1-\sigma_3)}{2C\ Cos\phi + 2\sigma_3 Sin\phi}\right]^2 KP_a\left(\frac{\sigma_3}{P_a}\right)^n \tag{5.5}$$

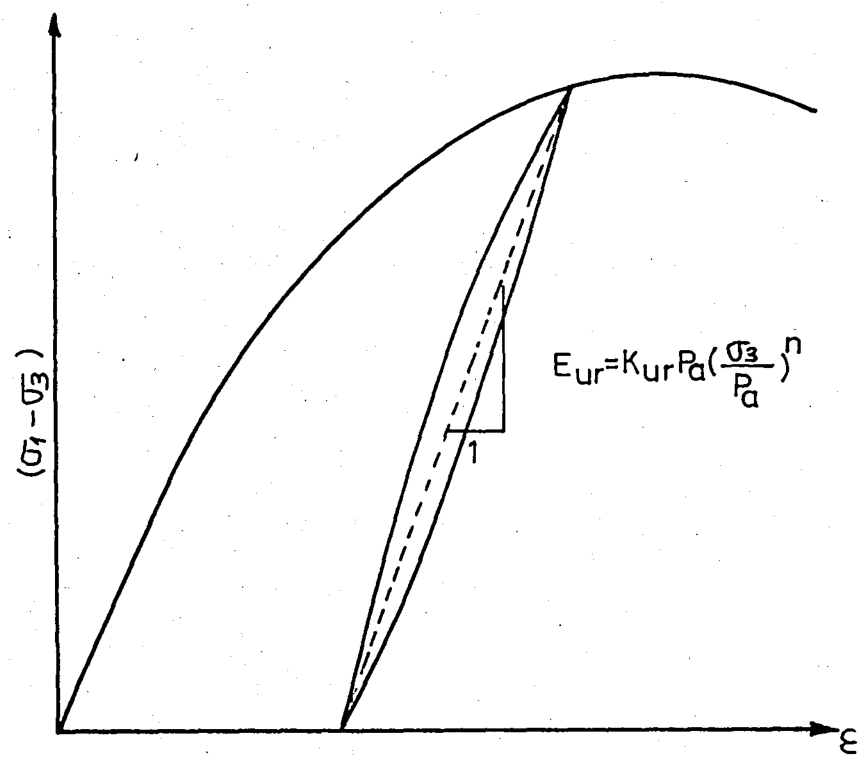


Fig. 5.4. Unloading-Reloading Modulus

This equation can be used to calculate the appropriate value of tangent modulus for any stress conditions, if the values of the parameters K, n, c, and R_f are known.

5.2.4. Inelastic Behavior of Soil

If a triaxial specimen is unloaded at some stage during a test, the stress-strain curve followed during unloading is steeper than the curve followed during primary loading, as shown in Fig. 5.4. If specimen is subsequently reloaded, the stress-strain curve followed is also steeper than the curve for primary loading and is quite similar in slope to the unloading curve. The soil behavior is inelastic since the strains occured during the primary loading are only partially recoverable on unloading.

In the hyperbolic stress-strain relationships, the same value of unloading-reloading modulus, E_{ur}, is used for both cases. The value of E_{ur} is related to the confining pressure by an equation of the sameform as equation (5.2)

$$E_{ur} = K_{ur} \, P_a \left(\frac{\sigma_3}{P_a}\right)^n \tag{5.6}$$

In this equation K_{ur} is the unloading-reloading modulus number. The value of K_{ur} is always larger than the value of K. Generally K_{ur} is 20% greater than K for stiff soils while it may be three times as large as K for soft soils (Puncan and Wong, 1984).

5.2.5. Nonlinear Volume Change

Many soils exhibit nonlinear and stress-dependent volume change Characteristics, as illustrated by the volume change curves shown in Fig. 5.5.

According to the theory of elasticity, the volume bulk modulus is defined by

$$B = \frac{\Delta\sigma_1+\Delta\sigma_2+\Delta\sigma_3}{3\varepsilon_v} \tag{5.7}$$

in which

B donetes the bulk modulus

$\Delta\sigma_1, \Delta\sigma_2, \Delta\sigma_3$ are the changes in the values of the pirincipal stresses

$\Delta\sigma_v$ is the corresponding change in volumetric strain

For a conventional triaxial test, in which the deviator stress $(\sigma_1-\sigma_3)$ increases while the confining pressure is held constant, equation 5.7 may be expressed as

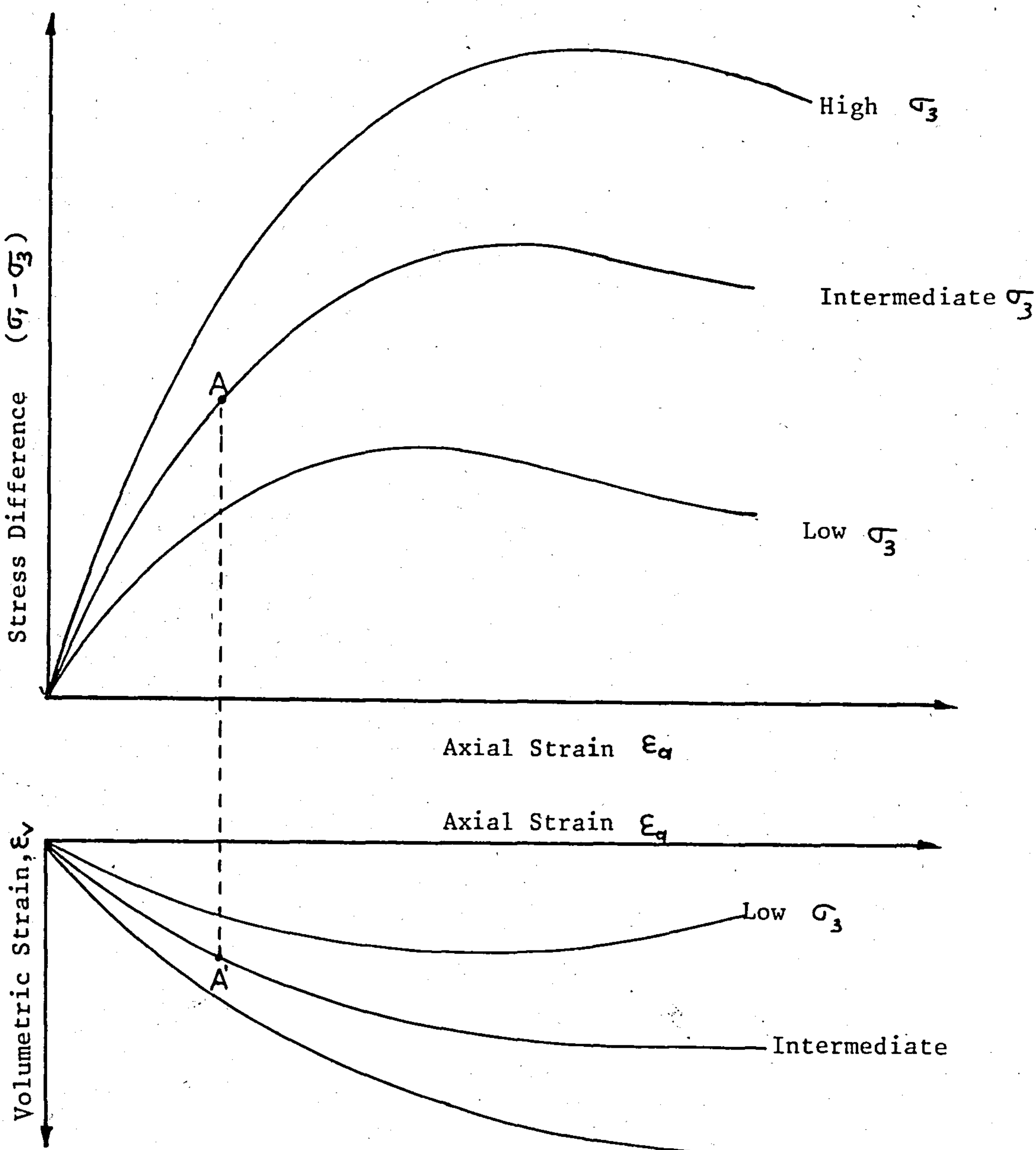


Fig. 5.5. Nonlinear And Stress-Dependent Stress-strain And Volume Change Curves

$$B = \frac{(\sigma_1 - \sigma_3)}{3\varepsilon_v} \tag{5.8}$$

The value of bulk modulus for a conventional triaxial compression test may be calculated using the value of $(\sigma_1-\sigma_3)$ corresponding to any point on the stress-strain curve, such as point A in Fig. 5.5 and the corresponding point on the volume change curve (A') when values of B are calculated for tests on the same soil at various confining pressures, the bulk modulus will usulally be found to increase with increasing confining pressure. As shown in Fig. 5.6 the variation of bulk modulus with confining pressure can be approximated by an equation of the form

$$B = K_b . P_a \left(\frac{\sigma_3}{P_a}\right)^m \tag{5.9}$$

where

K_b is the bulk modulus number

m is the bulk modulus exponent

P_a is atmospheric pressure, expressed in the same units

and B. For most soils the values of m vary between 0.0 and 1.0.

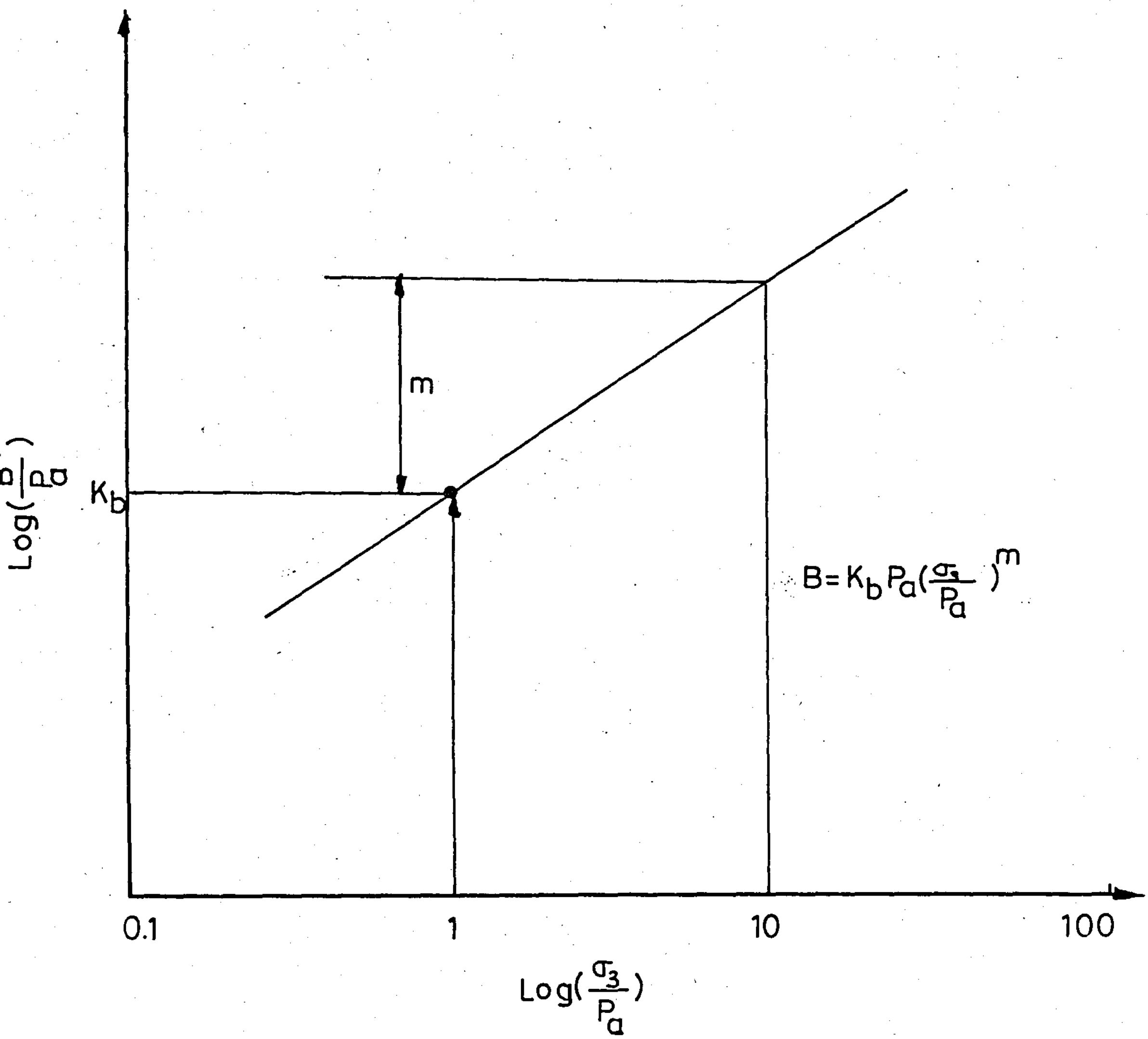


Fig. 5.6. Variation of Bulk Modulus with Confining Pressure

5.3. SUMMARY

Up to here, nine parameters are employed in the hyperbolic stress-strain relationships described in this chapter. These parameters and their functions within the relationships are listed in Table 5.1.

The hyperbolic relationships outlined previously have proven quite useful for a wide variety of practical problems for the following reasons,

1. The parameter values can be determined from the results of conventional triaxial compression tests.

2. The same relationships can be used for effective stress analyses, (using data from drained tests) and total stress analyses (using data from unconsolidated-undrained tests).

3. Values of the parameters have been calculated for many different types of soils and this information can be used to estimate reasonable values of the parameters in cases where the available data are in sufficient to define the parameters for all of the soils involved in a particular problem. The information is also quite useful for assesing the reliability of parameter values derived from laboratory test results.

However, the simple hyperbolic relationships have some significant limitations.

1. The relationships are most suitable for analysis of stresses and movement prior to failure. It is not reliable to continue the analyses after the stage where there is local failure in some elements. These relationship are not useful, therefore, for analyses extending upto, the stage of instability of a soil mass. They are useful for predicting movements in stable earth masses.

2. The hyperbolic relationships do not include volume changes due to changes in shear stress, or "shear dilatancy". They may therefore be limited in the accuracy with which they can be used to predict deformations in dilatant soils, such as dense sands under low confining pressures.

3. The values of the parameters depend on the density of the soil, its water content, the range of pressures used, in tenting, and the drainage conditions. In order that the parameters will be representative of the behavior of the soil, in the field condition, the laboratory test conditions must be correspond to the field conditions with regard to these factors.

TABLE 5.1 SUMMARY OF THE HYBERBOLIC PARAMETERS

Parameters	Name	Function
K, K_{ur}	Modulus Number	Relate E_i and E_{ur} to σ_3
n	Modulus Exponent	
c	Cohesion Intercept	Relate $(\sigma_1-\sigma_3)_f$ to σ_3
ϕ, $\Delta\phi$	Friction Angle parameters	
K_b	Bulk modulus number	Value of B/P_a at $\sigma_3 = P_a$
m	Bulk modulus exponent	Change in B/P_a for ten-fold increase in σ_3
R_f	Failure Ratio Relates	$(\sigma_1-\sigma_3)_{ult}$ to $(\sigma_1-\sigma_3)_f$

CHAPTER-VI

FINITE ELEMENT ANALYSIS OF REINFORCED EARTH SYSTEMS*

6.1. INTRODUCTION

There are two ways to analyze the Reinforced Earth Systems by Finite Element Method. The first is known as Soil Structure Interaction analysis. In this method reinforcing strip is represented by the bar element, while the soil is defined by a two dimensional finite element, and the interaction between the two is taken into consideration by choosing another type of element which is known as "Interface Element".

In the second method, The Reinforced Earth System is considered as composite material consisting of reinforcement and soil.

In this study the second type of analyses, which is explained in following sections, is utilized.

---

* Ref.1

6.2. FINITE ELEMENT REPRESENTATION OF R.E.S AS COMPOSITE MATERIAL

The theory of composite material behavior may be derived from several different points of view. The approach followed herein is to recognize that if reinforcing pattern is repeated a sufficiently large number of times, the material can be considered homogeneous. The consideration of the reinforced material as homogeneous at the structural level is analogous to the consideration of a microscropically crystalline material as macroscopically homogeneous. The reinforced material when viewed at the composite level will, in general, exhibit orthotropic behavior. Once the appropriate composite properties are determined, it is an easy matter to utilize the finite element procedure to analize complicated structures of reinforced material.

Thus, for reinforced earth structures the required step is to establish the appropriate composite properties. These relationships are defined by the concept of the "Unit Cell".

6.2.1. UNIT CELL CONCEPT

For a material that has a regular reinforcing pattern, one can, in general, isolate a small unit of material which compeletly exhibits the composite characteristics of the material. This fundamental building block is called the "Unit Cell", shown in Fig. 6.1. The average values of the stresses distributed over the cell faces

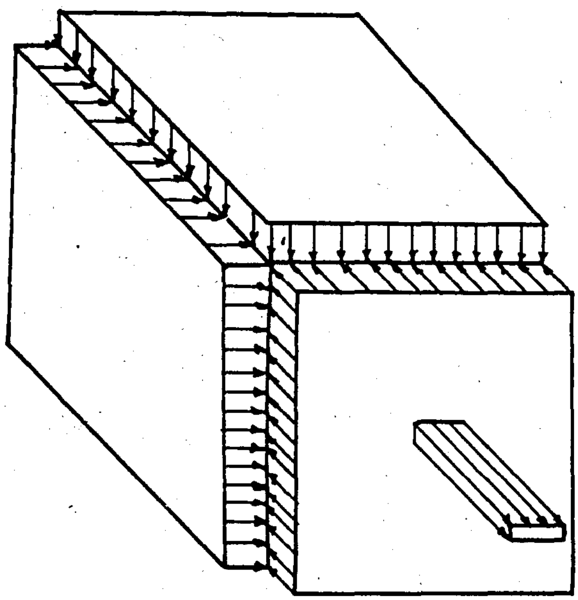


Fig. 6.1. Unit Cell Representation of Reinforced Earth

1. All unit cells will exhibit identical deformation and stress states.

2. The averages of the unit cell stresses and strains are equal to the phenomenological stresses and strains of the composite.

3. There must be continuity of the displacement and traction vectors across cell interfaces.

6.2.2. COMPOSITE STRESS-STRAIN RELATIONSHIPS

The most significant characteristic of the reinforced earth unit cell is that the percentage of reinforcement is extremely small. This characteristic leads to the assumption of the strains in the composite being equal to the strains in thesoil.

The other most significant assumptions utilized in the analysis of the unit cell are the use of an idealized nonlinear characterization for the soil, and that no slippage occurs between the soil and the steel. This latter assumption yields the assumption that the displacement of all points in the unit cell on any 2-3 plane are equal for both soil and strip.

For the determation of the composite properties of reinforced earth, first the composite stress state is considered.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & & \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{55} & \\ & & & & & C_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} \quad (6.1)$$

$$\sigma_1 = \sigma \; ; \sigma_2 = \sigma_3 = \sigma_{12} = \sigma_{13} = \sigma_{23} = 0 \qquad (6.2)$$

equation (6.1) yields

$$C_{11} = \frac{\varepsilon_1}{\sigma} \qquad (6.3)$$

$$C_{12} = \frac{\varepsilon_2}{\sigma} \qquad (6.3)$$

$$C_{13} = \frac{\varepsilon_3}{\sigma} \qquad (6.5)$$

The applied composite stress, σ, acting over the composite area, A^c, must equal the sum of the strip stress, σ^{st}, acting over the strip area, A^{st}, and the soil stress, σ^{so}, acting over the soil area, A^{so}. Fig. 6.1. The soil area and the composite area are essentially equal for the reinforced earth system under consideration ($bd = A^c \cong A^{so}$):

$$\sigma A^c = \sigma_1^{so} A^c + \sigma_1^{st} A^{st} \tag{6.6}$$

$$\sigma_2 = \sigma_2^{so} = 0 \tag{6.7}$$

$$\sigma_3 = \sigma_3^{so} = 0 \tag{6.8}$$

It follows from the previous assumptions that

$$\varepsilon_1 = \varepsilon_1^{so} = \varepsilon_1^{st} \tag{6.9}$$

$$\varepsilon_2 = \varepsilon_2^{so} \tag{6.10}$$

$$\varepsilon_3 = \varepsilon_3^{so} \tag{6.11}$$

Due to the assumption made before;

$$\varepsilon_1^{so} = \frac{\sigma_1^{so}}{E^{so}} \tag{6.12}$$

$$\varepsilon_1^{st} = \frac{\sigma_1^{st}}{E^{st}} \tag{6.13}$$

Substitution of Equations (6.12) and (6.13) into (Eq.69) gives

$$\sigma_1^{so} = \frac{E^{so}}{E^{st}} \sigma_1^{st} \tag{6.14}$$

which may be substituted into equation (6.6) to yield the relationship

$$\sigma_1^{st} = \frac{\sigma A^c E^{st}}{A^c E^{so} + A^{st} E^{st}} \tag{6.15}$$

or

$$\sigma_1^{so} = \frac{\sigma A^c E^{so}}{A^c E^{so} + A^{st} E^{st}} \tag{6.16}$$

Using Equations 6.3, 6.4 and 6.5, the composite material properties may then be solved as

$$C_{11} = \frac{\varepsilon_1}{\sigma} = \frac{\varepsilon_1^{so}}{\sigma} = \frac{\sigma_1^{so}}{\sigma E^{so}} = \frac{A^c}{A^c E^{so} + A^{st} E^{st}} \tag{6.17}$$

$$C_{12} = \frac{\varepsilon_2}{\sigma} = \frac{\varepsilon_2^{so}}{\sigma} = \frac{-\nu^{so}\sigma_1^{so}}{\sigma E^{so}} = \frac{-\nu^{so} A^c}{A^c E^{so} + A^{st} E^{st}} \tag{6.18}$$

$$C_{13} = \frac{\varepsilon_3}{\sigma} = \frac{\varepsilon_3^{so}}{\sigma} = \frac{-\nu^{so}\sigma_1^{so}}{\sigma E^{so}} = \frac{-\nu^{so} A^c}{A^c E^{so} + A^{st} E^{st}} \tag{6.19}$$

The other properties relating the composite normal streses and strains can be obtained in a similar manner and the final constitutive relationship for the composite becomes

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \frac{1}{E^{so}(1+\alpha)} \begin{bmatrix} 1 & -\nu^{so} & -\nu^{so} \\ -\nu^{so} & 1+\alpha(1-\nu^{so^2}) & -\nu^{so}|1+\alpha(1\ \alpha^{so}) \\ -\nu^{so} & -\nu^{so}|1+\alpha(1\ \nu^{so})| & 1+\alpha(1-\nu^{so^2}) \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}$$

$$\begin{bmatrix} 12 \\ 13 \\ 23 \end{bmatrix} = \begin{bmatrix} G^{so} & & & \sigma_{12} \\ & G^{so} & & \sigma_{13} \\ & & G^{so} & \sigma_{23} \end{bmatrix} ; \alpha = \frac{A^{st}E^{st}}{A^{c}E^{so}} \qquad (6.20)$$

This matrix is inverted to obtain the stress-strain relationship to be utilized in the Finite Element formulations i.e $|\sigma| = |D|\ |E|$.

Under certain conditions reinforced earth structures may be assumed to exhibit plane strain response where the strains, ε_3, γ_{13}, and γ_{23} are approximately zero. In the finite element study to be analyzed, these strains are assumed to be zero and the resultingincremental stress-strain relationship utilized is of the form.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & \varepsilon_1 \\ a_{21} & a_{22} & 0 & \varepsilon_2 \\ 0 & 0 & a_{44} & 12 \end{bmatrix} \qquad (6.21)$$

in which a_{iz} coefficients are directly obtained from the inversion of the C matrix, while considering the plane strain case.

6.3. SUMMARY

In this chapter, simulation of Reinforced Earth Structures by Finite Element Method has been studied. As it was explained in Chapter IV different type of elements and various material can be employed in Finite Element Analysis by selecting the content of the material poperty matrix and shape function appropriately. By utilizing this concept, Reinforced Earth Systems have been considered as composite material and the content of the material property matrix for the composite have been developed.

CHAPTER-VII

RESULTS OF FINITE ELEMENT ANALYSES

7.1. INTRODUCTION

For design purposes, distribution of stresses and deformations mobilized in reinforced earth structures must be predicted. In this chapter, therefore, attention is given to the evaluation of stresses and movements occuring in unreinforced and reinforced soils in order to determine the effect of reinforcement on stress and deformation. This was done by employing the Finite Element Method of analysis described previously, and the computer program developed, (FRSOIL) for this purpose.

The finite element computer program, FRSOIL, has the capability of making linear or nonlinear analysis considering plane strain and plane stress condition in homogeneous or nonhomogeneous, i.e. composite, media. Cross check of the program has been done in three ways. First it was checked by the Finite Element Computer program, M701, developed by S. Tezcan. In that way the part of the program which makes linear analyses has been tested. In order to verify whether or not the program makes a correct nonlinear analysis, the soil-structure interaction computer program, Soil-STRUCT, prepared by Clough and Duncan was utilized and a typical soil problem was solved by this program and FRSOIL, then the results were compared.

Finally, the last part of the program which was developed to analyze Reinforced Earth System was checked by solving a problem in Ref.1 which had been already solved and the results obtained by FRSOIL were almost exactly the same as the resilts in the reference.

The computer program FRSOIL was developed on CDC Cyber 170/815 system operating at Computer Center, Boğaziçi University, Istanbul. Program listing and User's manual are given in Appendix C.

7.2. DESCRIPTION OF THE PROBLEM

For the purpose of analyses, a typical excavation problem has been chosen. Since the system is symmetric, only the half of the structure is considered and cross section of this part is shown in Fig. 7.1. Although the main stduy is concentrated on the reinforced region, the structure is extended toward both directions in order to prevent the boundary effect on reinforced area,

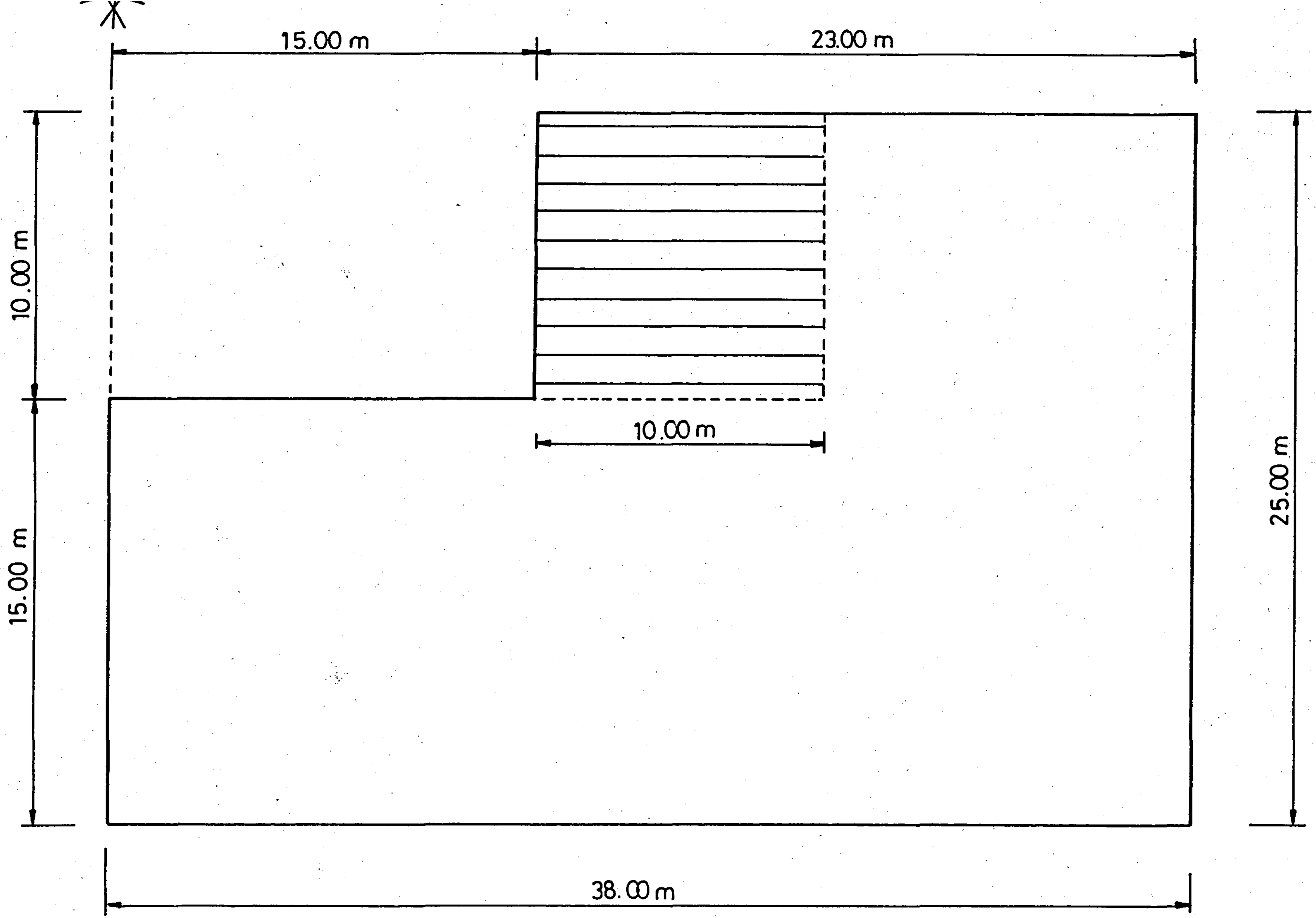


Fig. 7.1 Example Problem

The properties of the soil and that of the reinforcement are chosen according to soil which is frequently encountered in practice, and the reinforcement which can be easily provided in the market. The parameters for both components have been given in Table 7.1.

