EFFECT OF METACOGNITIVE STRATEGY BASED MATHEMATICS INSTRUCTION ON STUDENTS' SELF-REGULATION AND MATHEMATICS ACHIEVEMENT

by

Zehra Çoban

B.S., Integrated B.S. and M.S Program in Teaching Mathematics, Boğaziçi University, 2015

Submitted to the Institute for Graduate Studies in

Science and Engineering in partial fulfillment of

the requirements for the degree of

Master of Science

Graduate Program in Secondary School Science and Mathematics Education

Boğaziçi University

2019

EFFECT OF METACOGNITIVE STRATEGY BASED MATHEMATICS INSTRUCTION ON STUDENTS' SELF-REGULATION AND MATHEMATICS ACHIEVEMENT

APPROVED BY:

Assist. Prof. Dr. Yeşim İmamoğlu ....

(Thesis Supervisor)

Assoc. Prof. Dr. Sevda Yerdelen Damar ....

Assoc. Prof. Dr. Zeynep Çiğdem Özcan ....

DATE OF APPROVAL:

DEDICATION

I want to dedicate my thesis to my brother Mutlu Çoban since he always shared my feelings and supported my decisions.

ACKNOWLEDGEMENTS

I acknowledge that there are several people who endeavor for me to complete my thesis.

Firstly I would like to thank my supervisor, Assist. Prof. Dr. Yeşim İmamoğlu, since she continuously guided and motivated me to complete this thesis. I also would like to thank my thesis committee members Assoc. Prof. Dr. Sevda Yerdelen Damar and Assoc. Prof. Dr. Zeynep Çiğdem Özcan due to their guidance and assistance.

Secondly I would like to thank my parents, Nurperi Çoban and Hamza Çoban, my brother Mutlu Çoban and his wife İrem Çoban since they always trusted me to get success more and more. They were so understandable for me in master period. I sometimes neglect them because I had to work but I feel their existence even we were far from each other.

Lastly I would like to thank my friends Tuğba Aladağ and Betül İşcan. They have always encouraged me in this journey. They helped me to control my feelings and motivated me to continue to the period. Their support was so valuable for me and I feel lucky to have these wonderful people.

ABSTRACT

EFFECT OF METACOGNITIVE STRATEGY BASED MATHEMATICS INSTRUCTION ON STUDENTS' SELF-REGULATION AND MATHEMATICS ACHIEVEMENT

As student-centered learning have gained importance in education, many studies focused on students' thinking process and showed importance of students' ability to regulate their learning according to their own needs. Therefore, self-regulation and metacognition were needed to be studied to help students learn to develop their own strategies. This study focuses on developing students' questioning skills for their strategy construction via IMPROVE instruction method which has similar processes with self-regulated learning and problem-solving in mathematics lessons. Therefore, it is aimed to investigate whether there is a significant effect of metacognitive strategy based mathematics instruction on students' self-regulation and mathematics achievement, and how the IMPROVE instruction affects students' questioning skills. Mathematics instruction was on the unit of functions and lasted about two months. The study was designed using embedded mixed method strategy. While experimental research processes were conducted with 36 10^{th} grade students in total, qualitative data was also collected from seven voluntary students to better see the improvements of students on questioning. Think aloud sessions on function problems and reactions to the instruction were used as qualitative data and, an achievement test on functions and Metacognitive Strategy and Learning (MSLQ) questionnaire were used to collect quantitative data. The quantitative results showed that there was not a statistically significant difference between control and experimental group for both mathematics achievement and self-regulation although experimental group increased their mean on achievement and strategy use more than control group. Further, qualitative results showed all students use comprehension and strategy questions at some level; however, they were in difficulty to use connection and reflection questions. Additionally, students in the experimental group could develop their strategy using especially in terms of connection and reflection questions. Moreover, all of them stated that they could query more than before. On the other hand, it was observed that metacognitive skills of students from control group were more stable. Results of the study and implications on teaching are discussed, suggestions for further studies are given.

ÖZET

ÜSTBİLİŞSEL STRATEJİ TABANLI MATEMATİK ÖĞRETİMİNİN ÖĞRENCİLERİN ÖZ-DÜZENLEME BECERİLERİ VE MATEMATİK BAŞARILARINA ETKİSİ

Eğitimde öğrenci merkezli öğrenme önem kazandıkça, birçok çalışma öğrencilerin düşünme süreçleri üzerine odaklanmış ve öğrencilerin kendi ihtiyaçlarına göre öğrenmelerini düzenleyebilme becerilerinin önemini göstermiştir. Böylece, öz-düzenleme ve üst biliş, öğrencilerin kendi stratejilerini geliştirebilmelerine yardımcı olabilmek için çalışılmaya ihtiyaç duyulan bir alan olmuştur. Bu çalışma, matematik derslerinde öz-düzenleyici öğrenme ve problem çözme ile benzer süreçleri olan IMPROVE öğretim metodunun kullanılması aracılığıyla öğrencilerin strateji oluşturma becerilerini geliştirmeye odaklanmıştır. Böylece, üst bilişsel strateji tabanlı matematik öğretiminin öğrencilerin öz-düzenleme ve matematik başarılarına etkisinin araştırılmasının yanı sıra IMPROVE öğretim metodunun öğrencilerin soru sorma becerilerini nasıl etkilediğinin incelenmesi de amaçlanmıştır. Matematik öğretimi fonksiyonlar ünitesi ile sınırlıdır. Çalışmada gömülü karma araştırma deseni kullanılmıştır. Deneysel araştırma süreci toplamda 36 onuncu sınıf öğrencisi ile yürütülürken, öğrencilerin soru sorma becerileri üzerine gelişimlerini daha iyi gözlemleyebilmek için nitel veri de toplanmıştır. Fonksiyonlar üzerine problemlerin sesli düşünerek çözülmesi ve öğrencilerin öğretim metodu hakkındaki görüşlerinden oluşan görüşmeler nitel veri olarak kullanılırken, fonksiyonlar üzerine bir başarı testi ile Üst bilişsel Strateji ve Öğrenme Anketi (MSLQ) nicel veri edinmek için kullanılmıştır. Nicel araştırma sonuçlarına göre, deney grubu öğrencilerinin fonksiyonlar testindeki başarı ve strateji kullanımı ortalamalarını kontrol grubuna kıyasla daha çok artırmalarına rağmen, istatistiksel olarak anlamlı bir fark bulunamamıştır. Ayrıca, nitel sonuçlar tüm öğrencilerin anlama ve strateji sorularını bir düzeyde kullandıklarını ancak bağlantı ve değerlendirme sorularında güçlük çektiklerini göstermiştir. Ek olarak, deney grubu öğrencileri tüm soru türleri için ama özellikle bağlantı ve değerlendirme soruları konusunda strateji kullanımlarını geliştirmişlerdir. Ayrıca tüm deney grubu öğrencileri, eskiye kıyasla daha çok sorguladıklarını ifade etmişlerdir. Diğer yandan, kontrol grubu öğrencilerinin üst bilişsel becerilerinin daha durağan olduğu gözlenmiştir. Araştırma sonuçları ve öğretim uygulamaları tartışılarak, ileride yapılacak çalışmalar için öneriler sunulmuştur.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ACRONYMS/ABBREVIATIONS

IMPROVE	Metacognitive Strategy Instruction
MSLQ	Motivated Strategies and Learning Questionnaire
SRL	Self-Regulated Learning

1. INTRODUCTION

1.1. Context of the Problem

Mathematics education is necessary for both individuals to gain important knowledge and abilities to be applied in their personal lives, and the future of a country because it is a basis for many areas such as science, transportation, economics, medical conditions or technology, as emphasized in TIMSS Advanced Report (Mullis *et al.*, 2016). However, TIMSS Advanced International Reports (Mullis *et al.*, 2009; Mullis, *et al.*, 2016) showed that mathematics achievement in many countries were below the average score and many countries had a lower or the same achievement level in 2015 compared with 1995. Many people have a difficulty in learning mathematics and applying mathematical knowledge and reasoning in their life; however, there is a way of facilitating students' learning in mathematics. Thanks to self-regulated learning, not only students can manage their learning process, but also they can make use of life-long learning in their career and develop management skills in their personal lives (Zimmerman, 2010; Winne, 1997, Boekaerts, 1996). Its main idea is hidden in the well-known proverb that is "If you give a man a fish, you feed him for a day. If you teach a man to fish, you feed him for a lifetime."

According to Zimmerman, Bonner and Kovach (1996), high achievers have detailed and organized learning outcomes, make use of different learning strategies, observe their learning progress and systematically examine their learning outcomes, have high self-efficacy, and aware of personal responsibility for their learning. Therefore, Zimmerman gather these elements under the concept of self-regulated learning. Self-regulated learning (SRL) in his definition is that learners take their own responsibility during learning process. It has some components such as cognitive, emotional, behavioral, or environmental factors affecting learning process. Zimmerman developed a cyclical model for self-regulated learning that requires goal setting, strategic planning, continuously monitoring and evaluating the effectiveness of learners to adapt changing situations (Zimmerman, 2002; Schunk and Zimmerman, 1998). Studies showed that self-regulated learning helps students to improve their problem-solving skills and increase the persistency of knowledge and mathematical reasoning (Kılıç and Tanrıseven, 2012; Mevarech and Fridkin, 2006). Many other studies examined the effect of self-regulated learning on achievement and found a positive relation (Rosário *et al.*, 2013; Yıdızlı and Saban, 2016; Fadlelmula *et al.*, 2015).

One of the components of self-regulated learning is metacognition that refers to the knowledge about cognition and control of cognitive processes (Brown, 1978; Flavell, 1979). Knowledge about cognition includes knowledge and beliefs about the interaction among person, task and strategy (Flavell, 1979). Regulation of cognition, on the other hand, means planning, monitoring and evaluating the performance (Schraw, 1994). Many studies showed a significant relation among metacognition, self-regulated learning and achievement (Mevarech and Amrany, 2008; Ben-Eliyahu and Linnenbrink-Garcia, 2015; Maqsud, 1998).

There are many metacognitive instruction models in the literature to support mathematical problem solving and reasoning (Montague and Bos, 1986; Montague, 2007; Maccini and Hughes, 2000; Maccini and Ruhl, 2000; Hutchinson, 1993; Mevarech and Kramarski 1997) and IMPROVE is one them which was composed of the first letters of teaching steps: Introducing new concepts, metacognitive questioning, practicing, reviewing and reducing difficulties, obtaining mastery, verification, and enrichment. It uses mainly metacognitive self-questioning in a cooperative learning environment and evaluation of feedbacks (Mevarech and Kramarski 1997). As far as students can develop their capability of self-questioning, they can increase the comprehension of the material and display a higher order thinking (Palincsar and Brown, 1984; as cited in Blonder, 2015; Rosenshine *et al.*, 1996; Zoller, 1987).

1.2. Purpose of the Study

The purpose of this study is to investigate the effect of a metacognitive instruction based on developing students' questioning on their self-regulation level and mathematics achievement, in addition to investigate whether the instruction really develop students' questioning practices or not.

One of the main purposes of mathematics education as indicated in the Curriculum of Ministry of Turkish Education is to help students to deal with real life problems by gaining experience in mathematical problem solving. However, according to researcher's own observaitons, students are in difficulty to solve problems since they are inadequate to regulate their cognition. They can solve problems with leading questions of teachers; however, when they are alone, they cannot deal with the problems. This suggests that they have enough knowledge about the content but cannot regulate their knowledge cognitively. Ben-Eliyahu and Linnenbrink-Garcia's study (2015) also supports researcher's observation. They revealed that cognitive regulation affects achievement through the learning strategies.

In conclusion, this research focuses on developing students' metacognitive skills that hopefully will be resulted in increased self-regulation and mathematics achievement. IMPROVE instruction method will be used to support students' metacognitive skills because there are some similarities between Zimmerman's cyclical model of SRL and IMPROVE instruction method developed by Mevarech and Kramarski (1997), and this observation is supported by the study of Kramarski, Weisse and Kololshi-Minsker (2010). Therefore, it was hypothesized that if IMPROVE method is used in classrooms, then students can be helped to increase their self-regulatory skills and achievement, since it is in accordance with Zimmerman's definition of SRL.

1.3. Significance of the Study

Metacognitive instructions are emphasized in many studies (Veenman *et al.*, 2006; Schneider and Artelt, 2010) and supported in several empirical researches (Lee *et al.*, 2014; Mevarech, 1999). Most of these researches designed metacognitive instruction with the aim of developing students' self-questioning (Hutchinson, 1993; Maccini and Hughes, 2000; Maccini and Ruhl, 2000; Graesser and Olde, 2003; Otero and Graesser, 2001, Mevarech and Kramarski, 1997).

There are some studies revealing that students' questioning behavior is strongly related with mathematics achievement (Ge and Land, 2003; Byun, Lee and Cerreto, 2014). However, most studies focused on teacher questioning rather than students' questioning (Chin and Osborne, 2008) or students' questioning on reading text more than other areas (Yang, 2006). Singh *et al.* (2018) also stated that most of research investigating students' questioning are not recent. Therefore, this study will provide a contribution to the need of research about development of high school students' self-regulation with self-questioning strategy.

More specifically in Turkey context, IMPROVE metacognitive strategy was used in some educational studies (Pilten, 2008; Aziz 2016) and they revealed that it has a positive effect on mathematical reasoning, procedural and conceptual knowledge. This study examines the self-regulatory skills as distinct from the previous ones.

Another contribution of this research is to provide lesson plans and materials for 10^{th} grade mathematics courses, developed in accordance with IMPROVE method. Therefore, teachers can modify these materials according to their students' prior knowledge and thinking skills, and they can implement in their classrooms.

Furthermore, policy-makers can examine the results and materials of this study while developing the curriculum since this study can provide an insight of students' management skills on their learning and thinking processes. They may integrate some activities based on metacognitive strategy use to the curriculum in order to better support students' metacognitive skills.

In conclusion, this study provides an insight about the effect of IMPROVE model on students' self-regulatory skills and mathematics achievement in Turkey, in addition to being a model for teachers and administrators to be able to use in regular curriculum.

1.3.1. Research Questions

(i) Is there an effect of metacognitive strategy based mathematics instruction on 10^{th} grade Turkish students' achievement on the topic of functions?

(ii) Is there an effect of metacognitive strategy based mathematics instruction on 10^{th} grade Turkish students' self-regulation skills?

(iii) How does students' self-questioning practices develop during metacognitive strategy based instruction?

2. LITERATURE REVIEW

2.1. Self-Regulated Learning (SRL)

Zimmerman (2002) defined self-regulated learning as a process that learners self-directively make use of their mental abilities in achieving academic skills rather than a mental ability or academic skill itself. Students' responsibilities for their learning process consist of cyclical efforts classified in metacognitive, motivational, and behavioral aspects and students generate their own thoughts, feelings, and actions in learning process in accordance with their learning goals (Zimmerman and Schunk, 1998, 2009). When students fail to achieve their initial learning goals, they modify their behaviors and strategies in cyclical learning process (Zimmerman 1989).

Pintrich (2000b, p. 453) also emphasized the cognitive, motivational, and behavioral aspects of self-regulated learning process. Self-regulated learning in his definition is an active and constructive process in which learners define their own learning goals and then monitor, regulate, and control their learning process cognitively, motivationally and behaviorally in order to be compatible with their goals and the contextual features in the environment. Therefore, he identified three components for self-regulated learning. First one is metacognitive strategies including planning, monitoring and adjusting the learning process. Management skills to control learning on academic tasks is the second component. The last one is actual cognitive strategies used for understanding, learning and remember the information (Pintrich and De Groot, 1990).

Boekaerts (1999) drew attention to the transferability of knowledge and so defined self-regulation as being capable of developing academic knowledge, skills and attitudes that can be transferred from a learning context on which learners have acquired them to another context on which learners use this information as leisure or work necessities.

Boekaerts (1996) further identified the characteristics of self-regulated learners. She emphasized their capability of controlling the learning process by selecting, combining and coordinating their cognitive strategies in consideration of the context. Another characteristic she stated is that self-regulated learners are tended to allocate different resources for their learning process without including excursive aspects.

As an educational psychologist who emphasized the metacognitive, motivational and behavioral aspects of learning process, Zimmerman explained the characteristics of self-regulated learners in these contexts. They can plan, organize, self-instruct and self-evaluate their learning as metacognitive processes. As motivational beliefs, self-regulated learners have high self-efficacy, autonomy and intrinsic motivation. From behavioral aspect, self-regulated learners constitute social and physical environments which will optimize their learning. In other words, self-regulated learners can combine a number of cognitive process such as goal setting, self-monitoring or self-evaluation; task strategies such as study habits, time-management skills, and organizational strategies; and self-motivational beliefs such as self-efficacy or intrinsic interest (Zimmerman and Martinez-Pons, 1988; Cleary and Zimmerman, 2004).

The mental abilities and skills which learners gain with self-regulated instruction are important to get academic success and improve problem solving since it promotes learners to develop cognitive strategies, gain learning-to-learn strategies and increase self-motivation for learning. However, self-regulation is useful not only in school settings, but also after graduation since it's another important function is developing lifelong learning. Self-regulated learners continue to improve themselves after graduating from the school and update their knowledge in accordance with their needs. Therefore, they can easily be adapted to new positions in business settings or take responsibility in more creative projects such as art, literary or inventions. Additionally, they can regulate their leisure times in a more effective way (Zimmerman, 2010; Winne, 1997, Boekaets, 1996).

2.1.1. Models of Self-Regulated Learning

Self-regulated learning is a process containing monitoring and control process. Although all definitions come together about cognitive, motivational and behavioral aspects, different models appeared depending on the emphasis of various components (Efklides, 2011). For example, Pintrich's model (1999) focuses on strategies used in self-regulation process including cognitive learning strategies, self-regulatory strategies to control cognition, and resource management strategies. A 4-phased model of SRL proposed by Winne and his colleagues Hadwin and Perry (Winne, 2004) focuses on metacognitive monitoring and metacognitive control. Winne's model of SRL was expanded by Azevedo and colleagues to conceptualize the relationship between SRL and learning complex topics within hypermedia environments (Greene and Azevedo, 2009).

Zimmerman's model of SRL is the one that used more common. His cyclical model of self-regulated learning has three phases that are forethought, performance or volitional control, and self-reflection (Figure 2.1). The forethought phase involves some beliefs and strategic processes just before the actions of learning. The second phase refers to the process related to action including planning, concentration and performance. The latter one, self-reflection, is the process after learning a task and interested in learners' reactions about the learning experience. This phase affects the forethought process of the following learning efforts and so the cycle continues (Schunk and Zimmerman, 1998, p.2).

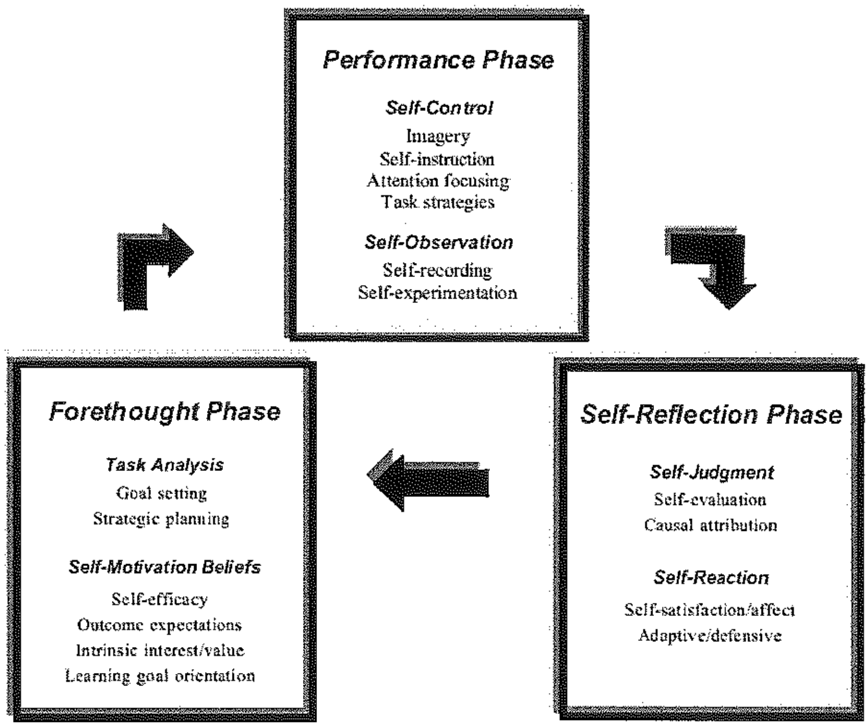


Figure 2.1. Zimmerman's cyclical model of SRL. From Barry J. Zimmerman (2002), Becoming a Self-Regulated Learner: An Overview, Theory Into Practice, 41:2, 64-70.

According to Zimmerman (2002), forethought process involves task analysis (goal setting and strategic planning) and self-motivation beliefs (self-efficacy, intrinsic interest/value, learning goal orientation). According to Locke and Latham's goal setting theory, higher specific goals lead to higher performance because it has an effect on effort and persistence. Learners' goals help them increase their focus on the performance and they will be engaged with the task until they are achieved (Latham *et al.*, 2008). Strategic planning means identifying the appropriate learning strategies and methods in order to realize the goals (Schunk and Zimmerman, 1998). Perceived self-efficacy is defined by Bandura (1997) as learners' beliefs about their learning capability to organize knowledge and produce expected outcomes. Goal orientation is classified as mastery or performance goals by Ames and Archer while Nicholls' classification named them as task-involvement and ego-involvement goals (Ames, 1992). Mastery or task-involvement goals focuses on personal development of competence level to achieve new skills, conceptually understanding the task, or having self-referenced standards. It values on the process of learning. On the other hand, performance or ego-involvement goal orientation values ability and outcomes, and individuals have concerns about being judged. It focuses on the outcome of learning depending on normative-based standards (Ames and Archer, 1987, 1988; Ames 1992). Lastly, Deci and Ryan (1985) defined intrinsic motivation or valuing task as continuous efforts for learning activity even if there is not any reward.

Schunk and Zimmerman (1998) studied performance or volitional control process under three major groups. First group is attention focusing referred to maintaining attention on the task by excluding distractions (Corno, 1993; Heckhausen, 2008). Second group is processes affecting how students apply the strategies or learning methods during working on a task, such as self-instructions or imaginal guidance. Self-instruction means deciding to use appropriate strategies during performing a task (Shunk and Zimmerman, 1998) and imaginal guidance includes mental pictures or images of modeling to facilitate recalling the learned material (Carney and Levin, 2002; Pressley, 1977). Self-monitoring is the third group of volition or performance control phase. It refers to cognitive process comparing the performance with some standards. Metacognitive monitoring used more likely during learning refers to the evaluation about the understanding of the subject matter and properties of their coginiton about the learning along with the standards (Winne, 2004).

According to Zimmerman (2002) self-reflection phase involves self-judgement (self-evaluation, causal attribution) and self-reaction (self-satisfaction, adaptivity). According to

Labuhn, Zimmerman and Hasselhorn (2010), self-evaluation refers to checking the accordance between learning goals and outcomes. Attributions according to Weiner (1979) means causal interpretations of results. As a result of attributions, self-reactions occur positively or negatively towards personal or contextual factors. Attributions also lead to adaptivity of learners' performances to new learning process (Schunk and Zimmerman, 1998, p.5). According to Labuhn, Zimmerman and Hasselhorn's study (2010), self-evaluative judgements can be supported by giving feedback.

Schunk and Zimmerman (1998) submitted two general organizing constructs that are knowledge or beliefs and strategies used for regulation. They also examined two general domains that are cognitive and motivational domains. Components of self-regulated learning are studied in these constructs and domains. Cognitive learning strategies have three categories. First one is rehearsal which involves memorizing, highlighting or underlining to help selecting important points of learned items. Weinstein and Mayer (1983) defined elaboration as a process including "paraphrasing, summarizing, creating analogies, generative note taking, and question answering." Lastly, organization involves searching the main idea, outlining the ideas in the material, use of different techniques to select and organize the ideas (Weinstein and Mayer, 1983).

In brief, Zimmerman' cyclical model including forethought, performance and self-reflection phase focuses on planning, implementation and monitoring, and evaluation of the strategy to solve problems. There are many studies supporting the self-regulated learning to increase students' achievement (Pintrich and De Groot, 1990; Labuhn *et al.*, 2010; Zimmerman and Martinez-Pons 1986, 1988; Cohors-Fresenborg *et al.*, 2010; Kayan *et al.*, 2013)

2.1.2. Self Regulated Learning in Mathematics Education

Self-regulation help learners develop cognitive strategies and get understanding of learning-to-learn (Winne, 1997). Learners can communicate mathematically to take active and constructive role in learning process thanks to self-regulatory knowledge and skills (Darr and Fisger, 2004).

Problem solving is an essential part of mathematics education since it supports developing understanding in mathematics (Brown, 2003). Schoenfeld (1992) advocated that mathematical problem solving is important not just because it helps learners to be a good problem solver in general, but also because it is valuable in its own right. Problem-solving is defined as an active process that includes making decisions and applying appropriate strategies directed to goals (Shuell, 1990). Therefore, students are required to engage in many cognitive and metacognitive activities during problem solving and they should be able to use their previous knowledge and skills (Fadlelmula, 2010 cited from Lester,1980). In other words, problem solving requires the use of self-regulatory skills (Schoenfeld, 1992). Pape and Smith (2002) emphasized the relationship between processes of problem solving and self-regulation as follows:

> "Within the realm of mathematical problem solving, self-regulation translates into careful decoding of the problem and analyzing the relationships between and among the problem's components to form a mental model for the problem. Given this mental representation, the problem solver chooses a mathematical algorithm, or procedure, to solve the problem. Once chosen, the individual must monitor how to carry out the algorithm toward solution. Finally, the problem solver must check her/ his solution in relation to the given problem. Each of these steps involves forethought and planning, monitoring the fidelity of solution process, and reflecting on the problem to determine whether the representation formed is accurate and whether the solution process is successful (p.94)."

Schoenfeld (1992) identified five components of problem solving process. First one is knowledge base meaning definitions, facts or prior knowledge. Second is problem solving strategies. Monitoring and controlling the process is the third one. The fourth is beliefs and affects including self-esteem and anxiety. Lastly, practices are integral of mathematical problem solving. As it can be seen, it includes similar processes with Zimmerman's cyclical phases of self-regulated learning (Zimmerman, 2002).

There are number of researches shoving that self-regulation skills can be developed via effective instructions. The study of Semana and Santos (2018), self-regulatory teaching intervention in a middle school had a positive effect on students' performance in class discussions, internalization of the assessment criteria and their self-regulatory capacity. It is also supported by Perels *et al.* (2009) that middle school students' self-regulation skills and mathematics achievement can be improved with self-regulation intervention. On the other hand, Dignath and Büttner (2008) showed that self-regulation capability can be fostered also for secondary school students.

Many researches emphasized that the increased competency in the components of self-regulated learning have a positive effect on mathematics achievement and problem solving (e.g., Pintrich and De Groot, 1990; Labuhn *et al.*, 2010; Zimmerman and Martinez-Pons 1986, 1988; Cohors-Fresenborg *et al.*, 2010; Fadlelmula *et al.*, 2013).

In the study of Sperling, Walls and Hill (2000), it was revealed that even preschool learners have self-regulation capability in different domains and levels, and they use self-regulatory strategies on problem solving tasks. Marcou and Philippou (2005) also studied self-regulatory strategies during problem solving. Their study demonstrated that elementary school students using self-regulatory strategies in cognitive, metacognitive and volitional aspects had a positively affected motivational beliefs and consequently increased problem solving performance.

Researchers additionally investigated how the components of self-regulated learning are related to problem solving. The study of Mohini and Tan (2005) revealed that there is six type of metacognitive behavior which can be associated with the achievement in problem solving performance of elementary school students: suggesting a plan, assessing difficulty of the task, reviewing progress, recognizing error, recognizing new development and self-questioning during solving process. Further, Cassel and Reid (1996) suggested that self-monitoring served as a motivational factor to increased performance on word problem solving for elementary school students with learning disability or mild mental retardation. Additionally, in the study of Long and Aleven (2013), it is revealed that Open Learner Model supported with self-assessment facilitated middle school students' learning processes and had a significant effect on their learning outcomes. Open Learner Model is a component of Intelligent Tutoring System that is used to improve domain-level learning. It is particularly focused on different types of visualization to present students' learning progress.

Schunk and Zimmerman (1998) wrote the processes of self-regulated learning in high schools. They emphasized that high school students may develop some general regulatory strategies which makes easier for them to benefit from a strategy instruction than elementary school students. Therefore, short-term instructions such as semester-long courses may also helpful in high schools. Cassidy's study (2011) has also some implications for self-regulated learning policy and practice in higher education. The range of individual differences is an important factor in practicing self-regulated learning in secondary level. He emphasized that self-regulated learning includes the capability of corresponding individual differences in learning, and changing normal practice that points to the individual needs of learners. Cassidy (2011) presented some key points of government-funded reports on guidance on the implementation of self-regulated learning. The reports written by Duckworth *et al.* (2009) and Meyer *et al.* (2008) emphasized that self-regulation can be improved with guidance, modelling, effective teaching and learning practices; and self-regulated learning environment should include physical setting, material resources, social interaction and positive support from teachers and peers.

Further, there are many studies showing the relationship between metacognitive instruction and students' self-regulated learning and achievement (Mevarech and Amrany, 2008; Ben-Eliyahu and Linnenbrink-Garcia, 2015; Maqsud, 1998). However, Joseph (2010) stated that many teachers focuses on the content rather than strategies to learn the content. One reason is that teachers do not need to think about the mental processes affecting students' performance on the material (Schoenbach *et al.*, 2003). Another reason is the state curriculum pressure but Joseph (2010) emphasized that reflective and critical thinking practices does not require extra content from teachers since it is a tool, not a content. He also stated that if students' metacognitive awareness increases, they can give more effective feedbacks so that teachers can regulate their instructions in a more appropriate way.

Several researchers used different metacognitive interventions oriented to developing self-regulated learning on the purpose of increasing mathematics achievement. For example, Montague's "Solve it!" model includes eight steps to promote secondary school students to read, comprehend, implement and check verbal mathematics problems. She designed this cognitive strategy intervention by combining some teaching techniques such as modeling, giving feedback, self-questioning, techniques of paraphrasing, visualizing, relating information, hypothesizing, interpreting, and checking. At the end of the study, students improved their strategy use and performance level (Montague and Bos, 1986; Montague, 2007).

Another model found out by Maccini uses a teaching sequence ranging from the concrete to abstract representation. Her "STAR" strategy is an acronym representing the steps: Search the word problem, Translate the words into an equation or picture, Answer the problem by using hints, and Review the solution. She revealed that students with learning disabilities increased their performance on representing and solving problems (Maccini and Hughes, 2000; Maccini and Ruhl, 2000).

Hutchinson (1993), focused on self-questions for the representation of the problem. She taught three types of problems (relational, proportion and two-variable two-equation problems) by using question prompts cards and concluded that students with learning disabilies who received the instruction had higher scores on the problems than students in the control group.

Another intervention which focused on developing self-questioning competency of students is IMPROVE method found out by Mevarech and Kramarski (1997). IMPROVE method uses four type of questions: comprehension, strategic, connection and reflection questions. The process of the implementation of these questions is similar with Zimmerman's cyclical model of SRL. The purpose of comprehension questions is to allow students to think about the problem and necessary strategies like that the task analysis in forethought phase of Zimmerman's cyclical model. Strategic questions help students apply the appropriate strategy and explain their reasoning, therefore students can monitor their learning process. Lastly, with connection and reflection questions, learners evaluate the problem and outcomes in order to explain similarities and differences between previous problems and the new one. Thus, there can be seen an analogy between IMPROVE model and Zimmerman's cyclical model of SRL (Mevarech and Kramarski 1997; Kramarski, Weisse and Kololshi-Minsker, 2010; Zimmerman, 2002)

Many researches supported the positive impact of IMPROVE model based on metacognitive strategies on mathematical reasoning, achievement, problem solving performance, and self-regulatory and metacognitive strategy using (Kramarski *et al.*, 2010; Kramarski *et al.*, 2002; Kramarski and Gutman, 2006; Mevarech, 1999).

2.2. Questioning

Generating questions foster the comprehension of the task (Palincsar and Brown, 1984) and higher level cognitive functions (Scardamalia and Bereiter, 1985; Garcia and Pearson, 1990). King (1992) also expressed that self-questioning provide learners control over their learning, feelings and behaviors.

Mayer (1980, 1984) argued that certain type of questions might lead students to create new representations of the knowledge in long term memory, and some other types of questions might help to relate the new knowledge with the preexisting knowledge so mental representations can be constructed in an increased complexity. Moreover, questions might help learners to link the knowledge to their everyday experiences and real world situations. Mental representations resulted from these construction activities help recall of the material in different ways (Kintsch, 1986).

Student generated questions are valued by many other authors. Self-questions promote the attention on the content and articulation on how much it is understood (Rosenshine *et al.*, 1996). Since they lead students to check what they know and what their deficiencies about the material are, questions are also significant for self- and peer-assessment (Black *et al.*, 2002). Additionally, some authors argued that self-questioning skill is useful for problem solving, decision making and productive and higher order thinking (Chin and Osborne, 2008). Chin and Osborne (2008) expressed that self- questioning in science learning help students direct their learning, enhance classroom discourse, monitor and self-evaluate their understanding on the task, and be motivated and interested in the topic. They also emphasized that questioning is valuable for teachers to assess the way of thinking of students, to plan future teachings, and to take a reflection on classroom practice.

There are many classifications on students' questions. Each classification includes factual knowledge questions, connection questions, questions to expand the knowledge, go beyond the content and examine topic in real world, and reflective questions. However, many authors separated them with some differences. Pizzini and Shepardson (1991) labeled questions as input that helps to recall prior knowledge, processing that helps seeking relationships and output questions that helps going beyond the tasks in new ways (Chin and Osborne 2008)

Scardamalia and Bereiter (1992) divided questions as text-based questions and knowledge-based questions. Text-based questions are related to understanding of a reading material whereas knowledge-based questions are related to thinking on the material to extend knowledge to the real world problems. The latter one might be classified according to the level of thought necessary to answer them and the level of thought might be determined with Bloom's taxonomy. It includes levels of comprehension, application, analysis, synthesis and evaluation. Scardamalia and Bereiter (1992) also seek the nature of the knowledge-based questions. They revealed that students mostly asked "basic information question" for the topic to which they are less familiar whereas "wonderment questions" for the topic to which they are more familiar. Wonderment questions promote curiosity, puzzlement, or scepticism, and so have a great potential to support conceptual understanding (Chin and Osborne 2008).

Watts *et al.* (1997) categorized questions as consolidation questions, exploration questions and elaboration questions. The first one refers to the explanations of the new knowledge and ideas. Exploration questions are to explore the knowledge in new constructs and elaboration questions helps resolving conflicts by examining both claims and counterclaims, testing circumstances and identifying their consequences (Chin and Osborne 2008).

Pedrosa de Jesus *et al.* (2003) preferred to use bipolar instead of uni-polar constructs to classify questions. These authors challenged uni-polar construct since it justifies that higher-level questions is always more valuable than low-level ones, and so they do not consider the context, task, preference, goals, intention, or strategies. On the other hand, bipolar construct promote to predicate the quality of the questions on the nature of the situation; students' learning style and the requirements of the tasks (p. 1028). Therefore, they did not categorize questions in different levels but on a continuum ranging from confirmation questions to transformation questions. "Confirmation questions seek to clarify information and detail, attempt to differentiate between fact and speculation, tackle issues of specificity, and ask for exemplification and/or definition. Transformation questions, on the other hand, involve some re-structuring or reorganization of the students' understanding. They tend to be hypothetic-deductive, seek extensions in knowledge, explore argumentative steps, identify omissions, examine structures in thinking, and challenge accepted reasoning. The authors emphasized that both kinds of questions are necessary and complement each other; with the type of question that is appropriate to ask depending on the nature of the situation and the requirements of the task in hand." (Chin and Osborne 2008).

Chin and Chia's study (2004) revealed that questions could be classified under four major categories. First one is information gathering questions which means to seek basic factual information. Second one is bridging questions to find connections among prior knowledge and new concepts. Third one is extension questions which includes application of new knowledge and expand it to creative situations. Reflective questions, the last one, means evaluating and decision-making.

Students have many personal, psychological or social barriers to ask questions although it makes learning experience more interesting and understandable. Biddulph and Osborne (1982) identified the factors affecting the number and type of questions that students ask as their age, experiences, preexisting knowledge and skills, teachers' teaching style and attitudes, nature of the knowledge and content structure, social climate in the classroom and peer interactions.

