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THE INSTITUTE OF SOCIAL SCIENCES

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DOCTORAL DISSERTATION

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A THESIS SUBMITTED TO
THE INSTITUTE OF SOCIAL SCIENCES
OF
YILDIRIM BEYAZIT UNIVERSITY

BY

Remzi GÖK

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
THE DEPARTMENT OF BANKING and FINANCE

2018

Approval of the Institute of Social Sciences

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WAVELET ANALYSIS OF STOCK RETURNS AND INTEREST RATE CHANGES: EVIDENCE FROM TURKEY

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Ph.D., Department of Banking and Finance

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July 2018, 348 pages

This thesis undertakes an attempt to re-examine the interdependence between 604 weekly observations of stock and bond returns in Turkey. The wavelet analysis provides a deeper understanding about the relationship considering the heterogeneous agents trading at different investment horizons. First, test findings reveal cointegration and causal relationships running from bond yields to stock prices for several indices. In line with these findings, time-domain tests suggest that bond yields Granger-cause stock returns, while the reverse does not hold for any indices in the short run. After implementing causality tests to the decomposed series, however, the paper shows that causal relationship is mostly concentrated on the higher frequencies, i.e. mid- and long-term horizons at the both sides. This finding implies that stock returns and changes in bond yields can be used as predictive power on each other. These results are also corroborated by the frequency causality test. Moreover, the asymmetric causality test reveals significant relationships between different return components. The positive component (shock) of bond returns, for example, leads the negative components of "RXU100" and "RXBANK", while, on the other hand, there are causal linkages from the negative components of "RXU100" and "RXBANK" to both positive and negative components of bond returns. Conversely, the wavelet-based outcomes indicate significantly negative relationships at varying significance and magnitudes between variables up to the fourth scale. Moreover, almost all stock indices are more volatile than bond market. “The higher scales, the lower volatility” finding suggests that short-term investors should respond to every variation in asset returns.

Keywords: Wavelets, causality, wavelet variance & correlation.

HİSSE SENEDİ GETİRİSİ VE TAHVİL FAİZLERİNDEKİ DEĞİŞMELER ARASINDAKİ İLİŞKİNİN DALGACIKLAR BAZLI ANALİZİ: TÜRKİYE ÖRNEĞİ

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Doktora, Bankacılık ve Finans

Tez Yöneticisi: Yrd. Doç. Dr. Erhan ÇANKAL

Temmuz 2018, 348 sayfa

Bu çalışmada 604 haftalık tahvil ve borsa getirisi arasındaki ilişki incelenmiştir. Dalgacıklar analizi hisse-tahvil ilişkisini heterojen piyasalar hipotezi doğrultusunda daha iyi anlaşılmasını sağlayan önemli bir metottur. Elde edilen ilk bulgulara göre bazı endeks fiyatları ile faiz oranı arasında anlamlı eşbütünleşme ve tek yönlü nedensellik ilişkisi mevcuttur. Bu sonuçlara paralel olarak, standart modeller de faiz oranı değişmelerinden hisse getirisine doğru kısa dönemli tek yönlü nedensellik ilişkisini ortaya koymaktadır. Değişkenler arasındaki gerçek ilişkiyi saptayabilmek, diğer bir ifadeyle farklı frekans boyutlarında saklı ilişkinin var olup olmadığını test etmek için dalgacıklar metodu ile elde edilen ölçek katsayılarına nedensellik testleri uygulanmıştır. Buna göre hemen hemen tüm modellerde orta ve uzun dönemde geçerli çift yönlü nedensellik ilişkisinin bulunması, cari dönem değerinin tahmin edilmesinde diğer değişkenin geçmiş değerinin kullanılmasının anlamlı sonuçlar verebileceğini ortaya koymaktadır. Bu bulgular, frekans nedensellik test sonuçlarıyla da desteklenmektedir. Asimetrik nedensellik testi, değişkenlerin farklı bileşenlerinin birbirinin Granger nedeni olduğunu göstermektedir. Örneğin, tahvil getirisindeki pozitif şoklardan "RXU100" ve "RXBANK" endekslerin negatif şoklarına ve bu iki endeksin negatif şoklarından tahvil getirisindeki hem pozitif hem de negatif şoklarına doğru nedensellik ilişkisi saptanmıştır. Nedensellik ilişkisinin yanı sıra dalgacık bazlı varyans değişimi ve değişkenler arasında basit ve çapraz korelasyon ilişkisi de incelenmiştir. Bulgular iki değişkenin dördüncü ölçeğe kadar istatistiksel olarak anlamlı ve zıt yönlü korelasyon ilişkisine sahip olduğunu göstermektedir. Ölçek düzeyi arttıkça varyansın azaldığı ve hisse borsasının tahvil borsasına göre daha yüksek düzeyde volatiliteye sahip olduğu görülmüştür. Bu sonuç, yatırımcıların kendi yatırım dönemlerindeki volatiliteye göre yatırım kararı vermesi gerektiğini ortaya koymaktadır. Diğer bir deyişle, varlık volatilitesinin kısa dönem yatırımcılar için daha büyük bir sorun olduğu söylenebilir.

Anahtar Kelimeler: Dalgacıklar, nedensellik, dalgacık korelâsyonu ve varyansı.

To My Parents, Wife & Son, Affan

The author wishes to express his deepest gratitude to his supervisor Asst. Prof. Erhan ÇANKAL for his guidance, advice, criticism, encouragements and insight throughout the research.

The author would also like to thank Prof. Dr. Nildağ Başak CEYLAN, Prof. Dr. Murat ATAN, Doç. Dr. Atilla GÖKÇE and Asst. Prof. Haroon Muzaffar for their suggestions and comments.

APT	Arbitrage Pricing Theory
CAPM	Capital Asset Pricing Theory
CML	Capital Market Line
CWT	Continuous Wavelet Transform
DWT	Discrete Wavelet Transform
EMH	Efficient Market Hypothesis
FFT	Fast Fourier Transform
FMH	Fractal Market Hypothesis
HMH	Heterogeneous Market Hypothesis
LFT	Loanable Funds Theory
LPT	Liquidity Preference Theory
MODWT	Maximal Overlap Discrete Wavelet Transform
MRA	Multiresolution Analysis
MRD	Multiresolution Decomposition
MSG	Market Segmentation Theory
PHT	Preferred Habitat Theory
SML	Security Market Line
STFT	Short-Time Fourier Transform
STFT	Short-Time Fourier Transform
TR2YGB	Turkey 2-Year Government Bond Rate
UET	Unbiased Expectations Theory

CHAPTER

1 INTRODUCTION

Academics, practitioners, and regulators have long debated the relationship between stock and debt market instruments since last decades in order to describe the strength and direction with a myriad of various approaches. Despite great efforts, however, there is no any consensus about the relationships among those participants because of differing results such as they are positive or negative correlated or they have predictive powers on each other's value estimation. The correlation relationship is of great importance for investors, for example, because it is the core input in portfolio optimization, risk management and asset valuation processes since those decisions are very sensitive to changes in interest rates. In theory, an asset value, whether a stock or bond, is determined by future cash flows that accrued to holders during the investment period. To obtain a fair price, investors use a discount rate to decide whether is worth to buy or hold it, where the discount rate is directly or indirectly affected by various factors.

The evidence about the impacts of interest rate changes on the stock prices is mixed. For example, on the one hand, according to test results of the first researchers, there should be a negative relationship between stock prices and bond yields. As widely known, the dividend discount model assumes that a stock or bond price is equal to sum of the discounted future cash flows. Asset price is calculated by two main inputs: cash flows and an appropriate discount rate –in general, risk-free rate plus real interest rate, i.e. nominal rates. Algebraically, when discount rate falls, all else being constant, asset value rises and the inverse is true as well. As noted by Shiller and Beltratti (1992), rising discount rates causes an increase in expected long-term interest rates and a decrease in stock prices, which in turn makes bond (stock) instruments more (less) attractive to investors. Accordingly, investors would shift their funds from stock markets to debt markets, which lead a change in portfolio composition in favor of bonds, therefore, a decrease in stock demand and an increase in bond demand in markets will be observed. The inverse situation holds when discount rate declines as well. However, it is not so simple because of the two arguments according to the authors (1992). The first argument is related to the fact that the cash stream of bond is radically different from the cash stream of stock, namely, the former is reasonably in nominal while, on the other hand, the latter is in real terms. When the economy faces a substantially high inflation rate –and a near zero real interest rate– then these two assets will be differently affected by interest rate changes. Because inflationary expectations are principally reflected in nominal long-term interest rates, its effect will be limited for stock prices. On the other hand, Panda (2008) remarks that asset allocation decisions between bonds and stocks are unlikely taken when high inflationary expectations are dominant among investors. Hence, there is no need to exist a negative relationship. The second argument, as noted by Shiller and Beltratti (1992), is regarding to relevant information about stock's future dividend cash flows. As observed on Black Monday, October 19, 1987, in the U.S., for example, bond prices did increase albeit the stock market, DJIA, shed its value of approximately 22% during that day. The fear of poor outlook about corporate profits underlined this biggest one-day loss of 508 points, which caused a positive linkage between stock prices and long-term interest rates. Equivalently speaking, movements in long-term bond rates might convey important information about movements in future dividends, offsetting a possible negative relationship between stock prices and bond yields.

In addition, Barsky (1989) states that there could be a positive linkage between stock and bond returns because of rising risk premium and precautionary savings and falling interest rates, accordingly, shifting funds from risky assets to less-risky assets such as bonds. On the other hand, the linkage can also be affirmative, as Panda (2008) notes, if an increase in interest rate is observed in response to the economy growing too quickly, and then both corporate profits and stock prices should increase quickly. Johnson et al. (2013) claim that both cyclical (short run) and long run correlation dynamics can be either positive of negative. In short-term, both asset classes may react differently to changes in investor risk appetite, for example during financial turmoil may cause decoupling, i.e., an inverse relationship. On the other hand, both returns may be similarly affected by macroeconomic factors in the long-term, therefore, inducing an affirmative linkage.

As mentioned above, there could be a positive or negative relationship between stock returns and bond yields. For example, the test results of Fama and French (1989), Schwert (1989b), Shiller and Beltratti (1992), Campbell and Ammer (1993), Fleming et al. (1998), and Stivers and Sun (2002) show statistically significant positive linkages. For example, Fama and French (1989) find that monthly stock prices and expected returns of the T-bill move in the same direction in the U.S., in addition, the default spread and dividend yield are good at forecasting bond and stock returns. Using annual data of 1871-1989 for the U.S. and 1918-1989 for UK, Shiller and Beltratti (1992) document a significantly positive relationship between the actual excess returns of both asset classes in both the full and postwar (1948-1989) samples, indicating an overreaction by stock market to bond market movements because changes in long-term bond may convey information about the future dividend stream on equities. However, they (1992) also report a significantly negative relationship between the growth rate of actual real log equity prices and the growth rate of actual long-term interest rate in the same periods. Schwert (1989b) finds evidence of positive linkage between interest rate and corporate bond volatility with stock market volatility. The paper shows that the higher financial leverage the more volatile the stock market particularly during financial turmoil and recessions. However, it is suggested that the stock market volatility is difficult to be explained by the variables under investigation –firm profitability, stock trading activity, financial leverage, and , default risk– over time. Campbell and Ammer (1993), on the other hand, find that an affirmative relation between bond and stock returns can be observed due to (i) changes in real interest rates (short- and long-term) via discount rate effect and (ii) common variations in future expected returns. A negative relationship, however, can also be witnessed due to changes in inflationary expectations since inflation has vague effect on stocks but negative impact on bonds. In line with these findings, Fleming et al. (1998) find that volatility linkages across the stock, bond, and money markets are strongly correlated because of a shift in volatility regimes or greater futures market liquidity since the 1987 stock market crash in the U.S. markets. Stivers and Sun (2002) report a positive co-movement between stock and bond returns particularly when stock markets face with lower uncertainty. Nonetheless, the correlation switches sign from positive to negative direction or loses its strength throughout periods of high stock market uncertainty, offering diversification advantages for portfolio allocations between stocks and bonds. Ismail et al. (2016)

reveal positive but insignificant result from interest rates, exchange rate and money supply to stock returns in Pakistan. Rankin and Idil (2014) state that a relatively long period of positive linkages observed over the past two decades was due to a substantial and persistent uncertainty about future economic activity.

In addition to positive relationships, the finance theory suggests a negative relationship between stock and bond returns. Fama and Schwert (1977), Flannery and James (1984), Campbell (1987), Thorbecke (1997), Gjerde & Sættem (1999), Gulko (2002), Li (2002), Ilmanen (2003), Connolly et al. (2005), and Cappiello et al. (2006) are among the authors that observed a negative linkage. For example, Campbell (1987) states that the conditional variances of monthly excess returns have an inverse relationship with the term structure of interest rate in the U.S. during both the sample 1959-1979 and 1979-1983 periods. The excess returns on stocks and bills can be strongly explained by the term structure while the evidence of predictability of bonds is weaker. Test findings of Thorbecke' (1997) paper shows that positive monetary innovations –changes in fed fund rates and central bank reserves– have positive effect on ex-ante and ex-post stock returns by means of decreasing the discount rate or increasing cash flow streams over time. The reverse holds as well. Gjerde & Sættem (1999), on the other hand, reveal that equity returns in Norway react instantaneously to movement in interest rates in the opposite direction. Gulko (2002) finds that the market witnessed decoupling, that is a significantly negative linkage is observed during financial turmoil, and a ″flight-to-quality″ phenomena is observed when implied stock volatility is high. The reverse is true as well, i.e. both markets are positively related when financial market is calm. Particularly, the bond market instruments provide effective diversification opportunities and enable investors to enhance their portfolio stabilities and resiliencies at the time of crisis. In fact, Treasury bonds are recognized as the global safe haven asset of choice by investors. These quite similar evidences are obtained by Baur and Lucey (2009) for developed countries and they (2009) conclude that financial markets that do not display flights during financial turmoil endure greater losses than markets with flights. Empirical test findings of Li' (2002) and Andersson et al.' (2008) papers highlights the uncertainty about inflation rate as the key driver behind negative correlation between treasuries and stocks for the G7 countries and the U.S., Germany, and UK, respectively. There is a positive relationship between inflation and stock-bond correlation, the higher inflation risk, the stronger correlation. According to Ilmanen (2003), the safe haven of choice might keep bond prices more expensive as long as the sign of correlation is negative. The paper finding shows that stock returns determinants –inflation environment, monetary policy stance, business cycle and volatility condition– offer important clues regarding to the sources of inverse relation. Moreover, an inverse linkage makes bond instruments outstanding substitutes, i.e. hedges, against systematic risk episodes –deflation, recession, financial crashes, etc.– Cappiello et al. (2006) reveal that conditional correlation noticeably declines during financial turbulence. Baele et al. (2010) highlight the fact that liquidity proxies have a greater power on explaining stock-bond correlations among a variety of macroeconomic factors.

Along with the correlation relationship, researchers also examine the direction of causal relationship between stocks and bonds. By conducting time-varying causality test, Jammazi et al. (2017) report a strong feedback mechanism that holds over the most of the period under investigation between changes in the long-term government bond rate and stock return of the S&P500 index. In particular, a strengthening causal relationship driven by the U.S. financial stress indices has been observed after the summer of 2007. In a similar vein, Hui et al. (2017) also document a significantly negative linkage in the long run and bidirectional causality in the short run between real interest rate and stock price in Malaysia, suggesting that the stock market stability is strongly related to the real interest rates. On the one hand, Alam and Rashid (2014) find that several macroeconomic variables including money supply and interest rate are negatively linked to the share returns however; there is not statistically significant causality between stock returns and interest rates. A similar finding is reported by Wong et al. (2006) in the U.S. and Singapore for the sample period 1982-2002. A one-way direction of causality, on the other side, running from stock return to interest rate changes is reported by Acikalin et al. (2008) for Turkey and Mohanamani and Sivagnanasithi (2012) for India. According to the authors (2008), an explanation for one-way causality is the signals on trading information sent by the dominant player of the markets to the domestic investors trading at debt markets. Bond market, on the other hand, can indeed forecast the future returns in the stock markets but the causal linkage does not hold from stocks to interest rates, namely, in the reverse direction according to Yilmaz et al. (2006) in Turkey. This finding is consistent with the empirical evidence highlighted in Hashemzadeh and Taylor (1988) for the U.S., in Shah et al. (2012) for Pakistan up to three month, in Chutang and Kumara (2008) for Sri Lanka. Gan et al. (2006) attempted to investigate the impacts of macroeconomic variables including money supply, inflation, exchange rate, (short- and long-term) interest rate, oil price, and real GDP on stock price movements for a sample period monthly observations from 1990-01 to 2003-01. Test findings reveal one-way causal linkages running from interest rates, exchange rates, real GPD, and oil price to stock price in New Zealand.

Muradoglu et al. (2000) find bidirectional causal linkages between inflation and interest rates with domestic share returns in Argentina; unidirectional relationship from stock returns to interest rates in Korea and Mexico. Furthermore, interest rates Granger-caused domestic share returns in Brazil, Zimbabwe and Pakistan. In line with our findings, the authors (2000) did not find any significant causal nexus between share returns and interest rates as well as production index, inflation rate, and exchange rate in Turkey from January 1987 through February 1996. By utilizing both the cointegration and causality tests (VECM), Panda's (2008) paper finds a long run relationship between interest rates and two stock indices –the BSE Sensex and NIFTY– in India. The significant error correction terms indicate long run causal linkage from short- and long-term interest rates towards stock markets, implying that both indices fall to correct disequilibrium relationship in 1.66 and 3.73 months, respectively. Moreover, a bidirectional causality is found between the long-term interest rate and NIFTY index, while, on the other hand, a one-way causal relationship is reported from short-term interest rates towards both indices in the short run. The significant coefficients in VECM models signify that the short-term interest rates positively while the five-period lagged difference of long-term interest rates negatively influence the stock prices in India. Verma and Jackson (2008) document an existence of volatility and price volatility spillovers from interest rates to three portfolio returns. Kaya et al. (2015) document a negative association between share returns and a positive linkage with money supply, while, on the other hand, there is no statistically significant relationship between stock returns and interest rate and industrial production in Turkey. Ferrer et al. (2010) researched the linear and nonlinear interest rate sensitiveness of Spain firms over a sample period between January 1993 and December 2008. Test results show that interest rate sensitivity of firms varies regarding to industry levels and the linear sensitivity is reasonably higher. The highly leveraged, regulated and banking industries are the most sensitive to movements in interest rates. As expected and in common in the previous papers, interest rate sensitivity is mostly negative; however, on the other hand, the exposure sign is surprisingly positive for the banking industry.

It is evident that findings are based on short- or/and long-term not medium-term time horizon. It is also assumed that stock markets are homogenous in terms of investors profile, risk appetite, and expectations. Mainly, it is because of the Efficient Market Hypothesis' (EMH) unrealistic assumptions where it says that all investors have similar expectation regarding to risk-return tradeoff and similar investment horizon. However, as observed in real world, it is not true. The Fractal Market Hypothesis (FMH) of Peters (1994) and the Heterogeneous Market Hypothesis (HMH) of Müller et al. (1993) are among the theories that disagree with the EMH. Both theories overall state that (i) financial markets are not homogenous but heterogeneous with many participants that have different time horizons, (ii) market participants with different investment horizons respond differently to information, i.e. pay attention only to suitable information regarding to their investment horizons, and (iii) both the long-term fundamental investing and short-term technical trading determine the market prices. Short-term trends in market are predominantly stemming from crowd behavior activities, while, on the other hand, long-term trends are the result of changing economic environment. Fluctuations in short-term periods, therefore, will be more volatile than long-term trends.

As pointed out by Gencay et al. (2010), financial markets are comprised of many investors that have different investment (holding) periods. In general, short-term investors trade with regard to trends (fluctuations) observed in markets and their sentiments. For example, short-term investors are grouped into four categories: investors with holding period equal to a couple of days, investors that carry their positions only overnight, intraday traders that change their position at the same day, and the market makers that operate at the highest frequencies. On the other hand, there are long-term investors that follow macroeconomic fundamentals. Accordingly, investors are expected to exhibit homogenous behaviors within their own habitat or class, leading heterogeneous market activities across investment horizons. A shock stemming from short-term activities would be lost its influence in a short span of time, therefore, it will have no significant effect outside their classes. Namely, their influence is restricted since short-term fluctuations have only a limited impact on the timing of fundamentalists' transactions not on long-term type traders' decisions. If so, it is expected that the linkage between debt and stock markets tend to differ across time horizons. To solve this puzzle, however, frequency based tests must be performed.

There are two main causal tests based on frequencies: wavelets and frequency causality test introduced by Breitung and Candelon (2006). Broadly speaking, a time series or signal is decomposed into different time scale components by using wavelet transform. Providing the frequency and time behavior concurrently, wavelets make it possible to uncover the true dynamics of relationship, which is hidden in time domain, thereby, impossible by standard econometric methods. Scale based results are important because they are of interest to heterogeneous market participants, for example, intraday traders, monetary policy authorities, or long-term investors. As Graps (1995) states, wavelets give a chance to see both the trees and forest simultaneously. Moreover, as Schleicher (2002) points out, it is possible to observe how investment horizons act relative to one another and reveal structure of time series at different time horizon. As mentioned above, the correlation relationship may also vary regarding to the investment horizons because, as Harrison and Zhang (1999) contend, short-term investments are likely affected by changes in investor risk appetite, asset allocation decisions, or unanticipated consumption needs. Accordingly, the true relationship in the long-term may deviate from its equilibrium due to this short-time noise. In a related paper, Dajcman (2012) investigates the comovement between sovereign bond yields and equity returns linkages in Eurozone countries –Germany, Italy, Spain, Portugal, and Ireland– by employing a DCC-GARCH model. The findings reveal that comovement between markets display a time-varying pattern. Except for Germany, all countries frequently observe a negative comovement during the European debt crisis of 2010-2011, namely, the flight-to-quality effect is only observed in Germany. Before 2010, however, all countries also show considerable flight-to-quality effects.

Using monthly long-term bond yields and stock price data consisting of 537 observations for the U.S., France, and Canada; 525, 496, 413, and 381 observations for Italy, the UK, Japan, and Germany, Kim and In (2007) reveal a scale-dependent findings. Apart from Japan, the other countries observe a negative correlation relationship between stock and bond yields. Besides, wavelet variance decomposition shows that stock returns are more volatile than bond yields in all countries, with only one exception for Japan. Dajcman (2015) reports the same results regarding to wavelet variance structure except for Portugal in ten European countries. The implication of this result for investors is that short-term traders should respond to every variation in asset returns to efficiently managing their portfolio risk. In addition, wavelet based correlations between changes in bond yield and stock prices are mostly positive, except for Portugal, at all scales. It is also proved that the comovement between financial markets in Germany and Portugal exhibit both scale-dependent and time-varying phenomenon during tested period. Tiwari (2012) examines the causal linkages between monthly stock prices and interest rates in India over the sample period between 1990-M01 and 2009-M03. Test findings of the cross-wavelet coherency approach show significant causality and both cyclical and anti-cyclical linkages over scales and periods. For example, a causality running from stock prices to interest rates is found at high frequencies corresponding to 1-4 years period. This finding implies that interest rates receive cyclical impact from stock prices. On the contrary, interest rates Granger-cause stock prices in 8-12 year horizon, indicating that stock prices receive cyclical impact from interest rates. Asgharian et al. (2015) investigate the factors that may have possible impacts on the correlation relationship between stocks and bonds in the long-term by conducting DCC-MIDAS models and wavelets. The authors (2015) reveal that the most factors including industrial production, inflation, short-term interest rates, trading volume, default spread, and producer and consumer confidence indices have significant power on the long run relation estimation. On the contrary, the effect of macro-finance variables on the long run (negative) correlation is found to be strong when the economy is weak. Ferrer et al. (2016) study the interdependence between share returns and movements in the long-term government bond yields by conducting wavelet coherency approach for several Eurozone countries. The main finding is that the interdependence considerably varies over time and frequencies and among countries. The strongest interdependence between markets is observed in the UK, while, on the other hand, markets in Portugal, Greece and Ireland show the weakest interdependence over time. In addition, the strongest relationship is predominantly intensified at lower frequencies corresponding to one to two yearly investment horizons. The empirical paper of Özer and Kamisli (2015), on the other hand, reveal significant spillover effects from equity returns to interest rates in the midst and lower frequencies in Turkey. More clearly, test findings report one-way causal linkages running from equity returns to macroeconomic factors including interest rates and exchange rates (in Dollar and Euro), however, the causality does not run in the inverse direction in the time domain. To uncover the hidden relationship that dispersed over frequencies, the authors (2015) implemented the Breitung and Candelon (2006) causality test and they find that the causality is concentrated on the medium- and long-term, driven mostly by trading activities of the foreign investors' pressure on stock market liquidity.

In this paper, the empirical relationship between stock returns and bonds yields is reinvestigated by using weekly observations of two-year government bond rates and the closing prices of the twenty-five stock market indices consisting the aggregate (1), financials (6), services (7), industrials (8), technology (2), and investment trust (1) index, by conducting two important complementary directions. In order to provide a deeper understanding for a bond-stock relationship, we implement both standard and frequency-based methodologies, wavelets, since we believe that the former method is incapable of uncovering true dynamics of the relationship. We compare our wavelet-based findings with the another's results, since it is only appropriate for short and/or long-term horizon. However, as observed in real world, each market is comprised by many participants with different investment horizon and degree of risk aversion. They have different characteristic dealing frequency, operate at multiple investment horizons and reacts differently to the same information in the same market than other components, even though they are included in the same investment horizons from daily to yearly. Because standard method cannot give an appropriate information to market components with, for example, 2-4 weekly period, this paper is motivated to provide each of market component valuable information for their trading strategies or monetary policy decisions.

First, according to standard method, all variables are found to be stationary at the first difference while only a few variables exhibit cointegration associations. Moreover, there exist one-way causal linkages running from bond yields to stock returns regarding to symmetric causality test, while, on the other hand, conventional models based on VAR models reveal a few causality results in the time domain. By implementing these causality tests on wavelet coefficients obtained by "MRA()" command, bidirectional significant relationships, which are in line with the existing studies' findings, are observed at higher investment horizons. Besides, another frequency-based causality test introduced by Breitung and Candelon (2006) corroborates our wavelet-based findings, suggesting that causality relationship is concentrated on the lower frequencies. To understand the direction of causality, we also conduct another important approach, the asymmetric causality test, introduced by Hatemi-J (2012). This method reveals significant results that are in common with the previous studies' findings, where evidence of negatively strong causal relationships between different components (negative and positive components) is revealed. This significantly inverse relationship is also validated by scale-dependent correlations. Namely, the higher investment horizon, the stronger but negatively co-movement between stock and bond returns. Finally, the results show that the volatility of short run dominates both the mid- and long run scales, indicating that short-term investors should realize their return because at least 76% of variation in asset returns is observed at the highest frequencies.

This doctoral thesis contains five chapters. The first chapter is the introduction. The second chapter, however, is devoted to (a) the risk-return and valuation fundamentals consisting of the major important theories regarding to stock valuation such as the Efficient Markets Hypothesis (EMH), Capital Asset Pricing Theory (CAPM) and Arbitrage Pricing Theory (APT) as well as the Behavioral Finance Theory. It is also related to interest rate fundamentals, namely, interest rate theories such as liquidity preference theory and loanable funds theory, the risk structure and term structure of interest rate. This chapter concludes with a comprehensive empirical and theoretical literature review, namely, test results of papers on stock-bond relationship in the time and frequency domains are discussed. On the other hand, the chapter 3 gives the details of the theoretical framework of wavelets and Fourier analysis and compares their advantageous over another one. The fourth chapter starts with the rationale for the paper and empirical financial series that used in the paper. Next, it sheds light on the relevant information about econometric and wavelet-based methodologies, such as unit root, cointegration and causality tests that used to study relationship between stock and bond returns. This chapter concludes with the empirical test results, where econometric outputs are explained and compared with the existing researchers' findings. The chapter 5, however, ended with conclusions, empirical and theoretical implications for investors and policy makers, and recommendation on the future studies.

CHAPTER

2 INTEREST RATE CHANGES AND STOCK INDEX RETURNS RELATIONSHIPS

The major aim of this paper is to study the relationship between the most important factors in financial markets: interest rate yields and stock returns. However, before delving into a related discussion, it is required to define their fundamentals both conceptually and algebraically. Firstly, we begin with a definition of characteristics of risk concepts for single assets and portfolios. After, the calculation of return on investments and the asset valuation theories are discussed. Lastly, the foremost concept required for valuation, interest rate and its historical theories are analyzed.

2.1 Risk Fundamentals

The first question to be answered should be related to the definition of risk. In finance and economics literature, there are numerous definitions, but the most accepted one is given by Megginson (1997) where the author succinctly defined it as the chance of financial loss and expressed as fluctuations of the fixed-income investments returns during a holding period. Its definition and measurement are important because it is the one of the main factors as well as return to being successful when faced by different investment opportunities. An investment asset or a portfolio with the possibility of higher loss is regarded as a more risky choice than those with a lower probability of financial loss by an ordinary investor, all else being equal. Hence, the probability of deviations in asset's or portfolio's return must be calculated before an investment decision made. The general statistical measurement for risk is a variance or its square root, standard deviation, of return fluctuations from an asset's average return series. If an asset has a greater variance or standard deviation value, it suggests that it might have a great probability for a financial loss or reward at the end of investment horizon due to its duration/maturity, volatility, liquidity and/or its issuer creditworthiness. Simko (2012) states that the fear of financial loss for a corporate bond consists of the inability to obtain periodic investment payments and/or take the principal amount back when due. The risk definition may vary according to different risk tolerance, for example, an ordinary investor is assumed to the risk-averse investor if she/he avoids investing in volatile instruments while a risk-lover investor seeks to invest in the higher volatile instrument for higher return probabilities. When given an option among several investment opportunities with the equal expected return but different volatilities, as noted by Megginson (1997), if one investor chooses the one with the lowest variance, then it is said that she/he is a risk-averse investor. Being indifference is to be risk neutral, namely, investment choices are not influenced by the degree of uncertainty; the expected return is the only important matter. A risk-lover investor is, however, an investor type that is willing to choose the more volatile investment when she/he is faced by several options with having an equal expected return. Evidently, the different choices are shaped by risk tolerances or utility functions. In finance, it is generally assumed that most investors are risk-averse, willing to maximize their wealth by maximizing their utility with the lowest risk preferences. Whether holding a single asset or constructing a portfolio, risk plays a critical role before investment decision is made. Thus, we need to understand how investment risk is calculated.

The risk calculation for a single asset is very straightforward. It is equal to variance or standard deviation of the possible returns deviation from the average value during a investment period. If a portfolio includes a single asset, then its variance is the same with the asset's variance. However, adding more than one asset into portfolio introduces a different risk framework: covariance. The variance and covariance of a single asset, A, or portfolio, P, is calculated as follow:

$$\sigma_A^2 = \sum_{i=1}^{N} \alpha_n [r_n - E(R_i)]^2 \tag{1}$$

$$\sigma_P^2(R_P) = E[R_P - E(R_P)]^2 \tag{2}$$

$$\sigma_{A,B} = E\{[R_A - E(R_A)][R_B - E(R_B)]\} \tag{3}$$

$$\sigma_P^2(R_P) = W_A^2\sigma_A^2 + W_A^2\sigma_B^2 + 2 * W_A W_B \sigma_{A,B} \tag{4}$$

where W_A represents the weight of the asset A in the portfolio and $E(R_A)$ denotes the average value of the asset A. It is evident that the variance of the single asset depends only on its value while it is cumbersome for portfolios.

Fabozzi and Drake (2009) note that portfolio variance is not simply a squared weighted sum of the single asset variances, but rather a sum of these variances plus a weighted measure of the covariance between the two asset returns. In equation (4), it is clear that portfolio variance is equal to the squared weighted of the each single asset and plus two times the weighted covariance. Note that, the covariance can be calculated as $\sigma_A\sigma_B\rho_{A,B}$ where the correlation coefficient, $\rho_{A,B}$, determines the co-movement direction and strength of the relationship. By including correlation coefficient into portfolio variance calculation, the new equation will be as given

$$\sigma_P^2(R_P) = W_A^2\sigma_A^2 + W_A^2\sigma_B^2 + 2 * W_A W_B \sigma_A \sigma_B \rho_{A,B} \tag{5}$$

Megginson (1997) documents that the fundamental importance of Equation (5) becomes important when an investor includes more assets in his portfolio. Because, including many assets leads Equation (5) become complex to variance calculation, namely, the covariance that must be calculated increases as the number of asset increases. Here, the contribution of single asset variance to total variance decreases quickly which leads covariance put forward. However, randomly adding assets into a portfolio increases the total variance of the portfolio although the measurable impact of the covariances between assets is small. To construct a well-diversified portfolio where the investor seeks to obtain a trade-off between risk and return, the most important thing to be concerned is to identify the covariance between the risky assets and market portfolio.

Focardi and Fabozzi (2004) assert that total risk that must be bore is broken into two main categories. The first and foremost part is the systematic risk stemming from fluctuation of macroeconomic factors. This risk component cannot be eliminated through portfolio diversification. Thus, it is also called as nondiversifiable risk factors, because no matter what an investor takes a step to eliminate, it is impossible to succeed. Megginson (1997) notes that the market risk pertains to a firm's sensitivity to business cycles, i.e., political and macroeconomic factors that can influence all investment instruments. On the other hand, the other risk component is called as the unsystematic risk that is related to a particular investment instrument rather than to the overall market forces. Hubbard and O'Brien (2011) state that these risk factors are unique to the firm, namely, they are affected by microeconomic factors such as scientific discoveries, unfavorable lawsuits, regulatory action, and/or worker strikes. With constructing an efficient portfolio, an investor can eliminate their effects on the portfolio returns. Hence, we can formulate the total portfolio risk as given

$$\sigma_A^2 = E[R_A - \bar{R}_A]^2 = E\{\beta_A[R_M - \bar{R}_M\} + \varepsilon_A]^2$$

$$\sigma_A^2 = \beta_A^2 \sigma_M^2 + \sigma_\varepsilon^2 \tag{6}$$

Mishkin (2007) state that the first component in equation (6), $\beta_i^2 \sigma_m^2$, refers to systematic (nondiversifiable) risk and σ_ε^2 denotes the unsystematic risk. Thus, it can be concluded that the risk of an efficient portfolio depends only upon the nondiversifiable risk factors. To a clear understanding of the relationship between these risk components, a graphical representation by Jordan and Miller (2009) is given in Figure **2-1** where the horizontal axis reports the number of assets and the vertical axis documents the total risk of the portfolio return.

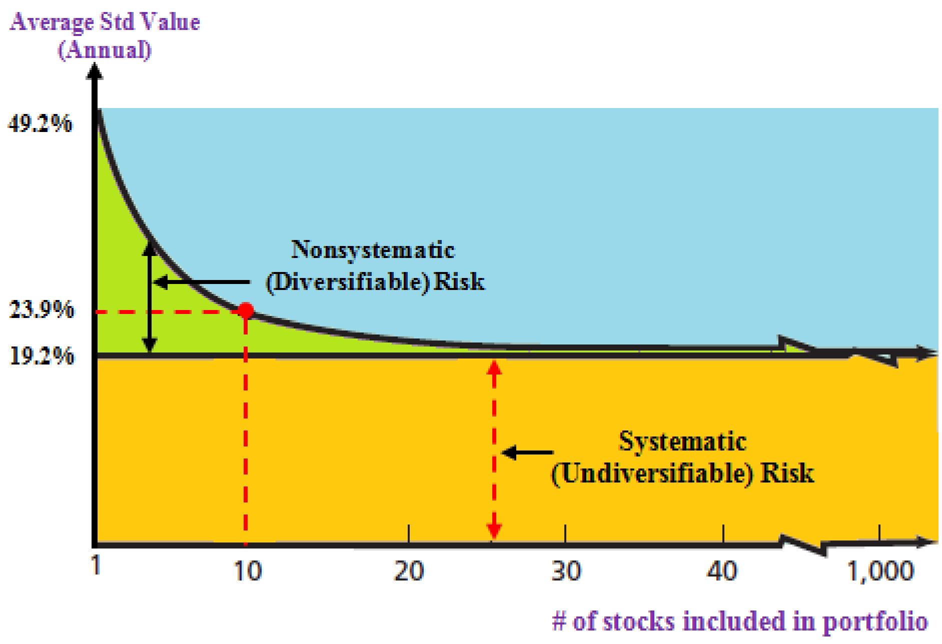


Figure 2-1 Effect of Portfolio Diversification

Source: Jordan and Miller (2009).

Figure **2-1** illustrates how diversification reduces the total portfolio risk, especially the diversifiable risk component related to nonsystematic risk forces. It is evident that when adding more assets, the total risk declines until it approaches a limit. According to the test results of different papers, a portfolio size of between 30 or 50 randomly selected stocks nearly eliminates the firm-specific risk factors leaving only nondiversifiable risk. Thus, it can be said that diversification shrinks total risk up to a point.

2.2 Return Fundamentals

After defining the risk concept in terms of single and portfolio, now we will give a brief detail about return calculation of individual assets and portfolios. The rule of thumb in the investment process is to select a security or portfolio with a higher return on alternatives. Thus, for a better choice, one must learn how to define and measure the historical (realized) and expected return over a holding period.

Reilly and Brown (2011) define return simply as the change in wealth of investor at the end of a period, deriving from a negative or positive change in the price of assets or cash flows, such as dividends from stocks or interest payment from bonds. Its mathematical expression is given as

$$R_A = \frac{P_1 - P_0 + D}{P_0} = \tag{7}$$

where P_0 and P_1 denote value (price) of the asset A at-the-beginning and at-the-end investment period, and D signifies the any cash flow obtained from the asset A during the holding period. Equivalently saying, the numerator shows the terminal wealth (ending value of investment) while denominator represents the initial wealth (beginning value of investment). Besides, the return rate of a portfolio can be calculated as

$$R_P = w_1 R_1 + w_2 R_2 + w_3 R_3 + \cdots + w_N R_N = \sum_{i=1}^{N} w_i R_i \tag{8}$$

where R_1 is the return rate of asset i during the investment period, w_i is the proportion of asset i and, N denotes the asset number in the portfolio R_P. Equation (8) implies that the portfolio return is equal to the weighted average of the individual assets' return or the overall percent change in value of the initial wealth. Here, the rate of return is also called as ex-post (historical/realized) return that presumes nothing about the probability distribution of different outcomes during the period. In the case of expected return, the formula will be as given

$$E(R_A) = p_1 R_1 + p_2 R_2 + p_3 R_3 + \cdots + p_N R_N = \sum_{i=1}^{N} p_i R_i \tag{9}$$

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2) + \cdots + w_N E(R_N) = \sum_{i=1}^{N} w_i E(R_i) \tag{10}$$

where p_i denotes the probability of occurrence of the ith outcome R_i for asset A, $E(R_A)$ and $E(R_P)$ indicate expected (ex-ante) return for asset A and portfolio, respectively, and N signifies the number of possible outcomes for both A and portfolio. Hence, Equation (10) shows that the expected rate of return for a portfolio is equal to the weighted average of the expected rate of returns of each single asset in the portfolio (Fabozzi and Drake, 2009).

2.3 Valuation Fundamentals

Fabozzi and Drake (2009) state that valuation can be described as the process of determining the fair value of any assets such as stocks, bonds or derivatives. In modern finance theory, regardless of the financial asset type, the fundamental value is equal to the present value of a stream of expected cash flows to the holder of the asset during the investment period discounted with an appropriate discount rate. The main purpose of the determining the fair value is, however, to find mispriced assets with different analyzing techniques such as technical analysis and fundamental analysis. In the analyzing process, there are three main inputs to determine. According to Megginson (1997), they are (a) the estimated the stream of cash flow(s), (b) the required rate of return to discount the cash flow(s), and (c) the timing of cash flow(s). If these are accurately quantified, then the valuation becomes very easy to calculate.

The value of a bond begins with the estimation of the stream of cash flows. If the target bond is a simple one, then both interest (coupon) payments and principal repayment are the stream of cash flows to be estimated. In this case, as Bodie et al. (2014) assert, the fundamental value is equal to present value of coupon payments expected to receive until the maturity date and par (face or principal) value of the bond. Hence, the required formula for bond valuation can be formulized as follows:

$$\text{Bond Value} = \sum_{t=1}^{T} \frac{C}{(1+k_d)^t} + \frac{M}{(1+k_d)^T}$$

$$\text{Bond Value} = C * \left(PVIFA_{k_d,T}\right) + M * \left(PVIF_{k_d,T}\right) \tag{11}$$

where T denotes the maturity date, C and M represent the coupons and the principal of the bond. Besides, $PVIFA$ is the present value interest factor of annuity and $PVIF$

is the present value interest factor. Note that, k_d is the required rate of return and it should be at least equal to the default-free rate. In the case of default and the other risks, then investors require a higher interest rate. Fabozzi and Drake (2009) remind that for a default-free bond the minimum interest rate is referred to as the base interest rate that varies according to different maturities. If investor perceives that the bond is not default-free, normally she/he will require a higher return rate for compensating additional risks. Hence, the required return will be the sum of base interest rate and risk premium (spread) caused by the creditworthiness of the bond issuer, options (if any), the taxability of interest rate and the expected liquidity.

Before proceeding further, it should be remarked that the bond price/value depends on the spread between the required rate and coupon interest rate. If the required rate is higher than the coupon rate, then it is said that the bond is sold at a discount where the bond value is less than the principal (par) value. If the former rate is lower than the latter, then it is said that the bond is sold at a premium, namely, the bond value is greater than the principal (par) value. As the time to maturity decreases, the fair value of the premium bond decreases until it gets close to the par value while the value of discount bond increases until it returns to the par value at the end of maturity.

Now it is time to focus on the stock valuation where the logic behind it is the same with the bond valuation. Namely, a stock value is equal to present value of the stream of cash flows, for example, dividends, to be received and stock price at the end of the investment horizon.

$$P_0 = \frac{D_1}{(1+k_s)^1} + \frac{D_2}{(1+k_s)^2} + \cdots + \frac{D_\infty}{(1+k_s)^\infty} = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_s)^t} \tag{12}$$

Equation (12) shows the basic valuation model for the common stock over an infinite time horizon. It should be noted that the last price of the stock is not included in the equation. Brigham and Ehrhardt (2013) and Megginson (1997) state that unless the firm's assets are liquidated or sold to another company, the value will be equal to the price calculated by a formula written in Equation (12). For example, at finite investment horizon the stock price, at "t" time point, will also be determined by the distant dividends that expected in the future. Thus from a valuation perspective, for all current and future investors in total, the only important thing to be concerned is the expected cash flows based on the future dividends, namely, the sale price is irrelevant.

The stock value can be calculated in the case of zero growth rate with the following equation:

$$P_0 = \sum_{t=1}^{\infty} \frac{D_1}{(1+k_s)^t} = D_1 * \left(PVIFA_{k_s,\infty}\right) = \frac{D_1}{k_s} \tag{13}$$

where D_1 is the estimated dividend amount at the end of year 1. This formula, as noted by Megginson (1997), is the simplest method to use for stock valuation because of non-growing, constant dividend stream. However, it is known that dividend amount might change because of different reasons, namely, it can be falling, or rising or fluctuating randomly in the subset of investment horizon. The most difficult part in valuation, actually, is the estimation of future dividend stream. To overcome this difficulty, Myron J. Gordon suggests assuming a constant growth rate for dividends. Thus, the formula will be as given (Brigham and Ehrhardt, 2013)

$$P_0 = \frac{D_0(1+\mathrm{g})^1}{(1+k_s)^1} + \frac{D_0(1+\mathrm{g})^2}{(1+k_s)^2} + \cdots + \frac{D_0(1+\mathrm{g})^\infty}{(1+k_s)^\infty} = \frac{D_1}{k_s - \mathrm{g}} \tag{14}$$

where D_1 is equal to $D_0(1+\mathrm{g})$. It should be pointed out that there is a necessary condition for the validity of formula in Equation (14) where k_s must be greater than dividend growth rate, g.

2.3.1 Stock Valuation Theories

In this section, we will describe the most important stock valuation theories that are widely used to determine the stock values. First, we begin with the main assumptions made to describe the investor expectations in financial markets. After, we will give brief information about some related theories such as the waiting theory of interest, the Q theory of investment. The most vital stock valuation theories of capital asset pricing model (CAPM) and arbitrage pricing model (APT) will be the next topics. We will conclude this section with the behavioral finance theory where the irrational behavior of investors is discussed.

2.3.1.1 Adaptive vs. Rational Expectations

In the previous section, we mentioned that an equity price is the present value of the expected stream of cash flows, dividends, in the future. Hubbard and O'Brien (2011) assert that regarding the constant growth model, to calculate a stock price Myron J. Gordon suggests using the current dividend payment, the required rate of return and the expected growth rate for dividend payments. Investors' expectations are a critical factor when determining stock valuation. In economics and finance, the model that assumes that an asset value depends only on the past values is called as adaptive expectations. The main reason behind this approach is that some certain patterns seen in the stock price history are likely to be repeated, hence, one can use the past price to forecast stock prices. However, using only past information and ignoring the available/current additional information to forecast may lead to inaccurate results. John Muth recommended a new approach called rational expectations in 1961, where he suggested using all available, past and current, information to forecast stock prices accurately (Hubbard and O'Brien, 2011). With rational expectations, people make forecasts using all available information. Muth remarks that if an investor does not use all available information, then it could be said that she/he had been acting irrationally, leading a mispricing in stock valuation. As noted by the authors (2011), one can apply this approach whenever she/he tries to forecast asset prices to obtain efficient and accurate valuation in financial markets. Therefore, in case of the application of rational expectation to financial markets, the market will be called efficient, where it is assumed that the investors' optimal forecast will lead to an equilibrium in stock prices.

2.3.1.2 Efficient Market Theory

The efficient market hypothesis (EMH) has been widely discussed by researchers and practitioners in the financial markets and by academicians in finance and economics world. As noted by Sharpe et al. (1999), this hypothesis is likely to have a great interest and to be a part of this debate, one must understand what it does mean to be efficient in the markets. Understanding it accurately, as stated by Hubbard and O'Brien (2011), then one can easily make investment strategies for different needs, such as portfolio allocation (diversification), trading and assessing the value of technical and fundamental analysis methods.

According to Sharpe et al. (1999), market efficiency is regarded to the allocation of funds, where funds must be channeled to the place for the best outcome. The authors (1999) called these markets as allocationally efficient market that should be encouraged by government policies. To be allocationally efficient, however, capital markets must be both internally and externally efficient. In internally efficient markets, the transaction fee is low but its speed is high due to fairly competition among brokerage houses. On the other hand, in external market efficiency, Reilly and Brown (2011) assert that the dissemination of information in the markets is very quick; therefore, equity prices are expected to fully and accurately reflect all available, past and current, information. If so, then it is said that the market is informationally efficient. In these markets, it is assumed that (a) there are many profit-maximizing and independently acting investors, (b) the new information randomly and unpredictably arrives to the markets, and (c) the investors' decisions lead stock prices to adjust rapidly to reflect the publicly available new information. Another way to phrase this efficiency is that, as Jordan and Miller (2009) remark, the market efficiency is caused by three economic factors of investor rationality, independent variations from rational decisions, and arbitrage. In fact, one of those factors enough makes a market to be efficient. Here, the rational term merely implies that the decisions of undervaluing or overvaluing stock prices are not made systematically with regards to all available information. Whenever stock price is different from its investment (fair, intrinsic) value, according to Hubbard and O'Brien (2011), rational expectation of investors will give a motivation to make money. Due to arbitrage opportunities, its price will be equal to the fundamental value, leading an existence for efficient markets. Namely, the driving force behind this efficiency, as Jordan and Miller (2009) remarks, is competition among market participants and the profit incentive with the aid of the most advanced analyzing tools.

Reilly and Brown (2011) contend that the earlier papers than Fama's (170) were largely based on the random walk hypothesis, popularized by Burton Malkiel, where it is assumed that a price change in stock is occurred by chance. Equivalently speaking, the equity prices cannot be forecasted, i.e., they are unpredictable. Sharpe et al. (1999) assert that the probability of an increase or decrease in stock prices is equal, namely, the new and unexpected information could be a negative or positive one.

Elton et al. (2009) contend that the hypothesis that stock prices rapidly and fully reflect all available information is very strong and extreme claim. To have an incentive to trading until the stock prices incorporate information depends on the equilibrium between the marginal cost and marginal benefit. In a related paper, Eugene Fama (1988) divides the market efficiency into three subhypotheses: (a) weak-form, (b) semi-strong-form, and (c) strong-form EMH. Dealing with different information set, the market efficiency can be illustrated in Figure **2-2**.

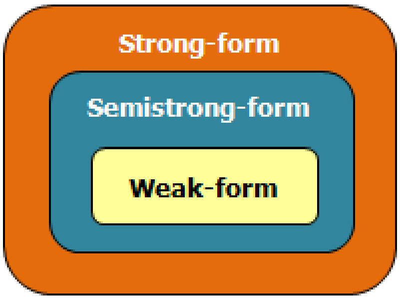


Figure 2-2 Information and Market Efficiency Levels in EMH

Source: Jordan and Miller (2009).

Evidently, the information set regarding the efficiency levels in Figure **2-2** are nested. Equivalently speaking, as noted by Sharpe et al. (1999), moving from weak-form to semi-strong form and to strong-form efficiency level, the extent of the information set increases. For instance, if a stock market is found to be efficient in strong-form, it implies that it is also efficient in semi- and weak-form.

Reilly and Brown (2011) contend that, in the first efficiency weak-form, it is assumed that equity prices fully reflect all available (current) information such as the historical price, return, volume and other security market-generated data. If so, security return is assumed independent, namely, there is not a relationship between past and current return data. Another way to phrase is given by Bodie et al. (2014), implementing trend analysis, depending on the past information to forecast the current prices, is a useless effort, i.e. fruitless because its related historical data is straightforwardly costless to obtain. Namely, if such information ever conveyed trustworthy signals regarding stock's future performance, it would be expected that all stock market participants already would have used to exploit them. Thus, earning high returns in the efficient markets is not possible, or it lasts very shortly because those signals lose their value.

Brigham and Ehrhardt (2013) assert that the all publicly available information, including past prices, management quality of the firm, earning and dividend announcements, news about the aggregate economy and political issues, etc., are incorporated in the current stock prices according to semi-strong market efficiency. Differently speaking, Megginson (1997), one of two implication of this assertion is that the level of stock prices should incorporate all relevant information (past, current and forecastable) whilst the second implication suggests that the new information is incorporated into security prices fully, accurately and instantaneously. Therefore as suggested by Brigham and Ehrhardt (2013), if a stock market is efficient in semi-strong, the fundamental analysis approach is of no use in beating the market. Note that if the semi-strong efficiency exists, then the stock market is also efficient in weak-form, but not in strong-form. The authors (2013) remark that return rate in the semi-strong efficient market is equal to the rate predicted by the security market line (SML) and the stock prices respond only the unexpected parts of new pertinent information announcements. Therefore, it is not possible to earn consistently above-average returns (other than by chance) and outwit the stock market in the semi-strong efficient markets.

According to the extreme and strong efficiency form, as noted by Radonjić and Kokotović (2012), all pertinent public and nonpublic (private information which is available only to company insiders) is accurately and fully incorporated in the current stock prices. An average individual or institutional investor cannot consistently beat the market and earn abnormal profits in the strong-efficient markets. On the average, however, it is highly possible that the return and profit earned will not exceed transaction costs. Namely, the marginal cost of acquiring new pertinent and useful information is equal to the marginal return, which is zero. Therefore, if a stock market is in the strong-efficient form, Brigham and Ehrhardt (2013) remind that it is not possible even for insiders to obtain consistently above-average returns.

Sharpe et al. (1999) remark that in a perfectly efficiency environment, the level of price changes are random, in a rational manner, however. Hubbard and O’Brien (2011) remind that this hypothesis does not require of having rational expectations for all efficient market participants. As long as there are enough well-informed and profit-maximizing rational investors, stock prices will be pushed to its fundamental value by their intensive competition and the profit motive. Namely, the pertinent new information will be digested by stock prices. Sharpe et al. (1999) state that those price changes are caused by traders’ reassessing stock valuations and adjusting their investment strategies. As a consequence of their efforts, a stock price will be equal to its fundamental (intrinsic) value, leaving no room for arbitrage opportunities.

In fact, Sharpe et al. (1999) list some observations seen in perfectly efficient markets both in the cases of with and without transaction costs. In the efficient markets, traders should only expect to earn a normal return rate, even though using either technical or fundamental analysis efforts for finding mispriced securities will be worthless. Besides, the efficiency holds if enough traders think that the market is not efficient because this belief will lead to reflect fair stock valuations. Successful investment strategies will not generate above-average returns anymore after they are disclosed to the public. Earning abnormal returns by some traders is totally due to chance, not skill. Professional traders, however, are not a cut above ordinary traders when it comes to picking misprices stocks and, therefore, earning above-average profits. Lastly, past performance is not guaranteed, namely, it cannot be accepted as an indicator of future performance because past infelicity and fortune do not tend to repeat themselves. On the other hand, if the transaction costs are taken into account, then it will be seen that identifying mispriced stocks is possible, but, in exchange for increased transaction costs. Besides, a performance of a passive buy-and-hold strategy is nearly the same with a professionally managed active portfolio due to high transaction costs.

Megginson (1997) states that E. Fama decomposed test of market efficiency into three different categories. The first group includes tests for yield predictability instead of weak-form tests while event studies for price adjustment instead of semistrong-form tests are included in the second group. The third group comprises of tests for private information. However, the author (1997) adds a fourth category including tests for rational fundamental valuation.

When looking at the tests for return predictability, there are several classifications by different researchers. For example, Elton et al. (2009) and Bodie et al. (2014) include the tests of (a) examining seasonal patterns (daily, weekly or monthly) in security prices, (b) the predictability of return using past trends (both short and long-term), and (c) return and firm characteristics into the weak-form hypothesis. On the other hand, Reilly and Brown (2011) assert that the weak-form tests are grouped into two categories: (a) statistical tests of independence between stock returns and (b) tests of trading rules concerning a comparison of risk-return results. Remark that we have mentioned that the stock prices are independent according to weak-form efficiency. In literature, the first group includes two components: autocorrelation tests and runs test. Bodie et al. (2014) document that some researchers find positive but small serial correlation findings for weekly returns of NYSE securities over the short time periods, implying that positive stock returns tend to follow positive returns. Besides, there is evidence of momentum effect in both the particular stocks and aggregate market index, implying that poorly- and well-performing stocks in one time period tends to follow this performance patterns in following periods.

One of the tests among trading rules is the filter tests, i.e., a timing strategy, where it is suggested, as noted by Megginson (1997), “if a stock’s price has risen by x percent, then buy it and hold until it has declined by y percent”. The author (1997) and Elton et al. (2009) remarks that the relevant papers' result showed that this strategy was not worth pursuing when transaction costs and taxes had been taken into account. Reilly and Brown (2011) state that some trading rules' –considering short sales, advanced-decline ratios, specialist activities, and short positions– results generally support the weak-form efficiency form.

Megginson (1997) and Reilly and Brown (2011) stated that the relevant studies for testing of semi-strong efficiency form could be divided into two categories. Some tests included in the first group of studies to forecast future performance of stocks are "risk premium proxies", "quarterly earnings reports", "anomalies", "predicting cross-sectional returns", "price-earnings ratios", "the size effect", "neglected firms and trading activity", and "book value-market value ratio". In literature, it has been seen that, according to Reilly and Brown (2011), using the aggregate dividend yields and yield spreads (default spread and the term structure spread for interest rates) can give a significant result for predicting stock returns, implying a negative support for semi-strong form efficiency. Besides, some studies considering quarterly earnings reports showed that a surprise in earnings is not immediately reflected in security prices; therefore, earnings revisions and surprises can be used to predict individual stock returns, implying a negative evidence against the semi-strong efficiency form.

Now, it is time to give information about the concept of anomalies observed in stock return patterns. They are accepted in literature as a contradiction to efficient market hypothesis because they are inexplicable and hard to reconcile with the EHM. Jordan and Miller (2009) list three important facts about anomalies: they are (a) generally small, (b) tend to vanish when found out, and (c) not easily used for trading strategy because of transaction costs. Some anomalies are the small-sized firm effect, the low price-earnings ratio, the neglected-firm effect, book-to-market value ratios, and various calendar effects. Fabozzi and Drake (2009) state the result of Banz (1981) where it was shown that portfolios including small-sized firms had outwitted portfolios including large-sized firms because small firms tended to have more risk than larger firms did. Besides, Elton et al. (2009) remark that the small-size effect generally seen in the first two weeks in January, implying a strong correlation between the January effect and the small-sized firm effect. Reilly and Brown (2011) assert two strongest explanations: higher risk measurements because of infrequent trading activities and higher transaction costs. On the other side, the neglected firms anomalies reveal that both small- and neglected firms' stocks generated higher returns, because, according to Bodie et al. (2014), they are generally neglected by large institutional investors and relevant information is less available, which make them riskier and thus enable to generate higher returns. Apart from the anomalies related to firm characteristics, there are several important calendar anomalies observed in financial markets: the month-of-the-year effect, January anomaly, weekday effect, and holiday effect. According to January effect, as stated by Reilly and Brown (2011), individual and institutional traders tend to sell out stocks before the end of year, and reacquire them or buy similar securities in the beginning of year, causing a downward and upward pressure on these stocks in late December and January, respectively, thus generating significant, in both economically and statistically abnormal returns in January. On the other hand, researchers documented a Monday effects, wherein Monday is found to be the only weekday having a negative average return. Lastly, the authors (2011) also contend that stocks with low P/E ratios will outwit stocks with high P/E ratios because of overestimation for the growth companies.

Another important tests for semi-strong efficiency is event studies that, as dictated by Sharpe et al. (1999), undertaken to see how fast stock prices actually respond to the information announcements, namely, how fast this information is reflected in security prices. It is an important tool in finance, Megginson (1997) state that, because of its simplicity when carrying out, flexibility and clarity of purposes. With this method, one can see whether stock prices reacted rapidly or slowly information releases (Sharpe et al. (1999). After the day of the new announcement, the stock return is abnormally high or low or it is just the normal rate determined by asset-pricing models.

Elton et al. (2009) gave the necessary eight steps for conducting event studies in their book. At the first step, one must determine the firm sample that had a surprise news announcement. At the second step, designate this announcement days as zero. Next, determine the sample period. At the fourth step, calculate daily returns for each firm included in the sample. After that, it is time to calculate the daily abnormal returns and the average abnormal return for each day for each firm. At the seventh step, calculate the cumulative abnormal return (CAR) from the starting day. At the final step, examine and discuss the obtained results.

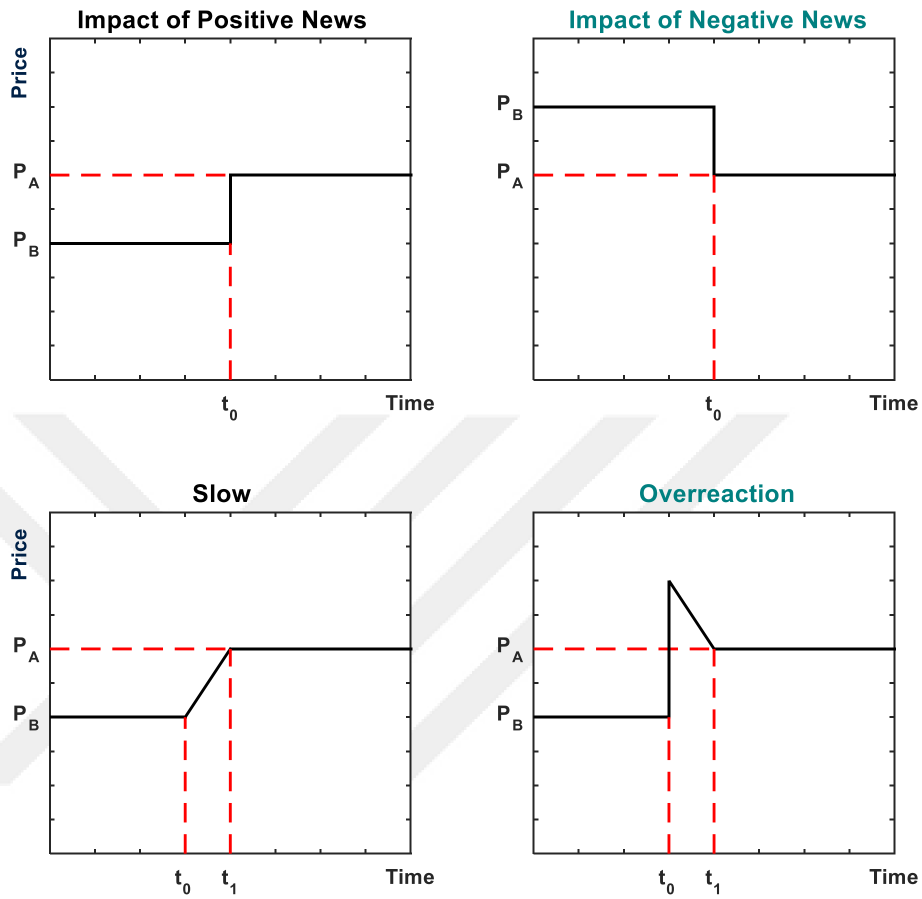


Figure 2-3 Possible Effects of Information on Equity Price in Efficient vs. Inefficient Markets

Source: Sharpe et al. (1999).

To a clear understanding event studies, see Figure **2-3** obtained from Sharpe et al. (1999) where the horizontal axis shows the timing of the announcement and the vertical axis is the stock price, to be precise, the abnormal return of the stock. In a perfectly efficient market, the stock price will react, as in the upper left-hand, instantaneously when good news information is released. The effect of bad news is illustrated in the upper left-hand, in which the price jumped to the new equilibrium price, P_A, at time point zero, t_0 and remains the same until new, additional, information releases. On the other hand, the effect of news announcement in inefficient markets is depicted in the lower part of the figure. As soon as good news is released, the reaction of the stock takes some time in those markets, namely, the equilibrium price occurs at point t_1. On the other hand, in the case of bad news the stock price overreacts at first, but then settles down to its equilibrium price at time point t_1.

At the strong-form market efficiency, studies are grouped into three categories: (a) corporate insider trading, (b) analysts' recommendations, and (c) performance of professional money managers (Reilly and Brown, 2011). At first group, it was observed that corporate insiders had earned supernormal returns, rejecting the strong-form efficiency. Besides, mimicking trades of the corporate insiders could not yield abnormal returns to the outside traders. According to the authors (2011), there is evidence in favor of analysts' recommendation. Besides, most of money fund managers' performances were found to be less than a buy-and-hold strategy. After considering risk factor into returns, without transaction costs, more than 50% of managers outperformed the overall market. When transaction costs were taken into account, the number of successful managers, however, decreased, namely, two out of three managers' performances were less than the overall market return rate.

2.3.1.3 Behavioral Finance Theory

Although it was suggested rational traders to follow expected utility theory, as Hens and Rieger (2010) state, it was observed in financial markets that they often neglect this rational decision model. Since this theory has been remained incapable to explain investors' irrational behaviors, therefore, a more connotative method called behavioral decision theory was theorized by researchers. According to one of the pioneer, Richard Thaler, this new approach can be simply described as open-minded finance.

Hubbard and O'Brien (2011) assert that the behavioral economics discipline is a special field that study people's irrational choices. It was also widely accepted in finance literature to understand investors' irrational behaviors such as the unwillingness of traders to realize their capital losses. The premise of this new theory, however, is that financial markets agents behave irrationally due to being motivated by emotions such as fear, greed, etc. when investment decisions are made. On the other side, Schindler (2007) states that some phenomena observed in financial markets can be better explained or understood with the aid of non-perfect rational behavioral models, in which some essential assumptions are relaxed. According to these models, investors are not capable to control their emotions easily and correctly, thus, their decisions are not allowedly consonant with the concept of subjective expected utility. The behavioral finance concept has three important cornerstones. The experimental evidence of conceptual psychology, one of the important cornerstones, has been consulted by practitioners and researchers to understand this irrationality. The results showed that the determination of traders' preferences and decisions have systematically yielded certain biases. In other word, cognitive errors caused by incomplete information, as noted by Jordan and Miller (2009), will cause inefficiency in markets. The other component, sociology, is of great importance – even though is often ignored by researchers– when studying individual investors' interactions on financial markets. The last cornerstone is, however, the traditional finance that compounds behavioral aspects from other two cornerstones. On the other hand, Reilly and Brown (2011) add a different component in behavioral finance model: neurofinance –the anatomy, mechanics, and functioning of the brain.

Hens and Rieger (2010) report that one of the important and particularly interesting model in behavioral finance is Prospect Theory developed by D. Kahneman and A. This theory is developed to describe investment decisions when faced with risky opportunities. According to Prospect Theory, traders/investors are assumed to make loss-averse decisions. In addition, they show risk-averse attitudes if they are told to compare two gains, and present risk-loving attitudes when comparing two losses. Putting differently by Jordan and Miller (2009), this theory is based on a fundamental idea that traders are much more affected by potential losses than prospective gains. In addition, the same situations are responded by traders differently, in which the difference depends on the presentations of gains and losses. For example, if an investor chooses a sure gain over a gamble that may result an increase or decrease in sure wealth, then it is said that this investors is a risk-averse trader. On the other hand, if this investor chooses a sure loss over a gamble that may yield a decrease or increase, then this trader is called as risk-taking investor. Note that, a fully rational trader's focus is the overall wealth, namely, the gains or losses are not taken into account. To a clear understanding, let's look at the graphs depicted in Figure 2-4.

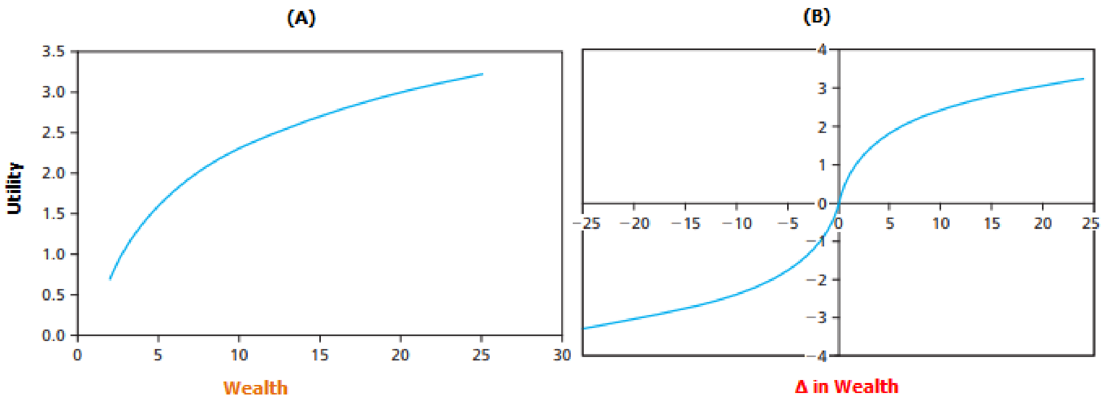


Figure 2-4 Prospect Theory

Source: Bodie et al. (2012).

Figure 2-4 illustrates both utility functions under conventional model and prospect theory (Bodie et al., 2012). In Panel A, conventional description of a risk-averse investor is illustrated. Evidently, there is a positive relationship between wealth in level and satisfaction/utility, namely, as wealth rises, also utility increases, but at a descending rate. For example, the satisfaction obtained from a \$100 gain is less than the utility obtained from a \$100 loss. In other words, risky alternatives will be rejected by traders if they do not offer a higher risk premium. On the other hand, Panel B depicts the utility function under "Prospect Theory". Note that the x-axis shows utility/satisfaction depended on changes in wealth from current levels. To the left of zero point, i.e., 3rd quadrant on this graph, the curve shape is convex. Namely, Elton et al. (2009) state that D. Kahneman and A. Tversky's utility function is convex below this zero point and concave above it. In the 3rd quadrant, traders are assumed to show risk-loving behaviors when faced by a potential loss. Apart from this theory, Bodie et al. (2014) list some investors' irrational, namely, befuddling examples of investor behavior in their book:

a) **Forecasting errors**: According to K&T's experiment results, when people are expected to forecast in financial markets, they are inclined to give too great importance recent experience. In addition, they tend to make too extreme forecasts given the uncertainty that subsistent in available information.

b) **Overconfidence**: In a word, this anomaly means that people tend to overestimate their abilities, but underestimates the faulty measurement of their decisions. In other words, it can be described, according to Elton et al. (2009), as a tendency to overvalue an investor's aptitude to precisely estimate the range of a gamble. Reilly and Brown (2011) state that growth firms are overconfidence in forecast, leading an overestimation by analyst in terms of growth rate. In addition, it also causes an overemphasizing of good news and ignoring the bad news for these companies.

c) **Regret avoidance**: This theory is related to study of investment tendency to refuse to admit a wrong investment decision that leads a poor performance. When an investor takes a wrong decision, with the intention of avoiding a feeling of regret she/he will continue to hold the stocks or portfolio too long in the hopes of recovering the losses (investopedia.com, 2018).

d) **Frame dependence:** Bodie et al. (2014) state that investment decisions seem to be affected by how different –in fact, they are equal– alternatives are presented, namely, they are framed. Jordan and Miller (2009) assert that the presentation of choices leads to wrong/irrational decisions made by traders. For example, a gamble may be rejected by an investor if it is presented with regard to potential losses, however, it may be accepted if it is posed in terms of possible gains.

e) **Mental accounting:** Briefly stated, it is the failure –as emphasized by Elton et al. (2009)– that occurs when investor does not consider that each investment account are a part of main portfolio. Namely, traders separate their funds into different account with different needs and treat them differently, because these accounts are assigned by different functions. It is also called as house-money affect. For example, Bodie et al. (2014) state that investors accept a gamble with the money come from their winner account, i.e., with the money that earned at the casino, leading be risk-taking investor.

Before proceeding further, we should mention the concept of noise trader. According to Megginson (1997), stock markets are populated by two investor types: informed (rational) traders and uninformed (noise or liquidity) traders. It has been aforementioned, rational investors always take into account both the risk and expected return of assets or portfolios, therefore accurately assess them. On the other hand, noise investors' trades are based only on their sentiments or beliefs that financially not meaningful, causing deviations from the fair value of stock prices, especially in the short run. Reilly and Brown (2011) contend that security prices will be higher than the intrinsic value determined by fundamentals in bullish markets. In the case of mispricing securities, rational investors are found to be reluctant, according to Megginson (1997), to utilize this arbitrage opportunity because they are afraid of facing two main risks: the fundamental risk and the future price risk. The first risk is related to a chance that the overall stock prices may increase or decrease in the following period while the second risk is the unpredictability of the resale price of security in the future. Reilly and Brown (2011) assert that during some periods, noise investors' trade cannot dominate stock markets because they are rather muted and inactive. Namely, they are allowed to survive during this period. On the other hand, they dominate financial markets when their sentiments are strong and trades are active.

2.3.1.4 Capital Asset Pricing Model (CAPM)

Focardi and Fabozzi (2004) state that the allocation of funds in investment portfolio has long been debated by researchers and practitioners. According to classical theory, an investor should choose the assets offering the highest expected returns but ignore their covariances. This investment policy depended on the notion that an investor should make an investment decision if they had a competitive advantage regarding information has lasted until the modern finance principles are theorized in the 1950s by Harry Markowitz. In his seminal work on portfolio selection problem, Markowitz suggested diversifying their resources among assets provided that being a risk-averse investor. An investor can manage security risks through diversification approach, therefore, a trade-off between the expected return and portfolio risk could be produced. When looking at the assumptions behind his ideas, it is seen that traders were suggested to order their preferences in terms of the utility function. Besides, it was assumed that portfolio returns were normally distributed because each security return was jointly normal. If the normal distribution assumption holds, then a utility function could be formulized in two moments/parameters: expected return (mean) and variance, therefore, his analysis method has been referred as mean-variance analysis. The process of constructing a portfolio, therefore, will entail maximizing utility of an individual investor in the space of portfolio weights, namely, when she/he is faced with different investment choices.

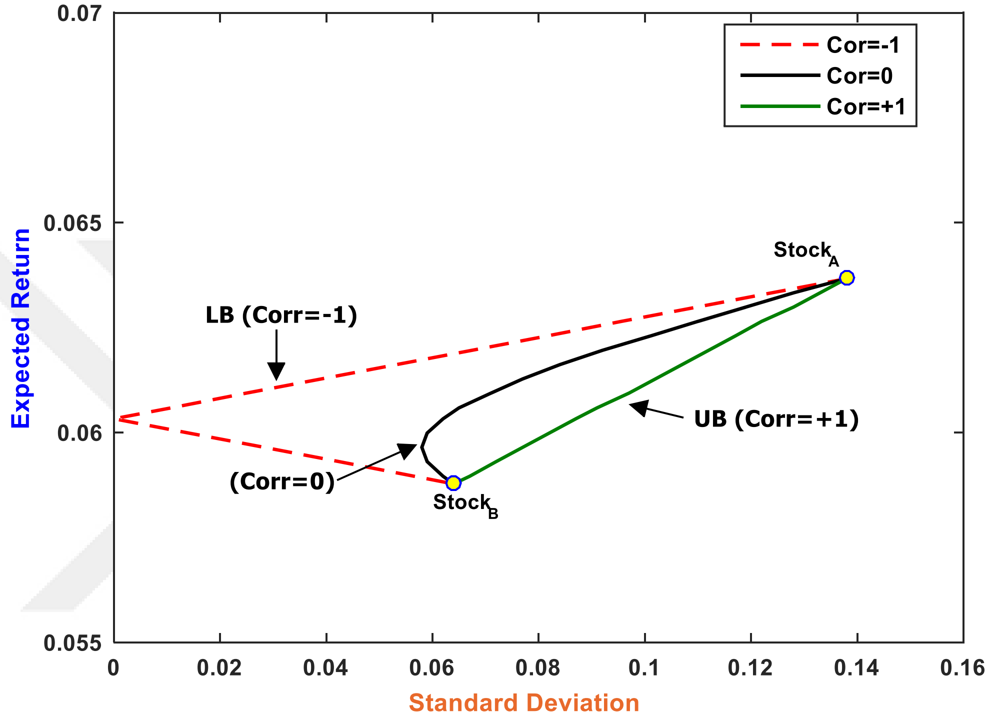


Figure 2-5 Efficient Frontier with Different Correlations

Source: Computed by the author.

As noted by Megginson (1997), the investment decision-making process for a portfolio constructing so as to minimizing portfolio risk for any given expected rate of return or maximizing expected return for any given risk level is called as the minimum variance (mean-variance efficient) portfolio, providing the best attainable combination of return and risk. To construct an optimal portfolio the necessary three inputs are the expected return and variance –measure of uncertainty– of each stock, and the covariances between the expected return of each two stocks in the target portfolio in the simple one-period. With these inputs, the investment opportunity set can be easily constructed, which can be seen in Figure 2-5, in which the x-axis shows the standard deviation of portfolio while y-axis illustrates the expected return of portfolio with the different weights and correlations.

Note that the shape of this opportunity set including two assets with the conditions of $R_A = 0.0637$ & $R_B = 0.0588$ and $\sigma_A = 13.806\%$ & $\sigma_B = 6.394\%$ varies with the degree of the correlation coefficient. In the case of a perfectly positive correlation coefficient, $\rho_{A,B} = +1$, the investment opportunity set is illustrated as a green straight line, which shows the upper bounds to combinations of those two assets. If the proportion of asset A in the portfolio is, $w_A = 0$, zero, then the expected return of the portfolio is $E(R_p) = R_B$ and risk is $\sigma_P = \sigma_B$. On the other hand, the expected return of portfolio and risk will be equal to $E(R_p) = R_A$ and $\sigma_P = \sigma_A$ if the $w_A = 100\%$ and $w_B = 0\%$. Evidently, there is no diversification effect in the case of $\rho_{A,B} = +1$, because the portfolio risk never declines for any combination of these assets. Note that if $\rho_{A,B} = +1$, then the standard deviation of the portfolio is equal to the weighted sum of the individual standard deviations of asset returns, $\sigma_P = w_A\sigma_A + w_B\sigma_B$. This is valid only when all stocks have a perfectly positive correlation between each other.

On the other hand, if the correlation is $\rho_{A,B} = -1$, then the shape of investment opportunity set is become two different red dashed lines (the lower bounds to combinations of those two assets). Evidently, the portfolio variance becomes zero ($\sigma_P = 4.7E\text{-}08$) when the correlation is $\rho_{A,B} = -1$. Besides, the optimal and efficient portfolios are constructed when correlation is $\rho_{A,B} = 0$. Jordan and Miller (2009) assert that as the correlation gets lower, the opportunity set gets more bowed to the left side. The necessary formulation for calculating the weight of asset A for a zero variance portfolio in the case of two-assets is given as follows:

$$w_A = \frac{\sigma_B(\sigma_B - \rho_{A,B}\sigma_A)}{\sigma_A^2 + \sigma_B^2 - 2\rho_{A,B}\sigma_A\sigma_B} \quad (15)$$

Actually, the calculation variance for a portfolio including two and three assets is not hard. However, it becomes very cumbersome when asset number increases. Focardi and Fabozzi (2004) state that the total inputs for a portfolio including 20 assets will be 20 (n) the expected returns, 20 variances, and $190 = (n/2) * (n-1)$ correlations or covariances coefficients. An example of portfolio consisting 20 Blue Chip firms' stock is depicted in Figure 2-6. Note that, efficient portfolio calculation is done with the codes available in MATLAB (2015a). The dataset spans from February 2013 to January 2018, totaling 60 monthly log-differenced of adjusted closing prices observations. The risk-free rate is (annual) 5% and the market data is SP500 return rate for the same data period.

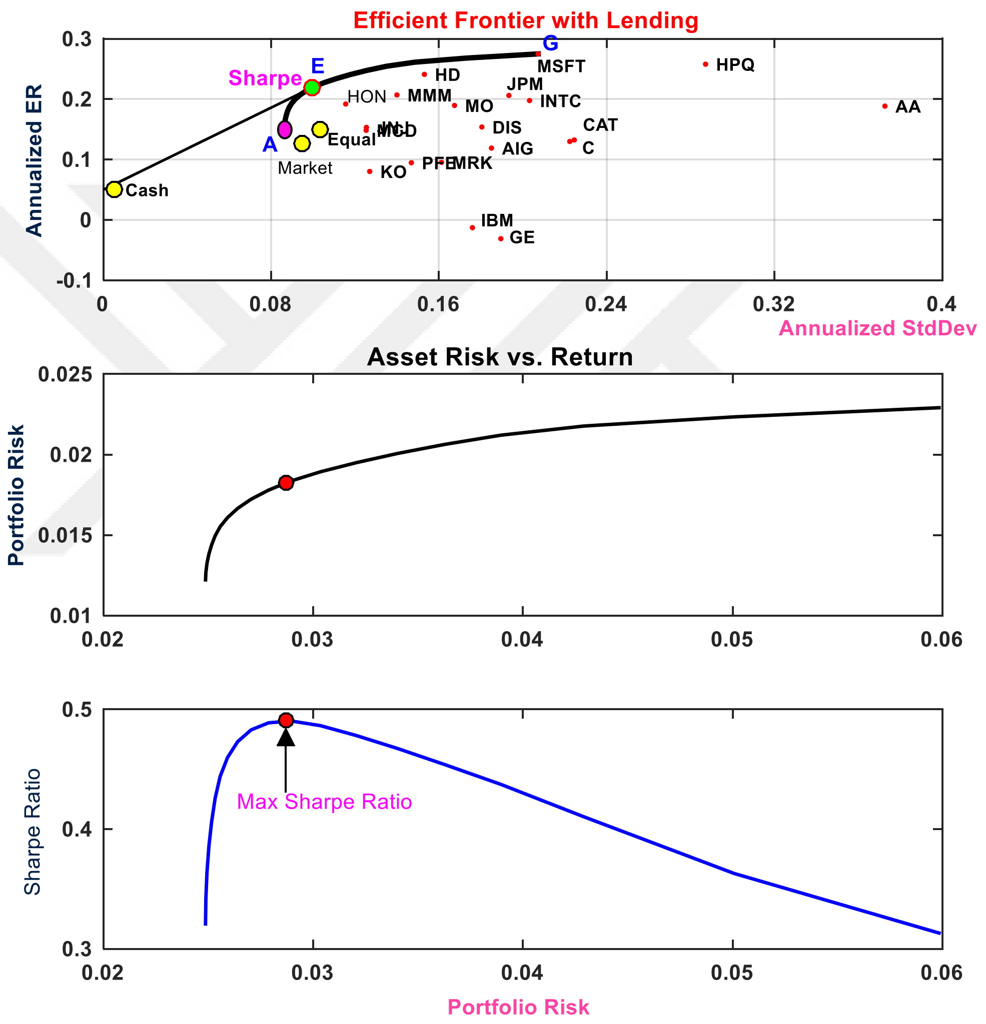


Figure 2-6 Efficient Frontier with Sharpe Ratio

Source: Calculated by the author.

Evidently, annualized standard deviations and expected returns of both of stocks and portfolios are depicted in the top panel of Figure 2-6. In the optimal portfolio, the selected stocks' weights are $w_{AA} = 5.5582\%$, $w_{HD} = 16.607\%$, $w_{HON} = 31.68\%$,

$w_{MMM} = 6.0821\%$, $w_{MO} = 17.564\%$ and $w_{MSFT} = 22.509\%$. Note that short sales are not allowed. Jordan and Miller (2009) state that efficient frontier just shows which portfolios are efficient, however, it does not specify the best one among all portfolios lying on the efficient frontier. Actually, according to mutual fund theory, the best portfolio is the Sharpe-optimal portfolio, which is determined by the Sharpe ratio. According to test results obtained by MATLAB (2015a), the portfolio return is 0.01825 and risk is 0.028725. Given a risk-free rate of (monthly) 0.004167, the optimal Sharpe ratio is 0.49038, which is illustrated in the bottom panel of Figure **2-6**. As noted by Alexander (2008), at this point, it can be said that there is not any other combination that offer the highest expected return for a given risk level, or the minimum risk level for a given expected return. Therefore, this bold and black line of curvature in the top panel of the figure is referred as efficient frontier, located in the left boundary of the opportunity set. The general characteristics of the efficient frontier can be summarized as follow:

- First of all, the shape of efficient frontier depends on the correlations between the rate of returns.
- Any portfolio that lie on the efficient frontier is a linear combination of any other two portfolios, including all the stocks in the opportunity set with different or same weights, either positive or negative (if short sales are allowed).
- In the case of allowance of short sales, there will be no upper limit to the portfolio risk bearing by an investor and the feasible set goes to infinity.

It should be noted that there exist some problems concerning with efficient frontier. According to Alexander (2008), the security returns that used in the portfolio construction are stationary and they do not have long-term memory, in fact, the level price of the stocks have a memory of the past. Obtaining return rates leads to a memory loss, namely, the information about cointegration relationship does not exist any longer. Therefore, it can be said that the optimality of a portfolio is valid only during the short term.

Given a risk-free rate of (annual) 5%, one can straightforwardly construct a set of minimum variance portfolios including both of the risk-free asset (bond) and a risky portfolio. This risky portfolio containing all available assets to invest is referred as the tangent or market portfolio. Alexander (2008) remarks that for a maximum risk reduction in the portfolio, where short sales are not allowed, it is required to highly negative correlations between assets, however, it turns out to required high positive correlations between the long and short positions.

After determining the expected returns and risks concepts, it is time to give a detailed information about how pertinent risks faced by investors are priced. Elton et al. (2009) assert that the equilibrium models generated by researchers permit investors to measure those risks accurately, thus, determine the optimal return rate for bearing them. The simplest and the most affective model used by investors to determine a trade-off between risk and return is the Capital Asset Pricing Model (CAPM). Fabozzi and Drake (2009) state that this first asset pricing model is independently and concurrently developed by four academicians: Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966) following the development of portfolio optimization theory introduced by H. Markowitz. The assumptions underlying this model can be summarized as given by Sharpe et al. (1999):

a) First of all, there are no taxes or transaction costs, namely, they are irrelevant in the capital markets.
b) It is possible to borrow or lend an unlimited amount of money at a risk-free rate.
c) All tradable individual assets are infinitely and perfectly divisible. It is possible, in other words, to buy a fraction of a security.
d) All traders are assumed to be Markowitz-efficient in that investment decisions are solely evaluated in terms of their expected returns and risk to maximize their terminal wealth over a one-period investment horizon.
e) Traders are rational and risk-averse, namely, when faced by two choices that have the same expected return but different standard deviation, they will choice the one with the lower risk, namely, they are mean-variance optimizers.
f) All traders have the same one-period investment horizon, such as one-year or a month.
g) There exists only a risk-free rate that same for all investors.

h) All pertinent information about securities and portfolios can be freely obtained, namely, it is publically and instantly available at no cost.

i) All traders have homogenous expectations regarding the probability of distributions of asset returns. That is to say, they are assumed to have identical expectations about the expected returns, standard deviations and covariances to construct a portfolio.

j) Each individual trader are price-taker, that is, they cannot affect the security prices. Namely, there is no any investor that has sufficient wealth to change a stock price.

Reilly and Brown (2011) state that, some of those assumptions behind the CAPM evidently untenable, namely, they are unrealistic. Relaxing, however, some of those assumptions would have only a negligible impact on the model because relaxing would not alter theory's conclusions. In addition, judging a theory solely on its assumptions is not suggested because the important things is that if a theory explains or helps to understand or predict investor behaviors in capital markets well, then it is said that this theory is useful even if it has unrealistic assumptions. Equivalently speaking, those assumptions give powerful insight, as noted by Bodie et al. (2012), into the nature of equilibrium in capital markets. They (2012) summarize the equilibrium that prevail in CAPM world as

a) By assumptions given above, all traders are willing to hold the same portfolio, called as market portfolio (M), including all tradable risky securities. Each stock included in the market portfolio is held in the proportion that its market value represents of the total market value of all included risky stocks.

b) The market, in fact the optimal risky, portfolio, M, lies on the efficient frontier (the opportunity locus). The straight line derived from the risk-free rate (as denoted cash in the top panel of Figure **2-6**) through the optimal portfolio, M, is called the capital market line (CML), which is the best achievable capital allocation line. Namely, all investors' portfolios will be a combination, differing according to their risk-return preferences, of the risky portfolio and risk-free assets.

c) If an investor includes a risky asset into his/her portfolio instead of investing only on riskless asset, then this investor will require a risk premium, which is determined by a beta coefficient.

