AN APPLICATION OF PALLET LOADING PROBLEM

Özlem AKMAN

Thesis Advisor: Asst.Prof.Dr.Adalet ÖNER

Department of Industrial Management and Information Systems

Bornova, İZMİR

2013

YASAR UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCE

AN APPLICATION OF PALLET LOADING PROBLEM

Özlem AKMAN

Thesis Advisor: Adalet ÖNER

Department of Industrial Management and Information Systems

Bornova, İZMİR

2013

This study titled “An Application of Pallet Loading Problem” and presented as Master’s Thesis by Özlem AKMAN has been evaluated in compliance with the relevant provisions of Y.U Graduate Education and Training Regulation and Y.U Institute of Science Education and Training Direction and jury members written below have decided for the defense of this thesis and it has been declared by consensus / majority of votes that the candidate has succeeded in thesis defense examination dated September 27, 2013.

Jury Members:	**Signature:**
Head:	…… ……...
Rapporteur Member:	……. ……
Member:	……. ……

ÖZET

PALET YÜKLEME PROBLEMİNİN BİR UYGULAMASI

AKMAN, Özlem

Yüksek Lisans Tezi, Endüstriyel Yönetim ve Bilişim Sistemleri
Tez Danışmanı: Asst. Prof. Adalet ÖNER

Eylül 2013, 34 sayfa

Kutuların palet üzerinde etkili biçimde yerleştirilmesi (istiflenmesi) problemi lojistikte pratik olarak büyük öneme sahiptir. Bu işlem aslında üç boyutlu bir yerleştirme problemidir. Bununla birlikte, pratikte kutular paletin kenarlarına paralel bir biçimde (orthogonal) yerleştirilir ve ayrıca paletin tabanında yapılan yerleşim ondan sonraki katlarda aynen tekrarlanır. Bu nedenle problem iki boyutlu yerleştirme problemine dönüşür. Diğer bir ifadeyle aynı uzunluk ve genişliğe sahip çok sayıda dikdörtgenin (kutuların) daha büyük bir dikdörtgenin (paletin) içine yerleştirilmesi problemine dönüşür ve literatürde Palet Yükleme Problemi (The Pallet Loading Problem (PLP)) olarak adlandırılır. Bu tezde palet yükleme problemi çalışılmıştır. İzmir'de bir kağıt ürünleri fabrikasında ortaya çıkan gerçek hayat problemidir. Çalışmanın hedefi, kutuların paletlere istiflenmesi işleminin etkinliğini arttırmaktır. Her biri farklı boyutlarda 95 ayrı ürün çeşidine ait kutuların farklı boyutlardaki paletlerin içine nasıl yerleştirileceği çalışıldı. Kullanılabilecek 6 farklı palet tipinin boyutları önceden tanımlanmıştır. Problem, her bir ürün çeşidine en uygun palet tipinin atanması ve belirlenen palet tipinde en iyi yerleşim planının belirlenmesidir. İki matematiksel model ve bir sezgisel yöntem incelenerek her bir ürün çeşidi için problem ayrı ayrı çözüldü. Yerleştirme etkinliği belirli oranda artırıldı. Sonuçlar ve tavsiyeler fabrikaya rapor edildi.

Anahtar Sözcükler: Palet yükleme problemi, depo yönetimi, optimizasyon

ABSTRACT

AN APPLICATION OF PALLET LOADING PROBLEM

AKMAN, Özlem

Master's Thesis, Department of Industrial Management and Information System
Supervisor: Asst.Prof. Adalet ÖNER

September 2013, 34 pages

The problem of packing boxes efficiently on a pallet has great practical importance in logistics. Stacking boxes on the pallets imposes a three-dimensional space utilization problem. However practical considerations suppose that the boxes must be placed orthogonally with respect to the edges of the pallet, and in layers in which the vertical orientation of the boxes is fixed. Therefore the problem is reduced to a two-dimensional one such that it becomes the problem of finding the best loading pattern of boxes on a pallet. In other words, it is the problem of filling a large rectangle, a pallet, with the maximum number of small identical rectangles, boxes. Formally it is called "The Pallet Loading Problem (PLP)". An application of PLP has been studied in this thesis. It emerged from a real life problem in a pulp factory in Izmir, Turkey. The goal of the study is to increase the efficiency of packing boxes on the pallets. There are 95 product types and the dimensions of the box which contains that product are fixed. Each product box has different dimensions. There are 6 distinct pallet types available to use and each of them has different dimensions. It is required first to assign the best pallet type for each product type, and then to find the best loading plan in order to maximize the number of boxes stowed on given pallet. Two mathematical models and a heuristics has been surveyed and used to solve the problem. A significant improvement has been achieved and recommendations have been reported to the factory.

Keywords: Pallet loading problem, warehouse management, optimization

ACKNOWLEDGEMENTS

This graduation project would not have been possible without the support of many people. The author wishes to express her gratitude to him supervisor, Asst. Prof. Adalet Öner, who was abundantly helpful and offered invaluable assistance, support and guidance.

Deepest gratitude are also due to the members of the supervisory committee, Prof. Dr. M.Fatih Taşgetiren, Doç. Dr. Önder Bulut without whose knowledge and assistance this study would not have been successful.

Special thanks also to all him graduate friends for sharing the literature and invaluable assistance.

The author wishes to express him love and gratitude to him beloved families; for their understanding & endless love, through the duration of him studies.

Table of Contents

Page

Table of Contents (continued)

APPENDICIES

LIST OF FIGURES

FIGURE PAGE

LIST OF TABLES

TEXT OF OATH

I declare and honestly confirm that my study titled “Pallet Loading Problem”, and presented as Master’s Thesis has been written without applying to any assistance inconsistent with scientific ethics and traditions and all sources I have benefited from are listed in bibliography and I have benefited from these sources by means of making references.

.. / .. / 20…

Özlem AKMAN

Signature

1. INTRODUCTION

Viking Pulp & Paper Mill Company was founded in 1969 in İzmir. The company imports pulp as raw material and produce its products under “Lily” and “Senso” brands with the product range including toilet paper, towel, napkins etc. It produces 95 different types of paper products for domestic and export markets.

Finished products are packed in batches first. The batches are then piled up onto some pallets and finally pallets are moved to some warehouses where they are stockpiled temporarily before they are shipped to customers. The scope this study is choosing proper pallet size for each product type.

When the products leave the manufacturing facilities, they are transferred to the warehouses before shipping to customers. There are 4 warehouses to accommodate the products. Basic flow of goods is shown in Figure 1.1.

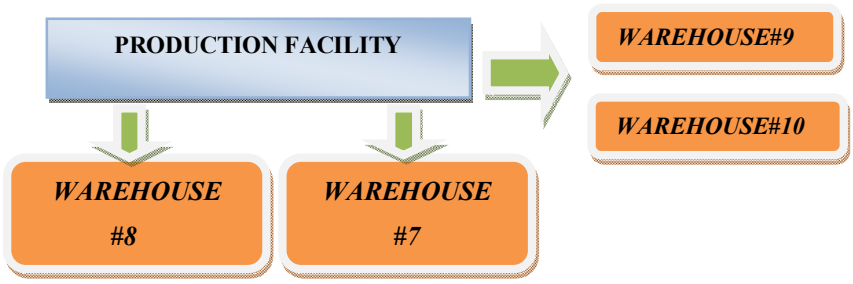


Figure 1.1. The Basic Layout and Flow of Goods

Warehouse #7 and # 8 are bigger then compared to Warehouse # 9 and #10. Moreover, they have 5 loading docks each whereas warehouse # 9 and # 10 have only one loading dock in total for shipping of goods to the customer. The dimensions and capacities of four warehouses are given in Table 1.1.

Table 1.1. Warehouse dimensions & capacities

	Area (square meter)	Length (mm)	Width (mm)	Height (mm)	Capacity (Boxes)
Warehouse # 7	1600	73560	22460	9000	100.000
Warehouse #8	1600	73560	22460	9000	100.000
Warehouse #9	700	16500	10240	5000	20.000
Warehouse #10	1200	22500	10240	5000	30.000

There are two fundamental groups of products with respect to packaging process. Some products are first put inside a covered box (cartons) whereas some others are packaged simply into nylon bags. We define the product types as Class-I and Class-II respectively depending on the packaging mode. This classification is important since each box or nylon bag is considered to be particular stocking units when they are put on the pallets. Based on 2012 data, there are 26 product types out of 95 to be in Class-I. Four sample products of this type and the details about their boxes are given in Table 1.2. The complete list of product types in Class-I is given in Appendix-1.

Table 1.2. Box dimensions of sample products from class-I

Product Code	Product	Dimensions of Box			Number of products in this box
		Width (mm)	Length (mm)	Height (mm)	
153603055	SensoEko.Peçete	330	540	250	32
153604967	BlumePeçete	340	370	540	48
153602256	Lily Extra Peçete	200	460	305	24
153605126	MorissonBeyazPeçete	410	600	463	36

Products which are not placed in the boxes are packaged in batches into nylon bags before stacking on pallets. Each batch has different number of products depending on the type of the product. Each batch has different number of products depending on the type of the product. Four sample products of this type

and the details about their bags are given in Table 1.3. The complete list of product types in Class-II is given in Appendix-2.

Table 1.3. Bag dimensions of sample products from class-II

Product Code	Product	Dimensions of Bag			Number of products in this bag
		Width (mm)	Length (mm)	Height (mm)	
153606057	Blume 12	420	690	200	4
153605946	Senso Bio	380	600	288	3
153604875	Queen 3 Katlı	420	650	200	6
153605107	SensoÇift Kat	400	560	192	4

The company produces finished goods for domestic and export markets. The volumes of sales for the year of 2012 are summarized in Table 1.4.

Table 1.4. Sales volumes of year 2012

(in terms of boxes or bags)	January	February	November	December
Domestic market	459.959	479.198	462.907	476.507
Export Market	20.000	22.000	26.000	26.000
Total	479.959	501.198	488.907	502.507

2. PROBLEM DEFINITION

Finished goods flow in the warehouses, stored for a while and then shipped to customers. Total number of boxes & batches in the warehouses fluctuate during the year. Inventory management faces frequently with the problem of insufficient space to store the products. It is a storage crisis and the management is forced either to store the finished goods in an open area or to rent some warehouses outside the factory when there is not space enough to store all finished goods. Furthermore, the production schedule is interrupted as a side- effect for the sake of not escalating the crisis. It is due to two main reasons. The first one is the long storage durations between production and shipping dates. The second reason is the inefficient utilization of storage space available.

The storage durations of finished goods are not in the scope of this study. Our basic concern is to choose the best size of pallet for each product type and therefore increasing the utilization of storage space and improve its efficiency.

Utilization of spaces in warehouses is not high because of the procedures that determine how finished goods are stored. Some products are first put inside a covered box whereas some others are packaged in batches into nylon bags. The dimensions of boxes or bags are not decision variables. They are determined earlier by sales department. The boxes or bags are then stacked on some wooden pallets. There are three standard types of pallets each having a square shape. The lengths of one side of pallets are 1240 mm, 1440 mm and 1540 mm respectively. When the boxes (or bags) are stacked onto a pallet, the height of the stack is not desired to exceed 2000 mm, otherwise it is hard to maintain the balance of the stack.

Stacking items on the pallets imposes a three-dimensional space utilization problem. However practical considerations suppose that the boxes must be placed orthogonally with respect to the edges of the pallet, and in layers in which the vertical orientation of the boxes is fixed. Therefore the problem is reduced to a two-dimensional one such that it becomes the problem of finding the best loading pattern of items on a pallet. In other words, it is the problem of filling a large

rectangle, a pallet, with the maximum number of small identical rectangles, boxes. Formally it is called “The Pallet Loading Problem (PLP)”

Currently, the company uses a predetermined standard pallet type for each product type. The following two tables show the pallet types used for some products. The first table includes a sample set of products from Class-I, and the second table shows a sample set of products from Class-II. The complete list of product-pallet mapping is given in Appendix 3.

Table 2.1 Product-pallet mapping for class-I products

Product Code	Name	Number of boxes in this pallet	1240x1240 (mm)	1440x1440 (mm)	1540x1540 (mm)
153603055	SensoEko.Peçete	56	**X**		
153604967	BlumePeçete	48		**X**	
153602256	Lily Extra Peçete	72	**X**		
153605126	Morisson Byz.Pç	24			**X**

Table 2.2. Product-pallet mapping for class-II products

Product Code	Name	Number of bags in this pallet	1240x1240 (mm)	1440x1440 (mm)	1540x1540 (mm)
153606057	Blume 12	88			**X**
153605946	Senso Bio 24	56	**X**		
153604875	Queen 3 Katlı 8	80			**X**
153605107	SensoÇift Kat 12	88		**X**	

This study is divided into two phases. The first phase is to assess the effectiveness of current loading patterns on predetermined pallet size for each product type. The goal is to investigate whether it is possible to increase the space utilization of pallets by changing loading patterns. Figure 2.1 shows how different loading patterns change the utilization of space on a particular pallet.

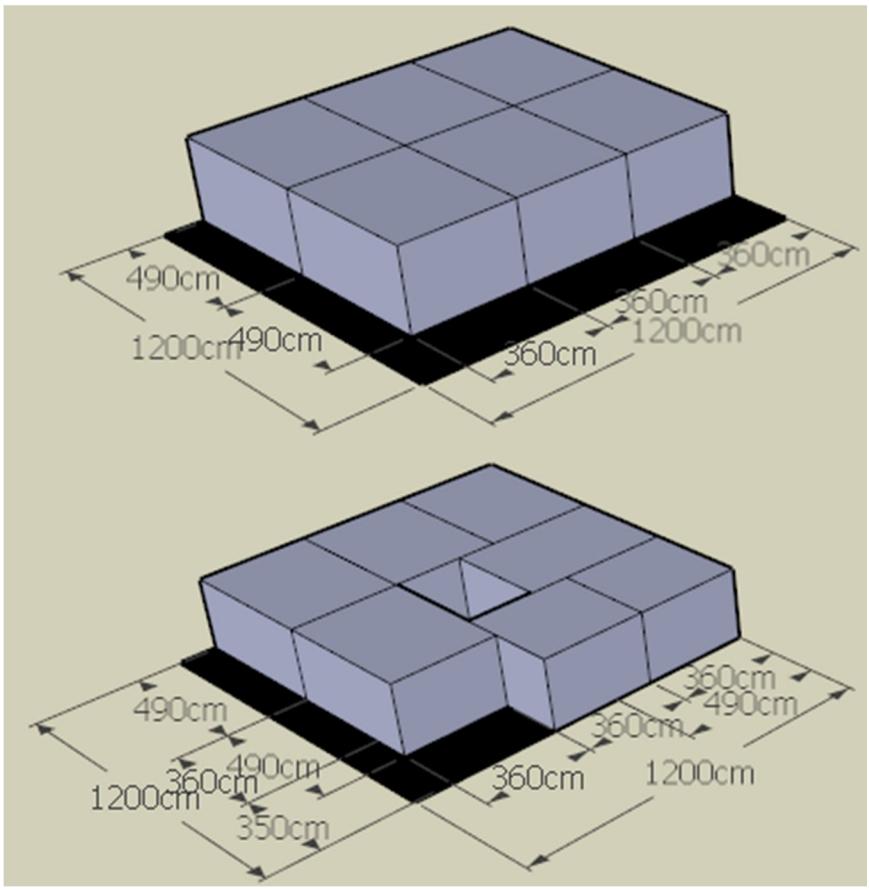


Figure 2.1. Different Pallet Loading Plan

There are two different loading patterns in Figure 2.1 above. It is possible to put 7 boxes on the pallet if we use the loading pattern which is seen at the bottom side in the figure. On the other hand, only 6 boxes can be loaded on the same pallet if we use a different pattern which is seen at the upper side in the Figure 2.1.

The second phase of the study is to identify the best pallet type for each product type. It may be more efficient if we use a different pallet type other than the predetermined one. Since the dimensions of the boxes (or bags) of products are known, the question is to determine the optimal pallet type for each product type. If the selection is not appropriate, you will end up more unused space. The goal is to minimize the unused space in palletization process. Figure 2.2 shows the unused space when the boxes (or bags) are loaded onto different types of pallet.

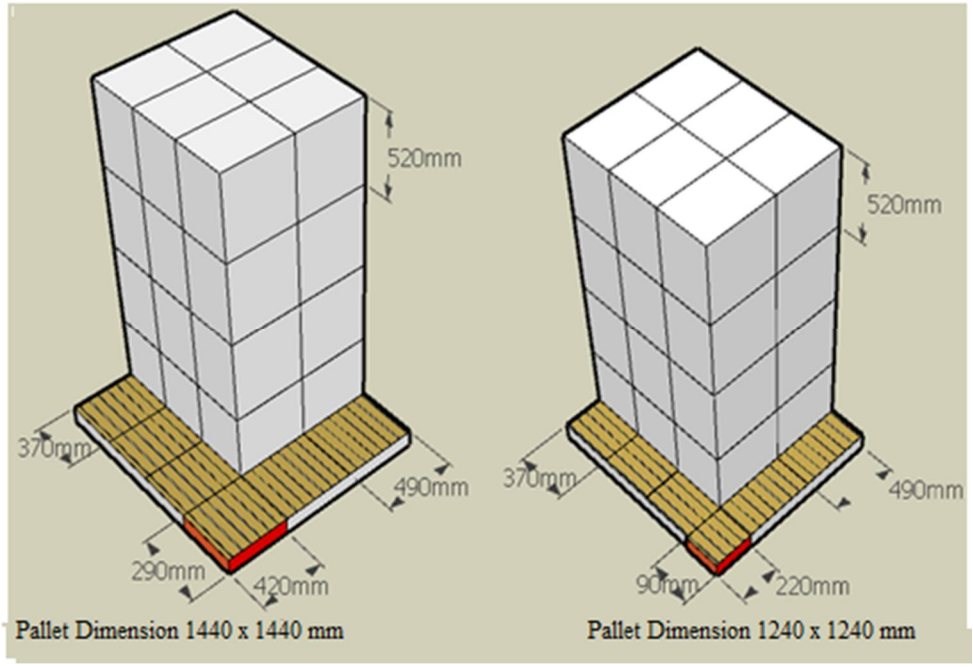


Figure 2.2. Wasted Space on Different Pallet Sizes

Furthermore, second phase of the study also includes investigating whether we can use new pallet dimensions which are not used before. The pallets with new different dimensions may increase space utilization. However, new options are limited by some technological constraints. A new list of pallet dimensions has been developed by discussing with the company. The complete list of available pallet dimensions is given in the following table.

Table 2.3. Dimensions and status of pallet types

Pallet Dimensions (mm)	Status
1240*1240	In Use
1440*1440	In Use
1540*1540	In Use
1240*1340	New Option
1240*1440	New Option
1240*1540	New Option

.

3. LITERATURE REVIEW

The problem of packing boxes (or bags) efficiently on a pallet is of great practical importance in logistics. If it is possible to increase the number of items on a pallet, the utilization of the warehouse is increased and transportation costs tend to decrease. Therefore the Pallet Loading Problem (PLP) has attracted a lot of research in the past, and numerous studies have been published.

