COMBINED LONGITUDINAL-TORSIONAL VIBRATION OF SANDWICHED PIEZOELECTRIC ULTRASONIC TRANSDUCERS

by

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ABSTRACT

COMBINED LONGITUDINAL-TORSIONAL VIBRATION OF SANDWICHED PIEZOELECTRIC ULTRASONIC TRANSDUCERS

In this thesis, combined longitudinal-torsional vibration of ultrasonic transducer using impedance matching method is studied. In combined vibration, two modes must resonate in the same frequency. Impedance matching method is applied the modes to find spatial solutions of the parts. Spatial solutions of horns are derived for each mode.

Dimensional parameters of the transducer are studied, and change of resonance frequencies is presented to give the idea how to synchronize the modes of the transducer. Changing the dimensions of the transducer, resonance frequencies of two modes are equalized. This shows that synchronization of two modes is an adjustment problem. For illustration of the application of theory, sample transducers are designed.

ÖZET

PİEZOELEKTRİK TRANSDUSERLERIN BOYLAMSAL VE BURULMA YÖNÜNDEKİ BİRLEŞİK TİTREŞİMİ

Bu tezde boylamsal ve burkulma yönündeki iki modlu transduserler empedans eşleştirmesi yöntemi kullanılarak incelenmiştir. Birleşik modlu titreşimde, her iki mod aynı frekansta titreşmek zorundadır. Uzaysal çözümleri bulmak için empedans eşleşmesi metodu kullanılmıştır. Her mod için, hornların uzaysal çözümleri çıkarılmıştır.

Ayrıca transduserin boyutsal parametreleri çalışılmış, transducerlerin modlarını nasıl senkronize edileceğini göstermek için resonans frekansların değişimleri verilmiştir. Transduserlerin boyutları değiştirilerek, iki modun resonans frekansları eşitlenmiştir. Bu senkronizasyonun bir ayarlama işlemi olduğunu gösterir. Teorinin uygulamasını göstermek için, farklı tipte hornlarla örnek transduserler tasarlanmıştır.

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LIST OF SYMBOLS

A_i^L	Constant of First Longitudinal Displacement of i’th part
A_i^T	Constant of First Torsional Displacement of i’th part
A_0	Amplitude at Resonance Frequency
B_i^L	Constant of Second Longitudinal Displacement of i’th part
B_i^T	Constant of Second Torsional Displacement of i’th part
b	The Efficiency of the Threaded Bolt
BW	Bandwidth
$\mathbf{C}$	Longitudinal Damping Operator
c	Sound of Speed
c_l	Speed of Sound in Longitudinal Vibration
c_t	Speed of Sound in Torsional Vibration
c^D	Elastic Coefficient for Constant D
C_0	Capacitor Capacitance
D	Complex Electric Displacement in Longitudinal Vibration
D^*	Electric Displacement in Longitudinal Vibration
d	Piezoelectric charge constants
d_c	Thickness of Piezoceramic
d_L	Longitudinal Piezoceramic Diameter
d_T	Torsional Piezoceramic Diameter
D_z	Complex Electric Displacement in Torsional Vibration
D_z^*	Electric Displacement in Torsional Vibration
E	Electrical Field
E_3	Electrical Field in LongitudinalVibration
$E_{\theta 3}$	Electrical Field in Torsional Vibration
$F(x,t)$	Force
F	Force Vector

f	Driving Frequency
$f(z,t)$	External Torque
F_0	Initial Force
F_R	Required Force to Clamp Bolt
${}_2F_1$	Gauss Hypergeometric Function
G	Shear Modulus
${}_2G_1$	Gauss Hypergeometric Function
$G_3(Z)$	Second Kind Bessel Function of Third Order
g	Piezoelectric voltage constant
$H_3(Z)$	First Kind Bessel Function of Third Order
h	Piezoelectric Coupling Coefficient
I	Current
$I(z)$	Mass Polar Moment of Inertia
I_i	Initial Acoustic Intensity
I_r	Reflected Acoustic Intensity
I_L	Acoustic Intensity
I_t	Transmitted Acoustic Intensity
$J(z)$	Polar Moment of Inertia
J_1	Initial Inertia Moments
J_2	End Inertia Moments
k	The Pitch of Bolt Thread
l	Length of Uniform Bar
$\mathbf{L}$	Longitudinal Stiffness Operator
L_i	Length of the i'th Part of a Continuous System
L_{2L}	Length of Exponential Horn in Longitudinal Vibration
L_{2T}	Length of Exponential Horn in Torsional Vibration
LR	Longitudinal Resonance Frequency
LF_L	Longitudinal Length Factor
LF_T	Torsional Length Factor

m	Mass of Transducer
$\mathbf{M}$	Longitudinal Time Operator
$M(z,t)$	Twisting Moment
M_0	Initial Moment
p	Sound Pressure
P	Power
$\mathbf{P}$	Torsional Stiffness Operator
P_{ac}	Acoustical Power
Q	Quality Factor
$\mathbf{Q}$	Torsional Damping Operator
r	Radius of Piezoceramic
$\mathbf{R}$	Torsional Time Operator
R_m	Mechanical Resistance
R_s	Longitudinal Damping Coefficient
R_v	Torsional Damping Coefficient
S	Surface Area
$S(x)$	Cross Sectional Area
s^D	The Elastic Coefficient for constant D
S_1	Initial Cross Sectional Area
S_2	End Cross Sectional Area
S_x	Tensile Strain
$S_{\theta z}$	Tangential Strain
T	Tensile Stress
T_B	Torque
$T_L(t)$	Longitudinal Temporal Solution
T_T	Torsional Temporal Solution
$T_{\theta 3}$	Tangential Stress
TR	Torsional Resonance Frequency
$u(x,t)$	Longitudinal Displacement

$\mathbf{U}$	Longitudinal Mode Shape Vector
V	Applied Voltage
V_x	Applied Voltage to Longitudinal Piezoceramics
V_z	Applied Voltage to Torsional Piezoceramics
$\mathbf{V}$	Torsional Mode Shape Vector
v	Particle Velocity
$v(z,t)$	Angle of Twisting of Cross Section
ω	Driving Angular Frequency
$X(x)$	Spatial Solution
X_C	Capacitive Reactance
Y	Young Modulus
Z	Acoustic Impedance
$[\mathbf{Z}]$	Longitudinal Impedance Matrix
Z_m	Mechanical Impedance
Z_R	Characteristic Impedance
Z_{sp}	Specific Acoustic Impedance
$[\mathbf{Z_t}]$	Torsional Impedance Matrix
ε	Electrical Permittivity
α	Tensile strain
$\theta(z)$	Angular Torsional Displacement
ρ	Density
β	Radius Decay Coefficient
β^S	Inverse of Electric Permitivity in Longitudinal Vibration
β^T	Inverse of Electric Permitivity in TorsionalVibration
λ	Wave Length
η	Poison Ratio
τ	Decay Time
β_{33}^S	Dielectric Impermeability Constants in T-E type piezoceramics
β_{11}^T	Dielectric Impermeability Constants in S-E type piezoceramics

$\Upsilon_i(x)$	First Part of the Longitudinal Spatial Solution
$\Psi_i(x)$	Second Part of the Longitudinal Spatial Solution
$\Phi_i(z)$	First Part of the Torsional Spatial Solution
$\Omega_i(z)$	Second Part of the Torsional Spatial Solution

1. INTRODUCTION

1.1. General

Ultrasonics has been of interest for several decades in industry. Although ultrasonics is generally considered a 20th century science, the foundations for this field were laid in the 19th century. In many application areas such as welding, cleaning, ultrasonics testing, and medical applications ultrasonic is used. Ultrasonic transducer is an electronic device, which converts electrical energy to mechanical or vice versa.

Ultrasonic welding is an industrial technique whereby two pieces of plastic or metal are joined together seamlessly through high-frequency acoustic vibrations. One component to be welded is placed upon a fixed anvil, with the second component being placed on top. A horn connected to a transducer is lowered down onto the top component, and a very rapid (~20,000 KHz), low-amplitude acoustic vibration is applied to a small welding zone. The acoustic energy is converted into heat energy by friction, and the parts are welded together in less than a second. Friction is localized at the interface of the assembly. The resultant heat quickly melts the plastic, which flows and joins parts. After it cools down, a solid homogeneous weld between the two components results.

Ultrasonic plastic welding has advantages over joining methods using solvents or adhesives. Production cycle times are usually improved with speeds around 2000 parts an hour being achieved in the most favorable cases. Additional advantages include clean exteriors of welded parts, potential manpower savings, absence of drying time in the material to be welded, gas tight and completely stable assemblies, possibility of welding in the presence of foreign bodies such as powders and liquids, and of welding materials which are incompatible using any other conventional assembly processes [4,15,17].

1.2. Overview of Ultrasonic Welding System

Ultrasonic welding equipment has both electrical and mechanical parts. These are generator, backing, piezoceramics, front part, booster and horn. In compound vibration front part and booster are not used, since uniform bars decrease torsional amplitudes. A compound mode transducer is shown in Figure 1.1.

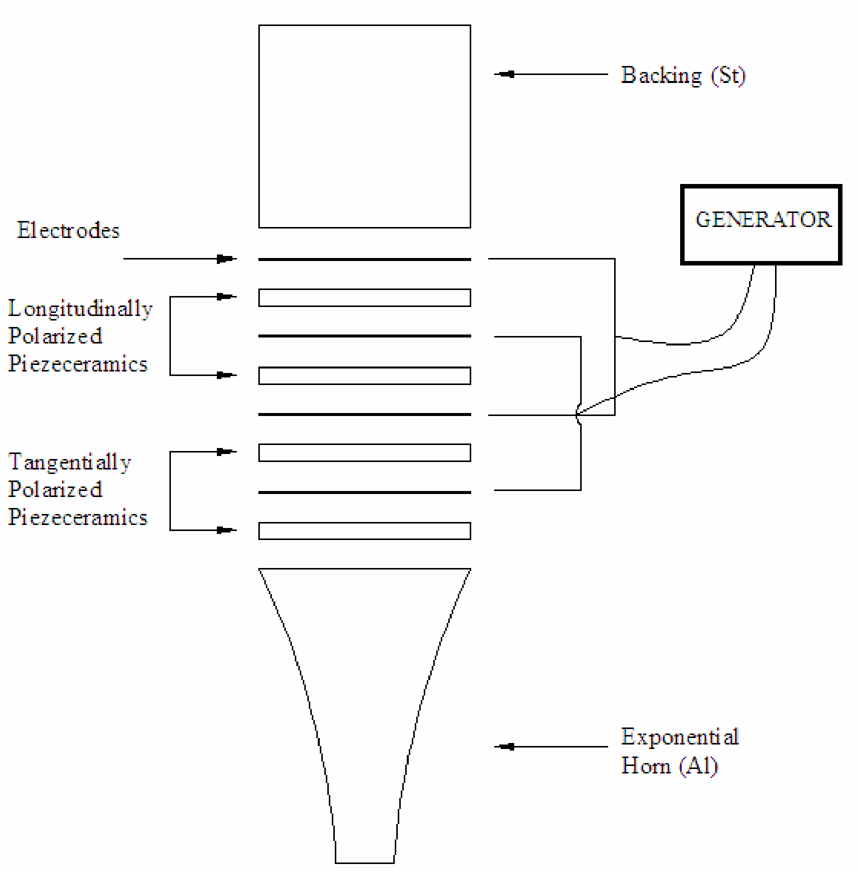


Figure 1.1. Schematic diagram of a compound mode transducer

Transducer is an electronic device that converts energy from one form to another. Common examples include microphones, loudspeakers, thermometers, position and pressure sensors, and antenna. Despite not generally being thought of as transducers, photocells, LEDs (light-emitting diodes), and even common light bulbs are transducers. It consists of materials such as piezoelectric crystals or ceramics, which show piezoelectric effect. Welding voltage is applied to the surfaces of piezoceramics [17].

In transducers, piezoceramics are clamped between two metal parts, which are horn and backing. Low and high tensile strength may cause crack in piezoceramics, so appropriate torque must be applied to bolt. And also, the backing part and horn should have certain characteristic impedance in order to reflect the oscillations in the direction of the parts to be welded. Steel has high impedance and is ideal to use as backing material. But horn material should have lower impedance, thus aluminium is ideal to use [15].

Boosters and horns are used to amplify the displacements or change resonance frequency of transducer. Booster can only change resonance frequency of longitudinal mode, but it also decreases amplitude of torsional mode. Horns are classified according to their cross-sectional areas. In this thesis, three types of horn are used, which are exponential, conical and catenoidal.

Generator is the electrical part that supplies energy to the system. 220 V- 50 kHz the signal is converted to high frequencies such as 20 kHz. The frequency of the generator is desired to coincide with the resonance frequency of the mechanical system. But during welding, resonance frequency of the system changes because of some reasons such as heating. Thus the generator tracks resonance frequency of the transducer in small ranges such as 18-22 kHz [17].

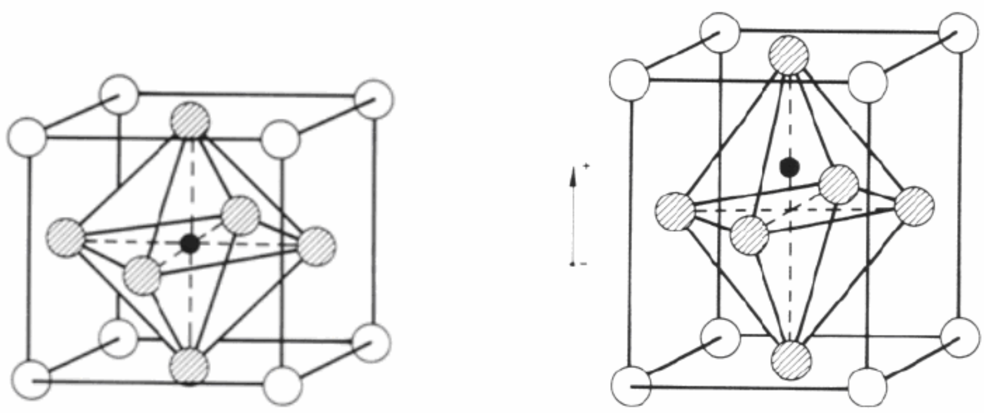
1.3. Piezoelectricity

Jacques and Pierre Curie discovered the piezoelectric effect in 1880. They found that if certain crystals were subjected to mechanical strain, they became electrically polarized and the degree of polarization was proportional to the applied strain. The Curies also discovered that these same materials deformed when they were exposed to an electric field. This has become known as the inverse piezoelectric effect.

The piezoelectric effect is exhibited by a number of naturally occurring crystals, for instance quartz, tourmaline and sodium potassium tartrate, and these have been used for many years as electromechanical transducers. A stress (tensile or compressive) applied to such a crystal will alter the separation between the positive and negative charge sites in each elementary cell leading to a net polarization at the crystal surface.

The effect is practically linear, i.e. the polarization varies directly with the applied stress, and is direction-dependent, so compressive and tensile stresses will generate electric fields and hence voltages of opposite polarity. It's also reciprocal, that is, if the crystal is exposed to an electric field, it will experience an elastic strain causing its length to increase or decrease according to the field polarity.

Besides the crystals mentioned above, an important group of piezoelectric materials are piezoelectric ceramics, to which PZT is an example. Above a temperature known as the Curie point, these crystallites exhibit simple cubic symmetry. The elementary cell of which is shown in Fig.1.2.a. Below the Curie point, however, the crystallites take on tetragonal symmetry in which the positive and negative charge sites no longer coincide Fig.1.2.b, so each elementary cell then has a built-in electric dipole, which may be reversed, and also switched to certain allowed directions by the application of an electric field. Such materials are termed ferroelectrics because this electrical behaviour presents a physical analogy with the magnetic behaviour of ferromagnetic materials. They don't necessarily contain iron as an important constituent. The analogy can, in fact, be carried further, since to some extent the polarization of ferroelectrics materials exhibits hysteresis, and their dielectric constants are very high and temperature-dependent [14,17].



a. Cubic Lattice (above Curie temp) b. Tetragonal Lattice (below Curie temp)

Figure 1.2. PZT Elementary Cell

1.4. Piezoelectric Constants

Since piezoceramics are anisotropic, their physical constants (elasticity, permittivity etc.) are tensor quantities and relate to both the direction of the applied stress, electric field etc., and to the directions perpendicular to them. For this reason the constants are generally given two subscript indices, which refer to the direction of the two related quantities (e.g. stress and strain for elasticity, displacement and electric field for permittivity). A superscript index is used to indicate a quantity that's kept constant. The direction of positive polarization is usually chosen to coincide with the Z-axis of a rectangular system of crystallographic axes X, Y, Z. If the directions of X, Y and Z are represented by 1, 2 and 3 respectively, and the shear about these axes by 4, 5 and 6 respectively, the various constants may be written with subscripts referring to them see Figure 1.3 [17].

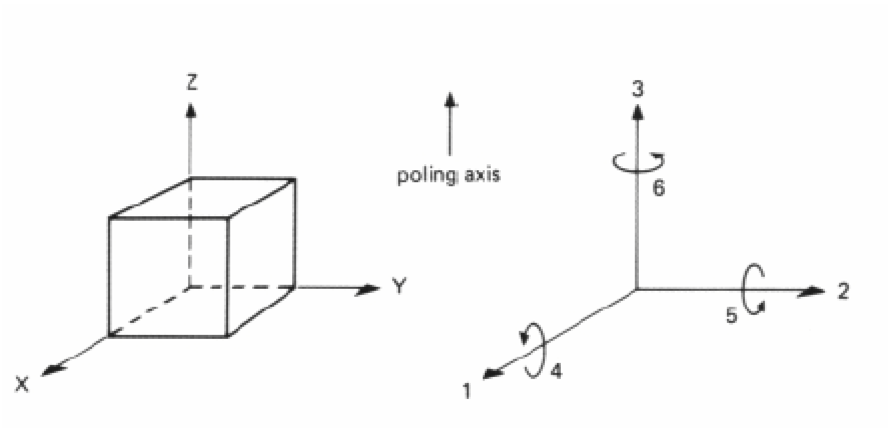


Figure 1.3. Design of the axes and directions of deformations

1.5. Subscripts and Indexes

If X is a property of the piezoceramic material, subscripts and indexes have following meanings;

$$X_{ab}^{Y}$$

- The property X is measured under constant Y,
- Letter a is electric displacement or electric field
- Letter b is stress or strain
- If **a** is electric field, then **b** is stress or vice versa.

1.5.1. Permittivity

The (absolute) permittivity (or dielectric constant) is defined as the dielectric displacement per unit electric field. The first subscript gives the direction of the dielectric displacement; the second gives the direction of the electric field. For example;

- ε_{11}^{T} is the permittivity for the dielectric displacement and electric field direction 1 under conditions of constant stress,

- ε_{33}^{S} is the permittivity for the dielectric displacement and electric field direction 3 under conditions of constant strain.

The ratio of absolute permittivity to the permittivity of free space is $8.85x10^{-12}$ F/m [17].

1.5.2. Compliance

The compliance of a material is defined as the strain produced per unit stress, shortly inverse of young modulus or shear modulus. It's the reciprocal of the modulus of elasticity. The first subscript refers to the direction of the strain, the second to the direction of the stress. For example:

- s_{11}^{E} is the compliance for a stress and accompanying strain direction 1 under the conditions of constant electric field,

- s_{36}^{D} is the compliance for a shear stress about axis 3 and accompanying strain in the direction 3 under the conditions of constant electric displacement.

1.5.3. Piezoelectric charge constants

The piezoelectric charge constant d is defined as the electric polarization generated in a material per unit mechanical stress applied to it. Alternatively, it is the mechanical strain experienced by the material per unit electric field applied to it. The first subscript refers to the direction of polarization generated in the material (at $E = 0$) or to the applied field strength, the second subscript refers to the direction of the applied stress or to the direction of the induced strain [17]. For example;

- d_{33} is the induced polarization per unit applied stress in the direction 3. Alternatively it is the induced strain per unit electric filed in the direction 3.

1.5.4. Piezoelectric voltage constant

The piezoelectric voltage constant g is defined as the electric field generated in a material per unit mechanical stress applied to it. Alternatively, it is the mechanical strain experienced by the material per unit electric displacement applied to it. The first subscript refers to the direction of the electric field generated in the material or to the applied electric displacement; the second refers respectively to the direction of the applied stress or to the direction of the induced strain [17]. For example;

- g_{31} is the induced electric field in the direction 3 per unit stress applied in the direction 1. Alternatively it is mechanical strain induced in material in the direction 1 per unit electric displacement applied in direction 3.

1.6. Constitutive Equations

Piezoelectricity is described mathematically within a material's constitutive equation, which defines how the piezoelectric material's stress (**T**), strain (**S**), charge-density displacement (**D**), and electric field (**E**) interact.

Four possible forms for piezoelectric constitutive equations are shown below. They were taken from the two dependent variables on the left-hand-side of each equation [6,14].

Strain-Charge Form

$$S = s_E.T + d.E$$

$$D = d.T + \varepsilon^T.E$$

Stress-Charge Form

$$T = c_E.S - e.E$$

$$D = e.S + \varepsilon^S.E$$

Strain voltage From

$$S = s_D.T + g.D$$

$$E = -g.T + \varepsilon_T^{-1}$$

Stress Voltage Form

$$T = c_D.S - h.D$$

$$E = -h.S + \beta^S.D$$

Matrix transformations for converting piezoelectric constitutive data from one form into another one are shown below.

Strain-Charge to Stress-Charge

$$c_E = s_E^{-1}$$

$$e = d.s_E^{-1}$$

$$\varepsilon_S = \varepsilon_T - d.s_E^{-1}$$

Strain-Charge to Strain-Voltage

$$s_D = s_E - d.\beta^T.d$$

$$g = \beta^T.d$$

Stress-Charge to Stress Voltage

$$c_D = c_E + e.\beta^S.e$$

$$h = \beta^S.e$$

Strain-Voltage to Stress Voltage

$$c_D = s_D^{-1}$$

$$h = g.s_D^{-1}$$

$$\beta^S = \beta^T + g.s_D^{-1}.g$$

1.7. Acoustic Impedance

Mathematically, acoustic impedance is sound pressure p divided by the particle velocity v divided by the surface area S through which an acoustic wave of frequency f propagates [4,15]

$$Z = \frac{p}{vS} \tag{1.1}$$

However there are other impedance definitions; such as specific acoustic impedance, mechanical impedance. Specific acoustic impedance is the sound pressure p divided by the particle velocity v, determined from boundary conditions;

$$Z_{sp} = \frac{p}{v} \tag{1.2}$$

And mechanical impedance ratio of sound force F to particle velocity that is;

$$Z_m = \frac{F}{v} \tag{1.3}$$

And characteristic impedance is given by;

$$Z_R = \rho c \tag{1.4}$$

1.8. Reflection and transmission of ultrasonic waves

Performing any operation with ultrasonic waves means transmitting them from one medium to another where the measurement or actuation is to be performed. In other cases, the objective may retain a wave in a given medium and prevent it from radiating out in the environment. For different environments reflection and transmissions will become different, and some of the cases are shown in Figure 1.4 .

The boundary conditions are easy to state, but their understanding is essential for posing and solving the problem correctly. They correspond to the conditions that must be met in order to obtain a perfectly defined interface for the problem at hand. The most general case is solid-solid interface. For this to be well defined there must be no net stress on the interface or displacement of one medium with respect to the other. This leads to boundary conditions of continuity of normal and tangential components of stress and displacement, i.e., four conditions corresponding to the four amplitudes to be determined are shown in Figure 1.4.

If these boundary conditions are satisfied at a given time everywhere along the interface, then the problem can be posed and solved. If, however, they are not respected locally at all times, the interface is no longer well defined and the conditions cannot be written down for all values of interface coordinates and so the problem cannot be solved straightforwardly [13].