7.3. FINITE ELEMENT IDEALIZATION OF THE STRUCTURE

The finite element grids for the simple reinforced earth wall subjected to its own weight is illustrated in Fig. 7.2. As seen in this figure, the discretization of the medium was represented by 160 rectangular elements with a total of 190 nodal points. The reinforcing strips are horizontally located throughout the mid-point of the grid elements. The mesh was chosen to give rather detailed information near the wall face, and at the back edge of the reinforced earth wall. The system was analyzed assuming one increment of construction and rollers along the back edge of the backfill and at the bottom of the system. Infact, this is not a realistic problem for simulating either the construction sequence or the boundary conditions. The purpose of the analysis is to investigate how the reinforcements change stresses, displacements and stress level developed in the structure during or after the construction and, to find out if the usage of reinforcement in soil medium is economical and advisable.

Table 7.1. Properties of the Soil and Reinforcement

Soil Properties		Reinforcement Properties	
ϕ, in degrees	30	E, in tons per square meters	$2.0\ 10^7$
c, in tons per square meters	2	Horizontal Spacing, in meters	1
γ, in tons per square meters	2	Vertical Spacing, in meter	1
v	0.3	Yield strength, in kilogram per square centimeter	3500
E, in tons per square meters	5000		

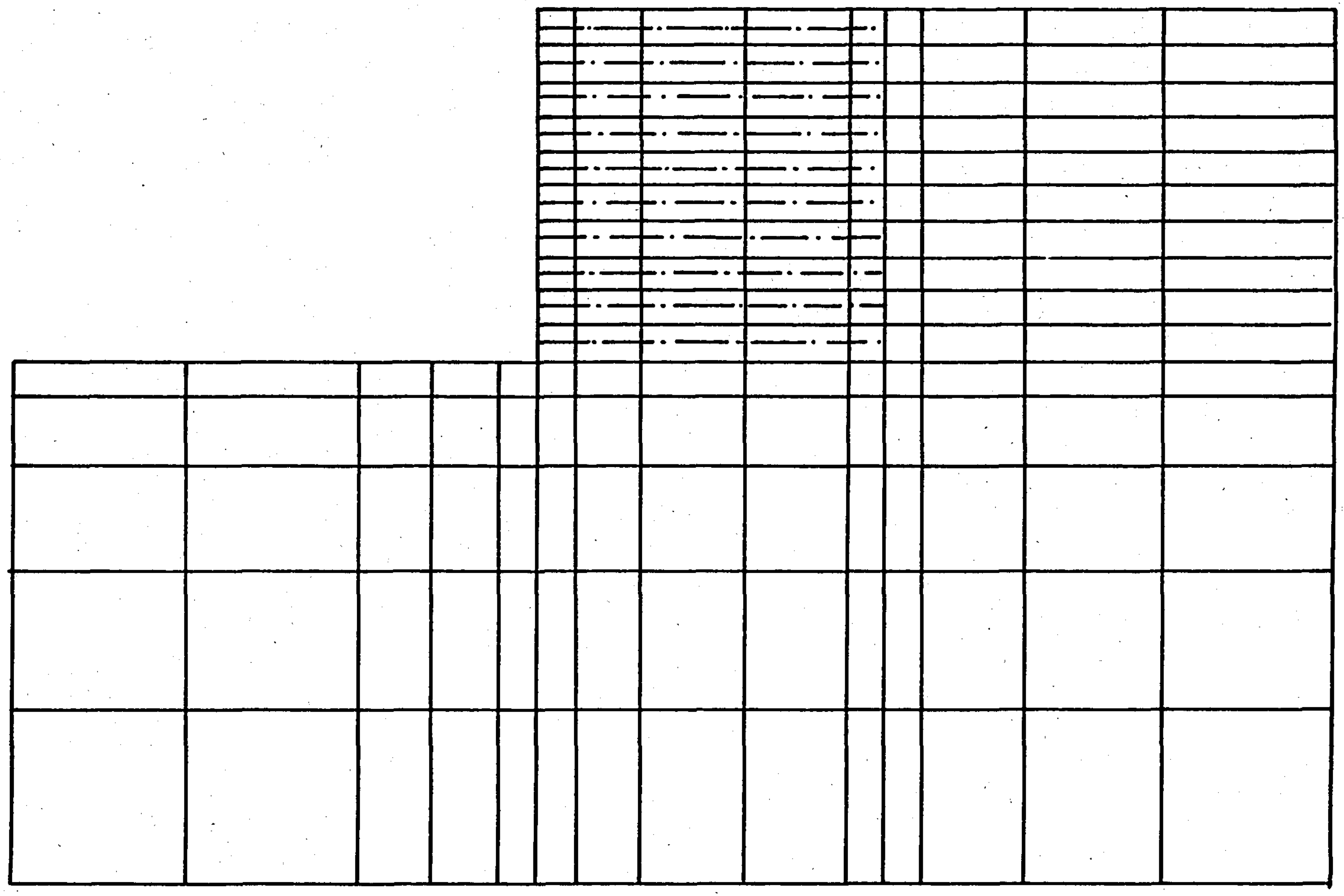


Fig. 7.2. Finite Element Mesh

The problem previously described is, therefore, first considered as a plane strain problem in homogeneous media, then the same problem is investigated taking into account effect of the reinforcements. The results obtained in two analyses and comparison of them are given in following sections.

7.4. THE EFFECT OF REINFORCEMENT ON HORIZONTAL STRESS

The contours of horizontal stresses developed in unreinforced soil is indicated in Fig. 7.3. As it is seen in the figure, there exists tension stresses causing tension crack at some points at the top of the wall. The maximum amount of horizontal stress reaches 20 t/m^2, and it occurs at the bottom of the structure. As Fig. 7.3 shows, the horizontal stresses developed in unreinforced soil converge toward the toe of the wall, which result in a stress concentration at this point.

The contours of horizontal stresses developed in reinforced soil is illustrated in Fig. 7.4. Comparison of Fig.7.3 and Fig.7.4 indicates that reinforcement causes the horizontal stresses to be distributed homogeneously and the contours of them to be seperated from each other. This means, there is no or a small amount of horizontal stress concentration. Infact, majority of these horizontal stresses are carried by the reinforcing strips, proportionally to the ratio of the elasticity modulus of reinforcement and soil.

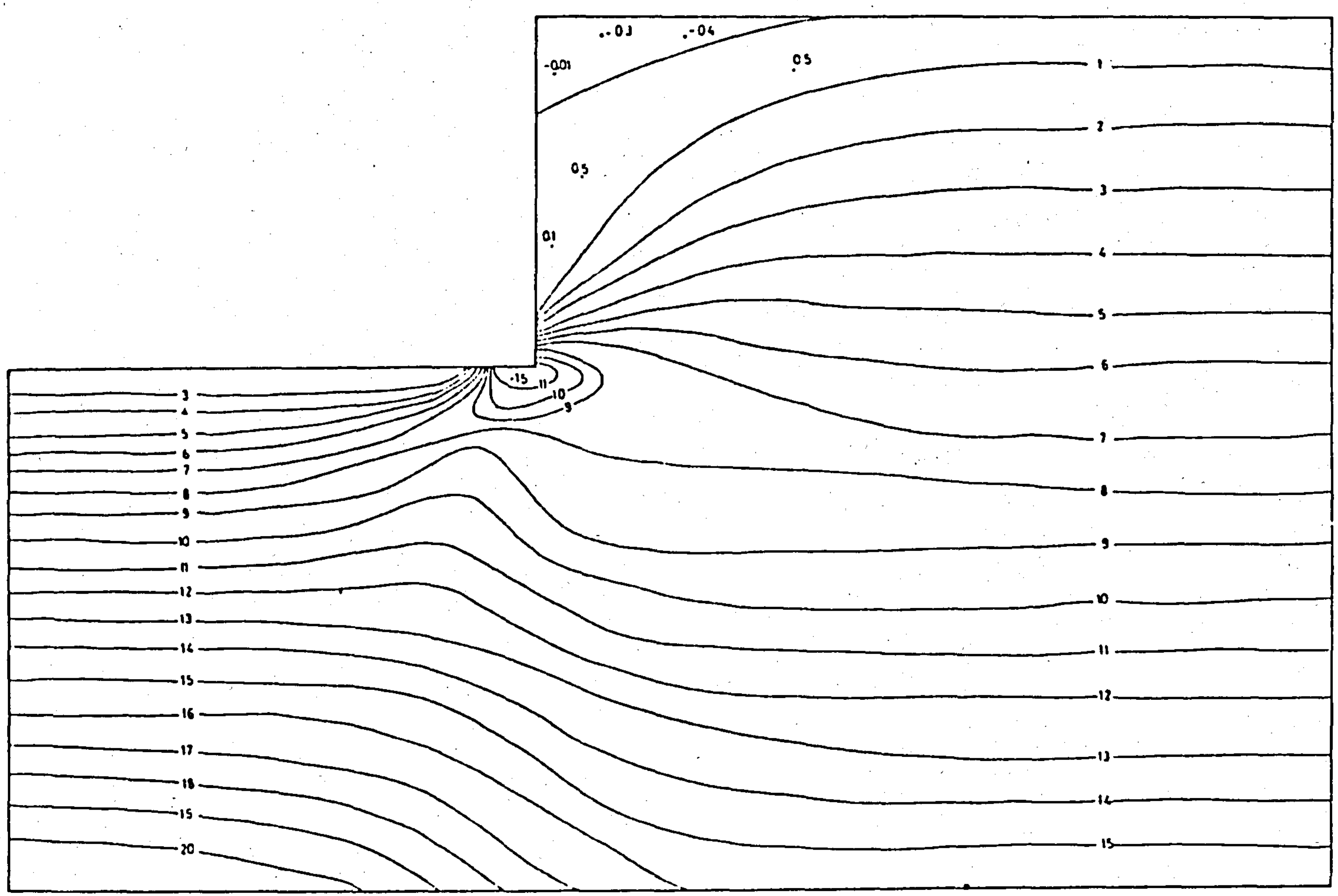


Fig. 7.3. Contours of Horizontal Stresses (t/m^2)

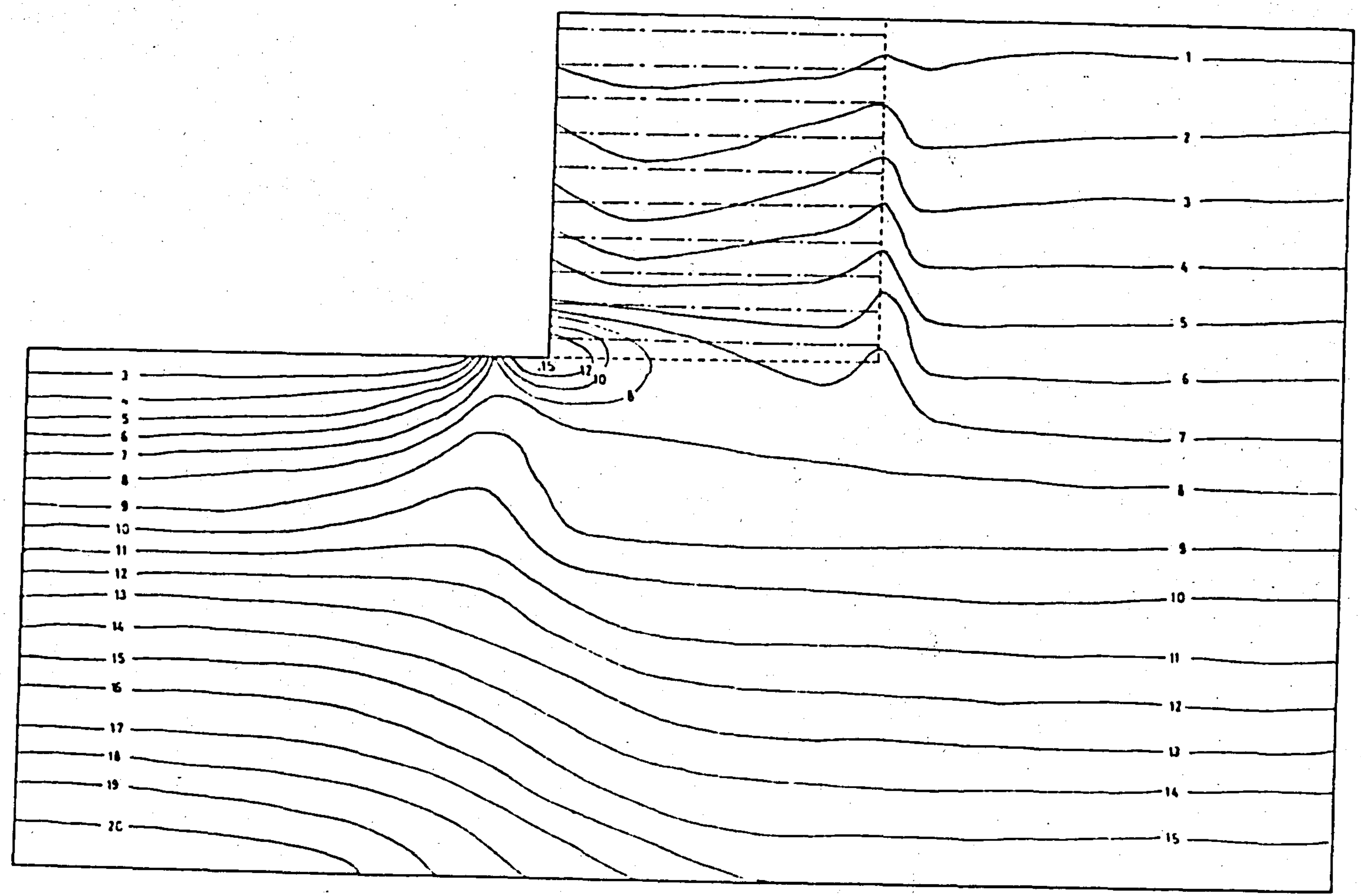


Fig. 7.4 Contours of Horizontal Stresses (t/m^2)

In reinforced case, the horizontal stresses behind the reinforced part sharply decrease and are leveled at a constant value, as shown in Fig. 7.4.

7.5. THE EFFECT OF REINFORCEMENT ON VERTICAL STRESS

The contours of vertical stresses in the case of unreinforced soil are illustrated in Fig. 7.5. As it is expected, except the toe of the wall, the contours of vertical stresses are levelled at a constant magnitude on a certain horizontal section. Due to discontinuity of the geometry, there is a stress concentration near the vicinity of the toe. The maximum value of vertical stresses developed in unreinforced case reaches the value of 47 t/m^2.

The contours of vertical stresses in the case of reinforced soil are indicated in Fig. 7.6. After comparing Fig. 7.5 and Fig. 7.6, it is clear that the influence of reinforcement on vertical stress is quite small.

7.6. THE EFFECT OF REINFORCEMENT ON MAXIMUM SHEAR STRESS

One of the main effects of the reinforcement on soil stability is confronted on contours of maximum shear stresses, which are shown in Fig. 7.7 and Fig. 7.8, for unreinforced and reinforced soil cases respectively. If these figures are overlaid on each other, it is easily noticed that reinforcement significantly decreases the shear

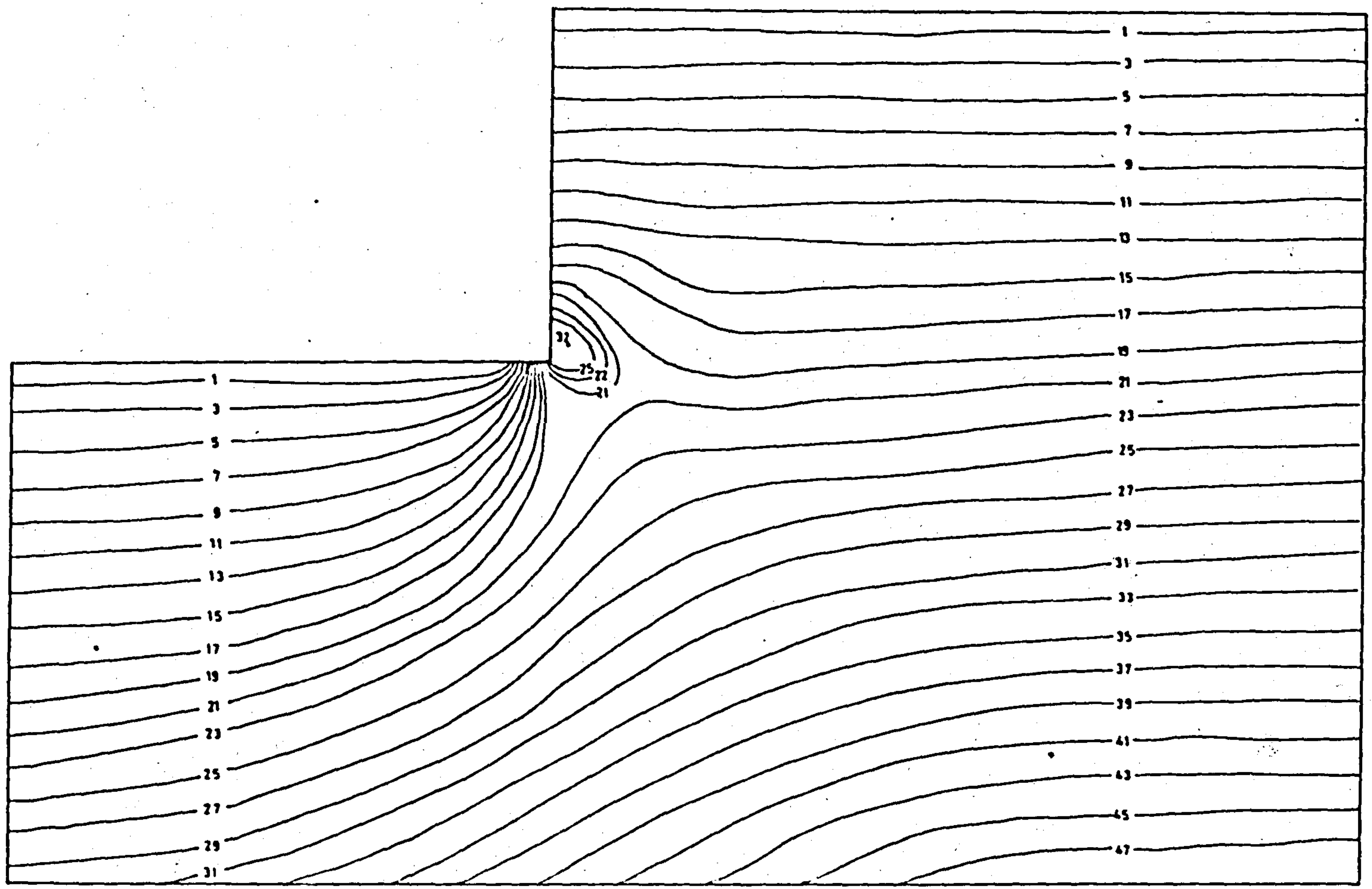


Fig 7.5. Contours of Vertical Stresses (t/m^2)

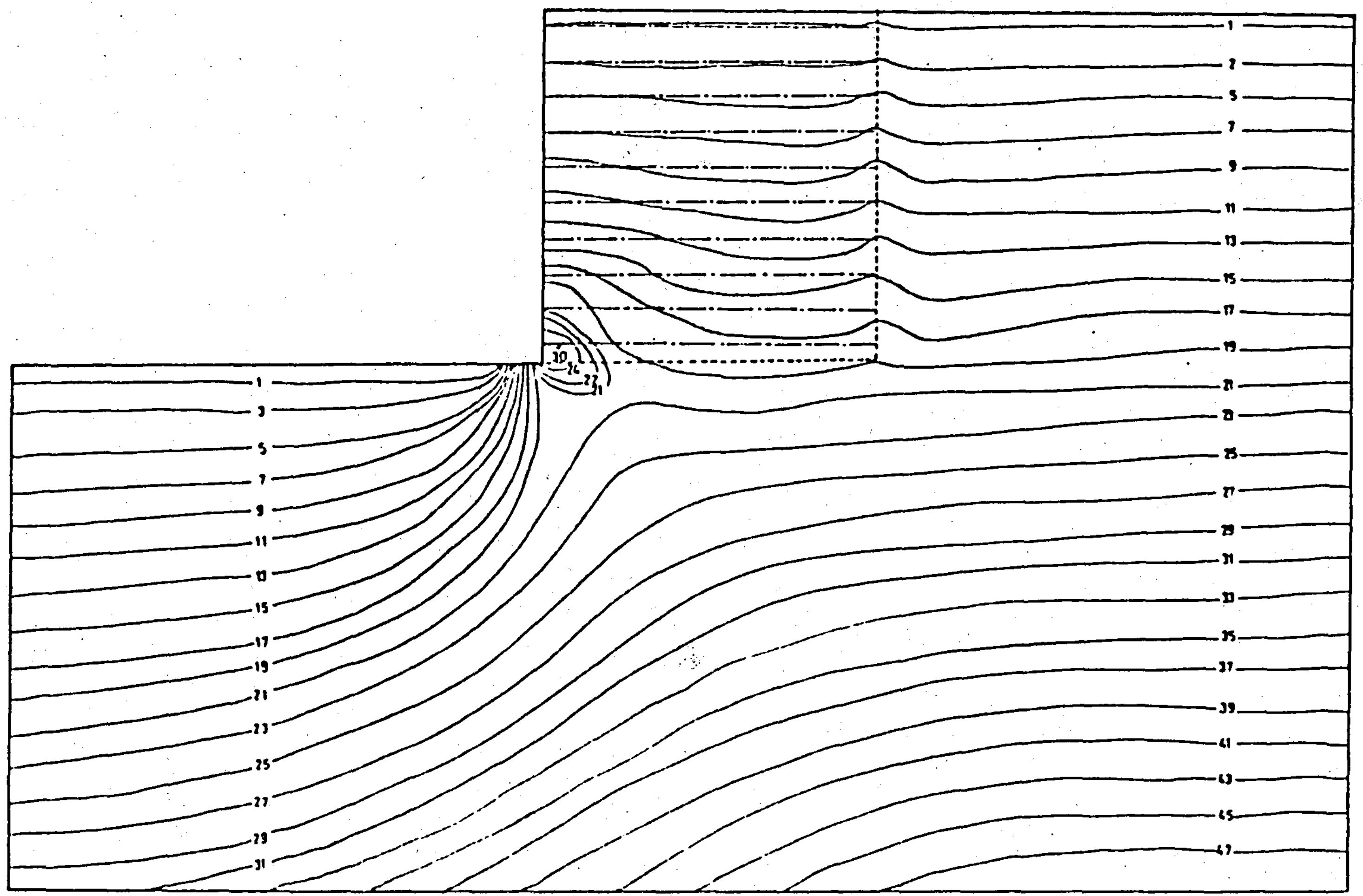


Fig 7.6 Contours of Vertical Stresses (t/m^2)

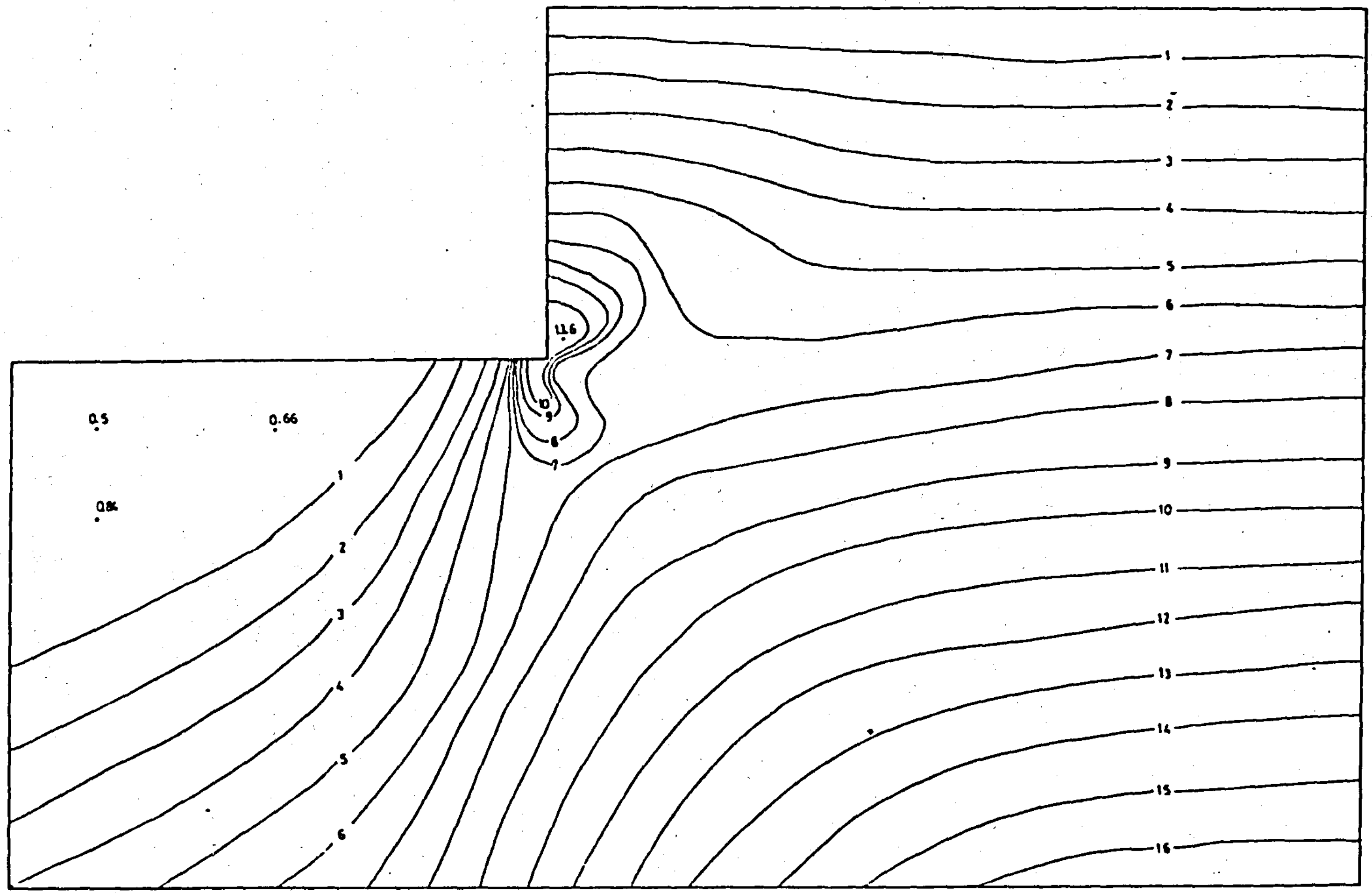


Fig. 7.7 Contours of Maximum Shear stresses (t/m^2)

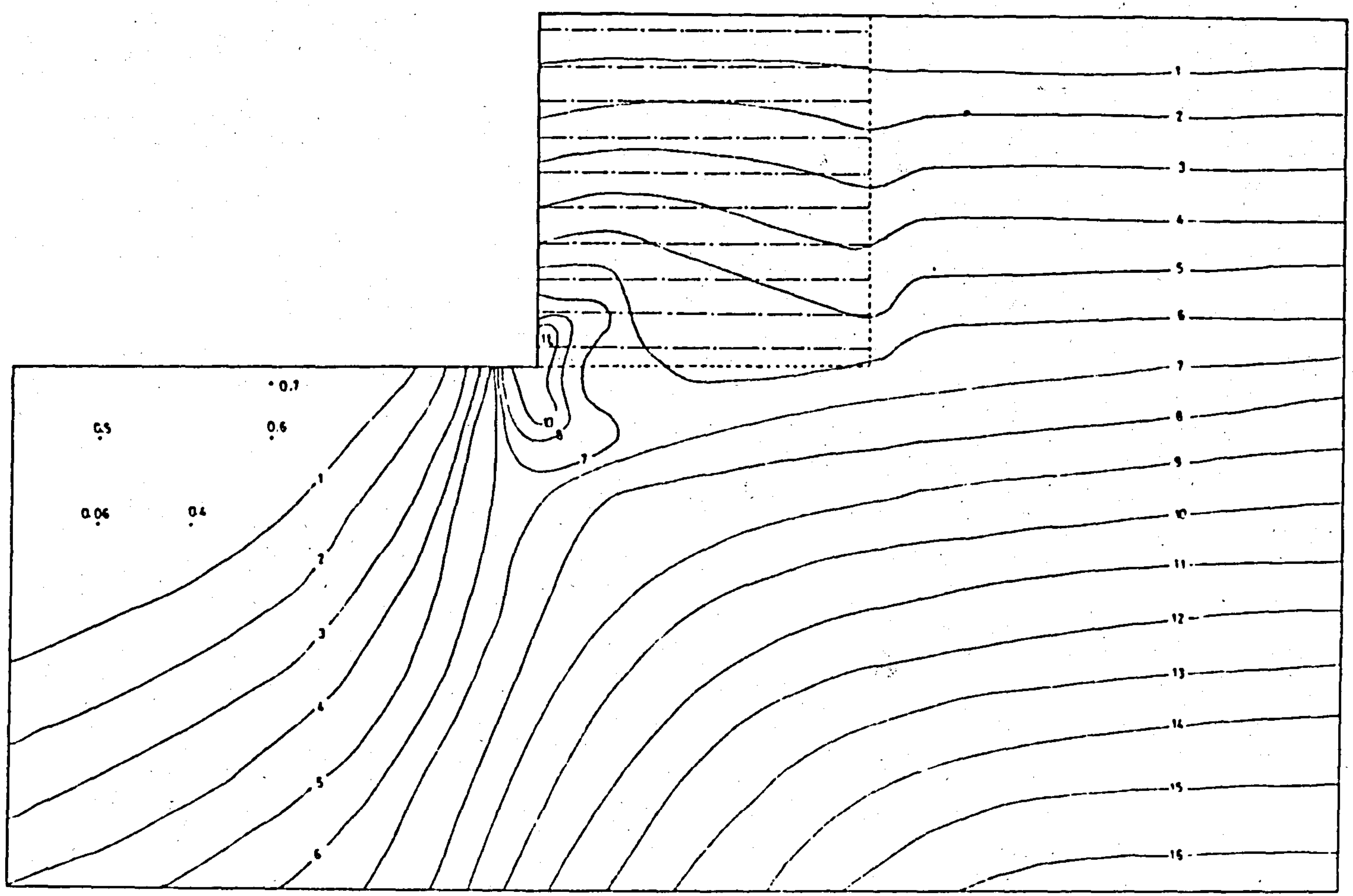


Fig. 7.8 Contours of Maximum Shear Stress (t/m^2)

stresses, which has an important role in the failure of the soil.

