THROUGHPUT AND INVENTORY ANALYSIS OF A FLEXIBLE MACHINE FOR DIFFERENT MAINTENANCE POLICIES

by

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ABSTRACT

THROUGHPUT AND INVENTORY ANALYSIS OF A FLEXIBLE MACHINE FOR DIFFERENT MAINTENANCE POLICIES

In this study, a manufacturing shop floor with two part types is examined and the impacts of different production and maintenance policies on the average throughput and average buffer levels are compared. Basically the comparisons are between a flexible machine with two different maintenance strategies and two independent ordinary machines, which are capable of producing only one type of product. A two-machine-one-buffer(2M1B) line is modelled as a Discrete State Continuous Time Markov Chain for two different layouts. First layout includes a flexible machine which is followed by a reliable assembly machine. Two types of parts, produced by the flexible machine, are assembled at the second workstation. In this layout, two different maintenance policies will be applied. First one is called Repair Together Policy; when a tool breaks down, the machine is not stopped and production goes on with the other part type until both tools are down. At that point, both tools are repaired together. Second maintenance policy is called Stop and Repair Policy; whenever a tool breaks down, the machine is stopped and that tool is repaired. Production goes on from the same point after the repair. The other layout to be used includes two independent/ordinary machines that are followed by a reliable assembly machine. After Markov anlaysis of the single machine cases for each system and maintenance policy, 2M1B lines are examined. Exact solutions are found by a software program using balance equations and used for comparison of the systems and maintenance policies. Results are investigated from a managerial point of view trying to decide which system and maintenance policy is better in which parameter set of the machines.

ÖZET

ESNEK BİR MAKİNEDE DEĞİŞİK ONARIM STRATEJİLERİ İÇİN ÜRETİM VE ENVANTER ANALİZİ

Bu çalışmada iki farklı parçanın üretilip bir araya getirildiği bir üretim atölyesi incelenmekte, farklı üretim ve onarım stratejilerinin üretimdeki verim ve envanter üzerindeki etkileri karşılaştırılmaktadır. Temel olarak karşılaştırmalar iki farklı onarım stratejisi uygulanan bir esnek makine ve sadece tek parça tipi üretebilen iki tane sıradan makineler arasındadır. iki makine bir arastok hattı, kesikli durumlu sürekli zamanlı bir Markov Zinciri olarak iki farklı makine yerleşim şekli için modellenmiştir. İlk yerleşim şekli bir esnek makine ve onu takip eden hatasız bir montaj makinesinden oluşmaktadır. İlk istasyonda esnek makine tarafından işlenen iki tip parça daha sonra ikinci istasyonda montaj makinesi tarafından birleştirilmektedir. Bu yerleşim şekli için iki farklı onarım stratejisi uygulanmıştır. Birincisi, Birlikte Tamir Et stratejisidir; makinenin iki farklı kesici ucundan biri bozulduğunda makine durdurulmaz ve üretime diğer parçayla devam edilir. Bu, iki kesici uç da bozulana kadar devam eder, ve sonunda ikisi birlikte tamir edilir. İkinci onarım stratejisi ise Dur ve Tamir Et uygulamasıdır; kesici uçlardan biri bozulduğunda makine durdurulur, o uç tamir edilir ve üretime kaldığı yerden devam edilir. Diğer yerleşim şekli ise iki bağımsız sıradan makineyi yine bir hatasız montaj makinesinin takip etmesiyle oluşmaktadır. Her sistem ve onarım stratejisinin tek makine halleri için Markov analizleri yapıldıktan sonra iki makine bir arastok hatları incelenir. Kesin çözümler geçiş denklemleri kullanılarak bir yazılım aracılığıyla elde edilir ve karşılaştırmalar için kullanılırlar. Sonuçlar yönetimsel bir bakış açısıyla incelenir ve hangi sistemin ve onarım stratejisinin hangi parametre setlerinde diğerlerinden daha iyi olduğu konusunda kararlara varılır.

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n_A	Current buffer level for A: 0 to N_A
n_B	Current buffer level for B: 0 to N_B
N_A	Buffer limit for A.
N_B	Buffer limit for B.
nu	Production rate of assembly machine.
p_A	Failure rate for tool A.
p_B	Failure rate for tool B.
Pr_A	Probability of flexible machine picking a part A from the infinite pool of parts.
Pr_B	Probability of flexible machine picking a part B from the infinite pool of parts.
r_A	Repair rate for tool A.
r_B	Repair rate for tool B.
r_{both}	Repair rate of both tools together for the flexible machine.
α_A	State of tool A: 1 or 0.
α_B	State of tool B: 1 or 0.
μ_A	Production rate of tool A.
μ_B	Production rate of tool B.

LIST OF ACRONYMS/ABBREVIATIONS

TH	Throughput.
WIP	Work in Process.
SR	Stop and Repair.
RT	Repair Together.
$2IM$	Two Independent Machines.

1. INTRODUCTION

Need for product variety is getting stronger in manufacturing systems with the increasing demands of customers. Simple machines that can produce one type of products only, are falling behind at satisfying this need. Therefore flexible machines are becoming more and more important for manufacturing lines.

Performance measures of manufacturing lines are mostly analyzed by using stochastic models or simulations in the literature. However, flexible machines add some complexity to these analysis by nature, since the number of product types increases. Having flexible machines at hand, we have a wide variety of policy options, especially about the maintenance regime of the tools. Let's say we have a flexible machine that is capable of producing ten different products using seperate tools and one of the tools breaks down during production. What should be the decision at that moment? Should we stop the machine and replace that tool or should we go on producing other types using the remaining tools? If we go on producing with the remaining tools, when should we stop? Is it better to wait until all tools are broken or should we repair when a certain amount of tools are broken?

These are common questions to be faced in a flexible manufacturing environment. Answering these questions gets harder with increasing number of product types. In this study, we will be investigating a manufacturing system with two types of products, which can be used as a basis for other multi-product systems. Our main goal will be to understand the effects of flexible machines and different maintenance policies in manufacturing systems. We will be searching for their effects on average throughput and average buffer levels in a two Machine one Buffer (2M1B) assembly line by using two different maintenance strategies for the flexible machine and also by replacing the flexible machine with two independent ordinary machines. We know that average throughput and average buffer levels are considered to be the two most important performance measures in a manufacturing system. So, checking these outputs will give us an idea of the effectivenes of each system and policy. First and second workstations in the 2M1B systems to be investigated will be as follows:

- One Flexible Machine - One Assembly Machine.
- Two Independent Ordinary Machines - One Assembly Machine.

The rest of the study will be in the following order. Literature review and background information will be given in the next chapter. Objectives of the study will be stated in Chapter 3. In Chapter 4, problem definition and models for each case will be introduced starting with the single machine cases. Assumptions for the models will be stated and Markov Analysis will be made so that steady-state solutions of each case can be reached. In the following chapter, exact solutions will be used to compare each of the three cases and to have insights about which system and maintenance policy is more effective in which scenario. That will be followed by the conclusions and future research directions in Chapter 6.

2. LITERATURE REVIEW AND BACKGROUND INFORMATION

There is a vast literature on transfer lines of manufacturing systems starting from early 50's. Since production is a key economic indicator all over the world, it is quite understandable that there have been serious amount of studies on the performance measures of production systems.

This study is based on a Markovian modelling approach which was popularised and mostly used after Buzacott uses a Markov process in his model (Buzacott, 1967). Some other approaches, used in performance evaluation of manufacturing systems, are Petri Net models, Bernoulli models using Queuing networks and brute force simulations.

Petri Nets are directed bipartite graph consisting of places, transitions and directed arcs. Arcs run from places to transitions or vice versa. Petri Net models are hierarchical models with well-developed mathematical and practical foundation. Bruno and Biglia were among the pioneers who used Generalized Stochastic Petri Nets for evaluating the performance of manufacturing systems (Bruno and Biglia, 1985).

In Bernoulli models, the state of the machine in each cycle time is determined by the process of Bernoulli trials. In other words, it is assumed that during each slot machine m_i, i = 1,...,M is up with probability p_i and down with probability 1-p_i. The state of the machine is determined at the beginning of each cycle, independent of the state of this machine in the previous cycle (Li and Meerkov, 2000).

In order to analyze production lines with a Markovian model, firstly two-machine-one-buffer (2M1B) systems need to be examined, because they are the building blocks for longer lines. Many different kinds of analytical models have been developed and exactly solved for (2M1B) lines (Dallery and Gershwin, 1992; Kim, 2004; Kim and Gershwin, 2005). These models can be divided into three main categories according to the state and time definition:

- Discrete State - Discrete Time
- Discrete State - Continuous Time
- Mixed State - Continuous Time.

The difference between a discrete and mixed state can be explained by the parts in the buffers. If these parts are discrete i.e. countable, then the model is called a discrete state model. If the buffer level is continuous on the other hand (e.g. fluid) it is called a mixed state model. It is called "mixed" and not "continuous" because in all models the state of the machines are represented discretely. State of the machines shows if the machine is in working condition (operational) or failed or in working condition but producing failed parts (for the models including quality aspect, as in Kim and Gershwin, 2005). In a discrete time model, time is divided into equal length periods which is usually picked as the production time of one part. In that case, events such as beginning and ending of a task are synchronised with the boundaries of these periods of time. In continuous time models on the other hand, stochastic processing times can be used for the machines. So the machines do not have to be synchronised in a continuous time model. (Bergeron *et al.*, 2010)

Another aspect that is used variably in manufacturing models is the failure types. The failures for unreliable machines are either Time-Dependant (they may fail after a certain amount of time) or Operation-Dependant (they may only fail after a certain amount of operation). In Operation-Dependant Failure models there is the assumption that a machine cannot fail while it is idle i.e. starved or blocked.

Most of the studies in this area are on single product type manufacturing lines. Although multi-product lines are commonly used in production plants, there are very few studies in the literature regarding the analysis of their performances. The paper of Colledani *et al.* is one of them, which presents an approximate analytical method for performance evaluation of a multi-product manifacturing system (Colledani *et al.*, 2007). Also in the study of (Nufer, 2006), 2M1B line is examined in a hybrid manufacturing environment with two part types. New and remanufactured products are processed together in the line and the study focuses more on the quality aspect.

In the literature of manufacturing systems, there exist many kinds of stochastic models such as two machines models, three or more machines (K machines) models, parallel machines and flow lines, assembly and disassembly models, complex manufacturing systems with rework loops, etc. These are dealing with the performance measures of the systems like the throughput and work-in-process (WIP) inventory. There is also another branch of the manufacturing systems literature, which is focusing on the maintenance and reliability concepts. The aim of maintenance optimization is to achieve a proper balance between maintenance costs and the benefit of increased process/tool availability (Van Rijn, 1987). In the review paper of Bergeron *et al.*, it is stated that almost any configuration of manufacturing systems can be modelled today with recent developments (Bergeron *et al.*, 2010). But, no consideration is made concerning maintenance strategies in a production model. What we will do in this study falls into that area. We will be investigating the effects of maintenance strategies on the main performance measures (throughput and WIP) of a manufacturing system. We will use a two-part-type product-assembly system for this study.

3. OBJECTIVES OF THE STUDY

The main objective of this study is to investigate the importance and effects of maintenance strategies on the throughput and work-in-process (WIP) inventory in manufacturing systems. For that purpose, a two-part-type Two Machine One Buffer (2M1B) production-assembly line is modelled as a Discrete State Continuous Time Markov Chain for two different layouts:

- *One Flexible Machine - One Assembly Machine.* At the first workstation there is one flexible machine which produces two types of products. Machine will be either producing A parts using its tool A or B parts using its tool B. So, there is no simultaneous production of parts A and B. These dedicated tools can break down with exponential failure rates. Two maintenance policies will be examined for these tools: First maintenance policy -Stop and Repair Policy- will be to stop the machine, repair the broken tool immediately and go on with the same part's production. Second policy - Repair Together Policy- will be to wait until both tools are broken and repair them together. So, when the first tool is broken, machine leaves that part and goes on production with the other part type until that tool is also broken. These two types of parts, leaving first workstation, go to their dedicated buffers: Buffer A and Buffer B. Later these parts are assembled in the second workstation, which consists of a reliable assembly machine, to have the final product.
- *Two Independent Ordinary Machines - One Assembly Machine.* At the first workstation there is no flexible machine this time. Instead of the flexible machine, there are two independent ordinary machines. One of the machines produces A parts only and the other one produces B parts only. Since each machine has only one tool, there is no special maintenance policy to consider here. Whenever the tool of a machine is broken, machine stops until that tool is repaired. So, each failure is immediately followed by repair. Then same as in the previous case; parts go to their dedicated buffers: Buffer A and Buffer B. Then they are assembled at the assembly machine to reach the final product.

In order to make comparisons, steady-state solutions will be found for each case. While comparing above systems with the defined policies, performance measures to be considered will be average throughput and average inventory. With those comparisons we will have a chance to understand the impacts of flexible machines and different maintenance policies in a manufacturing line.

One contribution of this study will be to create managerial insights for a manufacturing system with two part types, where different production layouts and policies can be applied. One will have an idea about which layout and policy is more efficient in which parameter set (production rates, failure rates, repair rates, buffer sizes). Even if this manufacturing system is a longer transfer line, 2M1B case constitutes the basis for that. So, one needs to study the 2M1B case first and later a decomposition technique can be used for longer lines as Gershwin states (Gershwin, 2002).

4. PROBLEM DEFINITION AND MODELS

In a Two Machine One Buffer (2M1B) manufacturing line, two types of parts are produced in the first workstation: A parts and B parts. Depending on the chosen layout, first workstation consists of either one flexible machine or two ordinary independent machines. Flexible machine has two seperate tools for producing these two different types of parts. The machine cannot use both tools at the same time. Thus the flexible machine either produces A parts or B parts at a given time. Ordinary machines on the other hand, have one tool each and they work independent of each other. One of them produces A parts only and the other one produces B parts only. In both cases, the first workstation is picking parts from the infinite pool of parts A and B. The flexible machine picks parts A and B from the infinite pool of parts after each production, with probabilities Pr_A and Pr_B. These probabilities should be balancing the throughput with respect to the assembly procedure. In our case one A and one B are assembled to reach the final product. So these probabilities should be such that:

$$\frac{TH_A}{TH_A + TH_B} = \frac{1}{2}$$

Three policies could be in effect for such a case:

(i) Process A parts only for a while and then process B parts only.

(ii) Process one A and one B in turns.

(iii) Pick parts A and B with certain probabilities and proceed accordingly.

(i) and (ii) are more meaningful in terms of real applications where (i) is better for systems with setup times and (ii) is better for systems without setup times. Justification of (iii), which is the policy that will be used in this study, is to find the average behaviour of the system using Markov Chains. So there will be certain probabilities Pr_A and Pr_B, both of which are equal to 0.5 in order to satisfy the assembly procedure.

After first workstation, these A and B parts leave for their dedicated finite buffers, waiting to be processed by the second workstation. Second workstation is a reliable assembly machine for all cases. So we will have one final product in the end by the assembly of parts A and B. The assembly machine needs to have non-zero levels of Buffer A and Buffer B to be able to start producing one final product, otherwise it will stay idle.

As mentioned above, flexible machine is capable of processing two different types of parts; A and B. After producing one unit of part A, it will continue producing the same part with probability Pr_A or switch to the other part type with probability $1 - Pr_A$, as long as both tools are in working condition. When a tool breaks down, we have two different maintenance policies to be applied. First one is to stop the machine and repair that broken tool immediately. The other policy is not to stop the machine and switch to produce the other part type until the other tool is also broken. This way, tools will be repaired together and the machine will be stopped less frequently. However, this policy might cause higher inventory and fluctuations in the buffer levels of type A and type B parts. For the ordinary machines, there is no need for a special maintenance policy. Whenever one of the tools fails, the machine has to stop and the tool needs to be repaired since there is no other tool to be used. Each flexible and ordinary machine has a parameter set of exponential rates. These are the production rates of parts A and B, failure rates of the tools and repair rates of the tools.

These different systems and policies will be investigated in detail and their efficiencies will be compared in the next chapters. Performance measures to be considered will be average throughput and average buffer levels. The comparisons will give us ideas about the impacts of flexible machines and different maintenance policies in a manufacturing line.

For the above systems and maintenance policies we will examine:

- Markov Analysis of Single Machine Cases.
- Markov Analysis of the Overall 2M1B Systems.

- Exact Numerical Solutions for 2M1B Cases.

It is interesting to investigate the performances of these cases and to observe which one is more efficient in which set up. For various repair, failure and production rates of the tools and for different buffer sizes, one can decide which case is preferable.

4.1. Fundamental Models and Assumptions

The processes in all models that will be investigated will consist of Discrete States, Continuous Time Markov Chains. States will be keeping the information given below:

- Number of parts in buffer A.
- Number of parts in buffer B.
- State of tool A: In working condition or broken.
- State of tool B: In working condition or broken.

Main assumptions that are used throughout this study are listed below:

- Dedicated tools and dedicated finite buffers for part types A and B.
- A machine is either producing good quality parts or not producing anything at all.
- There is one failure type: tool break down.
- Tool failures are independent of types. Failure of tool A does not effect the failure of tool B and vice versa.
- Operation Dependent Failures. A tool can only fail while working on a part.
- Assembly machine and its tools are completely reliable. Since the assembly machine is included in all cases that will be compared, this will not effect the results.
- We know beforehand the constant picking probability of part A and Part B for a flexible machine in the first workstation. If the machine is in working condition, it will start producing part A with Pr_A and part B with Pr_B.
- First Machine cannot be starved and last machine cannot be blocked.

We can now have a closer look at the systems that will be investigated.

4.1.1. Flexible Machine - Assembly Machine Case

As observed in Figure 4.1, A and B parts are picked by the first machine from an infinite pool of parts A and B with certain probabilities. These parts are processed by the flexible machine one by one and sent to their dedicated buffers afterwards. Unless one of the buffers is empty, assembly machine takes 1 piece from each buffer and produces one final product.

There are two maintenance policies for the flexible machine case. When one of the tools are broken, we either stop the machine immediately, repair and go on with the same part or we remove the part that we were working on, switch to the other part type and do not repair either until both tools are broken or starvation and blocking effect stops the production. This might happen in such a case: Say, tool A breaks down and flexible machine is producing B parts for a long time without failure. After a while, we may come up with a situation where Buffer A is depleted and Buffer B is full. Hence flexible machine cannot go on producing B parts because it is blocked and assembly machine cannot go on either, since it is starved due to depleted Buffer A. In such a case, system is stuck and we need to repair tool A alone.

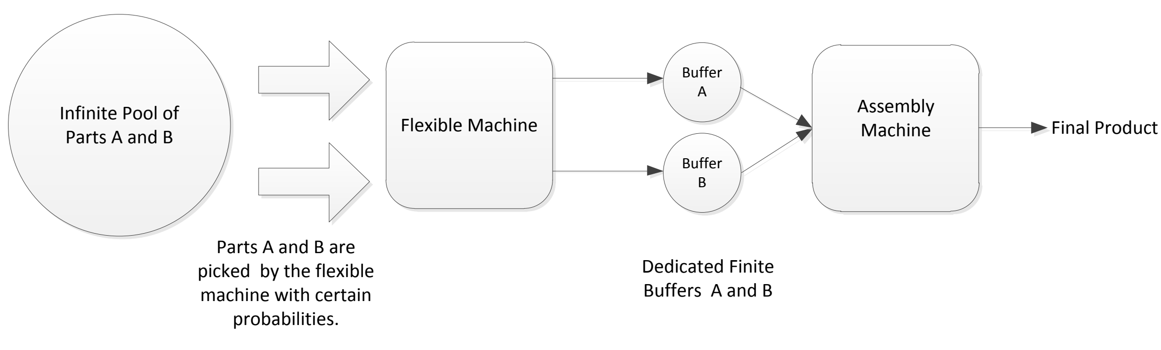

Figure 4.1. Flexible Machine Scheme.