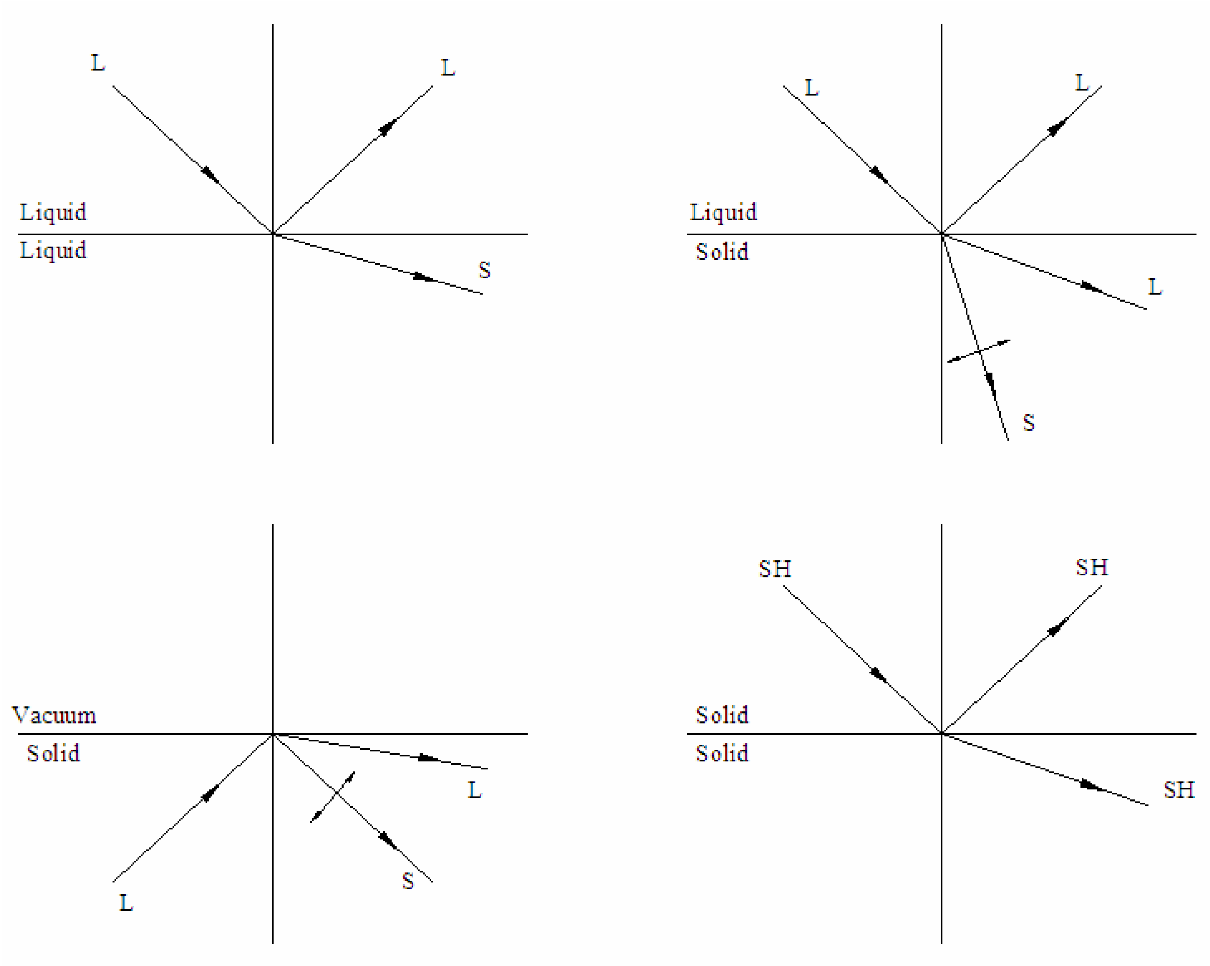


Figure 1.4. Reflection of waves through media

At normal incidence, consider two media in contact at perfect interface. The boundary conditions are continuity of pressure and velocity (displacement). Using the definition of acoustic impedance, it follows that;

$$R_p + 1 = T_p \tag{1.5}$$

$$T_p = \frac{2Z_2}{Z_1 + Z_2} \tag{1.6}$$

$$R_p = \frac{Z_2 - Z_1}{Z_1 + Z_2} \tag{1.7}$$

Where Z_1 and Z_2 are the characteristic impedances of two media. At normal incidence these can be obtained directly from the definition of acoustic intensity, which is explained in section 1.9. , $I_L = P_{ac}^2 / Z$.

$$\frac{I_t}{I_i} = \frac{Z_1}{Z_2} \left| T_p \right|^2 \tag{1.8}$$

$$\frac{I_r}{I_i} = \left| R_p \right|^2 \tag{1.9}$$

Where I_r and I_t are reflected and transmitted acoustic intensities and I_i is initial intensity. From Equation 1.8 and 1.9, it can be verified that the law of conservation of energy is met.

$$I_i = I_r + I_t \tag{1.10}$$

There is a lot of simple physics in this result. If $Z_1 \equiv Z_2$, then $T_p \equiv 0$ and $R_p \equiv 0$; it is as if there were one uniform medium so there is no reflection.

For $Z_2 \ll Z_1$, $R_p \approx -1$ and $T_p \to 0$. This is termed a free boundary, corresponding, for example, to medium 1 might be water or solid, and medium 2 might be air. There is a huge acoustic impedance mismatch, so nearly the entire acoustic wave is reflected. There is a phase change of π for the pressure at the interface. The transmitted acoustic intensity for this case is given by [13],

$$\frac{I_t}{I_i} = \frac{Z_1}{Z_2} \left| T_p \right|^2 \ll 1 \tag{1.11}$$

1.9. Acoustic Intensity

Acoustic intensity can be defined as the following way; consider some point in sound field and imagine a very small plane surface, normal to the direction of propagation, having an area, δA, about that point. If δP_{ac} is the rate of flow of energy (or acoustical power) through that surface, then ratio $\delta P_m / \delta A$ is defined as the mean acoustic intensity over the area, δA. Proceeding to the limit $\delta A \to 0$, then [12];

$$I_L = \frac{dP_{ac}}{dA} \tag{1.12}$$

I_L represents the acoustic intensity at that point. The acoustic intensity I_L in W/m^2 [Watt per square meter] of a plane progressive wave is:

$$I_L = P_{ac} v \tag{1.13}$$

$$I_L = \frac{P_{ac}^2}{Z} \tag{1.14}$$

1.10. Power and Prestressing

Prestressing is not important when calculating resonance frequency of a transducer theoretically, but it is important in practice. Power is also important when introducing a transducer and commonly used. In practice transducers are called not only for their resonance frequencies but also their power. One of the ways to decrease welding time is to increase the power of transducer. Power of transducer mainly depends on their piezoceramics. Thus, to design a transducer at desired properties, quality factor must be examined carefully. When prestressing the transducers, it is expected that acoustic impedances have a certain ratio.

1.10.1. Quality Factor

The Q factor or quality factor compares the time constant for decay of an oscillating physical system's amplitude to its oscillation period. Equivalently, it compares the frequency at which a system oscillates to the rate at which it dissipates its energy. A higher *Q* indicates a lower rate of energy dissipation relative to the oscillation frequency [12,13].

There are various ways to define the quality factor Q of the system. These can be summarized as follows;

1. The Q can be defined as the resonance frequency divided by the bandwidth BW, that is frequency difference between the upper and lower frequencies for which the power has dropped to half of its maximum value:

$$Q = \frac{\omega}{\omega_2 - \omega_1} \quad (1.15)$$

Hence high Q corresponds to a sharp resonance with narrow bandwidth.

2. The first form of Q can be rewritten in terms of mechanical constant.

$$Q = \frac{\omega}{R_m} \quad (1.16)$$

Thus high Q corresponds to small R_m or low loss, R_m is mechanical resistance.

3. In terms of the decay time τ of the free oscillator, which is the time for the amplitude to fall to $1/e$ of its initial value, $\tau = 2m / R_m$,

$$Q = \frac{1}{2}\omega\tau \quad (1.17)$$

4. Finally, a formal definition of Q, equivalent to above, is ;

$$Q = \frac{stored\ energy}{total\ energy\ dissipated} \tag{1.18}$$

Again, high Q oscillatory is low loss system.

5. Q can also be seen as amplification factor. As R_m decreases the displacement frequency curve gets sharper and the amplitude at resonance A_0 increases significantly. Direct calculation of Q from the definition leads to

$$Q = A_0(\frac{k}{F_0}) \tag{1.19}$$

F_0 / k is the amplitude at asymptotically low frequencies. This is the physical basis for the demonstrably high displacements attainable in mechanical systems at resonance.

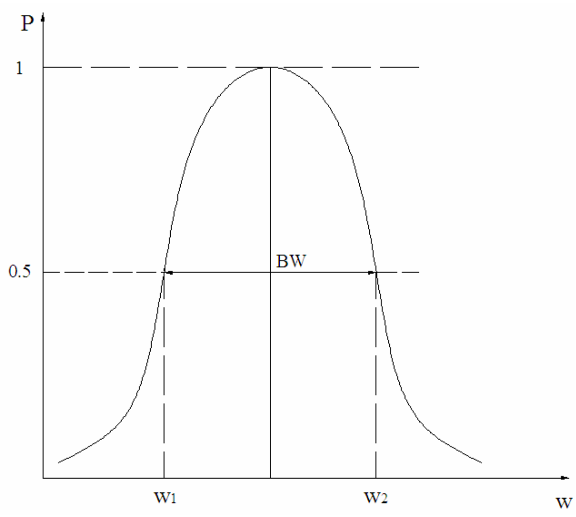


Figure 1.5. Mean power input, as a function of frequency

If Quality Factor is low at resonance frequency, amplitude becomes low and has high bandwidth. However, if a piezoceramic has high quality factor, this will result in narrow bandwidth, sharper and larger amplitudes, see Figure 1.6.

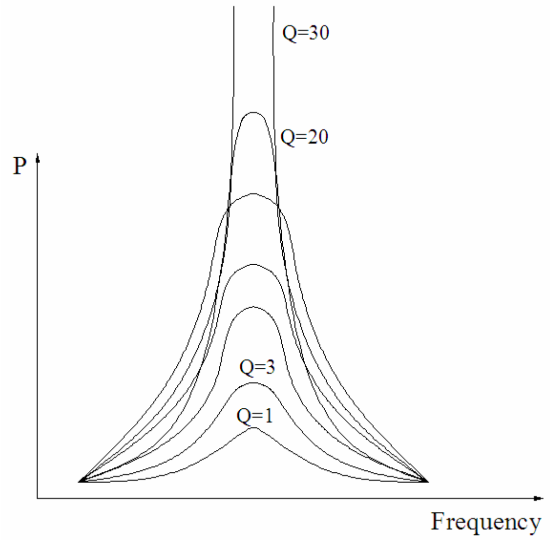


Figure 1.6. Mean power absorbed by a forced oscillator as a function of frequency

1.10.2. Power

Power of a compound mode transducer can be calculated in different ways. However output power of transducer has some loss that cannot be modeled exactly, thus the power of the transducer is calculated from input power. RMS electric power is simply equal to the multiplication of current and voltage [16,18].

$$P = \frac{VI}{2} \tag{1.20}$$

However average power cannot be calculated by the above formula, since current, which is time derivative of electric displacement, can be obtained only instantaneously.

$$I = \frac{\partial D}{\partial t} \tag{1.21}$$

In one mode, piezoceramics are parallel and sum of all current is equal to;

$$I = I_1 + I_2 + ... \tag{1.22}$$

Since piezoceramics are also capacitor, thus transducer power can be calculated using capacitor formula. Capacitance of a capacitor between two electrodes of area S, thickness d_C, and permittivity ε_C is given by;

$$C_0 = \frac{\varepsilon_C S}{d_C} \tag{1.23}$$

And also impedance of a capacitor;

$$Z_C = \frac{V_C}{I_C} = \frac{-j}{2\pi f C_0} = -jX_C \tag{1.24}$$

Where V_C and I_C are phasor voltage and current respectively. X_C is also called capacitive reactance.

So, the power in a piezoceramic can be calculated by;

$$P_C = \frac{V_C^{\ 2}}{2Z_C} \tag{1.24}$$

Longitudinal and torsional modes are assumed to be serially connected.

1.10.3. Prestressing

Piezoelectric transducers for power applications sometimes consist of plates or blocks of piezoelectric materials, which become a part of the resonant system, at the desired frequency. A typical half-wave transducer is shown schematically in Fig. 1.7. It is similar to the Langevin sandwich structure.

Here, two piezoelectric elements P_1 and P_2 are located between two identical metal blocks A and B. The elements P_1 and P_2 are separated by an electrode E connected to the high-voltage lead. The electrode is, therefore, located at a node and the elements P_1 and P_2 must be polarized in opposite directions for optimum activity.

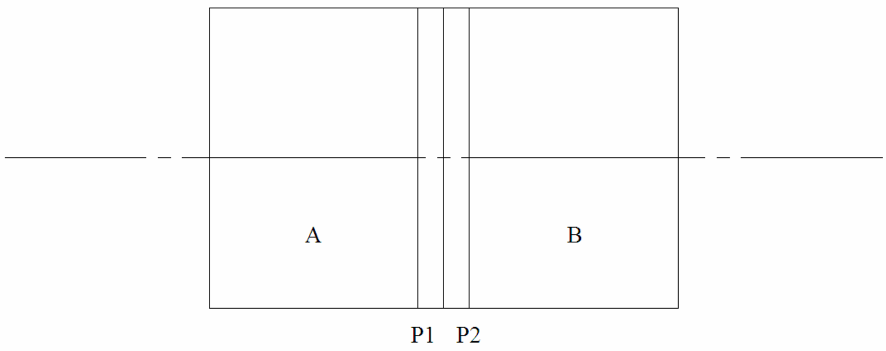


Figure 1.7. Typical half-wave piezoelectric transducer

If the mating surfaces are perfectly flat and highly polished, coupling without bonding between the piezoelectric elements and the metal end pieces may be accomplished by applying high pressure across the elements.

Clamping may be done in one of the two; a bolt may run through the centre of the elements, but is insulated from the piezoelectric elements; or a flange may be attached to the metal blocks and clamped externally by a series of bolts.
The proper torque to be applied to the bolts may be determined by,

1. Calculating the total force required to procedure the desired stress,
2. Determining the force F that each bolt must hold, and
3. Applying this value of F in the following equation:

$$T_{\mathbf{B}} = \frac{F_R k}{2\pi b} \tag{1.25}$$

Where T_B is the torque (Nm), F_R is the force per bolt (N), k is the pitch of bolt thread (m), b is the efficiency of the threaded combinations.

$$F_R = Y\delta S = Y\frac{\Delta x}{x}S \tag{1.26}$$

Required F_R may be calculated if displacements are known. δ is strain in vibration direction, S is the area.

The assembly described will resonate at a frequency below that of either piezoelectric element when it is unloaded by an amount of metal blocks or the piezoelectric elements.

2. COMPOUND MODE ULTRASONIC TRANSDUCER

2.1. General

The pre-stressed sandwiched piezoelectric ultrasonic transducers of longitudinal vibration mode have widely been used for many years in ultrasonic technologies. In recent years, the use of torsional vibration and longitudinal-torsional compound vibration for ultrasonic technologies has been of interest, particularly for ultrasonic motors and ultrasonic machining [1]. Schematic view of such a transducer is shown in Figure 2.1 [1,5].

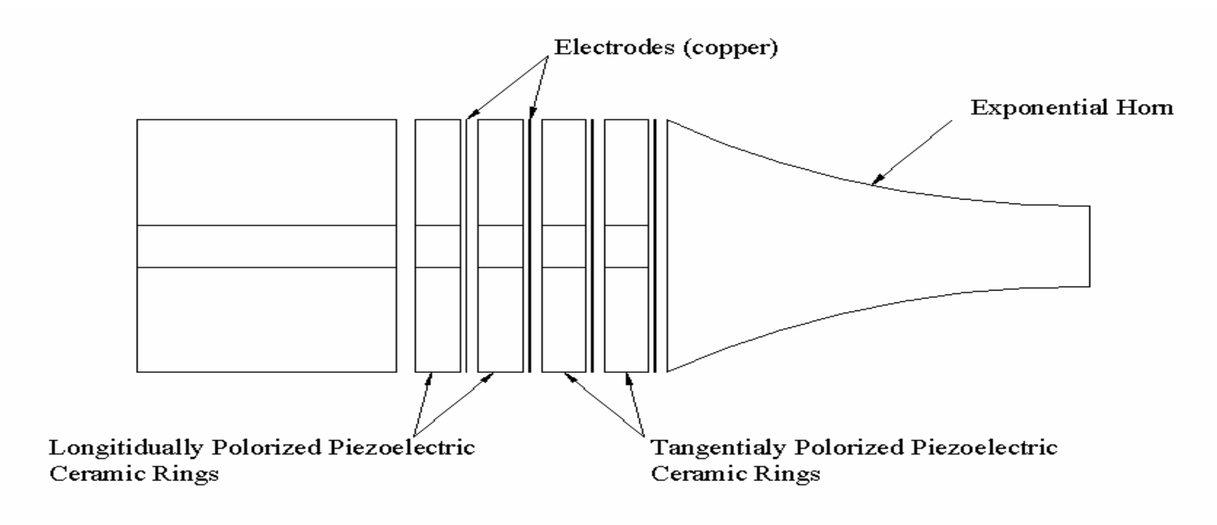


Figure 2.1. Schematic view of compound transducer

In the commonly used sandwiched ultrasonic transducer with untapered back metal rod, longitudinal and torsional speeds of sound, c_l and c_t, inside the material, are different. In the design of the longitudinal-torsional transducer, the geometrical shapes and dimensions must be carefully chosen and designed. In this thesis, exponential, conical and catenoidal metal horns are used. Front part is not used, since front part only shifts resonance frequencies and also decreases amplitudes of torsional mode.

The sound speeds of longitudinal and torsional vibrations in exponential metal horn depend not only on the material parameters, but also on the cross-sectional geometrical shape and dimensions. Therefore, it is possible that longitudinal and torsional vibration modes could be made resonate in the same frequency.

2.2. Vibration Modes

Vibration modes of compound transducers can be of different types; flexural, torsional, longitudinal etc. In this thesis torsional-longitudinal vibration modes of a transducer is analyzed. Each mode is assumed to be independent. Namely, when analyzing a mode, the other mode piezoceramics are neglected. Each part of the transducer is considered as bar or shaft according to driving mode. Backing part, longitudinal and torsional piezoceramics are uniform. However horn is non-uniform in shape. But all part of the transducer is assumed to be homogenous. In the following section, displacement and strain equations of longitudinal and torsional of a uniform bar or rod are given. Boundary conditions of the modes are given. These conditions are valid for exponential, conical and catenoidal horns, as well.

2.3. Longitudinal Vibrations of a Uniform Bar

A bar of uniform cross-sectional area S, density ρ, modulus of elasticity Y, and length l are fixed and free at the other end. The bar is subjected to an axial force F_0 at its free end, see Figure 2.2 [6,19].

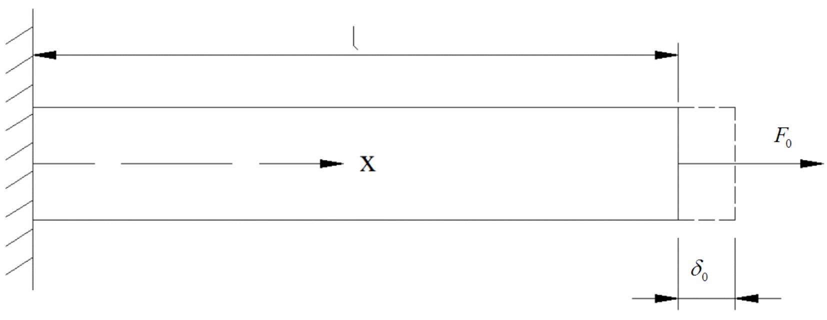


Figure 2.2. Bar subjected to an axial force at end

$$\alpha = \frac{F_0}{YS} \tag{2.1}$$

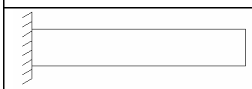
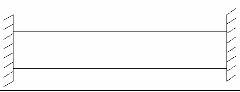
α is induced tensile strain in bar. As force F_0 is removed at that time, a displacement will occur at the end of bar, it is given by;

$$u(l,0) = \alpha l = \frac{F_0 l}{YS} \tag{2.2}$$

Y and S are young modulus and cross sectional area respectively.

In a bar, there could be three types of boundary conditions. In table 2.1 these conditions are given. The expression $\frac{\partial u}{\partial x}$ shows force condition at that point.

Table 2.1. Common boundary conditions for a bar in longitudinal vibration

Conditions of Bar	Boundary Conditions
 Fixed-free	$u(0,t) = 0, \frac{\partial u(l,t)}{\partial x} = 0$
 Free-free	$\frac{\partial u(0,t)}{\partial x} = 0, \frac{\partial u(l,t)}{\partial x} = 0$
 Fixed-fixed	$u(0,t) = 0, u(l,t) = 0$

2.4. Torsional Vibrations of a Uniform Rod or Shaft

A rod of uniform radius r, polar moment of inertia J, density ρ, torsional modulus G, and length l are fixed and free at the other end. The rod is subjected to a radial moment M_0 is at its free end as in Figure 2.3 [6].

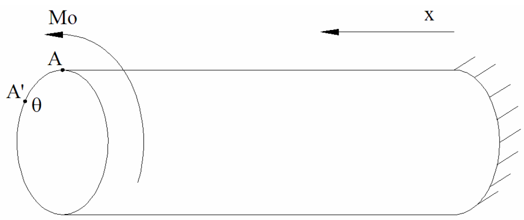


Figure 2.3. Rod is subjected to an axial moment

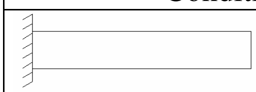
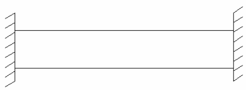
$$S_z(z,t) = r\frac{\partial v(z,t)}{\partial z} \tag{2.3}$$

S_z is torsional strain in rod. As M_0 moment is removed, a torsional deflection will occur at that time, at the end of the rod torsional deflection is given by;

$$v(l,0) = S_z(l,0)l = \frac{M_0 l}{GJ} \tag{2.4}$$

In a rod, there could be three types of boundary conditions. The expression $\frac{\partial v}{\partial x}$ shows moment condition at that point. In table 2.2 these conditions are given.

Table 2.2. Boundary conditions for a uniform rod in torsional vibration

Conditions of Rod		Boundary Conditions
	Fixed-free	$v(0,t)=0, \frac{\partial v(l,t)}{\partial z}=0$
	Free-free	$\frac{\partial v(0,t)}{\partial z}=0, \frac{\partial v(l,t)}{\partial z}=0$
	Fixed-fixed	$v(0,t)=0, v(l,t)=0$

2.5. Elastic Wave Equation for Longitudinal Vibration in Non Uniform Bar

Since the horn is non-uniform, displacements cannot be obtained as explained above. Thus non-uniform bar must be analyzed for longitudinal vibration. Consider a force F is applied to an elastic bar of length l and of varying cross sectional area $S(x)$, where $u(x,t)$ is the longitudinal displacement of the vibration in the x direction as shown Figure 2.4 .

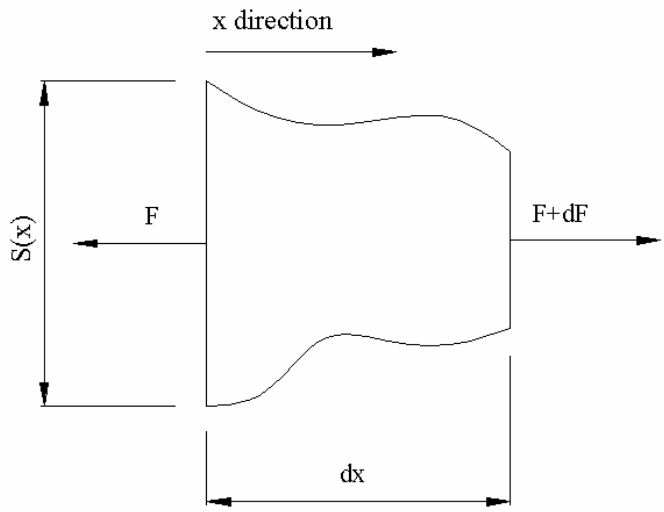


Figure 2.4. Schematic view of forces on a non-uniform bar

For the element the stress/strain equation gives [8],

$$F(x,t) = S(x)Y\frac{du(x,t)}{dx} \tag{2.5}$$

After applying Newton second law to the bar,

$$F + dF - F = \rho S(x)dx\frac{\partial^2 u}{\partial t^2} \tag{2.6}$$

Equation 2.6 doesn't include the damping forces. Putting Equation 2.5 into 2.6 following equation, one-dimensional elastic wave equation for varying cross-sectional area, is derived [9].

$$\frac{1}{c_l^2}\frac{\partial^2 u}{\partial t^2}-\frac{1}{S(x)}\frac{dS(x)}{dx}\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=0 \tag{2.7}$$

c_l is the speed of sound in longitudinal vibration and is equal to $\sqrt{\frac{Y}{\rho}}$. Equation 2.7 is also known as the horn equation. A change in cross sectional area results in an amplitude change at the tip of horn.