Teachers could deal with these barriers by prompts and scaffolding, so that they can promote students to ask questions as classroom activities. (Chin and Osborne, 2008). Chin (2004) presented some strategies to encourage students to ask (Chin and Osborne2008): (a) Presenting students an appropriate stimuli for them to ask questions, (b) Modelling question-asking, (c) Using question prompts or stems, (d) Providing a question taxonomy, (e) Requiring students to write questions through a weekly report, learning journal, question box, question board, or on-line technological systems, (f) Creating a question corner in the class to show 'questions of the week', (g) Adding a 'free question time' and 'brainstorm' sessions to lessons, (h) Giving 'question-making' homework, (i) Using question-asking in evaluation, (j) Preferring interactive teaching approaches so that students can collaborate with their classmates in a group work to generate questions, (k) Promoting a non-threatening classroom climate in order to allow students to ask questions without any shame.

Graesser *et al.* (2001, 2003) made another suggestion to promote students to ask questions. According to their PREG model, learners ask questions if they have a cognitive disequilibrium about the material. "Questions are asked when individuals are confronted with obstacles to goals, anomalous events, contradictions, discrepancies, salient contrasts, obvious gaps in knowledge, expectation violations, and decisions that require discrimination among equally attractive alternatives" (Chin and Osborne, 2008).

Rosenshine *et al.* (1996) argued the effect of question prompts as scaffolding strategy for questioning. They expressed that question prompts might help students pay attention to the task and monitor their learning by elaborating their questions. King (1991) classified these questions in three metacognitive process as planning, monitoring and evaluation (Ge and Land, 2003). This categorization is also in accordance with Zimmerman's cyclical model of SRL.

Ge and Land (2003) presented another categorization on question prompts: procedural, elaboration and reflection prompts. Procedural prompts direct learners to complete a certain task (Rosenshine *et al.*, 1996). Elaboration prompts promote learners to articulate knowledge and thoughts. Lastly, reflection prompts, according to Lin (2001), serve to self-monitoring and constructing new understanding (Ge and Land, 2004).

Ashmore *et al.* (1979) and Shepardson (1993) presented question asking not only as a stage in the problem-solving but also as a component of thinking skills during learning process (Dori and Herscovitz, 1999). Scardamalia and Bereiter (1985) and Garcia and Pearson (1990) expressed that question generation could be used to promote students think about issues requiring higher-level cognitive functions (Rosenshine and Meister, 1996) since students should play an active and initiating role in the learning process to compose questions (Rosenshine and Meister, 1996 cited from Collins *et al.*, 1990; King, 1994; Palincsar and Brown, 1984; Singer, 1978). Shodell (1995) also emphasized that self-questioning increase creativity and higher-order thinking skills (Dori and Herscovitz, 1999). Graesser and Person (1994) identified criteria for high-level questions that are including inferences, applying the new knowledge to other domains or situations, multi-step reasoning, the synthesis of the knowledge from multiple sources, or the evaluation of the new information (Chin and Osborne, 2008). Particularly, 'causal antecedent questions' (Why

…?), 'causal consequence questions' (What happens if …?), 'goal orientation questions' (What is the purpose of…?), or 'instrumental/procedural and enablement questions' (How …?) were considered by Graesser *et al.* (1993) and Graesser and Person (1994) to be deep reasoning questions, as well as they are indicator of higher levels of cognition (Chin and Osborne, 2008).

Several authors used question prompts during mathematical problem solving. Maccini's STAR strategy includes four steps in questioning during solving algebraic problems. Her strategy is to teach secondary students to solve algebraic problems including four operations by using representations from concrete to abstract. The name of the strategy comes from an acronym that means: (Maccini and Hughes, 2000; Maccini and Ruhl, 2000; as cited in Montague 2007)

S = Search the word problem (Read; Ask yourself questions, i.e., What facts do I know? What do I need to find?; Write down facts).

T = Translate the words into an equation in picture form (Choose a variable; Identify the operations; Represent the problem using concrete, semi-concrete, and abstract representations).

A = Answer the problem using cues and a work mat.

R = Review the solution (Reread the problem; Ask the question, i.e., Does the answer make sense? Why? Check the answer).

Hutchinson (1993) also prepared a set of self-questions for representing and solving algebraic word problems. S/he aimed to teach three types of problems which are relational, proportion and two-variable two-equation problems. Hutchinson's self-questions are listed below: (Montague, 2007)

Self-Questions for Representing Algebra Word Problems

(i) Have I read and understood each sentence? Are there any words whose meaning I have to ask

(ii) Have I got the whole picture, a representation, for the problem?
(iii) Have I written down my representation on the worksheet? (goal, unknown(s), known(s), type of problem, equation
(iv) What should I look for in a new problem to see it is the same kind of problem?

Self-Questions for Solving Algebra Word Problems

(i) Have I written an equation?
(ii) Have I expanded the terms?
(iii) Have I written out the steps of my solution on the worksheet? (collected like terms, isolated unknown(s), solved for unknown(s), checked my answer with the goal, highlighted my answer)
(iv) What should I look for in a new problem to see if it is the same kind of problem?

Kramarski (2010), presented some questions embedded in the task for the IMPROVE method. She included comprehension, connection, strategy and reflection questions and question prompts were elaborated to identify the type of the question and how to answer these questions. Comprehension questions help students understand the structure of the problem before solving it. Connection questions require students to find out the similarities and differences between solutions and to explain their reasons. Strategic questions promote students to choose appropriate strategy to solve the problem and explain their reasoning. Lastly, reflection questions are designed to prompt students to monitor and evaluate their understanding (Kramarski, 2010)

2.2.1. Empirical Researches on Questioning

There are some researches showing the positive effect of developing students' questioning on knowledge construction, problem-solving performance and achievement.

King (1994) examined three groups of fifth graders. In first group, students were guided by both lesson-based and experience-based questions organized to help learners make connections among different concepts. In the second group, students' discussions were promoted with only lesson-based questions and the last group was the control group receiving no guidance. The results showed that the first group outperformed on complex knowledge construction that others, and experience-based questions are more effective since they promote to access the prior knowledge or experiences.

King (1992) also compared the self-questioning, summarizing and note taking-review as learning strategies. She concluded that self-questioning is a more effective strategy to recall the content and to have more detailed lecture notes than summarizing or just reviewing the notes.

Some other studies investigated the effect of using question prompts on problem solving in ill-structured task. Ge and Land (2003) revealed that question prompts had a significant effect on students' problem solving performance on ill-structured tasks also. Byun *et al.* (2014), on the other hand, compared the effects of instructor-generated question prompts, peer-generated questions and individual question prompts on ill-structured problem solving. At the end of the study they conducted with undergraduate students, they indicated that students taking instructor-generated questions performed better than students allowed to construct their own question prompts.

As a conclusion, educators can use question prompts designed by themselves to support students' self-questioning and achievement in problem solving depending on the results of studies in this area.

2.3. Metacognitive Instruction (IMPROVE Method)

As learner centered approach gain an increasing importance in education area, many researchers concentrate on self-regulation skills and metacognition. The knowledge and regulation of cognition to choose appropriate strategies and understand how they work is referred as metacognition (Schraw, 2002). Many researches revealed that metacognition has a positive effect on the development of self-regulation (Kramarski 2010; Veenman, Van Hout-Wolters, and Afflerbach, 2006; Joseph, 2009) and mathematics achievement (Schneider and Artelt, 2010, review study; Özsoy, 2011; Özsoy and Ataman, 2009). Moreover, metacognitive skills can be taught with particular classroom methods and materials used by teachers (Jacobs, 2003; Paris and Paris, 2001; as cited in Joseph, 2009).

Metacognitive instructions can reinforce metacognitive awareness and learning of students (Veenman *et al.*, 1994). Metacognitive strategy instruction depends on the ways of using some particular strategies and it emphasizes when, why and how to use those strategies while engaging in a task (Baker and Brown, 1984; as cited in Huff and Nietfeld, 2009). Veenman (1998) called it as WWW&H rule (What to do, When, Why, and How). He and his workmates also stated particular features of a successful metacognitive instruction. First one is that metacognitive strategies are connected to the content. Secondly, learners are aware of the usefulness of metacognitive strategies and alerted to extra effort. Lastly, the instruction should be prolonged to ensure the maintenance of students' metacognitive knowledge and skills (Veenman *et al.*, 2006).

Nevertheless, educators concerns about that students' metacognition might be ignored since teachers focus on content more than strategies necessary to learn the content. Especially secondary school teachers are less likely to pay attention to mental process (Schoenbach *et al.*, 2003; as cited in Joseph, 2009). In addition to that they are not tended to consider it naturally, they also come up with curriculum pressure of the states and limited instruction time to focus on learning strategies. However, metacognitive strategy using do not extend the content; rather it is an instrument to enhance the content. Therefore, metacognitive skills can be implemented in ordinary learning activities (Joseph, 2009).

One of metacognitive instruction methods is IMPROVE developed by Mevarech and Kramarski (1997). IMPROVE is a multidimensional method including cooperative learning, metacognition and mastery learning. The aim of the method is to encourage students to pose questions by engaging in thinking and reflecting on their learning .The name of the model shows the first letters of the instruction steps: "introducing new concepts, metacognitive questioning, practicing, reviewing and reducing difficulties, obtaining mastery, verification, and enrichment" (Mevarechand and Kramarski, 1997).

The central part, self-questions, were composed so that students are able to aware of the learning progress and self-regulate their problem solving process. Cooperative learning was also included to model in the aim of that students can transfer their prior knowledge and skills to each other. Additionally, the model requires from teachers to give corrective feedbacks for low achievers and enrichment feedbacks for high achievers. Thus, students can improve their thinking skills (Mevarech and Kramarski, 1997). Information about the implementation of the model will be explained in methodology part in a more detail.

2.3.1. IMPROVE Method in Mathematics Education Context

There are several researches investigating the effect of metacognitive instruction, IMPROVE method on different variables. Mevarech and Kramarski (1997) revealed that IMPROVE instruction increased mathematics achievement and reasoning of students from grade 7 to 9. Another study investigated the effect of the metacognitive instruction on achievement of high school students in delayed or stressful situation and revealed that IMPROVE students had significantly higher scores than control group students (Mevarech and Amrany, 2008).

Kramarski *et al.* (2010) studied with third grade students with mathematics anxiety. They examined the effect of self-regulated learning supported by metacognitive questioning on problem solving performance. Their intervention group (MS students) had an instruction based on IMPROVE model and results showed that MS students outperformed on problem solving tests. The study of Kramarski and Gutman (2006) combined the e-learning environment with IMPROVE model for ninth graders and resulted that students taking the combined instruction significantly outperformed in problem solving that students taking instruction just including e-learning. In the study examining the effect of IMPROVE method on third and six grade students' solving word problems, it was revealed that IMPROVE students had significantly higher scores than their counterparts and also third graders could make use of the instruction more than six graders (Mevarech *et al.*, 2010).

There are some studied investigated the effect of metacognitive instruction embedded in cooperative learning environment. Mevarech (1999) studied with seventh grade students and examined three conditions based on IMPROVE project: metacognitive training including both making connections and strategy application, direct instruction with strategy application, and no special training. The results showed that students taking metacognitive training had significantly higher scores than other groups on problem solving performance. Kramarski *et al.* (2002) also studied metacognitive instruction combined with cooperative learning settings and they used IMPROVE method. They concluded that metacognitive instruction in cooperative setting had a significantly positive effect on the performance of seventh grade students on authentic and standard mathematical tasks. Kramarski and Mevarech (2003), on the other hand, chose mathematical reasoning as their dependent variable. They demonstrated that the students taking combined instruction significantly outperformed the students taking just metacognitive instruction, which in turn significantly outperformed the students taking just cooperative learning or individualized learning instruction on various aspects of mathematical explanations.

Mevarech and Fridkin (2006) also researched the effect of IMPROVE instruction on mathematical reasoning and knowledge. According to the study conducted with pre-college students, metacognitive instruction had a significant effect on both mathematical knowledge and reasoning. In another study, Mevarech and Kramarski (2003) compared the effect of metacognitive training and worked-out examples on mathematical reasoning of eight grade students. Worked-out examples were defined as showing all the steps of solving problems and presenting explanations about the sequence of actions. The results showed that students in metacognitive training group based on IMPROVE model outperformed students in worked-out example group.

There are some studies using IMPROVE method in Turkey also. Abdul Aziz (2016) studied the effect of IMPROVE method on mathematical procedural and conceptual knowledge, and metacognitive skills for 11th grade students as doctoral thesis. He stated that the metacognitive instruction had a positive effect on both mathematical procedural and conceptual knowledge, and metacognitive skills. Another doctoral thesis revealed that IMPROVE method was effective to develop 5th grade students' mathematical reasoning (Pilten, 2008).

In conclusion, IMPROVE method was designed to improve students' metacognitive and self-regulation skills. Therefore, they can gain metacognitive self-questioning which reinforce the understanding of the task, planning strategies and reflecting on applications during the learning process (Zimmerman, 2002)

2.4. Students' Understanding of Functions

Function topic is important to learn mathematics since it is a unifying theme in curricula (Steele *et al.*, 2013; Ayalon *et al.*, 2017). Some researchers emphasized the importance of deep understanding of functions to achieve higher mathematics (Eisenberg, 1992; as cited in Ratliff, 1990; Caldwell, 1996). Functional relationships also were needed in many areas in daily life such as financial plans, demographics and population growth, metric conversions or income and interest calculations (Kalchman and Koedinger, 2005). Further, it can be said that it only concept in mathematics that is important from kindergarten to graduate school (Harel and Dubinsky,1992; as cited in Santos, 2003).

Function has more extensive interpretation than its formal definition. It has multiple representations as algebraic symbols, tables, graphs, mappings or contextual situations and the essential point is to be able understand that these representations help us describe the same relationship (Kalchman and Koedinger, 2005, Steele *et al.*, 2013, Panaoura *et al.*, 2017). Therefore, function is a challenging topic for both teachers and students (Steele *et al.*, 2013; Kalchman and Koedinger, 2005). Another difficulty in learning functions comes from the complexity of its mathematical language including many special notations and symbols (Eisenberg, 1991; as cited in Santos, 2003; Wagner, 1981).

Many studies emphasized the lack of student knowledge and skills on function concept and necessity of improving instructions (Breidenbach *et al.*, 1992; Leinhardt *et al.*, 1990). Kieran (1992) examined the possible reasons of students' difficulties to understand functions conceptually such as its teaching or students' improper ways to study on functions (Panaoura *et al.*, 2017). Sajka (2003), on the other hand, stated that school tasks and standard procedures have an effect on students' abilities to solve function problems. Clements and Vaiyavutjamai (2006) also emphasized that traditional teaching and testing focus on only one correct answer, and so they restrict students' skills to make connections among different representations. Therefore, teachers may need additional support to teach the concept of function (Steele *et al.*, 2013).

An effective instruction is beyond to help students perform on basic operations such as calculations or graphing of given equations. It allows students to understand function conceptually and use in different contexts (Kalchman and Koedinger, 2005). According to Kalchman and Koedinger (2005), students fail to achieve algebra problems when they are inadequate in conceptual understanding and metacognitive monitoring.

Further, results of some studies also showed that students were in difficulty to understand function and students' ability to interpret function is related with the recognition and manipulation of the concept and their problem solving skills (Panaoura *et al.*, 2017). Sajka (2003) also concluded that insufficient understanding of the concept of function might be influenced by the selection of tasks during teaching process.

As a result, IMPROVE metacognitive instruction was implemented along function unit in the present study, since it is a challenging topic for both teachers and students, and further teaching opportunities are needed. As Steele and his colleagues (2013) stated, function, an important part of mathematics content, supports classroom practices of diverse teachers to engage in meaningful works. Moreover, it has multiple representations and so allows students to make interpretations and further investigations on it. IMPROVE method also aims to lead students to question more on the problems. Therefore, the present study aims to provide useful sources and materials for teachers and a new perspective for them to teach this unit.

3. METHOD

3.1. Design

Embedded mixed method design was used for this study since qualitative data were needed to be collected during the process while quantitative data were collected with pre and posttests. (Creswell and Piano Clark, 2007). In quantitative part, quasi-experimental research was conducted to investigate the effect of metacognitive strategy based instruction on students' achievement on functions and self-regulation. Therefore, there was one independent and two dependent variables. Independent variable was instruction method (IMPROVE method vs traditional teaching method) and dependent variables were students' self-regulation levels and mathematics achievement. There were one experimental and one control group. Pre and posttests were conducted to examine the effect of the intervention on the level of self-regulation and mathematics achievement. Self-regulatory skills were measured by Motivated Strategies and Learning Questionnaire (MSLQ) and mathematics achievement was measured by an achievement test. In qualitative part, on the other hand, basic qualitative processes was conducted to examine people's interpretations about their experiences and their attributions to their experiences (Merriam and Tisdell, 2016). Therefore, some students were examined using think aloud protocol to identify how students' questioning skills change. Their development in questioning was observed and compared with their self-regulation level and mathematics achievement. In addition to think aloud process, intervention group students were asked their ideas about the instruction via semi-structured interviews.

3.2. Participants

This study was conducted in a private school in Yenibosna, İstanbul. The school is chosen with convenient sampling since researcher had accessibility to those students and administration of the school. This school was a private science school. Science school is a school type where students take science lessons more than social science lessons and which takes relatively high achievers compared with other schools. Although the school where this study conducted is a science school, there were not students with very high scores in the central high school entrance exam. When the location of the school is considered, people were middle class around that part of İstanbul. The school had smart boards in each classroom but students did not have tablet etc individually in classrooms. Classes include 15 to 20 students. Teachers in that school generally teach with traditional methods including practice and drill, mathematical definitions without connecting to real life situations. Two tenth grade classes were chosen for the purpose of research. Totally 36 students participated to the study and 18 of them were female and 18 of them were male. One of the classroom, experimental group, had 20 students (8 female, 12 male) and the other, control group, had 16 students (10 female, 6 male). Classes included students from different academic levels according to the pretest of the study. Classes were assigned to two teachers by school administration and teachers were randomly assigned to experimental and control group. Both teachers had master degree but control group teacher was graduated from education faculty whereas intervention group teacher was graduated from mathematics department. Further, control group teacher had five years of teaching experiences and intervention group teacher had two years of teaching experiences. The intervention group teacher taught her lessons by using IMPROVE metacognitive strategy based mathematics instruction and the other teacher taught control group by using traditional learning methods. Furthermore, four voluntary students from experimental group and three voluntary students from control group were selected for interviews to examine their development on questioning skills. Two of the students from control group were high achievers and one of them was in average according to pretest results. In experimental group, on the other hand, one student was high achiever and the others were in average.

3.3. Data Collection

This mixed method study was included three types of data source. A survey and an achievement test were used to obtain quantitative data and semi-structured interviews were conducted to get qualitative data. MSLQ survey was used to measure the self-regulatory skills of students and an achievement test was developed to measure students' success on the topic of functions. Both survey and achievement test were given as pre and posttest to compare the results of two groups. Interviews also were done with voluntary students to observe their questioning skills during problem solving process. Interviews were included both function questions and questions about students' ideas on the implementation of the study.

3.4. Instruments

3.4.1. Motivated Strategies for Learning Questionnaire (MSLQ)

Students' self-regulation and metacognitive skills was measured by MSLQ developed by Pintrich and De Groot (1990). The questionnaire was adapted to Turkish by Erturan İlker *et al.* (2014) and validity and reliability tests were done by them. During the analysis, confirmatory factor analysis (CFA) and Cronbach Alfa were used. There are many fit index to use CFA. Chi-Square ($\chi 2$/sd), comparative fit index (CFI), goodness of fit index (GFI), adjusted goodness of fit index (AGFI), the normed fit index (NFI), the root mean square error of approximation (RMSEA), and standardized root mean square residual (SRMR) are used more commonly. After implementation of CFA, it is required that RMSEA and SRMR values should be smaller than 0.05; CFI, GFI, AGFI and NFI values should be over 0.90; and $\chi 2$/sd value should be smaller than 5 so that the model becomes valid ($\chi 2$/df: 3.93; RMSEA: 0.042; SRMR: 0.047; CFI: 0.95; GFI: 0.90; AGFI: 0.90; NFI:0.94). Further, Cronbach Alfa was used to test reliability of the model and it was found as 0.88. MSLQ is in likert type form ranging from 1 (it does not define me) to 7 (it completely defines me) and includes two parts: motivational strategies and learning strategies. It has totally 44 item and covers self-regulation strategies, cognitive strategy use, self-regulation, motivational beliefs, self-efficacy, inner value and exam anxiety (Erturan İlker *et al.*, 2014).

In this study, four dimension of MSLQ were used to examine students' self-regulation skills: self-regulation, metacognitive strategy use, self-efficacy and intrinsic value. Cronbach's alpha reliability tests were done for each subscale in the both pre and posttest. Metacognitive strategy use has alpha coefficient as .846 and .920 respectively. Self-efficacy alpha coefficient is .909 and .916 respectively. Intrinsic value has .808 and .853 reliability coefficient respectively. Lastly alpha coefficient for self-regulation subscale was found .559 and .556 in pre and posttest respectively, which was relatively low. Therefore, 3 items with low coefficients were removed from the measurement and alpha was calculated again. It was found as .755 and .757 for pre and posttest respectively. These values shows that the questionnaire is reliable for all subscales after 3 items were deleted. Detailed information about items in MSLQ was given in Appendix A.

3.4.2. Achievement Test

An achievement test was prepared to measure students' mathematics achievement. The test was covered all objectives of functions unit according to the curriculum of Ministry of Turkish Education. Further, there were some questions measuring the prerequisite knowledge for functions. The test included 20 open-ended problems that aimed to assess students' mathematics achievement on functions.

Four experts' opinions were taken during the preparation of the questions. Two of them were academicians and two of them were teachers who were participants in the school where data was collected. Experts were requested to examine whether each question measure the target objective, whether a question covers another objective except from assigned ones, and whether the number of questions or difficulty level of questions were appropriate (It is tried to add questions ranging from easy to difficult) (Appendix B). After experts' feedbacks and further ideas were taken, the test was arranged and finalized (Appendix C).

Different sources were used to prepare the test questions in addition to researcher's own questions. First and third questions were taken from Kalchman's study (2001). Second question was adapted from Postelnicu's study (2011). Fourth and fifth questions were chosen from You's study (2006). Sixth question was from Aviles-Garay (2001). Seventh, eighth and fifteenth questions were taken from Mercan's study (2015). Nineth, fourteenth, sixteenth and nineteenth questions were from Akçakın's study (2015). The researcher wrote 10^{th} and 13^{th} questions on her own. Santos' study (2003) was used for 11^{th} and 12^{th} questions. Lastly, Yağdıran' study (2005) was benefited for 17^{th}, 18^{th} and 20^{th} questions.

A rubric was also prepared by the researcher to define scores of each question on achievement test. Two mathematics teachers' opinions were taken to investigate whether the scoring way is appropriate or not. Further, the supervisor of this study examined the rubric and gave feedbacks. After expert's ideas were taken, the final rubric was prepared (Appendix D)

By conducting a pilot study in the same school with different 10th grade students from the main study, it is examined whether each question was understandable for students and the test was reliable or not. Reliability was measured by Cronbach's Alpha coefficient in present study. According to Gronlund (1988) reliability coefficient for achievement tests used in classrooms might be lower than standardized achievement tests and it ranges from .60 to .80. In this study, alpha coefficient was found as .677 for pilot study and .678 for posttest in the main study, which shows enough reliability of the test (Gronlund, 1988; Schmitt, 1996; Griethuijsen *et al.*, 2014).

Moreover, another one year experienced mathematics teacher who also has a master degree in mathematics education was consulted to provide interrater reliability. She did blind marking according to the rubric prepared by the researcher and agreement rate between the researcher and interrater teacher was found as 0.84. Then we met again to discuss on the items on which were not agreed. Then agreement rate increased to 0.97. There were some answers that we were confused to rate and so advisor of the study was also consulted and final decisions were taken to mark the items.

3.4.3. Interviews

In order to answer the third research question, interviews were completed by the participants who were voluntary to participate. Interviews were conducted at the beginning, in the middle and at the end of the study to observe the changes on students' questioning skills. Three students from control group and four students from intervention group participated to interviews. Participants were required to use think aloud strategy after the researcher showed a modeling of think aloud protocol. It is defined as to talk aloud while performing a task to point out cognitive and metacognitive processes (Ericsson and Simon, 1993; Pressley and Afflerbach, 1995). Students were asked mathematics questions on functions and the researcher just took audio recording without interfering the students' cognitive process. If the students stop talking aloud, the researcher gave prompts such as "keep talking", "please think aloud" or "tell me what you are thinking now". In the first interview, there were only function problems for students to solve with think aloud protocol. In the second and third interview, intervention group students are additionally asked their opinions on the method of teaching via semi-structured interviews. They were asked the possible advantages and disadvantages of the instruction in view of students. All interviews were recorded by audio-type and transcribed by the researcher. (Appendix E, F, G).

3.5. Procedure

All permissions about the implementation of the study were taken from school administration, teachers, parents and students. They were informed about the purpose of the study was to investigate the effect of metacognitive strategy based mathematics instruction students' mathematics achievement and self-regulation skills. They were told that the information obtained from this study would guide administrators and teachers to organize better educational programs for students to get better experiences. It was explained that all the personal information would hold in confidence and only investigator would access them. Both students and their parents signed a consent form to participate the study (Appendix H).

All instruction prepared for this study, training, practices and testing, was structured as part of regular mathematics curriculum of Ministry of Turkish Education. All activities implemented in the experimental classroom were managed by regular teacher under the superintendence of the researcher and her supervisor.

Both experimental and control classes used the same mathematics textbooks and other sources that have the similar exercises as in the textbook. They took mathematics course seven hours in a week. Lesson plans for both groups were prepared by the researcher. Appendix I is prepared to show an example unit plan for the content of the course. IMPROVE group students additionally took activities combined with metacognitive questioning, formative tests, corrective and enrichment activities. All tests, enrichment and correction activities were prepared by taking the opinions of the teachers participated to this study. All these activities apart from tests were designed for cooperative works. The control group took individualized practicing without metacognitive questioning and corrective or enrichment activities. The researcher participated to both control group and intervention group lessons as an observer.

The researcher worked with intervention group teacher in 5 sessions and each session was about 1 hour. In the first session, the research project was explained. A detailed information about IMPROVE instructional model was given and expectations from the teacher were clarified. A paper consisting of the parts explaining important components of IMPROVE from two article (Mevarech and Kramarski, 1997; Kramarski *et al.*, 2010) was given to the teacher (Appendix J). In the second session, lesson plans prepared by the researcher were examined and discussed with the teacher about the sequences of topics, introduction to the new concept, number of examples and difficulty level of examples. In the next two sessions, metacognitive questioning exercises were done with the teacher so that she can effectively use the question prompt cards. Lastly, teacher experienced how to ask leading questions for students to be able to direct them for the solution by using cues of the teacher. First three meeting were organized in two weeks before the intervention and last two meetings were organized in first two weeks of the intervention. In addition to these sessions, the teacher took feedback from the researcher at the end of each day.

The intervention was applied for two months in 10^{th} grades along functions unit. The reason of choosing function topic in this study is that it is an appropriate topic to use investigative questions and include many real life examples. Students had chance to produce creative reactions and also it is a long topic enough to be able to observe metacognitive changes. All students were given pretest in the first week of the intervention and posttest in the last week of the intervention. Further seven voluntary students were interviewed three times at beginning of the intervention, at the middle of the intervention and at the end of the intervention. The first interviews were done in the second week of the intervention and after 3 weeks, the second interviews were done. Last interviews were at the last week of the intervention.

3.6. Treatment

IMPROVE, the metacognitive strategy based instruction method, was developed by Mevarech and Kramarski in 1997 and this study was organized based on their model. It includes three interrelated parts: metacognitive questioning, cooperative learning and feedbacks to correct or enrich the students' learning.

In order to help learners develop metacognitive questioning skills, question prompt cards was designed as an individual hand-held strategy (Appendix K). Thus, they were allowed to use these cards while practicing or making discussions in the class.

Metacognitive questioning cards include four types of questions: comprehension, strategy, connection and reflection questions. It is aimed to enable students try to understand the problem and search the given and unknown in the problem with comprehension questions. Students are also required to identify the mathematical concepts related to the problem. Strategy questions are used to implement the appropriate strategy, tactic or principle to solve the problem. It is also required to reason the strategy and search how to apply the strategy. Connection questions, on the other hand, provide students to compare the problem with previous ones and assess the similarities and differences. Problems may be equivalent, similar, isomorphic or unrelated. Equivalent problems have both the same mathematical structure and the same story context. Similar problems have the same story context but with different structures whereas isomorphic problems share the same structure with different story context. Unrelated problems, on the other hand, have completely different structure and context. Thus, it was aimed that students can understand the surface and deep mathematical structures of a problem. Lastly, reflection questions are to justify the solution. Students are required to explain the reason of the solution or what can be the reason of that they cannot solve the problem. They are also directed to recognize the points they are in difficulty while solving the problem.

The teacher tried to emphasize the importance of using metacognitive questions so that students would be aware of that these questions would promote them to comprehend and remember the material learned in the class. Before students' applications of prompt cards, the teacher modeled the process.

A real life problem was prepared for each new concept in the lesson plan and so it was aimed that students could understand the logic of the topic by making more concrete it. The mathematical definitions were given after the discussion of the real life problem in the intervention class. However, control group teacher was not interfered how to apply the lesson plan as long as she used the same examples and so she preferred to give mathematical definitions firstly and then show the real life example. Intervention group teacher allowed students to ask questions or explain their ideas about the problem whereas control group teacher did not create such an interaction and solved the problem superficial. As a result, intervention group students were engaged with more creative questions on the given problem whereas control group students were engaged with lower level questions about the given problem according to Bloom's taxonomy (Bloom, 1956).

As an example for the discussion in the intervention group, when the teacher said that functions could be thought as a machine and it has inputs and outputs, one of the students asked what if f(x) is equal to just 3. After teacher listened to his classmates' ideas, she tried to explain. There was an example in the function machine including output as ayran while inputs as yoghurt and water. Teacher said that whatever they dropped into the machine, they always would get ayran. Then one student said even if they dropped pudding to the machine, they would get ayran. Another student interpreted that if the function f(x)=3x, then they could get pudding including ayran. On the other hand, students' questions in the control group were like "Will we draw these diagrams ourselves or will they be given to us?", or "Is that the result of multiplication of 15 and 3?"

After the discussions, the teacher started to solve example problems by using metacognitive questions. At the beginning of each lesson, the teacher modeled how to use metacognitive questions and then she required students to attend the solution process for the following problems. Students were encouraged to use these questions during their own work and group works. They were required to make reasoning and use appropriate mathematical language.

In the first week of the study, the teacher was in difficulty to ask the correct metacognitive questions for each problem. While she was asking the givens and the unknown of the question, she was focused on numbers or algebraic expressions. After the researcher's feedback, she began to express the meaning of those expressions. For example; instead of saying that the function f is from N to R, f(x)=2x-5 is given, she started to say that the domain and range of the function were given and also we know the rule this function uses to match the elements of two sets from the beginning of the second week. Further, when she asked the students "Which kind of problem is this?", the students responded as easy of difficult and she accepted this answer in the first week. Then she understood that the type of the question means which structures included in the question or which way it requires to proceed. For example, this question was oriented to find the image of an element, domain of a function or writing a function in terms of another function or the question included a relation between two functions instead of the direct rule of a function.

In the second week, the teacher correctly used the metacognitive questions; however, she could not emphasize to students that she used question prompt cards and they should follow them. Further, she could not use metacognitive questions concurrently with the solution of a problem. She explained the givens and unknown at the beginning of the problem, but how and why she used her strategy were clarified after the solution. An example can be seen below:

Question: For the function $f:\mathbb{R} \rightarrow \mathbb{R}$, it is given that f(x)=f(x+1)+4 and f(1)=3, find the value of f(12).

The teacher: What do we know about the question? We know the domain and range of f, the image of 3 under f, and also a relation between the image of x and x+1 under the function f is given to us. The unknown is the image of 12 under f. Here, I will start by writing 3 instead of x in the given equation.

For x=3, f(3)=f(4)+4. Now I should put 4 in place of x in the equation.

Students: Why do we do that?

The teacher: Wait for now and you will see why I am doing this. Now I will give 4 for x and find f(4)=f(5)+4. We see that the next value of x is 5.

(Then she writes the equation as below)

For x=3, f(1)=f(2)+4

For x=4, f(2)=f(3)+4

For x=5, f(3)=f(4)+4

....

For x=11, f(11)=f(12)+4

Now, if we add all the equations, we find:

f(1)+f(2)+...+f(11) = f(2)+f(3)+...+f(11)+f(12)+4+4+..+4

Then the same terms at different sides cancel each other, but how many 4's do we have here?

Students: 11

The teacher: Yes, we started with f(1) and finished with f(11) at left side, so we have eleven 4's here. Then:

f(1) = f(12)+4.11 here we know the value of f(1) as 3.

3 = f(12)+44, so f(12)=-41.

Here, what kind of strategy did I follow? Firstly since we knew the value of f(1), we aimed to use it in the equation, so we put 1 in place of x.

Students: Why don't we use 0 for x? We can find f(1) again.

The teacher: You are right, we can get f(1) with 0 also, but we are asked to find f(12). So, we should give increasing values for x, not decreasing. If we gradually increase x, we can arrive f(12), the unknown of the question.

In this example, the teacher could explain the aim of her strategy at the beginning of implication of strategy, and might ask students how they can obtain f(12) with a little class discussions. However, she required students to wait till the end of the solution and then she explained the reasons of her strategy.

Along the third week, she was better in applications of metacognitive questions in addition to class discussions on application of real life examples. She also emphasized the using of these questions. She often asked students whether they looked at question prompt cards or not. However, the fourth week was their exam week and the teacher did not use metacognitive questions in two review lessons for the exam and also after the exam, she solved exam questions in the lesson without metacognitive questioning. In another lessons of the week, she used questions in appropriate way mostly. There is an example below that she used all metacognitive questions in an appropriate way.

Question: If $f(x)=3^{x-1}$ is given, find f(2x-2) in terms of f(x).

The teacher: When we look at this question, what are givens and what is asked? Please put your metacognitive question cards on your front. Ok, now what are givens and unknown?

Students: The function f is defined.

Students: It requires us to write f(2x-2) depending on f(x).

The teacher: yes, now do we know the function f(2x-2)?

Students: No...

The teacher: No, so we firstly need to find f(2x-2), then we can write it in terms of f(x). So, what will be our strategy? The strategy should begin with finding f(2x-2). Right? Well. After that what will we do?

Students: It will include f(x)... There must be something common...

The teacher: Well done. Is there any other idea?... Ok. Let's do it. How can we find f(2x-2)?

Students: Writing 2x-2 in place of x in f.

The teacher: Yes, nice. Write it and tell me...

f(2x-2)= 3^{2x-2-1}=3^{2x-3}. Now, how can we continue the solution? can you separate 3^{2x-3}? Do you remember exponential numbers? $3^{a+b} = 3^a.3^b$. You learned this last year. Now for this question, $3^{2x-3} = 3^{2x}.3^{-3}$ I will continue to separate 3^{2x}. We can write it as $(3^x)^2$. Why do we do this? Because there is 3^x in f(x) also. We try to find some common terms. If we separate f(x) also, we can write f(x)= $3^x.3^{-1}$ Let's leave 3^x alone in this equation and then we can write it in another one:

3^x=3. f(x) and f(2x-2)=$(3.f(x))^2.3^{-3}$.

Then we find f(2x-2)=9.f^2(x).$\frac{1}{27}$=$\frac{f^2(x)}{3}$

Is there any problem with solution?... Is it reasonable?

Students: yeah… no problem…

The teacher: Do you look at your cards? Is there anyone who can solve with another way?

Students: nope…

The teacher: Ok, what kind of a problem was it?

Students: …

The teacher: We wrote a function in terms of another function. Then is it similar with the previous ones?

Students: Noo…

The teacher: No, it isn't. Why? Because, we found the image of an element under a function in previous examples but now, we relate a function with another one.

Until fifth week, she just asked students' different strategies for reflection question, she had forgotten to ask where students were in difficulty in any question. From the beginning of this week, she tried to concentrate on parts in which they have difficulty and further discussion were made at those points. Moreover, she had listened and allowed students to explain their different solution strategy if their strategy is true up to this point, but then she started to discuss students' wrong solutions. They talked about why that solution is wrong.

Question: If f(x)=3x-4 and (fog)(x) = 6x – g(x), then find the value of g(2)?

The teacher: Try to solve on yourself firstly… what did you find?

Students: 10... -10… -4 for x then…

The teacher: Ok, let's do it together. Firstly what are givens and unknown of the problem?

Students: There is a function f and composition of f and g are defined. It asks for g(2).

The teacher: Well, how should we start?

Students: We can start by rewriting the composition function as f[g(x)]

The teacher: And what is it equal?