4.1.2. Two Equivalent Independent Machines - Assembly Machine Case

Instead of using one flexible machine to produce two different products, one can also use two independent ordinary machines. One of the machines produces A parts and the other produces B parts. The rest is the same with the flexible machine case; assembly machine takes one piece from each buffer and produces one final product. The situation can be observed in Figure 4.2.

The advantage of having two independent machines is of course being able to produce two parts at the same time. Also whenever the tool of a machine is broken, we can repair it without interrupting the other machine's production.

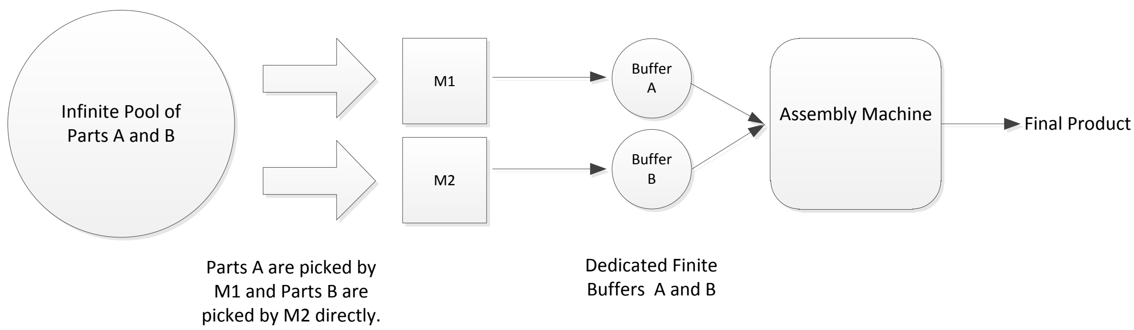

Figure 4.2. Two Machines Scheme.

4.2. Markov Analysis

As mentioned before, all cases are modeled as Continuous Time Markov Chains. Now let's examine the state spaces and transitions for each case seperately.

4.2.1. Single Machine CTMC Models

First we will start with single machine state spaces and transitions for both flexible and ordinary machines. Later the 2M1B model will be examined as a whole. In the single machine Markov Analysis, the given exponential rates and parameters are used as:

- p_A and p_B for the failure rates of tool A and B, respectively,
- r_A, r_B and r_{both} for the repair rate of tool A, repair rate of tool B and rate of repairing both tools together.

4.2.1.1. Single Flexible Machine with "Stop and Repair" Policy. States are represented by a two dimensional vector for the single machine cases: P(α_A, α_B), where α_A and α_B represent the conditions of tools A and B (1: up, 0: down). The flexible machine is capable of producing both A and B parts. Using Stop and Repair Policy, the system can be in three different states:

- $\alpha_A = 1$, $\alpha_B = 1$: In working condition,
- $\alpha_A = 1$, $\alpha_B = 0$: Tool B is broken down,
- $\alpha_A = 0$, $\alpha_B = 1$: Tool A is broken down.

Transitions between the states can be observed in Figure 4.3.

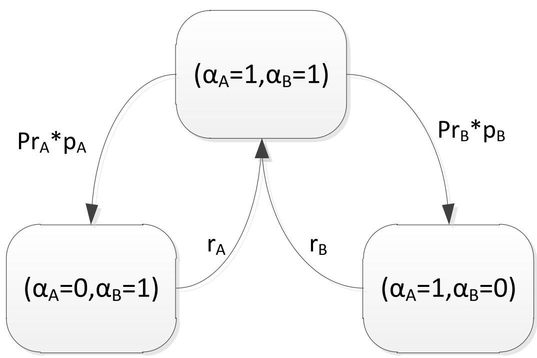

Figure 4.3. Single Flexible Machine States - Stop and Repair.

As observed in the state transitions, if one of the tools is broken the machine is stopped and that tool is repaired immediately. We can never observe the machine to have both tools broken. So we do not have the state [$\alpha_A = 0$, $\alpha_B = 0$].

The other policy is to wait until both tools are broken before any repair. Then the state space will of course change.

4.2.1.2. Single Flexible Machine with "Repair Together" Policy. The states for this policy will be as follows:

- $\alpha_A = 1$, $\alpha_B = 1$: Both tools are in working condition,
- $\alpha_A = 1$, $\alpha_B = 0$: Tool B is broken, tool A is producing part A,
- $\alpha_A = 0$, $\alpha_B = 1$: Tool A is broken, tool B is producing part B,
- $\alpha_A = 0$, $\alpha_B = 0$: Both tools are broken.

Transitions between these states are shown in Figure 4.4.

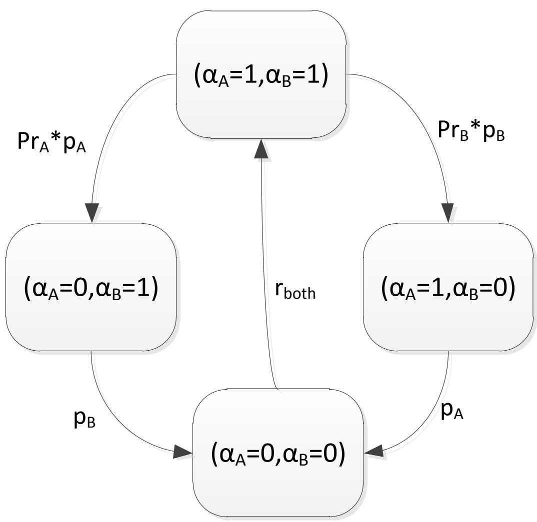

Figure 4.4. Single Flexible Machine States Repair Together.

With "Repair Together" policy, repairs will be done when both tools are broken. This repair rate is called r_{both}. Repair time for both tools together is slower than one tool's repair time alone but faster than the sum of two tools' individual repair times. This fact is a small advantage of this policy. However, there is also a down side of this policy: having unbalanced buffer levels depending on the initial parameters such as unbalanced tool failure rates.

In "Stop and Repair" policy, we have balanced flow of parts A and B since the first machine picks the parts with a certain probability and this order of the parts is never changed later due to break downs. In this case however, when a tool is broken we go on working with the other tool. This might create fluctuations in the buffer levels especially when parameters such as p (rate of break down) and r (repair rate) are different for each tool.

4.2.1.3. Two Equivalent Independent Machines. In the second system, there are two independent ordinary machines that are capable of producing one type of product only. For each of these two machines we have the below simple state space:

- $\alpha = 1$: In working condition.
- $\alpha = 0$: Broken.

When we consider two machines together, we come up with the following state space:

- $\alpha_A = 1$, $\alpha_B = 1$: Both tools (machines) are in working condition,
- $\alpha_A = 1$, $\alpha_B = 0$: Tool of Machine B is broken, Machine A is producing part A,
- $\alpha_A = 0$, $\alpha_B = 1$: Tool of Machine A is broken, Machine B is producing part B,
- $\alpha_A = 0$, $\alpha_B = 0$: Both Machines have broken tools.

Having two independent machines in the first workstation leaves us with a more straight-forward transition diagram as seen in Figure 4.5.

We have examined the state spaces of single machines for the first workstation. Now let's have a quick look at their throughput performances before examining the overall 2M1B system.

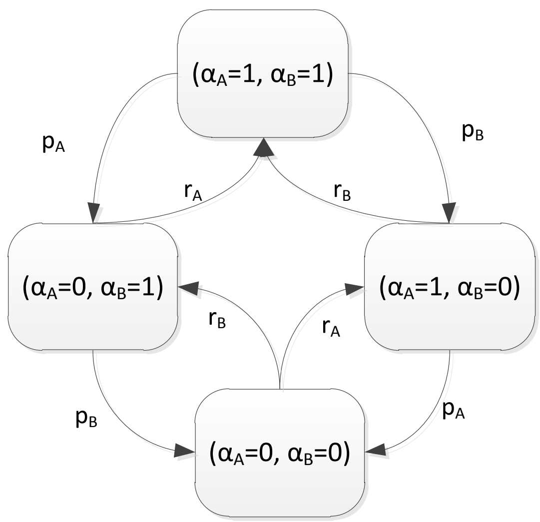

Figure 4.5. Two Independent Machines States.

4.2.2. Comparison of Single Machine Models

In order to compare these cases, we will first solve for each case, the balance equations, that we find using state transitions. For the parameter set in Table 4.1, we will see the throughput performances of each case.

As noticed in the parameter set, production rates of each independent machines are 0.5, while the rate for flexible machine is 1. This is simply to have a fair comparison between the cases since two independent machines are working at the same time. Resulting from this working speed difference, failure rates of the machines are also selected at the same ratio. Also please note that the rate for repairing both tools at the same time is selected in such a way, that it will not be faster than repairing only one tool and it will not be slower than repairing both tools at different times.

We can have a look at the steady state solutions and throughput results of each case in Table 4.2.

Table 4.1. Parameter Set.

	Repair Together	*Stop and Repair*	*2 Ind. Machs*
Pr_A	0.5	0.5	N/A
μ_A	1	1	0.5
μ_B	1	1	0.5
p_A	0.01	0.01	0.005
p_B	0.01	0.01	0.005
r_A	0.05	0.05	0.05
r_B	0.05	0.05	0.05
r_{both}	0.04	N/A	N/A

Table 4.2. Single Machine Performances.

Stop and Repair - Throughput = 0,833333			
P(1,1) = 0.833333	P(1,0) = 0.0833333	P(0,1) = 0.0833333	P(0,0) = 0
Repair Together - Throughput = 0.888889			
P(1,1) = 0.444444	P(1,0) = 0.222222	P(0,1) = 0.222222	P(0,0) = 0.111111
Two Independent Machines - Throughput = 0.909091			
P(1,1) = 0.826446	P(1,0) = 0.082645	P(0,1) = 0.082645	P(0,0) = 0.0082645

The outputs show us that with this specific parameter set, Two Independent Machines case is giving the best and Stop and Repair Policy gives the worst throughput results. This might also be explained by looking at the percentage of time when there is production for each case. Note that in Stop and Repair Policy, production is done only at the state $P(1,1)$.

For further comparison, we have obtained parametric solutions for each case. Throughput levels of each case, for part types A and B were found as in the following equations.

Stop And Repair Case:

$$TH_A = \mu_A \left(\frac{r_A r_B}{p_B r_A + p_A r_B + 2 r_A r_B} \right) \tag{4.2.1}$$

$$TH_B = \mu_B \left(\frac{r_A r_B}{p_B r_A + p_A r_B + 2 r_A r_B} \right) \tag{4.2.2}$$

2 Independent Machines Case:

$$TH_A = \mu_A \left(\frac{r_A}{p_A + r_A} \right) \tag{4.2.3}$$

$$TH_B = \mu_B \left(\frac{r_B}{p_B + r_B} \right) \tag{4.2.4}$$

Repair Together Case:

$$TH_A = \mu_A \left(\frac{p_B r_{both}}{p_A p_B + p_A r_{both} + p_B r_{both}} \right) \tag{4.2.5}$$

$$TH_B = \mu_B \left(\frac{p_A r_{both}}{p_A p_B + p_A r_{both} + p_B r_{both}} \right) \tag{4.2.6}$$

Using these equations, we can make the following comparisons between the cases:

Stop And Repair vs 2 Independent Machines:

Considering that the production rate of the flexible machine is two times faster than each of the independent machines, average throughput of parts A is higher for 2IM case if the following condition holds and vice versa:

$$p_B r_A > p_A r_B \tag{4.2.7}$$

For comparison of part type B, we can use the equation 4.2.8. If it holds, 2IM case achieves higher throughput levels than SR case.

$$p_A r_B > p_B r_A \tag{4.2.8}$$

Stop And Repair vs Repair Together: The following condition will let us know which case is giving better throughput A results. If the equation holds, SR case is giving better throughput A results and vice versa.

$$\frac{p_A}{r_{both}} + \frac{p_A}{p_B} - \frac{p_B}{r_B} - \frac{p_A}{r_A} > 1 \tag{4.2.9}$$

We observe from this equation that as p_B increases Repair Together policy is performing better than Stop and Repair policy for production of A parts. This makes sense. Whenever tool B fails, RT case switches to produce A parts. However, at each failure with SR policy, machine stops and waits until tool B is repaired. Equation 4.2.10 tells us about the throughput performances for B parts. If the equation holds, SR is giving better results than the RT policy.

$$\frac{p_B}{r_{both}} + \frac{p_B}{p_A} - \frac{p_B}{r_B} - \frac{p_A}{r_A} > 1 \tag{4.2.10}$$

Repair Together vs 2 Independent Machines: Following equation is for the throughput performances of each case for part type A. If that equation holds, 2IM case is giving better results than the RT case and vice versa.

$$\frac{p_A}{r_{both}} + \frac{p_A}{p_B} - \frac{2p_A}{r_A} > 1 \tag{4.2.11}$$

Throughput performances for part type B can be observed looking at the equation 4.2.12. If that equation holds, 2IM is giving better throuhput values compared to RT case.

$$\frac{p_B}{r_{both}} + \frac{p_B}{p_A} - \frac{2p_B}{r_B} > 1 \tag{4.2.12}$$

After investigating the single machine models and comparing the three cases, we can now go on with the overall 2M1B system.

4.2.3. Markov Analysis of 2M1B System

In 2M1B cases we have an assembly machine at the second workstation for each system. The state space representation is the same for all cases. States are represented by a four dimensional vector P(n_A, n_B, α_A, α_B) where,

- n_A: Current level in buffer A. [0 to N_A]
- n_B: Current level in buffer B. [0 to N_B]
- α_A: Working condition of tool A. [0 or 1]
- α_B: Working condition of tool B. [0 or 1]

Compared to the Single Machine cases, State Space is now enlarged by the additional information of Buffer Levels. Number of states for 2M1B cases will be 4xN_AxN_B. Representative state space transition diagrams for each case can be seen in Figure 4.6, 4.7 and 4.8.

In order to make comparisons between these cases, it is necessary to obtain mathematical outputs such as Average Throughput and Average Buffer Levels. For this purpose, equation sets, which are basically consisting of balance equations, will be solved and steady state probabilities will be calculated. After that, performance measures like Average Throughput and Average Buffer levels will be at hand for all cases. We can see the equations that will be used to find exact solutions of each case in the following subsections. The parameters and exponential transition rates that will be seen in the balance equations are listed below:

- μ_A, μ_B and nu are the production rates of parts A, B and assembly operation, respectively.
- N_A and N_B are buffer sizes for parts A and B.
- n_A and n_B are the current buffer levels for parts A and B.
- p_A and p_B are the failure rates of tools A and B.
- r_A, r_B and r_{both} are the repair rates of tool A, repair rate of tool B and rate of repairing both tools together.
- Pr_A is the probability of flexible machine picking a part A from the infinite pool after finishing a part.
- Pr_B is the probability of flexible machine picking a part B from the infinite pool after finishing a part.

4.2.3.1. Steady-State Solution for Stop and Repair Policy. For the Stop and Repair Policy, there are three sets of balance equations which are arranged according to the different situations of tool A and tool B (α_A and α_B). With this policy, we can never have both tools of the flexible machine broken since after each break down the machine is stopped and the broken tool is repaired. Therefore there are three different situations for α_A and α_B:

- $\alpha_A = 1$, $\alpha_B = 1$
- $\alpha_A = 1$, $\alpha_B = 0$
- $\alpha_A = 0$, $\alpha_B = 1$

Our balance equations will be shaped around these three sets. First let's have a look at the parameters that will be used in the equations: Pr_A and Pr_B are the constant probabilities of parts A and B entering the system. Production rates of the tools for part A and B (μ_A and μ_B), failure and repair rates of the tools (p_A, p_B ; r_A, r_B), production rate of the assembly machine(nu) are all exponential rates. Buffer Sizes for both parts(N_A and N_B) are the parameters which determine the number of equations since the number of unknowns increases with increasing buffer sizes.

Some of the transient states are due to the policy we are using here. For the Stop and Repair Policy, in the steady state, we cannot observe both tools to be broken at the same time. This is because, whenever a tool breaks down, the machine is stopped for repair and the other tool cannot break down while it is idle. Therefore, the steady state probabilities of such will be equal to 0: P(n_A, n_B, α_A=0, α_B= 0). These are not transient states for the other two cases. A more general thing, which is valid also for the other cases is that, if one of the buffers is full, corresponding tool cannot be broken. This is due to the assumption of "operational failures only". If a tool is blocked due to corresponding buffer being full, it cannot go on working and cannot break down. Hence, steady-state probabilities of such will also be 0: P(n_A, N_B, α_A, α_B= 0) or P(N_A, n_B, α_A=0, α_B).

We can write down the equations that will be used for exactly solving Stop and Repair case starting with the equations that steady-state probabilities of transient states are equal to zero. The equations to be solved are basically the collection of balance equations. These equations are divided into groups according to possible values of α_A and α_B. After adding our normalizing equation to this set of equations, we will be able to solve our problem using a software program to reach outputs like average

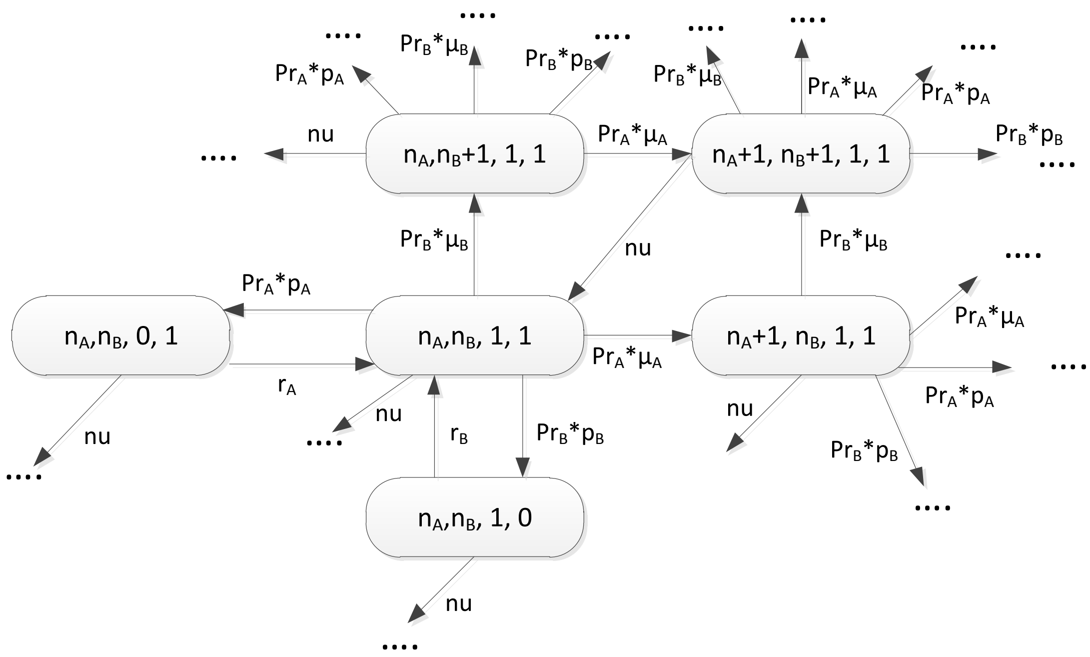

Figure 4.6. Representative State Transitions 2M1B - Stop and Repair.

throughput and average inventory levels. In Figure 4.6, a representative state transition diagram is given. We can observe that production and break down rates are multiplied by the picking probabilities, Pr_A and Pr_B, in the transitions. At any instant, the flexible machine starts working on one of the parts randomly. This picking probability is reflected in the transitions that way. But when a tool is broken, the next transiton is obvious in Stop and Repair case: r_A or r_B. Hence these rates are not multiplied with the picking up probabilities.