The studies on PLP can be divided into two main subgroups which include mathematical models and heuristic methods. There are many heuristic methods proposed for this problem whereas there are only few mathematical models based on mixed integer programming models.

Constructive heuristic methods focus on the "large rectangle (pallet)" and divide it into blocks. The "small rectangles (boxes)" are then put into those blocks (Smith and De Cani, 1980; Steudel, 1979). Nelissen(1993) reviews one-, two-, four-, and five-block heuristics. The one-block heuristic establishes all boxes (or bags) with the same direction (orientation). A constant-time computation determines the best direction. All n-block heuristics select a dimension for the first block. This is followed by for the second block through the last block is placed.

A special block structure, G-4, is defined by Scheithauer and Terno (1996). Another interesting block structure is proposed by Lins et al. (2003) based on L shaped structures.

Another block constructing approach is to use guillotine and non-guillotine cuts on the pallet area. A guillotine cut divides area in two smaller pieces with a cut from one side to the other (Gilmore and Gomory, 1965; Christofides and Whitlock, 1977; Wang, 1983; Oliveira and Ferreira, 1990). Beasley (1985) proposes an exact method based on non-guillotine cuts.

There are meta-heuristics based on tabu search, genetic algorithms and strategic oscillation (Amaral and Wright, 2001; Dowsland, 1996; Herbert and

Dowsland, 1996). In addition several upper bounds have been recommended (Nelissen, 1995; Dowsland, 1984, 1985; Letchford and Amaral, 2001), which regard the geometric structure of the problem and linearly relaxed integer programming formulations.

The exact algorithms are fundamentally tree search procedures that for at each node a partial layout of boxes on the pallet has been built. Different algorithms are defined by different ways of adding boxes, extending the partial solution, and different bounding procedures (De Cani, 1979; Iserman.1987; Exeler, 1988; Bhattacharya et al., 1988; Beasley, 1985). Alvarez-Valdes et al (2005) proposes an efficient exact algorithm based on branch and cut, to solve larger problems, of up to 100 boxes.

Mixed integer programming models proposed by Chen (1995) and Beasley (1985) are two of the few mathematical models that appeared in literature. Actually, Chen proposed a three dimensional model to be used for "Container Loading Problem". However it can be modified to be used for our "Pallet Loading Problem". Both models are reviewed in detail in following sections.

The computational complexity of the PLP is still unknown. Its NP-*completeness* has not yet been proven and is doubtful, as Nelissen (1995) and Letchford and Amaral (2001) point out. The size and the difficulty of an instance of the PLP depend on the ratio between the dimension of the pallet and the dimension of the box.

4. ANALYSIS

In this section, some selected mathematical models and heuristics are reviewed in detail. Actual instances of pallet loading problems in the factory will be solved using with selected methods.

4.1 Mathematical Models

Two specific mixed integer programming models are chosen which are developed by (Chen et al.,1995) and Beasley (1985) respectively.

4.1.1 Chen's Model

The model developed by (Chen et al., 1995) is actually for three dimensional truck (container) loading problem (CLP). However, it can be modified to be used for two dimensional pallet loading problem (2-D PLP). Modified version is described below.

A special two dimensional coordinate system is defined on which the pallet is placed such that its left top corner represents the origin. When a box is stowed on the pallet, its location is defined by the coordinates of left top corner of the box. The basic structure of the coordinate system is given in Figure 4.1.

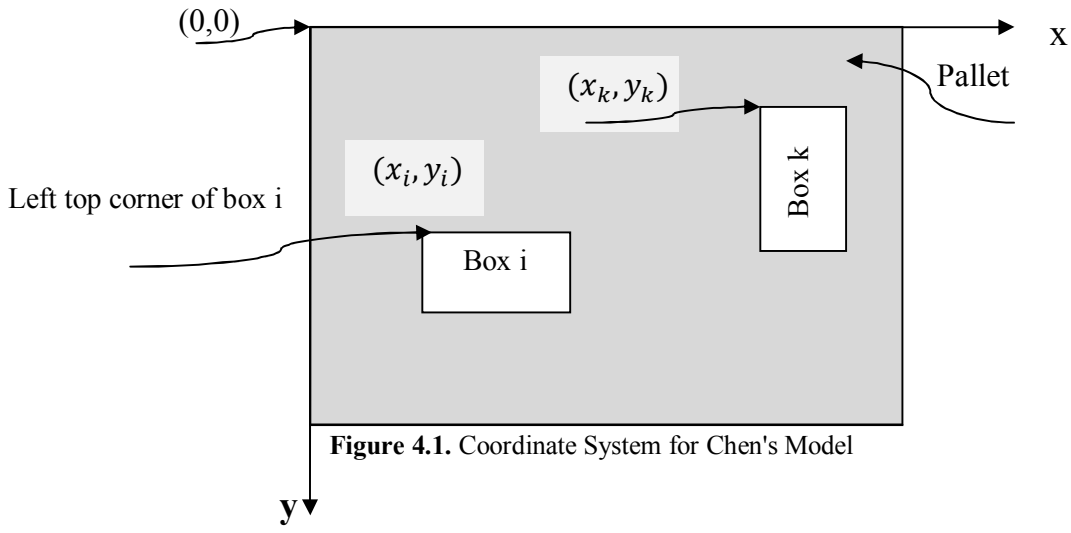


Figure 4.1. Coordinate System for Chen's Model

The objective is to minimize the unused area when boxes (batches) are stowed onto the pallets. The idea is to decide on the relative position of box pairs (say *i* and *k*), where *i* and *k* are the index sets over the boxes to be stowed. The relative positions define that the box *i* is left, right, behind or in front of box *k*. For example, box *i* is on the left side of box *k* in Figure 4.1.

The orientation of a box is also important and it is specified by determining which axis of pallet is parallel to the length and width of a box. For example, length of box *i* is parallel to X- Axis and its width is parallel to Y- Axis. On the other hand, the orientation of box *k* is just vice-versa as of box *i* in Figure 4.1.

Notation in Chen's Model

N	Total number of boxes to be stowed
M	An arbitrarily large number
s_i	A binary variable is equal to1 if box number *i* is placed on the pallet otherwise it is equal to 0.
(p_i, q_i)	Parameters indicating the length and width of box *i*.
(L, W)	Parameters indicating the length and width of the pallet
(x_i, y_i)	Continuous variables indicating the coordinates of the left bottom (LB) corner of box i.
(l_{xi}, l_{yi})	Binary variables indicating whether the length of box *i* is parallel to *X*- or *Y*- Axis. For example, the value of l_{xi} is equal to 1 if the length of box *i* is parallel to *X*- axis; otherwise it is equal to 0.
(w_{xi}, w_{yi})	Binary variables indicating whether the width of box *i* is parallel to *X*- *or Y*- Axis. For example, the value of w_{xi} equals to 1 if the width of box *i* is parallel to *X*- axis; otherwise it is equal to 0.

$(a_{ik}, b_{ik}, c_{ik}, d_{ik})$Binary variables indicating whether the placement of boxes relative to each other.

a_{ik} The a_{ik} is equal to 1 if box *i* is on the left side of box *k*

b_{ik} The b_{ik} is equal to 1 if box *i* is on the right side of box *k*

c_{ik} The c_{ik} is equal to 1 if box *i* is behind box *k*

d_{ik} The d_{ik} is equal to 1 if box *i* is in front of box *k*

The mathematical model is given in Figure 4.2.

Minimize $\sum_{j=1}^{m} L_j * W_j - \sum_{i=1}^{N} p_i * q_i * s_i$

$x_i + p_i * l_{xi} + q_i * w_{xi} \leq x_k + (1 - a_{ik}) * M \quad \forall\, i,k \quad i<k\,, \quad (1)$

$x_k + p_k * l_{xk} + q_k * w_{xk} \leq x_i + (1 - b_{ik}) * M \quad \forall\, i,k \quad i<k\,, \quad (2)$

$y_i + q_i * w_{yi} + p_i * l_{yi} \leq y_k + (1 - c_{ik}) * M \quad \forall\, i,k \quad i<k\,, \quad (3)$

$y_k + q_k * w_{yk} + p_k * l_{yk} \leq y_i + (1 - d_{ik}) * M \quad \forall\, i,k \quad i<k\,, \quad (4)$

$a_{ik} + b_{ik} + c_{ik} + d_{ik} \geq s_i + s_k - 1 \quad \forall\, i,k \quad i<k\,, \quad (5)$

$x_i + p_i * l_{xi} + q_i * w_{xi} \leq L + (1 - s_i) * M \quad \forall\, i \quad (6)$

$y_i + q_i * w_{yi} + p_i * l_{yi} \leq W + (1 - s_i) * M \quad \forall\, i \quad (7)$

$l_{xi}, l_{yi}, w_{xi}, w_{yi,} a_{ik}, b_{ik}, c_{ik}, d_{ik}, s_i = 0 \; or \; 1 \quad \forall\, i,k$

Figure 4.2. Chen's Model

$$x_i, y_i \geq 0 \qquad \forall\, i$$

$$l_{xi} + l_{yi} = s(i), \qquad \forall\, i \qquad (8)$$

$$w_{xi} + w_{yi} = s(i), \qquad \forall\, i \qquad (9)$$

$$l_{xi} + w_{xi} <= 1, \qquad \forall\, i \qquad (10)$$

$$l_{yi} + w_{yi} <= 1, \qquad \forall\, i \qquad (11)$$

Figure 4.2. Chen's Model (Continued)

The purpose of Chen's Model is to minimize the unused area on the pallet. Constraints (1) through (4) show locations and relationship between box *i* and box *k*. Constraints (6) and (7) determine that box *i* which is put into a pallet conforms the dimensions of the pallet. Constraints (8) through (11) provide that the dimensions of boxes *i* have to be parallel to only one of axis which is *X-axis and Y- axis* of the pallet.

The model is implemented in LINGO optimization software in order to solve the problem. A sample problem is chosen to show how the problem is solved by Chen's model. The problem is one the actual instances in the factory. The length (L) and width (W) of the pallet is 1240 mm. The length and width (p_i, q_i) of identical boxes are 540 mm and 330 mm respectively. The LINGO code is given in Figure 4.3.

```
MODEL:
SETS:
BOX/1..8/:S,X,Y,LX,LY,WX,WY,P,Q,R;
PAIRS(BOX,BOX): A,B,C,D;
ENDSETS
DATA:
M=1000000;
L W = 1240 1240;
P  Q  = 540 330 540 330 540 330 540 330 540 330 540 330 540 330 540 330;
ENDDATA
```

Figure 4.3. The LINGO Code of Chen's Model

```
MIN = L*W - @SUM(BOX: P*Q*S);

@FOR(PAIRS(I,K)|K #GT# I: X(I)+P(I)*LX(I)+Q(I)*WX(I) < X(K) + (1-A(I,K))*M);

@FOR(PAIRS(I,K)|K #GT# I: X(K)+P(K)*LX(K)+Q(K)*WX(K) < X(I) + (1-B(I,K))*M);

@FOR(PAIRS(I,K)|K #GT# I: Y(I)+Q(I)*WY(I)+P(I)*LY(I) < Y(K) + (1-C(I,K))*M);

@FOR(PAIRS(I,K)|K #GT# I: Y(K)+Q(K)*WY(K)+P(K)*LY(K) < Y(I) + (1-D(I,K))*M);

@FOR(PAIRS(I,K)|K #GT# I: A(I,K) + B(I,K)+ C(I,K)+ D(I,K) > S(I)+S(K) -1 );

@FOR(BOX(I): X(I)+P(I)*LX(I)+Q(I)*WX(I) < L+(1-S(I))*M);

@FOR(BOX(I): Y(I)+Q(I)*WY(I)+P(I)*LY(I) < W+(1-S(I))*M);

@FOR(BOX(I): LX(I)+WX(I) = S(I));

@FOR(BOX(I): WY(I)+LY(I) = S(I));

@FOR(BOX(I): LX(I)+LY(I) <= 1);

@FOR(BOX(I): WX(I)+WY(I) <= 1);

@FOR(PAIRS(I,K)|K #LE# I: A(I,K) =0);

@FOR(PAIRS(I,K)|K #LE# I: B(I,K) =0);

@FOR(PAIRS(I,K)|K #LE# I: C(I,K) =0);

@FOR(PAIRS(I,K)|K #LE# I: D(I,K) =0);

@FOR(PAIRS(I,K)|K #GT# I: @BIN(A(I,K)); @BIN(B(I,K)); @BIN(C(I,K)); @BIN(D(I,K)));

@FOR(BOX: @BIN (LX);@BIN (LY); @BIN (WX); @BIN (WY); @BIN (S) );

@FOR(BOX: @GIN (X);@GIN (Y));
END
```

Figure 4.3. The LINGO Code of Chen's Model (Continued)

The solution of the model proposes eight identical boxes can be stowed on the pallet. The orientation (either horizontal or vertical) and the coordinates of top left corners of the boxes are given below in Table 4.1.

Table 4.1. Solution of sample problem with modified Chen model

Variable	Value	Comment	Orientation
X(1)	700.0000		
X(2)	660.0000		
X(3)	910.0000		
X(4)	40.00000		
X(5)	330.0000		
X(6)	580.0000		
X(7)	0.000000		
X(8)	0.000000		
Y(1)	580.0000		
Y(2)	910.0000		
Y(3)	0.000000		
Y(4)	330.0000		
Y(5)	660.0000		
Y(6)	0.000000		
Y(7)	0.000000		
Y(8)	700.0000		
LX(1)	1.000000	length of box 1 is paralel to *X-axis*	Horizantal
LX(2)	1.000000	length of box 2 is parallel to *X-axis*	Horizantal
LY(3)	1.000000	length of box 3 is parallel to *Y-axis*	Vertical
LX(4)	1.000000	length of box 4 is parallel to *X-axis*	Horizantal
LY(5)	1.000000	length of box 5 is parallel to *Y-axis*	Vertical
LY(6)	1.000000	length of box 6 is parallel to *Y-axis*	Vertical
LX(7)	1.000000	length of box 7 is parallel to *X-axis*	Horizantal
LY(8)	1.000000	length of box 8 is parallel to *Y-axis*	Vertical

The loading plan (layout) created by the solution above is drawn and presented at Figure 4.4. On the other hand, a different layout is currently being used at the factory. Current layout is shown in Figure 4.5. It can be easily seen that mathematical model improves utilization of the pallet since it proposes a solution such that one more box can be stowed onto the same pallet.

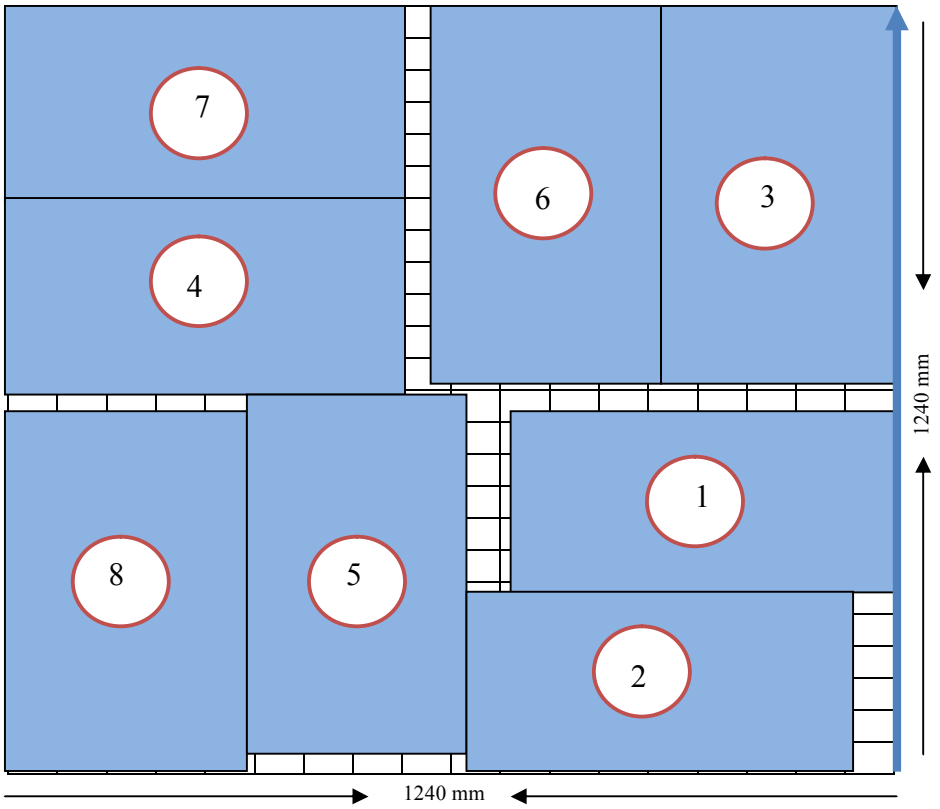


Figure.4.4. Loading Plan Imposed by Chen's Model (p=540, q=330)

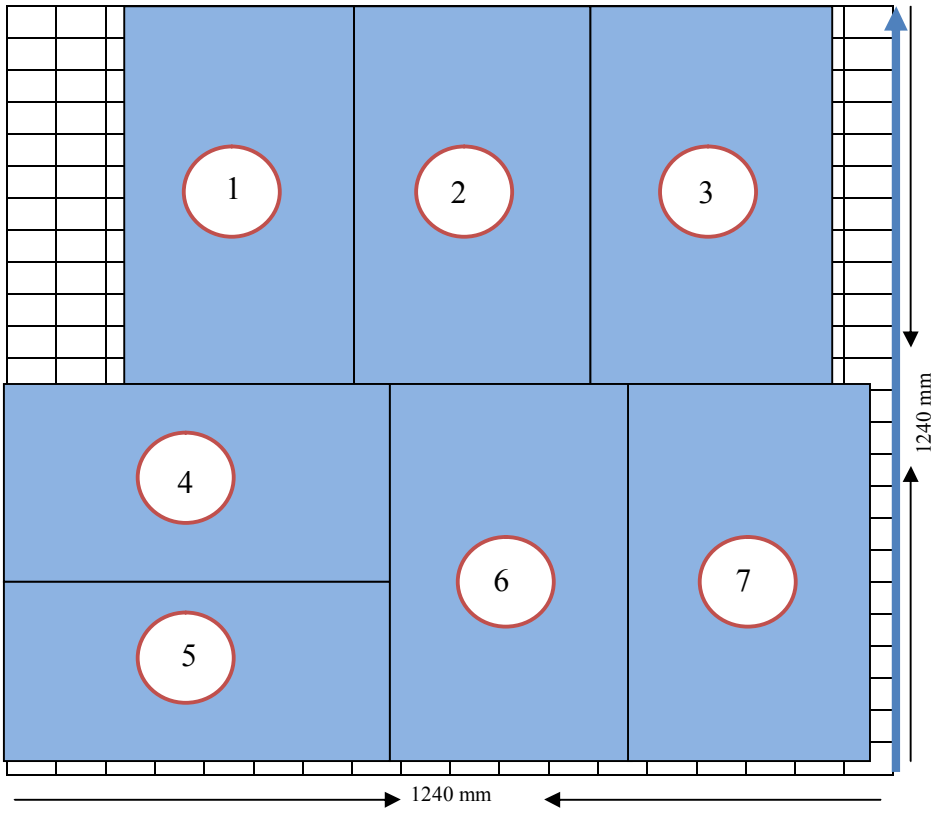


Figure 4.5. Current Loading Plan Used in Factory (p=540, q=330)

4.1.2 Beasley's Model

The Pallet Loading Problem (PLP) can also be formulated as a particular case of the two-dimensional non-guillotine cutting problem as shown in Beasley (1985).

