MATHEMATICAL QUALITY OF GEOMETRY INSTRUCTION OF A NOVICE HIGH SCHOOL TEACHER

by

Buket Semercioğlu Kapçak

Integrated BS & MS program in Teaching Mathematics, Boğaziçi University, 2016

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science

Graduate Program in Secondary School Science and Mathematics Education
Boğaziçi University
2019

MATHEMATICAL QUALITY OF GEOMETRY INSTRUCTION OF A NOVICE HIGH SCHOOL TEACHER

APPROVED BY:

Assist. Prof. Fatma Aslan Tutak . . . .
(Thesis Supervisor)

Assoc. Prof. Emine Adadan . . . .

Assoc. Prof. Dr. Rukiye Didem Taylan . . . .

DATE OF APPROVAL: 13.05.2019

ACKNOWLEDGEMENTS

First of all, I would like to thank to my thesis supervisor, Assoc. Prof. Fatma Aslan-Tutak, for her endless support, guidance and patience during my study. I appreciate her to widen my perspective and to provide motivation during this study. Her vision and knowledge in mathematics improved me a lot. Without her support this study would never be completed.

I also want to thank my thesis committee members, Assoc. Prof. Emine Adadan and Assoc. Prof. Dr. Rukiye Didem Taylan, for their unconditional support, valuable comments and critical feedbacks for this study. I appreciate their precious contributions to my thesis. I would also like to express my sincere thanks to the teacher who is the participant of the study. It was a privilege to observe her lessons. I am grateful for her willingness and support to my study. Additionally, I would also like to thank the expert who was involved in the validation process of this study.

I would like to express my deepest gratitude to my dear father, Kazım and my dear mother, Birgül for their love, continuous encouragement and belief throughout my life. They always tried to give me support in every stage of my life. In addition, special thanks to my sisters, Buse and Melis. They are gift for me and I thank them for cheering me up all the time. I always feel lucky to have such a family.

Last but not least, I owe thanks to a very special person, my beloved husband Halit for his endless love, support and understanding all the time. I thank him for making me feel strong, keeping me motivated and supporting me to achieve my goals. I could not have finished this thesis without his accompany to my life. I cannot find the words to express my deeply felt gratitude to my small family member.

ABSTRACT

MATHEMATICAL QUALITY OF GEOMETRY INSTRUCTION OF A NOVICE HIGH SCHOOL TEACHER

The aim of this study is to examine the quality of geometry instruction of a novice high school mathematics teacher and the factors that affect her instruction. The quality of the geometry instruction of a participant who is first year mathematics teacher is aimed to be examined. The data was collected by observing the mathematics teacher who worked at a private high school in Istanbul during the academic year 2017 -2018 through 6 lessons for 3 weeks. The observation form was designed by adapting the Mathematical Quality of Instruction tool. The observation data collected through the videotaped classroom observation was supported by lesson plans and materials along with the interviews. During the classroom observation, Mathematical Quality of Instruction scale was filled with additional notes. In the light of the data collected from different sources, the quality analysis of the geometry lesson was provided. First, the data was presented as a summary for each lesson. Afterwards, the data was given separately for each code of the Mathematical Quality of Instruction scale. It was specified that the Mathematics lesson was qualified as high for some of the codes. For instance, the ratio of the explanation code used in the lessons was very high. Taking all the codes into consideration, it was observed that the quality of mathematics lesson was limited. In addition to these results, among the factors that affect instruction of the teacher, it is claimed that besides the high school and college education of the teacher, the culture of the school where the teacher has been working has also an effect on the subject. It was revealed that the novice participant's knowledge of geometry instruction was based on her high school education. It can be suggested that geometry lessons should also be involved in college education.

ÖZET

ÖĞRETMENLİĞİNİN İLK YILINDAKİ BİR MATEMATİK ÖĞRETMENİN GEOMETRİ ÖĞRETİMİ BİLGİSİ

Bu çalışmada öğretmenlik mesleğine yeni başlamış bir matemaik öğretmeninin geometri öğretiminin niteliğinin incelenmesi ve öğretmenliğine etki eden unsurların incelenmesi amaçlanmaktadır. Matematik öğretiminin ilk yılında olan katılımcı öğretmenin geometri dersi öğretiminin niteliğinin incelenmesi amaçlanmıştır.2017-2018 Eğitim Öğretim yılında İstanbul'da özel bir lisede matematik öğretmenliği yapan katılımcının 3 hafta boyunca 6 dersi gözlemlenerek veri toplanmıştır. Gözlem sırasında kullanılan gözlem formu Mathematical Quality of Instruction aracını uyarlayarak niteliksel analize uygun olarak tasarlanmıştır. Video kaydı alınan sınıf içi gözlem verileri, ders plan ve materyalleri kullanarak analiz edilmiştir. Ders gözlemi sırasında hem Matematik Dersinin Niteliği (Mathematical Quality of Instruction) ölçeği uyarlanarak hazırlanan gözlem formu doldurulmuş hemde dersle ilgili ek gözlemler not alınmıştır. Farklı kaynaklardan toplanan bu verilerin analizi ışığında geometri dersinin niteliğinin analizi sağlanmıştır. Veriler, öncelikle her ders için kısa bir özet olarak sunulmuş. Ardından, Matematik Dersinin Niteliği (Mathematical Quality of Instruction) ölçeğinin her bir kodu için ayrı ayrı verilmiştir. Matematik dersinin bazı kodlar için niteliğinin yüksek olduğu belirlenmiştir. Örneğin açıklama (explanation) kodunun derslerde kullanım oranı çok yüksektir. Genel olarak tüm kodlar göze alındığında matematik dersinin niteliğinin yetersiz olduğu gözlemlenmiştir. Bu sonuçlara ek olarak, bu çalışmada öğretmenin öğretme tarzına etki eden faktörler arasında lise eğitimi ve üniversite eğitiminin yanında çalıştığı okulun kültürünün de etkisi olduğu ortaya konulmuştur. Mesleğe yeni başlamış katılımcının geometri öğretme bilgisinin lise temeline dayandığı ortaya çıkmıştır. Üniversite eğitiminde geometri derslerinin de yer alması gerektiği önerisinde bulunulabilir.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ACRONYMS/ABBREVIATIONS

CCK	Common Content Knowledge
HCK	Horizon Content Knowledge
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
MATH	Mathematics Major
MCK	Mathematics Content Knowledge
MEB	Milli Eğitim Bakanlığı / Ministry of National Education
MKT	Mathematical Knowledge for Teaching
MQI	Mathematical Quality of Instruction
PCK	Pedagogical Content Knowledge
SCK	Specialized Content Knowledge
SMK	Subject Matter Knowledge

1. INTRODUCTION

Albert Einstein said that he never teaches his pupils, he only attempts to provide the conditions in which they can learn; teaching is not only knowing content but also designing how to give knowledge to students. In subject matter knowledge, there are not only ideas and theories but also methods to understand discovery of knowledge, organizations, perspectives (McDiarmid and Ball, 1988). In addition to the subject matter knowledge, how teaching occurs in the class has an effect on students' learning (Hill, Rowan and Ball, 2005). Researchers who want to study about teachers' knowledge should focus not only on content knowledge but also on teaching during instruction.

There are relatively few studies that focus on novice mathematics teachers' knowledge during instruction. Thus, this study will focus on observing a first-year mathematics teacher's knowledge during instruction. I have been teaching for three years and I have experienced different situations during instruction from what we have learned in the University. Being a teacher is different from what one experiences in the University, because teaching is a complex concept. There are many externalinfluencesduring the teaching process. The importance of this research is to focus on a first-year mathematics teacher's knowledgeparticularly in teaching geometry andthe factors that have an effect onthe participant'steaching.

There are lots of research, which focus on teacher knowledge by using different research methods such as; survey, interviews, or different models. However, being a novice teacher with a research perspective, I believe, to observe teacher knowledge effectively, it should be studied in its natural environment. In this research, I am conducting a case study to observe a first-year high school mathematics teacher's knowledge during instruction. In this research, information about the teacher's knowledge were collected though individual interviews, articfact collection and classroom observations, which were conducted by using an adapted version of Mathematical Quality of Instruction instrument.

2. LITERATURE REVIEW

Researchers argue that better learning will result primarily from better teaching (Darling-Hammond and Rustique-Forrester, 2005) Thus, there is an ongoing interest on teachers' knowledge in the research area. There are many studies in the area of mathematics education, especially in the area of mathematical knowledge of teachers. Researchers have a continuing interest on this area because mathematics education hasalways been a controversial issue in the research world. Recently, research in mathematics teachers' knowledge has gained great importance. There are different approaches to teach and learn mathematics. It can be said that many models that focus on teachers' knowledge are coming from the idea of Shulman's "Teacher Knowledge" (1987). Thus, in the literature review of current study, we will discuss Shulman's work in detail (1987) and then Mathematical Quality of Instruction (MQI), which is a model to observe mathematics teachers' knowledge during instruction.

2.1. Teachers' Knowledge

The nature of teachers' knowledge can be seen in many fields of research. Shulman and his colleagues analyzed teachers' knowledge, and they made an important contribution to the research area. With their analysis, they supported data and information to researchers studying in this area. Their model has laid the foundations of today's research about teacher knowledge.

It should also be stated that, there is no clear agreement on the details of the components of teachers' knowledge (Ball *et al.*, 2001). However, Shulman and his colleagues' three categories of content knowledge are widely accepted. These three categories of content knowledge are Subject Matter Knowledge (SMK), Pedagogical Content Knowledge (PCK), and Curricular Knowledge (CK) (Shulman 1986).

Shulman and his colleagues (1987) proposed seven categories/kinds of teachers' knowledge.

- General pedagogical knowledge, such as classroom management principles and strategies;
- Knowledge of learners' characteristics;
- Knowledge of educational contexts;
- Knowledge of educational limits, purpose and values and their philosophical and historical grounds;
- Subject matter knowledge;
- Curricular knowledge
- Pedagogical content knowledge (Shulman, 1987, p. 8).

There are seven categories, and the first four of them do not depend on content. However, the last three categories are related to content, and they constitute the Content-specific Knowledge (Ball *et al.*, 2008; Shulman, 1986). These three are essential for the teachers' knowledge because the teacher who has all of them can have better teaching opportunities (Shulman, 1986). The Subject Matter Knowledge (SMK) is not only knowing facts, processes and concepts. SMK also requires a deep understanding of the underlying results. Second category is Curricular Knowledge (CK) that states all programs to be given in a grade level. Lateral Curriculum and Vertical Curriculum Knowledge are parts of Curriculum Knowledge (Shulman, 1987). The last one is Pedagogical Content Knowledge (PCK), which is defined as a combination of content and pedagogy. Teachers' professional understanding especially forms the knowledge (Shulman, 1986).

It is assumed that teachers' Subject Matter Knowledge and Pedagogical Content Knowledge are in relation. For improving teaching, better-subject-matter preparation is necessary. It is not only learning changing topics but also "designing constructivist views on teaching and learning and articulated understanding" (Even, 1993). There are similarities and differences of PCK and CK. Researchers investigate whether the association between PCK and CK depends on the level of mathematical experiences (Ball *et al.*, 2008). Shulman argued that the distinction between knowledge and pedagogy is important for development; however, knowledge only is not enough for a teaching certification.

Shulman (1987) emphasized that knowledge of subject matter should always come first for teachers. Although the combination of content-free and content-specific knowledge is important for teachers' knowledge, Shulman argued that content specific knowledge comes first. Knowledge of theories and methods of teaching also play an important secondary role. Teacher effectiveness, classroom management and content knowledge are the base of the general perspectives of knowledge.

2.2. Mathematics Teachers' Knowledge

Subject matter is more than context (Ball *et al.*, 2008). Knowing a subject for teaching needs more information than facts and concepts. Organization of principles, structures, preparation to lessons and rules should be done well. Teachers' aim is not ony understand meaning of mathematics but also they should have information about reasons inside of the mathematics (Shulman, 1986). There are many studies that emphasized both on the teaching of mathematics and the mathematics used in teaching as Learning to Teach a Project and Learning Mathematics for Teaching Projects (Ball *et al.*, 2008).

Experiencing mathematics from many perspectives is important for teachers (Ball, 2000). According to Usiskin (2001), teachers' mathematics knowledge should include the following competencies;

- Explanation of new ideas.
- Alternative definitions and their consequences.
- The wide range of applications of the mathematical ideas being taught.
- Responses to questions that learners have about what they are learning.
- Why concepts arose and how they have changed over time.
- How problems and proofs can be extended and generalized.
- How ideas studied in school relate to the ideas students may encounter in later mathematics study (Usiskin, 2001, p. 96).

2.2.1. Mathematical Knowledge for Teaching (MKT)

Shulman's model is also focused onthe area of mathematical knowledge. The Mathematical Knowledge for Teaching Model (MKT), which has been developed by Ball and her colleagues (2008) have been used in the Learning Mathematics for Teaching (LMT). The MKT model is the basis for other studies of mathematical knowledge of teachers. The MKT was built on Shulman's categorization. Mathematical Knowledge for Teaching Model and observation-based instrument has emerged by the study of Ball and her colleagues in the University of Michigan by 2000 (Ball *et al.*, 2008). Ball and her colleagues have developed an instrument to measure mathematical knowledge of teachers. In the MKT model, there are SMK and PCK categories based on Shulman's categorization. There are six sub-categories defined in these categories. These categories change from teachers to mathematicians. Figure 2.1 shows the MKT model based on Shulman's categorization.

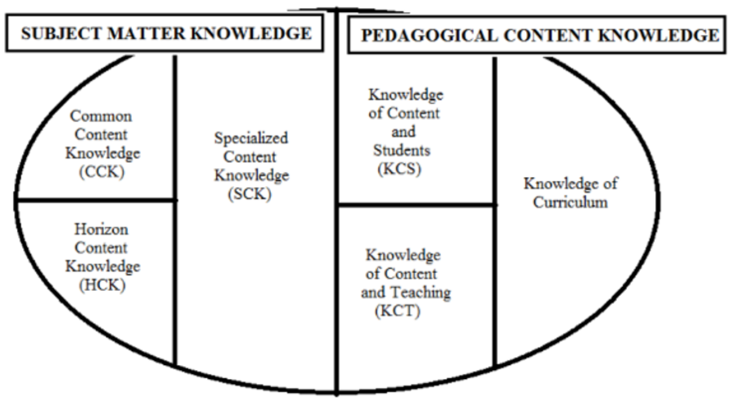


Figure 2.1. Domains of Mathematical Knowledge for Teaching (Ball *et al.*, 2008, p. 403).

MKT is a practice-based model. Practice-based categorization in MKT suggests three sub-domains of SMK. Common content knowledge CCK refers to the mathematical knowledge and skills. Individual ability is important here to solve mathematical problems. Specialized content knowledge SCK is the knowledge to use in mathematics teaching. SCK is not commonly used for the area outside of the teaching. Horizon content knowledge is teachers' knowledge about mathematics topics in the curriculum.

HCK can be thought as what teachers know and what they know beyond (Ball, 2008).

Another group of domains in MKT model is PCK, which is also defined as Shulman in Ball's view. One of its sub-categories is Knowledge of Content and Students (KCS). KSC includes knowledge about students and mathematics topics. To be aware of students' preconceptions and their understanding take part in this sub-category (Ball et al, 2001). The second one is Knowledge of Content and Teaching (KCT). KCT includes teacher knowledge about instructional design and instructional strategies. The last sub-category is the Knowledge of Curriculum. There is a strong relationship between the Knowledge of Curriculum and the Curricular Knowledge (CK) in Shulman's model. With MKT model, getting information about how teachers solve problems, answer students' questions, check students' work and understand the content of the school curriculum that constitute some parts of teaching is possible. Teachers' mathematical knowledge contains all these and more.

Although MKT model has been used in mathematics, there are some limitations of the model. For example, MKT is limited in studying secondary teachers' mathematical knowledge for teaching, because it is designed for elementary and middle school mathematics teachers (Usiskin, 2001). There has been a discussion about the inadequacy of the MKT model because this model does not include the effect of teachers' belief on teaching mathematics. In addition, the MKT model uses a pencil-and-paper measurement, which is not effective for studying teachers' knowledge during instruction (LMT, 2006).

2.2.2. Cognitively Activating Instruction (COACTIV)

MKT is one way of investigating teachers' knowledge. However, it is criticized as not describing the secondary mathematics teachers' knowledge due to its components and differences between its subcategories. Researchers conducted and implemented a new instrument called Cognitively Activating Instruction (COACTIV) to measure PCK and CK of secondary mathematics teachers (Krauss *et al.*, 2008). The project was conducted in Germany with teachers whose students attended to PISA in the years

2003-2004. One of the aims of the study was to understand the relationship between teachers' competencies and students' performance in PISA. Students' achievements in PISA and teachers' knowledge in COACTIV were measured. COACTIV used Schulman's knowledge categorization (Krauss *et al.*, 2008). The difference between COACTIV and MKT wasto haveopen-ended questions. One of the limitations of COACTIV is, its country specific planning just like MKT. Additionally, COACTIV measure was not completely published. This makes it difficult to use because it is not easy to be able to conduct a research that was done in a different curriculum, for a different country.

2.2.3. Teacher Education and Learning to Teach (TELT)

Teachers' mathematical knowledge and their development in the first year of teaching are arguable. TELT Study is conducted by the National Center for Research on Teacher Education (NCRTE). Learning to teach mathematics study was designed to examine the process of mid-experienced mathematics teachers and novice teachers in their first year of teaching (Borko, *et al.*, 1992). The aim of the study was to investigate how using different approaches can have an effect on teachers' education about their knowledge, skills and dispositions and to infer from teachers' responses (McDiarmid and Ball, 1988). Observing teachers work on different tasks can give an idea about their knowledge, their capacity to respond to instructional tasks and their pedagogical reasoning. To collect information from teachers, three instruments were used in the study. These were a self-administrative questionnaire, structured interviews, and observational guide. There were many tasks in the study, and researchers asked questions to teachers to have an idea about their responses in class. There is a limitation of TELT study though, because teachers tend to give responses according to situations. However, researchers cannot know what the teacher will actually do in action during an instruction. Thus, the study was conducted over time again. Questionnaire and interviews were the main sources of information (McDiarmid and Ball, 1988). Development of teachers' knowledge by writing scenarios to emphasize teachers' thinking and understanding was observed in different periods of the study (McDiarmid and Ball, 1988).