Equations for Transient States:

P(n_A, n_B, $\alpha_A = 0$, $\alpha_B = 0$) = 0 for n_A: [0 , N_A] and n_B: [0 , N_B]

P(n_A, N_B, $\alpha_A = 1$, $\alpha_B = 0$) = 0 for n_A: [0 , N_A]

P(N_A, n_B, $\alpha_A = 0$, $\alpha_B = 1$) = 0 for n_B: [0 , N_B]

Balance Equations for $\alpha_A = 1$ and $\alpha_B = 0$:

P(n_A, n_B, 1, 0)(nu + r_B) = P(n_A, n_B, 1, 1)$p_B Pr_B$ + P(n_A+1, n_B+1, 1, 0)nu, for n_A:[1,N_A-1], n_B:[1,N_B-1]

P(N_A, n_B, 1, 0)(nu + r_B) = P(N_A, n_B, 1, 1)p_B, for n_B:[1,N_B-1]

P(0, n_B, 1, 0)r_B = P(0, n_B, 1, 1)$p_B Pr_B$ + P(1, n_B+1, 1, 0)nu, for n_B:[0,N_B-1]

P(n_A, 0, 1, 0)r_B = P(n_A, 0, 1, 1)$p_B Pr_B$ + P(n_A+1, 1, 1, 0)nu, for n_A:[1,N_A-1]

P(N_A, 0, 1, 0)r_B = P(N_A, 0, 1, 1)p_B

Balance Equations for $\alpha_A = 0$ and $\alpha_B = 1$:

P(n_A, n_B, 0, 1)(nu + r_A) = P(n_A, n_B, 1, 1)$p_A Pr_A$ + P(n_A+1, n_B+1, 0, 1)nu, for n_A:[1,N_A-1], n_B:[1,N_B-1]

P(n_A, N_B, 0, 1)(nu + r_A) = P(n_A, N_B, 1, 1)p_A, for n_A:[1,N_A-1]

P(n_A, 0, 0, 1)r_A = P(n_A, 0, 1, 1)$p_A Pr_A$ + P(n_A+1, 1, 0, 1)nu, for n_A:[0,N_A-1]

P(0, n_B, 0, 1)r_A = P(0, n_B, 1, 1)$p_A Pr_A$ + P(1, n_B+1, 0, 1)nu, for n_B:[1,N_B-1]

P(0, N_B, 0, 1)r_A = P(0, N_B, 1, 1)p_A

Balance Equations for $\alpha_A = 1$ and $\alpha_B = 1$:

P(n_A, n_B, 1, 1)($Pr_A(\mu_A$+$p_A)$+$Pr_B(\mu_B$+$p_B)$ + nu) = P(n_A-1, n_B, 1, 1)$\mu_A Pr_A$ + P(n_A, n_B-1, 1, 1)$\mu_B Pr_B$ + P(n_A, n_B, 1, 0)r_B + P(n_A, n_B, 0, 1)r_A + P(n_A+1, n_B+1, 1, 1)nu, for n_A:[1,N_A-1], n_B:[1,N_B-1]

P(N_A, n_B, 1, 1)(μ_B+p_B+nu) = P(N_A-1, n_B, 1, 1)$\mu_A Pr_A$ + P(N_A, n_B-1, 1, 1)μ_B + P(N_A, n_B, 1, 0)r_B, for n_B:[1,N_B-1]

P(n_A, N_B, 1, 1)(μ_A+p_A+nu) = P(n_A, N_B-1, 1, 1)$\mu_B Pr_B$ + P(n_A-1, N_B, 1, 1)μ_A + P(n_A, N_B, 0, 1)r_A, for n_A:[1,N_A-1]

P(0, n_B, 1, 1)($Pr_A(\mu_A$+$p_A)$+$Pr_B(\mu_B$+$p_B)$) = P(0, n_B-1, 1, 1)$\mu_B Pr_B$ + P(1, n_B+1, 1, 1)nu + P(0, n_B, 1, 0)r_B + P(0, n_B, 0, 1)r_A, for n_B:[1,N_B-1]

P(n_A, 0, 1, 1)($Pr_A(\mu_A$+$p_A)$+$Pr_B(\mu_B$+$p_B)$) = P(n_A-1,0, 1, 1)$\mu_A Pr_A$ + P(n_A+1, 1, 1, 1)nu + P(n_A, 0, 1, 0)r_B +P(n_A, 0, 0, 1)r_A, for n_A:[1,N_A-1]

P(N_A, N_B, 1, 1)nu = P(N_A, N_B-1, 1, 1)μ_B + P(N_A-1, N_B, 1, 1)μ_A

P(0, 0, 1, 1)($Pr_A(\mu_A$+$p_A)$+$Pr_B(\mu_B$+$p_B)$) = P(1, 1, 1, 1)nu + P(0, 0, 1, 0)r_B + P(0, 0, 0, 1)r_A

P(0, N_B, 1, 1)(μ_A+p_A) = P(0, N_B-1, 1, 1)$\mu_B Pr_B$ + P(0, N_B, 0, 1)r_A

P(N_A, 0, 1, 1)(μ_B+p_B) = P(N_A-1, 0, 1, 1)$\mu_A Pr_A$ + P(N_A, 0, 1, 0)r_B

Normalizing Equation:

$$\sum_{\alpha_A=0}^{1}\sum_{\alpha_B=0}^{1}\sum_{n_A=0}^{N_A}\sum_{n_B=0}^{N_B}[P(n_A,n_B,\alpha_A,\alpha_B)] = 1$$

4.2.3.2. Steady-State Solution for Repair Together Policy. In the Repair Together Policy case, the parameters are not much different than the Stop and Repair policy case. There is an additional parameter r_{both} which is the rate of repairing both tools together. It makes sense that this rate is slightly slower than repairing only one tool, but a bit faster than repairing two tools individually at different times.

We need to mention here, the effect of buffer states on this policy. Eventhough the aim of this policy is to wait until both tools are broken, this might not be possible in some cases. For example, when tool A fails, B parts start to be produced immediately. However, at some point, if tool B does not fail soon afterwards, Buffer B will be full and Buffer A will be depleted. At that moment the system is stuck - assembly machine cannot produce anything because Buffer A is depleted and flexible machine cannot go on production since Buffer B is full. In that case, we need to repair the A tool alone without waiting for the B tool to be broken. So, we will have to use r_A and r_B when the first workstation is blocked and the assembly machine is starved at the same time. We can see this in Figure 4.7, representative state transition diagram. When we arrive at the state (N_A, 0, 1, 0) there is only one transition out of that state; r_B. Also note that, when both tools are up, production and break down transition rates are multiplied with the picking probabilities. However, when one of the tools is down, our policy tells us to use the other tool for sure. Therefore this time the rates are not multiplied with the picking probabilities.

There are no special transient states due to using this policy, only the general transient states that we have in all cases due to Operation Dependent Failures assumption. States of tools can be in four different situations, so the balance equations will be seperated into four groups:

- $\alpha_A = 1$, $\alpha_B = 1$

- $\alpha_A = 1$, $\alpha_B = 0$
- $\alpha_A = 0$, $\alpha_B = 1$
- $\alpha_A = 0$, $\alpha_B = 0$

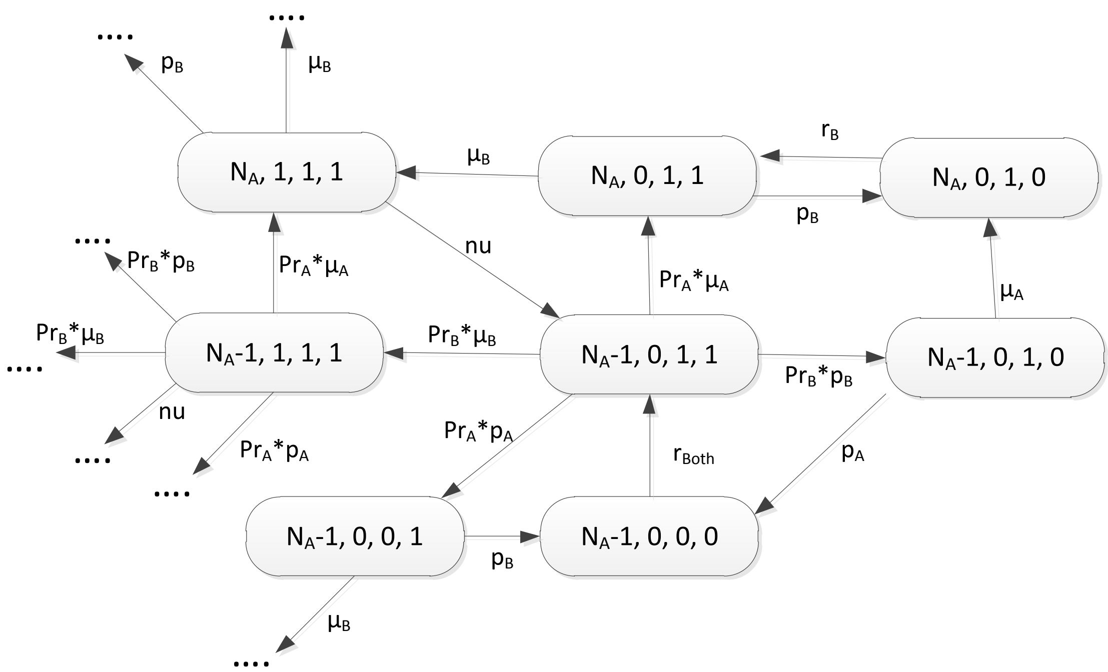

Figure 4.7. Representative State Transitions 2M1B - Repair Together.

Equations for Transient States:

P(N_A, n_B, $\alpha_A = 0$, $\alpha_B = 0$) = 0, for n_B: [0 , N_B]

P(n_A, N_B, $\alpha_A = 0$, $\alpha_B = 0$) = 0, for n_A: [0 , N_A-1]

P(n_A, N_B, $\alpha_A = 1$, $\alpha_B = 0$) = 0, for n_A: [0 , N_A]

P(N_A, n_B, $\alpha_A = 0$, $\alpha_B = 1$) = 0, for n_B: [0 , N_B]

Balance Equations for $\alpha_A = 0$ and $\alpha_B = 0$:

P(n_A, n_B, 0, 0)(nu + r_{both}) = P(n_A, n_B, 1, 0)p_A + P(n_A, n_B, 0, 1)p_B + P(n_A+1, n_B+1, 0, 0)nu, for n_A:[1,N_A-1], n_B:[1,N_B-1]

P(0, n_B, 0, 0)r_{both} = P(0, n_B, 1, 0)p_A + P(0, n_B, 0, 1)p_B + P(1, n_B+1, 0, 0)nu, for n_B:[0,N_B-1]

P(n_A, 0, 0, 0)r_{both} = P(n_A, 0, 1, 0)p_A + P(n_A, 0, 0, 1)p_B + P(n_A+1, 1, 0, 0)nu, for n_A:[1,N_A-1]

Balance Equations for $\alpha_A = 1$ and $\alpha_B = 0$:

P(n_A, n_B, 1, 0)(nu + μ_A + p_A) = P(n_A, n_B, 1, 1)$p_B Pr_B$ + P(n_A-1, n_B, 1, 0)μ_A + P(n_A+1, n_B+1, 1, 0)nu, for n_A:[1,N_A-1], n_B:[1,N_B-1]

P(N_A, n_B, 1, 0)nu = P(N_A, n_B, 1, 1)p_B + P(N_A-1, n_B, 1, 0)μ_A, for n_B:[1,N_B-1]

P(0, n_B, 1, 0)(μ_A + p_A) = P(0, n_B, 1, 1)$p_B Pr_B$ + P(1, n_B+1, 1, 0)nu, for n_B:[0,N_B-1]

P(n_A, 0, 1, 0)(μ_A + p_A) = P(n_A, 0, 1, 1)$p_B Pr_B$ + P(n_A-1, 0, 1, 0)μ_A + P(n_A+1, 1, 1, 0)nu, for n_A:[1,N_A-1]

P(N_A, 0, 1, 0)r_B = P(N_A, 0, 1, 1)p_B + P(N_A-1, 0, 1, 0)μ_A Note here that tool B has to be repaired alone or the sytem would be in deadlock

Balance Equations for $\alpha_A = 0$ and $\alpha_B = 1$: are symmetric with the previous case...

Balance Equations for $\alpha_A = 1$ and $\alpha_B = 1$:

P(n_A, n_B, 1, 1)(Pr_A(μ_A+p_A)+Pr_B(μ_B+p_B) + nu) = P(n_A-1, n_B, 1, 1)$\mu_A Pr_A$ + P(n_A, n_B-1, 1, 1)$\mu_B Pr_B$ + P(n_A, n_B, 0, 0)r_{both} + P(n_A+1, n_B+1, 1, 1)nu, for n_A:[1,N_A-1], n_B:[1,N_B-1]

P(N_A, n_B, 1, 1)(μ_B+p_B+nu) = P(N_A-1, n_B, 1, 1)$\mu_A Pr_A$ + P(N_A, n_B-1, 1, 1)μ_B, for n_B:[1,N_B-1]

P(n_A, N_B, 1, 1)(μ_A+p_A+nu) = P(n_A, N_B-1, 1, 1)$\mu_B Pr_B$ + P(n_A-1, N_B, 1, 1)μ_A, for n_A:[1,N_A-1]

P(0, n_B, 1, 1)(Pr_A(μ_A+p_A)+Pr_B(μ_B+p_B)) = P(0, n_B-1, 1, 1)$\mu_B Pr_B$ + P(1, n_B+1, 1, 1)nu + P(0, n_B, 0, 0)r_{both}, for n_B:[1,N_B-1]

P(n_A, 0, 1, 1)(Pr_A(μ_A+p_A)+Pr_B(μ_B+p_B)) = P(n_A-1,0, 1, 1)$\mu_A Pr_A$ + P(n_A+1, 1, 1, 1)nu + P(n_A, 0, 0, 0)r_{both}, for n_A:[1,N_A-1]

P(N_A, N_B, 1, 1)nu = P(N_A, N_B-1, 1, 1)μ_B + P(N_A-1, N_B, 1, 1)μ_A

P(0, 0, 1, 1)(Pr_A(μ_A+p_A)+Pr_B(μ_B+p_B)) = P(1, 1, 1, 1)nu + P(0, 0, 0, 0)r_{both}

P(0, N_B, 1, 1)(μ_A+p_A) = P(0, N_B-1, 1, 1)$\mu_B Pr_B$ + P(0, N_B, 0, 1)r_A

P(N_A, 0, 1, 1)(μ_B+p_B) = P(N_A-1, 0, 1, 1)$\mu_A Pr_A$ + P(N_A, 0, 1, 0)r_B

Normalizing Equation:

$$\sum_{\alpha_A=0}^{1}\sum_{\alpha_B=0}^{1}\sum_{n_A=0}^{N_A}\sum_{n_B=0}^{N_B}[P(n_A,n_B,\alpha_A,\alpha_B)] = 1$$

4.2.3.3. Steady-State Solution for Two Independent Machines. Parameters for the Two Independent Machines case are as in the Stop and Repair policy case of the flexible machine. This time there is no probability of parts A and B being picked by the first workstation. There are two machines working independently, they pick their parts from the infinite pool of parts A and B. Again there are four different situations for the tool states of Machine A and B.

- $\alpha_A = 1$, $\alpha_B = 1$
- $\alpha_A = 1$, $\alpha_B = 0$
- $\alpha_A = 0$, $\alpha_B = 1$
- $\alpha_A = 0$, $\alpha_B = 0$

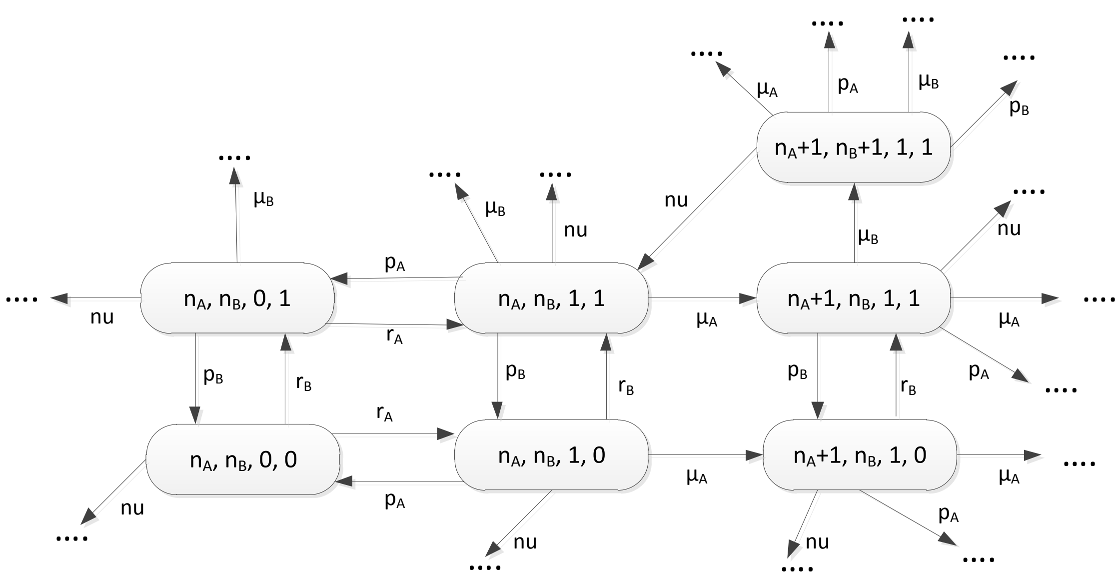

Figure 4.8. Representative State Transitions 2M1B - 2 Independent Machines.

Equations for Transient States:

P(N_A, n_B, $\alpha_A = 0$, $\alpha_B = 0$) = 0, for n_B: [0 , N_B]

P(n_A, N_B, $\alpha_A = 0$, $\alpha_B = 0$) = 0, for n_A: [0 , N_A-1]

P(n_A, N_B, $\alpha_A = 1$, $\alpha_B = 0$) = 0, for n_A: [0 , N_A]

P(N_A, n_B, $\alpha_A = 0$, $\alpha_B = 1$) = 0, for n_B: [0 , N_B]

Remaining equations are consisting of 4 sets, according to the states of tool A and tool B (α_A and α_B).

Balance Equations for $\alpha_A = 0$ and $\alpha_B = 0$:

P(n_A, n_B, 0, 0)($nu + r_A + r_B$) = P(n_A, n_B, 1, 0)p_A + P(n_A, n_B, 0, 1)p_B + P(n_A+1, n_B+1, 0, 0)nu, for n_A:[1,N_A-1], n_B:[1,N_B-1]

P(0, n_B, 0, 0)($r_A + r_B$) = P(0, n_B, 1, 0)p_A + P(0, n_B, 0, 1)p_B + P(1, n_B+1, 0, 0)nu, for n_B:[0,N_B-1]

P(n_A, 0, 0, 0)($r_A + r_B$) = P(n_A, 0, 1, 0)p_A + P(n_A, 0, 0, 1)p_B + P(n_A+1, 1, 0, 0)nu, for n_A:[1,N_A-1]

Balance Equations for $\alpha_A = 1$ and $\alpha_B = 0$:

P(n_A, n_B, 1, 0)($\mu_A + nu + p_A + r_B$) = P(n_A, n_B, 1, 1)p_B + P(n_A-1, n_B, 1, 0)μ_A + P(n_A+1, n_B+1, 1, 0)nu + P(n_A, n_B, 0, 0)r_A, for n_A:[1,N_A-1], n_B:[1,N_B-1]

P(N_A, n_B, 1, 0)($nu + r_B$) = P(N_A-1, n_B, 1, 0)μ_A + P(N_A, n_B, 1, 1)p_B, for n_B:[1,N_B-1]

P(0, n_B, 1, 0)($\mu_A + p_A + r_B$) = P(0, n_B, 1, 1)p_B + P(1, n_B+1, 1, 0)nu + P(0, n_B, 0, 0)r_A, for n_B:[0,N_B-1]

P(n_A, 0, 1, 0)($\mu_A + p_A + r_B$) = P(n_A, 0, 1, 1)p_B + P(n_A+1, 1, 1, 0)nu + P(n_A, 0, 0, 0)r_A + P(n_A-1, 0, 1, 0)μ_A, for n_A:[0,N_A-1]

P(N_A, 0, 1, 0)r_B = P(N_A, 0, 1, 1)p_B + P(N_A-1, 0, 1, 0)μ_A

Balance Equations for $\alpha_A = 0$ and $\alpha_B = 1$: are symmetric with the previous case...