Megginson (1997) state that the CAPM model had not been accepted by practitioners at first, but later, it was almost hailed by every practitioners and academicians because it was a simple and powerful tool. For the first time, in other words, a model for pricing assets in terms of risks and expected return was introduced. It is actually widely accepted a notion that an investor should be compensated for holding risky assets in well-functioning capital markets because of accepting a variety of risks, as aforementioned in the previous sections. As remarked by Fabozzi and Drake (2009), an asset pricing model can be expressed in terms of risk factors included in a portfolio.

$$E(R_A) \; = f(F_1, F_2, F_3, F_4, \dots F_N) \tag{16}$$

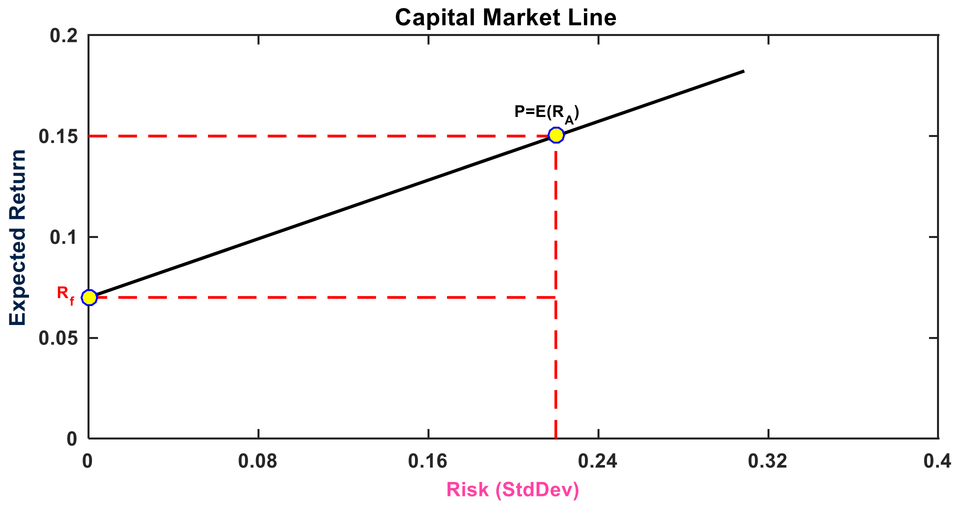
where $E(R_A)$, N and F_k, denote expected return for asset A, number of risk factors and risk factor k. According to Equation (16), the expected return for asset A is a function of N risk factors. The key part of this model is to determine which risk factors will be included and to describe the true relationship between those risk factors and expected return. For simplicity, researchers fine-tune the model by assuming a default-free asset offering the minimum expected return in capital markets. If an investor prefer a risky asset over this default-free asset, then the required (expected) return by investor is equal to $E(R_A) = R_f + RP$ where RP and R_f represent risk premium and riskless asset return rate. In the case of investing on risky asset, Equation (16) will be rewritten as

$$E(R_A) = R_f + f(F_1, F_2, F_3, F_4, \dots F_N) \tag{17}$$

It should be noted that, as Fabozzi and Drake (2009) state, the modern portfolio theory introduced by H. Markowitz is a normative theory that describes the standard behaviors that should be followed by investors when constructing a portfolio to maximize the expected return but minimize risk in terms of standard deviations of asset returns. Asset pricing theory, in contrast to the normative theory, is a positive theory that hypothesizes how traders behave. In addition, it provides a framework to gauge the pertinent risks.

Given all assumptions mentioned above, it is easy to understand, according to Bodie et al. (2012), why all traders invest in the same risky portfolios. If all stock market participants follow minimum variance approach for analyzing the whole security universe with same single-period horizon, then it is natural to have homogenous expectations for expected returns and risk, thus, hold the same combinations of risky portfolio. With arriving at the identical determination of the risky portfolio, the proportion of each asset, X, in this risky portfolio will be equal to its proportion in the optimal portfolio, including all individual portfolios. For constructing the optimal portfolio, each investor will use this market portfolio as a prototype.

In literature, the first step to derive CAPM model is the efficient frontier from Markowitz's portfolio optimization theorem according to Fabozzi and Drake (2009). Note that, the riskless investment is not considered in the portfolio selection theory, therefore, the efficient portfolios lying on efficient frontier can be created solely on both expected return and risk. Namely, in the case of absence of a risk-free rate, the optimal portfolio is the one portfolio that tangent to the trader's indifference curve. Remark that the optimal portfolio is the optimal Sharpe ratio, i.e., "E" (market) portfolio depicted in Figure **2-6**, where only risk-free lending is allowed. With the introduction of the riskless asset choice, however, the efficient frontier will change. Each investor's new portfolio will be a combination of risk-free asset and risky portfolios, according to two mutual fund theorem as noted by Elton et al. (2009), since all traders is satisfied with those two investment choices. A capital allocation line that derived from the risk-free rate, R_f, through the optimal portfolio (P) is called as the CML, as shown in Figure **2-6** and Figure **2-7**. Note that, all investors will hold a portfolio combination that somewhere along this line, however, not all stocks and portfolios except the efficient portfolios would lie on. Another way to phrase this is that non-efficient portfolios and stocks would lie below this line. The derivation of CML can be given as follows



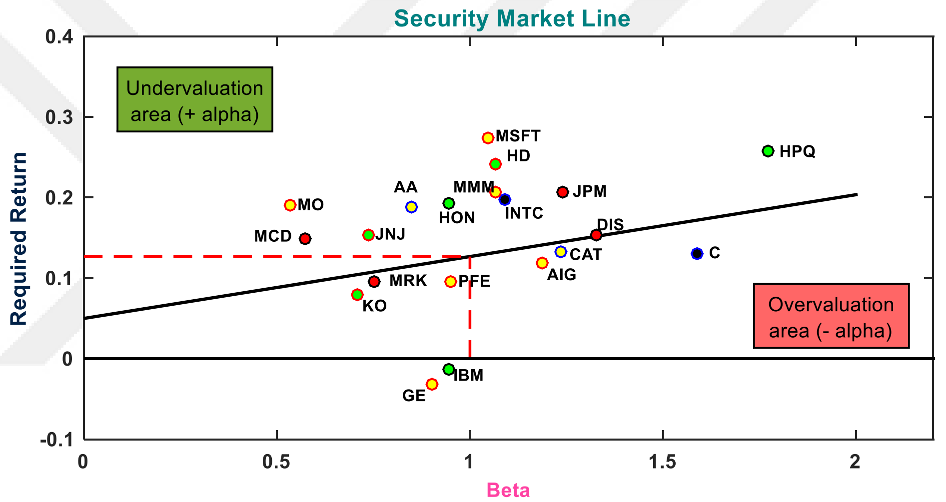


Figure 2-7 Capital Market Line (CML) vs. Security Market Line (SML)

Source: Calculated by the author.

$$\mathrm{E}(R_p) = (1 - w_M)R_f + w_M \mathrm{E}(R_M)$$

$$\mathrm{var}(R_p) = w_{\mathrm{M}}^2 * \mathrm{var}(R_M) \quad \& \quad \sigma(R_p) = w_M \sigma(R_M)$$

$$w_M = \frac{\sigma(R_p)}{\sigma(R_M)}$$

$$E(R_P) = R_f + \sigma(R_p)\left[\frac{\mathrm{E}(R_M) - R_f}{\sigma(R_M)}\right] \qquad (18)$$

It is evident that Equation (18) is the basic result, according Reilly and Brown (2011), of capital market theory. This equation can be interpreted as that if a trader allocate his/her fund between risky portfolio and riskless asset, then the expected return will be equal to sum of the riskless rate of return and compensation for increasing the portfolio risk for constructing of an efficient portfolio by one unit standard deviation. Namely, the first component is the price of time gained by lending at riskless rate while the second component is the market price of risk times amount of risk. Sharpe et al. (1999) that the slope of CML is equal to the market (equity) risk premium divided by market standard deviation. Let's assume that the expected return of market portfolio, $\mathrm{E}(R_M)$, is 0.15, the riskless rate of return, R_f, is 0.07, and standard deviation of market portfolio, $\sigma(R_M)$ is 22%. The equation for the expected return of efficient portfolio resulting from CML is found as $0.07 + 0.36\sigma(R_P)$. The intercept and the slope of the CML is 0.07 and 0.36, therefore, the reward for waiting (risk-free rate, the price of time) is 7% and the reward per unit of risk borne (the price of risk), i.e. the Sharpe ratio for the efficient portfolio is 36%.

According to Sharpe et al. (1999), the CML illustrates only the equilibrium association between the risk in terms of standard deviation and expected return for efficient portfolios. Equivalently speaking, holding a portfolio with only an individual risky stock makes it an inefficient portfolio, therefore, it will always fall below the CML. On the other hand, Reilly and Brown (2011) assert that capital market theory is inadequate model to explain the relationship between risk and return, namely, it is an incomplete explanation when dealing the risk and return association. Note that, an investor must be compensated with a higher return for higher risk that defined in terms of total volatility, namely, regarding standard deviation of investment returns. As aforementioned above, however, an investor is only rewarded for nondiversifiable risk, therefore, there is no compensation for the portion of diversifiable (specific) risk. According to the CML, the fully diversified portfolios are held by investors where the total risk is assumed to be the same of nondiversifiable risks, leading to a serious drawback for the CML. To be precise, CAPM does not say anything about the risk-return trade-off for single stocks because a large amount of specific risk is inherent in the total risk of standard deviations. Thus, this relationship requires a extensive analysis where the effect of unique risk is taken into account.

With extending capital market theory, the CAPM enables the investors, as noted by Reilly and Brown (2011), to assess the risk-return trade-off for both single stocks and diversified (efficient) portfolios. Instead of focusing on the total volatility of individual assets or portfolios, it suggests considering only the systematic risk, namely, the asset's beta. It is the amount of risk that contributed by an individual stock to the market portfolio or, as noted by Megginson (1997), it is the asset's covariance, $\sigma(R_{iM})$, with the overall market portfolio including risky assets. In fact, is the only relevant risk component for traders. In addition, a beta is a measure of the relationship between an individual stock's return and the market return or a measure of how sensitive an individual stock's return is to the aggregate market movements (Bailey, 2005). Thus, we can say that it depends on the correlation and standard deviations of both market return and asset return.

Bodie et al. (2012) state that the contribution of an individual stock to the total volatility of a well-diversified portfolio rest solely on the asset's beta, therefore, it can be said that its risk premium is proportional to its systematic risk factor, i.e. its beta. For example, a stock having a beta of 1.5 must offer 1.5 times a risk compensation for holding it or including it into a well-diversified portfolio. In words, the CAPM model postulates that, according to Cvitanić and Zapatero (2004), the expected return should be, in equilibrium, related to the asset beta. To be purchased by an investor, a stock with a higher beta should provide a higher expected return than a stock with lower beta. The beta coefficient for an individual stock can be calculated as given

$$\beta_A = \left(\frac{\sigma_A}{\sigma_M}\right)\rho_{M,A} \qquad \& \qquad \beta_A = \left(\frac{\rho_{M,A}\sigma_M\sigma_A}{\sigma_M^2}\right) = \frac{\mathrm{Cov}(R_M R_A)}{\mathrm{Var}(R_M)} \tag{19}$$

Evidently, a beta coefficient formulized in Equation (19) is a straight and linear estimate of the degree of co-movement between the market return and asset's return, given the CAPM's assumptions. Brigham and Ehrhardt (2013) state that the stock A having a high total volatility (σ_A) will cause a large amount of risk to an efficient portfolio because it will have a larger beta coefficient. In a similar vein, the stock A having a high correlation coefficient with the overall market movement will also tend to have a large amount of systematic risk, thus reducing the diversification effect on portfolio optimization. The fundamental association between asset return and market return, the CAPM's expected return-beta association, is provided by Sharpe et al. (1999) as given

$$\mathrm{E}(R_A) = R_f + \left[\frac{\mathrm{E}(R_A) - R_f}{\mathrm{var}(R_M)}\right]\mathrm{cov}(R_A, R_M) \tag{20}$$

$$\mathrm{E}(R_A) = R_f + \left[\mathrm{E}(R_A) - R_f\right]\beta_A \tag{21}$$

$$\mathrm{E}(R_A) - R_f = \beta_A\left[\mathrm{E}(R_A) - R_f\right] \tag{22}$$

where R_f denotes the vertical intercept while $\left[\mathrm{E}(R_A) - R_f/\mathrm{var}(R_M)\right]$ represents the slope coefficient in the CAPM. Note that both are positive. In addition, Equation (21) shows that the required return of a security, according to CAPM, is equal to the risk-free rate plus a market risk premium, $\left[\mathrm{E}(R_A) - R_f\right]$, times its beta. Sharpe et al. (1999) remind that stocks with larger covariance values will be priced in order to offer a higher expected return due to the positive slope coefficient. In finance literature, the graphical representation of expected return and beta is known as the security market line (SML), which is depicted in Figure **2-7**.

It is evident that the y-axis represents the expected return whilst the x-axis is represented in terms of the beta instead of standard deviation. Reilly and Brown (2011) assert that likewise in CML, this graph illustrates the risk-return trade-off as a linear line intersecting the y-axis (vertical) at a point that of equal to risk-free rate. However, there are two main differences between those market lines. The first one is mentioned above. The second one is that the SML model can be applied to any portfolio (efficient or inefficient) and an individual stock.

According to Bodie et al. (2012), the SML can be used as a benchmark to evaluate the expected return of investments. The SML gives the required return rate so as to compare it with the expected return to evaluate whether this investment is good or bad. Namely, it provides the fairly return rate of investment, therefore, all individual assets and portfolios would lie on the SML, meaning that the CAPM assumptions hold. Putting differently, Cvitanić and Zapatero (2004) assert that if the CAPM holds, it can be used as a benchmark to evaluate whether stocks are undervalued (underpriced) or overvalued. If a stock is overpriced, then it plots below the SML. Conversely, it would plot above the SML if it is underpriced. Figure **2-7** illustrates this relationship by plotting the required return and expected return vs. systematic risks of Blue Chip stocks. It is evident from Figure **2-7**, the only fairly valued stock is "DIS" because its require return is found to be, using the formula in Equation (21), 0.15195 while its expected (estimated) annualized return is 0.1538, indicating that it would line on the SML. On the other hand, the eight out of twenty stocks ("KO", "MRK", "PFE", "C", "AIG", "CAT", "GE", and "IBM") are overpriced because of low expected return, therefore, they plot below the SML. Note that, all stocks are positively and linearly connected with the overall stock market, "SP500" during the period.

Bodie et al. (2012) remind that the difference between the estimated return (from the historical prices) and the required rate (obtained with the CAPM) is known as alpha, namely, Jensen alpha (index). This parameter is calculated as given

$$\alpha_A = \mathrm{E}(R_A) - \left[R_f + \beta_A\left(\mathrm{E}(R_A) - R_f\right)\right] \tag{23}$$

With using this equation, we can find out whether those stocks are underpriced or overpriced. According to Fabozzi and Drake (2009), if an investor follows an active strategy, then he/she should buy or hold the undervalued stocks if they are included in the current portfolio. Conversely, the overvalued stocks must be sold out or avoided to buy if he/she believes that the CAPM is true, namely, it is a correct asset pricing model. Therefore, if $\alpha_A < 0$, it is said that this stock is overpriced while a positive $\alpha_A > 0$ means that this stock is underpriced, namely, it is performing better than predicted by the asset pricing model. They (2009) state that if the estimated (realized return rate) is higher (lower) than the required rate, then the stock is undervalued (overpriced), therefore, plots above (below) the SML. The Jensen index or ratios for the Blue-Chip stocks are summarized in Table **2-1**.

Table 2-1 Calculation of Alpha for Blue-Chip Stocks

Stocks	Beta	Required Return	Estimated Return	Alpha	Evaluation	It plots … the SML
AA	0.84879	0.115167	0.18842	0.07325	Undervalued	... above
AIG	1.18549	0.141018	0.11888	-0.02214	Overvalued	…below
C	1.58710	0.171852	0.12987	-0.04198	Overvalued	…below
CAT	1.23775	0.145030	0.13232	-0.01271	Overvalued	…below
DIS	1.32789	0.151951	0.15380	0.00185	Fairly valued	... above
GE	0.90279	0.119313	-0.03121	-0.15053	Overvalued	…below
HD	1.06397	0.131688	0.24100	0.10932	Undervalued	…above
HON	0.94336	0.122428	0.19190	0.06947	Undervalued	…above
HPQ	1.77089	0.185963	0.25786	0.07190	Undervalued	…above
IBM	0.94754	0.122749	-0.01294	-0.13569	Overvalued	…below
INTC	1.08806	0.133538	0.19761	0.06407	Undervalued	…above
JNJ	0.73973	0.106794	0.15313	0.04633	Undervalued	…above
JPM	1.24155	0.145322	0.20608	0.06076	Undervalued	…above
KO	0.70665	0.104254	0.08018	-0.02408	Overvalued	…below
MCD	0.57420	0.094085	0.14826	0.05418	Undervalued	…above
MMM	1.06581	0.131830	0.20693	0.07510	Undervalued	…above
MO	0.53265	0.090895	0.18951	0.09862	Undervalued	…above
MRK	0.75166	0.107710	0.09528	-0.01243	Overvalued	…below
MSFT	1.04498	0.130230	0.27489	0.14466	Undervalued	…above
PFE	0.95057	0.122982	0.09462	-0.02836	Overvalued	…below

Source: Calculated by the author.

Alexander (2008) asserts that if the stocks have a nonzero alpha then it is said that the stock market is not in equilibrium. The author (2008) reminds that those abnormal (excess) returns will not exist forever, namely, their prices will be equilibrated due to buying pressure to exploit the abnormal returns. To trade on securities, investors should accurately forecast the alphas with the means of regression models such as the CAPM, in order to decide whether to add a security in the current portfolio.

It is evident from Table 2-1, all beta coefficients for each individual stocks are positive. However, it is theoretically possible for a security to have a negative beta value. Should it be included in a well-diversified portfolio? Bailey (2005) contend that it is suggested to include it in order to control the portfolio risk as a whole because the beta of a portfolio is equal to the weighted average of the each individual asset's beta. Namely,

$$\beta_{p,M} = \sum_{\mathrm{i}=1}^{\mathrm{N}} w_{\mathrm{i}} * \beta_{i,M} \qquad (24)$$

With the aid of this formula, the beta for the Sharpe-optimal portfolio discussed above is as 1.16 and its monthly estimated return is 1.93%, where this portfolio comprises of only six securities with different proportions. Sharpe et al. (1999) remark that due to the simple calculation for portfolio beta, it is accepted that every portfolio will lie on the SML because each stock lie on the SML. Broadly speaking, not only every stock but also every assets' combinations must lies on the SML. Therefore, it should be pointed out that efficient well-diversified portfolios plot on both the SML and the CML, however, inefficient portfolios do not plot on the CML while they plot on the SML.

Although the CAPM theory is reasonably simple to be understood and it has had significant effects on academicians and practitioners, it has been still subject to, according to Merton (1990) and Megginson (1997), both empirical and theoretical criticisms since it was introduced by Sharpe in 1960s. Firstly, the CAPM is a static model, namely, for testing its validity, it is required to use historical (realized) return data even though the model generates the expected return. Here, there needs to make somewhat heroic rational expectations assumption about the investors' unbiased estimation. The second problem is related to the one-period investment horizon assumption. However, it is obliged to use longer periods, 52 weeks or 36-60 months, to test its validity, which its test procedure is very sensitive to the starting and ending points of tested periods. Thirdly, in order to test a one-period model with using longer time period data, some stationary assumptions about the model components must be made. For example, it is required to assume that the risk-free rate, the market risk premium, and stock's beta remains unchanged during the period. Lastly, in order to compute the market return rate, a proxy market index must be chosen because the true market portfolio in a real world is unobservable. In addition, Jordan and Miller (2009) assert that the true market for every risky asset of every type should contain all the securities, real estates, bonds (corporate and government), precious metals (gold, silver, platinum, oil, natural gas, etc), and the everything else in the all capital markets around the world. Because there is not such indices, investors should select a proxy stock price index for market portfolio.

Reilly and Brown (2011) and Brigham and Ehrhardt (2013) state that the validity tests are divided into two main categories: tests of (a) stability of beta coefficients and (b) the relationship between beta and return, namely, the slope of the SML. But before proceeding further, we must introduce the statistical test of the CAPM equilibrium. If some assumptions of the CAPM, the one-period investment horizon assumption and the invariance of beta, are relaxed for the sampled period, then the validity of this cross-sectional model may be tested based on a regression model as given according to Alexander (2008)

$$E(R_{it}) - R_{ft} = \alpha_i + \beta_i[E(R_{Mt}) - R_{ft}] + \varepsilon_{it} \tag{25}$$

$$Y_{it} = \alpha_i + X_{Mt}\beta_i + \varepsilon_{it} \tag{26}$$

where ε_{it} residual term is assumed to be normally distributed and independent, namely assert that $\varepsilon_{it} \sim \mathrm{i.i.d.}(0, \sigma_i^2)$.

The average realized return of stock should be positively and linearly dependent to the stock beta. The parameter of α_1 should be significantly different zero and positive, indeed, it should be equal to the market risk premium, $(R_M - R_f)$. Expressing the necessary regression model for the CAPM, the basic results can be expressed by mathematically and instinctively as given by Megginson (1997) and Fabozzi and Focardi (2004)

- ✰ The intercept parameter of α_i should not be significantly different zero. Namely, the prediction is that, in equilibrium, α_i is equal to zero. In graphical terms, it is equal to testing whether the SML cuts off the vertical y-axis at point risk-free rate, or somewhere below or above this R_f point.
- ✰ All parameters, α_0 and α_1, in equation above should be assumed to be time-invariant, namely, they should be stable over the period so as to produce the same conditional returns each tested period.

- ☆ Apart from dividend yields, beta squared, residual variance, etc., the only factor that systematically related to the asset beta should be the realized returns. Namely, the standard deviation or variance of the returns, or certain characteristics of firm such as the P/E ratios, firm size or MV/BV should not include any significant explanatory power to the CAPM equation.
- ☆ The relationship between the stock's beta and average realized return should be linear and positive. Namely, the parameters of b_0 and b_2 should not significantly different from zero for the estimated regression $R_p - R_f = b_0 + b_1\beta_p + b_1(\beta_p)^2 + \varepsilon_{it}$. In fact, b_1 should be equal to the observed market risk premium $[R_M - R_f]$.

According to Reilly and Brown (2011), when testing the validity of the CAPM, there exist two major questions to be answered. The first question is related to the stability of the beta. To know whether past betas estimated from the historical data can be used to forecasting the future beta is very important matter. The second question is, however, is related to the linearity and direction of the relationship between the asset return and beta. When looking at the papers' results for questioning the stability of the beta it is found out that the asset betas were unstable for single assets, however, it was stable for portfolios' beta. Differently speaking, individual securities' past betas could not be used to estimate for the future variability but portfolio with randomly selected ten or more stocks tended to be a good estimator for future volatility because the residuals for individual stocks were offset by another stock's beta error in a portfolio (Brigham and Ehrhardt, 2013). In addition, that the betas, according to Reilly and Brown (2011), tended to regress toward the average value, namely, the low-beta portfolios tended to rise to unity while the high-beta portfolios tended to decrease to 1.00 over tested period. On the other hand, some researchers found out that using the trading volume-adjusted beta would provide somewhat better estimations. Besides, the low-trading volume beta was biased downward.

On the other hand, according to tests' results for the slope of the SML, there were significant and positive relationships between beta and return, however, the slope was found not to be linear. Namely, as noted by Megginson (1997), stocks with high-beta values had significantly negative intercept parameter whilst it was significantly positive intercepts for the securities with low-beta. That is to say, the slope of the

SML is usually lower than that predicted by the theoretical model, $\beta_i < (R_M - R_f)$, which is depicted in the bottom panel of Figure 2-8.

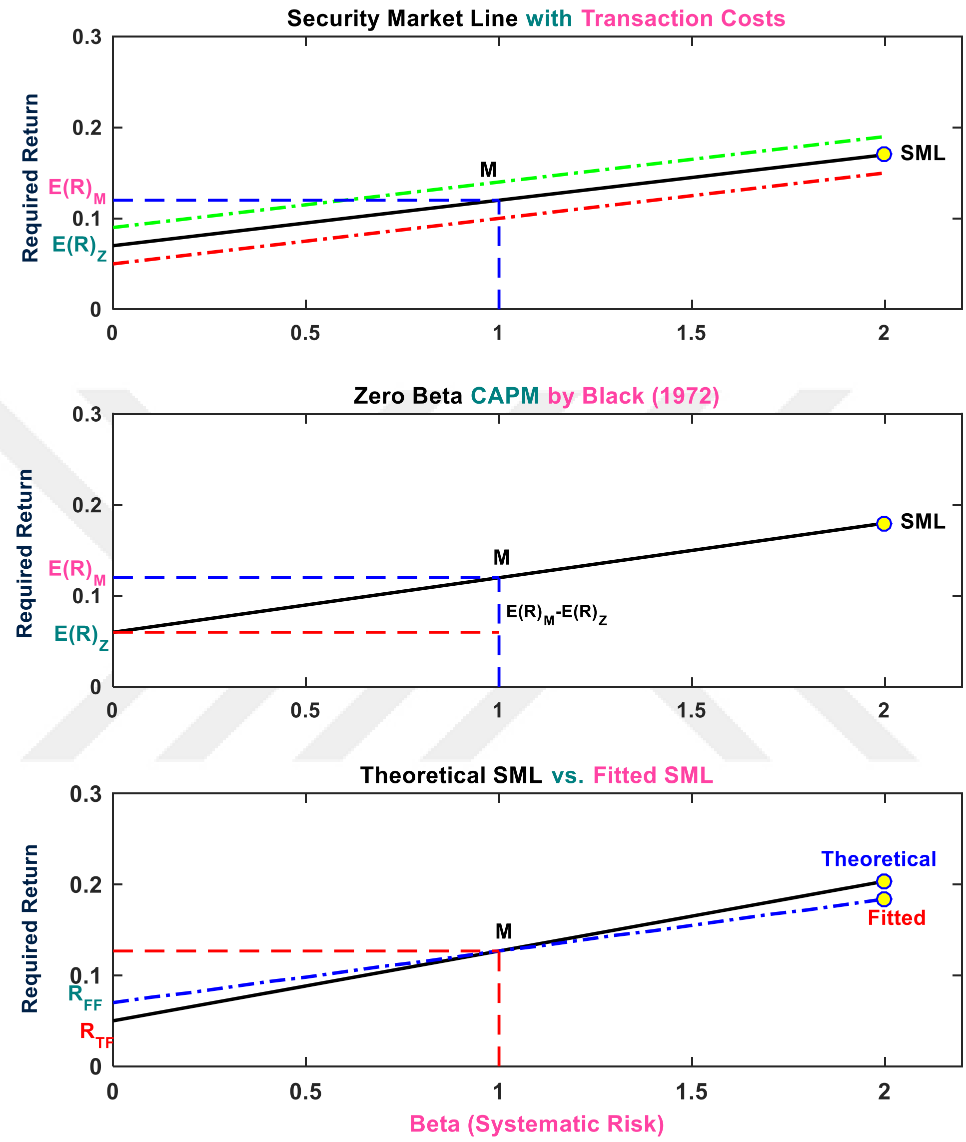


Figure 2-8 Test Results of CAPM Validity

Source: Brigham and Ehrhardt (2013) and Reilly and Brown (2011)

According to Megginson (1997), a possible explanation for the different results for the theoretical SML and observed SML was offered by Black (1972), which is based on an assumption that traders cannot borrow at the riskless-rate while it is valid for the lending. This model does not require a riskless-rate asset for borrowing, therefore, investors select a portfolio that it's return is uncorrelated with the other well-diversified portfolio and it's beta is zero with the market portfolio. Whenever all traders opt a combination of the well-diversified portfolio and the zero-beta portfolio, then an equilibrium in capital markets exists. If the market equilibrium occurs, then the zero-beta asset pricing model (Zero-Beta Model, CAPM) generates a lower β_i than the original parameter in Equation (26). In addition, the availability of a zero-beta portfolio, as noted by Reilly and Brown (2011), will not influence the CML, however, it enables traders to construct a linear SML, as illustrated in the middle panel of Figure **2-8**. In the case of $R_f < E(R_Z)$, the slope of the SML is not as steep as shown in the original SML, namely, the market risk premium is lower. The equation for the so-called zero-beta version of the CAPM is given as

$$E(R_i) = E(R_Z) + \beta_i[E(R_M) - E(R_Z)] \tag{27}$$

Bailey (2005) asserts that the prediction of the this zero-beta CAPM is not easy because the unknown parameter, $E(R_Z)$, must be calculated separately. However, its empirical analysis is possible. According to several papers' results, it was observed that this model would explain observed rate of returns better than did the original CAPM. Megginson (1997) states that a squared beta parameter and dividend yields were not significantly related to observed returns. In addition, Reilly and Brown (2011) contend that the paper of Stambaugh (1982) supported the validity of the zero-black CAPM, while it was rejected by the papers of Gibbons (1982) and Shanken (1985b).

The zero-beta model is a good example for the relaxing the assumption of risk-free asset for both lending and borrowing. In addition, if the assumption of no transaction cost is relaxed, then stocks will plot very close to the SML because the mispricing will not be corrected by investors' trades. Therefore, as noted by Reilly and Brown (2011), there will be a lower and an upper bound for the SML, as depicted in the top panel of Figure **2-8**, where the width of those bounds is described by the transaction costs. The paper of Degennaro and Robotti (2007) document that the slope of the SML declines as transaction costs are taken into account. According to after-tax CAPM introduced by Brennan (1970), as noted by Megginson (1997), stocks with high-yield must provide higher nominal return than those of offering low-yield to atone traders for the higher personal taxes. Namely, traders with a low tax bracket should hold more the stocks that offer high-dividend rates whilst stocks that offer low-dividend rate should be less held (Elton et al., 2009). Merton (1973) introduces the intertemporal CAPM (ICAPM) where the assumption of one-period of the standard CAPM is relaxed to a multi-period framework, namely, it is constructed under the assumption that investor's consumption and portfolio decisions are taken in continuous time. Note that the uncertainty for the future prices of stocks and consumption goods, future labor income, future investment opportunities, etc. affect the expected returns. Merton (1990) asserts that the standard CAPM is a static (single-period) model even though it is usually treated as if it is true in the multi periods, however this is not the case, i.e., the classical CAPM does not hold intertemporally. Lastly, Adler and Dumas (1983) introduces an extension of the CAPM known as international asset pricing model (IAPM). According to this model, some criteria must be met for a market equilibrium and totally integration in capital markets (Megginson, 1997).

Apart from those CAPM extensions, the most influential papers about the validity test for CAPM is presented by Richard Roll in 1977. Briefly stated, according to Megginson (1997) and Bailey (2005), it is not practically, even in theory, possible of testing its validity, because the true market is unobservable. If one uses proxies (stock indexes) instead of true market, it will generate serious implications when evaluating portfolio performance over the investment period. In addition, the one and merely testable hypothesis related to either of the standard CAPM or the Zero-beta (Black's CAPM) would be test whether the ex-ante true market portfolio is mean-variance efficient.

2.3.1.5 Arbitrage Pricing Theory (APT)

One of the most functional and influential model in finance the CAPM is, according to Sharpe et al (1999), an equilibrium model for asset pricing because this model explains why all stocks do not yield the same expected returns. As aforementioned, the required returns depend on solely the systematic risk of the relevant asset, namely, its beta. If stocks or efficient portfolios have different the systematic risks, then, it is natural to observe different required returns. However, as noted by Reilly and Brown (2011), this model had been widely criticized by different researchers due to its assumptions which were seen as untenable. As mentioned above, numerous papers showed that its main parameter, beta, was unstable for individual securities but stable for portfolios and documented different results for the linearity in the relationship between risk and return. Another way to phrase this is that the model was attacked mainly due to, as mentioned by Fabozzi and Drake (2009), its normality assumption for asset returns, homogenous expectation of investors for constructing a portfolio, the identification of the true market portfolio and the return-risk tradeoff that depends on only one risk, beta, factor. To remedy those limitations, researchers have searched for alternative models for pricing financial securities, namely, an alternative model that should be reasonable intuitive, require less assumptions, and permit to use more than one risk factor. This primary theoretical alternative model is the asset pricing model (APT) and introduced by Stephen Ross in the mid-1970s.

Before proceeding further, we must mention the main required assumptions for developing this model, which were given by Megginson (1997) and Reilly and Brown (2011):

a) Capital markets are perfectly frictionless and competitive.

b) The assumption of utilizing an efficient portfolio framework, according to Elton et al. (2009), is replaced by an assumption of the process generating asset yields. Namely, traders have homogenous expectations about the stochastic process generating individual stock returns.

c) More wealth is always preferred by traders to less wealth with certainty.

d) The expected return of each security is affected by F risk factors, namely, the return is linearly related to an unknown number of unknown systematic risk factors or indexes. Therefore, a trader is merely rewarded for bearing those the systematic risk factors.

e) The total return of a security is decomposed into two categories. The unexpected part caused by general economic conditions and the expected component that is linearly related to a set of systematic risk factors is determined by the following model

$$R_i = E(R_{\mathrm{i}}) + \beta_{i,1}\delta_1 + \beta_{i,2}\delta_2 + \cdots + \beta_{i,F}\delta_F + e_{it} \qquad i = 1, \dots, F \tag{28}$$

where, R_i represents the actual or random return and $E(R_{\mathrm{i}})$ is the expected return on asset i, $\beta_{i,F}$ (the factor loading for asset i) denotes the sensitivity of stock i's return on the Fth factor, δ_F stands for the value of the risk factor that common to all stocks under study, and e_{it} represents the random error term having zero mean and variance of σ_{ei}^2 which is assumed to be uncorrelated with each other cross-sectionally and completely diversifiable in large portfolios. The case of being uncorrelated each other is called as being orthogonal, thus, the number of relevant factor is determined by the statistically significant orthogonal factors.

Cvitanić and Zapatero (2004) assert that the starting point of this model is that the security returns deviate from their average value due to unexpected realizations of some systematic risk factors. On the other hand, Fabozzi and Drake (2009) mention that a fundamental principle of finance called the law of one price is the starting point of the APT. according to this law, each similar stock or asset must have the same trading price regardless of how it is packaged or created. Therefore, as noted by Bodie et al. (2012), arbitrage activities are precluded in well-functioning capital markets, because they can be eliminated by a limited number of investors' actions. Namely, as noted by Sharpe et al. (1999), if stocks or a set of stocks with the same factor sensitivities do not generate the same expected returns, then almost arbitrage opportunities materialize, but they will be eliminated by a few awareness investors' actions.

According to Sharpe et al. (1999) and Fabozzi and Drake (2009), with constructing an arbitrage portfolio, a trader can increase his/her current portfolio's expected return without change its risk. This portfolio is so attractive to any investors who does not concerned with the nonfactor risk but increasing its expected return due to this portfolio does not require additional funds and include zero factor exposure, and it offer a positive expected return. Therefore, the market equilibrium occurs if the possibility of constructing a portfolio in order to increase its expected return, on average, should not be existed without bearing more risk and adding extra funds.

It should be noted that if the APT's assumptions are met, then Equation (28) can be converted into a new multiple-factor model (Fabozzi and Drake, 2009) as given

$$E(R_i) = R_f + \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2 + \cdots + \beta_{i,F}\lambda_F \tag{29}$$

where $E(R_i)$ denotes the ex ante, expected return for asset i and λ represents the excess return of, $[E(R_{F2}) - R_F]$, jth common risk factor over the riskless rate of return, namely, it is the risk premium. Actually, Reilly and Brown (2011) assert that this equation shows the fundamental result for the APT. Moreover, it illustrates a relationship that is equivalent of the CAPM's SML, however, it has a security market plane (SMP) with $F + 1$ dimensions. Note that, the additional dimension is for the expected return of asset i.

The theoretical advantageous, as mentioned by Megginson (1997), of the APT can be listed as follows

1) The APTM model includes less restrictive assumptions about risk-return trade-off for investor preferences. Moreover, there is no any assumption related to the normality distribution for the expected returns and
2) Traders possess quadratic utility functions, namely, there is no any requirement to derive the APT in terms of trader's utility function.
3) Assets returns are determined by multiple risk factors
4) It is a multi-period model.
5) There is no special role for the determination of true market or risk-free rate.

On the other hand, the APT model has the following drawbacks as listed by Megginson (1997)

1) Accurately pricing individual assets is not ensured, namely, this model holds solely approximately.
2) The assumption that the risk factor structure generating yields is known with certainty is not true.
3) Lastly, according to the APT, the expected return is linearly related to an unknown number of unknown systematic risk factors that is they are not

described properly, it is not practically possible to operationalize this model for investment or financing decisions. It cannot make, however, a particular economic statement for the required return.

Evidently, the most important drawback is the specifying the pertinent systematic risk factors. Cvitanić and Zapatero (2004) remark that the methods that widely used by researchers to specify the risk factors are the principal component analysis (PCA) and factor analysis methods. Let's briefly examine those relevant papers' results.

Megginson (1997) asserts that the earliest papers had concentrated on empirical testing of the APT theory, namely, in term of econometrically. The first study was conducted by Roll and Ross in 1980, where the authors documented a partly supportive result for the APT predictions with identifying four influential risk factors on expected returns, because their results were inconclusive. Dhrymes, Friend and Gultekin (1984) acknowledged that they did not find any evidence of a significant relationship between risk factors for different sized portfolios. Being one of the first papers that dealt with the anomalies from the point of the APT theory, Chen (1983) asserted that size effect and assets' own residual variance did not have an significant effect on the expected returns, supporting Reinganum (1981a)'s findings. Burmeister and McElroy (1988) revealed a significant January effect on stocks returns. Gultekin and Gultekin (1987) reported a significant result for risk premium in only January. Besides, Chamberlain and Rothschild (1983) stated that the APT model would hold if the asset returns had conformed to an approximate risk factor structure, where the authors also laid the econometric groundwork for the PCA. Lehmann and Modest (1988) contended that the APT could explain the returns of portfolios that formed based on dividend yields but the size effect did not have an explanatory power. In addition, Connor and Korajczyk (1986, 1988) found out that their five-risk factor model would explain the cross-sectional deviation in asset returns better than the Sharpe's CAPM. A similar result was documented by Shukla and Trzcinka (1990), where the five-risk factor would explain nearly 40% of the cross-sectional deviation in stock returns using the PCA and factor analysis methods. On the other hand, Mei (1993b) mentioned that the stock returns was influenced by at least five risk factors and the author stated that he could explain the firm size effect on the security returns. According to Reilly and Brown (2011), one particularly important paper was prepared by Chen, Roll, and Ross (1986), where the authors introduced a model with macroeconomic factors. Broadly speaking, they contended that the stock prices were significantly determined by (a) the growth rate of industrial production, (b) the yield spread between term structure of long term and short-term, (c) the yield spread between low- and high-grade bond credit rates, and (d) changes in expected and unexpected level of inflation.

2.4 Interest Rates

The interest rate, as noted by Kidwell et al. (2016), is the rental price of fund that usually expressed as an percentage of the nominal amount of the fund that borrowed by market participants that need to use it. It is called as the price of fund or the penalty paid from borrower's viewpoint for the use of its purchasing power while it is the reward or compensation (coupon payment, etc.) from lender's viewpoint for deferring him/her current consumption or other needs over a specified period. This rate is determined via the interaction, according to Brigham and Ehrhardt (2013), of the fund providers and the fund users (borrowers) in capital markets. This price is called as the cost of equity for internal funds and it is the interest rate for debt, namely, for external funds.

2.4.1 Interest Rates Fundamentals

Brigham and Ehrhardt (2013) assert that the cost of fund (money) is affected by the four major factors: (a) production opportunities, (b) risk, (c) inflation, and (d) time preference for consumption. For example, lenders will require a higher interest rate if they currently have a strong preference for consumption needs which varies for different cultures, age groups or individuals. Besides, if the investment opportunity is described as risky, then the fund providers will require a higher interest rate. On the other hand, if inflation rate increases, then interest rates also increase due to declined purchasing power, etc.

On the other hand, the const of money is also affected by economic conditions and policies. For example, the central bank monetary policy, the central government's budget deficit or surplus, the level of business activity; the foreign trade balance, and exchange rates may have major effect on the cost of money. On the other hand, Hubbard and O'Brien (2011) contend that the cost of money may be the compensation for (a) inflation, (b) default risk, and (c) the opportunity cost of deferring the consumption needs.

2.4.1.1 Determinants of Asset Demand

Before studying the demand and supply analysis of the bond and money markets and the equilibrium in those market, as noted by Mishkin (2007), we must first comprehend what determine or affect an individual's quantity demanded for an asset, such as bonds, money, house or stocks.

Hubbard and O'Brien (2011) assert that different investors even they have the same income, wealth, and age will not give the same decisions whenever they are asked to evaluate different investment choices. According to the authors (2011) and Mishkin (2007), there exist five key factors of determinants of asset demand or portfolio choices that individuals must take into account:

a) The provider's total wealth, namely, all assets that savers have. If saver's wealth increases, it would be expected that, all else being constant, the quantity demanded for an asset or the size of the portfolio increases. Note that the rate of increase in demand may not be the same as the percentage increase in wealth. Therefore, we can say that there is a positive relationship between the quantity demanded of an asset and wealth.

b) Expected rate of return over the given period from an investment or one asset relative to expected rate of returns on other alternatives. Holding everything else fixed, if the expected return on an asset increases relative to other alternatives, then it would be expected an increase for the quantity demanded of this asset, namely, they move together in the same direction.

c) The degree of uncertainty in the rate of return on an investment or asset, namely, the risk of asset return. If the risk of return rises compared with the alternatives' risk, all else being equal, then its quantity demanded would be expected to decrease for an average investor, namely, for risk averse investors. Therefore, it is said that risk and demand are negatively correlated for an risk-averse individual.

d) Asset's liquidity relative to the alternatives' liquidity. This term is used to describe how easy and fast an asset is readily converted into cash. An asset is called as liquid asset if the market where it is traded has many sellers and buyers for it. Broadly speaking, the greater an asset's liquidity relative to alternatives' liquidity, all else being constant, the more desirable it is to traders, thus, the greater demand for its quantity will be. Note that, as in the case of risk and return relationship, there exists a trade-off between asset liquidity and return.

e) Last factor is the cost of acquiring information about an asset. Broadly speaking, an asset that bear a lower cost of information compared to alternatives, then it is said that this asset is more desirable to investors. Note that, as in the cases of risk-return and liquidity-demand relationships, there exists a trade-off between the cost of obtaining information and asset return.

Given those all determining factors, we can say that the desirable (undesirable) features of an asset lead to an increase (decrease) in the quantity demanded of this asset. Namely, all else being unchanged, the quantity demanded of an asset is positively related to (a) wealth, (b) its expected return and (c) its liquidity relative to alternative assets while the quantity demanded of an asset is negatively related to (d) the risk of its returns and (e) the cost of acquiring information relative to alternative assets.

2.4.1.2 Supply and Demand in the Bond Market

According to Hubbard and O'Brien (2011), we can use determinants of asset demand in markets to illustrate how the interaction of the supply and demand curves for bonds establishes the equilibrium interest rates. With drawing a figure where x-axis shows the quantity of bonds and y-axis (vertical axis) represents the bond price, we can determine the equilibrium price (also the equilibrium interest rate) and the quantity for bonds straightforwardly.

The first step for the analysis of interest-rate determination, as noted by Mishkin (2007), is to draw a bond demand curve in order to explore the relationship between the bond price and its quantity demanded by market participants. With this method to illustrate how interest rates are determined in capital market by market participants is known as, according to Hubbard and O'Brien (2011), the bond market approach, where it is assumed that the bond is being traded as good in those markets. The demand curve is depicted in Figure 2-9.

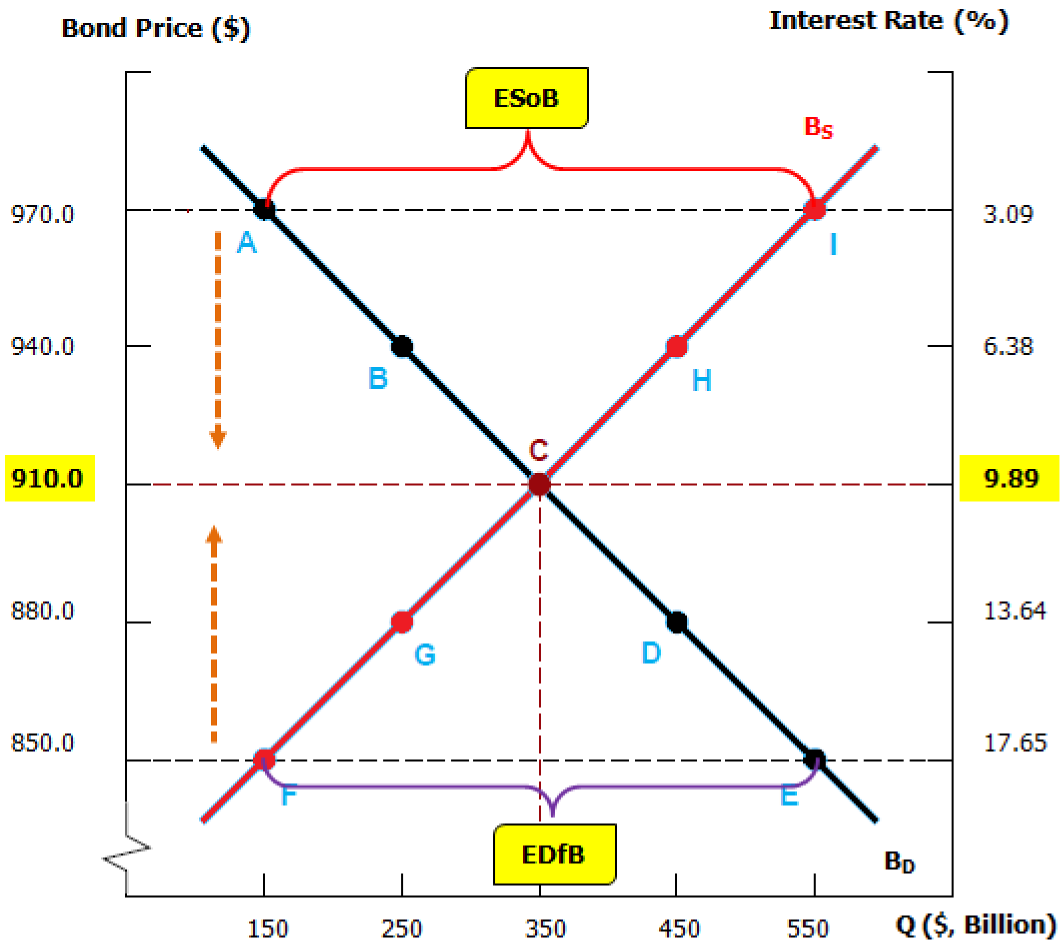


Figure 2-9 Equilibrium in Bond Markets

Source: Calculated by the author.

If the particular bond is selling at a price of $970, then the expected return or its interest rate is 3.09%[$= (1.000 - 970)/970$], assuming that its quantity demanded is $\$150B$. In addition, the expected return is 6.38% if the bond sell for $940. Note that, when bond price falls, the interest rate increases. The quantity demanded for the price $970 is $150 billion, however, it rises to $250 billion if the market interest rate increases to 6.38%. Point E in this figure shows that the market interest has risen to 17.65% where the quantity demanded has increased to $550 billion. Note that there exist a negative relationship between the quantity demanded by investors and bond prices, which is illustrated with the curve B^D provided that all else being equal.

On the other hand, the supply curve is also drawn in Figure **2-9**, with the same assumption. As remarked by Hubbard and O'Brien (2011), this curve shows the relationship between the quantity of bonds supplied by providers and the bond prices. Broadly speaking, as evident from the figure, as market interest rates fall, the price of bonds increases because (a) the holders of existing bonds will try to sell out them and (b) firms will issue new bonds to finance their projects at a lower interest rates, therefore, the quantity supplied will rise, all else being equal. Market equilibrium, however, occurs at the market interest rate of 9.89%, where both curves, B^D and B^S, intersect at the point C, namely, quantity of bond is $350 billion and the bond price is $910. Mishkin (2007) states that the equilibrium bond price is called the market-clearing price and the equilibrium interest rate is called the market-clearing interest rate.

2.4.1.3 Changes in Equilibrium Interest Rates

What will be when there exist a change in the market interest rates? What are the main factors behind those market interest changes? To answer these question, we can use the demand and supply framework for bonds according to Mishkin (2007). Before proceeding further, we must make the distinction between the shifts in a demand or supply curve and movements along a demand or supply curve in order to avoid confusion. To witness a movement along the demand or supply curve, there should be a change in the market interest rate or price of the bond, namely, the quantity demanded or supplied should change. A movement from point A to B or to C or to D or to E in Figure **2-9**, is an example of a movement along a demand curve. Conversely, if a change occurs due to some other factors such as wealth or the expected inflation rate in addition to the price and interest rate, then there exists a shift in demand or supply curve, namely, a new equilibrium occurs for the market interest rate.

According to Hubbard and O'Brien (2011) and Mishkin (2007), there exist some major factors that cause a shift in the demand or supply curve. These factors are grouped into five categories which are the same of the determinants of asset demand framework: wealth, expected return, risk, liquidity and information cost. The effect of changes in those factors is depicted in Figure **2-10**:

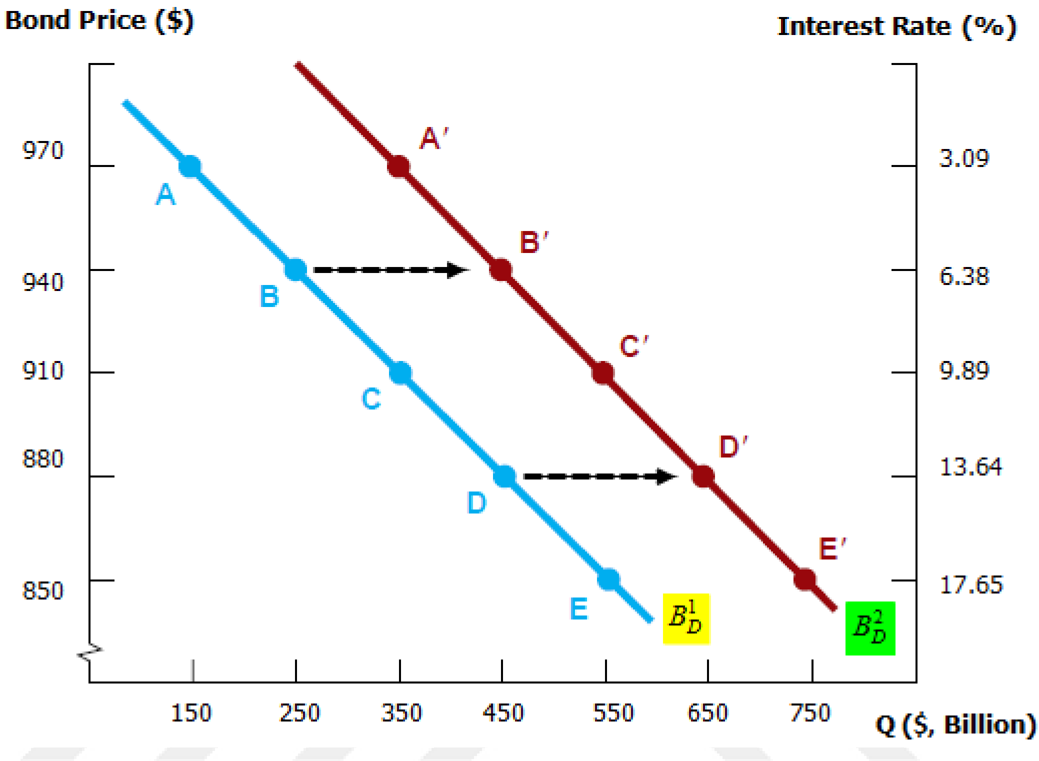


Figure 2-10 Shifts in the Demand Curve in Bond Markets

Source: Calculated by the author.

If an increase in wealth is observed due to an expansion in business cycle or savings, ceteris paribus, then, the demand curve will shift to the right owing to a rise in demand for bonds. The new equilibrium point will differ as a result of a shift in the demand curve to a lower interest rate and a higher bond prices. Similarly, if a recession in economy or a decrease in total saving is observed, then the demand curve shifts to the left due to falling wealth or income levels, causing a decline in both the equilibrium bond price and interest rate.

In a similar vein, as noted by Hubbard and O'Brien (2011), all else being equal, an increase (decrease) in expected returns on bonds causes the demand for bonds to increase (decrease), namely, it shifts to the right (left) because holding bond is relatively more (less) attractive to the investors. Note that, an increase observed in the expected inflation rate causes a decrease in the expected return, thus, leads a fall in demand, in turn, a shift to the left in the demand curve according to Mishkin (2007). Similarly, an increase (decrease) in liquidity of bonds relative to other financial assets leads the demand for bonds to increase (decrease). Because holding bond is relatively more (less) attractive to investors, a shift occurs to the right (left). On the other hand, an increase (decrease) in expected return on other assets or riskiness of bonds compared to other assets causes a decrease (increase), thus, a shift to the left (right) due to falling (rising) of attractiveness of holding bonds.

Hubbard and O'Brien (2011) contend that the supply curve for bonds shift to the right or to the left due to changes in some factors: business taxes, government borrowing (activities), expected inflation, and expected pretax profitability of investment opportunities.

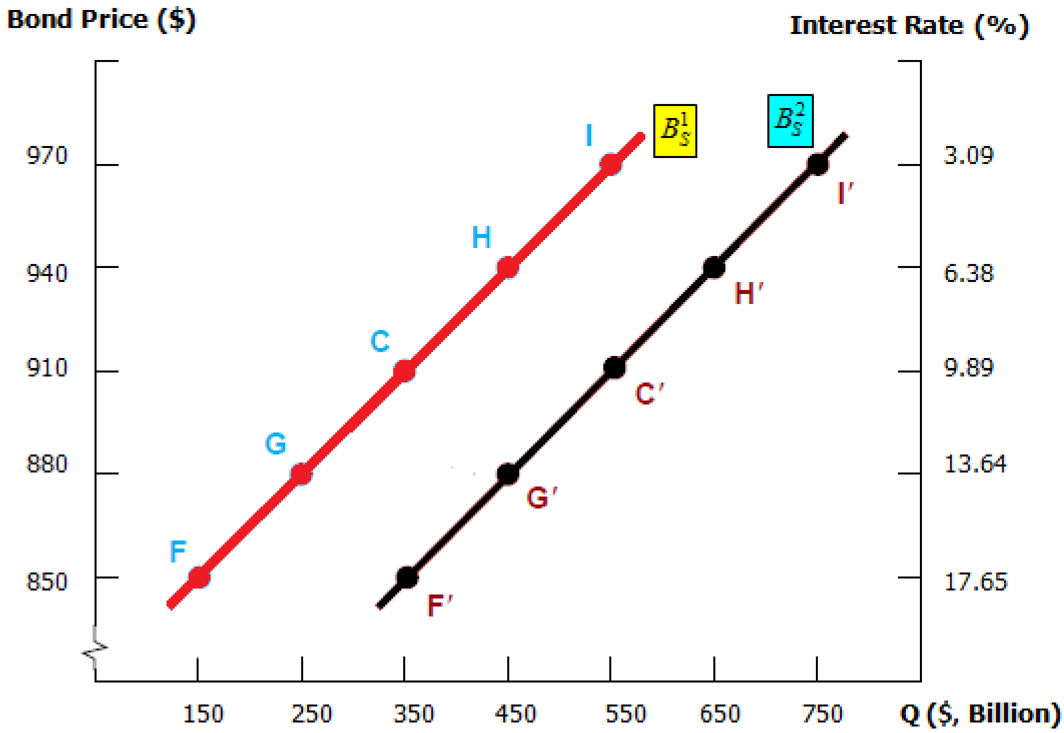


Figure 2-11 Shifts in the Supply Curve in Bond Markets

Source: Calculated by the author.

As noted by Mishkin (2007), all else being unchanged, an expansion (recession) in business cycle, the supply bond increases (decreases), in turn, the curve shifts to the right (left). Similarly, an increase (decrease) in expected profitability, investment tax credits, expected inflation and government borrowing leads the supply curves to the right (left). A shift to right occurs due to (a) business borrow to finance their profitable investment projects, (b) the cost of investment falls due to government tax credits, in turn, the profitability of investment increases, (c) the real cost of borrowing money declines at any given bond price, and (d) more bonds are supplied in the capital markets at any given rate of interests. Conversely, all else being constant, an increase (decrease) in business taxes shifts the supply curve to the right (left) because the profitability of investment decreases (increases).

2.4.2 Interest Rate Theories

In the previous section, we have looked at the general equilibrium in capital markets in terms of bond price and interest rate. Because investors make investing and/or financing decisions, as noted by Fabozzi and Drake (2009), in a dynamic financial atmosphere, it is required to understand the economy, financial markets and market participants that operate in or control the financial system. In this section, we will concentrate on the theories that explain the interest rate behaviors in the financial markets: the Fisher hypothesis, the loanable funds theory and the liquidity preference theory.

2.4.2.1 Fisher's Classical Approach

As mentioned by Hubbard and O'Brien (2011), the equilibrium observed in bond markets establishes the price and the cost of money, namely, interest rate but in nominal term. Here, an important detail is neglected: the purchasing power of the interest rate, namely, a protection against price-level changes during a period. Putting differently, to account the effect of price-level changes on the level of interest rates, as noted by Kidwell et al. (2016), there exist two key associations that should be drawn on: (a) the purchasing power of money and (b) price-level changes and the purchasing power. The value of money, i.e. purchasing power, decreases (increases) as price-level increases (decreases). Therefore, for a protection against the price-level changes, lenders should be compensated with an additional premium.

It is not surprising that, when looking at the investors' behaviors in real-world, it is noticed that both lenders and providers are solely concerned with the real interest rate of return, where the effect of inflation on purchasing power is taken into account. In fact, when deciding an investment decision, the most important thing concerned by investors and firms is the inflation rate of future, not the realized inflation rate. Therefore, the equilibrium interest rates should reflect the expectation of lenders and borrowers about the future real interest rate, which is simply equal to difference between the nominal interest rate and the expected inflation rate.

In finance and economics literature, the relationship among, as asserted by Jordan and Miller (2009), the nominal interest rate, the inflation rate and the real interest rate is commonly couched in terms of the Fisher effect, named by an economist Irvin Fisher in 1930s. Broadly stating, Fisher states that the nominal interest rates comes after the inflation rates. Equivalently saying, the nominal interest rate increases (decreases), as noted by Hubbard and O'Brien (2011), point-for-point with an increase (decrease) in the expected price-level changes. The Fisher hypothesis can be formulized as follows

$$i = r + \pi^e \tag{30}$$

where i, r, and π^e represent the nominal interest rate, the real rate of interest, and the expected inflation rate, respectively. Namely, Equation (30) shows that the real interest rate is equal to the nominal rate minus the inflation rate (expected). Kidwell et al. (2016) contend that there exist a couple of essential points about this equation. Notice that, Fisher suggests using the expected inflation rate instead of current (realized) rate to compensate the fund providers for the potential increase in expected inflation over the specified period. For a properly compensation, it is required to forecast changes in inflation rate in the future. Jordan and Miller (2009) remind that the short-term interest rates has already reflected the current price-level changes, therefore, the anticipated inflation rates are reflected in the long-term interest rates. Next, Kidwell et al. (2016) assert that the price-level changes is used in Equation (30) whether is positive or negative, namely, it could be inflationary or deflationary. Third, the actual interest, namely, the market interest rate that observed in capital markets is selected. In the case of zero inflation rate, nominal and real interest rate are equal. Fourth, it should not be surprising that the actual and expected inflation could be different because the latter is an expectation of market participants while the former is a realization at the beginning of the period.

According to Ball (2009), the expected inflation is determined with the adaptive expectations which was discussed in the previous sections. The author (2009) contends that the expected inflation is shaped with the recent realized inflation rate. For example, if the monthly inflation is 8%, then nearly the same inflation rate is expected in the future by markets. With the adaptive or backward-looking expectations, nominal interest rate is influenced by observed (realized) inflation rates. If an increase is observed in inflation rate, then expected inflation rate increases, in turn, the nominal interest rate increases.

2.4.2.2 The Loanable Funds Theory (LFT)

Fabozzi and Drake (2009) contend that there are two economic theories in order to explain interest rate and movements in interest rates: (a) the loanable funds theory and (b) the liquidity preference theory. According to the first theory introduced by Knut Wicksell in the 1900s, the level of interest rates (ex ante real interest rates) is determined by the demand and supply for loanable funds obtainable in the capital markets. More specifically, the level of long-term interest (observed in the credit markets) rates is determined by investment and savings plus net capital inflows in an economy. Conversely, short-term interest rates are determined by financial and monetary policies. In addition, as noted by Ball (2009), this theory assumes that (a) there is only type of loan, (b) one interest rates, and (c) funds are directly provided to investors by savers. Two important realities are neglected by the loanable funds theory, namely, it neglects the diversity of interest rates and the role of financial intermediaries in financial markets.

Figure **2-12** illustrates the demand for and supply of loanable funds in terms of the loanable funds theory. Notice that this figure resembles Figure **2-9**, where the equilibrium bond price and interest rates were determined by demand and supply curves. Those two figure differs in that the first axis is replaced by the third axis showing interest rate changes at upward slope in Figure 2-12. Furthermore, the supply curve for bonds is renamed as the demand for loanable funds while the demand curve for bonds is reinterpreted as the supply of loanable funds. Note that the horizontal axis is reidentified as loanable funds.

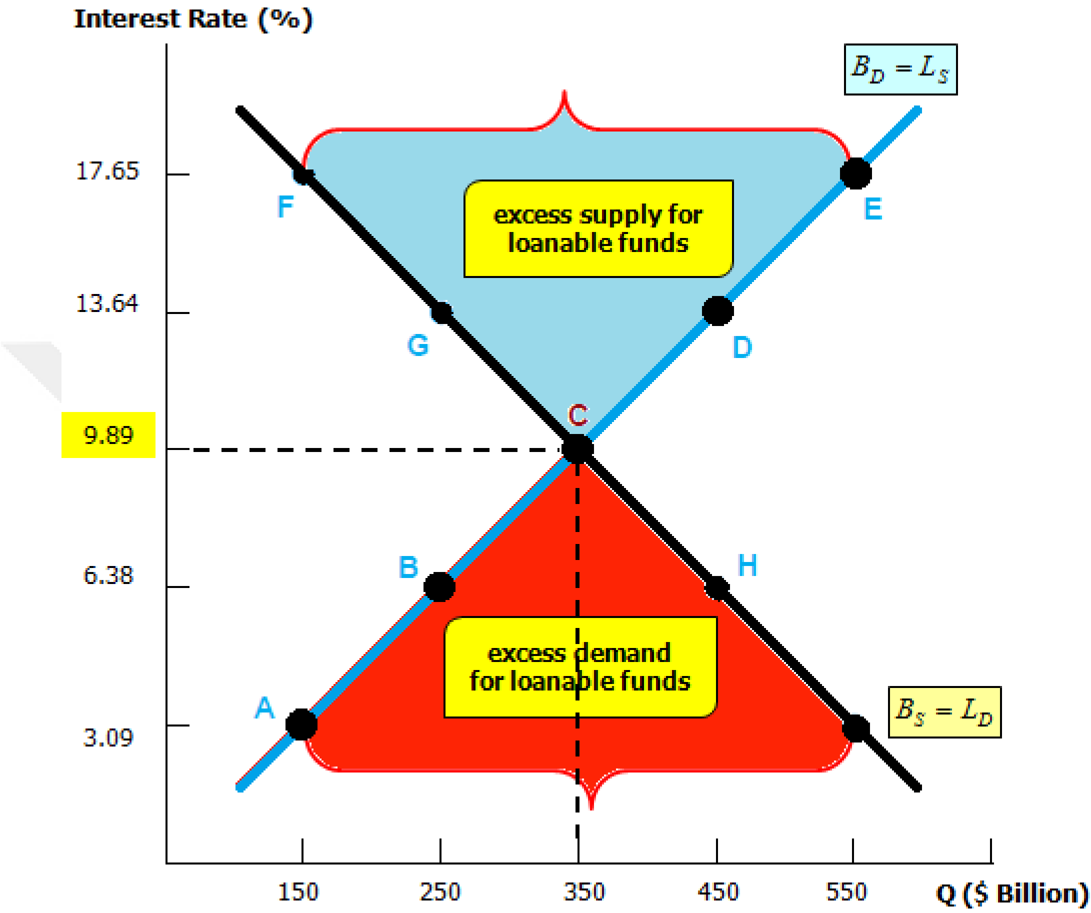


Figure 2-12 Interest Equilibrium in Bond Markets under the LFT

Source: Calculated by the author.

It is evident from Figure 2-12 that as interest rates, namely, ex ante real interest, increases the supply of loanable funds (the quantity of demanded for bonds) by financial market participants (consumers, governments, firms, and foreign investors). Equivalently speaking, as real interest rate rises, the sum of savings and net capital inflows and, in turn, quantity of loanable funds supplied (demand for bonds) rises. Conversely, in general as real interest rate increases, the level of investment and, in turn, quantity of loanable funds demanded (supply of bonds) decreases.

The equilibrium in interest rates occurs when the supply of loanable funds, as noted by Saunders and Cornett (2014), provided by the supplier of loanable funds (SSU) and the demand for loanable funds are equalized. Namely, the equilibrium exists when the demand curve, L_D, for loanable funds intersects with the supply curve, L_S, of loanable funds. As long as competitive forces are permitted operate freely in the financial systems, the forces of demand and supply bring the interest rates to the equilibrium point C as shown in Figure **2-12**. Whenever, there exists a higher interest rate in financial markets than this equilibrium rate, then, there will be a surplus (excess) of loanable funds. To absorb this excess funds, borrowers must be induced with a lower rate to purchase funds until the curves of L_S and L_D intersect at interest rate point 9.89%t. Conversely, if the interest rates is lower than 9.89%, there exists a shortage of loanable funds, namely, excess demand for loanable funds in the financial markets. To induce the borrowers to supply more funds, however, they should be offered by higher interest rates. With the aid of the market competitive forces, the quantity of funds supplied increases while the quantity of funds demanded decreases, therefore, this shortage of funds will disappears.

It should be noted that, according to Hubbard and O'Brien (2011), any of the factors that listed above will give the same effect on the curves. Broadly speaking, the supply curve shifts down and to the right in the case of (a) an increase in the total wealth of financial market participants, (b) monetary expansion to allow the economy expand, and (c) an improve in the underlying economic condition (for example a positive growth rate in GDP). Namely, interest rates fall since the supply of loanable funds increase. In addition, a decrease in risk of financial security and near-term consumption needs causes a shift down and to the right. Overall, there exists a positive relationship between interest rates and risk of financial security and near-term consumption while interest rates and wealth, monetary expansion and economic conditions have a negative relationship.

On the other hand, Saunders and Cornett (2014) contend that the demand for loanable funds is positively affected by the utility (satisfaction) derived from assets purchased with borrowed loans and an expansion in economic conditions while the restrictiveness of nonprice conditions on borrowing (such as fees, collateral, etc.) have a negative effect on demand for funds.

2.4.2.3 The Liquidity Preference Theory (LPT)

The loanable funds theory, according to Fabozzi and Drake (2009), was widely accepted by researchers and academicians until an alternative approach was introduced by Keynes in 1936. With introducing a new theory, Keynes stated that market interest rates (nominal) was determined based on the predilections of households regarding to hold money balances instead of investing or spending. Remark that the quantity of money held by households relies on their income level and, therefore, the quantity of money demanded will be directly related to total income in an economy.

This theory was known, according to Ball (2009), as the liquidity preference theory because the most liquid asset is money. The first difference between those two theories is that in the liquidity preference theory the equilibrium rate is the nominal interest rate. Note that the equilibrium rate is the real interest rate in the loanable funds theory and it is firstly required to find out the nominal interest rates to determine the real interest rates because this theory uses the Fisher equation (30).

As noted by Mishkin (2007), the key assumption in the liquidity preference theory is that there are only two asset categories for storing wealth: bonds and money. Namely, all wealth is shared between those assets. People's total wealth in the economy, therefore, will be equal to the sum of the total quantity of money and bonds. Equivalently saying, the quantity of money supplied, M_S, and the quantity of bonds supplied, B_S, is equivalent to total wealth in the economy. This relationship can be formulized as

$$B_S + M_S = B_D + M_D \tag{31}$$

$$B_S - B_D = M_D - M_S \tag{32}$$

where M_D and B_D represent the quantity of money and bonds demanded. Therefore, Equilibrium (31) states that the sum of the quantity of money and bonds demanded is equal to the sum of the quantity of money and bonds supplied. If we reorganize this equation to Equilibrium (32), an equilibrium in the money market ($M_D = M_S$) leads to another equilibrium in the bond market ($B_D = B_S$). In this sense, we can say that the liquidity preference theory is equal to the loanable funds theory even though they may differ in practice.

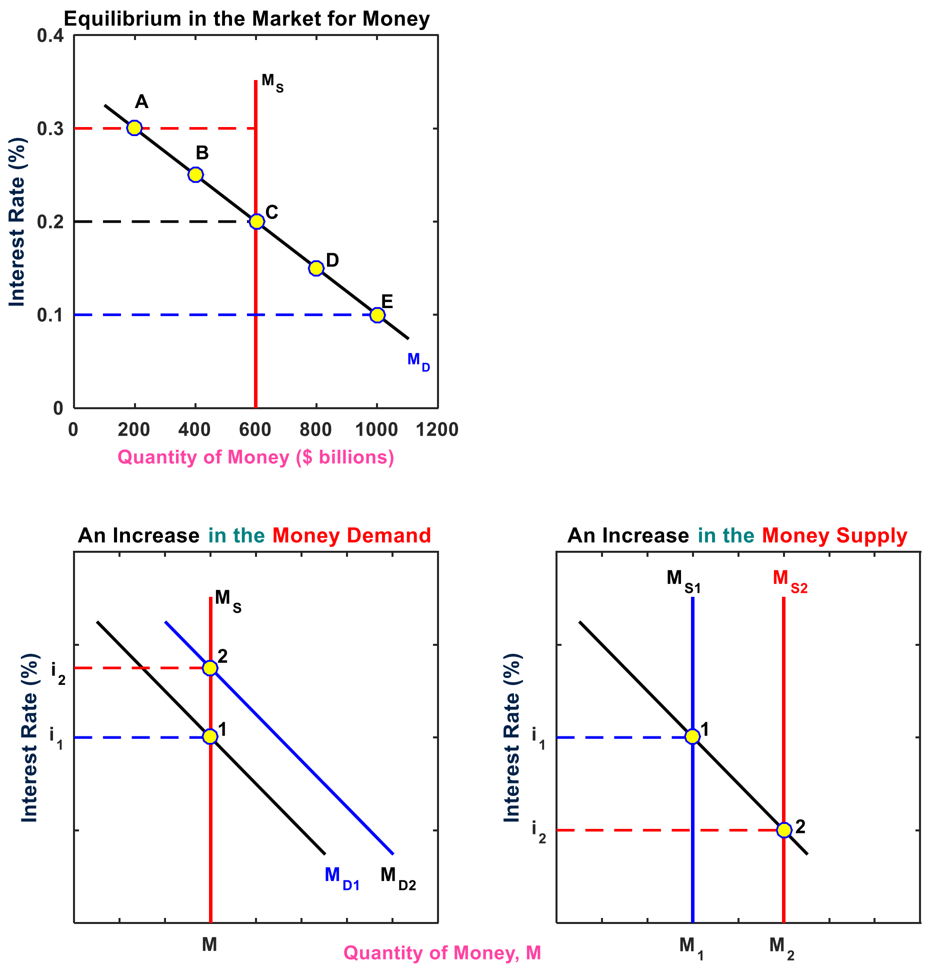


Figure 2-13 Interest Equilibrium under Liquidity Preference Theory

Source: Ball (2009).

Ball (2009) contends that there exist two main difference between money and demand. For example, (a) money is used as the medium of exchange while bonds do not have a similar feature. In addition, (b) money does not offer an interest yield because of its definition, however, bonds provide a yield that equals to interest rate. In the case of an increase in interest rates, all else being constant, the expected return on money (currency plus checking accounts) decreases compared to bonds' the expected returns, therefore, the quantity of money demanded will fall.

The equilibrium relationship between money demand and supply and expected return (interest rates) is depicted in upper panel of Figure **2-13**. Evidently, the demand for money is \$200 billion at the interest rate of 30%, where all other factors, such as the income and price level, are assumed to be unchanged. If the interest rates fall to 10%, the quantity of money demanded increases to \$1000 billion, namely, since the opportunity cost for holding liquidity decreases, the money demand rises. Broadly speaking, the quantity of money demanded and the interest rates are negatively related, therefore, the demand curve, M_D, is an downward-sloping curve. It should be noted that the quantity of money supplied is controlled by an central bank with the policy tools in the liquidity preference theory. In this economy, the quantity of supplied is fixed at \$600 billion. The market equilibrium, therefore, occurs at the intersection of the demand and supply curves at point C, where the equilibrium interest rate is 20% and quantity is \$600 billion, i.e., $M_S = M_D$.

As asserted by Mishkin (2007), a shift in the demand curve is caused by price and income level. Briefly stating, in the case of an increase in the level of income and the price level, the quantity of money demand increases, therefore, the demand curve shifts to the right. For a clear understanding, look at the bottom left panel of Figure **2-13**. When income level (price level) increases during an expansion of business cycle, the curve for quantity of money demanded shifts to the rightward, namely, from the demand curve M_{D1} to M_{D2}. The equilibrium interest rate, therefore, rises from point 1, i_1, to point 2, i_2. All else being constant, the new equilibrium occurs at the intersection of the demand, M_{D2}, and supply curves, M_S, at point 2, where the equilibrium quantity of money supplied is constant. On the other hand, a rise in the monetary base because of monetary policies causes a shift to the right, reducing the equilibrium (nominal) interest rate from i_1 to i_2. As is illustrated in the bottom right panel of Figure **2-13**, with an increase in the quantity of money supplied, the supply curve moves to the right, M_{S1} to M_{S2}, in turn, moving the equilibrium point 1 to 2.

Before concluding this section, as noted by Ball (2009), we should compare the loanable funds theory and the liquidity preference theory. According to the loanable funds theory, with the intersection of demand and supply curve for loans, the equilibrium for real interest occurs in the credit markets. Conversely, if the demand and supply curves for money intersects at a specific point, then the equilibrium for nominal interest occurs in the economy. In addition, the former theory is suitable for explaining the long run, 5 or 10 years, behavior of average real interest rate, however, the latter theory is most suitable for explaining for the short-term behavior of nominal interest rate. In conclusion, both theories assume that there exists only one interest rate.

2.4.3 The Risk Structure of Interest Rates

In the previous section, we describe the relationship between interest rate and bond price with an assumption that there exists only one type of interest and bonds available to market participants in order to simplify the topic. However, there are enormous number of bonds with different interest rates. With the following sections, we will try to explain why bonds have different interest rates and which factor(s) causes interest rates on bonds to vary according to maturity. In this section, we will focus on the risk structure of interest rates, namely, we will give the answer to the question of why might bonds with the same term to maturity, as noted by Hubbard and O'Brien (2011), have different yield to maturities or interest rates. The relationship among the interest rates on bonds –which their price affected by different characteristics such as default risk, liquidity and cost of information, and income tax rules– with the same maturity is called as the risk structure of interest rate by economists.

The difference between the interest rates, as noted by Fabozzi and Drake (2009), of a non-Treasury security and a Treasury security is called as spread, which is referred in terms of basis points. The reason behind the existence of spread is the additional risk(s) which a trader is exposed when she/he prefers to invest on the non-Treasury security. The spread or the risk premium of a non-Treasury security is influenced by its (a) credit (default) risk regarding to the market perception, (b) special provisions or covenants (taxability, callability and convertibility), and (d) liquidity.

2.4.3.1 Default (Credit) Risk

Default or credit risk is the most influential factors among risk components that affect securities' prices. It is derived from a risk that the bond issuer, as contended by Saunders and Cornett (2014), may not be able to make bond's promised coupon or principal payments when it is due. Logically, if the probability of a default risk rises, then, a higher interest rates should be offered to induce the investors to buy the security. In fact, not all securities encompass default risk, for instance, the U.S. Treasury bonds are regarded as non-default securities by investors, because they are issued by the full faith and credit of the U.S. government. When it is due, its coupon and principal payments are expected to be paid by the U.S. government thanks to the U.S. government's ability to print currency and its taxation powers. Having non-default risk makes the U.S. Treasury bonds a benchmark to gauge default risk of non-Treasury bonds.

The default risk premium is an additional premium that investors require to buy a non-Treasury bond which has the same maturity with a Treasury bond. The higher the default risk, as noted by Hubbard and O'Brien (2011), the higher a default risk premium will be required by investors. Namely, the cost of obtaining information about the issuer's creditworthiness or bond's rating determines the quantity of risk premium. A bond rating is a single statistic that assigned by the three major credit rating agencies (Moody's Investors Service, S&P's Corporation and Fitch Ratings) to provide investors useful information about the issuer's creditworthiness. In practice, there are two main rating categories even though they are denoted by different symbols: investment grade and non-investment grade. The first category is decomposed into four subcategories: highest credit quality (AAA), very high-grade quality (AA or Aa), upper premium or high credit quality (A), and lower-medium grade (BBB). Similarly, the second category is broken mainly into four subcategories: speculative (BB), highly speculative (B), substantial default risk (CCC), and default (D). Evidently, a high bond rating implies that there exist a low probability of default risk. Note that, as of March 29, 2018, Turkey's long-term bond ratings are assigned as non-investment (speculative) grade by Moody's as Ba2 (outlook is stable), S&P as BB (outlook is negative), and Fitch as BB+ (outlook is stable).

Mishkin (2007) reminds that if the possibility of a default for a country or firm operated in this country increases, the default risk on its bond will increase, therefore, its expected return will decrease but at different proportions. Particularly, during recessions in the economy, the default risk of firms naturally will increase, causing a flight-to-quality effect. According to Hubbard and O'Brien (2011), the existence of a flight-to-quality phenomenon implies that investors decrease (increase) their demand for bonds with higher (lower) risk. The price of corporate (Treasury) bonds will decrease (increase), therefore, the yield to maturity will rise (fall), causing a large default risk premium. In conclusion, a bond having default risk will always offer a positive risk premium compared to Treasury bonds, consequently, risk premium will rise if its default risk increases.

2.4.3.2 Liquidity and Information Costs

Mishkin (2007) contends that a highly liquid asset is one asset that can be cheaply and quickly sold at a predictable price, therefore, can be converted into cash whenever a need occurs. All else being constant, an increase in liquidity increases its attractiveness, therefore, investors would be willing to obtain it. The asset's interest rate, as remarked by Saunders and Cornett (2014), is a sign of it relative liquidity. If its liquidity is high, investors will require a lower interest rates compared to others with lower liquidity. An illiquid asset should offer a higher interest rates to compensate investors for its risk. Namely, investors demand an additional (liquidity) risk premium for its illiquidity, hence, they can be compensated for the bond's lack of liquidity and the potential lower price in the case of selling it before its maturity. In a similar vein, the another important risk is related to the costs of acquiring information about bond's characteristics (Hubbard and O'Brien, 2011).

In conclusion, we can say that the higher liquidity (costs of acquiring information) leads to a lower (higher) interest rates. Hubbard and O'Brien (2011) assert that if a bond's liquidity increases or its costs of acquiring information decreases, then, the demand curve will shift to the right due to falling interest rate. In a similar vein, a decrease in bond's liquidity or increase its costs of acquiring information causes a shift to the left, namely, the price declines and the interest rate rises.

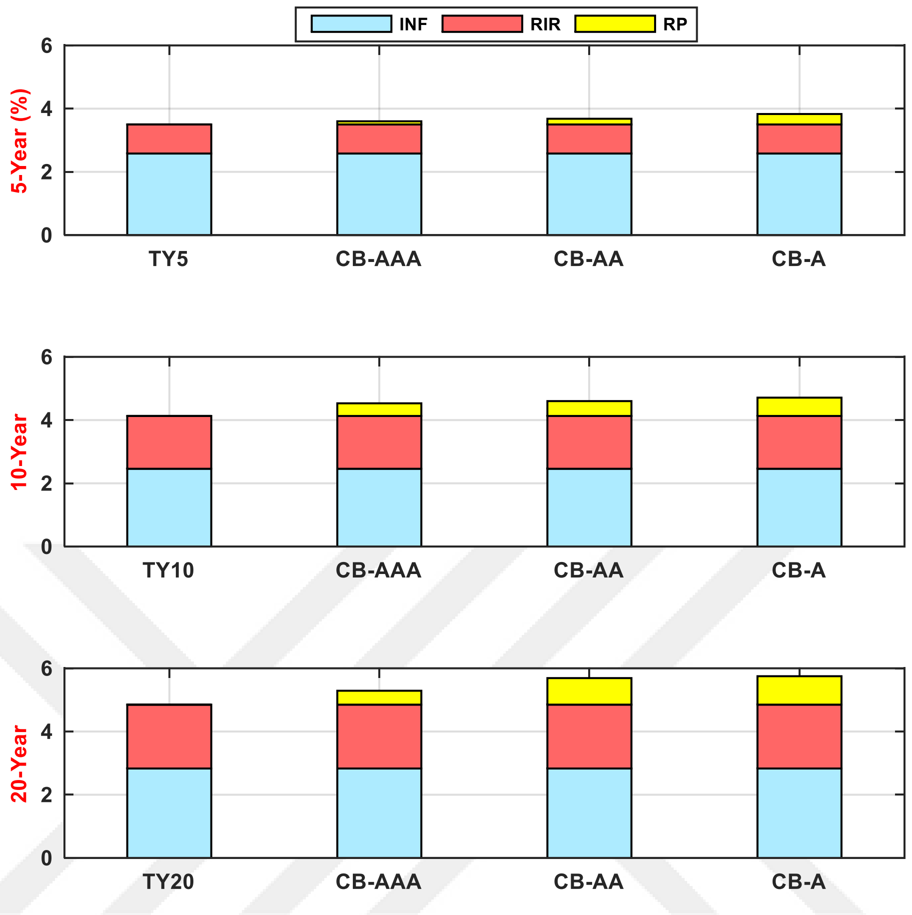


Figure 2-14 Determinant Factors of Nominal Interest Rates

Source: Calculated by the author.

2.4.3.3 Special Provisions or Covenants

As pointed out by Saunders and Cornett (2014), including some special provisions or covenants can affect a bond's price and its interest rate relative to other bonds' prices or interest rates. These covenants are a bond's callability, taxability or convertibility. For instance, if there exists a different taxability on bonds' yields, then investors require a higher interest rates for the bonds with the higher tax rules compared to bonds with the same maturity and interest rates but subject to a lower tax rules.

Before closing this section, we should describe the general equation that include these three factors to determine its fair value. Taken into account them, we can formulize the relationship between bond's price (interest rate) and the relevant factors as

$$i_i^* = f(IP, RIR, DRP_i, LRP_i, SCP_i, MP_i) \tag{33}$$

$$i_i^* = \text{Nominal Interest Rate} + \text{Risk Premium (Spread)} \tag{34}$$

where IP, RIR, DRP_j, LRP_j, SCP_j, and MP_j represent expected inflation, real interest rate, default risk premium, liquidity risk premium, special characteristic premium, and maturity risk premium on the ith security, respectively. Equation (33) shows that the market interest rate is functionally impacted by six factors. Similarly, Equation (34) contends that the market interest rate is decomposed into two components. All else being constant, interest rates differ according to these factors, as shown in Figure **2-14**. It evident from the figure, as maturity increases, risk premium increases at different proportions for non-Treasury bonds of firms that having different bond ratings.

2.4.4 Term Structure of Interest Rates

In the previous section, we analyzed the relationship among bonds that have same maturities to find out why they have different interest rates. Namely, we looked at the risk structure of interest rates. As noted by Hubbard and O'Brien (2011), we will turn to the another important concept for explaining different interest rate shapes: term structure of interest rates. This theory is related to the relationship among interest rates on bonds that have similar characteristics such as default risk, liquidity risk, and tax treatments but differ in their maturities. The main concern is to answer to the question of why bonds with different maturities have different interest rates. The easiest way to analyze this relationship is using the yield curve of Treasury bonds where their characteristics but maturity are hold constant.

As pointed out by Brigham and Ehrhardt (2013), a yield curve is formed by plotting a set of interest rates with different maturities for a specific date. Using this set of interest rates we can determine the term structure of interest rate of Treasury bond at any given point in time, as shown in Figure **2-15**, where the horizontal axis shows term to maturity and the vertical axis depicts the Treasury bond yields for different shaped curves.