2.3. Mathematics Teaching Practices

The presented studies until this part mostly were focused on teachers' quality and teachers' knowledge rather than teaching quality. When we talk about quality of mathematics instruction, quality is not about only teachers but also teaching. There are several studies that focus on improving teaching that is effective to improve classroom instruction (Hiebert and Morris, 2012). The main idea of improving teaching by improving teaching studies is to train teachers to acquire skills and knowledge to use in the classroom (Hiebert and Morris, 2012). This process is not easy to apply but, changing cultures and refining instruction would be more productive approaches. Teachers play a significant role on students' learning and engagement (Ellis, Özgür and Reiten, 2018). Thus, teachers' teaching methods should be developed to improve instruction quality in student-oriented classrooms (Ellis, Özgür and Reiten, 2018). To redefine teaching coming from college knowledge, in method courses, the aim of the course may be more on learning about instructional methods (Grossman, Hammerness and Mcdonald, 2009). By learning different instruction methods, teachers may develop their teaching during instruction. Learning about students' understanding is one of the important parts of teaching because there are many components inside of it such as being sensitive to students' errors, responding student questions effectively, leading classroom discussions and having strategies to make classroom environment productively and so on (Grossman, Hammerness and Mcdonald, 2009).

Overally, teaching is a professional job that happen in the classroom. Thus, teachers' knowledge and their teaching should be examined not only with paper-and-pencil tests but also in the classroom during instruction. It is my belief that, MKT and other methods, which are mentioned above are not enough to observe teachers' teaching during an instruction, in detail. There are other studies observing teachers' knowledge and teaching at the same time during instruction.

2.3.1. The Knowledge Quartet (KQ)

One of these studies, which examine mathematics teachers' knowledge in practice was conducted in the United Kingdom. The reserchers' aim was to investigate teachers' lessons and to analyze content-related knowledge (Rowland, Huckstep, and Thwaites, 2005). Researchers used grounded theory. Observations and video records were also used in these mathematics lessons. In the second step, the Knowledge Quartet (KQ) was used to provide information about content knowledge. The study showed that the KQ is primarily based on the foundation of teachers' knowledge, beliefs and understanding of mathematics and its teaching. The key parts of foundation were knowledge of mathematics, knowing how to teach and the reason and the way of learning mathematics (Rowland *et al.*, 2005). The transformation was thesecond unit of KQ. It focused on Shulman's transformation model. The other units of the KQ werethe connection and the contingency. Connections refered to relationships observed in teaching, and contingency wasthe teachers' way to respond appropriately to students to contribute their knowledge during instruction (Rowland *et al.*, 2005). The KQ had many components to investigate the teachers' content knowledge during instruction so it can be used effectively. There are studies that show that the KQ is an effective way to observe teachers' content knowledge, because it focuses on the classroom practice and instructional data together.

2.3.2. Mathematical Quality of Instruction (MQI)

Teachers' content knowledge has an effect on instruction (Fennema and Franke, 1992). Research shows that good instruction can make a difference on students' learning and achievements (Ball, 2003). Thus, teachers' knowledge should be measured with instruments, but there are only a few instruments to observe teachers' Mathematical Content Knowledge. One of them is the Mathematical Quality of Instruction that focuses on teachers' knowledge of the subject during instruction.

Teachers being graduated from the same education system that should be improved make so many teachers equipped with the same mathematics education. Hence, teachers making a difference for themselves by their own opportunities to learn mathematics can improve the quality of mathematics teaching (Ball, 2003). To give feedback and to correct students' error, teachers need to know more about content knowledge in mathematics. Teaching mathematics involves justifying, explaining, analyzing errors, generalizing, defining, and knowing ideas and procedures in detail (Ball, 2003). A respect for the integrity of the discipline, reasoning, awareness and understanding mathematical connections are necessary to teach effectively.

To improve students' opportunities to learn mathematics, teachers should know more than concepts besides knowing mathematics in detail. There are many parts of understanding of mathematics instruction such as using mathematical ideas, solutions and offering explanations well and posing effective problems (Ball, 2003). Learning opportunities of teachers improve the mathematics learning of children. Well designed-courses, workshops, materials and supports should be considered as alternatives for the development of teachers (Ball, 2003).

MKT may not be enough to have adequate information about teachers' mathematical knowledge in the classroom. Mathematical Quality of Instruction is one of the studies investigating quality of mathematics instruction in the classroom. The MQI is a 'Common Core-aligned observational rubric' that supplies a framework to analyze mathematics instruction with its several dimensions (Charalambous and Litke, 2018). Other studies have some limitations in studying teachers' mathematical knowledge for teaching. The aim of the MQI study is not only the mathematical quality of the lesson but also to have information about some factors that can affect mathematical quality as material, curriculum, content, and school culture (Hill *et al.*, 2008).

With the MQI model, observers can evaluate the quality of mathematics during instruction. To see the interactions between teacher-content, students-content and teacher-students separately, the MQI instrument is effective. In the study about MQI, it is also found that MKT is mostly associated with MQI, and this research also observes important factors in this relationship (Hill *et al.*, 2008).

MQI was designed not only with MKT but also with other influences (Hill *et al.*, 2008). There are 83 codes grouped into five dimensions of MQI. There were five sections in the instrument. These sections were instructional formats and content, knowledge of mathematical terrain of enacted lesson, use of mathematics with students, mathematical features of the curriculum and the teacher's guide, use of mathematics to teach equitably (LMT, 2006).

In 2014, the new version of MQI instrument; MQI 4-point version was designed. In the MQI 4-point version instrument, there were four dimensions; these were *richness of the mathematics, working with students and mathematics, errors and imprecision, common core aligned student practices* (Hill *et al.*, 2010). In addition, there were ten whole lesson codes to investigate teachers' knowledge in instruction. In each domain, there were many codes to observe teachers' knowledge during instruction. These codes are definedbriefly in the Tables below.

Table 2.1. Mathematical Quality of Instruction.

DIMENSIONS	SEGMENTS	BRIEF DEFINITION
	Classroom Work is Connected to Mathematics	The focus is on mathematical content in instruction.

The MQI was developed and applied between the years 2003 and 2012. To score each segment, each lesson was divided into equal-length (e.g., 5 or 7.5 minute) segments. In the MQI 4-point instrument, sub-categories of four dimensions were designed to score easily (Hill *et al.*, 2014). Classroom work that is connected to Mathematics should be answered in two ways 'yes' or 'no' (Hill *et al.*, 2014). The code was focused on the content and mathematics during instruction.

Table 2.2. Mathematical Quality of Instruction-Richness of the Mathematics.

RICHNESS OF THE MATHEMATICS	Linking between representation	This dimension is related to meaning of mathematics. The focus of the dimension is deepness of the mathematics that is given to students during instruction.
	Explanations	
	Mathematical Sense-Making	
	Multiple Procedures or Solution Methods	
	Patterns and Generalizations	
	Mathematical Language	
	Overall Richness of the Mathematics	

The richness of the mathematicsis the first dimension of MQI. Codes that focus on the meaning of facts and procedure link to representations, explanations and mathematical sense-making and multiple procedures or solution methods, patterns and generalization and mathematical language. In this section, teachers' use of language and examples given to students are important. Overall, this section is designed to capture the understanding of teachers' content knowledge that is taught. For all codes within this part, there are three aspects; low, middle, and high. If the element is not correct, it is not present (Hill *et al.*, 2014). These MQI dimensions are positioned on the relation between the teacher and the content (Charalambous and Litke, 2018).

Table 2.3. Mathematical Quality of Instruction - Working with Students and Mathematics.

DIMENSIONS	**SEGMENTS**	**BRIEF DEFINITION**
WORKING WITH STUDENTS AND MATHEMATICS	Remediation of Student Errors and Difficulties	This dimension reflects on teachers' understanding of students' mathematical errors, contributions as questions, explanations, ideas, etc.
	Teacher Uses Student Mathematical Contributions	
	Overall Working with students and Mathematics	

The second dimension of MQI is *working with students and mathematics.* In this dimension, the focus is whether teachers can understand and respond to students' mathematical contributions or mathematical errors or not. Student's contributions mean questions, claims, explanations, solution methods, ideas, etc. There are two categories in this dimension. These are remediation of student errors and difficulties, and how teachers use student mathematical contributions during instruction; with these codes, teachers' pedagogical choices can be captured. Mathematical understanding and resources with students are the key of the section. For all codes within this part, there are three aspects; low, middle, and high. If the element is not correct, it is not present. (Hill *et al.*, 2014). The MQI dimensions are positioned on the relation between the teacher and the students (Charalambous and Litke, 2018).

Table 2.4. Mathematical Quality of Instruction - Errors and Imprecision.

<table>
<tr><th>DIMENSION</th><th>SEGMENTS</th><th>BRIEF DEFINITION</th></tr>
<tr><td rowspan="4">ERRORS AND IMPRECISION</td><td>Mathematical Content Errors</td><td rowspan="4">The dimension focuses on teachers' mathematical errors or imprecision in language and notation while they are presenting the content.</td></tr>
<tr><td>Imprecision in Language or Notation</td></tr>
<tr><td>Lack of Clarity in Presentation of Mathematical Content</td></tr>
<tr><td>Overall Errors and Imprecision</td></tr>
</table>

In the third dimension, there are two parts; *errors and imprecision* capturing teacher errors in content or in language and notation, lack of clarity in the teachers' presentation of the content. This section addresses content, accuracy and materials. Teachers can have differences on language or materials to teach the same topics, these codes capture to observe teachers' presentation. For all codes within this part, there are three aspects; low, middle and high. If the element is not correct, it is not present (Hill, *et al.*, 2014). The MQI dimensions are positioned on the relation between the teacher and the content (Charalambous and Litke, 2018).

Table 2.5. Mathematical Quality of Instruction-Common Core Aligned Student Practices.

DIMENSION	**SEGMENTS**	**BRIEF DEFINITION**
COMMON CORE ALIGNED STUDENT PRACTICES	Student Provide Explanations	The dimension aims to investigate students' involvement to tasks. To observe the students' active participation, students' mathematical statements including reasoning, explanations and asking questions should be observed.
	Student Mathematical Questioning and Reasoning (SMQR)	
	Students Communicate about the Mathematics of the Segment	
	Task Cognitive Demand	
	Students Work with Contextualized Problems	
	Overall Common Core Aligned Student Practices	

Fourth dimension of the MQI instrument is *common core aligned student practices* (CCASP). This dimension focuses on how to do mathematics and the extent to which students participate in and contribute to meaning-making and reasoning. The CCASP dimension is related to the eight Standards of Mathematical Practices listed in the Common Core State Standards for Mathematics. These are not one to one correspondences codes. Parts of 4^{th} dimension can be listed as students providing explanations, mathematical questioning and reasoning (SMQR), students communicating about the mathematics of the segment, task cognitive demand, students working with contextualized problems, overall common core aligned student practices. For all codes within this part, there are three aspects; low, middle, and high. If the element is not correct, it is not present (Hill *et al.*, 2014). These MQI dimensions are positioned on the relation between the students and the content (Charalambous and Litke, 2018).

Table 2.6. Mathematical Quality of Instruction - Whole Lesson Codes.

DIMENSION	SEGMENTS	BRIEF DEFINITION
WHOLE LESSON CODES	Lesson Time is Used Effectively	Class is on task / Behaviour issues not disrupt the flow of the class
	Lesson is Mathematically Dense	The amount of Mathematics-Problems/ Tasks
	Students are Engaged	Engagement with the lesson
	Lesson Contains Rich Mathematics	The depth of mathematics given to students
	Teacher Attends to and Remediates Student Difficulty	Teachers' attends to students' difficulty and remediation of them
	Teacher Uses Students' Ideas	Using students' ideas to move the lesson forward
	Mathematics is Clear and not Distorted	Clear and well-defined lessons
	Tasks and Activities Develop Mathematics	Tasks done by the class support the development of the mathematics of the lesson
	Lesson Contains Common Core Aligned Student Practices	Participation to mathematics of the lesson in a substantive way.
	Whole-Lesson Mathematical Quality of Instruction	Overall MQI by the teachers' work during the lesson.

For the whole lesson codes completed at the end of the lesson; there are five levels to score codes. Not at all true of this lesson (1), (2), default score (3), (4), very true of this lesson (5). There are ten dimensions for the whole lesson codes (Hill *et al.*, 2014). These lesson codes are; lesson time is used efficiently, lesson is mathematically dense, students are engaged, lesson contains rich mathematics, teacher attends to and remediates student difficulty, teacher uses student ideas, mathematics is clear and not distorted, tasks and activities develop mathematics, lesson contains common core aligned student practices, whole-lesson mathematical quality of instruction. Mode of instruction can be direct instruction, whole-class discussion, working on applied problems. Whole lesson codes should be scored at the end of the lesson.

As a result, this study showed many kinds of information about mathematical quality of instruction. MQI is the observational rubric that provides a framework to analyze mathematics instruction with several codes. MQI captures the nature and the quality of the mathematical content available to students as expressed in teacher-student, teacher-content and student-content interactions. With all its aspects, MQI is associated with MKT (Hill, *et al.*, 2008). The instrument records not only the mathematical quality of the lesson but also provides information on factors that might affect mathematical quality, including the mathematical content and curriculum materials (Hill *et al.*, 2006).

The uniqueness of MQI is its ability to receive separate scores for each dimension as well as an overall score. MQI scores are significantly related to teacher mathematical knowledge for teaching (MKT) (Hill *et al.*, 2008). The other specificity of MQI's separate codes is to prioritize aspects of mathematics instruction that has an effect on student learning (Charalambous and Litke, 2018).

Like any other study, there are also some limitations of MQI. MQI does not capture some generic instructional aspects such as how teachers are structured and presented on information and management of the class (Charalambous and Litke, 2018). Some instructional aspects like the use of mathematical tools, appropriate use of tools, teaching mathematics equitably are also not captured in MQI. "With MQI, we can get information about errors that happened during instruction but cannot capture the quality of the presentation of procedures contained in the lesson" (Charalambous and Praetorius, 2018, p.446). Thus, the observer also took additional notes to use in the analysis. In addition, there is a large body of literature on teachers' knowledge, but there are relatively few studies, which are focused on MQI (Krauss *et al.*, 2008).

With all this information given in the literature, the Mathematical Quality of Instruction (MQI) is designed qualitatively for this study to observe mathematics teacher's knowledge during geometry instruction. Observation form is added to Appendix A.

2.4. Mathematics Teachers' Geometry Knowledge

Geometry derives from the Greek word geometria. Definition of the word is measurement of land (Hansen, 1998). Geometry is developed and evolved during the period of many cultures. Geometry teaching has evolved mostly after 1970 because of interest on mathematical thinking and problem solving (Panaoura, 2014). There are more geometry topics considerably known after 19th century (Jones, 2000). School geometry includes only some parts of them but a deep understanding of geometry can be developed during school period (Jones, 2000). There are many reasons for including geometry education into mathematics teaching. Some of the reasons can be stated as providing development of awareness, geometrical intuition, knowledge and understanding of geometrical properties and theorems, ICT skills in geometrical contexts, the contemporary applications of geometry (Jones, 2000).

For more than a century, geometry courses are given in high schools. The most difficult part of a mathematics curriculum is to design the geometry curriculum, because there are many interesting topics, which are reasonably included in curriculum (Jones, 2000). Geometry topics have continued to expand in the recent history. In schools, there is a geometry course, which includes Euclidean geometry from early years of education (Hansen, 1998). It is an abstract course and develops mathematical sensibility. Geometry course is integrating mathematical proofs and reasoning abilities of students as a part of mathematics curriculum (Weiss, 2009). Students meet with challenging and also mathematically dense questions during geometry courses. These parts should not be formalized while teaching geometry (Hansen, 1998). Thus, several mathematics educators developed some ways to observe students' geometrical reasoning such as VanHiele (Panaoura, 2014). Students have a conception about geometry that is the difficulty of the course because of theoretical or abstract parts of it (Barrantes *et al.*, 2006). Geometry is not only about proofs of theorems; there are many visualizations and spatial thinking inside of it (Jones, 2000). There are many geometry applications such as computer graphics, image reconstruction, robotics (Hansen, 1998). In addition, by using appropriate softwares, one can explore geometric relationships and interpretations and diagrams (Jones, 2000).

In Turkey, high school mathematics curriculum has changed the form of it many times on the recent history. In 2005-2006, paradigm shift has occurred in mathematics curriculum since constructivist idea was adapted to curriculum philosophy. The adaptation occurred in 2005 which began with primary school level and continued with secondary school level. The curriculum before 2005 was so dense especially regarding the objectives of high school mathematics. The adaptation starting from the year 2005 had affected high school mathematics curriculum especially after 2008. “Every child can learn mathematics” was the main philosophy of mathematics curriculum (Güzel, Karataş and Çetinkaya, 2010). Before the 2013-2014 education year, mathematics and geometry lessons had been given separately in the Turkish curriculum. In the 2013-2014 education year, the curriculum was changed, and these two subjects were added to curriculum together into one component as mathematics lesson (Sakallı *et al.*, 2016). In 2017-2018 education years, some changes in mathematics curriculum took place again. Curriculum was simplified in some parts, especially for mathematics topics. Geometry curriculum topics had not changed according to objectives even though algebra topics had changed.

Teachers’ experiences of geometry influence their conceptions of geometry and their ways of teaching, even if they do not want to imitate their past teachers (Barrantes and Blanco, 2006). The International Commission on Mathematical Instruction Study (ICMI) is a worldwide organization, which conducts studies to develop mathematical education for all grades. In one of the ICMI Study, Fujita and Jones (2002) argue that development of geometrical intuition by linking geometry’s theories should be reinforced to improve geometrical pedagogy. Knowing mathematics is not only about effectiveness of the teacher but also about how one knows the topics (Jones, 2002). To be a successful geometry teacher, there are some aspects such as knowing the geometry course in detailand how to teach it effectively (Jones, 2000). There are studies that show lack of teacher knowledge for beginner teachers about geometry (Aslan-Tutak and Adams, 2015). Teaching geometrical proofs to students are important, but it is not an easy task (Kunimune, Fujita, and Jones, 2009).