Solution of the Equation 2.7 can be obtained using separation of variables method, and then the solution has the following form [9].

$$u(x,t)=X(x)T_L(t) \tag{2.8}$$

2.6. Damped Wave Equation for Longitudinal Vibration in Non Uniform Bar

Equation 2.7 has no loss term, adding damping term can be in two ways, one of them is viscous damping, and however in this type modeling has no analytical solution not only to longitudinal vibration but also to torsional vibration. Other method is structural damping; structural damping terms are needed to add equations of motion of the system. Any type of damped continuous longitudinal vibration system can be expressed in terms of space and time operators as follows. [4, 10];

$$\mathbf{L}[u(x,t)]+\frac{\partial}{\partial t}\mathbf{C}[u(x,t)]+\mathbf{M}\frac{\partial^2 u(x,t)}{\partial t^2}=0 \tag{2.9}$$

By comparing Equation 2.7 and 2.9, it is obvious to define the operators as follows;

$$\mathbf{L} = -\frac{\partial^2}{\partial x^2} - \frac{1}{S}\frac{dS}{dx}\frac{\partial}{\partial x} \qquad (2.10)$$

$$\mathbf{M} = \frac{1}{c_l^2} \qquad (2.11)$$

It is assumed that the damping operator $\mathbf{C}$ is proportional to the operator $\mathbf{L}$.

$$\mathbf{C} = \frac{R_s}{\omega}\mathbf{L} \qquad (2.12)$$

R_s and ω are longitudinal damping coefficient and the driving angular velocity of the system respectively. Since the motion is harmonic, it can be assumed that solution of $u(x,t)$ has following form;

$$u(x,t) = X(x)e^{j\omega t} \qquad (2.13)$$

and derivation of Equation 2.13 with respect to time as follows;

$$\frac{\partial u(x,t)}{\partial t} = j\omega u(x,t) \qquad (2.14)$$

Using Equations 2.13 and 2.14, Equation 2.9 can be rearranged as;

$$(1 + jR_s)\mathbf{L}[u(x,t)] + \mathbf{M}\frac{\partial^2 u(x,t)}{\partial t^2} = 0 \qquad (2.15)$$

Expressing $\mathbf{L}$ and $\mathbf{M}$ operators with their initial forms as in Equation 2.10 and 2.11, the general horn equation with structural damping for longitudinal vibration will become as follows [9];

$$\frac{\partial^2 u(x,t)}{\partial x^2} + \frac{1}{S(x)}\frac{dS(x)}{dx}\frac{\partial u(x,t)}{\partial x} - \left(\frac{1}{1 + jR_s}\right)\frac{1}{c_l^2}\frac{\partial^2 u(x,t)}{\partial t^2} = 0 \qquad (2.16)$$

2.7. Elastic Wave Equation for Torsional Vibration in Non Uniform Shaft

The horn can be considered as a shaft for torsional vibration. If an elastic non-uniform shaft is subjected to an external torque $f(z,t)$ per unit length as represented in Figure 2.5. If $v(z,t)$ is the angle of the twisting of the cross section, the relation between the torsional deflection and the twisting moment $M(z,t)$ is given by [3];

$$M(z,t) = GJ(z)\frac{\partial v}{\partial z} \tag{2.17}$$

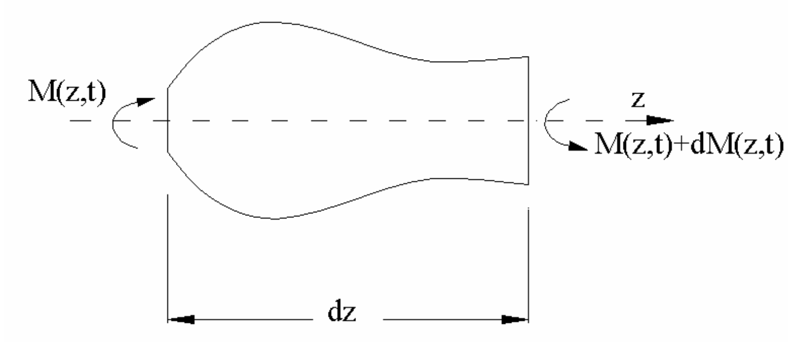


Figure 2.5. A non-uniform rod subjected to a twisting moment

G and J(z) are the shear modulus and polar moment of inertia in the case of a circular cross section respectively, and the *GJ*(*z*) is called torsional stiffness. Inertia torque acting on an element of length dz becomes;

$$I(z)dz\frac{\partial^2 v}{\partial t^2} \tag{2.18}$$

I(z) is the mass polar moment of inertia per unit length of the shaft, and for homogenous shafts the relation between J(z) and I(z) is given by;

$$I(z) = \rho J(z) \tag{2.19}$$

The application of Newton second law to the shaft yields the equation of motion:

$$(M + dM) + fdz - M = I(z)dz\frac{\partial^2 v}{\partial t^2} \tag{2.20}$$

In the case of free vibration, fdz term vanishes. Using Equation 2.17 and 2.19, Equation 2.20. can be rearranged as;

$$\frac{d}{dz}\left[GJ(z)\frac{\partial v}{\partial z}(z,t)\right] = I(z)\frac{\partial^2 v}{\partial t^2}(z,t) \tag{2.21}$$

$$\frac{\partial^2 v}{\partial z^2}(z,t) + \frac{1}{J(z)}\frac{\partial J(z)}{\partial z}\frac{\partial v}{\partial z}(z,t) - \frac{1}{c_t^2}\frac{\partial^2 v}{\partial t^2}(z,t) = 0 \tag{2.22}$$

c_t is the speed of sound in torsional vibration and equals to $\sqrt{\frac{G}{\rho}}$. The solution of Equation 2.22 can be separated into two parts in the following form;

$$v = \theta(z)T_T(t) \tag{2.23}$$

And harmonic solution of Equation 2.22 is assumed in the following form;

$$v = \theta(z)e^{j\omega t} \tag{2.24}$$

2.8. Damped Wave Equation for Torsional Vibration in Non Uniform Shaft

Equation 2.22 has no loss term, like longitudinal vibration, adding damping term can only be done by structural damping method. Similarly any type of damped continuous torsional vibration system can be expressed in terms of space and time operators as follows. [4, 10];

$$\mathbf{P}[v(z,t)] + \frac{\partial}{\partial t}\mathbf{Q}[v(z,t)] + \mathbf{R}\frac{\partial^2 v(z,t)}{\partial t^2} = 0 \tag{2.25}$$

By comparing Equation 2.22 and Equation 2.25, it is obvious to define the operators as follows;

$$\mathbf{P} = -\frac{\partial^2}{\partial z^2} - \frac{1}{J}\frac{dJ}{dz}\frac{\partial}{\partial z} \tag{2.26}$$

$$\mathbf{R} = \frac{1}{c_t^2} \tag{2.27}$$

It is assumed that the damping operator $\mathbf{Q}$ is proportional to the operator $\mathbf{P}$.

$$\mathbf{Q} = \frac{R_v}{\omega}\mathbf{P} \tag{2.28}$$

R_v is torsional damping coefficient. Putting operators $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$ in Equation 2.25, the general horn equation with structural damping for torsional vibration will become as follows:

$$\frac{\partial^2 v(z,t)}{\partial z^2} + \frac{1}{J}\frac{dJ}{dz}\frac{\partial v(z,t)}{\partial z} - \left(\frac{1}{1+jR_v}\right)\frac{1}{c_t^2}\frac{\partial^2 v(z,t)}{\partial t^2} = 0 \tag{2.29}$$

2.9. Wave Equations

In general there are four types of ultrasonic horn, which are uniform, exponential, conical, catenoidal. As explained before uniform horn cannot be used for synchronization. It is easy to synchronize exponential and catenoidal horns with respect to conical horn, since relation of dimensions is straight in conical horn. For torsional mode, solution of catenoidal horn is more complex with respect to exponential, conical horn.

2.9.1 Exponential Horn

Exponential horn has similar changing laws of cross sectional area and inertia moments in longitudinal and torsional vibration.

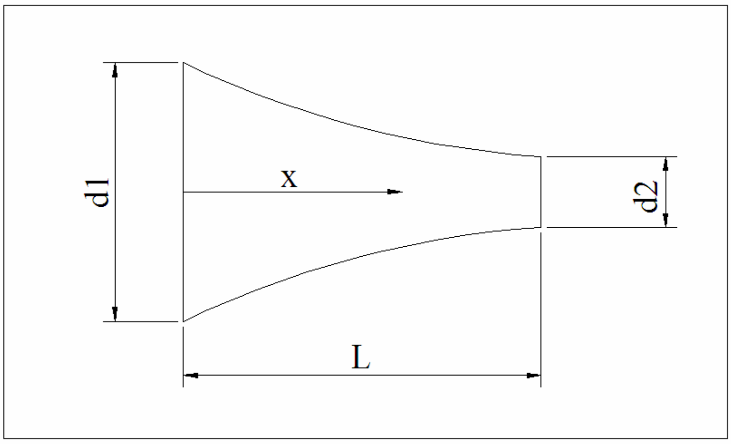


Figure 2.6. Exponential Horn

For exponential horn, cross sectional area changes in the following form;

$$S(x) = S_1 e^{-\beta x} \tag{2.30}$$

β is radius decay coefficient which is $\beta = \frac{1}{L}\ln(\frac{S_1}{S_2})$, and S_1 and S_2 are the initial and end cross sectional area. After putting $S(x)$ into Equation 2.16, the spatial solution of exponential horn for longitudinal vibration is obtained as follows;

$$X(x) = e^{\frac{\beta x}{2}}(A_1^L \cos\frac{\omega x}{c_l}\sqrt{\frac{1}{1+jR_s} - (\frac{\beta c_l}{2\omega})^2} + B_1^L \sin\frac{\omega x}{c_l}\sqrt{\frac{1}{1+jR_s} - (\frac{\beta c_l}{2\omega})^2}) \tag{2.31}$$

For exponential horn, inertia moment changes in the following form [11];

$$J(x) = J_1 e^{-\gamma x} \tag{2.32}$$

Where $\gamma = \frac{1}{L}\ln(\frac{J_1}{J_2})$, and J_1 and J_2 are the initial and end inertia moments, respectively. Putting $J(x)$ into 2.29, the spatial solution for torsional vibration is obtained as follows;

$$\theta(z) = e^{\frac{\gamma z}{2}}(A_1^T \cos\frac{\omega z}{c_t}\sqrt{\frac{1}{1+jR_v} - (\frac{\gamma c_t}{2\omega})^2} + B_1^T \sin\frac{\omega z}{c_t}\sqrt{\frac{1}{1+jR_v} - (\frac{\gamma c_t}{2\omega})^2}) \qquad (2.33)$$

Real parts of Equations 2.31 and 2.33 give the solution. Constants are determined from boundary conditions.

2.9.2 Conical Horn

Conical horn can also be used for synchronization. Typical form of the horn is given in Figure 2.7. Diameter of conical horn changes as follows;

$$d(x) = [d_1 + \frac{x}{L}(d_2 - d_1)]^2 \qquad (2.34)$$

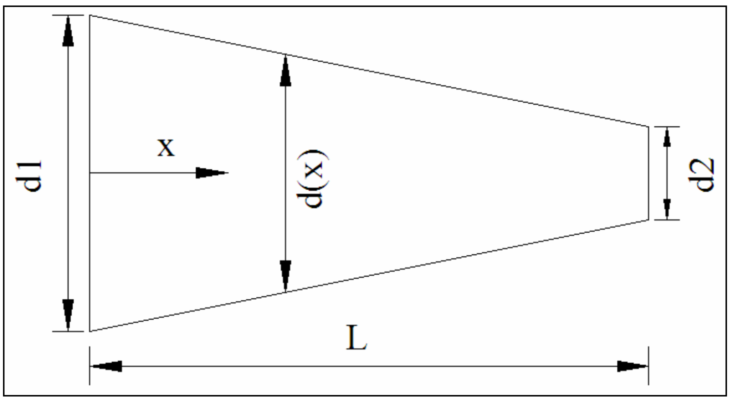


Figure 2.7. Conical Horn

Cross sectional area of conical horn is;

$$S(x)=\frac{\pi}{4}[d_1+\frac{x}{L}(d_2-d_1)]^2 \quad (2.35)$$

And polar moment of inertia is:

$$J_z=\frac{\pi d_x^2}{32} \quad (2.36)$$

Putting d_x into J_z;

$$J_z=\frac{\pi}{32}[d_1+\frac{z}{L}(d_2-d_1)]^2 \quad (2.37)$$

And the spatial solutions of two modes are obtained for conical horn as follows;

$$X(x)=\frac{1}{L+(N-1)x}(A_1^L\cos\frac{\omega x}{c_l}\sqrt{\frac{1}{1+jR_s}}+B_1^L\sin\frac{\omega x}{c_l}\sqrt{\frac{1}{1+jR_s}}) \quad (2.38)$$

Here $N=\frac{d_2}{d_1}$, and solution for torsional vibration;

$$\theta(z)=(\frac{d_2-d_1}{\sqrt{d_1L+zd_2-zd_1}})^3(A_1^T H_3(Z_3)+B_1^T G_3(Z_3)) \quad (2.39)$$

$H_3(Z)$ and $G_3(Z)$ are first and second kind Bessel functions of third order. Z_3 is equal to;

$$Z_3=\frac{2\omega}{c_t}\sqrt{(\frac{d_1L}{1+iR_v})(\frac{d_1L+zd_2-zd_1}{(d_2-d_1)^2})} \quad (2.40)$$

2.9.3 Catenoidal Horn

Typical catenoidal horn is seen in Figure 2.8. Cross sectional area of a catenoidal horn changes in following way;

$$S(x) = S_2 \cosh \mu (L - x) \tag{2.41}$$

Where $\mu = \dfrac{1}{L}\cosh^{-1}(\dfrac{d_1}{d_2})$ and $S_2 = \pi d_2^2 / 4$.

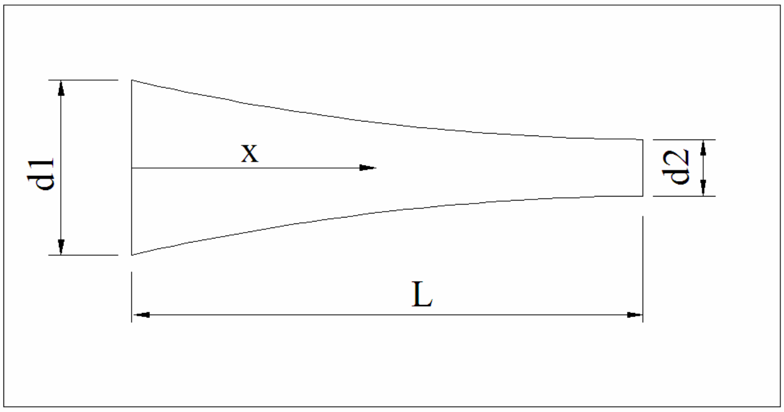


Figure 2.8. Catenoidal Horn

The polar moment of inertia takes following form;

$$J_x = J_2 \cosh^4 \frac{x}{a} \tag{2.42}$$

Where $a = d_2 / 2$ and $J_2 = \dfrac{\pi a^4}{2}$. Solutions of two modes are obtained as follows;

$$X(x) = \frac{1}{\cosh(-L + x)} (A_1^L \sin \frac{\omega}{c^{'}} x + B_1^L \cos \frac{\omega}{c^{'}} x) \tag{2.43}$$

$c^{'}$ is the modified speed of sound, and is equal to;

$$c^{'} = \frac{c_l}{\sqrt{1-(\frac{\mu c_l}{\omega})^2}} \tag{2.44}$$

The spatial solution of torsional vibration is obtained as follows;

$$\theta(z) = e^{-\frac{1}{2}a\ln(\tanh(z/a)-1)-\frac{1}{2}a\ln(\tanh(z/a)+1)}(A_1^T A + B_1^T B) \tag{2.45}$$

Where A and B are equal to;

$$A = \frac{(\frac{1}{2}(\tanh\frac{z}{a}-1))^{(\frac{c_t(-2i+2R_v)^{1/2}+\omega a+\omega ai}{2c_t(-2i+2R_v)^{1/2}})}(2\tanh(\frac{z}{a})+2)^{(\frac{c_t(-2i+2R_v)^{1/2}-a^{1/2}(-2ic_t^2R_v-2c^2+\omega^2 a)^{1/2}(1+i))}{2c_t(-2i+2R_v)^{1/2}})}{}_2F_1(a_x,b_x;c_x,d_x)}{(1-\tanh(z/a)^2)^{1/2}} \tag{2.46}$$

$$B = \frac{(\frac{1}{2}(\tanh\frac{z}{a}-1))^{(\frac{1}{2}+\frac{(1+i)\omega a}{2c_t(-2i+2R_v)^{1/2}})}(2\tanh(\frac{z}{a})+2)^{(\frac{1}{2}+\frac{(1+i)a^{1/2}(-2ic_t^2R_v-2c_t^2+\omega^2 a)^{1/2}}{2c_t(-2i+2R_v)^{1/2}})}{}_2G_1(a_y,b_y;c_y,d_y)}{(1-\tanh(z/a)^2)^{1/2}} \tag{2.47}$$

Where ${}_2F_1(a_x,b_x;c_x;d_x)$ and ${}_2G_1(a_y,b_y,c_y,d_y)$ are Gauss hypergeometric functions, which are;

$${}_2F_1 = \sum_{k=0}^{\infty}\frac{(a_x)_k(b_x)_k}{(c_x)_k}.\frac{(d_x)^k}{k!} \text{ and } {}_2G_1 = \sum_{k=0}^{\infty}\frac{(a_y)_k(b_y)_k}{(c_y)_k}.\frac{(d_y)^k}{k!} \tag{2.48}$$

a_x,b_x,c_x,d_x and a_y,b_y,c_y,d_y are equal to;

$$a_x = \frac{-a^{1/2}(-2ic_t^2R_v-2c_t^2+\omega^2 a)^{1/2}(1+i)+(1+2a)^{1/2}c_t(-2i+2R_v)^{1/2}+\omega a+\omega ai+c_t(-2i+2R_v)^{1/2}}{2c_t(-2i+2R_v)^{1/2}} \tag{2.49}$$

$$b_x = \frac{-a^{1/2}(-2ic_t^2R_v-2c_t^2+\omega^2 a)(1+i)-(1+2a)^{1/2}c_t(-2i+2R_v)+\omega a+\omega ai+c_t(-2i+2R_v)^{1/2}}{2c_t(-2i+2R_v)^{1/2}} \tag{2.50}$$

$$c_x = \frac{c_t(-2i + 2R_v)^{1/2} - a^{1/2}(-2ic_t^2 R_v - 2c_t^2 + \omega^2 a)^{1/2}(1+i)}{c_t(-2i + 2R_v)^{1/2}} \tag{2.51}$$

$$d_x = \frac{\tanh(z/a) + 1}{2} \tag{2.52}$$

$$a_z = \frac{a^{1/2}(-2ic_t^2 R_v - 2c_t^2 + \omega^2 a)^{1/2}(1+i) + (1+2a)^{1/2} c_t(-2i + 2R_v)^{1/2} + \omega a + \omega a i + c_t(-2i + 2R_v)^{1/2}}{2c_t(-2i + 2R_v)^{1/2}} \tag{2.53}$$

$$b_z = \frac{-a^{1/2}(-2ic_t^2 R_v - 2c_t^2 + \omega^2 a)(1+i) + (1+2a)^{1/2} c_t(-2i + 2R_v) - \omega a - \omega a i - c_t(-2i + 2R_v)^{1/2}}{-2c_t(-2i + 2R_v)^{1/2}} \tag{2.54}$$

$$c_z = \frac{c_t(-2i + 2R_v)^{1/2} + a^{1/2}(-2ic_t^2 R_v - 2c_t^2 + \omega^2 a)^{1/2}(1+i)}{c_t(-2i + 2R_v)^{1/2}} \tag{2.55}$$

$$d_z = \frac{\tanh(z/a) + 1}{2} \tag{2.56}$$

2.10. Constitutive Piezoelectric Equations

Piezoceramic equations used in transducer design are different for each mode. For longitudinal vibration, T-E type constitutive equations are used. When a voltage V is applied to electrodes of a piezoelectric disc of the cross sectional area S, and thickness l, the piezoelectric constitutive relations for thickness mode disc are given by [6,20];

$$T_3 = c_{33}^D S_{x3} - h_{33} D_3 \tag{2.57}$$

$$E_3 = -h_{33} S_{x3} + \beta_{33}^S D_3 \tag{2.58}$$

h_{33} can be expressed in terms of voltage constant of piezoceramic and elastic coefficient, $h_{33} = g_{33}c_{33}^D$. For torsional vibration S-E Type constitutive equations are used. The piezoelectric constitutive relations for torsional the disc are given by [6,7],

$$S_{\theta 3} = s_{55}^D T_{\theta 3} + g_{15} D_{z3} \tag{2.59}$$

$$E_3 = -g_{15} T_{\theta 3} + \beta_{11}^T D_{z3} \tag{2.60}$$

D, c^D, s^D, h, g are the electric displacements, the elastic coefficient, elastic compliance (stiffness), piezoelectric constants respectively. β_{33}^S and β_{11}^T are the dielectric impermeability constants (the inverse of the electric permittivity ε). Voltage V, which is applied to a piezoceramic between two electrodes, is the integral of the electric field E;

$$V = \int_0^l E dl \tag{2.61}$$

2.11. Boundary and Matching Conditions

Using Impedance matching method, boundary conditions are applied to transducer which made up of n subsystems has 2n boundary matching conditions both for longitudinal and torsional vibration. The first and last boundary conditions are usual for one-dimensional wave equation.

2.11.1. Boundary Conditions for Longitudinal Vibration

Boundary conditions can be free-free, fixed-fixed, fixed-free or vice versa as explained in sections 2.3. and 2.4. As an example, fixed-free boundary condition is given to build impedance matrix. Longitudinal boundary conditions of the parts of the transducer made up of N parts are assumed as follows: If first part of the transducer is fixed from end and the horn is free from tip, then following boundary conditions can be applied to the system [4,21]. At $x = 0$ since the first resonator is fixed;

$$u_1(0,t) = 0 \tag{2.62}$$

The second and third matching conditions come from the displacement $u(x,t)$ and force $F(x,t)$ continuity at the interface between the first and the second resonators.

$$u_1(L_1,t) = u_2(0,t) \tag{2.63}$$

$$F_1(L_1,t) = F_2(0,t) \tag{2.64}$$

and between the first two longitudinal piezoceramics

$$u_2(L_2,t) = u_3(0,t) \tag{2.65}$$

$$F_2(L_2,t) = F_3(0,t) \tag{2.66}$$

If there are more than two piezoceramics, boundary conditions will continue as follows.

$$u_{n-1}(L_{n-1},t) = u_n(0,t) \tag{2.67}$$

$$F_{n-1}(L_{n-1},t) = F_{n-1}(0,t) \tag{2.68}$$

Since at x=L the transducer is free to vibrate, then there will be no force at tip of the horn. So the last boundary condition is;

$$F_n(0,t) = 0 \tag{2.69}$$

The displacement $u(x,t)$ and the electric displacement D of the i'th part are assumed to be varying harmonically in time as follows;

$$u_i(x_i,t) = X_i(x_i)e^{j\omega t} \tag{2.70}$$

$$D_i(t) = D_i^* e^{j\omega t} \tag{2.71}$$

Therefore, the force $F(x,t)$ can be rearranged using Equations 2.57, 2.70 and 2.71 as:

$$F_i(x_i,t) = S(x)e^{j\omega t}(Y_i X_i{}' - h_i D_i^*) \tag{2.72}$$

D_i^* and h_i are the electric displacement and the piezoelectric constant respectively, which are zero for non-piezoelectric materials. When Equations 2.69 and 2.70 are put into Equations 2.62 -2.69, the equations will become time independent. Spatial solution of each part can be derived using Equation 2.47 given in section 2.9. After rearranging equations boundary conditions will take form as follows;

$$X(L_1) = X_2(0) \tag{2.73}$$

$$S_1(Y_1 X_1{}'(L_1) - h_1 D_1^*) = S_2(Y_2 X_2{}'(0) - h_2 D_2^*) \tag{2.74}$$

Expressing boundary conditions in general forms for two matching conditions between the (i-1)'th and i'th parts are:

$$X_{i-1}(L_{i-1}) = X_i(0) \tag{2.75}$$

$$S_{i-1}(Y_{i-1} X_{i-1}{}'(L_{i-1}) - h_{i-1} D_{i-1}^*) = S_i(Y_i X_i{}'(0) - h_i D_i^*) \tag{2.76}$$

And the last boundary condition is;

$$Y_n X_n{}'(L_n) - h_n D_n^* = 0 \tag{2.77}$$

2.11.2. Boundary Conditions for Torsional Vibration

A similar procedure can be followed for torsional vibration, as well. Boundary conditions of parts of the transducer, made up of N parts, are assumed to be as follows. If the first part of the transducer is fixed from end and the horn is free from the tip, following boundary conditions apply to the system. At $z = 0$ since the first resonator is fixed [3];

$$v_1(0,t) = 0 \tag{2.78}$$

Note that z is exactly the same with x, but it is used not to confuse torsional equations with longitudinal ones.

