THE EFFECTS OF MATHEMATICAL TASKS ON THE SEVENTH GRADE STUDENTS' ALGEBRAIC THINKING AND LEARNING

by

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ABSTRACT

THE EFFECTS OF MATHEMATICAL TASKS ON THE SEVENTH GRADE STUDENTS' ALGEBRAIC THINKING AND LEARNING

Algebra is one of the core branches of mathematics. However, many students fail in mathematics because they struggle with understanding algebraic concepts and procedures. Many scholars search for and propose effective ways of teaching and learning algebra to increase students' mathematics achievement and understanding. In the light of such research, the aim of this study was to investigate effects of task-assisted instruction on the seventh grade students' algebraic thinking and learning. This study was conducted in two of the seventh grade classes of a public middle school in Istanbul which taught by the same mathematics teacher. In one of the classes task-assisted instruction was applied for seven weeks while there was no intervention in the other class. The mathematics teacher taught in the same way in her regular mathematics course for both groups however the intervention took place in two-hour elective mathematics course of the experiment group. The test was administered before the study, just after the intervention and three months later the intervention to observe the retention. The tasks were developed to support three components of algebraic thinking: recognizing a pattern, writing algebraic expressions, setting up and solving equations. Students from the study group were grouped in threes or fours and one pre-service teacher was assigned for each group to manage the implementation of the tasks. The achievement test results showed that students in the study group performed significantly better than students in the control group. For post-achievement test $U=124.000$, $p=.001$ and for retention $U=159.500$, $p=.013$. Furthermore, video analysis revealed an improvement in at least two of three components of students' algebraic thinking. Therefore, task-assisted instruction might be taught as one of the effective ways to support students' algebraic thinking and learning.

ÖZET

MATEMATİK ETKİNLİKLERİNİN YEDİNCİ SINIF ÖĞRENCİLERİNİN CEBİRSEL DÜŞÜNME VE ÖĞRENMESİNE ETKİLERİ

Cebir, matematiğin en önemli dallarından biridir. Fakat birçok öğrenci cebirsel kavramlar ve işlemleri anlamakta zorlandığı için matematikte zorluk yaşamaktadır. Birçok araştırmacı öğrencilerin matematiksel başarısını ve kavrayışını attırmak amacıyla cebirde etkili olabilecek eğitim ve öğrenme metotlarını aramaktadırlar. Alanyazındaki çalışmaların ışığında, bu çalışmanın amacı etkinlik temelli öğretimin 7.sınıf öğrencilerinin cebirsel öğrenme ve düşünmelerine etkilerini araştırmaktır. Bu çalışma İstanbul'daki bir devlet okulunun zorunlu matematik derslerinin aynı matematik öğretmeni tarafından yapıldığı iki 7. sınıf şubesinde yürütülmüştür. Bir sınıfta 7 hafta boyunca etkinlik temelli öğretim uygulanırken diğer sınıfta bir uygulama yapılmamıştır. Öğretmen, zorunlu matematik dersinde her iki grupta da aynı şekilde öğretim yaparken deney grubunun 2 saatlik seçmeli matematik dersinde araştırmacı tarafından etkinlik temelli uygulamalar yapılmıştır. Testler çalışmaya başlanmadan, etkinlikler uygulandıktan hemen sonra ve uygulamadan üç ay sonra kalıcılığı ölçmek için uygulanmıştır. Etkinlikler cebirsel düşünmenin üç bileşenini desteklemek için geliştirilmiştir: örüntüler, cebirsel ifadeler, denklem kurma ve çözme. Deney grubundaki öğrenciler 3 veya 4 kişilik gruplara ayrılmış, her bir gruptaki etkinlik uygulama sürecini birer öğretmen adayı yönetmiştir. Başarı testlerinin sonuçları deney grubunun kontrol grubundan anlamlı ölçüde daha başarılı olduğunu göstermiştir. Son test sonuçlarında U=124.000, p=.001 ve kalıcılık testinde U=159.500, p=.013 bulunmuştur. Ayrıca, video analizleri öğrencilerin cebirsel düşünmesinde üç bileşenden en az ikisinde ilerleme olduğunu göstermiştir. Sonuç olarak, etkinlik temelli öğretimin öğrencilerin cebirsel düşünme ve öğrenimlerini desteklemek için etkili bir yol olarak görülebileceği söylenebilir.

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1. INTRODUCTION

Algebra has a great significance on both students' academic achievement in school and their future career in mathematics related jobs (Adelman, 2006; Knuth, *et al.*, 2006; Palabıyık and İspir, 2011). However, different studies revealed that students have difficulties and misconceptions in learning algebra around the world (e.g., Akkaya and Durmuş, 2006; Dede and Peker, 2007; Jupri *et al.*, 2014; Lucariello *et al.*, 2014; Welder, 2012). Therefore, scholars attempt to investigate causes of failure in understanding and learning algebra and figure out possible ways to eliminate them (e.g., Lucariello, *et al.*, 2014; İspir and Palabıyık, 2014; Tabach, Hershkowitz *et al.*, 2008).

The results of the Trends in International Mathematics and Science Study (TIMSS), held in 2011 showed that Turkish students performed poorly in mathematics comparing to the other countries since Turkey ranked 23 out of 41 countries. (TIMSS, 2013a). Turkish students performed better than the national average only in 13 items out of 31 algebra items (TIMSS, 2013b). 2015 TIMSS (TIMSS, 2017) scores revealed that Turkish students did not improve their mathematics achievement, since then Turkey ranked 24 out of 39 countries. Only in 7 items out of 64 algebra items, Turkish students got better scores comparing to the international average.

Not only the international assessment results but also the national studies showed that Turkish students' achievement in algebra is low (Ersoy and Erbaş, 2005). Turkish students have difficulties in understanding variables, usage of mathematical language in algebra, recognition of patterns, and solving equations (Akarsu, 2013; Akkan and Çakıroğlu, 2012; Akkaya and Durmuş, 2006; Baysal, 2010; Erbaş *et al.*, 2009). Similar difficulties of students were found in international studies as well (e.g., Jupri *et al.*, 2014; Welder, 2012). Furthermore, Baysal (2010) noted that such difficulties were frequently seen in 7th grade with respect to the other grades. For instance, Akarsu (2013) found that the seventh graders could not use mathematical language effectively. Similarly, Akkaya and Durmuş (2006) noted that students have difficulties especially in using and understanding the variables. In addition, Kalkan (2014) observed that 8th grade students had difficulties in using symbols and therefore, their algebraic thinking was weak.

To improve students' algebraic thinking and understanding, scholars make the following suggestions: to teach algebra from earlier grades (e.g. Carpenter *et al.*, 2003 ; Carraher *et al.*, 2006; Carraher *et al.*, 2008), use mathematical tasks (e.g. Lannin, 2005; Palabıyık and İspir, 2011; Yılmaz, 2015) use concrete manipulatives (e.g. Saraswati and Putri, 2016), use technology (e.g., Bills *et al.*, 2006; Tabach *et al.*, 2008) and use contexts from daily life of the students (Walkington *et al.*, 2013). The researchers revealed that teaching algebra in early grades could facilitate transition from arithmetic to algebra and contribute to students' mathematics achievement. Moreover, using well-designed tasks were likely to improve students' achievement in algebra as Lannin (2005), and Palabıyık and İspir (2011) who used pattern-based activities and Walkington *et al.* (2013) who used tasks including daily life contexts observed such an improvement in students' achievement. As a manipulative, some of the researchers used algebra tiles (e.g., Saraswati and Putri, 2016), while the others made use of computer spreadsheets (Bills *et al.*, 2006; Tabach *et al.*, 2008). Saraswati and Putri (2016) concluded that algebra tiles helped students understand equation with one variable better. Also, the researchers showed that computer spreadsheets provide opportunities to students to develop more strategies in solving equations and using algebraic notation. In addition, an online game intervention was added in early algebra instruction of 6th graders and stated to be effective on mathematical achievement of the students (Kolovou *et al.*, 2013). Therefore, we could conclude that to support students' understanding, mathematical tasks, technological tools, and manipulatives could be used while teaching algebra.

In Turkish mathematics curriculum, algebra is being taught formally at 6th grade such that students are expected to learn about patterns and algebraic expressions (Ministry of National Education, [MoNE], 2013; 2017). In the 7th grade, students begin to learn about setting up and solving linear equations. However, even in the 1st grade students learn about patterns in numbers and in the 4th grade they are expected to find the rule of a numerical pattern. These topics are aimed to prepare students for future topics in algebra. In that sense, students need to conceptualize what a variable is and how a relationship between two variables is represented to understand, write and solve a linear equation. Therefore, we need to support the seventh grade students' understanding of variables, algebraic expressions, equations in order to eliminate possible misunderstandings and prevent difficulties in algebra. As mentioned in the literature, mathematical tasks (e.g., Akkaya, 2006; Palabıyık and İspir, 2011; Yıldırım, 2016) and manipulatives (e.g., Saraswati & Putri, 2016) might help students to understand algebra better and make the transition from arithmetic to algebra easier (Gürbüz and Toprak, 2014).

Many researchers investigated the effects of using tasks on teaching different branches of mathematics such as probability (e.g. Gürbüz *et al.*, 2010), geometry (e.g., David and Tomaz, 2012; Günay, 2013) and algebra (e.g., Lannin, 2005; Rivera, 2010). They agreed that using tasks facilitate students' learning and understanding of that particular mathematical content. Moreover, tasks can be used as a tool for instruction as well as assessment. For instance, there are studies on algebra, where tasks were used for both assessment and instruction parts of the classroom to diagnose and eliminate the misconceptions of the students. Kospentaris, Spyrou and Lappas (2011), Becker and Rivera (2005), and Amit and Neria (2008) used pattern generalization tasks for assessment and they claimed that the students' strategies could be observed using these tasks. On the other hand, some researchers (e.g., Günay, 2013; Yüksel, 2014) made use of tasks during the instruction and observed that using tasks was effective for students' learning in different topics. Carpenter and Lehrer (1999) and Lin (2004) claimed that the mathematical tasks might give us opportunities to foster the students' thinking. Using these opportunities, teachers may elicit students' thinking and support students' learning. Moreover, teachers may use these opportunities to make inferences about students' misconceptions and take an action to eliminate these misconceptions. In that sense, the tasks need to be connected with the daily life of the students and create a cooperative learning environment where the students may share their thoughts and work collaboratively. Leatham and his colleagues (Leatham *et al.*, 2015) called the possible learning opportunities that help teachers to elicit students' mathematical thinking and address to their misunderstanding as Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST). They described MOST as having sequential and interrelated three components such that it should depend on students' mathematics, be mathematically significant, and be a pedagogical opportunity. They noted that mathematical tasks can be designed in a way to create MOST instances in a lesson so that teachers have chance to build on students' mathematics as well as assessing their mathematical understanding.

In the light of the literature, the effect of using tasks was investigated which can create learning opportunities for students' learning and teacher-student interaction, on the 7^{th} grade students' algebraic thinking and learning was investigated in this study. As suggested by Walkington *et al.*, (2013) the contexts from students' daily lives and close neighborhood were included in the tasks to increase their interests and achievement in algebra. In addition, the tasks were designed according to MOST Framework to create learning opportunities for the students to support their algebraic thinking.

2. LITERATURE REVIEW

2.1. Algebraic thinking

Algebra and algebraic thinking have been defined and understood differently from different perspectives. Malisani and Spagnolo (2009) stated that algebra is about using the symbols when expressing the relationship among different quantities. In that sense, understanding algebra requires to comprehend symbols. Moreover, Usiskin (1988) noted that there are different conceptions for algebra and algebra can be seen as 1) generalized arithmetic: expressing number patterns mathematically, 2) study of procedures for solving certain kinds of problems such as finding unknown from equations, 3) study of relationships among quantities such as representing the relationships among quantities with formulas, and 4) study of structures. Smith and Phillips (2000) explained algebra as a study about understanding and analyzing relationships between the covariating quantities. Some of those quantities might have linear whereas the others are in a nonlinear relationship. In here, different kind of relationships of quantities can be shown by using tabular, graphical, symbolic and verbal representations. Steele (2005) noted that algebraic thinking is about understanding variables and expressions and expressing relationships among quantities; whereas algebra is a language that includes symbols. Therefore, since algebra can be seen as a system and algebraic thinking is a process of using that system to make generalizations, we may think of algebraic thinking as something broader than algebra. In this case, the distinction of algebra and algebraic thinking needs to be done.

To make a distinction between algebra and algebraic thinking, Smith (2003) claimed that algebraic thinking is about the examination of patterns by understanding the relationship of variables between patterns whereas algebra is a study of symbol system. Moreover, algebraic thinking also deals with growing patterns, and understanding of extension of patterns. Furthermore, Kriegler (2008) identified algebraic thinking with two different components: the development of mathematical thinking and the study of basic algebraic ideas. In this definition, mathematical thinking requires analytical habits of mind including problem solving, representation skills and quantitative reasoning skills. In addition, basic algebraic ideas can be developed through three perspectives: algebra as generalized arithmetic, as a language and, as a tool for developing ideas for functions. Similar to Kriegler (2008), Driscoll (1999) noted that algebra is more than generalized arithmetic since algebraic thinking is also necessary for making generalizations. In that sense, algebraic thinking is used for generalization of functions, relations, and structures. In addition, Driscoll (1999) claimed that certain habits of mind support algebraic thinking such as; reversibility, recognizing and building rules for patterns, and abstracting form computation. In here, reversibility means to do operations backwards such as putting the roots of the equation to check that whether or not the number is a root of that equation. Moreover, abstracting from computation requires thinking independently from particular numbers while thinking about the computations.

According to Langrall and Swafford (1997) algebraic thinking is necessary to make transition from arithmetic to algebra. They defined algebraic thinking as “the ability to operate on an unknown quantity as if the quantity was known, in contrast to arithmetic reasoning which involves operations on known quantities” (p. 2). Contributing to Langrall and Swafford’s (1997) claim, Kieran (2004) noted that development of algebraic thinking depends on focusing relational aspects of the operations. Also, algebraic thinking needs to include: (i) focusing on relationships not just calculations, (ii) being able to do operations as well as their inverses, (iii) focusing on both representation and solution of problem rather than only solution, (iv) working with numbers and letters, means that understanding unknowns, variables and parameters, and (v) understanding the meaning of equal sign. From those definitions we can make a distinction between algebraic and arithmetic thinking, since algebraic thinking deals with the variables and generalizations rather than focusing on numbers and calculations.

Kaput (1999) used the term algebraic reasoning rather than algebraic thinking, and stated that algebraic reasoning is the process of making generalizations using formal ways. Kaput (1999) noted that algebraic reasoning includes four components: a) using arithmetic to generalize the mathematical ideas, b) generalizing the numerical patterns that lead to functional thinking, c) modeling to express the generalizations and d) generalizing the mathematical systems that comes from the relations. Even though Kaput (1999) preferred the term algebraic reasoning, he explained algebraic reasoning as similar to the algebraic thinking.

Briefly, from the literature we can conclude that algebra is about expressing the relationships among quantities using the symbols and making generalizations. Furthermore, algebraic thinking is claimed to be process of making generalizations of patterns, analyzing and representing the relationships. Presenting these relationships, variables, symbols and graphical representations can be used since algebraic thinking requires making transition from using known quantities to unknowns. Even though the researchers defined algebraic thinking differently, we can observe that there are common points in their statements. Therefore, using the definitions of Steele (2005), Driscoll (1999) and Kieran (2004), we can define algebraic thinking as ability to make generalizations and to understand variables, expressions and meaning of the equal sign.

2.2. Tasks in mathematics education

2.2.1. Definition of task

In the literature, there are different definitions and different classifications of mathematical tasks. Stein and her colleagues (Stein *et al.*, 1996) described mathematical tasks as “a classroom activity, the purpose of which is to focus students’ attention on a particular mathematical idea” (p. 460). They noted that a task can be a single complex problem or consist of related problems around a mathematical concept. Furthermore, Herbst (2006) defined a task as a unit where students can develop their thinking abilities on different mathematical ideas in a problem. Sierpinska (2004) stated that a task can be seen as a mathematical problem where the assumptions and questions were clear and such that problems might have more than one solution or interpretation. Watson and Mason (2007) stated that there is a distinction between an activity and a task. They defined an activity as all of the communication among the students whereas a task covers the activities that students perform, how students make sense of it and, how the teacher guides the students and in which extend the students engage and learn from it. In that distinction, Alexander (2000) claimed that task is about using cognitive demand to improve students' learning and an activity describes the usage of different teaching approaches for students' learning. Furthermore, according to Vershaffel *et al.* (2000) a task might be contextualized since it involves in word problems that connect mathematical situations with daily life. In that sense, the students are required to use their experiences from daily life while they are solving the word problems. Therefore, a task could be defined as the set of problems that might include real life contexts that aims to support students' learning.

2.2.2. Structure of tasks

In the literature, different types of tasks were used to promote students' learning and understanding. These tasks can be categorized according to their intellectual demands and structure. Stein and Smith (1998) defined *Mathematical Task Framework* to categorize the tasks in terms of their intellectual (cognitive) demands. In that framework, the tasks grouped as lower level demanding and higher level demanding tasks. If the tasks demand from students to perform the routine procedure they can lead students to one type of thinking and called lower level demand tasks. They identified two types of lower level demanding tasks as *memorization* tasks and *procedures without connection* tasks. Memorization tasks refer to the recall of memorized definitions, facts or formulas where students are expected to tell or write the answer immediately without doing calculations. Procedures without connections tasks entail appropriate and correct application of an algorithm to obtain correct answer. Students are not expected to know the reasoning behind the procedures but follow the procedure to get the answer. The purpose of higher level demanding tasks is to elicit students' conceptual understanding rather than reinforcing their procedural knowledge. There are two types of the tasks in high level demand: *procedures with connections* and *doing mathematics* tasks. In procedures with connections tasks, students have to conceptually understand the procedure behind the solutions to complete the tasks. For doing mathematics tasks there is no algorithmic solutions and students have to use higher order thinking skills to put cognitive effort into task. Moreover, Stein and Smith (1998) stated that implementation of the tasks creates student learning. Similar to Stein and Smith (1998), Liljedahl *et al.* (2007) claimed that good tasks can serve to understand specific mathematical ideas. In addition, how the tasks are implemented is important to be able to see whether or not the students' thinking was improved. In some cases, after the implementation the teachers need to make reflections and adjustments on the tasks. Therefore, student learning depends on both the design and the implementation of a task.

Tasks are also categorized according to their structure such that, among the other types, there are authentic tasks, heuristic tasks, and contextualized tasks used in mathematics education. Palm (2002) stated that even though in the literature the authentic tasks can be perceived as real world tasks, there is not a precise definition about the authentic tasks. For instance, Kramarski *et al.* (2002) defined authentic tasks as the tasks where there is no ready algorithm for the students and so they differentiated the real life tasks from authentic tasks. On the contrary, Beswick (2011) stated that depend on the problem solver, real life tasks can be simpler and there can be an algorithm to solve them.

To overcome the ambiguity in the definition, Reeves, Herrington and Oliver (2002) listed some criteria for authentic tasks. Authentic tasks:

- (i) need to have real world relevance
- (ii) are ill-defined: students make many interpretations on tasks rather than using an algorithm for the solution
- (iii) are complex tasks where the students have to spend time on them
- (iv) are required to use multiple sources to complete
- (v) give opportunities to students to work collaboratively
- (vi) make possible to do reflections on their learning both individually and socially

(vii) are appropriate for the interdisciplinary goals from different subjects: the perspectives from different areas for instance physics or chemistry knowledge can be used for the solution process of mathematical authentic tasks

(viii) are integrated with real world assessment

(ix) are about whole products process rather than the preparation of other tasks

(x) might have more than one solution.

About the implementation of the authentic tasks, Herrington (2006) claimed that they can be integrated into both instruction and assessment. The authentic tasks can be at the center of the course and the learning environment has to be appropriate for the authentic tasks. In order to do so, teachers need to practice scaffolding during the instruction and provide a collaborative environment to the students. In the assessment part of the authentic tasks, as similar to the instruction part the students need to be active to perform what they learned so far.

From a different perspective, Lee and Reigeluth (2003) separated the tasks into two categories: procedural and heuristic tasks. In those categories, procedural tasks are required to apply the steps whereas in the heuristic tasks higher order thinking skills needs to be used to decide which steps needs to be taken. Lee and Reigeluth's (2003) procedural tasks could be think of as Stein *et al*.'s (Stein & Smith, 1998) procedures without connections tasks while heuristic tasks could be matched with the procedures with connection tasks since higher order thinking skills are required to apply the steps. From a different perspective, Hoon *et al.* (2013) stated that heuristic tasks demand from students to make connections among different mathematical ideas, draw diagrams, examine specific problem cases and reach generalizations using these specific cases. In addition, Reiss and Renkl (2002) defined heuristic examples of the tasks as where students are expected to give their argumentation during their problem solving process to support their answers.

Moreover, Foster (2013) defined the contextualized tasks as the tasks which build on the real life contexts. Bates and Wiest (2004) and Walkington *et al.* (2013)

organized tasks according to the personal lives and interest of the students and specifically called these tasks as personalized tasks. Bates *et al.* (2004) stated that the organization was done by replacing some of the information with the students' personal information. Walkington *et al.* (2013) claimed that the personalized tasks give us opportunities to organize the instruction in accordance with the students' out of school experiences and interests. Therefore, the contextual tasks might be beneficial for motivating the students.

2.2.3. Task design

In the literature, the researchers give importance to different criteria while describing task design process. Some of the researchers stated the cognitive demand is the most important factor during the tasks design whereas some of them claimed the task design is an iterative process. Henningsen and Stein (1997) stated that the level of students' mathematical thinking depends on the task attributes. To support students' thinking, the teacher needs to prepare the tasks according to existing knowledge of the students and give enough time to student during the implementation of those tasks. Moreover, Henningsen and Stein (1997) and Stein *et al.* (1996) claimed that better support for students' learning can be provided with high level demand tasks. From another perspective, Liljedahl and his colleagues (Liljedahl *et al.*, 2007) noted that designing a good task is a cyclic process that includes predictive analysis, trial, reflective analysis, and adjustment. Predictive analysis phase refers to the initial phase where teachers use their experiences to develop a task, and then in trial phase teachers implement the task in the classroom. Then reflective analysis comes, where the implementation of the task is analyzed to see whether or not the task meets the intended mathematical and pedagogical affordances. Teacher rethinks and reorganizes the tasks in the final phase, which is adjustment; then the cycle restarts with predictive analysis. In this process, the tasks are checked over and over again to meet the best mathematical and pedagogical affordances.

Furthermore about defining criteria for good tasks, Foster (2013) claimed that the rich tasks are content-specific, open-ended, accessible by students to give them the opportunities to make range of mathematical actions, and contribute to learn important mathematical ideas. In that statement, good tasks were described as giving opportunities for mathematical actions that involve making decisions, facing with challenges, providing motivations. The researcher observed that many of the students stated that when the tasks are content specific rather than general, they could engage with the tasks more. Moreover, Shavelson and Stern (1981) noted that during the task design process the following components need to be taken into consideration: (i) the content: the topics that will be covered, (ii) materials (i.e. manipulatives), (iii) activities: what students are expected to perform, (iv) goal: the main aim of the task, (v) students' abilities and interest, (vi) social community: the students' need to belong in a community.

Mathematical actions taken during tasks can reveal important clues regarding students' reasoning processes and misconceptions. Some scholars emphasized teachers' noticing skills such that teachers should pay attention to what occurs in the classroom to create appropriate teaching learning environment for students as well as to elicit and support students' understanding (e.g., van Es and Sherin, 2002; Jacobs *et al.,* 2010). For instance, van Es and Sherin (2002) described three aspects of teachers' noticing as identifying what is important in a classroom situation, making connections between the particulars of the situation and broader educational principles, and reasoning about the situation in the context. However, Jacobs and his colleagues (Jacobs *et al*., 2010) focused more on students' thinking such that they identified teachers noticing as having three interrelated components: attending to students' strategies, interpreting students' mathematical understanding and deciding how to respond on the basis of students' understanding. Although there are some minor differences in description of teachers noticing, one commonality in them is *to pay attention to something important*. Leatham and his colleagues (Leatham *et al*., 2015) noted that there should be some criteria to identify the important instances to be noticed since it is not possible to attend every instance in a classroom. They named the instances that should be paid attention by teachers as Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST).

Leatham and his colleagues (Leatham *et al*., 2015) defined MOST as satisfying three attributes at the same time: depending on students' mathematics, being mathematically significant, and being pedagogical opportunity. Student thinking is the initial step to identify an instance as a MOST. However, student thinking should be based on *student mathematics*, that is, what a student does or says should be inferable and also it should convey a *mathematical point*. After the initial step is satisfied, the mathematics in students' thinking is analyzed in terms of being *appropriate* and *central* mathematics. The mathematical point in students thinking should be appropriate for students in terms of being discussed in the class or previously learned. In addition, it should be aligned with the goal of that particular lesson. As a final step it should have potential to create an intellectual need for students and it should be the appropriate time to take advantage of that opening. In other words, it should be a pedagogical opportunity that satisfies both *opening* and *timing* criteria. When an instance satisfies all of these six criteria in order, then it is named as a MOST instance. A simplified version of MOST Framework is given in Figure 2.1.

The MOST instances can be observed in any classroom where students actively engage in the lesson. Although students' wrong answers have more potential to be a MOST, students' correct answers can be counted as a MOST because there may be some flaws in students' thinking despite performing the correct answer. To identify when and how the MOST instances is more likely to occur, Zoest, Stockero, Leatham, Peterson, Atanga, and Ochieng (2017) conducted a study with students from 6^{th} to 12^{th} grades. They recorded 11 lessons to determine the MOST instances from the whole-class discussions during these lessons according to MOST Framework. The researchers categorized the context and student mathematics attributes of 278 MOST cases. They found that a MOST instance is more likely to be revealed when (i) teacher calls on a specific group of students, (ii) students' thought is elaborated at the moment of the conversation, (iii) the goal of the lesson is aligned with the objective of the course, (iv) the students are sure about their answers, (v) the students' answers are correct as well as they are incorrect, (vi) the student mathematics is obvious, and (vii) the students are asked to make sense about mathematical concepts.

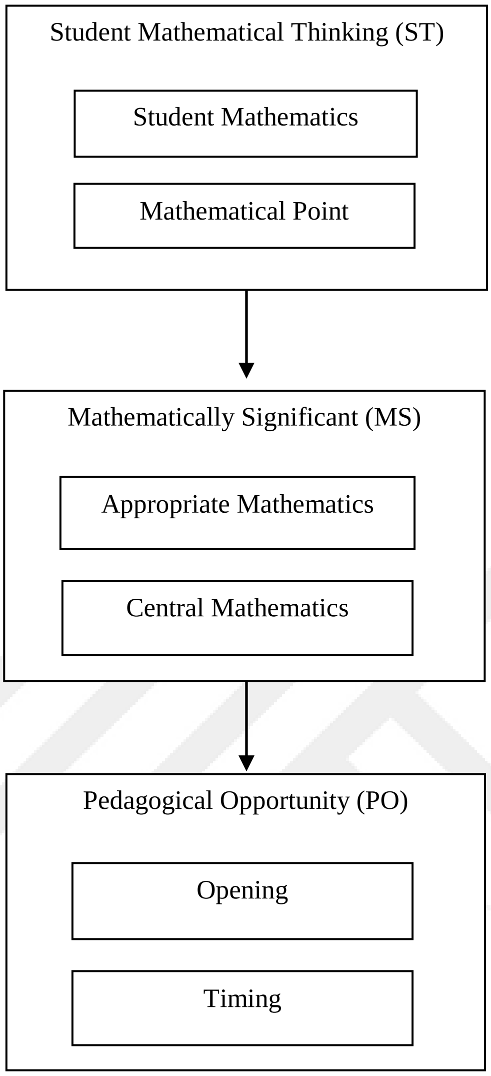

Figure 2.1. Simplified version of Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) Framework (Leatham *et al*., 2015, p.103).

MOST Framework was taken into consideration during task development phase of this study so that the tasks could reveal the MOST instances especially during the student-teacher interaction in the class in order to improve students' algebraic thinking.

2.2.4. Using tasks in mathematics education

The mathematical tasks were used for different purposes in research studies. In some of the research studies, the researchers used the tasks for assessment. Kendle and Northcote (2000) categorized assessment tasks as qualitative and quantitative tasks. The quantitative tasks include closed questions where there is only one correct solution. On the other hand, qualitative tasks demand from students to create their own solutions with open-ended questions. Boud (2007) claimed that assessment tasks might serve many purposes such as improving students' learning, assessment of students' understanding and giving feedback to students to foster their learning.

Kospentaris *et al.* (2011) examined the strategies and solutions of the students on area conservation tasks. In these tasks, students needed to develop the strategies to calculate and compare different areas. However, the results of the study showed that many of the students failed to give explanations and develop strategies for area conservation. Similar to Kospentaris *et al*. (2011), Becker and Rivera (2005) investigated the students' strategies about pattern generalization in algebra with growing patterns. Examined strategies of the students showed that most of the students developed numerical strategies and less figural or pragmatic strategies were developed. Amit and Neria (2008) found contradicting results with Becker and Rivera (2005) even though they worked with the same age group (11-13 year olds) with same growing pattern. In the study, the students drew numerical, verbal and pictorial representations while they were solving the tasks. Therefore, some of the researchers observed different types of representations used by the students while some of the researchers mostly encountered numerical strategies via assessment tasks.

To measure students' performance, some of the researchers make use of the tasks that include contexts to be close to daily life of the students. For instance, Bates and Wiest (2004) tried to measure problem solving ability of students via tasks that includes personalized word problems. The tasks designed according to the personal interest of the students depend on their daily lives and preferences. The results showed no significant difference in students' problem solving abilities comparing to their performance in non-personalized questions. In contrary to Bates and Wiest (2004), Walkington *et al*. (2013), found that the algebraic problem solving skills can be supported through personalized tasks. As different than other studies Cooper and Harries (2002) conducted a study with younger students on solving tasks including word (realistic) problems on early algebra and observed that even younger students made use of realistic problems since they could derive more realistic strategies while they were solving the tasks.

Integrating the tasks to instruction is another way of using tasks. In the studies tasks were applied different in terms of their ways of presentation and usage of manipulatives. David and Tomaz (2012) used visualization in geometry tasks and stated that the visualization can be helpful since the students developed new strategies as a result of the visualization of area tasks. In addition, Günay (2013) used different representations in task- assisted instruction on 7^{th} grades on the topic of rational numbers and angles. While representing tasks that involve different representations, the researcher gave the activity sheets in three ways: pictorial, verbal, and both verbal and pictorial. The achievement test results showed that even though the verbal and pictorial representations were most effective, each group of students performed better through usage of activities in instruction. Nathan and Kim (2007) conducted a similar study to compare the effect of representation of linear functions using verbal, graphical, and both verbal and graphical representations. Similar to Günay (2013), Nathan and Kim (2007) observed the students more benefited from when both of the representations were included in the tasks. As a result, using multiple representations and visualizations in tasks can be effective for students' learning and understanding.

Moreover *et al.* (2012) stated that use of representations may help to convert abstract concepts into concrete ones and make easier to understand different mathematical ideas. In that case, Chappell and Strutchens (2001) used algebra tiles to make abstract facts like algebraic expressions and operations more concrete ones. In this way, the students developed their reasoning on factorization of second degree polynomials. The other studies supported the advantage of using the algebra tiles. Saraswati and Putri (2016) observed that algebra tiles could be useful for improving understanding of linear equations with one variable, whereas Leitze and Kitt (2000) claimed that algebra tiles facilitates understanding of algebra such as algebraic expressions but also pre-algebra topics such as mathematical operations and monomials.

Furthermore, mathematical tasks were used to promote students' understanding and learning in different topics. Sullivan *et al.* (2011) claimed that mathematical tasks give students opportunities to explain their strategies and thinking to the teachers. In addition, the researchers observed that the students' engagement with the significant mathematical ideas increased by using the tasks. Similarly, Dimond and Walter (2006) stated that task-assisted instruction might be used to support students' learning since task-assisted instruction is beneficial to introduce mathematical ideas. Smith and Stein (1998) agreed on the benefits of using tasks since they claimed that if the students challenged by the appropriate tasks, they can engage in rich mathematical conversations. Investigating the effect of using tasks in mathematics instruction, Yüksel (2013) and Gürbüz (2010) conducted studies on geometry and probability, respectively. The researchers concluded that using tasks was likely to improve students' mathematics achievement in those subject matters.

From science perspective, the inquiry tasks were used to support students' scientific understanding. Furthermore, in science education there are inquiry tasks that requires problem solving skills from mathematics. Bell *et al.* (2005) defined the inquiry tasks as the tasks where the students have to solve the scientific research question through analyzing the relevant data. Similarly, Manlove *et al.* (2006) stated that in inquiry tasks, students need to make hypothesis, observations, collect the data and interpret the data. According to Edelson (2001) the inquiry tasks create opportunities for students to construct knowledge through discovery and so improve the understanding of the students.

The research studies implied that the tasks might be used for different topics including different representations and contexts. In addition, the research studies emphasized that using manipulatives especially algebra tiles during the implementation of the tasks is useful to introduce algebra concepts.

2.2.5. Using tasks in algebra instruction

The algebra tasks are mostly used to eliminate students' algebraic misconceptions, develop their understanding of variables, algebraic expressions and equations. Akkaya (2006) tried to diagnose learning difficulties and overcome these difficulties of 6th grade students using tasks in algebra and compared traditional teaching method with task-assisted instruction. Yıldırım (2016) conducted a similar study with 7th grade students on algebraic equations by using the task- assisted instruction. Although Akkaya (2006) observed that task-assisted instruction was effective to deal with the misconceptions of the students, Yıldırım (2016) did not find any significant difference between the students who were instructed by traditional instruction and by task-assisted instruction. Kaymakçı (2015) prepared algebra tasks by using 5E learning model including engagement, exploration, explanation, elaboration and evaluation steps. The researcher observed that the students' achievement improved after the implementation of the tasks. Moreover, Walkington *et al.* (2013) investigated the effect of personalized tasks, which involves in daily life issues that take students' interests, on students' achievement in linear functions in algebra. They observed that personalized tasks were useful especially for the students' who has low achievement in more challenging problems. Even though the researchers used the tasks with different purposes, they mostly agree on that using tasks helped to support students' achievement in algebra.

Yackel (1997) explained that the transition from numerical thinking to algebraic thinking can be supported with tasks since the tasks are beneficial for improvement of students' thinking on relationships. Steele (2005) stated that pattern tasks might be used to lead students to think about the relationships of variables. Walkowiak (2010) noted that the nature of the pattern tasks provides many approaches to students to make generalizations. In accordance with that claim, Driscoll (1999) stated that high visibility of the pattern tasks can be beneficial to introduce algebra to younger children. Amit and Neria (2007) preferred to use tasks that include growing pattern to introduce the algebraic topics to younger children (11-13 ages). The researchers concluded that pattern tasks were useful to improve students' algebraic thinking. In addition, Warren and Copper (2005) claimed that younger children could develop their reasoning in algebraic equations and equivalence as a result of pattern tasks. Using pattern in early grades, Store and Berenson (2010) made a teaching experiment with 5th graders. Even though at the beginning of the study the younger children had difficulties to construct schemes from patterns, at the end of the experiment they could both draw schemes and make justifications about their reasoning. Therefore, pattern tasks could be used to introduce ideas about algebra such as understanding the relationships and using variables.

While preparing the tasks that were implemented in this study, Leatham and his colleagues' (Leatham *et al*., 2015) ideas about MOSTs were taken into consideration. Tasks also included items that enable assessing students' misconceptions and difficulties in algebra. Furthermore, the students worked in groups so that they could learn about their peers' thoughts and attempt to make sense of algebraic concepts through group discussions.

3. SIGNIFICANCE OF THE RESEARCH STUDY

This study had a strong connection with the TUBITAK Project titled as “A University-School Collaboration Model for Promoting Pre-service Teachers’ Pedagogical Content Knowledge about Students” (Grant no: 215K049), conducted in Yeditepe University. In that project, pre-service teachers’ pedagogical content knowledge about students was described in terms of their noticing of students’ mathematical understanding. The researchers investigated pre-service teachers’ noticing by providing an environment for pre-service teacher-student interactions where mathematical tasks were used as a medium to initiate such interactions.

In this study, the tasks were discussed in terms of whether or not they could create learning opportunities for students and support their algebraic thinking. As different from other studies, this study analyzed the task-assisted instruction through MOST theoretical framework (Leatham *et al.*, 2015) lenses. Therefore, the tasks were designed in such a way that they both have potential to create MOST instances and to support students’ mathematical understanding. In order to support students’ mathematical understanding, the tasks were organized around a context emerged from either students’ close or far neighborhood. Some of the tasks entailed use of some concrete materials and require collaboration of students. Although such attributes might be common in other research studies on tasks, having a potential to create a MOST instance to provide an environment for understanding and supporting students’ mathematical thinking was a new attempt in this study. Thus, it has potential to open a window for further research on how to design tasks to create MOST instances in a classroom and take an advantage of strengthening students’ understanding as well as eliminating their misconceptions and difficulties.

Moreover, to what extent use of tasks in teaching algebra was beneficial for students’ algebraic learning and understanding in comparison to the traditional instruction was analyzed. Thus, this study has potential to suggest an alternative method to support students’ understanding of basic algebraic concepts and their achievement in algebra.

4. STATEMENT OF THE PROBLEM

4.1. Research questions

This study has emerged from a TUBITAK Project (Grant No: 215K049) that aimed to investigate the development of pre-service mathematics teachers' pedagogical content knowledge where mathematical tasks were used as a medium to set up an environment for teacher-student interaction. Collaborating with this TUBITAK Project, the study aimed to understand how students' algebraic thinking and achievement in algebra improved as a result of task-assisted instruction. In accordance with this aim, the following research questions were investigated:

- Is there any significant difference between algebra achievement of the 7th grade students who are exposed to traditional instruction and who are exposed to task-assisted instruction?
- How does students' algebraic thinking change during task-assisted instruction?
  - (i) How does students' performance in finding the rule and the terms of a pattern change during task-assisted instruction?
  - (ii) How does students' performance in writing algebraic expressions and converting verbal expressions to algebraic expressions change during task-assisted instruction?
  - (iii) How does students' performance in setting up and solving equations with one variable change during the task-assisted instruction?

