SOLUTION OF FRACTIONAL BURGER EQUATION AND STUDYING THE STASTICAL PROPERTIES OF THE SOLUTION

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SOLUTION OF FRACTIONAL BURGER EQUATION AND STUDYING THE STASTICAL PROPERTIES OF THE SOLUTION

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ABSTRACT

SOLUTION OF FRACTIONAL BURGER EQUATION AND STATISTICAL PROPERTIES OF THE SOLUTION

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The main purpose of this thesis is to find the solution of fractional Burger equation and study statistical properties of the solution. The solution was found in a truncated series form. As shown in the figure the waves are due to earth quake. The statistical concepts are used to ensure that the solution agrees with nature.

Keywords: Fractional Burger Equation, Statistical Properties.

ÖZ

KESİRLİ BURGER DENKLEMİNİN ÇÖZÜMÜ VE ÇÖZÜMÜN İSTATİSTİKSEL ÖZELLİKLERİ

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Ekim 2014 30 Sayfa

Bu tezin ana amacı kesirli Burger Denkleminin tam çözümünü bulmak ve çözümün istatistiksel özelliklerini incelemektir. Çözüm trunkat seriler formunda bulunmuştur. Şekilde gösterildiği gibi dalgalar deprem sebebiyle oluşmaktadır. İstatistiksel konseptler çözümün doğa ile anlaştığından emin olmak için kullanılmaktadır.

Anahtar Kelimeler: Kesirli Burger Denkleminin Çözümü, İstatistiksel Özellikleri.

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INTRODUCTION

Partial differential equations play an important role in Mathematics, Physics and the solution of these equations is very important because it explains the phenomena represented by these partial differential equations. Burger equation is the characteristic equation and often used in applications for example traffic flow, shock waves and ocean waves [4, 8]. Burger equation is solved numerically by using approximation, Nguyen[18] solved Burger equation numerically with finite spatial domain with boundary conditions; Kaya[9] used the decomposition method to construct the solution in the form of a convergent power series, Javidi [6] solved Burger equation by combination of method of lines and matrix free modified backward differential formula, Jawad[13] used non classical variational 'Cole-Hopf' transformation to solve Burger equation and Omer[16] solved Burger equation numerically by using the 'Variatianol Iteration method'. In this thesis we study and find the exact solution for Burger equation and calculate some statistical properties for this solution.

The main purpose of this thesis is to find the solution of fractional Burger equation and study statistical properties of the solution. The solution was obtained by four steps in the first step we give basic definitions and concepts concerning fractional calculus and statistics. In the second step we solve fractional Burger equation via Liouville definitions of fractional derivatives. In the third step we consider the solution of Burger equation with boundary conditions (Robin, Neumann and Dirchlet) for which the solution is probability density function to study their means, expected value, variance and correlation coefficient ...etc.

We use these statistical concepts to ensure that the solution agrees with the nature of ocean wave. We hope to get the beginning of the wave as a volcano or earth quake. In fourth step we gave conclusion and future work.

1.1 Fractional Calculus [18]

Fractional calculus is widely called as a generalized differ-integration which means arbitrary order (Real, Complex) derivatives and integrals.

1.2 Historical Background

The beginning of fractional calculus dates back to 1695 when G.W.Leibniz wrote a letter from Germany September 30, 1695 to G.A.L Hopital said that $d^{\frac{1}{2}}$ x $= x\sqrt{\frac{dx}{x}}$, which is an apparent paradox and this has been found in volume 2, pp, 301-302, Omls Verlag, Hildesheim, Germany 1962 and which was first published in 1849. After that Leibniz wrote to Wallis to discuss infinite product of π and this has been found in volume 4, pp, 25, Omls Verlag, Hildesheim, Germany 1962 and the first published in 1859. This letter Leibniz mentioned to differential calculus and used $d^{\frac{1}{2}}$ y to derivative of order $\frac{1}{2}$.

In the 18th century exactly in 1730 L. Euler raised this equation where he wrote d^n p, p is the function of x to dx^n can always be expressed algebraically and he asked what kind of ratio can be made if n is fraction . Lagrange's condition in 1772 is the law of exponents (indices): $\frac{d^m}{dx^m}\frac{d^n}{dx^n}$ y = $\frac{d^{m+n}}{dx^{m+n}}$ y, and they asked whether this remains true when m and n are fractional. Laplace in 1812 wrote expressions for certain fractional derivatives.Lacriox in 1819 page developed a formula for arbitrary order derivative through his 700 page text in which he devoted less than two pages to this topic starting with:

$$\frac{d^m\ x^n}{d\ x^m} = \frac{n!}{(n-m)!}x^{n-m} \quad ,n\epsilon N,$$

And replacing m by $\frac{1}{2}$, and n by any positive real number a to get:[2]

$$\frac{d^m\ \ x^n}{d\ \ x^{\frac{1}{2}}} = \frac{\Gamma(a+1)}{\Gamma(a+1)}x^{a-\frac{1}{2}}$$

$$And\ if\ a = 1, then \quad \frac{d^{\frac{1}{2}}x}{d\ x^{\frac{1}{2}}} = 2\sqrt{\frac{x}{n}}$$

The first application of fractional calculus is due to Abel in 1823 who used it in solving an integral equation which arises in the tautochrone problem. This problem is sometime called isochrones problem which is that of finding the shape of a frictionless wire lying in vertical plan such that the time of bead place on the wire slide to the lowest point of the wire is in the same time *regardless of where the bead is placed*. The integral he worked on:

$\int_0^x (x-t)^{\frac{-1}{2}}$ f(t)dt, is precisely the same which Riemann 1847 used for defining fractional integration. Liouvilles in 1832 gave the definition so called Liouvilles first definition. For any function f(x) expended in the series [7].

$$F(x) = \sum_{n=0}^{\infty} c_n\, e^{a_n x} \quad then: \ \frac{d^{\ v} f(x)}{d\ x^v} = \sum_0^{\infty} c_n\ \ a_n^v\, e^{a_n x}$$

Where v is an arbitrary number.