The second and third matching conditions come from the torsional deflection $v(z,t)$ and moment $M(z,t)$ continuity at the interface between the first and the second resonators.

$$v_1(L_1,t) = v_2(0,t) \tag{2.79}$$

$$M_1(L_1,t) = M_4(0,t) \tag{2.80}$$

and between the first two torsional piezoceramics ;

$$v_4(L_4,t) = v_5(0,t) \tag{2.81}$$

$$M_4(L_4,t) = M_5(0,t) \tag{2.82}$$

If there are more than two piezoceramics, boundary conditions will continue as follows.

$$v_{n-1}(L_{n-1},t) = v_n(0,t) \tag{2.83}$$

$$M_{n-1}(L_{n-1},t) = M_{n-1}(0,t) \tag{2.84}$$

Since at x=L the transducer is free to vibrate, then there will be no moment at the tip of the the horn. So the last boundary condition is;

$$M_n(0,t) = 0 \tag{2.85}$$

The torsional deflection $v(z,t)$ and the electric displacement D_z of the i'th part are assumed to be varying harmonically in time as follows;

$$v_i(z_i,t) = \theta_i(z_i)e^{j\omega t} \tag{2.86}$$

$$D_{zi}(t) = D_{zi}^{*}e^{j\omega t} \tag{2.87}$$

And in a circular disc with radius r, torsional moment can be expressed as [7];

$$M(z,t) = \iint_s rT_{\theta z}ds \tag{2.88}$$

ds is the circular unit area. From Equation 2.59;

$$T_{\theta 3} = \frac{1}{s_{55}^{D}}S_{\theta z} - \frac{g_{15}}{s_{55}^{D}}D_{z3} \tag{2.89}$$

Here $S_{\theta z}$ is torsional strain and equal to;

$$S_{\theta z} = r\frac{\partial \theta}{\partial z} \tag{2.90}$$

Using Equations 2.89 and 2.90, $M(z,t)$ can be rearranged as:

$$M(z,t) = Z_t\frac{\partial \theta}{\partial z} - Z_q D_{z3} \tag{2.91}$$

Where $Z_t = \frac{I_p}{s_{55}^D}$, $Z_q = \frac{W g_{15}}{s_{55}^D}$ and $I_p = \iint_s r^2 ds$ and $W = \iint_s r ds$.

$$M_i(z_i, t) = Z_{ti}\theta_i' e^{j\omega t} - Z_{qi} D_{zi}^* e^{j\omega t} \quad (2.92)$$

D_{zi}^* and g_{15} are the electric displacement and the piezoelectric constant respectively, which are zero for non-piezoelectric materials. Inserting Equations 2.86 and 2.87 into boundary conditions will become:

$$\theta(L_1) = \theta_2(0) \quad (2.93)$$

$$Z_{t1}\theta_1'(L_1) - Z_{q1} D_{z1}^* = Z_{t2}\theta_2'(0) - Z_{q2} D_{z2}^* \quad (2.94)$$

Spatial solutions of each part can be derived from Equation 2.33 given in section 2.9. Expressing boundary conditions in general forms for two matching conditions between the (i-1)'th and i'th parts are:

$$\theta_{i-1}(L_{i-1}) = \theta_i(0) \quad (2.95)$$

$$Z_{ti-1}\theta_{i-1}'(L_{i-1}) - Z_{qi-1} D_{zi-1}^* = Z_{ti}\theta_i'(0) - Z_{qi} D_{zi}^* \quad (2.96)$$

And the last boundary condition is;

$$Z_{tn}\theta_n'(L_n) - Z_{qn} D_{zn}^* = 0 \quad (2.97)$$

3. IMPEDANCE METHOD

3.1. General

An ultrasonic welding system excited through piezoelectric materials can be analyzed with the impedance method. The impedance method assumes harmonic solutions in time: $u(x,t) = u_0 e^{j\omega t}$ and $v(z,t) = v_0 e^{j\omega t}$. The spatial solutions to both systems include unknown constants such as A and B, which are determined by applying boundary conditions [4].

For a system with n parts, the 2n boundary and matching conditions can be expressed in matrix form. Consequently, all the terms in the equations are on the left hand side of the corresponding equations except $h_i D_i$ and $Z_{qi} D_{ti}$ terms, which serve as forcing in the system. The spatial solutions, $X_i(x_i)$ and $\theta_i(z_i)$, always have the following form [4]:

$$X_i = A_i^L \Upsilon_i(x_i) + B_i^L \Psi_i(x_i) \tag{3.1}$$

$$\theta_i = A_i^T \Phi_i(z_i) + B_i^T \Omega_i(z_i) \tag{3.2}$$

A_i^L, B_i^L, A_i^T and B_i^T are constants to be determined with boundary conditions.

3.2. Expressing the Boundary Conditions in Matrix Form

When boundary conditions are written beginning from the first part of the transducer of n parts, 2n equations will be obtained. These equations can be expressed in matrix form. Force and moment terms are formed by hD coefficients and gD coefficients respectively.

3.2.1. Longitudinal Mode

Matrix form of boundary conditions, given in section 2.11.1, can be written as follows. Let **U** be the mode shape vector, **F** the force vector, and **[Z]** be the 2n by 2n impedance matrix made up of $\Upsilon_i(x_i)$ and $\Psi_i(x_i)$ evaluated at the boundaries. **U**, **F** and **[Z]** are related by:

$$\mathbf{[Z]U=F} \tag{3.3}$$

Where:

$$\mathbf{U}=\begin{bmatrix} A_1^L \\ B_1^L \\ A_2^L \\ B_2^L \\ \vdots \\ A_i^L \\ B_i^L \\ \vdots \\ A_n^L \\ B_n^L \end{bmatrix}_{2nx1} \quad \text{and, } \mathrm{F}=\begin{bmatrix} 0 \\ 0 \\ h_1D_1^* - h_2D_2^* \\ 0 \\ h_2D_2^* - h_3D_3^* \\ \vdots \\ 0 \\ h_iD_i^* - h_{i+1}D_{i+1}^* \\ \vdots \\ 0 \\ h_{n-1}D_{n-1}^* - h_nD_n^* \\ 0 \end{bmatrix}_{2nx1} \tag{3.4}$$

The Matrix **F** can be separated into two parts and expressed as follows:

$$\mathbf{F = [H]D} \tag{3.5}$$

$$[\mathbf{H}]=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ h_1 & -h_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & h_2 & -h_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{i-1} & -h_i & 0 \\ 0 & \cdots & \cdots & \ddots & \vdots \\ 0 & 0 & 0 & h_{n-1} & -h_n \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{2nxn} \quad \text{and, } \mathbf{D}=\begin{bmatrix} D_1^* \\ D_2^* \\ \vdots \\ D_i^* \\ \vdots \\ D_{n_{piezo}}^* \end{bmatrix}_{nx1} \tag{3.6}$$

3.2.2. Torsional Mode

In the same manner, matrix form of boundary conditions, given in Section 2.11.2, can be written as follows; let $\mathbf{U_T}$ be the mode shape vector, **T** the moment vector, and $[\mathbf{Z_T}]$ be the 2n by 2n impedance matrix made up of $\Phi_i(z_i)$ and $\Omega_i(z_i)$ evaluated at the boundaries. $\mathbf{U_T}$, **T** and $[\mathbf{Z_T}]$ are related by:

$$[\mathbf{Z_T}]\,\mathbf{U_T}=\mathbf{T} \tag{3.7}$$

Where:

$$\mathbf{U_T}=\begin{bmatrix} A_1^T \\ B_1^T \\ A_2^T \\ B_2^T \\ \vdots \\ A_i^T \\ B_i^T \\ \vdots \\ A_n^T \\ D_n^T \end{bmatrix}_{2nx1} \quad \text{and, } \mathbf{T}=\begin{bmatrix} 0 \\ 0 \\ Z_{q1}D_{z1}^*-Z_{q2}D_{z2}^* \\ 0 \\ Z_{q2}D_{z2}^*-Z_{q3}D_{z3}^* \\ \vdots \\ 0 \\ Z_{q(i)}D_{z(i)}^*-Z_{q(i+1)}D_{z(i+1)}^* \\ \vdots \\ 0 \\ Z_{q(n-1)}D_{z(n-1)}^*-Z_{q(n)}D_{z(n)}^* \\ 0 \end{bmatrix}_{2nx1} \tag{3.8}$$

The matrix **T** can be separated into two parts and expressed as follows:

$$\mathbf{T} = [\mathbf{G_T}]\mathbf{D_z} \tag{3.9}$$

Integrating term W over cross sectional area, term Z_q can be expressed as;

$$Z_q = \frac{2\pi r^3 g_{15}}{3 s_{55}^D} \tag{3.10}$$

[G$_T$] and **D$_z$** are as follows;

$$[\mathbf{G_T}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ Z_{q1} & -Z_{q2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{q2} & -Z_{q3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{q(i-1)} & -Z_{q(i)} & 0 \\ 0 & \cdots & \cdots & \ddots & \vdots \\ 0 & 0 & 0 & Z_{q(n-1)} & -Z_{q(n)} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{2nxn} \quad \text{and, } \mathbf{D_z} = \begin{bmatrix} D_{z1}^* \\ D_{z2}^* \\ \vdots \\ D_{z(i)}^* \\ \vdots \\ D_{z(n_{piezo})}^* \end{bmatrix}_{nx1} \tag{3.11}$$

The impedance matrices [Z] and $[Z_T]$ are determined by expressing the boundary matching conditions in the matrix form.

3.3. Solution Method

Expressing coefficients of longitudinal and torsional spatial solutions, which are A_1^L, B_1^L, A_2^L, B_2^L, ... A_n^L, B_n^L and A_1^T, B_1^T, A_2^T, B_2^T, ... A_n^T, in terms of electric displacements, and using the expression $V = \int E dx$ given in Section 2.10, coefficients of spatial solutions can be determined. The equations $\det([Z]) = 0$ and $\det([Z_T]) = 0$ give the frequency equations, whose solution provides the longitudinal and torsional natural frequencies of the system.

3.3.1. Longitudinal Displacements:

The mode shape vector U can be obtained in terms of the electric displacements D_j^*'s as:

$$[U] = [Z]^{-1}[H][D] \tag{3.12}$$

Letting $Q = [Z]^{-1}[H]$, a row of Equation 3.12 is:

$$U_i = \sum_{j=1}^{n_{piezo}} q_{ij} D_i^* \tag{3.13}$$

Therefore, the spatial displacements $X_i(x_i)$ are given by:

$$X_i(x_i) = [\sum_{j=1}^{n_{piezo}} q_{(2i-1),j} D_j^*]\Upsilon_i(x_i) + \left[\sum_{j=1}^{n_{piezo}} q_{(2i),j} D_j^*\right]\Psi_i(x_i) \tag{3.14}$$

Which can be compactly expressed as:

$$X_i(x_i) = \sum_{j=1}^{n_{piezo}} p_j(x_i) D_j^* \tag{3.15}$$

In this equation, p_j's is equal to;

$$p_j(x_i) = q_{(2i-1),j}\Upsilon_i(x_i) + q_{(2i),j}\Psi_i(x_i) \tag{3.16}$$

The electric displacements D_i^* can be determined as follows. Inserting Equations 3.14, 3.15, and 3.16 into Equation 2.60 and taking the integral of the resulting expression we can obtain a relation between the electric displacement D_i and the voltage $V_{x(i)}$ applied to i'th piezoelectric disc:

$$V_{x(i)} = -h_i[u_i(L_i) - u_i(0)] + \beta_i^S D_i L_i \tag{3.17}$$

The voltage applied to the piezoelectric materials is of the form, $V_x = V_n e^{j\omega t}$. Inserting 3.14 into 3.17, replacing V_x and D with $V_n e^{j\omega t}$ and $D^* e^{j\omega t}$ the following expression is be obtained:

$$V_{x(i)} = -h_i \sum_{j=1}^{n_{piezo}} (p_j(L_i) - p_j(0))D_j^* + \beta_i^S L_i D_i^* \tag{3.18}$$

Expression 3.18 can be written in matrix form:

$$[Z_V]\begin{bmatrix} D_1^* \\ D_2^* \\ \vdots \\ D_{n_{piezo}}^* \end{bmatrix} = \begin{bmatrix} (V_n)_1 \\ (V_n)_2 \\ \vdots \\ (V_n)_{n_{piezo}} \end{bmatrix} \tag{3.19}$$

From 3.18, each element of the matrix Z_V can be determined as:

$$(Z_V)_{ij} = h_i(p_j(L_i) - p(0)) + \beta_i^S L_i \delta_{ij} \tag{3.20}$$

δ_{ij} is the Kronecker's delta. Therefore the electric displacements are obtained as:

$$[D] = [Z_V]^{-1}[V_n] \tag{3.21}$$

A compact expression for the spatial displacements can therefore be written as follows:

$$[U]=[Z]^{-1}[H][Z_V]^{-1}[V_n] \tag{3.22}$$

3.3.2. Torsional Displacements

In the same manner, the mode shape vector U_T can be obtained in terms of the electric displacements D_z^*'s as:

$$U_T = [Z_T]^{-1}[G_T]\,[D_z] \tag{3.23}$$

Letting $[W]=[Z_T]^{-1}[G_T]$, a row of Equation (3.23) is:

$$N_i = \sum_{j=1}^{m_{piezo}} r_{ij} D_{zi}^* \tag{3.24}$$

Therefore, the spatial displacements $\theta_i(z_i)$ are given by:

$$\theta_i(z_i)=[\sum_{j=1}^{m_{piezo}} r_{(2i-1),j} D_{zj}^*]\Phi_i(z_i)+\left[\sum_{j=1}^{m_{piezo}} r_{(2i),j} D_{zj}^*\right]\Omega_i(z_i) \tag{3.25}$$

This can be compactly expressed as:

$$\theta_i(z_i)=\sum_{j=1}^{m_{piezo}} t_j(z_i) D_{zj}^* \tag{3.26}$$

Where t_j's is equal to

$$t_j(z_i)=r_{(2i-1),j}\Phi_i(z_i)+r_{(2i),j}\Omega_i(z_i) \tag{3.27}$$

Calculating Equation 2.59 over cross-sectional area of torsional piezoceramics, it is found;

$$E_T = -Z_{hi}\frac{\partial\theta}{\partial z} + \overline{\beta}_{11} D_{zi} \tag{3.28}$$

Inserting Equation 3.28 into Equation 2.60 and taking the integral we obtain:

$$V_{z(i)} = -Z_{hi}[\theta(L_i) - \theta_i(0)] + \overline{\beta}_{11} D_{zi} L_i \tag{3.29}$$

Where $\overline{\beta}_{11} = \beta^T_{11} + \frac{g_{15}}{s^D_{55}}$ and $Z_{hi} = Z_{qi} / S_i$. Equation 3.29 can be expressed in matrix form as:

$$[Z_Z]\begin{bmatrix} D^*_{z1} \\ D^*_{z2} \\ \vdots \\ D^*_{zm_{piezo}} \end{bmatrix} = \begin{bmatrix} (V_m)_1 \\ (V_m)_2 \\ \vdots \\ (V_m)_{m_{piezo}} \end{bmatrix} \tag{3.30}$$

Each element of the matrix Z_Z can be determined as follows:

$$(Z_Z)_{ij} = Z_{qi}(t_j(L_i) - t(0)) + \overline{\beta}^T_{11} L_i \delta_{ij} \tag{3.31}$$

Therefore the electric displacements are obtained as follows:

$$[D_z] = [Z_Z]^{-1}[V_m] \tag{3.32}$$

A compact expression for the spatial displacements can therefore be written as follows:

$$U_T = [Z_T]^{-1}[H][Z_Z]^{-1}[V_m] \tag{3.33}$$

Putting coefficients of the spatial solutions into Equations 3.1. and 3.2 will give the displacements of the two modes. Using spatial solutions at a special frequency, displacement and force or moment graphs of a transducer with N parts can be plotted.

3.4. Determining the Length of the Exponential Horn

In this thesis, the length of the backing material is selected as a variable. Determination of the length of horns is an adjustment problem. In the simplest case, a uniform bar can be considered. In this case, full wavelength of the horn, λ, will be equal to [9];

$$L = \lambda = \frac{c}{f} = \frac{2\pi c}{\omega} \tag{3.34}$$

In Equation 3.34, f, ω are resonance frequency and angular velocity. c is the speed of sound inside the material, but in different vibration modes it is different. For exponential horn, the speed of sound in exponential horn is given by;

$$c_l' = \frac{c_l}{\sqrt{1-(\frac{\beta c_l}{2\omega})^2}} \tag{3.35}$$

$$c_t' = \frac{c_t}{\sqrt{1-(\frac{\gamma c_t}{2\omega})^2}} \tag{3.36}$$

c_l' and c_t' are the speeds of sound in exponential horn in longitudinal and torsional vibration, respectively.

Length factors are parameters to choose, for a half wave horn length factor will be 0.5. L_{2L}, L_{2T} are the lengths of exponential horn and LF_L, LF_T are length factors in longitudinal and torsional vibration. For longitudinal vibration, the length of horn is defined as:

$$L_{2L} = LF_L \frac{c_l'}{f} \tag{3.35}$$

For torsional vibration:

$$L_{2T} = LF_T \frac{c_t^{'}}{f} \tag{3.36}$$

When the two modes are synchronized, L_{2L} and L_{2T} should be equal. For conical and catenoidal horns the same procedure can be followed; however they can be adjusted by trial and error more easily. The length of the horn is a very important parameter to amplify displacements.

4. TRANSDUCER DIMENSIONAL PARAMETERS

4.1. General

It is difficult to synchronize two systems at the same frequency only by changing the dimensions of the transducer; however it could be facilitated by using graphical methods. In order to do this, a single parameter is changed while fixing other parameters. The change is represented graphically. An arrangement can be done to synchronize the two modes by this method. Amplitude changes must be paid attention to get better performance.

4.2. Parametric Studies on Exponential Horn

Parametric studies are done by using exponential horn with every boundary condition. In another transducer, these graphs could be different but they will change in the same manner with the same boundary conditions.

There are other parameters to be changed apart from β and L_4. These are backing length L_1, diameter of piezoceramics d_L, d_T and piezoceramics thickness L_2, L_3. Parameters of transducer parts are given in Figure 4.1.

To apply plane wave approach, however the diameters of piezoceramics must be larger than their length. Single parameter cannot be used to adjust the two modes. Combination of the parameters should be used.

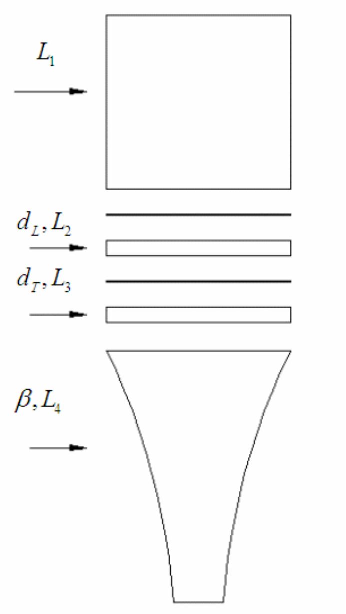


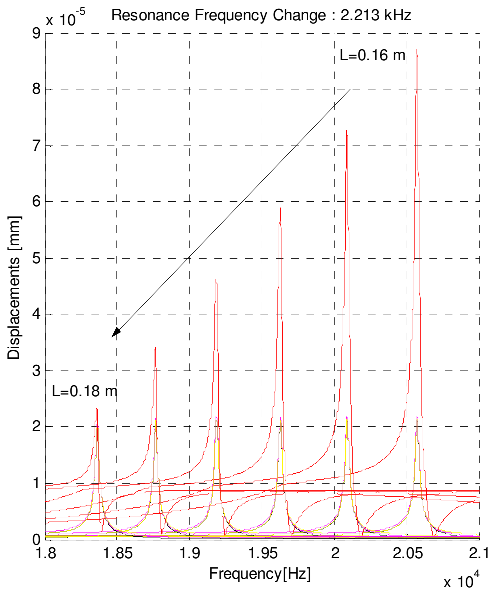
Figure 4.1. Dimensional parameters of transducers with exponential horn

The first three parametric studies are about the combination of the dimensional parameters of exponential horn. Horn length L_4 is changed according to dimensions given in Table 4.1 for constant end radius, d_2. In Figure 4.2 and 4.3 resulting resonance frequencies are shown. Natural frequencies of two systems are nearly the same.

Table 4.1. Increments of horn length

L_4 [m]	0.164	0.1642	0.1644	0.1646	0.1648	0.165

Direction of arrows shows increasing change of horn length. Similarly, in the other graphs, they will show increase of related parameter. As can be seen from the figures below, increasing length of exponential horn results in big decrease in resonance frequency. This may also cause a decrease in the amplitudes of displacement in both vibration modes. So, this parameter cannot be useful on its own, however when it is used with the other parameters it could be useful.



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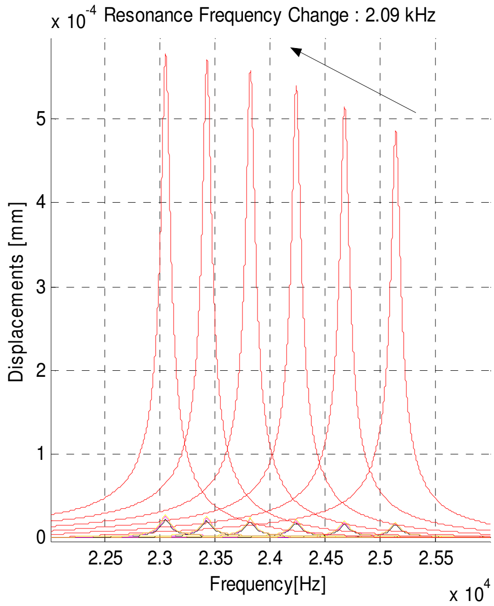

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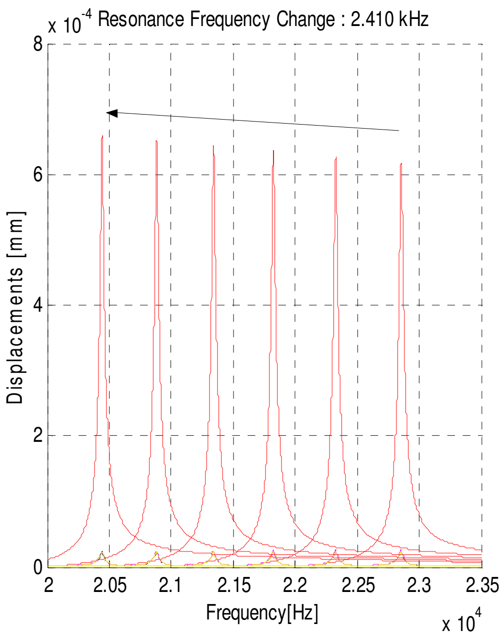

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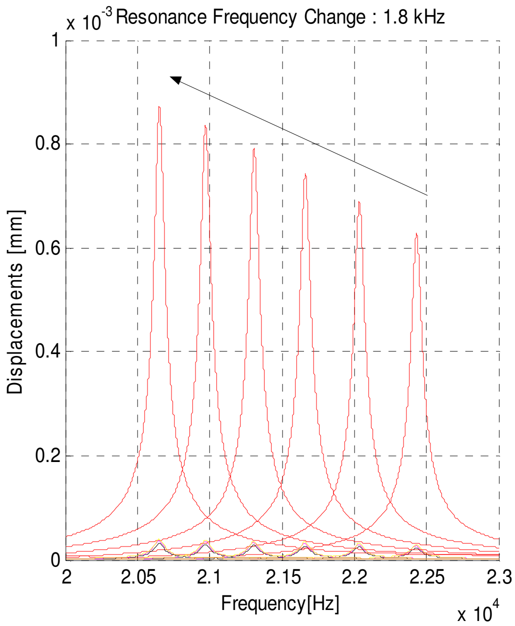

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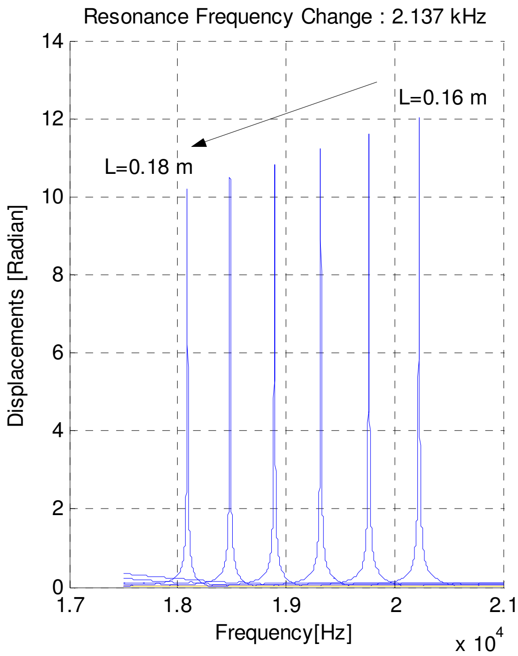
Figure 4.2. Change of longitudinal resonance frequencies for increasing L and constant d_2



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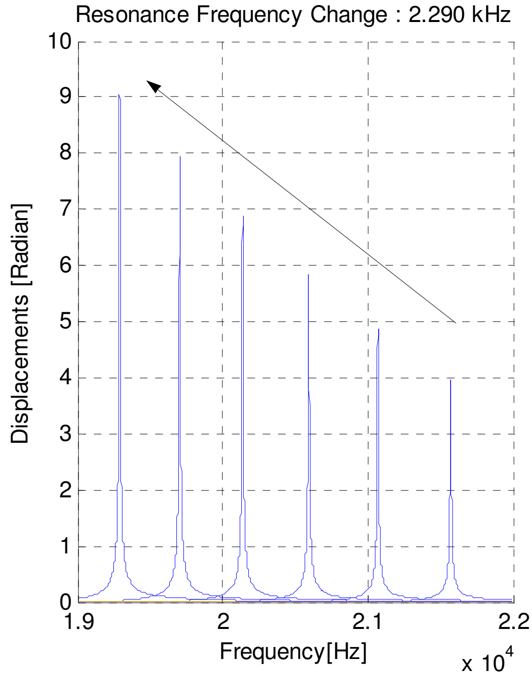

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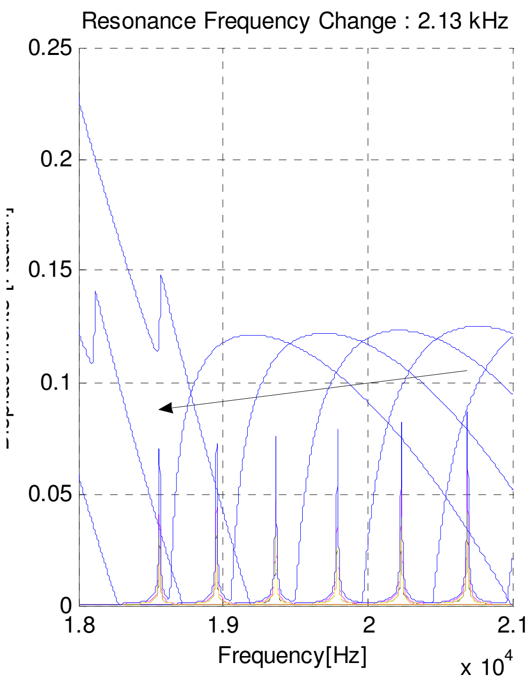

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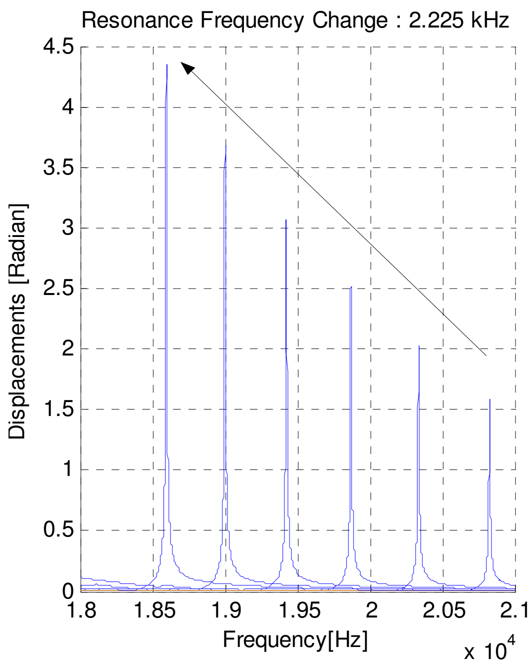

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Figure 4.3. Change of torsional resonance frequencies for increasing L and constant d_2

Another important parametric study is the relation between d_2 and β. When L and initial radius are constant and β is increased slightly, relation between end radius d_2 and β will become like in Table 4.2.

