AN INVESTIGATION OF MATHEMATICS PRESERVICE TEACHERS' NOTICING SKILLS

by

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B.S., Program in Primary Mathematics Education, Boğaziçi University, 2016

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science

Graduate Program in Secondary School Science and Mathematics Education
Boğaziçi University
2018

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DATE OF APPROVAL: 16.10.2018

ACKNOWLEDGEMENTS

Firstly, I would like to say that dreams are coming through step by step. Despite all the difficulties I have encountered, the sense of accomplishment is so nice. I prayed so much to complete my thesis in time and right now, I am offering my thanks. Completing this thesis is like raising a little baby, which requires continuously patience and interest. All my family members always supported me so much in good times and bad times. I wish my grandfather were here to witness my success. I would like to specifically thank all my family, especially my mother, Gülsüm, my father, Ziya and my aunt Ilkay.

Secondly, my advisors, Assistant Professor Fatma Aslan-Tutak and Assistant Professor Hülya Kılıç spent so much time to read my thesis and made great effort to organize my thesis. Special thanks for all their efforts and patience. In other words, every step of my study is an indicator of their patience and efforts. Assistant Professor Hülya Kılıç and Assistant Professor Fatma Aslan-Tutak are one of the best academicians who I have ever met.

I would like to also thank my committee members, Assistant Professor Oğuzhan Doğan, Associate Professor Fatih Çağlayan Mercan and Assistant Professor Engin Ader. I appreciate all your willingness to share your expertise and time with me. I would like to thank Scientific and Technological Research Council of Turkey for their financial support during my research. I was an assistant of a TUBITAK Project titled as "A University-School Collaboration Model for Promoting Preservice Teachers' Pedagogical Content Knowledge about Students" (Grant no: 215K049) for one year. This project expanded my knowledge on the field, and I also want to thank Assistant Professor Hülya Kılıç and Assistant Professor Oğuzhan Doğan for their contributions within this project.

I would like also to thank my participants of this study. They are the sweetest participants which I have ever seen. My dear friends, Seyhan, Meryem, Nil, Elif, Sibel and Tuğba, supported me throughout my thesis with their suggestions and affection.

They were always sure that I would finish my thesis in time.

I also would like to thank all Boğaziçi University staff and members and this university is my second home. I feel very proud to be a Boğaziçi graduate.

ABSTRACT

AN INVESTIGATION OF MATHEMATICS PRESERVICE TEACHERS' NOTICING SKILLS

As student centered teaching approaches have become popular in recent years, teachers' noticing skills in terms of attending to students' mathematical thinking, interpreting their thinking and responding to them have become one of the prominent issues investigated by scholars. Studies revealed that preservice teachers' noticing skills can be improved through analysis of videos of their own teaching. In this study, the progress in preservice teachers' noticing skills through and after an intervention was investigated. Throughout the 2016-2017 academic year, two preservice mathematics teachers worked with 7^{th} grade students on the mathematical tasks about numbers, algebra, data analysis and geometry. Data were collected through videos of preservice teachers' interactions with students during task implementations, videos of their oral reflections just after implementations and their written reflections about implementations. The researcher identified mathematical opportunities (MOST) occurred during the implementations and these MOST instances were used as a medium to investigate in the noticing skills of preservice teachers. The attending, interpreting and responding level of preservice teachers during the implementation of mathematical tasks were analyzed according to coding scheme. The analysis of data revealed that preservice teachers attended to majority of the mathematical opportunities (approximately %84) occurred during task implementations. In addition to this, they interpreted possible reasoning behind students' thinking in their written reports. However, their responding for such opportunities were mostly in the form of orienting students for correct answers rather than eliciting their conceptual understanding. Therefore, video analysis and written reports might be effective tools to improve preservice teachers' noticing skills.

ÖZET

MATEMATİK ÖĞRETMEN ADAYLARININ FARK ETME BECERİLERİNİN İNCELENMESİ

Son yıllarda öğrenci merkezli eğitimin önem kazanmasıyla birlikte öğrenci merkezli eğitimin önem kazanmasıyla birlikte matematik eğitiminde öğretmenlerin sınıf ortamında öğrenci düşüncesinin farkında olması, öğrenci düşüncesini yorumlayabilmesi ve öğrenci düşüncesine dayanarak rehberlik etmesi ön plana çıkmaktadır. Yapılan çalışmalar matematik öğretmen adaylarının kendi öğretim videolarını analiz etmelerinin fark etme becerilerini geliştirdiğini göstermiştir. Bu çalışmada matematik öğretmen adaylarının fark etme becerilerinin nasıl bir değişim sergilediği incelenmiştir. 2016-2017 akademik yılı süresince, iki matematik öğretmen adayı 7. sınıf öğrencileri ile sayılar, cebir, veri analizi ve geometri içerikli matematik etkinlikleri üzerinde çalışmışlardır. Çalışmanın veri toplama araçlarını öğretmen adaylarının etkinliklerin uygulanması sırasındaki öğrencilerle etkileşimi gösteren videolar, etkinlik sonrası yapılan yansıtıcı toplantıların videoları ve uygulamalara ilişkin yazılan yanstıcı raporlar oluşturmaktadır. Araştırmacı etkinlik uygulamaları sırasında ortaya çıkan matematiksel öğrenme fırsatlarını (MÖF) belirlemiş ve bu fırsatlar öğretmen adaylarının fark etme becerisini incelemek için araç olarak kullanılmıştır. Öğretmen adaylarının dikkat etme, yorumlama ve karşılık verme seviyeleri kodlama şemasına göre analiz edilmiştir. Bulgular öğretmen adaylarının etkinlik uygulama esnasında oluşan matematiksel öğrenme fırsatlarının yaklaşık %84'ünü yakaladığını göstermiştir. Ayrıca, öğretmen adayları yazılı raporlarında öğrenci düşüncesini uygun gerekçe göstererek yorumlamaya çalışmışlardır. Öğretmen adayları, öğrencilerle etkileşim sırasında öğrencilerin matematiksel düşüncesini ortaya çıkarıcı bir yaklaşımdan ziyade onları doğru yanıta yönlendirici bir yaklaşım sergilemiş olsalar bile sonuç olarak, video analizi ve yazılı raporların öğretmen adaylarının fark etme becerisini geliştirebilmek için etkili birer araç oldukları görülmüştür.

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LIST OF ACRONYMS/ABBREVIATIONS

ATT	Attending
CCK	Common Content Knowledge
CK	Curricular Knowledge
INT	Interpreting
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
KQ	Knowledge Quartet Model
MKT	Mathematical Knowledge for Teaching Model
MOST	Mathematically Significant Pedagogical Opportunity to Build on Student Thinking
MS	Mathematically Significant
PCK	Pedagogical Content Knowledge
PO	Pedagogical Opportunity
RES	Responding
SCK	Specialized Content Knowledge
SMK	Subject Matter Knowledge
ST	Student Thinking

1. INTRODUCTION

Studies on teachers' pedagogical content knowledge revealed that it affects students' learning and understanding (Campbell *et al.*, 2014; Darling-Hammond, 2010). For effective education, teachers should arrange a student-centered learning environment for students and encourage them to be active in lessons (Sowder, 2007). Therefore, any attempt to improve teachers' knowledge and skills specifically their knowledge of students is likely to contribute to the quality of mathematics education (Hill, Ball and Schilling, 2008). Teachers' knowledge about students involves understanding students' conceptions and assumptions, using appropriate teaching strategies to teach content and knowing the needs and characteristics of students (Shulman, 1986; Hill *et al.*, 2008). At this point, teachers are required to notice noteworthy situations such as students' misconceptions during their instruction. Being aware of what is happening in a classroom, attending to students' strategies which creates opportunity to support their learning or to overcome their misconceptions and improving students' thinking via questioning are significant components of teachers' noticing (Jacobs, Lamb and Philipp, 2010; Sherin and Star, 2011). Indeed, teachers' pedagogical content knowledge and noticing skills are two interrelated concepts feeding into each other. Attending to significant instances in a classroom environment and making decision about how to respond to them emerge from their pedagogical content knowledge (Star and Strickland, 2008).

Many researchers attempted to examine the nature of teachers' noticing skills and tried to improve preservice teachers' noticing skills (i.e. Erickson *et al.*, 1986; Jacobs *et al.*, 2010; Sherin, Jacobs, and Philipp, 2009; van Es and Sherin, 2002). Van Es and Sherin (2002) defined the nature of noticing skills based on three characteristics which consist of "identifying what is important or noteworthy about classroom situation, making connections between the specifics of classroom interactions and broader principles of teaching and learning they represent, using what one knows about the context to reason about classroom interactions" (p. 573). As different from van Es and Sherin's definition of noticing, Jacobs and her colleagues (2010) defined professional noticing of students' thinking. They identified noticing as having three interrelated components which are attending to students' strategies, interpreting students' understanding and responding to students' understanding. They emphasized students' thinking component in identifying what is important or noteworthy about classroom situation. Besides these definitions, it was required to identify the attributes of students' thinking component which provide opportunity to students' further understanding. To fill this gap in the literature, Leatham and his colleagues (Leatham, Peterson, Stockero, and van Zoest, 2015) described instances of students' thinking which are noteworthy to attend in classroom situations called as Mathematically Significant Pedagogical Opportunity to Build on Student Thinking (MOST).

In addition to studies about nature of noticing skills, many studies examined improvement of preservice teachers' noticing skills (Kılıç, 2016; Sun and van Es, 2015; Stockero, Rupnow, and Pascoe, 2017). Studies related to noticing skills demonstrated that preservice teachers' and novice teachers' noticing skills are insufficient however, both pedagogical content knowledge and noticing skills of teachers can be improved via training programs (Barnhart and van Es, 2015; Jacobs *et al.*, 2010). To improve preservice teachers' noticing skills, many of the studies on noticing examined preservice teachers' oral or written reflections based on analysis of their own or others' teaching videos (Amador, Carter, and Hudson, 2016; Sherin and van Es, 2005; Star, Lynch and Perova, 2009). Results of these studies indicated that preservice teachers developed their noticing skills under favour of oral or written reflections. Moreover, they paid more attention to students' thinking or ideas and responded more appropriately to students' ideas. However, very few studies investigated preservice teachers' in the moment noticing skills such that they attended to important instances and gave appropriate responses during their instruction (Sherin and Star, 2011). Therefore, in this study, I investigated preservice teachers' noticing skills of students' mathematical thinking and also search for any progress in preservice teachers' in-the-moment noticing after the intervention study. I used Jacobs and her colleagues' (2010) definition of professional noticing of students' thinking to describe preservice teachers' noticing skills and used Leatham and his colleagues' (2015) description of MOST instances as a unit of analysis of preservice teachers' noticing.

2. LITERATURE REVIEW

2.1. Knowledge for Teaching Mathematics

Shulman (1986) investigated knowledge growth in teaching and the sources of teacher knowledge. The main goal of his study was to examine transformation of subject matter knowledge into knowledge for teaching. The research focused on the high school novice teachers' development in the field of English, biology, mathematics and social studies. In 1986, Shulman's suggested model for teacher knowledge has become known when considering other models for teachers' knowledge (Aslan-Tutak, 2009).

2.1.1. Shulman's model for teachers' knowledge

According to Shulman (1986), content knowledge of teachers consists of three different categories which are subject matter knowledge, pedagogical content knowledge and curricular knowledge.

Subject matter knowledge (SMK): This refers to "the amount and organization of knowledge per se in the mind of the teacher" (p. 9). Not only knowledge of facts or concepts of a given domain but also understanding the structures of subject matter is required for teachers. Similarly, Shulman emphasized the significance of subject matter knowledge by giving explanations and definitions for students (Aslan-Tutak, 2009).

Pedagogical content knowledge (PCK): It is defined as a special kind of knowledge of teachers which emphasizes content knowledge for teaching comprising most appropriate forms of representation of those ideas, the most powerful analogies, illustrations, examples and explanations by providing comprehensible ways to others. It also includes an understanding of what makes learning of specific topics easy or difficult and also understanding of conceptions and preconceptions that students of different ages and backgrounds bring with them to learning of those most frequently taught topics and lessons (Shulman, 1986). Among the categories of content knowledge, pedagogical content knowledge has a tendency to distinguish teachers from scientists (Aslan-Tutak, 2009).

Curricular knowledge (CK): Shulman mentioned three aspects of content knowledge related to curricula including knowledge of alternative curriculum materials, lateral curriculum knowledge and vertical curriculum knowledge. He noted that "knowledge of alternative curriculum materials is required for teachers to understand materials such as alternative texts, software programs, visual materials, laboratory demonstrations" (p. 10). Likewise, lateral curriculum knowledge is defined as teachers' ability to establish connection between content of other subjects. Also, vertical curriculum knowledge is defined as teachers' familiarity to subject and materials related to this subject throughout the following years.

Based on this model, knowledge base of teaching, namely pedagogical content knowledge was situated in intersection of content and pedagogy. In addition, knowledge base of teaching was allowed to transformation of subject matter knowledge into structures being pedagogically powerful (Shulman, 1987). Figure 2.1 demonstrates the categories of Shulman's model for teachers' knowledge (Aslan-Tutak, 2009).

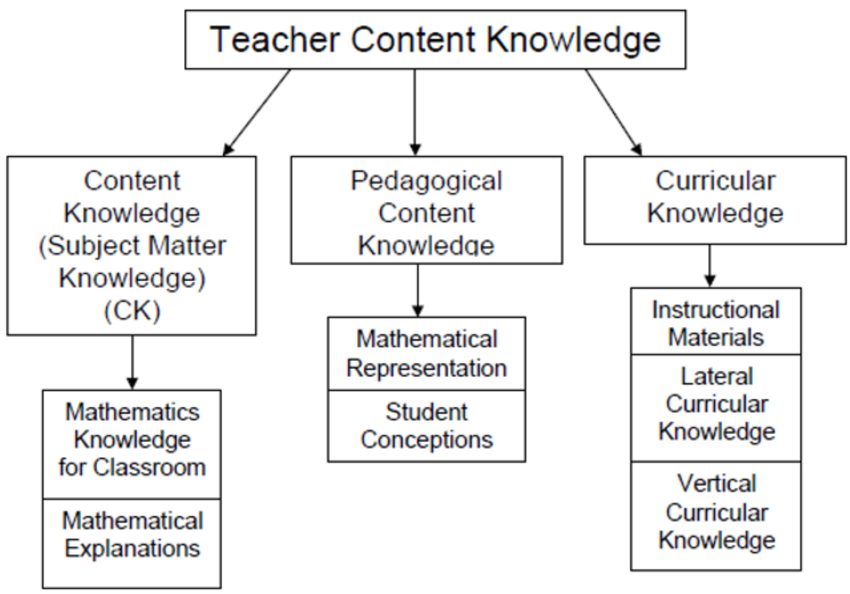


Figure 2.1. Shulman's model for teachers' knowledge.

2.1.2. Mathematical knowledge for teaching model

Ball, Thames and Phelps (2008) tried to develop practice-based theory related to content knowledge for teaching by building on Shulman's knowledge for teaching model. The main goal of the study was to investigate the nature of content knowledge needed for teaching by taking into consideration context of mathematics. Thus, Ball *et al.* (2008) developed Mathematical Knowledge for Teaching (MKT) Model. Mathematical knowledge for teaching is defined as "mathematical knowledge needed to carry out the work of teaching mathematics" (p. 395). Based on Shulman's knowledge types, the domains of teacher knowledge were improved in this model (Aslan-Tutak, 2009). There are six domains in MKT model and three of them correspond to Shulman's subject matter knowledge whereas other three categories correspond to Shulman's pedagogical content knowledge. Figure 2.2 shows the domains of Mathematical Knowledge for Teaching proposed by Ball and her colleagues (Ball *et al.*, 2008).

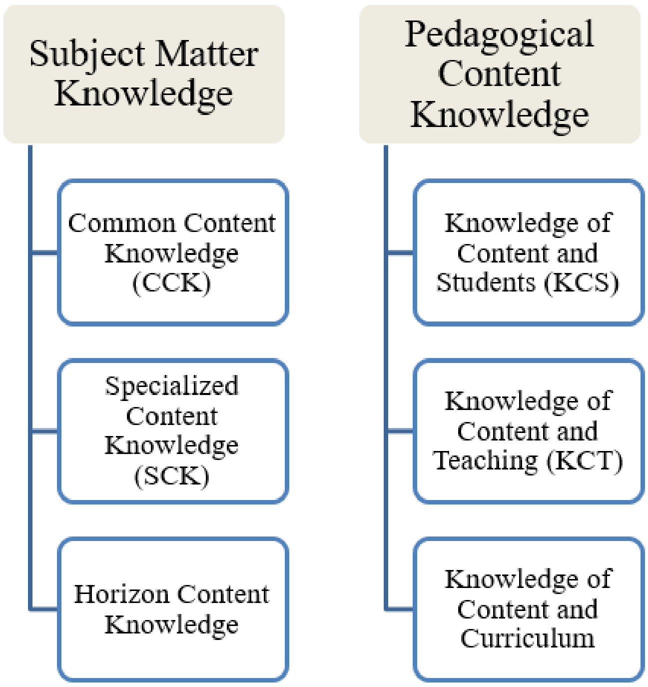


Figure 2.2. Domains of mathematical knowledge for teaching (Ball *et al.*, 2008).

Ball and her colleagues (Ball *et al.*, 2008) hypothesized that Shulman's subject matter knowledge could be subdivided into Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Horizon Content Knowledge whereas his pedagogical content knowledge could be subdivided into Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), Knowledge of Content and Curriculum (KCC).

The first domain, Common Content Knowledge (CCK) is defined as mathematical knowledge or skill which is not specific to teaching but should be possessed by all mathematics teachers. For instance, if a teacher can simply calculate the division of two fractions by using the invert and multiply rule, it means that the teacher uses common content knowledge. The second domain, namely, Specialized Content Knowledge (SCK) refers to mathematical knowledge that is special and necessary for teaching mathematics. If a teacher possesses knowledge about the reason behind using the invert and multiply rule in order to divide two fractions, it means that the teacher has specialized content knowledge. One of the significant characteristics of the SCK, which differentiates it from the PCK, is that it does not refer to knowledge of students or teaching (Morris, Hiebert and Spitzer, 2009). The third domain under subject matter knowledge in the MKT model is Horizon Content Knowledge. Ball and her colleagues (2008) defined horizon content knowledge as "an awareness of how mathematical topics" (p. 403). Having knowledge about how fractions are used in higher grades can be an example for horizon content knowledge. Horizon content knowledge corresponds to Shulman's vertical curriculum knowledge since it examines knowledge of teachers related to mathematics topics through the following years (Aslan-Tutak, 2009).

As for pedagogical content knowledge component, Knowledge of Content and Students (KCS) is defined as "knowledge that combines knowing about students and knowing about mathematics" (Ball *et al.*, 2008, p. 401). In this domain, teachers should predict students' mathematical thinking and interpret this thinking in their minds. Students' common conceptions and misconceptions about spesific mathematical content are central points for KCS. In Shulman' model for teachers' knowledge, student representation and conceptions play significant role under the term pedagogical content knowledge. In the MKT model, KCS demonstrates aspects of students' conceptions in Shulman's PCK. For example, recognizing students' confusion about addition of fractions such that adding up numerators and denominators of given fractions directly without finding the least common multiple of denominators is is a sign for teachers' KCS. In addition to this, teachers can easily interpret this misconception resulting from confusion about fractions and integers (Bingölbali and Özmantar, 2009).

Knowledge of Content and Teaching (KCT) is another domain under pedagogical content knowledge that is defined as the knowledge bringing together knowing about teaching and knowing about mathematics (Ball *et al.*, 2008). Teacher uses KCT to decide which examples or representations are appropriate to understand a particular mathematical content. For instance, KCT refers to making decision about which models can be useful to demonstrate fraction division concept to prevent students' difficulty about invert and multiply algorithm. The domain of KCT displays the aspect of using appropriate representations in Shulman's PCK model (Aslan-Tutak, 2009).

The last domain in the MKT model is Knowledge of Content and Curriculum (KCC). It refers to "how teachers use a given curriculum to support student learning of different mathematical topics" (Kieboom, 2013). By taking into consideration curriculum knowledge in Shulman's model, this knowledge is different from PCK and SMK. Furthermore, the MKT model situates knowledge of content and curriculum under PCK whereas horizon content knowledge is situated under SMK.

2.1.3. Other Models in Pedagogical Content Knowledge for Mathematics

Many researchers attempted to elaborate on definitions of pedagogical content knowledge (Depaepe *et al.*, 2013). To investigate pedagogical content knowledge of teachers, Marks (1990) presented a different approach to PCK in mathematics based on the results from interviews with eight 5th grade teachers. According to this approach, structure of PCK consists of four categories: subject matter for instructional purposes, students' understanding of the subject matter, media for instruction in the subject matter, and instructional processes for the subject matter. Those categories are intertwined to each other. For instance, "criticizing a textbook for its lack of examples of a specific mathematical process represents knowledge of both media and subject matter and to some extent students' understanding" (Marks, 1990). Figure 2.3 demonstrates structure for pedagogical content knowledge in 5th grade equivalance of fractions suggested by Marks (1990).

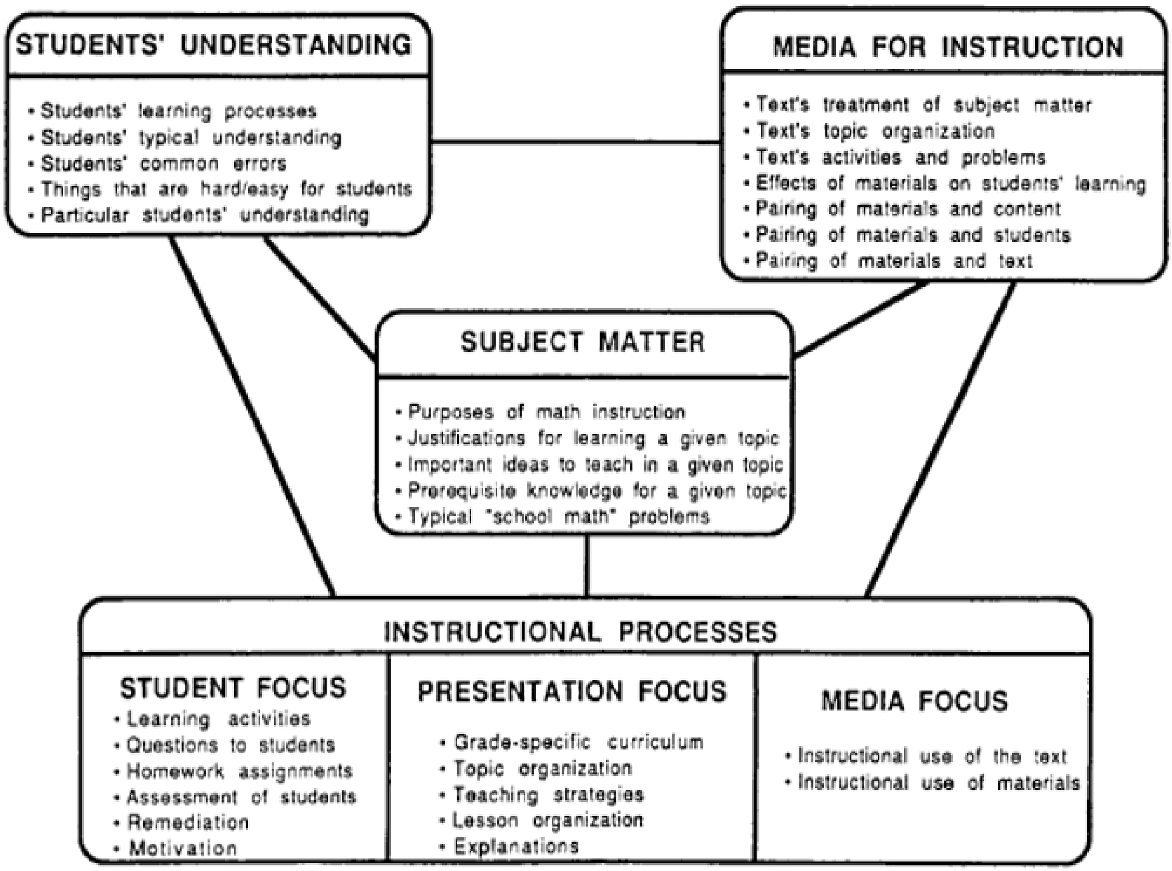


Figure 2.3. Structure for pedagogical content knowledge in 5th grade equilavance of fractions (Marks, 1990).

Marks' study (1990) focused on teachers' knowledge of students' understanding of subject matter. This study examined examples from interviews with mathematics teachers related to teachers' PCK focusing on how students understand mathematics content. In SMK, the umbrella term, students' understanding of the subject matter, had five categories: students' learning process, students' typical understanding, students' common errors, things that are hard or easy for students, particular students' understanding. For the first category, students' learning process, teachers expressed that understanding concepts rather than memorizing rule or facts and experiences with concrete materials were two significant requirements. When it comes to examples of students' understanding and students' common errors, teachers noted that "students confuse multiplying two fractions diagonally with multiplying straight across and they try to find common denominator for multiplication" (Marks, 1990, p. 6). Under the category of things that are easy or hard for students, most of the teachers revealed that many students had difficulty to understand several topics and tasks such as fractions and percent. Similarly, the teachers explained that students failed to visualize fractions and so they had a tendency to memorize rules rather than understanding the concept. As for particular students' understanding, teachers stated that students did not make an effort to understand problem situation and they just tried to apply formulas to problem situation. In the light of those instances given by teachers, this study elaborated on aspects of PCK based on teachers' views.

In addition to models related to teachers' knowledge, Rowland and his colleagues (Rowland, Huckstep and Thwaites, 2005) developed Knowledge Quartet (KQ) model consisting of four categories. These categories were foundation, transformation, connection and contingency. For the purpose of generating model, six lower primary and six upper primary preservice teachers were observed, and their 24 lessons were videotaped. Foundation refers to the knowledge of teachers gained in preservice teacher education program and "personal" education. It is originated from trainees' theoretical background and beliefs about descriptions of mathematical concepts and connections between them. The second category called as transformation is rooted in Shulman's pedagogical content knowledge. Unlike the first category, it "picks out behavior that is directed towards a pupil and which follows from deliberation and judgement informed by foundation knowledge (Rowland *et al.*, 2005). According to this model, teacher uses transformation knowledge to overcome students' misconceptions by using appropriate representations. The following category, connection, includes ordering the math topics, activities and exercises within and between instructions. Lastly, contingency is about classroom events which may not be foreseen in the plan of instruction. It is the "knowledge in interaction as revealed by the ability of the teacher to "think on her feet" and respond appropriately to the contributions made by her students during a teaching episode" (Rowland *et al.*, 2005).

Although these four models provide different approaches to teacher knowledge, they all share student component in PCK. In Shulman's model, knowledge about conceptions and preconceptions of students has significant place in PCK. Knowledge of content and students in MKT model (Ball *et al.*, 2008) contains teachers' knowledge related to students' conceptions and misconceptions. To overcome students' misconceptions, teacher uses knowledge of content and teaching via appropriate representations, illustrations and explanations. As for Marks' study (1990), teachers' knowledge about students' understanding of subject matter is similar to knowledge of content and student's domain under Ball's *et al.* (2008) model whereas it corresponds to Shulman's PCK. Furthermore, in KQ model, "transformation" component is originated from Shulman's PCK that contains mathematical representations and student conceptions. Additionally, teachers' knowledge about students' understanding of the subject matter in Mark's model (1990) can be associated to "transformation" component in KQ model.

In terms of responding appropriately to students during instruction, contingency component in KQ model is similar to KCT in MKT model. Unlike MKT model, KQ model is based on classroom practice and it can be improved according to classroom situations (Köklü, 2008). Even though there is no consensus on name or details of the model for PCK, it can be clearly seen that teachers being aware of students' mathematical thinking and using students' mathematical thinking in instruction are vital components of PCK.

Besides to four different teachers' knowledge models, several researchers share the idea that teachers should have strong pedagogical content knowledge for effective mathematics education (Ball *et al.*, 2008). However, the findings of many studies revealed that preservice and novice teachers' pedagogical content knowledge is weak (Morris *et al.*, 2009; Toluk-Uçar, 2011). Several studies were conducted to investigate the nature of preservice teachers' PCK as well as to support development of PCK, specifically their knowledge of content and students (Gökkurt, Şahin, Soylu, and Soylu, 2013; Hill *et al.*, 2008; Jenkins, 2010; Tanışlı, 2013). Furthermore, teachers' noticing of students' mathematical thinking is getting attention of researchers since it is thought to be related to teachers' PCK (McDuffie *et al.*, 2014). Therefore, studies on preservice teachers' PCK and noticing skills nourishes each other such that any intervention to one of these constructs is likely to have an influence on the other (Amador *et al.*, 2017).

2.2. Noticing

2.2.1. Definition of noticing

As a term in everyday language, noticing refers to observing or recognizing something (Sherin, Jacobs, and Philipp, 2011). Based on it is general meaning, scholars attributed to some characteristics to noticing so that it has become one of the constructs that has been studied in teacher education in recent years. Although there are different views about what involves in noticing (Mason, 2011), two main characteristics are common in definitions: attending to particular events and making sense of them (Sherin *et al.*, 2011). In other words, noticing refers to paying attention to noteworthy classroom events and then interpreting them based on teachers' own knowledge and previous experiences.