Students: 6x-g(x)… we can write t instead of g(x)…

The teacher: Do you write t in place of x? Ok, but here you find that f[g(x)]=6x – g(x), so is f(x) defined in the question? f[g(x)] means to write g(x) in place of x in the function f, right? You can put g(x) on the equation of f(x). Since f(x)=3x – 4, f[g(x)]=3g(x) – 4.

Students: I put x for g(x), couldn't I?... Me too… Yes…

The teacher: But, you don't know if g(x)=x.

Students: Isn't it the same thing?

The teacher: If you put x for g(x), you mean that g(x) is identity function, but there is no such a knowledge in the problem.

Students: We did the same in the previous question.

The teacher: But, in the previous one, f(x) was given only in terms of g(x), not x. Here, there is an x in addition to g(x). That is to say, the variable you want to use for g(x) is already here. You can't use the same variable. If there was no x there, you would be right. However, if there are both x and g(x) in the same equation, you can't say g(x) is equal to x. Ok?

The students: ahh… ok… yes…

The teacher: You can not use the same variable. Therefore, you should think another strategy. f(x) is defined in the problem, so we can write g(x) in place of x under the function f. Then, f[g(x)]=3g(x) – 4 = 6x – g(x). Now collect g(x) in the same side:

4g(x) = 6x + 4 then g(x) = (6x+4)/4.

So, g(2) = (6.2+4)/4=4.

Any question now?

Did you understand the solution? Is it reasonable?

Students: Yes, reasonable… ok…

The teacher: Could you tell me what is the difference of this question from the previous ones?

Students: We put a function instead of x for the first time… there are both a variable and a function in the image of composition function.

The teacher: Well done, you are right. So, is there any person who can use another strategy to solve the question?

Students: Nope…

In this example, students suggest a wrong strategy and the teacher discuss with them so that they can understand why it is wrong. After the clarification of wrong answer, the teacher continued to correct strategy. At the end of the solution, she also asked reasonability of the solution and connection of the question with other questions they have solved. Therefore, she exactly used the metacognitive questioning in a true way.

In the seventh week, comprehension, strategy and reflection questions were asked by the teacher; however, she skipped the connection questions. Since the researcher required her to ask that type of question also, she asked for a few problems but not for all. From the beginning of the seventh week, the teacher probably might be tired and distracted. In the eighth week, on one day, she did not use any of metacognitive questions and for the others, she asked comprehension and strategy questions for every problem but asked connection and reflection questions for some of them. In the ninth week, there were 4 lessons before the posttest and the teacher again asked comprehension and strategy questions regularly but connection and reflection questions rarely.

In the beginning of the study, the teacher was in difficulty to implement all the metacognitive questions to the given problem and so she was stressful. In time, she got used to the IMPROVE model and became more confident. Unfortunately, her implementations were deficient towards the end of the study since connection and reflection questions sometimes were skipped towards the end of the study. On the other hand, she changed her reactions to wrong answers of students. Although at the beginning, she emphasized only the reason behind the correct solution and why it was logical; in time, she started to discuss why the answer was wrong also with respect to feedbacks from the researcher. She tried to understand why students were in difficulty when they could not understand a specific point, then tried to lead students with metacognitive questions.

The aim of the cooperative settings was to provide students to work in heterogeneous groups and transfer their ideas with group discussions. Two heterogeneous group works were done along with this study. Each group included three or four students and groups were tried to be heterogeneous in terms of the academic level. Therefore, high, middle and low achieving students worked together. At the beginning of the semester, all students participated to the study were tested and they were assigned to these groups with respect to success level. Group members exchanged during the semester to maintain heterogeneity in the groups.

The researcher did not interfere to lessons but only in group discussions, the teacher and researcher joined to groups for 3-4 minutes to model metacognitive questions, and observed and promoted students to use them. Students could take help from teacher or researcher if team members could not agree on the problem. The researcher inference may decrease the external validity but it was needed to observe students to use metacognitive questioning. At the end of the lesson, the teacher emphasized the main ideas and presented additional explanations if it was needed.

Along with this study, the teacher gave two formative test including both similar with and different from the ones solved in the classroom (Appendix L). Students who gave 80% correct answers took enrichment activities about the learned unit so that they could go beyond whereas the others took corrective activities that are similar with the ones in the test as presented in the IMPROVE model (Mevarech and Kramarski, 1997). As different from the regular group works, students work in homogeneous groups during corrective or enrichment activities. After formative tests, two corrective and enrichment activities were done within this study (Appendix M).

The control group, on the other hand, learned with the traditional teaching method including question answering and individualized practices. Therefore, they learned the same topic with the same materials and examples; however they did not trained in metacognitive questioning or in a cooperative settings. The teacher was not interfered by the researcher as long as she used the same examples with experimental group. She generally began to lessons by reminding the previous lesson and taught new topic. She preferred to give mathematical definition of terms before real life examples in contrast to treatment group and also she and students did not discuss on the real life problem. The teacher just explained the example as that there is such a situation about this concept and that's was all. Then she solved some questions firstly and explained students how they should approach the problem. After she showed some example problems, she gave time to students for the followings so that they can work on the problems on their own individually. After students shared their answers with the teacher, she solved the problem on the board.

When she started to solve a problem, she examined the givens and unknown of the question but she did not emphasize to students to consider these factors for every question regularly. She also explained why she chose that solution for any question. However, she neither asked students whether they have different strategies or not and where they have difficulty for that question nor she asked the connection between questions. Although she sometimes used comprehension and strategy questions, this is not critical for the implementation of the IMPROVE method because Mevarech and Kramarski (1997) emphasized that even the control group teachers use some metacognitive questions, they do not this deliberately and systematically as in the experimental group and they do not call it as metacognitive questioning. Therefore, students are not aware of the advantages of metacognitive questioning and they are not leaded to use metacognitive questioning during their problem solving process. An example problem solving of control group teacher was given below.

> The teacher: In functions problems, I generally start from the unknown. What does (fog)(2) mean? It means to find f[g(x)]. Where is g(x)? Here, but which number do you look for? 2. Which number should you put in place of x to get g(2)?
>
> Students: 0.
>
> The teacher: 0, then let's write it. g(0+2)=2.0+1, so g(2)=1. Then the question now is what f(1) is. Now, f is a linear function and I showed you a short way before. F takes 2 and match with 4. If it takes 4, it matches with 8. Here the important thing is the difference between numbers. What is the difference between 2 and 4?
>
> Students: 2.
>
> The teacher: 2 and their images increases...?
>
> Students: 4.
>
> The teacher: So, how many does it increase for 1 difference?
>
> Students: 2.
>
> The teacher: What does it look for? f(1), so we should decrease it 1 and its image will decrease 2. Subtract 2 from 4 and it is found as 2. Any question?
>
> Students: Ok...

Further, control group students did not work in cooperative settings or have a chance to exchange their ideas. They also did not receive corrective or enrichment feedback after formative assessment. The control group teacher solved some questions from the correction or enrichment activities worksheet on the board as just part of her lesson.

3.7. Validity and Reliability

Quantitative reliability refers to consistency and stability of scores of participants over time according to Creswell and Clark's definition (2011). It can be assessed with alpha coefficient that is one of internal consistency methods (Fraenkel and Wallen, 2003). Therefore, a pilot study was conducted to examine the reliability of achievement test developed for this study. Reliability coefficients were calculated in pilot study and also in main study. Further, interrater reliability was examined and reliability of MSLQ questionnaire was calculated as presented in the instruments section.

Quantitative validity, on the other hand, means whether the scores obtained from the participants point out the construct to be measured (Creswell and Clark, 2011). In this study, experts' opinions were taken and necessary regulations were done to provide validity of the instruments as mentioned in the instruments section. The following paragraphs covers the internal and external validity issues related to this study.

External Validity refers to the generalizability of results of a study to other studies (Vaus, 2001). In this study, generalizability range is not so far since the study group is small; however, it can be generalized for the studies having similar constructs such as participants with similar characteristics or similar school climate and opportunities. Further extend of the external validity can be increased by replicating the study with different groups (Vaus, 2001).

Internal Validity, on the other hand, refers to that the difference on the dependent variable originates from the independent variable directly not from other external variables (Fraenkel and Wallen, 2003). This means whether the instruments used in the study really measure what it is wanted to measure. Therefore, the internal validity in this research refers to the degree to which instruction type made a difference on students' achievements on function topic and self-regulation skills. Threats to internal validity that are related to this study were subject characteristics, instrumentation, testing, history, statistical regression, implementation, diffusion of treatment, resentful demoralization and compensatory rivalry.

When participants of the study are not selected randomly, *subject characteristics* threat may occur because the difference between groups may come from the difference on participants' age, intelligence, attitude, fluency or reading ability etc (Fraenkel and Wallen, 2003). In this study, participants were selected with convenient sampling and quasi-experimental groups were used. However, the equalization of groups was examined with pretest analysis and it showed that the groups were equal on achievement test and MSLQ questionnaire at the beginning of the study. Therefore, the study was decided to be conducted with these groups.

Instrumentation threat refers in which way instruments are used. Data collector characteristics such as age, gender or experience may affect the results or data collector may have some unconscious bias to make certain outcomes (Fraenkel and Wallen, 2003). The researcher of this study has one year teaching experience and took lessons on conducting educational researches and data analysis in master education. Further, expert opinions were taken during data collection and analysis processes. Another mathematics teacher also was asked for scoring the achievement test again to get an agreement on scoring. Since MSLQ is a likert type test, its scoring was objective naturally. Moreover, the researcher did not interfere to students' answers during think aloud process on function problems.

Testing threat refers that participants may be alerted to the questions on pretest and they can remember the answers if the implementation period is too short (Fraenkel and Wallen, 2003). The period of implementation in this study was not too short for students to remember the answers. One of the reason to choose functions topic in this study was that it is long enough. There was two months period between pre and posttests.

Regression threat occurs when there are extreme scores on pre intervention measures. (Fraenkel and Wallen, 2003). Although the study groups were not selected randomly in this study, regression did not become a problem since there was no extreme scores on the tests according to data analysis results.

If some unanticipated events occur during the study and these events affect the responses of participants, then *history threat* takes place (Fraenkel and Wallen, 2003). Towards the end of the semester and this study, students had some school projects which might have caused distraction from the study. The intervention group teacher might also have been distracted towards the end because of her other responsibilities which was a threat to the implementation. The researcher provided necessary lesson plans and other materials for the teacher and continuously tried to support and motivate the teacher to prevent this treat. Additionally, students were often emphasized the importance of the study by teachers.

Implementation threat may occur in two ways. The first one come from the implementation of different methods by different individuals and the second occurs when individuals have some bias on methods (Fraenkel and Wallen, 2003). This threat was an important factor in this study since different teachers taught the intervention and the control group. The teaching experience of the control group teacher was 3 years more than the intervention group teacher. Further, control group teacher graduated from mathematics education department whereas intervention group teacher graduated from mathematics department and took pedagogical formation later. However, the school climate was close to traditional teaching strategies, and thanks to the unstructured meetings with teachers, it can be said that they were not so different. Further, despite their different characteristics, they are tried to be equalized by using the same lesson plans and materials prepared by the researcher. They were observed and often were given feedbacks by the researcher.

Diffusion of treatment refers to the transferring of knowledge about the implementation from intervention group students to control group (Creswell and Piano Clark, 2011). This threat was not serious for this study since metacognitive changes are difficult to observe and also the key point of the treatment was to imply it on participants as deliberately and systematically as emphasized by developers of the instruction method (Mevarech and Kramarski, 1997). Therefore, even if control group students took metacognitive question prompt card from their friends in intervention group, they would not see the systematic implementation of these questions and they would not attempt to use them effectively.

Resentful demoralization and compensatory rivalry occurs when control group students know that the other group take some favored implementation and expected that they would be higher achievers, so they may be resentfully demoralized towards the measurements (Creswell and Piano Clark, 2011). This threat was handled by giving an impression to students such that both groups receives a treatment since both them experienced some activities different from their usual lessons like examples including modeling real life situations or they had same quizzes and extra problems.

Treatment fidelity may come from that the implementer of the treatment could not exactly follow the procedures organized by the researcher (Mertens, 2005). There were some difficulties in the implementation of this study. The treatment in this study was not implemented adequately due to the history of the intervention teacher as mentioned in the treatment part. The researcher tried to prevent this treat by helping some other works of the teacher so that she could be focus on the intervention effectively. Further, the teacher was allowed to organize extra study hours with students after school so that the control and the experimental group teachers could proceed at an equal rate. However, the teacher did not use metacognitive questioning in this extra time.

Strength of the experimental treatment concerns about whether the duration of the treatment is enough to get effective results or not (Mertens, 2005). The treatment in this study implemented along function unit and so had continued for 2 months. Since metacognitive changes can be observed in long time periods (Schunk and Zimmerman, 1998), duration of the treatment in this study might not be adequate to observe changes.

3.8. Data Analysis

3.8.1. Pilot Study

Achievement test used in this study was developed by the researcher and in order to investigate the validity and reliability of the test, a pilot study was conducted. It implemented in another classroom in the same school so that students would have similar background with the sample of the main study. Seventeen 10^{th} grade students participated to the pilot study and further three 11^{th} grade students were asked whether the questions are clear enough since the test includes function questions which 9^{th} graders have not seen yet. Therefore, suggestions of students were considered during the adjustment of the test before the main study.

The instrument was examined by two academicians at Boğaziçi University and two mathematics teacher in the college. Two academicians have PhD degree in mathematics education and two teacher have master degree in mathematics and mathematics education. The test was included 20 open ended questions and the items were distributed to objectives according to the number of subtopics of each objective. The experts presented their ideas about both the number of questions and whether the questions were compatible with the objectives in the curriculum of Ministry of Turkish Education. After review and recommendations of experts, the test was revised and reconstructed. Then final approval of experts was taken. Further, Cronbach Alfa is used to test reliability of the instrument and it was found as .677 which shows the instrument is reliable enough (Schmitt, 1996; Griethuijsen *et al.*, 2014). The validity and reliability concerns were met by the researcher.

3.8.2. Quantitative Data Analysis

3.8.2.1 Mathematics Achievement. Achievement test on functions was given to students to search whether there is an effect of metacognitive strategy based instruction on students' achievement. Students' scores in pre and posttest were analyzed to compare the difference between control and intervention group.

In order to decide which inferential statistics is appropriate to use, assumptions of t-test were examined. Shapiro-Wilk test of normality was used to observe the distributions of scores on tests. Both pre and post test scores were tested for normality of data for both control and intervention groups and all they had normal distribution: control group pretest (W= .98, p>.05), intervention group pretest (W=.95, p>.05), control group posttest (W=.92, p>.05), intervention group posttest (W=.92, p>.05). Further skewness is another factor affecting normality; however, skewness values were found between -1.0 and +1.0 for all measures and so it can be said that scores on the test had normal distribution (Morgan *et al.*, 2013).

Further, second assumption t test is homogeneity of variances. It was examined with Levene's test for homogeneity of variances and the variances were found to be equal for control and intervention group pretest (F(1, 34)= .68, p=.417) and posttest (F(1, 34)=.48, p=.50)

Since the assumption of normality and homogeneity of variances were not violated, independent samples t test was conducted to define whether there is a statistically significant difference between the scores of two groups. Before conducting t test, reliability test was also run for both pre and posttest.

3.8.2.2 Self-Regulation Skills. In order to investigate the effect of the metacognitive strategy based instruction on students' self-regulatory skills, Motivated Strategy and Learning Questionnaire (MSLQ) was given to students. Its four components were used in this study related to self-regulated learning: self-regulation, metacognitive strategy use, self-efficacy and intrinsic value. All components analyzed separately with t test or Mann Whitney depending on the assumptions of t test. MANOVA could not be used to compare differences among these variables because the assumption of linearity between dependent variables could not be hold.

Shapiro Wilk test of normality was run and skewness was examined to define whether students' scores on the questionnaire had normal distribution for subscales of self-regulation, metacognitive strategy use, self-efficacy, intrinsic value and test anxiety. Further, Levene's test for homogeneity of variances was also used for each of these subscales.

Assumptions of normality and homogeneity of variances for t-test were examined for self-regulation after 3 items were deleted to increase reliability coefficient. Both control and intervention group scores on self-regulation had normal distribution: control group pretest (W=.89, p>.05), intervention group pretest (W=.95, p>.05), control group posttest (W=.95, p>.05), intervention group posttest (W=.93, p>.05). Further, second assumption t test, homogeneity of variances, was examined with Levene's test for homogeneity of variances and the variances were found to be equal for control and intervention group pretest (F(1, 34)= 1.57, p=.219) and posttest (F(1, 34)=.53, p=.472). Since the assumptions of t test was provided and there were two independent groups, independent samples t-test was run.

Assumptions of t-test were not provided for metacognitive strategy use subscale. Therefore, Mann Whitney U test was used to investigate the difference between control and intervention group.

Both control and intervention group scores on self-efficacy had normal distribution according to Shapiro Wilk test of normality: control group pretest (W=.92, p>.05), intervention group pretest (W=.96, p>.05), control group posttest (W=.89, p>.05), intervention group posttest (*W*=.97, *p*>.05). Further, second assumption t test, homogeneity of variances, was examined with Levene's test for homogeneity of variances and the variances were found to be equal for control and intervention group pretest ($F(1, 34)= 0.302$, $p=.587$) and posttest ($F(1, 34)=0.025$, $p=.875$) for self-efficacy. Therefore, assumptions of normality and homogeneity of variances for t-test were not violated for self-efficacy.

On the other hand, students' scores on intrinsic value had also normal distribution according to Shapiro Wilk test of normality: control group pretest (*W*=.96, *p*>.05), intervention group pretest (*W*=.97, *p*>.05), control group posttest (*W*=.89, *p*>.05), intervention group posttest (*W*=.96, *p*>.05). Further, Levene's test for homogeneity of variances was conducted for intrinsic value and variances were found to be equal for control and intervention group pretest ($F(1, 34)= 0.083$, $p=.775$) and posttest ($F(1, 34)=1.416$, $p=.242$). Therefore, assumptions of normality and homogeneity of variances for t-test were provided for intrinsic value.

As a result, independent sample t-test was used to analyze self-regulation, self-efficacy and intrinsic value whereas Mann Whitney U test was used to analyze metacognitive strategy use. The scores on each subscale of control and intervention group was compared in pre and posttest, therefore the difference between two groups was examined to define whether it statistically significant or not.

3.8.3. Qualitative Data Analysis

This study was implemented throughout functions unit and how students' question posing practices change during two-month period was measured by think aloud sessions with voluntary students from both experimental and control groups. These sessions were audio-recorded and transcripts of records were done by researcher. Content analysis was conducted to interpret the data. In order to examine students' questions, predetermined categories were used and these were the types of the metacognitive questions used in IMPROVE method: comprehension, strategy, connection and reflection questions. Therefore, researcher classified students' questions according to these four types of questions after the data was read and reviewed many times.

4. RESULTS

4.1. Students' Achievement on Functions

The first research question was whether mathematics achievement of 10th grade students who learned with metacognitive strategy instruction was statistically different from those who learned with traditional methods or not. Therefore, achievement test on functions was completed by students as pre and posttest.

Descriptive statistics about students' scores on achievement test showed that the control group had higher mean score (*M*=19.69; *SD*= 6.30) on pretest than intervention group (*M*=18.10; *SD*=5.25) whereas they had lower mean score on posttest (*M*=42.56; *SD*=13.85) than the intervention group. (*M*=48.75; *SD*=15.47) as can be seen from Table 4.1. This means that intervention group had showed a greater development than control group. Therefore, it can be concluded that intervention group students gained a higher development thanks to the instruction method.

Table 4.1. Descriptive statistics on achievement scores

Descriptive Statistics								
groups		N	Range	Min	Max	Mean	Std. Deviation	Variance
intervention group	Ach_pre	20	19	9	28	18.10	5.251	27.568
	Ach_post	20	54	15	69	48.75	15.468	239.250
	Valid N (listwise)	20						
control group	Ach_pre	16	23	8	31	19.69	6.300	39.696
	Ach_post	16	48	22	70	42.56	13.851	191.863
	Valid N (listwise)	16						

On the other hand, it was needed to examine the magnitude of the effect of the instruction and so effect size was examined. Cohen's d value was calculated as .42 for posttest results. It shows that the difference between control and intervention group has medium effect size. Therefore, it can be concluded that the intervention group gained higher development.

In order to examine the statistical significance, inferential statistics was also conducted. Since the assumptions of t test was provided and there were two independent groups, independent samples t-test was run. The results of t-test to examine whether there is a significant difference between the intervention group and control group pretest scores are given in Table 4.2. As can be seen from the table, t-test indicated that there is not a statistically significant difference between the control group pretest scores (*M*=19.69, *SD*= 6.30) and intervention group pretest scores (*M*=18.10, *SD*=5.25) conditions; $t(34)=.83$, $p=0.42$. Therefore, two groups were approximately equal for knowledge on functions at the beginning of the study.

Table 4.2. T-test results for achievement on functions test

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Ach_pre	Equal variances assumed	.676	.417	-.825	34	.415	-1.587	1.924	-5.498	2.323
	Equal variances not assumed			-.808	29.186	.426	-1.587	1.965	-5.604	2.429
Ach_post	Equal variances assumed	.475	.495	1.248	34	.220	6.188	4.956	-3.885	16.260
	Equal variances not assumed			1.264	33.520	.215	6.188	4.894	-3.764	16.139

At the end of the study, posttest was conducted to measure whether a significant difference occurred between two groups or not. Posttest results were analyzed with independent samples t-test again since the assumptions were not violated. T-test analyses showed that there is not a statistically significant difference between the control group posttest scores (*M*=42.56, *SD*=13.86) and the intervention group posttest scores (*M*=48.75, *SD*=15.47) conditions; $t(34)=1.25$, $p=0.22$. The results of t-test were given in the table 4.2.

Although, inferential analysis did not show a statistically significant difference between two groups, practical significance is more important in studies with small sample size (Fraenkel and Wallen, 2003) and so it can be concluded that intervention provided students a development on their achievement in practical terms depending on effect size.

4.2. Students' Self-Regulation Skills

The second research question was whether self-regulation level of 10th grade students who learned with metacognitive strategy instruction statistically different from those who learned with traditional methods or not. In order to seek this difference, students were given MSLQ questionnaire as pre and posttest. Four subscale of MSLQ were used in line with the aim of this study. These were self-regulation, metacognitive strategy use, self-efficacy and intrinsic value. Each subscale was analyzed separately.

4.2.1. Self-Regulation

Descriptive statistics about students' scores on self-regulation showed that the control group decreased their mean score from pretest (*M*=28.38; *SD*= 7.48) to posttest (*M*=27.06, *SD*=8.47) whereas intervention group increased their mean score from pretest (*M*=29.05; *SD*=6.01) to posttest (*M*=29.60; *SD*=6.97) as can be seen from Table 4.3. On the other hand, since it was needed to examine the magnitude of the effect of the instruction, effect size was examined. Cohen's d value was calculated as .33 for posttest results. It shows that the effect of the intervention is at medium size effect. This means that intervention group students gained a development in practical terms thanks to the instruction method.

Table 4.3. Descriptive statistics on self-regulation scores

Descriptive Statistics						
groups		N	Min	Max	Mean	Std. Deviation
intervention group	SRegulation_pre_6item	20	14	39	29.05	6.013
	SRegulation_post_6item	20	10	40	29.60	6.969
	Valid N (listwise)	20				
control group	SRegulation_pre_6item	16	12	37	28.38	7.482
	SRegulation_post_6item	16	6	39	27.06	8.465
	Valid N (listwise)	16				

In order to examine the statistical significance, t-test was also conducted since the assumptions of t test was provided. The results of t-test to examine whether there is a significant difference between the intervention group and control group pretest scores are given in Table 4.4. As can be seen from the table, there is not a statistically significant difference between the control group pretest scores (M=28.38, SD= 7.48) and intervention group pretest scores (M=29.05, SD=6.01) conditions; $t(34)$=.30, p=0.77. Therefore, two groups were approximately equal for self-regulation at the beginning of the study.

At the end of the study, posttest results were analyzed with independent samples t test again since the assumptions were not violated. T test analyses showed that there is not a statistically significant difference between the control group posttest scores (M=27.06, SD=8.47) and the intervention group posttest scores (M=29.60, SD=6.97) conditions; $t(34)$=.99, p=0.33. The results of t test were given in the table 4.4.

Although, inferential analysis did not show a statistically significant difference between two groups, intervention provided students a development on their self-regulation in practical terms depending on medium effect size.

Table 4.4. T-test results for the subscale of self-regulation

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
SRegulation_pre_6item	Equal variances assumed	1.567	.219	.300	34	.766	.675	2.248	-3.893	5.243
	Equal variances not assumed			.293	28.498	.772	.675	2.304	-4.040	5.390
SRegulation_post_6item	Equal variances assumed	.528	.472	.987	34	.331	2.538	2.571	-2.687	7.762
	Equal variances not assumed			.965	28.955	.342	2.538	2.628	-2.838	7.913

4.2.2. Metacognitive Strategy Use

According to the descriptive statistics about students' scores on metacognitive strategy use, the control group decreased their mean score from pretest (*M*=64.37; *SD*=13.96) to posttest (*M*=63.50, *SD*=18.54) whereas intervention group increased their mean score from pretest (*M*=67.50; *SD*=11.15) to posttest (*M*=68.35; *SD*=13.60) as can be seen from Table 4.5. On the other hand, effect size was also calculated to examine the magnitude of the effect of the instruction. Cohen's d value was found as .30 for posttest results, which is at medium size effect. Therefore, it can be concluded that intervention group students gained a development in practical terms thanks to the instruction method.

Table 4.5. Descriptive statistic on metacognitive strategy use

Descriptive Statistics						
groups		N	Min	Max	Mean	Std. Deviation
intervention group	StrategyUse_pre	20	44	83	67.50	11.152
	StrategyUse_post	20	33	86	68.35	13.601
	Valid N (listwise)	20				
control group	StrategyUse_pre	16	37	81	64.37	13.956
	StrategyUse_post	16	13	82	63.50	18.536
	Valid N (listwise)	16				

Mann Whitney U test also was run for statistical significance. The results showed that there is not a statistically significant difference on metacognitive strategy use between control group pretest score (*M*=64.37; *SD*=13.96) and intervention group pretest score (*M*=67.50; *SD*=11.15), *U*=145, *p*=.63. Posttest results also showed there is not a statistical significance between control group (*M*=63.50, *SD*=18.54) and intervention group (*M*=68.35; *SD*=13.60), *U*=142, *p*=.58. Nevertheless the difference had practical significance (Table 4.6)

Table 4.6. Mann Whitney Test for metacognitive strategy use

Test Statistics[a]		
	StrategyUse_pre	StrategyUse_post
Mann-Whitney U	145.000	142.500
Wilcoxon W	281.000	278.500
Z	-.479	-.558
Asymp. Sig. (2-tailed)	.632	.577
Exact Sig. [2*(1-tailed Sig.)]	.648[b]	.582[b]

a. Grouping Variable: groups

b. Not corrected for ties.

4.2.3. Self-efficacy

According to the descriptive statistics about students' scores on self-efficacy, both group experienced a decrease on their scores (Table 4.7). The mean score of control group students was 44.00 in pretest (*SD*=9.06) and 42.38 in posttest (*SD*=11.84). The mean score of intervention group students, on the other hand, was 42.40 in pretest (*SD*=10.64) and 39.85 in posttest (*SD*=10.48). Further, control group students had higher score in both pre and posttest than intervention group students.

Table 4.7. Descriptive statistics on self-efficacy

Descriptive Statistics						
groups		N	Min	Max	Mean	Std. Deviation
intervention group	Selfefficacy_pre	20	17	59	42.40	10.640
	Selfefficacy_post	20	18	60	39.85	10.484
	Valid N (listwise)	20				
control group	Selfefficacy_pre	16	28	59	44.00	9.063
	Selfefficacy_post	16	9	59	42.38	11.837
	Valid N (listwise)	16				

Table 4.8. T-test results for self-efficacy

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
Selfefficacy_pre	Equal variances assumed	.302	.587	-.478	34	.636	-1.600	3.346	-8.399	5.199
	Equal variances not assumed			-.487	33.837	.629	-1.600	3.285	-8.278	5.078
Selfefficacy_post	Equal variances assumed	.025	.875	-.678	34	.502	-2.525	3.724	-10.092	5.042
	Equal variances not assumed			-.669	30.311	.509	-2.525	3.775	-10.232	5.182

Independent samples t test was run to examine the significance of the difference on self-efficacy scores of two groups and according to the results of t-test, there is not a statistically significant difference on students' self-efficacy for both pretest conditions; $t(34)=.48$, $p=0.64$ and posttest conditions; $t(34)=.68$, $p=0.50$ (Table 4.8)

4.2.4. Intrinsic Value

Descriptive statistics about students' scores on intrinsic value showed that both groups experienced a decrease on their scores (Table 4.9). The mean score of control group students was 45.94 in pretest (*SD*=9.15) and 44.63 in posttest (*SD*=12.41). The mean score of intervention group students, on the other hand, was 45.90 in pretest (*SD*=7.72) and 44.40 in posttest (*SD*=7.93). Further, control group students had higher score in both pre and posttest than intervention group students.

Table 4.9. Descriptive statistics for intrinsic value

Descriptive Statistics						
groups		N	Min	Max	Mean	Std. Deviation
intervention group	IntrinsicValue_pre	20	32	60	45.90	7.718
	IntrinsicValue_post	20	33	60	44.40	7.930
	Valid N (listwise)	20				
control group	IntrinsicValue_pre	16	28	61	45.94	9.154
	IntrinsicValue_post	16	12	59	44.63	12.414
	Valid N (listwise)	16				

Independent samples t test also was run to examine the significance of the difference on intrinsic value scores of two groups and the results of t-test showed that there is not a statistically significant difference on students' intrinsic value for both pretest conditions; $t(34)=.01$, $p=0.99$ and posttest conditions; $t(34)=.07$, $p=0.95$ (Table 4.10)

Table 4.10. T-test results for intrinsic value

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
Intrinsic Value_pre	Equal variances assumed	.083	.775	-.013	34	.989	-.038	2.811	-5.751	5.676
	Equal variances not assumed			-.013	29.404	.990	-.038	2.866	-5.896	5.821
Intrinsic Value_post	Equal variances assumed	1.416	.242	-.066	34	.948	-.225	3.406	-7.147	6.697
	Equal variances not assumed			-.063	24.343	.950	-.225	3.574	-7.597	7.147

4.3. Students' Questioning Skills

Qualitative data was collected for the third research question of this study. Therefore, the change on students' questioning skills was examined via individual interviews with voluntary students. Four students from intervention group and 3 students from control group were interviewed at the beginning, middle and end of the study. During the interviews, students were asked to solve function problems thinking aloud in orders to observe their thinking process and also intervention students were asked about ideas about the instruction.

Students' solution process were examined according to the four type of metacognitive questions. First type is comprehension questions, which includes to search givens, unknown or concepts and also type of the problem. Strategy questions are "What is the appropriate strategy for this problem? Why and how I can imply it? I can do… since…"

Connection questions are to search whether there is a difference or similarity between the current problem and previous ones. Lastly, reflection questions includes "Is it reasonable?", "how can I explain my solution?", "how can I find another solution?" or "Where could I make a mistake?

Intervention group students were coded as PE# and control group students were coded as PC#. In the following sub-sections, development of questioning skills for each student is examined by giving excerpts from each think-aloud problem solving session. Phrases in the excerpts indicating four types of questions are given in italic type face.

4.3.1. Development in Questioning Skills of PE1

4.3.1.1 Comprehension Questions. First student from intervention group, PE1, asked comprehension questions during all interviews. However, she focused on just givens in the first interview before applying strategy whereas she paid attention to the unknown also in following interviews.

An excerpt from the first interview below shows she looked at givens of the problem since she used phrases like "it said…" or "It is given…" and so it can be said that she used comprehension questions:

> PE1: Now, I am thinking in that way. *It said me* that a sapling is growing 15 cm every year and time *was given* as month. I think one year is 12 months and can I do something from this. But… uh… 15 cm every year… It grows 15 cm every year… I am confused.

Then started to solve the problem without searching a relation with unknown. Before she paid attention to what the problem asks, she could not answer her strategy questions. After a long time, she looked the months asked and recognized that it was a multiplier of 12:

> PE1: Relation between the height of sapling and time passed… Its first height is 60 cm and *asks* what it will be after 4 months… for 12 months… 4 month, ah, I can do it. If it grows 15 cm in 12 months, how much does it grow in 4 months? I am calculating the proportion…

Another excerpt from first interview indicates that she carefully examine the givens of the problem:

PE1: I say that f(4)=3 was *given* and let me use it by putting 4 in place of x. I can find something from that. Mm... So, m. x is equals to 4m. Ah, *it says that part is equal to 3* also, isn't it? I am confused a little. I am ashamed.

Researcher: We already have started to unit, it is normal. Please do not be shy and continue.

PE1: Iı... *It gives* me such an information f(4)=3. If it says this, I think I should use it in some way. So, I think to put 4 for x...

On the other hand, she considered the unknown additionally before she was applied her strategy in second interview. She also defined the mathematical concept. Example excerpts for two questions were given below:

PE1: f goes from R to R. *It is defined* and a constant function. *It asks* me the value of k. I think that if it is *a constant function*, it should give the same output whatever I put in place of x.

Another example part from second interview:

PE1: *Solution set*... It gave me that f goes from R to R and it was defined. Let me product these firstly. x2-3x-2x+6 I guess. If I regulate it, it becomes x2-5x+6. Ok, it asks me the solution set of the equation f(x+1)=0...

In the third interview, she used comprehension questions by examining both givens and unknown of the problem again:

PE1: $f(3^x+1)=9^x+1$ and $f^{-1}(a)=4$, then what is a? Firstly, it *asks* a and *gives* the inverse of f for a is 4. I want to change it as f(4). So, f(4)=a.

Another excerpt from third interview for comprehension questions:

PE1: If they are *given, what is* $g^{-1}(3)$? Firstly, I start by rewriting this. $f(g^{-1}(x+1))=3x-2$. Mm... I think that... Should I calculate the composition function? Otherwise, I can try to make this 3 since it *asks* $g^{-1}(3)$.

4.3.1.2 Strategy Questions: After PE1 examined the comprehension questions, she thought about the solution and asked herself strategy questions like "What can I do now?" or "How can I use it?"

An excerpt from the first interview shows she used strategy questions:

> PE1: I think that it grows 15 cm every year and it was given as month. 1 year is 12 months but then I think it is nonsensical because 15 and 12 are irrelevant numbers. *What can I do now?...* Its height is 60 cm at the beginning. 15 cm every year... *What can I conclude here?* I guess I won't able to solve it.

In another excerpt from the first interview, she tried to develop her strategy depending on the given of the problem:

> PE1: Now, I put 4 for x, then 6 divided by 3 is 2. But... 4, I could not find 4 here. I need 4. *How can I get it,* I cannot understand. If I give 4 for x, then here $\frac{x+2}{x-1}$ should be equal to 4. I tried to continue such that but I could not find something I want. Therefore, *I should give such a value for x so that f take 4 as input.* However, I cannot find that value...

In second interview, she again asked strategy questions:

> PE1: I know but could not remember now, *how can I find k.* Let me put 1 for x. Maybe I remember then... $f(1)=(k-2).1^{5-k}$. Mm... can't remember... *How can I do*? Ah, yes, I need to wipe off the term including x. Ok, I am trying again. Since it is a constant function, the output should be constant. So, it should not include variable. Then I try to make variable 0...

Another example part for strategy question as:

> PE1: Ok, it asks me the solution set of the equation f(x+1)=0.... *So, what*? Mm, $f(x)= x^2-5x+6$... then I can put x+1 instead of x in the function. I can try...

In the third interview, she did not ask strategy questions clearly but it can be said that she used this type of question since she develop and explains her strategy:

> PE1: Then *what is it*? I can *try to make 4 the inside of given function* f. That is, $3^x+1=4$. $3^x=3$. So, x is 1. Then I put 1 in the function and $f(4)= 9^1+1$. So, a is equal to 10.

Another excerpt from third interview:

> PE1: If they are given, what is $g^{-1}(3)$? Firstly, *I start by* rewriting this. $f(g^{-1}(x+1))=3x-2$. Mm... I think that... *Should I calculate the composition function*? Otherwise, I can *try to make this 3* since it asks $g^{-1}(3)$. Then it equals to f(x)+f(2) but it is nonsensical. It is meaningless. Iı, $f(g^{-1}(x+1))$ equals, I will try to make 3 here. I say 2 for x, then , $f(g^{-1}(3))=6-2$. Is it 4? Yeah... I don't know. I guess it is nonsensical. Now, I think that I found f(x+2)=x-3. If I try to

make x-3 equal to 4, x will be 7. If I put there also, it becomes 9. That is, $g^{-1}(3)$ can be equal to 9. I did so…

4.3.1.3 Connection Questions. PE1 used connection questions once in each of first and second interview by stating the similarity or difference of problems but she did not clearly express these type of question in third interview. In the first interview, she connected the problem with a real life example showed in the lesson by the teacher as part of the lesson plan of this study. She remembered mathematical definition of functions depending on real life adaptation of functions:

> PE1: Firstly, it is not a function. I know this. There is "6" in the domain of the function but *it is similar with* the example of our teacher. One person cannot get on two bus at the same time, so we cannot match an element from the domain of the function to two elements of the range of the function. This is why it is not a function.

