PRICING AND REMANUFACTURING DECISIONS WITH SPECULATORS AND STRATEGIC CONSUMERS

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AUGUST 2017

PRICING AND REMANUFACTURING DECISIONS WITH SPECULATORS AND STRATEGIC CONSUMERS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES OF
ÇANKAYA UNIVERSITY

BY
SİMGE YOZGAT

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING
DEPARTMENT

AUGUST 2017

Title of the Thesis: **Pricing and Remanufacturing Decisions with Speculators and Strategic Consumers**

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

PRICING AND REMANUFACTURING DECISIONS WITH SPECULATORS AND STRATEGIC CONSUMERS

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August 2017, 116 pages

We investigate pricing and remanufacturing decisions with speculators and strategic consumers for a single type of a product over a two-period sales horizon. A monopolist manufacturer produces a fixed quantity in the first period. Some of the sold products are returned at the end of the first period, which are collected by the manufacturer and/or speculators. Returned products are remanufactured, and then sold in the second period, along with any new products remaining from the first period. Mathematical models take into account the behavioral patterns of different types of customers to maximize the manufacturer's expected total profit. Solution to the mathematical models show that one particular customer behavior is optimal. Specifically, the manufacturer should use a fixed-pricing policy for all products –new and remanufactured alike– and set the price at the maximum level that strategic customers are willing to buy. This will force customers to wait for the second period to buy any products, and hence, will yield the maximum profit for the manufacturer. Additionally, the manufacturer is better off remanufacturing. The sensitivity analysis has shown that the profit is most sensitive to the number of strategic customers.

Keywords: Dynamic Pricing, Fixed Pricing, Strategic Consumers, Remanufacturing

ÖZ

SPEKÜLATÖRLER VE STRATEJİK MÜŞTERİLER İLE FİYATLANDIRMA VE YENİDEN İMALAT KARARLARI

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Ağustos 2017, 116 sayfa

İki periyotluk bir satış çevreni boyunca tek bir ürün tipi için spekülatörler ve stratejik tüketicilerin bulunduğu bir piyasada fiyatlandırma ve yeniden imalat kararları çalışılmıştır. Tekelci bir üretici, ilk periyotta sabit miktarda ürün üretir. Satılan ürünlerin bir kısmı birinci periyodun sonunda iade edilir ve üretici ve/veya spekülatörler tarafından toplanır. İade edilen ürünler yeniden üretilir ve ardından ikinci periyotta, ilk periyottan kalan yeni ürünlerle birlikte satışa sunulur. Matematiksel modeller, üreticinin beklenen toplam kârını ençoklamak için farklı müşterilerin davranış biçimlerini hesaba katmaktadır. Matematiksel modellerin çözümü, belirli bir müşteri davranışının en iyi olduğunu göstermektedir. Özel olarak, üretici, yeni ve yeniden üretilen tüm ürünler için sabit fiyatlandırma politikası kullanmalı ve fiyatı stratejik müşterilerin verebileceği maksimum miktar olarak belirlemelidir. Bu, müşterileri herhangi bir ürün satın almak için ikinci periyoda kadar beklemek zorunda bırakacak ve böylece üretici için maksimum kazancı sağlayacaktır. Ayrıca, üretici yeniden üretim yaptığında daha çok kâr etmektedir. Duyarlılık analizi, kârın en çok stratejik müşterilerin sayısına duyarlı olduğunu göstermiştir.

Anahtar Kelimeler: Dinamik Fiyatlandırma, Sabit Fiyatlandırma, Stratejik Tüketiciler, Yeniden Üretim

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my advisor Assist. Prof. Dr. Gonca YILDIRIM for her supervision, special guidance, suggestions, interest, patience and encouragement through the development of this thesis. It was a great chance and opportunity to work with her.

I am also grateful to Assist. Prof. Dr. Ayyüce AYDEMİR KARADAĞ and Assist. Prof. Dr. Mükerrem Bahar BAŞKIR for accepting to examine this thesis and their valuable contributions.

It is a pleasure to express my special thanks to my family for their valuable support.

I would sincerely thank to Hüsnü Deniz BAŞDEMİR, Zehra Burçin KANIK and Mert BAYRAKTAR for their morale support and friendship throughout this study.

I would like special to thank the all department members of IE for their contributions during my educational life and assistantship.

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LIST OF ABBREVIATIONS

OEM	Original Equipment Manufacturer
CLSC	Closed-loop Supply Chain
I&PP	Inventory and Production Planning
LP	Linear Programming
MILP	Mixed Integer Linear Programming

CHAPTER 1

INTRODUCTION

A product to be sold can have different prices because of competition. Pricing depends on multiple factors. Product properties (such as quality, age, life cycle), market conditions, supply and demand are among factors that affect pricing decisions. Related with supply, who manufactures the product (i.e., Original Equipment Manufacturer (OEM), third party manufacturers) may be a major factor in the pricing of a product. On the demand side, customer profile (i.e., myopic, strategic and bargain hunter) has a significant impact on the pricing.

Pricing has been studied extensively in literature. Sellers charge the most profitable pricing policy for themselves. For example, they can offer a discount after a while during the sales horizon or they can continue to charge the same price (i.e., fixed pricing). Sales horizon can be divided into sub-periods. In most of the studies, sales horizon is examined in two periods. In the first period, products are put on the market at some initial price. After a while the firm decides whether or not to offer a discount. This second period can be called as salvage or sale period if a markdown price is applied. However, firms can continue to use same prices as they did in the first period. Thus, products are differentiated in terms of prices according to the sale period. In addition, a product can have different versions such as new and remanufactured ones in a particular period. This can result in a cannibalization of the new product. In other words, the remanufactured product may be preferred over new product by some customers. Due to reaching the end of useful lives or customer returns, some of the used or returned products are collected and restored to become as good as new condition. Then, they are sold as refurbished or remanufactured. Remanufacturing refers to reconstruction of a product to regain its working state through some processes such as cleaning, testing, repair, replacement of parts, etc. Version of the product has an important role for differentiation of the sales prices. For example, most customers may not be willing to pay money more for remanufactured products than for the new ones. In this thesis, pricing and remanufacturing decisions of a single type of a product are investigated. The product is manufactured by a monopolist having fixed production capacity. Sales horizon is divided into two periods. In the first period, only the new product is sold. At the end of the first period, some fraction of the sold products is returned. Some of the returns are collected by the manufacturer and the rest by the third party remanufacturers, who will be referred as speculators. The purpose of collection is to make profit from second-period sales of the remanufactured product in addition to continued sales of the new product.

Customer profile is also one the most critical factors in pricing. We investigate two types of customers in the market. The first one is myopic customers. They immediately buy a new product regardless of the product price. There are no myopic consumers in the second period. Thus, myopic consumers determine the first-period demand. The other type of consumers is strategic customers. They can buy the product in the first or the second period. There may be additional demand arriving I the second period. Therefore, strategic customers who wait and demand arriving in the second-period determine the second-period demand.

In this thesis, we investigate the pricing decisions for new products in both periods and remanufactured products in the second period while taking into account the myopic and strategic consumers and a random demand in the second period. We further examine remanufacturing decision in the presence of speculators.

1.1 Objectives

The primary aim of this study is to determine the optimal pricing policy for a manufacturer for new and remanufactured products over a two-period sales horizon.

1.2 Organization of the Thesis

This thesis contains six chapters.

Chapter 1 is an introduction part that involves motivation and objectives of the study.

Chapter 2 includes background information about dynamic pricing, speculation, strategic consumer behaviors.

Chapter 3 consists of pricing of new and remanufactured products.

Chapter 4 contains solution approach and numerical study.

Chapter 5 includes conclusion.

CHAPTER 2

LITERATURE REVIEW

We focus on the three main topics in the literature: dynamic pricing, remanufacturing decisions and customer behavior. Pricing decisions for a fixed quantity of products over a finite sales horizon is studied using dynamic pricing policy when demand is random and price sensitive [1]. There are some studies on pricing decisions which consider the presence of strategic consumers. Strategic consumers' behaviors may lead a firm to set prices dynamically. Dynamic pricing strategy allows the firm to act according to demand quantity and its timing, i.e., a firm may decide whether to make a discount, increase the price or keep it the same over time accordingly. For example, strategic consumers can immediately buy products when they are placed on the market or choose to wait hoping a discount. In addition, speculative behaviors may also affect a manufacturer's pricing decisions. Speculators can freely enter the market and set the price smaller than the manufacturer's price, in which case strategic customers prefer to buy products from them. As a result, pricing has become a critical decision in this competitive environment.

A model of pricing in the presence of speculators and strategic consumers is studied in [2]. There is a monopolist firm selling a fixed capacity of a product. The manufacturer sets a price. Speculators enter the market purely. There are two sales periods. Speculators can buy the new product in the first period and sell them in the second. Some major results are obtained. One of them is that the speculative resale can benefit the seller. Another conclusion is that the speculative resale can force a firm to apply dynamic pricing strategy. In addition, it affects a firm's long-run capacity decision. If speculators enter the market, the manufacturer keeps the product capacity low. Besides, if the firm uses a fixed pricing strategy, it is optimal for the firm to cut the speculators out of the market. Game-theoretic approach is used to determine equilibrium outcomes. For this purpose, Nash Equilibrium is used as the solution methodology to show all possible outcomes within the players. In the Nash Equilibrium, it is assumed that each player knows the equilibrium strategies. The players are speculators, manufacturers and consumers in this game. In summary, speculators behave like competitors when seller applies dynamic pricing but act as market makers when the seller uses fixed pricing. This study has vital conclusions that offer an insight into our thesis. We try to extend this study by taking into account not only the new products but also the remanufactured products, their pricing and remanufacturing decisions.

Dynamic pricing of products with random demand has received significant interest recently [3]. In particular, when strategic consumer behavior is involved, game-theoretic approaches are frequently used for pricing decisions. Dynamic pricing policies are also studied to maximize the total revenue over the selling horizon [4]. The results show that standard optimization techniques which obtain deterministic solutions, can be used to reach better approximations for problems when demand is stochastic. To reach an optimal pricing strategy, Hamilton-Jacobi-Bellman equation which is a complex differential equation, is used when demand is stochastic over a finite period of time. Besides, three main classes of research problems under dynamic pricing are reviewed with multiple types of products, competition and limited demand information [5].

Models involving limited demand information are analyzed using robust optimization and demand learning approaches. Monopolistic and competitive cases are also discussed based on the dynamic pricing [6]. Dynamic economics, new-product diffusion and game theory are studied. Moreover, factors effective on dynamic pricing policy are examined: increased availability of demand data, the ease of changing prices because of the new technology and the availability decision support systems for examining demand and dynamic pricing in [7]. Dynamic pricing in the presence of endogenous intertemporal demand is studied in [8]. The optimal pricing policy can be determined with both consumer valuation and impatience level of customers to buy a product. In this sense, customers are divided into four distinct categories: patient-high types, impatient-high types, patient-low types and impatient-low types. First, customers are divided into low or high types in terms of value the product. Then, they are grouped as patient and impatient from the point of waiting costs (i.e. customers spend time for monitoring the costs before purchasing). Thus, customer profile jointly specifies the optimal pricing policy. After determining the product price, consumers can buy the product if their valuation is enough to buy. Some portion of the sold products are returned because of reaching the end of useful lives.

In the presence of consumer returns policies, some complexities can be observed [9]. These complications can be resolved with the help of increasing the ability of monitoring of the manufacturer. There are two consumer return policies: full returns and partial returns. Two firms are considered: manufacturer and retailer. Manufacturer supplies products to the retailer and retailer sells the products to end consumers. To coordinate the supply chain, some contracts are made between the manufacturer and the retailer. There are three types of contracts if manufacturer has ability to distinguish the new and returned units. First one is buy-back contracts that consider the unsold units of product. Manufacturer guarantees to buy unsold units from retailer. The other one is differentiated buy-back policy that includes the unsold units and customer returns. The last one is direct-to-manufacturer returns. This contract guarantees that manufacturer buys back all customer returns. If new and returned products are indistinguishable, sales rebates can coordinate the supply chain but in this case retailer has full responsibility for unsold and returned units. In this case, seller makes price and quantity decisions, and regulates the suitable returns policy. As a result, two major conclusions are obtained in this study. One of them is that partial refunds optimize the supply chain performance by keeping it less than the selling price. Another conclusion is that consumer return policies are obtained with the supplier buy-backs. In general, manufacturers set a price for products themselves for the first sales period. Then, customers decide whether to buy or not in this period. Because of the customer returns, some collected portion of products are remanufactured for sale. Thus, remanufactured version of the product is on sale in the second period. Therefore, it is possible to see a mixture of products: remaining new products from the first period and remanufactured products. It is generally expected that remanufactured version of the product price should be less than that of the new product. Thus, this can force the manufacturer to offer a discount in the second period. Therefore, dynamic pricing strategy may be inevitable.

Remanufacturing has large economic, environmental and social potencies in itself [10]. Legislations on emissions can affect the remanufacturing decisions in order to reduce environmental damage, the high willingness to pay for avoiding the harm to the environment and reducing total emissions by remanufacturing. In this study, authors consider two pricing options: high priced remanufactured product and low priced remanufactured product. Two period model is used and two cases are examined: remanufacturing and no remanufacturing ones respectively. In the first period, there are new products only. Mixed product line is offered in the second period during which new and remanufactured products coexist. Because of the green market segment, cannibalization can occur within the new and remanufactured products. Green consumers do not want to reduce the value of remanufactured products. It is desired to find optimal emissions taxation policy. Moreover, the profitability of remanufacturing depends on the interaction between the green segment, manufacturer and the product life-cycle [11]. This study shows that remanufacturing decision depends on competition, cost savings, cannibalization and product life-cycle effects.

Remanufacturing decision requires firms to coordinate their closed-loop supply chain (CLSC) systems [12]. Some operations are necessary for inventory and production planning (I&PP) for CLSC systems. In this sense, demand is modeled in two categories which requires stochastic and deterministic processes. Reviews of models for I&PP for CLSC systems and research in I&PP are provided in this study. Remanufacturing activities can be profitable in accordance with the quantity and quality of the returns and quantity demanded of the remanufactured product [13]. A profit maximization model was developed. Optimal solutions were obtained if the sales revenue function is concave and the acquisition cost functions are convex. If the conditions are not satisfied, a heuristic solution is available. Pricing decision is directly related to customer behaviors as well as version of a product. Pricing of a product is a strategic decision for not only manufacturer but also retailer. Retailer can make optimal decisions according to behavior of the consumers [14]. There are three types of consumers: myopic, bargain-hunting and strategic consumers and they consider two periods. First period is considered as the full-price period and the second period is considered as the sale or salvage period. Retailer has two decisions: sales price and initial stocking quantity. At the beginning of the first period, retailer determines the stocking level. All myopic consumers and some of the strategic consumers can buy the product in the first period. At the end of the first period, retailer sets the sales price that retailer try to set an optimal markdown given the available inventory and initial sales using dynamic-pricing strategy. It is also dependent on the level of the initial inventory. Therefore, the retailer chooses an optimal order quantity in accordance with the consumers' behaviors. On the other hand, selling season determines the boundary of markdowns. Because strategic consumers' willingness to pay for a product acts upon the selling season. Besides, quick response (also considered as short-term replenishment) provides considerable value to a retailer when large portion of the consumers are myopic. Game-theoretic approach is also used in this study to determine equilibrium points of markdowns and stocking level. [15] also considers the customer behavior. Recently, most studies concentrate on dynamic pricing based on strategic-customer behavior [16]. Summary of existing studies about dynamic pricing, remanufacturing decision and customer behavior is given in Table 1.

Table 1. Summary of existing studies about dynamic pricing, remanufacturing decision and customer behavior.

Article	**Dynamic pricing**	**Remanufacturing**	**Customer behavior**
[1] Gallego & van Ryzin (1994, MS)	✓		
[2] Su (2010, MS)	✓		✓
[8] Su (2007, MS)	✓		✓
[9] Su (2009, MSOM)		✓	
[10] Yenipazarlı (2016, EJOR)	✓	✓	
[11] Atasu et al. (2008, MS)	✓	✓	
[14] Cachon & Swinney (2009, MS)	✓		✓
Survey papers	**[3]** den Boer (2015, SORMS) **[4]** Bitran & Caldentey (2003, MSOM) **[5]** Chen & Chen (2015,POMS) **[6]** Chenavaz et al.(2011,JESR) **[7]** Elmaghraby & Keskinocak (2003, MS)	**[12]** Akçalı & Çetinkaya (2011, IJPE) **[13]** Guide et al. (2003, MSOM)	**[15]** Gönsch et al. (2013,JBE) **[16]** Shen & Su (2007, POMS)

Dynamic pricing strategies have been investigated with respect to many factors. Nature of dynamic pricing is studied in [1]. Dynamic pricing strategies and their relations to the customer behaviors are considered in [2], [8] and [14]. Dynamic pricing and remanufacturing decisions are considered together in [10] and [11]. However; there is no study that takes into account the behavior of the customer and the remanufacturing decision for pricing policy. We mostly refer the studies in [2]. In this study, it is assumed a monopolist firm selling a fixed capacity of a product. Speculators are considered as a type of customer, and enter the market in the first sales period to purchase products for sale in the secondary market. There are also myopic and strategic type of customers. Fixed and dynamic pricing strategies are investigated separately.

The relevant literature examined so far considered pricing over a two-period sales horizon in the presence of speculators and strategic customers. We extend the current studies by including remanufacturing, which adds to the variety of the products considered. Consequently, we investigate pricing both new and remanufactured products. In addition, the speculators in this thesis are considered as third-party remanufacturers rather than being customers.

CHAPTER 3

PRICING OF NEW AND REMANUFACTURED PRODUCTS

We examine the pricing decisions of new and remanufactured products in a two-period sales horizon. There exists one monopolistic manufacturer who produces a fixed quantity of units at the beginning of the first period. During the first period, some product returns are observed due to failure or some other reasons. These returned products may be collected by both the manufacturer and speculators for remanufacturing. Speculators and the manufacturer individually collect and remanufacture the returned products at a cost for the purpose of making profit in the second period. Remaining unsold new products from the first period and remanufactured products are sold together in the second period. Quantity of remanufactured products depends on the number of returns.

On the demand side, there are customers who are segmented based on their product valuation and patience level. Valuation refers to the maximum amount that customers are willing to pay for the product. If a customer has high willingness to pay, that refers to high valuation for the product. Otherwise, customer valuation is low. Patience level is related to willingness to wait for purchase. In other words, the customer can be either patient or impatient to buy a product. Patient customers wait for a significant discount. They are referred to as strategic customers. Impatient customers buy immediately if the product price is not extremely high. This means that impatient customers buy the product if product price does not exceed their valuation when the product is put on the market. These customers do not wait for a discount.

Demand is segmented based on the arrival period. In the first period, a deterministic number of customers arrive, whereas a random demand is realized in the second period. These two segments have high willingness to pay for the product. There are also low-valuation consumers who only buy if the price is sufficiently low. As long as the price is low, it is assumed that there are many low-valuation consumers who buy the remaining capacity. Thus, initial demand constitutes the deterministic part and second-period demand constitutes the stochastic part of the demand.

Sales horizon is divided into two periods as shown in the Figure 1. First period is named as primary market and the second one is resale respectively.

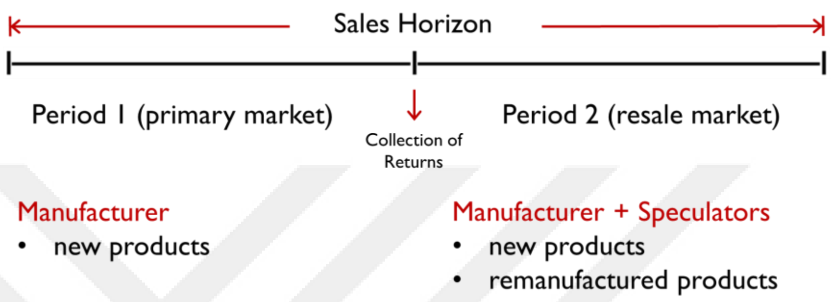

Figure 1. Supply during the two periods of the sales horizon.

We note that returned products are remanufactured to become as good as new, but sold with the label “remanufactured.” Although the new and remanufactured products are similar in quality, customers have different valuations for them, lower valuation for the remanufactured products compared to the new ones. In any case, we assume that a customer focuses on his or her benefit from a purchase. Customers choose the type of product and time to buy based on their valuation, and belief on future prices of products. In particular, a strategic consumer needs to decide on the following:

- Which item to purchase? (Remanufactured or new product)
- When to purchase? (First or second period)
- Purchase from whom? (Manufacturer or speculator)

Thus, a strategic consumer needs to compare his/her belief with the amount s/he is willing to pay. In period 1, while all myopic consumers buy the new product, strategic consumers need to decide on whether to buy the new product or wait for the second period. In period 2, existing and new strategic consumers need to decide on whether they will buy, and if they will, which product type. In other words, a strategic consumer's decision comprises of two basic stages:

1. First-period decision (i.e., buy or wait)
2. Second-period decision (i.e., buy or not, and which product to buy if at all)

We formulate mathematical models considering the above setting. Table 2 and Table 3 summarize the set of parameters and decision variables.

Table 2. Parameters.

Symbol	Explanation
K	Fixed capacity for new product
K_2	Number of units left
V_L	Low consumer valuation for new product
V_H	High consumer valuation for new product
V_L^r	Low consumer valuation for remanufactured product
V_H^r	High consumer valuation for remanufactured product
W	Number of consumers in the market
W_2	Number of strategic consumers who choose to wait
ϕ	Fraction of strategic consumers
$\bar{\phi}$ or ϕ'	Fraction of myopic consumers ($\bar{\phi} = 1 - \phi$)
f_M	Fraction of returns collected by the manufacturer
f_S	Fraction of returns that collected by the speculators
f	Fraction of returns, $f = f_M + f_S$
R_M	Number of remanufactured products by manufacturer
R_S	Number of remanufactured products by speculators
R	Total number of returned products, $R = (W - W_2)f$
c_r	Cost of remanufacturing
c_n	Cost of new product
X	Random number of new consumers who enter the market in the second period, with cumulative distribution function $F(x)$

Table 3. Decision variables.

Symbol	Explanation
p_1	First period price
p_2^n	Second period new product price
p_2^r	Manufacturer-remanufactured product price
p_2^s	Speculator-remanufactured product price

In addition to the above nomenclature, other symbols will be used to define some necessary equations or relations as follows.

Consumers act based on their benefit from buying the product, i.e., surplus. Surplus of a consumer can be thought as the gain from buying one unit of the product. In the first period, price, p, is known. Therefore, surplus of buying in the first period (S_1^n) is equal to consumer's willingness to pay for the product minus the first-period price. Both types of customers (myopic and strategic) are high-valuation consumers. Thus, their willingness to pay for the product is equal to V_H for the new product. Besides, strategic consumers maintain the same valuation level for the new product in the second period. On the other hand, they do not want to pay more than price p for the remanufactured product in period 2. Therefore, their maximum amount of the willingness to pay for a remanufactured item does not exceed the first period price. Before the second period prices are determined, surplus of consumers consists of their beliefs. We represent the surplus with hat symbol ($\hat{S}$). In this case, for new and remanufactured product surplus values with respect to consumer beliefs ($\hat{S}_2^n$ and $\hat{S}_2^r$) are calculated by subtracting the expected prices from valuations of products. Belief about the new product surplus for the second period ($\hat{S}_2^n$) is calculated by subtracting some fraction ($k : k \in [0,1]$) of the first-period price. In most cases, second-period price is expected to be smaller than the first-period price. Therefore, consumers' expectation for the new product price in the second period is at most its price p in the first period. However, in some special cases, seller may not prefer to make a discount. Thus, the coefficient k provides flexibility in the computations.

After second-period prices are set, surplus values from buying a product can be easily calculated by subtracting the prices set from valuations of products. When actual prices are set in the second period, customer compares the surpluses for buying new product (S_2^n), remanufactured product from manufacturer (S_2^r) or remanufactured product from speculators (S_2^s). Calculations of consumer surplus values for each type of product and each period are given as follows:

$$S_1^n = V_H - p$$
$$\hat{S}_2^n = V_H - p \times k, \quad k \in [0,1]$$
$$\hat{S}_2^r = p - \frac{V_L + p}{2}$$
$$S_2^n = V_H - p_2^n$$
$$S_2^r = p - p_2^r$$
$$S_2^s = p - p_2^s$$

In the following subsections, we formulate models for the cases first without, and then, with speculators present in the market.

3.1 Dynamic Pricing without Speculators

In this subsection, we determine prices which maximize the manufacturer's expected total profit. We assume that all returned products are collected by the manufacturer, i.e., there are no speculators present. Manufacturer remanufactures and sell them in the resale market along with remaining unsold new products from the first period. Additionally, all remaining products that remain unsold at the end of the period 2 are sold to a waste collector at a price of p_s. We try to maximize the manufacturer's expected total profit using dynamic pricing. Mathematical models are formulated for several cases formed by

- Which item to purchase? (Remanufactured or new product)
- When to purchase? (First or second period)

Cases considered are summarized in Table 4 based on the purchase period and seller preferences of the strategic customers as well as whether there is sufficient capacity to cover the first-period demand.

Table 4. Summary of possible cases in the absence of speculators (new product: N, remanufactured product: R).