The earlier studies of noticing that investigated students' interests and thinking process began with observing students while they were working on a given task such as block construction, drawing, speaking and writing (Biber, 1987; Johnson, 1933; Pratt, 1948; Stern and Cohen, 1958).According to Dewey (1904), there are two types of children's attention behaviors that teacher can notice via observations: inner attention and outer attention. Outer attention was described as "surface appearance of attending such as sitting up straight, looking where the child was supposed to be looking, sitting still and not talking to one's neighbor" (Erickson, 2011). Inner attention was described as "genuine interest of the child, which might or might not be displayed to the teacher in the child's behavior" (Erickson, 2011). It is important to discriminate inner attention from outer attention and novice teachers have a tendency to confuse students' inner attention and outer attention based on their noticing skills (Scheffler, 2014). Based on these earlier studies, Erickson and his colleagues (Erickson *et al.*, 1986) conducted intensive observational study about what early grade teachers paid attention during their teaching. They found that noticing was highly specific in the activity involving attending to some phenomena and inattentive to others. It had multiple dimensions such as attending to subject matter, behavior and other objects of attention. In addition, experience of teachers and their noticing skills were related to each other such that experienced teachers noticed more instances in a classroom with respect to novice teachers.

Van Es and Sherin (2002) conducted a study to investigate the nature of teachers' noticing. They defined three significant characteristics of noticing as: "a) identifying what is important or noteworthy about classroom situation, b) making connections between the specific of classroom interactions and broader principles of teaching and learning they represent and c) using what one knows about the context to reason about classroom interactions" (p. 573). The first characteristics of noticing is learning to identify what is significant in particular events. During an instruction, there are many aspects which teachers are required to attend such as students' actions and students' thinking about the subject matter. Naturally, it is impossible for a teacher to attend many different aspects that occur in a class. Therefore, teachers must select the instances to attend and respond during instruction. The second characteristic of noticing is to relate specific instances with the broader principles of teaching and learning. As an example, novice teachers have a tendency to describe events literally whereas expert teachers describe events by considering teaching and learning issues such as paying attention to students' ideas. The third aspect of noticing refers to "using what one knows about the context to reason about situations" (van Es and Sherin, 2002). For instance, physics teachers are likely to interpret students' thinking about buoyancy better than any biology teacher.

Although noticing is defined as a "process that teachers initially see, or perceive, different aspects of classroom activity (Sherin *et al.*, 2011, p. 80) some scholars put much more emphasis on noticing students' thinking rather than noticing different instances that occur in a classroom environment. For instance, Jacobs and her colleagues (2010) defined professional noticing of students' thinking as a composition of three interrelated components as attending to children's strategies, interpreting children's understanding and deciding how to respond on the basis of children's understanding. They noted that attending to students' verbal and written strategies is the first step for eliciting students' understanding. Once they pay attention to the students' strategies, teachers need to interpret these strategies by taking into consideration the evidence about students' thinking and understanding. Finally, teachers should decide how to respond to students in order to support their understanding and remedy their deficiencies.

In the recent studies on teachers' noticing, many scholars prefer to use Jacobs and her colleagues' definition of professional noticing of students' thinking (e.g., Barnhart and van Es, 2015; Simpson and Haltiwanger, 2016; Stockero *et al.*, 2017; Sun and van Es, 2015). Among the other reasons, scholars may prefer to use this definition because it gives a clearer idea about what is noteworthy in a classroom setting, namely students' thinking, and also it provides a flow of noticing that enables to analyze teachers' noticing skills (e.g., Simpson and Haltiwanger, 2016; Stockero *et al.*, 2017). Therefore, in this study, I used Jacobs and her colleagues' (2010) definition for noticing to describe preservice teachers' noticing skills.

2.2.2. Improving teachers' noticing skills

In many studies, preservice teachers' noticing skills were analyzed in terms of their oral or written reflections about the videos of their own or others' teaching. Among them, Barnhart and van Es (2015) examined the relationships in preservice science teachers' ability to attend, analyze and respond to students' thinking. In other words, they investigated the connection between components of preservice teachers' professional noticing of students' thinking. The participants of this study were two cohorts of secondary science teachers. One cohort was determined as intervention group and the other one was as a control group. In the former group, with a video-based course, the preservice teachers were asked to teach in a class and videotape their instruction and then write a reflection about their instruction. When their reflections were analyzed it was found that preservice teachers' highly sophisticated analyses depended on highly sophisticated attention to student ideas and highly sophisticated responses depended on highly sophisticated analyses of student ideas. Barnhart and van Es (2015) also noted that high level of attention to students' ideas was not always followed by a high level of analyzing or responding to students' ideas. However, they concluded that preservice teachers who enrolled in a video-based course demonstrated higher sophistication in attending, analyzing and responding to student thinking. Similarly, findings of the study by Sun and van Es (2015) showed that the use of video analyses provided an opportunity for preservice teachers to see noteworthy events in a classroom and interpret those events from different perspectives. Another significant result of their study was that preservice teachers who enrolled in a video-based course enacted more teaching practices than preservice teachers who did not. These teaching practices were composed of making space for students to think, to attend and to take up their' ideas, to pursue their thinking. Besides, it was found that preservice teachers focused on students' answers and procedures rather than focusing on reasoning behind those answers and students' conceptual understanding. In a similar manner, Simpson and Haltiwanger (2016) examined the ways in which secondary mathematics prospective teachers interpret and respond to students' mathematical thinking. They found that prospective teachers did not consider students' misconceptions or misunderstandings when interpreting students' mathematical thinking. As different from video-based courses, Krupa et al. (2017) designed a curricular module including pre-post interviews, class discussions and tasks related to equations to develop preservice teachers' noticing skills. Results of this study revealed that preservice teachers made progress in attending and interpreting aspects of students' thinking while they did not make progress in responding to students' thinking.

Likewise, Sherin and van Es (2005) found that in-service and preservice teachers' noticing skills have changed under favour of reflecting their own teaching practices. Many studies demonstrated that preservice teachers have difficulty to notice features of classroom instruction such as students' thinking and ideas. To overcome this issue, they suggested that teacher education programs should provide opportunities for preservice teachers to develop their noticing skills. In line with this recommendation Star, Lynch and Perova (2009) conducted a study to investigate the effects of using video in methods course to improve preservice teachers' noticing skills. They analyzed preservice teachers' noticing under five different categories: classroom environment, classroom management, tasks, mathematical content and communication. The findings revealed that preservice teachers' ability of noticing related to classroom environment, classroom management and communication improved. At the end of the methods course, preservice teachers noticed more features of the classroom environment such as the number of the students in classroom and noticed more about issues related to classroom communication category such as how the teacher was asking questions and responding to students' questions. In a similar vein, Males (2017) conducted a study to examine preservice teachers' noticing in method courses during two semesters by using videos of peer teaching. The findings revealed that preservice teachers mostly focused on teachers' talk and actions in the first semester while their focusing shifted to students' talk and actions in the second semester. The results of this study indicated that they mostly attended to communication, mathematics content and classroom management while attending less to classroom environment and tasks across two semesters.

Similar to Star and his colleagues' (Star *et al.*, 2009) study, van Es (2011) examined the development of teachers' skills to notice student thinking while teachers participated in a video club. The participants of this study were 7 fourth and fifth grade elementary school teachers. Data of the study were collected through videotapes and transcripts of 10 video-club meetings. She noted that video club contributed to development of teachers' noticing skills because it provided an opportunity for participants to discuss instances about student thinking and teacher practices that foster student thinking. She developed a coding scheme to analyze what teachers notice and how teachers notice. For what teachers notice category, she defined four levels of noticing; baseline, mixed, focused and extended. In baseline noticing, the participants focused on several issues such as whole class behavior, participation, classroom climate. They interpreted their observations as "I like how you set up the problem and the class is engaged". In mixed noticing, the focus shifted from noticing of whole class perspective to the particular students in the video clip. In focused noticing, the participants primarily focused on students' mathematical thinking rather than whole class perspective. Extended noticing included both noticing of students' mathematical thinking and teachers' pedagogy. For how teachers notice category, as in parallel to levels of what teachers notice, at level 1, participants wrote about general impressions like "that was a nice lesson. At level 2, the participants gave general impressions and highlighted significant events. At level 3, the participants' comments were highly discriminative and identified primarily significant events in the video clips. At level 4, the participants suggested alternative pedagogical solutions. Moreover, they made connections between particular events that they noticed and broader principles of teaching and learning.

Based on van Es's (2011) framework consisting of baseline, mixed, focused and extended noticing, Amador, Carter and Hudson (2016) aimed to determine what preservice teachers notice during observations of mathematics lessons. Twenty-four preservice teachers participated in their study and data was collected through video recordings of the six lesson study meetings and 24 video transcripts. The results of this study revealed that most of the preservice teachers focused on general, descriptive impressions of the lesson. In other words, they had a tendency to demonstrate baseline framework and most of the participants did not show extended noticing. The preservice teachers also neither produced pedagogical solutions to students' thinking, nor made any connection between specific events and principles of learning. In a similar way, a study conducted by Akyüz and Güner (2017) investigated prospective mathematics teachers' noticing of students' mathematical thinking by implementing lesson study professional development model. In this study, the framework by van Es (2011) was employed to make sense of their noticing skills. The findings revealed that prospective mathematics teachers mostly focused on teachers' behaviors, instructional materials and classroom environment rather than students' mathematical thinking throughout lesson study process.

On the other hand, a study by Taylan (2015) investigated experienced third grade teacher's noticing skills in her classroom via video recordings while she was teaching multiplication and division of numbers. The study indicated that the experienced teacher mostly focused on the students' thinking in the classroom environment rather than the classroom norms or the student characteristics. In addition to this, she made connections between students' thinking and broader principles of teaching and learning in video clips. However, she did not suggest any alternative pedagogical solution in this study because she mostly focused on students' understanding instead of focusing on students' misunderstandings. In a similar manner, Baş (2013) examined evolution of the teachers' noticing of students' mathematical thinking throughout a professional development program build on principles of modeling perspective. Four experienced secondary mathematics teachers were participated in this study and the framework developed by van Es (2011) was used in this study. It was found that teachers' attending level was baseline level at the beginning of intervention. They commented about students' responses as they were without interpreting them specifically. Within time, the teachers' level of attention and reasoning shifted to mixed and focused level. Thus, the results of this study demonstrated that noticing skills of experienced teachers developed over time thanks to the professional development program.

2.3. Mathematically Significant Pedagogical Opportunities

Leatham, Peterson, Stockero and van Zoest (2015) mentioned that it is not possible for teachers to attend to each situation or event occurring in a classroom. Similarly, it is difficult to achieve a consensus on what is noteworthy to attend. To address such difficulty Leatham and his colleagues (2015) attempted to identify the instances about students' thinking that are noteworthy to attend as Mathematically Significant Pedagogical Opportunity to Build on Student Thinking (MOST). They defined MOST instances as intersection of three characteristics:

(i) Student mathematical thinking,
(ii) Mathematically significant,
(iii) Pedagogical opportunity.

For each characteristic they defined two criteria that should be satisfied at the same time. They defined student mathematical thinking as the composition of student mathematics and mathematical point. Student mathematics refers to sufficient verbal or written evidence of what students say or do. For example, a student may add -8 and 10 as follows: and then he explains that he gets -18 when he adds -8 by 10. Based on this verbal evidence as well as students' written work, we may do following inference about student mathematics: Addition or subtraction operation with negative numbers is similar to addition or subtraction with positive numbers such that negative sign can be added in front of the result at the end. The second criterion was that "one can articulate a mathematical idea that is closely related to the student mathematics of the instance-what we call a mathematical point" (p. 92). In above example, mathematical point can be expressed as when adding a negative number with positive one, based on their absolute values, subtract smaller from the larger and insert the larger number's sign to the result. For any instance to be mathematically significant, appropriate mathematics and central mathematics are two requirements to be justified. The first criterion for being mathematically significant is that mathematical point should be appropriate for students' mathematical development level and the second criterion is mathematical point should be aligned with central mathematical goal in the instruction. In terms of this instance, addition and subtraction of integers is suitable for the 7^{th} grade mathematical development level and mathematical point is aligned with the 7th grade mathematical objective that any student should add or subtract positive and negative integers (Ministry of National Education [MoNE], 2013). As for pedagogical opportunity characteristics of MOST instance, opening and timing are the two key criteria. To meet the opening criteria, student mathematics should provide an opportunity to comprehend mathematically significant point for particular students as well as other students in the class. Timing is defined as the "right time to take advantage of the opening" (p. 102). The following Figure 2.4 demonstrates the MOST analytic framework through three different characteristics and their relevant criteria proposed by Leatham and his colleagues (Leatham *et al.*, 2015).

Leatham and his colleagues' (Leatham *et al.*, 2015) description of MOST provides more tangible descriptions for Jacobs and her colleagues' (Jacobs *et al.*, 2010) first two steps of noticing: what is noteworthy to attend and how to interpret that instance.

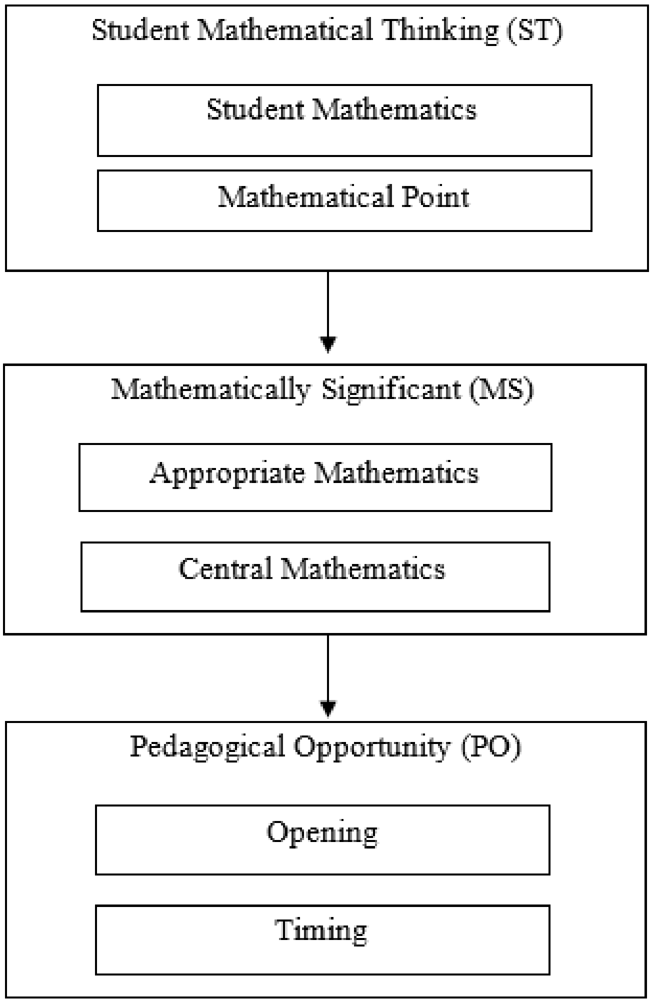


Figure 2.4. The MOST analytic framework-summarized version (Leatham *et al.*, 2015).

Indeed, Stockero and her colleagues (Stockero *et al.*, 2017) investigated improvement of preservice teachers' noticing skills in terms of their attention to MOST instances. They asked preservice teachers to videotape experienced teachers' lessons and then analyze those lessons. The participants of this study consisted of 17 prospective mathematics teachers enrolled in an early field experience course. The data were collected through preservice teachers' individual video analyses, which focused on the instances being identified and their corresponding explanations. Three coding categories were determined as agent- what the object of the noticing was, specificity of mathematics whether and how the mathematics of the instance is discussed-, focus of noticing -what students attended to. In terms of agent category results, intervention was successful in terms of focusing individual students' mathematical thinking. The preservice teachers attended more students' thinking rather than focusing on teacher and mathematics itself in the videos. In terms of specificity of mathematics results, non-mathematical noticing was disappeared at the end of the intervention and preservice teachers had a tendency to discuss more specific mathematics throughout the intervention. According to the results, teachers' noticing shifted from classroom interactions to noting and analyzing students' thinking. After intervention, the preservice teachers were able to notice and interpret MOST situations better. In addition, the study of Pascoe (2016) examined mathematics teaching assistants' noticing of MOST instances as a result of professional development intervention that included analyzing classroom videos with MOST framework. The results of this study revealed that attending level of participants to MOST instances developed over time and interpreting of students' understanding progressed throughout the intervention. Additionally, the mathematics teachers assistants developed responding aspect of their noticing skills such that they had a tendency to propose student-centered responses to MOST instances at the end of the intervention.

Briefly, a MOST is noteworthy instance to attend and can be used to interpret students mathematical understanding in a classroom setting. The use of video analyses supported preservice teachers noticing skills and also improved their attention and interpretation of MOST situations. Considering that PCK and noticing skills of preservice teachers are interrelated and feed each other, improvement of the preservice teachers noticing skills contributes to development of the preservice teachers PCK. Therefore, I used MOST instances as a medium to analyze preservice teachers noticing skills in this study.

3. SIGNIFICANCE OF THE STUDY

Although teacher training programmes in universities aims to improve preservice teachers' knowledge and skills, several studies conducted with preservice and novice teachers demonstrated that they have insufficient mathematics content knowledge and pedagogical content knowledge (Kılıç, 2011; Morris *et al.*, 2009; Tanışlı, 2013; Toluk-Uçar, 2011). Because teachers' knowledge and skills influence the quality of education (Knight *et al.*, 2015), preservice teachers should be given opportunity to improve their pedagogical content knowledge in teacher education programs. Scholars attempted to elicit and support preservice teachers' pedagogical content knowledge by making them to teach in a classroom setting or analyze students' work or reflect on videos of their own or others' teaching. Thus, preservice teachers can learn about students' possible difficulties and misconceptions, and get experience of how a teacher's instructional actions influence students' learning and understanding (e.g., Barnhart and van Es, 2015; Star and Strickland, 2008). In Turkey, there has been some research on evaluation of preservice teachers' current level of PCK (e.g., Akkoç and Yeşildere, 2010; Baştürk and Dönmez, 2011). However, the number of studies aiming to improve preservice teachers' pedagogical content knowledge is limited (Işıksal and Çakıroğlu, 2008; Tanışlı, 2013). In general perspective, this study contributed to the literature by examining two preservice teachers' noticing skills in the context of faculty-school collaboration model which is designed to support preservice mathematics teachers' pedagogical content knowledge.

In the recent years, student-centered teaching methods have become a focus of our education system such that the school curricula were reformed to address premises of student-centered teaching approach (MoNE, 2013). In a student-centered environment, teachers are expected to be aware of students' mathematical thinking and to use students' thinking to shape their instruction and support students' understanding. In other words, teachers should notice important issues about students' thinking and address them appropriately during the instruction. Indeed, awareness of students' mathematical thinking and use of students' mathematical thinking during instruction are both in the scope of pedagogical content knowledge and noticing skills of teachers (Stockero *et al.*, 2017). As similar to intervention studies on teachers' pedagogical content knowledge, having teachers analyze videos of their own or others' teaching and writing reflections about them contributes to development of the teachers' noticing skills (Jacobs *et al.*, 2010). Several studies were conducted to investigate both the nature and the ways of improving teachers' noticing skills (e.g., Sherin and van Es, 2005; Star *et al.*, 2009; Sun and van Es, 2015). One of such studies was conducted by Krupa et al. (2017). Accordingly, the researchers designed a curricular module including class discussions and pre-post interviews. However, majority of these studies were based on teachers' written or oral reflections about the instruction. Unlike these studies, the current study contributes to literature by investigating preservice teachers' in-the-moment noticing skills based on the video analysis of participants who work with students. Furthermore, data were supported by preservice teachers' written reflections about their own teaching. This study was planned and administered throughout a whole academic year as different from short-term intervention studies on teachers' noticing. Furthermore, in this study, MOST instances were used as a unit of analysis to investigate preservice teachers' noticing skills. Recently, Stockero *et al.* (2017) investigated how learning-to-notice intervention affects preservice teachers' recognition of MOST instances occurred in the videos. They found that initially a large percentage of the preservice teachers' noticing was on teachers however towards to the end of the intervention the percentage of instances where the preservice teachers focused on students in the videos had increased. Furthermore, the percentage of noticing specific mathematical issues increased while noticing of non-mathematical issues disappared at the end of the study. As different from Stockero et al.'s (2017) study, in this study, preservice teachers' noticing was defined in terms of attending to MOST instances, interpreting students' mathematics underlying those MOST instances and responding to students' mathematics. On the other hand, the patterns in their noticing is represented with regards to frequencies and percentages to enable making comparisons between the semesters or content areas or participants.

Moreover, this study provided opportunity to examine retention of preservice teachers' in-the-moment noticing skills about six months upon completion of the elective course. The retention part of this study was conducted with the preservice teachers to detect any progress in their in-the-moment noticing skills. Examining such a progress informed literature about nature of noticing skills including attending, interpreting and responding to students' thinking in terms of maintenance of the preservice teachers' noticing skills.

4. STATEMENT OF THE PROBLEM

This research study was connected within a TUBITAK Project called as A University-School Collaboration Model for Promoting Preservice Teachers' Pedagogical Content Knowledge about Students (Grant no: 215K049) conducted at a large-scale university in İstanbul. In this project, the researchers aimed to investigate preservice teachers' noticing skills of students' mathematical thinking throughout one year. In relation to this project, there were two aims of the study. The first aim was to investigate preservice teachers' noticing skills of students' mathematical thinking during one year and the second aim was to examine preservice teachers' noticing skills of students' mathematical thinking six months after the intervention study. In the light of these two aims, the following research questions were investigated.

- How did preservice teachers' noticing skills evolve throughout the year of intervention?

(i) How did preservice teachers' noticing skills evolve in terms of attending to students' strategies throughout the year of intervention?
(ii) How did preservice teachers' noticing skills evolve in terms of interpreting students' understanding throughout the year of intervention?
(iii) How did preservice teachers' noticing skills evolve in terms of responding to students' strategies throughout the year of intervention?

- How was preservice teachers' noticing skills in retention study compared to the intervention study for algebra tasks?

(i) How was preservice teachers' noticing skills in retention study in terms of attending to students' strategies compared to the intervention study in algebra tasks?
(ii) How was preservice teachers' noticing skills in retention study in terms of interpreting to students' understanding compared to the intervention study in algebra tasks?

(iii) How was preservice teachers' noticing skills in retention study in terms of responding to students' strategies compared to the intervention study in algebra tasks?

4.1. Variables of the study

In this study, preservice teachers' noticing skills was analyzed in terms of mathematical opportunities (MOST) that occurred during the implementation of mathematical tasks. Therefore, in this study, MOST instances could be thought of as independent variable and noticing as a dependent variable. Noticing: Jacobs and her colleagues' (Jacobs *et al.*, 2010) description of professional noticing of students thinking was followed in this study. Noticing is defined as a combination of three interrelated components: attending to children's strategies, interpreting children's understanding and deciding how to respond on the basis of children's understanding. Mathematically Significant Pedagogical Opportunity to Build on Student Thinking (MOST): Leatham and his colleagues' (Leatham *et al.*, 2015) description for significant mathematical opportunities was followed in this study. MOST is defined as an intersection of three characteristics such that it should be emerged from student mathematical thinking, be mathematically significant and be a pedagogical opportunity.

5. METHOD

5.1. Participants of the study

A total of 10 preservice teachers participated in this TÜBİTAK project. Seven of them attended the project for both semesters of 2016-2017 academic year however three of them attended only one semester. Four of the preservice teachers were sophomore students, four of them were juniors and two of them were senior students. Among ten preservice teachers two of them, Derya and Meltem (pseudonym), were selected as the sample of this current study. These two participants were sophomore students in the elementary mathematics education program in a large-scale university. These preservice teachers were taking elective courses (one for fall semester and one for spring semester) offered for the preservice teachers who participated in the project. The participants had already taken some pedagogy courses and they had been taking other pedagogy courses by the time of data collection. In addition, Derya's and Meltem's PCK level were similar as a result of the PCK test conducted by the research team. Due to their similar backgrounds, these two sophomore students were chosen among seven preservice teachers. These two participants were not randomly selected, rather chosen purposefuly based on the PCK test results. The Faculty of Education is located at Kayışdağı district in İstanbul. The participants were all female and their age were 21. Both participants were living in İstanbul with their families. In the line of project premise, preservice teachers were asked to work with a group of seventh graders in a local middle school with which Faculty of Education has a collaboration agreement. One of the seventh-grade classrooms was determined for this study by the school administration for the 2016-2017 academic year. Despite of students' poor performances in mathematics in the school, the students were grouped heterogeneously according to the achievement test adminestered by the project team. Each group consisted of three or four students. The participants of this study worked with four students. For the retention study which took place in the fall semester of the 2017-2018 academic year, four students for each participant were determined by the school administration. These students were from the same classrooms and they were high ability students by considering their math grades.

5.2. The design of the study

The design of the study was a case study since noticing skills of preservice teachers were investigated in depth and the selected sample consisting of two preservice teachers was used as a bounded system. The researcher investigated the preservice teachers' noticing skills via video records of the preservice teacher-student interactions, written and oral reflections about such interactions. By considering different types of case studies, exploratory case study was chosen for this study because it provided opportunity to gain insight into the structure of preservice teachers' noticing skills in order to build or develop Jacobs and her colleagues' theory of professional noticing of students' thinking (Yin, 2003).

5.3. Research Setting

The faculty-school collaboration study has been carried out since 2011. For this collaboration, an elective course was designed for preservice teachers in a way that they had opportunity to learn about the basics of task design and implementation process and different ways of scaffolding students' understanding. At the beginning of the semester, for a couple of weeks, it was discussed how they would implement the tasks in the school and how they should interact with students both to elicit their thinking and also scaffold their understanding. In this study, the project team took responsibility of the administration of the elective math course of one of the 7th grade classes in the school to implement mathematics tasks throughout the academic year. The tasks were implemented by the preservice teachers who took the elective course offered in the faculty. After starting to go to the school for implementation of the tasks, two meetings with preservice teachers were set up. One of them was set up just after the implementation to discuss how the implementation went, whether any unpredictable instance occurred and how they addressed to the students' difficulties (oral reflection). The other meeting was set up two days after the implementation to discuss the following week's tasks. Based on the students' performances on the earlier tasks, it was discussed whether we should revise the tasks or not (post-reflection). A 3-step process for task implementation was followed: i) students worked on the given tasks individually, ii) they discussed their solutions within their groups, and iii) preservice teacher joined in their discussion and asked for their solutions and reasoning. As students were working individually or making group discussions, preservice teachers observed them and took some notes about their performances. They occasionally managed the group discussion to justify each student's contribution to the discussion. It was expected that preservice teachers would respond to the MOST instances that they observed during individual work and group discussion.

For the retention study, four algebra tasks which were implemented in the 2016-2017 academic year were administered. Before the task implementation, the researcher asked the opinions of preservice teachers about possible difficulties of students in the algebra tasks. They remembered some of the MOST instances encountering in algebra tasks. The preservice teachers followed the 2-step process (individual working and discussion with preservice teacher) during the task implementation. In the intervention study, the researcher observed that several MOST instances were not mentioned by preservice teachers, since they were already discussed during discussion within their groups. Thus, this part of within group discussion done in the intervention study was removed from the retention study. The researcher observed the task implementation process and how preservice teacher responded to possible MOST instances. After the task implementation, the researcher conducted interview with each preservice teacher about students' difficulties, how they reacted to these difficulties and task implementation process. The duration of each interview was approximately 20-25 minutes. In the Table 5.1 and Table 5.2, respectively, the time schedule for the 2016-2017 academic year and for the retention study that took place in fall 2017 are presented.

Table 5.1. Time schedule for task implementation of 2016-2017.