Table 4.2. Increments of end diameter and beta

d_2 [m]	0.00204	0.00188	0.00174	0.00160	0.00148	0.00137
β	40	41	42	43	44	45

Similarly as seen in Figure 4.4 and 4.5, torsional mode frequency increases more than longitudinal, even if the same changes are applied. The difference is nearly half kHz which is really remarkable. But there are large changes at the end of horn, since torsional displacements depend on the circumference of end diameter which should be kept as small as possible to obtain higher displacements. As can be seen from figures, for free-free boundary conditions there is an intersection point.

In Table 4.3, dimensions of d_2 and L_4 are given for a constant β value of 40. As seen from Figures 4.6 and 4.7, longitudinal resonance frequency decreases more than torsional frequency. However, change in resonance frequency is less with respect to other parameters. With this parameter big differences cannot be decreased with small increments. The best possible synchronization point seems to exist for free-free boundary condition.

Table 4.3. Increments of horn length and end diameter

L_4	0.164	0.1642	0.1644	0.1646	0.1648	0.165
d_2 [m]	0.00261	0.00260	0.00259	0.00259	0.00257	0.00257

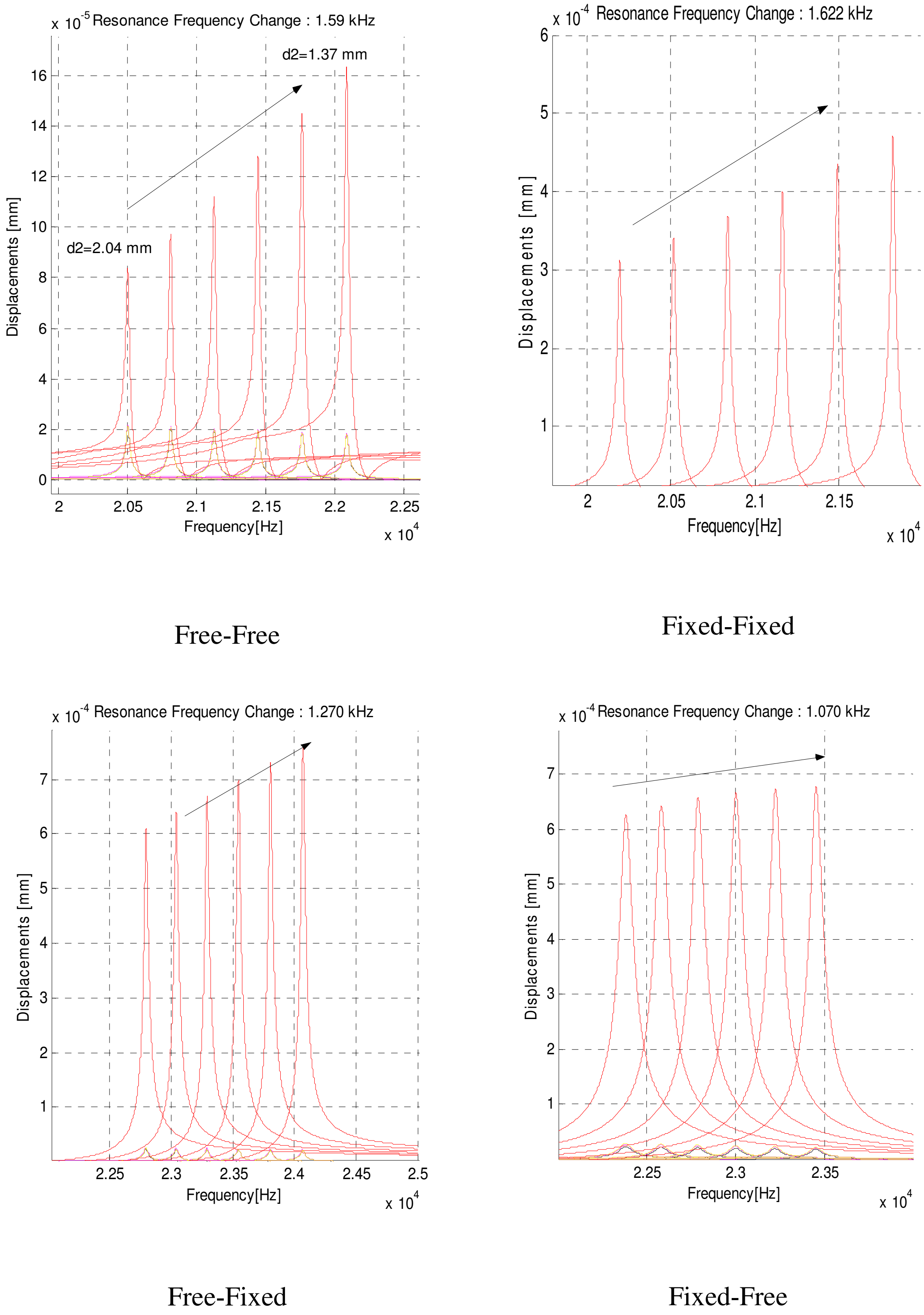


Figure 4.4. Change of longitudinal resonance frequencies for increasing β and constant L

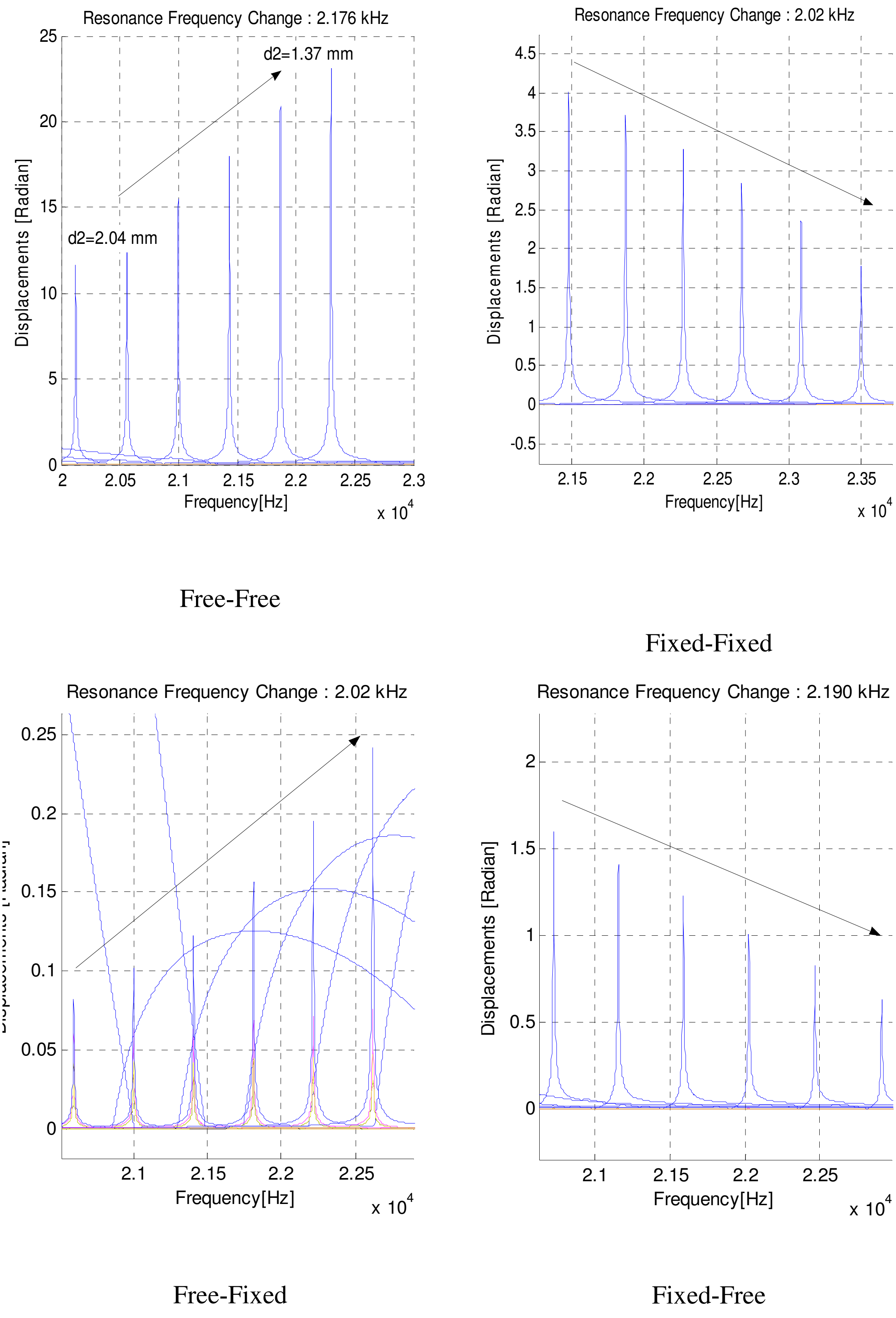
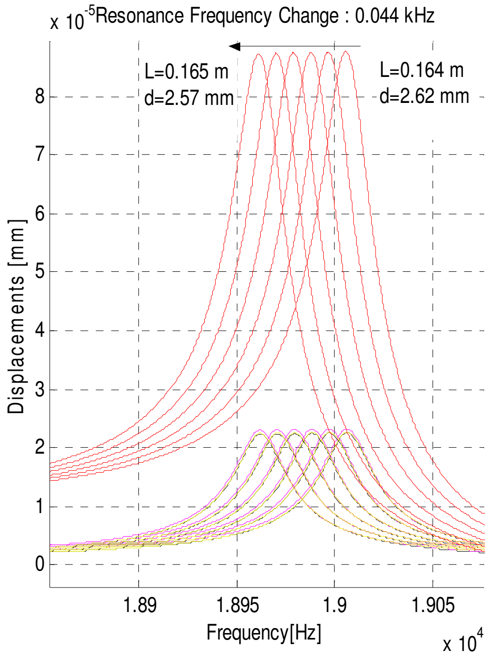


Figure 4.5. Change of torsional resonance frequencies for increasing β and constant L



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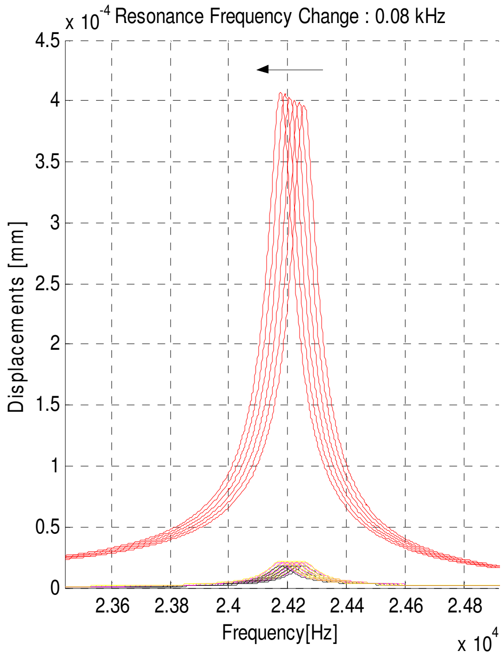

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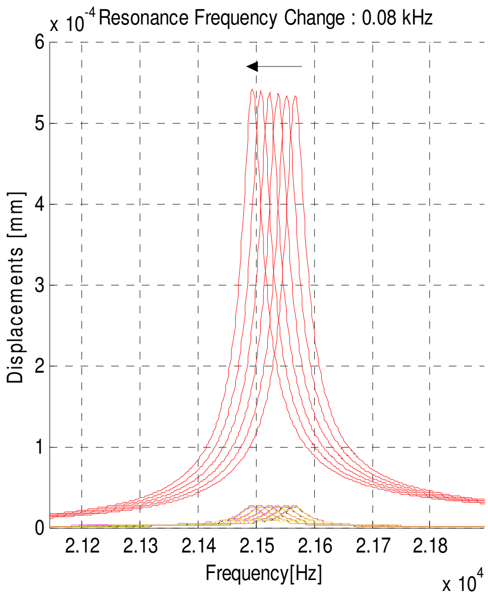

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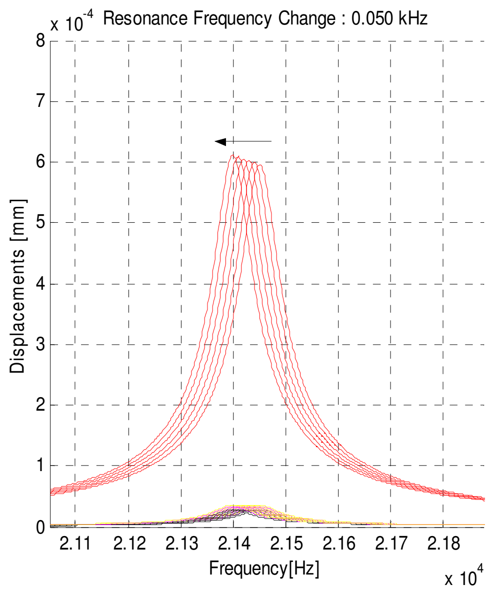

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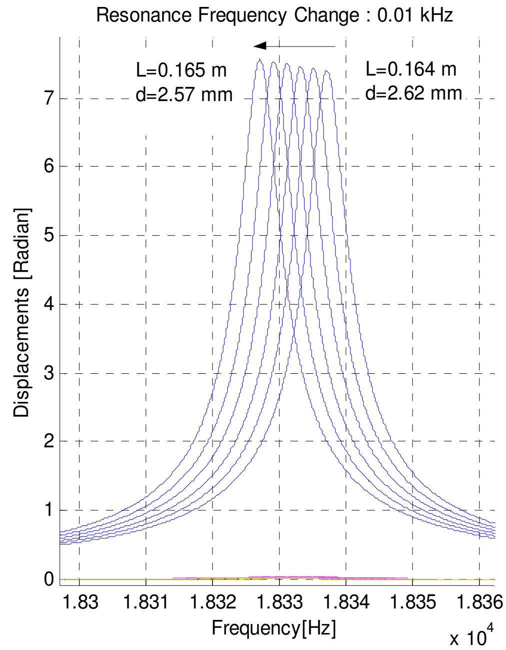
Figure 4.6. Change of longitudinal resonance frequencies for increasing L and constant β



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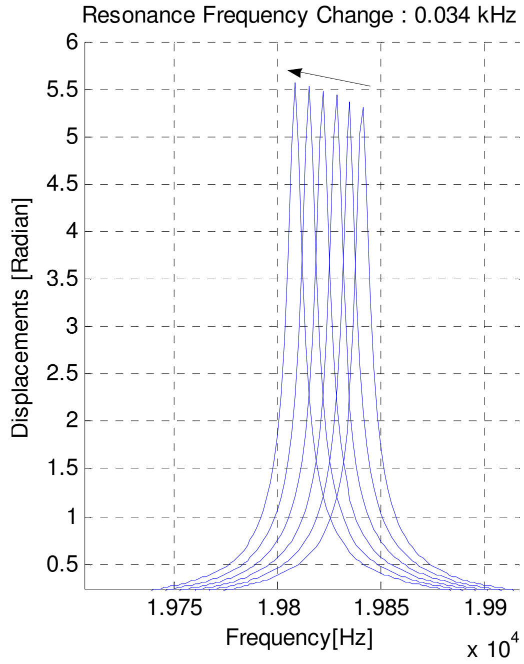

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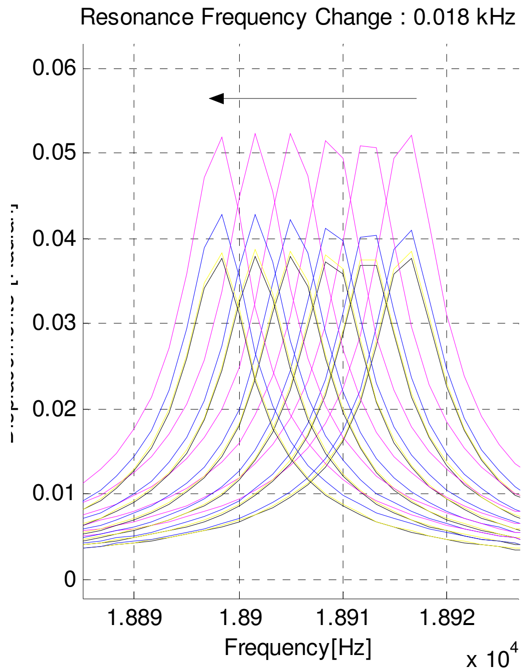

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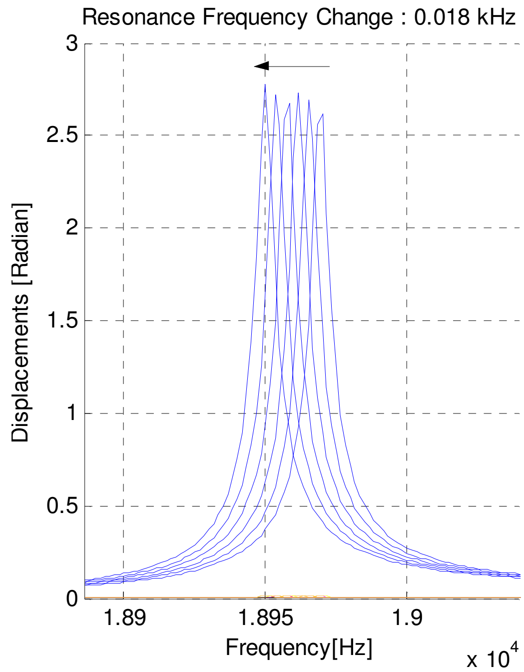

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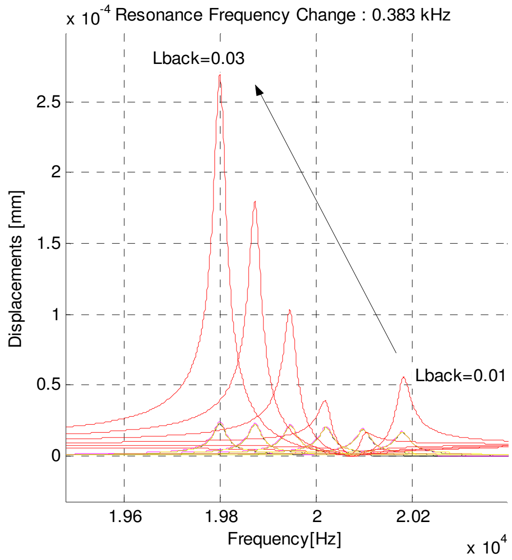
Figure 4.7. Change of torsional resonance frequencies for increasing L and constant β

Backing length may also be selected as a parameter. In Table 4.4 increments of backing length L_1 are given. Resulting changes in resonance frequencies are given in Figures 4.8 and 4.9.

