OPTIMAL MULTI-PERIOD PRICING STRATEGY FOR REMANUFACTURABLE LEASED GOODS

by

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ABSTRACT

OPTIMAL MULTI-PERIOD PRICING STRATEGY FOR REMANUFACTURABLE LEASED GOODS

The aim of this thesis is to determine in a multi-period setting the optimal pricing strategy for a profit-maximizing firm leasing new, durable, and remanufacturable products as well as selling remanufactured products. The resulting problem is nonlinear optimization problem and it is solved by a variant of Nelder-Mead simplex search method which can also handle the constraints.

We focus on a scenario where new products can only be leased and remanufactured products can only be sold after remanufacturing used equipments returned by the lessee at the end of the lease period. In this setting, if returned items in stock are not enough to meet the demand for remanufactured products, the manufacturer purchases the shortage in used products from the third-party core supplier. Two types of demand model are proposed in our work: In the base model, the customer preferences are explained through a maximum utility type approach. The second one is constructed as a linear function of prices of new and remanufactured products. We focus on the first one since it is more realistic in the marketing environment, and discuss attributes of the new and remanufactured products based on the experimental results. We characterize the roles that key product characteristics such as deterioration in age, cost of supplying used remanufacturable products from the third-party core supplier and initial inventory level, and key target market characteristics such as relative willingness-to-pay for buying a remanufactured product and relative willingness-to-pay for leasing a new product play in determining the optimal pricing strategy.

ÖZET

YENİDEN İMAL EDİLEBİLİR KİRALANAN ÜRÜNLER İÇİN ÇOK DÖNEMDE EN İYİ FİYATLANDIRMA STRATEJİSİ

Bu tezin amacı yeni, dayanıklı ve yeniden imal edilebilir ürünleri kiralayan ve yeniden üretilmiş ürünleri satan, karını en büyüklemeye çalışan bir firma için çoklu dönemde en iyi fiyatlandırma stratejisini belirlemektir. Ortaya çıkan problem karı en büyüklemeye çalışan doğrusal olmayan bir programlama modelidir ve kısıtları kontrol altında tutan farklı bir Nelder-Mead simleks arama işlemiyle çözülmüştür.

Senaryomuza göre yeni ürünler sadece kiralanabilir ve kiralama dönemi sonunda kiracının döndürdüğü kullanılmış ürünlerden imal edilen ürünler sadece satılabilir. Bu ortamda eğer eldeki eski ürünler yeniden üretilmiş ürüne olan talebi karşılamıyorsa eksik miktar üçüncü-parti eski ürün tedarikçisinden sağlanır. Çalışmamızda iki tip talep modeli öneriliyor: Ana modelde müşteri tercihleri en yüksek yarar tipi yaklaşıma göre anlatılmaktadır. İkinci talep modeli yeni ve yeniden üretilmiş ürünlerin fiyatlarının doğrusal fonksiyonu olarak kurulmuştur. Gerçek durumu daha iyi yansıttığı için ana model ele alınmaktadır ve deneysel sonuçlara dayanarak yeni ve yeniden üretilmiş ürünlerin nitelikleri tartışılmaktadır. Zaman içinde eskime payı, üçüncü-parti eski ürün tedarikçisinden yeniden üretilebilir eski ürün elde etme maliyeti ve dönem başında elde bulunan eski ürün miktarı gibi ürün özellikleri ile yeniden üretilmiş ürünü satın alma ve yeni ürünü kiralama için göreceli ödeme isteğinin en iyi fiyatlandırma stratejisi üzerindeki etkisi incelenmiştir.

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LIST OF SYMBOLS/ABBREVIATIONS

a_n	Potential size of new product's market
a_r	Potential size of remanufactured products's market
c_n	Cost to produce a new product
$c_r q_r$	Average cost to remanufacture q_r units
c'_r	Cost to acquire a used equipment from the third-party core supplier
d_m	Depreciation rate of the product over m periods
i	Annual interest rate
I_0	Initial inventory level
I_{t-1}	The inventory of remanufacturable products in stock at the beginning of period t
l_m	Relative willingness-to-pay for leasing a new product for m periods
L	The max. duration of the lease agreements
m	Index for lease periods expressed in year
M	Time horizon over which the product stays on the market
$p_{n,t}$	Selling price of the new product in period t
$p_{m,t}$	Lease payments over m periods determined in period t
$p_{r,t}$	Selling price of the remanufactured product in period t
$\mathbf{p}_t$	Price vector in period t
$PV(p_m)$	Present value of lease payments
$q_{n,t}$	Demand for new products in period t
$q_{No,t}$	The volume of consumers who prefer nothing in period t
$q_{m,t}$	The volume of new products leased for m periods in period t
$q_{r,t}$	Demand for remanufactured products in period t
$q0_{m,t}$	The volume of consumers who prefer nothing in the segment m in period t
$\mathbf{q}_t$	Demand vector in period t
r_β	Per-period revenue for yearly lease payment model

r_{β_m}	Per-period revenue for monthly lease payment model
R_t	The volume of used products that return in period t
t	Index for period over time horizon
T	Time horizon
$V_\beta(I)$	Optimal $\beta-$discounted multi-period profit under the initial condition $I_0 = I$
α	Reflection coefficient in the simplex search
α_m	Size of customer segments based on their lease expectations for m periods
α'_m	Return fraction of new products after m periods
α_n	Sensitivity of new product's demand to its price
α_r	Sensitivity of remanufactured product's demand to its price
β	Discount factor over time period
β_m	Monthly discount factor
γ	Expansion coefficient in the simplex search
δ	Relative willingness-to-pay for buying a remanufactured product
$\Delta_{r,t}$	The shortage in used products in period t
θ	Max. price any customer is willing to pay for buying the new product
θ_n	Sensitivity of remanufactured product's demand to new product's price
θ_r	Sensitivity of new product's demand to remanufactured product's price
λ	Contraction coefficient in the simplex search
Π_β	The optimal profit in a generic period for yearly lease payment model
Π_{β_m}	The optimal profit in a generic period for monthly lease payment model
χ	Shrinkage coefficient in the simplex search
ACC	Adjusted capitalized cost

ELA	Equipment leasing association
EPR	Extended producer responsibility
MSRP	Manufacturer suggested retail price
NCC	Net capitalized cost
NLP	Nonlinear programming
NPV	Net present value
OEM	Orginal equipment manufacturer
RV	Residual value

1. INTRODUCTION

Remanufacturing is the process of disassembling used items, inspecting and repairing their components and using these in manufacturing new products. A product is considered remanufactured if its primary components come from a used product. Recently, remanufacturing has been receiving growing attention for various reasons. For instance, consumer awareness, environmental activism and legislative pressure are forcing firms to design remanufacturable products. Moreover, the cost of remanufacturing is typically less than the cost of manufacturing a new product. Remanufacturing is practiced in many industries, including photocopiers, computers, telecommunication equipment, automotive parts, office furniture and tires. The problems faced by firms involved in remanufacturing are to take back used products before the end of their useful life to generate some revenue by remanufacturing or reusing and to forecast the time of used products returns. At this point, leasing strategy versus selling brand new products can help to manage the return process better.

Leasing is a widely used business strategy in United States, with 80 per cent of all US companies leasing some or all of their equipment and a estimated 226 billion worth of equipment leased in the US according to the research done by Equipment Leasing Association (ELA) in 1999. There are two main categories of leases: capital and operating. Only operating leases, not capital leases, increase the likelihood that the manufacturer will retain ownership of the product at the end of the lease and have responsibility for managing it. Furthermore, the company is more likely to get a consistent flow of feedstock for remanufacturing when used equipment is returned to the manufacturer. Also, leasing may help the manufacturer to forecast the quality of returned products and schedule the product line according to their return time. Besides the benefits of leasing for the manufacturer, consumers have also advantages from leasing versus buying. When you buy, you pay for the entire price of a equipment, but when you lease, you pay for only a portion of the equipment's price, which is the part that you "use up" during the lease period. Moreover, leases for products like computers make it easier for customers to upgrade to the newest technology.

Product characteristics affect the viability of leases. Some products are not suitable for leases like nondurable products consumed during ownership, so that little value remains at the end of a lease. This characteristic is important for both remanufacturing and leasing a product, since if it is consumed during leasing, it becomes impossible to remanufacture and generate revenue from a used product. So far, we can say that leasing allows firms to control the quality, quantity and timing of product returns, which are the primary concern of many remanufacturing initiatives.

On the other hand, determining leasing payments of new products and selling price of remanufactured products is an important tool since even small changes in price have a high impact on the profitability of the firm. Since the target market has heterogeneous customers who typically differ in their willingness-to-pay for new product and value remanufactured products less than new products, determining the price is a complex task for the firm.

The main objective of this thesis is to determine in a multi-period setting the optimal pricing strategy for a profit-maximizing firm leasing new, durable, and remanufacturable goods as well as selling remanufactured products in a market where consumers are heterogeneous in their willingness-to-pay and value remanufactured products less than new products. We consider a dynamic pricing policy where prices are time-dependent. Moreover, we do not only determine the selling price of a remanufactured product, but also we need lease payments of a new product since a new product can only be leased. In this setting, two types of lease payments are proposed: The first one is based on monthly payments and determined as a function of selling price of the new product taking into account the depreciation rate of the product in age and money factor. The second one is based on yearly payments and calculated as a function of selling price and life-cycle of the new product. In our analysis, we use monthly lease payment model since lease payments are generally arranged in terms of months in the real world. We initially assume that there is a remanufacturable product inventory at the beginning of the first period. We determine then optimal prices in each period to which corresponds to optimal path of demands for new and remanufactured products. There are two types of demand models proposed in our work: In the base model, the customer preferences are explained through a maximum utility type approach. In this setting, customers are segmented into sub-groups according to their lease expectations such that customers who wish to lease a new product for m periods are distributed with a known fraction in the potential market. In a given period, consumers in each segment determine which product to choose based on the utility that they derive in that period from this decision. If a consumer in a segment has utility neither from leasing a new product nor from buying a remanufactured product, he/she prefers nothing. This is a more realistic approach than forcing the total demand of a segment to be divided among products, i.e., new and remanufactured. Therefore, in this choice model, we obtain three regions in each segment: new product leased for m periods, remanufactured product and no product. In the second demand model, we consider a linear price-demand relation such that demand is a function of price of new and remanufactured products. In this setting, the potential market consists of a known segmentation of customers such that the fraction of customers in the population who lease the new product for m periods is known. Therefore, after generating the total demand for new products in a generic period, the volume of customers who lease a new product for m periods can be derived from the total demand for new products. We assume that leased products will return at the end of leasing agreement certainly, thus in each period we know how many cores (used product) will be returned. Therefore, the supply of used products depends on past lease volumes of new products. During the time, preceding inventory is added to the current inventory in each period. If returned products in stock are not enough to meet demand for remanufactured products in any period, we assume that the manufacturer buys the remaining from the third-party core supplier and sells these products to the customer after remanufacturing them in his facility. The overall profit function of the firm is obtained by aggregating the decisions of customers over the segments. Some of the questions that we address are:

- What are the optimal prices in each period?
- How do key product characteristics such as deterioration in age, cost of supplying used remanufacturable products from the third-party core supplier and initial inventory level influence the optimal price in each period?
- How do key market characteristics such as consumer acceptance of the remanufac-

tured product and consumer acceptance of the leased product play in determining the optimal pricing strategy?

- What is the effect of pricing strategy on the supply of used products?
- What is the effect of supplying used remanufacturable products from the third-party core supplier on the profitability of the firm?
- What is the effect of change in the duration of the lease agreement on the pricing and trend of optimal demands?

The study is organized as follows: The first chapter includes the literature review about pricing, leasing and remanufacturing. In the second chapter the problem description is presented and model formulations are explained in detail with corresponding assumptions. In the third chapter, solution procedure and its modifications are given. The experimental results which are grouped into two main categories such as single period and multi-period setting for the base model are presented in the fourth chapter. The final chapter includes the conclusion and suggestions for future research.

2. LITERATURE REVIEW

In this chapter, a brief review of published literature related to the various aspects of the research problem in this thesis is presented. Section 2.1 discusses literature on pricing, a popular research area for marketing science. Section 2.2 discusses literature on leasing which plays an increasingly important role in marketing durable goods. The main topics discussed are leasing versus selling, pricing lease contracts and reverse logistics in equipment leasing. Remanufacturing is an integral part of this research as well, and therefore, the growing literature on remanufacturing and pricing of new and remanufactured products are discussed in Section 2.3.

2.1. Pricing

The primary goal of most businesses is to make profit. Although there are many factors that affect the profitability of a business, such as management, location, cost of labor, quality of product or service, market demand and competition (U.S. Small Business Administration), the price is the only part of the marketing mix that generates revenue. Setting the right price is an important part of effective marketing. Price is also the marketing variable that can be changed most quickly, even in response to a competitor price change. Moreover, even small changes in price have a high impact on the profitability of the firm. It is reported that on average, a 5 per cent price increase leads to a 22 per cent improvement in operating profits which is far more than other tools of operational management (Hinterhuber, 2004).

By employing dynamic pricing, the act of changing prices over time in a marketplace, firms have the potential to increase their revenue by selling products to buyers “at the right time, at the right price”. Dynamic pricing can be implemented in several different ways. Price discrimination, or personalized pricing, is an intriguing area of dynamic pricing in which sellers charge different segments of customers different prices. While this area is rich, it also has greater risks of customer rejection. In contrast to dynamic pricing, Morris (2001) focuses on changing prices over time in a market.

This perspective on dynamic pricing focuses on how a seller can take advantage of the fluctuations in cumulative buyer demand over time taking into account a finite time horizon. He refers to this type of changing of prices over time as dynamic pricing and proposes that sellers should analyze dynamic pricing algorithms using a market simulator that is capable of simulating many different market scenarios with realistic models of buyer behavior. The author presents a tool, the Learning Curve Simulator, for modeling finite markets-a market with a finite time horizon, seller inventory and buyer population-and for testing dynamic pricing strategies. Using a market simulator, a seller could model its market's characteristics and the behavior of its customers to develop a pricing strategy that could capture more profit than fixed-price policies.

Any treatment of dynamic pricing must recognize that there may be a strong dependency of demand across periods. A promotion today that generates a spike in demand will typically be followed by a demand trough. Likewise, selling an airline seat today at a given price means that it will not be available later. Ideally, a pricing decision made today should optimally account for all future effects of the decision. Fleischmann *et al.* (2005) contributes to the literature on dynamic pricing by developing a deterministic, finite-horizon dynamic programming model that captures a price/demand effect as well as a stockpiling/consumption effect. The decision variable is the unit sales price in each period. They model full dynamic pricing in which the pricing decision in a given period affects demand, which in turn affects consumption. Hence, decisions in a given period explicitly depend on decisions in prior periods.

In terms of applications, dynamic pricing practices are particularly useful for those industries having high start-up costs, perishable capacity, short selling horizons, and a demand that is both stochastic and price sensitive. Therefore, sellers have an incentive to dynamically change the price to control the demand in order to maximize their total revenue, which is known as revenue management. Elmaghraby and Keskinocak (2003) provide a survey of the applications and theory of dynamic pricing with different set of angles such as pricing policies for short and long life cycle products, or combined inventory and pricing decisions, or pricing in markets with rational (strategic) customers. Bitran and Caldentey (2002) also examined the research and results of dynamic pricing policies, but they preferred to narrow the scope of their work to dynamic pricing models in revenue management context. Therefore, the survey is based on revenue management problem in which a perishable and non-renewable set of resources satisfy stochastic price-sensitive demand processes over a finite time horizon. In this framework where capacity is fixed, the seller is mainly interested in finding an optimal pricing strategy that maximizes the revenue collected over the selling horizon. One can refer to Adida and Perakis (2005) and Maglaras and Meissner (2006) for further information regarding dynamic pricing strategies. Although dynamic pricing has a wide application area for perishable and non-renewable products, studies for durable products are limited. There are three barriers to change prices of durables: one of them is the cost of implementing instantaneous price changes. The other one is challenge of developing an appropriate pricing strategy and the last one, which is the most important, is buyer acceptance of unpredictable price changes (Morris, 2001). The purchasing behavior of customers affects seller's pricing decisions over time. A myopic customer is one who makes a purchase immediately if the price is below his valuation, without considering future prices. Myopic (or non-strategic) customer behavior allows the seller to ignore any detrimental effect of future price cuts on current customer purchases. Nevertheless, a strategic (or rational) customer takes into account the future path of prices when making purchasing decisions. Generally, consumers consider durable goods such as cars as capital investments and wish to assess the asset's future value. Therefore, future value is affected by future price decreases and the possible introduction of a new version of a product. Dynamic pricing decisions of a seller facing strategic customers is more complex since the seller has to take into consideration the effects of future as well as current prices on customers' purchasing decisions (Elmaghraby and Keskinocak, 2003).

Theoretically, consumer reservation price (also called willingness-to-pay) has been instrumental in studying consumer purchase decisions and competitive pricing strategies. Managerially, knowledge of consumer reservation prices is critical for implementing many pricing tactics such as bundling, target promotions, nonlinear pricing, and one-to-one pricing, and for assessing the impact of marketing strategy on demand.

The reservation price refers to the maximum amount of money a consumer is willing to pay for a certain product. The consumer decision on whether or not to buy a product depends on the reservation price of the customer and product's price. He buys the product only if his reservation price is higher than or equal to the product's price. Another related concept with reservation price is the consumer surplus. Consumer surplus is the difference between the consumer's reservation price of the product and the price of the product. Consumer surplus measures the welfare that consumers derive from their consumption of goods and services, or the benefits they derive from the exchange of goods. The customer has benefit from the purchase if his surplus is greater than or equal to zero.

In many papers related to pricing (e.g., Ferguson and Toktay, 2004; Debo *et al.*, 2005; Ray *et al.*, 2005), the population of potential customers is characterized by distribution of reservation prices. In general, reservation prices have a continuous distribution over the population of potential customers. When making pricing decisions, the seller knows only the probability distribution of reservation prices. Therefore, he faces the trade-off of losing sales due to high prices and losing the customer surplus due to low price. The variance of the distribution of reservation prices depends on the heterogeneity of the market and the availability of information about customers' tastes and needs.

2.2. Leasing

Leasing as a means of transaction is playing an increasingly important role in marketing durable goods. On the other hand, leasing is also becoming a pervasive phenomenon in our ordinary life. For instance, many durable goods that are traditionally sold to consumers can now be leased too. The spectrum of leased durable goods is rapidly expanding. Examples of these include such daily necessities as cars, furniture, computers, and other electronic appliances (Huang and Yang, 2002).

There are two main categories of leases: capital and operating. Capital leases are basically finance arrangements and are treated as loans for accounting purposes.

Under such leases, ownership passes to the lessee automatically by the end of the lease term. End-of-life ownership of the product is no different under a capital lease than if the product were sold directly to the customer. Under operating leases (also called “true leases”), on the other hand, ownership is typically retained by the lessor to the end of the lease term (although the lessee is able to purchase the product at the end of the lease for its fair market value) (Fishbein *et al.*, 2000). Fishbein *et al.* (2000) have examined the practice of leasing products, rather than selling them, as a strategy for increasing resource productivity, particularly by preventing waste generation and moving to a pattern of closed-loop materials use. A closed-loop mimics natural systems, in which materials are continually reused so that waste from one application becomes the source of materials for another. In the case of products, this can be accomplished through reuse, remanufacturing and recycling. In this report, they mention about the extended producer responsibility (EPR) which requires that producers take back their products when consumers discard them, manage them at their own expense, and meet specified recycling targets.

Unless otherwise specified, the term “lease” will refer to an operating lease. Therefore, a lease can be defined as a contract in which the owner of property grants to a customer the right to use the property for a specified period of time in exchange for an agreed upon periodic payment. Fishbein *et al.* (2000) present many key findings about leasing: leasing increases the likelihood that the manufacturer will retain ownership of the product at end-of-life and have responsibility for managing it. It is important since products have to return to be recovered. Moreover, product characteristics affect the viability of leasing. Some products are not suitable for leasing since they may be consumed during the use so heavily that little value remains at end-of-lease period. Thus, we can say that leasing may provide an incentive for a company to make its products more durable. Furthermore, leasing is for the benefit of a company in terms of remanufacturing the used products. For instance, leasing can provide the manufacturer with a continual, predictable flow of post-consumer feed-stock for its remanufacturing activities. Also, it may help the manufacturer to forecast the quality of the returned product and schedule the product line according to their return time. Greater involvement with leasing also provides manufacturers with greater control over the resale market, because secondary market prices and equipment availability can impact new product sales and pricing. There are many companies that successfully acquire products through leasing and remanufacture returned products. One of the well known examples is Xerox, whose goal is to be the "leader in the global document market" with its document-processing products, systems and services. There are similar examples from industry sectors such as carpet (e.g., Interface Inc.), computers (e.g., Compaq, Dell, Gateaway, and IBM) (Fishbein *et al.*, 2000).

In the following, we discuss the papers that study the leasing, leasing versus selling, pricing lease contracts, and reverse logistics in equipment leasing.

Desai and Purohit (1998) analyze the problems associated with marketing a durable through leases and sales. This general choice between leases and sales is crucial because of three issues that affect the firm's marketing strategy over the long term. First, leases and sales lead to different forms of competition in the future. That is, once a firm sells a durable, the product exists in a competitive secondhand market that competes with the firm's sales of new products. However, if the product is leased, it is returned to the firm at end-of-lease; hence, the firm has more control over the secondhand market. Second, given the long life of a durable, consumers may perceive the depreciation of a durable to depend on whether it was originally leased or purchased, which then affects market prices. And the last one, in choosing durables, consumers try to forecast their long-term needs which have different implications for their willingness-to-pay for leases and purchases. Academic research in this area has argued that in a monopolistic environment, leasing dominates selling. Hence, the firm should not concentrate on both leasing and selling under this decision. In contrast to academic research in this area, Desai and Purohit (1998) show that the relative profitability of leasing and selling depends on the rates at which leases and sold units depreciate. The goals of this paper are to understand the strategic issues associated with concurrently leasing and selling a product and determine the conditions under which this concurrent strategy is optimal. They model a market in which both leases and sales are allowed, and a durable product is marketed in a two-period structure. Any product sold or leased in the first period, enter the market in the second period as either a used product or an ex-leased product. Either of these compete with any new products that the firm tries to sell or lease in the second period. The product in the problem is a car; therefore, they assume that there can potentially be three types of cars available in the market: new cars, ex-leased cars, and used cars. The difference between ex-leased and used cars is the form of prior "ownership," i.e., whether the car was leased or bought. This difference leads to leased and sold cars depreciating at different rates. In terms of the consumer side of the market, there are three commonly observed consumer usage patterns: a group of consumers who buy new cars and frequently replace them; another group of consumers who buy new cars and hold on to them as long as they last; and a third group of consumers who buy only not-new (i.e., used or ex-leased) cars. Consumers are heterogeneous in their willingness-to-pay. Based on this structure, they find that the firm's strategy to either lease or sell to any group of consumers depends on the relative depreciation rates of sold and leased cars. If leased cars are likely to depreciate more than sold cars, the firm should direct leases to the high willingness-to-pay consumers who tend to replace their car each period. On the other hand, if a leased car depreciates less, the firm should direct its leases to the lower willingness-to-pay consumers who are more inclined to purchase their ex-leased cars at the end of the lease period. Hence, if the depreciation rates are different for sold and leased cars, combination of leasing and selling is better for the manufacturer. Desai and Purohit (1999) also examine competition in a duopoly; they are interested in investigating firms' incentives to lease or sell its products. Moreover, their objective is to investigate a firm's rationale in choosing an optimal mix of leasing and selling and to understand how it is affected by the nature of competition in the market and the embedded quality in the product. They develop a two-period model in which consumers are indifferent between buying and leasing a durable product. Therefore, they can control for differential consumer preferences and focus on the effect of market competitiveness and product characteristics. They find that a competitive environment forces firms to adopt strategies where they only sell their products or use a combination of leasing and selling. In addition to this, the degree of competitive intensity between the competitors affects the extent of leasing that occurs in a market. As the competitive intensity increases, the competitors decrease their level of leasing. They find that the competitors choose the pure leasing strategy in extreme cases of competition. The extent of leasing also depends on the perceived rate of deterioration of the product. As product's rate of deterioration decreases, the firm chooses to increase its level of leasing.

While the effect of durability (or deterioration) on the profitability of firms has been studied by Desai and Purohit (1998, 1999), the interactions between durable goods and complementary products have been examined by Bhaskaran and Gilbert (2005). This aspect is important since the availability of such complements can stimulate demand for the durable good, increasing the manufacturer's profits. In many instances the lack of sufficient availability of complementary products can prevent the success of a durable good. For instance, in the automotive industry, complementarities affect the adoption of alternative fuel technologies such as hybrid and fuel cell vehicles, both of which depend on the availability of complements. For hybrids, batteries need to be replaced every couple of years, and greater availability of batteries decreases the overall cost of operating the vehicle. Fuel cell vehicles have an even stronger complementary dependence on the availability of hydrogen fuel since they cannot be operated without near-daily access to fuel. When a durable monopolist sells a product, it has an incentive to produce at a rate that will drive down the market price of the product over time. In anticipation of this opportunistic behavior, consumers are less willing to invest in ownership. This issue has been referred to as time inconsistency. One well-known way for a manufacturer to eliminate the problem of time inconsistency is to lease the product to the consumer instead of selling. Therefore, they investigate how a durable goods manufacturer's choice between leasing and selling is affected by complementary product that is produced by an independent firm. In conclusion, they show that in the case that the complementary product is produced by another firm and the extent of complementary is sufficiently strong, the manufacturer's preference for leasing will shift to selling. When complementary effects are weak, due to either a small individual marginal utility for the complementary product or to a low interaction between the two products, the firm should shift toward leasing.

A leasing company tries to maximize operating profits through key decisions associated with length of leases, efficient utilization of logistics facilities for material flow to and from customer sites, and equipment reuse, refurbishment and disposal actions (Sharma, 2004). The model proposed by Sharma (2004) allows decision-makers for electronic equipment leasing companies to simultaneously make optimal decisions about lease lengths, product flows and end-of-life product disposal. Pricing has not taken into account in this model since pricing is a strategic decision that is affected by many other factors like market competition, sales and marketing strategies, economic and political conditions, etc. Taking all these additional factors into account would not only make the model mathematically intractable, but would also detract from the main research focus of integrating reverse logistics and environmental issues with equipment replacement decisions. The model is deterministic, but the examination of uncertainty in problem parameters has been made by solving multiple scenarios (with different parameter values) using this model. It is shown that for asset purchase decisions and forward product flows, there exists a tradeoff between asset purchase costs and transportation costs. On the other hand, environmental legislation and transportation costs affect the reverse product flows. An increase in asset disposal costs due to a landfill ban in one location can lead to a significant increase in the disposal of assets at other locations. In addition, it is observed that rebuilding is a profitable activity, especially for high-end assets. Therefore, a leasing or asset manager for a large leasing company could apply the model and the insights to gain a competitive advantage by managing the business more efficiently.

In general, lease contracts consist of options that allow lessees the right but not the obligation to purchase the product at end-of-lease. This form of lease is very popular in automobile industry. Hence, Chen and Huang (2003) developed an experimental model adapted from the setting in Huang and Yang (2002) to examine the interaction between lease contracts that embed an option to purchase and an underlying used-goods market. This research is the first stage of collaboration between HP Labs and the Ford Motor company to create a general framework to address some of the unique issues in automobile marketing. One can refer to Chen and Huang (2003) for further information about experimental design and results.

In writing an operating lease, lessors must estimate what the equipment will be worth at the end of the lease term, but the estimation of residual values (RV) is one of the crucial determinants of profitability of equipment lessors. This value is often highly unpredictable, due to uncertainty about future market conditions, more specifically general economic conditions, competition, customer preferences, and innovation and new product development. Since manufacturers possess this often highly confidential knowledge, they will have an advantage over independent lessors in estimating the residual values for their equipment (Pierce, 2001).

The wholesale worth of a product at the end of its lease term, after it has depreciated, is called its residual value (http://www.leaseguide.com/lease07.htm). Residuals are usually stated as a percentage of manufacturer suggested retail price. Residual percentages decrease as the length of a lease, called the lease term usually expressed in months, increases. This is because the older a product gets, the less it's worth. For instance, a typical vehicle will lose 30 per cent of its value in the first year, far more than any other year, leaving 70 per cent of its original value, 17 per cent more in the second year, leaving 53 per cent, 8 per cent more in the third year, leaving 45 per cent, 6 per cent more in the fourth year, leaving 39 per cent, and 5 per cent more in the fifth year, leaving 34 per cent of its original value. As seen, residuals fall rapidly in the first 24 months, then more slowly in later months. This is why shorter term leases are more expensive than longer leases. Sharma (2004) also determined residual value of the electronic products used by the business enterprises. The most common lease periods offered by the company are 24, 30, 36 months where six months is considered as a one-period. Therefore, the minimum and maximum lease durations are respectively assumed to be four periods (24 months) and eight periods (48 months) for all orders. The residual value of an asset of age i periods is defined as the current value of an asset as a fraction of the original value (purchase price). Based on the information provided by the company, a residual value curve of the form exp (-0.2624 * i) is used to approximate residual values.

After this discussion on residual value, another issue is to determine lease payments. Lease payments are made up of two parts: a depreciation charge and finance (rent) charge (leaseguide.com/lease03.htm). The depreciation part of each monthly payment compensates the manufacturer for the portion of product's value that is lost during the lease. The finance part is interest on the money the manufacturer (lessor) has tied up in the product while the lessee is using it. In effect, the lessee borrows the money from the lessor and repays part of the money in monthly payments. At the end of the lease term, the lessee repays the remainder when she either buys or returns the product.

To understand how leasing works, we need to look at other components of leasing except from residual value. Manufacturer Suggested Retail Price (MSRP) is the full price for a product including optional packages and destination charges. Gross Capitalized Cost is the sum of selling price of the product, dealer acquisition fee, outstanding prior loan and lease balances which are lessee pays for over the lease term. Capitalized Cost Reduction is the amount of any net trade-in allowance, rebate, non-cash credit, cash down payment that the lessee pays to reduce the gross capitalized cost. When capitalized cost reductions are subtracted from gross capitalized cost, we obtain net capitalized cost (NCC), sometimes called adjusted capitalized cost (ACC). This amount is used so as to calculate depreciation part (DP) and finance part (FP) of leasing payments (http://carbuyingtips.com/regm.xls) given as follows.

$$DP = NCC - RV, \tag{2.1}$$

$$FP = (NCC + RV)MF12m, \tag{2.2}$$

where m denotes the year.

Money factor (MF) is a number often used by lessors to calculate the average monthly rent charge portion of the lease payment (Hyundai Motor Finance Company). Money factors can be converted to annual interest rate (AIR) by multiplying by 2400. It is always 2400 and is not related to the length of the loan in months. The lower the money factor, the lower your monthly lease payments. Lease term is the length of time the product is leased, usually expressed in months. Typical leases are 24, 36, 48 months, although “oddball” terms, such as 30, 39, 42 months are frequently seen in lease promotional ads. So far, lease terms have been given to understand how lease payments are calculated. Therefore, base monthly payments (BMP) can be stated mathematically as follows.

$$BMP = \frac{DP + FP}{12m}. \tag{2.3}$$

Sharma (2004) determines the prices for each different length of lease based on the residual value of assets. For a lease of t periods, the lease cost in each period equals:

$$Price = \left(\frac{residual(start) - residual(end)}{t} \right). \tag{2.4}$$

One can refer to (http://carprices.com, leaseguide.com, hmfcusa.com) for different payment calculators on the web.

2.3. Remanufacturing

Reverse logistics is the process of planning, implementing, and controlling the efficient, effective inbound flow and storage of secondary goods and related information opposite to the traditional supply chain direction for the purpose of recovering value or proper disposal (Fleischmann, 2001). Economic, marketing and legislative motives are commonly cited as reasons for companies to engage in reverse logistics. In the literature of reverse logistics, many authors have also pointed out that environmental consciousness of consumers is one of the driving factors of reverse logistics. Moreover, one can characterize a number of different categories of reverse logistics flows. Products are returned or discarded because they do not function properly or because they or their function are no longer needed. Within this scope it is differentiated between manufacturing returns, distribution returns, and customer returns. For detailed information about return reasons for reverse logistics, one can refer to Dekker *et al.* (2004).

Recovery is actually only one of the activities involved in the whole reverse logistics process. First there is collection, next there is combined inspection/selection

/sorting process, thirdly there is recovery (which may be direct or may involve a form of reprocessing), and finally there is redistribution. Collection refers to bringing the products from customer to a point of recovery. At this point the products are inspected and a decision is made on the recovery that follows. If the quality is as-good-as-new, products can be fed into the market almost immediately through reuse, resale, and redistribution. If not, another type of recovery may be involved that needs more action, i.e. a form of reprocessing. Reprocessing can occur at different levels: product level (repair), module level (refurbishing), component level (remanufacturing), selective part level (retrieval), material level (recycling), and energy level (incineration). If none of these recovery processes occur, products are likely to go to landfill. This section does not particularly address repair and refurbishing systems, which are a body of literature on their own. At component level (remanufacturing), products are dismantled, and used and/or new parts can be used either in the manufacturing of the same products or of different products (Dekker *et al.*, 2004). After performing these activities, the product can be sold as a remanufactured product. In material recovery, there is a series of activities by which discarded materials are collected, sorted, processed, converted to raw materials and used in the production of new products. Remanufacturing makes a much greater economic contribution per unit of product than recycling because it recaptures the value added to raw materials by the manufacturer: specifically, the costs of labor, energy, and manufacturing operations, which are typically greater than the value of the raw material constituents of the product. Because remanufacturing preserves the entire equipment components instead of returning them to raw materials (as recycling does), it allows processors to preserve the original value added by the manufacturer (Fishbein *et al.*, 2000). Finally, in energy level, products are burnt and the released energy is captured.