Week	Date	Procedure	Content
1	18.10.2016	Implementation of Task 1 Oral reflection for implementation	Number task
	21.10.2016	Post reflection and preparation	Number task
2	25.10.2016	Implementation of Task 2 Oral reflection for implementation	Number task
	28.10.2016	Post reflection and preparation	Number task
3	01.11.2016	Implementation of Task 3 Oral reflection for implementation	Number task
	04.11.2016	Post reflection and preparation	Number task
4	08.11.2016	Implementation of Task 4 Oral reflection for implementation	Number task
	11.11.2016	Post reflection and preparation	Number task
5	22.11.2016	Implementation of task 5 Oral reflection for implementation	Number task
	25.11.2016	Post reflection and preparation	Number task
6	29.11.2016	Implementation of task 6 Oral reflection for implementation	Algebra task
	01.12.2016	Post reflection and preparation	Algebra task
7	06.12.2016	Implementation of task 7 Oral reflection for implementation	Algebra task
	09.12.2016	Post reflection and preparation	Algebra task
8	13.12.2016	Implementation of task 8 Oral reflection for implementation	Algebra task
	16.12.2016	Post reflection and preparation	Algebra task
9	20.12.2016	Implementation of task 9 Oral reflection for implementation	Algebra task
	23.12.2016	Post reflection and preparation	Algebra task
10	27.12.2016	Implementation of task 10 Oral reflection for implementation	Algebra task
	30.12.2016	Post reflection and preparation	Algebra task
11	21.02.2017	Implementation of task 11 Oral reflection for implementation	Algebra task
	24.02.2017	Post reflection and preparation	Algebra task
12	28.02.2017	Implementation of task 12 Oral reflection for implementation	Algebra task
13	07.03.2017	Implementation of Task 13 Oral reflection for implementation	Number task
	10.03.2017	Post reflection and preparation	Number task
14	14.03.2017	Implementation of Task 14 Oral reflection for implementation	Geometry task
	17.03.2017	Post reflection and preparation	Geometry task
15	21.03.2017	Implementation of Task 15 Oral reflection for implementation	Geometry task
	24.03.2017	Post reflection and preparation	Geometry task
16	28.03.2017	Implementation of Task 16 and 17 Oral reflection for implementation	Geometry task and Algebra task
	31.03.2017	Post reflection and preparation	Geometry task and Algebra task
17	04.04.2017	Implementation of task 18 Oral reflection for implementation	Data Analysis task
	07.04.2017	Post reflection and preparation	Data Analysis task
18	11.04.2017	Implementation of task 19 and 20 Oral reflection for implementation	Data Analysis tasks
	14.04.2017	Post reflection and preparation	Data Analysis tasks
19	18.04.2017	Implementation of task 21 Oral reflection for implementation	Data Analysis task
	21.04.2017	Post reflection and preparation	Data Analysis task
20	25.04.2017	Implementation of task 22 Oral reflection for implementation	Geometry task
	28.04.2017	Post reflection and preparation	Geometry task
21	02.05.2017	Implementation of task 23 Oral reflection for implementation	Geometry task
	05.05.2017	Post reflection and preparation	Geometry task
22	09.05.2017	Implementation of task 24 Oral reflection for implementation	Geometry task
	12.05.2017	Post reflection and preparation	Geometry task
23	16.05.2017	Implementation of task 25 Oral reflection for implementation	Geometry task
	19.05.2017	Post reflection and preparation	Geometry task

Table 5.2. Time schedule for retention task implementation of 2017-2018.

	Date	Procedure	Content
1	05.12.2017	Implementation of Task 1	Algebra task
	05.12.2017	Post-interview of Task 1	Algebra task
2	12.12.2017	Implementation of Task 2	Algebra task
	12.12.2017	Post-interview of Task 2	Algebra task
3	19.12.2017	Implementation of Task 3	Algebra task
	19.12.2017	Post-interview of Task 3	Algebra task
4	26.12.2017	Implementation of Task 4	Algebra task
	26.12.2017	Post-interview of Task 4	Algebra task

As seen in the Table 5.1, tasks implemented during Week 1, Week 2, Week 3, Week 4, Week 5 and Week 13 were about numbers. Tasks implemented in Week 6, Week 7, Week 8, Week 9, Week 10, Week 11, Week 12 and Week 16 were about algebra. Tasks implemented during Week 17, Week 18, Week 19 were about data analysis. Tasks implemented during Week 14, Week 15, Week 16, Week 20, Week 21, Week 22 and Week 23 were about geometry. Indeed, two tasks were implemented in Week 16 and Week 18. For the retention study, as seen in the Table 5.2, tasks implemented through Week 1, Week 2, Week 3 and Week 4 were about algebra.

5.4. Data Collection

The data sources of this study were video records of preservice teachers' interactions with students during implementation of tasks, preservice teachers' oral reflections just after the implementations and their written reflections about the implementations. For the retention study, the data sources were video records of preservice teachers' interactions with students and video records of interviews with each preservice teacher about task implementation process. To investigate the state of preservice teachers' noticing skills after the intervention study, a new setting for two participants was set up. In 2017-2018 academic year, the participants were asked to work with a group of four students and re-implement four of the algebra tasks which were implemented during 2016-2017 academic year. Their interactions with students were videotaped and after implementations they were interviewed about task implementation.

5.4.1. Videos of preservice teachers' interactions with students

Preservice teachers implemented 25 tasks in their groups during the elective mathematics course throughout the year. They were asked to follow a 3-step process for task implementation. At first, students were given approximately 20 minutes to work individually on given tasks. Then they were asked to share their answers with their peers in the group that is, they made group discussion. They were given 15 to 20 minutes for group discussions. They were allowed to change their answers during the group discussions. And finally, preservice teachers involved in discussion to overview students' answers and address to their mistakes or misconceptions (approximately 35 to 40 minutes). The students were not allowed to change their answers on the worksheets as they were discussing with the preservice teachers but use another piece of paper to correct their answers if they were wrong. These videos were used to identify MOST instances and analyze preservice teachers' in-the-moment noticing. The purpose of data collected through videos of preservice teachers' interactions with students was to investigate the components of noticing skills of preservice teachers including attending and responding to students' mathematical thinking.

After implementation of tasks, students' worksheets and extra sheets of corrected answers were collected by researcher to analyze MOST situations. For each group of students, a total of 25 tasks were administered. Among 25 tasks, 20 of them were developed by the research team and they were common for all groups. However, preservice teachers developed 5 tasks for their own groups and these tasks were differing from one group to another. The tasks developed by the research team were piloted during the academic year of 2015-2016. Content of the tasks was categorized in terms of major content areas in the curriculum as numbers, algebra, geometry and measurement, and data analysis. Preservice teachers prepared at least one task for each content area but some of them prepared two tasks about algebra while others prepared two tasks for numbers. The tasks were developed based on several criteria such that i) tasks had potential to exert students' mathematical thinking and students' misconceptions ii) real life contexts were structured in tasks iii) discussion among students was encouraged by tasks.

The distribution of the tasks in terms of content areas was as follows: 6 tasks were about numbers, 9 of them were about algebra, 6 of them were geometry and measurement, and 4 of them were data analysis tasks. Because algebra tasks were used in retention study, algebra tasks are given as an example in Appendix A accompained with the MOST instances occured during the implementations of these tasks.

5.4.2. Videos of preservice teachers' oral reflections

After implementation of tasks, project team organized meetings with preservice teachers. In these meetings, preservice teachers mentioned about what they noticed during group work, how students performed on the tasks and students' difficulties and misconceptions occurred during the implementation. They gave brief information about how they responded to these difficulties and misconceptions during the oral reflections. Each oral reflection sessions were videotaped. The purpose of data collected through videos of preservice teachers' oral reflections was to examine components of noticing skills of preservice teachers including attending and interpreting of students' mathematical thinking.

5.4.3. Written reflections of preservice teachers about task implementations

After video recordings of preservice teachers, each preservice teacher wrote reflections about task implementation and their interactions with students in a week after the implementation. Written reflections consisted of six questions about mathematical tasks, group working with students and overall evaluation of implementation process. The items of written reflection are given in Appendix B. The video data was supported with written reflections. Furthermore, the purpose of data collected through written reflections was to investigate components of noticing skills of preservice teachers including attending and interpreting of students' mathematical thinking.

5.4.4. Videos of preservice teachers' interactions with students for retention study

For retention study, the researcher also collected data through video records of preservice teachers' interactions with students. The retention study was consisted of two parts; students' individual work on the task (20 minutes), and students' group discussion with preservice teacher (60 minutes). In this retention study, four algebra tasks which were implemented in 2016-2017 academic year were used as a medium for preservice teacher-student interactions.

5.4.5. Videos of preservice teachers' interview with researcher

After each video recordings of preservice teacher' interaction with students, the researcher collected data through follow-up interviews of preservice teachers. Each follow-up interview was video recorded by researcher. Follow-up interviews were consisted of five questions: (i) comments on each students' performance, (ii) what they have noticed about students' difficulties, (iii) what might be the reasons behind students' difficulties (iv) how they respond to students' misconceptions, (v) how they compare the last year's implementation with the recent one. The items of interview with preservice teachers are given in Appendix C.

5.5. Data Analysis

The main source of data was video recordings of preservice teachers' interactions with students throughout 2016-2017 academic year. The researcher transcribed each video recording to identify MOST instances and to analyze preservice teachers' in-the-moment noticing. The reseacher also transcribed the videos of oral reflections and read preservice teachers' written reflections. Then, the researcher used the coding scheme developed by the project team to analyze the state of preservice teachers' noticing skills. The coding scheme was formed in a hierarchical structure to identify whether or not preservice teachers attended to MOST instances and also their responding actions for those instances. In the coding scheme, for the in-the-moment noticing, preservice teachers' interpretation of students' thinking was evaluated under their responding actions because it was hard to observe preservice teachers' direct interpretations during the interactions. Because all these components of noticing are defined to be interrelated with each other (Jacobs *et al.*, 2010) focusing on preservice teachers' attention and responding actions during the implementations is acceptable. To ensure coding reliability, Kappa analysis was made by the researcher. Cohen's Kappa value was found as 0.84. In addition to this, meetings with research team were made to agree about coding of MOST instances for interrater reliability.

For in-the-moment noticing, there are two levels of attending and five levelsof responding to MOST instances are defined. For attending part of MOST instances, if the instance was missed then it was coded as "0", if it was attended then it is coded as "1". For responding part of MOST instances, levels 0, 1, and 2 were classified under answer-focused actions and levels 3 and 4 were classified under mathematical-understanding focused actions. When preservice teachers only told students that their answers were wrong, and they did not make any guidance for students their responding then it was coded as Level 0 (No Attempt). When preservice teachers or other students in the group explained correct answer or solution then it was coded as Level 1 (Explanation). When preservice teachers attempted to make students find out the correct answer through yes or no type questions, prompting correct answers like ..., isn't it, no follow up and non-specific type questions or asking them to re-read, re-do, re-think then their action was coded as Level 2 (Orientation). The actions were coded as at Level 3 (Exploration) when preservice teachers attempted to elicit students' thinking by asking probing questions (why, how, what if questions?) but either conversation was not concluded or in case of existence of misconceptions, preservice teacher failed to address gaps in students' mind because preservice teacher's guidance involved mathematically invalid issues such as lack of terminology, inappropriate examples or representations. Finally, an action was coded as Level 4 (Elaboration) when preservice teachers attempted to elicit students' thinking by asking probing questions and guiding students through appropriate examples, representations, connections between concepts and representations.

For the aspect of oral reflection of preservice teachers, coding scheme including 3 stages were defined. If the preservice teacher mentioned about MOST instance during oral reflection sessions, it was coded as “M” (mentioned). If the preservice teacher just gave general comment related to MOST instance during oral reflection sessions, it was coded as “NS” (none-specifically mentioned). If the preservice teacher did not mention about MOST instance during oral reflection sessions, it was coded as “NM” (not-mentioned).

The written reflections were used to evaluate and validate preservice teachers’ noticing in terms of attending and interpreting of students’ thinking. Furthermore, preservice teachers were asked to evaluate their own scaffolding in their reflections. There were 5 levels of coding scheme for attending to students’ mathematics, interpretation of students’ mathematics and evaluation of own scaffolding. In terms of attending to students’ mathematics, 4 cases were identified such that preservice teacher 1) neither noticed MOST during the interaction nor in written reflection (None IR), 2) missed the MOST during the interaction but attended to it in the report (R), 3) attended to the MOST during the interaction but not mentioned in the report (I), 4) attended to the MOST both during the interaction and also mentioned in the report (IR).

For the aspect of interpretation of students’ mathematics, it was coded as “0” if preservice teacher did not provide any interpretation or stated that she did not understand what the student did or thought about. When preservice teachers just rephrased students’ written procedures or pointed out students’ mistakes, their interpretation was coded as “1”. If preservice teachers commented on the possible reasoning behind students’ thinking (ST) or student mathematics (SM) but provided limited justification such as blaming student for lack of knowledge then it was coded as “2”. When the preservice teachers commented on the possible reasoning behind student mathematics by providing examples from students’ work or vignettes from student-teacher interaction but did not explain the justifications explicitly, it was coded as “3”. If preservice teachers gave a detailed explanation about possible reasoning behind student mathematics by providing valid justifications, it was coded as “4”. The framework for coding is summarized in Table 5.3.

Table 5.3. Classification of preservice teachers’ noticing skills.

Interaction (in-the-moment)	Attending	0	Missed the MOST
		1	Attended to the MOST
	Responding	Answer-focused	
		0	(No attempt) Only tells the students that their answers/solutions are wrong; no guidance for students
		1	(Explanation) S/he or other students tells/explains the procedure or solution
		2	(Orientation) Attempts to make students find out the correct answer through short-answer, Yes/No type, prompting (directs students to correct answer like ..., isn?t it?), no-follow up, non-specific type of questions or b) asking them to re-read, re-do, re-think
		Mathematical understanding-focused	
		3	(Exploration) Attempts to elicit students? thinking by asking probing questions (why, how, what if, ?) but either conversation is not concluded or in case of existence of misconceptions /misunderstandings she fails to address the gap in student?s mind because her guidance involves partially incorrect issues such as lack of terminology, inappropriate examples or representations
		4	(Elaboration) Attempts to elicit students? thinking by asking probing questions and guiding students through appropriate examples, representations, connections between concepts and representations
Written report	Attending	0	Missed the MOST
		IR	Attended to the MOST both during Interaction and in Report
		I	Attended to the MOST only during Interaction
		R	Attended to the MOST in Report
	Interpreting	0	Does not provide any interpretation or states that she did not understand what the student did/thought about
		1	Just rephrases students? written procedures and/or points out students? mistakes
		2	Comments on the possible reasoning behind student mathematics but provides limited justification such as laming student for lack of knowledge or her comments about student mathematics is partially correct
		3	Comments on the possible reasoning behind student mathematics by providing examples from students’ work or vignettes from student-teacher interaction but do not explain the justifications explicitly
		4	Gives a detailed explanation about possible reasoning behind student mathematics by providing valid justifications
Oral ref.	Attending	0	Missed the MOST
		1	Attended to the MOST
	Interpreting	NM	No interpretation
		NS	Non-specific about student mathematics
		M	Specifically mentioned about student mathematics

For the retention study, video recordings of preservice teachers’ interaction with students were transcribed then also possible MOST situations were detected by researcher. After doing this, video records of preservice teachers’ interactions with students were also analyzed. For this analysis, the coding scheme shown in Table 5.3 was used. Furthermore, follow up interview with preservice teachers was examined to detect what preservice teachers notice about students' misconceptions during interaction with students and reasoning behind students' misconceptions. The coding scheme consisting of attending and interpreting aspects of students' mathematics in written reflections were used to analyze follow-up interview of preservice teachers.

5.6. Role of researcher

The roles of the researcher in this study were as follows: i) transcribing interaction and oral reflection videos, ii) analyst for identifying MOST instances iii) coder for preservice teachers' noticing, iv) researcher to collect and analyze data for retention study in terms of coordinator of research setting, observer of preservice teacher-student interactions and interviewer with preservice teachers. During interview process, the researcher was also moderator to direct responses of each preservice teacher and to ask appropriate questions about task implementation.

The project team was consisted of one principal investigator, one researcher and two research assistants (me and another graduate student). Because this study was part of a three-year TÜBİTAK project, the principal investigator of the project received appropriate permissions to collaborate with public school. The mathematics tasks used this study were developed by the project team and they organized weekly meetings to discuss about tasks and to analyze MOST instances. Each member of the project team transcribed video recordings of students-teacher interaction then coded possible MOST instances. Different participants were assigned to each member of the project team and all codes were discussed at weekly meetings. Each of weekly meetings of discussion of coding lasted approximately three hours.

5.7. Trustworthiness of the study

Throughout the process of data collection and analysis of the study, a researcher should make sure that the findings and interpretations are accurate (Creswell, 2009).

The trustworthiness of the study was achieved in terms of credibility, transferability, dependability and confirmability. Credibility of this study was provided through triangulation of five different data sources. These data sources included videos of preservice teachers' interactions with students, videos of preservice teachers' oral reflections, written reflections of preservice teachers about task implementations, videos of preservice teachers' interactions with students for retention study, videos of preservice teachers' interview with researcher.

Theoretical triangulation and interrater reliability was appropriate for this study. The members of project team revised each other's codes for MOST instances and noticing skills. Furthermore, Jacobs *et al.* (2010) description of noticing and Leatham *et al.* (2015) definition for MOST instances were used to analyze data of this study.

Transferability is established with evidence that this research study can be applied to other contexts, situations and populations. Since the research context, participants, methods and analysis presented clearly, other researchers can extend to findings of this study in other contexts, populations and situations.

Dependability of research refers to establishment of the findings as consistent and repeatable. If other researchers conduct a similar study, it is likely to have similar findings with this study since this study was piloted by the project team previously. Furthermore, although there are several techniques to establish dependability, the technique which is called as "external audit" was used for this study (Creswell, 2009). Other researcher which is outside of research context coded to MOST instances to confirm accuracy of findings. Kappa analysis was made by researcher and Cohen's Kappa value was found as 0.84.

Confirmability or objectivity refers to present participants and findings of this study in unbiased way. To establish confirmability, the technique which is called as "audit trail" (Creswell, 2009) was used for this study. Video recordings of oral reflection, the researcher's notes related to coding schemes during the meetings with project team were satisfied the confirmability.

6. FINDINGS

Derya's and Meltem's noticing of MOST instances occuring during the implementation of tasks were classified in terms of content areas. Because the order of topics in the national mathematics curriculum (MoNE, 2013) was followed for the tasks, some of the tasks related with the same content area were not in successive order as shown in Table 5.1. For retention study, four of the algebra tasks which were implemented in successive order in previous year were re-implemented. Below, each preservice teachers' noticing of MOST instances for each content area is given and discussed separately.

6.1. Derya's Noticing of MOST Instances in Number Tasks

There were total of 9 MOST instances observed in Derya's implementation of number tasks. The analysis of Derya's noticing of MOST instances is given in Table 6.1.

Table 6.1. Derya's noticing of MOST instances in number tasks.

	MOST	Interaction		Written Report		Oral Reflection
		Att.	Res.	Att.	Int.	
Week 1	M1	0		R	4	NS
Week 2	M2	1	0	IR	4	NM
	M3	1	0	I		M
Week 3	M4	1	1	I		M
Week 4	M5	1	1	I		NS
	M6	0		NIR		NS
Week 5	M7	1	1	I		NM
	M8	0		R	1	NM
Week 13	M9	1	1	IR	4	M
Note: Att: Attenting; Int: Interpreting; Res: Responding						

As seen in Table 6.1, Derya was able to attend six of the MOST instances (M2, M3, M4, M5, M7 AND M9) while she missed three of them. When her responding actions were analyzed for those six MOST instances, it was seen that four of them were responded at Level 1 (Explanation) and two of them were responded at Level 0 (No Attempt).

For three of the MOST instances (M1, M2 and M9) she interpreted students' mathematics accurately and thoroughly in her report, which were coded as at Level 4. For one of the MOST instances (M8), she just rephrased students' written procedures or pointed out students' mistakes, which were coded as at Level 1.

During the oral reflection sessions, she directly talked about three MOST instances (M3, M4 and M9) that she attended in the interaction. However, for other instances, she either did indirectly mention about them or not. In four of the MOST cases (M3, M4, M5 and M7, coded as I in the table) she did not write anything in her report even though she noticed them during the implementation. In two of the MOST cases (M1 and M8, coded as R in the table), she mentioned about two of them which were missed during interaction in her written report. For two cases, namely M2 and M9 coded as IR, she mentioned about them in both written report and interaction. For one of the cases, which is M6 coded as NIR, she missed in her written report and also interaction.

In order to discuss Derya's noticing of MOST instances that occurred during the implementation of number tasks in depth, two MOST instances were chosen. One of the MOST instance was chosen from Week 2's task, namely M2 coded as (1, 4, 0, NM) given in Table 6.1. The task was about placement of rescue chambers into a mine. Students were asked to determine to the location of each chamber by following given directions. The aim of the task was to support students' understanding of operations with integers. The MOST instance observed in this task was that to write expression that "red rescue chamber is located twice of the depth of the blue one" as $-180 : 2 = -90$ as shown in Figure 6.1.

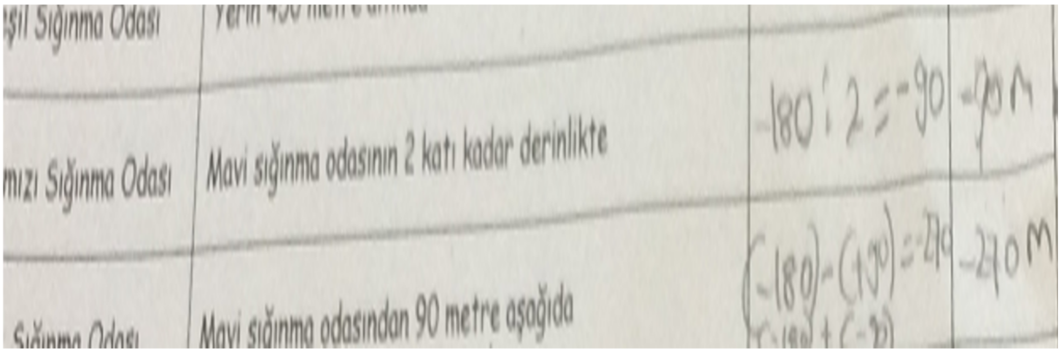


Figure 6.1. Example of a MOST instance related to integers (Ali's work).

For this MOST case we can verbalize the mathematical point (MP) as "verbal expressions or keywords that are used to represent operations with integers are common for both negative and positive integers". However, student thought that when doing operations with negative integers, arithmetic operations are reversed. That is, instead of multiplication, division operation is held. Derya realized student's mistake and began to interact with student as shown below.

- Ali: I found -90 for the depth of red chamber.
- Ayşe: I found 360 for the depth of red rescue chamber.
- Derya: How did you find 360?
- Ayşe: I multiplied 180 by 2.
- Derya: What did you think about this question Ali? Do you think that as you are going downwards?
- Ali: Yes, it may be wrong.
- Derya: Who will tell about the solution to Ali? Nobody, okey. Let's continue with next question.

In her written report Derya wrote the following about this MOST instance: Ali knows that he would multiply given numbers when it is asked to find "twice of a number". However, Ali might think that he should do division instead of multiplication because he is doing operations with negative integers.

Derya attended to this MOST instance during the interaction and also in her written report. However, she did not mention about it during the oral reflection. During the interaction she recognized Ali's mistake (Line 5 and Line 6) and attempted to interpret his thinking but neither did she attempt to eliminate his misunderstanding nor elaborate more of his thinking. Therefore, her respond for this MOST instance, in other words, her in-the-moment noticing was coded as No attempt (Level 0). However, in her written report, Derya provided an appropriate reasoning for student's mathematics such that a verbal expression for multiplication operation was utilized as division when dealing with negative integers. Therefore, her interpretation of student's mathematics, that is, her noticing in the written report was coded as at Level 4.

Another MOST instance was chosen from Week 5's tasks, namely M7 coded as (1, -, 1, NM). The task which was developed by Derya involved in exercises about integers, decimals and rational numbers. The aim of this task was to improve students' understanding of operations with integers, decimals and rational numbers. The MOST instance observed in this task was to find the result of the operation $\frac{5}{8}+\frac{1}{6}-\frac{5}{12}$ as $\frac{1}{2}$. For this MOST case, the student verbally expressed that "I have added to 5 and 1 then I have subtracted 5 from 6. In addition to this, I have added up 8 and 6 then subtracted 12 from 14". We can verbalize the mathematical point (MP) as "fractions with unlike denominators can be added after modifying them to have like denominators". However, the student thought that both denominators and numerators of fractions are added to each other directly. Derya realized student's mistake and began to interact with student as shown below.

- Atakan: I found $\frac{1}{2}$
- Derya: How did you find $\frac{1}{2}$?
- Atakan: I have added 5 to 1 then I have subtracted 5 from. After this operation, I have added up 8 to 6 then subtracted 12 from 14.
- Derya: However, there is a rule for the addition of fractions. Ayşe, please tell your solution to Atakan.
- Ayşe: I have enlarged the denominators to get a common denominator. I have enlarged this (pointed to $\frac{5}{8}$) by 3, this (pointed to $\frac{1}{6}$) by 4 and this (pointed to

$\frac{1}{12}$) by 2. I found the result as $\frac{9}{24}$.

- Derya: Can you simplify $\frac{9}{24}$?
- Ayşe: I can simplify 9/24 by 3 and I will get $\frac{3}{8}$.

Derya attended to this MOST instance during the interaction. However, she did not mention about it during oral reflection and in her written report. During the interaction, she recognized Atakan's mistake and she made one of the students to tell the solution to him (Lines 5 and 6). Therefore, her respond for this MOST instance was coded as Explanation, that is, her in-the-moment noticing was at Level 1. In Table 6.2, summary of Derya's noticing for number tasks was given.

Table 6.2. Summary of Derya's noticing of MOST instances in number tasks.

Interaction							**Written Report**									**Oral Reflection**		
Att.		Res.					Att.				Int.							
0	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**M**	**NM**	**NS**
3	6	2	4				1	4	2	2		1			3	3	3	3

The analysis of Derya's noticing of number tasks revealed that she was able to notice majority of the MOST instances (6 MOST instances) while she missed three of them. Furthermore, for three of the cases, she interpreted students' thinking by providing valid justifications for her explanations as discussed above at Level 4. One of the MOST instances was interpreted at Level 1. In terms of responding level, two of the MOST instances were responded at Level 0 while four of them were responded at Level 1. For four of the MOST instances she did not write anything in her report. Two of the MOST instances were just mentioned during interaction while one of them was missed in both interaction and written report. During the oral reflection, she directly mentioned about three of the MOST instances while she did not talk about three of them. In three of the cases she did not directly mention about the MOST instance by appointing specifically to the student mathematics but made a comment about students2 work. For instance, she did not specifically tell about students' mistakes in decimals in Week 4 but stated that students did not know about how to do operations with decimals.

6.2. Derya's Noticing of MOST Instances in Algebra Tasks

There were total of 19 MOST instances observed in Derya's implementation of algebra tasks. The analysis of Derya's noticing of MOST instances is given in Table 6.3.

Table 6.3. Derya's noticing of MOST instances in algebra tasks.

	MOST	Interaction		Written Report		Oral Reflection
		Att.	Res.	Att.	Int.	
Week 6	M1	1	2	I		NS
	M2	1	1	IR	2	M
Week 7	M3	0		R	4	M
	M4	1	2	IR	2	M
	M5	0		NIR		NM
	M6	0		NIR		M
Week 8	M7	1	1	IR	4	NM
	M8	1	1	IR	4	NS
Week 9	M9	1	2	IR	1	NM
Week 10	M10	0		R	1	NM
	M11	0		NIR		NS
Week 11	M12	1	1	IR	2	M
	M13	1	2	IR	4	M
	M14	0		R	3	M
	M15	1	1	IR	1	M
Week 12	M16	1	1	IR	4	M
	M17	1	1	IR	1	M
	M18	1	2	IR	4	M
Week 16	M19	1	1	IR	2	NM
Note: Att: Attenting; Int: Interpreting; Res: Responding						

As seen in Table 6.3, Derya was able to attend 13 of the MOST instances while she missed six of them. When her responding actions were analyzed, it was seen that eight of the MOST instances (M2, M7, M8, M12, M15, M16, M17 and M19) were responded at Level 1 (Explanation) and five of them (M1, M4, M9, M13 and M18) were responded at Level 2 (Orientation), that is her in the moment noticing skills.

In terms of analysis of written reports, for six of the MOST instances (M3, M7, M8, M13, M16 and M18), she interpreted students' mathematics accurately and throughly in her report, which were coded as at Level 4. In one of the MOST cases, namely M1, she did not write anything in her report even though she noticed them during the implementation. In her written report, for four of the MOST cases she commented possible reasoning behind students' mathematics by providing limited justification such as lack of attention or content which was coded as at Level 2 in terms of interpretation of students' mathematics. For four of the MOST instances (M9, M10, M15 and M17), she only wrote what the student did or noted that s/he made a mistake which were coded as Level 1.

During the oral reflection sessions, she directly talked about 11 of the MOST instances (M2, M3, M4, M6, M12, M13, M14, M15, M16, M17 and M18). For three of the MOST cases (M1, M8 and M11), she made general comments about students' performances. However, she did not mention about the rest of the instances during the oral reflection.

In one of the MOST cases (M1 which was coded as I in the table) she did not write anything in her report even though she noticed them during the implementation. In three of the MOST cases (M3, M10 and M14, coded as R in the table), she mentioned about three of them which were missed during interaction in her written report. In three of the MOST cases (M5, M6 and M11, coded as NIR in the table), she did not mention about them during interaction that is her in the moment noticing skills and also her written report. For other cases, coded as IR, she mentioned about them in both written report and interaction.

In order to discuss Derya's noticing of MOST instances occurred during the implementation of algebra tasks in depth two MOST instances were chosen, namely M7 and M3 as shown in Table 6.3. In one of the eight week's task, it was aimed to support students' understanding of algebraic expressions and also operations with algebraic expressions. In this task, the students found the the result of addition of algebraic expressions "(3x+5) + (2x+2)" as 12x 2 as shown in Figure 6.2.