4.2. Hypothesis of the first research question

H1: There is a significant difference between the algebraic achievement of the students who were exposed to the traditional instruction and who were exposed to task-assisted instruction in favor of the experimental group.

4.3. Variables of the study

The independent variable of the study was the type of instruction and the dependent variables of the study were algebraic thinking and algebra achievement of students.

4.4. Operational definition of the variables

Task-assisted instruction: An instruction (held in elective mathematics course) where mathematical tasks were used to explore students' algebraic thinking via group discussions managed by pre-service teachers.

Traditional instruction: An instruction (held in elective mathematics course) where mathematics teacher asked drill and practice problems, students worked individually and correct solutions of the problems mostly were provided by the teacher.

Algebra achievement: The achievement of the students measured by the pre and post-achievement tests developed by the researcher.

Algebraic thinking: The ability to make generalizations and understand variables, expressions and meaning of equal sign (Driscoll, 1999; Kieran, 2004; Steele, 2005). Algebraic thinking involves in a) finding the rule and the pattern of relationship among two variables, b) converting verbal expressions to algebraic expressions and algebraic expressions to verbal expressions, c) setting up and solving the equations.

5. METHOD

5.1. Participants

A total of 56 students from 7th grade classes of a public middle school in Kayışdağı district in İstanbul participated in this study. This school has university-school collaboration with Yeditepe University where TUBITAK Project had been carried out. Therefore, in this research study the convenient sampling technique was used. 7-A and 7-D classes were selected for the study, because the regular mathematics course of these two classes were carried out by the same mathematics teacher. In this way, the effect of the treatment can be observed more clearly while comparing to the control and experiment group. There were 28 students in each of the classes. Both in 7-A and 7-D there were 13 girls and 15 boys. From the study group a sub-group was created to examine the changes in their algebraic thinking. The students in the sub-group were selected according to their pre-achievement test scores and performances of two weeks of the study. The criteria of the selection are given in the data analysis part.

As a premise of TUBITAK project, 8 senior pre-service mathematics teachers were involved in this study. The pre-service teachers had similar academic background since all of them took fundamental pedagogy courses. The pre-service teachers were responsible for managing a group of 4 students while the students were working on the tasks. The pre-service teachers were instructed about task implementation procedure by the researcher before each implementation. Thus, the differences between groups in terms of implementation were tried to be diminished.

5.2. Tasks

Eight tasks were developed to investigate students' algebraic thinking and algebra achievement. Four of the tasks (Task 2, Task 3, Task 4, Task 5) were developed by the TUBITAK research team in previous year (Doğan and Dönmez, 2016) while others (Task 1, Task 6, Task 7, Task 8) were developed by the researcher in accordance with previous tasks. All of the tasks that were implemented are given in Appendix A. The tasks from Task 2 to Task 8 were implemented during 2016-2017 academic year. In that year, Task 2, Task 3, Task 4, Task 5, and Task 6 were used in the TUBITAK Project to observe the pre-service teachers' pedagogical content knowledge and noticing skills during their interactions with students. Task 7 and Task 8 were developed in 2016-2017 academic year and piloted by the researcher. Task 1 was developed as a result of analysis of the pilot study. The results of pilot study revealed that students had difficulties to understand the covariance of the variables since they paid attention to the difference between the numbers given in a single column rather than checking the relationship between numbers given in rows, in the given matrices of numbers. Therefore, Task 1 was developed to support students' understanding of covariance. The researcher revised the other tasks and made minor changes in the format of the tasks.

The tasks were developed based on some of the characteristics of authentic task defined by Reeves *et al.* (2002), the task design cycle that defined by Liljedahl, *et al.* (2007) and MOST Framework defined by Leatham, *et al.* (2015). The authentic tasks had real world relevance, and demanded from students to make interpretations rather than only apply an algorithm, and to reflect on their work. Moreover, the tasks were designed according to the 4 phases of the design cycle as follows: in the first phase, the researcher made a predictive analysis according to level of the students. Then, the pilot study was the trial phase and the reflective analysis was done from videos of the pilot study. Since the pilot study showed that some of the questions were too hard for the students or not clearly stated, in the adjustment phase the tasks were reorganized. In the line of characteristics of authentic tasks, task design cycle, the relevant literature, and MOST Framework the following criteria were determined to develop the tasks in this study. The tasks should 1) encourage students to think about given situation and follow problem solving procedures 2) allow using manipulatives and hands on aids 3) lead communication among students (collaborative work) 4) involve real life contexts 5) have potential to provoke students' misconceptions. The tasks were checked for alignment with criteria and the checklist is given in Appendix B.

In all of the tasks, the contexts were chosen from daily life of the students to increase their engagement. In the literature, inclusion of the students' interest was stated to be potentially beneficial for students' learning. Moreover, especially for algebra tiles some researchers (e.g., Saraswati and Putri, 2016) claimed that using manipulatives can be useful to support students' thinking. Furthermore, students should be encouraged to share their thoughts with their friends and the teacher to trigger the occurrence of MOST instances. In that sense, tasks should have a potential to create opportunity to reveal students' understanding. In other words, tasks might be used as a tool by teachers to explore students' difficulties and misconceptions as well as support their understanding. The tasks were developed according to the learning objectives of middle school mathematics curriculum as well (MoNE, 2013). The alignment of tasks with the mathematical objectives is given in Appendix C.

For the sequence of the tasks, the pilot study was taken into the consideration. In the previous year, all of the tasks except Task 1 were applied in one of the 7th grade class in the collaboration school in a similar setting. Each pre-service teacher was responsible from a group of 4 students. The pre-service teachers introduced the tasks to students and made some explanations when necessary. Then, the students worked on the tasks individually for 20 minutes and then they discussed their solutions with their peers in the group. Finally, the pre-service teachers involved in the discussion and asked students to explain their reasoning. The results of the pilot study showed that the students had difficulties especially in understanding covariation of the variables, and writing and solving the equations. In Task 8 and Task 2, students paid attention to change in one variable other than covariance of the variables. Moreover, in Task 8 and Task 3 students needed to decide which point has to be taken as reference point to decide the following paths. Many of the students took the reference point as the initial point without paying attention to the context of the problem. In addition, in Task 7 many of the students failed to set the equation since they could not decide the unknown and known values. Since Task 5 includes a simpler equation than Task 7, the tasks were ordered consecutively. Because Task 1 and Task 2 were quite similar to each other it was decided to be given in consecutively such that in Task 2 students were directly asked to find the rule of given patterns but in Task 1 students were expected to write about the pattern both arithmetically and verbally before algebraically. Therefore, based on the findings of the pilot study, the order of the tasks was re-organized to detect any changes in students' performances throughout the study. In that sense, Task 6 was designed as retention for Task 3 since students were asked to convert verbal expressions to algebraic expressions in both tasks.

5.3. Research setting

The design of the study was convergent parallel mixed method (Creswell, 2011) since both qualitative and quantitative data was collected simultaneously and interpreted together at the end of the study. The treatment group was randomly assigned by the researcher such that 7-A was assigned to be the study group and 7-D as the control group. The achievement test results of both of the groups were analyzed quantitatively and a representative sub-group was selected to analyze the change in the students' algebraic thinking qualitatively. Kaput (2010) also used a representative sample to understand the strategies of students on problem solving and analyzed their conversations qualitatively.

At the beginning of the study, pre-achievement test was administered in both experiment and control group to measure students' prior knowledge. Then for the following 7 weeks (dates between 08.11.2017 and 26.12.2017) 8 tasks were implemented in the elective mathematics course of 7-A. A typical implementation took two-lesson hours but Task 6 and Task 7 was applied in the same week such that one-hour lesson was allocated for the implementation of each task. The students were divided into groups and one of the pre-service teachers was assigned to each group. The student groups were formed heterogeneously in terms of their mathematics achievement during the sixth grade by their regular mathematics teacher. The appointment of pre-service teachers to each group was done randomly by the researcher. Before each implementation, the researcher met pre-service teachers to explain and discuss all of the tasks and inform them about how the tasks would be implemented in the school. To be sure that all pre-service teachers would follow the same procedure, the researcher gave a set of questions that would be asked to students during the discussion part of the implementation. Moreover, possible MOST instances were introduced to pre-service teachers to ensure that they would pay attention to those important cases. A list of possible MOST instances including *student mathematics* and *mathematical point* for each task is given in Appendix D.

During the implementation, the researcher checked all groups to make sure that the same procedures were followed by the pre-service teacher. At the beginning of the task implementation in the groups, pre-service teachers explained the tasks in the light of the discussion that had been held previously with the researcher. Then, the students were given approximately 20 minutes to work on the tasks individually and no interaction among the students was allowed. At the end of 20 minutes, pre-service teachers asked each student to explain his/her solutions and thoughts one by one. Pre-service teachers were asked to use probing questions such as "how do you know that?", "why did you do like this?" and "what if you do like this?" to elicit students' understanding. If none of the students figured out the correct answer of a particular question then the pre-service teacher would attempt to scaffold students' understanding by simplifying the given situation, using manipulative or visual aids or reviewing the problem statement and related mathematical concepts. After the discussions, the pre-service teacher asked for the thoughts of each student about the tasks and explains the solutions to check their understanding. Additionally, pre-service teachers were given suggestions about posing similar problems to the ones that have been discussed to justify students' understanding. A sample question for each task was as follows:

In task 1: How many hours and days are spent to track 200 rails?

In task 2: How many families will move during the 40^{th} year?

In task 3: How can you express algebraically the amount of the time spent in Dolmabahçe Palace if it is 1/5 of the time that is spent on Topkapı Palace?

In task 4: How can you represent 3x+2 by using algebraic tiles?

In task 5: How can you algebraically represent the weight of packages that Sırma holds?

In task 6: How can you express algebraically the amount of the steps are taken between the florist and stationer if it is 3 times of the steps that are taken between the house and the school?

In task 7: Find the amount of each ingredient that is needed to make 6 liters of the Tatlımtırak mixture.

In task 8: How can you algebraically represent the sum of numbers in the row?

It is aimed to elaborate and understand the students' algebraic thinking deeply with these questions and all of these questions were shared with the pre-service teachers to guide them for their discussions with the students.

5.4. Data Collection

The data was collected through students' written work during mathematical tasks, achievement tests and implementation videos. The tasks were implemented in study group for a 7-week period. The pre-service teachers videotaped whole implementation process and collected students' worksheets at the end of the implementation. Two forms of an achievement test were developed by the researcher and they were administered before and after the implementation. To provide the validity of the achievement tests, the researcher consulted the TUBITAK Project team members. The achievement tests involved in problems similar to the ones given in the tasks. There were five items in each test and they were evaluated out of 40 points. Pre and post-achievement tests were prepared as parallel forms to prevent the memorization of the questions and the answers. The achievement tests and their rubrics are given in Appendix E. In the pilot study the pre-achievement test was applied to both experimental and control group. The reliability coefficient of the achievement tests was 0.795. After 3 months from the post-achievement test, the pre-achievement test was administered as retention to both experiment and control group. After the conducting main study, parallel-forms reliability analysis was applied to determine the reliability coefficient of the tests. The reliability coefficient between pre and post test scores, post and retention scores are 0.71 and 0.954 respectively.

5.5. Data Analysis

Shapiro-Wilk test was used to check normality of the data since the sample size was small. The results of the test showed that the scores of achievement tests were not normally distributed. As a result, Mann Whitney U test method was applied on pre-achievement, post-achievement and retention test results of both experiment and control groups to check any changes in students' achievement in algebra due to intervention. Also effect size was calculated to measure the effect of task-assisted instruction on students' algebra achievement.

Students' answers and their explanations for the solutions of the tasks were analyzed to interpret students' algebraic thinking. To determine the depth and the accuracy of their solutions and explanations implementation videos were analyzed along with students' written work. A set of coding schemes were developed to evaluate each task. In general, students' algebraic thinking was classified into three levels. If the student was aware of algebraic concepts but failed to link them with the procedures or the problem context, then the depth of his/her thinking was coded as Level 1. If the student attempted to make links between the procedures and the concepts but failed to justify those links explicitly or give incomplete solution, then it was coded as Level 2. And finally, if the student explained the procedures and concepts clearly and justified his/her solution then his/her thinking was coded as Level 3. When students did not provide any answer to the questions or their answers were irrelevant or they stated that they did not know then their thinking was coded as No Attempt (NA). More specifically, to analyze students' algebraic thinking in terms of finding a rule for pattern, writing algebraic expressions and writing and solving equations, the following coding schemes given in Table 5.1 were used. To provide interrater reliability, the researcher consulted both of her advisors. Agreement rate between the researcher and the advisors was 0.97.

Table 5.1. Coding schemes for tasks

Algebraic thinking levels for finding the rule of the pattern	
NA	Student could not give answer or gave irrelevant answer
Level 1	Student attempts to find a rule by looking at changes in one (dependent) variable instead of relationship between two (independent and dependent) variables OR student only recognizes that letters n, m, a, b represent a variable
Level 2	Student finds the rule by trial and error process but does not recognize what leading coefficient or constant term represent for OR student recognizes what leading coefficient and constant term represent for but fails to write the rule
Level 3	Student finds the rule looking at the relationship between the independent and dependent variables and recognizes what leading coefficient and constant term represent for
Algebraic thinking levels for writing algebraic expression	
NA	Student could not give answer or gave irrelevant answer
Level 1	Student attempts to write algebraic expressions but neither pays attention to context of the problem nor mathematical operations OR only understands the concept of variable
Level 2	Student writes algebraic expressions without paying attention to context of the problem OR student understands the context of the problem but fails to apply order of operations /distributive property correctly
Level 3	Student writes algebraic expressions correctly by paying attention to context of the problem and applies correct mathematical operations

Algebraic thinking levels for setting up and solving equation	
NA	Student could not give answer or gave irrelevant answer
Level 1	Student only writes the algebraic expressions correctly OR recognizes that s/he needs to add the given amount of goods
Level 2	Student sets up the equation by adding the algebraic expressions but fails to solve the equation correctly OR sets up wrong equation but solves it correctly
Level 3	Student sets up correct equation and solves it completely

To understand the change in students' algebraic thinking better, a sub-sample group was constructed under experiment group. Out of 26 students, 9 of them were selected for further investigation. Those students were determined by the researcher and her advisors according to following criteria:

- At least one student from each group
- Differ in terms of algebraic achievement -excluding the highest and the lowest scored students
- Eagerness to learn and improve himself/herself
- Ability to clearly express his/her thinking both verbally and in written form.

To decide about students' communication skills and motivation, pre-achievement tests, and videos and students' written work of first two weeks were analyzed by the researcher and the advisors. Then Alper, Burak, Doruk, Erdem, Harun, Gonca, Mert, Tansu and Utku (pseudonym) were decided to be chosen for further analysis for algebraic thinking and learning.

6. FINDINGS

6.1. Algebra achievement of students

In this study, pre-achievement, post-achievement and retention tests were used to determine students' algebra achievement such that Mann Whitney U test was applied to analyze whether or not there was a significant difference between control and experiment group. According to pre-achievement test results, out of 40 points, the mean scores of the control group and study group were 3.82 and 6.29, respectively as seen in Table 1. Mann-Whitney U test showed that there was no significant difference between two groups (U=227.000, z=-1.102, p=0.27) for the pre-achievement test.

Table 6.1. Results of achievement tests

		N	**Mean**	**Std. dev.**	**P**
Pre-achievement	Experiment	24	6.29	7.11	0.270
	Control	23	3.82	2.90	
Post-achievement	Experiment	24	17.77	13.29	0.001*
	Control	23	6.39	5.71	
Retention	Experiment	24	14.06	12.74	0.013*
	Control	23	5.52	4.38	

* $p<0.05$

As seen in the Table 6.1, the mean score of post-test for experiment and control group was 17.77 and 6.39, respectively such that the mean score of study group was significantly higher than the control groups' score (U=124.000, z=-3.242, p=0.001). Similarly, the mean of retention test scores of study group was significantly higher than the mean scores of control group (U=159.500, z=-2.492, p=0.013). As seen in the table, the number of students who took the tests during the pre, post and retention implementation differed from each other because some students missed the tests. Therefore, their data was eliminated from the analysis. As a result, the size of the experimental group dropped to 24 students whereas the size of the control group became 23 students after the elimination.

The items in the achievement tests were about finding the rule of a pattern (item 1), writing algebraic expressions (item 2) and setting up and solving equations (items 3, 4 and 5). Each type of items was also analyzed by using Mann-Whitney U test. The item 1 was out of 5 points, the item 2 was out of 8 points and the rest three items were out of 27 points. The results obtained from the analysis are given in Table 6.2.

Table 6.2. Results of achievement tests in terms of items

Item 1		**N**	**Mean**	**Std. dev.**	**P**
Pre-achievement	Experiment	24	2.12	0.67	0.680
	Control	23	1.95	0.56	
Post-achievement	Experiment	24	3.08	1.58	0.043*
	Control	23	2.08	0.99	
Retention	Experiment	24	2.87	1.54	0.037*
	Control	23	1.95	1.02	
Item 2					
Pre-achievement	Experiment	24	2.50	3.07	0.164
	Control	23	1.21	2.23	
Post-achievement	Experiment	24	4.66	2.56	0.001*
	Control	23	2.00	2.27	
Retention	Experiment	24	3.95	2.98	0.041*
	Control	23	2.34	2.63	
Items 3, 4 and 5					
Pre-achievement	Experiment	24	1.66	4.38	0.496
	Control	23	0.65	1.36	
Post-achievement	Experiment	24	10.02	10.21	0.003*
	Control	23	2.30	3.66	
Retention	Experiment	24	6.77	8.92	0.014*
	Control	23	1.21	2.15	

* $p<0.05$

As seen from Table 6.2, there was no significant difference between experiment and control group in all items in terms of pre-test scores. However, p-values for post-test and retention test indicated that there was a significant difference between experiment and control group. Furthermore, effect size was calculated for both post-achievement and retention test. For post-achievement test the effect size was 0.472 and for retention test the effect size was 0.363. The effect size for the post-test might be interpreted as moderate level while for retention it was counted as small effect (Creswell, 2011). These results were parallel to p-values for both tests since the students were benefited from task-assisted instruction. However in the retention test the students' scores in retention test were lower than their scores from post-achievement test probably because they forgot some of the information learned from tasks. As a result the effect of task-assisted instruction decreased in the retention test.

The change in sub-sample group's achievement test scores were analyzed by using Wilcoxon signed rank test since the sample size was small. As seen in Table 6.3 the difference between the pre and posttest, and pre and retention test results were significant.

Table 6.3. Sub-sample group's achievement test results

	Pre-achievement	**Post-achievement**	**Retention**	**p-value pre-post**	**p-value pre-retention**	**p-value post-retention**
Alper	8	32	32	0.008*	0.013*	0.049*
Burak	2	10	5			
Doruk	8	24	17			
Erdem	2	19	11			
Gonca	6	13	18			
Harun	2	5	6			
Mert	2	9	4			
Tansu	8	29	9			
Utku	7	15	6			
$\bar{x}$	**5**	**17.33**	**12**			
$s_{\bar{x}}$	**2.92**	**9.34**	**9.06**			

* p<0.05

6.2. The components of algebraic thinking of students

In this study, algebraic thinking of students was defined as students' ability to find the rule of a given pattern, convert verbal expressions into algebraic expressions and vice versa and set up and solve equations. Indeed, setting up an equation and writing algebraic expressions are related to each other as well as solving equations entails knowledge of applying four operations into algebraic expressions correctly. However, some of the items in the tasks were quite specific to each components of algebraic thinking. Below, the findings about each component are discussed separately by providing results from sub-sample students' individual performances on the tasks and pre-service teacher-student discussions on the tasks.

6.2.1. Finding the rule of a pattern

In this study, Task 1, Task 2 and Task 8 included 7 questions in total about finding the rule and the terms of given patterns. In Task 1, there were two questions about finding the rules of given patterns. In one of the questions students were expected to find the relationship between the number of hours and the rails for roller coaster was built (the rule was $40n$). In the second problem, the students were expected to find the rule for the number of seats built up on roller coaster such that the locomotive had two seats and each day three more seats were added up to the coaster. Then the rule for the pattern was $3n+2$. In Task 2, students were given related problems about renovating their neighborhood such that each year three families had moving to another place for renovation (the rule was $3y$), the number of floors in new apartment buildings were getting higher by adding four flats each year (the rule was $4n+2$) and the number of renovated streets were increasing by two each year (the rule was $2m+1$). In Task 2, there was an additional problem whose rule was $2s+11$. Finally, in Task 8, the students were given a puzzle such that the number of squares was increasing by four in each step and the rule was $4n+1$ (see Appendix A for the tasks). The students' performances on these questions during both individual work and discussions were analyzed by using the coding scheme of algebraic thinking levels for finding the rule for patterns. The results of coding are given in Table 6.4.

Table 6.4. Sub-sample group's algebraic thinking levels for finding the rule of a pattern

<table>
<tr><th></th><th></th><th>Task 1 Q1</th><th>Task 1 Q2</th><th>Task 2 Q1</th><th>Task 2 Q2</th><th>Task 2 Q3</th><th>Task 2 Q Extra</th><th>Task 8 Q1</th></tr>
<tr><td rowspan="2">Alper</td><td>Ind.</td><td>-</td><td>-</td><td rowspan="2">L3</td><td>-</td><td>-</td><td>-</td><td>L2</td></tr>
<tr><td>Dis.</td><td>L1</td><td>L2</td><td>L2</td><td>L2</td><td>L3</td><td>L3</td></tr>
<tr><td rowspan="2">Burak</td><td>Ind.</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td rowspan="2">-</td><td>-</td></tr>
<tr><td>Dis.</td><td>L1</td><td>L3</td><td>L1</td><td>L2</td><td>L1</td><td>L2</td></tr>
<tr><td rowspan="2">Doruk</td><td>Ind.</td><td>-</td><td>-</td><td>-</td><td rowspan="2">L1</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td>Dis.</td><td>L1</td><td>L2</td><td>L1</td><td>L1</td><td>L2</td><td>L3</td></tr>
<tr><td rowspan="2">Erdem</td><td>Ind.</td><td>-</td><td>L1</td><td rowspan="2">L1</td><td rowspan="2">L3</td><td rowspan="2">L2</td><td rowspan="2">L2</td><td rowspan="2">L3</td></tr>
<tr><td>Dis.</td><td>L1</td><td>L2</td></tr>
<tr><td rowspan="2">Gonca</td><td>Ind.</td><td>-</td><td>-</td><td rowspan="2">L1</td><td>L1</td><td rowspan="2">L1</td><td>L1</td><td>L1</td></tr>
<tr><td>Dis.</td><td>L3</td><td>L2</td><td>L3</td><td>L3</td><td>L2</td></tr>
<tr><td rowspan="2">Harun</td><td>Ind.</td><td rowspan="2">L1</td><td rowspan="2">L1</td><td>L1</td><td>L1</td><td>-</td><td rowspan="2">-</td><td>L1</td></tr>
<tr><td>Dis.</td><td>L2</td><td>L2</td><td>L2</td><td>L3</td></tr>
<tr><td rowspan="2">Mert</td><td>Ind.</td><td>-</td><td rowspan="2">-</td><td>-</td><td>-</td><td>-</td><td rowspan="2">-</td><td>-</td></tr>
<tr><td>Dis.</td><td>L3</td><td>L2</td><td>L2</td><td>L1</td><td>L2</td></tr>
<tr><td rowspan="2">Tansu</td><td>Ind.</td><td rowspan="2">L3</td><td rowspan="2">L3</td><td rowspan="2">L1</td><td rowspan="2">L3</td><td>L1</td><td>-</td><td>L1</td></tr>
<tr><td>Dis.</td><td>L2</td><td>L3</td><td>L3</td></tr>
<tr><td rowspan="2">Utku</td><td>Ind.</td><td>-</td><td rowspan="2">L1</td><td rowspan="2">L1</td><td rowspan="2">L1</td><td>L1</td><td rowspan="2">L1</td><td rowspan="2">L1</td></tr>
<tr><td>Dis.</td><td>L1</td><td>L2</td></tr>
</table>

Note: L: Level, Ind: Individual work, Dis: Group discussion, -: No Attempt (NA)

In Table 6.4, if a student did not attempt to solve the question or s/he wrote something irrelevant then it is shown as (-) in the cell. Furthermore, (-) sign was used to show that the additional problem of Task 2 was not discussed during the implementation. Whenever the level of student's algebraic thinking changed during the group discussions then it is shown separately in rows of the table. However, when it was not the case then the code of student's thinking is shown in a single row.

To analyze the change in students' algebraic thinking in finding patterns, students' algebraic thinking levels during the individual work and group discussions are summarized in Table 6.5.

Table 6.5. Frequencies of students' algebraic thinking levels for finding the rule of a pattern

		Task 1				**Task 2**				**Task 8**			
		NA	L1	L2	L3	NA	L1	L2	L3	NA	L1	L2	L3
Alper	Ind.	2				3			1			1	
	Dis.		1	1				2	2				1
Burak	Ind.	2				4				1			
	Dis.		1		1	1	2	1				1	
Doruk	Ind.	2				3	1			1			
	Dis.		1	1			3	1					1
Erdem	Ind.	1	1				1	2	1				1
	Dis.		1	1			1	2	1				1
Gonca	Ind.	2					4				1		
	Dis.				2		2		2			1	
Harun	Ind.		2			2	2				1		
	Dis.		2			1		3					1
Mert	Ind.	2				4				1			
	Dis.	1			1	1	1	2				1	
Tansu	Ind.				2	1	2		1		1		
	Dis.				2		1	1	2				1
Utku	Ind.	1	1				4				1		
	Dis.		2				3	1			1		

From Table 6.4 and Table 6.5 it can be deduced that the students' prior knowledge was low since most of the students gave irrelevant or no attempt answers in individual work of Task 1. After the discussions however, the level of most of the students' algebraic thinking was Level 1 since they could only understand the meaning of the variable. In Task 2, again some of the students gave irrelevant or no attempt answers. For instance, Mert attained a number to "y" in the first question and wrote 21 instead of the rule of the pattern. As a result, his thinking was coded as NA. After the discussions in Task 2, only in some of the questions the students could reach Level 3 thinking. As an example, Tansu stated the number of the constant triangles in the flower will be the constant term of the formula and the coefficient terms will be the amount increase in the number of the leaves in the flower. The students' algebraic thinking rose to Level 3 only in the second and the forth questions of the task where the students were required to use manipulatives (pattern blocks). In this case using pattern blocks probably supported their algebraic thinking since they realized where the coefficient term and constant term of the rule of the pattern emerged from pattern blocks. Furthermore, in Figure 6.1, the distribution of the percentage of the frequencies of students' algebraic thinking levels during individual work and group discussions is summarized as a chart.

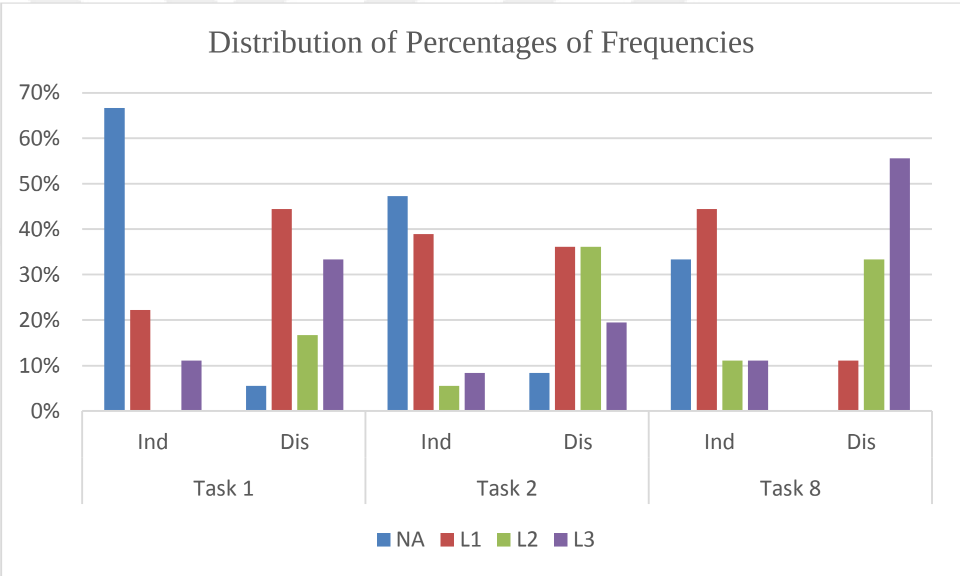

Figure 6.1. The change of frequencies of students' algebraic thinking in component 1

From Tables 6.4 and 6.5 and Figure 6.1, it can be concluded that most of the students' algebraic thinking levels increased after group discussions. Furthermore, students' algebraic thinking had developed through Task 1 to Task 8 since their algebraic thinking was mostly coded as at Level 2 and Level 3 in Task 8 whereas in Task 1 they gave irrelevant answers or could not give any answers as seen in the Table 6.5. Only 3 of the students' algebraic thinking level coded as NA in individual work of Task 8. The reason might be their lack of prior knowledge according to their pre-achievement test results. However, these students benefited from task-assisted instruction since the level of algebraic thinking became at least Level 2 at the end of the discussions.

6.2.2. Writing algebraic expressions

In this study Task 3, Task 5, Task 6 and Task 8 involved 4 questions in total about algebraic expressions. In Task 3, the students were given a route for historical tour such that the distance between the school and Ayasofya was given as ‘X’ and the time spent in the initial place (Ayasofya) as ‘T’. There were 10 directions to be followed for the route such that the students were expected to read the directions carefully, choose appropriate variable and write correct algebraic expression for the route.

In Task 5, there was one question related to algebraic expressions. In the context of the task, four kids went to Bazaar with their father and bought some fruits and bag of pickles. The weight of the bag of pickles was unknown. There was a question asking for writing an appropriate algebraic expression for the amount of weight that two of the kids were carrying on. It was written that Ayşe carried on 3 bags of pickles whereas Murat carried on one bag of pickle, a kilo of banana and half kilo of apple. Therefore, the algebraic expressions of weights that Ayşe and Murat carried on were $3x$ and $x + 1.5$, respectively. As similar to Task 3, in Task 6 a map showing the paths of the two siblings took for different places after school was given. The number of the steps taken for each place was written in terms of the number of steps between home and the school. For instance, Mehmet (one of the siblings) would walk as twice number of the steps that taken between the school and home. The algebraic expression for his steps was $2x$ when the number of steps taken between the school and home was accepted as x.

In Task 8 there was one question about writing algebraic expressions. In the task it was stated that Ali prepared a mathematics puzzle consisting of squares with different colors. He assigned an unknown value for the blue square and he gave directions about the values of other squares with respect to the blue square's value. For instance, the value of green square was 2 more than 5 times the value of the blue square. Then the value in the green square would be $5x+2$ if the value of blue square was assigned as to be x.

To analyze the change in students' algebraic thinking in converting verbal expressions into algebraic expressions, students' algebraic thinking levels during the individual work and group discussions are summarized in Table 6.6, Table 6.7 and Figure 6.2.

Table 6.6. Sub-sample group's algebraic thinking levels for algebraic expressions

<table>
<tr><th></th><th></th><th>Task 3
Q1</th><th>Task 5
Q2</th><th>Task 6
Q1</th><th>Task 8
Q2</th></tr>
<tr><td rowspan="2">Alper</td><td>Ind.</td><td>L2</td><td>-</td><td rowspan="2">L3</td><td>L2</td></tr>
<tr><td>Dis.</td><td>L3</td><td>L3</td><td>L3</td></tr>
<tr><td rowspan="2">Burak</td><td>Ind.</td><td>L1</td><td rowspan="2">L2</td><td>L2</td><td rowspan="2">L2</td></tr>
<tr><td>Dis.</td><td>L2</td><td>L3</td></tr>
<tr><td rowspan="2">Doruk</td><td>Ind.</td><td rowspan="2">L1</td><td rowspan="2">L3</td><td rowspan="2">L2</td><td rowspan="2">L2</td></tr>
<tr><td>Dis.</td></tr>
<tr><td rowspan="2">Erdem</td><td>Ind.</td><td>L2</td><td>-</td><td rowspan="2">L2</td><td rowspan="2">L2
L3</td></tr>
<tr><td>Dis.</td><td>L3</td><td>L3</td></tr>
<tr><td rowspan="2">Gonca</td><td>Ind.</td><td>L2</td><td rowspan="2">L2</td><td rowspan="2">L2
L3</td><td>L2</td></tr>
<tr><td>Dis.</td><td>L3</td><td>L3</td></tr>
<tr><td rowspan="2">Harun</td><td>Ind.</td><td rowspan="2">L2
L3</td><td rowspan="2">-
L2</td><td rowspan="2">L3</td><td>L2</td></tr>
<tr><td>Dis.</td><td>L3</td></tr>
<tr><td rowspan="2">Mert</td><td>Ind.</td><td>L1</td><td rowspan="2">L2</td><td>L3</td><td rowspan="2">L2</td></tr>
<tr><td>Dis.</td><td>L2</td><td>-</td></tr>
<tr><td rowspan="2">Tansu</td><td>Ind.</td><td rowspan="2">L2
L3</td><td rowspan="2">L3</td><td rowspan="2">L3</td><td rowspan="2">L2
L3</td></tr>
<tr><td>Dis.</td></tr>
<tr><td rowspan="2">Utku</td><td>Ind.</td><td rowspan="2">L2</td><td rowspan="2">L3</td><td rowspan="2">L2</td><td rowspan="2">L2</td></tr>
<tr><td>Dis.</td></tr>
</table>

Note: L: Level, Ind: Individual work, Dis: Group discussion, -: NA

Table 6.7. Frequencies of students' algebraic thinking levels for algebraic expressions

		Task 3				Task 5				Task 6				Task 8			
		NA	L1	L2	L3	NA	L1	L2	L3	NA	L1	L2	L3	NA	L1	L2	L3
Alper	Ind			1		1							1			1	
	Dis				1				1				1				1
Burak	Ind		1					1				1				1	
	Dis			1				1					1			1	
Doruk	Ind		1						1			1				1	
	Dis		1						1			1				1	
Erdem	Ind			1		1						1				1	
	Dis				1				1			1					1
Gonca	Ind			1				1				1				1	
	Dis				1			1					1				1
Harun	Ind			1		1							1			1	
	Dis				1			1					1				1
Mert	Ind		1					1					1			1	
	Dis			1				1		1						1	
Tansu	Ind			1					1				1			1	
	Dis				1				1				1				1
Utku	Ind			1					1			1				1	
	Dis			1					1			1				1	

Furthermore, in Figure 6.2, the distribution of the percentage of the frequencies of students' algebraic thinking levels during individual work and group discussions is summarized as a chart.

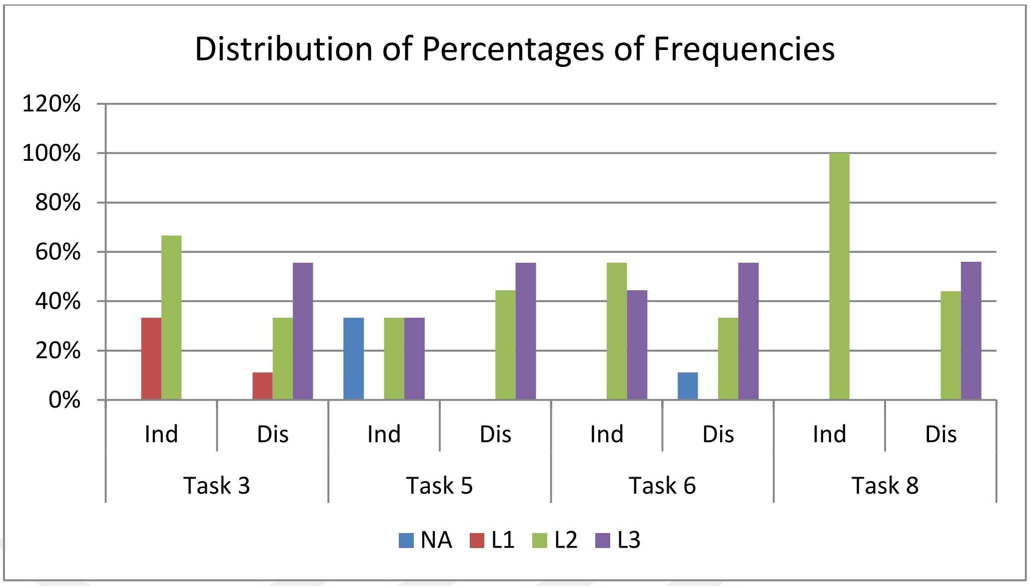

Figure 6.2.The change of frequencies of students' algebraic thinking in component 2

From Table 6.6, Table 6.7 and Figure 6.2 it can be concluded that all of the students' have some prior knowledge on algebraic expressions since from Task 3 their thinking levels were at least Level 1. Most of the students' algebraic thinking is improved at the end of the task. Except Doruk, the level of students' thinking became at least Level 2 at the end of the discussions.

Even though the students had some prior knowledge, 3 of the students (Alper, Erdem and Harun) gave irrelevant answers. As an example of their irrelevant answer, Alper wrote some random fractions instead of focusing algebraic expressions. As a result, his algebraic thinking was coded as NA (no attempt answer). After the discussions however the level of students' algebraic thinking became at least Level 2 since they could write correct algebraic expressions with the help of their teacher. All of the students could use what they learned the following task because even in the individual work the level of their algebraic thinking was at least Level 2. However, only in Mert's group the question cannot be discussed and his algebraic thinking cannot be interpreted and so coded as NA answer. In Task 6, 3 (Alper, Harun and Tansu) of the students' algebraic thinking was at Level 3 even in their individual work. The same students' algebraic thinking was at Level 2 in the individual work of Task 8. The reason might be that the complexity of the context since the students had difficulties in understanding the problem in Task 8. In Task 8, the values in different squares were sequentially depend on each other whereas in Task 6 all of the algebraic expressions were depend on the steps taken between home and the school. Therefore, students had to write harder algebraic expressions in Task 8. In this case, they especially made mistakes in writing the algebraic expressions of yellow and red squares. For instance in her individual work Tansu failed to use parenthesis for red and yellow squares. On the contrary she could write the correct algebraic expressions for the rest of the squares. As a result her algebraic thinking was coded as Level 2. The other students had the same difficulties with Tansu, but 5 of them could improve their algebraic thinking to Level 3 after the discussions. Therefore, most of the students could improve their algebraic thinking in algebraic expressions from Task 3 to Task 8.