According to the Remanufacturing Institute, a product is remanufactured if its primary components come from used products. First, the used product is dismantled to determine the condition of its components. Second, the used products components are thoroughly cleaned and free of rust and corrosion. Then, all missing, defective, broken and worn parts are either restored to functionally good condition or replaced with new, remanufactured, or functionally good used parts. Finally, the product is reassembled after ensuring that it will operate like a similar new product (Fishbein *et al.*, 2000). The cost of remanufactured items should be determined according to the reverse logistics, inventory control and production planning policies. Collection, testing, sorting, transportation and processing of the items may require cost, and the cost of remanufacturing increases as the quality of the remanufactured product increases (Çelebi, 2005). Product design is a key to improving the efficiency of resource use. Many companies are mandating take-back programs to encourage companies to make such design changes. Leasing, which allows manufacturers to retain ownership of their equipment at end of life, combined with recovery of valuable materials through reuse, remanufacturing, and recycling, can encourage manufacturers to reduce their use of virgin materials and can reduce post-consumer waste. However, the value obtainable from materials recovery programs is limited unless products are specifically designed with end-of-life processing in mind. Design for end of life maximizes the residual value of equipment returned to the manufacturer. Therefore, by incorporating end-of-life considerations into the product design phase, manufacturers can both reduce the environmental impacts of product disposal and increase the value of products taken back at end of life which results in decreasing remanufacturing cost of manufacturer (Fishbein *et al.*, 2000). The companies have also difficulties to gather the sufficient amount of used products of satisfactory quality to be overhauled. Furthermore, they have little control on the return flow in terms of quality and timing. Even though the collection of the used products is a difficult task, a successful remanufacturing firm must carefully manage its product acquisition process, i.e. buy the right quantities of the right qualities for the right prices, so as to maximize profits (Guide *et al.*, 2003). It is possible for a firm to manage the quality of product returns by offering financial incentives (Guide and Wassenhove, 2001). As can be seen from the cellular phone example by Guide and Wassenhove (2001), the remanufacturer may choose not to buy cellular phone of a lower quality. The seller may respond by offering the lower quality cellular phones for a lower price, or finding a buyer that will accept the lower levels of quality. Therefore, a seller of used products may grade the returned products and price the product accordingly (Guide and Wassenhove, 2001). Guide *et al.* (2003) have examined that the profitability of remanufacturing depends on the quantity and quality of product returns which can be influenced by varying quality-dependent acquisition prices and on the demand for remanufactured products which can be influenced by varying the selling price. Offering price incentives such as trade-in rebates can serve as a significant tool in order to influence quality, quantity, and timing of the product returns that are the primary concerns of remanufacturing initiatives (Çelebi, 2005). Trade-in rebate strategy hastens purchase decisions of customers who are willing to replace their existing product with a new one or purchase a second one (Ray *et al.*, 2005). In addition to trade-in rebate strategy, leasing can also provide the manufacturer with a continual, predictable flow of post-consumer feed-stock for its remanufacturing activities. Moreover, it may help the manufacturer to forecast the quality of the returned product and schedule the product line according to their return time. Greater involvement with leasing also provides manufacturers with greater control over the resale market, because secondary market prices and equipment availability can impact new product sales and pricing. There are many types of incentive systems used in U.S. remanufacturing sector, including deposits, credit toward a remanufactured or new unit, and cash for product returns except motives mentioned above.

The literature on remanufacturing focused on operational issues that arise in inventory management and production control as a result of the return flows of used products. Fleischmann *et al.* (1997) focus on three main areas, namely distribution planning, inventory control and production planning. For each of these, they discuss the implications of the emerging reuse efforts and review the mathematical models in the literature. Reverse logistics was a very young field at that time, and they conclude that many reuse or recycling activities required new planning methods and more comprehensive approaches than those that had been used up to that time.

There is a growing literature, which combines remanufacturing, pricing of new and remanufactured products, competition and marketing. These studies roughly seek optimal selling prices of remanufactured and new products that maximizes the profit of the company where remanufacturing is possible. Groenevelt and Majumder (2001) developed a two-period model to examine the effect of competition in remanufacturing considering one OEM and a local remanufacturer. When remanufacturable products are returned by the consumers, local remanufacturers can access used items before original equipment manufacturer (OEM). Therefore, the manufacturer can consider either to restrict the local remanufacturer's access to used items, or increase their cost of remanufacturing (or both). Groenevelt and Majumder (2001) consider one OEM and a local remanufacturer. In the first period, only the OEM manufactures and sells items. In the second period, a fraction of these items are returned for remanufacturing. However, some returned items are used up by the local remanufacturer. Thus, competition exists in the second period for remanufacturing returned items and selling them. The reverse logistics process is based on the "shell allocation mechanism" observed in the respective market. Four of these mechanisms are considered: whether each of the players (the OEM and the independent operator) can or cannot use the cores that are not utilized by the other company. The state of the world is determined by a single parameter- quantity of returns from the first period. This model captures the essential features of remanufacturing, a finite product lifetime, and competition in selling the product. The critical trade-offs for the OEM are between the lower cost of remanufacturing in the second period against the presence of increasing competition from the local remanufacturers. Results show that the presence of competition results in the OEM to manufacture less in the first period, and attempt to increase the local remanufacturer's cost of remanufacturing which reduces the competition in the second period. However, while the local remanufacturer competes with the OEM in selling the items, she also helps the OEM reduce his remanufacturing cost because any action which makes remanufacturing attractive to OEM induces him to manufacture more in the first period, and hence makes it possible for the local remanufacturer to produce more in the second period.

The effect of competition on recovery strategies has also examined by Ferguson and Toktay (2004) with some differences. Manufacturers often face a choice whether to recover the value in their end-of-life products through remanufacturing or not to recover driven by two concerns: cost and internal cannibalization. On the cost side, the cost to remanufacture plus the fixed cost needed to establish a remanufacturing operation may be too high to enter remanufacturing. However, even if the remanufactured product is independently profitable, firms may ignore this option due to concerns about cannibalization: if the remanufactured product is sold in the same market as the new product, it attracts the same customer population. In addition, if it is priced lower than the new product, customers may choose the remanufactured product instead. If the manufacturer chooses not to remanufacture due to concerns about internal cannibalization even though it is independently profitable, third-party remanufacturers may enter the market, resulting in external competition rather than internal cannibalization. To respond this threat, two-entry deterrent strategies are developed by Ferguson and Toktay (2004): remanufacturing and preemptive collection. Preemptive collection is a strategy to discourage competition so that manufacturer collects part or all of the items and does not recover the residual value of the used product, instead discards them. They find that a firm may prefer to remanufacture or preemptive collection to deter entry, even when the firm would not have chosen to do so under a pure monopoly environment. There are also cases where it is more beneficial for the firm to collect the used products but not remanufacture them. In this case, collection is used as a deterrent strategy to avoid competition from third-party remanufacturers. They conclude that the choice to remanufacture should be considered as part of an OEM's competitive strategy. They find that if collection is a major portion of the total remanufacturing fixed and/or variable cost, the manufacturer is better off remanufacturing. As the unit cost increases, the relative advantage of the remanufacturing strategy increases. In addition, as market acceptance (relative willingness-to-pay for remanufactured product) increases, the relative profitability of the remanufacturing strategy increases.

The study of Debo *et al.* (2005) has important insights since it is the first study that addresses the integrated market segmentation and production technology choice problem in a remanufacturing setting where the supply of used products that can be remanufactured depends on the past sales volumes of new products and the level of remanufacturability. Previous papers in the literature take the remanufacturability level as exogenously determined while this paper introduces the level of remanufacturability as a key variable. They solve the joint pricing and production technology selection problem faced by a manufacturer that considers introducing a remanufacturable product in a market that consists of heterogeneous customers. In the model, production technology selection determines the remanufacturability level. The customer preferences are explained through a maximum utility approach. Moreover, they try to answer the question how competition with independent remanufacturers should be taken into account when determining the remanufacturability level, because manufacturer can control the remanufacturability level, and therefore control the supply of remanufactured products to independent remanufacturers. In the decision making framework, the manufacturer's goal is to maximize the net present value of introducing a remanufacturable product, by determining level of remanufacturability and a sequence of prices for new and remanufactured products where the manufacturer is a monopolist in the market or there is a competition in the remanufactured product market. To model these, they develop a discrete-time, infinite-horizon, discounted-optimization problem. They demonstrate that the consumer profile plays a role in the determination of the profitability of remanufacturing. Therefore, it would be very useful in practice to invest in understanding the market well before launching a remanufacturable product. They characterize a specific role of the new product: New products may be sold below unit cost to generate a supply of remanufactured products. Furthermore, a decrease in the unit remanufacturing cost may lead to an increase in the new product sales volume, to supply remanufactured products in response to increased demand for them. When there is a competition on the remanufactured product market, the optimal level of remanufacturability offered by the manufacturer is lower than the monopoly model and decreases as the number of competing remanufacturers increases.

The work of Debo *et al.* (2005) resembles to the one of Esenduran (2004) because of the fact that Esenduran (2004) models the product line selection and pricing problem of an original equipment manufacturer with remanufacturing capability. She formulates a single constrained mixed-integer nonlinear programming model with the objective of maximizing profit using the probabilistic choice framework. The model assumes that the probability a customer selects a product is proportional to its utility and inversely proportional to its price. There exists a pool of predetermined candidate (manufactured and remanufactured and competitor's) products discriminated by their unit production costs and utilities and the population consists of a certain number of market segments of various sizes which are differentiated through the preferences of their customers. Each customer segment is homogenous within itself, namely the individuals within a segment have the same utility for a particular product in a product line and customers in each segment have the option of not buying any one of the products. This is a more realistic approach than forcing the total demand of a segment to be divided into among the products of the OEM and competitor. The problem has two sub-problems as pricing and product line selection. Therefore, she divides the problem into sub-problems and then solves them sequentially. The product line selection problem is solved via complete enumeration and genetic algorithm, whereas the pricing problem is solved a modified simplex search.

Most of the pricing models in the literature assume that the potential buyers buy the product for the first time. Although it is almost true for totally new products and consumable, most buyer make replacement purchases for durable goods. Replacement customers are influenced by the price as first-time buyers and what they perceive to be the “residual values” of their existing product. Therefore, firms usually adopt a price discrimination policy by offering a special discount referred to as a trade-in rebate only to replacement customers to hasten their purchase decisions. In addition to this, any return flow of products induced trade-in rebate may generate revenues through remanufacturing operations (Ray *et al.*, 2005). The optimal pricing /trade-in strategies for such durable and remanufacturable products were studied by Ray *et al.* (2005). Their framework integrates pricing decisions with the defining characteristics of such a durable good. These characteristics include the age profile of the current products in use, the durability of the product, the revenues associated with returns and relative size of the two customer segments which include replacement and first-time buyers. They study three pricing schemes: uniform prices referred to as a single price for all customers, age independent price differentiation between new and replacement customers offering a fixed trade-in rebate to all replacement customers and a fixed price for new customers, and age dependent price differentiation. When the firm has the option of charging age dependent prices, it would determine a unique optimal price for any given product age and therefore age dependent trade-in rebate for this product. In conclusion, there are some implications on the life-cycle pricing of durable, remanufacturable products. During the incubation phase, a product is likely to have large proportion of new customers and a very new age profile for existing products. Moreover, the design may be less stable for such a product. Therefore, customer segmentation and price differentiation are not critical under such conditions and uniform pricing to all customers is reasonable. One step further, during the growth phase, volume of replacement buyers increases and the firm also has more experience on the product technology, enabling it to improve durability as well as the efficiency of remanufacturing operations. Under such conditions, age-independent price discrimination is reasonable. When the product is in maturity phase, the firm is likely to have improved its design and operational efficiency, and it is appropriate to offer age-dependent price differentiation. Therefore, pricing strategy of the firm depends on the nature of the product/market as well as the characteristics of a durable good.

Çelebi (2005) has developed the single-period model of Ray *et al.* (2005) by considering a dynamic pricing policy where prices are determined periodically in multi-period setting. Hence, she has investigated the impact of past trade-in rebates on the future decisions of the firm. The potential market consists of a known population of first-time and replacement customers. In the first period, there are no replacement customers since the product is newly introduced one. In a certain period, the market size of the replacement customers depends on the purchase decisions of the first-time buyers up to that period. For insights obtained from experimental results, one can refer to Çelebi (2005).

The study of Mitra (2005) is really different from related literature so far. In all recent models, it was implicit that remanufactured products were sold along with new products in the primary markets at a price equal to or less than that of new products to satisfy customer demand. Mitra (2005) note that because of skepticism about the quality of remanufactured products, not all remanufactured products would be sold, and also there could be different quality levels of recovered products, which would draw different prices in the secondary market. They have discussed two pricing models in the context of recycled cellular phones in India to maximize the expected revenue from the recovered products. They have taken two quality levels, namely remanufactured which are "as good as new" and refurbished products which are of lower quality. The objective of this paper is to determine prices of the remanufactured and refurbished products such that the total revenue is maximized.

Ferrer and Swaminathan (2006) analyze a model where the remanufactured and the original products are not distinguishable to the customer. They analyze two-period and multi-period scenarios where the manufacturer only produces the new product in the first period, but has the option of making new and remanufactured products in subsequent periods. Next, they focus on the duopoly environment where an independent remanufacturer may intercept cores produced by the original manufacturer to sell remanufactured products in future periods. Their research differs from Groenevelt and Majumder (2001) in that they also consider a multi-period setting where the independent remanufacturer competes in the second and subsequent periods. In addition to this, in their core collection process, neither company can use the cores that are not used by its competitor (a situation similar to the third shell allocation mechanism introduced by Groenevelt and Majumder (2001)). They observe that as the marginal cost of remanufacturing decreases, the value of making new products in the first period increases, and the value of making new products in future periods decreases. In other words, if remanufacturing is very profitable, the firm tries to increase the available cores for remanufacturing later. This behavior does not change, whether the OEM is a monopolist or not, operating with any planning horizon. In addition to the two-period model, the optimal policy in the last period is similar in multi-period planning horizons.

The remanufactured product, which is cheaper substitute for the new product, is often put on the market during the life cycle of the new product and affects its sales dynamics. Debo *et al.* (2006) study the integrated dynamic management of a portfolio of new and remanufactured products that enter a potential market over the product life cycle. In order to study the joint diffusion of new and remanufactured products, they allow the product to have a finite residence time (duration of one use of the product by a customer) that is shorter in expectation than the life cycle (time horizon over which the product stays on the market) of the product. This cause a remanufacturing opportunity and possible repeat purchases of either new or remanufactured products by customers. The joint pricing of new and remanufactured products has been studied in Debo *et al.* (2005) in an infinite-horizon setting with instant diffusion. Groenevelt and Majumder (2001), Ferrer and Swaminathan (2006) and Ferguson and

Toktay (2004) develop models considering price competition between a manufacturer and an independent remanufacturer in a two-period setting, but these papers do not incorporate life-cycle dynamics. Debo *et al.* (2006) analyze life-cycle dynamics of new and remanufactured products and investigate the impact of various managerial levers (remanufacturability level, capacity structure and reverse channel responsiveness) on profitability.

3. PROBLEM DESCRIPTION AND MODEL FORMULATION

In this chapter two different models are described in detail. First, we explain the basic setup of our main model considering consumer's purchase decisions through a maximum utility type approach and then present the second model considering linear relation between price and demand.

3.1. Base Model

We study the optimal pricing strategy in a multi-period setting for a profit-maximizing firm leasing new, durable and remanufacturable products and selling remanufactured products. We state our assumptions about product, consumer, lease contract, decision criteria, remanufacturable product supply and cost structure.

3.1.1. Product

The product that we consider is durable and the manufacturer offers only one type of new durable product rather than a diversified product line. The assumption is reasonable since the competition between the new products in a product line is not our interest. The product is remanufacturable and must undergo a remanufacturing operation before being sold as a remanufactured product. Moreover, we assume that a remanufacturable product can be remanufactured only once.

A new product can only be leased by the original manufacturer, and when it is returned by the lessee, it becomes a used product. When a used product is remanufactured by the manufacturer, it can only be sold. In other words, a remanufactured product cannot be leased. We assume that the duration of lease agreements, L, cannot exceed the life cycle of the product. Here, we allow the product to have a shorter residence time (duration of one use of the product by a lessee) than the life cycle, M, (time horizon over which the product stays on the market) of the product in order to study remanufacturing of used products.

The durability of the product also suggests that second-hand markets may play an important role, because secondary market prices and equipment availability can impact new product sales and pricing, but leasing also provides manufacturers with greater control over the resale market. Therefore, we do not take into account the availability of the second-hand market. This assumption is appropriate since customers have to return their used products to the manufacturer at the end of the lease term and remanufactured products can only be produced by the original manufacturer under the assumption that the manufacturer has a proprietary remanufacturing technology that would limit the formation of a market for used remanufacturable products.

During lease agreements, products deteriorate by a factor d_m where m is the index for lease periods expressed in year. It should be pointed out that deterioration and depreciation are used interchangeably in this thesis. We only need depreciation rates while determining leasing payments. For a customer who leases a new product, the perceived residual value of the product at the end of m periods is assumed to be a function of selling price p_n of the new product and the depreciation rate of the product given by the term $p_n(1 - d_m)$. Note that if d_m increases, residual value decreases and since the returned product will have less value, payments will increase. In other words, a higher level of depreciation requires higher leasing payments. In this setting, it is important to note that these rates are exogenous to the system.

In a multi-period setting, we allow product prices to be time-dependent.

3.1.2. Consumer

We assume that consumers typically differ in their willingness-to-pay (valuations). For this reason, we associate with each consumer his or her willingness-to-pay θ for having a new product. We assume that θ is distributed uniformly in the interval $[0, 1]$ and that in any period, each consumer uses at most one unit. The uniform assumption represents a large degree of variability within customer market and has become a standard assumption in the marketing literature (e.g., Debo *et al.*, 2005; Debo *et al.*, 2006). Let $f(\theta)$ and $F(\theta)$ be the density and cumulative distribution function of θ, respectively.

Typically, consumers value remanufactured products less than new products. Therefore, we assume that each consumer's willingness-to-pay for a remanufactured product is a fraction δ of their willingness-to-pay for the new product. Note that if $\delta = 0$, consumers are not willing to pay anything for the remanufactured product; this eliminates the option of remanufacturing and selling remanufactured products. If $\delta = 1$, consumers view the new and remanufactured units as being identical and are willing to pay the same amount for either product. Most products fall between the two extremes. Therefore, we assume $0 < \delta < 1$.

The relative willingness-to-pay for remanufactured product is either due to customer concerns about quality or because of a "fair price" perspective – if it costs less for the manufacturer to remanufacture the product than to make it, the customer wants that reflected in the price (Ferguson and Toktay, 2004). In our model, we assume that new and remanufactured products are of equal quality; the lower willingness-to-pay is only due to consumer perception. Therefore, a consumer of type $\theta \in [0, 1]$ has a valuation of θ for a new product and $\delta\theta$ for a remanufactured product.

The potential market size is normalized to one in each period so that the volume of consumers who prefer new and remanufactured products is less than or equal to one. In other words, a consumer of type θ can choose either to lease a new product, or to buy a remanufactured product, or nothing. In our model, the potential market consists of a known segment of customers such that customers who desire to lease a new product for m periods are distributed with a known fraction denoted by $\alpha = (\alpha_1, \alpha_2, ..., \alpha_L)$ with $\sum_{m=1}^{L} \alpha_m = 1$ in the population. Then, in our demand model, each consumer in each segment decides whether to lease a new product or to buy a remanufactured product or to prefer nothing based on their net utility. In other words, in a given period, consumers determine which product to choose based on the utility that they derive in that period from this decision. So, in this choice model, we obtain three regions in each segment: new product leased for m periods, remanufactured product and no product. We obtain the quantity of new products by aggregating the quantity of leased products in each segment. Since there can be consumers who prefer nothing because of their negative utility in each segment, the demand may not reach the target market size in the aggregate.

So far, we explained that we associate with each consumer his or her willingness-to-pay for buying a new product, θ, and divide the set of potential customers into segments each one having its own set of attributes including different reservation prices. In this setting, it is important to note that each consumer's willingness-to-pay for leasing a new product for m periods will be a fraction l_m of their willingness-to-pay for buying it. Note that if a consumer has a valuation θ for buying a new product, then he/she has a valuation $l_m\theta$ for leasing this product for m periods. Therefore, we assume that if a product is leased throughout its life cycle, $l_m = 1$. Here, it is important to emphasize that these rates are exogenous to the system.

We also assume that customers are myopic, that is they do not take into account the future path of prices when making leasing or purchasing decisions. They have no knowledge about future price offers; so, they cannot anticipate future. Hence, myopic (or non-strategic) customer behavior allows us to ignore any detrimental effect of future price cuts on current customer preferences.

3.1.3. Leasing Contract

A lease can be defined as a contract in which the owner of property grants to a customer the right to use the property for a specified period of time in exchange for an agreed upon periodic payment. Recall that the duration of lease agreements, L, cannot exceed the life cycle of the product, M. In this setting, for instance, if $M = 5$, L can be at most five and the manufacturer offers lease contracts for $m = 1, ..., L$ periods. At the end of the lease term, customers have to return products to the owner. Hence, leasing provides the manufacturer a continual, predictable flow of post-consumer feed-stock for its remanufacturing activities.

3.1.3.1. Monthly Lease Payment Model. One way to calculate lease payments is to assume that they are paid monthly in each period (http://carbuyingtips.com/regm.xls). First of all, in writing a lease, manufacturer must estimate what the equipment will be worth at the end of the lease term, but the estimation of residual values is one of the crucial determinants of profitability of manufacturer.

Residual value, RV, is the wholesale worth of a product at the end of its lease term after it has depreciated and would be given by

$$RV = p_n(1 - d_m). \tag{3.1}$$

Lease payments are made up of two parts: a depreciation charge and finance (rent) charge. The depreciation part of each periodical payment compensates the manufacturer for the portion of product's value that is lost during the lease. The finance part is interest on the money the manufacturer (lessor) has tied up in the product while the lessee is using it. In effect, the lessee borrows the money from the lessor and repays part of the money in monthly payments. At the end of the lease term, the lessee repays the remainder when she either buys or returns the product, but, in our model, the lessee has to return the product.

With some modifications on the calculation method of monthly lease payments (carbuyingtips.com/regm.xls), the net capitalized cost used for calculating depreciation and finance part of leasing payments equals to the selling price p_n of the new product ignoring other charges, fees, rebates, credits, cash down payments etc. to simplify the calculation of leasing payments.

Therefore, depreciation part, DP, and finance part, FP, of leasing payments given in Equations (2.1) and (2.2) can be written as

$$DP = p_n d_m, \tag{3.2}$$

$$FP = (p_n + RV)MF12m. \tag{3.3}$$

As mentioned before, MF is a number often used to calculate the average monthly rent charge portion of the lease payment. It is important to note that the lower the money factor, the lower the monthly lease payments. If annual interest rate is i, then money factor can be written as

$$MF = \frac{i}{2400}. \tag{3.4}$$

Lease term m is the length of the time the product is leased and expressed in year. Since payments are usually calculated monthly, we multiply m by 12. In other words, one-period leasing offered by the manufacturer is 12 months.

As a result, we model lease payments as a function of selling price p_n taking into account depreciation rate d_m and money factor MF and obtain base monthly payments p_m for m periods as given in Equation (2.3).

The base monthly payments are mathematically given by

$$p_m = p_n \left(\frac{d_m}{12m} + (2 - d_m)\, MF \right) \qquad m = 1, ..., L. \tag{3.5}$$

Here, p_m refers to monthly payments of a customer who leases a new product for m periods. For instance, if he/she leases a product for one or two years, he/she pays, respectively, p_1 12 times or p_2 24 times.

Present value $PV\left(p_m\right)$ of lease payments is given by

$$PV\left(p_m\right) = \frac{\beta_m\left(1-\beta_m^{12m}\right)}{1-\beta_m} p_n \left(\frac{d_m}{12m} + \left(2-d_m\right) MF\right) \qquad m = 1, ..., L. \tag{3.6}$$

The term $\frac{\beta_m\left(1-\beta_m^{12m}\right)}{1-\beta_m}$ is the coefficient of present value of equal payments and β_m is the monthly discount factor in the interval $[0,1]$ given by

$$\beta_m = \frac{1}{1+\frac{i}{12}} \qquad m = 1, ..., L. \tag{3.7}$$

3.1.3.2. Yearly Lease Payment Model. Another way to calculate lease payments is to assume that they are paid yearly at the end of each lease period and to write p_m as

$$p_m = p_n \frac{1}{M} \qquad m = 1, ..., L, \tag{3.8}$$

where M is the life cycle of the product, namely, time horizon over which the product stays on the market. As can be seen easily, the same amount is paid for each period. This equation states that if a product is leased for M periods, lease payments, in total, will be equal to the selling price p_n of the product, but due to the discount factor, present value $PV(p_m)$ of lease payments is less than p_n and given by

$$PV(p_m) = \frac{\beta(1-\beta^m)}{1-\beta} p_n \frac{1}{M} \qquad m = 1, ..., L, \tag{3.9}$$

where β denotes the discount factor over this time period in the interval $[0,1]$ given by

$$\beta = \frac{1}{1+i}. \tag{3.10}$$

Unless otherwise stated, we use monthly lease payment model in our analysis.

3.1.4. Decision Criteria

We model the consumer's purchase decisions through a maximum utility type approach. Recall that we assume that each customer has an inherent maximum price in his mind that he would be willing to pay for the product, which we denote as willingness-to-pay, θ. θ differs from individual to individual, but does not change from one period to another since the product is a durable one. By letting p_n and p_r denote the prices of new and remanufactured products, respectively, we define $\mathbf{p} = (p_n, p_r)$, where $0 \leq p_n$ and $0 \leq p_r \leq \delta p_n$. This is because if p_r were larger than δp_n, no remanufactured products would be sold and the price could be reduced to the level δp_n without affecting the demand for either product.

Our demand model is inspired by Debo *et al.* (2005). They model the net utility that a customer of type θ derives from buying a new product, a remanufactured product, and no product by $\theta - p_N$, $\eta(\theta) - p_R$, and 0, respectively.

In our model, consumers do not have the option of buying a new product. They can only lease a new product for m periods or buy a remanufactured product. Hence, the set of consumers who lease a new product for m periods is given by

$$\Omega_{n,m}(\mathbf{p}) = \{\theta \in [0,1] : l_m\theta - PV(p_m) \geq \delta\theta - p_r\} \qquad m = 1, ..., L. \tag{3.11}$$

In this setting, any customer will make a lease agreement only if both his/her willingness-to-pay for leasing a new product $l_m\theta$ is higher than or equal to the present value $PV(p_m)$ of lease payments, and also the net utility from leasing a new product for m periods is higher than that from buying a remanufactured product. $\Omega_{r,m}(\mathbf{p})$ is defined analogously as the set of consumer types who purchase a remanufactured product instead of leasing a new product for m periods.

Let q_n and q_r denote the volume of consumers who lease new products and purchase remanufactured products, respectively, and define $\mathbf{q} = (q_n, q_r)$. Recall that α_m denotes the size of the customer segment in the population who desires to lease a new product for m periods or buy a remanufactured product. Then,

$$q_n = \sum_{m=1}^{L} \alpha_m \int_{\Omega_{n,m(\mathbf{p})}} dF(\theta), \tag{3.12}$$

and

$$q_r = \sum_{m=1}^{L} \alpha_m \int_{\Omega_{r,m(\mathbf{p})}} dF(\theta). \tag{3.13}$$

By construction, $\mathbf{q} \in D = \left\{(q_n, q_r) \in \mathbb{R}_+^2 : q_n + q_r \leq 1\right\}$.

The volume of consumers who lease a new product for m periods is given by

$$q_m = \alpha_m \int_{\Omega_{n,m(\mathbf{p})}} dF(\theta) \qquad m = 1, ..., L. \tag{3.14}$$

Recall that if a consumer in a segment has positive utility neither from leasing a new product nor from buying a remanufactured product, he/she prefers nothing and the volume of consumers who prefer no product in each segment is denoted by $q0_m$. In other words, $q0_m$ is the volume of customers in each segment not preferring any one of the new or remanufactured products. This is a more realistic approach than forcing the total demand of a segment, α_m, to be divided among products.

If $(l_m - \delta)$ is positive, we investigate the volume of customers who lease a new product for m periods according to $\theta \geq A_1$ where

$$A_1 = \frac{p_n S_m - p_r}{l_m - \delta}, \tag{3.15}$$

and

$$S_m = \frac{\beta_m \left(1 - \beta_m^{12m}\right)}{1 - \beta_m} \left(\frac{d_m}{12m} + (2 - d_m)\, MF\right) \qquad m = 1, ..., L. \tag{3.16}$$

If we use yearly lease payment model, S_m can be written as

$$S_m = \frac{\beta(1-\beta^m)}{1-\beta}\frac{1}{M} \qquad m = 1, ..., L. \tag{3.17}$$

The volume of customers who prefer leasing a new product for m periods or none of the products is given in Table 3.1 provided that $l_m - \delta > 0$. In this setting, there are customers who are indifferent between buying nothing and a remanufactured product (having $\theta_l(\mathbf{p})$) and indifferent between preferring a remanufactured product and new product (having $\theta_h(\mathbf{p})$) for each lease period. Therefore, $\Omega_{r,m}(\mathbf{p}) = [\theta_l(\mathbf{p}), \theta_h(\mathbf{p})]$ and $\Omega_{n,m}(\mathbf{p}) = [\theta_h(\mathbf{p}), 1]$.

If $(l_m - \delta)$ is negative, we investigate the volume of customers who lease a new product for m periods according to $\theta \le A_2$ where

$$A_2 = \frac{p_r - p_n S_m}{\delta - l_m}. \tag{3.18}$$

The volume of customers who prefer leasing a new product for m periods or none of the products is given in Table 3.2 provided that $l_m - \delta < 0$. In this setting, there are customers who are indifferent between buying nothing and a new product (having $\theta_l(\mathbf{p})$) and indifferent between preferring a new product and remanufactured product (having $\theta_h(\mathbf{p})$) for each lease period. Therefore, $\Omega_{n,m}(\mathbf{p}) = [\theta_l(\mathbf{p}), \theta_h(\mathbf{p})]$ and $\Omega_{r,m}(\mathbf{p}) = [\theta_h(\mathbf{p}), 1]$.

If $l_m - \delta = 0$, we analyze the set where

$$(l_m - \delta)\,\theta \ge p_n S_m - p_r. \tag{3.19}$$

Thus, the volume of customers who prefer leasing a new product or none of the products is given in Table 3.3 provided that $l_m - \delta = 0$.

Table 3.1. The volume of customers leasing a new product or no product under $l_m - \delta > 0$

Case				q_m	$q0_m$
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	$0 \leq \frac{p_n S_m}{l_m} \leq 1$		$\alpha_m(1 - \max\left\{A_1, \frac{p_n S_m}{l_m}\right\})$	$\alpha_m(\min\left\{\max\left\{A_1, \frac{p_n S_m}{l_m}\right\}, \frac{p_r}{\delta}\right\})$
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	$\frac{p_n S_m}{l_m} > 1$	$\frac{p_r}{\delta} > 1$	0	α_m
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	$\frac{p_n S_m}{l_m} > 1$	$\frac{p_r}{\delta} \leq 1$	0	$\alpha_m \frac{p_r}{\delta}$
$l_m - \delta > 0$	$A_1 < 0$	$\frac{p_n S_m}{l_m} \leq 1$		$\alpha_m(1 - \frac{p_n S_m}{l_m})$	$\alpha_m \frac{p_n S_m}{l_m}$
$l_m - \delta > 0$	$A_1 < 0$	$\frac{p_n S_m}{l_m} > 1$		0	α_m
$l_m - \delta > 0$	$A_1 > 1$	-	$\frac{p_r}{\delta} > 1$	0	α_m
$l_m - \delta > 0$	$A_1 > 1$	-	$\frac{p_r}{\delta} \leq 1$	0	$\alpha_m \frac{p_r}{\delta}$

Table 3.2. The volume of customers leasing a new product or no product under $l_m - \delta < 0$

Case					q_m	$q0_m$
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$A_2 < \frac{p_n S_m}{l_m}$		$\frac{p_r}{\delta} > 1$	0	α_m
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$A_2 < \frac{p_n S_m}{l_m}$		$\frac{p_r}{\delta} \leq 1$	0	$\alpha_m \frac{p_r}{\delta}$
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$A_2 \geq \frac{p_n S_m}{l_m}$	$\frac{p_r}{\delta} \leq A_2$		$\alpha_m(A_2 - \frac{p_n S_m}{l_m})$	$\alpha_m \frac{p_n S_m}{l_m}$
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$A_2 \geq \frac{p_n S_m}{l_m}$	$\frac{p_r}{\delta} > A_2$	$\frac{p_r}{\delta} > 1$	$\alpha_m(A_2 - \frac{p_n S_m}{l_m})$	$\alpha_m(\frac{p_n S_m}{l_m} + 1 - A_2)$
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$A_2 \geq \frac{p_n S_m}{l_m}$	$\frac{p_r}{\delta} > A_2$	$\frac{p_r}{\delta} \leq 1$	$\alpha_m(A_2 - \frac{p_n S_m}{l_m})$	$\alpha_m(\frac{p_n S_m}{l_m} + (\frac{p_r}{\delta} - A_2))$
$l_m - \delta < 0$	$A_2 < 0$	-		$\frac{p_r}{\delta} > 1$	0	α_m
$l_m - \delta < 0$	$A_2 < 0$	-		$\frac{p_r}{\delta} \leq 1$	0	$\alpha_m \frac{p_r}{\delta}$
$l_m - \delta < 0$	$A_2 > 1$	$\frac{p_n S_m}{l_m} > 1$			0	α_m
$l_m - \delta < 0$	$A_2 > 1$	$\frac{p_n S_m}{l_m} \leq 1$			$\alpha_m(1 - \frac{p_n S_m}{l_m})$	$\alpha_m \frac{p_n S_m}{l_m}$

Table 3.3. The volume of customers leasing a new product or no product under $l_m - \delta = 0$

Case			q_m	$q0_m$
$l_m - \delta = 0$	$p_n S_m - p_r \leq 0$		$\alpha_m(1 - \frac{p_n S_m}{l_m})$	$\alpha_m \frac{p_n S_m}{l_m}$
$l_m - \delta = 0$	$p_n S_m - p_r > 0$	$\frac{p_r}{\delta} > 1$	0	α_m
$l_m - \delta = 0$	$p_n S_m - p_r > 0$	$\frac{p_r}{\delta} \leq 1$	0	$\alpha_m \frac{p_r}{\delta}$

3.1.5. Remanufacturable Product Supply

We model the remanufacturable product supply in the same way as Debo *et al.* (2006). In order to study the joint diffusion of new and remanufactured products, they assume that the product has a finite residence time (duration of one use of the product by a consumer) that is shorter in expectation than the product's life cycle (time horizon over which the product stays on the market). They allow the residence time to be variable and assume that it has a distribution characterized by $\mathbf{h} = (h_1, h_2, ..., h_L)$ with $\sum_{\tau=1}^{L} h_\tau = 1$. After τ periods of use, a fraction h_τ of new or remanufactured products are returned by the customer. They consider the residence time distribution as a given characteristic of the product.

In each period, remanufactured product sales are constrained by the availability of returning remanufacturable products. They consider that (previously) new and remanufactured products return from the market at the beginning of period t. Since they assume that remanufactured products cannot be remanufactured a second time, they need to be disposed of when they return. Moreover, all (previously) new products that return are not remanufacturable, therefore, a fraction $1 - q$ of those also need to be disposed of. Moreover, they assume that used product enters a reverse channel before it becomes available for remanufacturing. With this model, a used product becomes available in period $t + \tau + \Delta$ where Δ denotes the time that the product spends in reverse channel. In each period, the total volume of remanufactured products and disposed remanufacturable products cannot exceed the available remanufacturable product inventory I_t.