All ideas mentioned in the literature, encouraged me to study about mathematics teachers’ knowledge during geometry instruction. The Mathematical Quality of Instruction (MQI) can be effective to be used as a measurement to investigate teachers’ knowledge during instruction.

3. SIGNIFICANCE OF THE STUDY

In today's world, teachers should develop themselves and manage their teaching effectively. It is accepted with research that teachers' subject-matter knowledge influences student's achievements (Hill, Rowan and Ball, 2005). Novice teachers are the newcomer teachers (Borko and Livingston, 1989) and they have great effects on students' achievements. Research on the comparison of the knowledge bases of experienced and novice teachers show that; experienced teachers know more than novice teachers and experienced teachers' knowledge is more structured and highly integrated (Krauss *et al.*, 2008). There is some research conducted to observe novice teachers' knowledge (Shulman, 1987). In this research, novice teacher knowledge will be observed during instruction to have more detailed information about novice mathematics teacher knowledge.

During the firstyear of teaching, teachers can encounter withdifferent principles, ideals, and experiences that they had before teaching and also some restrictions coming from the school environment (Losana, Fiorentini and Villarreal, 2018). A robust preservice training can have a big effect on novice teachers' development in their schools (Losana *et al.*, 2018). During the process of first year, professional identities are recreated by interactions with others socially and culturally (Losana *et al.*, 2018). There are many outside factors on the teaching process. During the first year, teachers encounter many challenges while they are developing themselves (Ruohotie-Lyhty, 2013).

The question "how novice teachers in their first year of teaching use their mathematical knowledge in instruction" is highly interesting for the research area since Shulman's studies, 1987. The findings show that first year of teachers' career is complex and contradictory but also a rich and essential part for the development of teachers' professional identities and knowledge (Losana, Fiorentini and Villarreal, 2018). To observe teachers' knowledge in instruction and to observe contributed factors to their teachers' knowledge is important. In addition, my motivation to study about novice mathematics teachers' knowledge is curiosity. I would like to explore novice mathematics teachers' knowledge during instruction, because I am also a novice mathematics teacher and an emerging researcher.

To study about the teachers' knowledge is not only having information about Pedagogical Content Knowledge and Subject Matter Knowledge. The information getting from tests give us only general idea about teacher knowledge. There are several research studies about teacher knowledge, but teacher knowledge during instruction is taking attention only recently. There are some approaches to investigate it with some limitationsfor each of them. Because teaching is dynamic and situated (Fennema and Franke, 1992), teacher's knowledge should be investigated during instruction. The first source of data should come from teachers' actual teaching practice in actual classroom setting (Fennema and Franke, 1992). One of the instruments that is named as Mathematical Quality of Instruction [MQI] was used in this research. Observations during lesson are supported by pre-observation and post-observation interviews, video records of instructions and lesson plans.

As stated below the purpose of the study is to conduct a case study to observe a novice high school mathematics teacher's knowledge during geometry instruction. The expectation of the study is to significantly contribute to the mathematics education research community, mathematics teachers and other educators.

4. STATEMENT OF THE PROBLEM

The study aimed to describe novice teachers' mathematical knowledge in action during instruction.

Therefore, the study explored the following research questions:

(i) How the first year of high school mathematics teacher's knowledge is observed during geometry instruction as investigated through MQI instrument?

(ii) How school environment and other factors have an influence on first-year high school mathematics teacher's knowledge observed during geometry instruction as reported by themselves?

5. METHOD

The focus of the current study is to observe novice mathematics teacher's knowledge on her first year of teaching during geometry instruction. In this study, qualitative single case study methodology used to provide information about novice mathematics teacher knowledge during instruction. Two key approaches guided this case study methodology; one proposed by Robert Stake and the second proposed by Robert Yin. Both Stake(1995) and Yin(2003) focused on the case study approach in depth. According to Yin (2003), a case study design should be used when; the focus of the research is to answer 'how' and 'why' questions, the behavior of individuals cannot be manipulated, researcher covers contextual conditions and the boundaries are not clear between phenomenon and context (Baxterand Jack, 2008). This study is one of the examples for single case study design as defined by Yin (2003).

Case study approach enable us to gather data from many sources; interviews, observations and lesson plans and to gain tremendous insight into mathematics teacher knowledge during instruction (Baxter and Jack, 2008) In the study, case study approach was used to observe novice mathematics teacher's knowledge during instruction.Data sources of study include interviews, observations and lesson plans to get information about teacher knowledge in depth. The data were collected from one teacher who is in her first year of teaching. She works in a private school in Istanbul. She is currently teaching geometry courses in the school, so all instructions are about geometry teaching.

Semi structured interviews were designed with open-ended questions. Unplanned follow-up questions were also asked during the interviews. To have more information that was needed for data analysis, the goal of the interviewer was to encourage the interviewee to share information in-depth comfortably (Dicicco-Bloom and Crabtree, 2006). All interviews were planned in the participant's school which was the comfortable place for her. The goal of each of the interviews was to gain a deep understanding of the novice mathematics teachers' knowledge during instruction, particularly taking attention to their common pedagogical content knowledge coming from college education and experiences coming from her high school education. To have audiotape-recording during interviews and video-recording during observations can prevent difficulties later in data analysis part(Dicicco-Bloom and Crabtree, 2006). All records put down on paper and analyzed carefully during data analysis. Observation notes are supported by video-records.

In the research, I started to collect data with pre-interview every day. Then, I observed and took notes on the MQI observation form during instruction while video recording the lesson at the same time. After each instruction finished, I have a semi-structured interview with her about the instruction. After six lessons, I collected all my data as an observation form, interview and lesson records and lesson plans. I had transcripted all pre-observation and post-observation interviews' voice records and also lesson records. While I was analyzing the data, I used transcribed records and sometimes took a fresh look to the records again. While I was doing the analysis, I used not only the MQI instrument but also interviews and lesson observation notes that I took myself to diverse viewpoints about the data. In this study, data collection and analysis has occurred concurrently.

Both Yin and Stake (2003) emphasized the importance of organizing data in an effective way. In this study, many methods were used to have consistent and detailed data. Firstly, I did not only complete the observation form but also took notes during instructions to check the consistency of findings generated by different data collection methods. I transcribed the video records and the interviews of each lesson. To make sure that I wrote all observations to the MQI form, I checked my records three times at different time periods. Then, I started to write each instruction one by one with the detailed explanation and by supporting examples for each segment and codes of MQI. Lesson notes consist of;the flow of the lessons, general information about instruction and observation form designed qualitatively by using MQI. Then, I redesigned the lesson examples for each segment and codes defined in MQI. With this detailed report, I started to write the data analysis part by taking all observations and interviews into account. Lesson observations were supported by post-observation and pre-observation interviews.

5.1. Participant

In this study, the participant was selected purposefully. The participant who accepted to participate in this study is in her 1st year of mathematics teaching. She was graduated from Secondary School Science and Mathematics Education at Public University in Istanbul in 2017. Her GPA was 3.16 when she graduated. During her university education, she took some compulsory courses for teaching mathematics such as seminar on practice teaching in mathematics, teaching methods in mathematics and practice teaching in mathematics. She also took selective courses such as teaching pre-algebra and pre-calculus and mathematical thinking. She had passed these courses with over avarage grade. She was above average student with her overall score. Additionally, her homeworks and projects were well-prepared and organized according to one of the course teacher's evaluation. She worked as a tutor from the first year of college education. By tutoring, she kept her mathematical knowledge alive.

She is working in the private school which is audited and supervised byMEB in a metropolitan city of Turkey, Istanbul. The information provided about the participant is limited for confidentiality. The main aim in choosing the participant is to observe how some factors (the school culture, personal traits, previous experiences etc.) influence novice mathematics teachers' knowledge during geometry instruction, because geometry lessons are not provided in the Mathematics Department in the University.

After she graduated from University, the participant started to work in a school which focuses on the Turkish University Entrance Exam. In this school, drill and practice is important because the Turkish University Entrance Exam has limited time and multiple-choice questions. When we look at the background of her education, the participant is also coming from a high school which is focused on practice and drill in mathematics. However, she had some courses to understand how to teach mathematics during her college education. In the University, these courses focus on conceptual instruction methods. She is affected from conceptual teaching methods also. The school that the participant was working is drill and practice focused, whereas her university education is focused on conceptual understanding methods; these two styles are of course in contradiction with each other. Thus, I was expecting to learn which method she is using mostly and how she transformed her experiences and college knowledge to teaching.

From the general interview, pre-observation and post-observation interviews, I had got information about Zeynep's professed view of Mathematics and Geometry and also her teaching of these disciplines. At the below, you can find her statements from the interviews.

5.1.1. Zeynep's Professed Views of Mathematics and Geometry

During the interviews, Zeynep gave information about her views of mathematics and geometry. I collected these statements and inferences below to have an idea about her views about mathematics and geometry.

- Mathematics is a collection of daily life examples.
- Geometry is not about what you see in the figure as everybody said; it is about knowing and understanding the properties and rules.

  Zeynep: If you can write all properties about a geometry topic on an empty piece of paper, it means that you can solve geometry questions about that topic as well.

  Zeynep : Geometry is more than mathematics for me because I always liked geometry during my high school education.

  Zeynep : I like to do puzzles. Solving geometry questions is like doing a puzzle for me. Geometry is an enjoyable part to teach in mathematics.
- She clearly held three views of mathematics that seemed to be the result of her experiences during her study in the University. The first one was about problem solving part of mathematics. The second one was about creative thinking in mathematics, and the other view was related to an importance of daily life examples in mathematics.
- The theorems in mathematics or geometry could not be learned through memorization, students should search for reasoning, understanding and the proof of them.

- Mathematics teachers are not the only sources of information. They also can do some mistakes.

5.1.2. Zeynep's Views of Mathematics Teaching

Statements and inferences which were gathered from the interviews about Zeynep's views of mathematics teaching were presented below.

- The role of teacher in a class is to guide the students.
- The role of teacher in the class is to support students' learning environment.
- Students learn best by attending to the lesson and applying their learning to other cases, relating the topics with other disciplines.
- The students should have a task related with daily life to focus on mathematical topic.

  Zeynep: It is the responsibility of the teacher to encourage students not only academically but also socially during mathematics classes (participation, self-expression)
- To love mathematics starts with loving the teacher of mathematics.
- Encouraging students to ask questions and giving importance to their ideas are also parts of teaching mathematics.

  Zeynep: Mathematics teachers should be well-equipped in their topics because students can ask different questions. Giving the correct answer to question at that time during the instruction is important. Knowing proofs and applications of topics are important in this manner.

  Zeynep: Mathematical content is the same all over the world, but the activities and lesson materials can be changeable according to the teacher. The important part of teaching starts during the instruction.

There will be more detailed information about the participant, in discussion part, supported by interviews.

5.2. Data Collection

The data was collected from the participant in three phases. Lesson observations and observation form designed by MQI completed during instruction is the first phase. Video records of the instructionswere also included. Pre-observation and post-observation interviews records were the second phase of the data. Lesson plans and materials were the last phase. The aim to gather data from different sources (triangulation) is validation for this study. The main data source of the study is lesson observations and MQI instrument which iscompleted during lesson. Lesson plans and interview records are used as an additional resource. Classroom observations were recorded once a week in the spring semester of 2017-2018 Education years. Interviews are effectively designed to get information about novice teachers' learning to teach and how beginning teachers use pre-training and preservice knowledge in their first year. For each of the observations, there are 15-20 minutes pre-observation interviews and post-observation interviews. Semi-structured interviews were recorded to get data from participant. Data were collected from April 2018 to May 2018 with the written permission of teacher and the private school she was working. The permission includes all pre-interviews, post-interviews and the full-video record of the instruction.

The observation form that is designed to use in this study is in Appendix A. Observation form was prepared by using MQI instrument. The semi-structured interview questions that are planned are in Appendix B, C and D. These questions are prepared to learn more information about participant, her experiences, her lesson objective, her perspective about teaching and so on. Data getting from lesson observations is supported with pre-observation and post observation interview questions.

5.2.1. General Interview

Researcher conducted an interview at the beginning of the data collection. General interview hold almost 40 minutes in the school that she worked. The aim of the interview was to get information about participant comprehensively. General interview was designed semi-structured to gain information about the teacher deeply. General interview questions are added in the Appendix B. Her perspective and opinion about teaching, mathematics teaching and her teaching experiences are added to interview questions extensively. Additional questions include information about her opinion, working school, her coteries, school administration, and students' achievement. One of the important questions was about college education and its effects on her teaching. There were many more questions about school environment and factors which influence her teaching.

These general interview questions gave more detailed information about participant's personal experiences and her opinions to support data that is collected during instruction.

Table 5.1. General Interview Questions and Rationale.

MAIN QUESTION	***RATIONALE***
1. Can you describe "really good mathematics teacher and mathematics teaching" in your opinion?	*The purpose of the question is to see teachers' thinking about mathematics teacher.*
2. What kind of mathematics lessons are really impressed you? Can you describe to me really good mathematics lesson?	*The purpose of the question is to understand how teacher explains the high-quality of mathematics instruction.*
3. Consider about mathematical quality of instruction and mathematics teachers' knowledge, how can you describe these two concepts?	*The purpose of the question is to see how the teacher describe and relate these concepts.*
4. How is mathematics as seen in your working school? Can you explain your division of labor and relationship in the mathematics coterie? How do you prepare your instruction materials in the school?	*The purpose of the question is to understand the school policy about mathematics instruction.*
5. What do you find challenging or difficult in this school? How can you contribute to develop these parts? What is the best part of working in this school?	*The purpose of the question is to have information about school culture and environment.*
6. How college education has an effect on your teaching? What is the effect of your high school mathematics teacher on your teaching?	*The aim of the question is to find a relationship between her teaching methods and her experiences coming from high school and college.*
7. What do you think about your instruction? How do you plan to develop your teaching as a novice teacher?	*The aim of the lesson is to learn about her own development plans.*
8. Is there anything that you have not mention so far during the process of the study?	*The aim of the question is to reinforce her additional ideas.*

5.2.2. Pre-Observation Interviews

Semi-structured pre-observation interviews were conducted in this study. The interviews, which were audio-recorded and then transcribed, took approximately ten minutes. The aim of the pre-interviews was to learn lesson objectives, teacher's plan for lesson and materials that teacher will use during instruction. In the interviews, participant was too attender and she eagerly answered all questions.

The pre-observation interview protocols and questions are added in the Appendix C. The interview transcripts were used for analysis.

Table 5.2. Pre-Observation Interview Questions and Rationale.

MAIN QUESTION	***RATIONALE***
1. What are the objectives / mathematics topic of today's instruction?	*The purpose of the question is to learn what topic will be taught during lesson.*
2. How would you address the topic mathematically?	*Teacher understanding of the mathematics topics to be taught.*
3. How did you get prepared for this lesson? *-Materials e,g. activities, problems* *-Student ideas* *-Mathematics representations*	*The purpose of the question is to learn how the teacher is prepared for the instruction and what her idea is about preparation to lesson*
4. How did you plan the lesson in terms of student's difficulties or understandings?	*The aim of the question is to learn her plan when she encounter with students' problems.*
5. Will you use specific teaching method for this lesson?	*The aim of the question is to learn her instruction method.*

5.2.3. Video Records of Classroom Instruction

The main aim of the study is to understand novice mathematics teacher knowledge. Hence, it is crucial to capture the instruction, observing teacher during instruction. The information that is collected during lessons was analyzed by using MQI instrument. Lesson was observed together with video recording. The purpose of the video recording of the instruction is to transcript the lesson later and use for triangulation method to confirm observer's findings. Video record is important to detail the data and watch again to reinforce findings. All instructions were recorded. I placed the camera sometimes close to teacher to capture the teachers' actions and teachers' voice during teaching and mostlyplaced at the back of the classroom. There are six lessons recorded in three days as a block session of two lessons. Two lesson records and transcribed data were used for validity of this study.

5.2.4. Observation Form - Mathematical Quality of Instruction

Mathematical Quality of Instruction is a Common Core-aligned observational rubric (Hill *et al.*, 2008). It's framework which helps to analyze mathematics instruction with its several dimensions. MQI study does not give information only about the mathematical quality of instruction. It gives information also about some factors that have effect on mathematical quality such as material, curriculum, content and school culture.

Mathematical Quality of Instruction is designed for quantitative studies. However, observation form was designed purposefully according to qualitative study by using MQI instrument's codes and dimensions for this study. When observation form was prepared, classroom setting, types of questions asked by both students and teacher and objective of lesson were added to observation form. Codes were given in original instrument by the rank of 1 to 5 to evaluate quantitatively. These rank orders were changed with explanation of each code and empty place to write comments and examples from lessons to use qualitative study. "With MQI, we can get information about errors that happened during instruction but cannot capture the quality of the presentation of procedures contained in the lesson" (Charalambous and Praetorius, 2018, p.365). Thus, observer took also additional notes to use in the analysis. If some different codes were emerged during observation, researcher was added the codes to analysis to get overall data. Researcher was broadened her perspective with MQI instrument. Researcher found answers to her questions about richness of mathematics of the lesson, students' involvement to lesson, both teacher and students' participation and errors occurred during instruction and so on. Observation form that is adapted from MQI instrument is added in Appendix A to be used for other studies.

5.2.5. Post-Observation Interviews

I conducted semi structured post-observation interviews with the teacher as the last step of the data collection. The interviews are conducted directly after each instruction finished. The interviews, which are audio recorded, took approximately fifteen minutes. The main aim of the post-observation interviews was to learn the teacher's opinion about her instruction. In the interviews, questions focused mostly to learn how instruction was according to her and what teacher would want to change if she had a chance and also what goes well or difficult.

The post-observation interviews were important to test and to confirm findings of lesson observations. The literature supports that only a researcher can be limited to describe instruction and to decide the aim of some parts of the lesson (Huberman, Miles and Matthew, 1984). With the data supported from post-observation interviews, researcher's interpretations can be more coherent. The post-observation interview questions are provided in the Appendix D. The interview protocols were used to get more detailed data about teacher's mathematics knowledge.