Balance Equations for $\alpha_A = 1$ and $\alpha_B = 1$:

P(n_A, n_B, 1, 1)($\mu_A + p_A + \mu_B + p_B + nu$) = P($n_A$-1, n_B, 1, 1)μ_A + P(n_A, n_B-1, 1, 1)μ_B + P(n_A, n_B, 1, 0)r_B + P(n_A, n_B, 0, 1)r_A + P(n_A+1, n_B+1, 1, 1)nu, for n_A:[1,N_A-1], n_B:[1,N_B-1]

P(N_A, n_B, 1, 1)(μ_B + p_B + nu) = P(N_A-1, n_B, 1, 1)μ_A + P(N_A, n_B-1, 1, 1)μ_B + P(N_A, n_B, 1, 0)r_B, for n_B:[1,N_B-1]

P(n_A, N_B, 1, 1)(μ_A + p_A + nu) = P(n_A, N_B-1, 1, 1)μ_B + P(n_A-1, N_B, 1, 1)μ_A + P(n_A, N_B, 0, 1)r_A, for n_A:[1,N_A-1]

P(N_A, N_B, 1, 1)nu = P(N_A, N_B-1, 1, 1)μ_B + P(N_A-1, N_B, 1, 1)μ_A

P(0, n_B, 1, 1)(μ_A + p_A + μ_B + p_B) = P(0, n_B-1, 1, 1)μ_B + P(1, n_B+1, 1, 1)nu + P(0, n_B, 1, 0)r_B + P(0, n_B, 0, 1)r_A, for n_B:[1,N_B-1]

P(n_A, 0, 1, 1)(μ_A + p_A + μ_B + p_B) = P(n_A-1, 0, 1, 1)μ_A + P(n_A+1,1, 1, 1)nu + P(n_A, 0, 1, 0)r_B + P(n_A, 0, 0, 1)r_A, for n_A:[1,N_A-1]

P(0, 0, 1, 1)(μ_A + p_A + μ_B + p_B) = P(1, 1, 1, 1)nu + P(0, 0, 1, 0)r_B + P(0, 0, 0, 1)r_A

P(0, N_B, 1, 1)(μ_A + p_A) = P(0, N_B-1, 1, 1)μ_B + P(0, N_B, 0, 1)r_A

P(N_A, 0, 1, 1)(μ_B + p_B) = P(N_A-1, 0, 1, 1)μ_A + P(N_A, 0, 1, 0)r_B

Normalizing Equation:

$\sum_{\alpha_A=0}^{1}\sum_{\alpha_B=0}^{1}\sum_{n_A=0}^{N_A}\sum_{n_B=0}^{N_B}[P(n_A, n_B, \alpha_A, \alpha_B)] = 1$

Outputs for all these cases will be compared in the Numerical Analysis. We will be investigating the performance measures of these systems and will be able to decide which system is preferable in which parameter set.

5. NUMERICAL ANALYSIS AND COMPARISONS

We will first have a look at the performance measures of three different cases under some initial parameter sets with changing buffer sizes. After that we will compare these systems and policies with each other for varying parameters such as; failure and repair rates. To start with, in Table 5.1, we have the first parameter set. The parameters in the table are respectively: Probability of an A part to arrive rather than a B part for the Flexible Machine cases, production rates of tools A and B for each case, production rate of the assembly machine, failure rates of tools A and B for each case, repair rates of tools A and B for each case and rate of repairing both tools for the Repair Together policy.

We should keep in mind that the rate of repairing both tools together is selected in such a way that it will not be faster than repairing only one tool and it will not be slower than repairing two individual tools seperately at different times. This is because the time lost for stopping and restarting the system is shared by two tools when we repair them together. Hence, we do gain some time by repairing the tools together.

Also, production and failure rates of the Two Independent Machine(2IM) case are taken as half the rates of flexible machine cases in order to make a fair comparison. A flexible machine operates at twice the speed of each independent machines since it can produce only one part at a time. Therefore, failure rates of the flexible machine are two times the failure rates of the independent machines.

Our performance measures for each case are average throughput and average inventory levels. We can see the results reached with the first parameter set we have chosen for different buffer sizes in Table 5.2. Also in Figure 5.1, we have the graph of Average Throughput levels with changing buffer sizes.

The results with this specific parameter set shows us first of all, that as the buffer sizes increase, average throughput and inventory levels tend to increase for all cases.

Table 5.1. Parameter Set No 1.

	Repair Together	*Stop and Repair*	*2 Ind. Machs*
Pr_A	0.5	0.5	N/A
μ_A	1	1	0.5
μ_B	1	1	0.5
nu	1	1	1
p_A	0.01	0.01	0.005
p_B	0.01	0.01	0.005
r_A	0.05	0.05	0.05
r_B	0.05	0.05	0.05
r_{both}	0.04	N/A	N/A

Considering the average throughput values, 2IM case stays behind for relatively small buffer sizes. However, after buffer sizes of 15, it gets ahead of the flexible machine cases. Repair Together(RT) policy gives better throughput results compared to Stop and Repair(SR) policy for mid and high buffer sizes. However, the advantage of repairing both tools together diminishes when the buffer size is small. We can see that for small buffer sizes, RT policy and SR policy give very close TH values. While observing the TH performances, we should also keep the bound values in mind. Maximum throughput values, that can possibly be achieved for each case, can be found by the equation coming from isolated efficiency formula (the prodcution rate of a machine when there is no blocking or starvation):

$$Maximum\,Throughput = \mu\left(\frac{r}{r+p}\right)$$

Here, the TH values reached with buffer size of 70 are 0.43 for RT, 0.42 for SR and 0.44 for 2IM and the bound levels for them respectively are 0.44, 0.42, 0.45. Meaning that SR case converges to its upper bound more quickly (it converges with buffer size 25). This makes sense, since with this policy there is no imbalance at the buffer levels, which minimizes the blocking effect of the flexible machine. Therefore after a while, increasing the buffer size doesn't have any positive effect on the performance. When

Table 5.2. Outputs for Set 1.

		Repair Together	*Stopping Policy*	*2 Ind. Machs*
N(A) = N(B) = 1	TH	**0.294245**	0.294118	0.227786
	Avg. Buffer A	0.499639	**0.470588**	0.498871
	Avg. Buffer B	0.499639	0.470588	0.498871
N(A) = N(B) = 2	TH	0.365700	**0.366135**	0.310371
	Avg. Buffer A	0.938700	**0.854008**	0.923719
	Avg. Buffer B	0.938700	0.854008	0.923719
N(A) = N(B) = 5	TH	**0.411577**	0.410984	0.384120
	Avg. Buffer A	2.016350	**1.747700**	1.978050
	Avg. Buffer B	2.016350	1.747700	1.978050
N(A) = N(B) = 10	TH	**0.419793**	0.416432	0.412501
	Avg. Buffer A	3.634940	**2.978620**	3.480050
	Avg. Buffer B	3.634940	2.978620	3.480050
N(A) = N(B) = 15	TH	0.422015	0.416654	**0.423575**
	Avg. Buffer A	5.371770	**4.185830**	4.907110
	Avg. Buffer B	5.371770	4.185830	4.907110
N(A) = N(B) = 20	TH	0.423610	0.416666	**0.429925**
	Avg. Buffer A	7.229690	**5.408960**	6.297560
	Avg. Buffer B	7.229690	5.408960	6.297560
N(A) = N(B) = 25	TH	0.424956	0.416667	**0.434102**
	Avg. Buffer A	9.135510	**6.642530**	7.656750
	Avg. Buffer B	9.135510	6.642530	7.656750
N(A) = N(B) = 30	TH	0.426129	0.416667	**0.437066**
	Avg. Buffer A	11.034500	**7.881780**	8.990980
	Avg. Buffer B	11.034500	7.881780	8.990980
N(A) = N(B) = 70	TH	0.432066	0.416667	**0.446452**
	Avg. Buffer A	24.791800	**17.852500**	19.275600
	Avg. Buffer B	24.791800	17.852500	19.275600

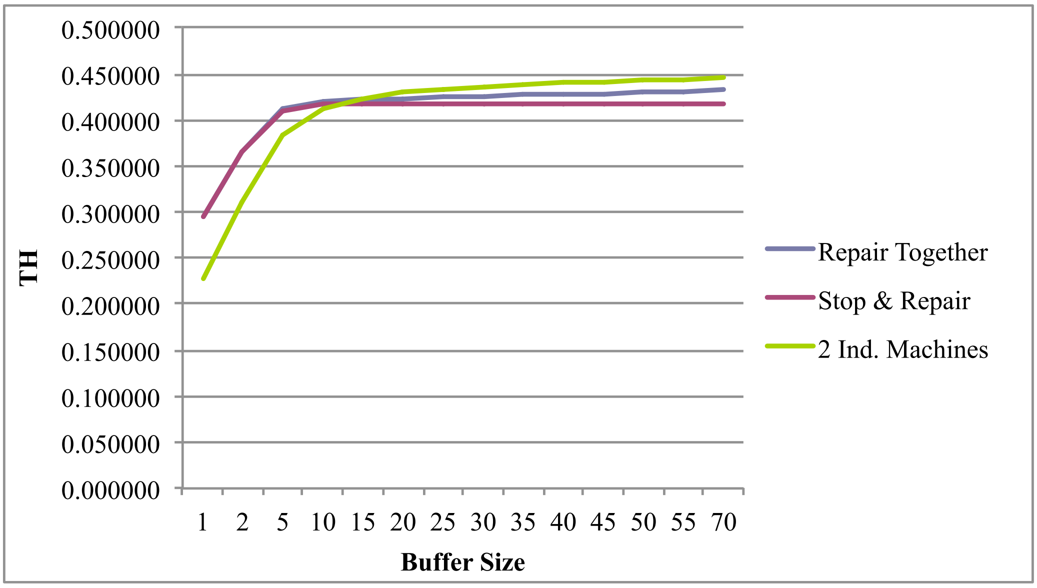

Figure 5.1. TH-Buffer Size Set1.

we look at the average inventory levels, as mentioned before, SR policy gives the best results since the order of parts never changes in this policy. When a part is picked by the flexible machine, that part is finished for sure and then another picking is made. Since the probability of picking each part is given as 0.5, it is understandable to have more balanced buffer levels with this policy, especially when compared to RT case, which gives the worst results. Order of parts doesn't change for 2IM case either, but in that case while one of the tools is repaired the other machine goes on producing the other part. Therefore, there is no such buffer balance as in SR case.

Now let's try another parameter set (Table 5.3). This time the failure rates are 5 times faster than the previous case. Outputs of Parameter Set 2 can be observed in Table 5.4 and Figure 5.2.

Some points that we reach from the outputs of set 2 can be stated as such: With increasing failure rates, TH levels of RT policy is outperforming SR policy. This is understandable since in this policy machine is not stopped for each failure. However, after buffer levels of 10, we see that 2IM case is again the best in TH performance. The TH values at buffer size of 50 and the bound values of each case are such: RT = 0.29/ 0.31, SR = 0.25/0.25, 2IM = 0.32/0.33. About inventory levels, again SR policy gives the best values and RT the worst.

Table 5.3. Parameter Set No 2.

	Repair Together	*Stop and Repair*	*2 Ind. Machs*
Pr_A	0.5	0.5	N/A
μ_A	1	1	0.5
μ_B	1	1	0.5
nu	1	1	1
p_A	0.05	0.05	0.025
p_B	0.05	0.05	0.025
r_A	0.05	0.05	0.05
r_B	0.05	0.05	0,05
r_{both}	0,04	N/A	N/A

In the Parameter set 3 (Table 5.5), repair rates are increased by five times. Results of parameter set 3 can be seen in Figure 5.3 and Table 5.6.

After observing the results of parameter set 3 we can say that, with increasing repair rates, the average throughputs of the 3 cases reach very high values and become very close to each other. All three cases go beyond the throughput rate of 0.48. SR case reaches the max. throughput value it can reach at early buffer levels of 20-25. The other two cases are also very close to their bound values at buffer levels of 45. For some small buffer sizes, TH performance of SR policy is interestingly outperforming RT policy. But in general, RT case is slightly better than the others, except for very high buffer levels, where 2IM case gets slightly ahead. However, considering the inventory levels, RT case is giving the worst results as observed in the previous parameter sets. This is expected, since production can turn one sided for quite long times in that policy, which causes fluctuations in the buffer levels.

In the next parameter set we will have unbalanced failure rates. Failure rate for tool A is 5 times the failure rate of tool B in Parameter Set 4. (Table 5.7) Results of parameter set 4 for different buffer sizes can be seen in Table 5.8 and Figure 5.4.

Table 5.4. Outputs for Set 2.

		Repair Together	*Stop and Repair*	*2 Ind. Machs*
N(A) = N(B) = 1	TH	**0.201373**	0.200000	0.168498
	Avg. Buffer A	0.494279	**0.400000**	0.494505
	Avg. Buffer B	0.494279	0.400000	0.494505
N(A) = N(B) = 2	TH	**0.234602**	0.231317	0.213465
	Avg. Buffer A	0.940244	**0.693950**	0.928879
	Avg. Buffer B	0.940244	0.693950	0.928879
N(A) = N(B) = 5	TH	**0.260298**	0.248209	0.255785
	Avg. Buffer A	2.115180	**1.455940**	2.062460
	Avg. Buffer B	2.115180	1.455940	2.062460
N(A) = N(B) = 10	TH	0.272766	0.249940	**0.279712**
	Avg. Buffer A	3.894830	**2.647060**	3.708340
	Avg. Buffer B	3.894830	2.647060	3.708340
N(A) = N(B) = 15	TH	0.279326	0.249997	**0.291961**
	Avg. Buffer A	5.571910	**3.855750**	5.210390
	Avg. Buffer B	5.571910	3.855750	5.210390
N(A) = N(B) = 20	TH	0.283730	0.250000	**0.299642**
	Avg. Buffer A	7.152650	**5.083120**	6.627500
	Avg. Buffer B	7.152650	5.083120	6.627500
N(A) = N(B) = 25	TH	0.286934	0.250000	**0.304917**
	Avg. Buffer A	8.658060	**6.319780**	7.993850
	Avg. Buffer B	8.658060	6.319780	7.993850
N(A) = N(B) = 30	TH	0.289377	0.250000	**0.308764**
	Avg. Buffer A	10.109500	**7.561120**	9.328960
	Avg. Buffer B	10.109500	7.561120	9.328960
N(A) = N(B) = 50	TH	0.295236	0.250000	**0.317395**
	Avg. Buffer A	15.613400	**12.544400**	14.520100
	Avg. Buffer B	15.613400	12.544400	14.520100

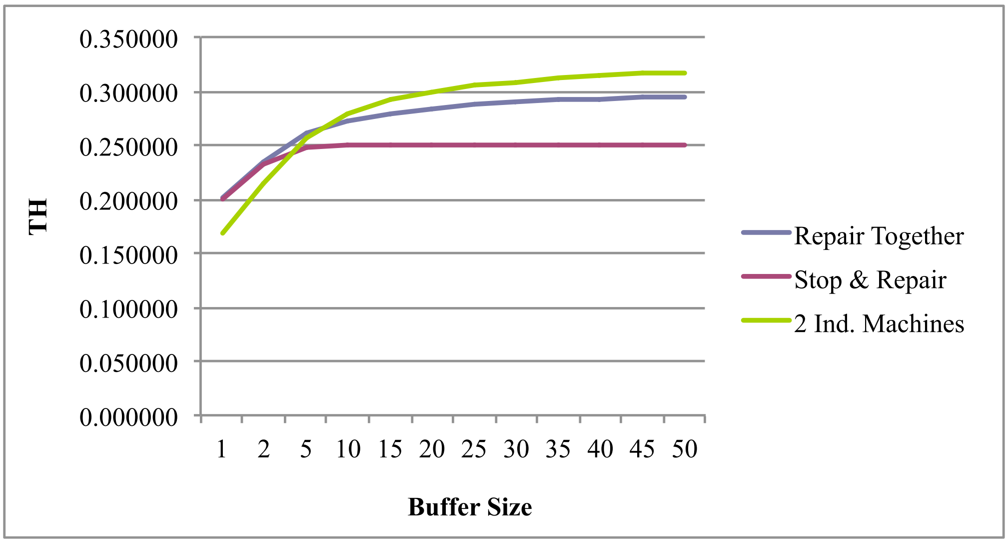

Figure 5.2. TH-Buffer Size Set2.

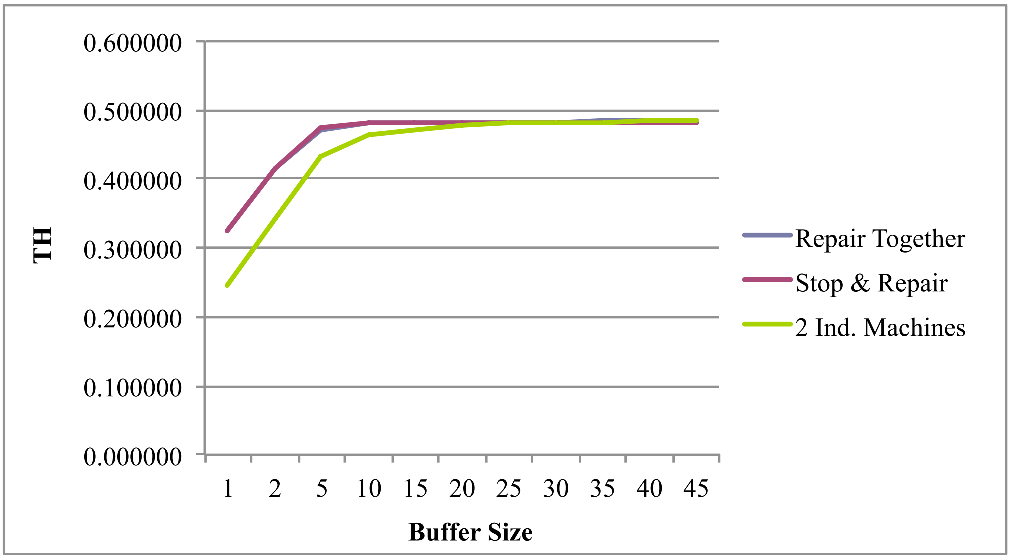

Figure 5.3. TH-Buffer Size Set3.

Looking at outputs of parameter set 4, we see that: With unbalanced failure rates, three cases give close throughput values. For small buffer sizes, 2IM case stays behind the others and after buffer sizes of 10 it gets slightly ahead of them. With this parameter set, SR policy is giving even better inventory results since it is not effected by the unbalanced failure rates. In SR case, picking probabilities Pr_A and Pr_B are the only factors effecting the buffer balance. For high buffer sizes of around 45, we observe that the average inventory level of SR policy is almost half of the other cases.

Table 5.5. Parameter Set No 3.