In the second interview, PE1 did a more complicated connection. It was not about to convert formal mathematics to a concrete one. She connected two abstract mathematics problem:

> PE1: Largest domain of a function… I don't know… *We found largest domain of functions at the beginning of the unit.* Can we *adapt* it to composition function here? But how? I could not remember exactly now but… Let me try to open it…There is nothing to put for x. I don't know… Maybe I can think in that way. Its denominator cannot be 0. So, I can exclude it from some things, or… …when we wanted to find largest domain, we excluded 0 from real numbers since the denominator cannot be 0. I consider this for largest domain of a function but we have not seen largest domain in composition function before, so I am a little confused…

4.3.1.4 Reflection Questions. If we examine reflection questions of PE1, she asked whether her solution is reasonable or not in all three interviews. Examples from three interview were given below respectively:

> PE1: It gave me that f(4)=3. I think to do this part such that I can put 4 in place of x and find some values. Then I can try f(0)… I must put 4 firstly for x. Then I will try to continue with respect to the result. I am not sure but I will try…
>
> Researcher: Can you think aloud?
>
> PE1: Ok, I will. *Do I bullshit*. It was not like that.

Researcher: Why do you think so?

PE1: I do because I found f(2) by doing that. *It is unrelated thing*. It must not be like that. Let me try again... *I cannot think something reasonable...*

In second interview:

PE1: It is a linear function in defined intervals. When I see a linear function, I write f(x)=ax+b... ...Then I can put 2 for x in linear function formula. So, it become 2a+b which *does not make sense* I think. Let me try another one... For example, it can be -2 also. Then I found the same thing... f(2)=2a+b... It goes to *nonsense*, I cannot understand.

Same passage above from the third interview:

PE1:...$f(g^{-1}(x+1))$ equals, I will try to make 3 here. I say 2 for x, then, $f(g^{-1}(3))$=6-2. Is it 4? Yeah... I don't know. I guess it won't *make sense...*

Further, she examined only whether the solution make sense or not in first interview whereas she added other sub questions in second and third interview. One of them is that she looked for multiple thinking ways to solve problems in the others. An excerpt from second interview shows that she thought about different ways to get correct solution in addition to reasonability. By conjunction "Otherwise", she started to explain another way to continue the solution:

PE1: ... Ok, f(0). If x is equal to 0, f sends it to 4. That is f(4) but there is no 4 in the table when I look it. I didn't understand. I am confused of the table. What if I think the reverse. x... When it is g(3)... No, it is not like that... ...there is 3 under the column of g(x), so I think here as x, that is g(x). *When it is 3, is g(x) equal to 0? Otherwise, should I look here for g(3)?...* but it does not make sense. I think the first way was more reasonable...

Another answer in third interview shows her different thinking ways before the solution:

PE1: If they are given, what is $g^{-1}(3)$? Firstly, I start by rewriting this. $f(g^{-1}(x+1))=3x-2$. Mm... I think that... *Should I calculate the composition function? Otherwise, I can try to make this 3 since it asks $g^{-1}(3)$.* Then it equals to f(x)+f(2) but it is nonsensical.

Furthermore, she focused on calculation errors in first interview after she applied her strategy whereas she searched the logical fallacies in the second interview. In the third interview, she looked for new ways in addition to recognizing her logic error. Her reactions to mistake in the first interview is as below:

> PE1: It doesn't make sense. I couldn't do this. Let me look again. 15 cm for 12 months, then how many months for 70 cm height for sapling? *I think I did incorrect multiplication.* Let me try but I guess it won't be solved. Again I did proportion but the number is so strange...

In second interview, she focused on the logic of her strategy rather than operations:

> PE1: What can I do now? I did a mistake possibly but... mm... I am trying to leave x alone. So, I have $4x=x^2+8$. I am dividing both sides by 4. Mm, *but I can't divide them when there is + sign here....*

In the third interview, from the previous passage, she tried to find another way after she recognized her first solution did not make sense:

> PE1: I think it is nonsense. Mm, I think *another way*. I found that $f(x+2)=x-3$. So, if I try to make x-3 take value 4, x will be 7. Then...

4.3.1.5 Semi-structured interview. Lastly, she identified her ideas about the instruction at the end of second and third interview. In both them, she stated that she queried problems more than before with metacognitive questions. She was tended to memorize problems before but now she changed the way she think in her opinion. Her thinking and interpretation skills were developed and she felt that she had to produce something on her own. An excerpt from the second interview is below:

> PE1: Now, we examine everything we do. With metacognitive questions or before, that is I think I am still like that but before for example this kind of question should be solved in that way, I am a little rote-learner. But now, I started to query more than before. Why I did such a thing or how I can do it in a different way, etc. That is, I try to make interpretations on the problem instead of just applying what is given to me.

When I asked her disadvantages of the instruction, she stated that there was not any disadvantage of the instruction method; however she was not sure about what if she starts to query too much and be confused. An excerpt from second interview again:

> PE1: We will enter an exam for university and we can encounter problems that we have not seen before. So, I need to interpret and produce something on my own. It is actually like not getting used to anything already prepared. This is an advantage for me. I cannot see any disadvantage. I think, yes really there is not. Maybe… I may make me confused while querying something but it is not so bad I think.

On the other hand, she said that she did not use metacognitive questions outside the class if she can solve the problem. She only used when she was in difficulty in solution since she thought that using question card is time-consuming. It can be seen from an excerpt from third interview below:

> PE1: I am not using question cards except from lessons. But, I think them if I see a difficult problem like that how I can find more appropriate strategy.
>
> Researcher: Why don't you use these questions outside the class?
>
> PE1: I don't know, it is difficult to ask them. I think If I have already solved the problem then why should I use them? I don't want to spent my time too much if I can. I query only when I cannot…

She also identified that it was difficult for her to find the relation between givens and unknown of the problem. When I asked her how she can deal with this problem, she said that she need to get it as habit to ask questions and she should not restrict herself as she cannot do. An excerpt later on third interview is below:

> PE1: I am in difficulty to find the relation between given and unknown. I may fail to notice why it gave me such a thing.
>
> Researcher: And what do you think to overcome this problem? Can you overcome?
>
> PE1: Yes, I can, by solving more problems. I should make it an habit. I should be open to different kind of problems. When I see a different kind of problem, I shouldn't say that I cannot do this. So, we should query now.

As a result, PE1 developed herself in terms of metacognitive questioning. She started to focus on unknown in addition to givens and she made higher level connections between concepts or problems in time. Further, her reflections to mistakes changed in a way that she considered on operational errors at the beginning and then she started to look for logical errors and find other ways for solution. Moreover, she emphasized that she benefited from the instruction since she was able to query more than before instead of memorizing problem types.

4.3.2. Development in Questioning Skills of PE2

4.3.2.1 Comprehension Questions. Second participant PE2 considers comprehension questions in all three interview. He examines the givens and unknown of the problem. An excerpt from first interview for comprehension questions was given below:

> PE2: *It said* that the height of the sapling at the beginning was 60 cm. *I also know* that it grows 15 cm every year. *It ask me* to find its height after 4 months, doesn't it?

In second interview, he said the givens but did not clearly stated the unknown as "It asks..." He also defined a mathematical concept. Related excerpt is below:

> PE2: *It is given* that $(k-2).x^{5-k}$ is a constant function. *In constant function*, I will leave only the terms including x. Ah, nope, I only want numbers and will try to get rid of x terms.

In third interview, PE2 focused unknown more than first two interviews:

> PE2: $f(3^x+1)=9^x+1$ and *it asks* $f^{-1}(a)=4$...since *it asks* the inverse of f, that is a, I try to make here a and I will take the inverse...9^x+1, ıı, *it asks* inverse but... hm...

4.3.2.2 Strategy Questions. After he asked comprehension questions, he tried to explain his strategy with reasons. An example passage from the first interview:

> PE2: I can think if 15 cm in 12 months then how many cm in 4 months? *I can use proportion.* 60 over 12, so it grows 5 cm in 4 months...

He again explained his strategy by focusing on concept definition in second interview:

> PE2: So, I try to *make the coefficient of x as 0*. Then k equals 2. Further, it becomes 1 *if the power of x is 0*. I try to make power 0 also. Then k equals 5. I found 2 and 5.

From another excerpt from third interview, it can be seen that he developed and explained his strategy:

PE2: I took the inverse of 3^x+1. While doing it, I equalized the expression in the function to y. here x terms… +1 goes as -1. I wrote x in place of y. So I equalized x-1 to 3^x. It equals to 4 and asks a. Mmm…

4.3.2.3 Connection Questions. PE2 did not make any connection in any interview.

4.3.2.4 Reflection Questions. He also did not ask any reflection question in any interview. He did not ask himself whether his solution is reasonable or not. When he did not find a solution, he did not seek other ways.

Nevertheless, his ability of expressing thoughts and ideas was improved. He were more confident in think aloud process in last interview. I asked too many times "What do you think now" in first interview and he gradually became better in explaining his ideas. An example from first interview:

PE2: … It grew 10 cm… We know it grows 15 cm in one year…

Researcher: What do you think?

PE2: Iı, I will make a proportion between 10 and 15. Iı, time and months. I want to change that into 3 things, 12 months. Since it says 15 cm in one year, here is 75…

Researcher: What are you thinking now?

PE2: To find months… I can draw graphs but now that… Height of the sapling is increasing in time. Iı, we can show it on graph… It grew 10 cm here. There are 12 months in a year. If it grows 15 cm in one year, I said 15/12. To find the amount of change in height for each month. 5/4…

Researcher: What do you think?

PE2: Hm, I am confused…

Researcher: You can say why you are confused?

PE2: Iı, It says 70 cm there. It grew 10 cm in a year. Iı, it grew 10 cm in a period given to us… That is 85…

4.3.2.5 Semi-structured interview. At the end of second interview, he said that the instruction method provided him comprehensive thinking and interpretation skills. He stated there was no disadvantage of the instruction; however, there were too many questions in the question cards and he was in difficulty to remember all them. He suggested that the number of questions should be decreased. Related excerpt is given below:

> PE2: It makes working easier since it has a systematic structure… It provides us a comprehensive thinking. Instead of memorizing one type of question, it supports to interpret many kind of questions, in addition to operational and thinking skills… As disadvantages, I can only say that I cannot remember the questions in the metacognitive questions card. Questions in the card is useful but I am in difficulty to remember them. There is no other disadvantage.
>
> Researcher: Ok, then what do you think to overcome this problem?
>
> PE2: ıı, it may be… decrease the number of items in the card. As I remember, there are 7 questions in the card and they can be unified and decreased to 4 questions maybe…

In third interview, he again stated that the instruction was effective since they could make connections between questions and find many different solutions for any question. They supported each other by sharing their viewpoints for questions. Lastly, he said that he did not use question prompt cards when he studied alone but if he was in difficulty for a question, then he thought questions as he can remember. Related excerpt is below:

> PE2: Iı, it has a certain sequence. Previously there were many problems we did not see in other mathematics topics. Now, they are similar. I don't want to say they are all same kind but they have some connections… We found different solutions with different point of views. There were some friends who can see the points that I cannot see. Maybe I see something that another one cannot see. So, we supported each other. We solved problems all together… and I think it doesn't has any disadvantage.
>
> Researcher: Do you use metacognitive question cards?
>
> PE2: I could not remember them so much while solving a problem. After a while, I can see solution ways better. Maybe this is thanks to that card since I looked at it at the beginning. But now I am not looking at it very much. If I have a difficulty, then I examine of course…

In conclusion, he developed himself in asking comprehension questions only by considering on the unknown better. Although he did not show a development in connection or reflection questions, he developed in think aloud process, which means he developed himself to make reasoning about his solutions. Further, he found the instruction beneficial for them since it helps them interpret the problem although he did not get a habit to use metacognitive questions out of the class. He also got benefit from cooperative works.

4.3.3. Development in Questioning Skills of PE3

4.3.3.1 Comprehension Questions. Third student in the intervention group, PE3, asked comprehension questions in all interviews. An example passage from the first interview shows he used comprehension questions:

> PE3: Let's look at the table firstly. It is **given** that its first height is 60 cm... It *asks* 4 months... It is *also known* that it grows 15 cm every year.

Another passage:

> PE3: *It is given* that $f\left(\frac{x+2}{x-1}\right) = \frac{mx+1}{x+1}$ and f(4)=3, then *it asks* the value of f(0)...

In second interview, PE3 again used comprehension questions:

> PE3: It is *given* that f(x+1)=0. f(x)=(x-2)(x-3). Now I *know* f(x) and f(x+1).

In third interview also, comprehension questions can be seen:

> PE3: Ok, composition function *was defined* here. An inverse function *was given* and it *asks* the inverse again. Since it asks g(3) and gave f composition g, it is more reasonable to try to solve by doing inverse.

4.3.3.2 Strategy Questions. He asked strategy questions also in all interview. An excerpt from first interview shows that he defined his strategy:

> PE3: So *15/12 is the amount of change* in its height in 1 month. I guess *I continue in that way.* In order to find how much it will grow in 4 months, ıı, *I can think there are three part as 4 months in a year.* So, then I can divide 15 by 3. The results is 5. That is, the height of the sapling will be 5 cm longer. Then, if it is 60 cm at the beginning, I can say it will be 65 cm after 4 months. Then, the height is given now and asks the time passing. *I can continue from my previous result.* I calculated that it grows 5 cm in 4 months. Since it grow up to 70 cm, that is, it increases from 65 to 70 and so it must be passed 4 months again. Therefore, I can say 8 months I guess...

Another excerpt from the first interview:

PE3: If I make this part 3 and that part f(4), that is the upper part must be 8 and the lower part must be 2 or it must be 12 and 3. So what is x? I try 6 but not. 11 is not. *How can I do*? I can do cross product I guess.

Second interview also has examples for his strategy using:

PE3: I can try that in equation of f(x+1) I can write (x-2)(x-3) in place of x. But it may not be as that. *Let I do the inverse and write x+1 in place of x*. So f(x+1)=ıı I will write x+1 in place of x. (x+1-2)(x+1-3)… ıı, if I do in such a way, x+1, but one minute, it cannot be like that. I can write 0 everywhere I see x+1. Iı, it is not. *I can try to do multiplication* but it seems very difficult.

He developed his strategy in the third interview as well:

PE3: Since it asks g(3) and gave f composition g, it is more reasonable to *try to solve by doing inverse*. Firstly I am taking the inverse of f composition the inverse of g. Iı, it becomes g composition the inverse of f. $(gof^{-1})(x+1)=3x-2$. Then open this composition as $g(f^{-1}(x+1))=3x-2$. Then I take the inverse of f defined in the problem so that f^{-1}can take x+1. Then I find $f^{-1}(x-3)=x+2$. So, I should equalize x-3 and x+1. To do it, I write x-3=x+1. But ıı, I should write t, then… Actually I can find it as t+4 to get x+1. So I will give x+4 for x…

4.3.3.3 Connection Questions. Although PE3 asked comprehension, strategy and reflection questions for most of the problems, he used connection questions for some problems but again it can be seen examples from all three interviews. Below passage is from first interview:

PE3: It is given that $f\left(\frac{x+2}{x-1}\right) = \frac{mx+1}{x+1}$ and f(4)=3, then it asks the value of f(0). I do not know what I can do in *that kind of questions* actually because I did not review the last lessons. So, I think I can equalize the f values…

Another example from second interview shows that he tried to connect one of the problems with those previously done in class although he could not reach a solution:

PE3:… Find the largest domain of the function. Mm… Actually I don't remember how to do it. Iı, how can it be?... *I guess we did by* drawing and then found some things but cannot remember now.

Researcher: What do you mean by drawing?

PE3: ıı, I am not sure whether its name is Venn schema or not. By drawing them, drawing domain and range of function and how they matched. But now, I cannot do it…

PE3 recognized the connection between concepts in third interview also:

PE3: mm… Actually it is a bit difficult because I have not *seen the composition function and graphs at the same time before.* Let me look what it wants me to do. f(x) and g(x) was given. The unknown, u, g(x) was defined on the graph but I am not sure about how I can interpret. There are both composition and inverse…

4.3.3.4 Reflection Questions. PE3 used reflection questions to investigate whether his solution make sense or not. Therefore, he understood his mistake and tried to correct it. Related excerpt from first interview is given below:

PE3: For the graphs showing relations, firstly let's write the values for the height of the sapling and time. At the beginning, it was 60 cm. then 65 cm, 70 cm, 75 cm and 80 and 85… Then I write months. Firstly, 4 months, then 8 months, then 12, 16 and lastly 20. Now, I can draw graph… I did it but…

Researcher: What do you think now?

PE3: I think *whether I made a mistake* because…

Researcher: Why?

PE3: There are one extra value. It did not match with any months. *I am trying to understand how I did.* Maybe I drew in a wrong way. Ah, ok, it did not give the first cm, I mistook that. I did *wrong since* it did not give 60 cm. If I delete 60 from the graph, I can continue in the same way.

Another example showing his reflection from first interview:

PE3: So, x+2 must be a product of 4. We can put 14 in place of x to make here 6. Then that part will be 13 and it *was not be correct.* I think *whether I mistake* while equalizing 4 or not. But the only way I think *sensible* is this. So I continue…

Besides investigating whether the solution make sense or not, he searched for new ways also. An excerpt from first interview again:

PE3: … When I have a problem at a point, I think that I am doing wrong from the beginning. I want *to try another way* but I cannot think anything else actually. So, I continue to think in the same way and if f(4)=m, then m=3… … It will be -1 and then I found the same result but since it *does not make sense*, I should *try another strategy*. That is, I did it wrong from the beginning but even so I needed to try to learn it. If I did not continue till the end, I would not be sure about that I did wrong.

In second interview, reflection questions can be seen again:

> PE3: ıı, I am looking for whether there is *anything else* I can. Except from doing multiplication, because it is so difficult really. I believe there is *another way*. It may be like that I equalize this part firstly. x and I do 1-2 then find -1. That is I found x-1 here. For the other, I subtract 3 from 1 and find -2. It is x-2.

In third interview, PE3 used reflection as well:

> PE3: Ok, composition function was defined here. An inverse function was given and it asks the inverse again. Since it asks g(3) and gave f composition g, it is *more reasonable to try to solve by doing inverse.* Firstly I am taking the inverse of f composition the inverse of g…

<u>4.3.3.5 Semi-structured Interview.</u> At the end of both second and third interviews, PE3 said the instruction made understanding easier because it provided to analyze the problem. It included questions to transfer verbal knowledge to mathematical language. He also stated that they often investigate different ways to solve problems. An excerpt from the second interview:

> PE3: We often investigate whether the problem has different solution ways. The teacher also often asks this to us. Previously, we did not focus on other ways after we solved it in a certain way. Further, in order to better understand the problems, especially some students may have a difficulty in, I think this comes from the fewness of reading books, since you read the problem but since you cannot understand it verbally, you cannot transfer it to mathematical operations. In order to overcome this, that is the instruction includes questions oriented to improve understanding and better solving skills. It provides us to deduce on the problem.

Additionally, he stated in the third interview that examining the reasonability of the problems was beneficial for them:

> PE3: We are solving problems in a more detail and take care of our understanding… It reinforces us to analyze the problem. Additionally, our teacher often asked us whether the solution make sense or not. This had a positive effect in my opinion.

He thought there were no disadvantage of the instruction. However, he expressed that they may be in difficulty to ask questions because they did not take such an instruction until now and they did not have such a habit to ask those questions. If they learned in that way from the beginning of the education, they may better apply in his opinion. An excerpt from the second interview is below:

> PE3: ... I think it would be better for us to take such a support from the beginning of our educational background. Now, we try this for the first time but it is more difficult to internalize something after a certain age. If we have learnt in this way since the beginning, for example first examine this and look at that etc, if we start to learn some basic things from the beginning, it would be more effective.

As a result, PE3 used all metacognitive questions at some level in correct ways. He used these questions in all three interviews, so it cannot be said that he developed metacognitive questioning. On the other hand, he was not aware of his using these questions. He was a low achiever student according to the both pre and posttest results. Therefore, he might think he could not ask these questions since he was not successful enough. He also stated in the third interview that he did nor review the topic so much. Therefore, he might need more practice to be more successful. Nevertheless, he thought that the instruction was effective to analyze the problem and recognizing other possible solutions to the problem.

4.3.4. Development in Questioning Skills of PE4

4.3.4.1 Comprehension Questions. Last participant from intervention group is PE4. She approached to problems in an investigating way. Although she did not clearly state the unknowns in some problems, she examined the givens and then developed her strategy to reach the unknown. If there are mathematical terms in the problem, she clearly defined those concepts. An excerpt from first interview:

> PE4: Ok. $f(x) = \frac{x+2}{x-1}$ was *given*...
>
> PE4: Since *it asks for* f(4), I must do $\frac{x+2}{x-1} = 4$. Then x=2. Yeah 2. 2+2 divided by 1 is 4. It is sensible. Then I can equalize this to 3. $\frac{2m+1}{2+1} = 3$. And 2m+3... I cannot continue. Mm, that is 9, $\frac{2m+1}{3} = \frac{9}{3}$. I guess I am not doing right. 9 times 3 is 27. 6m+3=27. Then 6m=24 and m=4. And it *wants* me to find f(0). Of too... f(0)... I found m as 4...

In second interview, she again used comprehension questions:

> PE4: $f: R \rightarrow R, f(x) = (k-2)x^{5-k}$ *given* as a constant function and *asks* the possible values of k. When I see *a constant function, I think* that f(x)=c or f(3)=c. that is to say, every f(x) goes to the same number…

Another part from third interview shows comprehension also:

> PE4: *If* f(x+1)= x-3, (fog^{-1})(x+1)=3x-2 then *what is* $g^{-1}(3)$? Ok, now firstly I will open the composition. $g^{-1}(x+1)$ and f. *We have such things*. I might be in difficulty for this kind of questions but *it says* $g^{-1}(3)$. I have a good idea. Since it *asks 3*, I should make here 3…

4.3.4.2 Strategy Questions. While applying her strategy, she asked herself many times why she did it or how she can deal with the problem. She clearly explained her strategies. An example from first interview:

> PE4: Ok. $f(x) = \frac{x+2}{x-1}$… and *what can I do*? The upper part may be 0 but the denominator cannot be 0. So, if x-1=0 then x=1 but ok I did. *What did I get now*? Assume I put it in place of x, 1+2 divided by… Why wasn't it? Mm, then, I should find m here. If I equalize that, but I equalized. It should give me another thing. f(4)… *What can I do to reach f(4)*? Denominator… I found. I don't know why I did such a thing. Since it asks for f(4), *I must do* $\frac{x+2}{x-1} = 4$. Then x=2…

In second interview, strategy questions can be seen again:

> PE4: $f: R \rightarrow R, f(x) = (k-2)x^{5-k}$ given as a constant function and asks the possible values of k. When I see a constant function, I think that f(x)=c or f(3)=c. that is to say, every f(x) goes to the same number. Then if *I think the logic* of that, I should do this x, mm… No, this k-2 is 0. *There should not be an x term since it creates changes*. So, I say k-2=0 and k=2. But *there is another thing*. X has a power also. K is 2. *What if* I put the value of k? If I put it as 2, 5-2=3. That is x^3 but the possible values of k… *I want to get rid of x* completely. k=2… x^{5-2} and x^3. I cannot think anything. *Why didn't x go*?

In the third interview, she explained her strategy as well:

> PE4: If f(x+1)= x-3, (fog^{-1})(x+1)=3x-2 then what is $g^{-1}(3)$? Ok, now *firstly I will open the composition*. $g^{-1}(x+1)$ and f. We have such things. I might be in difficulty for this kind of questions but it says $g^{-1}(3)$. I have a good idea. Since it asks 3, I should *make here 3*. g(3), that is, x+1=3 and x=2. Yeah, *after finding x, I will write 2 for f or put g(3) here* but I think it will be more complex…

4.3.4.3 Connection Questions. PE4 used connection once in first and third interview. Below part is from first interview and she connect the definition of a function with the example given in the class previously:

> PE4: 3 can match with 1. It is ok. But 6 must go either 3 or 2 because it cannot match with both. Then 9 can go to 4 since it is alone. 12 can go 6, yes. 15 can go 6 because *I think that there is one shuttle and two students. Both them can go to same place*. So, I think it is also ok. That is for 15 and 12. They can match with the same number.

She tried to connect what she found and what the question asks in the third interview:

> PE4: That is, it must be Z-{0}. There isn't also things such as + or -. I could think all these things but *I don't guess they are related to linear functions...*

4.3.4.4 Reflection Questions. She did not ask whether her solution is sensible or not in the first interview but she did for some solutions in second interview and for most solutions in third interview. Nevertheless, she did not ask where she can mistake. An excerpt from second interview is below:

> PE4: ... I found x^2-x equals 2. How can I use it? I think there should be another solution. I didn't make a *reasonable* thing. I did a simple operation. It may be *logical* but not too much. What will it give me?...

Another excerpt from second interview:

> PE4: x^0 is equal to 1. Yeah, it is 1. Then I can say 5-k=1 so k=5 or k=2. But if I put 5 in place of k, here is 3... I think both these because I cannot think anything else *reasonable*...

In the third interview:

> PE4: It goes from R^2 to R^2 and it is given for x and y. These things are equal. f(-2, 4). I think that f(x, y) is defined as 3x-2y+5 then f(-2, 4)=3.(-2)-2.4+5. It is -9. f(4, a)=3.4-2a+5. It is 17-2a. Then -9=17-2a yes. 2a=26 if I correctly add. A=13. Why did I do such a thing? I₁, here the function is defined from R^2 to R^2. It made me a little confused but I think *it make sense* because it gave x and y. So, I think it is 13.

4.3.4.5 Semi-Structured Interview. PE4 also stated at the end of interviews that they could investigate the concepts more than before. They understood the logic instead of memorizing.

Thanks to question prompt cards, they can try to develop strategies and new ways to solve problem, and they can make comparisons. Related excerpt from second interview is below:

> PE4: It does not depend on memorizing, instead, it provides deep understanding via metacognitive questions. We did comparisons among questions. ınstead of rote-learning, it helped permanent learning since it is based on a logical sequence.

Further, she found group works beneficial since they can share their ideas. An excerpt from the third interview:

> PE4: A person thinks different from me and I can take his/her ideas for myself. His/her way may make sense for me more than mine. Or, maybe I showed him/her a new view point. So, sharing our ideas was helpful.

The only disadvantage is that asking those question is a little time consuming as she expressed in second interview:

> PE4: If I focus on a problem and try to solve with these questions, I may lose time. This is a disadvantage for me. Maybe I can solve the problem by using these questions but the exam has a time restriction and I cannot. We are tended to memorize the problems for years and now we can have a comprehensive thinking and permanent learning, which is beneficial again.

In briefly, PE4 used comprehension and strategy questions at some level in all interviews; however she also developed herself to ask reflection questions by investigating the reasonability of her solutions at least. Additionally, she made a simple connection in first interview by relating a real life example used in the class and the problem asked in the interview. However, she tried to connect the definition of a mathematical term with the givens of the problem in the third interview although she could not find the correct answer. Furthermore, she explained that the instruction was beneficial for them since they were more tended to query the problems instead of memorizing them by using question prompt cards. Thanks to group works, they could exchange their ideas and searched new ways to solve problems.

4.3.5. Development in Questioning Skills of PC1

4.3.5.1 Comprehension Questions. PC1 is one of 3 participant from control group who were voluntary to interview. After examining the givens, he started his strategy and looked for unknown during solving process. Related excerpt from first interview:

> PC1: *It grew 15 cm every year.* Then each month, if we divide it to find the amount of extension for each month, we find 15/12. *Since it grows 4 months,* we multiply it by 4. It can be simplified and so it equals to 5. It grows 5 cm in four months…

Another part from second interview:

> PC1: I put the values into f. *since x+1 is given,* I will put x+1 here. If I add 1 to this, (x-1)(x-2)=0. That is, ıı… *Solution set is asked,* so one of these must be 0 so that the result of multiplication can be equal to 0. This one or the other. So, x can be 1 or x can be 2 as solution set.

The same process can be observed from third interview:

> PC1: $f(3^x+1)=9^x+1$ and $f^{-1}(a)=4$ are *given* then *find* a? Hmm, how can we interpret it? Iı, for example, this is the first power of 3. If we say the first power of 3 here mm… There is +1 here so that it can be 4.

PC1 also defined concepts if there is, before starting to solve:

> PC1: Hm, *constant function.* So, it will be like ax+b… …nope, it is not constant function. It was linear function. I was confused. So then it must be 0. 5-k must be 0 and k=5. Since it will be 0, whatever here is… ıı… 5-2 then 3. $3.x^0$ means that everything will go to 3…

4.3.5.2 Strategy Questions. PC1 used strategy questions in all interviews. The same examples above from the first and second interviews respectively:

> PC1: It grew 15 cm every year. Then each month, *if we divide it to find the amount of extension for each month,* we find 15/12. Since it grows 4 months, we multiply it by 4. It can be simplified and so it equals to 5. It grows 5 cm in four months…
>
> PC1: *I put the values into f.* since x+1 is given, I will *put x+1 here*. If I add 1 to this, (x-1)(x-2)=0. That is, ıı… Solution set is asked, so *one of these must*

be 0 so that the result of multiplication can be equal to 0. This one or the other. So, x can be 1 or x can be 2 as solution set.

Likewise in the third interview:

PC1: Hmm, *how can we interpret it*? I1, for example, this is the first power of 3. If we say the first power of 3 here mm… There is +1 here so that it can be 4. Then 3^x should be equal to 4. But it was not correct. So, *firstly let's do that function*. Mm, we can say it is multiplied by 3, yeah it can be multiplied by 3. That is, 3^x was multiplied by 3. +1 also must be multiplied since it has a parenthesis. So, it becomes -2 and then this can be find. Hm…

4.3.5.3 Connection Questions. He did not use any connection questions in any interview.

4.3.5.4 Reflection Questions. He did not asked himself whether his solution is reasonable or not; however, if he make a mistake, then he thought again on the solution. An example excerpt from first interview is below:

PC1: … I solved here by using proportion. Hm, it grows 5 cm in every month. Then it becomes multiple of 3 in 12 months and grows 15 cm. It cannot be 15 cm. *I did a mistake. Where? It is multiple of 3…*

Researcher: Why did you think to make a mistake?

PC1: Because it grows 70 cm in 14 months but hmm *I made a mistake here*. It says the height of the sapling is 60 cm. So, this must be 65. *But here is false again…* Hm, nope. *It is not false*. Because…

In the second interview also, he checked his answer:

PC1: … Then k can be 5 and also here can be 0. K must be 2 so that it can be 0. *Is it all that much?* Yes, it is…

In the third interview, he used reflection once but did not focus on that so much:

PC1: … But *I did a mistake there*. 3^x, 3 has a power there. Mm… How can we say? Hm, this a, ahh *I am drooling*. Nope, yes yes. 3 to the power, I will do same as before a-2. 3a=6 and a=2. It can be like that.

As a result, he used comprehension and strategy questions at some level and used reflection questions only when he recognized to have a mistake. He did not investigate the reasonability of solutions or did not search other ways when he did not solve the problem.

Further, he did not make connections between problems or concepts. He maintained his usability of metacognitive questions and did not show any change.

4.3.6. Development in Questioning Skills of PC2

4.3.6.1 Comprehension Questions. Second participant PC2 did not clearly stated the givens and unknown of the problem. Just after she read the problem, she started to solve it. Further, she could not explain the concepts correctly and did not think about that so much. An excerpt from first interview:

> PC2: This match is a function because every element in the domain has an image in the range. There can be 2 images of an element from the domain. So, it is ok, I think.

Another example from second interview is below:

> PC2: Linear function is ax+b. $\frac{ax-3b}{2}$, $\frac{ax+1}{x}$, $ax^2 + b$, $ax + b$ and this is also, $ax + b$. In defined intervals... this one is linear I think...

In the above example, the problem included functions $\frac{x-3}{2}$, $\frac{1}{x}$, x^2, $\sqrt{x}$ and |x| respectively but she could not think about the definition of the linear function deeply and she just tried to apply the formula by rote learning.

4.3.6.2 Strategy Questions. PC2 did so mechanical solutions. She did not investigate the problems as I said above and did not ask herself why to do it. While she was solving a problem, she was like that she memorized those types of problems. Therefore, if the problem is a little complex, she could not interpret it and easily quit up. In example below from the first interview, she starts to solve in a correct way but could not continue since she did not think deeply:

> PC2: If it grows 15 cm each year, in one month, since one year includes 12 months, if it 15 in 12 months, we can say x in 4 months. 12x=60 then x=5. 5 cm in 4 months. If 5 cm in 4 months, 70 cm in how many months? 280=5x and x=56. If 70 cm in 56 months, x cm in 12 months. 60=4x then 15 cm. If 5 cm in 4 months, 80 cm in how many months? 320=5x then 64 months. If 5cm in 4 months again, x cm in 20 months. It is 25 cm. We will draw graph in time...

Another example from the third interview shows she could not interpret the problem and just tried to solve by her memory. Since it was enough for this problem, she did not want to engage more and give up easily:

> PC2: f(x+2)=x-3 and (fog^{-1})=3x-2 the what is g^{-1}? f(x+2), let's equalize x+2 to x+1. Nope, I couldn't. If I try to find f(x), I must equalize x+2 to t. x becomes t-2. That is, if we write x-2, here will be cancelled. Let's write x-2 in place of x. x-2-3=x-5. I found f(x)=x-5. Now we find x+1. f(x+1)=x-4. 3x-2… x-4… hmm, I couldn't do. I want to pass this problem.

4.3.6.3 Connection Questions. She did not connect the concepts or problems in any interview.

4.3.6.4 Reflection Questions. As can be seen from above examples, she did not ask herself whether her solution is reasonable or not. She also did not search other ways to continue solving. When she could not solve a problem, she did not think about where she can mistake and wanted to quit up easily. Therefore, she never used reflection questions.

In brief, she was not tended to use metacognitive questions. Further, she did not query the problems and her solutions. It can be said that she approached problems based on memorization.

4.3.7. Development in Questioning Skills of PC3

4.3.7.1 Comprehension Questions. Last participant PC3 used comprehension questions in all interviews. He looked givens and unknowns and also defined concepts. An excerpt from first interview is below:

> PC3: *since* its height at the beginning is 60 cm, I will start with 60 for what it *asks*.

In second interview, he also used comprehension questions and explained correct definitions of constant functions:

> PC3: If *it is a constant function, I should* remove x from here *because* it creates changes in values…

Another example from second interview shows that he defined the mathematical concept before solving the problem:

> PC3: If it is a linear function, it should increase with the same ratio or it should be like ax+b...

Another excerpt about his using comprehension questions from third interview is given below:

> PC3: Since it *asks* the inverse, we can find the inverse if we exchanged these. So, I will exchange 9^x+1 and 3^x+1. Then *it want me to find the inverse function* for the value a. If I say a here, this part will be 4...

4.3.7.2 Strategy Questions. He used developed and explained his strategy in all interviews. An example from first interview is below:

> PC3: *It is proportion.* If it is 15 cm in one year, I can find for four months. But *if I find for one month firstly, it will be easier to find others.* I can find it for one month. 15/12 because one year includes 12 months. If I multiply it by 4 since it asks for four months, it becomes 60/12 and so 5. It grows 5 cm. *As the same way*, if we multiply it by x which will give how many months here, it will become 70 cm as given in the problem. It asks which number we should multiply 15/12 to find 70. Yes, Now I am trying to do this. 840/15 and I cannot divide it now. I can write it as 840/15. It asks 12 months and 15 cm as given. Again, after x months, it will become 80 cm. So, I should divide 80 by 15/12...

From the same passage above, he also showed his strategy in second interview:

> PC3: If it is a constant function, *I should remove x* from here *because* it creates changes in values. *To remove x, here must be 0.* I can give 2 to k. If k=2 then it will be 0. If I give another value to k, it will give different numbers, but I can give 5 to k also. Yes, it can be 5. If it is 5, I can get x^1. Since x^1 is 1, but one minute. Nope, it cannot be. I think only 2.