Case 1: Buy in period 1.						
1.1 Capacity is sufficient to cover first-period demand. ($W \leq K$)				**1.2** Capacity is not sufficient to cover first-period demand. ($W > K$)		
	Order of purchase				**Order of purchase**	
Case	**Period 1**	**Period 2**		**Case**	**Period 1**	**Period 2**
1.1.a	1) N	2) N	3) R	1.2.a	1) N	2) R
1.1.b	1) N	2) N	3) -	1.2.b	1) N	2) -
1.1.c	1) N	2) R	3) N			
1.1.d	1) N	2) R	3) -			
1.1.e	1) N	2) -	3) -			

Case 2: Wait for period 2.			
	Order of purchase		
Case	**Period 1**	**Period 2**	
2.1	No purchase.	1) N	2) R
2.2	No purchase.	1) N	2) -
2.3	No purchase.	1) R	2) N
2.4	No purchase.	1) R	2) -

Case 1: Buy in period 1

Initially, we assume that all strategic consumers buy the new product in the first period. Thus, first-period surplus is greater than or equal to the second-period surplus values with respect to consumer beliefs. Case 1 necessity conditions are formed as follows:

$$S_1^n \geq 0, \quad S_1^n \geq \hat{S}_2^n, \quad S_1^n \geq \hat{S}_2^r$$

1.1 Capacity is sufficient to cover the first-period demand ($W \leq K$)

It is assumed that there is sufficient capacity to meet the initial demand, which consists of myopic and strategic consumers. These customers buy the new product in the first period. Remaining new products and remanufactured products are sold in the second period to random demand X.

1.1.a. Prefer to buy new product first, and then, remanufactured product in period 2.

$$S_2^n \geq S_2^r, \quad S_2^r \geq 0$$

In this case, the surplus from buying the remanufactured product is less than that from a new product. Therefore, a consumer first chooses to buy the new product. If there are remaining customers, they may buy remanufactured product. The relationship between demand quantity and the number of units sold is explained from the manufacturer's point of view in Table 5.

Table 5. Case 1.1.a from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$X \leq K_2$	X	0	$K_2 - X$	R
$K_2 < X \leq K_2 + R$	K_2	$X - K_2$	0	$K_2 + R - X$
$K_2 + R < X$	K_2	R	0	0

Mathematical model for this case is given below:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{K_2} p_2^n x f(x) dx + \int_{x=K_2}^{\infty} p_2^n K_2 f(x) dx$$
$$+ \int_{x=0}^{K_2} p_s (K_2 - x + R) f(x) dx + \int_{x=K_2}^{K_2+R} p_2^r (x - K_2) f(x) dx$$
$$+ \int_{x=K_2+R}^{\infty} p_2^r R f(x) dx + \int_{x=K_2}^{K_2+R} p_s (K_2 - x + R) f(x) dx + Wp - c_n K$$
$$- c_r R$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$V_H - p_2^n \geq p - p_2^r$ (4)

$p - p_2^r \geq 0$ (5)

$p, p_2^n, p_2^r \geq V_L$ (6)

The objective function is linear in the decision variables p, p_2^n and p_2^r. Therefore, the optimal values of the decision variables should be at a boundary point. Therefore, we focus on the upper and lower bounds (UB and LB) to obtain the optimal values of prices by taking into consideration the constraint set. Constraint (1) ensures that the surplus of buying in the first period is nonnegative. Constraint (2) and (3) imply that surplus of buying in the first period is greater than or equal to the surplus from buying in the second period (new and remanufactured products) with respect to consumer beliefs. We take into account the consumer beliefs in this step because second-period prices are beliefs at best for consumers in the first period. Constraint (4) ensures that when random demand X is realized, customers prefer to buy the new product. Then, remaining customers, if any, can buy remanufactured products. Constraint (5) provides a nonnegative surplus and constraint (6) states that all prices are greater than or equal to the low valuation. On the other hand; decision variables have natural bounds with respect to valuation bounds:

- $V_L \leq p, p_2^n \leq V_H$
- $V_L \leq p_2^r \leq p$

We try to find tight bounds using the given constraint set. Here the coefficient k is a constant value that can take a value between 0 and 1. The second constraint forces k to be one. This can be thought of why strategic consumers prefer to buy in period 1. From constraint (3), we obtain $p \leq \frac{2V_H+V_L}{3}$. Using constraint (4), we observe that value of the equation $p + p_2^n - p_2^r$ cannot be greater than V_H (i.e., $V_H \geq p + p_2^n - p_2^r$). From all these observations, we reduce the original set of constraints to the following tighter set; which yield the lower and upper bounds on the decision variables:

- $V_L \leq p \leq \frac{2V_H+V_L}{3}$
- $V_L \leq p_2^n \leq V_H$
- $V_L \leq p_2^r \leq p$

Considering these bounds we have five feasible for the prices. Feasible prices are given in Table 6.

Table 6. Case 1.1.a. feasible prices.

p	p_2^n	p_2^r	Explanation
V_L	V_L	V_L	(LB, LB, LB)
V_L	V_H	V_L	(LB, UB, LB)
$\frac{2V_H + V_L}{3}$	V_L	V_L	(UB, LB, LB)
$\frac{2V_H + V_L}{3}$	V_L	$\frac{2V_H + V_L}{3}$	(UB, LB, UB)
$\frac{2V_H + V_L}{3}$	V_H	$\frac{2V_H + V_L}{3}$	(UB, UB, UB)

Our aim is maximizing the expected total profit, which is increasing in these prices. In this case, optimal values of decision variables are at their upper bounds.

$$p^* = \frac{2V_H + V_L}{3}, \qquad p_2^{n*} = V_H, \qquad p_2^{r*} = \frac{2V_H + V_L}{3}$$

1.1.b. Prefer to buy only the new product in period 2.

$$S_2^n \geq 0, \quad S_2^r \leq 0$$

In this setting, surplus from buying the new product in period 2 is nonnegative so it is preferable. Thus, random portion of consumers will buy only new products in period 2 as given in Table **7**.

Table 7. Case 1.1.b from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$X \leq K_2$	X	0	$K_2 - X$	R
$X > K_2$	K_2	0	0	R

The mathematical model to represent this case is given below:

$$\text{Max} \prod(p, p_2^n) = \int_{x=0}^{K_2} p_2^n x f(x)dx + \int_{x=K_2}^{\infty} p_2^n K_2 f(x)dx$$
$$+ \int_{x=0}^{K_2} p_s(K_2 - x + R)f(x)dx$$
$$+ \int_{x=K_2}^{\infty} p_s R f(x)dx + Wp - c_n K - c_r R$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L+p}{2}$ (3)

$V_H - p_2^n \geq 0$ (4)

$p - p_2^r \leq 0$ (5)

$p, p_2^n, p_2^r \geq V_L$ (6)

First three constraints are the same as in Case 1.1.a. Buying a remanufactured item is not preferred because its surplus is smaller than zero as seen in constraint (5). Therefore, consumers choose to buy the new product since it has nonnegative surplus as given in constraint (4). Constraint (5) ensures that the value of p_2^r is equal to its upper bound (p). When we take into the natural bounds of the prices with respect to customer valuations, we have four feasible cases given in Table 8.

Table 8. Case 1.1.b. feasible prices.

p	p_2^n	p_2^r	Explanation
V_L	V_L	V_L	(LB, LB, LB)
V_L	V_H	V_L	(LB, UB, LB)
$\frac{2V_H + V_L}{3}$	V_L	$\frac{2V_H + V_L}{3}$	(UB, LB, UB)
$\frac{2V_H + V_L}{3}$	V_H	$\frac{2V_H + V_L}{3}$	(UB, UB, UB)

For maximizing the expected total profit, we need to choose higher values of p and p_2^n which are observed in the last row of Table 8. While choosing the maximum value of

p, we have to choose the value of p_2^r as the upper bound of p_2^r because of constraint (5). Therefore, optimal values of the prices are given as follows:

$$p^* = \frac{2V_H + V_L}{3}, \qquad p_2^{n*} = V_H, \qquad p_2^{r*} = \frac{2V_H + V_L}{3}$$

1.1.c. Prefer to buy remanufactured product first, and then, new product in period 2

$$S_2^r \geq S_2^n, \quad S_2^n \geq 0$$

In this case, the surplus from buying the remanufactured product is more than that from buying a new product. Therefore, consumers first choose to buy the remanufactured product. If there are any remaining customers, they may buy the new product. The relationship between demand quantity and number of units sold is given in Table 9.

Table 9. Case 1.1.c from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$X \leq R$	0	X	K_2	$R - X$
$R < X \leq K_2 + R$	$X - R$	R	$K_2 + R - X$	0
$K_2 + R < X$	K_2	R	0	0

The mathematical model of this case is given as follows:

$$\text{Max} \prod(p, p_2^n, p_2^r)$$
$$= \int_{x=0}^{R} p_2^r x f(x)dx + \int_{x=R}^{\infty} p_2^r Rf(x)dx + \int_{x=0}^{R} p_s(K_2 + R - x)f(x)dx$$
$$+ \int_{x=R}^{K_2+R} p_2^n(x - R)f(x)dx + \int_{x=K_2+R}^{\infty} p_2^n K_2 f(x)dx$$
$$+ \int_{x=R}^{K_2+R} p_s(K_2 + R - x)f(x)dx + Wp - c_n K - c_r R$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L+p}{2}$ (3)

$p - p_2^r \geq V_H - p_2^n$ (4)

$V_H - p_2^n \geq 0$ (5)

$p, p_2^n, p_2^r \geq V_L$ (6)

First three constraints are the same as in the previous cases because of the necessity conditions of Case 1. Constraint (4) states that surplus of buying a remanufactured product is greater than or equal to the surplus of buying a new product ($p + p_2^n - p_2^r \geq V_H$). Constraint (5) implies that surplus of buying a new product is nonnegative. In this case, buying a new product is also feasible. For example, when demand is high, a customer can buy a new product. Combined with the natural bounds, we have three feasible cases as shown in Table 10 below.

Table 10. Case 1.1.c. feasible prices.

p	p_2^n	p_2^r	**Explanation**
V_L	V_H	V_L	(LB, UB, LB)
$\frac{2V_H + V_L}{3}$	V_H	V_L	(UB, UB, LB)
$\frac{2V_H + V_L}{3}$	V_H	$\frac{2V_H + V_L}{3}$	(UB, UB, UB)

For maximizing the expected total profit, we need to choose higher values of p, p_2^n and p_2^r which are observed in the last row of Table 10. Optimal values of the prices are given as follows:

$$p^* = \frac{2V_H + V_L}{3}, \quad p_2^{n*} = V_H, \quad p_2^{r*} = \frac{2V_H + V_L}{3}$$

1.1.d. Prefer to buy only the remanufactured product in period 2.

$$S_2^r \geq 0, \quad S_2^n \leq 0$$

In this case, buying a new product yields a negative surplus, therefore, it is not considered. The relationship between the demand quantity and the number of units sold is specified in Table 11.

Table 11. Case 1.1.d from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$X \leq R$	0	X	K_2	$R - X$
$R < X$	0	R	K_2	0

The mathematical model is given as follows:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R} p_2^r x f(x)dx + \int_{x=R}^{\infty} p_2^r\, R f(x)dx + \int_{x=0}^{R} p_s(K_2 + R - x)f(x)dx + \int_{x=R}^{\infty} p_s K_2 f(x)dx + Wp - c_n K - c_r R$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$p - p_2^r \geq 0$ (4)

$V_H - p_2^n \leq 0$ (5)

$p, p_2^n, p_2^r \geq V_L$ (6)

First three constraints are the same in the previous cases. In this setting, buying a remanufactured item is preferred because its surplus is nonnegative as seen in constraint (4). On the other hand, buying a new product is not considered due to its negative surplus given constraint (5). Therefore, consumers choose to buy the remanufactured products. According to constraint (5), value of p_2^n is equal to its upper bound (V_H). When combined with the natural bounds, we have three feasible cases for the prices as summarized in Table 12 below.

Table 12. Case 1.1.d. feasible prices.

p	p_2^n	p_2^r	Explanation
V_L	V_H	V_L	(LB, UB, LB)
$\frac{2V_H + V_L}{3}$	V_H	V_L	(UB, UB, LB)
$\frac{2V_H + V_L}{3}$	V_H	$\frac{2V_H + V_L}{3}$	(UB, UB, UB)

In this case, the objective function value depend on the values of p and p_2^r. Maximum values of p and p_2^r are found in last row of Table 12. Therefore, optimal values of the decision variables are as follows:

$$p^* = \frac{2V_H + V_L}{3}, \qquad p_2^{n*} = V_H, \qquad p_2^{r*} = \frac{2V_H + V_L}{3}$$

1.1.e. Prefer not to buy in period 2.

$$S_2^r \leq 0, \quad S_2^n \leq 0$$

In this case, neither buying remanufactured nor new product is feasible due to their negative surplus values. Therefore, there are no units sold in period 2. This case is explained in the Table 13 below.

Table 13. Case 1.1.e from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$0 \leq X < \infty$	0	0	K_2	R

The mathematical model is given as follows:

Max $\prod(p) = p_s(K_2 + R) + Wp - c_nK - c_rR$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L+p}{2}$ (3)

$p - p_2^r \leq 0$ (4)

$V_H - p_2^n \leq 0$ (5)

$p, p_2^n, p_2^r \geq V_L$ (6)

First three constraints are the same as in the previous cases because of the necessity condition of Case 1. Constraint (4) forces $p_2^r = p$. Value of p_2^n is equal to its upper bound V_H by constraint (5). Therefore, only two feasible cases which consist upper and lower bounds of p exist as given in Table 14.

Table 14. Case 1.1.d. feasible prices.

p	p_2^n	p_2^r	**Explanation**
V_L	V_H	V_L	(LB, UB, LB)
$\frac{2V_H + V_L}{3}$	V_H	$\frac{2V_H + V_L}{3}$	(UB, UB, UB)

Objective function has a positive linear relationship with the value of p. Therefore, last case which has the upper bound of p gives the optimal values of prices.

$$p^* = \frac{2V_H + V_L}{3}, \qquad p_2^{n*} = V_H, \qquad p_2^{r*} = \frac{2V_H + V_L}{3}$$

Optimal values of all combinations are the same as in other cases. Therefore, if strategic consumers prefer to buy in period 1 and there is sufficient capacity to meet the initial demand, optimal prices of new and remanufactured products are given as follows:

$$p^* = \frac{2V_H + V_L}{3}, \qquad p_2^{n*} = V_H, \qquad p_2^{r*} = \frac{2V_H + V_L}{3}$$

1.2 Capacity is not sufficient to cover the first-period demand (W > K)

There is no sufficient capacity to meet the initial demand. Myopic consumers buy first, and it is assumed that there is sufficient amount of capacity to meet myopic consumers' demand (i.e. $\overline{\phi}W \leq K$). Strategic consumers also prefer to buy in the first period. Thus, while some of them may be able to buy in period 1, remaining portion "$W - K$" can only buy in period 2. In the second period, there is a deterministic quantity of strategic consumers who are present from the first period, and a random portion of consumers who behave like strategic customers. Because there is no unsold new product from the first period, customers need to decide whether or not to buy remanufactured product in the second period. Thus, Case 1.2 consists of two subcases.

1.2.a. Prefer to buy remanufactured product in period 2.

$$S_2^r \geq 0$$

Quantity of units sold and leftover are given in Table 15 based on realized demand.

Table 15. Case 1.2.a from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$W-K+X \leq R$	0	$W-K+X$	0	$R-W+K-X$
$R < W-K+X$	0	R	0	0

Mathematical model for this case is a follows:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R-W+K} p_2^r(W-K+x)f(x)dx + \int_{x=R-W+K}^{\infty} p_2^r Rf(x)dx$$

$$+ \int_{x=0}^{R-W+K} p_s(R-W+K-x)f(x)dx + Kp - c_nK - c_rR$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L+p}{2}$ (3)

$p - p_2^r \geq 0$ (4)

$p, p_2^r \geq V_L$ (5)

Objective function includes p and p_2^r as decision variables. Amount of products sold is based on the demand. First three constraints are similar to those in the previous cases. We have three feasible cases as shown in Table 16.

Table 16. Case 1.2.a. feasible prices.

p	p_2^r	Explanation
V_L	V_L	(LB, LB)
$\frac{2V_H + V_L}{3}$	V_L	(UB, LB)
$\frac{2V_H + V_L}{3}$	$\frac{2V_H + V_L}{3}$	(UB, UB)

Objective function has a positive linear relationship with values of p and p_2^r. Therefore, optimal prices are found at upper bounds of variables.

$$p^* = \frac{2V_H + V_L}{3}, \qquad p_2^{r*} = \frac{2V_H + V_L}{3}$$

1.2.b. Prefer not to buy remanufactured product in period 2.

$$S_2^r \leq 0$$

Surplus from buying a remanufactured product is nonpositive, i.e., consumers do not prefer to buy remanufactured items. All remanufactured products are sold as salvage (see details in Table 17).

Table 17. Case 1.2.b from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$0 \leq X < \infty$	0	0	0	R

Mathematical model for this case is given below.

Max $\prod(p) = Kp + p_s R - c_n K - c_r R$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$p - p_2^r \leq 0$ (4)

$p, p_2^r \geq V_L$ (5)

First three constraints are similar to those in the previous cases. Constraint (4) forces the value of p_2^r to be equal the value of p. Thus, we have two feasible cases for prices as given in Table 18.

Table 18. Case 1.2.b. feasible prices.

p	p_2^r	Explanation
V_L	V_L	(LB, LB)
$\frac{2V_H + V_L}{3}$	$\frac{2V_H + V_L}{3}$	(UB, UB)

Objective function is an increasing linear function with respect to p. Therefore, the last case gives the optimal values of prices.

$$p^* = \frac{2V_H + V_L}{3}, \qquad p_2^{r*} = \frac{2V_H + V_L}{3}$$

Case 2: Wait for period 2.

We assume that myopic consumers buy in period 1, and all strategic consumers choose to wait for period 2. Strategic consumers believe that at least one of the second-period surplus values is greater than or equal to the first-period surplus. Thus, they prefer to wait for period 2. The necessary condition is as follows:

$$S_1^n \leq max\{\hat{S}_2^n, \hat{S}_2^r\}$$

Second-period demand consists of sum of all strategic consumers and random demand portion.

2.1. Prefer to buy new in period 2, then remanufactured product.

$$S_2^n \geq S_2^r, \quad S_2^r \geq 0$$

In this case, if we compare the surplus values of products, buying a new product is preferred to buying a remanufactured product. Therefore, consumers prefer to buy the new product first. For example, if demand is high, rest of them can buy remanufactured products due to nonnegative surplus. The relationship between the demand quantity and the number of units sold is given in Table 19.

Table 19. Case 2.1 from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$\phi W + X \leq K_2$	$\phi W + X$	0	$K_2 - \phi W - X$	R
$K_2 < \phi W + X \leq K_2 + R$	K_2	$\phi W + X - K_2$	0	$R - \phi W - X + K_2$
$K_2 + R < \phi W + X$	K_2	R	0	0

To construct the necessary conditions of Case 2, a binary variable w is needed. The mathematical model that represent this case is given below:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{K_{2-\phi W}} p_2^n(\phi W + x)f(x)dx + \int_{x=K_2-\phi W}^{\infty} p_2^n K_2 f(x)dx$$
$$+ \int_{x=0}^{K_{2-\phi W}} p_s(K_2 - \phi W - x + R)f(x)dx$$
$$+ \int_{x=K_{2-\phi W}}^{K_2+R-\phi W} p_2^r(\phi W + x - K_2)f(x)dx + \int_{x=K_2+R-\phi W}^{\infty} p_2^r R f(x)dx$$
$$+ \int_{x=K_{2-\phi W}}^{K_2+R-\phi W} p_s(R - \phi W - x + K_2)f(x)dx + \bar{\phi} W p - c_n K - c_r R$$

Subject to

$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w$ (1)

$(1 - w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1 - w)$ (2)

$p = x_1 + x_2$ (3)

$$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w) \quad (4)$$

$$V_H - p_2^n \geq p - p_2^r \quad (5)$$

$$p - p_2^r \geq 0 \quad (6)$$

$$w \in \{0,1\} \quad (7)$$

$$p, p_2^n, p_2^r \geq V_L \quad (8)$$

The value of $\frac{2V_H+V_L}{1+2k}$ is the intersection of $\hat{S}_2^n$ and $\hat{S}_2^r$ with respect to p in the first two constraints. In other words, from $V_H - pk = p - \frac{V_L+p}{2}$, we have $p = \frac{2V_H-V_L}{1+2k}$. Constraint (4) provides the necessary condition for Case 2, which is explained in Figure 2.

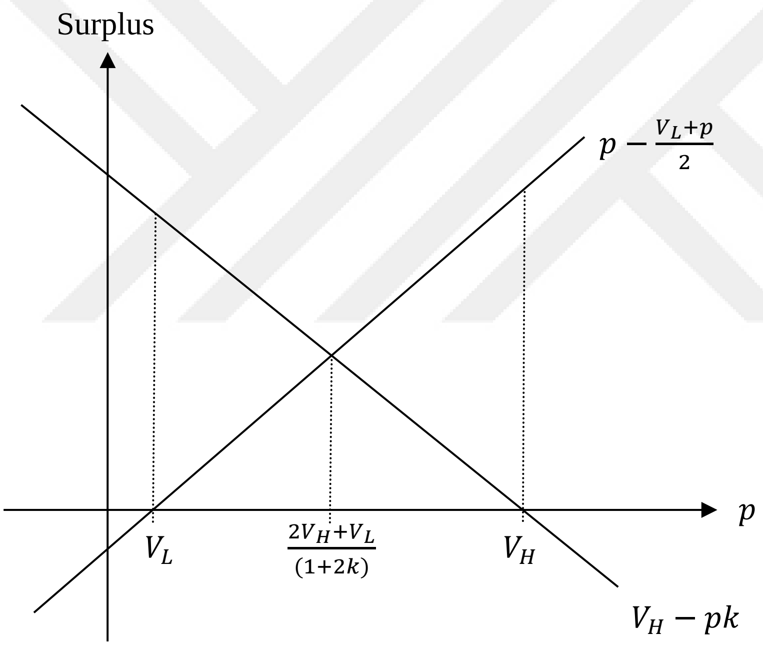

Figure 2. Second-period surplus based on beliefs versus first-period price.

When the first-period price $p \in [V_L, \frac{2V_H+V_L}{(1+2k)}]$, belief about the new product's surplus is greater than the remanufactured product's surplus. Therefore, S_1^n must be smaller than $\hat{S}_2^n$. When the first-period price $p \in [\frac{2V_H+V_L}{(1+2k)}, V_H]$, belief about the remanufactured product's surplus is greater than the new product's surplus. Thus, S_1^n must be smaller than $\hat{S}_2^r$. In summary, condition $S_1^n \leq max\{\hat{S}_2^n, \hat{S}_2^r\}$ is ensured through the first four constraints. Constraint (5) implies that buying a new product is preferable to buying a remanufactured product. Constraint (6) implies that buying a remanufactured product is also preferable due to its nonnegative surplus (i.e., when demand is high). Constraint (7) shows that w is a binary variable and constraint (8) indicates that all prices must be greater than or equal to low valuation. Objective function is an increasing linear function with respect to p, p_2^n, p_2^r. To maximize the objective function, upper bounds are checked first. When p is set at V_H, $w = 0$ and x_2 becomes V_H, x_1 becomes zero. Therefore, $p = V_H$. First four constraints are satisfied. When $p_2^n = p_2^r = V_H$ constraint (5) and (6) are satisfied. Therefore, optimal prices are given as:

$$p^* = V_H, \qquad p_2^{n*} = V_H, \qquad p_2^{r*} = V_H$$

2.2. Prefer to buy only new products in period 2.

$$S_2^n \geq 0, \quad S_2^r \leq 0$$

In this case, buying a new product is preferable due to its nonnegative surplus. However; buying a remanufactured product is not considered. Therefore, customers do not buy the remanufactured product even if the demand is high. Therefore, the relationship between the demand quantity and the number of units sold is summarized in Table 20.

Table 20. Case 2.2 from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$\phi W + X \leq K_2$	$\phi W + X$	0	$K_2 - \phi W - X$	R
$K_2 < \phi W + X$	K_2	0	0	R

The mathematical model that represent this case is given below:

$$\text{Max} \prod(p, p_2^n) = \int_{x=0}^{K_{2-\phi W}} p_2^n(\phi W + x) f(x) dx + \int_{x=K_2-\phi W}^{\infty} p_2^n K_2 f(x) dx$$
$$+ \int_{x=0}^{K_{2-\phi W}} p_s(K_2 - \phi W - x + R) f(x) dx$$
$$+ \int_{x=K_{2-\phi W}}^{\infty} p_s R f(x) dx + \bar{\phi} W p - c_n K - c_r R$$

Subject to

$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w$ (1)

$(1 - w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1 - w)$ (2)

$p = x_1 + x_2$ (3)

$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w)$ (4)

$V_H - p_2^n \geq 0$ (5)

$p - p_2^r \leq 0$ (6)

$w \in \{0,1\}$ (7)

$p, p_2^n, p_2^r \geq V_L$ (8)

First four constraint are the same as those in Case 2.1. Constraint (5) shows that consumers can choose to buy new product but they do not prefer to buy remanufactured product due to its negative surplus in constraint (6). Remaining constraints are also the same as those in Case 2.1. Objective function is increasing linearly with p and p_2^n. Therefore, if the constraints are satisfied, upper bounds of prices yields the maximum profit value. In other words, p and p_2^n are both set at V_H. Constraint (6) implies that $p \leq p_2^r$. Thus, $p_2^r = V_H$. Optimal prices are at the upper bound.

$$p^* = V_H, \qquad p_2^{n*} = V_H, \qquad p_2^{r*} = V_H$$

2.3. Prefer to buy the remanufactured product first, and then, the new product in period 2.

$$S_2^r \geq S_2^n, \quad S_2^n \geq 0$$

In this case, if we compare the surplus values of products, buying a remanufactured product is preferable compared to buying a new product. Therefore, consumers prefer to buy the remanufactured products first. Then, if there are remaining customers, they may buy the new product due to its nonnegative surplus. The relationship with the quantity of demand and the number of units sold is summarized in Table 21.