	Repair Together	*Stop and Repair*	*2 Ind. Machs*
Pr_A	0.5	0.5	N/A
μ_A	1	1	0.5
μ_B	1	1	0.5
nu	1	1	1
p_A	0.01	0.01	0.005
p_B	0.01	0.01	0.005
r_A	0.25	0.25	0.25
r_B	0.25	0.25	0.25
r_{both}	0.2	N/A	N/A

5.1. Performance Measures Under Changing Failure Rates

In this part, we will observe the effects of changing failure rates while other parameters are kept constant. In the following subsection, failure rates will be changed in a balanced way, i.e. failure rates for tools A and B will be the same in all cases. In the next subsection, results will be observed for unbalanced changes of failure rates. Failure rates for 2IMs will be kept as half of the flexible machine cases in all experiments, since they work at half speed.

5.1.1. Balanced Changes of Failure Rates

Figures 5.5 and 5.6 shows the effect of failure rate on average throughput and average inventory respectively, for small buffer sizes (N = 2). As observed in the figures; for small buffer sizes, RT policy and SR policy give very close TH results, whereas 2IM case stays slightly behind the two, until we have very high failure rates. In the end, when the failure rates become 0.1, TH value of each case is around 0.16. Results being so close to each other means that, small buffer sizes restricts 2IM and RT cases from showing their throughput superiority even at times of high failure rates. When we compare the inventory levels, by far the best results are obtained by SR policy and the

Table 5.6. Outputs for Set 3.

		Repair Together	*Stop and Repair*	*2 Ind. Machs*
N(A) = N(B) = 1	TH	**0.324706**	0.324675	0.245505
	Avg. Buffer A	0.499920	**0.493506**	0.499171
	Avg. Buffer B	0.499920	0.493506	0.499171
N(A) = N(B) = 2	TH	0.413647	**0.414668**	0.343485
	Avg. Buffer A	0.932265	**0.911919**	0.917563
	Avg. Buffer B	0.932265	0.911919	0.917563
N(A) = N(B) = 5	TH	0.471809	**0.473167**	0.432670
	Avg. Buffer A	1.956310	**1.859730**	1.920430
	Avg. Buffer B	1.956310	1.859730	1.920430
N(A) = N(B) = 10	TH	**0.480509**	0.480450	0.462893
	Avg. Buffer A	3.477690	**3.107710**	3.291070
	Avg. Buffer B	3.477690	3.107710	3.291070
N(A) = N(B) = 15	TH	**0.481614**	0.480752	0.472221
	Avg. Buffer A	5.154960	**4.316080**	4.577390
	Avg. Buffer B	5.154960	4.316080	4.577390
N(A) = N(B) = 20	TH	**0.482139**	0.480768	0.476788
	Avg. Buffer A	6.985480	**5.538800**	5.844670
	Avg. Buffer B	6.985480	5.538800	5.844670
N(A) = N(B) = 25	TH	**0.482536**	0.480769	0.479504
	Avg. Buffer A	8.882050	**6.771940**	7.104910
	Avg. Buffer B	8.882050	6.771940	7.104910
N(A) = N(B) = 30	TH	**0.482869**	0.480769	0.481305
	Avg. Buffer A	10.780100	**8.010860**	8.361700
	Avg. Buffer B	10.780100	8.010860	8.361700
N(A) = N(B) = 45	TH	0.483645	0.480769	**0.484289**
	Avg. Buffer A	16.246700	**11.743000**	12.123000
	Avg. Buffer B	16.246700	11.743000	12.123000

Table 5.7. Parameter Set No 4.

	Repair Together	*Stop and Repair*	*2 Ind. Machs*
Pr_A	0.5	0.5	N/A
μ_A	1	1	0.5
μ_B	1	1	0.5
nu	1	1	1
p_A	0.05	0.05	0.025
p_B	0.01	0.01	0.005
r_A	0.05	0.05	0.05
r_A	0.05	0.05	0.05
r_{both}	0.04	N/A	N/A

other two policies are giving very close results.

In Figures 5.7 and 5.8, buffer size is taken to be higher this time. We observe that performance results are different for higher buffer sizes. Throughput results are quite close for very small failure rates. However, as the failure rates increase, RT policy and 2IM give significantly better TH values than SR policy. (RT = 0.21, SR = 0.17, 2IM = 0.22) It is logical to observe the advantage of RT policy and 2IM case for higher failure rates, since in these policies production is stopped less frequently at failures. It also makes sense to observe this difference only for higher buffer sizes and not for sizes of around 1-5. For these buffer limits, in RT policy, the buffers are more quickly being full and the system is stuck, so even though both tools are not broken we often need to repair one of the tools seperately. 2IM case also cannot show its advantage for small buffer sizes. When a tool is broken, the other buffer is getting full quickly blocking that machine. Therefore, production needs to stop until failed tool starts production again. When we look at the avg. inventory results, RT policy shows us that good TH values are not achieved for free. RT policy gives the highest inventory values, while SR policy gives the best results.

Table 5.8. Outputs for Set 4.

		Repair Together	*Stop and Repair*	*2 Ind. Machs*
N(A) = N(B) = 1	TH	**0.238497**	0.238095	0.193460
	Avg. Buffer A	0.400879	0.380952	0.419620
	Avg. Buffer B	0.596308	**0.476190**	0.574388
N(A) = N(B) = 2	TH	0.283442	**0.283517**	0.251790
	Avg. Buffer A	0.701387	0.697707	0.724790
	Avg. Buffer B	1.197780	**0.813914**	1.135660
N(A) = N(B) = 5	TH	**0.311899**	0.309508	0.302256
	Avg. Buffer A	1.292440	1.494700	1.317970
	Avg. Buffer B	3.002840	**1.634140**	2.809300
N(A) = N(B) = 10	TH	0.319128	0.312389	**0.321675**
	Avg. Buffer A	1.698860	2.689360	1.657610
	Avg. Buffer B	6.494770	**2.846880**	6.034820
N(A) = N(B) = 15	TH	0.321722	0.312495	**0.328047**
	Avg. Buffer A	1.880270	3.892900	1.664080
	Avg. Buffer B	10.444600	**4.057560**	9.789140
N(A) = N(B) = 20	TH	0.323266	0.312500	**0.330820**
	Avg. Buffer A	1.981300	5.116900	1.550600
	Avg. Buffer B	14.607500	**5.284130**	13.905900
N(A) = N(B) = 25	TH	0.324266	0.312500	**0.332120**
	Avg. Buffer A	2.010120	6.351550	1.394840
	Avg. Buffer B	18.905700	**6.519950**	18.284000
N(A) = N(B) = 30	TH	0.324927	0.312500	**0.332744**
	Avg. Buffer A	1.981600	7.591590	1.236920
	Avg. Buffer B	23.313600	**7.760690**	22.851300
N(A) = N(B) = 45	TH	0.325865	0.312500	**0.333265**
	Avg. Buffer A	1.727180	11.325500	0.890330
	Avg. Buffer B	37.062500	**11.495700**	37.208500

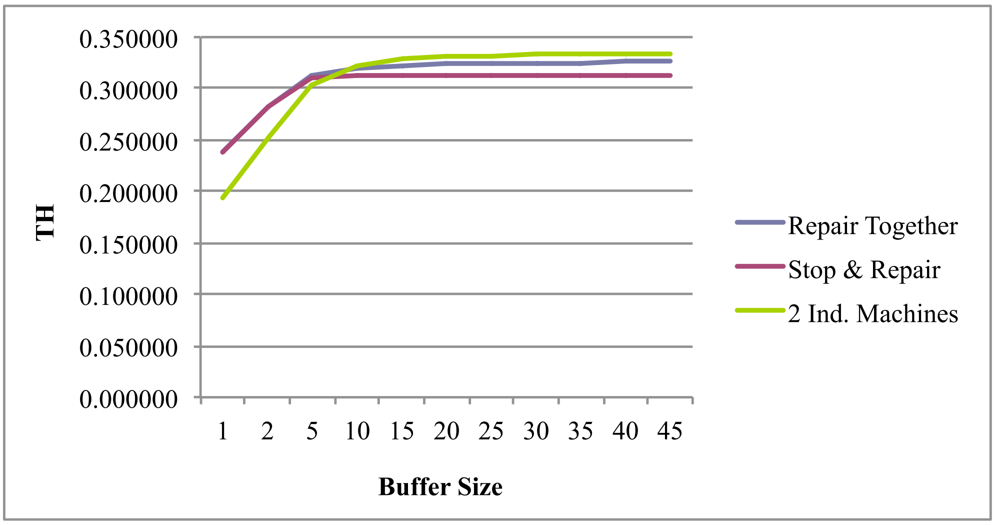

Figure 5.4. TH-Buffer Size Set4.

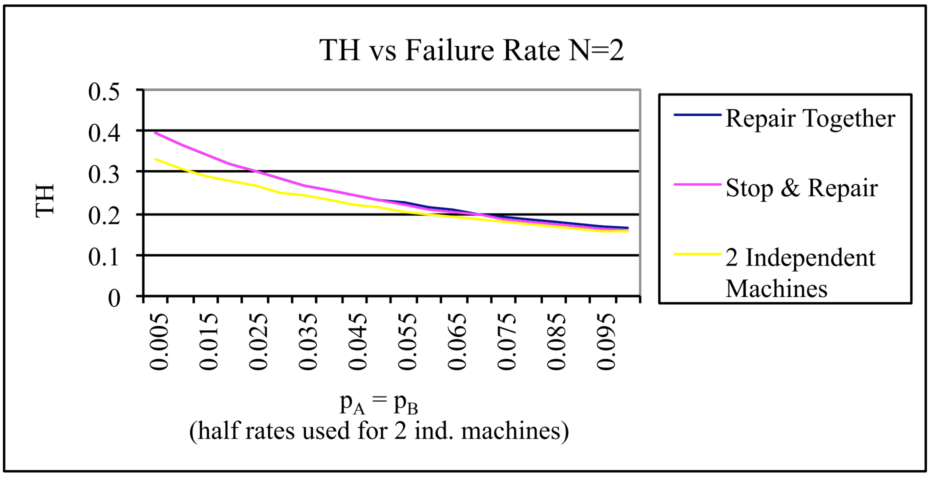

Figure 5.5. TH-Changing Failure Rate 1(Balanced).

5.1.2. Unbalanced Changes of Failure Rates

In this case, failure rates of the tools will not be identical. Our first results shown in figures 5.9, 5.10 and 5.11 are TH values, avg. inventory level for buffer A and avg. inventory level for buffer B respectively for changing failure rates, where the ratio between the rates is kept as 3/4. So, tool B is facing failures with a higher rate compared to tool A. When we check the TH results, we cannot see much of a different outcome compared to the balanced p failures. As the failure rates get higher, 2IM and RT cases are again giving better values then SR policy. This difference can be seen after buffer sizes of around 5-10. With smaller buffer sizes, TH performances of the cases are not much different. With increasing failure rates, for buffer size of 20, the

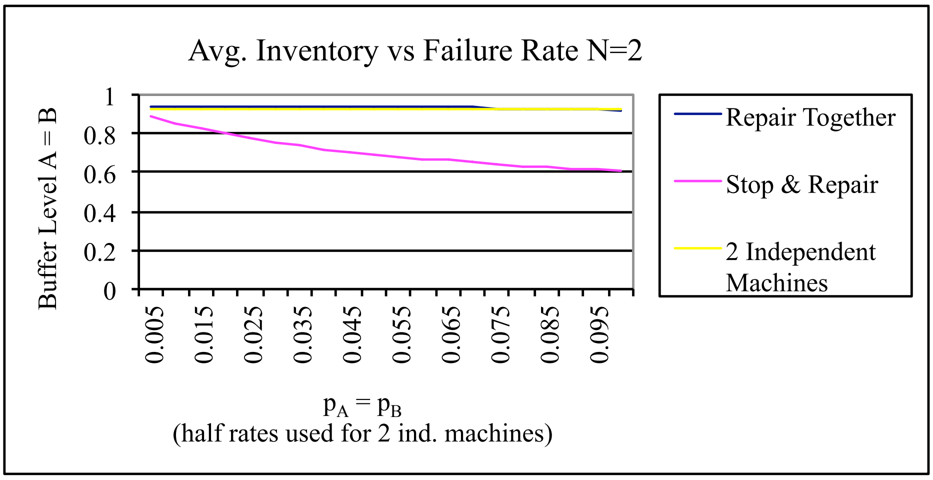

Figure 5.6. Average Inventory-Changing Failure Rate 1 (Balanced).

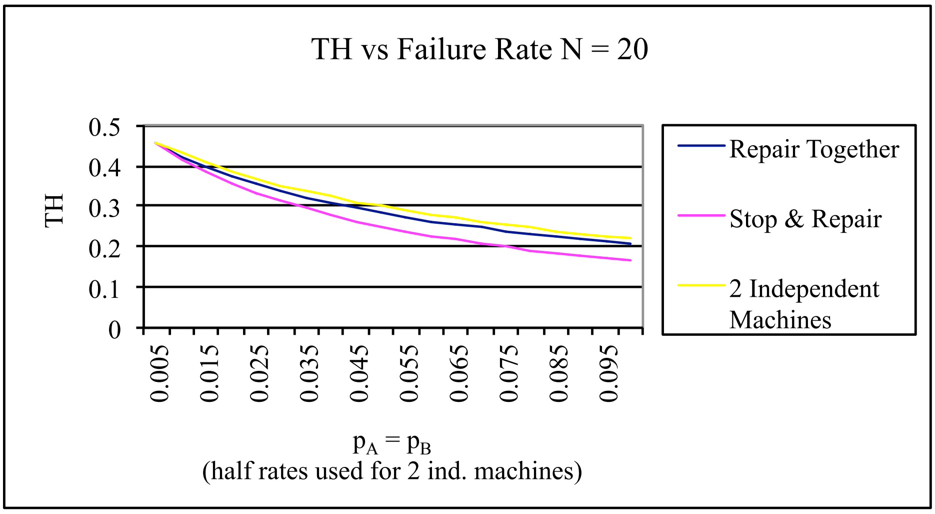

Figure 5.7. TH-Changing Failure Rate 2(Balanced).

TH values drop down to: RT = 0.19, SR = 0.15, 2IM = 0.20. Inventory levels can be observed in two seperate figures as mentioned. With unbalanced failure rates, RT policy and 2IM case cause higher buffer levels for part type A and much lower levels for part type B. However, SR case is not effected by this unbalanced failure rates thanks to the policy it uses. When we look at the overall inventory values, with unbalanced failure rates, SR case is even better than the others.

5.1.3. Changes in Failure Rates with Unbalanced Ratio

In this case the failure rates of the two tools will again be unbalanced but the ratio of the difference will not be kept at the same level. To do that, we keep p_A as

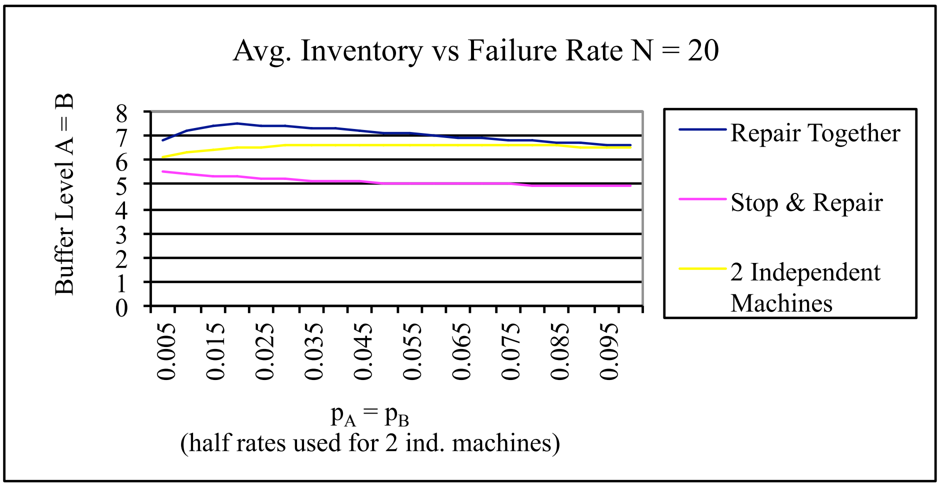

Figure 5.8. Average Inventory-Changing Failure Rate 2 (Balanced).

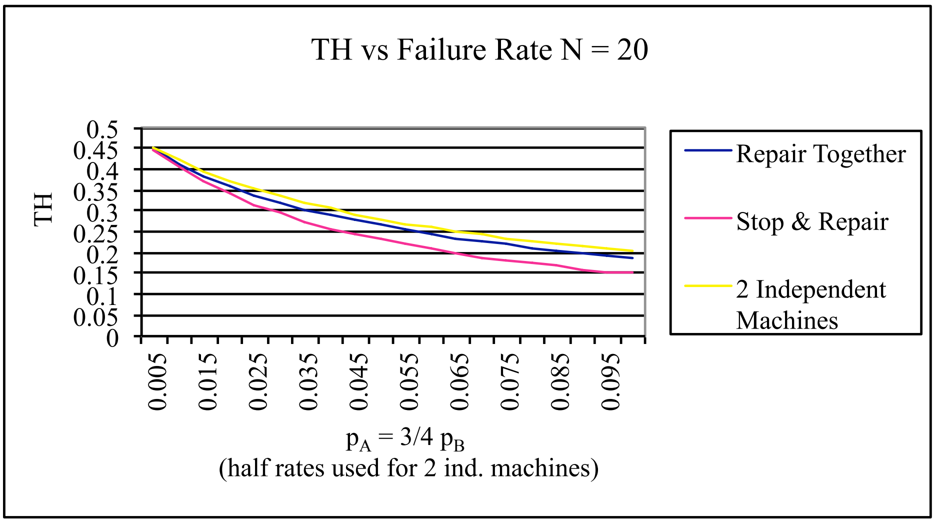

Figure 5.9. TH-Changing Failure Rate (Unbalanced).

0.01 and change p_B (For the 2IM case, half of these rates will be used). In figures 5.12, 5.13, 5.14 we can observe TH results, inventory of part A and inventory of part B respectively, for buffer size 20. The only major difference we see here compared to the constant ratio unbalanced failure case is, that SR policy is not falling behind at high failure rates here. Only very little TH difference is observed (RT = 0.34, SR = 0.33, 2IM = 0.35). This is meaningful, since now there is a big imbalance in the failure rates of the tools and SR is the most appropriate policy to deal with such imbalance. Other than being at almost the same TH levels with the other cases, SR policy also gives the best average inventory values as usual.

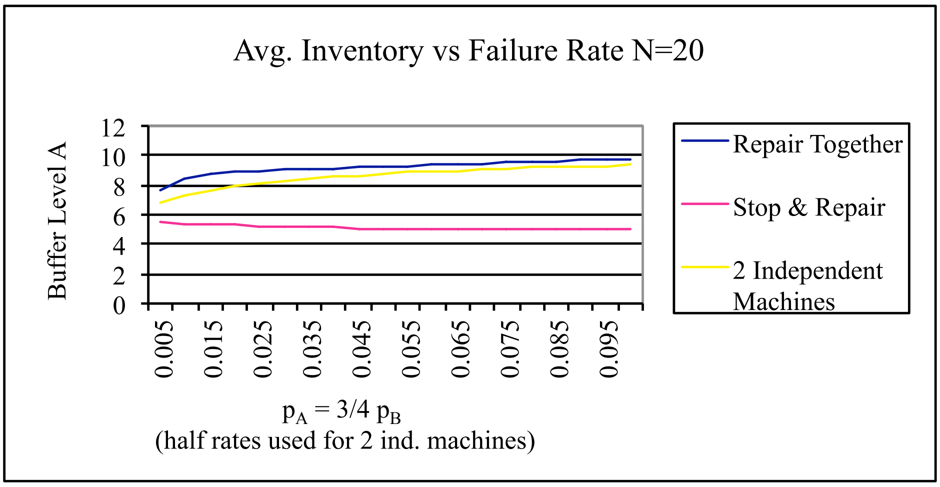

Figure 5.10. Average Inventory A-Changing Failure Rate (Unbalanced).