7.7. THE EFFECT OF REINFORCEMENT ON STRESS LEVEL

Fig. 7.9 and Fig. 7.10 indicate the contours of the stress level for unreinforced and reinforced soil, respectively. In the case of unreinforced Soil, stress level increases upto 283 percent, and the zone where the stress level is more than hundred percent is very large. After taking into account the influence of the reinforcement, stress level on reinforced zone sharply decreases, although there are some points where stress level is still more than hundred percent, which shows that the reinforcement placed in this area is not sufficient.

7.8. THE HORIZONTAL DISPLACEMENTS

The contours for horizontal displacements are drawn only in the reinforced case and illustrated in Fig. 7.11. As this figure indicates, the maximum horizontal movement is about 1.6 cm and takes place near the vicinity of the middle of the bottom line. As horizontal distance behind the wall face increases, the horizontal displacement steadily decreases, while it shqrply drops near the back line, which is reasonable owing to the fact that horizontal movement is restrained on this line. For the aim of investigating the effect of the reinforcements on horizontal displacement, depth

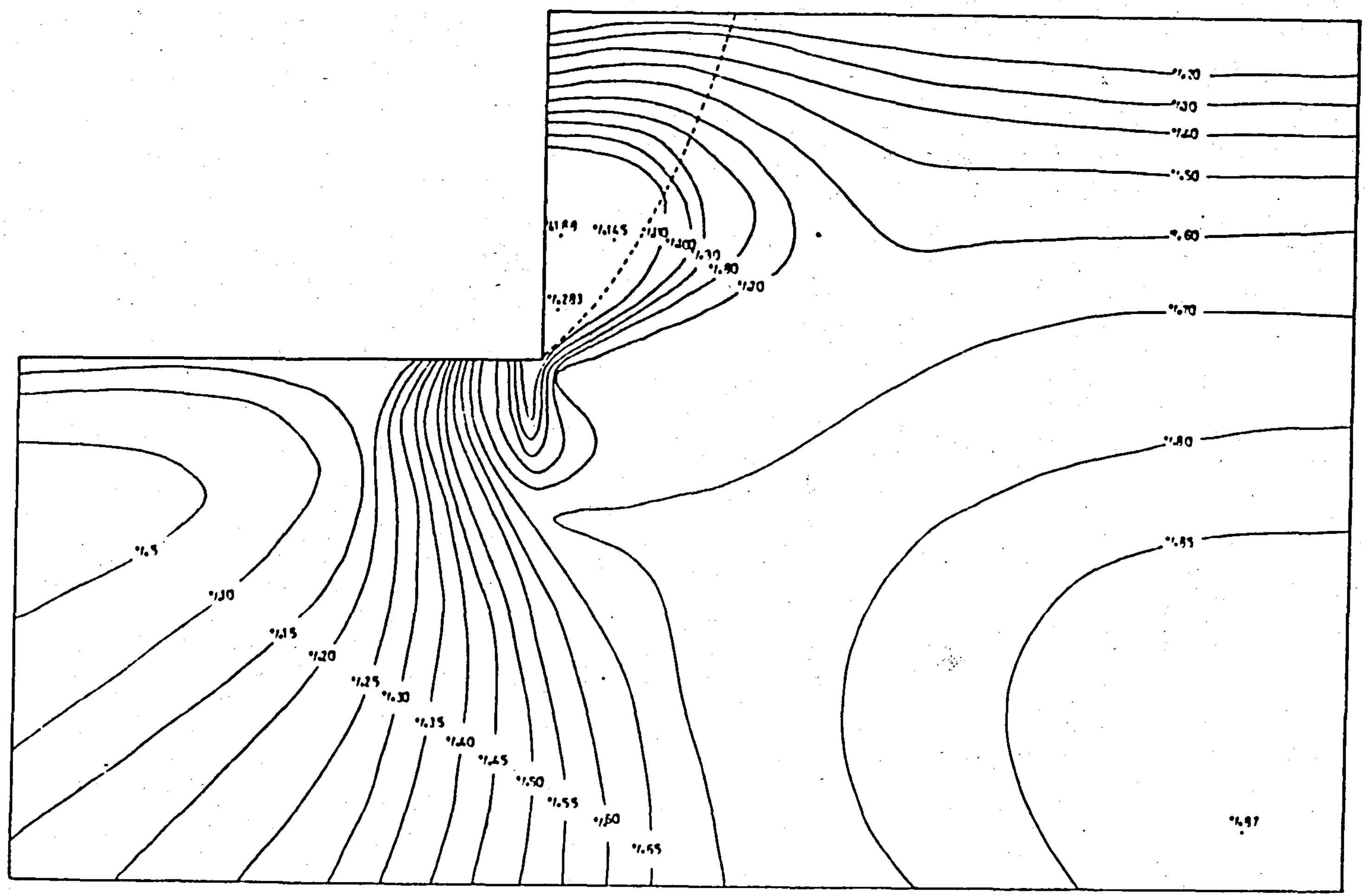


Fig. Contours of Stress Level

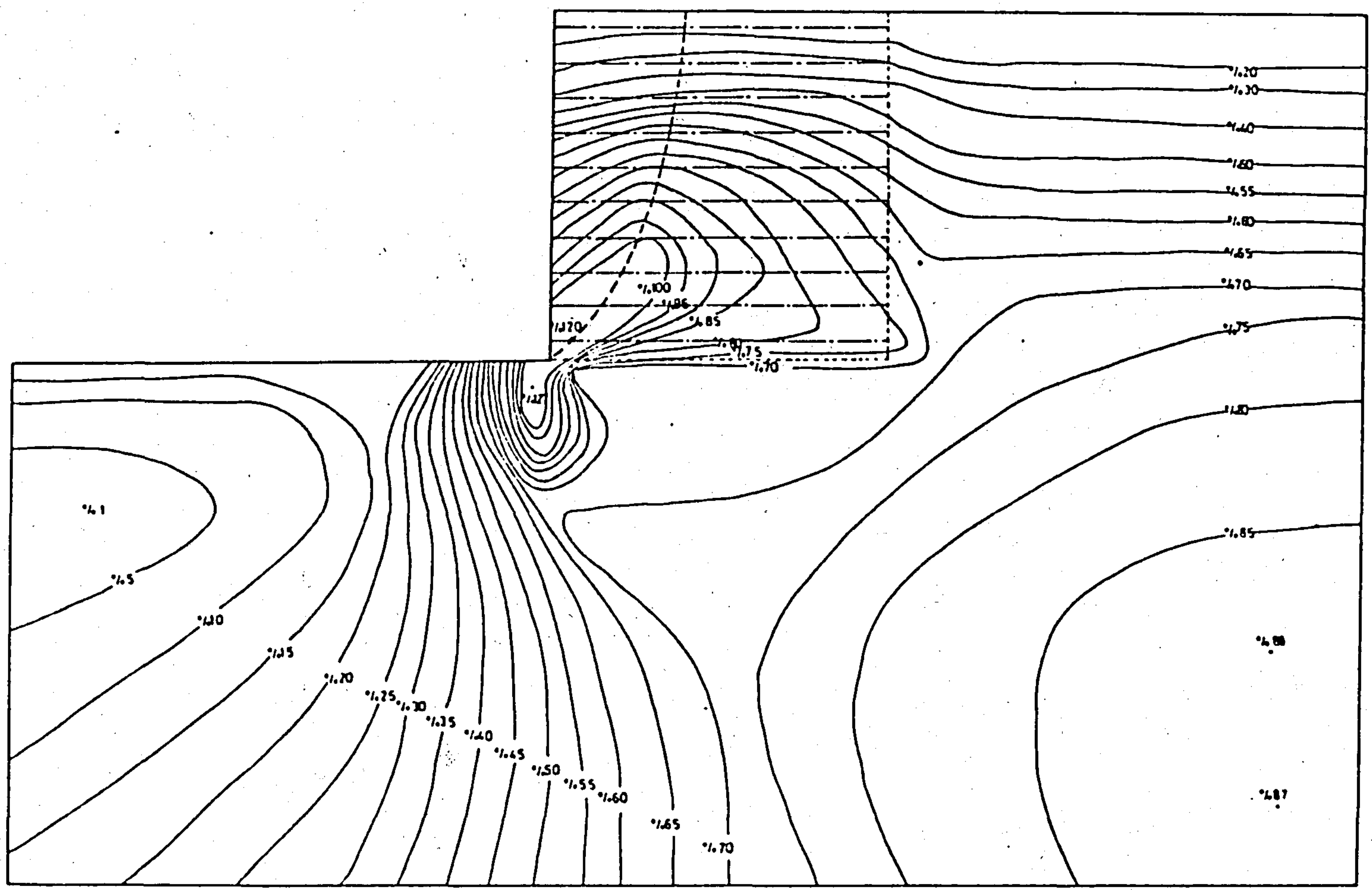


Fig. 7.10 Contours of Stress Level

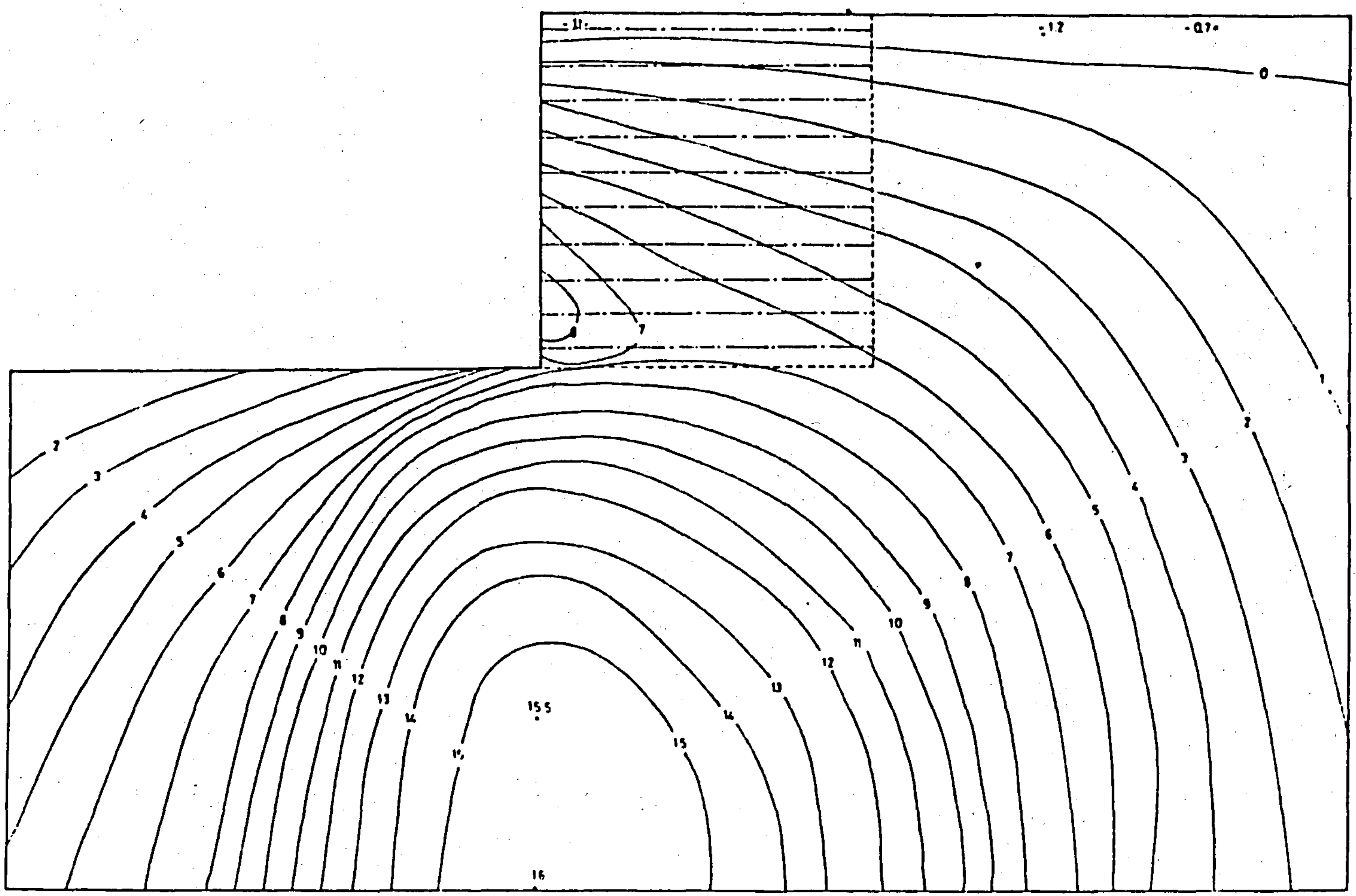


Fig. 7.11. Contours of Horizontal Displacement $(10^{-1} cm)$

below the ground surface versus horizontal movements toward the wall face in reinforced and unreinforced soil are plotted in Fig. 7.12. As this figure shows, reinforcements reduce the horizontal displacement on vertical section. In order to find out the effect of the reinforcement in horizontal movements on horizontal section, distance behind the wall face in both reinforced and unreinforced cases for two horizontal sections are plotted in Fig. 7.13. It is interesting to notice that although reinforcements cause the reduction of horizontal movement in reinforced part, they slightly increase it behind this region, as it is seen in Fig. 7.13.

7.9. VERTICAL DISPLACEMENT

As it was done for horizontal displacement, the contours of settlements are drawn only for the case of reinforced soil, and illustrated in Fig. 7.14. It is easy to notice that the maximum settlement is about 10 cm and occurs at the ground level. It is a fact that, contrary to the horizontal displacement, the vertical displacement steadily increases as the distance behind the wall face increases and is leveled at a constant value near the back line. In order to investigate the influence of reinforcement on vertical displacement on vertical section, depth below the ground surface versus, settlement in reinforced and unreinforced soil are plotted in Fig. 7.15. This figure shows that, reinforcement has no considerable effect on vertical displacement.

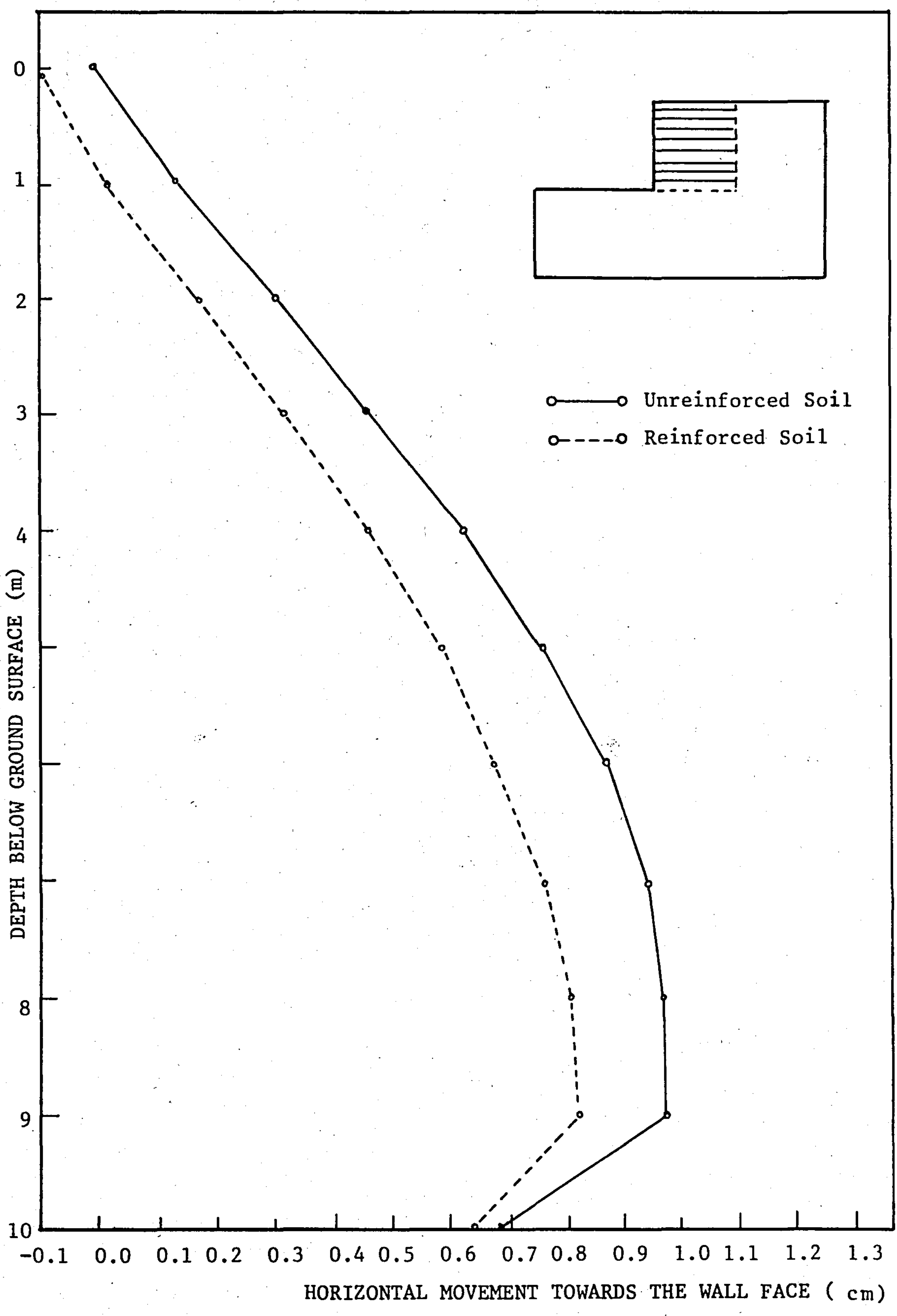


Fig. 7.12 Comparison of Horizontal Movements

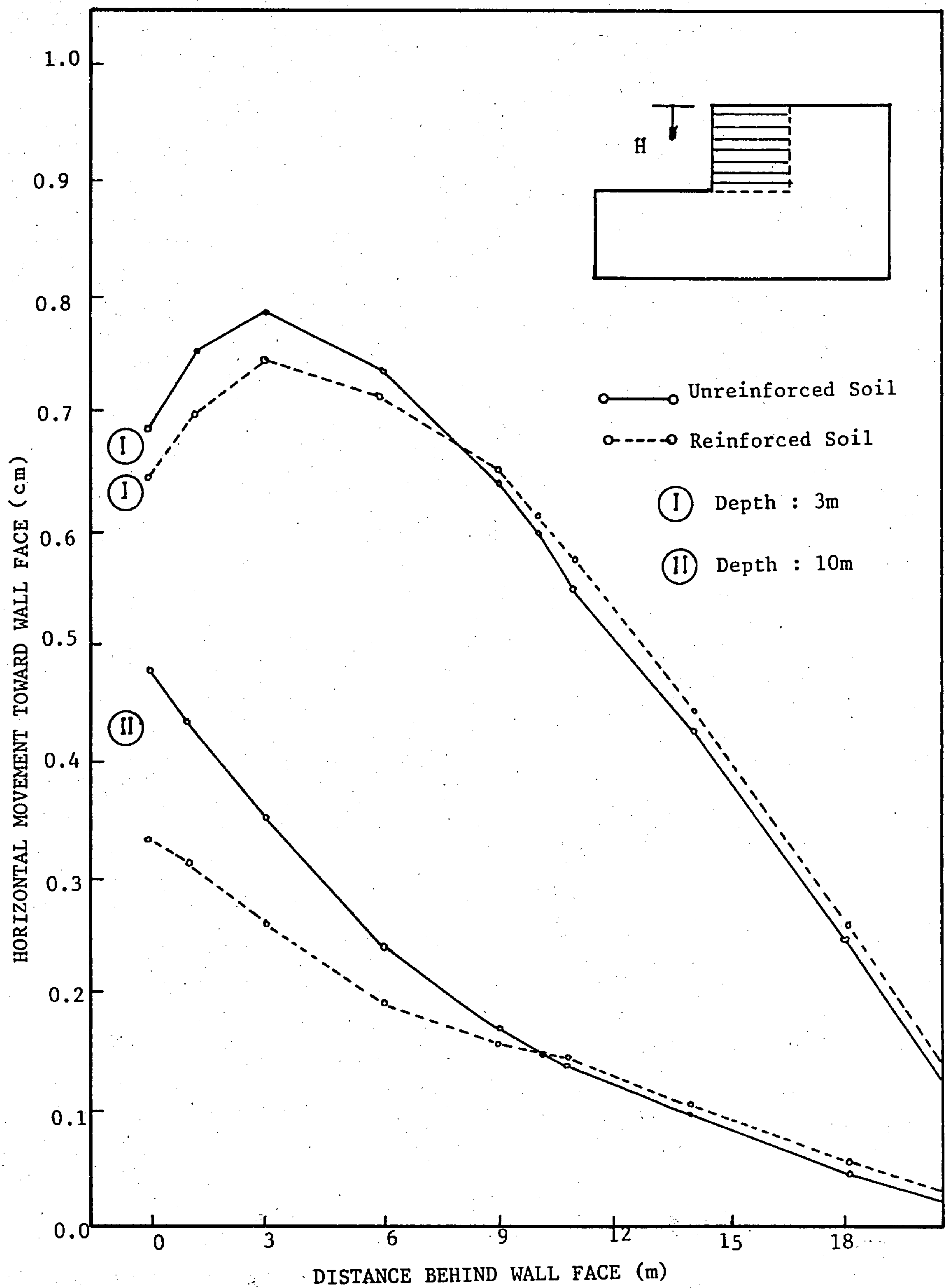


Fig. 7.13 Horizontal Movements on Horizontal section

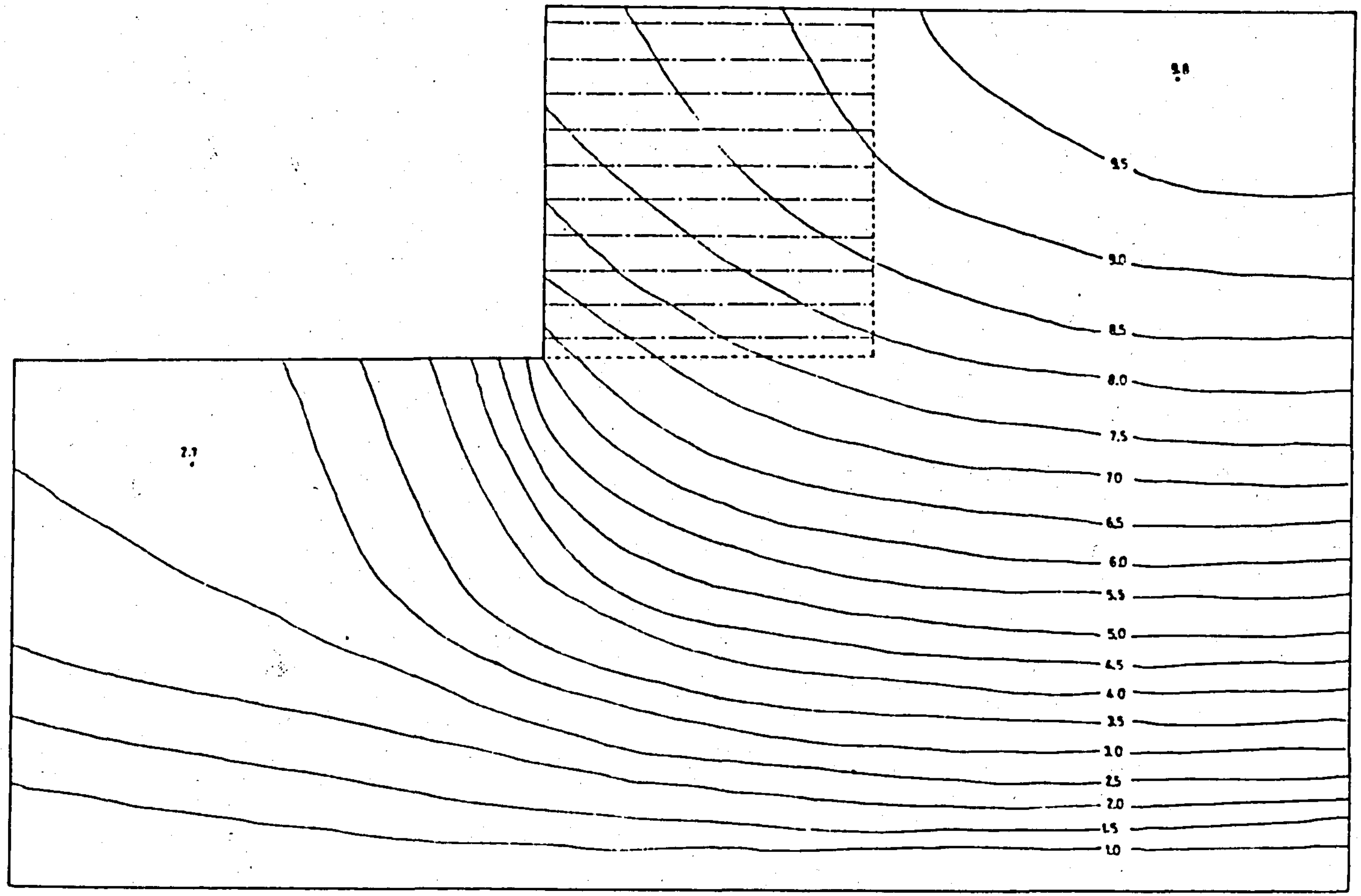


Fig. 7.14 Contours of Vertical Displacement (cm)

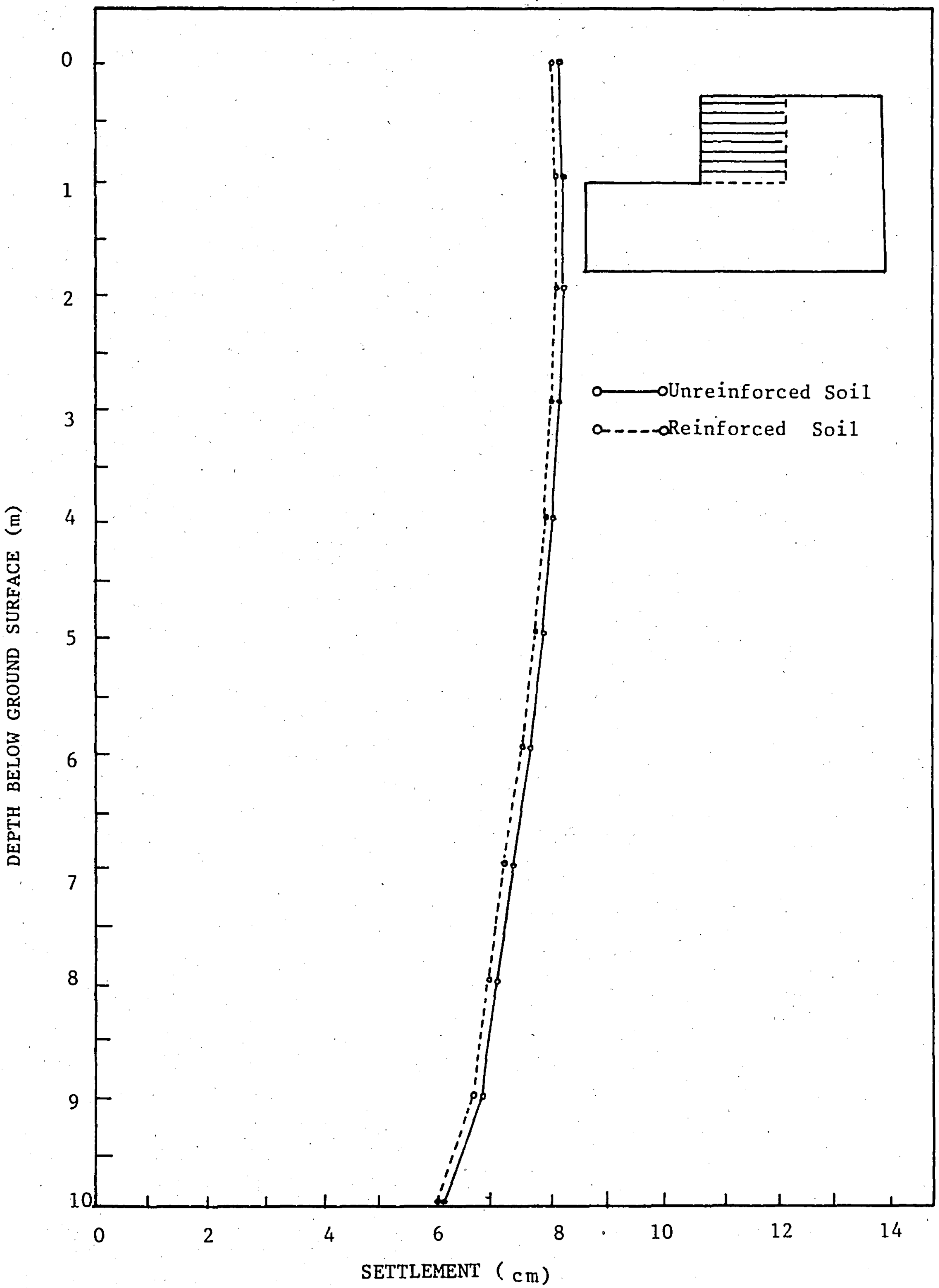


Fig. 7.15 Comparison of settlement

7.10 TENSILE FORCE IN REINFORCING STRIPS

The tensile force distribution along the reinforcements are illustrated in Fig. 7.16. As this figure shows, the maximum tensile force developed in this analysis is not more than 8.5t. This force causes the stress as much as

$$\sigma = \frac{P}{A} = \frac{8.500}{5} = 1700 \text{ kg/cm}^2$$

which is about half of the stress that a steel strip can carry. It means, it is more beneficial to decrease the spacing of reinforce-ents, instead of increasing tensile strength or area of them.

