YAŞAR UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

MASTERS THESIS

CALCULATING CONFIDENCE INTERVALS FOR IBNR WITH MONTE CARLO SIMULATION: A SOFTWARE FOR IBNR CALCULATIONS

Turgay DOĞUSAN

Thesis Advisor: Assist. Prof. Raif Serkan ALBAYRAK

Department of Actuarial Science

Presentation Date: 18.12.2014

Bornova-İZMİR
2014

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of master of science.

Assist. Prof. Dr. Raif Serkan ALBAYRAK (Supervisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of master of science.

Prof. Dr. Levent KANDİLLER

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of master of science.

Assist. Prof. Dr. Banu ÖZAKÇAN ÖZGÜREL

Prof. Dr. Behzat GÜRKAN
Director of the Graduate School

ABSTRACT

CALCULATING CONFIDENCE INTERVALS FOR IBNR WITH MONTE CARLO SIMULATIONS: A SOFTWARE FOR IBNR CALCULATIONS

DOĞUSAN, Turgay
MSc in Actuarial Science
Supervisor: Assist. Prof. Dr. Raif Serkan ALBAYRAK
June 2014, 157 pages

This thesis introduces two novel confidence interval construction techniques based on Monte Carlo Simulation that can be used by IBNR (Incured But Not Reported) estimation methods that are compatible with the Turkish Legislation. These techniques are coded in a web application that can be used by actuaries. The software computes these techniques permuted by three algorithms -where available, summing upto 15 alternatives. The software includes a widely known confidence interval construction method in literature that uses Bootstrap Algorithm. A performance measure metric is developed in order to compare performance of new confidence interval construction technique is better than the orthodox technique. Results show that our technique is better than the orthodox technique both in terms of length of the confidence interval and accuracy in estimation.

Keywords: IBNR, Monte Carlo Simulation, Bootstrap, IBNR Software, Confidence Interval for IBNR.

ÖZET

Bu tezde iki yeni güven aralığı belirme tekniği Monte Carlo simülasyon yöntemi ile, Türkiye kanunlarına uyumlu IBNR (gerçekleşmiş fakat raporlanmamış) tahminleme yöntemleri kullanılarak oluşturulmuştur. Bu teknikler web tabanlı bir yazılımda aktüerler için geliştirilmiştir. Yazılım iki teknik ve bir tekniğin üç ortalama algoritması için güven aralığı hesaplaması yapmakta ve hali hazırda 15 alternatif çözüm önermektedir. Yazılım ayrıca literatürde kullanılan Bootstrap yöntemi ile güven aralığı oluşturmaktadır. Geliştirilen performans ölçüm metriği yeni güven aralığı oluşturma yöntemleri ile literatürdeki yöntemin karşılaştırılmasını sağlamaktadır. Sonuç olarak tezde geliştirilmiş yöntemler, literatürdeki klasik yönteme göre hem güven aralığının genişliği hem de tahmin doğruluğu yönünden daha başarılı olmuştur.

Anahtar Kelimeler: IBNR, Monte Carlo Simülasyonu, Bootstrap, IBNR yazılımı, IBNR için güven aralığı.

TEXT OF OATH

I declare and honestly confirm that my study, titled “CALCULATING CONFIDENCE INTERVALS FOR IBNR WITH MONTE CARLO SIMULATIONS: A SOFTWARE FOR IBNR CALCULATIONS” and presented as a Master’s Thesis, has been written without applying to any assistance inconsistent with scientific ethics and traditions, that all sources from which I have benefited are listed in the bibliography, and that I have benefited from these sources by means of making references.

Tevfik Turgay DOĞUSAN

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF EQUATIONS

1 INTRODUCTION

IBNR corresponds to an estimation of total amount owed by the insurer that has not yet been reported by the claimants. Since neither the frequency of these losses that have occurred nor the severity of each loss can be known, the IBNR value is necessarily an estimate. Making accurate IBNR estimations are important for internal management of the insurance company, investors and regulators of the insurance sector.

The accuracy of the IBNR estimates is crucial in the route of decision making. Inaccurate IBNR estimates would endanger the financial condition of the company. From investors' perspective, inaccurate estimates of IBNR directly deteriorate financial reports such as balance sheets and income statements. Investors use key metrics in these reports to measure the financial strength of a company for investment purposes. Furthermore, miscalculated values (either severely under or above specified IBNR) will make a snowball effect on the financial system since regulators are responsible from the reliability of the information in these reports as actions taken by financial agents is based on these values.

Recently introduced The Solvency II regulations (European Commission, 2014) in Turkey forces the insurers to implement methods of risk management efficiently and there are 3 pillars in the Solvency II Directive framework as a way of grouping the requirements. The most challenging one is the Pillar 1 that focuses on the quantitative requirements and within this framework Insurance Company has to calculate the capital requirements. The Solvency Capital Requirement (SCR) is the new solvency standard for firms in Solvency II directives. SCR is based on the idea that an insurance firm should have the amount of capital that is sufficient with a 99.5% confidence-level to guarantee that the firm will have enough assets to cover its liabilities at the end of one year period. These requirements introduce new approach to establishing calculation of outstanding claims and premiums.

Due to the absence of a national software that is compatible with Turkish Legislations, actuaries have to calculate IBNR values using simple Excel Spreadsheets. They provide only point estimations for IBNR reserve calculations. Still, Actuaries demand confidence intervals associated with those techniques. In a number of countries (e.g., Australia, Singapore, the United Kingdom, and South Africa.), insurers are required to hold provisions (i.e., the estimate of unpaid claims) at a specified confidence level (75%) (Friedland, J. 2009).

This thesis aims to develop a national software compatible with IBNR methods regulated by Turkish Legislations. The developed software computes IBNR via four techniques permuted by three averaging algorithms thus yielding 15 alternatives. Furthermore we embeded three confidence interval construction techniques into the software. One of these algorithms is widely known in the literature and makes use the Bootstrap Algorithm. Two additional algorithms are developed for the first time in this thesis based on our knowledge . They use Monte Carlo Simulation techniques to sample from a high dimensional joint distribution space of estimates. The parametric form of the marginal distributions of estimations are borrowed from Mack (1993). Three techniques are compared with a performance metric also developed originally in this thesis.

This chapter is followed by a literature review of IBNR techniques. The techniques that are used in this thesis is explained in detail with examples. The third chapter starts with a review of confidence interval construction with the Bootstrap Technique and continues with the introduction of two novel techniques based on Manto Carlo Simulation. The fourth chapter explains the architecture of the software as well. This chapter includes a User's Manual for the software. The fifth chapter starts with introducing the performance metric developed and used in this thesis. The performance of confidence interval construction techniques are compared by means of 10 real world dataset. This thesis concludes with chapter 6.

2 LITERATURE REVIEW

“Unlike manufacturers, insurers may not know the true cost of goods sold during a financial reporting period until several years later.” (Friedland, 2010). Unlike some events where the insurer knows the exact cost of the settlements, for some other insured events the insurer may not know the ultimate cost. However as a financial institution, the company must report their financial claims on a regular basis. Since for those events that have not occurred yet, the settlements would be necessarily estimates and they need to be reported as estimates under current liabilities subheading as claims that occurred on or prior to the financial report date but have not yet been paid. These claims are called incurred but not reported claims.(IBNR).

Using the intuitive notation of Faculty and Institute of Actuaries Claims Reserving Manual v.1 (1997) IBNR claims are those that incurred before the reserving date but not reported yet. In symbols:

$$a] < v] < r]$$

where a] = accident date, v] = reserving date, r] = reporting date.

However the complications regarding the calculation of IBNR involve not only technical issues regarding the accuracy of the estimator but also from the nature of the reporting process. Following the manual;

> “…Claims will normally come in through branch offices, brokers or agents for the insurer concerned. There will be an interval between such actual first report and the later time when the claim is notified to head office and/or formally recorded in the insurer's main data base. For reserving purposes the latter event will be the more convenient one to take. Hence a ‘reported’ claim is normally one which has already been processed to the extent that a central record on it is held. The corollary is that, at any time, there are likely to be claims in the pipeline, already reported at branch level, but still counting for reserving purposes as IBNR.” (Claims Reserving Manual, vol.1: Section I: Methods for IBNR, p.I1.1 Christofides, S.1990)

Regarding the ambiguity of the meaning that the term IBNR refers as it may refer either to the claims in this category or to the reserves (sometimes IBNER - incurred but not enough reserved), we use the term IBNR for its former meaning.

2.1 Stakeholder Perspectives on IBNR

There are three distinguished insurance company stakeholder perspectives for IBNR (Friedland, 2009). These stakeholders are:

1. Internal management
2. Investors
3. Regulators

2.1.1 Internal Management

According to internal management the accuracy of the IBNR estimates are crucial in the route of decision making. This information is used in insurance company’s decisions including pricing, underwriting, strategic and financial. Furthermore, inaccurate IBNR estimates would endanger the financial condition of the company. For instance, under-estimated IBNR might urge the insurance company to lower its prices for risk not realizing that the unpaid claims were insufficient to cover historical claims.The inverse logic goes in the same manner: over-valued IBNR would motivate the company to unnecessarily increase its risk premiums and hence lose market share or even worse. Gaining unrealistic market share due to inaccuracy in IBNR calculations (or losing) could eventually lead to a situation where the future solvency of the insurance company is at risk.This flow of logic regresses to further consequences regarding misspecifications in reassurance needs and financial decision making such as capital management.

2.1.2 Investors

Inaccurate estimates of IBNR directly deteriorate financial reports such as balance sheets and income statements. Investors use the key metrics in these reports to measure the financial strength of a company for investment purposes. In return these metrics has substantial effect on the market value of a company.

2.1.3 Regulator

Regulators of the insurance sector carry out their supervisions on relying key metrics in financial statements. Regulator is responsible from the reliability of the information in these reports as actions taken by financial agents is based on these values. Miscalculated values (either severely under or above specified IBNR) will make a snowball effect on the financial system. Currently, all around the world the responsibility of appropriately calculating IBNR has been delegated to appointed actuaries of insurance companies through legislations.

2.2 Solvency II

Recently new regulations, called Solvency II (European Commission, 2014), are one the most important subject to the Insurance Sector in Turkey. The Solvency II Directive is a standard that forces the insurers to implement methods of risk management efficiently and there are three pillars in the Solvency II Directive framework as a way of grouping the requirements. The most challenging one is the Pillar 1 that focuses on the quantitative requirements and within this framework Insurance Company has to calculate the capital requirements. The Solvency Capital Requirement (SCR) is the new solvency standard for the firms in Solvency II directives. SCR is based on the idea that an insurance firm should have the amount of capital that is sufficient with a 99.5% confidence-level to guarantee that the firm will have enough assets to cover its liabilities at the end of one year period. This requirements introduce a new approach to establishing calculation of outstanding claims and premiums. This approach is driven by calculations on market consistent so risk orientated modelling has a great importance for claim calculations.

“Internal models” are substantial matter for insurance companies that find the standard model being infeasible their company or country like Turkey. Moreover, internal models can improve risk sensivity and ensure better risk management if the risk is determined in a better way. Therefore the calculation of claim reserves is important and a challenging job in SCR. We compare the performance of the Bootstrap confidence interval of the reserve with a new two approaches which are developed in the thesis.

2.3 Literature

There are various methods for estimating claim reserves in literature and some of them are useful in practice. Especially the chain ladder method is one of the widely used methods for these estimations. In this thesis, the deterministic chain ladder method is used. Many models and techniques have been presented to predict the sub-triangles. The chain ladder method was basically developed on past experience. Different methods have been suggested for IBNR reserve calculations since the original work of Tarbell in 1934. Alternative techniques in terms of classical statistics perspective were suggested by Taylor and Ashe (1983), De Jong and Zehnwirth (1983), Renshaw (1989, 1994), Verrall (1989, 1991, 1993, 1994, 1996), Hagerman and Renshaw (1996), and Renshaw and Verrall (1998). Jewell (1989), Verrall (1990), Makov et al. (1996), Haastrup and Arjas (1996), Alba et al. (1998), and Scollnik (2001) studied the in claim reserving with Bayesian approach. Murphy (1994) described the chain ladder technique within a normal linear regression framework and derived analytic formulas for the reserve risk. Also, various extensions are made by Barnett&Zehnwirth (1998). The Bornhuetter-Ferguson (1972) suggested a method that was built on exposure.

Despite its simplicity and being distribution free the chain ladder method results are very sensitive to variations in the last year's accident data. This situation makes error calculations important in the reserve estimation procedure (Mack, 1993). So it became a great deal to calculate the standard errors of the chain ladder reserves to deal with the data uncertainty. Many papers have been published that address this issue. Least square regression with assuming a lognormal distribution method is studied by Zehnwirth (1985), Renshaw (1989), Christofides (1990), Verrall (1990), Verrall (1990, 1991). Later Mack (1990), used the maximum likelihood estimation with assuming a gamma distribution for the same procedure. Also in the studies by Wright (1990) a generalized linear model was used for the calculation of the standard errors. Mack (1993) suggested a very simple formula for the standard error of chain ladder reserve estimates were developed. This study was by Schnieper (1991) who used a mixture of the Bornhuetter-Ferguson technique and the chain ladder method and estimated the standard errors via using a Taylor Series approximation.

We next review IBNR calculation methods analyzed in this thesis: Chain Ladder, Expected Loss Ratio, Borhhuetter-Ferguson and Average Cost Per Claim Method.

2.3.1 Chain Ladder

This method is the most globally accepted and implemented method due to its simplicity. Insurance and reassurance companies use different versions of chain ladder method. This method was originally developed by Tarbell in 1934. The method calculates IBNR by making use of the upper triangle of a matrix like the one given in Table 1.

	0	1	j	n-i	...	n-1	n
0	$X_{0,0}$	$X_{0,1}$	$X_{0,j}$	$X_{0,n-i}$	…	$X_{0,n-1}$	$X_{0,n}$
1	$X_{1,0}$	$X_{1,1}$	$X_{1,j}$	$X_{1,n-i}$	…	$X_{1,n-1}$	
...	…	…	…	…	…		
i	$X_{i,0}$	$X_{i,1}$	$X_{i,j}$	$X_{i,n-i}$			
...	…	…	…				
n-1	$X_{n-1,0}$	$X_{n-1,1}$					
n	$X_{n,0}$						

Table 1 Typical triangle of chain ladder

Parameter *i* refers to the occurrence date (year, month, etc.). For example, if a car accident happened in 2006, it takes the value of 2006. Parameter *j* refers to development or the payment date (year, month, etc.). For instance, the car accident payment occurred in 2008 is stating the value of j as 2008. Parameter $X_{i,j}$ refers to the incremental payment amount. For instance, $X_{2006,2008}$ denotes the car accident happened in 2006 but the payment was realized in 2008. Consider the following example depicted in Table 2. The blue cell having the value of 3,209 refers to the damage that happened in 2000 and the payment that realized in 2000, which is $X_{2000,0}$. Green backrounded of cell whose value`s 1678 refers to the damage that happaned in 2003 and the payment that occurred in 2004. That refers to $X_{2003,0}$. Yellow backrounded of cell which is value 10 refer to damage that happened in 2001 and payment that occurred in 2005. That refer to $X_{2001,4}$. The lower triangle part of the matrix given in Table 2 is reserved for the future payments. For example, the red cell refers to the damage happened in 2005 and the payment that will occur in 2009.