A two dimensional coordinate system is defined such that left bottom corner of the pallet represents the origin. When a box is stowed on the pallet, its location is defined by the coordinates of left bottom corner of the box. The basic structure of the coordinate system is given in Figure 4.2.

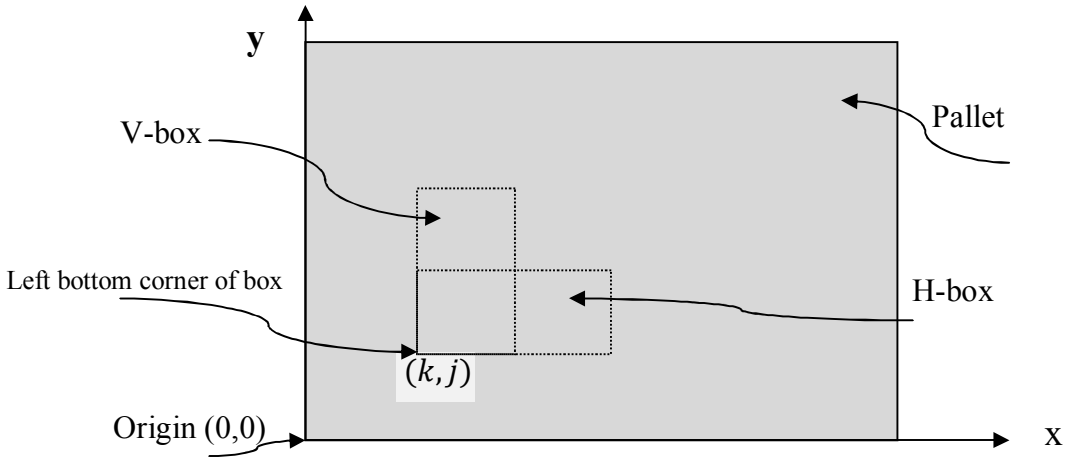


Figure 4.6. Coordinate System for Beasley's Model

Notation in Beasley's Model

$(a,\ b)$: Parameters indicating the length and width of box

(L, W) : Parameters indicating the length and width of the pallet

H-box : A horizontally oriented box

(a box is placed on the pallet such that its length is parallel to X-axis)

V-box : A vertically oriented box

(a box is placed on the pallet such that its length is parallel to Y-axis)

$$h_{kj} = \begin{cases} 1 & \text{if a H-box is placed in position (k,j)} \\ 0 & \text{otherwise} \end{cases}$$

$v_{kj}=$ $\begin{cases} 1 & \text{if a V-box is placed in position (k,j)} \\ 0 & \text{otherwise} \end{cases}$

k and j represent the position (coordinate) of the left bottom corner of a box with respect to the coordinate system.

The objective is to maximize the number of boxes to be stowed onto the pallet. The formulation is as follows:

$$max \sum_{k=0}^{L-a} \sum_{j=0}^{W-b} h_{kj} + \sum_{k=0}^{L-b} \sum_{j=0}^{W-a} v_{kj} \qquad (12)$$

subject to:

$$\sum_{k=\max\{0,r-a\}}^{\min\{r,L-a\}} \sum_{j=\max\{0,s-b\}}^{\min\{s,W-b\}} h_{kj} + \sum_{k=\max\{0,r-b\}}^{\min\{r,L-b\}} \sum_{j=\max\{0,s-a\}}^{\min\{s,W-a\}} v_{kj} \leq 1 \qquad (13)$$

$$(r = 0, \ldots . L-1; \qquad s = 0, \ldots ., W-1),$$

$$h_{kj} \in \{0,1\} \quad (0 \leq k \leq L-a; 0 \leq j \leq W-b) \qquad (14)$$

$$v_{kj} \in \{0,1\} \quad (0 \leq k \leq L-b; 0 \leq j \leq W-a) \qquad (15)$$

Constraints (13) are covering constraints avoiding box overlapping. For each pair (r,s) the constraint (13) guarantees that the corresponding unit square is covered by one box at most.

The number of constraints in this formulation is $(L - a) * (W - b)$, whereas the number of variables is $(L - a) * (W - b) + (L - b) * (W - a)$. If we consider a sample problem with L= 1240, W= 1240, a= 420 and b=220, the number of variables and constraints can be calculated as 1,672,800 and 836,400 respectively. It is clear that the problem needs quite large amount of computer memory in order to solve it. Furthermore please remember that all the variables are binary. We couldn't be able to solve the same sample problem using Beasley's model although we have a professional edition of LINGO software. However it is

possible to reduce the number of variables and constraints by taking a simple conversion: scale down all the dimensions by the same factor. For example, sample problem can be converted into a smaller one by scaling 1:20 of the original one. The new problem is now with L= 62, W= 62, a= 21 and b=11and the numbers of variables and constraints are reduced to 4,080 and 2,040 respectively. The model is implemented in LINGO optimization software and corresponding code is given in Figure 4.7.

```
SETS:
HORIZANTAL/1..62/;
VERTICAL/1..62/;
GRID(HORIZANTAL, VERTICAL) : H, V;
ENDSETS
DATA:
L=63;
W=63;
A=21;
B=11;
ENDDATA
! Objective function given in  (12) above;
MAX =  @SUM (HORIZANTAL(K) | K #LE# L-A :
            @SUM ( VERTICAL(J) | J #LE# W-B : H(K,J))) +
            @SUM (HORIZANTAL(K) | K #LE# L-B :
            @SUM ( VERTICAL(J) | J #LE# W-A : V(K,J))) ;

! Constraints given in (13) above ;
@FOR(HORIZANTAL(R):  @FOR (VERTICAL(S) :
@SUM(HORIZANTAL(K)| (K #GE# ( @IF((R-A) #GE# 1, R-A, 1))) #AND# (K #LE#
(@IF((L-A) #GE# R-1, R-1, L-A))) :
  @SUM(VERTICAL(J)| (J #GE# (@IF((S-B) #GE# 1, S-B, 1))) #AND# (J #LE# (@IF((W-B)
#GE# S-1, S-1, W-B))) : H(K,J))) +
@SUM (HORIZANTAL(K)| (K #GE# (@IF ((R-B) #GE# 1, R-B, 1))) #AND# (K #LE# ( @IF
((L-B) #GE# R-1, R-1, L-B))) :
@SUM (VERTICAL(J)| (J #GE# (@IF((S-A) #GE# 1, S-A, 1))) #AND# (J #LE# (@IF((W-A)
#GE# S-1, S-1, W-A))) : V(K,J))) <= 1));

! Binary constraints given in (14) and (15) above;
@FOR (HORIZANTAL (K) | K #LE# L-A : @FOR (VERTICAL ( J) | J #LE# W-B :
        @BIN( H(K,J))));

@FOR (HORIZANTAL(K) | K #LE# L-B :@FOR (VERTICAL(J) | J #LE# W-A :

      @BIN( V(K,J))));
```

Figure 4.7. The LINGO Code of Beasley's Model

The formulation delivers a solution such that 12 boxes can be placed on the pallet. The details of the solution are given below in Table 4.2, and corresponding loading plan (layout) is presented in Figure 4.8.

Table 4.2. Solution of sample problem with Beasley's model

Variable	Value	Description
H(1,1)	1.000000	A box is placed horizontally with its left bottom corner is at (1,1)
H(1,12)	1.000000	A box is placed horizontally with its left bottom corner is at (1,12)
H(1,23)	1.000000	A box is placed horizontally with its left bottom corner is at (1,23)
H(1,34)	1.000000	A box is placed horizontally with its left bottom corner is at (1,34)
H(1,45)	1.000000	A box is placed horizontally with its left bottom corner is at (1,45)
H(22,1)	1.000000	A box is placed horizontally with its left bottom corner is at (22,1)
H(22,12)	1.000000	A box is placed horizontally with its left bottom corner is at (22,12)
H(22,23)	1.000000	A box is placed horizontally with its left bottom corner is at (22,23)
H(22,34)	1.000000	A box is placed horizontally with its left bottom corner is at (22,34)
H(22,45)	1.000000	A box is placed horizontally with its left bottom corner is at (22,45)
V(43,1)	1.000000	A box is placed vertically with its left bottom corner is at (43,1)
V(43,22)	1.000000	A box is placed vertically with its left bottom corner is at (43,22)

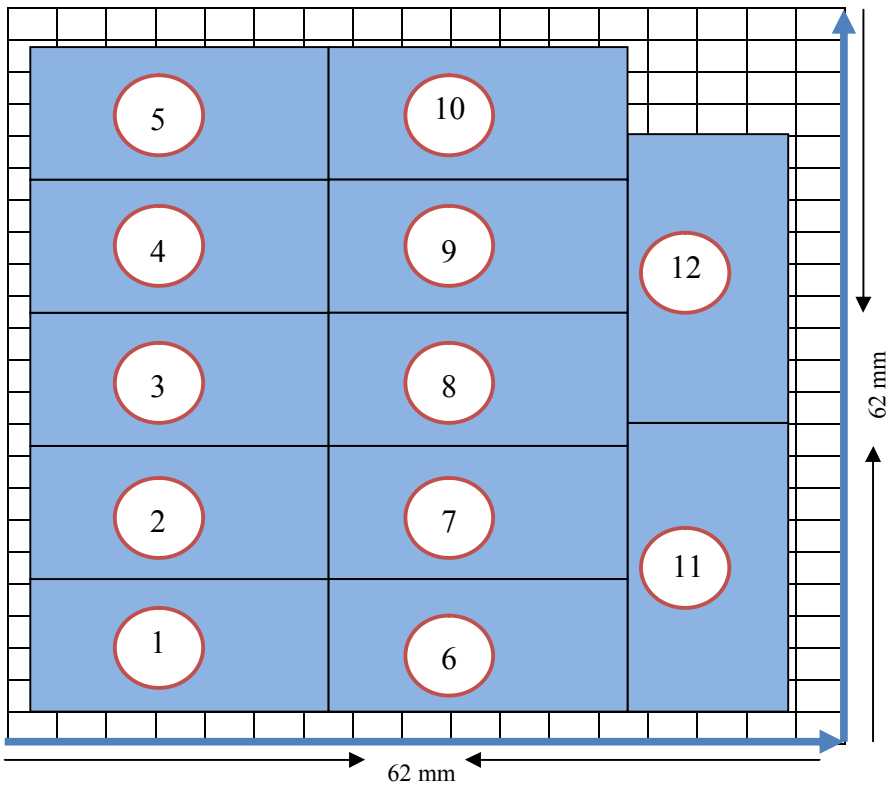


Figure 4.8. Loading Plan Imposed by Beasley's Model (a=21, b=11)

4.2 Bottom-Left Algorithm

In bottom-left algorithm, left bottom corner of the pallet should be designed as the origin of the xy-plane. There are three variables (x_i, y_i, v_i) for describe a placement of box i (i=1,2,...,n) in the corner. The coordinate of bottom-left corner point are $x_i, y_i \in R$ and $v_i \in \{0,1\}$ denotes its orientation, $v_i = 1$ means it is vertically placed, $v_i = 0$ otherwise.

Vector of 3n elements: $X = (x_1, y_1, v_1, x_2, y_2, v_2, \ldots, x_n, y_n, v_n)$ describe loading pattern of n boxes. Each box must be orthogonally loaded into the pallet and must not overstep each border of the pallet. The overlapping area between two boxes must be zero.

In a feasible loading, a box is bottom-left stable if and only if it can not move downwards or leftwards without overlapping others. A feasible loading is bottom-left stable just each box in this loading is bottom-left stable. See Figure 4.9, each box in the depicted loading has bottom-left stability and the loading is bottom-left stable.

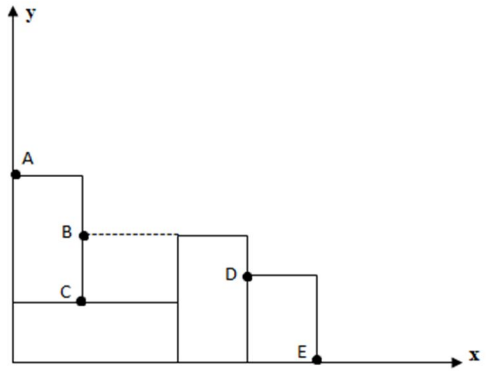


Figure 4.9. Bottom-Left Stability and Bottom-Left Corners

A bottom-left corner is an empty position where a large box has bottom-left stability. See Figure 4.9, there are in total five bottom-left corners: A, B, C, D and E. A bottom-left placement action is an action that places a box onto a bottom-left corner and makes that box bottom-left stable. Box *j* is over box *i* just there exists a

positive real number d such that if box *i* moves upwards y a distance of d. Then the overlapping area between boxes *i* and *j* is greater than zero.

See Figure 4.10, boxes 2 and 3 are over box 1, boxes 4 and 5 are not over box 1. Box *i* can move upwards freely just no box is over box *i*. Box *j* is on the right of box *i* just there exists a positive real number d such that if box *i* moves rightwards by a distance of d. Then the overlapping area between boxes *i* and *j* is greater than zero.

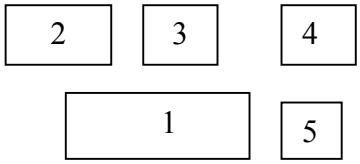


Figure 4.10. Boxes 2 and 3 are Over Box 1

See Figure 4.11, boxes 2 and 3 are on the right of box 1, boxes 4 and 5 are not on the right of box 1. Box *i* can move rightwards freely just no box is on the right of box *i*.

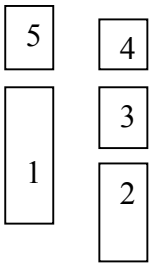


Figure 4.11. Boxes 2 and 3 are on the Right of Box 1

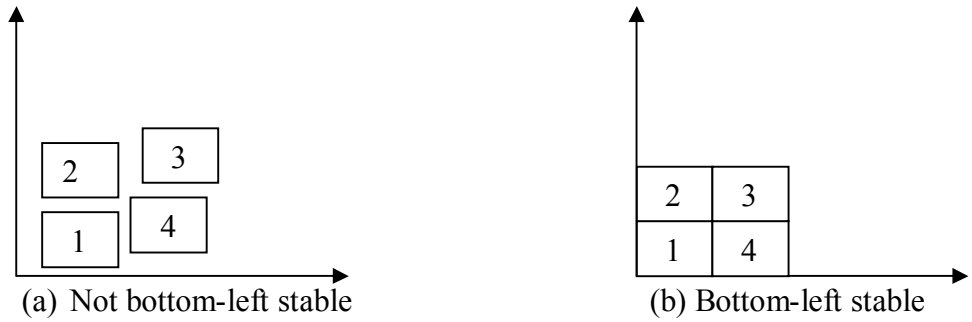


Figure 4.12. A feasible Loading and Its Equivalent Bottom-Left Stable Loading

Bottom-left placement theorem has two important lemmas. First lemma is any feasible loading can be replaced by another feasible loading where each box has bottom-left stability. Second lemma is in any feasible loading, if we take away the four borders of the pallet, then there is a box which can move upwards and rightwards freely.

There is also three theorem. First theorem is for any feasible, bottom-left stable loading, there exists a numbering of n boxes such that box i (i =1,2,...,n) locates on a bottom-left corner formed by boxes 1,2,..,i-1 and the four borders of the pallet. Second theorem is arbitrarily given N boxes and a rectangular pallet, if it is possible to orthogonally place all boxes into the pallet without overlapping, then we can find a feasible loading through a sequence of bottom-left placement actions. Third theorem is the box loading problem can be solved after finite times of bottom-left placement actions. That is to say, the box loading problem can be solved after finite times of basic arithmetic and logical operations on the real input parameters($W, H, \mathrm{w}_1, \mathrm{h}_1, \mathrm{w}_2, \mathrm{h}_2, \ldots, \mathrm{w}_\mathrm{n}, \mathrm{h}_\mathrm{n}$).

In Bottom-Left loading (BL) each item is moved as far as possible to the bottom of the object and then as far as possible to the left (Jakobs, 1996). A valid position is found when the box collides with the partial layout at its lower and left side. Figure.4.13 shows the placement of a sequence of boxes which is described by the permutation (2,6,4,3,0,1,5). The major disadvantage of the BL-routine consists of the creation of empty areas in the layout, when larger items block the movement of successive ones. Placement algorithm (Bottom-Left-Fill algorithm, BLF) has been developed that allows to place each item at the lowest available position of the object. BLF algorithm is based on the allocation of the lowest sufficiently large region in the partial layout rather than on a series of bottom-left moves.