Fig. 7.16 also shows that tensile forces mobilized in reinforcements are influenced by the boundary conditions. If slippage was allowed near the ends of the strips, the edge effect would probably be more significant with a probable increase in strip forces due to the decreased stiffness near the wall boundaries. However, detailed knowledge of edge effects and strip slippage is probably most important in determining required strip lengths and these effects, therefore, need to be understood more completely.

Fig. 7.17, on the other hand, indicates tensile force distribution with respect to depth from the ground surface. It is clear that tensile force increases as the depth from the ground surface increases. This force distribution recommends that the vertical

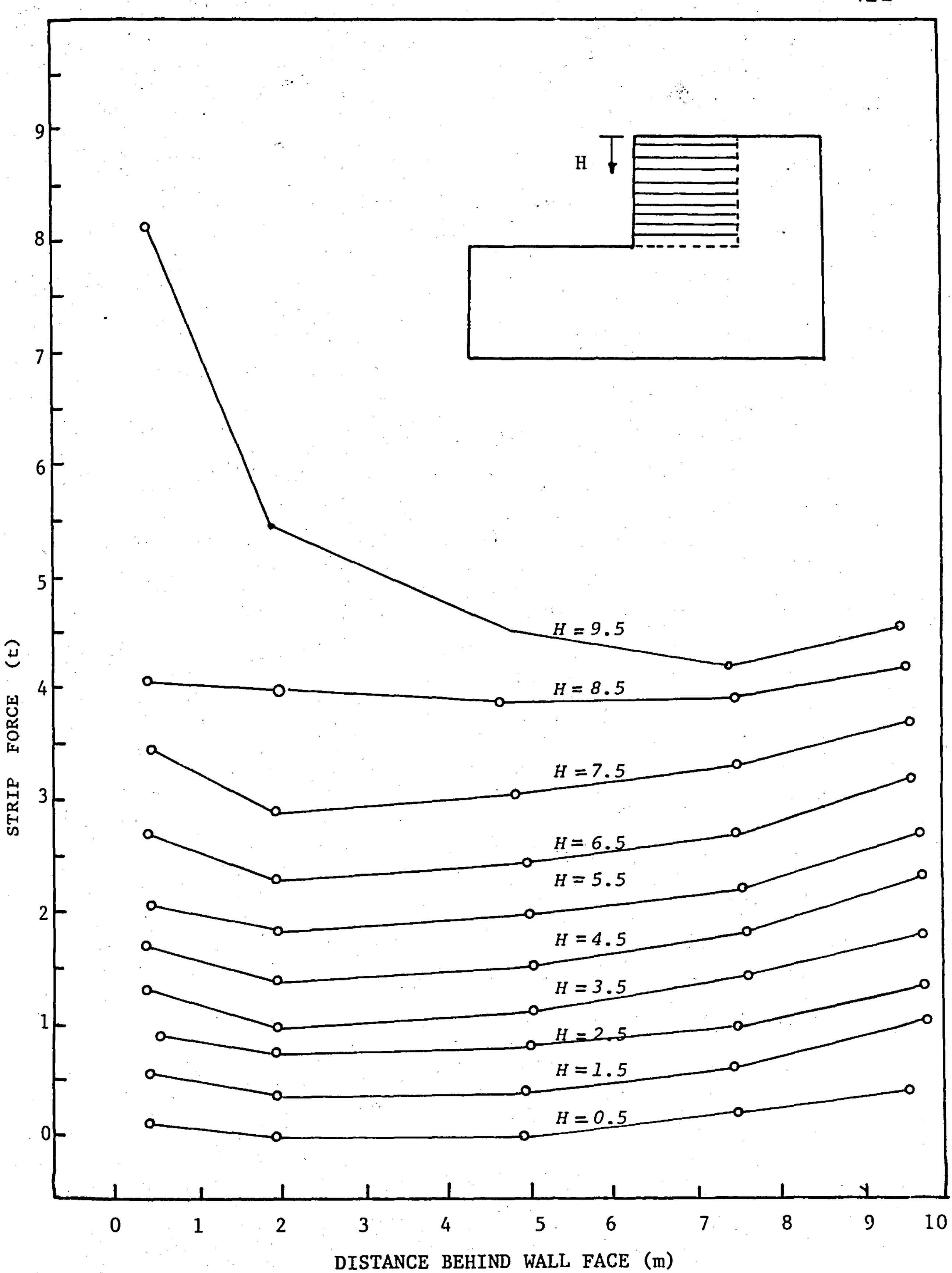


Fig. 7.16 Tensile Force

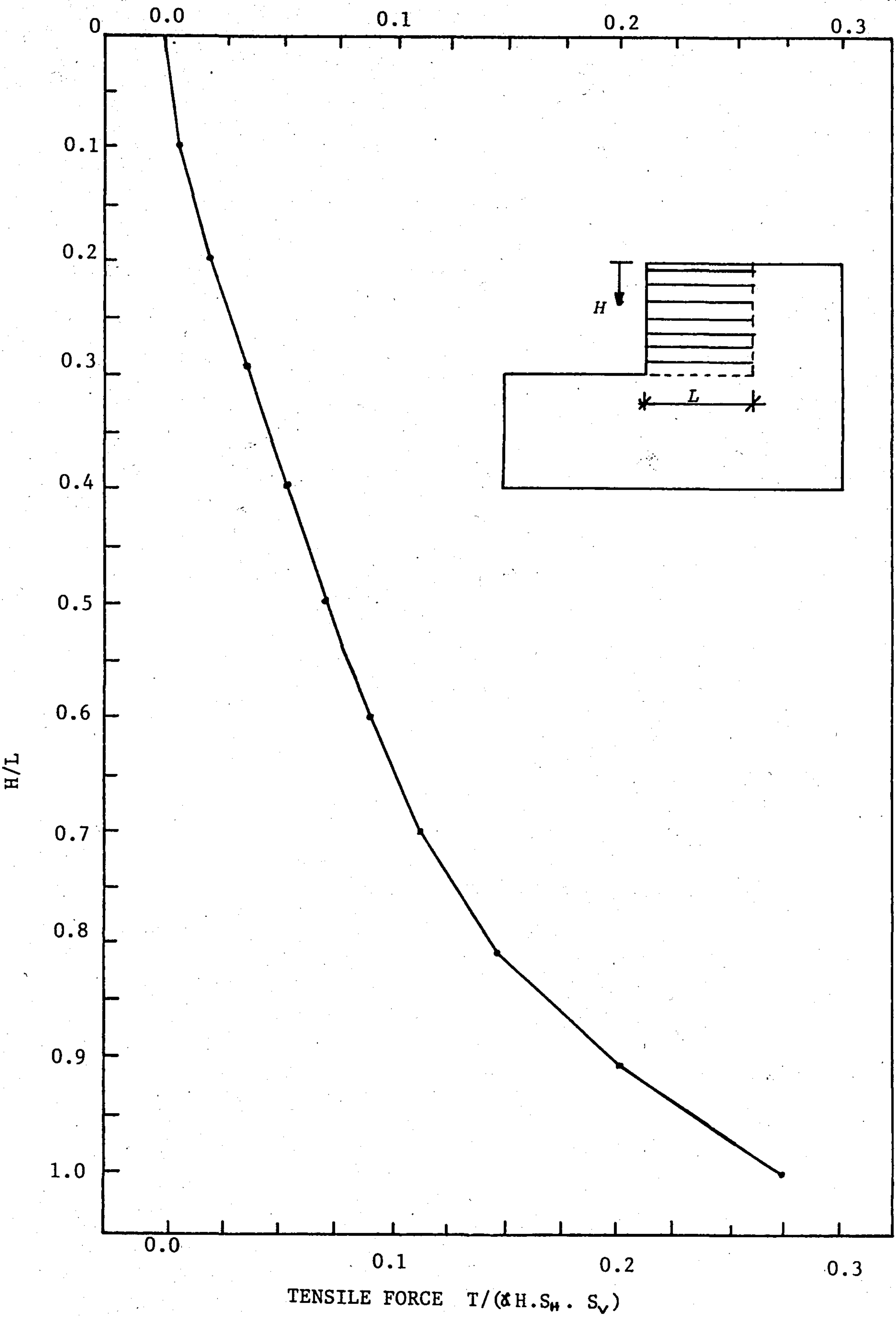


Fig. 7.17. Tensile Force

spacing of reinforcing strips should be decreased as the depth increases.

7.11 SUMMARY AND CONCLUSIONS

The studies presented in this chapter have demostrated distribution of stresses, movements and stress levels developed in unreinforced and reinforced soil. By comparing the results for unreinforced and reinforced cases, the influence of reinforcement on soil stability has been found out. Besides, tensile stress distribution on vertical section and along the reinforcing strips and corresponding design approach have been explained.

It has been revealed that reinforcing strips have following effects

- Reducing the shear stresses carried by the soil
- Redistributing the horizontal stresses and bearing the majority of them
- Releasing stress concentration
- Decreasing horizontal displacement.

CHAPTER-VIII

SUMMARY AND CONCLUSIONS

In this study, Reinforced Earth , which is rather different than conventional soil retaining structures has been investigated by various techniques. Infact the study can be divided into two parts. In the first part, principle of soil reinforcement interaction and design metodology were studied and selection of the components of the reinforced earth structures and construction method were briefly explained. In design method, the system was handled as a slope stability problem and multi-criterion analysis methods were employed to find out the allowable normal and shear forces for reinforcing bars. A computer program (SLOPER) which evaluates the forces mobilized in the bars, and estimates the safety factor, was prepared.

In the second part of this study, reinforced earth systems were analyzed by the finite element method, which gives more accurate anddetailed prediction about the stresses and strains mobilized in the system. For this purpose, the finite element technique was explained in detail and necessary formulations were derived. Since, reinforced earth system is a soil structure, it doesn't behave elastic. The hyperbolic stress-strain parameters, therefore, were utilized to take into account nonlinearity of soil.

A finite element computer program (FRSOIL) having various capacities was also developed in this study. For the aim of analyzing reinforced earth system by finite element method, this system was considered as composite material by using "Unit Cell Concept". Then, a typical excavation problem in unreinforced and reinforced soil were investigated by this technique. In order to find out the effect of reinforcement on soil stability, stresses, strains and stress levels mobilized in reinforced and unresforced soil were compared.

From the analysis of this study, following results have been obtained.

* Reinforcement carries the majority of horizontal Stresses
* Reinforcement decreases shear stresses
* Reinforcement Releases stress concentrations
* Reinforcement reduces stress level
* Reinforcement decreases horizontal displacements

On the contrary, reinforced earth structures have following shortcomings:

* Reinforcement has no considerable effect on vertical stresses.

* Effect of reinforcement depends very much on the orientation of reinforcing strips. But it is difficult to install the bars in appropriate direction in all cases.

* Reinforcing strips don't noticebly change the amount of settlement.

From the study conducted for the thesis the following recommendations are derived for the reinforced earth systems:

* Special attention should be given to the selection of reinforcing strips and backfill material.

* It is more beneficial to decrease the spacing of reinfo- reements instead of increasing tensile strength or area of them.

* Vertical spacing of reinforcing strips should be decreased as the depth from the ground surface increases.

* If it is possible, reinforcements should be oriented in the same direction as the principal tensile strain occuring in the unreinforced soil.

Thevalidity of the developed analytical procedure and recommendations for further Investigation are given as:

* The finite element method doesn't give any idea about the effect of decreased stiffness near the wall boundary.

* The validity of Unit Cell Concept and assumptions made in this concept should he investigated.

* In composite material concept, it is not possible to take into account neither the surface characteristics of reinforcing bars nor the intcraction between soil and the bars. In this case it is necessary to analyze this system by utilizing "Soil-structure interaction Method" where the bars are simulated by one dimensional elements and provide the mognitude of these effects.

REFERENCES

1. Karl M. Romstad, Leonard R. Herrmann, and Chih-Kang Shen *"Integrated Study of Reinforced Earth-I"*, Journal of the Geotechnical Engineering Division, ASCE, GT5, May 1976.

2. Duncan J.M., Chang C-Y, *"Nonlinear Analysis on Stress and Strain in Soils"*, Journal of the Soil Mechanics and Foundation Division, ASCE. Vol. 96, No. SM5, Semtember 1970.

3. Tezcan S., *"Finite Element Method, Lecture Notes"*, Boğaziçi University, 1985.

4. Bowles J.E., *"Physical and Geotechnical properties of soils"*, McGraw-Hill, 1979.

5. Zienkiewicz O.C., *"The Finite Element Method in Engineering science"*, McGraw-Hill, Condon, 1977.

6. Villet, W.C.B, Mitchel. J., *Reinforcement of Earth Slopes and Embankments"*, Draft Report, November 1984.

7. NCHRP PROJECT 24-2, *"Reinforcement of Earth Slopes and Embankments"*, May 1985.

8. *"Syposium On Earth Reinforcements"*, American Society of Civil Engineering, April 27, 1978.

9. Duncan J.M. and Wong K.S. (1984), *"Hyperbolic Stress-Strain Parameters For Nonlinear Finite Element Analyses of Stresses and Movements in Soil Masses"*, Report No. UCB/GT/80-01, Department of Civil Engineering, University of California, Berkely, California, August, 1980.

10. Rowe, R.K., *"Reinforced Embankments: Analysis and Design"*, Journal of the Geotechnical Engineering Division, ASCE, February, 1984.

11. Shen, C.K., Bang, S. and Hermann, L.R. (1981), *"Ground Movement Analysis of an Earth Support System"*, Journal of the Geotechnical Engineering Division, ASCE, Vol. 107, no. GT12, Dec.

12. Lec, K.L. *"Mechanism , Analysis and Design of Reinforced Earth"*, Proceeding of Symposium on Earth Reinforcement, ASCE, April, 1978.

13. Vidal, H. *"The Development and Future of Reinforced Earth"*, Proceeding of ASCE, April, 1978.

A P P E N D I C E S

APPENDIX A

HYPERBOLIC STRESS-STRAIN PARAMETERS

STRESS-STRAIN AND STRENGTH PARAMETERS FOR SOILS TESTED UNDER UNCONSOLIDATED-UNDRAINED CONDITIONS

Soil	Group	Soil Description	References	Grain Size, mm			LL	PI	Compaction					Degree Saturation	Rating	Particle Shape	Stress Range (TSF)	Number of Tests	c (TSF)	Friction Angle	K	n	R_f	K_b	m
				D60	D30	D10			Type	Max. Dry Unit Wt. (PCF)	Opt. w/c	Dry U. Wt. (PCF)	w/c												
GC	GC-2A	Sandy Gravel (Oroville Dam Core)	Dept. of Water Resources (21)	9.0	0.12	0.005	30	16	Std.AASHO	138.6	8.1	139.0	8.1		*		3.6-10.9	2	1.50	24	540	.51	.84		
GC	GC-2B	Sandy Gravel (Oroville Dam Core)	Dept. of Water Resources (21)	9.0	0.12	0.005	30	16	Std.AASHO	138.6	8.1	139.0	8.1		*		27.9-43.3	2	10.01	3	190	.95	.97		
SP	SP-8D	Poorly Graded Sand (Rodman Dam)	COE, Jacksonville District (16)	0.38	0.26	0.16	17	NP	Mod.AASHO	109.5	11.8	104.0	11.8	55	***	Sub-rounded	1.0- 3.0	3	0.	37 (4)	590	1.10	.89		
SP	SP-8E	Poorly Graded Sand (Rodman Dam)	COE, Jacksonville District (16)	0.38	0.26	0.16	17	NP	Mod.AASHO	109.5	11.8	98.6	11.8	47	*	Sub-rounded	1.0- 3.0	2	0.	37 (8)	770	- .14	.87		
SP	SP-8F	Poorly Graded Sand (Rodman Dam)	COE, Jacksonville District (16)	0.38	0.26	0.16	17	NP	Mod.AASHO	109.5	11.8	110.0	11.2	61	**	Sub-rounded	1.0- 3.0	3	0.	43 (9)	940	0.	.82		
SP	SP-9D	Poorly Graded Silty Sand (Rodman Dam)	COE, Jacksonville District (16)	0.16	0.14	0.084	23	NP	Mod.AASHO	101.1	13.6	101.3	13.4	57	***	Sub-rounded	1.0- 3.0	3	0.	44 (6)	420	.67	.76		
SP	SP-9E	Poorly Graded Silty Sand (Rodman Dam)	COE, Jacksonville District (16)	0.16	0.14	0.084	23	NP	Mod.AASHO	101.1	13.6	96.2	13.3	50	*	Sub-rounded	1.0- 2.0	2	0.	44 (11)	850	.79	.92		
SP	SP-9F	Poorly Graded Silty Sand (Rodman Dam)	COE, Jacksonville District (16)	0.16	0.14	0.084	23	NP	Mod.AASHO	101.1	13.6	92.0	12.4	42	***	Sub-rounded	1.0- 3.0	3	0.	40 (8)	470	.51	.86		
SM	SM-1	Gravelly Silty Sand (Ball Mountain Dam)	Linell & Shea (36)	0.80	0.074	0.05	NP	NP	Std.AASHO	122.9	10.0	124.0	9.4	71	**		1.1- 4.3	3	0.	42 (5)	430	.38	.57		
SM	SM-3A	Silty Sand (Somerville Dam)	COE, Fort Worth District (15)	0.108	0.095	0.004	NP	NP	Std.AASHO	109.1	13.4	109.3	13.4	70	**		.5- 6.0	4	0.	40 (2)	350	.91	.69		
SM	SM-3B	Silty Sand (Somerville Dam)	COE, Fort Worth District (15)	0.108	0.095	0.004	NP	NP	Std.AASHO	109.1	13.4	104.1	13.2	60	**		.5- 6.0	4	0.	40 (6)	420	.84	.75		
SM	SM-3C	Silty Sand (Somerville Dam)	COE, Fort Worth District (15)	0.108	0.055	0.004	NP	NP	Std.AASHO	109.1	13.4	103.6	16.7	75	**		.5- 6.0	4	0.	39 (4)	340	.64	.72		
SM-SC	SM-SC-2	Silty Clayey Sand (Hopkinton Dam)	Linell & Shea (36)	0.22	0.014	0.001	21	7	Std.AASHO	129.2	9.2	131.0	8.8	83	**		1.0- 6.0	3	.98	31	320	.35	.86		
SC	SC-2	Clayey Sand (Thomaston Dam)	Linell & Shea (36)	0.4	0.028	0.003	29	12	Std.AASHO	123.3	12.0	122.0	12.0	85	**		1.1- 4.3	3	.92	18	39	.61	.55		
SC	SC-3	Clayey Sand (New Don Pedro Dam Core)	Bechtel (1)	0.54	0.02	0.005	27	11	20,000	125.8	9.8	123.2	9.6	73	**		5.4-21.6	3	2.60	26	3900	- .08	.93	12000	- .99
SC	SC-5	Clayey Gravelly Sand (Proctor Dam)	COE, Fort Worth District (15)	0.25	0.08	-	28	18	Std.AASHO	120.1	11.2	126.0	8.3	70	*		.5- 1.5	2	1.80	4	510	.37	.64	250	0.
SC	SC-6A	Clayey Sand (Chatfield Dam)	COE, Omaha District (19)	0.24	0.04	-	22	7	Std.AASHO	122.0	11.7	116.2	14.7	90	*		6.0-10.0	2	1.30	0	52	0.	.76		
SC	SC-7B	Clayey Sand (Chatfield Dam)	COE, Omaha District (19)	0.11	0.01	-	32	18	Std.AASHO	115.0	15.0	110.0	17.0	88	*		6.0-10.0	2	1.10	0	250	0.	.97		
ML	ML-2A	Sandy Silt (Chatfield Dam)	COE, Omaha District (19)	0.09	0.03	0.003	25	4	Std.AASHO	115.0	12.8	108.7	15.6	77	***		6.0-10.0	3	1.80	19	200	.59	.86		
ML	ML-2B	Sandy Silt (Chatfield Dam)	COE, Omaha District (19)	0.09	0.03	0.003	25	4	Std.AASHO	115.0	12.8	109.3	12.7	63	***		6.0-10.0	3	.39	30	27	1.43	.72		
ML	ML-3A	Sandy Silt (Birch Dam Shell)	COE, Tulsa District (20)	0.070	0.045	0.013	19	1	Std.AASHO	108.8	13.6	104.0	11.6	53	***		.5- 6.0	4	.42	31	240	.31	.83		
ML	ML-3B	Sandy Silt (Birch Dam Shell)	COE, Tulsa District (20)	0.070	0.045	0.013	19	1	Std.AASHO	108.8	13.6	104.0	13.6	62	***		1.5- 6.0	3	.19	31	270	.38	.82		
ML	ML-3C	Sandy Silt (Birch Dam Shell)	COE, Tulsa District (20)	0.070	0.045	0.013	19	1	Std.AASHO	108.8	13.6	104.0	16.6	74	***		1.5- 6.0	3	.54	27	100	.84	.77		
CL	CL-1A	Silty Clay (Arkabutla Dam)	Casagrande et al (9)	0.023	0.01	-	40	20	Std.AASHO	110.0	18.0	108.7	16.7	81	***		1.0-12.3	4	.53	29	260	.60	.87		
CL	CL-1B	Silty Clay (Arkabutla Dam)	Casagrande et al (9)	0.023	0.01	-	40	20	Std.AASHO	110.0	18.0	107.0	19.5	89	*		1.0- 8.2	4	1.20	14	39	.48	.58		
CL	CL-2A	Lean Clay (Monroe Dam)	COE, Louisville District (18)	0.023	0.001	-	40	23	Std.AASHO	110.5	16.4	107.1	19.1	87	*		.7- 2.9	2	.95	0	66	0.	.75		
CL	CL-2B	Lean Clay (Monroe Dam)	COE, Louisville District (18)	0.023	0.001	-	40	23	Std.AASHO	110.5	16.4	104.0	21.2	89	**		.7- 2.9	3	.42	0	10	.03	.52		
CL	CL-3	Lean Clay (Monroe Dam)	COE, Louisville District (18)	0.015	0.0044	-	44	22	Std.AASHO	106.8	18.0	102.0	21.7	92	*		.7- 2.9	2	1.00	0	36	0.	.57		
CL	CL-5A	Pittsburg Silty Clay	Kulhawy, Duncan & Seed (32)	0.04	0.003	-	35	16	Mod.AASHO	118.9	13.5	105.4	11.5	52	**		1.0- 3.0	2	.92	31	650	- .68	.90	190	- .81
CL	CL-5B	Pittsburg Silty Clay	Kulhawy, Duncan & Seed (32)	0.04	0.003	-	35	16	Mod.AASHO	118.9	13.5	109.1	14.3	71	**		1.0- 6.0	3	1.50	17	760	- .14	.93	240	- .21
CL	CL-5C	Pittsburg Silty Clay	Kulhawy, Duncan & Seed (32)	0.04	0.003	-	35	16	Mod.AASHO	118.9	13.5	109.0	16.8	83	**		1.0- 6.0	2	1.30	6	430	.10	.93	115	.10
CL	CL-5E	Pittsburg Silty Clay	Kulhawy, Duncan & Seed (32)	0.04	0.003	-	35	16	Mod.AASHO	118.9	13.5	112.7	11.5	63	**		1.0- 6.0	3	1.80	24	2400	- .74	.92	740	- .96
CL	CL-5F	Pittsburg Silty Clay	Kulhawy, Duncan & Seed (32)	0.04	0.003	-	35	16	Mod.AASHO	118.9	13.5	114.7	14.5	84	**		1.0- 3.0	2	1.90	13	2000	- .30	.97	460	- .64
CL	CL-5H	Pittsburg Silty Clay	Kulhawy, Duncan & Seed (32)	0.04	0.003	-	35	16	Mod.AASHO	118.9	13.5	108.8	8.71	43	**		1.0- 6.0	3	1.50	32	8900	-1.10	.94	1900	-1.1
CL	CL-5I	Pittsburg Silty Clay	Kulhawy, Duncan & Seed (32)	0.04	0.003	-	35	16	Mod.AASHO	118.9	13.5	119.3	11.7	77	**		1.0- 6.0	3	3.30	18	5000	- .28	.95	1400	- .33
CL	CL-6A	Sandy Clay (Birch Dam Core)	COE, Tulsa District (20)	0.045	0.01	-	29	15	Std.AASHO	110.3	14.5	105.0	12.5	57	**		1.5- 6.0	3	.64	29	320	- .21	.80		
CL	CL-6B	Sandy Clay (Birch Dam Core)	COE, Tulsa District (20)	0.045	0.01	-	29	15	Std.AASHO	110.3	14.5	105.0	14.5	66	***		.5- 6.0	3	.50	25	190	.02	.81		
CL	CL-7A	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.06	0.003	-	43	30	Std.AASHO	107.5	17.2	107.9	17.2	87	**		.5- 6.0	4	1.00	2	74	.23	.87		
CL	CL-7B	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.06	0.003	-	43	30	Std.AASHO	107.5	17.2	107.2	17.0	74	**		.5- 6.0	4	1.00	1	68	- .05	.84		
CL	CL-7C	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.06	0.003	-	43	30	Std.AASHO	107.5	17.2	102.6	20.0	88	**		.5- 6.0	4	.45	1	27	.18	.85		
CL	CL-8B	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.085	0.0055	-	28	16	Std.AASHO	113.3	14.5	108.3	14.6	74	**		.5- 6.0	4	.57	25	320	.29	.85		
CL	CL-9A	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.052	0.0085	-	49	32	Std.AASHO	95.7	23.3	96.5	23.2	89	**		.5- 6.0	3	1.50	4	200	.29	.89		
CL	CL-9B	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.052	0.0085	-	49	32	Std.AASHO	95.7	23.3	91.7	23.3	77	**		1.5- 6.0	3	1.20	3	100	.18	.86		
CL	CL-9C	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.052	0.0085	-	49	32	Std.AASHO	95.7	23.3	90.8	26.7	87	**		.5- 6.0	3	.64	1	53	.14	[illegible]		
CL	CL-10A	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.085	0.004	-	29	16	Std.AASHO	110.7	15.0	111.8	15.1	86	**		.5- 6.0	4	.84	22	160	.34	.78		
CL	CL-10B	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.085	0.004	-	29	16	Std.AASHO	110.7	15.0	106.5	15.0	74	***		.5- 6.0	4	.55	22	290	.27	.91		
CL	CL-11A	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.06	0.002	-	25	12	Std.AASHO	107.5	16.8	100.3	13.5	58	**		.5- 6.0	4	.78	28	680	- .36	.84		
CL	CL-11B	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.06	0.002	-	25	12	Std.AASHO	107.5	16.8	106.5	13.3	66	**		.5- 6.0	4	1.50	25	600	.18	.68		