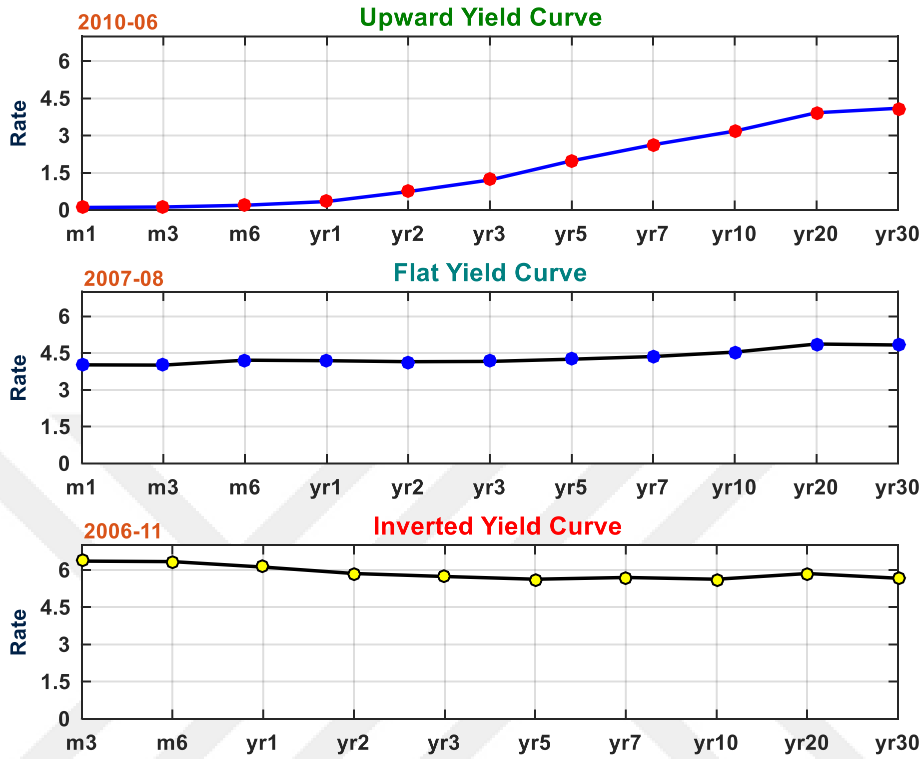


Figure 2-15 Monthly Treasury Yield Curve Rates for the US

Source: US Treasury Web Site (2018).

Before proceeding further, we must reveal the difference between the term structure of interest rate framework and yield curves. Jordan and Miller (2009) assert that in literature, the concept of term structure of interest rates and yield curve are used interchangeably as they are identical, however, they are not. In fact, the term structure framework is related to the relationship between the time maturity and default-free Treasury bond yields. However, the yield curve is related to Treasury bond offering coupon payments. Hence, we can say that term structure framework is based on default-free, i.e., pure discount securities, while yield curve is based on coupon Treasury bonds.

Figure **2-15** illustrates interest rate yields for Treasury bonds for different time to maturities ranging from 3-month to 30-years on three different months. Namely, it is a yield curve for Treasury bonds and shows the term structure of interest rates for those bonds on different time points. It is evident from the figure that yield curve shapes vary both in slope and in position during time. On June, 2010, it was observed that all interest rates on Treasury bonds were high due to the market expectation for high inflation rate in the future, as depicted in top panel of Figure **2-15**. Therefore, this yield curve is called as upward-sloping or positively sloped (normal) yield curve, because long-term rates were higher than short-term rates. Similarly, the market witnessed a nearly flat-yield curve on August, 2007, at the beginning of the global financial crisis, as shown in the middle panel. In addition, on November, 2006, according to the market, inflation rate was expected to decline in the future, therefore, long-term rates were lower than short-term rates, causing a downward (abnormal or inverted) sloping curve.

As noted by Mishkin (2007), in addition to explaining different shapes, a theory must account for three important empirical factors: (a) Treasury bond yields move together over time, namely, they are tend to increase and decrease together (b) long-term rates are usually above than short-term rates, namely, yield curves are almost always upward-sloping, (c) lastly, when short-term rates are low (high), yield curve are quite likely to have an positively (negatively) sloping shape.

As asked by Kidwell et al. (2016), one might wonder which economic factors can explain both the slope of the yield curves and changes in it during time. For a clear and better understanding of how financial markets work and why both the bond prices and yields changes over time, economist have introduced three major theories: (a) the unbiased expectations theory, (b) the segmented markets theory, and (c) the liquidity premium theory. In addition, Hubbard and O'Brien (2011) remark two useful criteria when evaluate these three theories: logical consistency of the theory and predictive power of the theory: Can the model offered by a theory explain the investor behaviors? And, can the underlying theory explain actual yield curve data?

2.4.4.1 The Unbiased Expectations Theory (UET)

Hubbard and O'Brien (2011) state that the unbiased (pure) expectations theory presents the basis for understanding the term structure of interest rates. This theory holds that the long-term interest rate of securities (spot rate) is equal to the (arithmetic) mean of interest rates (forward rates or expected future spot rates) investors expect on short-term securities over the investment horizon of the long-term securities. Namely, as noted by Kidwell et al. (2016), the slope of the yield curve for Treasury securities is determined by the marketplace believes (investors' average opinion) for the movement in the future expected short-term rates.

As mentioned by Hubbard and O'Brien (2011), there are two main assumption behind this theory: (I) investors (a) have the same homogenous expectations, (b) are profit-maximizers, and (c) risk neutral; (II) Investors view securities as being perfect substitutes for each other provided that they offer an equal expected returns. Note that neither of these assumptions of the idealized theory is completely correct, therefore, this theory is accepted as an incomplete explanation for different shape of yield curves.

To understand this theory accurately, Sharpe et al. (1999) suggest to consider an investment opportunity $1.000 with two investment strategies: (a) the maturity (the buy-and-hold) strategy and (b) the rollover strategy. For a market equilibrium, two strategies must offer the same expected return for investors. Assume that the current spot rate for one-year in 2018 is s_1 and the spot rate for one-year in 2019 is $f_{1,2}$ or the expected future spot rate in 2018 is $es_{1,2}$. Besides, the current spot rate for two-year in 2018 is s_2. According to the expectation theory, two strategies must yield an equal return at the end of investment horizon, 2019. If not, then a financial arbitrage opportunity exists which will be disappeared in a short span of time.

As noted by Saunders and Cornett (2014) and Sharpe et al. (1999), the current long-term interest rate (${}_1R_N$) is a geometric average of current (${}_1R_1$) short-term rates and a series of expected future $E({}_N r_1)$ or forward (${}_N f_1$) short-term rates. More formally, the mathematical equation for calculating the yield on a security that matures n years later is given as

$$[1+\ {}_1R_N] = \sqrt[N]{[1+\ {}_1R_1][1+E(\ {}_2r_1)]\ldots[1+E({}_N r_1)]} \tag{35}$$

Where ${}_1R_N$, N, ${}_1R_1$, and $E({}_Nr_1)$ represent the actual N-period rate today; term to maturity; actual current one-year interest rate today, and expected one-year interest rates for years observed at $i = 1, \dots, N$, respectively. Assume that the current one-year spot interest rate, ${}_1R_1$ is 5% and the current two-year spot rate is, ${}_1R_2$ is 7.5%. If the market's expectation for the future rate, $E({}_2r_1)$ is 10.06%, then, there exists a upward-sloping yield curve because the two-year rate is above the one-year rate (5% < 7.5%). In this case, the investor will be indifferent whether she/he must choice the maturity strategy or the rollover strategy, because both strategies yielded the same expected return, namely, the initial amount, $1.000, has grown to $1.155,62 in either strategies. More generally, the longer time to maturity, as noted by Sharpe et al. (1999), the higher the spot interest rate. Namely, an upward-sloping yield curve holds if the expected future spot rates are higher than the current spot rate. On the other hand, as noted by Hubbard and O'Brien (2011), if the expected future rates are lower than the current short-term rate, then a down-ward (inverted) yield curve occurs. Similarly, a flat yield curve exists if investors expect that future spot rates are equal in magnitude to the current spot rate.

Mishkin (2007) asserts that this theory is an elegant theory since it can explain why interest rates changes as term to maturity increases. In addition, it can provide an explanation of why long- and short-term interest rates on different bonds move together (fact a). Therefore, if short-term interest rates rise today, it is natural to expect that this increase will tend to continue for a considerable time period, causing a higher long-term rates. Similarly, this theory also explain fact (c) where an upward-sloping yield curve is the result of a higher long-term rate than short-term rate. Consequently, it can be said that this theory has an internally consistent, as pointed out by Hubbard and O'Brien (2011), explanation for the different shaped yield curves: upward-sloping, inverted, and flat yield curve. However, this theory is incapable to explain fact (b) where it is assumed that yield curves have usually positive shape (slope). If so, then, it is expected that short-term rates are usually lower than long-term rates most of the time. In reality, spot rates are just as likely to increase or decrease at any specific time. The reason behind this shortcoming is that the theory is condoning something important related to investors' behaviors in the markets.

2.4.4.2 The Market Segmentation Theory (MST)

As pointed out by Hubbard and O’Brien (2011), this theory deals with the weaknesses of the pure expectation theory by determining two important observations. First of all, theory assumes that investors do not have homogenous expectations. Secondly, bonds of different term to maturities are not seen as perfect substitutes for each other by investors.

Given those observations, debt markets for different maturity bonds are entirely segmented or separated according to investors’ (individuals and financial institutions) preferences or debt instruments available for investment (Jordan and Miller, 2009). The yield curve of spot rates for each particular maturity, therefore, will be determined by distinct demand and supply of bonds that particular segment’s maturity. Saunders and Cornett (2014) contend that debt markets with short-term maturities are mainly preferred by commercial banks due to their short-term nature of liabilities. Similarly, debt markets with long-term maturities are largely preferred by insurance companies and pension funds owing to long-term nature of their contractual liabilities. According to theory’s most restrictive form, individuals and financial institutions of a particular maturity do not enter a different segment unless they are offered with an adequate reward to leave their markets. Accordingly, spot rates or yield curve corresponding to each term to maturity are totally affected by its demand and supply conditions.

As mentioned above, segmented debt markets’ investment instruments are not perfect substitute each other, therefore, as noted by Mishkin (2007), the expected return is not affected by other markets’ conditions. Evidently, this observation shows that this theory is at the extreme to the unbiased expectation theory. The reason behind is that both individual and institutional investors have strong preferences in terms of the expected returns only for their particular markets. Most of investors generally prefers to invest in the short-term securities because, as noted by Hubbard and O’Brien (2011), they are (a) subject to less interest-rate risk exposure and (b) more liquid than long-term debt instruments. Because of these shortcomings, investors need to be induced by a higher expected return for an additional risk.

As Kidwell et al. (2016) note, if an investors with short-term (long-term) maturity invest in longer-term (short-term) securities, then she/he will be exposed to price (reinvestment) risk, indicating the reason why investors should be offered by a risk premium. In addition, this theory does not assume that investors are totally risk averse.

Remark that we described three facts about the term structure of interest rates. Mishkin (2007) contends that if the demand for short-term securities is greater than long-term securities, then the yield curve will be typically upward-sloping, as dictated by fact (b). Conversely, this theory cannot explain fact (a) and (c). Because debt markets of different term to maturities are totally separated, an increase in spot rate of a particular maturity does not have an influence on the spot rates of other maturities. Namely, this theory does a poor of job of explaining why, for example, short- and long-term interest rates tend to move together (fact a). In addition, Hubbard and O'Brien (2011) assert that it is difficult to consider an downward sloping yield curve, even though it occasionally exists, where the short-term spot rates is higher than long-term rates.

2.4.4.3 The Liquidity Premium (Preferred Habitat) Theory (PHT)

As can be seen from the previous sections, both theories cannot provide a complete explanation for the changes in spot rates, as Hubbard and O'Brien (2011) point out, due to their extreme assumptions. A logical step taken is to combine both theories under a new theory which ignores their extreme assumptions. According to Sharpe et al. (1999), this theory starts with the conception that lenders (borrowers) primarily tend to prefer short-term (long-term) securities to invest (issue) to evade interest rate risk; although, longer-term bonds might be purchased by some investors, there is a general tendency for short-term instruments.

In line with this theory, Hubbard and O'Brien (2011) document that debt markets of different term to maturities are accepted by investors as substitutes, note that it is not perfect substitutes. Therefore, the expected return of a particular maturity will be affected by the spot rate of different maturities. Saunders and Cornett (2014) remind that this theory holds that long-term securities will be hold as long as they offer a higher expected return in terms of an additional risk premium. Since short-term bonds have a greater liquidity and less price risk in practice, borrowers should pay a risk (term) premium to induce lenders to provide their fund over a long period. If lenders are compensated by an adequate rates, they will be willing to leave their particular maturity segments which is not possible in the segmentation theory. Namely, investors (lenders) are induced by a compensation called risk premium to hold long-term securities. Therefore, this theory holds if long-term spot rates are equivalent to geometric averages of the short-term spot rates and forward rates (i.e. the expected future spot rates) over the time horizon of the long-term bonds plus a risk (liquidity) premium which is positively related to term to maturity of the long-term bond.

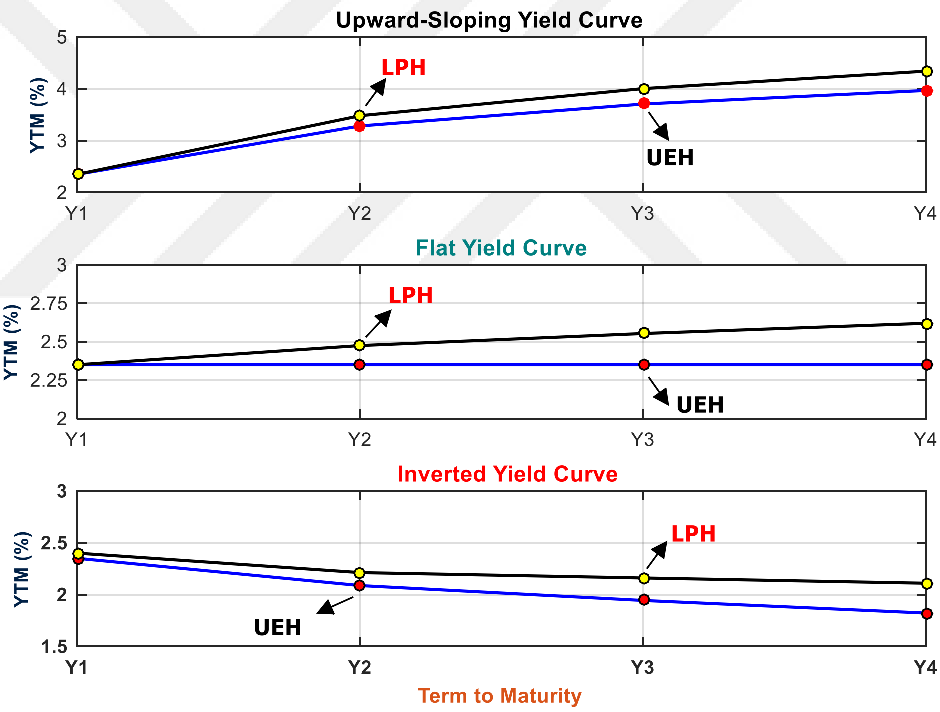


Figure 2-16 Yield Curve Comparisons under the Liquidity Premium Hypothesis (LPH) and the Unbiased Expectations Hypothesis (UEH)

Source: Calculated by the author.

The relationship among spot rates under the liquidity premium theory can be formulized as given

$$(1 + s_1)(1 + es_{1,2}) < (1 + s_2)^2 \tag{36}$$

where s_1, $es_{1,2}$, and s_2 represent the current short-term spot rate, the expected future spot rate one-year from now and two-year spot rate, respectively. Besides, the forward rate and expected future spot rate has a relationship as $f_{1,2} = es_{1,2} + L_{1,2}$ where $L_{1,2}$ denotes the risk (liquidity) premium.

According to Sharpe et al. (1999), the inequality in Equation (36) shows the key concept for understanding liquidity preference theory. For a clear comparison, let's look at Figure **2-16**. In top panel, there exists an upward sloping curve for both the unbiased expectations and liquidity preference theory. This panel illustrates that spot rates are expected to increase in the future, but, how large it does increase? First, it is evident that $s_1 < s_2$ and its shape of the yield curve is more steeply sloped because the average marketplace opinion is that spot rates are going to increase by a large amount in the future. When including the risk premiums, the yield curve under liquidity preference theory (LPH) will be above the yield curve under the expectations theory (UEH). Similarly, the middle panel depicts the flat curve where spot rates are equal, $s_1 = s_2$, under the UEH. This situation will hold for the LPH on condition that $es_{1,2} < s_1$, namely, the long-term rates are expected to decline. In the case of downward-sloping, the market expectation is that $s_1 > s_2$. However, the inequality in Equation (36) holds provided that the expected future rate is considerably lower than the current one-year interest rate. If occurs, then a downward-sloping yield curve will be observed because the marketplace expect that spot rates will decrease considerable in the future, as shown in bottom panel of Figure **2-16**.

2.5 Literature Review about Relationship between Economic Factors and Stock Prices

As aforementioned, the main purpose of security analysis is to discover the mispriced, underestimated or overestimated, bonds and/or stocks by conducting fundamental or technical analysis. The search for the identify the mispriced securities by investors has, as Elton et al. (2009) state, occupied a large amount of effort over a long time period, however, as the EMH posits, this efforts are doomed to failure because there is no way to systematically earn abnormal returns. When looking at the determinants of the securities, it is observed that they are specified in general terms. A common stock, for example, price can functionally be determined as the level of a firm's (a) earnings, (b) risk (specific), (c) dividends, (d) the cost of funds, and (e) future growth rate, etc.. Equivalently speaking, as Reilly and Brown (2011) mention, the valuation of a security depends on its profit potential and its quality. Evidently, it is easy to shows the determinants, however, it is a difficult task to construct a system, a valuation model, that uses those factors as input to value a security or pick the mispriced securities successfully (Elton et al., 2009). With the aid of a valuation model, a set of historical data related both to firm and economy is converted into, according to the authors (2009), an estimated price for a common stock. Broadly speaking, this model can be seen a formalization of the association that is expected between stock price and macro- and microeconomic variables.

Reilly and Brown (2011) assert that there exist two fundamental methods conducted by fundamentalist investors or technician investors for the valuation process: (a) the bottom-up, stock-picking method and (b) the top-down, three-step method. The supporters of the first approach claim that this approach enables investors to find the mispriced securities to earn abnormal returns irrespective of the aggregate market and sector outlook. Conversely, the supporters of the second approach assert that the total return of an individual stock is significantly influenced by both the aggregate market (economy) and sector outlook. It is clear that the main difference is related to the perceived importance of the aggregate market (economy) and the particular industry outlook on the stock valuation, thereby, the firm value. Accordingly, it is accepted a notion that the stock price and its return are directly or indirectly affected by the economic (macro) and sector environment (micro). To understand the effect of those factors, we should explain how a stock price changes when a factor changes by means of the empirical evidences.

2.5.1 Relationship with Macroeconomic Factors

In a related paper, Titman and Warga (1989) contended that they found a significantly positive linkage between equity returns and future inflation rate and future interest rate changes at the sample time period of October 1979 and October 1982 in the U.S. Besides, it was observed that the aggregate stock market produced a weaker result than real estate investment trusts (REITS) which was more sensitive to changes in interest and inflation rates. When looking at the reasons behind this positive (unexpected) result, the authors (1989) stated that they were somehow predictable in the underlying period, indicating that changes in those variables were more quickly incorporated into equity prices.

Mohanty et al. (2011) studied the effect of oil price exposure on both at the aggregate and sectoral levels of stock market returns in the six Gulf Cooperation Council (GCC) countries. Using weekly data spanning from 2005-06 to 2009-12, they (2011) revealed a significantly positive linkage between the underlying variables in Bahrain, Oman, UAE, Qatar, and Saudi Arabia. Besides, stock returns were negatively influenced by a decrease in the WTI price, namely, oil price exposure of equity markets (at aggregate level) were significantly positive to decreases in oil prices in all six GCC countries. Conversely, an asymmetric impact of oil price shocks on equity markets was observed only in two out of six GCC countries. Equivalently speaking, stock market returns of Saudi Arabia and UAE were positively affected by oil price increases. In addition, the paper showed that oil price exposure of 12 out of 20 sector indices in the GCC countries were significantly positive exposure to oil shocks, indicating a substantially different responds to oil price shocks in both the industries and countries. On the other hand, Akoum et al. (2012) presented an evidence of a strong cointegrating linkage between changes in oil prices and stock prices in the long run (at the lower frequencies) in the six GCC countries and Egypt and Jordan via conducting wavelet coherency approach. They (2012) also noted an increasing strength, in addition, market dependencies after global finance crisis of 2007, which indicates that investors can enhance their diversification benefits in the short term compared to long-term.

Kapusuzoglu (2011) studied short- and long-term dynamic relationship between Brent oil prices and stock market indices of "XU100", "XU050", and "XU030" in Turkey via daily closing prices consisting of 2437 days. The author (2011) documented a long run relationship between oil prices and stock market indices during the sample period of 2000-01-04 and 2010-01-04. Conducting causality tests, the study reached a unidirectional causality relationship from "XU100", "XU050", and "XU030" to oil prices, namely, all three stock indices were a powerful predictor for oil prices.

In order to examine whether exposure of oil price was systematically priced in stock prices, Demirer et al. (2015) used firm-level data (daily and monthly) of Gulf Arab region stock markets over the sample period of 2004-03-31 to 2013-03-31. According to test findings, they (2015) reported that stocks having higher sensitivity level for oil price shocks yielded significantly higher rate of returns than those of lower sensitivity, implying that risk exposure of oil price could be used as a return predictor by investors.

By conducting cointegration analysis and causality tests, Vuyyuri (2005) examined short and long run associations between a set of monthly financial and real sector variables –including inflation rate, interest rates, exchange rate, industrial production index, BSE index– over the sample period between 1992-06 and 2002-12. The author (2005) reported a long run relationship between the financial and real sector variables. There was no any evidence of causal linkage between industrial production and stock return. Similarly, a one-way causality was observed from exchange rate and inflation to stock returns. It was reported that interest rates did not have a predictable power on the monthly stock returns, however, the causality run in the reverse direction, namely, interest rates Granger-caused BSE sensitivity index in India.

Tezcan (2009) investigated the impacts of daily interest rate movements on stock index returns consisting of "XUSIN", "XUHIZ", "XUMAL", and "XUTEK" over the sample period between 2003-02-01 and 2008-10-17. By conducting TGARCH approach, there existed negatively relationships between the underlying variables. This negatively effect is most pronounced for "XUMAL" index returns.

In the related paper, Bhattacharya and Mukherjee (2002) aimed to determine causal relationships at lags and leads between stock market, the BSE Sensitive Index, and five major macroeconomic factors in India. To answer the research question of whether stock market could be used as a barometer for the economy, they employed the TY non-causality approach (1995) using monthly data for the period between 1992-04 and 2001-03. Accordingly, they (2002) revealed a bidirectional causal linkage between stock returns and inflation rate and a one-way causal relationship from industrial growth rate to stock market at 1% significance level. Conversely, the null hypothesis of non-causality was rejected even at 10% significance level between stock returns and 364-day Treasury bill interest yield, national income, and money supply, dictating that they could not be used as an indicator for the forecasting of stock returns and stock does not lead three key macroeconomic variables. A causal linkage from stock returns to an economic factor is the evidence of informationally inefficiency, therefore, it was said that the BSE Sensitive Index was inefficient market with respect to the inflation and industrial growth rate. According to the authors (2002), an investor can constantly obtain an above-average return by means of the lagged values of the inflation and industrial growth rate to forecast stock returns in Indian equity market.

Katechos (2011) revealed that there existed a strong linkage between global equity market returns and exchange rate, where the direction of the relationship depended on the features of the underlying currencies. Namely, the value of higher (lower) yielding currencies was positively (negatively) related to stock returns of global markets. Besides, the author (2011) reported that the higher relative large interest differentials, the higher explanatory powerful of the model. For instance, a lower relationship was observed when relative interest rate differentials were relatively narrow, signifying a decrease in the explanatory power of the model.

In their empirical paper, Sensoy and Sobaci (2014) observed a consistent and significantly negative relationship between stock market returns and interest rate changes, implementing the VAR(p)-FIAPARCH(1,d,1)-cDCC(1,1) method on the daily observations for the period of 2003-01-02 and 2013-09-05. Using dynamic conditional correlation approach, they found a time dependent linkage between stock and bond markets, i.e. this relationship is true only in the short run, therefore, there is no need to a reaction by policymakers to prevent a long run contagion effect between the stock and bond markets.

The empirical findings of Kasman et al. (2011) provided a significantly negative evidence of interest and exchange rate impacts on the conditional daily stock returns of "XBANK" index and 13 commercial banks listed on the Istanbul Stock Exchange in Turkey for the period between 1999-07-27 and 2009-04-09. It was observed that market return, moreover, played a more important role than exchange and interest rates in determining the dynamics of conditional daily stock returns. Both variables were found to be a primary predictive factor in the conditional return volatility of bank stocks, namely, changes in the returns of bank stocks could be explained by changes in the growth rates of exchange and interest rates. These results indicate that as exchange and interest rates had a predictable power on the conditional daily stock returns, investors should have followed them more closely to adapt their portfolio asset compositions.

Elyasiani and Mansur (1998) found a significantly negative impact of the long-term rate on the return of bank stocks in the US over the sample period of 1970-01 to 1992-12. It was observed that volatility of interest rates was a major determinant of the volatility and risk premium of bank stock returns. In addition, the authors (1998) revealed that the higher volatility in bank stock returns, the higher risk premium for the MCB and Large bank portfolios.

Saporoschenko (2002) studied to gauge the sensitivity of returns of bank stocks to (a) overall market returns, (b) short- and (c) long-term bond yields, (d) change in interest rate spread, and (e) exchange rate for a sample period of 1986-01 and 1992-12 using weekly data of 47 Japanese banks. The author (2002) aimed to measure the stock return sensitivity to unexpected changes, i.e., shocks, in those five factors. Broadly speaking, there was a significantly negative linkage between bank stock returns and shocks in long-term interest rates, in addition, general market returns and interest rate spread exhibited strong interest rate sensitivity during the tested period. Moreover, returns of bank stocks did not seem to be very sensitive to the return of foreign exchange rate.

Studying the impact of oil price shocks on stock markets at both the aggregate and industry levels in the Eurozone and US for the sample period 2000-06 to 2011-07, Reboredo and Rivera-Castro (2014) documented insignificant results for the aggregate and industry levels in the pre-crises period, with the exception of, however, stocks of oil and gas companies, which were positively affected by shocks in oil prices. The strength and direction of relationship was turned into a positive pattern since the onset of the GFC, namely, contagion and positive interdependence between equity and bond markets was observed.

Flannery and James (1984) asserted that interest rate movements were significantly related to return of bank stocks using weekly data for the sample period 1976-01-01 to 1981-11-01 in the US. Moreover, the maturity composition of nominal assets and liabilities, namely, the maturity mismatch was observed as a significant factor that had positive impact on bank stock returns.

As noted by Cenedese and Mallucci (2016), the major driver of international equity returns was found to be news related to future cash flows instead of discount rates in the US. Moreover, inflation rate was found to be the main factor of international bond returns. Conversely, it was observed that exchange rate movements had a little impact on the volatility of bond and unanticipated stock returns.

Chen et al. (1999) examined the impact of changes in discount rates on stock returns, market volatility, and trading volume implementing higher frequency over a sample period 1973-01 to 1996-01 in the US. The authors (1999) found that stock returns responded significantly but negatively to the unexpected announcements, i.e., policy changes, of discount rate movements. Equity returns, for instance, declined (rose) by approximately 50 basis points when discount rate rose by (declined) 10 basis points. Conversely, there was no evidence of a significant effect on equity returns by the expected changes in discount rates. Similar findings were observed when the sample period was broken into two parts: pre- and post-1979. In addition, stock prices were found to respond to the announcement within the same trading session for the whole sample. Moreover, the utility industry was also negatively affected by the announcement of discount rates but its effect lasted for two periods. On the other hand, the findings revealed that the foremost source of market return volatility was the announcement of public, not private, information, which dissipated in the short time. This result was accepted as the supportive of the efficient market hypothesis because the market incorporated the public information fully within a short time period.

In a recent study, Kontonikas et al. (2013) documented that equity prices was found to respond positively to unanticipated federal funds rate (FFR) cuts outside the crisis period. Throughout the crisis period, however, a different pattern was observed that is equity prices did not responded positively to unanticipated FFR cuts, indicating a signal of worsening economic conditions in the future. Accordingly, investors had shifted towards safe-haven assets due to falling stock prices.

Examining whether stock market uncertainty had a significant impact on the comovements of returns of equity and bonds, Connolly et al. (2005) employed an augmented GARCH(1,1) model using daily observations of VIX, 10-year U.S. Treasury notes, 30-year U.S. Treasury bonds, and the value-weighted NYSE/AMEX/NASDAQ over a sample period 1986 to 2000, consisting of 3755 observations for each variable. According to test results, there was a negative linkage between the future correlation of bond and stock returns and the uncertainty measures. Whenever a considerably increase (decrease) in implied volatility and an unpredictably high (low) stock turnover was observed during days, they (2005) witnessed a high (low) bond returns corresponding to equity returns. Accordingly, it was believed that benefit of bond-stock diversification rose whenever stock market uncertainty increased; suggesting that uncertainty in markets had important cross-market pricing influences.

Implementing a specified GARCH model, Flannery and Protopapadakis (2002) had sought to study the simultaneous effect of macroeconomic factors –such as balance of trade, consumer credit, CPI, new home sales, unemployment, industrial production index, etc.– on stock returns, the value-weighted NYSE/AMEX/NASDAQ market index, in terms of level and conditional volatility over the sample period 1980-01 to 1996-12. They (2002) found that six out of 17 macroeconomic variables were strong risk factor candidates and the CPI and the PPI variables influenced solely the return level of market portfolio. Three real factor candidates, in addition, had only a significant effect on the conditional volatility of returns. The money supply variable, M1, affected both stock returns and market volatility. The authors (2002) contended that these results had two direct benefits: (a) a hedging opportunities for investors and (b) constituting a priced factor for risk-averse investors.

Apergis and Eleftheriou (2002) examined the linkage between equity prices (ASE), interest (3-month T-bill yields) and inflation rates using monthly observations in Greece over the sample period 1988-99, where markets were characterized by declining inflation and interest rates. They (2002) found evidence in favor of the inflation-interest rate linkage over the period, namely, equity prices followed inflation rate rather than T-bill interest rate yields.

To capture the interactions between exchange rate, MYR-US, and five stock market indices, Ayub and Masih (2013) used daily variables over the sample period 2007-03-01 to 2012-12-31, consisting of 1523 observations. Briefly stating, stock markets and exchange rates were found to have a long run relationship. Largely, there were unidirectional causal linkages from stock markets to exchange rate at first wavelet scale, however, at the lowest frequencies (d3 and d5), they (2013) witnessed two-way relationships. In addition, negative correlations between the underlying variables observed at all wavelet scales, namely, the lower (higher) frequency (scales), the stronger correlation coefficients.

In a recent and a more extensive paper, Barragán et al. (2015) examined the effect of oil fluctuations and crashes of equity markets on correlations between debt and stock markets conducting wavelet-based approach for the U.S., UK, Germany, and Japan, over the sample period spanning from 1990-02-27 to 2011-11-22 comprising 5665 daily observations. First, the authors (2015) broke the sample period into two different period as pre- and post-crisis to compare wavelet correlations. It was found that the correlation of pre-crises period was close to zero, however, the magnitude of estimated correlation tended to change during the shock period in oil and equity markets. For instance, the correlation magnitude was significantly changed by oil shocks at the higher frequencies. Furthermore, they (2015) reported that correlation among stock markets of four countries was pushed up further by oil fluctuations. Broadly speaking, there was significant evidence in favor of contagion effect, namely, the correlation between two markets had significantly changed during the 2008 and 2011 stock market crashes. Moreover, the number of breakdowns in correlation during the crises period was higher at the lower frequencies, implying several shifts in market co-movements.

Employing a time-varying DCC-GARCH copula model, Jammazi et al. (2015) examined the dynamic comovements between weekly long-term interest yields and equity returns for 16 developed countries over the sample period starting at 1993-01 to 2013-04. Empirical findings showed that stock-bond comovement pattern had changed noticeably over time for most countries. During the 1990s, for example, a positive linkage for almost all developed countries was observed driven by declining inflation rates and strengthening economic prospects. From the early 2000s, a negative association was witnessed, however, between bond and stock returns. However, this negative relationship pattern was broken in late 2009 for some countries, such as Greece, Portugal, Spain, Belgium, and Ireland, because of the Eurozone sovereign debt crisis, supporting the presence of flight-to-quality effects. They (2015) also noted no evidence of tail dependence and asymmetric for most countries, indicating that (a) the stock–bond comovement was not different during market downturns (bearish markets) and upturns (bullish markets), (b) both markets did not tend to move together due to the absence of lack tail dependence, and (c) the dependence between bond and stock markets was present most of the time.

In the related empirical paper, Dar et al. (2014) investigated whether the yield spread between long-term and short-term government bond rates incorporated information about economic activity (industrial production) in the future employing wavelet methodology. The monthly data covered the sample period of 1996-10 to 2011-04 comprising 175 observations for each variable for India. They (2014) reported that there was no predictive power of yield spreads on future growth in economic activity in time domain. Conducting wavelet approach, the study presented a supportive evidence of time-varying predictive power across different frequencies, for example, the predictable power of yield spread on output growth held only in the lower frequencies, i.e. at the higher time scales.

Using monthly observations on stock market and exchange rates for India spanning from 1993-04 to 2009-02, Tiwari et al. (2015) found a one-way and two-way causal linkages at different frequencies. Stock prices lagged, for example, exchange rates at the higher time scales. In addition, it was observed that both variables were out- and in-phase (cyclical and anti-cyclical), namely, equity returns caused exchange rates at 4-6 monthly horizon because both the variables were found to be out of phase at the end of 1994 and beginning of 1995. Overall, a frequency dependent causal relationship was observed during different time horizons.

Bae (1990) investigated the interest rate sensitivity of common stocks of sixty-seven financial and thirty nonfinancial firms to interest rate changes for the sample period 1974-01 to 1985-12 for the U.S. It was observed that financial firms' common stock returns were negatively affected by the changes in current and unexpected interest rates. To be more precise, those stocks were more sensitive to long-term interest movements. On the contrary, common stocks of nonfinancial firms were found to be less, more clearly invariant, sensitive to unexpected interest rate movements because of their asset compositions. However, most subsectors of both nonfinancial and financial firms' stock returns tended to be invariant to expected interest rate changes. Market capitalizations of financial firms were, in addition, more affected by unexpected interest changes than current interest rate changes.

Doğukanli et al. (2010) examined the exchange rate of dollar and euro exposures on stock indices listed on the Istanbul Stock Exchange in Turkey for the sample period between 1999-01 and 2009-06. The paper revealed a long run relationship between equity returns and exchange rate changes. Exchange rate exposure, however, varied among the industrial stock prices. For example, "XUHIZ" index was adversely, while "XUMAL" and "XUSIN" indices were positively affected by the dollar exposure. Similarly, "XUSIN" index was positively, however, "XUMAL" and "XUSIN" indices negatively affected by the exposure of euro exchange rate over time. When looking at the degree of sensitiveness of industrial stocks, it was shown that both currencies had a little impact on "XUMAL" index due to the precautions taken by financial sectors, while "XUHIZ" index had the highest currency exposures among the industrial indices.

To investigate whether stock returns were influenced by exchange rates and interest rates (the UK 1-month T-bill), Joseph (2002) employed EGARCH model using weekly data prices of industrial stocks listed on the FTSE in the UK for the sample period between 1988-01-07 and 2000-08-31. According to test results, equity returns were more adversely affected by T-bill rate yields than by exchange rate. Up to 34% of all firms were significantly impacted by interest rate changes while T-bill rate yields affected nearly 28% of all firms. The electrical and engineering sectors were the two most negatively affected sectors while the pharmaceutical sector was the only sector that positively affected by changes in interest and exchange rates.

Aydemir and Demirhan (2009) studied the causal association between stock indices of "XU100", "XUHIZ", "XUMAL", "XUSIN", and "XUTEK" and exchange rate for Turkey during the sample period spanning from 2001-02-23 to 2008-01-11. The authors (2009) found feedback causalities between all stock indices and exchange rates via the T&Y Granger approach.

Employing a six-variable VAR model, Abugri (2008) tried to examine whether U.S. dollar-denominated market returns could be significantly explained by major four macroeconomic variables in four Latin American countries of Brazil, Argentina, Mexico, and Chile. The monthly data of nominal exchange rate, money supply, industrial production index, and nominal interest rate covered the sample period 1986-01 to 2001-08. The author (2008) documented that the global factors –the MSCI world index and the U.S. 3-month Treasury bill yield– were significantly and consistently explaining returns in all four domestic markets. In addition, market returns were also found to be affected by domestic variables at varying magnitudes and significance levels. These results might provide investors useful information about (a) portfolio diversification strategies, (b) achieving better return-risk tradeoff, (c) improving their portfolio performances by concentrating on the varying significance of the risk factors.

Dinenis and Staikouras (1998) used the weekly data of interest rate and stock prices of 153 financial intermediaries listed on the LSE to investigate the possible nexus for the UK case over the sample period starting from 1989-01 to 1995-12, comprising 365 weekly observations for each variable. The result of paper revealed (i) a negative linkage between the movement of interest rates and the common equity prices, (ii) the effect of interest rate was higher when the 3-month T-bill was used in model, (iii) even though financial institutions' stocks were significantly affected by the unexpected interest rate movement, this effect appeared to be substantially higher for nonfinancial firms, and (iv) interest rate volatility had a significantly positive impact on both nonfinancial and financial firm equity returns.

Assefa et al. (2017) investigated the dynamic relationship between macroeconomic variables and stock returns for 21 developed and 19 developing countries over the sample period 1999-Q01 to 2013-Q04. According to test result of the fixed effects panel model, “RGDP" variable had significantly positive effect on stock returns in only developed countries. A quarterly return on the world index, in addition, was priced in the domestic markets of both country groups. On the contrary, interest rate movements had a little but significant adverse effect on stock prices in both country groups while REER variable was only significantly and adversely related to equity returns in developing countries, indicating that currency appreciation (a decrease in exchange rate) caused a decrease in equity returns because of the adverse impact on balance of trade.

Using a STVEC-GARCH model, Liu and Chen (2016) examined the nonlinear interrelationship among monthly interest rate movements, changes in house and stock markets in Taiwan for the sample period of 1985-01 and 2009-03. The result of study provided an evidence to indicate causal linkage from house prices to stock market returns when interest rate movements were caused by either stock market returns or house price. It was observed that stock market volatility had significantly positive effect on interest rates. Besides, the lagged values of house prices and interest rates had significantly interactive impacts on the covariance between stock price and interest rate.

Ferrer et al. (2010) conducted an empirical analysis to investigate the linear and nonlinear interest exposure on stock returns at the industry level in Spain over the sample period 1993-2008. The empirical findings of the paper showed that the influence of interest rate risk was heterogeneous across industries, namely, the sign and its magnitude varied substantially during underlying period. In addition, it was observed that nonlinear exposure profile was reasonably less important than the linear one, namely, the linear exposure profile had prevailed over the asymmetric and nonlinear risk exposure patterns at the industry level. Accordingly, the paper revealed that industry returns generally did not react differently to changes in interest rate, with an exception for the food sector, namely, stock returns of food sector were less affected by falling interest rates rather than rising interest rates. Particularly, highly leveraged (indebted), regulated and financial sectors including real estate and construction; electrical and utilities; and banking industry were the most interest rate sensitive among Spanish industries.

In a recent study, González et al. (2017) examined the effects of the financial crisis and unanticipated movements in interest rate on monthly stock returns at the industry level in the US over the sample period 1989-11 to 2014-02 at different subperiods. According to test results, the authors (2017) found, in general, negative relationship between stock returns and changes in both nominal and real interest rates. However, it was observed that "Diversified Metals and Mining" sector was positively related to the unanticipated movements in both nominal and real interest rate while "Integrated Oil and Gas" industry was positively affected by the unanticipated movements in real interest rates, suggesting a hedging opportunity for investors. Finally, after taking account inflation rate into model, "Household Durables" and "Gold" showed a consistent negative response to unexpected inflation rate, indicating that they were significantly exposed to inflation risk.

In their more recent paper, Sancar et al. (2017) investigated the relationship between macroeconomic variables and stock prices in Turkey for the sample period between 2000-01 and 2016-12 using monthly observations for industrial production index, interbank interest rate, M1 money supply, exchange rate, consumer price index and stock market index. They (2017) contended that all variables were found to be stationary at the first differenced level, and cointegration test revealed a long run relationship. Accordingly, it was observed that money supply, consumer price index and industrial production index were significantly and positively; while exchange rate was negatively related to stock prices. However, both FMOLS and DOLS tests showed that stock price and interest rate were not related to each other in the long run.

Employing panel data approach, Sayilgan and Süslü (2011) investigated the effects of macroeconomic factors on stock returns using quarterly observations of inflation, money supply, real economic activity, exchange rate, interest rate, S&P500 and oil price in eleven countries including Argentina, Brazil, Indonesia, Hungary, Malaysia, Poland, Mexico, Russia, Chile, Jordan and Turkey over the sample period between 1999 and 2006. Test findings revealed significant impacts from inflation rate, S&P500 index and exchange rate; however, there was no evidence of significant effect from interest rates, real economic activity, oil price and money supply to stock returns. A one-unit increase, for instance, in inflation rate would increase share returns by 0.41 percentage, while, on the other side, the same increase in exchange rates would decrease share returns by 0.53 percentage, driven mostly by foreign portfolio investments.

The paper of Demir (2014) was conducted to study the impact of monetary policy rate decisions on monthly stock market index implementing cointegration and causality tests for Turkey case over the period from 2005-01 to 2015-06. According to test results, stock market index and policy rate were found to be cointegrated. A one-unit increase, for instance, in government bond rates would decrease stock index by nearly -0.66 percentage while the effect of policy rate cuts was -0.12 percentage. Test result of VECM model showed that both monetary policy rate decisions and dummy variable of 2006 year did not have significant impact on stock prices in the short-term. On the other hand, government bond rates had a similar effect on stock prices in the long-term where a one-unit increase caused approximately -0.40 percentage decrease in stock prices. Lastly, the coefficient of the estimated error correction term revealed that disequilibrium between underlying variables was corrected nearly in 19 months. Namely, the short run distortion in the model was converging to equilibrium in 1.5 years.

Simba (2016) investigated the relationship between stock market prices and various macroeconomic variables for Kenya over the sample period spanning from 2009-06 to 2015-06. The findings of the paper revealed that there was bidirectional causality between term deposit rate and stock index of "NASI" and term deposit rate and stock index of "NSE20". In addition, NSE20 stock index Granger-caused interest rate spread while a one-way causality from T-Bill rates to NSE20 stock index was detected.

By conducting both event study and econometric methodology consisting of cointegration and causality tests, Hu (2015) researched the relationship between daily observations of stock price and interest rate movements over the sample period 200-2014 in China. Test results showed that interest rates had a long run relationship with either Shanghai or Shenzhen stock index, indicating that stock price index and interest rate converged to equilibrium in the long-term. According to VECM results, stock index was significantly negative adhered to Shanghai stock index. The coefficient of the estimated error correction term (dependent variable was Shanghai) revealed that disequilibrium in system was corrected in 297 days, while short run distortion in system (dependent variable was interest rate) was corrected in 5 days. On the other hand, the same negative relationship was also true for Shenzhen stock index in the long run. Conversely, causality test findings revealed a feedback relationship between overnight interest rate and with either Shanghai or Shenzhen stock index in the short-term. These results suggested that both variables were a good indicator for estimation of another variable at 1% significance levels.

Brufatto (2016) examined the long run relationship monthly stock price index of DJIA and various macroeconomic variables including money supply, industrial production, crude oil price, consumer price index and short-term interest rates for the U.S. case. The dataset period was between January 1989 and August 2015. According to cointegration results, the author (2016) documented a long run relationship between stock index and selected macroeconomic variables. For example, change in interest rate was significantly negative related to stock return at one lag and six lag. Similarly, inflation rate was also significantly negative adherent to stock returns at lags 2, 4, 6, 8, and 11 while industrial production was connected to equity returns only at lag 10. Variables that significantly positive related to stock returns were growth rate of industrial production index at lag 1; change in money supply at lag 5; change in oil price at lags 8, 9, and 11. In terms of causality tests, it was revealed that there existed bidirectional causality relationship between industrial production index and T-bill 3-month interest rate with stock prices, while money supply Granger-caused stock prices, but the reverse linkage was not true. In addition, the null hypothesis of non-causality was not rejected for oil prices, namely, the contribution of past values of oil price to forecast stock prices and the reverse causality were insignificant. Finally, the lagged value of DJIA index had some impact in forecasting current value of consumer price index at 10 significant level.

Using low frequency (monthly) observations of the Australian stock market, 3-month interest rate, exchange rate, growth rate of the wholesale price index, the current account deficit, and industrial production index, Kearney and Daly (1998) aimed to identify the determinants of stock market volatility in Australia over the period 1970-06 to 1994-01. The paper revealed that the conditional volatilities of interest and inflation rate were found to be the two major determinants of the conditional volatility of stock market prices. These two determinants were directly, however, the conditional volatilities of the current account deficit, money supply, and industrial production were indirectly related to with conditional volatility of stock market. The strongest impact to stock market conditional volatility was, in fact, from the volatility of the money supply. Conversely, there was no evidence of significantly association between the conditional volatilities of the foreign exchange and stock markets during the period.

By suing an EGARCH(1,1) model, Erdem et al. (2005) examined the univariate price volatility spillovers in stock indices listed on the Istanbul Stock Exchange in Turkey for the period between 1991-01 and 2004-01. The empirical findings of the paper showed that the degree of volatility persistent was significant for interest rate, exchange rate, money supply, industrial production, and inflation. Similarly, it was true also for stock price indices; however, the length of persistence of stock indices was shorter compared to macroeconomic factors’ length of persistence. On the other hand, significant one-way spillovers were observed from macroeconomic factors to stock indices. For example, there were significantly negative spillovers from inflation to "XU100" and "XUSIN" indices; from interest rate to "XUHIZ" index; and from money supply to "XUMAL" index. On the contrary, the paper revealed significantly positive volatility spillover(s) from inflation to "XUHIZ" index; from interest rate to "XU100", "XUMAL", and "XUSIN" indices; and from exchange rate to "XU100" and "XUSIN" indices. Finally, there was no volatility spillover from industrial production index to any stock index.

In their paper, Dimic et al. (2016) investigated the effects of uncertainty of global financial markets and domestic macroeconomic variables on bond-stock correlation using monthly observations in the U.S. and ten emerging markets (Argentina, Brazil, Turkey, Bulgaria, Mexico, Colombia, Russia, Venezuela, the Philippines, and Peru). First, the authors (2016) reported that all emerging markets except Venezuela had significantly positive, while the U.S. had significantly negative unconditional stock-bond correlation relationship during the period. According to wavelet coherence approach, it was seen that bond-stock correlation did change significantly across frequency bands. Namely, for most of the domestic stock markets, the sing and magnitude of short run stock-bond correlation changed quickly from positive, however, to negative episodes corresponding to the crisis period, supporting the presence of flight-to-quality effects on markets in the short run. During the Dot-com market crash, (2001b to 2002e), for example, Argentina, Bulgaria, Russia, Venezuela, and Colombia observed sustainable negative bond-stock relationship episodes while Mexico, Turkey, Peru, and the Philippines observed higher negative correlation pattern. In addition, a positive correlation was observed only in Turkey, Argentina, Colombia, and the Philippines even though a considerable decrease in the magnitude was observed. These empirical results provided important explanations that investors did change their portfolio compositions in favor of bond market instruments during crises period, indicating that emerging debt markets provided a hedging opportunity in the short-term. On the other hand, all the emerging markets, apart from Venezuela, had positive and largely stable stock-bond correlations in the long-term, i.e., at the lower frequency bands, providing evidence against to flight-to-quality phenomenon. Namely, debt markets were not accepted as a hedging opportunity by long-term investors compared to equity markets in emerging countries throughout the entire sample period due to country-specific risk factors. In the short run, the most influential factor on bond-stock correlation was found to be the monetary policy decisions, on the contrary, in the long run the most influential factors were uncertainty in stock markets and inflation.

Andersson et al. (2008) investigated the effect of economic growth and inflation expectations and uncertainty in equity markets on the rolling correlation association between bond and stock returns using daily observations of the S&P500, FTSE 100 and DAX indices over the sample period between 1991-01 and 2006-06 for the U.S.

and between 1992-01 and 2006-06 for the UK and Germany. Conducting rolling window correlation (RWC) and the dynamic conditional correlation (DCC), it was observed that relationship changed considerably throughout period. Equivalently speaking, the relationship was affirmative most of the time, albeit the correlation changed the sign showing sustainable negative episodes, where all three markets exhibited similar patterns throughout the negative episodes.

Using monthly observations, Jareño (2008) investigated the stock market indices' sensitivity to inflation and interest rate movements via two-factor model and three-factor model in Spain over the sample period between 1993-02 and 2004-12. Test findings showed that industrial returns were influenced adversely and significantly by changes in real interest rates. On the contrary, there was no evidence of statistically significant relationship with the movements in expected inflation rate.

Korkeamäki (2011) studied risk exposure of interest rate on monthly equity returns of the European markets pre- and post-euro introduction for 13 developed countries. Test results illustrated significantly negative stock-bond correlation relationships for most of the EU countries but Germany and France earlier than 1999, which dissipated in the post-euro period for all sample countries, suggesting that interest risk was priced for global investors due to growth in European debt markets.

Including 57 financial intermediaries and 47 industrial corporations of France, the UK, Switzerland and Germany, Oertmann et al. (2000) investigated the interest rate sensitiveness of stock returns employing multifactor index models over the sample period from 1982-01 to 1995-03. It was observed that unanticipated interest rate movements had adverse impact on the equity returns of financial intermediaries partly because of their business activities. The multinational companies operating in the UK and Germany were the financial corporations that severely influenced by global interest rate movements. Nonfinancial companies, on the other hand, benefitted from changes in interest rate movements in the same direction, namely, there was significantly positive linkages. In addition, movements in both global and domestic interest rates could be accepted as driving forces of equity returns in the markets.

Employing the duration and convexity model, Udegbunam and Oaikhenan (2012) examined the effect of interest rates to stock price using annual observations spanning 1981 to 2006 in Nigeria. The empirical paper revealed a significantly negative impacts of net interest rate movements on the Nigerian stock market prices, namely, it was found that an increase in stock risk led a decrease in stock prices. Accordingly, these results revealed evidence in favor of the existence of a non-linear association between stock prices and interest rate risk regarding to duration and convexity hypothesis.

Uyar et al. (2016) studied the linkage between the 5-year government bond rate and stock indices of "XU100", "XU030", "XUTUM", "XUMAL", and "XBANK" for Turkey during the sample period spanning from 2006-01-02 to 2015-01-30. Employing simultaneous quantile regression technique, the authors (2016) found a negative impact of interest rate on stock indices at varying significance and magnitudes for low and high quantiles. In addition, these findings revealed that stock indices of "XUMAL" and "XBANK" were more sensitive to negative effect of interest movements. Putting differently, each stock index had responded differently to changes in the 5-year government bond rate, suggesting investors to adapt their portfolio compositions at periods of falling or rising stock prices.

In their related paper, Duran et al. (2010) examined the effect of monetary policy decisions on stock indices of "XUTUM", "XU100", "XU030", "XUSIN", "XUHIZ", "XTCRT", "XUMAL", and "XBLSM" using event study and GMM approaches over the period 2005 to 2009 for Turkey. The empirical findings of the paper showed that stock indices were found to be affected by policy rate shocks at varying magnitudes. A 25 basis-point increase in policy rate led 0.99%, 0.85%, 0.69%, 0.65%, and 0.85%, decrease in "XUMAL", "XUTUM", "XUSIN", "XUHIZ", and "XBLSM" stock indices, respectively, driven by different sensitiveness to interest rate movements. The authors (2010) claimed that the major reason behind the differential responses of stock indices to monetary policy decisions was due to their balance sheet compositions, namely, their differential sensitiveness to interest rate shocks.

By employing ARCH model, Gülec (2014) investigated the impacts of the monetary policy interest rates including policy rates (1-week repo), CBRT late liquidity window (LON) and CBRT overnight (O/N) rates on stock price volatility using closing price observations of the first session (SP), the second session (SR) and daily closing prices (ST) "XU100" over the sample period between 2002-01-02 and 2013-11-15 in Turkey. Test results indicated that the stock market volatilities in the ST were reduced by changes in the policy rates. In addition, including dummy variable denoting structural points caused a decline in persistence of the second session volatility. Namely, the magnitude of volatility increased with negative shocks stemming from policy rate decisions. Moreover, a negative linkage between stock prices and interest rates was found. Overall, the author (2014) ascertained significant results for the SR and ST but insignificant findings for the first session because of announcement hours related to the monetary policy committee meeting decisions.

Focusing on the linkage between stock prices and short-term interest rates, Michlian (2014) used GJR-GARCH-t-M model using daily observations of the PX Index (Prague Stock Exchange) and 2-week PRIBOR rates over the sample period of 2001-01-02 and 2014-02-19. According to test results, there was no significant effect of interest rates on the stock prices in either pre-crises of the post-crisis of 2007 GFC. Furthermore, it was noted that the GFC crisis had changed the linkage between stock prices and oil and gold prices, and exchange rate in the post-crisis era, signifying that global factors became important determinant factor for asset pricing.

The study of Çiftçi (2014) investigated whether four macroeconomic factors including interest rate, crude oil price, gold and exchange rate had significant effects on monthly returns of ten stock indices listed on the Dow Jones Index over the period between 1997-02 and 2007-11. In general, it was observed that the sign and the magnitude of the macroeconomic variables varied substantially across sector indices, namely, they had heterogeneous impacts on stock returns. There was significantly negative effect, for instance, from oil prices to “Financials”, “Consumer Services”, “Consumer Goods”, and “Healthcare” industries in the pre-crisis period, whereas the only sector that had positive relationship was “Oil and Gas” sector as expected by the author (2014) during the all three different periods. It was also revealed that the sign of the negative relationships of two indices switched to positive direction, namely, positive linkages appeared for the sectors of “Financials” and “Consumer Services” during the sample period between 2009-07 and 2014-09, i.e., post-crisis period.

Interestingly, there was no significant effect of interest rates on the stock prices during the all three periods due to enhanced tools for interest risk management, growth in derivative and corporate debt markets. Besides, the half of the sectors was positively related to movements in exchange rates during the different periods because of their export oriented natures. “Telecommunications” sector returns, for instance, had a positive effect from rising exchange rate in the pre-crisis period, whereas, “Basic Materials” and “Consumer Goods” sector returns were positively related to falling domestic currency in value over the whole period. On the other hand, rising exchange rate had a positively significant effect on “Consumer Services” and “Technology” index returns during the pre-crisis and whole period, namely, they were not affected by exchange rate movements during the post-crisis period. Lastly, the author (2014) presented negative relationships between stock returns and movements in gold prices during the post-crisis and whole periods, driving by the substitution effect from stocks to gold. “Financials” index, however, had significant negative effect from gold prices only during the post-crisis. Negatively significant results found for the returns of “Telecommunications” and “Technology” indices during the whole period, while the returns of “Industrials” and “Consumer Services” were significantly influenced by gold prices during the post-crisis and whole period. It was observed from the results that the only sector that affected by all factors at the same direction, i.e. positively, was “Basic Materials”.

By conducting multilinear regression model and causality tests, Mumcu (2005) studied the linkage between macroeconomic factors of T-Bond interest rate, money supply, industrial production index, inflation rate, exchange rate for Dollar, and gold prices and “XU100" index for Turkey over the sample period from January 1990 to December 2004. Empirical findings showed a negatively significant relationship between stock prices and T-Bond interest and industrial production index during the underlying period. Similarly, it was observed that there was evidence of significantly positive impact from exchange rate and money supply to "XU100" index. However, the relationship between stock prices and inflation rate and gold prices were statistically insignificant. According to causality test findings, the author (2005) found bidirectional causal relationship between stock price and T-Bond interest and stock price and gold prices, suggesting that both variables for each model could be used as a powerful predictor for each variable’s performance in the future. In a similar vein, a one-way causal linkage running from "XU100" exchange rate was observed. However, the null hypothesis of no-causality was not rejected for money supply, industrial production index and inflation rate, indicating that those variables and stock prices did not have significant impact on each other.

Using monthly observations of industrial production index, interest rate, inflation, exchange rate, gold, export, and oil prices, Bulut (2013) investigated the effects of macroeconomic variables on stock prices employing cointegration and causality tests for Turkey over the sample period from 1992-01 to 2012-06. According to cointegration tests, it was revealed that variables were cointegrated in the long run. More clearly, there was evidence of positively significant relationship between industrial production index, inflation, exchange rate, and oil with stock prices according to Model 1. Conversely, interest rate and oil prices were significantly negative related to stock prices in the long-term. A one-percentage increase, for instance, in interest rate and gold prices caused by -0.26% and by -0.39% decline in "XU100", respectively. After including export data into model, (Model 2), it was observed that significant relationship for interest rate and gold prices dissipated, while industrial production index, inflation, and oil prices were still significantly positive related to stock prices. Moreover, there existed a surprisingly negative relationship between export and stock prices, where "XU100" index would decrease by -0.48% if export had increased by one percentage. In fact, it was expected to be a positive relationship since rising export would positively affect the growth of economic activity, negatively affect exchange rate (an appreciation in local currency value compared to foreign currencies), and in turn, positively influence stock prices. On the other hand, causality test revealed significant results for the relationship between export and stock prices. By employing the T&Y Granger causality test approach, it was observed that there existed bidirectional causal linkage between exchange rate, and export with stock prices, namely, stock index was a powerful predictor for the estimation of export and exchange rate movement, vice versa. In addition, industrial production index and interest rate Granger-caused stock prices, namely, changes in those variables were found to stimulate stock prices while reverse, however, was not true. The null hypothesis of no-causality between stock prices and gold prices was not rejected, indicating that they were not related each other.

Using quarterly data of industrial production index, real GNP, real exchange rate, inflation rate, short-term interest rate, gold, exports, and changes in money supply, Zhang (2003) studied the effect of macroeconomic factors on stock prices conducting cointegration and causality tests for Canada over the sample period covering 1963 and 2001. The author (2003) reported long run relationships between macroeconomic variables and stock prices. There was positive association between exports and industrial production index with stock prices, indicating that increases in real economic activity had led to increases in future cash flows of firms and their stock prices. Conversely, real exchange and inflation rates were negatively related to stock prices. The author (2003) pointed out that the finding of positive relationship for inflation rate was contrary to the relevant economic theory. In addition to cointegration analysis, causality test results showed that the null hypothesis of no causal linkage from stock prices were not rejected for GNP, exports, money supply, inflation rate, interest rate, and exchange rate, however, it was only rejected for industrial production index. On the contrary, the author (2003) rejected the null hypothesis non-causal relationship running from stock prices in favor of GNP, exports, money supply, and exchange rate. These results indicated that the lagged stock index could reasonably be used to forecast current value of those macroeconomic variables. Unexpectedly, there was no Granger causal association between short-term interest rate and inflation rate with stock prices, suggesting that both variables did not have significant effects on stock prices and vice versa in Canada.

Demirel-Elitaş (2010) studied the causal linkage between stock prices and macroeconomic variables by employing Granger causality test and impulse-response function analysis for the sample period from 1998-Q01 to 2009-Q04. According to test results, GNP, money supply and exchange rate were cointegrated with stock prices in the long run, namely, GNP and money supply variables were positively while exchange rate was negatively related to stock prices. In terms of causality tests, there existed two-way causality results between GNP and exchange rate with stock price in the long term. Moreover, it was observed that stock prices Granger-caused two variables in the short term. Equity prices caused money supply in the long run and stock prices had significant predictor power on interest rate in the short-term. It was also observed that money supply Granger-caused stock prices in the short run.

Finally, the null hypothesis of non-causal association could not be rejected between stock prices and inflation rate, and from interest rate to stock prices at any conventional significance level in both short and long run.

Rastgeldi (2012) investigated the relationship between stock price index, "XU100", and macroeconomic variables of exchange rate, consumer confidence index, interest rate, and consumer price index by employing multilinear regression model, cointegration and causality tests over the sample period spanning from January 2004 to December 2009. According to multilinear regression model, approximately 60 percentage of variation in "XU100" could be explained by independent variables of exchange rate, CPI, consumer confidence index, and interest rate. It also was observed that stock prices were negatively related to exchange rate and interest rate, however, it was positively related to CPI and consumer confidence index. A one-unit increase, for instance, in exchange and interest rates would decrease stock index by nearly 42.265 and 854 points. However, "XU100" index would rise by 457 points if the CPI had increased by one-unit. Similarly, a one-unit increase in consumer confidence index would lead to an increase by 367 points in stock index. There was only two cointegrating vectors, indicating a long run relationship between stock index and macroeconomic variables. On the other hand, causality test revealed that exchange rate Granger-caused "XU100" index; however, the reverse was invalid. The null hypothesis of non-causality from caused "XU100" index to macroeconomic variables was rejected only for consumer price index. Furthermore, causality relationship was insignificant for consumer price index and stock prices, and interest rate and stock prices.

By employing cointegration and causality tests, Binici (2012) investigated the effects of macroeconomic variables on the low frequency (monthly) observations of industrial index, "XUSIN", listed on the Istanbul Stock Exchange over the sample period between 1997-01 and 2011-09. The empirical findings of the study found that exchange rates, interest rates, gold and oil prices, trade balance and money supply were cointegrated with the stock index. In terms of causality tests, there existed two-way causality associations gold prices and stock index in the short-term, but there was not any significant result for the long term. Exchange rate Granger-caused stock index in the short-term, the reverse was not valid. Similarly, stock index was a leading factor for trade balance in the long run, while the reverse was true only in the short run. The null hypothesis on non-causality was not rejected for stock index and interest rates in the short run, but, it was observed that stock index was a powerful indicator for interest rates in the long-term. The paper also revealed a long run causal linkage from money supply to stock index, although, there was not any causal relationship in the short-term. "XUSIN" index was a powerful predictor for oil prices in both the short and long term, while the reverse was only valid for short term.

Using monthly data of consumer price index, T-bill interest rate, exchange rate, money supply (M2), industrial production index, Bank of Tanzania reserves (BTR), income per capita index (GDP) and income tax index, Abdalla (2014) studied relationship between macroeconomic variables and stock market Dar Es Salaam Stock Exchange return over the sample period covering 2002 and 2013, consisting 144 observations. It was observed inflation were significantly positive while interest rate and money supply variables were negative related to stock index during the sample period. According to the author (2014), the reasons behind the positive relationship between inflation rate and stock index were (i) inexistence of money illusion effect, (ii) government's commitment to low inflation rate, (iii) hedging of stock returns against inflation, and (iv) money demand shocks. This result was also in line with the Fisher effect hypothesis where it was claimed that nominal equity returns and inflation were positively related. The negative relationship between interest rate and stock returns was explained by several reasons as (a) intrinsic impact of interest rates on the firm's profits, (ii) rising interest rates induce investors to buy debt market instruments causing decline in stock prices. In a similar vein, the author (2014) reasoned the negative relationship with money supply by the portfolio balance model where this theory assumed that an increase in money supply might cause a rise in stock price since investors would be motivated to buy more stocks. On the other hand, causality test results showed that stock market Granger-caused both exchange rate and money supply variables while the reverse situation was not significant. On the contrary, it was observed that the null hypothesis of non-causal linkage was not rejected for inflation and interest rates, namely, they were not a leading factor for stock returns estimation.

Conducting wavelet-based MGARCH approach, Khalfaoui et al. (2015) studied the volatility and mean spillovers of equity markets of the G-7 and oil prices over scaled time horizons. According to the findings, there existed a strong evidence of time-varying volatility in all stock and oil markets in the G-7 countries. Besides, market prices were directly impacted by their own volatilities and news, however, they were indirectly influenced by the volatilities of other market prices and wavelet time scales. Putting the same point in simpler terms, spillover effects of mean and volatilities were broken into many sub-spillovers on different wavelet scales in compliance with heterogeneous market participants. They (2015) also reported wavelet scale-based hedging ratios for the heterogeneous investors to take an optimal asset allocation decision, which allows investors to adapt their hedging strategies.

Özbay (2009) examined the causal linkage between monthly observations of macroeconomic variables including interest rate, exchange rates, inflation, money supply, industrial production index, foreign investor purchases and sales, current deficit to DGP, and central bank money and "XU030" index by utilizing causality tests for the sample period between 1998-01 and 2008-12. Test findings of the paper showed a bidirectional causality between share returns and foreign investors sales, while, on the other hand, stock returns Granger-caused foreign investor purchases. The null hypothesis non-causality from "RXU030" index hypothesis could be rejected for movements in interest rates, exchange rates, and central bank reserves, while, on the other hand, "RXU030" index was not a leading factor for inflation rate, growth rate of industrial production index, growth rate of money supply, and growth rate of current deficit to DGP variables. Conversely, the lagged values of inflation rate and growth rate of current deficit to DGP variables had predictive powers on current index returns, however, changes in interest rate, exchange rates, industrial production, money supply, foreign purchases, and central bank reserves did not Granger-cause "RXU030" index.

Using monthly observations of industrial production index (IPI), producer price index (PPI), consumer price index (CPI), money supply, (M1, M2, M3, and central bank reserves [CBR]), nominal interest rates (T-bill rate [T-Bill] and overnight rate [O/N]), transaction volume of "ISE100" index (VOL), current account deficit to GNP (CADGNP), foreign investors transactions (FIT) and foreign exchange basket (FEB), Öztürk (2008) investigated the linkage between those macroeconomic factors on stock market, "ISE100" index, return over the sample period of 1997-2006. The findings of the study revealed that the lagged value of "RISE100" index had significantly causal impacts on the DL_IPI at lag 6; on the DL_FEB at lag 1, 3, 6, 9, and 12; on the DL_T-Bill at lag 12; on the DL_O/N at lag 1, 3, 6, 9, and 12; on the DL_CBR at lag 1, 3, 6, 9, and 12; on the DL_CADGNP at lag 3; and on the DL_FIT at lag 1, 3, 6, 9, and 12. Conversely, DL_CPI, DL_PPI, DL_IPI, DL_M1, DL_M2, DL_FEB, DL_VOL, DL_T-Bill, DL_CBR, DL_M2Y, DL_CADGNP, and DL_FIT variables did not Granger-cause return rate of stock index at any lags. In addition, it was observed that the lagged values of DL_O/N led stock index return at lags of 3 and 9.

Şaşmaz (2011) researched the relationship between several macroeconomic variables of inflation, deposit interest rate and real exchange rate and stock returns for Turkey case using monthly observations over the sample period covering 2003-01 to 2010-05. The result of paper showed that there was a bilateral causal linkage between change in real exchange rate and stock return, while, on the other hand, the null hypothesis from stock return could be rejected for only inflation rate. In addition, direction of the causality between interest rate movements and stock return was only from interest rates to stock return. On the other hand, real exchange rate was significantly positive, while, interest rate was negative related to "ISE100".

To examine dynamic linkages between stock index, "ISE100", and various macroeconomic variables including interest rate, exchange rate, money supply, real GDP, and inflation, Doğan (2011) utilized causality tests, impulse response and variance decomposition approaches using quarterly observations covering 1987-Q01 to 2009-Q03 for Turkey case. According to the T&Y Granger test results, the null hypothesis could not be rejected for interest rate and stock prices. The lagged values of stock prices had significantly causal effects on the current value of exchange rates and money supply, M1. On the other hand, real GDP and inflation rates had predictive powers on current stock prices, while, exchange rates and money supply variables did not Granger-cause "ISE100". In addition to VAR model, the author (2011) also tested the relationship with VECM model due to cointegration vectors. The test result showed that there was only one long run causal linkage between inflation rate and stock prices, while, on the other side, there existed one-way causality running from stock price to real GDP among variables in the short run.

Mermer (2014) investigated the relationship between the CBRT-TSI consumer confidence index (CCI) and stock indices of "XU100", "XUHIZ", "XUMAL", "XUSIN", and "XUTEK", using monthly observations covering 2004-01 to 2012-12. The author (2014) found significantly positive relationship between the CCI and all stock indices. According to Granger causality test results, it was observed insignificant causal linkages from the CCI to all indices, while, on the other hand, "XU100", "XUMAL", "XUSIN", and "XUTEK" indices had predictive powers on the CCI.

Uyğur (2013) investigated the major determinants of stock returns within the framework of the APT using monthly observations of "XU100", Brent oil prices, exchange rate, and gold prices covering 2005-01 to 2012-12 periods. According to test results, there was not any significant causal linkage from variables to stock returns, while, on the other hand, "XU100" index led only gold prices. Namely, "XU100" index did not Granger-cause exchange rates and oil prices in the short run.

Jawaid and Ul Haq (2012) examined the impacts of short-term interest and exchange rates on monthly returns of banking industry in Pakistan over a sample period covering January 2004 and December 2010. Test results show significantly negative long run and significantly positive relationships between interest rate and exchange rate with equity prices. Moreover, test findings suggested a feedback causal relationship between stock prices and exchange rate and a one-way causality running from interest rates to banking stock prices. In overall, they (2012) concluded that both variables were reasonable indicators for investment decisions in the Pakistani banking index.

Using monthly data of trading volume, exchange rate, industrial production index, and real interest rate, Temiz (2012) study the relationship between those factors and stock prices over the sample period between 2005-01 and 2011-12 for Turkey case. It is observed from the results that there were bidirectional causal relationship between stock prices with exchange rate and industrial production index. In addition, there is only one-way causality from trading volume to stock prices while the reverse was not true.

In a related paper, Barakat et al. (2015) investigate the possible linkage between a set of macroeconomic variables –including exchange rate, interest rate, consumer price index, and money supply– and stock markets in Egypt and Tunisia over the sample period between 1998-01 and 2014-01. According to test results, there are cointegrating vectors and causal linkages between underlying variables in both countries. Exchange rate, interest rate, and money supply are significantly positive while interest rate is significantly negative long run relationship with stock index in Egypt. The authors (2015) find significantly positive with exchange rate and negative relationship in Tunisia. Although there is not strong relationship between stock returns and interest rate movements, the causality test results show a bidirectional causality association between them in Tunisia. In addition, the null hypothesis of no causality from interest rates to stock market cannot be rejected, implying that stock market does not have a predictive power on interest rate movements in Egypt.

To investigate possible linkages between several macroeconomic variables – consisting of industrial production, money supply, exchange rate, inflation and interest rate– and stock market index, Liu and Shrestha (2008) use heteroscedastic cointegration approach and find a long run relationship for the sample period between January 1992 and December 2001 consisting 120 monthly observations. More clearly, interest rate, inflation and exchange value are found to be significantly negative, while, on the other hand, money supply and industrial production are significantly positive linked to stock market, indicating that the Chinese stock market could offer better long-term returns and diversification opportunities.

Maysami and Koh (2000) investigate possible long run relationship between several macroeconomic variables and stock market in Singapore. The dataset used are monthly seasonally adjusted for the period between 1988-01 and 1995-01 consisting 85 observations. Test result of the paper shows that stock market is significantly positive related to short-term interest rates and significantly negative linked to long-term interest rates. This finding is interpreted as the long-term interest rates are a better proxy than short-term rates for the discount rate in equity valuation models. In addition, a negative but insignificant relationship between consumer price index and stock market, while a positive insignificant linkage is reported for money supply and stock market.

By utilizing cointegration and causality tests, Köroğlu (2009) examine the possible linkage between various macroeconomic variables of exchange rate, money supply, Brent oil prices, inflation rate, gold prices and stock price index over the sample period between 1998-01 and 2009-11. According to cointegration test results, the money supply and inflation rate variables was significantly negative, while, on the other hand, oil and exchange rate was significantly positive related to stock prices. In addition, all variables had predictive power on estimation of stock prices, while, on the other hand, the null hypothesis of non-causality relationship was rejected only for exchange rates, namely, there was a bidirectional causal linkage between stock price and exchange rate.

Andrieș et al. (2014) found significant relations among monthly observations stock prices, interest and exchange rates (REER), implementing the wavelet coherence approach, in India over the sample period 1997-07 to 2010-12. For instance, it was shown that return of stock market was lagging movements of exchange and interest rates. Stock market movements were caused by interest yields, namely, stock prices followed the interest rate movements.

By conducting wavelet coherence approach, Bayraci et al. (2018) used daily observations of the G-7 countries to investigate the dynamic relationship between stock and bond market movements and to identify the flight-to-quality effects over the sample period 2002-01-02 to 2014-09-03. The findings of paper ascertained empirical evidence of positive linkages varying in time and wavelet scales. In addition, a very little co-dependency between market prices was found at high frequency bands (short-term) while the linkage became stronger at the lower frequency bands (long-term) corresponding to an investment horizon between 128 and 512 days. Regarding to the global power spectrum (GPS) results, most of the significance with low power was concentrated in the higher frequency bands corresponding to a holding period that below 32 days, indicating of falling the power of coherence as holding period rose. In general, the findings showed a positive relationship between equity returns and long-term government bond yields in G7 countries, suggesting a hedging opportunity of bonds to stock prices. The strongly significant linkage revealed at the lower frequency bands corresponding to an investment horizon between 64 and 512 days while the linkage was weak but significant at the higher frequency bands. The authors (2018) interpreted these results as an evidence of heterogeneous behaviors of investors with different investment horizons. Investors with long-term investment horizons, for example, institutional investors are said to by and large follow macroeconomic fundamentals, on the other hand, investors, for instance, noise traders with short-term investment horizons are observed to pursue trends and respond to every events, good or bad it does not matter to them. Accordingly, it was natural to see a time-varying stock-bond linkage across holding periods. On the other side, they (2018) also investigated the dynamic relationship by rolling wavelet correlations and they found that correlation (a) linkage was highly volatile and (b) significantly rose across time scales during market downturns (bearish markets). These findings supported the presence of flight-to-quality effects where investors shifted from stocks to safer bond instruments due to change in their sentiments and risk preferences.

Şentürk et al. (2014) studied the causal linkage between market return and growth rate of economic activity for Turkey using the quarterly closing price of aggregate stock index ("XU100") and growth rate of GDP (RGDP) over the sample period 1998-Q02 to 2014-Q02. The result of paper revealed that there was no relationship between two variables when using the T&Y Granger causality approach. However, the frequency causality of Breitung and Candelon (2006) showed that the null hypothesis of no causality from "XU100" to "RGDP" was rejected at high frequency point, $\omega = 2.5$, and "RGDP" variable had significant causal effect on "XU100" variable at medium and high frequency points, $\omega = 2.5$ and $\omega = 1.0$.

2.5.2 Relationship with Microeconomic Factors

Allen and Rachim (1996) researched the relationship between dividend policy and share price of 173 Australian firms over the sample period covering 1972-1985. By conducting cross-sectional multiple regression, they (1996) reported a non-significant result for dividend yield and volatility in stock price, which was against the findings of Baskin's (1989) for the U.S. case. However, the paper revealed evidence of significantly, consistent with expectations, positive relationship between stock price volatility with earning volatility, leverage, and firm size. Empirical findings of the study also documented significantly negative relationship between stock price volatility and dividend payout ratio.

By conducting multiple regression analysis, Hussainey et al. (2011) investigated the linkage between stock price and dividend policy consisting, i.e., dividend payout and dividend yield, in the London Stock Exchange over the sample period from 1998 to 2007. The result of paper show that stock price volatility was significantly negative related to dividend yield and dividend payout, indicating that the less (higher) payout ratio, the higher (less) stock price volatility. It was also contended that the major determinant for stock price volatility was the payout ratio. In addition, firm size and debt were the main variables that had the highest correlation with volatility of stock prices among control variables. Equivalently speaking, debt was significantly positive, implying that the more (less) debt leverage, the more (less) volatility, while, on the other side, firm size was significantly negative related to volatility of stock prices, indicating that the larger a firm was, the less volatile stock price would be.

Profilet and Bacon (2013) investigated the effect of various financial variables of 599 firms including dividend yield, dividend payout ratio, growth, firm size, and leverage on stock price volatility by employing OLS regression on panel data analysis in the U.S. It was observed that dividend yield, firm size, leverage, and growth variables were significantly negative related to stock price volatility. Putting differently, the higher dividend yield, the less volatile stock price would be, suggesting that dividend cash flow could be used as a signaling device to investors.

Shah and Noreen (2016) studied the relationship between dividend policy and stock price volatility for Pakistan case including 50 firms over the sample period between 2005 and 2012. By employing random effect model and panel estimated generalized OLS method, they (2016) revealed that stock price volatility was significantly negative connected to dividend payout and dividend yield, while, on the other side, it was significantly positive adherent to control variables, i.e. asset growth, earning volatility, and earnings per share.

For Malaysian case, Zakaria et al. (2012) researched the effect of dividend policy on stock prices for firms in the construction and material sectors over the sample period between 2005 and 2010. The authors (2012) a significantly positive relationship between stock price volatility and dividend payout ratio, while, on the other hand, dividend yield, growth rate of investment, and earning volatility insignificantly, and negatively, affect changes in share prices. Firm size and leverage among control variables were the two financial variables that had high correlation (negative) relationship between stock return of underlying firms.

Buigut et al. (2013) studied the linkage between stock prices and capital structure of energy sector listed on the Nairobi Stock Exchange by utilizing panel data and multiple regression approaches over the sample period from 2006 to 2011. Test findings showed that financial variables of debt, equity and gearing (leverage) ratio were the major determinants of stock prices. It was also reported that debt and gearing ratio were significantly positive, while, on the other hand, equity negative related to share prices of energy sector.

Using panel regression, Tahmoorespour et al. (2015) researched the linkage between share returns and capital structures of 1082 firms in the Asian Pacific region consisting of Singapore, Australia, Hong Kong, China, Japan, Malaysia, Taiwan, and South Korea countries over a period between 1990 and 2012. The authors (2015) remarked that the impact of capital structure depends on both the nature of sector and market. It was reported that “debt to asset” ratio adversely influenced stock returns of Basic Materials in Japan; Consumer Services in Australia and Japan; Industrial Goods in Korea, China, Japan, and, Singapore; Consumer Goods in Hong Kong; Oil and Gas in Australia; while, on the other hand, Technology and Utilities were positively related in China. Similarly, ratio of “Debt to Capital” had positive relationship with stock returns of Industrial Goods in Korea and Consumer Goods in China; while, it was negatively related to stock returns of Consumer Goods in Hong Kong and in Japan; Oil and Gas in Australia, and Healthcare in China. They (2015) also found that “Long term debt to common equity” (LDCE) ratio positively influenced Basic Materials in Japan and Korea, Oil and Gas sector in Australia. It was also observed a negative relationship between LDCE ratio and stock returns of

Basic Materials in Japan; Industrial Goods in Japan and Singapore; Consumer Goods in China; and Healthcare in Japan.

Menon (2016) examined the impact of changes in capital structure on stock price of 113 firms listed on the Muscat Securities Exchange by employing correlation and multiple regression analysis. The result of paper showed a negative linkage between stock price and “amount of debt”, while, on the other hand, “debt to equity” and “amount of equity” ratios were significantly positive related to share prices at 1% significance level, suggesting evidence in favor of Net Income Approach.

In a related paper, Karcioğlu and Özer (2014) investigated the effect of macro and microeconomic factors on 113 firm ("XUSIN") stock returns over a time period 2002-Q01 to 2011-Q03. Employing dynamic and static panel data analysis, the authors (2014) revealed that firm’s “acid test ratio”, “current ratio”, “P/E ratio”, “size”, “beta”, “B/M ratio”, “D/E ratio”, “EVA”, “gross profit margin”, “value added intellectual capital”, “cash flow”, and macroeconomic factors of “internationalization”, “interest rate”, “exchange rate”, and “money supply” had significant impact on stock returns. For example, it was observed that “current ratio”, “B/M”, “size”, and “capital structure” ratios had an adversely significant relationship with stock returns, while “acid test ratio”, “P/E ratio”, “beta”, “cash flow”, “value added intellectual capital”, “internationalization (Export Sales/Net Sales)”, and “EVA” ratios were positively related to stock returns. On the contrary, the paper noted a insignificant effect of “ROA”, “free float”, “share of the largest shareholder”, “dividend per share”, “Tobin’s Q”, and “manager ownership” on stock returns during the studied period. Similarly, there existed significantly negative effects from “exchange rate”, “interest rate”, and “oil price” and significantly positive effect from “money supply” to stock returns. Moreover, it was found that “gold price”, “balance of trade”, “industrial production index” and “foreign portfolio investments” variables did not have any impact on returns of manufacturing companies.

By conducting one-way fixed effect panel data approach, Kanat (2011) investigated the effects of both micro- and macro-economic variables of money supply, inflation rate, exchange rate, gold and oil prices, current account balance, industrial production index, foreign portfolio investments and GDP on stock index "XU100" over the sample period covering 2002-Q01 to 2008-Q04. The firm-specific variables were financial structure, capital increases, dividend policy, firm size, stock beta and financial ratios. Test findings showed that approximately 60 percentage of variation in "XU100" could be explained by twelve independent variables. It was also observed that stock returns were significantly positive related to exchange rate, foreign portfolio investments, ROA, liquidity ratio, capacity utilization rate, and P/E ratio at varying significant levels. Conversely, inflation rates, Treasury interest rates, and beta (unexpected result) were significantly negative related to stock returns. Surprisingly, capital increases, dividend policy and MV/BV ratio did not have significant effect on stock returns.

In the recent study, Gautam (2017) investigated the effects of firm-specific factors on volatility of stock price and stock return in Nepal over the sample period between 2008-09 and 2015-12. By employing multiple regression models, the author (2017) found that share prices were significantly positive related to “dividend payout ratio”, “dividend yield”, “market capitalization”, and “leverage”, while, on the other hand, “earning price ratio”, “book to market ratio”, and “growth of assets” were adversely affected by stock returns. These results indicated that the higher “dividend payout ratio”, “dividend yield”, “market capitalization”, and “leverage”, the higher share prices. In a similar vein, the paper findings showed that “dividend payout”, “dividend yield”, and “leverage” ratios had significantly positive impact on volatility of stock prices, suggesting that the higher those ratios, the higher stock price volatility. On the contrary, the study illustrated that the higher “growth of assets”, “earning price ratio”, “book to market”, and “market capitalization”, the less volatility on stock prices.

Aveh et al. (2017) studied to identify the effects of firm-specific factors on share prices in the Ghana Stock Exchange utilizing paned regression analysis over the sample period between 2008 and 2014. The paper showed that approximately 55 percentage of variation in stock prices could be explained by independent variables of earnings per share (EPS), dividend per share (DS), dividend yield (DY), Book value of a share (BVS), Return on equity (ROE), Leverage (LEV), and Market capitalization (SIZE). It was observed that EPS, BVS, ROE, and SIZE were significantly positive related to share prices, while, on the other hand, DY had negatively impact on, indicating that investors were not induced by the dividend policy of the firm. The study also found that LEV and DS were negatively but insignificantly related to stock prices.

Nazir et al. (2010) examined the effect of dividend policy on stock price volatility of 73 firms listed on the Karachi Stock Exchange in Pakistan over the sample period between 2003 and 2008 by employing random effect and fixed effect models on panel data. They (2010) presented significant relationship between dividend policy variables and stock price volatility, indicating evidence in favor of validating the duration effect, arbitrage realization effects and information effect in KSE100 stock index. Besides, leverage and size variables were found to be insignificantly negative related to stock volatility.

By conducting systematic elimination method, Bahreini et al. (2013) investigated the linkage between the financial leverage and stock price of 145 firms listed on the Tehran Stock Exchange over the sample period between 2005 and 2006. The paper presented a significant association between stock price and economic leverage, namely, the higher amount of debt, the higher degree of relationship (inversely) between stock price and economic leverage. There was significantly negative linkage between the ratio of expenses to property and the change in economic leverage. In addition, change in economic leverage led to a decrease in the property efficiency.

Rjoub et al. (2017) studied the effects of macro and micro variables on the seven banks' stock prices listed on the Istanbul Stock Exchange in Turkey using quarterly data over the sample period covering 1995-Q03 and 2015-Q03. By conducting a fixed panel regression and granger causality tests, it was observed that interest rates, management quality and asset quality were significantly negative, while, on the other hand, size, earning, and money supply were significantly positive linked to stock prices. Although capital adequacy, liquidity, and exchange rate were negatively, on the other hand, inflation and industrial production were positively but insignificantly related to prices of bank stocks. The findings of the causality test show that, in addition, there existed a feedback causal linkage between money supply, bank asset quality, and size with stock prices. The results also provided a homogeneously one-way stock causal linkage from share returns to earnings and interest yields in the short run. These results, according to the authors (2017), manifested a statistically significant and positive relationship between earnings and stock returns, and the financial instability in the short term. Besides, a unidirectional causality from management quality from stock prices was reported. In overall, macro and micro variables were found to be statistically significant when explaining stock returns.

CHAPTER

3 THEORETICAL FRAMEWORK FOR THE METHODOLOGY EMPLOYED: WAVELET ANALYSIS

The aim of this chapter is to give a detailed theoretical framework for the methods implemented for the study. Before introducing wavelet analysis, this chapter starts with focusing on features of signals and their properties which is the starting point of the frequency methods. After presenting signals, the next topic will be the transform methods based on Fourier series such as the Fourier transform, the short-time Fourier transform, and the fast Fourier transform methods, which is a vital and first topic to understand wavelets accurately. Lastly, the rest of the chapter is dedicated to discuss wavelet transform methods which enable us to decompose signals or time series into different time periods to investigate multiscale relationships.

