PAPR REDUCTION OF OFDM SYMBOLS VIA OPTIMAL ROTATION OF INFORMATION SYMBOLS

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PAPR REDUCTION OF OFDM SYMBOLS VIA OPTIMAL ROTATION OF INFORMATION SYMBOLS

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ABSTRACT

PAPR REDUCTION OF OFDM SYMBOLS VIA OPTIMAL ROTATION OF INFORMATION SYMBOLS

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Orthogonal frequency division multiplexing (*OFDM*) is an effective multicarrier transmission for wireless communication systems. The demand for high data rate for multimedia applications made *OFDM* widely used in wireless communication. The main drawback of *OFDM* communication systems is their high peak-to-average power ratios (*PAPRs*) which limit their use in practical applications. There are two well-known *PAPR* reduction techniques known in the literature, partial transmit sequence (*PTS*), and selective mapping which are closely related to each other. In this thesis work, we propose a new *PAPR* reduction approach. Our method is based on rotation of the information symbols before inverse fast Fourier transform (*IFFT*) operation. The selection of the information symbols is performed inspecting their combinations in *OFDM* symbols, and this is achieved detailing decimation in frequency *IFFT* algorithm, i.e., inspecting the combination of information symbols while performing decimation in frequency *IFFT* algorithm. The proposed reduction method has better performance than that of the *PTS* method, and has much less complexity when compared to that of the *PTS* technique.

Keywords : *OFDM*, Peak-to-average power ratio (*PAPR*), Complexity, Distortion, *IFFT*, Partial Transmit Sequences, Decimation in Frequency Fast Fourier Transform, *BER*.

ÖZ

OFDM SEMBOLLERİNİN PAPR DEĞERLERİNİN VERİ SEMBOLLERİNİN EN İYİ MİKTARLARDA DÖNDÜRÜLMESİ İLE DÜŞÜRÜLMESİ

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Dik frekans bölmeli çoğullama (*OFDM*) çoklu taşıyıcı taşıma prensibine dayalı, kablosuz iletişimde kullanılan etkili haberleşme tekniklerinden birisidir. *OFDM* yöntemi özellikle yüksek hız gerektiren çoklu medya veri iletişimleri için zaruri hale gelmiştir. *OFDM* sistemlerinin en büyük dez avantajları bu sistemlerin sahip olduğu yüksek doruk-ortalama oran (*PAPR*) değerleridir. Yüksek PAPR değerleri bu sistemlerin pratikte kullanılabilirliğini zorlaştırmaktadır. Bu nedenden ötürü *PAPR* değerlerini düşürmek için bazı teknikleri önerilmiştir. Kısmı gönderim dizileri (*PTS*) ve seçmeli eşleştirme yöntemleri literatürde bilinen iki yöntemdir. Bu tez çalışmamızda PAPR değerlerini düşürmek için yeni bir yöntem öneriyoruz. Önerilen yöntem ters veri bilgilerinin ters Fourier dönüşüm işleminden önce döndürülmesi ilkesine dayanmaktadır, fakat bu döndürme işlemi rasgele olarak yapılmamaktadır. Bunun için frekans kırınım *IFFT* algoritması incelenerek, veri sembollerinin *OFDM* sembolleri içerisinde nasıl birleştirildiğine dikkat edilerek, hangi veri sembollerinin döndürülmesine gerek olduğuna karar verilmiştir. Önerilen yöntem *PTS* yönteminden daha iyi sonuç vermektedir ve toplam işlem miktarı *PTS* yönteminde gereken toplam işlem miktarından çok daha azdır.

Anahtar Kelimeler : *OFDM,* doruk-ortalama güç oranı, işlem miktarı, karmaşıklık, *IFFT*, kısmi dizi gönderme, frekans kırınım hızlı Fourier dönüşüm, bit hata oranı

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This thesis work is dedicated to my mother, who has been a constant source of support and encouragement during the challenges of graduate school and life. I am truly thankful for having you in my life. This work is also dedicated to my heartbeat my daughters, Shahhd and Shams. Finally, I would like to express my deepest gratitude to my supervisor Assoc. Prof. Dr. Orhan GAZİ for his guidance. I am very grateful for his leading role, advices and insight throughout the research.

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LIST OF ABBREVIATIONS

ADC/ DAC	Analog to Digital Converter/Digital to Analog Converter
BER	Bit Error Rate
BPSK	Binary Phase Shift Key
CCDF	Cumulative distribution function
CL	Clipping Level
DSR	Distortion to-Signal Power Ratio
DT	Discrete-time
M-PSK	M-ary phase shift key
M-QAM	M-ary Quadrature Amplitude Modulation
OFDM	Orthogonal Frequency Division Multiplexing
PAPR	Peak to Average Power Ratio
PRCs	Peak Reduction Carriers
QAM	Quadrature Amplitude Modulation
Q-PSK	Quadrature Phase-shift Keying
SLM	Selective Mapping
PA	Power Amplifier
TR	Tone Reservation
DFT	Discrete Fourier Transform
IDFT	Inverse Discrete Fourier Transform

CHAPTER I

1. INTRODUCTION

1.1 PAPR:

Peak to Average Power Ratio is a one of the drawbacks of *OFDM* communication systems. Large *PAPR* leads to both in band distortion and out band radiation. Large *PAPR* also increases the complexity in analog-to-digital and digital-to-analog convertors and minimize the efficiency of the radio frequency amplifier power tower used.

PAPR is a parameter calculated by dividing maximum power of the any given *OFDM* transmit to the average power of that *OFDM* symbol. When different sub-carriers are out of the phase with each other *PAPR* noise occurs in multi-carrier system. Their value is different at each phase at different instant. In an *OFDM* system, there are large numbers of sub-carriers, which are independent modulators, and due to these carriers; peak value of the system is very high as compared to the average of the whole system. When large of sub-carriers modulations are added coherently in an *OFDM*, large *PAPR* values are obtained.

1.2 PAPR Calculation:

Let $X = [X_1 X_2 \ldots \ldots . X_N]$ be a block of N symbols, where each symbol is modulated to one of the carrier frequencies $\{f_n, n = 1, 2, \ldots, N\}$. In *OFDM* the N subcarriers are chosen to be orthogonal to each other, i.e., $fn = n\Delta f, \ \ \Delta f = 1/NT$ where T signal time period . Then we can represent the transmitted signal by [1].

$$x(t) = 1/\sqrt{N} \sum_{n=1}^{N} X[n]\, e^{j2\pi f_n t}$$

For the above signal the *PAPR* is defined as

$$PAPR = max.power/mean\,power$$

$$= \frac{max|x(t)|^2}{E[|x(t)^2|]}$$

And *PAPR* in decibel unit is calculated as

$$PAPRdB = 10\ log10\ (PAPR)$$

PAPR is dimensionless random quantity, its probabilistic average can be calculated using the $E\{\cdot\}$, the expected value operation. Due to the random subcarrier's amplitudes (such as *QAM* modulation), the maximal value of an *OFDM* signal can become significant.

PAPR depends on the increasing of the number of total and used subcarriers (some of subcarriers used as nulls for synchronization), the maximum theoretical value for *PAPR* is calculated as [1]: as shown

$$PAPR\ =\ 10log10(N).$$

where *N* represents the number of subcarriers. Besides the number of used subcarriers, the *PAPR* is also very sensitive for the modulation scheme.

The peak power values for different *OFDM* symbols are graphically illustrated in Fig. 1.1 where it is seen that large differences between peak and average values are available.

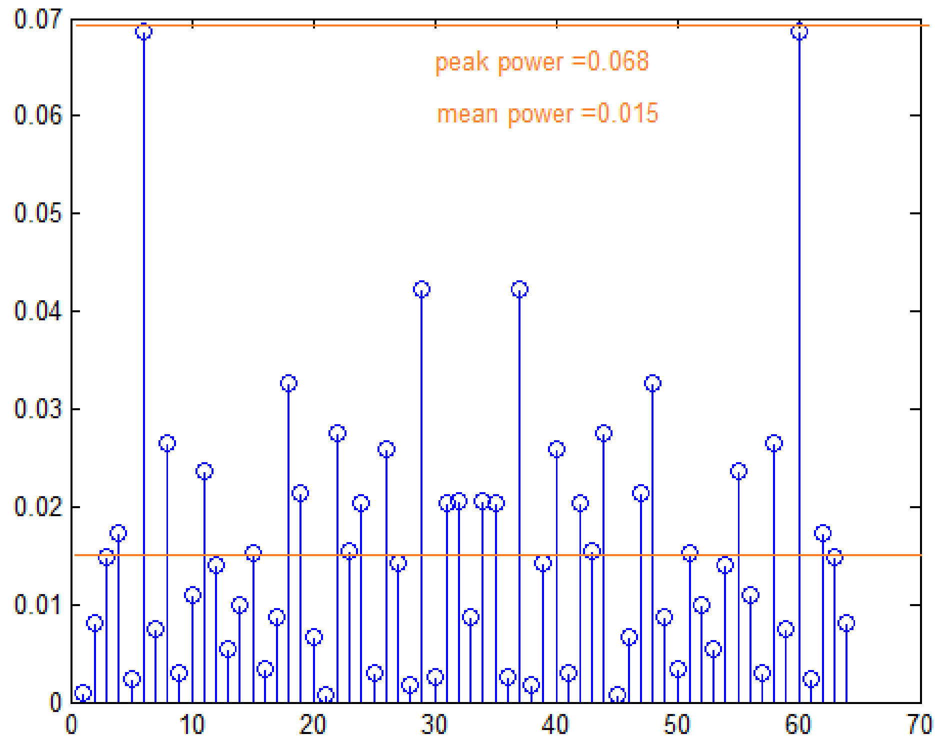


Fig. 1.1 Peak and Average Power Values

1.3 Effects of PAPR: The main effects of *PAPR* are:

1. It consumes more bandwidth due to *PAPR* noise in the *OFDM* system.

2. It causes inter-modulation between the subcarriers and distorts the transmit signal constellation

3. It increases *BER* in *OFDM.*

4. It causes low power efficiency at the power amplifier.

1.4 The Methods To Avoid the Effects of Large PAPR:

1- We can use high quality power amplifiers with wide working range.

2. We can reduce the *PAPR* itself by using one or more of the reduction techniques.

Practically, using a high quality power amplifier is not desired for devices, which use batteries as power supplies.

For this reason it is vital to reduce the *PAPR* of *OFDM* symbols before transmission.