Liouville definition may be applied to the function of the v form

$x^{-a},\ \ a \geq 0$

he considered:

$$\imath =]\ \int_0^{\infty} u^{a-1}\, e^{-xu}\, du \quad , \ \ the\ transformation\ xu\ =\ t\ gives$$

$$\imath = \int_0^{\infty} t^{1-a}\, x^{1-a}\ e^{-t}\, \frac{dt}{x} = x^{-a}\Gamma(a) \quad .$$

$Hence \quad x^{-a} = \frac{1}{\Gamma(a)}$ $\ .\ therefore$:[2]

$$\frac{d^{\ v}\, x^{-a}}{dx^v} = \frac{(-1)^v\Gamma(a+v)}{\Gamma(a)}\, x^{-a-v}$$

Liouville was successful in applying these definitions to problems in the potential theory. These concepts were too narrow because first, the definition is restricted to values of v such that the series converges and second it is not suitable to wide class of functions. Between 1835 and 1850 there was a controversy which centered on two definitions of fractional derivative. Some mathematician favored Lacroxs definition while others favored Liouvilles definition. William observed that the two definitions differ in the derivative of the constant where the derivative of the constant by Lacroxs definition is not zero while it is zero by Liouville 2nd definition. Riemann in 1847 while he was a student gave the definition for fractional integration by :

$$d^{-v}f(x) = \frac{1}{\Gamma(v)} \int_c^x (x-t)^{v-1} f(t)dt + \psi(t) \ .$$

$Letnikov\ in\ 1868\ proved\ that:$

$$(d^q\, d^p\,)f(x) \ = d^{q+p}\, f(x) \quad , p\&q\epsilon R$$

Nekrassov 1888 found the derivative of (x-a)^2 for any order[15]. Schuyler in 1918 asked what interpretation must be given to $\frac{d^{\frac{1}{2}}y}{dx^{\frac{1}{2}}}$ so that $\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}\frac{d^{\frac{1}{2}}y}{dx^{\frac{1}{2}}} = \frac{dy}{dx}$, and this problem was solve by post in 1919. Erdelyi, Kober and Osler, Okikioolu, Saxena, Kalla, Riesz, Bertram Ross, Nishimoto and Podlubny had good contributions to this concept.

1.3 Some Basic Definitions, Concepts:

1.3.1 Basic definitions

Riemann- Liouville fractional integral of order v:

$$f_v^{+}\ (a, x) = \frac{1}{\Gamma(v)} \int_a^x (x-t)^{v-1} f(t)dt \text{ , (right hand)}$$

(1.1)

$$f_v^{-}\ (b, x) = \frac{1}{\Gamma(v)} \int_x^b (t-x)^{v-1} f(t)d\text{, (left hand)}$$

(1.2)

Where a≤ x≤ b, v$> 0, \Gamma$ is the gamma function. Fractional derivative of order v is:

f_v^+ (a, x) $=\frac{d}{dx} f_{1-v}^+$(a,x), (right hand)

(1.3)

f_{-v}^- (x, b)$=\frac{d}{dx} f_{1-v}^-$(x,b), (left hand)

(1.4)

$where$ a≤x≤b,0 ≤v≤ 1.

(D2)Grunewald (Extended by post) Fractional l Derivative:

$$\frac{d^v\ f}{dx^v} = \lim_{N\to\infty}\left(\frac{\frac{X}{N}}{\Gamma(-V)}\sum_i^{N-1}\frac{\Gamma(j-v)}{\Gamma(j+1)}f\left(x-\frac{jx}{N}\right)\right)$$

(1.5)

(D3) Weyl Fractional Integral of order v:

f_v^+ (-∞, x)$=\frac{I}{\Gamma(v)}\int_{-x}^{x}(x-t)^{v-1}$f(t)dt

…………………………………………........…(1.6)

f_v^- (x,∞)$=\frac{I}{\Gamma(v)}\int_{-x}^{x}(x-t)^{v-1}$f(t)dt

……………………………………………….....(1.7)

$where$ f(t) is a periodic function and its mean value for one period is zero. But the formula (1.6) and (1.7) are used as the definition of the integral without any condition at the present time.

(D4) Erdelyi Fractional Integral of order v:

I_x^V f(x) =$\frac{1}{\Gamma(v)}\int_0^x (x-t)^{v-1}$f(t)dt , I_x^0 f(x)= f(x)

(1.8)

k_x^V f(x) =$\frac{1}{\Gamma(v)}\int_x^x (x-t)^{v-1}$f(t)dt , k_x^0 f(x)= f(x

(1.9)

(D5)Kober Fractional Integral of order v(Using Erdelyi s Notation):

$I_x^{\eta-v}$f(x)= $x^{-\eta-v} I_x^v x^{\eta}$ f(x), $\tau I_x^{\eta-O}$ f(x)= f(x)

(1.10)

$k_x^{\eta-V}$ f(x) =$\frac{1}{\Gamma(v)}\int_x^x (x-t)^{v-1}$f(t)dt , $k_x^{\eta-0}$ f(x)= f(x)

(1.11)

(D6) Okikiolu Fractional Integral of Order v:

H_v (f)(x) = $\frac{1}{\varphi(v)}\int_{-x}^{\infty}\frac{t-x^v}{t-x}$ f(t)dt,

(1.12)

k_v (f)(x) = $\frac{1}{\varphi(v)}\int_{-x}^{x} t-x^{v-1}$ f(t)dt

(1.13)

Where φ (v) = 2Γ(v)sin ($\pi v / 2$)

(D7)Saxena Fractional Integral of Order v:

$\mathcal{J}$ [f(x)] = $\mathcal{J}[v,\beta,\gamma,m;f(x)]$

= $\frac{x^{\gamma-1}}{\Gamma(1-v)}\int_0^x F(v,\ \beta+m,\beta,\frac{x}{t})t^{\gamma}f(t)dt$

(1.14)

$\mathcal{R}[f(x) = \mathcal{R}[v, \beta, \delta, m; f(x)]$

$= \frac{x^{\delta}}{\Gamma(1-v)} \int_{x}^{x} F(v,\ \beta + m, \beta, \frac{x}{t}) t^{-\delta-1} f(t) dt$

(1.15)

Where F(v, $\beta, \gamma, x)$ is the ordinary hyper geometric function and v, β, γ, δ are complex parameters .If m= 0, then they are reduced to Kober fractional integral.