Table 4.4. Increments of backing length

L_1 [m]	0.01	0.014	0.018	0.022	0.026	0.03



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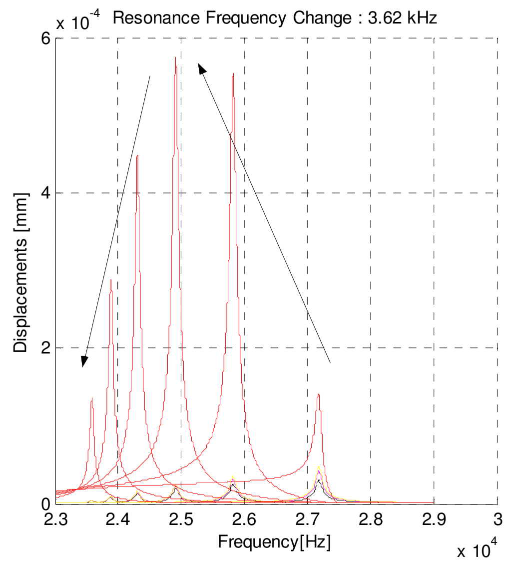

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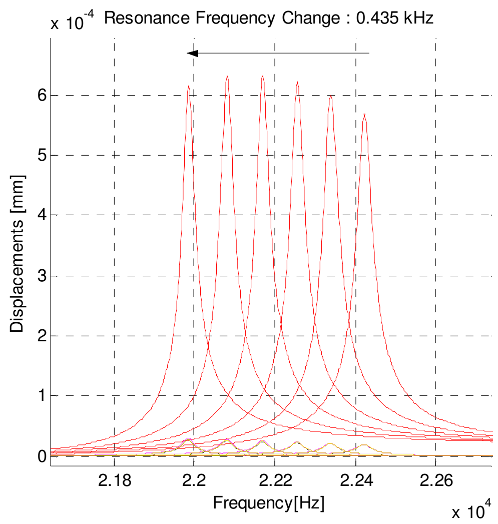

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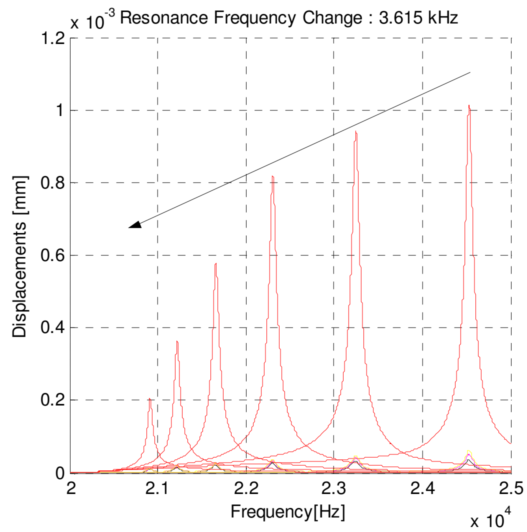

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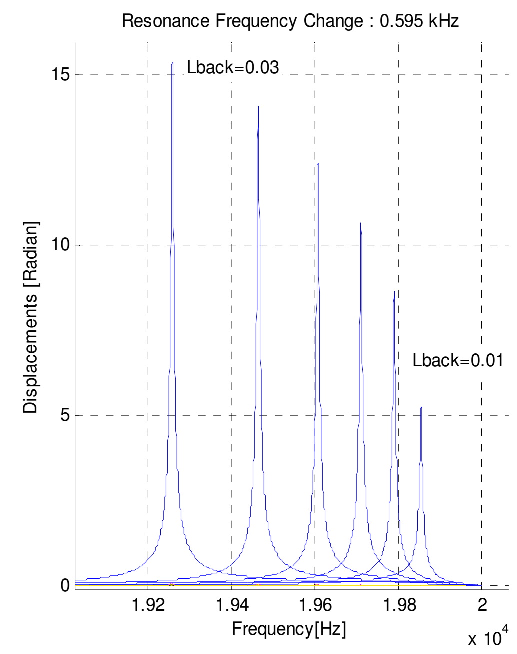
Figure 4.8. Change of longitudinal resonance frequencies for backing length



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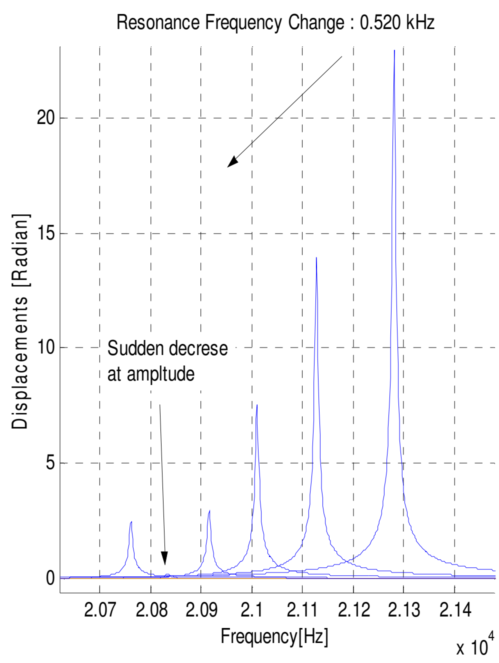

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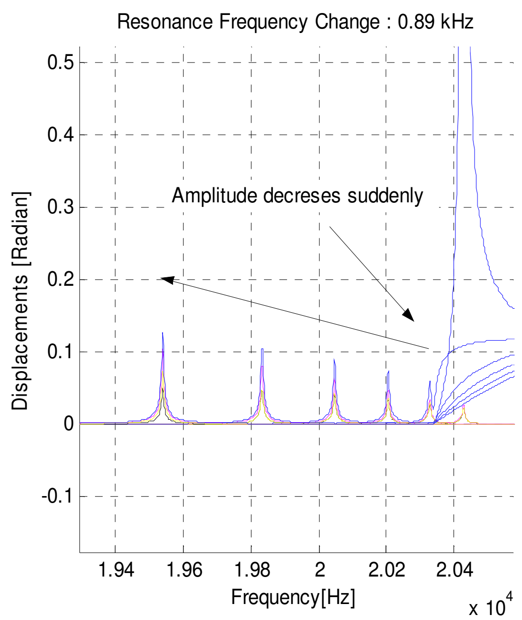

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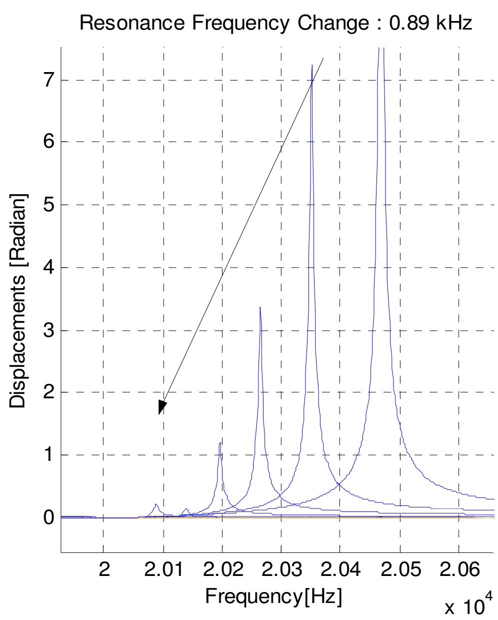

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Figure 4.9. Change of torsional resonance frequencies for backing length

Another parameter to be varied is the diameter of piezoceramics. In Table 4.5 increments of piezoceramic diameters are given. In Figures 4.10 and 4.11 the resulting resonance frequencies are shown. However in real, these frequencies could be of little difference than expected values due to some limitations.

Table 4.5. Increments of piezo diameters of two modes

d_L, d_T [mm]	3,5	3,6	3,7	3,8	3,9	4,0

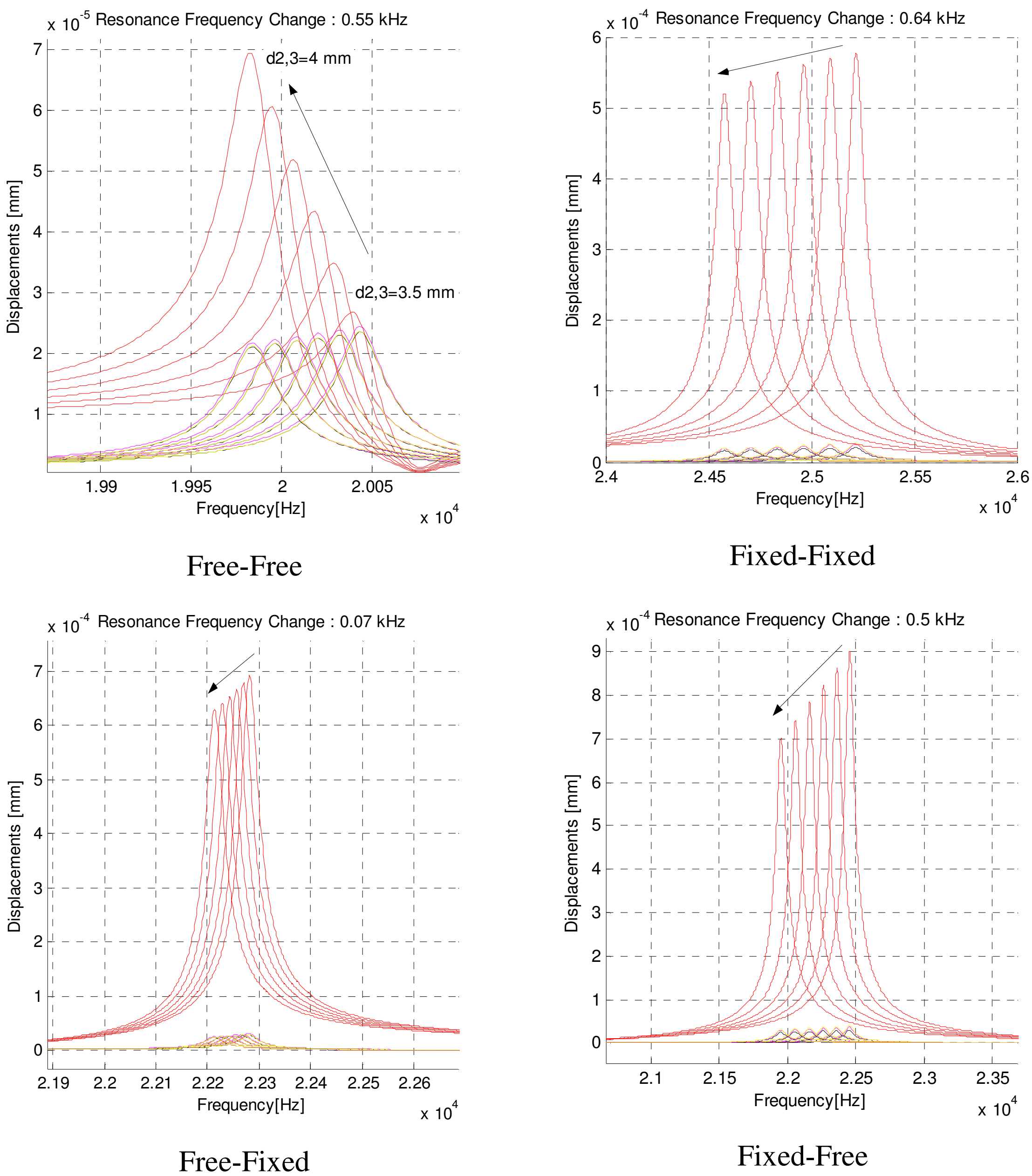


Figure 4.10. Change of longitudinal resonance frequencies for piezo diameter

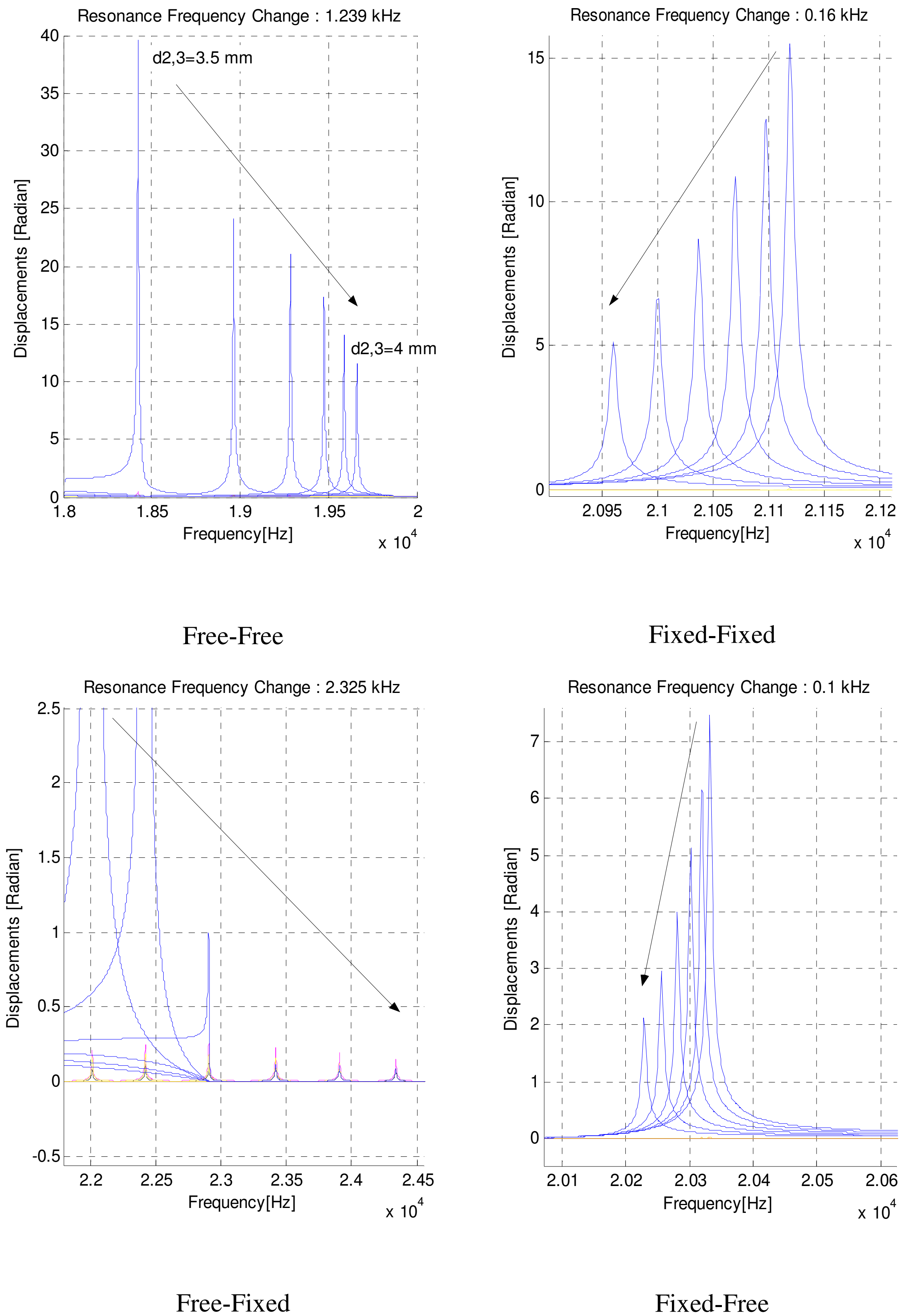


Figure 4.11. Change of torsional resonance frequencies for piezo diameter

Piezoceramic thickness is another parameter. In Figures 4.12 and 4.13, changes of resonance frequencies are shown. In Table 4.6, changes of piezoceramic thickness are given.

Table 4.6. Increments of piezo lengths of two modes

L_2, L_3 [mm]	4	4,2	4,4	4,6	4,8	5

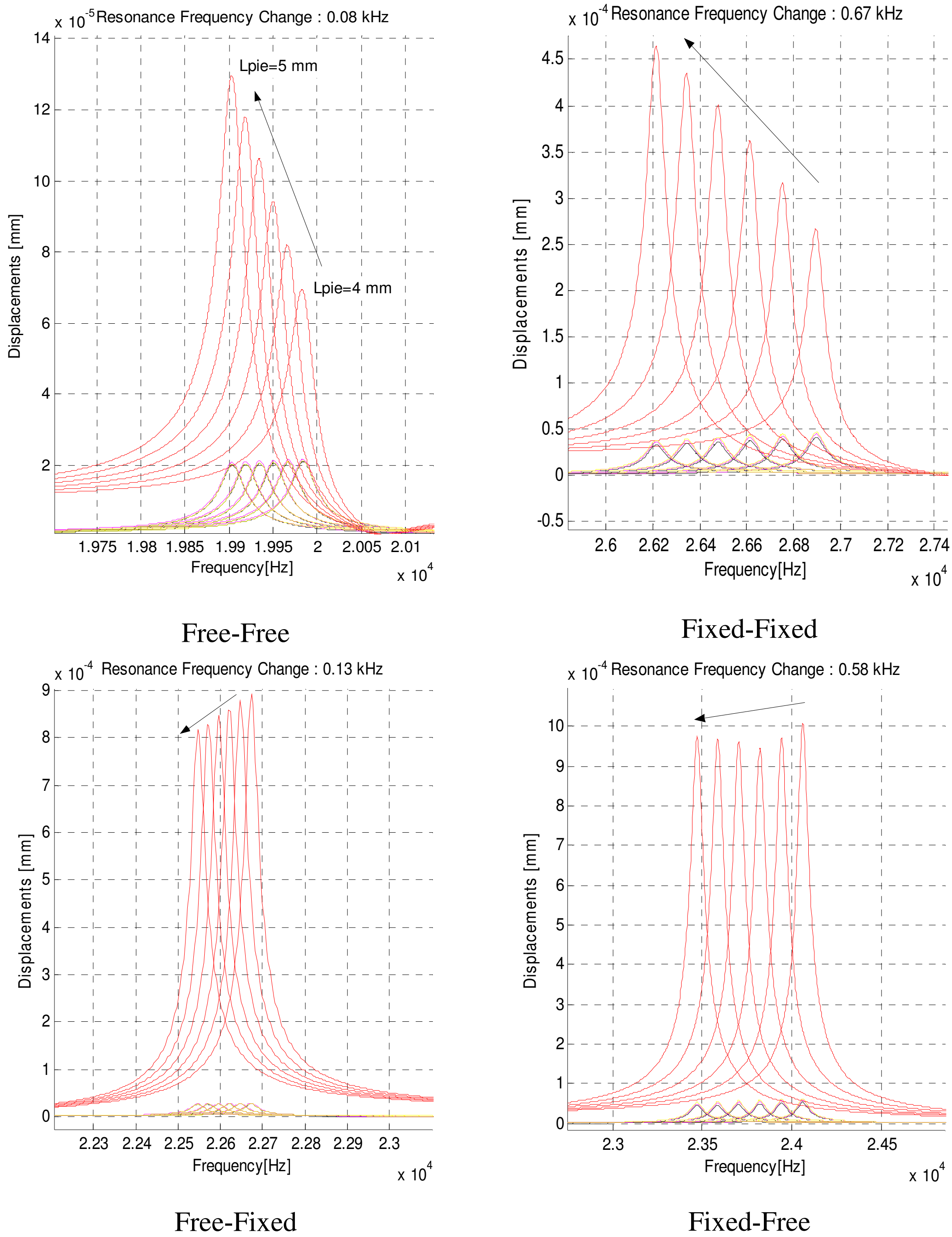
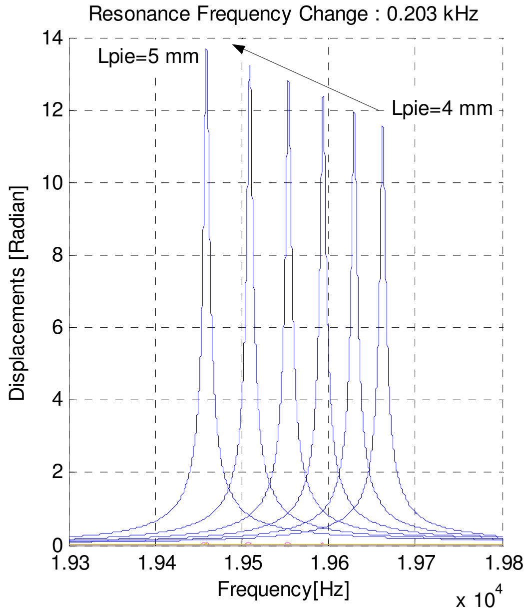


Figure 4.12. Change of longitudinal resonance frequencies for piezo thickness



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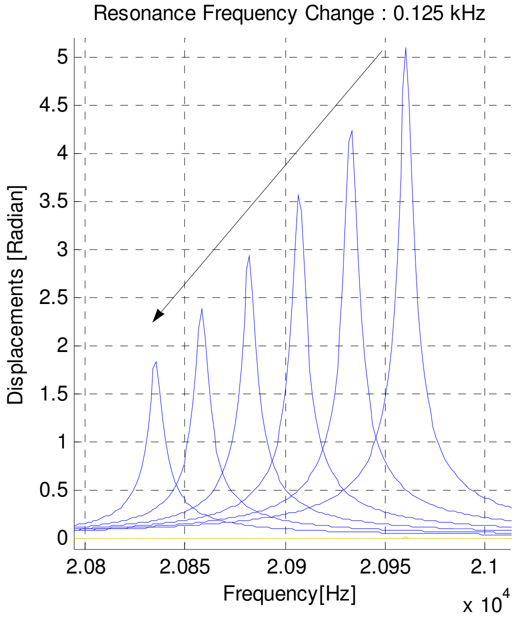

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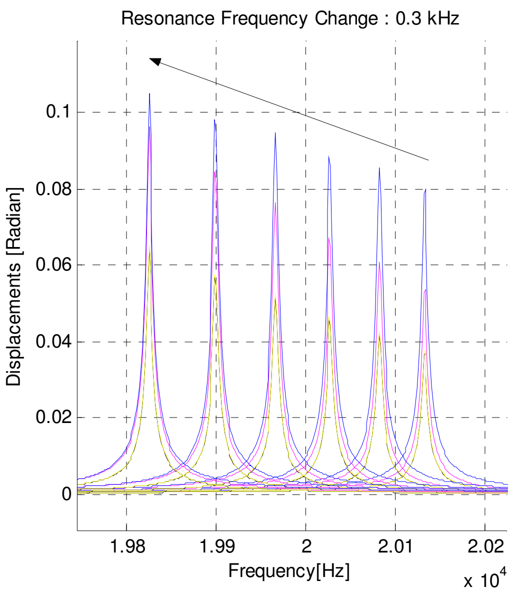

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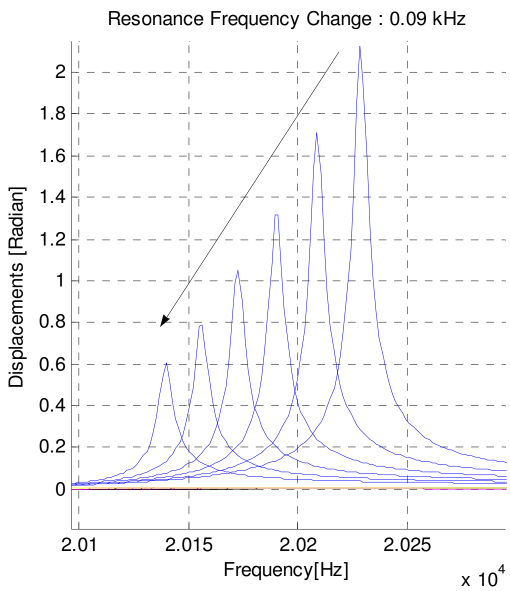

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Figure 4.13. Change of torsional resonance frequencies for piezo thickness

4.3. Material Thickness Effects on Modes of Transducer

Resonance frequencies of the two modes of the transducer may also change with the thickness of the material to be welded. In Figure 4.14 and 4.15, with increment of 2 mm, resonance frequencies are plotted for material length of 1 cm. Longitudinal resonance frequency does not change with material thickness, however torsional resonance frequency changes remarkably.

In this study, polythene (PE) is used as the material to be welded. For many materials torsional stiffness coefficient G is not given in written sources. Instead of G, using Young's module and Poisson ratio η, torsional stiffness can be stated as;

$$G = \frac{Y}{2(1+\eta)} \tag{4.1}$$

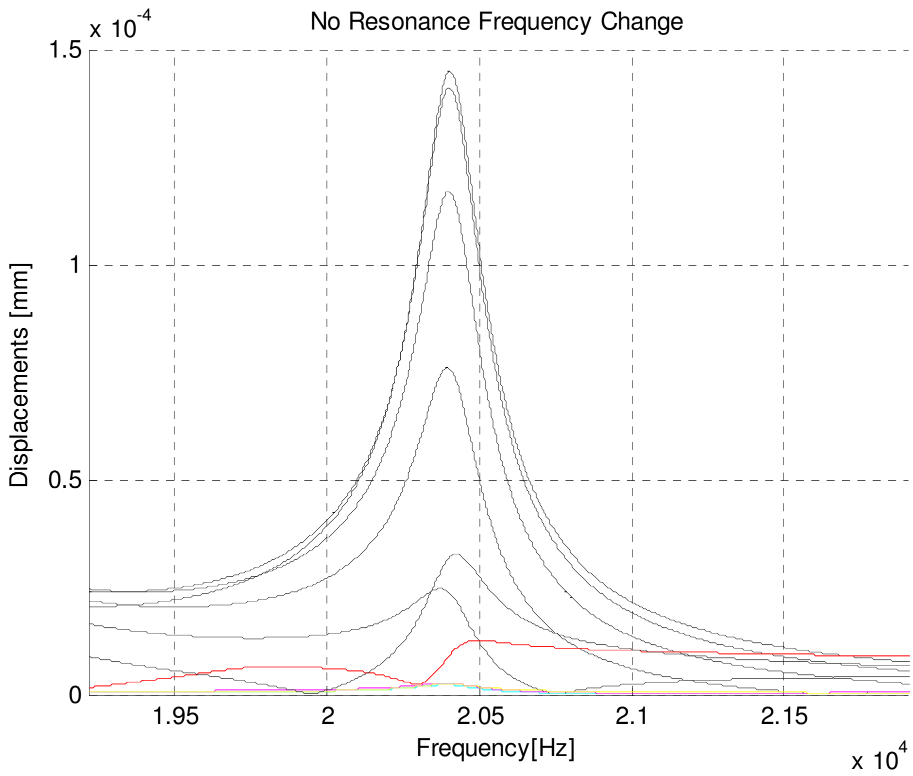

Figure 4.14. Change of longitudinal resonance frequencies for material thickness

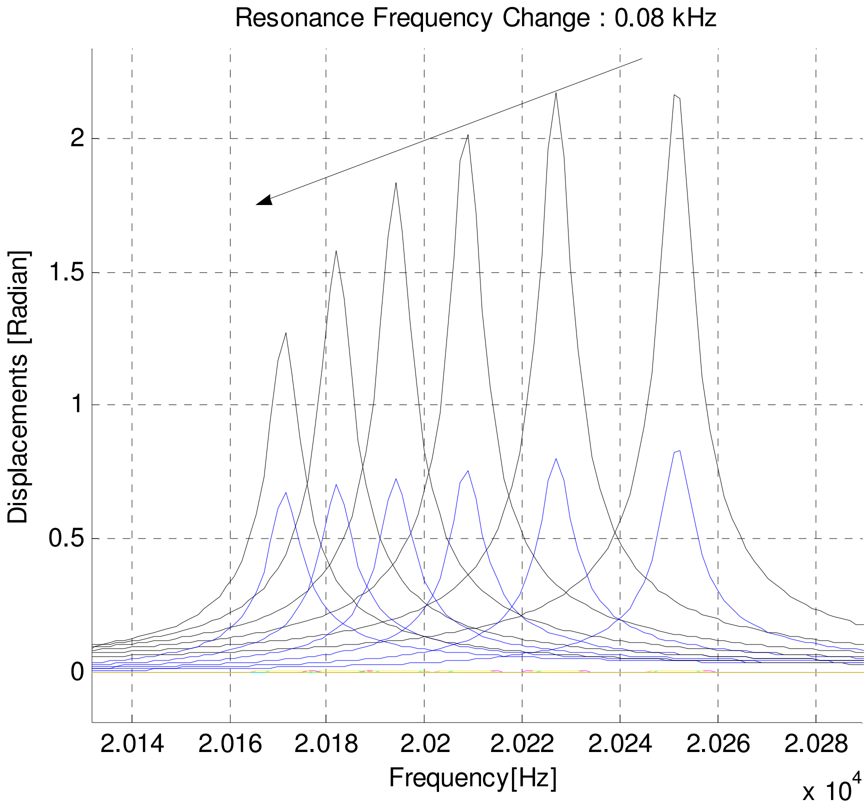

Figure 4.15. Change of torsional resonance frequencies for material thickness

4.4. Numerical Study

To synchronize two modes, numerical studies are carried out. If the resonance frequency of the transducer is given, the resonant length of a certain part of the transducer can be computed. The dimensions of the horn of the transducer can be computed as explained in section 3.4. However the cases studied are more complex than single mode.