6.2.3. Setting up and solving equations

In this study, Task 5, Task 6, Task 7 and Task 8 include 6 questions in total about setting up and solving equations. As explained the previous section, Task 5 was related to shopping at Bazaar. Four kids were helping their father to carry the fruits and bags of pickles. The weight of a bag of pickles was unknown and, in the question, total weight that one of the kids (Ali) was carrying on was given as 6 kilograms. Ali was carrying on two bags of pickles and two kilos of mandarin. Thus, the equation to find the weight of a bag of pickles would be as $2x + 2 = 6$ and the weight of the bag would be *2* kilograms. The context of the extra question was similar to that question since a recipe of a pickle was given such that the amount of ingredients was given with respect to the amount of the carrot would be put in pickle. The total amount of ingredients was given as 5.5 kilograms such that the equation to find the amount of the carrot would be as $x + 3x + 2x + 500 + \frac{x}{4} = 5000$. The solution of this equation (the amount of carrot) was *800* grams.

In Task 6, there was one question about setting up and solving equations. As described in the previous section, a map that showing the route of two siblings was given in Task 6. In one of the questions, the steps taken by Mehmet between the stationary and the school and the steps taken by Pelin between the school and volleyball court were given to be equal. The equation was $4x - 120 = 3x + 30$ such that x would be *150*. As similar to the additional question of Task 5, in Task 7 there were two recipes for juice mixtures and in both of them the amount of ingredients was given with respect to one of the ingredients in the recipe. The total amounts of two mixtures were 3000 ml and 2000 ml, respectively. The equation for the first one was $9x + 300 = 3000$ and the second one was $\frac{10x}{6} = 2000$.

In Task 8, there was one question about setting up and solving equations. As mentioned in the previous section, Ali prepared a mathematics puzzle which consisted of squares with different colors. Each color of square has a different value and these values were determined by some rules. The sum of the values written in two green squares, two purple squares and one blue square was given as 66. Then, the equation for the unknown value in blue square would be $21x + 24 = 66$ such that x would be *2*.

To analyze the change in students' algebraic thinking in setting up and solving equations, students' algebraic thinking levels during the individual work and group discussions are summarized in Table 6.8 and Table 6.9 and Figure 6.3.

Table 6.8. Sub-sample group’s algebraic thinking levels for setting up and solving equations

<table>
<tr><th></th><th></th><th>Task 5 Q1</th><th>Task 5 Extra Q</th><th>Task 6 Q2</th><th>Task 7 Q1</th><th>Task 7 Q2</th><th>Task 8 Q3</th></tr>
<tr><td rowspan="2">Alper</td><td>Ind.</td><td rowspan="2">L3</td><td rowspan="2">L3</td><td rowspan="2">L3</td><td rowspan="2">L3</td><td>L1</td><td>L2</td></tr>
<tr><td>Dis.</td><td>L2</td><td>L3</td></tr>
<tr><td rowspan="2">Burak</td><td>Ind.</td><td>-</td><td rowspan="2">-</td><td rowspan="2">-</td><td rowspan="2">-</td><td rowspan="2">L1</td><td rowspan="2">L1
L2</td></tr>
<tr><td>Dis.</td><td>L1</td></tr>
<tr><td rowspan="2">Doruk</td><td>Ind.</td><td rowspan="2">L2</td><td rowspan="2">-</td><td>-</td><td rowspan="2">L2</td><td>-</td><td>-</td></tr>
<tr><td>Dis.</td><td>L2</td><td>L3</td><td>L2</td></tr>
<tr><td rowspan="2">Erdem</td><td>Ind.</td><td>-</td><td rowspan="2">-</td><td rowspan="2">L1
L2</td><td rowspan="2">-</td><td rowspan="2">L1</td><td rowspan="2">L3</td></tr>
<tr><td>Dis.</td><td>L2</td></tr>
<tr><td rowspan="2">Gonca</td><td>Ind.</td><td>-</td><td>-</td><td rowspan="2">L2
L3</td><td>L1</td><td rowspan="2">L1
L3</td><td>-</td></tr>
<tr><td>Dis.</td><td>L2</td><td>L1</td><td>L2</td><td>L2</td></tr>
<tr><td rowspan="2">Harun</td><td>Ind.</td><td>-</td><td>-</td><td>L1</td><td rowspan="2">L2</td><td rowspan="2">-</td><td rowspan="2">-
L2</td></tr>
<tr><td>Dis.</td><td>L3</td><td>L1</td><td>L2</td></tr>
<tr><td rowspan="2">Mert</td><td>Ind.</td><td>L1</td><td rowspan="2">-</td><td>L2</td><td rowspan="2">L1</td><td rowspan="2">-</td><td rowspan="2">L1</td></tr>
<tr><td>Dis.</td><td>L2</td><td>L3</td></tr>
<tr><td rowspan="2">Tansu</td><td>Ind.</td><td rowspan="2">L3</td><td rowspan="2">-
L3</td><td rowspan="2">L3</td><td rowspan="2">-</td><td rowspan="2">L3</td><td>L2</td></tr>
<tr><td>Dis.</td><td>L3</td></tr>
<tr><td rowspan="2">Utku</td><td>Ind.</td><td>L1</td><td rowspan="2">L1</td><td rowspan="2">L1
L3</td><td rowspan="2">-
L1</td><td>-</td><td rowspan="2">-
L2</td></tr>
<tr><td>Dis.</td><td>L2</td><td>L1</td></tr>
</table>

Note: L: Level, Ind: Individual work, Dis: Group discussion, -: NA

Table 6.9. Frequencies of students' algebraic thinking levels for setting up and solving equations

F		**Task 5**				**Task 6**				**Task 7**				**Task 8**			
		NA	L1	L2	L3	NA	L1	L2	L3	NA	L1	L2	L3	NA	L1	L2	L3
Alper	Ind.				2				1		1		1			1	
	Dis.				2				1			1	1				1
Burak	Ind.	2				1				1	1				1		
	Dis.	1	1			1				1	1					1	
Doruk	Ind.	1		1		1				1		1		1			
	Dis.	1		1				1		1			1			1	
Erdem	Ind.	2					1			1	1						1
	Dis.	1		1				1		1	1						1
Gonca	Ind.	2						1			2			1			
	Dis.		1	1					1			1	1			1	
Harun	Ind.	2					1			1		1		1			
	Dis.		1		1			1		1		1				1	
Mert	Ind.	1	1					1		1	1				1		
	Dis.	1		1				1		1	1				1		
Tansu	Ind.	1			1				1	1			1			1	
	Dis.				2				1	1			1				1
Utku	Ind.		2				1			2				1			
	Dis.		1	1				1			2					1	

Furthermore, in Figure 6.3, the distribution of the percentage of the frequencies of students' algebraic thinking levels during individual work and group discussions is summarized as a chart.

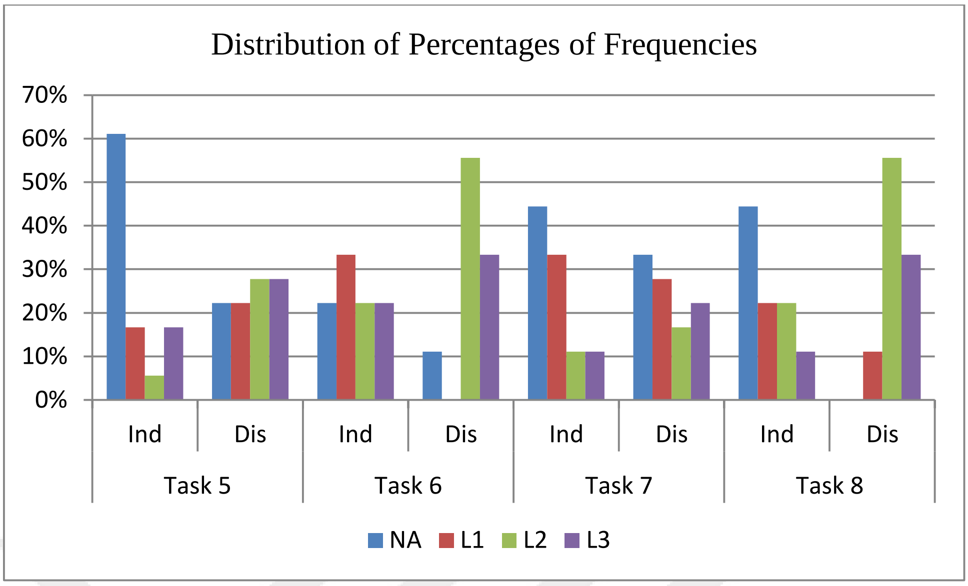

Figure 6.3.The change of frequencies of students' algebraic thinking in component 3

As seen in Tables 6.8 and 6.9 and Figure 6.3, most of the students had irrelevant answers or had no attempt to answer the questions in Task 5 in the individual work except two students (Alper and Utku). Alper had Level 3 thinking both in individual work and discussion part for both of the questions. That is, he could set up and solve the equation by himself. So, his results did not change as a result of the discussion. On the contrary, Utku could improve his thinking during the discussions since one of his answers in Level 1 became Level 2 in the discussion. In the individual work Utku could only write the algebraic expressions so the level of his thinking was Level 1 whereas in the discussion part Utku solved the equation after his teacher helped him to set up the equation. In Task 5, Burak, Harun, Gonca and Erdem failed to answer given question during individual work. At the end of the discussions Burak and Erdem were able to understand the problem to some extent but their thinking was at Level 1 and Level 2, respectively. On the other hand, they did not answer the extra question of the task. Harun's and Gonca's level of thinking changed for both questions. While Harun's thinking level became Level 1 and Level 3 during discussions, Gonca's thinking level became Level 1 and Level 2. Differently; during individual work Doruk, Mert and Tansu had one irrelevant answer and their thinking level were in Level 2, Level 1 and Level 3 respectively.

Doruk's thinking level did not change during the discussions whereas one of Mert's thinking level became Level 2. Moreover, at the end of the discussions Tansu had Level 3 thinking in both of the questions. That is, she was able to set up and solve the equation, and explain the steps of the solution by herself.

In Task 6, the levels of students' thinking were generally higher than the previous task even in their individual work. As a result, it can be deduced that they learned from previous task. In this case Burak was an exception since his answers were irrelevant both in individual work and discussion session. The group dynamics and his lack of prior knowledge probably effected Burak since two of his friends had difficulties in understanding setting up and solving the equations. Therefore, his pre-service teacher might have no enough time to discuss with Burak.

In Task 7 however, the students (except Alper and Gonca) failed to give correct answer at least one of the questions. The students could not apply what they learned from previous tasks probably because of the different context of the problems. A mentioned in the previous section the questions in Task 7 had a more complex nature since the amount of ingredients depend on the previous amount of ingredient so the students required to follow the context more carefully to understand the reference point. Also, there were more than 2 algebraic expressions needed to be added in this task different than the previous tasks. Therefore, they wrote wrong algebraic expressions by not paying attention to context of the problem or could not decide how to set up the equation so their algebraic thinking coded as NA (no attempt or irrelevant answer). After the discussions Doruk, Gonca and Tansu could reach Level 3 thinking that is, setting up and solving the correct equation by themselves, at least in one question. Moreover Alper and Harun had Level 2 thinking in one of the questions since they could set up and solve equations with the help of their pre-service teachers. On the contrary, nearly half of the students could not improve their algebraic thinking probably because of the complex nature of the problems in Task 7.

Some of the students could transfer what they learned from previous tasks since the number of students who gave NA answer decreased comparing to Task 7. In their individual work Burak and Mert could only write correct algebraic expressions and their algebraic thinking coded as Level 1. Within the task only Mert could not improve his algebraic thinking. The other students could improve their algebraic thinking at least one level and 3 of the students (Alper, Erdem and Tansu) could improve their thinking up to Level 3 since they could set up and solve correct equation.

6.3. Algebraic thinking of sub-sample students

In the following sub-sections, the students' algebraic thinking in each component is given in the tables and graphs for each student. Component 1 is used to indicate finding the rule and the terms of the pattern, Component 2 is for converting verbal expressions to algebraic expressions and algebraic expressions to verbal expressions and lastly Component 3 is for setting up and solving the equations. The first level indicates the students' algebraic thinking level in the individual work of the related task and the second level reflects the students' highest level in the last task of the related task in both tables and graphs. In graphs NA (no attempt or irrelevant answer) is represented with Level 0 to get a simplified graph. Moreover, description of a MOST instance is used to explain changes in students' algebraic thinking. In this case each dialogue between the researcher-student and pre-service teacher-student can be seen as the example of MOST instances that occurred during the implementations.

6.3.1. Alper's algebraic thinking

Alper's prior knowledge on all of the components of algebraic thinking was higher than most of the students in the sub-group since his algebraic thinking was mostly coded as at Level 2 even during the individual work even though it was not the case for Component 1. Changes in Component 1 are presented as a graph in Figure 6.4.

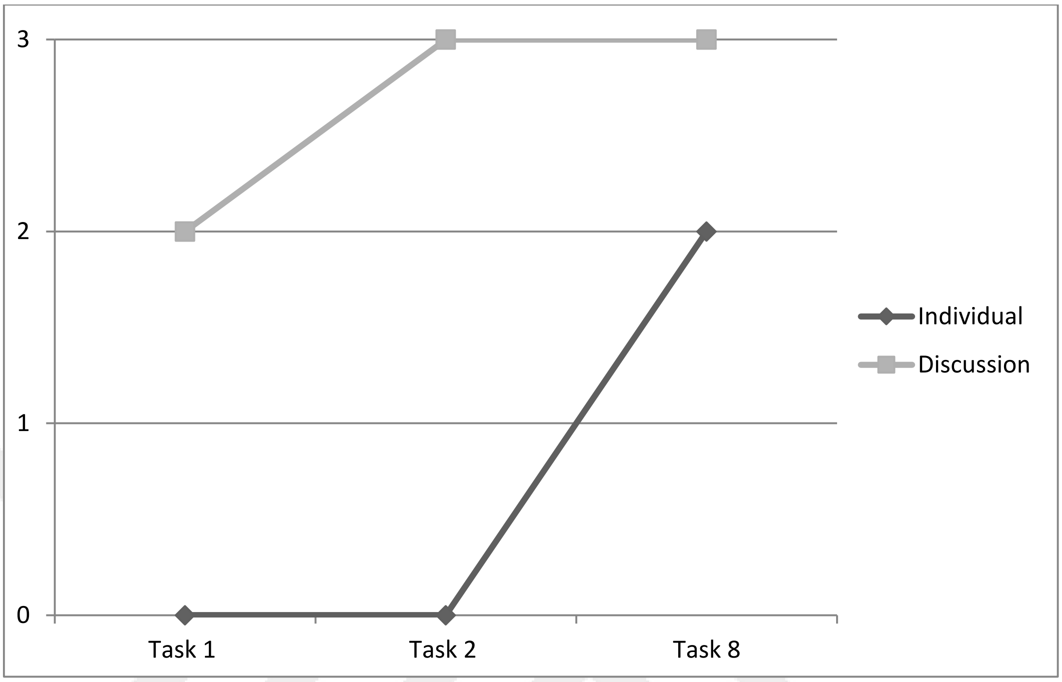

Figure 6.4. The change in Alper's algebraic thinking in component 1

For component 1, finding the rule and the terms of the patterns, Alper had two and three irrelevant answers at the beginning but he could develop his thinking up to Level 2 and Level 3 in Task 1 and Task 2, respectively. Afterwards, his algebraic thinking in Task 8 was coded as at Level 2 for the individual work but it increased to Level 3 after discussion as seen in Table 6.5. That is, he was able to find the rule by trial and error process (Level 2) or he was able to understand the relationship between the independent and dependent variables and recognized what leading coefficient and constant term represented for (Level 3). Similarly, in tasks related to algebraic expressions Alper could develop his algebraic thinking through Task 3 to Task 6 as seen in Figure 6.5 since his algebraic thinking was at Level 3 in the individual work at Task 6. Only in Task 8 the level of his algebraic thinking decreased to Level 2 probably because of the complexity of the problem.

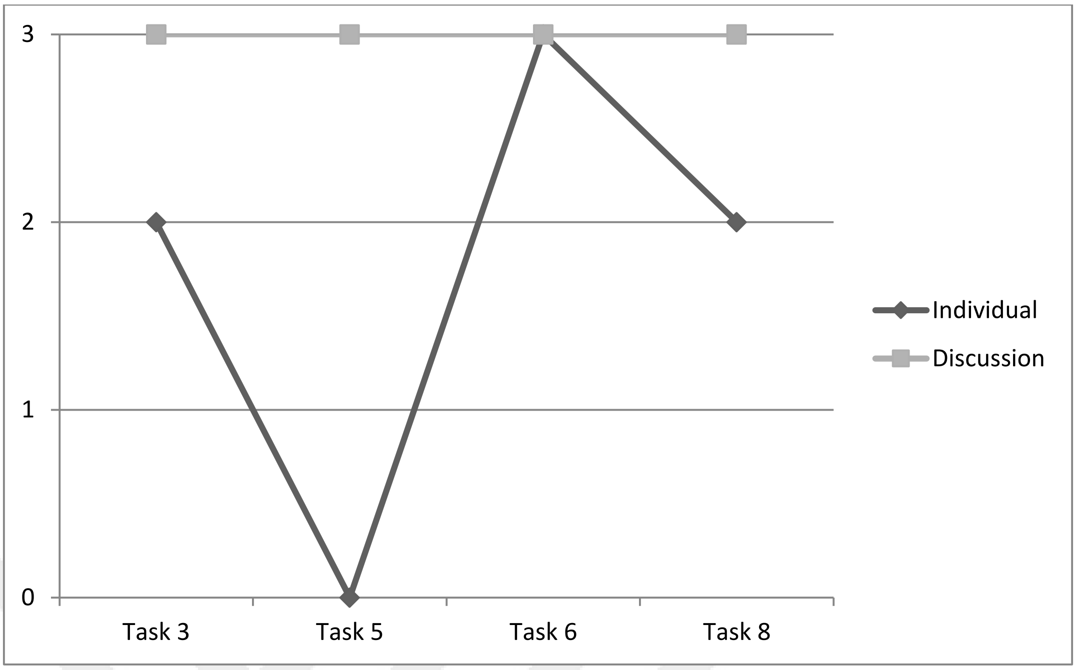

Figure 6.5. The change in Alper's algebraic thinking in component 2

In writing algebraic expressions we expected 1 MOST instance where students would have difficulties in using the correct operation. This MOST instances was observed in Alper's answer in Task 5. From Table 6.7 it can be observed that Alper had an irrelevant answer in Task 5 and the reason of his wrong answer was his difficulty on algebraic operations of verbal expressions such as "the one-fifth of the steps taken between the school and home". He was not sure if it is needed to use multiplication or division for "the one-fifth". For this case, he overcame his difficulty in the next task since he had Level 3 thinking because he used correct operation. In the last task, his thinking level was again in Level 2 individual work but it can be explained with the complex context of the problem since the problem required using many operations in one time such as addition, multiplication and parenthesis. After the discussions however, his thinking became Level 3 again since he realized that he confused the algebraic expressions of green and purple squares and wrote the correct algebraic expressions by himself. For the last component of algebraic thinking which is setting up and solving algebraic expressions, Alper's prior knowledge was in a high level since his algebraic thinking were in Level 3 even in his individual work as seen in Figure 6.6.

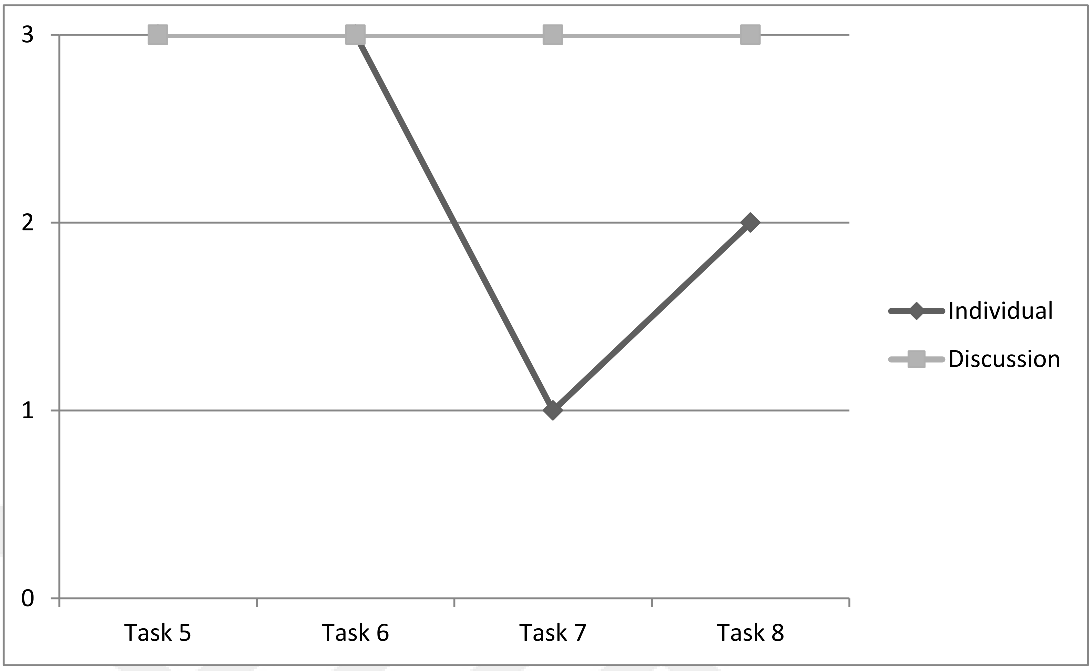

Figure 6.6. The change in Alper's algebraic thinking in component 3

Alper could explain his thinking in the solution of the equation with the following sentence: "We need to subtract 2 from 6 and since there are $2x$ we need to divide the result with 2" and he could find the unknown weight in this way. Only in Task 7 and Task 8 he had Level 1 and Level 2 thinking respectively since he did not know how to do operations on algebraic expressions. He could only write the correct equation in one question in Task 7, so his algebraic thinking was categorized as Level 1. Also in Task 8, he solved the wrong equation correctly because he wrote some of the algebraic expressions incorrectly. However, he realized that he made a mistake and wrote the correct equation by himself, solved it correctly and found all of the values of the squares. This case was thought as a sign for the improvement in his algebraic thinking since even though he made mistakes he managed to increase his thinking level to Level 3 afterwards.

6.3.2. Burak's algebraic thinking

Burak had mostly irrelevant or no attempt answers in the first tasks of each component, indicating that he had a lack of prior knowledge on all components of algebraic thinking. Changes in Component 1 are presented as a graph in Figure 6.7.

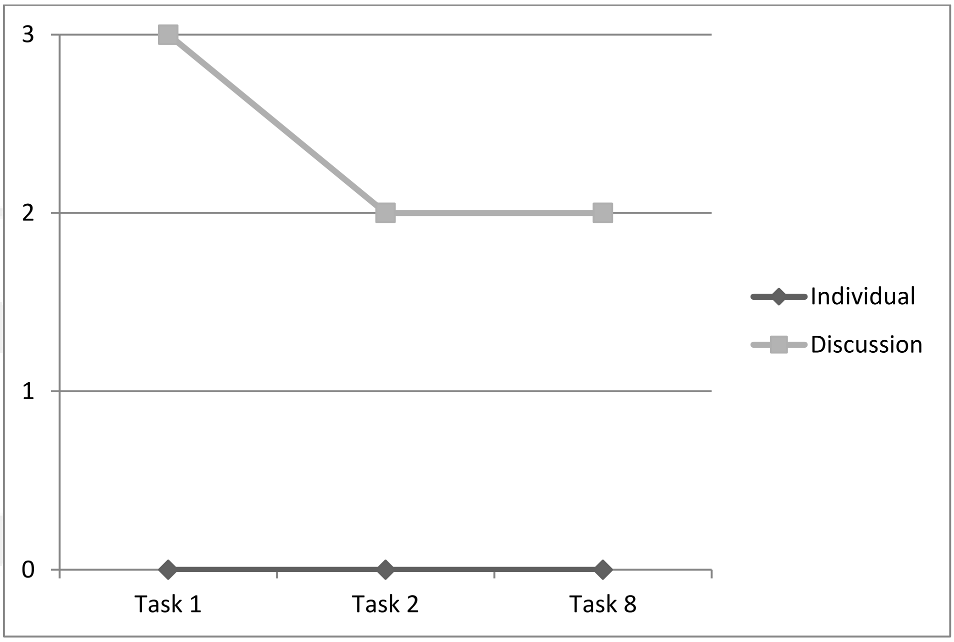

Figure 6.7. The change in Burak's algebraic thinking in component 1

Even though he had a lack of prior knowledge in all of the components, in the first task on finding the rule of the terms of the pattern he could understand the constant seats will be the constant term of the rule after the discussion of the rule in the 3rd question. Unfortunately, he could not transfer his knowledge neither Task 2 nor Task 8 but from Table 6.5 it can be concluded that he could develop his own algebraic thinking within the discussions. The reason behind this fact might be his lack of prior knowledge or the group dynamics since in his group there were two students who had lowest scores. Therefore, the pre-service teacher had to focus on the other two students in the discussions. On the contrary to first component, in the second component Burak had some prior knowledge since his algebraic thinking was at Level 1 in the first task of related component as seen in Figure 6.8.

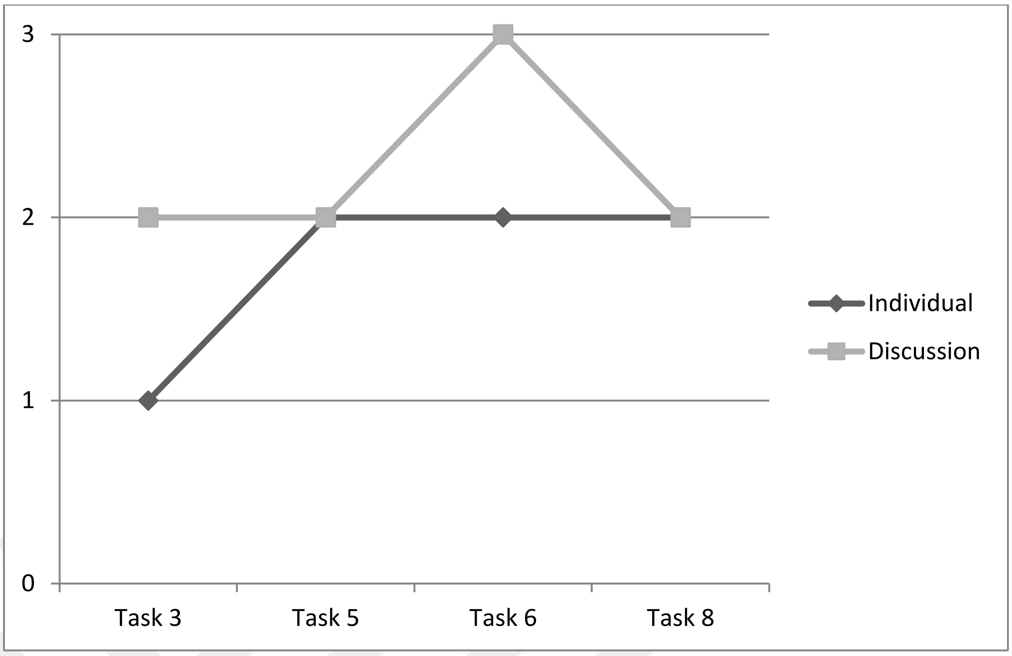

Figure 6.8. The change in Burak's algebraic thinking in component 2

The student's performance on converting verbal expressions to algebraic ones was low since in the first task (Task 3), Burak only could write 3 out of 10 algebraic expressions correctly in his individual work. He failed to use variable in some of the questions and wrote the fractions such as one third of the previous road incorrectly. Therefore, he both failed to use mathematical operations correctly and understand the concept of variable. Only in some of the questions he could identify the appropriate variable. As a result, his individual algebraic thinking is coded as Level 1. Because of the time limitation, during the discussion the pre-service teacher discussed some of the algebraic expressions. The dialogue between the pre-service teacher and the student is given below:

Teacher: What did we say for the algebraic expression between the school and Ayasofya?

Burak: x.

Then, the researcher came and posed questions.

Teacher: How can we describe the twice of this road?

Burak: 2x…

Researcher: How we can take the twice of the 2x+6?

Burak wrote 2(2x+6) and used the parenthesis correctly.

At the end of the discussions, Burak could use the mathematical operations and variable correctly in 8 questions out of 10. Since he met two of the criteria, his thinking level became Level 2.

In Task 5, in individual work, Burak misunderstood the question and wrote the algebraic expression for total amount of weights that Murat and Ayşe carry.

Teacher: We have 3 bags, how can we say the amount of weights that Ayşe carries?

Burak: 3x…

Burak could tell the weights separately but still fails to add half kilo of banana, so his algebraic expressions were not totally correct and his algebraic thinking was coded as Level 2. Even though in Task 6 he wrote one of five algebraic expressions incorrectly, he could realize his own mistake and changed his answer. Therefore, his algebraic thinking level increased from Level 2 to Level 3. As seen in Figure 6.9. Burak could develop his algebraic thinking from Task 5 to Task 8 in setting up and solving equations.

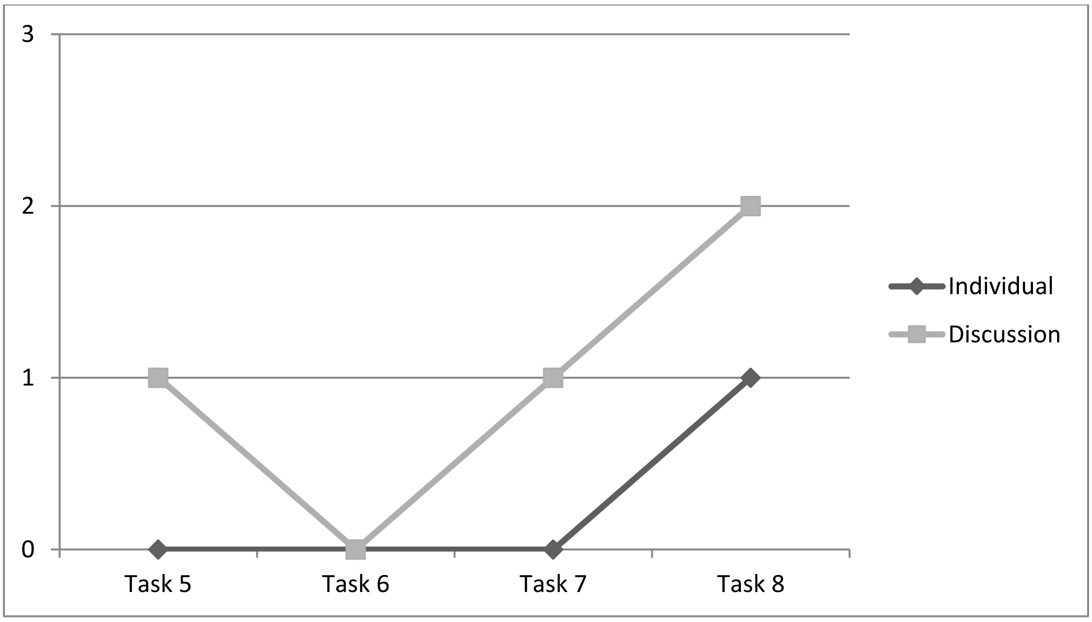

Figure 6.9. The change in Burak's algebraic thinking in component 3

Also, for setting up and solving equations, Burak had difficulties in the first task since he had two irrelevant answers at the beginning. After the questions of his teacher, he could only understand setting up the equations. In the Task 6 and Task 7 his friends and teacher explained the correct equation to Burak and applied what he learned from previous tasks to Task 8 since his algebraic thinking was in Level 1 in his individual work.

6.3.3. Doruk's algebraic thinking

Similar to Burak, Doruk had prior knowledge only in component 2 (writing algebraic expressions). Changes in Component 1 are presented as a graph in Figure 6.10.

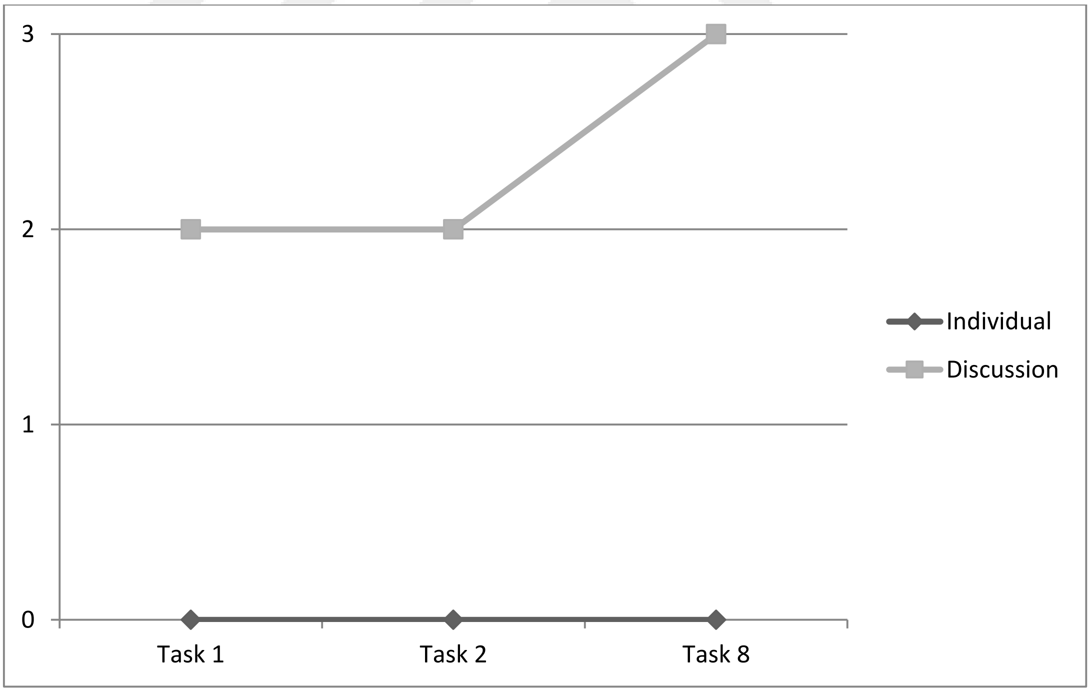

Figure 6.10. The change in Burak's algebraic thinking in component 1

Burak had two no attempt or irrelevant answers related to finding the rule and the terms of the pattern, indicating that he had lack of prior knowledge in this component. After the discussions he understood the meaning of the variable in one question, and in the other question he could state that the amount of increase in the dependent variable will be the coefficient of the variable in the rule. In the second task, he again focused on the change of one variable or gave irrelevant answers in his individual work. However, with the help of the discussions in the extra question; he could also realize that the amount of increase would be the leading coefficient and the rule must start with *2s*. The reason of his Level 2 answer might be the similarity between this question and the question in the previous task that where his algebraic thinking was in Level 2. In both of the questions there were pictures that showed the difference in the dependent variable in days or weeks. In the individual work of the last task, Doruk gave an incorrect answer by focusing on the change in one variable again. In finding rules we expected 1 MOST instances where students would focus on the change in one variable rather than covariation. This MOST instance was observed in Doruk's individual work of the last task. After the discussion with the researcher however he realized the coefficient and the constant term of the rule with the following dialogue:

Researcher: What is the constant?

Doruk pointed the square in the middle of the shape, which is the constant square.

Researcher: You said the red square is constant, how many squares I add to this square in that shape? (She pointed the second shape).

Doruk: 8?

Researcher: What n represents in here?

Doruk: The unknown…

Researcher: There are 4 in the first shape, 8 in the second shape.

Doruk: 4×2…

Researcher: What can you say about third shape then?

Doruk: 3×4…

Researcher: What is our rule then?

Doruk: 4n+1…

He actually derived the rule of the pattern by himself with the previous discussion, and then his thinking level became Level 3 at the end of the discussions. Therefore, his algebraic thinking was developed from irrelevant answers to Level 3 through the tasks. The change in Doruk's algebraic thinking is given in Figure 6.11. As seen in the Figure 6.11 Doruk's algebraic thinking did not changed within the tasks since the levels of his algebraic thinking were the same for his individual work and after group discussion.

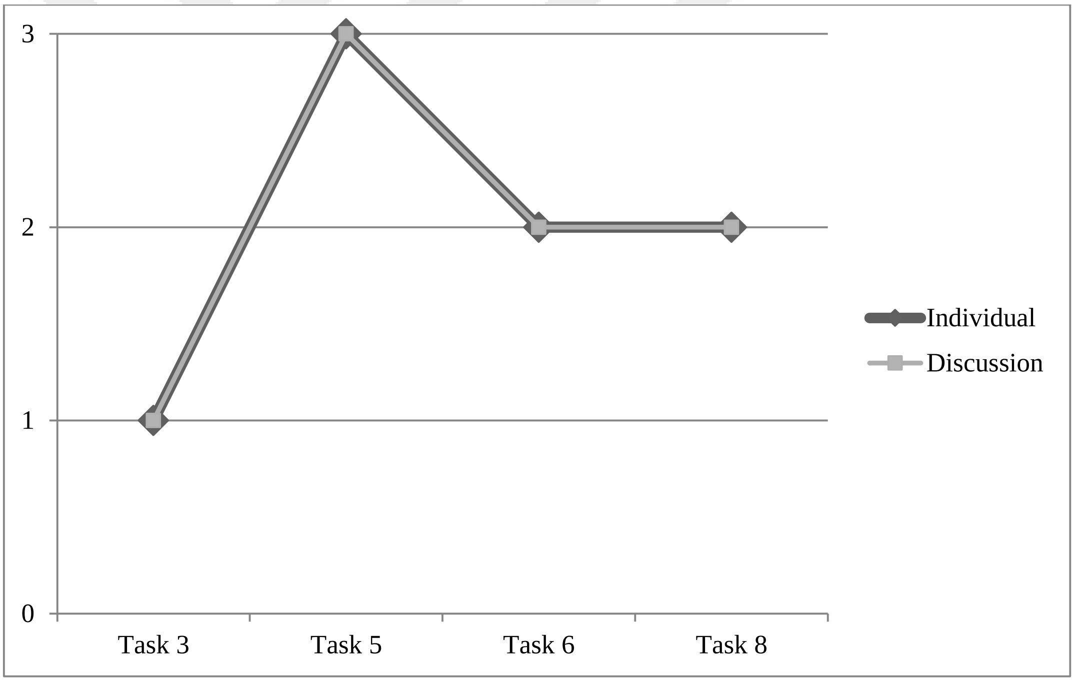

Figure 6.11.The change in Doruk's algebraic thinking in component 2

Doruk could not improve his algebraic thinking within the Task 3 since both in individual work and after discussion his algebraic thinking was in Level 1. He failed to understand the concept of the problem since he always took the reference as the previous road. His teacher and group friends explained him the correct algebraic expressions but he failed to explain the reasoning behind the answers so the level of his algebraic thinking did not change. On the contrary, he could show that he learned from previous task since he had Level 3 thinking in his individual work in Task 5. He wrote both of the algebraic expressions correctly. Different than Task 5, in Task 6 Doruk's algebraic thinking was in Level 2 since he had difficulties to follow the context of the problem. The same difficulty can be seen in his algebraic thinking in the first task of algebraic expressions (Task 3) because he also took the reference point as the steps taken in previous road instead of the steps taken in initial road (between the home and the school). In the last task also, Doruk's algebraic thinking on writing algebraic expressions remained in Level 2 since he failed to use parenthesis correctly by ignoring the order of the operations. The reason behind his difficulty might be the complexity of the problem since the students were expected to write algebraic expressions including many operations (such as addition, multiplication) at the same time. Therefore, different than finding the rule and the terms of the pattern Doruk's algebraic thinking for writing algebraic expressions increased to Level 2 through the tasks. Doruk's algebraic thinking related to the third component is given in Figure 6.12. As seen in the Figure 6.12 different than the second component, Doruk could develop his algebraic thinking within the tasks since the level of his algebraic thinking was higher after the group discussion.