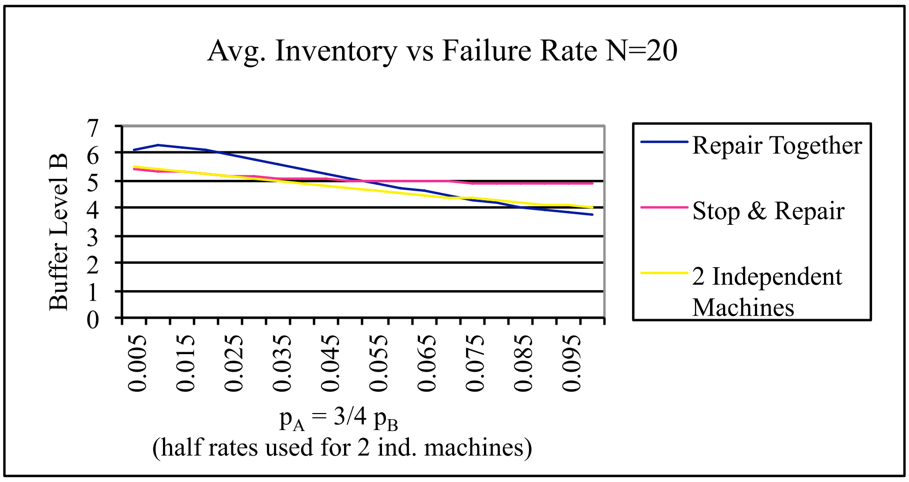

Figure 5.11. Average. Inventory B-Changing Failure Rate (Unbalanced).

5.2. Performance Measures Under Changing Repair Rates

In this part we will be investigating the effects of repair rates on the performance measures. Firstly repair rates will be changed in a balanced way, i.e. the rates for two tools will be the same. In the next part we will be changing them in an unbalanced way.

5.2.1. Balanced Changes of Repair Rates

We can observe TH and Inventory situations of the cases under changing repair rate parameters in Figures 5.15 and 5.16. Throughput behaviours of the three cases

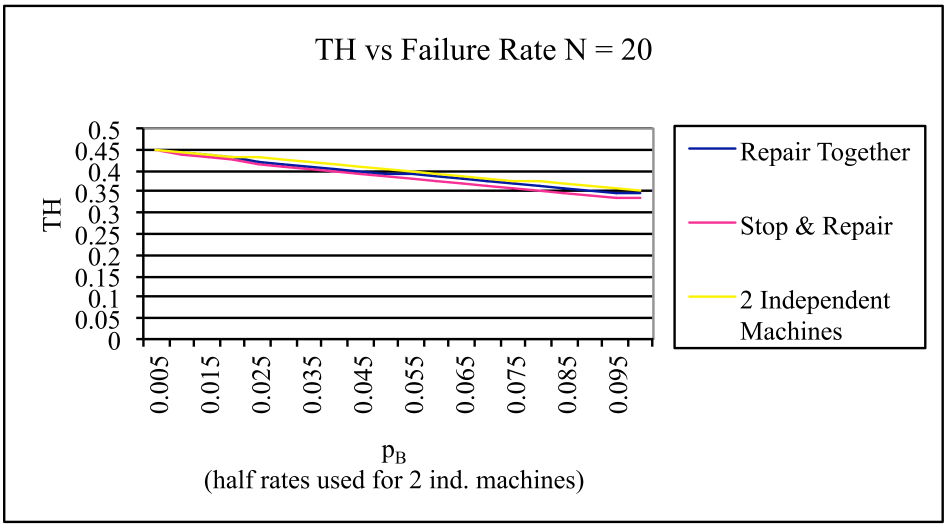

Figure 5.12. TH-Changing Failure Rate Ratio(Unbalanced).

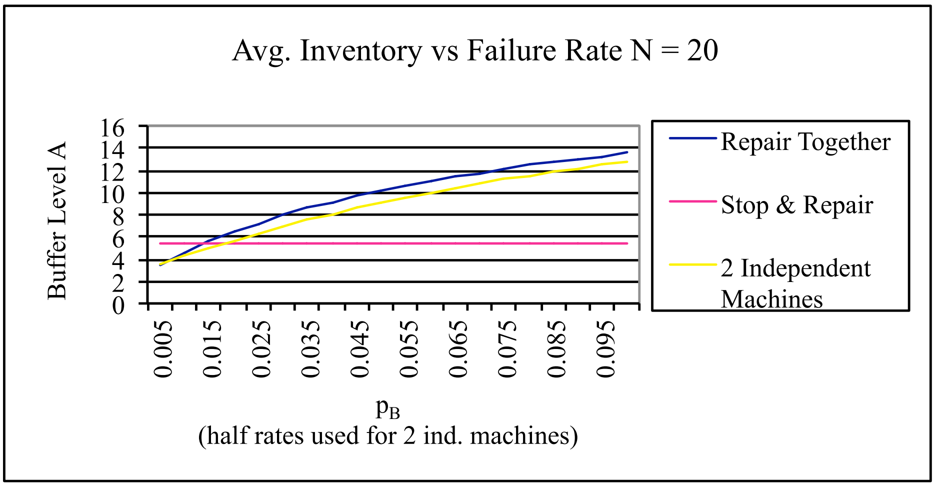

Figure 5.13. Average Inventory A-Changing Failure Rate Ratio (Unbalanced).

are quite similar. TH values increase for each of them together with higher repair rates and almost converging at the same 0.46 value for high repair rates like 0.1. However, it is interesting to note that for SR policy, average inventory level tends to increase with increasing repair rates while for the other two cases that is the opposite. This might be explained by the fact, that in SR policy an increase in TH can only be achieved with an increase in the buffer levels because this policy already keeps the buffers at the lowest levels possible. However in the other two cases, TH might also be increased by having more balanced buffer levels (here, by increasing the repair rates), since in these cases buffer levels might be quite high and that might not be effectively reflected to TH values.

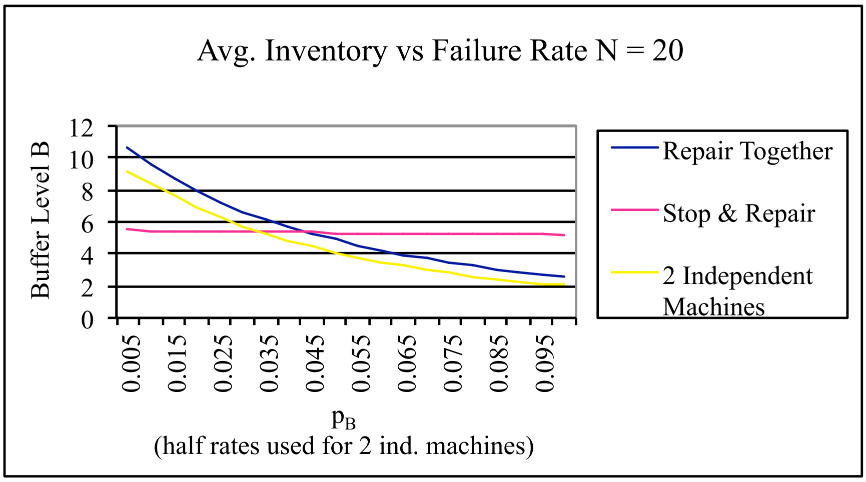

Figure 5.14. Average Inventory B-Changing Failure Rate Ratio (Unbalanced).

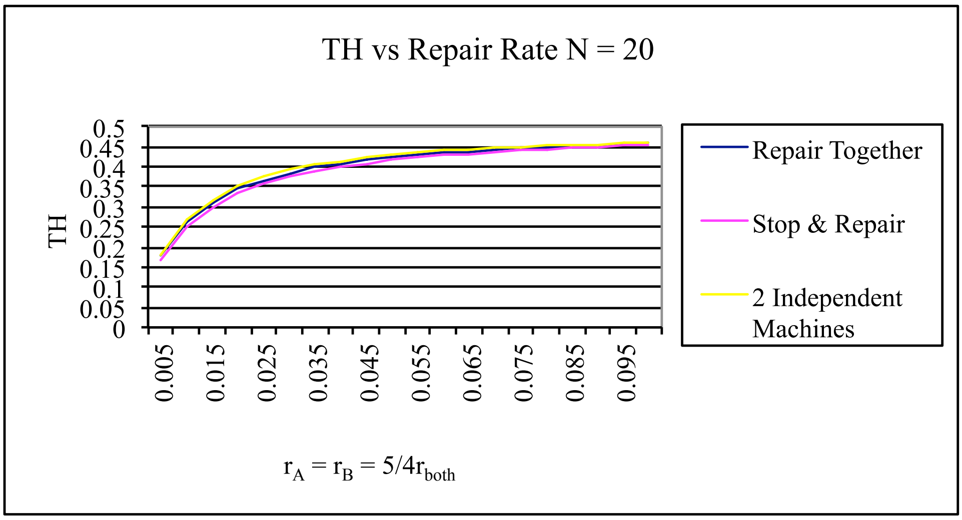

Figure 5.15. TH-Changing Repair Rate (Balanced).

5.2.2. Unbalanced Changes of Repair Rates

For this case, we will be changing repair rates, keeping a constant ratio between the tools. While changing r_A between $(0.05\, and\, 0.1)$, r_B will be $\frac{4}{3}r_A$ and r_{both} will be $\frac{4}{5}r_A$.

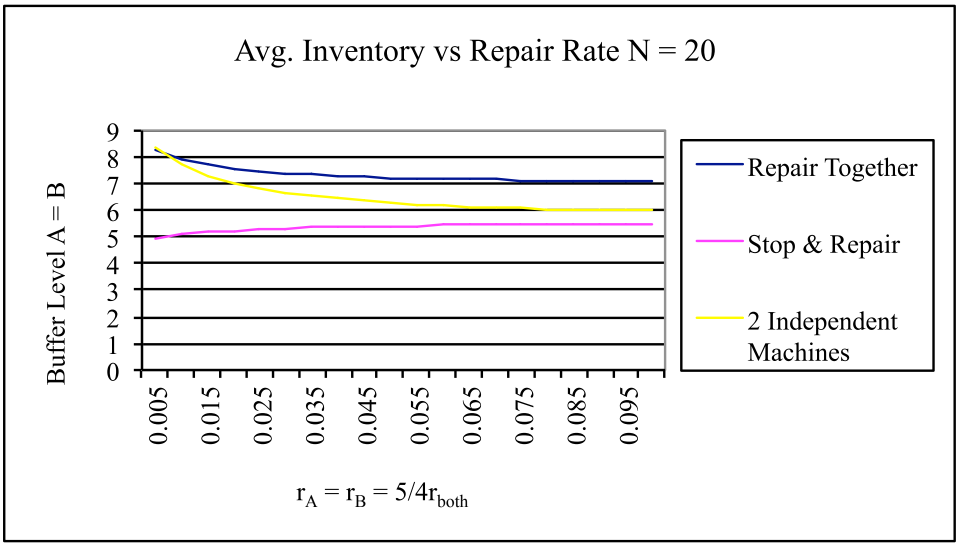

Figure 5.16. Average Inventory-Changing Repair Rate (Balanced).

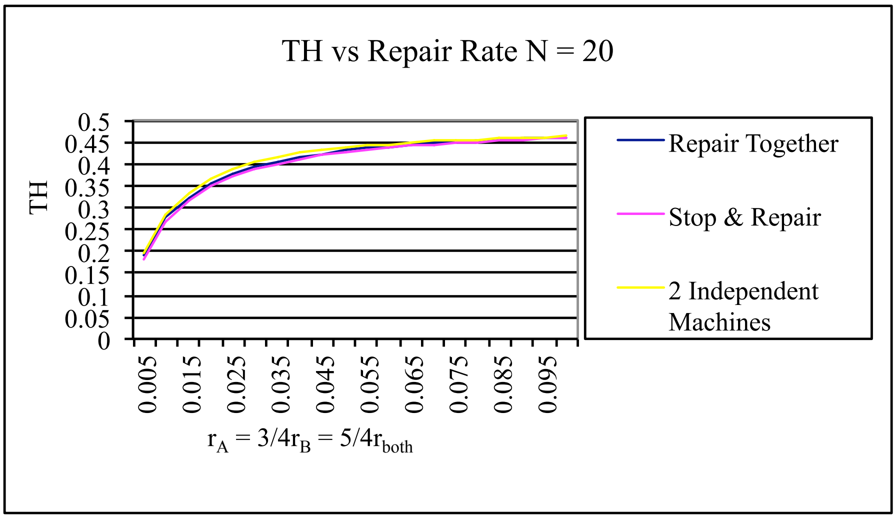

Figure 5.17. TH-Changing Repair Rate (Unbalanced).

Looking at Figure 5.17, 5.18 and 5.19, we see that throughput performances of the cases are very similar to each other. Especially as the repair rates increase, they all converge at 0.46. Inventory graphs show us, that unbalanced repair rates does not cause as much imbalance in the buffer levels as does the failure rates. Especially when the repair rates get higher, there is almost no imbalance. SR case is of course not effected from this unbalanced structure due to its policy, but the other cases are also only slightly effected. Worst inventory levels are observed in RT policy and as usual the best levels are observed in SR policy.

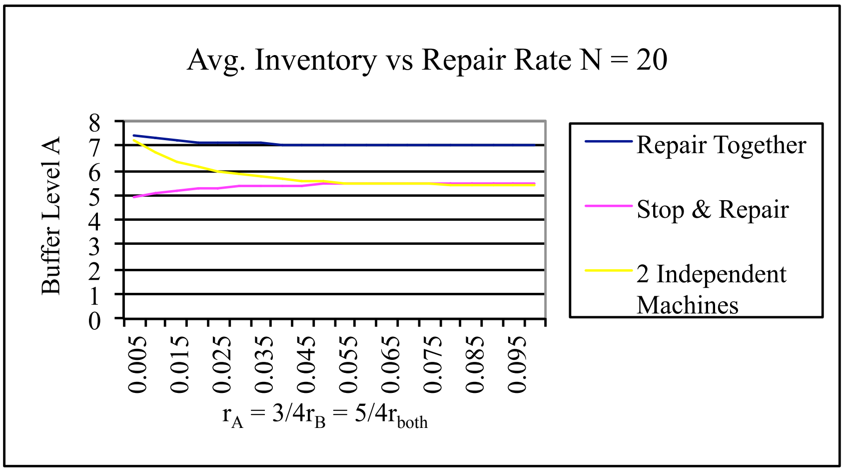

Figure 5.18. Average Inventory A-Changing Repair Rate (Unbalanced).

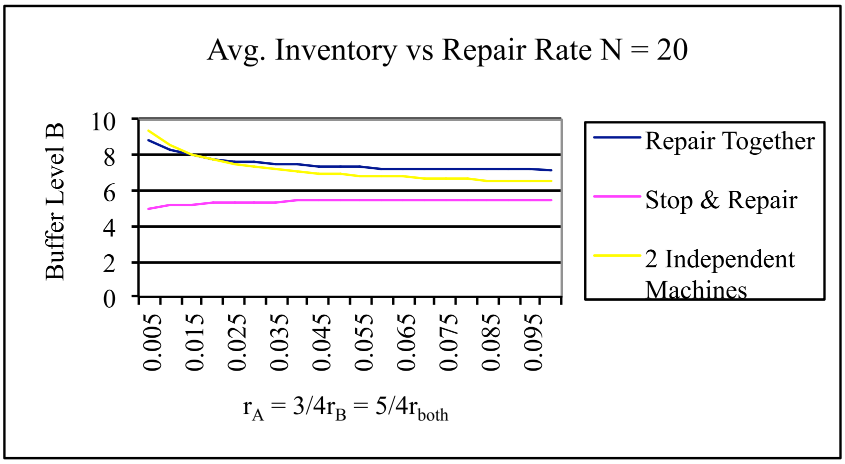

Figure 5.19. Average Inventory B-Changing Repair Rate (Unbalanced).

5.2.3. Changes in Repair Rates with Unbalanced Ratio

For this case we willl keep r_A and r_{both} constant and increase r_B alone. We can see the outputs in Figures 5.20, 5.21 and 5.22.

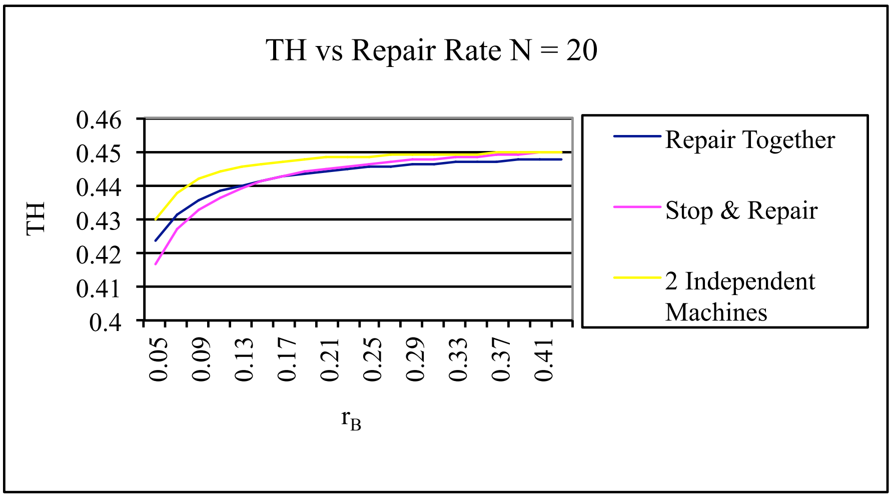

Figure 5.20. TH-Changing Repair Rate Ratio(Unbalanced).

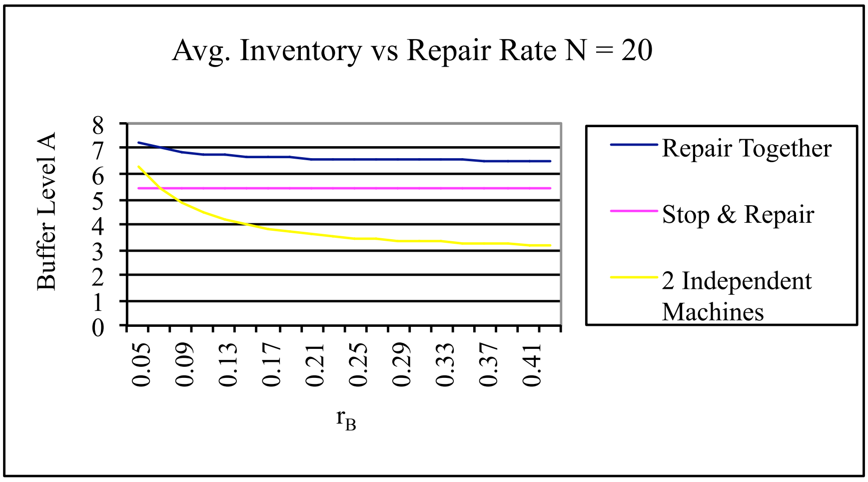

Figure 5.21. Average Inventory A-Changing Repair Rate Ratio (Unbalanced).

