IMPLEMENTATION OF DETERMINISTIC INVENTORY MODELS WITH BACKORDERS AND LOST SALES IN A RETAIL COMPANY

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IMPLEMENTATION OF DETERMINISTIC INVENTORY MODELS WITH BACKORDERS AND LOST SALES IN A RETAIL COMPANY

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STATEMENT OF NON-PLAGIARISM PAGE

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ABSTRACT

IMPLEMENTATION OF DETERMINISTIC INVENTORY MODELS WITH BACKORDERS AND LOST SALES IN A RETAIL COMPANY

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In this thesis, a study is carried out on inventory management for a retail company which sells electronics and computer parts. Deterministic inventory models are used with a consideration of backorders and lost sales. In stockout, some sales are completely backordered, some are completely lost, and some are partially backordered while the rest is lost. Each of these cases are analyzed separately with the aim of determining minimum cost order quantities. Also, quantity discounts offered by suppliers for some products are examined.

Keywords: Economic Order Quantity (EOQ), Backorder, Lost Sale, Deterministic Inventory Models, Quantity Discounts.

ÖZ

PERAKENDE SATIŞ YAPAN BİR FİRMADA SONRADAN KARŞILAMA VE SATIŞ KAYIPLARI İLE DETERMİNİSTİK ENVANTER MODELLERİ ÜZERİNE BİR UYGULAMA

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Bu tez kapsamında, elektronik ve bilgisayar malzemelerin perakende satışını yapan bir firmanın envanter yönetimi üzerine bir çalışma yapılmıştır. Sonradan karşılama ve satış kayıplarını içeren deterministik envanter modelleri kullanılmıştır. Stoksuz kalma durumunda, bazı satışlar tamamen sonradan karşılanmakta, bazıları tamamen kayıp olmakta ve bazıları ise kısmen sonradan karşılanmakta ve kısmen kayıp olmaktadır. Bu durumların her biri en düşük maliyetli sipariş miktarlarını belirlemek amacıyla ayrı ayrı ele alınmıştır. Ayrıca, tedarikçilerin bazı ürünler için sunduğu miktar indirimleri de incelenmiştir.

Anahtar Kelimeler: Ekonomik Sipariş Miktarı (ESM), Sonradan Karşılama, Satış Kaybı, Deterministik Envanter Modelleri, Miktar İndirimleri.

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CHAPTER 1

INTRODUCTION

Almost every company from manufacturing, merchandising, or even service sector should maintain inventories to run its business. A manufacturing company usually has three types of inventories; raw materials, work in process, and finished goods inventories. For a merchandising company who is buying and selling products, inventories are simply the merchandise stocked in order to meet the demand. Since some amount of capital is invested in inventories, and there are some costs associated with inventories, inventory management is an important problem for the companies where the objective is usually to minimize the total cost.

In this thesis, we consider inventory problem of a retail company operating in merchandising sector, and use deterministic inventory models to compute economic order quantity (EOQ) under various situations. We make use of the existing deterministic inventory models, and especially the work of Montgomery et al. [15] in which they developed a model and solution methodology for deterministic inventory problems with backorders and lost sales. We present the application of such models on the Company. A summary of the Company and the problem is given below, whereas more details are explained later in Chapter 4.

The Company buys computer and electronics parts from suppliers and sells them to two types of customers: dealers and individual customers. Some products are demanded by both types of customers, whereas the others are purchased by one type of customer only (some products are purchased by dealers and the rest is demanded by individuals). In stockout situations, that is, when demands are not covered sufficiently, three different cases arise depending on behavior of customers. The shortages for sales through individual customers are thought to be a case of lost sales. However, the shortages resulting from sales to dealers are supposed to be backordered. Additionally, for the products sold to both customer types a mixture or backorders (for dealers) and lost sales for (individuals) are considered. The firm receives most of the orders from suppliers on the same day, whereas for some items there is a lead time which is usually constant. There are also some quantity discounts offered by suppliers for some products. Therefore, four cases are considered in this study. Three of them are related to stockout situations: (i) pure backorders, (ii) pure lost sales, (iii) mixture of backorders and lost sales. In addition, considering price discounts from suppliers, we define an additional case: (iv) quantity discounts.

When we have a look at the last five years' demands for the products of the Company, we observe close values for a group of products. Therefore, we focus on that kind of products where the demand is recognized to be deterministic. Hence, the deterministic models of inventory are taken into consideration.

Although studies about inventories take place frequently in literature, many of them usually consider backorders or lost sales only under deterministic or stochastic demand, whereas the studies handling both backorders and lost sales are limited. Therefore, due to the above explained different situations, we had the opportunity of implementing inventory models with a mixture of backorders and lost sales, as well as pure backorders and pure lost sales cases. Hence, this thesis study involves a combination of various EOQ models. Since the main decisions related to inventory management are the order quantity and order point, analysis of suitable inventory models and answering these questions will be the ultimate goal of the study under the objective of reducing inventory costs of the Company to the lowest level.

This thesis consists of five sections. In Chapter 2 an overview of inventory models is given. Literature review is presented in Chapter 3. Chapter 4 involves detailed definition of the problems, solution methodologies and numerical results. Finally, conclusion is given in Chapter 5 which covers the summary of the study and addresses possible future work.

CHAPTER 2

OVERVIEW OF INVENTORY MODELS

Inventory management emerged for the need of controlling system to hold the inventory at a certain level. Determination of a real policy for inventory management is a vital decision for the firms. Holding more items than it is required in inventory causes wasting free space in limited inventory place, a high cost of inventory and a financial loss. However, holding less items in inventory than the demands of customers could causes unsatisfied demand, increase of unsatisfied customers and unsatisfied demand fine.

Aims of inventory management are stated as follow (Silver, 2008) [20]:

(i) Cost minimization

(ii) Profit maximization

(iii) Maximization of rate of return on stock investment

(iv) Determination of a feasible solution

(v) Meeting the satisfactory level by using minimum workforce in the management and control of inventories.

Various kinds of modeling that were published have concentrated only the first aim that give before. A few possible constraints involve;

(i) Keeping the size of order at minimal level and bringing a limit to certain pack sizes.

(ii) Limitations at marketing, keeping the service levels of a tolerable customer at minimal level (iii) Limitations for storage space, maximum budget for purchases to be used during a special period, a maximum amount of work to be done (number of replenishments per period) and the number of staff involved.

The firms dealing with the business of selling goods are required to determine new ways to overcome their inventory level issues whether they resell them as retailers or produce them.

Components of Typical Inventory Problems

Holding Costs

The cost of holding in hand is used to determine on measure the financial rise from the products in inventories.

The variations of holding cost in inventory are related to duration holding the product in inventory. This main cost has five different components. (Silver, 2008) [20] These are;

- Cost of capital
- Cost of inventory and handling
- Cost of inventory at risk
- Cost of inventory services
- Cost of taxes

Demand

It is thought that demand expresses how many goods and services we bought at different prices during a certain period of time. A demand shows the consumers' requirement or will to buy the product or the experience he/she gains by service.

Ordering Costs

Ordering cost comes out while giving an order. Expenses such as selecting supplier, transferring information, getting necessary documents are considered under ordering cost. Ordering cost is not related with quantity of items in an order, it is related to number of giving order. This cost is fixed and it is constituted when an order is given.