3.1 Signals

Before describing a signal, “data” term definition should be done first. Forouzan (2006) says that it is divided into two components. The first component is analog data, which stores continuous information, another component is digital data, and it has information in the discrete form. The author (2006) gives a simple example of both analog and digital data. His example is a simple one: an analog clock. It shows information with second, minute and hour hands in a continuous form. For a one minute change, the whole movement of the second hand is observable. On the other hand, a digital clock that shows only the minutes and the hours, a change appears suddenly from 11.06 to 11.07. Besides, a speech made by a person is an analog data but it can be converted to signal by a microphone. The author (2006) declares that this signal is in continuous form but with sampling, it later can be transformed into a digital signal which has discrete value.

Now we learned data types, we can continue to describe signals. According to Yang (2009), a signal represented by mathematical function normally is defined as information conveyer. Ingle and Proakis (2016) demonstrate that, such as data, signals are usually classified into analog and discrete forms. An analog signal is symbolized by $x(t)$ and the variable "t" represent any physical quantity, but it is generally assumed to be in seconds. A discrete signal, on the other side, is denoted by $x[n]$ and in this function, “n" has integer value. For this reason, they call this function which is arranged by a number sequence as a discrete-time signal. For a better explanation and the simplest way is to show them by plotting as illustrated in Figure 3-1.

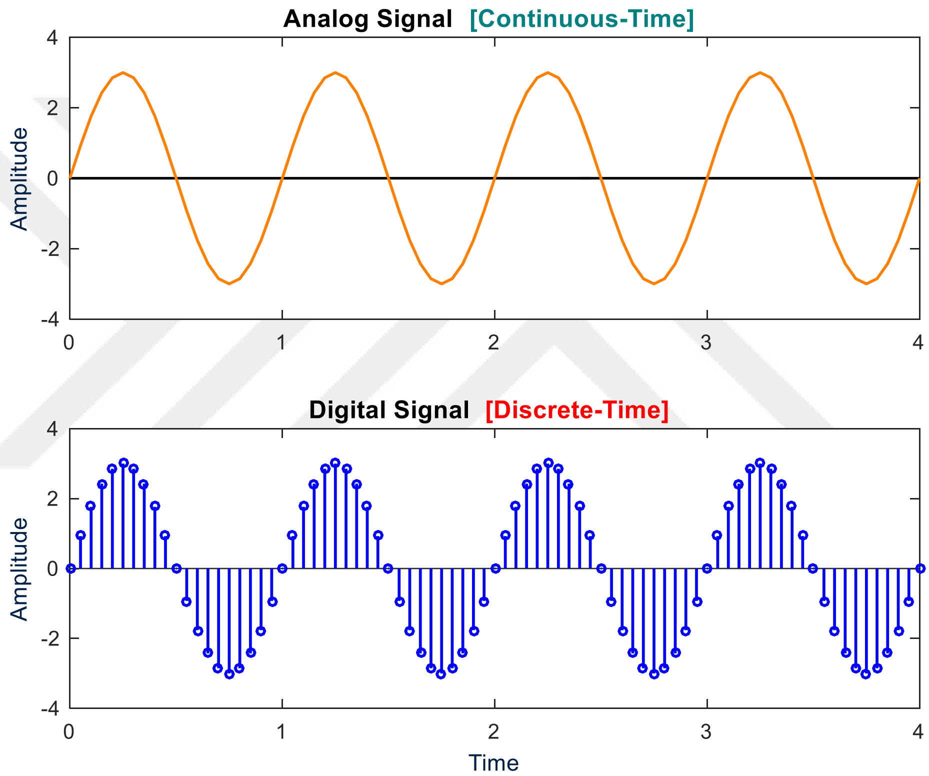


Figure 3-1 Analog [Continuous] and Digital [Discrete] Signals

Source: Calculated by the author.

In the figure above, the vertical "y" axis gives the amplitude which represents the size or strength of a signal while “x" axis shows time. One can clearly see that, in the first graph of Figure 3-1, an analog signal is depicted by continuous value. Looking at the second graph of this figure, a digital signal is shown in the discrete/finite values. It is obvious that both signals have the same pattern but different values in that digital signal is a sampled of an analog signal. The signals depicted in this figure are periodic signals which will be explained below.

Both signal types can be periodic or nonperiodic signal (Lessard, 2006). He states that a periodic signal repeats its pattern in a specific period, say T, and can perpetually continue. Here, T, in the case of the smallest positive value, is equal to a full oscillation to complete, namely, it is called a cycle. A nonperiodic signal or an aperiodic signal, conversely, is described as a signal that does not have a period and does not repeat the sequences of values exactly after a fixed length of time, T. A nonperiodic signal's pattern is not a cycle because it is lacking in finishing a period where it is equivalent to 2π or 360^0. It is easy to see in Figure 3-2.

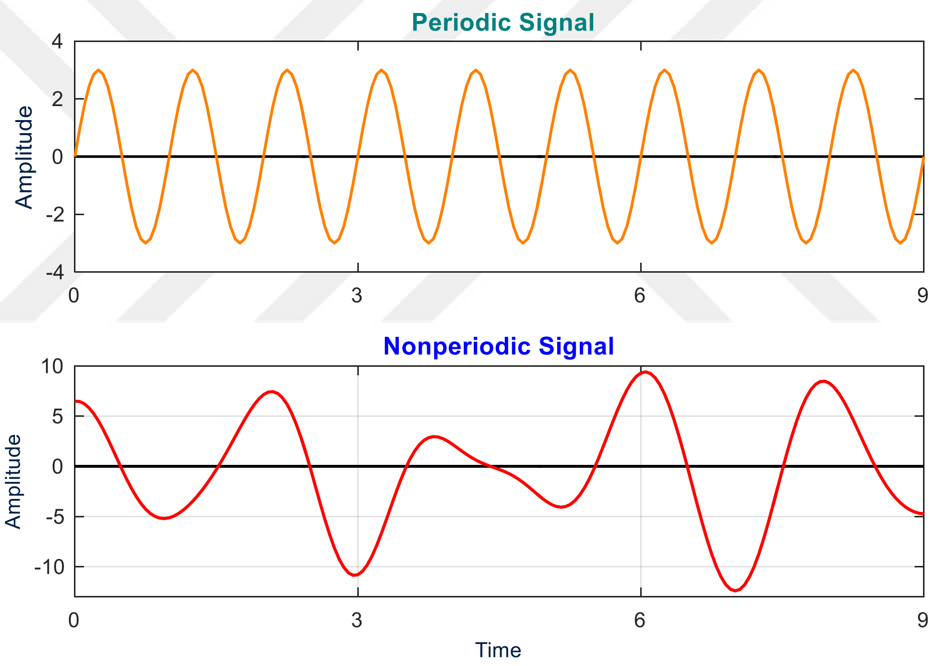


Figure 3-2 Periodic and Nonperiodic Signals

Source: Calculated by the author.

It should be emphasized that, so far, the signals depicted are sine waves and they are represented in the following format according to Weeks (2010):

$$x(t) = A * sin(2\pi * f * t + \mathfrak{p}) \quad (37)$$

Equation (37) represents a sinusoid wave. “A", "f", "t" and "$\mathfrak{p}$" parameters stand for amplitude, frequency, time and phase angle. Weeks (2010) says that because they are frequently studied with regards to these parameters, if one knows these all parameter, then it is very easy to find the value of $x(t)$function. For a better understanding, these five parameters will be explained in the following with two $x(t)$ functions depicted in Figure **3-3**.

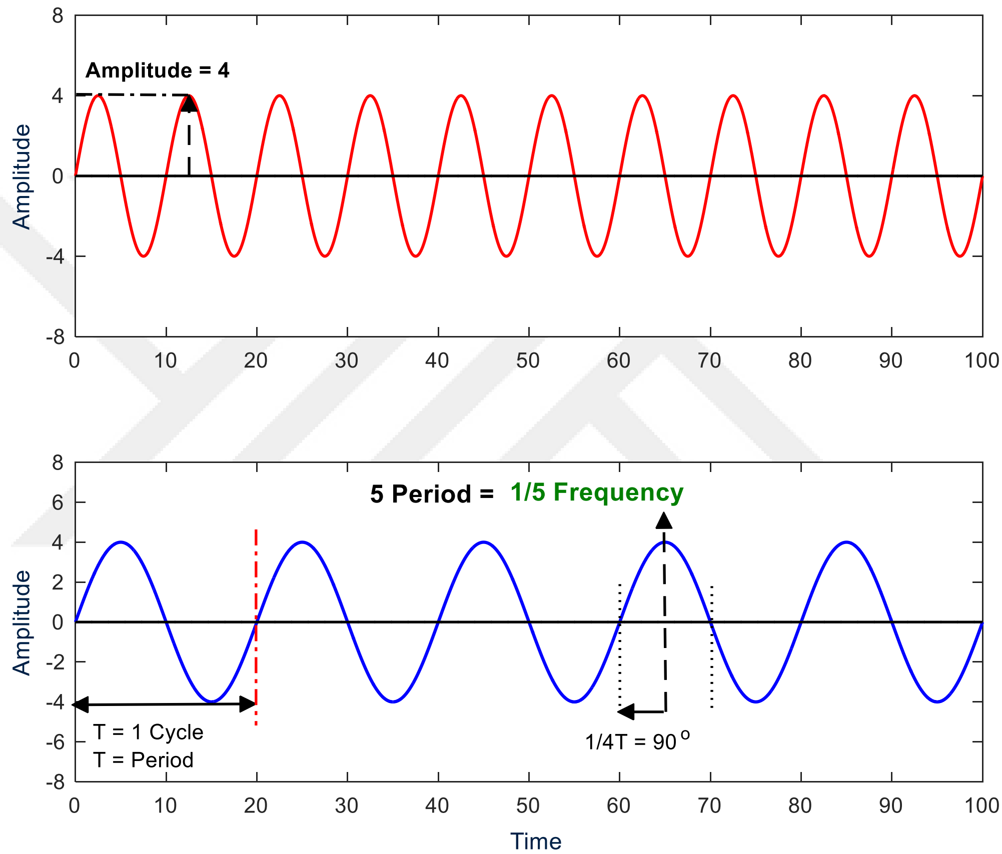


Figure 3-3 Frequency, period and different phase shifts of a sine wave

Source: Calculated by the author.

In the figure above, there are two sine functions with the same size but different pattern. First of all, they have the same maximum time, S, 100. Their sampling time is 0.001 seconds meaning that they increase by this. In equation (37), "t" has an interval of $0{:}\,0.001{:}\,S$. Also, they have the same amplitude, 4. This parameter shows the biggest (absolute) size of the signal. They are both periodic function but with different frequencies meaning that they complete one period with different time. For example, in the first graph, the signal completes one oscillation with 10 seconds while it is 20 seconds for the signal in the below graph. Hence, the first signal has 10 cycles and the second has 5 cycles to complete at the same time. The mathematical representations of the two signals are:

$$4 * sin\left(2\pi * \frac{1}{10} * t + 0\right) \qquad \& \qquad 4 * sin\left(2\pi * \frac{1}{20} * t + 0\right)$$

Forouzan (2006) describes a phase as the position of the signal corresponding to time "zero". It is clear that both signals' phases are zero meaning that they are not shifted to both sides. For a better understanding, a sine signal and its four phase shifts are depicted in Figure 3-4.

According to Elahi and Arjeski (2014), two signals with the same amplitude or frequency can be different due to phase point. They clarify this statement as one of the signals in a figure may possibly start at a different point in time axis from the other ones. In other words, a phase is used to measure timing differences when these signals have the same frequency. Moreover, they conclude that these moving from a different point in time are measured by radians π, or in degrees from 0^0 to 360^0 where 360^0 is equal to 2π.

In this figure, the vertical "y" axis presents the amplitude while "x" axis gives time. There are one sine signal and four different phase shift examples. In the top-left of the figure, the signal in green line is not shifted so it's phase is 0 but the other one in orange line moves to the right by 90^0 or $\pi/2$. In "B" part, the signal in blue line shifts to the right side by π radian. This move is, by the way, equal to a half oscillation. In the bottom part, the signal in purple' phase shift is 270^0 and the signal in red is 2π radians which is equivalent to a full oscillation or a cycle.

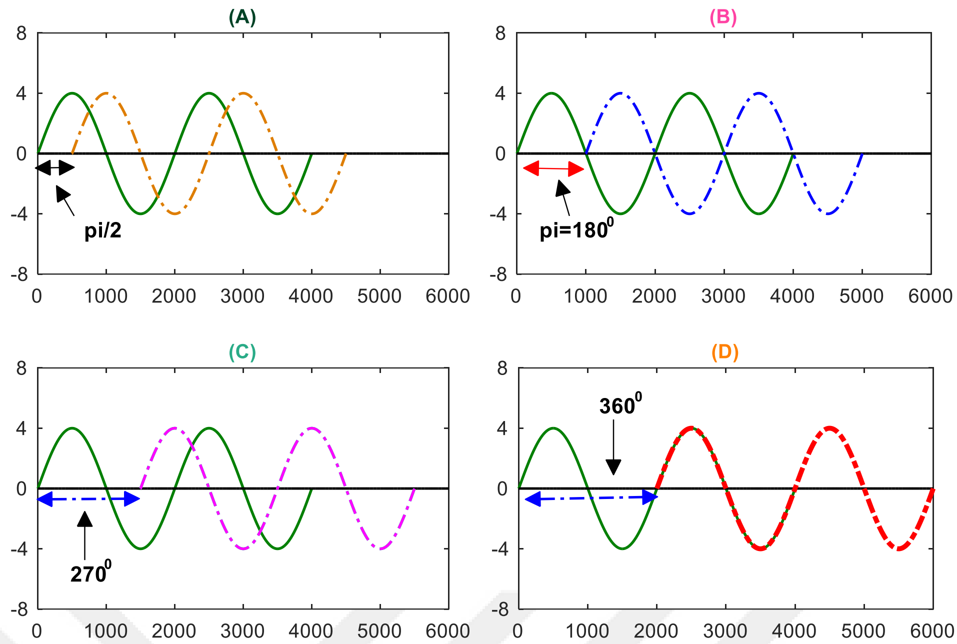


Figure 3-4 Four different phase shifts

Source: Calculated by the author.

When analyzing signals, period and frequency are defined differently due to a different meaning. Roughly speaking, a period is measured with time quantity while frequency is defined with the rate of change quantity. Lessard (2006) defines a period "T", as a duration of one complete cycle and frequency is illustrated as the reciprocal of "T", in other words as $T = 1/f$. Put it differently, a period is described as duration of a signal to complete a cycle in seconds and frequency having opposite meaning with a period refers to the number of periods in 1 s by Forouzan (2006). However, he also gave another different definition as a measurement for the frequency where it is defined as the rate of change in the time-domain. For example, the signals in Figure 3-3, they have 10 periods and 5 periods to complete in 100 seconds, therefore their frequencies will be $100 * 1/10 = 10$ Hz and $100 * 5/10 = 20$ Hz, respectively. Here, the frequencies are defined as the rate of change. Well, which one of these signals is in the high frequency? Or in the other words, which signal does occur in the short time? The answer is the second signal with 5 periods but 20 Hz, which stand for Hertz.

Up to now, the signals analyzed and plotted are a single one, especially sine waves. They were usually periodic and analog ones. The sine wave is the simplest signal that can be analyzed and plotted in the time domain. But in daily life, there are composite signals that comprise more than one signal, sine or cosine waves. Weeks (2010) says that these composite signals can be broken down into their simple parts.

$$x(t) = A * sin(2\pi * f_1 * t + \mathfrak{p}_1) + \; B * cos(2\pi * f_2 * t + \mathfrak{p}_2) \tag{38}$$

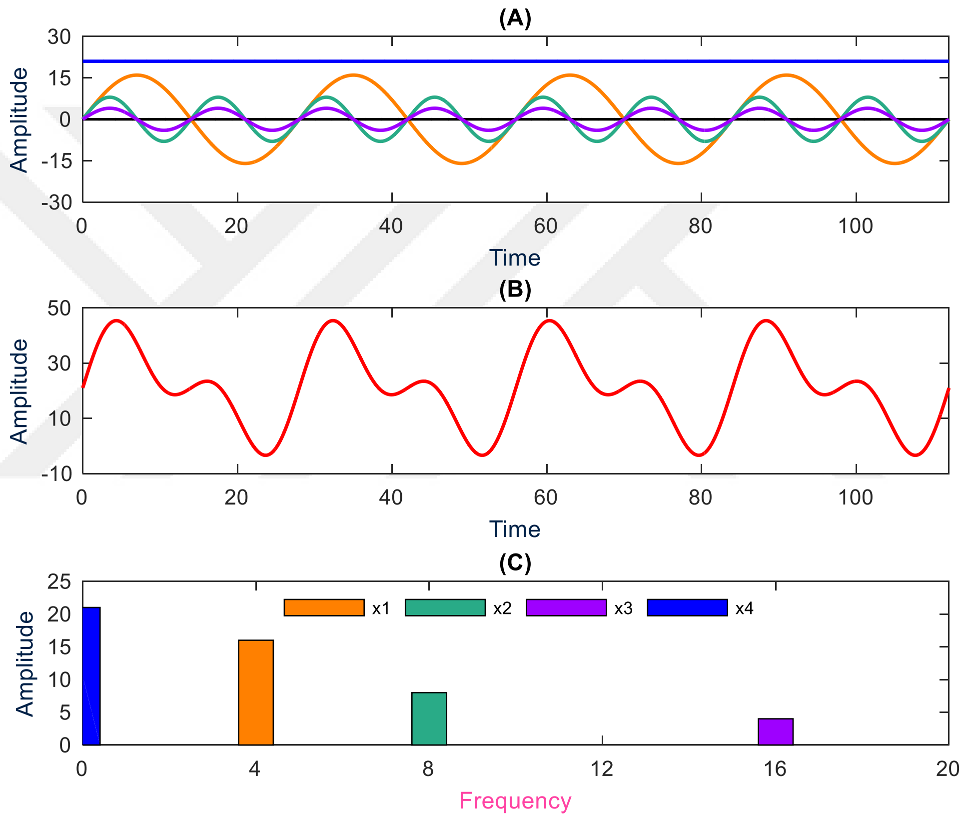


Figure 3-5 Time-domain vs. frequency domain of sine waves

Source: Calculated by the author.

Like a sine wave, a composite signal also can be periodic or aperiodic (Forouzan, 2006). He states that a periodic composite signal consists of many of simple sine waves with discrete frequencies while an aperiodic composite signal includes many sine waves. Furthermore, the former's frequencies have integer values such as 1, 2 and so on; and the latter's have real values. For creating composite signals, it is all need to know the amplitudes, frequencies, and phases according to Weeks (2010). When one has all the information, then he/she can calculate the value of this signals at time (t).

Figure **3-5** includes both the time-domain and the frequency-domain plots. First of the two graphs are in the time domain that is the horizontal axis is presented with time and the vertical axis is presented with the size of the signal, amplitude. With this type plotting, it can be seen only the signal in the time domain with amplitude namely it displays only changes in amplitude with respect to time.

To overcome this problem, one should use frequency-domain plotting instead of time-domain. In the top of the figure, there are four waves with different peak, amplitude, and frequency while in the middle graph a composite signal which is the summation of these is illustrated. Observing these two first graphs is not easy to figure out the amplitudes and frequencies because these parameters are not shown. In the bottom graph of the figure, a frequency-domain plot is displayed. It can be seen that this plot is very useful to convey the information of the frequencies of the signals. The biggest amplitude is 21 for x_4 signal with 0 frequency meaning its position is zero 0 in the frequency-domain plot; namely it is unchanged along the time axis. The other signals x_1, x_2 and x_3 have $4, 8$ and 16 frequencies with $16, 8,$ and 4 amplitudes, respectively. Here, the composite signal's frequency is not plotted due to implementation difficulties which are solved with a different method named Fast Fourier Transform.

3.2 Frequency-Domain Methods

In the previous section, the timing of frequency domain has been told. But, the main reasons behind that are not answered yet: Why would one use frequency-domain plotting? or, how will one accomplish a transform needed for a that? These questions are the natural and the simple ones. Weeks (2010) gave answers for the need for transforms. For analyzing a signal, such as human voice or time series, transform process is a generally necessary step. The author (2010) gave an excellent example of transform process which one is needed to solve a math problem with Roman numerals. For example, you are told to subtract $MCMXXIII$ from $MMXVII$. It is not easy to solve this question because it is not in the Latin alphabet. Weeks says that one should first transform these numbers from Roman to Latin form. After converting, it is found that the first figure $MCMXXIII$ is equal to 1923 which is the date of the declaration of the Turkish Republic and the second one is 2017. After converting the Roman figures to decimal numbers, and subtracting them, 94 will be the answer which is the inverse-transformed to XCIV. To find an answer in Roman alphabet with a great effort, Weeks (2010) concludes that, it seems not easy but a necessary step.

What is the importance of frequency-domain plot? Is it not enough just using time-domain? The answer to the second question is no. Weeks (2010) answers the first question and says that these two types domains give the same information to different views. The relationship between frequency and time is explained with an example by Weeks (2010). How long is it taking to fill up a tank of a car? Or, What is it's density in a year? The answer will be, naturally, in terms of weeks or months. For example, let's say it is four weeks or a month. The answer to the first question is a time-based one. On the other hand, the second question will be answered in frequency-based. Due to a reverse relationship, the frequency will be four weeks in 52 weeks, $1/13$, or one month in a year $1/12$.

According to finance view, another example should be the turnover rates. Inventory turnover, for example, presents a rate which shows how rapidly a firm sells its goods in a given period, say a year. If AEG Co's ratio is found as 2, this means that the firm sells its whole inventory in 6 months according to time-domain, namely its frequency is twice per year.

Without diving into too much literature, we will give some examples to clarify why frequency-domain analysis is used in this thesis. The main advantage of the frequency analysis, which is mentioned before, is that it is used to detect the key point of changing in the signal such as seasonalities, trends, business cycles, and unexpected changes. Hence, it is obliged to give the definitions of these effects briefly in the following.

Hyndman (2011) defines seasonal patterns as a time series which it is affected by some parameters like summer, winter or commercial seasons, the month (the Ramadan), and day of the week (Monday or Friday) or holidays. He also says seasonality is the characteristic of a time series where anticipated changes take place in certain business areas for every calendar time, say in a year. Due to predictability, it is generally named as periodic time series. However, sometimes, some changes irregularly happen in a given period, longer than $3/2$ year which is called a cyclic by OECD (2007). Unlike seasonal patterns, these type patterns occur irregularly and its volatility variations have not of the fixed period. Hyndman (2011) gives business cycles as an example of a cyclic pattern which lasts for several and its duration is not known in advance. The author (2011) also declares that these two terms are often interchanged but they have really different meanings. If the variations last less than a calendar year, then they are called as seasonal patterns; conversely, if they have not a specific, fixed time period and last more than a calendar year, they are described as cyclic patterns. He reports that the main difference is that cyclic pattern duration is generally longer and fluctuations are more volatile than seasonal patterns.

Now, it is time to find seasonalities and cyclic patterns with frequency-domain analysis with some examples. Aforementioned, the frequency is the rate of change in a given time period. Selçuk (2005) gives an excellent example of a better understanding. The author (2005) says that a monthly macroeconomic time series, such as monthly GDP growth rate may include a seasonality component which its cycle is completed in 12 months or 120 months. The author (2005) asks that if one of those cycles, say 120 months, is found to be more important than the other, then it is said that if an economy is now in “trough” phase, this economic phases all are completed in 10 years. Gencay et al. (2002) clarify this and say that this cycle completes itself in 120 months, namely each month has $f = 1/120$ oscillation because of the inverse relationship between time and frequency. This definition undeniably helps a lot to understand and interpret. Selçuk (2005) concludes that investors or individuals can forecast what would be in the following 10 years in order that to make their investment and consumption decisions accurately.

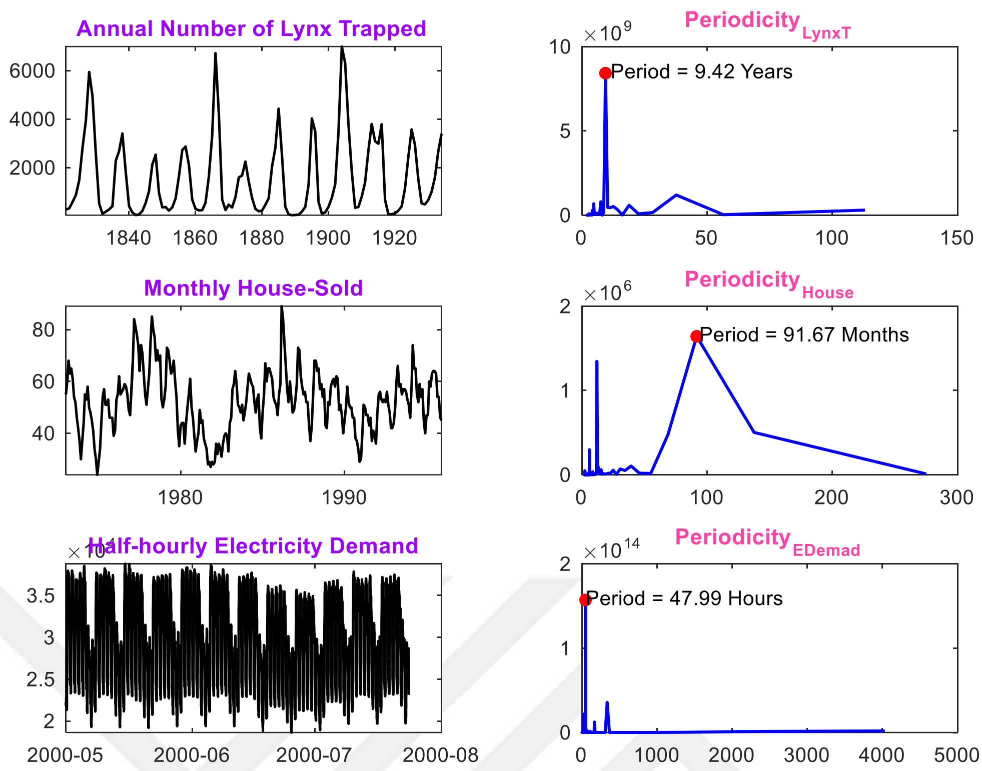


Figure 3-6 Calculation of cyclic and seasonal magnitudes of some time series

Source: Hyndman (2011) and calculated by the author.

Hyndman (2011) presents three time series which has seasonality or cyclic patterns and are depicted in Figure 3-6. In-the-left-hand side of the figure, the three time series is illustrated. The first time series is about the yearly number of lynx trapped in Canada between 1821 and 1934. He claims that it confirms nonperiodic cycles of roughly less or more than 10 years. In the second and the third graph, the monthly house sold time series spanning from 1973 to 1995 and half-hourly electricity demand between 2000-06-05 and 2000-08-27 in Great Britain are plotted respectively. Hyndman (2011) finds a strong cyclic behavior for the second graph with a period of between $6-10$ years and a daily pattern for the third time series. We can say that it is not hard to find the same results with him because that's as plain as the nose on your face. Ultimately, in-the-right-hand sides of the graph, the results of him are confirmed with the Fast Fourier Transform method. The approximate results are 9.4 years for annual lynx trapped, 7.7 years for the monthly house sold and 2 days for electricity demand. It should be noted that these calculations are done with Matlab (2015a) and the time series are stationary.

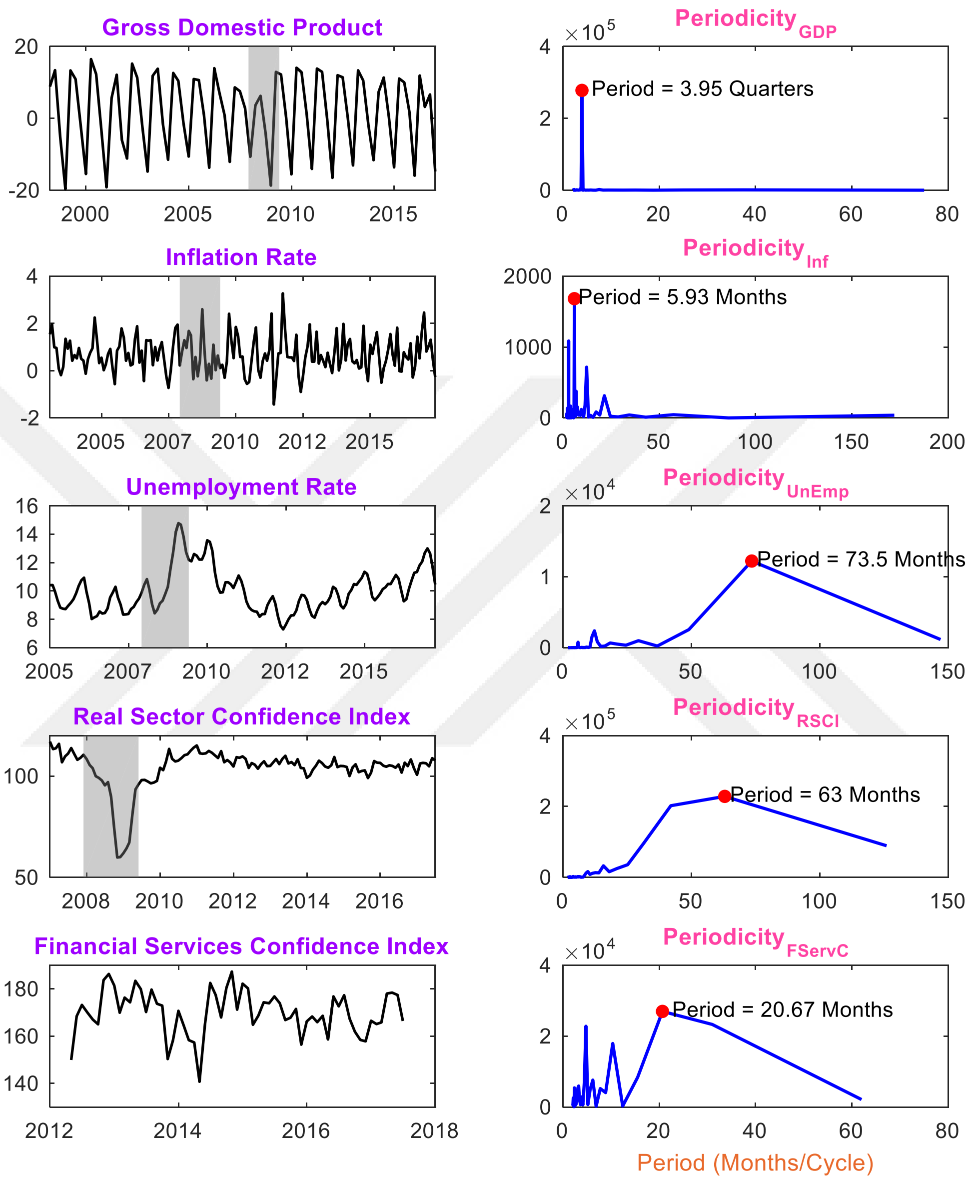


Figure 3-7 Calculation of cyclic and seasonal magnitudes of the time series of Turkey

Source: Calculated by the author.

For more clear understanding, a several financial and economic time series are used for frequency analysis detecting seasonality and cyclical patterns. These time series are GDP growth rate (quarterly), inflation rate (monthly), unemployment rate (monthly), the real sector confidence index (RSCI, monthly) and the financial services confidence index (FSCI, monthly) for Turkey. These series are illustrated in Figure **3-7**. They have different time patterns, but it is not easy to notice them at first glance in a figure. Which one does has a seasonal pattern or cyclical behavior? Here, FFT comes to help us. Firstly, they are checked for stationarity. The standard Augmented Dickey-Fuller (ADF, 1979) test is used. The test results showed that the unemployment rate, the RSCI, and the FSCI series are found stationary on the level, namely they are integrated of order zero, $I(0)$. The other series, by the way, have unit roots in level, i.e. non-stationary. After taking their monthly changes, they are stationary.

Subsequent to solving the unit root problem, we can explore their pattern shown in Figure **3-7**. It can be seen that the GDP growth rate has seasonality and this means that the Turkey economy will complete its business cycles approximately in 4 quarters. Besides, the inflation rate also has seasonality and its periodicity is nearly 6 months which is an expected result. The rest of the series have cyclic behaviors because their magnitudes are longer than 1.5 years; 6, 5.2, and 1.7 years respectively. It should be pointed out that these time series' seasonality and cyclic patterns depend on the time interval. If one changes time intervals, then it would not be a surprise when a time series will have a cyclic pattern comparing the current result with seasonality.

3.2.1 Fourier transform (FT)

At the beginning of the 19th century, Jean Baptiste Joseph Fourier, the French mathematician, presented a memoir on the study of heat diffusion. In this memoir, he gave a detailed study of trigonometric series and he asserted that any periodic waveform which repeats its cycle in a given time, can be written in the sine and/or cosine functions (Mallat, 2008). It should be noted that Fourier's ideas were not welcomed by academics, scientist and researchers studying at different areas because of his unconfirmed claims and overstated results despite his profound impact on (Boggess and Narcowich, 2009). Nevertheless, after a century and a half, his results are widely acclaimed and Mallat (2008) reports that the major reason behind is that of the method's simplicity.

A function $f(t)$ with a finite duration, T_0, and broken down into a sum of trigonometric or complex exponential functions is called Fourier series (Corinthios, 2009). The author (2009) gives an example for this series plotted in the figure below. $f(t)$ is a time function and its value is spanning from minus infinity to plus infinity that is $-\infty < t < \infty$. A specific part of this function is also shown in this figure. This section's duration is $(t_0, t_0 + T_0)$ which can be formulized in the following trigonometric form:

$$\hat{f}(t) = f(t) \tag{39}$$

where

$$t_0 < t < t_0 + T_0$$

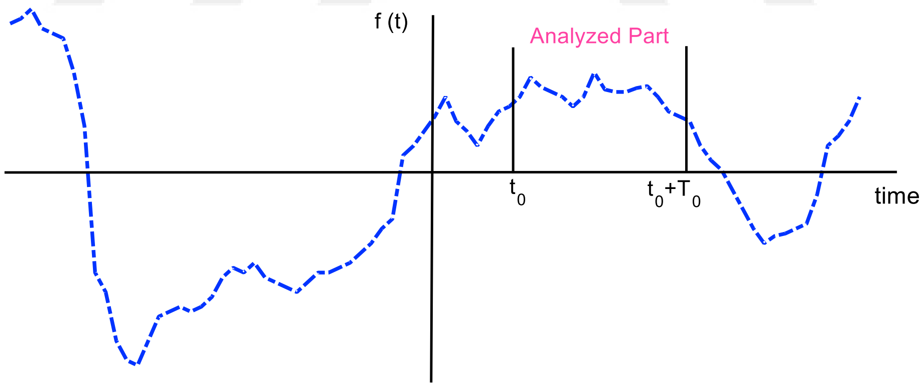


Figure 3-8 A Signal and analysis time interval

Source: Corinthios (2009).

The Fourier series $\hat{f}(t)$ of $f(t)$ function can be represented in the trigonometric terms such that (Lessard, 2006):

$$\hat{f}(t) = \frac{1}{2}a_0 + b_1 sin\omega_0 t + b_2 sin2\omega_0 t \ldots + b_n sinn\omega_0 t +$$

$$a_1 cos\omega_0 t + a_2 cos2\omega_0 t \ldots + a_n cosn\omega_0 t \qquad (40)$$

with the conditions as follows:

$$n = 1,2,3,4,5,6,7, \ldots \infty, \quad \& \quad \omega_0 = \frac{2\pi n}{T_0}, \qquad \& \qquad T_0 = \frac{1}{f_0}$$

As it can be seen in Equation (40), a Fourier is represented by, generally, the sum of periodic signals. The frequencies included are said to be harmonically related (Mallat, 2008) meaning that each sinusoid has an integer frequency value that is the multiple of the first sine/cosine function (Weeks, 2010). The amplitude of this signal is calculated as the sum of the amplitude of each of its sine and cosine components (Chaparro, 2010). Being harmonically related both sine and cosine trigonometric terms should have the form in Equation (40) (Lessard, 2006).

In Equation (40), the constant, a_0, is the average component of $f(t)$. Firstly, both coefficients a_1 and b_1 stand for the fundamental/basic frequency element ω_0. Additionally, the second, $2\omega_0$, and the third harmonic, $3\omega_0$, components are represented by the coefficients a_2 and b_2 and a_3 and b_3, respectively. Briefly summarizing, the $n\omega_0$ harmonic components are indicated by the coefficients a_n and b_n (Karris, 2003).

The function $f(t)$ mentioned before, can be also formulized in the exponential form such that (Corinthios, 2009):

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \qquad (41)$$

with the conditions as follows:

$$t_0 < t < t_0 + T_0, \quad \& \quad \omega_0 = \frac{2\pi}{T_0}, \quad \& \quad F_n = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(t)e^{-jn\omega_0 t}dt$$

Corinthios (2009) remarks that T_0 is the Fourier series expansion analysis section depicted in the figure above and ω_0 is the fundamental frequency of this expansion.

Briefly stated, this method basically converts a signal from one domain (time or frequency) to another (frequency or time) where one can obtain many characteristics of this signal (Goswami and Chan, 2011). The Fourier analysis includes both the Fourier series and the Fourier transform. By the way, these components are related to functions defined on the real line, $\mathbb{R}$, and periodic, respectively. Likewise, Chui (1992) defines the Fourier analysis from a practical point of view and states that when talking about it, one usually refers to its integral Fourier series. Let's assume a f function, say a digital signal. The spectral domain of this function is $\hat{f}$ and it is obtained using the Fourier transform. Chui (1992) says since this domain describes the spectral characteristic of the signal, it is shown in the terms of frequency, i.e. in the frequency domain. Roughly speaking, one can compute $\hat{f}$ from f and then can restructure f from $\hat{f}$ easily.

To a clear understanding of transform, Raj (2015) gives some excellent examples. He says that to change a radio station to another, one should change frequencies or to obtain a good quality for sounds in a radio, one should adjust the equalizers. He also gives the history of spectrum term which leads to Newton's studies. He continues that, Newton's aim was to build a lens for a telescope. But his efforts remained inconclusive, in other words, his results were to obtain rainbow instead of white light. Eventually, Newton formulized these results meaning that white light comprises all colors which are illustrated in Figure **3-9**. By using two prisms, Newton managed to gain all colors from white and reunite all colors to white. These colors are called as "specter" (ghosts) and its spread is described as the spectra of white light. Raj (2015) concludes that Newton, unfortunately, could not tie in with frequencies.

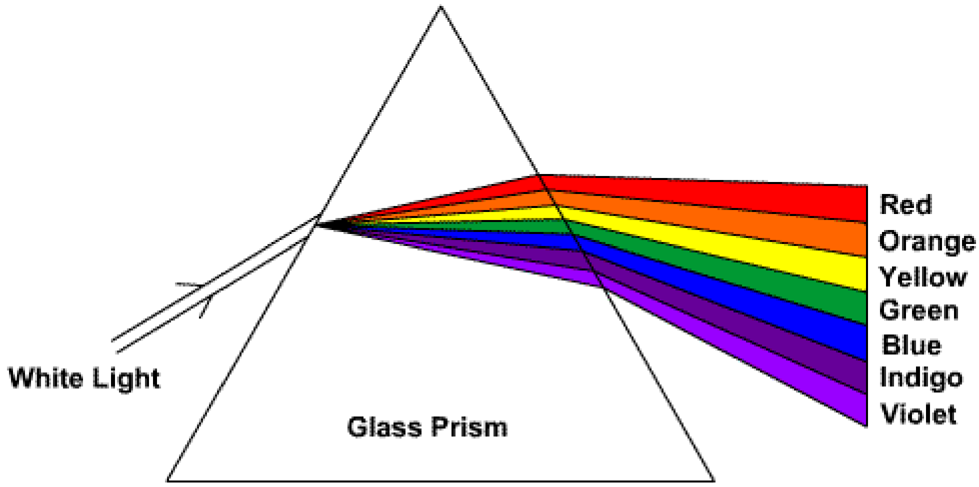


Figure 3-9 Newton's Prism

Source: Raj (2015).

Hubbard (2005) declares that a function f and its Fourier transform $\hat{f}$ are two different aspects of the same information. The f function shows the signal with regarding time to neglect frequency information. The author (2005) gives an example corresponding to a musical recording. When playing this recording, one recognizes how the sound waves vary with time, but it is not possible to delineate the frequencies, i.e., notes, that compose the music. In the other hand, the author (2005) says when listening to the transformed signal, one can tell what notes are played, however, it is not easy to delineate its time information, in other words, when these notes are played. Simply stated by Chaparro (2010), the Fourier transform quantifies a signal's frequencies included. Due to the inverse relationship, the demonstration of this signal in one domain gives the information which cannot be clearly observed in the other domain.

It is known what the difference between periodic and non-periodic signals is. The term of spectra described above can be generalized for both finite-power and finite energy signals (Chaparro, 2010). If one decides to use the Fourier Transform, the signal's form will not create difficulties. The author (2010) says that in the case of the Fourier transform, for a periodic signal, its frequency representation, i.e. its spectrum, is in the discrete form and for a nonperiodic signal, it is in the continuous form. These frequency representations of the both display how their amplitudes are distributed to different frequency components. The author (2010) remarks that in practice there are no periodic signals; hence signals in practice are generally treated as non-periodic signals having an infinite fundamental period.

Mentioned before, the Fourier transform for periodic or non-periodic results cannot give the time information of a signal in that it is lost in the frequency representation. This is the first drawback of the transform. The main reason is stated by Miner (1998). The author (1998) says that it is mainly driven by the fact that the basis functions of the Fourier transform have infinite support, in other words, they are non-zero across an infinite interval. With the Fourier transform, one will only obtain a global picture of this signal. In other words, the sinusoids are localized in frequency, not in time (Cascio, 2007). For this reason, this transform is suitable for a signal or a time series that does not enclose local irregularities, that is it should be stationary (Gencay et al., 2002) and this is the second drawback.

3.2.2 Short-Time Fourier Transform (STFT)

In the light of all the drawbacks mentioned above, one needs to a transform method that is appropriate for both periodic and non-periodic signal and local picture of a signal. At first step, D. Gabor introduced a new transform method using the Gaussian function named short-time Fourier transform (STFT) or windowed Fourier transform due to the poor time-localization of the Fourier transform (Burrus et al., 1997). Gabor's attempt was to simultaneously attaining a good resolution in the time-frequency plane. The idea was a simple one, for analyzing a signal one should study its frequencies by different small sections (Cascio, 2007). The author (2007) says that it is a studying of frequencies by section, and multiplying it by a fixed window. When one section's analysis is finished, the window is moved along the time axis in order to analyze the other sections. This process is continued until the whole sections are analyzed. Goswami and Chan (2011) remarks that these small sections can be read as the components of the function $f(t)$ function regarding in the time-frequency plane. After the analysis is finished, one gets a decomposition of two parameters (τ, ω), that are time and frequency parameters given in the equation below.

Let's assume a signal $f(t)$. Firstly, one should multiply $f(t)$ with a preferred analysis window coefficient, $\gamma^*(t - \tau)$, and then calculate this windowed signal's Fourier transform which is formulized as (Mertins, 1999):

$$\mathcal{F}_x^{\gamma}(\tau, \omega) = \int_{-\infty}^{\infty} f(t)\, \gamma^*(t - \tau) e^{j\omega t} dt \qquad (42)$$

The function $\gamma^*(t)$ in the equation is called the window function that is the responsible for the accuracy of the information and its width is decided by the user. This is the main reason for calling it with the other name that is the windowed Fourier transform (Chui et al., 1998). With this simple method, one suppresses $f(t)$ outside of a certain region and then gets a local spectrum (Mertins, 1999). In other words, unlike the Fourier transform, this method needs to know $f(t)$ only in the interval selected (Goswami and Chan, 2011).

Fugal (2009) demonstrates that a possible solution to providing information in the time-frequency plane is to split the total time period into several shorter periods and after that continue to transforming every time periods. Norsworthy et al. (2000) pronounce, with this small section's transforming, one should assume that it is stationary. By locating the window and moving it along the time axis, one will localize the frequency and acquire a time-frequency picture. The authors (2000) remark that these transformation coefficients obtained are called the amplitudes that are spread out in the different frequencies and at different time intervals.

While the STFT provides information in the time-frequency plane, the accuracy is limited by the size and shape of the window. For example, using many time intervals would give a good time resolution but the very short time of each window would not give us a good frequency resolution, especially for lower frequency signals (Fugal, 2009).

Although the STFT gives both time and frequency representation, it has some disadvantages because of the accuracy of the transform. The first problem is related to the quantity of the data generated with various window shapes and sizes compared with data generated using simple Fourier transform. The second problem is that fixing the width of the window means that one accepts compromises (Cascio, 2007). Fugal (2009), here, remarks the inverse relationship between the frequency and the window's shape and size. Put it differently, for obtaining high-frequency components, a small window should be used which makes impossible to obtaining information for low frequency by the way. In short, good frequency picture leads to poor time representation while good time representation leads to poor frequency picture.

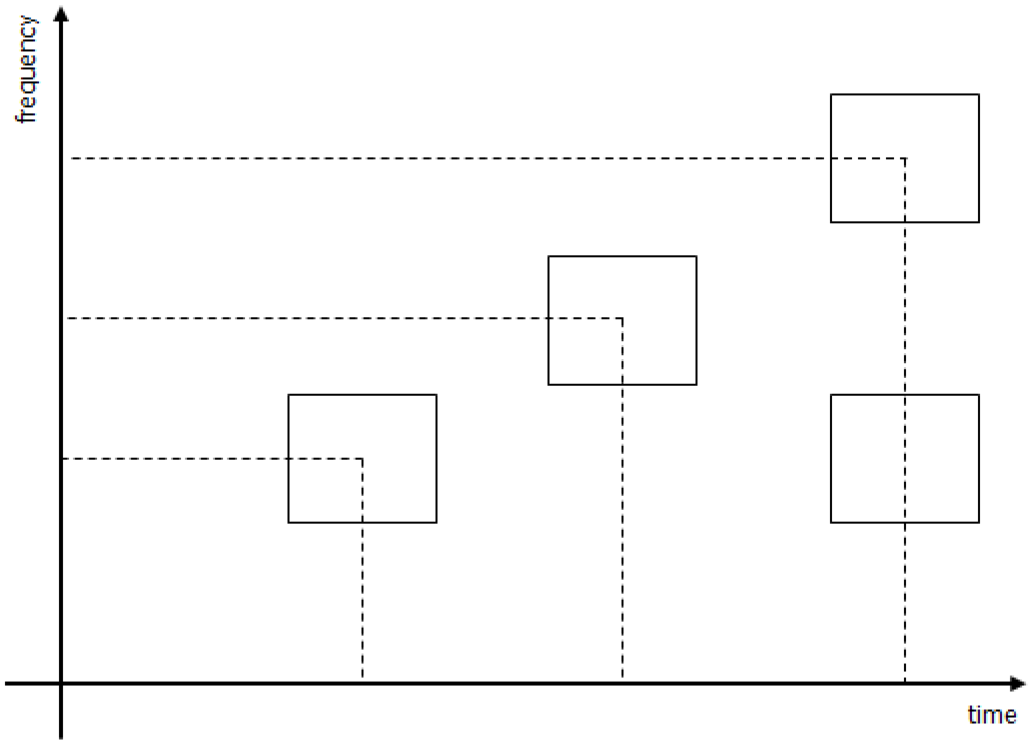


Figure 3-10 Time-frequency windows for windowed Fourier transform

Source: Chui et al. (1998).

The effect of using a fixed window for STFT is depicted in Figure **3-10** (Chui et al., 1998). It can be seen that the window widths are the same for the all frequency and time period. Put it differently by the authors (1998), once the width is chosen, it is fixed in the time-frequency plane because its shape and size are independent of the axis parameters. This characteristic leads to an important drawback for the STFT since this transform type cannot give the detail of the information took place outside of the width of the window (Kaiser, 2010).

3.2.3 The Fast Fourier Transform (FFT)

Rather than jumping straight to the theory of the Fast Fourier Transform (FFT), it is required to first give some details of the Discrete-Time Fourier Transform which is the slow version of it. The STFT of a discrete-time signal $x(\mathrm{n})$ in question is formulized as given equation below (Mertins, 1999):

$$\mathcal{F}_x^{\gamma}\left(m, e^{j\omega}\right) = \sum_{n} x(n)\gamma^*(n - mN)e^{j\omega n} \tag{43}$$

As discussed by Mertins (1999), the short-time spectrum is a function of the two parameters of m and ω which are the discrete and continuous parameter, respectively. In practice, however, this formula takes a different form:

$$X(m,k) = \sum_{n} x(n)\gamma^*(n - mN)W_M^{kn} \tag{44}$$

with the following conditions:

$$\omega_k = 2\pi k/M\,, \qquad k = 0, 1, 2, 3, \dots, M - 1$$

Let's assume a complex-valued input signal, $x(n)$, in question and its length is N. To compute its Discrete Fourier Transform, one should create a matrix, W, that entails $N * N = N^2$ complex multiplications. When implementing signal analysis, it is frequently noticed that the parameter N is generally very big and hence the execution's cost very expensive even for high-speed computers (Wong, 2011). The author (2011) questions the need for N^2 complex multiplications in a matrix. The key answer lies in the structure of this matrix, W, wherein the single complex number, ω_N, is formulized as $\omega_N = e^{-2\pi i/N}$. Wong (2011) remarks that this matrix structure enables us to decompose N into factors with many zeros. This factorization, the fundamental idea behind the FFT, is pictured firstly by Carl Friedrich Gauss in 1805, just two years before J.B. Joseph Fourier's presentation in Paris. Hubbard (2005) denotes that its discovery and usage as a computer program is done by John Tukey and James Cooley in 1965.

Considering the idea behind the FFT, Mertins (1999) defines it as a fast implementation of the DFT rather than a different method which lead to the conclusion that, as stated by Broughton and Bryan (2011) one does not need to a knowledge for the FFT for using or understanding the DFT computations.

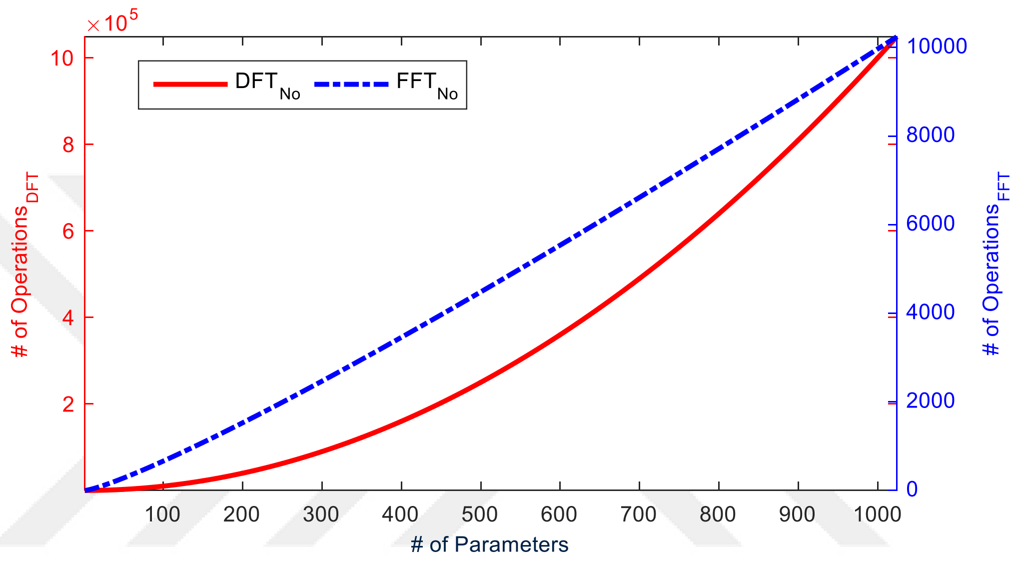


Figure 3-11 Comparison of DFT and FFT calculation cost

Source: Calculated by the author.

The FFT is a smart and faster algorithm to execute the DFT that is it yields the same results as the DFT does but much faster than it thanks to the efficiency of the algorithm (Weeks, 2010). Figure 3-11 illustrates the computation costs. It should be pointed out that this figure has two y-axes, where the left y-axis belongs to the DFT and the right one belongs to the FFT. Implementing the DFT, the number of computation is calculated with the formula mentioned before, N^2, while it is equal to $N\log_2(N)$ for the FFT. Hubbard (2005) highlights that the larger N value is, the more remarkable the gain in computation speed. For instance, if the value N is $= 2^5 = 32$, then its is $N^2 = 2^{5*2} = 1.024$ for the DFT, while 160 for the FFT ($n\log_2 n = 32 * \log_2(32) = 160$). Hubbard (2005) states that if one takes $N = 2^{20} = 1.048.576$, then a huge difference, nearly 52.000 will arise. From the computation cost, Weeks (2010) enunciates that the DFT completes a data sample of 100.000 in 5 seconds however it takes 2 months to compute 100.000.000 for a data samples. On the other side, the time span of this sample is lower than 1.5 seconds in the case of the FFT.

Given that the calculation drawbacks of Fourier transform, such as the accuracy and the timing, a need for a transform method that overcomes these problems arise. The details are given in the following section.

3.3 Time-Frequency Domain Methods

Before giving the details of time-frequency domain, it should be summarized the Fourier transform methods to refresh the memories of readers. Goswami and Chan (2011) define the Fourier transform, $\hat{f}(t)$, of a function, $f(t)$, as:

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\, dt \tag{45}$$

The Fourier transform method breaks down this function, $f(t)$, into different frequencies by means of Fourier series that are sine and cosine waves. In this sense, it is called a frequency domain representation, namely a decomposition of a function on a frequency-by-frequency basis (Gallegati, 2008). Raihan et al. (2005) remind that this method is a proper tool to study the cyclical behaviors of a time series just only in the frequency domain. But we have learned that under this method, one cannot reach the information on time. Tiwari et al. (2015) mention that, however, losing the time information it makes difficult to study a macroeconomic time series in question having structural changes and the results of the Fourier transform will not reliable. In other words, this method is not suitable for non-stationary signals, i.e. with varying frequency behavior. To overcome this, one needs to use a different method suitable for local analysis rather than global analysis, introduced by Gabor (1946) called Short-Time FT (Abramovich et al., 2000).

Goswami and Chan (2011) define the local analysis as achieving local behavior of a function or a signal where one can acquire both the frequency and the time domain. To know the local frequency components of this function, one should first select a section and then apply Fourier transform to this removed section until whole the sections are transformed via FT. Here, it is assumed that this small section is stationary. But here a problem arises due to the width of the window used by SFTF. Raihan et al. (2005) mention that the width of the window can be selected freely but it cannot be changed anymore after it is exogenously selected. The importance of this lies in the fact that to acquire good time-frequency resolution, one should choose different window sizes according to low or high frequencies. Put it differently, Mallat (2008) states that this resolution is determined by the spread of the window in time and frequency.

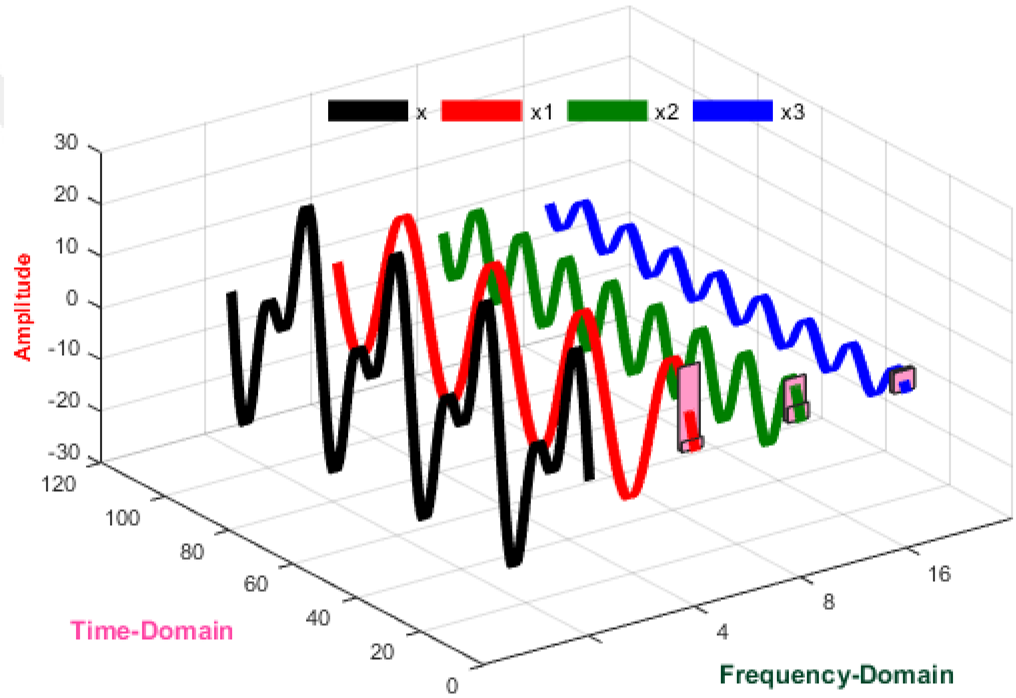


Figure 3-12 Time-domain vs. frequency-domain of sine signals

Source: Author’s calculation with MATLAB (2015a).

But, unfortunately, this method has a problem which is presented by a physicist, Werner Heisenberg, in 1927. In his uncertainty principle, Heisenberg declared that theoretically, the speed and the location of an object cannot be measured concurrently and precisely (Soman et al., 2010). Weeks (2010) states this theory as if one gets a good resolution in the time-domain but a poor representation in the frequency-domain or vice versa. Put it differently, it is not possible to know the frequency and the time in a signal or a function simultaneously (Boggess and

Narcowich, 2009). These problems require a new method that is suitable for both stationary and non-stationary signals and has different window sizes for lower and higher frequencies. This method is called wavelet transform which is widely used in the different science fields for different aims.

It should be underlined that the Fourier analysis is the origin of the wavelet transform. It is based on the idea that a signal or a time series can be represented by the time- and frequency-domain simultaneously. This method gives a different perspective to analyze a composite or harmonic signal which is depicted in the figure above. Figure 3-12 has three axes: x, y, and z. These axes represent amplitude, time domain, and frequency domain respectively. When one needs to get time information, he/she should look at the x-axis and y-axis, i.e. the left-hand side of the figure, while the right-hand side of the figure is suitable for frequency information. Generally speaking, Subbey et al. (2008) state that these two domains, time and frequency, are similar; however they give two different perspectives to see the same signal or the time series.

3.3.1 Wavelets

Despite its great success in analyzing, the STFT method has an important drawback which based on the length of the window. Raihan et al. (2005) outline that if one needs to improve the frequency resolution of a signal, he/she should opt a long window which leads to a loss of information whereas to get a better time information, one should opt a short window size which causes a poor frequency quality.

As dictated by Gallegati (2008), once the length of the window is selected, it cannot be changed anymore. Here, the choice depends inversely on the timing of the window function, and the stationarity is presumed. After the choice of the window size, the frequency and time information resolutions are the same for all different frequency and time points. Therefore, this method is unsuitable for achieving a good quality resolution in terms of frequency or time. Abramovich et al. (2000) state that to overcome these challenges is one of most essential motivations to find a different method which is called wavelet transform.

Subbey et al. (2008) remark that a sine signal can be considered a big wave because it gets a value between $-\infty$ and $+\infty$. The word wavelet, by the way, is a translation of the French word "ondelette" and it means "short wave" or "small wave". This term firstly has been introduced by Grossman and Morlet in 1984 (Cascio, 2007) and it is evident from its name, the adjective term "small" means that wavelet grows and decays in a given finite time period (Masset, 2008). The wavelet function is formulized on the real axis satisfying two important conditions that are given as

$$\int_{-\infty}^{+\infty} \psi(x)dx = 0 \tag{46}$$

$$\int_{-\infty}^{+\infty} |\psi(x)|^2 dx = 1 \tag{47}$$

These conditions in both equations, altogether, signify that some coefficients are different from zero and that deviations from zero must cancel out. Masset (2008) illustrates that these two requirements are met by a vast quantity of functions apart from sin functions. For a particular and practical purpose, one should meet an extra condition namely admissibility condition. This is illustrated as follows

$$R_g = \int_0^{+\infty} \frac{\left|\hat{\psi}(f)\right|^2}{f} df < +\infty \tag{48}$$

where $\hat{\psi}(f)$ is the Fourier transform of $\psi(f)$ function.

Addison (2002) states the admissibility constant, R_g, value changes according to wavelet desired. Also, the author (2002) defines the conditions as with no zero frequency component, $\hat{\psi}(0) = 0$ or, namely $\psi(t)$ function's mean is zero. Chui (1992) conveys that Equation (46) guarantees for any basic wavelet's graph is a small wave.

According to wavelet literature, Crowley (2007) says that wavelets have two types or genders which are called as father and mother wavelets shown in Greek alphabets, phi, ϕ, and psi, ψ, respectively. These functions are given as follows:

$$\int \phi(t)dt = 1 \tag{49}$$

$$\int \psi(t)dt = 0 \tag{50}$$

Firstly, one can say that father wavelet (or scaling function) in Equation (49), integrates to one (1), and mother wavelet integrates to zero (0). Ramsey and Lampart (1998) express that the former wavelets are utilized for the lower-frequency, while the latter one used for the higher-frequency. These wavelets are employed for the smooth and the detail components of a time series or a function, respectively. The authors (1998) mentioned that the second wavelet is used for all deviations from trend. In diagrammatic terms, these wavelets can be depicted for the Symmlet and Daubechies wavelets as in the figure below:

Looking at the wavelet history, one can see that the first example is of Haar Wavelet. Lepik and Hein (2014) point out that this type wavelet is introduced by Alfred Haar in 1910. Its functions are the simplest among the all the other wavelet families. By the way, Strang (1989) points out that the Haar is orthogonal to its own dilations and translations functions. Differently speaking, this wavelet is the historically simplest example of an orthonormal wavelet basis and they are supported on small subintervals, $[0, 1)$, on the time axis, having compact support (Addison, 2002). Walnut (2013) says that the Haar wavelet basis represents the functions having smooth and slowly varying sections efficiently.

Starting with Haar wavelets, scientists and researchers generated lots of wavelet types, some are unintentionally created such as Morlet wavelets by Jean Morlet. Soman et al. (2010) assert that Morlet actually didn't want to discover a wavelet type while trying to provide an efficient search method analyzing the seismic signals for oil. His method was working to decompose a signal into components and later compile these components into the original signal. With the aid of the physicist Alex

Grossmann, Morlet proved that this method was mathematically sound and works better than the Fourier transform. They published their results in the paper in 1984 where the term wavelet is used firstly in the literature. At the same year, Yves Meyer, widely accepted as one of the founders of wavelet theory, found a link between Morlet's and the other previous mathematical wavelet studies. Later he discovered a new wavelet type, called with his name, having orthogonality characteristics. Soman et al (2010) assert that a former student of Meyer's, Stephane Mallat pioneered in the multiresolution analysis where one can explore the different scale of a signal in 1986.

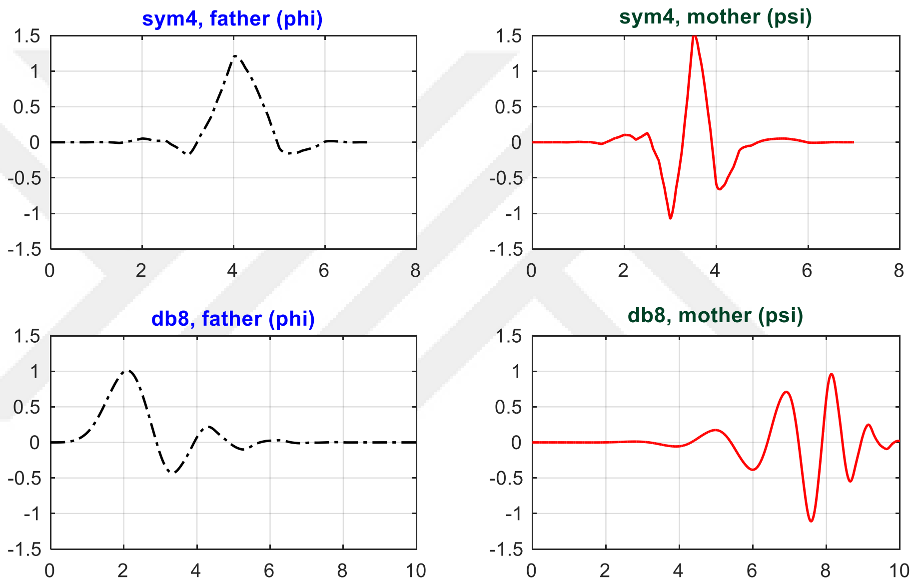


Figure 3-13 Father and mother wavelets for Symmlet(4) and Daubechies(8)

Source: Author’s calculation with MATLAB (2015a).

The last and the most important wavelet family detailed by Soman et al (2010) is the Daubechies wavelets. This wavelet family is discovered by Ingrid Daubechies in 1987. The most contribution of Daubechies wavelets was that they were not only orthogonal as seen in Meyer wavelets but also gave an option to using simple digital filtering ideas. Soman et al (2010) declared that they were analogue to Haar wavelet in that simplicity in usage but they had a different characteristic such as smoothness. In other words, they can be easily used in analysis by any researchers without having mathematical training. Lepik and Hein (2014) evoke that the Daubechies wavelets have a drawback in that they cannot be drawn clearly in a mathematical expression; therefore, calculation of the coefficients is not simple.

Boggess and Narcowich (2009) remark that the Daubechies wavelets are categorized according to the number of vanishing moments. When this number increases, the smoothness of the wavelet does. Addison (2002) mentions that this family has $N_g/2$ vanishing moments and its smoothness in terms of the non-zero scaling coefficients, N_g, can be formulized as:

$$\sum_{g=0}^{N_g-1}(-1)^g\, c_g g^k = 0 \tag{51}$$

With the condition $k = 0, 1, 2, 3, \ldots, N_g/2 - 1$. By the way, this family is represented by "DN" or "dbN" abbreviations. When one defines a wavelet of this family with its support length, he/she must subtract these numbers. For instance, a wavelet with 1 support length is equal to "$D2$" or "$db2$" which is remarked by Boggess and Narcowich (2009) that it actually is the Haar wavelet. From this point of view, "$db3$", "$db4$", "$db5$", "$db6$", and "$db7$" wavelets will have a length support of $2, 3, 4, 5,$ and 6 respectively.

Apart from these family members, dbN, Daubechies introduced another wavelet group called "Least Asymmetric, $LA[L]$" with the condition as $7 < L \leq 20$ (Percival and Mofjeld, 1997) where $LA(8)$ is the most used by researchers particularly in the field of finance. The other wavelet families used in finance are depicted in the following figure.

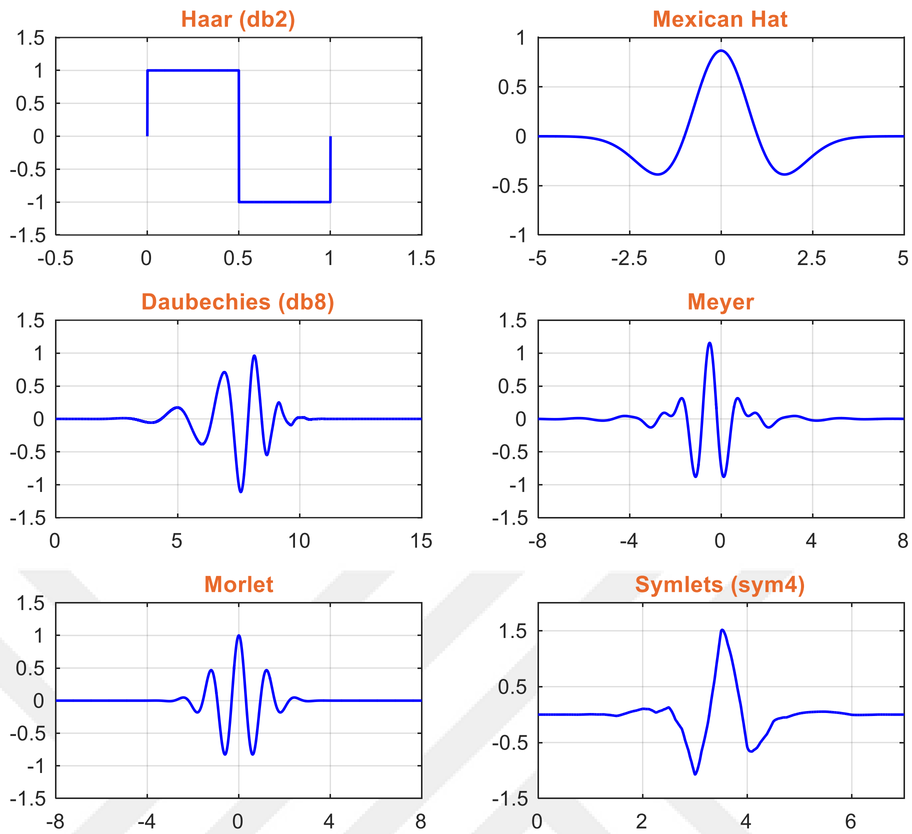


Figure 3-14 Some wavelet families used in finance

Source: Author's calculation with MATLAB (2015a).

Crowley (2007) comments that wavelets depicted in the figure come in different shapes. In-the-top and the left side, the Haar wavelet is in discrete form, the Mexican hats are in symmetric form. The wavelet in the bottom and the right side there is Symmlet wavelet which is almost in symmetric form. The wavelets shown are all mother wavelets. The wavelet that the mostly used by researchers in the finance, is the Daubechies wavelets and they are in asymmetric form.

3.3.2 Wavelet Transform

Before describing wavelet transform, we should first define "transform" term. Weeks (2010) delineates it's an operation which is performed by a system. When it comes to wavelet transform, Sahu and Sanjeev (2008) say that a transformation process is done to signals or time series data to attain more information from them. The key reason is that one cannot see or acquire these extra data from original data which is not easily observable. When the subject is the wavelet transform, this data may be stationary or nonstationary.

Daubechies (1992) states that this transform method is a tool that breaks a time series or a function into different frequency parts, resulting on two variables that are time information and scale information (the inverse of the frequency) at the same time. Differently stated by Raihan et al. (2005), a time series under examination is divided into shifted and scaled versions of a mother wavelet function which is defined before.

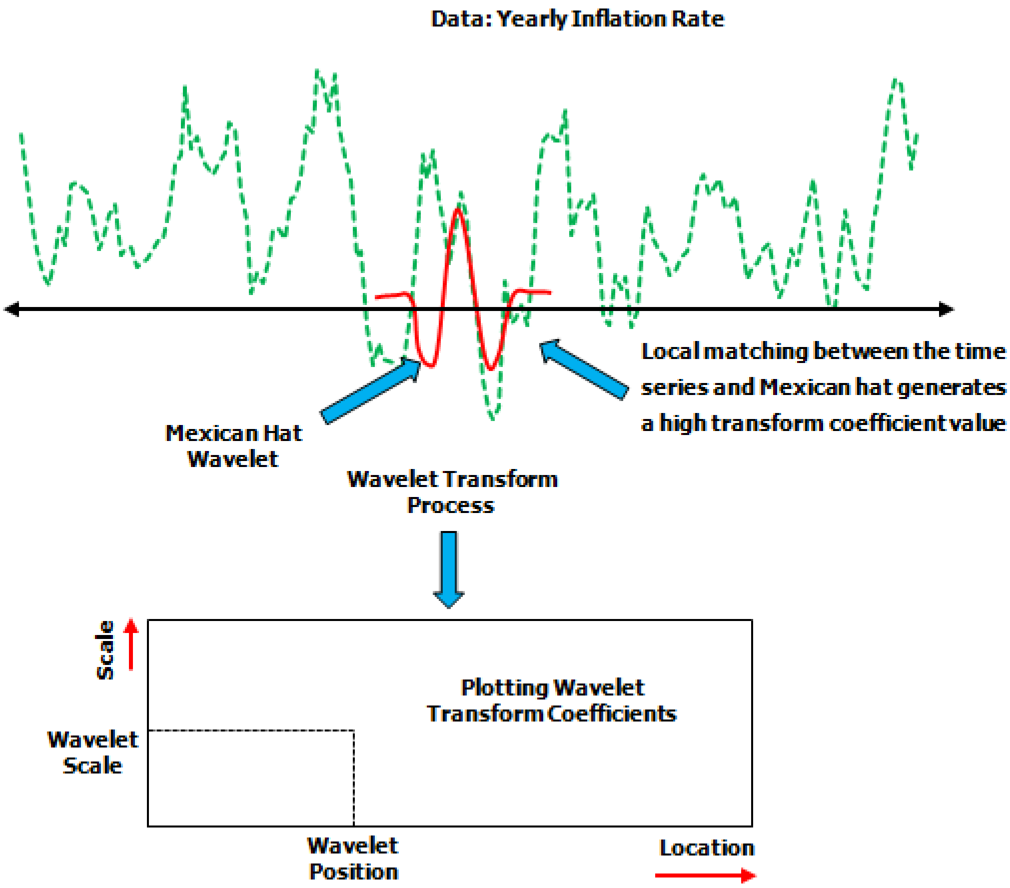


Figure 3-15 Wavelet Transform of A Time Series

Source: Soman et al. (2010).

Addison (2002) reveals that one can manipulate a wavelet by two aspects: shifting and dilation. A wavelet can be shifted to the right or to the left on the axis without changing its size or its size can be widened or squeezed to analyze a signal. The author (2002) says that if the wavelet function and the shape of a time series match in the time axis using the scale and shifting factors, then the transform coefficient value will be large. In contrast, if these two do not move together, in other words, they are not matched, and then the transform coefficient value will be small. For a representation of wavelet shifting and dilation is depicted in the following figure.

The wavelet used in the figure above is a Mexican hat wavelet and is formulized as follows (Arı et al., 2008):

$$\psi_{t,a,b} = \frac{(\mathcal{M}^2 - 1)e^{-0.5\mathcal{M}^2}}{\sqrt{2\pi}b^3} \tag{52}$$

where

$$\mathcal{M} = \left(\frac{t-a}{b}\right)$$

Said before, wavelet analysis is a modification of the Fourier transform. This transform method breaks down a time series into components which show its frequencies and its energies exists in the raw time series, namely it presents the time series in the frequency domain. The lack of this method is that it does not yield the time information, i.e. where one cannot reach a specific frequency's timing. Norsworthy et al. (2000) comment that if this time series is in the stationary form, then one does not require the time information, in other words, "location" information is not required, but, unfortunately in the real world, the vast majority of the time series are non-stationary. Here, the wavelet transform comes to the rescue where this transform method has important advantages over the former transform method (Baruník et al., 2014). This wavelet method is divided into two different forms: continuous and discrete alike the Fourier transform.

3.3.2.1 Continuous Wavelet Transform (CWT)

Burrus et al. (1997) declare that having compact support and being orthonormal for a wavelet yields an opportunity to desired time localization information. When it comes to the STFT method, this opportunity is not valid according to them even if this method is orthogonal. Using the STFT, the resolution quality results for the frequency and the time will be low, differently saying the outcome will be rigid which is not seen for the wavelet transform. Tkacz (2001) remarks that because the most financial time series –the interest rates or stock markets– do not follow the smooth rhythmic cycles suggested by the sinusoid functions, it makes this transform more effective and useful for researchers.

Miner (1998) mentions that the wavelets work for both stationary and non-stationary time series because this method is successful in capturing the time-varying behaviors. Unlike the Fourier transform, the wavelets transform preserve all the scale and timing information owing to a finite duration which yields better results than the classical econometric methods.

As described by Jensen (1999), its ability to localize a process in time and in reverse of frequency, i.e. scale, at the same time is of the most advantage. Firstly speaking, there is an opposite relationship between scale and time information. Differently expressed, a large-time support is possible at low scales while a small-time support is possible at high scales. Small time supports enable one to discover short periodic period behavior points as large time supports give an opportunity to the researchers to discover long periodic behaviors. By shifting the window from low scales to high scales, in other words, zooming in or zooming out, the wavelet reveals the rough or the smooth features of a time series, respectively. On the other hand, Lai (2015) classifies the wavelets as "mathematical microscope" because the wavelets give an opportunity to analyze the periodic variations in the scales of this time series. To a clear understanding, Graps (1995) gives a simple example. If one look at a time series with a small window, he/she will see fine details, however, a large window will lead to coarse details. The result acquired with the wavelet analysis is, in a manner of speaking, to see both the trees and the forest, respectively. Saâdaoui (2013) illustrates the wavelet transform as a prism, which helps the researchers to uncover seasonalities, trends, periodicities or cycles a time series. In wavelet transform, the wavelets should be orthonormal that leads a decrease in redundancy of information and guarantees that the results of wavelet transform are not correlated.

As mentioned before, the two functions of wavelet transform are father wavelet and mother wavelets and they are formally formulized as (Reboredo and Rivera-Castro, 2014):

$$\phi_{j,k}(\mathrm{t}) = 2^{-\frac{\mathrm{j}}{2}}\phi\left(\frac{t-2^{j}k}{2^{j}}\right) \qquad \text{for } j = 1, 2, 3, \ldots, J \tag{53}$$

$$\psi_{j,k}(\mathrm{t}) = 2^{-\mathrm{j}/2}\psi(\frac{t-2^{j}k}{2^{j}}) \qquad \text{for } j = 1, 2, 3, \ldots, J \tag{54}$$

Soman et al. (2010) declare that if scale and location of wavelets change smoothly, then the transform method will be called as continuous transform, it is called as discrete wavelet transform while they vary in discrete form. The continuous wavelet transform, i.e. integral wavelet form, of a function $f(t) \in L^2$ is formulized as (Goswami and Chan, 2011):

$$W_{\psi}f(h,c) \coloneqq \int_{-\infty}^{\infty} f(t)\,\overline{\psi_{h,c}(t)}dt \tag{55}$$

In equation, $\psi_{h,c}(t)$ is

$$\psi_{h,c}(t) = \frac{1}{\sqrt{|c|}}\psi\left(\frac{t-h}{c}\right); \quad c \in \mathbb{R}^{+} \,\&\, h \in \mathbb{R} \tag{56}$$

where c and h are scaling/dilation and location parameters. Ferrer et al. (2016) state that the former parameter, c, controls the length of the wavelet as the other one, h, provides the position of the wavelet. Easily produce a wavelet, Dempster (2001) remarks that one can use these parameters. By decreasing "c" value, the width of wavelet decreases, in other words, it squeezes while changing "h" value, the position of wavelet changes, in other words, it translates.

The term $1/\sqrt{|c|}$ in the equation above is called normalization factor which ensures that the norm of $\psi(.)$ is equal to one (Ramsey and Lampart, 1998). Torrence and

Compo (1998) cite that it is for energy conservation, put it differently, it is to guarantee the wavelet transforms generated at each scale, "c", are comparable to each other scales, the wavelet functions are normalized by this factor so that each scale has unit energy. Mathematically expressing, it is $\|\psi_{\alpha,b}(t)\| = \|\psi\| = 1$. With this, Crowley (2007) comments that the energy will be intensified in a neighborhood of h with size proportional to c.

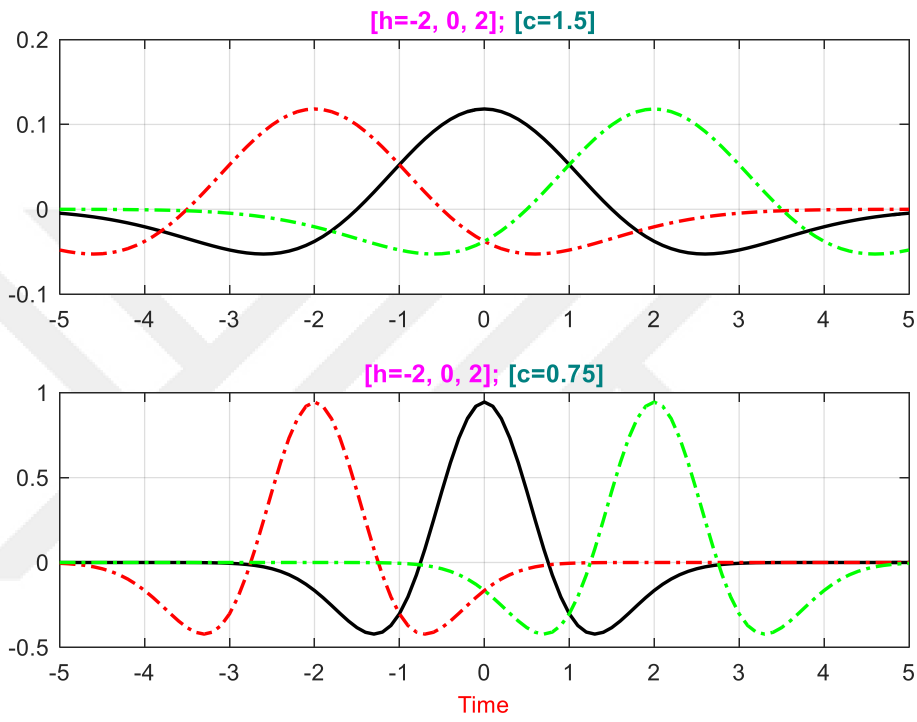


Figure 3-16 Wavelet Shifting and Scaling (Mexican Hat)

Source: Author's calculation with MATLAB (2015a).

Mentioned above, the mother wavelet function integrates to "0" while the father wavelet integrates to 1. The mother wavelet has zero mean. Yalamova (2004) remarks that these two functions can be written as follows by the way:

$$\int \phi\,(t)dt = 1 \tag{57}$$

$$\int \psi\,(t)dt = 0 \tag{58}$$

To a clear understanding, dilation and translation of a wavelet function are exampled as in the figure below. The wavelet used is Mexican hat; the reason for this name is that it looks like a Mexican hat. Daubechies (1992) says that this function is well localized in both scale/frequency and time domain.

In Figure **3-16**, the Mexican hat wavelet is manipulated in two ways. Firstly, in the top graph of the figure, it is stretched by scaling factor. The red line represents a moving to the left-hand-side while the red one shows a movement to the right in the time axis. Looking at the bottom graph, evidently, the two movements are the same with the first one except for the sizes. Strictly speaking, these all wavelet shifts and dilations can be shown with using $\psi_{h,c}(t)$ formulation such as $\psi_{-2;1.5}(t)$, $\psi_{0;1.5}(t)$, and $\psi_{2;1.5}(t)$ for the upper graph; $\psi_{-2;0.75}(t)$, $\psi_{0;0.75}(t)$, and $\psi_{2;0.75}(t)$ for the lower graph.

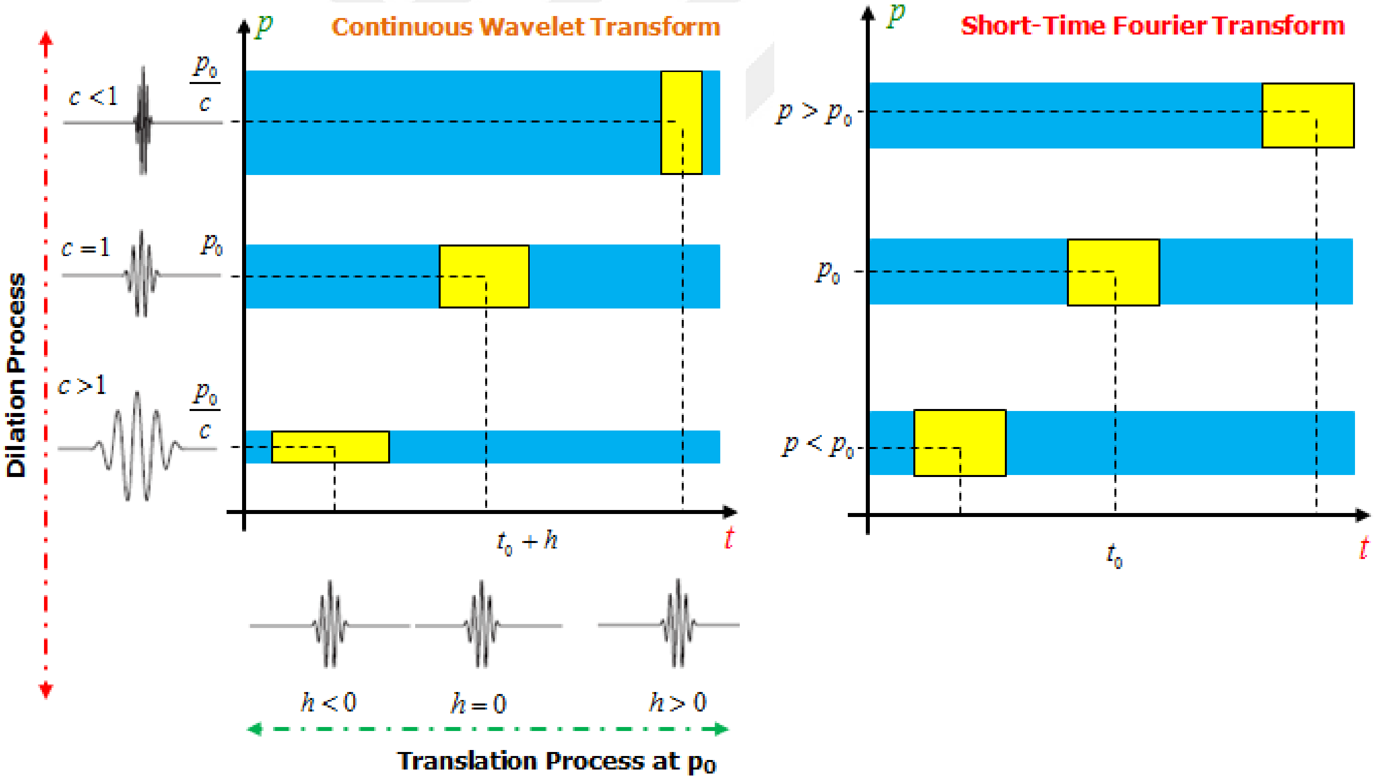


Figure 3-17 The Continuous Wavelet Transform vs. The Short-Time Fourier Transform

Source: Soman et al. (2010).