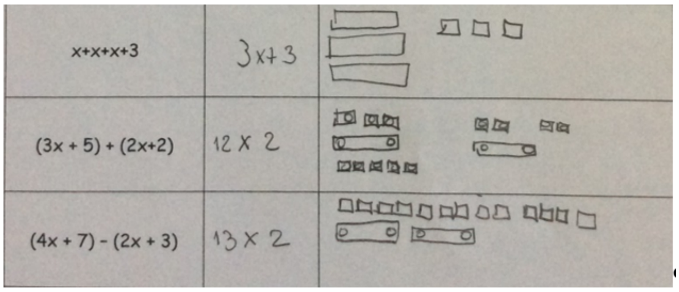


Figure 6.2. Example of MOST instance related to operations with algebraic expressions (Atakan's work).

For this MOST case we can verbalize the mathematical point (MP) as "similar terms can be added or subtracted". However, student thought that coefficients of algebraic terms are added or subtracted with each other without paying attention to whether they are similar terms or not. Then, unknown, in this case x, is added to result of the operation. Derya realized student's mistake and began to interact with student as shown below.

- Derya: Atakan, how did you find 12x to result of this operation?
- Atakan: I added 3 and 5 got 8 then I added 2 and 2. 8 plus 4 is equal to 12.
- Derya: Right now, think that one apple and one pear. What can you say about the result if one apple and one pear are added. One apple is added with one apple, isn't it? In other words, x square is added with x square and x is added with x. Similar terms are added with similar ones. In that algebraic expression, which ones are similar terms?
- Atakan: These (pointed to 3x and 2x) and these (pointed to 5 and 2)
- Derya: (by demostrating algebra tiles) there are 3 tiles and how many algebra tiles are there?
- Atakan: Two.
- Derya: Ok. Please take two tiles. How many x are there?
- Atakan: Five.
- Derya: Ok. Why did you write 12x? Please, write this operation and then write the result.
- Derya: How many x are there when you added 3x and 2x?

- Atakan: Five.
- Derya: Please, write 5x. Okey, please take plus two and five (showing algebra tiles).
- Derya: What is the result when you add five and two?
- Atakan: Seven.
- Derya: Please write seven. Now, show 5x plus 7 by using algebra tiles.

In her written report, she noted that: Atakan solved correctly first example. However, in following examples, both x terms and constant terms were added with each other.

Derya attended to this MOST instance during the interaction. However, she did not mention about it during oral reflection. During the interaction, she recognized Atakan's mistake and she told the solution way and following steps to Atakan (Lines 5, 6, 15 and 17). Therefore, her respond for this MOST instance was coded as Explanation, that is, her in-the-moment noticing was at Level 1. However, in her written report, Derya provided an appropriate reasoning for student's mathematics such that all x terms and all constant terms are added with each other in algebraic expressions then unknown term which is x is added at the end of the result. Therefore, her interpretation of student's mathematics, that is, her noticing in the written report was coded as at Level 4.

Another MOST instance was chosen from Week 7's tasks, namely M3. The task was about that students were given a route for historical tour such that the distance between the school and Ayasofya was given as X and the time spent in the initial place (Ayasofya) as T. There were 10 directions to be followed for the route such that the students were expected to read the directions carefully, choose appropriate variable and write correct algebraic expression for the route. The aim of this task was to write appropriate algebraic expression by taking into consideration given situations in the problem. The MOST instance observed in this task was to insert the x variable for all both distance and time as shown in Figure 6.3.

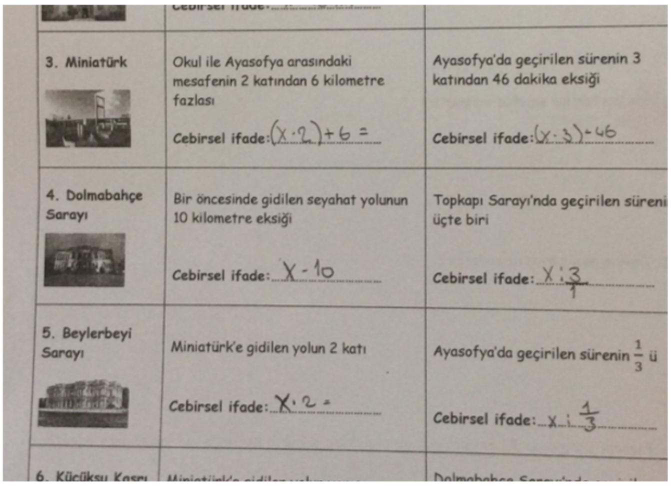


Figure 6.3. Example of MOST instance related to algebraic expressions in given contexts (Atakan's work).

For this MOST case, the mathematical point (MP) can be expressed as to write appropriate algebraic expressions by taking into consideration problem context. However, the student thought that for any given written expression, the unknown always to be x in regardless of the context of the statement. Derya did not realize student's mistake during the interaction with Atakan and she focused on operation rather than using different variables in algebraic expressions. Therefore, her respond for this MOST instance was coded as No Attempt, that is, her in-the-moment noticing was at Level 0. In her written report, she noted that:

The time spent between places was represented with unknown "T" would be substituted for unknown terms. Atakan made mistake because he took unknown as "X" in all algebraic expressions without considering context.

Although Derya missed this MOST instance during interaction with Atakan, she interpreted Atakan's mistake correctly by providing appropriate reasoning such that using unknown X for all algebraic expressions. Therefore, her interpretation of student's mathematics, that is, her noticing in the written report was coded as at Level 4. During the oral reflection, Derya told that Atakan used X unknown for time variable and confused his mind. Also, she mentioned that he used X unknown for all of the algebraic expressions. In the light of this result, her reflection about the MOST was coded as M (specifically mentioned). In Table 6.4, summary of Derya's noticing for algebra tasks is given.

Table 6.4. Summary of Derya's noticing of MOST instances in algebra tasks.

Interaction							**Written Report**									**Oral Reflection**		
Att.		Res.					Att.				Int.							
0	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**M**	**NM**	**NS**
6	13		8	5			3	1	3	12		4	4	1	6	11	5	3

To sum up, the analysis of Derya's noticing of algebra tasks demonstrated that she was able to notice the MOST instances. However, Derya missed many of the MOST instances during the interaction and attended them in her written report. In addition to this, during the interaction, she had a tendency to make students apply correct procedures to reach the correct answer like number tasks. Also, she was able to attend to the MOST instances that she missed during the interaction as in the case of M3 shown in Table 6.3. In addition to this, she interpreted students' thinking by providing valid justifications for her explanations as discussed above. However, for four of the MOST instances she did not write anything in her report. During the oral reflection, she directly mentioned about four of the MOST instances while she did not talk about four of them. In three of the cases she did not directly mention about the MOST instance by giving general comment about students' work. For example, she did not specifically tell about students' mistakes in making rectangular shape with algebra tiles in Week 8 but stated that students had difficulty about it.

6.3. Derya's Noticing of MOST Instances in Data Analysis Tasks

There were total of 7 MOST instances observed in Derya's implementation of data analysis tasks. The analysis of Derya's noticing of MOST instances is given in Table 6.5.

Table 6.5. Derya's noticing of MOST instances in data analysis tasks

	MOST	Interaction		Written Report		Oral Reflection
		Att.	Res.	Att.	Int.	
Week 17	M1	1	1	IR	2	M
	M2	1	1	IR	2	NS
	M3	1	1	IR	3	M
Week 18	M4	1	2	I		NM
	M5	0		R	3	NS
Week 19	M6	1	1	IR	4	M
	M7	1	3	I		M
Note: Att: Attenting; Int: Interpreting; Res: Responding						

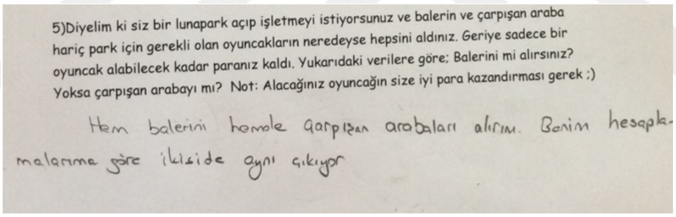
As demonstrated in Table 6.5, Derya was able to attend six of the MOST instances while she missed one of them. When her responding actions were analyzed, it was seen that four of the MOST instances (M1, M2, M3 and M6) were responded at Level 1 (Explanation), one of them (M4) were responded at Level 2 (Orientation) and one of the MOST instances (M7) were responded at Level 3 (Exploration).

In terms of analysis of written reports, one of the MOST instances, namely M6, she interpreted students' mathematics accurately and thoroughly in her report, which were coded as at Level 4. Two of the MOST cases were interpreted at Level 2 because she commented possible reasoning behind students' mathematics by providing limited justification such as lack of attention or content. She commented two of the MOST instances (M3 and M5) at Level 3 in her written report because she interpreted possible reasoning behind students' mathematics. However, she could not explain justifications behind their reasoning in detail way.

During the oral reflection sessions, she directly talked about four MOST instances (M1, M3, M6, M7). For two of the MOST cases (M2 and M5), she mentioned generally about them during oral reflection sessions. However, for other instances, she either did indirectly mention about them or not.

In one of the MOST cases (M5, coded as R in the table), she mentioned about one of them which were missed during interaction in her written report. In two of the MOST cases (M4 and M7, coded as I in the table), she mentioned only in her interaction with students, that is her in the moment noticing skills. For other MOST instances, she mentioned them during interaction and in her written report.

To examine Derya's noticing of MOST instances occurred during the implementation of data analysis tasks in depth, two MOST instances were chosen, namely M7 and M3. During Week 19, Derya prepared a task by considering objectives of data analysis. She aimed to support students' understanding of charts specifically column and pie charts. In addition to this, it was aimed to interpret given data by considering mean, mode and median values. The MOST instance, namely M7, observed during the implementation of in this task was about interpretation of given data as shown in Figure 6.4.


5)Diyelim ki siz bir lunapark açıp işletmeyi istiyorsunuz ve balerin ve çarpışan araba hariç park için gerekli olan oyuncakların neredeyse hepsini aldınız. Geriye sadece bir oyuncak alabilecek kadar paranız kaldı. Yukarıdaki verilere göre; Balerini mi alırsınız? Yoksa çarpışan arabayı mı? Not: Alacağınız oyuncağın size iyi para kazandırması gerek ;)

Hem balerini hemde çarpışan arabaları alırım. Benim hesaplamalarıma göre ikiside aynı çıkıyor

Figure 6.4. Example of MOST instance related to data analysis (Esin's work).

- Esin: I preferred both of them.
- Ayşe: I preferred bumper car because youth were preferred this.
- Derya:
- Ali: I said that you should look at given data above. You did not look at given data. You preferred ballerina. Already, you gave lowest point to ballerina.
- Derya: We should look at the means of given data.
- Ali: It was not necessary to look at median, mode values of given data. Seven is larger than six-point five so ballerina should be preferred.
- Derya: Which one did you look at when the means of two data were equal?
- Ali: We have looked at all of them.
- Derya: For instance, how were range of given data given to you an idea?

- Ali: If the range of given data was small so this means that you preferred more.
- Derya: To find range you subtract minimum value from the maximum value. The difference that is, range is four in this data. The maximum value for the one you like is nine and the least value is five. The difference between them is four.
- Ali: The given data that has smaller range should be more preferred.
- Derya: Nine and five are approximate values. What did you say in terms of means of given data. You should not say both of them. You have money for purchasing only one of them. Esin, please write *blue one* because we looked at means of given data.

For this MOST case we can verbalize the mathematical point (MP) as to interpret population's tendency for making decision, measures of central tendency (mean, median and mode) are taken into consideration. However, student thought that mean, mode and median values of given data can be ignored while comparing two data set to make a decision. Derya realized student's mistake and began to interact with student as shown below.

Derya attended MOST instance during the interaction however, she did not mention about MOST instance in her written report. In oral reflection, she mentioned about MOST instance such that:

- Derya: They decided to look at mean of given data. Ali responded to look at mean of given data. I asked to him that what did you do when means of given data are equal. He said that he decides to prefer his favorite one. Then, I asked to him whether range would help him for his decision. Ali predicted that data whose range is smaller then it is preferable. He knows discrepancy between data however, he does not know the reasoning behind it. Additionally, others said that they preferred the one whose range is the largest.
- Ms. Ece: Have you ever discussed about this subject with your group?
- Derya: Not, exactly. I said that the maximum value is 9 and the minimum is five.

During the interaction, Derya recognized Esin's mistake and she attempted to elicit students' thinking by asking probing questions (how, which one) (Line 9, Line 11). Due to her effort to eliminate students' misunderstanding, the conversation between group and Derya was not concluded. Therefore, her responding for this MOST instance was coded as Level 3 (Exploration). She did not mention about MOST instance in her written report, so it was coded as *I*. In addition to this, she mentioned about this MOST instance during the oral reflection.

Another MOST instance was chosen from Week 17's tasks, namely M3 as given in Figure 6.5. The aim of this task was to support students' understanding of column chart and pie chart. The task was also designed to improve students' understanding of ratio and proportion. The MOST instance observed during the implementation of in this task was that students failed to differentiate between the degrees and percentages in a pie chart as shown in Figure 6.5.

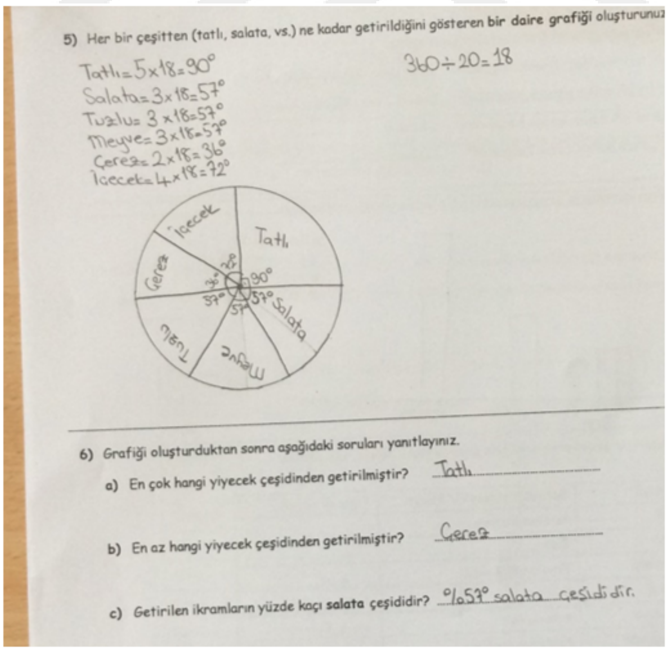


Figure 6.5. Example of MOST instance related to ratio and proportion (Ayşe's work).

The mathematical point is that percentage is found by ratio and proportion with mathematical data in given problem. However, student thought that the degree of slice of pie chart is equal to percentage ratio. Derya realized student's mistake and began to interact with student as shown below.

- Ayşe: Teacher, I found 54 percent to this question.
- Ali: You said 54 so the salads among goods are more than 10. How many salads are there?
- Ayşe: We are multiplying 3 and 18. There are 54 percent salads.
- Ali: Ayşe, look! You said that there are 3 salads among 20 goods. You should figure out proportion such that if there are 3 salads among 20 goods, what percent you have? You should do like that (by demonstrating proportion).
- Derya: Did you understand Ayşe?
- Ayşe: Teacher, setting up proportion is difficult for me.
- After Derya and Ali told solution way to Ayşe, Ayşe found %15 as the answer.

In her written report, she noted that: Ayşe directly wrote degree measures of angles of pie chart as the percentage of salads.

In her oral reflection, she mentioned about MOST instance such that:

- Ms. Ece: Derya, did you encounter any problem related to percentage?
- Derya: Ayşe had difficulty about percentages. She wrote directly degree measures of angle. She found 54 degree of salads so she wrote 54 percent.
- Ms. Ece: Did you have any opportunity to discuss it?
- Derya: She discussed with Ali. Ali found 15 percent and he said to Ayşe that she wrote degree measure of angle rather than percentage. He said that pie chart was divided into one hundred. Ayşe said that she has understood.

During the interaction, Derya recognized Ayşe's mistake (Line 9). However, Ali attempted to tell solution way to Ayşe and tried to reach right answer to Ayşe. Therefore, her respond for this MOST instance, in other words, her in-the-moment noticing was coded as Explanation (Level 1). However, in her written report, Derya commented on possible reasoning behind student's mathematics by providing examples from students' work such that total degree of pie chart is 100. Due to this interpretation in her written report, she did not explain students' understanding explicitly such that students cannot figure out ratio and proportion and she did not give also give detail about students' understanding of ratio and proportion. Therefore, her interpretation of student's mathematics, that is, her noticing in the written report was coded as at Level 3. In addition to this, she mentioned about this MOST instance during the oral reflection. In Table 6.6, summary of Derya's noticing for data analysis tasks is given.

Table 6.6. Summary of Derya's noticing of MOST instances in data analysis tasks.

Interaction							**Written Report**										**Oral Reflection**		
Att.		Res.					Att.				Int.							
0	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**M**	**NM**	**NS**
1	6		4	1	1		-	2	1	4	-	-	2	2	1	4	1	2

To sum up, the analysis of Derya's noticing of data analysis tasks demonstrated that she was able to notice most of the MOST instances (six of them) and missed one of the MOST instances. During the interaction, she had a tendency to tell solution process to her students. Four MOST instances were responded at Level 4, one of them were responded at Level 2 however, in Week 19, her responding action in terms of in the moment noticing skill developed through Level 3. In terms of interpretation level of Derya, two MOST instances were interpreted at Level 3 while one of them were interpreted at Level 4. In addition to this, two MOST instances were interpreted at Level 2. She was able to attend to the MOST instances and interpreted MOST instances in higher level and also interaction moment with group of students. In addition to this, she interpreted students' thinking by providing valid justifications or did not explain justifications explicitly as discussed above. During the oral reflection, she directly mentioned about four of the MOST instances while she did not talk about one of them. In one of the cases she did not directly mention about the MOST instance by giving general comment about students' work. During oral reflection, preservice teachers did not have opportunity to tell about MOST cases in detail since they were given at most 10 minutes to reflect on how the implementation went and what took their attention most.

6.4. Derya's Noticing of MOST Instances in Geometry Tasks

There were total of 10 MOST instances observed in Derya's implementation of geometry tasks. The analysis of Derya's noticing of MOST instances is given in Table 6.7.

Table 6.7. Derya's noticing of MOST instances in geometry tasks.

	MOST	Interaction		Written Report		Oral Reflection
		Att.	Res.	Att.	Int.	
Week 16	M1	0	-	R	2	NM
Week 20	M2	1	1	IR	3	M
	M3	1	2	IR	1	M
	M4	1	1	IR	2	NS
Week 21	M5	1	1	I	-	M
	M6	1	3	IR	2	NM
Week 22	M7	1	1	IR	1	M
	M8	1	1	IR	4	M
	M9	1	1	IR	3	M
Week 23	M10	1	1	IR	3	M
Note: Att: Attenting; Int: Interpreting; Res: Responding						

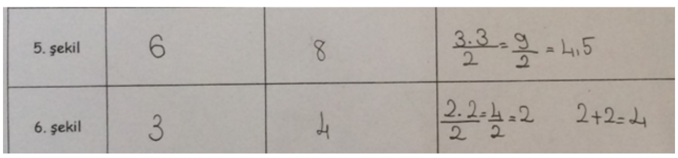
As given in Table 6.7, Derya was able to attend nine of the MOST instances and missed one of the MOST instances among 10 MOST instances. When her responding actions were analyzed, it was seen that eight of the MOST instances were responded at Level 1 (Explanation), one of them were responded at Level 2 (Orientation) and one of the MOST instances were responded at Level 3 (Exploration).

Moreover, one of the MOST instances, namely M8, she interpreted students' mathematics accurately and thoroughly in her report, which were coded as at Level 4. Three of the MOST cases (M1, M4 and M6) were interpreted at Level 2 because she commented possible reasoning behind students' mathematics by providing limited justification such as lack of attention or content. Two of the MOST instances (M3, M7) were interpreted at Level 1 because she rephrased students' written procedures or pointed out students' mistake. She commented three of the MOST instances (M2, M9 and M10) at Level 3 in her written report because she interpreted possible reasoning behind students' mathematics. However, she could not explain justifications behind their reasoning in detail way.

During the oral reflection sessions, she directly talked about seven MOST instances. For one of the MOST cases, she mentioned generally about them during oral reflection sessions. However, for other instances, she either did indirectly mention about them or not.

For eight of the MOST instances (M2, M3, M4, M6, M7, M8, M9 and M10, coded as IR in Table 6.7, she attended to them in both her interaction with other students and her written report). In one of the MOST instances, namely M1 (coded as R in Table 6.7), she mentioned only in her written report. In one of the MOST instances, namely M5 (coded as I in table), she only attended to it during interaction with students.

To examine Derya's noticing of MOST instances occurred during the implementation of geometry tasks in depth, two of the MOST instances, M6 and M9 were chosen. Aim of this task implemented in Week 21 was to support students' understanding about circumference and area of two-dimensional geometric figures. The MOST instance, namely M6, observed during the implementation of in this task was that applying area formula of triangle to any other two-dimensional geometric figures as shown in Figure 6.6. In the question shown in Figure 6.6., the 5th figure was a paralleogram whose base and height were 3 units and the 6th figure was a rhombus with diagonals 2 units by 4 units.



5. şekil	6	8	$\frac{3.3}{2} = \frac{9}{2} = 4,5$
6. şekil	3	4	$\frac{2.2}{2} = \frac{4}{2} = 2$ $2+2=4$

Figure 6.6. Example of MOST instance related to area of parallelogram (Ayşe's work).

The mathematical point (MP) of this MOST case is verbalized as that area of any parallelogram is found by multiplying its base with its height. However, student thought that area of parallelogram is found by using area formula of triangle. Derya realized student's mistake and began to interact with student as shown below.

- Derya: Which method did you prefer to find area of parallelogram?
- Ayşe: I counted gaps between points.
- Derya: You found its height, and did you find base of this parallelogram?
- Ayşe: (by demonstrating sides of parallelogram) Teacher, I counted this one. I multiplied with 3 and 3 then divided into 2. I got 4,5.
- Derya: Why did you divide 9 into 2?
- Ayşe: Teacher, actually we divided into 2.
- Derya: Is this formula to find the area of triangle? If this geometric figure is triangle, you are right. Is there any need to divide into two?
- Ayşe: The base of this parallelogram is multiplied with this side.
- Derya: I do not know the side of parallelogram however, it is enough to multiply base with height of parallelogram. Ayşe tried to multiply base with height. This one is the area of parallelogram.

In her written report, she noted that: Firstly, Ayşe found the area of triangle by dividing this parallelogram into two. After that, she noticed this error. She said that there is no need to divide into two and she found the area of parallelogram by multiplying base with height.

During the interaction, Derya recognized Ayşe's mistake and she attempted to elicit students' thinking by asking probing questions (why) (Line 1 and Line 6). Although her effort to eliminate students' misunderstandings and to explore her understandings, she failed to address the gap in students' mind because her guidance involves partially incorrect issues such as the question which is "Is there any need to divide into two?". Therefore, her responding for this MOST instance was coded as Level 3 (Exploration). She mentioned about MOST instance in her written report, so it was coded as IR. In addition to this, she did not mention about this MOST instance during the oral reflection. In her written report, she emphasized that Ayşe divided the area of parallelogram into two. However, Ayşe's mathematical understanding was to apply area of triangle to find area of each geometric figure. Therefore, Derya's this interpretation was attributed that there is no need to divide into two such that her interpretation is consisted of limited justification. Derya's noticing in her written report was coded at Level 2.

Another MOST instance was chosen from Week 22's tasks, namely M9. The aim of this task was to support students' understanding of symmetry, translation and rotation of any geometric figure. The MOST instance observed during the implementation of in this task was that students failed to translate given figure by 4 units to the right as shown in Figure 6.7.

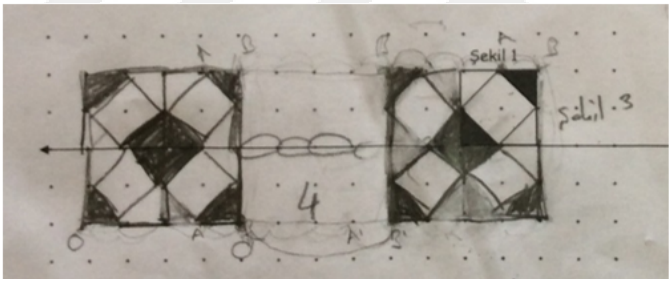


Figure 6.7. Example of MOST instance related to translation of geometric figure (Ali's work).

The mathematical point (MP) of this MOST case is verbalized as that translation is any geometric transformation which moves every point of geometric figure by the same distance in a given direction vector. However, student thought that translation is copying the figure by leaving given amount of distane between the original and copied figures. Derya realized student's mistake and began to interact with student as shown below.

- Derya: Let's pass to translation. Many of you made mistake and you spaced out four For example, Ali spaced out four between figures. Translation is moving of every point of geometric figure by same distance. For example, please take B point then this is the new B point. Actually, this side of given geometric figure moves in here. Every point is translated by four units. For instance, you could think going to cinema. You assumed to sit down different place. You shifted four seats to the left. In the same way, each point of this figure should shift four units to left. This side of geometric figure shifted here. I assembled points of geometric figure and it moved to this position.

- Esin: We did not understand.
- Derya: Each point of this geometric figure shifted four units.
- Ali: Exactly! Look Esin, 1, 2, 3, 4.
- Derya: For instance, I counted gaps of your figure. 1, 2, 3, 4, 5, 6, 7, 8. You translated eight units. The distance between B and B' should be four units. Let's draw new figure in here and C will be shift 4 units to left. Then, A and B will be shifted. The distance between points will be 2 units however, all points will be translated.

In her written report, she noted that: In translation, Ali drew new figure which its distance from new figure as four units. Then, she continues to comment students' thinking by providing vignettes from student-teacher interaction. In her oral reflection, she mentioned about MOST instance such that: In last question, all of the students made mistake and translated eight units. Ali said that figure will translate in this way. I asked to how many units will be translated then he answered four units. During the interaction, Derya recognized Ali's mistake (Line 1). Derya attempted to tell solution way to Ali (in Line 4, 5, 6, 7, 8 and Line 12). Therefore, her respond for this MOST instance, in other words, her in-the-moment noticing was coded as Explanation (Level 1). However, in her written report, Derya commented on possible reasoning behind student's mathematics by providing vignettes from student-teacher interaction such that they spaced out four units within student-teacher interaction. Due to this interpretation in her written report, she did not explain students' understanding explicitly such that students cannot figure out translation and she did not tell much about why students have a tendency toward moving four units between figures. Therefore, her interpretation of student's mathematics, that is, her noticing in the written report was coded as at Level 3. In addition to this, she mentioned about this MOST instance during the oral reflection. In Table 6.8, summary of Derya's noticing for geometry tasks is given.

Table 6.8. Summary of Derya's noticing of MOST instances in geometry tasks.

Interaction							**Written Report**										**Oral Reflection**		
Att.		Res.					Att.				Int.							
0	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**M**	**NM**	**NS**
1	9	-	7	1	1		-	1	1	8	-	2	3	3	1	7	2	1

To sum up, the analysis of Derya's noticing of geometry tasks demonstrated that she was able to notice most of the MOST instances and missed one of the MOST instances. During the interaction, she had a tendency to tell solution process to her students. However, in Week 21, her responding action in terms of in the moment noticing skill progressed to Level 3. She was able to attend to the MOST instances and interpreted MOST instances in higher level. In addition to this, she interpreted students' thinking by providing valid justifications or did not explain justifications explicitly as discussed above. During the oral reflection, she directly mentioned about seven of the MOST instances while she did not talk about two of them. In one of the cases she did not directly mention about the MOST instance by giving general comment about students' work. During oral reflection, preservice teachers did not have opportunity to tell about MOST cases in detail since they were given at most 10 minutes to reflect on how the implementation went and what took their attention most. The summary of Derya's noticing of MOST instances for all tasks was given in Table 6.9.

Table 6.9. Summary of Derya's noticing of MOST instances in for all tasks during intervention study.

	Interaction (%In-the-moment)							**Written Reports**											**Oral Reflection**	
	Attending		Responding					Attending				Interpreting								
Content area	**0**	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	0	**1**	**2**	**3**	**4**	**NM**		**M**	**NS**
Numbers	3 (%33)	6 (%67)	2 (%33)	4 (%67)	-	-	-	1 (%11)	4 (%44)	2 (%22)	2 (%22)	-	1 (%25)	-	-	3 (%75)	3 (%33)		3 (%33)	3 (%33)
Algebra	6 (%31)	13 (%69)	-	8 (%62)	5 (%38)	-	-	3 (%16)	1 (%5)	3 (%16)	12 (%63)	-	4 (%27)	4 (%27)	1 (%6)	6 (%40)	5 (%26)		11 (%58)	3 (%16)
Data Analysis	1 (%14)	6 (%86)	-	4 (%66)	1 (%17)	1 (%17)	-	-	2 (%29)	1 (%14)	4 (%57)	-	-	2 (%40)	2 (%40)	1 (%20)	1 (%14)		4 (%57)	2 (%29)
Geometry	1 (% 10)	9 (%90)	-	7 (%78)	1 (%11)	1 (%11)	-	-	1 (%10)	1 (%10)	8 (%80)	-	2 (%22)	3 (%33)	3 (%33)	1 (%11)	2 (%20)		7 (%70)	1 (%10)
Total	**10 (%23)**	**34 (%77)**	**2 (%6)**	**23 (%68)**	**7 (%20)**	**2 (%6)**	**-**	**4 (%9)**	**8 (%18)**	**7 (%15)**	**26 (%58)**	**-**	**7 (%21)**	**9 (%27)**	**6 (%18)**	**11 (%33)**	**11 (%24)**		**25 (%56)**	**9 (%20)**

In terms of all tasks out of retention study, Derya was attended to 34 MOST instances (%77) while missing 10 MOST instances (%23) within 44 MOST instances as shown in Table 6.9. In terms of responding level, two MOST instances were responded at Level 0 (%6) while 23 of them were responded at Level 1 (%68). Seven MOST instances were responded at Level 2 (%20) while two of them were responded at Level 3 (%6).