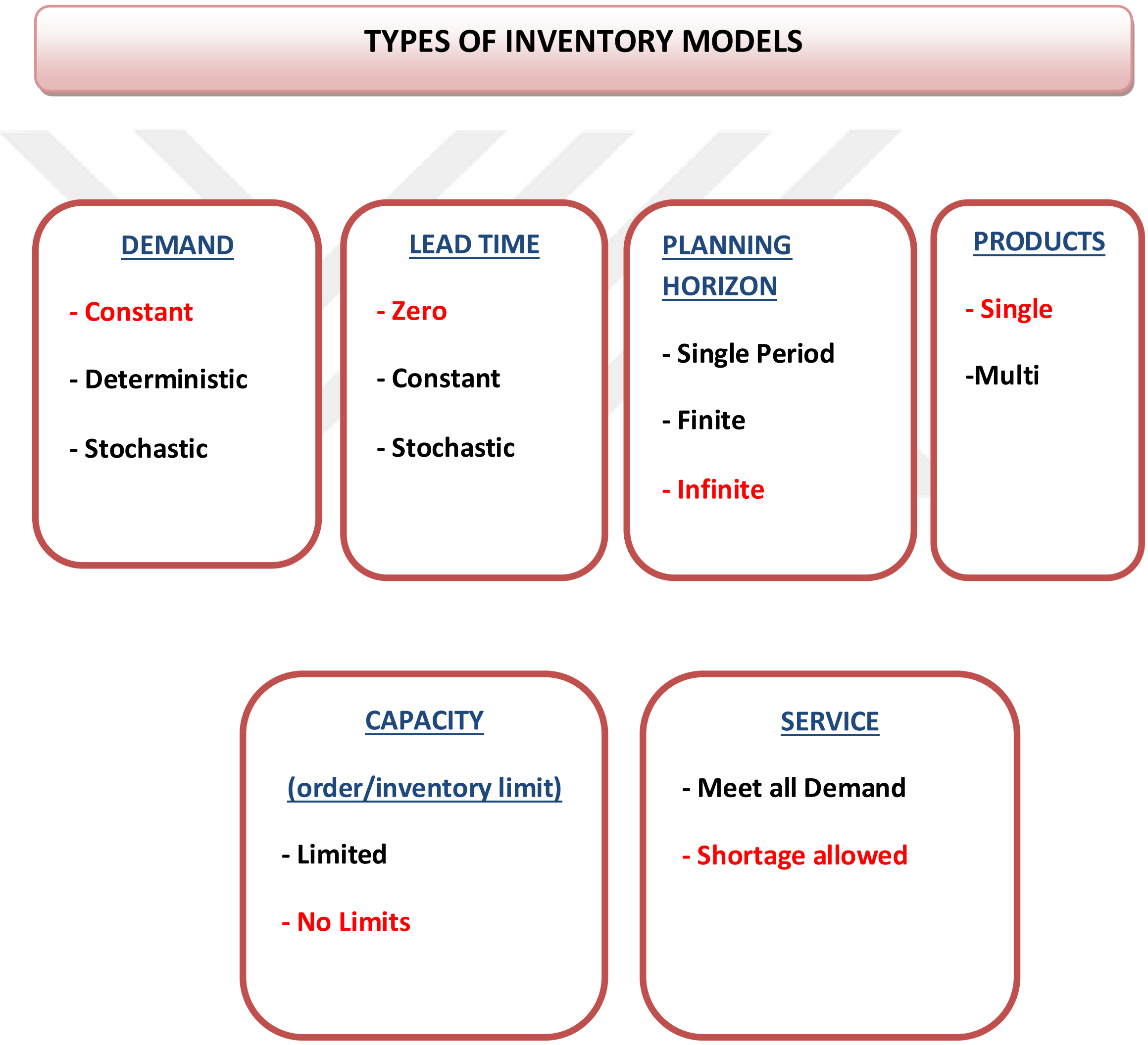

Figure 1 Types of inventory models

Inventory models are divided into many classifications. According to one classification, we choose red colored specifications in this study.

There are some problems as economic order quantity (EOQ) because of the shortage. We considered inventory problem of a retail company operating in merchandising sector and use deterministic inventory models to compute EOQ under various situations.

<u>Other issues:</u>

Demand Behavior

The demand behavior is one of the most important components of inventory policy that will be used.

As to the demands, when we examine the sales data we observe that a group of products were sold in almost the same or similar quantities over the last five years and therefore we assume demands for those are deterministic. On the other hand, demands for other products vary from year to year, so they can be considered as stochastic. In this study, we focus on the products having deterministic demand.

Lead Time

A supply chain management reveals the amount of time between a supplier that receives an order and the distributor or customer that delivers goods. In this study, we assumed that the lead time is "0".

CHAPTER 3

LITERATURE REVIEW

In this chapter this thesis is submitted as a summary of literature in relation with our field involving incorporated studies about inventory control system. The studies are defined in the literature in respect of is (i) the number of echelons (single vs. multi, echelon), (ii) number of items (single vs. multiple), (iii) type of inventory policies considered and (iv) noticing hour hostages and losses are presented lost sales and backordering. Other reviews will be done on inventory control literature beforehand.

We have taken more importance on deterministic inventory models since we are studying on a deterministic inventory during this thesis study we realized our literature search in four headlines. Firstly, inventory problems with pure backordering in discuss. Secondly, we have searched articles about inventory problems with pure lost sales in detail. And next, studies on inventory problems with backordering and lost sales play important role for the aim of our study. Therefore, a detailed literature search was conducted. We have examined various articles with an opinion which could be useful in the content of our study.

3.1 Inventory Problems with Pure Backordering

Some products which are sold to dealers are usually backordered. Therefore, we consider inventory models with backorders for such products. We did a literature search on inventory problems with pure backordering.

Ghalebsaz-Jeddi et al. (2004) [9] spend great efforts on a single echelon inventory system with multi items. They attempt to find out a system with backordering when the estimation of marginal backorder cost is available and the payment is received due upon order arrival. The problem is solved with Lagrange multiplier technique.

Kocağa and Şen (2004) [10] study $(S-1,\ S)$ spare parts service system with backordering. The system has rationing for their customers; if the down orders, the orders are supplied immediately for the equipment failures of the customers and when the lead time orders, the orders are supplied a future date for the scheduled maintenance activities. They develop an approximation model and a simulation model to analyze and optimize the critical levels.

3.2 Inventory Problems with Pure Lost Sales

Many of the products which are demanded by individual customers are usually lost when there is no available inventory at the time of a demand arrival request. For these kinds of products, we consider inventory models with lost sales. Because of this reason we did a literature search for inventory problems with pure lost sales.

Cohen et al. works (1988) [5] (R, Q) inventory system with two priority customer classes with lost sales for single echelon inventory system. They make use of a model developed by Markov Chain which is derived as approximate renewal based models. They examine the performance of two models.

Frank et al. (1999) [7] works to realize a periodic review system for deterministic and stochastic demands. The system is thought to provide a deterministic demand. On the contrary, it occurs lost sales for stochastic demand when there are not enough demands to respond. A dynamic programming model is formulated in order to characterize optimal policies.

Melchiors (1999) [13] studies on (R, Q) inventory model with unit Poisson demand, including several demand classes and lost sales. Critical levels check the demand classes themselves with a formulation $c \leq s+1$ for single item. Two policies such as simple and optimal are improved and finally they decide to accept the simple policy because it is much easier to practice than optimal policy after solving the policies.

Melchiors et al. (2000) [14] examines a continuous review (R, Q) model with lost sales and two demand classes which have priorities. Different critical levels of a such as $c < s$ and $c \geq s$ are considered and later an exact formulation for the average inventory cost with numerical examples are proposed by the use of Poisson demand and deterministic lead times.

Frank et al. (2003) [8] work a periodic review inventory system with two priority demand classes such as deterministic and stochastic. When deterministic demand is supplied, the stochastic demand is not satisfied during the period; that's why, it is defined as lost sales in their study. An optimal policy and a simple heuristic policy are indicated. They create a numerical result for these policies and prove that the simple heuristic policy operates absolutely well and is very easy to calculate.

Kranenburg and Van Houtum (2006) [11] study single item and continuous review model for multiple demand classes for the $(S - 1,\ S)$ lost sales inventory model. The system has been modeled with several classes of critical levels, and therefore different penalty cost parameters are defined when there are occurring lost sales. Three accurate and efficient heuristic algorithms are defined to optimize critical levels while minimizing inventory holding and penalty costs.

3.3 Studies on Inventory Problems with Backordering and Lost Sales

Some products are demanded by both dealers and individuals. Examining the attitude of different types of customers, we assume that demands from dealers are backordered and demands from individual customers are lost. Hence, we use the model that allows both backorders and lost sales. Because of this reason we conducted a literature review of the studies on inventory problems with backordering and lost sales.