In transducer design, dimensions and materials should be chosen appropriately. Difference in resonance frequencies should not be very large at the beginning of numerical study.

Not only dimensions but also material parameters change the resonance frequency. Material parameters are piezoceramic constants, Young modulus or torsional stiffness and density. Using the following procedure, the two modes can be synchronized [1].

1- Resonance frequency of the transducer must be determined. The resonance frequency is equal to that of the longitudinal or torsional vibration of the transducer.

2- Materials of backing and exponential horn used must be determined. Low mechanical loss and large elastic modulus must be taken into account when choosing materials. Light front material and heavy backing material are more preferable in order to increase the ratio of the displacement of the front end of the exponential horn.

3- The type of longitudinal and torsional piezoceramics must be chosen, including the number of the piezoceramics, and the material, the geometrical shape and dimensions. And the length and radius of backing part must be assigned. To simplicity, all radiuses can be chosen the same.

4- If appropriate piezoceramics are chosen, they will have close resonance frequencies. The important point in synchronization is that torsional mode is more sensitive to radial changes, and longitudinal mode is more sensitive to longitudinal changes.

5- For exponential horn, choosing appropriate length factor, length of the horn can be adjusted the same in both modes, but their resonance frequencies will be different. When the lengths of exponential horn are the same in the two modes, then γ will be twice radius decay coefficient β. A certain end radius and length factor of exponential horn is chosen. According to these values, resonance frequencies are computed.

6- Dependent upon above step, it is seen that there is only one end diameter and length factors to synchronize two modes for certain length of backing part. For example if the length of backing material is changed, then other parameters must be studied again.

4.5. An Example Transducer

As a simple example, a transducer which has four piezoceramics, two longitudinally and two tangentially polarized piezoceramics is designed by using exponential horn. As seen in Figure 4.16, piezoceramics are clamped between two metal parts by a bolt, which goes through piezoceramics without touching an electrode. Contact between the bolt and the electrodes can cause short circuit problem.

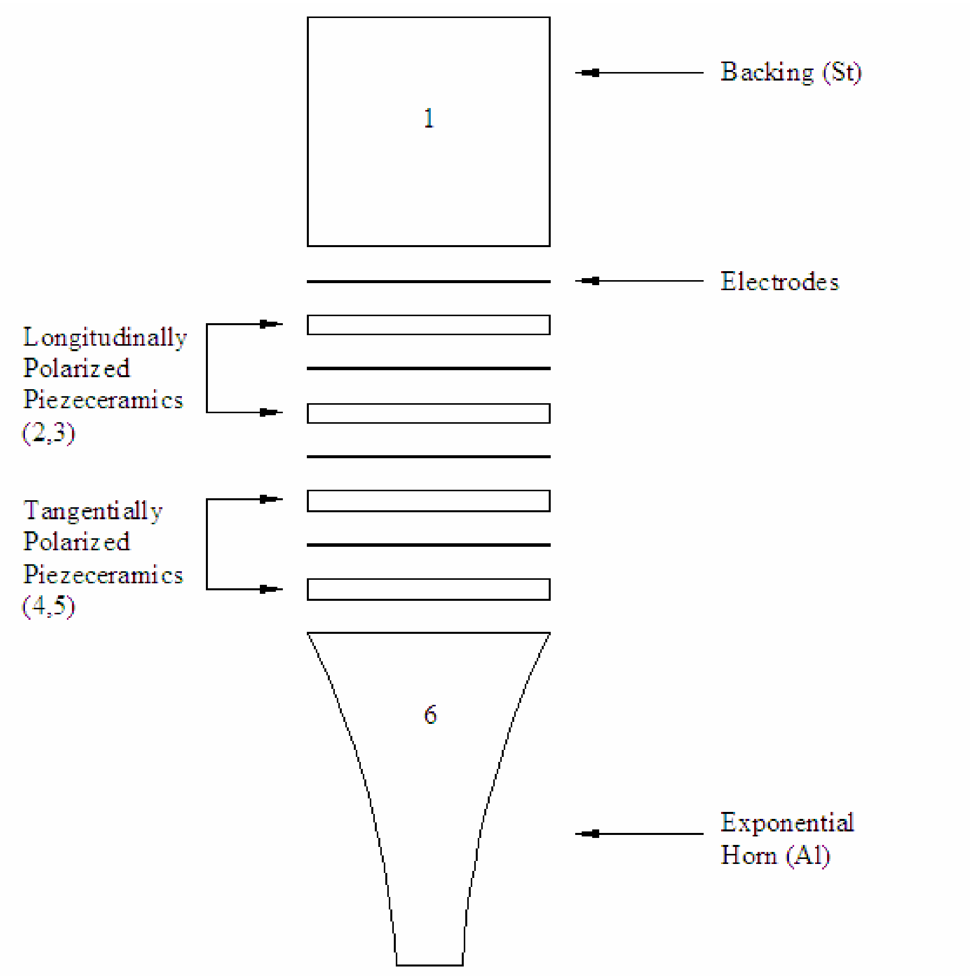

Figure 4.16. Schematic view of a compound mode transducer

Because of good acoustic properties, steel and aluminium are used as backing and horn materials respectively, see Table 4.7. Its geometrical dimensions are given in table 4.3. The piezoceramic is the equivalent of PZT 4. Material parameters used are density and compliances $\rho_0 = 7.5x10^3 \ kg/m^3 \ s_{33}^E = 15.5x10^{-12} \ m^2/N \ s_{55}^E = 39x10^{-12} \ m^2/N$.

Table 4.7. Properties of the example transducer

	Material	d_i [cm]	d_f [cm]	d_{inner} [cm]
Backing	St	5	5	2
L.P.P.	PZT 4	5	5	2
T.P.P.	Equivalent of PZT 4	5	5	2
Horn	Al	5	0.2	-

Number of Longitudinally Polarized Piezoceramics	2
Number of Tangentially Polarized Piezoceramics	2
Lengths of Piezoceramics [mm]	5
Voltage [V]	2000

When designing a compound mode transducer, it is assumed that the two modes do not affect each other. As the longitudinally polarized piezoelectric ceramic elements are excited by an external electric field while the tangentially polarized piezoelectric ceramic elements are open circuited, the transducer will vibrate only in longitudinal mode. In this case, the tangentially polarized piezoelectric ceramic elements can be regarded as a pure mechanical vibrating body in longitudinal vibration. For longitudinal and torsional vibrations, free-free boundary conditions are given below respectively.

$$T_1(0) = 0 \tag{4.2}$$

$$u_1(L_1) = u_2(0) \tag{4.3}$$

$$T_1(L_1) = T_2(0) \tag{4.4}$$

$$u_2(L_2) = u_3(0) \tag{4.5}$$

$$T_2(L_2) = T_3(0) \tag{4.6}$$

$$u_3(L_3) = u_6(0) \tag{4.7}$$

$$T_3(L_3) = T_6(0) \tag{4.8}$$

$$T_6(L_6) = 0 \tag{4.9}$$

For simplicity, piezoceramic and metal parts are assumed to have same diameters, so stress can be used instead of force. And boundary conditions for torsional vibration;

$$M_1(0) = 0 \tag{4.10}$$

$$\phi_1(L_1) = \phi_4(0) \tag{4.11}$$

$$M_1(L_1) = M_4(0) \tag{4.12}$$

$$\phi_4(L_4) = \phi_5(0) \tag{4.13}$$

$$M_4(L_4) = M_5(0) \tag{4.14}$$

$$\phi_5(L_5) = \phi_6(0) \tag{4.15}$$

$$M_5(L_5) = M_6(0) \tag{4.16}$$

$$M_6(L_6) = 0 \tag{4.17}$$

As mentioned in section 4.4, dimensions of the compound mode transducer are arranged with numeric study. Their lengths are designed to vibrate in the same frequency with a small error. In Figure 4.17 and 4.18, displacements are plotted versus frequency. Longitudinal vibration displacements are in millimeter, however torsional vibration displacements are in radians.

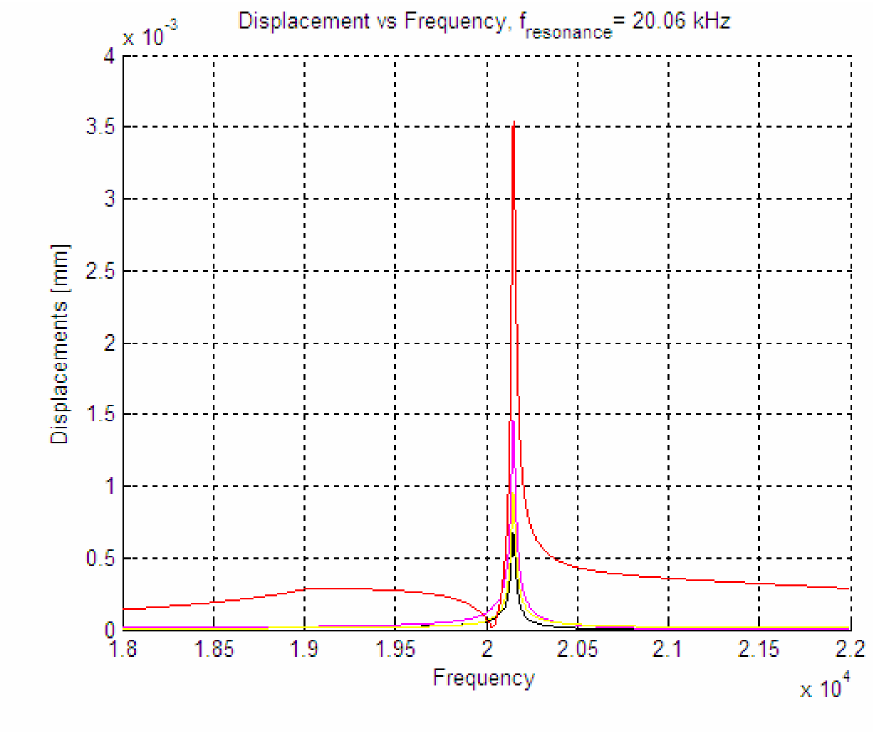

Figure 4.17. Displacements vs. frequency of longitudinal vibration

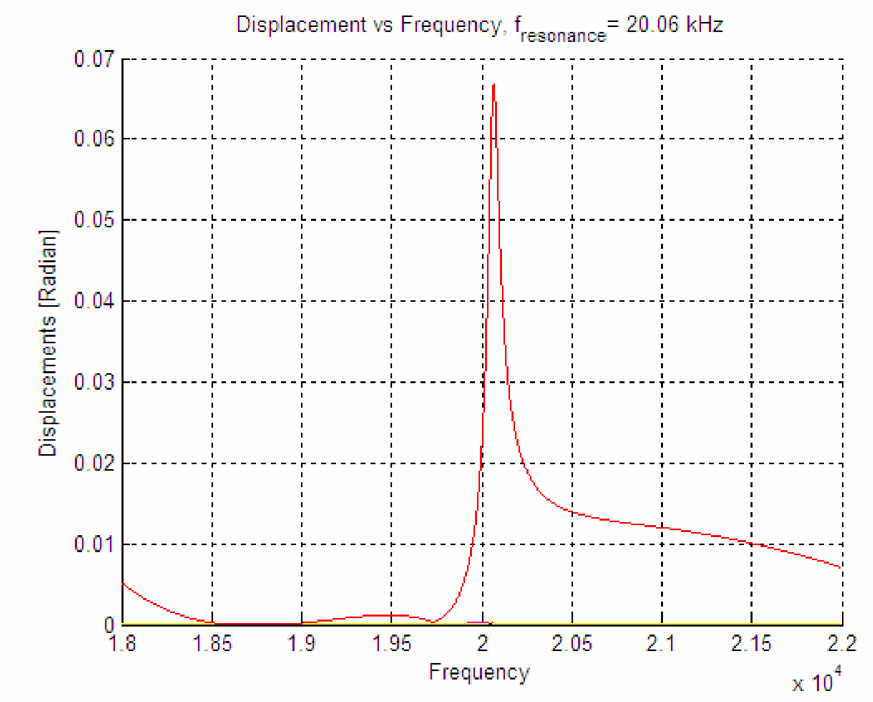

Figure 4.18. Displacements vs. frequency of torsional vibration

5. MULTIPLE MODELLING AND EXAMPLES

A transducer which has multiple longitudinal and torsional piezoceramics does not differ from a transducer that has two longitudinal and two torsional piezoceramics as those in the previous chapters. In Figure 6.1, a transducer which has multiple piezoceramics is shown. Longitudinal piezoceramics should always be at the back and torsional piezoceramics should always be at front. Theoretically it is not important, however in practice, since there is a slipping risk between the two surfaces of piezoceramics, displacements may get smaller. To avoid this risk an adhesive substance must be used between all surfaces.

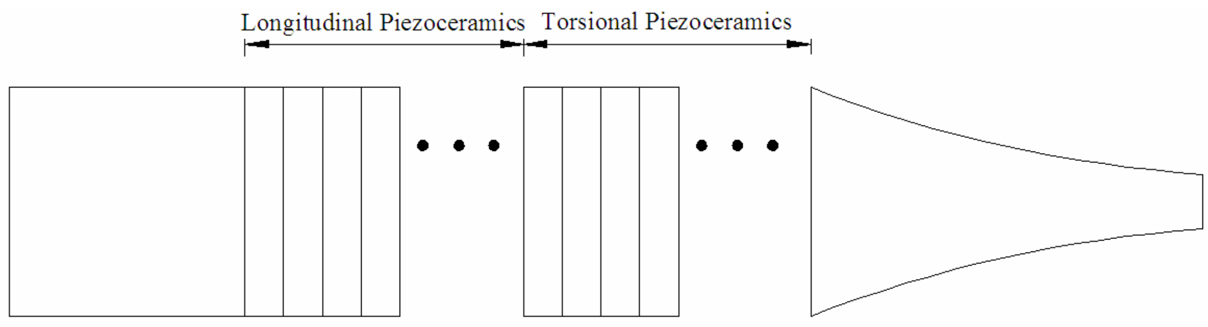

Figure 5.1. Displacements vs. frequency in torsional vibration

Using the procedures given before, a transducer can be designed step by step. Again a transducer which has two longitudinal and two torsional piezoceramics are used. Duralumin is used for backing and exponential horn. Longitudinal piezoceramics, which are made by THORNTON-INPEC given in the article of Arnold&Muhlen [20,21], are used for thickness mode, and Morgan Piezo custom made PZT 407 piezoceramics are used for torsional mode. In Table 5.1, dimensions of transducer are given:

Table 5.1. Dimensions of the example transducer

	Material	d_i [cm]	d_f [cm]	d_{inner} [cm]
Backing	St	5	5	2
L.P.P.	PZT 4	5	5	2
T.P.P.	Equivalent of PZT 4	5	5	2
Horn	Al	5	-	-

Notice that initially some dimensions, which are end diameter of horn, length of piezoceramics and backing length, are not given. These values will become definite after design. For these dimensions, initial values are assigned. These dimensions will be changed as required. In Table 5.2, initial dimensions are given.

Table 5.2. Initial dimensions of example transducer

End Diameter of Exponential Horn [cm]	0.18
Lengths of Longitudinal Piezoceramics [mm]	4
Lengths of Torsional Piezoceramics [mm]	4
Backing Length[m]	0.03

With this assumption, obtained resonance frequencies are given below.

Table 5.3. First values of resonance frequencies

Longitudinal Resonance Frequency [kHz]	22.15
Torsional Resonance Frequency [kHz]	20.25

At first, longitudinal resonance frequency is more than torsional resonance frequency. To decrease the difference, parametric studies given in section 4.2 can be used. Difference is nearly 2 kHz, so a good parameter such as the backing length is needed. Decreasing backing length 10 cm gives closer resonance frequencies.

Table 5.4. Second values of resonance frequencies

Longitudinal Resonance Frequency [kHz]	20.44
Torsional Resonance Frequency [kHz]	19.86

Difference is now less then 0.6 kHz. As told before, combined parametric study must be carried out. Decreasing end diameter of the horn 2 mm and increasing piezoceramic thickness of longitudinal mode 4.2 mm, then the transducer will be synchronized. In Table 5.5, final resonance frequencies are given.

Table 5.5. Final values of resonance frequencies

Longitudinal Resonance Frequency [kHz]	19.692
Torsional Resonance Frequency [kHz]	19.69

And final values of design dimensions are given in Table 5.6.

Table 5.6. Final dimensions of the transducer

End Diameter of Exponential Horn [cm]	0.2
Lengths of Longitudinal Piezoceramics [mm]	8.2
Lengths of Torsional Piezoceramics [mm]	4
Backing Length[m]	0.02

It is not the best performance but fast approach. In Figure 5.2 and 5.3 resonance frequency response graphs are given and also from Figure 5.4 to Figure 5.9, nodes & antinodes and admittance change are given respectively.

At resonance real part of impedance of the system Z_s goes to infinity. For each mode impedance is found in following way;

$$Z_s = \frac{V_{\max}}{I_1 + I_2 + ... + I_n} \tag{5.1}$$

For longitudinal vibration $I_i = j\omega D_i$ and D_i is obtained from Equation 3.21. Electrical admittance is equal to;

$$Y_s = 1/Z_s \tag{5.2}$$

Similar procedure can be followed for torsional mode. In Figure 5.8 and 5.9, Electrical admittance is plotted versus frequency. At resonance, admittance goes to infinity.

