RADAR CROSS SECTION OF A CONDUCTING FLAT PLATE BY THE METHOD OF MODIFIED THEORY OF PHYSICAL OPTICS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
ÇANKAYA UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
ELECTRONIC & COMMUNICATION ENGINEERING

SEPTEMBER 2010

Title of the Thesis : **Radar Cross Section of a Conducting Flat Plate by the Method of Modified Theory of Physical Optics**

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STATEMENT OF NON-PLAGIARISM

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

RADAR CROSS SECTION OF A CONDUCTING FLAT PLATE BY THE METHOD OF MODIFIED THEORY OF PHYSICAL OPTICS

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September 2010, 51 pages

In this thesis, reflected scattered fields from a conducting rectangular flat plate, and its edges and corners are observed, calculated and analyzed by using the Method of Modified Theory of Physical Optics. In the calculations, two methods, named “Stationary Phase Point Method” and “Edge Point Method”, are used to solve scattering integrals manually. Numerical results of the calculations are evaluated by plotting them in the MATLAB tool.

Keywords: Electromagnetic Fields, Electromagnetic Diffraction, Electromagnetic Scattering, Modified Theory of Physical Optics, Conducting Rectangular Flat Plate

ÖZ

FİZİKSEL OPTİĞİN DEĞİŞTİRİLMİŞ TEORİSİ METODU İLE İLETKEN DÜZ PLAKANIN RADAR KESİT ALANININ İNCELENMESİ

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Yüksek Lisans, Elektronik ve Haberleşme Mühendisliği Anabilim Dalı
Tez Yöneticisi : Doç.Dr. Yusuf Ziya Umul

Eylül 2010, 51 sayfa

Bu tezde, bir iletken dikdörtgen düz plakadan ve onun köşeleri ile kenarlarından yansıyan saçılan alanlar, Fiziksel Optiğin Değiştirilmiş Teorisi Metodu kullanılarak gözlemlenmiş, hesaplanmış ve incelenmiştir. Bu hesaplamalarda, saçılma integrallerini elle çözmek için "Stasyonel Faz Noktası Metodu" ve "Köşe Noktası Metodu" isimli iki metod kullanılmıştır. Hesaplamaların nümerik sonuçları Matlab aracında çizdirilerek değerlendirilmiştir.

Anahtar Kelimeler: Elektromanyetik Alanlar, Elektromanyetik Kırınım, Elektromanyetik Saçılım, Fiziksel Optiğin Değiştirilmiş Teorisi, İletken Dikdörtgen Düz Plaka

ACKNOWLEDGMENTS

I would like to thank my supervisor Assoc. Prof. Dr. Yusuf Ziya Umul for his invaluable supervision, guidance and suggestions throughout the development of this thesis. His uninterrupted support always encouraged me to continue my studies.

Thanks to my friends, because their patience, friendships and understanding behaviors made me relaxed and quiet during the study.

I would also like to thank sincerely TÜBİTAK for the great support and help during my research about the studies.

My special thanks are for my family. I am deeply grateful to them for their encouragement, trust and endless support in all conditions.

Finally, thanks to all people who helped me during this thesis but I unfortunately forgot to mention about them here.

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LIST OF ABBREVIATIONS

EP	Edge Point
GO	Geometrical Optics
GTD	Geometrical Theory of Diffraction
MATLAB	Matrix Laboratory
MTPO	Modified Theory of Physical Optics
PO	Physical Optics
PTD	Physical Theory of Diffraction
RCS	Radar Cross Section
SPP	Stationary Phase Point
UAT	Uniform Asymptotic Theory
UTD	Uniform Theory of Diffraction

CHAPTER 1

INTRODUCTION

1.1 Background of the study

Diffraction is the process of breaking up or bending of waves (e.g. light waves) when they encounter an aperture or an obstacle. The problem of diffraction of a conducting half plane was first studied by Sommerfeld in 1896 [1]. Sommerfeld's study can be accepted as the basis of the electromagnetic scattering literature. After his studies on the subject, many have been made about this problem since 19^{th} century; some examples can be given as [2, 3, 4, 5 and 19]. Sommerfeld's exact solution is the expression of Fresnel integrals [15, 26] and it consists of the sum of two waves, Geometrical Optics (GO) fields and diffracted fields.

In Physics, when an object in space is assumed, its boundary conditions become considerable. They can be taken into account by using accurate geometry of the object. It can be accepted as an obstacle for electromagnetic waves. The observed electromagnetic waves without any obstacle in the space are named as the "incident waves". When electromagnetic waves encounter an obstacle in space, "reflections" occur from its surface, and "diffractions" occur at its edges, and more, "edge diffraction" occurs at its tangential regions.

"Diffracted fields" appear from the interactions of discontinuities, in other words, diffracted fields are discontinuous.

"GO fields" are the sum of all electromagnetic fields except diffracted fields, i.e. GO fields are the sum of incident, reflected and transmitted fields. The GO method explains that waves go ahead linearly along the rays, therefore GO fields are discontinuous.

"Scattered fields" are the sum of GO fields and diffracted fields, and therefore continuous. This is due to the characteristic of the Fresnel function. Diffracted fields compensate the discontinuity of GO fields. The amplitude value of diffracted fields is always the half of the amplitude value of GO fields in transition regions.

In the electromagnetic scattering literature, there are two types of high frequency methods to evaluate electromagnetic diffracted fields: One is ray based methods including Geometrical Optics (GO), Geometrical Theory of Diffraction (GTD), Uniform Theory of Diffraction (UTD), and Uniform Asymptotic Theory (UAT) [2, 5, 10, 11, 20]; and the other is current based methods known as Physical Optics (PO), Physical Theory of Diffraction (PTD), and Modified Theory of Physical Optics (MTPO) [3, 4, 6, 13, 15].

The GO method, a ray based method, can be simply shown as the amplitude of wave multiply by an exponential term [26]. Despite this method is able to find the fields in illuminated areas, it can not solve the fields in shadow regions. To calculate the fields in a shaded area, diffracted waves must be considered. Keller noticed the lack of this method when applying it to the edges, corners and boundary surfaces [5]. According to him, the GO method could not express the fields at these regions, but the method he developed, named GTD, does [5]. In spite of being very well-known and usage of some successful asymptotic techniques, according to GO method, fields become invalid in transition regions in the evaluation of scattered fields and in the asymptotic evaluation of the diffracted fields. Since the denominator of the equation becomes zero, fields go to the infinity. Keller in 1952 tried to clarify this point [5]. According to him, GTD coefficients are non-uniform, and therefore they become zero in transition regions. The advantage of his clarification is that it makes easy to understand features of scattered fields. However, GTD method can not calculate the fields on transition regions [4, 20]. The UTD method formulated by Pathak and Kouyoumjian in 1974 [2], and the UAT method developed by Ahluwalia, Lewis and Boersma [10, 11] can deal with this problem. While UAT obtains uniform solution by modifying the GO fields, UTD modifies the diffracted fields for the continuity across shadow and reflection boundaries by multiplying the diffraction coefficient by a Fresnel integral [16]. But the UTD and UAT methods are still insufficient to calculate the diffracted fields at foci, caustics and shadow regions, because the amplitude of approximate solution becomes infinity at caustics and there are no real rays passing through shadow regions [18]. To cope with this insufficiency, the PO and PTD methods, current based methods, are suggested by Macdonald and Ufimtsev, respectively [27, 3].