	0	1	2	3	4	5
2000	3209	1163	39	17	7	21
2001	3367	1292	37	24	10	
2002	3871	1474	53	22		
2003	4239	1678	103			
2004	4929	1865				
2005	5217				?	

Table 2 Incremental payments of triangle

The first step of the chain ladder method transforms the incremental payments triangle into the cumulative payments triangle. The parameter $C_{i,j}$ refers to the cumulative payment due to the damage happened in year *i* and the cumulative payment that occurred in year *j*. The cumulative triangle whose values are calculated by means of Equation 1 is shown in Table 3.

$$C_{i,j} = X_{i,0} + X_{i,1} + X_{i,2} + \ldots + X_{i,j}$$

Equation 1 Transforms incremental to cumulative triangle

	0	1	j	n-i	...	n-1	n
0	$C_{0,0}$	$C_{0,1}$	$C_{0,j}$	$C_{0,n-i}$	$\ldots$	$C_{0,n-1}$	$C_{0,n}$
1	$C_{1,0}$	$C_{1,1}$	$C_{1,j}$	$C_{1,n-i}$	$\ldots$	$C_{1,n-1}$	
...	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$		
i	$C_{i,0}$	$C_{i,1}$	$C_{i,j}$	$C_{i,n-i}$			
...	$\ldots$	$\ldots$	$\ldots$				
n-1	$C_{n-1,0}$	$C_{n-1,1}$					
n	$C_{n,0}$						

Table 3 Cumulative payment triangle of chain ladder

Now we calculate the cumulative payments of the values reported in Table 2.

First iteration:

$$C_{2000,1} = X_{2000,0} + X_{2000,1}$$

$$C_{0,1} = 3209 + 1163$$

$$C_{0,1} = 4372$$

	0	1	2	3	4	5
2000	3209	4372				
2001	3367					
2002	3871					
2003	4239					
2004	4929					
2005	5217					

Table 4 First iteration

Second iteration:

$$C_{2000,2} = X_{2000,0} + X_{2000,1} + X_{2000,2}$$

$$C_{2000,2} = 3209 + 1163 + 39$$

$$C_{2000,2} = 4411$$

	0	1	2	3	4	5
2000	3209	4372	4411			
2001	3367					
2002	3871					
2003	4239					
2004	4929					
2005	5217					

Table 5 Second iteration and Third iteration:

$$C_{2000,3} = X_{2000,0} + X_{2000,1} + X_{2000,2} + X_{2000,3}$$

$$C_{2000,3} = 3209 + 1163 + 39 + 17$$

$$C_{2000,3} = 4428$$

	0	1	2	3	4	5
2000	3209	4372	4411	4428		
2001	3367					
2002	3871					
2003	4239					
2004	4929					
2005	5217					

Table 6 Third iteration

Now we apply the same procedure to all the cells.

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794				
2005	5217					

Table 7 Full cumulative payments triangle

The second step of the method is about the prediction of the future claims. Cells which have red background below of the ladder are used for the future claims. The method uses an estimator to predict the future claims, λ_j, which is calculated based on historical data. λ_j is the growth ratio of between consecutive years yielding the calculating of future claims:

$$\lambda_j = \frac{\sum_{i=1}^{n-j} C_{i,j+1}}{\sum_{i=1}^{n-j} C_{i,j}}$$

Equation 2 Calculation of estimator

First iteration:

$$\lambda_0 = \frac{C_{2000,1+}C_{2001,1+}C_{2002,1+}C_{2003,1+}C_{2004,1}}{C_{2000,0+}C_{2001,0+}C_{2002,0+}C_{2003,0+}C_{2004,0}}$$

$$\lambda_0 = \frac{4372+4659+5345+5917+6794}{3209+3367+3871+4239+4929}$$

$$\lambda_0 = \frac{27087}{19615}$$

$\lambda_0 = 1.380933$

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794				
2005	5217					
λ_j	1.380933					

Table 8 First iteration

Second iteration:

$$\lambda_1 = \frac{C_{2000,2+}C_{2001,2+}C_{2002,2+}C_{2003,2}}{C_{2000,1+}C_{2001,1+}C_{2002,1+}C_{2003,1}}$$

$$\lambda_1 = \frac{4411+4696+5398+6020}{4372+4659+5345+5917}$$

$$\lambda_1 = \frac{20525}{20293}$$

$\lambda_1 = 1.011433$

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794				
2005	5217					
λ_j	1.380933	1.011433				

Table 9 Second iteration

Now we apply the same procedure to all of the remaning cells.

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794				
2005	5217					
	λ_j	**1.380933**	**1.011433**	**1.004343**	**1.001858**	**1.004735**

Table 10 Full estimators

Finally, we can calculate the future claims by multiplying the cumulative payment ($C_{i,j}$ with the growth ratio (λ_j):

$$\hat{C}_{i,j} = \left[\hat{\lambda}_{n+1-i} \ldots \hat{\lambda}_{j-1}\right] C_{i,n+1-i}$$

Equation 3 Prediction of future claims

Now we calculate the future claims of the values reported in Table 7 and Table 10.

First iteration:

$\hat{C}_{2005,1} = C_{2005,0} \times \lambda_1$

$\hat{C}_{2005,1} = 5214 \times 1.380933$

$\hat{C}_{2005,1} = 7204.33$

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794				
2005	5217	**7204.33**	**?**			

Table 11 First iteration

Second iteration:

$\hat{C}_{2005,2} = C_{2005,1} \times \lambda_2$

$\hat{C}_{2005,2} = 7204.33 \times 1.011433$

$\hat{C}_{2005,2} = 7286,69$

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794	?			
2005	5217	7204,33	**7286,69**			

Table 12 Second iteration

Third iteration:

$$\hat{C}_{2004,2} = C_{2004,1} \times \lambda_2$$

$$\hat{C}_{2004,2} = 6794 \times 1.011433$$

$$\hat{C}_{2004,2} = 6871.67$$

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794	**6871.67**			
2005	5217	7204.33	7286.69			

Table 13 Third iteration

Now we apply the same procedure to all the cells.

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	4752.40
2002	3871	5345	5398	5420	5430.07	5455.78
2003	4239	5917	6020	6046.15	6057.38	6086.06
2004	4929	6794	6871.67	6901.52	6914.34	6947.08
2005	5217	7204.33	7286.69	7318.34	7331.94	7366.66

Table 14 Full future claims payment

The third step of the method computes the most important parameter, IBNR RESERVE, referring to the funds for the future claims. IBNR RESERVE is equal to the difference of the ultimate of prediction claims and the actual claims. The actual claims are the maximum payment $max_j \{ C_{i,j} \}$ in each row.

	0	1	2	3	4	5	**Actual Ultimate Claim**
2000	3209	4372	4411	4428	4435	4456	$\max_j \{C_{2000,j}\}$
2001	3367	4659	4696	4720	4730		$\max_j \{C_{2001,j}\}$
2002	3871	5345	5398	5420			$\max_j \{C_{2002,j}\}$
2003	4239	5917	6020				$\max_j \{C_{2003,j}\}$
2004	4929	6794					$\max_j \{C_{2004,j}\}$
2005	5217						$\max_j \{C_{2005,j}\}$

Table 15 Calculation of actual ultimate claims

Now we calculate the actual ultimate claim of the values reported in Table 15

First iteration:

$$U^a_{2000} = \max(C_{2000,0}, C_{2000,1}, C_{2000,2}, C_{2000,3}, C_{2000,4}, C_{2000,5})$$

$$U^a_{2000} = \max(3209, 4372, 4411, 4428, 4435, 4456)$$

$$U^a_{2000} = 4456$$

	0	1	2	3	4	5	**Actual Ultimate Claim**
2000	3209	4372	4411	4428	4435	4456	**4456**
2001	3367	4659	4696	4720	4730		
2002	3871	5345	5398	5420			
2003	4239	5917	6020				
2004	4929	6794					
2005	5217						

Table 16 First iteration

Second iteration:

$$U^a_{2001} = \max(C_{2001,0}, C_{2001,1}, C_{2001,2}, C_{2001,3}, C_{2001,4})$$

$$U^a_{2001} = \max(3367, 4659, 4696, 4720, 4730)$$

$$U^a_{2001} = 4730$$

	0	1	2	3	4	5	Actual Ultimate Claim
2000	3209	4372	4411	4428	4435	4456	4456
2001	3367	4659	4696	4720	4730		**4730**
2002	3871	5345	5398	5420			
2003	4239	5917	6020				
2004	4929	6794					
2005	5217						

Table 17 Second iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5	Actual Ultimate Claim
2000	3209	4372	4411	4428	4435	4456	4456
2001	3367	4659	4696	4720	4730		4730
2002	3871	5345	5398	5420			5420
2003	4239	5917	6020				6020
2004	4929	6794					6794
2005	5217						5214

Table 18 All ultimate claims

The second step of the method is about the prediction of the ultimate claims. We calculate the prediction of ultimate claim of the values reported in Table 14. Prediction of ultimate claim is equal to last column of the future claims.

	0	1	2	3	4	5	Prediction of Ultimate Claims
2000	3209	4372	4411	4428	4435	4456	**4456**
2001	3367	4659	4696	4720	4730	4752.40	**4752.40**
2002	3871	5345	5398	5420	5430.07	5455.78	**5455.78**
2003	4239	5917	6020	6046.15	6057.38	6086.06	**6086.06**
2004	4929	6794	6871.67	6901.52	6914.34	6947.08	**6947.08**
2005	5217	7204.33	7286.69	7318.34	7331.94	7366.66	**7366.66**

Table 19 Prediction of ultimate claims

Now we calculate the IBNR RESERVE of the values reported in Table 18 and Table 19. The related formula is given below:

$$\mathrm{R}_i = U_i^{\,p} - U_i^{\,a}$$

Equation 4 Calculation of Reserve

First iteration:

$$\mathrm{R}_{2000} = U^{p}_{2000} - U^{a}_{2000}$$
$$R_{2000} = 4456 - 4456$$
$$R_{2000} = 0$$

	0	1	2	3	4	5	**Actual Ultimate Claim**	**Prediction of Ultimate Claim**	**IBNR RESERVE**
2000	3209	4372	4411	4428	4435	4456	4456	4456	0
2001	3367	4659	4696	4720	4730	4752.40	4730	4752.40	
2002	3871	5345	5398	5420	5430.07	5455.78	5420	5455.78	
2003	4239	5917	6020	6046.15	6057.38	6086.06	6020	6086.06	
2004	4929	6794	6871.67	6901.52	6914.34	6947.08	6794	6947.08	
2005	5217	7204.33	7286.69	7318.34	7331.94	7366.66	5217	7366.66	

Table 20 First iteration

Second iteration:

$$\mathrm{R}_{2002} = U^{p}_{2002} - U^{a}_{2002}$$
$$R_{2002} = 4752.40 - 4730$$
$$R_{2002} = 22.40$$

	0	1	2	3	4	5	**Actual Ultimate Claim**	**Prediction of Ultimate Claim**	**IBNR RESERVE**
2000	3209	4372	4411	4428	4435	4456	4456	4456	0
2001	3367	4659	4696	4720	4730	4752.40	4730	4752.40	22.40
2002	3871	5345	5398	5420	5430.07	5455.78	5420	5455.78	
2003	4239	5917	6020	6046.15	6057.38	6086.06	6020	6086.06	
2004	4929	6794	6871.67	6901.52	6914.34	6947.08	6794	6947.08	
2005	5217	7204.33	7286.69	7318.34	7331.94	7366.66	5217	7366.66	

Table 21 Second iteration

Third iteration:

$$R_{2003} = U^{p}_{2003} - U^{a}_{2003}$$
$$R_{2003} = 5455.78 - 5420$$
$$R_{2003} = 66.06$$

	0	1	2	3	4	5	**Actual Ultimate Claim**	**Prediction of Ultimate Claim**	**IBNR RESERVE**
2000	3209	4372	4411	4428	4435	4456	4456	4456	0
2001	3367	4659	4696	4720	4730	4752.40	4730	4752.40	22.40
2002	3871	5345	5398	5420	5430.07	5455.78	5420	5455.78	35.78
2003	4239	5917	6020	6046.15	6057.38	6086.06	6020	6086.06	
2004	4929	6794	6871.67	6901.52	6914.34	6947.08	6794	6947.08	
2005	5217	7204.33	7286.69	7318.34	7331.94	7366.66	5217	7366.66	

Table 22 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5	**Actual Ultimate Claim**	**Prediction of Ultimate Claim**	**IBNR RESERVE**
2000	3209	4372	4411	4428	4435	4456	4456	4456	0
2001	3367	4659	4696	4720	4730	4752.40	4730	4752.40	22.40
2002	3871	5345	5398	5420	5430.07	5455.78	5420	5455.78	35.78
2003	4239	5917	6020	6046.15	6057.38	6086.06	6020	6086.06	66.06
2004	4929	6794	6871.67	6901.52	6914.34	6947.08	6794	6947.08	153.08
2005	5217	7204.33	7286.69	7318.34	7331.94	7366.66	5217	7366.66	2149.66

Table 23 Result of IBNR RESERVE

Finally, we calculate the funds of future claims payment which is the summation of all IBNR RESERVE values:

$$\text{Total Reserve} = \sum_{i}^{n} R_i$$

Equation 5 Calculation of total reserve

In our example calculation, we have

Total Reserve= 0+22.40+35.78+66.06+153.08+2149.66= 2426.99.

2.3.2 The Expected Loss Ratio Methods

This method is one of the simple methods for calculating the claims. It is used when there is no past data available for analysis. It is usually applied in new insurance branches or small-scaled businesses. The method calculates IBNR by using the triangle like the one given in Table 24.

	0	1	j	n-i	...	n-1	n
0	$X_{0,0}$	$X_{0,1}$	$X_{0,j}$	$X_{0,n-i}$	...	$X_{0,n-1}$	$X_{0,n}$
1	$X_{1,0}$	$X_{1,1}$	$X_{1,j}$	$X_{1,n-i}$	...	$X_{1,n-1}$	
...	…	…	…	..	…		
i	$X_{i,0}$	$X_{i,1}$	$X_{i,j}$	$X_{i,n-i}$			
...	…	…	…				
n-1	$X_{n-1,0}$	$X_{n-1,1}$					
n	$X_{n,0}$						

Table 24 Typical triangel of chaind ladder

Parameter i refers to the occurance date (year, month, etc.). For example. if a car accident happened in 2006. takes the value of 2006. Parameter j refers to then development or the payment date (year, month, etc.). For instance, the car accident payment occurred in 2008 is stating the value of j as 2008. Parameter $X_{i.j}$ refers to the incremental payment amount. For instance, $X_{2006,2008}$ denotes the car accident happened in 2006 but the payment was realized in 2008. Consider the following example depicted in Table 25.