Another example passage about his using strategy from third interview is given below:

> PC3: Since it asks the inverse, we can find the inverse *if we exchanged these*. So, I will exchange 9^x+1 and 3^x+1. Then it want me to find the inverse function for the value a. *If I say a here, this part will be 4...*

4.3.7.3 Connection Problems. PC3 mostly did not try to make connections except from a problem in the second interview. In that problem, he identified the difference between the problem and those he solved before; however, he could not connect his knowledge from different topics:

> PC3: What is the solution set? Then let me find x values. I should find x. I1, x^2-2x-x then -3x +2=0. x^2-3x=-2. So, what can x be? Not 4, 3, hm, 4 no. It doesn't an integer. I don't understand what it asks. It asks solution set but *I see solution set in a function problem for the first time.* I could not think it now…

4.3.7.4 Reflection Questions. PC3 used reflection to check his solution. He looked whether he found what the problem asks for or he checked his operation errors. An example from the second interview shows he looked again what he should do:

> PC3: If it is a linear function, it should increase with the same ratio or it should be like ax+b. If I give a random number for x, for example 5, here will be equal to 1. So, it should be 2 when I give 10 to it. When I give 10 here, it becomes 7/2. So it is not a linear function because it should increase with the same ratio. But, *one minute I didn't find the increase.* Yes… So, let me find for 10 and 15. When it is 5, the image is 1. When it is 10, the image is 7/2 which is equal to 3,5. When it is equal to 15, it will be 6. So, it increases 2,5 and 2,5 again. Then it is a linear function.

On the other hand, if he felt that he made a mistake, he did not search where he did the mistake and he stated that he wanted to give up. An example excerpt from second interview again:

> Researcher: You say only 2, don't you?
>
> PC3: Yes.
>
> Researcher: So, do you want to pass?
>
> PC3: Yeah. Most likely there are other values but I won't be able to do it now.
>
> Researcher: You can continue if you want?
>
> PC3: I couldn't do it so I want to pass.

Therefore, it can be seen that PC3 mostly used comprehension and strategy questions appropriately but he was inadequate in asking reflection questions. He did not focus on his mistakes and did not search other ways to solve the problem. He only checked the unknown and operations. Further, he was in difficulty to make connections between concepts even if he recognized the differences.

4.3.8. Overall Interpretation about Students' Questioning Skills

As can be seen from the think aloud session analysis, there is a stability of metacognitive questioning skills of students from control group. If they could do it instinctively in first interview, they were able to continue in the following interviews. However, if they are weak in first interview, they could not develop themselves in terms of metacognitive process. Although they generally examine the comprehension and strategy questions at some level, they were weak in asking connection and reflection questions.

On the other hand, some differences were observed for intervention group students like that some started to look for new ways, some started to check unknown of the problem, some looked for connection or some investigated reasonability of the solution. Although some of them were not observed to change, it might come from their habits since they might not change their studying way due to some other reasons. These issues will be discussed in the next section in detail.

Further, all them stated that they could query more than before and so they could follow the logical sequence. Although they said that they did not use metacognitive questions so much when they were alone, they liked to use them in the classroom. The reasons of their not using questions alone were that they were a little time consuming and also students were in difficulty to ask some of them but again they accepted that these questions make easier to follow the solution. Furthermore, they expressed that they could not get a habit to use metacognitive questions. If they were educated in this way from the beginning of their school life, they would get more benefit and use these questions easier. PE3 also stated that he did not review the entire topic so much. If his posttest result was examined, it could be seen that he was really lack of some necessary knowledge.

In addition to interviews, intervention group students also were asked the difference of the instruction than before and the advantages and disadvantages of the instruction in the last lesson of the intervention. They commonly said that they queried problems more than before and emphasized the logic behind them. Only two students stated a different thing from the others. One said that they used real life examples more than before and this helped him understand the topic better. The other stated that he could not understand why it was wrong when he had a mistake before, but along this study, he took answers to all questions that made him confused.

5. CONCLUSION AND DISCUSSION

The purpose of this study was to examine the effect of metacognitive strategy based mathematics instruction (IMPROVE) on students' achievement on functions unit and self-regulatory skills. The study also examined whether students' questioning skills really develop via the IMPROVE instruction. Mixed method research design was used to answer all research questions. Students' achievement and self-regulatory skills were analyzed with quantitative data and questioning skills were analyzed with qualitative data. Results showed that although there was not found a statistically significant difference on students' achievement on functions and self-regulation skills, the instruction was effective in practice for students both on their performance on functions and self-regulation skills. It was also concluded that they developed their questioning skills and gain an awareness of their thinking process. This section was prepared to present discussions on results of the study. It will be examined in three parts for each research questions, and then limitations and recommendations for future researches will be mentioned.

5.1. Students' Achievement on Functions

There are several researches emphasizing the benefits of students' questioning. When students ask questions, the comprehension of the task and higher order thinking skills can be fostered (Palincsar and Brown, 1984; Scardamalia and Bereiter, 1985; Garcia and Pearson, 1990). Questions help students recognizing knowledge gaps, use their preexisting knowledge and consider carefully on the content and main ideas (Schmidt, 1993; Rosenshine *et al.*, 1996). Some studies showed that self-questioning has a significant effect on learning, conceptual understanding and problem solving skills (Ge and Land, 2003; Ge and Land, 2004; King, 1992; King 1994; Jesus 2009).

In this study, students are aimed to develop their questioning skills via IMPROVE metacognitive strategy based instruction and then it was expected to increase their achievement. The intervention group teacher modelled four types of questions and allowed students to practice them. These were comprehension, strategy, connection and reflection questions. According to several research, IMPROVE instruction method has a positive effect on students' achievement (Mevarech *et al.*, 2010; Michalsky *et al.*, 2009; Aziz, 2016; Pilten, 2008) In this study, students' understanding on the function unit measured with an achievement test improved for the study. In line with literature, students took metacognitive strategy based instruction gained higher improvement than students in the other group with medium effect size. For both pre and posttest results, there was not found a statistically significant difference between groups; however, control group students had higher mean on pretest than intervention group whereas they had lower mean on posttest. Effect size analysis also supported the higher development of students in intervention group. However, there might be some reasons not to get high effect size such as implementation of treatment and duration of the study.

According to Schunk and Zimmerman (1998), short term programs for a few weeks are not enough to develop cognitive and metacognitive strategies but as students get older, the time needed can be shorter. Since older students have already developed some metacognitive strategies, they can even benefit from shorter-term programs. In this study, the treatment was applied for two months because it was aimed to investigate the development of students on function unit. However, Schunk and Zimmerman (1998) also presented another aspect for older students. They are getting some habits in k-12 education process and if they get a certain level success, they could be more confident in their knowledge and skills, so they can be more resistant to change their habits. The study groups of this study may have more confidence on their own since they were science school students and the results of the study might be influenced from this effect.

Beyond timeframe of the study, effect of the treatment might have been affected from the threats in the implication. Two different teacher taught the intervention and control group and they had different features and characteristics. The effect of these differences were tried to be minimized by using same lesson plans and observing teachers as mentioned in validity section. Nevertheless, it might lead to medium effect size. The study of Wang and Cai (2016) also indicated that novice teachers' questioning tendency were lower than qualified and experienced teachers' questioning tendency. Although both teachers in this study can seem as in the same level in terms of experience, three years difference might create an effect.

Furthermore, the intervention group teacher had difficulty to follow study procedures as mentioned in the internal validity part. Towards the end of the semester, she was distracted by other school works and projects and so her motivation for this study was decreased. On the other hand, the implication of IMPROVE method requires deliberate systematic using of metacognitive questions (Mevarech and Kramarski, 1997). Although necessary support was given to the teacher and she was tried to be motivated continuously, distractions of the intervention teacher might have broken the systematic implication.

In conclusion, medium effect size rather than high effect size might be affected from the implementation and time period. However, practical importance is more valuable for studies with such a small sample size. In this study, intervention group students gained higher development despite the duration and difficulties arose during implementation. If it was implemented with a larger sample in longer period, the possibility of getting a statistical significance and higher effect size would increase. A similar study where IMPROVE method was implemented for ten weeks with 704 11^{th} grade students also obtained medium effect size for students' procedural and conceptual knowledge (Aziz, 2016). Nevertheless, this study showed the effectiveness of the IMPROVE metacognitive strategy instruction on students' achievement in practice.

5.2. Students' Self-Regulation Skills

Metacognitive strategy based instruction is aimed to teach using some particular strategies by emphasizing when, why and how to use those strategies, and it is effective on self-regulation (Baker and Brown, 1984). Many other studies emphasized the positive effect of metacognitive instruction on self-regulation skills (Joseph, 2009; Arslan and Gelişli, 2017; Nash-Ditzel, 2010). Students can be aware of their thinking and learning via metacognitive instruction and methods and materials used by teachers can contribute to students' metacognitive awareness (Joseph, 2009). IMPROVE developed by Mevarech and Kramarski (1997), one of the metacognitive strategy based instructions, was chosen in this study to examine its effect on 10^{th} grade students' self-regulatory skills because there are many studies which shows its benefits on self-regulation (Aziz; 2016; Kramarski and Gutman, 2006; Mevarech and Amrany, 2008).

In this study, before and after intervention, MSLQ questionnaire was conducted to define students' self-regulation level. Its four subscale were included to analysis as self-regulation, metacognitive strategy use, self-efficacy and intrinsic interest. All components analyzed separately and there were not found a statistically significant difference between groups for any of subscales. Nevertheless, self-regulation and metacognitive strategy use level increased more in intervention group than control group when cohen's d value is considered. Although self-efficacy and intrinsic interest were decreased a little bit without statistical significance in both groups, intervention group could be said to gain more development compared with control group overall scales. However, medium effect sizes were found for these differences and the possible reasons for not getting high effect size may be because students needed more feedback since they were not accustomed to these strategies, and they probably were not able to internalize the strategies in addition to duration and treats to internal validity as stated in the previous section.

Firstly, feedback is very important for students to develop self-regulated learning (Butler and Winne, 1995; Winne, 1997; Labuhn *et al.*, 2010) since it provide learners information about their learning quality and learners may have a chance to monitor their learning process (Butler and Winne, 1995). Further, it enriches students' self-reflection as part of cyclical model of self-regulation (Zimmerman, 2000). Depending on the literature, students were tried to give feedbacks in this study both during lessons and after quizzes. Students in intervention group took enrichment and correction activities after quizzes so that they can correct their faults in quizzes and go beyond their current achievement level. However, because of the duration of the study as mentioned before, the number of these activities were limited for students to evaluate their learning and change their strategies.

On the other hand, it was also important to give students feedbacks during lessons. The intervention group teacher was in difficulty to give feedbacks on students' wrong answers about why they were wrong at the beginning of the study. Towards to middle, she started to give more effective feedbacks to students' answers through researcher observation and feedbacks for her.

Furthermore, strategy instructions should allow students to personalize strategies as Borkowski stated (1992). In this study, the teacher was modeled to use metacognitive questions for different problems and gave homework to students to use these questions for few times; however, most of students did not bring homework although they were often motivated to do them. This shows that they could not internalize the strategies. This might be because the teacher had some difficulties following the procedures

Nevertheless, this study provide some development and awareness on students' self-regulatory skills in practice. As mentioned in the previous part, this study indicated practical importance of the IMPROVE method. Students benefited from the instruction in spite of possible barriers.

5.3. Students' Questioning Skills

There are some studies investigating students' questioning tendencies and they concluded that students asked very few questions and many of them were lower level thinking questions (Dillon, 1988; White and Gunstone, 1992, p.170; Chin and Brown, 2002). Therefore, many researchers were interested in possible ways to develop students' questioning capability (Jesus, 2009; Zoller, 1987; Marbach-Ad and Sokolove, 2000; Toledo, 2006). In this study, students' questioning skills were aimed to be developed via metacognitive strategy based instruction and examined with think aloud protocol. Students' ideas were taken also with semi-structured interviews.

As a result, there was found a stability on metacognitive strategy based questions of students from control group. They continued to solve problems in the same way with the first interviews. Although they used comprehension and strategy questions at some level, they were particularly weak to ask connection and reflection questions.

On the other hand, some differences were observed for intervention group students in line with the literature. Most of them have already been using comprehension questions and strategy questions at some level; however, half of them developed some other skills with the intervention process. They started to look for new ways and connections or investigated reasonability of the solution and where they did mistakes. Further, all them stated that they could query more than before and so they could follow the logical sequence and gained benefits of the instruction.

Nevertheless, two students from intervention group were not observed improvement in terms of questioning. When they are asked whether the instruction was beneficial for them or not, they said that they queried why they did what and could follow the logical sequence easier than before. However; they stated that they did not have a habit to use metacognitive questions and could not change it. Their reflections were in line with Schunk and Zimmerman' (1998) statement that is older students might have some habits in k-12 education process and they could be in difficulty to change their habits more than others. Borkowski (1992) also highlighted the importance of internalization of strategies. However; since students thought metacognitive strategy thinking is a little time consuming, it might be difficult for them to change their habits in two months of implementation.

Chin and Osborne (2008) identified possible barriers for students to ask questions and they added some other factors to students' age and personal experiences. One of them is lack of domain specific knowledge that might affect the results of this study also since one student who had a stability on his metacognitive strategic questioning skills stated that he did not review the entire topic so much and his posttest score also supported his explanation.

As a result, some differences were observed on students' achievement on functions test, self-regulation and questioning skills in this study, although the differences were in medium effect size. The reasons may originate from the duration and implementation of the study, students' previous habits and domain specific knowledge, and insufficient number of feedbacks. Although some precautions were taken to decrease the effect of these factors, higher effect size could not obtained. However, students gained benefits in all three skills even in the conditions of this study. Therefore, it revealed the importance of the implementation of metacognitive strategy instruction for students.

6. LIMITATIONS AND SUGGESTIONS

This study aimed to investigate the effect of metacognitive strategy based mathematics instruction on students' achievement, self-regulation and questioning skills. It was conducted in a private school in Yenibosna, İstanbul and participants were students from two classes in that school selected by convenient sampling. Therefore, the results of this study was limited for that school. Although experimental design was used, 36 students participated to the study in total. In order to generalize the results, it is recommended to imply the instruction method with a larger sample and different schools.

As mentioned before, it might be needed to implement a metacognitive instruction for a long period of time to be able to observe changes. Since this study was focused on the functions unit, it was conducted along two months. However, some students stated during interviews that they could not get a habit to use metacognitive strategies. Therefore, the study can be repeated for longer time period and extended in terms of mathematics topics in order to increase the effectiveness of the intervention.

Moreover, teacher characteristics is a limitation of the study although they were tried to be equalized with same lesson plans and materials. Further, the researcher met with intervention group teacher in five sessions to talk about the implementation of the method and gave continuous feedback. However, in future studies, it is suggested to give teachers more qualified education before the application of the instruction to obtain more effective results.

Additionally, students could be given more homework with more detailed feedbacks for effective use of strategies. As some researchers emphasized, question posing also can be used in assessment process (Jesus, 2009; Dori and Herscovitz, 1999). Both homogeneous and heterogeneous group works can be developed by including more challenging or non-routine mathematics problems.

In conclusion, the method can be adapted to other mathematics topics like the materials on function concept in this study. Teachers may have a curriculum pressure or some other difficulties; but if the implementation these kind of works is increased, students can benefit from it even in a short period of time.

REFERENCES

Akçakın, V. (2015). *The effects of using geometric functions approach in dynamic mathematics environment on 9th grade students' academic achievement on functions and motivation towards mathematics learning*. (Doctoral Dissertation), accessed on Ulusal Tez Merkezi. 397472.

Alcı, B. & Altun, S. (2007). Lise Öğrencilerinin Matematik Dersine Yönelik Öz-düzenleme ve Biliş üstü Becerileri, Cinsiyete, Sınıfa ve Alanlara Göre Farklılaşmakta mıdır? *Çukurova Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, 16:1, 33-44.

Alpaslan, M. M., Yalvac, B., Loving, C. C., Willson, V. (2016). Exploring the Relationship Between High School Students' Physics-Related Personal Epistemologies and Self-regulated Learning in Turkey. *International Journal of Science and Mathematics Education*, 14:2, 297–317. doi:10.1007/s10763-015-9685-7

Ames, C. (1992). Classrooms: Goals, structures, and student motivation. *Journal of Educational Psychology*, 84:3, 261-271. http://dx.doi.org/10.1037/0022-0663.84.3.261

Ames, C., & Archer, J. (1987). Mothers' beliefs about the role of ability and effort in school learning. *Journal of Educational Psychology*, 79:4, 409-414. http://dx.doi.org/10.1037/0022-0663.79.4.409

Ames, C., & Archer, J. (1988). Achievement goals in the classroom: Students' learning strategies and motivation processes. *Journal of Educational Psychology*, 80:3, 260-267. http://dx.doi.org/10.1037/0022-0663.80.3.260

Arsal, Z. (2009). The Impact of Self-Regulation Instruction on Mathematics Achievements and Attitudes of Elementary School Students. *Education and Science*, 34:152, 3-14.

Arslan, S., Gelişli, Y. (2017). Üstbilişsel Öğretim Stratejilerinin Fen ve Teknoloji Dersinde Öğrencilerin Üstbilişi Yönetme, Öz Düzenleme Becerilerine ve Akademik

Başarılarına Etkisi. *FSM İlmî Araştırmalar İnsan ve Toplum Bilimleri Dergisi,* 10:Güz, 49-60.

Aviles-Garay, E. J. (2001). *Using multiple coordinated representations in a technology - intensive setting to teach linear functions at the college level.* (Doctoral Thesis), accessed on ProQuest Dissertations Publishing. 3023012.

Ayalon, M., Watson, A. & Lerman, S. (2017) Students' conceptualisations of function revealed through definitions and examples. *Research in Mathematics Education,* 19:1, 1-19, DOI: 10.1080/14794802.2016.1249397

Aziz, A. T. (2016). *The Effect of Metacognitive Instructional Method on Eleventh Grade Students' Metacognitive Skill and Mathematical Procedural and Conceptual Knowledge* (Doctoral Dissertation), accessed on Ulusal Tez Merkezi. (442119).

Baker, L., & Brown, A. L. (1984). Metacognitive skills and reading. In P. D. Pearson, R. Barr, M. Kamil, & P. Mosenthal (Eds.), Handbook of reading research (pp. 353–394). New York: Longmans.

Bandura, A. (1997). *Self Efficacy, The Exercise of of Control.* W. H. Freeman and Company, USA New York. Pp 3.

Ben-Eliyahu, A. & Linnenbrink-Garcia, L. (2015). Integrating the regulation of affect, behavior, and cognition into self-regulated learning paradigms among secondary and post-secondary students. *Metacognition and Learning,* 10:1, 15–42. doi:10.1007/s11409-014-9129-8

Biddulph, F., & Osborne, R. (1982). *Some issues relating to children's questions and explanations.* LISP(P) Working paper no. 106. Waikato, New Zealand: University of Waikato.

Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2002). *Working inside the black box: Assessment for learning in the classroom.* London: King's College London.

Bloom, B. S. (1956). *Taxonomy of educational objectives. Handbook I: Cognitive domain.* New York: Longman.

Boekaerts, M. (1996). Self-regulated Learning at the Junction of Cognition and Motivation. *European Psychologist,* 1:2, 100-112.

Boekaerts, M. (1999). Self-regulated learning: where we are today. *International Journal of Educational Research*, 31:6, 445-457. Doi: 10.1016/S0883-0355(99)00014-2

Borkowski, J. G. (1992). Metacognitive Theory: A Framework for Teaching Literacy, Writing, and Math Skills. *Journal of Learning Disabilities*, 25:4, 253-257.

Boujaoude, S. B. (1992). The relationship between students' learning strategies and the change in their misunderstandings during a high school chemistry course. *Journal of Research in Science Teaching*, 29:7, 687–699. DOI: 10.1002/tea.3660290706

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational Studies in Mathematics*, 23(3), 247-285.

Brown, A. L. (1978). Knowing when, where, and how to remember: A problem of metacognition. In R. Glaser (Ed.), *Advances in instructional psychology*, 1, 77–165.

Brown, Nancy Mullins. (2003). *A study of elementary teachers' abilities, attitudes, and beliefs about problem –solving* (Doctoral dissertation), accessed on ProQuest Dissertations Publishing. (3108818)

Butler, D. L., & Winne, P. H. (1995). Feedback and self-regulated learning: A theoretical synthesis. *Review of Educational Research*, 65:3, 245-281.

Byun, H., Lee, J., Cerreto, F. A. (2014). Relative effects of three questioning strategies in ill-structured, small group problem solving. *Instructional Science*, 42:2, 229–250. https://doi.org/10.1007/s11251-013-9278-1

Caldwell, F. (1996). *Bring functions and graph to life with the CBL.* Paper presented at the annual meeting of the American Mathematical Association of Two-Year Colleges, Long Beach, CA.

Carney, R. N., Levin, J. R. (2002). Pictorial Illustrations Still Improve Students' Learning from Text. *Educational Psychology Review*, 14:1, 5-26.

Cassel, J. & Reid, R. (1996). Use of a Self-Regulated Strategy Intervention to Improve Word Problem-Solving Skills of Students with Mild Disabilities. *Journal of Behavioral Education*, 6:2, 153-172.

Cassidy, S. (2011). Self-regulated learning in higher education: identifying key component processes. *Studies in Higher Education*, 36:8, 989-1000. http://dx.doi.org/10.1080/03075079.2010.503269

Chatzistamatiou, M., Dermitzaki, I., Efklides, A. & Leondari, A. (2013). Motivational and affective determinants of self-regulatory strategy use in elementary school mathematics. *Educational Psychology*, 35:7 835-850. http://dx.doi.org/10.1080/01443410.2013.822960

Chin, C. (2004). Students' questions: Fostering a culture of inquisitiveness in science classrooms. *School Science Review*, 86:314, 107–112.

Chin, C. & Brown, D. E. (2002). Student-generated questions: A meaningful aspect of learning in science. *International Journal of Science Education*, 24:5, 521-549, DOI: 10.1080/09500690110095249

Chin, C., & Chia, L.G. (2004). Problem-based learning: Using students' questions to drive knowledge construction. *Science Education*, 88, 707–727.

Cleary, T. J. & Zimmerman, B. J. (2004). Self-Regulation Empowerment Program: A School-Based Program to Enhance Self-Regulated and Self-Motivated Cycles of Student Learning. *Psychology in the Schools*, 41:5, 537-550. DOI: 10.1002/pits.10177

Clements, M., & Vaiyavutjamai, P. (2006). Effects of classroom instruction on student performance. *Mathematical Thinking and Learning*, 8:2, 113-147.

Cohors-Fresenborg, E., Kramer, S., Pundsack, F., Sjuts, J., Sommer, N. (2010). *ZDM Mathematics Education*, 42, 231–244. DOI 10.1007/s11858-010-0237-x

Corno, L. (1993). The best laid plans: modern conceptions of volition and educational research. *Educational Researcher*, 22:2, 14-22.

Creswell, J. W. & Piano Clark, V. L. (2007). *Designing and Conducting Mixed Methods Research*. California: Sage Publications.

Creswell, J. W. & Piano Clark, V. L. (2011). *Designing and Conducting Mixed Methods Research*. California: Sage Publications.

Cross, D. I. (2008). Creating Optimal Mathematics Learning Environments: Combining Argumentation And Writing to Enhance Achievement. *International Journal of Science and Mathematics Education*, 7:5, 905-930. doi:10.1007/s10763-008-9144-9

Darr, C. & Fisher, J. (2004). Self-Regulated Learning in the Mathematics Class. Proceedings from *NZARE Conference, Turning the Kaleidoscope*. Wellington

Deci, E. L., Ryan, R. M. (1985). *Intrinsic Motivation and Self-Determination in Human Behavior*. New York: Plenum Press.

Dignath, C. & Büttner, G. (2008). Components of fostering self-regulated learning among students. A meta-analysis on intervention studies at primary and secondary school level. *Metacognition Learning*, 3, 231–264. DOI 10.1007/s11409-008-9029-x

Dillon, J. T. (1988). The remedial status of student questioning. *Journal of Curriculum Studies*, 20:3, 197–210.

Duckworth, K. R., Ackerman, A., MacGregor, E., Salter, and J. Vorhaus (2009). Self-regulated learning: A literature review. Centre for Research on the Wider Benefits of

Learning, Institute of Education, University of London. http://www.dcsf.gov.uk/research/data/ uploadfiles/WBL33.pdf

Efklides, A. (2011). Interactions of Metacognition with Motivation and Affect in Self-Regulated Learning: The MASRL Model. *Educational Psychologist*, 46:1, 6-25. http://dx.doi.org/10.1080/00461520.2011.538645

Eisenberg, T. (1991). Functions and associated difficulties. In D. Tall (Ed.), *Advanced mathematical thinking*. Boston, MA: Kluwer Academic Publishers.

Eisenberg, T. (1992). On the development of a sense for functions. In G. Harel, & E. Dubinsky (Eds.). *The concept of function: Aspects of epistemology and pedagogy, MAA notes 25*, (pp. 153-174). Washington, D.C.: Mathematical Association of America.

Ericsson, K. A., & Simon, H. A. (1993). *Protocol analysis: Verbal report as data.* Cambridge, Massachusetts. London, England: The MIT Press

Erturan-İlker, G., Arslan, Y., Demirhan, G. (2014). Öğrenmeye İlişkin Motivasyonel Stratejiler Ölçeği'nin geçerlik ve güvenirlik çalışması. *Kuram ve Uygulamada Eğitim Bilimleri*, 14:3, 821-833.

Fadlelmula, F.K., Cakiroglu, E. & Sungur, S. (2015). Developing a Structural Model on the Relationship among Motivational Beliefs, Self-Regulated Learning Strategies, and Achievement in Mathematics. *International Journal of Science and Mathematics Education*, 13:6, 1355-1375. doi:10.1007/s10763-013-9499-4

Finau, T., Treagust, D.F., Won, M., Chandrasegaran, A. L. (2016). Effects of a Mathematics Cognitive Acceleration Program on Student Achievement and Motivation. *International Journal of Science and Mathematics Education*, 1-20. doi:10.1007/s10763-016-9763-5

Flavell, J. H. (1979). Metacognition and cognitive monitoring. *American Psychologist*, 34, 906–911.

Fraenkel, J. R. & Wallen, N. E. (2003). *How to Design and Evaluate Research in Education*. New York: McGraw-Hill Companies.

Garcia, G. E., & Pearson, P. D. (1990). *Modifying reading instruction to maximize its effectiveness for all students* (Tech. Rep. No. 489). Champaign, IL: University of Illinois, Center for the Study of Reading.

Ge, X., Land, S. M. (2003). Scaffolding Students' Problem-Solving Processes in an Ill-Structured Task Using Question Prompts and Peer Interactions. *Educational Technology Research and Development*, 51:1, 21–38. https://doi.org/10.1007/BF02504515

Graesser, A.C., & Olde, B.A. (2003). How does one know whether a person understands a device? The quality of the questions the person asks when the device breaks down. *Journal of Educational Psychology*, 95, 524–536.

Greene, J. A. & Azevedo, R. (2009). A macro-level analysis of SRL processes and their relations to the acquisition of a sophisticated mental model of a complex system. *Contemporary Educational Psychology*, 34:1, 18–29. https://doi.org/10.1016/j.cedpsych.2008.05.006

Griethuijsen, R. A. L. F., Eijck, M. W., Haste, H., Brok, P. J., Skinner, N. C., Mansour, N., *et al*. (2014). Global patterns in students' views of science and interest in science. *Research in Science Education*, 45(4), 581–603. doi: 10.1007/s11165-014-9438-6.

Gronlund, N. E. (1988). *How to Construct Achievement Tests*. New Jersey: Prentice-Hall, Inc.

Harel, G., & Dubinsky, E. (1992). Foreward. In E. Dubinsky & G. Harel (Eds.), *The concept of function: Aspects of epistemology and pedagogy* (p. vii). Washington, DC: Mathematical Association of America.

Heckhausen, H. (2008). *Motivation and Action* (electronic resource, edited by Jutta Heckhausen, Heinz Heckhausen). New York : Cambridge University Press.

Heikkilä, A. & Lonka, K. (2006). Studying in higher education: students' approaches to learning, self-regulation, and cognitive strategies. *Studies in Higher Education*, 31:1, 99-117.

Hutchinson, N. L. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. *Learning Disability Quarterly*, 16, 34–63.

Joseph, N. (2009) Metacognition Needed: Teaching Middle and High School Students to Develop Strategic Learning Skills. *Preventing School Failure: Alternative Education for Children and Youth*, 54:2, 99-103, DOI: 10.1080/10459880903217770

Kahraman, N., Sungur, S. (2011). The Contribution of Motivational Beliefs to Students' Metacognitive Strategy Use. *Education and Science*, 36:160, 3-10.

Kalchman, M. S. (2001). *Using a Neo-Piagetian Framework for Learning and Teaching Mathematical Functions*. (Doctorol Thesis), accessed on ProQuest Dissertations Publishing. NQ63668.

Kalchman, M., & Koedinger, K. R. (2005). Teaching and learning functions. In National Research Council (Ed.), *How students learn* (pp. 351-393). Washington, DC: National Academies Press

Kılıç, Ç., Tanrıseven, I. (2012). Öz-düzenleme stratejileri ve motivasyonel inançlar ile standart olmayan sözel problem çözme arasındaki ilişkiler. *Abant İzzet Baysal Üniversitesi Eğitim Fakültesi Dergisi*, 12(1), 167-180.

Kieran, C. (1992). The learning and teaching of school algebra. In A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp.390–419). New York, NY: Macmillan Publishing Co.

King, A. (1992). Comparison of Self-Questioning, Summarizing, and Notetaking-Review as Strategies for Learning from Lectures. *American Educational Research Journal*, 29:2, 303-323.

King, A. (1994). Guiding Knowledge Construction in the Classroom: Effects of Teaching Children How to Question and How to Explain. *American Educational Research Journal*, 31:2, 338-368.

Kintsch, W. (1986). Learning from text. *Cognition and Instruction*, 3(2), 87-108.

Kitsantas A. & Zimmerman, B. J. (2009). College students' homework and academic achievement: The mediating role of self-regulatory beliefs. *Metacognition and Learning*, 4:2, 97-110. doi:10.1007/s11409-008-9028-y

Kramarski, B. & Gutman, M. (2006). How can self-regulated learning be supported in mathematical E-learning environments? *Journal of Computer Assisted Learning*, 22, 24–33.

Kramarski, B., Mevarech, Z. R. (2003). Enhancing Mathematical Reasoning in the Classroom: The Effects of Cooperative Learning and Metacognitive Training. *American Educational Research Journal*, 40:1, 281–310.

Kramarski, B., Mevarech, Z. R., Arami M. (2002). The Effects of Metacognitive Instruction on Solving Mathematical Authentic Tasks. *Educational Studies in Mathematics,* 49, 225–250.

Kramarski, B.,Weisse, I., Kololshi-Minsker, I. (2010). How can self-regulated learning support the problem solving of third-grade students with mathematics anxiety? *ZDM Mathematics Education,* 42, 179–193. DOI 10.1007/s11858-009-0202-8

Kuhl, J. (1985). Volitional mediators of cognitive behavior consistency: Self-regulatory processes and action versus state orientation. In J. Kuhl & J. Beckman (Eds.), *Action control* (pp. 101-128). New York: Springer.

Labuhn, A.S., Zimmerman, B.J. & Hasselhorn, M. (2010) Enhancing students' self-regulation and mathematics performance: the influence of feedback and self-evaluative standards. *Metacognition and Learning*, 5:2, 173-194. doi:10.1007/s11409-010-9056-2

Latham, G. P., Borgogni, L. & Petitta, L. (2008). Goal Setting and Performance Management in the Public Sector. *International Public Management Journal*, 11:4, 385-403. http://dx.doi.org/10.1080/10967490802491087

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, 60(1), 1-64.

Long, Y. & Aleven, V. (2013). Active Learners: Redesigning an Intelligent Tutoring System to Support Self-regulated Learning. In: Hernández-Leo D., Ley T., Klamma R., Harrer A. (eds) Scaling up Learning for Sustained Impact. EC-TEL 2013. Lecture Notes in Computer Science, vol 8095. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-40814-4_44

Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research & Practice*, 15, 10–21.

Maccini, P., & Ruhl, K. L. (2000). Effects of a graduated instructional sequence on the algebraic subtraction of integers by secondary students with learning disabilities. *Education and Treatment of Children*, 23, 465–489.

Marbach-Ad, G., & P.G. Sokolove (2000). Can undergraduate biology students learn to ask higher level questions? *Journal of Research in Science Teaching,* 37:8, 854–870.

Marcou, A. & Philippou, G. (2005). Motivational beliefs, self-regulated learning and mathematical problem solving. *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education*, *Australia*, *3*, 297-304.

Maqsud, M. (1998). Effects of metacognitive instruction on mathematics achievement and attitude towards mathematics of low mathematics achievers. *Educational Research*, 40:2, 237-243. http://dx.doi.org/10.1080/0013188980400210

Mayer, R. E. (1980). Elaboration techniques that increase the meaningfulness of technical text; An experimental test of the learning strategy hypothesis. *Journal of Educational Psychology*, 770-784.

Mayer, R. E. (1984). Aids to prose comprehension. *Educational Psychologist*, 19, 30-42.

Mercan, E. (2015). *Analyzing the effect of argumentation-based learning approach in teaching functions in terms of different variables*. (Doctoral Thesis), accessed on Ulusal Tez Merkezi. 418246.

Merriam, S. B. & Tisdell, E. J. (2016). *QUALITATIVE RESEARCH: A Guide to Design and Implementation*. (4th ed.) San Francisco: John Wiley & Sons, Inc.

Mertens, D. M. (2005). *Research and Evaluation in Education and Psychology: Integrating Diversity with Quantitative, Qualitative and Mixed Methods*. California: Sage Publications, Inc.

Mevarech, Z. R. (1999) Effects of Metacognitive Training Embedded in Cooperative Settings on Mathematical Problem Solving. *The Journal of Educational Research*, 92:4, 195-205, DOI: 10.1080/00220679909597597

Mevarech, Z. R. & Amrany, C. (2008). Immediate and delayed effects of meta-cognitive instruction on regulation of cognition and mathematics achievement. *Metacognition Learning*, 3, 147–157. DOI 10.1007/s11409-008-9023-3

Mevarech, Z., Fridkin, S. (2006). The effects of IMPROVE on mathematical knowledge, mathematical reasoning and meta-cognition. *Metacognition and Learning*, 1:1, 85–97. doi:10.1007/s11409-006-6584-x

Mevarech, Z. R., Kramarski, B. (1997). IMPROVE: A Multidimensional Method for Teaching Mathematics in Heterogeneous Classrooms. *American Educational Research Journal*, 34:2, 365-394.

Mevarech, Z. R. & Kramarski, B. (2003). The effects of metacognitive training versus worked-out examples on students' mathematical reasoning. *British Journal of Educational Psychology*, 73, 449–471.

Mevarech, Z. R., Terkieltaub, S., Vinberger, T., Nevet, V. (2010). *ZDM Mathematics Education,* 42, 195–203. DOI 10.1007/s11858-010-0244-y

Meyer, B., N. Haywood, D. Sachdev, and S. Faraday. (2008). Independent learning: A literature review. Learning and Skills Network. http://www.dcsf.gov.uk/research/data/uploadfiles/ DCSF-RR051.pdf.

Mohini, M., & Tan, T. (2005). *The use of metacognitive process in learning mathematics.* Paper presented at the annual meeting of The Mathematics Education into the 21st Century Project, Johor Bahru, Malaysia.

Montague, M. (2007). Self-Regulation and Mathematics Instruction. *Learning Disabilities Research & Practice*, 22(1), 75–83.

Montague, M. & Bos, C. (1986). The effect of cognitive strategy training on verbal math problem solving performance of learning disabled adolescents. *Journal of Learning Disabilities*, 19, 26–33.

Morgan, G. A., Leech, N. L., Gloeckner, G. W. & Barrett, K. C. (2013). IBM SPSS for Introductory Statistics. New York: Routledge

Mullis, I.V.S., Martin, M.O., Robitaille, D.F., & Foy, P. (2009). *TIMSS Advanced 2008 International Report: Findings from IEA's Study of Achievement in Advanced Mathematics and Physics in the Final Year of Secondary School*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS Advanced 2015 International Results in Advanced Mathematics and Physics*, accessed on Boston College, TIMSS & PIRLS International Study Center website: http://timssandpirls.bc.edu/timss2015/international-results/advanced/

Nalçalkar, Y., Demir, H., Kaya, M. C., Kaplan, V., Odabaşı, E. (2016). Final Yayınları 10. Sınıf Matematik Soru Bankası. İstanbul: Neşe Basım Yayın

Nancy Joseph (2009) Metacognition Needed: Teaching Middle and High School Students to Develop Strategic Learning Skills. *Preventing School Failure: Alternative Education for Children and Youth*, 54:2, 99-103, DOI: 10.1080/10459880903217770

Nash-Ditzel, S. (2010). Metacognitive Reading Strategies Can Improve Self-Regulation. *Journal of College Reading and Learning*, 40:2, 45-63, DOI: 10.1080/10790195.2010.10850330

Otero, J., & Graesser, A.C. (2001). PREG: elements of a model of question asking. *Cognition and Instruction*, 19(2), 143–175.