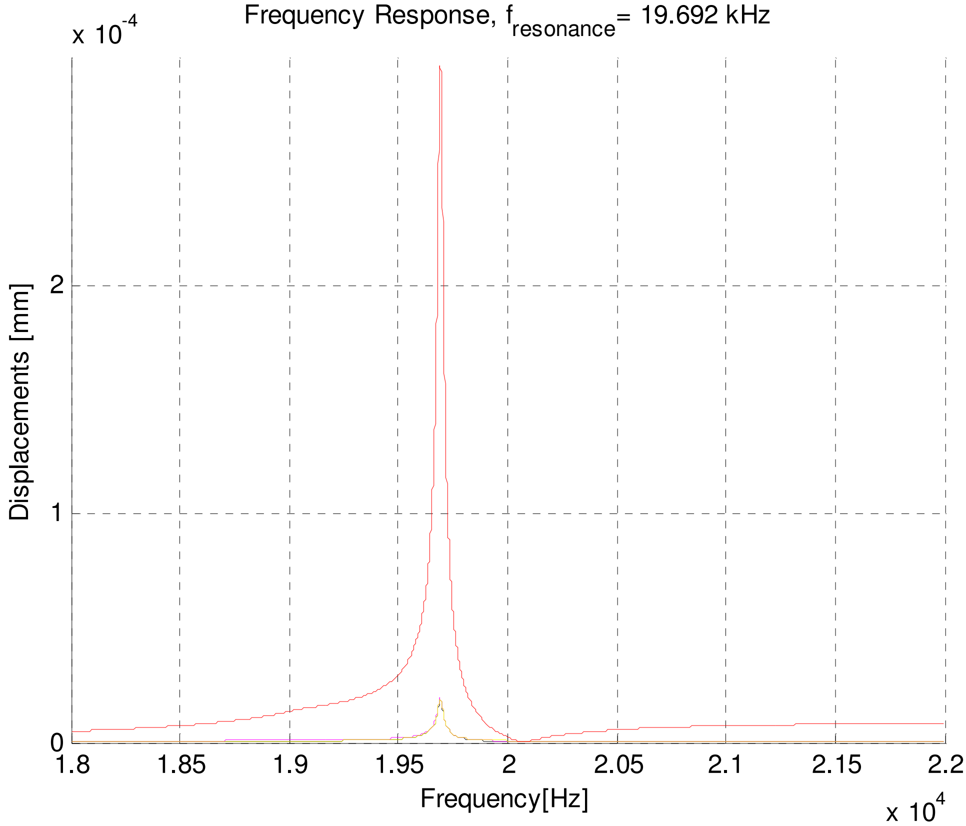

Figure 5.2. Resonance frequency of the longitudinal mode

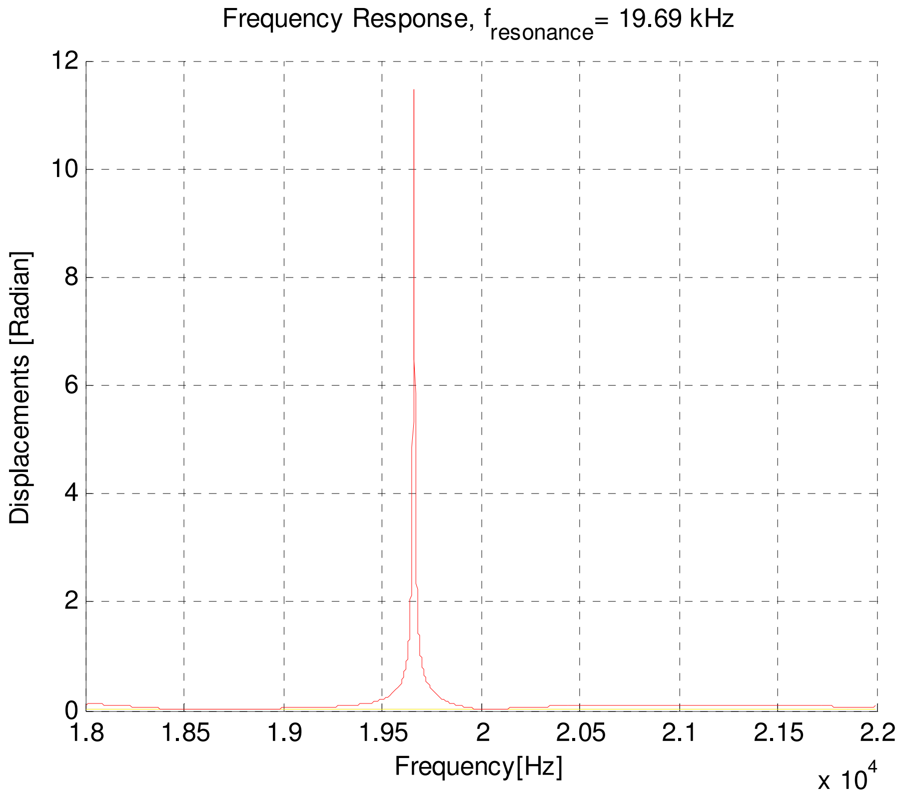

Figure 5.3. Resonance frequency of the torsional mode

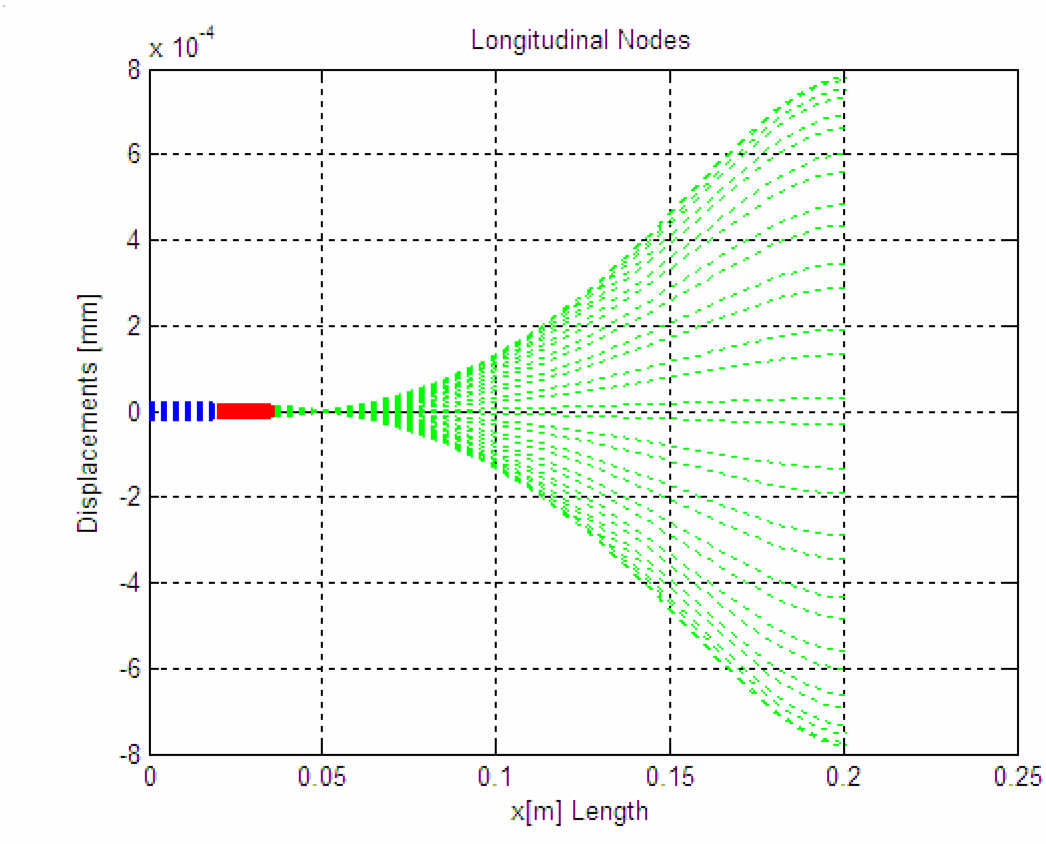

Figure 5.4. Nodes of the longitudinal mode

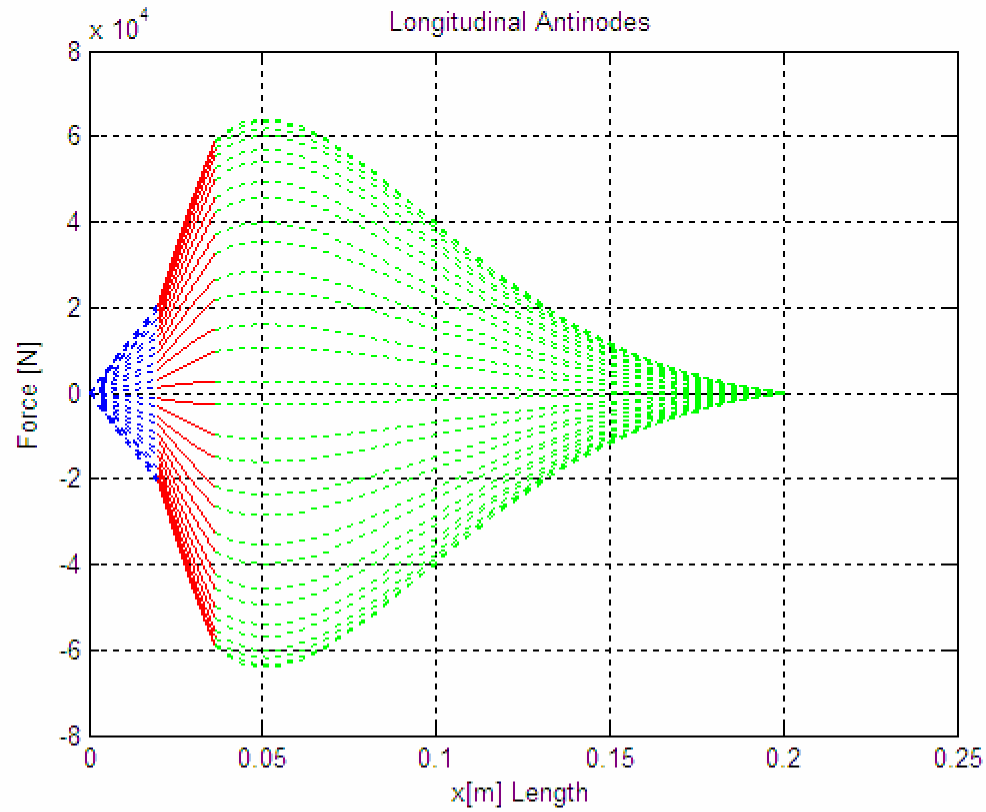

Figure 5.5. Antinodes of the longitudinal mode

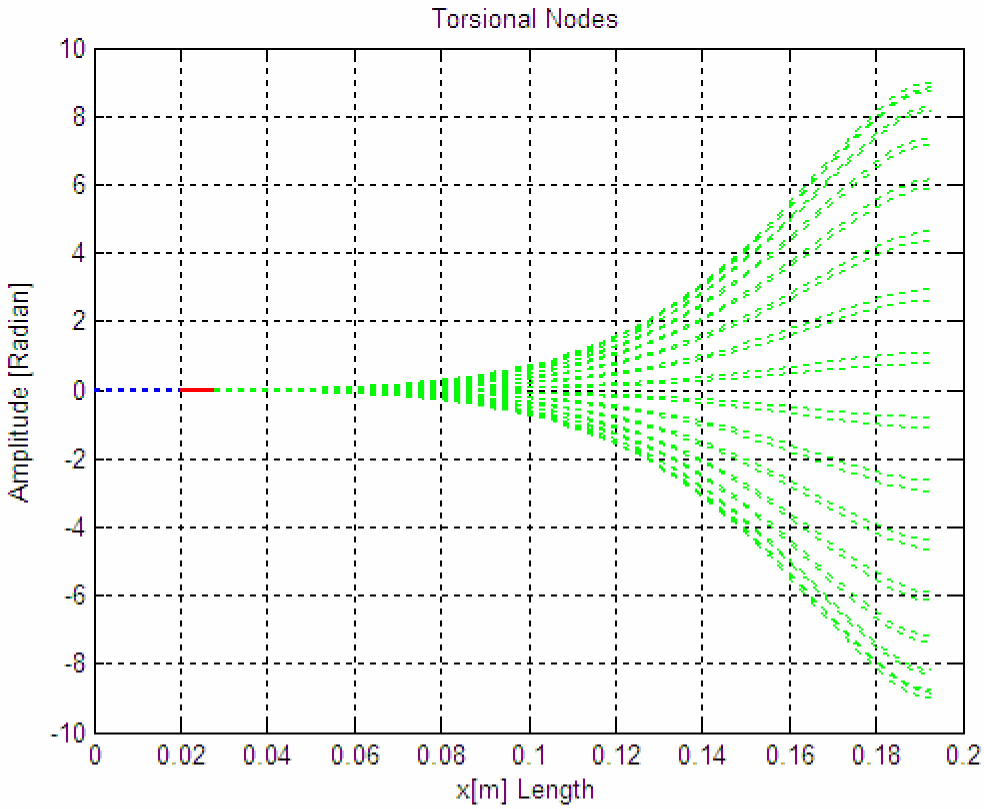

Figure 5.6. Nodes of the torsional mode

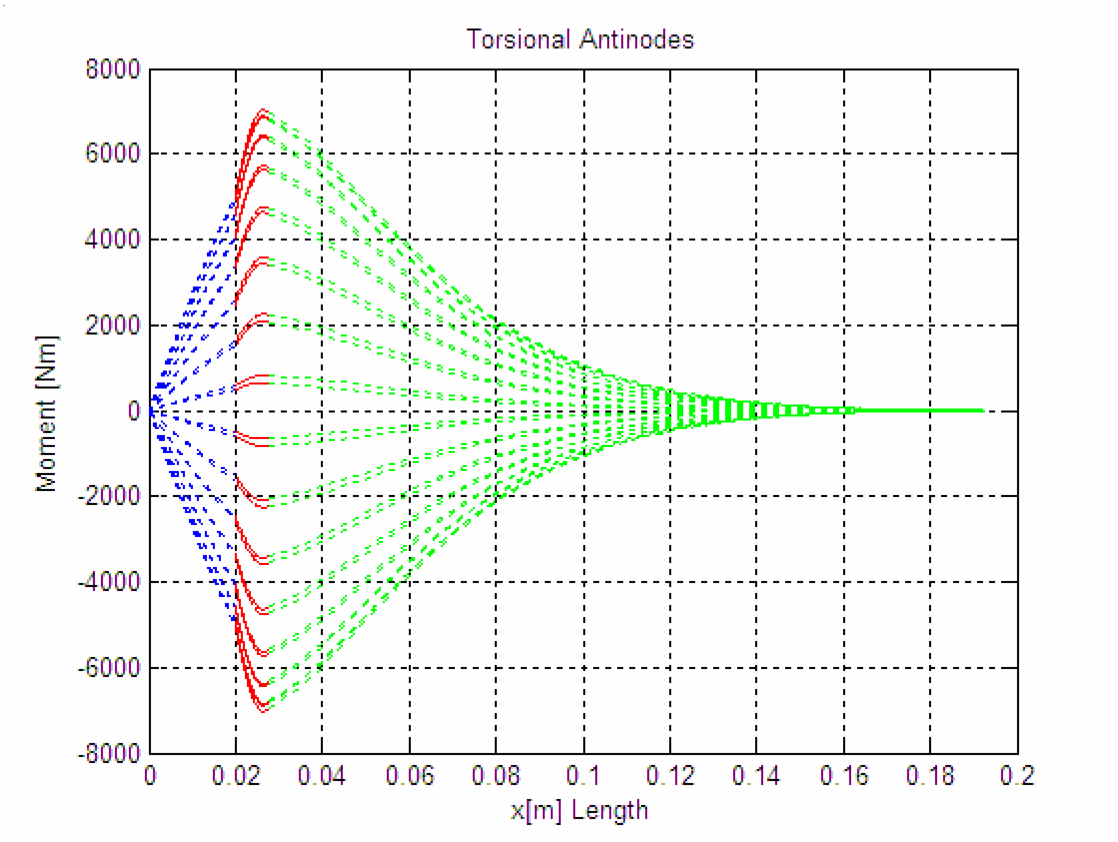

Figure 5.7. Antinodes of the torsional mode

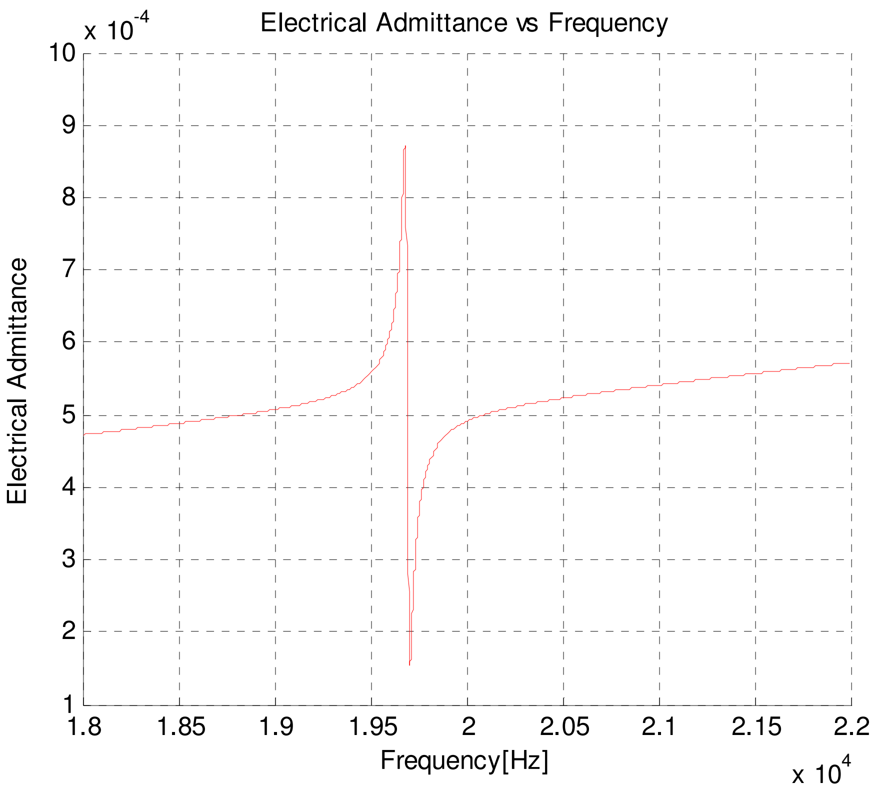

Figure 5.8. Electric admittance of the longitudinal piezoceramics

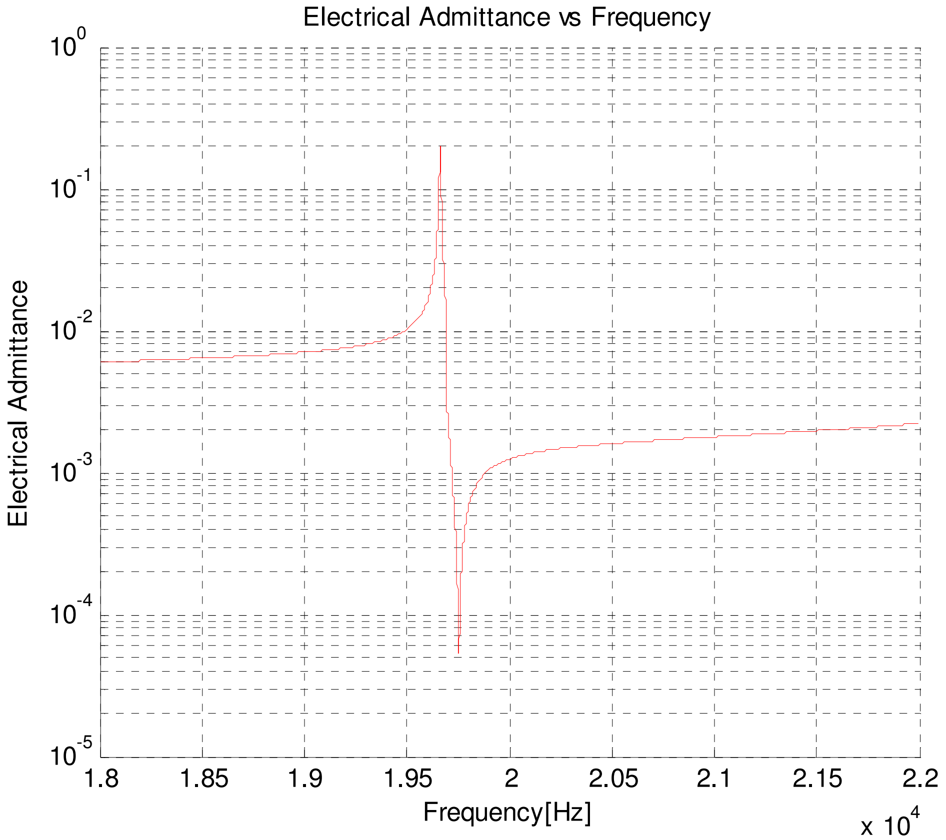

Figure 5.9. Electric admittance of the torsional piezoceramics

Using same type and number of piezoceramics, another transducer is designed with conical horn. Similar procedure is applied to the transducer. After synchronization, final dimensions are given in Table 5.7. Frequency responses are seen in Figure 5.10 and 5.11. The difference of resonance frequencies of the modes is 7 Hz.

Table 5.7. Final dimensions of the transducer

	Material	d_i [cm]	d_f [cm]	d_{inner} [cm]
Backing	St	5	5	2
L.P.P.	PZT 4	5	5	2
T.P.P.	Equivalent of PZT 4	5	5	2
Horn	Al	5	2.17	-
Lengths of Longitudinal Piezoceramics [mm]			10	
Lengths of Torsional Piezoceramics [mm]			5	
Backing Length [m]			0.015	

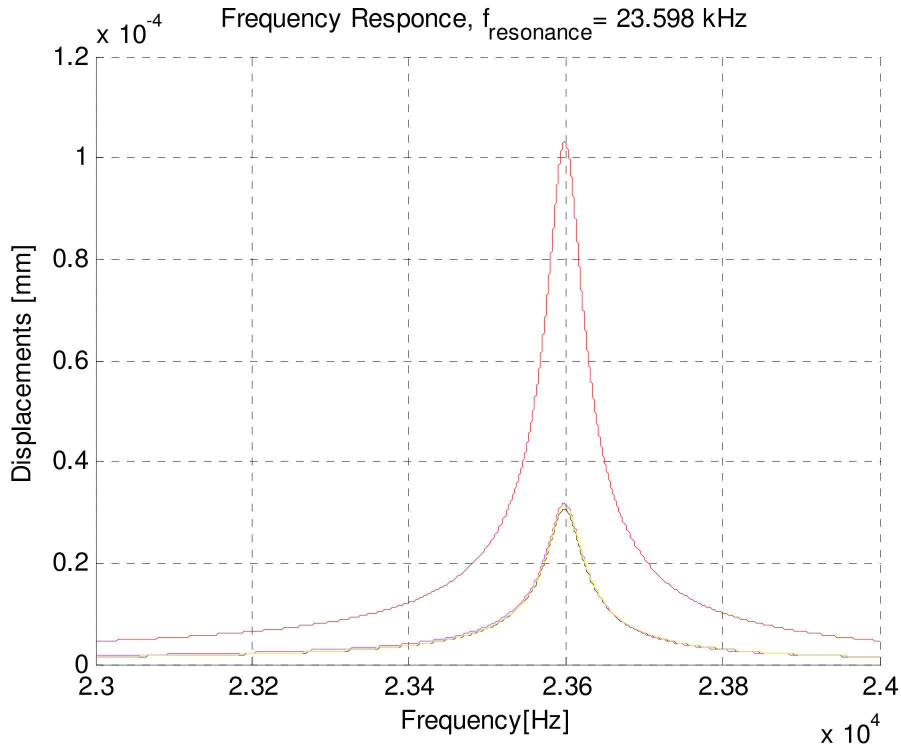

Figure 5.10. Resonance frequency of longitudinal mode of conical horn

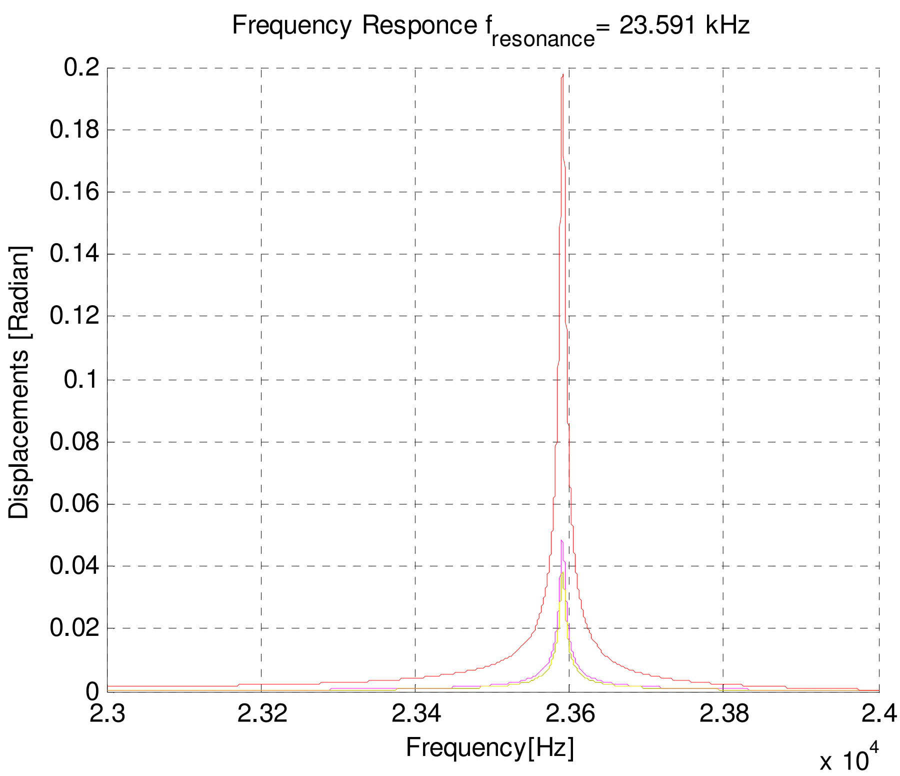

Figure 5.11. Resonance frequency of torsional mode of conical horn

Catenoidal horn is also used to synchronize the two modes with the same type and numbers of piezoceramics. Final dimensions are given in Table 5.8. Frequency responses are seen in Figure 5.12 and 5.13. Difference of resonance frequencies of the modes is 10 Hz.

Table 5.8. Final dimensions of the transducer

	Material	d_i [cm]	d_f [cm]	d_{inner} [cm]
Backing	St	5	5	2
L.P.P.	PZT 4	5	5	2
T.P.P.	Equivalent of PZT 4	5	5	2
Horn	Al	5	2.17	-
Lengths of Longitudinal Piezoceramics [mm]			4	
Lengths of Torsional Piezoceramics [mm]			5	
Backing Length [m]			0.063	

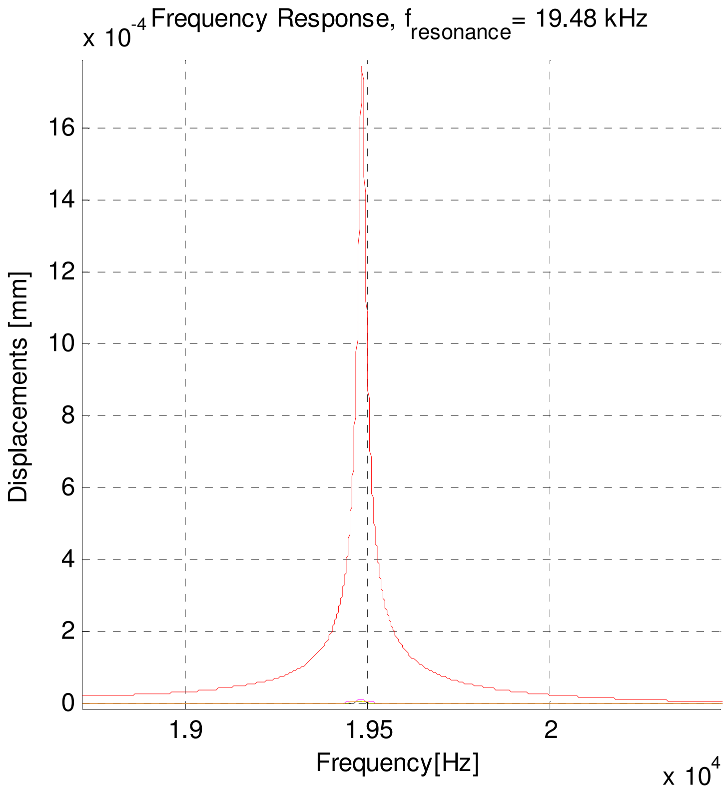

Figure 5.12. Resonance frequency of longitudinal mode of catenoidal horn

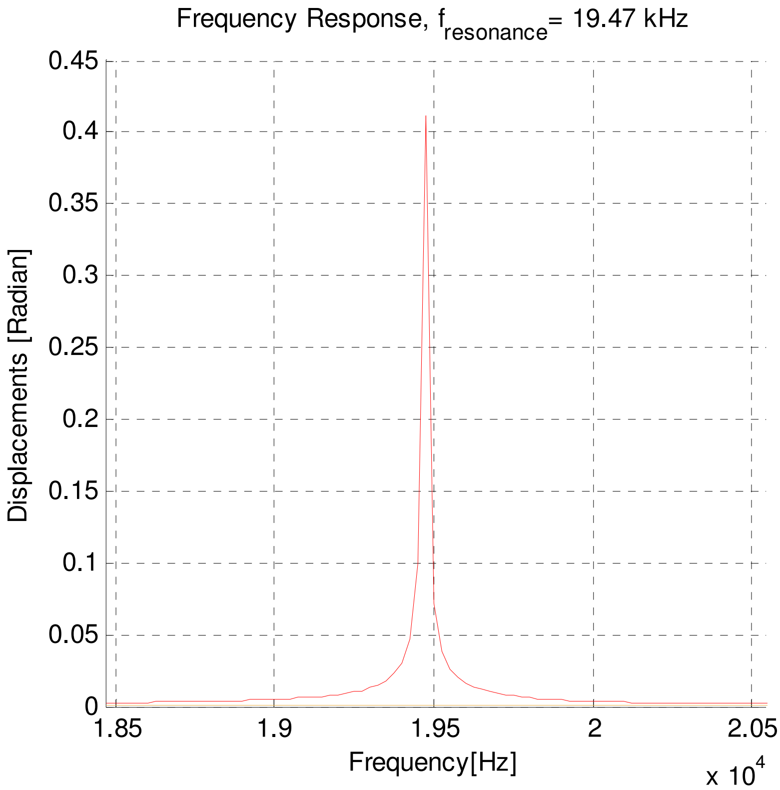

Figure 5.13. Resonance frequency of torsional mode of catenoidal horn

SUMMARY AND CONCLUSIONS

In this thesis, longitudinal and torsional vibration modes of an ultrasonic transducer for welding were examined. A matlab code is developed to analyze the welding system. Impedance matching method was used to analyze the system. Coefficients of the spatial solutions of the ultrasonics transducer were found by applying boundary conditions.

Using exponential, conical and catenoidal horns, longitudinal and torsional vibration modes were synchronized. Using the Matlab code and applying the concepts given in Chapter 4, dimensions of transducer were changed and found a frequency at which two modes were synchronized. To solve the synchronization problem of the transducer, length-beta (L-β) parameters have been examined widely for exponential horn. Through the studies, it was shown that when a dimension of a transducer is changed, resulting resonance frequency can be guessed.

The plane-wave assumption was used in the analysis. That is lateral dimensions should be less than a quarter of the wavelength of the transducer. In this thesis thermal effects were neglected. However, as a result temperature length of horn may change during ultrasonic welding, and the synchronization of horn could be lost. This is one of the realistic problems, and needs a more detailed study.

It is assumed that theoretically torsional vibration has no slipping problem. However this assumption may not be valid in practice. Expected amplitudes could not be obtained. Thus to glue the parts of transducers with a filling material, such as epoxy resin, can be used to avoid this problem.

It is assumed that longitudinal and torsional vibrations are independent. The interaction between the longitudinal and torsional vibrations in the compound transducer, the impedance characteristics, and vibration characteristics of the transducer for high power applications can be studied further.

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