(D8)Kalla and Saxena Fractional Integral of Order v:

$\mathcal{J}$ [f(x)] = $\mathcal{J}[v, \beta, \gamma, m, \mu, \eta, , a; f(x)]$

$= \frac{\mu\ x^{-\eta-1}}{\Gamma(1-v)} \int_{0}^{x} F(v, \beta + m, \frac{at^{\mu}}{x^{\mu}}) t^{\eta} f(t) dt$

(1.16)

$\mathcal{R}$ [f(x)] =$\mathcal{R}$ $[v, \beta, \gamma, m, \mu, \delta, a; f(x)]$

$= \frac{\mu\ x^{\delta}}{\Gamma(1-v)} \int_{x0}^{x} F(v, \beta + m, \frac{ax^{\mu}}{t^{\mu}}) t^{-\delta-1} f(t) dt$

(1.17)

Where $v, \beta, \gamma, \eta, \delta$ and *a* are complex parameters.

(D9)M. Riesz Fractional Integral of Order v:

$f_v(x) = \int_{-x}^{x} x - t^{v-1} f(t) dt, 0 < v < 1$

(1.18)

(D10) Thomas J. Osler fractional integral of order v:

$$D_{z-a}^{v} = \frac{\Gamma(v+1)}{2\pi i} \int_{a}^{z+} (t-z)^{-v-1} f t) dt, v \notin z^{-}$$

(1.19) where he made *a* branch cut from *z* to a and the integral curve is an open contour which starts from *a* and encloses *z* in positive sense and return to a.

(D11) Bertram Ross Fractional Integral of Order v:

$$\frac{d^{v}}{dz^{v}} f(z) = \frac{\Gamma(+1)}{2\pi i} \int_{C} \frac{f(t)}{(t-z)^{v+1}} dt$$

(1.20)

Where he made a branch cut from ***z*** to infinity through the origin and integral curve C is an open contour which encloses ***z*** in positive sense and z$\notin C(i.e. C$ is an integral curve along that cut).

(D12) Nishinmoto Definition for Derivative of Order v:

IF f(z) is analytic function and it has no branch point on and inside C (=C,C-) and :

$$\text{C-} f_{v} = C - f_{v}(z) = \frac{\Gamma(v+1)}{2\pi i} \int_{C-} \frac{f(t)}{(t-z)^{v+1}} dt$$

$$= \frac{\Gamma(v+1)}{2\pi i} \int_{C-}^{o+} \eta^{-(V+1)} f(z+\eta) d\eta$$

(1.21)

Where η= t –z; t$\neq z, -\pi \leq \arg(t-z) \leq \pi, v \notin z^{-}$

$$C\_f_{v} = C\_f_{v}(z) = \frac{\Gamma(v+1)}{2\pi i} \int_{C-} \frac{f(t)}{(t-z)^{v+1}} dt$$

$$= \frac{\Gamma(v+1)}{2\pi i} \int_{x}^{o+} \eta^{-(v+1)} f(z+\eta) d\eta$$

(1.22)

Where η = t-z; t$\neq z, -O \leq (t-z) \leq 2\pi, v \notin z^{-}$.

If nϵ positive integers (z) ,C= { C_{-}, C},then:

$$f_{-a} = Cf_{-} \lim_{v \to -n} Cf_{v}$$

Where $C_{-} and\ C_{+}$ are integral curves

1.3.2 Some typical PDE [10,12]

The typical partial differential equation of linear model and nonlinear model as follows:

1.3.2.1 Some linear model equations:

1-The wave equation

$$u_{tt} - \nabla^2 u = 0$$

(1.23)

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

2- The heat or diffusion equation

$$u_t - k\nabla^2 u = 0$$

(1.24)

3 - The Laplace equation

$$\nabla^2 u = 0$$

(1.25)

4- The Poisson equation

$$\nabla^2 u = f(x, y, z)$$

(1.26)

5- The telegraph equation

$$u_{tt} - c^2 u_{xx} + au_t + bu = 0$$

(1.27)

6- The Helmholtz equation

$$\nabla^2 u + \lambda u = 0$$

(1.28)

7- The liner Korteweg-de Vries (or KdV) equation

$$u_{tt} + \alpha u_x + \beta u_{xxx} = 0$$

(1.29)

8- The linear Boussinesq equation

$$u_{tt} - \alpha^2 \nabla^2 u - \beta^2 \nabla^2 u_{tt} = 0$$

(1.30)

9- The inharmonic Wave equation

$$u_{tt} + c^2 \nabla^4 u = 0$$

(1.31)

1.3.2.2 Some nonlinear model equation:

1. The simplest first- order wave (or kinematic wave) equation

$$u_t + c(u) u_x = 0 \quad , \quad x \in \mathbb{R}, t > 0$$

(1.32)

2. The Nonlinear Klein - Gordon equation

$$u_{tt} - c^2 \nabla^2 u + v'(u) =$$

(1.33)

3. The sine – Gordon equation

$$u_{tt} - c^2 u_{xx} + k sinu = 0 \ , \ x \in \mathbb{R}, t > 0$$

(1.34)

4. The Burgers equation

$$u_t + u u_x = v u_{xx} \, , \quad x \in \mathbb{R}, t > 0$$

(1.35)

5. The Fisher equation

$$u_t - vu_{xx} = k(u - \frac{u^2}{k})$$

(1.36)

6. The Boussinesq equation

$$u_{tt} - u_{xx} + (3u^2)_{xx} - u_{xxx} = 0$$

(1.37)

7. The Korteweg-de Vries (KdV) equation

$$u_t + \alpha uu_x + \beta u_{xxx} = 0 , \quad x \in \mathbb{R}, t > 0$$

(1.38)

8. The modified KdV (mKdV) equation

$$u_t - \sigma u^2 u_{xxx} = 0 , \quad x \in \mathbb{R}, t > 0$$

(1.39)

9. The Burgers- Huxley (BH) equation

$$u_t + \alpha uu_x - vu_{xx} = \beta(1 - u)(u - y)u , \quad x \in \mathbb{R}, t > 0$$

(1.40)

1.3.3 Burger equation

For more than sixty years, Burger Equation has been studied and used as a simple model for many physically interesting problems and for convection-diffusion phenomena such as shock waves, turbulence, decaying free turbulence, traffic flows, flow-related problems, etc. The quasilinear parabolic equation first appeared in 1915 paper by Bateman [1], who used the equation as a model for the motion of a viscous fluid when the viscosity approaches zero, and derived two types of steady state solutions for infinite domain problem. More than thirty years later, Johannes Martinis Burgers introduced the equation in his attempt to formulate a simple mathematical model that would show the fundamental features exhibited by the turbulence in hydrodynamic flows.]