Table 21. Case 2.3 from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$\phi W + X \leq R$	0	$\phi W + X$	K_2	$R - \phi W - X$
$R < \phi W + X \leq K_2 + R$	$\phi W + X - R$	R	$K_2 - \phi W - X + R$	0
$K_2 + R < \phi W + X$	K_2	R	0	0

The mathematical model that represent this case is given as follows:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{R-\phi W} p_2^r(\phi W + x)f(x)dx + \int_{x=R-\phi W}^{\infty} p_2^r\, Rf(x)dx$$

$$+ \int_{x=0}^{R-\phi W} p_s(K_2 - \phi W - x + R)f(x)dx$$

$$+ \int_{x=R-\phi W}^{K_2+R-\phi W} p_2^n(\phi W + x - R)f(x)dx + \int_{x=K_2+R-\phi W}^{\infty} p_2^n K_2 f(x)dx$$

$$+ \int_{x=R-\phi W}^{K_2+R-\phi W} p_s(R - \phi W - x + K_2)f(x)dx + \bar{\phi} W p - c_n K - c_r R$$

Subject to

$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w$ (1)

$(1 - w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1 - w)$ (2)

$p = x_1 + x_2$ (3)

$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w)$ (4)

$p - p_2^r \geq V_H - p_2^n$ (5)

$V_H - p_2^n \geq 0$ (6)

$w \in \{0,1\}$ (7)

$p, p_2^n, p_2^r \geq V_L$ (8)

First four constraints provide Case 2 necessary conditions. Constraint (5) implies that buying a remanufactured product is more preferable than buying a new product due to its larger surplus. Buying a new product is also feasible but it gives less surplus. Remaining constraints are the same as in the previous cases. Objective function is increasing linearly in p, p_2^n and p_2^r. Therefore, optimal prices are

$$p^* = V_H, \qquad p_2^{n*} = V_H, \qquad p_2^{r*} = V_H$$

2.4. Prefer to buy only the remanufactured product in period 2.

$$S_2^r \geq 0, \quad S_2^n \leq 0$$

Buying a remanufactured product is preferable because it has nonnegative surplus. Consumers do not buy the new product due to its negative surplus. The relationship between the demand quantity and the number of units sold is summarized in Table 22.

Table 22. Case 2.4 from the manufacturer's point of view.

Demand	Number of Units Sold		Leftover Amounts	
	New Product	Remanufactured Product	New Product	Remanufactured Product
$\phi W + X \leq R$	0	$\phi W + X$	K_2	$R - \phi W - X$
$R < \phi W + X$	0	R	K_2	0

The mathematical model that represent this case is given below:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R-\phi W} p_2^r(\phi W + x) f(x) dx + \int_{x=R-\phi W}^{\infty} p_2^r \, R f(x) dx$$
$$+ \int_{x=0}^{R-\phi W} p_s(K_2 - \phi W - x + R) f(x) dx + \int_{x=R-\phi W}^{\infty} p_s K_2 f(x) dx$$

Subject to

$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w$ (1)

$$(1-w)\times\frac{2V_H+V_L}{1+2k}\le x_2\le V_H\times(1-w) \qquad (2)$$

$$p=x_1+x_2 \qquad (3)$$

$$V_H-(x_1+x_2)\le V_H\times w-x_1\times k+\frac{x_2}{2}-\frac{V_H}{2}\times(1-w) \qquad (4)$$

$$p-p_2^r\ge 0 \qquad (5)$$

$$V_H-p_2^n\le 0 \qquad (6)$$

$$w\in\{0,1\} \qquad (7)$$

$$p,p_2^n,p_2^r\ge V_L \qquad (8)$$

First four constraints provide Case 2 necessary conditions. Constraint (5) implies that buying a remanufactured product is preferable. On the other hand, buying a new product is not preferable due to its negative surplus. Remaining constraints are same as in the above cases. The objective function is increasing in p and p_2^r and upper bound satisfies all constraints. Optimal prices are given below, followed by a summary in Table 23.

$$.\,p^*=V_H,\qquad p_2^{n*}=V_H,\qquad p_2^{r*}=V_H$$

Table 23. Optimal prices without speculators.

Case	p^*	p_2^{n*}	p_2^{r*}
1.1.a	$\frac{2V_H+V_L}{3}$	V_H	$\frac{2V_H+V_L}{3}$
1.1.b	$\frac{2V_H+V_L}{3}$	V_H	$\frac{2V_H+V_L}{3}$
1.1.c	$\frac{2V_H+V_L}{3}$	V_H	$\frac{2V_H+V_L}{3}$
1.1.d	$\frac{2V_H+V_L}{3}$	V_H	$\frac{2V_H+V_L}{3}$
1.1.e	$\frac{2V_H+V_L}{3}$	V_H	$\frac{2V_H+V_L}{3}$
1.2.a	$\frac{2V_H+V_L}{3}$	-	$\frac{2V_H+V_L}{3}$
1.2.b	$\frac{2V_H+V_L}{3}$	-	$\frac{2V_H+V_L}{3}$
2.1	V_H	V_H	V_H
2.2	V_H	V_H	V_H
2.3	V_H	V_H	V_H
2.4	V_H	V_H	V_H

3.2 Dynamic Pricing with Speculators

We determine optimal prices considering the presence of speculators in the market. We initially investigated a game-theoretic approach for the fixed-pricing policy with speculators in the market which is included in Appendix A. However, the solution resulted in similar conclusions as provided in [2] with the exception that we consider remanufacturing which leads to more cases. Furthermore, the game-theoretic approach does not give a conclusive result for pricing. Therefore, we continued with the dynamic pricing policy, which resulted in fixed pricing to be optimal. Some portion of the returned products from the first period are collected by the manufacturer, and the rest by the speculators. The manufacturer and speculators remanufacture and sell them in the resale market. The manufacturer sells also any remaining unsold new products from the first period. All remaining products that are unsold in period 2 are sold as salvage at a price of p_s similar in the situation without speculators. Cases considered are summarized in Table 24 and Table 25 based on the purchase period and seller preferences of the strategic customers as well as whether there is sufficient capacity to cover the first-period demand.

Table 24. Summary of possible cases in the presence of speculators when strategic customers buy in period 1 (new product: N, remanufactured product from the manuacturer: RM, remanufactured product from speculators: RS).

Case 1: Buy in period 1.								
1.1 Capacity is sufficient to cover first-period demand. ($W \leq K$)					**1.2** Capacity is not sufficient to cover first-period demand. ($W > K$)			
	Order of purchase					**Order of purchase**		
Case	**Period 1**	**Period 2**			**Case**	**Period 1**	**Period 2**	
1.1.a	1) N	2) N	3) RM	4) RS	1.2.a	1) N	2) RM	3) RS
1.1.b	1) N	2) N	3) RS	4) RM	1.2.b	1) N	2) RM	3) -
1.1.c	1) N	2) N	3) RM	4) -	1.2.c	1) N	2) RS	3) RM
1.1.d	1) N	2) N	3) RS	4) -	1.2.d	1) N	2) RS	3) -
1.1.e	1) N	2) N	3) -	4) -	1.2.e	1) N	2) -	3) -
1.1.f	1) N	2) RM	3) N	4) RS				
1.1.g	1) N	2) RM	3) RS	4) N				
1.1.h	1) N	2) RM	3) N	4) -				
1.1.i	1) N	2) RM	3) RS	4) -				
1.1.j	1) N	2) RM	3) -	4) -				
1.1.k	1) N	2) RS	3) N	4) RM				
1.1.l	1) N	2) RS	3) RM	4) N				
1.1.m	1) N	2) RS	3) N	4) -				
1.1.n	1) N	2) RS	3) RM	4) -				
1.1.o	1) N	2) RS	3) -	4) -				
1.1.p	1) N	2) -	3) -	4) -				

Table 25. Summary of possible cases in the presence of speculators when strategic customers wait for period 2 (new product: N, remanufactured product from the manuacturer: RM, remanufactured product from speculators: RS).

Case 2: Wait for period 2.				
	Order of purchase			
Case	**Period 1**	**Period 2**		
2.1.	No purchase	1) N	2) RM	3) RS
2.2	No purchase	1) N	2) RS	3) RM
2.3	No purchase	1) N	2) RM	3) -
2.4	No purchase	1) N	2) RS	3) -
2.5	No purchase	1) N	2) -	3) -
2.6	No purchase	1) RM	2) N	3) RS
2.7	No purchase	1) RM	2) RS	3) N
2.8	No purchase	1) RM	2) N	3) -
2.9	No purchase	1) RM	2) RS	3) -
2.10	No purchase	1) RM	2) -	3) -
2.11	No purchase	1) RS	2) N	3) RM
2.12	No purchase	1) RS	2) RM	3) N
2.13	No purchase	1) RS	2) N	3) -
2.14	No purchase	1) RS	2) RM	3) -
2.15	No purchase	1) RS	2) -	3) -
2.16	No purchase	1) -	2) -	3) -

Case 1: Buy in period 1

Strategic consumers prefer to buy the new product in the first period if the first-period surplus is greater than or equal to the second-period surplus values determined by their beliefs. Case 1 necessity conditions are formed as follows:

$$S_1^n \geq 0, \quad S_1^n \geq \hat{S}_2^n, \quad S_1^n \geq \hat{S}_2^r$$

1.1. Capacity is sufficient to cover the first-period demand ($W \leq K$)

We assume there is sufficient capacity to meet the initial demand. All myopic and strategic customers may buy the new product in the first period and there is no strategic consumers who wait for period 2 ($W_2 = 0$). Unsold new products (K_2) and remanufactured products ($R = R_M + R_S$) are sold in the second period to random demand X. Some useful formulations are given as follows:

Demand in Period 2: X

$K_2 = K - W$

$W_2 = 0$

$R_M = W \times f_M$

$R_S = W \times f_S$

1.1.a. Customers prefer to buy the new product first. Afterwards, remaining customers prefer to buy the remanufactured product from the manufacturer. Lastly, any customers prefer to buy the remanufactured product from speculators. Therefore, their surplus relationships are given as follows:

$$S_2^n \geq S_2^r, \quad S_2^r \geq S_2^s, \quad S_2^s \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is summarized in Table 26.

Table 26. Case 1.1.a demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$X \leq K_2$	X	0	0	$K_2 - X$	R_M	R_S
$K_2 < X \leq K_2 + R_M$	K_2	$X - K_2$	0	0	$R_M - X + K_2$	R_S
$K_2 + R_M < X \leq K_2 + R$	K_2	R_M	$X - K_2 - R_M$	0	0	$R_S - X + K_2 + R_M$
$X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model for this case is given as follows:

$$\text{Max} \prod(p, p_2^n, p_2^r) = \int_{x=0}^{K_2} p_2^n x f(x)dx + \int_{x=0}^{K_2} p_s(K_2 - x + R_M)f(x)dx + \int_{x=K_2}^{\infty} p_2^n K_2 f(x)dx + \int_{x=K_2}^{K_2+R_M} p_2^r(x - K_2)f(x)dx + \int_{x=K_2}^{K_2+R_M} p_s(R_M - x + K_2)f(x)dx + \int_{x=K_2+R_M}^{\infty} p_2^r R_M f(x)dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$$V_H - p \geq p - \frac{V_L + p}{2} \qquad (3)$$

$$V_H - p_2^n \geq p - p_2^r \qquad (4)$$

$$p - p_2^r \geq p - p_2^s \qquad (5)$$

$$p - p_2^s \geq 0 \qquad (6)$$

$$p, p_2^n, p_2^r, p_2^s \geq V_L \qquad (7)$$

Constraint (1) ensures that the surplus of buying in the first period is greater than or equal to zero. Constraints (2) and (3) imply that the surplus from buying in the first period is greater than or equal to the surplus from second period new and remanufactured products with respect to consumer beliefs. Constraint (4) ensures that customers prefer to buy the new product. Remaining consumers can buy the remanufactured products, first from the manufacturer, and then, from speculators. Thus, constraint (5) ensures this order of preference. Constraint (6) provides nonnegative surplus from buying a remanufactured product from speculators and constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.b. In this case, customers prefer to buy the new product first. Remaining customers prefer to buy remanufactured product from speculators. If there are still remaining customers, then they prefer to buy remanufactured products from manufacturer. Their surplus relationships are given as follows:

$$S_2^n \geq S_2^s, \quad S_2^s \geq S_2^r, \quad S_2^r \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 27.

Table 27. Case 1.1.b demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$X \leq K_2$	X	0	0	$K_2 - X$	R_M	R_S
$K_2 < X \leq K_2 + R_S$	K_2	0	$X - K_2$	0	R_M	$R_S - X + K_2$
$K_2 + R_S < X \leq K_2 + R$	K_2	$x - K_2 - R_S$	R_S	0	$R_M - X + K_2 + R_S$	0
$X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model for this case is given below:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{K_2} p_2^n x f(x)dx + \int_{x=0}^{K_2} p_s(K_2 - x + R_M)f(x)dx$$

$$+ \int_{x=K_2}^{\infty} p_2^n K_2 f(x)dx + \int_{x=K_2}^{K_2+R_S} p_s R_M f(x)dx$$

$$+ \int_{x=K_{2+R_S}}^{K_2+R} p_2^r(x - K_2 - R_S)f(x)dx$$

$$+ \int_{x=K_2+R_S}^{K_2+R} p_s(R_M - x + K_2 + Rs)f(x)dx + \int_{x=K_2+R}^{\infty} p_2^r R_M f(x)dx$$

$$+ Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$V_H - p_2^n \geq p - p_2^s$ (4)

$p - p_2^s \geq p - p_2^r$ (5)

$p - p_2^r \geq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints explain why customers previously buy in the first period. Constraint (4) ensures that customers prefer to buy remaining new products later. Remaining consumers can buy remanufactured products first from speculators, and then, from the manufacturer given by constraint (5). Constraint (6) provides nonnegative surplus from buying remanufactured product from the manufacturer and constraint (7) implies that all prices are greater than or equal to the lower bound.

1.1.c. Customers firstly prefer to buy the new product. Later, they prefer to buy remanufactured product from the manufacturer. Buying a remanufactured product from the speculators is not feasible. Therefore, their surplus relationships are given as follows:

$$S_2^n \geq S_2^r, \quad S_2^r \geq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 28.

Table 28. Case 1.1.c demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$X \leq K_2$	X	0	0	$K_2 - X$	R_M	R_S
$K_2 < X \leq K_2 + R_M$	K_2	$X - K_2$	0	0	$R_M - X + K_2$	R_S
$X > K_2 + R_M$	K_2	R_M	0	0	0	R_S

The mathematical model for this case is given as follows:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{K_2} p_2^n x f(x)dx + \int_{x=0}^{K_2} p_s(K_2 - x + R_M)f(x)dx + \int_{x=K_2}^{\infty} p_2^n K_2 f(x)dx + \int_{x=K_2}^{K_2+R_M} p_2^r(x - K_2)f(x)dx + \int_{x=K_2}^{K_2+R_M} p_s(R_M - x + K_2)f(x)dx + \int_{x=K_2+R_M}^{\infty} p_2^r R_M f(x)dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$V_H - p_2^n \geq p - p_2^r$ (4)

$p - p_2^r \geq 0$ (5)

$p - p_2^s \leq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints explain why customers previously buy in the first period. Constraint (4) ensures that customers prefer to buy the remaining of the new product from the first period later. Then, remaining consumers can buy remanufactured products from the manufacturer in constraint (5). Due to negative surplus of buying remanufactured product from speculators, customers do not prefer to buy from speculators as given in constraint (6). Constraint (7) ensures that all prices are greater than or equal to the lower bound.

1.1.d. In this case, customers prefer to buy the new product first. Later, they prefer to buy the remanufactured product from speculators. Buying remanufactured product from manufacturer is not feasible. Therefore, their surplus relationships are given as follows:

$$S_2^n \geq S_2^s, \quad S_2^s \geq 0, \quad S_2^r \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 29.

Table 29. Case 1.1.d demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$X \leq K_2$	X	0	0	$K_2 - X$	R_M	R_S
$K_2 < X \leq K_2 + R_S$	K_2	0	$X - K_2$	0	R_M	$R_S - X + K_2$
$X > K_2 + R_S$	K_2	0	R_S	0	R_M	0

The mathematical model for this case is given below:

$$\text{Max} \prod(p, p_2^n) = \int_{x=0}^{K_2} p_2^n x f(x) dx + \int_{x=0}^{K_2} p_s (K_2 - x + R_M) f(x) dx$$
$$+ \int_{x=K_2}^{\infty} p_2^n K_2 f(x) dx + \int_{x=K_2}^{\infty} p_s R_M f(x) dx + Wp - c_n K - c_r R_M$$

Subject to

$$V_H - p \geq 0 \qquad (1)$$

$$V_H - p \geq V_H - pk \qquad (2)$$

$$V_H - p \geq p - \frac{V_L + p}{2} \qquad (3)$$

$$V_H - p_2^n \geq p - p_2^s \qquad (4)$$

$$p - p_2^s \geq 0 \qquad (5)$$

$$p - p_2^r \leq 0 \qquad (6)$$

$$p, p_2^n, p_2^r, p_2^s \geq V_L \qquad (7)$$

First three constraints explain why customers previously buy in the first period. Constraint (4) ensures that second-period customers prefer to buy left overs of the new product from the first period. Then, remaining consumers can buy remanufactured products from speculators due to nonnegative surplus as given by constraint (5). However; customers do not prefer to buy remanufactured product from the manufacturer as enforced by constraint (6). Constraint (7) ensures all prices to be greater than or equal to the lower bound.

1.1.e. In this case, customers prefer to buy the new product only. Thus, surplus relationships are given as follows:

$$S_2^n \geq 0, \quad S_2^r \leq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 30.

Table 30. Case 1.1.e demand relationships.

Demand	**Number of Units Sold**			**Leftover Amounts**		
	New Product	**Remanufactured Product from Manufacturer**	**Remanufactured Product from Speculator**	**New Product**	**Remanufactured Product from**	
					Manufacturer	**Speculators**
$X \leq K_2$	X	0	0	$K_2 - X$	R_M	R_S
$X > K_2$	K_2	0	0	0	R_M	R_S

The mathematical model for this case is given on the next page:

$$\text{Max} \prod(p, p_2^n) = \int_{x=0}^{K_2} p_2^n x f(x)dx + \int_{x=0}^{K_2} p_s(K_2 - x + R_M)f(x)dx + \int_{x=K_2}^{\infty} p_2^n K_2 f(x)dx + \int_{x=K_2}^{\infty} p_s R_M f(x)dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$V_H - p_2^n \geq 0$ (4)

$p - p_2^s \leq 0$ (5)

$p - p_2^r \leq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints explain why customers previously buy in the first period. Constraint (4) ensures that second-period customers can buy left overs of the new product from the first period due to nonnegative surplus. However; remaining consumers do not prefer to buy remanufactured products due to negative surplus values as given by constraints (5) and (6). Constraint (7) ensures that all prices are greater than or equal to the lower bound.

1.1.f. Customers firstly prefer to buy remanufactured product from manufacturer. Later, they prefer to buy the new product. Lastly, any remaining customers prefer to buy the remanufactured product from the speculators. Therefore, their surplus relationships are given as follows:

$$S_2^r \geq S_2^n, \quad S_2^n \geq S_2^s, \quad S_2^s \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 31.

Table 31. Case 1.1.f demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$X \leq R_M$	0	X	0	K_2	$R_M - X$	R_S
$R_M < X \leq K_2 + R_M$	$X - R_M$	R_M	0	$K_2 - X + R_M$	0	R_S
$K_2 + R_M < X \leq K_2 + R$	K_2	R_M	$X - K_2 - R_M$	0	0	$R_S - X + K_2 + R_M$
$X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model for this case is given below:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{R_M} p_2^r x f(x)dx + \int_{x=0}^{R_M} p_s(K_2 + R_M - x)f(x)dx + \int_{x=R_M}^{\infty} p_2^r R_M f(x)dx + \int_{x=R_M}^{K_2+R_M} p_2^n(x - K_2)f(x)dx + \int_{x=R_M}^{K_2+R_M} p_s(K_2 - x + R_M)f(x)dx + \int_{x=K_2+R_M}^{\infty} p_2^n K_2 f(x)dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$p - p_2^r \geq V_H - p_2^n$ (4)

$V_H - p_2^n \geq p - p_2^s$ (5)

$p - p_2^s \geq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints provide Case 1 conditions. Constraint (4) ensures that customers prefer to buy remanufactured product from manufacturer because it has the maximum amount of surplus. Then, remaining consumers can buy new products. After, buying new product, they can buy remanufactured products from speculators. Thus, constraint (5) supports this preference. Constraint (6) provides nonnegative surplus of buying remanufactured product from speculators, and constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.g. In this case customers prefer to buy remanufactured product from manufacturer first. Later, they prefer to buy remanufactured product from speculators. Lastly, any remaining customers prefer to buy the new product. Therefore, their surplus relationships are given as follows:

$$S_2^r \geq S_2^s, \quad S_2^s \geq S_2^n, \quad S_2^n \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 32.

Table 32. Case 1.1.g demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$X \leq R_M$	0	X	0	K_2	$R_M - X$	R_S
$R_M < X \leq R$	0	R_M	$X - R_M$	K_2	0	$R_S - X + R_M$
$R < X \leq K_2 + R$	$X - R_M - R_S$	R_M	R_S	$K_2 - X + R$	0	0
$X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model that represents this case from the manufacturer's point of view is given below:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{R_M} p_2^r x f(x)dx + \int_{x=0}^{R_M} p_s(K_2 + R_M - x)f(x)dx$$

$$+ \int_{x=R_M}^{\infty} p_2^r R_M f(x)dx + \int_{x=R_M}^{R} p_s K_2 f(x)dx$$

$$+ \int_{x=R}^{K_2+R} p_2^n(x - R_M - R_S)f(x)dx + \int_{x=R}^{K_2+R} p_s(K_2 - x + R)f(x)dx$$

$$+ \int_{x=K_2+R}^{\infty} p_2^n K_2 f(x)dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L+p}{2}$ (3)

$p - p_2^r \geq p - p_2^s$ (4)

$p - p_2^s \geq V_H - p_2^n$ (5)

$V_H - p_2^n \geq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints represent Case 1 conditions. Constraint (4) guarantees that customers prefer to buy remanufactured product from manufacturer due to highest amount of surplus. Then, remaining consumers prefer to buy from speculators. After buying from speculators, any remaining customers can buy new products remaining from the first period. Constraint (5) shows this criterion with respect to surplus values. Constraint (6) provides nonnegative surplus from buying a new product and constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.h. Customers prefer to buy remanufactured product from the manufacturer first. Later, they prefer to buy new product. They do not prefer to buy from speculators because of nonpositive surplus. Therefore, their surplus relationships are given as follows:

$$S_2^r \geq S_2^n, \quad S_2^n \geq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 33.

Table 33. Case 1.1.h demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$X \leq R_M$	0	X	0	K_2	$R_M - X$	R_S
$R_M < X \leq K_2 + R_M$	$X - R_M$	R_M	0	$K_2 - X + R_M$	0	R_S
$X > K_2 + R_M$	K_2	R_M	0	0	0	R_S

The mathematical model for this case is given below:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{R_M} p_2^r x f(x)dx + \int_{x=0}^{R_M} p_s(K_2 + R_M - x)f(x)dx + \int_{x=R_M}^{\infty} p_2^r R_M f(x)dx + \int_{x=R_M}^{K_2+R_M} p_2^n(x - R_M)f(x)dx + \int_{x=R_M}^{K_2+R_M} p_s(K_2 - x + R_M)f(x)dx + \int_{x=K_2+R_M}^{\infty} p_2^n K_2 f(x)dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$p - p_2^r \geq V_H - p_2^n$ (4)

$V_H - p_2^n \geq 0$ (5)

$p - p_2^s \leq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints represent Case 1 conditions. Constraint (4) guarantees that customers prefer to buy remanufactured product from manufacturer because it gives maximum amount of surplus. Remaining consumers prefer to buy new product because it has nonnegative surplus as shown in constraint (5). However; buying remanufactured product from speculators is not feasible due to nonpositive surplus given by constraint (6). Constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.i. Customers prefer to buy remanufactured product from manufacturer first. Later, they prefer to buy remanufactured product from speculators. They do not prefer to buy new product due to nonpositive surplus. Therefore, their surplus relationships are given as follows:

$$S_2^r \geq S_2^s, \quad S_2^s \geq 0, \quad S_2^n \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 34.