It is interesting to see that SR policy is ahead of RT policy in TH values after a little increase in r_B, even though the difference is rather small.(After r_B is above 0.19, SR gets slightly above RT and all three cases converge to very close values around 0.45 as r_B gets higher.) This is because the advantage of repairing the tools together is becoming ineffective in this case. We increase r_B to quite high values but r_{both} cannot be high because it has to be slower than repair rate of any individual tool. So it has to be kept slower than r_A. Therefore the difference between r_B and r_{both} increases and it looks like there is no point in not changing r_B alone while its repair rate is so fast. When we look at the inventory levels, the first thing to notice, is the terrible imbalance of the 2IM case as r_B increases. After a while buffer B is higher than three times Buffer A levels. But when we check the overall inventory levels, RT case stays with

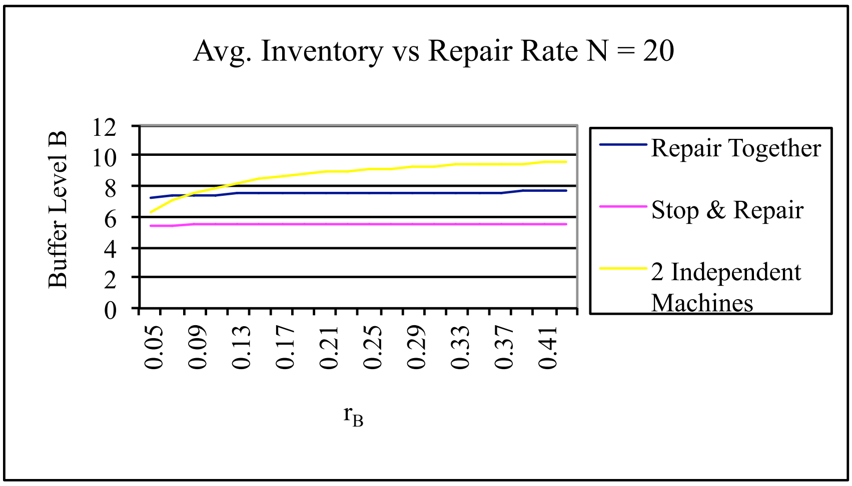

Figure 5.22. Average Inventory B-Changing Repair Rate Ratio (Unbalanced).

the biggest inventory for all r_B values. In this setup we can easily say that SR policy dominates the other cases by inventory levels it reaches and by not staying behind in the TH values immediately after r_B gets a bit higher.

5.3. Performance Measures When The Bottleneck is The Assembly Machine

In our analyses, the production rates of the assembly machine and the flexible machine are assigned as 1. For the 2IM case, each machine has production rate of 0.5, since they work at the same time. With these production rates, the bottleneck has always been the first workstation since the assembly machine doesn't have any failures and it can never be blocked. It is also interesting to investigate the results when the bottleneck turns out to be the assembly machine. To do that, we need higher production rates for the first workstation. Outputs of the three cases for relatively low and high buffer sizes (2 and 20) are in the following figures starting with figure 5.23. While obtaining these outputs, production rates of the flexible machine tools are increased from 1 to 10. (Half rates used for each of the independent machines) Failure rates of the machines are also changed with the same ratio as production rates.

The most important result that we obtain is, that when the bottleneck is the second workstation, SR policy gives better TH results compared to RT policy. This is

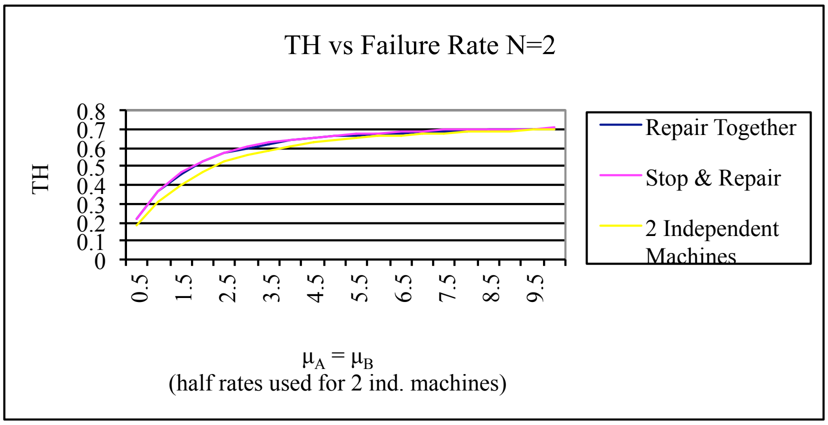

Figure 5.23. TH-Changing Production Rate 1.

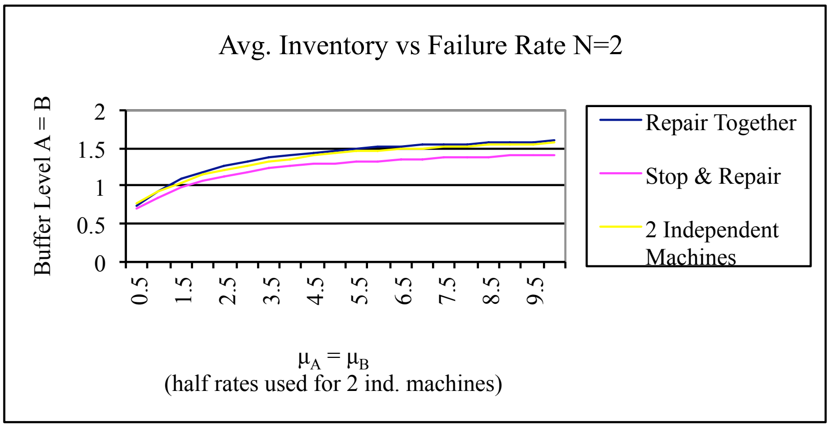

Figure 5.24. Average Inventory - Changing Production Rate 1.

due to the buffer balance of SR case. Since the bottleneck is the assembly machine, total TH can only be increased if we can increase the probability of having both buffers non-zero. This probability is for sure much higher in SR case than RT case. When we examine the figures for buffer sizes of 2, SR dominates the others in both TH and inventory performance. With production rates of 10, SR reaches TH values of 0.70 and the others follow with 0.69. Inventory level of each buffer at the same point is 1.4 for SR, which is followed by 2IM and RT with 1.56 and 1.59 respectively. When the buffer sizes increase to 20, TH performance of RT case stays far behind the others. 2IM leads with 0.85, SR follows with 0.84 and RT case stays behind with 0.74. There are no surprising results in the inventory values: SR case has the lowest level with 14.8 for each buffer; RT has 15.9 and 2IM has the worst result with 17.2.

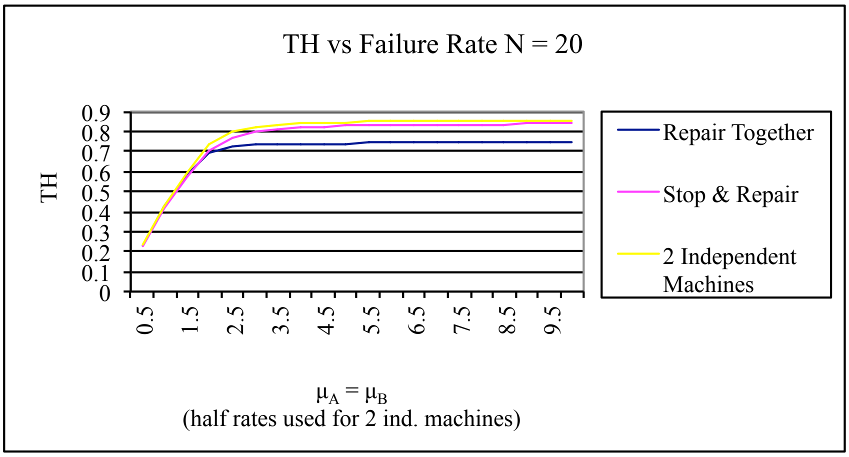

Figure 5.25. TH-Changing Production Rate 2.

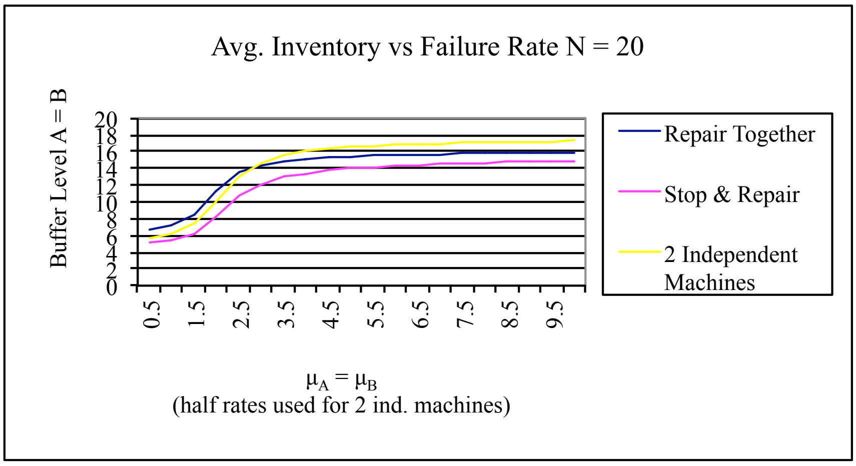

Figure 5.26. Average Inventory - Changing Production Rate 2.

5.4. Performance Measures Under Changing Failure Rates of 2IM

In the previous analyses, we have always kept the failure rates of each independent machine as half of the flexible machine rate. This is sensible since the production rate of the machines were also half of the flexible machine rate and slower tools should deteriorate more slowly compared to the faster ones. However, this rate of 2IM might not always be exactly half of the flexible machine case. So, it might be useful to investigate the results with changing failure rates of the independent machines. We have used failure rates between 0.005 to 0.01 for the 2IM case. Applying these rates to our common parameter set, we obtained the following TH results as seen in Table 5.10 and Figure 5.27.

Table 5.9. Parameter Set For Changing Failure Rates of 2IM.

	Repair Together	*Stop and Repair*	*2 Ind. Machs*
Pr_A	0.5	0.5	N/A
μ_A	1	1	0.5
μ_B	1	1	0.5
nu	1	1	1
p_A	0.01	0.01	changing
p_B	0.01	0.01	changing
r_A	0.05	0.05	0.05
r_B	0.05	0.05	0.05
r_{both}	0.04	N/A	N/A

Table 5.10. TH Results For Changing Failure Rates of 2IM.

			Two Independent Machines					
N	*RT*	*SR*	*p = 0.005*	*p = 0.006*	*p = 0.007*	*p = 0.008*	*p = 0.009*	*p = 0.01*
N = 1	**0.29425**	0.29412	0.22779	0.22382	0.21999	0.21629	0.21272	0.20927
N = 2	0.36570	**0.36614**	0.31037	0.30334	0.29664	0.29025	0.28413	0.27828
N = 5	**0.41158**	0.41098	0.38412	0.37430	0.36501	0.35622	0.34788	0.33996
N = 10	**0.41979**	0.41643	0.41250	0.40237	0.39280	0.38373	0.37514	0.36697
N = 15	0.42202	0.41665	**0.42358**	0.41370	0.40435	0.39547	0.38703	0.37900
N = 20	0.42361	0.41667	**0.42993**	0.42030	0.41116	0.40246	0.39418	0.38628
N = 25	0.42496	0.41667	**0.43410**	0.42467	0.41570	0.40715	0.39899	0.39120
N = 30	0.42613	0.41667	**0.43707**	**0.42779**	0.41895	0.41052	0.40246	0.39475
N = 35	0.42717	0.41667	**0.43928**	**0.43013**	0.42139	0.41305	0.40507	0.39743
N = 40	0.42809	0.41667	**0.44100**	**0.43194**	0.42330	0.41503	0.40711	0.39952
N = 45	0.42892	0.41667	**0.44236**	**0.43339**	0.42482	0.41661	0.40875	0.40121
N = 50	0.42967	0.41667	**0.44348**	**0.43458**	0.42607	0.41791	0.41009	0.40259

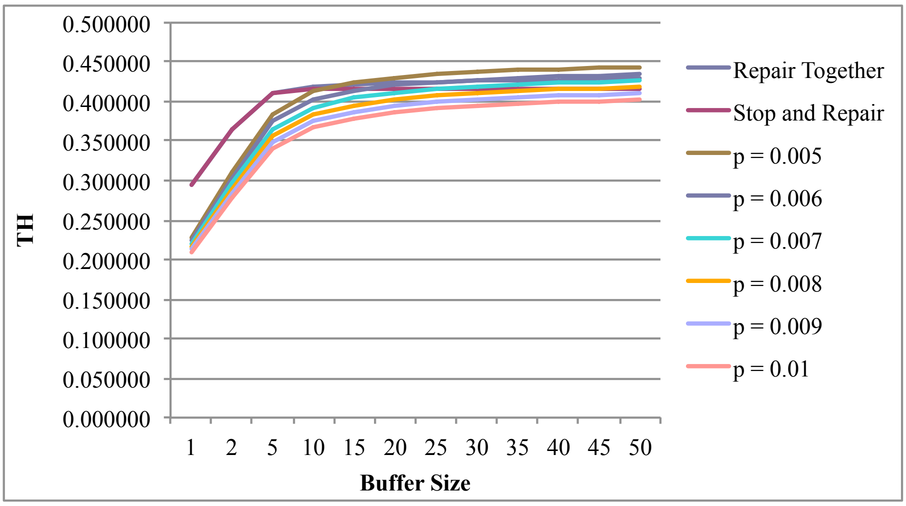

Figure 5.27. TH - Buffer Changing Failure Rates for 2IM.

The outputs of Table 5.10 proves the importance of failure rates of each independent machine on TH performance. When it is 0.005 (half of the flexible machine cases as we have used in the previous sections) 2IM case gives the best TH values after buffer sizes of 15. However, if the failure rate is above 0.006, 2IM case never gives the best TH values for any buffer size. This means that failure rates of the independent machines can change the results upside down; 2IM case might as well be the worst case considering the TH performances. We can also have a look at the inventory performances for these rates in Table 5.11 and Figure 5.28. Changing the failure rates of 2IM does not cause much difference in terms of average inventory performance. 2IM case remains in between RT and SR for all rates and buffer sizes.

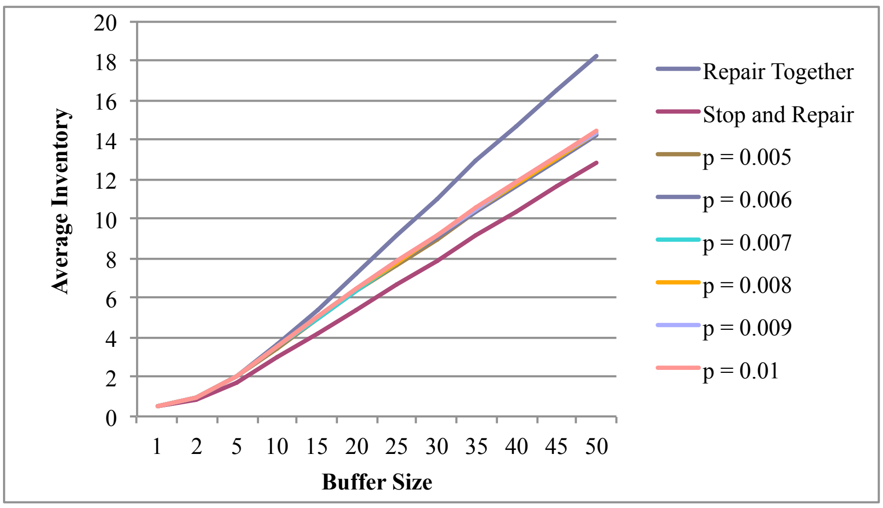

Figure 5.28. Average Inventory - Buffer Changing Failure Rates for 2IM.

Table 5.11. Inventory Results For Changing Failure Rates of 2IM.

			Two Independent Machines					
N	*RT*	*SR*	*p = 0.005*	*p = 0.006*	*p = 0.007*	*p = 0.008*	*p = 0.009*	*p = 0.01*
N = 1	0.49964	**0.47059**	0.49887	0.49865	0.49842	0.49820	0.49798	0.49776
N = 2	0.93870	**0.85401**	0.92372	0.92443	0.92507	0.92565	0.92617	0.92663
N = 5	2.01635	**1.74770**	1.97805	1.98682	1.99486	2.00222	2.00895	2.01512
N = 10	3.63494	**2.97862**	3.48005	3.50746	3.53199	3.55396	3.57364	3.59128
N = 15	5.37177	**4.18583**	4.90711	4.94993	4.98717	5.01961	5.04790	5.07259
N = 20	7.22969	**5.40896**	6.29756	6.35014	6.39476	6.43275	6.46519	6.49291
N = 25	9.13551	**6.64253**	7.65675	7.71506	7.76367	7.80442	7.83869	7.86758
N = 30	11.03450	**7.88178**	8.99098	9.05276	9.10366	9.14587	9.18104	9.21041
N = 35	12.89890	**9.12424**	10.30650	10.37060	10.42300	10.46610	10.50180	10.53140
N = 40	14.71880	**10.36870**	11.60830	11.67410	11.72750	11.77130	11.80740	11.83720
N = 45	16.49280	**11.61440**	12.90010	12.96710	13.02140	13.06570	13.10210	13.13210
N = 50	18.22350	**12.86100**	14.18430	14.25240	14.30740	14.35220	14.38890	14.41900

6. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

In this study, the impact of maintenance policies on a 2M1B system has been investigated using a Markovian model. After obtaining steady-state solutions, three different cases were compared considering average throughput and average inventory levels. Second workstation for all cases was composed of a totally reliable assembly machine, whereas first workstation differed in these three cases as follows:

- 1 flexible machine with Stop and Repair(SR) maintenance policy,
- 1 flexible machine with Repair Together(RT) maintenance policy,
- 2 independent ordinary machines (2IM).

Before going into 2M1B system analyses, single machine systems were investigated. If we had infinite buffer levels, the results obtained for the single machine systems could also be used for the 2M1B system. However, this is not always the case in reality. Having finite buffer capacities causes blockings and therefore it has a big influence on the overall performance of the system. This blocking effect is especially important for the RT policy because the production strategy is not well balanced as in the other cases. Hence, the study on 2M1B system was essential to reach healthy results.

In the last chapter of the study, some numerical outputs were obtained, which can be useful for a managerial decision maker and for future research in this area. Main results that were obtained in the Numerical Analysis chapter can be stated as such:

- As expected, SR policy is dominating the other two, considering the average inventory levels. Especially when the tools have unbalanced failure rates, the buffer levels can be as low as half of the other cases.
- Except for small buffer sizes, 2IM case shows the advantage of being the simpler model by outperforming the others in average throughput performance. However, this superiority is diminished when very large repair rates are used, because that

reduces the effect of failure.

- For small buffer sizes, SR policy can give very close average throughput results compared to the RT policy. But for higher buffer sizes, RT policy achieves better throughput results, although that results in high buffer levels. In general, RT policy causes the worst inventory levels within the three.
- Increased failure rates create an advantage for RT and 2IM cases, because in SR policy every failure causes the machine to stop. Therefore the TH performance of SR is effected at a higher level, compared to the other cases.
- When we have high production rates for the first workstation which causes the assembly machine to be the bottleneck, TH results of SR policy are better than the RT policy. This means that SR policy totally dominates RT policy in both of the performance measures if the bottleneck is the second workstation.
- The failure rates of the independent machines, which are working at half speed of the flexible machine, is an important factor effecting the TH performance results. It is sensible to think that a tool working at half speed will deteriorate at half speed. However, this might not always be the case in practice and TH performance of 2IM case higly depends on this rate.

After looking at these results, we can say that a preference between these cases can only be made according to the facilities and primary needs of the decision maker and rates of the machines available in the related market. A study with a cost function can contribute for that kind of preference decisions. Also for future work, longer transfer lines could be examined by a decomposition technique using our 2M1B models and analysis. As we mentioned in the introduction, these types of maintenance strategy problems arise quite often in multi-product manufacturing lines. However, our model includes two-types of products only and specifically for a production-assembly line. In order to contribute to a wider range of systems, further studies should be made with higher number of products and for different manufacturing lines.