(CONTINUED)

Soil	Group	Soil Description	References	Grain Size, mm D60	D30	D10	LL	PI	Compaction Type	Max. Dry Unit Wt. (PCF)	Opt. w/c	Dry U. Wt. (PCF)	w/c	Degree Saturation	Rating	Particle Shape	Stress Range (TSF)	Number of Tests	C (TSF)	Friction Angle	K	n	R_f	K_b	m
CL	CL-11C	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.06	0.002	-	25	12	Std.AASHO	107.5	16.8	102.6	19.3	87	*		.5- 6.0	4	.74	6	23	.32	.61		
CL	CL-11D	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.06	0.002	-	25	12	Std.AASHO	107.5	16.8	106.7	16.7	85	*		.5- 6.0	4	.91	18	280	.60	.93		
CL	CL-11E	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.06	0.002	-	25	12	Std.AASHO	107.5	16.8	101.5	16.3	72	*		.5- 6.0	4	.66	20	220	.23	.90		
CL	CL-12A	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.065	0.0055	0.001	38	25	Std.AASHO	106.1	17.2	105.0	18.6	89	*		.5- 6.0	4	1.30	8	140	.20	.84		
CL	CL-12C	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.065	0.0055	0.001	38	25	Std.AASHO	106.1	17.2	101.9	17.1	75	**		.5- 6.0	4	1.00	13	120	.09	.83		
CL	CL-12D	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.065	0.0055	0.001	38	25	Std.AASHO	106.1	17.2	103.0	19.7	89	*		.5- 6.0	4	.80	2	47	.33	.82		
CL	CL-12E	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.065	0.0055	0.001	38	25	Std.AASHO	106.1	17.2	106.5	13.9	70	**		.5- 6.0	4	1.50	24	950	-.15	.90		
CL	CL-12F	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.065	0.0055	0.001	38	25	Std.AASHO	106.1	17.2	108.3	16.9	89	*		.5- 6.0	4	1.50	8	470	0.	.95		
CL	CL-13A	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.046	0.0045	-	36	23	Std.AASHO	104.9	17.6	98.7	20.8	86	**		.5- 6.0	4	.67	4	75	.44	.88		
CL	CL-13B	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.046	0.0045	-	36	23	Std.AASHO	104.9	17.6	104.9	14.8	72	*		.5- 6.0	4	1.80	23	840	-.19	.84		
CL	CL-13C	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.046	0.0045	-	36	23	Std.AASHO	104.9	17.6	101.2	17.4	76	*		.5- 6.0	4	1.20	12	270	.06	.87		
CL	CL-13D	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.046	0.0045	-	36	23	Std.AASHO	104.9	17.6	100.5	14.2	62	*		.5- 6.0	4	1.40	29	1100	-.36	.83		
CL	CL-13E	Sandy Clay (Somerville Dam)	COE, Fort Worth District (15)	0.046	0.0045	-	36	23	Std.AASHO	104.9	17.6	104.4	17.5	84	*		.5- 6.0	3	1.40	13	410	.15	.87		
CL	CL-14	Lean Clay (Clinton Dam)	COE, Kansas City District (17)	-	-	-	46	27	Std.AASHO	103.0	21.2	98.0	24.0	92	**		1.0- 5.0	3	.77	2	57	.43	.86		
CL	CL-16C	Lean Clay (Clinton Dam)	COE, Kansas City District (17)	-	-	-	37	18	Std.AASHO	105.0	20.2	99.7	22.9	91	**		1.0- 3.0	2	.97	1	110	.43	.90		
CL	CL-17A	Lean Clay (Clinton Dam)	COE, Kansas City District (17)	-	-	-	43	24	Std.AASHO	101.0	20.1	99.1	22.7	90	**		2.0- 6.0	3	1.10	2	100	.27	.89		
CL	CL-17B	Lean Clay (Clinton Dam)	COE, Kansas City District (17)	-	-	-	43	24	Std.AASHO	101.0	20.1	98.1	23.9	90	**		2.0- 6.0	3	.99	1	160	.54	.97		
CL	CL-17C	Lean Clay (Clinton Dam)	COE, Kansas City District (17)	-	-	-	43	24	Std.AASHO	101.0	20.1	98.9	22.7	90	**		2.0- 6.0	3	1.10	3	130	.46	.91		
CL	CL-19A	Lean Clay (Clinton Dam)	COE, Kansas City District (17)	-	-	-	42	26	Std.AASHO	102.0	19.9	96.8	22.7	83	**		2.0- 6.0	3	.78	2	53	.41	.85		
CL	CL-24A	Sandy Clay (Chatfield Dam)	COE, Omaha District (19)	0.016	-	-	43	24	Std.AASHO	104.0	19.3	97.6	23.4	90	*		6.0-10.0	2	1.20	0	240	0.	.95		
CL	CL-25A	Sandy Clay (Chatfield Dam)	COE, Omaha District (19)	0.09	0.007	-	34	18	Std.AASHO	113.0	15.1	107.4	18.1	86	*		6.0-10.0	2	.95	0	160	0.	.93		
CL	CL-28	Sandy Clay (Proctor Dam)	COE, Fort Worth District (15)	0.033	0.002	-	31	20	Std.AASHO	115.0	14.6	114.8	12.2	72	**		1.5- 6.0	2	1.60	12	150	.16	.79		
CL	CL-29A	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (8)	0.037	0.008	-	34	19	Harvard	116.2	15.2	110.9	13.0	67	*		1.0-14.3	5	2.00	20	440	.17	.85		
CL	CL-29B	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (8)	0.037	0.008	-	34	19	Harvard	116.2	15.2	115.8	13.1	77	*		1.0-14.3	4	2.50	20	440	.34	.86		
CL	CL-30A	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (8)	0.037	0.008	-	34	19	Harvard	112.8	16.7	111.0	16.2	84	*		1.0- 6.3	4	1.00	16	110	.94	.91		
CL	CL-30B	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (8)	0.037	0.008	-	34	19	Harvard	112.8	16.7	112.2	16.6	88	*		1.0- 4.1	3	1.40	11	67	.71	.77		
CL	CL-30D	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (8)	0.037	0.008	-	34	19	Harvard	112.8	16.7	110.3	17.3	88	*		1.1- 4.1	3	1.00	9	37	.37	.65		
CL	CL-33B	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (7)	0.037	0.008	-	34	19	Harvard	108.8	18.0	106.3	16.2	75	**		4.1-13.5	4	2.20	3	71	1.06	.98		
CH	CH-1	Fat Clay (Clinton Dam)	COE, Kansas City District (17)	-	-	-	60	38	Std.AASHO	94.0	26.5	90.0	28.8	90	**		1.0-3.0	2	.61	4	92	.21	.89		
CH	CH-3A	Fat Clay (Monroe Dam)	COE, Louisville District (18)	0.0067	-	-	61	36	Std.AASHO	95.5	26.5	89.3	31.1	93	*		.7- 2.9	2	.37	0	21	0.	.65		
CH	CH-3B	Fat Clay (Monroe Dam)	COE, Louisville District (18)	0.0067	-	-	61	36	Std.AASHO	95.5	26.5	92.6	28.6	93	**		.7- 2.9	3	.51	1	67	.02	.79		
CH	CH-4	Fat Clay (Monroe Dam)	COE, Louisville District (18)	0.018	-	-	69	45	Std.AASHO	100.0	22.7	96.4	26.5	94	**		1.1- 2.9	2	.63	1	65	.14	.77		
CH	CH-5A	Fat Clay (Chatfield Dam)	COE, Omaha District (19)	0.0095	-	-	54	36	Std.AASHO	95.0	24.4	90.3	27.4	84	*		6.0-10.0	3	1.20	0	36	.72	.91		
CH	CH-5B	Fat Clay (Chatfield Dam)	COE, Omaha District (19)	0.0095	-	-	54	36	Std.AASHO	95.0	24.4	90.7	24.4	76	***		6.0-10.0	3	1.50	2	52	.66	.89		

STRESS-STRAIN AND STRENGTH PARAMETERS FOR SOILS TESTED UNDER DRAINED CONDITIONS

Soil	Group	Soil Description	References	Grain Size, mm			LL	PI	Compaction					Init. Void Ratio	Relative Density	Degree Saturation	Rating	Particle Shape	Stress Range (TSF)	Number of Tests	C (TSF)	Friction Angle	K	n	R_f	K_b	m
				D60	D30	D10			Type	Max. Dry Unit Wt. (PCF)	Opt. w/c	Dry U. Wt. (PCF)	w/c														
GW	GW-1	Conglomerate Rockfill (Netzahu. Dam)	Marsal et al (38)	47.	7.5	0.9						118.9		0.39	70		••	Sub-angular	1.9- 25.5	3	0.	50 (10)	540	.43	.64	135	.34
GW	GW-2	Granitic Gneiss Rockfill (Mica Dam)	Casagrande (10)/Marsal (39)	79.	24.	4.						123.7		0.32	95		••	Sub-angular	5.1- 25.6	3	0.	44 (9)	210	.51	.64	100	.34
GW	GW-3	Quartzite Rockfill (Furnas Dam Shell)	Casagrande (10)	10.	-	-											•••	Sub-rounded	4.1- 36.9	4	0.	49 (6)	560	.48	.65	330	.33
GW	GW-4	Quartzite Rockfill (Furnas Dam Transit.)	Casagrande (10)	25.													•••	Sub-rounded	4.1- 36.9	4	0.	53 (7)	950	.52	.59	470	.52
GW	GW-5	Furnas Dam Transition	Casagrande (10)	10.													•••	Sub-rounded	4.1- 36.9	4	0.	50 (7)	690	.57	.51	360	.57
GW	GW-6	Pinzandapan Gravel	Marsal et al (38)	21.	2.7	0.25						132.1		0.34	65		••	Sub-rounded	.4- 26.5	6	0.	51 (9)	690	.45	.59	170	.22
GW	GW-7	Diorite Rockfill (El Infiernillo Dam)	Marsal et al (38)	93.	42.	17.						105.7		0.56	50		••	Angular	.4- 17.0	7	0.	46 (9)	340	.28	.71	52	.18
GP	GP-2	Sandy Gravel (Mica Dam Shell)	Casagrande (10)	22.	1.2	0.23									50		••	Sub-angular	7.2- 32.5	3	0.	41 (3)	420	.50	.78	125	.46
GP	GP-3	Basalt Rockfill	Casagrande (10)/Marsal (39)	19.	3.6	1.						133.8		0.3	95		•••	Angular	5.1- 25.6	3	0.	52 (10)	450	.37	.61	255	.18
GP	GP-6	Silty Sandy Gravel (Oroville Dam)	Hall & Gordon (25)	18.	4.8	0.4	21	.3				148.0		0.21	100		•••	Rounded	9.0- 46.8	4	0.	53 (8)	1300	.40	.72	900	.22
GP	GP-7	Amphibolite Gravel (Oroville Dam Shell)	Marachi (37)	13.2	4.6	0.36						152.0		0.2	100		••	Rounded	2.2- 28.6	4	0.	51 (6)	1780	.39	.67	1300	.16
GP	GP-11	Crushed Basaltic Rock (Round Butte Dam)	Shannon & Wilson (41)	15.	12.	6.				91.6		99.0	3.2		99		••	Angular	2.0- 14.1	3	0.	51 (14)	410	.21	.71	195	0
GP	GP-13	Sandy Gravel (Rowallan Dam)	Boughton (5)	10.	3.	0.6						135.0		0.233	100		••	Rounded	1.8- 10.8	4	0.	58 (10)	2500	.21	.75	1400	0
GC	GC-1	Clayey Gravel (New Hogan Dam Core)	Bird (3)	12.	0.6	-	51	30		113.0	10.8	107.0	10.8			51	••		1.1- 4.3	3	.28	19	99	.70	.86	45	0
SW	SW-1	Argillite Rockfill (Pyramid Dam Shell)	Marachi (37)	4.1	1.8	0.6						111.6		0.46	100		•	Angular	2.2- 46.8	4	0.	53 (9)	1600	.08	.72	600	0
SW	SW-2	Crushed Olivine Basalt	Marachi (37)	4.1	1.8	0.6						125.4		0.43	100		•	Angular	2.2- 46.8	4	0.	55 (10)	1000	.22	.70	390	.14
SW	SW-3	Silty Sand, Some Gravel (Round Butte Dam)	Shannon & Wilson (41)	1.7	0.09	0.009	NP	NP	16,450	120.0	13.2	108.7	13.5				•	Sub-rounded	2.0- 14.0	3	0.	38 (3)	260	.50	.76	100	.5
SW	SW-5	Venato Sandstone (0.5 in. max. size)	Becker, Chan & Seed (2)	0.17	0.07	0.025	NP	NP		118.3		117.5		0.47	93		•	Angular	2.2- 28.6	4	0.	43 (4)	330	.46	.51	110	.46
SP	SP-3	Glacial Cutwash Sand	Hirschfeld & Poulos (26)	0.03	0.4	0.14						112.3		0.5	80		•••	Sub-rounded	1.0- 41.1	6	0.	44 (4)	190	.70	.57	190	.35
SP	SP-4A	Sacramento River Sand	Lee (34)	0.22	0.17	0.15						89.5		0.87	38		•	Rounded	1.0- 41.1	8	0.	35 (2)	430	.27	.84	230	.02
SP	SP-4B	Sacramento River Sand	Lee (34)	0.22	0.17	0.15						94.0		0.78	60		•	Rounded	1.0- 13.0	4	0.	37 (2)	410	.69	.90	260	.15
SP	SP-4C	Sacramento River Sand	Lee (34)	0.22	0.17	0.15						97.8		0.71	78		•	Rounded	1.0- 41.1	8	0.	41 (5)	1100	.36	.85	900	0
SP	SP-4D	Sacramento River Sand	Lee (34)	0.22	0.17	0.15						103.9		0.61	100		•	Rounded	1.0- 41.1	6	0.	45 (7)	1200	.48	.85	1500	0
SP	SP-5A	Ham River Sand	Bishop (4)	0.25	0.17	0.15								0.82	Loose		••	Rounded	7.2-287.9	4	0.	31 (2)	890	.26	.78	360	.11
SP	SP-5B	Ham River Sand	Bishop (4)	0.25	0.17	0.15								0.64	Dense		••	Rounded	7.2- 71.3	3	0.	47 (9)	1100	.57	.86	2250	0
SP	SP-7A	Poorly Graded Sand (Port Allen Lock)	Sherman & Trahan (44)	0.2	0.17	0.12	NP	NP		100.0	13.0	95.5		0.73	49		••	Rounded	.9- 3.9	3	0.	39 (0)	410	.65	.84		
SP	SP-7B	Poorly Graded Sand (Port Allen Lock)	Sherman & Trahan (44)	0.2	0.17	0.12	NP	NP		100.0	13.0	100.0		0.65	73		••	Rounded	.9- 3.9	3	0.	40 (1)	400	.49	.77		
SP	SP-7C	Poorly Graded Sand (Port Allen Lock)	Sherman & Trahan (44)	0.2	0.17	0.12	NP	NP		100.0	13.0	105.1		0.57	98		••	Rounded	.9- 3.9	3	0.	44 (3)	750	.77	.83		
SP	SP-12	Coarse to Fine Sand (Round Butte Dam)	Shannon & Wilson (41)	minus No. 4 sieve			NP	NP				74.8		1.22	70		••	Angular	2.0- 14.0	3	0.	39 (6)	280	.37	.71	95	.21
SP	SP-13	Pumicecus Sand (Round Butte Dam)	Shannon & Wilson (41)	0.85	0.41	0.24				87.4		84.2	18.0		77		•	Angular	2.0- 14.1	3	0.	48 (10)	340	.45	.70	230	.06
SP	SP-14	Pumicecus Sand (Round Butte Dam)	Shannon & Wilson (43)	1.0	0.5	0.24				80.7		76.9	25.0		71		•	Angular	2.0- 14.1	3	0.	49 (12)	650	.38	.77	380	.05
SP	SP-16A	Fine Silica Sand (Loose)	Duncan & Chang (22)	0.27	0.2	0.165								0.65	38		•••	Rounded	1.0- 5.1	3	0.	30 (0)	280	.65	.93	110	.65
SP	SP-16B	Fine Silica Sand (Dense)	Duncan & Chang (22)	0.27	0.2	0.165								0.54	100		•••	Rounded	1.0- 5.1	3	0.	37 (0)	1400	.74	.90	1080	.15
SP	SP-17A	Monterey No. 0 Sand (Cylind. specimen)	Lade (33)	0.43	0.37	0.29	NP	NP						0.78	27		•••	Rounded	.3- 1.2	3	0.	35 (0)	920	.79	.96	465	.32
SP	SP-17B	Monterey No. 0 Sand (Cubical specimen)	Lade (33)	0.43	0.37	0.29	NP	NP						0.78	27		••	Rounded	.3- 1.2	3	0.	39 (0)	510	.51	.97	370	.22
SP	SP-17C	Monterey No. 0 Sand (Cylind. specimen)	Lade (33)	0.43	0.37	0.29	NP	NP						0.57	98		•••	Rounded	.3- 1.2	3	0.	45 (3)	3200	.78	.92	1400	.45
SP	SP-17D	Monterey No. 0 Sand (Cubical specimen)	Lade (33)	0.43	0.37	0.29	NP	NP						0.57	98		••	Rounded	.3- 1.2	3	0.	47 (5)	1500	.76	.91	1100	.52
SP	SP-18	Basaltic Sand (Round Butte Dam)	Shannon & Wilson (42)	3.	9.	0.13				120.1	9.5	120.0	9.5				••	Angular	2.0- 14.0	3	0.	39 (13)	1600	.08	.63	750	0
SM	SM-4	Silty Sand (Chatfield Dam)	COE, Omaha District (19)	0.62	0.16	0.026	20	0	Std.AASHO	123.0	9.5	116.7	9.4				••	Sub-rounded	6.0- 10.	3	0.	37 (0)	100	1.07	.62		
SM	SM-5	Silty Gravelly Sand (Chatfield Dam)	COE, Omaha District (19)	1.15	0.28	0.05	NP	NP	Std.AASHO	132.0	8.1	124.5	7.53				•	Sub-rounded	6.0- 10.0	3	0.	41 (0)	530	.51	.62	640	0
SM	SM-6	Silty Sand w/Pebbles (Round Butte Dam)	Shannon & Wilson (41)	0.31	0.1	0.04	NP	NP	16,450	110.6	17.5	108.1	17.5				•••	Angular	2.0- 14.0	3	0.	46 (8)	700	.35	.75		
SM	SM-9	Silty Sand w/Pumice (Round Butte Dam)	Shannon & Wilson (41)	0.15	0.054	0.013	NP	NP	16,450	91.7	19.5	88.4	19.0				••	Angular	2.0- 13.7	3	0.	43 (8)	670	.25	.72	500	0
SM	SM-13	Silty Sand (Round Butte Dam)	Shannon & Wilson (43)	0.27	0.027	0.0022				105.6	16.4	104.5	15.				•	Sub-angular	2.0- 14.1	3	0.	36 (5)	530	.28	.74	470	0
SM	SM-16	Silty Sand & Gravel (Round Butte Dam)	Shannon & Wilson (42)	0.45	0.052	0.012				109.3	12.9	109.	12.				••	Sub-angular	2.0- 14.0	3	0.	36 (11)	800	.20	.67	600	0
SM-SC	SM-SC-1A	Silty Clayey Sand (Mica Dam Core)	Casagrande (10)/Insley & Hillis (27)	0.34	0.03	0.002	21	4	Std.AASHO	136.0	9.8	131.1	7.7				••		3.6- 32.4	6	.31	33	700	.37	.80	280	.19
SM-SC	SM-SC-1B	Silty Clayey Sand (Mica Dam Core)	Casagrande (10)/Insley & Hillis (27)	0.34	0.03	0.002	21	4	Std.AASHO	136.0	9.8	134.0	9.7				•••		3.6- 18.0	4	.85	34	425	.58	.70	205	.44
SM-SC	SM-SC-1C	Silty Clayey Sand (Mica Dam Core)	Casagrande (10)/Insley & Hillis (27)	0.34	0.03	0.0002	21	4	Std.AASHO	136.0	9.8	128.2	11.9				•••		3.6- 32.4	6	.40	34	160	.81	.63	65	.81
ML	ML-1	Cannonsville Silt (Undisturbed)	Hirschfeld & Poulos (26)	0.033	0.018	0.005						108.0		0.57			•		1.5- 7.4	4	0.	45 (6)	200	1.07	.57	200	.89
ML	ML-4	Sandy Silty w/Pumice (Round Butte Dam)	Shannon & Wilson (41)	0.078	0.032	0.0064	NP	NP	16,450	97.0	19.0	92.8	17.7				•		2.0- 13.9	2	0.	42 (7)	500	.45	.82	400	0
ML	ML-5	Sandy Silty w/Pumice (Round Butte Dam)	Shannon & Wilson (41)	0.1	0.025	0.0052	NP	NP	16,450	102.5	16.5	99.2	17.0				••		2.0- 13.9	3	0.	36 (1)	530	.35	.71	520	.23
CL	CL-29C	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (8)	0.037	0.008	-	34	19	Harvard	116.2	15.2	111.2	13.1			69	••		1.0- 8.2	4	.17	30	550	-.05	.82		
CL	CL-29D	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (8)	0.037	0.008	-	34	19	Harvard	116.2	15.2	116.2	13.3			79	••		1.0- 8.2	4	.59	29	690	.10	.71		
CL	CL-30E	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (8)	0.037	0.008	-	34	19	Harvard	112.8	16.7	115.1	15.2			88	••		1.0- 8.2	4	.51	33	150	.62	.61	360	0
CL	CL-30F	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (8)	0.037	0.008	-	34	19	Harvard	112.8	16.7	110.0	17.4			88	••		1.0- 4.0	4	.39	30	160	.50	.63	210	0
CL	CL-34E	Silty Clay (Canyon Dam)	Casagrande & Hirschfeld (7)	0.037	0.008	-	34	19	Harvard	105.6	19.8	106.3	19.0			87	•		.5- 8.0	5	.26	31	130	.59	.72	45	.59

APPENDIX B

FINITE ELEMENT MATRICES

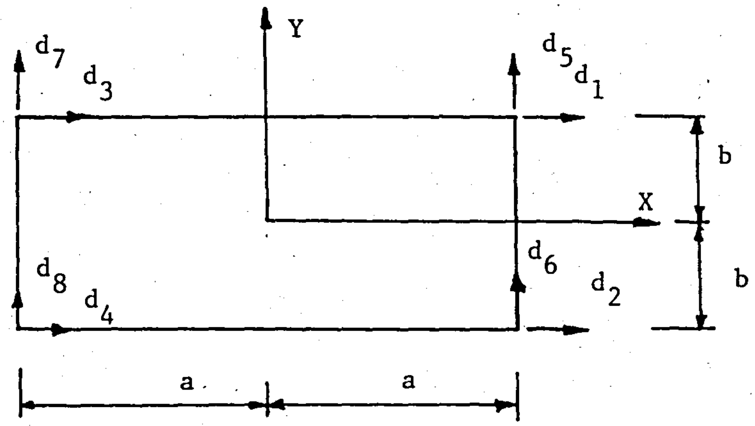
FOR

RECTANGULAR ELEMENT

RECTANGULAR ELEMENTS IN PLANE STRESS

In this study, rectangular elements are utilized to make stress and strain analysis of various structures, including any homogeneous media and reinforced earth systems. In this section, therefore, Finite Element matrices for rectangular elements are given in detail.

The sign convention and dimensions are as follows :



Material Properties Matrix :

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Shape Function

$$[N] = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_1 & N_2 & N_3 & N_4 \end{bmatrix}$$

$$[G] = \frac{1}{4ab} \begin{bmatrix} b(1+\eta) & b(1-\eta) & -b(1+\eta) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a(1+\rho) & -a(1+\rho) & a(1-\rho) & -a(1-\rho) & -a(1-\rho) \\ a(1+\rho) & -a(1+\rho) & -a(1-\rho) & b(1+\eta) & b(1-\eta) & -b(1+\eta) & -b(1-\eta) & -b(1-\eta) \end{bmatrix}$$

stiffness Matrix $[K] = \int_v [6]^T [D][G] \, dv$

$$[K] = \begin{array}{|c|c|c|c|c|c|c|c|} \hline K_{11} & & & & & & & \\ \hline K_{21} & K_{11} & & & & & & \\ \hline K_{31} & K_{41} & K_{11} & & & & & \\ \hline K_{41} & K_{31} & K_{21} & K_{11} & & & & \\ \hline K_{51} & -K_{61} & -K_{71} & K_{81} & K_{55} & & & \\ \hline K_{61} & -K_{51} & -K_{81} & K_{71} & K_{65} & K_{55} & & \\ \hline K_{71} & -K_{81} & -K_{51} & K_{61} & K_{75} & K_{85} & K_{55} & \\ \hline K_{81} & -K_{71} & K_{71} & K_{51} & K_{85} & K_{75} & K_{65} & K_{55} \\ \hline \end{array}$$

where

$$K_{11} = \frac{t}{3} \left(D_{11} \frac{b}{a} + D_{33} \frac{a}{b} \right)$$

$$K_{55} = \frac{t}{3} \left(D_{22} \frac{a}{b} + D_{33} \frac{b}{a}\right)$$

$$K_{21} = \frac{t}{3} \left(D_{11} \frac{b}{2a} - D_{33} \frac{a}{b}\right)$$

$$K_{31} = \frac{t}{3} \left(- \frac{D_{11}b}{a} + \frac{D_{33}a}{2b}\right)$$

$$K_{41} = - \frac{t}{2} \left(\frac{D_{11}b}{a} + \frac{a\, D_{33}}{b} \right)$$

$$K_{51} = \frac{t}{4} (D_{12} + D_{33})$$

$$K_{61} = \frac{t}{4} (-D_{12} + D_{33})$$

$$K_{71} = \frac{t}{4} (D_{12} - D_{33})$$

$$K_{81} = - \frac{t}{4} (D_{12} - D_{33})$$

$$K_{65} = \frac{t}{3} \left(- \frac{D_{22}a}{b} + \frac{D_{33}b}{2a}\right)$$

$$K_{75} = \frac{t}{3} \left(\frac{D_{22}a}{2b} - \frac{D_{33}b}{a} \right)$$

$$K_{85} = \frac{t}{2} \left(-D_{11} \frac{a}{b} - D_{33} \frac{b}{a} \right)$$

INITIAL STRAINS $\{f\}_{\varepsilon_o} = -\frac{1}{2}\left(\int_V [G]^T[D]dv\right)\{\varepsilon_o\}$

$$\{f\}_{\varepsilon_o} = -\frac{t}{2}\begin{bmatrix} D_{11}b\,\varepsilon_{x_o} + D_{12}b\varepsilon_{y_o} + D_{33}a\,\gamma_{xy_o} \\ D_{11}b\varepsilon_{x_o} + D_{12}b\varepsilon_{y_o} - D_{33}a\gamma xy_o \\ -D_{11}b\varepsilon_{x_o} - D_{12}b\varepsilon_{y_0} + D_{33}a\gamma xy_o \\ -D_{11}b\varepsilon_{x_o} - D_{12}b\varepsilon_{y_o} - D_{33}a\gamma xy_o \\ D_{12}a\varepsilon_{x_o} + D_{22}a\varepsilon_{y} + D_{33}b\gamma xy_o \\ -D_{12}a\varepsilon_{x_o} - D_{22}a\varepsilon_{y_o} + D_{33}b\gamma xy_o \\ D_{12}a\varepsilon_{x_o} + D_{22}a\ y_o - D_{33}b\gamma xy_o \\ -D_{12}a\ y_o - D_{22}a\ y_o - D_{33}b\gamma xy_o \end{bmatrix}$$