Table 34. Case 1.1.i demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from: Manufacturer	Remanufactured Product from: Speculators
$X \leq R_M$	0	X	0	K_2	$R_M - X$	R_S
$R_M < X \leq R$	0	R_M	$X - R_M$	K_2	0	$R_S - X + R_M$
$X > R$	0	R_M	R_S	K_2	0	0

The mathematical model for this case is given below:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R_M} p_2^r x f(x) dx + \int_{x=0}^{R_M} p_s (K_2 + R_M - x) f(x) dx + \int_{x=R_M}^{\infty} p_2^r R_M f(x) dx + \int_{x=R_M}^{\infty} p_s K_2 f(x) dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$p - p_2^r \geq p - p_2^s$ (4)

$p - p_2^s \geq 0$ (5)

$V_H - p_2^n \leq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints represent Case 1 conditions. Constraint (4) ensures that customers prefer to buy remanufactured product from manufacturer due to highest amount of surplus. Remaining consumers prefer to buy remanufactured product from speculators because of its nonnegative surplus given in constraint (5). Buying new product is not preferable due to nonpositive surplus given by constraint (6). Constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.j. Customers prefer only to buy remanufactured product from manufacturer. They do not prefer to buy other products due to nonpositive surplus. Therefore, their surplus relationships are given as follows:

$$S_2^r \geq 0, \quad S_2^n \leq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 35.

Table 35. Case 1.1.j demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$X \leq R_M$	0	X	0	K_2	$R_M - X$	R_S
$X > R_M$	0	R_M	0	K_2	0	R_S

The mathematical model that represents this case is given below:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R_M} p_2^r x f(x)dx + \int_{x=0}^{R_M} p_s(K_2 + R_M - x)f(x)dx + \int_{x=R_M}^{\infty} p_2^r R_M f(x)dx + \int_{x=R_M}^{\infty} p_s K_2 f(x)dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$p - p_2^r \geq 0$ (4)

$V_H - p_2^n \leq 0$ (5)

$p - p_2^s \leq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints represent Case 1 conditions. Constraint (4) ensures that customers prefer to buy remanufactured product from manufacturer due to nonnegative surplus. Afterwards, remaining consumers do not prefer to buy remanufactured product from speculators and new product from manufacturer because they have nonpositive surplus values given by constraints (5) and (6). Constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.k. Customers prefer to buy remanufactured product from speculators first. Later, they prefer to buy the new product. Lastly, any remaining customers prefer to buy remanufactured product from the manufacturer. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq S_2^n, \quad S_2^n \geq S_2^r, \quad S_2^r \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 36.

Table 36. Case 1.1.k demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$X \leq R_S$	0	0	X	K_2	R_M	$R_S - X$
$R_S < X \leq K_2 + R_S$	$X - R_S$	0	R_S	$K_2 - X + R_S$	R_M	0
$K_2 + R_S < X \leq K_2 + R$	K_2	$X - K_2 - R_S$	R_S	0	$R_M - X + K_2 + R_S$	0
$X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model that represents this case is given below:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{R_S} p_s(K_2 + R_M)f(x)dx + \int_{x=R_S}^{K_2+R_S} p_2^n(x - R_S)f(x)dx$$

$$+ \int_{x=R_S}^{K_2+R_S} p_s(K_2 - x + R_S + R_M)f(x)dx + \int_{x=K_2+R_S}^{\infty} p_2^n K_2 f(x)dx$$

$$+ \int_{x=K_2+R_S}^{K_2+R} p_2^r(x - K_2 - R_S)f(x)dx$$

$$+ \int_{x=K_2+R_S}^{K_2+R} p_s(R_M - x + K_2 + R_S)f(x)dx + \int_{x=K_2+R}^{\infty} p_2^r R_M f(x)dx$$

$$+ Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$p - p_2^s \geq V_H - p_2^n$ (4)

$V_H - p_2^n \geq p - p_2^r$ (5)

$p - p_2^r \geq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints represent Case 1 conditions. Constraint (4) guarantees that customers prefer to buy remanufactured product from speculators due to highest amount of surplus. Then, remaining consumers prefer to buy new form the manufacturer, and remanufactured products from speculators according to surplus amounts shown in constraints (5) and (6), respectively. Constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.l. In this case, customers prefer to buy remanufactured product from speculators first. Later, they prefer to buy remanufactured product from manufacturer. Lastly, any remaining customers prefer to buy new product. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq S_2^r, \quad S_2^r \geq S_2^n, \quad S_2^n \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 37.

Table 37. Case 1.1.l demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$X \leq R_S$	0	0	X	K_2	R_M	$R_S - X$
$R_S < X \leq R$	0	$X - R_S$	R_S	K_2	$R_M - X + R_S$	0
$R < X \leq K_2 + R$	$X - R_M - R_S$	R_M	R_S	$K_2 - X + R$	0	0
$X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model for this case is given on the next page:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{R_S} p_S(K_2 + R_M) f(x)dx + \int_{x=R_S}^{R} p_2^r(x - R_S) f(x)dx$$

$$+ \int_{x=R_S}^{R} p_S(K_2 + R_M - x + R_S) f(x)dx + \int_{x=R}^{\infty} p_2^r R_M f(x)dx$$

$$+ \int_{x=R}^{K_2+R} p_2^n(x - R_M - R_S) f(x)dx + \int_{x=R}^{K_2+R} p_S(K_2 - x + R) f(x)dx$$

$$+ \int_{x=K_2+R}^{\infty} p_2^n K_2 f(x)dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$p - p_2^s \geq p - p_2^r$ (4)

$p - p_2^r \geq V_H - p_2^n$ (5)

$V_H - p_2^n \geq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints support Case 1 conditions. Constraint (4) shows that customers prefer to buy remanufactured product from speculators due to maximum amount of surplus. Then, remaining consumers prefer to buy remanufactured product from manufacturer and rest of them prefer to buy new product given by constraints (5) and (6), respectively. Constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.m. Customers prefer to buy remanufactured product from speculators first. Later, they prefer to buy new product, but they do not prefer to buy remanufactured product from manufacturer. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq S_2^n, \quad S_2^n \geq 0, \quad S_2^r \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 38.

Table 38. Case 1.1.m demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$X \le R_S$	0	0	X	K_2	R_M	$R_S - X$
$R_S < X \le K_2 + R_S$	$X - R_S$	0	R_S	$K_2 - X + R_S$	R_M	0
$X > K_2 + R_S$	K_2	0	R_S	0	R_M	0

The mathematical model to express this case is given below:

$$\text{Max} \prod(p, p_2^n) = \int_{x=0}^{R_S} p_s(K_2 + R_M) f(x)dx + \int_{x=R_S}^{K_2+R_S} p_2^n (x - R_S) f(x)dx + \int_{x=R_S}^{K_2+R_S} p_s(K_2 - x + R_S + R_M) f(x)dx + \int_{x=K_2+R_S}^{\infty} p_2^n K_2 f(x)dx + \int_{x=K_2+R_S}^{\infty} p_s R_M f(x)dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \ge 0$ (1)

$V_H - p \ge V_H - pk$ (2)

$V_H - p \ge p - \frac{V_L + p}{2}$ (3)

$p - p_2^s \ge V_H - p_2^n$ (4)

$V_H - p_2^n \ge 0$ (5)

$p - p_2^r \le 0$ (6)

$p, p_2^n, p_2^r, p_2^s \ge V_L$ (7)

First three constraints support Case 1 conditions. Constraint (4) shows that customers prefer to buy remanufactured product from speculators due to highest surplus. Then, remaining consumers prefer to buy new product due to its nonnegative surplus given by constraint (5). Customers never prefer to buy remanufactured product according to constraint (6). Constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.n. Customers prefer to buy remanufactured product from speculators first. Later, they prefer to buy remanufactured product from manufacturer but they do not prefer to buy new product. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq S_2^r, \quad S_2^r \geq 0, \quad S_2^n \leq 0$$

The relationship between the interval of demand quantity, number of units sold, and leftover amounts is explained in Table 39.

Table 39. Case 1.1.n demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$X \leq R_S$	0	0	X	K_2	R_M	$R_S - X$
$R_S < X \leq R$	0	$X - R_S$	R_S	K_2	$R_M - X + R_S$	0
$X > R$	0	R_M	R_S	K_2	0	0

The mathematical model for this case is given follows:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R_S} p_s(K_2 + R_M)f(x)dx + \int_{x=R_S}^{R} p_2^r(x - R_S)f(x)dx$$
$$+ \int_{x=R_S}^{R} p_s(K_2 + R_M - x + R_S)f(x)dx + \int_{x=R}^{\infty} p_2^r R_M f(x)dx$$
$$+ \int_{x=R}^{\infty} p_s K_2 f(x)dx + Wp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L+p}{2}$ (3)

$p - p_2^s \geq p - p_2^r$ (4)

$p - p_2^r \geq 0$ (5)

$V_H - p_2^n \leq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints correspond to Case 1 conditions. Constraint (4) shows that customers prefer to buy remanufactured product from speculators due to highest surplus. Then, remaining consumers prefer to buy remanufactured product due to nonnegative surplus given in constraint (5). Customers never prefer to buy new product according to constraint (6). Constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.o. In this case, customers only prefer to buy remanufactured product from speculators. They do not prefer to buy new product and remanufactured product from manufacturer. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq 0, \quad S_2^n \leq 0, \quad S_2^r \leq 0$$

The relationship between the interval of demand quantity, number of units sold, and leftover amounts is explained in Table 40.

Table 40. Case 1.1.o demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$X \leq R_S$	0	0	X	K_2	R_M	$R_S - X$
$X > R_S$	0	0	R_S	K_2	R_M	0

The mathematical model that represents this is given on the next page:

Max $\prod(p) = p_s(K_2 + R_M) + Wp - c_nK - c_rR_M$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$p - p_2^s \geq 0$ (4)

$V_H - p_2^n \leq 0$ (5)

$p - p_2^r \leq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints correspond to Case 1 conditions. Constraint (4) shows that customers prefer only to buy remanufactured product from speculators due to nonnegative surplus, but they never prefer to buy new and remanufactured products from manufacturer given by constraint (5) and (6), respectively. Constraint (7) ensures that all prices are greater than or equal to low valuation.

1.1.p. Customers never prefer to buy in the second period. Therefore, surplus relationships are given as follows:

$$S_2^n \leq 0, \quad S_2^r \leq 0, \quad S_2^s \leq 0$$

The relationship between the interval of demand quantity, number of units sold, and leftover amounts is explained in Table 41.

Table 41. Case 1.1.p demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$0 \leq X \leq \infty$	0	0	0	K_2	R_M	R_S

The mathematical model that represents this case is given below:

Max $\prod(p) = p_s(K_2 + R_M) + Wp - c_nK - c_rR_M$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$V_H - p_2^n \leq 0$ (4)

$p - p_2^r \leq 0$ (5)

$p - p_2^s \leq 0$ (6)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (7)

First three constraints contain Case 1 conditions. Constraint (4), (5) and (6) show that buying in the second period is not feasible for customers. Constraint (7) ensures that all prices are greater than or equal to low valuation.

1.2. Capacity is not sufficient to cover the first-period demand (W > K)

Suppose that there is no sufficient capacity to meet the initial demand. Firstly, myopic consumers buy the new product. There is always enough capacity to meet myopic consumers' demand (i.e., $\bar{\phi}W \leq K$). Rest of new products are bought by strategic consumers. Nevertheless, some strategic consumers have to wait for second period. It is clear that the new product will not be left for the second period (i.e., $K_2 = 0$). Thus, p_2^n is no longer a decision variable in this setting. Besides, consumers have to decide about whether to buy the remanufactured product. If they decide to buy, then, they have to decide about from whom. Second-period demand consists of strategic consumers who have not succeeded in buying in the first period and random demand realized in the second period. Some useful formulations are given as follows:

Demand in Period 2: $W - K + X$

$K_2 = 0$

$W_2 = W - K$

$R_M = K \times f_M$

$R_S = K \times f_S$

1.2.a. Customers prefer to buy remanufactured product from the manufacturer first. Later, they prefer to buy from speculators. Therefore, their surplus relationships are given as follows:

$$S_2^r \geq S_2^s, \quad S_2^s \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 42.

Table 42. Case 1.2.a demand relationships.

Demand	Number of Units Sold		Leftover Amounts	
	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	Remanufactured Product from	
			Manufacturer	Speculators
$W-K+X \le R_M$	$W-K+X$	0	$R_M - W + K - X$	R_S
$R_M < W-K+X \le R$	R_M	$W-K+X - R_M$	0	$R_S - W + K - X + R_M$
$W-K+X > R$	R_M	R_S	0	0

The mathematical model that represents this case is given below:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R_M-W+K} p_2^r(W-K+x)f(x)dx + \int_{x=0}^{R_M-W+K} p_s(R_M - W + K - x)f(x)dx + \int_{x=R_M-W+K}^{\infty} p_2^r R_M f(x)dx + Kp - c_n K - c_r R_M$$

Subject to

$$V_H - p \ge 0 \qquad (1)$$

$$V_H - p \ge V_H - pk \qquad (2)$$

$$V_H - p \ge p - \frac{V_L+p}{2} \qquad (3)$$

$$p - p_2^r \ge p - p_2^s \qquad (4)$$

$$p - p_2^s \ge 0 \qquad (5)$$

$$p, p_2^r, p_2^s \ge V_L \qquad (6)$$

Constraint (1) ensures that surplus from buying in the first period is nonnegative Constraints (2) and (3) imply that surplus from buying in the first period is greater than or equal to the surplus from buying second period’s remanufactured products according to consumer beliefs. Constraint (4) ensures that customers prefer to buy remanufactured product from manufacturer at first. Then, remaining consumers can buy remanufactured products from speculators given by constraint (5). Constraint (6) ensures that all prices are greater than or equal to low valuation.

1.2.b. In this case, customers only prefer to buy remanufactured product from the manufacturer. They never prefer to buy from speculators. Therefore, their surplus relationships are given as follows:

$$S_2^r \geq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 43.

Table 43. Case 1.2.b demand relationships.

Demand	Number of Units Sold		Leftover Amounts	
	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	Remanufactured Product from	
			Manufacturer	Speculators
$W - K + X \leq R_M$	$W - K + X$	0	$R_M - W + K - X$	R_S
$W - K + X > R_M$	R_M	0	0	R_S

The mathematical model that represents this case is given below:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R_M-W+K} p_2^r(W - K + x)f(x)dx + \int_{x=0}^{R_M-W+K} p_s(R_M - W + K - x)f(x)dx + \int_{x=R_M-W+K}^{\infty} p_2^r R_M f(x)dx + Kp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L+p}{2}$ (3)

$p - p_2^r \geq 0$ (4)

$p - p_2^s \leq 0$ (5)

$p, p_2^r, p_2^s \geq V_L$ (6)

First three constraints imply that surplus from buying in the first period is greater than or equal to the surplus from buying second period's remanufactured products according to consumer beliefs. Constraint (4) ensures that customers can prefer to buy remanufactured product from manufacturer due to nonnegative surplus; however, they never prefer to buy from speculators given by constraint (5). Constraint (6) ensures that all prices are greater than or equal to low valuation.

1.2.c. In this case, customers prefer to buy remanufactured product from speculators. Later, they prefer to buy from manufacturer. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq S_2^r, \quad S_2^r \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 44.

Table 44. Case 1.2.c demand relationships.

Demand	Number of Units Sold		Leftover Amounts	
	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	Remanufactured Product from	
			Manufacturer	Speculators
$W - K + X \leq R_S$	0	$W - K + X$	R_M	$R_S - W + K - X$
$R_S < W - K + X \leq R$	$W - K + X - R_S$	R_S	$R_M - W + K - X + R_S$	0
$W - K + X > R$	R_M	R_S	0	0

The mathematical model for this case is given follows:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R_S-W+K} p_s R_M f(x)dx + \int_{x=R_S-W+K}^{R-W+K} p_2^r(W - K + x - R_S)f(x)dx$$
$$+ \int_{x=R_S-W+K}^{R-W+K} ps(R_M - W + K - x + R_S) + \int_{x=R-W+K}^{\infty} p_2^r R_M f(x)dx$$
$$+ Kp - c_n K - c_r R_M$$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L + p}{2}$ (3)

$p - p_2^s \geq p - p_2^r$ (4)

$p - p_2^r \geq 0$ (5)

$p, p_2^r, p_2^s \geq V_L$ (6)

Firs three constraints provide Case 1 conditions. Constraint (4) ensures that customers prefer to buy remanufactured product from speculators first. Then, remaining consumers can buy from manufacturers given by constraint (5). Constraint (6) ensures that all prices are greater than or equal to low valuation.

1.2.d. Customers prefer to buy only remanufactured product from speculators. They never prefer to buy from manufacturer. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq 0, \quad S_2^r \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 45.

Table 45. Case 1.2.d demand relationships.

Demand	Number of Units Sold		Leftover Amounts	
	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	Remanufactured Product from	
			Manufacturer	Speculators
$W - K + X \leq R_S$	0	$W - K + X$	R_M	$R_S - W + K - X$
$W - K + X > R_S$	0	R_S	R_M	0

The mathematical model that represents this case is given follows:

Max $\prod(p) = p_s R_M + Kp - c_n K - c_r R_M$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L+p}{2}$ (3)

$p - p_2^s \geq 0$ (4)

$p - p_2^r \leq 0$ (5)

$p, p_2^r, p_2^s \geq V_L$ (6)

Firs three constraints provide Case 1 conditions. Constraint (4) shows that customers can only prefer to buy remanufactured product from speculators. They never prefer to buy from manufacturers due to nonpositive surplus as shown in constraint (5). Constraint (6) ensures that all prices are greater than or equal to low valuation.

1.2.e. In this case, customers do not prefer to buy in the second period. Therefore, their surplus relationships are given as follows:

$$S_2^r \leq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 46.

Table 46. Case 1.2.e demand relationships.

Demand	Number of Units Sold		Leftover Amounts	
	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	Remanufactured Product from	
			Manufacturer	Speculators
$0 \leq W - K + X \leq \infty$	0	0	R_M	R_S

The mathematical model for this case is given below:

Max $\prod(p) = p_s R_M + Kp - c_n K - c_r R_M$

Subject to

$V_H - p \geq 0$ (1)

$V_H - p \geq V_H - pk$ (2)

$V_H - p \geq p - \frac{V_L+p}{2}$ (3)

$p - p_2^r \leq 0$ (4)

$p - p_2^s \leq 0$ (5)

$p, p_2^r, p_2^s \geq V_L$ (6)

First three constraints correspond to Case 1 conditions. Constraints (4) and (5) show that buying in the second period is not preferable for customers due to their surplus values. Constraint (6) ensures that all prices are greater than or equal to low valuation.

Case 2: Wait for period 2.

Strategic consumers prefer to wait for the second-period sales. At least one of the second-period surplus values should be greater than or equal to the first-period surplus in beliefs. Thus, Case 2 necessity conditions are formed as follows:

$$S_1^n \leq \max\{\hat{S}_2^n, \hat{S}_2^r\}$$

We assume that there is sufficient capacity to meet myopic customers' demand. Strategic consumers prefer to wait for period 2 because they believe that buying in second period will result in more surplus. Unsold new products (K_2) and remanufactured products ($R = R_M + R_S$) are sold in the second period to random demand X and strategic consumers who waited (W_2). Some useful formulations are given as follows:

Demand in Period 2: $\phi W + X$

$K_2 = K - \bar{\phi} W$

$W_2 = \phi W$

$R_M = \bar{\phi} W \times f_M$

$R_S = \bar{\phi} W \times f_S$

2.1. Customers prefer to buy new product first. Later, they prefer to buy remanufactured product from manufacturer. Lastly, any remaining customers prefer to buy remanufactured product from speculators. Therefore, their surplus relationships are given as follows:

$$S_2^n \geq S_2^r, \quad S_2^r \geq S_2^s, \quad S_2^s \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 47.

Table 47. Case 2.1. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$\phi W + X \leq K_2$	$\phi W + X$	0	0	$K_2 - \phi W - X$	R_M	R_S
$K_2 < \phi W + X \leq K_2 + R_M$	K_2	$\phi W + X - K_2$	0	0	$R_M - \phi W - X + K_2$	R_S
$K_2 + R_M < \phi W + X \leq K_2 + R$	K_2	R_M	$\phi W + X - K_2 - R_M$	0	0	$R_S - \phi W - X + K_2 + R_M$
$\phi W + X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model for this case is given below:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{K_2-\phi W} p_2^n(\phi W + x) f(x)dx$$

$$+ \int_{x=0}^{K_2-\phi W} p_s(K_2 - \phi W - x + R_M) f(x)dx + \int_{x=K_2-\phi W}^{\infty} p_2^n K_2 f(x)dx$$

$$+ \int_{x=K_2-\phi W}^{K_2+R_M-\phi W} p_2^r(\phi W + x - K_2) f(x)dx$$

$$+ \int_{x=K_2-\phi W}^{K_2+R_M-\phi W} p_s(R_M - \phi W - x + K_2) f(x)dx$$

$$+ \int_{x=K_2+R_M-\phi W}^{\infty} p_2^r R_M f(x)dx + \bar{\phi} W p - c_n K - c_r R_M$$

Subject to

$V_L \times w \leq x_1 \leq \frac{2V_H+V_L}{1+2k} \times w$ (1)

$(1-w) \times \frac{2V_H+V_L}{1+2k} \leq x_2 \leq V_H \times (1-w)$ (2)

$p = x_1 + x_2$ (3)

$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1-w)$ (4)

$$V_H - p_2^n \geq p - p_2^r \quad (5)$$

$$p - p_2^r \geq p - p_2^s \quad (6)$$

$$p - p_2^s \geq 0 \quad (7)$$

$$w \in \{0,1\} \quad (8)$$

$$p, p_2^n, p_2^r, p_2^s \geq V_L \quad (9)$$

The reason behind using binary variable of w is same as in the case without speculators. Thus, first four constraints ensure that waiting for the second period is preferred for customers in beliefs. Constraint (5) ensures that customers buy new product first, and then remanufactured products from manufacturer. Any remaining customers buy remanufactured products from speculators. Thus, constraint (6) supports this preference. Constraint (7) ensures nonnegative surplus from buying remanufactured product from speculators. Constraint (8) shows that w is binary. Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.2. Customers prefer to buy new product first. Later, they prefer to buy remanufactured product from speculators. Lastly, any remaining customers prefer to buy remanufactured product from manufacturer. Their surplus relationships are:

$$S_2^n \geq S_2^s, \quad S_2^s \geq S_2^r, \quad S_2^r \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 48.

Table 48. Case 2.2. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$\phi W + X \leq K_2$	$\phi W + X$	0	0	$K_2 - \phi W - X$	R_M	R_S
$K_2 < \phi W + X \leq K_2 + R_S$	K_2	0	$\phi W + X - K_2$	0	R_M	$R_S - \phi W - X + K_2$
$K_2 + R_S < \phi W + X \leq K_2 + R$	K_2	$\phi W + X - K_2 - R_S$	R_S	0	$R_M - \phi W - X + K_2 + R_S$	0
$\phi W + X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model for this case is given below:

$$\text{Max} \prod(p, p_2^n, p_2^r)$$

$$= \int_{x=0}^{K_2-\phi W} p_2^n(\phi W + x) f(x) dx$$

$$+ \int_{x=0}^{K_2-\phi W} p_s(K_2 - \phi W - x + R_M) f(x) dx + \int_{x=K_2-\phi W}^{\infty} p_2^n K_2 f(x) dx$$

$$+ \int_{x=K_2-\phi W}^{K_2+R_S-\phi W} p_s R_M f(x) dx$$

$$+ \int_{x=K_2+R_S-\phi W}^{K_2+R-\phi W} p_2^r(\phi W + x - K_2 - R_S) f(x) dx$$

$$+ \int_{x=K_2+R_S-\phi W}^{K_2+R-\phi W} p_s(R_M - \phi W - x + K_2 + R_S) f(x) dx$$

$$+ \int_{x=K_2+R-\phi W}^{\infty} p_2^r R_M f(x) dx + \bar{\phi} W p - c_n K - c_r R_M$$

Subject to

$$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w \quad (1)$$

$$(1 - w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1 - w) \quad (2)$$

$$p = x_1 + x_2 \quad (3)$$

$$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w) \quad (4)$$

$$V_H - p_2^n \geq p - p_2^r \quad (5)$$

$$p - p_2^r \geq p - p_2^s \quad (6)$$

$$p - p_2^s \geq 0 \quad (7)$$

$$w \in \{0,1\} \quad (8)$$

$$p, p_2^n, p_2^r, p_2^s \geq V_L \quad (9)$$

Constraint (5) shows that customers firstly prefer to buy new product. Then, remaining consumers can buy remanufactured products from manufacturer. Lastly, any remaining customers prefer to buy remanufactured products from speculators given by constraint (6). Constraint (7) ensures nonnegative surplus from buying remanufactured product from speculators.

2.3. Customers prefer to buy new product first. Later, they prefer to buy remanufactured product from manufacturer. They do not prefer to buy remanufactured product from speculators. Therefore, their surplus relationships are given as follows:

$$S_2^n \geq S_2^r, \quad S_2^r \geq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 49.