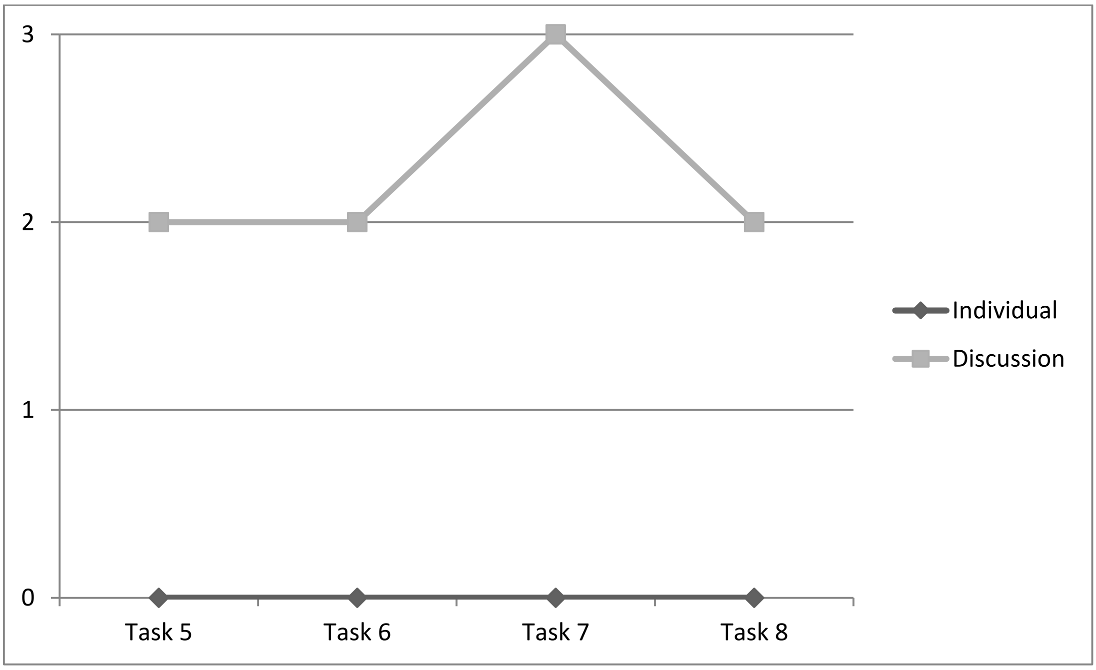

Figure 6.12.The change in Doruk's algebraic thinking in component 3

For setting up and solving equations, at the beginning of the tasks Doruk did not know how to solve the equations but he could set up the equation. Unfortunately, because of the time restriction in his group this question was not discussed efficiently so it is hard to understand whether or not his algebraic thinking was developed within this task (Task 5). In the following task, the teacher of the group interfered the individual work of the student, and they set up and solved the equation together so his algebraic thinking categorized as Level 2 since he got the help of his teacher in each step of the problem. Similar to Task 5, in Task 7 Doruk could set up one of the equations by himself but since some of his algebraic expressions were incorrect he failed to solve the equation. In the other question he had difficulties in comprehending the problem, so the researcher created a discussion with the student by skipping his individual work part. However, after the discussion he totally understood the step of setting up and solving the equation since he could tell these steps to his friends in the group and persuade them to the solution. Therefore, his algebraic thinking level became Level 3 in one of the problems. In Task 8, the group had a time limitation so similar to Gonca's situation (Gonca and Doruk were in the same group for this task) so he stated the steps of the solution orally but failed to complete his solution. Therefore, because of the missing steps his algebraic thinking was evaluated as Level 2 even though he knew the steps.

6.3.4. Erdem's algebraic thinking

At the beginning of the tasks Erdem had mostly irrelevant or no attempt answers in the first tasks of first and third component, indicating that he had a lack of prior knowledge on these components of algebraic thinking. However in the second component that is writing algebraic expressions, the student's algebraic thinking was at Level 2 even in his individual work. Changes in Component 1 are presented as a graph in Figure 6.13.

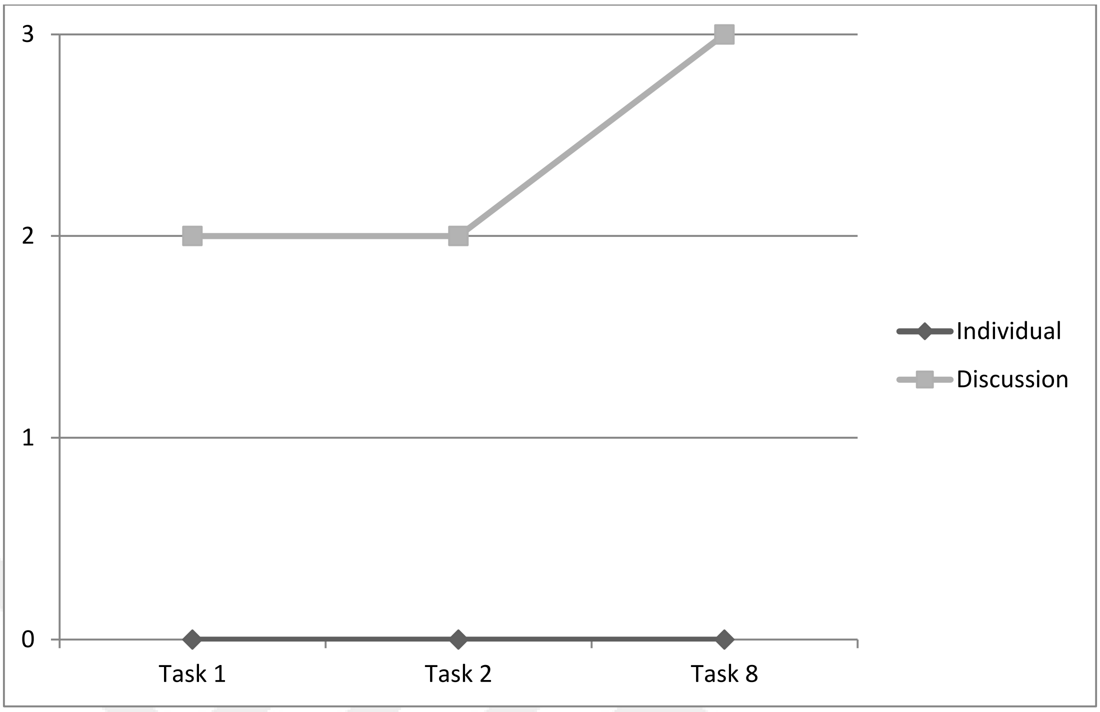

Figure 6.13. The change in Erdem's algebraic thinking in component 1

Erdem's expressions can be seen as an evidence for his improvement since he was one of the students who made the huge progress in his algebraic thinking. For finding the rule and the terms of pattern component, he only focused on the change in one variable and defined the rule with that change in the first task. He stated that the difference between two consecutive terms of dependent variable is the constant term in the pattern rule so his algebraic thinking level was Level 1. In the second task, the student found the rule by trial and error process while in the last task he understood where the constant term and the coefficient of unknown come from in the rule. For instance, he said that we can find the number of renovated streets by multiplying number of weeks and adding one. The students' expressions from Task 8 categorized as Level 3 since the student understands that the constant square in the pattern will be the constant term of the rule, and the amount of increase will be the coefficient of the variable. Therefore, the student recognizes what the leading coefficient and the constant term represent in the rule of the pattern and showed his improvement in finding the rule and the terms of the pattern. Also, for the second and third component of algebraic thinking Erdem could develop his thinking up to Level 3 as seen in Figure 6.14 and Figure 6.15.

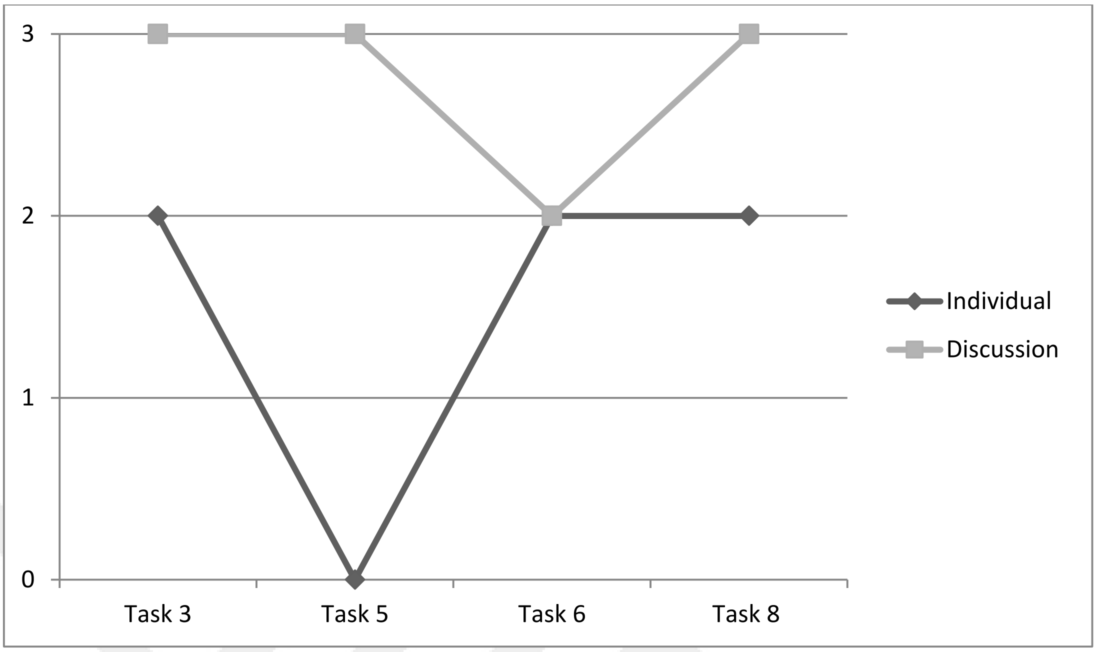

Figure 6.14.The change in Erdem's algebraic thinking in component 2

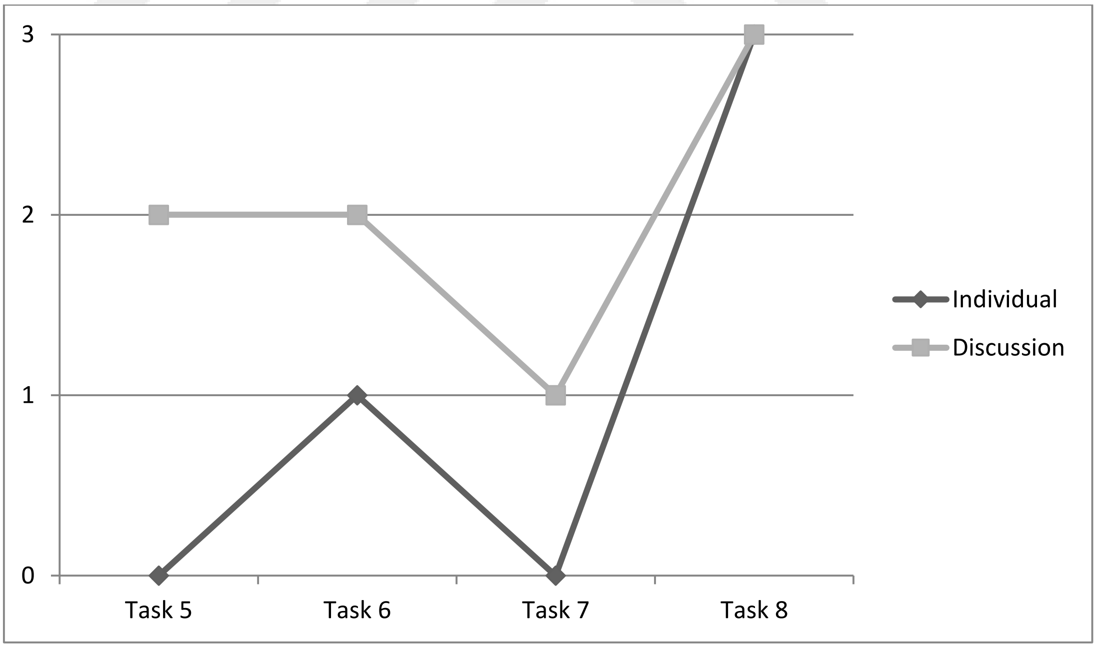

Figure 6.15.The change in Erdem's algebraic thinking in component 3

In Task 3, Erdem's paper for individual work indicated that the failed to understand the context of the problem. Moreover, in Task 5 Erdem could not write the algebraic expressions and in the following task he could not write one of the six algebraic expressions. In the last task, Erdem could understand the context and used the algebraic operations except parenthesis so his thinking was in Level 2. Then with the following dialogue his algebraic thinking was developed:

Teacher: Are these algebraic expressions same with or without using parenthesis?

Erdem: We wrote in a wrong way teacher.

Teacher: Did we? How we should write?

Erdem: We need to use parenthesis from 5 to 10. Without using parenthesis we are taking the two fifths of the 10. It must be the same for the yellow square *(The algebraic expression was (5x+10).2/5 for the red square and the yellow square was related to red square).*

In this case he could realize his own mistake and achieved an improvement in writing algebraic expressions. Erdem also failed to set up an equation in Task 5, Task 6 and Task 7 but in these tasks his friends and teacher explained to Erdem the correct equation and steps of the solution. In the last task he could use what he learned from previous tasks since the level of his algebraic thinking was Level 3 even in his individual work. Erdem could set up and solve the equation correctly and also could find the values of different squares.

6.3.5. Gonca's algebraic thinking

Gonca's prior knowledge was similar to Erdem's prior knowledge in all of the components of algebraic thinking since she gave irrelevant answers or no attempt answers except Component 2. Additionally, changes in Component 1 are presented as a graph in Figure 6.16.

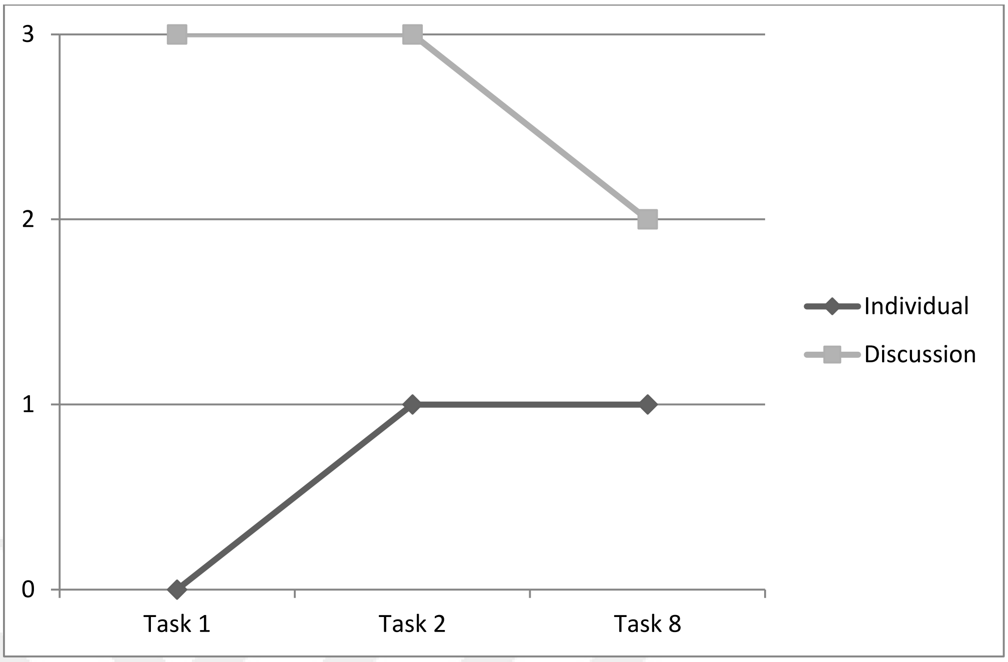

Figure 6.16.The change in Gonca’s algebraic thinking in component 1

For finding the rule and the terms of the pattern, Gonca improved her algebraic thinking from irrelevant answers up to Level 3 even in the first task. When the researcher asked the meaning of “n” in the rule she could explain her rule of pattern. Also, she realized where the constant term and coefficient of variable come from. In the second task, Gonca had Level 3 thinking in two questions out of four questions after the discussions. The questions where pattern blocks were used as manipulatives probably supported her algebraic thinking better since she realized the constant term and the coefficient by means of these manipulatives. In the last task also, she focused on the change in dependent variable shows that she forgot some of the information when time passed. Even though she failed to derive the rule, when the researcher asked to check her rule and constant term she realized that her rule was wrong. After the discussions, Doruk explained the rule to her and she could understand the logic behind the rule (he was in her group for this task) so her thinking level became Level 2. In component 2, Gonca could develop algebraic thinking up to Level 3 in group discussions except Task 5. The change in her algebraic thinking is given in Figure 6.17.

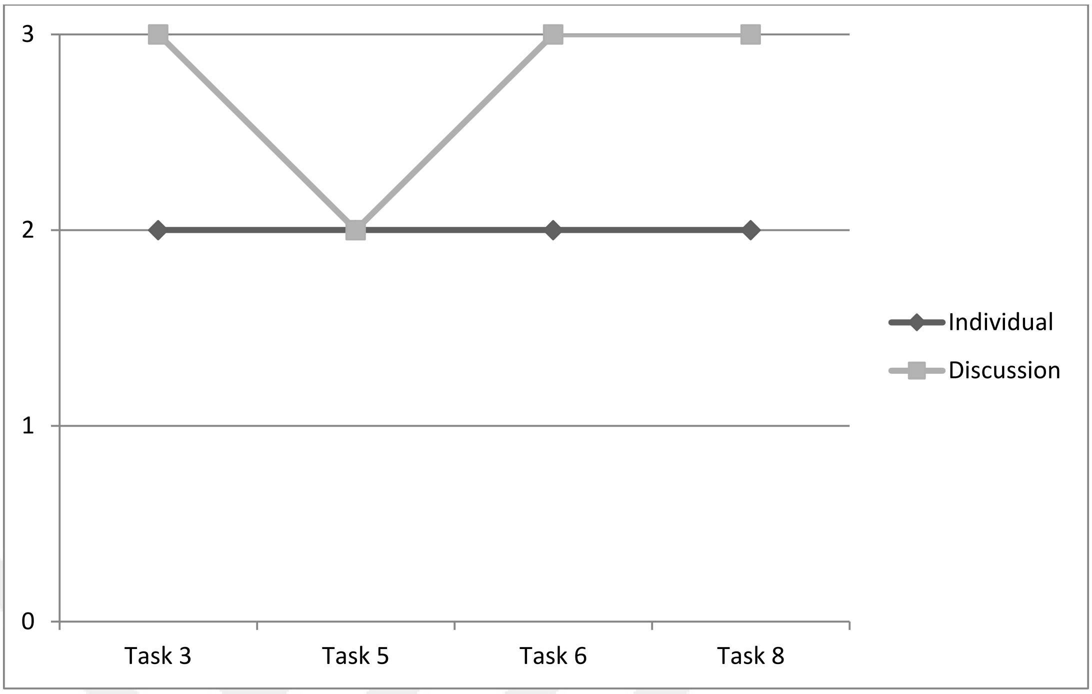

Figure 6.17. The change in Gonca's algebraic thinking in component 2

As the second component of algebraic thinking, Gonca mostly answered questions from Task 3 correctly but fails to understand the context of the problem in some of the questions. For instance, she wrote $x - 10$ instead of $2x + 6 - 10$ and $T.\frac{1}{3}$ instead of $2T.\frac{1}{3}$. Since she missed one of the criteria in algebraic expressions, her thinking level was Level 2. When the researcher asked, Gonca stated that her answer as x.2 is wrong and wrote correct algebraic expression using the parenthesis correctly. In this case, she could both use mathematical operations and follow the context of the problem at the end of the discussions and her algebraic thinking was categorized as Level 3. In the following tasks, her answers were in Level 2 in her individual work for different reasons. In Task 5, she misunderstood the question while in Task 6 she was not sure one of the algebraic expressions and in the last task she failed to use parenthesis. In the following dialogue in the last task she understood the order of operations:

Gonca: Since it is connected with the value of the green square it must be x.(2+5) and I add 8.

Researcher: If we want to do the operations, what is the order of operations?

Gonca: Inside the parenthesis…

Researcher: If we do the operations inside the parenthesis how many x's we get?

Gonca: 7…

Researcher: Do we have 7x here?

Gonca: I would do 5x.

Researcher: You told me that we have 7x, are 5x and 7x equal to each other?

Gonca: No…

Researcher: What will you write then?

Gonca: 5x+2+8…

Researcher: I warned you that you multiplied 2/5 with only 10, do we multiply 2/5 with only 10?

Gonca: No teacher, we need to use parenthesis *(she pointed at the correct place of the parentheses).*

Therefore, the dialogue above indicates that Gonca understood her mistake and remembered order of the operations. Therefore, her thinking level became Level 3 at the end of the task. Moreover, the change in Gonca's algebraic thinking in component 3 is given in Figure 6.18.

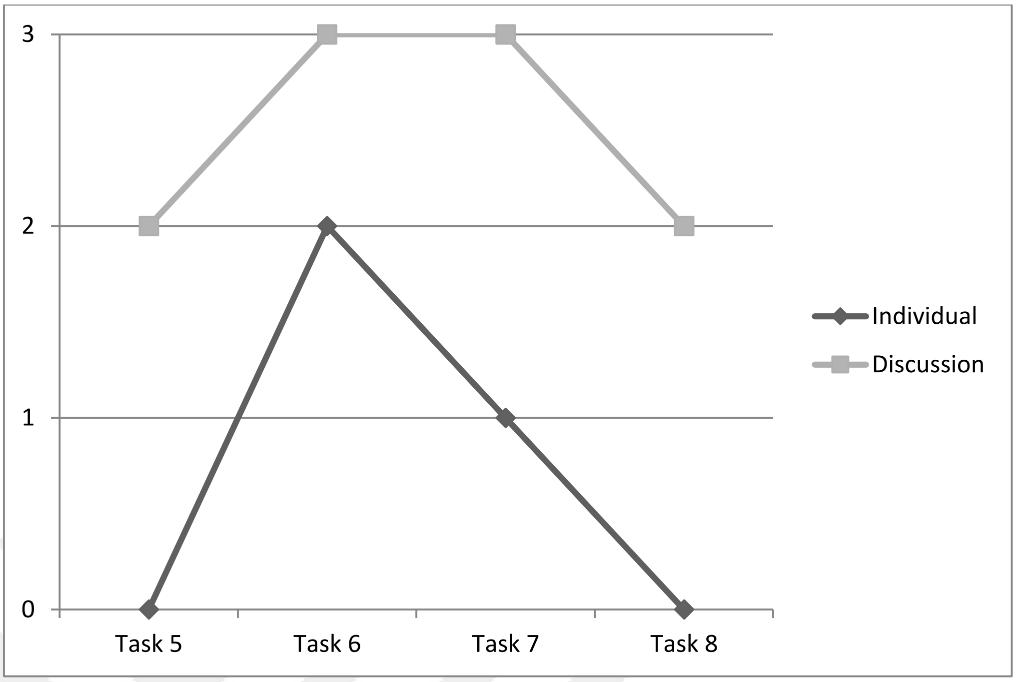

Figure 6.18. The change in Gonca's algebraic thinking in component 3

As the last component of the algebraic thinking, in setting up and solving the equations Gonca initially had a wrong answer in the first task since her algebraic expressions were wrong. Similar to the previous dialogue, when Gonca recognized correct algebraic expressions she could set and solve the equation. She also could transfer her knowledge into the next task and her thinking level in her individual work was also Level 2. Even though in Task 7, her answers during individual work were at Level 1, she stated that she did not know how to add the algebraic expressions. As a result, it can be concluded that her problem was not about setting up and solving the equations but about writing algebraic expressions. In the last task, Gonca failed to understand the question since she did not comprehend which squares need to be added. Therefore, she failed to set up the equation. However, during the discussions she interpreted that two green, one blue and two purple squares would be added and expressed some steps of the solution. Because of time limitation she could not complete her solution even though she stated the steps of the whole solution. As a consequence, her thinking level remained as in Level 2.

6.3.6. Harun's algebraic thinking

Harun's prior knowledge on all of the components of algebraic thinking was higher than most of the students in the sub-group since his algebraic thinking was mostly coded as at Level 1 and Level 2 even during the individual work even though it was not the case for Component 3. Additionally changes in Component 1 are presented as a graph in Figure 6.19.

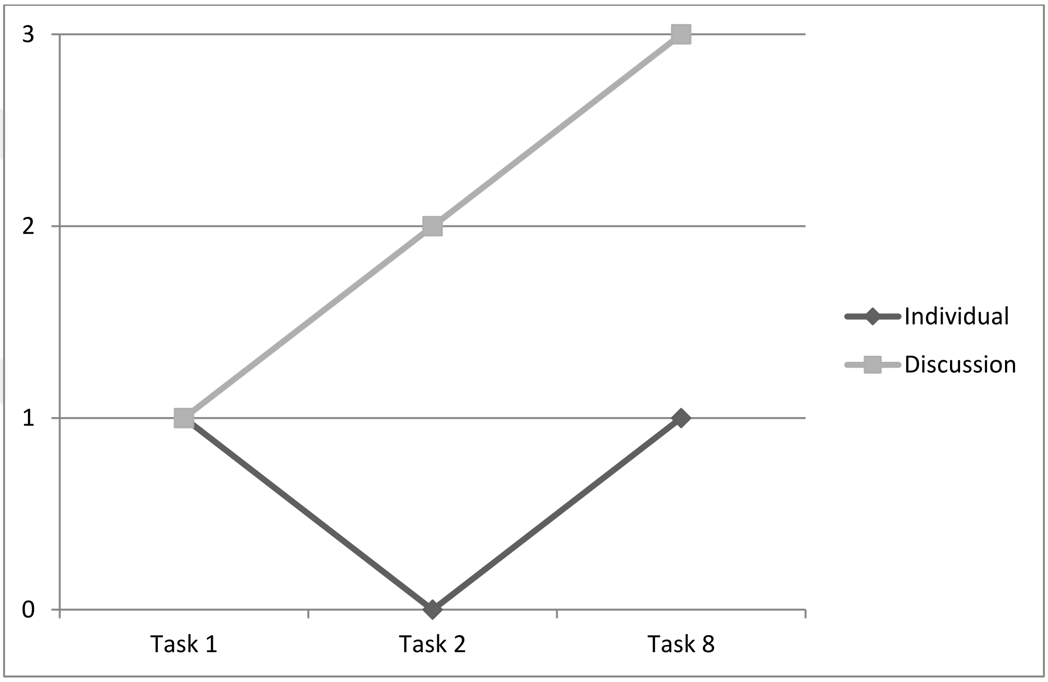

Figure 6.19.The change in Harun's algebraic thinking in component 1

At the beginning of the tasks, Harun failed to interpret the covariation since he only paid attention to the change in one variable to write the rule in both of the question. Due to the time restriction, one of the questions cannot be discussed and his algebraic thinking could not be developed within this task. In other question, Harun only comprehended the meaning of the variable so his algebraic thinking was remained as Level 1. In the next task, Harun also gave Level 1 answers mostly by looking the change in the dependent variable and writing the amount of increase as the constant term of the rule. The following dialogues show improvement in Harun's algebraic thinking:

For the first question;

Harun: Everything is multiplied by 3, multiplied since each year 3 flat families are moving to another place.

The researcher wants him to try his rule.

Researcher: You said it is multiplied, how is it going to be?

Harun: This is multiplied by 3 and it is multiplied by 3 *(he pointed different years).*

Researcher: If each year is multiplied by 3 what is the rule?

Harun: y.3

For the second question:

Researcher: The roof is constant right? If we don't think the roof there is 8 apartments in here, 12 apartments in here, 16 apartments in here right? 4, 8, 12, 16 if want to express the relationship the number of the years and the number of apartments, how I can express?

Harun: 4 apartments are added up.

Researcher: If 4 apartments are added up and if you think like in the first question, how can you express?

Harun: 4.n…

Researcher: How can I include the roof to our rule?

Harun: Isn't it 6.n?

The researcher asked him to try his rule.

Researcher: You said 4n, it is going 4 times how can I add the roof then? Which operation should we apply?

Harun: Addition…

Researcher: Addition, there are 4 apartments in the first year and what?

Harun: +2…

Researcher: What is the rule then?

Harun: 4.n+2…

Then, Harun realized the constant term of the third rule since he stated all of the number of the streets obtained by adding 1 to *2m*. From the dialogues above, it can be observed that Harun could transfer what the learned from the first dialogue (question) to the second and third question since he could understand the constant term and the coefficient of the rule and his algebraic thinking was categorized as Level 2. On the contrary, he probably forgot some of the information from Task 1 and Task 2 since in the last task; he again defined the rule by paying attention to increase in the dependent variable. After the discussion with his teacher however, he realized the constant term and the coefficient. Therefore, his algebraic thinking was improved to Level 3 at the end of the three tasks. Furthermore, the change in Harun's algebraic thinking in component 2 is given in Figure 6.20.

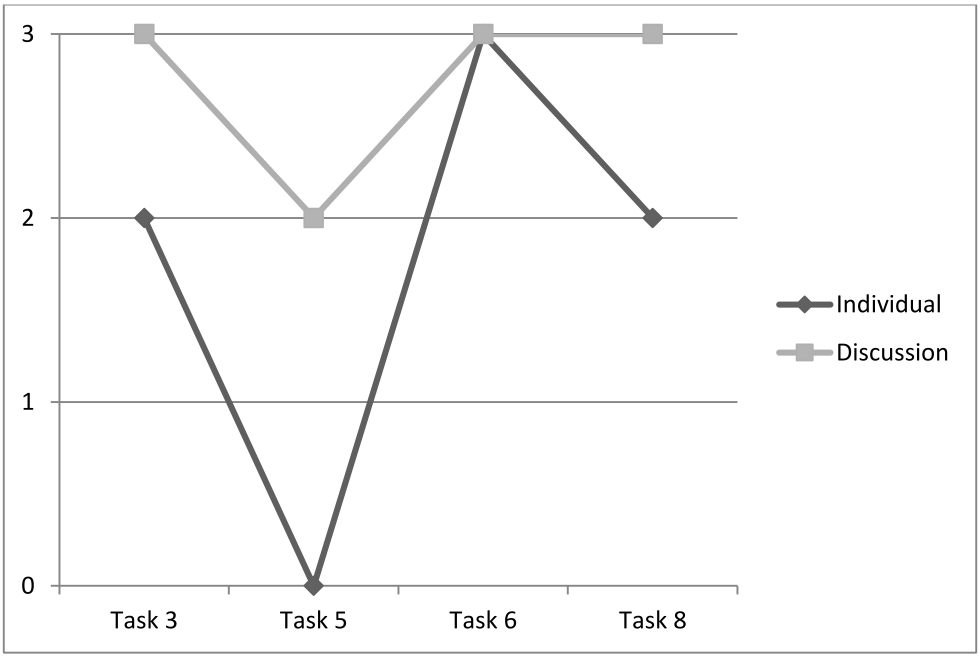

Figure 6.20.The change in Harun's algebraic thinking in component 2

In the second component of algebraic thinking, Harun took the reference point wrongly and in one question he could not apply mathematical operation correctly. In this case, his teacher guided him and asked the previous road to lead him to the correct reference road. After the teacher's questions, Harun realized his mistakes by himself and wrote correct algebraic expressions. In the next task, he could write some of the algebraic expressions but failed to add the amount of fruits correctly since he expressed the weights as unknown. Therefore, even though he could transfer his knowledge into the next task, he failed to write algebraic expressions including the known and unknown weights at the same time. Using what he learned from previous tasks, Harun increased the level of his algebraic thinking to Level 3 even in his individual work. In parallel to Task 3, Harun had difficulties to follow the context of the problem in Task 8 since he wrote the value of the yellow square incorrectly. After the discussions he stated that the algebraic expression was missing and could correct his own mistake. As a result, even though his algebraic thinking was in Level 2 in his individual work, by correcting his mistake his thinking became Level 3 after the discussions. Different than component 1 and 2, Harun had a lack of prior knowledge in component 3 as seen in Figure 6.21.

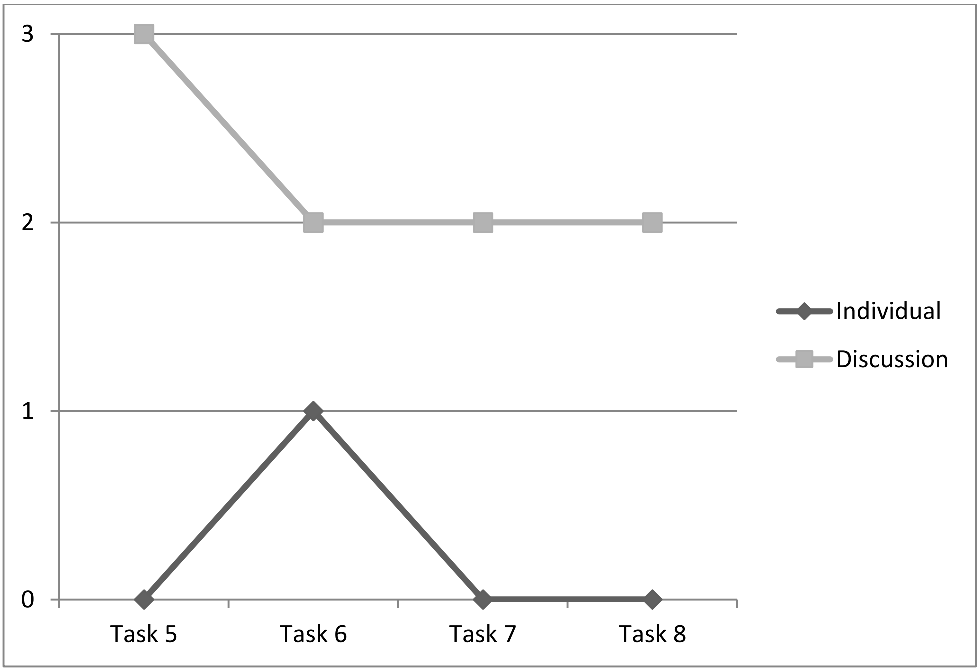

Figure 6.21.The change in Harun's algebraic thinking in component 3

For setting up and solving the equations, Harun could not give answers for any of the questions in Task 5. After the discussion with his teacher, he was able to explain the solution with his own words in the first question. The next question was the extra question where students are expected to add different ingredients. Harun could only understand that these ingredients must be added probably because of the complexity of the problem, so his algebraic thinking was categorized as Level 1. On the contrary to second component, Harun could transfer only some of his knowledge into the next task since his algebraic thinking was at Level 1 in his individual work. In his individual work he could set up the equation but failed to solve it. Then, after the discussions, he could explain some of the steps so his algebraic thinking level was increased to Level 2. In Task 7 his algebraic thinking was at Level 2 since he set up equation but could not solve it. After the discussions, he could solve the equation only with the help of his teacher. The next question could not be discussed because of time limitation. Therefore, his algebraic thinking became Level 2 at the end of the tasks.

6.3.7. Mert's algebraic thinking

Mert had a lack of prior knowledge on all of the components of algebraic thinking except component 2. Changes in Component 1 are presented as a graph in Figure 6.22.

Similar to his friends, Mert could not answer both of the questions related to finding the rule and the terms of the pattern in the first task. Unfortunately, only one of the questions could be discussed but, in that question, Mert could realize the constant term and chancing value of *n* and therefore, his algebraic thinking was coded as Level 3. However Mert failed to figure out the rule of the pattern given in Task 8 during individual work. Then the researcher asked him to tell about the constant and he said the square in the middle is constant. Therefore, his algebraic thinking was consistent with his algebraic thinking from the previous task. Furthermore, Mert's algebraic thinking in second component is given in Figure 6.23.