	0	1	2	3	4	5
2000	3209	1163	39	17	7	21
2001	3367	1292	37	24	10	
2002	3871	1474	53	22		
2003	4239	1678	103			
2004	4929	1865				
2005	5217				?	

Table 25 Incremental payments of triangle

The blue cell having the value of 3209 refers to the damage that happened in 2000 and the payment that realized in 2000, which is $X_{2000,0}$. Green backround of cell which is value 1678 refer to damage that happaned in 2003 and payment that occurred in 2004. That refer to $X_{2003,0}$. Yellow backround of cell which is value 10 refer to damage that happened in 2001 and payment that occurred in 2005. That refer to $X_{2001,4}$. The lower triangler part of the matrix given in Table 26 is reserved for future payments. For example, the red cell refers the damage happened in 2005 and the payment that will occur in 2009.

The first step of the chain ladder method transforms the incremental payments triangle into the cumulative payments triangle. The parameter $C_{i.j}$ refers to the cumulative payment due to the damage happened in year *i* and the cumulative payment that occurred in year *j*. The Cumulative triangle whose values are calculated by means of Equation 6 is shown in Table 26.

$$C_{i,j} = X_{i,0} + X_{i,1} + X_{i,2} + ... + X_{i,j}$$

Equation 6 Transforms incremental to cumulative triangle

	0	1	j	n-i	...	n-1	n
0	$C_{0,0}$	$C_{0,1}$	$C_{0,j}$	$C_{0,n-i}$	$\cdots$	$C_{0,n-1}$	$C_{0,n}$
1	$C_{1,0}$	$C_{1,1}$	$C_{1,j}$	$C_{1,n-i}$	$\cdots$	$C_{1,n-1}$	
...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$		
i	$C_{i,0}$	$C_{i,1}$	$C_{i,j}$	$C_{i,n-i}$			
...	$\cdots$	$\cdots$	$\cdots$				
n-1	$C_{n-1,0}$	$C_{n-1,1}$					
n	$C_{n,0}$						

Table 26 Cumulative payment triangle of chain ladder

Now we calculate the cumulative payments of the values reported in Table 25.

First iteration:

$$C_{2000,1} = X_{2000,0} + X_{2000,1}$$
$$C_{0,1} = 3209 + 1163$$
$$C_{0,1} = 4372$$

	0	1	2	3	4	5
2000	3209	1163	39	17	7	21
2001	3367	1292	37	24	10	
2002	3871	1474	53	22		
2003	4239	1678	103			
2004	4929	1865				
2005	5217					

Table 27 Incremental payment of triangle

	0	1	2	3	4	5
2000	3209	4372				
2001	3367					
2002	3871					
2003	4239					
2004	4929					
2005	5217					

Table 28 First iteration

Second iteration:

$$C_{2000,2} = X_{2000,0} + X_{2000,1} + X_{2000,2}$$
$$C_{2000,2} = 3209 + 1163 + 39$$
$$C_{2000,2} = 4411$$

	0	1	2	3	4	5
2000	3209	4372	4411			
2001	3367					
2002	3871					
2003	4239					
2004	4929					
2005	5217					

Table 29 Second iteration

Third iteration:

$$C_{2000,3} = X_{2000,0} + X_{2000,1} + X_{2000,2} + X_{2000,3}$$
$$C_{2000,3} = 3209 + 1163 + 39 + 17$$
$$C_{2000,3} = 4428$$

	0	1	2	3	4	5
2000	3209	4372	4411	4428		
2001	3367					
2002	3871					
2003	4239					
2004	4929					
2005	5217					

Table 30 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794				
2005	5217					

Table 31 Full cumulative payments triangle

The second step of the method calculates the transition ratio of cumulated payments, which is called ldf in the literature. The related formula is given below:

$$f_{i,j} = \frac{C_{i,j+1}}{C_{i,j}}$$

Equation 7 Calculation of transition ratio

All ldf values establish the ldf triangle as shown in Table 32.

	0	1	…	n-1
0	$f_{0,0}$	$f_{0,1}$	…	$f_{0,n-1}$
1	$f_{1,0}$	$f_{1,1}$	…	$f_{1,n-1}$
..			…	
i	$f_{i,0}$	$f_{i,1}$		
n-1	$f_{n-1,0}$			

Table 32 Triangle of ldf.

Now we calculate the ldf of the values reported in Table 31.

First iteration:

$$f_{0,0} = \frac{C_{0,1}}{C_{0,0}}$$

$$f_{0,0} = \frac{4372}{3209}$$

$$f_{0,0} = 1.36242$$

	0	1	2	3	4
0	**1.36242**				
1					
2					
3					
4					

Table 33 First iteration

Second iteration:

$$f_{0,1} = \frac{C_{0,2}}{C_{0,1}}$$
$$f_{0,1} = \frac{4411}{4372}$$
$$f_{0,1} = 1.00892$$

	0	1	2	3	4
0	1.36242	**1.00385**			
1					
2					
3					
4					

Table 34 Second iteration

Third iteration:

$$f_{1,0} = \frac{C_{1,1}}{C_{1,0}}$$
$$f_{1,0} = \frac{4659}{3367}$$
$$f_{1,0} = 1.38372$$

	0	1	2	3	4
0	1.36242	1.00385			
1	**1.38372**				
2					
3					
4					

Table 35 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				

Table 36 Triangle of ldf

The third step of the method is about the calculation of the estimators by using the ldf. There are three types of the estimators.

Type 1: Arithmetic Mean.

Type 1 is equal to the arithmetic mean of each column into the ldf triangle. The related formula is given below:

$$f_j^{\,a} = \frac{1}{n-j} \sum_{i=0}^{n-j-1} f_{i,j}$$

Equation 8 Calculation of Type 1

Now we calculate the Type 1 of the values reported in Table 36.

First iteration:

$$f_0^{\,a} = \frac{1.36242 + 1.38372 + 1.38078 + 1.39585 + 1.37837}{5}$$

$$f_0^{\,a} = 1.380229$$

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
$f_j^{\,a}$	**1.380229**				

Table 37 First iteration

Second iteration:

$$f_1^{\,a} = \frac{1.00892 + 1.00794 + 1.00992 + 1.01741}{4}$$

$$f_1^{\,a} = 1.011046$$

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
f_j^a	1.380229	**1.011046**			

Table 38 Second iteration

Third iteration:

$$f_2^a = \frac{1.00385 + 1.00511 + 1.00408}{3}$$

$$f_2^a = 1.004347$$

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
f_j^a	1.380229	1.011046	**1.004347**		

Table 39 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
f_j^a	1.380229	1.011046	1.004347	1.00185	1.004735

Table 40 Full Type 1 estimators.

Type 2: Geometric Mean

Type 2 is equal to geometric mean of the each column into the ldf triangle. The related formula is given below:

$$f_j^g = \left(\prod_{i=0}^{n-j-1} f_{i,j} \right)^{1/(n-j)}$$

Equation 9 Calculation of Type 2

Now we calculate Type 2 of the values reported in Table 36.

First iteration:

$$f_0^g = \sqrt[5]{1.36242 \times 1.38372 \times 1.38078 \times 1.39585 \times 1.37837}$$
$$f_0^g = 1.380187$$

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
f_j^g	**1.380187**				

Table 41 First iteration

Second iteration:

$$f_1^g = \sqrt[4]{1.00892 \times 1.00794 \times 1.00992 \times 1.01741}$$
$$f_1^g = 1.011039$$

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
f_j^g	1.380187	**1.011039**			

Table 42 Second iteration

Third iteration:

$$f_2^g = \sqrt[3]{1.00385 \times 1.00511 \times 1.00408}$$
$$f_2^g = 1.004347$$

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
f_j^g	1.380187	1.011039	**1.004347**		

Table 43 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
f_j^g	1.380187	1.011039	1.004347	1.00185	1.004735

Table 44 Full Type 2 estimators

Type 3: Weighted Mean

Type 3 is equal to weighted mean of the each column into ldf triangle. The related formula is given below:

$$f_j^w = \frac{\sum_{i=0}^{n-j-1} f_{i,j} \times (j+1)}{\sum_{i=0}^{n-j-1} j+1}$$

Equation 10 Calculation of Type 3

Now we calculate Type 3 of the values reported in Table 36.

First iteration:

$$f_0^{\,w} = \frac{(1.36242\times 1)+(1.38372\times 2)+(1.380878\times 3)+(1.39585\times 4)+(1.37837\times 5)}{(1+2+3+4+5)=15}$$

$$f_0^{\,w} = 1.383164$$

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
$f_j^{\,w}$	**1.383164**				

Table 45 First iteration

Second iteration:

$$f_1^{\,w} = \frac{(1.00892\times 1)+(1.00794\times 2)+(1.00992\times 3)+(1.01741\times 4)}{(1+2+3+4)=10}$$

$$f_1^{\,w} = 1.012418$$

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
$f_j^{\,w}$	1.383164	**1.012418**			

Table 46 Second iteration

Third iteration:

$$f_2^{\,w} = \frac{(1.00385\times 1)+(1.00511\times 2)+(1.00408\times 3)}{(1+2+3)=6}$$

$$f_2^{\,w} = 1.004384$$

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
f_j^w	1.383164	1.012418	**1.004384**		

Table 47 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4
0	1.36242	1.00892	1.00385	1.00158	1.00474
1	1.38372	1.00794	1.00511	1.00212	
2	1.38078	1.00992	1.00408		
3	1.39585	1.01741			
4	1.37837				
f_j^w	1.383164	1.012418	1.004384	1.001939	1.004735

Table 48 Full Type 3 estimators.

The fourth step of the method estimates the future payments by using one of the ldf estimators. The future payments is equal to multiplication cumulative payments with ldf estimators. The formula of the future payments ($\hat{C}_{i,j}$) is shown in the following equation:

$$\hat{C}_{i,j} = C_{i,n+1-i} \times f_j^a$$

Equation 11 Calculation of future payments

We can estimate $\hat{C}_{i,j}$ (future payments) by using one of the ldf estimators. $\hat{C}_{i,j}$ is calculated by using Type 1 values given in Table 31 and Table 40.

First iteration:

$$\hat{C}_{2005,1} = C_{2005,0} \times f_0^a$$

$$\hat{C}_{2005,1} = 5217 \times 1.380229$$

$$\hat{C}_{2005,1} = 7200.65$$

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794				
2005	5217	**7200.65**				

Table 49 First iteration

Second iteration:

$$\hat{C}_{2005,2} = C_{5,1} \times f_1^a$$

$$\hat{C}_{2005,2} = 7200.65 \times 1.011046$$

$$\hat{C}_{2005,2} = 7280.19$$

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794				
2005	5217	7200.65	**7280.19**			

Table 50 Second iteration

Third iteration:

$$\hat{C}_{2004,2} = C_{2004,1} \times f_1^a$$

$$\hat{C}_{2004,2} = 6794 \times 1.011046$$

$$\hat{C}_{2004,2} = 6872.14$$

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794	**6872.14**			
2005	5217	7200.65	7280.19			

Table 51 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	4752.40
2002	3871	5345	5398	5420	5430.03	5455.74
2003	4239	5917	6020	6046.17	6057.35	6086.03
2004	4929	6794	6869.05	6898.91	6911.67	6944.40
2005	5217	7200.65	7280.19	7311.84	7325.36	7360.05

Table 52 Full future payments.

Final step of the method computes the most important parameter IBNR RESERVE, referring to the funds for the future claims. IBNR RESERVE is equal to the difference of the ultimate of prediction claims and the actual claims. The actual claims are the maximum payment $max_j \{ C_{i,j} \}$ in each row.

	0	1	2	3	4	5	**Actual Ultimate Claim**
2000	3209	4372	4411	4428	4435	4456	$\max_j \{C_{2000,j}\}$
2001	3367	4659	4696	4720	4730		$\max_j \{C_{2001,j}\}$
2002	3871	5345	5398	5420			$\max_j \{C_{2002,j}\}$
2003	4239	5917	6020				$\max_j \{C_{2003,j}\}$
2004	4929	6794					$\max_j \{C_{2004,j}\}$
2005	5217						$\max_j \{C_{2005,j}\}$

Table 53 Calculation of actual ultimate claims

Now we calculate the actual ultimate claim of the values reported in Table 53.

First iteration:

$$U_{2000}^{a} = \max(C_{2000,0}, C_{2000,1}, C_{2000,2}, C_{2000,3}, C_{2000,4}, C_{2000,5})$$

$$U_{2000}^{a} = \max(3209, 4372, 4411, 4428, 4435, 4456)$$

$$U_{2000}^{a} = 4456$$

	0	1	2	3	4	5	**Actual Ultimate Claim**
2000	3209	4372	4411	4428	4435	4456	**4456**
2001	3367	4659	4696	4720	4730		
2002	3871	5345	5398	5420			
2003	4239	5917	6020				
2004	4929	6794					
2005	5217						

Table 54 First iteration

Second iteration:

$$U_{2001}^{a} = \max(C_{2001,0}, C_{2001,1}, C_{2001,2}, C_{2001,3}, C_{2001,4})$$

$$U_{2001}^{a} = \max(3367, 4659, 4696, 4720, 4730)$$

$$U_{2001}^{a} = 4730$$

	0	1	2	3	4	5	**Actual Ultimate Claim**
2000	3209	4372	4411	4428	4435	4456	4456
2001	3367	4659	4696	4720	4730		**4730**
2002	3871	5345	5398	5420			
2003	4239	5917	6020				
2004	4929	6794					
2005	5217						

Table 55 Second iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5	**Actual Ultimate Claim**
2000	3209	4372	4411	4428	4435	4456	4456
2001	3367	4659	4696	4720	4730		4730
2002	3871	5345	5398	5420			5420
2003	4239	5917	6020				6020
2004	4929	6794					6794
2005	5217						5214

Table 56 All ultimate claims

The second step of the method is about the prediction of the ultimate claims. We calculate the prediction of ultimate claim of the values reported in Table 52. Prediction of ultimate claim is equal to last column of the future claims.