Özsoy, G. (2011). An investigation of the relationship between metacognition and mathematics achievement. *Asia Pacific Educ. Rev.*, 12, 227–235. DOI 10.1007/s12564-010-9129-6

Özsoy, G. & Ataman, A. (2009). The effect of metacognitive strategy training on mathematical problem solving achievement. *International Electronic Journal of Elementary Education,* 1:2, 67-82.

Palincsar, A. S., & Brown, A. L. (1984). Reciprocal teaching of comprehension-fostering and comprehension-monitoring activities. *Cognition and Instruction*, 2, 117-175.

Pamuk, S., Sungur, S., Oztekin, C. (2016). A Multilevel Analysis of Students' Science Achievements in Relation to their Self-Regulation, Epistemological Beliefs, Learning Environment Perceptions, and Teachers' Personal Characteristics.

International Journal of Science and Mathematics Education, 1–18. doi:10.1007/s10763-016-9761-7

Pape, S. J. & Smith, C. (2002). Self-Regulating Mathematics Skills. *Theory Into Practice*, 41:2, 93-101. https://doi.org/10.1207/s15430421tip4102_5

Panaoura, A., Michael-Chrysanthou, P., Gagatsis, A., Elia, I. & Philippou, A. (2017). A Structural Model Related to the Understanding of the Concept of Function: Definition and Problem Solving. *International Journal of Science and Mathematics Education*, 15, 723–740. DOI 10.1007/s10763-016-9714-1

Pedrosa de Jesus, H., Teixeira-Dias, J.J.C., & Watts, M. (2003). Questions of chemistry. *International Journal of Science Education*, 25(8), 1015–1034

Perels, F., Dignath, C., Schmitz, B. (2009). Is it possible to improve mathematical achievement by means of self-regulation strategies? Evaluation of an intervention in regular math classes. *European Journal of Psychology of Education*, Vol. XXIV, n° 1, 17-31

Pierce, Rod. (24 Mar 2016). "About Math is Fun". Math Is Fun. http://www.mathsisfun.com/aboutmathsisfun.html, accessed on 12 Sep 2018.

Pilten, P. (2008). *Üstbiliş Stratejileri Öğretiminin İlköğretim Beşinci Sınıf Öğrencilerinin Matematiksel Muhakeme Becerilerine Etkisi*. (Doctoral Dissertation), accessed on Ulusal Tez Merkezi. (214521).

Pintrich, P. R. (1999). The role of motivation in promoting and sustaining self-regulated learning. *International Journal of Educational Research*, 31:6, 459–470. https://doi.org/10.1016/S0883-0355(99)00015-4

Pintrich, P. R. (2000b). The role of goal orientation in self-regulated learn - ing. In M. Boekaerts, P. R. Pintrich, & M. Zeidner (Eds.), Handbook of self-regulation (pp. 451–502). San Diego, CA: Academic.

Pintrich, P. R. & De Groot, E. V. (1990). Motivational and Self-Regulated Learning Components of Classroom Academic Performance. *Journal of Educational Psychology,* 82:1, 33-40.

Pizzini, E.L., & Shepardson, D.P. (1991). Student questioning in the presence of the teacher during problem solving in science. *School Science and Mathematics*, 91, 348–352.

Postelnicu, V. (2011). *Student difficulties with linearity and linear functions and teachers' understanding of student difficulties.* (Doctorol Thesis), accessed on ProQuest Dissertations Publishing. 3450231.

Pressley, M. (1977). Imagery and Children's Learning: Putting the Picture in Developmental Perspective. *Review of Educational Research,* 47(4), pp. 585-622. DOI: 10.2307/1170001

Pressley, M., & Afflerbach, P. (1995). *Verbal protocols of reading: The nature of constructively responsive reading.* Hillsdale: Lawrence Erlbaum Associates

Ratliff, B. K. (1990). *Student Understanding of Function Composition and the Effect of Dynamic Visualization.* (Doctoral Thesis), accessed on ProQuest Dissertations Publishing. 3400904.

Rosário, P., Núñez J. C., Valle, A., González-Pienda J., Lourenço, A. (2013). Grade level, study time, and grade retention and their effects on motivation, self-regulated learning strategies, and mathematics achievement: a structural equation model. *European Journal of Psychology of Education,* 28:4, 1311–1331. doi:10.1007/s10212-012-0167-9

Rosenshine, B., Meister, C., & Chapman, S. (1996). Teaching students to generate questions: A review of the intervention studies. *Review of Educational Research*, 66, 181–221.

Sajka, M. (2003). A secondary school students' understanding of the concept of function—A case study. *Educational Studies in Mathematics*, 53, 229-254.

Santos, F. F. (2003). *Examining the effects of a writing to learn mathematics approach on students' understanding of the function concept*. (Doctoral Thesis), accessed on ProQuest Dissertations Publishing. 3091289.

Scardamalia, M., & Bereiter, C. (1985). Fostering the development of self-regulation in children's knowledge processing. In S. F. Chipman, J. W. Segal, & R. Glaser (Eds.), *Thinking and learning skills: Vol. 2. Research and open questions* (pp. 563-577). Hillsdale, NJ: Erlbaum.

Schmidt, H.G. (1993). Foundations of problem-based learning: Rationale and description. *Medical Education*, 17, 11–16.

Schmitt, N. (1996). Uses and abuses of coefficient alpha. *Psychological Assessment*, 8(4), 350–353. doi: 10.1037/1040-3590.8.4.350.

Schmitz, B. & Perels, F. (2011). Self-monitoring of self-regulation during math homework behaviour using standardized diaries. *Metacognition and Learning*, 6:3, 255–273. doi:10.1007/s11409-011-9076-6

Schoenbach, R., Braunger, J., Greenleaf, C., & Litman, C. (2003). Apprenticing adolescents to reading in subject-area classrooms. Phi Delta Kappan, 85, 133–138.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), Handbook for Research on Mathematics Teaching and Learning (pp. 334-370). New York: MacMillan.

Schraw, G. (1994). The effect of metacognitive knowledge on local and global monitoring. *Contemporary Educational Psychology*, 19, 143–154.

Schraw, G. (1998). Promoting general metacognitive awareness. *Instructional Science*, 26:1-2, 113-125.

Schunk, D. H. (2005). Self-Regulated Learning: The Educational Legacy of Paul R. Pintrich. *Educational Psychologist,* 40:2, 85-94. http://dx.doi.org/10.1207/s15326985ep4002_3

Schunk, D. H. & Zimmerman, D. J. (1998). *Self Regulated Learning: From Teaching to Self-Reflective Practice*. New York, NY : The Guilford Press.

Schunk, D. H. & Zimmerman, D. J. (2009). *Self-Regulated Learning and Academic Ahievement*. New York, NY : Routledge

Shuell, T. J. (1990). Teaching and Learning as Problem Solving. *Theory Into Practice,* 29:2, 102-108.

Semana, S. & Santos, L. (2018). Self-regulation capacity of middle school students in mathematics. *ZDM*, 50, 743–755. https://doi.org/10.1007/s11858-018-0954-0

Sperling, R. A., Walls, R. T., Hill, L. (2000). Early relationships among self-regulatory constructs: theory of mind and preschool children's problem solving. *Child Study Journal*, 30(4), 233-253.

Steele, M., Hillen, A. & Smith, M. (2013). Developing mathematical knowledge for teaching in a methods course: the case of function. *Journal of Mathematics Teacher Education*, 16, 451–482.

Stephen, J. P. & Cynthia, S. (2002). Self-Regulating Mathematics Skills. *Theory Into Practice*, 41:2, 93-101.

Sungur, S. & Tekkaya, C. (2006). Effects of Problem-Based Learning and Traditional Instruction on Self-Regulated Learning. *The Journal of Educational Research*, 99:5, 307-320, DOI: 10.3200/JOER.99.5.307-320

Toledo, C.A. (2006). 'Does your dog bite?' Creating good questions for online discussions. *International Journal of Teaching and Learning in Higher Education*, 18:2, 150–154.

Van Meter, P. N., Firetto, C. M., Turns, S.R., Litzinger, T. A., Cameron, C.E., Shaw, C. W., (2016). Improving Students' Conceptual Reasoning by Prompting Cognitive Operations. *The Research Journal For Engineering Education*, 105:2, 245–277. DOI: 10.1002/jee.20120

De Vaus, D. (2001). *Research Design in Social Research.* London: Sage Publications Ltd.

Veenman, M. V. J., Elshout, J. J., & Busato, V. V. (1994). Metacognitive mediation in learning with computer-based simulations. *Computers in Human Behavior*, 10, 93–106

Veenman, M. V. J., Van Hout-Wolters, B. H. A. M., & Afflerbach, P. (2006). Metacognition and learning: Conceptual and methodological considerations. *Metacognition and Learning*, 1(1), 3–14. DOI 10.1007/s11409-006-6893-0

Velayutham, S., Aldridge, J. M., Fraser, B. (2012). Gender Differences in Student Motivation and Self-Regulation in Science Learning: A Multi-Group Structural Equation Modeling Analysis. *International Journal of Science and Mathematics Education*, 10:6, 1347–1368. doi:10.1007/s10763-012-9339-y

Wagner, S. (1981). Conservation of equation and function under transformations of variable. *Journal for Research in Mathematics Education*, 12(2), 107-118.

Watts, M., Gould, G., & Alsop, S. (1997). Questions of understanding: Categorising pupils' questions in science. *School Science Review*, 79(286), 57–63.

Weiner, B. (1979). A theory of motivation for some classroom experiences. *Journal of Educational Psychology*, 71:1, 3-25.

Weinstein, C. E. & Mayer, R. E. (1983). The Teaching of Learning Strategies. *Innovation Abstracts*, 5:32, 1-4.

Winne, P. H. (1997). Experimenting to Bootstrap Self-Regulated Learning. *Journal of Educational Psychology,* 89:3, 397-410.

Winne, P. H. (2004). Students' calibration of knowledge and learning processes: Implications for designing powerful software learning environments. *International Journal of Educational Research,* 41:6, 466–488. https://doi.org/10.1016/j.ijer.2005.08.012

White, R. T., & Gunstone, R. F. (1992). *Probing understanding.* London, New York: Falmer Press.

Yağdıran, E. (2005). *Teaching functions in the 9th grade maths class of a secondary school with worksheets, vee diagrams and concept maps.* (Master Thesis), accessed on Ulusal Tez Merkezi. 198284

Yıdızlı, H. & Saban, A. (2016). (Bronwyn Frances Ewing (Reviewing Editor) The effect of self-regulated learning on sixth-grade Turkish students' mathematics achievements and motivational beliefs. *Cogent Education,* 3:1. http://dx.doi.org/10.1080/2331186X.2016.1212456

You, Z. (2006). *Preservice teachers' knowledge of linear functions within multiple representation modes.* (Doctoral Thesis), accessed on ProQuest Dissertations Publishing. 3280530.

Zimmerman, B. J., Bonner, S., & Kovach, R. (1996). *Developing self-regulated learners : beyond achievement to self-efficacy.* Washington, DC : American Psychological Association.

Zimmerman, B.J. (1989). A social cognitive view of self-regulated academic learning. *Journal of Educational Psychology*, 81, 329–339.

Zimmerman, B. J. (2002). Becoming a Self-Regulated Learner: An overview. *Theory into Practice*, 41(2): 64–70. http://dx.doi.org/10.1207/s15430421tip4102_2

Zimmerman, B. J. & Martinez-Pons, M. (1988). Construct Validation of a Strategy Model of Student Self-Regulated Learning. *Journal of Educational Psychology*, 80:3, 284-290.

Zimmerman, B. J., & Martinez-Pons, M. (1986). Development of a structural interview for assessing student use of self-regulated learning strategies. *American Educational Research Journal*, 23, 614–628.

Zoller, U. 1987. The fostering of question-asking capability: A meaningful aspect of problem solving in chemistry. *Journal of Chemical Education,* 64:6, 510–512.

APPENDIX A: EXTRA INFORMATION ON MSLQ

There are 44 items in this questionnaire. All them are likert type questions ranging from 1 (I never agree) to 7 (I completely agree). It has five subscale as self-regulation, metacognitive strategy use, self-efficacy, intrinsic value and test anxiety. In purpose of this study, four of them were analyzed except from test anxiety.

Items related to self-regulation focuses on students study habits such as preparing a "to do list". Metacognitive strategy use items are related to students' understanding and learning style about a certain topic like that focusing on main ideas or recalling techniques. Self-efficacy items measure students' self-confidence on learning a certain topic; for example, students' thinking about being successful on any topic. Lastly, intrinsic value is about students' special interest to the lesson without gaining some prizes or being punished. Related items are as "I like to learn this lesson".

APPENDIX B: FONKSİYONLAR BAŞARI TESTİ DEĞERLENDİRME YÖNERGESİ

Fonksiyonlar Başarı Testi Yönergesi

Bu test, 10. sınıf öğrencilerinin fonksiyonlar konusundaki başarısını ölçmek amacıyla hazırlanmıştır. Başarı için Milli Eğitim müfredatında yer alan kazanımları edinip edinmediği ya da ne kadarını edindiği dikkate alınacaktır. Kazanımlara ait soru sayısı belirlenirken kazanım içeriğinin yoğunluğu, sahip olduğu alt başlıklar ve diğer kazanımlar için önkoşul olup olmadığı dikkate alınmıştır. Bazı sorular birden çok kazanımı ölçmeye yöneliktir.

Testi inceleyecek uzmanlardan ricamız, her bir sorunun amaçlanan kazanımı ölçüp ölçmediği, belirlenen kazanımlar dışında bir kazanımı da içerip içermediği, soru sayısı ve zorluk derecesinin uygunluğunu (her düzeyde soru konmaya çalışılmıştır) incelemeleridir. Diğer yorum ve önerilerinizi de tabloya ekleyebilir, veya test üzerinde değişiklik/yorum şeklinde belirtebilirsiniz.

Katkılarınız araştırmamız açısından çok değerli olacaktır, zamanınızı ayırıp görüşlerinizi paylaştığınız için şimdiden çok teşekkür ederiz.

Aşağıdaki tabloda kazanımları ve her kazanıma ait yazılan soruları görebilirsiniz. Yorumlarınızı ilgili sütuna ekleyiniz.

Konu	Kazanımlar	Alt Başlıklar	Soru Sayısı	Soru Numaraları	Kazanımlar ve sorular doğru eşleştirilmiş mi?	Yorum ve önerileriniz
10.2.1. Fonksiyon Kavramı ve Gösterimi	10.2.1.1. Fonksiyonlarla ilgili problemler çözer.	a) Fonksiyon kavramı açıklanır. b) Fonksiyonun özel bir bağıntı olduğu vurgulanır. c) İçine fonksiyon, örten fonksiyon, bire bir fonksiyon, eşit fonksiyon, birim (özdeşlik) fonksiyon, sabit fonksiyon, doğrusal fonksiyon, tek fonksiyon, çift fonksiyon ve parçalı tanımlı fonksiyon açıklanır. ç) İki fonksiyonun eşitliği örneklerle açıklanır. d) f ve g fonksiyonları kullanılarak $f+g$, $f-g$, $f.g$, $f:g$ işlemleri yapılır. e) Gerçek hayat problemlerine ve tablo-grafik kullanımına yer verilir.	11	1, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15		
	10.2.1.2. Fonksiyonların grafiklerini çizer.	a) f(x) = ax + b şeklindeki fonksiyonların grafikleri ile ilgili uygulamalar yapılır. b) $f(x) = ax^n$ ($n \in \{1, 2, 3, -1\}$) fonksiyonlarının grafikleri değer tablosu ve/veya dinamik geometri programları kullanılarak çizdirilir. c) Parçalı tanımlı şekilde verilen fonksiyonların grafikleri çizilir. ç) f(x) = ax + b tipindeki fonksiyonların grafiği bilgi ve iletişim teknolojileri yardımıyla çizilerek a ve b katsayıları ile fonksiyon grafiği arasındaki ilişki ele alınır.	2	3, 4, 10		
	10.2.1.3. Fonksiyonların grafiklerini yorumlar.	a) Grafiği verilen fonksiyonların tanım, görüntü ve ters görüntü kümeleri gösterilir. Tanım kümesinin bir alt kümesinin görüntüsü ve değer kümesinin bir alt kümesinin ters görüntüsü bulunur. b) Bir fonksiyon grafiğinde, fonksiyonun x ekseni üzerinde tanımlı olduğu her bir noktadan y eksenine paralel çizilen doğruların, grafiği yalnızca bir noktada kestiğine (düşey/dikey doğru testi) işaret edilir. c) Bir f fonksiyonunun grafiğinin x eksenini kestiği noktaların f(x) = 0 denkleminin kökleri olduğu gösterilir, grafik kullanılarak $f(x) > 0$ ve $f(x) < 0$ eşitsizliklerinin çözüm kümeleri bulunur.	8	2, 3, 5, 6, 13, 15, 19,20		
	10.2.1.4. Gerçek hayat durumlarından		4	4, 5, 6, 14		

	doğrusal fonksiyonlarla ifade edilebilenlerin grafik gösterimlerini yapar.					
10.2.2. İki Fonksiyonun Bileşkesi ve Bir Fonksiyonun Tersi	10.2.2.1. Bire bir ve örten fonksiyonlar ile ilgili uygulamalar yapar.	a) Bir fonksiyonun bire bir ve örtenliği grafik üzerinde yatay doğru testiyle incelenir ve cebirsel olarak ilişkilendirilir. b) Bilgi ve iletişim teknolojileri yardımıyla bir fonksiyonun bire bir ve örten olup olmadığı belirlenir.	1	16		
	10.2.2.2. Fonksiyonlarda bileşke işlemiyle ilgili işlemler yapar.	a) Bileşke işlemi, fonksiyonların cebirsel ve grafik gösterimleri ile ilişkilendirilerek ele alınır. b) Fonksiyonlarda bileşke işleminin birleşme özelliğinin olduğu belirtilir, değişme özelliğinin olmadığı örneklerle gösterilir. c) Parçalı tanımlı fonksiyonların bileşkesine girilmez.	2	17, 20		
	10.2.2.3. Verilen bir fonksiyonun tersini bulur.	a) Bir fonksiyonun tersinin de fonksiyon olması için gerekli şartlar belirtilir. b) Sadece bire bir ve örten doğrusal fonksiyonun tersinin grafiği çizilir; fonksiyonun grafiği ile tersinin grafiğinin y=x doğrusuna göre simetrik olduğu gösterilir.	2	18, 20		

APPENDIX C: ACHIEVEMENT TEST ON FUNCTIONS

Fonksiyonlar Konu Ölçme Testi

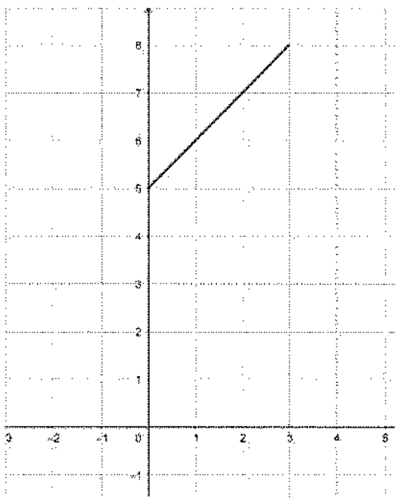
Adı Soyadı:

1. Aşağıda verilen örüntüye göre soruları cevaplayınız.
   2, 5, 8, 11, 14, 17,...

   a) Örüntünün 15. terimini bulunuz.
   b) Örüntüdeki n. terimi bulmak için bir kural yazınız.

2. (2, -8) noktası, y = 3x – 14 doğrusunun grafiği üzerinde midir? Açıklamanızı ayrıntılı olarak yazınız?

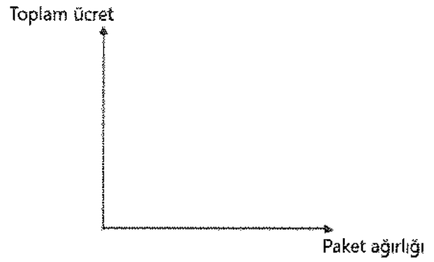
3. Aşağıdaki grafikte bir parçası verilen doğruya ait denklemi yazınız. Her bir adımı açıkça gösteriniz.



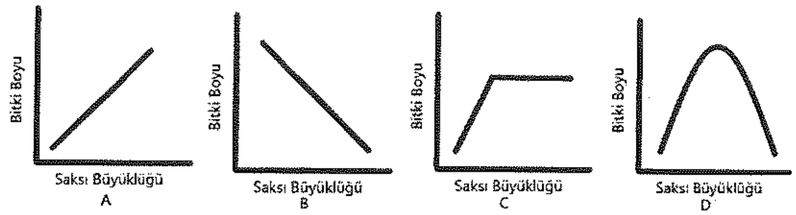
4. Bir kargo şirketinde, bir paket göndermenin sabit ücreti 5 liradır ve her bir kilogram için 3 lira alınmaktadır. Paket ağırlığı ve ödenmesi gereken ücret ile ilgili aşağıdaki tablonun boşluklarını doldurunuz. Çözümünüzün her adımını açıkça gösteriniz.

Paket ağırlığı (kg)	Toplam ücret (tl)
2	?
?	14
4	?
?	20

Paket ağırlığı ve toplam ücret arasındaki ilişkiyi gösteren grafiği çiziniz.

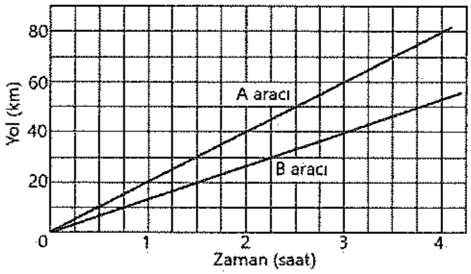


5. Aşağıdaki cümleleri en iyi tanımlayan grafiği seçiniz.



a. Saksı büyüklüğü arttıkça, bitki boyu kısalır: ....
b. Saksı büyüklüğü arttıkça, bitki boyu belirli bir noktaya kadar artar. Daha büyük saksılarda bitki boyu aynı kalır: ....

6. Aşağıdaki grafiği inceleyerek soruları yanıtlayınız.



a. B aracının 50 km gidebilmesi, A aracının 50 km gidebilmesinden ne kadar daha uzun sürer? Her bir adımınızı açıkça gösteriniz.
b. Harekete başladıktan 3 saat sonra A aracı, B aracının kaç km önündedir? Çözümünüzü açıklayınız.

7. $f(x) = \frac{2x-a}{x-4}$ fonksiyonu sabit fonksiyon olduğuna göre a değerini bulunuz. Adımlarınızı açıkça gösteriniz.

8. f(x) = 3x + (a + b)x + a – 2 fonksiyonu birim fonksiyon olduğuna göre a ve b değerlerini bulunuz. Adımlarınızı açıkça gösteriniz.

9. Gerçek sayılar kümesi üzerinde tanımlı doğrusal f(x) fonksiyonu için f(1) = 7 ve $f^{-1}(1) = -1$'dir. Buna göre f(0) değeri kaçtır? Adımlarınızı açıkça gösteriniz.

10. $f(x) = \begin{cases} x+1, & x<2 \\ -2x+1, & x \geq 2 \end{cases}$ fonksiyonunun grafiğini çiziniz ve $A = \{-1, 2, 3\}$ kümesinin verilen fonksiyon altındaki görüntü kümesini bulunuz. Adımlarınızı açıkça gösteriniz.

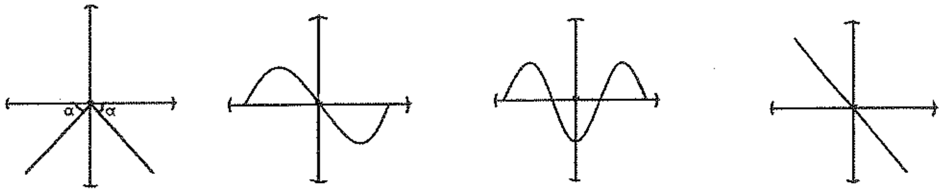
11. $f(x) = x^2 + 5$ ve $g(x) = \sqrt{1-x}$ olmak üzere;

a. (f + g)(x) fonksiyonunu yazınız. Çözümünüzü açıklayınız.
b. f + g fonksiyonunun tanım kümesini bulunuz. Çözümünüzü açıklayınız.

12. $g(x) = x\sqrt{1-x^2}$ fonksiyonunun tanımlı olduğu aralıklarda tek mi, çift mi, yoksa bu konuda bir yargıya varılamaz mı olduğunu açıklayınız.

13. Aşağıdaki grafiklerde verilen fonksiyonların tek mi çift mi olduğuna karar veriniz.

Kararınızın nedenini her bir grafik için açıklayınız.

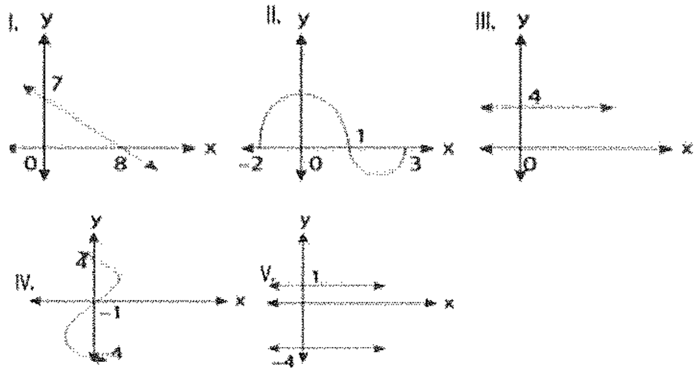


....

14. İdeal bir kekin yapımında kullanılması gereken un miktarını (y), şeker miktarına (x) bağlı olarak veren fonksiyon f(x) = 4x + 3 şeklindedir.

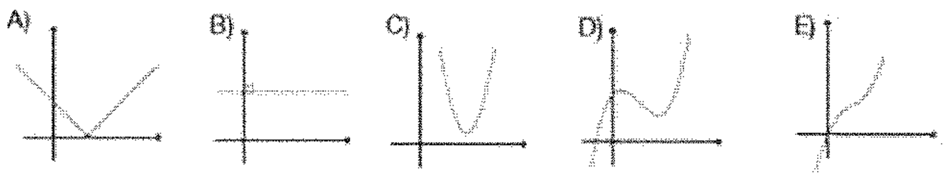
Buna göre, toplam 23 kg un ve şeker bulunan bir kekte, kaç kg un vardır? Adımlarınızı açıkça gösteriniz.

15. Aşağıda verilen grafiklerden hangisi/hangileri bir fonksiyon grafiği olabilir? Düşündüğünüz seçenekleri daire içine alınız ve nedenini açıklayınız.



Neden:

16. Aşağıdaki grafiklerden hangisi/hangileri birebir ve örten bir fonksiyon grafiği olabilir? Nedenini açıklayınız.



Neden:

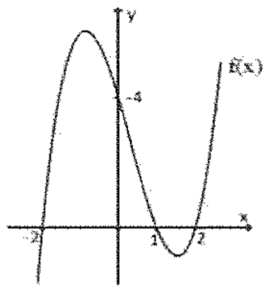
17. $f(x) = x^2 + 2x$ ve $(fog)(x) = x^2 + 6x + 8$ olduğuna göre $g(x)$ fonksiyonunu bulunuz. Adımlarınızı açıkça gösteriniz.

18. $f(x): \left[-\frac{1}{2}, \infty\right) \to [0, \infty)$ $f(x) = \sqrt{2x+1}$ fonksiyonu veriliyor.

a. $f(x)$ fonksiyonunun tersi alınabilir mi inceleyiniz. Çözümünüzü açıklayınız.

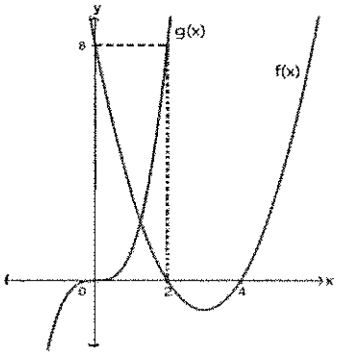
b. Tersi alınabiliyorsa $f^{-1}(x)$ fonksiyonunu bulunuz ve çözümü açıklayınız. Tersi alınamıyorsa fonksiyonu tersini alabileceğiniz şekilde düzenleyiniz. Çözümünüzü açıklayınız.

19. Aşağıdaki şekilde f(x) fonksiyonunun grafiği verilmiştir.



Buna göre, f(x – 2) fonksiyonunu sıfır yapan x değerlerini bulunuz ve çözümü açıklayınız.

20.



Yukarıdaki şekilde, f(x) ve g(x) fonksiyonlarının grafikleri verilmiştir.
Buna göre,

a) $(g^{-1})(8)$ değeri kaçtır? Çözümünüzü açıklayınız.

b) $(fof)(2)$ değeri kaçtır? Adımlarınızı açıkça gösteriniz.

APPENDIX D: RUBRIC FOR ACHIEVEMENT TEST

Başarı testi için Puanlama Cetveli

Öğrenci:

Soru No	Toplam Puan	Verilmesi gereken puan	Yönergeler
1a	2	0	Tamamen yanlış bir yol izlemiş veya boş bırakmıştır.
		1	Sayarak veya örüntü kuralını izleyerek sonuç bulmayı denemiş ancak işlem hatası yapmıştır.
		2	Sayarak veya örüntü kuralıyla doğru cevabı bulmuştur
1b	2	0	Tamamen yanlış bir yol izlemiş veya boş bırakmıştır.
		1	Kurala yaklaşmış ama hatalı bulmuştur.
		2	Örüntü kuralını tam ve doğru bir şekilde yazmıştır.
2	3	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	Noktanın denklemi sağlaması gerektiğini düşünmüş ancak noktanın koordinatlarını denkleme ters yerleştirmiştir.
		2	Noktanın denklemi sağlaması gerektiğini düşünmüş ve noktanın koordinatlarını denkleme doğru yerleştirmiştir, ancak işlem hatası yapmıştır.
		3	Noktanın denklemi sağlaması gerektiğini düşünmüş ve noktanın koordinatlarını denkleme doğru yerleştirmiştir. Noktanın grafik üzerinde olduğuna karar vermiştir.
3	3	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	• Noktaların koordinatlarını gösteren tabloyu oluşturmuş veya eğimi incelemiştir. • Hiç açıklama olmadan bir şekilde doğru denklemi yazmıştır.

		2	Noktaların koordinatlarının birbirleriyle ilişkisini keşfetmiştir veya eğim ile denklemin ilişkisini kurmuştur. Ancak çeşitli sebeplerle denklemi doğru yazamamıştır.
		3	Denklemi, koordinatların ilişkisi veya eğim yardımıyla tam ve doğru şekilde yazmıştır.
4tablo	5	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	Toplam ücret ve paket ağırlığı arasındaki ilişkiyi matematiksel olarak doğru kavramış ancak işlem hatalarından dolayı tabloyu dolduramamıştır.
		2	Toplam ücret ve paket ağırlığı arasındaki ilişkiyi matematiksel olarak doğru kavramış ancak işlem hatalarından dolayı sadece 1 boşluğu doğru bulmuştur.
		3	Toplam ücret ve paket ağırlığı arasındaki ilişkiyi matematiksel olarak doğru kavramış ancak işlem hatalarından dolayı sadece 2 boşluğu doğru bulmuştur.
		4	Toplam ücret ve paket ağırlığı arasındaki ilişkiyi matematiksel olarak doğru kavramış ancak işlem hatalarından dolayı 3 boşluğu doğru bulmuştur.
		5	Toplam ücret ve paket ağırlığı arasındaki ilişkiyi matematiksel olarak doğru kavramış ve tabloyu tam ve doğru şekilde doldurmuştur.
4grafik	3	0	Grafik çizimini boş bırakmış veya soruyla ilgisiz karalamalar yapmıştır.
		1	• Noktaları eksenlere doğru yerleştirmiş ancak doğru çizimi yapamamıştır. • Doğru çizimi yapmasına rağmen noktaları ters eksene yerleştirmiş veya hiç yerleştirmemiştir veya yanlış noktalar yerleştirmiştir.
		2	Noktaları eksenlere doğru yerleştirmiş, noktaları birleştiren doğruyu çizmiş fakat başlangıç noktasını doğru gösterememiştir.
		3	Grafiği tam ve doğru şekilde çizmiştir.
5a	1	0	Yanlış seçeneği yazmıştır.
		1	Doğru seçeneği yazmıştır
5b	1	0	Yanlış seçeneği yazmıştır.
		1	Doğru seçeneği yazmıştır.

6a	7	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	X = v.t formülüyle araçlardan **birinin hızını** bulmuştur veya grafik üzerinden bir aracın 50 km gidebilmesi için **tahmini** süre aralığını yazmıştır.
		2	• X = v.t formülüyle araçların **ikisinin de hızını** bulmuştur veya grafik üzerinden iki aracın da 50 km gidebilmesi için **tahmini** süre aralığını yazmıştır. • X = v.t formülüyle bir aracın 50 km gidebilmesinin kaç saat süreceğini hesaplamıştır veya doğrudan grafik üzerinden bu süreyi işaretlemişlerdir ancak ondalıklı sayı **gösterim hatası** yapmıştır.
		4	• X = v.t formülüyle **iki aracın** da 50 km gidebilmesinin kaç saat süreceğini hesaplamıştır ancak ondalıklı sayı **gösterim hatası** yapmıştır. • İki aracın da 50 km gidebilmesinin kaç saat süreceğini doğrudan grafik üzerinden işaretlemişlerdir ancak ondalıklı sayı **gösterim hatası** yapmıştır. • X = v.t formülüyle bir aracın 50 km gidebilmesinin kaç saat süreceğini hesaplamıştır veya doğrudan grafik üzerinden bu süreyi işaretlemişlerdir ve de doğru gösterimle yazmıştır.
		5	• X = v.t formülüyle **iki aracın** da araçların 50 km gidebilmelerinin kaç saat süreceğini hesaplamışlar veya doğrudan grafik üzerinden bu süreleri işaretlemişlerdir ve **doğru gösterimle** yazmışlardır. • X = v.t formülüyle iki aracın da araçların 50 km gidebilmelerinin kaç saat süreceğini hesaplamışlar veya doğrudan grafik üzerinden bu süreleri işaretlemişlerdir ancak ondalıklı sayı **gösterim hatası** yapmıştır. Devamında sürelerinin farkını da bu yanlış gösterimle hesaplamıştır.
		6	Sürelerin farkını doğru gösterimlerle hesaplamayı denemişler ancak işlem hatası vb yapmışlardır.
		7	Soruyu tam ve doğru çözmüşlerdir.
6b	3	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	Açıklama yapmadan doğru sonucu yazmıştır.
		2	3 saat sonra A ve B araçlarının aldıkları yolları grafik üzerinden doğru bulmuşlardır.
		3	3 saat sonra A ve B araçlarının aldıkları yolların farkını doğru bulmuşlardır.
7	5	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	Sabit fonksiyon tanımını bilir.
		2	Sabit fonksiyon olabilmesi için payın, paydanın tam katı olması gerektiğini fark etmiştir.
		3	Sabit fonksiyon olabilmesi için payın, paydanın 2 katı olması gerektiğini fark etmiştir veya x'in katsayıları oranı ile sabit terimlerin oranını eşitlemiştir.

		4	a bilinmeyeninin 4'ün 2 katı olması gerektiğini anlamıştır veya doğru sonucu bulmasına karşın sabit fonksiyonu net olarak tanımlayamamıştır.
		5	Soruyu tam ve doğru şekilde çözerek a bilinmeyenini 8 olarak bulmuştur.
8	5	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	Birim fonksiyon olması için x'in katsayısının 1 olması veya sabit terimin sıfır olması gerektiğini anlamıştır.
		2	Birim fonksiyon olması için x'in katsayısının 1 olması ve sabit terimin sıfır olması gerektiğini anlamıştır.
		3	a'yı doğru hesaplamıştır veya x'in katsayılarını paranteze almıştır.
		4	a'yı doğru hesaplamıştır ve x'in katsayılarını paranteze almıştır.
		5	Hem a hem b'yi doğru hesaplamıştır.
9	8	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	• Doğrusal fonksiyon denkleminin f(x) = ax + b şeklinde olması gerektiğini bilir. • Fonksiyonun eşit tanım aralıklarında eşit oranda değişmesi gerektiğini bilir.
		2	• f(1) = 7 bilgisini doğru kullanabilmiştir. • $f^{-1}(1)=-1$ bilgisini doğru kullanabilmiştir.
		3	f(1) = 7 ve $f^{-1}(1) = -1$ bilgilerini doğru kullanabilmiştir.
		4	• Önceki adımda elde ettiği denklemleri birbirleriyle ilişkilendirmek istemiş fakat hatalı sonuç bulmuştur. • Fonksiyonun eşit tanım aralıklarında hangi oranda değişmesi gerektiğini doğru bulmuştur.
		5	Önceki adımda elde ettiği denklemleri birbirleriyle ilişkilendirmiş ve a ve b'den birini doğru bulmuştur.
		6	• Önceki adımda elde ettiği denklemleri birbirleriyle ilişkilendirmiş ve a ve b'nin her ikisini de doğru bulmuştur. • Bulduğu oranı f(0) için kullanmaya çalışmıştır.
		7	f(0) değerini doğru yolla bulmaya çalışmış ancak a veya b için hatalı işlem yaptığından ötürü sonucu yanlış hesaplamıştır.
		8	f(0) değerini doğru hesaplamıştır.