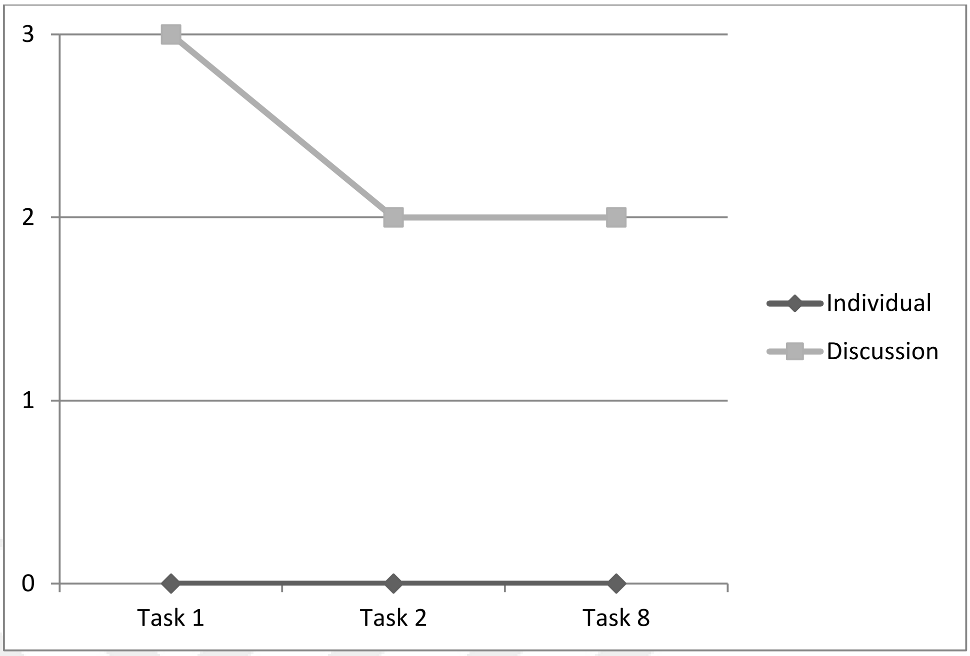

Figure 6.22.The change in Mert's algebraic thinking in component 1

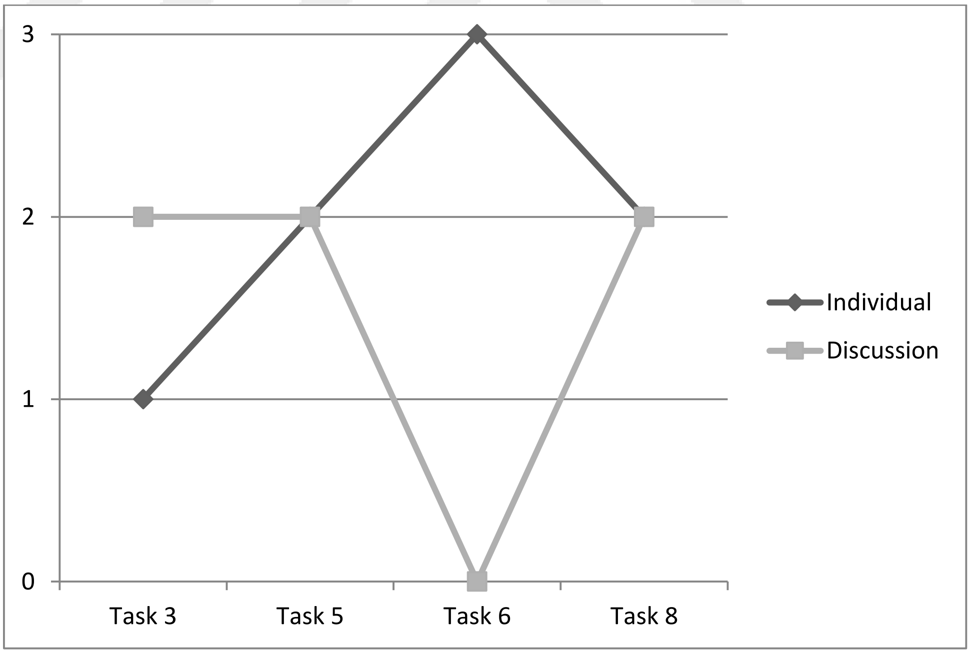

Figure 6.23: The change in Mert's algebraic thinking in component 2

In writing algebraic expressions, Mert had limited prior knowledge in the first task (Task 3) because he could write only one algebraic expression correctly. During the discussions, Mert could understand 6 out of 10 algebraic expressions with the given dialogue below:

Teacher: What is the distance between home and school?

Mert: x…

Teacher: How we can express twice of that distance?

Mert: 2x…

Teacher: It says this road is 6 kilometers more than it.

Mert: +6…

Teacher: So what is the algebraic expression?

Mert: Is it 8x?

Teacher: What we said before 2x?

Mert: 2x+6…

With this discussion, Mert could write the similar algebraic expressions and his algebraic thinking level became Level 2. It can be concluded that he benefited from the discussions within the task.

He also could transfer his knowledge to the next task since his algebraic thinking level was at Level 2. He wrote algebraic expressions partially correct but failed to add known weights to algebraic expressions. Even after the discussions, he could not write exactly correct algebraic expressions because he had difficulties in writing fractions. Consistently, Mert could write all of the algebraic expressions correctly in the following task and his algebraic thinking was in Level 3 probably because there were no algebraic expressions including unknown and known weights at the same time. In the last task, Mert had difficulties in understanding the context of the problem and failed to pay attention to the order of the operations. However, Mert could not understand how to use parenthesis even at the end of the discussions so his algebraic thinking was remained at Level 2. Moreover, Mert's performance in setting up and solving equations is given in Figure 6.24 with related tasks.

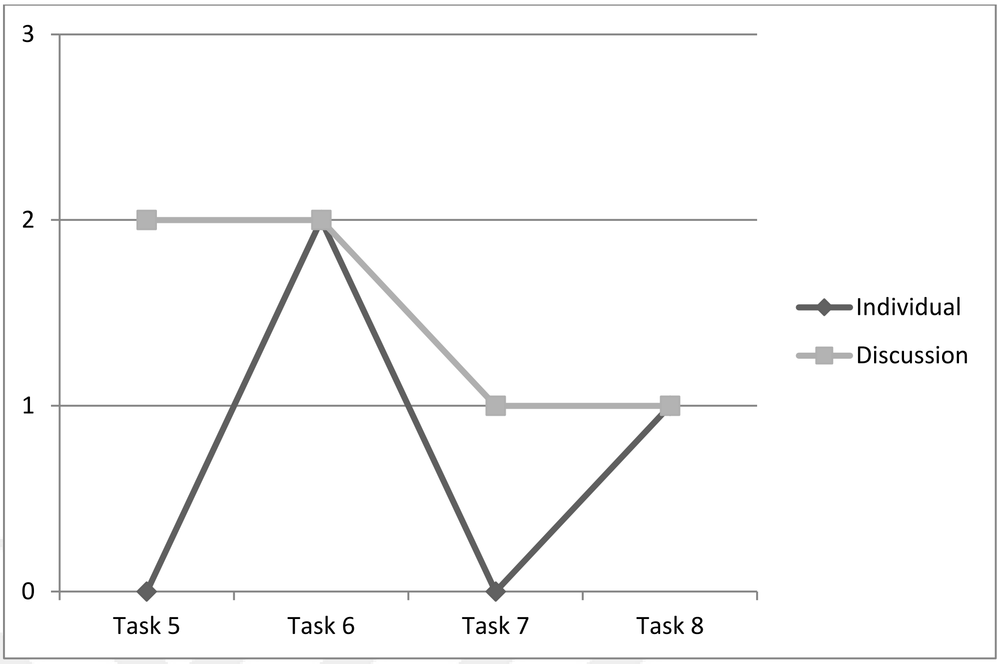

Figure 6.24. The change in Mert's algebraic thinking in component 3

In Task 5, Mert could only write algebraic expressions for setting up and solving the equations. Mert failed to solve the equation individual work but could understand the steps of the solution after the individual work. Therefore, the level of his algebraic thinking increased from Level 1 to Level 2 with discussions. In parallel to the previous task, Mert could set up the equation in his individual work. Moreover, he solved the equation together with his teacher. In the Task 7 only one question could be discussed, in the other question the teacher tried to guide Mert to the solution. Even though the teacher tried to give simpler example, Mert could not understand this example and his algebraic thinking level was remained at Level 1.

6.3.8. Tansu's algebraic thinking

Tansu's prior knowledge on all of the components of algebraic thinking was higher than most of the students in the sub-group since his algebraic thinking was mostly coded as at Level 2 and Level 3 even during the individual work even though it was not the case for Component 3. Additionally changes in Component 1 are presented as a graph in Figure 6.25.

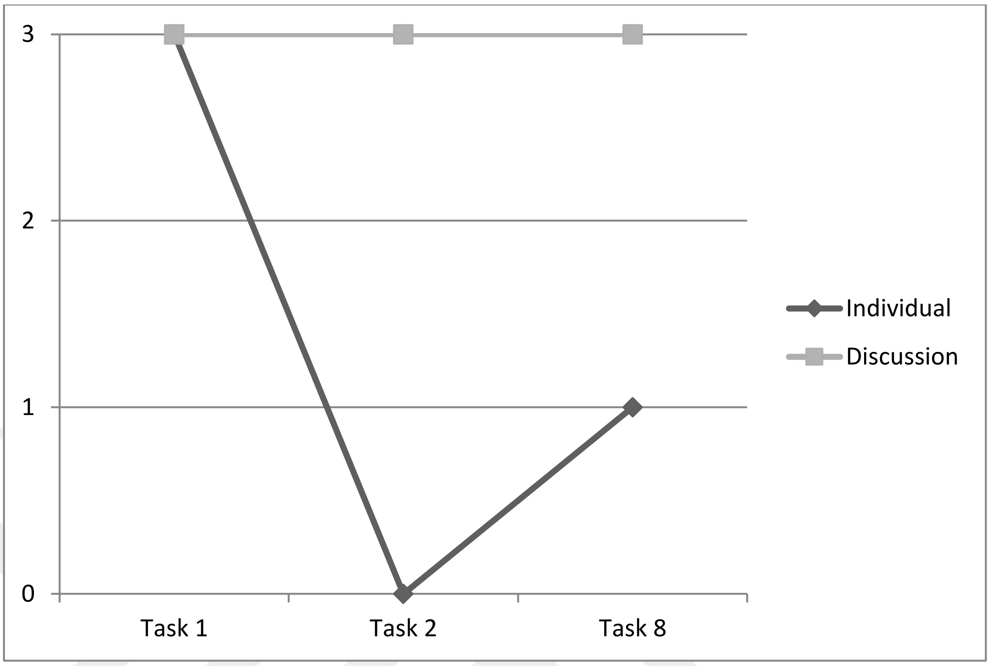

Figure 6.25.The change in Tansu's algebraic thinking in component 1

Tansu's algebraic thinking in finding the rule and the terms of the pattern was mostly coded at Level 3. One of the possible reasons behind her high level thinking is that her prior knowledge was high in the patterns. In the first task she realized that constant seats will be the constant term and the amount of increase in the number of seats will be the coefficient of the variable. In her paper from individual work she wrote that the first two seats are constant and the seats increased by 3 in each day. Therefore the rule must be *3n+2* for the 4th question. Therefore, she knew the coefficient and constant term of the variable at the beginning of the implementation of the tasks. In this case, when her teacher poses the questions she could realize her mistakes or find the correct rule as a result of the questions. For instance; in the Task 2 Tansu paid attention to the difference between dependent variable in her individual work. As a result, she wrote *n+2* for the third question and her algebraic thinking coded as Level 1 since she only knows the meaning of the variable but fails to understand the covariation. After the discussion of the first and the second questions, Tansu figured out that the rule for the third question was incorrect because the coefficient must be two. She said that "It is increasing by two, so I wrote *2m* for the rule. When we replace 1 for the variable, the number of the streets must be equal to 3, and when we replace 3 for the variable the number of the streets must be equal to the 7. Since then, we need to add 1 to the rule". Thus, based on her previous work Tansu rethought about her rule and revised it. It can be inferred that the previous questions in the task helped her to understand how to figure out the rule of a given pattern. As seen in Figure 6.26, Tansu's prior knowledge was also high for the second component since the level of her algebraic thinking was at least Level 2 in her individual work.

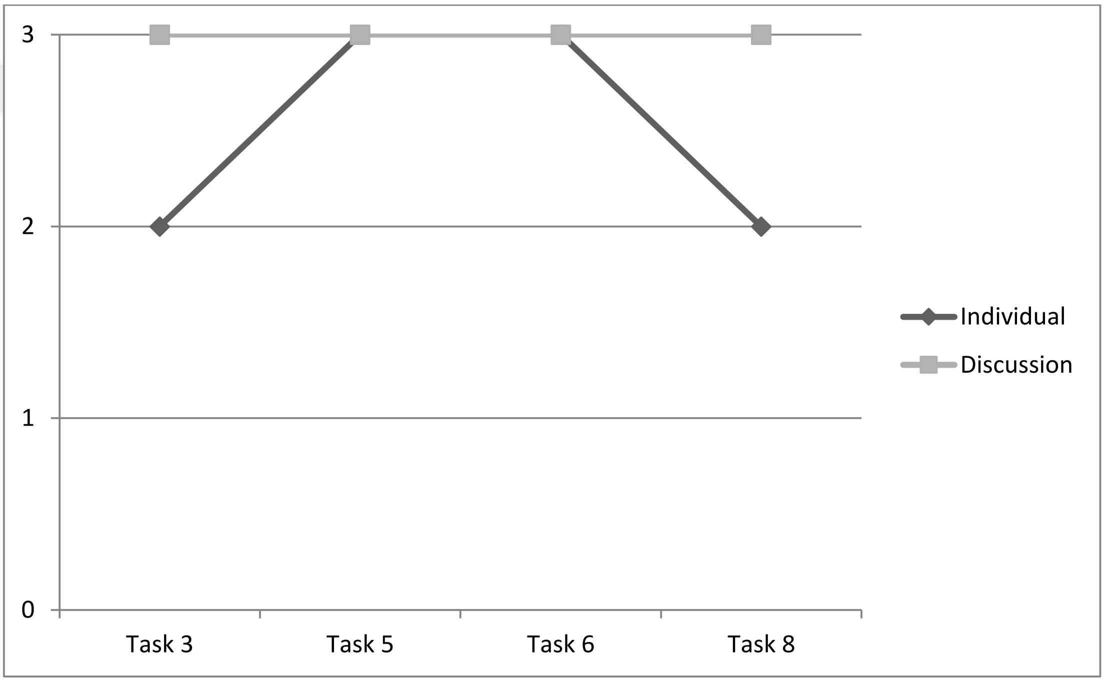

Figure 6.26. The change in Tansu's algebraic thinking in component 2

Tansu's algebraic thinking in writing algebraic expressions was at Level 2 in her individual work since she failed to follow the context of the problem and apply order of the operations. When the researcher asked the order of operations and the previous road taken she realized her mistakes. Then, in Task 5 and Task 6 she could benefit from previous tasks and started to increase her thinking level to Level 3. Only in last task, her algebraic thinking was at Level 2 probably because this task included the questions with more complex context. When the teacher asked, she again could understand her own mistakes and give correct answers by herself. On the contrary to component 1 and 2, Tansu had a lack of prior knowledge in the third component (setting up and solving equations). The level of her algebraic thinking in related tasks is given in Figure 6.27.

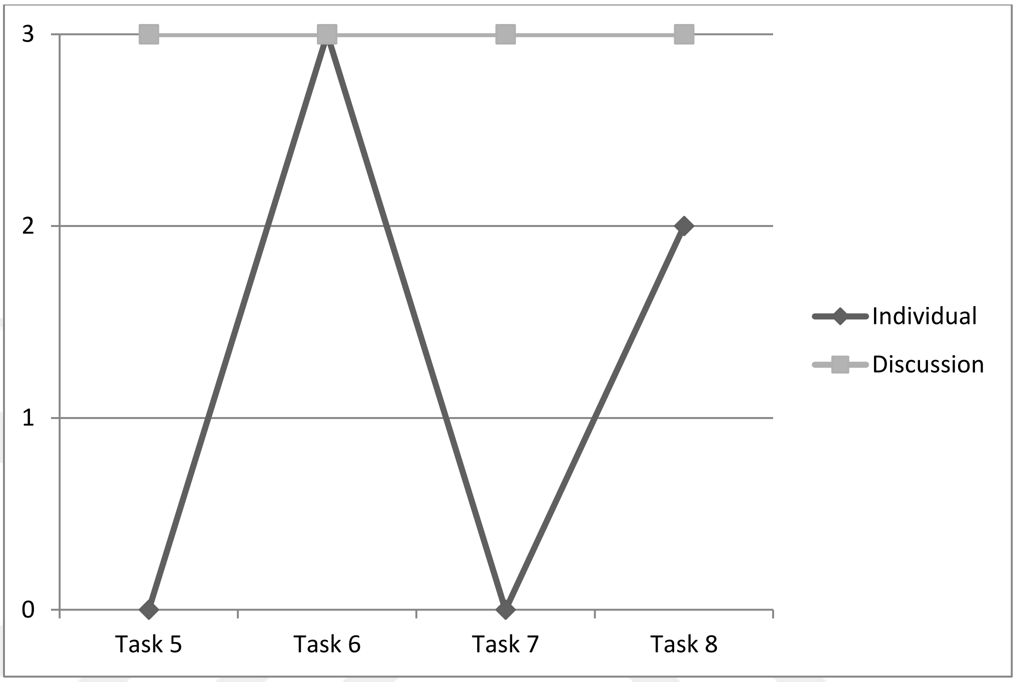

Figure 6.27. The change in Tansu's algebraic thinking in component 3

For setting up and solving equations Tansu had already Level 3 thinking even in her individual work except last task. In the last task, she found some algebraic expressions wrongly so she had difficulties in the solution of the equation. Then, like previous components of the algebraic thinking she found the correct answer by herself and her level of thinking became Level 3.

6.3.9. Utku's algebraic thinking

Utku had a lack of prior knowledge only in component 1 since he had no attempt or irrelevant answers in his individual work at the beginning of related tasks. However at the first tasks the level of his algebraic thinking was Level 1 and Level 2 in component 2 and 3. Additionally changes in Component 1 are presented as a graph in Figure 6.28.

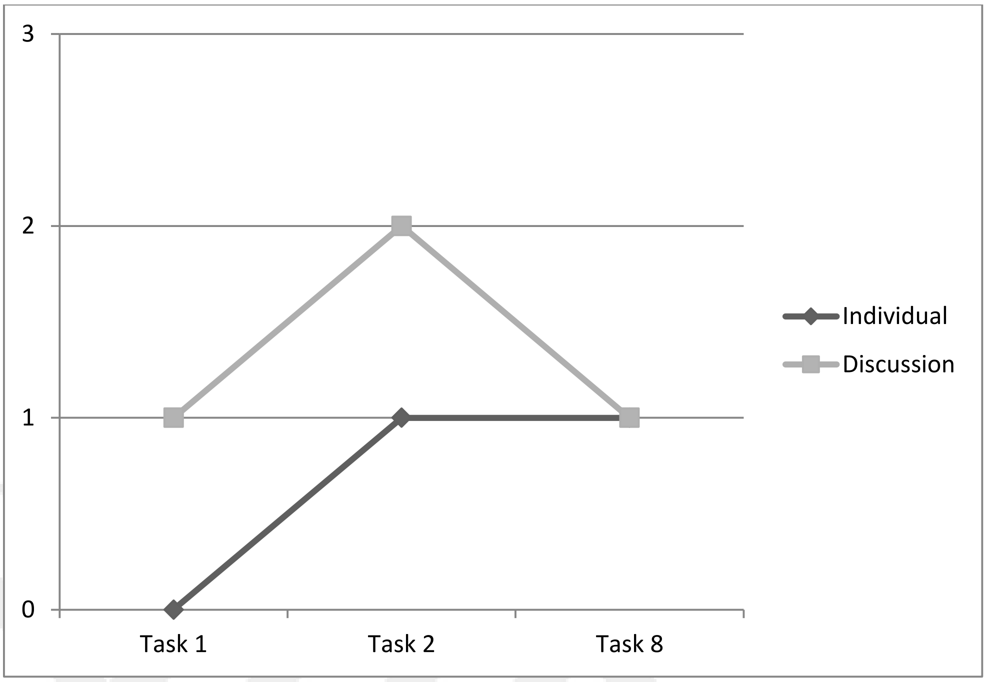

Figure 6.28. The change in Utku's algebraic thinking in component 1

As the first component of the algebraic thinking, Utku's pre-knowledge on finding the rule and the terms of the pattern was weak since he only understood the meaning of the variable. In the discussion part, Utku insisted on looking the difference in dependent variable. In Task 2, Utku still paid attention to the difference between the dependent variable in his individual work. After the discussions he was convinced about the rules of the first and the second questions. Then he was able to transfer his knowledge to the third question with the following dialogue:

Utku: The rule must be a.2+1…

Teacher: Why it must be a.2+1?

Utku: Because the number of weeks goes up 1 by 1 and the number of the streets goes up 2 by 2.

Therefore, Utku found the rule by trial and error process and his algebraic thinking level became Level 2. On the contrary to his friends, Utku could not transfer his knowledge to the last task since he defined the rule by looking the change in only one variable. As seen in the Figure 6.29, Utku's algebraic thinking did not change within the tasks since the level of his algebraic did not change from his individual work to group discussion.

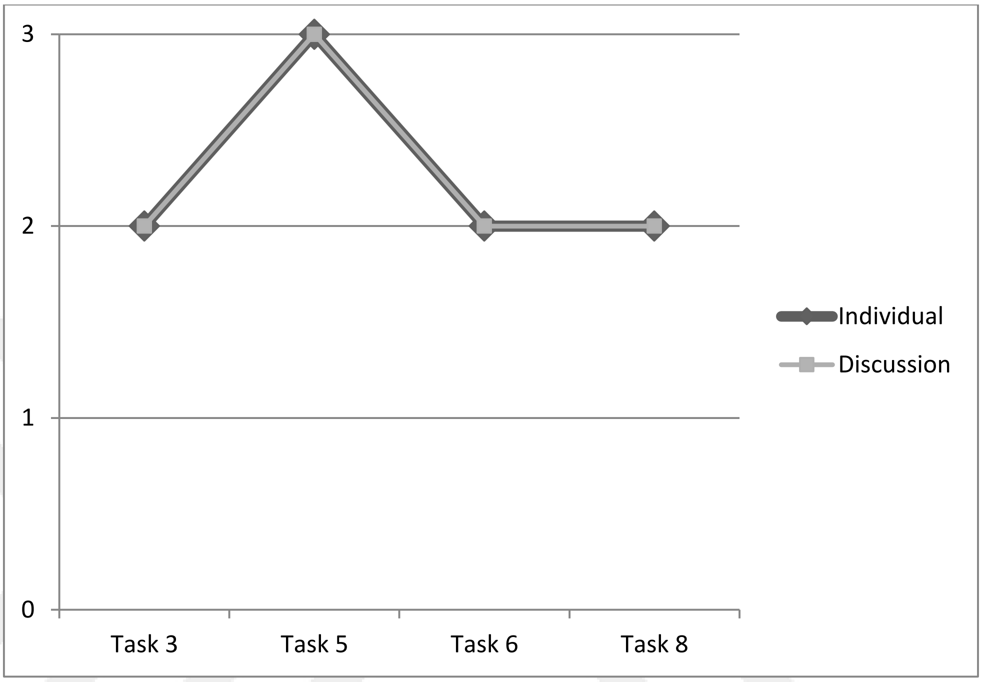

Figure 6.29.The change in Utku's algebraic thinking in component 2

For writing algebraic expressions Utku had some prior knowledge but on the other hand he had some difficulties in terms of order of operations such as using parenthesis. In the discussion part, the teacher tried to show the difference between dividing a number by 1/3 versus by 3 to Utku but he insisted on that there was no difference between these operations. As a consequence, Utku could not improve his algebraic thinking because he had problems with applying the order of operations in the fractions and the level of his algebraic thinking was still at Level 2. On the contrary to Task 3, in Task 5 Utku's algebraic thinking was at Level 3 probably because there were no algebraic expressions with fractions in the question. In Task 6, Utku had the same mistake with Task 3 since he wrote $x \div \frac{1}{5}$ while expressing "one fifth of the road between home and school". In this case, his teacher tried to ask questions to lead him to figure out difference between the division and multiplication with the following dialogue:

Teacher: Utku what does 1/5 mean?

Utku: Dividing by 5 and multiplying with 1.

Teacher: What do you do when you divide by 1/5? Could you all try to find 1/5 of 120?

Utku: 24…

Teacher: What is 2/5 of 120?

Utku: 48…

Teacher: I said the one-fifth and then you divided it by 5 and multiplied by 1, when I said two fifths you divided by 5 and multiplied by 2. All right. Please divide 120 by 1/5, how we can do division with fractions?

Another student said 1/5 must be inverted.

Teacher: Are 600 and 24 are equal? What we are doing when we say 1/5?

Utku: We are multiplying.

Therefore, Utku could write algebraic expressions with the help of his teacher. As a result, the level of his algebraic thinking did not change. Similar to the previous task, in the last task Utku had Level 2 thinking since he failed to use parenthesis correctly. In the discussions also he learned how to use parenthesis from his teacher so his algebraic thinking was at Level 2 at the end of the tasks. Moreover, the change in Utku's algebraic thinking in the third component is given in Figure 6.30 in below.

Similar to finding the rule and the terms of the pattern, Utku had a lack of prior knowledge in terms of setting up and solving the equations since in the first task he could only write algebraic expressions. Therefore, his algebraic thinking level was at Level 1 at the beginning of the tasks.

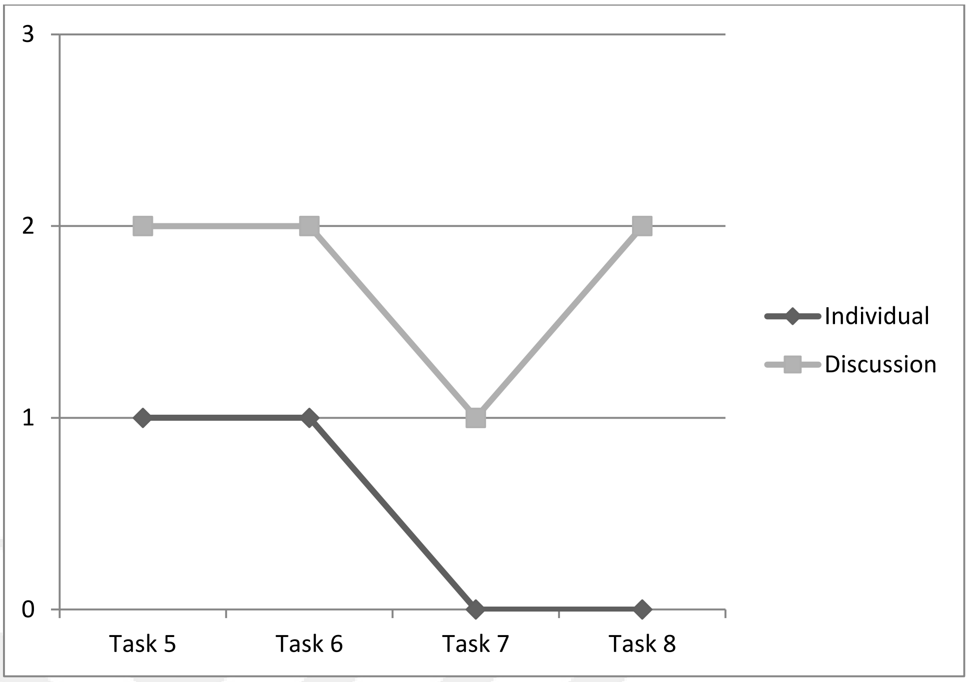

Figure 6.30. The change in Utku's algebraic thinking in component 3

With the discussion below his teacher supported his algebraic thinking and guided him to solution of the problem:

Teacher: 4 equals to what? When I take away 2 kilos of mandarin, what would I have?

Utku: Pickles…

Teacher: 2 bags of pickles?

Utku: 2x…

Teacher: We can write 2x for them. If 2x equals to 4, one x equals to what?

Utku: 2…

This conversation shows that Utku could understand the solution of the equation to some extent. Therefore, his algebraic thinking level became Level 2. In the extra question Utku also could write the algebraic expressions but he had no chance to improve his thinking since because of the difficulty of the problem, the teacher had to teach the steps of the solution of the equation. In Task 6, he showed a similar improvement in his algebraic thinking with the Task 5 after the discussions. Differently, in Task 7 he could not write the algebraic expressions since he failed to pay attention to order of the operations. In the discussions, the teacher asked questions for order operations and because of the time limitation he could only write the algebraic expressions correctly at the end of the discussions and the level of his algebraic thinking was Level 1. Finally, in Task 8 Utku tried to find the values of different squares by trial and error process. At the end of the discussions he could partially understand the steps of the solution since he could solve 21x=42 and found the values of the other squares using this equation. As a result, through the discussions his algebraic thinking mostly became Level 2 and he could not transfer his knowledge to the other tasks probably because he could not internalize what he learned from the previous tasks.

The change in all students' algebraic thinking is summarized in Table 6.10. As seen in Table 6.10, task-assisted instruction was effective to support students' algebraic thinking since the students could improve their algebraic thinking at least in two of three components. Except Utku, all of the students could improve their algebraic thinking in all of the components even though they had weak prior knowledge especially in component 1 and component 3.

Table 6.10. The change in the students' algebraic thinking in each component

	Component 1	**Component 2**	**Component 3**
Alper	NA-Level 3	Level 2-Level 3	Level 3-Level 3
Burak	NA-Level 2	Level 1-Level 2	NA-Level 2
Doruk	NA-Level 3	Level 1-Level 2	NA-Level 2
Erdem	NA-Level 3	Level 2-Level 3	NA-Level 3
Gonca	NA-Level 2	Level 2-Level 3	NA-Level 2
Harun	Level 1-Level 3	Level 2-Level 3	NA-Level 2
Mert	NA-Level 2	Level 1-Level 2	NA-Level 1
Tansu	Level 3-Level 3	Level 2-Level 3	NA-Level 3
Utku	NA-Level 1	Level 2-Level 2	Level 1-Level 2

7. DISCUSSION AND CONCLUSION

In this study, the change in students' algebraic thinking through task-assisted instruction was examined. For this purpose, 8 tasks were applied during 7 weeks. Algebraic thinking was defined as consisting of 3 components: finding the rules and the terms of the pattern, converting verbal expressions to algebraic expressions and algebraic expressions to verbal expressions, and lastly setting up and solving the equations. In this section, change in students' algebraic thinking will be discussed by taking into consideration the previous studies in the literature.

The design of this study was convergent parallel design. As Creswell (2011) suggested the qualitative and quantitative data were separately collected. In this section results obtained from two different sources of data will be examined to understand to what extend these two data sources are merged. The pre, post and retention achievement test results and the change in students' algebraic thinking levels will be compared. At the end of the discussion the limitations of this study will be mentioned.

7.1. Effect of task-assisted instruction on algebra achievement

In the literature, many of the international (e.g. Rivera, 2009) and Turkish studies (e.g. İspir and Palabayık, 2011; Yüksel, 2013) revealed that using tasks in algebra instruction can be useful to support students' algebraic thinking. In this study when the post-test and retention test results were compared, significant differences were observed between experiment and control groups in terms of algebra achievement. These results showed that the students could benefit from task-assisted instruction more than traditional instruction. Moreover, it can be concluded that the students from study group could remember what they learned after three months since there was a significant difference between the results of the two groups in the retention test. Another benefit of task-assisted instruction is its effect on eliminating misconceptions. When the student-teacher interactions were analyzed from that perspective, it can be seen that students' some of the misconceptions were eliminated. For instance, in converting verbal expressions to algebraic expressions many of the students had misconceptions while writing the algebraic expression of "twice of the road that is taken to arrive Miniatürk". In this question the road is taken to arrive Miniatürk was $2x + 6$. The students wrote $2x + 6.2$ instead of $(2x + 6).2$ without paying attention to order of the operations. The teacher-student interactions were beneficial to eliminate this kind of misconceptions since the students could correct their own mistakes. During the discussions, Alper and his friend realized their own mistakes and stated the necessity of the parenthesis. When their teacher asked the reason of putting parenthesis, Alper said that in the order of operations the multiplication has the priority. Furthermore, Gonca stated that the expression is the twice of 2x+6 not the twice of only 6 after the discussions of the previous algebraic expressions. As a result, Gonca understood the necessity of the parenthesis in algebraic expressions. In parallel to current study's conclusion Akkaya (2006) reported that task-assisted instruction was more effective to eliminate these misconceptions then traditional instruction since the number of students who had misconceptions decreased more in task-assisted instruction.

When change in students' algebraic thinking in finding the rule and terms of the pattern (see Table 6.4) and setting up and solving equations (see Table 6.8.) was examined it can be seen that especially students who were lacked of prior knowledge could benefit from task-assisted instruction. For instance, Harun and Burak were low-achieving students since both had irrelevant or no attempt answers in their individual work at the beginning of the tasks. Also, they both took 2 points out of 40 points in pre-achievement test. After the discussions, the level of their thinking became Level 2. The students with lacked of prior knowledge comparing to the other students, had a chance to improve their thinking mostly since their thinking level starts from lower levels. Yüksel (2013) also stated that such low-achieving students mostly benefited from task-assisted instruction. That is, the findings of the current study are compatible with the results of the previous studies.

Moreover, in this research the tasks include the contexts that were from the close environment of the students such as going to bazaar, taking paths to school from home. In the literature also, some of the scholars suggested that personalized tasks which is inclusion of the contexts that are related to the interest and daily lives of the students. Walkington *et al.* (2013) claimed that using personalized contexts in the tasks are likely to contribute students' algebraic thinking especially in learning new contexts. Since the tasks in this study were designed in the light of this perspective, the findings of this study can be seen as a support of the previous study conducted by Walkington and her colleagues (2013).

In terms of application of the tasks, Saraswati and Putri (2016) preferred to use algebra tiles. The discussion on Gonca's answers in section 6.3.5 could be thought as an evidence of benefits of using manipulatives. In finding the rule and the terms of the pattern, Gonca achieved Level 3 thinking after the discussions in two questions out of four questions in Task 2. Her teacher used manipulatives in these two questions in accordance with the suggestions of the researcher. During the discussions Gonca stated that "the roofs are constant so we need to add 2 to the rule, the rule become 4n+2". In the last question, she said "There are 11 pieces that are constant in the flower and the leaves increased twice each time so the rule includes multiplication by 2." Her expressions were proof of how she made use of manipulatives to improve her algebraic thinking since she realized the constants will be the constant term in the rule of the pattern. The current studies' findings are consistent with Saraswati and her colleagues' (2016) since they claimed that using algebra tiles can be useful to support students' understanding in one variable equation.

A student-centered teaching model was preferred during the implementation of tasks. The pre-service teachers were scaffolding students by asking questions to them during the discussions. During the implementation of the tasks, the researcher asked questions such as "What is constant?", "You said the red square is constant, how many squares we add to here?". With these questions Doruk could derive his rule of the pattern by himself and he could change his wrong answer in his individual work. In accordance with the aim of the study, the level of his thinking became Level 3.

Therefore, these discussions and student-centered teaching could support his algebraic thinking. On the other hand, the control group was instructed by traditional instruction where the students were less active comparing to the student-centered teaching model. Since there was a significant difference between control and experiment group, it could be concluded that student-centered teaching model was effective for supporting students' algebraic achievement. Similarly Schukajlow *et al*. (2012) investigated effects of different teaching models in implementation of the tasks. The researchers claimed that student-centered teaching model was more effective than teacher-centered model on students' achievement. Billings *et al*. (2007) preferred similar questions to the questions of pre-service teachers during the discussions to enhance students' thinking. The researchers noted that the students could reach the relationship between the patterns by means of growing patterns and the question. Moreover, Billings *et al*. (2007) and Moyer-Packenham (2005) used growing patterns as in the first question of the last task of this study. Moyer-Packenham (2005) claimed that this pattern helps students to identify the rule of the pattern since in the growing pattern students can comprehend the amount of increase from one shape to another. Briefly, the findings of previous studies support the current studies' findings about the effects of task-assisted instruction on students' algebra achievement.

Many of the scholars (e.g. Lannin, 2005) preferred pattern tasks to elaborate students' thinking on finding the rule of the pattern. Lannin (2005) observed that students could reach the generalizations especially in small group discussions. In the current study some of the students could learn from their friends in small group discussions. For instance, Burak's group was changed because in his previous group there were two low-achieving students who had difficulties in understanding. In the last task he was in the same group with Erdem and he could make use of the discussions with Erdem since Erdem explained the rule of the pattern to him. With small group discussions Burak could understand the logic behind the rule of the pattern. It can be deduced that the small group discussions and pattern based teaching supported Burak's algebraic thinking in finding the rule and the terms of the pattern. Moreover, Palabıyık and İspir (2011) compared the effect of pattern based and algebra teaching which was not based on patterns with an experimental study. Palabıyık and İspir (2011) noted that the study group performed significantly better in the algebra achievement test consisting questions from all algebra topics including both conceptual and procedural questions. In the current study there was a significant difference between the study and the control groups since p-values were 0.014 for post-achievement test and 0.021 for retention test in related item. Therefore, the p-values indicate that the results of this study overlapped with the previous studies since the students could make use of pattern tasks in finding the rule and the terms of the pattern.

7.2. Effect of task-assisted instruction on algebraic thinking

As the first component of algebraic thinking the task-assisted instruction supported finding the rule and the terms of the pattern. In the individual work of Task 8, the students defined the rule with the amount of change between the numbers of squares in consecutive figures. Afterwards, the researcher demanded them to focus on the relationship between the number of the figures and squares. Also, the researcher asked “What was changed?” and “What was stable in the figures?”. Then, the students realized that the stable squares will be the constant term and change between the consecutive figures will be the coefficient of the variable. Billings *et al*. (2007) claimed that one of the students could observe the increase in the number of dots (dots in the pattern) in consecutive figures but he failed to understand what is remained unchanged and which dots were changed among the figures. The researchers claimed that with this task the student started to understand the covariation of the variables rather than the change in the dependent variable. Therefore, current study supports Billings *et al*.’s study (2007) in terms of eliminating students’ difficulties about covariation.

As another component of algebraic thinking, it was observed that the students who were instructed by task-assisted instruction performed better in setting up and solving equations as shown in Table 6.2. Similarly, Gürbüz and Toprak (2014) conducted an experimental study to investigate the effects of task-assisted instruction compared to traditional instruction. In accordance with the results of the current study, the results of achievement tests indicated that task-assisted instruction was more effective.