In our model, remanufactured product sales also depend on the availability of returning used products in each period, but we allow the manufacturer to obtain used products from the third-party core supplier when the resulting demand is greater than the available inventory. Therefore, there is no condition in our model as given in Debo *et al.* (2006). As remanufactured products are sold and cannot be remanufactured a second time, we assume that they do not return to the manufacturer, and, perhaps, they are disposed of by the user at the end of the usage. Only, the leased new products return at the end of the lease term. Let R_t denote the volume of used remanufacturable products that return from the market at the beginning of period t and given by

$$R_t = \sum_{m=1}^{\min(L,t)} q_{m,t-m}, \tag{3.20}$$

where $R_1 = 0$. The indices m and $t-m$ denote the lease duration and the beginning of the lease period of a new product, respectively. The product leased in time $t-m$ becomes available in period t.

Recall that all leased products have to return at the end of the lease term which results in a known stock of used remanufacturable products available (on hand) in period t. In contrast to the assumption given by Debo *et al.* (2006), we assume that used products are available for remanufacturing as soon as they return.

Let I_{t-1} be the volume of used products that remain in stock at the beginning of period t. Then,

$$I_t = \max(I_{t-1} + R_t - q_{r,t}, 0). \tag{3.21}$$

The inventory of remanufacturable products at the end of the current period is equal to the inventory at the beginning of the current period, plus the supply of remanufacturable products that become available at the beginning of period t, minus the amount remanufactured. If the prices in period t are chosen such that the resulting demand $q_{r,t}$ for remanufactured products is greater than the available inventory $(I_{t-1} + R_t)$, the shortage $\Delta_{r,t}$ in used products is obtained from the third-party core supplier. $\Delta_{r,t}$ is given by

$$\Delta_{r,t} = \max(q_{r,t} - I_{t-1} - R_t, 0). \tag{3.22}$$

In our model, the manufacturer holds a monopoly in the markets for new and remanufactured products, but recall that we allow the manufacturer to meet the shortage in used products from the third-party core supplier. This assumption is reasonable if the manufacturer has a proprietary remanufacturing technology that would limit the formation of a market for used remanufacturable products. We assume that the manufacturer buys only used products, and then remanufactures them in his facility.

3.1.6. Cost Structure

We assume that the average cost of remanufacturing increases in the quantity of the products remanufactured. This assumption is reasonable since used products arrive in different quality levels, so an increase in q_r forces to firm to remanufacture cores of decreasing quality levels (Ferguson and Toktay, 2004). The case of remanufacturing cost being convex increasing in the quantity has been identified in several studies (e.g., Guide and Wassenhove, 2001). To model this phenomenon, we assume that the total cost to remanufacture q_r units is $c_r q_r^2$ such that an average cost of remanufacturing q_r units becomes $c_r q_r$. In our model, as q_r increases, the manufacturer has to remanufacture cores of decreasing quality levels and therefore, average cost of remanufacturing increases. One of the reasons why average cost increases is that we allow the manufacturer to buy used products from third-party core supplier, so these products may be of much lower quality than those returned by lessee at the end of the lease term. The other reason is that there are different lease options. If products are used for a longer time, they depreciate more and hence residual value decreases. When q_r increases, the manufacturer has to use these low-quality products to remanufacture, which causes an increase in remanufacturing cost. The manufacturer can produce new units at a price of c_n each.

The cost of acquiring a used product from the third-party core supplier is given by c_r'. All returned and purchased remanufacturable products are remanufactured at an average cost of $c_r q_r$. In this setting, we assume that the unit cost of manufacturing, c_n, the unit cost of remanufacturing, $c_r q_r$, and the unit purchasing cost of used products, c_r', are constant over the life cycle of the product.

We allow the manufacturer to carry inventory of used products, but we do not consider associated holding costs to keep the focus on returns from lease agreements.

3.1.7. The Decision Making Framework

Our main analysis is for an industry in which the manufacturer holds a monopoly in the markets for new and remanufactured products. Our goal is to maximize the net present value of leasing a new product and selling a remanufactured product by determining a sequence of prices for these products. To this end we develop a discrete-time, multi-period, discounted profit optimization model. Each period corresponds to a period of lease agreement where lease period m expressed in year is the length of the time the product is used by the lessee. We assume that one-period leasing offered by the manufacturer is 12 months (one year) and lease agreements can differ from one to L periods, after which product needs to be remanufactured for further use.

The model consists of continuous variables, $\mathbf{p}_t = (p_{n,t}, p_{r,t})$ and $\mathbf{q}_t = (q_{n,t}, q_{r,t})$. Product prices are allowed to be time-dependent. The manufacturer chooses $\mathbf{p}_t = (p_{n,t}, p_{r,t})$ in period $t \geq 0$. Let β denote the discount factor over this time period. Thus, the longer the time on the market, the lower the discount factor should be (Debo *et al.*, 2005).

It is important to note that we omit t in our revenue and profit functions for the sake of convenience in understanding.

3.1.7.1. Formulation of the Manufacturer's Optimization Problem. If we use monthly lease payment model, the per-period revenue is given by

$$r_{\beta_m}(\mathbf{p}) = \left[\sum_{m=1}^{L} \frac{\beta_m \left(1 - \beta_m^{12m}\right)}{1 - \beta_m} q_m(\mathbf{p}) p_m \right] + q_r(\mathbf{p}) p_r. \tag{3.23}$$

The profit obtained in a generic period under the decision $\mathbf{p} = (p_n, p_r)$ is given by

$$\Pi_{\beta_m}(\mathbf{p}) = r_{\beta_m}(\mathbf{p}) - c_n q_n(\mathbf{p}) - c_r q_r^2(\mathbf{p}) - c_r' \Delta_r. \tag{3.24}$$

Let $V_\beta(I)$ denote the optimal β-discounted multi-period profit of the manufacturer under the initial condition $I_0 = I$.

$$V_\beta(I) = \max \sum_{t=1}^{T} \beta^{t-1} \Pi_{\beta_m}(\mathbf{p}_t). \tag{3.25}$$

The above model is our main model; however, recall that we have another lease payment model formulated according to yearly payments. If we use this model type for lease payments, the per-period revenue is formulated as

$$r_\beta(\mathbf{p}) = \left[\sum_{m=1}^{L} \frac{\beta \left(1 - \beta^{m}\right)}{1 - \beta} q_m(\mathbf{p}) p_m \right] + q_r(\mathbf{p}) p_r, \tag{3.26}$$

and the profit obtained in a period under the decision $\mathbf{p} = (p_n, p_r)$ is written as

$$\Pi_\beta(\mathbf{p}) = r_\beta(\mathbf{p}) - c_n q_n(\mathbf{p}) - c_r q_r^2(\mathbf{p}) - c_r' \Delta_r, \tag{3.27}$$

and the optimal β-discounted multi-period profit of the manufacturer with the initial condition $I_0 = I$ becomes as below

$$V_\beta(I) = \max \sum_{t=1}^{T} \beta^{t-1} \Pi_\beta(\mathbf{p}_t). \tag{3.28}$$

The optimal solution to these maximization problems is the price path for new and remanufactured products, $\mathbf{p}_t^*$, to which corresponds to an optimal path of demands, $\mathbf{q}_t^*$.

3.2. The Model Considering Linear Price-Demand Relation

The difference of this model from the previous model is due to the demand model. The assumptions stated in Sections 3.1.1, 3.1.3, and 3.1.6 are valid for this model as well.

3.2.1. Consumer

As mentioned before, we assume that customers are myopic. Each consumer's willingness-to-pay for a remanufactured product is a fraction δ of their willingness-to-pay for the new product. Consumer willingness-to-pay is heterogeneous and uniformly distributed in the interval $[0, 1]$. In any period, each consumer prefers at most one unit. The market size is normalized to 1.

In the previous model, we discussed that the potential market consists of a known segmentation of customers. In this setting, customers who *wish* to lease a new product for m periods are distributed with a known fraction denoted by $\alpha = (\alpha_1, \alpha_2, ..., \alpha_L)$ with $\sum_{m=1}^{L} \alpha_m = 1$ in the population. Therefore, α_m denotes the potential customers in a segment. In this framework, if a customer in a segment derives higher utility from buying a remanufactured product than from leasing a new product or negative utility from leasing, he/she does not prefer leasing. For instance, if we assume that 20 per cent of potential market desires to lease a new product for one period and 40 per cent for two periods, we can obtain that only 50 per cent of customers in the first segment and 20 per cent of customers in the second segment decide to lease a new product and others choose remanufactured product or nothing. From this example, we can see that 10 per cent of the population leases a new product for one period, 8 per cent leases for two periods.

In this linear model there is not any segmentation of consumers. We assume that customers use the product for a lease period and return at end-of-lease period. Return fractions are exogenous to the system and have a distribution characterized by $\alpha'_m = (\alpha'_1, \alpha'_2, ..., \alpha'_L)$ with $\sum_{m=1}^{L} \alpha'_m = 1$. In this setting, α'_m denotes the fraction of customers who lease a new product for m periods, and therefore return the product after m periods.

3.2.2. Decision Criteria

Linear demand model was used in previous studies (e.g., Tsay and Agrawal, 2000; Gallego *et al.*, 2006). Gallego *et al.* (2006) illustrate commonly used linear demand model in Equation (3.29).

$$d_i(\mathbf{p}) = (a_i - b_i + c_i \sum_{j \neq i} p_j)^+, \tag{3.29}$$

where $a_i > 0, b_i > 0, c_i \geq 0$. They let n be the number of firms, which are indexed by $i = 1, ..., n$. The demand for each firm is specified as a function of prices. They assume that firm i's demand is strictly decreasing in its price and that products are gross substitutes.

Tsay and Agrawal (2000) consider a system of one manufacturer and two retailers selling the same product and propose a demand model for the retailers. It is assumed that each retailer i chooses its price p_i, and service level s_i and each retailer's demand is a function of it's price and service level as well as the difference of prices and the service levels of the two retailers.

We also propose a linear demand function as follows

$$q_n = a_n - \alpha_n p_n + \theta_r p_r, \tag{3.30}$$

and

$$q_r = a_r - \alpha_r p_r + \theta_n p_n, \tag{3.31}$$

where a_n and a_r are the potential size of new and remanufactured product's market, respectively. α_n and θ_r are the sensitivity of new product's demand to its price and remanufactured product's price, respectively, and α_r and θ_n are the sensitivity of remanufactured product's demand to its price and new product's price, respectively. $a_n + a_r$ is the total demand if the price for both products is zero.

The demand functions given in Equations (3.30) and (3.31) are not related to the relative willingness-to-pay for remanufactured product, δ. Ferguson and Toktay (2004) assume that if a consumer of type $\phi \in [0, 1]$ has a valuation of ϕ for a new product, his valuation for a remanufactured product is $\delta\phi$. They propose a linear inverse demand function which is derived from consumers' utility functions. This construction leads to the following formulation:

$$p_n = 1 - q_n - \delta q_r, \tag{3.32}$$

$$p_r = \delta\left(1 - q_n - q_r\right). \tag{3.33}$$

These functions capture the competition between the new product and the remanufactured product. As relative willingness-to-pay for remanufactured products increases, the price of the remanufactured product increases, but the price of the new product decreases as the two products become closer substitutes.

First, by using Equations (3.32) and (3.33), we can observe if the condition $p_r \leq \delta p_n$ is provided. For this reason, we replace p_n and p_r with price functions proposed by Ferguson and Toktay (2004).

$$\delta\,(1 - q_n - q_r) \leq \delta(1 - q_n - \delta q_r). \tag{3.34}$$

If this inequality is satisfied, $p_r \leq \delta p_n$. After some simplifications, the inequality can be written equivalently as follows:

$$q_r \geq \delta q_r. \tag{3.35}$$

Since we assume $0 < \delta < 1$, it is obvious that the condition is provided. Therefore, we conclude that the linear inverse demand functions include the price constraint in their model.

Second, in order to relate to the relative willingness-to-pay for remanufactured product in our demand models, we obtain linear demand functions from Equations (3.32) and (3.33).

$$q_n + \delta q_r = 1 - p_n, \tag{3.36}$$

$$\delta q_n + \delta q_r = \delta - p_r. \tag{3.37}$$

After solving these two equations with respect to q_n and q_r, we obtain

$$q_n = \frac{1 - \delta - p_n + p_r}{(1 - \delta)}, \tag{3.38}$$

and

$$q_r = \frac{\delta p_n - p_r}{\delta(1 - \delta)}. \tag{3.39}$$

After some algebraic manipulations, we obtain linear demand functions as

$$q_n = 1 - \frac{1}{1-\delta}p_n + \frac{1}{1-\delta}p_r, \tag{3.40}$$

and

$$q_r = -\frac{1}{\delta(1-\delta)}p_r + \frac{1}{1-\delta}p_n. \tag{3.41}$$

Note that if we compare our initial demand functions given in Equations (3.30) and (3.31) with their new versions, $a_n + a_r$ corresponds to 1,

$$\alpha_n = \frac{1}{1-\delta}, \tag{3.42}$$

$$\alpha_r = \frac{1}{\delta(1-\delta)}, \tag{3.43}$$

$$\theta_n = \frac{1}{1-\delta}, \tag{3.44}$$

$$\theta_r = \frac{1}{1-\delta}. \tag{3.45}$$

It is seen from Equations (3.40) and (3.41) that if the prices for both products are zero, the market size equals to 1, and $q_n = 1$, $q_r = 0$. This is because if new and remanufactured products were sold free of charge, then all consumers would prefer new product instead of remanufactured one.

Since $q_n + q_r$ is the total demand, the exact formulation of total demand can be written as

$$q_n + q_r = 1 - \frac{1}{\delta} p_r. \tag{3.46}$$

Note that if $p_r = 0$, each consumer surely chooses a product in the population. If the price of the remanufactured product increases, total demand decreases, and the volume of consumers who prefer nothing increases.

In this demand model type, we obtain the total demand for new products in a generic period. Using the new products' quantity, the volume of customers who lease the product for m periods is calculated as follows:

$$q_m = \alpha'_m q_n \qquad m = 1, ..., L. \tag{3.47}$$

3.2.3. Remanufacturable Product Supply

In the model which is set by considering price-demand relation, R_t at the beginning of period t is given by

$$R_t = \sum_{m=1}^{\min(L,t)} \alpha'_m q_{n,t-m}, \tag{3.48}$$

or

$$R_t = \sum_{m=1}^{\min(L,t)} q_{m,t-m}, \tag{3.49}$$

where $R_1 = 0$.

As discussed before, we assume that after m periods of use, a fraction α'_m of new products are returned by the lessee. Moreover, there is no difference in the formulation of I_t which is given in Equation (3.21). Recall that if the price $\mathbf{p}_t$ in period t is chosen such that the resulting demand $q_{r,t}$ for remanufactured products is greater than the available inventory $(I_{t-1} + R_t)$, the shortage $\Delta_{r,t}$ in used products is obtained from the third-party core supplier. $\Delta_{r,t}$ is given by Equation (3.22).

3.2.4. The Decision Making Framework

The main difference occurs due to the type of demand model in the revenue part generated through leasing new products. Except this, all settings are the same as the previous formulation of the manufacturer's optimization problem.

3.2.4.1. Formulation of the Manufacturer's Optimization Problem. If we use monthly lease payment model, the per-period revenue is given by

$$r_{\beta_m}(\mathbf{p}) = \left[\sum_{m=1}^{L} \frac{\beta_m \left(1 - \beta_m^{12m}\right)}{1 - \beta_m} \alpha'_m q_n(\mathbf{p}) p_m \right] + q_r(\mathbf{p}) p_r. \tag{3.50}$$

The profit obtained in a generic period under the decision $\mathbf{p} = (p_n, p_r)$ is given by

$$\Pi_{\beta_m}(\mathbf{p}) = r_{\beta_m}(\mathbf{p}) - c_n q_n(\mathbf{p}) - c_r q_r^2(\mathbf{p}) - c'_r \Delta_r. \tag{3.51}$$

Let $V_\beta(I)$ denote the optimal β-discounted multi-period profit of the manufacturer with the initial condition $I_0 = I$.

$$V_\beta(I) = \max \sum_{t=1}^{T} \beta^{t-1} \Pi_{\beta_m}(\mathbf{p}_t). \tag{3.52}$$

If we use yearly lease payment model, the per-period revenue becomes

$$r_\beta(\mathbf{p}) = \left[\sum_{m=1}^{L} \frac{\beta\,(1-\beta^m)}{1-\beta} \alpha'_m q_n(\mathbf{p}) p_m\right] + q_r(\mathbf{p}) p_r, \tag{3.53}$$

and the profit obtained in a period under the decision $\mathbf{p} = (p_n, p_r)$ is written as

$$\Pi_\beta(\mathbf{p}) = r_\beta(\mathbf{p}) - c_n q_n(\mathbf{p}) - c_r q_r^2(\mathbf{p}) - c'_r \Delta_r, \tag{3.54}$$

and the optimal β-discounted multi-period profit of the manufacturer with the initial condition $I_0 = I$ becomes as below

$$V_\beta(I) = \max \sum_{t=1}^{T} \beta^{t-1} \Pi_\beta(\mathbf{p}_t). \tag{3.55}$$

The optimal solution to these maximization problems is the price path for new and remanufactured products, $\mathbf{p}_t^*$, to which corresponds to an optimal path of demands, $\mathbf{q}_t^*$.

<u>3.2.4.2. Exploiting Properties of the Objective Function.</u> The profit in a generic period is given in Equation (3.51). If we write the function with respect to p_n and p_r, it becomes as follows:

$$\begin{aligned}
\Pi_{\beta_m}(\mathbf{p}) = &\left[\sum_{m=1}^{L} \frac{\beta_m\,(1-\beta_m^{12m})}{1-\beta_m} \alpha'_m (a_n - \alpha_n p_n + \theta_r p_r) p_n \left(\frac{d_m}{12m} + (2-d_m)\,MF\right)\right] \\
&+ (a_r - \alpha_r p_r + \theta_n p_n) p_r - c_n (a_n - \alpha_n p_n + \theta_r p_r) - c_r (a_r - \alpha_r p_r + \theta_n p_n)^2 \\
&- c'_r \max(0, (a_r - \alpha_r p_r + \theta_n p_n - I)).
\end{aligned} \tag{3.56}$$

For simplification, we use

$$U_m = \frac{\beta_m\,(1-\beta_m^{12m})}{1-\beta_m} \alpha'_m \left(\frac{d_m}{12m} + (2-d_m)\,MF\right) \qquad m = 1, ..., L. \tag{3.57}$$

Therefore,

$$\Pi_{\beta_m}(\mathbf{p}) = \left[\sum_{m=1}^{L} U_m(a_n p_n - \alpha_n p_n^2 + \theta_r p_n p_r)\right] + (a_r p_r - \alpha_r p_r^2 + \theta_n p_n p_r)$$
$$- c_n(a_n - \alpha_n p_n + \theta_r p_r) - c_r(a_r - \alpha_r p_r + \theta_n p_n)^2$$
$$- c_r' \max(0, (a_r - \alpha_r p_r + \theta_n p_n - I)). \tag{3.58}$$

$$\frac{\partial \Pi_{\beta_m}}{\partial p_n} = \left[\sum_{m=1}^{L} U_m(a_n - 2\alpha_n p_n + \theta_r p_r)\right] + \theta_n p_r + c_n \alpha_n - 2c_r \theta_n (a_r - \alpha_r p_r + \theta_n p_n) - c_r' \theta_n. \tag{3.59}$$

$$\frac{\partial \Pi_{\beta_m}}{\partial p_r} = \left[\sum_{m=1}^{L} U_m \theta_r p_n\right] + (a_r - 2\alpha_r p_r + \theta_n p_n) - c_n \theta_r + 2c_r \alpha_r (a_r - \alpha_r p_r + \theta_n p_n). \tag{3.60}$$

$$\frac{\partial^2 \Pi_{\beta_m}}{\partial p_n^2} = \left[\sum_{m=1}^{L} U_m(-2\alpha_n)\right] - 2c_r \theta_n^2. \tag{3.61}$$

$$\frac{\partial^2 \Pi_{\beta_m}}{\partial p_r^2} = -2\alpha_r - 2c_r \alpha_r^2. \tag{3.62}$$

$$\frac{\partial^2 \Pi_{\beta_m}}{\partial p_n \partial p_r} = \left[\sum_{m=1}^{L} U_m \theta_r\right] + \theta_n + 2c_r \theta_n \alpha_r. \tag{3.63}$$

The Hessian is $\begin{pmatrix} \left[\sum_{m=1}^{L} U_m(-2\alpha_n)\right] - 2c_r\theta_n^2 & \left[\sum_{m=1}^{L} U_m\theta_r\right] + \theta_n + 2c_r\theta_n\alpha_r \\ \left[\sum_{m=1}^{L} U_m\theta_r\right] + \theta_n + 2c_r\theta_n\alpha_r & -2\alpha_r - 2c_r\alpha_r^2 \end{pmatrix}$, whose leading coefficient is negative due to the negative sign. Since all parameters are positive, diagonal entries are positive, and the last element of Hessian is negative. When we assign numerical values to parameters, determinant is positive. Thus, the Hessian is negative definite and the profit function is concave in (p_n, p_r).

The Lagrangean is

$$
\begin{aligned}
L(p_n, p_r) = {} & \left[\sum_{m=1}^{L} U_m(a_n p_n - \alpha_n p_n^2 + \theta_r p_n p_r)\right] + a_r p_r - \alpha_r p_r^2 + \theta_n p_n p_r \\
& - c_n(a_n - \alpha_n p_n + \theta_r p_r) - c_r(a_r - \alpha_r p_r + \theta_n p_n)^2 \\
& - c_r' \max(0, (a_r - \alpha_r p_r + \theta_n p_n - I)) \\
& + \mu_1(a_n - \alpha_n p_n + \theta_r p_r) + \mu_2(a_r - \alpha_r p_r + \theta_n p_n).
\end{aligned} \tag{3.64}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial p_n} = {} & \left[\sum_{m=1}^{L} U_m(a_n - 2\alpha_n p_n + \theta_r p_r)\right] + \theta_n p_r + c_n \alpha_n \\
& - 2c_r \theta_n(a_r - \alpha_r p_r + \theta_n p_n) - c_r' \theta_n - \mu_1 \alpha_n + \mu_2 \theta_n.
\end{aligned} \tag{3.65}
$$

$$
\frac{\partial L}{\partial p_r} = \left[\sum_{m=1}^{L} U_m \theta_r p_n\right] + (a_r - 2\alpha_r p_r + \theta_n p_n) - c_n \theta_r + 2c_r \alpha_r(a_r - \alpha_r p_r + \theta_n p_n) + \mu_1 \theta_r - \mu_2 \alpha_r. \tag{3.66}
$$

Since the profit function is concave, necessary conditions and sufficient conditions for optimality are zero as well as

$$
\mu_1(a_n - \alpha_n p_n + \theta_r p_r) = 0, \tag{3.67}
$$

$$
\mu_2(a_r - \alpha_r p_r + \theta_n p_n) = 0, \tag{3.68}
$$

$$
\mu_1 \geq 0, \tag{3.69}
$$

$$
\mu_2 \geq 0. \tag{3.70}
$$

4. SOLUTION PROCEDURE

Since the resulting problem is a nonlinear constrained optimization problem (NLP), the optimal solution cannot be obtained easily. Moreover, due to the complex structure of the problem, we are not able to analytically determine the unique optimal prices and payments and it is not practical to work with the derivatives during the solution step. In general, when the objective function is nonlinear and non-differentiable, or it is not convenient to use the information obtained from differentiation, some direct search methods are preferred. One of them is the Nelder-Mead Simplex Search Method (Nelder and Mead, 1965). In summary, this method starts with a set of solutions and at each iteration a new candidate is generated, which is to be accepted or not. Most of the time a greedy decision is utilized so that the candidate is accepted if and only if it results in an improvement in the objective function, which brings the risk of being trapped into a local optimum. One common strategy to avoid this risk is to restart with different initial points.

4.1. Simplex Search

Since the simplex search is originally developed to solve unconstrained problems, we modify it in order to handle price constraint. We first give the steps of the simplex search algorithm and we will then explain the details of our implementation in the solution of the resulting problem.

1. Construction of the initial simplex: Choose points $\mathbf{p}^1, \mathbf{p}^2, ..., \mathbf{p}^{n+1}$ to form a simplex. Choose a reflection coefficient $\alpha > 0$, an expansion coefficient $\gamma > 1$, a contraction coefficient $0 < \lambda < 1$, and a shrinkage coefficient $\chi > 0$. Go to Step 2.
2. Initilization: Let $\mathbf{p}^{\min}, \mathbf{p}^{\max} \in \{\mathbf{p}^1, ..., \mathbf{p}^{n+1}\}$ such that $\Pi(\mathbf{p}^{\max}) = \max_{1 \leq h \leq n+1} \Pi(\mathbf{p}^h)$, $\Pi(\mathbf{p}^{\min}) = \min_{1 \leq h \leq n+1} \Pi(\mathbf{p}^h)$. Let $\bar{\mathbf{p}} = \frac{1}{n} \sum_{h=1 | \mathbf{p}^h \neq \mathbf{p}^{\min}}^{n+1} \mathbf{p}^h$. Go to Step 3.
3. Reflection: Let $\mathbf{p}^r = \bar{\mathbf{p}} + \alpha(\bar{\mathbf{p}} - \mathbf{p}^{\min})$.

If $\Pi(\mathbf{p}^r) \geq \Pi(\mathbf{p}^{\max})$, go to Step 4.

If $\Pi(\mathbf{p}^r) < \Pi(\mathbf{p}^{\max})$, but $\Pi(\mathbf{p}^r) \geq \min_{h|\mathbf{p}^h \neq \mathbf{p}^{\min}} \Pi(\mathbf{p}^h)$, then replace $\mathbf{p}^{\min}$ by $\mathbf{p}^r$ to form a new set of $n+1$ points and go to Step 6.

4. Expansion: Let $\mathbf{p}^e = \bar{\mathbf{p}} + \gamma(\mathbf{p}^{\mathbf{r}} - \bar{\mathbf{p}})$.

   Replace $\mathbf{p}^{\min}$ by $\mathbf{p}^e$ if $\Pi(\mathbf{p}^r) < \Pi(\mathbf{p}^e)$ and by $\mathbf{p}^r$ if $\Pi(\mathbf{p}^r) \geq \Pi(\mathbf{p}^e)$ to yield a new set of $n+1$ points and go to Step 6.
5. Contraction: Let $\mathbf{p}^c = \bar{\mathbf{p}} + \lambda(\hat{\mathbf{p}}^{\min} - \bar{\mathbf{p}})$,

   where $\hat{\mathbf{p}}^{\min}$ is defined as $\Pi(\hat{\mathbf{p}}^{\min}) = \max\left\{\Pi(\mathbf{p}^{\min}), \Pi(\mathbf{p}^r)\right\}$. If $\Pi(\mathbf{p}^c) \geq \Pi(\hat{\mathbf{p}}^{\min})$, replace $\mathbf{p}^{\min}$ with $\mathbf{p}^c$. If $\Pi(\mathbf{p}^c) < \Pi(\hat{\mathbf{p}}^{\min})$, replace $\mathbf{p}^h$ with $\mathbf{p}^h + \chi(\mathbf{p}^{\max} - \mathbf{p}^h)$ for $h = 1, ..., n+1$. Go to Step 6.
6. Termination: If $\left\{\frac{1}{n+1} \sum_{h=0}^{n} \left[\Pi(\mathbf{p}^h) - \Pi(\bar{\mathbf{p}})\right]^2\right\}^{1/2} < \varepsilon$, then stop and set $\mathbf{p}^{best} \leftarrow \mathbf{p}^{\max}$, else go to Step 2.

Simplex search uses a polyhedron with $n+1$ vertices for a problem with n variables to define the current simplex. Each vertex is represented by an n-dimensional vector. New candidate vectors are generated by reflections of some of the vectors and contractions around the vectors which correspond to a higher objective function value. The decision for the candidate vectors are made according to their objective value. By following this procedure, the simplex expands and contracts during the solution step and finally contracts to a single vector, which is a local optimum. We modify the simplex search so that after any update of the vertices of the simplex, the constraints are not violated. Therefore, the feasibility is preserved throughout the search.

4.1.1. Construction of the Initial Simplex

Construction of the initial simplex is done by defining a price vector $\mathbf{p}^h$ for each vertex of the simplex. Since we have two variables in our problem as p_n and p_r, we have to obtain $2T+1$ points $\mathbf{p}^1, \mathbf{p}^2, ..., \mathbf{p}^{2T+1}$ to form a simplex. For instance, if we solve a 5-period problem, we have 10 variables, therefore we will use 11 vertices to form the initial simplex. Moreover, each point consists of $p_{n,t}$ and $p_{r,t}$ for $t = 1, ..., T$ where the first T component of the price vector is constructed by $p_{n,t}$'s, and the rest of the vector by $p_{r,t}$'s. Therefore, we have $2T$ components at each vertex.

The first price vector is selected by generating prices arbitrarily in interval $[0, 1]$ considering also the price constraint which requires that $p_{r,t} \leq \delta p_{n,t}$ for $t = 1, ..., T$. The rest of the simplex is constructed by using Bazaraa *et al.* (1993)'s suggestion with some modifications since we have to also satisfy the price constraint for the rest of the vertices. Bazaraa *et al.* (1993) suggest that

$$\mathbf{p}^{h+1} = \mathbf{p}^h + \mathbf{d}^h \qquad h = 1, ..., n. \tag{4.1}$$

where n denotes the number of variables. Here $\mathbf{d}_h$ is a vector with h^{th} component is equal to a and all other components equal to b with

$$a = \frac{s}{n\sqrt{2}}\left(\sqrt{n+1} + n - 1\right), \tag{4.2}$$

$$b = \frac{s}{n\sqrt{2}}\left(\sqrt{n+1} - 1\right), \tag{4.3}$$

where s is a scalar.

Thus, after our modification, the first part of the vertices is constructed as follows

$$\mathbf{p}^{h+1} = \mathbf{p}^h + \mathbf{d}^h \qquad h = 1, ..., T, \tag{4.4}$$

where $\mathbf{d}_h$ is a vector with h^{th} component is equal to a and all other components equal to b and

$$\mathbf{p}^{h+1} = \mathbf{p}^h + \mathbf{d}^h \qquad h = T + 1, ..., 2T, \tag{4.5}$$

where $\mathbf{d}_h$ is a vector with $h^{th} - T$ component is equal to a and all other components equal to b with

$$a = \frac{s}{T\sqrt{2}}\left(\sqrt{T+1} + T - 1\right), \tag{4.6}$$

$$b = \frac{s}{T\sqrt{2}}\left(\sqrt{T+1} - 1\right). \tag{4.7}$$

Although we have $2T$ variables in our model, we set n to T while constructing components of $\mathbf{d}_h$ since we need only T components for the first part of each vertex.

After we obtain the first part of each vertex, we construct the second part by multiplying each $p_{n,t}$ with a number generated randomly in interval $[0, \delta]$ to satisfy the price constraint.

To avoid the risk of being trapped into a local optimum, we restart the simplex run 100 times with different step sizes, s. The values of $s = \{0.4, 0.5, 0.6, 0.7, 0.8\}$ works well. Here, we use $s = 0.4$ for initial 20 runs, and $s = 0.5$ for the following 20 runs, and etc. We select the best solution among the results of these 100 runs. The frequency of the local optima obtained during the search can be observed from standard deviation of the solution.

4.1.2. Reflection Step

After calculating the reflected point $\mathbf{p}^r$ $(\bar{\mathbf{p}}+\alpha(\bar{\mathbf{p}} - \mathbf{p}^{\min}))$, the feasibility of $\mathbf{p}^r$ should be checked. $\bar{\mathbf{p}}$ is already a feasible point since it is the centroid of a feasible simplex. However, while reflecting the worst point of the simplex through the feasible centroid, it is possible to fall out of the feasible region. If this occurs, we modify the reflected point moving the $p_{r,t}$ to δ times $p_{n,t}$. Thus, the infeasible remanufacturing price p_r of the reflected vector $\mathbf{p}^r$ is adjusted so that it becomes equal to δp_n. Here, the reflection coefficient α is set to one as suggested by Nelder and Mead (1965).

4.1.3. Expansion Step

After calculating the expansion point $\mathbf{p}^e$ $(\bar{\mathbf{p}}+\gamma(\mathbf{p}^{\mathbf{r}} - \bar{\mathbf{p}}))$, the feasibility of $\mathbf{p}^e$ should be checked. If $\mathbf{p}^e$ causes infeasibility with respect to price constraint, $p_{r,t} \leq \delta p_{n,t}$, we set $p_{r,t} = \delta p_{n,t}$. In this setting, the expansion coefficient γ is set to two as suggested by Nelder and Mead (1965).

4.1.4. Contraction Step

After calculating the contraction point $\mathbf{p}^c$ $(\bar{\mathbf{p}}+\lambda(\hat{\mathbf{p}}^{\min}-\bar{\mathbf{p}}))$, the feasibility of $\mathbf{p}^c$ should be checked. If $\mathbf{p}^c$ causes infeasibility with respect to price constraint, $p_{r,t} \leq \delta p_{n,t}$, we set $p_{r,t} = \delta p_{n,t}$. Here, the contraction coefficient λ is set to 0.5 as suggested by Nelder and Mead (1965).

As mentioned before in the contraction step, if the contracted point is not replaced with $\mathbf{p}^{\min}$, then every point is shrank by using $\mathbf{p}^h = \mathbf{p}^h + \chi(\mathbf{p}^{\max} - \mathbf{p}^h)$. Here, χ is the shrinkage coefficient and is chosen to be 0.5 as suggested by Nelder and Mead (1965).

Finally, note that in our experiments $\varepsilon = 10^{-14}$ is chosen to stop the algorithm.

5. EXPERIMENTAL RESULTS

In our experiments we will focus on the base model which is formulated according to the maximum utility type approach because of the fact that it is more realistic in the marketing environment. The experiments can be grouped into two main categories. In the first group, we solve single period problems where there is only 1-period option of leasing a new product in addition to buying a remanufactured product. Moreover, the examination of uncertainty in problem parameters is possible by solving multiple scenarios (with different parameter values). We perform sensitivity analysis with respect to the following parameters: consumers' relative willingness-to-pay for a remanufactured product, relative willingness-to-pay for leasing a new product, deterioration of the product in age, initial inventory level, and cost of supplying used products from the third-party core supplier. In the second group, we solve multi-period problems in order to analyze the effects of returns of previously leased products on the manufacturer's decision and profit. The effect of problem parameters mentioned above is also presented. In addition, multi-period problem will also be considered with the two-period lease option in order to investigate the trends on demands and prices.

Since we are not able to analytically determine the unique optimal prices and quantities, the problems are solved by the simplex search method explained in detail in the previous chapter. Small sized problems such as the single period problems can be solved by exhaustive search in terms of p_n and p_r. We note that simplex search algorithm produces the same results as the exhaustive search does. However, when the problem size increases, exhaustive search fails to find the optimum since even small changes in prices have an impact on the optimum profitability, and requires very long run times. Since we solve a maximization problem, we generate Hessian matrix in single period problem to examine the concavity of the objective function. The examination of jointly concavity of the objective function for single period problem is performed by using the first and second derivatives given in Appendix A.