Diffraction phenomenon has been popular for many researchers and has been studied in many aspects for many years. Satterwhite [21], for example, obtained diffraction from angular sector of a perfectly conducting quarter plane in spheroconal coordinates. He firstly derived the dyadic Green's function by using the method of the separation of variables [22]. He used the "eigenfunction expansion method" to get the exact solution. Hansen obtained a corner diffraction coefficient based on exact solution of a canonical scattering problem [23]. He obtained induced surface current density near a corner on a scatterer by calculating physical optics current, fringe wave current and corner current. Zhang et. al. [24] dealt with the corner diffraction formula which can be derived by evaluating the radiation integral asymptotically by using (ordinary) equivalent edge currents. Different from ordinary equivalent edge currents, he used the UTD method, and therefore he got the result that was uniformly continuous through the entire far field region. Sıkta et al. [25] used the modified equivalent current concept to find corner diffraction scattering from flat plate structures. As another example, Hill and Pathak [20] used the UTD method to evaluate the electromagnetic diffraction in the corner of a plane angular sector. In this study, apart from incident, reflected and edge diffracted fields, they have defined a uniform corner diffraction term and by calculating this term, they have found the UTD corner diffracted field term. As it is seen, the function of the diffracted fields that all above researchers have been interested in is to compensate the discontinuities of the GO fields at the transition regions [9].

Two different edge diffracted waves occur in a diffraction problem of a conducting surface [26]: One of them is incident diffracted waves compensating the discontinuities of the incident fields at the shadow boundary. The other is the reflected diffracted fields compensating the discontinuities of the reflected fields at the reflection boundary.

According to the PO method, incident fields form a surface current on surface body and this current is integrated over the surface body. The current on the surface body acts as a secondary source of electromagnetic fields and this results a current on lit sides, but not on shaded sides. The PO method is defined for perfectly conducting surfaces, but it does not give the exact solution for diffraction problems [3]. An enhanced version of the PO method, named as PTD, is firstly discussed by Ufimtsev [3], and is able to evaluate the fields at foci, caustics and shadow boundaries and to give the asymptotic solution of diffraction. To calculate and evaluate diffraction effects on the fields, a non-uniform current, called *fringe current*, is added to the ordinary uniform current. The PTD method uses fringe current (a non-uniform current) addition to real physical optics current (an ordinary uniform current) appearing on the surface of a scatterer.

MTPO, developed by Umul [4], is a method capable of finding the exact diffracted waves for scattering problems caused by conducting geometries. The MTPO method eliminates the erroneous edge effect on the scattering integral [4]. Since a complementary aperture surface is taken into account in addition to the surface of the scatterer, the MTPO method can evaluate the total scattered fields.

Radar Cross Section (RCS) is a reflecting factor of electromagnetic radar waves of a body [8]. The σ term is the RCS which is equal to

$$\sigma = \lim_{r\to\infty} 4\pi r^2 \frac{S_s}{S_i} = \lim_{r\to\infty} 4\pi r^2 \frac{|E_s|^2}{|E_i|^2} \tag{1.1}$$

where S_i, S_s, E_i, E_s and r are the incident power density, scattered power density, incident electric field, scattered field and the distance from target to observation point, respectively [8].

1.2 Objectives of the thesis

The aim of this thesis is to obtain the electromagnetic scattered fields at all points of a conducting rectangular flat plate by using the MTPO method. The reflected scattered fields at the edges, corners and at the surface of the plate are firstly observed, then by using Matlab tool, these fields are plotted and evaluated.

1.3 Organization of the thesis

This thesis contains five chapters and is organized as follows:

Chapter 1 provides the introduction of the thesis. It consists of three subparts; a background of the study, objectives of the thesis and organization of the thesis.

Chapter 2 is an overview of the study. It has three subparts. The first mentions about the PO method and the second is the MTPO method used in this thesis, and the third is about the geometry of this study; the fields, the angles etc.

Chapter 3 discusses the reflected scattered fields. It is composed of four subparts. The first three parts mention about the reflected scattered fields at the surface of the plate, at the edges and at the corners, respectively. The last subpart is a comparison of this study with literature.

Chapter 4 contains numerical results. This chapter evaluates the results given in Chapter 3. In other words, it has Matlab results and comments on the figures.

Chapter 5 is a conclusion part. This chapter summarizes the main points and results of the study together.

CHAPTER 2

OVERVIEW

2.1 Physical optics

According to the PO method, incident fields form a surface current on surface body and this current $\vec{J}_{PO}$ is integrated over the surface body. Figure 1 gives the PO structure. $\vec{H}_i$ is the incident magnetic field at the surface.

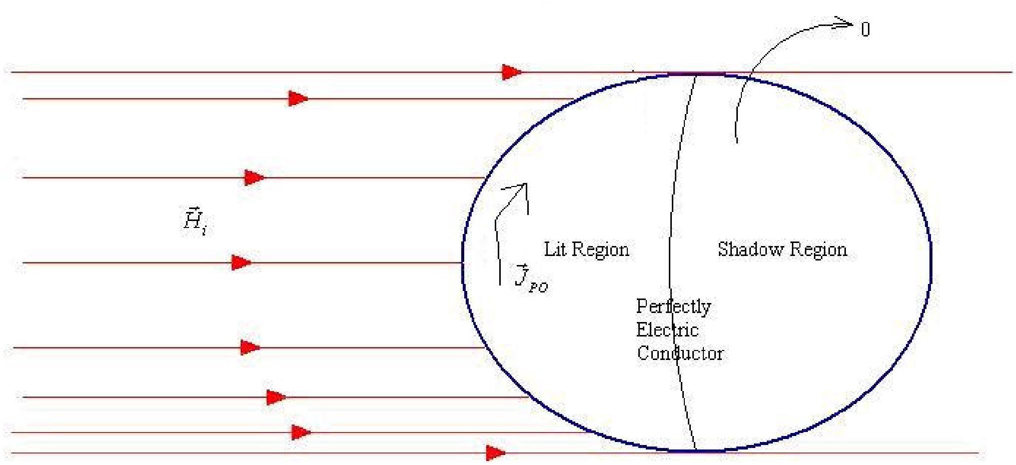

Figure 1 Physical optics

The current on the surface body acts as a secondary source of electromagnetic fields and this results a current on lit sides, but not on shaded sides.

The first condition is given as

$$\vec{n}x\vec{E}\Big|_S = 0 \tag{2.1}$$

where $\vec{n}$ is the unit vector of the surface and inside the conductor there is neither magnetic fields nor electric fields, so

$$\vec{H}_{inside} = \vec{E}_{inside} = 0 \tag{2.2}$$

Since this is a perfectly electric conductor, reflection occurs. The fields reflect to space as if the source is this perfectly electric conductor. This reflection fields can be found by using this method. In the perfectly electric conductor there is no charge density, but there is current density. This current density is evaluated, because this is the reason of radiation. For perfectly electric conductor second boundary condition is

$$\vec{J}_{PO} = \vec{n}x\vec{H}_T\Big|_S \tag{2.3}$$

where

$$\vec{H}_T = \vec{H}_i + \vec{H}_r \tag{2.4}$$

which gives the total magnetic fields.

$$\vec{H}_i\Big|_S = \vec{H}_r\Big|_S \tag{2.5}$$

where $\vec{H}_r$ is the reflected field at the surface. According to Eqn. (2.4), one obtains

$$\vec{H}_T\Big|_S = 2\vec{H}_i\Big|_S \tag{2.6}$$

which means that total magnetic fields are twice of magnetic fields' tangential component at the surface. Eqn. (2.3) can be rewritten as

$$\vec{J}_{PO} = 2\vec{n}x\vec{H}_i\Big|_S . \tag{2.7}$$

In the shadow region, there are no fields. But in the lit region, the current given in Eqn. (2.7) is considered. The physical optics integral can be written as

$$\vec{A} = \frac{\mu_0}{4\pi}\iiint_{V'}\vec{J}(\vec{r}\,')\frac{e^{-jkR}}{R}dV' \tag{2.8}$$

where $\vec{J}(\vec{r}\,')$ is the induced current on the surface, and $\frac{e^{-jkR}}{R}$ is Green's function. By using Eqn. (2.7) one can rearrange Eqn. (2.8) as

$$\vec{A} = \frac{\mu_0}{2\pi}\iiint_{V'}\vec{n}x\vec{H}_i\Big|_S\frac{e^{-jkR}}{R}dV' . \tag{2.9}$$

In PO method, stationary phase of the scattering integrals gives exact geometrical optics fields. But at the edges, EP method gives the edge diffracted fields incorrectly [7]. Also, in the scattering integral only the perfectly conducting surface is considered and the aperture part is neglected. Besides, when reflected and reflected diffracted fields are evaluated, there will not be any information about incident and incident diffracted fields [4].