1.3.3.1 Forms of burger equation [11, 21]

There are many forms of Burger equation some of them as follow:

1- Burger equation with no viscosity

$$u_t + uu_x = 0$$

(1.41)

2- Burger equation with viscosity

$$u_t + uu_x = \mathrm{v}u_{xx}$$

(1.42)

3- Burger equation with external force F(x, t)

$$u_t + uu_x vu_{xx} + F(x,t) = 0$$

(1.43)

4- Burger equation with fractional derivative

$$u_t + uu_x - \lambda \frac{\partial^\alpha u}{\partial x^\alpha}$$

(1.44)

$$\frac{\partial^\alpha u}{\partial x^\alpha} + uu_x - \lambda u_{xx} = 0$$

(1.45)

$$u_t + \frac{1}{2}\frac{\partial y}{\partial x}\left(\frac{\partial^{1-\alpha}}{\partial x^{1-\alpha}}\right)^2 - \lambda u_{xx} = 0$$

(1.46)

Where α is any positive non integer number.

1.3.3.2 Some properties of burger equation [7, 15, 19, 20]

1. Burger equation is a simple model of nonlinear parabolic differential equation.

2. Burger equation arises in various areas of applied mathematics especially in physical such as Modeling of gas dynamics, propagation of wave in shallow water.
3. Burger equation has been used for convection-diffusion flows, shock waves, etc.
4. The positive constant "λ" is called the kinematic viscosity.
5. The terms "uu_x&λu_{xx} are called convective, diffusive terms respectively.
6. Burger equation is a model equation for the balance between the nonlinear convective term and the diffusive term.
7. We get the "Heat equation" if we omit the term "uu_x " from Burger equation
8. Burger equation is called "hyperbolic" if the term is " λu_{xx}=0".
9. We get the elliptic "Steady Stats" equation from Burger equation if the term is"$u_t = 0$.

1.3.4 Statistical concepts [3,4,5]

In this section we define and discuss some statistical concepts which are needed through this thesis

1.3.4.1 Statistics:

The Statistics is a branch of mathematics which represents the phenomenon in life. In Statistics we represent any phenomenon in population or life as a function and this function is a real number set and its counter domain is between zero and one.

F(x):$\mathbb{R} \to [0,1]$

In Statistics the probability density function is also called distribution.,Every distribution contains two things, the first one is a random variables denoted by x's and the second thing is parameters denoted by $\lambda_{\kappa,}(k = 1,2,3, \dots..)$

1.3.4.2 Probability of density function

Any function f(x) with domain of real numbers set and counter domain [0, 1] is defined to be a probability density function iff:

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)dx = 1.$ If x's are continuous random variable

iii) $\sum au_x f(x) = 1$ are discrete random variable.

Where x's are by any random variables representing the phenomenon in life. f(x) is the distribution or function represents this phenomenon.

1.3.4.3 Expected value

Let x be a random variable. The Mean of x denoted by M_1 or E(x) is defined by:

a- $M_1 = E(x) = \sum au_x f(x)\ if\ x\ is\ discrete\ random\ variable.$

b- $M_1 =$ E(x) = $\int x$f(x) dx if x is continuous random variable.

Where $M_1 is\ cauterize$ and focus all values in the center of data.

1.3.4.4 Second moment

Let x be a random variable. The second moment of x denoted by $M_2 or E(x^2) is\ defined\ by:$

a- $M_2 = E(x^2) = \sum au_x x^2 f(x)$ if x is discrete random variable.

b- $M_2 = E(x^2)f(x)dx$ if x is continuous random variable.

1.3.4.5 Variance

a- $\sigma^2 = var(x) = \sum a\tau\tau_x (x - M)^2 f(x)$ if x is discrete random variable.

$\sigma^2 = var(x) =$

b- $\int (x - M)^2 f(x)dx\ \ if\ x\ is\ continuous\ random\ variable.$

c- $\sigma^2 = var(x) = E(x)^2 - (M_{1)}{}^2$

Where σ^2 is the separation or variation between the value of random variables and the mean of this random variable.

1.3.4.6 Covariance

Let x and y be two random variables of any phenomenon in life, the covariance of x and y denoted by $\sigma_{xy} = E(xy) - E(x)E(y) = E(XY) - M_x M_y$

Where σ_{xy} is measures the separation or variation between x and y.

1.3.4.7 Correlation coefficient

Let x and y be two random variables of any phenomenon in life, the Correlation Coefficient of x and y denoted by ρ_{xy} is defined by:

$$\rho_{xy} = \frac{cov(x,y)}{\sqrt{var(x)var(y)}} = \frac{\rho_{xy}}{\rho_x \rho_y}$$

The correlation coefficient measures the correlation and relationship between the two phenomenon's x and y. The correlation coefficient lies in [-1, 1], where $\rho_{xy} = +1$ is positive correlation between x and y while ρ_{xy} = -1 is negative correlation between x and y.

CHAPTER 2

SOLUTION OF FRACTIONAL BURGER EQUATION

2.1 The Solution

Burger equation has the form

$$\frac{\partial^{\alpha} u}{\partial t} + uu_x - \lambda u_{xx} = 0$$

(2.1)

Let the solution be

u(x, t) =$c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 xt + \; c_5 xt^2 + c_6 x^2 t + c_7 t + c_8 t^2 + \cdots$

(2.2)

This infinite series converge uniformly on the domain of convergence. Therefore a few terms will attain the maximum accuracy [17].

Therefore we consider the solution in this form

u(x, t) =$c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 xt + \; c_5 xt^2 + c_6 x^2 t + c_7 t + c_8 t^2$ (2.3)

And we will see that this choice of u(x, t) *is reasonable*

Now we will substitute $u(x,t)$ *and its derivatives in*(2.1) *fractional Burger equation*