Table 49. Case 2.3. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$\phi W + X \leq K_2$	$\phi W + X$	0	0	$K_2 - \phi W - X$	R_M	R_S
$K_2 < \phi W + X \leq K_2 + R_M$	K_2	$\phi W + X - K_2$	0	0	$R_M - \phi W - X + K_2$	R_S
$\phi W + X > K_2 + R_M$	K_2	R_M	0	0	0	R_S

The mathematical model for this case is given below:

$$\text{Max} \prod(p, p_2^n, p_2^r)$$
$$= \int_{x=0}^{K_2-\phi W} p_2^n(\phi W + x) f(x) dx$$
$$+ \int_{x=0}^{K_2-\phi W} p_s(K_2 - \phi W - x + R_M) f(x) dx + \int_{x=K_2-\phi W}^{\infty} p_2^n K_2 f(x) dx$$
$$+ \int_{x=K_2-\phi W}^{K_2+R_M-\phi W} p_2^r(\phi W + x - K_2) f(x) dx$$
$$+ \int_{x=K_2-\phi W}^{K_2+R_M-\phi W} p_s(R_M - \phi W - x + K_2) f(x) dx$$
$$+ \int_{x=K_2+R_M-\phi W}^{\infty} p_2^r R_M f(x) dx + \bar{\phi} W p - c_n K - c_r R_M$$

Subject to

$$V_L \times w \le x_1 \le \frac{2V_H+V_L}{1+2k} \times w \quad (1)$$

$$(1-w) \times \frac{2V_H+V_L}{1+2k} \le x_2 \le V_H \times (1-w) \quad (2)$$

$$p = x_1 + x_2 \quad (3)$$

$$V_H - (x_1 + x_2) \le V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1-w) \quad (4)$$

$$V_H - p_2^n \ge p - p_2^r \quad (5)$$

$$p - p_2^r \ge 0 \quad (6)$$

$$p - p_2^s \le 0 \quad (7)$$

$$w \in \{0,1\} \quad (8)$$

$$p, p_2^n, p_2^r, p_2^s \ge V_L \quad (9)$$

Customers firstly prefer to buy new product as shown in constraint (5). Then, remaining consumers can buy remanufactured products from manufacturer according to constraint (6). Customers do not prefer to buy remanufactured products from speculators due to nonpositive surplus shown in constraint (7). Constraint (9) ensures that all prices are greater or equal to low valuation.

2.4. In this case, customers prefer to buy new product first. Later, they prefer to buy remanufactured product from speculators. They do not prefer to buy remanufactured product from manufacturer. Therefore, their surplus relationships are given as follows:

$$S_2^n \ge S_2^s, \quad S_2^s \ge 0, \quad S_2^r \le 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 50.

Table 50. Case 2.4. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$\phi W + X \le K_2$	$\phi W + X$	0	0	$K_2 - \phi W - X$	R_M	R_S
$K_2 < \phi W + X \le K_2 + R_S$	K_2	0	$\phi W + X - K_2$	0	R_M	$R_S - \phi W - X + K_2$
$\phi W + X > K_2 + R_S$	K_2	0	R_M	0	R_M	0

The mathematical model that represents this case is given follows:

$$\text{Max} \prod(p, p_2^n) = \int_{x=0}^{K_2-\phi W} p_2^n(\phi W + x) f(x) dx$$

$$+ \int_{x=0}^{K_2-\phi W} p_s(K_2 - \phi W - x + R_M) f(x) dx + \int_{x=K_2-\phi W}^{\infty} p_2^n K_2 f(x) dx$$

$$+ \int_{x=K_2-\phi W}^{\infty} p_s R_M f(x) dx + \bar{\phi} W p - c_n K - c_r R_M$$

Subject to

$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w$ (1)

$(1 - w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1 - w)$ (2)

$p = x_1 + x_2$ (3)

$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w)$ (4)

$V_H - p_2^n \geq p - p_2^s$ (5)

$p - p_2^s \geq 0$ (6)

$p - p_2^r \leq 0$ (7)

$w \in \{0,1\}$ (8)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (9)

Customers prefer to buy new product first as shown in constraint (5). Then, remaining consumers can buy remanufactured products from speculators according to constraint (6). Customers do not prefer to buy remanufactured products from manufacturer due to nonpositive surplus as shown in constraint (7). Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.5. Customers prefer to buy new product only. They do not prefer to buy remanufactured product. Therefore, their surplus relationships are given as follows:

$$S_2^n \geq 0, \quad S_2^r \leq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 51.

Table 51. Case 2.5. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$\phi W + X \leq K_2$	$\phi W + X$	0	0	$K_2 - \phi W - X$	R_M	R_S
$\phi W + X > K_2$	K_2	0	0	0	R_M	R_S

The mathematical model that represents this case is given below:

$$\text{Max} \prod(p, p_2^n) = \int_{x=0}^{K_2-\phi W} p_2^n(\phi W + x) f(x)dx$$

$$+ \int_{x=0}^{K_2-\phi W} p_s(K_2 - \phi W - x + R_M) f(x)dx + \int_{x=K_2-\phi W}^{\infty} p_2^n K_2 f(x)dx$$

$$+ \int_{x=K_2-\phi W}^{\infty} p_s R_M f(x)dx + \bar{\phi} W p - c_n K - c_r R_M$$

Subject to

$$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w \quad (1)$$

$$(1-w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1-w) \quad (2)$$

$$p = x_1 + x_2 \quad (3)$$

$$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1-w) \quad (4)$$

$$V_H - p_2^n \geq 0 \quad (5)$$

$$p - p_2^r \leq 0 \quad (6)$$

$$p - p_2^s \leq 0 \quad (7)$$

$$w \in \{0,1\} \quad (8)$$

$$p, p_2^n, p_2^r, p_2^s \geq V_L \quad (9)$$

In this setting, customers only prefer to buy the new product in period 2 because of its nonnegative surplus as shown in constraint (5). Buying remanufactured product is not rational due to their surplus values as shown in constraints (6) and (7). Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.6. In this case, customers prefer to buy remanufactured product from the manufacturer first. Later, they prefer to buy new product. Lastly, any remaining customers prefer to buy remanufactured product from speculators. Their surplus relationships are given below, followed by the relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 52.

$$S_2^r \geq S_2^n, \quad S_2^n \geq S_2^s, \quad S_2^s \geq 0$$

Table 52. Case 2.6. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$\phi W + X \leq R_M$	0	$\phi W + X$	0	K_2	$R_M - \phi W - X$	R_S
$R_M < \phi W + X \leq K_2 + R_M$	$\phi W + X - R_M$	R_M	0	$K_2 - \phi W - X + R_M$	0	R_S
$K_2 + R_M < \phi W + X \leq K_2 + R$	K_2	R_M	$\phi W + X - K_2 - R_M$	0	0	$R_S - \phi W - X + K_2 + R_M$
$\phi W + X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model for this case is given as follows:

$$\begin{aligned} \text{Max} \prod(p, p_2^n, p_2^r) &= \int_{x=0}^{R_M-\phi W} p_2^r(\phi W + x) f(x) dx \\ &+ \int_{x=0}^{R_M-\phi W} p_s(K_2 + R_M - \phi W - x) f(x) dx + \int_{x=R_M-\phi W}^{\infty} p_2^r R_M f(x) dx \\ &+ \int_{x=R_M-\phi W}^{K_2+R_M-\phi W} p_2^n(\phi W + x - R_M) f(x) dx \\ &+ \int_{x=R_M-\phi W}^{K_2+R_M-\phi W} p_s(K_2 - \phi W - x + R_M) f(x) dx \\ &+ \int_{x=K_2+R_M-\phi W}^{\infty} p_2^n K_2 f(x) dx + \bar{\phi} W p - c_n K - c_r R_M \end{aligned}$$

Subject to

$$V_L \times w \le x_1 \le \frac{2V_H + V_L}{1+2k} \times w \quad (1)$$

$$(1-w) \times \frac{2V_H + V_L}{1+2k} \le x_2 \le V_H \times (1-w) \quad (2)$$

$$p = x_1 + x_2 \quad (3)$$

$$V_H - (x_1 + x_2) \le V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1-w) \quad (4)$$

$$p - p_2^r \ge V_H - p_2^n \quad (5)$$

$$V_H - p_2^n \ge p - p_2^s \quad (6)$$

$$p - p_2^s \ge 0 \quad (7)$$

$$w \in \{0,1\} \quad (8)$$

$$p, p_2^n, p_2^r, p_2^s \ge V_L \quad (9)$$

First four constraints ensure that waiting for the second period is preferred for customers in belief. Constraint (5) shows that customers firstly prefer to buy remanufactured product and remaining consumers can buy new products. Then, any remaining customers prefer to buy remanufactured products from speculators. Thus, constraint (6) supports this preference. Constraint (7) provides nonnegative surplus of buying remanufactured product from speculators. Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.7. Customers prefer to buy remanufactured product from manufacturer first. Later, they prefer to buy remanufactured product from speculators. Lastly, any remaining customers prefer to buy new product. Therefore, their surplus relationships are given as follows:

$$S_2^r \ge S_2^s, \quad S_2^s \ge S_2^n, \quad S_2^n \ge 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 53.

Table 53. Case 2.7. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$\phi W + X \leq R_M$	0	$\phi W + X$	0	K_2	$R_M - \phi W - X$	R_S
$R_M < \phi W + X \leq R$	0	R_M	$\phi W + X - R_M$	K_2	0	$R_S - \phi W - X + R_M$
$R < \phi W + X \leq K_2 + R$	$\phi W + X - R_M - R_S$	R_M	R_S	$K_2 - \phi W - X + R$	0	0
$\phi W + X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model for this case is given as follows:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{R_M-\phi W} p_2^r(\phi W + x) f(x) dx$$

$$+ \int_{x=0}^{R_M-\phi W} p_s(K_2 + R_M - \phi W - x) f(x) dx + \int_{x=R_M-\phi W}^{\infty} p_2^r R_M f(x) dx$$

$$+ \int_{x=R_M-\phi W}^{R-\phi W} p_s K_2 f(x) dx$$

$$+ \int_{x=R-\phi W}^{K_2+R-\phi W} p_2^n(\phi W + x - R_M - R_S) f(x) dx$$

$$+ \int_{x=R-\phi W}^{K_2+R-\phi W} p_s(K_2 - \phi W - x + R) f(x) dx$$

$$+ \int_{x=K_2+R-\phi W}^{\infty} p_2^n K_2 f(x) dx + \bar{\phi} W p - c_n K - c_r R_M$$

Subject to

$$V_L \times w \leq x_1 \leq \frac{2V_H+V_L}{1+2k} \times w \quad (1)$$

$$(1-w) \times \frac{2V_H+V_L}{1+2k} \leq x_2 \leq V_H \times (1-w) \quad (2)$$

$$p = x_1 + x_2 \quad (3)$$

$$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1-w) \quad (4)$$

$$p - p_2^r \geq p - p_2^s \quad (5)$$

$$p - p_2^s \geq V_H - p_2^n \quad (6)$$

$$V_H - p_2^n \geq 0 \quad (7)$$

$$w \in \{0,1\} \quad (8)$$

$$p, p_2^n, p_2^r, p_2^s \geq V_L \quad (9)$$

Constraint (5) shows that customers firstly prefer to buy remanufactured product from manufacturer. Then, remaining consumers can buy remanufactured products from speculators. Lastly, any remaining customers prefer to buy new products as given by constraints (6) and (7), respectively. Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.8. Customers prefer to buy remanufactured product from manufacturer first. Later, they prefer to buy new products. They do not prefer to buy remanufactured product from speculators. Therefore, their surplus relationships are given as follows:

$$S_2^r \geq S_2^n, \quad S_2^n \geq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 54.

Table 54. Case 2.8. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$\phi W + X \leq R_M$	0	$\phi W + X$	0	K_2	$R_M - \phi W - X$	R_S
$R_M < \phi W + X \leq K_2 + R_M$	$\phi W + X - R_M$	R_M	0	$K_2 - \phi W - X + R_M$	0	R_S
$\phi W + X > K_2 + R_M$	K_2	R_M	0	0	0	R_S

The mathematical model for this case is given below:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{R_M-\phi W} p_2^r(\phi W + x) f(x) dx$$

$$+ \int_{x=0}^{R_M-\phi W} p_s(K_2 + R_M - \phi W - x) f(x) dx + \int_{x=R_M-\phi W}^{\infty} p_2^r R_M f(x) dx$$

$$+ \int_{x=R_M-\phi W}^{K_2+R_M-\phi W} p_2^n(\phi W + x - R_M) f(x) dx$$

$$+ \int_{x=R_M-\phi W}^{K_2+R_M-\phi W} p_s(K_2 - \phi W - x + R_M) f(x) dx$$

$$+ \int_{x=K_2+R_M-\phi W}^{\infty} p_2^n K_2 f(x) dx + \bar{\phi} W p - c_n K - c_r R_M$$

Subject to

$V_L \times w \leq x_1 \leq \frac{2V_H+V_L}{1+2k} \times w$ (1)

$(1-w) \times \frac{2V_H+V_L}{1+2k} \leq x_2 \leq V_H \times (1-w)$ (2)

$p = x_1 + x_2$ (3)

$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1-w)$ (4)

$p - p_2^r \geq V_H - p_2^n$ (5)

$V_H - p_2^n \geq 0$ (6)

$p - p_2^s \leq 0$ (7)

$w \in \{0,1\}$ (8)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (9)

Constraint (5) shows that customers firstly prefer to buy remanufactured product from manufacturer. Then, remaining consumers can prefer to buy new products from speculators as given by constraint (6). However, customers do not prefer to buy remanufactured product from speculators as shown in constraint (7). Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.9. Customers prefer to buy remanufactured product from manufacturer first. Later, they prefer to buy remanufactured products from speculators. They do not prefer to buy new product. Therefore, their surplus relationships are given as follows:

$$S_2^r \geq S_2^s, \quad S_2^s \geq 0, \quad S_2^n \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 55.

Table 55. Case 2.9. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$\phi W + X \leq R_M$	0	$\phi W + X$	0	K_2	$R_M - \phi W - X$	R_S
$R_M < \phi W + X \leq R$	0	R_M	$\phi W + X - R_M$	K_2	0	$R_S - \phi W - X + R_M$
$\phi W + X > R$	0	R_M	R_S	K_2	0	0

The mathematical model for this case is given below:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R_M - \phi W} p_2^r(\phi W + x) f(x) dx + \int_{x=0}^{R_M - \phi W} p_s(K_2 + R_M - \phi W - x) f(x) dx + \int_{x=R_M-\phi W}^{\infty} p_2^r R_M f(x) dx + \int_{x=R_M-\phi W}^{\infty} p_s K_2 f(x) dx + \bar{\phi} W p - c_n K - c_r R_M$$

Subject to

$$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w \quad (1)$$

$$(1-w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1-w) \quad (2)$$

$$p = x_1 + x_2 \tag{3}$$

$$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w) \tag{4}$$

$$p - p_2^r \geq p - p_2^s \tag{5}$$

$$p - p_2^s \geq 0 \tag{6}$$

$$V_H - p_2^n \leq 0 \tag{7}$$

$$w \in \{0,1\} \tag{8}$$

$$p, p_2^n, p_2^r, p_2^s \geq V_L \tag{9}$$

Constraint (5) shows that customers firstly prefer to buy remanufactured product from manufacturer. Then, remaining consumers can prefer to buy remanufactured products from speculators as given by constraint (6). However, customers do not prefer to buy new product as given by constraint (7). Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.10. In this case, customers only prefer to buy remanufactured product from manufacturer. They do not prefer to buy new and remanufactured product from speculators. Therefore, their surplus relationships are given as follows:

$$S_2^r \geq 0, \quad S_2^n \leq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 56.

Table 56. Case 2.10. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$\phi W + X \leq R_M$	0	$\phi W + X$	0	K_2	$R_M - \phi W - X$	R_S
$\phi W + X > R_M$	0	R_M	0	K_2	0	R_S

The mathematical model for this case is given on the next page:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R_M - \phi W} p_2^r(\phi W + x) f(x) dx$$
$$+ \int_{x=0}^{R_M - \phi W} p_s(K_2 + R_M - \phi W - x) f(x) dx + \int_{x=R_M - \phi W}^{\infty} p_2^r R_M f(x) dx$$
$$+ \int_{x=R_M - \phi W}^{\infty} p_s K_2 f(x) dx + \bar{\phi} W p - c_n K - c_r R_M$$

Subject to

$$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w \quad (1)$$

$$(1-w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1-w) \quad (2)$$

$$p = x_1 + x_2 \quad (3)$$

$$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1-w) \quad (4)$$

$$p - p_2^r \geq 0 \quad (5)$$

$$V_H - p_2^n \leq 0 \quad (6)$$

$$p - p_2^s \leq 0 \quad (7)$$

$$w \in \{0,1\} \quad (8)$$

$$p, p_2^n, p_2^r, p_2^s \geq V_L \quad (9)$$

Constraint (5) shows that customers only prefer to buy remanufactured product from manufacturer because it gives nonnegative surplus. They do not prefer to buy other products given by constraints (6) and (7). Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.11. Customers firstly prefer to buy remanufactured product from speculators. Later, they prefer to buy new product. Lastly, any remaining customers prefer to buy remanufactured product from the manufacturer. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq S_2^n, \quad S_2^n \geq S_2^r, \quad S_2^r \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 57.

Table 57. Case 2.11. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$\phi W + X \leq R_S$	0	0	$\phi W + X$	K_2	R_M	$R_S - \phi W - X$
$R_S < \phi W + X \leq K_2 + R_S$	$\phi W + X - R_S$	0	R_S	$K_2 - \phi W - X + R_S$	R_M	0
$K_2 + R_S < \phi W + X \leq K_2 + R$	K_2	$\phi W + X - K_2 - R_S$	R_S	0	$R_M - \phi W - X + K_2 + R_S$	0
$\phi W + X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model for this case is given as follows:

Max $\prod(p, p_2^n, p_2^r)$

$$
\begin{aligned}
&= \int_{x=0}^{R_S-\phi W} p_s(K_2 + R_M) f(x)dx \\
&+ \int_{x=R_S-\phi W}^{K_2+R_S-\phi W} p_2^n(\phi W + x - R_S) f(x)dx \\
&+ \int_{x=R_S-\phi W}^{K_2+R_S-\phi W} p_s(K_2 - \phi W - x + R_S + R_M) f(x)dx \\
&+ \int_{x=K_2+R_S-\phi W}^{\infty} p_2^n\, K_2 f(x)dx \\
&+ \int_{x=K_2+R_S-\phi W}^{K_2+R-\phi W} p_2^r(\phi W + x - K_2 - R_S) f(x)dx \\
&+ \int_{x=K_2+R_S-\phi W}^{K_2+R-\phi W} p_s(R_M - \phi W - x + K_2 + R_S) f(x)dx \\
&+ \int_{x=K_2+R-\phi W}^{\infty} p_2^r R_M f(x)dx + \bar{\phi} W p - c_n K - c_r R_M
\end{aligned}
$$

Subject to

$$V_L \times w \leq x_1 \leq \frac{2V_H+V_L}{1+2k} \times w \qquad (1)$$

$$(1-w) \times \frac{2V_H+V_L}{1+2k} \leq x_2 \leq V_H \times (1-w) \qquad (2)$$

$$p = x_1 + x_2 \qquad (3)$$

$$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w) \qquad (4)$$

$$p - p_2^s \geq V_H - p_2^n \qquad (5)$$

$$V_H - p_2^n \geq p - p_2^r \qquad (6)$$

$$p - p_2^r \geq 0 \qquad (7)$$

$$w \in \{0,1\} \qquad (8)$$

$$p, p_2^n, p_2^r, p_2^s \geq V_L \qquad (9)$$

Constraint (5) shows that customers firstly prefer to buy remanufactured product from speculators and remaining consumers prefer to buy new products. Then, customers prefer to buy remanufactured products from the manufacturer. Thus, constraint (6) supports this preference. Constraint (7) ensures nonnegative surplus of buying remanufactured product from the manufacturer. Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.12. Customers prefer to buy remanufactured product from speculators first. Later, they prefer to buy remanufactured product from speculators. Lastly, any remaining customers prefer to buy new product. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq S_2^r, \quad S_2^r \geq S_2^n, \quad S_2^n \geq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 58.

Table 58. Case 2.12. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$\phi W + X \leq R_S$	0	0	$\phi W + X$	K_2	R_M	$R_S - \phi W - X$
$R_S < \phi W + X \leq R$	0	$\phi W + X - R_S$	R_S	K_2	$R_M - \phi W - X + R_S$	0
$R < \phi W + X \leq K_2 + R$	$\phi W + X - R_M - R_S$	R_M	R_S	$K_2 - \phi W - X + R$	0	0
$\phi W + X > K_2 + R$	K_2	R_M	R_S	0	0	0

The mathematical model for this case is given as follows:

Max $\prod(p, p_2^n, p_2^r)$

$$= \int_{x=0}^{R_S-\phi W} p_s(K_2 + R_M) f(x) dx + \int_{x=R_S-\phi W}^{R-\phi W} p_2^r(\phi W + x - R_S) f(x) dx$$

$$+ \int_{x=R_S-\phi W}^{R-\phi W} p_s(K_2 + R_M - \phi W - x + R_S) f(x) dx$$

$$+ \int_{x=R-\phi W}^{\infty} p_2^r R_M f(x) dx$$

$$+ \int_{x=R-\phi W}^{K_2+R-\phi W} p_2^n(\phi W + x - R_M - R_S) f(x) dx$$

$$+ \int_{x=R-\phi W}^{K_2+R-\phi W} p_s(K_2 - \phi W - x + R) f(x) dx$$

$$+ \int_{x=K_2+R-\phi W}^{\infty} p_2^n K_2 f(x) dx + \bar{\phi} W p - c_n K - c_r R_M$$

Subject to

$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w$ (1)

$(1 - w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1 - w)$ (2)

$p = x_1 + x_2$ (3)

$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w)$ (4)

$p - p_2^s \geq p - p_2^r$ (5)

$p - p_2^r \geq V_H - p_2^n$ (6)

$V_H - p_2^n \geq 0$ (7)

$w \in \{0,1\}$ (8)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (9)

Constraint (5) shows that customers firstly prefer to buy remanufactured product from speculators and remaining consumers continue to prefer to buy remanufactured products from manufacturer. Then, customers prefer to buy new product given by constraints (6) and (7). Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.13. Customers prefer to buy remanufactured product from speculators first. Later, they prefer to buy new product. They do not prefer to buy remanufactured product from manufacturer. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq S_2^n, \quad S_2^n \geq 0, \quad S_2^r \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 59.

Table 59. Case 2.13. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$\phi W + X \leq R_S$	0	0	$\phi W + X$	K_2	R_M	$R_S - \phi W - X$
$R_S < \phi W + X \leq R_S + K_2$	$\phi W + X - R_S$	0	R_S	$K_2 - \phi W - X + R_S$	R_M	0
$\phi W + X > R_S + K_2$	K_2	0	R_S	0	R_M	0

The mathematical model for this case is given below:

$$\begin{aligned} \text{Max} \prod(p, p_2^n) = & \int_{x=0}^{R_S - \phi W} p_s(K_2 + R_M) f(x) dx \\ & + \int_{x=R_S-\phi W}^{R_S+K_2-\phi W} p_2^n(\phi W + x - R_S) f(x) dx \\ & + \int_{x=R_S-\phi W}^{R_S+K_2-\phi W} p_s(K_2 - \phi W - x + R_S + R_M) f(x) dx \\ & + \int_{x=R_S+K_2-\phi W}^{\infty} p_2^n K_2 f(x) dx + \int_{x=R_S+K_2-\phi W}^{\infty} p_s R_M f(x) dx + \bar{\phi} W p \\ & - c_n K - c_r R_M \end{aligned}$$

Subject to

$$V_L \times w \leq x_1 \leq \frac{2V_H+V_L}{1+2k} \times w \qquad (1)$$

$$(1-w) \times \frac{2V_H+V_L}{1+2k} \leq x_2 \leq V_H \times (1-w) \qquad (2)$$

$$p = x_1 + x_2 \qquad (3)$$

$$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1-w) \qquad (4)$$

$$p - p_2^s \geq V_H - p_2^n \qquad (5)$$

$$V_H - p_2^n \geq 0 \qquad (6)$$

$$p - p_2^r \leq 0 \qquad (7)$$

$$w \in \{0,1\} \qquad (8)$$

$$p, p_2^n, p_2^r, p_2^s \geq V_L \qquad (9)$$

Constraint (5) shows that customers firstly prefer to buy remanufactured product from speculators. Remaining consumers can prefer to buy new products as given by constraint [6]. Customers do not prefer to buy remanufactured product from manufacturer as given in constraint (7). Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.14. In this case, customers prefer to buy remanufactured product from speculators. Later, they prefer to buy remanufactured product from manufacturer. They do not prefer to buy new product. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq S_2^r, \quad S_2^r \geq 0, \quad S_2^n \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 60.

Table 60. Case 2.14. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$\phi W + X \leq R_S$	0	0	$\phi W + X$	K_2	R_M	$R_S - \phi W - X$
$R_S < \phi W + X \leq R$	0	$\phi W + X - R_S$	R_S	K_2	$R_M - \phi W - X + R_S$	0
$\phi W + X > R$	0	R_M	R_S	K_2	0	0

The mathematical model for this case is given as follows:

$$\text{Max} \prod(p, p_2^r) = \int_{x=0}^{R_S-\phi W} p_s(K_2 + R_M)f(x)dx + \int_{x=R_S-\phi W}^{R-\phi W} p_2^r(\phi W + x - R_S)f(x)dx$$
$$+ \int_{x=R_S-\phi W}^{R-\phi W} p_s(K_2 + R_M - \phi W - x + R_S)f(x)dx$$
$$+ \int_{x=R-\phi W}^{\infty} p_2^r R_M f(x)dx + \int_{x=R-\phi W}^{\infty} p_s K_2 f(x)dx + \bar{\phi} W p - c_n K$$
$$- c_r R_M$$

Subject to

$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w$ (1)

$(1 - w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1 - w)$ (2)

$p = x_1 + x_2$ (3)

$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w)$ (4)

$p - p_2^s \geq p - p_2^r$ (5)

$p - p_2^r \geq 0$ (6)

$V_H - p_2^n \leq 0$ (7)

$w \in \{0,1\}$ (8)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (9)

Constraint (5) shows that customers firstly prefer to buy remanufactured product from speculators. Remaining consumers prefer to buy remanufactured products from manufacturer as given by constraint (6). Customers do not prefer to buy new product as given by constraint (7). Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.15. Customers prefer only to buy remanufactured product from speculators. They do not prefer to buy new and remanufactured product from manufacturer. Therefore, their surplus relationships are given as follows:

$$S_2^s \geq 0, \quad S_2^n \leq 0, \quad S_2^r \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 61.