2.2 Modified theory of physical optics

The MTPO has been proposed by Umul [4] as a new method to find the exact diffracted waves for scattering problems by conducting geometries. It has the advantage of the eliminating of the erroneous edge effect of the PO method on the scattering integral. So, it can be said that the MTPO method is an improved version of the PO method.

By using the MTPO method, one can directly derive the exact scattered fields from the asymptotic evaluation of the surface integral of the PO method [4]. It directly gives the exact edge-diffracted waves. This method was also applied to wedge diffraction problems by Umul [13]. The MTPO integrals which are uniform asymptotic evaluation techniques are divided into two integrals according two their boundaries: Exact diffracted fields and geometrical optics fields constitute the asymptotic evaluation of the MTPO integrals. This method helps to express the integrals in terms of the GO fields and Fresnel functions [12]. Umul has defined three assumptions in this method [4]: One of them is that the reflection angle is a function of integral variables, i.e., it is calculated inside the integral as a variable function. Secondly, he has defined a new unit vector which is not constant and divides the angles between the incident and reflected fields into two equal parts. This variable unit vector does not affect geometrical optics fields since this unit vector is equal to the actual normal vector at the stationary phase point. MTPO gives the exact diffracted fields at the edge point [7]. Finally, he has considered the perfectly conducting surface with an aperture part together.

2.3 Rectangular plate geometry

The reflected scattered fields from a rectangular plate can be calculated and evaluated by using the method of the MTPO. The 3-dimensional geometry of the rectangular plate is given in Figure 2. As it is seen in the figure, the rectangular plate lies down on z axis. Its height in the z axis is assumed as negligible. It has a length from –a to +a in x axis, and similarly width from –b to +b in y axis. λ is the wavelength and there is a condition that the lengths of the rectangular plate which are 2a and 2b should be greater than the value of this wavelength, λ. Also, the distance to the observation point should be greater than the wavelength, λ.

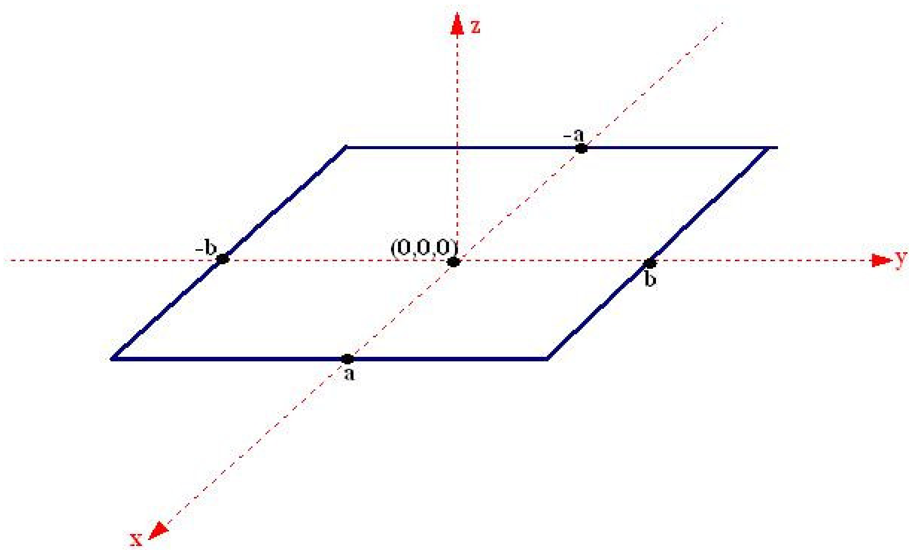

Figure 2 Geometry of rectangular flat plate

In this study, the surface of the rectangular plate is assumed as "soft surface" (providing Dirichlet Condition; the condition that the total of fields over a surface is zero). For a soft surface, a minus sign (-) can be put in front of the scattering integral. If a surface was accepted as "hard surface" (providing Neumann Condition; the condition that the derivative of the total fields with respect to unit vector over the surface is zero), a plus sign (+) would be put in front of the scattering integral.

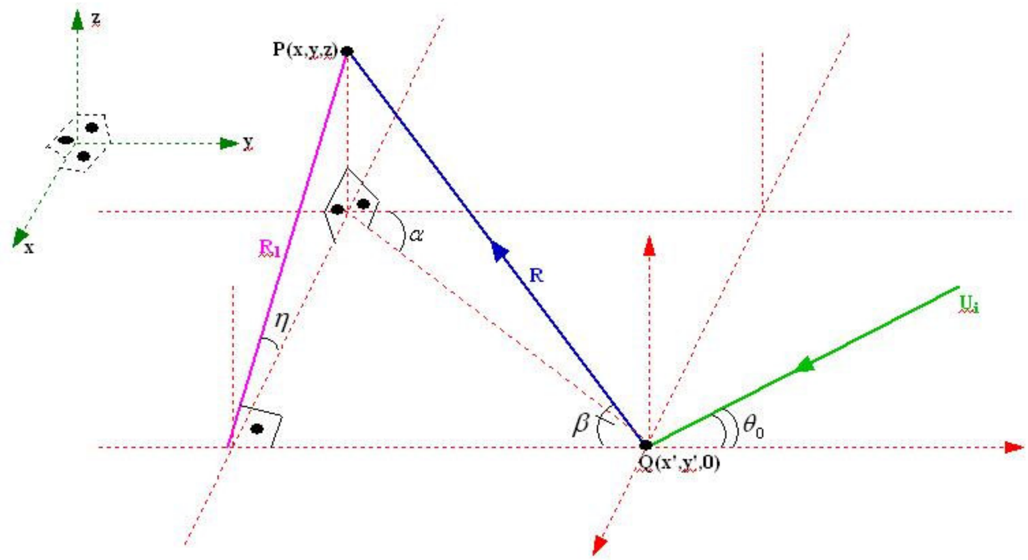

Figure 3 Geometry of incident and reflected scattered fields

Here, θ_0 is the incident angle, η is the angle between x and z axis, and similarly, β is the angle between y and z axis. From the geometry we can calculate R as

$$R = (y - y')\cos\beta + R_1 \sin\beta \tag{2.10}$$

depends on R_1, which is

$$R_1 = (x - x')\cos\eta + z\sin\eta \tag{2.11}$$

in terms of angles that will be explained in Chapter 3 Section 3.1. In Figure 3, P(x,y,z) is the observation point, and U_i is the incident field. It encounters x-y plane at x=x' and y=y'. This point is called Q as shown in Figure 3. The Q point changes on the surface. When R projects to x-y plane, α can be easily found between this projection and y axis which is depicted in Figure 3. The incident electric field is

$$\vec{E}_i = \vec{e}_x E_i e^{jk(y\cos\theta_0 + z\sin\theta_0)} \tag{2.12}$$

and the reflected electric field is

$$\vec{E}_r = \vec{e}_\rho E_r e^{-jk(z\sin\beta + x\cos\beta\cos\alpha - y\cos\beta\sin\alpha)} \tag{2.13}$$

where

$$\vec{e}_\rho = \vec{e}_y \cos\alpha + \vec{e}_x \sin\alpha\,. \tag{2.14}$$

The incident field's direction is

$$\vec{s}_i = -\cos\theta_0 \vec{e}_y - \sin\theta_0 \vec{e}_z \tag{2.15}$$

and the reflected field's direction is

$$\vec{s}_r = \sin\beta\cos\eta\vec{e}_x - \cos\beta\vec{e}_y + \sin\beta\sin\eta\vec{e}_z \tag{2.16}$$