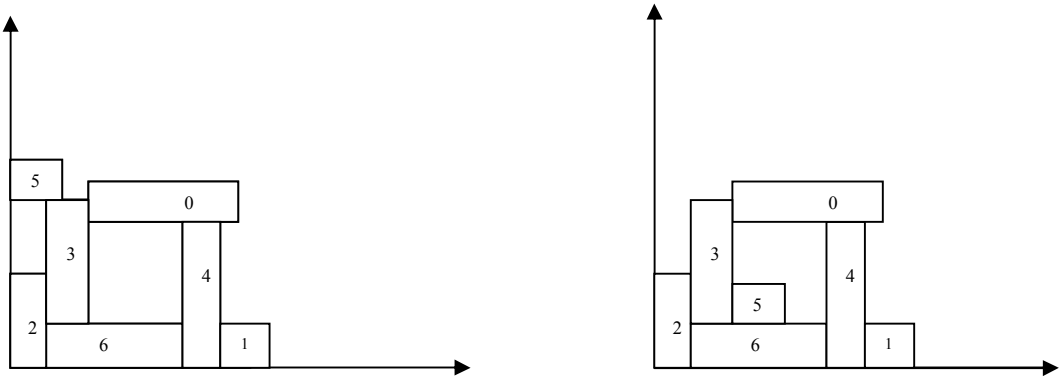


Figure 4.13. BL-rule (left) and BLF-rule (right)

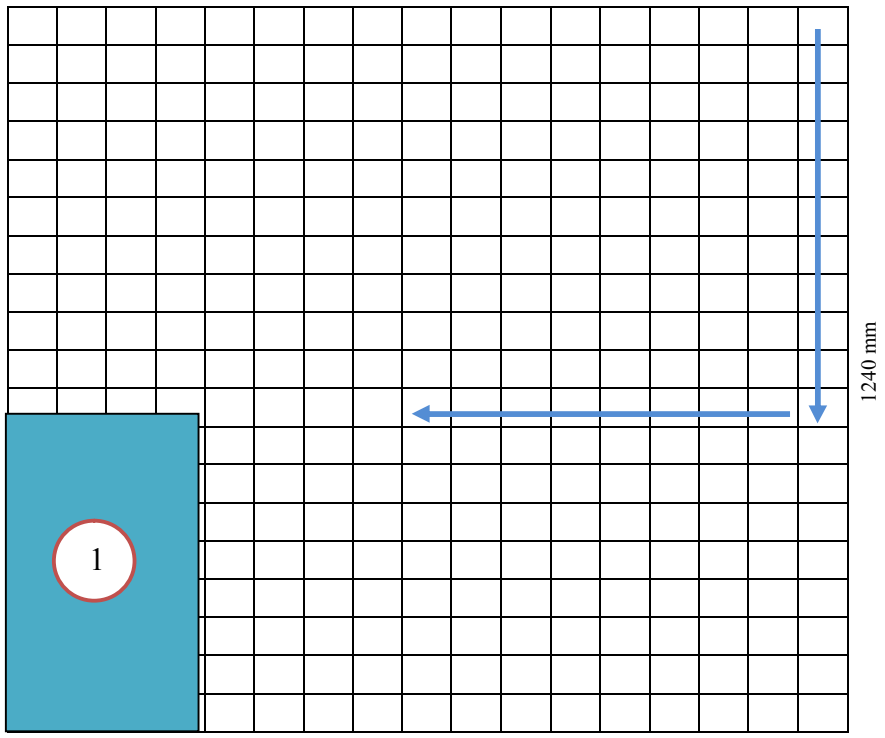


Figure 4.14. First Step of Bottom-Left Algorithm Example

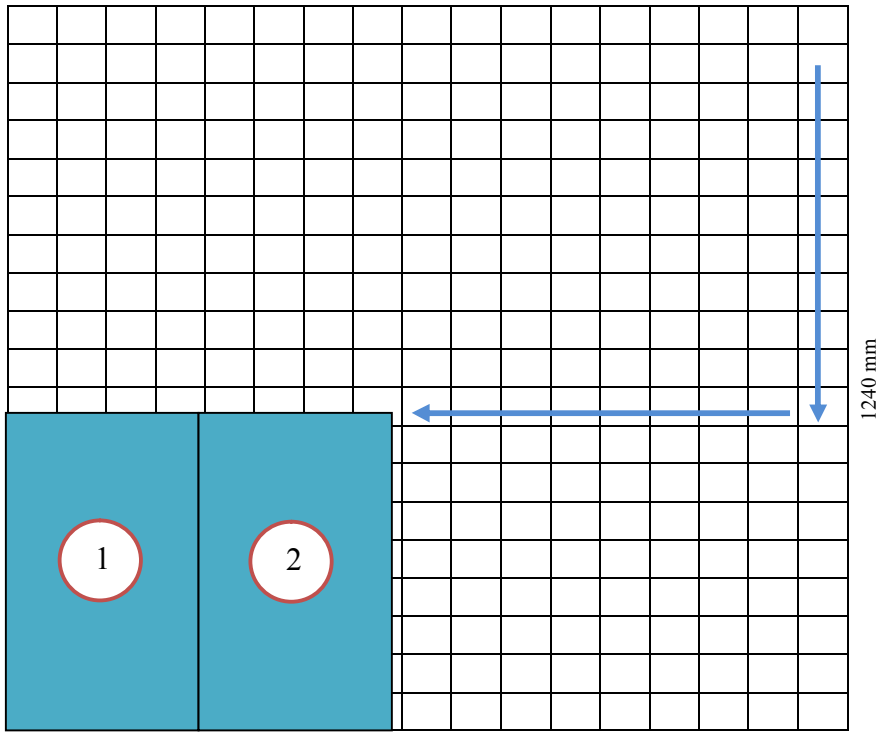


Figure 4.15. Second Step of Bottom-Left Algorithm Example

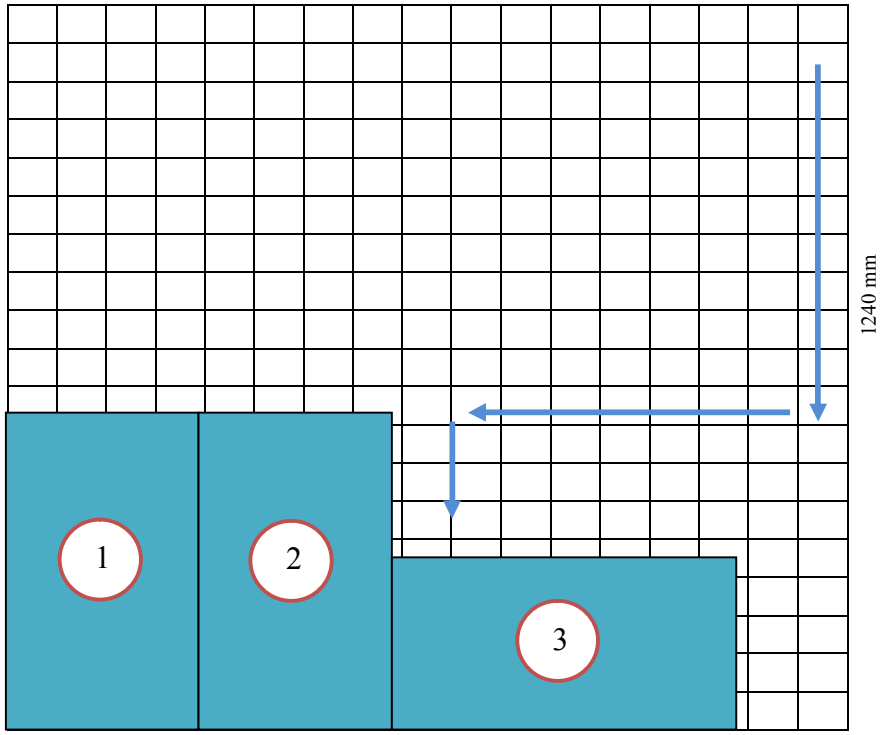


Figure 4.16. Third Step of Bottom-Left Algorithm Example

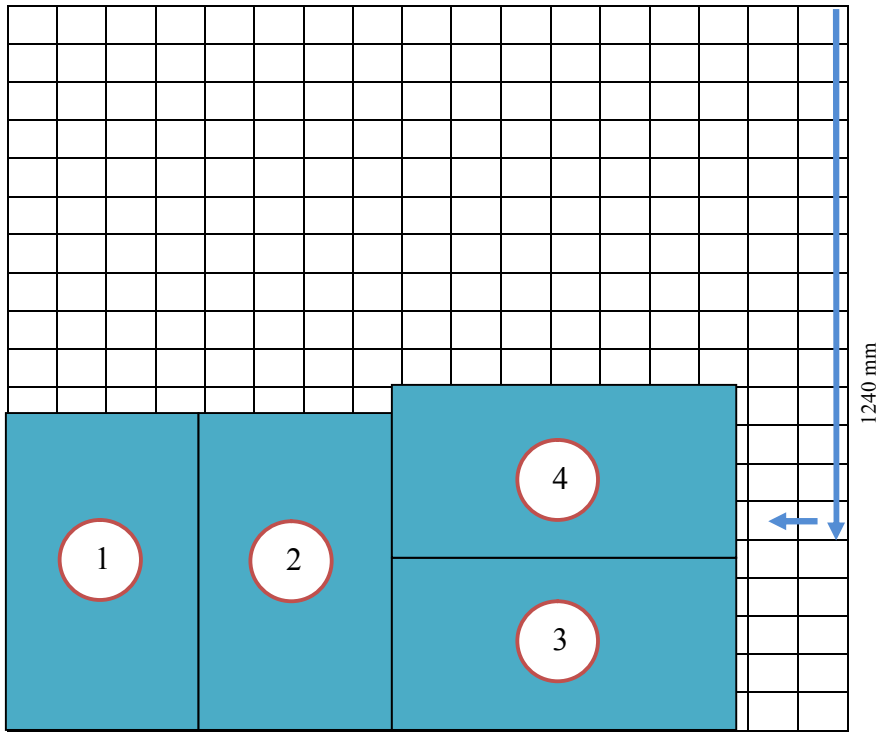


Figure 4.17. Fourth Step of Bottom-Left Algorithm Example

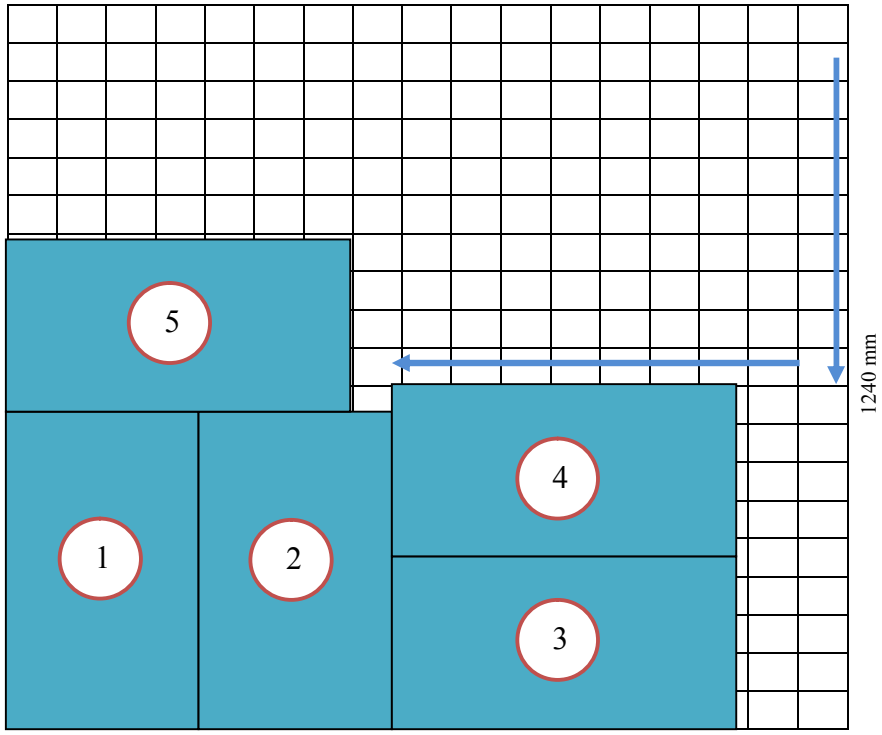


Figure 4.18. Fifth Step of Bottom-Left Algorithm Example

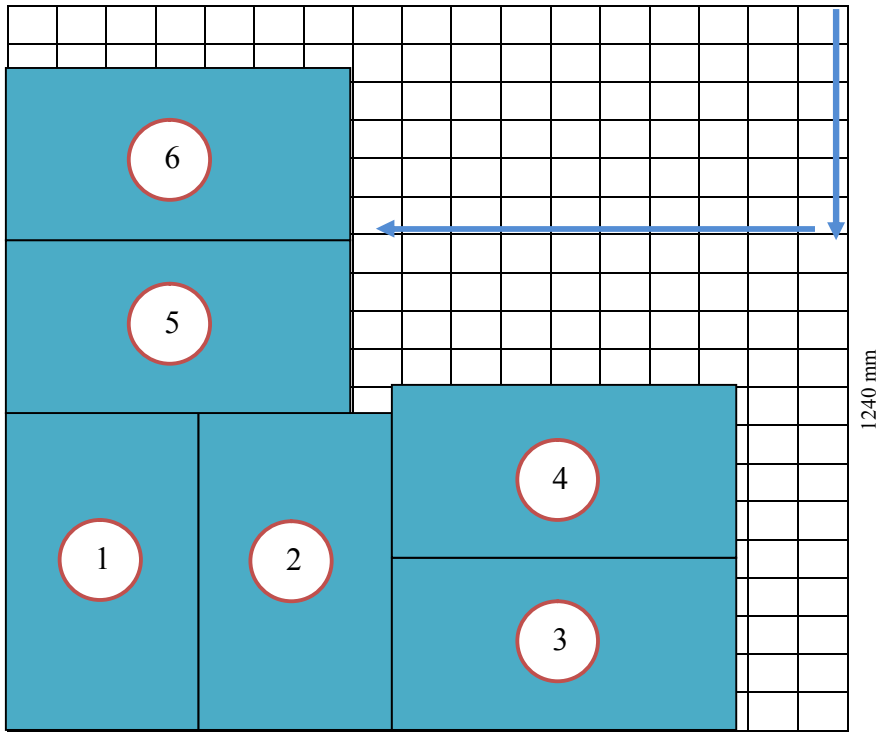


Figure 4.19. Sixth Step of Bottom-Left Algorithm Example

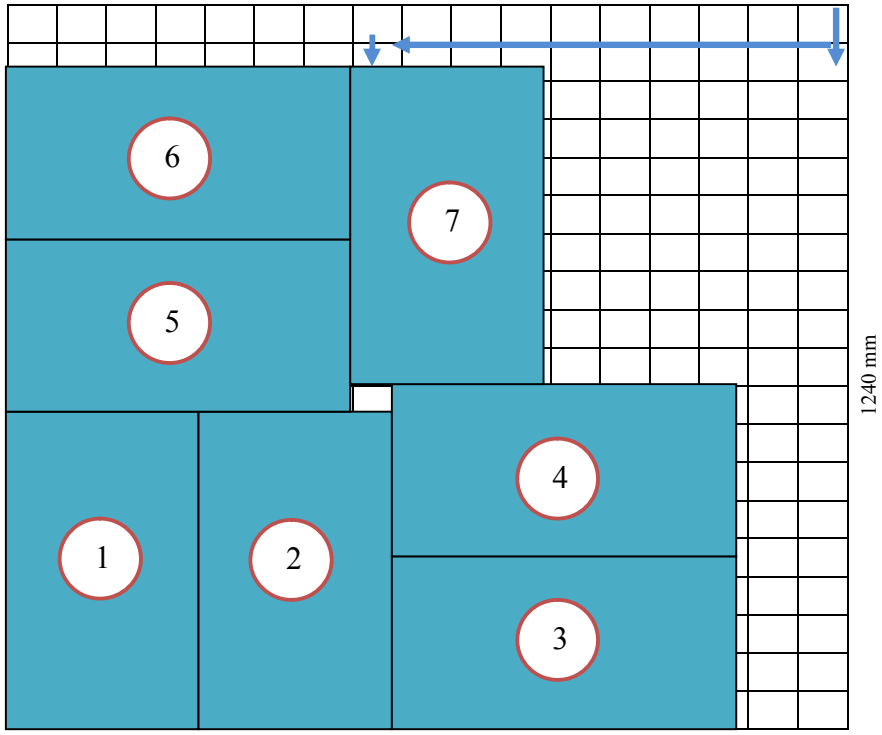


Figure 4.20. Seventh Step of Bottom-Left Algorithm Example

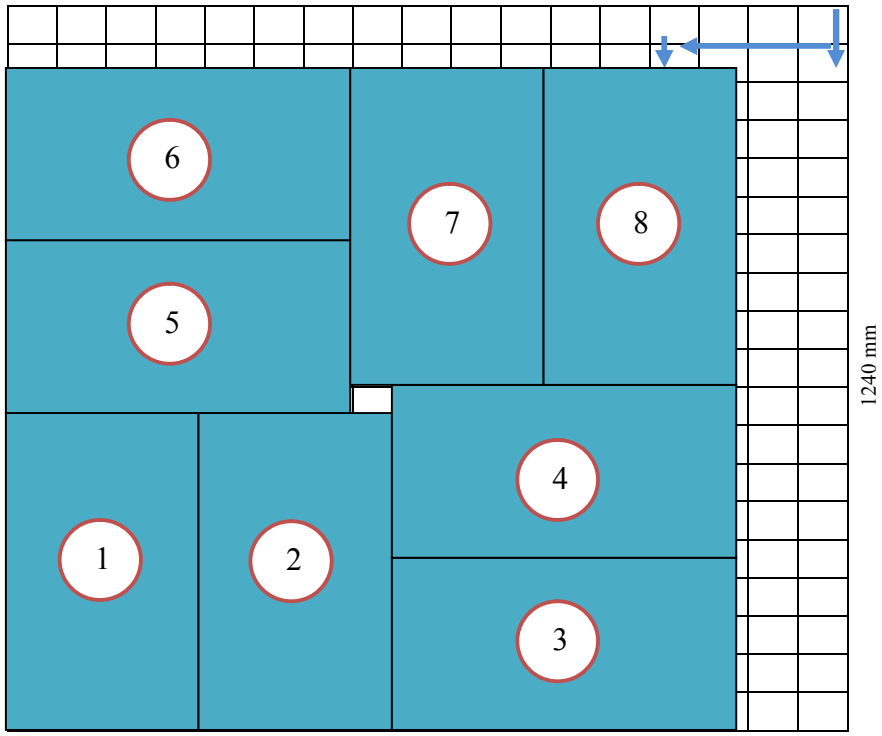


Figure 4.21. Eighth Step of Bottom-Left Algorithm Example

5. RESULTS

There are 95 product types produced in the plant. Currently, the company uses a predetermined standard pallet type for each product type. A predetermined pallet type has fixed dimensions. In the first phase of the study, we have evaluated the effectiveness of current loading pattern on the predetermined pallet for each product type. The goal was to investigate whether it is possible to increase the space utilization of the pallet by changing the pattern without changing the pallet dimensions.

Each product type represents a different instance of pallet loading problem (PLP). The problem has been solved for each instance (product type) given that the boxes are loaded on the pallet type predetermined by the company. It is found that there are 21 instances out of 95 in which area utilization of the pallet can be improved. In other words, it is possible to load more box(es) on the same pallet. Such an instance has already been presented in section 4.1.1 through Figure 4.4 – Figure 4.5. The complete list of such instances is presented in Table 5.1. Corresponding layout drawings are presented in Appendix 4.

Table 5.1. List of loading patterns that can be improved on the same pallet

	Product Dimensions		Predetermined Pallet Dimensions		Number of Boxes That Can Be Placed on the Pallet as the First Layer	
Product Code	Width (mm)	Length (mm)	Width (mm)	Length (mm)	Current Loading Plan Used by Company	Proposed Loading Plan by Math Models
153602242	370	490	1240	1240	6	8
153602256	200	460	1240	1240	12	14
153602262	200	460	1240	1240	12	14
153604199	340	540	1240	1240	7	8
153604555	380	750	1440	1440	4	5
153605647	330	540	1240	1240	7	8
153605937	220	420	1240	1240	10	12
153605940	320	460	1440	1440	12	13
153606426	340	510	1440	1440	8	10
153606430	320	450	1440	1440	12	13
153606438	340	510	1440	1440	8	10
153606439	340	510	1440	1440	8	10
153606479	340	510	1440	1440	8	10
153606480	340	510	1440	1440	8	10

List of loading patterns that can be improved on the same pallet (Continued)

153606655	380	600	1440	1440	6	8
153606685	330	540	1240	1240	7	8
153604754	250	465	1240	1240	10	11
153604874	250	465	1240	1240	10	11
153605118	250	465	1240	1240	10	11
153605475	250	465	1240	1240	10	11
153606006	250	465	1240	1240	10	11

In the second phase of the study, we have focused on identifying the best pallet dimensions for each product type. In other words, the question is now to decide whether utilization can be improved further if we change the dimensions of the pallet. However, pallet dimensions are limited by some technological constraints and there are only 6 distinct options available which are already listed in Table 2.3.