Table 61. Case 2.15. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$\phi W + X \leq R_S$	0	0	$\phi W + X$	K_2	R_M	$R_S - \phi W - X$
$\phi W + X > R_S$	0	0	R_S	K_2	R_M	0

The mathematical model that represents this case is given below:

Max $\prod(p) = p_s(K_2 + R_M) + \bar{\phi} W p - c_n K - c_r R_M$

Subject to

$V_L \times w \leq x_1 \leq \frac{2V_H + V_L}{1+2k} \times w$ (1)

$(1 - w) \times \frac{2V_H + V_L}{1+2k} \leq x_2 \leq V_H \times (1 - w)$ (2)

$p = x_1 + x_2$ (3)

$V_H - (x_1 + x_2) \leq V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w)$ (4)

$p - p_2^s \geq 0$ (5)

$V_H - p_2^n \leq 0$ (6)

$p - p_2^r \leq 0$ (7)

$w \in \{0,1\}$ (8)

$p, p_2^n, p_2^r, p_2^s \geq V_L$ (9)

Constraint (5) shows that customers can only prefer to buy remanufactured product from speculators. Remaining consumers do not prefer to buy other products according to constraint (6) and (7). Constraint (9) ensures that all prices are greater than or equal to low valuation.

2.16. Customers do not prefer to buy in the second period. Therefore, their surplus relationships are given as follows:

$$S_2^n \leq 0, \quad S_2^r \leq 0, \quad S_2^s \leq 0$$

The relationship between the demand quantity, number of units sold, and leftover amounts is explained in Table 62.

Table 62. Case 2.16. demand relationships.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from	
					Manufacturer	Speculators
$0 \le \phi W + X < \infty$	0	0	0	K_2	R_M	R_S

The mathematical model that represents this is given below:

Max $\prod(p) = p_s(K_2 + R_M) + \bar{\phi} W p - c_n K - c_r R_M$

Subject to

$V_L \times w \le x_1 \le \frac{2V_H + V_L}{1+2k} \times w$ (1)

$(1 - w) \times \frac{2V_H + V_L}{1+2k} \le x_2 \le V_H \times (1 - w)$ (2)

$p = x_1 + x_2$ (3)

$V_H - (x_1 + x_2) \le V_H \times w - x_1 \times k + \frac{x_2}{2} - \frac{V_H}{2} \times (1 - w)$ (4)

$V_H - p_2^n \le 0$ (5)

$p - p_2^r \le 0$ (6)

$p - p_2^x \le 0$ (7)

$w \in \{0,1\}$ (8)

$p, p_2^n, p_2^r, p_2^s \ge V_L$ (9)

Constraints (5), (6), and (7) show that buying in the second period are not preferred according to surplus values. Constraint (9) ensures that all prices are greater than or equal to low valuation.

CHAPTER 4

SOLUTION APPROACH AND NUMERICAL STUDY

Linear Programming (LP) and Mixed Integer Linear Programming (MILP) were used to formulate mathematical models presented in the previous section. MATLAB is used to solve each of the models because of the stochastic objective function. MATLAB Optimization Toolbox functions of *linprog* is used for LP models, and *intlinprog* is used to solve MILP models. Each run was conducted on a computer that has the following properties:

CPU: Intel® Core™ i7-4700HQ CPU with 2.40 GHz
RAM: 16 GB
System Type: 64 Bit

Solution time was approximately 13 hours for each model considering all instances. Computers were operated in parallel to speed up the process. We provide a numerical study to find the optimal solution. The following provides details of the numerical study. In what follows, we describe the numerical setup used.

Capacity of new products, K, is fixed at a value of 100. Number of units left K_2 depends on this value and customer behavior. Also, consumer valuations V_L and V_H values are assumed as 10 and 20, respectively. Fraction of returns f is set between 0 and 1 with 0.25 increments. Fractions of returns collected by the manufacturer and speculators are determined as ratios with respect to f. Suppose that f is equal to 0.5. If f_m: f_s ratio is taken as 4 : 2, value of these ratios become 0.33 and 0.17, respectively. Cost of the manufacturing new product c_n is taken as 10, where cost of remanufacturing is taken as a fraction of c_n (i.e., $c_r = \alpha \times c_n$). Multiplier α is selected between 10% and 50% with 10% increments. Thus, c_r can take the values of 1, 2, 3, 4 and 5. Random demand X is assumed *Uniformly* distributed over the interval $[a, b]$. Lower bound of this interval is taken as 0 (i.e., $a = 0$). Upper bound of this interval is taken as a function of W (i.e., $b = \beta \times W$). Multiplier β is taken between 0.05 and 0.45 with 0.20 increments. Number of consumers who are present in the market W is assumed between 25 and 150 with 25 increments. We include both cases (i.e., $W \leq K$ and $W > K$) in the numerical experiments. Myopic customers' ratio $\bar{\phi}$ is considered as a small portion of whole customers. Thus, this ratio is chosen within the interval of 0.025 to 0.1 with 0.025 increments. By subtracting the number of myopic customers from the total number of consumers who are present in the market, number of strategic customer can be easily computed (i.e., $\phi W = W - \bar{\phi} W$). Generated parameter set is shown in Table 63.

Table 63. Parameter values.

Parameter	Values					
K	100					
W	25	50	75	100	125	150
$\bar{\phi}$	0.025	0.05	0.075	0.1		
f	0	0.25	0.5	0.75	1	
f_m: f_s	6 : 0	4 : 2	3 : 3	2 : 4	0 : 6	
c_n	10					
α	10%	20%	30%	40%	50%	
a	0					
β	0.05	0.25	0.45			
V_L	10					
V_H	20					

9,000 different problem instances are constructed using Excel VBA that consist of all combinations of assigned values of parameters. Each instance is solved for each model.

4.1 Discussion of the Results

After solving each model with each individual data instance, the maximum objective function value is observed in Case 2.11 and Case 2.12. This result implies that waiting for period 2 should be made more attractive by the manufacturer for strategic consumers. Case 2.11 has the following consumer behavior for purchase priorities as:

customers prefer to buy remanufactured product from speculators first, then new product, and any remaining customers prefer to buy remanufactured product from the manufacturer. In this case, optimal prices are given below.

$$p^* = p_2^{n^*} = p_2^{r^*} = 20, \quad p_2^{s^*} = 10$$

In other words, the manufacturer should set initial prices very high to make waiting for second-period sales to become more attractive for strategic consumers. Therefore, first-period price is equal to high valuation (i.e., $p^* = V_H$). Our objective is maximizing the expected total profit from the manufacturer's point of view. Therefore, objective function does not include speculators' profit. Speculators' sales price p_2^s is not considered in the profit function. Therefore, speculators set their prices very low to attract consumers for buying from them first. Therefore, $p_2^{s^*}$ is set at low valuation. Consumers prefer to buy remanufactured product from speculators first because buying from them gives the highest surplus amount (i.e., $S_2^s = p - p_2^s$). Remaining consumers continue to buy from manufacturer if their surplus values (i.e., $S_2^r = p - p_2^r$ and $S_2^n = V_H - p_2^n$) are nonnegative. Only S_2^s gives nonnegative surplus for customers. Both S_2^r and S_2^n give zero surpluses for customers. After buying from speculators, any remaining customers are indifferent between buying new or remanufactured products due to zero surplus from buying either. This result was observed in some instances where the best objective function value is found in Case 2.12. This case implies that customers firstly prefer to buy remanufactured products from speculators as in Case 2.11. The only difference is that remaining consumers prefer to buy new product, then, remanufactured product from the manufacturer. In some instances, optimal prices are the same as in Case 2.11 (i.e., $p^* = p_2^{n^*} = p_2^{r^*} = 20, \quad p_2^{s^*} = 10$). However, for some instances, optimal prices are found as follows:

$$p^* = p_2^{n^*} = 20, \quad p_2^{r^*} = p_2^{s^*} = 10$$

This results arises when fraction of remanufactured products collected and remanufactured by manufacturer is equal to zero (i.e., $f_m = 0$). In other words, the manufacturer does not collect returned products at the end of the first period, so lower bound V_L is assigned for optimal values of p_2^r. There is also an alternative optimal solution. Case 2.12 and 2.13 give the same total expected profit with the same optimal prices. According to Case 2.13, customers firstly prefer to buy remanufactured products from speculators, and then, remaining consumers prefer to buy the new product. Consumers do not prefer to buy remanufactured products from manufacturer if surplus is not sufficiently high enough or if there is no product that is remanufactured by manufacturer. Latter case supports this pricing policy.

In the optimal cases, it is clearly seen that the manufacturer prefers to apply a fixed pricing policy instead of a dynamic pricing policy by setting new product prices in the first and second periods equal to high valuation V_H. Due to limited space, we provide partial results of the numerical study for the most sensitive parameters and base values in Appendix B.

4.2 Sensivity Analysis

In this section, we investigate how sensitive the expected total profit is to parameters; $W, f, \bar{\phi}, \beta$. We do not consider c_r or c_n since they do not affect the expected total profits. As it can be observed in Figure 3, the expected total profit increases in an exponential fashion with respect to the number of customers in the market.

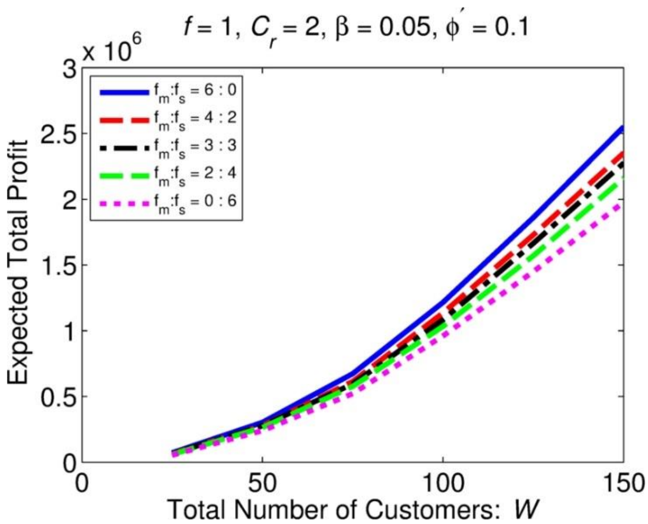

Figure 3. Sensitivity of expected total profit to W when $f = 1, c_r = 2, \beta = 0.05, \bar{\phi} = 0.1$.

In this setting, it is assumed that all products sold in period 1 are returned (i.e., $f = 1$). Cost of remanufacturing is taken as 2. Multiplier of maximum demand is taken as 0.05 (i.e., $b = 0.05W$). Ratio of the myopic customers taken as 0.1 (i.e., $\bar{\phi} = 0.1W$). The maximum expected total profit can be observed when manufacturer collects all returned products that are represented in blue color on the graph. All combinations with different parameter values are analyzed with respect to W and they are shown in Appendix C.

Sensivity of the expected total profit to f when demand is moderate can be observed in Figure 4.

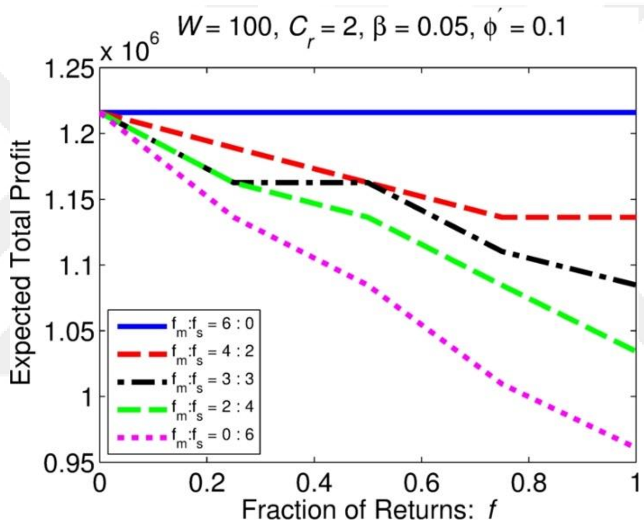

Figure 4. Sensitivity of expected total profit to f when demand is moderate and $W = 100, c_r = 2, \beta = 0.05, \bar{\phi} = 0.1$.

According to Figure 4, maximum profit can be observed when there are no returns and when demand is moderate. When there are returns, the expected total profit shows decreasing behavior except for the scenario when manufacturer collects all returned products. Similar behavior can be seen when second-period maximum demand multiplier β takes upper limit value of 0.45. When W takes the upper limit value of 150, behavior of the expected total profit is seen in Figure 5.

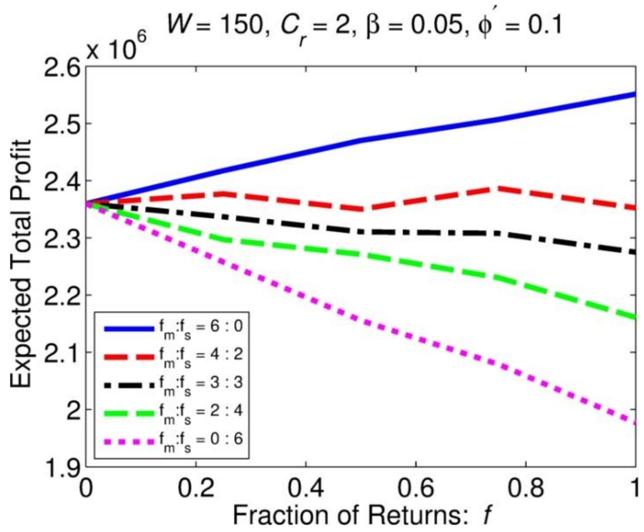

Figure 5. Sensitivity of expected total profit to f when demand is high and $W = 150, c_r = 2, \beta = 0.05, \bar{\phi} = 0.1$.

When demand is high, increasing fraction of returns increases the expected total profit when manufacturer collects all returned products as represented in blue color. However, when collected portion of returned products by speculators increases, expected total profit for the manufacturer starts to decrease. Sensitivity of the expected total profit to f when demand is low can be observed in Figure 6.

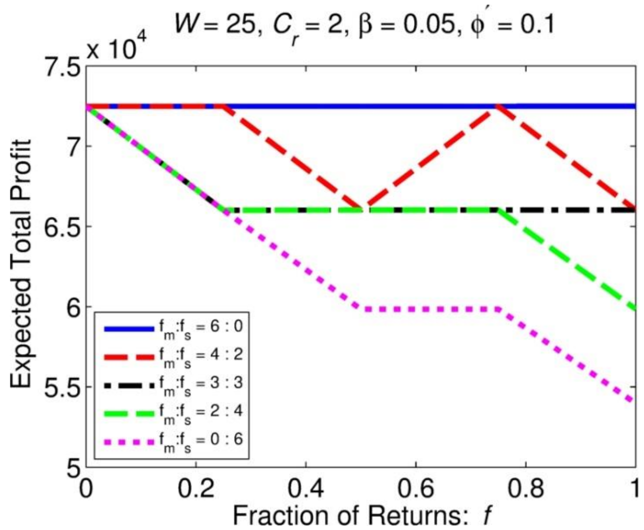

Figure 6. Sensitivity of expected total profit to f when demand is low and $W = 25, c_r = 2, \beta = 0.05, \bar{\phi} = 0.1$.

If demand is low, expected total profit is the maximum if manufacturer collects all returned products. Expected total profit will be high as long as the ratio of the collected products by the manufacturer is high. When f becomes 0.5, manufacturer can collect products with ratios of 4 : 2 or 2 : 4. Both of the scenarios give the same value of the expected profit. Therefore, manufacturer's strategy should take into account the fraction of returns and level of demand. Other combinations with different parameter values are analyzed with respect to f and they are shown in Appendix D.

Sensitivity of the expected total profit with regard to the maximum second-period demand coefficient β when demand is high can be observed in Figure 7.

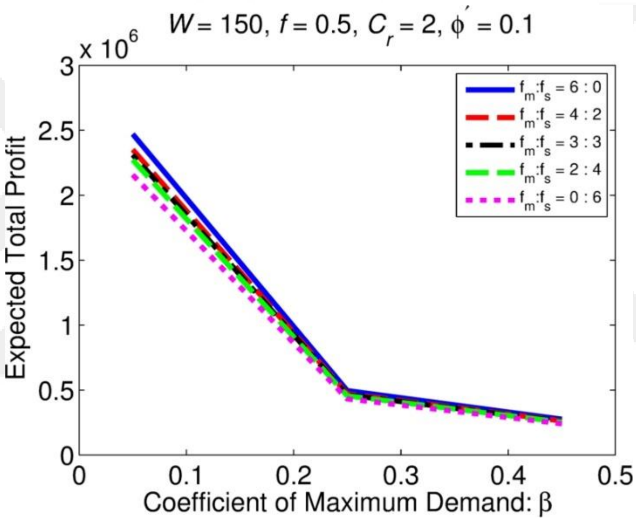

Figure 7. Sensitivity of expected total profit to β when demand is high and $W = 150, c_r = 2, f = 0.5, \bar{\phi} = 0.1$.

As seen in Figure 7, expected total profit takes higher values when manufacturer collects all returned products. In general, when β is taken at its lower limit of 0.05, the manufacturer can make higher profits for all ratios of the collection of returns. When β is taken at its base value of 0.25, rapid decrease is observed in the expected total profit. However, when β is taken at its upper limit of 0.45, milder decrease can be observed over expected profit. In general, increasing β value leads to a decline the in expected profit. Similar tendency is observed at other combinations of the parameters with respect to β and one shown in Appendix E. Sensitivity of the expected total profit to $\bar{\phi}$ when demand is high can be observed in Figure 8.

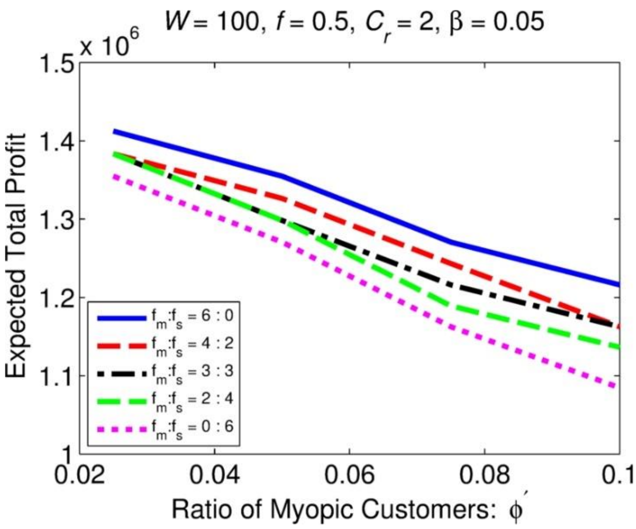

Figure 8. Sensitivity of expected total profit to $\bar{\phi}$ when demand is moderate and $W = 100, c_r = 2, \beta = 0.05, f = 0.5$.

As seen in Figure 8, expected total profit decreases when ratio of myopic customers increases. Similar behavior can be seen when demand is high. In addition, some levels on the ratio of myopic customers display the same value of expected profit as seen in Figure 9.

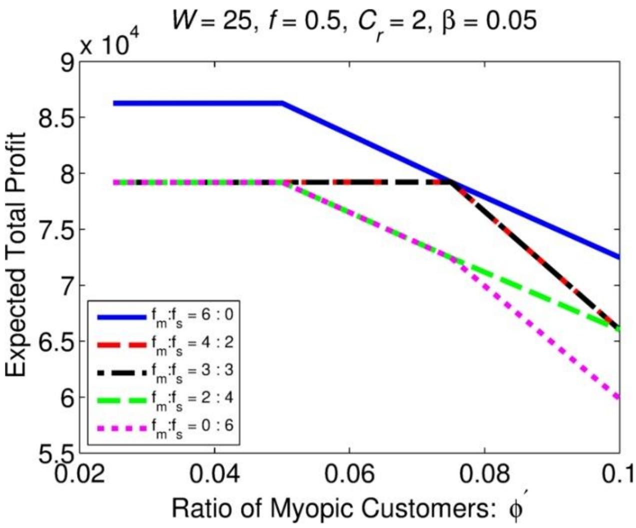

Figure 9. Sensitivity of expected total profit to $\bar{\phi}$ when demand is low and $W = 25, c_r = 2, \beta = 0.05, f = 0.5$.

The reason for constant level in Figure 9 is that values assigned to number of myopic customers should be integer. Thus, multiplied values of $W \times \bar{\phi}$ are rounded up (i.e., $25 \times 0.025 \cong 1$). This results in the same levels of expected profit on the graph but behavior does not change. In general, an increase in the number of myopic consumers results in a decline of the expected total profit. This can also be interpreted as follows: an increase in the number of strategic customer results in an increase in the expected total profit because total number of customers is directly proportional to the expected total profit. Similar tendency can be observed with other combinations of the parameters with respect to $\bar{\phi}$ and are shown in Appendix F.

Spider plots are used to show the effects of all parameters on the expected total profit together and to compare their effects relative to each other. In order to do so, we hypothetically set a base level value for each parameter as shown in Table 64.

Table 64. Base values of parameters.

Parameter	**Base Value**
W	100
$\bar{\phi}$	0.05
f	0.5
c_r	2
β	0.25

According to base values, percent increase and decrease values are determined and they are represented in Table 65.

Table 65. Percent increase/decrease and values of parameters.

		Increase / Decrease					
$\boldsymbol{W}$	**Value**	25	50	75	100	125	150
	% Change	-75	-50	-25	0	25	50
$\boldsymbol{\bar{\phi}}$	**Value**	0.025	0.05	0.75	0.1		
	% Change	-50	0	50	100		
$\boldsymbol{f}$	**Value**	0	0.25	0.5	0.75	1	
	% Change	-100	-50	0	50	100	
$\boldsymbol{c_r}$	**Value**	1	2	3	4		
	% Change	-50	0	50	100		
$\boldsymbol{\beta}$	**Value**	0.05	0.25	0.45			
	% Change	-80	0	80			

Value of 5 for c_r is ignored to make more sensitive computation. A spider plot is drawn for the expected total profit with respect to percent change values from the base case shown in Figure 10.

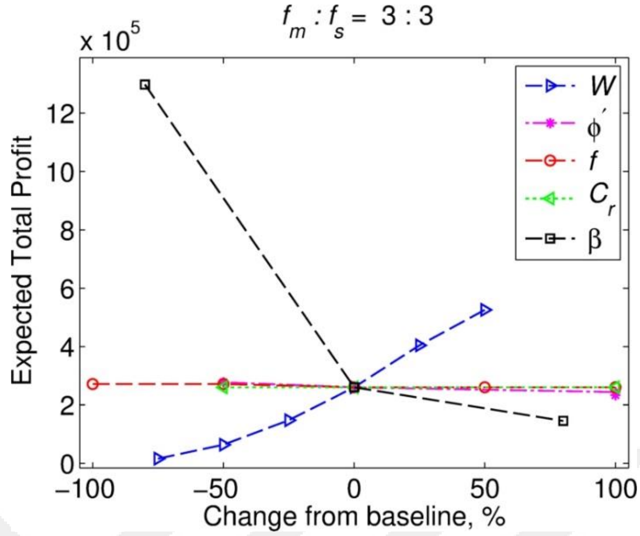

Figure 10. Spider plot showing sensitivity of the expected total profit with equal collection ratios.

According to the spider plot in Figure 10, it is clearly seen how expected total profit is influenced from each parameter. Parameters that affect the expected total profit most are W and β due to larger slope. Increase in W will also result in an increase in the profit. On the contrary, increase in β will lead to decrease in the profit. Besides, other parameters seem to have no effect on the expected total profit. Ratio of 3 : 3 is selected for illustration purpose in Figure 10. Spider plots with other ratios are presented in Appendix G. In all plots, similar behavior can be observed.

CHAPTER 5

CONCLUSION

In this thesis, pricing and remanufacturing decisions with speculators and strategic consumers are analyzed in a two-period setting. Manufacturer sets the initial selling price for the new product in the first period. A portion of the customers is myopic and buy the product immediately. Speculators can enter the market to collect returns to remanufacture them to be sold in the second period for profit. The manufacturer may also collect some portion of the returns, and hence, needs to decide on the selling price for the remanufactured products s/he collected. Possible variation in the prices set by the both parties creates a competition and customers need to decide from whom to purchase the remanufactured items. In addition, strategic consumers act with respect to their surplus values when buying a product. Thus, speculators will rationally set the price sufficiently low enough to attract strategic consumers while ensuring profit. Within this scope, remanufactured and new products have different consumer valuations.