So far, we have discussed the continuous wavelet transform. Now, in the following figure, we will compare this method with the STFT. Having stated before, Heisenberg's uncertainty principle states that one cannot know the exact time and the frequency of a signal. The boxes used according to this principle are shown in the following figure. These boxes have different shapes in case of wavelet transform than the STFT method. Remembering, the STFT method has fixed boxes which lead the same time resolution and frequency resolution for a time series. Soman et al. (2010) remark that moving the boxes will result the same time information for all frequency bands. If one intends to acquire the information more accurately, then the boxes' shape should be changed. The wavelet transform method is a solution to overcome this drawback.

It can be seen that Figure **3-17** includes two transform methods. In-the-right hand, the STFT method has a fixed window for all time points and frequencies leading to constant resolutions. Unlike the STFT, the wavelet transform method yields varying-quality resolution according to frequencies. Broadly speaking, the wide windows used to look at low frequencies (high scale) are precise about frequency, but they are vague about time. Stating before, the transform process is done by two parameters depicting in-the-left-hand of the figure. It should be pointed out that y-axis in-the-left side represents the scale, the inverse of frequency. However, as noted by Ramsey and Lampart (1998), the inverse relationship between frequency and scale holds if the signal is stationary; in other words, the wavelet transform becomes meaningful when no oscillations are found in this signal. Cazelles et al. (2007) outline these two transform process. By decreasing scale, c, one will get a better time resolution but a poorer frequency resolution, i.e. with a higher frequency and a longer height and a shorter width of box leads a good quality of time information. On the contrary, by increasing scale, c, but decreasing frequency, one will obtain a better frequency resolution but a poorer time resolution. Raihan et al. (2005) abridge this process as that the wavelet transform has a fine time resolution but a coarse frequency resolution at low scales (high frequency), while it yields a coarse time resolution but a fine frequency resolution at high scales, i.e. low frequency.

Torrence and Compo (1998) declares that a wavelet function is either orthogonal or nonorthogonal form while wavelet basis is just an orthogonal form. These two forms determine which wavelet transform method, the continuous or the discrete, will be used. In the continuous wavelets, only nonorthogonal wavelet functions are used but for the discrete wavelet, both orthogonal and nonorthogonal forms are suitable. Masset (2008) remarks that, in empirical applications, the continuous wavelet transform method has some drawbacks. Using all wavelet coefficients to analyze a signal makes an expensive and impracticable computation for researchers making itself an inappropriate tool for time-series. And the second drawback is about the highly large redundant information due to the two parameters, scale and time. Because of these drawbacks, especially for the computational burden, researchers generated the discrete wavelet transform method which is detailed in the following section.

3.3.2.2 Discrete Wavelet Transform (DWT)

Cascio (2007) comments that if one needs to obtain a perfect rebuilding of a signal, then the wavelet transform method should be in continuous form. To get a perfect reconstruction, one should analyze the signal in all its possible resolutions with the wavelets which all are dislocated by an integer and non-integer translations. As stated previously, the CWT method has two main parameters –dilation and translation– which real numbers and these the two parameters produce an outcome with a mass of extra information, i.e. a redundancy in a number of wavelet coefficients. As noted by Hubbard (2005), in the CWT method, the majority of information is enciphered by the adjacent wavelets simultaneously, i.e. it is over-sampled and is redundant.

To make an efficient transformation, one should minimize the number of coefficients. Vidakovic (2009) remarks that the solution is to select discrete values instead of continuous values for scaling, c, and location, h, parameters ensuring the transform is still invertible. In other words, instead of all wavelet coefficients, a subsample preserving all information will be enough to represent the signal. This procedure is called the discrete wavelet transform, DWT. Gencay et al. (2002) state that the DWT method can be seen as either deriving or not from the CWT formulation. However, they comment that they see this method as a discretization of the CWT by means of subsampling of wavelet coefficients. As noted by Vidakovic (2009), a critical sampling is acquired as

$$c = 2^{-j} \quad \& \quad h = k2^{-j}$$

In the equation above j and k integers stand for the set of discrete dilations and translations. With $j, k \in \mathbb{Z}$ condition, the critical sampling will yield the minimal basis. As dictated by Vidakovic (2009), any coarser sampling will not produce a unique inverse transformation but produces an orthogonal basis as shown below

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi\left(2^{j}t - k\right), \;\; j, k \in \mathbb{Z}$$

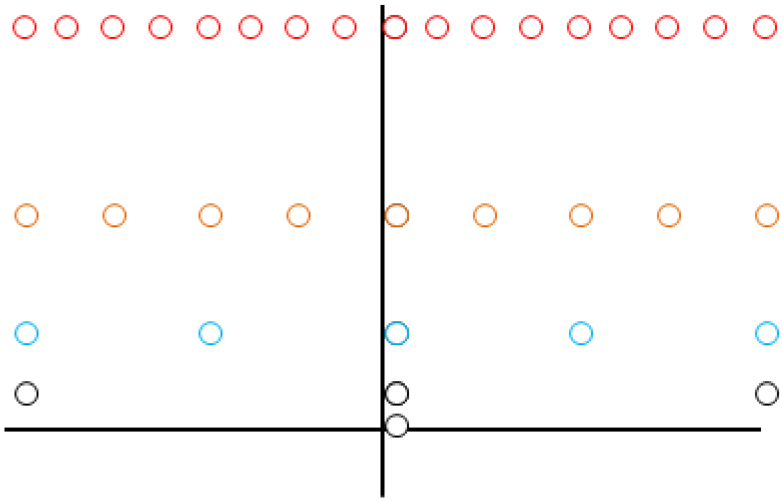


Figure 3-18 Critical sampling of the CWT

Source: Soman et al. (2010).

The time-scale (frequency) tilling by discretizing the CWT parameters is plotted in Figure 3-18 (Vidakovic, 2009). In case of the CWT, it is not computationally cheap, as noted before, to calculate coefficients for each possible scale. For more efficient and just accurate as the CWT, one should pick merely a subset of scales and translations based on powers of 2, i.e. discretize the CWT parameters (Alexandridis and Zapranis, 2014). With this discretizing process by means of $c = 2^{-j}$ and $h = k2^{-j}$, one can reach a different analysis so-called the Discrete Wavelet Transform (DWT) method (Gencay et al., 2002). They state that decreasing j value leading to an increase in scale (c), seen in the figure above, the number of coefficients in each subset throughout time axis are multiplied by two according to Nyquist's rule (Norsworthy et al., 2000). By using these coefficients, one can faultlessly restructure a function (Soman et al., 2010).

As noted by Soman et al. (2010), upon the replacement of dilation and translation parameters $-c = 2^{-j}$ & $h = k-$ the wavelet function in equation (20) evolves to a non-decimated (stationary) wavelet transforms, i.e. maximal overlap DWT discussed in next section.

Let's assume a signal data, $x(t)$, where the number of observation, N, is equal to $N = 2^J$. As said by Mitra et al. (2007), it's the DWT with regard to ψ, is given as follows:

$$W_{j,k}^{\psi} = \sum_{t=1}^{N} x(t)\, \psi_{j,k}(t) \tag{59}$$

or it can be written in matrix form as $W = \mathcal{W}x$, where $\mathcal{W}$ is an orthogonal matrix of size $2^J x 2^J$ satisfying $\mathcal{W}^T\mathcal{W} = I$ condition (Percival and Mofjeld, 1997). They remark that the W vector includes the transform coefficients where its first $N - N/2^J$ and the last $N/2^J$ components are defined wavelet and scaling coefficients.

Here, one main question arises: how can one decompose a signal and then reconstruct a signal using its wavelet coefficients. It is answered by Strang (1989). According to him, it is the pyramid algorithm generated by Mallat (1989b) which makes available a straightforward and fast computation to attain a perfect approximation (Misiti et al., 2013). A particular attractive characteristic of this algorithm, as Daubechies (1992) remarks, is that it enables to zoom in scaling function. The steps of a discrete decomposition with the pyramid algorithm are detailed in the figure below. To note, the filters and down-sampling operator denoted by $\downarrow 2$ ($\downarrow$ decimation) do not rely on the level j leading to a fast and efficient process (Boggess and Narcowich, 2009).

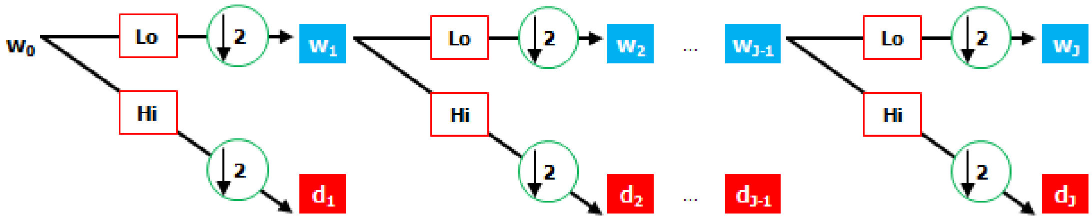


Figure 3-19 A Pyramid Algorithm for Discrete Wavelets

Source: Misiti et al. (2013).

The pyramid algorithm is calculated by a sequence of application of filters (Mitra et al., 2007). These filters are low-pass filters and high-pass filters, with filter coefficients $g_k c$ and $h_k c$ and they generate an approximation and detail series, respectively. The low-pass filters and high-pass filters represent the high-scale and the low-scale elements of a signal or time series, in other words, the low-frequency and the high-frequency parts.

$$g_k = \sqrt{2} \int_{-\infty}^{\infty} \phi(t)\, \phi(2t - k) dt \tag{60}$$

$$h_k = \sqrt{2} \int_{-\infty}^{\infty} \psi(t)\, \phi(2t - k) dt \tag{61}$$

Wavelets with filters are associated with multi-resolution orthogonal or biorthogonal analyses; discrete transform and fast calculations using the Mallat algorithm are then possible. Wavelets without a filter, on the other hand, are useful for the continuous wavelet transform (Mitra et al., 2007).

As said by Masset (2008), the discrete wavelet method has two main wavelet filters; the father wavelet filter in a sequence of $g_l = (g_0,\ g_1, g_2, \dots, g_{L-1})$ and the mother wavelet filter in a sequence of $h_l = (h_0, h_1, h_2, \dots, h_{L-1})$ where L is an integer value. The mother wavelet filter is comprised of the following three fundamental features (Walden, 2001):

$$\sum_{l=0}^{L-1} h_l = 0 \quad (62)$$

$$\sum_{l=0}^{L-1} h_l^2 = 1 \quad (63)$$

$$\sum_{l=0}^{L-1} h_l h_{l+2n} = \sum_{l=-\infty}^{\infty} h_l h_{l+2n} = 0 \quad (64)$$

Wu (2006) states that the wavelet filter (i) integrates to zero, i.e. have zero-sum, (ii) has unit energy and (iii) is orthogonal to its even shifts for all non-zero integers. Crowley (2007) says that the last two properties in Equation (63) and (64) are the orthonormality conditions, characterizing the father wavelet in filter terms. According to Gencay et al. (2010), the first equation determines the changes in the data under investigation; one can make sure that the coefficients preserve energy with the second property, i.e. they will have the same overall variance alike the original data has. The last property in Equation (64) ensures that one can obtain a perfect multiresolution decomposition. Based upon the aim, the desired wavelet filter should meet the following conditions (Masset, 2008):

a) Orthogonality: It means that both the wavelet and the scaling coefficients' information are different. Besides, it guarantees that the wavelet decomposition and the original series have the same energy, i.e. variance. By the way, the mostly used wavelet family, the Daubechies (both dbN and $LA(N)$) satisfy this condition.

b) Symmetry: This property guarantees that there will be no phase shift in the wavelet transform components. Putting differently by Lindsay et al. (1996), its important stems from reducing the phase shift of features during the decomposition process. Except the Haar wavelets, the majority of wavelets do not meet this condition. It should be emphasized that this is more prominent for the DWT but it is less important for MODTW.

c) Smoothness: The number of continuous derivatives of the basis function determines the smoothness of a wavelet filter degree. The simplest wavelet family, the Haar, is the least smooth wavelet example and it is suitable for a data which is less smooth and has pure jumps.

d) Number of vanishing moments

Masset (2008) remarks that the smoothness and the number of vanishing moments conditions are both determined by the wavelet filter and its length number, N, such as dbN where N value specifies the fitting. But, unfortunately, increasing N to better a fit increases the need for boundary conditions more severe according to him. By the way, as noted by Misiti et al. (2013) the requirement of having a filter is not necessary for the CWT.

3.3.2.3 Maximal Overlap Discrete Wavelet Transform (MODWT)

Before delving into the maximal overlap DWT (MODWT) method, it should be noted that it is an alternative method to the DWT method, i.e. a modified version (Gencay et al., 2002). This method has several different names in literature such as "undecimated DWT" by Shensa (1992), "stationary DWT" by Nason and Silverman (1995), "translation-invariant DWT" by Coifman and Donoho (1995), and "time-invariant DWT" by Pesquet et al. (1996). Actually, the widely used term "maximal overlap" by researchers is firstly mentioned in the paper of Percival and Guttorp (1994) due to the relationship between the estimators and Allan variance (Percival and Mofjeld, 1997). Gencay et al. (2001) note that this term is chosen for clarifying that for computation process all possible shifted time intervals were used.

The main reasons behind this method are given by (Gencay et al., 2002). They state that by using DWT method, one can obtain an approximate but efficient decorrelating certain processes in a finite time duration where the Mallat's pyramid algorithm is used. It is already known that the DWT computation is done by subsampling the filtered output depicted in Figure **3-19**. Although the MODWT method possesses some important attributes that the DWT does not have, it forgoes orthogonality property, i.e. it is not an orthogonal basis according to Jensen and Whitcher (2014) due to using moving averages (Hacker et al., 2014).

Actually, it is worth giving the general differences between the DWT and the MODWT methods provided by Percival and Mofjeld (1997) as follows:

1) The most challenging problem, researchers have to contend with, in the DWT is that the sample size must be an integer multiple of 2^J, i.e. it should be dyadic. Conversely, this requirement is not necessary for the MODWT, that is to say that for any sample size, N, is well defined for the MODWT of level J. They remark that it takes $O(N)$ multiplications for the DWT whilst it is $O(Nlog_2N)$ for the MODWT which is the same computational burden as in case of the FFT.

2) Just as in the DWT method, the MODWT can be used for a multiresolution analysis, variance, covariance and correlation analysis. The difference is that this modified method guarantees that both wavelet and scaling coefficients are shift-invariant. Strictly speaking, when one circularly shifts a time series by an integer value to the right direction, then the MODWT coefficients will be the same which is not valid for the DWT.

3) The MODWT coefficients, the detail and the smooths, are related to zero-phase filters. The significance of the zero phase attribute in a transform process is that the MODWT coefficients approximately will be lined up with the original time series under investigation. Put it differently, Jensen and Whitcher (2014) say that the number of MODWT coefficients for each scale is equal to N because the method does not decimate these coefficients which is the reason why this method is also called as non-decimated DWT by Shensa (1992).

4) Both the DWT and the MODWT can be used for variance analyzing. But, as noted by Crowley (2007), under a stationarity assumption, the MODWT yields a more statistically efficient wavelet variance than the DWT does.

A clear comparison of these two methods can be summarized in the following table which is given by Kang et al. (2011).

Table 3-1 Difference between the DWT and MODWT

Method Property	DWT	MODWT
Data size	$N = 2^J$	Any sample size N
Decomposition (MRA) procedure	Downsampling	Zero-phase filters
Circularly shifting condition	Not hold	Holds and Invariant
Efficiency in Variance	Less efficient	Asymptotically Efficient
Number of coefficients ($\boldsymbol{n_j}$)	$n_j < N$	$n_j = N$

Source: Kang et al. (2011).

After the comparing of wavelet transform methods, it is time to distinguish the Fourier and the wavelet methods.

3.3.3 Comparison between Fourier Transform and Wavelet Transform

In this section, the two main transform methods will be discussed and the reasons for choosing depending on the need one of these methods are summarized. Firstly, the similarities between them will be discussed.

Having mentioned before that the wavelet analysis is derived from the Fourier transform analysis. Graps (1995) lines up the similarities and the differences in their paper. The author (1995) says that both the transform methods are linear operations where these operations produce a data structure storing $log_2 N$ parts. The second similarity is about their mathematical property that is to say their inverse transform matrixes are equal to their transpose of original data. Putting differently by In and Kim (2012), this similarity is about their reversibility property. Equivalently saying, these two methods allow one to decompose a time series and then reconstruct this original data using these transformed data. The last similarity is "basis" functions they have. Graps (1995) notes that because these basis functions are localized in the frequency domain, this property makes possible to pick out frequencies and calculate power distributions.

Despite their similarities, they also have some discrepancies which are the reasons behind the preferring the wavelet methods to Fourier methods. In and Kim (2012) present three advantages of wavelet transform over Fourier transforms. The first one is that the wavelet analysis makes available to break a data given into the scales instead of the frequencies. According to Crowley (2007), this property allows one to identify the events and fluctuations hidden and plot these in the time and frequency domain, simultaneously.

The second advantage is about the time-frequency resolution differences as shown in the figure below. The two upper plots of this figure show frequency and time domain representations which have mentioned before. Strictly remarking, the left-hand side plot shows only frequency resolution where time information is lost and the right-hand side plot depicts only time resolution where frequency information is lost. On the other hand, the two lower plots illustrate the Short-Time FT (Gabor) and Wavelet Transform where the window size is fixed for the former transform and varies for the latter transform method.

Graps (1995) makes clear this advantageous. By wavelet transform method, the author (1995) states, one can obtain detailed frequency analysis and isolate signal discontinuities at the same time. This method is done by long low-frequency and short high-frequency basis functions, respectively, which is depicted in the figure above.

The last advantage is the capability to use both the stationary and non-stationary data. As noted by Ramsey (2014), the Fourier analysis requires a data to be covariance stationary whilst it is not obliged for the wavelets because the majority of time series have a unit root, i.e. these show quite complex patterns over time. Put differently by Gallegati (2005), the wavelet analysis can handle both the global movements (in case of Fourier) and local movements of the data that makes it an ideal analysis tool for both stationary and non-stationary time series.

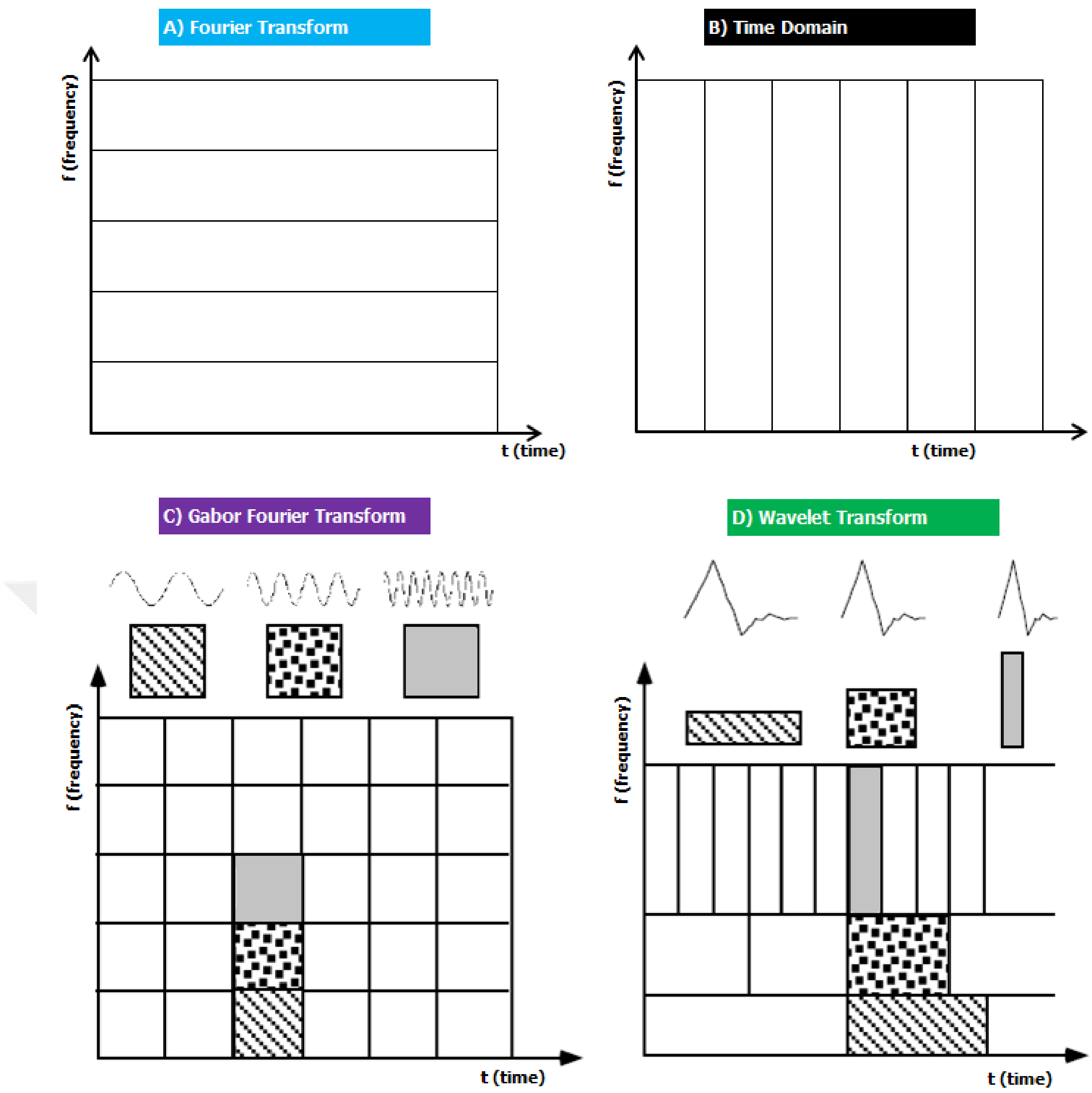


Figure 3-20 Comparison of the Fourier and Wavelet Techniques

Source: Gencay et al. (2002).

According to Strang (1993), roughly, the Fourier transform method is the right choice for music decompositions while the wavelet transform methods are better for image decompositions. The reason for this task sharing in transformation process is explained as music is in sinusoidal form whereas images have sharp edges.

3.3.4 Wavelet Analysis

Baruník et al. (2014) assert that the core feature of wavelet analysis is that one can decompose a time series in order to analyze the relationship according to time periods, i.e. scales which are determined as investment horizons. By using these scales, one can study the short, the medium or the long economic relationships individually which is not easy in the classical econometric analyzing methods. They point out that if one expects different economic relationship patterns at various investment horizons, then it would be a good decision to implement wavelet analysis to unmask the hidden interesting characteristics of the data.

In literature, the analysis types that are generated by wavelet method are classified into two groups according to Percival and Mofjeld (1997). The authors (1997) state that the first type of coefficients gives researchers an ability to implement an additive decomposition method namely multireso1ution analysis (MRA). Using this method, one can decompose a time series into several segments, which are called “details” and “smooth”. Each detail, w_j, component refers to high and low-frequency variations at a specific time scale while the smooth, s_J, component gives information about the low-frequency variations. The second type of analysis according to them is done via the sample variances, covariances and correlations of the time series across different time scales and over time.

3.3.4.1 Multiresolution Analysis (MRA)

Before delving into the multiresolution analysis, at first, the time scale and multiresolution terms should be defined. Gencay et al. (2010) describe “time scale” as a “resolution”. The authors (2010) mention that via using wavelet analysis (DWT or MODWT), one can achieve a coarse and a fine resolution simultaneously. In wavelet language, these coarse resolutions and the fine resolutions are defined as the wavelet scale (Jensen and Whitcher, 2014). Remarking the inverse relationship between frequency and scale, for instance, a coarse resolution of a time series is obtained at high time scales (long-term), i.e. at low frequencies, while a fine resolution is obtained at high frequencies, i.e. at low time scale (short-term)., there exists a high resolution. Roughly speaking, moving from low frequencies (high time scales) to high frequencies (low time scales) one can get a more fine resolution of the time series because of the averaging process.

Briefly defining, using multiresolution analysis, one can obtain both the details of the time series and the overall structure as if he brought the camera closer and then moved it back (Hubbard, 2005). As noted by Koenderink (1984), a multiresolution representation renders a simple hierarchical framework for a signal or an economic data, namely one can obtain information of an image at all scales at the same time. A multiresolution decomposition method, according to Gallegati (2012), provides a scale-invariant interpretation of the economic data or an image. Mallat (1989a) states that the scale of an image depends on the distance between the optical center of the camera and the scene, i.e. the interpretation of the scene varies if the image scale changes due to resolution parameters $(r_j)_{j \in \mathbb{Z}}$. For instance, if the camera gets "a" times nearer to the scene, then each object of the scene is measured at a resolution "a" times bigger where $a \in \mathbb{R}$ for all integers $j = r_j = a^j$. Mallat (1989a) also dictate that at different resolution level, say at a coarse resolution level, the details of an image presents the image “context”. Hence, it can be said that the analyzing process should follow a path from coarser to finer resolution levels gradually.

For a clear understanding, Gencay et al. (2010) give a simple example. In this example, they analyze the volatility of the Dow Jones Industrial Index (the DJIA) with different sized windows. At the end of the day, (2001-01-03), a 2.82% daily increase was reported which is classified as volatile. To find out when this volatility is observed, one should inspect the day at the different time intervals. At the 5-minutes windows, they could not observe this volatility but when they study the data from an hourly scale, they witnessed two high volatility examples at the beginning of the session and around 13:30. Apart from these two examples, they conclude that the market was not volatile according to hourly scale point of view. Upon this result, they remark that need for a successful method to reveal the market dynamics at different frequencies arises. Fortunately, wavelet method fulfills this need.

As given by Cascio (2007), the multiresolution decomposition can be described as a set of wavelets related to the cells of a grid depicted in the figure below. Besides, the partitioning of the cells defined as Mosaic Diagram is done according to discrete wavelet transform method, namely conventional dyadic multiresolution analysis. The example size, N, in this figure is $128 = 2^7$ and is restricted to be a dyadic form, thereby, the scale resolution level is $s = 2^j$.

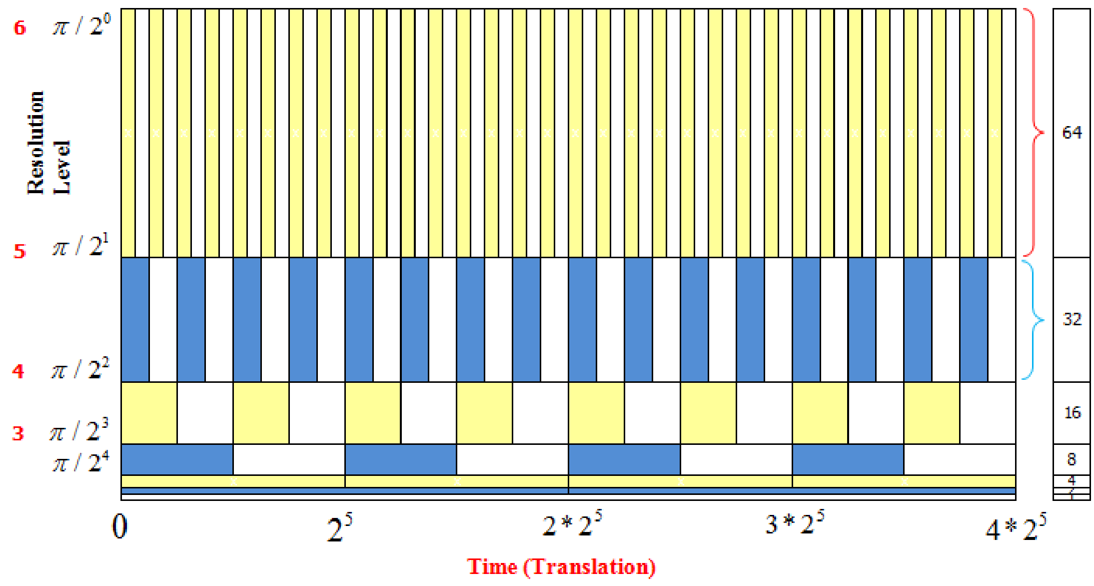


Figure 3-21 DWT Multiresolution Decomposition (MRD)

Source: Cascio (2007).

Moreover, unlike the classical DWT which has fewer coefficients at coarser scales, MODWT has a number of coefficients equal to the sample size at each scale and thus is over-sampled at coarse scales (Saâdaoui, 2013).

It is seen in the figure above that y-axis represents the resolution level while the x-axis represents time (translation). In particular, it should be firmly noted that at this stage instead of the frequency term, the level of resolution term is chosen because the concept of frequency is related merely to sinusoids (Priestley, 1996). In this figure, the highest observable resolution level is the Nyquist frequency of $\pi/2^0$ radians per observation interval (Cascio, 2007). According to conventional dyadic multiresolution analysis, the frequency intervals in this figure range from $\pi/2^{j-1}$ to $\pi/2^j$ where $j = 1, 2, \ldots, J$, i.e. descending from high frequencies to low frequencies (Cascio, 2007). At $\pi/2^0$ resolution level, the number of wavelet coefficient is equal to $64 = 2^6$ and their frequency range is the biggest one. The other coefficient numbers can be found via using a 2^j formula where $1 < j \leq 7$. The method used here is known as Pyramid Algorithm method by Mallat (1989a). The point here is that when resolution level decreases, then the coefficient number also decreases by a dyadic factor. If one sum these coefficients by frequencies using $N_{DWT} = 1 + 2^j + 2^{j+1} + 2^{j+2} + \cdots + 2^{J-1}$, then the total number of coefficients is found as 127 which is smaller than $2^j = 2^7 = 128$. The missing coefficient is not depicted in the figure because of the resolution level. When resolution level becomes 0, then the pyramid algorithm stops calculating process where only one coefficient is produced (Nason, 2010). Putting differently by Addison, 2002), the reason for one single smooth coefficient remaining is of interest the signal mean.

Let us turn our attention to MRD in case of MODWT. Jensen and Whitcher, (2014) dictate that in wavelet vernacular, multiresolution analysis refers to decomposition a time series into weighted moving average values and the information necessary to restructure the signal from these average values. Furthermore, Gallegati (2005) mentions that these "smooth" coefficients, $s_{J,k}$, and "details" coefficients, $d_{j,k}$, are formulized as follows:

$$s_{J,k} = \int X(t)\phi_{J,k}(t)dt \tag{65}$$

$$d_{j,k} = \int X(t)\psi_{j,k}(t)dt \tag{66}$$

With these coefficients, wavelet representation of a time series, $x(t)$, can be constructed as (Gallegati et al., 2017):

$$\begin{aligned} x(t) = \sum_k s_{J,k}\, \phi_{J,k}(t) + \sum_k d_{J,k}\, \psi_{J,k}(t) + \sum_k d_{J-1,k}\, \psi_{J-1,k}(t) + \cdots \\ + \sum_k d_{1,k}\, \psi_{1,k}(t) \end{aligned} \tag{67}$$

or, equivalently is rewritten (Yang and Hamori, 2015):

$$S_J(\mathrm{t}) = \sum_k S_{J,k}\, \phi_{J,k}(\mathrm{t}) \tag{68}$$

$$D_j(\mathrm{t}) = \sum_k d_{j,k}\psi_{j,k}(\mathrm{t}) \tag{69}$$

where $S_J(\text{t})$ component stands for the smooth behavior of the $x(t)$ time series, whereas $D_j(\text{t})$ component stands for the scale deviations from the smooth process (Alzahrani et al., 2014), in other words, it is "degree of difference" of the observations at each scale and location (Nason, 2010).

Crowley (2007) states that the wavelet coefficient refers to an "atom" and the set of coefficients stored at each scale are defined as "crystals" according to wavelet terminology. The multiresolution decomposition (MRD) of the $x(t)$ variable is then given by the formula below (Crowley and Hallett, 2014):

$$x(t) = S_J(\text{t}) + \sum_{j=1}^{J} D_j(\text{t}) \tag{70}$$

or

$$x(t) = D_J(t) + D_{J-1}(t) + D_{J-2}(t) + \cdots + D_1(t) + S_J(t) \tag{71}$$

where $j = 1, 2, 3, \ldots, J$ represents the number of multiresolution decomposition scales.

Hacker et al. (2014) indicates that since this decomposition process evidently is an additive decomposition, one can attain the original $x(t)$ time series by summing up all components which can be detailed in the following example.

Let us assume a financial time series $r_{Bist}(t)$ with $N_{\text{Bist}} = 1939$ daily return observations. The wavelet multiresolution level of decomposition of r_{Bist} is chosen 7 arbitrarily where the maximum level is $J_{Max} = round\big(log_2(1989)\big) = 10$ (Barragán et al., 2015). This MRD is comprised of 7 wavelet details coefficients and a single wavelet smooth coefficient:

$$r_{Bist}(t) = d_1 + d_2 + d_3 + d_4 + d_5 + d_6 + d_7 + s_7$$

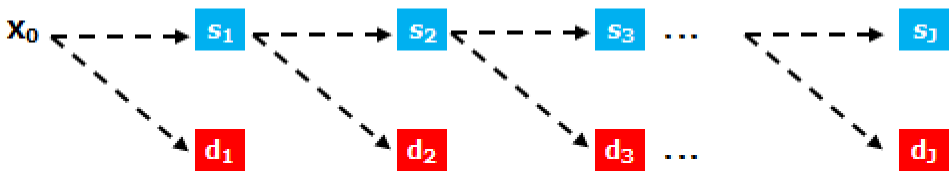


Figure 3-22 MODWT Multiresolution Decomposition Scheme

Source: Miner (1998).

It is convenient at this point to remark that if the wavelet used has orthogonality property, then the reconstruction of this $r_{Bist}(t)$ time series must be perfect. Let us prove this with different MRD levels. For example, $r_{Bist}(t)$ times series is decomposed with $J = 2, 3, 4, 5, 6,$ and 7 multiresolution levels. For brevity, only MRD of the first observation, 1.3898%, is shown in the table below.

Table 3-2 Multiresolution decomposition with different levels

MRA	d_1	d_2	d_3	d_4	d_5	d_6	d_7	s_j
J = 2	0.0073445	0.0035887					[s_2]	0.0029649
J = 3	0.0073445	0.0035887	0.0042019				[s_3]	-0.001237
J = 4	0.0073445	0.0035887	0.0042019	-0.0002594			[s_4]	-0.000978
J = 5	0.0073445	0.0035887	0.0042019	-0.0002594	0.0007934		[s_5]	-0.001771
J = 6	0.0073445	0.0035887	0.0042019	-0.0002594	0.0007934	-0.0014542	[s_6]	-0.000317
J = 7	0.0073445	0.0035887	0.0042019	-0.0002594	0.0007934	-0.0014542	-0.0007273	0.0004106

Source: Author's calculation.

It is evident from the Table 2-2 that when $J = 2$ is chosen, the necessary formula will be like that

$$r_{Bist}(t) = \text{d1} + \text{d2} + \text{s2}$$

and then summing these three coefficients,

$$r_{Bist}(t) = 0.73445\% + 0.35887\% + 0.29649\%$$

the result will be 1.3898%, which is the same for the first observation. Hence, the result will be the same for all the different J levels due to orthogonality. Besides, it should be easily seen in this table that, the smooth coefficient is equal to the sum of the following the detail and the smooth coefficient, d_J and s_J respectively. Upon this equation, the other smooth coefficients can be calculated as:

$$S_2 = D_3 + S_3 \Rightarrow S_2 = 0.0042019 + (-0.001237) = 0.0029649$$

$$S_3 = D_4 + S_4 \Rightarrow S_3 = -0.0002594 + (-0.000978) = -0.001237$$

$$\dots$$

$$S_6 = D_7 + S_7 \Rightarrow S_6 = -0.0007273 + (0.0004106) = -0.000317$$

The interpretation of the MRD using the DWT or MODWT method is important because it refers to the time period at which point an activity occurs. For example with a daily return time series, $r_{Bist}(t)$, Table 2-3 demonstrates these time periods according to scale level, J (Crowley, 2005).

Table 3-3 Frequency interpretation of MRD scale levels

Scale Level, J	**Scale Crystals (Details and Smooth)**	**Period (A, Q, M, W, D)**
1	d1	2 – 4
2	d2	4 – 8
3	d3	8 – 16
4	d4	16 – 32
5	d5	32 – 64
6	d6	64 – 128
7	d7	128 – 256
	s7	> 256

Source: Gallegati (2005).

According to Gallegati (2014), time periods in wavelet transforms are changed at scale levels via 2^{j-1} where this scale corresponds to frequencies in the interval $\left[{}^{1}/_{2^j}, {}^{1}/_{2^{j+1}}\right]$. For the example given above, scale 1, d1, corresponds to $2 - 4$ days; scale 2, d2, corresponds to $4 - 8$ days, and scale 7, d7, corresponds to $128 - 256$ days. The smooth coefficient, s7, by the way, corresponds to $+256$ days. However, note that, by employing this MRD interpretation method, the scale time intervals will be the same for the different time periods, saying monthly, d1 will correspond to 2 –

4 months; d2 will correspond to $4 - 8$ months, and so on for the signal under examination.

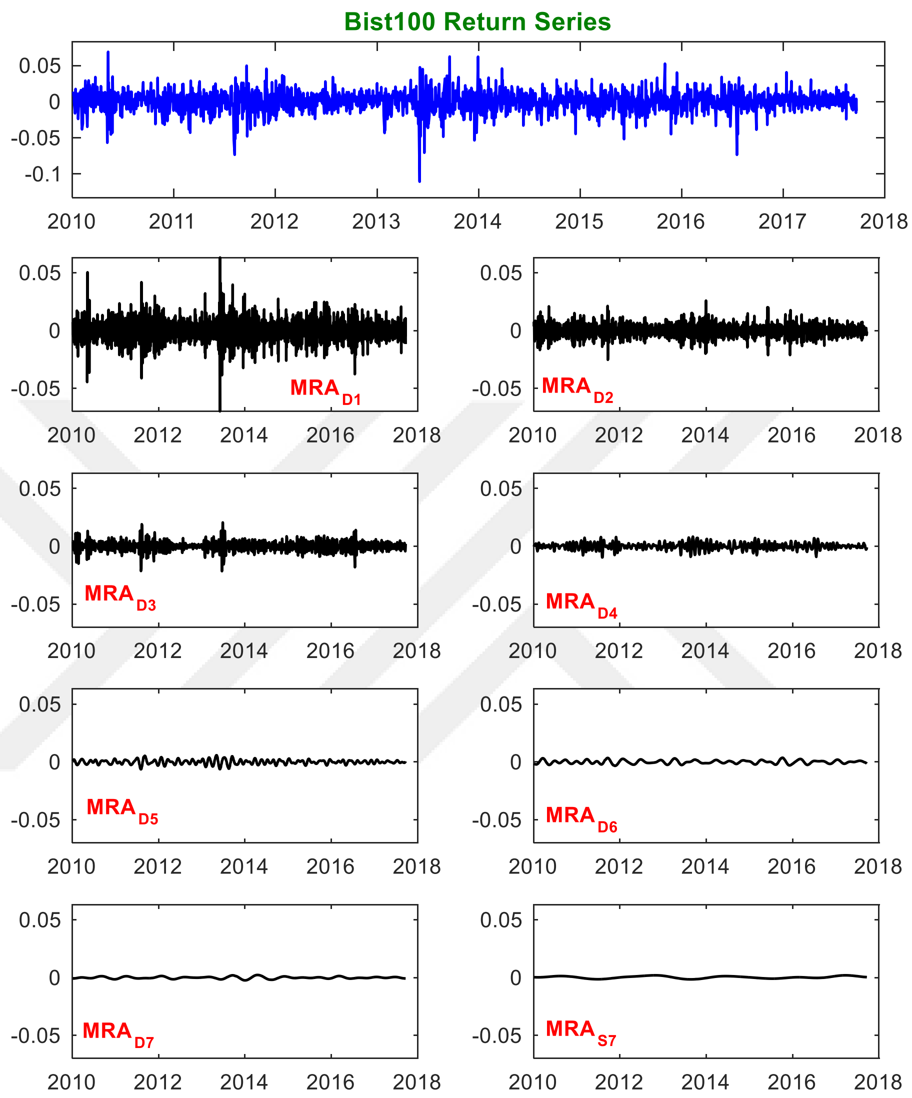


Figure 3-23 Multiresolution Decomposition of Time Series by Scale Levels

Source: Author's calculation.

The MRA coefficients using MODWT methods according to these scales are depicted in the figure below. Apart from the top plot, the others represent the details and the smooth coefficients, namely, scale crystals in Figure 3-23. It can be easily seen that as the scale level increases, the variation of each scale decreases where the y-limit interval values are the same. In other words, the higher scale level, the lower variation in scale will occur, namely a smoother time series becomes. As mentioned above, if one sums all these scale crystals, then the result will be exactly as the return series. Moreover, the numbers of each crystal are the same of the observation number of the original return series.

It should be pointed out that if the MODWT method had not been available, then one might have modified the observation number to the 2^J condition. In this case, the observation number of the example above, $r_{Bist}(t)$, should be reduced to $N = 2^{10} = 1024$ or should be increased to $N = 2^{11} = 2048$ which leads an information lost or an extra cost to find the observations, respectively. Incidentally, in case of the Fourier transform, this condition is not necessary according to Weeks (2010). For completing N to 2^{11}, for example, Matlab adds $59[= 2^{11} - 1989]$ observations which are equal to zero. Weeks (2010) says that appending zeroes called zero-padding will not affect the results because it does not add any information.

3.3.4.2 Wavelet Variance, Covariance and Correlation by Scale

In this section, we will define the wavelet variances, covariance, and correlation in the case of the DWT and the MODWT, respectively. These concepts will be discussed by using two different financial time series; the daily returns of Bist100 Index and the Dollar/TL exchange rates.

Percival (1995) defines the wavelet variances as a variance decomposition of a time series into several components related to scales. Differently stating, a variance, for instance, $\sigma^2_{RBist}(\lambda)$, shows a changing in the variance of a time series from one scale λ to the next scale, i.e., time period. Besides, Lindsay et al. (1996) say that thanks to the orthogonality of DWT, one can partition the variance of a time series on a scale-by-scale basis, leading to the notion of the scale-dependent wavelet variance like frequency-dependent Fourier power spectrum. Percival and Walden (2000) present three reasons with respect to why the scale-dependent wavelet variance is so important, especially in physical sciences. The most important reason is that the wavelet variance gives an opportunity to researchers to analyze this variance scale-by-scale just as the spectrum decomposes the variance across frequencies.

Having defined the concept of wavelet variance, let us start with the sample variance of the $r_{Bist}(t)$ series in the DWT expressing with

$$\hat{\sigma}^2_{RBist} = \frac{1}{N}\sum_{i=1}^{N}[r_{Bist}(x_i) - \overline{r_{Bist}}]^2 = \frac{1}{N}\sum_{i=1}^{N}[r_{Bist}(x_i)]^2 - \overline{r_{Bist}}^2 \tag{72}$$

where M and $\overline{r_{Bist}}$ represent the observation number and the average value of the daily returns of Bist100 Index, $\overline{r_{Bist}}$. Lindsay et al. (1996) assert that the sum of the squares of this returns series can be rewritten as the sum of the squares of the wavelet coefficients owing to the wavelet basis orthonormality

$$\sum_{i=1}^{N}[r_{Bist}(x_i)]^2 = \sum_{j=1}^{L}\sum_{k=1}^{n_j} D^2_{j,k} + N\overline{r_{Bist}}^2 \tag{73}$$

Consequently, the sample variance can be rewritten in terms of the wavelet coefficients as follows:

$$\hat{\sigma}^2_{RBist} = \frac{1}{N}\sum_{i=1}^{L}\left\{\sum_{k=1}^{n_j} D^2_{j,k}\right\} \tag{74}$$

And then each scale variance is

$$\hat{\sigma}^2_{RBist,j} = \frac{n_j}{N}\left(\frac{1}{n_j}\sum_{k=1}^{n_j} D^2_{j,k}\right) = \frac{\hat{\sigma}^2_{D,j}}{2^j} \tag{75}$$

where $n_j = N/2^j$ and the sample wavelet variance for each scale (crystals), $\hat{\sigma}^2_{D,j}$, is presumed to be zero mean. Before proceeding further, an important restriction topic which determines the DWT coefficient numbers for each scale should be discussed carefully. This restriction is called boundary condition. Masset (2008) says that the filters mentioned before are related to the particular aim of analysis and they differ according to property and ability to match with the characteristics of the time series under investigation. Besides, for transforming with wavelet filters, it is necessary to decide which the filter length will be used in both cases of the DWT and MODWT. The author (2008) states that the filter length is of interest to the length of the time series under study. To put it in another way, roughly, the longer time series, the longer filter is required.

Masset (2008) asserts that the boundary conditions occur because of two main reasons. The first problem is of interest in the DWT where this method requires the time series to be dyadic. If the time series is not dyadic, e.g. $2^j < N < 2^{(j+1)}$, then one will be confronted with two choices. If the data length is $N < 2^{(j+1)}$, for instance $N(r_{Bist}) = 1939$, one has to complete this series by 109 observations to $2^{(10+1)} = 2048$, or to remove some observations, 915 observations to 1024. The second alternative is not preferable due to an information loss.

According to Masset (2008), the reason for the second problem is related both the DWT and the MODWT. The transforming process is done for all observations, i.e. from $t = 1$ to N. The second problem occurs if the number of observations before t required by the convolution operator is lower than $L - 1$. If so, then the second choice mentioned above, i.e. removing data, is worthless. Hence, the first choice remains to implement, namely completing data to $2^{(j+1)}$. Here, two methods arise. The first one is related to zero-padding. The second method is called "reflection" or "mirror", i.e., extending the series to length $2N(r_{Bist})$ that is (Cornish et al., 2006):

$$x_0, x_1, x_2, \dots, x_{N-2}, x_{N-1}, x_{N-1}, x_{N-2}, \dots, x_2, x_1, x_0$$

According to Gencay et al. (2002), implementing this method does not change the sample mean or the sample variance because in this method all coefficients are duplicated once. Hence, to form an unbiased estimator for wavelet variance, covariance, and correlation, according to Gencay et al. (2001), one has to remove all coefficients that affected by the boundary condition.

If one implements the boundary condition in the DWT, then the DWT based sample variance will be (Lindsay et al., 1996)

$$\hat{\sigma}^2_{RBist,j} = \frac{1}{2^j \, \overset{\frown}{n}_j} \left(\sum_{k=1}^{\overset{\frown}{n}_j} D^2_{j,k} \right) = \frac{\hat{\sigma}^2_{D,j}}{2^j} \tag{76}$$

where $\overset{\frown}{n}_j$ represents the DWT coefficient numbers after boundary condition imposed for each scale and they are calculated as follows:

$$\overset{\frown}{n}_j = \left\lfloor \frac{N}{2^j} \right\rfloor - \left(\frac{L-2}{2} \right), \qquad for\ j = 1; \tag{77}$$

$$\overset{\frown}{n}_j = \left\lfloor \frac{N}{2^j} \right\rfloor - \left(\frac{L-2}{2} + \left\lfloor \frac{L}{2^j} \right\rfloor \right), \qquad for\ j = 2; \tag{78}$$

$$\overset{\frown}{n}_j = \left\lfloor \frac{N}{2^j} \right\rfloor - \left(\frac{L-2}{2} + \left\lfloor \frac{\frac{L}{2} + \left\lfloor \frac{L}{4} \right\rfloor}{2} \right\rfloor \right), \qquad for\ j = 3; \tag{79}$$

$$\overset{\frown}{n}_j = \left\lfloor \frac{N}{2^j} \right\rfloor - (L-2), \qquad for\ j \geq 4; \tag{80}$$

where $\overset{\frown}{n}_j$ represent the number of the DWT coefficients with a brick wall boundary condition and $\lfloor x \rfloor$ stands for the greatest integer value lower than or equal to x value. However, by employing this method, Cornish et al. (2006) state that the number of unaffected coefficients decreases while scale level increases. With $N = 1024$ a time series, the first 1024 observations of $N(r_{Bist})$, is the number of DWT coefficients for each scale after implementing boundary condition will be $\overset{\frown}{n}_1 = 512 - 3 = 509$, $\overset{\frown}{n}_2 = 256 - 5 = 251$, $\overset{\frown}{n}_3 = 128 - 6 = 122$, $\overset{\frown}{n}_4 = 64 - 6 = 58$, $\overset{\frown}{n}_5 = 32 - 6 = 26$, $\overset{\frown}{n}_6 = 16 - 6 = 10$, and $\overset{\frown}{n}_7 = 8 - 6 = 2$. Note that, unlike MODWT, the DWT coefficient numbers for each scale decreases by the 2^j factor. For example, for the scale level $j = 3$, the observation number of this crystal is 128.

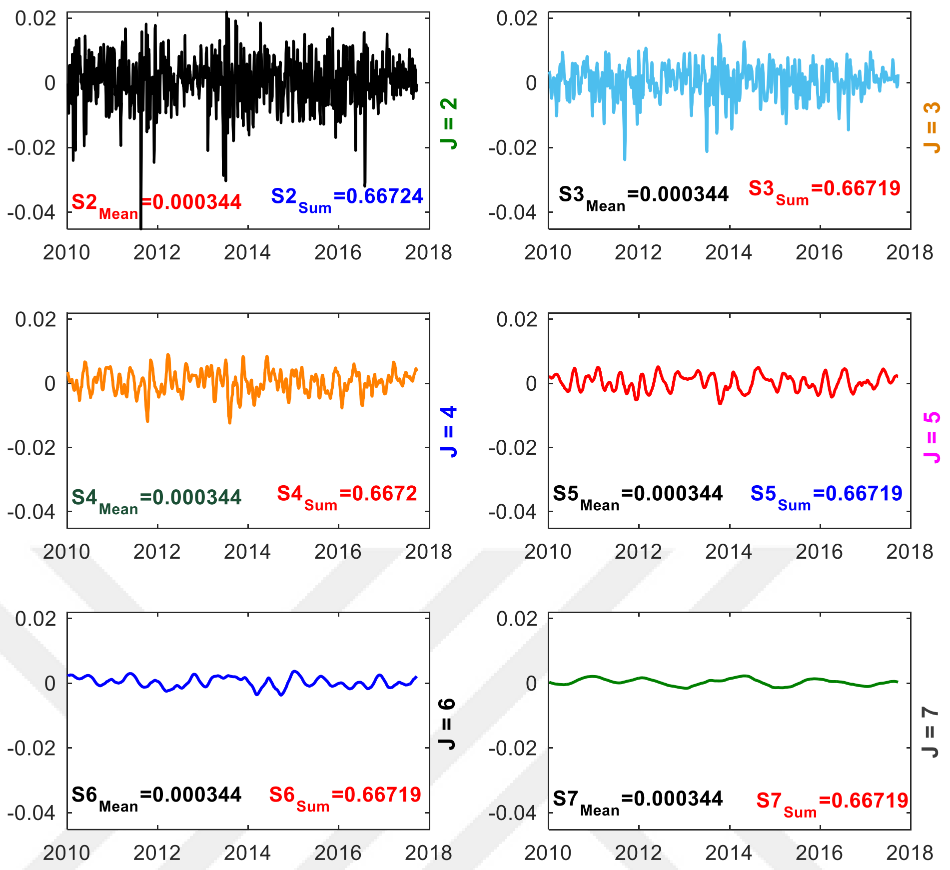


Figure 3-24 MODWT Scaling Coefficients by Different Scale

Source: Author’s calculation.

In the case of MODWT, the wavelet coefficients and scaling coefficients for computing variance and correlation estimations will be slightly different according to Whitcher (1998). Before proceeding further, it is required to mention about the energy decomposition analysis, which is another important characteristic of the wavelets for the analysis of signal or time series (Gallegati and Ramsey, 2013). Due to energy preserving property the sum of the energies of the two crystals, i.e., wavelet and scaling coefficients, is equivalent to the total energy of the original data. Besides, Percival and Mofjeld (1997) show that for any sample size, N, with an integer scale level, $J \geq 1$, the MODWT coefficients for computing wavelet variance result in an energy decomposition:

$$\|RBist\|^2 = \sum_{j=1}^{J} \left\|\widetilde{\mathcal{W}}_j\right\|^2 + \left\|\widetilde{\mathcal{V}}_j\right\|^2; \quad for\ j \geq 1; \tag{81}$$

where $\widetilde{\mathcal{W}}_j$ and $\widetilde{\mathcal{V}}_j$ refers to wavelet coefficients and scaling coefficients generated by MODWT function for wavelet variances. It is evident that, as stated by Percival and Mofjeld (1997), this equation will allow one to partition the sample variance by resolution level. Roughly speaking, these wavelet coefficients capture the deviations of $RBist$ time series from its long run mean value at the different resolution levels. Hence, as dictated by Masset (2008), the mean and the sum value of this time series are equal to the scaling coefficient's mean and the sum for each decomposition process, i.e. for each scale j.

It can be seen from the figure above that the variation in each scale decreases while frequency decreases, namely the crystals become smoother while the scale level increases. The point here is that the mean and the sum values are not changed. For instance, if one chooses $j = 2$ as decomposition level, then the scaling coefficient's mean value and sum value will be $\bar{s}_2 = 0.000344$ and $\sum_{i=1}^{N} s_2(i) = 0.66724$, respectively. These two values will be the same for each scale decomposition level. On the other hand, the wavelet coefficients have, not given in this figure, zero mean for each decomposition process if the filter length is $L \geq 2$ according to Percival (1995). Besides, Crowley (2007) remarks that having zero means leads to variance analysis process to be regarded as energy decomposition.

The wavelet variance for each scale, $\lambda_j = 2^{j-1}$, in the case of MODWT is defined by Whitcher (1998)

$$\tilde{v}_{RBist}^2(\lambda_j) \equiv \frac{1}{\widetilde{N}_j} \sum_{l=L_j-1}^{N-1} \widetilde{w}_{j,t}^2 \tag{82}$$

where L stands for the filter length. Gencay et al. (2002) state that $\widetilde{N}_j$ and L_j parameters in the equation above are the number of coefficients unaffected and the number of coefficients affected by the boundary conditions for each scale. These two parameters are defined as

$$\widetilde{N}_j = N - L_j + 1 \text{ where } L_j = \left[(L-1) * (2^j - 1)\right] + 1$$

Gencay et al. (2002) remark that the normalization factor, $2\lambda_i$ is not required here because the wavelet filter of MODWT is a rescaled version of the DWT wavelet filter.

Briefly remarking, one of the differences between the DWT and the MODWT is that the MODWT coefficients and MRA coefficients are equal to the length of the original time series. However, due to boundary conditions, these numbers, only for MODWT coefficients, are not equal anymore. Using the equation above, we obtain a result for $RBist$ time series as:

$$\widetilde{N}_1 = 1939 - [(8-1) * (2^1 - 1) + 1] + 1 = 1939 - 8 + 1 = 1932$$

$$\widetilde{N}_2 = 1939 - [(8-1) * (2^2 - 1) + 1] + 1 = 1939 - 22 + 1 = 1918$$

...

$$\widetilde{N}_7 = 1939 - [(8-1) * (2^7 - 1) + 1] + 1 = 1939 - 890 + 1 = 1050$$

It should be noted that the calculation process is done for LA(8) wavelet, which filter length is 8. After determining the non-boundary wavelet coefficients, one can easily compute the wavelet covariance and correlations scale-by-scale basis. The wavelet variances by scale for $RBist$ and $RDollar$ time series are depicted in the following figure.

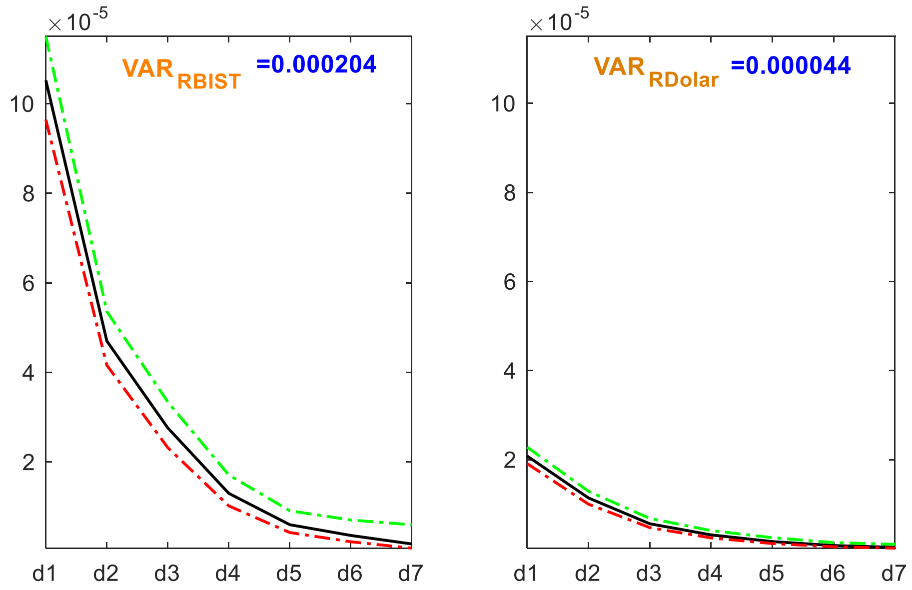


Figure 3-25 Wavelet Variances by Scale

Source: Author's calculation.

This figure depicts wavelet variances of two different time series where red and green dashed lines represent the lower and upper bounds at 95% confidence intervals, respectively. Besides, the estimated wavelet variances are drawn with a black line. Evidently, both of the two wavelet variances decline by scale.

After calculating wavelet variances by scale, it is straightforward to describe the wavelet covariance between two different time series, $RBist$ and $RDollar$. Roughly describing, the wavelet covariance is a measure of the degree of simultaneous correlation between two different wavelet crystals for each scale (Cornish et al., 2006). Besides, Gencay et al. (2001) state that the wavelet covariance can be described as the covariance relationship using the DWT between the two time series' scales λ_i, such as

$$\gamma_{RBist,RDollar}(\lambda_i) \equiv \frac{1}{2\lambda_i} Cov\left\{\widetilde{W}_{j,t,Bist}, \widetilde{W}_{j,t,RDollar}\right\} \tag{83}$$

Removing the affected coefficients by boundary conditions, Gencay et al. (2001) assert that an unbiased estimator of the wavelet covariance according to the MODWT will be as given

$$\gamma_{RBist,RDollar}(\lambda_i) \equiv Cov_{RBist,RDollar}(\lambda_i) = \frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} \widetilde{W}_{j,t,X} \widetilde{W}_{j,t,Y} \quad (84)$$

Just as the wavelet variance by scales, the wavelet covariances of the two time series are depicted in the following figure.

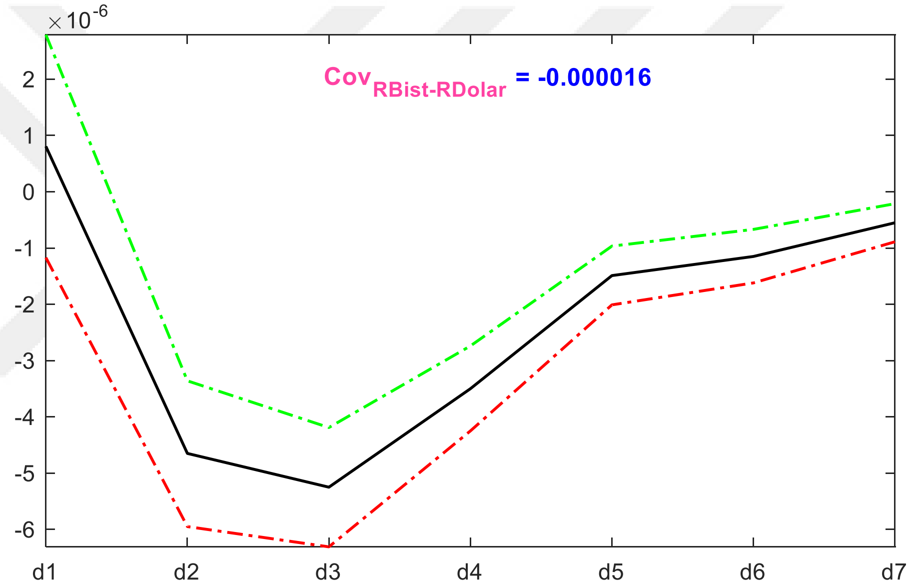


Figure 3-26 Wavelet Covariance by Scale

Source: Author's calculation.

Gencay et al. (2001) assert the wavelet covariance gives an ability to determine which scale level (time horizon) are significantly contributing to the covariance relationship between $RBist$ and $RDollar$ time series. The covariance coefficient for the original time series written in the figure is very small and negative. Looking at Figure **3-26**, one can see that the wavelet covariance decreases until scale d3, after this point it sharply increases.

Now we can turn our attention to the correlation analysis of wavelet scales. Baruník et al. (2014) reveal that the scale decomposition of a time series refers to an investment horizon. In other words, studying wavelet correlation analysis at each scale is equal to studying correlations of two times series at different investment periods. Hence, the wavelet correlation method provides an alternative way of studying the dependence between two time series. Not only in correlation analysis but also in wavelet variance and covariance analysis, one can reveal or show the relationships exactly which is usually not obvious in the original time series data. Because the wavelet correlation is calculated via wavelet variance and wavelet covariance for two time series, Whitcher (1998) presents this correlation coefficient using the MODWT non-boundary coefficients as

$$\hat{\rho}_{RBist,RDollar}(\lambda_i) = \frac{Cov_{RBist,RDollar}(\lambda_i)}{\sigma_{RBist}(\lambda_j) * \sigma_{RDollar}(\lambda_j)} \tag{85}$$

After defining the correlation coefficients by scale, Gencay et al. (2001) state that due to the inherent non-normality for small sample sizes, one should construct a confidence interval using a nonlinear transformation, namely Fisher's z transform for the wavelet correlation $\hat{\rho}_j$ (Lark and Webster, 2001) as given

$$\mathrm{h}(\hat{\rho}) = \frac{1}{2} log\left(\frac{1+\tilde{\rho}}{1-\hat{\rho}}\right) = tanh^{-1}(\hat{\rho}) \tag{86}$$

where $\hat{\rho}$ is a transformed, an unbiased estimated correlation coefficient, based on N independent samples. Whitcher et al. (2000) remark that for the $\hat{\rho}$, $\sqrt{N-3}[h(\hat{\rho}) - h(\rho)]$ has approximately an $N(0,1)$ distribution, i.e. it is distributed as a Gaussian with mean zero and unit variance. Whitcher (1998) notes that the $\sqrt{N-3}$ factor is used for a better approximation of the distribution. An approximate $100(1-2p)\%$ confidence interval for MODWT is given as

$$\tanh\left\{h\left[\hat{\rho}_{XY}\left(\lambda_j\right)\right]-\frac{\Phi^{-1}(1-p)}{\sqrt{\hat{N}_j-3}}\right\},$$
$$\tanh\left\{h\left[\hat{\rho}_{XY}\left(\lambda_j\right)\right]+\frac{\Phi^{-1}(1-p)}{\sqrt{\hat{N}_j-3}}\right\} \tag{87}$$

where $\tanh$ maps the confidence interval back to between $[-1]$ and $[+1]$ to generate confidence interval at 95% (Kang et al., 2011).

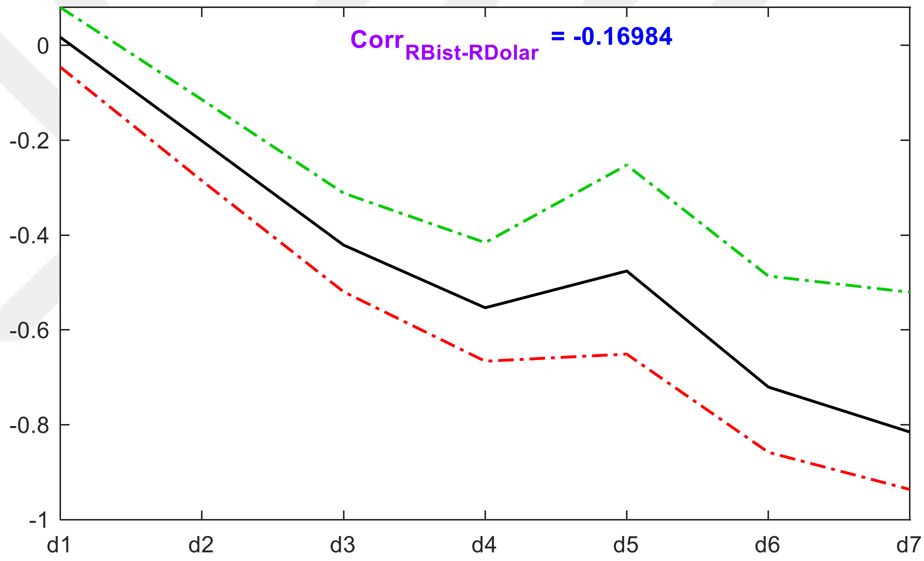


Figure 3-27 Wavelet Correlation by Scale

Source: Author's calculation.

Gencay et al. (2001) say that this random interval obtains the true wavelet correlation. In the equation above, the quantity $\hat{N}_j$ stands for the wavelet coefficient number in the case of DWT, not MODWT, scale-by-scale basis. Whitcher (1998) reminds that the validity of the assumption of uncorrelated observations for using the Fisher's z-transformation in the equation above depends only if it is believed that the wavelet coefficients at each scale do not have any non-stationary features and systematic trends. Since the DWT offers a reasonable measure of the scale-dependent sample size, their wavelet coefficients are used in the confidence interval computation. Putting differently, Ranta (2013) states that one can obtain more realistic confidence intervals using the DWT coefficients in the Fisher's z-transformation instead of the MODWT. Besides, Gencay et al. (2001) note that the reason for not given any adjustment about the approximate confidence interval is that this confidence interval does not exploit any information about the distribution condition, i.e. whether it is distributed by Gaussian or non-Gaussian condition.

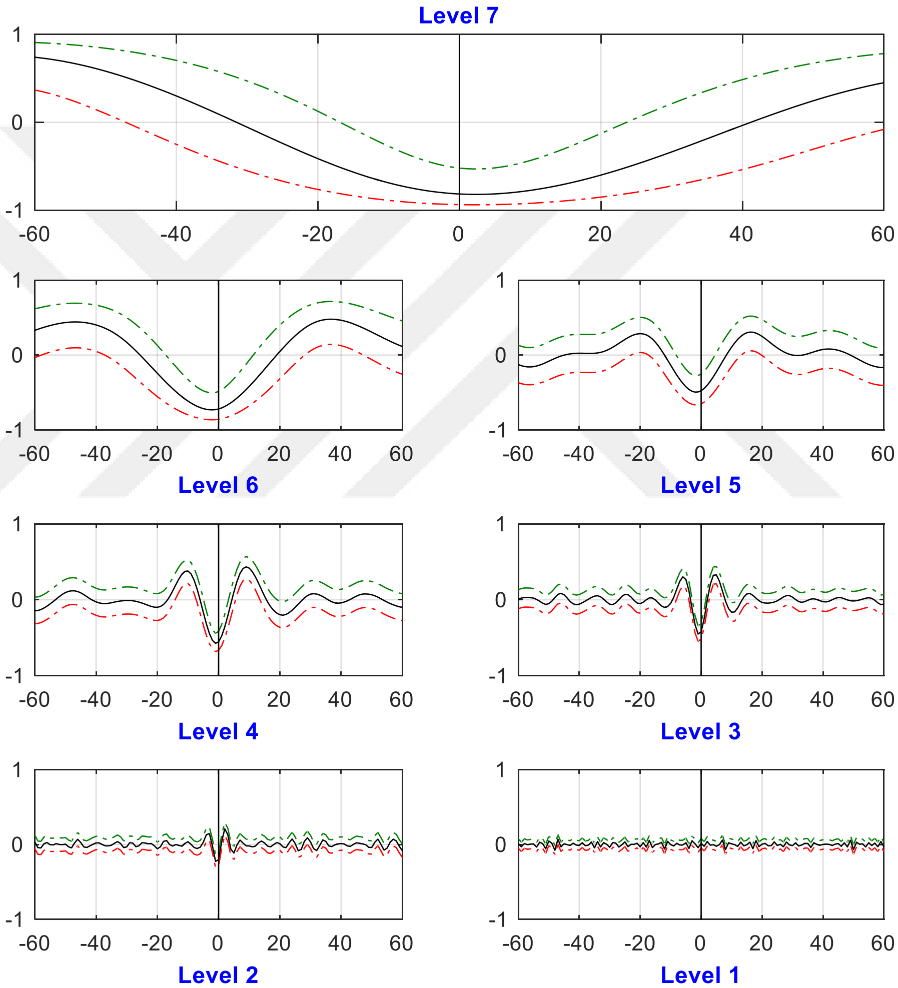


Figure 3-28 Wavelet Cross-Correlation by Scale Levels

Source: Author's calculation.

The correlation coefficients by wavelet scale of the two time series are illustrated in the following figure. It is seen that the correlation coefficient, −0.16984, for the original data is negative but statistically significant. Looking at the figure, it is evident that the correlation coefficients decrease as scale level increases apart from the scale d4.

Evidently, Figure **3-27** depicts wavelet correlation coefficients scale-by-scale basis, where the correlation coefficients range between $-1 \leq \hat{\rho}_{RBist,RDollar}(\lambda_i) \leq 1$ for all scales, as with the standard correlation coefficient. It is convenient at this point to derive the wavelet cross-correlation coefficients for a lead-lag analysis which is the standard cross-correlation statistic value (Kang et al., 2011).

Differently stated by Saâdaoui (2013), the cross-correlation analysis is similar to the standard time domain measure of a relationship between two different time series by scale levels, namely, it is related to bands of frequencies. Likewise, Whitcher et al. (2000) remark that this analysis method can be used to measure lead/lag relationships between two time series on the scale-by-scale basis. This coefficient value is developed by Whitcher (1998) as

$$\hat{\rho}_{\tau,RBist,RDollar}(\lambda_i) = \frac{Cov_{\tau,RBist,RDollar}(\lambda_i)}{\sigma_{RBist}(\lambda_j) * \sigma_{RDollar}(\lambda_j)} \tag{88}$$

where τ coefficient represents the lag value. At lag $\tau = 0$, the cross-correlation function is equal to basic wavelet correlation coefficient, $\hat{\rho}_{0,RBist,RDollar}(\lambda_i) = \hat{\rho}_{RBist,RDollar}(\lambda_i)$. Whitcher (1998) reminds that the magnitude of the wavelet cross-correlation is bounded $\left|\hat{\rho}_{\tau,RBist,RDollar}(\lambda_i)\right| \leq 1$. According to the equation described above the wavelet cross-correlation of two time series, $RBist, RDollar$, by scales are depicted in the following figure.Dajcman (2013) asserts that in the cross-correlation analysis, one can shift $RBist$ time series while does not move the second time series, $RDollar$, and then calculate the correlation coefficient between these two time series for each scale. This analysis method is usually called as lead-lag analysis in the wavelet literature, where "lag" means negative and lead refers positive value. Roughly, this method gives an ability to identify which time series is leading and which time series is lagging. Differently saying, the magnitude and significance value will show if the leading time series, for instance, $RBist$, has predictive power for the lagging time series, $RDollar$ for each scale level. In the light of the information given, the cross-correlation coefficients are almost the same for both time series for each scale level according to Figure **3-28**. It is noteworthy that the correlation coefficient gets bigger as scale level increases.

CHAPTER

4 DATA AND EMPIRICAL RESULTS

It is widely accepted that the stock price is influenced by many macroeconomic factors in long-term as well as some firm-specific and micro-events in short run. Moreover, the different behaviors of the different market participants are another important factor that also determines the stock prices. However, generally speaking, one of the main factors that have a great impact on the stock prices is of interest rates changes via in direct or indirect ways. For the aim of revealing these impacts hidden at different time horizons, there exists a special analytical tool that based on frequencies, namely, wavelets.

Regarding the scope of this research, first, we will introduce the required econometric methods. Later, we will discuss variance and correlation estimations by scale decompositions. Lastly, the empirical results of the causal relationship in time and frequency-domain will be compared.

4.1 Rationale for the Research Method

In this thesis, we aimed to study the interrelationship between the interest rate changes and the stock market index returns, at both the aggregate market and the sectoral levels by implementing time-domain and frequency-domain (wavelet-based causality and standard frequency causality test) methods. The main reason behind using the wavelet-based methods is that this method provides both a broad and narrow/detailed landscape of the time series which is not possible in time-domain methods. In this respect, we can answer the question of whether there exists a time-varying significant relationship between the underlying variables. In case of the existence of a significant relationship, we will shed light on the hidden relationship to uncover the possible impacts of one variable to another across different time scale periods (investment horizons).

There are several contributions of this thesis to the current literature for Turkey case. Besides the paper of Moya-Martínez et al. (2015) for Spain case, the main contribution is that the sample of the study (the aggregate and sectoral levels) and the econometric tools are more comprehensive than the studies conducted in previous years in the finance literature. Not only standard tests, but also the new and widely used by researchers in the econometric literature causality tests of symmetric, asymmetric, and frequency methods will be implemented to uncover the interrelationship between the variables under investigation.

4.2 Empirical Data and Their Collection

For this study, the end-of-week values of two-year government bond rates and stock market indices derived from various data sources are used. Stock market indices and the government bond yields (2-year) depicted in Table 4-1 are drawn from the Borsa Istanbul A.Ş database and the CBRT Bloomberg Terminal, respectively. The weekly dataset spans from April 1, 2005 to December 30, 2016, totaling 605 observations.

4.2.1 Descriptive Statistic

To conduct our analysis, all time series data are transformed into natural logarithms to remedy potential heteroskedasticity problems. Besides, the returns of time series are calculated as $r_t = \ln(P_t/P_{t-1})$ where P_t is the weekly closing price. Table 4-2 reveals the results of the basic descriptive statistics for the weekly returns.

It is evident from the table that both the growth rates of the aggregate market and sectoral indices and the benchmark government bond rates are close to zero in this study sample. It is worth stating that the average weekly stock market index returns, the first moment, are positive whereas the benchmark government yields are negative, suggesting a poor performance for the bond market over time. It should be underlined that the largest average weekly returns to be found in the stock market indices of "RXTCRT", "RXUTEK", "RXMANA", "RXULAS", and "RXMESY", starting from 0.356%, 0.316%, 0.281%, 0.278%, and 0.259%, respectively. However, the mean weekly return value is 0.184% for the aggregate stock market index, "RXU100", and −0.098%, the lowest value, for the two-year benchmark yields, "LTR2YGB". The highest top five variability values represented by greater standard deviation, the second moment, are to be found in the stock market indices of “RXULAS" [5.24%], "RXSPOR" [5.09%], "RXELKT" [4.91%], "RXBANK" [4.87%], and "RXTRZM" [4.80%], whereas the lowest standard deviations are observed for “RXUHIZ" [3.05%], "RXTAST" [3.16%], "RXYORT" [3.17%], "RXUSIN" [3.28%], and "LTR2YGB" [3.39%], indicating a partly validity of the risk-return tradeoff. On the other side, the most weekly decreases and increases are observed for the same stock indices, namely the peak values (min and max) are related to "RXELKT" [−35.15% & 35.41%], "RXFINK" [−35.98% & 24.66%], and "RXSPOR" [−45.80% & 22.46%].

Table 4-1 Borsa Istanbul Stock Indices and TR 2-Year GB

Variables	Variable names in Turkish	Variable Codes
Turkey 2-Year Government Bond Yield	Türkiye 2 Yıllık Devlet Tahvil Faizi	TR2YGB
BIST 100	Bist 100	XU100
BIST BANKS	Bist Banka	XBANK
BIST INF. TECHNOLOGY	Bist Bilişim	XBLSM
BIST ELECTRICITY	Bist Elektrik	XELKT
BIST LEASING FACTORING	Bist Fin. Kir. Faktöring	XFINK
BIST FOOD BEVERAGE	Bist Gıda İçecek	XGIDA
BIST REAL EST. INV. TRUSTS	Bist Gayrimenkul Y.O.	XGMYO
BIST HOLD. AND INVESTMENT	Bist Holding ve Yatırım	XHOLD
BIST TELECOMMUNICATION	Bist İletişim	XILTM
BIST WOOD PAPER PRINTING	Bist Orman Kağıt Basım	XKAGT
BIST CHEM. PETROL PLASTIC	Bist Kimya petrol Plastik	XKMYA
BIST BASIC METAL	Bist Metal Ana	XMANA
BIST METAL PRODUCTS MACH.	Bist Metal Eşya Makine	XMESY
BIST INSURANCE	Bist Sigorta	XSGRT
BIST SPORTS	Bist Spor	XSPOR
BIST NONMETAL MIN. PRODUCT	Bist Taş Toprak	XTAST
BIST W. AND RETAIL TRADE	Bist Ticaret	XTCRT
BIST TEXTILE LEATHER	Bist Tekstil Deri	XTEKS
BIST TOURISM	Bist turizm	XTRZM
BIST SERVICES	Bist Hizmetler	XUHIZ
BIST TRANSPORTATION	Bist ulaştırma	XULAS
BIST FINANCIALS	Bist Mali	XUMAL
BIST INDUSTRIALS	Bist Sınaî	XUSIN
BIST TECHNOLOGY	Bist Teknoloji	XUTEK
BIST INVESTMENT TRUSTS	Bist Menkul Kıymetler Y.O.	XYORT

Table 4-2 Descriptive Statistic for Return Series

Variables	Mean	SD	Min	Max	Skewness	Kurtosis	JB	n
RTR2YGB	-0.0010	0.0339	-0.1361	0.1953	0.7012	6.9793	447.99***	604
RXU100	0.0018	0.0380	-0.1927	0.1576	-0.4538	5.1638	138.56***	604
RXUMAL	0.0017	0.0447	-0.2169	0.2035	-0.3100	5.1215	122.94***	604
RXBANK	0.0017	0.0487	-0.2059	0.2151	-0.1724	4.6049	67.81***	604
RXFINK	0.0021	0.0463	-0.3598	0.2466	-0.7402	12.3350	2248.23***	604
RXGMYO	0.0008	0.0382	-0.1955	0.1109	-0.9653	5.8977	305.10***	604
RXHOLD	0.0015	0.0420	-0.2450	0.1967	-0.5723	6.4615	334.52***	604
RXSGRT	0.0025	0.0438	-0.2885	0.1648	-1.0315	8.9631	1001.97***	604
RXUSIN	0.0022	0.0328	-0.2012	0.1182	-1.0299	6.8019	470.54***	604
RXGIDA	0.0022	0.0375	-0.1720	0.1201	-0.3345	4.9350	105.49***	604
RXKAGT	0.0010	0.0388	-0.2297	0.1343	-0.5219	5.8779	235.86***	604
RXKMYA	0.0024	0.0385	-0.1772	0.1616	-0.4629	5.1143	134.07***	604
RXMANA	0.0028	0.0471	-0.2442	0.2092	-0.6311	6.0115	268.34***	604
RXMESY	0.0026	0.0398	-0.2664	0.1462	-1.0675	7.8306	701.96***	604
RXTAST	0.0016	0.0316	-0.1592	0.1128	-0.7356	4.9985	154.99***	604
RXTEKS	0.0020	0.0360	-0.2255	0.1102	-0.9645	6.8707	470.69***	604
RXUHIZ	0.0021	0.0305	-0.1310	0.1573	-0.3166	4.9074	101.64***	604
RXELKT	0.0004	0.0491	-0.3515	0.3541	-0.3333	12.2637	2170.90***	604
RXILTM	0.0008	0.0397	-0.1422	0.1426	-0.1267	3.9193	22.88***	604
RXSPOR	0.0014	0.0509	-0.4580	0.2246	-1.0546	16.3996	4630.60***	604
RXTCRT	0.0036	0.0369	-0.2351	0.2793	-0.0189	10.9406	1586.87***	604
RXTRZM	0.0003	0.0480	-0.2237	0.1844	-0.4723	5.4039	167.89***	604
RXULAS	0.0028	0.0524	-0.2973	0.2029	-0.4437	5.6122	191.53***	604
RXUTEK	0.0032	0.0400	-0.1958	0.1345	-0.6553	5.0615	150.17***	604
RXBLSM	0.0020	0.0421	-0.1875	0.1786	-0.4018	5.9064	228.83***	604
RXYORT	0.0007	0.0317	-0.1923	0.0950	-1.1748	7.7277	701.44***	604

*,**, and *** indicate 10%, 5%, and 1% significance level, respectively.

Regarding the third and fourth moment about the mean value, the skewness and kurtosis coefficient values are listed in Table 4-2. It is obvious that "RTR2YGB" is the only variable having a positive skewness coefficient, [0.7012], during the time period, implying a right-skewed distribution. In addition, all stock market indices have a left-skewed distribution, the most negative values are observed for the stock indices of "RXYORT" [−1.17], "RXMESY" [−1.08], "RXSPOR" [−1.05], "RXSGRT" [−1.031], and “RXUSIN" [−1.03], indicating that the right tail is short corresponding to the left tail. On the other side, the fourth moment of kurtosis coefficient for all variable is higher than 3. Putting differently, it is obvious that both growth rate of stock market and bond market data possess a leptokurtic behavior, namely they have fat tails and peakedness during the sample period. Notable, for a normal distribution, the third and the fourth moments of a time series should have coefficients of 0 and 3, respectively. Hence, the findings above indicate of nonexistence of normality for all variables. The results of a formal test for normality are reported in the seventh column in the Table **4-2**. It can be concluded that the Jarque–Bera test results indicate that the null hypothesis of normality is rejected at 1% significance level for all variables.

4.3 Methodology

In this section, we will delve into the literature of integration and cointegration tests, and the causality tests. Besides, the results of wavelet-based estimations and causality in the frequency-domain and time-domain will be discussed.

4.3.1 Unit Root Tests

Before analyzing a relationship between two or more variables, the time series used should meet a critical necessity to have efficient and proper statistical properties. This property is called stationary which is rarely found in the financial variables.

The stationarity is crucial in econometric analysis. A variable that does not meet this condition, then it is said that this variable has unit root problem, namely, it follows a random walk. Putting differently, Pindyck and Rubinfeld (1998) claim that if one variable does not revert to long run trend after a shock, then it is a random walk. For example the price of natural gas should be tied to its marginal production cost in the long-term, namely this commodity's de-trended price should return to its normal prices so that no investor could earn an extra return after temporary fluctuations.

Enders (2014) declares that the unit root process found in financial or economics time series is related to characteristic roots which are equivalent to unity. Broadly speaking, Heij et al. (2004) define a time series as stationary where its statistical properties remain unchanged over time. On the other side, Brooks (2014) gives three explanations for a testing of whether a time series has a unit root or is stationary:

a) The first reason is about the effect of a time series on its behavior and properties when it is stationary or non-stationary.

Brooks (2014) presents an example of this effect. The author (2014) asserts that "shocks" in a time series should gradually die away for a stationarity condition. In other words, he states, a shock from a variable during time point, t, will have a smaller impact in time point, $t + 1$, a smaller effect in time pint, $t + 2$, and so on. If this data is not stationary, on the other hand, then the effect of interest rate will always be infinite, namely, the effects will not dissipate during the time. Gujarati and Porter (2004) state that it is the reason why random walk process is called to have an infinite memory, i.e., this process never forgets the shock. Differently stated by Pindyck and Rubinfeld (1998), the stationarity requirement has consequences for the understanding of the relationship between an economy and macroeconomic variables appropriately and for prediction. If a variable follows a random walk process, the effects of a temporary shock, the FED rate rising for instance, on another variable, such as stock market or FX rate, will not disperse a few days later, but instead, its effect will be permanent.

b) The second problem is related to spurious results due to using non-stationary data.

The spurious problem is firstly mentioned by George Udny Yule. In his article, Yule (1926) studied the correlation between two different time series, the rate of marriage and mortality per 1000 person in England for 1966-1911. Yule (1926) states that he found a highly significant correlation between these two variables unexpectedly. According to him (1926), this result was surprising since these two time series are totally irrelevant, hence he called the output as nonsense. At the end, he concluded that the reasons behind the fall of marriage rate and the mortality rate were due to "Spread of Scientific Thinking" and "Progress of Science", respectively. Differently speaking, they are mostly influenced by a common factor and the correlation coefficient had no significant meaning and could not be interpreted that they were causally related to each other. On the other hand, Granger and Newbold (1974) also studied this topic and they conclude that if one uses non-stationary data, then the estimated coefficients and the forecasting will be inefficient and sub-optimal, respectively. Besides, using unrelated and non-stationary data in regression analysis would lead significant results seemingly, but instead insignificant, i.e., spurious results. Pindyck and Rubinfeld (1998) claim that the Gauss-Markov theorem does not hold any longer when it comes to non-stationary time series including into a regression model. In other words, due to absent of finite variance in random walk process, ordinary least squares would give inconsistent parameter estimator. Brooks (2014) reveals that when two stationary but independent time series are used in a regression model, then the R^2 value is expected to be very low. If two variables trending over time, however, are used in a regression model, then the R^2 value will be very high, i.e., it is inflated, although these two variables are totally irrelevant. Hence, such a model would be called as a “spurious regression” which yields worthless results. Studenmund (2013) clarifies that using non-stationary data leads to an incorrect model specification where the regression estimation results were actually caused by other factors, such as trend that affects all the variables used in a model. Hence, the degree of relationship between the variables, for example, correlation coefficients, will be higher than expected due to the nonstationarity. To check for whether the model results are spurious or not, Granger and Newbold (1974) suggest comparing the Durbin-Watson (DW) d value and the R^2 value. If $R^2 > DW$, then it is said that the estimated regression is spurious.

c) The standard assumptions for asymptotic analysis are invalid when non-stationary data are employed in a regression model.

Brooks (2014) states that the usual t and F ratios will not follow a t-distribution and an F-distribution. In other words, when the data used are non-stationary, therefore one cannot undertake hypothesis tests about the regression parameters.

Broadly speaking for the importance of stationarity, Gujarati and Porter (2004) stress that in the case of nonstationarity, the examination of a time series’ behavior is restricted to the time period under investigation. That is, the implications of this period are not valid for a generalization to other periods.