In order to decide which pallet option is the best for a particular product type, first we need to use mathematical models to find the best loading pattern for each pallet option. Therefore, for a particular product type, we need to determine 6 different best loading patterns that correspond for 6 different pallet options. For example, let's consider the product with width w= 370 mm and length l=490 mm. The following table summarizes the best loading patterns in terms of max number of boxes that can be stowed on given pallet option.

Table 5.2. Alternative pallet options for a particular product type

Pallet Options	Max. number of boxes stowed
Pallet 1 1240 x 1240 mm	8
Pallet 2 1440 x 1440 mm	9
Pallet 3 1540 x 1540 mm	12
Pallet 4 1240 x 1340 mm	8
Pallet 5 1240 x 1440 mm	9
Pallet 6 1240 x 1540 mm	10

The next step is to decide which pallet option yields the best solution. It is not an easy decision since a proper performance measure should be defined first

for comparison. We use the "wasted space per stowed box" criteria to make comparisons between pallet options. This concept can be understood easier with the following definitions.

A : area of the pallet floor

B : area of a single box (length x width)

n: max number of boxes that can be stowed on the pallet

U : Utilized area by the stowed boxes = (n x B)

W : Wasted area = (A-U)

WPB : Wasted area per box = W/n

Therefore we now ready to extend the Table 5.2 above such that it can include the definitions above. The resulting new table in given in Table 5.3

Table 5.3. Deciding alternative pallet options for a particular product type

Pallet Options	Area of the Pallet (mm^2) (A)	Max. Number of Boxes Stowed (n)	Occupied Area by Boxes (mm^2) (U)	Wasted Area (mm^2) (W)	Wasted Area per box (mm^2) (WPB)
Pallet 1 1240 x 1240 mm	1,537,600	8	1,450,400	87,200	10,900
Pallet 2 1440 x 1440 mm	2,073,600	9	1,631,700	441,900	49,100
Pallet 3 1540 x 1540 mm	2,371,600	12	2,175,600	196,000	16,333
Pallet 4 1240 x 1340 mm	1,661,600	8	1,450,400	211,200	26,400
Pallet 5 1240 x 1440 mm	1,785,600	9	1,631,700	153,900	17,100
Pallet 6 1240 x 1540 mm	1,909,600	10	1,813,000	96,600	9,660

The best pallet option should be the one with the minimum wasted space per box (WPB). For the sample product stated above, it is recommended to use the pallet option with the dimensions 1240 x 1540 since it delivers the minimum WPB. Currently the pallet option with the dimensions 1240 x 1240 is used in the factory.

The analysis has been repeated for each product type which means that 570 (=95 product types * 6 pallet options) instances of pallet loading problem have been solved to complete the study. The details of outcomes are presented in Appendix 5 and 6.

The solutions yield the best pallet option for each product type. Proposed pallet options have been compared with the ones that are currently in use at the factory. The outcomes show that proposed pallet options coincide with the predetermined pallet types for some product types. In other words, there is no need to change the pallet type for some product types. However, it is recommended to change the pallet dimensions for some other product types. There are 48 product types such that a different pallet option is recommended instead of predetermined pallet dimensions. The following table presents the list of product types whose pallet dimensions are recommended to be changed.

Table 5.4. The list of products types that a different pallet type should be used

Product Code	Pallet			
	Currently in Use		Proposed	
	Dimensions (mm)	WPB (mm^2)	Dimensions	WPB (mm^2)
153602245	1240 x 1240	44.094	1440 x 1440	11,490
153602246	1240 x 1240	37.594	1540 x 1540	14,975
153602257	1440 x 1440	101.800	1540 x 1540	19.711
153604455	1440 x 1440	101.800	1540 x 1540	19.711
153604555	1440 x 1440	129.720	1540 x 1540	11.450
153604728	1540 x 1540	64.450	1240 x 1240	24.267
153604936	1440 x 1440	41.200	1240 x 1240	38.267
153604937	1440 x 1440	82.350	1440 x 1440	33.200
153605105	1240 x 1240	44.094	1440 x 1440	11.490
153605476	1440 x 1440	55.800	1540 x 1540	6.650
153605645	1540 x 1540	39.511	1240 x 1240	32.267
153605646	1440 x 1440	79.100	1540 x 1540	29.950
153605745	1440 x 1440	35.200	1240 x 1240	32.267
153605933	1440 x 1440	55.800	1540 x 1540	6.650
153605935	1440 x 1440	42.000	1240 x 1240	3.920

The list of products types that a different pallet type should be used (Continued)

153605955	1440 x 1440	45.200	1240 x 1240	26.160
153605961	1440 x 1440	55.800	1540 x 1540	6.650
153605963	1440 x 1440	55.800	1540 x 1540	6.650
153605979	1440 x 1440	55.800	1540 x 1540	6.650
153606210	1440 x 1440	81.600	1540 x 1540	32.450
153606369	1440 x 1440	78.400	1240 x 1240	18.844
153606425	1440 x 1440	55.800	1540 x 1540	6.650
153606426	1440 x 1440	33.960	1240 x 1240	18.800
153606429	1440 x 1440	55.800	1540 x 1540	6.650
153606434	1440 x 1440	35.200	1240 x 1240	32.267
153606438	1440 x 1440	33.960	1240 x 1240	18.800
153606439	1440 x 1440	33.960	1240 x 1240	18.800
153606479	1440 x 1440	33.960	1240 x 1240	18.800
153606480	1440 x 1440	33.960	1240 x 1240	18.800
153606583	1440 x 1440	57.600	1540 x 1540	8.450
153606655	1440 x 1440	31.200	1240 x 1240	28.267
153606658	1440 x 1440	31.200	1240 x 1240	28.267
153606660	1440 x 1440	28.800	1240 x 1240	26.844
153606663	1240 x 1240	347.200	1440 x 1440	96.800
153606666	1440 x 1440	79.100	1540 x 1540	29.950
153606716	1540 x 1540	92.867	1440 x 1440	43.200
153606729	1440 x 1440	46.400	1540 x 1540	24.360
153606750	1440 x 1440	129.720	1540 x 1540	11.450
153606788	1440 x 1440	31.200	1240 x 1240	28.267
153606791	1440 x 1440	79.100	1540 x 1540	29.950
153606793	1440 x 1440	55.800	1540 x 1540	6.650
153606797	1440 x 1440	55.800	1540 x 1540	6.650
153606799	1440 x 1440	57.600	1540 x 1540	8.450
153606935	1440 x 1440	55.800	1540 x 1540	6.650
153606936	1440 x 1440	42.000	1240 x 1240	3.920
153606939	1440 x 1440	60.000	1540 x 1540	10.850
153606985	1440 x 1440	81.600	1540 x 1540	32.450
153607045	1440 x 1440	27.600	1240 x 1240	25.644
TOTAL WPB		3.034.300		1.963.866

Bibliography

Smith, A. and De Cani, P., 1980, An algorithm to optimize the layout of boxes in pallets, *Journal of the Operational Research Society*, 31: 573-578 pp

Steudel, H.J., 1979, Generating pallet loading patterns: A special case of the two-dimensional cutting stock problem, *Management Science*, 25: 997-1004 pp

Scheithauer, G. and Terno, J., 1996, The G4-Heuristic for the Pallet Loading Problem, *European Journal of Operational Research*, 47: 511-522 pp

Lins, L., Lins, S. and Morabito, R., 2003, An L-approach for packing (l,w)-rectangles into rectangular and L-shaped pieces, Journal of the Operational Research Society 54: 777-789 pp

Amaral, A. and Wright, M., 2001, Experiments with a strategic oscillation algorithm for the pallet loading problem, International Journal of Production Research 39-11: 2341-2351 pp

Dowsland, K.A., 1996, Simple tabu thresholding and the pallet loading problem. In: Osman, I.H. and Kelly, J.P. (Eds). Metaheuristics: theory and applications, Kluwer Academic Publishers, 379-405 pp

Herbert, A and Dowsland, W., 1996, A family of genetic algorithms for the pallet loading problem, *Annals of Operations Research* 63: 415-436 pp

Nelissen, J., 1995, How to use structural constraints to compute an upper bound for the pallet loading problem, *European Journal of Operational Research* 84: 662-680 pp

Dowsland, K.A., 1984, The three-dimensional pallet chart: an analysis of the factors affecting the set of feasible layouts for a class of two-dimensional packing problems, *Journal of the Operational Research Society* 35: 895-905 pp

Bibliography (continued)

Dowsland, K.A., 1985, Determining an upper bound for a class of rectangular packing problems, *Computers and Operations Research* 12: 201-205 pp

Letchford, A. and Amaral, A., 2001, Analysis of upper bounds for the Pallet Loading Problem, *European Journal of Operational Research* 132: 582-593 pp

De Cani, P., 1979, Packing problems in theory and practice, Ph.D.Thesis Department of Engineering Production, University of Birmingham.

Iserman, H., 1987, Ein Planungsssytem zur Optimierung der Palettenbeladung mit kongruenten rechteckigen Versangebinden, *OR Spektrum* 9: 235-249 pp

Exeler, H., 1988, Das homogene Packproblem in der betriebswirtschaftlichen Logistik, Physica-Verlag, Heidelberg.

Bhattacharya, R., Roy, R. and Bhattacharya, S., 1998, An exact depth-first algorithm for the pallet loading problem, *European Journal of Operational Research* 110: 610-625 pp

Christofides, N., Whitlock, C., 1977, An algorithm for two-dimensional cutting problems, *Operations Research* 25: 30–45 pp

Gilmore, P.C., Gomory, R.E., 1965, Multistage cutting stock problems of two and more dimensions, *Operations Research* 13: 94–120 pp

Nelissen, J., 1993, New Approaches to the Pallet Loading Problem, Working paper, RWTH Aachen ftp://ftp.informatik. rwth-aachen.de/pub/reports/pallet.ps.Z.

Bibliography (continued)

Oliveira, J., Ferreira, J., 1990, An improved version of Wang's algorithm for two-dimensional cutting problems, *European Journal of Operational Research* 44: 256–266 pp

Wang, P.Y., 1983, Two algorithms for constrained two-dimensional cutting stock problems, *Operations Research* 31: 573– 586 pp

Beasley, J. E., 1985, An exact two-dimensional non-guillotine cutting tree search procedure, *Operations Research* 33(1) : 49-64 pp

Chen, C.S., Lee, S.M., Shen, Q.S., 1995, An analytical model for the container loading problem, *European Journal of Operational Research* 80(1) : 68-76 pp

R.Alvarez, Valdes, 2005, A branch and cut algorithm fort the pallet loading problem, *Computers&Operations Research* 32(11): 3007-3029 pp

Appendix 1 Class-I Products and Their Properties

Product Code	Product	Dimensions of Box			Number of products in this box
		Width (mm)	Length (mm)	Height (mm)	
153602256	LİLY EXTRA PEÇETE (30X30)	200	460	305	24
153602257	LİLY TOMBUL PEÇETE	460	530	305	24
153602259	SENSO DESENLİ PEÇETE	310	580	305	24
153602262	SELECT EXPERT PEÇETE (30X30)	200	460	305	24
153602265	SELECT STANDART DİSPENSER PEÇETE	285	380	275	18
153602271	P&P 500'LÜ PAKET PEÇETE	305	650	305	10
153602272	SAINSBURY'S 30X30 PEÇETE	310	310	270	12
153602320	SELECT AMBLEMLİ PEÇETE (30X30)	310	550	305	24
153602322	SELECT EXP.AMB. PEÇETE (30X30)	200	460	305	24
153602369	SELECT EKONOMİK DİSP. PEÇETE	300	360	190	12
153603055	SENSO EKO PEÇETE 32 Lİ 24*26,5	330	540	250	32
153604188	SENSO 150'Lİ PEÇETE 24*26,5	340	540	370	24
153604199	SELECT 200'LÜ PEÇETE 25,5*26,5	340	540	260	16
153604455	SELECT TOMBUL(YENİ DOKU) PEÇ. (30X30)	460	530	305	24
153604754	SWAN SCDS 21X26,5cm. DİSPENSER PEÇETE	250	465	265	20
153604874	CAFE COSTA AMB. DİSPENSER PEÇ.	250	465	265	20
153604967	BLUME EKONOMİK PEÇETE 24x26,5	340	370	540	48
153605118	CAFE NERO AMB. DİSPENSER PEÇ.	250	465	265	20
153605126	MORRİSON BEYAZ PEÇETE	410	600	463	36
153605340	İHR. 30X30 356 LI PEÇETE	325	460	305	9
153605475	TESCO 21x26,5cm. DİSPENSER PEÇ.	250	465	265	20
153605588	İHRACAT SENSO TOMBUL PEÇ. (30X30)	460	530	305	24
153605647	PUFLA 100'LÜ PEÇETE 24*26,5	330	540	250	32
153605741	DMO SELECT DİSPENSER PEÇETE	285	505	275	24
153605772	TABLESMART 24cm. 300LÜ DİSP.PEÇ.	270	450	265	20
153606006	TABLESMART 21x26,5cm. 300LÜ DİSP.PEÇ.	250	465	265	20

Appendix 2 Class- II Products and Their Properties

Product Code	Product	Dimensions of Box			Number of products in this box
		Width (mm)	Length (mm)	Height (mm)	
153602223	SELECT DOUBLE RULO 12 TK	360	720	200	3
153602225	THY 2'Lİ TUVALET KAĞIDI	400	580	200	24
153602242	SELECT EXTRA MUAYENE MASA ÖRT.	370	490	520	6
153602247	SELECT JUMBO ENDÜSTRİYEL HAV.	265	265	223	1
153603043	DMO SELECT EXTRA TUV.K. (KOLİ)	400	560	200	4
153603045	DMO SELECT EXTRA 8'Lİ HAVLU (KOLİ)	440	680	230	3
153603277	SELECT EXTRA ACTIVE HAVLU	360	540	223	6
153603636	SELECT EXPERT 6 HV	420	680	226	4
153603637	SELECT SMART 6 HV	410	650	226	4
153604196	LİLY PLUS 6'LI HV	480	650	226	4
153604217	LİLY PLUS 24 TK	410	690	300	3
153604490	LİLY PLUS 3'LÜ HV	640	690	226	12
153604555	EDA 24 TK (İHRACAT)	380	750	300	**4**
153604728	SEVİNÇ 12 TK	400	580	200	4
153604875	QUEEN 3 KATLI 8'Lİ TK	420	650	200	6
153604886	EDA 3'LÜ HV	590	430	230	8
153604895	SEVİNÇ 6'LI HV	410	650	230	4
153604966	BLUME EXTRA 4'LÜHV	420	680	226	6
153605071	SELECT SMART ACTIVE HAVLU 150MT	360	540	223	6
153605356	BLUME32'Lİ EXTRA TK	420	450	400	2
153605455	BLUME EXTRA12'Lİ HV	320	460	450	2
153605476	SPAR 16'LI 3 KAT TK (İHRACAT)	420	690	200	3
153605587	İHRACAT SENSO2'LİHV	420	680	226	12
153605592	İHRACAT SENSO ÇİFTKAT 12 TK	400	560	192	4
153605593	SELECT EKONOMİK ACTIVE HV. 207MM.	340	510	207	6
153605594	SELECT İÇTEN ÇEKMELİ HAVLU (5)	340	510	207	6
153605595	SELECT MİNİ JUMBO TUV. KAĞ.(5)	340	510	192	12
153605645	SENSO PUFLA 12 TK	400	560	192	4
153605646	SENSO PUFLA 4 HV	410	650	226	6
153605743	DMO SELECT MİNİ JUMBO TUV. KAĞ.	390	570	192	12
153605744	İHRACAT SENSO 2'Lİ HV ETİKETLİ	420	680	226	12
153605747	FS SELECT MİNİ JUMBO TUV. KAĞ.(5)	340	510	200	12
153605785	FS SELECT EKONOMİK ACTIVE HAVLU 207MM.	340	510	207	6
153605889	İHR. 230MT İÇ.ÇEK. HV.	350	520	207	6
153605933	LİLY 16 TK	420	690	200	3

Class – II Products and Their Properties (Continued)

153605935	LİLY 6 HV	460	660	226	4
153605936	LİLY 32 TK	420	450	400	2
153605937	LİLY 40 TK	220	420	500	1
153605940	LİLY 12 HV	320	460	450	2
153605942	LİLY DOLGUN 16 TK	430	670	200	3
153605943	LİLY DOLGUN 32 TK	430	460	400	2
153605945	LİLY 16 HV	460	460	450	2
153605946	SENSO BIO 24 TK	380	600	288	3
153605947	SENSO BIO 32 TK	360	400	384	2
153605951	SENSO BIO 3 HV	620	680	226	12
153605952	SENSO BIO 6 HV	410	650	226	4
153605955	SENSO PUFLA 12 HV	290	440	450	2
153605957	LİLY DOLGUN 6 HV	450	650	226	4
153605958	LİLY DOLGUN 12 HV	320	450	450	2
153605961	SELECT EXPERT 24 TK	420	690	300	3
153605963	SELECT SMART 24 TK	420	690	300	3
153605979	SPAR 8'LI 3 KAT TK (İHRACAT)	420	690	200	6
153605997	ÖZEL 180 metre EXTRA ACTIVE HAVLU	360	540	223	6
153606057	BLUME 12 (19g)	420	690	200	4
153606162	SENSO PUFLA 2 HV	420	680	226	12
153606170	DMO SELECT EXPERT 24 TK	420	690	300	3
153606196	PROMOSYON LİLY DOLGUN TK RULO	430	670	200	1
153606210	SELECT EXPERT ACTIVE TUV. KAĞ. 16'LI	400	660	192	3
153606238	DKK SELECT EXPERT 24 TK	430	690	300	3
153606295	LİLY DOLGUN 32 TK PROMOSYONLU	430	460	400	2
153606341	LİLY DOLGUN 12 HAVLU PRO	320	450	450	2
153606366	LİLY 32 TK PROMOSYONLU	420	450	400	2
153606368	LİLY 12 HAVLU PROMOSYONLU	320	460	450	2
153606369	SENSO PUFLA 32	400	560	192	2
153606395	SELECT EXPERT 2	420	680	226	12
153606405	SELECT EXTRA ACTIVE 180m. HAVLU	360	540	223	6