It is mentioned in [4, 6] that the defect of the PO method consists of a *constant* normal vector between the incident field and the diffracted field. The MTPO method eliminates this defect by using a *variable* normal vector $\vec{n}_1$, dividing the directions of incident and reflected fields into two equal angles. It is defined as

$$\vec{n}_1 = a\vec{e}_x + b\vec{e}_y + c\vec{e}_z\,. \tag{2.17}$$

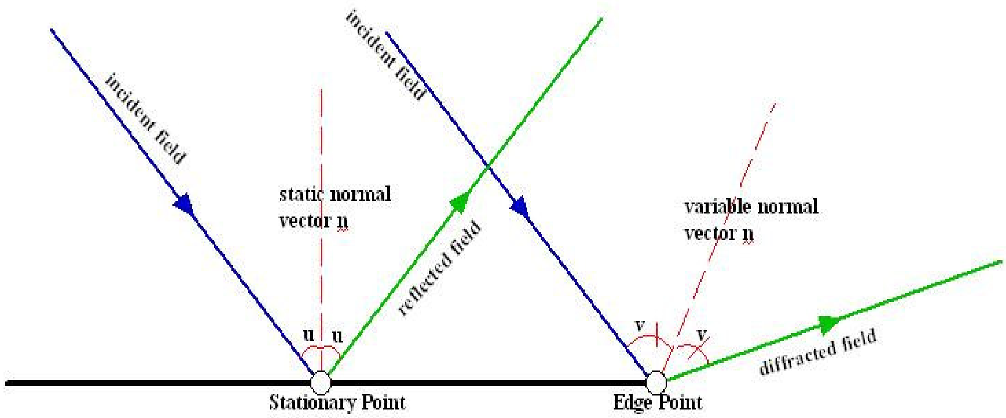

Figure 4 Reflection geometry

The geometry about those directions is depicted in Figure 4 [7]. By using the following cosine equations

$$2\cos^2 u - 1 = \cos 2u = 1 - 2\sin^2 u \,, \tag{2.18}$$

and also considering Figure 5, some equations are found.

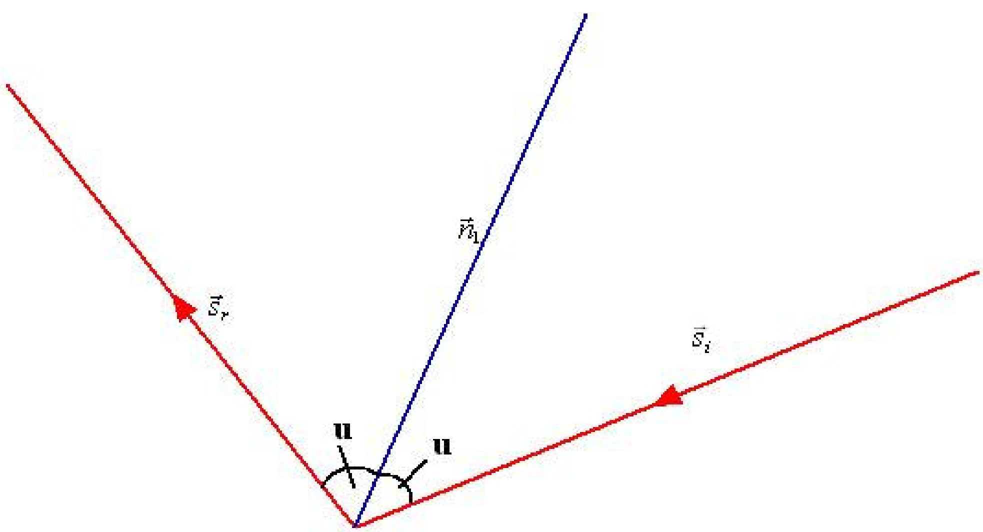

Figure 5 Variable unit vector

These equations consist of the dot product and the cross product of incident and reflected rays with the variable unit vector. These are

$$\vec{n}_1 \cdot \vec{s}_r = \cos u \tag{2.19}$$

and

$$\vec{n}_1 \cdot \vec{s}_i = -\cos u \tag{2.20}$$

and also

$$\vec{s}_i \cdot \vec{s}_r = -\cos 2u \tag{2.21}$$

which are the dot product equations. Also the cross product can be written as

$$\vec{n}_1 \times (\vec{s}_i - \vec{s}_r) = 0\,. \tag{2.22}$$

By solving the Eqns. (2.18), (2.19), (2.20), (2.21) and (2.22) together, a, b and c coefficients in the $\vec{n}_1$ unit vector can be found as

$$a = \frac{-\vec{e}_x(\sin\beta\cos\eta)}{\sqrt{2(1-\cos\beta\cos\theta_0 + \sin\beta\sin\eta\sin\theta_0)}}\,, \tag{2.23}$$

and

$$b = \frac{-\vec{e}_y(\cos\theta_0 - \cos\beta)}{\sqrt{2(1-\cos\beta\cos\theta_0 + \sin\beta\sin\eta\sin\theta_0)}} \tag{2.24}$$

and

$$c = \frac{-\vec{e}_z(\sin\theta_0 + \sin\eta\sin\beta)}{\sqrt{2(1-\cos\beta\cos\theta_0 + \sin\beta\sin\eta\sin\theta_0)}}. \tag{2.25}$$

By inserting Eqns. (2.23), (2.24) and (2.25) into $\vec{n}_1$, we can clearly get $\vec{n}_1$ as

$$\vec{n}_1 = -\frac{\vec{e}_x(\sin\beta\cos\eta) + \vec{e}_y(\cos\theta_0 - \cos\beta) + \vec{e}_z(\sin\theta_0 + \sin\eta\sin\beta)}{\sqrt{2(1-\cos\beta\cos\theta_0 + \sin\beta\sin\eta\sin\theta_0)}}. \tag{2.26}$$

Exact diffraction coefficients can be found from the asymptotic evaluation of the MTPO integrals. The general formula of reflected scattered fields given in [15] is

$$U_{rs}(P) = -U_i \frac{jk}{2\pi} \int_{x'=-a}^{a} \int_{y'=-b}^{b} f(\beta,\eta,\theta_0) \frac{e^{-jkR}}{R} e^{jky'\cos\theta_0} dy' dx'. \tag{2.27}$$

Eqn. (2.27) is a double integral and depends on the angles given in Figure 3 and variable R. Beside of Eqn. (2.10), R can be expressed as

$$R = \sqrt{(x-x')^2 + (y-y')^2 + z^2} \tag{2.28}$$

which has a dependency of Cartesian coordinates. By using the Eqn. (1.1), one can easily find the RCS of the fields given in Eqn. (2.27).

In the Eqn. (2.27), $f(\beta,\eta,\theta_0)$ is equal to $f(\beta,\eta,\theta_0) = \vec{s}_i \cdot \vec{n}_1$ when Figure 3 and Figure 4 are observed. The crucial point of the MTPO method is this scalar product [15] given in Eqn. (2.29). Finally, $f(\beta,\eta,\theta_0)$ becomes

$$f(\beta,\eta,\theta_0) = -\frac{\sqrt{1-\cos\theta_0\cos\beta+\sin\theta_0\sin\beta\sin\eta}}{\sqrt{2}}\,. \qquad (2.29)$$

The calculation of the integral given in Eqn. (2.27) gives us 9 separate integrals' solution. One of them is taken into account for the surface of the rectangular plate whose height is too small to consider. Two integrals are taken into account for two edges of this rectangular plate and other two integrals are considered for its other two edges. The remaining four integrals are measured for the corners which are the points of (a, b), (a,-b), (-a, b) and (-a,-b).

In these calculations, we deal with two types of integral solving method. First, named Stationary Phase Point (SPP) method, is

$$I = \int_a^b f(x)e^{jkg(x)}dx \approx \sqrt{\frac{2\pi}{jk}}\frac{f(x_s)}{\sqrt{g''(x_s)}}e^{jkg(x_s)} \qquad (2.30)$$

where f(x) and g(x) are the real-valued amplitude and phase function, respectively. Also, f(x) indicates the surface current induced on the surface and g(x) is the ray path of the scattered fields [17]. x_s can be found when derivative of g(x) is equal to 0. This method is used to measure the GO fields.