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} = \; (c_0 + c_1 x + c_2 x^2 + c_3 x^3) \frac{t^{-\alpha}}{\Gamma(1-\alpha)} + c_4 x \frac{\Gamma(2)}{\Gamma(2-\alpha)} \; t^{1-\alpha} + c_5 x \frac{\Gamma(3)}{\Gamma(3-\alpha)} +$$

$$c_6 x^2 \frac{\Gamma(2)}{\Gamma(2-\alpha)} t^{1-\alpha} + \qquad c_7 \frac{\Gamma(2)}{\Gamma(2-\alpha)} + \; c_8 \frac{\Gamma(3)}{\Gamma(3-\alpha)} + \; c_9 \frac{\Gamma(4)}{\Gamma 2-\alpha)} \tag{2.4}$$

and

$$u_x = c_1 + 2c_2x + 3c_3x^2 + c_4xt + c_5xt^2 + 2c_6xt \qquad (2.5)$$

$$\lambda u_{xx} = 2\lambda c_2 + 6\lambda c_3x + 2\lambda c_6t \qquad (2.6)$$

We compensate $\frac{\partial^\alpha u}{\partial t^\alpha}$, u, u_x , u_{xx} in the equation

$$(c_0 + c_1x + c_2x^2 + c_3x^3)\frac{t^{-\alpha}}{\Gamma(1-\alpha)} + c_4x\frac{\Gamma(2)}{\Gamma(2-\alpha)} t^{1-\alpha x} + c_5x\frac{\Gamma(3)}{\Gamma(3-\alpha)} + c_6x^2\frac{\Gamma(2)}{\Gamma(2-\alpha)} t^{1-\alpha} +$$

$$+ c_7\frac{\Gamma(2)}{\Gamma(2-\alpha)} + c_8\frac{\Gamma(3)}{\Gamma(3-\alpha)} + c_9\frac{\Gamma(4)}{\Gamma 2-\alpha)}$$

$$+ 3c_3x^2 + c_4xt + c_5xt^2 + 2c_6xt) - \lambda\ (2c_2 + 6c_3x + 2c_6t) = 0 \qquad (2.7)$$

$$(c_0 + c_1x + c_2x^2 + c_3x^3) \frac{t^{-\alpha}}{\Gamma(1-\alpha)} + c_4x\frac{\Gamma(2)}{\Gamma(2-\alpha)} t^{1-\alpha} + c_5x \frac{\Gamma(3)}{\Gamma(3-\alpha)} +$$

$$c_6x^2\frac{\Gamma(2)}{\Gamma(2-\alpha)}t^{1-\alpha} + c_7\frac{\Gamma(2)}{\Gamma(2-\alpha)} + c_8\frac{\Gamma(3)}{\Gamma(3-\alpha)} + c_9\frac{\Gamma(4)}{\Gamma 2-\alpha)}$$

$$+c_0c_1 +$$

$$c_1^2 x + c_1c_2x^2 + c_1c_3x^3 + c_1c_4xt + c_1c_5xt^2 + c_1c_6x^2t + c_1c_7t + c_1c_8t^2 + c_1c_9t^3$$

$$+2c_0c_2x + 2c_1c_2x^2 + 2c_2^2x^3 + 2c_2c_3x^4 + 2c_2c_4x^2t + 2c_2c_5x^2t^2+2$$

$$c_2c_6x^3t + 2c_2c_7xt + 2c_2c_8xt^2 + 2c_2c_9xt^3$$

$$+3c_0c_3x^2 + 3c_1c_3x^3 + 3c_2c_3x^4 + 3c_3^2x^5 + 3c_3c_4x^3t +3$$

$$c_3c_5x^3t^2 + 3c_3c_6x^4t + 3c_3c_7x^2t + 3c_3c_8x^2t^2 + 3c_3c_9x^2t^3$$

$$+c_0c_4t +$$

$$c_1c_4xt + c_2c_4x^2t + c_3c_4x^3t + c_4^2xt^2 + c_4c_5xt^3 + c_4c_6x^2t^2 + c_4c_7t^2 + c_4c_8t^3 + c_4c_9t^4$$

$$+$$

$$c_0c_5t^2 + c_1c_5xt^2 + c_2c_5x^2t^2 + c_3c_5x^3t^2 + c_4c_5xt^3 + c_5^2xt^4 + c_5c_6x^2t^3 + c_5c_7t^3 + c_5c_8t^4 +$$

$$c_5c_9t^5$$

$$+2c_0c_6xt + 2c_1c_6x^2t + 2c_2c_6x^3t + 2c_3c_6x^4t + 2c_4c_6x^2t^2 + 2c_5c_6x^2t^3 + 2c_6^2$$

$$x^3t^2 + 2c_6c_7xt^2 + 2c_6c_8xt^3 + 2c_6c_9xt^4 - \lambda\ (2c_2 + 6c_3x + 2c_6t) = 0 \qquad (2.8)$$

By substituting in Burger equation and equating coefficients with zero we get the following system of algebraic equations.

$c_0c_1 - 2\lambda c_2 = 0$ (1) Constant term

$2c_0c_2 + c_1^2 - 6\lambda c_3 = 0$ (2) Coefficient of x

$3c_1(c_2 + c_3) = 0$ (3) Coefficient of x^2

$c_1c_3 + 2c_2^2 + c_2c_3 = 0$ (4) Coefficient of x^3

$3c_2c_3 + 3c_3^2 = 0$ (5) Coefficient of x^4

$c_1c_7 + 4c_0c_4 - 2\lambda c_6 = 0$ (6) Coefficient of t

$c_1c_8 + c_1c_9 + 4c_4c_7 + c_0c_5 = 0$ (7) Coefficient of t^2

$c_4(c_8 + c_9) = 0$ (8) Coefficient of t^3

$c_5c_8 = 0$ (9) Coefficient of t^4

$c_1c_4 + c_2c_7 + c_0c_6 = 0$ (10) Coefficient of xt

$2c_1c_5 + 2c_2c_8 + c_4^2 + 2c_6c_7 = 0$ (11) Coefficient of xt^2

$c_2c_9 + c_4c_5 + c_6c_8 = 0$ (12) Coefficient of xt^3

$2c_{5+}^2c_6c_9 = 0$ (13) Coefficient of xt^4

$c_1c_6 + c_2c_4 + c_3c_7 = 0$ (14) Coefficient of x^2t

$4c_3c_4 + 2c_2c_6 = 0$ (15) Coefficient of x^3t

$2c_3c_6 = 0$ (16) Coefficient of x^4t

$c_2c_5 += c_3c_8 + c_4c_6 = 0$ (17) Coefficient of x^2t^2

$4c_3c_5 + 2c_6^2 = 0$ (18) Coefficient of x^3t^2

$c_3c_9 + c_5c_6 = 0$ (19) Coefficient of x^2t^3

$c_5c_8 = 0$ (20) Coefficient oft^4

Because of the uniformly convergence property of the infinite series, a few terms will attain the maximum accuracy.