INITIAL STRESSES $\{f\}\sigma_o = \frac{1}{2}\left(\int [G]^T dv\right)\{\sigma_o\}$

$$\{f\}_{\sigma_o} = \frac{1}{2}\, t \begin{bmatrix} b\sigma x_o + a\tau xy_o \\ b\sigma x - a\tau xy \\ -b\sigma_x + a\tau xy \\ -b\sigma_x - a\tau xy \\ a\sigma_y + b\tau xy \\ -a\sigma_y + b\tau xy \\ a\sigma_y - b\tau xy \\ -a\sigma_y - b\tau xy \end{bmatrix}$$

Body Forces $\{f\}_X = - \int_V [N]^T \{x\} dv$

$$\{f\}_X = - \frac{w}{4} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

A P P E N D I X   C

COMPUTER PROGRAM , SLOPER

USER'S MANUAL FOR PROGRAM, SLOPER
INPUT DATA INFORMATION*

1. TITLE

TITLE - Title card for program identification

2. Definition of Slope

NSPTS, NCIRC, - NSPTS Number of slope

NCIRC Number of Circle

3. Slope Point Coordinates

B(N), C(N), B(N) - X - Coordinate of slope points

C(N) - Y - Coordinate of slope points

repeat this card as many as NSPTS

4. Definition of different material zones

GY, GAMMAD, GAMMAF - GY - Elavation of boundary between material zones

* All data are in free format

GAMMAD - Unit Weight in Upper Zone

GAMMAF - Unit Weight in lower zone

5. Properties of the soil for upper zone

CO, PHIDAM - CD - Cohesion intercept

PHIDAM - Friction angle for upper zone

6. Properties of the soil for lower zone

CF, CFGAMH, PHIFND - CF - Cohesion intercept

CFGAMH - Rate of increase of cohesion intercept with depth

PHIFND - Friction angle for lower zone

7. Circle Data

KCIRCL, Z1, Z2, CRU, KCIRCL = 1, Radius of circle Z1

KCIRCL = 2, Circle passes through x = Z1, Y = Z2

KCIRCL = 3, circle is tangent to Y = Z1

8. Reinforcement

IRF - IRF = 1, reinforcement exists

IRF ≠ 1, no-reinforcement

9. Properties of the reinforcement

This card is required when IRF = 1

OPT, OPT = 1, Only tensile force is allowed

OPT = 2, Tensile and Shearing Force are allowed

FMAX, PMAX, LO - FMAX = Limit Skin friction

P_{max} = Maximum passive pressure

L_o = Transfer length

NR, SH, RD, RN - NR = Number of reinforcement

SH = Horizontal spacing of reinforcement

RD = Diameter of Reinforcement

RN = Rupture strength of reinforcement

RNX(I,J), RNY(I,J) , I = Reinforcement Number

J = 1, first end

J = 2, other end

RNX(I,J) - x - Coordinates of bar ends

KNY(I,J) - Y - Coordinates of bar ends

```
      PROGRAM SLOPER (INPUT,SOUT,OUT,TAPE2=INPUT,TAPE6=SOUT)
C**********************************************************************
      COMMON /GENERA/  A(10),B(10),C(10),Y(100),YS(100),RADCON
      COMMON /GENERB/  NSPTS,XC,YC,R,ARCLEN,GY,NS,CRU
      COMMON /MBSG/ SFORCE(100),BETA(100),ALPHA(100),WT(100)
      COMMON /MBS/  NSP1,FS,ITER,ACC,ITERL
      COMMON /MGB/ X(100),NEGEQ,S(100)
      COMMON /DAM/ CD,CDGAMF,PHIDAM,PHIDMC
      COMMON /FND/ CF,CFGAMF,PHIFND,PHIFDC
      COMMON/BB/RNX(20,2),RNY(20,2),RD,FMAX,NR,RN,SH,PMAX,LC,CPT

      CHARACTER TITLE*52,IDP*1

      DIMENSION AREA(100),TOPSLP(100)

      INTEGER CPT
      REAL LO

      RADCON=ACOS(-1.)/180.0
C**********************************************************************
C     READ  TITLE
C**********************************************************************
      ICN=0
    1 READ(2,1040,END=9999) TITLE
C**********************************************************************
C     READ  NUMBER OF SLOPE POINTS, NUMBER OF CIRCLES
C**********************************************************************
      READ(2,*) NSPTS,NCIRC
      A(NSPTS)=0.0
C**********************************************************************
C     READ  SLOPE POINT COORDINATES
C**********************************************************************
      N2=2*NSPTS
      READ(2,*) (B(N),C(N),N=1,NSPTS)
      DO 10 I=1,NSPTS
      C1=B(I)-B(I-1)
      IF(C1.EQ.0.) THEN
        A(I-1)=9999.
      ELSE
        A(I-1)=(C(I)-C(I-1))/C1
      ENDIF
   10 CONTINUE
C**********************************************************************
C     READ  ELEVATION OF BOUNDARY BETWEEN MATERIAL ZONES,
C     UNIT WEIGHT IN UPPER ZONE, UNIT WEIGHT IN LOWER ZONE
C**********************************************************************
      READ(2,*) GY,GAMMAD,GAMMAF
C**********************************************************************
C     READ  COHESION INTERCEPT, FRICTION ANGLE FOR UPPER ZONE
C**********************************************************************
      READ(2,*) CD,PHIDAM
      PHIDMO=PHIDAM
      PHIDAM=PHIDAM*RADCON
C**********************************************************************
C     READ  COHESION INTERCEPT, RATE OF INCREASE OF COHESION
C     INTERCEPT WITH DEPTH, FRICTION ANGLE FOR LOWER ZONE
C**********************************************************************
      READ(2,*) CF,CFGAMF,PHIFND
      PHIFDO=PHIFND
      PHIFND=PHIFND*RADCON
C**********************************************************************
C     READ  CIRCLE DATA AND PORE PRESSUR COEFFCIENT
C**********************************************************************
      READ(2,*) KCIRCL,Z1,Z2,CRL
      READ(2,*) IDP
```

```
      OUTPUT

      WRITE(6,100) TITLE
      WRITE(6,113)
      WRITE(6,111)
      WRITE(6,113)
      DO 27 IK=1,NSPTS
      WRITE(6,122) IK,B(IK),C(IK)
   27 CONTINUE
      WRITE(6,113)
      WRITE(6,114)
      WRITE(6,131)
      WRITE(6,150)
      WRITE(6,131)
      IL=1
      C1=C.
      WRITE(6,160) IL,GAMMAC,PHIDMO,CC,C1
      IL=2
      WRITE(6,160) IL,GAMMAF,PHIFCG,CF,CFGAMH
      WRITE(6,131)
      WRITE(6,114)
      WRITE(6,170)
      WRITE(6,180)
      WRITE(6,170)
      WRITE(6,190) GY,GY
      WRITE(6,170)
      WRITE(6,114)
      WRITE(6,201)
      WRITE(6,210) Z1,Z2,KCIRCL
      WRITE(6,114)
      *******************************
      REINFORCEMENT
      *******************************
      READ(2,*) IRF
      IF(IRF.EQ.1) THEN
      READ(2,*) OPT
      READ(2,*) FMAX,PMAX,LO
      READ(2,*) NR,SH,RD,RN
      DO 7 IJ=1,NR
      READ(2,*) (RNX(IJ,J),RNY(IJ,J),J=1,2)
    7 CONTINUE
      RC=RN*0.5

      OUTPUT

      WRITE(6,114)
      WRITE(6,250)
      WRITE(6,260)
      WRITE(6,250)
      DO 9 IK=1,NR
      XX=ABS(RNX(IK,2)-RNX(IK,1))
      YY=ABS(RNY(IK,2)-RNY(IK,1))
      TL=SQRT(XX*XX+YY*YY)
      WRITE(6,270) IK,(RNX(IK,J),RNY(IK,J),J=1,2),TL
    9 CONTINUE
      WRITE(6,250)
      WRITE(6,114)
      WRITE(6,280) RD,RN,RC,FMAX,SH
      WRITE(6,114)
      WRITE(6,281) LO,PMAX
      WRITE(6,114)
      WRITE(6,282) OPT
      ENDIF

      DO 2 NNTS=1,NCIRC
```

```
      INITIALIZE VARIABLES

      DO 3 INIT=1,100
      X(INIT) =0.
      Y(INIT) = 0.
      WT(INIT) =0.
      AREA(INIT) = 0.
      SFORCE(INIT) = 0.
      S(INIT) = 0.
      TOPSLP(INIT) = 0.
      BETA(INIT) = 0.
      YS(INIT) = 0.
      ALPHA(INIT) = 0.
    3 CONTINUE
      PHIINT=0.0
      XC=0.
      YC=0.
      R=0.
      KILLG=0

      READ X AND Y COORDINATES OF CIRCLE CENTER

      READ(2,*) XC,YC

      CALCULATE RADIUS AND PRINT CENTER COORDINATES AND RADIUS

      IF(KCIRCL.EQ.1) R=Z1
      IF(KCIRCL.EQ.2) R=SQRT((XC-Z1)**2+(YC-Z2)**2) -0.001
      IF(KCIRCL.EQ.3) R=YC-Z1
      IF(R.EQ.0.0) STOP1

      CALL GENER TO CALCULATE WHERE CIRCLE INTERSECTS SLOPE,
      AND COORDINATES OF SLICE BOUNDARIES ALONG CIRCLE

      CALL GENER(KILLG)
      IF(KILLG.EQ.0) GO TO 20
      GO TO 2
   20 CONTINUE

      CALCULATE SLICE DIMENSIONS AND WEIGHTS,AND STRENGTH FOR EACH SLICE

      NSP1 = NS + 1
      DO 99   J = 1, NSP1
      JM1 = J - 1
      IF(J .EQ. 1) GO TO 33
      DELTAX = X(J) - X(JM1)
      DELTAY = Y(J) - Y(JM1)
   35 ALPHA(J) = ATAN(-1. * (DELTAY/DELTAX ))
   33 DO 40 K=1,NSPTS
      IF (X(J) - B(K)) 41,41,40
   41 KM1 = K - 1
      YS(J)=A(KM1)*(X(J)-B(KM1))+C(KM1)
      TOPSLP(J)=ATAN(A(KM1))
      GO TO 43
   40 CONTINUE
      YS(J) = GY
      KM1=NSPTS
   43 CONTINUE
      IF(J .EQ. 1) GO TO 99
      BASE = X(J) - X(JM1)
      CHORD=BASE/COS(ALPHA(J))
      YA=(Y(J-1)+Y(J))/2.0
      YSA=(YS(J-1)+YS(J))/2.0
      HTD=AMIN1((YSA-YA),(YSA-GY))
      HTF=AMAX1((GY-YA),0.)
```

```
      AREA(J)=BASE*(HTB+HTF)
      IF(HTF .GT. 0.) GO TO 75
   70 CINTER=CD
      PHIINT=PHIDAM
      GO TO 80
   75 CINTER=CF+HTF*CFGAMF
      PHIINT=PHIFND
   80 CONTINUE
   83 S(J)=CINTER
      SFORCE(J)=PHIINT
   99 CONTINUE
      IF(IDP.EQ.'Y') THEN
      WRITE(6,114)
      WRITE(6,291)
      WRITE(6,292)
      WRITE(6,291)
      DO 17 IJ=2,NSP1
      BB=X(IJ)-X(IJ-1)
      ALP=ALPHA(IJ)*180./ACOS(-1.)
      SF=SFORCE(IJ)*180./ACOS(-1.)
      WRITE(6,293) (IJ-1),X(IJ-1),X(IJ),BB,WT(IJ),ALP,S(IJ),SF
   17 CONTINUE
      WRITE(6,291)
      ENDIF
      ****************************************************************
      CALCULATE AND PRINT ORDINARY METHOD OF SLICES FACTOR OF SAFETY
      ****************************************************************
      CALL OMS(KILOM)
      FSO=FS
      ****************************************************************
      CALCULATE AND PRINT BISHOPS MODIFIED METHOD FACTOR OF SAFETY
      ****************************************************************
      CALL BISHOP (KILLB,IRF,IDP)
      IF(KILLB.EQ.0) GO TO 200
      GO TO 600
  200 ICN=ICN+1
      WRITE(6,114)
      WRITE(6,220)
      WRITE(6,230)
      WRITE(6,220)
      WRITE(6,240) ICN,XC,YC,R,FS
      WRITE(6,220)
      WRITE(6,221) FSO

      DETERMINE AND PRINT TIME REQUIRED FOR CIRCLE

  600 CONTINUE
      CALL SECOND(TIME2)
      TIME=TIME2-TIME1
    2 CONTINUE
      WRITE(6,222) TIME
      IF(RADCON.GT.0.0) GO TO 1
  999 STOP
      ****************************************************************
                        INPUT FORMAT STATEMENTS
      ****************************************************************
   00 FORMAT(2I10)
   10 FORMAT(2F10.0)
   15 FORMAT(3F10.0)
   20 FORMAT(3F10.0,I10)
   30 FORMAT(I10)
   40 FORMAT (A52)
   50 FORMAT(3I10,2F10.0)
   60 FORMAT(5F10.0)
   70 FORMAT(I10)
   80 FORMAT(5F10.0)
      ****************************************************************
                        OUTPUT FORMAT   STATEMENTS
      ****************************************************************
  100 FORMAT(1H1,/////,15X,46('-'),/,15X,':  SLOPE STABILITY ANALYSIS DA
     *TA INPUT ECHO  :',/15X,46('-'),/,15X,72('-'),/15X,':  PROJECT TI
     *TLE : ',A52,':',/15X,72(
  113 FORMAT(15X,45('-'))
```

```
      ****************************************************************
  100 FORMAT(1H1,/////,15X,46('-'),/,15X,':  SLOPE STABILITY ANALYSIS DA
     *TA INPUT ECHO  :',/15X,46('-'),///,15X,72('-'),/15X,':  PROJECT TI
     *TLE : ',A52,':',/15X,72('-'),///)
  113 FORMAT(15X,45('-'))
  114 FORMAT(////)
  111 FORMAT(15X,':',' POINT  :  X-COORDINATE  :  Y-COORDINATE  :')
  122 FORMAT(15X,':',4X,I2,3X,':',2(F12.2,4X,':'))
  131 FORMAT(15X,63('-'))
  150 FORMAT(15X,':  SOIL  :  UNIT WEIGHT  :  PHI  :  COHESION  :  INCR.
     *OF.COH  :')
  160 FORMAT(15X,':',I5,3X,':',F10.1,5X,':',F4.1,'   :',F7.1,5X,':',F8.2
     *,7X,':')
  170 FORMAT(15X,36('-'))
  180 FORMAT(15X,':  SOIL  :   FROM   :    UPTO    :')
  190 FORMAT(15X,':    1   :  BOTTOM  :  Y=',F6.1,'  :',/15X,':    2
     * :  Y=',F5.1,'  :  TOP      :')
  201 FORMAT(///,15X,50('-'),/15X,':         TYPE OF CIRCLE',28X,':',/,15X,
     *50('-'),/15X,':  1- RADIUS OF CIRCLE IS  Z1',20X,':',/15X,':  2- C
     *IRCLE PASSES THROUGH   X=Z1, Y=Z2',9X,':',/15X,':  3- CIRCLE IS TA
     *NGENT TO    Y=Z1',15X,':',/15X,50('-'))
  210 FORMAT(/30X,'Z1=',F8.2,/30X,'Z2=',F8.2,//15X,'OPTION  ====>> ',I1)
  220 FORMAT(15X,83('-'))
  230 FORMAT(15X,':  CIRCLE  :  X-COORDINATE  :  Y-COORDINATE  :    RADI
     *US    :  FACTOR  OF SAFETY  :')
  240 FORMAT(15X,':',5X,I2,3X,':',2(F10.2,6X,':'),F10.2,4X,
     *':    ====>>>  ',F4.2,3X,'  :')
  221 FORMAT(////,15X,'FACTOR OF SAFETY OBTAINED BY ORDINARY SLICE METHO
     *D  ====>>> ',F8.2)
  250 FORMAT(15X,87('-'))
  260 FORMAT(15X,':       :      F I R S T   E N D              :    S E C O
     *N D   E N D          :   TOTAL  :',/,15X,':',6X,69('-'),10X,':'
     *,/,15X,':  NO  :  X-COORDINATE  :  Y-COORDINATE  :  X-COORDINATE
     *:  Y-COORDINATE  :  LENGTH  :')
  270 FORMAT(15X,':  ',I2,'  :',4(F10.2,'      :'),F7.2,'   :')
  280 FORMAT(15X,34('-'),/15X,':     R E I N F O R C E M E N T   :',/15X
     *,34('-'),/15X,':  DIAMETER            :',F8.2,'  :',/15X,':  TENSIL
     *E STRENGTH   :',F8.2,'  :',/15X,':  SHEAR STRENGTH    :',F8.2,'
     *:',/15X,':  COEF.OF FRICTION   :',F8.2,'  :',
     */15X,':  HORIZONTAL SPACING :',F8.2,'  :',/15X,34('-'))
  281 FORMAT(15X,35('-'),/15X,':              S   O   I   L         :',/15X
     *,35('-'),/15X,':  EFFECTIVE LENGTH       :',F7.2,'  :',/15X,':  MAX.
     *PASSIFF PRESSURE :',F7.2,'  :',/15X,35('-'))
  282 FORMAT(15X,38('-'),/15X,':           A S S U M P T I O N       :',/
     *15X,38('-'),/15X,':   1  ONLY TENSILE STRENGTH         :',/15X,
     *':   2  TENSILE & SHEARING STRENGTH   :',/15X,38('-'),//15X ,
     *'OPTION  --->> ',I3)
  291 FORMAT(15X,100('-'))
  292 FORMAT(15X,': SLICE :  X-BEGIN   :   X-END   :    WIDTH    :    WEI
     *GHT   :    ALPHA   :  COHESION  :     PHI     :')
  293 FORMAT(15X,':',I4,'    :',7(F9.2,'   :'))
  222 FORMAT(/////,15X,'TIME  REQUIRED  =====>>> ',F8.2,' SECS',/,1H1)
      END
      SUBROUTINE GENER(KILLG)
      ****************************************************************
      COMMON /GENERA/  A(10),B(10),C(10),Y(100),YS(100),RADCON
      COMMON /GENERB/  NSPTS,XC,YC,R,ARCLEN,GY,NS,CRU
      COMMON /MBSG/ SFORCE(100),BETA(100),ALPHA(100),WT(100)
      COMMON /MBS/  NSP1,FS,ITER,ACC,ITERL
      COMMON /MGB/ X(100),NEGEG,S(100)
      COMMON /DAM/ CD,CDGAMH,PHIDAM,PHIDMC
      COMMON /FND/ CF,CFGAMH,PHIFND,PHIFDC
```

```
*************************************************************************
DETERMINE WHERE CIRCLE INTERSECTS SLOPE
*************************************************************************
XMIN=0.
XMAX=0.
IMIN=0
IMAX=0
DO 100 I=1,NSPTS-1
SM=A(I)
SN=C(I)-B(I)*SM
IF(SM.NE.0.) THEN
 AA=1+SM*SM
 BC=2*SM*SN-2*XC-2*YC*SM
 CC=SN*SN-2*YC*SN-R*R+XC*XC+YC*YC
 X1=(-BC-SQRT(BC*BC-4.*AA*CC))/(AA+AA)
 Y1=SM*X1+SN
 X2=(-BC+SQRT(BC*BC-4.*AA*CC))/(AA+AA)
 Y2=SM*X2+SN
ELSE
 IF(SM.EQ.0.) THEN
   Y1=C(I)
   X1=XC-SQRT(R*R-(Y1-YC)**2)
   Y2=C(I)
   X2=XC+SQRT(R*R-(Y2-YC)**2)
 ENDIF
 IF(SM.EQ.9999.)THEN
   X1=B(I)
   Y1=YC-SQRT(R*R-(X1-XC)**2)
   X2=B(I)
   Y2=YC+SQRT(R*R-(X2-XC)**2)
 ENDIF
ENDIF
PRINT*,'X1=',X1,'   Y1=',Y1
IF(X1.GE.B(I).AND.X1.LT.B(I+1).OR.X2.GE.B(I).AND.X2.LT.B(I+1))THEN
  IF(X1.GE.B(I).AND.X1.LT.B(I+1)) THEN
    X0=X1
    Y0=Y1
  ELSE
    X0=X2
    Y0=Y2
  ENDIF
  PRINT*,'X0,Y0',X0,Y0
  IF(XMIN.EQ.0.) THEN
    XMIN=X0
    YMIN=Y0
    IMIN=I
  ELSE
    XMAX=X0
    YMAX=Y0
    IMAX=I
  ENDIF
PRINT*,'JHSDGFSADHJFK=',XMIN,XMAX,YMIN,YMAX
ENDIF
IF(XMAX.NE.0.AND.XMIN.NE.0.) GOTO 201
00 CONTINUE
PRINT*,'XMAX',XMIN,XMAX,YMIN,YMAX
*************************************************************************
RETURN IF CIRCLE DOES NOT INTERSECT SLOPE
*************************************************************************
00 IF(XMIN.EQ.0..AND.XMAX.EQ.0.) GOTO 300
*************************************************************************
RETURN IF CENTER IS BELOW LOWER INTERSECTION POINT
*************************************************************************
IF(YMAX .GT. YC) GO TO 400
*************************************************************************
CALCULATE SLICE BOUNDARY COORDINATES
*************************************************************************
01 X(1)=XMIN
Y(1)=YC-SQRT(ABS(R**2-(XC-X(1))**2))
IF(YMIN .GT. YC) Y(1)=YC
IF(YMIN .GT. YC) X(1)=XC-R
BETA(1)=CRU
THETA(1)=1.5707963
IF((YC-Y(1)) .GT. 0.000001) THETA(1)=ATAN((XC-X(1))/(YC-Y(1)))
```

```
C     ********************************************************************
C     CALCULATE SLICE BOUNDARY COCRDINATES
C     ********************************************************************
201   X(1)=XMIN
      Y(1)=YC-SQRT(ABS(R**2-(XC-X(1))**2))
      IF(YMIN .GT. YC) Y(1)=YC
      IF(YMIN .GT. YC) X(1)=XC-R
      BETA(1)=CRU
      THETA(1)=1.5707963
      IF((YC-Y(1)) .GT. 0.0000001) THETA(1)=ATAN((XC-X(1))/(YC-Y(1)))
      CHORD=SQRT ((XMIN-XMAX)**2+(YMIN-YMAX)**2)
      AA=0.5*(CHORD/R)
      CENANG=(ASIN(AA))/15.0
210   CONTINUE
      K=1
      N=1MIN
220   K=K+1
      BETA(K)=CRU
      IF(K .GE. 100) GO TC 235
      KM1=K-1
      THETA(K)=THETA(KM1)-CENANG
      IF(THETA(K) .LT. 0.0 .AND. THETA(KM1) .GT. 0.0) THETA(K)=0.0
      X(K)=XC-R*SIN(THETA(K))
      IF(X(K).GT.B(N+1).AND.X(KM1).LT.B(N+1)) X(K)=B(N+1)
      IF(X(K) .GT. XMAX) X(K)=XMAX
      Y(K)=YC-SQRT(ABS(R**2-(XC-X(K))**2))
      THETA(K)=ATAN((XC-X(K))/(YC-Y(K)))
      IF(Y(K-1) .LT. GY .AND. Y(K) .GT. GY ) GO TC 225
      IF(Y(K-1) .GT. GY .AND. Y(K) .LT. GY) GC TC 226
      GO TO 230
225   Y(K)=GY
      X(K)=XC + SQRT(R**2-(YC-Y(K))**2)
      GO TO 227
226   Y(K)=GY
      X(K)=XC - SQRT(R**2-(YC-Y(K))**2)
227   THETA(K)=ATAN((XC-X(K))/(YC-Y(K)))
230   CONTINUE
      ABSTA=ABS(THETA(K))
      IF(ABSTA.LT.0.00001) THETA(K)=0.0
      IF(X(K).EQ.B(N+1)) N=N+1
      IF(X(K) .NE. XMAX) GO TC 220
      GO TO 240
235   CENANG=2.0*CENANG
      GO TO 210
240   NS=K-1
      GC TO 310
300   WRITE(6,301)
      KILLG=1
310   CONTINUE
      RETURN
400   KILLG=1
      RETURN
301   FORMAT(///,15X,40('*'),/,15X,
     *'*    CIRCLE DOES NOT INTERSECT SLCPE    *',/15X,40('*'))
      END

      SUBROUTINE CMS(KILCMS,RFORCE)

      COMMON /GENERA/  A(10),B(10),C(10),Y(100),YS(100),RADCON
      COMMON /GENERB/  NSPTS,XC,YC,R,ARCLEN,GY,NS,CRU
      COMMON /MBSG/ SFORCE(100),BETA(100),ALPHA(100),WT(100)
      COMMON /MBS/  NSP1,FS,ITER,ACC,ITERL
      COMMON /MGB/ X(100),NEGEG,S(100)
      COMMON /DAM/ CD,CDGAMF,PHIDAM,PHIDMC
      COMMON /FND/ CF,CFGAMF,PHIFND,PHIFDC
      KILCMS=0
```

```
      SUMNUM=0.
      SUMDEN=0.
      DO 1 I=2,NSP1
      DELTAX=X(I)-X(I-1)
      UU=BETA(I)*WT(I)/DELTAX
      POS=WT(I)*COS(ALPHA(I))-(UU*DELTAX)/COS(ALPHA(I))
      IF(POS .LT. 0.0) POS=0.
      SUMNUM=SUMNUM+S(I)*DELTAX/COS(ALPHA(I))+TAN(SFORCE(I))*POS
      SUMDEN=SUMDEN+WT(I)*SIN(ALPHA(I))
    1 CONTINUE
      FS=SUMNUM/(SUMDEN-RFORCE)
      RETURN
      END

      SUBROUTINE BISHOP(KILLB,IRF,IDP)
C***********************************************************************
      COMMON /GENERA/  A(10),B(10),C(10),Y(100),YS(100),RADCON
C***********************************************************************
      COMMON /GENERB/  NSPTS,XC,YC,R,ARCLEN,GY,NS,CRU
      COMMON /MBSG/ SFORCE(100),BETA(100),ALPHA(100),WT(100)
      COMMON /MBS/  NSP1,FS,ITER,ACC,ITERL
      COMMON /MGB/ X(100),NEGEQ,S(100)
      COMMON /DAM/ CD,CDGAMF,PHIDAM,PHIDMC
      COMMON /END/ CF,CFGAMF,PHIFND,PHIFDC
      DIMENSION RFT(100),RFR(100)
      CHARACTER IDP*1
C***********************************************************************
C     INITIALIZE FOR FIRST ITERATION
C***********************************************************************
      ITERL=20
      ITER=1
      F0=FS
      GO TO 3
C***********************************************************************
C     CHECK TO SEE IF ITERATIONS EXCEED ALLOWABLE
C***********************************************************************
    1 IF(ITER.LT.ITERL) GO TO 2
      WRITE(6,204) ITERL
      KILLB=1
      RETURN
    2 ITER=ITER+1
      F0=F1
    3 CONTINUE
      CALL SECOND(TIME5)
      DO 5 IK=1,100
      RFT(IK)=0.
      RFR(IK)=0.
    5 CONTINUE
C*********************
C     REINFORCEMENT
C*********************
      IF(IRF.EQ.1) THEN
      CALL REINFOR(F0,RFT,RFR)
      ENDIF
C***********************************************************************
C     INITIALIZE SUMMATION TERMS
C***********************************************************************
      IF(IDP.EQ.'Y'.AND.ITER.EQ.1) THEN
      WRITE(6,200)
      WRITE(6,201)
      WRITE(6,202)
      WRITE(6,201)
      ENDIF
      SUM1=0.
      SUM2T=0.
      SUM2B=0.
```

```
 SUM3T=0.
 SUM3B=0.
 SUM3=0.
 DO 4 K=2,NSP1
 RR=RFR(K)
 RT=RFT(K)
 KM1=K-1
 CEE=S(K)
 PHEE=SFORCE(K)
 RL=BETA(K)
 BB=X(K)-X(K-1)
 SUM1=SUM1+WT(K)*SIN(ALPHA(K))-RT
 SUM2T=(1+TAN(PHEE)*TAN(ALPHA(K))/FO)
 SUMTT=(WT(K)+RR*COS(ALPHA(K))-RT*SIN(ALPHA(K)))*TAN(PHEE)
 SUM3T=CEE*BB+SUMTT
 SUM2=SUM2+(SUM3T/SUM2T)/COS(ALPHA(K))
4 CONTINUE
 *****************************************************************
 CALCULATE FACTOR OF SAFETY
 *****************************************************************
 F1=SUM2/SUM1
 F1=FO*(1.-(SUM1-SUM2)/(SUM1-SUM3-RFORCE))
 *****************************************************************
 CHECK CONVERGENCE AND REPEAT IF NECESSARY
 *****************************************************************
 IF(IDP.EQ.'Y') THEN
 WRITE(6,203) ITER,FO,F1
 ENDIF
 IF(ABS(F1-FO).GT.0.001) GO TO 1
 FS=F1
 KILLB=0
 IF(IDP.EQ.'Y')    THEN
 WRITE(6,201)
 WRITE(6,200)
 ENDIF
 RETURN
0 FORMAT(///)
1 FORMAT(15X,40('-'))
2 FORMAT(15X,': ITERATION   :    INITIAL   : CALCULATED :')
3 FORMAT(15X,':',I8,'     :',2(F9.2,'    :'))
4 FORMAT(///15X,45('*'),/15X,
 *'BISHOP SOLUTION DID NOT CONVERGE IN',I5,2X,
 *'ITERATIONS',/15X/45('*'),///)
 END

 CALCULATION OF THE FORCES    *
 IN REINFORCEMENTS            *

 SUBROUTINE REINFOR(FS,RFT,RFR)

 COMMON /GENERB/  NSPTS,XC,YC,R,ARCLEN,GY,NS,CRU
 COMMON /ABS/  NSP1,FK,ITER,ACC,ITERL
 COMMON /NGB/ X(100),NEGEQ,S(100)
 COMMON/BB/RNX(20,2),RNY(20,2),RC,FMAX,NR,RN,SF,PMAX,LC,OPT
 DIMENSION RFT(100),RFR(100)
 INTEGER OPT
 REAL LO

 IF(ITER.EQ.1) THEN
 WRITE(6,201)
 ENDIF
 DO 1 I=1,NR
 DM=(RNY(I,1)-RNY(I,2))/(RNX(I,1)-RNX(I,2))
 DN=RNY(I,1)-RNX(1,1)*DM
 C1=DN-YC
```

```
      BB=-2.*XC+2.*DM*C1
      CC=XC*XC+C1*C1-R*R
      XP=(-BB-SQRT(BB*BB-4.*AA*CC))/(2.*AA)
      YP=DM*XP+DN
      RLGTH=SQRT((XP-RNX(I,1))**2+(YP-RNY(I,1))**2)
      P1=RNY(I,2)-RNY(I,1)
      P2=RNX(I,2)-RNX(I,1)
      IF(P2.EQ.0.) THEN
        ALP1=90.
      ELSE
        ALP1=ATAN(P1/P2)
      ENDIF
      C1=XC-XP
      C2=YC-YP
      IF(C2.EQ.0.) THEN
        ALP2=90.
      ELSE
        ALP2=ATAN(C1/C2)
      ENDIF
      ALPR=ALP1+ALP2
      ALPD=ALPR*180./ACOS(-1.)
      IF(CPT.EQ.1) THEN
        TEN=(RLGTH*FMAX*ACOS(-1.)*RD)/FS
        IF(TEN.GT.RN) TEN=RN
        SHEAR=0.
        TALO=RN
        VALO=0.
      ELSE
        C1=TAN(90.-ALPD)
        VAL=RN/(2.*SQRT(1.+4.*C1*C1))
        TAL=4.*VAL*C1
        TALO=TAL
        VALO=VAL
        VSOIL=PMAX*RD*LO/2.
        IF(VSOIL.LT.VAL) THEN
          VAL=VSOIL
          TAL=SQRT(RN*RN-4.*VAL*VAL)
          TALO=TAL
          VALO=VAL
        ENDIF
        TEN=(RLGTH*FMAX*ACOS(-1.)*RD)/FS
        IF(TEN.GT.TAL) TEN=TAL
        SHEAR=VAL
      ENDIF
      RR=TEN*SIN(ALPR)+SHEAR*COS(ALPR)
      RT=TEN*COS(ALPR)+SHEAR*SIN(ALPR)
      DO 3 II=1,NSP1-1
      IF(XP.GT.X(II).AND.XP.LE.X(II+1)) THEN
        RFT(II)=RFT(II)+RT/SH
        RFR(II)=RFR(II)+RR/SH
      ENDIF
    3 CONTINUE
      IF(ITER.EQ.1) THEN
      WRITE(6,202) I,TALO,VALO,RLGTH,ALPD,TEN,SHEAR
      ENDIF
    1 CONTINUE
      IF(ITER.EQ.1) THEN
      WRITE(6,203)
      WRITE(6,204)
      ENDIF
      RETURN
  201 FORMAT(////,15X,79('-'),/15X,
     *': NO : TEN-AL : SHEAR-AL : EFF-LENGTH : ALPHA : TENSIO
     *N : SHEAR :',/15X,79('-'))
  202 FORMAT(15X,':',I4,' :',F7.2,' :',F7.2,' :',F9.3,' :',F7
     *.2,' :',F8.2,' :',F7.2,' :')
  203 FORMAT(15X,79('-'))
  204 FORMAT(////)
  115 FORMAT(///,20X,'FACTOR OF SAFETY IS ASSUMED AS ',F7.2)
  130 FORMAT(///,20X,'FORCE CARRIED BY REINFORCEMENT :',F10.2,//)
  100 FORMAT(///,15X,'ALLOWABLE STRENGTH',10X,'APPLIED STRENGH'
     *,//13X,'REINFORCEMENT',5X,'SOIL',//12X,'SHEAR',2(4X,
     *'TENSILE',5X,'SHEAR'),5X,'R-DIAM',5X,'E-LENGTH'
     *,5X,'ANGLE',5X,'TENSILE',5X,'SHEAR',5X,'TOTAL',/)
  120 FORMAT(7X,10F10.2)
      END
```