CHAPTER 2

2. PAPR Reduction Techniques:

PAPR reduction techniques are categorized into three categories:

1. Signal Distortion Techniques
2. Signal Scrambling Techniques
3. Coding Techniques

2.1 Signal Distortion Techniques: Signal Distortion is a problem in which signal quality decrease due to interferences to the signal. The signal distortion techniques reduce the *PAPR* by distortion the transmitted *OFDM* signal and these techniques introduce distortion in both in-band and out-band leading to increment in *BER*. There are some researchers suggested techniques to reduce the distortion. These type of techniques have low complexity compared to other types.

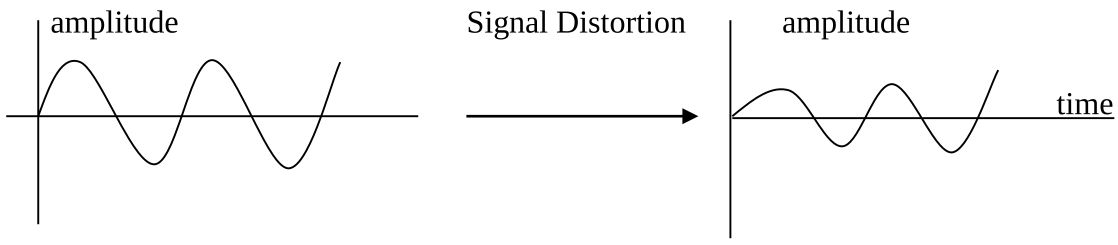


Fig. 2.1 Signal distortion

Signal Distortion Techniques to Reduce the PAPR of OFDM Symbols

Main techniques of signal distortion are:

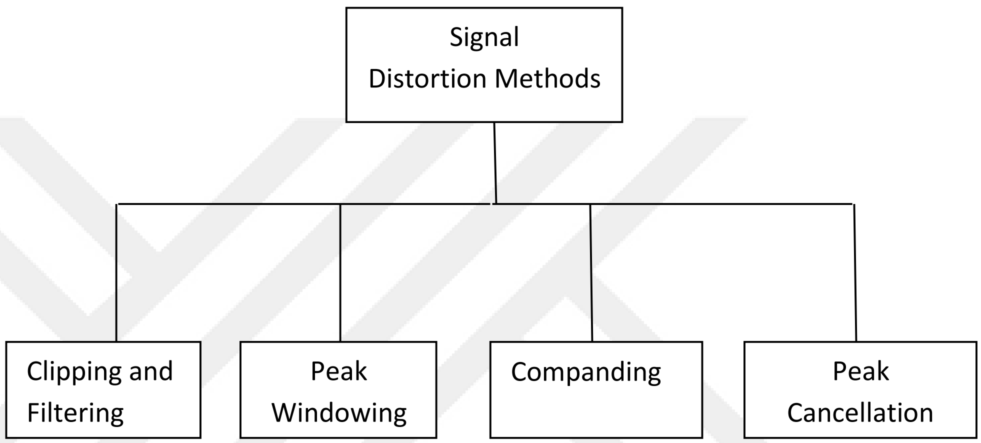


Fig. 2.2 *PAPR* Reduction Signal Distortion Techniques.

2.1.1 Clipping and Filtering:

Clipping and filtering is the simplest technique to reduce *PAPR* methods in *OFDM*. Clipping method is done before the signal passing through the power amplifier. In this process a threshold value for the amplitudes is set. The sub-carrier amplitude which has more value than the threshold are clipped and filtered to get lower *PAPR* value.

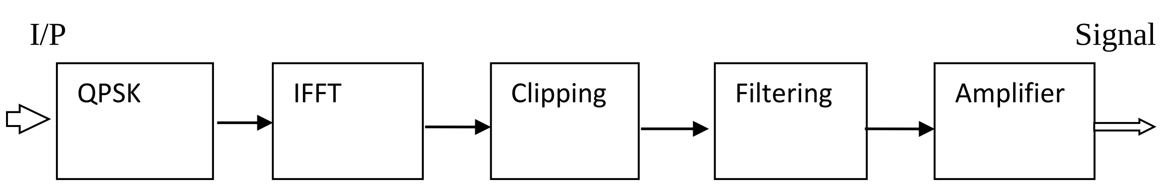


Fig. **2.3** Signal Clipping for Lower *PAPR*

Clipper limits the value to the predetermined value known as clipping level (*CL*) if and only if the value is beyond to the *CL* otherwise no changes are required. Clipping method is a non-linear process, which causes signal distortion, resulting in in-band and out-bands distortions. In band distortion reduce *BER* performance and cannot be reduced by filtering. In addition to it, out of band distortion which causes spectral spreading and it can be eliminated by filtering the clipped *OFDM* signal and improve *BER* performance and it preserve spectral efficiency.[2-3]

2.1.2. Peak Windowing: In peak windowing *PAPR* reduction approach, increment in *BER* can be reduced at the cost of increasing and out-of-band radiation [4]. In this system, we multiply large signal peak with a specific windowing. Hamming window, Gaussian shaped window etc. Ideally, the window should be as narrow band as possible to reduce the *BER*.

2.1.3. Companding: It is one of the best techniques to decrease audio signal data rate using unequal quantization level and *PAPR* of the *OFDM* signals. It is a compressing technique. Best-known ones are the A-law and μ-law compression methods. Its complexity is not effected by the number of subcarriers. No side information required to send. It is suitable for speech processing where high peaks accrue infrequently [5]. However, this method suffers from increased *BER* and increased transmission bandwidth.

2.1.4 Peak Cancellation: It is one of the categories of *PAPR* reduction technique for *OFDM* systems. It can control out-of-band radiation and *PAPR* simultaneously in additional interferences. It depends on subtracting spectrally shaped pulses from signal peaks that exceed a specified threshold to reduce the *PAPR*. In this technique we first define all signal peaks and then pick up the maximum to cancellation pulse generator (*CPG*) as shown in Fig. 2.4

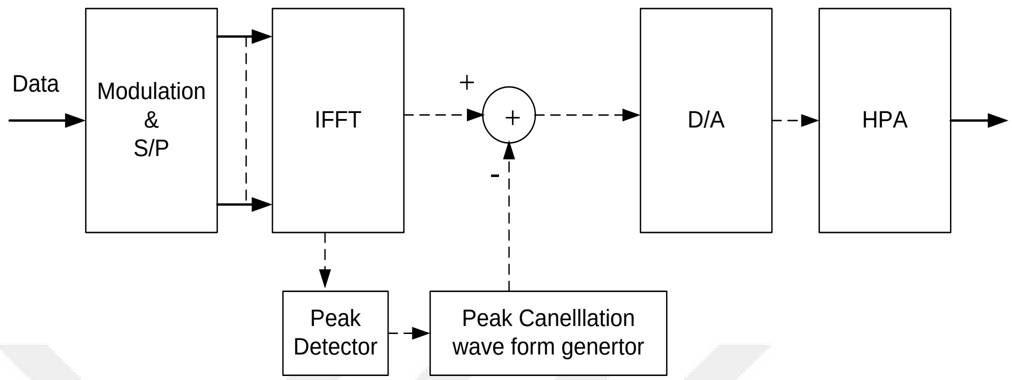


Fig. 2.4 Stages of the Peak Cancellation Technique

2.2. Signal Scrambling Techniques: Signal Scrambling techniques are used to scramble the signal to reduce *PAPR*.

Techniques for Signal Scrambling: There are many signal scrambling techniques basic one of them are summarized below:

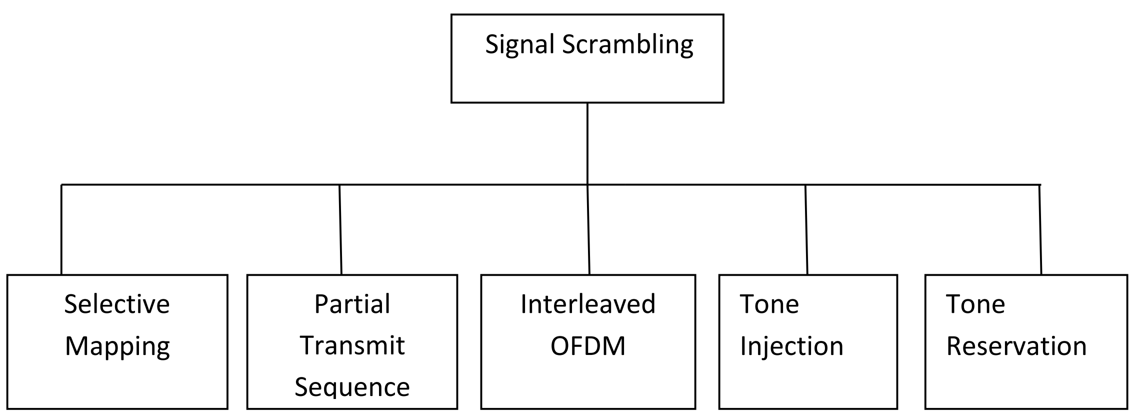


Fig. 2.5 Signal scrambling techniques

2.2.1 Selective Mapping: In selective information symbols are multiplied by exponential signals and those exponential signals which gives lower *PAPR* are selected for rotation operations. This method is described in [7]. The disadvantage of this method is that, the complexity is high at the transmitter side, which can cause serious latency during transmission. The receiver needs the side information and this caused an increase in the transmission bandwidth. In Fig. 2.6 *SLM* technique is explained block wise where V is number of generated phase sequences and the performance of *PAPR* depends on V [8]. It is obvious from the block diagrams that the complexity of the transmitter increases in parallel with V.

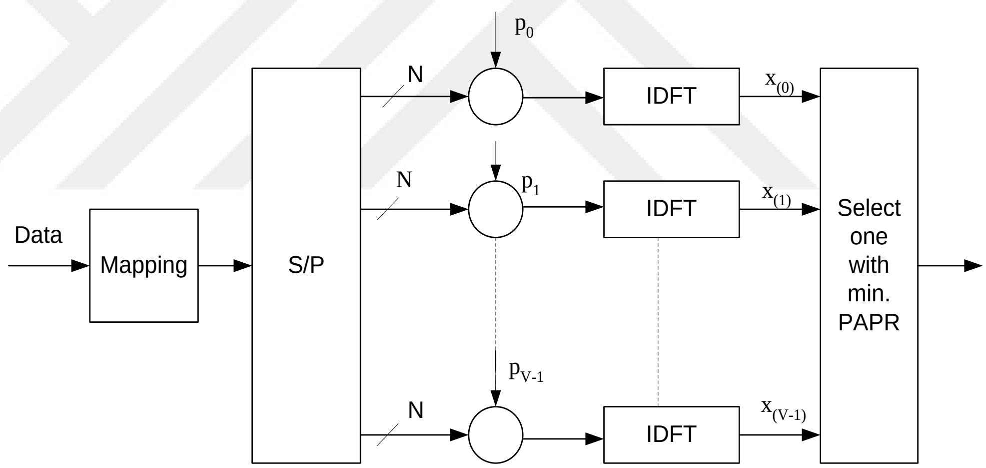


Fig. **2.6** Selective Mapping *PAPR* Reduction Technique

2.2.2 Partial Transmit Sequence: In this approach *OFDM* symbol is divided into frames and the frames are rotated by some exponentials and combined in an optimum manner. We will inspect *PTS* in details in Chapter 3.