7.3. Change in algebraic thinking of students in the sub-sample

In this study, the tasks were designed to reveal MOST instances to support students' algebraic thinking. As seen in teacher-student and researcher-student interactions MOST cases helped to elicit and improve students' algebraic thinking (see section 6.3). The items in the tasks were prepared to elicit students' common misconceptions to give an opportunity for pre-service teachers to eliminate those misconceptions and support students' algebraic thinking and learning. For instance, one of the MOST instances described for Task 2 was to focus on the change in one variable rather than the covariation (see Appendix E for the list of MOST instances). During the implementation of this task, this MOST instances occurred in many groups. For instance, Harun focused on the change in the number of apartments and defined the rule as n+4 although the rule was 4n+2. Then the researcher asked questions to make him to recognize the relationship between variables. He eventually found the answer by help of the researcher (see section 6.3.6). Harun also made use of what he learned in the next question with this MOST instance. He probably easily transferred his information to the next question since the contexts of the problems were very similar. In both of the questions there were constant terms in the rule and the amount of the increase were the coefficient term. In other words, MOST instances not only provide a platform for promoting students' thinking through scaffolding practices but also create an opportunity for students to correct their own mistakes and understand mathematical concepts and procedures better.

On the other hand, in some of the cases students could not transfer what they learned from previous tasks probably because of the more complex nature of the following tasks. As the second component of algebraic thinking, in writing algebraic expressions some of the students gave irrelevant answers even though their algebraic thinking was at least Level 1 in the individual work of Task 3. Even though the contexts of the Task 3 and Task 5 were similar, the number of questions was less than in Task 5. For instance, in Task 5 only in 1 question students needed to choose multiplication or division. Therefore, when they could not use the correct operation even in this question they could not meet one criterion in algebraic thinking. Not only writing algebraic expressions but also in setting up and solving equations the level of students' algebraic thinking in their individual work were lower in Task 7 with respect to Task 6 most probably because of the complex nature of the problems. In Task 7 for both of the problems students were expected to add more than two algebraic expressions to solve the equation and they need to understand there were more than one reference points different than the previous tasks. As a result, the students might not apply what they learned from the previous tasks because of the complexity of the problem in the following tasks.

Both qualitative and quantitative data collected from sub-sample group is compared to understand the nature of changes in students' algebraic thinking. According to pre-test results Alper, Doruk and Tansu had highest scores since they had 8 points out of 40 points and other students had lower scores. At the beginning of the tasks, Alper had Level 3 thinking in component 3 (setting up and solving equations) whereas Tansu had Level 3 thinking in both component 1 (finding the rule and the terms of the pattern) and component 3. In that case, Tansu and Alper's algebraic thinking in the tasks and their pre-achievement test scores overlapped since they had highest levels at the beginning of the tasks and highest test scores in pre-achievement test. Moreover, Doruk had Level 2 thinking in component 3 and Level 1 thinking in component 1. Therefore, his high score in pre-achievement test overlapped with his Level 1 and Level 2 thinking levels in his individual work.

In terms of the difference between the pre-test and post-test scores; Alper, Tansu, Erdem's scores increased by 24, 21, and 17 points, respectively. That is, their scores increased up more than the other students'. In the tasks, same students could reach Level 3 thinking in all of the components of algebraic thinking. However, in the retention test only Alper could get the same score as in the post-test. In the beginning of the tasks the level of his thinking was also high and he might remember what he learned easily because of his prior knowledge. Harun could improve his score only 3 points whereas Mert and Gonca increased their scores 7 points. On the contrary to test scores, Harun's algebraic thinking improved to some extent in all of the components. Even though he could not enhance his score in post-test, he could use his knowledge in the retention test. Similar to Harun's situation, Gonca could improve her algebraic thinking through tasks but increased her test score less than her friends. Different than the other students Gonca could increase her test score in retention test comparing to the post-test. The reason behind Gonca and Harun's lower scores might be timing of the tests. Since the post-test was administered just after the implementation of the last task, students might not have time to repeat what they learned. Different than Gonca and Burak, Mert could not increase his test score from post-test to retention test and also he had a lower increase from pre-test to post-test comparing to his friends in the sub-group. The results from the achievement tests overlapped with Mert's performance in the tasks since the level of his algebraic thinking changed in two components (see Table 6.10). In the second component his algebraic thinking changed only one level and in the first component his algebraic thinking became Level 2 at the end of the discussions. As a result, there was a consistency between students' performance in achievement tests and the change in students' algebraic thinking.

Moreover, some of the groups had more suitable dynamics to discussions since in some of the groups the students were more eager to express their thinking via their expressions or eager to help their friends even though the researcher tried to equalize the groups. In this case, the other students in that group (especially the students who were lacked of prior knowledge) could learn from other friends. For instance, Erdem was in a group of three students where one student was the highest achieving student of the class (Sena) and Sena helped Erdem during the discussions. Starting from Task 1, Sena used pattern blocks to illustrate the logic behind the rule of the pattern. Therefore, Erdem might increase the level of his thinking up to Level 3 probably with the help of his friend and his teacher. On the other hand, Mert and Burak were in the same group where two of the students had difficulties and misconceptions. The teacher of this group had to focus on these two students rather than Mert and Burak and the discussions in this group were shorter than the other groups'. Therefore, Mert and Burak had fewer opportunities to improve their algebraic thinking.

On the contrary, Utku was one of the students who got benefit from task-assisted instruction less than his friends since he was able to improve his algebraic thinking only one level in two of the components. Also, his score increased up 8 points from pre-test to post-test however in the retention test his score decreased down. Utku’s teacher asked many questions during the discussions to elaborate his thinking. Moreover, his group friends tried to help them because both of them were successful according to pre-test results and answers in the individual work of the tasks. However, he consistently repeated his mistakes in parallel tasks. For instance he wrote $x \div \frac{1}{3}$ instead of $x.\frac{1}{3}$ while writing one third of given road in Task 3 and he did the same mistake in Task 5 even though this question was discussed in Task 3. Therefore, he could not comprehend writing algebraic expressions and revealed less improvement in algebraic thinking.

As seen in Table 6.10, all of the students were able to achieve an improvement in their algebraic thinking at least in 2 out of 3 components. In total the increase in students’ scores from pre-test and post-test verifies the improvement in students’ algebraic thinking. However, some of the students benefited more from task-assisted instruction. The reasons behind their improvement might be their prior knowledge and group dynamics.

7.4. Limitations and implications for future research

This study was conducted in two classrooms of a public middle school in Kayışdağı district, İstanbul. Therefore, the results of this study limited for that school. The students did not possess prior knowledge that was supposed to be taught in earlier grades, such as finding the rule and the terms of a given pattern. The patterns are taught in the 6th grade, however, the students could hardly remember what they learned and even failed to transfer their knowledge while implementing such tasks during the study. In that sense, it might be thought that some students could not get benefit from the tasks because of their lack of prior knowledge.

Although there was a significant difference between pre-test and post-test results, for some of the students such a change was not observed. Therefore, it could be concluded that an improvement of algebraic thinking might not be achieved for all students although they were exposed to the same intervention. For instance, Utku was one of the students whose algebraic thinking shown less improvement than other students' most probably because of deficiencies in his prior knowledge. Similarly, Burak was in a group where two of the students had serious learning difficulties. Therefore, not only gaps in students' prior knowledge but also group dynamics were likely to influence students' performance.

Another limitation of this study was that each group of students was managed by different pre-service teachers. Although the researcher provided written instructions for the implementation of each task and discussed possible misconceptions of the students before the sessions, during the implementation the pre-service teachers asked some additional questions or gave clues to students during their individual work. During the individual work some of the students had difficulties especially in setting up and solving the equations. At that point, some of the pre-service teachers gave some clues about how to set up equation without giving the right equation such as "How we can show the equality of 5 and 5?" or "How can we express the total amount of weight of 2 bags of pickles and 2 kilos of banana?" in Task 5. As a consequence, in some cases, the level of students' algebraic thinking might be affected from those clues. To eliminate these differences, in the future studies the researchers might restrict student-teacher interaction in the individual work or might give the exact questions to pre-service teachers during the discussion of instructions.

When the student-teacher interactions are examined, it can be concluded that some of the students benefited from task-assisted instruction probably because of probing questions of their teachers. When pre-service teachers could make use of the opportunities that defined by MOST Framework, they posed questions to students to enhance their algebraic thinking. Leatham *et al*. (2015) claimed that timing and opening are important to define a case as MOST. In that case, the pre-service teachers were expected to open the students' thinking with additional questions. For instance, in two of three questions in Task 2 Alper focused the change in dependent variable. Then, his teacher realized the student's mathematics from his individual work and tried to pose questions about the covariance of the variables in both of the questions. As a result, Alper could interpret the logic behind the rule of the pattern with the help of his teacher and his thinking level became Level 2. Therefore, it can be concluded that the tasks could reveal the students' mathematics and made use of a MOST case to promote students' algebraic thinking. On the contrary, some of pre-service teachers missed the opportunities even though the researcher informed them about the possible MOST instances in the discussions. Some of the pre-service teachers missed the opportunities because of the time limitation. Since they had no time to discuss all of the questions, they skipped some of the questions or demand from higher achieving students to teach solution method instead of initiating discussions. Specifically in Task 1, many of the groups had no chance to discuss the last question (the question about converting verbal expressions to algebraic expressions and algebraic expressions to verbal expressions). As a result, it might be useful to reduce the number of the questions in some of the tasks to create more time to discussions.

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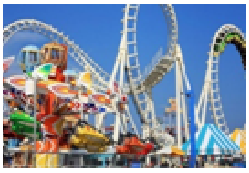
APPENDIX A: TASKS AND THEIR RUBRICS

ETKİNLİK 1

Lunapark



Mahallemizde yeni açılacak bir Lunaparkta çeşitli oyuncaklar olacaktır. Bu oyuncaklardan biri de hızlı trendir.

1. Treni inşa etmek için de Lunapark görevlileri önce tren rayları döşeyeceklerdir. 1 saatte 5 tane ray döşenmektedir. Lunapark görevlileri günde 8 saat çalıştıklarına göre

a) Bir günde kaç tane ray döşenir? İşlemlerinizi gösteriniz.

b) İki günde kaç tane ray döşenir? İşlemlerinizi gösteriniz.

c) Üç günde kaç tane ray döşenir? İşlemlerinizi gösteriniz.

2. Trenin kurulabilmesi için 400 tane rayın döşenmesi gerekmektedir. Bu rayların döşenmesi için kaç saat ve kaç gün çalışılması gerekmektedir? Açıklayınız.

3. Döşenen ray miktarı ile saat sayısı arasında nasıl bir ilişki vardır? Bu ilişkiyi gösteren örüntü kuralını bulunuz. Bu kurala nasıl ulaştığınızı açıklayınız.

Figure A.1. Etkinlik 1

4. Ray kurulumu bittikten sonra ilk 3 günde yerleştirilen koltuklar resimlerdeki gibi gösterilmiştir. İlk 3 günde yerleştirilen koltukları örüntü bloklarıyla gösterip, **gün sayısı** ile **yerleştirilen koltuk sayısı** arasındaki ilişkiyi gösteren örüntü kuralını bulunuz. Bu kurala nasıl ulaştığınızı açıklayınız.

1. gün

2. gün

Koltuk sayısı:……

Koltuk sayısı: ……

3. gün

Koltuk sayısı: ………

5. Lunapark sahibi hızlı trene koltuklar eklemeye devam ediyor. Koltukları yerleştirmek için toplam 15 gün harcanıyor ise, hızlı trende kaç koltuk vardır? Açıklayınız.

6. Cem, Lunapark açılışı için arkadaşlarıyla beraber davetiyeleri dağıtmıştır. Dağıttıkları davetiye miktarlarını şöyle ifade ediyorlar:

- Aylin, Cem'in dağıttığı davetiye sayısının 2 katı davetiye dağıtmıştır.
- Merve ise Aylin'in dağıttığı davetiye sayısının 3 fazlası kadar davetiye dağıtmıştır.
- Burak ise, Turgay'ın dağıttığı davetiye sayısının 3 katının 5 eksiği kadar davetiye dağıtmıştır.

Verilen bilgilere göre, dağıtılan davetiye sayısını cebirsel ifade olarak kutucuklara yazınız.

Cem	Aylin	Merve
a		

Turgay	Burak
b	

Figure A.1. Etkinlik 1 (cont.)

ETKİNLİK 2

Kentsel Dönüşüm

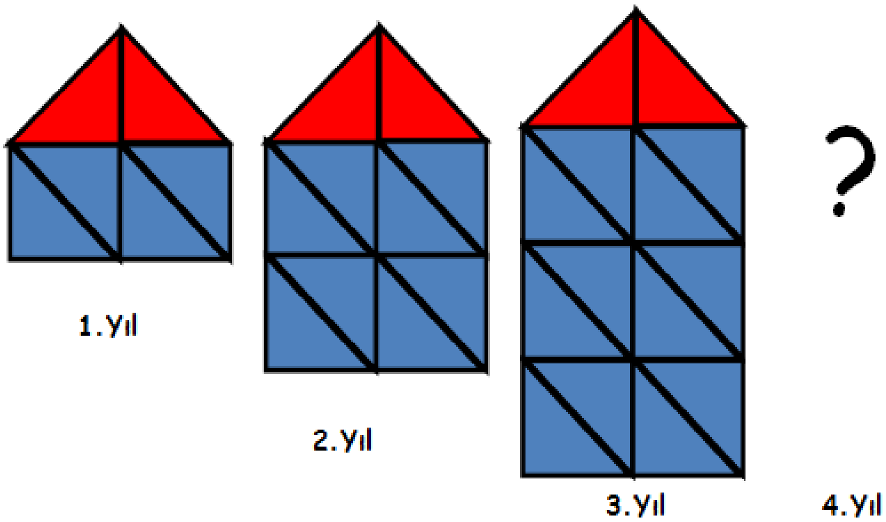
Mevcut depreme dayanıksız, ekonomik ömrünü tamamlamış binaların yaşanabilir, depreme dayanıklı, sosyal donatıları, otoparkı, yeşil alanları olan kaliteli yaşam alanlarına dönüştürme sürecine (projesine) kentsel dönüşüm denir. Kentsel dönüşümün bu tür yararları olmasına rağmen bu süreç içinde bazı ailelerin mahallelerinden başka yerlere taşınması da gerekir.

1) İstanbul'da kentsel dönüşüme girecek mahallelerden biri de Kayışdağı mahallesidir. Bu süreçte mahalleden taşınacak aile sayısı aşağıdaki tabloda verilmiştir. **Yıl sayısı** ile **Taşınan Aile Sayısı** arasındaki ilişkiyi (örüntü kuralını) bulabilir misiniz?

Yıl sayısı	**Taşınan aile sayısı**
1	3
2	6
3	9
4	12
5	15
⋮	⋮
x	?

Figure A.2. Etkinlik 2

2) Kentsel dönüşüm projesi kapsamında inşa edilen apartmanlar şekildeki gibidir. **Yeni yapılan apartmanların yüksekliği her yıl biraz daha artmaktadır.**



Şekildeki üçgenler (çatı katındakiler de dâhil) apartmandaki daireleri göstermektedir. **Örneğin**, 1. yıl yapılan apartmanda 6 daire vardır.

a) **4. yılda** yapılan bir apartman nasıl olacaktır? Örüntü blokları yardımıyla oluşturunuz.

b) Apartmanların her yıl **bir kat** yükseldiğini düşünürsek, 2., 3. ve 4. yılda apartmanda kaç daire olacaktır? Peki ya "**n** yıl" sonra? (Örüntü kuralını bulunuz.)

Yıl sayısı	**Daire sayısı**
1	6
2	
3	
4	
⋮	⋮

Figure A.2. Etkinlik 2 (cont.)

3) Ataşehir Belediyesi tarafından hazırlanan kentsel dönüşüm planına göre hangi haftada toplam kaç sokakta dönüşüm gerçekleşeceği tabloda gösterilmektedir.

a) Bu plana göre 4. ve 5. haftada düzenlenen sokak sayısı kaç olacaktır?

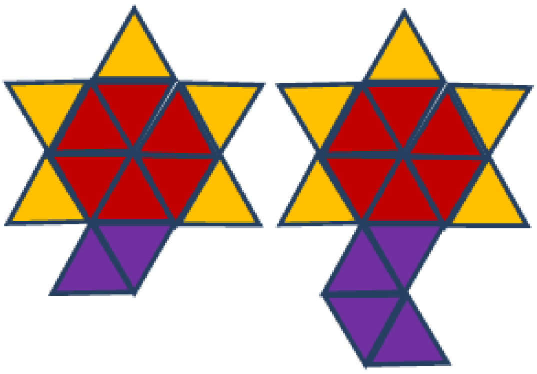
b) Peki, "**m** hafta" sonra kaç olacaktır? (Örüntü kuralını bulunuz.)

Hafta sayısı	**Sokak sayısı**
1	3
2	5
3	7
4	
5	
⋮	⋮
m	....

Figure A.2. Etkinlik 2 (cont.)

Ek sorular:

1)



1.gün 2.gün

Kentsel dönüşüm projesinde bahçe düzenlemesi için çiçeklendirme yapılacaktır. Ekilen çiçeğin sapının günlük uzama miktarı yanda gösterildiği gibidir. 3. ve 4. günde çiçeği oluşturmak için kaç üçgen kullanmak gerekir? Peki ya "**s** gün" sonra?

Gün sayısı	Kullanılan üçgen sayısı
1	
2	
3	
4	
⋮	⋮
S	

2) Örüntü bloklarıyla bir örüntü oluşturarak arkadaşınızdan oluşturduğunuz örüntünün kuralını bulmasını isteyiniz.

Figure A.2. Etkinlik 2 (cont.)

ETKİNLİK 3

Gezelim Görelim

Celal Yardımcı Ortaokulunda gezi kulübü bir gezi organizasyonu hazırlamıştır. Bu gezi kapsamında sırasıyla gezilecek yerler, yaklaşık uzaklık ve ne kadar zaman geçirileceğine dair yönergeler aşağıdaki kutucuklarda verilmiştir. Bu bilgilendirmelere göre kutucuklardaki eksikleri uygun cebirsel ifadeler ile tamamlayınız. **Örneğin, Okul ile Ayasofya arasındaki mesafe X, Ayasofya'da geçirilen süre ise T'dir.** *Gördüğünüz yerlerin yanına bir artı işareti koyunuz.*

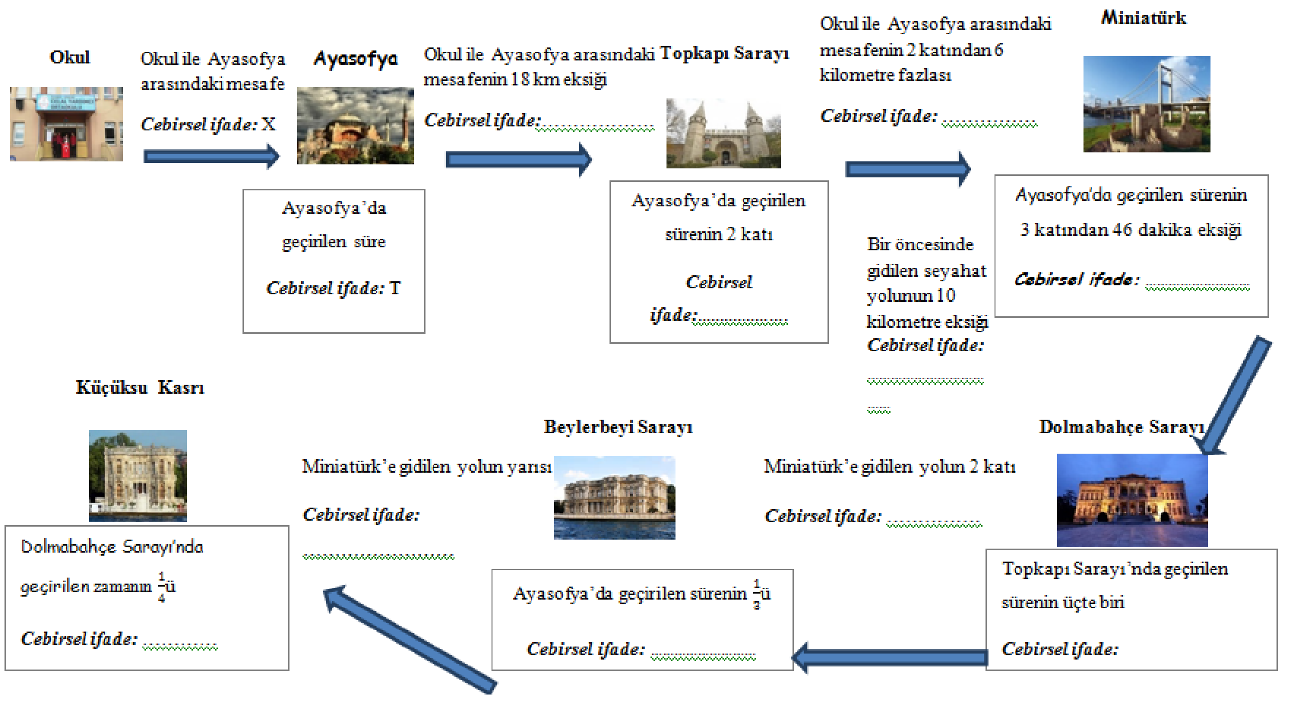


Figure A.3. Etkinlik 3

Aşağıdaki soruları cevaplamak için öğretmeninizin Okulunuzla Ayasofya arasındaki mesafeyi ve Ayasofya'da geçirilecek süreye karar vermesini bekleyiniz.

- Okulunuzla Ayasofya arasındaki mesafe (X) =
- Ayasofya'da geçirilecek süre (T) =

Verilen bilgiler ışığında aşağıdaki soruları yanıtlayınız.

1) Topkapı Sarayı'ndan Dolmabahçe Sarayı'na gitmek için kaç km seyahat edilmiştir?

2) Ayasofya ve Topkapı Sarayı'nda geçirilen toplam süre ne kadardır?

3) Toplam kaç km seyahat edilmiştir?

4) Toplam gezi süresi ne kadardır?

5) **Topkapı Sarayında 90 dakika** geçirmiş olsaydınız, **Küçüksu Kasrında** kaç dakika geçirirdiniz?

Ek sorular:

1) Gezi kulübü ayrıca **Büyükada**, **Yıldız Parkı** ve **Emirgan Korusu**'na da gezi düzenleyecektir. Yukarıdaki yerlerin **aralarındaki mesafeleri** ve buralarda **geçirilecek süreleri** belirten ifadeler yazıp bu ifadeleri cebirsel olarak gösteriniz.

Gezilecek yer	Mesafe (km)	Geçirilecek zaman (dk)
1. Büyükada		
2. Yıldız Parkı		
3.Emirgan Korusu		

2) Verileri kullanarak bir problem oluşturunuz ve denklem kurarak çözünüz.

3) Toplam gezi süreniz **470 dakika olsaydı**, her bir ziyaret yerinde ne kadar vakit geçirmeniz gerekirdi?

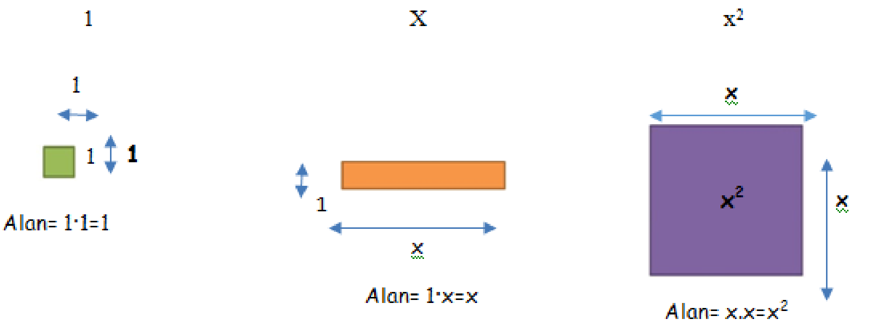
Figure A.3. Etkinlik 3 (cont.)

ETKİNLİK 4

Cebirsel Oyunlar

Lego parçalarıyla çeşitli yapılar oluşturan Cenk, bu yapıları matematiksel olarak nasıl ifade edebileceğini düşünmüş ve ilk olarak parçaları aşağıda görüldüğü gibi isimlendirmiştir.

Cebir Karoları



1. Cebirsel ifadeleri, cebir karolarıyla (lego) gösteriniz ve işlemlerin sonucunu yazınız.

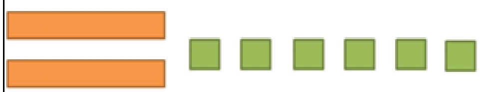
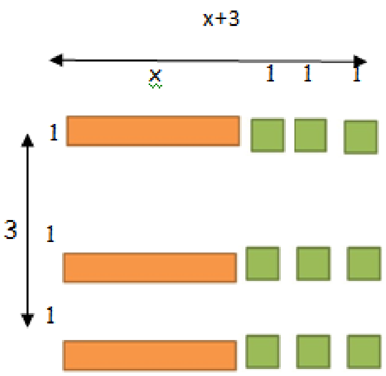
İşlem	Sonuç	Gösterim
x+x+6	2x+6	
x+x+x+3		
(3x + 5) + (2x+2)		
(4x + 7) – (2x + 3)		
$\frac{6x + 4}{2}$		

Figure A.4. Etkinlik 4

2. Aşağıda lego parçalarıyla gösterilen cebirsel ifadeleri işlem ve sonuç olacak şekilde yazınız.

Cebir karoları gösterimi	İşlem	Sonuç
	3·(x+3)	3x+9

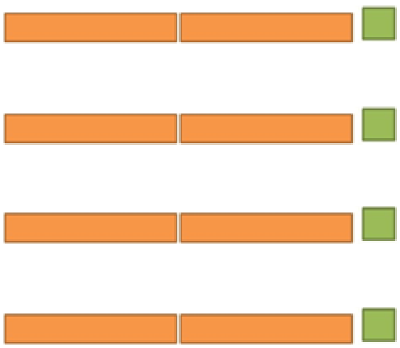
		
	2·(2x+1)	
		2x+6

Figure A.4. Etkinlik 4 (cont.)

Ek soru:

1. Cebir Karolarını kullanarak bir dikdörtgen oluşturunuz. Sıra arkadaşınızdan bu dikdörtgenin alanını cebirsel olarak ifade etmesini isteyiniz.

Figure A.4. Etkinlik 4 (cont.)

ETKİNLİK 5

Pazara Gidelim

Ali, Ayşe, Murat ve Sırma babalarına yardım etmek için onunla pazar alışverişine çıkmışlardır. O hafta evde turşu kurulacak olduğundan alınması gereken kilolarca malzeme vardır.

Pazardan **eşit ağırlıkta 7 poşet** turşu malzemesi ve birkaç kilo meyve almışlardır.

- Ali 2 kilo mandalina ve 2 turşu poşetini taşımaktadır.
- Ayşe turşu poşetlerinden 3 tane taşımaktadır.
- Murat ise 1 turşu poşeti, 1 kilo muz ve yarım kilo elma taşımaktadır.
- Sırma 1 turşu poşeti ve 3 kilo portakal taşımaktadır.

Eve dönüş yolunda kardeşler arasında kimin yükünün daha ağır olduğuna dair bir tartışma başlar.

Ali elindeki tüm poşetleri pazarın çıkışındaki terazide tartar ve ağırlığın **6 kilo** olduğunu görür. (**Unutmayın!** Turşu poşetleri eşit ağırlıktadır.)

1. Turşu poşetlerinin tek bir tanesinin ağırlığını denklem kurarak bulunuz.

2. Ayşe ve Murat'ın taşıdıkları ağırlıkları cebirsel olarak gösteriniz ve ağırlık miktarını hesaplayınız.

3. Pazardan alınan turşuluk malzemenin ve meyvenin toplam ağırlığını bulunuz.

4. Ali, Ayşe, Murat ve Sırma'nın taşıdıkları ağırlık miktarlarını karşılaştırınız.

Figure A.5. Etkinlik 5

Ek Soru:

Fatma Hanım eve gelen turşuluk malzemelerden turşu kuracaktır. Fatma Hanım'ın karışık turşu tarifi aşağıdaki gibidir.

- Bir miktar havuç
- Havuç miktarının 3 katı kadar beyaz lahana
- Havuç miktarının 2 katından 500 gr daha fazla salatalık
- Havuç miktarının dörtte biri kadar sivri biber

Fatma Hanım, toplam **5,5 kilo** turşuluk malzeme kullanarak turşu yapacaktır. Yukarıdaki tarife göre her bir malzemeden ne kadar kullanması gerekir? Denklem kurarak çözünüz. (1 kilo=1000 gr)

Figure A.5. Etkinlik 5 (cont.)

ETKİNLİK 6

Sağlıklı Adımlar

Dünya Sağlık Örgütü'nün (WHO) kuruluş günü olan **7 Nisan** her yıl **Dünya Sağlık Günü** olarak kutlanmaktadır. Dünya Sağlık Örgütüne göre sağlıklı bir yaşam için günde en az 5000 adım atmak gereklidir.

Yaklaşan günün önemini vurgulamak için Banu Hanım, çocukları Pelin ve Mehmet'ten gün içinde farklı yerlere giderken attıkları adımları saymalarını istemiştir. Bir gün içerisinde farklı yerlere giderek en çok adım atan çocuk annesinden bir ödül alacaktır. Kardeşlerin gittikleri yerler aşağıda verilmiştir.

(Kardeşler ev ile okul arasında aynı sayıda adım atmıştır.)

Aynı okula giden kardeşler sabah evden beraber çıkmışlar ve okula gitmişlerdir.

Okuldan sonra; Mehmet önce okul malzemelerini almak için kırtasiyeye, daha sonra doğum günü olan annesine çiçek almak için çiçekçiye gitmiştir. Ablası Pelin ise, önce voleybol oynamak için voleybol sahasına daha sonra da annesine pasta almak için pastaneye gitmiştir.

Mehmet ve Pelin en son olarak eve dönmüşlerdir.

1. Aşağıdaki şekilde Mehmet ve Pelin'in izledikleri yollar verilmiştir.
2. Aşağıdaki şekilde Mehmet ve Pelin'in adım sayılarını hesaplayarak kutucuklara yazınız.
3. Mehmet çiçekçiden eve dönerken 300 adım; Pelin ise; pastaneden eve dönerken 250 adım atmıştır. Günün sonunda hangi çocuk annesinden ödül kazanacaktır; bulunuz.

Ek soru:

1. Siz de gün içinde gittiğiniz yerleri düşünerek her mesafe için adımlarınızı tahmin etmeye çalışınız. Toplam attığınız adımları belirleyiniz.

Figure A.6. Etkinlik 6

Ad Soyad: ……………………………………………………………………………………………

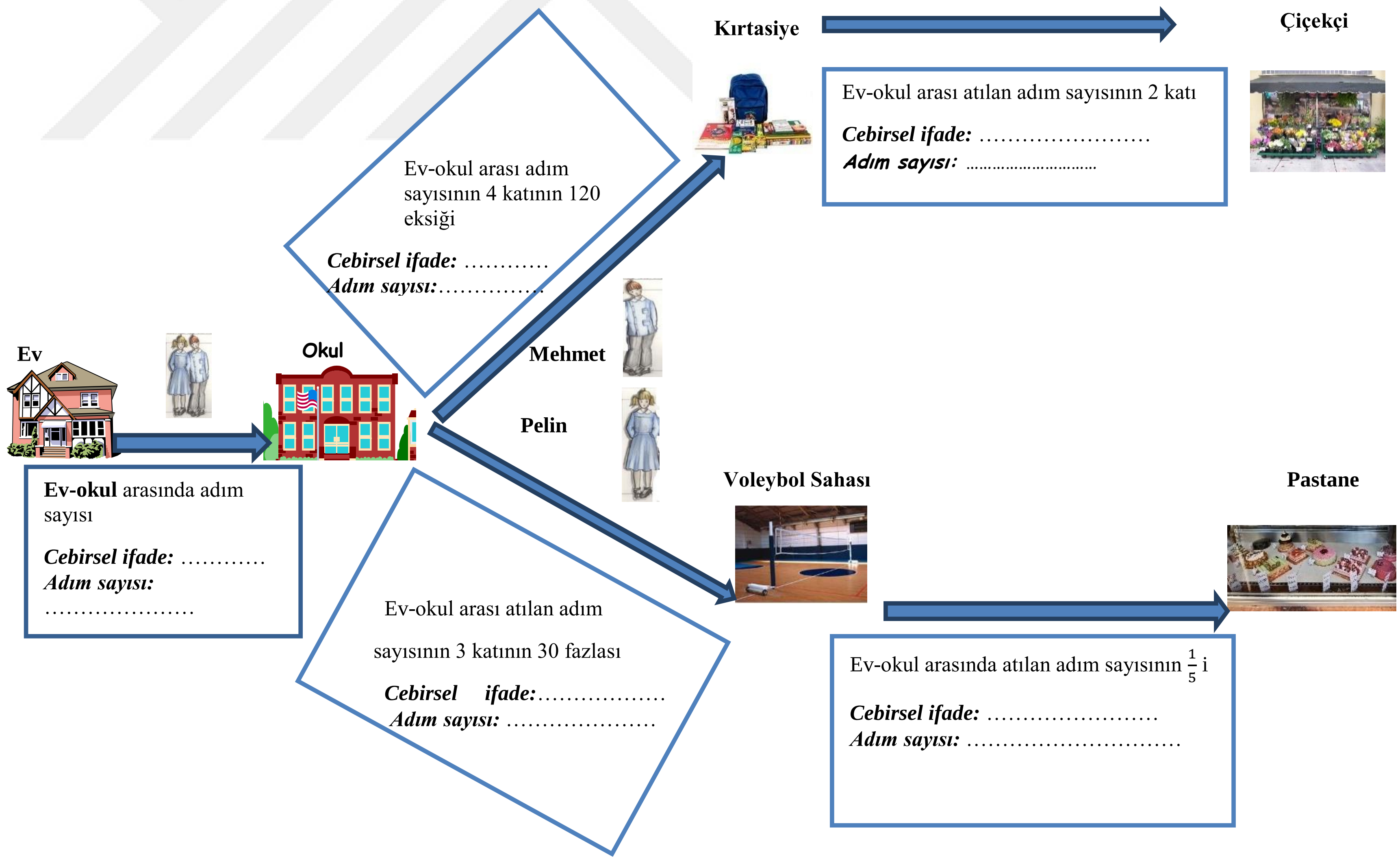


Figure A.6. Etkinlik 6 (cont.)

ETKİNLİK 7

Karışık Meyve Suyu

Selin annesiyle birlikte taze meyveler almak için pazara gitmiştir. Haftasonu arkadaşlarını evine davet etmiş ve onlara annesinin vitamin deposu olan lezzetli meyve suyu karışımlarından ikram etmek istemektedir.

Selin'in hazırlamayı düşündüğü 2 çeşit tarif vardır:

Tarif 1: Tatlımtrak

- Bir miktar nar suyu
- Nar suyunun 4 katından 200 ml fazla elma suyu
- Elma suyunun yarısı kadar armut suyu
- Nar suyunu 2 katı kadar portakal suyu

Tarif 2: TurunçSu

- Bir miktar portakal suyu
- Portakal suyunun yarısı kadar limon suyu
- Limon suyunun üçte biri kadar mandalina suyu

1. Selin, **3 litre Tatlımtrak** karışımından yapmak isterse her bir meyve suyundan kaç ml. kullanması gerekir? Denklem kurarak çözünüz ve çözümünüzü açıklayınız. (1 litre=1000 ml)
2. Selin, **2 litre TurunçSu** karışımından yapmak isterse her bir meyve suyundan kaç ml. kullanması gerekir? Denklem kurarak çözünüz ve çözümünüzü açıklayınız.

Ek soru:

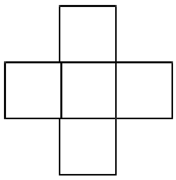
Siz de arkadaşınız için bir meyve suyu karışımı tarifi hazırlayın. O tariften toplam ne kadarlık karışım hazırlayacağını söyleyerek her bir meyve suyundan ne kadar kullanması gerektiğini bulmasını isteyin.

Figure A.7. Etkinlik 7

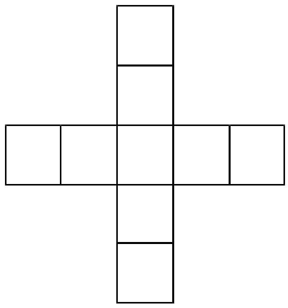
ETKİNLİK 8

Gizli Sayı

Ali kareler ile aşağıdaki şekilleri oluşturmuştur.



1.Şekil



2.Şekil

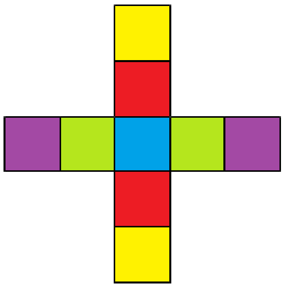
1) a) Ali'nin şekillerindeki örüntüye göre 3. ve 4. şeklin nasıl olacağını çizerek gösteriniz.

b) Şekillere bakarak Ali'nin n. şeklindeki kare sayısını gösteren örüntü kuralını tabloyu tamamlayarak bulunuz. Yazdığınız kurala nasıl ulaştığınızı açıklayınız.

Şekil sayısı	**Şeklindeki kare sayısı**
1. Şekil	5
2. Şekil	9
3. Şekil	
4. Şekil	
5. Şekil	
⋮	⋮
n. Şekil	?

Figure A.8. Etkinlik 8

2) Ali matematik bulmacalarını çok sevmektedir ve 2. şeklini arkadaşlarına bir bulmaca olarak hazırlamıştır. Bulmacadaki kareleri şekildeki gibi boyamıştır ve karelerin her birine gizli sayılar koymaya karar vermiştir. Bu sayıları ise aşağıdaki kurallara göre belirlemiştir.



- Yeşil karedeki sayı, mavi karedeki sayının 5 katının 2 fazlasıdır.
- Mor karedeki sayı, yeşil karedeki sayının 8 fazlasıdır.
- Kırmızı karedeki sayı, mor karedeki sayının 2/5 idir.
- Sarı karedeki sayı, kırmızı karedeki sayının 4 eksiğidir.

Buna göre kırmızı, mor, mavi, yeşil ve sarı karelerdeki sayıların cebirsel ifadesini yazınız.

Mavi:

Yeşil:

Mor:

Kırmızı:

Sarı:

3) Bulmacada yukarıdan aşağı doğru (sütunda) olan sayıların toplamı 26 dır. Buna göre mavi, sarı, kırmızı, mor ve yeşil karelerdeki sayıları denklem kurarak bulunuz. Çözümünüzü açıklayınız.

Renk	**Karedeki Sayı**
Sarı	
Kırmızı	
Yeşil	
Mavi	
Mor	

Ek soru: Siz de kendi bulmacanızı hazırlayın. Bulmacanın her bir karesine bir kurala göre sayı yerleştirerek arkadaşınızdan bu sayıları bulmasını isteyin.