Optimization on the pricing and remanufacturing decisions is performed covering all possible customer behavior patterns in separate cases. A numerical study is conducted to solve mathematical models numerically in each of these cases. We observed a unique optimal customer behavior as a result: customers prefer to buy from the speculator first, then they prefer to buy new products from the manufacturer, and finally, if any customers remain, they prefer to buy the remanufactured product prom the manufacturer. In order to ensure such customer behavior, the manufacturer should use a fixed-pricing policy in which the optimal prices should be set at the upper bound of consumer valuations (i.e., $p^* = p_2^{n*} = p_2^{r*} = V_H$). Therefore, strategic customers are motivated to wait for the second period, hoping for a markdown. The manufacturer maintains the same pricing policy in the second period. This result is contrary to the common intuition, i.e., instead of a dynamic pricing policy, we found out that the manufacturer should use a fixed-pricing policy. Moreover, manufacturer obtains higher profit for collecting, remanufacturing, and selling the returns. Thus, speculators should be kept out of the market.

This thesis contributes to the literature by incorporating remanufacturing activities that can be performed by the OEM as well as speculators. The manufacturer not only has to decide on the prices to set for the new products over the two periods, but also the price for the remanufactured products in the second period. While doing so, s/he needs to take into account the behavior of customers, and may need to consider the existence of speculators in order to ensure maximum profit.

A possible extension to the current study may consider different remanufacturing capabilities for the remanufacturer and the speculator, hence, remanufacturing costs, which may affect the pricing decisions. Another extension could look into using probability distributions other than uniform distribution for the second period in the numerical study. A worthwhile extension would be to investigate the models and their solutions when new and remanufactured products are perceived as of equal value by some or all of the customers.

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APPENDIX A

FIXED PRICING WITH SPECULATORS

Manufacturer sets the second period price of new product as $p_2^n = p.$ However; remanufactured product price can be changeable or may remain the same price as p and p_2^n. We assume that strategic consumers are indifferent between purchasing in period 1 and waiting to buy in period 2. Rationing is concerned only in the second period decision. Strategic consumers prefer to buy new product at first if its surplus is sufficiently enough to buy. Then, if demand exists, they buy remanufactured product if its surplus is nonnegative. However, they have to decide on to buy from which seller. This decision depends on some critical amounts such as number of demand, surpluses when they buy, number of remanufactured products and number of speculators. To understand better, the relationship with demand and number of units sold is characterizes from over the manufacturer's point of view with speculators in Table A.1.

Table A. 1. Fixed pricing from the manufacturer's point of view with speculators.

Demand	Number of Units Sold			Leftover Amounts		
	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculator	New Product	Remanufactured Product from Manufacturer	Remanufactured Product from Speculators
$X + W_2 \leq K_2$	$X + W_2$	0	0	$K_2 - X - W_2$	R_M	R_S
$K_2 < X + W_2 \leq K_2 + R_S$	K_2	0	$X + W_2 - K_2$	0	R_M	$R_S - X - W_2 + K_2$
$K_2 + R_s < X + W_2 \leq K_2 + R$	K_2	$X + W_2 - K_2 - R_s$	R_S	0	$R_M - X - W_2 + K_2 + R_S$	0
$K_2 + R < X + W_2$	K_2	R_M	R_S	0	0	0

In Table A.1., if demand is low that means sum of the strategic consumers who wait for period 2 and random consumers is smaller or equal to the number of new products which are unsold from the first period, customers buy new product. Leftover from new and remanufactured products are sold as salvage that shown in the first line in Table A.1. If overall demand is greater than the number of unsold new product and smaller or equal to the sum of the number of unsold new and remanufactured products of speculators, consumers buy new at first and remaining demand buy remanufactured products from speculator. This occurs when demand falls in the second range in Table A.1. The reason behind this preference of consumers to buy from the speculators at first of remanufactured products is that speculator have a chance to set their prices after manufacturer has determined the prices of his products. Speculators set their prices as smaller or equal to the manufacturer's prices. Thus, consumers buy speculators preferably. In the meantime, our setting indicates a noncooperative game within the speculators as mentioned in [2] because manufacturer's price is given in advance. In addition, when demand falls in the third range, after buying new from manufacturer and remanufactured products from speculators, remaining consumers buy from remanufactured products from manufacturers and leftover amounts are sold as salvage. When demand is high as shown in the last row in Table A.1., all products are sold. The following lemma shows the equilibrium remanufactured product price.

LEMMA 1: Suppose that manufacturer has K_2 units left, W_2 strategic and X new consumers are observed in period 2. There are R_S speculators who remanufacture some fraction of returned products each priced at $p_2^r \in [V_L, p]$. The equilibrium remanufactured product price is

$$\tilde{p}_2^{r*} = \begin{cases} V_L & if\ X + W_2 < K_2 + R_S \\ p & if\ X + W_2 \geq K_2 + R_S \end{cases}$$

There are two possible outcomes. When demand is low (i.e., $X + W_2 < K_2 + R_S$) speculators undercut one another and derive the remanufactured product price reduce to V_L. When demand is high, there is sufficient demand to remain the same price with p. Thus, expected remanufactured price is observed as follows:

$$E\tilde{p}_2^{r*} = V_L \times F(K_2 + R_S - W_2) + p \times \bar{F}(K_2 + R_S - W_2)$$

We observe that increasing competition with the number of speculators (R_S) becomes larger causes a lower expected price of remanufactured products. Speculators will enter to the market till remanufacturing is no more cost-effective to do so. Expected profit from speculation is:

$$E\tilde{p}_2^{r*} - c_r$$

As seen in the above equation, number of speculators increases from zero until $E\tilde{p}_2^{r*} = c_r$. In equilibrium, $V_L \times F(K_2 + R_S - W_2) + p \times \bar{F}(K_2 + R_S - W_2) = c_r$ Then we have;

$$p = \frac{c_r - V_L F(K_2 + {R_S}^* - {W_2}^*)}{\bar{F}(K_2 + {R_S}^* - {W_2}^*)}$$

${R_S}^*$ and ${W_2}^*$ means that equilibrium number of speculators and strategic consumers who wait for period 2, respectively. On the other hand, if the cost of remanufacturing of speculators exceeds the value of $p - (p - V_L) \times F(K_2 + {R_S}^* - {W_2}^*)$, speculators should not enter the market because remanufacturing will not be profitable. Manufacturer's expected profit function is given by:

$$\begin{aligned}\prod(p, p_2^n, p_2^r) = &\int_{x=0}^{K_2-W_2} p_2^n(W_2 + x)f(x)dx + \int_{x=0}^{K_2-W_2} p_s\,(K_2 - x - W_2 + R_M)f(x)dx \\ &+ \int_{x=K_2-W_2}^{\infty} p_2^n K_2 f(x)dx + \int_{x=K_2-W_2}^{K_2+R_S-W_2} p_s R_M f(x)dx \\ &+ \int_{x=K_2+R_S-W_2}^{K_2+R-W_2} p_2^r(W_2 + x - K_2 - R_S)f(x)dx \\ &+ \int_{x=K_2+R_S-W_2}^{K_2+R-W_2} p_s(R_M - x - W_2 + K_2 + R_S)f(x)dx \\ &+ \int_{x=K_2+R-W_2}^{\infty} p_2^r R_M f(x)dx + p(W - W_2) - c_n \times K - c_r \times R_M\end{aligned}$$

Substituting "K-W" to "K_2-W_2" profit becomes:

$$pW - pW_2 + p_2^n W_2 F(K-W) + p_2^n \int_{x=0}^{K-W} xf(x)dx + p_s K_2 F(K-W)$$
$$- p_s \int_{x=0}^{K-W} xf(x)dx + p_s W_2 F(K-W) + p_s R_M F(K-W) + p_2^n K_2$$
$$- p_2^n K_2 F(K-W) + p_2^r \int_{x=K-W+R_S}^{K-W+R} xf(x)dx + p_2^r W_2 F(K-W+R)$$
$$+ p_2^r W_2 F(K-W+R_S) - p_2^r K_2 F(K-W+R)$$
$$+ p_2^r K_2 F(K-W+R_S) - p_2^r R_S F(K-W+R)$$
$$+ p_2^r R_S F(K-W+R_S) + p_s R_M F(K-W+R)$$
$$- p_s \int_{x=K-W+R_S}^{K-W+R} xf(x)dx - p_s W_2 F(K-W+R)$$
$$+ p_s W_2 F(K-W+R_S) + p_s K_2 F(K-W+R)$$
$$- p_s K_2 F(K-W+R_S) + p_s R_S F(K-W+R)$$
$$- p_s R_S F(K-W+R_S) + p_2^r R_M - p_2^r R_M F(K-W+R) - c_n K$$
$$- c_r R_M$$

In order to understand which one is profitable (i.e., whether to wait for period 2), first order derivative is taken with respect to W_2.

$$\frac{d\prod(p, p_2^n, p_2^r)}{dW_2} = -p + p_2^n F(K-W) + p_s F(K-W) + p_2^r F(K-W+R)$$
$$+ p_2^r F(K-W+R_S) - p_s F(K-W+R) + p_s F(K-W+R_S)$$

$$= -p + (p_2^n + p_s)F(K-W) + (p_2^r - p_s)F(K-W+R) + (p_2^r + p_s)F(K-W$$
$$+ R_S)$$

Manufacturer charges fixed pricing strategy to new products so substituting "p" to "p_2^n" we have:

$$\frac{d\prod(p, p_2^n, p_2^r)}{dW_2} = -p + (p + p_s)F(K-W) + (p_2^r - p_s)F(K-W+R)$$
$$+ (p_2^r + p_s)F(K-W+R_S)$$

If the derivative is positive, this shows that manufacturer's profit function is an increasing function with regard to W_2. In such a case, manufacturer should set the first period price as V_H. Thus, waiting for second period becomes more interesting for strategic consumers because they may find the opportunity to buy it cheaper in the second period. Therefore, all strategic consumers wait for the second period sales (i.e., $W_2 = \phi W$). On the contrary, if the derivative is negative, manufacturer should set a first period price as smaller than V_H to attract strategic consumers' attention. To analyze a simple case, assume that random variable X has a uniform distribution, denoted $U(a, b)$. Let $b \leq K - W$. It is known that cumulative distribution function (CDF) of a uniform random variable X is:

$$F(x) = \frac{(x-a)}{(b-a)}$$

In such case, $F(K - W) = 1$. Thus, the derivative becomes nonnegative. Waiting for the second period should be made more attractive to strategic consumers. Note that all myopic customers will buy the product. Therefore, manufacturer should set the first period price very high. According to these assumptions, manufacturer's optimal first period price is $p^* = V_H$. When considering the case of $\frac{d\prod(p,p_2^n,p_2^r)}{dW_2} = 0$, the profit function does not show increasing or decreasing tendency because of the zero slope at some value of W_2. In such case, first period price becomes;

$$p = (p + p_s)F(K - W) + (p_2^r - p_s)F(K - W + R) + (p_2^r + p_s)F(K - W + R_S)$$

This does not tell us which one makes more sense. For example, while some portion of the strategic consumers buy in the first period, remaining part can choose to wait for period 2. On the other hand, if the derivative of $\frac{d\prod(p,p_2^n,p_2^r)}{dW_2}$ is negative, this implies that profit function is a decreasing with respect to W_2. It is possible to see that all strategic consumers buy in the first period when price is set low necessarily enough. In this case, manufacturer can charge initial prices as $p = V_L$. Therefore, all strategic consumers buy in the first period, if capacity is enough.

APPENDIX B

PARTIAL RESULTS OF THE NUMERICAL STUDY

Table B. 1 Numerical results at all values of the most influential parameters (*W*, β).

Instances	p	p_2^n	p_2^r	p_2^s	Best Obj Func	Best Case	Alt.Case1	Alt.Case2	K	W	$\bar{\phi}$	f	f_m	f_s	c_r	c_n	α	β
655	20	20	20	10	86278.375	case211	case212	case213	100	25	0.05	0.5	0.5	0	2	10	0	0.05
656	20	20	20	10	331606.375	case211			100	50	0.05	0.5	0.5	0	2	10	0	0.05
657	20	20	20	10	756781.375	case211			100	75	0.05	0.5	0.5	0	2	10	0	0.05
658	20	20	20	10	1354759.375	case211			100	100	0.05	0.5	0.5	0	2	10	0	0.05
659	20	20	20	10	2052604	case211			100	125	0.05	0.5	0.5	0	2	10	0	0.05
660	20	20	20	10	2708272	case211			100	150	0.05	0.5	0.5	0	2	10	0	0.05
775	20	20	20	10	79210.375	case211			100	25	0.05	0.5	0.33	0.17	2	10	0	0.05
776	20	20	20	10	317638.375	case211			100	50	0.05	0.5	0.33	0.17	2	10	0	0.05
777	20	20	20	10	735613.375	case211			100	75	0.05	0.5	0.33	0.17	2	10	0	0.05
778	20	20	20	10	1326391.375	case211			100	100	0.05	0.5	0.33	0.17	2	10	0	0.05
779	20	20	20	10	2017036	case211	case212		100	125	0.05	0.5	0.33	0.17	2	10	0	0.05
780	20	20	20	10	2665804	case211			100	150	0.05	0.5	0.33	0.17	2	10	0	0.05
895	20	20	20	10	79210.375	case211			100	25	0.05	0.5	0.25	0.25	2	10	0	0.05
896	20	20	20	10	317638.375	case211			100	50	0.05	0.5	0.25	0.25	2	10	0	0.05
897	20	20	20	10	735613.375	case211			100	75	0.05	0.5	0.25	0.25	2	10	0	0.05
898	20	20	20	10	1298323.375	case211	case212	case213	100	100	0.05	0.5	0.25	0.25	2	10	0	0.05
899	20	20	20	10	2017036	case211	case212		100	125	0.05	0.5	0.25	0.25	2	10	0	0.05
900	20	20	20	10	2623636	case211			100	150	0.05	0.5	0.25	0.25	2	10	0	0.05
1015	20	20	20	10	79210.375	case211			100	25	0.05	0.5	0.17	0.33	2	10	0	0.05
1016	20	20	20	10	317638.375	case211			100	50	0.05	0.5	0.17	0.33	2	10	0	0.05
1017	20	20	20	10	735613.375	case211			100	75	0.05	0.5	0.17	0.33	2	10	0	0.05
1018	20	20	20	10	1298323.375	case211	case212	case213	100	100	0.05	0.5	0.17	0.33	2	10	0	0.05
1019	20	20	20	10	1981768	case211	case212		100	125	0.05	0.5	0.17	0.33	2	10	0	0.05
1020	20	20	20	10	2581768	case211			100	150	0.05	0.5	0.17	0.33	2	10	0	0.05
1135	20	20	20	10	79210.375	case211			100	25	0.05	0.5	0	0.5	2	10	0	0.05
1136	20	20	20	10	303970.375	case211			100	50	0.05	0.5	0	0.5	2	10	0	0.05
1137	20	20	20	10	714745.375	case211			100	75	0.05	0.5	0	0.5	2	10	0	0.05
1138	20	20	20	10	1270555.375	case211	case212	case213	100	100	0.05	0.5	0	0.5	2	10	0	0.05
1139	20	20	20	10	1946800	case211			100	125	0.05	0.5	0	0.5	2	10	0	0.05
1140	20	20	20	10	2540200	case211			100	150	0.05	0.5	0	0.5	2	10	0	0.05
3655	20	20	20	10	17159.875	case211			100	25	0.05	0.5	0.5	0	2	10	0	0.25
3656	20	20	20	10	66527.875	case211	case212	case213	100	50	0.05	0.5	0.5	0	2	10	0	0.25
3657	20	20	20	10	151862.875	case211	case212	case213	100	75	0.05	0.5	0.5	0	2	10	0	0.25
3658	20	20	20	10	271760.875	case211	case212		100	100	0.05	0.5	0.5	0	2	10	0	0.25
3659	20	20	20	10	411364	case211			100	125	0.05	0.5	0.5	0	2	10	0	0.25
3660	20	20	20	10	542512	case211			100	150	0.05	0.5	0.5	0	2	10	0	0.25
3775	20	20	20	10	15731.875	case211	case212	case213	100	25	0.05	0.5	0.33	0.17	2	10	0	0.25
3776	20	20	20	10	63719.875	case211			100	50	0.05	0.5	0.33	0.17	2	10	0	0.25
3777	20	20	20	10	147614.875	case211	case212	case213	100	75	0.05	0.5	0.33	0.17	2	10	0	0.25
3778	20	20	20	10	266072.875	case211	case212	case213	100	100	0.05	0.5	0.33	0.17	2	10	0	0.25
3779	20	20	20	10	404236	case211	case212		100	125	0.05	0.5	0.33	0.17	2	10	0	0.25
3780	20	20	20	10	534004	case211	case212		100	150	0.05	0.5	0.33	0.17	2	10	0	0.25

Table B.1. Numerical results at all values of the most influential parameters (W, β) (continued).

Instances	p	p_2^n	p_2^r	p_2^s	Best Obj Func	Best Case	Alt.Case1	Alt.Case2	K	W	$\bar{\phi}$	f	f_m	f_s	c_r	c_n	a	β
3895	20	20	20	10	15731.875	case211	case212	case213	100	25	0.05	0.5	0.25	0.25	2	10	0	0.25
3896	20	20	20	10	63719.875	case211			100	50	0.05	0.5	0.25	0.25	2	10	0	0.25
3897	20	20	20	10	147614.875	case211	case212	case213	100	75	0.05	0.5	0.25	0.25	2	10	0	0.25
3898	20	20	20	10	260444.875	case211			100	100	0.05	0.5	0.25	0.25	2	10	0	0.25
3899	20	20	20	10	404236	case211	case212		100	125	0.05	0.5	0.25	0.25	2	10	0	0.25
3900	20	20	20	10	525556	case211			100	150	0.05	0.5	0.25	0.25	2	10	0	0.25
4015	20	20	20	10	15731.875	case211	case212	case213	100	25	0.05	0.5	0.17	0.33	2	10	0	0.25
4016	20	20	20	10	63719.875	case211			100	50	0.05	0.5	0.17	0.33	2	10	0	0.25
4017	20	20	20	10	147614.875	case211	case212	case213	100	75	0.05	0.5	0.17	0.33	2	10	0	0.25
4018	20	20	20	10	260444.875	case211			100	100	0.05	0.5	0.17	0.33	2	10	0	0.25
4019	20	20	20	10	397168	case211	case212		100	125	0.05	0.5	0.17	0.33	2	10	0	0.25
4020	20	20	20	10	517168	case211	case212		100	150	0.05	0.5	0.17	0.33	2	10	0	0.25
4135	20	20	20	10	15731.875	case211	case212	case213	100	25	0.05	0.5	0	0.5	2	10	0	0.25
4136	20	20	20	10	60971.875	case211			100	50	0.05	0.5	0	0.5	2	10	0	0.25
4137	20	20	20	10	143426.875	case211	case212	case213	100	75	0.05	0.5	0	0.5	2	10	0	0.25
4138	20	20	20	10	254876.875	case211	case212	case213	100	100	0.05	0.5	0	0.5	2	10	0	0.25
4139	20	20	20	10	390160	case211	case212	case213	100	125	0.05	0.5	0	0.5	2	10	0	0.25
4140	20	20	20	10	508840	case211	case212	case213	100	150	0.05	0.5	0	0.5	2	10	0	0.25
6655	20	20	20	10	9481.375	case211			100	25	0.05	0.5	0.5	0	2	10	0	0.45
6656	20	20	20	10	37076.04167	case211	case212		100	50	0.05	0.5	0.5	0	2	10	0	0.45
6657	20	20	20	10	84651.04167	case211			100	75	0.05	0.5	0.5	0	2	10	0	0.45
6658	20	20	20	10	151429.0417	case211			100	100	0.05	0.5	0.5	0	2	10	0	0.45
6659	20	20	20	10	229004	case212			100	125	0.05	0.5	0.5	0	2	10	0	0.45
6660	20	20	20	10	301872	case211			100	150	0.05	0.5	0.5	0	2	10	0	0.45
6775	20	20	20	10	8680.041667	case211			100	25	0.05	0.5	0.33	0.17	2	10	0	0.45
6776	20	20	20	10	35508.04167	case211	case213		100	50	0.05	0.5	0.33	0.17	2	10	0	0.45
6777	20	20	20	10	82283.04167	case211			100	75	0.05	0.5	0.33	0.17	2	10	0	0.45
6778	20	20	20	10	148261.0417	case211	case213		100	100	0.05	0.5	0.33	0.17	2	10	0	0.45
6779	20	20	20	10	225036	case211	case212		100	125	0.05	0.5	0.33	0.17	2	10	0	0.45
6780	20	20	20	10	297137.3333	case211	case212		100	150	0.05	0.5	0.33	0.17	2	10	0	0.45
6895	20	20	20	10	8680.041667	case211			100	25	0.05	0.5	0.25	0.25	2	10	0	0.45
6896	20	20	20	10	35508.04167	case211	case213		100	50	0.05	0.5	0.25	0.25	2	10	0	0.45
6897	20	20	20	10	82283.04167	case211			100	75	0.05	0.5	0.25	0.25	2	10	0	0.45
6898	20	20	20	10	145126.375	case211			100	100	0.05	0.5	0.25	0.25	2	10	0	0.45
6899	20	20	20	10	225036	case211	case212		100	125	0.05	0.5	0.25	0.25	2	10	0	0.45
6900	20	20	20	10	292436	case211	case212		100	150	0.05	0.5	0.25	0.25	2	10	0	0.45
7015	20	20	20	10	8680.041667	case211			100	25	0.05	0.5	0.17	0.33	2	10	0	0.45
7016	20	20	20	10	35508.04167	case211	case213		100	50	0.05	0.5	0.17	0.33	2	10	0	0.45
7017	20	20	20	10	82283.04167	case211			100	75	0.05	0.5	0.17	0.33	2	10	0	0.45
7018	20	20	20	10	145126.375	case211			100	100	0.05	0.5	0.17	0.33	2	10	0	0.45
7019	20	20	20	10	221101.3333	case211			100	125	0.05	0.5	0.17	0.33	2	10	0	0.45
7020	20	20	20	10	287768	case211	case212		100	150	0.05	0.5	0.17	0.33	2	10	0	0.45
7135	20	20	20	10	8680.041667	case211			100	25	0.05	0.5	0	0.5	2	10	0	0.45
7136	20	20	20	10	33973.375	case211	case212	case213	100	50	0.05	0.5	0	0.5	2	10	0	0.45
7137	20	20	20	10	79948.375	case211			100	75	0.05	0.5	0	0.5	2	10	0	0.45
7138	20	20	20	10	142025.0417	case211			100	100	0.05	0.5	0	0.5	2	10	0	0.45
7139	20	20	20	10	217200	case211	case212	case213	100	125	0.05	0.5	0	0.5	2	10	0	0.45
7140	20	20	20	10	283133.3333	case211			100	150	0.05	0.5	0	0.5	2	10	0	0.45

APPENDIX C

SENSITIVITY OF EXPECTED TOTAL PROFIT TO W

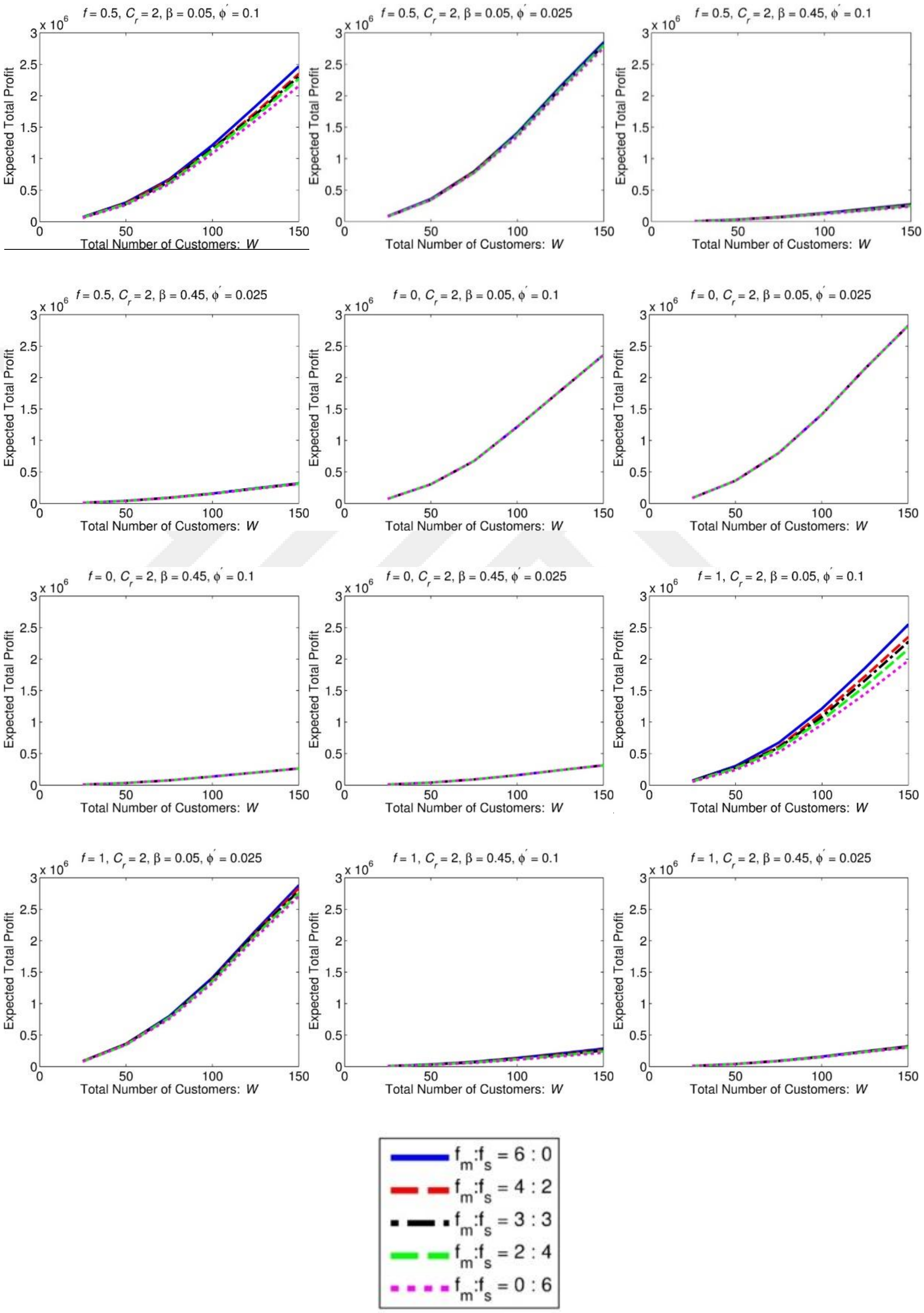

Figure C. 1 Sensitivity of expected total profit to W for different values of $f, \beta, \bar{\phi}$.