Appendix 3 Products and Pallet Dimensions Currently in Use

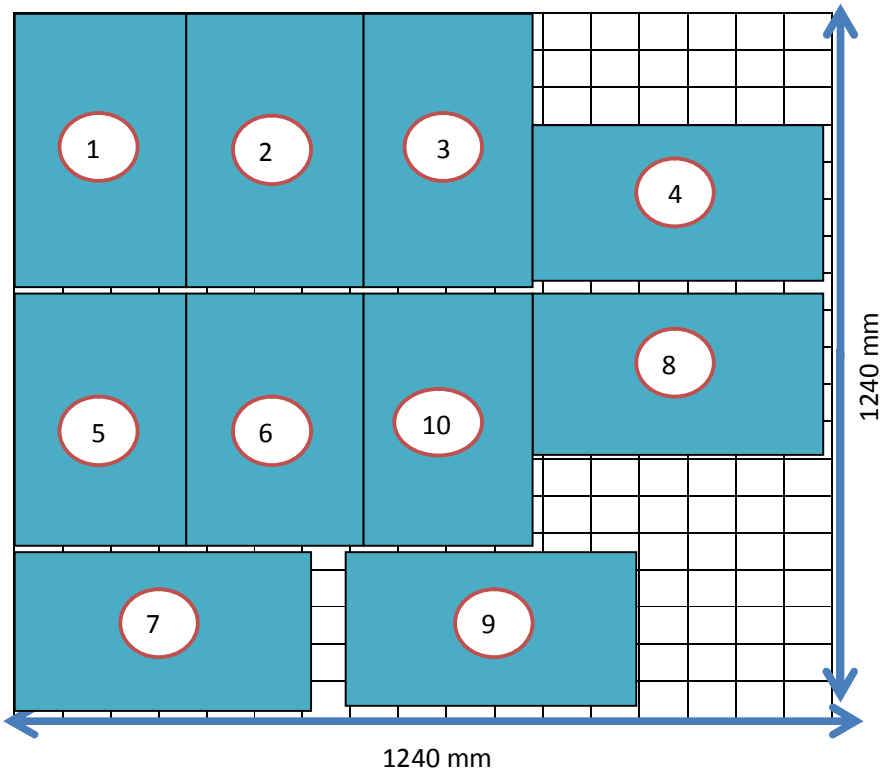
Product Code	Name	Number of boxes in this pallet	120x120 (cm)	140x140 (cm)	150x150 (cm)
153602223	SELECT DOUBLE RULO 12 TK	80		X	
153602242	SLC. EXTRA MUAYENE MASA ÖRT.	22	X		
153602245	SELECT SMART DELUX DİSPEN. HV	42	X		
153602246	SELECT EXPERT DELUX DİSP. HV.	42	X		
153602247	SELECT JUMBO ENDÜSTRİYEL HV..	128	X		
153602256	LİLY EXTRA PEÇETE (30X30)	72	X		
153602257	LİLY TOMBUL PEÇ.	48		X	
153602259	SENSOx1 DESENLİ PEÇ.	48	X		
153602262	SELECT EXPERT PEÇ. (30X30)	72	X		
153602265	SELECT STANDART DİSPENSER PEÇ.	72	X		
153602369	SELECT EKONOMİK DİSPENSER PEÇ.	108	X		
153603055	SENSO EKONOMİK PEÇ. 24*26,5 32'Lİ	56	X		
153603636	SELECT EXPERT HAV 6'LI	80			X
153603637	SELECT SMART HAV 6'LI	80			X
153604188	SENSO EKONOMİK PEÇ. 150'Lİ	40	X		
153604199	SELECT EKONOMİK PEÇ. 200' LÜ	56	X		
153604455	SELECT TOMBUL PEÇ.	48		X	
153604555	EDA TUV.KAĞ. 20+4' LÜ	56		X	
153604728	SEVİNÇ 12 TK	88			X
153604936	EDA TUV.KAĞ. 8' Lİ	80		X	
153604937	EDA TUV.KAĞ. 4' LÜ	80		X	
153605105	SELECT STANDART DELUX HAV 200 YP.	64	X		
153605476	SPAR 16'LI TK 3 KATLI	80		X	
153605594	SELECT İÇTEN ÇEKMELİ HAV(5)	72	X		
153605595	SELECT MİNİ JUMBO TK(5)	80	X		
153605645	PUFLA TK 12'Lİ	100			X
153605646	PUFLA HAV 4'LÜ	80		X	
153605647	PUFLA PEÇ. 24*26,5 100'LÜ	56	X		
153605745	PUFLA TK 4'LÜ	88		X	
153605933	LİLY TK 13+3'LÜ	80		X	
153605935	LİLY HAV 6'LI	80		X	
153605936	LİLY TK 26+6'LI	45		X	
153605937	LİLY TK 40'LI	40	X		
153605938	PREMIA 20+4 TK	56			X
153605939	PREMIA 26+6 TK	45		X	
153605940	LİLY HAV 12'Lİ	60		X	
153605945	LİLY HAV 16' LI	45		X	
153605955	PUFLA HAV 12'Lİ	60		X	
153605961	SELECT EXPERT TK 24'LÜ	56		X	
153605963	SELECT SMART TK 24'LÜ	56		X	
153605979	SPAR 8'Lİ TK 3 KATLI	80		X	
153606162	PUFLA HAV 2'Lİ	80			X
153606210	SELECT EXPERT ACTİVE TK 16'LI	88		X	
153606238	DKK SELECT EXPERT TK 24'LÜ	42		X	
153606369	PUFLA TK 32'Lİ	60		X	
153606395	SELECT EXPERT 2'Lİ HAV	80			X
153606396	LİLY TOMBUL DESENLİ PEÇ.	48	X		

Products and Pallet Dimensions Currently in Use (Continued)

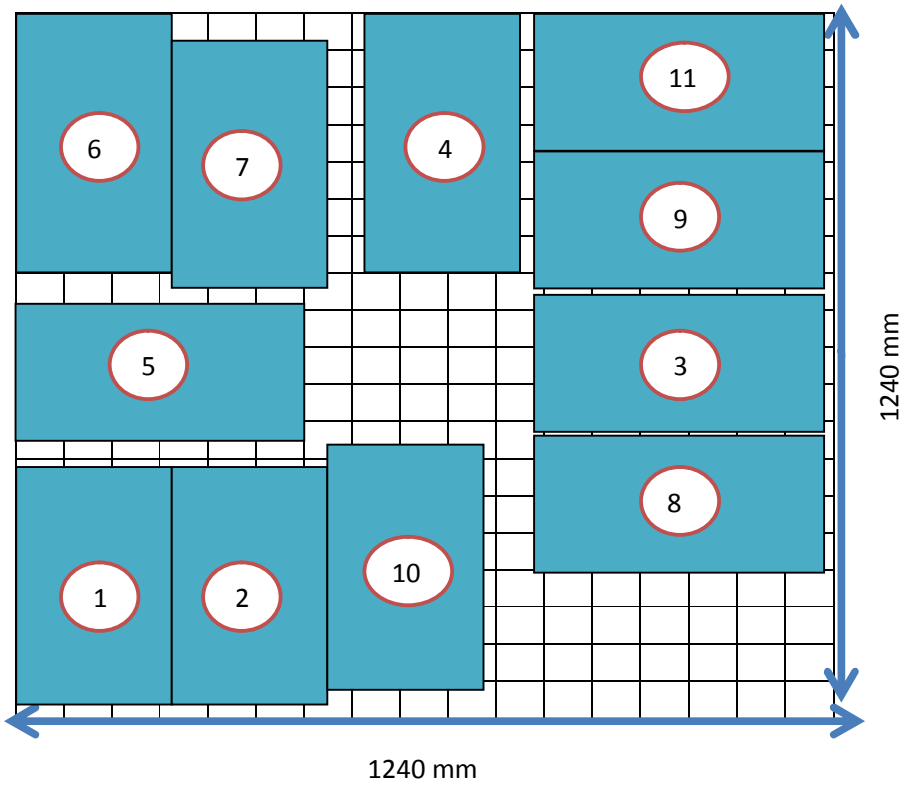
153606407	ÖZEL HANDY STANDART DİSPENSER PEÇETE	72	X		
153606408	ÖZEL HANDY EKO. DİSP. PÇ.	108	X		
153606425	PREMIA 13+3'LÜ TK	80		X	
153606426	SELECT İÇTEN ÇEKMELİ	56		X	
153606429	PREMIA 8'Lİ HAV	80		X	
153606430	PREMIA 12'Lİ HAV	60		X	
153606434	PUFLA TK 24'LÜ	56		X	
153606437	SELECT SMART ACTİVE HV 150 M	72		X	
153606438	SELECT SMART ACTİVE HV 80 M	72		X	
153606439	SELECT İÇTEN ÇEKMELİ HAV(5)_yeni	72		X	
153606441	SELECT EXPERT ACTİVE HV 180 M	72		X	
153606479	SELECT İÇTEN ÇEKMELİ TK-2	72		X	
153606480	SELECT EXPERT ACTİVE HV 88 M	72		X	
153606583	PREMIA 6'LI HAV	80		X	
153606655	SENSO TK 13+3'LÜ	88		X	
153606658	SENSO TK 20+4'LÜ	56		X	
153606660	SENSO TK 26+6'LI	60		X	
153606663	SENSO HAV 3'LÜ	40	X		
153606666	SENSO HAV 6'LI	80		X	
153606685	SELECT 24x26,5 100'LÜ PEÇETE	56	X		
153606715	SPAR 8'Lİ EXTRA 2 KAT TK	80		X	
153606716	SPAR 12'Lİ EXTRA 2 KAT TK	80			X
153606729	MANANA STANDART TK 12'Lİ	88		X	
153606750	PETU TK 24'LÜ	56		X	
153606788	SENSO TK 12'Lİ	88		X	
153606790	SENSO HAV 2'Lİ	80			X
153606791	SENSO HAV 4'LÜ	80		X	
153606793	PREMIA 4'LÜ TK	80		X	
153606797	PREMIA 8'Lİ TK	80		X	
153606798	PREMIA 2'Lİ HAV	80			X
153606799	PREMIA 4'LÜ HAV	80		X	
153606935	LİLY TK 24'LÜ	56		X	
153606936	LİLY HAV 8'Lİ	80		X	
153606939	SELECT EXPERT HAV 8'Lİ	80		X	
153606985	SELECT EXPERT ACTİVE TK 8'Lİ	88		X	
153607045	SENSO HAV 12'Lİ	60		X	
153604754	SWAN DİSPENSER PEÇETE	70	X		
153604874	CAFE COSTA AMB. DİSPENSER PEÇ.	70	X		
153605118	CAFE NERO AMB. DİSPENSER PEÇ.	70	X		
153605475	TESCO BASKISIZ DİSP. PEÇETE	70	X		
153606006	TABLESMART DİSPENSER PEÇETE 26,5	70	X		

Appendix 4 Layout Drawings

200 x 465 1240 x 1240 mm (factory)

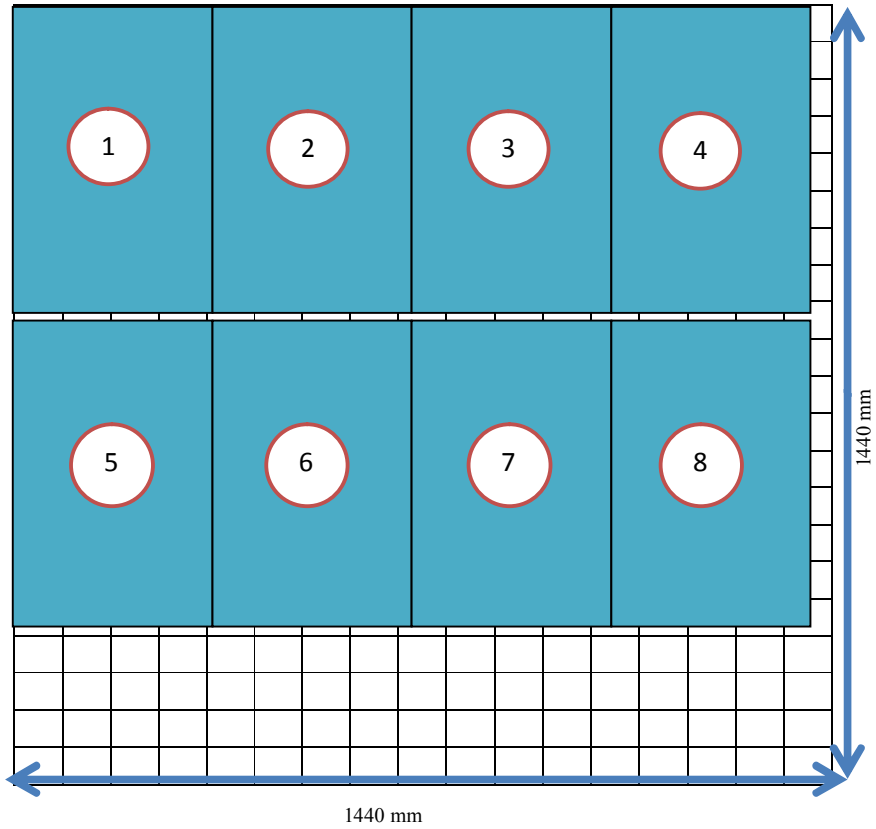


200 x 465 1240 x 1240 mm (Lingo)

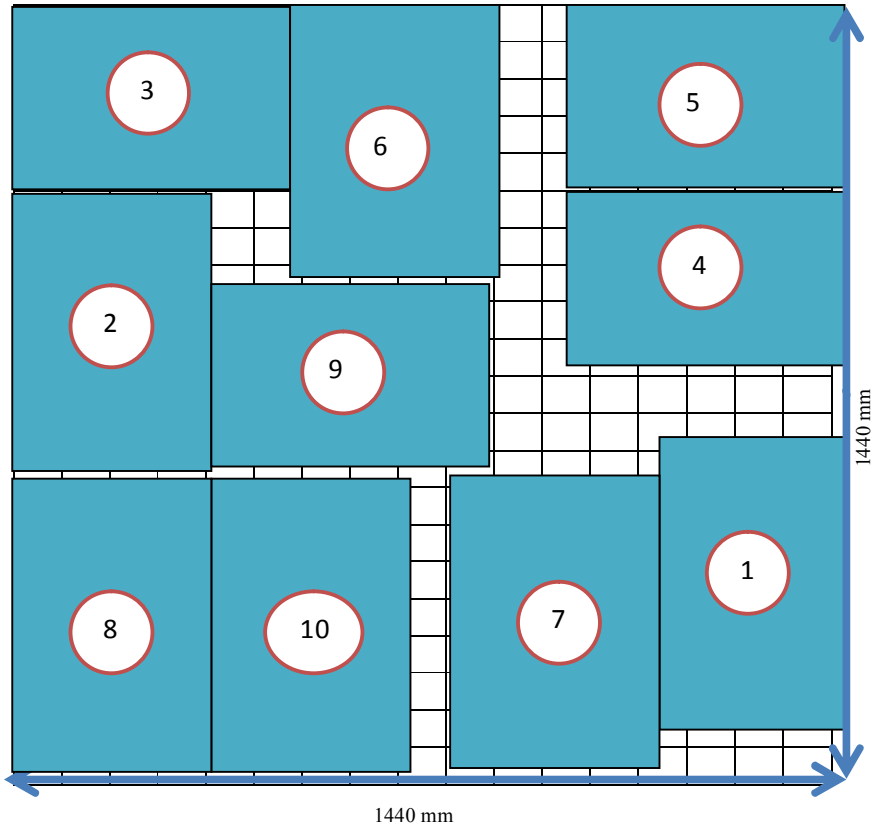


Layout Drawings (Continued)

340 x 510 1440 x 1440 mm (Factory)

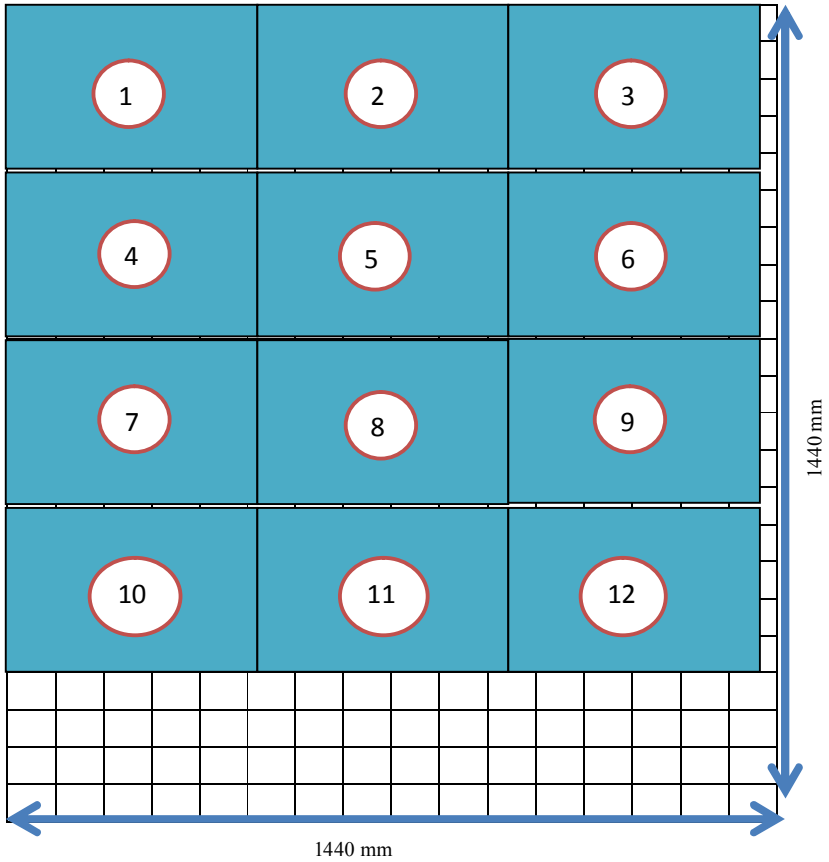


340 x 510 1440 x 1440 mm (Lingo)

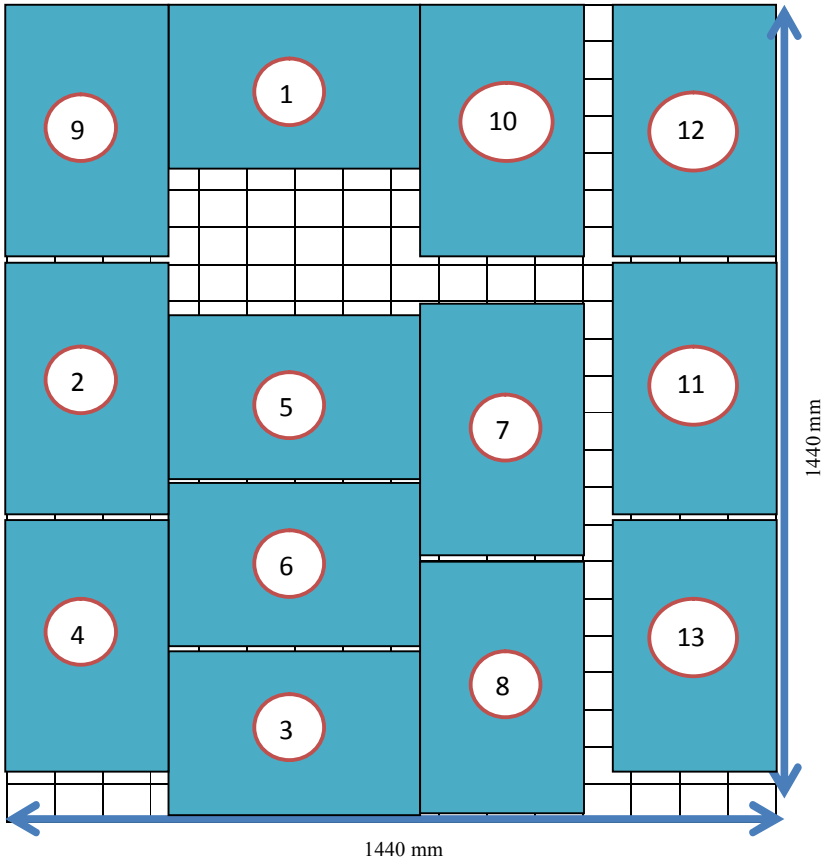


Layout Drawings (Continued)