Table 5.3. Post-Observation Interview Questions and Rationale.

MAIN QUESTION	***RATIONALE***
1. Do you have any comment about your mathematics instruction in your classroom? Something is going well, or going difficult for you? Which part of the lesson did you like/dislike more? (Try to get the teacher to talk about their own mathematics teaching)	*The purpose of the question is to direct the teacher to reflect on her own mathematics instruction.*
2. Was there anything that you struggle or your students struggle with any part of the lesson? Which parts? Why do you think so?	*The aim of the question is to learn most difficult or unplanned parts of the lesson.*
3. Howyour mathematics instruction contributes to mathematics of students'	*The purpose of the question is to understand how she comments on her own instruction according to mathematical quality.*
4. What were you hoping that students would learn from this lesson?	*The purpose of the question is to understand what the teacher thinks about students' learning.*
5. What would you have done differently if you had taught the lesson again? Why?	*The purpose of the question is to understand what the teacher wants to change from her lesson and why.*

5.2.6. Lesson Plans and Materials

Lesson plans and materialsare used as an additional data source if researcher needs more detailed information about lesson such as theorems or rules. Artifact collections are also used to understand how participant prepare her lesson before instruction. To get detailed information about questions and to use questions on data analysis, lesson materials are used mostly. You can see pictures of questions on findings. It should be noted that lesson materials were mostly questions from test books.

There are test books of the schools in Turkey and teachers tend to use them while they are teaching. These publications are designed according to questions asked before in the Turkish University Entrance Exam.

5.3. The Role of the Researcher

The researcher was a non-participant observer in the classroom during the study. Researcher tried to observe quality of teacher's knowledge during instruction by using observation form designed by adapting MQI instrument. Thus, the researcher tried not to disturb the natural setting of the classroom as much as possible. The interviews with teacher were semi-structured. While asking questions, the researcher didnot judge or lead the participant in order to have an idea about participant's own ideas.

5.4. Trustwhortiness of the Study

To increase the quality of the study, validity and reliability are considered in the qualitative study. There are different methods and terms for these concepts. There are terms such as *trustworthiness* for validity, *credibility* for internal validity and *transferability* for external validity and *confirmability* for obsectivity (Merriam, 2015). For trustworthiness, researcher should design the study detailed by taking into consideration of scientific community. To increase believability of the research, gathering data from different sources and supporting the finding with multiple sources are necessary.

In this study, *triangulation* method was used to gather the data via observations, interviews and video records of both instructions. Also lesson materials, artifact collection, were used when it was necessary. Data retrieved from all these different sources were analyzed carefully with the thesis advisor. Triangulation is a state of mind (Huberman (Miles and Matthew, 1984). Triangulation of data collection allowed researcher to study the research questions by using various data sources. Trustworthiness also depends on crediability of the researcher (Merriam, 2015). To strengthen the credibility, the observations are recorded as videos with notes. Lessons were observed throughout three weeks. By using triangulation, not only credibility but also confirmability of the study was strengthened. To increase the transferability, the data was collected in detail without any personal comments. Participant's views and statements are shared directly for many cases to be transferable to other studies.

For the reliability of this study, the term dependability, a term used by Lincoln and Guba (1985) for reliability, is ensured by taking video records and transcribing all the observed lessons. In order to strengthen the depedenability of the study, I also asked another researcher, an expert in MQI to code two lesson obervations. She analyzed the lessons (video recordings, transcripts, and observation notes) according to the oberservation form which was developed by the researcher by adapting MQI dimensions. In Table 5.4 summarizes the coders (the researcher and the expert in MQI) answers for each of the 4 MQI dimension, 31 segments under them. In the table, the symbol $\times$ shows that a segment is not presented during instruction. The symbol $\surd$ means that a segment is observed during the same part of the instruction supported by the same example from lesson.

Among 31 segments, only 4 segments (87 percent) of MQI are coded differently by coders. The segments with* next to their name was coded differently. For example, "teacher uses student mathematical contributions" segment was coded dissimilarly for two observers. The researcher counts answer of calculations as a students' contribution. However, expert in MQI did not observe any contribution of students to the lesson. Our codes were open to be discussed. To resolve the differences, we discuss differenceses with reasons.

After one year passed from the observations of participant, member check and follow-up study were conducted by sharing all findings with the participant to improve validity and accuracy of the research (Guba and Lincoln, 1985). During interview, firstly, all findings of study were shared with participant then, the same questions which had asked during general interview one year before about her mathematics teaching were asked again. With the information getting from her statements, she was working at the same school but, she was searching to find a school to improve herself and also apply the practices that she was mentioning yet could not find chance to implement.

She did not stop to study and develop herself for teaching mathematics.

Table 5.4. Comparison of Analysis Results of the Researcher and the MQI Expert.

DIMENSIONS	SEGMENTS (totally 31 segments)	Expert	Resch.
RICHNESS OF THE MATHEMATICS	Classroom Work is Connected to Mathematics	×	×
	Linking between representation*	×	√
	Explanations	√	√
	Mathematical Sense-Making	×	×
	Multiple Procedures or Solution Methods	×	×
	Patterns and Generalizations	×	×
	Mathematical Language	×	×
	Overall Richness of the Mathematics	×	×
WORKING WITH STUDENTS AND MATHEMATICS	Remediation of Student Errors and Difficulties	√	√
	Teacher Uses Student Mathematical Contributions*	×	√
	Overall Working with students and Mathematics	√	√
	Mathematical Content Errors	×	×
ERRORS AND IMPRECISION	Imprecision in Language or Notation	×	×
	Lack of Clarity in Presentation of Mathematical Content	×	×
	Overall Errors and Imprecision	×	×
STUDENT PRACTICES	Student Provide Explanations*	×	√
	Student Mathematical Questioning and Reasoning (SMQR)	×	×
	Students Communicate about the Mathematics of the Segment	√	√
	Task Cognitive Demand	√	√
	Students Work with Contextualized Problems	×	×
	Overall Common Core Aligned Student Practices	×	×
	Lesson Time is Used Effectively	×	×
	Lesson is Mathematically Dense	×	×
	Students are Engaged	√	√
WHOLE LESSON CODES	Lesson Contains Rich Mathematics	×	×
	Teacher Attends to and Remediates Student Difficulty	√	√
	Teacher Uses Students Ideas*	×	√
	Mathematics is Clear and not Distorted	√	√
	Tasks and Activities Develop Mathematics	×	×
	Lesson Contains Common Core Aligned Student Practices	×	×
	Whole-Lesson Mathematical Quality of Instruction	×	×

For ethical issues, permission from the private school of the participant was taken at the beginning of the study. Students who are attending the lesson are informed about the study and confidentiality of the study. The teacher and the principal were informed about the research process in detail and asked to sign an informed consent, containing information about the study and the ethical issues. Since the focus of the study was teacher's knowledge, the participat was only the teacher. The confidentiality of the participant such as her name or school's name was preserved. All kinds of data were protected from other people. The reason to not share the names of the teacher, student and textbooks is to increase confidentiality. The name of teacher was assigned randomly as Zeynep.

6. FINDINGS

Generally, the class environment that she gave instruction was bright and huge. There were seventeen students in the class. Five students of the class were female. When she came to class, she firstly opened the smart board. Then, she talked with students about their issues, problems or other lessons. The class was settled one by one sitting plan. Preparation to lesson took almost 7 minutes mostly. During most of the lessons, students are assigned to solve questions. The aim of the lesson was to solve test book questions.

Zeynep was organized during all instructions. She has planned her lesson by selecting and solving the geometry questions to solve in class. She put her lesson into practice in 4 out of 6 lessons. She finished everything she planned before lesson. For other two lessons, she listened students' sharing about out of mathematical topics. In almost all lessons, disciplinary or behavior issues disrupt the flow of the instruction. She warned some students many times to listen to the instruction. The classroom environment was positive for both teacher and students.

Students started the lesson by solving questions mostly. Most of them were interested in questions at the beginning. When they could not solve some questions or were bored, they started to talk with friends about different topics.

6.1. General Flow of the Courses

During three days of Zeynep's teaching, six lessons were observed. First and second sessions, third and fourth sessions and fifth and sixth sessions were observed at the same day consecutively. These two sessions were block scheduling with 10 minutes break between them. In this part of findings, general flow of the courses will be explained shortly.

6.1.1. First Session

This was first video recorded lesson with Zeynep's 9th grade Geometry class. The number of students was seventeen. The objectives planned for the lesson were "to solve questions about trigonometry on right triangle?? and "to understand some properties of isosceles triangles".

She came to class and opened the smart board. The class was settled one by one sitting plan. Preparation of students to lesson (e.g opening their noteboooks, stoping to talk with friends) took almost 9 minutes.

Lesson started with a question from the test that teacher gave them in last lesson. They discussed and solved the questions. It took almost 10 minutes. Then, students took note to their notebooks both for questions and examples. Then, she started new topic "isosceles triangle" after trigonometry on right triangle questions. She first asked 'What is the definition of isosceles triangle' 'How can we define properties of it' Then, she asked for the properties of perfect four in isosceles triangle.

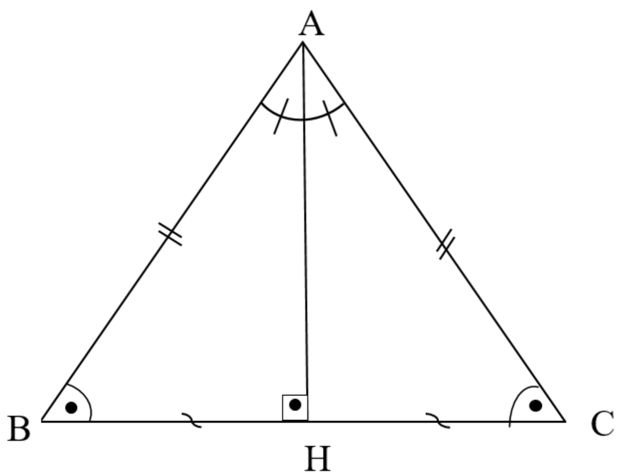


Figure 6.1. Perfect Four in Isosceles Triangle.

She drew the given picture. She wrote the four properties of the isosceles triangle rule (YAKI rule as a mnemonic): isosceles triangle, height, median line, angle bisector.

Then, she wrote a question to the board and solved on the board after students drew it to their notebooks. They used the properties of Euclid's theorem. One of the students pleased to review the rules of Euclid again. She said to explain in other lesson.

Table 6.1. Frequency of codes observed during 1^{st} lesson.

1^{ST} LESSON	**1**	**2**	**3**	**4**	**5**	**6**
Richness Of Mathematics	0	2	1	1	1	1
Working With Students & Mathematics	2	1				
Errors And Imprecision	0	0	0			
Student Practices	2	1	1	0	0	

6.1.2. Second Session

She came to class and firstly started with the rules of Euclid. She explained the rules upon the student's request.

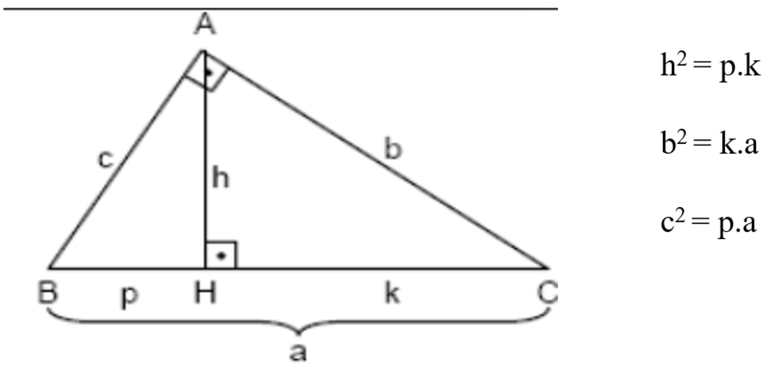


Figure 6.2. Euclid's Theorem.

She asked properties to students. She confirmed properties given from students and wrote to the board if it is true. If students gave wrong explanation, she remediated information and explained the correct one.

Then she continued with isosceles triangle questions about properties. She firstly drew the question on the board. She wrote questions to do practice about the rules.

Table 6.2. Frequency of codes observed during 2^{nd} lesson.

24^{ND} LESSON	**1**	**2**	**3**	**4**	**5**	**6**
Richness Of Mathematics	1	2	1	0	1	0
Working With Students & Mathematics	1	2				
Errors And Imprecision	0	0	0			
Student Practices	1	2	1	0	0	

6.1.3. Third Session

Zeynep settled the class in one by one sitting plan. She asked for a reason of many students who are absent. Then, the lesson started five minute late. She had planned to do quiz but she changed her mind because of many absent students.

Students are assigned to solve questions. Some students wanted to study with their project about science lesson. She warned them to start the given test and to focus on the geometry questions. Some pairs of students started to talk about questions and solution methods.

To see what students are doing, Zeynep moved around the class, worked with individual questions. Zeynep remediated students' errors or solution methods and reminded them some rules to solve questions.

When some students asked for the questions, she also showed them examples from notebook and reminded them median or Pythagorian theorem. Some students continued to solve questions and others talked with teacher.

Table 6.3. Frequency of codes observed during 3^{rd} lesson

3^{RD} LESSON	**1**	**2**	**3**	**4**	**5**	**6**
Richness Of Mathematics	5	5	2	1	0	0
Working With Students & Mathematics	2	2				
Errors And Imprecision	1	0	0			
Student Practices	2	2	0	0	0	

6.1.4. Fourth Session

They continued with the test material on the lesson. Students continued to solve questions from the test and asked to Zeynep.

Some students finished the test so they wanted to check their results. She gave them answer key at the beginning of lesson if they finished the test totally. These students asked for permission to solve mathematics test questions from different book, she gave them permission after they checked their answers.

Zeynep continued to talk with others who are solving the test. Some students did not want to solve and they wanted to leave the rest of the test. Zeynep encouraged and motivated them to finish the test during this lesson before general test exam. She assigned Ömer to teach some questions to Batuhan.She emphasized that this lesson was opportunity for students to review last topics about right and isosceles triangle.

Table 6.4. Frequency of codes observed during 4^{th} lesson.

4^{TH} LESSON	**1**	**2**	**3**	**4**	**5**	**6**
Richness Of Mathematics	0	2	0	0	0	0
Working With Students & Mathematics	2	3				
Errors And Imprecision	0	0	0			
Student Practices	2	0	3	0	0	

6.1.5. Fifth Session

Lesson started with talking about science projects for first 10 minutes.She started the lesson by asking what they studied in last lesson. She reviewed quickly and verbally last properties that they proved in the other lesson. She drew a question about properties that they learned. She encouraged students to write the question to their notebooks.

She checked notebooks of some students. Zeynep asked to a student to solve the question on the board to show calculation when she was checking notebooks.

She explained special right triangles as 30-60-90, 45-45-90 for some questions. Actually, she reminded quickly by drawing two different triangle. She drew a triangle to show a new property and then opened a video that explains the rules by lecturing. They talked about the proof. Then, they solved example about that proof.

Table 6.5. Frequency of codes observed during 5^{th} lesson.

5^{TH} **LESSON**	**1**	**2**	**3**	**4**	**5**	**6**
Richness Of Mathematics	1	2	1	0	1	0
Working With Students & Mathematics	1	2				
Errors And Imprecision	0	0	0			
Student Practices	2	0	0	0	0	

6.1.6. Sixth Session

After 8 minutes, she started the lesson by drawing example on the board. She explained another property after first example. She explained the property then solved examples about these properties.

Zeynep: When you see parallel lines, you can remove the angle to the other parallel line passing through the same line. Thus, angle B is equal to angle E. When you *remove the angle, you can get isosceles triangles BED and EFC. Distance of one side of isosceles triangle is equal to* $|DE| + |EF| = |AC|$.

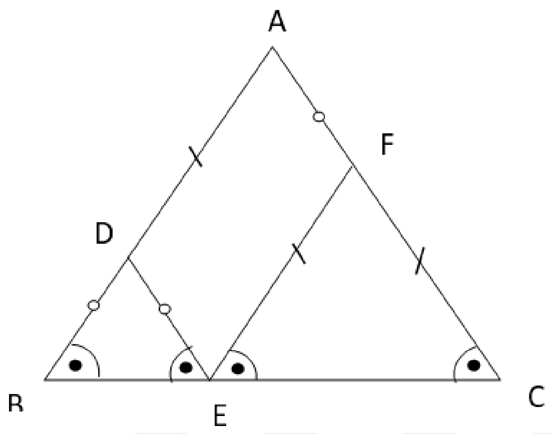


Figure 6.3. Property of Isosceless Triangle.

Table 6.6. Frequency of codes observed during 6^{th} lesson

6^{TH} LESSON	**1**	**2**	**3**	**4**	**5**	**6**
Richness Of Mathematics	1	2	0	0	0	0
Working With Students & Mathematics	1	2				
Errors And Imprecision	0	0	0			
Student Practices	1	0	1	0	0	

I introduce four main and one final dimension of Mathematical Quality of Instruction and gave general characteristics of it in the literature review. Now, I am going to offer my interpretations and observations of Zeynep's instructions according to each dimension of MQI.

6.2. Findings from the Lesson Observations

In this part of the findings section, each MQI codes will be analyzed one by one for all data getting from observation forms.

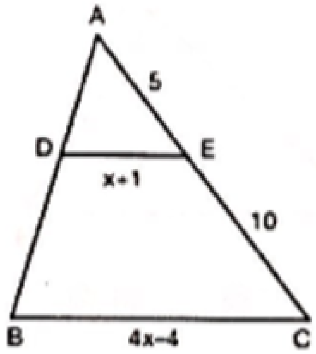
6.2.1. Richness of Mathematics

This section captures the depth of mathematics that is offered to students. This dimension is related to meaning of mathematics and how mathematics instruction is given to students during instruction. When compared to other dimensions of MQI, there are more data which support richness of mathematics during instruction.

It consists of seven categories. First six categories of richness of mathematics can be observable during lesson. The last category is about overall evaluation of Zeynep's lessons. Some of the categories were observed more frequently compared to others. Linking between representations and explanations are the most observed sections of richness of mathematics.