2.2.3 Interleaving *OFDM*: The data containing high correlation units has large *PAPR* which can be reduced if long correlation can be reduced down. In Adaptive interleaving starting threshold value must be set [9]. As shown in Fig. 2.7 its block diagram is similar to *SLM* technique but instead of phases in *SLM* there are interleavers employed .

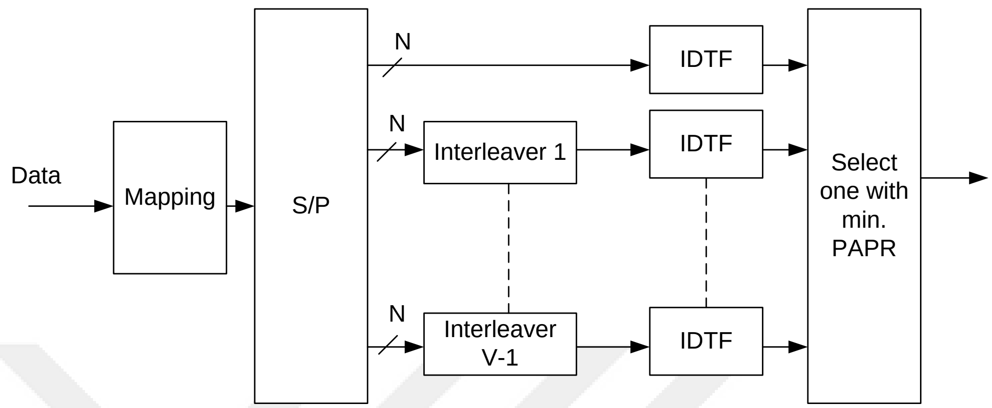


Fig. 2.7 Interleaving Technique

2.2.4 Tone Reservation: To reduce *PAPR* there is a need to keep small set of tones. This is known as convex problem and can be solved accurately. Time domain signal is added in to the original signal. The aim of this addition is to minimize high peak values. In other hand, the transmitter reserves a small number of unused subcarriers. These subcarriers are referred to as peak reduction carriers (*PRCs*) [10] and these *PRCs* do not carry data.

2.2.5 Tone Injection: In tone injection method for *PAPR* reduction method, with the help of adaptive method without the loss of data rate, *PAPR* reduction can be achieved.

2.3. Coding: Coding techniques can be used to scramble signals. Barker codes, Glory complimentary sequences, M sequence, Shapiro-Rudin sequences can be used to reduce *PAPR* efficiently. Coding technique in *OFDM* is also of two types:

1. Block Coding

2. Pre-coding

2.3.1 Block coding: In this coding approach, the message, which has symbols, containing low peak power is chosen for coding as a valid code for transmission.

2.3.2 Pre-coding Technique: Pre-coding technique's main goal is to obtain a lower *PAPR* signals as compared to *OFDM* and to minimize the obstruction created by multiple users. In pre coding, data which is modulated is multiply by shaping matrix before the formation *OFDM* symbols. First of all, signals are modulated in baseband as a input using *M-QAM*, *M-PSK* etc. After that baseband modulation is transferred by pre-coding matrix. Pulse coding, *DCT* methods can be used for pre-coding [11].

CHAPTER 3

3. OPTIMUM COMBINING OF PARTIAL TRANSMIT SEQUENCES FOR PAPR REDUCTION

PTS technique is one of the distortion less methods to reduce *PAPR* in *OFDM* systems. The concept of the PTS scheme is illustrated in Fig. 3.1 where it is seen that phase rotation is introduced for the divided *OFDM* frames combined in an optimum manner such that the resulting signal has low *PAPR* [12]. Fig. 3.1 shows the stages of *PTS*. Below we explain this method in details.

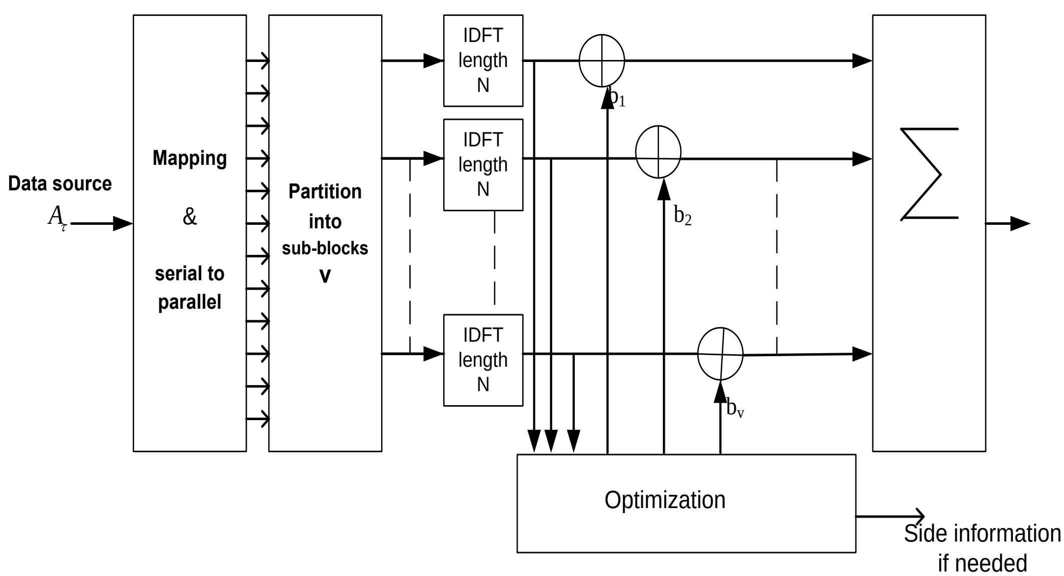
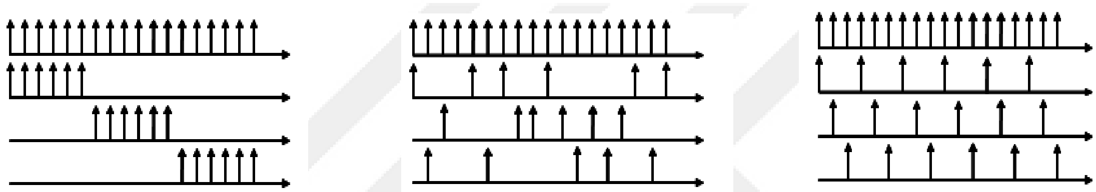


Fig. 3.1 Partial Transmit Sequence *PAPR* Reduction Method

3.1 Partitioning:

It is the first step in (*PTS*) partial transmits sequence *PAPR* reduction technique. The data block of length *N* is partitioned into a number of disjoint sub-blocks. There are three types of partitioning schemes; adjacent, interleaved and pseudo-random partitioning. The partitioning method affects the *PAPR* performance and its reduction complexity [13]. Interleaving method has low complexity compared to adjacent and pseudo- random but has worst *PAPR* performance among them. In Fig. 3.2 partitioning methods are summarized.



Adjacent Random Interleaving

Fig. 3.2 Partitioning Methods

3.2 Mathematical Development of the PTS Technique:

First the *OFDM* block is divided into sub-blocks and the inverse discrete Fourier transform (*IDFT*) of each sub-block is computed separately. Next, these sub-blocks are multiplied by $e^{j\theta_i}$ exponentials and summed. The phase terms in these exponentials are selected in such a way to reduce the *PAPR* of the *OFDM* block [14] [15].

Let A_τ be the data block and it is divided into V pairwise disjoint carrier sub-blocks, i.e.,

$$A_\tau = \sum_{v=1}^{V} A_\tau^{(v)} \tag{1}$$

Let's introduce a rotation factor as

$$b_\tau^{(v)} = e^{j\theta_t(v)} \quad ,\theta \in \{0, 2\pi\} \tag{2}$$

$b_\tau^{(v)}$ is also called side information and $v = 1, 2, \ldots, V$ Let's rotate the sub-blocks by the above defined phase terms and sum them as in

$$\tilde{A}_\tau = \sum_{v=1}^{V} b_\tau^{(v)} \ A_\tau^{(v)} \tag{3}$$

A_τ & $\tilde{A}_\tau$ represent the same information.

A_τ is the vector consisting of all discrete subcarrier amplitudes $A_{\tau,v}$ for $0 \le v \le D$.

Let $$a_\tau = IDFT\{A_\tau\} \tag{4}$$

Where a_τ is the vector consisting of the samples $a_{\tau,\rho}$ $0 \le \rho \le D$ let's introduce the following terms

$$\tilde{a}_\mu = IDFT\{\tilde{A}_\tau\} \tag{5}$$

$$\tilde{a}_\mu = \sum_{v=1}^{V} b_\tau^{(v)} \ IDFT\{A_\tau^{(v)}\} \tag{6}$$

$$= \sum_{v=1}^{V} b_\tau^{(v)} \ a_\tau^{(v)} \qquad a_\tau^{(v)} = IDFT\{A_\tau^{(v)}\} \tag{7}$$

The optimum phase factors for the *OFDM* symbol τ are determined using

$$\{b_\tau^{(1)}, \ldots\ldots \mathrm{b}_\tau^{(V)}\} = \arg\min\left(\max_{0\le\rho\le D} \left| \sum_{v=1}^{V} b_\tau^{(v)} \ a_{\tau,\rho}^{(v)} \right| \right) \tag{8}$$

which can be written in matrix form as

$$\{b_\tau^{(1)}, \ldots\ldots \mathrm{b}_\tau^{(V)}\} = \arg\min\left(\max_{0\le\rho\le D} \left| \left[b_\tau^{(1)} \ldots\ldots b_\tau^{(V)} \right] \begin{bmatrix} b_{\tau,1}^{(1)} & \mathrm{b}_{\tau,2}^{(1)} \ldots \mathrm{b}_{\tau,Z}^{(1)} \\ b_{\tau,1}^{(2)} & b_{\tau,1}^{(2)} \ldots b_{\tau,Z}^{(2)} \\ . & . \quad \ldots . \\ . & . \quad \ldots . \\ b_{\tau,1}^{(V)} & b_{\tau,1}^{(V)} \ldots b_{\tau,1}^{(V)} \end{bmatrix} \right| \right) \tag{9}$$

Using (9), we calculate the optimum $\mathrm{b}_\tau^{(V)}$ sequence that gives the minimum of maximums and use that phase sequence for the rotation of information sub-blocks.