10grafik	7	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		2	$x<2$ veya $x\geq2$ durumları için x ve y'nin birbirine bağlı değerlerini gösteren tabloları oluşturabilmiştir.
		4	Tabloda bulunan koordinatlar eksenlere doğru yerleştirilmiş ancak doğruların başlangıç veya bitiş noktaları doğru gösterilmemiştir.
		7	Grafik tam ve doğru şekilde çizilmiştir.
10görüntü kümesi	7	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	A kümesinin elemanlarını x yerine koyarak y değerlerini bulması gerektiğini anlamıştır.
		4	A kümesinin elemanlarını doğru aralıklarda x yerine koyabilmiştir.
		6	İşlem yapmamasına rağmen görüntü kümesini doğru yazmıştır.
		7	Görüntü kümesi elemanlarını doğru hesaplayabilmiş ve açıklamasını yazmıştır.
11a	2	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	Fonksiyonu doğru şekilde yazmış ancak açıklama getirememiştir.
		2	Fonksiyonu doğru şekilde yazmış ve açıklamıştır.
11b	2	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	Kökün içinin 0'dan büyük veya eşit olması gerektiğini göstermiş ancak işlem hatası yapmıştır.
		2	Tanım kümesini tam ve doğru şekilde gösterebilmiştir.
12	5	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		2	Tek ve çift fonksiyon olma koşulu ifade etmiştir.
		4	g(x) fonksiyonunu her iki koşul için de doğru incelemiştir.
		5	İncelemeleri sonucu tek fonksiyon olduğuna karar vermiştir.
13	8	0	Boş bırakılmış, yanlış cevap verilmiş veya doğru cevabın dayanağı olmadığını söylemiştir.

		4	Her doğru cevap 1 puan
		4	Her doğru açıklama 1 puan
14	4	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	x ve f(x) toplamının 23 olduğu fark edilmiştir.
		2	Önceki adıma yazılan denklemden x doğru olarak hesaplanmıştır.
		3	x değeri yerine koyarak f(x) bulunmuş ancak bu adımda veya önceki adımda yapılan işlem hatasından ötürü yanlış sonuç bulunmuştur.
		4	f(x) doğru olarak hesaplanmıştır.
15	7	0	Boş bırakmış veya tamamen yanlış seçenekler işaretlenmiştir.
		1	1 doğru seçenek ile yanlış seçenekler işaretlenmiştir.
		2	2 doğru seçenek ile yanlış seçenekler işaretlenmiştir veya sadece 1 doğru seçenek işaretlenmiştir.
		3	3 doğru seçenek ile yanlış seçenekler işaretlenmiştir veya sadece 2 doğru seçenek işaretlenmiştir.
		4	Sadece 3 doğru seçenek işaretlenmiştir.
		7	Neden için fonksiyon tanımı tam ve doğru ifade edilmiştir. (doğru cevap seçenekleri eksik olmasına rağmen doğru açıklama yapılmışsa +1 puan eklenir. Doğru cevapları vermiş ve nedenlerini de açıklamaya çalışmış ancak eksik açıklama yapılmışsa +2 puan eklenir.)
16	4	0	Boş bırakmış veya tamamen yanlış seçenekler işaretlenmiştir.
		1	1 doğru seçenek ile yanlış seçenekler işaretlenmiştir.
		2	Sadece doğru seçenek işaretlenmiştir.
		4	Neden için birebir ve örtenlik şartları tam ve doğru ifade edilmiştir.
17	4	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	Bileşke fonksiyon doğru tanımlanmıştır.
		2	Sorunun bir kısmı doğru çözülmüş ancak devamı getirilememiştir.

		3	Sorunun tamamı doğru yolla çözülmüş ancak işlem hatası vb sebeplerden doğru sonuca ulaşılamamıştır.
		4	Soru tam ve doğru şekilde çözülmüştür.
18	6	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	• Fonksiyonun birebir veya örten olduğunu incelemiş ancak yanlış sonuçlara varmıştır. • Birebir ve örten olması gerektiğini söylemesine rağmen soru üzerinde incelememiştir.
		2	Fonksiyonun birebir veya örten olduğunu incelemiş ve doğru sonuçlara varmıştır. Fonksiyonun birebir ve örten olduğunu incelemiş ancak yanlış sonuçlara varmıştır.
		3	Fonksiyonun birebir ve örten olduğunu incelemiş ancak sadece biri için doğru sonuca varmıştır.
		4	Fonksiyonun birebir ve örten olduğunu incelemiş ve doğru sonuçlara varmıştır.
		5	Fonksiyonun tersini bulmayı denemiş ancak çözüm yarım kalmış veya işlem hatası yapılmıştır.
		6	Fonksiyonun tersini tam ve doğru şekilde bulmuştur.
19	5	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	Verilen fonksiyonun 0 olabileceği durumu doğru yorumlamıştır.
		2	Şartı sağlayan bir x değeri bulunmuştur.
		3	Şartı sağlayan iki x değeri bulunmuştur.
		4	Şartı sağlayan üç x değeri de bulunmuştur.
		5	Çözüme mantıklı bir açıklama yazabilmiştir.
20a	2	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.
		1	Cevap doğru bulunmuş ancak açıklanamamıştır.
		2	Cevap doğru bulunmuş ve açıklanmıştır.
20b	2	0	Boş bırakmış veya tamamen soruyla ilgisiz cevaplar yazmıştır.

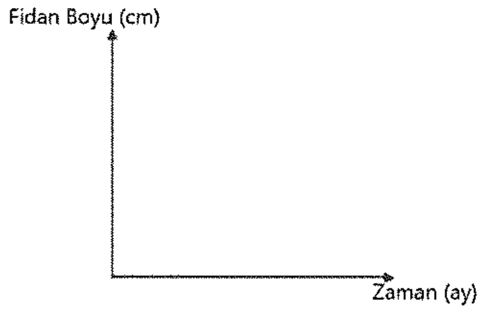
		1	Sorunun bir kısmı doğru çözülmüştür.
		2	Soru tam ve doğru olarak çözülmüştür.

APPENDIX E: FIRST INTERVIEW PROTOCOL

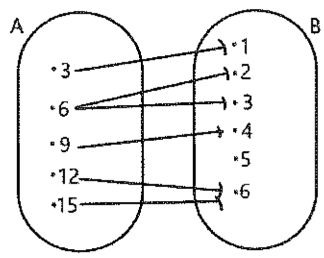
1. İlk alındığında boyu 60 cm olan bir fidanın her yıl 15 cm uzadığı gözlenmiştir. Fidan boyu ve geçen zaman arasındaki ilişkiyi gösteren aşağıdaki tabloyu doldurunuz. Çözümünüzü açık ve net olarak anlatınız.

Zaman (ay)	Fidan boyu (cm)
4	?
?	70
12	?
?	80
20	?

Geçen zaman ve fidan boyu arasındaki ilişkiyi gösteren grafiği çiziniz.



2.



A ve B kümeleri arasındaki eşleşme bir fonksiyon mudur? Neden? Neden değildir?

a. Fonksiyon olduğunu düşünüyorsanız tanım, değer ve görüntü kümelerini yazınız.

b. Fonksiyon olmadığını düşünüyorsanız fonksiyon olabilmesi için neyi değiştirirdiniz? Değiştirdikten sonra yazdığınız fonksiyona ait tanım, değer ve görüntü kümelerini yazınız.

3. $f\left(\frac{x+2}{x-1}\right) = \frac{mx+1}{x+1}$ olduğuna göre $f(4)=3$ ise $f(0)$'ın değerini bulunuz.

4. $f(3x - 5) = x^2 + 1$ olduğuna göre $f(4)$ değerini bulunuz.

APPENDIX F: SECOND INTERVIEW PROTOCOL

1. $f: R \rightarrow R, f(x) = (k-2)x^{5-k}$ sabit fonksiyondur. Buna göre k'nın alabileceği değer/değerleri bulunuz.
2. $f: R \rightarrow R, f(x) = (x-2).(x-3)$ şeklindedir. Buna göre $, f(x+1) = 0$ denkleminin çözüm kümesi nedir?
3. $f(x) = \frac{1}{x}$ ve $g(x) = x + 3$ şeklinde tanımlı fonksiyonlar için;

   a. $(fog)(x)$ fonksiyonunu bulunuz.

   b. $(fog)(2)$ değerini bulunuz.

   c. fog fonksiyonunun en geniş tanım kümesini bulunuz.

4. Aşağıdaki tabloya dayanarak $(fog)(3)$ değerini bulunuz.

x	f(x)	g(x)
-2	0	5
-1	6	3
0	4	2
1	-1	1
2	3	-1
3	-2	0

5. Aşağıdaki fonksiyonlardan hangisi/hangileri tanımlı olduğu aralıklarda doğrusal bir fonksiyondur?

   a. $f(x) = \frac{x-3}{2}$

   b. $f(x) = \frac{1}{x}$

   c. $f(x) = x^2$

   d. $f(x) = \sqrt{x}$

   e. $f(x) = |x|$

Deney grubu öğrencileri için ayrıca:

6. Bu çalışma ile birlikte gördüğün matematik öğretiminin daha önceden gördüğün matematik öğretimi arasında ne tür farklar veya benzerlikler var?
7. Bu çalışma ile birlikte gördüğün matematik öğretiminin sana göre avantaj veya dezavantajları var mıydı? Bunlar neler?

APPENDIX G: THIRD INTERVIEW PROTOCOL

1. $f(3^x + 1) = 9^x + 1$ ve $f^{-1}(a) = 4$ olduğuna göre, a kaçtır?

2. $f : R^2 \rightarrow R$

   $f(x, y) = 3x - 2y + 5$

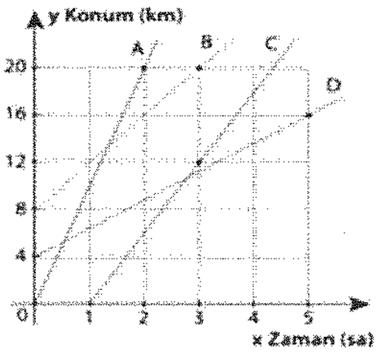
   $f(-2, 4) = f(4, a)$ olduğuna göre, a kaçtır?

3. $f(x + 2) = x - 3$

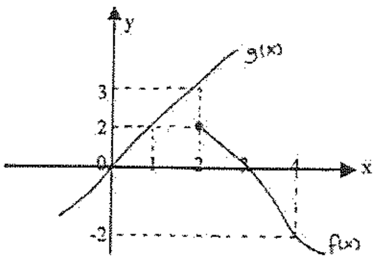
   $(fog^{-1})(x + 1) = 3x - 2$

   olduğuna göre, $g^{-1}(3)$ kaçtır?

4. Aşağıda A, B, C ve D koşucularının konum-zaman grafiği verilmiştir. Buna göre koşucuların zamana bağlı değişim oranlarını (hızlarını) büyükten küçüğe doğru sıralayınız.



5.



Yukarıda $f(x)$ ve $g(x)$ fonksiyonlarının grafikleri verilmiştir. Grafikteki bilgilere göre,

$\frac{(fog)(1)}{f^{-1}(-2)}$ değeri kaçtır?

Deney grubu öğrencileri için ayrıca:

6. Bu çalışma ile birlikte gördüğün matematik öğretiminin daha önceden gördüğün matematik öğretimi arasında ne tür farklar veya benzerlikler var?
7. Bu çalışma ile birlikte gördüğün matematik öğretiminin sana göre avantaj veya dezavantajları var mıydı? Bunlar neler?
8. Size dağıttığım üst bilişsel soru kartlarındaki soruları sürekli kullanıyor musun?
9. Bu soru kartlarında tam olarak nasıl kullanacağımı anlamadım dediğin bir soru var mıydı?

APPENDIX H: CONSENT FORM

Ebeveyn Onam Formu

Araştırma Projesine Katılım için İzin Belgesi

Proje Başlığı: Üst bilişsel strateji tabanlı matematik öğretiminin öğrencilerin öz-düzenleme becerileri ve matematik dersi başarılarına etkisi.

Araştırmacı: Zehra Çoban, Boğaziçi Üniversitesi

Sayın gönüllü,

Ben Boğaziçi Üniversitesi Ortaöğretim Fen ve Matematik Alanları Eğitimi Bölümü yüksek lisans öğrencisiyim. Çocuğunuz yukarıda adı geçen projeye katılmak üzere davetlidir. Bu onam formu size araştırmanın amacı, nasıl yürütüleceği, sizlerden neler beklendiği, faydaları ve riskleri konusunda bilgi vermek için düzenlenmiştir. Çocuğunuzun katılımı gönüllülük esasına dayanmaktadır. Aşağıdaki bilgileri lütfen dikkatlice okuyunuz, sorularınız olursa sorunuz ve açık yanıtlar isteyiniz.

Amaç:

Öğrencilere üst bilişsel strateji tabanlı matematik öğretimi verilirse, onların öz-düzenleme becerilerinin, soru sorma alışkanlıklarının ve başarılarının nasıl etkileneceğini belirlemek.

Üstbiliş kişilerin öğrenme süreçlerini gözlemlemesi ve düzenlemesine yardımcı olur. İki ana ögesi vardır: biliş bilgisi ve bilişi kontrol etme. Biliş bilgisi, öğrenenin kuvvetli ve zayıf yanlarını farkederek kendi öğrenme süreci, çözüm stratejileri ve bu stratejileri nerede nasıl kullanacağı hakkındaki bilgilerini içerir. Bilişi kontrol etme ise planlama, gözlemleme ve performansı değerlendirmeyi içerir.

Öz-düzenleyici öğrenme ise kişinin kendi öğrenme sorumluluğunu alabilmesidir. Akademik becerileri edinmek için kendi öğrenme süreçlerini yöneterek zihinsel becerilerinden faydalandığı süreci ifade eder.

Uygulama Prosedürü:

Araştırma kapsamında fonksiyonlar ünitesi boyunca MEB müfredatını takip ederek konunun daha iyi anlaşılmasını sağlayacağı düşünülen etkinlikler yapılacaktır. Bu ünite boyunca MEB'in hazırlamış olduğu müfredattan fazla ya da eksik konu işlenmeyecektir. Bu çalışmada kullanılacak bilgi kaynakları, öz-düzenleme becerisi anketi ve konu bilgisi için ön test ve son test, etkinlikler sırasında alınacak ses kayıtları, sınıf içi araştırmacı gözlem notları ve bazı öğrencilerle yapılacak birebir görüşmelerden oluşmaktadır.

Projenin Faydaları:

Yurtdışında yapılmış benzer çalışmalar incelendiğinde, bu araştırma sonucunda öğrencilerin soru sorma ve muhakeme yeteneklerinin geliştirileceği, problem çözme için gerekli strateji üretme becerilerinin artacağı ve bunun sonucunda öz-düzenleme ve başarılarının da artacağı düşünülmektedir.

Projenin Riskleri:

Bu çalışmada çocuklar için öngörülen herhangi bir risk bulunmamaktadır.

Gizlilik ve Güvenlik:

Bu projede okul ve öğrencilerin gerçek isimleri yer almayacaktır, bunun yerine kod isimler kullanılacaktır. Kişisel bilgiler, sadece araştırmacının kendisinde bulunacaktır. Bu çalışmada yer almak tümüyle sizin isteğinize bağlıdır. Kararınız çocuğunuzun normal program içinde alacağı eğitim hizmetlerini etkilemeyecektir. Siz çocuğunuzun çalışmaya katılmasına izin verseniz dahi kendisi bunu reddetme hakkına sahiptir. Siz ve çocuğunuz bu formu imzalayarak çalışmaya katılmayı onaylasanız dahi çocuğunuz çalışmanın herhangi bir aşamasında sebep sunmaksızın çekilme hakkına sahiptir. Araştırma süresi içinde araştırmadan çekilmeniz halinde öğrenciyle ilgili veriler kullanılmayacaktır. Sizden elde edilen tüm bilgiler gizli tutulacaktır.

Bu araştırma Boğaziçi Üniversitesi tez jürisi tarafından uygulanmaya uygun bulundu ve Arel Koleji yönetimi ile ders öğretmenlerinden de onay alındı.

İletişim Bilgileri:

Bu projeyle ilgili herhangi bir sorunuz olması durumunda benimle .... adresinden mail yoluyla veya .... telefon numarasıyla ulaşabilirsiniz. Size mümkün olan en kısa zamanda dönüş sağlanacaktır.

Onay Bildirimi ve İmza:

Bu onam formunu okudum ve anladım, projeyle ilgili sorularım giderildi. Çocuğumun projeye katılmasına izin veriyorum. Bu izin formunun bir kopyasının bana sağlanabileceğini biliyorum.

- Araştırmacının, sadece çocuğumla ilgili gözlem, ses kaydı, anket ve test yanıtlarını kullanmasına izin veriyorum.

☐ Evet ☐ Hayır

- Araştırmacının, gözlem, ses kaydı, anket ve test yanıtlarına ek olarak çocuğumla birebir görüşme yapmasına da izin veriyorum.

☐ Evet ☐ Hayır

Ebeveyn

Adı Soyadı Tarih

İmzası

Öğrenci Onam Formu

• Ailem bu projeye katılmama izin verse dahi benim reddetme hakkına sahip olduğumu ve bu formda onaylasam dahi birebir görüşmeyi istediğim anda bırakabileceğimi biliyorum.

• Bu araştırmadaki anket ve testlere katılıp katılmamamın matematik dersi notumu etkilemeyeceğini biliyorum.

• Araştırmacının anket ve testlere vereceğim cevapları kullanacağını biliyorum. Araştırmacının cevaplarıma dayanarak benimle birebir görüşme yapmak isteyebileceğini biliyorum.

• Gerçek ismimin anket ve testlerden silineceğini, bu notlarda sadece kod isim kullanılacağını ve gerçek kimlik bilgilerimin hiç kimseyle paylaşılmayacağını biliyorum.

• Görüşme sırasında alınan ses kayıtlarının Nisan 2019'a kadar yazıya geçirilip sonra silineceğini biliyorum.

- Araştırmacı tarafından tanımlanan projeye katılmayı kabul ediyorum.

☐ Evet ☐ Hayır

- Araştırmacı ile yapılacak birebir görüşmelere katılmayı kabul ediyorum.

☐ Evet ☐ Hayır

Öğrenci
Adı Soyadı Tarih
İmzası

APPENDIX I: EXAMPLES FOR LESSON PLAN ON FUNCTIONS

FONKSİYONLAR

Bugün başlayacağımız konu eşleştirmeler yapmaya dayanıyor. Örneğin; her ayı bir mevsimle eşleştirmemiz gibi veya 7 farklı gömleği olan bir insanın haftanın her bir gününde birini giymesi gibi –her günü bir gömlekle eşleştiriyor- veya her bir öğrencinin bir şubeye yerleştirilmesi gibi. Sizin günlük hayatta gözlemlediğiniz ne tür eşleştirmeler var? (Beyin fırtınası ile sınıf tartışması yapılır ve farklı örnekler verilmiş olur.) Fonksiyonları bizim için bu eşleştirmeyi yapan makineler gibi düşünebilirsiniz. Ancak herhangi bir eşleştirmeye fonksiyon diyebilmemiz için birinin diğerine bağlı olarak değişmesi gerekiyor. Örneğin;

1. Zamanla bir bebeğin ağırlığının değişmesi – burada belli zaman aralıklarıyla bebeğin ağırlığını veren sayıları eşleştirmiş oluyoruz ancak bebeğin ağırlığı zamana bağlı bir değişken olma durumunda. Bu yüzden zaman bağımsız değişken, ağırlık bağımlı değişken olarak adlandırılır.
2. Yapılan işin işçi sayısıyla ilişkisi örneğinde bağımlı ve bağımsız değişkenler öğrencilerle tartışılarak bulunur.

Bir eşleştirmenin fonksiyon olabilmesi için sahip olması gereken diğer özellikleri bir örnek üzerinde görelim. Okulumuzun 10. Sınıf öğrencileri için bir gezi düzenleyeceğimizi varsayalım. Sizi oraya götürmek için otobüslere ihtiyacımız var. Yani tüm öğrencilerin bir araçla eşleşmesi gerekiyor:

Öğrenciler	Araçlar
a	
b	A
c	B
.	C
.	
x	

Şimdi birlikte bu eşleştirme ile ilgili sonuçları yazalım:

1. Herhangi bir öğrenciyi geride bırakmak ister miyiz?
2. Bir öğrenci aynı anda iki araçta olabilir mi?
3. Birkaç öğrenci aynı araçta olabilir mi?
4. Bir araçta sadece bir öğrenci olabilir mi? (Belki bir öğrenci bir öğretmenin arabasına biner)
5. Boşta araç kalabilir mi?

Bu soruların cevabı öğrencilerle tartışılır ve fonksiyonların bu özellikleri taşıyan eşleştirmeler olduğu söylenir. Daha sonra matematiksel tanım verilir.

Figure I.1. Example Lesson Plan: Beginning part of definiton of functions

Örnek.

Bir fonksiyon makinesine giren x'ler 3x – 5 olarak çıkmaktadır. Buna göre fonksiyon makinesinde $A = \left\{-3, 0, \frac{2}{3}\right\}$ kümesinin elemanları girdi için kullanıldığında makineden elde edilen çıktıları bulunuz.

Öğretmen modeller. Sorulması gereken sorular:

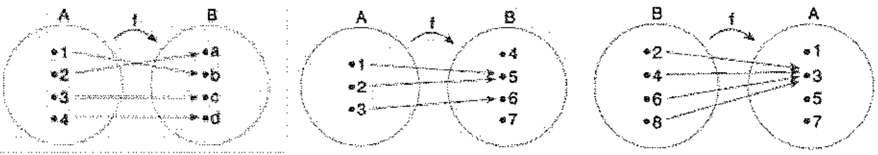
1. Kavramlar :Girdi ve çıktı nedir? -Makinenin içine atılan ve makinenin dışarıya verdiği ürün
2. Ne verilmiş, ne isteniyor? – fonksiyonun kuralı ve girdiler verilmiş çıktılar isteniyor.
3. Bu ne tür bir soru? – Girdiler verildiğinde çıktıların bulunması
4. Girdileri bildiğimiz zaman çıkanları bulmak için nasıl bir strateji izleyebiliriz?

   Her sayıyı 3 katının 5 eksiğine götürüyorsa verilen kümedeki elemanların da 3 katının 5 eksiğini bulmalıyız.

   $f(-3) = 3(-3) - 5 = -14$, $f(0)=3.0-5=-5$, $f(\frac{2}{3})=3.\frac{2}{3}-5=-3$
5. Çözümümüz mantıklı mı? –Bize verilen kuralı izlediğimiz için mantıklı
6. Daha önce çözdüğümüz sorulardan farkı nedir? – Daha önce bir bağıntının fonksiyon olup olmadığını görmüştük şimdi ise fonksiyon olan bir bağıntının çıktılarını bulduk.

Cevaplar öğrencilerle soru cevap yapılarak bulunur.

Örnek: Aşağıdaki fonksiyonların tanım, değer ve görüntü kümelerini bulunuz.



Birincisini metebilişsel soruları modelleyerek öğretmen çözer, diğerlerini aynı şekilde öğrencilerin çözmesi istenir:

1. Tanım, değer ve görüntü kümesi neydi?
2. Ne verilmiş ne isteniyor? – Verilen şekillerdeki kümelerin hangisinin tanım hangisinin değer kümesi olduğuna karar vermemiz gerekiyor.
3. Nasıl bir strateji izleyebilirim? – Ok yönüne bakabiliriz. Geldiği yer tanım, gittiği yer değer kümesi. Değer kümesinde okların karşısına düşen elemanlar ise görüntü kümesi
4. Ne tür bir soru? – tanım ve görüntü kümesi bulma
5. Çözüm mantıklı mı?
6. Önceki sorularla benzerliği/farklılığı nedir? – öncekilerden farklı olarak girenler tanım kümesi olarak adlandırıldı, çıkanlar görüntü kümesi, çıkanların diğer elemanlarla bulunduğu küme değer kümesi olarak adlandırıldı. Önceki sorularla girdiler yani tanım kümesi elemanlarıyla görüntü kümesinin elemanlarını bulma yönünden benzer, değer kümesinin de eklenmesi bakımından farklı bir soru.

Figure I.2. Example Problem Solving

Örnek: A = {0,1,2} ve B = {a,b,c,d} kümeleri için aşağıda verilen fonksiyonların görüntü kümelerini bulunuz.

F: {(0,b),(1,a),(2,d)}   G: {(0,c),(1,c),(2,c)}

Öğrencilerin metabilişsel sorularla çözmesi beklenir:

1. Görüntü kümesi neydi? – fonksiyondan çıkanlar
2. Nasıl bir strateji izleyebiliriz? – eşleştirmelerde ilk eleman tanım kümesindeki girenleri, ikinci eleman çıktıları yani görüntü kümesini ifade eder.
3. Bu ne tür bir soru?
4. Önceki sorularla benzerliği/farkı nedir?
5. Çözüm mantıklı mı?

Örnek: f fonksiyonu tanımlı olduğu değerlerde $y = f(x) = x^2 + 1$ ise f(3), f(2x), f(x -1) ve f(t) ifadelerini bulunuz.

1. Tanımını düşünmemiz gereken kavram var mı? – fonksiyon
2. Ne verilmiş? Ne isteniyor? –Fonksiyon kuralı verilmiş, tanımlanan değerler için görüntü kümesindeki karşılıkları isteniyor.
3. Bu ne tür bir soru? –görüntü kümesi bulma sorusu
4. Verilen ile istenen arasında ilişki kurmak için nasıl bir strateji izleyebiliriz?- girdilere kuralı uygulayarak çıktılarını yani görüntülerini bulabiliriz.
5. Çözüm mantıklı mı? Anlamadığımız bir nokta var mı?
6. Önceki sorularla benzerliği farklılığı nedir? Kuralını ve girdilerini verip görüntülerini istemesi bakımından benzer, kuralın şekilsel verilişi farklı.

Grup Çalışması:

Aşağıda verilen bağıntıların fonksiyon olup olmadığını inceleyiniz. (10 dk)

a) $f:\mathbb{N}\rightarrow\mathbb{N}, f(x)=2x+3$

b) $g:\mathbb{Z}\rightarrow\mathbb{N}, g(x)=\frac{2x}{3}$

c) $h:\mathbb{Z}\rightarrow\mathbb{Z}, h(x)=x^2$

ç) $u:\mathbb{R}\rightarrow\mathbb{R}, u(x)=\frac{1}{x-2}$

Öğrencilerin metabilişsel soruları birbirlerine sorarak tartışarak çözmeleri beklenir. Onlar çalışırken öğretmen arlarında dolaşır ve hiçbir soru ve cevap bulamayan öğrenciler için kendisi modeller. Grup çalışması bittikten sonra öğretmen soruyu gözden geçirir ve tüm sınıf için toparlamasını yapmış olur.

Figure I.2. Example Problem Solving (cont.)

Örnek:

$f: A \rightarrow \mathbb{R},\ f(x) = x^2 + 5x + 6$ fonksiyonunun en geniş tanım kümesini bulunuz.

Öğretmen metabilişsel soruları modelleyerek çözer:

1. Kavramlar: fonksiyon, tanım kümesi
2. Ne verilmiş ne isteniyor? – Kural ve değer kümesi verilmiş tanım kümesi isteniyor
3. Ne tür bir soru? –tanım kümesi bulma
4. Verilen ile istenen arasında bağlantı kurmak için geliştireceğim strateji nedir? – x için vereceğimiz değerler sonucunda $x^2 + 5x + 6$ ifadesinin reel sayı olmasını istiyoruz. Demek ki bu ifadeyi hangi sayılar reel sayı yapabilir, bu sayıları ifade edebileceğim en geniş küme nedir üzerinde yoğunlaşmalıyım. X için hangi değeri verirsek verelim sonuç reel sayı çıkacağı için A = ℝ olur.
5. Çözüm mantıklı mı? Anlamadığım bir şey var mı?
6. Önceki sorularla benzerliği, farklılığı nedir? –Daha önceki sorularda kuralı ve tanım kümesini verip görüntülerini bulmamız isteniyordu ancak bu soruda kural ve değer kümesi verilmiş tanım kümesi isteniyor.

Örnek:

$f: \mathbb{R} \rightarrow \mathbb{R}$ ve $g: \mathbb{R} \rightarrow \mathbb{R}$ fonksiyonları f(x)=7x-4, g(x)=2x+8 olarak veriliyor. f(2a) = g(a) ise a değeri nedir bulunuz.

Öğrenciler üstbilişsel sorularla çözer.

1. Tanımlanması gereken bir kavram var mı? –hayır
2. Ne verilmiş, ne isteniyor?- iki fonksiyonun kuralı verilmiş, görüntü kümesinde bir elemanın eşit olduğu verilmiş, bu eşitliği sağlayan a değeriisteniyor.
3. Bu ne tür bir soru? – tanım kümesi elemanı bulma
4. Verilen ile istenen arasında bağlantı kurmak için geliştireceğim strateji nedir? – görüntü elemanının eşitliği için denklem kurabiliriz. Girdileri kuralı veren denklemde yerine yazarsak istediğimiz eşitliği elde ederiz:

   F(2a) = 7.2a – 4 = 14a – 4

   G(a) = 2.a + 8 = 2a + 8

   f(2a) = g(a) ise 14a – 4 = 2a + 8

   a = 1 bulunur.
5. Çözüm mantıklı mı? Hangi noktalarda zorlandık?
6. Daha önceki sorularla benzerliği/farklılığı nedir? – Daha önce de görüntü kümesinde eleman bulma çözmüştük ama bu soruda iki fonksiyon var ve ikisindeki görüntüleri eşitledik.

Figure I.2. Example Problem Solving (cont.)

Örnek:

$f:\mathbb{R}\rightarrow\mathbb{R}, f(x+2)=3x+1$ olduğuna göre, $f(3x-1)$ ifadesini bulunuz.

Metabilişsel sorular modellenerek bulunur.

1. Tanımlanması gereken bir kavram var mı? -hayır
2. Ne verilmiş, ne isteniyor?- fonksiyonun kuralı verilmiş, .tanım kümesinde başka bir eleman soruluyor. (x+2)'leri (3x+1)'lere götüren f fonksiyonu 3x -1 'i neye götürür?
3. Bu ne tür bir soru? – görüntü kümesi elemanı bulma
4. Verilen ile istenen arasında bağlantı kurmak için geliştireceğim strateji nedir? – f'in girdi olarak 3x – 1 almasını istiyoruz. Acaba x yerine ne yazarsak f'in içini 3x – 1 yapabiliriz?
   X*+2 = 3x – 1 dersek f içindeki x*= 3x -3 bulunur.
   Denklemin her iki tarafında da x yerine 3x – 3 yazmalıyız:
   F(3x – 1) = 3.(3x-3) + 1 = 9x – 8 bulunur.
5. Çözüm mantıklı mı? Hangi noktalarda zorlandık?
6. Daha önceki sorularla benzerliği/farklılığı nedir? – öncekilere anafikir olarak benziyor, biçimsel farklar var.

Örnek:

$f:\mathbb{R}\rightarrow\mathbb{R}, f(x-1)=x^2-x-5$ olduğuna göre, f(x) değerini bulunuz.

Öğrenciler akıl yürütür.

1. Tanımlanması gereken bir kavram var mı? -hayır
2. Ne verilmiş, ne isteniyor?- fonksiyonun kuralı verilmiş, .tanım kümesinde başka bir eleman soruluyor. (x - 1)'leri (x^2 – x - 5)'lere götüren f fonksiyonu x 'i neye götürür?
3. Bu ne tür bir soru? – görüntü kümesi elemanı bulma
4. Verilen ile istenen arasında bağlantı kurmak için geliştireceğim strateji nedir? – f'in girdi olarak x almasını istiyoruz. Acaba x yerine ne yazarsak f'in içini x yapabiliriz?
   X* - 1 = x dersek f içindeki x*= x +1 bulunur.
   Denklemin her iki tarafında da x yerine x + 1 yazmalıyız:
   F(x) = $(x + 1)^2$ – (x +1) – 5 = ... bulunur.
5. Çözüm mantıklı mı? Hangi noktalarda zorlandık?
6. Daha önceki sorularla benzerliği/farklılığı nedir? – öncekilere anafikir olarak benziyor, biçimsel farklar var.

Figure I.2. Example Problem Solving (cont.)

Örnek:

$f:\mathbb{N}\rightarrow\mathbb{N},\ f(x)=x+1$ fonksiyonunun örten olup olmadığını inceleyiniz.

Öğretmen önce öğrencilerin fikirlerini alır, bu sırada üstbilişsel soruları kullanmaya teşvik eder daha sonra bu sorularla kendisi çözer.

1. Kavramlar:
   örten fonksiyon, bu durumda değer kümesinde boşta eleman olmamalı
2. Ne verilmiş, ne isteniyor?
   – fonksiyonun tanım ve değer kümesi ile kuralı verilmiş. Örtenliğini incelememiz isteniyor.
3. Bu ne tür bir soru?
   – değer ve görüntü kümesini karşılaştırmaya yönelik bir soru.
4. Nasıl bir strateji geliştirebiliriz?
   – tanım kümesi elemanlarına kuralı uygulayarak görüntü kümesini bulabiliriz ve bu küme değer kümesine eşit mi diye bakabiliriz. En küçük doğal sayı olan 0 ile başlarsak görüntüsü 1 olacak. Tanım kümesinden seçtiğimiz elemanı bir artırdıkça görüntü kümesi de hep 1 artacağından görüntüsü 0 olan bir eleman olmayacak. Değer kümesinde 0 boşta kalacağı için örten olamaz.
5. Çözüm mantıklı mı? Anlamadığım bir şey var mı? Neredelerde zorlandık?
6. Önceki sorularla benzerliği/farklılığı nedir?
   –önceki sorularda tanım veya görüntü kümesini bulmamız isteniyordu ama bu soruda farklı olarak ikisi arasındaki eşleşmenin örtüşüp örtüşmediğini incelememiz gerekiyor. Tanım kümesini fonksiyon kuralına uygulayarak görüntü kümesi bulmamız bakımından ise öncekilere benziyor.

Figure I.2. Example Problem Solving (cont.)

Örnek:

$f:\mathbb{R}\rightarrow\mathbb{R},\ f(x)=x^2$ fonksiyonunun birebir olup olmadığını inceleyiniz.

Öğretmen üstbilişsel sorularla çözer.

1. Kavramlar: birebir fonksiyon, bu durumda her elemanın farklı bir görüntüsü olmalı.
2. Ne verilmiş, ne isteniyor?
   – fonksiyonun tanım ve değer kümesi ile kuralı verilmiş. Birebir olup olmadığını incelememiz isteniyor.
3. Bu ne tür bir soru?
   – değer ve görüntü kümesini karşılaştırmaya yönelik bir soru.
4. Nasıl bir strateji geliştirebiliriz?
   – tanım kümesi elemanlarına kuralı uygulayarak görüntüsü aynı olan elemanlar var mı diye araştırabiliriz. Tanım kümesi reel sayılar olduğu için mutlak değeri aynı olan bir negatif ve pozitif sayı seçilip görüntülerine bakılır. Aynı elemanla eşleştikleri için birebir değildirler.
5. Önceki sorularla benzerliği/farklılığı nedir?
   – önceki sorulara tanım ve görüntü kümesi elemanlarını karşılaştırmamız bakımından benziyor ancak örtenlik değil birebirlik incelememiz istenmiş.
6. Çözüm mantıklı mı? Anlamadığım bir şey var mı? Nerelerde zorlandık?
7. Farklı bir strateji de düşünebilir miydik?
   -Evet, tersten bakmayı deneyebilirdik. Görüntü kümesindeki her eleman tanım kümesindeki eleman veya elemanların karesi olacağına göre, görüntü kümesinden bir eleman alıp kareköküne bakarak kaç elemanla eşleştiğini inceleyebilirdik.

Figure I.2. Example Problem Solving (cont.)

Örnek:

$f:\mathbb{R}\rightarrow\mathbb{R},\ f(x)=(a-2)x^2+(b+3)x+c-5$ fonksiyonu, birim fonksiyon olduğuna göre a, b, c değerlerini bulunuz.