	0	1	2	3	4	5	Prediction of Ultimate Claims
2000	3209	4372	4411	4428	4435	4456	**4456.00**
2001	3367	4659	4696	4720	4730	4752.40	**4752.40**
2002	3871	5345	5398	5420	5430.03	5455.74	**5455.74**
2003	4239	5917	6020	6046.17	6057.35	6086.03	**6086.03**
2004	4929	6794	6869.05	6898.91	6911.67	6944.40	**6944.40**
2005	5217	7200.65	7280.19	7311.84	7325.36	7360.05	**7360.05**

Table 57 Prediction of ultimate claims

Now we calculate the IBNR RESERVE of the values reported in Table 56 and Table 57. The related formula is given below:

$$\mathrm{R}_i = U_i^{\,p} - U_i^{\,a}$$

Equation 12 Calculation of Reserve

First iteration:

$$R_i = U_i^p - U_i^a$$

$$R_{2000} = U_{2000}^p - U_{2000}^a$$

$$R_{2000} = 4456 - 4456$$

$$R_{2000} = 0$$

	0	1	2	3	4	5	Actual Ultimate Claim	Prediction of Ultimate Claims	IBNR RESERVE
2000	3209	4372	4411	4428	4435	4456	4456	4456.00	**0**
2001	3367	4659	4696	4720	4730	4752.40	4730	4752.40	
2002	3871	5345	5398	5420	5430.03	5455.74	5420	5455.74	
2003	4239	5917	6020	6046.17	6057.35	6086.03	6020	6086.03	
2004	4929	6794	6869.05	6898.91	6911.67	6944.40	6794	6944.40	
2005	5217	7200.65	7280.19	7311.84	7325.36	7360.05	5217	7360.05	

Table 58 First iteration

Second iteration:

$$R_{2002} = U_{2002}^p - U_{2002}^a$$

$$R_{2002} = 4752.40 - 4730$$

$$R_{2002} = 22.40$$

	0	1	2	3	4	5	Actual Ultimate Claim	Prediction of Ultimate Claims	IBNR RESERVE
2000	3209	4372	4411	4428	4435	4456	4456	4456.00	0
2001	3367	4659	4696	4720	4730	4752.40	4730	4752.40	**22.40**
2002	3871	5345	5398	5420	5430.03	5455.74	5420	5455.74	
2003	4239	5917	6020	6046.17	6057.35	6086.03	6020	6086.03	
2004	4929	6794	6869.05	6898.91	6911.67	6944.40	6794	6944.40	
2005	5217	7200.65	7280.19	7311.84	7325.36	7360.05	5217	7360.05	

Table 59 Second iteration

Third iteration:

$$R_{2003} = U^{p}_{2003} - U^{a}_{2003}$$
$$R_{2003} = 5455.74 - 5420$$
$$R_{2003} = 66.03$$

	0	1	2	3	4	5	Actual Ultimate Claim	Prediction of Ultimate Claims	IBNR RESERVE
2000	3209	4372	4411	4428	4435	4456	4456	4456.00	0
2001	3367	4659	4696	4720	4730	4752.40	4730	4752.40	22.40
2002	3871	5345	5398	5420	5430.03	5455.74	5420	5455.74	**66.03**
2003	4239	5917	6020	6046.17	6057.35	6086.03	6020	6086.03	
2004	4929	6794	6869.05	6898.91	6911.67	6944.40	6794	6944.40	
2005	5217	7200.65	7280.19	7311.84	7325.36	7360.05	5217	7360.05	

Table 60 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5	Actual Ultimate Claim	Prediction of Ultimate Claims	IBNR RESERVE
2000	3209	4372	4411	4428	4435	4456	4456	4456.00	0.00
2001	3367	4659	4696	4720	4730	4752.40	4730	4752.40	22.40
2002	3871	5345	5398	5420	5430.03	5455.74	5420	5455.74	35.74
2003	4239	5917	6020	6046.17	6057.35	6086.03	6020	6086.03	66.03
2004	4929	6794	6869.05	6898.91	6911.67	6944.40	6794	6944.40	150.40
2005	5217	7200.65	7280.19	7311.84	7325.36	7360.05	5217	7360.05	2143.05

Table 61 Result of IBNR RESERVE

Finally. we calculate the funds of future claims payment which is the summation of all IBNR RESERVE values:

$$\text{Total Reserve} = \sum_{i}^{n} R_i$$

Equation 13 Calculation of total reserve

In our example calculation. we have
Total Reserve= 0+22.40+35.74+66.03+150.40+2143.05= 2417.61

The other two estimators (Type 2 and Type 3) defined at the beginning of this subsection are calculated in the same manner. The calculation steps are omitted to save space in this manuscript.

2.3.3 Borhhuetter-Ferguson Method

This method is combination of expected loss ratio and chain ladder method. The method is used as instant loss ration within unpaid and not reported claims. In addition it is assumed that the past experiences do not project the future properly. The method needs to two type of triangle that cumulative payment shown in Table 62 and cumulated premium shown in Table 63 to calculate IBNR RESERVE.

Parameter i refers to occurrence date (year, month, etc.). For example, if car accident happened in 2006, takes the value of 2006. Parameter *j* refers to development or payment date (year, month, etc.). For instance, the car accident cumulative payment occurred in 2008 is stating the value of *j* as 2008. The parameter $C_{i,j}$ refers to the cumulative payment due to the damage happened in year *i* and the cumulative payment that occurred in year *j*. The method calculates IBNR by using the triangle like the one given in Table 62.

	0	1	j	n-i	...	n-1	n
0	$C_{0,0}$	$C_{0,1}$	$C_{0,j}$	$C_{0,n-i}$	…	$C_{0,n-1}$	$C_{0,n}$
1	$C_{1,0}$	$C_{1,1}$	$C_{1,j}$	$C_{1,n-i}$	…	$C_{1,n-1}$	
...	…	…	…	…			
i	$C_{i,0}$	$C_{i,1}$	$C_{i,j}$	$C_{i,n-i}$			
...	…	…	…				
n-1	$C_{n-1,0}$	$C_{n-1,1}$					
n	$C_{n,0}$						

Table 62 Cumulative payment triangle

Parameter *i* refers to occurrence date (year, month, etc.). For example. if car accident happened in 2006, takes the value of 2006. Parameter of *j* which refers to development or payment date (year, month, etc.). For instance, the car accident cumulative payment occurred in 2008 is stating the value of *j* as 2008. The parameter $Pr_{i,j}$ refers to cumulative premium due to the damage happened in year i and the cumulative premium that occurred in year *j*. The amount of premium that happened in *i* and cumulative payment that occurred in *j*. The method calculates IBNR by using the triangle like the one given in Table 63.

	0	1	j	n-i	...	n-1	n	**Ultimate Premium**
0	$Pr_{0,0}$	$Pr_{0,1}$	$Pr_{0,j}$	$Pr_{0,n-i}$	…	$Pr_{0,n-1}$	$Pr_{0,n}$	$Pr_{0,n}$
1	$Pr_{1,0}$	$Pr_{1,1}$	$Pr_{1,j}$	$Pr_{1,n-i}$	…	$Pr_{1,n-1}$		$Pr_{1,n-1}$
...	…	…	…	…	…			
i	$Pr_{i,0}$	$Pr_{i,1}$	$Pr_{i,j}$	$Pr_{i,n-i}$				$Pr_{i,n-i}$
...	…	…	…					
n-1	$C_{n-1,0}$	$Pr_{n-1,1}$						
n	$Pr_{n,0}$							$Pr_{n-1,1}$

Table 63 Cumulative premium triangle

Consider the following example depicted in Table 64 and Table 65.

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794				
2005	5217					

Table 64 Cumulative payments triangle

	0	1	2	3	4	5
2000	4563	4589	4590	4591	4591	4591
2001	4618	4664	4671	4672	4672	
2002	4836	4861	4861	4863		
2003	5140	5168	5173			
2004	5633	5668				
2005	6389					

Table 65 Premium triangle

The first step of the method calculates loss premium ratio ($Lpr_{i,j}$) triangle. $Lpr_{i,j}$ is equal to dividing of the cumulative payment to premium ultimate. The premium ultimate are the maximum premium in each row.

	0	1	2	3	4	5	**Ultimate**
2000	4563	4589	4590	4591	4591	4591	$\max_j\{\Pr_{2000,j}\}$
2001	4618	4664	4671	4672	4672		$\max_j\{\Pr_{2001,j}\}$
2002	4836	4861	4861	4863			$\max_j\{\Pr_{2002,j}\}$
2003	5140	5168	5173				$\max_j\{\Pr_{2003,j}\}$
2004	5633	5668					$\max_j\{\Pr_{2004,j}\}$
2005	6389						$\max_j\{\Pr_{2005,j}\}$

Table 66 Calculation of premium ultimate

Now we calculate the premium ultimate of the values reported in Table 65.

First iteration:

$$U_{2000}^{p} = \max(\Pr_{2000,0}, \Pr_{2000,1}, \Pr_{2000,2}, \Pr_{2000,3}, \Pr_{2000,4}, \Pr_{2000,5})$$
$$U_{2000}^{p} = \max(4563, 4589, 4590, 4591, 4591, 4591)$$
$$U_{2000}^{p} = 4591$$

	0	1	2	3	4	5	**Ultimate**
2000	4563	4589	4590	4591	4591	4591	**4591**
2001	4618	4664	4671	4672	4672		
2002	4836	4861	4861	4863			
2003	5140	5168	5173				
2004	5633	5668					
2005	6389						

Table 67 First iteration

Second iteration:

$$U_{2001}^{p} = \max(\Pr_{2001,0}, \Pr_{2001,1}, \Pr_{2001,2}, \Pr_{2001,3}, \Pr_{2001,4})$$

$$U_{2001}^{p} = \max(4616, 4664, 4671, 4672, 4672)$$

$$U_{2001}^{p} = 4672$$

	0	1	2	3	4	5	**Ultimate**
2000	4563	4589	4590	4591	4591	4591	4591
2001	4618	4664	4671	4672	4672		**4672**
2002	4836	4861	4861	4863			
2003	5140	5168	5173				
2004	5633	5668					
2005	6389						

Table 68 Second iteration

Third iteration:

$$U_{2002}^{p} = \max(\Pr_{2002,0}, \Pr_{2002,1}, \Pr_{2002,2}, \Pr_{2002,3})$$

$$U_{2002}^{p} = \max(4836, 4861, 4861, 4863)$$

$$U_{2002}^{p} = 4863$$

	0	1	2	3	4	5	**Ultimate**
2000	4563	4589	4590	4591	4591	4591	4591
2001	4618	4664	4671	4672	4672		4672
2002	4836	4861	4861	4863			**4863**
2003	5140	5168	5173				
2004	5633	5668					
2005	6389						

Table 69 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5	**Ultimate**
2000	4563	4589	4590	4591	4591	4591	4591
2001	4618	4664	4671	4672	4672		4672
2002	4836	4861	4861	4863			4863
2003	5140	5168	5173				5173
2004	5633	5668					5668
2005	6389						6389

Table 70 Full premium ultimates

Now we calculate the loss premium ration of the values reported in Table 64 and Table 70. The related formula is given below:

$$Lpr_{i,j} = \frac{C_{i,j}}{U_i^p}$$

Equation 14 Calculation of loss premium ratio.

First iteration:

$$Lpr_{2000,0} = \frac{C_{2000,0}}{U_{2000}^p}$$

$$Lpr_{2000,0} = \frac{3209}{4591}$$

$$Lpr_{2000,0} = 69.90\%$$

	0	1	2	3	4	5
2000	**69.90%**					
2001						
2002						
2003						
2004						
2005						

Table 71 First iteration

Second iteration:

$$Lpr_{2000,1} = \frac{C_{2000,1}}{U_{2000}^p}$$

$$Lpr_{2000,1} = \frac{4372}{4591}$$

$$Lpr_{2000,1} = 95.23\%$$

	0	1	2	3	4	5
2000	69.90%	**95.23%**				
2001						
2002						
2003						
2004						
2005						

Table 72 Second iteration

Third iteration:

$$Lpr_{2001,0} = \frac{C_{2001,0}}{U^{p}_{2001}}$$

$$Lpr_{2001,0} = \frac{3367}{4672}$$

$$Lpr_{2001,0} = 72.07\%$$

	0	1	2	3	4	5
2000	69.90%	95.23%				
2001	**72.07%**					
2002						
2003						
2004						
2005						

Table 73 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5
2000	69.90%	95.23%	96.08%	96.45%	96.60%	97.06%
2001	72.07%	99.72%	100.51%	101.03%	101.24%	
2002	79.60%	109.91%	111.00%	111.45%		
2003	81.94%	114.38%	116.37%			
2004	86.96%	119.87%				
2005	81.66%					

Table 74 Triangle of loss premium ratio.

The second step of the method is about the calculation of the ultimate loss premium ratio. The ultimates are the maximum loss premium ratio in each row.

	0	1	2	3	4	5	**Ultimate**
2000	69.90%	95.23%	96.08%	96.45%	96.60%	97.06%	$\max_j\{Lpr_{2000,j}\}$
2001	72.07%	99.72%	100.51%	101.03%	101.24%		$\max_j\{Lpr_{2001,j}\}$
2002	79.60%	109.91%	111.00%	111.45%			$\max_j\{Lpr_{2002,j}\}$
2003	81.94%	114.38%	116.37%				$\max_j\{Lpr_{2003,j}\}$
2004	86.96%	119.87%					$\max_j\{Lpr_{2004,j}\}$
2005	81.66%						$\max_j\{Lpr_{2005,j}\}$

Table 75 Calculation of loss premium ratio ultimate

First iteration:

$$Lpr_{2000}^{U} = \max(Lpr_{2000,0}, Lpr_{2000,1}, Lpr_{2000,2}, Lpr_{2000,3}, Lpr_{2000,4}, Lpr_{2000,5})$$
$$Lpr_{2000}^{U} = \max(69.90\%, 95.23\%, 96.08\%, 96.45\%, 96.60\%, 97.06\%)$$
$$Lpr_{2000}^{U} = 97.06\%$$

	0	1	2	3	4	5	**Ultimate**
2000	69.90%	95.23%	96.08%	96.45%	96.60%	97.06%	97.06%
2001	72.07%	99.72%	100.51%	101.03%	101.24%		
2002	79.60%	109.91%	111.00%	111.45%			
2003	81.94%	114.38%	116.37%				
2004	86.96%	119.87%					
2005	81.66%						

Table 76 First iteration

Second iteration:

$$Lpr_{2001}^{U} = \max(Lpr_{2001,0}, Lpr_{2001,1}, Lpr_{2001,2}, Lpr_{2001,3}, Lpr_{2001,4})$$
$$Lpr_{2001}^{U} = \max(72.07\%, 99.72\%, 100.51\%, 101.03\%, 101.24\%)$$
$$Lpr_{2001}^{U} = 101.24\%$$

	0	1	2	3	4	5	**Ultimate**
2000	69.90%	95.23%	96.08%	96.45%	96.60%	97.06%	97.06%
2001	72.07%	99.72%	100.51%	101.03%	101.24%		101.24%
2002	79.60%	109.91%	111.00%	111.45%			
2003	81.94%	114.38%	116.37%				
2004	86.96%	119.87%					
2005	81.66%						

Table 77 Second iteration

Third iteration:

$$Lpr^U_{2002} = \max(Lpr_{2002,0}, Lpr_{2002,1}, Lpr_{2002,2}, Lpr_{2002,3})$$
$$Lpr^U_{2002} = \max(79.60\%, 109.91\%, 111.00\%, 111.45\%)$$
$$Lpr^U_{2002} = 111.45\%$$

	0	1	2	3	4	5	**Ultimate**
2000	69.90%	95.23%	96.08%	96.45%	96.60%	97.06%	97.06%
2001	72.07%	99.72%	100.51%	101.03%	101.24%		101.24%
2002	79.60%	109.91%	111.00%	111.45%			111.45%
2003	81.94%	114.38%	116.37%				
2004	86.96%	119.87%					
2005	81.66%						

Table 78 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5	**Ultimate**
2000	69.90%	95.23%	96.08%	96.45%	96.60%	97.06%	97.06%
2001	72.07%	99.72%	100.51%	101.03%	101.24%		101.24%
2002	79.60%	109.91%	111.00%	111.45%			111.45%
2003	81.94%	114.38%	116.37%				116.37%
2004	86.96%	119.87%					119.87%
2005	81.66%						81.66%

Table 79 Full loss premium ratio ultimate

The second step of the method transforms cumulative loss premium ratio triangle into incremental triangle. The related formula is given below:

$$lpr_{i,j} = Lpr_{i,j} - Lpr_{i,j-1}$$

Equation 15 Transforms cumulative into incremental triangle

Now we calculate the incremental loss premium ratio of the values reported in Table 74.