320 x 460 1440 x 1440 mm (Factory)

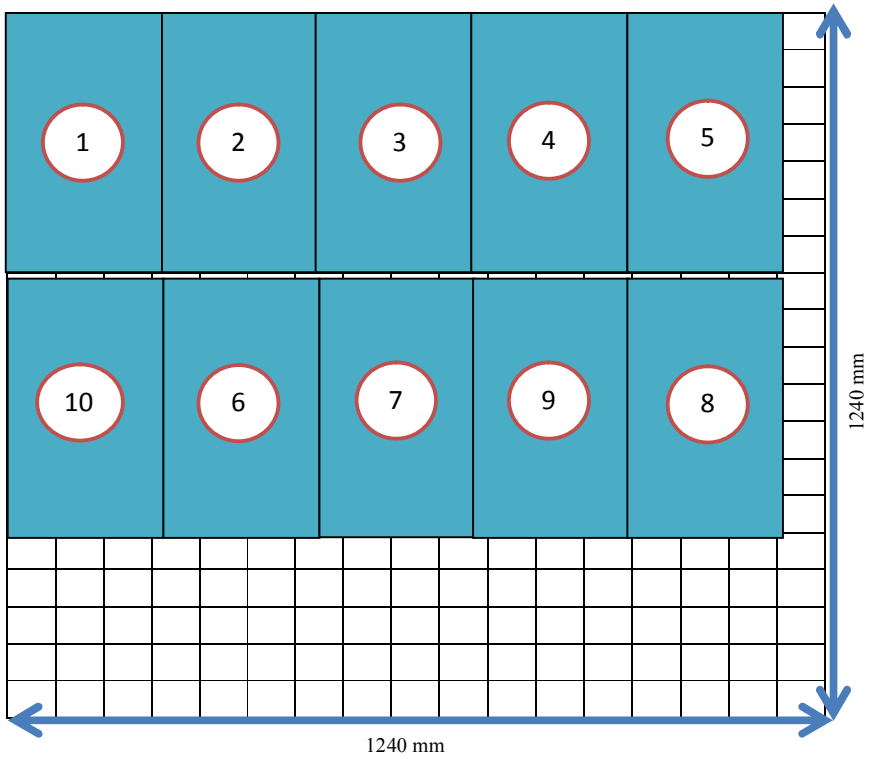


320 x 460 1440 x 1440 mm (Lingo)

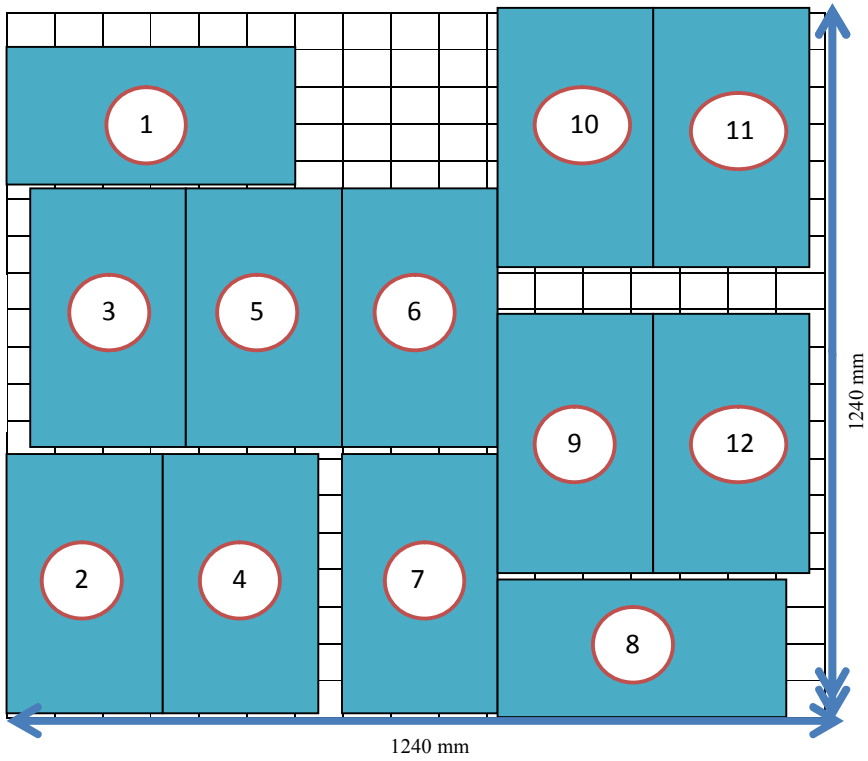


Layout Drawings (Continued)

220 x 420 1240 x 1240 (Factory)

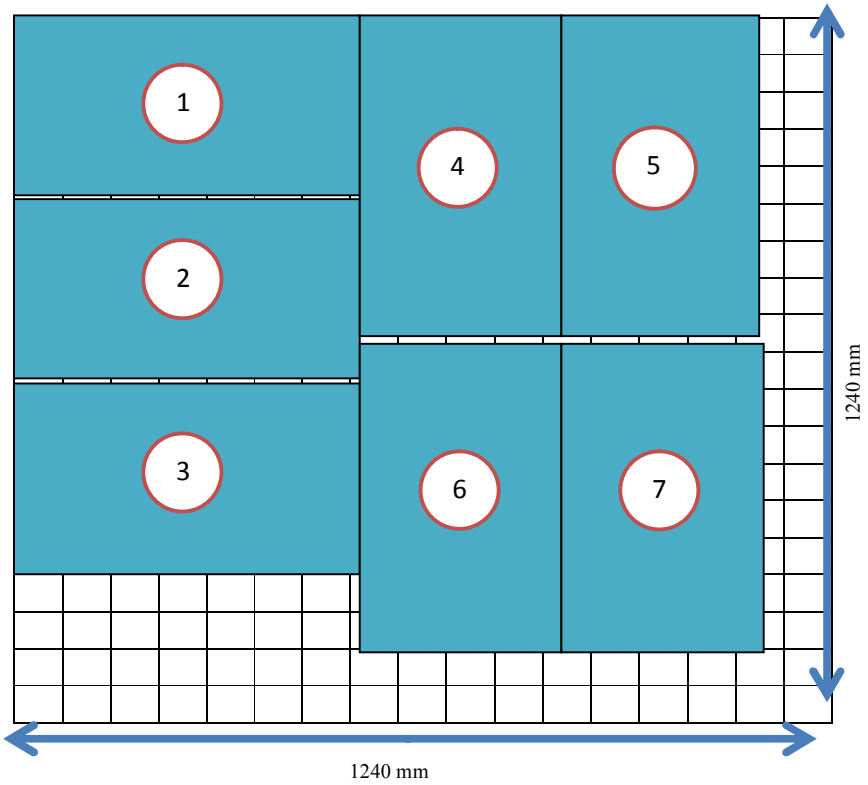


220 x 420 1240 x 1240 (Lingo)

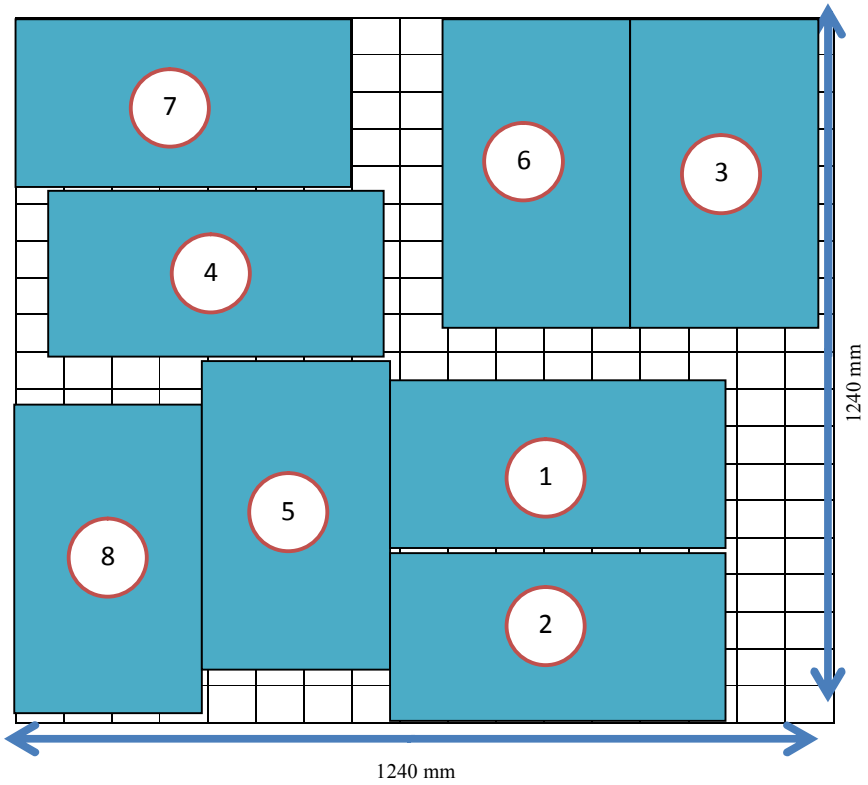


Layout Drawings (Continued)

330 x 540 1240 x 1240 (Factory)

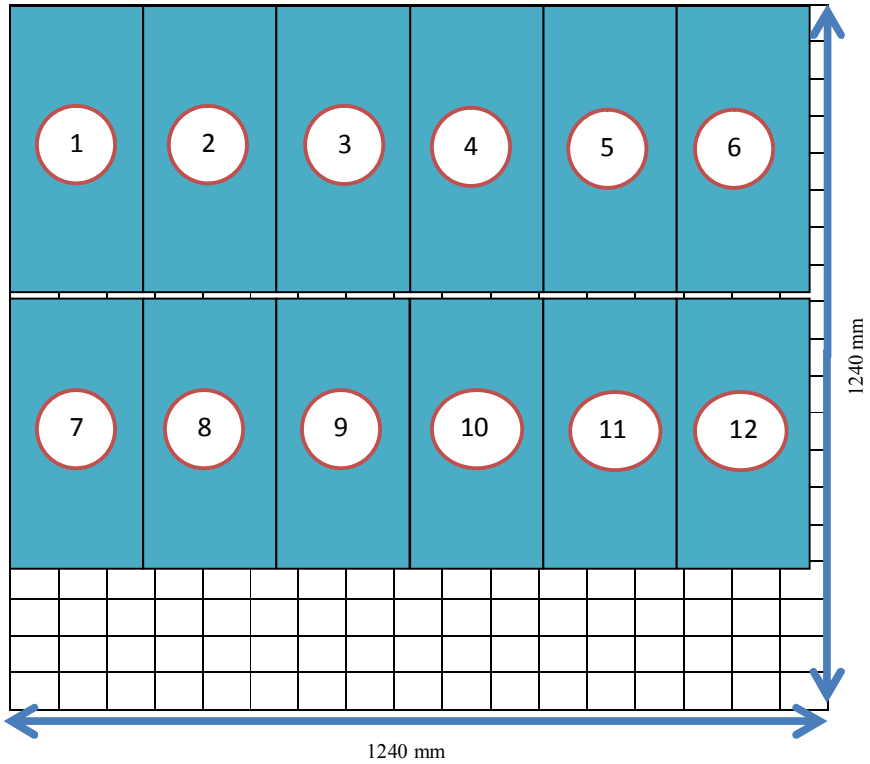


330 x 540 1240 x 1240 (Lingo)

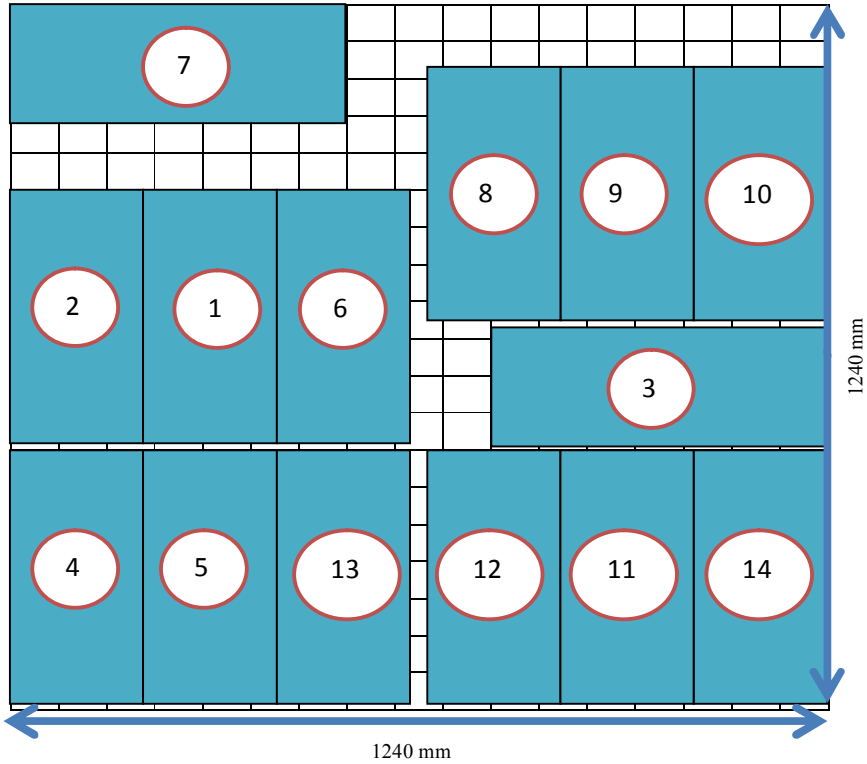


Layout Drawings (Continued)

200 x 460 1240 x 1240 (Factory)



200 x 460 1240 x 1240 (Lingo)



Appendix 5 Determining Best Pallet Type Based on the Minimum Wasted Space per Box

Product			Wasted Area Per Box (mm^2)					
			Pallet 1	Pallet 2	Pallet 3	Pallet 4	Pallet 5	Pallet 6
Product Code	Width	Length	1240*1240	1440*1440	1540*1540	1240*1340	1240*1440	1240*1540
153602223	360	720	48.320	0	37.250	156.200	38.400	59.067
153602242	370	490	10.900	49.100	16.333	26.400	17.100	9.660
153602245	325	390	44.094	11.490	21.475	24.305	22.050	32.383
153602246	325	410	37.594	26.258	14.975	5.217	15.550	25.883
153602247	265	265	25.875	59.375	69.281	33.625	34.810	35.864
153602256	200	460	17.829	23.200	32.821	26.686	27.040	35.307
153602257	460	530	63.720	101.800	19.711	171.600	53.800	74.467
153602259	310	580	12.400	79.400	17.833	27.900	43.400	11.160
153602262	200	460	17.829	23.200	32.821	26.686	27.040	35.307
153602265	285	380	19.833	21.300	23.456	10.386	29.054	28.100
153602369	300	360	20.133	30.240	50.107	30.467	29.354	38.892
153603055	330	540	14.000	29.160	58.960	29.500	20.200	33.978
153603636	420	680	98.800	60.000	10.850	129.800	71.520	96.320
153603637	410	650	117.900	79.100	29.950	10.433	31.100	51.767
153604188	340	540	8.600	23.760	53.560	24.100	14.800	28.578
153604199	340	540	8.600	23.760	53.560	24.100	14.800	28.578
153604455	460	530	63.720	101.800	19.711	171.600	53.800	74.467
153604555	380	750	22.520	129.720	11.450	130.400	161.400	33.267
153604728	400	580	24.267	27.200	64.450	44.933	23.086	40.800
153604936	400	545	38.267	41.200	45.511	19.371	37.086	20.700
153604937	450	585	44.270	82.350	33.200	152.150	34.350	55.017
153605105	325	390	44.094	11.490	21.475	24.305	22.050	32.383
153605476	420	690	17.720	55.800	6.650	125.600	67.320	92.120
153605594	340	510	18.800	33.960	42.200	34.300	25.000	17.560
153605595	340	510	18.800	33.960	42.200	34.300	25.000	17.560
153605645	400	560	32.267	35.200	39.511	52.933	31.086	14.700
153605646	410	650	117.900	79.100	29.950	10.433	31.100	51.767
153605647	330	540	14.000	29.160	58.960	29.500	20.200	33.978
153605745	400	560	32.267	35.200	39.511	52.933	31.086	14.700
153605933	420	690	17.720	55.800	6.650	125.600	67.320	92.120
153605935	460	660	3.920	42.000	91.667	111.800	-6.000	14.667
153605936	420	450	118.520	41.400	74.511	87.933	108.600	129.267
153605937	220	420	35.733	45.840	39.356	46.067	44.954	34.907
153605938	410	690	24.620	62.700	13.550	49.420	14.700	35.367
153605939	410	450	71.767	45.900	79.011	23.200	13.900	27.678
153605940	320	460	23.644	12.308	35.231	18.960	15.127	26.400
153605945	460	460	172.800	18.800	51.911	203.800	86.000	106.667
153605955	290	440	26.160	45.200	41.800	23.455	34.727	31.533
153605961	420	690	17.720	55.800	6.650	125.600	67.320	92.120
153605963	420	690	17.720	55.800	6.650	125.600	67.320	92.120
153605979	420	690	17.720	55.800	6.650	125.600	67.320	92.120
153606162	420	680	98.800	60.000	10.850	129.800	71.520	96.320
153606210	400	660	43.520	81.600	32.450	12.933	33.600	54.267
153606238	430	690	87.700	48.900	98.567	118.700	60.420	85.220
153606369	380	400	18.844	78.400	45.633	32.622	46.400	7.133
153606395	420	680	98.800	60.000	10.850	129.800	71.520	96.320
153606396	310	600	6.200	73.200	29.600	21.700	37.200	4.960
153606407	285	380	19.833	21.300	23.456	19.515	29.054	28.100
153606408	300	360	20.133	30.240	50.107	30.467	29.354	38.892
153606425	420	690	17.720	55.800	6.650	125.600	67.320	92.120
153606426	340	510	18.800	33.960	42.200	34.300	25.000	17.560
153606429	420	690	17.720	55.800	6.650	125.600	67.320	92.120
153606430	320	450	26.844	15.508	38.431	22.160	18.327	15.133
153606434	400	560	32.267	35.200	39.511	52.933	31.086	14.700
153606437	360	540	61.867	-5.891	21.200	42.971	28.800	44.300
153606438	340	510	18.800	33.960	42.200	34.300	25.000	17.560

Determining Best Pallet Type Based on the Minimum Wasted Space per Box (Continued)