Rachev et al. (2007) assert that stationarity of a time series guarantees that the fundamental characteristics do not change over time. This stationarity is broken into two groups where this differentiation depends on the number of characteristics that a time series has. A time series, y_t, is called strictly stationary if all moments of its probability distributions do not vary over time, namely, if the joint distribution of y_t, y_{t-1}, y_{t-2}, … y_{t-s} are invariant over time where $s \geq 1$. Strictly noted by Montgomery et al. (2015), if a time series' properties are invariant over time and its probability distribution is unchanged for all time periods, then it is called as strictly stationary. Evidently, this type of stationary condition is not very easy, i.e. it is often too restrictive. Rachev et al. (2007) stress a less restrictive stationary condition is required: weak stationary.

Gujarati and Porter (2004) emphasize that a time series, x_t, is called stationary if both mean and variance values are invariant over time and their covariance value between two time points depends only on the time intervals. In the time series terminology, weakly stationary is also known as covariance stationary or second-order stationary. Montgomery et al. (2015) state that stationarity refers to statistical stability in this data, x_t. To be stationary x_t must satisfy the following conditions as noted by Hill et al. (2008):

$$E(x_t) = E(x_{t+k}) = \mu \qquad \forall\, t \text{ and } k \geq 1; \quad |\mu| < \infty$$

$$var(x_t) = var(x_{t+k}) = E[(x_t - \mu)^2] = \sigma_x^2 < \infty$$

$$cov(x_t, x_{t-k}) = cov(x_t, x_{t+k}) = \gamma_k \qquad |\gamma_k| < \infty \tag{89}$$

It is evident from the equations that this stationarity focuses only on the first (μ) and the second (σ_x^2) moments of x_t. Wooldridge (2012) clarifies that the mean and variance are the same for all time points and the covariance between x_t and x_{t-k} or x_t and x_{t+k} does not depend on the location of the initial time point, t, but k. In other words, Patterson (2010) reminds that, if a time series fails to satisfy these three conditions, i.e. its mean and its variance varies over time and the kth order auto-covariances is variant to an arbitrary shift in the time basis. Box et al. (2015) affirm that with the covariance stationarity and an assumption of normality conditions are enough to generate strict stationarity. Incidentally, Wooldridge (2012) says that the correlation coefficient between x_t and x_{t-k} is also based only on k value.

Let us assume that x_t is a stationary time series. Heij et al. (2004) state that if γ_k represents the auto-covariance function between two adjacent values of X, one can easily define the autocorrelation function as $\rho(k) = \gamma_k/\gamma(0)$ where $k = 0, 1, 2, 3, \ldots$. Rachev et al. (2007) remark that γ_k, $\gamma(0)$ and $\rho(k)$ are time independent for a weakly (second-order) stationarity. Wooldridge (2012) mentions that for a stationary process, both the auto-covariance $\gamma(.)$, and auto-correlation functions $\rho(.)$ should decay to zero fast enough as $k \to \infty$. Rachev et al. (2007) note that this is short-term memory behavior of x_t. Besides, as denoted by Wooldridge (2012), the correlation coefficient is independent of the starting time point, t, differently speaking, since the variables get further apart eventually, gradually the coefficient degree will become smaller and smaller.

Recall that the most time series in finance, business or economics follow a random walk, i.e., they are non-stationary. Acquiring stationarity for modeling and forecasting, Bisgaard and Kulahci (2011) state that it can be guaranteed by control actions taken at regular intervals, namely, taking a difference between the consecutive observations. But before taking differences, initially one should transform the data where the rationale behind of transformation is listed by Bisgaard and Kulahci (2011). They state that the first reason is to stabilize the variability with dissociating the variance and the mean values. The second motivation is to make the model simple. And the last rationale is related to the residuals to get normally distributed with constant variance and zero mean value. According to them, the general class of transformation, such as Box–Cox power transformation method to cope with mainly the time-varying constant variance problem is given by

$$x^{(\eta)} = \begin{cases} \dot{x}\ln(x) & \eta = 0 \\ \left(\dfrac{x^{\eta} - 1}{\eta \dot{x}^{\eta-1}}\right) & \eta \neq 0 \end{cases} \qquad \dot{x} = exp\left[\left(\frac{1}{N}\right)\sum_{n=1}^{N} lnx_t\right] \tag{90}$$

where N denotes the total observations and $\dot{x}$ stands for the geometric mean of the observations. Montgomery et al. (2015) state that if $\eta = 1$, then there is no need for transformation. Moreover, the usual values such as $\eta = 0.5$, $\eta = -0.5$ and $\eta = -1$ are required to a square root, reciprocal square root and inverse transformation, respectively. If $\eta = 0$, then the log transformation widely used for business and economic time series is chosen. In addition to transformation, Montgomery et al. (2015) give some information about the adjustment methods such as seasonal adjustment and trend adjustment. For stationarity, a method to remove trend in a data is differencing this data. Putting differently, Pindyck and Rubinfeld (1998) mention that if a time series, x_t, is differenced one time, $t-1$, then this new data, $\Delta x_t = x_t - x_{t-1}$, may be stationary. They state that the differencing number required of being stationary is called as the order of homogeneity and it is denoted as $I(d)$. If a time series is stationary in the first-difference, then it is said that x_t first-order homogeneous non-stationarity and it is denoted as $I(1)$. Actually, d can take a value of 0 or 1, and sometimes 2 (Box et al., 2015).

In literature terminology, there are several ways to find a stationarity in a time series. These methods are (i) graphical (visual inspection) analysis and (ii) the correlogram tests. For example, Bisgaard and Kulahci (2011) state that if a time series is found as non-stationary, then the autocorrelation function (ACF) will not die out for big lags. On the other hand, they assert that using the variogram method, one can test whether a time series stationary or not where the stationarity depends on the variance of between observations. If the variances stabilize and the variogram becomes smooth, then it is said that the data is stationary. Instead of using these two methods, however, a formal method, the unit root tests will be preferred.

4.3.1.1 The Dickey-Fuller and Augmented Dickey-Fuller Unit Root Test

The first and pioneering unit root test is introduced by Dickey and Fuller (1979). According to them, the simplest model for the non-stationary testing is the AR(1) process, namely, it is a random walk

$$X_t = \xi X_{t-1} + \epsilon_t \qquad t = 1,2,3,\ldots \tag{91}$$

where $\epsilon_t \sim N(0, \sigma^2)$ and $cov[\epsilon_t, \epsilon_M] = 0$ for all $t \neq M$. If $X_0 = 0$ and $|\xi| < 1$, then this time series will be stationary due to $t \to \infty$. If $|\xi| = 1$, conversely, it is said that X_t is a non-stationary or random walk, which leads to the variance is $t\sigma^2$. Note that σ^2 is the variance of X_t conditional on X_{t-1}. Therefore, the hypothesis of nonstationarity for X_t is:

$$H_0: X_t \text{ is random walk } \Rightarrow \xi = 1 \tag{92}$$

$$H_1: X_t \text{ is stationary} \Rightarrow |\xi| < 1 \tag{93}$$

For the hypothesis testing, the first thing to do is to find the ordinary least squares (OLS) estimate of parameter ξ. Next, perform a t-test for ξ whether it is significantly different from 1 or not. However, if $\xi = 1$ is found, there exists a problem; this estimation will be biased toward zero (0) and the tabulated significance levels of the t-statistic will be invalid. Lastly, t-statistic will not have an approximately standard normal distribution even though for large-sized time series.

To solve this problem, Dickey and Fuller (1979) suggest subtracting X_{t-1} from the equation above. Thus, the new equation will be

$$X_t - X_{t-1} = \xi X_{t-1} - X_{t-1} + \epsilon_t \qquad t = 1,2,3, \ldots$$

$$\Delta X_t = (\xi - 1)X_{t-1} + \epsilon_t$$

$$\Delta X_t = \zeta X_{t-1} + \epsilon_t \tag{94}$$

and the hypothesis will change to:

$$H_0: \xi - 1 = \zeta = 0 \text{ [unit root or random walk]} \Leftrightarrow H_0: \xi = 1 \tag{95}$$

$$H_1: \xi - 1 = \zeta < 0 \text{ [stationary]} \qquad \Leftrightarrow H_1: |\xi| < 1 \tag{96}$$

Because t-statistic is not valid for this model, they (1979) computed the critical values through using Monte Carlo simulations. In econometric literature, these new critical values are called as Dickey-Fuller test or τ (tau) test, where it is comprised of

τ, τ_κ, and $\tau_\daleth$ parameters. The models under the null and the alternative hypothesis are given as

$$H_0 : \Delta X_t = \epsilon_t \tag{97}$$

$$H_1 : \Delta X_t = \zeta X_{t-1} + \kappa + \daleth t + \epsilon_t \tag{98}$$

where κ and $\daleth$ represent constant and deterministic trend parameters, respectively. Putting differently DF test can be estimated for these three cases

$$\Delta X_t = \zeta X_{t-1} + \epsilon_t \tag{99}$$

$$\Delta X_t = \zeta X_{t-1} + \epsilon_t + \kappa \tag{100}$$

$$\Delta X_t = \zeta X_{t-1} + \epsilon_t + \kappa + \daleth t \tag{101}$$

where X_t is a pure random walk for the first case, a random walk with constant drift for the second case, and a random walk with a deterministic linear trend for the third case, respectively. The null hypothesis is that $\zeta = 0$ or $\xi = 1$, namely X_t is a random walk or non-stationary. Conversely, the alternative hypothesis is that $|\xi| < 1$, that is X_t is stationary. After implementing the DF unit root test, compare the right critical values in line with the significance level and the tau statistic values. If the $|\tau| > |t_t|$, where t_t represents the DF or MacKinnon critical values, the data is stationary. Conversely, if the $|\tau| < |t_t|$, on the contrary, the null hypothesis cannot be rejected, and it is said that the data under investigation is a random walk or non-stationary. Equivalently speaking, if the null hypothesis is not accepted, then it is said that X_t is stationary with zero mean for Equation (99), X_t is stationary with nonzero mean $[= \kappa/(1-\xi)]$ for Equation (100), and lastly X_t is stationary around a deterministic trend for Equation (101).

The authors (1979) state that this unit root test is also appropriate for the AR(2), AR(3), and so on. The appropriate equation for test, $X_t = \xi X_{t-1} + \epsilon_t$, can be rewritten as

$$X_t = \xi_1 X_{t-1} + \xi_2 X_{t-2} + \xi_3 X_{t-3} + \cdots + \xi_n X_{t-n} + \epsilon_t \tag{102}$$

Implementing this model, however, leads a serious problem: the disturbances, ϵ_t, will not be white noise since it is the violation of the DF test assumptions. Differently speaking, the residuals from the model can be autocorrelated, where the DF test will be oversized. The reason behind this autocorrelation problem is that one can think that the error term, ϵ_t, is equal to $(= \xi_2 X_{t-2} + \xi_3 X_{t-3} + \cdots + \xi_n X_{t-n} + u_t)$. To deal with the serial correlation problem, one solution is to add sufficiently many lagged values of the dependent variable to the right-hand side of the equation until the residuals become white noise. Specifically the augmented model will become to

$$\Delta X_t = \zeta X_{t-1} + \epsilon_t + \sum_{j=1}^{n} \zeta_i \, \Delta X_{t-j} \tag{103}$$

$$\Delta X_t = \zeta X_{t-1} + \epsilon_t + \kappa + \sum_{j=1}^{n} \zeta_i \, \Delta X_{t-j} \tag{104}$$

$$\Delta X_t = \zeta X_{t-1} + \epsilon_t + \kappa + \daleth t + \sum_{j=1}^{n} \zeta_i \, \Delta X_{t-j} \tag{105}$$

Knowing as the augmented DF test or the ADF test in econometric literature, it has the same critical values for the DF test. The main problem here is that one should determine the optimal lag order for the dependent variable to guarantee that the error term is not serial correlated. However, the researcher should be very careful when deciding the true lag order (p) because when (p) is too small, then the autocorrelation problem would not be removed, however, conversely, if the (p) is too higher, then standard errors of coefficient would be increased. There exist two ways for determining optimal lag orders (i) depending on the frequency of the data and (ii) the information criterion methods. Since there is not an obvious choice for the first choice, it is recommend to use the second method of the information criterion such as

Akaike information criterion (AIC), the Bayesian information criterion (SIC), Hannan-Quinn information criterion (HQC), Hatemi-J information criterion (HJC).

In literature, researchers are not contented, mostly, a single unit root test due to several reasons. The main reason is mainly related to the accuracy of the test applied. Literally, this problem is discussed by Gujarati and Porter (2004). In their book, they (2004) affirm that the main reasons are the power and size of the unit root test implemented. The size of a test refers to the level of significance and the power of a test infers the probability of not accepting the null when it is actually not true. For instance, we decided to implement a unit root test for a time series. If this series is appropriate for a model, where it includes only a drift but misguidedly a wrong regression is chosen such as a pure random walk model, then the result obtained will be incorrect. The explanation is that the true level of significance varies according to model types. Besides, the excluding moving average (MA) components from a model could also affect the result of the unit root test.

On the other hand, the most vital criticism made for the unit root tests is about their power. Brooks (2014) pronounces that when a time series is found as stationary with a root value close to the non-stationary boundary of "1", then the power of test falls. For instance, if the ξ, a parameter from a purely random process, is 0.95, then it is said that the data is stationary, namely, the null hypothesis should be rejected. Particularly, the problem arises for the small-sized samples because the parameter value is very close to the non-stationary boundary and the tests are not enough powerful for deciding whether the data is stationary or a random walk is. Apart from this reason, Gujarati and Porter (2004) enumerate several reasons. First of all, that the frequency of the data is considered more important than the size of the data for the power of the test. Secondly, according to these tests, the data comprises only a single unit root, namely after one-differencing, it will be stationary, $I(1)$. If the data is non-stationary after one-differencing, then it is said that it has more than one unit root, i.e. it is $I(2)$ is $I(3)$. The last reason is about the structural breaks. If a data, say oil, comprise structural breaks such as embargoes imposed by different authorities or countries, then the test may be unsuccessful to identify them.

4.3.1.2 Phillips and Perron Unit Root Test (1988)

This method was introduced by Phillips and Perron (1988) for testing the presence of unit root in time series, which was an extension of the Phillips (1987) nonparametric approach. It includes simply (i) a constant, and (ii) a constant and a linear trend in the specification. As noted by the author (1988), its method is asymptotic and depend on the theory of functional weak convergence. Expressing as functional of standard Brownian motion, the limit distribution of the test statistics are the same as those ADF approach used while their computing methods are quite different in how they deal with autocorrelation and non-constant variance problems. In other words, Phillips and Perron (1988) ignore any serial correlation in the test regression, i.e. they correct it by employing a nonparametric factor.

In order to test the null hypothesis of nonstationarity, i.e. y is $I(1)$, against the alternative hypothesis of stationarity, i.e. y is $I(0)$, The application of this test is simply based on the OLS parameter estimate, such as given

$$\Delta y_t = \beta' A_t + \pi y_{t-1} + \epsilon_t \tag{106}$$

where A_t is a vector of drift and trend, and $\pi = \theta - 1$. Under the null hypothesis, Δy_t is $I(0)$ which signifies that $\pi = 0$ or $\theta = 1$. Besides, the error term, ϵ_t, is assumed to be $I(0)$ and homoskedastic. For eliminating the nuisance parameter dependencies asymptotically in the error terms, they (1988) suggest using the modified test statistics of Z_π and Z_t, which are given as follows

$$Z_\pi = N\hat{\pi} - \frac{(\hat{\lambda}^2 - \hat{\sigma}^2)}{2} \frac{N^2 * SE(\hat{\pi})}{2} \tag{107}$$

$$Z_t = \sqrt{\frac{\hat{\sigma}^2}{\hat{\lambda}^2}} * t_{\pi=0} - \left(\frac{\hat{\lambda}^2 - \hat{\sigma}^2}{\hat{\lambda}^2}\right) * \frac{1}{2}\left(\frac{N * SE(\hat{\pi})}{\hat{\sigma}^2}\right) \tag{108}$$

where λ^2 and σ^2 are estimators of the long and short run variances of error terms, respectively. Under the null of $\pi = 0$, modified test statistics described in Equation

(107) and (108) both have the same asymptotic distributions. Since the critical values for testing are the same as those for the ADF, i.e. MacKinnon critical values, they generally give the same results for the nonstationarity.

4.3.1.3 KPSS Unit Root Test (1992)

Remarking the criticism of the tests for their power outlined above, a researcher may have a decision of stationary for a data where the unit root value is very close to the non-stationary boundary. Brooks (2014) demonstrates that if the parameter of the unit root is less than the critical value of "1", then, especially with small-sized samples, the unit root test will fail to decide whether this data is a random walk or stationary due to the null hypothesis of a unit root. In econometric literature, the classical unit root methods have a null hypothesis of non-stationary: if the absolute value of test statistic is greater than t-table value, then the null hypothesis is rejected. Broadly speaking, the null should either be rejected or not be rejected according to the test result. Brooks (2014) claims that if the absolute value of test statistic is less than the t-table value that is to say if one fails to reject the null, it is said that the null hypothesis is correct or the sample information is not enough to reject it. Arltova and Fedorova (2016) state that the solution is to use the null of stationary and the alternative of non-stationary tests besides the classical unit root tests.

In their paper, Kwiatkowski et al. (1992, KPSS hereafter) suggest a test of the hypothesis of stationarity against the hypothesis of non-stationary, where the time series under investigation is assumed to be stationary around a deterministic trend and it is written as the sum of a deterministic trend, a random walk, and a stationary random error term. The null hypothesis is of trend stationary because the time series are detrended. Besides, they (1992) parameterize the alternative hypothesis as random walk, where its variance is equal to zero.

Kwiatkowski et al. (1992) recommend a modified version of the one-sided LM statistics, where its validity depends on the assumptions of normality of random walk and white noise of stationary error term and the asymptotic distribution is not standard. They (1992) test the stationary of null hypothesis with a time series, X_t, wherein it is comprised of three components

$$X_t = \vartheta t + \omega_t + \epsilon_t \tag{109}$$

$$\omega_t = \omega_{t-1} + \upsilon_t \tag{110}$$

where ϑt, ω_t, and ϵ_t stand for deterministic trend, random walk, and stationary error term. The error term of a random walk, υ_t and the stationary error term, ϵ_t are iid$(0, \sigma_\upsilon^2)$ and iid$N(0, \sigma_\epsilon^2)$, respectively. The drift parameter, however, is the initial value of a random walk, ω_0. Moreover, the null hypothesis is $H_0 : \sigma_\upsilon^2 = 0$, equivalently saying, the error term of a random walk is zero. If it is assumed that the error term of the time series is stationary, then the null hypothesis would be trend-stationary against the alternative hypothesis of a random walk, $H_0 : \sigma_\upsilon^2 > 0$. If $\vartheta = 0$ for a special case of the model above, then the null hypothesis will be determined as X_t is stationary around a level, ω_0. The statistic of the KPSS test can be parameterized as follows

$$LM = \hat{\varkappa}_\tau = \frac{1}{T^2} \sum_{t=1}^{T} \frac{\hat{S}_t^2}{\hat{\sigma}_\epsilon^2} \tag{111}$$

where T^{-2} is the normalization factor, and $\hat{S}_t^2 = \sum_{t=1}^{T} \epsilon_t$.

4.3.1.4 Lee and Strazicich Unit Root Test (2003)

According to Brooks (2014), if a test does not include structural breaks, then its power will be lower especially for a larger break and a small-sized sample, equivalently saying, the test tends to reject the nulls easily when the null hypothesis is correct. In unit root literature, the Perron test (1989) is known as the first test that permits a one-time change in level or trend parameters. The breakpoint date, however, is not known from the time series under study, namely, the researchers determine it exogenously. After the Perron test, different researchers implement several unit root tests allowing one or more structural breaks. However, when interpreting these unit root tests results, some important issue arises according to Lee and Strazicich (2003). The authors (2003) declare it is the main problem of not allowing for breaks both in the null and alternative hypothesis. Especially in the case of endogenous break test, if a test presumes no break(s) under the null hypothesis, then the test statistic will diverge and cause significant rejections for a unit root with structural breakpoints. For that, Lee and Strazicich (2003) suggest a unit root test allowing for two structural breaks where the alternative hypothesis is that the data is trend-stationary. The authors (2003) introduce two different models as given

$$M_t = [1, t, YK_{1t}, YK_{2t}]' \qquad \text{(Model A)} \tag{112}$$

$$M_t = [1, t, YK_{1t}, YK_{2t}, TD_{1t}, TD_{2t}]' \qquad \text{(Model C)} \tag{113}$$

where M_t represents a vector of exogenous variables. And

$$X_t = \vartheta' M_t + \epsilon_t \tag{114}$$

$$\epsilon_t = \delta \epsilon_{t-1} + \upsilon_t \tag{115}$$

where error term is $\upsilon_t \sim \text{iid} N(0, \sigma^2)$. Lee and Strazicich (2003) state that "Model A" permits two breaks in the level and "Model C" allows two breaks in the level and in the trend of a time series. The dummy variable of YK_{jt} is "1" when $t \geq BT_{Bj} + 1$ and it is "0" when $t < BT_{Bj} + 1$. Besides, the dummy variable in "Model C", TD_{jt}, is equal to $= t - BT_{Bj}$ when $t \geq BT_{Bj} + 1$ and it is "0" when $t < BT_{Bj} + 1$. It should be emphasized that BT_{Bj} represents the structural breakpoint in time series. The null hypothesis for this test is that $\delta = 1$ and the alternative hypothesis is $\delta < 1$. Using the parameters determined before to specify the nulls and the alternatives for two different models as given

Model A

$$H_0: X_t = \kappa_0 + g_1 B_{1t} + g_2 B_{2t} + X_{t-1} + u_{1t} \tag{116}$$

$$H_1: X_t = \kappa_1 + \psi t + g_1 B_{1t} + g_2 B_{2t} + u_{2t} \tag{117}$$

Model C

$$H_0: X_t = \kappa_0 + g_1 B_{1t} + g_2 B_{2t} + X_{t-1} + u_{1t} + YK_{1t} + YK_{2t} \tag{118}$$

$$H_1: X_t = \kappa_1 + \psi t + g_1 B_{1t} + g_2 B_{2t} + u_{2t} + TD_{1t} + TD_{2t} \tag{119}$$

where $g = (g_1, g_2)'$ and u_{1t} and u_{2t} are stationary error terms. B_{jt} is equal to "1" when $t = BT_{Bj} + 1$ and $B_{jt} = 0$ when $t \neq BT_{Bj} + 1$. As noted by Perron (1989), to guarantee that the asymptotic distribution of the test statistic under the null hypothesis is invariant to the size of structural breakpoints (d), it is required to include B_{jt} parameters to the regression.

4.3.2 Cointegration Tests

In unit root testing, we have tested whether a time series follows a random walk or it is stationary. If a time series is non-stationary in level, it means that it follows a random walk. If two or more non-stationary variables are used in an OLS regression, then the result may be spurious, meaning that it is invalid. To remove the problems including spurious or nonconstant variance, it is required to use stationary variables via transformation methods. If a time series becomes stationary after detrending it, then it is said that this time series is trend stationary while the nonstationarity problem is removed by differencing, then it is called as difference stationary. Let us assume a simple regression model with/without a constant parameter. Having learned before, if this model is stationary after one-time differencing, then it is called as integrated of order "1". If this time series is still not stationary despite a one-time differencing, it is required to differentiate it twice. Thus, if it is stationary, then it said that this model is integrated of order "2", i.e., it is $I(2)$. Broadly speaking, if the series is stationary after "d" times, it is said that the time series is $I(d)$.

In economics and finance, the majority of the variables are generally non-stationary, i.e. they are $I(1)$. Despite this reality, how can one make regression to analyze a possible relationship between variables? It is possible to get spurious results when non-stationary variables are used in a regression. Hill et al. (2008) demonstrate that such as a regression model should not be used due the spurious problem. However, the authors (2008) also remind that an exception exists in the econometric literature. Sevüktekin and Çınar (2014) state that there are two ways for avoiding the spurious problem. The first one is to take a difference of time series and the second one is to use cointegration analysis to unmask the relationship. As noted by Pindyck and Rubinfeld (1998), however, the first method leads a loss of the information. Fabozzi et al. (2014) state that if two and more financial or economic variables are cointegrated, then one can study the long run relationship and short run dynamics of them. Putting differently, if a cointegration relationship exists, it is said that they have a long run relationship although they may diverge from each other in the short-term, but, it is possible to move together closely.

The variables that have possible cointegration relationships are, according to Davidson and MacKinnon (2004), interest rates of bonds on assets of different maturities, such as one-year, two-year or ten-year, prices of similar commodities, grains, natural gas or silver or foreign currencies, in different countries or places, tax revenues and government spending, futures and spot prices of assets or the price level and interest rates or money supply. On the other hand, Pindyck and Rubinfeld (1998) give an example of the stock market. If stock value is rational, namely it is priced by the present value method where the expected dividend payments are discounted by a discount factor. They (1998) state that even though stock prices and dividend payments are expected to follow a random walk, their linear combination may move together. The discount rate used for the calculation value of a stock is, by the way, the same the cointegration parameter.

In literature, cointegration, a long run relationship, the framework is firstly mentioned by Granger (1981). The author (1981) states that although two series, such as the births and deaths of people in a city where immigration and emigration are not allowed, may not be equal in the short-time period, they are expected to move together in the long-term period. On the other hand, a deviation from a relationship is called as cointegration by Engle and Granger (1987) where they state that the deviation from equilibrium is stationary although these two series are non-stationary and they have infinite variance due to the contribution of the low frequencies for $I(1)$.

On the other hand, Rachev et al. (2007) demonstrate that the concept of cointegration can be described in terms of reduction of the order of integration. The authors (2007) mention that if $y_t \sim I(d)$ and $x_t \sim I(d)$, that is they are integrated of order $d \geq 1$, they are considered to be cointegrated given that their linear combination, $\varepsilon_t = y_t - \delta x_t$, is stationary of b order, where $d > b > 0$ and δ is cointegrating parameter. To enlighten this framework, an example is given by Wooldridge (2012). Two financial variables, x_t and y_t are a two-year and ten-year interest rate of government bonds. Assume that they are non-stationary, i.e. $I(1)$ but a new parameter generated by their linear combination is, $sp_t(= y_t - x_t) > 0$, found as stationary, meaning that x_t and y_t are cointegrated. What happens if it is non-stationary? Wooldridge (2012) declares that due to arbitrage opportunities they are forced to move together in the long run by market participants, equivalently saying, sp_t converges to its mean value and the arbitrage opportunities disappear. Thus, it can be said that the basic idea of cointegration, as noted by Faliva and Zoia (2008), is to give a picture of stable links for financial variables.

Focardi and Fabozzi (2004) highlight the importance of cointegration tests for portfolio management in their book. The authors (2004) remark that this method permits investors to locate assets or commodities, which are mispriced according to conventional methods in finance theory. If two assets are found as cointegrated, then an investor can make a profit since their return series are predictable due to the autocorrelation of returns. Putting differently, even though individual asset price follows a random walk, i.e. unpredictable, The authors (2004) declare that investors can make a profit via tracking portfolios revealing a stationary behavior instead of individual assets. The authors (2004) also emphasize that these variables can move together on different frequencies.

4.3.2.1 The Engle-Granger Cointegration Test (1987)

In econometric literature, there are several methods for cointegration, where some of the methods include breaks or some do not. The first method not including breaks is the cointegration method by Engle and Granger (1987). Remark that if two different time series are both found stationary after d times, $I(d)$, then any linear combination, normally, of them will be also stationary. If a vector of α exists in the regression of $Y_t = \alpha X_t + \epsilon_t$, the integration order of the error term, ϵ_t, would be $I(d - b)$ provided that $b > 0$. The two variables are defined by Engle and Granger (1987) as cointegrated of order (d, b). Differently stating, if $Y_t \sim I(1)$ and $X_t \sim I(1)$ and the error term ϵ_t is $I(0)$, the cointegration of order is written as $CI(I, I)$. The idea based on the this cointegration test is very straightforward. The required steps are:

a) Stationary test is the first step; determine whether the time series are stationary or a random walk. If they are integrated order of "1", then proceed to the second step.

b) Save the residuals of the regression, $Y_t = \mu + hX_t + \varepsilon_t$, to test whether it is stationary or not. The null hypothesis is that ε_t is not cointegrated while the alternative is of cointegration relationship exists.

c) Third step is to test whether the error term, $\hat{\varepsilon}_t = \hat{Y}_t - \hat{\mu} - \hat{h}X_t$, is a random walk or stationary. It should be stated that the critical values of the DF or the ADF tests are not appropriate. Instead, the critical value of Engle and Yoo (1987) should be used to whether it is stationary or not.

d) The last step entails estimating the error correction model if a cointegration relationship exists. Engle and Granger (1987) indicate that if the null of nonstationarity is rejected, equivalently saying, if a cointegration relation exists, then the following equation parameterized by these two different variables is used for hypothesis testing

$$\Delta Y_t = \beta_{10} + \sum_{i=1}^{n} \beta_{1i} \Delta Y_{t-i} + \sum_{j=1}^{n} \mu_{1j} \Delta X_{t-j} + h_1 (Y_{t-1} - \Omega X_{t-1}) + \epsilon_{1t} \tag{120}$$

$$\Delta Y_t = \beta_{20} + \sum_{i=1}^{n} \beta_{2i} \Delta Y_{t-i} + \sum_{j=1}^{n} \mu_{2j} \Delta X_{t-j} + h_2 (Y_{t-1} - \Omega X_{t-1}) + \epsilon_{2t} \tag{121}$$

These two equations reveal that the value of Y_t depends on the past of value of Y_t and X_t and the disequilibrium between the parameters of Y_{t-1} and X_{t-1}, namely the cointegration relation, $(Y_{t-1} - \Omega X_{t-1})$. Besides, h_1 and h_2 parameters are called as the adjustment coefficients and they capture the speed of adjustment of dependent and independent variables to the prior period's disequilibrium. Equivalently saying, they assert how the independent and dependent variables are adjusted when these variables are out of equilibrium. If $h < 0$ and $\mathrm{Y}_{t-1} > \Omega \mathrm{X}_{t-1}$, then a downward adjustment occurs in the direction of equilibrium, namely ϵ_t induces a negative change in the dependent variable, Y_t, back toward the equilibrium.

4.3.2.2 The Johansen Cointegration Test

Due to the drawback mentioned above for E-G cointegration test (1987), a new cointegration method introduced by Johansen (1988), Johansen and Juselius (1990), Johansen (1991), and Johansen (1995), which works in a case of multivariate framework. The general VAR(p) model for cointegration test can be given as

$$\Delta y_t = \Pi y_{t-p} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_j \tag{122}$$

Where $k = 1, \dots, p$, y_t is an nx1 vector of variables that are $I(1)$ and u_j is an nx1 vector of innovations. Besides, $\Pi = \sum_{\mathrm{j}=1}^{\mathrm{p}} \mathrm{A_j} - I$, $\Gamma_j = -\sum_{\mathrm{k}=\mathrm{j}+1}^{\mathrm{p}} \mathrm{A_j}$, and x_t is a d- vector of deterministic variables. If the coefficient matrix, Π, is equal to zero, then it is said that all of the coefficients are zero and model in Equation (122) turns to a VAR(p) model with first-differenced data. In the case of $\mathrm{rank}(\Pi) = 0$, there is no cointegration relationship since all of the Δy_t series are not stationary. If, on the other hand, Π has full rank, i.e. $\mathrm{rank}(\Pi) = n$, then all y_t must be stationary because all sequences in the right-hand and left-hand side of Equation (122) are stationary. However, it does not mean that there exists any cointegration vectors. Similarly, if $\mathrm{rank}(\Pi) \neq 0$ but has less than full rank, $1 < r < n$, then there are nxr matrices β and α each with rank r. In the case of cointegration relationship $\Pi = \alpha\beta'$ and $\beta' y_t$ is integrated of order zero, the $\mathrm{rank}(\Pi) > 1$, Πy_{t-p} indicate the error correction term. Furthermore, α and β represent the speed of adjustment towards equilibrium and cointegration vectors, a matrix of long run coefficients, in the error correction model (VEC) as given in the equation, respectively.

The maximum likelihood estimator Johansen cointegration test is calculated by looking at the eigenvalues, i.e. characteristic roots, of Γ matrix. Namely, the cointegration test depends on the rank of Γ matrix, i.e. the number of the characteristic roots, λ_i. If they are different from zero, there exists a cointegration relationship between underlying variables. For a given $\mathrm{rank}(\Pi) = z$ there exist z stationary linear combinations of the underlying variables, and the maximum likelihood estimator of β gives the maximum z canonical correlations of y_{t-j} with

Δy_t. The eigenvalues of Π are put in ascending order such as $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$ where $|\lambda_i| < 1$, $(\lambda_{max} = \lambda_1)$ and $(\lambda_{min} = \lambda_n)$. It should be noted that λ_{max} is the closest to one while λ_{min} is the closest to zero. Therefore, if the rank$(\Pi) = 0$, then it is said that $\lambda_i = 0$ and there does not exit any cointegrating vectors. Johansen (1991) suggests two different likelihood ratios for significance test for the canonical correlations given by

$$\lambda_{max}(z, z+1) = -N \ln(1 - \hat{\lambda}_{z+1}) \tag{123}$$

$$\lambda_{trace}(z) = -N \sum_{i=z+1}^{s} \ln(1 - \hat{\lambda}_i) \tag{124}$$

where λ_{max} and λ_{trace} are the maximum eigenvalue and trace test, respectively. Besides, z is equal to the total cointegrating vectors, N is the sample size and $\hat{\lambda}_i$ is the i-th estimated largest canonical correlation. λ_{max} in Equation (123) tests the null hypothesis of z cointegrating vectors against the alternative of $z+1$ cointegrating vectors. λ_{trace}, on the other hand, in Equation (124) tests the null hypothesis that the number of cointegrating vectors is $\leq z$ against the alternative hypothesis that the independent cointegrating vectors is $> z$. If the calculated value of λ_{max} and λ_{trace} are bigger than the critical value obtained from Johansen and Juselius (1990), then it is said that there exists a cointegration relationship between variables under study. According to Johansen (1995), there are special five deterministic trend cases, which can be found in most econometric software packages

$$H_2(r): \quad \alpha\beta' y_{t-1} = \Pi y_{t-1} + Bx_t \tag{125}$$

$$H_1^*(r): \quad \alpha(\beta' y_{t-1} + \kappa_0) = \Pi y_{t-1} + Bx_t \tag{126}$$

$$H_1(r): \quad \alpha\left(\beta' y_{t-1} + \kappa_0\right) + \alpha_\perp \delta_0 = \Pi y_{t-1} + Bx_t \tag{127}$$

$$H_1^*(r): \quad \alpha\left(\beta' y_{t-1} + \kappa_0 + \kappa_1 t\right) + \alpha_\perp \delta_0 = \Pi y_{t-1} + Bx_t \tag{128}$$

$$H(r): \quad \alpha\left(\beta' y_{t-1} + \kappa_0 + \kappa_1 t\right) + \alpha_\perp(\delta_0 + \delta_0 t) = \Pi y_{t-1} + Bx_t \tag{129}$$

where y_t and cointegrating equations do not have deterministic trends and intercepts, respectively, in Equation (125). Similarly, Equation (126) shows that cointegrating equations have intercepts but y_t do not have deterministic trends. In Equation (127), y_t have linear trends however cointegrating equations have only intercepts. Conversely, y_t and cointegrating equations both include linear trends according to Equation (128). Lastly, cointegrating equations have linear trends while y_t have quadratic trends as shown in Equation (129).

4.3.2.3 Hatemi-J Cointegration Test (2008)

Previously, the cointegration tests without allowing structural breaks are defined. But, as in the case of unit root testing, if the data includes structural breaks, then the conventional cointegration, akin to unit root, the test will tend to not reject the null hypothesis of non-cointegration because these tests are biased, thus, it is possible to obtain spurious results.

The Gregory and Hansen (1996) cointegration test is the one of first cointegration test with allowing structural breaks endogenously determined. In their paper, the authors (1996) state that they built a new and a more general cointegration test. The motivation of this test is based on the standard concept of regime change. Whenever a conventional test is implemented, it is likely to achieve incorrect results because they are unable to detect the cointegration vector shift at an unknown point in the data. The reason why the standard tests are not suitable is that under the null hypothesis it is assumed that the cointegrating vector is time-invariant during the period of study. Differently saying, it is assumed, as noted by Hatemi-J (2008), that the cointegrating vectors remain the same. However, especially in the case of long time span, the long run relationship may change due to structural breaks, such as economic or financial crises, changes in policy and regime, wars, strikes, embargoes or technological shocks. Within this framework, Gregory and Hansen (1996) offer a new test where the null hypothesis of no cointegration is the same as the standard cointegration tests while the alternative hypothesis differs from the conventional tests, namely it suggests an endogenously determined structural break in the data under investigation. The three different models are listed as given

$$Y_{1t} = \mu + \theta^T Y_{2t} + \varepsilon_t \qquad 1 \le t \le s \qquad \text{Model 1} \tag{130}$$

$$Y_{1t} = \mu_1 + \mu_2 \kappa_{t\tau} + \theta^T Y_{2t} + \varepsilon_t \qquad 1 \le t \le s \qquad \text{Model 2: C} \tag{131}$$

$$Y_{1t} = \mu_1 + \mu_2 \kappa_{t\tau} + \gamma t + \theta^T Y_{2t} + \varepsilon_t \qquad 1 \le t \le s \qquad \text{Model 3: C/T} \tag{132}$$

$$Y_{1t} = \mu_1 + \mu_2 \kappa_{t\tau} + \theta_1^T Y_{2t} + \theta_2^T Y_{2t} + \varepsilon_t \qquad \text{Model 4: C/S} \tag{133}$$

Gregory and Hansen (1996) describe the first model as standard cointegration, the second model as level shift (C), the third model as level shift with a trend (C/T), and the last model as regime shift (C/S). They (1996) assume that Y_{2t} is integrated of order one and the error term, ε_t, is integrated of order zero.

“Model 1” it is a standard cointegration test with no structural change. On the other hand, in the case of structural change, they built three different models where unknown change points, "τ", are denoted by dummy variable with the conditions as follows

$$\kappa_{t\tau} = \begin{cases} 0 & \text{if} \quad t \le [s\tau] \\ 1 & \text{if} \quad t > [s\tau] \end{cases}$$

where $0 < \tau < 1$. The structural change point,$\kappa_{t\tau}$, generally represents a one-time change in the intercept parameter, μ, and/or to the slope, θ.

The first model with structural change is “Model 2” where it includes a level shift, namely a one-time change in the intercept parameter is allowed but for the slope parameter is not permitted. μ_1 parameter, in the last three models, stands for the intercept before the shift while μ_2 parameter denotes the change in the intercept when a shift occurs. “Model 3” is the extended version of “Model 2” because it also includes a time trend. Thus this model is called as “level shift the trend” and denoted as C/T. The last model is also the extended version of “Model 2” and call as regime shift where it permits both a change in the level and in the slope. In this “Model 4”, θ_1 and θ_2 represent the cointegration slope coefficients before the regime shift and represent a change in the slope coefficients, respectively.

Gregory and Hansen (1996) calculate the test statistics for each possible structural breakpoint in the interval $([15\%n], [85\%n])$ for the data. Using the last three models by OLS method, the authors (1996) obtained the error terms, $\hat{\varepsilon}_t$ and conducted unit root test on these estimated error terms. For deciding whether a cointegration relationship exists, the decision is made according to the smallest value of the unit root test statistic under the null hypothesis. If the null hypothesis is not rejected, it is said that no cointegration vector exist. Conversely, if the null hypothesis is rejected, then it is decided that there is a cointegration vector with an endogenous structural break. It should be noted that this test allows only a structural break. In the case of two or more breaks, this test will not be appropriate.

Hatemi-J (2008), on the other hand, extends the G-H cointegration test by allowing for two structural breaks. The author (2008) states that the new critical values are provided for the cointegration test. For testing of the long run relationship, the ADF test by Engle and Granger (1987) and the extensions of Z_a and Z_t tests provided by Phillips (1987) are used. As noted by the author (2008), the structural breakpoints are determined endogenously, namely, the timing of changes are not known before testing, they depend on the data under study. A model for standard cointegration test and for two regime shifts allowed both in the intercept and in the slope parameters are parameterized as given in the equation below:

$$Y_t = \mu + \theta_1' X_t + e_t \qquad 1 \le t \le s \tag{134}$$

$$Y_t = \mu_0 + \mu_1 SB_{1t} + \mu_2 SB_{2t} + \theta_0' X_t + \theta_1' SB_{1t} X_t + \theta_2' SB_{2t} X_t + e_t \tag{135}$$

where the timing of the breakpoints is denoted by SB dummy variables. SB_{1t} and SB_{2t} are described as given

$$SB_{1t} = \begin{cases} 0 & \text{if } t \le [s\tau_1] \\ 1 & if\ t > [s\tau_1] \end{cases}$$

$$SB_{2t} = \begin{cases} 0 & \text{if } t \le [s\tau_2] \\ 1 & \text{if } t > [s\tau_2] \end{cases}$$

Where with the unknown parameters $\tau_1 \in (0,1)$ and $\tau_2 \in (0,1)$, the dummy variables imply the timing of breaks.

Hatemi-J (2008) remarks that the ADF and the modified test of Z_a and Z_t tests by Philips (1987) have nonstandard distribution, and the asymptotic distribution of the ADF and the Z_t test are the same. The modified Philips test statistics values, on the other hand, depend on the calculation of the coefficient value below.

$$\hat{\rho}^* = \left[\frac{\sum_{t=1}^{s-1}\left(\hat{e}_t\hat{e}_{t+1} - \sum_{j=1}^{B}\omega\left(\frac{j}{B}\right)\hat{\delta}(j)\right)}{\sum_{t=1}^{s-1}\hat{e}_t^2}\right] \tag{136}$$

where $\hat{\rho}^*$ represents the estimated first-order serial correlation coefficient value. Hence, the Philips test statistics, Z_a and Z_t, are calculated as given

$$Z_a = s(\hat{\rho}^* - 1)$$

$$Z_t = \frac{\hat{\rho}^* - 1}{\left(\frac{\hat{\delta}(0) + 2\sum_{j=1}^{B}\omega\left(\frac{j}{B}\right)\hat{\delta}(j)}{\sum_{1}^{s-1}\hat{e}_t^2}\right)} \tag{137}$$

After running the regressions, the smallest value of the ADF and Philips tests and the critical values given in his (2008) paper are compared. If the calculated value is smaller than the critical value, then the null hypothesis of no-cointegration is rejected. Equivalently stating, it implies that there is a long run relationship between the underlying variables with two endogenous structural breaks.

4.3.3 Causality Tests

Having introduced the cointegration relationship, it is time to explain the causality tests. If the variables under study have the same integration order $(d \geq 1)$, then it is required to test the hypothesis of long run relationship. If the variables are found to be cointegrated, the following step is to seek the direction of a relationship between variables via ECM or VECM allowing for the question whether there is short run or/and long run relationship. If the null of a non-cointegration hypothesis is not rejected, it is said that the variables are not moving together in the long run. But, for causality relationship between the variables, a different method is required after a non-cointegration output, namely vector autoregressive model (VAR). Generally speaking, for causality test based on VAR, both the stationarity and the non-cointegration conditions are required. In his book, Enders (2014) enumerates the consequences in the case of using the variables, not cointegrated, in levels for Granger causality. The first consequence is that test power will fall due to an estimation of one extra lag for each variable. And, the second consequence is, Granger test will not have a standard F distribution.

Granger (1969) describes the causality as in terms of the predictability and precedence of time series. Putting differently, the Granger causality refers to a linear causality in mean with regard to a specified data set. For a clear understanding, let us assume two different variables, X_t and Y_t which are both stationary:

$$X_t = \mu_1 + \sum_{i=1}^{s} \beta_i X_{t-i} + \sum_{i=1}^{s} \lambda_j Y_{t-i} + u_t \tag{138}$$

$$Y_t = \mu_2 + \sum_{i=1}^{s} A_i X_{t-i} + \sum_{i=1}^{s} \theta_i Y_{t-i} + v_t \tag{139}$$

where both the error terms are uncorrelated white-noise. The test output will have the following different cases:

a) The first outcome is a Granger causality from Y_t to X_t and it is denoted as $Y_t \Rightarrow X_t$. It means that Y_t Granger-causes X_t provided that λ_j parameters are different from zero. If it is the case and the opposite is not true, then it is said that there is a univariate or one-way causal relation from Y_t to X_t. Here, Y_t is sufficiently exogenous.

b) The second outcome is a Granger causality from X_t to Y_t and it is denoted as $X_t \Rightarrow Y_t$. More specifically, it is said that X_t Granger-causes Y_t provided that A_i parameters significantly different from zero. If it is the case and the opposite is not true, then it is said that there is a one-way causality from X_t to Y_t,

c) The third outcome is a situation where bi-directional (feedback) causality occurs. If both λ_j and A_i parameters are significantly different from zero, then it is said that Y_t Granger-causes X_t and X_t Granger-causes Y_t, i.e. $Y_t \Leftrightarrow X_t$. Namely, both variables contain equal amounts of information about each other.

d) The last outcome is that if both λ_j and A_i parameters are not different from zero, namely they are not significant, it is said that X_t and Y_t are independent. In this case, there does not exit any causal relation between X_t and Y_t.

It is should be noticed that for the causality test, "Granger" term is used. For avoiding misunderstanding, Granger (1980) advice of using "Granger causality" because, it refers to a simple (cross) correlation between X_t and Y_{t-1}, equivalently saying, it does not mean that a movement in Y_t causes a movement of X_t.

4.3.3.1 Hacker & Hatemi-J Symmetric Causality Test (2006)

To remark, in the case of causality tests, the underlying variables must be stationary and not cointegrated in VAR models. If this is not the case, the causality tests result based on VAR will be biased. Differently speaking, Granger and Newbold (1974) found via Monte Carlo simulations that the regression outcomes are spurious as the asymptotic distribution theory is invalid for hypothesis testing. Hence, as stated by Sims et al. (1990), whenever the data under study are cointegrated and non-stationary, one cannot implement a VAR model where they are used in level form because the standard distributions are not reliable.

To remedy the problems mentioned, Toda and Yamamoto (1995) suggest a new method where the cointegration and integration order is not regarded. Lach (2010) affirms that this method is a modification of the Wald test and it has been frequently implemented by researchers due to its simplicity and non-necessity for pretesting of cointegration and integration order. The author (2010) reminds that the central idea behind is based on estimating causality relationship with an augmented VAR($p + d$) model where d represents the maximum integration order of the underlying data and p represents the lag length for a model. Toda and Yamamoto (1995) claim that the need for including extra d parameters in the VAR is for ensuring that the asymptotic theory holds. However, as noted by Hacker and Hatemi-J (2006), the T-Y method is poorly performed such as nonnormality and time-varying volatility in the case of small-sized samples due to its asymptotical distribution. The authors (2006) propose an extended version of the T-Y method where the critical value of MWALD test is calculated to remedy its size distortions by Monte Carlo simulations.

It should be emphasized that Hacker and Hatemi-J (2006) follow the same way of the T-Y method. Hence, let us assume a VAR(p) model as given Toda and Yamamoto (1995):

$$Y_t = b_0 + b_1 t^1 + b_2 t^2 + \cdots + b_q t^q + v_t \tag{140}$$

where assumed that v_t is integrated of order (d) and $CI(d, b)$. With the condition of $\varepsilon_t \sim iid(0, \sigma^2)$, v_t can be parameterized as

$$v_t = B_1 v_{t-1} + B_2 v_{t-2} + \cdots + B_k v_{t-k} + \varepsilon_t \tag{141}$$

If one substitutes $v_t = Y_t - b_0 - b_1 t^1 - b_2 t^2 - \cdots - b_q t^q$ into equation above, then Y_t will be

$$Y_t = b_0 + b_1 t^1 + \cdots + b_q t^q + B_1 Y_{t-1} + \cdots + B_k Y_{t-k} + \varepsilon_t \tag{142}$$

or equivalently written for the augmented VAR($p + d$) where $\hat{\delta} = \hat{b}_0 + \hat{b}_1 t^1 + \cdots + \hat{b}_1 t^q$ according to Hacker and Hatemi-J (2006)

$$Y_t = \hat{\delta} + \hat{B}_1 Y_{t-1} + \cdots + \hat{B}_k Y_{t-k} + \cdots + \hat{B}_{k+d} Y_{t-k-d} + \hat{\varepsilon}_t \tag{143}$$

Before hypothesis testing, the authors (2006) give the following definitions

$Y := (Y_1, Y_2, Y_3, \ldots, Y_N)$ an (n x N) matrix

$$\widehat{D} := (\hat{\delta}, \widehat{B}_1, \widehat{B}_2, \dots, \widehat{B}_k, \widehat{B}_{k+1}, \dots, \widehat{B}_{k+d}) \text{ an } (n \text{ x } (1 + n(k + d))) \text{ matrix}$$

and

$$M_t := \begin{bmatrix} 1 \\ Y_t \\ Y_{t-1} \\ Y_{t-2} \\ \vdots \\ Y_{t-k-d+1} \end{bmatrix} \text{is a } \left((1 + n(k + d)) \text{ x } 1\right) \text{ matrix, for } t = 1,2, \dots M$$

where

$$M := (M_0, M_1, M_2, \dots, M_{N-1}) \text{ a } \left((1 + n(k + d)) \text{ x } N\right) \text{ matrix}$$

For estimation of the augmented VAR$(p + d)$ via using the definitions above, the model can be described as given

$$Y = \widehat{D}M + \hat{\epsilon} \tag{144}$$

where $\hat{\epsilon} := (\hat{\varepsilon}_1, \hat{\varepsilon}_2, \hat{\varepsilon}_3, \dots, \hat{\varepsilon}_N)$ an $(n$ x N) matrix and it is bootstrapped residuals. The authors (2006) proceed by estimation C_U parameter which is the variance-covariance matrix of residuals from the unrestricted augmented model. It is calculated as follows

$\mathrm{C}_U = \hat{\epsilon}_U' \hat{\epsilon}_U / N$, and $\vartheta = vec(\delta, B_1, B_2, \dots, B_k, 0_{n \text{ x } nd})$ and $\hat{\vartheta} = vec(\widehat{D})$

Hence, the modified Wald (MWALD) test statistic for testing the null hypothesis of non-Granger causality based on the T-Y method is given as Toda and Yamamoto (1995)

$$\text{MWALD}_{\text{HH}} = (R\hat{\vartheta})' [R((M'M)^{-1} \oplus + \mathrm{C}_U) R']^{-1} (R\hat{\vartheta}) \tag{145}$$

where R is $\left(k \text{ x } n\left(1+n(k+d)\right)\right)$ matrix and $\oplus$ represents the Kronecker product. As a result, the null hypothesis is

$$H_0: R\hat{\vartheta} = 0 \tag{146}$$

They (2006) claim that the MWALD test statistic for this method is asymptotically χ^2 distributed and the error terms are normally distributed where k parameter is equivalent to the number of degrees of freedom.

4.3.3.2 Hatemi-J Asymmetric Causality Test (2012)

This causality method (2012) is actually based on Granger and Yoon's (2002) hidden cointegration method where the authors (2002) used the positive and negative components of the underlying time series instead of the original data. Granger and Yoon (2002) define the cointegration of two time series, for example, as responding to (such as oil) shocks together. But, if they do not respond to shocks in a similar way, putting equivalently, if their responding differs to kind of shocks, the authors (2002) ask what would happen then? For example, as noted by the authors (2002), central banks may respond differently to interest rising shocks than interest falling shocks. As a result, the authors (2002) clarify the main reason behind their hidden cointegration theory as the macroeconomic variables might have valuable information hidden in their positive or negative parts although they are not cointegrated at the original form.

On the other hand, Hatemi-J (2012) enumerates the motives, such as Granger and Yoon's (2002), behind the asymmetric causality tests. Firstly, the author (2012) states that in literature, the possible effect of negative and positive shocks are usually neglected, namely their effects are presumed to be the same. For example, as noted by the author (2012), it is generally agreed that investors (institutional or individual) and practitioners incline to respond less to positive news than the negative news. Thus, the asymmetric nature of shocks can be defined as the first reason for nonlinear causality test form.

On the other hand, the existence of asymmetric information fact, especially, in financial markets, can be seen as the second reason. The author (2012) proposes taking into account the effect of this asymmetric information fact as cumulative sums of negative and positive shocks for causality tests. Due to non-normality and time-varying volatilities of financial variables, the author (2012) follows the same way of the symmetric causality test where the critical values are obtained by Monte Carlo simulations.

Let us assume that there are two time series for the causality relationship. According to the author (2012), they are non-stationary:

$$Y_{1t} = Y_{1t-1} + \epsilon_{1t} = Y_{10} + \sum_{i=1}^{t} \epsilon_{1i} \tag{147}$$

$$Y_{2t} = Y_{2t-1} + \epsilon_{2t} = Y_{20} + \sum_{i=1}^{t} \epsilon_{2i} \tag{148}$$

where $Y_{1,0}$ and $Y_{2,0}$ stand for the initial values, ϵ_{1i} and ϵ_{2i} represent white noise error terms and $1 \leq t \leq T$. The author (2012) describes the negative and the positive shocks of the underlying variable as $\epsilon_{S,i}^{-} = \min(\epsilon_{S,i}, 0)$ and $\epsilon_{S,i}^{+} = \mathrm{mak}(\epsilon_{S,i}, 0)$ where "S" is the integer value of "1" and "2", respectively. Thus, the white noise disturbance or error terms is equal to a sum of $\epsilon_{S,i} = \epsilon_{S,i}^{-} + \epsilon_{S,i}^{+}$. Then

$$Y_{S,t} = Y_{S,t-1} + \epsilon_{S,t} = Y_{S,0} + \sum_{i=1}^{t} \epsilon_{S,i}^{-} + \sum_{i=1}^{t} \epsilon_{S,i}^{+} \tag{149}$$

Similarly, the cumulative sum of negative and positive shocks having permanent impact can be formulized as $Y_{S,i}^{-} = \sum_{i=1}^{t} \epsilon_{S,i}^{-}$ and $Y_{S,i}^{+} = \sum_{i=1}^{t} \epsilon_{S,i}^{+}$. After calculating components, the next step is to focus on causality relationship. If one assumes that $Y_t^{-} = Y_{1t}^{-} + Y_{2t}^{-}$ for the causality between negative shocks, then the VAR(k) model will be

$$Y_t^- = \delta + B_1 Y_{t-1}^- + \cdots + B_k Y_{t-1}^- + \varepsilon_t^- \tag{150}$$

where Y_t^-, δ and ε_t^- parameters are the 2 x 1 vector of the variables, intercepts and error terms, respectively. As noted by the author (2012), the B_r variable, on the other hand, is a 2 x 2 matrix of parameters for the length of lag r where $1 \leq r \leq k$ and the optimal lag order is selected via the HJC method robust to ARCH effect suggested by Hatemi-J (2003). Hence, the null hypothesis is

$$H_0\text{: [the row } \omega\text{, column } c\text{, element in } B_r] = 0 \text{ where } 1 \leq r \leq k \tag{151}$$

Similarly, in the symmetric causality, the author (2012) defines the necessary denotations for the negative and positive components as follows

$$Y := (Y_1^-, Y_2^-, Y_3^-, \ldots, Y_N) \quad \text{an } (n \text{ x N) matrix}$$

$$D := (\delta, B_1, B_2, \ldots, B_k) \text{ an } (n \text{ x } (1 + nk)) \text{ matrix}$$

and

$$M_t := \begin{bmatrix} 1 \\ Y_t^- \\ Y_{t-1}^- \\ Y_{t-2}^- \\ \vdots \\ Y_{t-k+1}^- \end{bmatrix} \text{is a } \big((1 + nk) \text{ x } 1\big) \text{ matrix, for } t = 1,2, \ldots M$$

where

$$M := (M_0, M_1, M_2, \ldots, M_{N-1}) \text{ a } \big((1 + nk) \text{ x } N\big) \text{ matrix}$$

For estimation of the VAR(p) via using the definitions above, the model can be described as given

$$Y = DM + \epsilon \tag{152}$$

where $\epsilon := (\varepsilon_1^+, \varepsilon_2^+, \varepsilon_3^+, \ldots, \varepsilon_N^+)$ an $(n \text{ x N})$. The required test statistic, on the other hand, for asymmetric causality relationship is

$$\text{MWALD}_{\text{HJ}} = (R\vartheta)'[R((M'M)^{-1} \oplus +\text{C}_U)R']^{-1}(R\vartheta) \tag{153}$$

where $\vartheta = \text{vec}(D)$ and $\oplus$ implies the Kronecker product. The variance-covariance matrix, on the other hand, of the unrestricted VAR regression is calculated as $\text{C}_U = (\hat{\epsilon}_U' \hat{\epsilon}_U)/(N - q)$, where "$q$" signifies the parameter numbers for each VAR regression. The null hypothesis, consequently, is parameterized as follows

$$H_0: R\vartheta = 0 \tag{154}$$

The author (2012) claims that the MWALD test statistic for this method is asymptotically χ^2 distributed and the error terms are normally distributed where k parameter is equivalent to the number of degrees of freedom. However, to remedy the drawbacks of non-normality and ARCH effects, the author (2006) proposes the bootstrapping simulation methods. Hence, if the absolute value of the bootstrap critical value is lower than the calculated statistic value, then the null hypothesis is rejected, implying a Granger-causality between the underlying components.

4.3.3.3 Breitung & Candelon Frequency Causality Test (2006)

As stated above, the conventional causality tests based on VAR or VECM are related to the time domain, namely, they illustrate the relationship between the underlying variables not only over the short run but also over the long run. The main drawback of them is that they are unable to detect the link between the short and long-term. Hence, a need for causality test depended in the frequency-domain arises for the underlying relationship.

In his article, Granger (1969) proposes not only conventional causality test but also describes a relationship called cross-spectrum based on the frequency domain which should remark the wavelets. The author (1969) states that the link between the underlying variables, however, can be defined as according to their frequency parts. To a clear understanding of the relationship, two different functions, the coherence and the phase, are described by the author. The first term refers to the square of the correlation while the second term, on the other hand, gauges the degree of the time lag in the context of cross-correlation between related frequency parts. In the beginning of 1980's and 1990's, Geweke (1982) and Hosoya (1991) both suggest a causality test based on frequency-domain. Breitung and Candelon (2006), on the other hand, propose a frequency causality test built on work of Geweke (1982) where the authors state that their method is based on imposing linear restrictions on the parameters of VAR model.

The measure of frequency-based causality test proposed by Geweke (1982) and Hosoya (1991) is formulized as given

$$M_{Y\to X}(\omega) = \log\left[\frac{2\pi f_X(\omega)}{|\mathcal{F}_{11}(e^{-i\omega})|^2}\right] = \log\left[1 + \frac{\left|\mathcal{F}_{12}(e^{-i\omega})\right|^2}{|\mathcal{F}_{11}(e^{-i\omega})|^2}\right] \tag{155}$$

where $M_{Y\to X}(\omega)$ and $f_X(\omega)$ signify the causality at a frequency (ω) and spectral density of X_t, respectively. In the case of $\left|\mathcal{F}_{12}(e^{-i\omega})\right| = 0$, $M_{Y\to X}(\omega)$ is equal to zero and it is said that "Y" variable does not cause "X" variable at frequency (ω) point. On the other side, in the case of stationarity, the measure of frequency-based causality formulation using the orthogonalized moving average representation is the same of the equation above

$$M_{Y\to X}(\omega) = \log\left[1 + \frac{\left|\tilde{\mathcal{F}}_{12}(e^{-i\omega})\right|^2}{\left|\tilde{\mathcal{F}}_{11}(e^{-i\omega})\right|^2}\right] \tag{156}$$

Breitung and Candelon (2006) describe the VAR regression model for frequency-based causality test having the linear restrictions as

$$X_t = \kappa_1 X_{t-1} + \cdots + \kappa_k X_{t-k} + G_1 Y_{t-1} + \cdots + G_k Y_{t-k} + \epsilon_{1t} \tag{157}$$

or equivalently

$$X_t = \sum_{s=1}^{k} \kappa_s X_{t-s} + \sum_{s=1}^{k} G_s Y_{t-s} + \epsilon_{1t} \tag{158}$$

The authors (2006) define the null hypothesis as $M_{Y\rightarrow X}(\omega) = 0$ which is equal to the linear restriction

$$H_0: A(\omega)G = 0 \tag{159}$$

where $G = [G_1, G_2, \ldots, G_k]'$ and $A(\omega)$ is defined as

$$A(\omega) = \begin{bmatrix} \cos(\omega) & \cos(2\omega) & \cos(3\omega) & \cdots & \cos(k\omega) \\ \sin(\omega) & \sin(2\omega) & \sin(3\omega) & \cdots & \sin(k\omega) \end{bmatrix} \tag{160}$$

The authors (2006) assert that the null hypothesis at (ω) point is tested via F statistics which is approximately distributed as $F(2, N-2k)$ for $\omega \in (0, \pi)$.

4.4 Wavelet-based Analysis

In this section, wavelet estimations for variance, correlation and cross-correlations and causal relationships will be discussed in order to explore true dynamics between the underlying variables across frequency bands.

4.4.1 Wavelet Multiresolution Analysis

In literature, the wavelets can be calculated via using statistical programs such as "R" (2006) or "MATLAB" (2015a). It should be mentioned, with regarding the aim of this thesis, "Waveslim" R (2006) package created by Whitcher (2005) is chosen. No matter which statistical software is chosen, the main functions for wavelet calculation are the same: MRA() and DWT() or MODWT(). The first function is used for multiresolution analysis coefficients (MRA coefficients, hereafter) and the second function is applied for the coefficients used for wavelet (MODWT coefficients) statistics. For this thesis, the MODWT method is applied to weekly log-differenced data at $J = 5$ decomposition scale, although, the maximum integer level is $9 \leq \log_2(604)$. Wavelet decomposition, however, is related to the observation number, namely the wavelet levels (J) are determined via $J \leq \log_2(N)$. But, unfortunately, due to the boundary condition, the feasible coefficients number for wavelet statistics decreases gradually by scale. For example, at scale 6, the observation number unaffected by boundary condition is 163 while at scale 7, the number drops to below zero. It should be mentioned that the boundary condition works only for the MODWT coefficients, i.e. calculation of variance and correlations. Putting differently, the observation number for MRA decomposition is the same while it changes gradually for MODWT decomposition.

For this empirical application, LA(8) wavelet filter (least asymmetric function, Daubechies) with periodic boundary condition is chosen. Due to boundary condition problems, we decompose the underlying time series into $J = 5$ levels, which gives us five levels of detail components and a smooth component: $\mathrm{MRA}_5 = d1 + d2 + d3 + d4 + d5 + s5$ and wavelet components and a scaling component $\mathrm{MODWT}_5 = w1 + w2 + w3 + w4 + w5 + s5$. These decomposition levels are described as $d1\,[2-4)$ weeks, $d2\,[4-8)$ weeks, $d3\,[8-16)$ weeks, $d4\,[16-32)$ weeks, $d5\,[32-64)$ weeks, $s5\,[+64)$ weeks. It is evident that decomposition levels are defined as the inverse of frequency. Unlike frequencies, as decomposition level increases, so time-interval increase. The wavelet decomposition by scale is depicted in the following pages.

The first figure is Figure **4-1**, which depicts the MRA coefficients of the weekly 2-year government bond returns. One can clearly see that, at the top of the figure, the original data of return series is illustrated. The three lines below the original return series illustrate the five wavelet detail crystals and the wavelet smooth crystal which are an additive decomposition of the original data. Differently speaking, the MRA coefficient crystals completely capture the volatility of the original data. It is evident that the original return data includes several extreme values, namely the highest volatilities. The first of highest weekly values is 14.3% seen on June 23, 2006, where it was expected that the Federal Reserve (the FED) would raise the federal funds rates. The second of highest weekly values is 13.06% observed on November 24, 2008, during the global financial crisis (the GFC, hereafter) began in the US in 2007 but was triggered after the Lehman Brothers filed for bankruptcy. In June 2013, however, two highest weekly increase, 14.57% and 19.52% in "TR2YGB" rates, are observed.

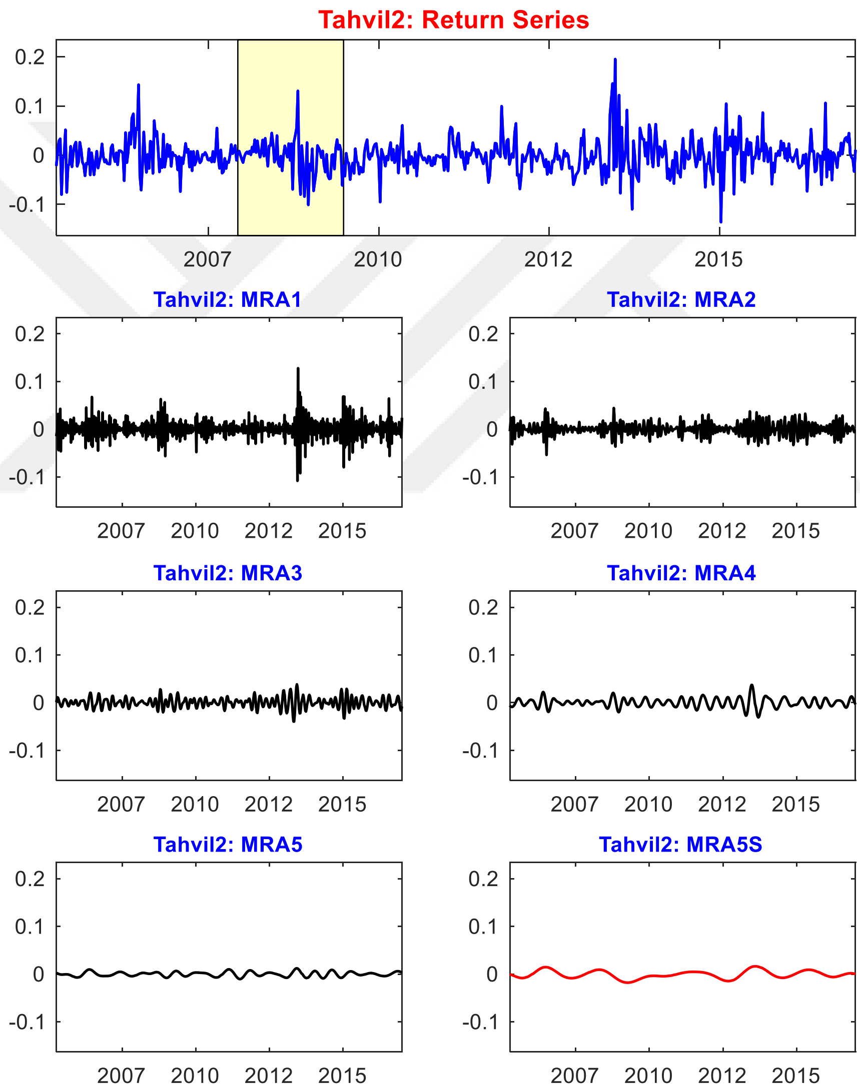


Figure 4-1 Multiresolution Analysis of TR 2-Year Bond Return Series

Besides the impact of domestic development, the most important reason behind was that the FED announced that it would begin soon to taper its $85 billion monthly bond purchases. On the other hand, on September 20, 2013 and on January 9, 2015 "TR2YGB" rate decreased by 10.99% and 13.61%, because the CBRT decided to not change the fund rates at its meeting on September 2013 while it decided to cut rate at its meeting in January 2015 owing to the positive improvement in domestic and foreign markets largely caused by falling oil price. Hence, when looking at the graph, it can be seen that the effect of development in domestic and foreign markets mainly caused by FED rate decisions decreases by scale, namely, all levels get smoother as they increase.

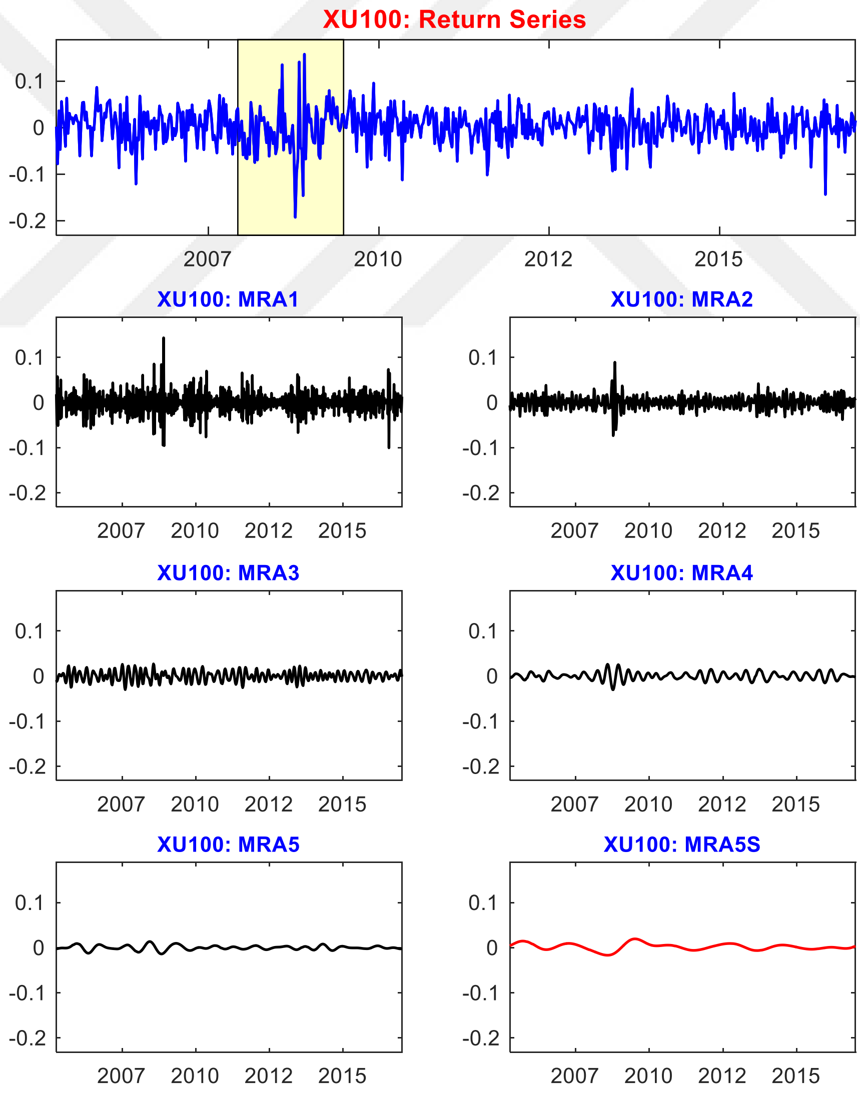


Figure 4-2 Multiresolution Analysis of XU100 Return Series

In literature, it is widely accepted that there is a negative relationship between interest rates and stock markets. Broadly speaking, whenever an increase in interest is observed, then the stock market starts to fall because they are seen as alternative investment assets to each other. Thus, when looking at the Figure **4-2**, it can be observed that the weekly "XU100" return series has lower volatilities than "TR2YGB" return series.

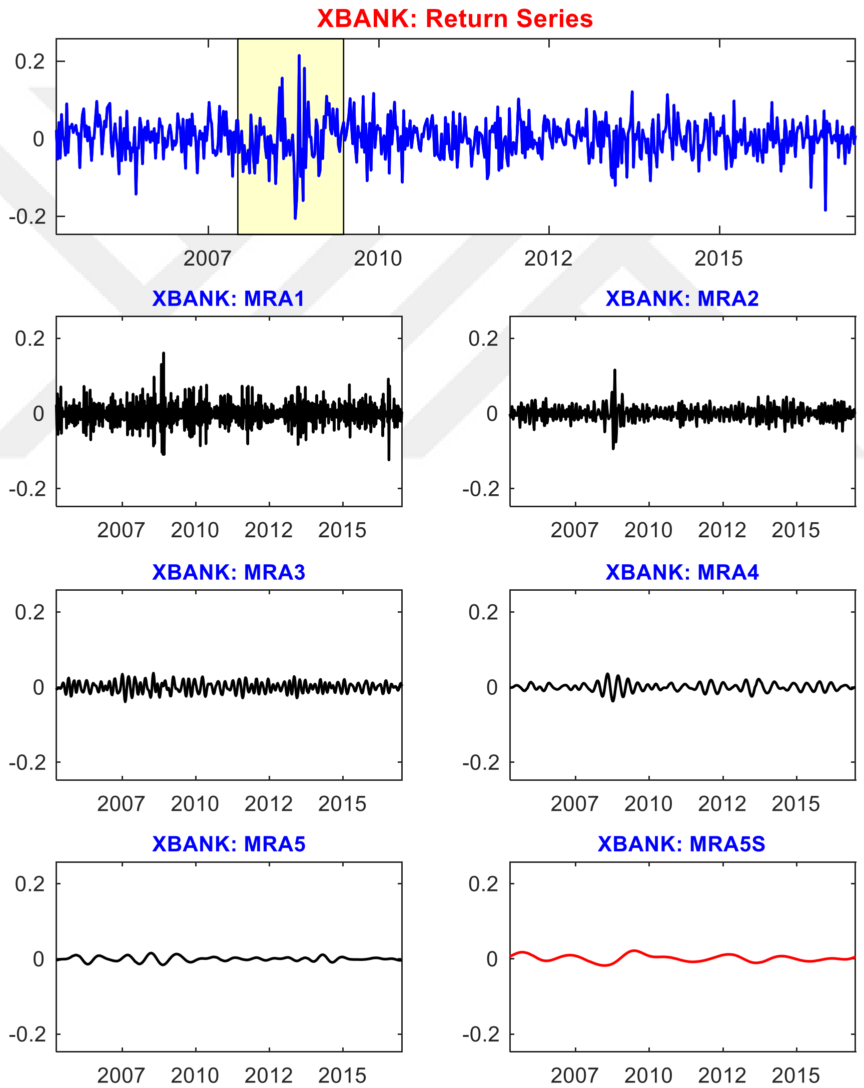


Figure 4-3 Multiresolution Analysis of XBANK Return Series

Apart from the volatilities of "TR2YGB" return series, mainly caused by the same factors, "XU100" return series has higher weekly volatility value, -19.27%, observed on October 10, 2008. At the same month, "TR2YGB" series, however, had increased by 13.06%. It should be mentioned that the highest increase in "XU100" series, 15.76%, is seen in December 2008. On the other hand, on May 6, 2010, the DJIA decreased sharply by 9% in a matter of minutes, at about 2:45 due to likely "fat-finger" error or high frequency trading and this free-fall is called as the Flash Crash. In global financial markets, this largest intraday loss in history had a great impact on stock prices. The weekly drop of "XU100" index was higher than 11%. Conversely, the most prominent volatility is, -14.37%, seen on July 22, 2016, which is observed after a week later of the failed coup attempt on July 15, 2016, in Turkey. According to the MRA coefficients, these all volatilities are approximately captured by the decomposition levels of d1 [2 – 4], d2 [4 – 8], and d3 [8 – 16] week periods.

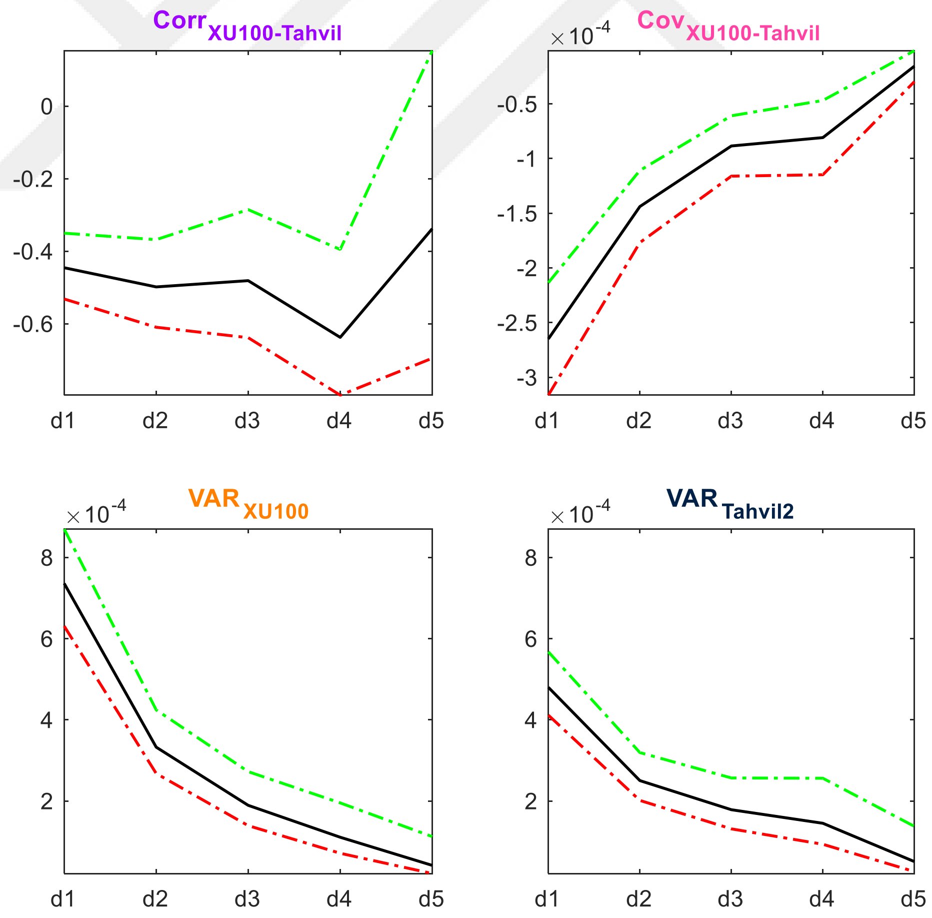


Figure 4-4 Wavelet Covariance, Correlation and Variance by Scale: TR2YGB and XU100 Return Series

Beside the main index, "XU100", the sector indices are also decomposed by "MODWT MRA" function. The first sector index is the BIST Banks index ("XBANK") which is the leading index of "XU100". Whenever "XU100" index, for example, increases by 10%, "XBANK" index rises more and vice versa. Actually, the main factor behind fluctuation in "XU100" index is the movement of "XBANK" index. Hence, it can be easily seen that their volatilities based on MRA decomposition levels are “nearly” the same. However, there is actually a slight difference from their volatilities: the reaction of "XBANK" is higher than "XU100", i.e. generally the beta of "XBANK" is higher than 1.00, implying higher risk and hence higher profit opportunity. On the other hand, in the Flash Crash week, this index declined by 11% which is almost the same as "XU100" index had experienced.

When looking for the maximum and minimum return values for the other indices, it is observed that their reactions differ from "XBANK" and "XU100". Within the same specific time intervals, for example, the GFC, the Flash Crash or July 15, 2016, the failed-coup attempts, some stock indices’ reaction was the same while the other reacted differently. For example, on October 10, 2008, "XU100" and "XBANK" declined by -19.27% and -20.58%. However, at the same time, "XSPOR" went down sharply by -45.79% while the indices of "XFINK", "XELKT", "XSGRT", "XULAS" and "XMANA" decreased by -35.97%, -35.14%, -28.84%, -27.97%, and -24.42%, respectively, which were higher than "XU100" and "XBANK" decreases. But, after a couple of weeks, these losses were recovered by the stock indices, for example, the largest weekly increase, 35.41%, for "XELKT" was observed after a week later for its largest decrease of -35.14%. On the other hand, on July 15, 2016, all of the stock indices declined at different percentages. The highest decreases were observed at "XBANK", "XULAS", "XELKT", and "XUMAL" indices, namely they declined at least by -17%, but the recovery was not easy in the short run. When looking at the wavelet decomposition of "MRA", it can be stated that the effect of volatilities has continued until the scale d4, i.e. 32 week periods.

4.4.2 Wavelet-based Estimations

In this section, wavelet-based statistics of wavelet variance, covariance, correlations and cross-correlations by scale decomposition are analyzed.

4.4.2.1 Wavelet Variance and Covariance

Having mentioned before, these statistics are calculated via "MODWT" coefficients, where the feasible coefficients number decreases by scale. For example, the total number of observations is 604, but, after "MODWT" decomposition with the periodic condition, the number of non-boundary coefficients drops to 597 for d1, 583 for d2, 555 for d3, 499 for d4, and 387 for d5 and s5. Notable, this is due to obtain unbiased estimations for wavelet variance, covariance and correlation by scale decomposition plotted in Figure **4-4**.

Figure **4-4** shows "MODWT" based covariance, correlation, and variance of "TR2YGB" and "XU100" return series scale-by-scale basis. The straight and black lines signify the estimated wavelet-based statistics while the red-dotted and the green-dotted lines indicate the lower and the upper bounds for the 95% approximate confidence interval. Evidently, their variances decrease, as the correlation coefficient increases negatively from scale d1 to d2, flattens between d2 and d3, sharply increases at scale d3, and then aggressively closes to zero at scale d5. It should be noted, as expected the correlation is negative for all scales and statistically significant, except the scale d5.Figure **4-5** displays “MODWT" based variance of the stock market indices and "TR2YGB" returns series according to the scale of weeks. It is evident that all of the time series’ variances decrease by scale decompositions. Putting differently, the change in variance by scale decomposition is quite similar for the time series under study, but their magnitude shows different patterns by scale. For example, the first five highest volatilities are observed for "RXULAS", "RXELKT", "RXBANK", "RXMANA", and "RXSPOR" stock indices at scale d1, where the variance values are 0.131%, 0.126%, 0.123%, 0.115%, and 0.113%, respectively.

On the other hand, the first five lowest variances are seen for "RXTAST", "RTR2YGB", "RXYORT", "RXUHIZ", and "RXUSIN" time series at scale d1, where the scale contribution to overall energy is 0.0440%, 0.0480%, 0.0481%, 0.0490%, and 0.057%, respectively. When looking at the scale d2, it is seen that the first five highest and lowest volatilities for the scale d1 are nearly the same, except for "RXMANA" and "RTR2YGB", but with a different ranking. For the scale d1 and d2, the total contribution to overall energy distribution is the higher than 60%. Differently speaking, the short run period explains at least 60% of the variance for all the underlying time series between the two and eight weeks. Interestingly, "RXGIDA" and "RXILTM" indexes' total energy decompositions are the two highest value for the scale $d1 + d2$ and $d1 + d2 + d3$, where their short runs explain at least 80% and 91% of volatilities, respectively. Furthermore, at least 76% of volatilities of all time series is explained by short run, i.e. the short run dominates the long run in case of energy distribution.