A P P E N D I X D

COMPUTER PROGRAM , FRSOIL

USER'S MANUAL FOR PROGRAM, FRSOIL

INPUT DATA INFORMATION*,**

1. Heading

1-80 HEAD - Title card for program identification

2. Type of material

ITM - ITM = 1 Elastic Media

ITM = 2 Soil

3. Type of soil and type of analysis

This card is required when ITM = 2.

ITS, ITA - ITS = 1, homogeneous soil

ITS = 2, reinforced soil

- ITA = 1, linear analysis

ITA = 2, Non-linear analysis

---

* All data are in free format

** Any unit system can be chosen by the user

4. Automatic mesh generation

IMESH - IMESH = 0, nodal point's coordinates are given as input data

IMESH ≠ 0, nodal point's coordinates are calculated automatically

5. Nodal point's coordinates

This card is required when IMESH = 0

NJ, NM - NJ = Number of Joints (Maximum = 500)

NM = Number of member (Maximum = 250)

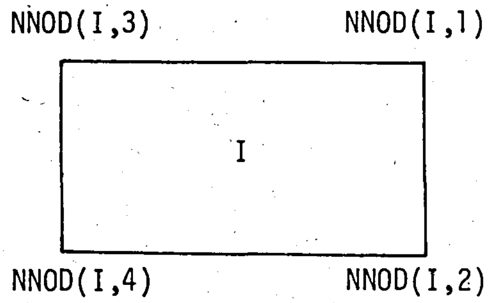
NNUM, x(I), Y(I), DX(I), DY(I) -

NNUM	Nodal number
X(I)	x-coordinate of this node
Y(I)	y-coordinate of this node
DX(I)	restrain in x-direction DX = 0, fixed DX ≠ 0, free
DY(I)	Restrain in y-direction DY = 0, fixed DY ≠ 0, free

Repeat this card as many as NJ

NNOD(I,J) - I   Member number

J   Joint number



Repeat this card as many as NM,

OMIT 6, 7, 8 and 9 when IMESH = 0

6. <u>Automatic Mesh Generation</u>

EST, VST, THK - (Required when ITM = 1)

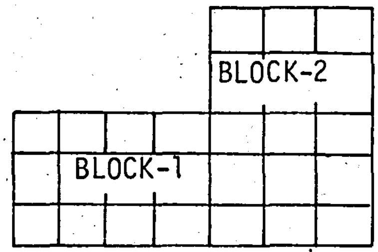
- EST   Elasticity modulus of the considered material

VST   Poisson's ratio of this material

THK   Thickness of the member

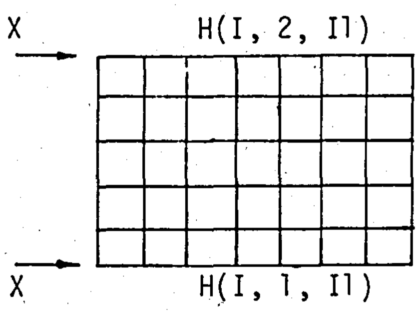
7. <u>Definition of Blocks</u>

NOB - Number of Block (Maximum = 30)

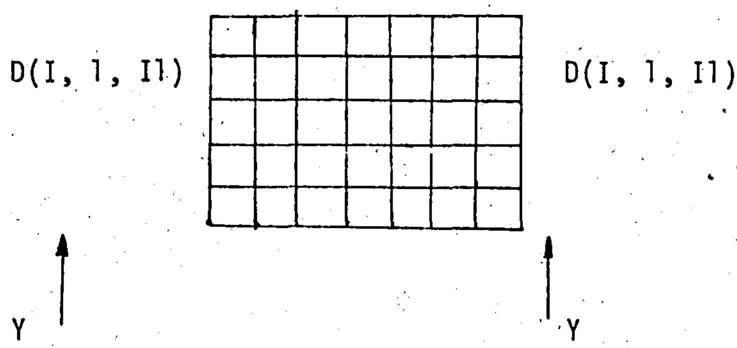


8. <u>Horizontal and vertical spacing</u>

NH(I), NV(I) - I		Block number
	NH(I)	Number of nodal points in horizontal direction
	NV(I)	Number of nodal points in vertical direction
	H(I, 1, I1)	X - coordinates of nodal points located on bottom of the block
	H(I, 2, I1)	X - coordinates of nodal points located on top of the block



D(I, 1, I1)	Y - coordinates of nodal points located on right edge of the block
D(I, 2, I1)	Y - coordinates of nodal points located on lefth edge of the block



ESO, VSO, PI, COH, DEN, THK, UKM, RN, PA, RF - (This card is required when ITM = 2)

ESO	Elasticity modulus of the soil
VSO	Poissson's ratio of the soil
COH	Cohesion of the soil
DEN	Density of the soil
THK	Thickness
UKM	Modulus number
RN	Modulus exponent
PA	Atmospheric Pressure
RF	Failure ratio

Repeat this card as many as NOB

9. <u>Restrains</u>

NP - Number of nodal points that are restrained.

I1, DX(I1), DY(I1) - I1 Nodal number

DX(I1) = 0 Fixed in x - direction

DX(I1) $\neq$ 0 Free in x - direction

DY(I1) = 0 Fixed in Y - direction

DY(I1) $\neq$ 0 Free in Y - direction

Repeat this card as many as NP

10. Concentrated load

NLOAD - Number of concentrated load MEM(I), DIR(I), LOAD(I)