3.3 PTS Example

Let $N = 16$ be the number of information symbols and $V = 4$ be the number of sub-blocks. We use BPSK modulation before *IDFT* calculation. Let the information block be as

$$A_\tau = [1\ 10\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0]$$

After *BPSK* modulation we get

$$A_\tau = [\ 1\ 1\ -1 \quad 1\ \ -1\ -1\ \ 1\ 1\ 1\ 1\ -1\ 1\ 1\ -\ 1\ -1\ -1]$$

which is partitioned into 4 sub-blocks as

$$A_t^{(1)} = [\ 1\ 0\ 0\ 0\ -1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ ]$$

$$A_t^{(2)} = [\ 0\ 1\ 0\ 0\ 0\ -1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ -1\ 0\ 0\ ]$$

$$A_t^{(3)} = [\ 0\ 0\ -1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ -1\ 0\ 0\ 0\ -1\ 0\ ]$$

$$A_t^{(4)} = [\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ -1]$$

which when summed gives

$$A_\tau = A_t^{(1)} + A_t^{(2)} + A_t^{(3)} + A_t^{(4)}$$

Calculating the *IDFT* of each sub-block separately we get

$IDFT\{A_t^{(1)}\}$ = [0.1250 , 0 - 0.1250i , 0.1250 , 0 + 0.1250i, 0.1250 , 0 - 0.1250i, 0.1250 ,
0 + 0.1250i , 0.1250 , 0 - 0.1250i , 0.1250 , 0 + 0.1250i, 0.1250 ,
0 - 0.1250i , 0.1250 , 0 + 0.1250i]

$IDFT\{A_t^{(2)}\}$ = [0 , 0 , 0.1768 + 0.1768i , 0 , 0 , 0 , -0.1768 + 0.1768i , 0 , 0 , 0
-0.1768 - 0.1768i , 0 , 0 , 0 , 0.1768 - 0.1768i , 0]

$IDFT\{A_t^{(3)}\}$ = [-0.1250, -0.0884 + 0.0884i, 0 - 0.1250i , 0.0884 + 0.0884i , 0.1250,

0.0884 - 0.0884i , 0 + 0.1250i, -0.0884 - 0.0884i, -0.1250 , -0.0884 + 0.0884i,

0 - 0.1250i, 0.0884 + 0.0884i , 0.1250 , 0.0884 - 0.0884i , 0 + 0.1250i ,

-0.0884 -0.0884i]

$IDFT\{A_t^{(4)}\}$ = [0.1250 , -0.1155 + 0.0478i, -0.0884 + 0.0884i, -0.0478 + 0.1155i, 0 - 0.1250i,

0.0478 + 0.1155i, 0.0884 + 0.0884i, 0.1155 + 0.0478i, -0.1250, 0.1155 - 0.0478i,

0.0884 - 0.0884i, 0.0478 - 0.1155i, 0 + 0.1250i, -0.0478 - 0.1155i, -0.0884 - 0.0884i,

-0.1155 - 0.0478i]

We choose the phase factor set as {1, -1, j, -j} and them write all possible phase sequences. We can generate up to 24 phase sequences as below:

{1 -1 –j j , 1 -1 j –j , 1 -j -1 j , 1 –j j -1, 1 j -1 –j, 1 j -j -1, -1 1 -j j, -1 1 j –j, -1 j -j 1 , -1 j 1 –j, -1 -j 1 j, -1 -j j 1, j -1 -j 1, j -1 1 –j, j -j -1 1, j -j 1 -1, j 1 -j -1, j 1 -1 –j, -j -1 1 j, -j -1 j 1, -j 1 j -1, -j 1 -1 j, -j j -1 1, -j j 1 -1}

Now multiplying phase sequences by *IDFT* matrix and computing the absolute value of the elements, we obtain :

$$
\begin{bmatrix}
1,-1,-j,j \\
1,-1,\ j,-j \\
1,-j,-1,\ j \\
1,-j,\ j,-1 \\
1,\ j,-1,-j \\
1,\ j,-j,-1 \\
-1,1,-j,\ j \\
-1,1\ ,\ j,-j \\
-1,\ j,-j,1 \\
-1,\ j,1,-j \\
-1,-j,1,\ j \\
-1,-j,\ j,1 \\
j,-1,-j,1 \\
j,-1,1,-j \\
j,-j,-1,1 \\
j,-j,1,-1 \\
j,1,-j,-1 \\
j,1,-1,-j \\
-j,-1,1,\ j \\
-j,-1,\ j,1 \\
-j,1,\ j,-1 \\
-j,1,-1,\ j \\
-j,\ j,-1,1 \\
-j,\ j,1,-1
\end{bmatrix}
\begin{bmatrix}
IDFT\{A_{\tau}^{(1)}\} \\
IDFT\{A_{\tau}^{(2)}\} \\
IDFT\{A_{\tau}^{(3)}\} \\
IDFT\{A_{\tau}^{(4)}\}
\end{bmatrix}
$$

$$
\begin{bmatrix}
0.2795 & 0.1574 & 0.3750 & 0.0293 & 0.2795 & 0.2626 & 0.3497 & 0.3560 & 0.1250 & 0.1574 & 0.3750 & 0.2207 & 0.1250 & 0.2626 & 0.1842 & 0.1060 \\
0.2795 & 0.1060 & 0.1842 & 0.2626 & 0.1250 & 0.2207 & 0.3750 & 0.1574 & 0.1250 & 0.3560 & 0.3497 & 0.2626 & 0.2795 & 0.0293 & 0.3750 & 0.1574 \\
0.2795 & 0.3314 & 0.2553 & 0.2042 & 0.1250 & 0.2042 & 0.2553 & 0.3314 & 0.2795 & 0.1678 & 0.3919 & 0.0887 & 0.1250 & 0.0887 & 0.3919 & 0.1678 \\
0.1250 & 0.2626 & 0.5794 & 0.1060 & 0.2795 & 0.1574 & 0.1250 & 0.0293 & 0.2795 & 0.2626 & 0.2656 & 0.3560 & 0.1250 & 0.1574 & 0.1250 & 0.2207 \\
0.2795 & 0.1678 & 0.3919 & 0.0887 & 0.1250 & 0.0887 & 0.3919 & 0.1678 & 0.2795 & 0.3314 & 0.2553 & 0.2042 & 0.1250 & 0.2042 & 0.2553 & 0.3314 \\
0.1250 & 0.2207 & 0.1250 & 0.1574 & 0.1250 & 0.3560 & 0.2656 & 0.2626 & 0.2795 & 0.0293 & 0.1250 & 0.1574 & 0.2795 & 0.1060 & 0.5794 & 0.2626 \\
0.2795 & 0.1060 & 0.1842 & 0.2626 & 0.1250 & 0.2207 & 0.3750 & 0.1574 & 0.1250 & 0.3560 & 0.3497 & 0.2626 & 0.2795 & 0.0293 & 0.3750 & 0.1574 \\
0.2795 & 0.1574 & 0.3750 & 0.0293 & 0.2795 & 0.2626 & 0.3497 & 0.3560 & 0.1250 & 0.1574 & 0.3750 & 0.2207 & 0.1250 & 0.2626 & 0.1842 & 0.1060 \\
0.1250 & 0.2626 & 0.5794 & 0.1060 & 0.2795 & 0.1574 & 0.1250 & 0.0293 & 0.2795 & 0.2626 & 0.2656 & 0.3560 & 0.1250 & 0.1574 & 0.1250 & 0.2207 \\
0.2795 & 0.3314 & 0.2553 & 0.2042 & 0.1250 & 0.2042 & 0.2553 & 0.3314 & 0.2795 & 0.1678 & 0.3919 & 0.0887 & 0.1250 & 0.0887 & 0.3919 & 0.1678 \\
0.2795 & 0.1678 & 0.3919 & 0.0887 & 0.1250 & 0.0887 & 0.3919 & 0.1678 & 0.2795 & 0.3314 & 0.2553 & 0.2042 & 0.1250 & 0.2042 & 0.2553 & 0.3314 \\
0.1250 & 0.2207 & 0.1250 & 0.1574 & 0.1250 & 0.3560 & 0.2656 & 0.2626 & 0.2795 & 0.0293 & 0.1250 & 0.1574 & 0.2795 & 0.1060 & 0.5794 & 0.2626 \\
0.2795 & 0.1678 & 0.3919 & 0.0887 & 0.1250 & 0.0887 & 0.3919 & 0.1678 & 0.2795 & 0.3314 & 0.2553 & 0.2042 & 0.1250 & 0.2042 & 0.2553 & 0.3314 \\
0.1250 & 0.2207 & 0.1250 & 0.1574 & 0.1250 & 0.3560 & 0.2656 & 0.2626 & 0.2795 & 0.0293 & 0.1250 & 0.1574 & 0.2795 & 0.1060 & 0.5794 & 0.2626 \\
0.2795 & 0.1060 & 0.1842 & 0.2626 & 0.1250 & 0.2207 & 0.3750 & 0.1574 & 0.1250 & 0.3560 & 0.3497 & 0.2626 & 0.2795 & 0.0293 & 0.3750 & 0.1574 \\
0.2795 & 0.1574 & 0.3750 & 0.0293 & 0.2795 & 0.2626 & 0.3497 & 0.3560 & 0.1250 & 0.1574 & 0.3750 & 0.2207 & 0.1250 & 0.2626 & 0.1842 & 0.1060 \\
0.2795 & 0.3314 & 0.2553 & 0.2042 & 0.1250 & 0.2042 & 0.2553 & 0.3314 & 0.2795 & 0.1678 & 0.3919 & 0.0887 & 0.1250 & 0.0887 & 0.3919 & 0.1678 \\
0.1250 & 0.2626 & 0.5794 & 0.1060 & 0.2795 & 0.1574 & 0.1250 & 0.0293 & 0.2795 & 0.2626 & 0.2656 & 0.3560 & 0.1250 & 0.1574 & 0.1250 & 0.2207 \\
0.1250 & 0.2626 & 0.5794 & 0.1060 & 0.2795 & 0.1574 & 0.1250 & 0.0293 & 0.2795 & 0.2626 & 0.2656 & 0.3560 & 0.1250 & 0.1574 & 0.1250 & 0.2207 \\
0.2795 & 0.3314 & 0.2553 & 0.2042 & 0.1250 & 0.2042 & 0.2553 & 0.3314 & 0.2795 & 0.1678 & 0.3919 & 0.0887 & 0.1250 & 0.0887 & 0.3919 & 0.1678 \\
0.2795 & 0.1678 & 0.3919 & 0.0887 & 0.1250 & 0.0887 & 0.3919 & 0.1678 & 0.2795 & 0.3314 & 0.2553 & 0.2042 & 0.1250 & 0.2042 & 0.2553 & 0.3314 \\
0.1250 & 0.2207 & 0.1250 & 0.1574 & 0.1250 & 0.3560 & 0.2656 & 0.2626 & 0.2795 & 0.0293 & 0.1250 & 0.1574 & 0.2795 & 0.1060 & 0.5794 & 0.2626 \\
0.2795 & 0.1574 & 0.3750 & 0.0293 & 0.2795 & 0.2626 & 0.3497 & 0.3560 & 0.1250 & 0.1574 & 0.3750 & 0.2207 & 0.1250 & 0.2626 & 0.1842 & 0.1060 \\
0.2795 & 0.1060 & 0.1842 & 0.2626 & 0.1250 & 0.2207 & 0.3750 & 0.1574 & 0.1250 & 0.3560 & 0.3497 & 0.2626 & 0.2795 & 0.0293 & 0.3750 & 0.1574
\end{bmatrix}
$$