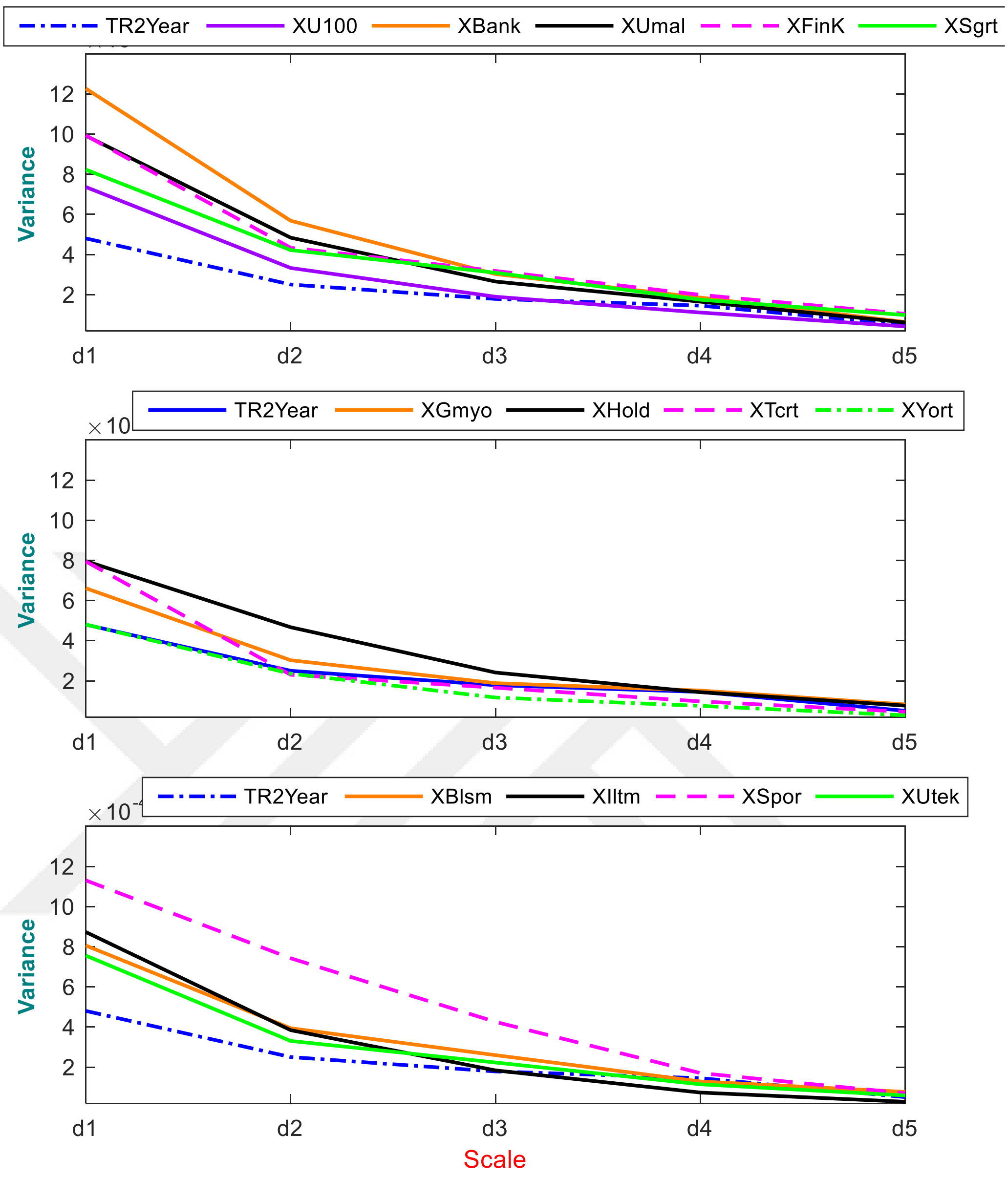


Figure 4-5 Variance by Scale Decomposition

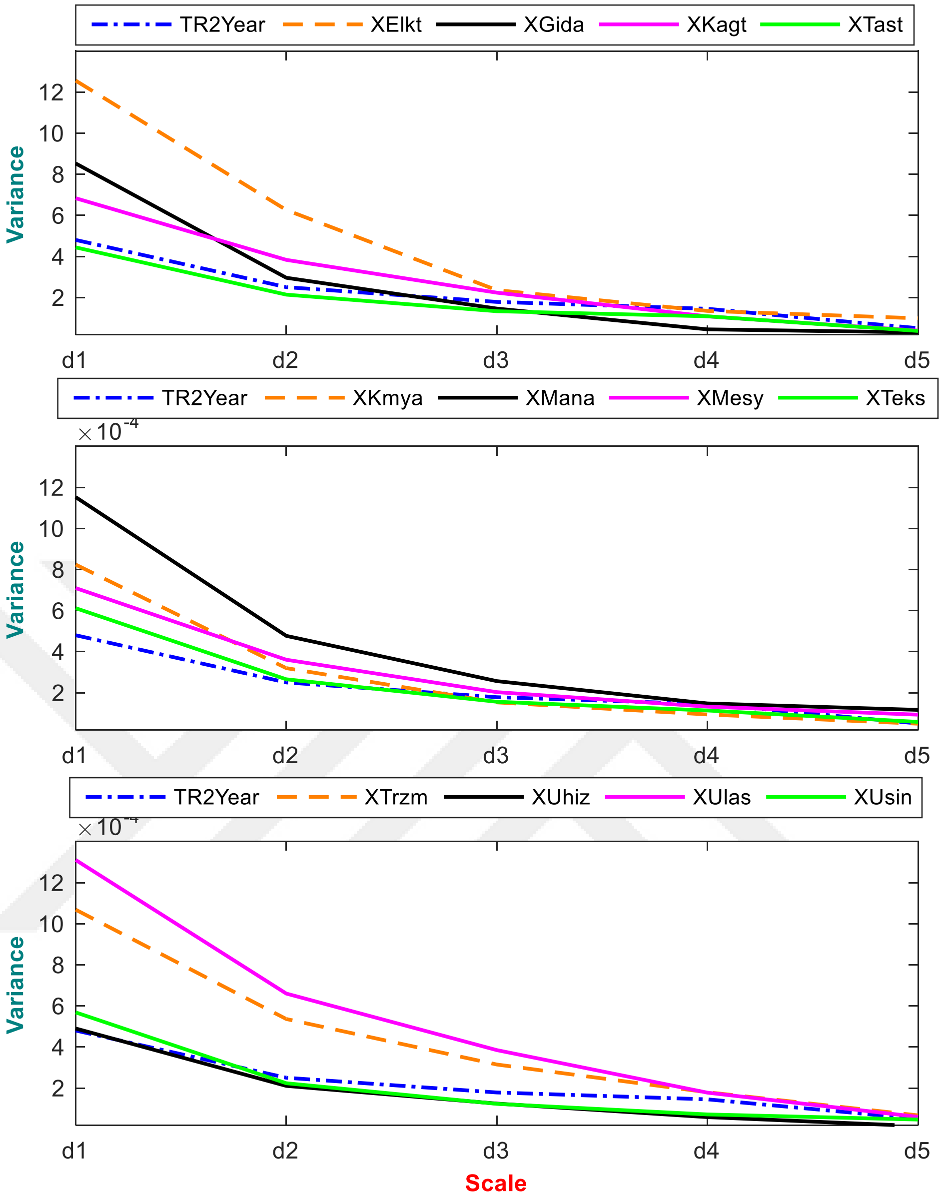


Figure 4-5 Variance by Scale Decomposition (cont.)

The evidence of an inverse but a cogent linear link between the wavelet variance and the wavelet time scale decomposition consists with that of earlier empirical papers of Kim and In (2007), Çifter and Özün (2007b), Moya-Martínez et al. (2015). For example, Çifter and Özün (2007b) state that the wavelet variance of "IMKB100", "IMKB30" and some individual stocks decreases while the wavelet scale (investment horizon) increases. On the other hand, Kim and In (2007) report the same results for the wavelet variance of the stock markets and bond markets of G-7 countries, besides, it is said that the observed variances of the former market are higher than the latter market variances. Dajcman (2015) reports the same results for the paper including Eurozone countries' markets, except Portugal case where the bond market volatility is higher than the stock market volatility for all wavelet scales. Moya-Martínez et al. (2015), generally speaking, confirm our wavelet variance results of stock market indices and "RTR2YGB", namely, the bond market exhibits less volatile than the stock market. Notable, the changes in bond rates is found less than all sector indices of Spain stock market regardless of the wavelet scales which this result is slightly different from the Turkey stock market case, equivalently saying the wavelet variance of "RTR2YGB" variable is the 25th, 21th, 18th, 10th, and 16th lowest one among the other variables at scale d1, d2, d3, d4, and d5, respectively. It should be pointed out that a lower wavelet variance of "RTR2YGB" is observed for the last two scales of d4 and d5, than "RXU100".

These results intimate that, according to the heterogeneous market hypothesis, investors having short-time investment horizons, as dictated by Kim and In (2007), should react to every variation in the realized returns to avoid facing higher risks such as uncertainty or volatilities in both stock and bond markets to manage these risks efficiently. Conversely, the investors having long-time investment horizons should not respond to every variation in the financial markets because the risk or uncertainty in long-term has considerably less important.

As in case of wavelet variance, the wavelet covariance based on "MODWT" coefficient unaffected by boundary condition can be derived by scale decomposition to calculate the degree of association between the stock index and "RTR2YGB" time series. The following two figures illustrate how the wavelet covariance changes by scale decomposition. Evidently, the whole wavelet covariance between "RTR2YGB" and the stock indices are negative at every scale and they decrease to zero by scale decomposition. Putting differently, the relationship between "RTR2YGB" and the stock index is negative, and they do not move in the same direction. Moving inversely implies that an increase in "RTR2YGB" growth rate leads a decrease in the underlying time series return movement.

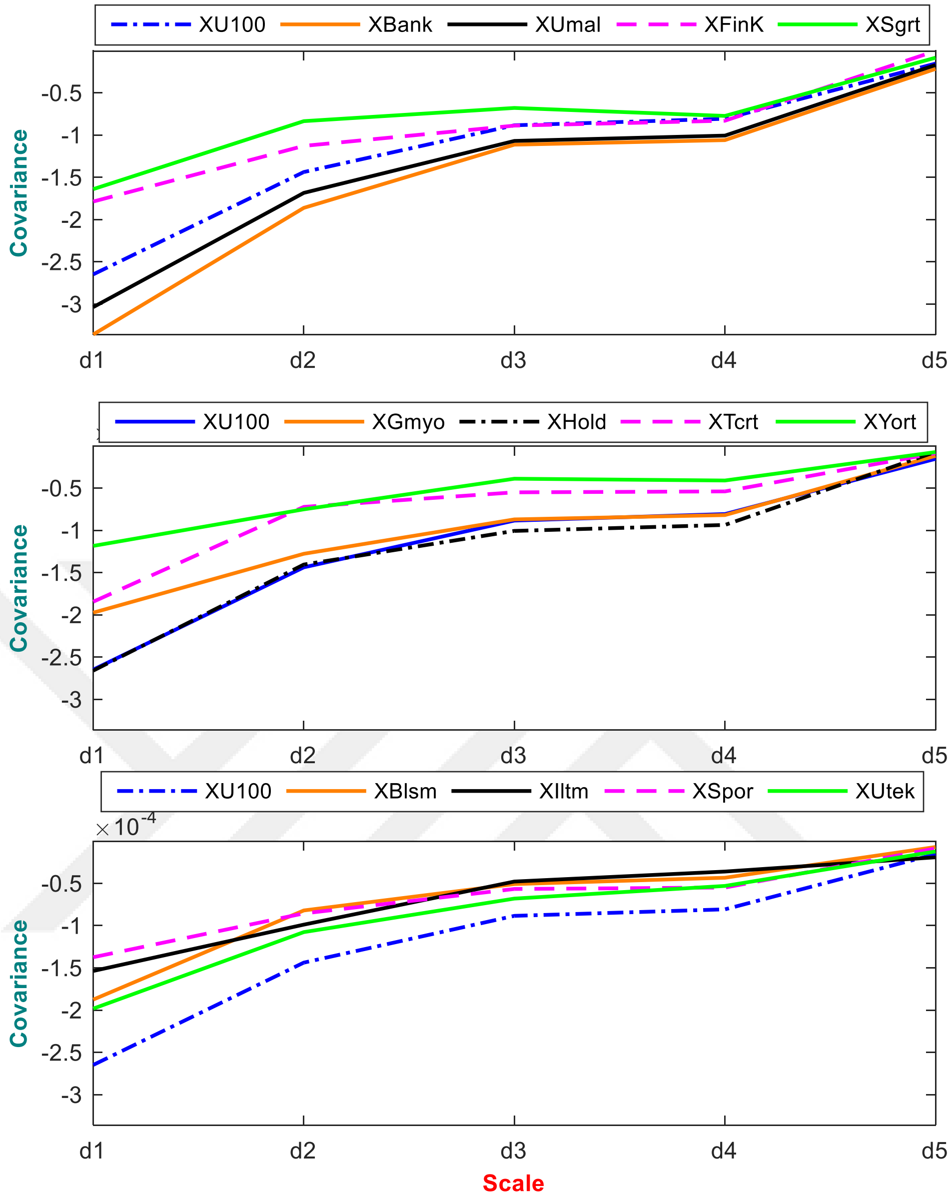


Figure 4-6 Covariance by Scale Decomposition

When looking at the figures below, it is shown that the highest negative covariance coefficients observed are for the indices of "RXBANK", "RXUMAL", "RXULAS", "RXHOLD", and "RXU100" at scale d1, ranking as −0.0336%, −0.0304%, −0.0274%, −0.0266%, and −0.0265%, respectively. The indices of "RXBANK" and "RXUMAL" are the two indices of having the highest negative covariance value for all scale, except the scale d3. This result implies that they have a stronger covariance relationship with “RTR2YGB" than the main stock market index, “RXU100", suggesting a lower volatility for portfolio diversification with stock indices than the aggregate stock market index for the earlier scales. At the last scales, the opportunity of reduction in the overall risk of the portfolio decreases for all stock market indices as both stock and bond markets move independently in the long run.

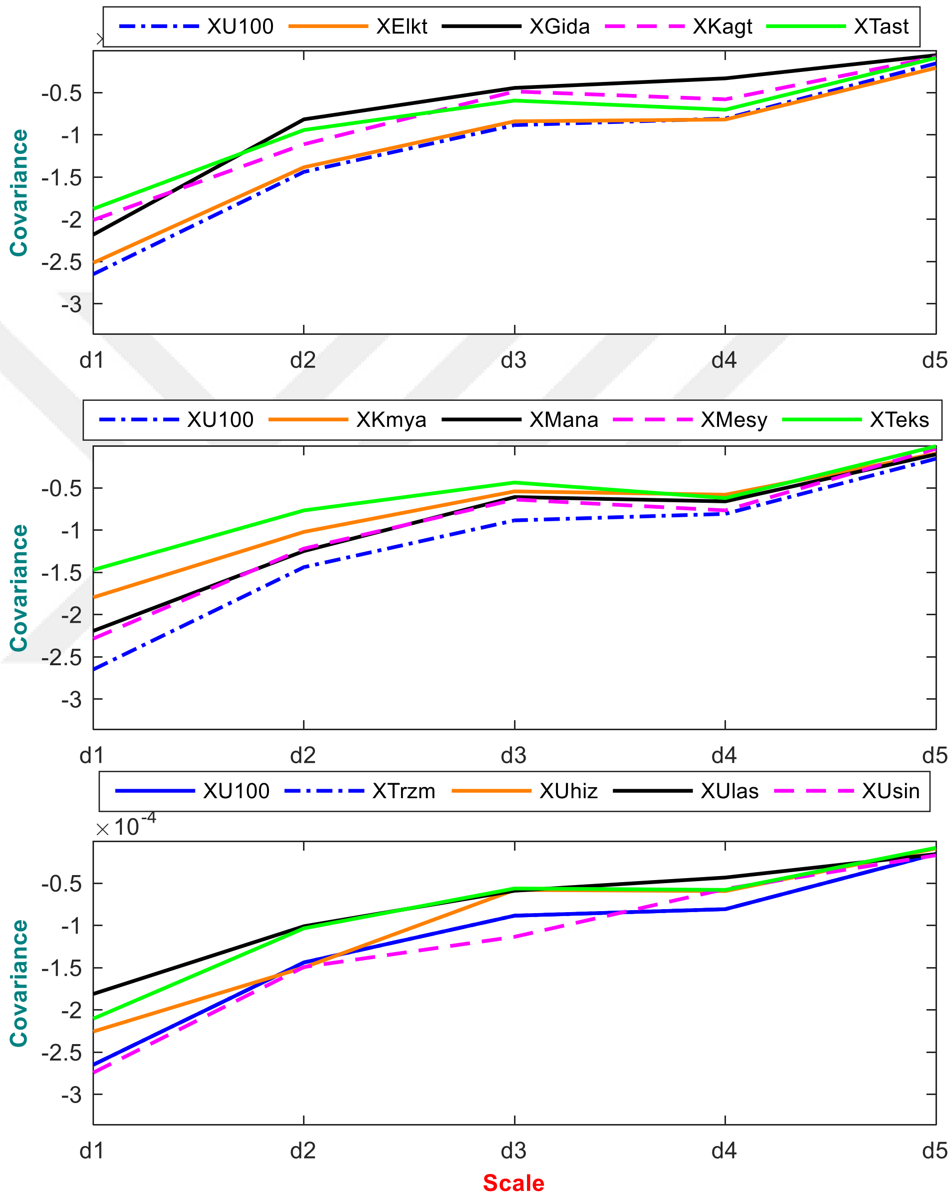


Figure 4-6 Covariance by Scale Decomposition (cont.)

It can be concluded that the covariance relationship becomes weaker as scale increases for all the underlying time series. It should be emphasized that although the wavelet covariance shows a relationship between the underlying variables, one cannot measure how close or how far “RTR2YGB" and a stock index move together. Thus, one should standardize the wavelet covariance values dividing by their variances to compare the relationship across the scales.

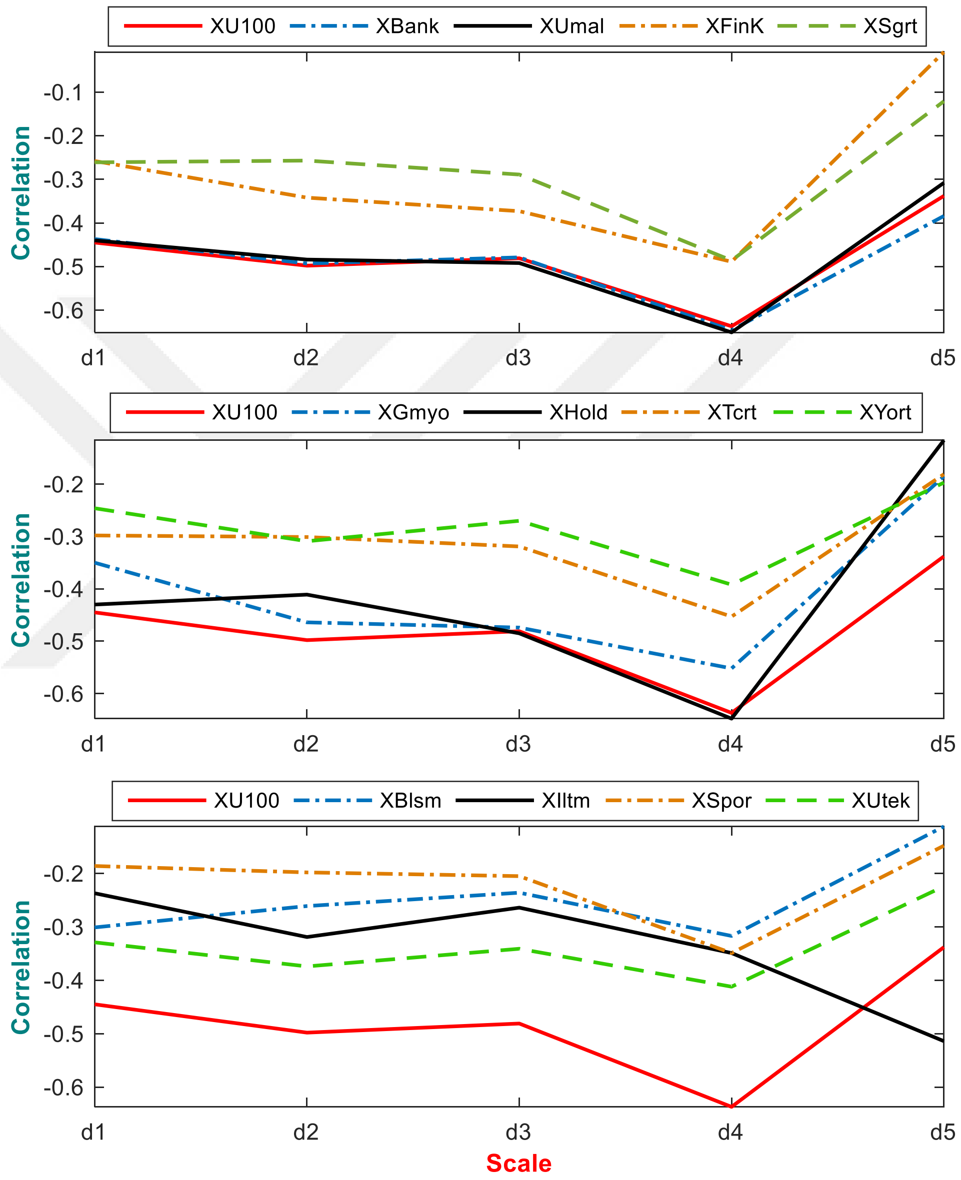


Figure 4-7 Correlation by Scale Decomposition

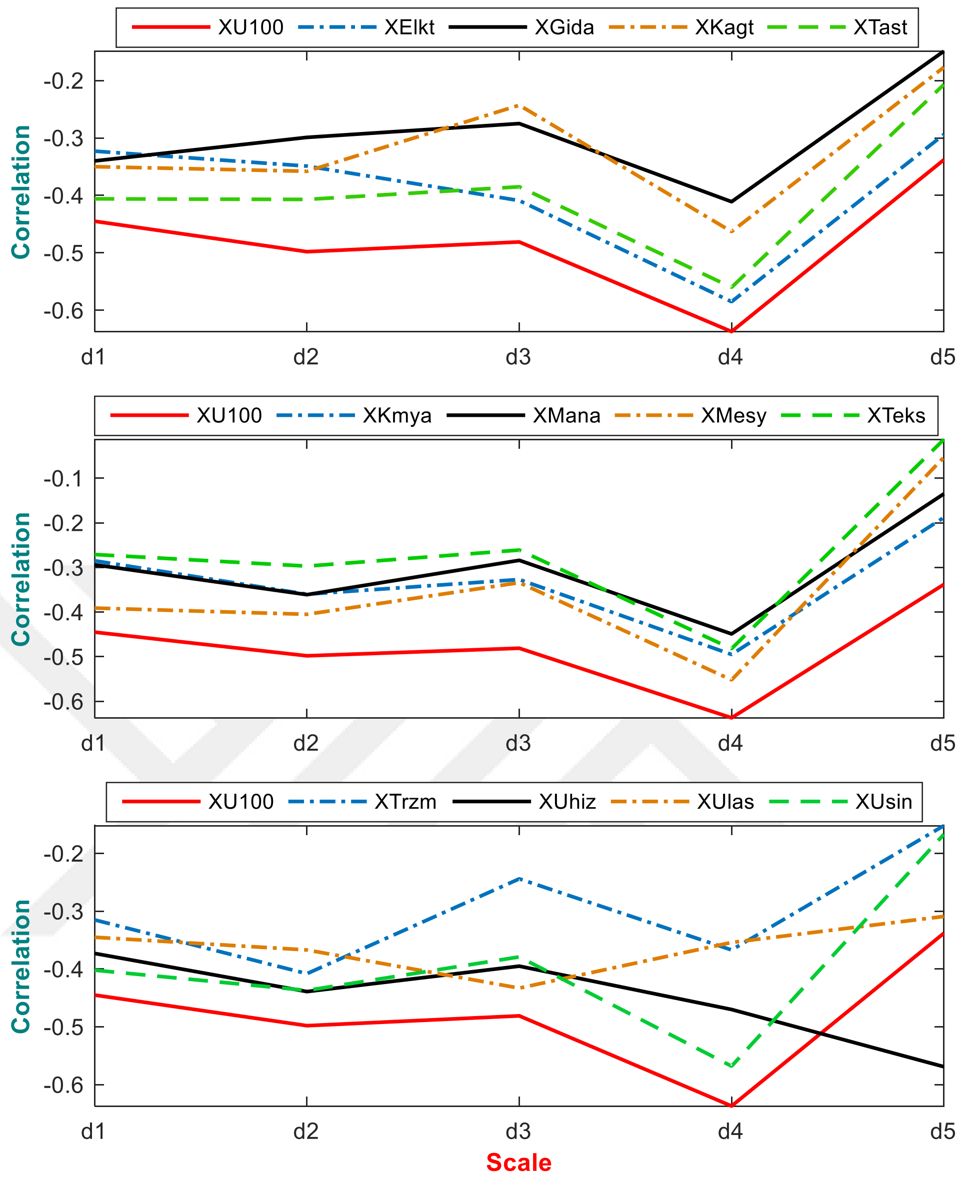


Figure 4-7 Correlation by Scale Decomposition (cont.)

4.4.2.2 Wavelet Correlation and Cross-Correlation

Using the MODWT based wavelet coefficients for scale decompositions, it can be calculated the wavelet correlation between two different time series. Thus, calculating correlations by scale enable to detect whether the relationship is time dependent or not. Figure 4-7 displays the correlation coefficient between "RTR2YGB" and stock indices at different scale decompositions.

Evidently, as expected, the wavelet correlations are negative and they show different pattern regarding to wavelet scales. The minimum and maximum estimated correlation coefficients are -0.0082 and -0.6512 over time-scales.

When looking at Figure **4-7**, it can be seen that there is dependence between "RTR2YGB" and stock indices for all wavelet scales. Putting the same point in simpler terms, "RTR2YGB" and stock indices either at the aggregate or sectoral levels, has a negative but strong correlation relationship across the scales. Notable, the stock indices of the financial group, such as "RXBANK" and "RXUMAL", exhibit a perfect correlation pattern with the aggregate stock index, "RXU100", for all wavelet scales where the underlying time series move together with "RTR2YGB". Until scale d4, the correlation coefficients increase negatively but decrease to zero in absolute value at scale d5. It should be noted that the significant test of wavelet correlations are done by "Brainwaver" R-Package created by Achard (2012).

On the other hand, the non-financial indices also have a similar relationship between "RTR2YGB" time series. Except for "RXBLSM", "RXGIDA", "RXHOLD", "RXILTM", "RXKAGT", "RXSGRT", "RXTRZM", and "RXUHIZ" stock indices, the wavelet correlations increase negatively from scale d1 to d2, slightly decreases at scale d3, and radically increase negatively at scale d4. Using "Brainwaver R-Package", it is observed that all the wavelet correlation coefficients are significantly different from zero for scales d1, d2, d3, and d4, namely these coefficients are strongly, but negatively, significant at 1% for all time scales shorter than 32 weeks. The wavelet correlation coefficient, as shown in Table 4-3, for all stock indices decreases to zero at scale d5 (32-64 weeks), except "RXILTM" and "RXUHIZ" which have a strong and significant relationship at 5%. Roughly speaking, the significant relationship is not different from zero at the investment period of 32-64 weeks that is the growth rate of "TR2YGB" and stock index returns move independently in the long run.

From the table below, highly significant interdependences between "TR2YGB" and the stock indices are evident at 1% of the raw data. With using wavelet decomposition, one can uncover the relationship hidden in the time scales, equivalently saying, discover whether the degree of the relationship is permanent or temporal or the relationship remains nearly the same across the investment period. It is obvious that the negative correlation relationship lasts until the scales d4, then it is not different from zero, i.e. the relationship is insignificant for all case of stock indices, except for "RXILTM" and "RXUHIZ". Broadly speaking, both "TR2YGB" and stock indices have a negative, significant and a strong impact on each other between the scales of d1 and d4, which is a reality that should be taken into account for the short run investment decisions.

A negative, as expected, but strong correlation relationship results partially corroborates with that of earlier empirical paper of Moya-Martínez et al. (2015). The authors (2015) advocate that the wavelet correlation between the sectoral index returns and 10-year government bond rate suggest that this relationship is a multiscale fact. In addition, a heterogeneity relationship is revealed among the industries, putting the same point in simpler terms, the sign of wavelet-based correlation is negative for the most of industries regardless of the time scales. For example, the firms that benefitted from a decrease in an interest rate for all wavelet scales are the indices of "Consumer Goods", "Technology & Telecom", "Real Estate", "Financial Services", "Utilities", "Construction", and "Food& Beverages". Conversely, "Banking" index return move in tandem with the change in 10-year government bond rate at only scale d1 while they move apart after this time scale. On the other hand, a strong evidence of the flight-to-quality phenomenon is observed for some stock indices, such as "Consumer Services" for scales of d5 and d6; "Chemicals & Paper" for scales of d3, d5, and d6; "Basic Resources" for scales of d3, d4, d5, and d6; "Health Care" for scales of d5, and d6; "Industrials" for only scale d3; and "Energy" for scales of d2, and d3. The authors (2015) state that the reason behind these positive relationships observed at longer time horizon is related to their pro-cyclical nature, namely, their performance depends on economic growth in the long-term

On the hand side, Kim and In (2007) report the same results for the wavelet correlation of the aggregate stock market index of Turkey. For example, the relationship between the changes in stock prices and bond rates in Canada, France, Germany, Italy, the UK, and the US is significantly negative for all scale while they ascertain a significant positive link for scales of d2 and d4 in Japan. Besides, an obvious evidence of the flight-to-quality fact is reported by Dajcman (2015) for the Eurozone countries in the sample period between 2000-01-01 and 2011-08-30 measured in daily returns. Apart from Portugal, all countries observed a cogent affirmative relationship for all wavelet scales, implying a safe haven role seen in the bond market during the last financial turmoil periods of the GFC. These all result, in line with our findings, suggests that tactical asset allocation strategy is valid for countries where the stock returns and bond yields move apart..

Table 4-3 Significance of Correlation Coefficients by Wavelet Scale

Variable	Return	d1	d2	d3	d4	d5
RXU100	-0.484***	-0.445***	-0.498***	-0.481***	-0.637***	-0.338
RXBANK	-0.484***	-0.437***	-0.493***	-0.479***	-0.646***	-0.384
RXBLSM	-0.291***	-0.301***	-0.261***	-0.236**	-0.317*	-0.112
RXELKT	-0.369***	-0.323***	-0.349***	-0.409***	-0.585***	-0.294
RXFINK	-0.325***	-0.258***	-0.342***	-0.373***	-0.489***	-0.008
RXGIDA	-0.324***	-0.34***	-0.299***	-0.275**	-0.411**	-0.149
RXGMYO	-0.427***	-0.35***	-0.464***	-0.474***	-0.552***	-0.185
RXHOLD	-0.450***	-0.43***	-0.411***	-0.485***	-0.648***	-0.116
RXILTM	-0.276***	-0.237***	-0.319***	-0.264**	-0.349**	-0.514**
RXKAGT	-0.362***	-0.35***	-0.358***	-0.243**	-0.463***	-0.177
RXKMYA	-0.321***	-0.285***	-0.36***	-0.327***	-0.495***	-0.188
RXMANA	-0.301***	-0.294***	-0.361***	-0.284**	-0.449***	-0.135
RXMESY	-0.395***	-0.391***	-0.405***	-0.334***	-0.552***	-0.053
RXSGRT	-0.298***	-0.261***	-0.257***	-0.289**	-0.486***	-0.121
RXSPOR	-0.206***	-0.186***	-0.198**	-0.205*	-0.349**	-0.148
RXTAST	-0.423***	-0.406***	-0.407***	-0.385***	-0.56***	-0.207
RXTCRT	-0.310***	-0.298***	-0.301***	-0.319***	-0.453***	-0.181
RXTEKS	-0.311***	-0.271***	-0.297***	-0.261**	-0.482***	-0.013
RXTRZM	-0.329***	-0.315***	-0.408***	-0.244**	-0.367**	-0.152
RXUHIZ	-0.401***	-0.373***	-0.439***	-0.395***	-0.47***	-0.569**
RXULAS	-0.380***	-0.345***	-0.367***	-0.433***	-0.354**	-0.309
RXUMAL	-0.485***	-0.44***	-0.484***	-0.492***	-0.651***	-0.308
RXUSIN	-0.413***	-0.402***	-0.437***	-0.379***	-0.568***	-0.167
RXUTEK	-0.352***	-0.329***	-0.374***	-0.341***	-0.412**	-0.224
RXYORT	-0.300***	-0.246***	-0.309***	-0.270**	-0.392**	-0.197

*,**, and *** indicate 10%, 5%, and 1% significance level, respectively.

Next, as suggested by Ashley et al. (1980), it is time to explore a possible lead/lag (causal) relationship between variables' "MODWT" based wavelet coefficients by cross-correlation method which is the analogue of the conventional cross-correlation method in time domain. As in case of correlation, the non-boundary wavelet coefficients are used to determine the relationship between scale decomposition.

Remark that a causality relationship is defined by Brooks (2014) as the correlation between the lagged price of X and the current value of Y. It should be noted that it is exactly the same in cross-correlation method. For example, let us give the cross-correlation function for "RTR2YGB" and "RXU100" variables as given

$$\mathrm{XCor}_{\mathrm{T} \Rightarrow \mathrm{XU100}} = \mathrm{crosscorr}(\mathrm{XU100}_t, \mathrm{Tahvil}_{t-k})$$

$$\mathrm{XCor}_{\mathrm{XU100} \Rightarrow \mathrm{T}} = \mathrm{corr}(\mathrm{Tahvil}_t, \mathrm{XU100}_{t-k})$$

where $\Longrightarrow$ denotes a causality relationship from "RTR2YGB" to "RXU100" variables, and vice versa, given that $\mathrm{RTahvil}_{t-k}$ and $\mathrm{RXU100}_{t-k}$ signify the lagged values of the underlying variables.

In Figure **4-8**, the MODWT based wavelet cross-correlation coefficients are depicted scale-by-scale basis, where the black-straight lines signify the estimated cross-correlation coefficients while the red-dotted and the green-dotted lines indicate the lower and the upper bounds for the 95% approximate confidence interval.

It should be emphasized that the left-side of the graph is "RTR2YGB" $\nRightarrow$ "RXU100" relationship and the right-side is of "RXU100" $\nRightarrow$ "RTR2YGB" relationship. Differently speaking, the correlation coefficient of "TR2YGB" returns rate at time t point is displayed against the stock index returns on $\pm$ 24 weekly lags, i.e. $t - k$ and $t + k$ lags where $1 \leq k \leq 24$. The first thing to note is that the lag zero, 0, is equal to standard correlation coefficient for all wavelet scales, d1, d2, d3, d4, and d5. It is evident that the finest wavelet decomposition levels do not show evidence of significant cross-correlations coefficients between the underlying variables, except for lags lower than 6 weeks for wavelet scales of d1, d2, and d3.

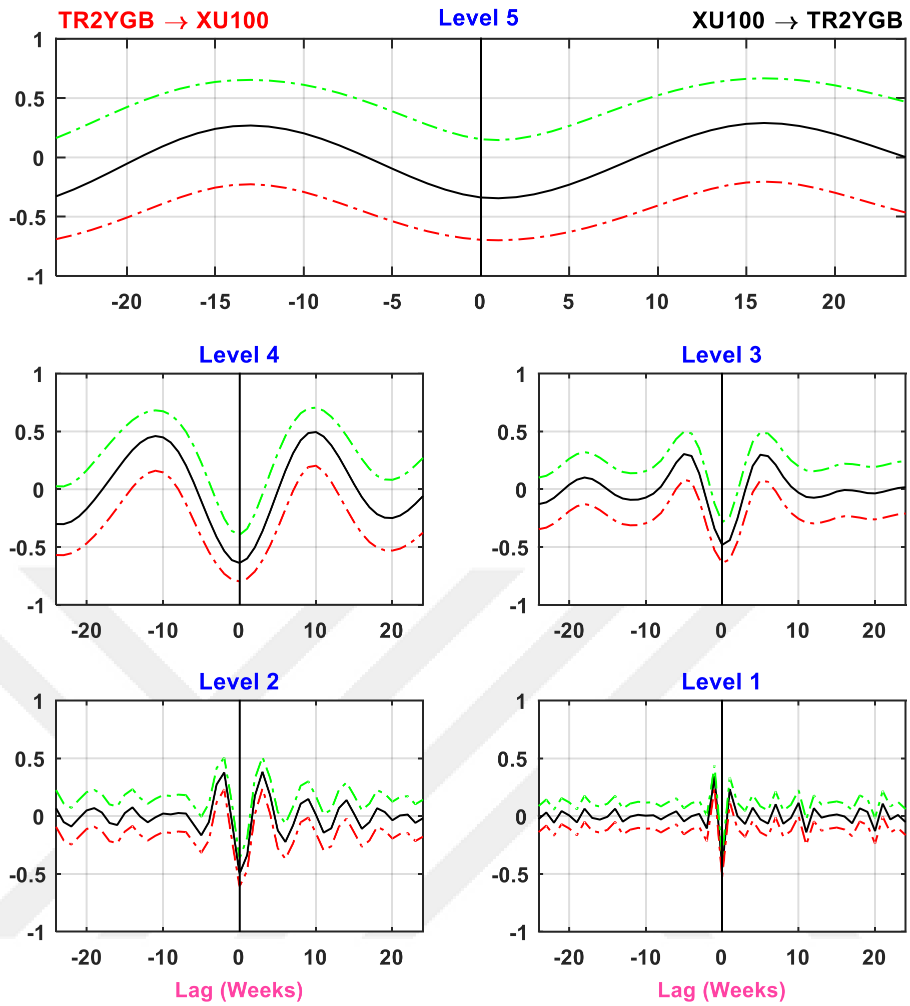


Figure 4-8 Wavelet cross-correlation between TR2YGB and XU100 Return Series

For the left-side of figures, the largest positive correlation values in the first three wavelet scales are at a lag of 2 weeks for d2, a lag of 1 weeks for d1, and a lag of 5 weeks for d3, implying a causality from "TR2YGB" returns to "XU100" stock index returns. Furthermore, the largest negative peak value is observed at a lag of 1 weeks for d3, meaning that "TR2YGB" return series leads "XU100" stock index return series. On the other hand, "RXU100" series leads "RTR2YGB" series at a lag of 3 weeks for d2 and lags of 5 and 6 weeks for d3, namely in the [4-8) and [8-16) week periods, respectively. Broadly speaking, as scale decomposition increases, the correlation coefficients increases for both cases, except for the scale d5 where the most correlation coefficients are not significantly different from zero. For scale from d1 to d4, the largest positive peak values are observed at lags of between 7 and 12 weeks for d4, while the largest negative peak values are observed at lags of 1, 2, and

3 weeks for d4 and d5. Briefly speaking, there is a clear asymmetry relationship between the variables under study for all wavelet scales, i.e. bidirectional causality at different lags for different investment periods.

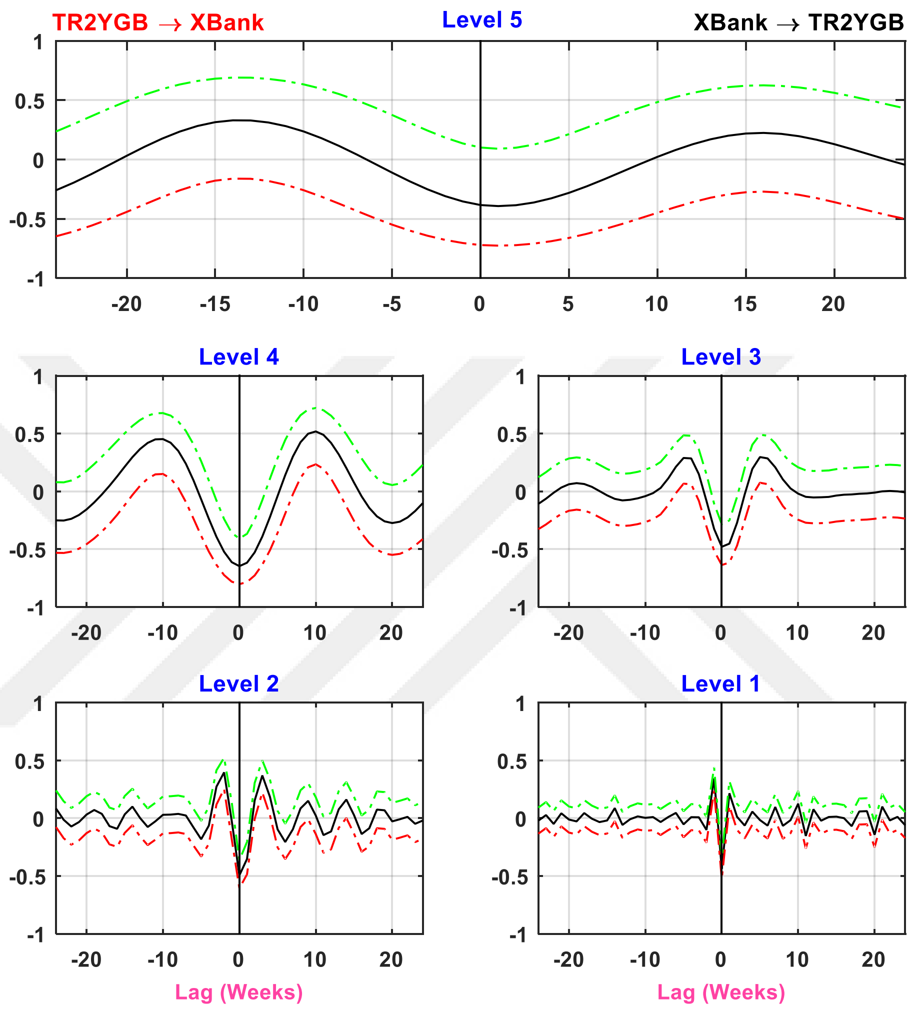


Figure 4-9 Wavelet cross-correlation between the TR2YGB and XBANK Return Series

Figure **4-9** depicts "MODWT" based wavelet cross-correlation coefficients between "RTR2YGB" and "RXBANK" series. The black-straight lines denote the estimated cross-correlation coefficients as the red-dotted and the green-dotted lines indicate the lower and the upper bounds for the 95% approximate confidence interval. The left-side of the graph is related to "RTR2YGB" ⇏ "RXBANK" relationship and the right-side is related to "RXBANK" ⇏ "RTR2YGB" relationship. It is worth mentioning, the cross-correlation relationship is nearly the same as with "RTR2YGB" and

"RXU100" relationship where the negative and positive cross-correlation coefficients are significantly different from zero for all scales both in the left-side and right-side of the graph. According to wavelet cross-correlation analysis, symmetry causality relationship arises between the underlying variables, confirming a clear evidence of the lead-lag relationship in the bond-stock market relationship. As wavelet scale increases, the causality relationship becomes stronger, especially at the lower lags of different scales.

The wavelet cross-correlation results for "TR2YGB" growth rate and stock index returns are in accordance with the papers of Hamrita and Trifi (2011), Abdullah et al. (2014), and Moya-Martínez et al. (2015). For instance, Hamrita and Trifi (2011) report an insignificant relationship between monthly interest rate changes and stock market returns for the higher frequencies of d1, d2, and d3, suggesting that these two variables were moving independently until 16 month periods. Besides, an affirmative leading relationship was observed at scale d5, implying a causality running from stock to interest rates. On the other hand, Abdullah et al. (2014) explore positive leading relationships from the short-term interest rates to stock market and running from the stock market to government bond rate at the lower frequencies, namely, in the long run.

Corroborating our findings, Moya-Martínez et al. (2015) document bidirectional lead/lag relationships between the alternative investment instruments of stock index returns and long-term interest rates in the long run for Spain case. In particular, an affirmative leading relationship from interest rate changes to stock market indices and a negative causal relationship from stock market returns to interest rate changes are found at the lower wavelet frequencies, suggesting that stock market is mainly driven by macroeconomic factors, such as interest rates, instead of short-term dynamics, such as changes in investor sentiments etc.

Apart from wavelet-based cross-correlation, Alaganar and Bhar (2003) document lead/lag causality-in-mean and causality-in-variance test results in the time-domain between the financial sector returns (the total and subsectors of banking and insurance) and interest rate changes of the G7 countries. The authors (2003) say that the causality results are more widespread at the mean level than at the variance level for all countries. In four out of seven countries, a bidirectional causal relationship is reported for the financial sectors and its subsectors of banking and insurance industries. At the mean level causation, all countries show evidence of a strong relationship for banking and insurance sectors, except for Japan because, according to them, monetary policy (interest rates) had been unsuccessful to stimulate the economy. In addition, they (2003) demonstrate that significant causality from industry returns to interest rate yields was surprising given that it was the violation of the efficient market hypothesis.

In wavelet literature, the causality relationship based on "MODWT" coefficients is not a widely accepted method because it can be misleading according to Tiwari et al. (2013). Instead, the author suggests implementing Granger causality test based on VAR model because this method is more successful for detecting a possible information flow or causality relationship between variables. Hence, for robustness check of causality relationship, several Granger causality methods are also applied.

4.5 Wavelet-based Econometric (Empirical) Tests Results

In this section, we analyze the causality relationship between "TR2YGB" rate and the aggregate and sectoral stock market indices of Borsa Istanbul based on "MODWT" wavelet coefficients. At first step, we apply the unit root tests without and with structural breaks. After determining the integration order of variables, the next step is to conduct the cointegration test without and with structural breaks for the variables of $I(1)$. If the data in VAR models have a long run relationship, then the next step is to use a VECM model to reveal both short run and long run relationships. If the data in VAR models do not move together in the long run, i.e. there is no a long run relationship, the standard causality test, then, has to be applied.

For the causality test, in our study, VAR Granger and symmetric causality (Hacker and Hatemi-J, 2006) tests are applied for both time-domain and frequency-domain (wavelet scales). Last, the asymmetric causality test of Hatemi-J (2008) and the frequency causality test of Breitung and Candelon (2006) are conducted for comparing the results of these tests with time-domain (VAR and symmetric) and frequency-domain (wavelet-based) causality tests.

4.5.1 Unit Root Test Results

Having mentioned before that avoiding for the spurious results, the data in question should be stationary. Hence, to determine the integration order of each variable, several unit root tests are implemented for this research. Unit root testing of data under study is the pretesting of cointegration and causality tests according to the time series analysis in literature.

It should be mentioned that the test with structural breaks is used to confirm the results of the test without structural breaks. In other words, the final decision is related to test with structural breaks. For example, if the ADF test statistic does not fail to reject the null of nonstationarity while the LS unit root test result implies that the data is stationary, then it is concluded the data is stationary. This method also will be used for the cointegration results for this research.In this study, we prefer the conventional unit root tests of the ADF, PP and KPSS tests that are widely used to test for nonstationarity of the variables. Notable, the null hypotheses of them differ. The null hypothesis of both the ADF and PP test is the same that is the variable under investigation is non-stationary while the KPSS test claims that the data is stationary when the null cannot be rejected. The main reason for applying these three tests with two different null hypotheses is to avoid the possible conflicting results.

For detecting the integration order of variable, the tests are conducted on the natural logarithms of the closing prices of the time series. If they are not found as stationary, then their first-differenced values are used to stationarity. The lag length for the ADF test is determined by the t-test, general-to-specific criterion (GTOS), method from eighteen lags for the weekly data. In this method, the lag length is calculated as "=trunc(12*(T/100)^(1/4))=18" according to Schwert (1989a) where "trunc" is the same of "rounddown" function in Excel and *T* is the observation number. On the other hand, the bandwidth for the PP and KPSS test is the Newey-West using the Bartlett kernel. For all unit root tests, a constant and a linear trend are included in the model.

Table 4-4 Standard Unit Root Tests Results

Variable	Level (Log)			Return (Log-Differenced)		
	ADF	**KPSS**	**PP**	**ADF**	**KPSS**	**PP**
LTR2YGB	-1.765	0.353***	-2.014	-13.99***	0.057	-21.551***
LXU100	-2.822	0.133*	-2.991	-15.932***	0.042	-25.225***
LXUMAL	-2.896	0.134*	-3.05	-15.941***	0.042	-24.694***
LXBANK	-2.815	0.195**	-2.964	-16.287***	0.04	-25.548***
LXFINK	-2.275	0.133*	-2.494	-14.744***	0.052	-23.086***
LXGMYO	-2.344	0.212**	-2.638	-14.803***	0.044	-23.272***
LXHOLD	-2.604	0.296***	-2.687	-16.194***	0.045	-22.956***
LXSGRT	-2.738	0.232***	-2.945	-14.349***	0.051	-22.229***
LXUSIN	-2.529	0.303***	-2.756	-16.056***	0.042	-26.047***
LXGIDA	-1.477	0.321***	-1.53	-18.617***	0.071	-29.935***
LXKAGT	-2.367	0.202**	-2.622	-15.95***	0.045	-22.861***
LXKMYA	-2.27	0.327***	-2.539	-17.378***	0.04	-26.889***
LXMANA	-2.299	0.308***	-2.683	-16.068***	0.032	-25.286***
LXMESY	-1.661	0.649***	-1.598	-14.911***	0.046	-23.753***
LXTAST	-2.448	0.119	-2.697	-14.883***	0.064	-22.831***
LXTEKS	-2.342	0.243***	-2.479	-15.347***	0.061	-24.106***
LXUHIZ	-2.543	0.136*	-2.499	-16.98***	0.042	-26.57***
LXELKT	-2.014	0.401***	-2.324	-17.34***	0.037	-25.986***
LXILTM	-2.988	0.252***	-3.151*	-18.313***	0.028	-28.636***
LXSPOR	-2.139	0.493***	-2.006	-17.232***	0.081	-22.147***
LXTCRT	-2.51	0.288***	-2.65	-17.075***	0.046	-27.996***
LXTRZM	-2.818	0.103	-3.191*	-15.208***	0.065	-24.294***
LXULAS	-1.707	0.245***	-1.744	-16.463***	0.087	-23.955***
LXUTEK	-0.218	0.554***	-0.502	-14.934***	0.058	-24.08***
LXBLSM	-1.625	0.3***	-1.907	-14.964***	0.057	-23.376***
LXYORT	-2.136	0.265***	-2.317	-16.966***	0.059	-23.991***

*,**, and *** indicate 10%, 5%, and 1% significance level, respectively.

Table 4-4 shows the unit root test results of the three different test methods for all time series both in log-level and log-differenced forms. Evidently, the three tests give the same result for the most of the variables in log-level. However, there are several exceptions for stationarity results; to be precise "LXTAST", "LXILTM", and "LXTRZM" indices' test results are conflicting. For example, according to the ADF (1979) and KPSS (1992) tests, "LXILTM" follows random walk while the PP test shows that it is stationary. Furthermore, "LXTAST" index has a unit root according to the ADF and PP (1988) test results while the KPSS test indicates that it is stationary, i.e. it is $I(0)$. Besides, the KPSS and PP show that "LXTRZM" index is $I(1)$ as the ADF test suggests that it has unit root. Except for these three variables, all variables have a unit root, namely, they are not stationary. On the contrary, however, after taking first-difference of the variables, all variables become stationary according to the three different unit root tests. In other words, they are integrated of order one, $I(1)$ at first differences.

It can be seen that with the exception of three variables, the remaining variables are found to be integrated of the order one according to unit root tests of without structural breaks. To avoid spurious and biased results in the presence of structural breaks, one should implement the test methods taking account the effect of possible breaks. Hence, as mentioned above, the test results of the conventional methods should be confirmed with the LS unit root test with two unknown structural breaks. The results of this unit root test are shown in Table **4-5**.

The null hypothesis, however, is that the variable has unit root with two structural breaks while the alternative hypothesis states that the variable is stationary with two structural breaks such as economic or financial crises, or technological shocks in the underlying time series. The lag length is determined by the GTOS method which is equal to 18. The unit root testing is conducted on the two models, “Model A (Crash)” and “Model C (Break)” where the first model captures a change in the level while the second model captures a change in the slope of the trend.

Two versions of the Lee and Strazicich unit root (2003) tests have been adopted in this study: firstly, the Crash or A model, which picks up a one-time abrupt change in the level of the series, and secondly, the Break or C model that captures a change in the slope of the trend. It should be emphasized that the time-points for structural breaks in this model are determined with endogenously.

According to Table **4-5**, six out of twenty-six variables are found to be non-stationary for both models, namely, "LXGMYO", "LXUSIN", "LXKMYA", "LXMESY", "LXELKT", and "LXILTM" indices are stationary at log-level. Notable, the unit root test on differenced-level via the LS is found an unnecessary step, hence, it is concluded all variables are stationary in first difference.

Table 4-5 The Lee & Strazizich (LS) Unit Root Test

Variable	Model A			Model C		
	LM test	**BP1**	**BP2**	**LM test**	**BP1**	**BP2**
LTR2YGB	-2.343 [5]	2009-12-31	2013-05-24	-4.239 [12]	2009-06-12	2013-09-13
LXU100	-3.190 [17]	2007-12-28	2009-11-25	-5.048 [17]	2008-08-08	2009-11-25
LXUMAL	-3.105 [17]	2008-05-16	2009-11-25	-4.935 [17]	2008-08-08	2009-11-25
LXBANK	-3.102 [17]	2008-11-21	2014-09-05	-4.785 [17]	2008-07-25	2009-11-25
LXFINK	-2.577 [16]	2007-08-10	2009-02-13	-4.367 [16]	2008-05-02	2010-04-16
LXGMYO	-2.753 [11]	2009-01-16	2010-05-07	-5.454 [17]*	2008-08-08	2009-12-31
LXHOLD	-3.089 [15]	2007-08-10	2009-02-06	-4.900 [15]	2008-07-25	2009-10-02
LXSGRT	-2.971 [14]	2008-05-02	2009-10-23	-5.139 [14]	2008-05-02	2009-09-04
LXUSIN	-3.439 [15]	2008-01-11	2009-03-06	-5.542 [15]*	2008-08-08	2009-11-25
LXGIDA	-2.491 [17]	2008-10-10	2015-07-15	-4.571 [17]	2008-10-10	2013-05-31
LXKAGT	-3.002 [17]	2007-02-23	2012-01-20	-4.647 [17]	2008-08-01	2012-12-07
LXKMYA	-3.596 [15]*	2008-01-11	2009-03-06	-5.310 [17]*	2008-08-08	2009-12-31
LXMANA	-3.476 [13]	2009-03-06	2011-11-18	-4.967 [17]	2007-06-01	2008-10-24
LXMESY	-2.854 [15]	2007-08-10	2009-02-06	-6.488 [17]***	2008-08-01	2009-10-02
LXTAST	-2.461 [15]	2009-02-13	2013-12-20	-4.022 [15]	2008-05-02	2010-01-22
LXTEKS	-2.586 [15]	2010-05-07	2011-08-19	-5.030 [15]	2008-08-01	2011-01-14
LXUHIZ	-2.876 [17]	2008-11-21	2015-03-06	-3.899 [15]	2008-07-25	2013-01-25
LXELKT	-3.758 [14]*	2008-11-21	2011-11-18	-4.797 [13]	2009-07-24	2011-11-18
LXILTM	-3.972 [1]**	2007-09-28	2015-04-03	-5.618 [0]*	2008-05-02	2014-12-05
LXSPOR	-1.675 [16]	2008-06-27	2012-05-11	-4.26 [15]	2010-08-20	2013-09-13
LXTCRT	-3.052 [14]	2008-11-21	2010-05-07	-4.433 [17]	2008-09-26	2010-04-30
LXTRZM	-2.605 [15]	2009-02-06	2011-11-18	-3.717 [15]	2007-07-06	2009-10-02
LXULAS	-2.256 [15]	2008-07-25	2012-01-27	-3.418 [15]	2009-07-03	2015-10-30
LXUTEK	-2.257 [18]	2007-02-23	2013-09-13	-4.459 [18]	2008-05-09	2009-12-31
LXBLSM	-2.436 [18]	2007-06-01	2009-02-20	-3.479 [18]	2008-05-02	2010-01-22
LXYORT	-2.697 [17]	2007-06-01	2010-02-26	-4.962 [17]	2008-05-16	2010-01-22

*,**, and *** indicate 10%, 5%, and 1% significance level, respectively. The lag number for unit root test is in brackets.

After finding that the variables are not stationary in the log-level, it is time to proceed to the long run relationship. Differently speaking, being integrated of order one, it allows to explore the possible long run relationship between "TR2YGB" and stock indices.

4.5.2 Cointegration Test Results

After determining the integration order of variables, the next step is to check whether the dependent and independent variables move together in the long run via cointegration tests without and with structural breaks. According to the table above, except the model of "LTR2YGB~LXILTM", all model are subject to cointegration test. For the cointegrating relationship, VAR models are described with an optimal lag length according to AIC. However, for an unbiased result, the model should be checked by diagnostic tests such as serial-correlation test, heteroskedasticity test, normality, and stability test. According to these tests, all models are suitable for the cointegration tests, except normality test.

The first cointegration test is Johansen Cointegration Test (1990) and its result is reported in Table **4-6**. It can be seen that there is a long run relationship between "LTR2YGB" and stock indices, with exceptions for "LXTRZM~LTR2YGB", "LXUTEK~LTR2YGB", and "LXBLSM~LTR2YGB" models, i.e. there is no any cointegration relationships between these stock indices and interest rates.

It should be noted that "LXGMYO~LTR2YGB", "LXUSIN~LTR2YGB", "LXKMYA~LTR2YGB", "LXMESY~LTR2YGB", "LXELKT~LTR2YGB", and "LXILTM~LTR2YGB" are excluded from the cointegration tests due to different integration order of variables. Hence, the test results are denoted as "NA" in Table **4-6** and Table **4-7**. Apart from this, it is evident that there is at least one cointegrating vector between the underlying variables according to λ_{trace} and λ_{max} test statistics at 10% and 5% significance level.

As observed in the unit root tests, this cointegration method assumes that a possible relationship, if exists, does not change during the period of study. In other words, it ignores the effect of structural breaks, which may lead to unreliable results. Hence, in the next, we implement a cointegration method with structural breaks to confirm our Johansen test results. Again, if these results differ, then the result of the test with structural breaks will be preferred to Johansen test results.

Table 4-6 Johansen Cointegration (1990) Test Results

Model			λ_{trace}		λ_{max}	
			$r = 0$	$r \geq 1$	$r = 0$	$r \geq 1$
LXU100	~	LTR2YGB	15.5882**	4.0831**	11.5052	4.0831**
LXUMAL	~	LTR2YGB	17.5688**	3.7687*	13.8002*	3.7684*
LXBANK	~	LTR2YGB	19.0818**	3.2361*	15.8458**	3.2361*
LXFINK	~	LTR2YGB	15.8957**	3.8802**	12.0156	3.8802**
LXGMYO	~	LTR2YGB	NA	NA	NA	NA
LXHOLD	~	LTR2YGB	15.0606*	5.9604**	9.1002	5.9604**
LXSGRT	~	LTR2YGB	15.3019*	3.3417*	11.9602	3.3417*
LXUSIN	~	LTR2YGB	NA	NA	NA	NA
LXGIDA	~	LTR2YGB	19.3402**	2.9297*	16.4105**	2.9297*
LXKAGT	~	LTR2YGB	14.199*	4.4431**	9.7681	4.431**
LXKMYA	~	LTR2YGB	NA	NA	NA	NA
LXMANA	~	LTR2YGB	7.541	3.4698*	4.0712	3.4698*
LXMESY	~	LTR2YGB	NA	NA	NA	NA
LXTAST	~	LTR2YGB	19.7611**	4.551**	15.2102**	4.551**
LXTEKS	~	LTR2YGB	16.3748**	3.3702*	13.0047*	3.3702*
LXUHIZ	~	LTR2YGB	14.3016*	3.1666*	11.1351	3.1666*
LXELKT	~	LTR2YGB	NA	NA	NA	NA
LXILTM	~	LTR2YGB	NA	NA	NA	NA
LXSPOR	~	LTR2YGB	8.5508	3.4895*	5.0614	3.4895*
LXTCRT	~	LTR2YGB	12.5338	3.0987*	9.4352	3.0987*
LXTRZM	~	LTR2YGB	10.5704	2.1392	8.4312	2.1392
LXULAS	~	LTR2YGB	14.2061*	2.5782	11.6279	2.5782
LXUTEK	~	LTR2YGB	9.9802	1.5301	8.4501	1.5301
LXBLSM	~	LTR2YGB	8.4494	2.461	5.9884	2.461
LXYORT	~	LTR2YGB	13.1574	4.4576**	8.6998	4.4576**

*,**, and *** indicate 10%, 5%, and 1% significance level, respectively.

The long run relationship test results via the Johansen (1990) method for the stock market and bond market in Turkey are in line with the papers of Alp et al. (2016) for Turkey; Ratanapakorn and Sarma (2007) for the US; Humpe and Macmillan (2009) for the US and Japan; Maysami and Koh (2000) for Singapore, Japan and the US; Gunasekaragea et al. (2004) for Sri Lanka; Panda (2008) for India; Das (2005) for India and Pakistan, and with those of Hasan and Javed (2009) and more recently with Ahmed et al. (2017) for Pakistan. For instance, Alp et al. (2016) report significant long run relationships between macroeconomic variables and stock market indices of

"XU100", "XU030", "XU050", "XUHIZ", "XGMYO", "XUMAL", "XUSIN", and "XUTEK" at 10% significance level, where the interest rate is the most prominent and negatively affecting factor. Ratanapakorn and Sarma (2007), on the other hand, highlight the evidence of cointegration as a potential arbitrage profit to earn.

Table 4-7 Hatemi-J Cointegration Test Results (2008)

Model			Modified ADF Test			Model			Modified ADF Test		
			Stat	**BP1**	**BP2**				**Stat**	**BP1**	**BP2**
LXU100	~	LGB2	-5.25	2008-08-29	2012-01-06	LGB2	~	LXU100	-4.87	2008-09-26	2011-11-04
LXUMAL	~	LGB2	-5.75*	2008-08-29	2010-08-06	LGB2	~	LXUMAL	-4.76	2008-08-08	2011-12-23
LXBANK	~	LGB2	-5.54	2008-08-29	2010-08-13	LGB2	~	LXBANK	-4.54	2008-09-26	2011-11-25
LXFINK	~	LGB2	-5.37	2008-05-23	2008-10-03	LGB2	~	LXFINK	-4.38	2008-06-20	2013-01-11
LXGMYO	~	LGB2	NA			LGB2	~	LXGMYO	NA		
LXHOLD	~	LGB2	-5.84*	2008-08-29	2011-12-02	LGB2	~	LXHOLD	-5.07	2008-08-15	2011-12-16
LXSGRT	~	LGB2	-5.37	2008-08-29	2011-01-07	LGB2	~	LXSGRT	-5.31	2008-09-19	2011-07-29
LXUSIN	~	LGB2	NA			LGB2	~	LXUSIN	NA		
LXGIDA	~	LGB2	-4.84	2008-11-28	2010-02-05	LGB2	~	LXGIDA	-4.94	2009-05-08	2011-11-11
LXKAGT	~	LGB2	-5.41	2008-10-17	2010-07-30	LGB2	~	LXKAGT	-5.39	2009-02-13	2011-11-04
LXKMYA	~	LGB2	NA			LGB2	~	LXKMYA	NA		
LXMANA	~	LGB2	-4.35	2007-04-20	2012-04-27	LGB2	~	LXMANA	-5.01	2009-05-08	2011-12-09
LXMESY	~	LGB2	NA			LGB2	~	LXMESY	NA		
LXTAST	~	LGB2	-5.36	2008-08-01	2008-08-08	LGB2	~	LXTAST	-4.39	2008-06-20	2013-01-25
LXTEKS	~	LGB2	-5.77*	2008-06-13	2008-10-31	LGB2	~	LXTEKS	-4.51	2009-05-08	2011-12-09
LXUHIZ	~	LGB2	-4.95	2007-02-09	2011-11-04	LGB2	~	LXUHIZ	-5.66*	2009-05-15	2011-11-04
LXELKT	~	LGB2	NA			LGB2	~	LXELKT	NA		
LXILTM	~	LGB2	NA			LGB2	~	LXILTM	NA		
LXSPOR	~	LGB2	-4.23	2008-01-04	2010-09-03	LGB2	~	LXSPOR	-4.81	2009-06-12	2011-11-25
LXTCRT	~	LGB2	-4.93	2008-12-05	2011-11-04	LGB2	~	LXTCRT	-5.38	2009-04-24	2011-11-25
LXTRZM	~	LGB2	-6.42**	2008-06-13	2010-02-05	LGB2	~	LXTRZM	-4.83	2009-06-12	2011-11-25
LXULAS	~	LGB2	-5.8*	2008-08-08	2011-10-27	LGB2	~	LXULAS	-5.57	2009-04-24	2011-11-25
LXUTEK	~	LGB2	-4.2	2008-10-24	2012-05-18	LGB2	~	LXUTEK	-4.92	2008-09-12	2011-11-25
LXBLSM	~	LGB2	-3.92	2008-08-01	2013-01-25	LGB2	~	LXBLSM	-4.59	2008-09-12	2009-06-05
LXYORT	~	LGB2	-5.35	2008-03-07	2010-02-26	LGB2	~	LXYORT	-5.22	2008-10-03	2011-12-09

*,**, and *** indicate 10%, 5%, and 1% significance level, respectively.

Conversely, Yildiz (2014) and Chan et al. (1997) document insignificant long-haul affiliations between stock and bond markets for Turkey and the US, respectively. Chan et al. (1997) assert that the tactical allocation strategy in managing both investment assets for portfolio diversification is valid when these two markets do not move in tandem in the long run. Putting the same point in different terms by Schwarz and Szakmary (1994), in the case of a no-cointegration relationship, the arbitrage activity is not any longer valid.

Now, we will implement Hatemi-J cointegration test (2008) to uncover whether there is a long run relationship between the underlying variables under the conditions of structural breaks. For analyzing the optimal lag length is determined by Hatemi-J Information Criterion (2003, hereafter HJC) method, which is robust to ARCH effects. To remedy the small size distortions, the critical values are obtained by Monte Carlo simulations and the author suggests using three different test statistics; ADF^*, Z_a^*, and Z_t^*. This method is actually an extended version of Gregory and Hansen (1996) cointegration method with one unknown structural break. Hatemi-J (2008) modified this method by adding one extra structural break into cointegration model, namely with two unknown/possible structural breaks determined endogenously.

As reported in Table **4-7**, there are only several cointegrating vectors between the underlying data. It is not surprising that the total cointegration vectors decreased when implementing cointegration method with structural breaks. The empirical results reveal that choosing Model C/S, the null hypothesis of no cointegration between variables is strongly rejected for "LXTRZM~LTR2YGB" models at 5% significance level and for "LXUMAL~LTR2YGB", "LXHOLD~LTR2YGB", "LXTEKS~LTR2YGB", "LXULAS~LTR2YGB", and "LTR2YGB~LXUHIZ" at 10% significance level using ADF^* test critical values, corroborating some results of Johansen test (1990). Differently speaking, only six model test results show strong evidence of the existence of the long run relationship with two structural breaks.

In line with our findings from the cointegration test, Evrim-Mandaci et al. (2011) and Akbas (2013) state that the aggregate stock market and bond market do not move together in the long run in Turkey. Evrim-Mandaci et al. (2011), however, document significant long run relationships between the ISE Government Debt Securities (GDS) Price Index and stock market indices of "XUHIZ" and "XUTEK" over the sample period of May 2001 to August 2009. Besides, despite a cogent relationship was expected for "XBANK" and "XU100" with the GDS, the authors (2011) did not explore any significant link. Having invested in their funds to bond market instruments, it is not unusual that the bond market might affect their performance in the long run. The authors state that, during the sample period, banks might adjust their asset composition or individual investors prefer to change their investment sentiment or implement the do-nothing strategy. On the other hand, Akbas (2013) report a nonlinear long run relationship between "XU100" and interest rate due to heterogeneous market hypothesis.

From these results, "LTR2YGB" and stock market indices move together in the long run. Putting differently, there should be at least one-way causality relationship from one variable to another variable. Evidently, we did fail to give which variable is endogenous or exogenous, namely, it is not clear which variable leads to another. Hence, as mentioned above, in the following section, we will use VECM model for variables having cointegrating vectors, while we will use VAR model for non-cointegration results.

Besides the VECM and VAR model for standard Granger causality test, the modified version of TY causality tests and frequency causality test will be implemented for causality relationship.

4.5.3 Causality Test Results

To remark the definition of causality, Granger (1969) states that if X granger-causes Y, it means that the predictability of Y is improved with the inclusion of X’s past information into the model in addition to Y’s past information. The first causality test in this study is based on VECM model and test results are reported in Table **4-8**.

In VECM causality tests, the ECT_{t-1} term denotes a long run causality between the underlying variables while χ^2 represents a short run causality. It is quite apparent that there are several causalities from "LTR2YGB" to stock market indices but there is not a leading relationship from stock market index to "LTR2YGB" variable. Differently speaking, results show that there are several one-way causality relationships running from "LTR2YGB" to stock market indices in the long run but there is only one-causality running from "LTR2YGB" to index in the short run. The predictability of "LXHOLD" stock index performance, for example, can be improved via using the TR2YGB variable's past information at 10% significance level in the short run of empirical study's period.

Table 4-8 Standard Granger Causality – VECM

MODEL	LX ⇏ LTR2YGB		MODEL	LTR2YGB ⇏ LX	
	χ^2	ECT_{t-1}		χ^2	ECT_{t-1}
LXU100 ~ LTR2YGB			LTR2YGB ~ LXU100		
LXUMAL ~ LTR2YGB	0.96	-0.036***	LTR2YGB ~ LXUMAL		
LXBANK ~ LTR2YGB			LTR2YGB ~ LXBANK		
LXFINK ~ LTR2YGB			LTR2YGB ~ LXFINK		
LXGMYO ~ LTR2YGB			LTR2YGB ~ LXGMYO		
LXHOLD ~ LTR2YGB	6.806*	-0.019***	LTR2YGB ~ LXHOLD		
LXSGRT ~ LTR2YGB			LTR2YGB ~ LXSGRT		
LXUSIN ~ LTR2YGB			LTR2YGB ~ LXUSIN		
LXGIDA ~ LTR2YGB			LTR2YGB ~ LXGIDA		
LXKAGT ~ LTR2YGB			LTR2YGB ~ LXKAGT		
LXKMYA ~ LTR2YGB			LTR2YGB ~ LXKMYA		
LXMANA ~ LTR2YGB			LTR2YGB ~ LXMANA		
LXMESY ~ LTR2YGB			LTR2YGB ~ LXMESY		
LXTAST ~ LTR2YGB			LTR2YGB ~ LXTAST		
LXTEKS ~ LTR2YGB	0.619	-0.017***	LTR2YGB ~ LXTEKS		
LXUHIZ ~ LTR2YGB			LTR2YGB ~ LXUHIZ	0.505	-0.001
LXELKT ~ LTR2YGB			LTR2YGB ~ LXELKT		
LXILTM ~ LTR2YGB			LTR2YGB ~ LXILTM		
LXSPOR ~ LTR2YGB			LTR2YGB ~ LXSPOR		
LXTCRT ~ LTR2YGB			LTR2YGB ~ LXTCRT		
LXTRZM ~ LTR2YGB	0.553	-0.014**	LTR2YGB ~ LXTRZM		
LXULAS ~ LTR2YGB	3.083	-0.012***	LTR2YGB ~ LXULAS		
LXUTEK ~ LTR2YGB			LTR2YGB ~ LXUTEK		
LXBLSM ~ LTR2YGB			LTR2YGB ~ LXBLSM		
LXYORT ~ LTR2YGB			LTR2YGB ~ LXYORT		

*,**, and *** indicate 10%, 5%, and 1% significance level, respectively.

For being significant, on the other hand, ECT_{t-1} term value should be below zero. The significant speed of adjustment coefficients, ECT_{t-1}, range between (-0.012) and (-0.036). The required time period lengths for a significant adjustment toward long run equilibrium are 27.8 weeks for "LXUMAL"; 52.6 weeks for "LXHOLD"; 58.8 weeks for "LXTEKS"; 71.4 weeks for "LXTRZM" and 83.3 weeks for

"LXULAS" sector indices. Evidently, the shortest period to correct the magnitude of disequilibrium in the long run is observed for financial sector of "LXUMAL".

Thus, it can be said that the predictability of "LXUMAL", "LXHOLD", "LXTEKS", and "LXULAS" stock index performances can be improved via using "LTR2YGB" variable's past information at 1% significance level while it is 5% significance level for "LXTRZM" stock index in the long run. However, despite a negative value for ECT_{t-1} term, there is no causality from "LXUHIZ" to "LTR2YGB" in the long run because the null hypothesis of non-causality cannot be rejected.

The evidence of significant corrections in disequilibrium in stock market indices (or long run) and short run causal relationships are consistent with those papers' results of Rahman and Mustafa (1997) for the US, Wongbangpo and Sharma (2002) for Indonesia, Malaysia, Philippines, Singapore and Thailand; Ratanapakorn and Sarma (2007) for the US, Pyeman and Ahmad (2009) for Malaysia, Sohail and Zakir (2010) for Pakistan, Kumar and Puja (2012) for India; Amata et al. (2016) for Kenya, and more recently with Li et al. (2017) for Malaysia. For example, Sohail and Zakir (2010) report a positive significant relationship between 3-month T-Bill rate and stock prices in Pakistan, while Amata et al. (2016) and Li et al. (2017) document a negative significant association between interest rate and stock market prices in Kenya and Malaysia in the long run, respectively. For strengthening the stability of the economy and the stock market, a forward-looking monetary policy is strongly recommended by Li et al. (2017). Conversely, Kumar and Puja (2012) explore a one-way causal relationship between the short-term treasury bills rate and BSE Sensex stock price, namely, interest rate Granger-cause stock prices only in the long run in India. Besides, Rahman and Mustafa (1997) and Ratanapakorn and Sarma (2007) find evidence of significant causal relationships from short-term and long-term interest rate to S&P500 in the long run. Pyeman and Ahmad (2009), on the other hand, document a significant causal relationship running from interest rates to sectoral indices of "KLSECON" [Construction], "KLSECOP" [Consumer Product], "KLSEPRP" [Property], "KLSEFIN" [Finance], "KLSEINP" [Industrial Production], and "KLSETAS" [Trading and Services] where the required time period lengths to achieve long run equilibrium are 15.4, 6.5, 4.6, 7.5, 7.1, and 8.5 months for sector indices, respectively. Besides, Lean and Smyth (2012) report a one-way causality from interest rate changes to "REIT" stock market index in the long run for Malaysia. On the other hand, in a related paper, Wongbangpo and Sharma (2002) document feedback relationships between two variables for Indonesia, Malaysia, and Thailand while interest rates lead stock markets in the short-term in Philippines and Singapore.

More importantly, Özün and Çifter (2006) and Alp et al. (2016) document significant causal relationships between interest rate and sector indices in the short- and long run in Turkey. For instance, the authors found (2016) significant causal relationships in the long run from interest yields to stock market indices of "BIST100" [XU100], and "BISTFIN" [XUMAL]. In addition, interest rate changes Granger-cause stock market indices of "BISTREIT" [XGMYO], "BISTIND" [XUSIN], and "BISTTECH" [XUTEK] in the short- and long run. On the other hand, Özün and Çifter (2006) state that the null hypothesis of non-causality can be rejected at 10% significance level, suggesting that interest rate leads "XBANK" in the short and long run.

From this point, we will compare the results of time domain and frequency domain using wavelet scales. The causality relationship based on VAR model is summarized in Table **4-9**. VAR model causality tests are computed by the "MSBVAR" package developed by Brandt (2009). Before conducting causality test on return series, all models are checked via diagnostic tests. According to the results, all models are appropriate for causality tests. The optimal lag length is chosen by AIC for each "RX~RTR2YGB" and "RTR2YGB~RX" models as well as wavelet scale based models. As Table **4-9** exhibits, there is only one causality relationship from “RTR2YGB" to "RXELKT" stock index at 5% significance level while "RXHOLD" and "RXUMAL" stock indices Granger-cause "TR2YGB" at 10% significance level.

Table 4-9 Standard Granger Causality Test by Scale – VAR Model

Variable	RTR2YGB does not Granger-Cause RX						RX does not Granger Cause RTR2YGB					
	Return	d1 [2-4)	d2 [4-8)	d3 [8-16)	d4 [16-32)	d5 [32-64)	Return	d1 [2-4)	d2 [4-8)	d3 [8-16)	d4 [16-32)	d5 [32-64)
RXU100	0.189	0.78	1.086	2.258***	1.536	3.6***	1.286	1.834**	1.359	2.127**	2.634***	3.09***
RXUMAL							2.391*	2.084**	1.548	2.207**	3.133***	3.233***
RXBANK	0.322	1.16	1.42	2.136**	2.183**	4.316***	2.016	2.169**	1.719*	2.216**	3.355***	3.496***
RXFINK	1.741	1.398	1.586*	1.285	1.572*	1.881**	0.713	1.686*	1.973**	1.665*	1.383	1.473
RXGMYO	1.437	0.673	1.099	2.11**	1.46	2.208**	1.893	1.35	1.767*	1.347	2.108**	1.174
RXHOLD							2.354*	1.53	1.023	1.973**	2.282***	2.518***
RXSGRT	1.318	0.592	0.876	1.842**	1.241	3.08***	1.659	1.833**	2.11**	2.402***	1.381	2.254***
RXUSIN	0.242	0.775	1.079	2.781***	1.192	4.462***	0.067	1.37	1.192	2.2**	1.153	2.733***
RXGIDA	1.53	1.202	1.294	2.329***	1.467	5.617***	0.711	1.526	1.382	1.535	1.124	4.829***
RXKAGT	1.081	0.918	1.156	2.776***	1.555	2.451***	1.295	1.36	1.589*	2.41***	1.93**	2.085**
RXKMYA	1.061	0.717	1.005	2.723***	2.354***	5.63***	0.359	1.398	1.339	2.336***	1.628*	3.76***
RXMANA	0.218	0.741	1.476	1.906**	1.956**	3.336***	0.236	0.712	0.83	1.078	1.051	1.883**
RXMESY	0.298	0.94	1.295	2.383***	1.505	3.519***	0.443	1.643*	1.197	2.207**	1.417	1.485
RXTAST	0.654	0.571	0.876	1.988**	1.934**	2.312***	0.478	1.173	1.382	2.56***	2.195**	3.554***
RXTEKS							1.553	0.957	1.072	1.922**	2.223***	1.142
RXUHIZ	0.686	0.364	0.604	1.617*	1.589*	2.649***						
RXELKT	3.007**	1.074	1.357	3.127***	3.143***	2.847***	0.404	1.342	0.968	1.847**	1.368	2.033**
RXILTM	1.443	0.291	0.587	1.295	1.863**	3.723***	0.164	1.21	2.048**	1.908**	2.882***	2.794***
RXSPOR	0.331	1.184	0.936	2.601***	2.436***	1.726*	0.845	1.817**	1.559*	2.257***	2.658***	2.382***
RXTCRT	0.889	0.644	0.928	2.136**	1.836**	2.984***	0.17	0.914	0.719	1.373	1.257	2.614***
RXTRZM							1.569	0.679	0.74	2.444***	2.304***	1.846**
RXULAS							0.884	1.706*	1.688*	1.843**	3.278***	4.677***
RXUTEK	0.055	1.006	1.499	1.398	1.478	1.282	0.834	0.75	0.989	1.015	1.88**	1.208
RXBLSM	0.784	1.491	2.34***	1.133	1.63*	1.555	0.304	0.344	0.814	0.561	1.163	1.883**
RXYORT	1.333	0.686	0.887	2.505***	2.316***	1.783**	0.765	0.974	1.361	2.72***	2.375***	2.506***

*,**, and *** indicate 10%, 5%, and 1% significance level, respectively.

However, thanks to wavelet's ability to decompose data into several scale components, one can uncover the possible relationships between variables which are not possible in case of using time-domain analysis tools. For more understanding, let us look at the results obtained via wavelet scales of time series. For example, it was observed that there were only a few causality relationships from "RTR2YGB" to stock indices and from the stock indices to "RTR2YGB". When looking at frequency-based causalities, it is obvious that the causality relationship is not found at scale d1, [2-4) week periods, and scale d2, [4-8) week periods, from "RTR2YGB" to "RXELKT" and from "RXHOLD" to "RTR2YGB".

On the other hand, from "RXUMAL" to "RTR2YGB" causality relationship started from scale d1 and continued after scale d3 to d5. Owing to wavelets, an investor can predict when interest rate changes affect the stock prices or vice versa which is impossible with traditional methods. In addition, wavelet scales based on "MODWT-MRA" coefficients also come to help for other models. It can be seen from the table; there are bi-directional causality relationships between wavelet scales of "RTR2YGB" and stock indices, except a case from "RTR2YGB" to "RXUTEK" where the null hypothesis of "RTR2YGB" does not Granger-cause "RXUTEK" is not rejected for all wavelet scales. On the other hand, for both financial indices and non-financial indices, there are very strong causality results from "RTR2YGB" variable, namely, via monitoring the performance of "RTR2YGB", both short-time and long-time investors can predict the stock price changes. Moreover, from stock indices to "RTR2YGB" variable, causality relationship occurs at earlier wavelet scales. For instance, "RXBANK", "RXSPOR", and "RXULAS" stock index returns lead to "RTR2YGB" return series from scale d1 to scale d5, perpetually at strong significance level while this relationship is valid at scale d1 but disappears at scale d2, and again becomes evident between scale d3 to scale d5 for the aggregate stock index, "RXU100", and financial stock indices of "RXFINK" and "RXUMAL". Briefly speaking, the results in Table **4-9** show evidence of a more pronounced link running from the stock indices to "RTR2YGB" than from "RTR2YGB" to stock indices. Besides, bi-directional causality in the medium-run and long run is a clear investment opportunity for investors, which is not evident with conventional analyzing tools.

Table 4-10 Symmetric Causality Test by Scale

Variable	RTR2YGB does not Granger-Cause RX						RX does not Granger Cause RTR2YGB					
	Return	d1 [2-4)	d2 [4-8)	d3 [8-16)	d4 [16-32)	d5 [32-64)	Return	d1 [2-4)	d2 [4-8)	d3 [8-16)	d4 [16-32)	d5 [32-64)
RXU100	6.93**	0.724	0.307	0.628	0.445	0.19	0.198	0.028	0.05	3.24*	0.002	6.725**
RXUMAL	7.309**	0.783	0.038	0.726	0.763	0.434	0.211	0.066	0.207	2.512	0.128	7.158***
RXBANK	8.198***	0.977	0.157	0.248	0.404	0.301	0.167	0.1	0.34	1.358	0.033	6.762***
RXFINK	17.485***	0.066	0.002	0.6	0.802	0.03	0.569	0.737	0.0	2.277	1.083	1.419
RXGMYO	4.178**	1.054	1.152	2.434	2.022	1.207	0.881	0.495	0.34	4.382**	0.439	0.157
RXHOLD	3.529*	0.08	0.14	3.546*	3.736	1.184	0.108	0.024	0.05	5.804**	1.263	6.706**
RXSGRT	8.501***	1.611	0.603	0.011	2.07	2.317	0.014	0.302	0.047	5.663**	0.74	4.393**
RXUSIN	7.193**	0.344	0.722	0.771	0.436	0.121	0.146	0.037	3.666*	3.234*	0.022	3.965**
RXGIDA	4.318**	1.043	0.704	5.762**	0.541	0.191	2.077	1.413	10.593***	1.467	0.126	2.65
RXKAGT	1.784	1.411	0.095	0.065	1.066	0.012	0.642	0.29	3.889**	5.421**	0.148	1.313
RXKMYA	7.079***	0.469	1.023	0.006	0.276	0.078	0.11	0.346	0.145	3.822*	0.052	7.41***
RXMANA	10.069***	0.002	0.001	0.024	0.701	0.074	0.066	0.011	1.878	0.264	0.001	1.859
RXMESY	4.212**	0.023	0.104	3.907**	0.114	0.519	0.152	0.157	1.847	9.041***	0.001	2.284
RXTAST	7.884***	0.02	0.017	0.532	0.03	0.541	0.0	0.0	0.572	1.684	0.0	0.01
RXTEKS	12.005***	1.226	0.004	0.023	0.478	0.119	0.197	0.269	1.167	2.333	0.101	1.912
RXUHIZ	4.756**	0.047	0.028	0.427	1.533	1.318	0.068	0.258	0.638	4.028**	0.473	2.335
RXELKT	10.279***	3.017	0.36	1.245	0.484	0.33	0.201	2.219	0.176	1.504	0.472	0.016
RXILTM	2.126	0.0	0.412	0.007	1.209	2.227	0.001	0.523	1.501	1.162	0	1.441
RXSPOR	1.545	0.184	0.008	0.886	0.563	0.058	0.426	0.915	0.637	2.864*	0.355	0.004
RXTCRT	8.156***	0.267	0.28	0.049	0.458	0.251	0.401	1.296	0.018	3.284*	1.957	3.203*
RXTRZM	5.034**	0.637	0.461	0.012	0.671	1.505	0.247	0.541	2.586*	2.844*	0.865	0.011
RXULAS	1.436	0.857	0.576	2.529	0.243	0.077	0.478	0.682	0.673	5.481**	0.207	1.264
RXUTEK	2.234	1.122	0.213	0.117	0.012	0.241	0.394	0.755	0.875	0.968	1.088	1.43
RXBLSM	4.672**	0.499	0.299	3.081*	0.112	0.095	0.008	0.363	0.585	1.148	0.217	0.514
RXYORT	9.658***	0.278	1.31	0.283	0.419	0.542	0.204	1.067	0.104	2.442	0.003	0.179

*,**, and *** indicate 10%, 5%, and 1% significance level, respectively.

Until now, the determinant factor for causality relationship was based on the cointegration results. But, from this point, the return series of underlying variables will be used because the pre-testing for cointegration relationship is not a necessary step with the symmetric and asymmetric causality tests. For our study, in the symmetric causality, 5.000 bootstrap simulations are carried out to calculate the critical values, the HJC (2003) information criterion is used to determine the optimal lag length for VAR models and finally “intorder” is selected as zero due to stationary level, $I(0)$. The symmetric causality test results are reported in Table **4-10**.

When looking at symmetric causality results, it is evident that there are one-way causality relationships running from "RTR2YGB" to stock market indices for time-domain analysis. These results differ from the VAR Granger causality test results where it was observed a few causality relationships in both ways. For time-domain, it can be said that this symmetric causality test is more powerful than standard VAR tests for this study.

Table **4-10** illustrates that there does not exist any causal relationship from "RTR2YGB" to some nonfinancial stock indices of "RXILTM", "RXKAGT", "RXSPOR", "RXULAS", and "RXUTEK" for both in time-domain and frequency-domain, implying that the interest rate changes are not more pronounced for nonfinancial sectors.

On the other side, for wavelet-based analysis, the null hypothesis of non-causality is rejected for only four out of twenty-five sectors of "RXBLSM", "RXGIDA", "RXHOLD", and "RXMESY" at only scale d3, confirming the VAR causality results. Therefore, it can be stated that this method is lack of finding any wavelet-based causal relationship running from "RTR2YGB" to stock indices for this study. Contrary to any causality found in time-domain, there are some significant causal relationships from stock indices to "RTR2YGB" in frequency-domain. For example, stock market indices Granger-cause "RTR2YGB" starting at scale d2 for sixteen out of twenty-five sectors. Putting differently, the predictability of "RTR2YGB" performance in the future can be improved via using these stock indices’ past information in the frequency-domain at different significance levels, which is not possible in the time-domain analysis.

The significant causal relationships test results from interest rate changes to the aggregate stock market index returns for both VAR and symmetric test in the time domain agree with the results from Özer et al. (2011) and Yildiz (2014) for Turkey, Hasan and Javed (2009) and Ahmed et al. (2017) for Pakistan in time domain. On the other hand, Herve et al. (2011) and Gunasekaragea et al. (2004) document a feedback relationship between the underlying variables for Cote d'Ivoire and Sri Lanka in the time-domain. Conversely, the papers that consist of both time and wavelet-based causal relationship results are Özün and Çifter (2006) paper of between "XBANK" and short-term bond rates and Çifter and Özün (2007a) paper of examining the relationship between "XU100" and compounded interest rates in Turkey. These two papers conclude that the causal relationship is a time-dependent phenomenon in wavelet-domain. For example, interest rate leads "XBANK" at scales of d1, d2, d3, and d4 in Özün and Çifter (2006)'s paper while Çifter and Özün (2007a) report an insignificant result in time-domain but a cogent link concentrated between scales of d3 and d6, namely, the bond market have strong effects on the aggregate stock market in the medium and long-term. In addition, Hamrita and Trifi (2011) and Tiwari (2012) present bi-directional relationship for the US and India cases, respectively. For instance, a two-way strong causal relationship is found by Hamrita and Trifi (2011) at scales of d4 and d5 using monthly observations.