To find the coefficient $c_0, c_1, \dots\dots\dots\dots, c_8$ *we consider the following system:* −

$c_0c_1 - 2\lambda c_2 = 0$ (1) Constant term

$2c_0c_2 + c_1^2 - 6\lambda c_3 = 0$ (2) Coefficient of x

$3c_1(c_2 + c_3) = 0$ (3) Coefficient of x^2

$c_1c_3 + 2c_2^2 + c_2c_3 = 0$ (4) Coefficient of x^3

$3c_2c_3 + 3c_3^2 = 0$ (5) Coefficient of x^4

$c_1c_7 + 4c_0c_4 - 2\lambda c_6$ (6) Coefficient of t

$c_1c_8 + c_1c_9 + 4c_4c_7 + c_0c_5 = 0$ (7) Coefficient of t^2

$c_4(c_8 + c_9) = 0$ (8) Coefficient of t^3

$This\ system\ has\ infinite\ number\ of\ solutions. From\ equation\ (8)\ we\ want\ c_4 \neq 0$

so we take

$c_8 = c_9 = 0$

from equation (3) $c_1 = 0\ or c_2 = -c_3$

If $c_1 = 0$, then from this equation you get (1) $c_2 = 0$ & c_3= 0

But if $c_1 \neq 0$, then from equation (4) we get $c_2 = 0, and\ then\ c_3 = 0$.

and also from equation (1) we get

$c_0 = 0$.

Therefore we neglect the case $c_1 \neq 0, i.e$

We consider $c_1 = 0$ which implies that $c_2 = c_3 = 0$

Then

u(x, t) = $c_0 + c_4xt + c_5xt^2 + c_6x^2t + c_7t$

Where $c_6 = \frac{2c_0c_4}{\lambda}$ from equation 6

and $c_7 = \frac{c_0c_5}{4c_4}$ from equation 7

$For\ simplicity\ we\ take\ 0 \le x \le 1\ \&\ 0 \le t \le 1\ , instead\ of\ 0 \le x < \infty\ \&\ 0 \le t < \infty$

2.2 Condition on $c_0, c_4\ and\ c_5$

For the first time (mathematically) $c_0, c_4 and\ c_5$ are arbitrary but they are not so because nature imposes conditions on them as follows.

2.2.1 Condition on c_0

1) From statistics we want to show that the first wave is greater than the second i.e.

$$E(x) > E(x^2) \quad , E(t) > E(t^2)$$

2) The variance must be positive

$$\sigma_x^2 = E(x^2) - (E(x))^2 > 0$$

$$and \;\; \sigma_t^2 = E(t^2) - (E(t))^2 > 0$$

2.2.2 Conditions on c_4:

Without loss of generality we take $c_4 = 1$. Therefore $c_0 > 0$,

and we take $c_0 = \frac{1}{2}$

2.2.3 Condition on c_5

2.2.3.1 $c_5 < 0$ [We need non trivial critical point]

Proof:

u(x, t) = $c_0 + c_4xt + c_5xt^2 + \frac{2c_0c_4}{\lambda}x^2t - \frac{c_0c_5}{4c_4}t$, let $c_4 = 1$ then

u(x, t) = $c_0 + xt + c_5xt^2 + \frac{2c_0}{\lambda}x^2t - \frac{c_0c_5}{4}t$

We derive u(x, t) for x

$$u_x = t + c_5t^2 + \frac{4c_0}{\lambda}xt$$

$$u_x = t(1 + c_5t + \frac{4c_0}{\lambda}x) = 0$$

We derive u(x, t) for t

$$u_t = x + 2c_5xt + \frac{2c_0}{\lambda}x^2 - \frac{c_0c_5}{4}$$

The critical points determined by $u_x = u_t = 0$

By solving these two equations we get from $u_x = 0$

$t_c = -\frac{\lambda+4c_0x}{\lambda c_5}$ or t =0

Substituting

$t_c = -\frac{\lambda+4c_0x}{\lambda c_5}$ in $u_t = 0$ we get

$$x^2 + \frac{\lambda}{6c_0}x + \frac{\lambda c_5}{24} = 0$$

$$x - 2c_5x + \frac{\lambda+4c_0x}{\lambda c_5} + \frac{2c_0c_5}{4} = 0$$

$$x^2 + \frac{\lambda}{6c_0}x + \frac{\lambda c_5}{24} = 0$$

$$x_{1,2} = \frac{-1\mp\sqrt{1-\frac{6c_0c_5}{\lambda}}}{\frac{12c_0}{\lambda}}$$

Since we want real critical point then

$c_5 < 0$

Since

$x_c > 0$ then we consider

$$x_c = \frac{-1\mp\sqrt{1-\frac{6c_0c_5}{\lambda}}}{\frac{12c_0}{\lambda}}$$

The critical point is $(x_c, t_c) = \left(\frac{-1\mp\sqrt{1-\frac{6c_0c_5}{\lambda}}}{\frac{12c_0}{\lambda}}, -\frac{\lambda+4c_0x}{\lambda c_5}\right)$

Note: If we take t_c=0 then from $u_t = 0$

We get

$$x + \frac{2c_0c_5}{\lambda}x^2 - \frac{c_0c_5}{4} = 0$$

$$x_{1,2} = \frac{-1\mp\sqrt{1+\frac{2c_0^2c_5}{\lambda}}}{\frac{4c_0}{\lambda}}$$

The critical point is

$(x_c, t) = (x_c, 0)$

Which are trivial, so we neglect them.

2.2.3.2 $-6 < c_5 < 0$

Since $\lambda > 0$ and we want u(x, t) to be probability density function

<u>Proof:</u>

u(x, t) is probability density function

u(x, t) = 1+xt+$c_5 x t^2 - \frac{2}{3\lambda} x^2 t - \frac{c_5}{4} t$

$\int_0^1 \int_0^1 u(x,t)\, dx dt = 1$

1= $\int_0^1 [\ 1 + \frac{1}{2} t + \frac{c_5}{2} t^2 - \frac{2}{3\lambda} t - \frac{c_5}{4} t] dt$

= 1+ $\frac{1}{4}$ + $\frac{c_5}{6}$ - $\frac{1}{3\lambda}$ $- \frac{c_5}{8}$

$\therefore \lambda = \frac{8}{6+c_5}$

2.2.3.3 - 2.9$< c_5 < 0$

As we shall see in chapter three during evaluating the expected values and the second moment we get