First iteration:

$$lpr_{2000,5} = Lpr_{2000,5} - Lpr_{2000,4}$$
$$lpr_{2000,5} = 97.06\% - 96.60\%$$
$$lpr_{2000,5} = 0.46\%$$

	0	1	2	3	4	5
2000	69.90%					**0.46%**
2001	72.07%					
2002	79.60%					
2003	81.94%					
2004	86.96%					
2005	81.66%					

Table 80 First iteration

Second iteration:

$$lpr_{2000,4} = Lpr_{2000,4} - Lpr_{2000,3}$$
$$lpr_{2000,4} = 96.60\% - 96.45\%$$
$$lpr_{2000,4} = 0.15\%$$

	0	1	2	3	4	5
2000	69.90%				**0.15%**	0.46%
2001	72.07%					
2002	79.60%					
2003	81.94%					
2004	86.96%					
2005	81.66%					

Table 81 Second iteration

Third iteration:

$$lpr_{2000,3} = Lpr_{2000,3} - Lpr_{2000,2}$$
$$lpr_{2000,3} = 96.45\% - 96.08\%$$
$$lpr_{2000,4} = 0.37\%$$

	0	1	2	3	4	5
2000	69.90%			**0.37%**	0.15%	0.46%
2001	72.07%					
2002	79.60%					
2003	81.94%					
2004	86.96%					
2005	81.66%					

Table 82 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%
2001	72.07%	27.65%	0.79%	0.51%	0.21%	
2002	79.60%	30.31%	1.09%	0.45%		
2003	81.94%	32.44%	1.99%			
2004	86.96%	32.90%				
2005	81.66%					

Table 83 Incremental loss premium ratio

The third step of the method is about prediction of the future incremental loss premium ratio. There are two types of future loss premium ratio.

Type 1: Arithmetic Mean

Type 1 is equal to arithmetic mean of $lpr_{i,j}$ for each column. The related formula is given below:

$$lpr_{i,j}{}^{*} = \frac{1}{n-j}\sum_{i=0}^{n-j-1} lpr_{i,j}$$

Equation 16 Prediction of incremental loss premium ratio

Now we calculate the Type 1 of values reported in Table 83.

First iteration:

$$lpr^*_{2005,1} = \frac{lpr_{2000,1} + lpr_{2001,1} + lpr_{2002,1} + lpr_{2003,1} + lpr_{2004,1}}{6-1}$$
$$lpr^*_{2005,1} = \frac{25.33\% + 27.65\% + 30.31\% + 32.44\% + 32.90\%}{5}$$
$$lpr^*_{2005,1} = 29.73\%$$

	0	1	2	3	4	5
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%
2001	72.07%	27.65%	0.79%	0.51%	0.21%	
2002	79.60%	30.31%	1.09%	0.45%		
2003	81.94%	32.44%	1.99%			
2004	86.96%	32.90%				
2005	81.66%	**29.73%**				

Table 84 First iteration

Second iteration:

$$lpr^*_{2004,2} = \frac{lpr_{2000,2} + lpr_{2001,2} + lpr_{2002,2} + lpr_{2003,2}}{6-2}$$
$$lpr^*_{2004,2} = \frac{0.85\% + 0.79\% + 1.09\% + 1.99\%}{4}$$
$$lpr^*_{2004,2} = 1.18\%$$

	0	1	2	3	4	5
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%
2001	72.07%	27.65%	0.79%	0.51%	0.21%	
2002	79.60%	30.31%	1.09%	0.45%		
2003	81.94%	32.44%	1.99%			
2004	86.96%	32.90%	**1.18%**			
2005	81.66%	29.73%				

Table 85 Second iteration

Third iteration:

$$lpr^{*}_{2005,2} = \frac{lpr_{2000,2} + lpr_{2001,2} + lpr_{2002,2} + lpr_{2003,2} + lpr_{2004,2}}{6-1}$$

$$lpr^{*}_{2005,2} = \frac{0.85\% + 0.79\% + 1.09\% + 1.99\% + 1.18\%}{5}$$

$$lpr^{*}_{2005,2} = 1.18\%$$

	0	1	2	3	4	5
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%
2001	72.07%	27.65%	0.79%	0.51%	0.21%	
2002	79.60%	30.31%	1.09%	0.45%		
2003	81.94%	32.44%	1.99%			
2004	86.96%	32.90%	**1.18%**			
2005	81.66%	29.73%				

Table 86 Third iteration

Now we apply the same procedure to all cells.

	0	**1**	**2**	**3**	**4**	**5**
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%
2001	72.07%	27.65%	0.79%	0.51%	0.21%	0.46%
2002	79.60%	30.31%	1.09%	0.45%	0.18%	0.46%
2003	81.94%	32.44%	1.99%	0.45%	0.18%	0.46%
2004	86.96%	32.90%	1.18%	0.45%	0.18%	0.46%
2005	81.66%	29.73%	1.18%	0.45%	0.18%	0.46%

Table 87 Full future incremental loss premium ratio

The fourth step of the method is about calculation of the growth estimation. The related formula is given below:

$$lpr_i^{\wedge} = \sum_{i=0}^{n-i-1} lpr_{i,j}$$

Equation 17 Calculation of growth estimation

Now we calculate the growth estimation of the values reported in Table 87.

	0	1	2	3	4	5	Growth Estimation
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%	0
2001	72.07%	27.65%	0.79%	0.51%	0.21%	**0.46%**	$lpr_{i,n}$
2002	79.60%	30.31%	1.09%	0.45%	**0.18%**	**0.46%**	$\sum_{i=0}^{n-i-1} lpr_{i,j}$
2003	81.94%	32.44%	1.99%	**0.45%**	**0.18%**	**0.46%**	
2004	86.96%	32.90%	**1.18%**	**0.45%**	**0.18%**	**0.46%**	
2005	81.66%	**29.73%**	**1.18%**	**0.45%**	**0.18%**	**0.46%**	

Table 88 Triangle of growth estimation

First iteration:

$$lpr_1^{\wedge} = lpr_5$$
$$lpr_1^{\wedge} = 0.46\%$$

	0	1	2	3	4	5	Growth Estimation
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%	0
2001	72.07%	27.65%	0.79%	0.51%	0.21%	**0.46%**	**0.46%**
2002	79.60%	30.31%	1.09%	0.45%	**0.18%**	**0.46%**	
2003	81.94%	32.44%	1.99%	**0.45%**	**0.18%**	**0.46%**	
2004	86.96%	32.90%	**1.18%**	**0.45%**	**0.18%**	**0.46%**	
2005	81.66%	**29.73%**	**1.18%**	**0.45%**	**0.18%**	**0.46%**	

Table 89 First iteration

Second iteration:

$$lpr_2^{\wedge} = 0.18\% + 0.46\%$$
$$lpr_2^{\wedge} = 0.64\%$$

	0	1	2	3	4	5	Growth Estimation
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%	0
2001	72.07%	27.65%	0.79%	0.51%	0.21%	**0.46%**	**0.46%**
2002	79.60%	30.31%	1.09%	0.45%	**0.18%**	**0.46%**	**0.64%**
2003	81.94%	32.44%	1.99%	**0.45%**	**0.18%**	**0.46%**	
2004	86.96%	32.90%	**1.18%**	**0.45%**	**0.18%**	**0.46%**	
2005	81.66%	**29.73%**	**1.18%**	**0.45%**	**0.18%**	**0.46%**	

Table 90 Second iteration

Third iteration

$$\hat{lpr_3} = 0.45\% + 0.18\% + 0.46\%$$
$$\hat{lpr_3} = 1.086\%$$

	0	1	2	3	4	5	Growth Estimation
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%	0
2001	72.07%	27.65%	0.79%	0.51%	0.21%	**0.46%**	**0.46%**
2002	79.60%	30.31%	1.09%	0.45%	**0.18%**	**0.46%**	**0.64%**
2003	81.94%	32.44%	1.99%	**0.45%**	**0.18%**	**0.46%**	**1.086%**
2004	86.96%	32.90%	**1.18%**	**0.45%**	**0.18%**	**0.46%**	
2005	81.66%	**29.73%**	**1.18%**	**0.45%**	**0.18%**	**0.46%**	

Table 91 Third iteration

We apply the same procedure to all cells.

	0	1	2	3	4	5	Growth Estimation
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%	0
2001	72.07%	27.65%	0.79%	0.51%	0.21%	**0.46%**	**0.46%**
2002	79.60%	30.31%	1.09%	0.45%	**0.18%**	**0.46%**	**0.64%**
2003	81.94%	32.44%	1.99%	**0.45%**	**0.18%**	**0.46%**	**1.086%**
2004	86.96%	32.90%	**1.18%**	**0.45%**	**0.18%**	**0.46%**	**2.27%**
2005	81.66%	**29.73%**	**1.18%**	**0.45%**	**0.18%**	**0.46%**	**31.99%**

Table 92 Full growth estimation

The final step is consolidating all figures we have calculated so far in the format shown in Table 93

Premium Ultimates	Ultimate Loss Premium Ratio	Growth Estimation	Estimators	IBNR Reserve
$\Pr_i$	Lpr_i^U	$lpr_i^{\wedge}$	$Lpr_i^U + lpr_i^{\wedge}$	$\Pr_i \times (Lpr_i^U + lpr_i^{\wedge})$
…	…	…	…	
			Total IBNR Reserve	

Table 93 Consolidating all figures

First column comes from Table 70. Second column comes from Table 79 and third column comes from Table 92. Now we calculate the IBNR RESERVE of the values reported in Table 70, Table 79 and Table 92.

First iteration:

$$R_{2001} = \Pr_{2001} \times (Lpr_{2001}^U + lpr_{2001}^{\wedge})$$
$$R_{2001} = 4672 \times (101.24\% + 0.46\%)$$
$$R_{2001} = 21.37$$

	Premium Ultimates	Ultimate Loss Premium Ratio	Growth Estimation	Estimators	IBNR Reserve
2000	4591	97.06%	0.00%	97.06%	**0**
2001	4672	101.24%	0.46%	101.70%	**21.37**
2002	4863	111.45%	0.64%	112.09%	**31.16**
2003	5173	116.37%	1.09%	117.46%	**56.19**
2004	5668	119.87%	2.27%	122.13%	**128.48**
2005	6389	81.66%	31.99%	113.65%	**2044.12**

Table 94 First iteration

Second iteration:

$$R_{2002} = \Pr_{2002} \times (Lpr^U_{2002} + lpr^\wedge_{2002})$$
$$R_{2002} = 4863 \times (111.45\% + 0.64\%)$$
$$R_{2002} = 31.16$$

	Premium Ultimates	Ultimate Loss Premium Ratio	Growth Estimation	Estimators	IBNR Reserve
2000	4591	97.06%	0.00%	97.06%	**0**
2001	4672	101.24%	0.46%	101.70%	**21.37**
2002	4863	111.45%	0.64%	112.09%	**31.16**
2003					
2004					
2005					

Table 95 Second iteration

Third iteration:

$$R_{2003} = \Pr_{2003} \times (Lpr^U_{2003} + lpr^\wedge_{2003})$$
$$R_{2003} = 5173 \times (116.37\% + 1.09\%)$$
$$R_{2003} = 56.19$$

	Premium Ultimates	Ultimate Loss Premium Ratio	Growth Estimation	Estimators	IBNR Reserve
2000	4591	97.06%	0.00%	97.06%	**0**
2001	4672	101.24%	0.46%	101.70%	**21.37**
2002	4863	111.45%	0.64%	112.09%	**31.16**
2003	5173	116.37%	1.09%	117.46%	**56.19**
2004					
2005					

Table 96 Third iteration

Now we apply the same procedure to all cells.

	Premium Ultimates	Ultimate Loss Premium Ratio	Growth Estimation	Estimators	IBNR Reserve
2000	4591	97.06%	0.00%	97.06%	**0**
2001	4672	101.24%	0.46%	101.70%	**21.37**
2002	4863	111.45%	0.64%	112.09%	**31.16**
2003	5173	116.37%	1.086%	117.46%	**56.19**
2004	5668	119.87%	2.27%	122.13%	**128.48**
2005	6389	81.66%	31.99%	113.65%	**2044.12**

Table 97 Result of IBNR RESERVE

Finally. we calculate the fund of the future claims payment which is the summation of all IBNR RESERVE.

$$\text{Total Reserve} = \sum_{i}^{n} R_i$$

Equation 18 Calculation of total reserve

In our example calculation. we have

Total Reserve=0+21.37+31.16+56.19+128.48+2044.12=2281.32

Type 2 = Min – Max Approach

In this approach lpr values are calculated with the formula shown below.

$$lpr_{i,j}{}^{*} = \frac{\sum_{i=0}^{n-j-1} lpr_{i,j} - \min(lpr_{i,j}) - \max(lpr_{i,j})}{n-2}$$

Equation 19 Calculation of Min-Max Approach

Now we calculate the future loss premium ratio of the values reported in Table 83.