In the same vein, Moya-Martínez et al. (2015) investigate causal relationship in the wavelet-based for Spain. According to test results, the authors (2015) reach cogent bidirectional causal relationships between the most of stock market index returns and the long-term interest rate changes, except "Healthcare" industry, concentrated in the higher wavelet scales. For instance, "Banking" index return Granger-causes and caused by interest rate changes at scales of d4 and d6 and scale d6, respectively. On the other side, changes in long-term interest rates lead the growth rate of "Food& Beverages" between scale d3–d6 while the opposite is valid up to the scale d4. As noted by them (2015), these causal results corroborate the notion that the long-term market participants in financial markets follow macroeconomic fundamentals because the relationship between the underlying variables is investment horizon dependent.

After analyzing the symmetrical relationship between the underlying variables, it is time to examine non-symmetrical relationship via asymmetric causality by Hatemi-J (2012) for this study. The reason behind using this method is that it is widely accepted that investors in financial markets assign less weight to positive news; hence, a suitable econometric method is required to investigate the causal relationship between negative and positive components of the underlying time series.

It should be outlined that for asymmetric causality, the natural logarithms of the variables are used. The optimal lag length is endogenously determined via HJC (2003) while in order to account for possible nonstationarity in true VAR model, "intorder" value is selected as one. The research findings are provided in Table **4-11**. The first column of the table shows the nonsymmetrical causal relationship between positive changes of the underlying data. It is evident that the positive changes of "LTR2YGB" variable have a significant impact on positive changes of stock indexes of "LXBLSM", "LXHOLD", "LXMESY", "LXTRZM", "LXULAS", and "LXUMAL" at 5% significance level for first four indices and at 10% level of significance for last two indices. On the other hand, there also exits causal relationship between negative components, namely negative changes of "LTR2YGB" have a significant effect on negative changes of stock indexes of "LXTCRT", "LXTRZM", and "LXYORT" at different significant level. Notable, the only stock index that affected by same shocks is "LXTRZM", i.e. the predictability of "LXTRZM" negative or positive performance in the future can be improved via using negative or positive performance of "LTR2YGB" in the short run.

On the other side, the results obtained between the different components are in line with the results in the literature due to the inverse relationship between the bond market and stock market. For instance, positive component of "LTR2YGB" Granger-causes negative component of both the aggregate market index, "LXU100", and sector indices of "LXBANK", "LXELKT", "LXHOLD", "LXMANA", "LXMESY", "LXULAS", and "LXUMAL" at different significance levels. Putting differently, a possible increase in "LTR2YGB" variable gives a decrease in these stock indices, suggesting a flight of funds from equity markets to bond markets. In addition, a possible decrease in "LTR2YGB" variable gives an increase in just the two stock indices of "LXFINK" and "LXKAGT", implying a flight of funds from bond markets to equity markets. It can be concluded that the impact of negative changes in "LTR2YGB" variable on both negative and positive changes in stock market indices is less pronounced for this study.

The second part of Table **4-11** provides asymmetric causality from stock indices to "LTR2YGB" variable in short run. The first column exhibits a causal relationship between the positive components of two underlying data. It is evident that there exists only a significant relationship from the positive component of "LXSGRT" to positive component of "LTR2YGB" variable. Besides, the null hypothesis of no causality can be rejected at 1% and 5% significance level for positive changes in "LXSGRT" and "LXTCRT" stock indices to negative changes in "LTR2YGB". Differently speaking, positive changes in these indices Granger-causes negative changes in "LTR2YGB". On the other hand, the causal relationship from negative shock in stock indices to both negative and positive shocks in "LTR2YGB" is more pronounced. Putting differently, a flight of funds from equity markets to bond markets is valid for both the aggregate market index, "LXU100", and sector indices. From negative shock in "LXU100", "LXBANK", "LXHOLD", "LXKMYA", "XSGRT", "LXTCRT", "LXULAS", and “LXUMAL” stock indices to a negative change in "LTR2YGB" is observed at 5% or 10% significance level. If a decrease is observed in "LTR2YGB" time series, it can be said that this decrease is caused by a negative change in some stock indices. Lastly, causal relationship between different components is reported in the fourth column. The null hypothesis of no a causality from negative change in stock indices to positive change in "LTR2YGB" is strongly rejected for all stock indices; except negative shock in stock indices of "LXELKT", "LXILTM", "LXMANA", "LXTCRT", and "LXUHIZ" where they do not Granger-causes "LTR2YGB" in short run.

Next, the test result of frequency causality test of Breitung and Candelon (2006) method will be discussed. To remark, this method enables us to separate the impact of one variable on another variable in time-domain to frequency-domain. Putting differently, one can reveal the link investigated over the short run, medium and long run. It should be underlined that the highest frequency point in frequency causality is 3.14 while the lowest point is 0.01, which can be transformed to time-domain with the formula of $T = 2pi/\omega$. Using this formula, the lowest investment period is found to be as 2 weeks for this study.

Table 4-11 Hatemi-J (2012) Asymmetric Causality Test

MODEL			LTahvil2 does not Granger Cause LX				MODEL			LX does not Granger Cause LTahvil2			
			$T^+ \nRightarrow X^+$	$T^+ \nRightarrow X^-$	$T^- \nRightarrow X^-$	$T^- \nRightarrow X^+$				$T^+ \nRightarrow X^+$	$T^+ \nRightarrow X^-$	$T^- \nRightarrow X^-$	$T^- \nRightarrow X^+$
LXU100	~	LTR2YGB	2.647	5.172**	0.427	0.077	LTR2YGB	~	LXU100	3.668	0.195	2.883*	15.385***
LXUMAL	~	LTR2YGB	5.167*	7.461**	0.268	0.015	LTR2YGB	~	LXUMAL	4.776	0.266	4.256**	20.23***
LXBANK	~	LTR2YGB	4.421	7.219**	0.297	0.02	LTR2YGB	~	LXBANK	4.664	0.6	4.51**	18.449***
LXFINK	~	LTR2YGB	1.052	0.925	4.749	4.815**	LTR2YGB	~	LXFINK	0.882	5.662	2.791	34.091***
LXGMYO	~	LTR2YGB	2.059	0.345	1.66	0.945	LTR2YGB	~	LXGMYO	0.182	1.069	1.801	26.353***
LXHOLD	~	LTR2YGB	9.149**	5.485**	0.032	0.2	LTR2YGB	~	LXHOLD	3.002	0.347	2.907*	20.509***
LXSGRT	~	LTR2YGB	3.116	2.228	0.567	1.091	LTR2YGB	~	LXSGRT	7.052**	16.418***	2.817*	34.273***
LXUSIN	~	LTR2YGB	1.408	2.234	1.008	0.251	LTR2YGB	~	LXUSIN	1.657	0.597	0.651	9.868**
LXGIDA	~	LTR2YGB	0.81	0.325	0.492	0.115	LTR2YGB	~	LXGIDA	0.796	3.188	0.351	8.31**
LXKAGT	~	LTR2YGB	4.669	0.355	0.465	3.499*	LTR2YGB	~	LXKAGT	3.048	4.02	0.827	20.93***
LXKMYA	~	LTR2YGB	0.069	1.267	0.442	0.659	LTR2YGB	~	LXKMYA	0.5	0.059	2.77*	9.268***
LXMANA	~	LTR2YGB	2.918	8.37***	0.584	0.075	LTR2YGB	~	LXMANA	1.018	1.497	0.09	3.166
LXMESY	~	LTR2YGB	9.052**	3.661*	1.761	1.034	LTR2YGB	~	LXMESY	2.198	0.659	1.835	17.277***
LXTAST	~	LTR2YGB	0.75	0.044	1.446	0.109	LTR2YGB	~	LXTAST	0.254	0.63	0.832	11.755***
LXTEKS	~	LTR2YGB	4.473	0.006	1.957	0.735	LTR2YGB	~	LXTEKS	4.365	3.539	0.159	22.202***
LXUHIZ	~	LTR2YGB	2.687	0.567	1.217	0.778	LTR2YGB	~	LXUHIZ	0.639	0.782	0.961	2.915
LXELKT	~	LTR2YGB	0.837	3.068*	1.902	1.418	LTR2YGB	~	LXELKT	2.128	1.713	2.607	1.663
LXILTM	~	LTR2YGB	2.673	0.032	0.763	0.424	LTR2YGB	~	LXILTM	0.264	0.242	1.491	1.542
LXSPOR	~	LTR2YGB	0.146	0.33	0.029	1.395	LTR2YGB	~	LXSPOR	1.21	1.692	0.074	14.983***
LXTCRT	~	LTR2YGB	0.322	1.818	3.149*	0.114	LTR2YGB	~	LXTCRT	0.495	10.661**	4.757**	2.763
LXTRZM	~	LTR2YGB	9.533**	0.21	4.265**	0.013	LTR2YGB	~	LXTRZM	0.89	0.181	0.525	5.098*
LXULAS	~	LTR2YGB	6.8*	4.403**	0.004	0.655	LTR2YGB	~	LXULAS	0.765	0.259	3.148*	11.001***
LXUTEK	~	LTR2YGB	3.848	0.694	0.369	0.01	LTR2YGB	~	LXUTEK	2.115	0.378	0.703	11.788***
LXBLSM	~	LTR2YGB	7.567**	0.042	1.318	0.771	LTR2YGB	~	LXBLSM	0.473	0.488	0.385	12.357***
LXYORT	~	LTR2YGB	3.365	0.682	2.962*	0.341	LTR2YGB	~	LXYORT	0.981	0.647	2.006	12.594***

*,**, and *** indicate 10%, 5%, and 1% significance level, respectively.

Notable, the returns series, i.e. stationary, are performed for frequency causality test. The optimal lag length that has to be higher than "2" is determined via AIC method. Besides, for comparing our findings with the results obtained in literature, we also highlighted some specific frequency points to study the temporary and permanent effects as suggested by Ciner (2011). The author (2011) states the test statistic calculated at $\omega = 0.01$ implies long run causal dynamics while the frequency at $\omega = 2.5$ to identify short-term dynamics of the underlying variables. These causal dynamics are called as permanent shocks (long-term) and transitory shocks (short-term) by the author. The overall test results are exhibited in Figure **4-10** and Figure **4-11**.

According to test results in Figure **4-10**, the null hypothesis of no causality from "LTR2YGB" to stock indices is rejected for "LXELKT", "LXFINK", "LXHOLD", "LXSGRT", "LXTCRT", and "LXYORT" at 10% significance level. Putting differently, "LTR2YGB" Granger-causes "LXELKT" variable both at low and high frequencies $[0.01 < \omega < 0.89]$ and $[2.82 \leq \omega \leq 3.14]$ while it has a significant causal impact on "XSGRT" and "LXYORT" variables at low-frequency intervals, $[0.01 \leq \omega \leq 1.04]$ and $[0.01 \leq \omega \leq 0.49]$, respectively. In addition, "LTR2YGB" variable leads "LXFINK", "LXHOLD", and "LXTCRT" at high-frequency level intervals, $[2.07 \leq \omega \leq 3.14]$, $[1.43 \leq \omega \leq 3.14]$, and $[2.28 \leq \omega \leq 2.5]$, respectively.

The causal relationship of frequency domain from stock indices to "LTR2YGB" variable is shown in Figure **4-11**. It is evident that the stock indices of "LXGMYO", "LXHOLD", "LXSGRT", and "LXUMAL" Granger-causes "LTR2YGB" at all frequencies, namely, at $[0.01 \leq \omega \leq 3.14]$. Besides, "LXBANK" has a significant causal impact on "LTR2YGB" at low and high frequency level intervals, i.e., $[0.01 \leq \omega \leq 1.3]$ and $2.72 \leq \omega \leq 3.14$ while "LXTEKS" stock index is a powerful predictor for "LTR2YGB" at high-frequency intervals, $1.89 \leq \omega \leq 2.17$, suggesting a significant causal impact at a 2.89–3.32 week periods for this study.

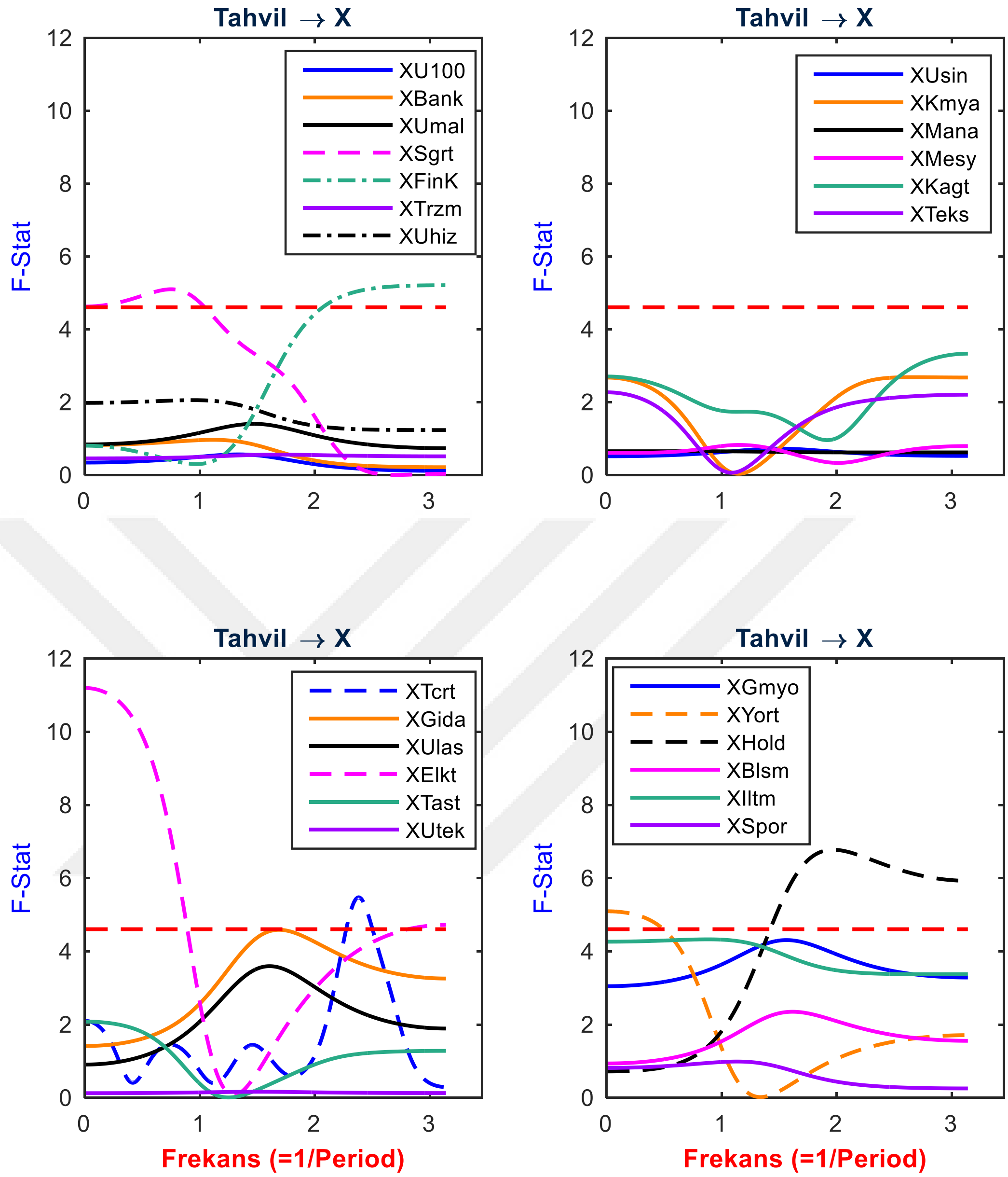


Figure 4-10 Frequency-domain causality test (DLTR2YGB to DLX)

Overall, there exists a strong feedback causal relationship between stock indices of "LXHOLD", "LXSGRT" and "LTR2YGB" at different frequency intervals. However, there is no any causality between the aggregate stock market and "LTR2YGB". In respect of Ciner's (2011) method, there exists transitory (short run) causality from "LTR2YGB" to "LXFINK", "LXELKT", "LXTCRT", and "LXHOLD" while a permanent causality is observed running from "LTR2YGB" to

"LXELKT", "LXSGRT", and "LXYORT". On the other hand, according to Ciner's (2011) method, there are evident for both transitory and permanent causalities from stock indices of "LXGMYO", "LXHOLD", "LXSGRT", and "LXUMAL" to "LTR2YGB" variable. However, the predictability power of "LXBANK" on "LTR2YGB" is permanent if only if the exact frequency points are stable, namely if $\omega = 2.5$ frequency point is made an obligatory condition for a permanent causal relationship.

In the same vein, Özer and Kamisli (2015) investigate causal relationship between the weekly data of macroeconomic factors and stock market index, "XU030", both in the time (TY) and frequency-domain method for Turkey case, spanning from 2003 to 2015. The authors state that the null hypothesis of non-causality is not rejected for the general notion for the causality from interest rates to stock prices, while an evidence of significant causal relationship is obtained from stock market changes to interest yields over the sample period, both in the time and frequency-domain (in the medium and long run). Corroborating the test result of traditional tests, however, this method gives more an accurate detail about relationship across investment horizons.

To remark, until now, both the log-level and return levels are used for causality tests, namely the former series is for asymmetric, the latter series is for other causality tests. For comparing the power of methods implemented, wavelets scale-based and frequency test results will be discussed. Before, it is required to determine the appropriate the frequency intervals for wavelet scales. More concretely, wavelet scale d1 [2-4), d2 [4-8), d3 [8-16), d4 [16-32), and d5 [32-64) are equal to the frequency point intervals of [$1.57 < \omega \leq 3.14$], [$0.79 < \omega \leq 1.57$], [$0.39 < \omega \leq 0.79$], [$0.2 < \omega \leq 0.39$], and [$0.10 < \omega \leq 0.19$], respectively. Let's denote these frequency intervals as $f1$, $f2$, $f3$, $f4$, and $f5$. Remarking that nineteen out of twenty-five sectors are caused by "LTR2YGB" variable according to wavelet-based causality test results. On the other hand, the total number of stock indices that caused by "LTR2YGB" variable is just 6 (six). Considering causal relationship at frequencies, both methods do not differ with each other. For example, "LTR2YGB" variable Granger-causes "LXELKT" at scale d1, d2, and d3 while it has a significant impact on this sector index at all frequency intervals. In addition, "LXFINK" is Granger-caused by "LTR2YGB" at only $f1$, but at wavelet scales of d2, d4, and d5.

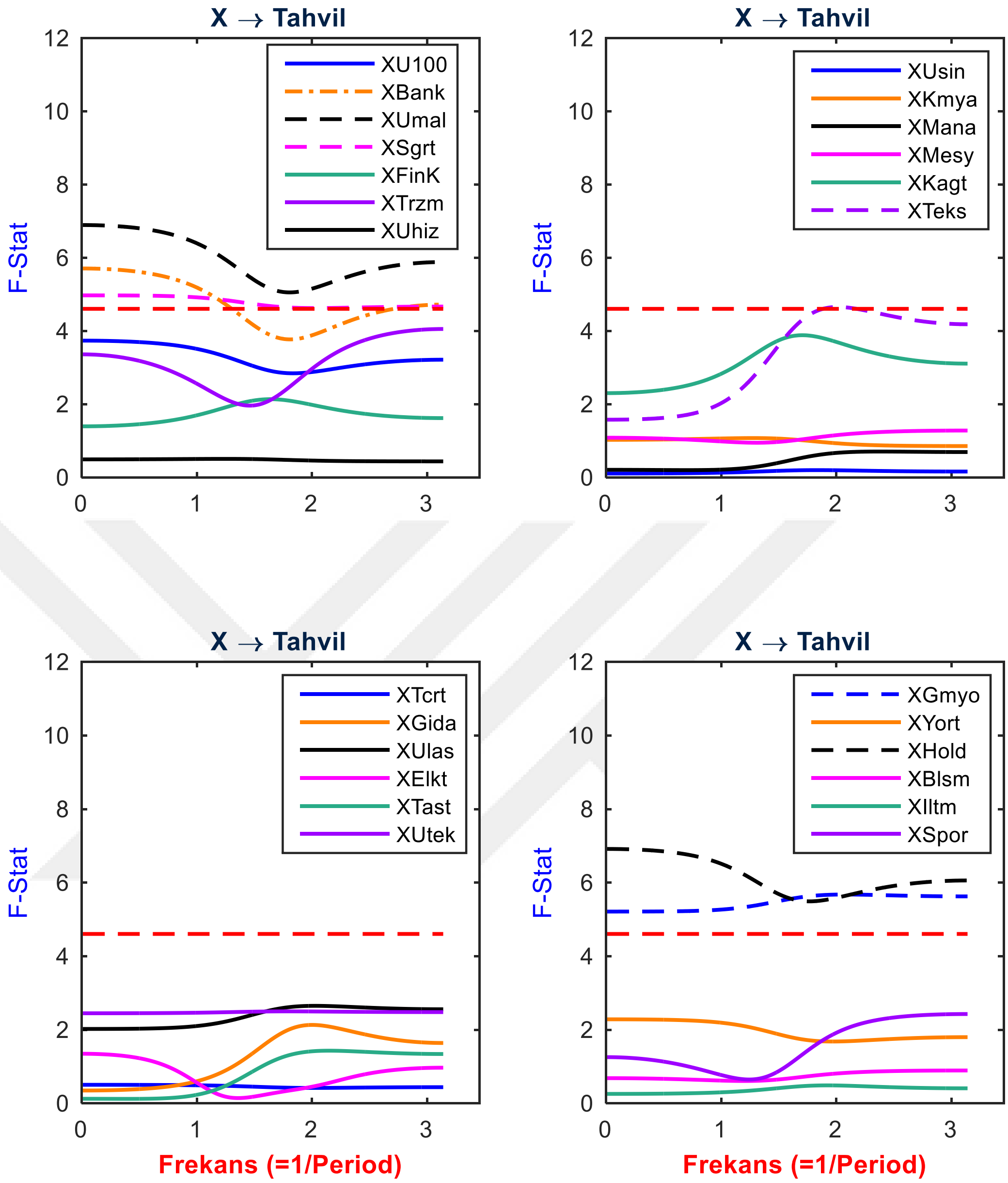


Figure 4-11 Frequency-domain causality test (DLX to DLTR2YGB)

On the other foot, the stock indices that significantly Granger-causes "LTR2YGB" variables are "LXBANK", "LXGMYO", "LXHOLD", "LXSGRT", "LXTEKS", and "LXUMAL" in case of frequency causality test, but, using a wavelets scale based method, it is found an evidence of statistically significant causal relationship from all stock market indices, including the aggregate market index, to "LTR2YGB". However, the frequency causality method is more pronounced than wavelets based causality tests regarding the link at frequencies. For instance, all stock indices mentioned above, except "LXTEKS", Granger-causes "LTR2YGB" at $f1$ to $f5$ while the wavelets based results vary according to scales. The null hypothesis of no causality from "LXSGRT" to "LTR2YGB" is rejected for scale d1, d2, d3, and d5 as the only scale that "LXUMAL" does not have an impact on "LTR2YGB" is scale d2, namely 4-8 week periods. These all results suggest that evidently, the wavelet method is more powerful than frequency causality test of Breitung and Candelon (2006).

CHAPTER

5 CONCLUSIONS

This study undertakes an attempt to examine the linkage between the 2-year government bond yields, "LTR2YGB", and stock indices listed on the Istanbul Stock Exchange in Turkey using weekly data over the sample period covering 2005-04-01 and 2016-12-30, consisting 605 observations. By conducting both standard econometric tools and a novel approach, wavelet analysis, the interdependence between two major assets is re-investigated for Turkey case at the aggregate and sectoral levels to offer implications for heterogeneous agents of markets trading at different investment horizons.

In the first step, it is investigated by the conventional tests without structural breaks and modern approach, the Lee and Strazicich unit root (2003) considering structural breaks whether the underlying data is stationary or not. Test findings reveal that all variables are found to be stationary at first difference, namely, they are integrated at $I(1)$. Observing stationarity after taking log-difference of variables, next step is to analyze whether there exists long run relationship between financial variables. Equivalently speaking, evidence of cointegrating vector between stock index and "LTR2YGB" entails that the long run relationship is not purely spurious. Accordingly, in the case of "LTR2YGB" being independent, the Hatemi-J cointegration test (2008) approach with two structural breaks shows that there exist cointegration associations between interest rate and six stock indices, suggesting a theoretical linkage in the long run. On the other hand, the test findings of the paper present only one long run relationship between "LTR2YGB" and "LXUHIZ" being independent, indicating a possible long run causal linkage between underlying variables in the long-term. As Abdullah et al. (2014) remark, this result suggests that consistently earning above-average return is restricted for those variables when adjusting portfolio composition. After determining cointegrating vectors between variables, it is required to implement VECM model to identify direction of causality for only those models. According to test results, there exist causal linkages running from "LTR2YGB" to "LXGMYO", "LXHOLD", "LXTEKS", "LXTRZM", "LXULAS", and "LXUMAL" in the long run, namely, those indices are significantly affected by movements in "LTR2YGB". In addition, there is a unidirectional causality from “LTR2YGB" to “LXHOLD" index in the short run, while, on the other hand, the null hypothesis of non-causality from "LXUHIZ" to "LTR2YGB" cannot be rejected both in the short- and long-term.

Apart from cointegrated models, Granger causality test results based on VAR model reveal that there does not exist any causal relationship from "RTR2YGB" to index returns, except "RXELKT", implying that changes in interest rate do not have any predictive power on estimation of share returns. On the contrary, the outcomes of the paper show that only two variables, "RXHOLD" and "RXUMAL", Granger-causes changes in "RTR2YGB". The results of the Hacker and Hatemi (2006) symmetric causality test, however, suggests one-way causalities running from the change in interest rates to share returns, exceptions for "RXILTM", "RXKAGT", "RXSPOR", "RXULAS", and "RXUTEK". To illustrate how share returns and interest yields concurrently relate in time and frequency domains, as noted by Rua and Nunes (2009), Granger causality tests are conducted on the wavelet coefficients obtained by MRA() function. With this method, we can observe how and when one variable significantly affect another variable across all frequencies (inversely, time scales). Accordingly, wavelet-based causality test findings display strong evidence of feedback mechanism at the higher scales "d3", "d4", and "d5", namely, the dynamic leading relationship strengthens at the lower frequencies. A remarkable finding of the wavelet analysis is that this method divulges hidden significant causal relationships that cannot be observed by using standard method. By conducting the Hacker and Hatemi (2006) test, it is found that neither of the share returns Granger-causes the changes in "RTR2YGB" in the time domain, however, wavelet method reveals that "RXU100", for example, have significant explicative power on "RTR2YGB" variable at scales "d3" and "d5". In addition, wavelet-based causality test is able to show the dispersed linkage over frequencies. For example, the significant causality finding from "RTR2YGB" to "RXELKT" in the time domain is dispersed at scales from 8 weeks to 64 weeks.

Test findings of the Hatemi-J (2012) approach, on the other hand, reveal some significant causal relationship between shock components. For example, the positive component of "RTR2YGB" has predictive powers on positive shocks of "RXBLSM", "RXHOLD", "RXMESY", "RXTRZM", "RXULAS", and "RXUMAL" indices, while, on the other hand, there exists only one causal linkage running from the positive component of "RXSGRT" to positive component of "RTR2YGB". In addition, "RTR2YGB(+)" has significant explicative powers on "RXU100(-)", "RXBANK(-)", "RXELKT(-)", "RXHOLD(-)", "RXMANA(-)", "RXMESY(-)", "RXULAS(-)", and "RXUMAL(-)", whereas, there are only two significant results for causality running from "RXSGRT(+)" and "RXTCRT(+)" to "RTR2YGB(-)", i.e. negative shocks in government bond yields. The null hypothesis of non-causality from "RTR2YGB(-)" to positive shocks in stock returns can be rejected only for two cases, "RXFINK(+)" and "RXKAGT(+)", while, on the other hand, the causality does hold for twenty out of twenty-five cases in the reverse direction. Equivalently speaking, the negative shocks of index returns, (20/25), can be used as a leading factor for the positive shock estimation of bond yields.

Along with conventional methods, a widely used method introduced by Breitung and Candelon (2006) is also utilized to compare tests results. This method also shows somehow significant results, however, its significant results are not as much as wavelet-based reveals.

In addition to econometric analysis, wavelet-based statistics calculated by MODWT() non-boundary coefficients in which the number of efficient wavelet coefficients decreases as wavelet scales increases. The results from the analysis of wavelet variance, covariance, correlation, and cross-correlation coefficients can be summarized as given. For example, as expected and in common in literature, an inverse linear linkage is observed, that is the higher (longer) wavelet scale (time horizon), the lower wavelet variances. The greatest variability, alike energy decomposition, of all financial variables is intensified at lower wavelet scales, i.e. short investment horizons, which is in accordance with results of Kim and In (2007) for G7 countries, Gallegati (2008) for the US, Dajcman (2015) for Eurozone countries, and Moya-Martínez (2015) for Spain. This higher energy decomposition manifest that changes in bond and stock market returns are mostly driven by short-term fluctuations in those markets. The highest and lowest energy decomposition, for instance, are found to be 60.52% and 40.60% for "RXGIDA" and "RTR2YGB" at scale d1. Those figures rise to 81.54% and 61.76% at scale d2, and 91.90% and 76.85% at scale d3. Overall, at least 76% of volatilities of all returns is explained by short run, i.e. the short run dominates the long run in case of energy distribution. Accordingly, it implies that, as noted by the authors (2015), short-term investors confront higher investment risk compared to long-term investors. In line with the findings of Moya-Martínez (2015), Kim and In (2007), and Çifter and Özün (2007b), wavelet-based variance of most sector equity return rates are found to be higher than changes in 2-year government bond yields at higher frequencies over all time horizons, indicating that the debt market is less volatile than the stock markets regardless of the wavelet scales. For example, the wavelet variance of "RTR2YGB" is in the first lowest place, in terms of energy decomposition, at scale d1 and d2, while, it is on the second lowest, sixth lowest, and eighth lowest rank at scale d3, d4, and d5, respectively.

Besides, as expected, and in line with that of previous empirical papers on financial return, it is found that all stock returns are significantly negative related to movements in bond yields up to the scale d4 with the exception of "RXILTM" and "RXUHIZ" variables where negative relationship holds at all wavelet scales. The highly significant relationship (at 1% significance level) indicates that both markets are tightly related to each other, which is quite similar to the findings of Kim and In (2007), Ferrer et al. (2010), and Reilly et al. (2007). As expected, highly leveraged (indebted), regulated and financial sectors are the most interest rate sensitive in Turkey. In general, the financials, industrials, and utilities indices are among the top six most sensitive to interest rate movements at all wavelet scales, at increasing magnitudes up to scale d4. At highest wavelet scale d5, there exist negatively low magnitudes but insignificant correlation relations with the exceptions of "RXILTM" and "RXUHIZ" variables significant at 5% level.

Test results of wavelet cross-correlation reinforce the findings of Hamrita and Trifi (2011) for the U.S, Abdullah et al. (2014) for Malaysia, Moya-Martínez (2015) for Spain. In overall, as wavelet scales increases, the magnitude and significance level also increases. The causal relationship in term of lead-lag correlation becomes clear beyond scale level d3, which is in line with the findings from Granger causality tests, namely, the relation is restricted to those scale levels. More clearly, there occur feedback causal linkages between share returns and interest rate movements at the coarse scales.

As mentioned above, there is significantly negative relationship induced by the flight-to-quality phenomenon between debt and stock market indices, and bidirectional causalities at intermediate and long-term scales. In line with those findings, we also presented frequency causality test, introduced by Breitung and Candelon (2006), results. Moreover, asymmetric causality linkages between different data components ($T^{-}, T^{+}, X^{-}, X^{+}$) are provided, mostly from X^{-} to T^{+} which corroborates the adverse relationship between stock returns and bond yields. Evidently, all these results not only supportive of existing theory and evidence on the significantly negative in terms of correlation and powerful predictor in terms of causal linkage between bond yields and share returns, but also offer a plausible interpretation of the association across investment horizons.

Attempts to deepen our understanding of driving dynamics of relationship between debt and stock markets are of high importance for investors and policy makers. As noted by Andersson et al. (2008) in their relevant paper, for example, the linkage between these two markets directly affects investors' risk management strategies and asset allocation decisions. Note that, investment strategies and standard econometric models assume a steady linkage between variables over period. Taking into account the dynamic relation across frequencies may provide better portfolio diversifications and investment strategies as such given

- ⇒ It is shown that bond and stock returns are negatively correlated with each other across wavelet scales. This result implies that both instruments can be used as hedging instruments, namely, whenever a market falls, there exists a safe haven for investors that holds true for all investors with having different investment strategies. Moreover, negative relationship between bond yield and all stock indices does not allow investors to follow tactical asset allocation strategy.

- ⇒ As wavelet scale increased, the strength (magnitude) of relationship inversely increased as well, providing a better portfolio diversification at lower frequencies. Wavelets, evidently, enable investors to adjust their portfolio compositions at periods of falling or rising stock prices.

- ⇒ Wavelet variance results show that energy decomposition and scale are inversely related. The higher time horizon, the less variation in bond and stock market returns. The evidence of an approximately linear link observed suggests that, as noted by Kim and In (2007), short-term traders must react to every variation in realized returns, while, on the other side, fluctuations are less important for long-term investors. Accordingly, the true risk-return association between variables occurs in the long-term after eliminating the effects of short-time noise arising from changes in unexpected consumption needs and portfolio rebalancing activities.

- ⇒ In addition to wavelet statistics, granger causality tests reveal bidirectional relationship at intermediate and higher scales, suggesting that both variables can be used as forecasting tools, i.e. barometer, by investors to adjust their portfolio compositions. In line with the classical wisdom observed in previous studies, long-term investors are largely related to macroeconomic fundamentals for their investment strategies. The absence of causal relationship at high frequencies, therefore, implies that investor may consistently gain abnormal returns regardless of stock indices at higher frequencies, while it is not possible at other frequencies.

- ⇒ Monetary policy decisions should be closely followed by investors to adjust their portfolio compositions or hedging strategies for a better risk-return trade-off since macroeconomic factors, especially interest rates, are reasonable indicators for stock returns and vice versa in the medium- and long-term.

From the policy-making standpoint, on the other hand, these results also propose some significant implications for monetary policy. For example, as observed above, since stock markets are found to be very sensitive to interest rate movements across scales, the authorities should take into account this interrelationship when implementing decisions and regulations. Having regard to time-varying association, the authorities should be patient for consequences. In addition, as noted by Bayraci et al. (2018), the authorities that responsible for implementing monetary policy, central banks, should utilize the relevant information to secure the resiliency and durability of the financial system through manipulating the investors' perception regarding to the economic outlook in the future. On the other hand, further papers should take into account the possible effects of other factors –firm specific and macroeconomic– by using other modern techniques, such as wavelet coherence approach and nonlinear causality tests, to shed lights on stock-bond interdependence at both aggregate and industries.

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A. MRD FIGURES

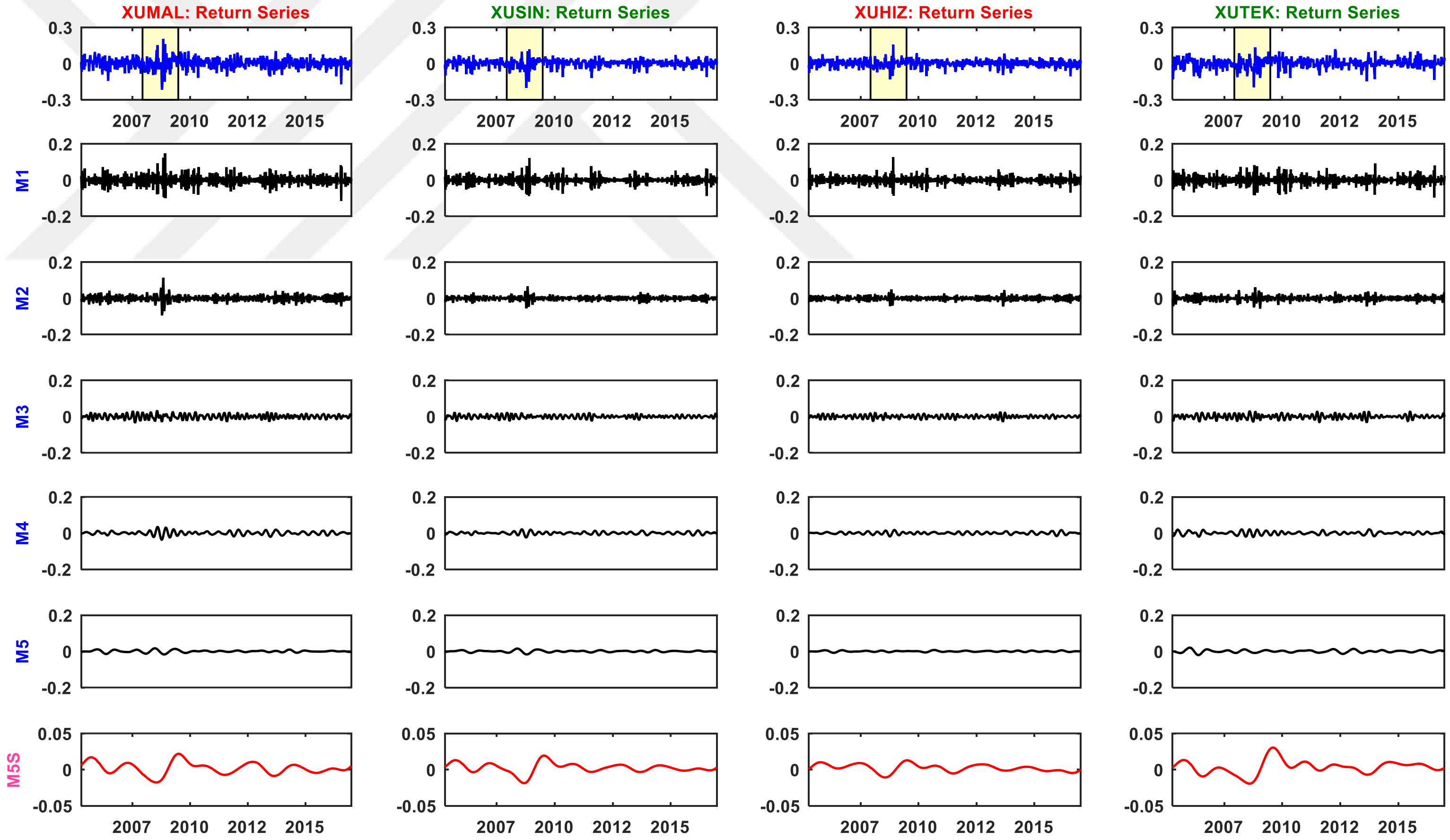

Figure Multiresolution Analysis for "XUMAL", "XUSIN", "XUHIZ", "XUTEK"

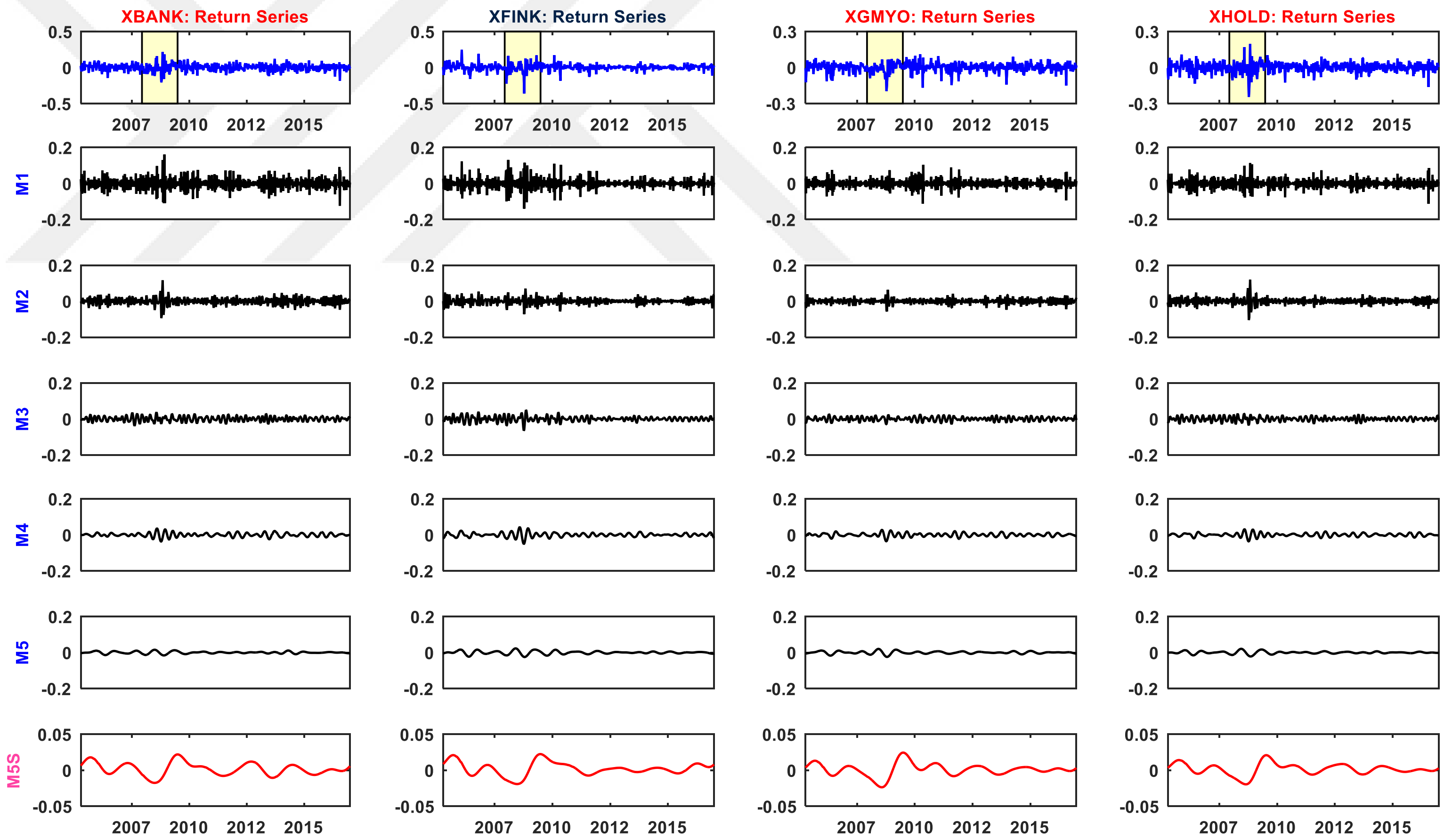

Figure Multiresolution Analysis for "XBANK", "XFINK", "XGMYO", "XHOLD"

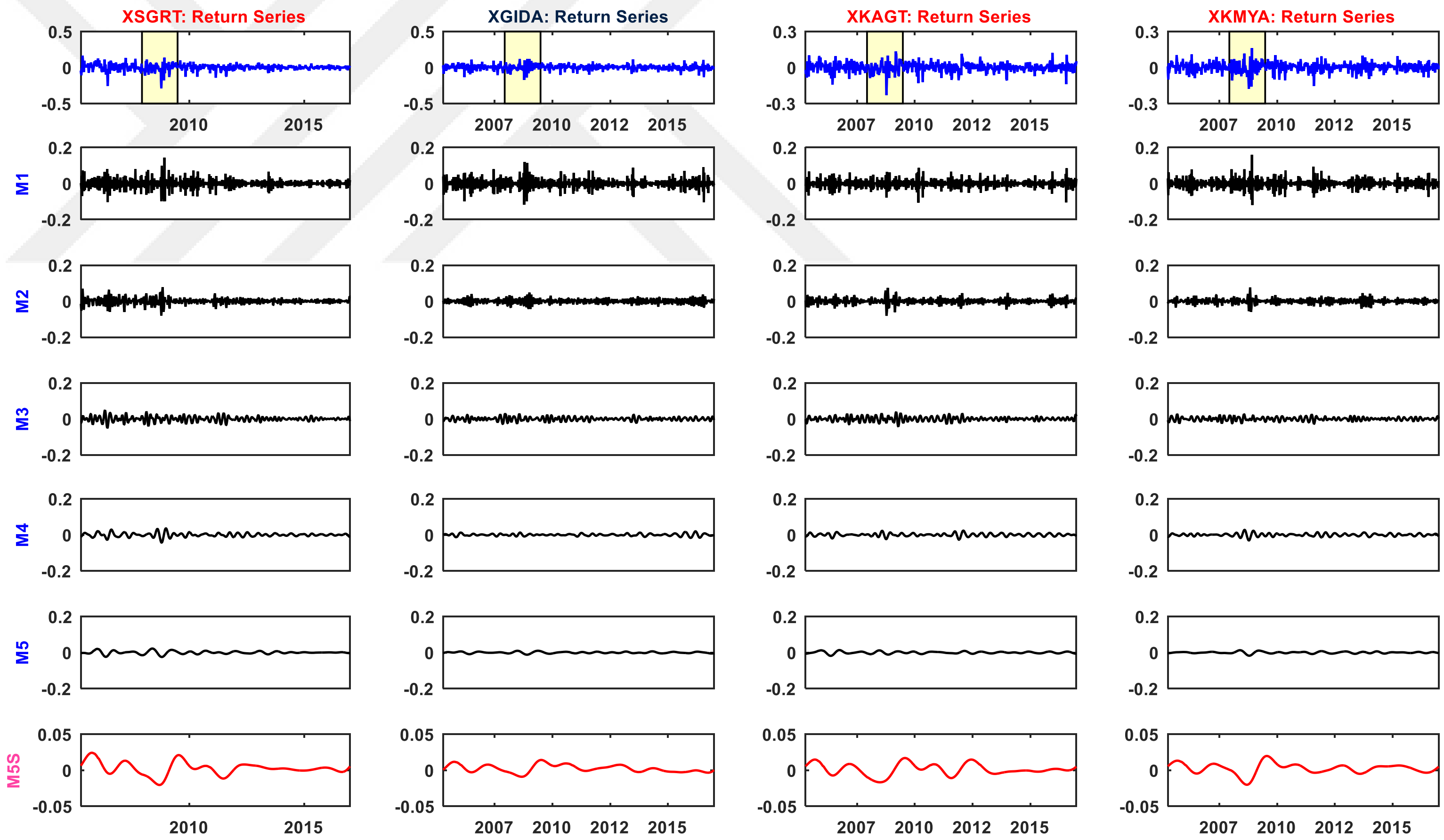

Figure Multiresolution Analysis for "XSGRT", "XGIDA", "XKAGT", "XKMYA"

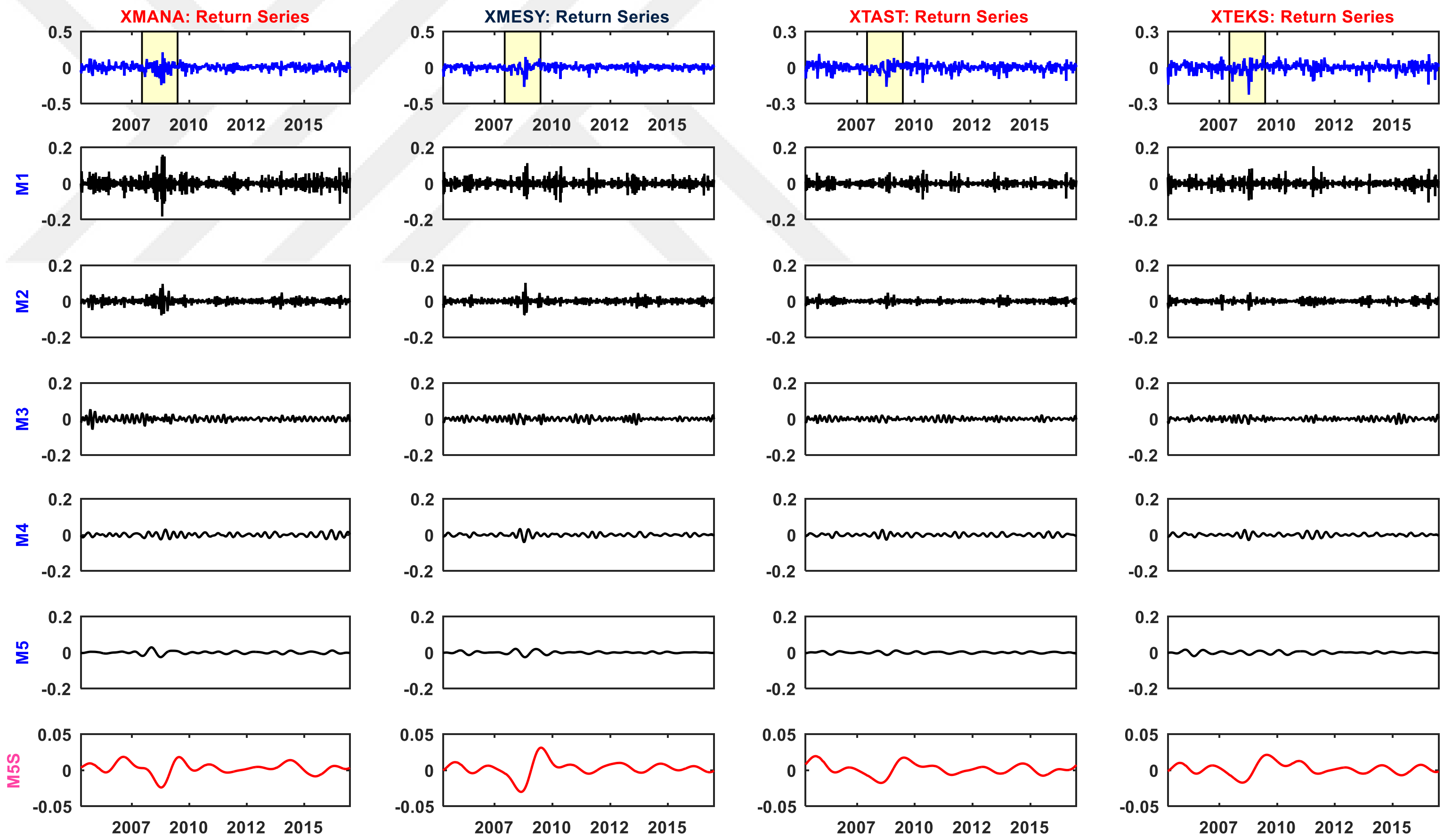

Figure Multiresolution Analysis for "XMANA", "XMESY", "XTAST", "XTEKS"

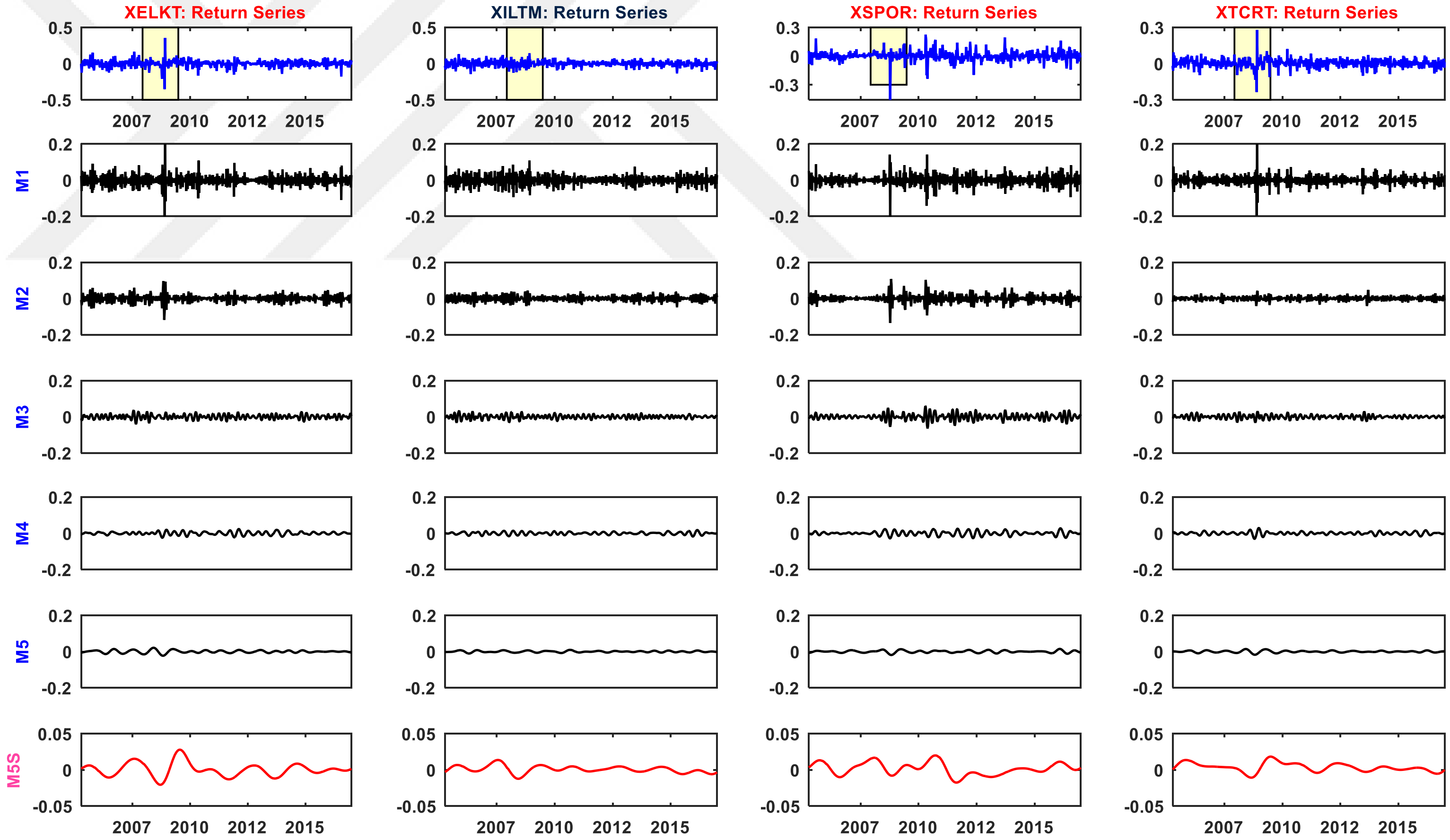

Figure Multiresolution Analysis for "XELKT", "XILTM", "XSPOR", "XTCRT"

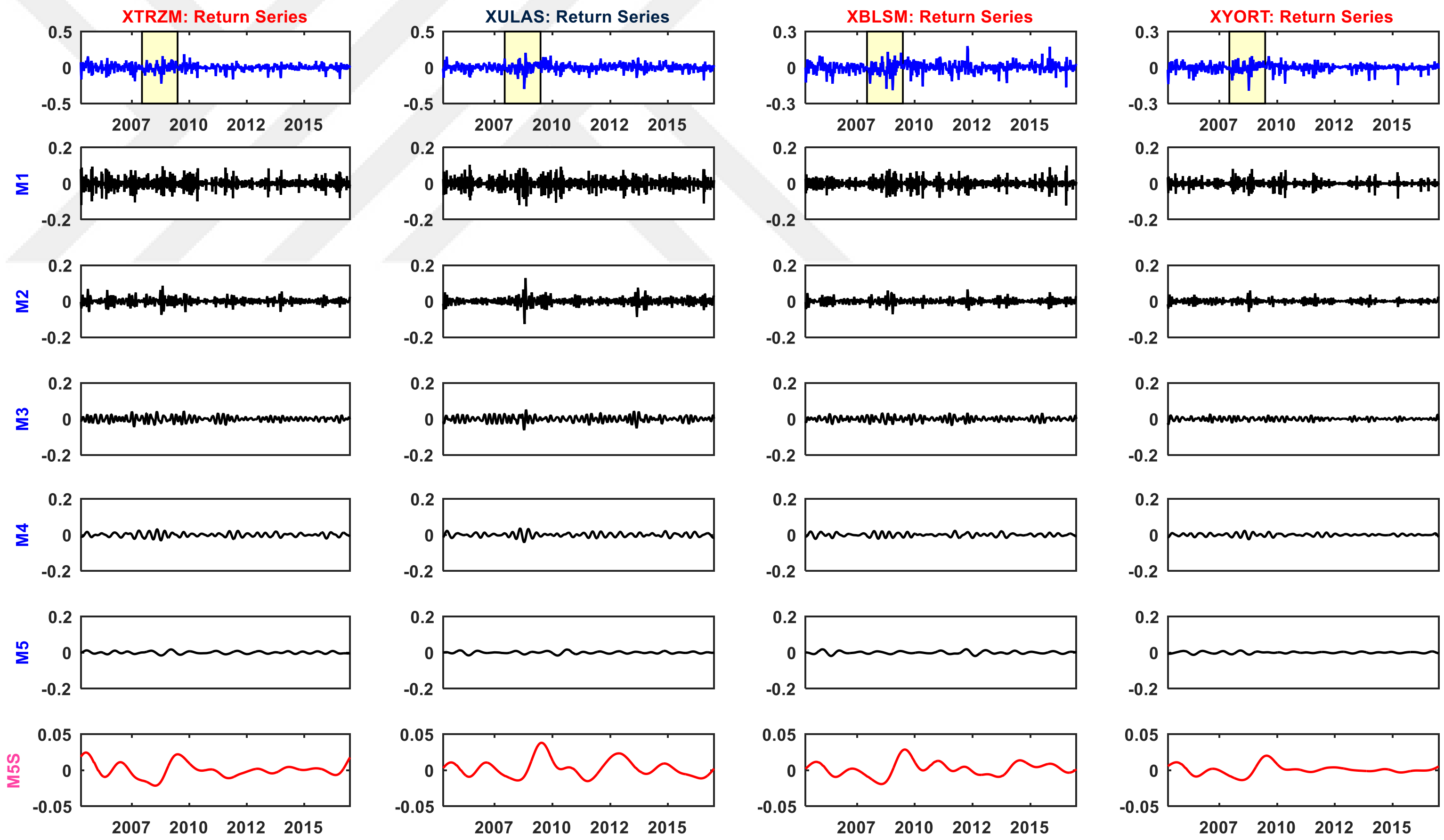

Figure Multiresolution Analysis for "XTRZM", "XULAS", "XBLSM", "XYORT"

B. CROSS-CORRELATION FIGURES

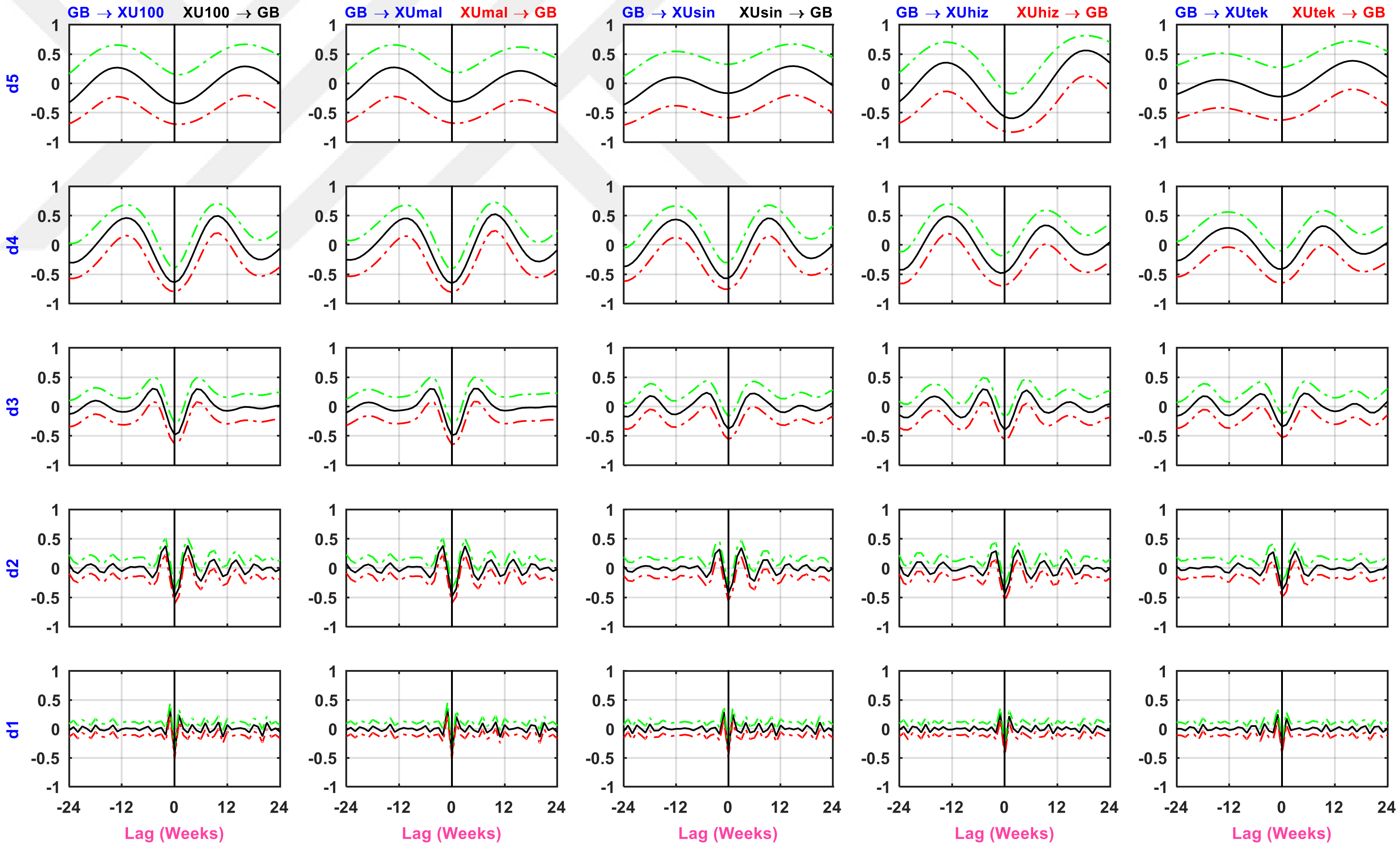

Figure Wavelet cross-correlation for "XU100", "XUMAL", "XUSIN", "XUHIZ", "XUTEK"

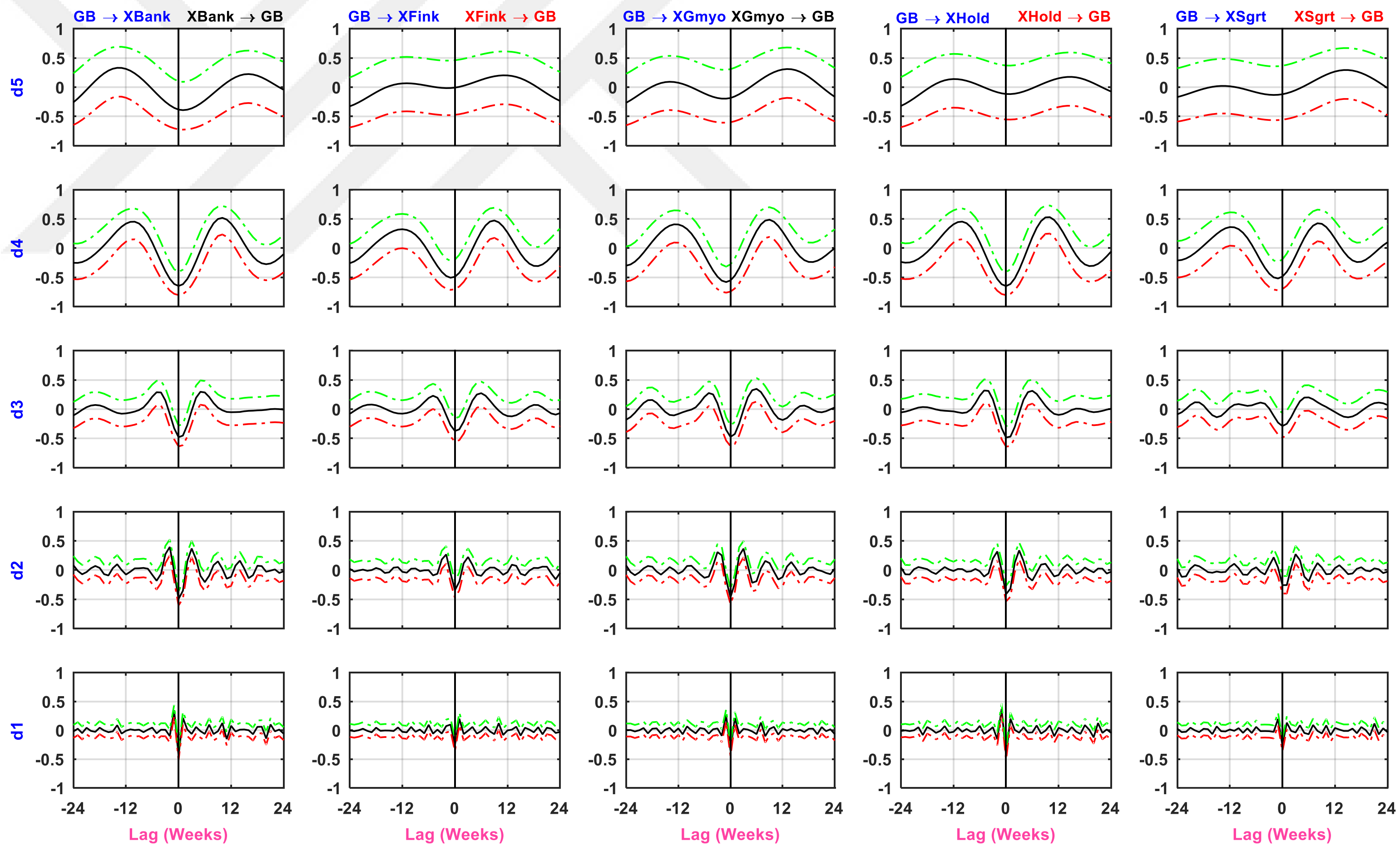

Figure Wavelet cross-correlation for "XBANK", "XFINK", "XGMYO", "XHOLD", "XSGRT"

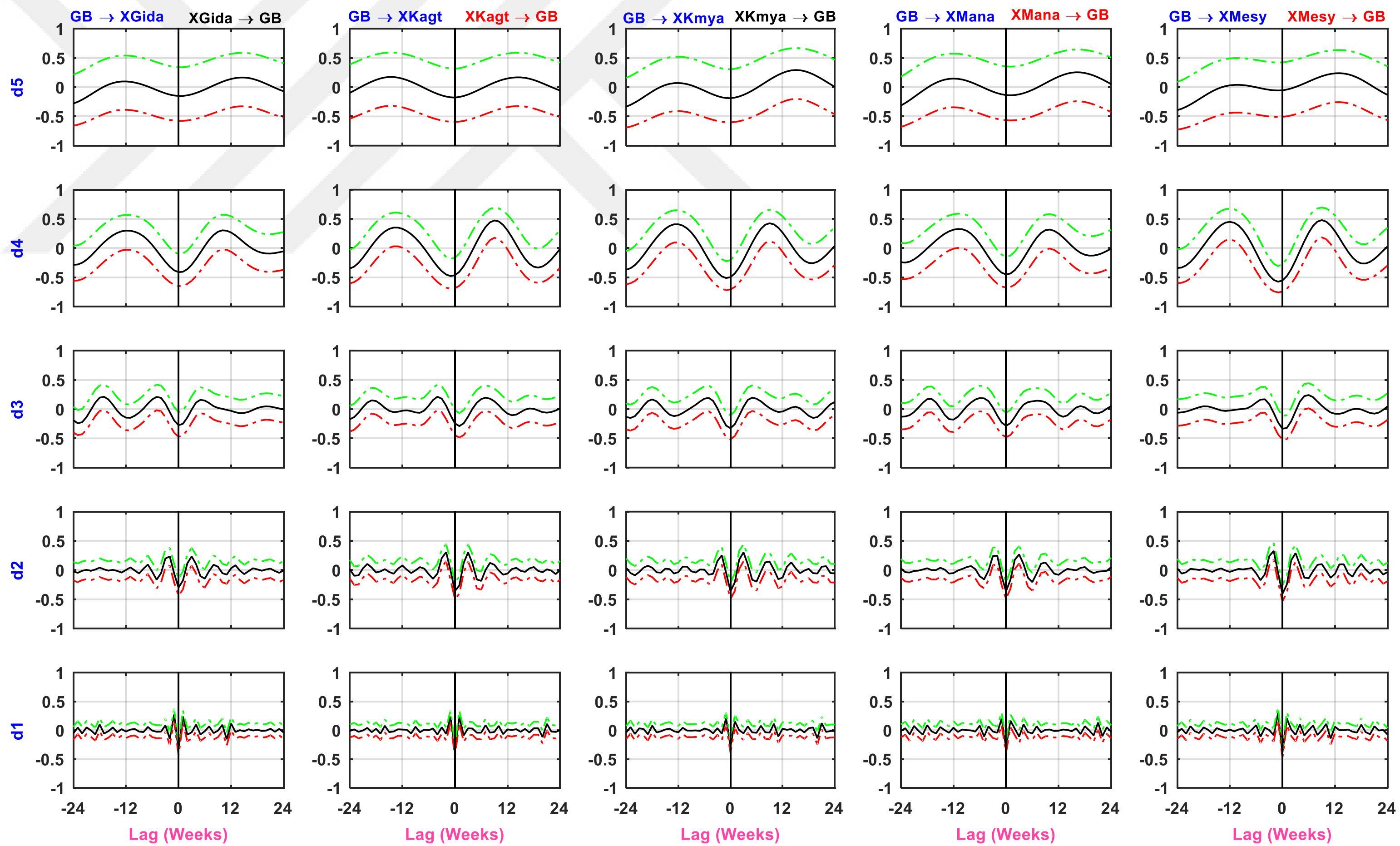

Figure Wavelet cross-correlation for "XGIDA", "XKAGT", "XKMYA", "XMANA", "XMESY"

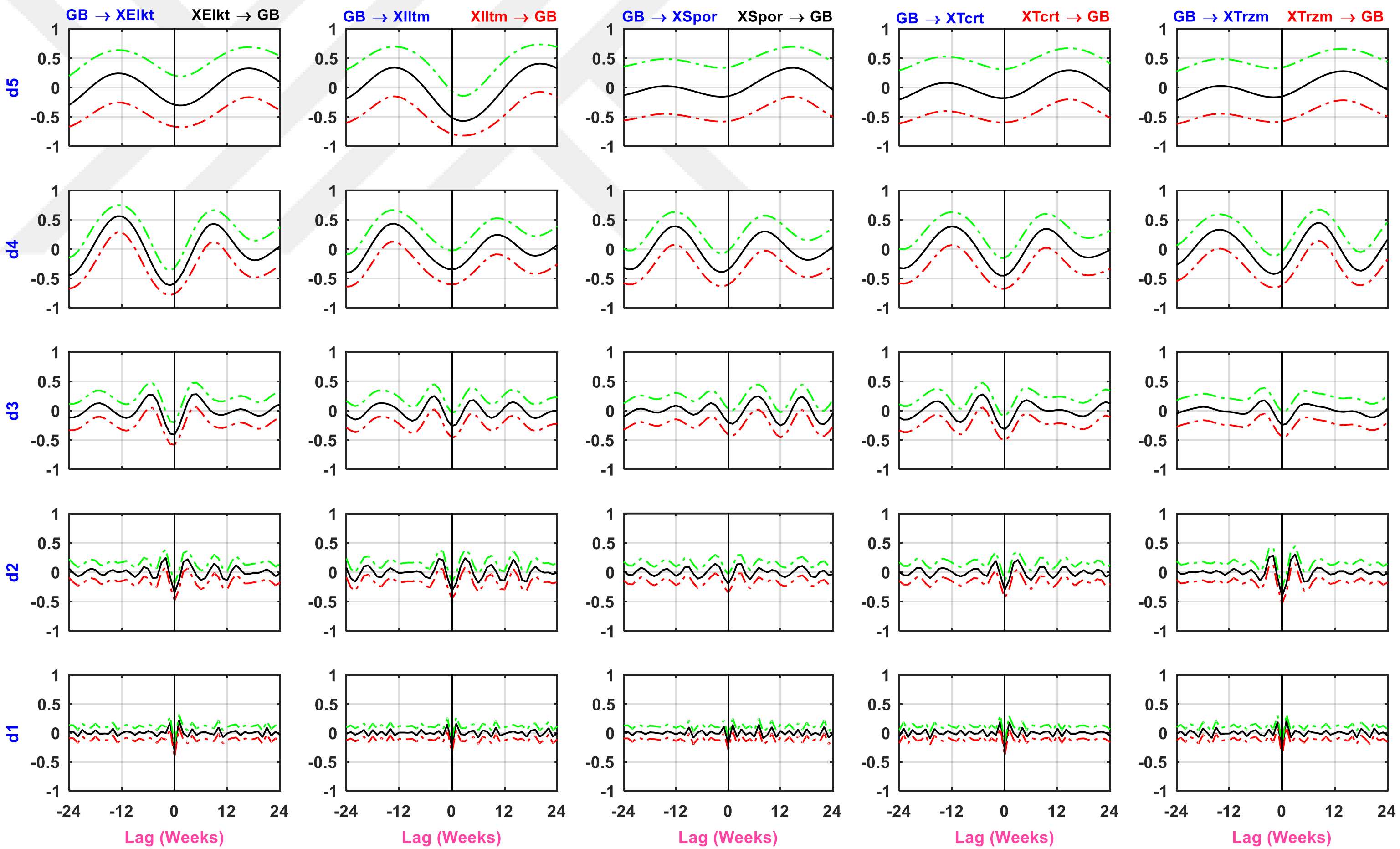

Figure Wavelet cross-correlation for "XELKT", "XILTM", "XSPOR", "XTCRT", "XTRZM"

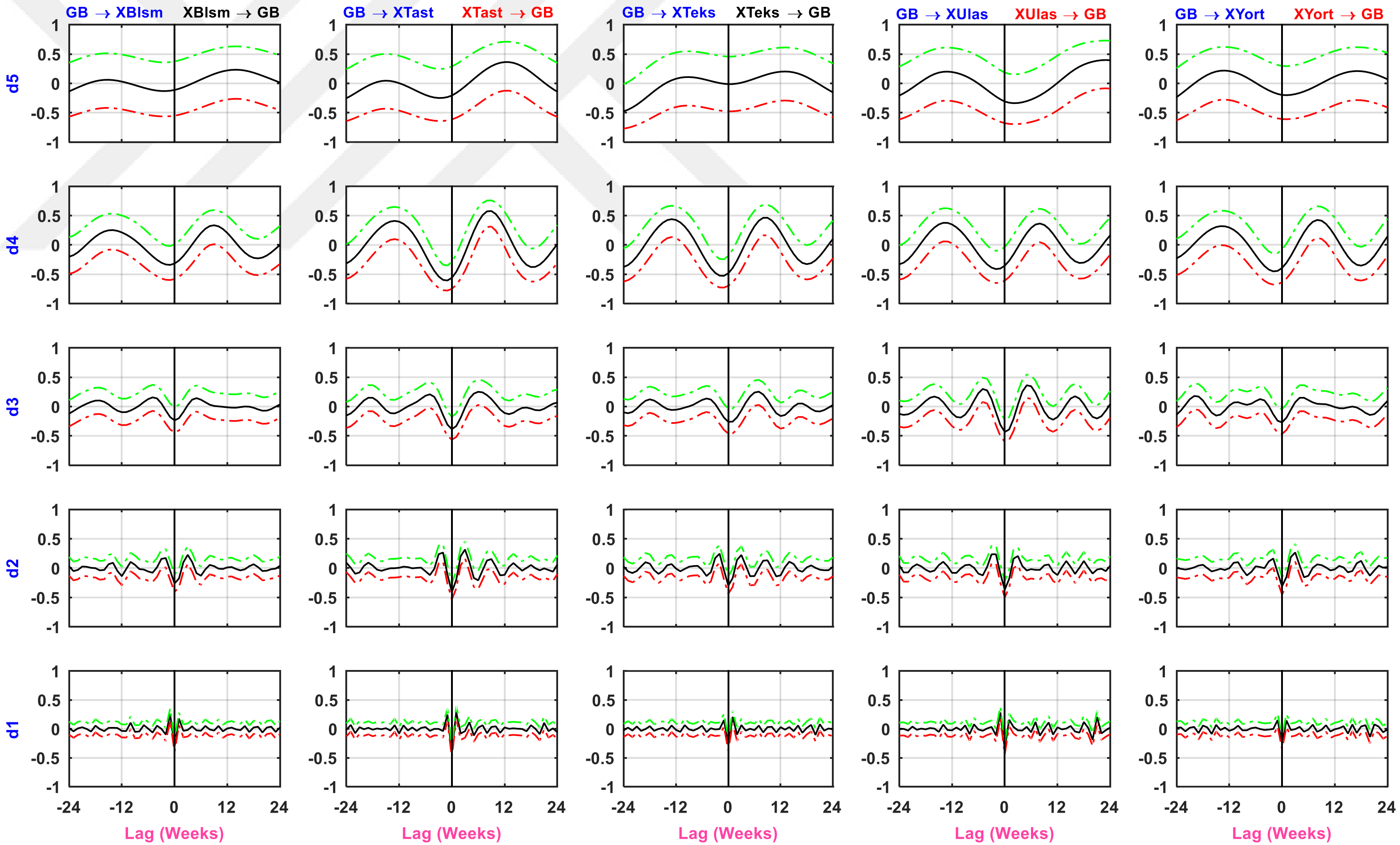

Figure Wavelet cross-correlation for "XBLSM", "XTAST", "XTEKS", "XULAS", "XYORT"

D. DATA FOR SAMPLE MAP GENERATION

ENSTİTÜ

Fen Bilimleri Enstitüsü ☐

Sosyal Bilimler Enstitüsü ☐

YAZARIN

Soyadı : GÖK

Adı : Remzi

Bölümü : Bankacılık ve Finans

TEZİN ADI (İngilizce) : Wavelet analysis of stock returns and interest rate changes: evidence from Turkey

TEZİN TÜRÜ : Yüksek Lisans ☐ Doktora ☐

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
3. Tezimden bir (1) yıl süreyle fotokopi alınamaz. ☐

TEZİN KÜTÜPHANEYE TESLİM TARİHİ

E. CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name	: Gök, Remzi
Nationality	: Turkish (TC)
Date of Birth	: 05 May 1984
Place of Birth	: Diyarbakır
Phone	: 0412 241 10 00 / 8177
Email	: remzi.gok@dicle.edu.tr

EDUCATION

Degree	Institution	Year of Graduation
MS	DU Business Administration	2012
BS	MU Banking	2006

PROFESSIONONAL EXPERIENCE

Year	Place	Enrollment
2010- Present	Dicle University	Research Assistant
2008-2010	Turkish Finance Participation B.	Banker

FOREIGN LANGUAGES

Intermediate English

PUBLICATIONS

◊ Bildiri: Türkiye ve ABD örneğinde zaman ve frekans bazlı nedensellik ilişkisi (2017). I. Uluslararası ekonomi, siyaset ve yönetim sempozyumu (Diyarbakır).

◊ Article: Dynamic wavelet-based causal relationship between equity returns and aggregate economic activity in G7 and E7 countries [ID 1410, it is on refereeing in the Anadolu University Journal of Social Sciences].

F. TURKISH SUMMARY

Bu çalışmada Türkiye örnekleminde 604 haftalık tahvil ve borsa getirisi arasındaki ilişki incelenmiştir. Bu ilişkinin analizinde güncel metotların yanı sıra son dönemlerin popüler analiz türü olan dalgacıklar analizi kullanılmıştır. Bunun temel nedeni, borsa-tahvil ilişkisinin kısa, orta ve uzun vadeli yatırım dönemine sahip heterojen yapıdaki katılımcılar için geçerli anlamlı sonuçlar sunabilmesidir. Bilindiği gibi geleneksel ekonometri analiz teknikleri, değişkenler arasında kısa ve/veya uzun dönemlik ampirik sonuçlar ortaya koyarken söz konusu daha spesifik yatırım dönemine sahip yatırımcılar ve politika yapıcılar için yetersiz kalmaktadır. Bu eksiklik frekans bazlı tekniklerin ortaya çıkmasıyla giderilmeye çalışılırken, dalgacıklar tekniği araştırmacılar tarafından sıklıkla tercih edilen etkin bir yöntem olarak ön plana çıkmıştır. Tüm bu gelişmeler doğrultusunda heterojen yapıdaki yatırımcılara ve politika yapıcılara daha etkin kararlar alınması için bu çalışmada hem standart hem de frekans bazlı metotlar tercih edilmiştir. Elde edilen bulgular, dalgacıklar metodunun diğerlerine göre daha sağlıklı ve tutarlı sonuçlar verdiğini göstermektedir.

Analizin ilk aşamasında hem standart hem de yapısal kırılmaları dikkate alan ADF, PP, KPSS ve Lee-Strazizich birim kök ve Johansen ve Hatemi-J eşbütünleşme testleri kullanılmıştır. Standart birim kök test sonuçlarına göre tüm değişkenler logaritmik düzeyde birim köklü, birinci farkta ise durağandır. Yapısal kırılmalı Lee-Strazizich (2003) testine göre ise 6 değişkenin logaritmik düzeyde durağan olduğu görülmektedir. Birinci farkta durağan değişkenler arasında yapılan Johansen eşbütünleşme (1990) testine göre tahvil faizi ile 20 borsa endeksi istatistiksel düzeyde anlamlı uzun dönem ilişkisine sahip iken, yapısal kırılmaları göz önüne alınca anlamlı ilişki sayısı oldukça azalmaktadır. Hatemi-J (2008) eşbütünleşme test sonuçları dikkate alınarak yapılan VECM analizinde, tahvil faizinden beş hisse endeks fiyatına ("LXUMAL", "LXHOLD", "LXTEKS", "LXTRZM" ve "LXULAS") doğru uzun dönemde geçerli nedensellik ilişkisi bulunurken, faiz oranının "LXHOLD" değişkeninin kısa dönemde de Granger nedeni olduğu tespit edilmiştir. Eşbütünleşme ilişkisine sahip olmayan değişkenler dikkate alınarak yapılan VAR analizine göre sadece üç adet anlamlı Granger nedensellik ilişkisi ("DL_TR2YGB" ⇒ "DL_XELKT", ve "DL_XUMAL" ⇒ "DL_TR2YGB", "DL_XHOLD" ⇒ "DL_TR2YGB") saptanmıştır. Ancak, daha önce belirtildiği üzere, standart modellerin değişkenler arasındaki gerçek ilişkiyi saptamakta yetersiz kaldıklarını ve piyasada farklı yatırım vadelerine sahip katılımcılarının ihtiyaçlarının da dikkate alınması gerektiğini göz önüne alarak orijinal getiri değişkenleri dalgacık dönüşüm metodu ile ölçeklerine ayrılmıştır. Elde edilen anlamlı ilişkinin hangi zaman periyotlarında, yani frekans boyutlarında, geçerli olduğunu ya da standart yöntemlerin sunduğu anlamsız sonuçların gerçekte herkes için geçerli olup olmadığını, eğer geçerli değilse hangi tür yatırımcı ve/veya politika yapıcılarının ihtiyacını giderebildiğini görmek için dalgacık ölçeklerine nedensellik testi uygulanmıştır. VAR model test sonuçlarına göre hem orijinal serilerde geçerli nedensellik ilişkisinin hangi zaman ölçeklerinde yoğunlaştığını, hem de standart metoda göre anlamsız ilişkinin hangi frekans aralığında anlam kazandığını ortaya konmuştur. Buna göre faiz değişmeleri ile yukarıda adı belirtilen endeks getirileri ve diğer değişkenlerin getirileri arasında orta ve uzun dönemde (ölçek 3, 4 ve 5) çift taraflı nedensellik ilişkisi bulunmuştur. Diğer taraftan frekans bazlı nedensellik test sonucuna göre sadece iki değişken ("DL_XSGRT" ve "DL_XHOLD") ile "DL_TR2YGB" arasında tüm frekans noktalarında geçerli çift taraflı, bazı endeks getirileri ve faiz değişmeleri arasında ise tek taraflı nedensellik sonucu elde edilmiştir. Bu sonuçlar, beklenildiği gibi, dalgacıklar metodunun diğer yöntemlere daha avantajlı sonuçlar verdiğini ortaya koymaktadır.

Değişkenler arasındaki nedensellik ilişkisinin yanı sıra ölçek bazlı dalgacık varyansı ve korelasyonu da analiz edilmiştir. Literatürde elde edilen sonuçlara benzer şekilde ölçek düzeyi arttıkça tüm endeks ve faiz getirisindeki volatilite azalmaktadır. Ayrıca, tahvil borsasının hisse senedi borsasından daha az oynaklığa sahip olduğu görülmektedir. Bu sonuç, kısa vadeli piyasa katılımcıları yaşanan oynaklığa karşı tedbir alması gerektirirken, uzun vadelilerin fazla endişe etmesine gerek olmadığını ifade etmektedir. Diğer bir deyişle, bu sonuç kısa vadeli yatırımcıların piyasadaki kısa dönemli gelişmelere göre hareket ettiğini, uzun vadeli yatırımcıların ise faiz oranı, enflasyon ve döviz kuru gibi piyasa dinamiklerine göre aksiyon aldığını teyit etmektedir. Literatürdeki temel beklentiye paralel olarak faiz oranı değişmeleri ile endeks getirileri arasında zıt yönlü korelasyon ilişkisi tespit edilmiştir. İki değişkenin hem orijinal seride hem de ölçek bazında istatistiksel olarak çok güçlü ancak ters yönde hareket ettiği görülmektedir. İki değişken hariç ("DL_XILTM" ve "DL_XUHIZ"), anlamlı ilişki sadece ilk dört ölçekte (2-32 haftalık periyot) geçerli olmaktadır. Tüm bu sonuçlar, iki yatırım enstrümanına birbirinin alternatifi olarak portföyde yer verilmesi gerektiğini, yani, borsa düşünce faiz enstrümanın güvenli liman olarak görülmesi gerektiğini ortaya koymaktadır.