6.2.1.1. Linking Between Representations. Linking between representation focuses on explicitness about how two or more representations are related. It is related with details and elaborations of representations.In this study, this code was occurred five times of the six lessons. The lesson that the code is not observed was not suitable to observe linking between representations. She related pictorial and verbal explanations by using linking between representations for all five lessons. For example, she showed picture or shape of triangle and explained given properties in the text also. When she used pictures, she used text to explain the picture at the same time. It was the only way that she used to link representations.

She did not use linking of representations as connecting real life situations to a pictorial representation. Examples from daily life or pictures showing mathematical applications from daily life situations could be used in geometry lessons. This kind of representations helps also students' learning process.



ABC bir üçgen
[DE] // [BC]
|DE| = (x + 1) br
|BC| = (4x − 4) br
|AE| = 5 br
|EC| = 10 br

Yukarıdaki verilere göre, |BC| kaç br dir?

A) 20 B) 22 C) 24 D) 26 E) 28

Figure 6.4. Example from lecture three.

For each question on the test, she gave representation both on triangle and in the text. One of the examples is given on the left.

She used questions that are prepared by some publications. She chose questions from test books of the school. The only material which she offered to students during instruction was test book questions. In these questions, she used representation only to support picture with text. Thus, she did not use representation to relate geometry with real life situations. Actually, publications that she used do not promote real life example questions. These test books have the same kind of test questions that are not related with daily life examples.

During interviews, she explained why she did not use representations. She honestly said that it is easy to choose prepared questions and these questions which are taken from test books are in the test question format. She stated that choosing test questions is easy because preparing real life problems takes time and energy. Also, she stated that there is no expectation from administration and parents, so it is not the priority for her. However, she is willing to prepare materials if they will use actively during instruction.

Zeynep: When we are in the meeting with mathematics teachers of our school, I said that I am willing to prepare materials to support conceptual learning that I have learned from University. I want to use my University knowledge to support students' learning. Our direct teaching method aims only to do practice and solving many questions. School administration needs time for this change because they have some doubts about parents' opinion.

She also warned students about representations.

Zeynep: Some pictures cannot have all explanations so you should read given text carefully.

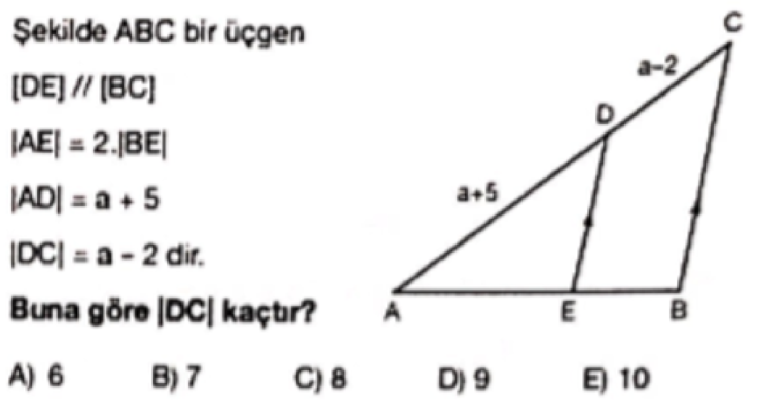


Figure 6.5. Example from lecture three.

Linking between picture and text was mostly observed one. Parallel lines showed in the picture are also given in the text.

Zeynep: As you can see in the picture, distance of AD and DC is given on the picture. You should read the text carefully because ratio of the distance AE and BE is not given on the picture.

6.2.1.2. Explanation. This code is the most applied part of the segment during observation. For each lesson, explanation was observed at least two times. It occurred in two ways; explaining solving method of questions or explaining definition and properties.

Explanations during instructions were based on the question that teacher is aimed to solve. Her explanations mostly focused to solve and give the answer of questions. Considering methods of teaching, teacher focused on teaching through solving questions. Thus, it was expected that instruction focuses on problem solving methods and explanations of solution methods of questions mostly.

Given example is an understanding relationships and differences between Euclid's theorem and perfect four (YAKI).

Zeynep: There is a difference between two rules. Euclid's theorem can be used if you have right triangle and height inside of that. Perfect four can be used if you have isosceles triangle and height inside of it. In the second, it is not necessary to have right triangle to apply the rule.

This explanation is superficial and procedural according to question. When she is explaining, she is not using conceptual methods. She is giving explanation to reach the solution of questions. During problem solving, students asked for help to find answer of questions and she explained the way how to solve them.

Actually, explanations should not be to memorize the solution of questions. However, teacher had directing role instead of guiding for students to construct their own knowledge. The quality of explanations was limited because of instruction method. Her aim was to direct students to find solution of given questions even while explaining the process of the questions.

However, she stated in the interview that I am trying to show them proofs but students are answer-oriented rather than processes. They quickly want to learn the way of solution. The steps and meaning of procedure are not interesting for them.Role of teacher during instruction was directing and students did not do explanation mostly. Teacher oriented explanations managed all the lessons.

Secondly, explanations are observed to define terms and properties at the beginning of instruction. These were also teacher-oriented. Students only contribute to move lesson forward by giving answers to her questions. Zeynep was the reason of explanation mostly at the beginning of the lesson also by defining terms or properties.

Zeynep: Today, we will start to learn properties of isosceles triangle. Firstly, who wants to define isosceles triangle?
Student: The distance of two sides of isosceles triangle is equal.
Zeynep: Is that enough to define isosceles triange? What is the definition of isosceles triangle detailed?, How can we define properties of it?

Then, she said that the most important rule of isosceles triangle is YAKI rule. She asked for the properties of YAKI rule in isosceles triangle.

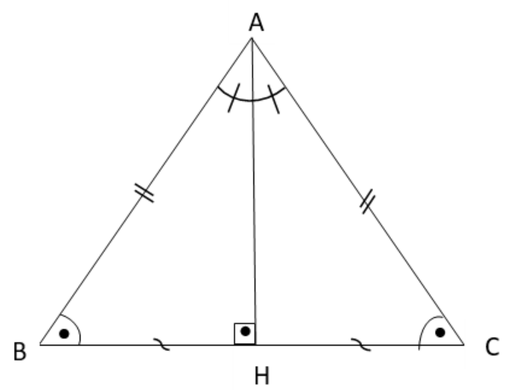


Figure 6.6. Example from lecture three.

She drew the given picture. She asked and wrote the four properties of the isosceles triangle rule (YAKI).

Student: YAKI have four properties; isosceles triangle, height, median line, angle bisector at the same time.

Then, students took note of these properties to their notebook as a review. Students were the writer mostly to the board mostly. When they come to board to solve the question, Zeynep explained the reason of the method most of the time. Students did not explain how they solved; they drew and calculated the answer of questions. Role of students were to solve the given questions and listen to teacher carefully to get explanations well.

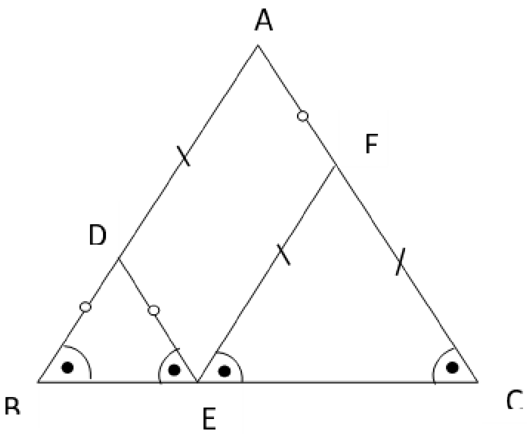


Figure 6.7. Example from lecture six.

She explained the property then solved examples about these properties mostly.

6.2.1.3. Mathematical Sense-Making. Mathematical sense making focuses on understanding relationships between concepts. It supports to find connections between mathematical ideas and representations. This code searches for meaning to mathematical ideas during lesson. In geometry, the code includes making sense of definitions, formulas by elaborating and applying them. Finding counter-examples rather than just stating them can be counted as mathematical sense-making.

She mostly used explanation rather than mathematical sense making. Actually, proofs and questions were totally correct; there was not wrong or deficient part. However, quality of sense making and frequency of it were low. Quality of sense making was mostly directed to find the solution of questions, not to give the meaning of it. She mostly stated definitions and formulas as it is mentioned in explanation. When she proved the formulas, it makes sense for students. However, she mostly used direct explanation for proofs, not application or elaboration to prove properties. The examples that are given in this code are limited because of instruction method. She solved examples after she proved the properties. There could be proofs that are mathematically rich and applicable for real-life examples. Richness of the instruction could be enhanced in terms of providing mathematical proofs.

During instruction, she wanted to prove properties by using different proof methods. She showed students some videos which explain origin of properties. Examples and videos were limited to make properties sense because videos were used also direct teaching method to prove properties. She used the direct instruction when proving properties on the board.

Her questions were not enough to let students think about proofs. She asked questions but then mostly gave the answer herself. She stated in interviews that students get used to memorize laws and rules and they are answer-oriented, they focus on the final part. Thus, how to find this rule or theorem does not have a meaning for them. Thus, she mostly chose direct explanation method and direct proving according to instruction method.

Zeynep: Who wants to say why addition of two parallel lines in isosceles triangle gives the one equal sides of isosceles triangle.
Student: Because we draw inside and divide into two parts.
Student: Is this the same all the time?
Zeynep: Yes, it is.

Students were impatient to wait for proof. They only focused on the result of the proof not the process. She gave a clue but they asked for the result and she explained the proof by making sense of it on the board or opening video about the proof. According to purpose of instruction, she explained the property by drawing lines inside of triangle then solved related examples about this property.

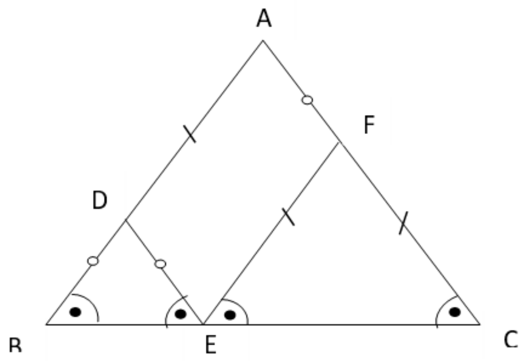


Figure 6.8. Example from lecture six.

Zeynep: When you see parallel lines, you can remove the angle to the other parallel line passing through the same line. Thus, angle B is equal to angle E. When you remove the angle, you can get isosceles triangles BED and EFC. Distance of one side of isosceles triangle is equal to IDEI + IEFI=—AC—.

It was teacher-oriented proof; making sense of it is given to students directly, not by doing. She stated in the post interview of the instruction that time is not restriction for her but students were not willing to do activity and they do not get used to think before solving. They were assuming information as a pill and they are not thinking about process. At the end, they are memorizing everything, not taking into account of process to make sense properties.

There could be learning by doing activities to increase richness of the instruction in terms of providing mathematical proofs. She was willing to search for methods that make sense the properties and proofs by using handiwork or activities supporting that. In the interviews, she said that it will be helpful to do learning deeply but teachers need time to change for conceptual learning.

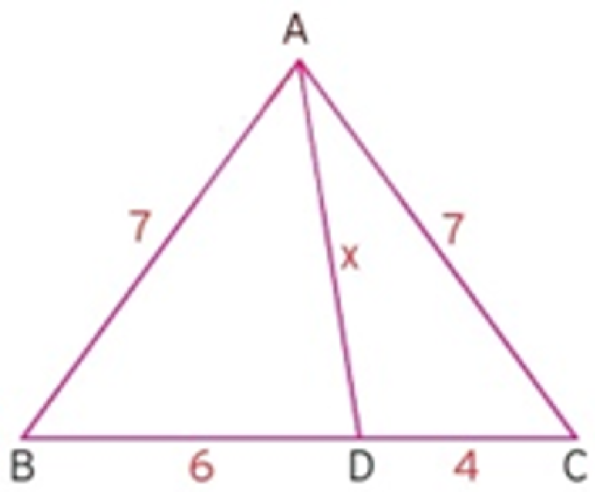


Figure 6.9. Example from lecture two.

Zeynep asked who can solve the question for x. Then, she gave a clue that we should draw additional line.

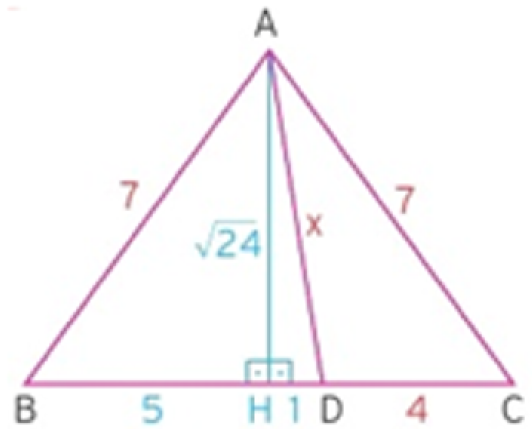


Figure 6.10. Example from lecture two.

They drew additional line to triangle and she asked why we need to draw this line. Teacher helped students to understand and make sense of need of right triangle inside of triangle to solve the problem.

Zeynep : If you see isosceles triangle, you can look for perfect four rule. Thus, it is necessary to draw height of isosceles triangle. When you have two properties, you *can add the other two.*

In the properties of isosceles triangle, she showed the symmetry line in the middle and asked why it should be there. They discussed for a while. One student said that it is median because it divides triangle in the middle. Teacher explained that it is symmetrical because when you fold the paper, they have the same amount and size on both sides of line.

In the first lesson, she explained the perfect three rules directly. She explained the rule without reasoning or critical thinking. Students accepted rule and used for some questions. Then, she remembered another way to prove the perfect three rules by using circle during interview after instruction. She explained and proved again the rule in the sixth instruction while students were solving questions.

When I compare the first and sixth lesson, I can obviously say that the quality of mathematical making sense and proving was rich in sixth lesson. Above you can see perfect three rule that is proved by using circle.

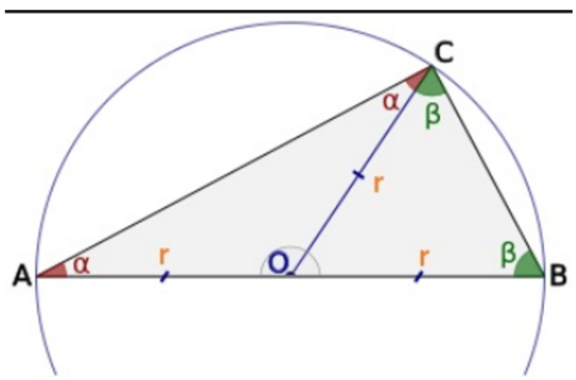


Figure 6.11. Example from lecture six.

In the properties, she asked firstly the proof of it to students, *Zeynep: How can we explain the property? Zeynep: Why one side of triangle is equal to the sum of two normal line?*

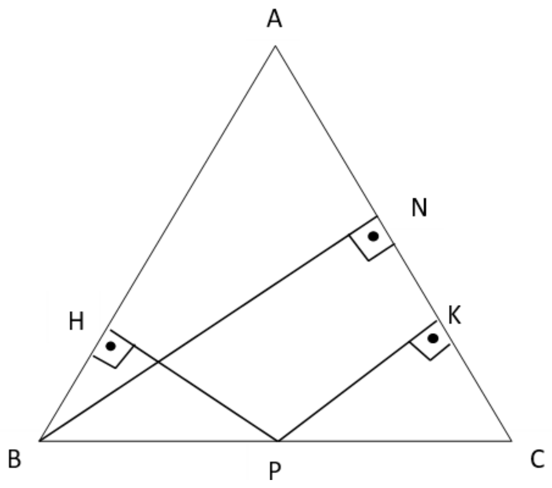


Figure 6.12. Example from lecture six.

For the new property as shown in figure 6.12, she used area formula of triangle. She showed the video of proof and then wrote the board explanation and the proof.

Total of two areas in triangles ABP and PAC is equal to total area of ABC triangle.

$A : \frac{|AB|.|HP|}{2} + \frac{|PK|.|CA|}{2} = \frac{|AB|.|BN|}{2}$ because it is isosceles triangle; $|AB| = |AC|$

$A : \frac{|AB|.|HP|}{2} + \frac{|PK|.|AB|}{2} = \frac{|AB|.|BN|}{2}$ by using simplification.

$A : |HP| + |PK| = |BN|$.

<u>6.2.1.4. Multiple Solution Methods.</u> It is recommended that teachers should present different solution methods for students to solve problems (Grobe, 2014). Multiple solution methods widen students' mathematical perspective. Her instruction was based on solving various questions regarding the topic of lesson. However, multiple procedures or solution methods did not observed frequently during her instruction.

She did not solve questions with other methods. The routine of the lessons were such that when the classroom reached the correct answer, they moved to another question. Yet, considering the topic of the lesson, there could be other solution methods, other ways to reach correct answer for the questions. The reason can be because of the fact that she did not give importance or consider about multiple solution methods. In the interview, she stated that students are also answer-oriented. It is enough for them to find the correct choice in the given test. Students want to move forward to another question quickly.

She might not even get prepared for lesson, thinking about other solution methods, because of her expectation regarding purpose of teaching in this particular school. She did not adopt the lessons by using multiple solution methods. She did not ask for other solution methods. As a result, quality of multiple procedures or solution methods was low.

There is only one mathematics question that she showed a second method of solving it. It was because she had to show the methods to solve different types of questions afterwards. The aim of multiple solution method was to prepare students to solve other question types which can be solved only one solution way of given multiple solutions.

For question 5, she used two different solution methods. They are amount of increase to find ratio and drawing lines to complete the shape to triangle.

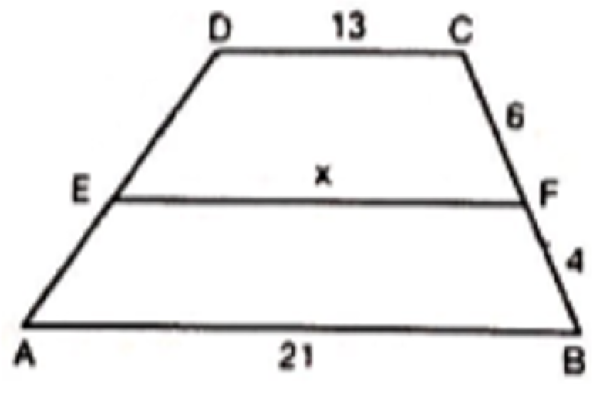


Figure 6.13. Example from lecture three.