For the interpretation level of Derya, seven MOST instances were interpreted at Level 1 (%21) while nine MOST instances were interpreted at Level 2 (%27). Six MOST instances were interpreted at Level 3 (%18) while 11 of them were interpreted at Level 4 (%33). Considering oral reflections of Derya, 11 MOST instances were not mentioned (%24) while 25 MOST instances were mentioned during meetings with researchers. Nine MOST instances were not specifically mentioned (%20) because Derya provided general comments about students' difficulties.

In terms of written reports of Derya, four MOST instances were not mentioned in interaction and written reports (%9) while eight of them were just mentioned during the interaction with students (%18). In addition to this, seven MOST instances were only mentioned in her written reports (%15) while 26 of them were attended in interaction with students and written reports (%58).

6.5. Meltem's Noticing of MOST Instances in Number Tasks

There were total of 12 MOST instances observed in Meltem's implementation of number tasks. The analysis of MOST instances is given in Table 6.10.

Table 6.10. Meltem's noticing of MOST instances in number tasks.

	MOST	Interaction		Written Report		Oral Reflection
		Att.	Res.	Att.	Int.	
Week 1	M1	1	1	IR	2	M
Week 2	M2	1	1	I		NM
	M3	1	1	IR	1	NM
Week 3	M4	0		R	1	NS
Week 4	M5	0		R	1	NS
	M6	1	1	IR	1	NS
	M7	1	1	I		M
Week 5	M8	1	1	IR	2	M
	M9	1	2	IR	2	NM
Week 13	M10	1	1	IR	3	M
	M11	1	2	IR	3	M
	M12	1	3	IR	3	NS
Note: Att: Attenting; Int: Interpreting; Res: Responding						

As seen in Table 6.10 , Meltem only missed two of the MOST instances but noticed majority of them (10 MOST instances). However, except M9, M11 and M12, her responding action for MOSTs was in the form of Explanation, that is, her in-the-moment noticing was at Level 1. For M9 and M11 instances, her responding action for MOSTs that is her in-the-moment noticing was at Level 2 while for M12 instances, her in-the-moment noticing skills was at Level 3.

In terms of her interpretation skills, four MOST instances were interpreted at Level 1 while three of them were interpreted at Level 2. Three MOST instances were interpreted at Level 3. That is, out of twelve interpretations, in four of them she only wrote what the student did or noted that s/he made a mistake. In three of the MOST instances (M1, M8 and M9) she attempted to interpret student mathematics however her justifications were limited such that she noted that students did not know about the content or they had some misunderstandings. In three of MOST instances at Week 13, she interpreted reasoning behind students' thinking by providing examples from students' work, but she did not explain justifications explicitly. Apart from five weeks, the interpretation level of Meltem in number tasks had tendency to develop based on Week 13.

During the oral reflections, she mostly talked about the MOST instances occurred during the implementations. However, in three of the cases (M4, M5, M6 and M12), she made a general comment about students' performances rather than specifically discussing students' mathematics.

In two of the MOST cases (M2 and M7 which were coded as I in the table) she did not write anything in her report even though she noticed them during the implementation. In two of the MOST cases (M4 and M5, coded as R in the table), she mentioned about two of them which were missed during interaction in her written report. For other cases, coded as IR, she mentioned about them in both written report and interaction.

To discuss Meltem's noticing of MOST instances occurred during the implementation of number tasks in depth, two MOST instances were chosen. In the fifth week, preservice teachers developed their own tasks. In her task, Meltem aimed to support students' understanding of operations with integers and rational numbers by giving them to solve some real-life problems related to numbers. The MOST instance, namely M9, observed during the implementation of in this task was to write mathematical expression "the number of floors went down from the 4th floor to -3^{rd} floor" as 4-3= -1 as shown in Figure 6.8.

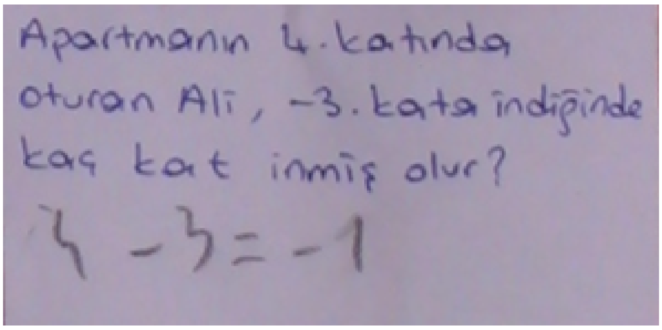


Figure 6.8. Example of a MOST instance related to integers (Nihat's work)e.

For this MOST case the mathematical point (MP) might be verbalized as "distance between two points equals to the absolute value of the difference between them". However, student thought that going downwards can be mathematically expressed by operation of addition instead of subtraction. Meltem realized student's mistake and began to interact with student as shown below.

- Meltem: Right now, you are supposed to be in the 4th floor and you want to go down -3rd floor. How many floors that you're going downwards?
- Nihat: I'm going down 4 floors.
- Meltem: You are going downwards to -3rd floor. How much you need to go further to reach -3rd floor?
- Nihat: OK. I found -1 by going down three floors.
- Meltem: You are supposed to be in the 4th floor. Show me the 4th floor in your paper. He constructs a vertical number line with numbers.
- Meltem: Are you going down from the 4th floor?
- Nihat: Yes.
- Meltem: Now, show me -3rd floor and count how many floors are between them.
- Nihat: One, two, three, four, five, six, seven, eight, nine.
- Meltem: You're supposed to be on the 4th floor. If you go down one floor, at which floor will you be?
- Nihat: 3rd floor.
- Meltem: How many floors you have went down?
- Nihat: One floor.

- Meltem: If I went down one more, it means I went down 2 floors. One more so?
- Nihat: Three.
- Meltem: Please, continue to count down.
- Nihat: Five, six and seven.
- Meltem: Yes, seven is the right answer. We'll pass next question.

During the interaction, Meltem recognized Nihat's mistake and she tried to direct student to correct answer by asking him to draw a number line and count the distance between 4 and -3. She asked yes/no type of question (Line 8) and short answer type of questions (Lines 12, 15 and 17). Therefore, her noticing for this MOST instance was coded as at Level 2 (Orientation). Meltem attended to this MOST in her report. Meltem wrote that Nihat made a mistake while adding and subtracting negative integers. So, her attention was coded as IR. However, she stated that he did not know operations with negative numbers, so he made such a mistake. Therefore, her interpretation of student mathematics, that is, her noticing in the report was coded as at Level 2. On the other hand, she did not mention about this MOST during the oral reflection.

Another MOST instance was chosen from Week 4's task, namely M5. The task was about the operations with rational numbers, ratio and proportion. The students failed to set up proportion between given quantities, but they thought proportion as the difference between given quantities. For instance, Nuray wrote that she would find the difference between 14 (MWh) and 1 (ton) and then she would add up that amount to 16 (ton) as shown in Figure 6.9.

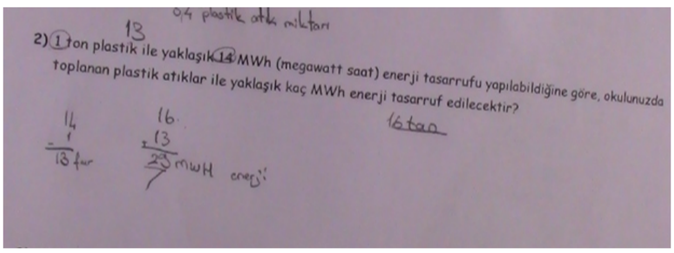


Figure 6.9. Example of MOST instance related to proportion and ratio (Nuray's work).

The mathematical point (MP) of this MOST instance can be stated as "a multiplicative relationship between two quantities is written as a proportion". However, Nuray used addition and subtraction to represent the proportional relationship between given quantities. Meltem missed this MOST instance during the interaction but she mentioned about it in her report. In her written report she noted that:

> As a solution for the second question, Nuray told me that she found the difference between 1 ton and 14 MWh then I added that sum to 16 ton. I found 29 MWh.

In her written report, Meltem rephrased Nuray's written procedures and pointed out student's mistake. She did not give any mathematical reasoning behind students' mathematics. This situation was coded as at Level 1 in terms of noticing student mathematics. During the oral reflection, Meltem told that students in her group could not solve the second problem so she explained the solution to them. Since Meltem did not specifically refer to Nuray's work, her reflection about the MOST was coded as NS (none - specifically mentioned).

Table 6.11. Summary of Meltem's noticing of MOST instances in number tasks

Interaction							**Written Report**									**Oral Reflection**		
Att.		Res.					Att.				Int.							
0	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**M**	**NM**	**NS**
2	10		7	2	1			2	2	8		4	3	3		5	3	4

The analysis of Meltem's noticing of number tasks revealed that she was able to notice ten MOST instances while she missed two of them. In terms of her responding level, seven MOST instances were responded at Level 1 while two of them were responded at Level 2. One of them was responded at Level 3. Although Meltem was able to notice MOST instances that she missed during the implementation, her interpretation of student mathematics was either at Level 1 or Level 2. That is, she only noted that what the student did, or she wrote that the student made such a mistake because of his/her lack of knowledge. During the oral reflection, Meltem directly mentioned about three of the MOST instances while she did not talk about three of them.

In three of the cases, she made an overall evaluation for students' performance rather than pointing out the MOST instances specifically.

6.6. Meltem's Noticing of MOST Instances in Algebra Tasks

As shown below in Table 6.6.1, Meltem was able to attend 16 of the MOST instances during interaction with group of students while she missed three of them. In terms of her responding actions, it could be seen that five of the MOST instances were responded at Level 1 (Explanation) and 10 of them were responded at Level 2 (Orientation). In addition to this, one of the MOST instances was responded at Level 3.

Table 6.12. Meltem's noticing of MOST instances in algebra tasks.

	MOST	**Interaction**		**Written Report**		**Oral Reflection**
		Att.	Res.	Att.	Int.	
Week 6	M1	1	2	IR	2	NS
	M2	1	1	IR	3	NM
Week 7	M3	1	1	IR	2	M
	M4	1	2	IR	3	M
	M5	1	2	IR	3	NM
Week 8	M6	1	2	IR	4	M
	M7	1	2	IR	4	M
	M8	1	1	IR	3	M
Week 9	M9	1	2	IR	4	M
	M10	1	2	IR	2	M
Week 10	M11	1	2	IR	4	NM
	M12	1	1	IR	3	M
Week 11	M13	0		R	3	M
	M14	0		R	3	NM
	M15	1	2	IR	4	M
	M16	0		R	2	NM
Week 12	M17	1	2	IR	4	M
	M18	1	1	IR	2	M
	M19	1	3	IR	2	M
Note: Att: Attenting; Int: Interpreting; Res: Responding						

Likewise, for seven of the MOST instances (M2, M4, M5, M8, M12, M13 and M14), she commented on students' mathematics by providing examples from students' work however, she did not explain justifications in detail way which were coded as at Level 3. In terms of six MOST instances, namely (M6, M7, M9, M11, M15 and M17), she interpreted students' mathematics accurately and thoroughly in her report by giving detail about students' mathematics which were coded as at Level 4. In six of the MOST cases, namely (M1, M3, M10, M16, M18 and M19) were interpreted at Level 2 because she interpreted students' mathematics by providing limited justification such as lack of attention or content.

During the oral reflection sessions, she directly mentioned about twelve MOST instances (M3, M4, M6, M7, M8, M9, M10, M12, M13, M15, M17, M18 and M19). For one of the MOST cases, namely M1, she talked generally about them during oral reflection sessions. However, for other instances, she either did indirectly mention about them or not at all.

In three of the MOST cases (M13, M14 and M16, coded as R in the table), she mentioned about three of them which were missed during interaction in her written report. For other MOST instances, she mentioned them during interaction and in her written report.

Two MOST instances, namely M2 and M7 were selected to discuss Meltem's noticing of students' thinking in algebra tasks in depth. In one of the first week's task, it was aimed to support finding the rule of any pattern by using pattern blocks. The MOST instance, namely M2, observed during the implementation of in this task was to find the rule of pattern as "m+2" instead of 2m+1. In this task students focused on the change in dependent variable rather than thinking about the relationship between independent and dependent variable, that is, the relationship between number of weeks and the number of streets renovated in each week as shown in Figure 6.10.

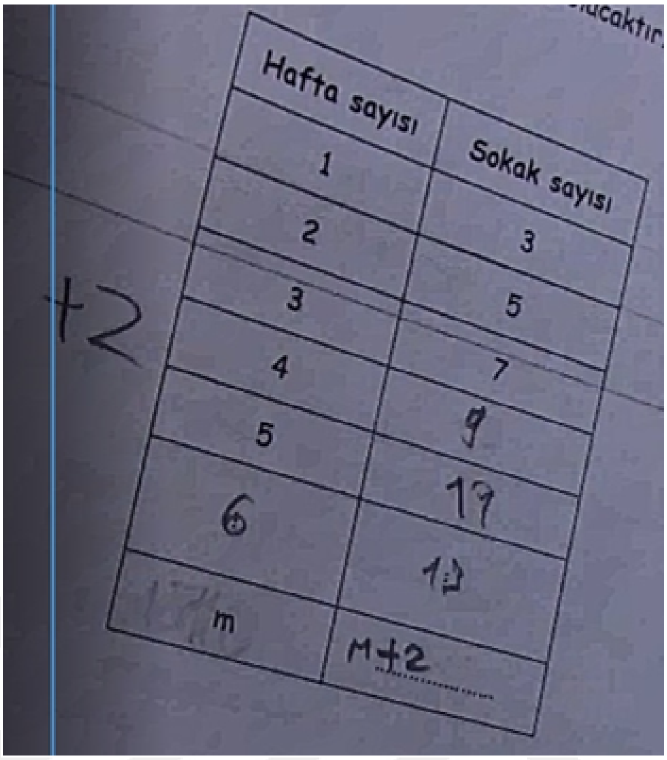


Figure 6.10. Example of MOST instance related to finding the rule of pattern (Emre's work).

For this MOST case, the mathematical point (MP) can be expressed as when finding the rule of the pattern, it is needed to check the relationship between two variables rather than the change in one variable. However, the student thought that it can be found the rule of the pattern by looking the difference between the number of streets and thus the answer is m+2. Meltem attended Emre's mistake during interaction with him and began to interact with Emre as shown below.

- Meltem: Ok. Let's pass to third question. What is asked in this question?
- Emre: This question wants me to find the number of streets in 4th and 5th weeks.
- Meltem: Now, there are three streets in the first week, five streets in the second week, seven streets in the third week. What did you do to find the rule of pattern?
- Nuray: I have found it. It can be multiplied by two and one is added to sum.
- Meltem: It might be. Try it. The camera focused on Emre's work and he found the result as "m+2".
- Nuray: We multiplied by two.

- Emre: We will add one to the result and I find it!

In her written report, she noted that: In below question, Emre said that we multiplied with four or added to four. While writing the result, he decided to add four. When it came to n, he assigned number to n which is 100 then he found 401. In third question, he thought in the same way and wrote answer as +2. In the third question:

- Emre: The pattern is "increasing up by two from previous week".
- Nuray: The sum of this week and one gives the new week but I cannot write this pattern.
- Emre: You cannot add up number of streets and the number of weeks.
- Nuray: Emre, why did you write "m+2"?
- Emre: Because, the pattern was 'increasing up by 2'.
- Nuray: If m was equal to 10, the result could be 25.

In addition to this, she added to interaction with group of students in her written report. She noted that: In third question:

- Meltem: In above question, your friend multiplied by four and then add two. In here, there is similar operation with this one.
- Elif: I have found it. We should multiply by two and add one.
- Meltem: Is it valid for each value in the pattern? My students were happy when they saw that is valid. However, Nihat did not understand the rule of the pattern. Elif told to him about the rule of pattern.

Meltem attended this MOST instance during the interaction and in her written report. However, she did not mention about it during the oral reflection. During the interaction, she recognized Emre's mistake (Lines 3 and 4). However, Nuray attempted to tell solution to Emre and tried to convince Emre about the correct answer. Therefore, her respond for this MOST instance, in other words, her in-the-moment noticing was coded as Explanation (Level 1). However, in her written report, Meltem commented on possible reasoning behind student's mathematics by providing examples from students' work such that while writing the result, he decided to add four and he thought the same logic with above question finding two. Due to this interpretation in her written report, she did not explain students' understanding explicitly such that looking at only change in one variable rather than relationship between two variables. Therefore, her interpretation of student's mathematics, that is, her noticing in the written report was coded as at Level 3.

Another MOST instance was chosen from Week 3's tasks, namely M7. In this task, it was aimed to support students' understanding of algebraic expressions and operations with algebraic expressions. The MOST instance observed during the implementation of in this task was to find the result the operation of algebraic expression "(6x+4)/2" as 6x+2 demonstrated in Figure 6.11.

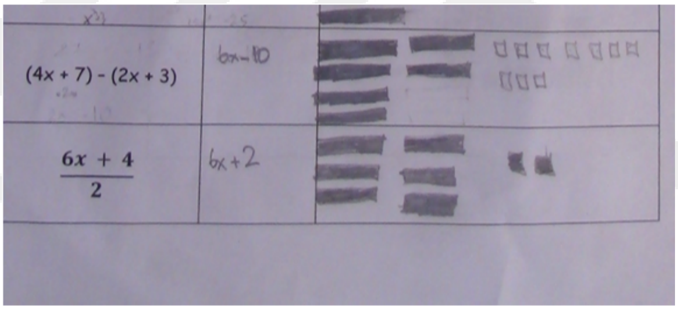


Figure 6.11. Example of MOST instance related to operations with algebraic expressions (Nuray's work).

For this MOST case we can verbalize the mathematical point (MP) as "when an algebraic expression is divided by a constant term, only the coefficients are divided or simplified by that constant". However, student thought that only the constant terms are divided by each other and the unknown term (x) is added to result of division operation. Meltem realized student's mistake and began to interact with student as shown below.

- Meltem: Please, pass to next question. You have found 6x plus two. So, what is the result of the operation that is (4+2)/2?
- Nuray: Six is divided by two.

- Meltem: Is this three, right?
- Nuray: Yes.
- Meltem: In here, numbers were added then divided by 2. Four and two were divided by 2 then the results were added up.
- Meltem: How will you divide (2x+2)/2?
- Emre: I don't know.
- Nuray: 2x plus one.
- Meltem: How did you find?
- Nuray: This number was divided into this number.
- Meltem: Why did you divide only this number?
- Emre: I have found it (by writing x plus one to activity sheet)
- Meltem: Nihat, how did you operate 6x plus four?
- Nihat: I added.
- Meltem: After adding them up, you divided into two, is it right?
- Nihat: Six was divided into two so I have got 3x.
- Meltem: Yes, you have got 3x. So, what is the function of two?
- Emre: Operation of division.
- Meltem: There is operation of division under 6x plus four. So, you should divide each term by two.
- Emre: I have got it.

In her written report, she noted that:
Nuray divided both known and unknown terms into two in any other question. However, in this question, known term only was divided into two because denominator included known term. Emre only divided into known term by two. In fifth question, students did not divide into unknown term by two.

In addition to this, Meltem mentioned the same dialog in interaction with her students in her written report.

In her oral reflection, she mentioned about MOST instance such that: In below question, students found 6x plus two. I said that what did you find (4+2)/2 and they reached correct result by addition. Actually, I said that both four and two were divided by two, so they understood.

During the interaction, Meltem recognized Nuray's mistake and she tried to direct student to correct answer by asking her to (2x+2)/2 or (4+2)/2. She asked yes/no type of questions (Line 4 and 16) and short answer type of questions (Lines 1, 8, 11, 14 and 17). Therefore, her noticing for this MOST instance was coded as at Level 2 (Orientation). Meltem attended to this MOST in her report. Meltem wrote that students made a mistake related to dividing only known term into two. So, her attention was coded as IR. She stated that Nuray divided only known term into two because denominator included known term. When her interpretation was evaluated, reasoning behind students' mathematics such that dividing only known term into two was appropriate and was justified explicitly via interaction with students. Therefore, her interpretation of student mathematics, that is, her noticing in the report was coded as at Level 4. Moreover, she mentioned about this MOST during the oral reflection.

Table 6.13. Summary of Meltem's noticing of MOST instances in algebra tasks

Interaction							**Written Report**									**Oral Reflection**		
Att.		Res.					Att.				Int.							
0	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**M**	**NM**	**NS**
3	16		5	10	1		-	-	3	16			6	7	6	13	5	1

To sum up, the analysis of Meltem's noticing of algebra tasks demonstrated that she was able to notice all MOST instances. During the interaction, she had a tendency to make students apply correct procedures to reach the correct answer through short-answer questions or yes/no type of questions. In addition to this, her responding level progressed to Level 3 in one of the MOST instances. In terms of her interpretation level, she mostly commented on the possible reasoning behind students' thinking by providing examples from students' work or vignettes from student-teacher interaction but do not explain the justifications explicitly. In addition to that, she gave a detailed explanation about possible reasoning behind students' thinking by providing valid justifications for six MOST instances in her written report. During the oral reflection, she directly mentioned about eight of the MOST instances while she did not talk about three of them. In one of the cases she did not directly mention about the MOST instance by giving general comment about students' work.

6.7. Meltem's Noticing of MOST Instances in Data Analysis Tasks

There were total of 7 MOST instances observed in Meltem's implementation of data analysis tasks. The analysis of Meltem's noticing of MOST instances is given in Table 6.14.

Table 6.14. Meltem's noticing of MOST instances in data analysis tasks.

	MOST	**Interaction**		**Written Report**		**Oral Reflection**
		Att.	Res.	Att.	Int.	
	M1	1	1	IR	1	M
Week 17	M2	1	3	IR	4	M
	M3	1	2	IR	4	M
Week 18	M4	1	3	IR	2	M
	M5	1	2	IR	4	M
Week 19	M6	1	2	IR	3	NM
	M7	1	3	IR	1	NS
Note: Att: Attenting; Int: Interpreting; Res: Responding						

As demonstrated in Table 6.14, Meltem was able to attend all MOST instances. When her responding actions were analyzed, it was seen that one of the MOST instances were responded at Level 1 (Explanation), three of them were responded at Level 2 (Orientation) and three of the MOST instances were responded at Level 3 (Exploration).

Moreover, three of the MOST instances, namely M2, M3 and M5, she interpreted students' mathematics accurately and thoroughly in her report, which were coded as at Level 4. One of the MOST cases were interpreted at Level 2 because she commented possible reasoning behind students' mathematics by providing limited justification such as lack of attention or content. Two of the MOST instances were interpreted at Level

1 because she rephrased students' written procedures or pointed out students' mistake. She commented one of the MOST instances (M6) at Level 3 in her written report because she interpreted possible reasoning behind students' mathematics. However, she could not explain justifications behind their reasoning in detail way.

During the oral reflection sessions, she directly talked about five MOST instances (M1, M2, M3, M4 and M5). For one of the MOST cases (M7), she mentioned generally about them during oral reflection sessions. However, for other instances, she either did indirectly mention about them or not.

For all MOST instances, Meltem mentioned about them in both her written report and interaction with students, that is her in the moment noticing skills. To examine Meltem's noticing of MOST instances occurred during the implementation of data analysis tasks in depth, one of the MOST instances, M4 was chosen. The aim of this task was to support students' understanding of mean, median and mode. The task was also designed to improve students' understanding of data interpretation. The MOST instance, namely M4, observed during the implementation of in this task was about data analysis and interpretation of this data.

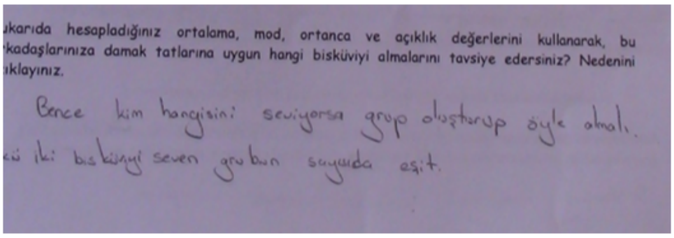


Figure 6.12. Example of MOST instance related to interpretation of data (Nuray's work).

The mathematical point (MP) of this MOST case is verbalized as that the mean, mode, median and range values of data set are taken into consideration while comparing two data sets. However, student thought that data set which includes the maximum value is preferred while comparing two data set and it is enough to look at the mean of data set while comparing them. Meltem realized student's mistake and began to interact with student as shown below.

- Meltem: In second question, I want to give you several reasoning. Which one did you prefer in this question?
- Emre: It can be B.
- Meltem: Why?
- Emre: Its taste is good.
- Meltem: You should look at the values. What did others say about this question?
- Elif: I preferred both of them.
- Meltem: Why?
- Elif: Because, taste of each person is different.
- Nuray: We should only look at the means of data set.
- Nihat: I cannot decide to this question because the means of data set are equal.
- Meltem: Yes, the means of two data set are equal. If the means of data sets are equal, which one will you prefer?
- Nuray I will to prefer my favorite.
- Meltem: Which value will you look at among median, mode and range if means are equal?
- Nuray: I will look at range of data sets.
- Meltem: Why?
- Nuray: I don't know.
- Nihat: I will look at the maximum values for biscuit A and biscuit B.
- Meltem: Nihat said that he will prefer biscuits in terms of maximum values so which one will you prefer?
- Nihat: I will prefer to biscuit A.
- Meltem: Which biscuit will you prefer?
- Emre: I will look at range of each data set and also median and mode. I will prefer the highest of them.
- Meltem: Now, what can you say about range of data set? The smallest value is subtracted from the largest one. There is five and also one. There are persons who would give five and one. There are persons who would like the most and, the least. In here, there are four and two. Which one will you prefer biscuit A or

biscuit B?

- Nuray: Teacher, I did not understand.
- Meltem: In other words, you will look at data set. What can you say about mode of data set?
- Emre: Mode is any number which is repeating frequently.
- Meltem: Which one will you prefer if you are imperative?
- Nihat: I will prefer biscuit which has largest value.
- Nuray: It is non-sense.

In her written report, she noted that:
In second question, Nuray said that everybody should prefer favorite ones because means of data sets were equal. When it comes to interpretation, everybody preferred to look at the means of data set. I said that the means of data set are equal, so I asked to look at other values of data sets. Nuray said to look at range of data set however she cannot explain the reasoning behind it. Again, I asked for the range of data set then I asked Nuray that which biscuit was preferred. However, she did not give full answer and time for discussion was finished.

In her oral reflection, she mentioned about MOST instance such that: In interpretation part, both biscuit A and biscuit B were preferred. I said that the means of data sets are equal then Nuray said to look at range of data set. However, she did not know the reasoning behind it. My group did not understand interpretation part.

During the interaction, Meltem recognized Nuray's mistake and she attempted to elicit students' thinking by asking probing questions (why, which one) (Line 4, Line 8, Line 13, Line 18, Line 21 and Line 36). Although her effort to eliminate students' misunderstanding and explore her understandings, the conversation between group and Meltem was not concluded. Therefore, her responding for this MOST instance was coded as Level 3 (Exploration). She mentioned about MOST instance in her written report, so it was coded as IR. In addition to this, she mentioned about this MOST instance during the oral reflection. In her written report, she mentioned such that Nuray told to look at rate of data set different from other students in the group and she did not know the reasoning behind it. Therefore, Meltem's this interpretation was attributed to Nuray's lack of knowledge. Meltem's noticing in her written report was coded at Level 2. In Table 6.15, summary of Meltem's noticing for data analysis tasks is given.

Table 6.15. Summary of Meltem's noticing of MOST instances in data analysis tasks

Interaction							**Written Report**									**Oral Reflection**		
Att.		Res.					Att.				Int.							
0	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**M**	**NM**	**NS**
-	7	-	1	3	3		-	-	-	7		2	1	1	3	5	1	1

To sum up, the analysis of Meltem's noticing of data analysis tasks demonstrated that she was able to notice all MOST instances while interacting with group of students. During the interaction, she had a tendency to attempt to elicit students' thinking by asking probing questions (why, how, what if) however, the conversation was not concluded, or she failed to address the gap in students' mind. Unlike to number and algebra activities, Meltem's responding action in terms of in-the-moment noticing skill develop into Level 3. In terms of Meltem's written reports, she interpreted students' thinking in higher level because she gave detailed explanation about possible reasoning behind students' mathematics by providing valid justifications. During the oral reflection, she directly mentioned about five of the MOST instances while she did not talk about one of them. In one of the cases she did not directly mention about the MOST instance by giving general comment about students' work.