Park (1983) [16] studies on another inventory is submitted for situations during the stock out period. In that case, a friction b of the demand is backordered and the remaining fraction $1 - b$ is lost. He gets a unimodal objective function which stands for the average cost of the inventory system by the way of describing exact time proportional backorder cost and a fixed penalty cost per unit lost. The calculations of the variables for the optimal operating policy are carried out directly.

The classical EOQ inventory is explored under the effect of time-limited free back-orders by Abboud and Sfairy (1997) [1]. Customers are claimed to have a wish to wait for a limited time until their orders are exposed at no extra charge during a stock out period. But, when the delay time takes too long, they will lose a certain number of customers. Moreover, the orders of the rest will fall in a case of backordered and a time-weighted penalty cost will be undertaken. It is not forgotten that their purpose is to find the optimal ordering quantity and back ordering level in order that the total average cost per unit time keeps at a minimal level.

The management of inventory in connection with stochastic demand is studied by Risa and DeCroix (1997) [17]. In their exploration, they found out that the products supply is randomly interrupted for periods of random duration and demands that reach when the inventory system is temporarily out of stock lead to a mix of backorders and lost sales. They struggled and succeed in the stock according to the following (s, S) policy. When the inventory level is at or below and it is estimated that the supply is appropriate, request an order to bring the inventory level up to S. It is believed that their analysis produces the optimal values of the policy parameters and does us enable to perceive the optimal inventory strategy if there are changes in seriousness of supply flows or interruptions or in the behavior of unfilled demand.

Chu et al. (2004) [4] studies include a large number of implementations related to a backordering and lost sales into their systems. They improved the criterion for the optimal solution for the total cost function. When they do not get a satisfactory conclusion from the criterion, it is claimed that this model will fall to a low condition into one cycle inventory model within a limited inventory period. This indicates an extension of shortage period as long as possible to produce lower cost. In contrast, it is understood that time is another, important factor in company competitiveness. Customers will not unclearly wait for backorders, they will make a tradeoff between the two most important factors: time and cost. The evaluation of the minimum total cost is carried out under the differences of cycle time and visual pictures are used to explain the calculation process. It is pointed out that this work makes up a reference for decision makers.

Enders et al. (2008) [6] make a vast observation of (S, c) inventory policy on multiclass customers, backorders for primarily distinguished customers and lost sales for the others. An efficient algorithm for each item is improved to make out the optimal critical level policy.

Chang and Lo (2009) [3] an approach is developed to fight against the drawback of classical methods for enhancing the continuous and separate lead time with mixture backorders and lost sales. By doing this, they can only obtain a local optimal solution. Moreover, decision makers are permitted to add appropriate limitations to their model in accordance with their exact business environments by the help of this proposed model. Finally, an actual life case some digital examples and detailed analysis are also included so that the precision and efficiency of the suggested model can be perceived.

San-Jose et al. (2009) [18] indicated a mathematical model with a common deterministic inventory systems with partial backlogging. Shortage costs backorder cost and lost sales cost are formed from both a fixed cost and a variable cost based on the length of the waiting time for the next supplies. They also define the optimal policy through a sequential optimization procedure.

Benjaafar et al. (2010) [2] studied in a situation of items produced as one unit at a time they consider the optimal control of a production inventory system with only product and two customer types. When a customer demands an order, it can be responded from the existing inventory whether it is short off, backordered or refused. They attempt to describe and show the differences of two classes by their backorder and lost sales costs. At each stage of decision, it must be taken a step to determine if an item is produced or not and if so, whether this item is used to increase the inventory or to decrease unfilled orders. Before each stage of decision is made, it must also be fixed whether a demand from an unusual class that makes unsatisfactory (should one arise) may be backordered or refused it. By considering these sorts of situations, they think to constitute a balance at inventory holding costs against the costs of backordering and lost sales. Therefore, they make a formulation of the problem as a process of Markov decision and benefit from it to describe the framework of the optimal policy. It is pointed out that the optimal policy can be defined by three-state dependent views a production base stock level and two order admission levels, one for each class. The state of the production base-stock level expresses to carry out the determination of the production when it puts in a specific position and how to share items that are produced. The state of the base- stock level also defines to perform the determination of the time when the orders being received from the (class 2) are backordered and not responded from inventory. The stage of the order-admission levels implies the determination of the time when orders should be refused. It is informed that these threshold of levels at different stages are dull and static either non-increasing or non-decreasing) in the backorder level of class 2 according to their parameters. It is compared the performance of the optimal policy against several heuristics by using digital results and also indicated that those don't allow for the possibility of both backordering and refusing orders which can perform, weakly.

3.4 Other Related Studies

The company may purchase some products at a discounted price if the order quantity is within some specific ranges which are offered by the supplier. Also, quantity discounts offered by suppliers for some products are examined. For this reason we did a literature search on inventory problems with quantity discounts.

Lu and Qui (1994) [12] generated the worst-case performance of a power of two policies in an all-unit quantity discounts model with one price break point. This model also helps to extend the sensitivity analysis for the classical EOQ model. It is indicated that the worst case performance will be based on the discounts rate a and 7.66%. of optimality when $a < 7.51\%$. and nearly within 100 a % of optimality when $a \geq 7.51\%$.

Shinn et al. (1996) in their report struggled to overcome the problem of the retailer's optimal price and lost size simultaneously occurring under the conditions of permissible delay in payment. They also estimated that the ordering cost a fixed set up cost and a freight cost in which evolves cost has a quantity discounts offered owing to the economics of scale. They accepted the constant price elasticity demand to have a connection with a decreasing function of retail price.

Silver (2008) [20] shows an overview of inventory management. He mentioned about inventory problems and associated models. So, substantial evidence is supplied of widespread Canadian applications of inventory management.

In this thesis, we made use of the existing deterministic inventory models, and especially the work of Montgomery et al. (1973) [15] in which they developed a model and solution methodology for deterministic inventory problems with backorders and lost sales. When we have a look at the last five years' demands for the products of the company, we observe close values for a group of products. Therefore, we focus on that kind of products where the demand is recognized to be deterministic. Hence, the deterministic models of inventory are taken into consideration.

In study we assumed some sales are completely backordered, some are completely lost, and some are partially backordered while the rest is lost. Each of these cases are analyzed separately with the aim of determining minimum cost order quantities. In addition to this, we obtained a different aspect in the literature adding to the quantity discounts in combination with the case of lost sales and backorders. Also, observing that some products with usually high selling prices have low demand values, we addressed the issue of determining integer valued order quantities for non-integer amounts.

Table 1 Taxonomy of inventory policies

Authors	Demand Distribution	Number of Items	Number of Echelons	Back order	Lost Sales	Quantity Discounts
Abboud and Sfairy (1997)	Deterministic Demand	Single		X	X	
Benjaafar et al. (2010)	Stochastic Demand	Single		X	X	
Chang and Lo (2009)	Deterministic Demand			X	X	
Chu et al. (2004)	Deterministic Demand	Single	Single	X	X	
Cohen et al. (1988)		Single	Single		X	
Enders et al. (2008)	Poisson	Multi	Single	X	X	
Frank et al. (1999)	Deterministic and Stochastic Demand	Multi	Single		X	
Frank et al. (2003)	Deterministic and Stochastic Demand		Single		X	
Ghalebsaz-Jeddi et al. (2004)	Stochastic Demand	Multi	Single	X		

Table 1 (continued)

Authors	Demand Distribution	Number of Items	Number of Echelons	Back order	Lost Sales	Quantity Discounts
Kocağa and Şen (2004)	Poisson	Single	Single		X	
Kranenburg and Van Houtum (2006)	Poisson	Single	Single		X	
Lu and Qiu (1994)	Deterministic Demand					X
Melchiors (1999)	Poisson	Single	Single		X	
Melchiors et al. (2000)	Poisson	Multi	Single		X	
Montgomery et al. (1973)	Deterministic and Stochastic Demand	Single	Single	X	X	
Park (1983)	Deterministic Demand			X	X	
Risa and DeCroix (1998)	Stochastic Demand			X	X	

CHAPTER 4

PROBLEM DEFINITION AND SOLUTION METHODOLOGY

4.1 Problem Definition and Basic Assumptions

In this study, we carry out an analysis using deterministic inventory models with backorders and lost sales. The study is based on inventory management of a retail company. The Company deals with selling electronic and computer equipment to customers. It purchases these items from some suppliers. Mainly the customers of the Company are classified into two categories as dealers and individual clients. Based on the sales data, we assume three different kinds of products depending on the customer types who purchase them. A group of products is generally demanded by dealers, whereas some other products are usually sold to individuals. Also there are some products which are sold to both dealers and individuals. For the last type of products demanded by both customer types, usually most of the sales are made to dealers. It is assumed that the percentage of sales to dealers for these common products changes between 80% and 95%.