Second method, used in this study, is Edge Point (EP) method equal to

$$II = \int_{x_e}^{\infty} f(x)e^{jkg(x)}dx \approx \frac{1}{jk}\frac{f(x_e)}{g'(x_e)}e^{jkg(x_e)} \qquad (2.31)$$

and it is used for edge points or corners. The EP method gives us the contribution of the edges and corners to the total fields.

According to the Figure 6 ;

(1) can be found by the SPP method twice, it gives the GO fields of the plate (since when the PO fields are evaluated by using the SPP method, the GO fields are obtained);

(2) is the corner point of (-a,-b) and it can be found by using the EP method twice;

(3) is the corner point of (-a,b) and it can be found by using the EP method twice;

(4) is the corner point of (a,-b) and it can be found by using the EP method twice;

(5) is the corner point of (a,b) and it can be found by using the EP method twice;

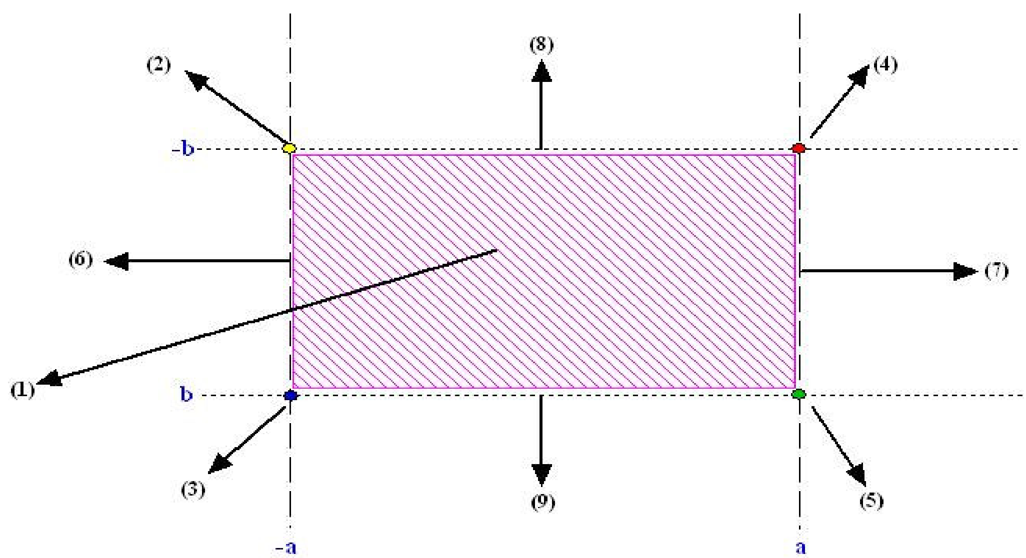

Figure 6 Separation each of the diffraction parts

(6) is the edge of “-a”, it can be found by applying the SPP method to y’ integrand, and by applying the EP method to x’ integrand (at x’=-a point), respectively;

(7) is the edge of “a”, it can be found by applying the SPP method to y’ integrand, and by applying the EP method to x’ integrand (at x’=a point), respectively;

(8) is the edge of “-b”, it can be found by applying the SPP method to x’ integrand, and by applying the EP method to y’ integrand (at y’=-b point), respectively and

(9) is the edge of “b”, it can be found by applying the SPP method to x’ integrand, and by applying the EP method to y’ integrand (at y’=-b point), respectively.

CHAPTER 3

REFLECTED SCATTERED FIELDS

3.1 Reflected scattered fields at the surface the rectangular plate

For the surface of the rectangular plate, the double integral is solved in Eqn. (2.27) by using the SPP method twice.

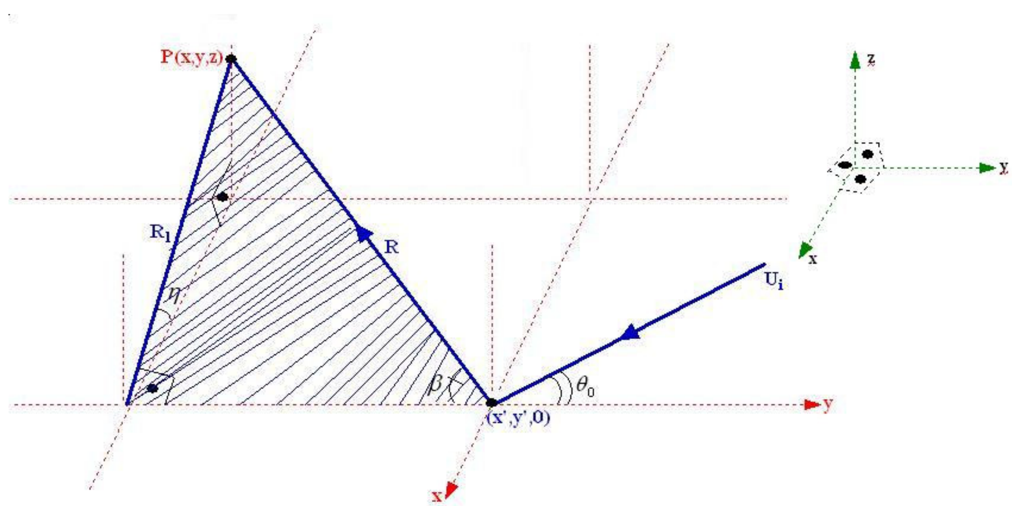

Figure 7 First triangle inside the geometry

As depicted in Figure 7, the equality of

$$\frac{R_1}{\sin\beta} = R = \frac{y - y'}{\cos\beta} \tag{3.1}$$

can be found from sine theorem. Similarly, from Figure 8, it can be seen that

$$R_1 = \frac{z}{\sin\eta} = \frac{x - x'}{\cos\eta}. \tag{3.2}$$

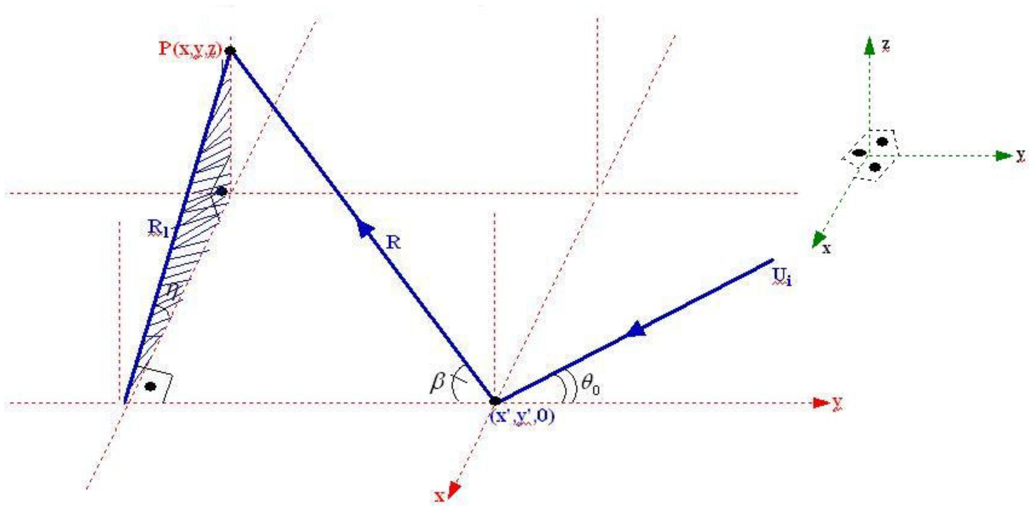

Figure 8 Second triangle inside the geometry

In order to use the SPP method for x' integral, the sine theorem gives us R as

$$R = (y - y')\cos\beta + R_1 \sin\beta \tag{3.3}$$

and

$$R_1 = (x - x')\cos\eta + z\sin\eta \tag{3.4}$$

as mentioned in Chapter 2.3. According to Eqn. (2.27),

$$g(x') = -R = -[(y - y')\cos\beta + (x - x')\cos\eta\sin\beta + z\sin\eta\sin\beta]. \tag{3.5}$$