6.8. Meltem's Noticing of MOST Instances in Geometry Tasks

There were total of 10 MOST instances observed in Meltem's implementation of geometry tasks. The analysis of Meltem's noticing of MOST instances is given in Table 6.16.

Table 6.16. Meltem's noticing of MOST instances in geometry tasks.

	MOST	Interaction		Written Report		Oral Reflection
		Att.	Res.	Att.	Int.	
Week 14	M1	1	1	IR	2	NM
Week 16	M2	1	1	IR	2	M
Week 20	M3	1	2	IR	4	M
	M4	1	2	IR	2	M
Week 21	M5	1	2	IR	4	M
	M6	1	2	IR	1	M
Week 22	M7	1	2	IR	1	M
	M8	1	1	IR	4	M
Week 23	M9	1	4	IR	4	M
	M10	1	2	IR	4	M
Note: Att: Attenting; Int: Interpreting; Res: Responding						

As demonstrated in Table 6.16, Meltem was able to attend all of the MOST instances. When her responding actions were analyzed, it was seen that three of the MOST instances were responded at Level 1 (Explanation), six of them were responded at Level 2 (Orientation) and one of the MOST instances were responded at Level 4 (Elaboration).

Moreover, five of the MOST instances, namely M3, M5, M8, M9 and M10, she interpreted students' mathematics accurately and thoroughly in her report, which were coded as at Level 4. Three of the MOST cases (M1, M2 and M4) were interpreted at Level 2 because she commented possible reasoning behind students' mathematics by providing limited justification such as lack of attention or content. Two of the MOST instances (M6, M7) were interpreted at Level 1 because she rephrased students' written procedures or pointed out students' mistake.

During the oral reflection sessions, she directly talked about nine MOST instances. For one of the MOST cases, she mentioned generally about them during oral reflection sessions.

For all MOST instances (coded as IR in Table 6.16), she mentioned about them during interaction with other students and in her written report.

To examine Meltem's noticing of MOST instances occurred during the implementation of geometry tasks in depth, two of the MOST instances, namely M3 and M9 were chosen. The aim of this task implemented in Week 20 was to support students' understanding about surface area and volume of geometric solids. The MOST instance, namely M3, observed during the implementation of in this task was that students did not find volume of given rectangular prism. The MOST instance was based on verbal expressions of Nuray and Emre. The mathematical point (MP) of this MOST case is verbalized as that volume of any rectangular prism is found by multiplying width, length and height of it. However, student thought that volume of any rectangular prism is found by multiplying any two dimensions of it. Meltem realized student's mistake and began to interact with student as shown below.

- Meltem: What could you say about formula of volume of rectangular prism?
- Nuray: I found six.
- Meltem: How did you find six? What can you say about the width and height of rectangular prism?
- Nuray: The width of rectangular prism is two and the height of it is one. The length of it is three.
- Meltem: The volume of it is 6. What can you say about this volume?
- Nuray: The width of it is 1, the length of it is 6 and the height of it is one.
- Meltem: How will you find the volume of this rectangular prism?
- Emre: I will add it.
- Meltem: Is it valid for all volume of rectangular prisms?
- Emre: No.
- Meltem: How will you find by using other method?
- Emre: I can multiply.
- Meltem: Is it valid formula for all volume of rectangular prisms if you multiply of it?
- Emre: I will say all of them.

- Meltem: If you multiply of it, you will get six. What can you say about formula of volume of rectangular prisms?
- Nuray: The width is multiplied by the length.
- Emre: The length is multiplied by the height.
- Meltem: We have three dimensions for this rectangular prism. What will we do?
- Emre: I will multiply it.
- Meltem: What can you say about the formula of volume?
- Emre: The width is multiplied by the height.
- Meltem: Is it true? What can you say about the width, length and height of this rectangular prism?
- Nuray: The width, length and height of it are 1 cm.
- Meltem: The volume of it is 1 cm. The width, length and height of it are 1, 2, 3 and we multiplied of them and got 6.
- Nuray: We will add all of dimensions.
- Meltem: Will you multiply all dimensions or add all of them?
- Esin: We will multiply.
- Nuray: The width multiplied by the length multiplied by the height.

In her written report, she noted that:

Emre said to add all dimensions to find volume. In other rectangular prism, we tried to add all dimensions to find volume of rectangular prism. We did not find volume of it while adding all dimensions. As other idea, they said to multiply dimensions to find volume of it. Firstly, they suggested that it can be multiplied width and length of rectangular prism. If the length of rectangular prism is 1, they suggested that it can be multiplied the height and width of rectangular prism. We tried to all of those suggestions. Then, we saw that multiplying the width, the length and the height of rectangular prism gives the formula to find the volume of rectangular prism. In oral reflection, the MOST instance was mentioned such that: Emre said to add all of dimensions to find volume. Another student suggested that it can be multiplied two dimensions. Nuray said that it was not any solution to multiply just two dimensions because she thought that the height and the length of rectangular prism are identical. Then, Nuray understood the formula to find volume of rectangular prism.

During the interaction, Meltem recognized Nuray's and Emre's mistakes and she tried to direct student to correct answer by asking him yes/no type of questions (Line 11, Line 15 and Line 27) and short answer type of questions (Line 3, Line 4, Line 17, Line 25, Line 33). Therefore, her noticing for this MOST instance was coded as at Level 2 (Orientation). Meltem attended to this MOST in her report. Meltem wrote adding all dimensions to find volume or multiplying two dimensions to find volume of it. So, her attention was coded as IR. She stated that firstly, they multiplied only two dimensions of rectangular prism, so they did not find the formula of rectangular prism. When her interpretation was evaluated, reasoning behind students' mathematics such that multiplying two dimension or adding all of the dimensions to find volume was appropriate and was justified explicitly via interaction with students. Therefore, her interpretation of student mathematics, that is, her noticing in the report was coded as at Level 4. Moreover, she mentioned about this MOST during the oral reflection.

Another MOST instance was chosen from Week 23's tasks, namely M9. This task was prepared by Meltem and the aim of this task was to support students' understanding of transformation geometry and area of geometric figures. In given task, the question was asked to determine symmetry of "d" triangle with respect to "m" line. The MOST instance observed during the implementation of it in this task was about that students could not determine the symmetry of a given figure with respect to given line as shown in Figure 6.13.

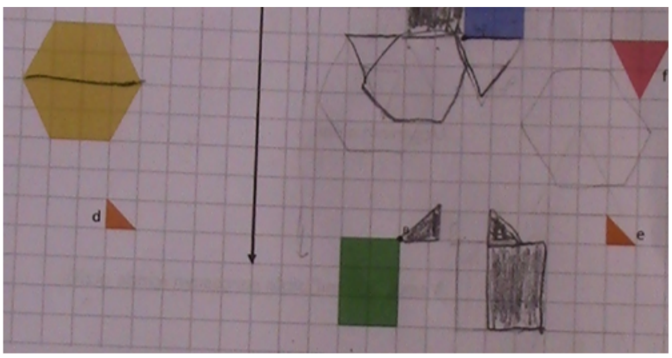


Figure 6.13. Example of MOST instance related to symmetry of geometric figure (Nihat's work).

The mathematical point (MP) of this MOST case is verbalized as that the symmetry of any geometric figure is flipped figure with the same distance to the given symmetry axis. However, student thought that the symmetry of any geometric figure is formed away from axis of the symmetry. Meltem realized student's mistake and began to interact with student as shown below.

- Meltem: Nihat, how did you find the symmetry of this given triangle?
- Nihat: Teacher, I showed it here. I drew the symmetry of given triangle considering line of m.
- Meltem: What is symmetry? Why did you draw symmetry of it in here?
- Nihat: Hmm... his symmetry will be in this way.
- Meltem: What is the reason behind it?
- Nihat: Because, this is not a symmetry. It should be on a line.
- Meltem: What did you think while drawing symmetry here? For example, your friend said to count 4 units from symmetry axis. Why did you construct symmetry in here?
- Nihat: I sketched it randomly.
- Meltem: When you constructed this one, this one is "m" line. How did you draw the
- Nihat: symmetry of this point? This was rotated 90 degrees. This will move on four
- Meltem: units. Is this a line?
- Nihat: Yes.
- Meltem: How did you draw symmetry of this point considering to symmetry axis? For example, you supposed to a point instead of this triangle. What can you say about the symmetry of this point according to line of m? Why did you choose this one? Sorry... 1, 2 and 1, 2 for this point. 1, 2, 3, 4 and 1, 2, 3, 4 for this triangle.

In her written report, she noted that:

In orange triangles, Esin made a mistake about symmetry of d triangle. While she told her solution, she corrected to her mistake. Nihat also could not construct symmetry of d triangle. However, I asked to him that why he did but he did not give a respond. Because Nihat did not understand the symmetry concept, I asked different examples to him. Firstly, I drew half triangle and he was able to draw half of it in symmetry axis. After doing this, I drew a point whose distance from symmetry axis was 2 units. He drew point of symmetry whose distance from symmetry axis 3 units. I asked to him that why he draw in that way. Then, he realized that he makes a mistake and he said that the point of symmetry should be equidistant from symmetry line by counting the distance between the original point and symmetry line. Then, he checked accuracy of symmetry of d triangle.

In oral reflection, the MOST instance was mentioned such that: Nihat made a mistake about symmetry and he drew symmetry of figures randomly. Then, I drew a point closer to symmetry line. He drew its symmetry randomly and lastly, he realized that he makes a mistake. Finally, he understood how to find the symmetry of a point.

During the interaction, Meltem recognized Nihat's mistake (Line 1 and Line 4). Meltem attempted to elicit students' thinking by asking probing questions (Line 4, Line 6, Line 8, Line 10, Line 12, Line 19) and guiding students through appropriate examples, representations (Line 16,17,18,19). Therefore, her respond for this MOST instance, in other words, her in-the-moment noticing was coded as Elaboration (Level 4). However, in her written report, Meltem commented on possible reasoning behind student's mathematics by providing vignettes from student-teacher interaction such that she drew a point which its distance from symmetry axis to 2 units then he drew point of symmetry which its distance from symmetry axis to 3 units. Therefore, her interpretation level was coded as 4 because her reasoning was appropriate for students' thinking. In addition to this, she mentioned about this MOST instance during the oral reflection. Table 6.17, summary of Meltem's noticing for geometry tasks is given.

Table 6.17. Summary of Meltem's noticing of MOST instances in geometry tasks

Interaction							**Written Report**									**Oral Reflection**		
Att.		Res.					Att.				Int.							
0	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**M**	**NM**	**NS**
-	10	-	3	6	-	1	-	-	-	10	-	2	3	-	5	9	1	-

Shortly, the analysis of Meltem's noticing of geometry tasks demonstrated that she was able to notice all MOST instances. During the interaction, she had a tendency to direct students through short-type of questions/ yes-no type of questions. However, in Week 23, her responding action in terms of in the moment noticing skill developed into Level 4. In terms of her interpretation level, five of the MOST instances were interpreted at Level 4 while three of them were interpreted at Level 2. She was able to attend to the MOST instances and interpreted MOST instances in higher level. In most of the MOST instances, she interpreted students' thinking by providing valid justifications or did not explain justifications explicitly as discussed above. During the oral reflection, she directly mentioned about nine of the MOST instances while she did not talk about one of them. In Table 6.18, the summary of Meltem's noticing of MOST instances in all all tasks out of retention study is given.

Table 6.18. Summary of Meltem's noticing of MOST instances in all tasks of intervention study.

	Interaction (%In-the-moment)							**Written Reports**						**Oral Reflection**					
	Attending		Responding					Attending				Interpreting							
Content area	**0**	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**NM**	**M**	**NS**
Numbers	2 (%16)	10 (%84)	-	7 (%70)	2 (%20)	1 (%10)	-	-	2 (%17)	2 (%17)	8 (%66)	-	4 (%40)	3 (%30)	3 (%30)	-	3 (%25)	5 (%41)	4 (%34)
Algebra	3 (%16)	16 (%84)	-	5 (%31)	10 (%62)	1 (%7)	-	-	-	3 (%16)	16 (%84)	-	-	6 (%32)	7 (%36)	6 (%32)	5 (%26)	13 (%69)	1 (%5)
Data Analysis	-	7 (%100)	-	1 (%14)	3 (%43)	3 (%43)	-	-	-	-	7 (%100)	-	2 (%29)	1 (%14)	1 (%14)	3 (%43)	1 (%14)	5 (%72)	1 (%14)
Geometry	-	10 (%100)	-	3 (%30)	6 (%60)	-	1 (%10)	-	-	-	10 (%100)	-	2 (%20)	3 (%30)	-	5 (%50)	1 (%10)	9 (%90)	-
Total	**5 (%10)**	**43 (%90)**	**-**	**16 (%37)**	**21 (%49)**	**5 (%12)**	**1 (%2)**	**-**	**2 (%4)**	**5 (%10)**	**41 (%86)**	**-**	**8 (%17)**	**13 (%28)**	**11 (%24)**	**14 (%31)**	**10 (%21)**	**32 (%66)**	**6 (%13)**

In terms of all tasks in the intervention study, Meltem attended to 43 MOST instances (%90) MOST instances while missing five MOST instances (%10) within 48 MOST instances as shown in Table 6.18. In terms of responding level, 16 MOST instances were responded at Level 1 (%37) while 21 of them were responded at Level 2 (%49). Five MOST instances were responded at Level 3 (%12) while one of them were responded at Level 4 (%2).

For the interpretation level of Meltem, eight MOST instances were interpreted at Level 1 (%17) while 13 MOST instances were interpreted at Level 2 (%28). Eleven MOST instances were interpreted at Level 3 (%24) while 14 of them were interpreted at Level 4 (%31). Considering oral reflections of Meltem, ten MOST instances were not mentioned (%21) while 32 MOST instances were mentioned (%66) during meetings with researchers. Six MOST instances were not specifically mentioned (%13) because Meltem gave general comment about students' difficulties.

In terms of written reports of Meltem, two MOST instances were just mentioned in interaction with students of Meltem (%4). In addition to this, five MOST instances were only mentioned in her written reports of Meltem (%10) while 41 of them were attended in interaction with students and written reports of Meltem (%86).

6.9. Derya's Noticing of MOST Instances in Algebra Tasks in Retention Study

There were total of 16 MOST instances observed in Derya's implementation of algebra tasks for retention study. The analysis of Derya's noticing of MOST instances is given in Table 6.19.

Table 6.19. Derya's noticing of MOST instances in algebra tasks for retention study.

	MOST	**Interaction**		**Interview**	
		Att.	Res.	Att.	Int.
Week 1	M1	1	2	IR	2
	M2	1	2	IR	3
	M3	1	1	I	
	M4	1	2	IR	4
	M5	1	2	IR	4
Week 2	M6	1	1	IR	2
	M7	1	1	IR	2
	M8	1	1	I	
	M9	1	2	IR	2
Week 3	M10	1	2	IR	4
	M11	1	1	IR	2
	M12	1	1	I	
	M13	0		NIR	
Week 4	M14	1	1	IR	1
	M15	1	1	IR	3
	M16	0		R	3

As demonstrated in Table 6.19, Derya was able to attend 14 MOST instances while she missed two of them during interaction. When her responding actions were analyzed, it was seen that eight MOST instances were responded at Level 1 (Explanation), six of them were responded at Level 2 (Orientation).

Moreover, three of the MOST instances, namely M4, M5 and M10, she interpreted students' mathematics accurately and thoroughly during interview, which were coded as at Level 4. Three of the MOST instances were interpreted at Level 3, namely M2, M15 and M16, she commented on possible reasoning behind students' thinking however, she did not justify explicitly. Five of the MOST cases (M1, M6, M7, M9, M11) were interpreted at Level 2 because she commented possible reasoning behind students' mathematics by providing limited justification such as lack of attention or content. One of the MOST instances (M14) were interpreted at Level 1 because she rephrased students' written procedures or pointed out students' mistake. For three

MOST instances, namely M3, M8 and M12 coded as I in Table 6.19, she only mentioned to them during interaction with students that is, her in-the-moment noticing skills. For one of the MOST instances, namely M16 coded as R in Table 6.19 , she mentioned only of it in her interview with researcher. In one of the MOST instances, namely M13 coded as NIR in Table 6.19, she missed this MOST instance during interaction and interview with researcher. For other MOST instances coded as IR in Table 6.19, she attended to them during interaction with students and interview with researcher.

To discuss Derya's noticing of MOST instances occurred during the implementation of algebra tasks for retention study in depth, two MOST instances were chosen. One of the MOST instances was M10 and the other one was M15. In one of the third week's task, it was aimed to support students' understanding of algebraic expressions and operations with algebraic expressions. The MOST instance, namely M10, observed during the implementation of in this task was to find the result of addition of algebraic expressions "(3x+5) + (2x+2)" as 8x+4x and to find the result of operation "6x+4/2" as 10x/2 demonstrated in Figure 6.14.

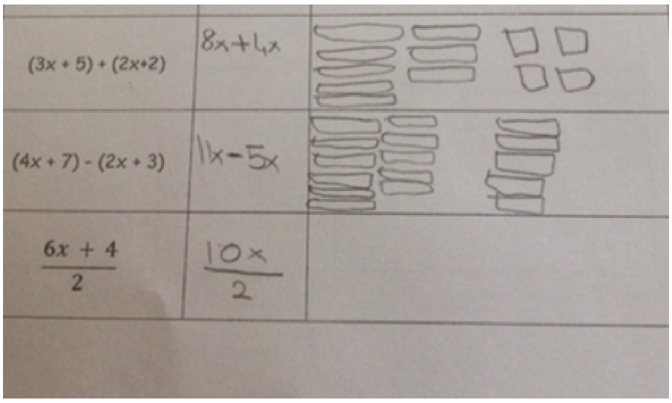


Figure 6.14. Example of MOST instance related to algebraic expressions (Kerem's work).

For this MOST case we can verbalize the mathematical point (MP) as "similar terms of algebraic expressions can be added or subtracted". However, student thought that when algebraic expressions given in parenthesis he can add the coefficient of the unknown with the constant term and the unknown would leave as itself. Derya realized student's mistake and began to interact with student as shown below.

- Canan:Teacher, I added to 3x plus 2x and then 5 plus 2. I got 5x plus 7.
- Kerem: I did not do that.
- Derya: How did you solve this?
- Kerem: I added 3x plus 5 then got 8x.
- Derya: Are you adding 3x plus 5? Please, show me to 3x then 5. You can represent 5 by using small algebra tiles. There is five added to 3x. What will be added to 3x+5?
- Kerem: 2x plus 2.
- Derya: 2x plus 2 (pointed at algebra tiles). Do you think that you will get 8x by adding 3x and this one (pointed at "5")? But, we should add similar terms to similar ones. The x term should be added with x ones.
- Esin: In algebraic expressions, unknown terms should be added with unknown ones.
- Derya: That's right! Terms with unknowns should be added to the terms with the same unknown and constant terms should be added with constant ones.
- Kerem: Teacher, we are adding 3 plus 2.
- Derya: Okey, you got 5x.
- Kerem: Known terms are added. Then five plus two is seven.
- Derya: Five and two are constant terms. So what is the result?
- Kerem: 5x plus 2.

In her interview, Derya mentioned about MOST instance such that:

- Researcher: What can you say about difficulties of students while implementing the task?
- Derya: Kerem had difficulty about algebraic operations. He forgot that similar terms can be added or subtracted. He directly added 3x to 5 and wrote 8x. For another parenthesis, he wrote 4x.
- Researcher: What is the reason behind it?
- Derya: I wanted to show this operation by using algebra tiles. He used 1 by x

tiles when I asked him to show 3x and he used the same tiles to represent 5. Most probably he did not pay attention to my explanations about algebra tiles. Then his friends reminded him which of algebra tile represents 1, which one represent x.

- Researcher: How did you respond it?
- Derya: I showed the operation by using algebra tiles. In second line, he solved correctly. I responded that he should take 3x and five constant. Generally, they made mistakes about addition of x and constants.

During the interaction, Derya recognized Kerem's mistakes (Line 3) and she tried to direct student to correct answer by asking him yes/no type of questions (Line 5 and Line 9) and short answer type of questions (Line 7 and Line 19). She directed to explain solution way to students however; she mostly used algebra tiles to understand operations with algebraic expressions. Therefore, her noticing for this MOST instance was coded as at Level 2 (Orientation). Derya attended to this MOST in during her interview. Derya mentioned about Kerem's difficulty related to algebraic operations. So, her attention was coded as IR. She stated that Kerem directly added 3x with 5 and 2x with 2. In addition to this, she mentioned about he did not know the knowledge which similar terms are added with similar ones and unlike terms are added with unlike ones. When her interpretation was evaluated, reasoning behind students' mathematics such that adding known terms with unknown ones in algebraic expressions was appropriate and was justified explicitly via interaction with students. Therefore, her interpretation of student mathematics, that is, her noticing during the interview was coded as at Level 4.

Another MOST instance was chosen from fourth week's tasks, namely M15 as given in Figure 6.15. The aim of this task was to support students' understanding of writing verbal expressions in algebraic form and solving them. The MOST instance observed during the implementation of it in this task was to find the result of 6x+x/4 as 7x/4 which is demonstrated in Figure 6.15.

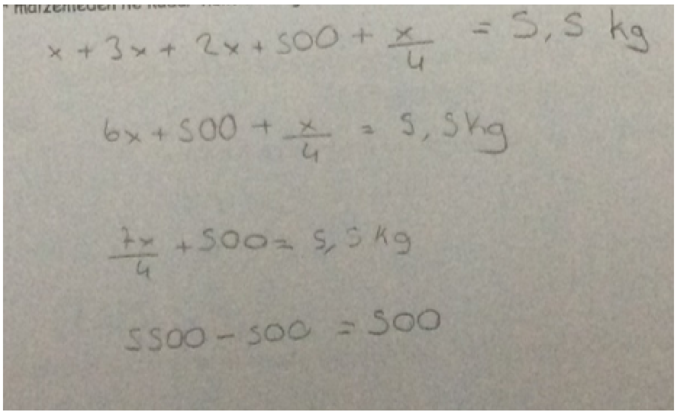


Figure 6.15. Example of MOST instance related to algebraic expressions (Melis' work).

For this MOST case we can verbalize the mathematical point (MP) as "while adding algebraic expressions involving rational coefficients, denominators are enlarged to get common denominators and then similar terms are added". However, student thought that while adding two algebraic expressions consisting of fractions, numerators of them directly are added without getting common denominators. Derya realized Melis' mistake and began to interact with student as shown below.

- Derya: What did you do in this question?
- Melis: I wrote total of goods and wrote 5,5 kilos. There were 6x plus 500 gram plus x/4. This was equal to 5500 gram. 6x plus x /4 was equal to 7x/4.
- Derya: 6x is whole number but x/4 is a fraction, there is division line here. Are you adding them up directly?
- Melis: In a sense...
- Derya: Just few minutes ago, we added 1 plus $\frac{1}{2}$. Kerem mentioned about hidden one under whole number 1. Like addition with fractions you should find the common denominator of given algebraic terms should find the common denominator of given algebraic terms any number instead of x. Have you convinced about that?
- Melis: Teacher, I found common denominator by multiplying by 2. I got 24x /4 and it was added with x/4. Then the result was 25x /4.

In her interview, Derya mentioned about MOST instance such that:

- Researcher: Let's look at this question.
- Derya: Melis thought addition of all terms in algebraic expressions. She added x terms and found 6x plus 500. She only made a mistake while adding 6x plus x/4. She found 7x /4.
- Researcher: What is the reason behind this mistake?
- Derya: I asked to her and she responded that I added directly. However, I said that one of the terms is fraction and another one is not fraction I asked that how can you add all of them.
- Researcher: How can you interpret students' thinking in this problem?
- Derya: Melis might think such that we always remember the rule which terms with unknowns are added with each other and constant terms are added to each other. If she thinks that way, she might add one and and four so she might get five. She did not make so much reasoning and she directly added terms with x's with each other.

During the interaction, Derya recognized Melis' mistake (Line 4 and Line 7). Derya attempted to tell solution way to Melis and tried to reach right answer to Melis (Line 9, Line 10 and Line 11). Therefore, her respond for this MOST instance, in other words, her in-the-moment noticing was coded as Explanation (Level 1). However, in her interview, Derya commented on possible reasoning behind student's mathematics by providing vignettes from student-teacher interaction such that she added directly similar terms with similar ones. Due to this interpretation in her interview, she did not explain students' understanding explicitly such that Melis does not make so much reasoning. To justify for students' thinking, Derya should mention about finding common denominators of fractions with algebraic expressions in detail. Therefore, her interpretation of student's mathematics, that is, her noticing during the interview was coded as at Level 3.

Table 6.20. Summary of Derya's noticing of MOST instances in algebra tasks for retention study.

Interaction							Interview								
Att.		Res.					Att.				Int.				
0	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**
2	14	-	8	6	-	-	1	3	1	11	-	1	5	3	3

Briefly, the analysis of Derya's noticing of algebra tasks for retention study revealed that she was able to notice the MOST instances despite of missing two MOST instances. However, her major concern was to make students apply correct procedures to get the answer or orient students to reach correct answers via short answer type questions and yes/no type of questions. As similar 2016-2017 algebra task implementation process, Derya had a tendency to tell solution way or direct students to reach correct answer during implementation. In terms of interview with researcher, three MOST instances were interpreted at Level 4 while three of them were interpreted at Level 3. Thus, she was able to interpret students' thinking by providing valid justifications for her explanations as discussed above or she commented on possible reasoning behind students' thinking by providing vignettes from student-teacher interaction. In below Table 6.21, the summary of Derya's noticing of MOST instances in retention study for algebra tasks and algebra tasks implemented in 2016-2017 were given.

Table 6.21. Summary of Derya's noticing of MOST instances in algebra tasks during both intervention and retention studies.

	Interaction (%In-the-moment)							Written Reports or Interview											
	Attending		Responding					Attending				Interpreting					Oral Reflection		
Content area	**0**	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**NM**	**M**	**NS**
Algebra	6 (%31)	13 (%69)	-	8 (%62)	5 (%38)	-	-	3 (%16)	1 (%5)	3 (%16)	12 (%63)	-	4 (%27)	4 (%27)	1 (%6)	6 (%40)	5 (%26)	11 (%58)	3 (%16)
Retention for Algebra	2 (%12)	14 (%88)	-	8 (%57)	6 (%43)	-	-	1 (%6)	3 (%19)	1 (%6)	11 (%69)	-	1 (%8)	5 (%42)	3 (%25)	3 (%25)	-	-	-

In terms of retention study for algebra tasks, Derya attended to 14 MOST instances (%88) while missing two MOST instances (%12) within 16 MOST instances as shown in Table 6.21. In terms of responding level, eight MOST instances were responded at Level 1 (%57) while six of them were responded at Level 2 (%43).

For the interpretation level of Derya in retention study for algebra tasks, one MOST instances were interpreted at Level 1 (%8) while five MOST instances were interpreted at Level 2 (%42). Three MOST instances were interpreted at Level 3 (%25) while three of them were interpreted at Level 4 (%25).

During interview of Derya for retention study in algebra tasks, three MOST instances were just mentioned in interaction with students of Derya (%19) coded as I in Table 6.21. In addition to this, one MOST instance was only mentioned in her interview of Derya (%6) coded as R in Table 6.21 while 11 of them were attended in interaction with students and interview of Derya (%69) coded as IR in Table 6.9.3. One of the MOST instances were missed during interaction with students and interview with Derya (%6), coded as NIR in Table 6.21.

By comparing findings of algebra tasks implemented during intervention, the attending level of Derya progressed to %88 from %69. In addition to this, the percentage of missing MOST instances regressed (from %31 to %12) in retention study for algebra tasks. In terms of responding level of Derya, the findings of algebra tasks implemented in 2016-2017 were similar to findings of retention study which she responded to MOST instances as an answer focused. In interpretation level of Derya, the percentage of Level 2 developed (from %27 to %42) while percentage of Level 1 (from %27 to %8) regressed during interview with researcher. Moreover, the percentage of Level 4 regressed (from %4 to %25) while percentage of Level 3 developed (from %6 to %25).

6.10. Meltem's Noticing of MOST Instances in Algebra Tasks in Retention Study

There were total of 17 MOST instances observed in Meltem's implementation of algebra tasks for retention study. The analysis of Meltem's noticing of MOST instances is given in Table 6.22.

Table 6.22. Meltem's noticing of MOST instances in algebra tasks for retention study.

	MOST	Interaction		Interview	
		Att.	Res.	Att.	Int.
Week 1	M1	1	4	IR	4
	M2	1	3	IR	2
	M3	1	3	IR	2
	M4	1	2	IR	1
Week 2	M5	1	2	IR	2
	M6	1	3	IR	2
	M7	1	2	I	
	M8	1	1	IR	3
Week 3	M9	1	2	IR	2
	M10	1	2	IR	4
	M11	1	2	IR	4
	M12	1	2	IR	4
	M13	1	2	IR	2
Week 4	M14	1	2	IR	4
	M15	1	2	IR	2
	M16	1	1	I	
	M17	1	1	IR	3

As demonstrated in Table 6.22, Meltem was able to attend all MOST instances. When her responding actions were analyzed, it was seen that three MOST instances were responded at Level 1 (Explanation), ten MOST instances were responded at Level

2 (Orientation), three MOST instances were responded at Level 3 (Exploration), one MOST instance was responded at Level 4 (Elaboration).