Now we find the maximum values of above matrix

$$
\begin{bmatrix}
0.3750 \\
0.3750 \\
0.3919 \\
0.5794 \\
0.3919 \\
0.5794 \\
0.3750 \\
0.3750 \\
0.5794 \\
0.3919 \\
0.3919 \\
0.5794 \\
0.3919 \\
0.5794 \\
0.3750 \\
0.3750 \\
0.3919 \\
0.5794 \\
0.5794 \\
0.3919 \\
0.3919 \\
0.5794 \\
0.3750 \\
0.3750
\end{bmatrix}
$$

Then from those maximum values, we choose the minimum value, which is 0.3750 whose position indexes are $\{1, 2, 7, 8, 15, 16, 23, 24\}$. Then, we can choose any of the phase sequences corresponding to these locations. And these phase sequences are found as

Position	phase sequence
1	$1, -1, -j, \ j$
2	$1, -1, \ j, -j$
7	$-1, \ 1, -j, \ j$
8	$-1, \ 1, \ j, -j$
15	$j, \ j, -1, \ 1$
16	$j, -j, \ 1, -1$
23	$-j, \ j, -1, \ 1$
24	$-j, \ j, \ 1, -1$

The above table includes the suitable phase sequence that gives us minimum *PAPR* when we used for the rotation of partial sequences

Let's now calculate the *PAPR* when *PTS* method is employed and note the improvement in *PAPR* reduction.

First let's calculates *PAPR* without *PTS* as follows. Let the *BPSK* modulated signal be

$$A_\tau = [1\ 1\ -1\ 1\ \ -1\ -1\ 1\ 1\ 1\ 1\ -1\ 1\ 1\ -1\ -1\ -1]$$

The *IDFT* of the above sequence is calculated as

$|IDFT\ [1\ 1\ -1\ 1\ \ -1\ -1\ 1\ 1\ 1\ 1\ -1\ 1\ 1\ -1\ -1\ -1]\ |$ =

$$[0.1250 \quad 0.2042 \quad 0.2553 \quad 0.3314 \quad 0.2795 \quad 0.1678 \quad 0.3919 \quad 0.0887 \quad 0.1250 \quad 0.0887 \quad 0.3919 \quad 0.1678 \quad 0.2795 \quad 0.3314 \quad 0.2553 \quad 0.2042].$$

And the power of the *OFDM* signal is calculated as

The power of output signal = $(|IDFT\ [1\ 1\ -1\ 1\ \ -1\ -1\ 1\ 1\ 1\ 1\ -1\ 1\ 1 - 1\ -1\ -1]|)^2$ =

[0.0156 0.0417 0.0652 0.1098 0.0781 0.0281 0.1536 0.0079 0.0156 0.0079 0.1536 0.0281 0.0781 0.1098 0.0652 0.0417]

Peak power value is 0.1536 and the mean power value is 0.0622, then *PAPR* of the *OFDM* signal is

$$PAPR = \frac{peakpower}{meanpower} = \frac{0.1536}{0.0622} = 2.46$$

But in *PTS* power output signal =
[0.0781 0.0248 0.1406 0.0009 0.0781 0.0690 0.1223 0.1267 0.0156 0.0248 0.1406 0.0487 0.0156 0.0690 0.0339 0.0112]

whose peak and mean powers are found as, peak power = 0.1406, mean power = 0.0625 then *PAPR* is calculated as

$$PAPR= \frac{peakpower}{meanpower} = \frac{0.1406}{0.0625} = 2.25$$

3.4 Computational complexity

Generally more complex techniques have better *PAPR* reduction capabilities. Many researchers worked to reduce the computational of *PTS* and yet maintain substantial reduction in *PAPR*. Most of these techniques focuses on choosing phase sequences [20] that maximize the similarity of the input and output of the power amplifier using the cross correlation as an optimized metric to reduce the complexity. In [20] researchers proposed a two-step research procedure to fınd a subset of phase rotatıng vector wıth good *PAPR* performance. In the first step, sequences with low correlation such as quaternary sequences [19] or Kasami sequences [18] are used as initial phase rotating vector for *PTS* and then in the second step local search is performed based on the initial vector to introduce additional suitable phase vectors. The artificial bee colony scheme [17] and parallel tabu search scheme [16] are used to search a good subset of rotating vectors for *PTS* to get good *PAPR* performance with low complexity. The distortion which introduced by the power amplifiers while in non-linearity region is predicted and then used to select the suitable phase sequence for the *PTS* in [21].

CHAPTER 4

4. Fast Fourier Transform Algorithms

4.1 Decimation in Time Fast Fourier Transform

Discrete Fourier Transform (*DFT*) is a mapping from *N*-point Discrete-time (*DT*) signal x[n] to a number of complex discrete harmonics.

N-point *DFT* formula is given as

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-\frac{jk2\pi n}{N}} \qquad (4.1.1)$$

Let's denote the exponential function $e^{-j\frac{2\pi}{N}}$ in the above formula by e_N, i.e., $e_N = e^{-j\frac{2\pi}{N}}$ The function e_N has the following properties :

1) $e_N^2 = e_{N/2}$

This property comes from the definition directly, i.e.,

$$e_{N/2}^2 = e^{-j2\left(\frac{2\pi}{N}\right)}$$

which can be written

$$e_{N/2}^2 = e^{-\frac{j2\pi}{N/2}} = e_{N/2} \longrightarrow e_N^2 = e_{N/2}$$

2) $e_N^N = 1$, or in general $e_N^{mN} = 1, m \in Z$

Again starting by the definition

$$e_N = e^{-\frac{j2\pi}{N}} \longrightarrow e_N^{mN} = e^{-jm\left(\frac{2\pi N}{N}\right)} \longrightarrow e_N^{mN} = e^{-jm2\pi} \longrightarrow e_N^{mN} = 1$$

3) $e_N^{m+N} = e_N^m$

$$e_N^{m+N} = e_N^m \underbrace{e_N^N}_{=1} \longrightarrow e_N^{m+N} = e_N^m$$

This means that $f(m) = e_N^m$ is a periodic function , and its period equals to N, i.e., $f(m) = f(m + N)$.

Let's now derive the decimation in time *FFT* algorithm as outlined below. Let's write the *DFT* formula in terms of the defined function e_N as :

$$X[k] = \sum_{n=0}^{N-1} x[n] e_N^{kn}, \mathrm{k} = 0, 1 \cdots, \mathrm{N} - 1 \tag{4.1.2}$$

which can be partitioned for even and odd n values as :

$$X[k] = \sum_{n=0}^{N/2-1} x[2n] e_N^{2kn} + \sum_{n=0}^{N/2-1} x[2n+1] e_N^{k(2n+1)} \tag{4.1.3}$$

Where the first term on the right side using the property $e_N^2 = e_{N/2}$ can be written as

$$\sum_{n=0}^{N/2-1} x[2n]e_N^{2nk} \quad \rightarrow \quad \sum_{n=0}^{N/2-1} x[2n](e_N^2)^{nk} \quad \rightarrow \quad \sum_{n=0}^{N/2-1} x[2n]\left(e_{N/2}\right)^{nk}$$

and the similarly the second term on the right side using the property $e_N^2 = e_{N/2}$ can be written as

$$\sum_{n=0}^{N/2-1} x[2n+1]e_N^{(2n+1)k} \quad \longrightarrow \sum_{n=0}^{N/2-1} x[2n+1]e_N^{2nk}\ e_N^k \quad \longrightarrow$$

$$e_N^k \sum_{n=0}^{N/2-1} x[2n+1]e_N^{2nk} \longrightarrow e_N^k \sum_{n=0}^{N/2-1} x[2n+1]e_{N/2}^{nk} \tag{4.1.4}$$

Then the *DFT* formula can be written as

$$X[k] = \underbrace{\sum_{n=0}^{N/2-1} x[2n]e_{N/2}^{nk}}_{\text{G[k]}} + e_N^k \underbrace{\sum_{n=0}^{N/2-1} x[2n+1]e_{N/2}^{nk}}_{\text{H[k]}} \tag{4.1.5}$$

k = 0 ,1 ,. ….,N – 1.

where the terms $G[k]$ and $H[k]$ are periodic with period $N/2$. Since $G[k]$ and $H[k]$ are calculated for $k = 0, 1, \cdots, N-1$ in $X[k]$ then $G[k]$ and $H[k]$ have repeated values for for $k = 0, 1, \cdots, N-1$ as given below

$$G[k] = \left[\underbrace{g_0 \quad g_1 \quad g_2 \cdot \cdot \cdot \cdot \cdot \cdot}_{\text{the first N/2 samples}} \ \underbrace{g_0 \quad g_1 \quad g_2 \cdot \cdot \cdot \cdot \cdot \cdot}_{\text{the second N/2 samples}}\right] \tag{4.1.6}$$

$$H[k] = \Big[\underbrace{h_0 \quad h_1 \quad h_2 . \; . \; . \; . \; . \; .}_{\textit{the first } N/2 \textit{ samples}} \; \underbrace{h_0 \quad h_1 \quad h_2 . \; . \; . \; . \; . \; .}_{\textit{the second } N/2 \textit{ samples}}\Big] \qquad (4.1.7)$$

and $G[k]$ $k = 0, 1, \cdots, N/2 - 1$ is the $N/2$ point DFT of the even numbered samples of $x[n]$ and $H[k]$ $k = 0, 1, .. N/2 - 1$ is the $N/2$ point DFT of the odd samples of $x[n]$.