First iteration:

$$\min = \min(lpr_{2000,1}, lpr_{2001,1}, lpr_{2002,1}, lpr_{2003,1}, lpr_{2004,1})$$
$$\max = \max(lpr_{2000,1}, lpr_{2001,1}, lpr_{2002,1}, lpr_{2003,1}, lpr_{2004,1})$$
$$lpr^*_{2005,1} = \frac{(lpr_{2000,1} + lpr_{2001,1} + lpr_{2002,1} + lpr_{2003,1} + lpr_{2004,1}) - \min - \max}{5-2}$$
$$\min = \min(25.33\%, 27.65\%, 30.31\%, 32.44\%, 32.90\%)$$
$$\max = \max(25.33\%, 27.65\%, 30.31\%, 32.44\%, 32.90\%)$$
$$lpr^*_{2005,1} = \frac{(25.33\% + 27.65\% + 30.31\% + 32.44\% + 32.90\%) - \min - \max}{5-2}$$
$$lpr^*_{2005,1} = 30.13\%$$

	0	1	2	3	4	5
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%
2001	72.07%	27.65%	0.79%	0.51%	0.21%	
2002	79.60%	30.31%	1.09%	0.45%		
2003	81.94%	32.44%	1.99%			
2004	86.96%	32.90%				
2005	81.66%	**30.13**				

Table 98 First iteration

Second iteration:

$$\min = \min(lpr_{2000,2}, lpr_{2001,2}, lpr_{2002,2}, lpr_{2003,2})$$
$$\max = \max(lpr_{2000,2}, lpr_{2001,2}, lpr_{2002,2}, lpr_{2003,2})$$
$$lpr^*_{2004,2} = \frac{(lpr_{2000,2} + lpr_{2001,2} + lpr_{2002,2} + lpr_{2003,2}) - \min - \max}{4-2}$$
$$\min = \min(0.85\%, 0.79\%, 1.09\%, 1.99\%)$$
$$\max = \max(0.85\%, 0.79\%, 1.09\%, 1.99\%)$$
$$lpr^*_{2004,2} = \frac{(0.85\% + 0.79\% + 1.09\% + 1.99\%) - \min - \max}{4-2}$$
$$lpr^*_{2004,2} = 0.97\%$$

	0	1	2	3	4	5
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%
2001	72.07%	27.65%	0.79%	0.51%	0.21%	
2002	79.60%	30.31%	1.09%	0.45%		
2003	81.94%	32.44%	1.99%			
2004	86.96%	32.90%	**0.97%**			
2005	81.66%	30.13				

Table 99 Second iteration

Third iteration:

$$\min = \min(lpr_{2000,2}, lpr_{2001,2}, lpr_{2002,2}, lpr_{2003,2}, lpr_{2004,2})$$

$$\max = \max(lpr_{2000,2}, lpr_{2001,2}, lpr_{2002,2}, lpr_{2003,2}, lpr_{2004,2})$$

$$lpr^{*}_{2005,2} = \frac{(lpr_{2000,2} + lpr_{2001,2} + lpr_{2002,2} + lpr_{2003,2}, + lpr_{2004,2}) - \min - \max}{5-2}$$

$$\min = \min(0.85\%, 0.79\%, 1.09\%, 1.99\%, 0.97\%)$$

$$\max = \max(0.85\%, 0.79\%, 1.09\%, 1.99\%, 0.97\%)$$

$$lpr^{*}_{2005,2} = \frac{(0.85\% + 0.79\% + 1.09\% + 1.99\% + 0.97\%) - \min - \max}{5-2}$$

$$lpr^{*}_{2005,2} = 0.97\%$$

	0	1	2	3	4	5
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%
2001	72.07%	27.65%	0.79%	0.51%	0.21%	
2002	79.60%	30.31%	1.09%	0.45%		
2003	81.94%	32.44%	1.99%			
2004	86.96%	32.90%	0.97%			
2005	81.66%	30.13	**0.97%**			

Table 100 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%
2001	72.07%	27.65%	0.79%	0.51%	0.21%	0.46
2002	79.60%	30.31%	1.09%	0.45%	0.18%	0.46
2003	81.94%	32.44%	1.99%	0.45%	0.18%	0.46
2004	86.96%	32.90%	0.97%	0.45%	0.18%	0.46
2005	81.66%	30.13	0.97%	0.45%	0.18	0.46

Table 101 Full future incremental loss premium ratio

The fourth step of the method is about calculation of the growth estimation. The related formula is given below:

$$lpr_i^{\wedge} = \sum_{i=0}^{n-i-1} lpr_{i,j}$$

Equation 20 Calculation of growth estimation

	0	1	2	3	4	5	Growth Estimation
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%	0
2001	72.07%	27.65%	0.79%	0.51%	0.21%	**0.46%**	$lpr_{i,n}$
2002	79.60%	30.31%	1.09%	0.45%	**0.18%**	**0.46%**	$\sum_{i=0}^{n-i-1} lpr_{i,j}$
2003	81.94%	32.44%	1.99%	**0.45%**	**0.18%**	**0.46%**	
2004	86.96%	32.90%	**0.97%**	**0.45%**	**0.18%**	**0.46%**	
2005	81.66%	**30.13%**	**0.97%**	**0.45%**	**0.18%**	**0.46%**	

Table 102 Triangle of growth estimation

Now we calculate the growth estimation of the values reported in Table 101.

First iteration:

$$lpr_1^{\wedge} = lpr_5$$

$$lpr_1^{\wedge} = 0.46\%$$

	0	1	2	3	4	5	Growth Estimation
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%	0
2001	72.07%	27.65%	0.79%	0.51%	0.21%	**0.46%**	**0.46%**
2002	79.60%	30.31%	1.09%	0.45%	**0.18%**	**0.46%**	
2003	81.94%	32.44%	1.99%	**0.45%**	**0.18%**	**0.46%**	
2004	86.96%	32.90%	**0.97%**	**0.45%**	**0.18%**	**0.46%**	
2005	81.66%	**30.13%**	**0.97%**	**0.45%**	**0.18%**	**0.46%**	

Table 103 First iteration

Second iteration:

$$lpr_2^{\wedge} = 0.18\% + 0.46\%$$

$$lpr_2^{\wedge} = 0.64\%$$

	0	1	2	3	4	5	Growth Estimation
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%	0
2001	72.07%	27.65%	0.79%	0.51%	0.21%	**0.46%**	**0.46%**
2002	79.60%	30.31%	1.09%	0.45%	**0.18%**	**0.46%**	**0.64%**
2003	81.94%	32.44%	1.99%	**0.45%**	**0.18%**	**0.46%**	
2004	86.96%	32.90%	**0.97%**	**0.45%**	**0.18%**	**0.46%**	
2005	81.66%	**30.13%**	**0.97%**	**0.45%**	**0.18%**	**0.46%**	

Table 104 Second iteration

Third iteration:

$$lpr_3^{\wedge} = 0.45\% + 0.18\% + 0.46\%$$

$$lpr_3^{\wedge} = 1.09\%$$

	0	1	2	3	4	5	Growth Estimation
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%	0
2001	72.07%	27.65%	0.79%	0.51%	0.21%	**0.46%**	**0.46%**
2002	79.60%	30.31%	1.09%	0.45%	**0.18%**	**0.46%**	**0.64%**
2003	81.94%	32.44%	1.99%	**0.45%**	**0.18%**	**0.46%**	**1.093%**
2004	86.96%	32.90%	**0.97%**	**0.45%**	**0.18%**	**0.46%**	
2005	81.66%	**30.13%**	**0.97%**	**0.45%**	**0.18%**	**0.46%**	

Table 105 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5	Growth Estimation
2000	69.90%	25.33%	0.85%	0.37%	0.15%	0.46%	0
2001	72.07%	27.65%	0.79%	0.51%	0.21%	**0.46%**	**0.46%**
2002	79.60%	30.31%	1.09%	0.45%	**0.18%**	**0.46%**	**0.64%**
2003	81.94%	32.44%	1.99%	**0.45%**	**0.18%**	**0.46%**	**1.09%**
2004	86.96%	32.90%	**0.97%**	**0.45%**	**0.18%**	**0.46%**	**2.06%**
2005	81.66%	**30.13%**	**0.97%**	**0.45%**	**0.18%**	**0.46%**	**32.19%**

Table 106 Growth Estimation

The final step is consalidating all figures we have calculated so far in the format shown in Table 107.

Premium Ultimates	Ultimate Loss Premium Ratio	Growth Estimation	Prediction of Loss Premium Ratio	IBNR Reserve
$\Pr_i$	Lpr_i^U	$\hat{lpr_i}$	$Lpr_i^U + \hat{lpr_i}$	$\Pr_i \times \hat{lpr_i}$
…	…	…	…	…
			Total IBNR Reserve	

Table 107 Consolidating all figures

First column comes from Table 70. Second column comes from Table 79 and third column comes from Table 101. Now we calculate the IBNR RSERVE of the values reported in Table 70, Table 79 and Table 101.

First iteration:

$$R_{2001} = \Pr_{2001} \times \hat{lpr}_{2001}$$
$$R_{2001} = 4672 \times 0.46\%$$
$$R_{2001} = 21.37$$

	Premium Ultimates	Ultimate Loss Premium Ratio	Growth Estimation	Estimators	IBNR Reserve
2000	4591	97.06%	0.00%	97.06%	**0**
2001	4672	101.24%	0.46%	101.70%	**21.37**
2002	4863	111.45%	0.64%	112.09%	**31.16**
2003	5173	116.37%	1.09%	117.46%	**56.19**
2004	5668	119.87%	2.27%	122.13%	**128.48**
2005	6389	81.66%	31.99%	113.65%	**2044.12**

Table 108 First iteration

Second iteration:

$$R_{2002} = \Pr_{2002} \times \hat{lpr}_{2002}$$
$$R_{2002} = 4863 \times 0.64\%$$
$$R_{2002} = 31.16$$

	Premium Ultimates	Ultimate Loss Premium Ratio	Growth Estimation	Prediction of Loss Premium Ratio	IBNR Reserve
2000	4591	97.06%	0.00%	97.06%	**0**
2001	4672	101.24%	0.46%	101.70%	**21.37**
2002	4863	111.45%	0.64%	112.09%	**31.16**
2003					
2004					
2005					

Table 109 Second iteration

Third iteration:

$$R_{2003} = \Pr_{2003} \times \hat{lpr}_{2003}$$
$$R_{2003} = 5173 \times 1.09\%$$
$$R_{2003} = 56.19$$

	Premium Ultimates	Ultimate Loss Premium Ratio	Growth Estimation	Prediction of Loss Premium Ratio	IBNR Reserve
2000	4591	97.06%	0.00%	97.06%	**0**
2001	4672	101.24%	0.46%	101.70%	**21.37**
2002	4863	111.45%	0.64%	112.09%	**31.16**
2003	5173	116.37%	1.093%	117.47%	**56.54**
2004					
2005					

Table 110 Third iteration

Now we apply the same procedure to all cells.

Premium Ultimates	Ultimate Loss Premium Ratio	Growth Estimation	Prediction of Loss Premium Ratio	IBNR Reserve
4591	97.06%	0.00%	97.06%	**0.00**
4672	101.24%	0.46%	101.70%	**21.37**
4863	111.45%	0.64%	112.09%	**31.16**
5173	116.37%	1.093%	117.47%	**56.54**
5668	119.87%	2.06%	121.92%	**116.52**
6389	81.66%	32.19%	113.85%	**2056.61**

Table 111 Result of IBNR Reserve

Finally. we calculate the fund of the future payments which is the summation of all IBNR Reserve values:

$$\text{Total Reserve} = \sum_{i}^{n} R_i$$

Equation 21 Calculation of total reserve

In our example calculation, we have

Total Reserve=0+21.37+31.16+56.54+116.52+2056.61=2282.21

2.3.4 Average Cost Per Claim Methods

As claim numbers and the average cost per claim develop over different origin years. this technique for estimating reserves dissolves around analyzing both of them. The idea follows to study distinctly frequency as well as severity of claims by using delay triangles both claim numbers and average cost per claim. In addition to this, the idea is also to run-off the triangles to obtain estimates of ultimate claim numbers and mean costs. At that point; the result are multiplied together to give estimates of future payments. as a result the necessary reserve is determined. The method calculates to IBNR by using triangle like in Table 112 and Table 119

	0	1	j	n-i	...	n-1	n
0	$X_{0,0}$	$X_{0,1}$	$X_{0,j}$	$X_{0,n-i}$	$\dots$	$X_{0,n-1}$	$X_{0,n}$
1	$X_{1,0}$	$X_{1,1}$	$X_{1,j}$	$X_{1,n-i}$	$\dots$	$X_{1,n-1}$	
...	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$		
i	$X_{i,0}$	$X_{i,1}$	$X_{i,j}$	$X_{i,n-i}$			
...	$\dots$	$\dots$	$\dots$				
n-1	$X_{n-1,0}$	$X_{n-1,1}$					
n	$X_{n,0}$						

Table 112 Incremental payments of triangle

Parameter *i* refers to the occurrence date (year, month, etc.). For example. if a car accident happened in 2006, takes the value of 2006. Parameter *j* refers to then development or the payment date (year, month, etc.). For instance. the car accident payment occurred in 2008 is stating the value of *j* as 2008. Parameter $X_{i,j}$ refers to the incremental payment amount. For instance, $X_{2006,2008}$ denotes the car accident happened in 2006 but the payment was realized in 2008. Consider the following example depicted in Table 113.

	0	1	2	3	4	5
2000	3209	1163	39	17	7	21
2001	3367	1292	37	24	10	
2002	3871	1474	53	22		
2003	4239	1678	103			
2004	4929	1865				
2005	5217				?	

Table 113 Incremental payments of triangle

The blue cell having the value of 3209 refers to the damage that happened in 2000 and the payment that realized in 2000. which is $X_{2000,0}$. Green backround of cell which is value 1678 refer to damage that happaned in 2003 and payment that occurred in 2004. That refer to $X_{2003,0}$. Yellow backround of cell which is value 10 refer to damage that happened in 2001 and payment that occurred in 2005. That refer to $X_{2001,4}$. The lower triangler part of the matrix given in Table 113 is reserved for future payments. For example, the red cell refers the damage happened in 2005 and the payment that will occur in 2009.

The first step of the chain ladder method transforms the incremental payments triangle into the cumulative payments triangle. The parameter $C_{i,j}$ refers to the cumulative payment due to the damage happened in year *i* and the cumulative payment that occurred in year *j*. The Cumulative triangle whose values are calculated by means of Equation 22 is shown in Table 114.

$$C_{i,j} = X_{i,0} + X_{i,1} + X_{i,2} + ... + X_{i,j}$$

Equation 22 Transforms incremental to cumulative triangle

	0	1	j	n-i	...	n-1	n
0	$C_{0,0}$	$C_{0,1}$	$C_{0,j}$	$C_{0,n-i}$	$\dots$	$C_{0,n-1}$	$C_{0,n}$
1	$C_{1,0}$	$C_{1,1}$	$C_{1,j}$	$C_{1,n-i}$	$\dots$	$C_{1,n-1}$	
...	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$		
i	$C_{i,0}$	$C_{i,1}$	$C_{i,j}$	$C_{i,n-i}$			
...	$\dots$	$\dots$	$\dots$				
n-1	$C_{n-1,0}$	$C_{n-1,1}$					
n	$C_{n,0}$						

Table 114 Cumulative payment triangle of chain ladder

Now we calculate the cumulative payments of the values reported in Table 113.