153606439	340	510	18.800	33.960	42.200	34.300	25.000	17.560
153606441	360	540	61.867	-5.891	21.200	42.971	28.800	44.300
153606479	340	510	18.800	33.960	42.200	34.300	25.000	17.560
153606480	340	510	18.800	33.960	42.200	34.300	25.000	17.560
153606583	450	640	96.400	57.600	8.450	127.400	69.120	93.920
153606655	380	600	28.267	31.200	68.450	48.933	27.086	10.700
153606658	380	600	28.267	31.200	68.450	48.933	27.086	10.700
153606660	360	400	26.844	28.800	38.431	40.622	18.327	15.133
153606663	620	680	347.200	96.800	171.300	132.267	-64.480	-39.680
153606666	410	650	117.900	79.100	29.950	10.433	31.100	51.767
153606685	330	540	14.000	29.160	58.960	29.500	20.200	33.978
153606715	480	630	82.000	43.200	92.867	251.467	54.720	79.520
153606716	480	630	82.000	43.200	92.867	113.000	54.720	15.867
153606726	420	580	63.920	15.600	52.850	33.333	54.000	74.667
153606727	420	580	63.920	15.600	52.850	33.333	54.000	74.667
153606728	420	590	136.600	11.400	48.650	29.133	49.800	70.467
153606729	380	560	43.467	46.400	24.360	24.571	42.286	25.900
153606730	440	680	85.200	46.400	96.067	116.200	57.920	19.067
153606731	330	540	14.000	29.160	58.960	29.500	20.200	33.978
153606733	460	490	159.000	5.000	38.111	190.000	72.200	92.867
153606734	310	540	24.800	39.960	30.233	69.971	11.160	23.560
153606750	380	750	22.520	129.720	11.450	130.400	161.400	33.267
153606788	380	600	28.267	31.200	68.450	48.933	27.086	10.700
153606790	410	650	117.900	79.100	29.950	10.433	31.100	51.767
153606791	410	650	117.900	79.100	29.950	10.433	31.100	51.767
153606793	420	690	17.720	55.800	6.650	125.600	67.320	92.120
153606797	420	690	17.720	55.800	6.650	125.600	67.320	92.120
153606798	450	640	96.400	57.600	8.450	127.400	69.120	93.920
153606799	450	640	96.400	57.600	8.450	127.400	69.120	93.920
153606935	420	690	17.720	55.800	6.650	125.600	67.320	92.120
153606936	460	660	3.920	42.000	91.667	111.800	-6.000	14.667
153606939	420	680	98.800	60.000	10.850	129.800	71.520	96.320
153606985	400	660	43.520	81.600	32.450	12.933	33.600	54.267
153607045	330	440	25.644	27.600	52.433	20.960	17.127	28.400
153604754	250	465	23.532	31.864	41.857	22.217	21.104	20.150
153604874	250	465	23.532	31.864	41.857	22.217	21.104	20.150
153605118	250	465	23.532	31.864	41.857	22.217	21.104	20.150
153605475	250	465	23.532	31.864	41.857	22.217	21.104	20.150
153606006	250	465	23.532	31.864	41.857	22.217	21.104	20.150

Appendix 6 Best Pallet Options for Each Product Type

A Pallet Loading Problem (PLP) is solved for deciding best loading plan for a particular product type given a particular pallet option. The solution can be expressed in terms maximum number of boxes that can be stowed on that pallet option.

Since we have 6 pallet options with different dimensions, we need to solve 6 different instances of PLP for each product type. The next step is to compare the solutions and deciding the best pallet option. “Wasted area per box (WPB)” is used as the performance criteria for comparison, and the best pallet option is decided as the one with minimum WPB.

The following table represents the details of the outcomes. The entries in each row represent solutions of different pallet options for the product types. In order to follow the table, please remember the following definitions:

A : area of the pallet floor

B : area of a single box (length x width)

n: max number of boxes that can be stowed on the pallet

U : Utilized area by the stowed boxes = (n x B)

W : Wasted area = (A-U)

WPB : Wasted area per box = W/n

Best Pallet Options for Each Product Type (Continued)

Product Code	Pallet 1			Pallet 2			Pallet 3			Pallet 4			Pallet 5			Pallet 6		
	n	W	WPB	n	W	WPB	n	W	WPB	n	W	WPB	n	W	WPB	n	W	WPB
153602223	5	241.600	48.320	8	0	0	8	298.000	37.250	4	624.800	156.200	6	230.400	38.400	6	354.400	59.067
153602242	8	87.200	10.900	9	441.900	49.100	12	196.000	16.333	8	211.200	26.400	9	153.900	17.100	10	96.600	9.660
153602245	9	396.850	44.094	15	172.350	11.490	16	343.600	21.475	11	267.350	24.305	12	264.600	22.050	12	388.600	32.383
153602246	9	338.350	37.594	13	341.350	26.258	16	239.600	14.975	12	62.600	5.217	12	186.600	15.550	12	310.600	25.883
153602247	16	414.000	25.875	16	950.000	59.375	17	1.177.775	69.281	16	538.000	33.625	17	591.775	34.810	18	645.550	35.864
153602256	14	249.600	17.829	18	417.600	23.200	19	623.600	32.821	14	373.600	26.686	15	405.600	27.040	15	529.600	35.307
153602257	5	318.600	63.720	6	610.800	101.800	9	177.400	19.711	4	686.400	171.600	6	322.800	53.800	6	446.800	74.467
153602259	8	99.200	12.400	8	635.200	79.400	12	214.000	17.833	8	223.200	27.900	8	347.200	43.400	10	111.600	11.160
153602262	14	249.600	17.829	18	417.600	23.200	19	623.600	32.821	14	373.600	26.686	15	405.600	27.040	15	529.600	35.307
153602265	12	238.000	19.833	16	340.800	21.300	18	422.200	23.456	14	145.400	10.386	13	377.700	29.054	14	393.400	28.100
153602369	12	241.600	20.133	15	453.600	30.240	15	751.600	50.107	12	365.600	30.467	13	381.600	29.354	13	505.600	38.892
153603055	8	112.000	14.000	10	291.600	29.160	10	589.600	58.960	8	236.000	29.500	9	181.800	20.200	9	305.800	33.978
153603636	4	395.200	98.800	6	360.000	60.000	8	86.800	10.850	4	519.200	129.800	5	357.600	71.520	5	481.600	96.320
153603637	4	471.600	117.900	6	474.600	79.100	8	239.600	29.950	6	62.600	10.433	6	186.600	31.100	6	310.600	51.767
153604188	8	68.800	8.600	10	237.600	23.760	10	535.600	53.560	8	192.800	24.100	9	133.200	14.800	9	257.200	28.578
153604199	8	68.800	8.600	10	237.600	23.760	10	535.600	53.560	8	192.800	24.100	9	133.200	14.800	9	257.200	28.578
153604455	5	318.600	63.720	6	610.800	101.800	9	177.400	19.711	4	686.400	171.600	6	322.800	53.800	6	446.800	74.467
153604555	5	112.600	22.520	5	648.600	129.720	8	91.600	11.450	4	521.600	130.400	4	645.600	161.400	6	199.600	33.267
153604728	6	145.600	24.267	8	217.600	27.200	8	515.600	64.450	6	269.600	44.933	7	161.600	23.086	7	285.600	40.800
153604936	6	229.600	38.267	8	329.600	41.200	9	409.600	45.511	7	135.600	19.371	7	259.600	37.086	8	165.600	20.700
153604937	5	221.350	44.270	6	494.100	82.350	8	265.600	33.200	4	608.600	152.150	6	206.100	34.350	6	330.100	55.017
153605105	9	396.850	44.094	15	172.350	11.490	16	343.600	21.475	11	267.350	24.305	12	264.600	22.050	12	388.600	32.383
153605476	5	88.600	17.720	6	334.800	55.800	8	53.200	6.650	4	502.400	125.600	5	336.600	67.320	5	460.600	92.120

Best Pallet Options for Each Product Type (Continued)

153605594	8	150.400	18.800	10	339.600	33.960	11	464.200	42.200	8	274.400	34.300	9	225.000	25.000	10	175.600	17.560
153605595	8	150.400	18.800	10	339.600	33.960	11	464.200	42.200	8	274.400	34.300	9	225.000	25.000	10	175.600	17.560
153605645	6	193.600	32.267	8	281.600	35.200	9	355.600	39.511	6	317.600	52.933	7	217.600	31.086	8	117.600	14.700
153605646	4	471.600	117.900	6	474.600	79.100	8	239.600	29.950	6	62.600	10.433	6	186.600	31.100	6	310.600	51.767
153605647	8	112.000	14.000	10	291.600	29.160	10	589.600	58.960	8	236.000	29.500	9	181.800	20.200	9	305.800	33.978
153605745	6	193.600	32.267	8	281.600	35.200	9	355.600	39.511	6	317.600	52.933	7	217.600	31.086	8	117.600	14.700
153605933	5	88.600	17.720	6	334.800	55.800	8	53.200	6.650	4	502.400	125.600	5	336.600	67.320	5	460.600	92.120
153605935	5	19.600	3.920	6	252.000	42.000	6	550.000	91.667	4	447.200	111.800	6	-36.000	-6.000	6	88.000	14.667
153605936	5	592.600	118.520	9	372.600	41.400	9	670.600	74.511	6	527.600	87.933	6	651.600	108.600	6	775.600	129.267
153605937	12	428.800	35.733	15	687.600	45.840	18	708.400	39.356	12	552.800	46.067	13	584.400	44.954	15	523.600	34.907
153605938	5	123.100	24.620	6	376.200	62.700	8	108.400	13.550	5	247.100	49.420	6	88.200	14.700	6	212.200	35.367
153605939	6	430.600	71.767	9	413.100	45.900	9	711.100	79.011	8	185.600	23.200	9	125.100	13.900	9	249.100	27.678
153605940	9	212.800	23.644	13	160.000	12.308	13	458.000	35.231	10	189.600	18.960	11	166.400	15.127	11	290.400	26.400
153605945	4	691.200	172.800	9	169.200	18.800	9	467.200	51.911	4	815.200	203.800	6	516.000	86.000	6	640.000	106.667
153605955	10	261.600	26.160	12	542.400	45.200	14	585.200	41.800	11	258.000	23.455	11	382.000	34.727	12	378.400	31.533
153605961	5	88.600	17.720	6	334.800	55.800	8	53.200	6.650	4	502.400	125.600	5	336.600	67.320	5	460.600	92.120
153605963	5	88.600	17.720	6	334.800	55.800	8	53.200	6.650	4	502.400	125.600	5	336.600	67.320	5	460.600	92.120
153605979	5	88.600	17.720	6	334.800	55.800	8	53.200	6.650	4	502.400	125.600	5	336.600	67.320	5	460.600	92.120
153606162	4	395.200	98.800	6	360.000	60.000	8	86.800	10.850	4	519.200	129.800	5	357.600	71.520	5	481.600	96.320
153606210	5	217.600	43.520	6	489.600	81.600	8	259.600	32.450	6	77.600	12.933	6	201.600	33.600	6	325.600	54.267
153606238	4	350.800	87.700	6	293.400	48.900	6	591.400	98.567	4	474.800	118.700	5	302.100	60.420	5	426.100	85.220
153606369	9	169.600	18.844	9	705.600	78.400	12	547.600	45.633	9	293.600	32.622	9	417.600	46.400	12	85.600	7.133
153606395	4	395.200	98.800	6	360.000	60.000	8	86.800	10.850	4	519.200	129.800	5	357.600	71.520	5	481.600	96.320
153606396	8	49.600	6.200	8	585.600	73.200	11	325.600	29.600	8	173.600	21.700	8	297.600	37.200	10	49.600	4.960
153606407	12	238.000	19.833	16	340.800	21.300	18	422.200	23.456	13	253.700	19.515	13	377.700	29.054	14	393.400	28.100
153606408	12	241.600	20.133	15	453.600	30.240	15	751.600	50.107	12	365.600	30.467	13	381.600	29.354	13	505.600	38.892

Best Pallet Options for Each Product Type (Continued)

153606425	5	88.600	17.720	6	334.800	55.800	8	53.200	6.650	4	502.400	125.600	5	336.600	67.320	5	460.600	92.120
153606426	8	150.400	18.800	10	339.600	33.960	11	464.200	42.200	8	274.400	34.300	9	225.000	25.000	10	175.600	17.560
153606429	5	88.600	17.720	6	334.800	55.800	8	53.200	6.650	4	502.400	125.600	5	336.600	67.320	5	460.600	92.120
153606430	9	241.600	26.844	13	201.600	15.508	13	499.600	38.431	10	221.600	22.160	11	201.600	18.327	12	181.600	15.133
153606434	6	193.600	32.267	8	281.600	35.200	9	355.600	39.511	6	317.600	52.933	7	217.600	31.086	8	117.600	14.700
153606437	6	371.200	61.867	11	-64.800	-5.891	11	233.200	21.200	7	300.800	42.971	8	230.400	28.800	8	354.400	44.300
153606438	8	150.400	18.800	10	339.600	33.960	11	464.200	42.200	8	274.400	34.300	9	225.000	25.000	10	175.600	17.560
153606439	8	150.400	18.800	10	339.600	33.960	11	464.200	42.200	8	274.400	34.300	9	225.000	25.000	10	175.600	17.560
153606441	6	371.200	61.867	11	-64.800	-5.891	11	233.200	21.200	7	300.800	42.971	8	230.400	28.800	8	354.400	44.300
153606479	8	150.400	18.800	10	339.600	33.960	11	464.200	42.200	8	274.400	34.300	9	225.000	25.000	10	175.600	17.560
153606480	8	150.400	18.800	10	339.600	33.960	11	464.200	42.200	8	274.400	34.300	9	225.000	25.000	10	175.600	17.560
153606583	4	385.600	96.400	6	345.600	57.600	8	67.600	8.450	4	509.600	127.400	5	345.600	69.120	5	469.600	93.920
153606655	6	169.600	28.267	8	249.600	31.200	8	547.600	68.450	6	293.600	48.933	7	189.600	27.086	8	85.600	10.700
153606658	6	169.600	28.267	8	249.600	31.200	8	547.600	68.450	6	293.600	48.933	7	189.600	27.086	8	85.600	10.700
153606660	9	241.600	26.844	12	345.600	28.800	13	499.600	38.431	9	365.600	40.622	11	201.600	18.327	12	181.600	15.133
153606663	2	694.400	347.200	4	387.200	96.800	4	685.200	171.300	3	396.800	132.267	5	-322.400	-64.480	5	-198.400	-39.680
153606666	4	471.600	117.900	6	474.600	79.100	8	239.600	29.950	6	62.600	10.433	6	186.600	31.100	6	310.600	51.767
153606685	8	112.000	14.000	10	291.600	29.160	10	589.600	58.960	8	236.000	29.500	9	181.800	20.200	9	305.800	33.978
153606715	4	328.000	82.000	6	259.200	43.200	6	557.200	92.867	3	754.400	251.467	5	273.600	54.720	5	397.600	79.520
153606716	4	328.000	82.000	6	259.200	43.200	6	557.200	92.867	4	452.000	113.000	5	273.600	54.720	6	95.200	15.867
153606726	5	319.600	63.920	8	124.800	15.600	8	422.800	52.850	6	200.000	33.333	6	324.000	54.000	6	448.000	74.667
153606727	5	319.600	63.920	8	124.800	15.600	8	422.800	52.850	6	200.000	33.333	6	324.000	54.000	6	448.000	74.667
153606728	4	546.400	136.600	8	91.200	11.400	8	389.200	48.650	6	174.800	29.133	6	298.800	49.800	6	422.800	70.467
153606729	6	260.800	43.467	8	371.200	46.400	10	243.600	24.360	7	172.000	24.571	7	296.000	42.286	8	207.200	25.900
153606730	4	340.800	85.200	6	278.400	46.400	6	576.400	96.067	4	464.800	116.200	5	289.600	57.920	6	114.400	19.067
153606731	8	112.000	14.000	10	291.600	29.160	10	589.600	58.960	8	236.000	29.500	9	181.800	20.200	9	305.800	33.978

Best Pallet Options for Each Product Type (Continued)

153606733	4	636.000	159.000	9	45.000	5.000	9	343.000	38.111	4	760.000	190.000	6	433.200	72.200	6	557.200	92.867
153606734	8	198.400	24.800	10	399.600	39.960	12	362.800	30.233	7	489.800	69.971	10	111.600	11.160	10	235.600	23.560
153606750	5	112.600	22.520	5	648.600	129.720	8	91.600	11.450	4	521.600	130.400	4	645.600	161.400	6	199.600	33.267
153606788	6	169.600	28.267	8	249.600	31.200	8	547.600	68.450	6	293.600	48.933	7	189.600	27.086	8	85.600	10.700
153606790	4	471.600	117.900	6	474.600	79.100	8	239.600	29.950	6	62.600	10.433	6	186.600	31.100	6	310.600	51.767
153606791	4	471.600	117.900	6	474.600	79.100	8	239.600	29.950	6	62.600	10.433	6	186.600	31.100	6	310.600	51.767
153606793	5	88.600	17.720	6	334.800	55.800	8	53.200	6.650	4	502.400	125.600	5	336.600	67.320	5	460.600	92.120
153606797	5	88.600	17.720	6	334.800	55.800	8	53.200	6.650	4	502.400	125.600	5	336.600	67.320	5	460.600	92.120
153606798	4	385.600	96.400	6	345.600	57.600	8	67.600	8.450	4	509.600	127.400	5	345.600	69.120	5	469.600	93.920
153606799	4	385.600	96.400	6	345.600	57.600	8	67.600	8.450	4	509.600	127.400	5	345.600	69.120	5	469.600	93.920
153606935	5	88.600	17.720	6	334.800	55.800	8	53.200	6.650	4	502.400	125.600	5	336.600	67.320	5	460.600	92.120
153606936	5	19.600	3.920	6	252.000	42.000	6	550.000	91.667	4	447.200	111.800	6	-36.000	-6.000	6	88.000	14.667
153606939	4	395.200	98.800	6	360.000	60.000	8	86.800	10.850	4	519.200	129.800	5	357.600	71.520	5	481.600	96.320
153606985	5	217.600	43.520	6	489.600	81.600	8	259.600	32.450	6	77.600	12.933	6	201.600	33.600	6	325.600	54.267
153607045	9	230.800	25.644	12	331.200	27.600	12	629.200	52.433	10	209.600	20.960	11	188.400	17.127	11	312.400	28.400
153604754	11	258.850	23.532	14	446.100	31.864	15	627.850	41.857	12	266.600	22.217	13	274.350	21.104	14	282.100	20.150
153604874	11	258.850	23.532	14	446.100	31.864	15	627.850	41.857	12	266.600	22.217	13	274.350	21.104	14	282.100	20.150
153605118	11	258.850	23.532	14	446.100	31.864	15	627.850	41.857	12	266.600	22.217	13	274.350	21.104	14	282.100	20.150
153605475	11	258.850	23.532	14	446.100	31.864	15	627.850	41.857	12	266.600	22.217	13	274.350	21.104	14	282.100	20.150