The company experiences stockout situations from time to time resulting in backorders and/or lost sales. At this point it is assumed, in case of shortages, sales to dealers are backordered whereas sales to individuals are lost. This yields the following three situations. Sales to dealers are completely backordered, sales to individuals are completely lost, and finally sales to both dealers and individuals are partially backordered whereas the rest is lost. As to the demands, when we examine the sales data we observe that a group of products were sold in almost the same or similar quantities over the last five years and therefore we assume demands for those are deterministic. On the other hand, demands for other products vary from year to year, so they can be considered as stochastic. In this study, we focus on the products having deterministic demand and keep the items with stochastic demand out of scope. Table 2 gives the demand values for 20 items in the last 5 years.

Table 2 Demand values of last five years

Item	**2013**	**2014**	**2015**	**2016**	**2017**	**Average of last 5 years**
1	1250	1487	1360	1330	1540	1353
2	809	850	900	980	960	899
3	500	580	510	600	540	546
4	2100	2500	2568	2589	2235	2398
5	2800	3100	3000	3305	3800	3201
6	300	387	410	500	529	425
7	890	785	647	702	750	754
8	2690	2800	2900	2750	3000	2828
9	600	642	678	709	647	655
10	780	769	899	969	1050	893
11	1000	1250	1124	1179	1200	1150
12	2470	2247	2190	2345	2437	2337
13	3890	3685	3569	3157	3750	3610
14	379	350	368	309	367	354
15	798	850	1000	880	690	843
16	1285	1409	1168	1679	1478	1403
17	695	580	438	569	600	576
18	6075	5907	5500	5800	6000	5856
19	379	364	570	609	780	540
20	1048	1460	1205	1309	1032	1210

The aim of our study is to determine the optimal lot sizes for each order while minimizing the total cost in the existence of backorders and/or lost sales. Since we restrict our analysis to the products having deterministic demands, we focus on deterministic inventory models. In the remaining part of this chapter after giving basic assumptions and our notation, we introduce the basic economic order quantity (EOQ) model, which is followed by formulation and solutions for four cases including backorders and/or lost sales and quantity discounts.

The basic assumptions concerning our study are summarized below.

- The system is a single echelon system
- Demand is continuous and it is known and constant per year
- Demands for different items are independent
- Single item is considered in each case
- Year is considered as the planning period
- All costs and selling prices are known and constant
- Lead time is zero

Other additional assumptions are given for the specific cases that are analyzed later in this text.

4.2 Definition and Analysis of Cases

In this section, firstly we introduce the notation we use in the EOQ models. Then we present our analysis separately for each of the following cases.

(i) All shortages are backordered

(ii) All shortages are lost

(iii) Inventory model with both backorders and lost sales

(iv) Inventory model with quantity discounts

In each case, we first define the problem, then give the mathematical formulation of the related EOQ model. After that, we present the calculations and numerical results for some selected products. The first three cases are related to handling backorders and/or lost sales, whereas the fourth one is based on the idea of considering quantity discounts together with the third case, which allows a mixture of backorders and lost sales.

Notation (symbols and explanations):

The common notation used in modeling and analyzing the cases are explained below.

D: demand per year

S: total demand per cycle during the stock out period

Q: order quantity

TC: total cost per year

C: unit cost of each item (purchasing price)

h: holding cost (cost of carrying one unit of item in inventory per year)

I: interest rate

K: fixed ordering cost per inventory cycle

γ_s: shortage cost per unit period

γ_b: backorder cost per unit per year

γ_l: lost sales cost per unit per year (profit per unit)

b: fraction of unmet demand which is backordered during stockout period

Before explaining our analysis of the cases, we briefly mention the basic EOQ model. The first derivation of EOQ formula was made by Harris (2013), which is also known as classical square root EOQ model as given below. In his study, he presents the well known square root EOQ formula that minimizes the cost. The optimal order size is the same as the EOQ. The EOQ calculations are the most acceptable analysis and yields important results in any inventory management.

Later, many extensions of EOQ formula have been derived which are used to determine optimal order quantities under various situations. Now we begin with the basic model that assumes no shortages under a constant demand and zero lead time.

Figure 2 shows a typical evolution of such a process over time. Here T_1, T_2, … are the points in time where orders are placed and received immediately.

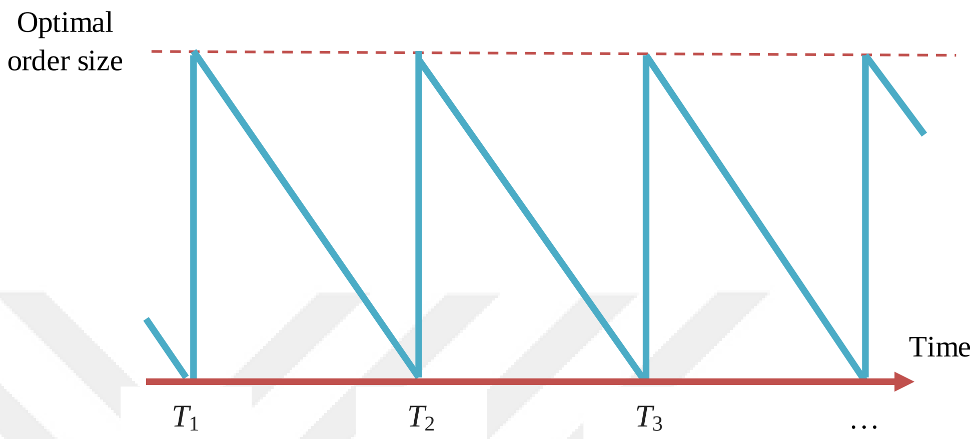

Figure 2 Stock levels with constant order size (Harris, 2013)

The total annual cost (TC) is composed of purchasing cost, ordering cost and average holding cost. Since there is no production, and no shortages are allowed, total purchasing cost is simply unit purchasing cost multiplied by order quantity, CD. Since all demand must be met, number of cycles (or orders) is obtained by dividing total demand by the order quantity, D/Q. Therefore, total ordering cost is ordering cost per cycle multiplied by number cycles, which is $\frac{KD}{Q}$. Finally, average inventory during each cycle is $Q/2$ and therefore total holding cost per cycle is obtained by multiplying average inventory and the holding cost per unit as $hQ/2$. Putting these components together, total cost is expressed as follows.

$$TC = CD + \frac{KD}{Q} + \frac{hQ}{2} \tag{4.1}$$

To find the optimal order quantity (Q^*) we set $\frac{d(TC)}{dQ} = 0$ and solve for Q, and obtain the following result.

$$Q^* = \sqrt{\frac{2KD}{h}} \quad \text{(Optimal order quantity)} \tag{4.2}$$

In our study, we make use of the existing deterministic inventory models, and especially the work of Montgomery et al. [15] in which they developed a model and solution methodology for inventory problems with backorders and lost sales under deterministic and stochastic demand. For the deterministic model, they develop a formulation and a solution methodology that guarantees the optimal solution. They describe the inventory geometry for this system as in Figure 3. Their approach is summarized below.