APPENDIX A: CODE FOR THE SOLUTION OF SR POLICY

A.1. A Set of Constants

```
(*Constants*)

(*Probabilities of part A and B*)
ρA = 0.5;
ρB = 1 - ρA;
(*1st machine's rates*)
(*Rates of tools A and B*)
μA = 1;
μB = 1;
(*Rates of failure and repair for tools A & B*)
pA = 0.01;
pB = 0.01;
rA = 0.05;
rB = 0.05;
(*Assembly machine's rates*)
(*Production rate of 2nd machine*)
nu = 1.0;
(*Buffer Capacity*)
NA = 5;
NB = 5;


Table[P[nA, nB, 0, 0] = 0, {nA, 0, NA}, {nB, 0, NB}];(*0 Probabilities*)
Table[P[nA, NB, 1, 0] = 0, {nA, 0, NA}];
Table[P[NA, nB, 0, 1] = 0, {nB, 0, NB}];
```

Figure A.1. A Set of Constants For Stop and Repair Policy.

A.2. Equations

```
eqnset = Join[{
   (* αA=1, αB=0*)
   Table[P[nA, nB, 1, 0] (nu + rB) == P[nA, nB, 1, 1] pB * ρB + P[nA + 1, nB + 1, 1, 0] nu,
    {nA, 1, NA - 1}, {nB, 1, NB - 1}],
   Table[P[NA, nB, 1, 0] (nu + rB ) == P[NA, nB, 1, 1] pB , {nB, 1, NB - 1}],
   Table[P[0, nB, 1, 0] ( rB) == P[0, nB, 1, 1] ρB * pB + P[1, nB + 1, 1, 0] nu,
    {nB, 0, NB - 1}],
   Table[P[nA, 0, 1, 0] (rB) == P[nA, 0, 1, 1] ρB * pB + P[nA + 1, 1, 1, 0] nu,
    {nA, 1, NA - 1}],
   P[NA, 0, 1, 0] (rB) == P[NA, 0, 1, 1] pB,
   (* αA=0, αB=1*)
   Table[P[nA, nB, 0, 1] ( nu + rA) == P[nA, nB, 1, 1] ρA * pA + P[nA + 1, nB + 1, 0, 1] nu,
    {nA, 1, NA - 1}, {nB, 1, NB - 1}],
   Table[P[nA, NB, 0, 1] ( nu + rA) == P[nA, NB, 1, 1] pA , {nA, 1, NA - 1}],
   Table[P[nA, 0, 0, 1] (rA) == P[nA, 0, 1, 1] ρA * pA + P[nA + 1, 1, 0, 1] nu,
    {nA, 0, NA - 1}],
   Table[P[0, nB, 0, 1] (rA) == P[0, nB, 1, 1] ρA * pA + P[1, nB + 1, 0, 1] nu,
    {nB, 1, NB - 1}],
   P[0, NB, 0, 1] (rA) == P[0, NB, 1, 1] pA,
   (* αA=1, αB=1*)
   Table[P[nA, nB, 1, 1] (ρA (μA + pA) + ρB (μB + pB) + nu ) ==
     P[nA - 1, nB, 1, 1] ρA * μA + P[nA, nB - 1, 1, 1] ρB * μB + P[nA, nB, 1, 0] rB +
      P[nA, nB, 0, 1] rA + P[nA + 1, nB + 1, 1, 1] nu, {nA, 1, NA - 1}, {nB, 1, NB - 1}],
   Table[P[NA, nB, 1, 1] (μB + nu + pB) ==
     P[NA - 1, nB, 1, 1] ρA * μA + P[NA, nB - 1, 1, 1] μB + P[NA, nB, 1, 0] rB, {nB, 1, NB - 1}],
   Table[P[nA, NB, 1, 1] (μA + nu + pA) ==
     P[nA, NB - 1, 1, 1] ρB * μB + P[nA - 1, NB, 1, 1] μA + P[nA, NB, 0, 1] rA, {nA, 1, NA - 1}],
   P[NA, NB, 1, 1] (nu ) == P[NA, NB - 1, 1, 1] μB + P[NA - 1, NB, 1, 1] μA,
   Table[P[0, nB, 1, 1] (ρA (μA + pA) + ρB (μB + pB)) ==
     P[0, nB - 1, 1, 1] ρB * μB + P[1, nB + 1, 1, 1] nu + P[0, nB, 1, 0] rB + P[0, nB, 0, 1] rA,
    {nB, 1, NB - 1}],
   Table[P[nA, 0, 1, 1] (ρA (μA + pA) + ρB (μB + pB)) ==
     P[nA - 1, 0, 1, 1] ρA * μA + P[nA + 1, 1, 1, 1] nu + P[nA, 0, 1, 0] rB + P[nA, 0, 0, 1] rA,
    {nA, 1, NA - 1}],
   P[0, 0, 1, 1] (ρA (μA + pA) + ρB (μB + pB)) ==
    P[1, 1, 1, 1] nu + P[0, 0, 1, 0] rB + P[0, 0, 0, 1] rA,
   P[0, NB, 1, 1] (μA + pA) == P[0, NB - 1, 1, 1] ρB μB + P[0, NB, 0, 1] rA,
   (*P[NA,0,1,1] (μB + pB) ==P[NA-1,0,1,1]ρAμA + P[NA,0,1,0]rB,*)
   Sum[P[nA, nB, α1, α2], {α1, {0, 1}}, {α2, {0, 1}}, {nA, 0, NA}, {nB, 0, NB}] == 1
  }];
```

Figure A.2. Equations For Stop and Repair Policy.

APPENDIX B: CODE FOR THE SOLUTION OF RT POLICY

B.1. A Set of Constants

```
(*Constants*)
(*Probabilities of part A and B*)
ρA = 0.5;
ρB = 1 - ρA;
(*1st machine's rates*)
(*Production rates of tools A and B*)
μA = 1.0;
μB = 1.0;
(*Rates of failure and repair for 1st machine's tools*)
pA = 0.01;
pB = 0.01;
rA = 0.05;
rB = 0.05;
rboth = 0.04;
(*2nd machine's rates*)
(*Production rate of 2nd machine*)
nu = 1.0;
(*Buffer Capacity*)
NA = 5;
NB = 5;

Table[P[NA, nB, 0, 0] = 0, {nB, 0, NB}];    (*  0 Probabilities   *)
Table[P[nA, NB, 0, 0] = 0, {nA, 0, NA - 1}];
Table[P[nA, NB, 1, 0] = 0, {nA, 0, NA}];
Table[P[NA, nB, 0, 1] = 0, {nB, 0, NB}];
```

Figure B.1. A Set of Constants For Repair Together Policy.

B.2. Equations

```
eqnset = Join[{
   (* αA=0, αB=0*)
   Table[P[nA, nB, 0, 0] (nu + rboth) ==
     P[nA, nB, 1, 0] pA + P[nA, nB, 0, 1] pB + P[nA + 1, nB + 1, 0, 0] nu, {nA, 1, NA - 1},
    {nB, 1, NB - 1}],
   Table[P[0, nB, 0, 0] (rboth) == P[0, nB, 1, 0] pA + P[0, nB, 0, 1] pB +
      P[1, nB + 1, 0, 0] nu, {nB, 0, NB - 1}],
   Table[P[nA, 0, 0, 0] (rboth) == P[nA, 0, 1, 0] pA + P[nA, 0, 0, 1] pB +
      P[nA + 1, 1, 0, 0] nu, {nA, 1, NA - 1}],
   (* αA=1, αB=0*)
   Table[P[nA, nB, 1, 0] (μA + nu + pA) ==
     P[nA, nB, 1, 1] ρB pB + P[nA - 1, nB, 1, 0] μA + P[nA + 1, nB + 1, 1, 0] nu ,
    {nA, 1, NA - 1}, {nB, 1, NB - 1}],
   Table[P[NA, nB, 1, 0] ( nu ) == P[NA - 1, nB, 1, 0] μA + P[NA, nB, 1, 1] pB , {nB, 1, NB - 1}],
   Table[P[0, nB, 1, 0] (μA + pA) == P[0, nB, 1, 1] ρB pB + P[1, nB + 1, 1, 0] nu ,
    {nB, 0, NB - 1}],
   Table[P[nA, 0, 1, 0] (μA + pA) == P[nA, 0, 1, 1] ρB pB + P[nA + 1, 1, 1, 0] nu +
      P[nA - 1, 0, 1, 0] μA , {nA, 1, NA - 1}],
   P[NA, 0, 1, 0] (rB) == P[NA, 0, 1, 1] pB + P[NA - 1, 0, 1, 0] μA,
   (* B needs to be repaired alone *)
   (* αA=0, αB=1*)
   Table[P[nA, nB, 0, 1] (μB + nu + pB) ==
     P[nA, nB, 1, 1] ρA pA + P[nA, nB - 1, 0, 1] μB + P[nA + 1, nB + 1, 0, 1] nu ,
    {nA, 1, NA - 1}, {nB, 1, NB - 1}],
   Table[P[nA, NB, 0, 1] ( nu ) == P[nA, NB - 1, 0, 1] μB + P[nA, NB, 1, 1] pA ,
    {nA, 1, NA - 1}],
   Table[P[nA, 0, 0, 1] (μB + pB) == P[nA, 0, 1, 1] ρA pA + P[nA + 1, 1, 0, 1] nu,
    {nA, 0, NA - 1}],
   Table[P[0, nB, 0, 1] (μB + pB) == P[0, nB, 1, 1] ρA pA + P[1, nB + 1, 0, 1] nu +
      P[0, nB - 1, 0, 1] μB, {nB, 1, NB - 1}],
   P[0, NB, 0, 1] (rA) == P[0, NB, 1, 1] pA + P[0, NB - 1, 0, 1] μB,
   (* A needs to be repaired alone *)
   (* αA=1, αB=1*)
   Table[P[nA, nB, 1, 1] (ρA * (μA + pA) + ρB * (μB + pB) + nu ) ==
     P[nA - 1, nB, 1, 1] ρA μA + P[nA, nB - 1, 1, 1] ρB μB + P[nA, nB, 0, 0] rboth +
      P[nA + 1, nB + 1, 1, 1] nu, {nA, 1, NA - 1},
    {nB, 1, NB - 1}],
   Table[P[NA, nB, 1, 1] (μB + nu + pB) == P[NA - 1, nB, 1, 1] ρA μA + P[NA, nB - 1, 1, 1] μB,
    {nB, 1, NB - 1}],
   Table[P[nA, NB, 1, 1] (μA + nu + pA) == P[nA, NB - 1, 1, 1] ρB μB + P[nA - 1, NB, 1, 1] μA,
    {nA, 1, NA - 1}],
   P[NA, NB, 1, 1] (nu) == P[NA, NB - 1, 1, 1] μB + P[NA - 1, NB, 1, 1] μA,
   Table[P[0, nB, 1, 1] (ρA * (μA + pA) + ρB * (μB + pB)) ==
     P[0, nB - 1, 1, 1] ρB μB + P[1, nB + 1, 1, 1] nu + P[0, nB, 0, 0] rboth,
    {nB, 1, NB - 1}],
   Table[P[nA, 0, 1, 1] (ρA * (μA + pA) + ρB * (μB + pB)) ==
     P[nA - 1, 0, 1, 1] ρA μA + P[nA + 1, 1, 1, 1] nu + P[nA, 0, 0, 0] rboth,
    {nA, 1, NA - 1}],
   P[0, 0, 1, 1] (ρA * (μA + pA) + ρB * (μB + pB)) == P[1, 1, 1, 1] nu + P[0, 0, 0, 0] rboth,
   (*P[0,NB,1,1] (μA + pA) ==P[0,NB-1,1,1]ρBμB + P[0,NB,0,1]rA,*)
   P[NA, 0, 1, 1] (μB + pB) == P[NA - 1, 0, 1, 1] ρA μA + P[NA, 0, 1, 0] rB,
   Sum[P[nA, nB, α1, α2], {α1, {0, 1}}, {α2, {0, 1}}, {nA, 0, NA}, {nB, 0, NB}] == 1
```

Figure B.2. Equations For Repair Together Policy.

APPENDIX C: CODE FOR THE SOLUTION OF 2IM CASE

C.1. A Set of Constants

```
(*Constants*)
(*Rates of Machines A and B*)
μA = 0.5;
μB = 0.5;
(*Rates of failure and repair for Machines A & B*)
pA = 0.01;
pB = 0.01;
rA = 0.05;
rB = 0.05;
(*Assembly machine's rates*)
(*Production rate of 2nd machine*)
nu = 1.0;
(*Buffer Capacity*)
NA = 5;
NB = 5;

Table[P[NA, nB, 0, 0] = 0, {nB, 0, NB}];   (*0 Probabilities*)
Table[P[nA, NB, 0, 0] = 0, {nA, 0, NA - 1}];
Table[P[nA, NB, 1, 0] = 0, {nA, 0, NA}];
Table[P[NA, nB, 0, 1] = 0, {nB, 0, NB}];
```

Figure C.1. A Set of Constants For Two Independent Machines.

C.2. Equations

```
eqnset = Join[{
   (* αA=0, αB=0*)
   Table[P[nA, nB, 0, 0] (nu + rA + rB) ==
     P[nA, nB, 1, 0] pA + P[nA, nB, 0, 1] pB + P[nA + 1, nB + 1, 0, 0] nu, {nA, 1, NA - 1},
    {nB, 1, NB - 1}],
   Table[P[0, nB, 0, 0] (rA + rB) == P[0, nB, 1, 0] pA + P[0, nB, 0, 1] pB +
      P[1, nB + 1, 0, 0] nu, {nB, 0, NB - 1}],
   Table[P[nA, 0, 0, 0] (rA + rB) == P[nA, 0, 1, 0] pA + P[nA, 0, 0, 1] pB +
      P[nA + 1, 1, 0, 0] nu, {nA, 1, NA - 1}],
   (* αA=1, αB=0*)
   Table[P[nA, nB, 1, 0] (μA + nu + pA + rB) ==
     P[nA, nB, 1, 1] pB + P[nA - 1, nB, 1, 0] μA + P[nA + 1, nB + 1, 1, 0] nu +
      P[nA, nB, 0, 0] rA, {nA, 1, NA - 1}, {nB, 1, NB - 1}],
   Table[P[NA, nB, 1, 0] ( nu + rB ) == P[NA - 1, nB, 1, 0] μA + P[NA, nB, 1, 1] pB ,
    {nB, 1, NB - 1}],
   Table[P[0, nB, 1, 0] (μA + pA + rB) ==
     P[0, nB, 1, 1] pB + P[1, nB + 1, 1, 0] nu + P[0, nB, 0, 0] rA, {nB, 0, NB - 1}],
   Table[P[nA, 0, 1, 0] (μA + pA + rB) ==
     P[nA, 0, 1, 1] pB + P[nA + 1, 1, 1, 0] nu + P[nA, 0, 0, 0] rA + P[nA - 1, 0, 1, 0] μA,
    {nA, 1, NA - 1}],
   P[NA, 0, 1, 0] (rB) == P[NA, 0, 1, 1] pB + P[NA - 1, 0, 1, 0] μA,
   (* αA=0, αB=1*)
   Table[P[nA, nB, 0, 1] (μB + nu + pB + rA) ==
     P[nA, nB, 1, 1] pA + P[nA, nB - 1, 0, 1] μB + P[nA + 1, nB + 1, 0, 1] nu +
      P[nA, nB, 0, 0] rB , {nA, 1, NA - 1}, {nB, 1, NB - 1}],
   Table[P[nA, NB, 0, 1] ( nu + rA) == P[nA, NB - 1, 0, 1] μB + P[nA, NB, 1, 1] pA ,
    {nA, 1, NA - 1}],
   Table[P[nA, 0, 0, 1] (μB + pB + rA) ==
     P[nA, 0, 1, 1] pA + P[nA + 1, 1, 0, 1] nu + P[nA, 0, 0, 0] rB, {nA, 0, NA - 1}],
   Table[P[0, nB, 0, 1] (μB + pB + rA) ==
     P[0, nB, 1, 1] pA + P[1, nB + 1, 0, 1] nu + P[0, nB, 0, 0] rB + P[0, nB - 1, 0, 1] μB,
    {nB, 1, NB - 1}],
   P[0, NB, 0, 1] (rA) == P[0, NB, 1, 1] pA + P[0, NB - 1, 0, 1] μB,
   (* αA=1, αB=1*)
   Table[P[nA, nB, 1, 1] (μA + pA + μB + pB + nu ) ==
     P[nA - 1, nB, 1, 1] μA + P[nA, nB - 1, 1, 1] μB + P[nA, nB, 1, 0] rB +
      P[nA, nB, 0, 1] rA + P[nA + 1, nB + 1, 1, 1] nu, {nA, 1, NA - 1}, {nB, 1, NB - 1}],
   Table[P[NA, nB, 1, 1] (μB + nu + pB) ==
     P[NA - 1, nB, 1, 1] μA + P[NA, nB - 1, 1, 1] μB + P[NA, nB, 1, 0] rB, {nB, 1, NB - 1}],
   Table[P[nA, NB, 1, 1] (μA + nu + pA) ==
     P[nA, NB - 1, 1, 1] μB + P[nA - 1, NB, 1, 1] μA + P[nA, NB, 0, 1] rA, {nA, 1, NA - 1}],
   (*P[NA,NB,1,1](nu )==P[NA,NB-1,1,1]μB+P[NA-1,NB,1,1]μA,*)
   Table[P[0, nB, 1, 1] (μA + pA + μB + pB) ==
     P[0, nB - 1, 1, 1] μB + P[1, nB + 1, 1, 1] nu + P[0, nB, 1, 0] rB + P[0, nB, 0, 1] rA,
    {nB, 1, NB - 1}],
   Table[P[nA, 0, 1, 1] (μA + pA + μB + pB) ==
     P[nA - 1, 0, 1, 1] μA + P[nA + 1, 1, 1, 1] nu + P[nA, 0, 1, 0] rB + P[nA, 0, 0, 1] rA,
    {nA, 1, NA - 1}],
   P[0, 0, 1, 1] (μA + pA + μB + pB) == P[1, 1, 1, 1] nu + P[0, 0, 1, 0] rB + P[0, 0, 0, 1] rA,
   P[0, NB, 1, 1] (μA + pA) == P[0, NB - 1, 1, 1] μB + P[0, NB, 0, 1] rA,
   P[NA, 0, 1, 1] (μB + pB) == P[NA - 1, 0, 1, 1] μA + P[NA, 0, 1, 0] rB,
   Sum[P[nA, nB, α1, α2], {α1, {0, 1}}, {α2, {0, 1}}, {nA, 0, NA}, {nB, 0, NB}] == 1
  }];
```

Figure C.2. Equations For Two Independent Machines.

APPENDIX D: PERFORMANCE MEASURES CALCULATIONS

```
varset = Join[{
    Table[P[nA, nB, 0, 0], {nA, 0, (NA - 1)}, {nB, 0, (NB - 1)}],
    Table[P[nA, nB, 1, 0], {nA, 0, (NA)}, {nB, 0, (NB - 1)}],
    Table[P[nA, nB, 0, 1], {nA, 0, (NA - 1)}, {nB, 0, (NB)}],
    Table[P[nA, nB, 1, 1], {nA, 0, (NA)}, {nB, 0, (NB)}]
   }];

eqns = Flatten[eqnset]
vars = Flatten[varset]
sol = Solve[eqns, vars];

(*Performance measures*)
InvA = Sum[nA P[nA, nB, α1, α2], {α1, {0, 1}}, {α2, {0, 1}}, {nA, 1, NA}, {nB, 0, NB}] /. sol
InvB = Sum[nB P[nA, nB, α1, α2], {α1, {0, 1}}, {α2, {0, 1}}, {nA, 0, NA}, {nB, 1, NB}] /. sol
Throughput =
 nu (Sum[P[nA, nB, α1, α2], {α1, {0, 1}}, {α2, {0, 1}}, {nA, 1, NA}, {nB, 1, NB}]) /. sol
```

Figure D.1. Performance Measures Calculations.

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