Zeynep: For the first way, look at the increasing amount from right side. The upside increased 6units and down side increased 4units. This amount should be correlated with the increase of intermediate base. Rate of increase should be the same. Right side *increased 10 units totally and bases increased 8 units.*

<u>6.2.1.5. Patterns and Generalization.</u> Teachers build a definition or notice and extend a pattern that works for all cases during lesson. It is one of the important parts of mathematics. However, she mostly preferred to show properties and then to solve questions about that. When introducing properties, she stated them to use in other questions/examples. We cannot accept this method as a part of generalization because generalization of properties was method-oriented

In the lecture 1, Zeynep remediate students' error by using generalization. To prove.

cosx.cosx = $\cos^2$x, she started with some examples like cos30.cos30=$\cos^2 30 \neq$cos900. Then, she generalized the examples to the rule cosx.cosx = $\cos^2$x.

Some students got difficulty to solve questions because students confused the equality cosx.cosx$\neq\cos^2$x.

Student: I could not find the answer of question 5.

Zeynep: How did you solve that question?

Student: I used equality cosx.cosx=cosx^2 and simplified the equation.

Zeynep noticed and emphasize that cosx^2 has a different meaning than $\cos^2$x. To support this notice;

Zeynep: The equality that you used is not correct. I will give numerical example as cos30=$\sqrt{3}/2$ to find correct equivalent form of cosx.cosx

Zeynep: When you take the square of cos30, it is not equal to cos30.cos30$\neq$cos900.

cos30.cos30=$\sqrt{3/2}$. $\sqrt{3/2}$ = 3/4 but it is not equal to cos900=-1. But it is easy to say that the square of cos30 is 3/4 as well.

She used patterns and generalization to prove cosx.cosx = $\cos^2$x in this lesson. She used this generalization because students had difficulty to calculate square of cosx in one of the questions.

This generalization was not the aim of the instruction. It was written in one of the test questions. Quality of generalization was limited because of her instruction method. She gave the numerical example and then wrote the general rule. She used mostly examples to support generalization, and also used counter-examples.

6.2.1.6. Mathematical Language. Her mathematical language was fluent. Her use of geometric notation, representation and presentation of mathematical content were clear. She used geometric terminology (e.g. isosceles triangle, equality, triangle, height, median, distance) explicitly for all lessons. She used geometrical terms and correct notations for mathematical concepts such as isosceles triangle, angle bisector, and inequality. She encouraged students to use mathematical terms by asking them questions about terms and checking their notebooks to understand their usage of geometrical terms.

6.2.1.7. Overall Richness of the Mathematics. A general evaluation of Zeynep's instruction in terms of richness of mathematics can be considered to be limited to solving geometry questions that can be solved by application of procedures. In addition, those questions were not suitable for solving with multiple methods or linking topics frequently. Mathematics teaching should not only consist of solving mathematics and memorizing properties. There could be more applications on mathematics to give a broad perspective to students. Linking between representations during instruction were used in many lessons to relate picture and given text only. It could be because of the nature of geometry instruction. Her instruction was focused on solving questions, which are from various tests. Her explanations were limited with answers of test questions. She did not use multiple methods to solve questions in different ways. She used generalization and construction to enhance richness of mathematics. She used mostly examples to support generalization, not used counter-examples. She could support her lessons with rich mathematics materials. As a result, richness of mathematics was limited during observed lesson. There could be many restrictions on instructions because of instruction method, school culture related choices such as students' and parents? expectation.

6.2.2. Working with Students and Mathematics

This dimension reflects on teachers' understanding of students' mathematical errors, contributions as questions, explanations, ideas, etc. There are two categories inside of this section, remediation of students' errors and students' mathematical contributions.

6.2.2.1. Remediation of Student's Errors and Difficulties. This section is about teachers' capability to interpret and respond to students' ideas and errors during instruction. Zeynep remediated students' errors frequently during instruction. She used remediation for all lessons in two ways, procedural and conceptual remediation. These are defined in MQI as a sub-category of remediation of students' errors. She mostly used procedural remediation for calculation mistakes or solution methods. She solved extra examples to remediate students' errors. Actually, it can be explained due to type of questions used in the class. Since questions were not appropriate for conceptual discussion or other solution methods, she needed to solve another procedural question for remediating students' difficulties.

During the interview after the lesson, she said that she is reviewing some properties many times, but students are asking them again and again. Students are not totally learning; because they memorize rules, they get confused and forget in short periods. Thus, I am reminding them properties mostly.

Student: I used equality $(1+\text{sinx})^2=1+\sin^2 x$ but I could not find the solution of question 2.

Zeynep: It is not the correct identity. Do you remember identity of $(a+b)^2 = a^2 + 2ab + b^2$ You should use the same for the equality of $(1+\text{sinx})^2=(1+\text{sinx})(1+\text{sinx})$. We can prove this equality by using distributive law. Multiply 1 with 1 and sinx and then multiply sinx with 1 and sinx to get $1+2\text{sinx}+\sin^2 x$.

Another example of that can be given as
Student: I couldn't find the answer of question 4.
Zeynep: Which rule did you use to solve the question?
Student: I used Euclid's theorem for the question.
Zeynep: If we have only one 90 degree- right triangle, it is not enough to use Euclid's theorem. Right triangle and height of hypotenuse are also necessary to use the rule. Thus, you cannot use Euclid's theorem for question 4. Check again the question to understand which rule you should use.

Furthermore, almost in all instructions, it is observed that she remediates calculation errors of students. When students found wrong answer of question because of calculation mistake, she solved question by explaining calculation steps. Thus, it can be said that she used only procedural remediation of student errors and mistakes.

6.2.2.2. Student Mathematical Contributions. Teacher used student mathematical contributions to move lesson forward in almost all observed lessons. Main contributions of students were results of mathematics calculations and solution ways of students. Mathematical contributions of students occurred in two ways, student-initiated and teacher-initiated. These are defined in MQI as sub-categories of student mathematical contributions. She sometimes asked results of calculations to students or definition of terms. Sometimes students said the result directly, and she used that information to move lesson forward. Generally, students contributed to lesson with the initiation of teacher. These observed contributions were limited to procedural or calculation contributions.

Zeynep: Yes, we can use parallel lines for the question to find the distance.

Student: Can we use similarity rule also?

Zeynep: Of course, the second step is this.

Student: 3 is directly proportional to 7 in the triangle. Thus, 6 is directly proportional to 14 by using similarity. The result is 14 by the way.

Another Example

Zeynep: The last step is to find square root 96

Student: It is 4 square root 6

Zeynep: Yes, it is the answer.

6.2.2.3. Overall Working with Students and Mathematics. Overall, the teacher - student interactions around the content were high. Zeynep mostly gave importance to remediate students' errors. When students are solving test questions, she walked around students and asked for their answers. If she realized any error or difficult part for students, she helped students to solve questions. She used their contributions to move lesson forward.

6.2.3. Errors and Imprecision

This section is related with teachers' errors about content, language or notation. Lack of clarity during instruction is also accepted in this section. There are three main categories in the section.

6.2.3.1. Mathematical Content Errors. Zeynep explained geometrical terms and content correctly. She prepared her instruction material before lesson, and she used these materials during instruction. Some deficient part of her instruction can be small calculation mistakes. She corrected them quickly when she realized during solution.

6.2.3.2. Imprecision in Language and Notation. Zeynep used mathematical language properly to teach mathematics. The language of lesson was in Turkish, native language of both students and the teacher. Students are familiar to all terms and notations, so she used language effectively to convey mathematical contents and talking about mathematics. She used mathematical language mostly to define terms such as isosceles triangle, angle and equilateral triangle or to prove properties about geometry. There was not language error observed during instruction.

6.2.3.3. Lack of Clarity of Mathematical Content. Zeynep explained all properties and content clearly. She gave more than one example to make content clear. She sometimes used counter-examples also. She did not ignore students' difficulties about content. If she realized students' error, she remediated it by using examples. Her language and solution methods were explicit.

6.2.3.4. Overall Errors and Imprecision. Error was not observed during instructions. She solved problems correctly and defined terms correctly. Her solution methods and equations were correct. Her notation, representation and presentation of mathematical content were clear. Her notation consists of mathematical symbols. Her mathematical language includes technical geometrical terms such as isosceles triangle, angle bisector, and equation. Teacher did not neglect to solve questions clearly. It should be noted that she did not make any errors during instruction for teaching procedures.

She prepared her instruction materials and notes before lesson. She was organized during instruction. She wrote questions to the board from her notebook that is prepared with mathematics department head of the school.

6.2.4. Student Practices

This section depends on students' involvement to activities. There are five categories in the section. Students' explanations, communications and reasoning, task cognitive demand and conceptualized problems are parts of student practices

6.2.4.1. Students Provide Explanation. Students provide explanations for many cases such as talking about solution methods, defining terms, naming triangles and writing properties about questions. During instructions, students' explanations and contributions were based on Zeynep's initiation. Zeynep asked students to give definitions mostly at the beginning of lessons and used student answers to ask them more questions to build the definition. An example of this case is as follows: The following example is an excerpt from a lesson with the topic of the property of equilateral triangles.

Zeynep: What is the definition of equilateral triangle?

Student1: If all sides of triangle are equal, we can say that it is equilateral triangle.

Zeynep: Who wants to give more detailed explanation?

Students: All angles are 60 degree also in equilateral triangle.

Zeynep: Yes, it is correct.

Zeynep: Let's write the definition while I am writing to board.

In this process, students wrote on their notebooks while the teacher writes on the board to the definition: An equilateral triangle is a triangle in which all three sides are equal and also all three internal angles are congruent to each other - 60°.

Another example can be given for isosceles triangle rule as followed.

Zeynep: What is the most important rule in isosceles triangle that we learned before?

Student1: It was the rule of YAKI [mnemonics]

Zeynep: Yes, this is correct. First letters of Turkish words of Height, Angle Bisector, Median and Isosceles Triangle - YAKI [mnemonics]. If we have any two of the properties in the triangle, we can add other two properties to triangle to solve the question.

6.2.4.2. Students' Interest in the Instruction. Students were generally interested during the instruction. Students attending the lesson were limited to answering teacher's question rather than providing counterexamples or using ideas from a different mathematics topic to reason. Students used the same rule for triangle in different type of questions, but they did not connect different rules of triangles. They asked mathematical questions, described terms that are asked by Zeynep, offered explanation solution methods of questions. It is observed that most of the students did not comment on the reasoning questions or properties. They were not interested in engaging multiple solution methods of questions or counter-examples of properties. They made less conjectures about the mathematics. In the following example, the mathematical rule of Euclid was used.

Student: Teacher, how was the Euclid's theorem to find the height of the triangle?
Zeynep: We do not have enough time in this lesson, I can prove it in the next lesson.
Student: Can you say the rule directly please? I will not remember the proof anyway.
Zeynep: The height of the triangle will be equal to multiplication of distances that are completing the base side of the triangle as p.k=h2.

6.2.4.3. Communication about Mathematics of the Segment. Most of the students do not pay attention to ask for mathematical questions including reasoning of answers or theorems. They discuss about solution ways of questions and calculation process mostly. Students' communication about mathematics was limited because they want to talk more about non-mathematical topics (e.g. disciplinary issues, other lesson topics or their life) during instruction as given above.

Student1: Can we talk about different-issues rather than mathematics today?
Student2: Can we do counseling course today to talk about our problems?
Student3: Miss Zeynep, did you know information about Mathematics Olympiads in our school? Can you explain us how we will compete?

6.2.4.4. Task. Almost in all lessons, test questions were the task thatwere used about geometry content. Students got used to solve test questions as a task of lesson. She stated in interviews that she prepared materials and activities for her lesson, but students were not interested in them. They were bored quickly, and they did not want to continue. She chose questions from different publications focused on University Entrance Exam questions. She decided on the level of the questions by discussing them with the mathematics department head of the school.

She sometimes gave permission to do peer work when students were dealing with test questions. She said that they could study with their pair to solve questions during lesson. This method engaged students to solve questions.

6.2.4.5. Conceptualized Problems. Zeynep did not come to class with conceptualized problems (real-life examples).She used examples or questions from test books. Also, level of tasks that were used in instruction was low. However, students' engagement of task was enough to continue and solve all of the problems. Students got used to solve test book questions.

6.2.4.6. Overall Evaluation of Student Practices. When we look at the section overall, students were involved the lesson in some parts. They provided explanations and definitions about the content. Zeynep used that to move the lesson forward. They calculated answers of questions and discussed solution ways for questions. They were interested in mathematics mostly, but they got bored quickly. Their communication about mathematical concepts was restricted because they were focused on the answers of questions. Thus, they did not give more importance to talk and discuss about relations of concepts critically. Question types of both students and teacher were closed ended. Student responses were invited. Answers of students were key words, not elaboration of them. Task and problems that are used during instruction were prepared from Zeynep. Her materials were mostly test questions.

7. DISCUSSION

The purpose of the study was to investigate first year teacher's mathematics knowledge during instruction by using MQI instrument. Teacher's mathematical knowledge during geometry lesson was investigated qualitatively. In findings section, Zeynep's answers to general interviews, post-observation and pre-observation interviews were discussed with observation data. Discussion part was provided in two sections. The first one is about Zeynep's experiences. The second one is about knowledge for teaching geometry. All data getting from lesson observations were supported by interviews to have consistent interpretations about mathematical teacher knowledge of participant.

7.1. Zeynep's First Year Teaching Experiences

"Beginning teachers are involved in a complex process of identity" (Losano, Fiorentini, and Villarreal, 2018). The participant, Zeynep, is graduated from highest ranking secondary school mathematics teaching, and she is a first year beginning teacher. She stated that this year is her transition period from being a student to be a teacher. The first year of teaching is a period which teachers can use their experiences and learning into practice (Losano *et al*, 2018). She is teaching geometry during her instructions even if geometry lessons are not given during University Education of Mathematics Teaching in the University which she graduated. According to Zeynep's view, how to teach mathematics is given generally in the university. Applying them for each topic totally depends on teachers. Thus, teachers should develop themselves more after graduation as well. Development of both teachers and students is an ongoing process.

Most of the views inferred from Zeynep were about the main parts of the conceptual learning and modeling. This is the effect of her University education on her ideas. However, she could not plan her lessons well according to what she has learned about conceptual teaching. She stated that it was because the school environment did not let her to do what she plans. She emphasized that she tried to prepare materials and activities, but students were not interested in them. Thus, she took steps backward because of students' pressure. There is also a reality about Turkish University Entrance Exam and the question styles of that exam. Thus, school administration expects teachers to solve different type of questions and do more practices. Thus, she applies the teaching method as solving test questions rather than rediscoveries and discussion of mathematical ideas.

Procedural knowledge indicates how to use information during problem solving such as rules, procedures and algorithms. The knowledge mostly was used to calculate problems (Alexander, Schallert and Hare, 1991). When Zeynep is solving questions during instruction, she emphasized mostly solution ways of questions. Students know rules, theorems and strategies to complete tasks. This is procedural knowledge (Long, 2005). It also shows the limited quality of lesson. Mathematics lesson can be a part of anything such as daily life or other disciplines. Mathematics should be given to students as a nature of it rather than only solving some types of test questions. Everything we encounter in the natural, social and mental worlds created mathematical concepts, structures and ideas. Real life situations can be in different context than school curriculum about mathematics. Yet, real life situations are easy to solve if students learn how to apply their knowledge with understanding (Ojose, 2011).

By memorizing facts and procedures, students cannot learn mathematics and algorithms by understanding. Learning should be seen as exploring, reasoning and testing approaches in mathematics (Schifter, 1998). Zeynep is also emphasized that following teachers' step is not the only way of learning mathematics. Other teaching methods should be added to instruction. However, she could not do it thoroughly because of her instruction method, school culture, parent pressure and many other reasons.

In addition, data analysis shows that Zeynep is qualified teacher in practice with her ideas and knowledge. However, she does not have proper conditions to bring her knowledge into the practiceduring instruction. There are many reasons such as school administration, school policy or students' /parents' pressure. Nevertheless, she stated that she will find a condition to use all her teaching ability one day. She stated that she is waiting for the proper conditions. She is developing herself from many areas and preparing herself to teach mathematics much more meaningful.

During general interview, she also stated that she tried to improve herself from many perspectives. She mentioned about the seminar that she attended last summer. Seminar was about how to use Texas Instrument Calculator. After the seminar, she started to work with the team to develop her abilities to use TI calculator. Then, they started to record videos to introduce TI to the new users in Turkish language.

Zeynep: Teachers should develop themselves continuously. I am learning TI and working about TI deeply. I could not use it in my class yet because preparing this kind of lessons is difficult for now on. This is for my career development. Consistently, Zeynep indicated in the interviews that she regarded mathematics as everything we do in our life. It is not only the mathematics of the school curriculum, it is all around us. Findings showed that she used explanation mostly rather than sense making during geometry instruction. Explanations were procedural according to question. She used explanations to focus on computational parts of problems. Making sense in geometry requires reasoning and conceptual understanding. It is inferred from interviews that the conceptual instruction which he want to apply is contradicting with her actual class applications.

7.1.1. Geometry Instruction

Teachers teach geometry as the way they have learned in high school (Jones, 2003). She confirmed the idea during interviews that she is mostly using her high school geometry knowledge to teach geometry. Her geometry teacher during her high school years has influence on her way of teaching.

To have an information about her geometry learning, we should focus on her high school years. 2007-2011 education years were her high school period. Before the 2013-2014 education year, mathematics and geometry lessons were given separately in the Turkish curriculum. These were Zeynep's high school years as well. Thus, she learned geometry lesson as a different subject than mathematics lesson. During high school years, she solved mostly questions related with University Entrance Exam focusing on drill and practice. Conceptual understanding of students about geometry is at comprehension level (Kılıç, 2013). Students' geometry abilities are low because they only learn test questions and theorems as we learned in high school. Thus, they do not have in-depth information about geometry topics. Students cannot apply their knowledge for a high level (Kılıç, 2013). The actual problem about geometry prejudice in Turkey is coming from the way that students learned geometry in high school.