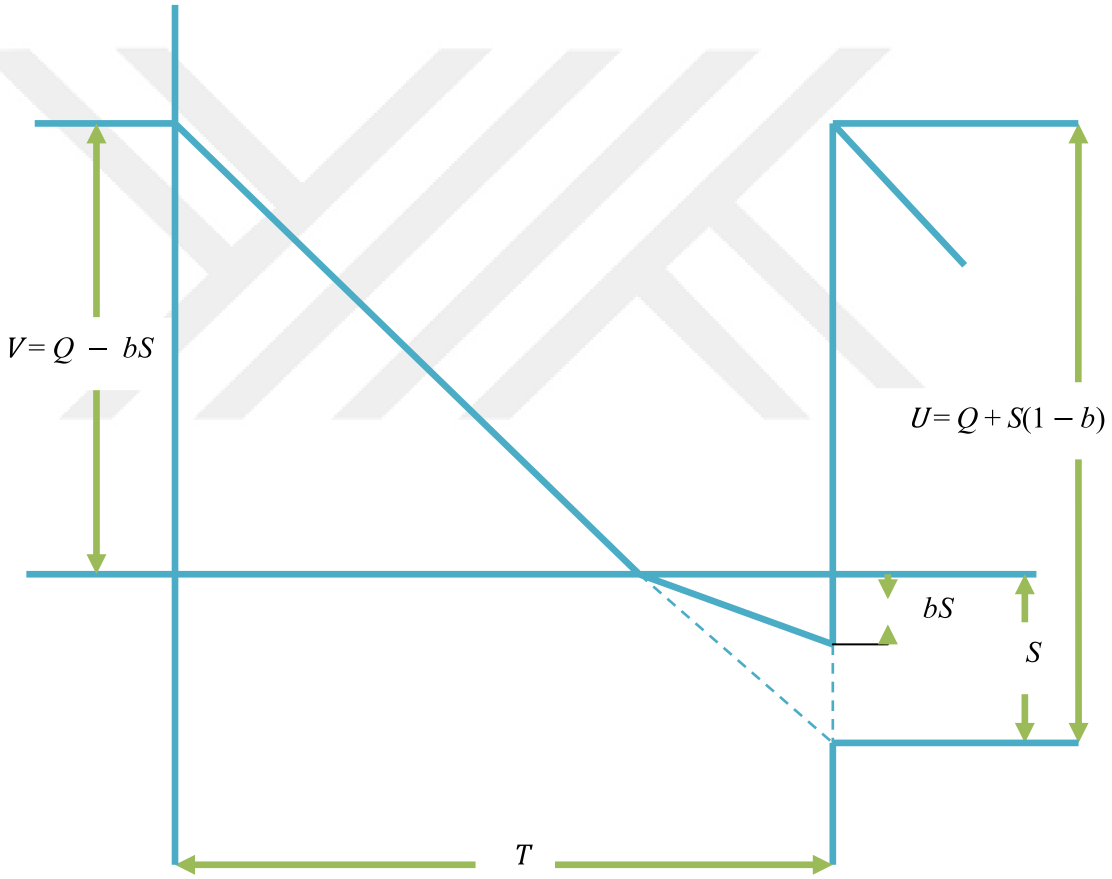

Figure 3 An inventory cycle with backorders and lost sales together (Montgomery et al., 1973) [15]

The notation used in Figure 3 is explained below.

U = total demand during cycle

V = on-hand inventory at the beginning of cycle

Q = order quantity

S = total demand per cycle during stockout

Based on the geometry given in Figure 3 and the assumption that mixture of backorders and lost sales is constant, they obtained a total cost function that gives average annual cost. Using our notation, the cost function is as follows.

$$TC(Q,S) = \frac{KD}{Q+S(1-b)} + \frac{IC(Q-bS)^2}{2[Q+S(1-b)]} + \frac{\gamma_s SD}{Q+S(1-b)} + \frac{\gamma_b\, bS^2}{2[Q+S(1-b)]} + \frac{\gamma_l SD(1-b)}{Q+S(1-b)} \tag{4.3}$$

Right side of Equation (4.3) includes five cost components which are ordering, carrying, stockout penalty, backorder, and lost sale costs respectively. Ordering cost is obtained by multiplying by the fixed cost per order (K) by the number of cycles (orders) in a year. Carrying or holding cost is based on the cost of capital tied in inventory and therefore computed by multiplying unit purchasing cost (C), interest rate (I) and average inventory hold during year. Costs related to shortages can be explained as follows. Stockout penalty cost is obtained from the multiplication of stockout penalty per unit short (γ_s), the number of units which are short during a cycle (S), and the number of cycles in a year. Cost of backordering is unit backordering cost (γ_b) multiplied by number of units backordered and number of cycles in a year. Finally, cost of lost sales is obtained from the product of unit lost sales cost (γ_l), number of units lost during cycle and number of cycles in a year, and it represents the profit lost due to lost sales.

Pointing out that the cost function is not convex and necessary conditions ($\frac{\partial TC}{\partial Q} = 0$ and $\frac{\partial TC}{\partial S} = 0$) yield two simultaneous nonlinear equations which are hard to solve and may not yield global minimum of TC, they developed a solution methodology that guarantees the global minimum. They started by making the following transformation (Equations (4.4) and (4.5)), and obtained the total cost function in terms of U and V as given in Equation (4.6).

$$U = Q + S(1-b) \tag{4.4}$$

$$V = Q - bS \tag{4.5}$$

$$\widehat{TC}(U,V) = \frac{1}{U}[a_1 + a_2(U-V) + a_3(U-V)^2 + a_4V^2] \tag{4.6}$$

where,

$a_1 = KD$

$a_2 = \gamma_s D + \gamma_l D(1-b)$

$a_3 = \gamma_b b/2$

$a_4 = IC/2$

Firstly they found the optimal U and V values (U^* and V^*) that minimizes $\widehat{TC}$ indicating that, since the transformation is nonsingular, optimal Q and S values (Q^* and S^*) minimizing TC could be obtained from the following inverse transformation (Equations (4.7) and (4.8)).

$$Q^* = bU^* + (1-b)\, V^* \tag{4.7}$$

$$S^* = U^* - V^* \tag{4.8}$$

Later in their analysis they defined and used the following additional parameters.

$$a_5 = 4a_1 a_3 \,/\, a_2^2$$

$$a_6 = 4a_1 a_4 \,/ a_2^2 = \frac{2ICK}{D[\gamma_s + \gamma_l\,(1-b)]^2}$$

Finally, they described the optimal solution based on two conditions depending on the value of a_6 as explained below.

Condition 1:

If $a_6 = \dfrac{2ICK}{D[\gamma_s + \gamma_l(1-b)]^2} \leq 1$,

no shortages are allowed in the optimal solution and total demand during cycle is equal to on-hand inventory at the beginning of cycle ($U^* = V^*$).

Then, from Equations (4.7) and (4.8), optimal order quantity becomes equal to total demand during cycle (Q^*=U^*) whereas demand during stockout period is zero (S^*=0) since no shortages are allowed. Therefore, optimal order quantity is computed using basic EOQ formula.

$$Q^* = U^* = \sqrt{\frac{2KD}{IC}} \tag{4.9}$$

Hence, the total cost function becomes

$$TC(Q,S) = \frac{KD}{Q} + \frac{ICQ}{2} \tag{4.10}$$

Condition 2:

If $a_6 = \dfrac{2ICK}{D[\gamma_s+\gamma_l(1-b)]^2} > 1$,

then $U^*$and V^* are obtained from the following equations.

$$\beta^* = \frac{a_5}{a_5 + a_6} + \frac{1}{a_5 + a_6}\sqrt{\frac{a_5 a_6}{a_5 + a_6 - 1}} \tag{4.11}$$

$$U^* = \sqrt{\frac{2KD}{\gamma_b b(1-\beta)^2 + IC\beta^2}} \tag{4.12}$$

$$V^* = \beta^* U^* \tag{4.13}$$

Finally, optimal order quantity and total demand during stockout period is computed from inverse transformation given by Equations (4.7) and (4.8).

$$Q^* = bU^* + (1-b)V^*$$

$$S^* = U^* - V^*$$

Interested readers may see the details of their work in [15]. In the following cases, using this solution procedure, we explain the determination of optimal order quantities for the selected items of the company.