By equalizing the derivative of g (x') to zero we get η as $\frac{\pi}{2}$.Then the double integral given in Eqn. (2.27) turns into a single integral which is

$$U_{rs}(P) = -\frac{U_i}{2}\sqrt{\frac{jk}{\pi}}\int_{y'=-b}^{b}\sqrt{1-\cos\theta_0\cos\beta+\sin\theta_0\sin\beta}\,\frac{e^{-jkR(x_s)}}{R(x_s)}\,e^{jky'\cos\theta_0}\,dy' \qquad (3.6)$$

where

$$R(x_s) = (y-y')\cos\beta + z\sin\beta\,. \qquad (3.7)$$

The SPP method is applied to remaining y' integral given in Eqn. (3.6) after the first integral is solved. In this calculation, we get

$$g(y') = -z\sin\beta - y\cos\beta - y'(\cos\beta - \cos\theta_0)\,. \qquad (3.8)$$

When we make equal Eqn. (3.8) to zero, it gives $\beta = \theta_0$. By using Eqn. (2.27), we find U_{GO} as

$$U_{GO} = -U_i e^{jk(y\cos\theta_0 - z\sin\theta_0)} \qquad (3.9)$$

where

$$\beta = \arcsin\left[\frac{r\cos\theta}{r^2 + y^2 + 2yr\sin\theta}\right] \qquad (3.10)$$

according to Figure 9 given below. Both the incident and the reflected fields are plane waves in this situation.

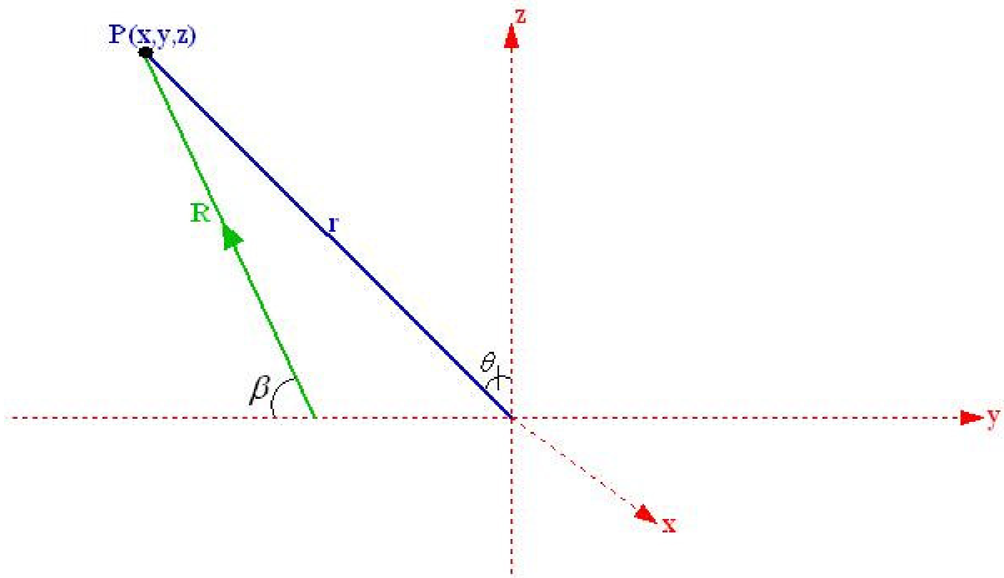

Figure 9 Angles for GO fields for the geometry

3.2 Reflected scattered fields at the edges of the rectangular plate

For the edge of the rectangular plate, the double integral in Eqn. (2.27) is solved by using both the SPP and EP methods, respectively. By following the same way in Section 3.1, using the SPP method to find the fields in the “-b” edge of the rectangular plate, we can find η as $\frac{\pi}{2}$. In the second integral, we use the EP method, since $g'(y_e) = \cos\theta_0 - \cos\beta_e$, in this case $\beta_e \neq \theta_0$. As a result, the fields at the edge “-b” is

$$U_{rs}(P) = U_i(Q_e) D_e \frac{e^{-jkR_1}}{\sqrt{kR_1}} \tag{3.11}$$

where

$$R_1 = (y\cos\beta_e + z\sin\beta_e) + b\cos\beta_e \tag{3.12}$$

and

$$\beta_e = \arcsin[\frac{r\cos\theta}{r^2 + b^2 + 2br\sin\theta}] \quad (3.13)$$

as shown in Figure 10.

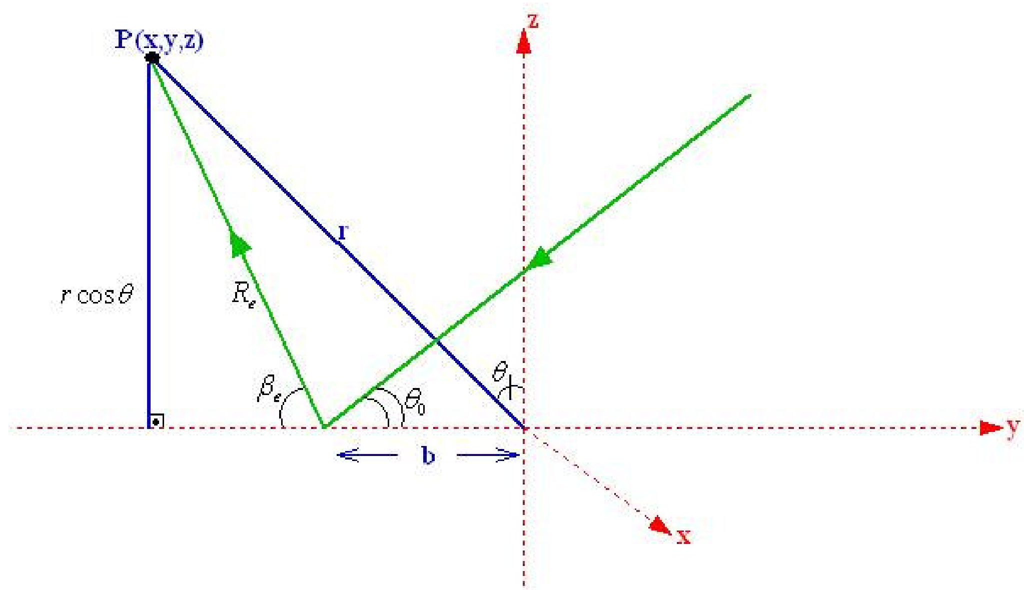

Figure 10 Edge diffraction geometry

In Eqn. (3.11),

$$U_i(Q_e) = -U_i e^{jkb\cos\theta_0} \quad (3.14)$$

and D_e is a coefficient

$$D_e = \frac{\cos(\theta_0 + \beta_e) - 1}{2(\cos\theta_0 - \cos\beta_e)\sqrt{j\pi[1 - \cos(\theta_0 - \beta_e)]}} \quad (3.15)$$

and the remaining coefficient $\frac{e^{-jkR_1}}{\sqrt{kR_1}}$ is cylindrical wave factor. It means that the incident plane wave is scattered as a cylindrical wave causing Keller's Cone [5] in a Q_e point in this situation. Edge diffracted fields exist for the line discontinuities; while corner diffracted fields exist for a point discontinuity [26]. The main reason of Keller's Cone formed from the edge diffracted fields is these two types of discontinuities [5].