Moreover, five of the MOST instances, namely M1, M10, M11, M12 and M14, she interpreted students' mathematics accurately and thoroughly during interview, which were coded as at Level 4. Two of the MOST instances were interpreted at Level 3, namely M8 and M17, she commented on possible reasoning behind students' thinking however, she did not justify explicitly. Seven of the MOST cases (M2, M3, M5, M6, M9, M13 and M15) were interpreted at Level 2 because she commented possible reasoning behind students' mathematics by providing limited justification such as lack of attention or content. One of the MOST instances (M4) were interpreted at Level 1 because she rephrased students' written procedures or pointed out students' mistake. Meltem attended to 15 MOST instances, coded as IR in Table 6.10.1, in interview with researcher and during the interaction with students. For two of the MOST instances, namely M7 and M16 coded as I in Table 6.22, she attended only during interaction with students and missed in interview with researcher.

To discuss Meltem's noticing of MOST instances occurred during the implementation of algebra tasks for retention study in depth two MOST instances, namely M1 and M6 were chosen. In one of the first week's task, it was aimed to support students' understanding of finding pattern rule via pattern blocks. The MOST instance, namely M1, observed during the implementation of in this task was to find the rule of pattern as "y+3" instead of 3y. In this task students focused on the change in dependent variable rather than thinking about the relationship between independent and dependent variable, that is, the relationship between number of years and the number of families in each year as shown in Figure 6.16.

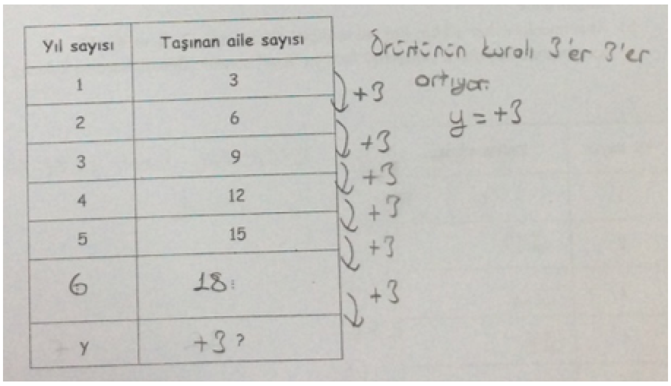


Figure 6.16. Example of MOST instance related to find pattern rule (Alara's work).

For this MOST case we can verbalize the mathematical point (MP) such that when finding the rule of the pattern, it is needed to check the relationship between two variables rather than the change in only one variable. However, student thought that unknown value y is perceived as increasing amount and thus the answer is plus three. Meltem realized Alara's mistake and began to interact with student as shown below.

- İpek: I found that y equals to plus three because the difference between number of families is three.
- Meltem: What did you find?
- Havva: I understood that this point equals to y.
- Alara: Me, too.
- Emel: I thought that any number equals to y.
- Meltem: You thought different values for y.
- Emel: Yes, teacher.
- Meltem: You found +3, right?
- İpek: Yes.
- Meltem: Here, three shows the amount of increase but there are a lot of numbers in here. For instance, how will you find in 100th year? I will find the number of families by operating to number of years. For instance, I will look at 65th year then you will say the number of families by finding rule. How will you respond to it? The number of families is increased by three. How does the relationship between the number of years and the number of families change?

- Havva: By multiplies of three.
- Meltem: Actually, you multiplied by three. You did not understand because, you focused on the change in one variable What can you say about the relationship between number of years and families?
- Alara: I did not understand.
- Meltem: There are number of years and number of families which are moving. I have found the number of families by operating the number of years. What can you say about this operation?
- Alara: I can multiply.
- Meltem: How many times?
- İpek: I don't know.
- Meltem: Up to now, you have looked at the change in one variable. Also, you can look at the relationship between number of years and number of families. What can you say about the first year?
- İpek: Three.
- Meltem: What can you say about the second year?
- Havva: Six.
- İpek: We multiplied one by three. Then, two is multiplied by three
- Meltem: What can you say about this operation? What can you say about the number of families in 60th year?
- İpek: 180.
- Meltem: What can you say about operation?
- İpek: Multiplication.
- Meltem: There is y variable instead of any number.
- İpek: y is multiplied by 3.
- Meltem: What can you say about the result?
- Havva: 3y.

In her interview, Meltem mentioned about MOST instance such that:

- Researcher: What can you say about difficulties of students in terms of first question in the task?

- Meltem: In first question, pattern is increased by three. It was found to the sixth pattern and seventh pattern. However, many students thought y as any number rather than a variable.
- Researcher: Can you give any example to this?
- Meltem: For instance, students wrote that 7y is equal to 21. Students have a tendency for looking at the increasing amount.
- Researcher: What can be the reason behind this?
- Meltem: Students had a tendency to focus on one variable rather than looking at the relationship between number of families and number of years. They have only looked at number of families.
- Researcher: How did you respond it?
- Meltem: I responded that the question is asked to the relationship between number of years and number of families. In the end, they found three times relationship.

During the interaction, Meltem recognized Alara's mistake (Line 7 and Line 9) during the interaction. She attempted to elicit students' thinking by asking probing questions (Line 12, Line 16, Line 22, Line 27 and Line 41) and she guided them through appropriate connections between concepts (Line 31, Line 32 and Line 38). Therefore, her noticing for this MOST instance was coded as at Level 4 (Elaboration). Meltem attended to this MOST in during her interview (Line 3) hence, her attention was coded as IR. When her interpretation was evaluated, reasoning behind students' mathematics such that focusing on the relationship between one variable rather than the relationship between two variables was appropriate and was justified explicitly via interaction with students. Therefore, her interpretation of student mathematics, that is, her noticing during the interview was coded as at Level 4.

Another MOST instance was chosen from second week's tasks, namely M6, as given in Figure 6.17. The aim of this task was to support students' understanding about converting verbal expressions into algebraic expressions. In this task, it was given historical places and following paths. In this figure, it was asked to write the amount of time spent in Küçüksu Palace which is one fourth of the time spent in Dolmabahçe

Palace. The MOST instance, namely M3, observed during the implementation of in this task was to one fourth of 2T/3 as 2T/3/1/4 instead of (2T/3). (1/4) as demonstrated in Figure 6.17.

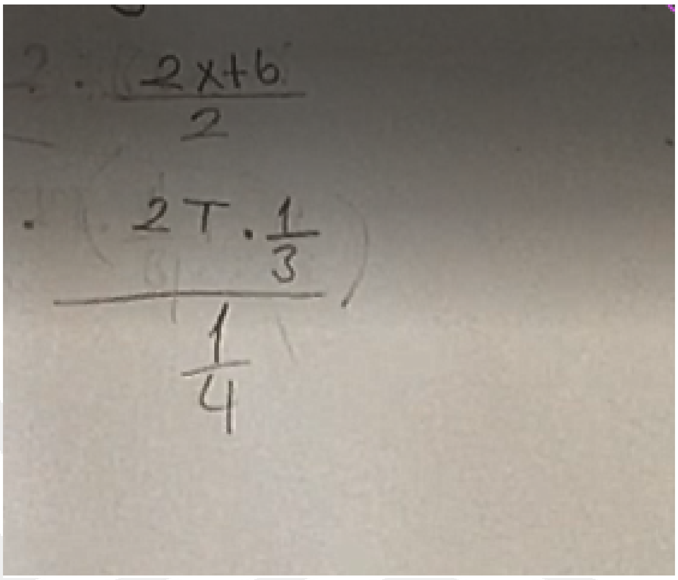


Figure 6.17. Example of MOST instance related to convert verbal expressions into algebraic expressions (Havva's work).

For this MOST case we can verbalize the mathematical point (MP) such that when finding fraction of a fraction given numbers, fractions are multiplied. However, student thought that when finding the fraction of a fraction, the division needs to be done. Meltem realized Havva's mistake and began to interact with student as shown below.

- Meltem: Let's pass to question related to Küçüksu Kasrı.
- Emel: The question asks about writing one fourth of the time spent in Dolmabahçe Palace.
- Meltem: What can you say about time spent in Dolmabahçe Palace?
- İpek: 2T is multiplied by 1/3.
- Meltem: Let's look at Havva's response.
- Havva: I divided 2T/3 by 1/4.
- Meltem: Did you mean dividing by 1/4?

- Havva: Yes.
- Meltem: If you operate 2T/3 divided by 1/4, what can you say about the result?
- Havva: 2T is divided by 3.
- Meltem: Divided by 1/4.
- Havva: Yes.
- Meltem: Why did you divide by 1/4?
- Havva: I need to multiply 2T/3 by 4.
- Meltem: How did you do that?
- Havva: I found 8T/3. I multiplied 2T/3 by 4.
- Meltem: If this question asked to me four times of time spent in Dolmabahçe Palace, what can you say about the result?
- Havva: 2T is multiplied by 1/3 then multiplied by 4.
- Meltem: This question does not ask for four times of spent time. It asks for 1/4 of time spent in Dolmabahçe Palace.
- Havva: The operation will be changed. 2T/3 will be multiplied by 1/4.
- Meltem: Yes. Why did you firstly divide 2T/3 by 1/4?
- Havva: I thought multiplication operation instead of division operation.
- Meltem: You will do this operation to get the result. Havva again multiplied 2T/3 by four. 26 Meltem: Did you find again 8T/3? However, you multiplied by four and the question asks for multiplying by 1/4.
- Havva: Hmm, OK.

In her interview, Meltem mentioned about MOST instance such that:

- Researcher: What did students do in Dolmabahçe Palace?
- Meltem: They thought that 2T/3 is divided by 1/4 instead of multiplication. I gave many examples because one of the students did not understand. She wrote that T is divided by 1/3. I asked to result of this operation. She said to result of it as 3T. Then I asked to three times of T. She said to 3T.
- Researcher: What can you say about the reason behind it?
- Meltem: She found 3T because she needed to find any result. She directly wrote that T is divided by 1/3. That is to say, she did not understand the concept.

During the interaction, Meltem recognized Havva's mistake (Line 7) during the interaction. She attempted to elicit students' thinking by asking probing questions (Line 13, Line 15 and Line 23). However, she failed to address students' gap in her mind because her guidance involves inappropriate examples such as three times of T variable rather than focusing on 1/3 of any variable. Therefore, her noticing for this MOST instance was coded as at Level 3 (Exploration). Meltem attended to this MOST in during her interview (Line 2) hence, her attention was coded as IR. When her interpretation was evaluated, her reasoning behind students' mathematics such that students wrote 3T because, she needed to find any result. Also, she added that she did not understand the concept. Therefore, her interpretation of student mathematics, that is, her noticing during the interview was coded as at Level 2.

Table 6.23. Summary of Meltem's noticing of MOST instances in algebra tasks for retention study

Interaction							**Interview Report**								
Att.		Res.					Att.				Int.				
0	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**
-	17	-	3	10	3	1	-	2	-	15	-	1	7	2	5

Briefly, the analysis of Meltem's noticing of algebra tasks for retention study revealed that she was able to notice all MOST instances. In terms of her responding level, three MOST instances were responded at Level 1 while ten MOST instances were responded at Level 2. In addition to this, three MOST instances were responded at Level 3 while one of them were responded at Level 4. When it was compared to algebra tasks implemented during intervention study, it was seen that her noticing skills showed an improvement in terms of responding to her students. She began to focus on students' mathematical understanding instead of only focusing on correct procedures and results. In terms of her interview analysis, one of the MOST instances was interpreted at Level 1 while seven MOST instances were interpreted at Level 2. Moreover, two MOST instances were interpreted at Level 3 while five of them were interpreted at Level 4. Thus, she interpreted students' thinking in algebra by providing valid justifications for her explanations as discussed above or she commented on possible reasoning behind students' thinking by providing vignettes from student-teacher interaction. Therefore, interpretation stage of her noticing skills was high level because she was aware of possible students' MOST instances and reasoning behind them. As shown in below table Table 6.24, the summary of Meltem's noticing of MOST instances in retention study for algebra tasks and algebra tasks implemented in 2016-2017 were given.

Table 6.24. Summary of Meltem's noticing of MOST instances in retention study for algebra tasks and algebra tasks implemented in 2016-2017.

	Interaction (%In-the-moment)							**Written Reports or Interview**										**Oral Reflection**		
	Attending		Responding					Attending				Interpreting								
Content area	**0**	**1**	**0**	**1**	**2**	**3**	**4**	**NIR**	**I**	**R**	**IR**	**0**	**1**	**2**	**3**	**4**	**NM**	**M**	**NS**
Algebra	3 (%16)	16 (%84)	-	5 (%31)	10 (%62)	1 (%7)	-	-	-	3 (%16)	16 (%84)	-	-	6 (%32)	7 (%36)	6 (%32)	5 (%26)	13 (%69)	1 (%5)
Retention for Algebra	-	17 (%100)	-	3 (%17)	10 (%59)	3 (%17)	1 (%6)	-	2 (%12)	-	15 (%88)	-	1 (%7)	7 (%47)	2 (%13)	5 (%33)	-	-	-

In terms of retention study for algebra tasks, Meltem attended to all MOST instances (%100) as shown in Table 6.24. In terms of responding level, three MOST instances were responded at Level 1 (%17) while 10 of them were responded at Level 2 (%59). In addition to this, three MOST instances were responded at Level 3 (%17) while one of them was responded at Level 4 (%6).

For the interpretation level of Meltem in retention study for algebra tasks, one of the MOST instances was interpreted at Level 1 (%7) while seven MOST instances were interpreted at Level 2 (%47). Three MOST instances were interpreted at Level 3 (%13) while three of them were interpreted at Level 4 (%33).

During interview of Meltem for retention study in algebra tasks, two MOST instances were just mentioned in interaction with students of Meltem (%12) coded as I in Table 6.24. In addition to this, 15 of them were attended in interaction with students and interview of Meltem (%88) coded as IR in Table 6.24.

By comparing findings of algebra tasks implemented in 2016-2017, the attending level of Meltem progressed to %100 from %84. In terms of responding level, the percentage of Level 1 and Level 2 regressed (from %31 to %17 for Level 1 and from %62 to %59 for Level 2) while the percentage of Level 3 and Level 4 developed in retention study for algebra tasks (from %7 to %17 for Level 3 and from %0 to %6 for Level 4). In terms of interpretation level of Meltem, the percentage of Level 2 and Level 1 progressed (from %0 to %7 for Level 1 and from %32 to %47 for Level 2) while percentage of Level 3 regressed (from %36 to %13) during interview with researcher. Moreover, the percentage of Level 4 progressed in interview with researcher (from %32 to %33).

7. DISCUSSION

The purpose of this study was to examine progress in preservice teachers' noticing skills of students' mathematical thinking for one year and after intervention study. For this purpose, a total of 25 tasks about numbers, algebra, data analysis and geometry were implemented for 23 weeks. Also, to detect any progress in preservice teachers' noticing skills, four algebra tasks were re-implemented for four weeks in a year after the intervention study. The definition of professional noticing of students' thinking (Jacobs *et al.*, 2010) was used as a medium to investigate preservice teachers' noticing skills. Their noticing skills were analyzed under the four categories consisting of number tasks, algebra tasks, data analysis tasks and geometry tasks. In this section, the progress in preservice teachers' noticing skills over time is discussed in the light of the findings and literature.

7.1. Progress in preservice teachers' noticing skills over time

The studies on preservice teachers' noticing revealed that preservice teachers' noticing showed an improvement through the intervention studies such that they began to pay attention to noteworthy instances occurred in the classroom rather than non-specific issues (Barnhart and van Es, 2015; Krupa *et al.*, 2017; Stockero *et al.*, 2017; Sun and van Es, 2015). Similarly, in this study, during implementation of tasks in 2016-2017, the attending level of Derya showed a progress throughout time. In the number tasks she attended to %67 of tasks, while she attended to %69 of algebra tasks. Then, her attention to data analysis tasks was %86, while it was %90 for geometry tasks. Although Meltem's attention level did not yield such a result with respect to content areas, it followed a higher path throughout the study such that it was %84 for number tasks, %84 for algebra tasks, %100 for data analysis tasks and %100 for geometry tasks.

In terms of responding level of Derya, she had a tendency to tell the solution way or direct students towards the correct result in all tasks. Even though of her responding pattern was Level 1 for most of the MOST instances, responding level shifted to Level

3 from Level 1 in two MOST instances including data analysis and geometry tasks. In terms of responding level of Meltem, she had a tendency to tell the solution way or attempt to make students find out correct answer through short-answers or yes/no type of questions in number tasks and algebra tasks. For data analysis tasks of Meltem, the responding level was developed into Level 3 such that she attempted to elicit students' thinking by asking probing questions but either she did not conclude the conversation, or she failed to address the gap in her students' mind. In one of the MOST instances in geometry tasks, her responding level was observed as to be Level 4.

In terms of interpretation level of Derya, she mostly interpreted MOST instances at Level 4 in number and algebra tasks. For geometry and data analysis tasks, Derya mostly interpreted students' thinking at Level 2 or Level 3. Thus, there was not a progress in the interpretation level within time. According to the interpretation level of Meltem, she mostly interpreted students' thinking at Level 1 in number tasks while she mostly interpreted students' thinking at Level 4 in algebra, geometry and data analysis tasks. Thus, there is a development in Meltem's interpretation level within time. In terms of oral reflections, Derya mostly mentioned about MOST instances in all tasks while Meltem also did.

In terms of attending pattern in Derya's written reports, Derya attended to the MOST instances which were missed during the interaction in her written report. However, the percentage of MOST instances which were coded as "R" did not change much within time. There is a development in her percentage of MOST instances which were coded as "IR" within time. With regarding to attending pattern of Meltem's written reports, there is an improvement in her percentage of MOST instances which were coded as "IR" within time. In addition to this, Meltem attended all MOST instances in geometry and data analysis tasks.

In the light of findings of this study, there is a gradually progression in percentage of attending level of preservice teachers in all tasks during interaction with students, in other words, in their in-the-moment noticing. This finding is compatible with results of the study conducted by Barnhart and van Es (2015). They found that a preservice teacher who enrolled video-based course demonstrated higher sophistication in attending and analyzing students' thinking. Similarly, in her study, Pascoe (2016) was examined to mathematics teaching assistants' noticing skills who analyzed classroom videos via MOST instances. Additionally, this finding is parallel to findings of the study conducted by Pascoe (2016) because Pascoe (2016) found that attending level of mathematics teaching assistants who analyzed classroom videos via MOST instances progressed within time.

From the point of interpretation aspect of preservice teachers in all tasks, preservice teachers interpreted students' thinking at Level 3 or Level 4. Preservice teachers watched their own teaching videos so, this situation provided opportunity to analyze students' work critically. Thus, they were able to attend to the MOST instances and interpreted MOST instances in higher level in contrast to their responding actions while interacting with students. Watching their own videos before writing reflection reports and oral reflection sessions to discuss students' misconceptions might lead to interpret students' thinking in higher level. Most of the researchers used teachers' oral and written reflections to improve their noticing skills. They found that noticing skills of teachers were developed through oral and written reflections (Amador *et al.*, 2016; Sherin and van Es, 2005; Star *et al.*, 2009). Similarly, this study found that interpretation aspect of Meltem's noticing skills developed throughout the study. The study conducted by Barnhart and van Es (2015) demonstrated that highly sophisticated analyses depend on highly sophisticated attending. This result is compatible with findings of Meltem in algebra such that she mostly interpreted students' thinking at Level 3 or Level 4 while she attended to most of the MOST instances. Additionally, this finding is parallel to the result of the study conducted by Krupa et al. (2017), in which preservice teachers, who participated class discussions, developed in attending and interpreting aspects of students' thinking.

In terms of responding level of Derya to MOST instances, she had a tendency to tell solution process or the correct result to her students in all tasks. As shown in Table 6.9, her responding to students' thinking was mostly answer-focused. This result is complied with the findings of the study of Sun and van Es (2015) such that preservice teachers had tendency to focus on students' answers rather than students' conceptual understanding. One of the reasons behind such an attitude might be her tendency to tell solution process to her students. Krupa *et al.* (2017) designed curricular module including pre-post interviews and class discussions. This finding of Derya's responding level is also parallel with Krupa's (2017) study that preservice teachers did not show a progress in responding to students' thinking.

On the other hand, Meltem attempted to orient students towards correct answers in numbers and algebra tasks while she responded to MOST instances in data analysis and geometry tasks at Level 3 or Level 4. In other words, her responding level gradually evolved throughout the study. The discussion of students' possible difficulties before implementation of tasks, analyzing her own videos and getting familiar to the students and implementation process might support such a shift in Meltem's responding actions. This result is compatible with the study of Sun and van Es (2015) because they found that preservice teachers participating video-based course demonstrated more teaching practicing including making space for students' thinking, attending and pursuing to students' thinking. This finding of Meltem also builds upon of existing study conducted by Pascoe (2016) such that mathematics teaching assistants developed their responses to students' thinking gradually.

In addition to this, for retention study in algebra, findings of Derya demonstrated that there is a development in her percentage of attending level in comparison to the study of 2016-2017 for algebra tasks as shown in Table 6.21. As in Meltem's case, the discussion of students' possible difficulties before implementation of tasks might be a trigger for preservice teachers to attend students' mistakes while they are working on tasks. Van Es and Sherin (2002) found that the video club provided opportunity to improve teachers' noticing skills because this created an environment to discuss instances about students' thinking and teacher practices. Similarly, oral reflection sessions supported preservice teachers' noticing skills, because they provided an environment to negotiate possible instances related to students' thinking. Additionally, the findings of Derya build upon of existing studies conducted by Pascoe (2016) and Krupa (2017) in terms of attending aspect.

For the retention study in algebra, the percentage of attending level of Meltem and percentage of responding level of Meltem were gradually evolved in comparison to 2016-2017 data as shown in Table 6.24. In one of the MOST instances, the responding level of Meltem shifted to Level 4 however, it can be said that noticing skills of Meltem were open for improvement. In this situation, prior knowledge of current students and oral reflections implemented in 2016-2017 had an influence on Meltem's responding actions. Due to the fact that students possed required prior knowledge in algebra, preservice teachers had opportunity to focus on mathematical understanding of students via why or how type of questions. This result is compatible with study of Barnhart and van Es (2015) such that preservice teachers who attended video-based course demonstrated higher progression in attending and responding aspects of noticing skills. In addition to this, Meltem already had knowledge about students' possible difficulties in algebra since it was discussed in oral reflections in the previous year. The researcher observed that Meltem and Derya were aware of students' possible difficulties before implementing algebra tasks for the retention study. Sun and Van Es (2015) found that preservice teachers participating video-based course enacted more teaching practices including making space for students' thinking, attending and pursuing to students' thinking. This result supports to the findings of preservice teachers' noticing in algebra tasks. Besides to them, Sun and van Es (2015) concluded that preservice teachers attended to students' answers and procedures instead of the reasoning behind those answers and students' conceptual understanding. In a similar manner, Derya and Meltem mostly focused on students' answers and procedures during the interaction for retention study in algebra tasks.

7.2. Limitations and implications for future research

Qualitative method is appropriate for my research study because noticing skills of preservice teachers was investigated in depth and two preservice teacher got involved in as a bounded system for this study. Thus, it can be said that generalizability of findings and selected sample have several limitations. First of all, it might be said that the findings of this study cannot be generalized to all preservice teacher as large population. These two preservice teachers cannot be representative for all entire population because they have been selected by considering their class level and MKT test scores. Secondly, this study was conducted in one of the 7th grade classrrom in Kayışdağı district. At this class, researcher observed that prior knowledge of students was at very low level, especially, in number tasks and geometry tasks. Thus, preservice teachers had difficulty into implementing these tasks because students failed to respond to the questions of the tasks. In addition to this, preservice teachers made great effort to fill the gaps in students' prior knowledge and they invested much of their time for this purpose during the implementations. Thus, they failed to complete the tasks on time during implementations. So, they missed several MOST instances during interaction with students because of time limitation.

Thirdly, most of the groups had heteregonous structure and high-achieving students had a tendency to tell solution to other students during group discussions. Thus, preservice teacher did not require to attend MOST instances because these MOST instances were mentioned by those high-achieving students. For example, in Derya's group, Atakan was one of the low-achieving students while Ali was the high-achieving student. Ali had a tendency to tell solution way to Atakan in most of the tasks. Because of this situation, responding level of preservice teacher was coded as Level 1 for most of the MOST instances.

Another limitation of this study was time limitation in oral reflections. Each preservice teacher had five or seven minutes to reflect about MOST instances. Most of the MOST instances were not mentioned specifically during oral reflections. In addition to this, preservice teacher did not watch their teaching videos to interpret students' thinking in the retention study. The researcher gave worksheets of students to preservice teacher to make reasoning of students' thinking during interview. However, because they failed to remember details of interaction with their students, they were not able to give detailed information about students' thinking while interviewing with the researcher.

For the further studies, the researchers might select students who possess required prior knowledge to investigate preservice teachers' noticing skills in depth. Preservice teachers might be selected among senior preservice teachers. To minimize the effect of pedagogical content knowledge on preservice teachers' noticing skills, it might be a good choice to select sample as higher MKT scores. In addition to this, the effect of task selection on preservice teachers' noticing skills might be a good choice for future studies because, researcher observed that using of manipulatives in some tasks might be improved to the responding level of noticing skills (Kılıç, Doğan, Arabacı, and Tün, 2018). As an example, pattern blocks were used to find the pattern rule in task for first week of the retention study while the paper and pencil were used in task for the fourth week of the retention study. Responding level of Meltem was coded as Level 4 for the tasks implemented during the first week of retention study while her responding level was Level 2 or Level 1 during the fourth week of the retention study.

Furthermore, findings of the study by Coddington (2014) demonstrated that there is a significant relationship between mathematical knowledge for teaching and teachers' level of noticing skills. In Coddington's study, she mentioned that teachers' MKT and their noticing skills are two constructs of nourishing to each other. Therefore, the relationship between teachers' MKT and their noticing skills in the future studies can be discussed.

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APPENDIX A: ALGEBRA TASKS

A.1. Etkinlik 1

A.1.1. Kentsel Dönüşüm

Mevcut depreme dayanıksız, ekonomik ömrünü tamamlamış binaların yaşanabilir, depreme dayanıklı, sosyal donatıları, otoparkı, yeşil alanları olan kaliteli yaşam alanlarına dönüştürme sürecine (projesine) kentsel dönüşüm denir. Kentsel dönüşümün bu tür yararları olmasına rağmen bu süreç içinde bazı ailelerin mahallelerinden başka yerlere taşınması da gerekir.

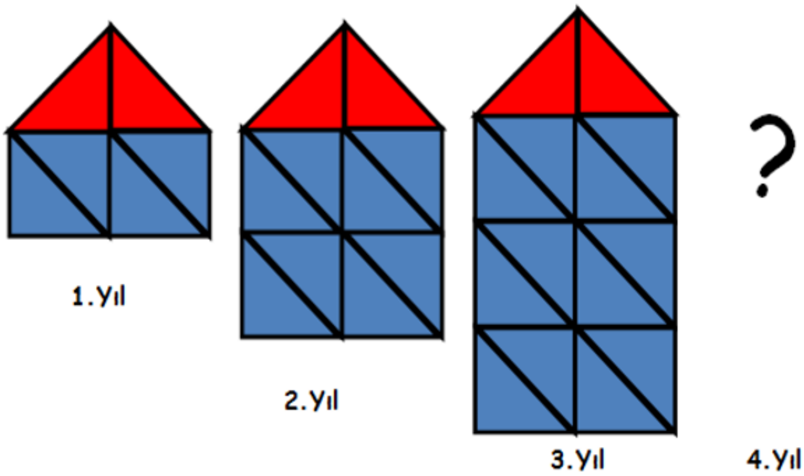
Figure A.1. Etkinlik 1.1. Kentsel Dönüşüm etkinliği.

1) İstanbul'da kentsel dönüşüme girecek mahallelerden biri de Kayışdağı mahallesidir. Bu süreçte mahalleden taşınacak aile sayısı aşağıdaki tabloda verilmiştir. **Yıl sayısı** ile **Taşınan Aile Sayısı** arasındaki ilişkiyi (örüntü kuralını) bulabilir misiniz?

Yıl sayısı	**Taşınan aile sayısı**
1	3
2	6
3	9
4	12
5	15
⋮	⋮
x	?

Figure A.2. Etkinlik 1.2. Kentsel dönüşüm etkinliği 1.sorusu.

2) Kentsel dönüşüm projesi kapsamında inşa edilen apartmanlar şekildeki gibidir. **Yeni yapılan apartmanların yüksekliği her yıl biraz daha artmaktadır.**



Şekildeki üçgenler (çatı katındakiler de dâhil) apartmandaki daireleri göstermektedir. **Örneğin**, 1. yıl yapılan apartmanda 6 daire vardır.