4.2.1 Case I: All Shortages are Backordered

"A backorder is an occurrence of a case that an item is out of stock when a customer demands and then waits to receive the item from the next delivery from suppliers" [15].

As explained in Chapter 1, in case of stockout, the products sold to dealers are usually backordered. Therefore, for such products we make the following additional assumptions.

- All shortages are backordered (no lost sales)

- Lead time is zero

Figure 4 expresses the stock level during one cycle if shortages are backordered.

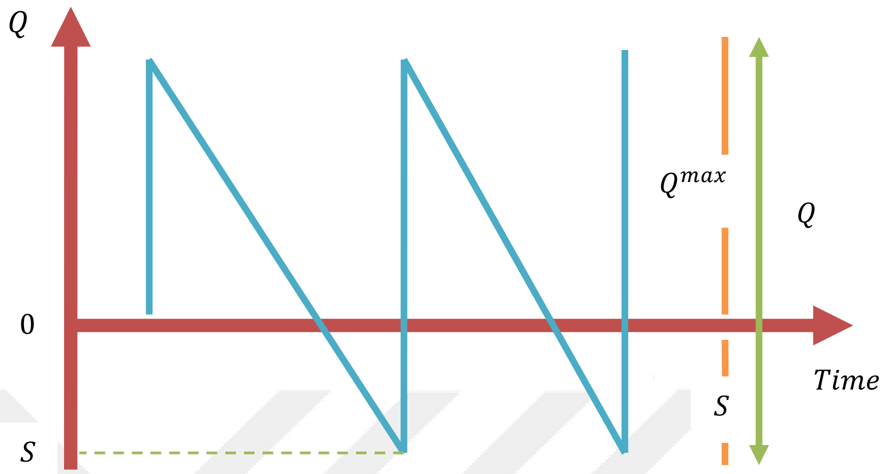

Figure 4 EOQ with pure backorders (Silver, 2008) [20]

Setting b=1 (the fraction of shortages backordered), the problem becomes pure backordered problem and the cost function (given by Equation (4.3)) can be written as in Equation (4.14) below.

$$TC(Q,S) = \frac{KD}{Q} + \frac{IC(Q-S)^2}{2Q} + \frac{\gamma_s SD}{Q} + \frac{\gamma_b S^2}{2Q} \tag{4.14}$$

Now, the total cost consists of four components, which are ordering, carrying, stockout penalty, and backorder costs.

The selected ten items, which are sold to dealers and subject to backordering, are named as 1A, 1B, …, 1J. These items are analyzed using the above procedure. Input data and solution results are summarized in Table 3.

Data consists of demand (D), purchasing cost (C), ordering cost (K), interest rate (I), stockout penalty (γ_s), backorder cost per unit short (γ_b) and lost sale cost per unit short (γ_l). For each product, we assume that the Company makes a profit of 20% of purchasing cost and we use it as lost sale cost per unit. That is, γ_l = 0.20C.

Table 3 Input data and results for case I (pure backorders)

Item	Data							Results					
	D	C	K	I	γ_s	γ_b	γ_l	a_6	Q	S	TC	Number of Cycles
1A	5000	3.93	50	0.1	0.08	0.2	0.79	1.23	1317.82	198.82	439.76	3.79
1B	3800	1.43	50	0.1	0.08	0.2	0.29	0.59	1630.14	0.00	233.11	2.33
1C	3580	1.26	50	0.1	0.08	0.2	0.25	0.55	1685.61	0.00	212.39	2.12
1D	3200	2.80	50	0.1	0.08	0.2	0.56	1.37	1254.02	198.18	295.64	2.55
1E	3180	1.29	50	0.1	0.08	0.2	0.26	0.63	1570.07	0.00	202.54	2.03
1F	3160	1.26	50	0.1	0.08	0.2	0.25	0.62	1583.65	0.00	199.54	2.00
1G	3155	1.62	50	0.1	0.08	0.2	0.32	0.80	1395.54	0.00	226.08	2.26
1H	3000	1.47	50	0.1	0.08	0.2	0.29	0.77	1428.57	0.00	210.00	2.10
1I	2800	1.87	50	0.1	0.08	0.2	0.37	1.04	1247.29	23.88	228.78	2.24
1J	2700	1.00	50	0.1	0.08	0.2	0.20	0.58	1643.17	0.00	164.32	1.64

Detailed calculations for two selected items, 1A and 1B, are shown below.

<u>Item 1A</u>

Annual demand for this item is 5000 units, purchasing cost is 3.93 ₺ per unit, ordering cost is 50 ₺ per order, whereas stockout penalty and backorder cost per unit per unit are 0.08 ₺ and 0.20 ₺ respectively. Interest rate is assumed as 10% per year in all calculations. First, value of a_6 is computed as follows.

$$a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l\,(1\text{-}b)]^2} = \frac{2(0.1)(3.93)(50)}{5000[0.08+0]^2} = 1.23$$

Since $a_6 > 1$, the procedure defined in condition 2 is applied. Necessary calculations are shown below.

$a_1 = KD = (50)(5000) = 250000$

$a_2 = \gamma_s D + \gamma_l D(1-b) = (0.08)(5000) + 0 = 400$

$a_3 = \gamma_b b/2 = (0.2)(1)/2 = 0.1$

then,

$a_5 = 4a_1 a_3 / a_2^2 = 4(250000)(0.1) / 4002 = 0.625$

$$\beta^* = \frac{a_5}{a_5 + a_6} + \frac{1}{a_5 + a_6}\sqrt{\frac{a_5 a_6}{a_5 + a_6 - 1}}$$

$$= \frac{0.625}{0.625 + 1.228} + \frac{1}{0.625 + 1.228}\sqrt{\frac{(0.625)(1.228)}{0.625 + 1.228\text{-}1}} = 0.849$$

$$U^* = \sqrt{\frac{2KD}{\gamma_b b(1-\beta)^2 + IC\beta^2}} = \sqrt{\frac{2(50)(5000)}{(0.2)(1)(1-0.849)^2 + 0.1(3.93)(0.849)^2}} = 1317.99$$

$$V^* = \beta^* U^* = (0.849)(1317.82) = 1118.98$$

Finally, optimal order quantity and backordered units (demand during stockout) are obtained using Equations (4.7) and (4.8).

$$Q^* = bU^* + (1 - b)\, V^* = (1)(1317.99) + 0 = 1317.99$$

$$S^* = U^* - V^* = 1317.99 - 1118.98 = 199.01$$

When these values are substituted into Equation (4.14), minimum total cost is found as 439.76₺. Also, number of cycles is obtained as D/Q^* = 5000 / 1317.99 = 3.79.

Item 1B

This item has an annual demand of 3800 units and a purchasing cost of 1.43 ₺ per unit. Other cost data is the same as in Item 1A. First, value of a_6 is computed as

$$a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l\,(1-b)]^2} = \frac{2(0.1)(1.43)(50)}{3800[0.08 + 0]^2} = 0.59$$

Since $a_6 < 1$, condition 1 is realized. Hence, no shortages are allowed and the optimal order quantity is obtained as follows.

$$Q^* = \sqrt{\frac{2KD}{IC}} = \sqrt{\frac{2(50)(3800)}{(0.1)(1.43)}} = 1630.14$$

In addition, using Equation (4.10) minimum cost is computed as 233.11 ₺. Number of cycles in a year is D/Q^* = 3800 / 1630.14 = 2.33.

According to the solution, the Company experiences backorders from three items (1A, 1D, 1I) which have relatively high unit purchasing costs. For the other items, all demand is met on time and hence no shortages occur.

4.2.2 Case II: All Shortages are Lost

If shortages occur, then some customers may not wait for backorders to arrive. Therefore, they will probably prefer another supplier and this leads to lost sales. Stock model under lost sales is shown in Figure 5.