In order to find the fields at "+b" edge, the same procedure is applied; firstly SPP method to first integral and EP method to the second at "+b" edge. The reflected scattered fields become

$$U_{rs}(P) = U_i(Q_e) D_e \frac{e^{-jkR_2}}{\sqrt{kR_2}}. \tag{3.16}$$

In Eqn. (3.16),

$$U_i(Q_e) = -U_i e^{jkb\cos\theta_0} \tag{3.17}$$

and D_e is a coefficient

$$D_e = \frac{\cos(\theta_0 + \beta_e) - 1}{2(\cos\theta_0 - \cos\beta_e)\sqrt{j\pi[1 - \cos(\theta_0 - \beta_e)]}}. \tag{3.18}$$

In this case,

$$R_2 = (y - b)\cos\beta_e + z\sin\beta_e \tag{3.19}$$

and

$$\beta_e = \arcsin[\frac{r\cos\theta}{r^2 + b^2 - 2br\sin\theta}] \tag{3.20}$$

and the remaining coefficient $\dfrac{e^{-jkR_2}}{\sqrt{kR_2}}$ is cylindrical wave factor. It means that the incident plane wave is scattered as a cylindrical wave.

The remaining edges "-a" and "a" are calculated by using the same methods, the SPP and EP methods. But for the edge of "-a", the SPP method is applied to y' integral firstly, and

$$g(y') = y'\cos\theta_0 - R \tag{3.21}$$

and also the derivative of g(y') can be found as

$$g'(y') = \cos\beta - \cos\theta_0 = 0 \tag{3.22}$$

Eqn. (3.22) gives us $\beta = \theta_0$. By using this result in the remaining integral, the reflected scattered fields become

$$U_{rs}(P) = U_i(Q_e)D_e \frac{e^{-jkR_3}}{\sqrt{kR_3}} \tag{3.23}$$

where

$$R_3 = (y_s - y)\cos\theta_0 + (x + a)\cos\eta\sin\theta_0 + z\sin\eta\sin\theta_0 \tag{3.24}$$

and

$$y_s = y - \cot\theta_0 \sqrt{(x+a)^2 + z^2}\ . \tag{3.25}$$

In Eqn. (3.23), $U_i(Q_e)$ term is

$$U_i(Q_e) = -U_i e^{jky_s \cos\theta_0} \tag{3.26}$$

and the coefficient D_e is equal to

$$D_e = -\frac{\sqrt{1+\sin\eta}}{2\sin\theta_0 \cos\eta \sqrt{j\pi}} \tag{3.27}$$

and the remaining coefficient $\dfrac{e^{-jkR_3}}{\sqrt{kR_3}}$ is cylindrical wave factor. It means that the incident plane wave is scattered as a cylindrical wave. Similarly, for the edge “a”,

$$U_{rs}(P) = U_i(Q_e) D_e \frac{e^{-jkR_4}}{\sqrt{kR_4}} \tag{3.28}$$

and also R_4, y_s are altered as

$$R_4 = (y_s - y)\cos\theta_0 + (x-a)\cos\eta\sin\theta_0 + z\sin\eta\sin\theta_0 \tag{3.29}$$

and

$$y_s = y - \cot\theta_0 \sqrt{(x-a)^2 + z^2}\ . \tag{3.30}$$

In Eqn. (3.28), $U_i(Q_e)$ term is

$$U_i(Q_e) = -U_i e^{jky_s \cos\theta_0} \tag{3.31}$$

and the coefficient D_e is equal to

$$D_e = -\frac{\sqrt{1+\sin\eta}}{2\sin\theta_0 \cos\eta\sqrt{j\pi}} \tag{3.32}$$

and the remaining coefficient $\dfrac{e^{-jkR_4}}{\sqrt{kR_4}}$ is cylindrical wave factor. It means that the incident plane wave is scattered as a cylindrical wave.

3.3 Reflected scattered fields at the corners of the rectangular plate

In order to find the reflected scattered fields at the corners of the rectangular plate, the double integral in Eqn. (2.27) is solved by using the EP method twice. The integrals at the corners of (a, b), (a,-b), (-a, b) and (-a,-b) should be calculated. In the first corner, (a, b), the fields can be calculated as

$$U_{rs}(P) = U_i(Q_e) D_e \frac{e^{-jkR_5}}{kR_5} \tag{3.33}$$

where

$$U_i(Q_e) = -U_i e^{jkb\cos\theta_0} \tag{3.34}$$

and

$$D_e = \frac{\sqrt{1+\sin\eta\sin\beta\sin\theta_0 - \cos\beta\cos\theta_0}}{2\sqrt{2}\,j\pi\sin^2\beta\cos^2\eta}. \tag{3.35}$$

In Eqn. (3.33),

$$R_5 = (y-b)\cos\beta + (x-a)\cos\eta\sin\beta + z\sin\beta\sin\eta \tag{3.36}$$

The coefficient $\frac{e^{-jkR_5}}{kR_5}$ in Eqn. (3.33) is spherical wave factor. The incident plane wave is scattered as a spherical wave in a Q_e point in this situation.

For the corner of (-a,-b), the reflected scattered fields can be expressed as

$$U_{rs}(P) = U_i(Q_e)D_e\frac{e^{-jkR_6}}{kR_6} \tag{3.37}$$

where

$$U_i(Q_e) = -U_i e^{-jkb\cos\theta_0} \tag{3.38}$$

and

$$D_e = \frac{\sqrt{1+\sin\eta\sin\beta\sin\theta_0 - \cos\beta\cos\theta_0}}{2\sqrt{2}\,j\pi\sin^2\beta\cos^2\eta}. \tag{3.39}$$

In Eqn. (3.37),

$$R_6 = (y+b)\cos\beta + (x+a)\cos\eta\sin\beta + z\sin\beta\sin\eta \tag{3.40}$$

The coefficient $\dfrac{e^{-jkR_6}}{kR_6}$ in Eqn. (3.37) is spherical wave factor. The incident plane wave is scattered as a spherical wave in a Q_e point in this situation.

For the corner of (a,-b), the reflected scattered fields can be expressed as

$$U_{rs}(P) = U_i(Q_e)D_e\frac{e^{-jkR_7}}{kR_7} \tag{3.41}$$

where

$$U_i(Q_e) = -U_i e^{-jkb\cos\theta_0} \tag{3.42}$$

and

$$D_e = \frac{\sqrt{1+\sin\eta\sin\beta\sin\theta_0 - \cos\beta\cos\theta_0}}{2\sqrt{2}j\pi\sin^2\beta\cos^2\eta}. \tag{3.43}$$

In Eqn. (3.41),

$$R_7 = (y+b)\cos\beta + (x-a)\cos\eta\sin\beta + z\sin\beta\sin\eta \tag{3.44}$$

The coefficient $\dfrac{e^{-jkR_7}}{kR_7}$ in Eqn. (3.41) is spherical wave factor. The incident plane wave is scattered as a spherical wave in a Q_e point in this situation.

For the corner of (-a, b), the reflected scattered fields can be expressed as

$$U_{rs}(P) = U_i(Q_e) D_e \frac{e^{-jkR_8}}{kR_8} \quad (3.45)$$

where

$$U_i(Q_e) = -U_i e^{jkb\cos\theta_0} \quad (3.46)$$

and

$$D_e = \frac{\sqrt{1+\sin\eta\sin\beta\sin\theta_0 - \cos\beta\cos\theta_0}}{2\sqrt{2}\, j\pi \sin^2\beta\cos^2\eta}. \quad (3.47)$$

In Eqn. (3.45),

$$R_8 = (y-b)\cos\beta + (x+a)\cos\eta\sin\beta + z\sin\beta\sin\eta \quad (3.48)$$

The coefficient $\frac{e^{-jkR_8}}{kR_8}$ in Eqn. (3.45) is spherical wave factor. The incident plane wave is scattered as a spherical wave in a Q_e point in this situation.

3.4 Comparison with literature

In a similar study, published in 2010 by Moschovitis et. al. [14], the fields scattered from a rectangular plate by using Stationary Phase method approximation are considered.

In this three-dimensional scattering electromagnetic problem, “vector potential A” and “electric field E” are found. In this study, far field, Fresnel field and near field, which can be changed according to the observation distance, are examined. Similar to this thesis, they used the SPP and EP methods. But it is different from this thesis, because they find the fields by subtracting the areas which are not a part of this geometry from the whole space. Moreover, they do not take into account the effect of all angles of the rectangular plate during the double integral calculation. In this thesis, all angles are considered to calculate the reflected scattered fields at all corners and edges, and at the surface of the rectangular plate.