The simplex algorithm is coded in Visual C++ 6.0 environment and the experiments are run on a Pentium M, 1.7 MHz machine with 512 MB of RAM. As the run times are considerably short, no result is given for the CPU times.

5.1. Single Period Problem

Single period optimization gives us general information about the problem parameters which affect the manufacturer's decision. Moreover, we can observe whether the objective function is jointly concave or not with respect to p_n and p_r. Due to the problem structure, it is not possible to determine the concavity of the objective function taking derivatives directly. Thus, we generate Hessian matrix H using second order derivatives and evaluate H numerically with the values obtained from the simplex search.

In this case, if determinant is positive and leading element is negative, then the Hessian is negative definite and the profit function is jointly concave in (p_n, p_r).

Since we are not able to equalize first derivatives of profit function to zero due to the model structure, we do not analytically determine the unique optimal prices and quantities. Therefore, we prove that we obtain the optimal prices solving the same problem by exhaustive search which produces the same solution as simplex search does.

In some cases, only one of the remanufactured or new products exists. In such cases, since Hessian is meaningless, we consider the first and second derivative of the profit function with respect to the price of the resulting product, i.e. remanufactured or new, such that if the second derivative of the profit function is negative, it is concave. Therefore, when we equalized the first derivative to zero, we would obtain optimal price which corresponds to optimal demand. However, as mentioned before, since we are not able to analytically determine the unique optimal price, we find it by numerical search. When we analyze the first derivative of the objective function with respect to this optimal price, it is almost zero in all cases.

Since we design our experiments considering only 1-period leasing option, the potential market consists of a segment of customers who desire to lease a new product for only one period, or buy a remanufactured product, or nothing, thus $\alpha_1 = 1$. Unless otherwise stated, some parameter values shown in Table 5.1 have been fixed for the analysis.

Table 5.1. Fixed parameter values common for all experiments

c_n	c_r	i
0.1	0.05	0.08

5.1.1. The Effect of Changes in the Relative Willingness-to-pay for Remanufactured Product

In this section, we keep all parameters fixed except δ to analyze the effect of its changes. Some parameter values have already been given in Table 5.1, and d_1, l_1, c_r' and I_0 are given in Table 5.2.

Table 5.2. Fixed parameter values to analyze the effect of changes in δ

d_1	l_1	c_r'	I_0
0.1	0.5	0.08	0

These values implies that a product loses 10 percent of its value in the first year ($d_1 = 0.1$), and a customer's valuation for leasing a new product for one period is half of his valuation for buying a new product ($l_1 = 0.5$). If the inventory of used products is not adequate to meet the demand for remanufactured products, the manufacturer purchases the necessary cores from the third-party supplier with cost $c_r' = 0.08$. We assume that there exists no inventory at the beginning of time horizon, $I_0 = 0$. We conduct our experiments in this section for different δ values ranging from 0.2 to 0.9.

Figure 5.1 illustrates the change in the optimal profit as δ increases. If δ is high, customers view the new and remanufactured products almost as being identical and are willing to pay almost the same amount for either product which results in an increase in the profit. When $l_1 > \delta$, the increase in this parameter does not affect the profit since there is no demand for remanufactured products. In this case profit does not change because q_n and q_r remain the same. When $l_1 \leq \delta$, the profit of the manufacturer increases at an increasing rate due to the increase of q_r and p_r. Optimal prices and demands are given in Table 5.3.

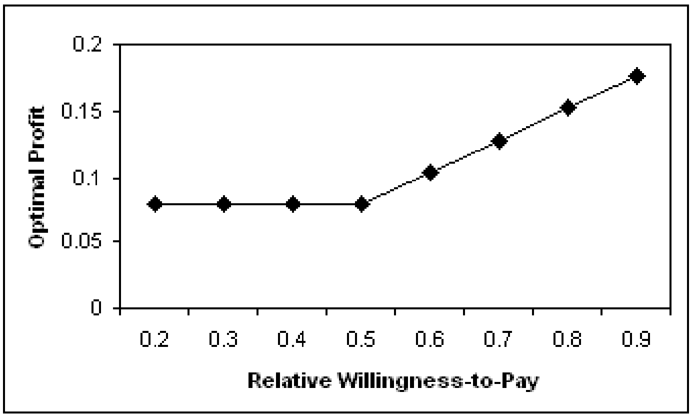

Figure 5.1. The effect of δ on the profit

Table 5.3. Optimal prices and demands for different δ values

δ	p_n	p_r	q_n	q_r	q_{No}
0.2	1.7793	0.1468	0.4000	0.0000	0.6000
0.3	1.7793	0.3159	0.4000	0.0000	0.6000
0.4	1.7793	0.6624	0.4000	0.0000	0.6000
0.5	2.1676	0.3091	0.0000	0.3818	0.6182
0.6	1.9000	0.3600	0.0000	0.4000	0.6000
0.7	2.3063	0.4107	0.0000	0.4133	0.5867
0.8	1.8197	0.4612	0.0000	0.4235	0.5765
0.9	1.9095	0.5116	0.0000	0.4316	0.5684

As long as $\delta < l_1$, there is no demand for remanufactured products as seen in Figure 5.2 since customers value remanufactured products less than new products which prevents the manufacturer from having profit from the remanufactured product. Thus, for low δ values, it is more profitable to encourage consumers to lease new products than to buy remanufactured products. When $\delta = l_1$, there is no demand for new products

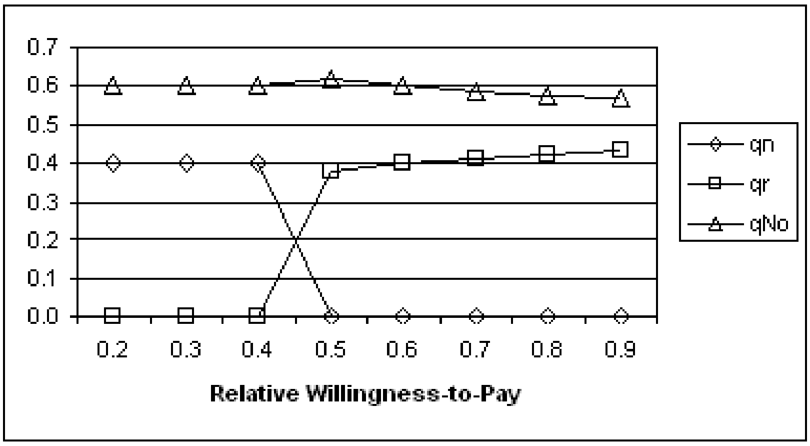

Figure 5.2. The effect of δ on the demands

and most of the consumers shift towards the remanufactured products. Thus, at this point, the volume of consumers who prefer nothing increases. From this point on, it is seen that as δ increases, the amount of remanufactured products increases and the volume of consumers who prefer nothing decreases.

If $\delta < l_1$, the price of new products does not change and price of remanufactured products is charged higher with respect to δ in order to create demand for only new products. However, if $\delta \geq l_1$, the price of remanufactured products increases to take advantage of the increased willingness-to-pay. In this case we observe that the profit of the manufacturer increases due to the increase of q_r and p_r as illustrated in Figure 5.3.

Finally, our experiments show that the manufacturer obtains higher profit from customer segments with higher relative willingness-to-pay for remanufactured products since higher prices are charged to take advantage of the increased willingness-to-pay. Moreover, the volume of customers who prefer nothing decreases.

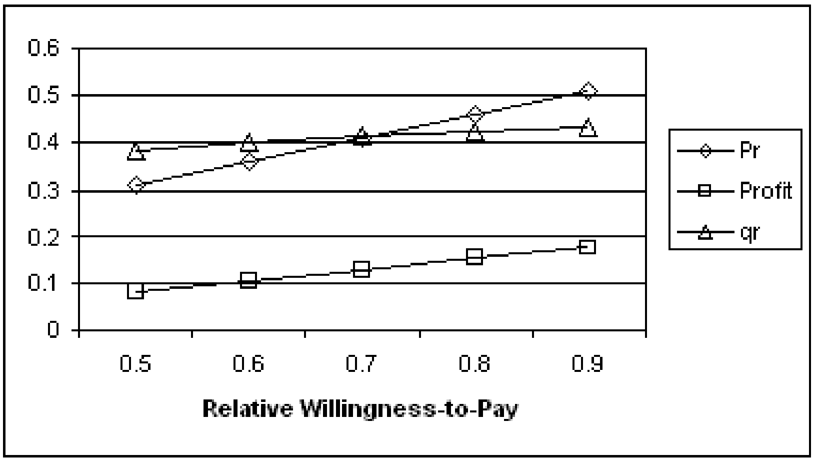

Figure 5.3. The effect of δ on q_r, p_r and profit if $\delta \geq l_1$

5.1.2. The Effect of Changes in the Relative Willingness-to-pay for Leasing a New Product

Each consumer's willingness-to-pay for leasing a new product for m periods is a fraction l_m of their willingness-to-pay for buying it. Since we only consider 1-period leasing in this section, $m = 1$ in our experiments. We keep all parameters fixed except l_1 to analyze the effect of its changes. We can observe the effect of l_1 in those cases when there is demand for new products. For instance, if $\delta = 0.8$, the volume of new products is zero for all values of l_1 from 0.1 to 0.5. Therefore, we set δ to 0.2 to be able to see the effects of l_1. Moreover, l_1 is at most set to 0.5, because if l_1 is greater than 0.5, consumers value 1-period leasing much more than it's worth which is not possible in real cases. The fixed parameter values to analyze the effect of consumer acceptance of leased product are given in Table 5.4.

Table 5.4. Fixed parameter values to analyze the effect of changes in l_1

d_1	δ	c_r'	I_0
0.1	0.2	0.08	0

When the effect of changes in l_1 is analyzed for $\delta = 0.2$, similar results are found to the cases while analyzing the effect of δ with $l_1 = 0.5$. This is because each consumer's willingness-to-pay for leasing a new product for one period and for buying a remanufactured product are fractions l_1 and δ, respectively, of their willingness-to-pay for buying a new product. Similar to the effect of δ, the price of the new product increases as l_1 increases to take advantage of increased willingness-to-pay for leasing. Therefore, the higher the l_1 value, the higher is the profit as illustrated in Figure 5.4. However, if $l_1 \leq \delta$, the increase in l_1 does not affect the profit since there is no demand for new products. Optimal prices and demands are given in Table 5.5.

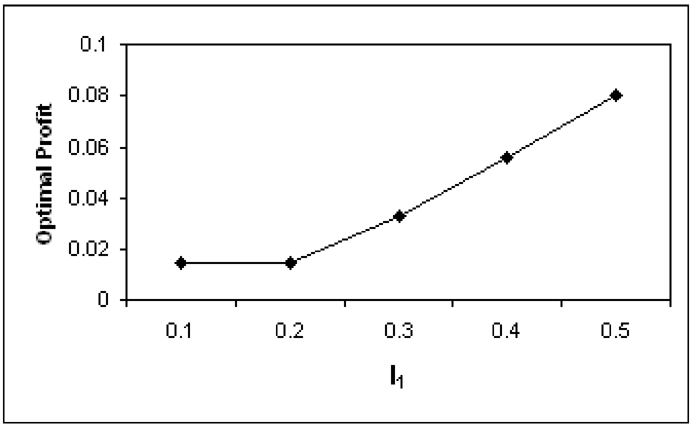

Figure 5.4. The effect of l_1 on the profit

Table 5.5. Optimal prices and demands for different l_1 values

l_1	p_n	p_r	q_n	q_r	q_{No}
0.1	1.5200	0.1520	0.0000	0.2400	0.7600
0.2	1.2400	0.1520	0.0000	0.2400	0.7600
0.3	1.1862	0.2372	0.3333	0.0000	0.6667
0.4	1.4828	0.2040	0.3750	0.0000	0.6250
0.5	1.7793	0.1926	0.4000	0.0000	0.6000

For $l_1 \leq \delta$, there is no demand for the new product since consumers have more utility from buying remanufactured product than leasing a new product due to the low perception on leasing. However, when $l_1 > \delta$, namely a customer's valuation for leasing a new product for one period is at 30 per cent of his valuation for buying a new product and thus greater than his valuation for buying a remanufactured product, new product is preferred by customers. From this point on, it is seen that as l_1 increases, the amount of new products increases and the volume of consumers who prefer nothing decreases. Figure 5.5 exhibits the trends on demands as l_1 increases.

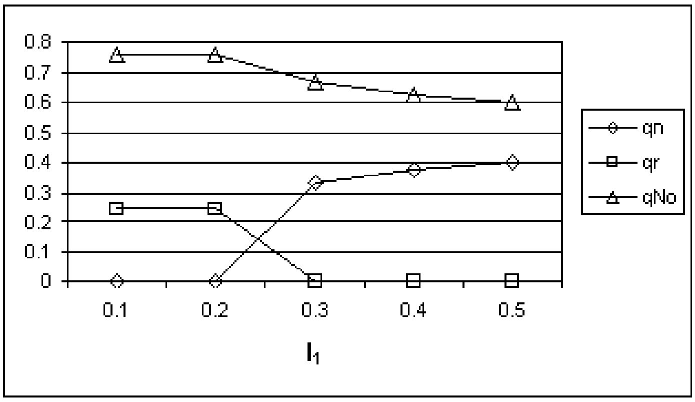

Figure 5.5. The effect of l_1 on the demands

In this phenomenon, it is important to note that if both l_1 and δ decrease, the volume of customers who prefer nothing increases which results in a decrease in the manufacturer's profit since both demands and prices decrease.

If we only analyze l_1 values where $l_1 > \delta$, we can see that as l_1 increases, the volume and the price of the new product increase to grow the profit which increases at an increasing rate as presented in Figure 5.6.

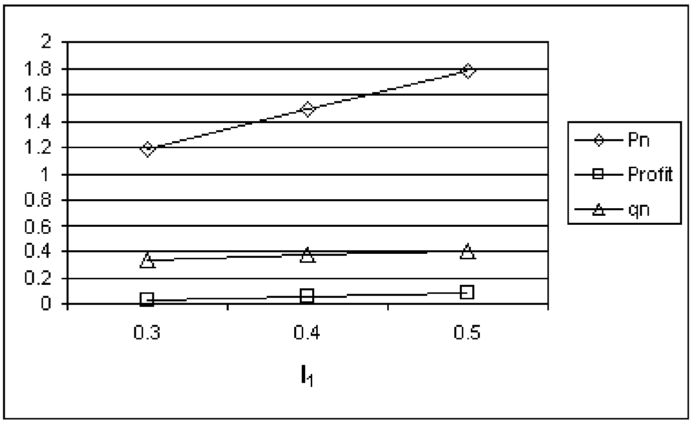

Figure 5.6. The effect of l_1 on q_n, p_n and profit

Combined with the relative willingness-to-pay δ for remanufactured product, we plot the optimal profit against l_1 for different values of δ (ranging from 0.2 to 0.5) in Figure 5.7.

When the joint effect of l_1 and δ is analyzed, it is seen that the profit increases

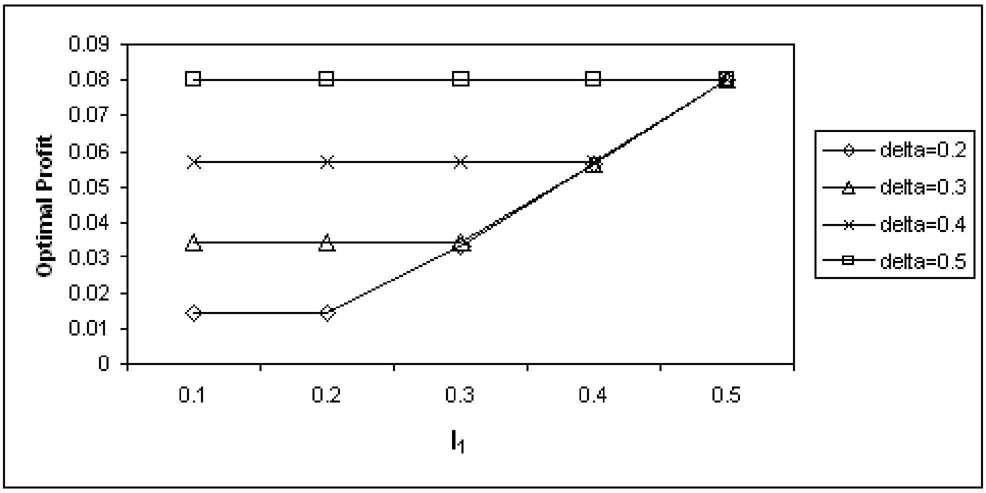

Figure 5.7. The effect of l_1 on the profit for different δ values

only if l_1 is greater than δ as l_1 increases. This is because the volume of the new products increases as l_1 increases only if $l_1 > \delta$. If l_1 is still less than or equal to δ when it increases, consumers do not change their preferences because of the fact that they derive more utility from buying a remanufactured product than from leasing a new product. For instance, in Figure 5.7, when δ=0.5, the profit does not change as l_1 increases since all of the l_1 values are less than or equal to δ. In this case, consumers prefer to buy remanufactured product. Therefore, it can be said that if $l_1 \leq \delta$, consumers are willing to pay for the remanufactured product, whereas they choose new product if $l_1 > \delta$ in the single period problem.

5.1.3. The Effect of Changes in the Deterioration of the Product

Residual value of the product decreases as the length of a lease increases. This is because the older a product gets, the less its remaining value. Therefore, residual value is the wholesale worth of a product at the end of its lease term.

Products deteriorate by a factor d_m where m is the index for lease periods. As mentioned before, since we only consider 1-period lease in this section, products are used one year and return at the end of the year. For a customer who leases a new product, the perceived residual value at the end of the year is given by Equation

(3.1). In other words, the depreciation part of the product is given by Equation (3.2). Note that if d_m increases, residual value decreases and since the returned product will have less value, payments will increase. In other words, a higher level of depreciation requires higher leasing payments. In the light of this information, we analyze the effect of changes in d_m for $m = 1$.

We conduct the experiments for the set of $d_1 = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6\}$ considering all other parameters fixed as given in Table 5.6.

Table 5.6. Fixed parameter values to analyze the effect of changes in d_1

l_1	δ	c'_r	I_0
0.5	0.2	0.08	0

We suggested that a higher level of depreciation requires higher level of leasing payments. Leasing payments given in Equation (3.5) can be simplified as

$$p_m = p_n K_m \qquad m = 1, ..., L, \tag{5.1}$$

where

$$K_m = \left(\frac{d_m}{12m} + (2 - d_m)\, MF \right). \tag{5.2}$$

In our numerical experiments, we observe that the increasing part is K_m where $m = 1$. Table 5.7 illustrates K_1 values for different d_1 values.

Table 5.7. K_1 values for different d_1 values

d_1	0.1	0.2	0.3	0.4	0.5	0.6
K_1	0.014667	0.022667	0.030667	0.038667	0.046667	0.054667

Table 5.8 illustrates the optimal prices and demands for different d_1 values. As d_1 increases, p_n decreases whereas K_1 increases for d_1 values from 0.1 to 0.4. Therefore, p_1 values do not change which results in no change in demands and profit. As p_n decreases,

p_r also decreases, but since δ is low, namely consumers see remanufactured products almost worthless, q_r does not change. Therefore, it is obvious that there is a threshold for depreciation rate such that for values less than this threshold, there is no change in the demands even though d_1 increases. On the other hand, when the threshold is exceeded, q_n decreases and q_r increases as d_1 increases. This is because the decrease in p_n forces p_r to decrease due to the constraint $p_r \leq \delta p_n$. Although consumers with high willingness-to-pay for leasing a new product still prefer leasing, consumers with low willingnes-to-pay shift towards the remanufactured product due to the decrease in the price of the remanufactured product. This decrease of prices affects the profit negatively. In Figure 5.8, we exhibit the effect of d_1 on the prices and demands.

Table 5.8. Optimal prices and demands for different d_1 values - $l_1 = 0.5$, $\delta = 0.2$

d_1	p_n	p_r	q_n	q_r	q_{No}	$PV(p_1)$	p_1
0.1	1.7793	0.2411	0.4000	0.0000	0.6000	0.3000	0.0261
0.2	1.1513	0.2231	0.4000	0.0000	0.6000	0.3000	0.0261
0.3	0.8510	0.1702	0.4000	0.0000	0.6000	0.3000	0.0261
0.4	0.6749	0.1350	0.4000	0.0000	0.6000	0.3000	0.0261
0.5	0.5526	0.1105	0.3803	0.0672	0.5526	0.2964	0.0258
0.6	0.4488	0.0898	0.3590	0.1922	0.4488	0.2821	0.0245

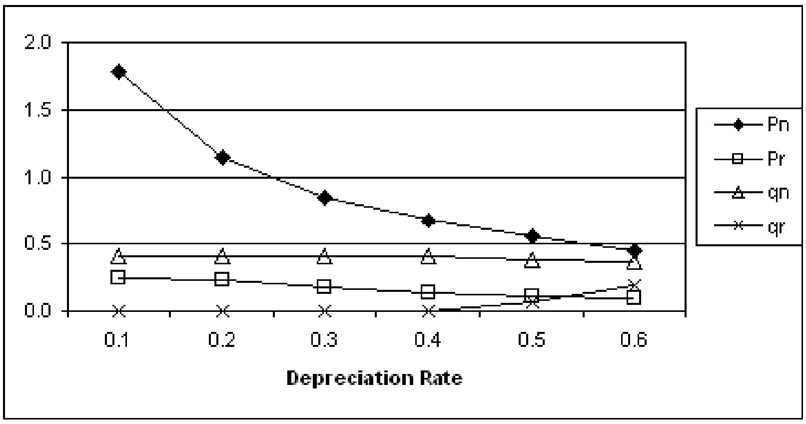

Figure 5.8. The effect of d_1 on prices and demands - $l_1 = 0.5$, $\delta = 0.2$

Combined with the relative willingness-to-pay δ for remanufactured product, we plot the optimal profit against the depreciation rate for different values of δ (ranging from 0.2 to 0.5) considering $\delta \leq l_1$ in Figure 5.9. This is because if $\delta > l_1$, no demand exists for new products.

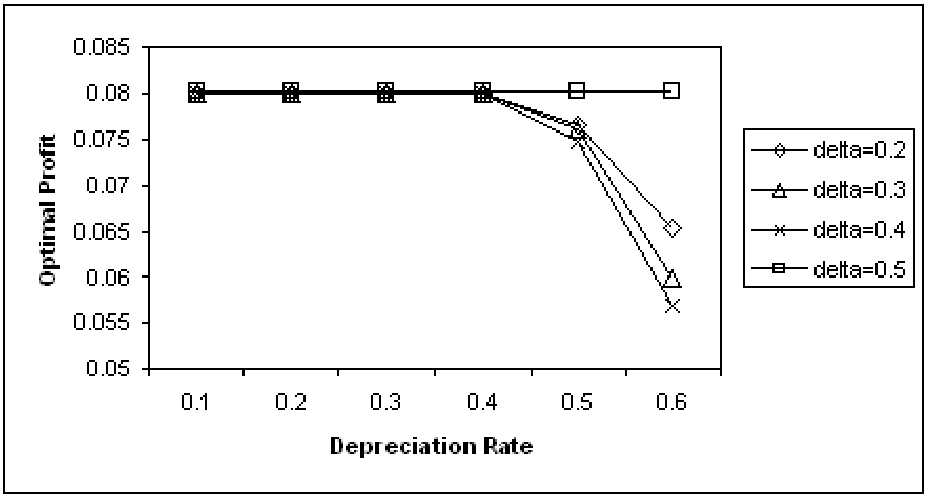

Figure 5.9. The effect of d_1 on the profit for different δ values

Note that when d_1 exceeds its threshold value, the profit curves are downward sloping for $\delta < l_1$. Moreover, when d_1 is higher than its threshold value (e.g., $d_1 = 0.5$), the profit decreases as δ increases. This is because even though the volume of consumers who prefer buying a remanufactured product increases as δ increases, the price and the amount of new products leased decreases. On the other hand, when $\delta = l_1$, from this point on, there is no effect of depreciation rate on the profit since there is no demand for new products. Table 5.9 illustrates the optimal demands, prices and profit for $\delta = l_1$.

Table 5.9. Optimal prices and demands for different d_1 values - $l_1 = 0.5$, $\delta = 0.5$

d_1	Profit	p_n	p_r	q_n	q_r	q_{No}	$PV(p_1)$	p_1
0.1	0.0802	3.3839	0.3091	0.0000	0.3818	0.6182	0.5705	0.0496
0.2	0.0802	1.4117	0.3091	0.0000	0.3818	0.6182	0.3678	0.0320
0.3	0.0802	1.8480	0.3091	0.0000	0.3818	0.6182	0.6515	0.0567
0.4	0.0802	0.7861	0.3091	0.0000	0.3818	0.6182	0.3494	0.0304
0.5	0.0802	3.3532	0.3091	0.0000	0.3818	0.6182	1.7989	0.1565
0.6	0.0802	1.2947	0.3091	0.0000	0.3818	0.6182	0.8136	0.0708

Finally, we find the relationship between δ, l_1 and d_1 such that if d_1 is less than threshold value and $\delta < l_1$, consumers prefer leasing a new product. As long as $\delta < l_1$, an increase in d_1 has no effect on decisions and profit. This is because consumers still prefer leasing a new product due to the constant payments provided by decreasing prices as d_1 increases. When d_1 is greater than threshold value while $\delta < l_1$, consumers shift towards buying a remanufactured product. In this case, if d_1 increases, the volume of remanufactured products increases and the volume of new products decreases which results in the decrease of profit. Moreover, if $\delta \geq l_1$, consumers prefer buying remanufactured products, therefore the change in d_1 has no impact on decisions and profit.

5.1.4. The Effect of Changes in the Initial Inventory Level

Until now, we assumed that no used products exist initially. In this section, we will analyze the case that the period begins with an initial inventory of used remanufacturable products. This is important because of the fact that remanufactured products' sales depend on the availability of returned used products in each period. We allow the manufacturer to meet the shortage in used products from the third-party core supplier. Note that average cost to produce a remanufactured product is $c_r q_r$ when supplied from our returns, but the unit cost becomes $c'_r + c_r q_r$ when we purchase used cores from the third-party supplier. This is because the cost of acquiring a used product from the third-party core supplier is c'_r and to produce a remanufactured product in manufacturer's facility costs $c_r q_r$ on the average. Therefore, if initial inventory of used cores decreases, manufacturer's profit decreases due to the higher costs.

First, we conduct the experiments for the values of $I_0 \in [0, 1]$ with increments of 0.1 considering all other parameters fixed as given in Table 5.10.

Table 5.10. Fixed parameter values to analyze the effect of changes in I_0

l_1	d_1	c'_r
0.5	0.1	0.08

Recall that if $l_1 \leq \delta$, consumers prefer remanufactured products, whereas they choose new products if $l_1 > \delta$ for one-period problem. It is important to note that if there is no available stock of used products when $\delta < l_1$, demand for remanufactured products does not exist. This is because when δ is low, the price of the remanufactured product is low due to the low willingness-to-pay of consumer and, in that case, if the used cores are supplied from the third-party core supplier with cost c_r', the manufacturer's profit obtained from each unit decreases. When we look from the consumer's side, they see remanufactured product worthless if δ is low and thus, they are not willing to pay high prices for it. When their net utility obtained from buying a remanufactured product is less than leasing a new product, they choose the new product. Therefore, we obtain different insights under the cases where $l_1 \leq \delta$ and $l_1 > \delta$.

If there were no dependence between stock of used products and remanufactured products' sales, namely all demand for remanufactured products were provided, there would be an optimum q_r^* for each δ value when all parameters are held constant. In our experiments, $I_0 = 1$, in fact, represents this assumption in single period setting. Thus, for all δ values, when I_0 increases, q_r increases up to q_r^*, but there are some differences between the cases where $l_1 \leq \delta$ and $l_1 > \delta$.

Table 5.11 exhibits optimal prices and demands for different I_0 values when $l_1 = 0.5$ and $\delta = 0.2$. If $l_1 > \delta$, the increase of q_r depends on I_0 such that q_r does not exceed the available inventory of used products. If $q_r^* \geq I_0$ for a δ value, then, as I_0 increases, q_r increases as illustrated in Figure 5.10. The price of the remanufactured product decreases such that the demand for remanufactured products increases which results in the decrease of the amount of new products. However, the increase in the amount of remanufactured products makes the profit increase as seen in Figure 5.11. When q_r reaches the optimum q_r^*, it does not change even if I_0 increases. Therefore, the profit remains the same since both prices and demands for new and remanufactured products do not change as illustrated in Figure 5.11.

Table 5.11. Optimal prices and demands for different I_0 values - $l_1 = 0.5$, $\delta = 0.2$

I_0	p_n	p_r	q_n	q_r	q_{No}
0	1.7793	0.1395	0.4000	0.0000	0.6000
0.1	1.7793	0.1080	0.3600	0.1000	0.5400
0.2	1.7793	0.1059	0.3529	0.1176	0.5294
0.3	1.7793	0.1059	0.3529	0.1176	0.5294
0.4	1.7793	0.1059	0.3529	0.1176	0.5294
0.5	1.7793	0.1059	0.3529	0.1176	0.5294
0.6	1.7793	0.1059	0.3529	0.1176	0.5294
0.7	1.7793	0.1059	0.3529	0.1176	0.5294
0.8	1.7793	0.1059	0.3529	0.1176	0.5294
0.9	1.7793	0.1059	0.3529	0.1176	0.5294
1	1.7793	0.1059	0.3529	0.1176	0.5294

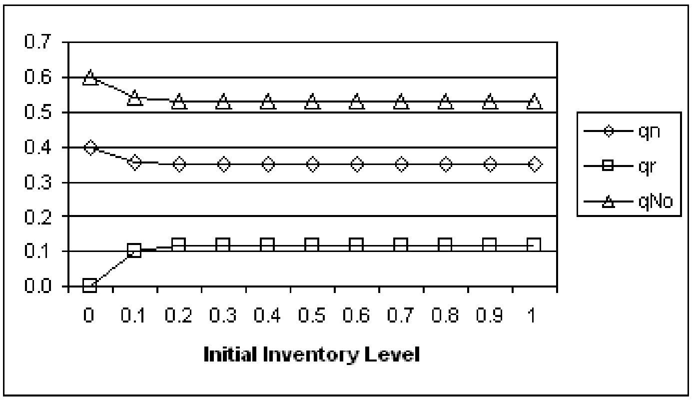

Figure 5.10. The effect of I_0 on the demands - $l_1 = 0.5$, $\delta = 0.2$

When $\delta \geq l_1$, q_r is positive although there is no stock of used products. This is because the company earns more from customers who prefer buying a remanufactured product even though used cores are supplied at a cost of c_r'. In such cases, even though p_r and q_r do not change, profit continually increases as I_0 increases, because the part of used cores supplied from manufacturer's returned products is remanufactured with less cost compared to the case where all used cores are bought from the third-party core supplier. In other words, as I_0 increases, the cores supplied with an extra cost decrease.

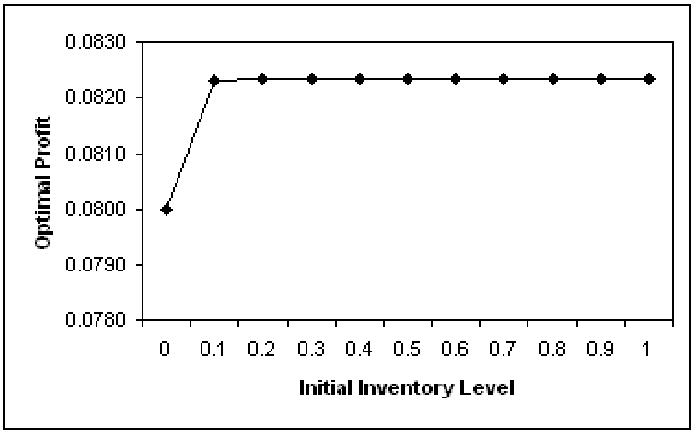

Figure 5.11. The effect of I_0 on the profit - $l_1 = 0.5$, $\delta = 0.2$

When I_0 becomes equal to q_r, there is no need to purchase from the third-party core supplier. From this point on, if $q_r^* \geq I_0$, q_r continues to increase as I_0 increases. The profit increases due to the increase of q_r. When $I_0 > q_r^*$, the increase of I_0 does not affect the volume and price of remanufactured products and profit does not change. Table 5.12 illustrates optimal prices and demands for different I_0 values when $l_1 = 0.5$ and $\delta = 0.5$. Figure 5.12 exhibits the trend of optimal demands with respect to I_0.

Table 5.12. Optimal prices and demands for different I_0 values - $l_1 = 0.5$, $\delta = 0.5$

I_0	p_n	p_r	q_n	q_r	q_{No}
0	2.2569	0.3091	0.0000	0.3818	0.6182
0.1	2.9626	0.3091	0.0000	0.3818	0.6182
0.2	1.9237	0.3091	0.0000	0.3818	0.6182
0.3	2.0985	0.3091	0.0000	0.3818	0.6182
0.4	2.9214	0.3000	0.0000	0.4000	0.6000
0.5	2.3456	0.2727	0.0000	0.4545	0.5455
0.6	1.6411	0.2727	0.0000	0.4545	0.5455
0.7	3.1878	0.2727	0.0000	0.4545	0.5455
0.8	2.1641	0.2727	0.0000	0.4545	0.5455
0.9	1.7303	0.2727	0.0000	0.4545	0.5455
1	1.8665	0.2727	0.0000	0.4545	0.5455

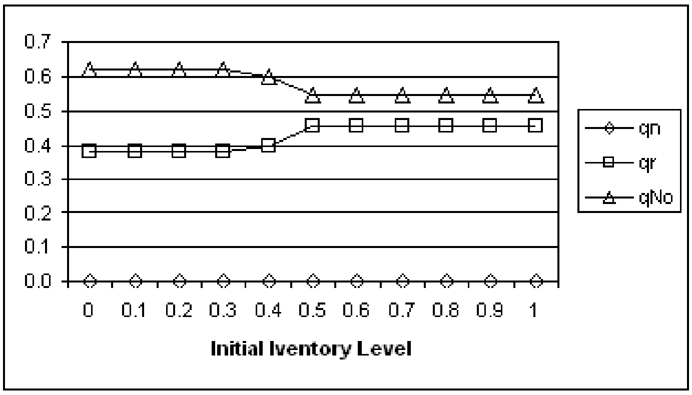

Figure 5.12. The effect of I_0 on the demands - $l_1 = 0.5$, $\delta = 0.5$

Combined with the relative willingness-to-pay δ for remanufactured product we plot the optimal profit against the initial inventory level for different values of δ (ranging from 0.2 to 0.5) in Figure 5.13.