With the change of curriculum in 2013-2014 education years, geometry subject and mathematics subject are added to curriculum together as mathematics lesson. In 2017-2018 education years, mathematics curriculum was simplified although geometry part of it stayed almost the same. Zeynep started her job during these years with the same objectives of her high school years. She is giving geometry instruction separately. It is different from Turkish high school mathematics curriculum. It is because of school policy which she works.

7.2. Knowledge for Teaching Geometry

In Turkey, there are faculties of education in Universities since 1982. The participant of the study is graduated from Secondary School Mathematics Education Departmant of Public University in Istanbul. During pre-service education, Zeynep did not have geometry lesson in the University. She emphasized that college education focused more on teaching process rather than mathematical knowledge during University education. This suggests that pre-service education does not support her to teach Geometry lesson in her first year of teaching. In other words, Zeynep was not able to provide an instruction about geometry by taking into consideration of her University Education.

To teach geometry lessons effectively, she is receiving support from an experienced teacher in the school, head of the mathematics department. In addition, she is studying geometry herself before instruction. Generally, she was prepared to her lessons and she was willing to give mathematics instruction. Even if she is happy to teach Geometry for 9th graders, this situation has some negative effects on teaching. Students think that geometry is difficult to solve, all questions have a different method and it is hard to understand how to solve geometry questions as she mentioned. Zeynep said that most students who are bad at mathematics are also reluctant to learn geometry. She emphasized that geometry teaching is a barrier to her teaching because students have prejudice about geometry in Turkey.

During Mathematics courses, mostly Geometry, teachers use some techniques to teach theorems, postulates or rules. For simple-knowledge objectives, teachers can construct knowledge by using exposition, explication, mnemonics, monitoring and feedback or overlearning (Cangelosi, 2003). Mnemonics have advantages on students' performance (Mocko *et al.*, 2017). It is found that students increased their motivation to begin study with Mnemonics and it helps to reduce anxiety. She prefers mostly to use mnemonics during her mathematics instruction. For example, YAKI is Turkish Initials of Height, Angle Bisector, Median Line and Isosceles Triangle. If two of the properties exist in the triangle, others can be added to the triangle. Her aim is to help students to remember the meaning of rules.

How concepts are presented and elaborated during lesson is important (Davis and Simmt, 2006). Materials give a way to present the concepts. In Turkey, there are textbooks for each level prepared by MEB. It is given to all students freely. However, many teachers do not use the textbooks very frequently during instruction (Altun, Arslan and Yazgan, 2004). The reason for that is the type of University Entrance Exam Questions in Turkey. Authors discussed that teachers are using extra test books to prepare students for the exam. The main teaching materials of Zeynep's mathematics instruction were test books mostly as well. She used not only the school's own test book but also different sources to prepare materials. But, in addition to text books, there are many advanced materials for mathematics lessons (Fan, 2014) To develop students' conceptual learning, teachers can use different methods during instruction. According to MQI instrument findings, quality of instruction was limited generally.

There are many reasons to have limited quality, but instruction method can be the main reason. Direct instruction methods were outweighed in her instruction. She used videos during instruction. Videos can be used to initiate inquiry about concepts (Borko *et al.*, 2011). In this way, they can foster learning mathematics with understanding. However, these videos were also in the direct teaching way.

Mathematical richness of her instruction is analyzed by using MQI framework. The MQI instrument findings indicated that Zeynep used representations mostly during instructions by linking the given text and picture which assist students' understanding of problem context. Proof representations and other methods can be used to do geometry education different fromtraditional classroom teaching (Wong, 2011). Many representations can be used to define a problem, enrich the reasoning process and represent proofs.

7.2.1. Using the MQI to Explore Mathematical Quality of Geometry Instruction

During this study, observation form which had been adapted from Mathematical Quality of Instruction was used to get information about quality of instruction. The use of framework developed in many aspects to this study. This instrument opened researcher perspective with its framework and codes. All codes and dimensions were analyzed separately and together for observed lessons (Hill, 2010). The information getting in this study developed with this uniqueness of MQI. Data getting from interviews was supported findings of MQI. Thus, liability of the framework was high. However, there were more overlapped codes and categorization for one code was difficult. Additionally, there were some missing parts of MQI to apply in Turkey. Thus, MQI instrument was not used directly in this study. MQI study was applied quantitatively in some studies (Hangül, 2018). Thus, MQI instrument was adapted in accordance with quantitative analysis in this study.

Listening to students has arisen as important characteristics of teachers. By using information getting from observation form adapted from MQI, Zeynep's interactions with students around the content were strong during instruction. Students contribute to lesson by solving questions and calculating results. She used all contributions of students to move lesson and remediate their errors if they have misunderstanding about the content. According to data analyzed, remediation or interactions with students was procedural. She only focused on the remediation to solve the problems. She did not focus on reasoning of students and discussion about content. It was the nature of the lesson. However, quality of mathematics can be higher if teacher gives more importance to students' ideas and contributions to find out meaning of mathematics.

Findings showed that students' reasoning has not been developed to find out or discuss about mathematical content. Thus, they were asked mostly procedural questions during lesson. Their information was not enough to do reasoning about mathematics or geometry. For successful mathematics teaching, teachers' professional knowledge should build different learning environments for students (Neubrand, 2018). Investigations and representations in geometry will reinforce students' learning more than solving test questions.

As a nature of her instruction, students involved the lesson in some parts procedurally such as calculating solutions, defining terms. Results showed that question types of both students and teacher were closed ended. Close ended questions require response that does not allow students to say new opinions. However, open-ended questions and discussions about context are the main parts of conceptually designed lessons. Open-ended questions motivate students to explore concepts (Sullivan, Warren and White, 2000) and promote thinking and understanding (Yee, 2002).

Students' communication about mathematics was limited, because they wanted to talk more about non-mathematical topics (e.g., disciplinary issues, other lesson topics or their life) during instruction. Her interaction with students was high also for out of mathematical content because students shared their problems or happiness with her most of the time during lessons. Results supported by interviews show that this could be because she is friendly and giving students' problems importance, and she is responsible teacher of the class.

8. LIMITATIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

Like any study, this study is not without limitations. The number of the lessons analyzed, the content and mathematical topics observed as well as the single grade level and the single teacher considered may have influenced the results of this analysis and thus the conclusions drawn.

The purpose of the study was to observe only a novice high school mathematics teacher's knowledge during geometry instruction. The number of observation was limited to 3 days, 6-class hours. The reason for that was data saturation of the study. These observed lessons gave enough information about teacher's mathematical knowledge during geometry instruction. This study does not aim to observe the development of the teacher. However, if there would be a possibility to observe her first semester lessons, there could have been a developmental study. It was not possible because of observer's time restriction. For further studies, the same teacher may be observed and her mathematical knowledge can be investigated to study whether there are any developments. It may be a longitudinal study of following a novice high school mathematics teacher from pre-service years.

In this study, the data collection tool was adapted from MQI, which was developed by Heather Hill to observe and measure many dimensions of instructions in the US. Observation form was adapted in a way to ensure it has the same dimensions and segments, but with some differentiation to capture the Turkish classrooms' nature of instruction. Thus, the findings of this study are shaped by MQI and its framework, MKT. Future studies can combine in different lenses to understand instructional quality in Turkey more comprehensively. "Through better instructional quality, we can improve student learning" (Charalambous and Praetorius, 2018). In addition, conducting a longitudinal study may provide information in other aspects. I can claim that the data in this study can provide opportunity for researchers to make comparisons for studies in the future.

Purposefully, the participant of the study had been chosed from Public University in Istanbul. During the study, it is inferred that she did not take any geometry course in the University for teaching geometry. Even though some collages are not offering geometry courses for secondary school mathematics teaching departments, Turkish curriculum has geometry lessons. Thus, there should be courses in Universities for this manner. Mathematics departments of Universities should give importance to prepare future teachers to teach geometry effectively (Jones, 2000). In addition, it is deduced that teachers can teach mathematics and geometry courses more comfortable and equipped if they learned how to teach each topic in college with reasoning and application. Thus, teacher training programs in colleges should include more courses to teach prospective teachers how to teach each topic during their instruction.

REFERENCES

Adam, R.D., “Teacher Development: A Look At Changes In Teacher Perceptions and Behavior Across Time”,*Journal of Teacher Education*, Vol. 33, No. 4, pp. 40-43, 1982.

Alexander, P.A., D.L. Schallert, V.C. Hare, “Coming To Terms: How Researchers in Learning And Literacy Talk About Knowledge”, *Review of Educational Research*, Vol. 61, No. 3, pp. 315-343, 1991.

Altun, M., C. Arslan, Y. Yazgan, “Lise Matematik Ders Kitaplarının Kullanım Şekli ve Sıklığı Üzerine Bir Çalışma”, *Eğitim Fakültesi Dergisi*, Vol. 17, No. 2, pp. 131-147, 2004.

Arbor, A., “Learning Mathematics for Teaching: Coding rubric for Measuring the Mathematical Quality of Instruction”, *University of Michigan: School of Education*, pp.1-21, 2006.

Areti, P., “Using Representations in Geometry: a Model of Students’ Cognitive and Affective Performance”, *International Journal of Mathematical Education in Science and Technology*, Vol. 45, N0. 4, pp. 498-511, 2014.

Aslan-Tutak, F. and T.L. Adams, “A Study of Geometry Content Knowledge of Elementary Preservice Teachers”, *International Journal of Elementary Education*, Vol. 7, No. 3, pp.301-318, 2015.

Ball, D.L., “Bridging practices: Intertwining Content and Pedagody in Teaching and Learning to Teach”, *Journal of Teacher Education*, Vol. 51, No. 3, pp. 241-247, 2000.

Ball, D.L., 2003, “What Mathematical Knowledge is Needed for Teaching Mathematics?”, *Secretary’s Summit on Mathematics*, US Department of Education.

Ball, D.L., and G.W. Mcdiarmid, "The Teacher Education and Learning to Teach Study: an Occasion for Developing a Conception of Teacher Knowledge", *The National Center for Research on Teacher Education*, ERIC, 1989.

Ball, D.L., S.T. Lubienski, and D.S. Mewborn, "Research on Teaching Mathematics: The Unsolved Problem of Teachers' Mathematical Knowledge", *Handbook of research on teaching*, Vol. 4, pp. 433-456, 2001.

Ball, D.L., M.H. Thames, and G. Phelps, "Content Knowledge for Teaching: What Makes It Special", *Journal of Teacher Education*, Vol. 59, No. 5, pp. 389-407, 2008.

Barrantes, M. and L.J. Blanco, "A Study of Prospective Primary Teachers' Conceptions of Teaching and Learning School Geometry", *Journal of Mathematics Teacher Education*, Vol. 9, No. 1, pp. 411-438, 2006.

Baxter, P., and S. Jack, "Qualitative Case Study Methodology: Study Design and Implementation for Novice Researchers", *The qualitative report*, Vol. 13, No. 4, pp. 544-559, 2008.

Borko, H., and C. Livingston, "Cognition and Improvisation: Differences in Mathematics Instruction by Expert and Novice Teachers", *American Educational Research Journal*, Vol. 26, No. 4, pp. 473-498, 1989.

Borko, H., M. Eisenhart, C. Brown, R. Underhill, D. Jones, and P. Agard, "Learning to Teach Hard Mathematics: Do Novice Teachers and Their Instructors Give up Too Easily", *Journal for research in Mathematics Education*, pp. 194-222, 1992.

Borko. H., K. Koellner, J. Jacobs, N. Seago, "Using Video Representations of Teaching In Practice-Based Professional Development Programs", *ZDM Mathematics Education*, Vol. 43, No. 1 , pp. 175-187, 2010.

Cangelosi, J.S., "Teaching Mathematics in Secondary and Middle School", *Pearson,* pp. 207-228, 2014.

Charalambos, C.Y., and E. Litke, "Studying Instructional Quality by using a Content-Specific Lens: the Case of the Mathematical Quality of Instruction Framework", *Zentralblatt f r Didaktik der Mathematik*, Vol. 50, No. 3, pp. 445-460 , 2018.

Charalambos, C.Y., and A.K. Praetorius , "Studying Mathematics Instruction Through Different Lenses: Setting the Ground for Understanding Instructional Quality More Comprehensively", *Zentralblatt f r Didaktik der Mathematik*, Vol. 50, No. 3, pp. 355-366, 2018.

Cohen, D.K., W.R. Stephen, and D.L. Ball, "Resources, Instruction, and Research", *Educational Evaluation and Policy Analysis*, Vol. 25, No. 2, pp. 119-142, 2003.

Çiftçi, Z.B., L. Akgül , and D. Deniz, "Teachers' Opinions and Solution Suggestions Regarding Encountered Issues on the Ninth Grade Mathematics Curriculum", *Anadolu Journal of Educational Sciences International*, Vol. 3, No. 1, pp. 1-21, 2013.

Darling-Hammond L., and E. Rustique-Forrester , "The Consequences of Student Testing for Teaching and Teacher Quality", *Yearbook of the National Society for the Study of Education* , 2005.

Davis B., E. Simmt, "Mathematics for Teaching: an Ongoing Investigation of Mathematics that Teachers (Need to) Know", *Educational Studies in Mathematics*, Vol. 61 No. 3, pp. 293-319, 2006.

Dicicco-Bloom, B., and B.F. Crabtree, "The Qualitative Research Interview", *Medical Education*, Vol. 40 No. 4, pp. 314-321, 2006.

Duru A., and H. Korkmaz, "Teachers' Views About A New Mathematics Curriculum and Difficulties Encountering Curriculum Change", *H. U. Journal of Education*, Vol. 38 No.1, pp. 67-81 , 2010.

Ellis, A., Z. Özgür, and L. Reiten, "Teacher moves for supporting student reasoning", *Mathematics Education Research Journal*, pp. 1-26, 2018.

Even, R., "Subject-Matter Knowledge and Pedagogical Content Knowledge: Prospective Secondary Teachers and the Function Concept", *Journal for Research in Mathematics Education*, pp. 94-116, 1993.

Fan, L., "The Pedagogy of Mathematics: How do Teachers Develop Their Knowledge", *Imperial College Press*, U.S. , 2014.

Fennema E., and M. Franke, "Teachers' Knowledge and Its Impact", *Handbook of Research on Mathematics Teaching and Learning*, New York , 1992.

Fujita T., and K. Jones, "The Bridge between Practical and Deductive Geometry: Developing the Geometrical Eye", *Proceeding of the 26th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2 No. 1, pp. 384-391, 2002.

Güzel, I., I. Karataş, B. Çetinkaya, "A Comparison of Secondary School Mathematics Curriculum Guidebooks: Turkey, Germany and Canada", *Turkish Journal of Computer and Mathematics Education*, Vol. 1, No. 3, pp. 309-325, 2010.

Globe, C.S. , "Mathematics Learning with Multiple Solution Methods; Effect of Types of Solutions and Learners' Activity", *Instructional Sciences*, Vol. 42, No. 1, pp. 715-745, 2014.

Grossman, P., K. Hammerness, and M. Mcdonald, "Redefining Teaching, Re-Imagining Teacher Education", *Teachers and Teaching: Theory and Practice*, Vol. 15, No. 2, pp. 273-289, 2009.

Hangül, T., *Öğretmen Eğitimcilerinin Öğretimin Matematiksel Kalitesine Yönelik Değerlendirmeleri Üzerine Bir İnceleme*, Ph.D., Sakarya Üniversitesi, Matematik Bölümü, 2018.

Hansen, V.L., "Everlasting Geometry in Mammana Perspectives on The Teaching of Geometry for 21st Century: An ICMI Study", *Kluwer Academic Publishers*, pp. 9-18, 1998.

Herbst, P., K. Kosko, "Mathematical Knowledge for Teaching High School Geometry", *North American Chapter of the International Group for the Psychology of Mathematics Education*, pp. 438-444, 2012.

Hiebert, J., and A.K. Morris, "Teaching, rather than Teachers, as a Path toward Improving Classroom Instruction", *Journal of Teacher Education*, Vol. 63, No. 2, pp. 92-102, 2012.

Hill, H., "Mathematical Quality of Instruction", *The National Science Foundation*, U.S., 2010.

Hill, H., 2014, *Mathematical Quality of Instruction 4-point version*, National Science Foundation, 2014.

Hill, H.C., M.L. Blunk, C.Y. Charalambous, J.M. Lewis, G.C. Phelps, L. Sleep, and D.L. Ball, "Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An exploratory Study", *Cognition and Instruction*, Vol. 26, No. 4, pp. 430-511, 2008.

Hill, H., D. Ball, H. Bass, S. Schiling, M. Blunk , C. Branch, G. Phelps, and A. Arbor, "A Coding Rubric for Measuring the Mathematics Quality of Instruction", *Learning mathematics for teaching project*, 2006.

Hill, H., M. Blunk, C. Charalambos, J. Lewis, G. Phelps, L. Sleep, and D.L. Ball, "Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An Exploratory Study", *Routledge*, pp. 430-511, 2008.

Hill, H., B. Rowan, and D.L. Ball, "Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement", *American Educational Research Journal*, pp.

371-406, 2005.

Huberman, M. and M.B. Miles, "Qualitative Data Analysis: a Sourcebook of New Methods", *Sage Publications*, Beverly Hill , 1984.

Jones, K., "Critical Issues in the Design of the Geometry Curriculum", *Readings in Mathematics Education*, pp.75-90, 2000.

Jones, K., "Teacher Knowledge and Professional Development in Geometry", *Proceedings of the British Society for Research into Learning Mathematics*, Vol. 20, No. 3, pp. 109-114, 2000.

Jones, K., C. Mooney, and T. Harries, "Trainee Primary Teachers' Knowledge of Geometry for Teaching", *Proceedings of the British Society for Research into Learning Mathematics*, Vol. 22, No. 2, pp. 95-100, 2002.

Jones, K., "Issues in the Teaching and Learning of Geometry", *Aspects of Teaching Secondary Mathematics*, pp.137-155, 2003.