CHAPTER 4

NUMERICAL RESULTS

In this chapter, numerical analysis is evaluated. The total reflected scattered fields are plotted by using MATLAB tool.

In the numerical analysis in Figure 11 the incident angle is taken as $\pi/6$, and r=6λ, where λ is the wavelength of the diffracted wave. The limits of the rectangular plate are taken as λ.

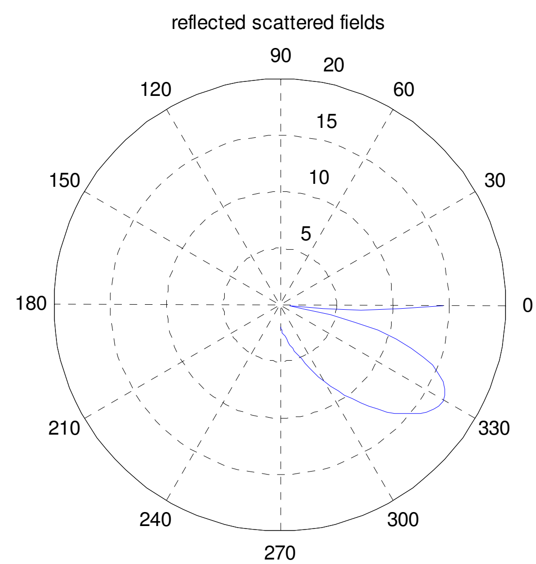

Figure 11 Reflected scattered fields for teta0=pi./6

The plotting is considered for the quarter of the plate, at the region of $\frac{-\pi}{2} < \theta < 0$. As it is seen in Figure 11, incident angle is taken as $\pi / 6$, the reflected scattered field is also formed at vicinity of $\pi / 6$ at the forth region.

In the numerical analysis in Figure 12, the incident angle is taken as $\pi / 3$, and r=6λ, where λ is the wavelength of the diffracted wave. The limits of the rectangular plate are taken as λ. The plotting is considered for the quarter of the plate, at the region of $\frac{-\pi}{2} < \theta < 0$. As it is seen in Figure 12, incident angle is taken as $\pi / 3$, the reflected scattered field is also formed at vicinity of $\pi / 3$ at the forth region.

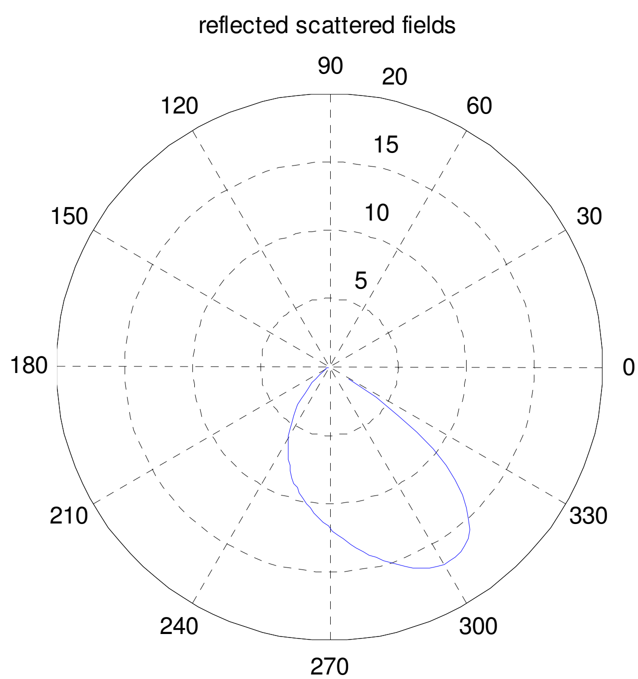

Figure 12 Reflected scattered fields for teta0=pi./3;

In the numerical analysis in Figure 13, the incident angle is taken as $\pi / 4$, and r=6λ, where λ is the wavelength of the diffracted wave. The limits of the rectangular plate are taken as λ. The plotting is considered for the quarter of the plate, at the region of $\frac{-\pi}{2} < \theta < 0$. As it is seen in Figure 13, incident angle is taken as $\pi/4$, the reflected scattered field is also formed at vicinity of $\pi/4$ at the forth region.

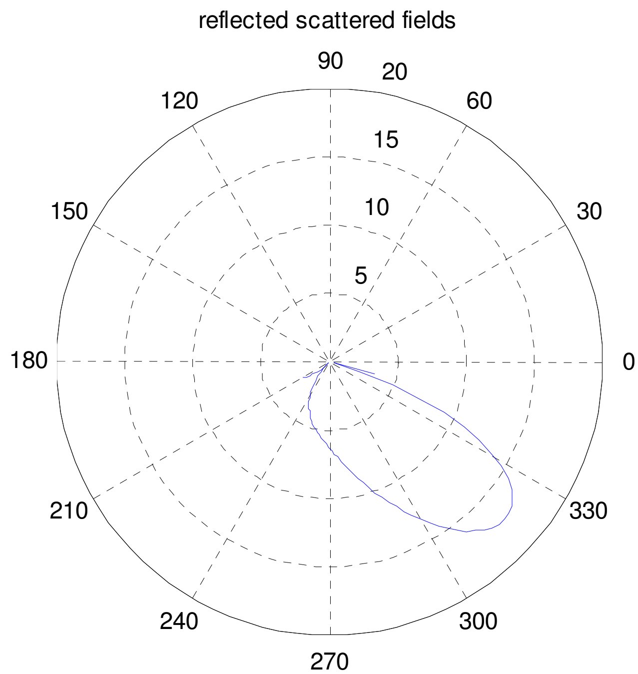

Figure 13 Reflected scattered fields for teta0=pi./4;

We finally evaluated that the numerical analysis results are harmonious with the physical behaviors of the fields.

CHAPTER 5

CONCLUSIONS

In this study, we obtained the solution of the reflected scattered fields from a rectangular flat plate by using the MTPO method. As we have discussed in Section 3.1, at the surface of the plate, we obtained Geometrical Optics field. At the edges given in Section 3.2, we faced with cylindrical waves, and at the corners given in Section 3.3, we obtained spherical waves as expected. Then, these waves are plotted by using MATLAB tool and the reflected scattered fields are analyzed numerically. According to the MATLAB plotting, we finally observed that the results of the numerical analysis are harmonious with the physical behaviors of the fields above.

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APPENDIX

The Matlab code used for the plot of reflected scattered fields is given below;

```
clc
clear all;
l=0.1;
k=2.*pi./l;
teta=-pi./2:pi./100:0;
teta0=pi./6;
fi=-pi./2;
r=6*l;

x=r.*sin(teta).*cos(fi);
y=r.*sin(teta).*sin(fi);
z=r.*cos(teta);

N=100;
sum1=0;
asinir1=-l;
usinir1=l;
delta1=(usinir1-asinir1)./N;
sum2=0;
asinir2=-l./2;
usinir2=l./2;
delta2=(usinir2-asinir2)./N;

for p=1:N
x1=asinir1+(p.*delta1);
for q=1:N
x2=asinir2+(q.*delta2);
R=sqrt(((x-x1).^2)+((y-x2).^2)+(z.^2));
```

```
R1=sqrt(((x-x1).^2)+(z.^2));
beta=asin(R1./R);
eta=asin(z./R1);
A=(-j.*k)./(2.*pi);
K=sqrt((1-
cos(beta).*cos(teta0)+sin(beta).*sin(eta).*sin(teta0))./2);
G=exp(-j.*k.*R)./R;
T=exp(j.*k.*x2.*cos(teta0));
m=A.*G.*T.*K;
sum1=sum1+m;
sum2=sum2+sum1;
end
end
f=sum2.*delta1.*delta2;
polar(teta,abs(f))
title('reflected scattered fields')
```