First iteration:

$$C_{2000,1} = X_{2000,0} + X_{2000,1}$$
$$C_{0,1} = 3209 + 1163$$
$$C_{0,1} = 4372$$

	0	1	2	3	4	5
2000	3209	4372				
2001	3367					
2002	3871					
2003	4239					
2004	4929					
2005	5217					

Table 115 First iteration

Second iteration:

$$C_{2000,2} = X_{2000,0} + X_{2000,1} + X_{2000,2}$$
$$C_{2000,2} = 3209 + 1163 + 39$$
$$C_{2000,2} = 4411$$

	0	1	2	3	4	5
2000	3209	4372	4411			
2001	3367					
2002	3871					
2003	4239					
2004	4929					
2005	5217					

Table 116 Second iteration

Third iteration:

$$C_{2000,3} = X_{2000,0} + X_{2000,1} + X_{2000,2} + X_{2000,3}$$
$$C_{2000,3} = 3209 + 1163 + 39 + 17$$

$C_{2000,3} = 4428$

	0	1	2	3	4	5
2000	3209	4372	4411	4428		
2001	3367					
2002	3871					
2003	4239					
2004	4929					
2005	5217					

Table 117 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5
2000	3209	4372	4411	4428	4435	4456
2001	3367	4659	4696	4720	4730	
2002	3871	5345	5398	5420		
2003	4239	5917	6020			
2004	4929	6794				
2005	5217					

Table 118 Full cumulative payments triangle

Parameter i refers to occurrence date (year, month, etc.). For example. if a car accident happened in 2006, takes the value of 2006. Parameter *j* refers to then development or the payment date (year, month, etc.). For instance, the car accident payment occurred in 2008 is stating the value of *j* as 2008. The parameter $Nl_{i,j}$ refers to the number of claims due to the damage happened in year i and the number of claims that occurred in year *j*.

	0	1	j	n-i	...	n-1	n
0	$Nl_{0,0}$	$Nl_{0,1}$	$Nl_{0,j}$	$Nl_{0,n-i}$	…	$Nl_{0,n-1}$	$Nl_{0,n}$
1	$Nl_{1,0}$	$Nl_{1,1}$	$Nl_{1,j}$	$Nl_{1,n-i}$	…	$Nl_{1,n-1}$	
...	…	…	…	…	…		
i	$Nl_{i,0}$	$Nl_{i,1}$	$Nl_{i,j}$	$Nl_{i,n-i}$			
...	…	…	…				
n-1	$Nl_{n-1,0}$	$Nl_{n-1,1}$					
n	$Nl_{n,0}$						

Table 119 Cumulative number of claims

Consider the following example depicted in Table 118 and Table 120.

	0	1	2	3	4	5
2000	414	460	482	488	492	494
2001	453	506	526	536	539	
2002	494	548	572	582		
2003	530	588	615			
2004	545	605				
2005	557					

Table 120 Number of claims

The first step of the method calculates the transition ratio of number of claims which is called ldf in literature. The related formula is given below:

$$f_{i,j} = \frac{Nl_{i,j+1}}{Nl_{i,j}}$$

Equation 23 Calculation of $f_{i,j}$

All ldf values establish the ldf triangle as shown in Table 121.

	0	1	...	n-1
0	$f_{0,0}$	$f_{0,1}$	…	$f_{0,n-1}$
1	$f_{1,0}$	$f_{1,1}$	…	$f_{1,n-1}$
..	…	…	…	
i	$f_{i,0}$	$f_{i,1}$		
n-1	$f_{n-1,0}$			

Table 121 Triangle of ldf

Now we calculate ldf of the values reported in Table 120.

First iteration:

$$f_{0,0} = \frac{Nl_{2000,1}}{Nl_{2000,0}}$$

$$f_{0,0} = \frac{460}{414}$$

$$f_{0,0} = 1.111111$$

	0	1	2	3	4
0	**1.111111**				
1					
2					
3					
4					

Table 122 First iteration

Second iteration:

$$f_{0,1} = \frac{Nl_{2000,2}}{Nl_{2000,1}}$$

$$f_{0,1} = \frac{482}{460}$$

$$f_{0,1} = 1.047826$$

	0	1	2	3	4
0	1.111111	**1.047826**			
1					
2					
3					
4					

Table 123 Second iteration

Third iteration:

$$f_{1,0} = \frac{Nl_{2001,1}}{Nl_{2001,0}}$$

$$f_{1,0} = \frac{506}{453}$$

$$f_{1,0} = 1.116998$$

	0	1	2	3	4
0	1.111111	1.047826			
1	**1.116998**				
2					
3					
4					

Table 124 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				

Table 125 Triangle of ldf

The second step of the method is about calculation of the estimators by using the ldf. There are three types of the estimators.

Type 1: Arithmetic Mean

Type 1 is equal to arithmetic mean of the each column into ldf triangle. The related formula is given below:

$$f_j^a = \frac{1}{n-j} \sum_{i=0}^{n-j-1} f_{i,j}$$

Equation 24 Calculation of Type 1

Now we calculate Type 1 of the values reported in Table 125.

First iteration:

$$f_0^a = \frac{1.111111 + 1.116998 + 1.109312 + 1.109434 + 1.110092}{5}$$

$$f_0^a = 1.111389$$

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
f_j^a	**1.111389**				

Table 126 First iteration

Second iteration:

$$f_1^a = \frac{1.047826 + 1.039526 + 1.043796 + 1.045918}{4}$$

$$f_1^a = 1.044266$$

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
f_j^a	1.111389	**1.044266**			

Table 127 Second iteration

Third iteration:

$$f_2^{\,a} = \frac{1.012448 + 1.019011 + 1.017483}{3}$$

$$f_2^{\,a} = 1.016314$$

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
$f_j^{\,a}$	1.111389	1.044266	**1.016314**		

Table 128 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
$f_j^{\,a}$	1.111389	1.044266	1.016314	1.006897	1.004065

Table 129 Full Type 1 estimators.

Type 2: Geometric Mean

Type 2 is equal to geometric mean of the each column into ldf triangle. The related formula is given below:

$$f_j^{\,g} = \left(\prod_{i=0}^{n-j-1} f_{i,j} \right)^{1/(n-j)}$$

Equation 25 Calculation of Type 2

Now we calculate Type 2 of the values reported in Table 125.

First iteration:

$$f_0^g = \sqrt[5]{1.111111 \times 1.116998 \times 1.109312 \times 1.109434 \times 1.110092}$$

$$f_0^g = 1.111386$$

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
f_j^g	**1.111386**				

Table 130 First iteration

Second iteration:

$$f_1^g = \sqrt[4]{1.047826 \times 1.039526 \times 1.043796 \times 1.045918}$$

$$f_1^g = 1.044263$$

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
f_j^g	1.111386	**1.044263**			

Table 131 Second iteration

Third iteration:

$$f_2^g = \sqrt[3]{1.012448 \times 1.019011 \times 1.017483}$$

$$f_2^g = 1.016311$$

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
f_j^g	1.111386	1.044263	**1.016311**		

Table 132 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
$f_j^{\,g}$	1.111386	1.044263	1.016311	1.006896	1.004065

Table 133 Full Type 2 estimators

Type 3: Weighted Mean

Type 3 is equal to weighted mean of the each column into ldf triangle. The related formula is given below:

$$f_j^{\,w} = \frac{\sum_{i=0}^{n-j-1} f_{i,j} \times (j+1)}{\sum_{i=0}^{n-j-1} j+1}$$

Equation 26 Calculation of Type 3

Now we calculate Type 3 of the values reported in Table 125.

First iteration:

$$f_0^{\,w} = \frac{(1.111111 \times 1) + (1.116998 \times 2) + (1.109312 \times 3) + (1.109434 \times 4) + (1.110092 \times 5)}{(1+2+3+4+5) = 15}$$

$$f_0^{\,w} = 1.110749$$

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
f_j^w	**1.110749**				

Table 134 First iteration

Second iteration:

$$f_1^w = \frac{(1.047826 \times 1) + (1.039526 \times 2) + (1.043796 \times 3) + (1.045918 \times 4)}{(1 + 2 + 3 + 4) = 10}$$

$$f_1^w = 1.044194$$

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
f_j^w	1.110749	**1.044194**			

Table 135 Second iteration

Third iteration:

$$f_2^w = \frac{(1.012448 \times 1) + (1.019011 \times 2) + (1.017483 \times 3)}{(1 + 2 + 3) = 6}$$

$$f_1^w = 1.017153$$

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
f_j^w	1.110749	1.044194	**1.017153**		

Table 136 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4
0	1.111111	1.047826	1.012448	1.008197	1.004065
1	1.116998	1.039526	1.019011	1.005597	
2	1.109312	1.043796	1.017483		
3	1.109434	1.045918			
4	1.110092				
f_j^w	1.110749	1.044194	1.017153	1.006464	1.004065

Table 137 Full Type 3 estimators

The third step of the method estimates the future number of claims by using one of the ldf estimators. The future number of claims is equal to the multiplication cumulative number of claims with ldf estimators. The formula of the future number of claims $(\hat{N}_{i,j})$ shown below the equation:

$$\hat{N}l_{i,j} = Nl_{i,n+1-i} \times f_j^a$$

Equation 27 Calculation of $\hat{N}_{i,j}$

We can estimate $\hat{N}_{i,j}$ by using by one of the ldf estimators. $\hat{N}_{i,j}$ is calculated by using Type 1 values given in Table 120 and Table 129.

First iteration:

$$\hat{N}l_{2005,1} = Nl_{2005,0} \times f_0^a$$

$$\hat{N}l_{2005,1} = 557 \times 1{,}111389$$

$$\hat{N}l_{2005,1} = 619{,}04$$

	0	1	2	3	4	5
2000	414	460	482	488	492	494
2001	453	506	526	536	539	
2002	494	548	572	582		
2003	530	588	615			
2004	545	605				
2005	557	**619.04**				

Table 138 First iteration

Second iteration:

$$\hat{Nl}_{2004,2} = Nl_{2004,1} \times f_1^a$$

$$\hat{Nl}_{2004,2} = 605 \times 1.044266$$

$$\hat{Nl}_{2004,2} = 631.78$$

	0	1	2	3	4	5
2000	414	460	482	488	492	494
2001	453	506	526	536	539	
2002	494	548	572	582		
2003	530	588	615			
2004	545	605	**631.78**			
2005	557	619.04				

Table 139 Second iteration

Third iteration:

$$\hat{Nl}_{2005,2} = Nl_{2005,1} \times f_1^a$$

$$\hat{Nl}_{2005,2} = 619,04 \times 1.044266$$

$$\hat{Nl}_{2005,2} = 646.45$$

	0	1	2	3	4	5
2000	414	460	482	488	492	494
2001	453	506	526	536	539	
2002	494	548	572	582		
2003	530	588	615			
2004	545	605	631.78			
2005	557	619.04	**646.45**			

Table 140 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5
2000	414	460	482	488	492	494
2001	453	506	526	536	539	541.19
2002	494	548	572	582	586.01	588.40
2003	530	588	615	625.03	629.34	631.90
2004	545	605	631.78	642.09	646.52	649.14
2005	557	619.04	646.45	656.99	661.52	664.21

Table 141 Full number of claims

The other two estimators (Type 2 and Type 3) defined at the beginning of this subsection are calculated in the same manner. The calculation steps are omitted to save space in this manuscript.

Now we calculate the claim number of the values in Table 141. The claim numbers are the maximum number of claims in each row.

	0	1	2	3	4	5	**Ultimate**
2000	414.00	460.00	482.00	488.00	492.00	494.00	$\max_j\{\hat{Nl}_{2000,j}\}$
2001	453.00	506.00	526.00	536.00	539.00	541.19	$\max_j\{\hat{Nl}_{2001,j}\}$
2002	494.00	548.00	572.00	582.00	586.01	588.40	$\max_j\{\hat{Nl}_{2002,j}\}$
2003	530.00	588.00	615.00	625.03	629.34	631.90	$\max_j\{\hat{Nl}_{2003,j}\}$
2004	545.00	605.00	631.78	642.09	646.52	649.14	$\max_j\{\hat{Nl}_{2004,j}\}$
2005	557.00	619.04	646.45	656.99	661.52	664.21	$\max_j\{\hat{Nl}_{2005,j}\}$

Table 142 Calculation of ultimate

First iteration:

$$U_{2000}^{Nl} = \max(Nl_{2000,0} Nl_{2000,1}, Nl_{2000,2}, Nl_{2000,3}, Nl_{2000,4}, Nl_{2000,5})$$
$$U_{2000}^{Nl} = \max(414, 460, 482, 488, 492, 494)$$
$$U_{2000}^{Nl} = 494$$

	0	1	2	3	4	5	**Ultimate**
2000	414.00	460.00	482.00	488.00	492.00	494.00	494.00
2001	453.00	506.00	526.00	536.00	539.00	541.19	
2002	494.00	548.00	572.00	582.00	586.01	588.40	
2003	530.00	588.00	615.00	625.03	629.34	631.90	
2004	545.00	605.00	631.78	642.09	646.52	649.14	
2005	557.00	619.04	646.45	656.99	661.52	664.21	

Table 143 First iteration

Second iteration:

$$U_{2001}^{Nl} = \max(Nl_{2001,0} Nl_{2001,1}, Nl_{2001,2}, Nl_{2001,3}, Nl_{2001,4}, Nl_{2001,5})$$

$$U_{2001}^{Nl} = \max(453, 506, 526, 536, 539, 541.19)$$

$$U_{2001}^{Nl} = 541.19$$

	0	1	2	3	4	5	**Ultimate**
2000	414.00	460.00	482.00	488.00	492.00	494.00	494.00
2001	453.00	506.00	526.00	536.00	539.00	541.19	**541.19**
2002	494.00	548.00	572.00	582.00	586.01	588.40	
2003	530.00	588.00	615.00	625.03	629.34	631.90	
2004	545.00	605.00	631.78	642.09	646.52	649.14	
2005	557.00	619.04	646.45	656.99	661.52	664.21	

Table 144 Second iteration

Third iteration:

$$U_{2002}^{Nl} = \max(Nl_{2002,0} Nl_{2002,1}, Nl_{2002,2}, Nl_{2002,3}, Nl_{2002,4}, Nl_{2002,5})$$

$$U_{2002}^{Nl} = \max(494, 548, 572, 582, 586.01, 588.40)$$

$$U_{2002}^{Nl} = 588.40$$

	0	1	2	3	4	5	**Ultimate**
2000	414.00	460.00	482.00	488.00	492.00	494.00	494.00
2001	453.00	506.00	526.00	536.00	539.00	541.19	541.19
2002	494.00	548.00	572.00	582.00	586.01	588.40	**588.40**
2003	530.00	588.00	615.00	625.03	629.34	631.90	
2004	545.00	605.00	631.78	642.09	646.52	649.14	
2005	557.00	619.04	646.45	656.99	661.52	664.21	

Table 145 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5	**Ultimate**
2000	414.00	460.00	482.00	488.00	492.00	494.00	494.00
2001	453.00	506.00	526.00	536.00	539.00	541.19	541.19
2002	494.00	548.00	572.00	582.00	586.01	588.40	588.40
2003	530.00	588.00	615.00	625.03	629.34	631.90	631.90
2004	545.00	605.00	631.78	642.09	646.52	649.14	649.14
2005	557.00	619.04	646.45	656.99	661.52	664.21	664.21

Table 146 Full ultimates

The fourth step of the method is about the calculation of the unit claim-loss ratio. The unit claim-loss ratio values are calculated by means of Equation 28 is shown in Table 147.

$$ul_{i,j} = \frac{C_{i,j}}{Nl_{i,j}}$$

Equation 28 Calculation of unit claim-loss ratio

	0	1	j	n-i	...	n-1	n
0	$ul_{0,0}$	$ul_{0,1}$	$ul_{0,j}$	$ul_{0,n-i}$		$ul_{0,n-1}$	$ul_{0,n}$
1	$ul_{1,0}$	$ul_{1,1}$	$ul_{1,j}$	$ul_{1,n-i}$		$ul_{1,n-1}$	
...							
i	$ul_{i,0}$	$ul_{i,1}$	$ul_{i,j}$	$ul_{i,n-i}$			
...							
n-1	$ul_{n-1,0}$	$ul_{n-1,1}$					
n	$ul_{n,0}$						

Table 147 Triangle of unit claim-loss

Now we calculate the unit claim-loss ratio of the values reported in Table 118 Table 120.