hence for the computation of G[k] and H[k] the k index range is first taken as k = 0 , 1,…,N/2- 1. and G[k] and H[k] are calculated for $k = 0, 1, .. N/2 - 1$. Let's denote the calculation results as :

$$G[k] = [g_0 \;\; g_1 \; \cdots \; g_{\frac{N}{2}-1}] \qquad H[k] = [h_0 \;\; h_1 \cdots h_{\frac{N}{2}-1}], \qquad k = 0, 1 \; .... N/2 - 1$$

then $G[k]$ and $H[k]$ values for $k = 0, 1, .. N - 1$ are obtained using

$$G[k] = [g_0 \;\; g_1 \cdots g_{\frac{N}{2}-1} \;\; g_0 \;\; g_1 \;\cdots g_{\frac{N}{2}-1}] \quad H[k] = [\, h_0 \; h_1 \; \cdots h_{\frac{N}{2}-1} \;\; h_0 \;\; h_1 \;\; \cdots h_{\frac{N}{2}-1}]$$

then they are combined in $X[k]$ using

$$X[k] = G[k] + W_N^k H[k], \quad k = 0, 1, .., N - 1 \qquad (4.1.8)$$

The partition performed for $X[k]$ can be done for $G[k]$ and $H[k]$ also . the calculation of $G[k]$ can be written as :

$$G[k] = G1[k] + W_N^k G2[k], \;\; k = 0, 1, .., N/2 - 1. \qquad (4.1.9)$$

where G_1[k] is the $N/4$ point *DFT* of the even numbered samples of $x[2n]$ and G_2[k] is the $N/4$ point DFT of the odd numbered samples of $x[2n]$.

and the calculation of H[k] can be written as :

$$H[k] = H_1[k] + W_N^k H_2[k] \quad , k = 0, 1, \dots, \frac{N}{2} - 1 \qquad (4.1.10)$$

where H_1[k] is the $N/4$ point *DFT* of the even numbered samples of $x[2n + 1]$ and H_2[k] is the $N/4$ point *DFT* of the odd numbered samples of $x[2n + 1]$.

this procedure can be carried out until we calculate 2-point *DFT* of the sequences obtained from $x[n]$.

4.2 Decimation in Frequency

In decimation in frequency algorithm, the even and odd indexed *DFT* coefficients are calculated separately. This operation is explained as follows.

The *DTF* coefficient are calculated using

$$X[k] = \sum_{n=0}^{N-1} x[n]e_N^{kn}\,, \qquad k = 0\,,1,\dots,N-1 \tag{4.2.1}$$

from which even indexed coefficients can be obtained via

$$X[2k] = \sum_{n=0}^{N-1} x[n]e_N^{2kn}, \qquad k = 0\,,1,\dots,N/2-1 \tag{4.2.2}$$

where the summation term can be divided into two parts as :

$$X[2k] = \sum_{n=0}^{\frac{N}{2}-1} x[n]e_N^{2kn} + \underbrace{\sum_{n=N/2}^{N-1} x[n]e_N^{2kn}}_{\sum_{n=0}^{N/2-1} x\left[n+\frac{N}{2}\right]e_N^{2k(n+\frac{N}{2})}}\,, k = 0\,,1,\dots,\frac{N}{2}-1 \tag{4.2.3}$$

By changing the frontiers of the second summation expression in the above equation we obtain the following expression for $X[2k]$.

$$X[2k] = \sum_{n=0}^{N/2-1} x[n]e_N^{2kn} + \sum_{n=0}^{N/2-1} x\left[n+\frac{N}{2}\right] e_N^{2k(n+\frac{N}{2})} , k$$

$$= 0 ,1, \dots , \frac{N}{2} - 1 \qquad (4.2.4)$$

where the exponential term $e_N^{2k\left(n+\frac{N}{2}\right)}$ can be simplified as :

$$e_N^{2k\left(n+\frac{N}{2}\right)} = e_N^{2kn} \underbrace{e_N^{kN}}_{=1} \longrightarrow e_N^{2k\left(n+\frac{N}{2}\right)} = e_N^{2kn}$$

and making use of the $e_N^2 = e_{N/2}$ the expression for $X[2k]$ can be written as :

$$X[2k] = \sum_{n=0}^{N/2-1} x[n]e_N^{2kn} + \sum_{n=0}^{N/2-1} x\left[n+\frac{N}{2}\right] e_{N/2}^{kn} , k$$

$$= 0 ,1, \dots , \frac{N}{2} - 1 \qquad (4.2.5)$$

which is further simplified as :

$$X[2k] = \sum_{n=0}^{\frac{N}{2}-1} (x[n]+x[n+(\frac{N}{2})]e_{N/2}^{kn} , k$$

$$= 0 ,1, \dots , \frac{N}{2} - 1 \qquad (4.2.6)$$

which can be written in more compact from as :

$$X[2k] = \sum_{n=0}^{N/2-1} x_1[n]e_{N/2}^{kn} , k$$

$$= 0 ,1, \dots , \frac{N}{2} - 1 \qquad (4.2.7)$$

where $x1[n] = \left(x[n] + x\left[n + \frac{N}{2}\right]\right) \quad n = 0,1,\ldots,N/2 - 1$.

the odd indexed coefficients of $X[k]$ can be obtained via

$$X[2k+1] = \sum_{n=0}^{N-1} x[n] e_N^{(2k+1)n}, k = 0,1,\ldots,\frac{N}{2} - 1 \qquad (4.2.8)$$

and proceeding as in the case of even indexed coefficients we obtain

$$X[2k+1] = \sum_{n=0}^{N/2-1} (x[n] - x\left[n + \frac{N}{2}\right]) e_N^{(2k+1)n}, k = 0,1,\ldots,\frac{N}{2} - 1 \qquad (4.2.9)$$

which can also be written as :

$$X[2k+1] = \sum_{n=0}^{\frac{N}{2}-1} (x[n] - x[n + \frac{N}{2}]) e_N^{n} \ e_{\frac{N}{2}}^{kn}, k = 0,1,\ldots,\frac{N}{2} - 1 \qquad (4.2.10)$$

which can be written in more compact from as :

$$X[2k+1] = \sum_{n=0}^{N/2-1} x_2[n] e_{\frac{N}{2}}^{kn}, k = 0,1,\ldots,N/2 - 1 \qquad (4.2.11)$$

where $x2[n] = (x[n] - x[n + N/2]) \, e_N^n \quad, n = 0,1,\ldots,N/2 - 1$.

To sum it up

$$X[2k] = \sum_{n=0}^{N/2-1} x_1[n]e_{N/2}^{kn}\,, k = 0\,,1, \dots , \frac{N}{2} - 1 \qquad (4.2.12)$$

$$X[2k+1] = \sum_{n=0}^{N/2-1} x_2[n]e_{N/2}^{kn}\,, k = 0\,,1, \dots , \frac{N}{2} - 1 \qquad (4.2.13)$$

where $x_1[n] = (x[n] + x[n + N/2])$ $\qquad$ $x_2[n] = (x[n] - x[n + N/2])\, e_N^n$

$$n = 0\,,1\,, \dots , N/2 - 1, \quad e_N^n = e_N^{-\frac{j2\pi n}{N}}$$

Note : if the signal x[n] is written as $x[n] = [A \quad B]\,, n = 0{,}1, \dots , N - 1$. Where A is first half and B is the second half of $x[n]$ then

$$x[n] + x\left[n + \frac{N}{2}\right] = [A + B],\; n = 0\,,1, \dots , N/2 - 1.$$

and

$$x[n] - x\left[n + \frac{N}{2}\right] = [A - B\,],\; n = 0\,,1\,, \dots , N/2 - 1.$$

and e_N^n for $n = 0\,,1\,, \dots , N/2 - 1$ equals to

$$e_N^n = \left[e^{-j0\left(\frac{2\pi}{N}\right)} \quad e^{-j1\left(\frac{2\pi}{N}\right)} \dots e^{-jN/2\left(\frac{2\pi}{N}\right)}\,\right]$$

4.3 PARTICAL EXAMPLE:

If $x[n] = [a \;\; b \;\; c \;\; d]$ find 8-point *DFT* coefficients of x[n] using decimation in frequency method .

At first we split $x[n]$ into first and second half

$$x[n] = [\underbrace{a \qquad b}_{\text{first half}} \quad \underbrace{c \qquad d}_{\text{second half}}]$$

By applying eq. (1) we obtain:

$$x_1[n] = x[n] + x[n+ N/2] = [a \quad b] + [c \quad d] = [a + c \qquad b + d]$$

$$x_2[n] = x[n] - x[n + N/2] = [a \quad b] - [c \quad d] = [a - c \quad b - d]$$

For $n = 0, 1$

$$\mathrm{e}_4^{\mathrm{n}} = [e^{-\frac{j2\pi\, 0}{4}} \qquad e^{-\frac{j2\pi\, 1}{4}} \quad] = [1 \qquad e^{-\frac{j\pi}{4}} \quad]$$

Then

$$x_1[n] = [a + b \qquad b + d]$$

$$x_2[n] = (x[n] \;-\; x[n + N/2])\, e_4^n = [a - c \quad b - d].* [1 \quad e^{-\frac{j\pi}{2}}\,]$$

$$= [a - c \qquad (b - d) * e^{-\frac{j\pi}{2}} \quad]$$

Using $e^{-\frac{j\pi}{2}}$ = -j we get the signals

$x_1[n] = [a + b \qquad b + d]$, and $x_2[n] = [a - c \quad -j(b - d) \;\;]$

The *DFT* coefficients of $x_1[n]$ and $x_2[n]$ can be found using the decimation in frequency algorithm. let's denote the *DFT* coefficients of $x_1[n]$ and $x_2[n]$ as $X_1[k]$ and $X_2[k]$ which can be found as

$$X_1[k] = [a + b + c + d \quad a + c - b - d]$$

$$X_2[k] = [a - c - jb + jd \quad a - c + jb - jd]$$

where $X_1[k]$ and $X_2[k]$, $k = 0,1$ corresponding to $X[2k + 1]$ and $X[2k]$ respectively , then we get

$$X[2k + 1] = \left[\underbrace{a + b + c + d}_{\mathrm{X[1]}} \quad \underbrace{a + c - b - d}_{\mathrm{X[3]}}\right]$$

$$X[2k] = \left[\underbrace{a - c - jb + jd}_{\mathrm{X[0]}} \quad \underbrace{a - c + jb - jd}_{\mathrm{X[2]}}\right]$$

Then the coefficients $X[k]$ are found as

$$[\, a - c - jb + jd \quad a + c + b + d \quad a - c + jb - jd \quad a + c - b - d]$$

4.4 General Parameters:

We consider five parameters for the evaluation of system performance, and these parameters are:

1- **Distortion**: Refers to the change between the transmitted and received data as shown in Fig 2.1/ Ch.1.

2- **Complexity**: Refers to the size of the operations to treat a data in both sides transmitting and receiving. Increasing the complexity leads to increased power consumption and increased delay time in addition size of the device.