Many of the Company's products demanded by individual customers are usually lost when there is no available inventory at the time of a demand arrival. For these kinds of products, we consider inventory models with pure lost sales under the following additional assumptions.

- All shortages are lost (no backorders)
- Lead time is zero

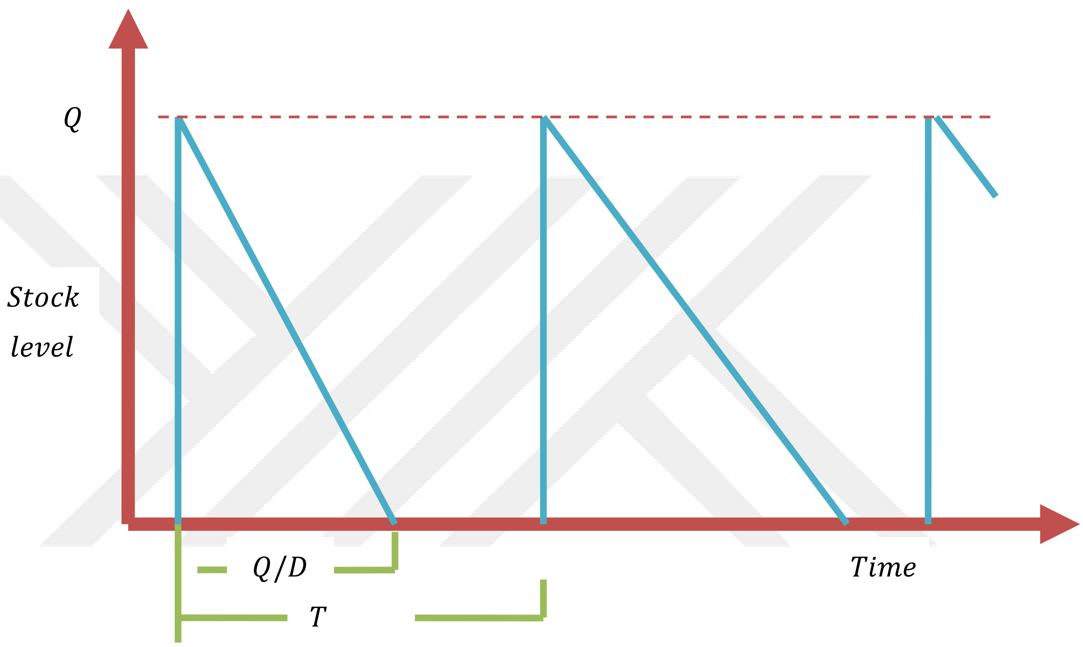

Figure 5 Stock level with lost sales (Silver, 2008) [20]

Setting the fraction of shortages backordered equal to zero (b = 0) yields the problem with lost sales. The cost function is given in Equation (4.15) below.

$$TC(Q,S) = \frac{KD}{Q+S} + \frac{IC(Q)^2}{2[Q+S]} + \frac{\gamma_s SD}{Q+S} + \frac{\gamma_l SD}{Q+S} \tag{4.15}$$

Among the products which are sold to individuals, and therefore assumed to be lost when there is no on-hand inventory at the time of demand, we select ten items and name as 2A, 2B, …, 2J. Input data and solution results of our analysis are given in Table 4.

Table 4 Input data and results for case II (pure lost sales)

Item	Data							Results				
	D	C	K	I	γ_s	γ_b	γ_l	a_6	Q	S	TC	Number of Cycles
2A	1000	2.53	50	0.1	0.1	0.2	0.51	0.07	628.69	0.00	159.06	1.59
2B	950	3.42	50	0.1	0.1	0.2	0.68	0.06	527.05	0.00	180.25	1.80
2C	700	3.16	50	0.1	0.1	0.2	0.63	0.08	470.66	0.00	148.73	1.49
2D	600	2.07	50	0.1	0.1	0.2	0.41	0.13	538.38	0.00	111.45	1.11
2E	890	2.10	50	0.1	0.1	0.2	0.42	0.09	651.01	0.00	136.71	1.37
2F	750	3.34	50	0.1	0.1	0.2	0.67	0.08	473.87	0.00	158.27	1.58
2G	580	2.40	50	0.1	0.1	0.2	0.48	0.12	491.60	0.00	117.98	1.18
2H	900	1.42	50	0.1	0.1	0.2	0.28	0.11	796.12	0.00	113.05	1.13
2I	1000	1.51	50	0.1	0.1	0.2	0.30	0.09	813.79	0.00	122.88	1.23
2J	2700	1.00	50	0.1	0.08	0.2	0.20	0.58	633.78	0.00	151.47	1.51

As seen in the table, $a_6 < 1$ for all items. Therefore, we apply condition 1 and use basic EOQ formula to obtain the order quantities. That is, for the given items we have no stockout. As an example, some calculations for product 2A are given below.

$$\gamma_l = 0.20C = 0.20(2.53) = 0.51 \text{ ₺}$$

$$a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l(1-b)]^2} = \frac{2(0.1)(2.53)(50)}{1000[0.1 + 0.51]^2} = 0.068 < 1$$

Then, optimal order quantity is

$$Q^* = \sqrt{\frac{2KD}{IC}} = \sqrt{\frac{2(50)(1000)}{(0.1)(2.53)}} = 628.69$$

This yields a total cost of 159.06 ₺, where number of cycles in a year is $D/Q^* = 1000 / 628.69 = 1.59$.

4.2.3 Case III: Inventory Model with Both Backorders and Lost Sales

Some products are demanded by both dealers and individuals. Examining the attitude of different types of customers, we assume that demands from dealers are backordered and demands from individual customers are lost. Hence, we use the model that allows both backorders and lost sales. Additional assumptions for this model are:

- A fraction b of shortages is backordered whereas the rest ($1–b$) is lost
- Lead time is zero

Ten of such items are selected for Case III and named from 3A to 3J. Problem data related to these items and solution results are presented in Table 5. Since the percentage of sales to dealers changes between 80% and 95% for these kinds of products, we assume that 90% of shortages are backordered and 20% are lost. So, we set b = 0.9 for all products.

Table 5 Input data and results for case III (mixture of backorders and lost sales)

Item	Data							Results				
	D	C	K	I	γ_s	γ_b	γ_l	a_6	Q	S	TC	Number of Cycles
3A	1489	4.53	50	0.1	0.1	0.2	0.91	0.84	573.32	0.00	259.71	2.60
3B	1263	3.42	50	0.1	0.1	0.2	0.68	0.95	607.70	0.00	207.83	2.08
3C	1028	3.27	50	0.1	0.1	0.2	0.65	1.16	620.98	69.64	182.57	1.66
3D	884	2.05	50	0.1	0.1	0.2	0.41	1.17	702.70	53.25	134.23	1.26
3E	1200	2.03	50	0.1	0.1	0.2	0.41	0.86	768.85	0.00	156.08	1.56
3F	500	3.22	50	0.1	0.1	0.2	0.64	2.38	542.85	197.10	117.68	0.92
3G	3000	0.5	50	0.1	0.1	0.2	0.10	0.14	2449.49	0.00	122.47	1.22
3H	2920	0.45	50	0.1	0.1	0.2	0.09	0.13	2547.33	0.00	114.63	1.15
3I	2500	0.48	50	0.1	0.1	0.2	0.10	0.16	2282.18	0.00	109.54	1.10
3J	2400	0.49	50	0.1	0.1	0.2	0.10	0.17	2213.13	0.00	108.44	1.08

Below, we demonstrate the calculations on two selected items, 3A and 3C.