Figure A.3. Etkinlik 1.3. Kentsel dönüşüm etkinliği 2. sorusu.

a) **4. yılda** yapılan bir apartman nasıl olacaktır? Örüntü blokları yardımıyla oluşturunuz.

b) Apartmanların her yıl **bir kat** yükseldiğini düşünürsek, 2., 3. ve 4. yılda apartmanda kaç daire olacaktır? Peki ya "**n** yıl" sonra? (Örüntü kuralını bulunuz.)

Yıl sayısı	**Daire sayısı**
1	6
2	
3	
4	
⋮	⋮

Figure A.4. Etkinlik 1.4. Kentsel dönüşüm etkinliği 2.sorusu devamı.

3) Ataşehir Belediyesi tarafından hazırlanan kentsel dönüşüm planına göre hangi haftada toplam kaç sokakta dönüşüm gerçekleşeceği tabloda gösterilmektedir.

a) Bu plana göre 4. ve 5. haftada düzenlenen sokak sayısı kaç olacaktır?

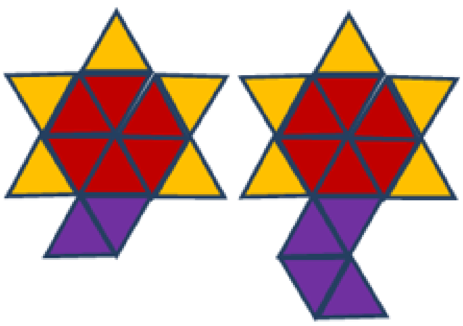
b) Peki, "**m** hafta" sonra kaç olacaktır? (Örüntü kuralını bulunuz.)

Hafta sayısı	**Sokak sayısı**
1	3
2	5
3	7
4	
5	
⋮	⋮
m	....

Figure A.5. Etkinlik 1.5.Kentsel dönüşüm etkinliği 3.sorusu.

Ek sorular:

1) Kentsel dönüşüm projesinde bahçe düzenlemesi için çiçeklendirme yapılacaktır. Ekilen çiçeğin sapının günlük uzama miktarı yanda gösterildiği gibidir. 3. ve 4. günde çiçeği oluşturmak için kaç üçgen kullanmak gerekir? Peki ya "**s gün**" sonra?



1.gün **2.gün**

Figure A.6. Etkinlik 1.6. Kentsel dönüşüm etkinliği ek sorusu.

Gün sayısı	Kullanılan üçgen sayısı
1	
2	
3	
4	
⋮	⋮
S	....

2) Örüntü bloklarıyla bir örüntü oluşturarak arkadaşınızdan oluşturduğunuz örüntünün kuralını bulmasını isteyiniz.

Figure A.7. Etkinlik 1.7. Kentsel dönüşüm etkinliği ek soru devamı.

A.2. Etkinlik 2

A.2.1. Gezelim Görelim

Celal Yardımcı Ortaokulunda gezi kulübü bir gezi organizasyonu hazırlamıştır. Bu gezi kapsamında sırasıyla gezilecek yerler, yaklaşık uzaklık ve ne kadar zaman geçirileceğine dair yönergeler aşağıdaki kutucuklarda verilmiştir. Bu bilgilendirmelere göre kutucuklardaki eksikleri uygun cebirsel ifadeler ile tamamlayınız. **Örneğin, Okul ile Ayasofya arasındaki mesafe X, Ayasofya'da geçirilen süre ise T'dir.** *Gördüğünüz yerlerin yanına bir artı işareti koyunuz.*

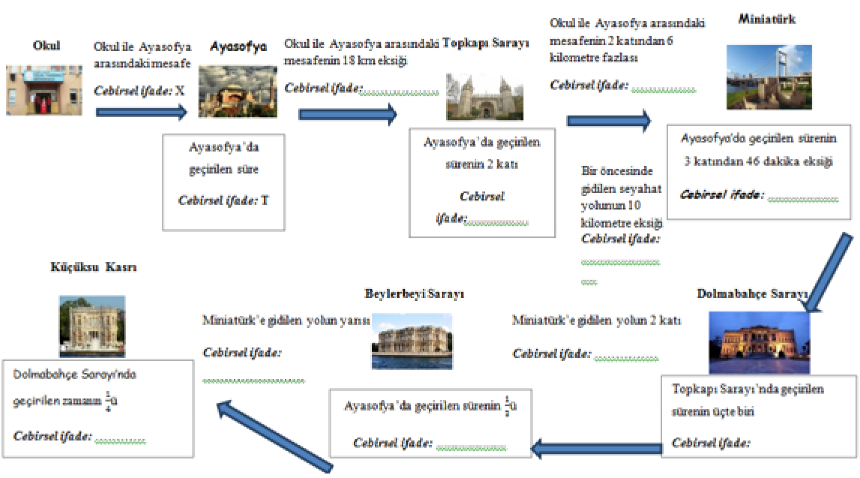


Figure A.8. Etkinlik 2.1. Gezelim görelim etkinliği.

Aşağıdaki soruları cevaplamak için öğretmeninizin Okulunuzla Ayasofya arasındaki mesafeyi ve Ayasofya'da geçirilecek süreye karar vermesini bekleyiniz.

⊛ Okulunuzla Ayasofya arasındaki mesafe (X) =

⊛ Ayasofya'da geçirilecek süre (T) =

Verilen bilgiler ışığında aşağıdaki soruları yanıtlayınız.

1) Topkapı Sarayı'ndan Dolmabahçe Sarayı'na gitmek için kaç km seyahat edilmiştir?

2) Ayasofya ve Topkapı Sarayı'nda geçirilen toplam süre ne kadardır?

3) Toplam kaç km seyahat edilmiştir?

4) Toplam gezi süresi ne kadardır?

5) **Topkapı Sarayında 90 dakika** geçirmiş olsaydınız, **Küçüksu Kasrında** kaç dakika geçirirdiniz?

Figure A.9. Etkinlik 2.2. Gezelim görelim etkinliği soruları.

Ek sorular:

1) Gezi kulübü ayrıca **Büyükada**, **Yıldız Parkı** ve **Emirgan Korusu**'na da gezi düzenleyecektir. Yukarıdaki yerlerin **aralarındaki mesafeleri** ve buralarda **geçirilecek süreleri** belirten ifadeler yazıp bu ifadeleri cebirsel olarak gösteriniz.

Gezilecek yer	Mesafe (km)	Geçirilecek zaman (dk)
1. Büyükada		
2. Yıldız Parkı		
3.Emirgan Korusu		

2) Verileri kullanarak bir problem oluşturunuz ve denklem kurarak çözünüz.

3) Toplam gezi süreniz **470 dakika olsaydı**, her bir ziyaret yerinde ne kadar vakit geçirmeniz gerekirdi?

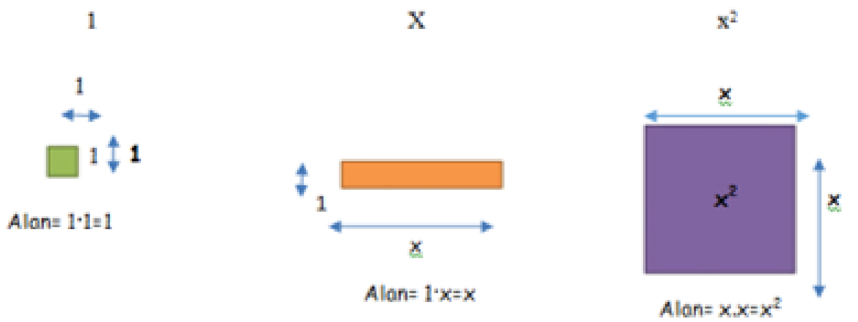
Figure A.10. Etkinlik 2.3. Gezelim Görelim etkinliği ek sorusu.

A.3. Etkinlik 3

A.3.1. Cebirsel Oyunlar

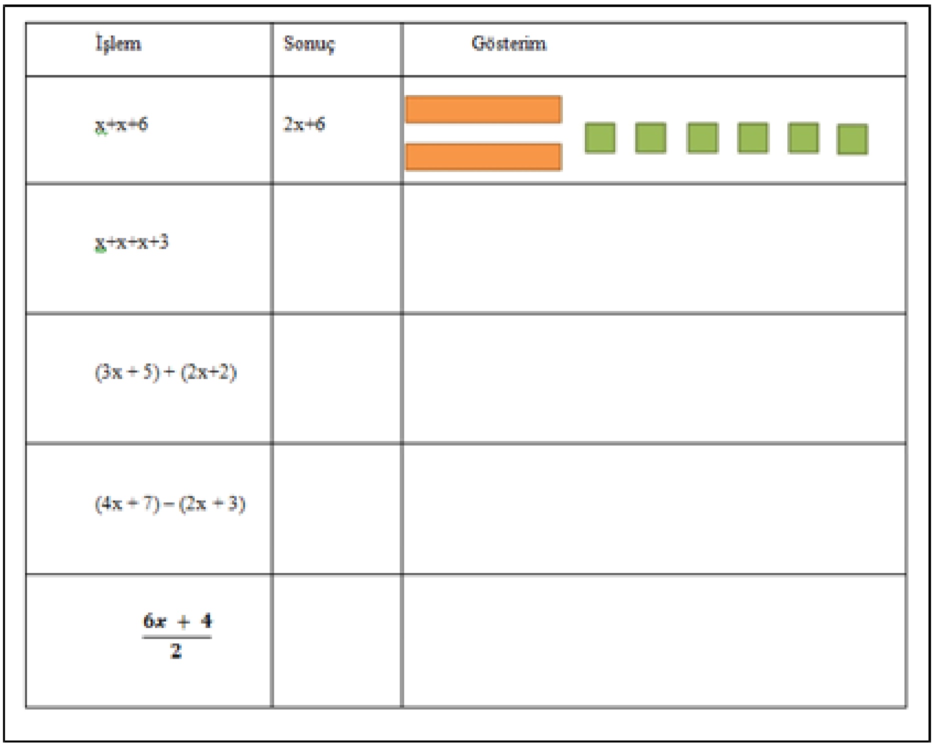
Lego parçalarıyla çeşitli yapılar oluşturan Cenk, bu yapıları matematiksel olarak nasıl ifade edebileceğini düşünmüş ve ilk olarak parçaları aşağıda görüldüğü gibi isimlendirmiştir.

Cebir Karoları



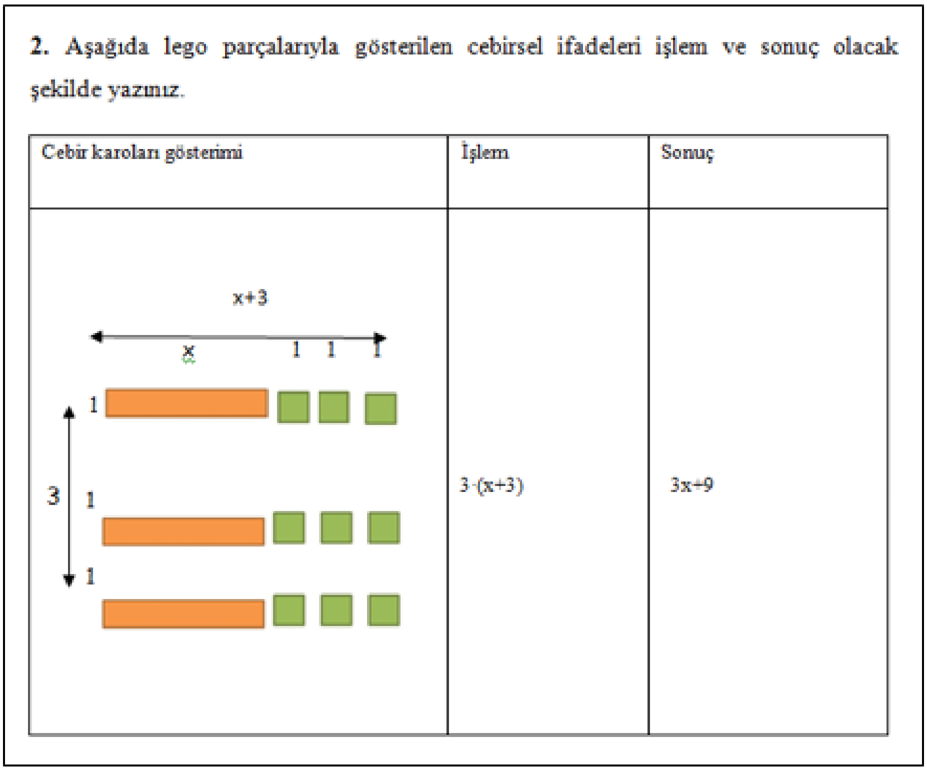
1. Cebirsel ifadeleri, cebir karolarıyla (lego) gösteriniz ve işlemlerin sonucunu yazınız.

Figure A.11. Etkinlik 3.1. Cebirsel Oyunlar etkinliği ilk sorusu.



İşlem	Sonuç	Gösterim
x+x+6	2x+6	
x+x+x+3		
(3x + 5) + (2x+2)		
(4x + 7) − (2x + 3)		
$\frac{6x + 4}{2}$		

Figure A.12. Etkinlik 3.2. Cebirsel oyunlar etkinliği ilk sorusu devamı.



2. Aşağıda lego parçalarıyla gösterilen cebirsel ifadeleri işlem ve sonuç olacak şekilde yazınız.

Cebir karoları gösterimi	İşlem	Sonuç
x+3; x 1 1 1; 3; 1 1 1	3·(x+3)	3x+9

Figure A.13. Etkinlik 3.3. Cebirsel oyunlar etkinliği ikinci sorusu.

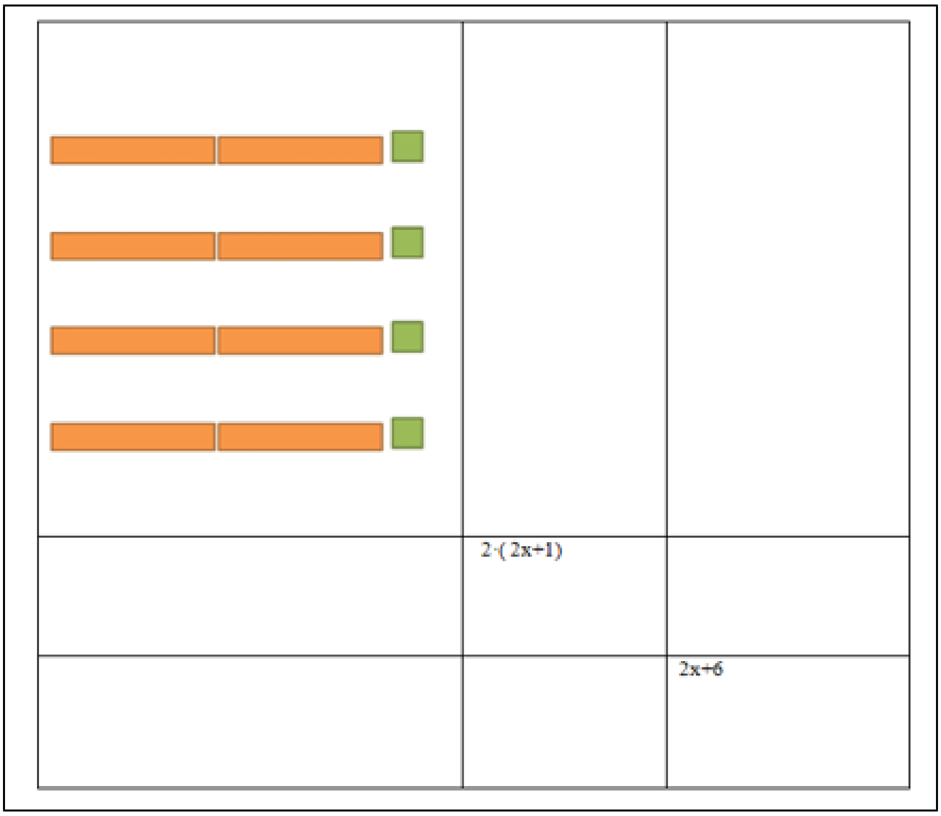


Figure A.14. Etkinlik 3.4. Cebirsel oyunlar etkinliği ikinci sorusu devamı.

Ek soru:

1. Cebir Karolarını kullanarak bir dikdörtgen oluşturunuz. Sıra arkadaşınızdan bu dikdörtgenin alanını cebirsel olarak ifade etmesini isteyiniz.

Figure A.15. Etkinlik 3.5. Cebirsel oyunlar etkinliği ek sorusu.

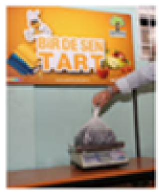
A.4. Etkinlik 4

A.4.1. Pazara Gidelim

Ali, Ayşe, Murat ve Sırma babalarına yardım etmek için onunla pazar alışverişine çıkmışlardır. O hafta evde turşu kurulacak olduğundan alınması gereken kilolarca malzeme vardır.

Pazardan **eşit ağırlıkta 7 poşet** turşu malzemesi ve birkaç kilo meyve almışlardır.

- Ali 2 kilo mandalina ve 2 turşu poşetini taşımaktadır.
- Ayşe turşu poşetlerinden 3 tane taşımaktadır.
- Murat ise 1 turşu poşeti, 1 kilo muz ve yarım kilo elma taşımaktadır.
- Sırma 1 turşu poşeti ve 3 kilo portakal taşımaktadır.



Eve dönüş yolunda kardeşler arasında kimin yükünün daha ağır olduğuna dair bir tartışma başlar.

Ali elindeki tüm poşetleri pazarın çıkışındaki terazide tartar ve ağırlığın **6 kilo** olduğunu görür. (**Unutmayın!** Turşu poşetleri eşit ağırlıktadır.)

Figure A.16. Etkinlik 4.1. Pazara Gidelim etkinliği.

1. Turşu poşetlerinin tek bir tanesinin ağırlığını denklem kurarak bulunuz.

2. Ayşe ve Murat'ın taşıdıkları ağırlıkları cebirsel olarak gösteriniz ve ağırlık miktarını hesaplayınız.

3. Pazardan alınan turşuluk malzemenin ve meyvenin toplam ağırlığını bulunuz.

4. Ali, Ayşe, Murat ve Sırma'nın taşıdıkları ağırlık miktarlarını karşılaştırınız.

Figure A.17. Etkinlik 4.2. Pazara Gidelim etkinliği soruları.

Ek Soru:

Fatma Hanım eve gelen turşuluk malzemelerden turşu kuracaktır. Fatma Hanım'ın karışık turşu tarifi aşağıdaki gibidir.

- Bir miktar havuç
- Havuç miktarının 3 katı kadar beyaz lahana
- Havuç miktarının 2 katından 500 gr daha fazla salatalık
- Havuç miktarının dörtte biri kadar sivri biber

Fatma Hanım, toplam **5,5 kilo** turşuluk malzeme kullanarak turşu yapacaktır. Yukarıdaki tarife göre her bir malzemeden ne kadar kullanması gerekir? Denklem kurarak çözünüz. (1 kilo=1000 gr)

Figure A.18. Etkinlik 4.3. Pazara Gidelim etkinliği ek sorusu.

A.5. Most Instances in Algebra Tasks (Retention Study)

A.5.1. Task 1: Kentsel Dönüşüm (Derya)

MOST instance 1: To find the rule of pattern as "y+3" instead of 3y.
Mathematical point (MP): When finding the rule of the pattern, it is needed to check the relationship between two variables rather than the change in only one variable.
Student Mathematics (SM): Unknown value "y" is perceived as increasing amount and thus the answer is "y+3".
MOST instance 2: To find the rule of the pattern as "6n or 4n" instead of "4n+2".
Mathematical point (MP): When finding the rule of the pattern, it is needed to check the relationship between two variables rather than the change in only one variable.
Student Mathematics (SM): When finding the rule of the pattern, it is needed to check the change in only one variable.
MOST instance 3: To find the result of "4n plus 2" as "6n".
Mathematical point (MP): Similar terms of algebraic expressions can be added or sub-

tracted.
Student Mathematics (SM): It can be added the coefficient of the unknown with the constant term and the unknown would leave as itself.
Task 1: Kentsel Dönüşüm (Meltem)
MOST instance 1: To find the rule of pattern as “+3” instead of 3y.
Mathematical point (MP): When finding the rule of the pattern, it is needed to check the relationship between two variables rather than the change in only one variable.
Student Mathematics (SM): Unknown value “y” is perceived as increasing amount and thus the answer is “+3”.
MOST instance 2: To find the rule of the pattern as “plus four” instead of “4n+2”.
Mathematical point (MP): When finding the rule of the pattern, it is needed to check the relationship between two variables rather than the change in only one variable.
Student Mathematıcs (SM): When finding the rule of the pattern, it is needed to check the change in only one variable.
MOST instance 3: To find the rule of the pattern as “plus two” instead of “3s plus 1”.
Mathematical point (MP): When finding the rule of the pattern, it is needed to check the relationship between two variables rather than the change in only one variable.
Student Mathematics (SM): When finding the rule of the pattern, it is needed to check the change in only one variable.

A.5.2. Task 2: Gezelim Görelim (Derya)

MOST instance 1: Students might insert the x variable in expressing the distance without paying attention to context.
Mathematical point (MP): The algebraic expressions were written according to given context.
Student Mathematics (SM): In all of the algebraic expressions we can take unknown as x without paying attention to context.
MOST instance 2: When finding fraction of a fraction, division operation was done.
Mathematical point (MP): When finding fraction of a fraction in given numbers, fractions are multiplied.

Student Mathematics (SM): When finding the fraction of a fraction, the division needs to be done.
MOST instance 3: It was performed firstly operation of addition and performed secondly operation of division in terms of order of operations.
Mathematical point (MP): It was performed division operation first then addition operation
Student Mathematics (SM): It was performed addition operation firstly then division operation.

A.5.3. Task 2: Gezelim Görelim (Meltem)

MOST instance 1: Students might insert the x variable in expressing the distance without paying attention to context.
Mathematical point (MP): The algebraic expressions were written according to given context.
Student Mathematics (SM): In all of the algebraic expressions we can take unknown as x without paying attention to context.
MOST instance 2: When finding fraction of a fraction, division operation was done.
Mathematical point (MP): When finding fraction of a fraction in given numbers, fractions are multiplied.
Student Mathematics (SM): When finding the fraction of a fraction, the division needs to be done.
MOST instance 3: When finding half of the algebraic expression, two is dividided into algebraic expression.
Mathematical point (MP): When finding half of the algebraic expression, given algebraic expression is didided into two.
Student Mathematics (SM): When finding half of the algebraic expression, two is dividided into algebraic expression.
MOST instance 4: Given number in problem context were not substituted into unknown value.
Mathematical point (MP): Given number in problem context were substituted into

unknown value so it can reached the correct result.
Student Mathematics (SM): Given number in problem context is multiplied with unknown value.

A.5.4. Task 3: Cebirsel oyunlar (Derya)

MOST instance 1: To find the result of (3x+5) + (2x+2) as 8x+4x and to find the result of (6x+4) /2 as 10x/2.
Mathematical point (MP): Similar terms of algebraic expressions can be added or subtracted.
Student Mathematics (SM): When algebraic expressions given in paranthesis he can add the coefficient of the unknown with the constant term and the unknown would leave as itself.
MOST instance 2: To find the result of (4x+7)- (2x+3) as 6x+4 Mathematical point (MP): Distributive property was applied to each term of algebraic expression.
Student Mathematics (SM): Distributive property was applied to only one term and other term was written as constant term.
MOST instance 3: The student cannot demonstrate the algebraic representation of (2x+6) and (3x+2) by using algebra tiles.
Mathematical point (MP): The algebra tile which long side is x unit and short side is 1 unit represents “x” algebra tile and algebra tile which sides are 1 units represents unit algebra tile.
Student Mathematics (SM): The algebra tile which represents “x” can be substituted into unit algebra tile.

A.5.5. Task 3: Cebirsel oyunlar (Meltem)

MOST instance 1: To find the result of (4x+7)- (2x+3) as 2x-4 Mathematical point (MP): Distributive property was applied to each term of algebraic expression.
Student Mathematics (SM): Distributive property was applied to only one term and

other term was written as constant term.
MOST instance 2: To find the result of (6x+4) /2 as 10x/2.
Mathematical point (MP): Similar terms of algebraic expressions can be added or subtracted.
Student Mathematics (SM): When algebraic expressions given in paranthesis he can add the coefficient of the unknown with the constant term and the unknown would leave as itself.

A.5.6. Task 4: Pazara Gidelim (Derya)

MOST instance 1: When solving the linear equation which is "2+2x=6", the sign of the operation will not be changed when passing opposite side of the equation.
Mathematical point (MP): Linear equation was made by considering problem context and linear equation was solved by operating appropriately.
Student Mathematics (SM): The sign of the operation was not changed when passing opposite side of the equation.
MOST instance 2: To find the result of 6x+(x/4) as 7x/4
Mathematical point (MP): While adding algebraic expressions involving rational coefficients, denominators are enlarged to get common denaminators and then similar terms are added.
Student Mathematics (SM): While adding two algebraic operations consisting offractions, numerators of them directly are added without getting common denaminators.
MOST instance 3: Although the weight of the pickle bag is known, it is expressed as unknown value which is "x".
Mathematical point (MP): Once the unknown value of algebraic expression is found, unknown value is substituted into algebraic expression and the result is calculated.
Student Mathematics (SM): Once the unknown value of algebraic expression is found, unknown value is substituted into algebraic expression then "x" is added to the result.

A.5.7. Task 4: Pazara Gidelim (Meltem)

MOST instance 1: Students cannot state relationships in problem context algebraically Mathematical point (MP): Expressions and relationships in problem sentence are key words for operations.

Student Mathematics (SM): The problem can be solved by applying random operations to the numbers given in the problem statement.

MOST instance 2: To find the result of 6x+(x/4) as 7x/4

Mathematical point (MP): While adding algebraic expressions involving rational coefficients, denominators are enlarged to get common denaminators and then similar terms are added.

Student Mathematics (SM): While adding two algebraic operations consisting offractions, numerators of them directly are added without getting common denaminators.

MOST instance 3: When finding one fourth of unknown value, unknown value was added to $\frac{1}{4}$.

Mathematical point (MP): When finding one fourth of unknown value in given numbers, one fourth is multiplied with unknown value.

Student Mathematics (SM): When finding one fourth of unknown value, one fourth is added to unknown value.

MOST instance 4: The student cannot solve the result of operation "2x=4"

Mathematical point (MP): In the mathematical equation, unknown value which is x left alone and equation is divided by same number.

Student Mathematics (SM): In the mathematical equation, unknown value which is x left alone and equation is divided by different number.

APPENDIX B: ITEMS OF WRITTEN REFLECTION

(i) a) Eğer grup tartışması öncesi bireysel bir çalışma gerektiren etkinlik ise grup tartışmasından önce her bir öğrenci etkinliği nasıl çözdü? Öğrenci çözümünün resmini ekleyerek, çözümün doğru olup olmadığını açıklayınız.
b) Eğer grup çalışması gerektiren bir etkinlik ise öğrenciler etkinliğe nasıl başladılar, görev dağılımı yaptılar mı?

(ii) Grup tartışması sırasında neler konuşuldu? Öğrenciler birbirlerini dinledi mi? Her biri etkinliğe/tartışmaya katkıda bulundu mu? Nasıl katkılarda bulundular? Karşılıklı konuşmalardan örnek vererek yazınız.

(iii) Grup tartışması sonrası veya etkinlik üzerinde çalışırlarken öğrencilerle neler konuştun, neler yaptın, hangi soruları sordun? Neden? Onların tepkileri/yanıtları ne oldu? (Gerekirse karşılıklı konuşmalardan örnek veriniz.)

(iv) Öğrencilerin etkinlik sırasındaki tartışmaları çalışma kâğıtlarındaki yanıtlara veya oluşturdukları ürünlere nasıl yansıdı? Doğru bir yöntem izleyip sonuca ulaşabilmişler mi? Öğrencilerin çalışma kâğıtlarından örnek vererek açıklayınız.

(v) Öğrenciler etkinliği anlayabildiler mi? Etkinliği tamamlayabildiler mi? Sence bu etkinlik amacına ulaşabildi mi? Neden ulaştı veya neden ulaşamadı?

(vi) Uygulama sırasında yapmak veya sormak istediğin başka herhangi birşey oldu mu? Aynı etkinliği tekrar uygulama şansınız olsaydı ne yapmak veya sormak isterdiniz?

APPENDIX C: ITEMS OF INTERVIEW FOR RETENTION STUDY

(i) Bugün öğrencilerle yapmış olduğun etkinlik çalışmasını düşündüğünde öğrencilerin performanslarını değerlendirir misin?
   - Değerlendirmelerinde öğrencilerin yapabildikleri ve zorlandıkları yerleri açıklar mısın?
   - Öğrencilerin yapabildikleri ve zorlandıkları yerler için örnek verir misin?

(ii) Bahsetmiş olduğun öğrencilerin zorlandıkları yerleri düşündüğünde onların zorlanmalarının nedeni neler olabilir?

(iii) Öğrencilerin bu bahsettiğin zorlukları konusunda neler yaptın?

(iv) Bu etkinliği geçen yılda uygulamıştın. Geçen yılki ve bu yılki uygulamalarını karşılaştırır mısın?

NOTLAR: İlk 3 soru her bir öğrenci için tek tek sorulmuş son soru self- reflection sorusudur.