Figure A.8. Etkinlik 8 (cont.)

ANSWER KEYS OF THE TASKS

ETKİNLİK 1

Lunapark

Mahallemizde yeni açılacak bir Lunaparkta

çeşitli oyuncaklar olacaktır.

Bu oyuncaklardan biri de hızlı trendir.

1. Treni inşa etmek için de Lunapark görevlileri önce tren rayları döşeyeceklerdir. 1 saatte 5 tane ray döşenmektedir. Lunapark görevlileri günde 8 saat çalıştıklarına göre

a) Bir günde kaç tane ray döşenir? İşlemlerinizi gösteriniz.

1 günde 8 saat var ve her saatte 5 ray döşeniyor ise, 1 günde 1.8.5=40 tane ray döşenir.

b) İki günde kaç tane ray döşenir? İşlemlerinizi gösteriniz.

2 günde 1 günde döşenen ray sayısının 2 katı ray döşenmektedir, yani 2.5.8=80 tane ray döşenir.

c) Üç günde kaç tane ray döşenir? İşlemlerinizi gösteriniz.

3 günde 1 günde döşenen ray sayısının 3 katı ray döşenmektedir, yani 3.5.8=120 tane ray döşenir.

2) Trenin kurulabilmesi için 400 tane rayın döşenmesi gerekmektedir. Bu rayların döşenmesi için kaç saat ve kaç gün çalışılması gerekmektedir? Açıklayınız.

Her gün 40 tane ray döşeniyor ise 400 tane ray döşenmesi için $\frac{400}{40} = 10$ gün harcanması gerekmektedir. Her gün 8 saat çalışılıyor ise, 10 günde 10×8=80 saat çalışılmıştır. 400 tane ray döşenmesi için 10 gün ve 80 saat gerekmektedir.

3) Döşenen ray miktarı ile saat sayısı arasında nasıl bir ilişki vardır? Bu ilişkiyi gösteren örüntü kuralını bulunuz. Bu kurala nasıl ulaştığınızı açıklayınız.

Ray döşemek için harcanan saat sayısı ile döşenen ray miktarı arasında bir doğru orantı vardır çünkü saat sayısı arttıkça döşenen ray miktarı da artmaktadır.

Örüntü ise; 1.8.5; 2.5.8; 3.5.8 terimleri ile ilerlemektedir. İlk 3 terime bakıldığında 5.8 yani 40 sabit kalmakta başındaki katsayı ise sürekli değişmektedir. Bu nedenle örüntü kuralı 40.n olmalıdır.

Figure A.9. Etkinlik 1 Cevap Anahtarı

4) Ray kurulumu bittikten sonra ilk 3 günde yerleştirilen koltuklar resimlerdeki gibi gösterilmiştir. İlk 3 günde yerleştirilen koltukları örüntü bloklarıyla gösterip, **gün sayısı** ile **yerleştirilen koltuk sayısı** arasındaki ilişkiyi gösteren örüntü kuralını bulunuz. Bu kurala nasıl ulaştığınızı açıklayınız.

1. gün

Koltuk sayısı: 5

2. gün

Koltuk sayısı: 8

3. gün

Koltuk sayısı: 11

Koltuk sayısı 5, 8, 11 şeklinde ilerlemektedir. Şekillerde ilk 2 koltuk sabit konumdadır. Her gün bu 2 koltuğa 3 tane koltuk eklenmiştir. Değişim miktarı, bilinmeyen gün sayısını temsil eden "n" nin önüne katsayı olarak gelecek; sabit olan koltuklar ise örüntü kuralına toplam olarak eklenecektir. Bu nedenle örüntü kuralı 2+3n olmalıdır.

5) Lunapark sahibi koltuklar eklemeye devam ediyor. Koltukları yerleştirmek için toplam 15 gün harcanıyor ise, hızlı trende kaç koltuk vardır? Açıklayınız.

Toplam 15 gün harcanıyor ise örüntü kuralında bilinmeyeni temsil eden "n" yerine 15 konulmalıdır. Böylece vagon sayısı 3×15+2=47 olacaktır.

6) Cem, Lunapark açılışı için arkadaşlarıyla beraber davetiyeleri dağıtmıştır. Dağıttıkları davetiye miktarlarını şöyle ifade ediyorlar:

- Aylin, Cem'in dağıttığı davetiye sayısının 2 katı davetiye dağıtmıştır.
- Merve ise Aylin'in dağıttığı davetiye sayısının 3 fazlası kadar davetiye dağıtmıştır.
- Burak ise, Turgay'ın dağıttığı davetiye sayısının 3 katının 5 eksiği kadar davetiye dağıtmıştır.

Verilen bilgilere göre, dağıtılan davetiye sayısını cebirsel ifade olarak kutucuklara yazınız.

Cem	Aylin	Merve
a	2a	2a+3

Turgay	Burak
b	3b-5

Figure A.9. Etkinlik 1 Cevap Anahtarı (cont.)

ETKİNLİK 2

Kentsel Dönüşüm

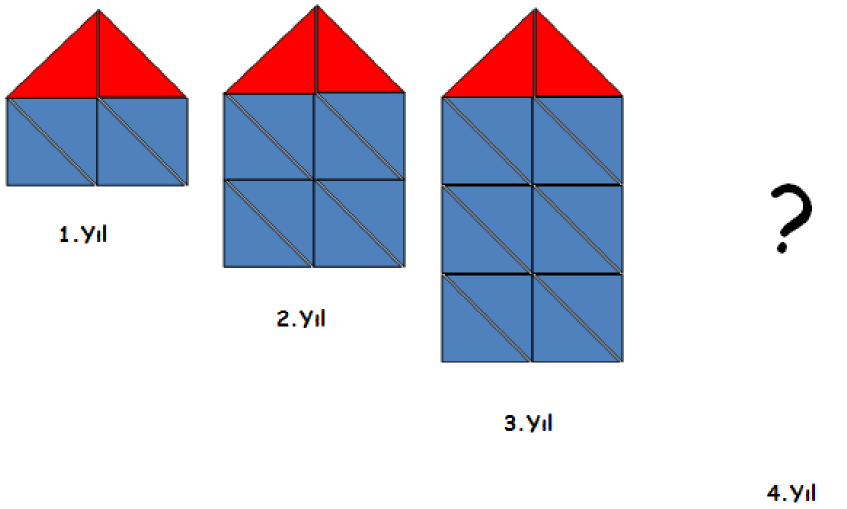
Mevcut depreme dayanıksız, ekonomik ömrünü tamamlamış binaların yaşanabilir, depreme dayanıklı, sosyal donatıları, otoparkı, yeşil alanları olan kaliteli yaşam alanlarına dönüştürme sürecine (projesine) kentsel dönüşüm denir. Kentsel dönüşümün bu tür yararları olmasına rağmen bu süreç içinde bazı ailelerin mahallelerinden başka yerlere taşınması da gerekir.

1) İstanbul'da kentsel dönüşüme girecek mahallelerden biri de Kayışdağı mahallesidir. Bu süreçte mahalleden taşınacak aile sayısı aşağıdaki tabloda verilmiştir. **Yıl sayısı** ile **Taşınan Aile Sayısı** arasındaki ilişkiyi (örüntü kuralını) bulabilir misiniz?

Yıl sayısı	**Taşınan aile sayısı**
1	3=3.1
2	6=3.2
3	9=3.3
4	12=4.3
5	15=5.3
⋮	⋮
Y	? 3y

Figure A.10. Etkinlik 2 Cevap Anahtarı

2) Kentsel dönüşüm projesi kapsamında inşa edilen apartmanlar şekildeki gibidir. **Yeni yapılan apartmanların yüksekliği her yıl biraz daha artmaktadır.**



Şekildeki üçgenler (çatı katındakiler de dâhil) apartmandaki daireleri göstermektedir. **Örneğin**, 1. yıl yapılan apartmanda 6 daire vardır.

a) **4. yılda** yapılan bir apartman nasıl olacaktır? Örüntü blokları yardımıyla oluşturunuz.

b) Apartmanların her yıl **bir kat** yükseldiğini düşünürsek, 2., 3. ve 4. yılda apartmanda kaç daire olacaktır? Peki ya "**n** yıl" sonra? (Örüntü kuralını bulunuz.)

Yıl sayısı	**Daire sayısı**
1	6=4.1+2
2	10=4.2+2
3	14=4.3+2
4	18=4.4+2
⋮	⋮
n	.........4.n+2............

Figure A.2. Etkinlik 2 Cevap Anahtarı (cont.)

3) Ataşehir Belediyesi tarafından hazırlanan kentsel dönüşüm planına göre hangi haftada toplam kaç sokakta dönüşüm gerçekleşeceği tabloda gösterilmektedir.

a) Bu plana göre 4. ve 5. haftada düzenlenen sokak sayısı kaç olacaktır?

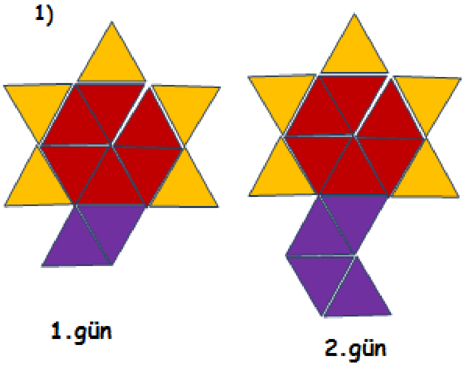
b) Peki "**m** hafta" sonra kaç olacaktır? (Örüntü kuralını bulunuz.)

Hafta sayısı	**Sokak sayısı**
1	3=2.1+1
2	5=2.2+1
3	7=2.3+1
4	9=2.4+1
5	
⋮	⋮
m	2.m+1

Figure A.2. Etkinlik 2 Cevap Anahtarı (cont.)

Ek sorular:

1)



Kentsel dönüşüm projesinde bahçe düzenlemesi için çiçeklendirme yapılacaktır. Ekilen çiçeğin sapının günlük uzama miktarı yanda gösterildiği gibidir. 3. ve 4. günde çiçeği oluşturmak için kaç üçgen kullanmak gerekir? Peki ya "s gün" sonra?

Gün sayısı	**Kullanılan üçgen sayısı**
1	13=2.1+11
2	15=2.2+11
3	17=2.3+11
4	19=2.4+11
⋮	⋮
s	2.s+11

2) Örüntü bloklarıyla bir örüntü oluşturarak arkadaşınızdan oluşturduğunuz örüntünün kuralını bulmasını isteyiniz.

Figure A.2. Etkinlik 2 Cevap Anahtarı (cont.)

ETKİNLİK 3

Gezelim Görelim

Celal Yardımcı Ortaokulunda gezi kulübü bir gezi organizasyonu hazırlamıştır. Bu gezi kapsamında sırasıyla gezilecek yerler, yaklaşık uzaklık ve ne kadar zaman geçirileceğine dair yönergeler aşağıdaki kutucuklarda verilmiştir. Bu bilgilendirmelere göre kutucuklardaki eksikleri uygun cebirsel ifadeler ile tamamlayınız. **Örneğin, Okul ile Ayasofya arasındaki mesafe X, Ayasofya'da geçirilen süre ise T'dir**. *Gördüğünüz yerlerin yanına bir artı işareti koyunuz.*

Okul

Okul ile Ayasofya arasındaki mesafe

Cebirsel ifade: X

Ayasofya

Ayasofya'da geçirilen süre

Cebirsel ifade: **T**

Okul ile Ayasofya arasındaki mesafenin 18 km eksiği

Cebirsel ifade: X-18

Topkapı Sarayı

Ayasofya'da geçirilen sürenin 2 katı

Cebirsel ifade:**2T**

Okul ile Ayasofya arasındaki mesafenin 2 katından 6 kilometre fazlası

Cebirsel ifade: **2X+6**

Miniatürk

Ayasofya'da geçirilen sürenin 3 katından 46 dakika eksiği

Cebirsel ifade: 3T-46

Bir öncesinde gidilen seyahat yolunun 10 kilometre eksiği ***Cebirsel ifade:*** **(2X+6)-10=2X-4**

Dolmabahçe Sarayı

Topkapı Sarayı'nda geçirilen sürenin üçte biri

Cebirsel ifade: $\frac{2T}{3}$

Miniatürk'e gidilen yolun 2 katı

Cebirsel ifade: **2(2X+6)=4X+12**

Beylerbeyi Sarayı

Ayasofya'da geçirilen sürenin $\frac{1}{3}$ü

Cebirsel ifade: $\frac{T}{3}$

Miniatürk'e gidilen yolun yarısı

Cebirsel ifade: $\frac{2X+6}{2} = x + 3$

Küçüksu Kasrı

Dolmabahçe Sarayı'nda geçirilen zamanın $\frac{1}{4}$ü

Cebirsel ifade: $\frac{2T}{3} \times \frac{1}{4} = \frac{T}{6}$

Figure A.11. Etkinlik 6 Cevap Anahtarı

Aşağıdaki soruları cevaplamak için öğretmeninizin Okulunuzla Ayasofya arasındaki mesafeyi ve Ayasofya'da geçirilecek süreye karar vermesini bekleyiniz.

- Okulunuzla Ayasofya arasındaki mesafe (X) = 20 kilometre
- Ayasofya'da geçirilecek süre (T) = 60 dakika

Verilen bilgiler ışığında aşağıdaki soruları yanıtlayınız.

1) Topkapı Sarayı'ndan Dolmabahçe Sarayı'na gitmek için kaç km seyahat edilmiştir?

Topkapı Sarayı'ndan Miniatürk'e gitmek için 2x+6 kilometre, Miniatürk'ten Dolmabahçe Sarayı'na gitmek için ise 2x-4 kilometre yol gitmek gerekmektedir. Toplam yol 2x+6+2x-4=4x+2 kilometredir, bilinmeyen x değeri yerine 20 yi yerleştirirsek, $4\times20+2=82$.

2) Ayasofya ve Topkapı Sarayı'nda geçirilen toplam süre ne kadardır?

Ayasofya'da geçirilen sure T, Topkapı Sarayı'nda geçirilen sure 2T olduğundan toplam sure T+2T=3T dir, bilinmeyen yerine 60 koyarsak toplam sure $3\times60=180$ dakika olur.

3) Toplam kaç km seyahat edilmiştir?

Toplam gidilen seyahat yolu tüm seyahat yolları toplanarak bulunabilir. Bu durumda x+x-18+2x+6+2x-4+4x+12+x+3=11x-1 toplam yol miktarı olur. Bilinmeyen yerine 20 koyarsak toplam seyahat yolu, $11\times20-1=119$ kilometre olur.

4) Toplam gezi süresi ne kadardır?

Toplam gezi süresi tüm gezi süreleri toplanarak bulunabilir. Toplam gezi süresi, $T+2T+3T-46+\frac{2T}{3}+\frac{T}{3}+\frac{T}{6}=\frac{43T}{6}-46$ dır. Bilinmeyen yerine 60 ı yerleştirirsek, $\frac{43\times60}{6}-46=384$ dakika tüm gezi süresi olur.

5) Topkapı Sarayında 90 dakika geçirmiş olsaydınız, **Küçüksu Kasrında** kaç dakika geçirirdiniz?

Topkapı Sarayı'nda geçirilen sure 2T=90 dakika olmuş olsaydı T= 45 olurdu. Küçüksu Kasrı'nda geçirilen süre $\frac{T}{6}$ ise bilinmeyen yerine 45 koyarsak $\frac{45}{6}=$ 7,5 dakika olur.

Figure A.11. Etkinlik 3 Cevap Anahtarı (cont.)

Ek sorular:

1) Gezi kulübü ayrıca **Büyükada**, **Yıldız Parkı** ve **Emirgan Korusu**'na da gezi düzenleyecektir. Yukarıdaki yerlerin **aralarındaki mesafeleri** ve buralarda **geçirilecek süreleri** belirten ifadeler yazıp bu ifadeleri cebirsel olarak gösteriniz.

Gezilecek yer	**Mesafe (km)**	**Geçirilecek zaman (dk)**
1. Büyükada		
2. Yıldız Parkı		
3.Emirgan Korusu		

2) Verileri kullanarak bir problem oluşturunuz ve denklem kurarak çözünüz.

3) Toplam gezi süreniz **470 dakika olsaydı**, her bir ziyaret yerinde ne kadar vakit geçirmeniz gerekirdi?

Toplam gezi süresi, $\frac{43T}{6} - 46 = 470$ olsaydı, $\frac{43T}{6} = 516$ dan T=72 olurdu. Bilinmeyen T yerine 72 koyarsak, Ayasofya'da geçirilen süre T=72 dakika, Topkapı Sarayı'nda geçirilen süre T×2=72×2=144 dakika olur. Miniatürk'te geçirilen süre 3T-46 olduğu için 3×72-46 dan 170 dakika olur. Dolmabahçe Sarayı'nda geçirilen süre $\frac{2\times72}{3} = 48$ dakika olur, Beylerbeyi Sarayı'nda geçirilen süre ise $\frac{T}{3} = \frac{72}{3}$ten 24 dakikadır. Aynı şekilde bilinmeyen yerine 72 yi yerleştirirsek, Küçüksu Kasrı'ndan geçirilen süre $\frac{T}{6}$ ise, $\frac{72}{6} = 12$ dakikadir.

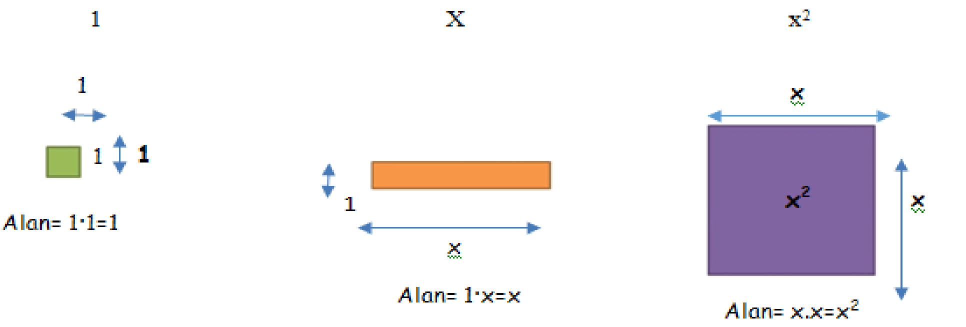
Figure A.11. Etkinlik 3 Cevap Anahtarı (cont.)

ETKİNLİK 4

Cebirsel Oyunlar

Lego parçalarıyla çeşitli yapılar oluşturan Cenk, bu yapıları matematiksel olarak nasıl ifade edebileceğini düşünmüş ve ilk olarak parçaları aşağıda görüldüğü gibi isimlendirmiştir.

Cebir Karoları



1. Cebirsel ifadeleri, cebir karolarıyla (lego) gösteriniz ve işlemlerin sonucunu yazınız.

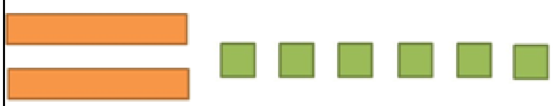
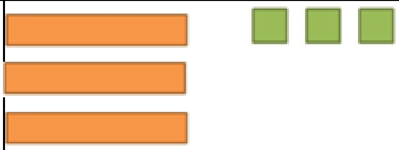
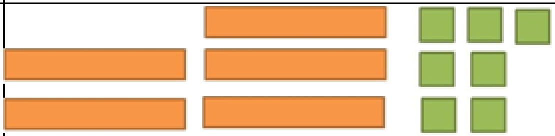
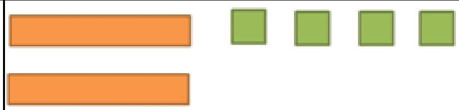
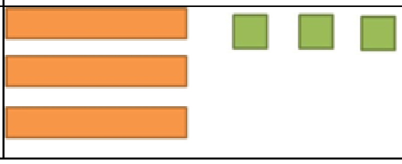
İşlem	Sonuç	Gösterim
x+x+6	2x+6	
x+x+x+3	3x+3	
(3x + 5) + (2x+2)	5x+7	
(4x + 7) – (2x + 3)	2x+4	
$\frac{6x + 4}{2}$	2x+3	

Figure A.12. Etkinlik 4 Cevap Anahtarı

2. Aşağıda lego parçalarıyla gösterilen cebirsel ifadeleri işlem ve sonuç olacak şekilde yazınız.

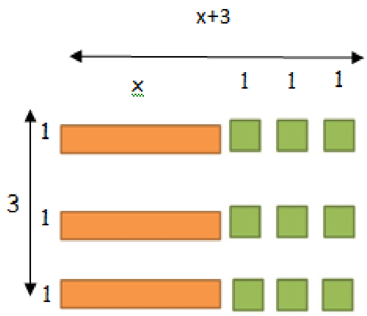
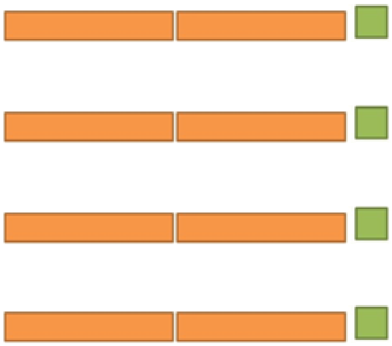
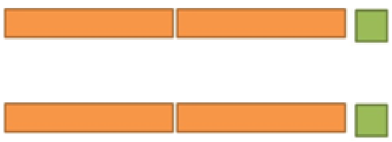
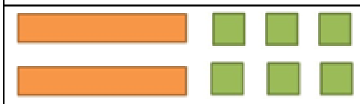
Cebir karoları gösterimi	İşlem	Sonuç
	3·(x+3)	3x+9
	4.(2x+1)	8x+4
	2·(2x+1)	4x+2
	2.(x+3)	2x+6

Figure A.12. Etkinlik 4 Cevap Anahtarı (cont.)

Ek soru:

1. Cebir Karolarını kullanarak bir dikdörtgen oluşturunuz. Sıra arkadaşınızdan bu dikdörtgenin alanını cebirsel olarak ifade etmesini isteyiniz.

Figure A.12. Etkinlik 4 Cevap Anahtarı (cont.)

ETKİNLİK 5

Pazara Gidelim

Ali, Ayşe, Murat ve Sırma babalarına yardım etmek için onunla pazar alışverişine çıkmışlardır. O hafta evde turşu kurulacak olduğundan alınması gereken kilolarca malzeme vardır.

Pazardan **eşit ağırlıkta 7 poşet** turşu malzemesi ve birkaç kilo meyve almışlardır.

- Ali 2 kilo mandalina ve 2 turşu poşetini taşımaktadır.
- Ayşe turşu poşetlerinden 3 tane taşımaktadır.
- Murat ise 1 turşu poşeti, 1 kilo muz ve yarım kilo elma taşımaktadır.
- Sırma 1 turşu poşeti ve 3 kilo portakal taşımaktadır.

Eve dönüş yolunda kardeşler arasında kimin yükünün daha ağır olduğuna dair bir tartışma başlar.

Ali elindeki tüm poşetleri pazarın çıkışındaki terazide tartar ve ağırlığın **6 kilo** olduğunu görür. (**Unutmayın!** Turşu poşetleri eşit ağırlıktadır.)

1. Turşu poşetlerinin tek bir tanesinin ağırlığını denklem kurarak bulunuz.

Ali 2 kilo mandalina ve 2 tane turşu poşeti taşıyor ve toplam ağırlığın 6 kilo olduğunu görüyor. Turşu poşetinin ağırlığını bilmediğimiz için bir turşu poşetinin ağırlığına x diyelim, 2 turşu poşetinin ağırlığı 2x olur. Buradan denklem kuracak olursak 2x+2=6

2x=6-2

2x=4 x=2 olur denklemden görülebileceği gibi bir turşu poşetinin ağırlığı 2 kilodur.

2. Ayşe ve Murat'ın taşıdıkları ağırlıkları cebirsel olarak gösteriniz ve ağırlık miktarını hesaplayınız.

Ayşe turşu poşetlerinden 3 tane taşıdığı için elindeki ağırlık 3x olur. Murat ise 1 turşu poşeti yani x, ve yanında 1 kilo muz ve yarım kilo elma taşımaktadır. Bu nedenle Murat'ın taşıdığı toplam miktar x+1+0,5 ten x+1,5 olacaktır. Bilinmeyen x değerini 1. soruda 2 bulduk, bunu yerine yazacak olursak, Ayşe=2×3= 6 kilo taşımış; Murat ise 2+1,5 olmak üzere 3,5 kilo taşımıştır.

Figure A.13. Etkinlik 5 Cevap Anahtarı

3. Pazardan alınan turşuluk malzemenin ve meyvenin toplam ağırlığını bulunuz.

Pazardan alınan turşuluk malzeme her bir çocuğun taşıdığı ağırlıkları toplanarak bulunabilir. Toplam 2+3+1+1=7 tane turşu poşeti taşınmıştır, her bir turşu poşeti 2 kilo olduğuna göre toplam turşuluk malzeme miktarı 2×7=14 kilodur. Meyve ise yine aynı yol izlenerek tüm çocukların taşıdıkları meyvenin ağırlıkları toplanarak bulunabilir. 2+1+0,5+3=6,5 kilo toplam meyvenin ağırlığıdır.

4. Ali, Ayşe, Murat ve Sırma'nın taşıdıkları ağırlık miktarlarını karşılaştırınız.

Ali=6 kg

Ayşe=6 kg

Murat=3,5 kg olduğu yukarıdaki sorularda bulunmuştur. Sırma'nın taşıdığı ağırlık ise x+3 olduğu için bilinmeyen yerine 2 yazılarak toplam ağırlık 2+3=5 kilo olarak bulunabilir. Bu nedenle sıralama; Ali=Ayşe>Sırma>Murat olacaktır.

Ek Soru:

Fatma Hanım eve gelen turşuluk malzemelerden turşu kuracaktır. Fatma Hanım'ın karışık turşu tarifi aşağıdaki gibidir.

- Bir miktar havuç
- Havuç miktarının 3 katı kadar beyaz lahana
- Havuç miktarının 2 katından 500 gr daha fazla salatalık
- Havuç miktarının dörtte biri kadar sivri biber

5.Fatma Hanım, toplam **5,5 kilo** turşuluk malzeme kullanarak turşu yapacaktır. Yukarıdaki tarife göre her bir malzemeden ne kadar kullanması gerekir? Denklem kurarak çözünüz. (1 kilo=1000 gr)

Havuç miktarı bilinmediği için ona x diyebiliriz. Bu durumda beyaz lahana 3.x olur, salatalık ise 2.x+500 dur. Sivri biber ise havuç miktarının dörtte biri olduğu için x i ¼ ile çarpmak gerekmektedir, çünkü bütünün bir miktarı bulunurken çarpma işlemi yapılır. Böylelikle sivri biber miktarı $x.\frac{1}{4}=\frac{x}{4}$ bulunur. Salatalık miktarında kullanılan birim gram olduğu için toplam turşuluk malzemenin birimini de grama çevirmeliyiz. Toplam malzeme $5{,}5\times 1000 = 5500$ olur.

$x+3x+2x+500+\frac{x}{4} = 5500$

$\frac{25x}{4} = 5500 - 500$ $\frac{25x}{4} = 5000$ bulunur, içle dışlar çarpımı yapılarak 25x=5500×4, $x=\frac{5500\times 4}{25} = 800$ çıkar yani havuç miktarı 800 gramdır, yukarıdaki cebirsel ifadelere bilinmeyen yerine 800 gram yazarsak; Beyaz lahana=3x=800×3=2400 gram olur. Salatalık=2x+500=800×2+500 den 2100 gram, sivri biber de $\frac{x}{4} = \frac{800}{4}$=200 gram bulunur.

Figure A.13. Etkinlik 5 Cevap Anahtarı (cont.)

ETKİNLİK 6

Sağlıklı Adımlar

Dünya Sağlık Örgütü'nün (WHO) kuruluş günü olan **7 Nisan** her yıl **Dünya Sağlık Günü** olarak kutlanmaktadır. Dünya Sağlık Örgütüne göre sağlıklı bir yaşam için günde en az 5000 adım atmak gereklidir.

Yaklaşan günün önemini vurgulamak için Banu Hanım, çocukları Pelin ve Mehmet'ten gün içinde farklı yerlere giderken attıkları adımları saymalarını istemiştir. Bir gün içerisinde farklı yerlere giderek en çok adım atan çocuk annesinden bir ödül alacaktır. Kardeşlerin gittikleri yerler aşağıda verilmiştir.

(Kardeşler ev ile okul arasında aynı sayıda adım atmıştır.)

- Aynı okula giden kardeşler sabah evden beraber çıkmışlar ve okula gitmişlerdir.
- Okuldan sonra; Mehmet önce okul malzemelerini almak için kırtasiyeye, daha sonra doğum günü olan annesine çiçek almak için çiçekçiye gitmiştir. Ablası Pelin ise, önce voleybol oynamak için voleybol sahasına daha sonra da annesine pasta almak için pastaneye gitmiştir.
- Mehmet ve Pelin en son olarak eve dönmüşlerdir.

1. Aşağıdaki şekilde Mehmet ve Pelin'in izledikleri yollar verilmiştir. Kutucuklara doğru cebirsel ifadeyi yazınız.
2. Mehmet'in okul ile kırtasiye arası attığı adım sayısı, Pelin'in okul ile voleybol sahası arası attığı adım sayısına eşit ise ev ile okul arası kaç adımdır? Denklem kurarak çözünüz.

Mehmet'in okul ile kırtasiye arası attığı adım sayısı 4x-120, Pelin'in okul ile voleybol sahası arası attığı adım sayısı 3x+30 dur, bunların eşit olduğu biliniyorsa,

$4x-120=3x+30$

$4x-3x=120+30$

$x=150$ bulunur. x ev ile okul arası adım sayısına eşit olduğuna göre, ev ile okul arası atılan adım sayısı 150 dir.

Figure A.14. Etkinlik 6 Cevap Anahtarı

3. Aşağıdaki şekilde Mehmet ve Pelin'in adım sayılarını hesaplayarak kutucuklara yazınız.

Mehmet'in okul ile kırtasiye arasında attığı adım sayısı 4x-120, kırtasiye ile çiçekçi arası attığı adım sayısı 2x tir. Bilinmeyeni temsil eden x değeri yerine 150 yazacak olursak; okul ile kırtasiye arası: 4×150-120=480 adım, kırtasiye-çiçekçi=2×150=300 adım olur. Pelin ise, okul ile voleybol sahası arasında 3x+30; voleybol sahası ile pastane arasında ise $\frac{x}{5}$ adım atmıştır. Bilinmeyen yerine yine 150 yazacak olursak okul ile voleybol sahası: 3×150+30=480 adım, voleybol sahası ile pastane arası $\frac{150}{5} = 30$ adım bulunur.

4. Mehmet çiçekçiden eve dönerken 300 adım; Pelin ise; pastaneden eve dönerken 250 adım atmıştır. Günün sonunda hangi çocuk annesinden ödül kazanacaktır; bulunuz.

Mehmet'in toplam adım sayısını tüm yollar için attığı adım sayılarını toplayarak bulabiliriz. Buradan, Mehmet 150+480+300+300=1230 adım atmış olur. Benzer bir yolla Pelin ise, 150+480+30+250=910 adım atmıştır. Mehmet daha çok adım attığı için günün sonunda ödülü kazanan Mehmet olacaktır.

Figure A.14 Etkinlik 6 Cevap Anahtarı (cont.)

Ad Soyad: ....

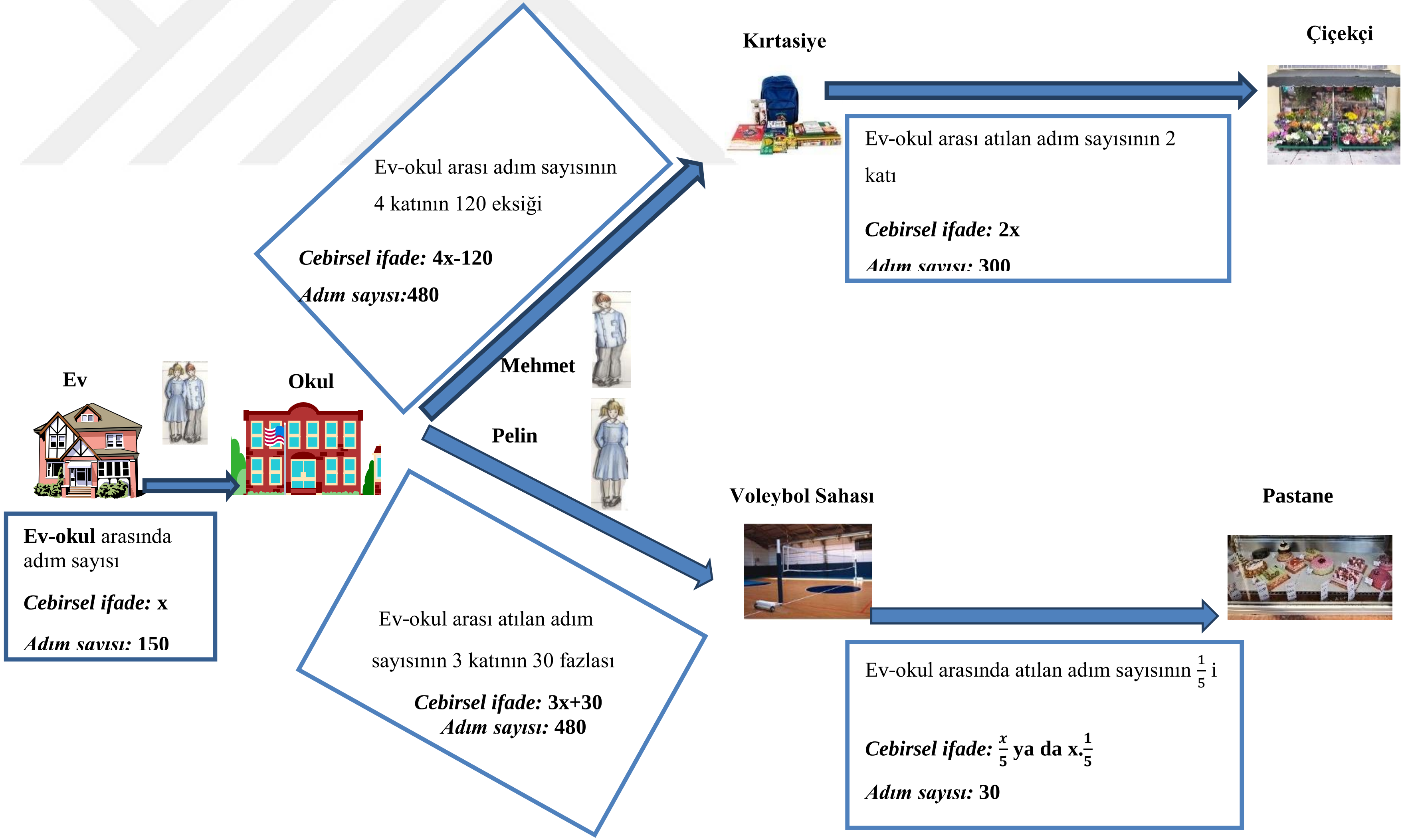


Figure A.14. Etkinlik 6 Cevap Anahtarı (cont.)

Ek soru:

Siz de gün içinde gittiğiniz yerleri düşünerek her mesafe için adımlarınızı tahmin etmeye çalışınız. Toplam attığınız adımları belirleyiniz.

Figure A.14. Etkinlik 6 Cevap Anahtarı (cont.)

ETKİNLİK 7

Karışık Meyve Suyu

Selin annesiyle birlikte taze meyveler almak için pazara gitmiştir. Haftasonu arkadaşlarını evine davet etmiş ve onlara annesinin vitamin deposu olan lezzetli meyve suyu karışımlarından ikram etmek istemektedir.

Selin'in hazırlamayı düşündüğü 2 çeşit tarif vardır:

Tarif 1: Tatlımtrak

- Bir miktar nar suyu
- Nar suyunun 4 katından 200 ml fazla elma suyu
- Elma suyunun yarısı kadar armut suyu
- Nar suyunu 2 katı kadar portakal suyu

Tarif 2: TurunçSu

- Bir miktar portakal suyu
- Portakal suyunun yarısı kadar limon suyu
- Limon suyunun üçte biri kadar mandalina suyu

1. Selin, **3 litre Tatlımtrak** karışımından yapmak isterse her bir meyve suyundan kaç ml. kullanması gerekir? Denklem kurarak çözünüz ve çözümünüzü açıklayınız. (1 litre=1000 ml)

Elma suyu ml cinsinden verildiği için tüm karışımı da aynı birimde almak gerekmektedir, 1 litre 1000 ml ise 3 litre 3000 ml olur.

Nar suyu bilinmediği için nar suyuna x diyebiliriz. Elma suyu da nar suyunun 4 katından 200 ml fazla olduğu için 4.x+200 olur. Armut suyu ise elma suyunun yarısı olduğu için $\frac{4x+200}{2} = 2x + 100$ bulunur. Portakal suyunun miktarı belirlenirken nar suyu referans noktası olarak alınmalıdır, portakal suyu da nar suyunun 2 katı olduğu için 2x olur.

Toplam karışım 3000 ml olduğu için, x+4x+200+2x+100+2x=3000 ml olur.

9x+300=3000

9x=2700

x=300 ml olur.

Cebirsel ifadelerde bilinmeyen yerine 300 yazacak olursak,

Nar suyu: 300 ml,

Elma suyu: 4x+200=4×300+200=1400 ml,

Armut suyu: 2x+100=2×300+100=700 ml,

Portakal suyu 2x=2×300=600 ml bulunur.

Figure A.15. Etkinlik 7 Cevap Anahtarı

2. Selin, **2 litre TurunçSu** karışımından yapmak isterse her bir meyve suyundan kaç ml. kullanması gerekir? Denklem kurarak çözünüz ve çözümünüzü açıklayınız.

Portakal suyunun miktarı bilinmediği için bilinmeyene x diyebiliriz. Limon suyu portakal suyunun yarısı olduğu ve bir bütünün bir miktarı çarpma yoluyla bulunduğu için, limon suyu $x \times \frac{1}{2} = \frac{x}{2}$ olur. Mandalina suyu limon suyunun üçte biri kadar olduğu için, mandalina suyunu hesaplarken limon suyunu referans almak gerekmektedir. Bir bütünün belli bir kısmını bulurken çarpma işlemi yapılması gerektiğinden mandalina suyu $\frac{x}{2} \times \frac{1}{3} = \frac{x}{6}$ olur.