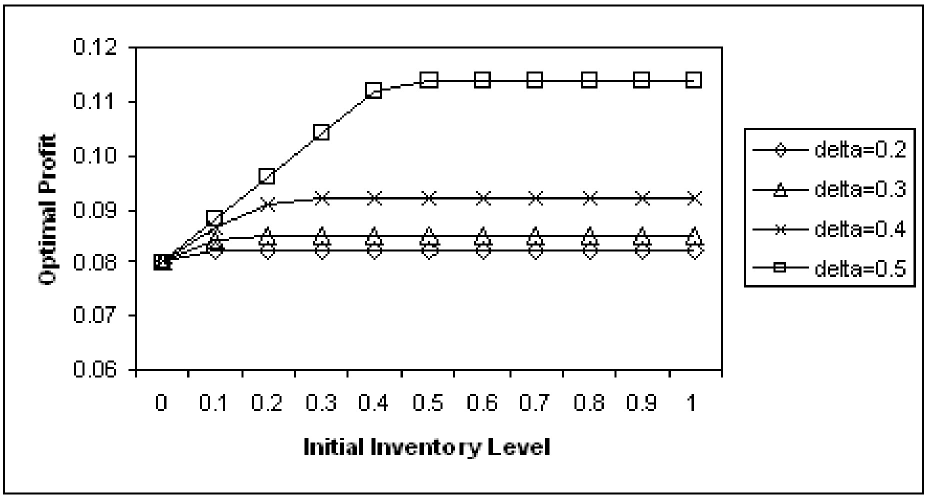

Figure 5.13. The effect of I_0 on the profit for different δ values

When I_0 is held fixed at some value greater than zero, the profit increases as δ increases because of the fact that p_r increases to take advantage of the increased willingness-to-pay for $\delta < l_1$, but q_r does not always increase as δ increases. Recall that when $I_0 = 0$, the increase in δ does not affect q_r. Moreover, when $I_0 > 0$, this is also seen in some experiments. As mentioned before, this is because q_r does not exceed I_0 due to the extra cost c_r'. Therefore, when $\delta < l_1$, q_r increases only if I_0 is enough to meet the increase in q_r. However, if $\delta \geq l_1$, the profit increases as δ increases when I_0 is held fixed due to the increase in both price and amount of the remanufactured product.

In conclusion, there are two reasons of the increase in the profit as I_0 increases: The first one is the increase in q_r; the second is the decrease in the amount supplied from the third-party core supplier. For all δ values, if $q_r^* < I_0$, there is no effect of I_0 neither on the volume of remanufactured product nor on the price of the remanufactured product.

5.1.5. The Effect of Changes in the Cost of Supplying Used Products from Third-party Core Supplier

The cost c_r' of acquiring a used product includes unit transportation cost from the third-party core supplier to manufacturer's facility. In this section, we keep all parameters fixed except c_r' to analyze its effect. Table 5.13 gives the fixed parameter values used in the analysis of the effect of c_r'.

Table 5.13. Fixed parameter values to analyze the effect of changes in c_r'

l_1	d_1	δ	I_0
0.5	0.1	0.2	0

$c_r' = 0$ looks like the case where there is no constraint with respect to remanufactured products' sales, namely $I_0 = 1$. Thus, the average cost of remanufacturing used cores is only $c_r q_r$. On the other hand, an increase in c_r' makes a remanufactured product less attractive with respect to a new product. Thus, the volume of remanufactured products sold decreases as c_r' increases and optimal pricing becomes such that the demand for remanufactured products decreases so as to reduce the detrimental effect of the increase in c_r' on the profit. The increase in the price of the remanufactured product in such a way that demand for remanufactured products decreases and demand for new products increases makes the profit decrease as illustrated in Figure 5.14. This is because the profit gain obtained from the increase in the amount of new products leased is less than the profit loss which occurs due to the decrease in the amount of remanufactured products sold and the increase in the volume of consumers who prefer nothing.

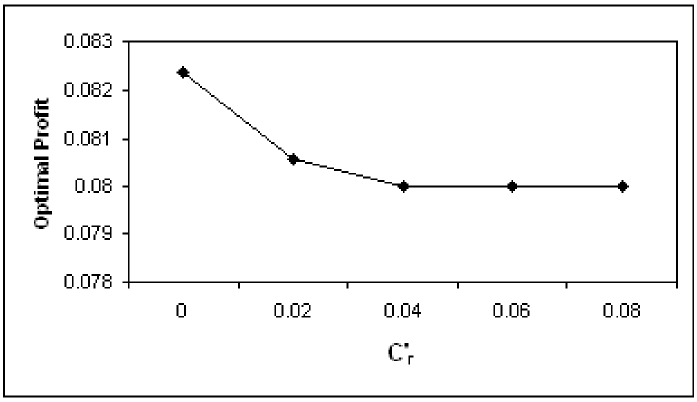

Figure 5.14. The effect of c_r' on the profit

When the demand for remanufactured products does not exist due to the high c_r', from this point on, the increase of this parameter does not affect decisions on the demands and profit. This is because the optimal pricing is such a way that demand for remanufactured products is zero while demand for new products remains the same as given in Table 5.14.

Table 5.14. Optimal prices and demands for different c_r' values - $l_1 = 0.5$, $\delta = 0.2$

c_r'	p_n	p_r	q_n	q_r	q_{No}
0.00	1.7793	0.1059	0.3529	0.1176	0.5294
0.02	1.7793	0.1129	0.3765	0.0588	0.5647
0.04	1.7793	0.2567	0.4000	0.0000	0.6000
0.06	1.7793	0.2038	0.4000	0.0000	0.6000
0.08	1.7793	0.1705	0.4000	0.0000	0.6000

When the effect of changes in c_r' is analyzed for different δ values , similar results are obtained. Figure 5.15 presents the behavior of the optimal profit as c_r' increases for different δ values.

Finally, if c_r' increases, the manufacturer makes the price of the remanufactured product increase to decrease the demand for remanufactured products. Thus, the amount of used products supplied from the third-party core supplier decreases. An increase in p_r makes a new product more attractive, but the profit decreases. This is because the profit loss which occurs due to the increase in the volume of consumers who

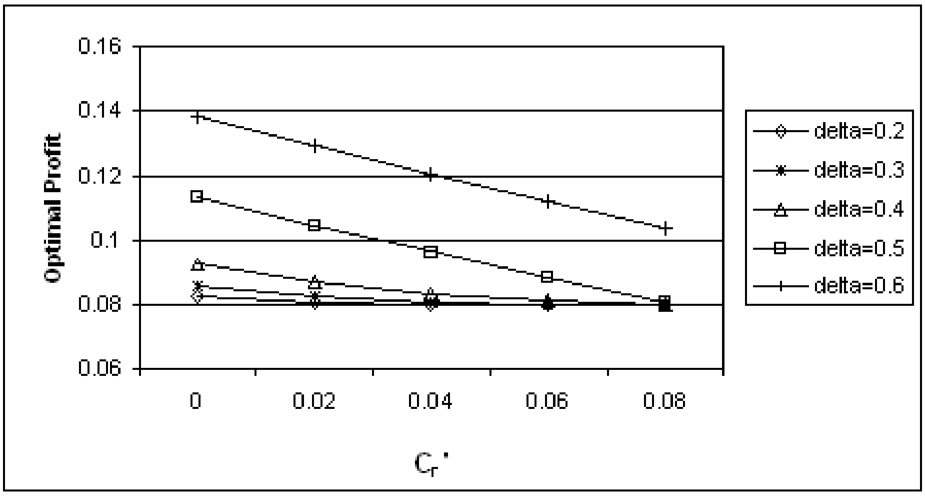

Figure 5.15. The effect of c_r' on the profit for different δ values

prefer nothing and the decrease in the amount of remanufactured products is relatively higher with respect to the profit gain obtained from the increase in the amount of new products leased.

5.2. Multi-Period Problem

Due to the interdependence of new and remanufactured products, a decrease in demand for new products results in a decrease in the availability of used products that are remanufactured. In the previous section, we observed the effect of changes in initial inventory level, but since there is no supply of used products that become available at the beginning of the current time, we could not investigate the impact of past lease decisions on the future decisions of the firm. In the multi-period setting, it is possible to investigate the interdependence of new and remanufactured products since all previously leased products have to return at the end of the lease term, which are further used for remanufacturing.

In this section, we also investigate the implications of this dependency on the pricing strategy. For instance, the manufacturer may choose to produce some new products only for the future value that they generate through their sale as remanufactured products, although these products are sold at a loss currently.

The analysis is considered up to 5 periods, and in each period consumers decide whether to lease a new product or to buy a remanufactured product based on their net utility that they derive in that period. As mentioned before, we offer only one-period lease option in each period and present the effect of different problem parameters in this setting. However, we also analyze the trend on demands and prices if two-period lease is offered by the manufacturer. In the single period setting we did not analyze this option, because decisions are the same in the first and second periods due to returns of used products at the beginning of the third period.

5.2.1. The Effect of Changes in the Relative Willingness-to-pay for Remanufactured Product

In this section, we keep all parameters fixed except δ to analyze the effect of its changes on the marketing strategy of the manufacturer in the multi-period setting. Some parameter values have already been given in Table 5.1, and d_1, l_1, c_r' and I_0 are given in Table 5.2.

Before the effect of changes in δ, we investigate the distribution of volume of new and remanufactured products among periods due to the interdependence between them. For the one-period lease option, new products return at the end of their lease period. As mentioned before, the supply of used products that become available at the beginning of time t is R_t. The volume of used products that remain in stock from returns in previous periods at the beginning of period t is I_{t-1}. $I_{t-1} + R_t$ gives us the available inventory at the beginning of the current period.

As mentioned in the single period problem, if there is inventory of used products when $\delta < l_1$, demand for remanufactured products may be positive such that q_r does not exceed the available inventory of used products. If the maximum q_r value q_r^* for a given δ is greater than I_0, then, as I_0 increases, q_r increases as well. Thus, in a multi-period problem, if returns are adequate for remanufacturing, q_r becomes positive in the second period as illustrated in Table 5.15 for two-period problem when $l_1 = 0.5$ and $\delta = 0.2$. Since no used products exist initially, the demand for remanufactured products is zero. However, in the second period since used products return, the manufacturer produces remanufactured products. The left of the used products remains in stock.

Table 5.15. Optimal prices and demands - 2 periods, $l_1 = 0.5$, $\delta = 0.2$

Periods	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.7793	0.2982	0.4000	0.0000	0.6000	0.0000	0.3000	0.0261
2	1.7793	0.1059	0.3529	0.1176	0.5294	0.2824	0.3000	0.0261

In the single period model, when $l_1 = \delta$, there is demand only for remanufactured products, but in the multi-period setting, there is not due to the threat of supply of remanufacturable products in the next period. Thus, in the first period, the demand for new products exists, and the manufacturer starts the second period with the available $q_{n,1}$ cores to recover from the products that were leased in the previous period. If the problem were solved period by period without considering the interdependence of new and remanufactured products, selling more remanufactured products would seem to be more profitable without considering the next period. However, if the manufacturer chooses to produce some new products only for the future value that they generate through their sale as remanufactured products, he makes more profit. In this framework, the optimal pricing is such that the demand for new products is positive in the first period, while the demand for remanufactured products is positive in the second period. This comparison is given in Table 5.16.

Table 5.16. Profit comparison for two-period problem - $l_1 = 0.5$, $\delta = 0.5$

	Demands			
	Simultaneous Decision		Step by Step	
	$q_{n,t}$	$q_{r,t}$	$q_{n,t}$	$q_{r,t}$
1. Period	0.4275	0	0	0.3818
2. Period	0	0.4275	0	0.3818
Profit	0.184468		0.154424	

Table 5.17 illustrates the optimal prices and demands if $l_1 = 0.5$ and $\delta = 0.6$. Recall that in the single period problem if $l_1 < \delta$, there is demand only for remanufac-

tured products. However, in the multi-period setting, the demand for both new and remanufactured products is positive in the first period so as to decrease the amount supplied from the third-party core supplier in the next period. The volume of remanufactured products in the second period is the same as the results obtained from step by step optimization. But since the volume of used products supplied from the third-party core supplier is less, the profit is higher.

Table 5.17. Optimal prices and demands - 2 periods, $l_1 = 0.5$, $\delta = 0.6$

Periods	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.5596	0.3477	0.3210	0.1531	0.5259	0.0000	0.2630	0.0229
2	1.7794	0.3600	0.0000	0.4000	0.6000	0.0000	0.3000	0.0261

In Figures from 5.16 to 5.23, we exhibit the effect of δ on the demands for new and remanufactured products in the first and last period of the two-period, three-period, four-period and five-period problems, respectively. The effect of δ in the first period becomes different from the effect of it in the rest of the time horizon due to the availability of used products. In the first period, when $\delta < l_1$ and $I_0 = 0$, q_r does not increase as δ increases because of the fact that q_r does not exceed I_0 due to the extra cost c_r'. We also see from Figure 5.16 that there is no demand for remanufactured products since customers value remanufactured products less than new products. This causes the manufacturer not to charge high p_r values and, therefore not to obtain profit by selling the remanufactured product. To create demand for new products in the first period is beneficial from the perspective of both manufacturer's profit and the supply of used products in the next period. When $l_1 = \delta$, consumers are undecided between leasing a new product and buying a remanufactured product because of the fact that they value both of them equal. Therefore, they prefer the one which has less price. In this case, manufacturer charges higher prices for remanufactured product to shift consumers towards new products in the first period considering the supply of used products in the future. q_r is positive for the first time when $\delta > l_1$ and, from this point on, q_n decreases (q_r increases) as δ increases.

The manufacturer starts the last period with the opportunity to remanufacture used products that were leased in previous periods and become available at the beginning of the last period. q_r increases (q_n decreases) in δ such that q_r does not exceed the volume of returns when $\delta < l_1$. For $\delta \geq l_1$, q_r may exceed the stock of used products since there are consumers who are willing to pay high prices for remanufactured products which creates profit although the shortage in used products is supplied from the third-party core supplier with an extra cost c_r'.

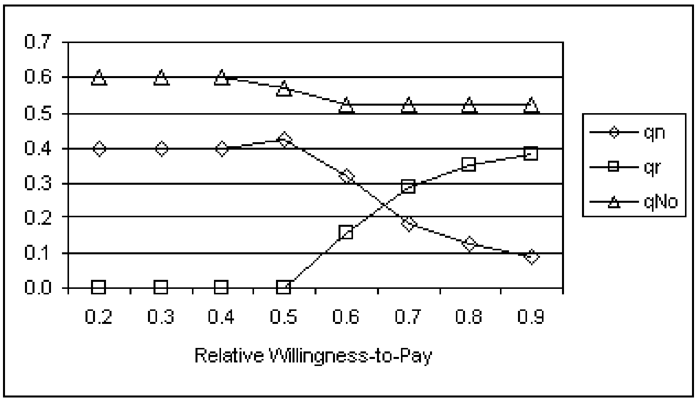

Figure 5.16. The effect of δ on the demands in the first period - 2 periods

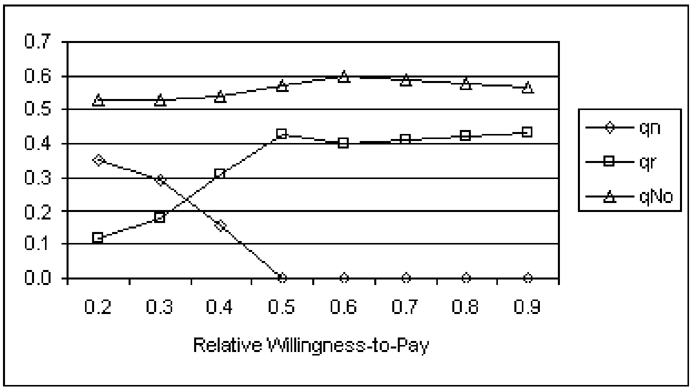

Figure 5.17. The effect of δ on the demands in the last period - 2 periods

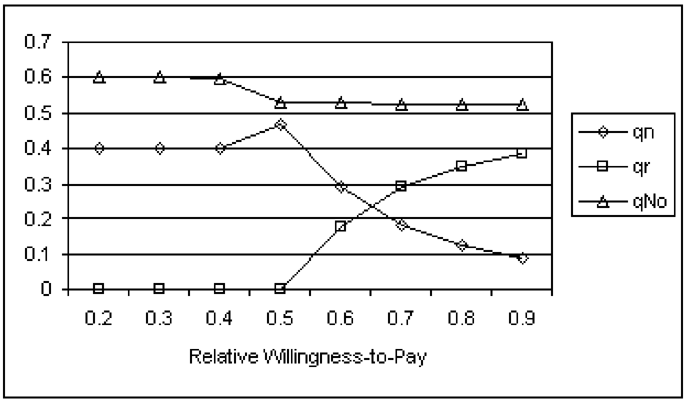

Figure 5.18. The effect of δ on the demands in the first period - 3 periods

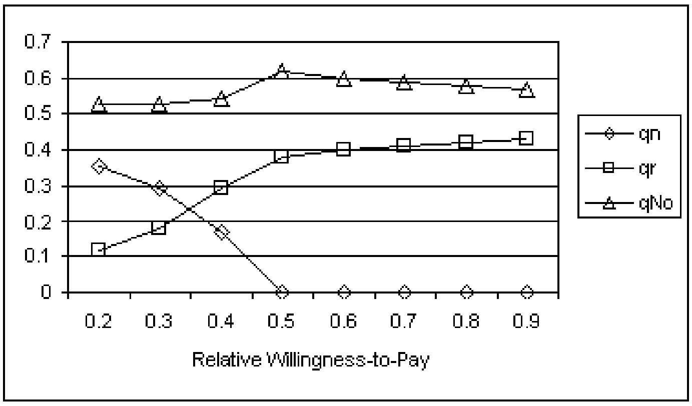

Figure 5.19. The effect of δ on the demands in the last period - 3 periods

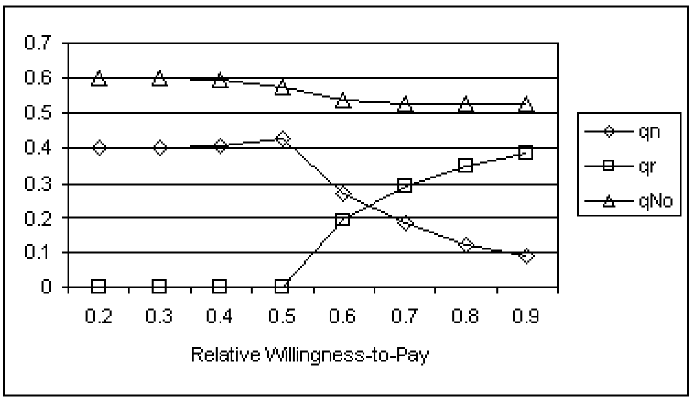

Figure 5.20. The effect of δ on the demands in the first period - 4 periods

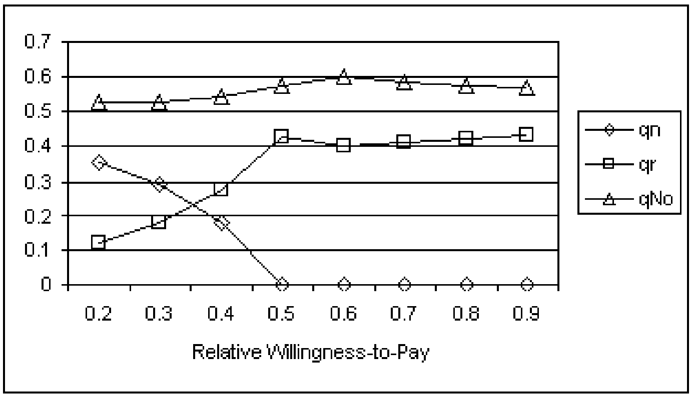

Figure 5.21. The effect of δ on the demands in the last period - 4 periods

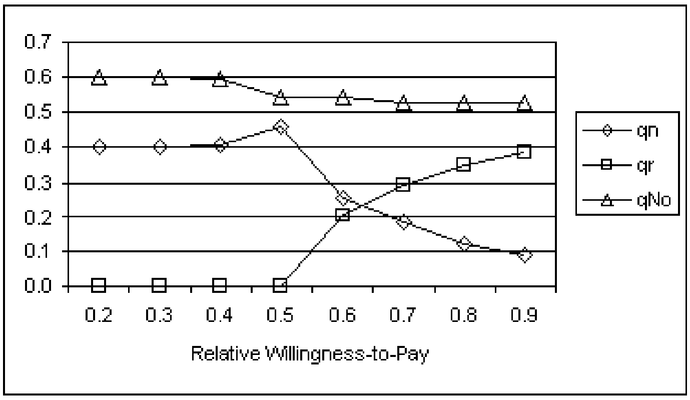

Figure 5.22. The effect of δ on the demands in the first period - 5 periods

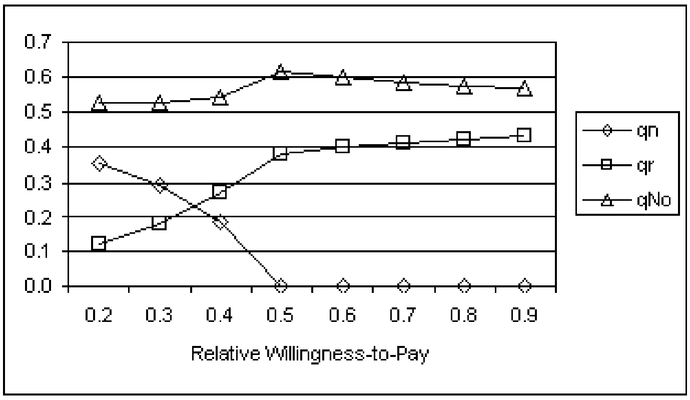

Figure 5.23. The effect of δ on the demands in the last period - 5 periods

Until now, we present the effect of changes in δ on demands for the first and last period of the multi-period problems for different T values. In periods stated between first and last period of the time horizon, when $\delta < l_1$ and $\delta > l_1$, q_r also increases as δ increases. But when $\delta = l_1$, this trend changes such that q_r increases too much due to the returns. However, when $\delta > l_1$, since the demand for remanufactured products exists in each period, the volume of new products decreases and also causes the volume of the remanufactured products to decrease in the next period. From this point on, q_r continues to increase with respect to δ. Figure 5.24 illustrates this phenomenon.

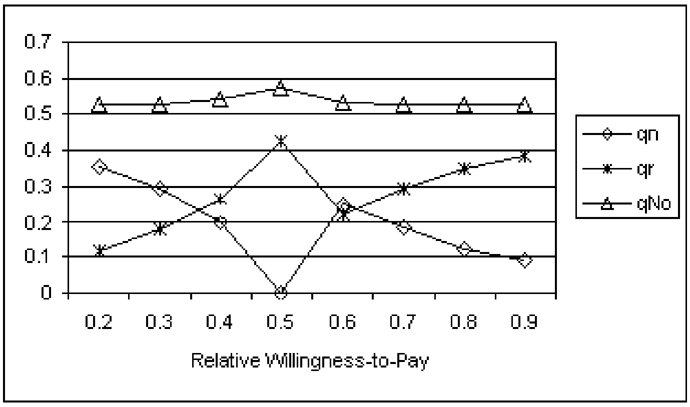

Figure 5.24. The effect of δ on the demands in the second period - 4 periods

As δ increases, p_r also increases to take advantage of increased willingness-to-pay which makes the profit increase. In Figure 5.25, we present the optimal profit against the relative willingness-to-pay δ for remanufactured products for different time horizons up to 5 periods. Note that profit curves are upward sloping: When consumers shift towards the remanufactured product, the price and the volume of remanufactured products increase.

The another issue in the multi-period setting is the inventory of used products that remains in stock. As δ increases, I_t decreases because of the fact that q_n decreases and q_r increases. Moreover, if δ is high, there is no stock of used cores at the beginning of any period due to the excess demand for remanufactured products. In Tables from 5.18 to 5.20, the changes in I_t for 5-period problem are illustrated for δ values 0.2, 0.6 and 0.8, respectively.

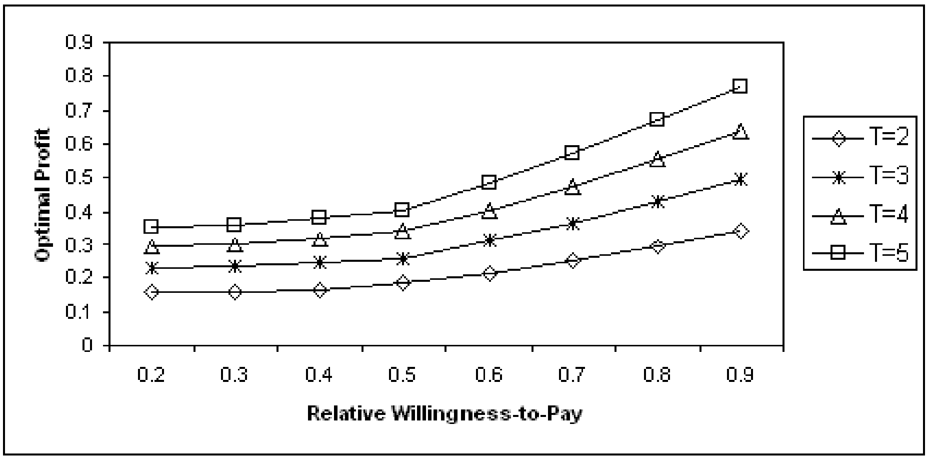

Figure 5.25. Change in the optimal profit as δ increases for different multi-period problems

Table 5.18. Inventory of used cores at the beginning of each period - 5 periods, $l_1 = 0.5$, $\delta = 0.2$

Periods	q_n	q_r	q_{No}	I_t
1	0.4000	0.0000	0.6000	0.0000
2	0.3529	0.1176	0.5294	0.2824
3	0.3529	0.1176	0.5294	0.5176
4	0.3529	0.1176	0.5294	0.7529
5	0.3529	0.1176	0.5294	0.9882

Table 5.19. Inventory of used cores at the beginning of each period - 5 periods, $l_1 = 0.5$, $\delta = 0.6$

Periods	q_n	q_r	q_{No}	I_t
1	0.2535	0.2051	0.5414	0.0000
2	0.2184	0.2447	0.5369	0.0088
3	0.2581	0.2107	0.5312	0.0164
4	0.3000	0.1744	0.5256	0.1001
5	0.0000	0.4001	0.5999	0.0000

Table 5.20. Inventory of used cores at the beginning of each period - 5 periods, $l_1 = 0.5$, $\delta = 0.8$

Periods	q_n	q_r	q_{No}	I_t
1	0.1228	0.3513	0.5259	0.0000
2	0.1228	0.3513	0.5259	0.0000
3	0.1228	0.3513	0.5259	0.0000
4	0.1228	0.3513	0.5259	0.0000
5	0.0000	0.4235	0.5765	0.0000

Figures from 5.26 to 5.45 exhibit the trend of new and remanufactured products in the multi-period setting analyzed up to 5 periods for different δ values.

When $\delta < l_1$, there is no demand for remanufactured products whereas the demand for new products is positive in the first period as presented in Figures from 5.26 to 5.33. With respect to the returns of used products at the beginning of the second period, q_r becomes positive and q_n decreases. Compared with the first period, the volume of consumers who do not prefer any one of the products decreases in the second period. The way of creating demand for remanufactured products in the second period is to decrease its price even though this affects the demand for new products in a negative way. We note that the volume of new and remanufactured products do not change in the following periods. In other words, the experiments present that from second period on, the behavior of consumers does not change in the rest of the time horizon.

For $\delta < l_1$, the amount of remanufactured products increases in the second period, but if δ is too low, the volume of new products may be still higher than the volume of remanufactured products. As seen in Figures from 5.30 to 5.33, when δ comes close to l_1, the volume of remanufactured products becomes higher than the volume of new products in the second period, and in the following periods as well.

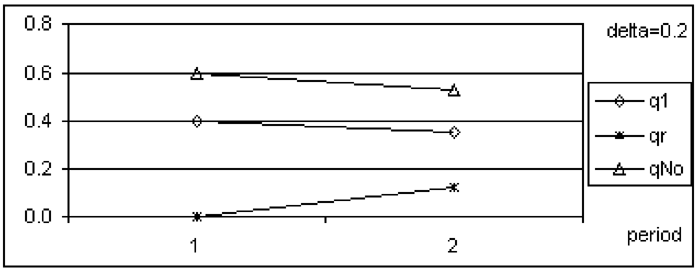

Figure 5.26. The trend of optimal demands - 2 periods, $\delta = 0.2$

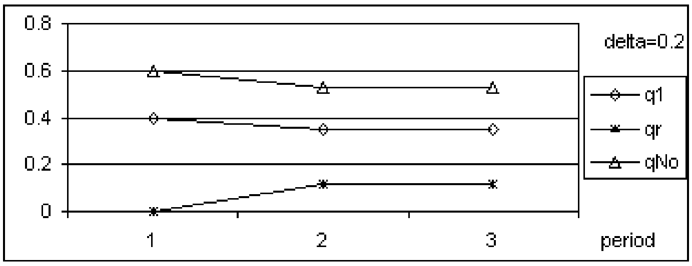

Figure 5.27. The trend of optimal demands - 3 periods, $\delta = 0.2$

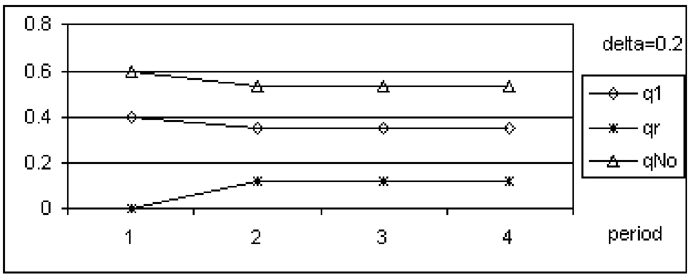

Figure 5.28. The trend of optimal demands - 4 periods, $\delta = 0.2$

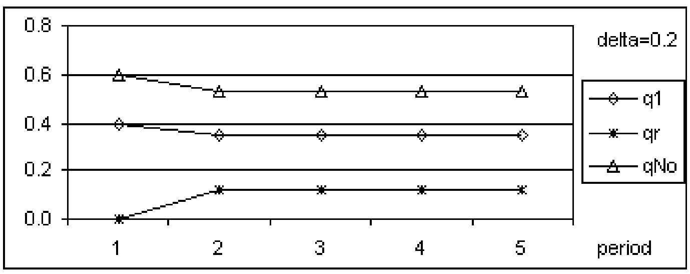

Figure 5.29. The trend of optimal demands - 5 periods, $\delta = 0.2$

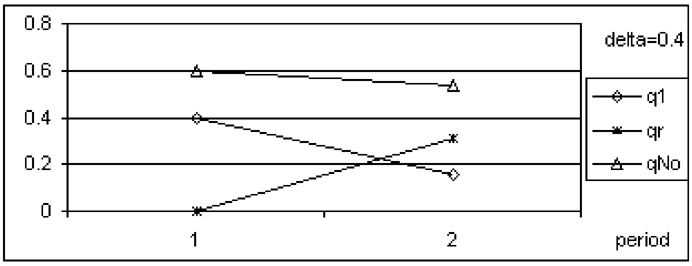

Figure 5.30. The trend of optimal demands - 2 periods, $\delta = 0.4$

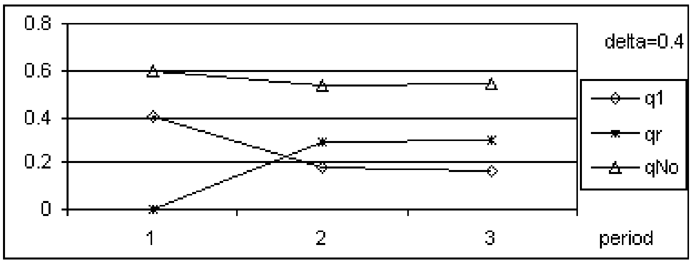

Figure 5.31. The trend of optimal demands - 3 periods, $\delta = 0.4$

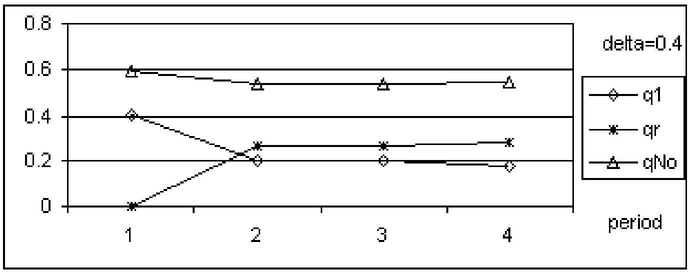

Figure 5.32. The trend of optimal demands - 4 periods, $\delta = 0.4$

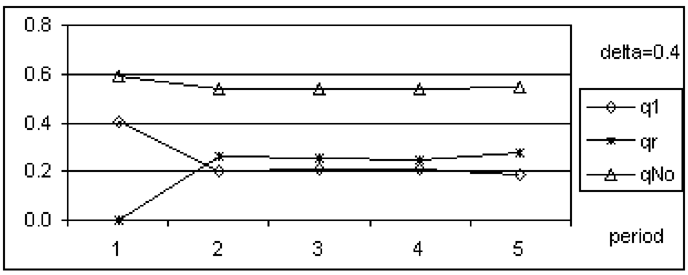

Figure 5.33. The trend of optimal demands - 5 periods, $\delta = 0.4$

If $\delta = l_1$, an interesting phenomenon occurs. In each period the demand exists for only one type of the products as seen in Tables from 5.34 to 5.37, and the pricing is such that the demand for new products exists in the first period and it is zero in the last period. As mentioned before, consumers are indifferent between leasing a new and buying a remanufactured product in this case. Therefore, this type of strategy states that these two products present the characteristics of complementary products even though they are substitutes. This is because producing remanufactured products depends on the used products obtained through leasing new products in previous periods.