3- **Delay time**: Refers to the amount of delay in transmitting and receiving sides, mainly it is associated with the complexity. Large delay time means the system unsuitable for real time applications.

4- **PAPR performance**: Refers to the Peak to Average Power Ratio as explained in Ch.1

5- **Side information**: Refers to the amount of additional information sent to the receiving side to recover the data. It may include phase shift and position information. Large amounts leads to reduction in data rate.

Considering the above factors, we note that the *PAPR* performance does not represent the system performance exactly. Therefore, we have to take the others parameters when we introduce a proposed method.

4.5 The proposed method

After modulation operation, the data symbols are grouped according to their index values, i.e., the data symbols with odd indexes are put into a set, and the data symbols with even indexes are put into another set. Next, subsets of the data symbols from even and odd data symbol sets are formed. Moreover, the data symbols in these subsets are rotated with the same phase amount, i.e., the data symbols in odd indexed data subsets are all rotated with a phase factor; similar operation is done for the data symbols in the even indexed data subset. Next, the rotated data symbols are placed into a vector according to their index order, and then sent to *IFFT* block. After *IFFF* calculation, *PAPR* value is measured, if low *PAPR* is observed, the phase values used for rotation operation are recorded.

The above procedure is repeated for a number of phase candidates and the one giving the minimum *PAPR* is chosen for rotation of the information symbols before transmission operation. The flow chart of the proposed scheme is depicted in Fig. 4.1. The candidate phases for the rotation of the information symbols are chosen as

$$0, \quad \frac{\pi}{2}, \quad -\frac{\pi}{2}, \quad \pi$$

In addition, we can choose some elements in odd and even subset for example if we have $\boldsymbol{N}$ elements then there are $\boldsymbol{N/2}\ \boldsymbol{odd}\ (\boldsymbol{x} :\ \boldsymbol{x}+\boldsymbol{2} :\ \boldsymbol{N})$ where $\boldsymbol{x}$ represents the first element in the odd sub-group and $\boldsymbol{N/2}\ \boldsymbol{even}\ (\boldsymbol{y} :\ \boldsymbol{y}+\boldsymbol{2} :\ \boldsymbol{N})$ where $\boldsymbol{y}$ represents the first element in the even sub-group to get a good performance and in this case, we have to send the position (side information) of these elements $(\boldsymbol{x}\,, \boldsymbol{y})$ to the receiving side for discovering data. In this way, the complexity will be minimum but a little increasing in the side information. In Fig. 4.1 flowchart of the proposed scheme is depicted.

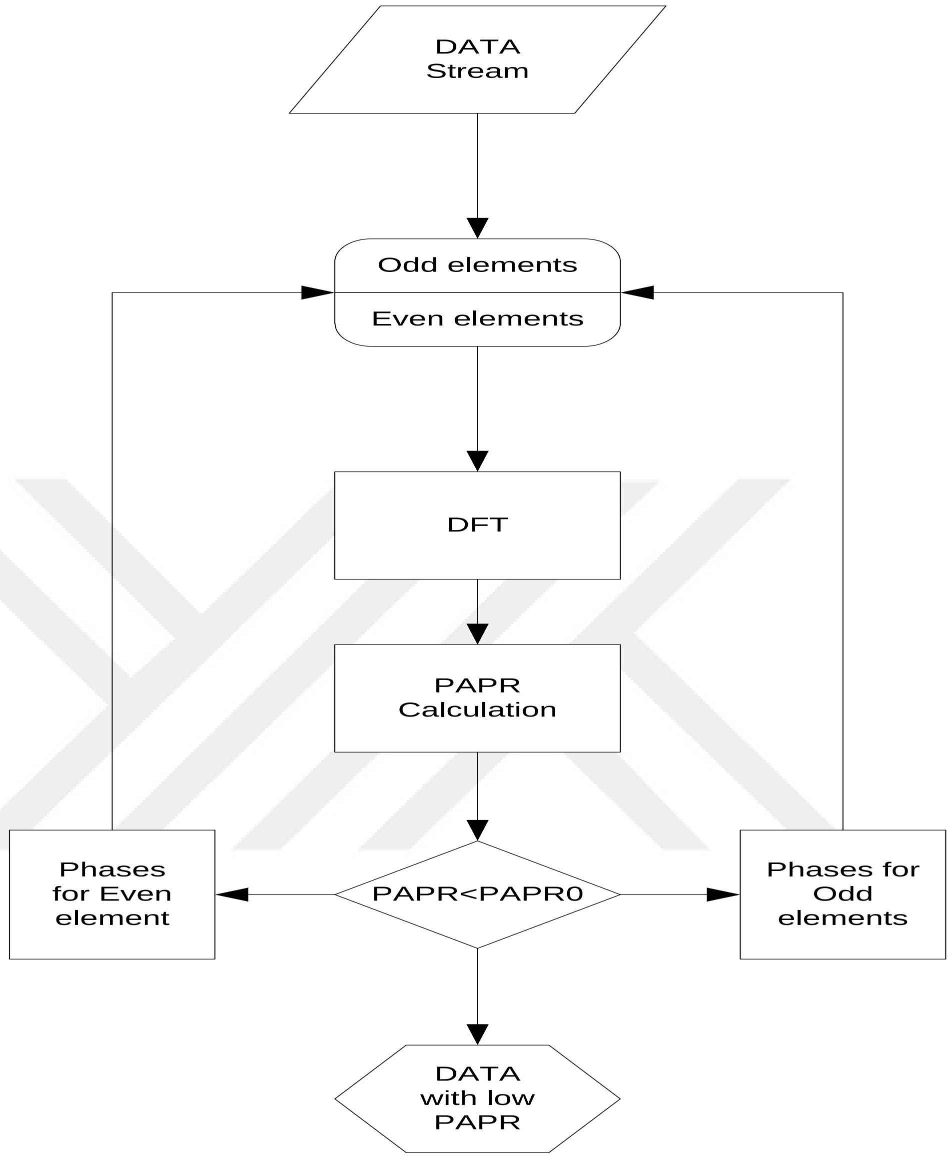


Fig. 4.1 The Flow Chart of the Proposed Scheme

CHAPTER 5

5. Simulation Results and Discussion

5.1 Simulation specifications

Partial Transmit Sequence (*PTS*) and the proposed technique are simulated and their *PAPR* improvement is compared to the original *PAPR* .The complementary cumulative distribution function (*CCDF*) of *PAPR* is used as performance measure. All the simulations are done in MATLAB. The standard IEEE802.11a wireless local area network (*WLAN*) frame structure, parameters of which are given below, is used for the simulations.

Parameter	**Value**
FFT size (n*FFT*)	64
Number of subcarriers (nDSC)	52
FFT sampling frequency	20 MHz
Subcarrier spacing	312.5 KHz
Subcarrier index	{-26 to -1 ,+1 to +26}
Data Subcarriers	48
Pilot Subcarriers	4(-21,-7, 7,21)
DC Subcarrier	Null (0 Subcarrier)
Coding Rate	1/2 , 2/3 ,3/4
Data symbol duration	3.2 µs
Cyclic prefix duration	0.8 µs
Total symbol duration	4 µs
Modulation	*BPSK* , *QPSK* ,16-*QAM* ,64-*QAM*

Frame Parameters of IEEE802.11a Standard

The standard 802.11a has 52 used subcarriers, 4 of them are used as pilots and the positions of these pilots are (-7,-21,7,21) and they are set to (1 -1 1 1) respectively. There are 48 data subcarriers used in this frame format. *MPSK* modulation is used before *IFFT* operation .

At first we simulated our proposed approach and compared it with the original signal *PAPR* as shown in Fig. 5.1 we note the *PAPR* performance is improved greatly with the proposed technique.

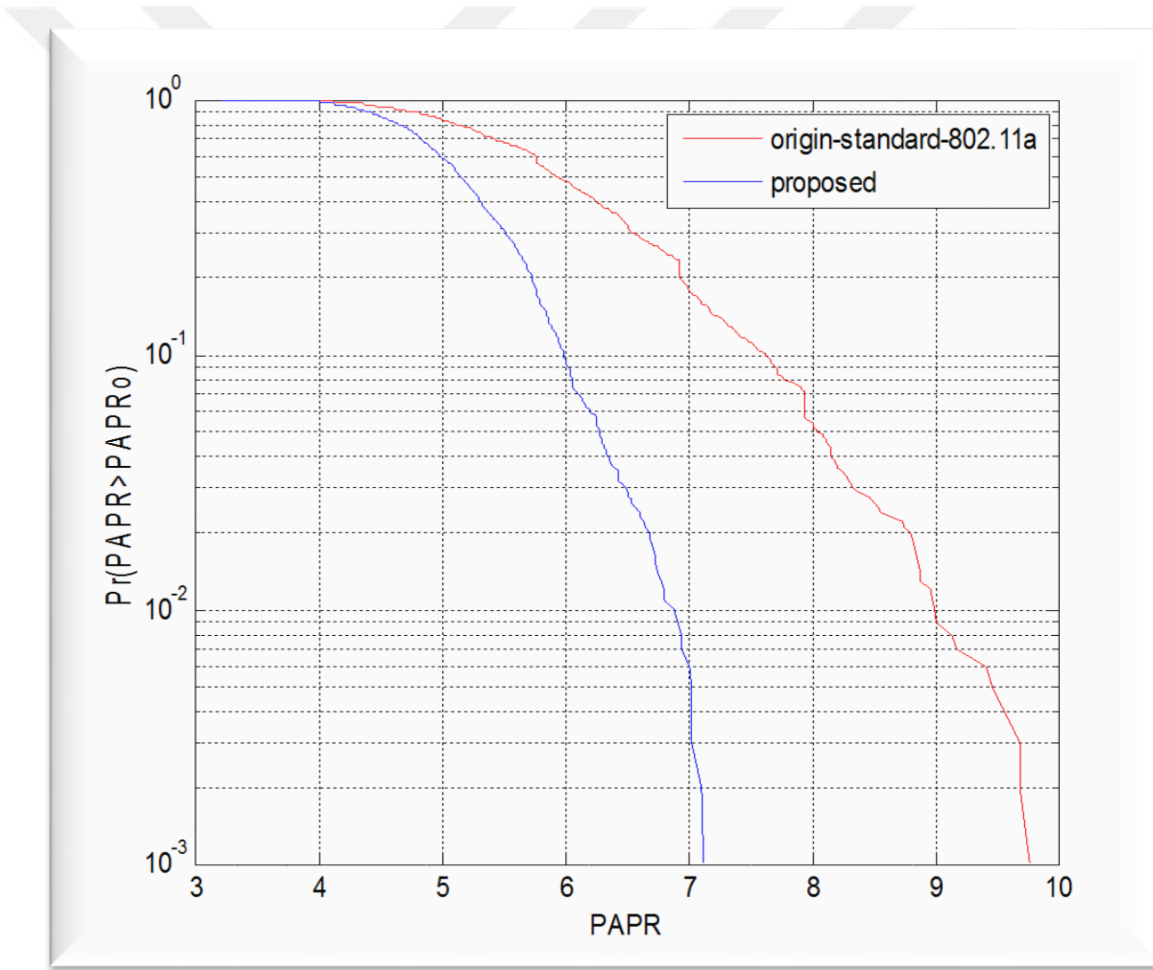


Fig. 5.1 Shows the proposed technique PAPR performance

Next, our method is compared to that of the PTS with $V = 4$ reduction approach. The results are depicted in Fig. 5.1 where it is clear that our proposed approach provides better *PAPR* reduction performance compared to PTS with $V = 4$.