E(x) = 0.59+ 0.2c_5

E(x^2) $= 0.441 + 0.145$

Since E(x)$> 0 \Rightarrow c_5 > -2.9$

E (x^2))$> 0 \Rightarrow c_5 > -3.04$

So $c_5 > -2.9$

Also we can find condition on c_5 from E (t) and E (t^2) and we shall take $c_5 = -1$

Therefore the solution will be

$$u\,(x,\,t) = 1+xt-xt^{2}+1.25x^{2}t+\frac{1}{4}t$$

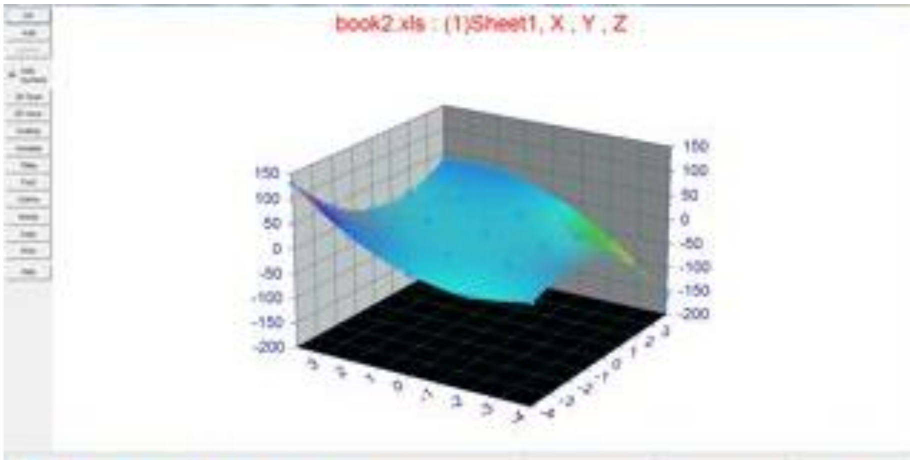

Figure 1 Earth quake

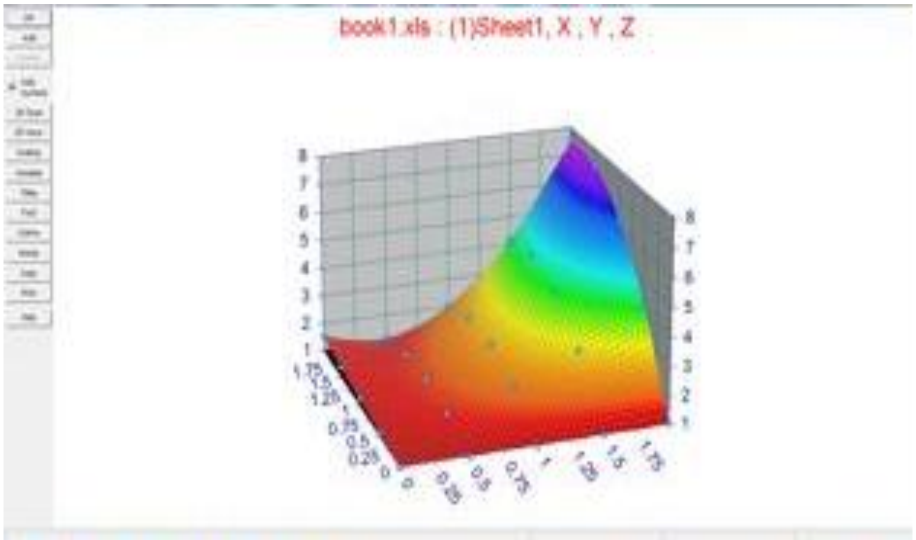

Figure 2 Earth quake

These two figures show that the waves are due to the earth quake

CHAPTER 3

THE STATISTICAL PROPERTIES OF THE SOLUTION

3.1 Introduction

The proximity to reality is considered one of the most important things in statistics, it has been defined, characterized as a random occurrence issue. It is well known through statistics if the case is a study of wave in oceans then it occurs in random way and the solution of the equation which describes the waves is probability density function.

Throughout this chapter we study this and we prove the formula which we found in the second chapter is probability density function. From statistics the first moment or expected value is greater than the second moment when the values are fractional. Our solution agree with these facts as we shall see through this chapter.

3.2 Probability of Density Function

As mentioned in chapter two we take

$0 \leq x \leq 1 \; and \; 0 \leq t \leq 1$

u(x, t) = $1 + xt - xt^2 + 1.25x^2 t + \frac{1}{4}t$

This solution is p.d.f as mentioned in 2.2.3.2 of chapter two

3.3 The Moments

To evaluate the expected values E(x), E(t), E(xt) and the second moments E(x^2), $E(t^2)$ we need

$u^*(x) and \; u^*(t)$. We find them as follows.

$u^*(x) = \int_0^1 u^*(x,t)dt$

$= \int_0^1 (1 + xt - xt^2 + 1.25x^2t + \frac{1}{4}$ t) dt

= 1.125+$\frac{1}{6}x + \frac{5}{8}x^2$

$u^*(t) = \int_0^1 u^*(x,t)\,dx$

$= \int_0^1 (1 + xt - xt^2 + \frac{5}{4}x^2t + \frac{1}{4}$t) dx

$= [x + \frac{x^2t}{2} - \frac{x^2t^2}{2} + \frac{5x^3t}{12} + \frac{xt}{4}]_0^1$

= [1+$\frac{t}{2} - \frac{t^2}{2} + \frac{5t}{12} + \frac{t}{4}$]

3.3.1 Expected value of x

E(x) = $\int_0^1 x\,u^*(x)dx$

$= \int_0^1 x\left(1.125 + \frac{1}{6}x + \frac{5}{8}x^2\right) dx$

$=\int_0^1 (1.125x + \frac{x^2}{6} + \frac{5x^3}{8})dx$

$=[\frac{1.125x^2}{2} + \frac{x^3}{18} + \frac{5x^4}{24}]_0^1$

$=\frac{1.125}{2} + \frac{1}{18} + \frac{5}{24}$

$=\frac{49.5}{72}$ = o.687

It means that the first expected length of the wave is concentrated at the power point 0.687 which means that the power of length wave is large (high)

3.3.2 Expected value of t

E(t) = $\int_0^1 t\,u^*(t)dt$

$=\int_0^1 t\,(1 + \frac{t}{2} - \frac{t^2}{2} + \frac{5t}{12} + \frac{t}{4})\,dt$