<u>Item 3A</u>

This item has an annual demand of 1489 units, and purchasing cost of 4.53 ₺ per unit. Therefore lost sale cost per unit is

$$\gamma_l = 0.20C = 0.20(4.53) = 0.91 \text{ ₺}.$$

Then, value of a_6 is computed as follows.

$$a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l\,(1-b)]^2} = \frac{2(0.1)(4.53)(50)}{1489[0.1 + 0.91(1-0.9)]^2} = 0.84 < 1$$

Since a_6 < 1, we follow condition 1. No shortages occur and the optimal order quantity is obtained basic EOQ formula as follows.

$$Q^* = \sqrt{\frac{2KD}{IC}} = \sqrt{\frac{2(50)(1489)}{(0.1)(4.53)}} = 573.32$$

Using the total cost formula, minimum cost is computed as 259.71 ₺. Number of cycles for this item is D/Q^* = 1489 / 573.32 = 2.60.

Item 3C

Annual demand for this item is 1028 units and purchasing cost is 3.27 ₺ per unit. Hence,

γ_l = 0.20C = 0.20(3.27) = 0.65 ₺.

Next, we compute the value of a_6.

$$a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l\,(1-b)]^2} = \frac{2(0.1)(3.27)(50)}{1028[0.1 + 0.65(1-0.9)]^2} = 1.16 > 1$$

Since a_6 > 1, we apply the procedure in condition 2. Necessary calculations are shown below.

$a_1 = KD$ = (50)(1028) = 51400
$a_2 = \gamma_s D + \gamma_l D(1-b)$ = (0.1)(1028) + 0.65(1028)(1 – 0.9) = 170.00
$a_3 = \gamma_b b/2$= (0.2)(0.9)/2 = 0.09

then,
$a_5 = 4a_1 a_3 / a_2^2 = 4(51400)(0.09) / 170^2 = 0.640$

$$\beta^* = \frac{a_5}{a_5 + a_6} + \frac{1}{a_5 + a_6}\sqrt{\frac{a_5 a_6}{a_5 + a_6 - 1}}$$

$$= \frac{0.640}{0.640 + 1.16} + \frac{1}{0.640 + 1.16}\sqrt{\frac{(0.640)(1.16)}{0.640 + 1.16 - 1}} = 0.889$$

$$U^* = \sqrt{\frac{2KD}{\gamma_b b(1-\beta)^2 + IC\beta^2}} = \sqrt{\frac{2(50)(1028)}{(0.2)(0.9)(1-0.889)^2 + 0.1(3.27)(0.889)^2}} = 627.94$$

$$V^* = \beta^* U^* = (0.889)(627.94) = 558.30$$

Finally, optimal order quantity and backordered units (demand during stockout) are obtained using Equations (4.7) and (4.8).

$$Q^* = bU^* + (1-b)V^* = 0.9(627.94) + 0.1(558.30) = 620.98$$

$$S^* = U^* - V^* = 627.94 - 558.30 = 69.64$$

Finally, total cost is obtained as 182.57 ₺, and number of cycles is found as D/Q^* = 1028 / 620.98 = 1.66. For this item, the Company has a shortage of 69.64 ≅ 70 units. 90% of these items (63 units) are backordered whereas 10% of them (7 units) are lost. According to the results shown in Table 5, the Company has shortages in three items (3C, 3D, 3F) and meets all demands on time for the other seven items.

4.2.4 Case IV: Inventory Model with Quantity Discounts

The studies in relation with quantity discounts and EOQ are frequently mentioned in the literature. In this section we make an analysis demonstrating the effect of possible price discounts for different order sizes. The Company may purchase some products at some discounted prices offered by suppliers if the order quantity is within some specific ranges. For this purpose, we consider an extension of previous cases, by allowing quantity discounts for three different prices. A numerical example for one of the items (item 3A used in case (iii)) is given. Therefore, additional assumptions considered in this case are as follows.

- A fraction of shortages is backordered whereas the rest is lost
- Suppliers offer discounted prices for different order quantities
- Lead time is zero

The unit purchasing costs depending on the quantity ordered are as follows.

Range of order quantity	Purchasing cost
$Q < 500$	$C_1 = 4.53$ ₺
$500 \leq Q < 1000$	$C_2 = 4.00$ ₺
$1000 \leq Q$	$C_3 = 3.70$ ₺

The optimal order quantities for C_1, C_2 and C_3 are computed and given in Table 6. As seen in the table, only the quantity for a unit cost of C_2 = 4.00 ₺ is valid since it is within the given range for that price ($500 \leq Q = 610.12 < 1000$) whereas the other quantities are invalid as they are out the ranges defined for C_1 and C_3 . For each of the three calculations, we find no stockout ($S = 0$), therefore no stockout exists and we have basic EOQ problem.

Table 6 Input data and results for case IV (quantity discounts with backorders and lost sales)

Item	Data							Results				
	D	C	K	I	γ_s	γ_b	γ_l	a_6	Q	S	TC	Number of Cycles
3A	1489	4.53	50	0.1	0.1	0.2	0.91	0.84	573.32	0.00	259.71	2.60
	1489	4.00	50	0.1	0.1	0.2	0.80	0.95	610.12	0.00	244.05	2.44
	1489	3.70	50	0.1	0.1	0.2	0.74	1.16	634.38	0.00	234.72	2.35

Starting from the lowest unit cost, we proceed as follows.

- For C_3 = 3.70 ₺ (lowest unit cost).

  Optimal order quantity is Q = 634.38 units with a cost of TC = 234.72 ₺. Clearly this quantity is not within the range defined for C_3 and therefore invalid. Then, considering the behavior of total cost curve, we calculate the cost for 1000 units which is the value at the lower end of the valid range.

  Using Equation (4.3) with the given data and with Q = 1000 and S = 0, total cost is found as 259.45 ₺.

- For C_2 = 4.00 ₺ (next lowest unit cost).

  Q = 610.12 units and TC = 244.05 ₺. This quantity is within the range defined for C_2 so it is valid.

Since for the highest unit cost (C_1 = 4.53 ₺) we cannot have a lower cost value, we do not need to consider it and we just compare the following two alternatives to determine optimal order quantity.

for Q = 1000, TC = 259.45 ₺

for Q = 610.12, TC = 244.05 ₺

Therefore the decision is to place an order or size 610 since it yields the minimum annual cost (244.05 ₺) among the alternatives.

For all products where quantity discounts are applicable, we can perform similar analysis and determine optimal order quantities.

CHAPTER 5

CONCLUSION

In this thesis, we carried out a study on inventory management for a retail company, which sells electronics and computer parts. Customers of the Company are categorized as dealers and individual customers. Although there are some products demanded by both types of customers, some products are sold to one type of customers, dealers or individuals. When there are not sufficient items upon arrival of demand, stockout occurs and the Company faces different situations depending on customers' attitude. During stockout, usually demands from individuals are lost whereas demands of retailers are backordered. This yields three types of inventory problems which are pure lost sales for products demanded by individuals only, pure backorders for products sold to dealers, and mixture of backorders and lost sales resulting from the items demanded by both customer types.

In our study, we focus on deterministic inventory models, since demands for a number of products are observed to have close values over last five years. Therefore, we consider such products assuming deterministic demand for each. We define four cases and use the existing deterministic inventory models and solution approaches to determine the optimal order quantities that minimize total annual cost. Our analysis and results are demonstrated on ten items in each of the first three cases. In addition, considering possible price discounts for some products, we present an analysis on a selected product for which optimal order quantity is obtained based on some quantity discounts.

We identify the following issues as possible extensions of our study.

- A rationing policy can be taken into consideration to determine the stock level where for example we stop meeting the demands of individuals while continuing to meet the demand of dealers.
- Forecasting methods can be used to determine demand for products whose demands vary over time.
- The effect of different cost types (holding, ordering, shortage, backorder, lost sales costs) can be analyzed in detail.
- Based on the observation that some products with usually high selling prices have low demand values, the issue of determining integer valued order quantities for non-integer amounts can be considered.
- Stochastic inventory models can be used to determine the inventory policy for the products whose demands are not deterministic.
- The demands of both customer types (dealers and individuals) for some products can be backordered or lost, with different backordering or lost sale costs for each.
- Finally, development of a decision support system, including demand forecasts and integration of inventory models, can be considered that that allows dynamic planning in accordance with the changing situations.

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