Jöhannsdottir, B., *The Mathematical Content Knowledge of Prospective Teachers in Iceland*, Ph.D., Teachers College, Colombia University, 2013.

Krauss, S., M. Brunner, M. Kunter, J. Baumert, W. Blum, A. Jordan, and M. Neubrand, "Pedagogical Content Knowledge and Content Knowledge of Secondary Mathematics Teachers", *Journal of Educational Psychology*, Vol. 100, No. 3, pp. 716-725, 2008.

Kılıç, H., "High School Students' Geometric Thinking, Problem Solving and Proof Skills", *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, Vol. 7, No. 1, pp. 222-241, 2013.

Lockheed Martin, LMT, "A Coding Rubric for Measuring the Mathematical Quality of Instruction.", *University of Michigan*, 2006.

Lincoln, Y.S., and E.G. Guba, *Naturalistic Inquiry*, Sage, Newbury Park, CA, 1985.

Lincoln, Y.S., and E.G. Guba, "But is it rigorous? Trustworthiness and Authenticity in Naturalistic Evaluation", *New Directions for Program Evaluation*, Vol. 30, No. 1, pp. 73-84, 1986.

Long, C., "Maths Concepts in Teaching: Procedural and Conceptual Knowledge", *Pythagoras*, pp. 59-65, 2005.

Losano, L., D. Fiorentini, and M. Villarreal, "The Development of a Mathematics Teacher's Professional Identity during her First Year Teaching", *Journal of Mathematics Teacher Education*, Vol. 21, No. 3, pp. 287-315, 2018.

Merriam, S.B., and E.J. Tisdell, "Qualitative Research: A Guide to Design and Implementation", *John Wiley and Sons*, pp. 238-266, 2015.

McDiarmid, G.W., and D.L. Ball, 1989, "The Teacher Education and Learning to Teach Study: as Occasion for Developing a Conception of Teacher Knowledge", *National Center for Research on Teacher Learning*, Vol. 89, No. 1, pp. 1-26

Mocko, M., L.M. Lesser, A.E. Wagler, and W.S. Francis, "Assessing Effectiveness of Mnemonics for Tertiary Students in a Hybrid Introductory Statistics Course", *Journal of Statistics Education*, Vol. 25, No. 1, pp. 2-11, 2017.

Neubrand, M., "Conceptualizations of professional knowledge for teachers of mathematics", *Zentralblatt f r Didaktik der Mathematik*, Vol. 50, No. 1, pp. 601-612, 2018.

Ojose, B., "Mathematics Literacy: Are We Able to Put the Mathematics We Learn into Everyday Use", *Journal of Mathematics Education*, Vol. 4, No. 1, pp. 89-100, 2018.

Panaoura, A., "Using Representations in Geometry: a model of Students' Cognitive and Affective Performance", *International Journal of Mathematical Education in*

Science and Technology, Vol. 45, No. 4, pp. 498-551, 2014.

Richardson, B., and J. Thompson, “Strategic and Competitive Success: Towards a Model of the Comprehensively Competent Organization”, *Management Decision*, Vol. 34, No. 2, pp. 5-19, 1996.

Rowland, T., P. Huckstep, and A. Thwaites, “Elementary Teachers’ Mathematics Subject Knowledge: the Knowledge Quartet and the Case of Naomi”, *Journal of Mathematics Teacher Education*, Vol. 8, No. 3, pp. 255-281, 2005.

Ruohotie-Lyhty, M., “Struggling for a Professional Identity: Two newly Qualified Language Teachers’ Identity Narratives during the First Years at Work”, *Teaching and Teacher Education*, Vol. 30, No. 1, pp. 120-129, 2013.

Sakallı, A.F., C. Çalan, A. Borazan, and E. Korkmaz, “Lise Mathematik Öğretmenlerinin Yeni Ortaöğretim Matematik Program ile İlgili Değerlendirmeleri”, *Uluslararası Eğitim Bilimleri Dergisi*, Vol. 3, No. 7, pp. 65-81, 2016.

Schifter, D., “Learning Mathematics For Teaching: From A Teachers’ Seminar To The Classroom”, *Journal of mathematics teacher education*, Vol. 1, No. 1, pp. 55-87, 1998.

Shulman, L.S., “Those Who Understand: Knowledge Growth in Teaching”, *Educational Research, Stanford University*, Vol. 15, No. 2, pp. 4-14, 1986.

Shulman, L.S., “Knowledge and Teaching: Foundations of the New Reform” *Harvard Educational Review*, Vol. 57, No. 1, pp. 1-22, 1987.

Skott, J., R. Mosvold, and C. Sakonidis, “Classroom Practice and Teachers’ Knowledge, Beliefs and Identity”, *Developing Research in Mathematics Education: Twenty Years of Communication, Cooperation and Colloboration in Europe*, pp.162-180, 2018.

Stein C.C., “ Promoting Mathematical Discourse in the Classroom”, *The National*

Council of Mathematics Teachers, Vol. 101, No. 4, pp. 285-289, 2007.

Stigler, J.,W., and J. Hiebert, "Improving Mathematics Teaching", *Journal of the Department of Supervision and Curriculum Development*, Vol. 61, No. 5, pp. 12-17, 2004.

Sullivan, P., E. Warren, and P. White, "Students' Responses to Content Specific Open-Ended Mathematical Tasks", *Mathematics Education Research Journal*, Vol. 12, No. 1, pp. 2-17, 2000.

Tatto, M.T., J. Schwille, L. Ingvarson, R. Peck, R. Glenn, and S. Senk, *Teacher Education and Developmental Study in Mathematics (TEDS-M)*, IEA, Amsterdam, 2008.

Tchoshanov, M.A., "Relationship between Teacher Knowledge of Concepts and Connections, Teaching Practice, and Student Achievement in Middle Grades Mathematics", *Educational Studies in Mathematics*, Vol. 76, No. 2, pp. 141-164, 2011.

Usiskin, Z., "Teachers' Mathematics: a Collection of Content Deserving to be a field", *The Mathematics Educator*, Vol. 6, No. 1, pp. 86-98, 2001.

Watson, A., "Developing and deepening mathematical knowledge in teaching: being and knowing", *MKi 6, Nuffield Seminar Series*, University of Loughborough, 2008.

Weiss, K.M., *Mathematical Sense, Mathematical Sensibility: The Role of the Secondary Geometry Course in Teaching Students to be like Mathematicians*, Ph.D. Thesis, The University of Michigan, 2009.

Wong, W.,K., S.K. Yin, H.H. Yang, and Y.H. Cheng, "Using Computer-Assisted Multiple Representations in Learning Geometry Proofs", *Journal of Educational Technology and Society*, Vol. 14, No. 3, pp. 43-54, 2011.

Yee, F.P., "Using Short Open-ended Mathematics Questions to Promote Thinking and Understanding", *National Institute of Education*, pp.135-140, 2002.

Yin, R.K., *Case Study Research*, Sage Publications, London,1994.

Yin, R.K., *Case Study Research: Design and Methods*, CA:Sage, 2009.

APPENDIX A: OBSERVATION PROTOCOLMATHEMATICAL

FIRST SECTION: This section is completed before the lesson begins. Some of the answers of the questions can be found in the pre-observation interview questions.

Day Month Year

1.DATE OF OBSERVATION (DD/MM/YY)

2. DURATION OF OBSERVATION Minutes

3. NAME & ADDRESS OF SCHOOL____________

4. SCHOOL CODE

5. STATE/REGION______________________

6. OBSERVER NAME ______________________

7. OBSERVER CODE

8. COUNTRY NAME ______________________

9. TEACHER CODE

10. GRADE OBSERVED To

11. NAME OF MATH CLASS__________________

12. AGE RANGE OF STUDENTS IN YEARS To

13. NO. OF STUDENTS ENROLLED IN CLASS Total Of Which Girls

14. NO. OF STUDENTS PRESENT IN THE CLASS Total Of Which Girls

15. PERCENTAGE OF STUDENTS USING TEXTBOOKS

16. WHAT TIME WAS CLASS SUPPOSED TO BEGIN

17. WHAT TIME DID CLASS ACTUALLY BEGIN

18. WHAT TIME WAS CLASS SUPPOSED TO END

19. WHAT TIME DID CLASS ACTUALLY END

20. WHAT LANGUAGE IS PREDOMINANTLY USED DURING THE OBSERVATION? ____________

21. IS THIS THE NATIVE LANGUAGE OF THE MAJORITY OF THE STUDENTS IN THE CLASSROOM? Y/N

22. Log: Mention any special events, problems with coding this sheet, abnormal activities etc.

Figure A.1. Pre-observation interview questions.

Draw the classroom arrangement in the box below. Your drawing should include following: student seats, teacher desk, door(s), windows, place(s) where books, supplies, etc are stored, computer (s) and any other classroom elements that are important for this observation. For each students, show their gender by using ‘B’ for boys, ‘G’ for girls.

Figure A.2. Classroom order of seating.

Observations: (Please, write any comment about classroom management)

Figure A.3. Comment about classroom management.

Lesson Topic and level(s) taught: (Topic of the lesson and grade of the students).

Lesson Objectives: (Please indicate here that the objectives of the lesson stated by teacher at the begining of the lesson or not)

SECOND SECTION:

CLASSROOM OBSERVATION:

This section is completed during the instruction by observer.

SEGMENT CODES		
Classroom Work is Connected to Mathematics	**YES**	**NO**
A)Focus is on non-mathematical topics or student activities that have no clear connections for developing mathematical content. ❖ Gathering or distributing materials ❖ Disciplinary issues ❖ Students doing an activity B) Ex:		

	Not Present	**Low**	**Mid**	**High**
RICHNESS OF THE MATHEMATICS				
1-Linking between representation				
A)Focus is on ❖ Explicitness about how two or more representations are *related* ❖ Detail and elaboration of the topic B) Ex:				

Figure A.4. Classroom Observation.

2-Explanations				
A) Focus is on ❖ Justification using a definition and statements. ❖ "Why questions" are asked by teacher e.g why an object is symmetrical?, why a second figure is a transformation of the first one? ❖ B)Ex:				
3-Mathematical Sense-Making				
A) Focus is on ❖ Understanding relationships between numbers ❖ Connections between mathematical ideas or between ideas and representations ❖ Giving meaning to mathematical ideas ❖ In geometry, include making sense of definitions (what counts as a polygon, what does not count as a polygon), formulas, by elaborating them, applying them, finding counter-examples, etc. rather than just stating/executing them. -Do not count "Give me examples of a circle" – instead, count cases where the definition or formula has meaning made around it. B) Ex				
4-Multiple Procedures or Solution Methods				
A) Focus is on ❖ Multiple solution methods for a single problem e.g generalizing to an n-sided regular polygon to use multiple ways. B) Ex:				

Figure A.5. Classroom Observation 1.

5- Patterns and Generalizations				
A) Focus is on ❖ Examining particular cases and then noticing and extending a pattern ❖ Saying whether mathematical procedures work in all case ❖ "Building up" a mathematical definition or deriving a mathematical property e.g., defining "polygons" after considering different examples and non-examples of polygons B) Ex:				
6- Mathematical Language				
A) Focus is on ❖ Fluent use of technical language ❖ Explicitness about mathematical terminology ❖ Encouraging students to use mathematical terms B) Ex:				
7- Overall Richness of the Mathematics				
A) Focus is on ❖ This code captures the depth of the mathematics offered to students. B) Ex:				

Figure A.6. Classroom Observation 2.

WORKING WITH STUDENTS AND MATHEMATICS				
1-Remediation of Student Errors and Difficulties				
A)Focus is on ❖ *Conceptual remediation e.g misconceptions* ❖ *Procedural remediation e.g calculations* *B)*				
2-Teacher Uses Student Mathematical Contributions				
A) Focus is on ❖ Student mathematical contributions to move instruction forward B) Ex:				
3-Overall Working with students and Mathematics				
A) Focus is on ❖ The teacher-student interactions around the content. B) Ex:				

Figure A.7. Classroom Observation 3.

ERRORS AND IMPRECISION				
4-Mathematical Content Errors				
A) Focus is on ❖ Solving problems incorrectly ❖ Defining terms incorrectly ❖ Forgetting a key condition in a definition ❖ Equating two non-identical mathematical terms B) Ex:				
5-Imprecision in Language or Notation				
A) Focus is on ❖ ***Notation*** includes conventional mathematical symbols (such as +, -, =) ❖ ***Mathematical language*** includes technical mathematical terms, such as "angle," "equation, ❖ Teachers often use "***general language***" to convey mathematical concepts B)Ex:				
6-Lack of Clarity in Presentation of Mathematical Content				
A) Focus is on ❖ Mathematical point is muddled, confusing, or distorted ❖ Language or major errors make it difficult to discern the point ❖ Teacher neglects to clearly solve the problem or explain content B) Ex:				

Figure A.8. Classroom Observation 4.

7-Overall Errors and Imprecision				
COMMON CORE ALIGNED STUDENT PRACTICES				
1-Student Provide Explanations				
A) Focus is on ❖ Student explain... B) Ex:				
2-Student Mathematical Questioning and Reasoning (SMQR)				
A) Focus is on ❖ Students provide counter-claims in response ❖ Students make conjectures about the mathematics ❖ Students use ideas from a different mathematical topic to reason B) Ex:				

Figure A.9. Classroom Observation 5.

3-Students Communicate about the Mathematics of the Segment				
A) Focus is on ❖ asking mathematical questions, describing the meaning of a term, offering an explanation, discussing solution methods, commenting on the reasoning B) Ex:				
4-Task Cognitive Demand				
A) Focus is on ❖ Student engagement in tasks B) Ex				
5-Students Work with Contextualized Problems				
A) Focus is on ❖ story problems, real-world applications, experiments that generate data : work with them B) Ex				

Figure A.10. Classroom Observation 6.

6-Overall Common Core Aligned Student Practices				
A) Focus is on ❖ During ***active instruction segments***, this mainly occurs through student mathematical statements: reasoning, explanations, question-asking. ❖ During ***small group/partner/individual work time***, this mainly occurs through work on a non-routine task. B) Ex				

	1	2	3	4	5
WHOLE LESSON CODES					
1-Lesson Time is Used Effectively					
A) Focus is on ❖ This code captures the extent to which lesson time is used efficiently; class is on task, and behavior issues do not disrupt the flow of the class B) Ex:					
2-Lesson is Mathematically Dense					
A) Focus is on ❖ the amount of mathematics – problems, tasks or concepts –worked on relative to the length of the lesson B) Ex:					

Figure A.11. Classroom Observation 7.

3-Students are Engaged					
A) Focus is on ❖ The classroom environment is characterized by student engagement. ❖ Students are eager to participate in the lesson B) Ex:					
4-Lesson Contains Rich Mathematics					
A) Focus is on ❖ The depth of the mathematics offered to students. B) Ex					
5-Teacher Attends to and Remediates Student Difficulty					
A) Focus is on ❖ The teacher attends to student difficulty with the material. B) Ex					
6-Teacher Uses Students Ideas					
A) Focus is on ❖ The teacher uses student ideas and solutions to move the lesson forward. B) Ex					

Figure A.12. Classroom Observation 8.

7-Mathematics is Clear and not Distorted					
A) Focus is on ❖ clear and not distort B) Ex:					
8-Tasks and Activities Develop Mathematics					
A) Focus is on ❖ The tasks and activities done by the class contribute to the development of the mathematics of the lesson. In other words, this code refers to the *architecture* of the lesson, B) Ex.					
9-Lesson Contains Common Core Aligned Student Practices					
A) Focus is on ❖ Students ask mathematically-motivated questions ❖ Students notice patterns and form conclusions based on them ❖ Students make connections across content areas ❖ Students provide mathematical explanations ❖ Students comment on the reasoning of others ❖ Students work on cognitively demanding tasks ❖ Students engage in sense-making					

Figure A.13. Classroom Observation 9.

B) Ex:					
10-Whole-Lesson Mathematical Quality of Instruction					

Figure A.14. Classroom Observation 10.

Question Types		Responses		Answers	
Open Ended	Yes/ No	Invited	Voluntary	Key Word	Elaboration

Figure A.15. Classroom Observation 11.

APPENDIX B: GENERAL INTERVIEW QUESTIONS

During the first meeting, general interview questions were asked to the participant.

Table B.1. General interview questions.

MAIN QUESTION
1) Can you describe "really good mathematics teacher and mathematics teaching" in your opinion?
2) What kind of mathematics lessons are really impressed you? Can you describe to me really good mathematics lesson?
3) Consider about mathematical quality of instruction and mathematics teachers' knowledge, how can you describe these two concepts?
4) How is mathematics as seen in your working school? Can you explain your division of labor and relationship in the mathematics coterie? How do you prepare your instruction materials in the school?
5) What do you find challenging or difficult in this school? How can you contribute to develop these parts? *What is the best part of working in this school?*
6) How college education has an effect on your teaching? What is the effect of your high school mathematics teacher on your teaching?
7) What do you think about your instruction? How do you plan to develop your teaching as a novice teacher?
8) Is there anything that you have not mention so far during the process of the study?

APPENDIX C: PRE-OBSERVATION INTERVIEW QUESTIONS

Before all of the observations, the researcher was asked these questions to the participant.

Table C.1. Pre-observation Interview Observation questions.

MAIN QUESTION
1) What are the objectives / mathematics topic of today's instruction?
2) How would you address the topic mathematically?
3) How did you get prepared for this lesson? *-Materials e,g. activities, problems* *-Student ideas* *-Mathematics representations*
4) How did you plan the lesson in terms of student's difficulties or misunderstandings?
5) Will you use different teaching method for this lesson?

APPENDIX D: POST-OBSERVATION INTERVIEW QUESTIONS

After all of the observations, the researcher was asked these questions to the participant.

Table D.1. Post-observation interview questions.

MAIN QUESTION
1) Do you have any comment about your mathematics instruction in your classroom? Something is going well, or going difficult for you?Which parts of the lesson did you like/dislike more?
2) Was there anything that you struggle or your students struggle with any part of the lesson? Which parts? Why?
3) What about the mathematics instruction in your classroom that contributes to mathematical quality of instruction?
4) What were you hoping that students would learn from this lesson?
5) What would you have done differently if you had taught the lesson again? Why?