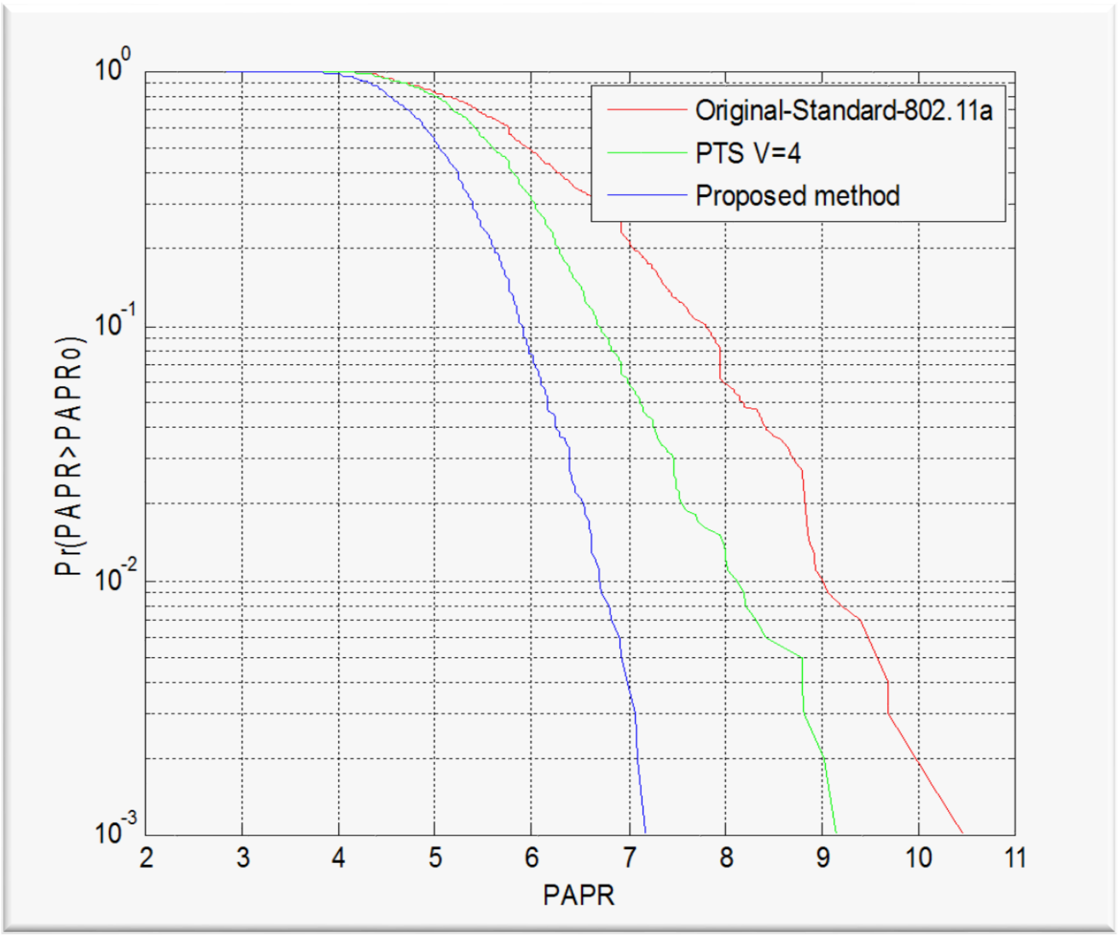


Fig. 5.2 PTS with (V= 4) and the proposed technique PAPR performance

For *PTS* technique with *N* sub-carriers and 4 sub-blocks we need 4**N IFFT* operation because this technique needs *N IFFT* for each sub-block. But in our proposed scheme just we need *N IFFT*. We get better *PAPR* performance than *PTS* technique, in addition the complexity of the proposed approach is rather low when compared to the *PTS* method achieving similar improvement in *PAPR*.

5.2 Conclusion

OFDM is a very attractive technique for wireless communications due to its channel robustness and spectrum efficiency. The peak to average power ratio (*PAPR*) is one of the serious drawbacks for *OFDM* technique. Many *PAPR* reduction techniques are available in the literature *IFFT* algorithm. In this thesis work, we introduced a new method for the reduction of PAPR of OFDM symbols. The proposed method is based on the rotation of information symbols in an optimum manner. For this purpose, we considered decimation in time FFT technique and inspected the combination of data symbols in decimation in time FFT algorithm and decided on the values of the phases to be used for the rotation of the information symbols so that PAPR is reduced when OFDM symbols are formed.

The proposed method is simple to implement, and has very low computational complexity. The proposed method is compared to that of the *PTS* technique well known in the literature. And via the computer simulations, it is seen that the proposed method is much better than that of the *PTS* technique proposed in the literature. And the complexity of our proposed method is much less than that of the *PTS* technique. In addition, the proposed method can easily be integrated with those of the other PAPR reduction methods to obtain joint PAPR reduction systems.

5.3 Future Work

An analytic technique can be inspected to determine the optimum phase values to be used for the rotation of information symbols, or better phase values can be searched via computer simulations for lower PAPRs. Joint PAPR systems can be designed considering the proposed method, and high performance, lower complexity systems can be designed.

REFERENCES

[1] **S. Kaiser**, "*OFDM code-division multiplexing in fading channels,*" *Communications, IEEE Transactions on,* vol. 50, pp. 1266-1273, 2002.

[2] **J. Armstrong,** "*Peak-to-average power reduction for OFDM by repeated clipping and frequency domain filtering*," *Electronics letters,* vol. 38, p. 1, 2002.

[3] **H. Ochiai and H. Imai,** "*On clipping for peak power reduction of OFDM signals,*" in *Global Telecommunications Conference, 2000. GLOBECOM'00. IEEE,* 2000, pp. 731-735.

[4] **D. Kim, D. Shi, Y. Park, and B. Song,** *"New peak-windowing for PAPR reduction of OFDM systems,*" in *Proc. Asia-Pacific Conference on Wearable Computing Systems (APWCS),* 2005, pp. 169-173.

[5] **X. Huang, J. Lu, J. Zheng, J. Chuang, and J. Gu,** "*Reduction of peak-to-average power ratio of OFDM signals with companding transform,*" *Electronics letters,* vol. 37, p. 1, 2001.

[6] **S. H. Müller, R. W. Bäuml, R. F. Fischer, and J. B. Huber**, "*OFDM with reduced peak-to-average power ratio by multiple signal representation,*" in *Annales des télécommunications,* 1997, pp. 58-67.

[7] **B. Robert, F. Robert, and B. Johannes,** "*Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping,*" *Electron. lett,* vol. 32, pp. 2056-2057, 1996.

[8] **M. V. Malode and D. B. Patil,** "*PAPR reduction using modified selective mapping technique,*" *Int. J. of Advanced Networking and Applications,* vol. 2, pp. 626-630, 2010.

[9] **A. Jayalath and C. Tellambura**, "*Reducing the peak-to-average power ratio of orthogonal frequency division multiplexing signal through bit or symbol interleaving*," *Electronics Letters*, vol. 36, pp. 1161-1163, 2000.

[10] **D.-W. Lim, H.-S. Noh, J.-S. No, and D.-J. Shin,** "*Near optimal PRT set selection algorithm for tone reservation in OFDM systems*," *Broadcasting, IEEE Transactions on*, vol. 54, pp. 454-460, 2008.

[11] **M. Shin, S. Kim, J. Kang, and C. Lee**, "*An efficient multimode quantized precoding technique for MIMO wireless systems*," *Vehicular Technology, IEEE Transactions on*, vol. 58, pp. 733-743, 2009.

[12] **C. Johri, P. Asthana, and S. Mishra,** " *Review Paper on PAPR Reduction Techniques in OFDM System*," 2014.

[13] **S. H. Müller and J. B. Huber**, "*A novel peak power reduction scheme for OFDM*," in *Personal, Indoor and Mobile Radio Communications, 1997. Waves of the Year 2000. PIMRC'97., The 8th IEEE International Symposium on*, 1997, pp. 1090-1094.

[14] **S. H. Müller and J. B. Huber**, "*OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences*," *Electronics letters*, vol. 33, pp. 368-369, 1997.

[15] **C. Tellambura,** "*Improved phase factor computation for the PAR reduction of an OFDM signal using PTS*," *Communications Letters, IEEE*, vol. 5, pp. 135-137, 2001.

[16] **N. Taşpınar, A. Kalınlı, and M. Yıldırım**, "*Partial transmit sequences for PAPR reduction using parallel tabu search algorithm in OFDM systems*," *Communications Letters, IEEE*, vol. 15, pp. 974-976, 2011.

[17] **Y. Wang, W. Chen, and C. Tellambura,** "*A PAPR reduction method based on artificial bee colony algorithm for OFDM signals*," *Wireless Communications, IEEE Transactions on*, vol. 9, pp. 2994-2999, 2010.

[18] **T. Kasami,** "*Weight distribution formula for some class of cyclic codes*," *Coordinated Science Laboratory Report no. R-285,* 1966.

[19] **J.-S. Chung, J.-S. No, and H. Chung**, "*A Construction of a New Family of-ary Sequences With Low Correlation From Sidel'nikov Sequences*," *Information Theory, IEEE Transactions on,* vol. 57, pp. 2301-2305, 2011.

[20] **E. Al-Dalakta, A. Al-Dweik, A. Hazmi, C. Tsimenidis, and B. Sharif**, "*PAPR reduction scheme using maximum cross correlation*," *Communications Letters, IEEE,* vol. 16, pp. 2032-2035, 2012.

[21] **E. Al-Dalakta, A. Al-Dweik, A. Hazmi, C. Tsimenidis, and B. Sharif**, "*Efficient BER reduction technique for nonlinear OFDM transmission using distortion prediction*," *Vehicular Technology, IEEE Transactions on,* vol. 61, pp. 2330-2336, 2012.

APPENDICES A

CURRICULUM VITAE

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