- MEM(I) = Nodal Number

DIR(I) = 1 X - direction

DIR(I) = 2 Y - direction

LOAD(I) = Magnitude of the load

11. Gravity Forces

IGF IGF = 1, Gravity force exists

IGF $\neq$ 1, No-Gravity force

12. Definition of layers for incremental analysis

This card is required when ITA = 2

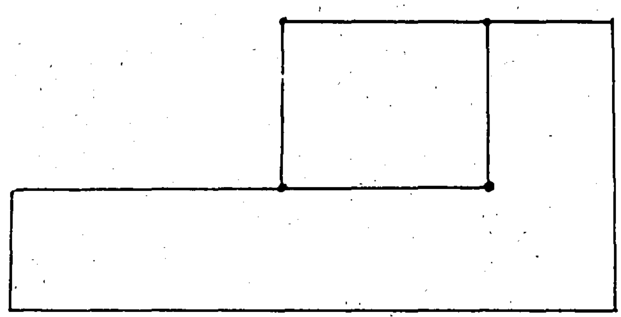
NIL Number of layers

NIM(IK) The last element number of added layer

13. Reinforced Soil

This card is required when ITS = 2

XC(IK), YC(IK) - Corner coordinates of reinforced region



ER, AR - ER = Elasticity modulus of the reinforcing strip

AR = Area of the reinforcing strip

```
*****************************************************************
*        F I N I T E    E L E M E N T     A N A L Y S I S
*****************************************************************
*
*    THIS PROGRAM SOLVES   PLAIN STRESS  AND   PLAIN STRAIN  PROBLEM
*    IN HOMOGENEOUS OR NON-HOMOGENEOUS    MEDIA, USING  EITHER LINEAR
*    OR  NON-LINEAR ANALYSING METHOD   (   INCLUDING   ITERATIVE AND
*    INCREMENTAL LOADING METHOD  )...
*
*****************************************************************
*
*                    F I K R E T  E Y G O R E N
*
*              B O G A Z I C I     U N I V E R S I T Y
*
*                        NOVEMBER-1985
*
*****************************************************************
      PROGRAM RSOIL(DFINE,OFINE,TAPE5=DFINE,TAPE6=OFINE)
C
      PARAMETER(ND1=250,ND2=550,ND3=15000)
C
      COMMON/B1/X(ND2),Y(ND2),DX(ND2),DY(ND2)
      COMMON/B2/NNOD(ND1,4),ICOD(ND1,8),PLOAD(ND2)
      COMMON/B3/ITM,ITS,ITA,ITOM,IGF,EE(ND1,5),E(ND1)
      COMMON/B4/THK,EST,VST,ESO,VSO,PI,DEN,COH,PA,ER,AR
      COMMON/B5/SIG(ND1,4,3),EPS(ND1,4,7),AVS(ND1,9)
      COMMON/B6/SS(3,8),G(3,3),S(8,8)
      COMMON/B7/UNKN(ND2),BANTH(ND3)
      COMMON/B8/RTYPE
      CHARACTER RTYPE(ND1)*2
      DIMENSION DD(3),XC(4),YC(4),NL(ND1),DK(ND1)
      DIMENSION SL(ND1,10),AVSR(ND1,10)
      CHARACTER*1,DFS ,CC1*80,CC2*80
C
      REAL LOAD
      INTEGER DX,DY,DIR
C
      DATA MM,LN,IN,ITR,IWR/5*0/
C
      WRITE(6,100) DATE(),TIME()
      CALL SREAD(MM,NJ,NN,NB,LN,XC,YC,NL)
****************************************
*     START TO ESTIMATE DISPLACEMENTS    *
****************************************
      DO 1 I=1,MM
      CALL ESM(I,1,0,XC,YC)
      CALL SYSTEM(I,NN,NB)
    1 CONTINUE
      WRITE(6,360) NB,LN
      CALL GSELFB(NN,NB,LN)
      WRITE(6,340)
      DO 85 II=1,NJ
      IF(DX(II).EQ.0) THEN
       DDX=0.
```

```
      ELSE
       DDX=UNKN(DX(II))
      ENDIF
      IF(DY(II).EQ.0) THEN
       DDY=0.
      ELSE
       DDY=UNKN(DY(II))
      ENDIF
      WRITE(6,341)II,X(II),Y(II),DDX,DDY
   65 CONTINUE
      IP=2
      IF(IP.EQ.1) THEN
      WRITE(6,300)
      ELSE
      WRITE(6,301)
      ENDIF
      DO 64 I=1,MM
      DO 66 I4=1,4
      DO 68 IK=1,8
      IF(ICOD(I,IK).EQ.0) THEN
      DD(IK)=0.
      ELSE
      DD(IK)=UNKN(ICOD(I,IK))
      ENDIF
      CALL ESM(I,2,I4,XC,YC)
   68 CONTINUE
      CALL STRESS(I,I4,DD)
      IF(IP.EQ.1) THEN
      IF(I4.EQ.1) THEN
      WRITE(6,310)I,I4,(SIG(I,I4,I3),I3=1,3),(EPS(I,I4,I3),I3=1,7)
      ELSE
      WRITE(6,320)I4,(SIG(I,I4,I3),I3=1,3),(EPS(I,I4,I3),I3=1,7)
      ENDIF
      ENDIF
   66 CONTINUE
      IF(IP.EQ.2) THEN
      AVS(I,1)=X(NNOD(I,3))+(X(NNOD(I,1))-X(NNOD(I,3)))*0.5
      AVS(I,2)=Y(NNOD(I,2))+(Y(NNOD(I,1))-Y(NNOD(I,2)))*0.5
      A1=0.
      A2=0.
      A3=0.
      DO 78 IK=1,4
      A1=A1+SIG(I,IK,1)
      A2=A2+SIG(I,IK,2)
      A3=A3+SIG(I,IK,3)
   78 CONTINUE
      AVS(I,3)=-A1*0.25
      AVS(I,4)=-A2*0.25
      AVS(I,5)=-A3*0.25
      BB1=AVS(I,3)
      BB2=AVS(I,4)
      BB3=AVS(I,5)
      AVS(I,6)=(BB1+BB2)/2+SQRT((((BB1-BB2)/2)*(BB1-BB2)/2)+BB3*BB3)
      AVS(I,7)=(BB1+BB2)/2-SQRT((((BB1-BB2)/2)*(BB1-BB2)/2)+BB3*BB3)
      AVS(I,8)=(AVS(I,6)-AVS(I,7))*0.5
      IF(BB1.EQ.BB2) THEN
        IF(BB3.GT.0) THEN
```

```
          AVS(I,9)=45.0
          ELSE
          AVS(I,9)=-45.0
          ENDIF
        ELSE
          BB=2.*BB3/(BB1-BB2)
          CB=ATAN(BB)*0.5
          AVS(I,9)=ATAN(BB)*90./ACOS(-1.)
        ENDIF
C
C       CALCULATE THE AVERAGE AND PRINCIPAL STRAINS
C
        AA1=0.
        AA2=0.
        AA3=0.
        DO 79 IK=1,4
        AA1=AA1+EPS(I,IK,1)
        AA2=AA2+EPS(I,IK,2)
        AA3=AA3+EPS(I,IK,3)
     79 CONTINUE
        AVSR(I,1)=-AA1*0.25
        AVSR(I,2)=-AA2*0.25
        AVSR(I,3)=-AA3*0.25
        BC1=AVSR(I,1)
        BC2=AVSR(I,2)
        BC3=AVSR(I,3)*0.50
        AVSR(I,4)=(BC1+BC2)/2+SQRT((((BC1-BC2)/2)*(BC1-BC2)/2)+BC3*BC3
        AVSR(I,5)=(BC1+BC2)/2-SQRT((((BC1-BC2)/2)*(BC1-BC2)/2)+BC3*BC3
        AVSR(I,6)=(AVS(I,4)-AVS(I,5))
        IF(BC1.EQ.BC2) THEN
          IF(BC3.GT.0) THEN
          AVSR(I,7)=45.0
          ELSE
          AVSR(I,7)=-45.0
          ENDIF
        ELSE
          BC=BC3/(BC1-BC2)
          AVSR(I,7)=ATAN(BC)*90./ACOS(-1.)
        ENDIF
C
C       CALCULATE THE STRESS LEVEL AND FACTOR OF SAFETY
C
        SL(I,1)=AVS(I,6)
        SL(I,2)=AVS(I,7)
        SL(I,3)=AVS(I,8)
        SL(I,4)=0.
        IF(RTYPE(I).EQ.'RS') THEN
          AC=THK*(Y(NNOD(I,1))-Y(NNOD(I,2)))
          AAC=AC*LE(I,1)+AR*ER
          SXS=BB1*AC*EE(I,1)/AAC
          SXR=BB1*AC*ER/AAC
          BB1=SXS
          SL(I,1)=(BB1+BB2)/2+SQRT((((BB1-BB2)/2)*(BB1-BB2)/2)+BB3*BB3
          SL(I,2)=(BB1+BB2)/2-SQRT((((BB1-BB2)/2)*(BB1-BB2)/2)+BB3*BB3
          SL(I,3)=(AVS(I,6)-AVS(I,7))*0.5
          SL(I,4)=AR*SXR
        ENDIF
```

```
      PI=EE(I,3)
      SIGU=2*(EE(I,4)*COSD(PI)+AVS(I,7)*SIND(PI))/(1-SIND(PI))
      SL(I,5)=(SL(I,1)-SL(I,2))/SIGU
      WRITE(6,*) I,RTYPE(I),SIGU,SL(I,1),SL(I,2),SL(I,5)
      SL(I,5)=1/SL(I,5)
      ENDIF
   64 CONTINUE
      IF(IP.EQ.2) THEN
        WRITE(6,302)
        DO 65 J=1,MM
        WRITE(6,326) J,RTYPE(J),(AVS(J,K),K=1,2),(AVSR(J,L),L=1,7)
   65   CONTINUE
      ENDIF
      WRITE(6,327)
      DO 71 I=1,MM
      WRITE(6,328) I,RTYPE(I),(AVS(I,IK),IK=1,2),(SL(I,J),J=1,6)
   71 CONTINUE
*****************************
*     FORMAT STATEMENTS     *
*****************************
  100 FORMAT(/////,1X,14('*'),
     +' F.E.M ANALYSIS USING RECTANGULAR ELEMENTS ',13('*')/
     +           1X,20('*'),A10,10('*'),A10,20('*'),///)
  103 FORMAT(///,20X,A50,//,20X,A50,//)
  110 FORMAT(/,20X,'NO.OF ELEMENTS=',I4,/20X,'NO.OF UNKNOWNS=',I4,/
     +,20X,'NO.OF LAYERS  =',I4)
  300 FORMAT(///,1X,'ELEMENT',1X,'JOINT',5X,'GX',8X,'GY',11X,
     +'TXY',9X,'EPSX',9X,'EPSY',9X,'GAMA',8X,'MAX G',7X,'MIN G'
     +,7X,'MAX T',7X,'TETA',/)
  261 FORMAT(15X,I4,7X,F10.2)
  301 FORMAT(///,3X,'EL. NO.',2X,'EL.TYPE',5X,'X-COORD',3X,'Y-COORD',
     *3X,'X-STRESS',3X,'Y-STRESS',3X,'XY-STRESS',4X,'MAX STRESS',
     *3X,'MIN STRESS',3X,'MAX SHEAR',2X,'ANGLE',/)
  302 FORMAT(///,3X,'EL. NO.',2X,'EL.TYPE',5X,'X-COORD',3X,'Y-COORD',
     *3X,'X-STRAIN',3X,'Y-STRAIN',3X,'XY-STRAIN',4X,'MAX STRAIN',
     *3X,'MIN STRAIN',3X,' MAX GAMA',2X,'ANGLE',/)
  325 FORMAT(/,3X,I3,7X,A2,7X,F8.2,2X,F8.2,3F11.3,1X,3F13.3,F8.2)
  326 FORMAT(/,3X,I3,7X,A2,7X,F8.2,2X,F8.2,3E11.3,1X,3E13.3,F8.2)
  327 FORMAT(1H1,/////,
     &'  E.N.  E.T          X              Y         MAX-STRESS      MIN-STRESS
     &   MAX-SHEAR          STRIP          STRESS          SAFETY',//,
     &'
     &                      FORCE          LEVEL           FACTOR')
  328 FORMAT(/,I4,4X,A2,6F13.3,2F13.2)
  310 FORMAT(2X,I3,3X,I3,3(3X,F9.5),3(3X,F10.8),3(2X,F10.6),2X,F7.3)
  320 FORMAT(8X,I3,3(3X,F9.5),3(3X,F10.8),3(2X,F10.6),2X,F7.3)
  340 FORMAT(////,15X,'N.P',5X,'X-COORD',5X,'Y-COORD',5X,'DISPL-X',
     *5X,'DISPL-Y',/)
  341 FORMAT(15X,I3,2F12.2,2E12.4)
  350 FORMAT(13F10.0)
  360 FORMAT(//,25X,'BAND  WIDTH =',I6,//,25X,'TOTAL N.O.E =',I6)
************************************************************************
  900 STOP
      END
```

```
TA INITIAL    74/810   OPT=0,ROUND= A/ S/ M/-D,-DS       FTN 5.1+628          86
/-OT,ARG=-COMMON/-FIXED,CS= USER/-FIXED,DB=-TB/-SB/-SL/ ER/-ID/-PMD/-ST,-AL
COILR2,L=L,B=B.

****************************************
*      INITIAL VALUE OF THE ARRAYS      *
****************************************
      BLOCK DATA INITIAL
      PARAMETER(ND1=250,ND4=ND1*12,ND5=ND1*28)
      PARAMETER(ND2=550,ND6=ND1*5)
      PARAMETER(ND3=15000)
      INTEGER DX,DY
      COMMON/B1/X(ND2),Y(ND2),DX(ND2),DY(ND2)
      COMMON/B2/NNOD(ND1,4),ICOD(ND1,8),PLOAD(ND2)
      COMMON/B3/ITM,ITS,ITA,ITOM,IGF,EE(ND1,5),E(ND1)
      COMMON/B5/SIG(ND1,4,3),EPS(ND1,4,7),AVS(ND1,9)
      COMMON/B6/SS(3,8),G(3,8),S(8,8)
      COMMON/B7/UNKN(ND2),BANTH(ND3)
      DATA SIG,EPS/ND4*0.0,ND5*0.0/
      DATA BANTH/ND3*0.0/
      DATA DX/ND2*0/,DY/ND2*0/,PLOAD/ND2*0./
      DATA EE/ND6*0.0/
      DATA G,S,SS/112*0.0/
      END
```

```
******************************************
*       DATA READING                     *
******************************************
*       THIS  SUBROUTINE   READS         *
*       THE DATA  AND EVALUATES          *
*       NODE  AND  CODE NUMBERS          *
******************************************
        SUBROUTINE SREAD(NM,NJ,NF,NB,LN,XC,YC,NL)
        PARAMETER(ND1=250,ND2=550,ND3=15000)
        COMMON/B1/X(ND2),Y(ND2),DX(ND2),DY(ND2)
        COMMON/B2/NNOD(ND1,4),ICOD(ND1,8),PLOAD(ND2)
        COMMON/B3/ITM,ITS,ITA,ITOM,IGF,EE(ND1,5),E(ND1)
        COMMON/B4/THK,EST,VST,ESO,VSO,PI,DEN,COH,PA,ER,AR
        DIMENSION MEM(ND2),DIR(ND2),LOAD(ND1)
        DIMENSION NH(30),NV(30),H(30,2,30),D(30,2,30)
        DIMENSION HB(30),HE(30),DB(30),DE(30),SLOPE(30,2)
        DIMENSION NL(ND1),XC(4),YC(4)
        CHARACTER HEAD*80 ,CC*80,UNI*1,EX*1,TOA*1
        REAL LOAD
        INTEGER DX,DY,DIR
        NF=0
        READ(5,143) HEAD
        READ(5,*) ITM
        IF(ITM.EQ.2) READ(5,*) ITS
        READ(5,*) IMESH
        IF(IMESH.EQ.0) THEN
        DO 2 IR=1,NJ
        READ(5,*)  NNUM, X(IR),Y(IR),DX(IR),DY(IR)
      2 CONTINUE
        DO 32 IC=NM
        READ(5,*)(NNOD(IC,JC),JC=1,4)
     32 CONTINUE
        ELSE
*********************************************
*       AUTOMATIC MESH GENERATION           *
*********************************************
        NJ=0
        NM=0
        K=0
        KN=1
        IF(ITM.EQ.1) THEN
        READ(5,*) EST,VST,THK
        ENDIF
        READ(5,*)  NOB
        DO 1 I=1,NOB
C       HORIZONTAL & VERTICAL SPACING
        READ(5,*) NH(I),NV(I)
        READ(5,*) (H(I,1,I1),I1=1,NH(I))
        READ(5,*) (H(I,2,I1),I1=1,NH(I))
        READ(5,*) (D(I,1,I1),I1=1,NV(I))
        READ(5,*) (D(I,2,I1),I1=1,NV(I))
        IF(ITM.EQ.2) THEN
        READ(5,*) ESO,VSO,PI,COH,DEN,THK
        ENDIF
        DO 3 I1=1,NV(I)
```

```
      HX=(H(I,2,1)-H(I,1,1))*D(I,1,I1)/
     &(D(I,1,NV(I))-D(I,1,1))
      HB(I1)=H(I,1,1)+HX
    3 CONTINUE
      DO 4 I1=1,NV(I)
      HX=(H(I,2,NH(I))-H(I,1,NH(I)))*(D(I,2,I1)-D(I,2,1))/
     &(D(I,2,NV(I))-D(I,2,1))
      HE(I1)=H(I,1,NH(I))+HX
    4 CONTINUE
      DO 5 I2=1,NH(I)
      DDX=(D(I,2,1)-D(I,1,1))*H(I,1,I2)/
     &(H(I,1,NV(I))-H(I,1,1))
      DB(I2)=D(I,1,1)+DDX
    5 CONTINUE
      DO 6 I2=1,NH(I)
      DDX=(D(I,2,NV(I))-D(I,1,NV(I)))*(H(I,2,I2)-H(I,2,1))/
     &(H(I,2,NH(I))-H(I,2,1))
      DE(I2)=D(I,1,NV(I))+DDX
    6 CONTINUE
      DO 11 I2=1,NH(I)
      BC=H(I,1,I2)-H(I,2,I2)
      IF(BC.EQ.0.) THEN
      SLOPE(I2,1)=0.
      ELSE
      SLOPE(I2,1)=(DB(I2)-DE(I2))/(H(I,1,I2)-H(I,2,I2))
      IF(SLOPE(I2,1).EQ.0.) GOTO 9
      ENDIF
      SLOPE(I2,2)=DB(I2)-SLOPE(I2,1)*H(I,1,I2)
   11 CONTINUE
      IF(I.EQ.1) THEN
      IB=1
      ELSE
      IB=2
      ENDIF
      DO 7 I1=IB,NV(I)
      BB=HB(I1)-HE(I1)
      IF(BB.EQ.0.) THEN
      GOTO 9
      ELSE
      BB1=(D(I,1,I1)-D(I,2,I1))/BB
      ENDIF
      BB2=D(I,1,I1)-BB1*HB(I1)
      DO 8 I2=1,NH(I)
      C1=SLOPE(I2,1)
      C2=SLOPE(I2,2)
      NJ=NJ+1
      IF(BB1.EQ.0.) THEN
        Y(NJ)=D(I,1,I1)
        IF(C1.EQ.0.) THEN
          X(NJ)=H(I,1,I2)
        ELSE
          X(NJ)=(Y(NJ)-C2)/C1
        ENDIF
      ELSE
        IF(C1.EQ.0) THEN
          X(NJ)=H(I,1,I2)
        ELSE
```

```
          X(NJ)=(C2-BB2)/(BB1-C1)
        ENDIF
        Y(NJ)=BB1*X(NJ)+BB2
      ENDIF
    8 CONTINUE
    7 CONTINUE
      IF(I.EQ.1) THEN
        K=1
      ELSE
        NT=NM-NH(I-1)+1
        DO 16 IJ=NT,NM
        DO 17 IK=1,4,2
        IF(X(NNOD(IJ,IK)).NE.H(I,1,1))THEN
        GOTO 17
        ELSE
        K=NNOD(IJ,IK)
        GOTO 18
        ENDIF
   17   CONTINUE
   16   CONTINUE
      ENDIF
   18 IF(I.EQ.1) THEN
        LL=0
        LL1=0
      ELSE
        LL=NNOD(NM,1)-K-NH(I)+1
        LL1=0
      ENDIF
      DO 12 L=1,NV(I)-1
      DO 13 J=1,NH(I)-1
      NM=NM+1
      NNOD(NM,1)=K+NH(I)+1+LL
      NNOD(NM,2)=K+1+LL1
      NNOD(NM,3)=K+NH(I)+LL
      NNOD(NM,4)=K+LL1
      IF(ITM.EQ.2) THEN
      EE(NM,1)=ESO
      EE(NM,2)=VSO
      EE(NM,3)=PI
      EE(NM,4)=COH
      EE(NM,5)=DEN
      ENDIF
      K=K+1
   13 CONTINUE
      LL1=LL
      K=K+1
   12 CONTINUE
    1 CONTINUE
      IF(NJ.GT.ND2) THEN
        WRITE(6,*) 'NUMBER OF JOITS IS ',NJ
        WRITE(6,*) 'PLEASE CHECK PARAMETER "ND2" '
        STOP
      ENDIF
      IF(NM.GT.ND1) THEN
        WRITE(6,*) 'NUMBER OF MEMBERS IS ',NM
        WRITE(6,*) 'PLEASE CHECK PARAMETER "ND1" '
        STOP
```

```
      ENDIF
      GOTO 19
    9 PRINT*,'***  W A R N I N G  ***'
      PRINT*,'*** UNSUITABLE MESH GENERATION ***'
      STOP
   19 READ(5,*) NP
      DO 20 II=1,NP
   20 READ(5,*) I1,DX(I1),DY(I1)
      ENDIF
***************************************************
*     END OF AUTOMATIC MESH GENERATION            *
***************************************************
      READ(5,*) NGOAD
      DO 22 IK=1,NLOAD
      READ(5,*) MEM(IK),DIR(IK),LOAD(IK)
   22 CONTINUE
      READ(5,*) IGF
      IF(ITS.EQ.2) THEN
      DO 24 IK=1,4
      READ(5,*) XC(IK),YC(IK)
   24 CONTINUE
      READ(5,*) ER,AR
      ENDIF
******************************
*     CODE NUMBERING         *
******************************
      NF=0
      DO 23 IK=1,NJ
      IF(DX(IK).EQ.1) THEN
        DX(IK)=0
      ELSE
        NF=NF+1
        DX(IK)=NF
      ENDIF
      IF(DY(IK).EQ.1) THEN
        DY(IK)=0
      ELSE
        NF=NF+1
        DY(IK)=NF
      ENDIF
   23 CONTINUE
      DO 25 IK=1,NM
      DO 26 IJ=1,4
      ICOD(IK,IJ)=DX(NNOD(IK,IJ))
   26 CONTINUE
      DO 25 IJ=1,4
      ICOD(IK,4+IJ)=DY(NNOD(IK,IJ))
   25 CONTINUE
      DO 27 IK=1,NLOAD
      IF(DIR(IK).EQ.1) THEN
      NN=DX(MEM(IK))
      IF(NN.GE.1) PLOAD(N)=LOAD(IK)
      ELSE
      KK=DY(MEM(IK))
      IF(KK.GE.1) PLOAD(KK)=LOAD(IK)
      ENDIF
   27 CONTINUE
```

```
      DO 31 IK=1,NM
      E(IK)=EE(IK,1)
   31 CONTINUE
      DO 28 I=1,NM
      DO 28 II=1,7
      DO 28 IJ=II+1,8
      IF(ICOD(I,II).EQ.0.OR.ICOD(I,IJ).EQ.0) GOTO 28
      NQ=ABS(ICOD(I,II)-ICOD(I,IJ))
      IF(NQ.GT.NB) NB=NQ
   28 CONTINUE
      NB=NB+1
      DO 29 II=1,NB
   29 LN=LN+II+1
      LN=LN+(NF-NB)*(NB+1)
      IF(LN.GT.ND3) THEN
         WRITE(6,*) 'NUMBER OF ELEMENTS IS ',LN
         WRITE(6,*) 'PLEASE CHECK PARAMETER "ND3" '
         STOP
      ENDIF
      IF(ITM.EQ.1) THEN
      CC='  ***  STELL / WOOD  *** '
      ELSE
      IF(ITS.EQ.1) THEN
      CC='***  HOMOGENEOUS SOIL  ***'
      ELSE
      CC='***  REINFORCED SOIL  ***'
      ENDIF
      ENDIF
*************************
*     INPUT FORMATS     *
*************************
  100 FORMAT(3I5)
  110 FORMAT(I5,2F10.0,2I5)
  120 FORMAT(6I3)
  130 FORMAT(F10.0)
  140 FORMAT(2F10.0)
  150 FORMAT(3I5)
  160 FORMAT(I5)
  170 FORMAT(I5,I5,F10.0)
  143 FORMAT(A80)
********************
*     OUTPUT       *
********************
      WRITE(6,200)HEAD
      WRITE(6,197) CC
      IF(ITS.EQ.2) WRITE(6,238) (XC(IK),YC(IK),IK=1,4),ER,AR
      WRITE(6,198)
      WRITE(6,236)
      IF(NLOAD.EQ.0) THEN
      WRITE(6,235)
      ELSE
      WRITE(6,230)
      WRITE(6,240) (MEM(IJ),DIR(IJ),LOAD(IJ),IJ=1,NLOAD)
      ENDIF
      WRITE(6,220) NM,NJ,THK
      IF(ITM.EQ.1) THEN
      WRITE(6,221) EST,VST
```

```
      ENDIF
      WRITE(6,199)
      WRITE(6,201)
      DO 30 IW=1,NJ
      WRITE(6,210)  IW,X(IW),Y(IW),DX(IW),DY(IW)
   30 CONTINUE
      WRITE(6,270) NF
      WRITE(6,222)
      IF(ITM.EQ.1) THEN
      WRITE(6,250)
      ELSE
      WRITE(6,251)
      ENDIF
      DO 40 IK=1,NM
      IF(ITM.EQ.1) THEN
      WRITE(6,260)IK,(NNOD(IK,IJ),IJ=1,4),(ICOD(IK,IJ),IJ=1,8)
      ELSE
      WRITE(6,261)IK,(NNOD(IK,IJ),IJ=1,4),(ICOD(IK,IJ),IJ=1,8),
     +(EE(IK,IJ),IJ=1,5)
      ENDIF
   40 CONTINUE
*************************
*     OUTPUT FORMATS    *
*************************
  200 FORMAT(1H1,/////20X,A40)
  197 FORMAT(//,20X,40('*'),/,20X,'*',38X,'*',/20X,'*',
     +6X,A26,6X ,'*',/20X,'*',38X,'*',/20X,40('*'))
  198 FORMAT(///,20X,'*** LOADING CASES ***')
  199 FORMAT(1H1,///,20X,'*** NODAL DATA ***')
  201 FORMAT(///,20X,'JOINT',5X,'X-COORD.',5X,'Y-COORD.',
     +5X,'DX',5X,'DY',/)
  210 FORMAT(20X,I3,5X,F8.2,5X,F8.2,5X,I3,4X,I3)
  220 FORMAT(6(/),25X,'NUMBER OF ELEMENTS =',I4,/,25X,
     +'NUMBER OF JOINTS   =',I4,/,25X,'THICKNESS           =',F5.2)
  221 FORMAT(/,25X,'ELASTICITY MODULUS ='
     +,F12.0,/,25X,20HPOISON'S RATIO     =,F5.2)
  222 FORMAT(1H1,////,20X,'*** ELEMENT DATA ***',///)
  235 FORMAT(///,20X,' * THERE IS NO EXTERNAL LOAD *')
  236 FORMAT(//,20X,' 1-GRAVITY FORCES ',/)
  230 FORMAT(/,20X,' 2- EXTERNAL LOADS',
     +///,26X,'NODE',6X,'DIRECTION',5X,'MAGNITUDE',/)
  233 FORMAT(//////,15X,'COORDINATES OF THE REINFORCED PART',/,15X,
     *'X-COORDINATE          Y-COORDINATE',/,4(15X,F11.2,8X,F11.2,/),///,
     *15X,'ELASTICITY MODULUS OF REINFORCEMENT :',E10.2,//,15X
     *,'AREA OF THE REINFORCEMENT           :',E10.2,////)
  240 FORMAT(25X,I4,10X,I2,8X,F8.2)
  250 FORMAT(///,20X,'ELEMENT',10X,'NODES',20X,'CODE NUMBERS',/)
  251 FORMAT(///,15X,'ELEMENT',10X,'NODES',20X,'CODE NUMBERS',
     +14X,'ESO',6X,'VSO',6X,'PI',6X,'COH',7X,'DEN',/)
  260 FORMAT(20X,I4,6X,4I4,5X,8I4)
  261 FORMAT(15X,I4,6X,4I4,5X,8I4,3X,F8.2,3X,F5.2,3X,F5.2,3X,F5.2,
     +3X,F5.2)
  270 FORMAT(//,20X,'NUMBER OF UNKNOWNS=',I4)
      PRINT*,'DO YOU WANT TO PLOT THE MESH ?'
      READ*,UNI
      IF(UNI.EQ.'Y') THEN
      CALL PLOTING(NM)
```

```
      PRINT*,'DO YOU WANT TO CONTINUE ?'
      READ*,EX
      IF(EX.EQ.'N') STOP
      ENDIF
      RETURN
      END
```

```
LE ESM          74/810   OPT=0,ROUND=A/S/ M/-D,-DS        FTN 5.1+628        8
/-OT,ARG=-COMMON/-FIXED,CS= USER/-FIXED,DB=-TB/-SB/-SL/ ER/-ID/-PMD/-ST,-A
SOILR2,L=L,B=B.


********************************
*      ELEMENT STIFFNESS MATRIX        *
********************************
*                                      *
*      THIS SUBROUTINE CALCULATES      *
*      THE  STIFFNESS MATRIX  FOR      *
*      RECTANGULAR ELEMENT             *
********************************
      SUBROUTINE ESM(I,IR,I4,XC,YC)
      PARAMETER(ND1=250)
      PARAMETER(ND2=550)
      COMMON/B1/X(ND2),Y(ND2),DX(ND2),DY(ND2)
      COMMON/B2/NNOD(ND1,4),ICOD(ND1,8),PLOAD(ND2)
      COMMON/B3/ITM,ITS,ITA,ITOM,IGF,EE(ND1,5),E(ND1)
      COMMON/B4/T,EST,VST,ESO,VSO,PI,DEN,COH,PA,ER,AR
      COMMON/B5/SIG(ND1,4,3),EPS(ND1,4,7),AVS(ND1,9)
      COMMON/B6/SS(3,8),G(3,8),S(8,8)
      COMMON/B8/RTYPE
      CHARACTER RTYPE(ND1)*2
      DIMENSION XC(4),YC(4)
      INTEGER DX,DY
      A=(X(NNOD(I,1))-X(NNOD(I,3)))
      B=(Y(NNOD(I,1))-Y(NNOD(I,2)))
      COF=B/A
      IF(IR.EQ.2) GOTO 3
************************
*      STEEL OR WOOD       *
************************
      IF(ITM.EQ.1) THEN
      RTYPE(I)='ST'
      CC=EST/(1.-VST*VST)
      D11=CC
      D12=VST*CC
      D21=VST*CC
      D22=CC
      D33=((1.-VST)*0.5)*CC
      ELSE
***************
*      SOIL      *
***************
      AC=2.*A*T
      IG=0
      ESO=EE(I,1)
      VSO=EE(I,2)
      IF(ITS.EQ.2) THEN
      DO 2 I1=1,4
      X1=X(NNOD(I,I1))
      Y1=Y(NNOD(I,I1))
      IF((X1.LE.XC(1).AND.X1.LE.XC(2).AND.X1.GE.XC(3).AND.
     &X1.GE.XC(4)).AND.(Y1.LE.YC(1).AND.Y1.LE.YC(3).AND.Y1.
     &GE.YC(2).AND.Y1.GE.YC(4))) THEN
      IG=IG+1
      ENDIF
    2 CONTINUE
      ENDIF
```

```
      IF(ITS.EQ.1.OR.IG.LT.3)THEN
*****************************
*     HOMOGENOUS SOIL OR    *
*     UNREINFORCED PART     *
*****************************
      RTYPE(I)='HS'
      ALFA=0.
      ENDIF
*************************
*     REINFORCED  SOIL  *
*************************
      IF(ITS.EQ.2.AND.IG.GE.3) THEN
      RTYPE(I)='RS'
      ALFA=AR*ER/(AC*ESO)
      ENDIF
      A1=1./(ESO*(1.+ALFA))
      C1=1.+ALFA*(1.-VSO*VSO)
      C2=1+ALFA*(1+VSO)
      A7=A1*(1-VSO*VSO/C1)
      B7=A1*(-VSO-VSO*VSO*C2/C1)
      D7=A1*(C1-VSO*VSO*C2*C2/C1)
      CC=A7*D7-B7*B7
      D11=D7/CC
      D12=-B7/CC
      D21=D12
      D22=A7/CC
      D33=ESO/(2*(1+VSO))
      ENDIF
**********************
*     GRAVITY FORCES *
**********************
      IF(IGF.EQ.1) THEN
      DO 15 I1=1,4
      IF(DY(NNOD(I,I1)).GE.1) THEN
         PLOAD(DY(NNOD(I,I1)))=PLOAD(DY(NNOD(I,I1)))-EE(I,5)*T*A*B
      ENDIF
   15 CONTINUE
      ENDIF
      ENDIF
*************************************
*     STIFFNESS MATRIX FOR ALL CASES *
*************************************
      S(1,1)=T*(D11*COF+D33/COF)/3
      S(2,1)=T*(D11*COF/2-D33/COF)/3
      S(3,1)=T*(-D11*COF+D33/(2*COF))/3
      S(4,1)=T*(-D11*COF/2-D33/(2*COF))/3
      S(5,1)=T*(D12+D33)*0.25
      S(6,1)=T*(-D12+D33)*0.25
      S(7,1)=T*(D12-D33)*0.25
      S(8,1)=T*(-D12-D33)*0.25
      S(5,5)=T*(D22/COF+D33*COF)/3.
      S(6,5)=T*(-D22/COF+D33*COF/2)/3.
      S(7,5)=T*(D22/(2*COF)-D33*COF)/3.
      S(8,5)=T*(-D22/(2*COF)-D33*COF/2)/3.
      S(2,2)=S(1,1)
      S(3,2)=S(4,1)
      S(4,2)=S(3,1)
```

```
      S(5,2)=S(7,1)
      S(6,2)=S(8,1)
      S(7,2)=S(5,1)
      S(8,2)=S(6,1)
      S(3,3)=S(1,1)
      S(4,3)=S(2,1)
      S(5,3)=S(6,1)
      S(6,3)=S(5,1)
      S(7,3)=S(8,1)
      S(8,3)=S(7,1)
      S(4,4)=S(1,1)
      S(5,4)=S(8,1)
      S(6,4)=S(7,1)
      S(7,4)=S(6,1)
      S(8,4)=S(5,1)
      S(6,6)=S(5,5)
      S(7,6)=S(8,5)
      S(8,6)=S(7,5)
      S(8,7)=S(7,5)
      S(7,7)=S(5,5)
      S(8,7)=S(6,5)
      S(8,8)=S(5,5)
      DO 1 II=1,8
      DO 1 IJ=II,8
      S(II,IJ)=S(IJ,II)
    1 CONTINUE
      IF(IR.EQ.1) GOTO 7
C************************
    3 IF(I4.EQ.1) THEN
      CC=1
      DD=1
      ELSE
      IF(I4.EQ.2) THEN
      CC=-1
      DD=1
      ELSE
      IF(I4.EQ.3) THEN
      CC=1
      DD=-1
      ELSE
      CC=-1
      DD=-1
      ENDIF
      ENDIF
      ENDIF
      C1=1/(4*A*B)
      G(1,1)=B*(1+CC)*C1
      G(1,2)=B*(1-CC)*C1
      G(1,3)=-G(1,1)
      G(1,4)=-G(1,2)
      G(2,5)=A*(1+DD)*C1
      G(2,6)=-G(2,5)
      G(2,7)=A*(1-DD)*C1
      G(2,8)=-G(2,7)
      G(3,1)=G(2,5)
      G(3,2)=G(2,6)
      G(3,3)=G(2,7)
```

```
      G(3,4)=G(2,8)
      G(3,5)=G(1,1)
      G(3,6)=G(1,2)
      G(3,7)=G(1,3)
      G(3,8)=G(1,4)
      SS(1,1)=D11*G(1,1)
      SS(1,2)=D11*G(1,2)
      SS(1,3)=D11*G(1,3)
      SS(1,4)=D11*G(1,4)
      SS(1,5)=D12*G(2,5)
      SS(1,6)=D12*G(2,6)
      SS(1,7)=D12*G(2,7)
      SS(1,8)=D12*G(2,8)
      SS(2,1)=D12*G(1,1)
      SS(2,2)=D12*G(1,2)
      SS(2,3)=D12*G(1,3)
      SS(2,4)=D12*G(1,4)
      SS(2,5)=D22*G(2,5)
      SS(2,6)=D22*G(2,6)
      SS(2,7)=D22*G(2,7)
      SS(2,8)=D22*G(2,8)
      DO 11 II=1,8
      SS(3,II)=D33*G(3,II)
   11 CONTINUE
  100 FORMAT(///,20X,'*** EXECUTION IS COMPLITED ***',///)
    7 RETURN
      END
```

```
INE SYSTEM      74/810   OPT=0,ROUND= A/ S/ M/-D,-DS      FTN 5.1+628
G/-OT,ARG=-COMMON/-FIXED,CS= USER/-FIXED,DB=-TB/-SB/-SL/ ER/-ID/-PMD/-ST,-
FSOILR2,L=L,B=B.

C*****************************
C      SYSTEM STIFFNESS MATRIX
C*****************************
C
C      THIS SUBROUTINE ASSEMBLES
C      THE   SYSTEM   STIFFNESS
C      MATRIX  IN  BANDED  FORM.
C*****************************
      SUBROUTINE SYSTEM(I,N,NB)
      PARAMETER(ND1=250)
      PARAMETER(ND2=550)
      PARAMETER(ND3=15000)
      COMMON/B2/NNOD(ND1,4),ICOD(ND1,8),PLOAD(ND2)
      COMMON/B6/SS(3,3),G(3,3),S(8,8)
      COMMON/B7/UNKN(ND2),BANTH(ND3)
      DO 6 K=1,8
      IF(ICOD(I,K).NE.0) THEN
      M=ICOD(I,K)
      IF(M.LE.(N-NB+1)) THEN
        I1=(NB+1)*(M-1)+1
      ELSE
        I1=(NB+1)*(N-NB+1)+1
        DO 10 I2=M-1,N-NB+2,-1
   10   I1=I1+(N-I2)+2
      ENDIF
      DO 7 L=1,8
      IF(ICOD(I,L).EQ.0) GO TO 7
      J=ICOD(I,L)
      IF(J.LT.M) GO TO 7
      IF(M.LE.(N-NB).AND.J.GT.(M+NB-1)) GO TO 7
      J=J-M
      BANTH(I1+J)=BANTH(I1+J)+ S(K,L)
    7 CONTINUE
      ENDIF
    6 CONTINUE
      RETURN
      END
```

```
C*********************************
C     GAUSS ELIMINATION
C*********************************
C
C     SOLUTION OF  EQUATIONS USING
C     GAUSS ELIMINATION METHON FOR
C     BANDED MATRIX FORM.
C*********************************

      SUBROUTINE GSELFB(N,NB,LN)
      IMPLICIT INTEGER (S)
      PARAMETER(ND1=250)
      PARAMETER(ND2=550)
      PARAMETER(ND3=15000)
      COMMON/B2/NNOD(ND1,4),ICOD(ND1,8),B(ND2)
      COMMON/B7/C(ND2),A(ND3)
      DO 1 I=1,N
      IF(I.LE.(N-NB+1)) THEN
      S5=(NB+1)*I
      ELSE
      S5=(NB+1)*(N-NB+1)
      DO 2 I2=I,N-NB+2,-1
    2 S5=S5+(N-I2+2)
      ENDIF
      A(S5)=B(I)
    1 CONTINUE
      DO 10 I=1,N-1
      IF(I.LE.(N-NB+1)) THEN
      S1=(NB+1)*(I-1)+1
      S2=S1+NB
      ELSE
      S1=(NB+1)*(N-NB+1)+1
      DO 11 I3=I-1,N-NB+2,-1
   11 S1=S1+(N-I3)+2
```

```
      S2=S1+(N-I)+1
      ENDIF
      DO 20 J=I+1,N
      IF(J.LE.(N-NB+1)) THEN
      S3=(NB+1)*(J-1)+1
      S5=S3+NB
      ELSE
      S3=(NB+1)*(N-NB+1)
      DO 13 I3=J-1,N-NB+2,-1
   13 S3=S3+(N-I3)+2
      S3=S3+1
      S5=S3+N-J+1
      ENDIF
      K1=J-I+1
      S4=S3+S2-S1-K1
      T=A(S1+J-I)/A(S1)
      K2=J-I
      DO 25 K=S3,S4
   25 A(K)=A(K)-T*A(S1+K-S3+K2)
      IF(S3.GT.S4) GO TO 20
      A(S5)=A(S5)-T*A(S2)
   20 CONTINUE
   10 CONTINUE
      DO 34 I=N,1,-1
      IF(I.LE.(N-NB+1)) THEN
      S1=(NB+1)*(I-1)+1
      S2=S1+NB
      ELSE
      S1=(NB+1)*(N-NB+1)+1
      DO 35 I3=I-1,N-NB+2,-1
   35 S1=S1+(N-I3)+2
      S2=S1+(N-I)+1
      ENDIF
      DD=0
      DO 32 L=I+1,I+S2-S1-1
      DD=DD+C(L)*A(S1+L-I)
   32 CONTINUE
      C(I)=(A(S2)-DD)/A(S1)
   34 CONTINUE
      RETURN
      END
```

```
************************************************
      CALCULATION OF STRESSES AND STRAINS
************************************************
      SUBROUTINE STRESS(I,I4,DD)
      PARAMETER(ND1=250,ND2=550,ND3=15000)
      COMMON/B1/X(ND2),Y(ND2),DX(ND2),DY(ND2)
      COMMON/B2/NNOD(ND1,4),ICOD(ND1,8),PLOAD(ND2)
      COMMON/B6/SS(3,8),G(3,8),S(8,8)
      COMMON/B5/SIG(ND1,4,3),EPS(ND1,4,7),AVS(ND1,9)
      DIMENSION DD(8)
      DO 1 II=1,3
      DO 1 JJ=1,8
      SIG(I,I4,II)=SIG(I,I4,II)+SS(II,JJ)*DD(JJ)
      EPS(I,I4,II)=EPS(I,I4,II)+G(II,JJ)*DD(JJ)
    1 CONTINUE
      BB1=SIG(I,I4,1)
      BB2=SIG(I,I4,2)
      BB3=SIG(I,I4,3)
      EPS(I,I4,4)=(BB1+BB2)/2+SQRT((((BB1-BB2)/2)*(BB1-BB2)/2)+BB3*BB3)
      EPS(I,I4,5)=(BB1+BB2)/2-SQRT((((BB1-BB2)/2)*(BB1-BB2)/2)+BB3*BB3)
      EPS(I,I4,6)=(EPS(I,I4,4)-EPS(I,I4,5))/2
      IF(BB1.EQ.BB2) THEN
        IF(BB3.GT.0) THEN
        EPS(I,I4,7)=45.0
        ELSE
        EPS(I,I4,7)=-45.0
        ENDIF
      ELSE
        BB=2.*BB3/(BB1-BB2)
        CB=ATAN(BB)*0.5
        EPS(I,I4,7)=ATAN(BB)*90./ACOS(-1.)
      ENDIF
      RETURN
      END
```

```
C
C
C
      SUBROUTINE PLOTING(M)
      PARAMETER(ND1=250)
      PARAMETER(ND2=550)
      COMMON/B1/X(ND2),Y(ND2),DX(ND2),DY(ND2)
      COMMON/B2/NNOD(ND1,4),ICOD(ND1,8),PLOAD(ND2)
      CHARACTER*1 IUNI,DFS*1,HEAD*20,NAME*4
      PRINT*,' ENTER XLLIM,YLLIM  '
      READ*, XLLIM,YLLIM
      PRINT*,' ENTER XULIM,YULIM '
      READ*, XULIM,YULIM
C     PRINT *,'  ENTER CHARACTER SIZE SCALE FACTOR, SCZ'
C     READ *,SCZ
      PRINT *,' DO YOU WANT A HARD COPY FILE TO BE GENERATED ? '
      READ(*,183) IUNI
  183 FORMAT(A1)
      IF(IUNI.EQ.'Y') THEN
      PRINT *,'  ENTER PLOTTING SCALE FACTOR FAC '
      READ *,FAC
      ENDIF
      PRINT*,'  ENTER FILE NAME '
      READ*,NAME
      CALL INITIG(.TRUE.,.TRUE.,4HNAME)
      CALL SPLIM(XLLIM,YLLIM,XULIM,YULIM)
      CALL SPPORT(XLLIM,YLLIM,XULIM,YULIM)
      IF(IUNI.EQ.'Y') THEN
      CALL UNION
      CALL FACTOR(FAC)
      ENDIF
C*******************
C     PLOTTING
C*******************
C     CALL SMCSIZ(.00725*10.*SCZ,.0125*SCZ)
C     CALL MOVEA(0.,YULIM-2.)
C     CALL TEXT(6,'FINITE')
C     CALL SMSTYL(LL)
      DO 33 IL=1,M
      CALL MOVEA(X(NNOD(IL,1)),Y(NNOD(IL,1)))
      CALL DRAWA(X(NNOD(IL,2)),Y(NNOD(IL,2)))
      CALL DRAWA(X(NNOD(IL,4)),Y(NNOD(IL,4)))
      CALL DRAWA(X(NNOD(IL,3)),Y(NNOD(IL,3)))
      CALL DRAWA(X(NNOD(IL,1)),Y(NNOD(IL,1)))
   33 CONTINUE
C     CALL MOVEA(0.,-1.)
C     CALL SMSYM(5)
C     DO 8 I=1,NH
C     BB(I)=-1.
C   8 HH(I)=H(I,1)
C     CALL PLOTA(NH,HH,BB,.TRUE.)
C     DO 6 I=1,NH-1
C     DF=H(1,I+1)-H(1,I)
C     CALL MOVEA(H(1,I)+DF/2.,-0.5)
C   6 CALL TEXT(4,DF)
```

```
      IF(IUNI.EQ.'Y') THEN
      CALL UNIOFF
      END IF
      CALL AWTKEY(1,ITIRG,1,NCHAR,ICHAR)
      CALL CLRPT
      CALL QUITIG(.TRUE.)
      RETURN
      END
```