$=\int_0^1 \left(t + \frac{5}{4}t^2 - \frac{1}{2}t^3 + \frac{5t^3}{12} + \frac{t^2}{2}\right)dt$

$=\frac{1}{2} + \frac{1}{6} - \frac{1}{8} + \frac{5}{36} + \frac{1}{12}$

$=\frac{43}{72}$

$= 0.597$

The first expected of the wave is concentrated at time (0.597), which means that the wave takes long time

3.3.3 The second moment of x

$E(x^2) = \int_0^1 x^2 u^*(x)dx$

$= \int_0^1 x^2(1.125+ \frac{1}{6}x + \frac{5}{8}x^2)$ dx

$= \int_0^1 (1.125x^2 + \frac{1}{6}x^3 + \frac{5}{8}x^4)dx$

$= [\frac{1.125}{3}x^3 + \frac{1}{24}x^4 + \frac{1}{8}x^5]_0^1$

$= \frac{13}{24}$

$= 0.541$

$E(x^2) < E(x)$,which shows that the first wave is stronger than the second.

The second expected length of wave is concentrated at the power point (0.541), which means that the length begins to disperse (scatter).

3.3.4 The second moment of t

$E(t^2) = \int_0^1 t^2 u^*(t)dt$

$= \int_0^1 t^2 [1+\frac{t}{2} - \frac{t^2}{2} + \frac{5t}{12} + \frac{t}{4}]dt$

$= \int_0^1 (t^2 + \frac{13.5}{12}t^3 - \frac{1}{2}t^4)dt$

$= [\frac{t^3}{3} + \frac{t^4}{8} - \frac{t^5}{10} + \frac{5t^4}{48} + \frac{t^4}{16}]_0^1$

$=\frac{126}{240}$ $= 0.525$

The second expected time of wave is concentrated at time point (0.525), which means that the wave stays for a short time.

3.3.5 The expected value of xt

E(xt)= $\int_0^1 \int_0^1 xt(1 + xt - xt^2 + 1.25x^2t + \frac{1}{4}$t) dxdt

$= \int_0^1 \int_0^1 (xt + x^2t^2 - x^2t^3 + 1.125x^3t^2 + \frac{1}{4}xt^2)dxdt$

$= \int_0^1 [\frac{x^2t}{2} + \frac{x^3t^2}{3} - \frac{x^3t^3}{3} + \frac{1.125x^4t^2}{4} + \frac{x^2t^2}{8}]_0^1$dt

$=\int_0^1 [\frac{t}{2} + \frac{t^2}{3} - \frac{t^3}{3} + \frac{1.125t^2}{4} + \frac{t^2}{8}]$

$=[\frac{t^2}{4} + \frac{t^3}{9} - \frac{t^4}{12} + \frac{1.125\,t^3}{12} + \frac{t^3}{24}]_0^1$

$=\frac{18+8-6+6.75+3}{72}$

= 0.468

The joint expected value for length of the wave and time of the wave is (0.468), which is very small.

3.3.6 The variance

3.3.6.1 variance of x:

$\sigma_x^2 = E(x^2) - [E(x)]^2$

=0.541 -0.471

=0.070

Therefore the amplitude of the waves is 0.64∓ 0.070, which is acceptable similarly for variance of t.

The variation for length of wave is (0.070), so the separation is very small. This means the power of wave is focused in the middle of the wave and separated after the second wave.

3.3.6.2 Variance of t

$\sigma_t^2 = E(t^2) - [E(t)]^2$

$\sigma_t^2 =$ 0.525- 0.356

= 0.169

The variation for time is (0.169), so that the separation is very small, this means that the time of the separated wave begins after second wave and so on.

3.3.7 The covariance and correlation coefficients

Cov (x, t) = E (xt) - E(x) E (t)

= 0.468- 0.410

= 0.058

The range of deviation of length and time of the wave from its expected value is small which is a good property.

3.3.8 The Correlation Coefficients

$$\rho = \frac{cov(x,t)}{\sqrt{var(x)\sqrt{var(t)}}}$$

$$= \frac{0.041}{0.149} = 0.537$$

This means that the relation between the amplitude of the wave with length and time is opposite and this relation is strong (high amplitude corresponding to beginning of the wave in terms of length and time) and vice versa

CHAPTER 4

CONCLUSIONS AND FUTURE WORK

4.1.Conclusion:

1) Our solution agrees with nature in the calculation as mentored in chapter three.
2) If we take $\alpha = 1, the\ solution$ from this relation[14]

$$u(x, t) = \frac{1}{\frac{1}{2(\lambda+k)} + \alpha e^{\left(1+\frac{k}{\lambda}\right)(x-\lambda t)}} - k \quad , 0 \leq x < \infty, 0 \leq t < \infty$$

Where α is an arbitrary constant, and k = $-\lambda \mp \sqrt{\lambda^2 - 2\lambda c}$

And our solutions agree qualitatively with this solution, but different quantitatively.

3) The proposed procedure is very simple for solving Burger equation compared with [9, 16 and 6]
4) The situation is volcano or earthquake
5) As three dimensional surface (x, t, u(x, t)) the wave u(x, t) will settle down with the 'x t- plane'
6) We can modify the solution by taking the best values of c_5
7) If we use the concept of the series solution then this is the same if we take the definition of Caputo or Nishimoto.

4.2. Future Work

1) $Solving\ fractional\ Burger\ equation\ which\ forms\ [17{,}23]\ are$

$$u_t + uu_x - \lambda \frac{\partial^{\alpha} u}{\partial^{\alpha}} = 0$$

$$u_t + \frac{1}{2}\frac{\partial^{\alpha}}{\partial^{\alpha}}\left(\frac{\partial^{\alpha-1}}{\partial^{1-\alpha}}\right) - \lambda u_{xx} = 0$$

2) *Solving these fractional Burger equations with (Neumann, Dirichlet, and Robin conditions) where*

Neumann condition

$u_x(0,t) = g(t)$

$u_x(1,t) = h(t)$

Dirichlet condition

$u_x(0,t) = g(t)$ $\qquad$ $00 \leq t \leq t_c$

$u_x(1,t) = h(t)$

Robin condition

$u(0,t) + a(t)u_x(0,t) = g(t)$

$u(1,t) + b(t)u_x(1,t) = h(t)$ $\qquad$ $0 \leq t \leq t_c$

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APPENDICES A

CURRICULUM VITAE

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FOREIGN LANGUAGES

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