Öğretmen üstbilişsel sorularla çözer.

1. Kavramlar: birim fonksiyon, bu durumda her elemanın görüntüsü kendisine eşit olmalı.
2. Ne verilmiş, ne isteniyor?

   – fonksiyonun birim fonksiyon olduğu ve kuralı verilmiş. A, b, c bilinmeyenleri soruluyor.
3. Bu ne tür bir soru?

   – birim fonksiyon tanımına yönelik bir soru
4. Nasıl bir strateji geliştirebiliriz?

   –tanıma dayalı olarak fonksiyonun tanım ve görüntü kümesini birbirine eşitleyebiliriz.
5. Önceki sorularla benzerliği/farklılığı nedir?

   –önceki sorularda örtenliği ya da birebirliği bizim incelememiz gerekiyordu, bu soruda ise birim fonksiyon olduğu bilgisi kesin ve bizim bu bilgiyi doğru şekilde kullanmamız bekleniyor.
6. Çözüm mantıklı mı? Anlamadığım bir şey var mı? Nerelerde zorlandık?

Figure I.2. Example Problem Solving (cont.)

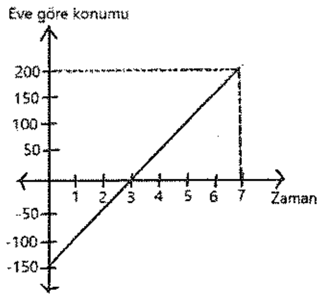
I. Doğrusal Fonksiyon

Giriş etkinliği:

Yuvasından 150 metre güneyde olan bir karınca yuvasına doğru yürüyüşe geçerek yuvasından 200 metre kuzeye ulaşmıştır. Bu karınca dakikada 50 metre gidebildiğine göre karıncanın evine göre konumu ve geçen süre arasındaki ilişkiyi gösteren tabloyu oluşturunuz ve grafiğini çiziniz.

Öğrencilerle tartışılarak tablo ve grafik oluşturulur.

Zaman (dk)	0	1	2	3	4	5	6	7
Eve uzaklık (m)	-150	-100	-50	0	50	100	150	200



Grafiği incelersek birim zaman aralıklarında konumunun değişim oranının hep aynı olduğunu görürüz. Değişim oranı aynı olduğu zaman grafiğimiz bir doğru belirtiyor. Ortaokul yıllarından da hatırlarsanız değişim oranını eğim olarak adlandırıyorduk.

Düzlemde bu şekilde doğru bildiren eşleşmelere doğrusal fonksiyon diyoruz.

Matematiksel Tanım:

f:R → R ve a,b ∈ R, olmak üzere $f(x)$ = ax + b biçimindeki fonksiyonlara **doğrusal fonksiyon** denir. Bu fonksiyonların görüntü kümeleri analitik düzlemde doğru belirtir.

Figure I.3. Example Lesson Plan: Beginning of linear functions

Örnek:

f :R → R bir doğrusal fonksiyon olmak üzere $f(3) = 15$ ve $f(5) = 23$ olduğuna göre $f(9)$ değerini bulunuz.

Öğretmen üstbilişsel sorularla çözer.

1. Kavramlar: doğrusal fonksiyon, bu durumda f(x) = ax + b formunda olmalı.
2. Ne verilmiş, ne isteniyor?
   – fonksiyonun doğrusal fonksiyon olduğu ve iki elemanın görüntüleri verilmiş, başka bir elemanın görüntüsü soruluyor.
3. Bu ne tür bir soru?
   – doğrusal fonksiyon tanımına yönelik bir soru.
4. Nasıl bir strateji geliştirebiliriz?
   –görüntüsü verilen elemanları doğrusal fonksiyon tanımına yerleştirerek fonksiyon kuralını bulup daha sonra 9'un görüntüsünü bulabiliriz.
5. Önceki sorularla benzerliği/farklılığı nedir?
   –önceki sorulardan farklı olarak doğrusal fonksiyon kavramı var.
6. Çözüm mantıklı mı? Anlamadığım bir şey var mı? Nerelerde zorlandık?
7. Farklı bir strateji de düşünebilir miydik?

Örnek:

f bir doğrusal fonksiyon olmak üzere f(x – 3) + f(x + 2) = 6x + 7 ise f(7) değerini bulunuz.

1. Kavramlar: doğrusal fonksiyon, bu durumda f(x) = ax + b formunda olmalı.
2. Ne verilmiş, ne isteniyor?
   – fonksiyonun doğrusal fonksiyon olduğu ve iki elemanın görüntüleri arasındaki ilişkiyi veren denklem verilmiş, başka bir elemanın görüntüsü soruluyor.
3. Bu ne tür bir soru?
   – doğrusal fonksiyon tanımına yönelik bir soru.
4. Nasıl bir strateji geliştirebiliriz?
   –görüntüsü verilen elemanları doğrusal fonksiyon tanımına yerleştirerek fonksiyon kuralını bulup daha sonra 7'nin görüntüsünü bulabiliriz.
5. Önceki sorularla benzerliği/farklılığı nedir?
   –önceki soruyla doğrusal fonksiyon kullanmamız açısından benzer ancak belirli elemanlar yerine bilinmeyen içeren elemanların görüntüsünü bulma açısından farklı.
6. Çözüm mantıklı mı? Anlamadığım bir şey var mı? Nerelerde zorlandık?
7. Farklı bir strateji de düşünebilir miydik?

Figure I.4. Example Problem Solving

Örnek:

$f : R \rightarrow R$ doğrusal fonksiyon olmak üzere (fof)(x) = 9x + 16 olduğuna göre f(0) değerlerinin toplamını bulunuz.

Öğretmen öğrencilerin üstbilişsel sorularla çözmesini bekler:

1. Hangi matematiksel kavramlar var ve ne demek? – doğrusal fonksiyon kavramı var ve doğrusal fonksiyon f(x) = ax + b şeklindedir.
2. Ne verilmiş, ne isteniyor? - f fonksiyonunun doğrusal fonksiyon olduğu ve kendisiyle bileşkesi verilmiş. F fonksiyonunun 0 elemanı için görüntüsü isteniyor.
3. Bu ne tür bir soru? – doğrusal fonksiyon ve bileşke tanımını kullanmaya yönelik bir soru
4. Nasıl bir strateji geliştirebiliriz? – doğrusal ve bileşke fonksiyonun tanımından yola çıkmalıyız.

   (fof)(x) = f[f(x)] ve f(x) = ax+ b olmalıdır:

   $(fof)(x) = f[f(x)] = f(ax + b)$ = a(ax + b)

   $a^2x + ab = 9x + 16$

   $a^2 = 9$ ise a = 3 veya a = -3 bulunur.

   a = 3 olursa b = 16/3

   a = -3 olursa b = -16/3

   f(0) = b olacağından f(0) = 16/3 veya f(0) = -16/3 bulunur.
5. Çözüm mantıklı mı? Anlamadığım ya da zorlandığım noktalar var mı, neler?
6. Bu sorunun bir önceliyle benzerliği/farklılığı nedir? – bileşke fonksiyon içine başka bir fonksiyon türü yerleştirilmiş olması bakımından farklı, görüntü elemanı bulma bakımından benzer.

Figure I.4. Example Problem Solving (cont.)

Örnek:

A = {1, 3, 5, 7} ve B = {2, 4, 6, 8} olmak üzere

$f : A \rightarrow B, f = \{(1, 2), (3, 4), (5, 6), (7, 8)\}$ fonksiyonu veriliyor.

f fonksiyonunun tersi varsa bulunuz.

Öğretmen üstbilişsel sorularla çözer:

1. Ne verilmiş, ne isteniyor? – bir fonksiyonun tanım ve görüntü kümesi arasındaki eşleşme verilmiş. Tersinin olup olmadığı soruluyor ve varsa tersini bulmamız isteniyor.
2. Bu ne tür bir soru? – bir fonksiyonun tersini incelemeye yönelik bir soru.
3. Verilen ile istenen arasında nasıl bir ilişki var, çözüm için nasıl bir strateji geliştirebilirim? – fonksiyonun elemanları nasıl eşleştirdiği verilmiş. Ters fonksiyonu olabilmesi için de birebir ve örten olması gerektiğini biliyoruz. Bu yüzden tanım kümesindeki her eleman farklı bir elemana gidiyor mu ve değer kümesinde açıkta eleman kalmış diye incelemeliyiz. Bu incelemeyle fonksiyonun birebir ve örten olduğunu görüyoruz. Bu durumda tersi vardır. Bir fonksiyonun tersi tanım ve görüntü kümesi yer değiştirilerek bulunuyordu. Bu durumda:
   $f^{-1} : B \rightarrow A, f = \{(2, 1), (4, 3), (6, 5), (8, 7)\}$ olur.
4. Çözüm mantıklı mı? Anlamadığım bir şey var mı?
5. Bu sorunun daha önce çözdüklerimizle benzerliği / farklılığı nedir? – fonksiyonda tanım, değer, görüntü kümelerini, birebir ve örtenliği incelemeyi daha önceden biliyorduk, tersini almayı gördük.

Figure I.4. Example Problem Solving (cont.)

Örnek:

$f : R - \{-1\} \to R - \{-3\},\ f(x) = \frac{ax-2}{x-b}$ olduğuna göre $f(a-b)$ değerini bulunuz.

Öğretmen üstbilişsel sorularla çözer:

1. Ne verilmiş, ne isteniyor? – bir fonksiyona ait tanım ve değer kümesi ile fonksiyonun kuralı verilmiş, kuralındaki bilinmeyenlerin farkının görüntüsü soruluyor.
2. Bu ne tür bir soru? –fonksiyon ve ters fonksiyon tanımına yönelik bir soru.
3. Verilen ile istenen arasında nasıl bir ilişki var, çözüm için nasıl bir strateji geliştirebilirim?
   - Bir fonksiyonun tanım kümesi belirlenirken fonksiyonu tanımsız yapan değerler çıkarılıyordu. Bu soruda tanım kümesinden -1 çıkarıldığına göre -1 fonksiyonu tanımsız yapan değer olmalı. Rasyonel ifadeli bir fonksiyon sadece payda 0 olursa tanımsız olabilir. Yani x=b değerinde tanımsız olacağı için b=-1 olmalıdır.
   - Bu fonksiyonun görüntü kümesinden de -3 çıkrıldığına göre tersi alındığında -3 tanımsız yapan değer olmalı.

     $f^{-1}(x) = \frac{bx-2}{x-a}$ olduğundan x=a değerinde tanımsızdır ve

     a=-3 bulunur.

     $f(a-b) = f(-3+1) = f(-2) = \frac{-3.(-2)-2}{-2+1} = -4$

4. Çözüm mantıklı mı? Anlamadığım bir şey var mı?
5. Bu sorunun daha önce çözdüklerimizle benzerliği / farklılığı nedir? – önceki soru ile rasyonel ifadeli fonksiyonun tersinin düşünülmesi açısından benzer. Ayrıca en geniş tanım kümesi bulma sorularıyla benzerliği de var. Ancak bu soruda tanımsız yapan değerler ters fonksiyonla da ilişkilendirilmiş.

Figure I.4. Example Problem Solving (cont.)

Örnek:

$f(x)$ birebir, örten ve $f(x) = \frac{3x-f(x)}{x+2}$ olduğuna göre $f^{-1}(x)$ fonksiyonunun kuralını bulunuz.

1. Ne verilmiş, ne isteniyor? – bir fonksiyonun kendisi ve bir değişken türünden eşiti verilmiş, fonksiyonun tersinin kuralı soruluyor.
2. Bu ne tür bir soru? – ters fonksiyon tanımına yönelik bir soru
3. Verilen ile istenen arasında nasıl bir ilişki var, çözüm için nasıl bir strateji geliştirebilirim? Fonksiyonun tersini bulabilmek için x'i yalnız bırakmamız gerekiyordu. Öncelikle çapraz çarpım yapıp x'leri yan yana getirerek yalnız bırakmaya çalışacağız.

$$f(x).(x+2) = 3x - f(x)$$

$$f(x).x + 2f(x) = 3x - f(x)$$

$$2f(x) + f(x) = 3x - x.f(x)$$

$$3f(x) = x.(3 - f(x))$$

$$x = \frac{3f(x)}{3 - f(x)}$$

Olduğundan:

$$f^{-1}(x) = \frac{3x}{3-x}$$

4. Çözüm mantıklı mı? Anlamadığım bir şey var mı?
5. Bu sorunun daha önce çözdüklerimizle benzerliği / farklılığı nedir? – daha önce çözdüklerimizde hep f fonksiyonu x cinsinden veriliyordu ama bu soruda x değişkeni ile f fonksiyonu birlikte verilmiş.

Figure I.4. Example Problem Solving (cont.)

Örnek:

$f(2x+3)=g(x-1)$ olduğuna göre $(g^{-1}of)(5)$ değerini bulunuz.

Öğrenciler üstbilişsel sorularla çözer:

1. Ne verilmiş, ne isteniyor? – bir fonksiyonun diğer bir fonksiyonla ilişkisi verilmiş, birinin tersi ile diğerinin bileşkesi soruluyor.
2. Bu ne tür bir soru? – ters fonksiyon özelliklerine yönelik bir soru
3. Verilen ile istenen arasında nasıl bir ilişki var, çözüm için nasıl bir strateji geliştirebilirim?
   - f'in g^{-1} ile bileşkesi sorulduğundan verilen bilgide de bunu elde etmeyi deneyebiliriz. Bu durumda her iki tarafın da g^{-1} ile soldan bileşkesini alırsak:

$$(g^{-1}of)(2x+3) = (g^{-1}og)(x-1)$$
$$(g^{-1}of)(2x+3) = I(x-1) = x-1$$

   $(g^{-1}of)(5)$ için $2x+3=5$ dersek, x=1 yazmamız gerektiğini buluruz:

$$(g^{-1}of)(5) = 1-1 = 0$$

4. Çözüm mantıklı mı? Anlamadığım bir şey var mı?
5. Başka bir strateji geliştirebilir miydik?
   - $(g^{-1}of)(5) = g^{-1}(f(5))$ demek olduğu için öncelikle f(5) değerini bulmaya çalışırız:

     X = 1 olduğunda

     $f(5) = g(0)$ bulunur. Bunu bileşkede yerine yazarsak:

     $(g^{-1}of)(5) = g^{-1}(g(0)) = I(0) = 0$ bulunur.

6. Bu sorunun daha önce çözdüklerimizle benzerliği / farklılığı nedir? – iki fonksiyonun birbiri cinsinden verildiği sorular çözmüştük daha önce ama bu soruda verilen ilişkiyi ters fonksiyonun kendisiyle bileşkesinin birim fonksiyon olması özelliğini kullanarak çözdük.

Figure I.4. Example Problem Solving (cont.)

Örnek:

$f(x) = \frac{ax-2}{x}$, $g(x) = \frac{x+4}{x-2}$ olmak üzere $(fog^{-1})(-1) = 6$ olduğuna göre a değerini bulunuz.

Öğretmen öğrencilerin üstbilişsel sorularla çözmesi için zaman verir:

1. Ne verilmiş, ne isteniyor? – iki fonksiyonun kuralı ve bileşkelerinin -1 için görüntüsü verilmiş, kuraldaki bilinmeyen soruluyor.
2. Bu ne tür bir soru? – rasyonel ifadeli ters fonksiyon bulmaya yönelik bir soru
3. Verilen ile istenen arasında nasıl bir ilişki var, çözüm için nasıl bir strateji geliştirebilirim?

$$(fog^{-1})(-1) = f(g^{-1}(-1))$$

$g^{-1}(x) = \frac{2x+4}{x-1}$ ve $g^{-1}(-1)) = \frac{2.(-1)+4}{(-1)-1} = -1$

$f\left(g^{-1}(-1)\right) = f(-1) = \frac{a(-1)-2}{-1} = 6$ ise

$-a - 2 = -6$ ve $a = 4$ bulunur.

4. Çözüm mantıklı mı? Anlamadığım bir şey var mı?
5. Bu sorunun daha önce çözdüklerimizle benzerliği / farklılığı nedir? – bileşke, ters fonksiyon ve görüntü bulma bakımından benzer, ancak görüntünün verilip kuralda bilinmeyen olması bakımından farklı.

Figure I.4. Example Problem Solving (cont.)

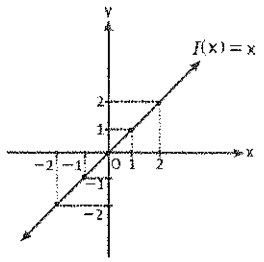
Örnek 1:

$I: R \rightarrow R$ olmak üzere $I(x) = x$ birim fonksiyonunun grafiğini çiziniz.

1. Ne verilmiş, ne isteniyor? – Birim fonksiyon bilgisi verilmiş, grafiği soruluyor.
2. Bu ne tür bir soru? – doğrusal fonksiyon grafiği çizimine yönelik bir soru
3. Nasıl bir strateji izlemeliyiz? – Fonksiyonu sağlayan noktaları koordinat düzlemine yerleştirip birleştirmeliyiz.

Birim fonksiyonu sağlayan birkaç nokta tabloda verilmiştir.

x	−2	−1	0	1	2
I(x)	−2	−1	0	1	2



4. Grafik çizimi ile ilgili zorlandığımız bir nokta oldu mu?
5. Bu sorunun öncekilerle benzerliği/farklılığı nedir? –ilk defa fonksiyon kuralı ile grafik çizimi yaptık.

Örnek:

$f: R \rightarrow R$ olmak üzere $f(x) = 3x - 9$ fonksiyonunun grafiğini çiziniz.

Öğretmen modeller:

1. Ne verilmiş, ne isteniyor? – Fonksiyon kuralı verilmiş, grafiği soruluyor.
2. Bu ne tür bir soru? – doğrusal fonksiyon grafiği çizimine yönelik bir soru
3. Nasıl bir strateji izlemeliyiz? – Fonksiyonu sağlayan noktaları koordinat düzlemine yerleştirip birleştirmeliyiz. Özellikle eksenleri kestiği noktaları bulmaya çalışmalıyız.
4. Grafik çizimi ile ilgili zorlandığımız bir nokta oldu mu?
5. Bu sorunun öncekilerle benzerliği/farklılığı nedir? –öncekilerle ile benzer ancak onlar orijinden geçiyordu, bu grafik orijinden geçmiyor, eksenleri kesiyor.

Figure I.4. Example Problem Solving (cont.)

Örnek:

$f: R \rightarrow R$ olmak üzere $f(x) = -3x - 9$ fonksiyonunun grafiğini çiziniz.

Öğrencilerin çözmesi istenir.

Örnek:

$f: R \rightarrow R$ olmak üzere

$$y = f(x) = \begin{cases} x, & x < -1 \; ise \\ x + 1, & -1 \leq x < 2 \; ise \\ -x + 2, & 2 \leq x \; ise \end{cases}$$

fonksiyonunun grafiğini çiziniz.

Öğretmen modeller:

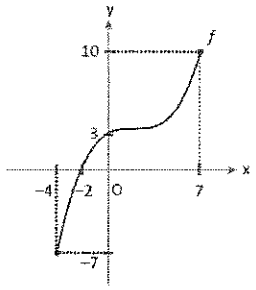
1. Ne verilmiş, ne isteniyor? – Parçalı fonksiyon verilmiş, grafiği soruluyor.
2. Bu ne tür bir soru? – fonksiyon grafiği çizimine yönelik bir soru
3. Nasıl bir strateji izlemeliyiz? – Fonksiyonu, parçalandığı her bir bölge için ayrı ayrı incelemeliyiz. Üç bölge için üç fonksiyon grafiği çizmeliyiz. Daha sonra her birinin verilen aralık dışında kalan bölümleri silinmeli.
4. Grafik çizimi ile ilgili zorlandığımız bir nokta oldu mu?
5. Bu sorunun öncekilerle benzerliği/farklılığı nedir? –öncekilerle doğrusal grafik çizimi konusunda benzer ancak bu kez tek bir doğru çizmek yerine üç doğruyu aynı grafik üzerinde çizdik.

Figure I.4. Example Problem Solving (cont.)

Fonksiyon Grafiklerini Yorumlama

Örnek:

Aşağıda grafiği verilen f fonksiyonunun tanım ve görüntü kümelerini bulunuz.



1. Ne verilmiş, ne isteniyor? – grafik verilmiş, tanım ve görüntü kümeleri soruluyor.
2. Matematiksel kavramlar – tanım ve görüntü kümesi tanımları hatırlatılır.
3. Nasıl bir strateji izlemeliyiz? – Tanım kümesi elemanlarını x ile görüntü kümesi elemanlarını y=f(x) ile temsil ediyorduk. Bu durumda tanım kümesi bulmak için x eksenini, görüntü kümesi bulmak için y eksenini incelemeliyiz.
4. Nerede zorlandık?
5. Bu sorunun öncekilerle benzerliği/farklılığı nedir? – Öncekilerde fonksiyon kuralı verilip grafik soruluyordu, bu kez grafik verilmiş fonksiyon hakkında soru var.

Figure I.4. Example Problem Solving (cont.)

APPENDIX J: TEACHER WORKSHEET

IMPROVE: A Multidimensional Method for Teaching Mathematics in Heterogeneous Classrooms

IMPROVE is a multidimensional instructional method based on metacognitive strategy using. It aims to enhance mathematical reasoning. The method involves three interrelated components: (a) metacognitive processes for strategy acquisition; (b) cooperative works including four students with different prior knowledge: one high, two middle, and one low-achieving student; and (c) provision of feedback-corrective-enrichment The method is implemented in heterogeneous classrooms in terms of their backgrounds and prior knowledge. The method is called as IMPROVE, because it consists of the first letters of the teaching steps:

Introducing new concepts,
Metacognitive questioning,
Practicing,
Reviewing and reducing difficulties,
Obtaining mastery,
Verification, and
Enrichment.

After the teacher introduces the new concepts to class and modeled the metacognitive questions, students work in small heterogeneous groups. They are expected to use four metacognitive questions: comprehension, strategic, connection and reflection questions.

Comprehension questions oriented the students to understand the main ideas in the problem (e.g., "Describe . . . in your own words"), classify the problem into an appropriate category (e.g., "This is a rate problem of the form cost-per unit rate"; "This is a simplification problem with a negative multiplier"), and elaborate the new concepts (e.g., "The definition of... is ..."; "The meaning of. . . is . . ."; "The given are . . .";"The unknown is . . . ").

Strategic questions are used to find an appropriate strategy during problem solving process. Students have to select the suitable strategy, justify their decision, and describe the application of the strategy to the given problem. (e.g., "What can I do now?"; "How can I continue?")

Figure J.1. Teacher Worksheet

Connection questions help students recognize the similarities and differences between the problem at hand and the problems previously solved. Using connection questions, students learned to distinguish between equivalent problems sharing the same mathematical structure and the same story context, similar problems sharing the same story context but having different mathematical structures, isomorphic problems sharing the same mathematical structures but having a different story context, and unrelated problems sharing neither the mathematical structure nor the story context. In addition, students learned to distinguish between different kinds of quantities, propositions, and procedures. (e.g., What are the differences between this problem and the previous ones?)

Reflection questions were designed to prompt students to self-regulate their problem solving. They monitored and evaluated their understanding by examining the reasonability of their solutions, as well as they search different ways to solve problems and different teaching approaches. (e.g., "Do I understand?", "Is the solution reasonable?", "What is a good mathematical argument?", "Can I solve the task differently?", "Which point was difficult for me to solve the problem?)

The process of the implementation of these metacognitive questions were similar with the 4-stage model of the problem-solving process: orientation and problem identification, organization, execution, and evaluation. The questions were deliberately designed to help students to be aware of the problem-solving process and to self-regulate their progress. Self-regulation involves (a) understanding what the problem is all about before attempting a solution, (b) planning the solution, (c) monitoring or keeping track of how well things are going during the solution, and (d) allocating resources, or deciding what to do while working on the problem.

IMPROVE also includes cooperative-mastery learning based on peer interaction. The systematic provision of corrective/enrichment feedback enhances mathematical thinking. First, peer interaction provides opportunities for students to articulate their thoughts and explain their mathematical reasoning. It can also enhance the strategic management of cognitive resources for students with different ability and prior knowledge Lastly, feedback-corrective-enrichment helps students define their own needs and deepen their mathematical thinking.

At the end of each unit, students were given **a formative test** that focused on the main ideas taught in the unit. Students who did not attain mastery (80% correct) on the formative tests were given **corrective activities** to do, whereas others worked on **enrichment activities** related to the unit. The enrichment activities includes challenging tasks that fosters mathematical reasoning rather than lower order skills. In contrast to others, at the corrective-enrichment session, students work in relatively homogeneous teams. Students who have to correct their learning are given a parallel form of the formative test and they are allowed to correct their learning just once.

Figure J.1. Teacher Worksheet (cont.)

APPENDIX K: QUESTION PROMPTS CARD

Üst bilişsel Soru Kartı

Anlama Soruları:

Anafikir – Sayılardan bağımsız olarak problemi kendi cümlelerinizle **tanımlayınız**. Problem ne hakkında? Soruda hangi **matematiksel kavramlar** var ve bu kavramların anlamı nedir?

Sınıflandır – Bu ne **tür** bir problem?

Verilen/İstenen - Soruda ne **verilmiş**? Ne **isteniyor**? Verilen ile istenen arasında nasıl bir **ilişki** görebiliriz?

Strateji Soruları:

Soruyu çözmek için uygun strateji **ne** olabilir? **Neden**?

Bu stratejiyi **nasıl** uygulayabilirim?

Bağlantı Soruları:

Bu sorunun daha önce çözdüğümüz sorulardan **farkı/benzerliği** nedir? Neden?

Değerlendirme Soruları:

Nasıl çözdüm? / Neyi yanlış yapmış olabilirim?

Çözümü anladım mı?

Çözüm mantıklı mı?

Çözümü nasıl açıklayabilirim?

Soruyu çözerken zorlandığım noktalar neler?

Farklı bir yolla da çözebilir miydim?

Figure K.1. Question Prompts Card

APPENDIX L: FORMATIVE TESTS

10. Sınıf Matematik Küçük Sınav 1

1. Aşağıdakilerden hangileri fonksiyon belirtir?

I. f: Z → Z, $f(x) = \frac{3x}{2} - 1$
II. f: Q → Z, $f(x) = x + 1$
III. f: R → R^+, $f(x) = \sqrt{x}$
IV. f: R^+ → R, $f(x) = \frac{x-1}{x+2}$
V. f: N → N, $f(x) = x - 1$
VI. f: Z^- → Q, $f(x) = 2^x$

2. Aşağıdaki fonksiyonlardan hangileri birebirdir?

I. f: R → R , $f(x) = -3x + 1$
II. f: Z → Z , $f(x) = 2x^2 + 5$
III. f: Z → Z, $f(x) = -x^3$
IV. f: Z^- → Z, $f(x) = 4x^4 + 4$
V. f: Z → Q, $f(x) = 3^x$
VI. f: Z → Z, $f(x) = -|x - 1|$

3. Aşağıdaki fonksiyonlardan hangileri örten hangileri içinedir?

I. f: R → R , $f(x) = -3x + 1$
II. f: Z → Z, $f(x) = -x^3$
III. f: Z → Z^+, $f(x) = |x|$

4. $f: A \rightarrow B$, $f(x) = x^2 - 1$

   $A = \{-2, -1, 0, 1, 2\}$ olduğuna göre f(A) kümesinin eleman sayısı kaçtır?

5. $f(x) = 2^{x+1}$ olduğuna göre,

   $f(x-3)$ ifadesinin f(x) cinsinden değerini bulunuz.

6. $\forall x \in \mathbb{N}$ için

   $f(x+1) = \frac{2x}{3} + f(x)$ tir.

   $f(10) = 60$ olduğuna göre, f(1) kaçtır?

7. $f(0) = 5$

   $f(x+1) - f(x) = x^2$ olduğuna göre, f(6) değeri kaçtır?

8. $f(x) = x.f(x+1)$ ve $f(1) = 10!$ Olduğuna göre, f(10) kaçtır?

9. Bir ürünün alış fiyatı x, satış fiyatı y olmak üzere,

   $y = -x^2 + 8x + 23$ bağıntısı bulunmaktadır.

   Buna göre, alış fiyatı 6 olan bu ürünün satışından elde edilen kar kaç liradır?

10. $f(x^x) = x^{2x} + 2x^x + 1$ olduğuna göre, $f(2)$ nin eşiti nedir?

11. $A = \{1, 2, 3\}$

    $B = \{1, 2, 3, 4, 5\}$ kümeleri veriliyor. A'dan B'ye kaç tane fonksiyon yazılabilir?

12. Tanımlı olduğu aralıklarda $f(x^2 + 2x - 5) = (a-2)x^2 + 2(b+1)x - c$ fonksiyonu birim fonksiyon olduğuna göre, a + b+ c toplamı kaçtır?

13. Tanımlı olduğu aralıklarda $f(x) = (2^a - 8)x^2 - (\sqrt{b} - 2)x + 3$ sabit fonksiyon olduğuna göre, a ve b değerlerini bulunuz.

10. Sınıf Matematik Küçük Sınav 2

1. $f(x) = 2x - 1,$

   $g(x) = 3x + k$ fonksiyonları veriliyor.

   $(fog)(x) = (gof)(x)$ olduğuna göre, k kaçtır?

2. f = {(1, 2), (2, 5), (3, 7), (4, 12)}

   g = {(2, 3), (4, 7), (3, 18)} olduğuna göre, $(fog)(2)$ değeri kaçtır?

3. $f\colon R \to R$ fonksiyonu veriliyor.

   $f(x) = \begin{cases} 3 - x, & x < 2 \\ 2x - 5, & x \geq 2 \end{cases}$ olduğuna göre, (fofof)(2) değeri kaçtır?

4. f doğrusal bir fonksiyondur.

   $f(2) = 4$

   $f(4) = 8$

   $g(x + 2) = 2x + 1$ olduğuna göre, $(fog)(2)$ kaçtır?

5. $f\colon R \to R$ ve $g\colon R \to R$

   $(f + g)(x) = 3x$

   $(f - g)(x) = x - 2$ olduğuna göre, $(f.g)(1)$ kaçtır?

6. f ve g doğrusal fonksiyonlardır.

   $(fog)(x) = 3g(x) + 2$

   olduğuna göre, $f(5)$ değeri kaçtır?

7. $f(x) = 2^x$

   $g(x) = 2x - 3$

   $(gof)(x) = 13$ olduğuna göre, x değeri kaçtır?

8. $g(x) = x - 4$

   $(fog)(x) = 9x^2 - 16$ olduğuna göre, $f(x)$ fonksiyonunu bulunuz.

9. $f: R \rightarrow R$ ve $g: R \rightarrow R$

   f(x) çift fonksiyon, g(x) tek fonksiyondur.

   $f(x) - g(x) = f(-x) - g(-x) + 2x^3 + x$ olduğuna göre, g(2) kaçtır?

10. $f: R - \{4\} \rightarrow R - \{4\}$ fonksiyonu birebir ve örtendir.

    $f(x) = \frac{4x-16}{x-4}$ olduğuna göre $\underbrace{(fofofo \ldots of)}_{10 \text{ tane}}(8)$ kaçtır?

BONUS ☺

f ve g gerçek sayılarda tanımlı oldukları aralıklarda birebir ve örten fonksiyonlardır.

$g(2x + 1) = f^{-1}(4 + x)$ olduğuna göre $(fog)(5)$ kaçtır?

APPENDIX M: ENRICHMENT AND CORRECTION SHEETS

First Correction Sheet

1. Ayda x tane ürün üreten bir şirketin aylık gideri y ile gösterilmektedir.
   $y = 5000 + 7x + x^2$ liradır.
   Buna göre, Kasım ayında 100 adet, Aralık ayında 150 adet üretim yapan bu şirketin, Aralık ayındaki gideri, Kasım ayındaki giderinden kaç lira fazladır?

2. $f(x^x + 1) = x^{2x} + 2x^x - 1$ olduğuna göre, $f(2)$ nin eşiti nedir?

3. $f: R \rightarrow R$, $f(x^2) = 2x^6 - x^4 + x^2 + 1$ fonksiyonu veriliyor. Buna göre f(3) değeri kaçtır?

4. $K = \{x, y, 1, 2\}$ kümesi veriliyor.
   $f: K \rightarrow K$ kaç tane fonksiyon tanımlanabilir?

5. $f: K \rightarrow K$ tanımlanan birebir fonksiyon sayısı kaçtır?

6. $f: R \rightarrow R$, ve $f(x) = (17 - a^2)x + b - a$ fonksiyonu birim fonksiyon olduğuna göre, b'nin alabileceği değerler çarpımı kaçtır?

7. Tanımlı olduğu aralıklarda $f(x) = 2^{(2a-b)x+3b-6}$ sabit fonksiyon olduğuna göre, a ve b değerlerini bulunuz.

Figure M.1. First Correction Sheet

First Enrichment Sheet

1. $f(x) = 2^{x+1}$ olduğuna göre,

$\frac{f(x+1)+f(x-2)}{18}$ ifadesinin f(x) cinsinden değerini bulunuz.

2. $f\left(\frac{x}{2}\right) + 2f\left(\frac{2}{x}\right) = 2x + 2$ olduğuna göre, f(1) + f(2) toplamı kaçtır?

3. $f(x^x + 1) = x^{2x} + 2x^x - 1$ olduğuna göre, $f\left(\frac{1}{x} + x\right)$ in eşiti nedir?

4. f: R → R, $f(x^2) = 2x^6 - x^4 + x^2 + 1$ fonksiyonu veriliyor. Buna göre f(2x) değeri kaçtır?

5. K = {x, y, 1, 2} kümesi veriliyor.
   f: K → K kaç tane fonksiyon tanımlanabilir?

6. f: K → K tanımlanan birebir fonksiyon sayısı kaçtır?

7. Tanımlı olduğu aralıklarda $f(x) = \sqrt{4x - (a+2)x} - \sqrt[3]{a+6}$ sabit fonksiyon olduğuna göre $f(\sqrt{2})$ değerini bulunuz.

8. Tanımlı olduğu aralıklarda $f(x) = 2^{(2a-b)x^3+(3b-6)x}$ sabit fonksiyon olduğuna göre, a ve b değerlerini bulunuz.

Figure M.2. First Enrichment Sheet

Second Correction Sheet

1. $f(x) = \begin{cases} 2x^2 - 3, & x \leq 4\ ise \\ 3x + 5, & x > 4\ ise \end{cases}$ olduğuna göre, $(fof)(2)$ değeri kaçtır?

2. $f(x) = x + 1$

   $(gof)(x) = \frac{x+1}{x}$ olduğuna göre, g(x) fonksiyonunu bulunuz.

3. f(x) fonksiyonunun grafiği orijine göre simetrik noktalardan oluşmaktadır.

   $f(x) = ax^3 - (2a - 4)x^2 + x$ olduğuna göre, f(-1) kaçtır?

4. Uygun koşullarda tanımlı f(x) tek fonksiyon, g(x) çift fonksiyondur.

   $\frac{f(1)+g(2)}{f(-1)-g(-2)}$ oranının en sade halini bulunuz.

5. $(gof)(x) = 3.f^2(x) - 5$

   $(fog)(x) = 2.g^2(x) + 1$

   olduğuna göre f(1) + g(1) kaçtır?

6. $f : R \rightarrow R$ ve $g : R \rightarrow R$

   $f(x) = 2x - 9$ ve $g(x) = x + 3$ fonksiyonları veriliyor.

   $(f + g)(a) = 3$ olduğuna göre, a kaçtır?

Figure M.3. Second Correction Sheet

Second Enrichment Sheet

1. $(fof)(x) + f(x) = f^2(x)$

   $(gog)(x) - 3g(x) = 2$ olduğuna göre, $(f + g)(-3)$ ifadesinin değeri kaçtır?

2. f doğrusal bir fonksiyondur.

   $(fofof)(x) = 8x - 7$

   olduğuna göre $f(2)$ değerini bulunuz.

3. $f(x) = x + 1$

   $f(2x + g(x)) = (fog)(x) + 8$

   olduğuna göre, x kaçtır?

4. $f : R \to R$ tanımlanan f fonksiyonu, $f_n(x) = \underbrace{(fofofo \ldots of)}_{n\ tane}(x)$ şeklinde verilmiştir.

   $f(x) = x - 1$ olduğuna göre $f_1(x) + f_2(x) + f_3(x) + \cdots + f_{12}(x)$ ifadesinin eşitini bulunuz.

5. $f, g : R^+ \to R,\ (fog)(x) = f(x) + g(x)$ ve $f(x) = 3x + 1$ olduğuna göre $g(2)$ değerini bulunuz.

6. $f(x) = |x - 9|$

   $g(x) = x^2 + 3$ olduğuna göre, 2f(-2) – 3g(1) kaçtır?

Figure M.4. Second Enrichment Sheet