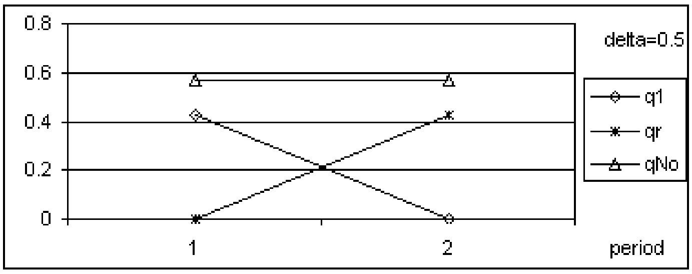

Figure 5.34. The trend of optimal demands - 2 periods, $\delta = 0.5$

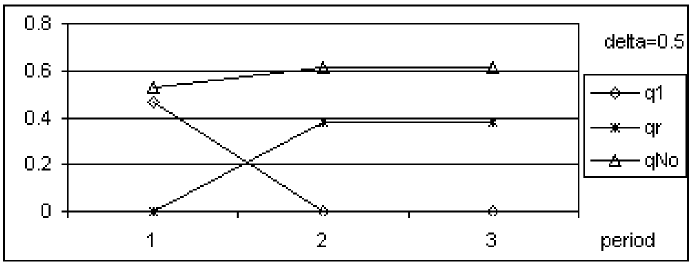

Figure 5.35. The trend of optimal demands - 3 periods, $\delta = 0.5$

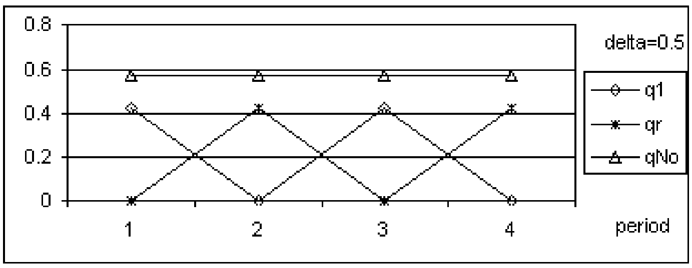

Figure 5.36. The trend of optimal demands - 4 periods, $\delta = 0.5$

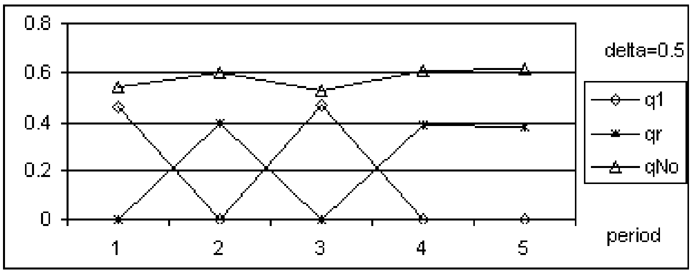

Figure 5.37. The trend of optimal demands - 5 periods, $\delta = 0.5$

If $\delta > l_1$, the demand exists for both new and remanufactured products in the first period as illustrated in Figures from 5.38 to 5.45. Up to the last period, the volume of new and remanufactured products do not change, but in the last period q_r increases whereas q_n becomes zero. This decrease in leasing new products states that, in fact, new products are leased for the future value that they generate through their sale as remanufactured products. This implies that marginal profit for new products leased in the last period is less than that for remanufactured products sold. Therefore, only remanufactured products are sold at the end of the time horizon. It is interesting to note that even though the price of the remanufactured product increases in the last period, a higher demand occurs for it. This is because the prices of both new and remanufactured products increase in such a way that the demand for only remanufactured products increases. If the increase in p_r due to an increase in p_n is relatively low, both consumers choose a remanufactured product and also the manufacturer takes advantage of increased price of remanufactured product. This is illustrated in Table 5.21 for $\delta = 0.8$.

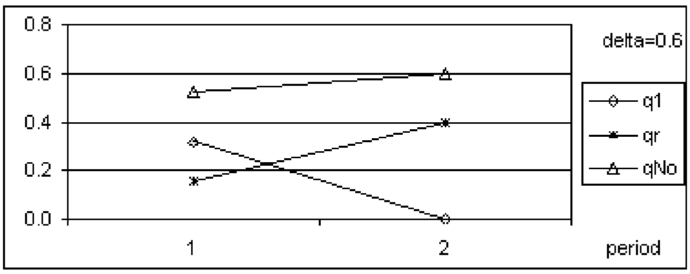

Figure 5.38. The trend of optimal demands - 2 periods, $\delta = 0.6$

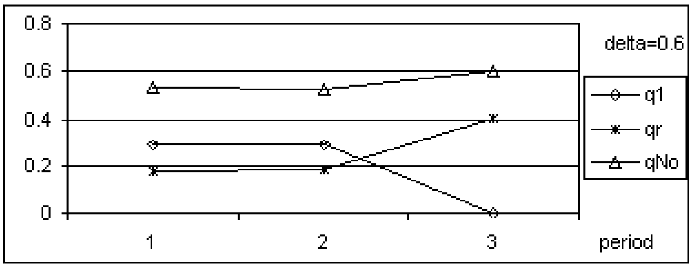

Figure 5.39. The trend of optimal demands - 3 periods, $\delta = 0.6$

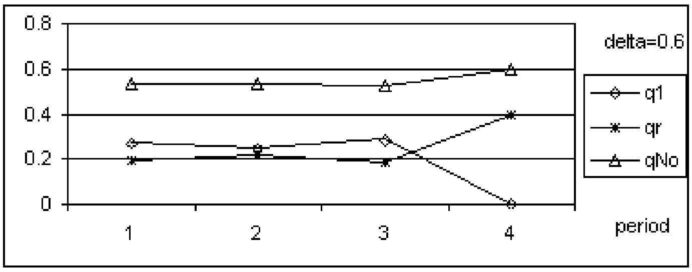

Figure 5.40. The trend of optimal demands - 4 periods, $\delta = 0.6$

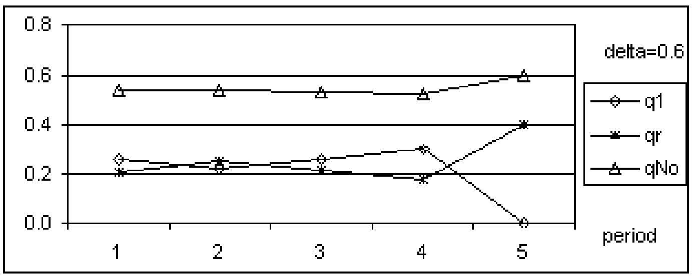

Figure 5.41. The trend of optimal demands - 5 periods, $\delta = 0.6$

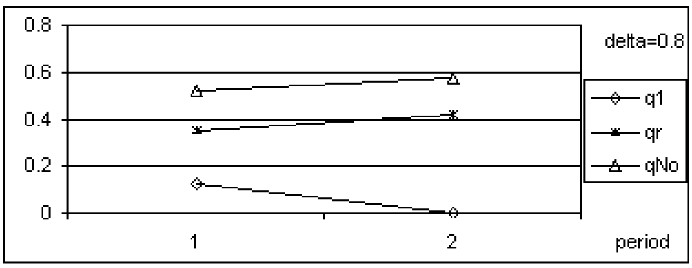

Figure 5.42. The trend of optimal demands - 2 periods, $\delta = 0.8$

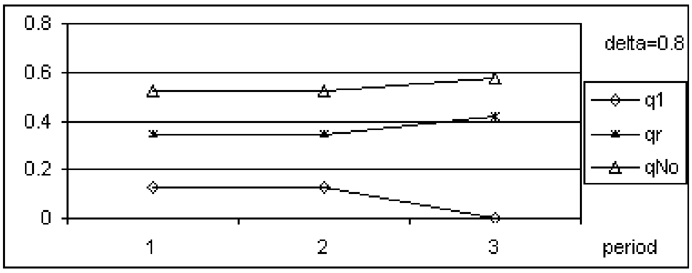

Figure 5.43. The trend of optimal demands - 3 periods, $\delta = 0.8$

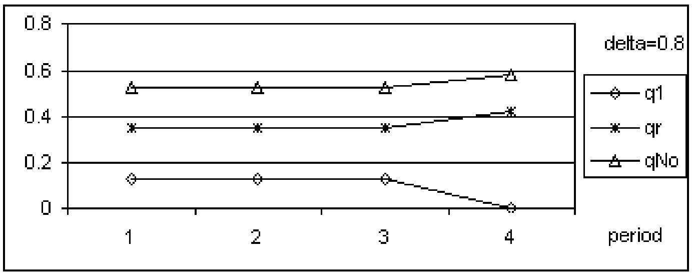

Figure 5.44. The trend of optimal demands - 4 periods, $\delta = 0.8$

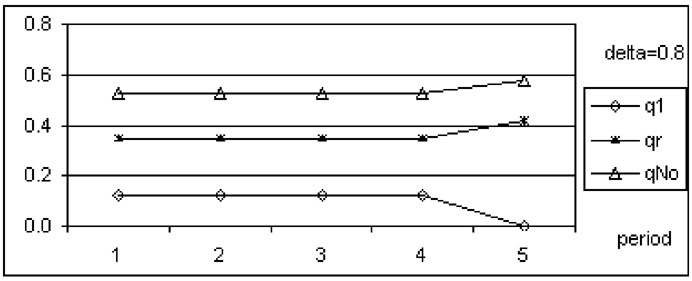

Figure 5.45. The trend of optimal demands - 5 periods, $\delta = 0.8$

Table 5.21. Optimal prices and demands - 5 periods, $l_1 = 0.5$, $\delta = 0.8$

Periods	p_n	p_r	q_n	q_r	q_{No}
1	1.5596	0.4576	0.1228	0.3513	0.5259
2	1.5596	0.4576	0.1228	0.3513	0.5259
3	1.5596	0.4576	0.1228	0.3513	0.5259
4	1.5596	0.4576	0.1228	0.3513	0.5259
5	2.8159	0.4612	0.0000	0.4235	0.5765

5.2.2. The Effect of Changes in the Relative Willingness-to-pay for Leasing a New Product

In this section, we will first investigate the differences that occur in the multi-period setting compared with the single period problem. Then, we will present the effect of changes in l_1. In fact, when the effect of changes in l_1 is analyzed for fixed δ, similar results are found to the cases while analyzing the effect of δ for fixed l_1. As mentioned before, this is because each consumer's willingness-to-pay for leasing a new product for one period and for buying a remanufactured product are fractions l_1 and δ, respectively, of their willingness-to-pay for buying a new product. Some parameter values have already been given in Table 5.1, and d_1, δ, c_r' and I_0 are given in Table 5.4.

We know from the analysis done for the single period problem that if $l_1 \leq \delta$, consumers are willing to pay for the remanufactured product, whereas they choose a new product if $\delta < l_1$. Moreover, when the joint effect of l_1 and δ is analyzed, it is seen that the profit increases only if $l_1 > \delta$ when l_1 increases. In the multi-period setting, Table 5.22 exhibits the optimal prices and demands for two-period problem when $l_1 = 0.1$ and $\delta = 0.2$.

Table 5.22. Optimal prices and demands - 2 periods, $l_1 = 0.1$, $\delta = 0.2$

Periods	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.2878	0.1520	0.00	0.24	0.76	0.00	0.2171	0.0189
2	0.9687	0.1520	0.00	0.24	0.76	0.00	0.1633	0.0142

As seen in Table 5.22, the demand for new products is zero either in the first or in the last period of the time horizon. While we analyzed the effect of changes in δ, q_n was positive in the first period when $\delta > l_1$, but in this example, the demand for new products is zero. This phenomenon brings the matter into the open such that when we choose low values of δ and l_1 while investigating their effects or comparing the results, the volume of customers who prefer nothing becomes larger. Therefore, the demand for one of the two products turns out to be zero. However, when these values are taken high, the volume of consumers who prefer nothing decreases and others make their decisions with respect to their net utility. In the single period problem, the product which makes more profit is marketed, but in the multi-period setting, the pricing is such that the demand for new products is positive only for the future profit they generate by their sales as remanufactured products. Finally, if both δ and l_1 are low, only q_r is positive in the first period and q_n is zero in the rest of the time horizon, either. However, if δ is high, q_n is positive in the first period due to the threat of supply of remanufactured products in the rest of the time horizon.

In the single period problem, if $\delta = l_1$, q_r is positive. But, in the multi-period setting, q_n is positive due to the threat of supply of used products in the next period as mentioned before. This situation is presented in Table 5.23.

Table 5.23. Optimal prices and demands - 2 periods, $l_1 = 0.2$, $\delta = 0.2$

Periods	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	0.7942	0.1588	0.3305	0.0000	0.6695	0.0000	0.1339	0.0116
2	1.3511	0.1339	0.0000	0.3305	0.6695	2.84E-14	0.2278	0.0198

If $l_1 > \delta$, q_n is positive due to the high consumer acceptance of leasing compared with δ in the first period. Since used items return at the beginning of the next period, q_r increases in the second period. From this point on, as l_1 increases, q_n increases and q_r decreases. Therefore, the amount of used products that remains at the end of the period increases as illustrated in Tables 5.24 and 5.25.

Table 5.24. Optimal prices and demands - 2 periods, $l_1 = 0.3$, $\delta = 0.2$

Periods	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.1862	0.1584	0.3333	0.0000	0.6667	0.0000	0.2000	0.0174
2	1.1862	0.1143	0.1429	0.2857	0.5714	0.0476	0.2000	0.0174

The effect of changes in l_1 is analyzed for $T = 5$. Figures 5.46 and 5.47 give the effects of l_1 on demands at the beginning and at the end of the time horizon, respectively. If $l_1 < \delta$, q_n increases in l_1 in the first period, but q_r decreases. Recall that q_n did not increase in the single period problem. But, since new products are the

Table 5.25. Optimal prices and demands - 2 periods, $l_1 = 0.4$, $\delta = 0.2$

Periods	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.4828	0.1865	0.3750	0.0000	0.6250	0.0000	0.2500	0.0217
2	1.4828	0.1083	0.2917	0.1667	0.5417	0.2083	0.2500	0.0217

source of remanufactured products in the future, this increase in l_1 results in an increase of q_n in the multi-period setting. p_n increases in l_1 since the manufacturer takes the advantage of the increased willingness-to-pay for leasing a new product. However, p_r decreases not to lose consumers due to the increased willingness-to-pay for leasing a new product. We conclude that if $\delta > l_1$, q_n increases and q_r decreases in the first period as l_1 increases , because leasing more new products provides more profit, and more used products in the future. When $\delta = l_1$, the manufacturer charges higher price for remanufactured products to shift consumers towards new products in the first period considering again the supply of used products in the future. If $l_1 > \delta$, q_r is zero whereas q_n increases in l_1. While p_n increases, p_r also increases so as to prevent the demand for remanufactured products.

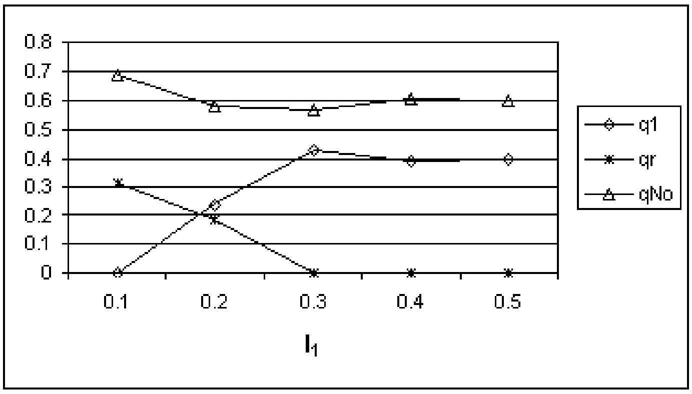

Figure 5.46. The effect of l_1 on the demands in the first period - 5 periods, $\delta = 0.3$

In the last period, if $l_1 \leq \delta$, q_n is zero since the manufacturer leases new products in order to provide used cores for remanufacturing in the future. In this case, as l_1 increases, p_n increases to hold q_n at zero, and p_r decreases to create q_r. If $l_1 > \delta$, q_n becomes positive, and it increases in l_1 whereas q_r decreases. Like in the case where $l_1 < \delta$ in the first period, p_n increases in l_1, and p_r decreases.

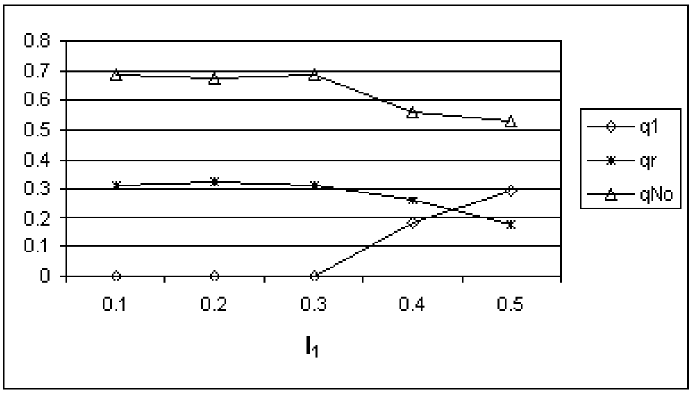

Figure 5.47. The effect of l_1 on the demands in the last period - 5 periods, $\delta = 0.3$

We discussed that the price of the new product increases as l_1 increases to take advantage of increased willingness-to-pay for leasing a new product. Therefore, the higher the l_1 value, the higher the profit is obtained. We present the optimal profit against l_1 for different time horizons up to 5 periods in Figure 5.48.

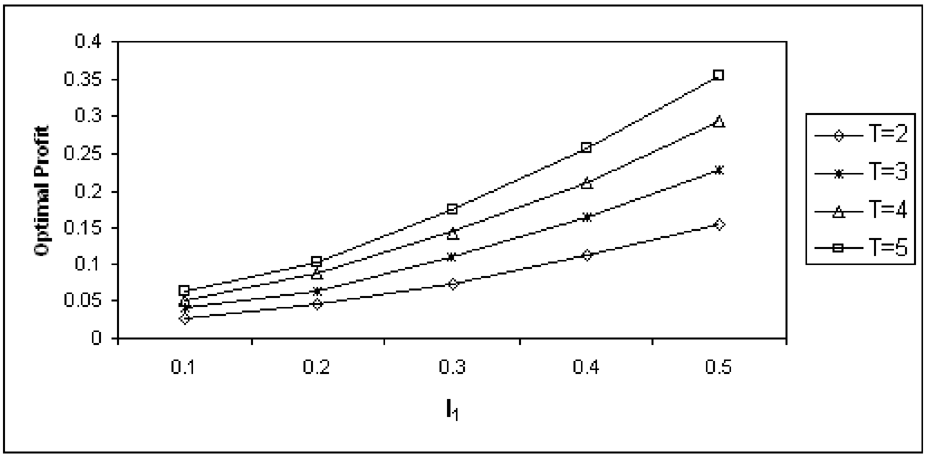

Figure 5.48. The effect of l_1 on the profit for different multi-period problems

Since we present the trend of new and remanufactured products in the previous section in detail, we will only illustrate the trend in five periods for different l_1 values. This is because when time horizon grows, the trend for demands does not change. Figures from 5.49 to 5.53 exhibit demands for l_1 values ranging from 0.1 to 0.5, respectively.

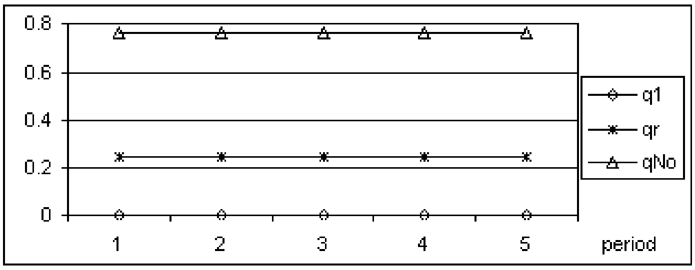

Figure 5.49. The trend of optimal demands - 5 periods, $l_1 = 0.1$

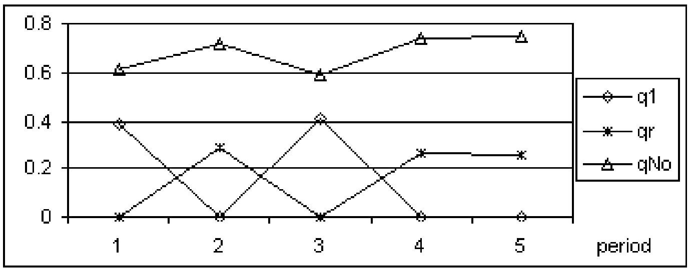

Figure 5.50. The trend of optimal demands - 5 periods, $l_1 = 0.2$

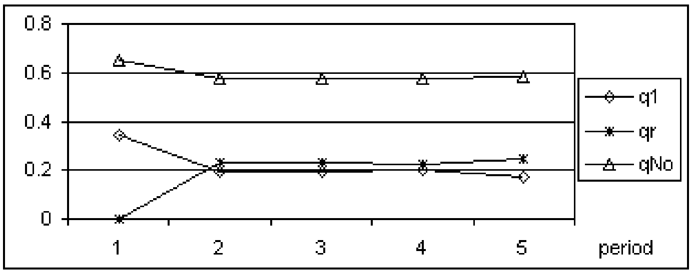

Figure 5.51. The trend of optimal demands - 5 periods, $l_1 = 0.3$

As seen in Figure 5.49, when $\delta > l_1$, q_r is positive, and remains constant in the rest of the time horizon. In fact, we have observed before that q_n exists up to the last period so as to supply used cores in the next period, but in this case l_1 is significantly low to create the demand for new products. If it were higher, q_n would be positive in the first period up to the last period, and it would be zero whereas q_r would increase in the last period. If $\delta = l_1$, consumers are indifferent between leasing a new product and buying a remanufactured product as presented in Figure 5.50. When $l_1 > \delta$, q_n exists in the first period. With returns in the second period, q_r becomes positive, and q_n decreases. Demands do not change in the following periods as given in Figures from 5.51 to 5.53.

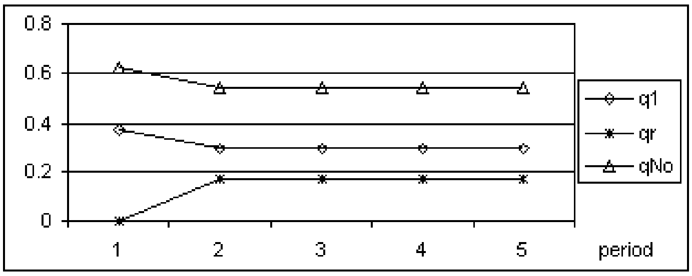

Figure 5.52. The trend of optimal demands - 5 periods, $l_1 = 0.4$

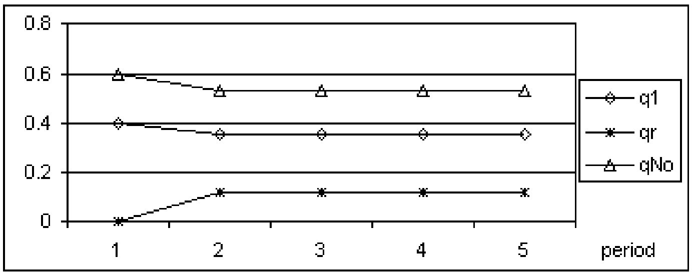

Figure 5.53. The trend of optimal demands - 5 periods, $l_1 = 0.5$

In conclusion, the effects of l_1 and δ are similar on the results. In other words, a decrease in δ for fixed l_1 generates similar trends on demands and prices as an increase in l_1 for fixed δ.

5.2.3. The Effect of Changes in the Deterioration of the Product

In the single period problem, the effect of changes in d_1 is different for $\delta < l_1$ and $\delta \geq l_1$. As mentioned before, when $\delta < l_1$, an increase in d_1 makes the new product less attractive, but with some price modifications, the demand for new products exists as d_1 increases. If $\delta \geq l_1$, q_n is zero even if d_1 is low. Therefore, there is no effect of increase in d_1 on the demands and prices of the remanufactured product. In the multi-period setting, we investigate the effect of changes in d_1 for $\delta < l_1$, $\delta = l_1$, and $\delta > l_1$, respectively. Fixed parameter values are given in Table 5.26.

Table 5.26. Fixed parameter values to analyze the effect of changes in d_1

l_1	c_r'	I_0
0.5	0.08	0

We observed that when $\delta < l_1$ and d_1 is low, q_r is zero whereas q_n is positive in the first period. With respect to the returns of used products at the beginning of the second period, q_r becomes positive and q_n decreases. Compared with the first period, the volume of consumers who do not prefer any one of the products decreases in the second period. We note that the volume of new and remanufactured products do not change in the following periods. Figures from 5.54 to 5.59 illustrates the increase in d_1 if $\delta = 0.2$ and $l_1 = 0.5$ so as to analyze the effects of it in the five-period problem. We choose only a five-period problem because of the fact that the increase in d_1 affects the first and second periods, and the rest of the time horizon is the repetition of the second period.

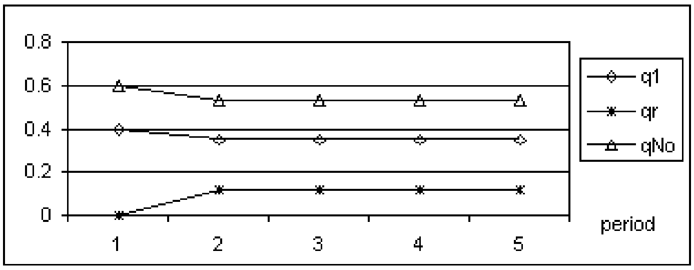

Figure 5.54. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.2$, $d_1 = 0.1$

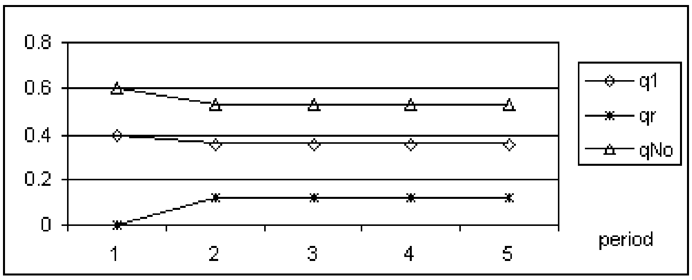

Figure 5.55. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.2$, $d_1 = 0.2$

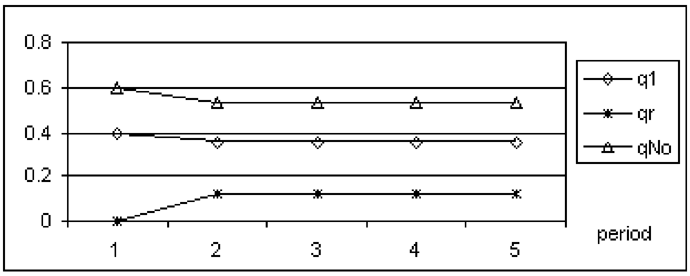

Figure 5.56. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.2$, $d_1 = 0.3$

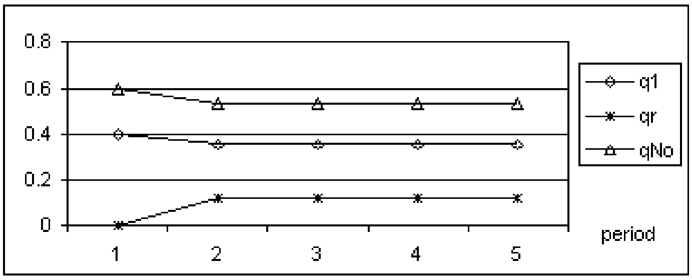

Figure 5.57. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.2$, $d_1 = 0.4$

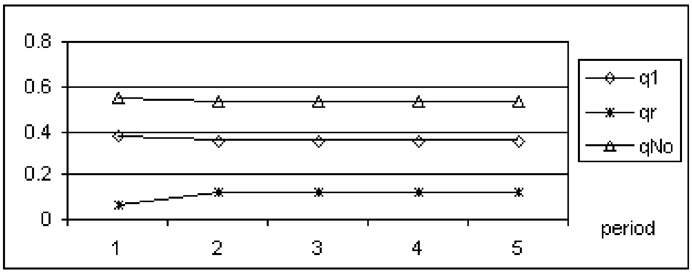

Figure 5.58. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.2$, $d_1 = 0.5$

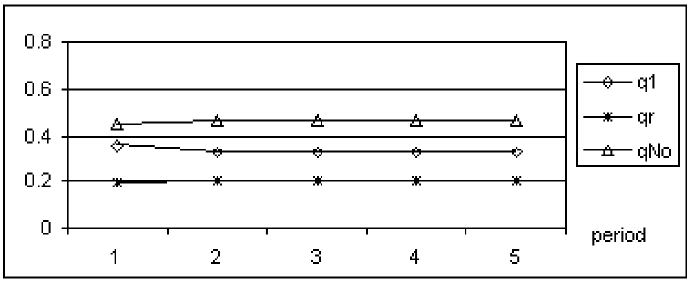

Figure 5.59. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.2$, $d_1 = 0.6$

As mentioned before, as d_1 increases, there is not always a decrease in the volume of new products. This is because the optimal pricing is such a way that the demand for new products does not change. Therefore, it is obvious that there is a threshold for depreciation rate such that for values less than the threshold, there is no change in the demand even though d_1 increases as seen in Figures from 5.54 to 5.57. This is because p_n decreases to keep the present value of payments the same whereas p_r does not change. However, when the threshold is exceeded, the volume of new products decreases and the volume of remanufactured products increases as d_1 increases as shown in Figures 5.58 and 5.59. Since the prices of new and remanufactured products decrease, the profit decreases from this point on. Figure 5.60 illustrates the effect of d_1 on the profit.

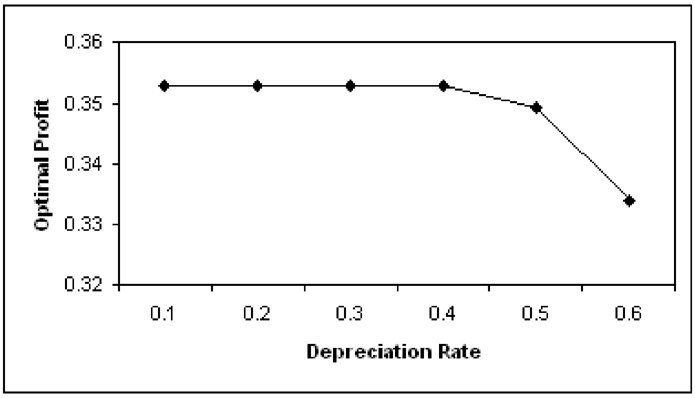

Figure 5.60. Behavior of the optimal profit as d_1 increases - 5 periods, $l_1 = 0.5$, $\delta = 0.2$

If $\delta = l_1$ and d_1 is low, we found that the demand exists for only one type of products in each period, and the pricing is such that q_n is positive in the first period and zero in the last period. This trend is valid for d_1 values less than the threshold value as shown in Figure 5.61. In this setting, when the threshold is exceeded, q_n becomes zero and q_r becomes positive in the first period. This phenomenon is illustrated in Figure 5.62. Since q_n is zero in the first period, the amount of the remanufactured products is less in the second period when compared with the case where d_1 is lower. Moreover, the volume of remanufactured products does not change in the rest of the time horizon. From the threshold value on, the profit decreases because of the fact that the volume of total demand decreases, and the volume of consumers who prefer nothing increases. In this setting, the effect of change in the price of the remanufactured product is not significant on the profit. However, the most important effect on the profit is the decrease in the volume of consumers in each period, and the increase in the amount of used products supplied from the third-party core supplier.

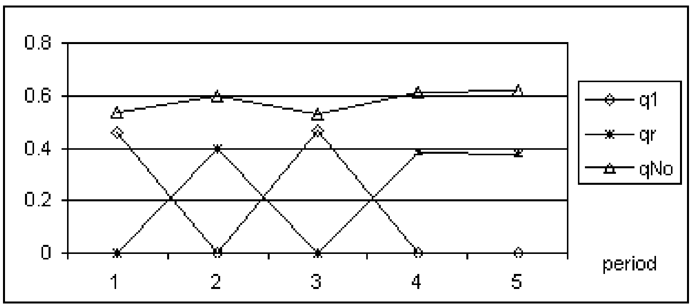

Figure 5.61. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.5$, $d_1 = 0.1$

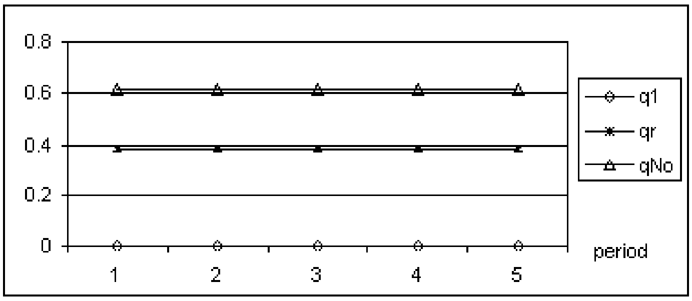

Figure 5.62. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.5$, $d_1 = 0.5$

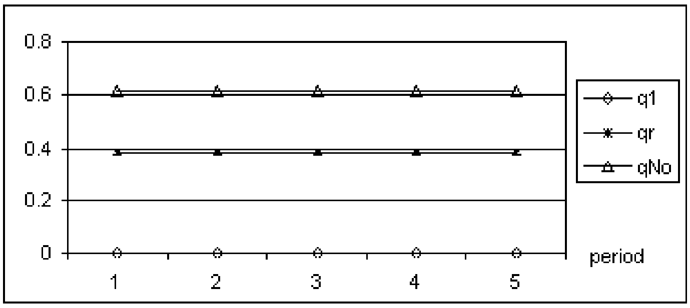

Figure 5.63. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.5$, $d_1 = 0.6$

It is important to note that when the volume of new products becomes zero, from this point on, the increase in d_1 has no impact on the results. This can be seen when Figure 5.63 is compared with Figure 5.62. Therefore, from this point on, the profit remains the same as illustrated in Figure 5.64.

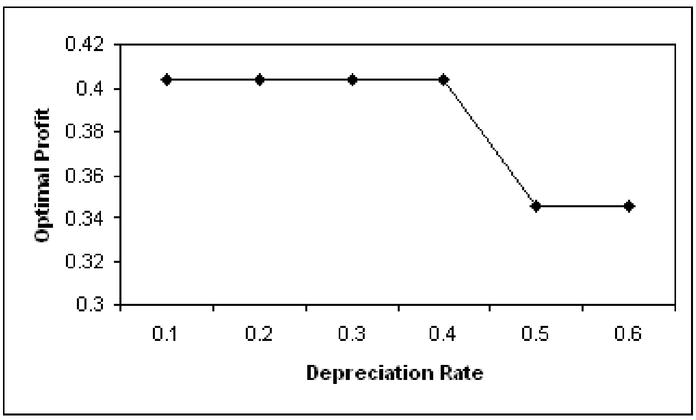

Figure 5.64. Behavior of the optimal profit as d_1 increases - 5 periods, $l_1 = 0.5$, $\delta = 0.5$

When $\delta > l_1$, the way of generating demand for new products is to provide d_1 to be low. Figure 5.65 presents the optimal demands in the five-period problem if $\delta = 0.7$, $l_1 = 0.5$ and $d_1 = 0.1$. In this case, we observe that both q_n and q_r are positive in the first period, and also in the following periods up to the last period. Recall that new products are produced for the future profit they generate through their sales as remanufactured products, therefore they are not produced in the last period. If d_1 increases, p_n decreases to keep the present value of payments the same whereas p_r does not change. But, when it exceeds the threshold, q_n becomes zero and q_r increases as shown in Figure 5.66. This is because, from this point on, the manufacturer makes consumers shift towards remanufactured product by charging higher price for the new product, and increases p_r to take advantage of remanufactured products. Even though p_r and q_r increase, the profit decreases because of the fact that all used cores are supplied from the third-party supplier at a cost of c_r', and the volume of consumers who prefer nothing increases. From this point on, the profit remains the same since the increase in d_1 has no impact on the results. Figure 5.67 exhibits the effect of the increase in depreciation rate on the profit.