APPENDIX D

SENSITIVITY OF EXPECTED TOTAL PROFIT TO f

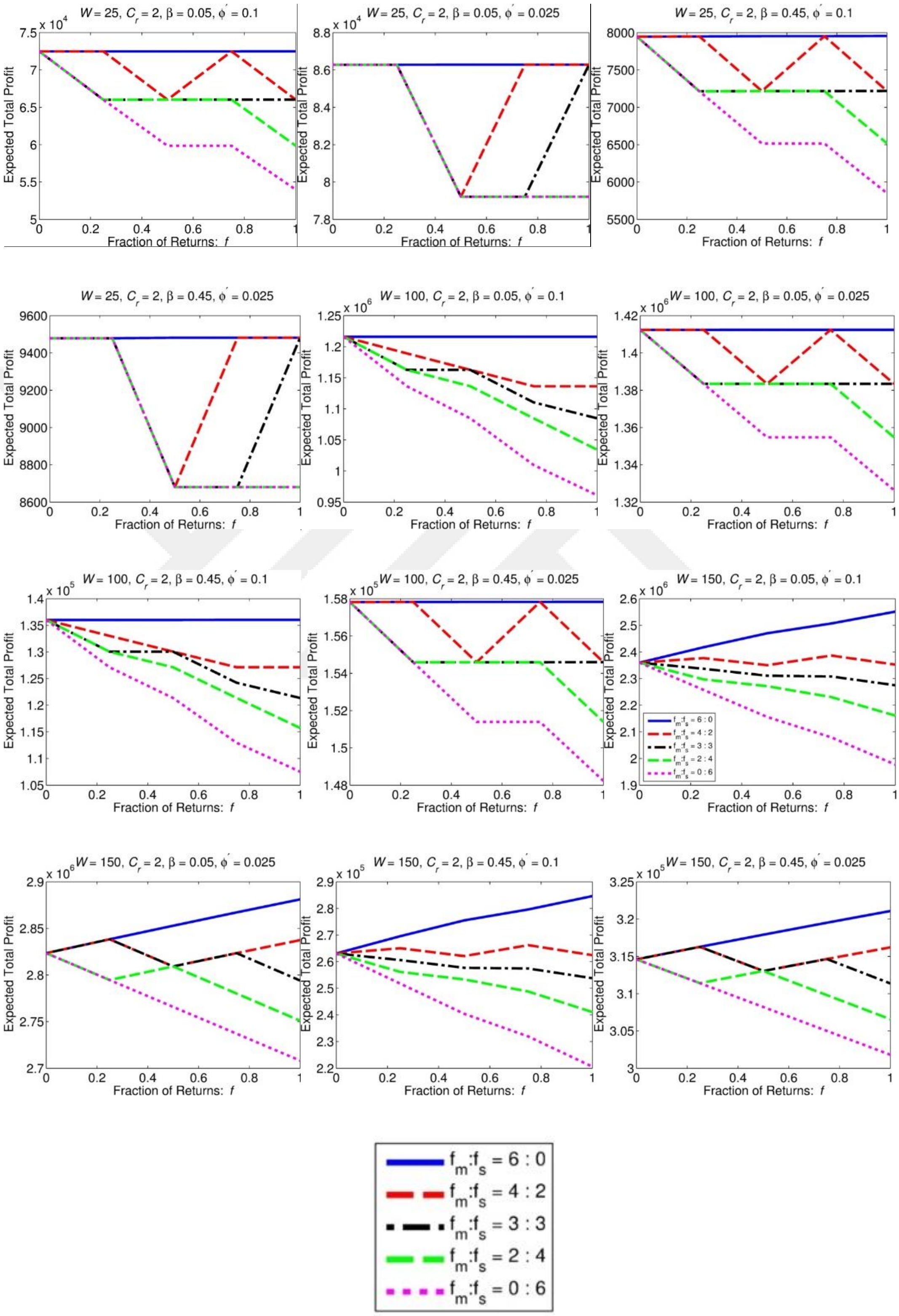

Figure D. 1 Sensitivity of expected total profit to f for different values of $W, \beta, \bar{\phi}$.

APPENDIX E

SENSITIVITY OF EXPECTED TOTAL PROFIT TO $\boldsymbol{\beta}$

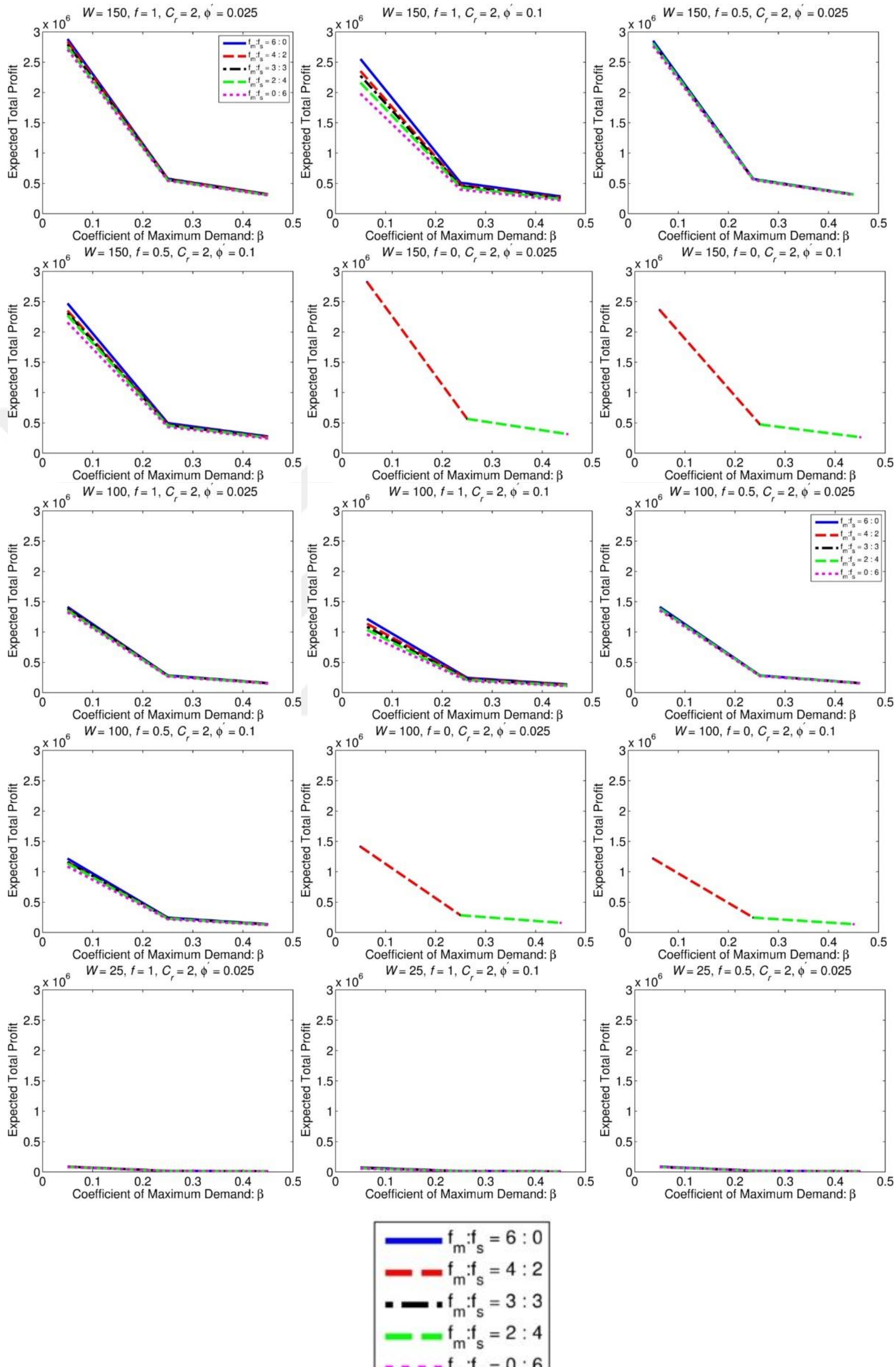

Figure E. 1 Sensitivity of expected total profit to β for different values of $W, f, \bar{\phi}$.

APPENDIX F

SENSITIVITY OF EXPECTED TOTAL PROFIT TO $\bar{\phi}$

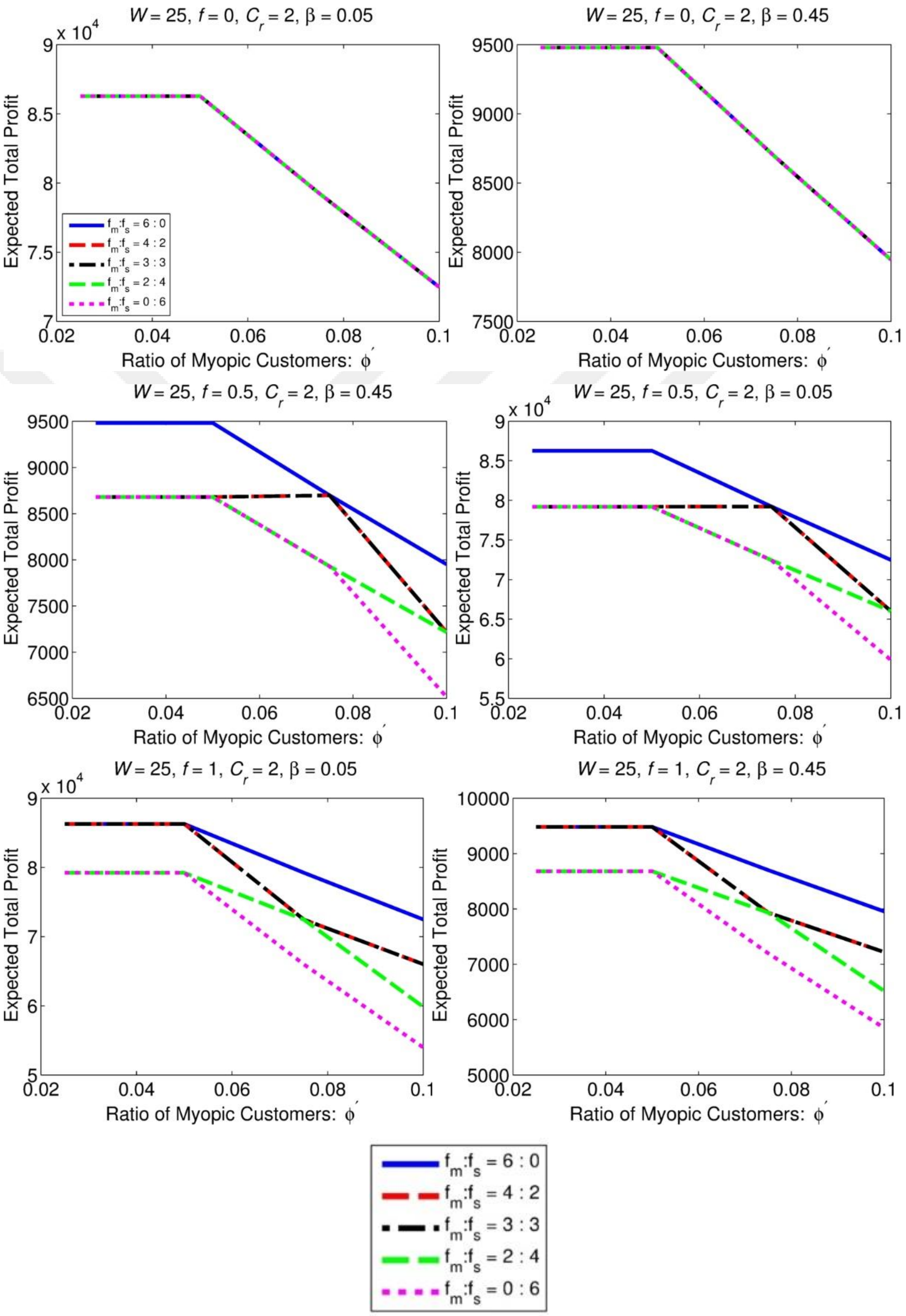

Figure F. 1 Sensitivity of expected total profit to $\bar{\phi}$ for $W = 25$ and different values of f, β.

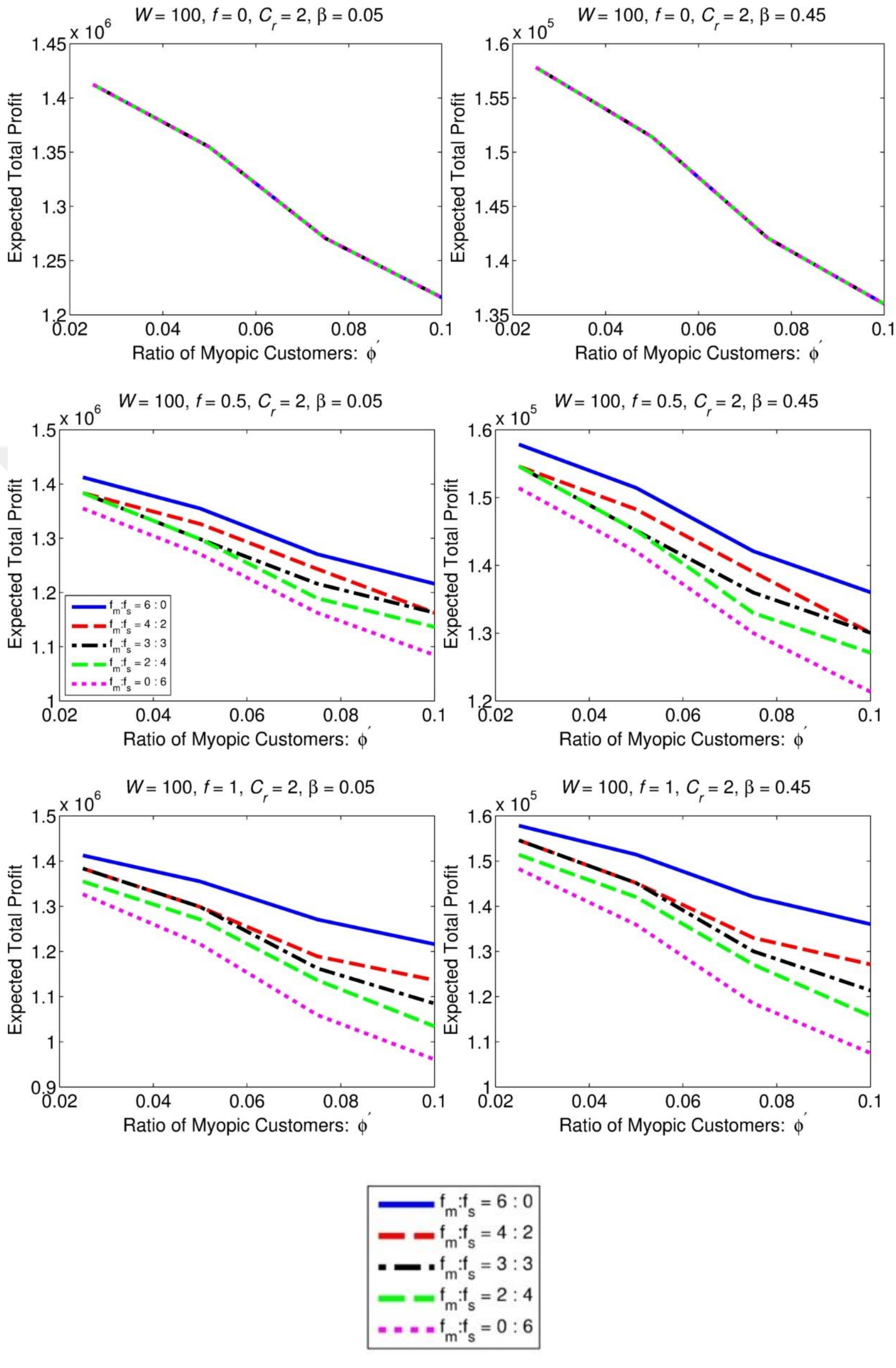

Figure F. 2 Sensitivity of expected total profit to $\bar{\phi}$ for $W = 100$ and different values of f, β.

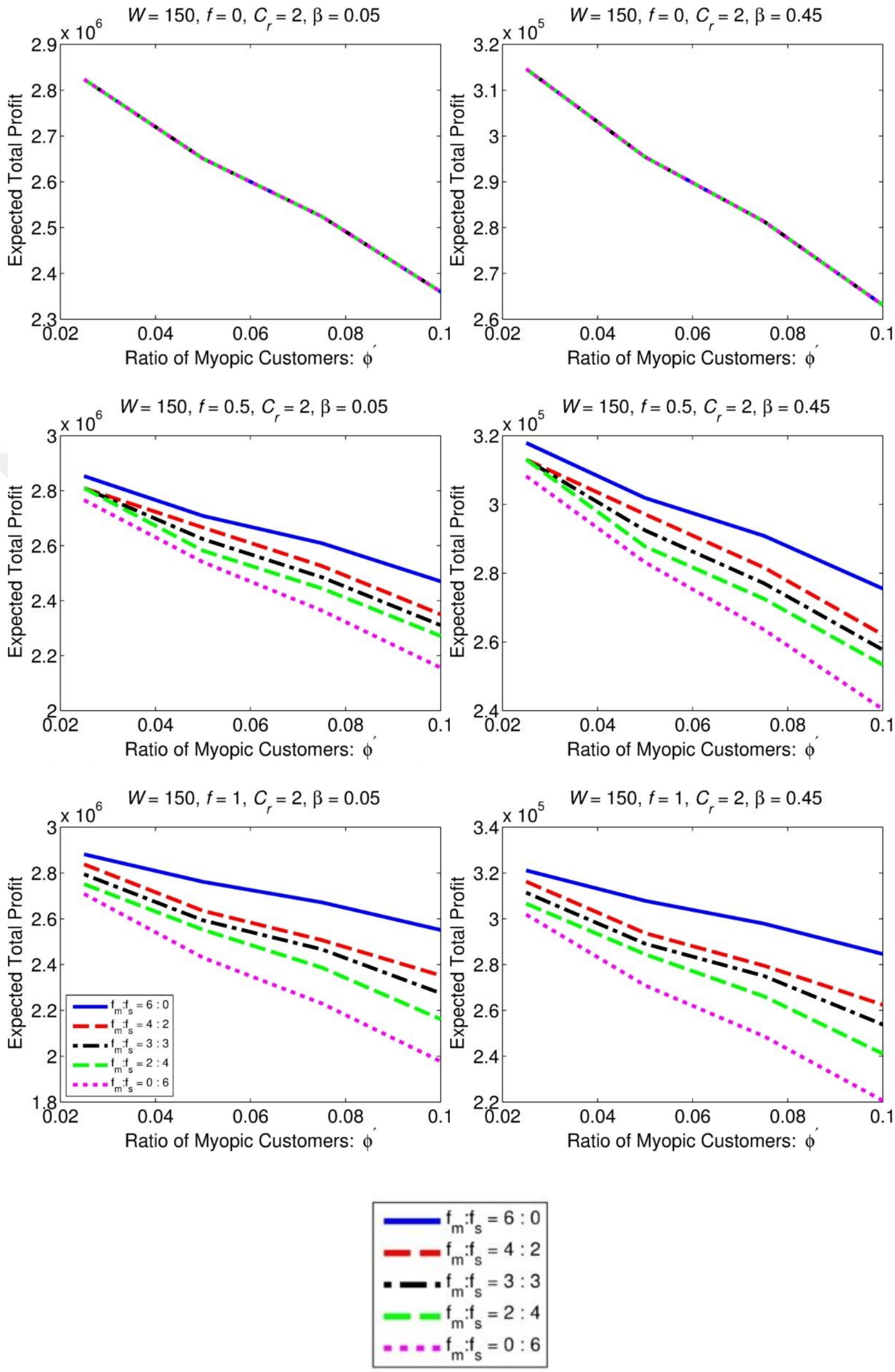

Figure F. 3 Sensitivity of expected total profit to $\bar{\phi}$ for $W = 150$ and different values of f, β.

APPENDIX G

SPIDER PLOTS

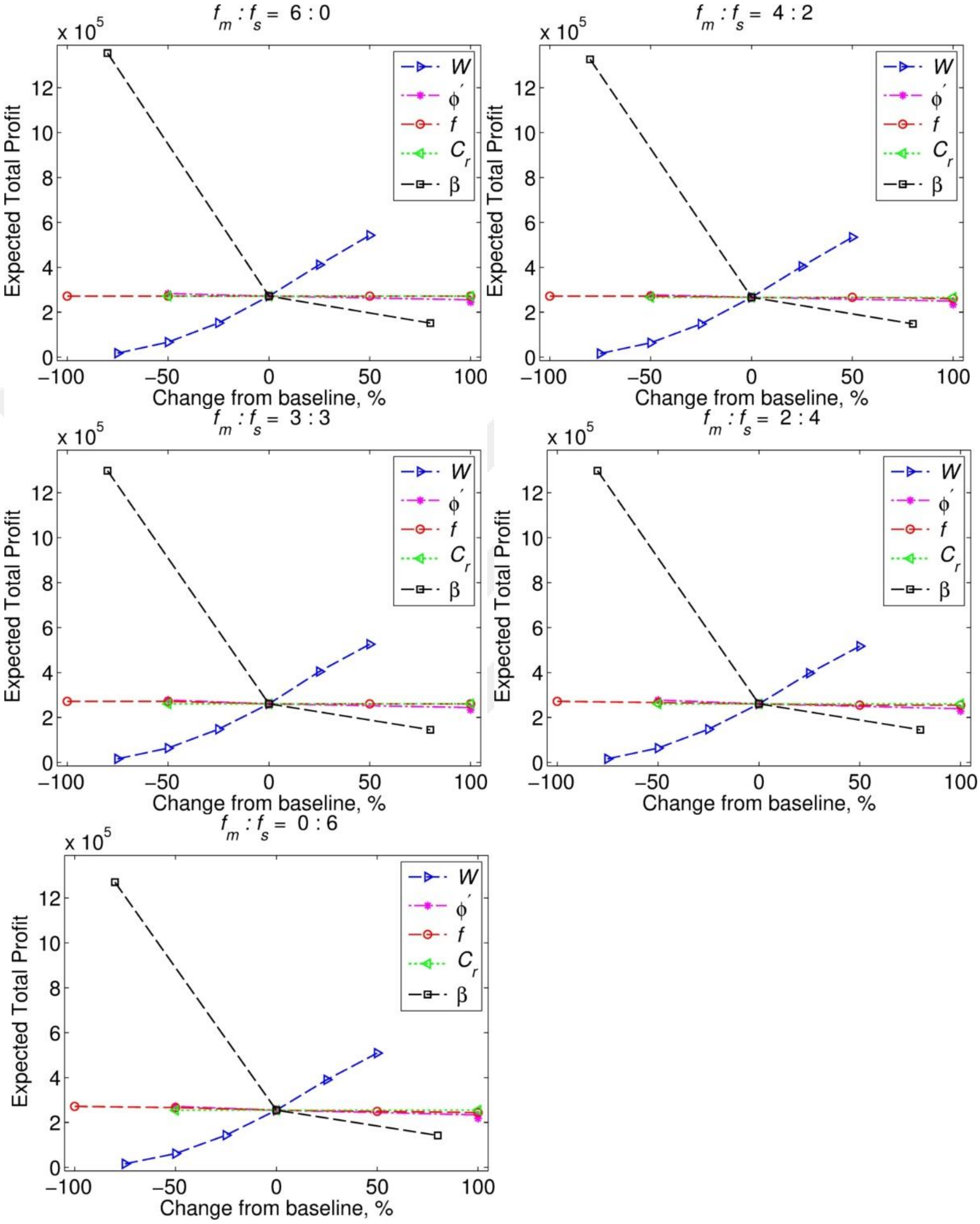

Figure G. 1 Spider plot showing sensitivity of the expected total profit to different parameters.

APPENDIX H

CURRICULUM VITAE

PERSONAL INFORMATION

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EDUCATION

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M.Sc.	Çankaya Univ., Industrial Engineering, Ankara	2017
B.Sc.	Çankaya Univ., Industrial Engineering, Ankara	2014
High School	Süleyman Demirel Science High School, Isparta	2009

WORK EXPERIENCE

Year	**Place**	**Enrollment**
2014- Present	Çankaya Univ. Department of Industrial Engineering	Research Assistant
2013 August	Ortadoğu Rulman Sanayi ve Tic. A.Ş.	Intern Student
2012 August	Eti Alüminyum A.Ş.	Intern Student

FOREIN LANGUAGES

Advanced English, Beginner German.

HOBBIES

Sport, Travel, Shopping, Reading.