Toplam meyve suyu karışımını 2 litreyi işlem kolaylığı için ml ye çevirebiliriz, karışım 2000 ml olur.

$x + \frac{x}{2} + \frac{x}{6} = \frac{10x}{6} = 2000$ ml olur, içler dışlar çarpımı yaparsak $10x = 6 \times 2000$, $x = \frac{6 \times 2000}{10} =$ 1200 ml olur. Cebirsel ifadelerde bilinmeyen yerine 120 yazacak olursak, portakal suyu 1200 ml, limon suyu $\frac{120}{2} = 600$ ml, mandalina suyu ise $\frac{600}{3} = 200$ ml olur.

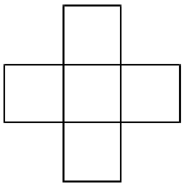
Ek soru:
Siz de arkadaşınız için bir meyve suyu karışımı tarifi hazırlayın. O tariften toplam ne kadarlık karışım hazırlayacağını söyleyerek her bir meyve suyundan ne kadar kullanması gerektiğini bulmasını isteyin.

Figure A.15. Etkinlik 7 Cevap Anahtarı (cont.)

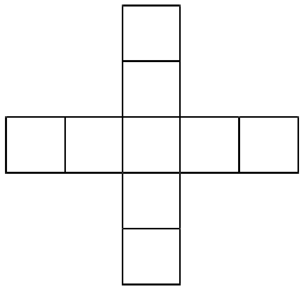
ETKİNLİK 8

Gizli Sayı

Ali kareler ile aşağıdaki şekilleri oluşturmuştur.

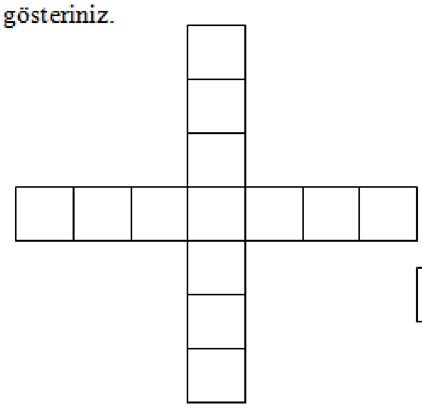


1.Şekil

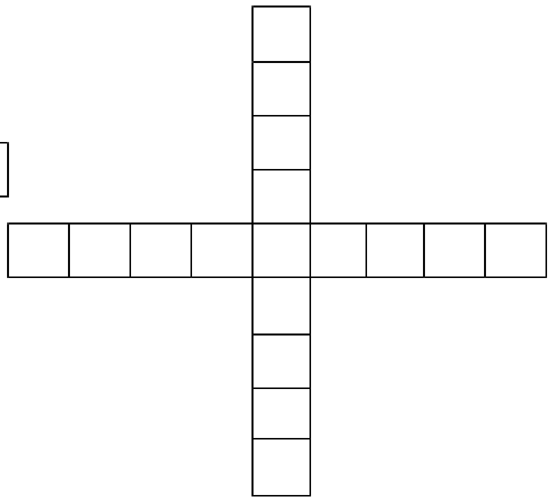


2.Şekil

1) **a)** Ali'nin şekillerindeki örüntüye göre 3. ve 4. şeklin nasıl olacağını çizerek gösteriniz.



3. şekil



4. şekil

Figure A.16. Etkinlik 8 Cevap Anahtarı

b) Şekillere bakarak Ali'nin n. şeklindeki kare sayısını gösteren örüntü kuralını tabloyu tamamlayarak bulunuz. Yazdığınız kurala nasıl ulaştığınızı açıklayınız.

Şekil sayısı	**Şeklindeki kare sayısı**
1. Şekil	5
2. Şekil	9
3. Şekil	13
4. Şekil	17
5. Şekil	21
⋮	⋮
n. Şekil	?

Her bir şekilde merkezdeki 1 kare sabittir. Şekil sayısı arttıkça şeklin kenarlarına 4 er kare eklenmektedir. Bu durumda kare sayısı şekil sayısının 4 katıdır ve ortadaki kare sabit olduğu için örüntü kuralına sabit olarak eklenmelidir. n şekil sayısını ifade ediyorsa, 4n kare sayısını ifade etmelidir, ortadaki sabit kareyi de +1 olarak örüntü kuralına eklersek örüntü kuralı 4n+1 olur.

Figure A.16. Etkinlik 8 Cevap Anahtarı (cont.)

2) Ali matematik bulmacalarını çok sevmektedir ve 2. şeklini arkadaşlarına bir bulmaca olarak hazırlamıştır. Bulmacadaki kareleri şekildeki gibi boyamıştır ve karelerin her birine gizli sayılar koymaya karar vermiştir. Bu sayıları ise aşağıdaki kurallara göre belirlemiştir.

- Yeşil karedeki sayı, mavi karedeki sayının 5 katının 2 fazlasıdır.
- Mor karedeki sayı, yeşil karedeki sayının 8 fazlasıdır.
- Kırmızı karedeki sayı, mor karedeki sayının 2/5 idir.
- Sarı karedeki sayı, kırmızı karedeki sayının 4 eksiğidir.

Buna göre kırmızı, mor, mavi, yeşil ve sarı karelerdeki sayıların cebirsel ifadesini yazınız.

Mavi:x

Yeşil:5x+2

Mor:5x+2+8=5x+10

Kırmızı:(5x+10).$\frac{2}{5}$ =2x+4

Sarı:2x+4-4=2x

3) Bulmacada yukarıdan aşağı doğru (sütunda) olan sayıların toplamı 26 dır. Buna göre mavi, sarı, kırmızı, mor ve yeşil karelerdeki sayıları denklem kurarak bulunuz. Çözümünüzü açıklayınız.

Renk	**Karedeki Sayı**
Sarı	4
Kırmızı	8
Yeşil	12
Mavi	2
Mor	20

Yukarıdan aşağıya kadar olan sayıların toplamını, her bir sayıyı ifade eden cebirsel ifadeyi toplayarak bulabiliriz. Yukarıdan aşağıya iki tane kırmızı, iki tane sarı, bir tane de mavi kare olduğuna göre bu sayıları ifade eden cebirsel ifadeleri toplayabiliriz.

2.(2x+4)+2.(2x)+x=26

4x+8+4x+x=26

9x+8=26

9x=26-8

9x=18 x=2 bulunur.

Bilinmeyen x yerine 2 yazacak olursak, mavi karedeki sayı 2, yeşil karedeki sayı 5x+2=2×5+2=12, mor karedeki sayı 5x+10=5×2+10=20, kırmızı karedeki sayı 2x+4=2×2+4=8, sarı karedeki sayı 2x=2×2=4 bulunur.

Figure A.16. Etkinlik 8 Cevap Anahtarı (cont.)

Ek soru: Siz de kendi bulmacanızı hazırlayın. Bulmacanın her bir karesine bir kurala göre sayı yerleştirerek arkadaşınızdan bu sayıları bulmasını isteyin.

Figure A.16. Etkinlik 8 Cevap Anahtarı (cont.)

APPENDIX B: THE TASK CHECKLIST

	Task1/Lunapark	Task 2/Kentsel Dönüşüm	Task 3/Gezelim Görelim	Task 4/Cebirsel Oyunlar	Task 5/Pazara Gidelim	Task 6/Gizli Sayı	Task 7 /Karışık Meyve Suyu	Task 8/Sağlıklı Adımlar
Encourages students to think about given situation and follow problem solving procedures	Students need to understand the relationship among the quantities and write algebraic expressions	Students need to understand the pattern	Students need to understand the sequence of the places and follow the paths	Students need to demonstrate the formulas and area of the rectangle	Students need to understand the context to set the equation	Students have to realize the relationship among different color of puzzles	Students have to understand the relationship Among different juices (follow the sequence)	Students need to understand the path of the children
Using manipulative and hands on aids	✓ Pattern blocks	✓ Pattern blocks	✓ Map for the route of the places.	✓ Algebraic tiles	✓ Representation of the bags (concrete material)	✓ Pattern blocks can be used	✓ Real juice mixture	✓ Map for the routes of the children is given
Leading communication among students (collaborative work)	✓ Group discussion	✓ Group discussion	✓ Group discussion	✓ Group discussion	✓ Group discussion	✓ Group discussion	✓ Group discussion	✓ Group discussion
Involves in real life contexts	✓	✓	✓	✓ Algebraic tiles were called as lego.	✓	✓ Puzzle pieces	✓	✓

Potential to provoke students' misconception s	Students might have difficulties about the covariation of the quantities.	Students can focus on changes in one variable rather than the relationship between dependent and independent variable	Students might have difficulties about converting verbal expressions into algebraic expressions (e.g. 1/10 of x= x÷ 1/10.) Also they might have problems on determination of reference point /criteria.	Students may have misconceptions about; doing operations with algebraic expressions, making distinction between variable and constants, applying arithmetic operations correctly. (e.g. 2x+3=5x, (4x+7)-(2x+3)=2x-4	Students may have difficulties on determining unknown value, setting up appropriate equation, and solving equations by following correct procedures.	Students may have misconceptions about determination of reference point/ criteria and writing algebraic expressions.	Students might have difficulties on determination of the reference point /criteria and setting up the equations.	Students may have misconcepti ons about setting up the equation and determining the real value of algebraic expressions.

APPENDIX C: ALIGMENT OF OBJECTIVES WITH THE CURRICULUM OBJECTIVES

Etkinlik 1- Lunapark

Kazanım: 6.2.1.1. Aritmetik dizilerin kuralını harfle ifade eder; kuralı harfle ifade eden dizinin istenilen terimini bulur.

6.2.1.2 Sözel olarak verilen bir duruma uygun cebirsel ifade ve verilen bir cebirsel ifadeye uygun sözel bir durum yazar.

Etkinlik 2- Kentsel Dönüşüm

Kazanım: 6.2.1.1 Aritmetik dizilerin kuralını harfle ifade eder, kuralı harfle ifade edilen dizinin istenilen terimini bulur.

Etkinlik 3- Gezelim Görelim

Kazanım: 6.2.1.2 Sözel olarak verilen bir duruma uygun cebirsel ifade ve verilen bir cebirsel ifadeye uygun sözel bir durum yazar (ilk sayfa soruları).

1. 2. 3. ve 4. Sorular:

Kazanım: 6.2.1.3. Cebirsel ifadenin değerlerini değişkenin alacağı farklı doğal sayı değerleri için hesaplar.

Ek sorular: 2. soru: Kazanım: 7.2.1.1. Gerçek yaşam durumlarına uygun birinci dereceden bir bilinmeyenli denklemleri kurar.

7.2.1.4 Birinci dereceden bir bilinmeyenli denklemleri çözer.

Etkinlik 4: Cebirsel oyunlar

1.soru: Kazanım: 6.2.1.5 Cebirsel ifadelerle toplama ve çıkarma işlemleri yapar.

2. soru: Kazanım: 6.2.1.6 Bir doğal sayı ile bir cebirsel ifadeyi çarpar.

Etkinlik 5: Pazara gidelim

1. ve 5. soru: Kazanım: 7.2.1.1 Gerçek yaşam durumlarına uygun 1. dereceden bir bilinmeyenli denklemleri kurar.

Kazanım: 7.2.1.4 Birinci dereceden bir bilinmeyenli denklem kurmayı gerektiren problemleri çözer.

2.soru: Kazanım: 6.2.1.2 Sözel olarak verilen bir duruma uygun cebirsel ifade ve verilen bir cebirsel ifadeye uygun sözel bir durum yazar.

Etkinlik 6: Gizli sayı

1.soru: Kazanım: 6.2.1.1 Aritmetik dizilerin kuralını harfle ifade eder, kuralı harfle ifade edilen dizinin istenilen terimini bulur.

2.soru: Kazanım: 6.2.1.2 Sözel olarak verilen bir duruma uygun cebirsel ifade ve verilen bir cebirsel ifadeye uygun sözel bir durum yazar.

3.soru: Kazanım: 7.2.1.1 Gerçek yaşam durumlarına uygun 1. dereceden bir bilinmeyenli denklemleri kurar.

Etkinlik 7: Karışık meyve suyu

1.ve 2.soru: Kazanım: 7.2.1.1 Gerçek yaşam durumlarına uygun 1. dereceden bir bilinmeyenli denklemleri kurar.

Kazanım: 7.2.1.4 Birinci dereceden bir bilinmeyenli denklem kurmayı gerektiren problemleri çözer.

Etkinlik 8: Sağlıklı adımlar

1.soru: Kazanım: 6.2.1.2 Sözel olarak verilen bir duruma uygun cebirsel ifade ve verilen bir cebirsel ifadeye uygun sözel bir durum yazar.

2. ve 4.soru: Kazanım: 7.2.1.2 Denklemlerde eşitliğin korunumu ilkesini anlar.

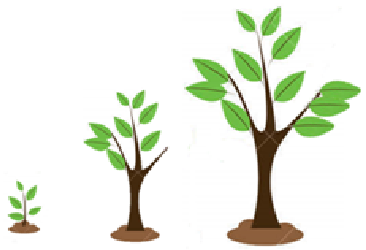
7.2.1.1. Gerçek yaşam durumlarına uygun 1. dereceden bir bilinmeyenli denklemleri kurar.

3.soru: Kazanım: 6.2.1.3. Cebirsel ifadenin değerlerini değişkenin alacağı farklı doğal sayı değerleri için hesaplar.

APPENDIX D: ACHIEVEMENT TEST and THEIR RUBRICS

AKLIMDA KALANLAR – CEBİR ÖN TESTİ

1)



Ayşe bahçelerindeki ağacın büyümesini haftalar içinde gözlemlemiştir ve ağaçtaki yaprak sayılarını gösteren yukarıdaki fotoğrafları çekmiştir. Buna göre aşağıdaki tabloyu doldurarak Ayşe'nin n. haftada gözlemleyeceği yaprak sayısını gösteren örüntü kuralını bulunuz. **(5p)**

Hafta sayısı	**Ağaçtaki yaprak sayısı**
1. hafta	5
2. hafta	8
3. hafta	11
4. hafta	....
5. hafta	....
⋮	⋮
n. hafta	....

Figure A.17. Cebir Ön Test

2) Aşağıda verilen sözel veya cebirsel ifadelerin uygun karşılıklarını yazınız. **(8p)**

Sözel ifade	**Cebirsel ifade**
Elif'in 5 yıl sonraki yaşı	
Bir sayının 2 katının 3 eksiği	
	$\frac{x}{2}+8$
Elimdeki cevizlerin 15 fazlasının 3 katı	

3) Mehmet elindeki 42 parçalık yapbozu kuzeni Ayla ile yapacaktır. Yapbozu tamamladıktan sonra, Ayla'nın koyduğu parça sayısının Mehmet'in koyduğu parça sayısının 3 katından 6 fazla olduğunu görmüşlerdir. Mehmet ve Ayla'nın yapboza kaç parça eklemiş olduklarını denklem kurarak bulunuz. **(8p)**

4) Emre'nin **evi** ile **okulu** arasındaki mesafe, **evi** ile **park** arasındaki mesafenin 4 katından 50 m daha fazladır. Emre'nin **evi** ile **market** arasındaki mesafe ise **ev-park** mesafesinden 30 m daha kısadır.

Emre bu sabah okula gidiyor, okuldan geldikten sonra annesi marketten ekmek almasını söylüyor. Emre marketten ekmek alıp eve bıraktıktan sonra parka gidiyor. Akşam eve döndüğünde o gün toplam 520 metre yürüdüğünü hesaplıyor.

Buna göre, Emre'nin **evi** ile **okul** arasındaki mesafe ne kadardır? **(12p)**

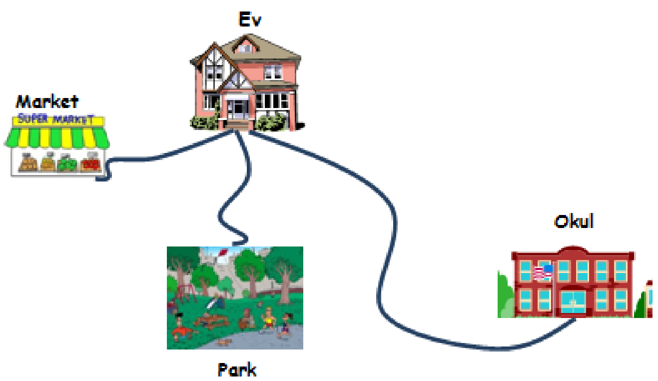


Figure A.17. Cebir Ön Test (cont.)

5) Aşağıda verilen problem için Hande, Nur ve Semih farklı denklemler yazmışlardır.

Doğru yazılan denklemi bularak çözünüz. (7p)

Problem: Bir sayının 3 katının 8 eksiğinin 1/5i, 2 ise bu sayı kaçtır?

Hande	Nur	Semih
$(3x-8)\div\frac{1}{5}=2$	$\frac{3x-8}{5}=2$	$3x-8\div\frac{1}{5}=2$

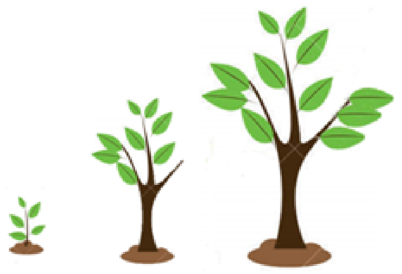
Doğru denklem:

Çözüm:

Figure A.17. Cebir Ön Test (cont.)

AKLIMDA KALANLAR – CEBİR ÖN TESTİ
ÇÖZÜMLER VE PUANLAMA CETVELİ

1)



Ayşe bahçelerindeki ağacın büyümesini haftalar içinde gözlemlemiştir ve ağaçtaki yaprak sayılarını gösteren yukarıdaki fotoğrafları çekmiştir. Buna göre aşağıdaki tabloyu doldurarak Ayşe'nin n. haftada gözlemleyeceği yaprak sayısını gösteren örüntü kuralını bulunuz. **(5p)**

Hafta sayısı	**Ağaçtaki yaprak sayısı**
1. hafta	5
2. hafta	8
3. hafta	11
4. hafta	**14 (1p)**
5. hafta	**17 (1p)**
⋮	⋮
n. hafta	**3n+2 (3p)**

Puanlama: Yukarıda gösterildiği şekilde olacaktır.

Figure A.18. Cebir Ön Test Cevap Anahtarı

2) Aşağıda verilen sözel veya cebirsel ifadelerin uygun karşılıklarını yazınız. **(8p)**

Sözel ifade	**Cebirsel ifade**
Elif'in 5 yıl sonraki yaşı	**e+5 (2p)**
Bir sayının 2 katının 3 eksiği	**2a-3 (2p)**
Bir sayının yarısının 8 fazlası (2p)	$\frac{x}{2}+8$
Elimdeki cevizlerin 15 fazlasının 3 katı	**3(c+15) (2p)**

Puanlama: Yukarıda gösterildiği şekilde olacaktır.

3) Mehmet elindeki 42 parçalık yapbozu kuzeni Ayla ile yapacaktır. Yapbozu tamamladıktan sonra, Ayla'nın koyduğu parça sayısının Mehmet'in koyduğu parça sayısının 3 katından 6 fazla olduğunu görmüşlerdir. Mehmet ve Ayla'nın yapboza kaç parça eklemiş olduklarını denklem kurarak bulunuz. **(8p)**

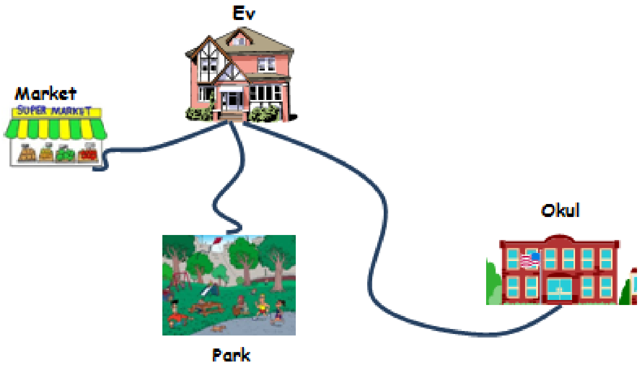
Çözüm	**Puanlama**
Mehmet: x Ayla: 3x+6 $3x+6+x=42$ $4x+6=42$ $4x=36$ $x=9$ Mehmet: 9 Ayla: 3·9+6=33	**8p:** Doğru denklemi kurup, doğru sonuca ulaşıp, istenilen değerleri bulduysa **7p:** Doğru denklemi kurup, doğru sonuca ulaşmış ancak Ayla'nın yapboz sayısını bulmamışsa **5p-6p:** Doğru denklemi kurmuş ancak çözümün sonunu getirememişse **5p:** Denklemi 3x+6=42 şeklinde kurup, doğru çözmüş, hem x değerini hem de Ayla'nın yapboz sayısını bulmuşsa **4p:** Denklemi 3x+6=42 şeklinde kurup, doğru çözmüş ancak Ayla'nın yapboz sayısını bulmamışsa **1p:** Sadece Ayla için 3x+6 yazmışsa veya sadece 3x+6=42 yazmışsa Önemsiz işlem hatasından **0.5p** kırılacaktır.

Figure A.18. Cebir Ön Test Cevap Anahtarı (cont.)

4) Emre'nin **evi** ile **okulu** arasındaki mesafe, **evi** ile **park** arasındaki mesafenin 4 katından 50 m daha fazladır. Emre'nin **evi** ile **market** arasındaki mesafe ise **ev-park** mesafesinden 30 m daha kısadır.

Emre bu sabah okula gidiyor, okuldan geldikten sonra annesi marketten ekmek almasını söylüyor. Emre marketten ekmek alıp eve bıraktıktan sonra parka gidiyor. Akşam eve döndüğünde o gün toplam 520 metre yürüdüğünü hesaplıyor.

Buna göre, Emre'nin **evi** ile **okul** arasındaki mesafe ne kadardır? **(12p)**



Çözüm

Ev-park: x

Ev-okul: 4x+50

Ev-market: x-30

$$(4x+50)+(4x+50)+(x-30)+(x-30)+x+x=520$$
$$12x+40=520$$
$$12x=480$$
$$x=40$$

Ev-okul: 4·40+50=210 m

Puanlama

12p: Doğru denklemi kurup, doğru sonuca ulaşıp, istenilen değeri bulduysa

8p: Denklemi kurarken mesafeleri sadece bir kez yazıp ona göre çözmüş ve istenilen değeri ona göre bulmuşsa

6p: Mesafeleri yanlış şekilde ifade etmiş ama doğru yöntemi izleyerek sonuca ulaşmışsa

2p-4p: Birkaç doğru cebirsel ifade var ancak çözüm yoksa

Önemsiz işlem hatasından **0.5p** kırılacaktır.

Figure A.18. Cebir Ön Test Cevap Anahtarı (cont.)

5) Aşağıda verilen problem için Hande, Nur ve Semih farklı denklemler yazmışlardır. **Doğru yazılan denklemi bularak çözünüz. (7p)**

Problem: Bir sayının 3 katının 8 eksiğinin 1/5i, 2 ise bu sayı kaçtır?

Hande	Nur	Semih
$(3x-8)\div\frac{1}{5}=2$	$\frac{3x-8}{5}=2$	$3x-8\div\frac{1}{5}=2$

Doğru denklem: **Nur**

Çözüm:

$$\frac{3x-8}{5}=2$$
$$3x-8=10$$
$$3x=18$$
$$x=6$$

Puanlama

7p: Doğru denklemi seçip denklemi doğru şekilde çözüp sonuca ulaşmışsa

4p: Yanlış denklemi seçip doğru bir şekilde çözüp sonuca ulaşmışsa

Veya

Denklemi bulmadan ters işlem yoluyla doğru sonucu bulmuşsa

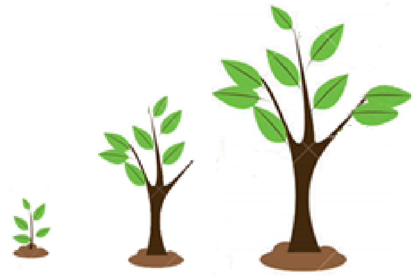
2p: Sadece denklemin hangisi olacağını bulmuş ama denklemi çözmemişse

1p-3p: Rastgele birkaç doğru işlem varsa

Figure A.18. Cebir Ön Test Cevap Anahtarı (cont.)

AKLIMDA KALANLAR – CEBİR SON TESTİ
ÇÖZÜMLER VE PUANLAMA CETVELİ

1)



1.hafta 2.hafta 3.hafta

Ayşe bahçelerindeki ağacın büyümesini haftalar içinde gözlemlemiştir ve ağaçtaki yaprak sayılarını gösteren yukarıdaki fotoğrafları çekmiştir. Buna göre aşağıdaki tabloyu doldurarak Ayşe'nin n. haftada gözlemleyeceği yaprak sayısını gösteren örüntü kuralını bulunuz. **(5p)**

Hafta sayısı	**Ağaçtaki yaprak sayısı**
1. hafta	5
2. hafta	7
3. hafta	9
4. hafta	
5. hafta	
⋮	⋮
n. hafta	

Figure A.19 Cebir Son Test

2) Aşağıda verilen sözel veya cebirsel ifadelerin uygun karşılıklarını yazınız. **(8p)**

Sözel ifade	**Cebirsel ifade**
Fulya'nın 6 yıl önceki yaşı	
Bir sayının 4 katının 10 eksiği	
	$\frac{x}{5}+2$
Elimdeki balonların 7 fazlasının 2 katı	

3) Mehmet, kuzeni Ayla ile kelime türetme oyunu oynamıştır. Oyunu tamamladıktan sonra, Ayla'nın türettiği kelime sayısının Mehmet'in türettiği kelime sayısının 4 katından 9 eksik olduğunu görmüşlerdir. Toplam türetilen kelime sayısı 21 ise, Mehmet ve Ayla'nın kaç kelime türetmiş olduklarını denklem kurarak bulunuz. **(8p)**

4) Emre'nin **evi** ile **okulu** arasındaki mesafe, **evi** ile **park** arasındaki mesafenin 3 katından 100 m daha fazladır. Emre'nin **evi** ile **market** arasındaki mesafe ise **ev-park** mesafesinden 40 m daha kısadır.

Emre bu sabah okula gidiyor, okuldan geldikten sonra annesi marketten ekmek

almasını söylüyor. Emre marketten ekmek alıp eve bıraktıktan sonra parka gidiyor.

Akşam eve döndüğünde o gün toplam 720 metre yürüdüğünü hesaplıyor.

Buna göre, Emre'nin **evi** ile **okul** arasındaki mesafe ne kadardır? **(12p)**

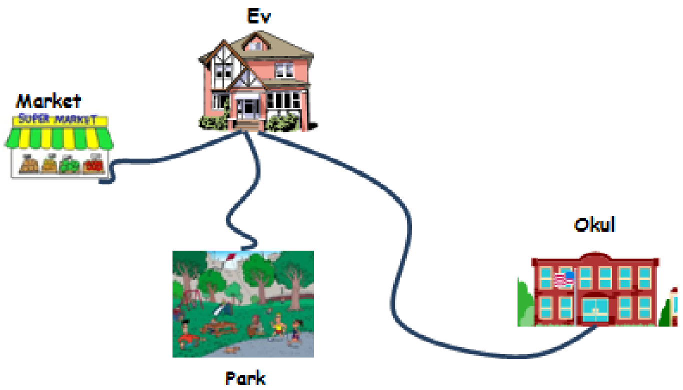


Figure A.19. Cebir Son Test (cont.)

5) Aşağıda verilen problem için Hande, Nur ve Semih farklı denklemler yazmışlardır.

Doğru yazılan denklemi bularak çözünüz. (7p)

Problem: Bir sayının 5 katının 4 fazlasının 1/3 ü, 8 ise bu sayı kaçtır?

Hande	Nur	Semih
$(5x+4)\div\frac{1}{3}=8$	$\frac{5x+4}{3}=8$	$5x+4\div\frac{1}{3}=8$

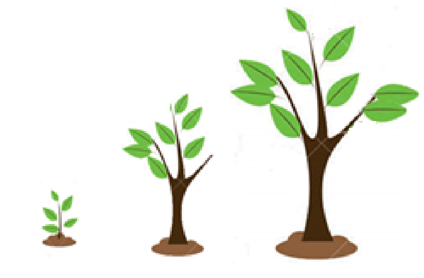
Doğru denklem:...

Çözüm:

Figure A.19. Cebir Son Test (cont.)

AKLIMDA KALANLAR – CEBİR SON TEST
ÇÖZÜMLER VE PUANLAMA CETVELİ

1)



Ayşe bahçelerindeki ağacın büyümesini haftalar içinde gözlemlemiştir ve ağaçtaki yaprak sayılarını gösteren yukarıdaki fotoğrafları çekmiştir. Buna göre aşağıdaki tabloyu doldurarak Ayşe'nin n. haftada gözlemleyeceği yaprak sayısını gösteren örüntü kuralını bulunuz. **(5p)**

Hafta sayısı	**Ağaçtaki yaprak sayısı**
1. hafta	5
2. hafta	7
3. hafta	9
4. hafta	**11 (1p)**
5. hafta	**13 (1p)**
⋮	⋮
n. hafta	**2n+3 (3p)**

Puanlama: Yukarıda gösterildiği şekilde olacaktır.

Figure A.20. Cebir Son Test Cevap Anahtarı

2) Aşağıda verilen sözel veya cebirsel ifadelerin uygun karşılıklarını yazınız. **(8p)**

Sözel ifade	**Cebirsel ifade**
Fulya'nın 6 yıl önceki yaşı	**f-6** (2p)
Bir sayının 4 katının 10 eksiği	**4a-10** (2p)
Bir sayının 1/5 nin 2 fazlası (2p)	$\frac{x}{5}+2$
Elimdeki balonların 7 fazlasının 2 katı	**2(b+7)** (2p)

Puanlama: Yukarıda gösterildiği şekilde olacaktır.

3) Mehmet, kuzeni Ayla ile kelime türetme oyunu oynamıştır. Oyunu tamamladıktan sonra, Ayla'nın türettiği kelime sayısının Mehmet'in türettiği kelime sayısının 4 katından 9 eksik olduğunu görmüşlerdir. Toplam türetilen kelime sayısı 21 ise, Mehmet ve Ayla'nın kaç kelime türetmiş olduklarını denklem kurarak bulunuz. **(8p)**

Çözüm

Mehmet: x

Ayla: 4x-9

$$4x-9+x=21$$
$$5x-9=21$$
$$5x=30$$
$$x=6$$

Mehmet: 6

Ayla:4·6-9=15

Puanlama

8p: Doğru denklemi kurup, doğru sonuca ulaşıp, istenilen değerleri bulduysa

7p: Doğru denklemi kurup, doğru sonuca ulaşmış ancak Ayla'nın kelime sayısını bulmamışsa

5p-6p: Doğru denklemi kurmuş ancak çözümün sonunu getirememişse

5p: Denklemi 4x-9=21 şeklinde kurup, doğru çözmüş, hem x değerini hem de Ayla'nın kelime sayısını bulmuşsa

4p: Denklemi 4x-9=21 şeklinde kurup, doğru çözmüş ancak Ayla'nın kelime sayısını bulmamışsa

1p: Sadece Ayla için 4x-9 yazmışsa veya sadece 4x-9=21 yazmışsa

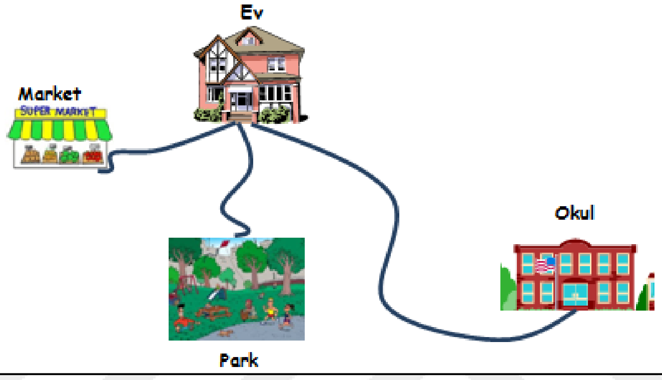
Önemsiz işlem hatasından **0.5p** kırılacaktır.

Figure A.20. Cebir Son Test Cevap Anahtarı (cont.)

4) Emre'nin **evi** ile **okulu** arasındaki mesafe, **evi** ile **park** arasındaki mesafenin 3 katından 100 m daha fazladır. Emre'nin **evi** ile **market** arasındaki mesafe ise **ev-park** mesafesinden 40 m daha kısadır.

Emre bu sabah okula gidiyor, okuldan geldikten sonra annesi marketten ekmek almasını söylüyor. Emre marketten ekmek alıp eve bıraktıktan sonra parka gidiyor. Akşam eve döndüğünde o gün toplam 720 metre yürüdüğünü hesaplıyor.

Buna göre, Emre'nin **evi** ile **okul** arasındaki mesafe ne kadardır? **(12p)**



Çözüm

Ev-park: x

Ev-okul: 3x+100

Ev-market: x-40

$$(3x+100)+(3x+100)+(x-40)+(x-40)+x+x=720$$
$$10x+120=720$$
$$10x=600$$
$$x=60$$

Ev-okul: $3\cdot 60+100=280$ m

Puanlama

12p: Doğru denklemi kurup, doğru sonuca ulaşıp, istenilen değeri bulduysa

8p: Denklemi kurarken mesafeleri sadece bir kez yazıp ona göre çözmüş ve istenilen değeri ona göre bulmuşsa

6p: Mesafeleri yanlış şekilde ifade etmiş ama doğru yöntemi izleyerek sonuca ulaşmışsa

2p-4p: Birkaç doğru cebirsel ifade var ancak çözüm yoksa

Önemsiz işlem hatasından **0.5p** kırılacaktır.

Figure A.20. Cebir Son Test Cevap Anahtarı (cont.)

APPENDIX E: EXPECTED MOST INSTANCES

Task 1: Lunapark

Expected MOST instances;

- Instance 1: Students might pay attention to the difference between the amount of rails over years rather than look for the relationship between the amount of hour and rails that were tracked.

  Student Mathematics: We can find the rule for the pattern by looking the difference between the amount of rails and thus the answer is n+5.

  Mathematical Point: When finding the rule of the pattern, it is needed to check the relationship between to variables rather than the change in one variable.

- Instance 2: Students might algebraically express the amount of invitation cards as 3(x-5) rather than 3x-5.

  Student Mathematics: When writing algebraic expressions follow the same order of operations as given in verbal expression.

  Mathematical Point: When converting verbal expressions to algebraic expressions, the order of operations is taken into consideration.

Task 2: Kentsel Dönüşüm

Expected MOST instances;

- Instance 1: Students might pay attention to the difference between the number of families that they move among years rather than look for the relationship between number of families and number of years.

  Student Mathematics: We can find the rule of the pattern by looking the difference between the number of families and thus the answer is n+4.

  Mathematical Point: When finding the rule of the pattern, it is needed to check the relationship between two variables rather than the change in one variable.

- Instance 2: Students might confuse about and unknown and a variable.

Student Mathematics: A variable has a unique numeric value.

Mathematical Point: In a one-variable equation a letter is used for an unknown value. Otherwise a letter may represent a varying value.

Task 3: Gezelim Görelim

Expected MOST instances;

- Instance 1: Students might insert the x variable in expressing the distance without paying attention to context.

Student Mathematics: In all of the algebraic expressions we can take unknown as x.
Mathematical Point: In one problem situation same unknown is represented with the same variable.

- Instance 2: Students might write "one third of a number" as x÷1/3 instead of x/3.

Student Mathematics: When finding the fraction of a fraction the division needs to be done.

Mathematical Point: When finding fraction of a fraction given numbers (fractions) are multiplied.

Task 4: Cebirsel Oyunlar

Expected MOST instances;

- Instance 1: Students might distribute the minus sign over parenthesis in a wrong way, for instance 4x+7-2x+3 in (4x+7)-(2x+3).

Student Mathematics: The minus sign in front of the parenthesis will affect only the first term of the parenthesis.

Mathematical Point: The minus sign in front of the parenthesis will affect each term of the parenthesis.

- Instance 2: Students might take the 2x+6 parentheses as 2(x+6).

Student Mathematics: Factorization of two terms can be done by dividing only first term with coefficient.

Mathematical Point: Factorization of two terms can be done by diving both terms with coefficient.

Task 5: Pazara Gidelim

Expected MOST instances;

- Instance 1: Students might have fail to solve the equations such as 2x+2=6 divide only one side with two and write x+1=6.

Student Mathematics: In solving equations we can divide only one side with a number and equation is still held.

Mathematical Point: In solving equations we need to do same operations to both sides to hold the equation.

Task 6: Sağlıklı Adımlar

Expected MOST instances;

- Instance 1: Students might write "one fifth of a number" as x÷1/5.

Student Mathematics: When taking fraction of a fraction the division needs to be done.

Mathematical Point: When finding fraction of a fraction given numbers (fractions) are multiplied.

Task 7: Karışık Meyve Suyu

Expected MOST instances;

- Instance 1:Students may fail to understand and determine the reference point since in some cases the juice depend on the previous amount and in some other cases depend on the initial point.

Student Mathematics: Converting the verbal expressions into algebraic expressions we can take initial amount as reference point.

Mathematical Point: Depend on the context the reference point needs to be determined.

- Instance 2: Dividing 4x+200 with two students might divide only 4x or 200 with 2 and find 2x+200 or 4x+100 rather than 2x+100 (in pear juice).

  Student Mathematics: When we are dividing binomials with a constant it is enough to divide one term with that number.

  Mathematical Point: Dividing binomials with a constant requires the division of both terms with the same constant.

  Task 8: Gizli Sayı

  Expected MOST instances;

- Instance 1: Students might pay attention to the difference between the amount of squares in consecutive shapes rather than look for the relationship between the amount of squares and the number of shapes.

  Student Mathematics: We can find the rule of pattern by looking the difference between the amount of squares and thus the answer is n+4.

  Mathematical Point: When finding the rule of the pattern, it is needed to check the relationship of the two variables rather than the change in one variable.

- Instance 2: While taking the 2/5 of 5x+10 students may divide only 5x or 10 with 5 rather than both of the terms.

  Student Mathematics: When we are dividing binomials with a constant it is enough to divide one term with that number.

  Mathematical Point: Dividing binomials with a constant requires the division of both terms with the same constant.