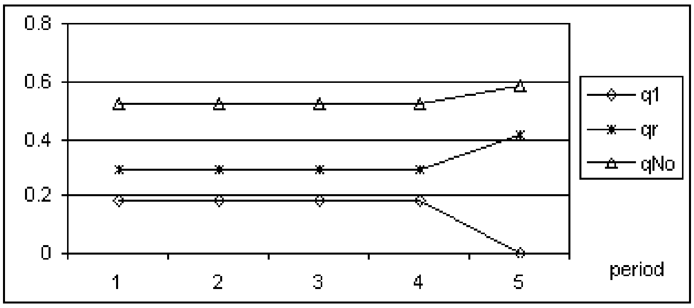

Figure 5.65. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.7$, $d_1 = 0.1$

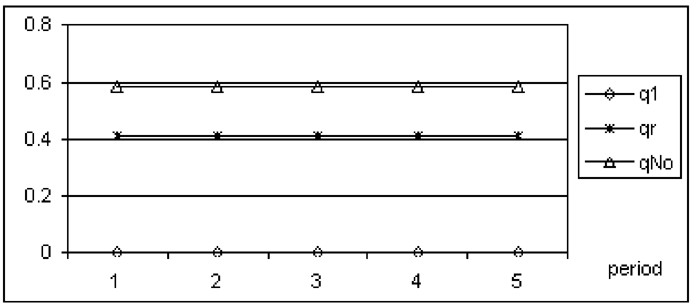

Figure 5.66. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.7$, $d_1 = 0.5$

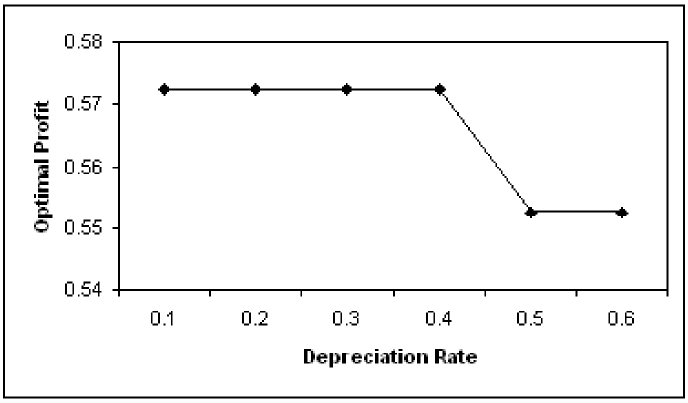

Figure 5.67. Change in the optimal profit as d_1 increases - 5 periods, $l_1 = 0.5$, $\delta = 0.7$

5.2.4. The Effect of Changes in the Initial Inventory Level

We will analyze the effect of initial inventory level I_0 for $\delta < l_1$, $\delta = l_1$ and $\delta > l_1$, respectively, in this section. Since the trend of optimal demands does not change in the two, three, four, and five-period problems, we only exhibit the five-period setting. Fixed parameter values are given in Table 5.27.

Table 5.27. Fixed parameter values to analyze the effect of changes in I_0

l_1	d_1	c_r'
0.5	0.1	0.08

Figures from 5.68 to 5.70 exhibit the optimal demands in the five-period problem as I_0 increases if $l_1 = 0.5$, and $\delta = 0.2$. We find that if $\delta < l_1$ and $I_0 = 0$, q_r is zero in the first period, but it becomes positive due to the returns of used products in the following periods. Therefore, we expect the demand for remanufactured products to be positive if I_0 increases. The expected increase in q_r occurs, but the volume of remanufactured products does not exceed the available inventory as discussed in the single period problem. The increase in q_r occurs due to the decrease in the price of the remanufactured product which causes the decrease in q_n in the first period. Moreover, the profit increases due to the increase in q_r. However, if $I_0 > q_r^*$, the increase in I_0 has no impact on q_r and p_r. Therefore, the profit does not change as illustrated in Figure 5.71.

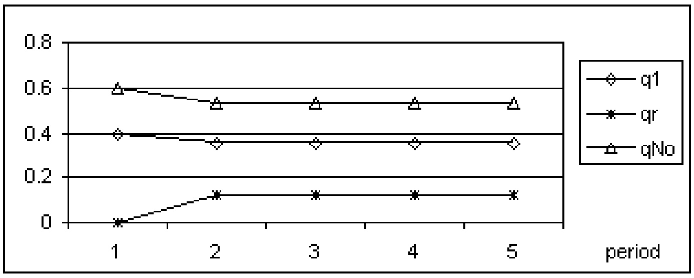

Figure 5.68. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.2$, $I_0 = 0$

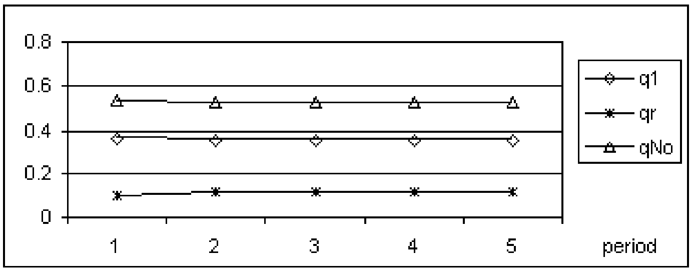

Figure 5.69. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.2$, $I_0 = 0.1$

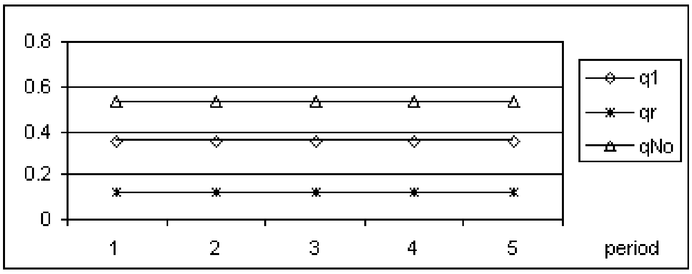

Figure 5.70. The trend of optimal demands - 5 periods, $l_1 = 0.5$, $\delta = 0.2$, $I_0 = 0.2$

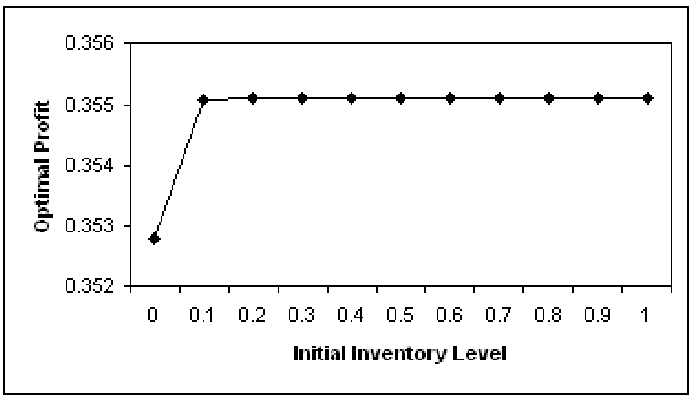

Figure 5.71. Behavior of the optimal profit as I_0 increases - 5 periods, $l_1 = 0.5$, $\delta = 0.2$

If $\delta = l_1$ and $I_0 = 0$, we find that the demand for one type of the products is positive in each period as presented in Table 5.28. If I_0 increases, there is not always an increase in the volume of remanufactured products as illustrated in Table 5.29. This is because the optimal pricing is such a way that q_n is positive unless I_0 is adequate for q_r in the first period. Therefore, it is clear that I_0 should be enough to create the demand for remanufactured products in the first period. Even though there is not an increase in q_r in the first period as I_0 increases, the profit can increase. This is because the amount of used cores supplied from the third-party core supplier decreases over time horizon.

Table 5.28. Optimal prices and demands - 5 periods, $\delta = 0.5$, $I_0 = 0$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.6049	0.7206	0.4588	0.0000	0.5412	0.0000	0.2706	0.0235
2	1.8453	0.3016	0.0000	0.3968	0.6032	0.0620	0.3111	0.0271
3	1.5759	0.7144	0.4686	0.0000	0.5314	0.0620	0.2657	0.0231
4	2.5567	0.3064	0.0000	0.3872	0.6128	0.1434	0.4311	0.0375
5	2.9037	0.3091	0.0000	0.3818	0.6182	0.0000	0.4896	0.0426

Table 5.29. Optimal prices and demands - 5 periods, $\delta = 0.5$, $I_0 = 0.1$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.6049	0.5376	0.4588	0.0000	0.5412	0.1000	0.2706	0.0235
2	1.9572	0.3016	0.0000	0.3968	0.6032	0.1620	0.3300	0.0287
3	1.5759	0.6613	0.4686	0.0000	0.5314	0.1620	0.2657	0.0231
4	1.9053	0.3064	0.0000	0.3872	0.6128	0.2434	0.3212	0.0279
5	2.5229	0.3091	0.0000	0.3818	0.6182	0.0000	0.4254	0.0370

When I_0 is enough to meet the demand for remanufactured products in the first period, the firm produces remanufactured products. Table 5.30 illustrates this phenomenon. In the second period, the manufacturer has to produce some new products in order to provide available inventory of used products in the future. This is because remanufactured products cannot be produced without supplying used cores. Recall that in the two-period problem, the second period represents the last period, therefore q_r becomes positive in the second period. This is because the manufacturer does not worry about the supply of used products in the future. However, the manufacturer produces new products in the second period of the three, four and five-period settings. Moreover, from the second period on, new and remanufactured products are produced in sequence. But, only q_r is positive in the last period.

Table 5.30. Optimal prices and demands - 5 periods, $\delta = 0.5$, $I_0 = 0.4$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	2.5806	0.3000	0.0000	0.4000	0.6000	0.0000	0.4351	0.0378
2	1.6977	0.7335	0.4275	0.0000	0.5725	0.0000	0.2862	0.0249
3	2.8719	0.2862	0.0000	0.4275	0.5725	0.0000	0.4842	0.0421
4	1.6977	0.7425	0.4275	0.0000	0.5725	0.0000	0.2862	0.0249
5	1.7538	0.2862	0.0000	0.4275	0.5725	0.0000	0.2957	0.0257

When I_0 is enough to meet the demand for remanufactured products in the first and also in the second period as seen in Table 5.31, the manufacturer produces remanufactured products in both two periods. However, for instance, the demand for remanufactured products does not exist in the second period in the three-period problem as shown in Figure 5.72. This is because if used products required for two periods are supplied from the available inventory, the manufacturer has to provide used products from the third-party core supplier in the last period.

Table 5.31. Optimal prices and demands - 5 periods, $\delta = 0.5$, $I_0 = 0.8$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.8532	0.2995	0.0000	0.4011	0.5989	0.3989	0.3125	0.0272
2	2.3252	0.3016	0.0000	0.3968	0.6032	0.0021	0.3920	0.0341
3	1.5759	0.2813	0.4686	0.0000	0.5314	0.0021	0.2657	0.0231
4	2.3894	0.3064	0.0000	0.3872	0.6128	0.0835	0.4029	0.0350
5	2.1161	0.3091	0.0000	0.3818	0.6182	0.0000	0.3568	0.0310

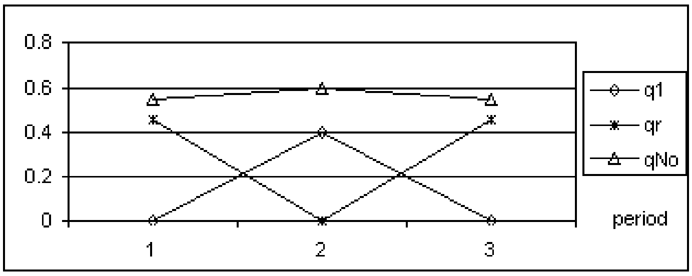

Figure 5.72. The trend of optimal demands - 3 periods, $l_1 = 0.5$, $\delta = 0.5$, $I_0 = 0.8$

As a result, if $\delta = l_1$, q_r can be positive in the initial periods of the time horizon if the stock of used products is enough. Moreover, if the next period is not the last period of the time horizon, q_n becomes positive in order to supply the used products in the future. In the most of the cases, the profit increases in I_0 because of the fact that either q_r increases or the amount of used cores supplied from the third-party core supplier decreases. The increase in I_0 sometimes does not affect the profit since the volume of demands, and prices do not change. Figure 5.73 illustrates the effect of I_0 on the profit for the five-period problem when δ and l_1 are fixed at 0.5.

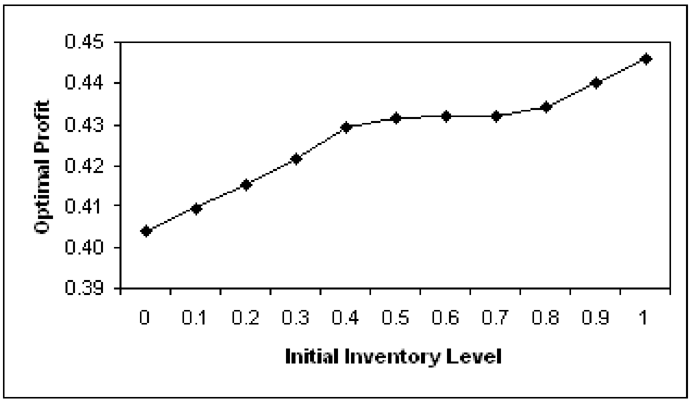

Figure 5.73. The effect of I_0 on the profit - 5 periods, $l_1 = 0.5$, $\delta = 0.5$

When $\delta > l_1$, the manufacturer produces remanufactured products even if $I_0 = 0$. The effect of the increase in I_0 is presented in Tables from 5.32 to 5.36. The increase in I_0 sometimes has no impact on the demands and prices, but it affects the profit in a positive way. This is because the amount of used products supplied from the third-party core supplier decreases. In the first period, $q_{r,1}$ increases only if I_0 is enough to meet the demand for remanufactured products, or higher than the resulting demand for remanufactured products. When $q_{r,1}$ increases, $q_{n,1}$ decreases such that the manufacturer charges less price for remanufactured products and higher price for new products. In the following, if $I_{t-1}+R_t$ exceeds the volume of remanufactured products created in the case where I_0 is lower, $q_{r,t}$ also increases and $q_{n,t}$ decreases. In the last period, the volume of remanufactured products does not depend on the stock of used products, and q_n is zero.

As mentioned before, if the initial inventory of used products is enough to meet the demand for remanufactured products over the time horizon, the new product does not exist in any period. For instance, if $I_0 = 1$, the manufacturer produces only the remanufactured product both in the first and last periods of the two-period setting. In the five-period problem, it is also enough to meet the demand for remanufactured products in the first and second periods, but the manufacturer takes into consideration the future demand of remanufactured products, and therefore, produces new products. In Tables from 5.32 to 5.36, it is obvious that the volume of new products decreases in such periods where q_r increases in I_0, but does not become zero. Moreover, as I_{t-1} decreases due to the remanufacturing used cores, the manufacturer increases $q_{n,t}$ in period t considering the future value that they generate through their sales as remanufactured products.

Table 5.32. Optimal prices and demands - 5 periods, $\delta = 0.7$, $I_0 = 0$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.5596	0.4046	0.1822	0.2919	0.5259	0.0000	0.2630	0.0229
2	1.5596	0.4046	0.1822	0.2919	0.5259	0.0000	0.2630	0.0229
3	1.5596	0.4046	0.1822	0.2919	0.5259	0.0000	0.2630	0.0229
4	1.5596	0.4046	0.1822	0.2919	0.5259	0.0000	0.2630	0.0229
5	5.3231	0.4107	0.0000	0.4133	0.5867	0.0000	0.8975	0.0781

Table 5.33. Optimal prices and demands - 5 periods, $\delta = 0.7$, $I_0 = 0.3$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.5596	0.4030	0.1741	0.3000	0.5259	0.0000	0.2630	0.0229
2	1.5596	0.4046	0.1822	0.2919	0.5259	0.0000	0.2630	0.0229
3	1.5596	0.4046	0.1822	0.2919	0.5259	0.0000	0.2630	0.0229
4	1.5596	0.4046	0.1822	0.2919	0.5259	0.0000	0.2630	0.0229
5	2.1343	0.4107	0.0000	0.4133	0.5867	0.0000	0.3599	0.0313

Table 5.34. Optimal prices and demands - 5 periods, $\delta = 0.7$, $I_0 = 0.5$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.5759	0.4006	0.1429	0.3257	0.5314	0.1743	0.2657	0.0231
2	1.5596	0.4022	0.1704	0.3037	0.5259	0.0136	0.2630	0.0229
3	1.5596	0.4046	0.1822	0.2919	0.5259	0.0000	0.2630	0.0229
4	1.5596	0.4046	0.1822	0.2919	0.5259	0.0000	0.2630	0.0229
5	9.7437	0.4107	0.0000	0.4133	0.5867	0.0000	1.6428	0.1429

Table 5.35. Optimal prices and demands - 5 periods, $\delta = 0.7$, $I_0 = 0.7$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.5889	0.3993	0.1211	0.3432	0.5358	0.3569	0.2679	0.0233
2	1.5737	0.4008	0.1467	0.3226	0.5307	0.1553	0.2653	0.0231
3	1.5596	0.4026	0.1720	0.3021	0.5259	0.0000	0.2630	0.0229
4	1.5596	0.4046	0.1822	0.2919	0.5259	0.0000	0.2630	0.0229
5	2.4393	0.4107	0.0000	0.4133	0.5867	0.0000	2.0973	0.1824

Table 5.36. Optimal prices and demands - 5 periods, $\delta = 0.7$, $I_0 = 0.9$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.5969	0.3985	0.1075	0.3540	0.5385	0.5460	0.2693	0.0234
2	1.5823	0.3999	0.1321	0.3343	0.5336	0.3192	0.2668	0.0232
3	1.5666	0.4015	0.1587	0.3131	0.5283	0.1382	0.2641	0.0230
4	1.5596	0.4036	0.1772	0.2969	0.5259	0.0000	0.2630	0.0229
5	1.7715	0.4107	0.0000	0.4133	0.5867	0.0000	0.2987	0.0260

As seen in Figure 5.74, the profit increases in I_0 because of the fact that either q_r increases or the amount of used items supplied from the third-party core supplier decreases.

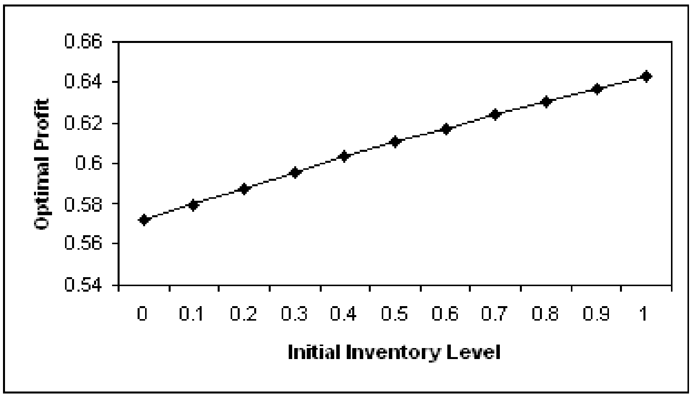

Figure 5.74. The effect of I_0 on the profit - 5 periods, $l_1 = 0.5$, $\delta = 0.7$

5.2.5. The Effect of Changes in the Cost of Supplying Used Products from Third-party Core Supplier

We conduct the experiments for the set of $c'_r = \{0, 0.02, 0.04, 0.06, 0.08\}$ for $\delta < l_1$, $\delta = l_1$ and $\delta > l_1$, respectively, considering all other parameters fixed as given in Table 5.37. Since the trends do not change in the multi-period settings from two to five periods, we present results only for the five-period problem.

Table 5.37. Fixed parameter values to analyze the effect of changes in c'_r

l_1	d_1	I_0
0.5	0.1	0

Tables 5.38 and 5.39 give the optimal prices and demands for $l_1 = 0.5$, and $\delta = 0.2$ if $c'_r = 0$ and $c'_r = 0.02$, respectively. When $\delta < l_1$, the results obtained in the multi-period setting show that the volume of demands for new and remanufactured products in the first period are the same as in the single period problem. If $c'_r = 0$, the manufacturer acquires used cores without any extra cost. In other words, this assumption reveals that the inventory of used products is enough to meet the demand in each period, and the average cost of remanufacturing used products is only $c_r q_r$.

For this reason, the remanufactured products exist although δ is low. However, as discussed in the single period setting, an increase in c_r' makes a remanufactured product less attractive with respect to a new product, therefore q_r decreases gradually as c_r' increases and becomes zero for some c_r' value. In the multi-period setting, when c_r' is too high, since the company earns more from customers who lease a new product than those who buy a remanufactured product, he charges initially higher prices for remanufactured products to shift customers towards new products, and then reduces the price because of the fact that used products return. From this point on, there is no impact of c_r' on the demands.

Table 5.38. Optimal prices and demands - 5 periods, $\delta = 0.2$, $c_r' = 0$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.7793	0.1059	0.3529	0.1176	0.5294	0.0000	0.3000	0.0261
2	1.7793	0.1059	0.3529	0.1176	0.5294	0.2353	0.3000	0.0261
3	1.7793	0.1059	0.3529	0.1176	0.5294	0.4706	0.3000	0.0261
4	1.7793	0.1059	0.3529	0.1176	0.5294	0.7059	0.3000	0.0261
5	1.7793	0.1059	0.3529	0.1176	0.5294	0.9412	0.3000	0.0261

Table 5.39. Optimal prices and demands - 5 periods, $\delta = 0.2$, $c_r' = 0.02$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.7793	0.1129	0.3765	0.0588	0.5647	0.0000	0.3000	0.0261
2	1.7793	0.1059	0.3529	0.1176	0.5294	0.2588	0.3000	0.0261
3	1.7793	0.1059	0.3529	0.1176	0.5294	0.4941	0.3000	0.0261
4	1.7793	0.1059	0.3529	0.1176	0.5294	0.7294	0.3000	0.0261
5	1.7793	0.1059	0.3529	0.1176	0.5294	0.9647	0.3000	0.0261

When c_r' is too high, q_r becomes zero in the first period and the increase in c_r' does not affect decisions as well as the optimal profit as seen in Figure 5.75. This is because there is no need for used cores supplied from the third-party supplier in the second period due to the returns.

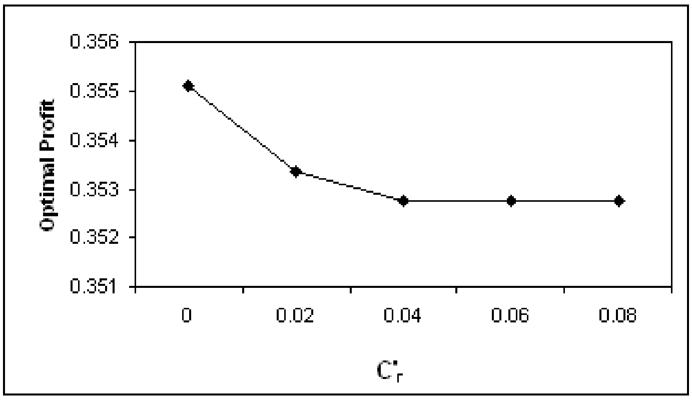

Figure 5.75. The effect of c_r' on the profit - 5 periods, $l_1 = 0.5$, $\delta = 0.2$

We conclude that when $\delta < l_1$, remanufactured products exist in the first period only if c_r' is low or initial inventory is enough since otherwise selling of remanufactured products is not profitable for the firm. In the next period, returns of new products leased in the first period make the remanufactured product attractive.

When $\delta = l_1$, as discussed before, if c_r' is high and there is no inventory in the first period, the remanufactured product does not exist in the first period. In Table 5.40, we present optimal prices and demands in the five-period problem for $c_r' = 0$. In that situation, q_r becomes positive in all periods. Therefore, it is important to note that, if consumers view leasing a new product and buying a remanufactured products as being identical, and $c_r' = 0$, or I_t is enough in each period, the manufacturer never leases new products. This is because remanufactured products' sales are more profitable than leasing new products due to the less recovery cost. As c_r' increases, q_r decreases in each period as illustrated in Table 5.41. When the manufacturer has less profit from selling a remanufactured product over the time horizon for some c_r' value, he charges higher price for remanufactured products in the first period in order to shift all consumers towards the new products. In the second period, he creates the demand for remanufactured products by reducing price for remanufactured products and increasing for new products as presented in Table 5.42. In this phenomenon, it is obvious that new products are produced only for generating used products in the next period, therefore q_n becomes zero in the last period. Figure 5.76 exhibits the behavior of the optimal profit as c_r' increases for $\delta = l_1$.

Table 5.40. Optimal prices and demands - 5 periods, $\delta = 0.5$, $c_r' = 0$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.7915	0.2727	0.0000	0.4545	0.5455	0.0000	0.3021	0.0263
2	2.1522	0.2727	0.0000	0.4545	0.5455	0.0000	0.3629	0.0316
3	2.0542	0.2727	0.0000	0.4545	0.5455	0.0000	0.3464	0.0301
4	2.2522	0.2727	0.0000	0.4545	0.5455	0.0000	0.3797	0.0330
5	1.8687	0.2727	0.0000	0.4545	0.5455	0.0000	0.3151	0.0274

Table 5.41. Optimal prices and demands - 5 periods, $\delta = 0.5$, $c_r' = 0.02$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	2.1868	0.2818	0.0000	0.4364	0.5636	0.0000	0.3687	0.0321
2	1.6734	0.2818	0.0000	0.4364	0.5636	0.0000	0.2821	0.0245
3	2.9649	0.2818	0.0000	0.4364	0.5636	0.0000	0.4999	0.0435
4	1.7487	0.2818	0.0000	0.4364	0.5636	0.0000	0.2948	0.0256
5	2.6008	0.2818	0.0000	0.4364	0.5636	0.0000	0.4385	0.0381

Table 5.42. Optimal prices and demands - 5 periods, $\delta = 0.5$, $c_r' = 0.06$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.6485	0.8243	0.4441	0.0000	0.5559	0.0000	0.2779	0.0242
2	3.0922	0.2944	0.0000	0.4112	0.5888	0.0329	0.5214	0.0454
3	1.6268	0.8090	0.4514	0.0000	0.5486	0.0329	0.2743	0.0239
4	1.8826	0.2980	0.0000	0.4040	0.5960	0.0803	0.3174	0.0276
5	2.8996	0.3000	0.0000	0.4000	0.6000	0.0000	0.4889	0.0425

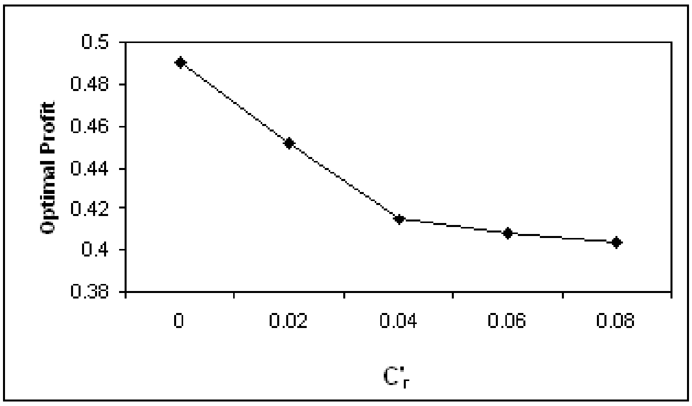

Figure 5.76. Change in the optimal profit as c_r' increases - 5 periods, $l_1 = 0.5$, $\delta = 0.5$

When $\delta > l_1$, as mentioned before, although c_r' is high and $I_0 = 0$, demands for both new and remanufactured products exist. Up to the last period, the volume of new and remanufactured products do not change, but q_r increases and q_n becomes zero in the last period. However, we observe that if $c_r' = 0$, or it is very low, the manufacturer creates demand for only remanufactured products. Table 5.43 illustrates the optimal prices and demands if $c_r' = 0$. As c_r' increases, q_r decreases in each period as illustrated in Table 5.44. Therefore, the volume of consumers who do not prefer nothing increases. When the manufacturer has less profit from selling only remanufactured products over the time horizon for some c_r' value, he charges higher price for remanufactured products and less price for new products up to the last period in order to shift some consumers who buy a remanufactured product and none of the products towards the new product as presented in Table 5.45. This behavior of the manufacturer to maximize the profit over the time horizon also states that new products are leased for the future value that they generate through their sales as remanufactured products. From this point on, if c_r' continues to increase, the optimal pricing will be such a way that q_n increases whereas q_r decreases, and only q_r is positive in the last period. Figure 5.77 exhibits the behavior of the optimal profit as c_r' increases for $\delta > l_1$.

Table 5.43. Optimal prices and demands - 5 periods, $\delta = 0.7$, $c_r' = 0$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	2.1172	0.3733	0.0000	0.4667	0.5333	0.0000	0.3570	0.0311
2	1.6067	0.3733	0.0000	0.4667	0.5333	0.0000	0.2709	0.0236
3	1.5913	0.3733	0.0000	0.4667	0.5333	0.0000	0.2683	0.0233
4	2.7685	0.3733	0.0000	0.4667	0.5333	0.0000	0.4668	0.0406
5	3.3420	0.3733	0.0000	0.4667	0.5333	0.0000	0.5635	0.0490

Table 5.44. Optimal prices and demands - 5 periods, $\delta = 0.7$, $c_r' = 0.02$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	2.6393	0.3827	0.0000	0.4533	0.5467	0.0000	0.4450	0.0387
2	2.1007	0.3827	0.0000	0.4533	0.5467	0.0000	0.3542	0.0308
3	1.6398	0.3827	0.0000	0.4533	0.5467	0.0000	0.2765	0.0240
4	1.6260	0.3827	0.0000	0.4533	0.5467	0.0000	0.2741	0.0238
5	7.6453	0.3827	0.0000	0.4533	0.5467	0.0000	1.2890	0.1121

Table 5.45. Optimal prices and demands - 5 periods, $\delta = 0.7$, $c_r' = 0.06$

Period	p_n	p_r	q_n	q_r	q_{No}	I_t	$PV(p_1)$	p_1
1	1.6146	0.3984	0.0867	0.3689	0.5444	0.0000	0.2722	0.0237
2	1.6146	0.3984	0.0867	0.3689	0.5444	0.0000	0.2722	0.0237
3	1.6146	0.3984	0.0867	0.3689	0.5444	0.0000	0.2722	0.0237
4	1.6146	0.3984	0.0867	0.3689	0.5444	0.0000	0.2722	0.0237
5	3.6296	0.4013	0.0000	0.4267	0.5733	0.0000	0.6120	0.0532

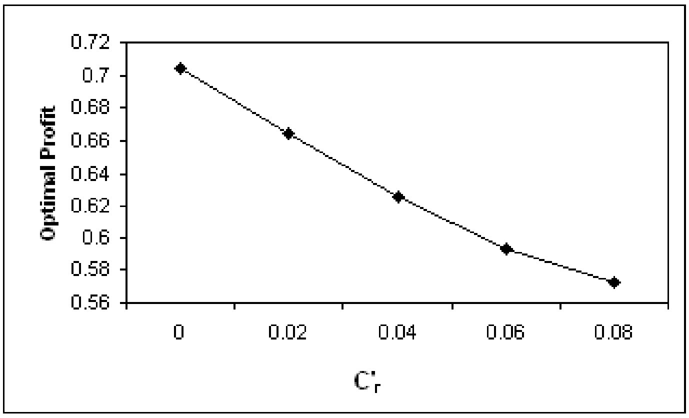

Figure 5.77. Behavior of the optimal profit as c'_r increases - 5 periods, $l_1 = 0.5$, $\delta = 0.7$

5.2.6. The Effect of Change in the Duration of Lease Agreement

In this section, we will analyze the effect of change in the duration of the lease agreement offered by the manufacturer. Until now, we observed the trend on demands and prices of new and remanufactured products and behavior of the optimal profit by solving different scenarios for 1-period option of leasing a new product. When the manufacturer offers 2-period leasing, new products leased currently return two periods later. In this setting, consumers prefer either leasing a new product for two periods or buying a remanufactured product or nothing in each period. Therefore, there is only one customer segment, and $\alpha_2 = 1$. We set $l_2 > l_1$ since a customer's valuation for leasing a new product for two periods is greater than his valuation for one period. In addition, if a product loses 10 per cent of its value in the first year, we assume that it loses 20 per cent more in the second year, leaving 70 per cent of its original value. As discussed before, this is because the older a product gets, the less it's remaining value. We keep all parameters fixed except δ to compare with the results obtained in Section 5.2.1. The parameter values given in Table 5.1 are also valid in this section, and others are presented in Table 5.46.

Table 5.46. Fixed parameter values to analyze the effect of changes in δ

l_2	d_2	c'_r	I_0
0.7	0.3	0.08	0

Figures from 5.78 to 5.85 exhibit the trend of new and remanufactured products in the multi-period setting analyzed up to 5 periods for $\delta < l_2$. When $\delta < l_2$, in the initial two periods, there is no demand for remanufactured products whereas q_n is positive. With respect to the returns of used products in the beginning of the third period, q_r becomes positive and q_n decreases. Compared with the initial two periods, consumers who do not prefer any one of the products decrease in the third period. The optimal pricing is such a way that the price of the remanufactured products decreases even though this affects the demand for new products in a negative way. We note that the volume of new and remanufactured products do not change in the following periods. In other words, the experiments present that from third period on, the behavior of consumers does not change. For one and two-period problems, we can not analyze the effects of returns since used products return at the beginning of the third period for the first time. As seen in Figures 5.78 and 5.82, only new products exist in the initial two periods.

For $\delta < l_2$, the volume of remanufactured products increases in the third period, but the volume of new products may be still higher than the volume of remanufactured products if δ is too low as illustrated in Figures from 5.79 to 5.81. When δ comes close to l_2, the volume of remanufactured products becomes higher than the volume of new products in the third period as shown in Figures from 5.83 to 5.85.

In fact, the behavior of the optimal demands in the 2-period lease scenario is similar in the 1-period lease scenario. As we discussed before, if $\delta < l_1$ and there is no inventory of used products initially, then q_r is zero in the first period. In the 2-period leasing setting, we also see that q_r is zero in the first period. Moreover, since used cores return two periods later, the demand for remanufactured products does not exist in the second period, either. After used cores arrive to the facility, remanufactured products can be produced. Since the manufacturer decreases the remanufactured product's price, q_r becomes positive which results in the decrease in q_n. From this point on, the behavior of demands does not change in the rest of the planning horizon. Therefore, we conclude that the behavior of the optimal demands for $\delta < l_2$ is consistent with the results obtained in the 1-period lease scenario for $\delta < l_1$.

Figure 5.78. Behavior of the optimal demands - 2 periods, $l_2 = 0.7$, $\delta = 0.2$

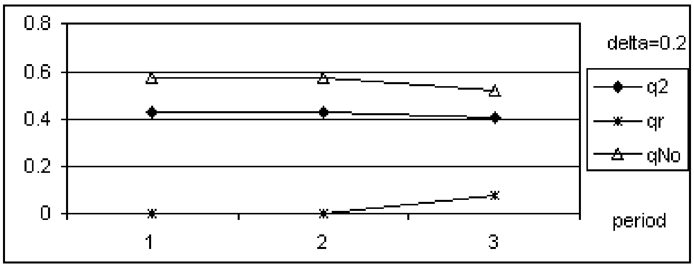

Figure 5.79. Behavior of the optimal demands - 3 periods, $l_2 = 0.7$, $\delta = 0.2$

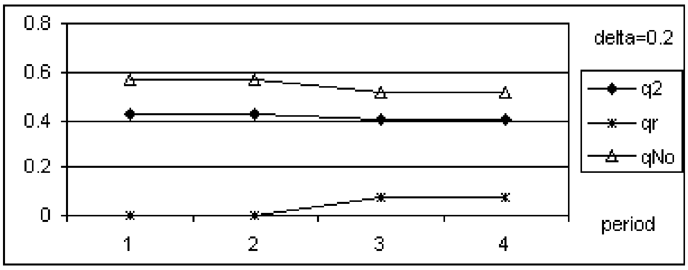

Figure 5.80. Behavior of the optimal demands - 4 periods, $l_2 = 0.7$, $\delta = 0.2$

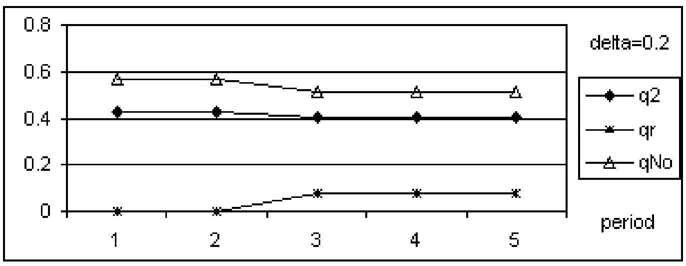

Figure 5.81. Behavior of the optimal demands - 5 periods, $l_2 = 0.7$, $\delta = 0.2$

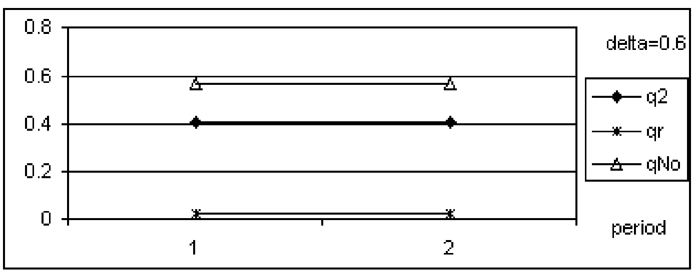

Figure 5.82. Behavior of the optimal demands - 2 periods, $l_2 = 0.7$, $\delta = 0.6$

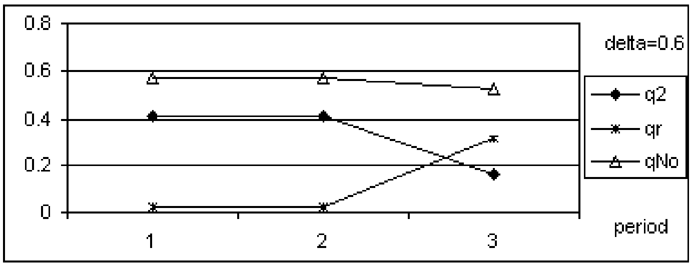

Figure 5.83. Behavior of the optimal demands - 3 periods, $l_2 = 0.7$, $\delta = 0.6$

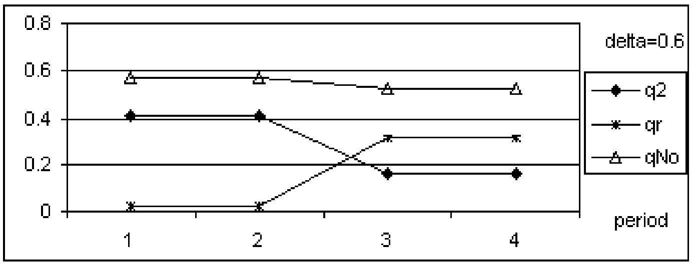

Figure 5.84. Behavior of the optimal demands - 4 periods, $l_2 = 0.7$, $\delta = 0.6$

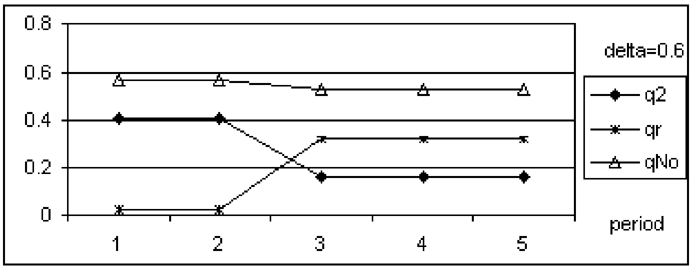

Figure 5.85. Behavior of the optimal demands - 5 periods, $l_2 = 0.7$, $\delta = 0.6$

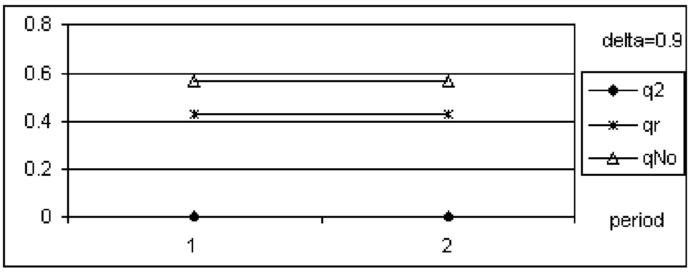

Figure 5.86. Change in the optimal demands - 2 periods, $l_2 = 0.7$, $\delta = 0.9$

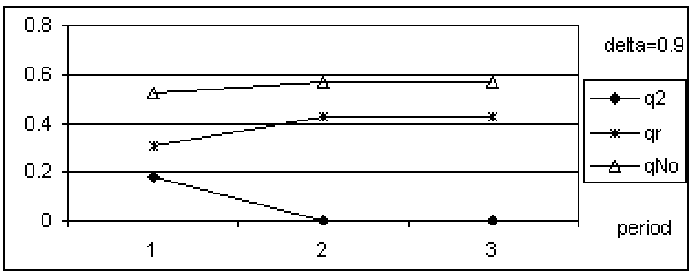

Figure 5.87. Change in the optimal demands - 3 periods, $l_2 = 0.7$, $\delta = 0.9$

Recall that if $\delta > l_1$, demands for both new and remanufactured products exist up to the last period in the scenario where 1-period lease is offered. However, we observed that the demand for only remanufactured products exists in the last period. When the manufacturer offers 2-period lease, q_n and q_r are positive to the last two periods, but only remanufactured products exist in the last two periods. This reveals that new products are marketed for the threat of supply of used cores in the future, and in this scenario, since products return two periods later, the new products produced just before the last period cannot come in handy throughout the planning horizon. For this reason, new products are not produced in the last two periods as seen in Figures from 5.86 to 5.89.

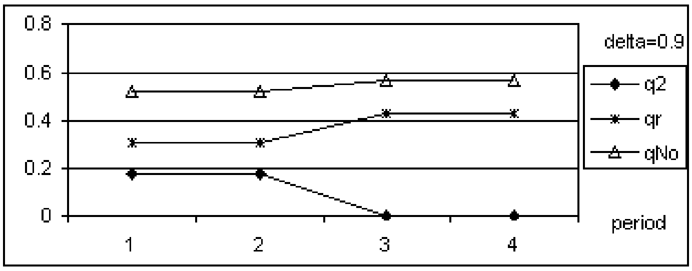

Figure 5.88. Change in the optimal demands - 4 periods, $l_2 = 0.7$, $\delta = 0.9$

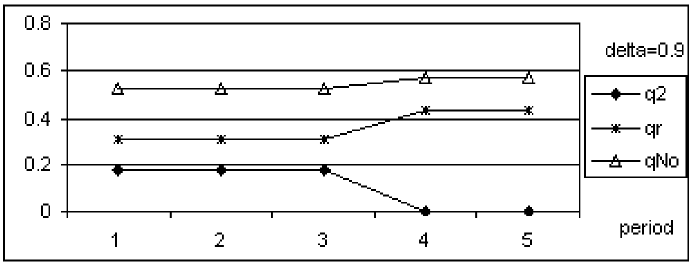

Figure 5.89. Change in the optimal demands - 5 periods, $l_2 = 0.7$, $\delta = 0.9$

If $\delta = l_1$, only new products are produced in the initial two periods. Since returns occur for the first time in the third period, remanufactured products are not sold in the initial two periods. With respect to the length of the time horizon, demands are created in the following periods considering the threat of supply of used products. However, in the last period, the manufacturer sells remanufactured products whatever the length of the time horizon is. These statements are illustrated in Figures from 5.90 to 5.93.

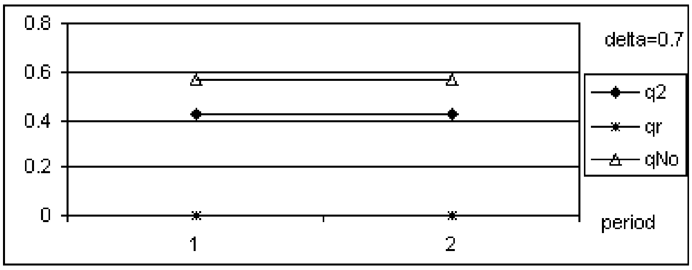

Figure 5.90. Change in the optimal demands - 2 periods, $l_2 = 0.7$, $\delta = 0.7$

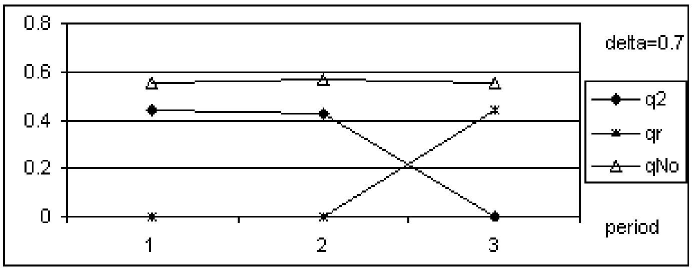

Figure 5.91. Change in the optimal demands - 3 periods, $l_2 = 0.7$, $\delta = 0.7$

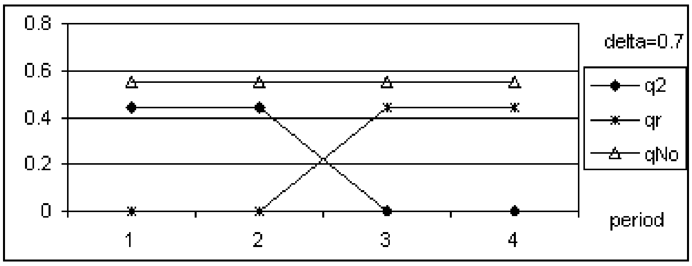

Figure 5.92. Change in the optimal demands - 4 periods, $l_2 = 0.7$, $\delta = 0.7$

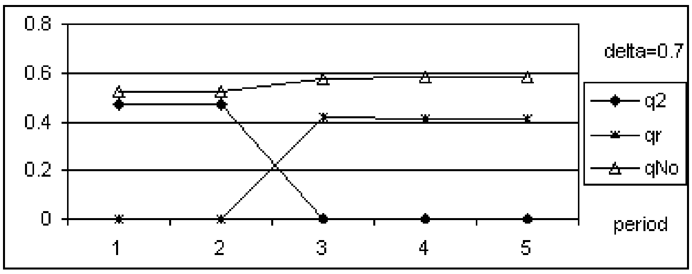

Figure 5.93. Change in the optimal demands - 5 periods, $l_2 = 0.7$, $\delta = 0.7$

6. CONCLUSIONS

The aim of this thesis was to determine the optimal pricing strategy in a multi-period setting for a profit-maximizing firm leasing new, durable, and remanufacturable products as well as selling remanufactured products to a customer base that has a lower willingness-to-pay for the remanufactured product. We allow the manufacturer to lease new products and sell remanufactured products. In this setting, if available used products are not enough to meet the demand for remanufactured products, the manufacturer acquires the remaining from the third-party core supplier with an extra cost. We formulate demand functions based on the consumer preferences and linear price relation. In the base model, consumer preferences are explained through maximum utility type approach. In the model formulated according to linear price-demand relation, we assume that the demand is a linear function of prices of new and remanufactured products. The resulting problem is solved by a variant of Nelder-Mead simplex search method which can also handle the constraints. In our experiments we focus on the base model since it is more realistic in practice.

Experimental results are performed for both single period and multi-period problems with respect to the different problem parameters such as relative willingness-to-pay δ for remanufactured products, relative willingness-to-pay l_m for leasing new products for m periods, depreciation rate d_m of the product over lease period m, initial inventory level I_0, and cost c_r' of supplying used products from the third-party core supplier .

In the single period problem, first we focus on the market characteristics such as consumer acceptance of a remanufactured product and a leased product. We observe that if there are customers with higher relative willingness-to-pay for remanufactured products with respect to leasing new products for one period in a customer segment, the manufacturer produces remanufactured products by charging high prices to take advantage of high willingness-to-pay. Moreover, since the increase in relative willingness-to-pay for remanufactured products implies that consumers' valuation for the remanufactured product increases, the manufacturer increases the remanufactured product's price to earn from customers with higher willingness-to-pay. The effect of relative willingness-to-pay for leasing a new product is the same as a remanufactured product. Therefore, similar results are found to the changes in this parameter such that if there are customers with higher relative willingness-to-pay for leasing a new product for one period with respect to buying a remanufactured product in a customer segment, the manufacturer produces new products charging high prices to take advantage of high willingness-to-pay. Moreover, as this parameter increases, since the relative advantage of leasing strategy increases, the profit that the manufacturer can make from leasing also increases. If consumers are indifferent between new and remanufactured products, the manufacturer sells remanufactured products which provides relatively more profit. It is important to note that the decrease in the both relative willingness-to-pays makes the volume of consumers who prefer nothing increase and the optimal profit decrease. Second, we find that the effect of deterioration rate of the new product depends on the threshold value, above which the manufacturer decreases the quantity of leased products and increases the quantity of remanufactured products. It also depends on relative willingness-to-pays such that if depreciation rate is less than threshold value and the customers have higher willingness-to-pay for leasing a new product with respect to buying a remanufactured product, the manufacturer produces new products. Moreover, the increase in this parameter up to the threshold value has impact on the new product's price such that manufacturer try to attract the lessees by lowering prices in order to keep lease payments constant. However, from threshold value on, the decrease in the new product's price results in the decrease in the remanufactured product's price due to the price costraint which induces customers to switch from the new product to the remanufactured product. In the scenario where consumers view buying a remanufactured product and leasing a new product as being identical or value a remanufactured product more than leasing a new product, there is no effect of the increase in the depreciation rate on the decisions since the manufacturer does not produce new products even if depreciation rate of the product is low in such cases. Finally, we find that there are two reasons of the increase in the profit as initial inventory of used products increases: The first one is because of the increase in the volume of remanufactured products; the second one is due to the decrease in the volume of used cores supplied from the third-party core supplier. If there is no available stock of used cores initially, the decrease in the cost of acquiring used cores from the third-party core supplier makes the remanufactured product more attractive.

In the multi-period setting, we investigate the interdependence of new and remanufactured products such that a decrease in the demand for new products in prior periods results in a decrease in the availability of used products. This is because decisions in a given period explicitly depend on decisions in previous periods in such a scenario where the manufacturer acquires used cores through leasing new products. In this framework, we find that the manufacturer may choose to produce some new products only for the future value that they generate through their sales as remanufactured products. If there is no available stock of used cores at the beginning of the time horizon, and consumers have higher willingness-to-pay for leasing a new product with respect to buying a remanufactured product, the manufacturer does not produce remanufactured products in the first period due to the extra cost of supplying used items from the third-party core supplier. This is because of the fact that remanufacturing results in a significant drop in the profit in the scenario where consumers are willing to pay low prices for remanufactured products. However, in the subsequent periods, the manufacturer starts the period with the opportunity to recover used cores that become available at the beginning of the period and remain in stock from returns in previous periods. Therefore, it favors remanufacturing in the rest of the time horizon. Moreover, as relative willingness-to-pay for remanufactured products increases, remanufacturing becomes more attractive option with respect to producing new products on the manufacturer's side. This observation also provides the marketing strategy of the manufacturer over periods. We conclude that the manufacturer chooses to produce new products in the first period, and returns of used cores make remanufacturing attractive in the subsequent periods such that the volume of new and remanufactured products do not change up to the last period. If consumers have higher willingness-to-pay for buying a remanufactured product with respect to leasing a new product, we observe that the manufacturer produces only remanufactured products to take advantage of increased willingness-to-pay for remanufactured product in the single period setting, but in the multi-period setting he chooses to produce some new products due to the threat of supply of used items in the following periods up to the last period. In the last period, the manufacturer sells only remanufactured products without any concern about the future demand for remanufactured products. If consumers are indifferent between leasing a new product and buying a remanufactured product, we find that two products present the characteristics of complementary products even though they are substitutes. This is because producing remanufactured products depends on the used products obtained through leasing new products in previous periods.

We conclude that if consumers have higher willingness-to-pay for leasing a new product with respect to buying a remanufactured product, leasing becomes more valuable both in its own right and as source of used cores. However, if consumers have higher willingness-to-pay for remanufactured products with respect to leasing new products, leasing becomes a strategy only to generate available used products in the future. The another issue is the effect of the depreciation rate on decisions. The deterioration rate above the threshold value makes the relative advantage of the remanufacturing strategy increases. However, the profit of the firm decreases because of three reasons: The first one is the decrease in the new product's price, and thus the decrease in the price of the remanufactured product due to the price constraint. The other one is the increase in the amount of used products supplied from the third-party core supplier. This is because less new product is produced in previous periods due to the high depreciation rate. The last one is the increase in the volume of consumers who prefer nothing, and so the decrease in the total demand. We find that if the initial inventory of used cores is enough to meet the demand for remanufactured products over the time horizon, the relative advantage of the remanufacturing also increases due to the less recovery cost of a used product with respect to producing a new product. In other words, if the manufacturer acquired used items without any extra cost, he would be better off remanufacturing since remanufacturing is cheaper than manufacturing a new product.

The change in the duration of lease agreements illustrated by two-period lease option shows that the trend on demands and prices of new and remanufactured products, and behavior of the optimal profit are the same as one-period lease option. The only difference in this scenario is the return time of the new products. Therefore, the manufacturer manage remanufacturing activities according to the time of the returning items. In this case, we find that if consumers have higher willingness-to-pay for leasing a new product with respect to buying a remanufactured product, the manufacturer produces only new products in the initial two periods under the condition that there is no inventory of used cores initially and the cost of supplying used cores from the third-party core supplier is high. With respect to the returns of used items at the beginning of the third period, remanufacturing becomes attractive for the manufacturer. If consumers have higher willingness-to-pay for buying a remanufactured product with respect to leasing a new product, the relative advantage of remanufacturing increases, but still manufacturer chooses to produce some new products for the future value that they generate through their sale as remanufactured product in the following periods up to the last two periods. In the last two periods, the manufacturer produces only remanufactured products since there is no threat of supply of used cores in the future, and remanufacturing makes more profit with respect to producing a new product.

As a future work, it can be interesting to extend this model by considering variable costs which depend on the technological development. We assume that the costs of manufacturing and remanufacturing are constant over the time horizon, but in practice, they can be reduced by a new technology. Moreover, we allow the manufacturer to carry inventory of used products, but we do not consider associated holding costs. Therefore, it can be an extension of our model. Finally, we assume that the manufacturer holds a monopoly in the markets for new and remanufactured products. To capture the impact of competition in the remanufactured product market, the model can be extended by considering an industry in which the manufacturer holds a monopoly in the new product market and independent remanufacturers compete on the remanufactured product market. Since we assume new products are only leased, and return at the end of the lease period, the selling option of new products can be added to our scenario by considering that they can be collected by independent remanufacturers.

APPENDIX A: HESSIAN FOR THE BASE MODEL

In this section, the Hessian H will be given for the base model. Our aim is to show that one-period model is jointly concave with respect to p_n and p_r. Recall that the profit obtained in a generic period under the decision $\mathbf{p} = (p_n, p_r)$ is given by

$$\Pi_{\beta_m}(\mathbf{p}) = \left[\sum_{m=1}^{L} \frac{\beta_m \left(1 - \beta_m^{12m}\right)}{1 - \beta_m} q_m(\mathbf{p}) p_m\right] + q_r(\mathbf{p}) p_r - c_n q_n(\mathbf{p}) - c_r q_r^2(\mathbf{p}) - c_r' \max(0, \Delta_r)$$

Taking the derivative of this objective function with respect to $\mathbf{p} = (p_n, p_r)$ gives

$$\begin{aligned}\frac{\partial \Pi_{\beta_m}}{\partial p_n} &= \left[\sum_{m=1}^{L} \frac{\beta_m \left(1 - \beta_m^{12m}\right)}{1 - \beta_m} \left(\frac{\partial q_m}{\partial p_n} p_m + \frac{\partial p_m}{\partial p_n} q_m\right)\right] + \left(\frac{\partial q_r}{\partial p_n} p_r + \frac{\partial p_r}{\partial p_n} q_r\right) \\ &\quad - c_n \frac{\partial q_n}{\partial p_n} - 2 c_r q_r \frac{\partial q_r}{\partial p_n} - c_r' \max(0, \frac{\partial q_r}{\partial p_n}) \end{aligned} \tag{A.1}$$

$$\begin{aligned}\frac{\partial \Pi_{\beta_m}}{\partial p_r} &= \left[\sum_{m=1}^{L} \frac{\beta_m \left(1 - \beta_m^{12m}\right)}{1 - \beta_m} \left(\frac{\partial q_m}{\partial p_r} p_m + \frac{\partial p_m}{\partial p_r} q_m\right)\right] + \left(\frac{\partial q_r}{\partial p_r} p_r + \frac{\partial p_r}{\partial p_r} q_r\right) \\ &\quad - c_n \frac{\partial q_n}{\partial p_r} - 2 c_r q_r \frac{\partial q_r}{\partial p_r} - c_r' \max(0, \frac{\partial q_r}{\partial p_r}) \end{aligned} \tag{A.2}$$

Here, $\frac{\partial p_m}{\partial p_n} = \frac{d_m}{12m} + (2 - d_m)MF$, $\frac{\partial p_m}{\partial p_r} = 0$, $\frac{\partial p_r}{\partial p_n} = 0$ and partial derivatives of q_m and $q0_m$ with respect to p_n and p_r are presented under the conditions $l_m - \delta > 0$, $l_m - \delta < 0$ and $l_m - \delta = 0$ in Tables from A.1 to A.3, respectively.

Recall that

$$q_n = \sum_{m=1}^{L} q_m$$

Table A.1. Partial derivatives of q_m and $q0_m$ with respect to p_n and p_r under $l_m - \delta > 0$

Case				$\frac{\partial q_m}{\partial p_n}$	$\frac{\partial q_m}{\partial p_r}$	$\frac{\partial q0_m}{\partial p_n}$	$\frac{\partial q0_m}{\partial p_r}$
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	$A_1 \geq \frac{p_n S_m}{l_m}$	$A_1 \leq \frac{p_r}{\delta}$	$\alpha_m \frac{-S_m}{l_m-\delta}$	$\alpha_m \frac{1}{l_m-\delta}$	$\alpha_m \frac{S_m}{l_m-\delta}$	$\alpha_m \frac{-1}{l_m-\delta}$
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	$A_1 \geq \frac{p_n S_m}{l_m}$	$A_1 > \frac{p_r}{\delta}$	$\alpha_m \frac{-S_m}{l_m-\delta}$	$\alpha_m \frac{1}{l_m-\delta}$	0	$\alpha_m \frac{1}{\delta}$
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	$\frac{p_n S_m}{l_m} > 1$	$\frac{p_r}{\delta} > 1$	0	0	0	0
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	$\frac{p_n S_m}{l_m} > 1$	$\frac{p_r}{\delta} \leq 1$	0	0	0	$\alpha_m \frac{1}{\delta}$
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	$A_1 \leq \frac{p_n S_m}{l_m} \leq 1$	$\frac{p_n S_m}{l_m} \leq \frac{p_r}{\delta}$	$\alpha_m \frac{-S_m}{l_m}$	0	$\alpha_m \frac{S_m}{l_m}$	0
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	$A_1 \leq \frac{p_n S_m}{l_m} \leq 1$	$\frac{p_n S_m}{l_m} > \frac{p_r}{\delta}$	$\alpha_m \frac{-S_m}{l_m}$	0	0	$\alpha_m \frac{1}{\delta}$
$l_m - \delta > 0$	$A_1 < 0$	$\frac{p_n S_m}{l_m} \leq 1$		$\alpha_m \frac{-S_m}{l_m}$	0	$\alpha_m \frac{S_m}{l_m}$	0
$l_m - \delta > 0$	$A_1 < 0$	$\frac{p_n S_m}{l_m} > 1$		0	0	0	0
$l_m - \delta > 0$	$A_1 > 1$	-	$\frac{p_r}{\delta} > 1$	0	0	0	0
$l_m - \delta > 0$	$A_1 > 1$	-	$\frac{p_r}{\delta} \leq 1$	0	0	0	$\alpha_m \frac{1}{\delta}$

Table A.2. Partial derivatives of q_m and $q0_m$ with respect to p_n and p_r under $l_m - \delta < 0$

Case					$\frac{\partial q_m}{\partial p_n}$	$\frac{\partial q_m}{\partial p_r}$	$\frac{\partial q0_m}{\partial p_n}$	$\frac{\partial q0_m}{\partial p_r}$
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$A_2 < \frac{p_n S_m}{l_m}$		$\frac{p_r}{\delta} > 1$	0	0	0	0
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$A_2 < \frac{p_n S_m}{l_m}$		$\frac{p_r}{\delta} \leq 1$	0	0	0	$\alpha_m \frac{1}{\delta}$
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$A_2 \geq \frac{p_n S_m}{l_m}$	$\frac{p_r}{\delta} \leq A_2$		$\alpha_m \frac{-\delta S_m}{l_m(\delta-l_m)}$	$\alpha_m \frac{1}{\delta-l_m}$	$\alpha_m \frac{S_m}{l_m}$	0
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$A_2 \geq \frac{p_n S_m}{l_m}$	$\frac{p_r}{\delta} > A_2$	$\frac{p_r}{\delta} > 1$	$\alpha_m \frac{-\delta S_m}{l_m(\delta-l_m)}$	$\alpha_m \frac{1}{\delta-l_m}$	$\alpha_m \frac{\delta S_m}{l_m(\delta-l_m)}$	$\alpha_m \frac{-1}{(\delta-l_m)}$
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$A_2 \geq \frac{p_n S_m}{l_m}$	$\frac{p_r}{\delta} > A_2$	$\frac{p_r}{\delta} \leq 1$	$\alpha_m \frac{-\delta S_m}{l_m(\delta-l_m)}$	$\alpha_m \frac{1}{\delta-l_m}$	$\alpha_m \frac{\delta S_m}{l_m(\delta-l_m)}$	$\alpha_m \frac{-l_m}{\delta(\delta-l_m)}$
$l_m - \delta < 0$	$A_2 < 0$	-		$\frac{p_r}{\delta} > 1$	0	0	0	0
$l_m - \delta < 0$	$A_2 < 0$	-		$\frac{p_r}{\delta} \leq 1$	0	0	0	$\alpha_m \frac{1}{\delta}$
$l_m - \delta < 0$	$A_2 > 1$	$\frac{p_n S_m}{l_m} > 1$			0	0	0	0
$l_m - \delta < 0$	$A_2 > 1$	$\frac{p_n S_m}{l_m} \leq 1$			$\alpha_m \frac{-S_m}{l_m}$	0	$\alpha_m \frac{S_m}{l_m}$	0

Table A.3. Partial derivatives of q_m and $q0_m$ with respect to p_n and p_r under $l_m - \delta = 0$

Case			$\frac{\partial q_m}{\partial p_n}$	$\frac{\partial q_m}{\partial p_r}$	$\frac{\partial q0_m}{\partial p_n}$	$\frac{\partial q0_m}{\partial p_r}$
$l_m - \delta = 0$	$p_n S_m - p_r \leq 0$		$\alpha_m \frac{-S_m}{l_m}$	0	$\alpha_m \frac{S_m}{l_m}$	0
$l_m - \delta = 0$	$p_n S_m - p_r > 0$	$\frac{p_r}{\delta} > 1$	0	0	0	0
$l_m - \delta = 0$	$p_n S_m - p_r > 0$	$\frac{p_r}{\delta} \leq 1$	0	0	0	$\alpha_m \frac{1}{\delta}$

$$q_{No} = \sum_{m=1}^{L} q0_m$$

$$q_r = 1 - q_n - q_{No}$$

Therefore, $\frac{\partial q_n}{\partial p_n} = \sum\limits_{m=1}^{L} \frac{\partial q_m}{\partial p_n}$, $\frac{\partial q_n}{\partial p_r} = \sum\limits_{m=1}^{L} \frac{\partial q_m}{\partial p_r}$, $\frac{\partial q_r}{\partial p_n} = -\sum\limits_{m=1}^{L} \frac{\partial q_m}{\partial p_n} - \sum\limits_{m=1}^{L} \frac{\partial q0_m}{\partial p_n}$, $\frac{\partial q_r}{\partial p_r} = -\sum\limits_{m=1}^{L} \frac{\partial q_m}{\partial p_r} - \sum\limits_{m=1}^{L} \frac{\partial q0_m}{\partial p_r}$.

Taking the derivative of $\frac{\partial \Pi_{\beta_m}}{\partial p_n}$ and $\frac{\partial \Pi_{\beta_m}}{\partial p_r}$ with respect to p_n and p_r, we obtain the elements of the Hessian H:

$$\begin{aligned}\frac{\partial^2 \Pi_{\beta_m}}{\partial p_n^2} = & \left[\sum_{m=1}^{L} \frac{\beta_m\,(1-\beta_m^{12m})}{1-\beta_m} \left(\frac{\partial^2 q_m}{\partial p_n^2} p_m + \frac{\partial q_m}{\partial p_n}\frac{\partial p_m}{\partial p_n} + \frac{\partial^2 p_m}{\partial p_n^2} q_m + \frac{\partial p_m}{\partial p_n}\frac{\partial q_m}{\partial p_n}\right)\right] \\ & + \left(\frac{\partial^2 q_r}{\partial p_n^2} p_r + \frac{\partial q_r}{\partial p_n}\frac{\partial p_r}{\partial p_n}\right) - c_n \frac{\partial^2 q_n}{\partial p_n^2} - 2c_r(\frac{\partial^2 q_r}{\partial p_n^2} q_r \\ & + \frac{\partial q_r}{\partial p_n}\frac{\partial q_r}{\partial p_n}) - c_r' \max(0, \frac{\partial^2 q_r}{\partial p_n^2})\end{aligned}$$

Here, $\frac{\partial^2 q_m}{\partial p_n^2} = 0$, $\frac{\partial^2 p_m}{\partial p_n^2} = 0$, $\frac{\partial^2 q_r}{\partial p_n^2} = 0$, $\frac{\partial^2 q_n}{\partial p_n^2} = 0$, $\frac{\partial p_r}{\partial p_n} = 0$. Therefore, the second derivative of profit function with respect to p_n is

$$\frac{\partial^2 \Pi_{\beta_m}}{\partial p_n^2} = \left[\sum_{m=1}^{L} \frac{\beta_m\,(1-\beta_m^{12m})}{1-\beta_m} (2\frac{\partial p_m}{\partial p_n}\frac{\partial q_m}{\partial p_n})\right] - 2c_r(\frac{\partial q_r}{\partial p_n})^2 \qquad \text{(A.3)}$$

$$\begin{aligned}\frac{\partial^2 \Pi_{\beta_m}}{\partial p_r^2} = &\left[\sum_{m=1}^{L} \frac{\beta_m \left(1-\beta_m^{12m}\right)}{1-\beta_m} \left(\frac{\partial^2 q_m}{\partial p_r^2} p_m + \frac{\partial q_m}{\partial p_r}\frac{\partial p_m}{\partial p_r}\right)\right] \\ &+ \left(\frac{\partial^2 q_r}{\partial p_r^2} p_r + \frac{\partial q_r}{\partial p_r}\frac{\partial p_r}{\partial p_r} + \frac{\partial q_r}{\partial p_r}\right) - c_n \frac{\partial^2 q_n}{\partial p_r^2} \\ &- 2c_r\left(\frac{\partial^2 q_r}{\partial p_r^2} q_r + \frac{\partial q_r}{\partial p_r}\frac{\partial q_r}{\partial p_r}\right) - c_r' \max(0, \frac{\partial^2 q_r}{\partial p_r^2})\end{aligned}$$

Here, $\frac{\partial^2 q_m}{\partial p_r^2} = 0$, $\frac{\partial p_m}{\partial p_r} = 0$, $\frac{\partial^2 q_r}{\partial p_r^2} = 0$, $\frac{\partial^2 q_n}{\partial p_r^2} = 0$. Thus, the second derivative of profit function with respect to p_r is

$$\frac{\partial^2 \Pi_{\beta_m}}{\partial p_r^2} = 2\frac{\partial q_r}{\partial p_r}\left(1 - c_r \frac{\partial q_r}{\partial p_r}\right) \tag{A.4}$$

$$\begin{aligned}\frac{\partial^2 \Pi_{\beta_m}}{\partial p_n \partial p_r} = &\left[\sum_{m=1}^{L} \frac{\beta_m \left(1-\beta_m^{12m}\right)}{1-\beta_m} \left(\frac{\partial^2 q_m}{\partial p_n \partial p_r} p_m + \frac{\partial q_m}{\partial p_n}\frac{\partial p_m}{\partial p_r} + \frac{\partial^2 p_m}{\partial Pn \partial p_r} q_m + \frac{\partial p_m}{\partial Pn}\frac{\partial q_m}{\partial p_r}\right)\right] \\ &+ \left(\frac{\partial^2 q_r}{\partial p_n \partial p_r} p_r + \frac{\partial q_r}{\partial p_n}\frac{\partial p_r}{\partial p_r}\right) - c_n \frac{\partial^2 q_n}{\partial p_n \partial p_r} - 2c_r\left(\frac{\partial^2 q_r}{\partial p_n \partial p_r} q_r + \frac{\partial q_r}{\partial p_n}\frac{\partial q_r}{\partial p_r}\right) \\ &- c_r' \max(0, \frac{\partial^2 q_r}{\partial p_n \partial p_r})\end{aligned}$$

Here, $\frac{\partial^2 q_m}{\partial p_n \partial p_r} = 0$, $\frac{\partial p_m}{\partial p_r} = 0$, $\frac{\partial^2 p_m}{\partial Pn \partial p_r} = 0$, $\frac{\partial^2 q_r}{\partial p_n \partial p_r} = 0$, $\frac{\partial^2 q_n}{\partial p_n \partial p_r} = 0$. Therefore, diagonal entries of the Hessian are as below:

$$\frac{\partial^2 \Pi_{\beta_m}}{\partial p_n \partial p_r} = \left[\sum_{m=1}^{L} \frac{\beta_m \left(1-\beta_m^{12m}\right)}{1-\beta_m} \frac{\partial p_m}{\partial Pn}\frac{\partial q_m}{\partial p_r}\right] + \frac{\partial q_r}{\partial p_n} - 2c_r \frac{\partial q_r}{\partial p_n}\frac{\partial q_r}{\partial p_r} \tag{A.5}$$

In this case, if $|H| > 0$ and $\frac{\partial^2 \Pi_{\beta_m}}{\partial p_n^2} < 0$, then Π_{β_m} is jointly concave. If we use yearly lease payment model, $\frac{\beta_m\left(1-\beta_m^{12m}\right)}{1-\beta_m}$ will be replaced with $\frac{\beta(1-\beta^m)}{1-\beta}$ and $\frac{\partial p_m}{\partial p_n}$ will be equal to $\frac{1}{M}$.

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