First iteration:

$$ul_{2000,0} = \frac{C_{2000,0}}{Nl_{2000,0}}$$

$$ul_{2000,0} = \frac{3209}{414}$$

$$ul_{2000,0} = 7.75121$$

	0	1	2	3	4	5
2000	**7.75120**					
2001						
2002						
2003						
2004						
2005						

Table 148 First iteration

Second iteration:

$$ul_{2000,1} = \frac{C_{2000,1}}{Nl_{2000,1}}$$

$$ul_{2000,1} = \frac{4372}{460}$$

$$ul_{2000,1} = 9.50435$$

	0	1	2	3	4	5
2000	7.75120	**9.50435**				
2001						
2002						
2003						
2004						
2005						

Table 149 Second iteration

Third iteration:

$$ul_{2001,0} = \frac{C_{2001,0}}{Nl_{2001,0}}$$

$$ul_{2001,0} = \frac{3367}{453}$$

$$ul_{2001,0} = 7.43267$$

	0	1	2	3	4	5
2000	7.75120	9.50435				
2001	**7.43267**					
2002						
2003						
2004						
2005						

Table 150 Third iteration

Now we apply the same procedure to all cells.

	0	1	2	3	4	5
2000	7.75121	9.50435	9.15145	9.07377	9.01423	9.02024
2001	7.43267	9.20751	8.92776	8.80597	8.77551	
2002	7.83603	9.75365	9.43706	9.31271		
2003	7.99811	10.06293	9.78862			
2004	9.04404	11.22975				
2005	9.36625					

Table 151 Full unit claim-loss ratio

The fifth step of the method is about the calculation of the transition ratio. The transition ratio values are calculated by following steps.

First iteration:

$$\imath lp_0^* = ul_0$$

Then calculate by:

$$ulp_{0,j} = \frac{ul_{0,j}}{ulp_0^*}$$

First row is completed. In the second row, first iteration:

$$ulp_{1,(n-1)} = \frac{1}{1}\sum_{i=0}^{0} ulp_{i,n-1}$$

For this is simply

$$ulp_{1,(n-1)} = ulp_{0,(n-1)}$$

Next iteration:

$$ulp_1^* = \frac{ul_{1,(n-1)}}{ulp_{1,(n-1)}}$$

For the third row (2^{nd} year),

$$ulp_{2,(n-2)} = \frac{1}{2}\sum_{i=0}^{1} ulp_{i,(n-2)}$$

This value is $ulp_{2,(n-2)} = \frac{ulp_{2000,3} + ulp_{2000,4}}{2}$

Next iteration:

$$ulp_2^* = \frac{ul_{2,(n-2)}}{ulp_{2,(n-2)}}$$

Then calculate

$$ulp_{2,j} = \frac{ul_{2,j}}{ulp_2}$$

In general the algorithm can be summarized as follows (without the peculiarities)
For $i \geq 1$

Step 1: Calculate $ulp_{i,(n-i)} = \frac{1}{i} \times \sum_{m=0}^{i-1} ulp_{m,(n-i)}$

There are two alternative calculation for Step 1.

First alternative:

$$ulp_{i,(n-i)} = \left(\prod_{m=0}^{i-1} ulp_{m,(n-i)} \right)^{1/(n-i)}$$

Second alternative:

$$ulp_{i,(n-i)} = \frac{\sum_{m=0}^{i-1} ulp_{m,(n-i)} \times (m+1)}{\sum_{m=0}^{i-1} m+1}$$

Step 2: Calculate $ulp_i^* = \frac{ul_{i,(n-i)}}{ulp_{i,(n-i)}}$

Step 3: Calculate $ulp_{i,j} = \frac{ul_{i,j}}{ulp_i^*}$

Thus we obtain.

The final step is consolidating all figures we have calculated to so far in the format shown in Table 152.

Actual Ultimate	Prediction Number of Claims	Prediction of ultimate loss-claim ratio	IBNR Reserve
U_i^a	U_i^{Ul}	ulp_i^*	$(U_i^{Ul} \times ulp_i^*) - U_i^a$
		Total IBNR Reserve	

Table 152 Consolidating all figures

Now we calculate the first iteration of the IBNR RESERVE.

First iteration:

$$R_{2001} = (U_{2001}^{Ul} \times ulp_{2001}^*) - U_{2001}^a$$

$$R_{2001} = (541.19 \times 8.78) - 4730$$

$$R_{2001} = 22.40$$

Actual Ultimate	Prediction Number of Claims	Prediction of ultimate loss-claim ratio	IBNR Reserve
4456	494.00	9.02	**0**
4730	541.19	8.78	**22.40**

Table 153 First iteration

Second iteration:

$$R_{2002} = (U_{2002}^{Ul} \times ulp_{2002}^*) - U_{2002}^a$$

$$R_{2002} = (588.40 \times 9.27) - 5420$$

$$R_{2002} = 35.73$$

Actual Ultimate	Prediction Number of Claims	Prediction of ultimate loss-claim ratio	IBNR Reserve
4456	494.00	9.02	**0**
4730	541.19	8.78	**22.40**
5420	588.40	9.27	**35.73**

Table 154 Second iteration

Third iteration

$$R_{2003} = (U_{2003}^{Ul} \times ulp_{2003}^{*}) - U_{2003}^{a}$$
$$R_{2003} = (631.90 \times 9.63) - 6020$$
$$R_{2003} = 66.05$$

Actual Ultimate	Prediction Number of Claims	Prediction of ultimate loss-claim ratio	IBNR Reserve
4456	494.00	9.02	**0**
4730	541.19	8.78	**22.40**
5420	588.40	9.27	**35.73**
6020	631.90	9.63	**66.05**

Table 155 Third iteration

Now we apply the same procedure to all cells.

Actual Ultimate	Prediction Number of Claims	Prediction of ultimate loss-claim ratio	IBNR Reserve
4456	494.00	9.02	**0**
4730	541.19	8.78	**22.40**
5420	588.40	9.27	**35.73**
6020	631.90	9.63	**66.05**
6794	649.14	10.70	**150.37**
5217	664.21	11.08	**2142.42**

Table 156 Final Table

Finally. we calculate the fund of the future claims payment which is the summation of all IBNR Reserve.

$$\text{Total Reserve} = \sum_{i}^{n} R_i$$

Equation 29 Calculation of total reserve

In our example calculation we have

Total Reserve = 0+22.40+35.73+66.05+150.37+2142.42=2416.98

The other two estimators of number of claims (Type 2 and Type 3) defined at the beginning of this subsection are calculated in the summer. Furthermore. the other two estimators of prediction of ultimate loss-claim ratio (Type 2 and Type 3) defined at the previous of this subsection are calculated in the summer. The calculation steps are omitted to save in this manuscript

2.4 IBNR calculation in Turkey

According to Article 16(6) of the Insurance Law No. 5684, which was published via the Official Gazette No. 26552 of 14.06.2007:

Outstanding claims reserve; consist of the amount of claims that has been reported but not yet paid, estimated amount of claims that has been incurred but not reported and reserves for expenses arising from such claims. and of additional reserves allocated according to the principles set by the Undersecretariat for adequacy when such amounts prove to be inadequate.

Accordingly it will be equal to the sum of:

- Amounts of claims that have accrued and determined by approximation but have not paid actually during preceding account periods or current account period; or estimated amounts if such amounts have not been calculated.
- Amounts of claims that have been incurred but not yet reported (IBNR)
- Amounts determined in connection with all shares of expenditure that are required for maturation of claim files. including calculated or estimated fees payable to experts, surveyors, consultants, and action and communication charges.

Based on the authorization granted by Article 16(9) of the Insurance Law No. 5684 of 03.06.2007. the Undersecretariat of Treasury published the Regulation on Technical Reserves of Insurance and Reinsurance and Pension Companies, and the Assets in Which Such Reserves are to be Invested. via the Official Gazette No. 28.07.2010 of 28.07.2010. The principles and methods of setting aside outstanding claim reserves are regulated in Article 7 of said Regulation. Accordingly:

Outstanding claim reserves can be calculated using five different methods. These methods are Standard Chain. Loss/Premium, Cape Code, Frequency/Severity and Munich Chain. Companies are allowed to pick one of these five methods for each branch. In 2010, companies were granted a trial period to pick their own periods. In the last quarter of 2010, it was told that a company which picked its method may not change the method for a period of three years. Methods used for each branch will be explained in footnotes to financial statements.

Reinsurance companies will select the data sets that are to be used for study. perform correction activities, select appropriate methods from among proposed methods, and also select development factors using actuarial methods on the basis of each branch as a result of an analysis of data obtained by reinsurance companies from insurance companies. On the other hand, the reasons and the results of these selections will be evaluated by actuary units in relevant sections of actuarial reports in a detailed manner. Reinsurance companies may generate data on the basis of year where businesses were accepted instead of on the basis of applicable accident period at the time of performance of ACLM (Actuarial Chain Ladder Method) calculations due to nature of reinsurance deals, and may perform ACLM calculations once a year as of yearend. When choosing a method, a company must choose the method giving results that are nearest to actual figures depending upon the nature of branch in question as well as the portfolio structure of the company. Methods used by companies for branches will be monitored by the Undersecretariat, and it will be reviewed whether the method used by a company for a branch is different from that is generally used across the sector for that branch. At the time of performance of ACLM calculations. methods will be calculated based on actual damages (e.g. the sum of outstanding and paid claims). In order to enable companies to make calculations based on a more homogenous set of data. it is possible to eliminate extreme damages in a separate table / file using statistical methods (Box Plot). Elimination can be performed by companies' actuary units for branches where the number of files contained within relevant main branch is at or below one per thousand of the number of claim files other than health branch. Furthermore. elimination can also be performed by companies' actuary units for branches where the total number of claim files is below 300 for any period of time which is subject to ACLM calculations.

For files that have remained outstanding with different amounts for several periods. it is required to subject the final claim amount to the elimination involving large claims.

If the final amount is over the limit that is specified in the Box-Plot, then any amounts remaining outstanding in connection with the file during interim periods must also be eliminated (even if they are below the limit). If the final amount is below the limit that is specified in the Box-Plot. and if there are any amounts reflected in outstanding amounts of interim periods that exceed the limit applicable for large claims, then the portion of said outstanding amounts of interim periods up to the final amount must be eliminated.

The amount determined as a result of ACLM calculations will be compared to the outstanding claim provision that has accrued and determined by approximation. and the difference will be defined as the Incurred But Not Reported (IBNR) Claim. Out of outstanding amounts reflected in the balance sheet; the amounts not covered by ACLM calculations. and the claim amounts put out of scope of ACLM calculations since they are called as large claims. will be added to the amount determined as a result of ACLM calculations. and this amount will then be compared to the outstanding file amounts as reflected in the balance sheet.

ACLM calculations will be performed over gross amounts. and net figures will be determined based on current or relevant reinsurance agreements of company. For ACLM calculations. no consideration will be given to calculations associated with indirect deals for which no association can be made with file.

It is required to perform ACLM calculations on the basis of main branches. and to pay consideration to the weights. in relevant main branches. of claim amounts that have accrued with respect to sub-branches in connection with the distribution to sub-branches of such amounts that have been found for main branches.

If ACLM calculations give a negative result. then the result will be taken as zero. If the amount obtained from ACLM calculations is less than claim amounts that have accrued and determined by approximation. then IBNR will be taken as zero.

The companies are required to submit to the Undersecreteriat a report of functions of their actuary divisions as well as software products used by them (in CD format) at least three (3) months in advance of the period when they wish to intervene in development coefficients. The Undersecretariat will grant a right to intervene in development coefficients to those companies who are found eligible as a result of respective evaluations. The reports must contain at least the following facts:

- The number of employees employed at the actuarial reporting unit
- Educational statuses. foreign language backgrounds. and experiences of employees employed at relevant units
- International professional eligibility certificates held by employees employed at relevant units (e.g. FRM, PRM, CFA, SOA)
- Actuarial software products used by company. and technical specifications and features of these software products
- All studies (reports) prepared and submitted by actuarial unit to management with respect to pricing and reserve calculations during the last two years, together with explanations thereof.

3 METHODOLOGY

In this chapter, generation of confidence interval for IBNR reserves will be explained. Bootstrap approach and parametric approaches will be expalined respectively. Bootstrap method is popular technique in literature however Parametric approach has been developed in this thesis using standard error formulation developed my Mack (1993) and by applying Monte Carlo Simulation technique to sample from the prescribed distribution.

3.1 Bootstrap Approach

This method was originally developed by Julien Lowe in 1994. The method algorithm shown below. Consider the following example depicted in Table 157.

	1	2	3	4	5	6	7	8	9	10
1	357848	1124788	1735330	2218270	2745596	3319994	3466336	3606286	3833515	3901463
2	352118	1236139	2170033	3353322	3799067	4120063	4647867	4914039	5339085	
3	290507	1292306	2218525	3235179	3985995	4132918	4628910	4909315		
4	310608	1418858	2195047	3757447	4029929	4381982	4588268			
5	443160	1136350	2128333	2897821	3402672	3873311				
6	396132	1333217	2180715	2985752	3691712					
7	440832	1288463	2419861	3483130						
8	359480	1421128	2864498							
9	376686	1363294								
10	344014									

Table 157 Cumulative payment triangle

i	1	2	3	4	5	6	7	8	9
$\hat{f}$	3.490607	1.747333	1.457413	1.173852	1.103824	1.086269	1.053874	1.076555	1.017725

Table 158 Development Factors

In order to generate bootstrap intervals following steps are required:

1. Calculate development factors ($\hat{f}$) and IBNR values using Chain Ladder method.
2. Using diagonal values and estimated values in the lower triangle matrix and further $\hat{f}$ values calculated in Step 1, estimate upper triangle values using Equation 30.

$$\hat{C}_{i.j} = \frac{C_{i.I+1-i}}{f_{I-i} \times \ldots \times f_j}$$

Equation 30 Generate prediction value.

For instance. We have already predicted $C_{1.8}$ value (3606286) using the Chain Ladder parameters.

$$\hat{C}_{1.8} = \frac{C_{1.10+1-1}}{f_9 \times f_8} = \frac{3901463}{1.0177247 \times 1.076555} = 3560908.978$$

This calculations apply to all cells in the upper triangle matrix except for diagonal values.

3. The third step of the algorithm calculates the Pearson Residual of upper triangle values except for diagonal values. This calculation mimics a typical chi-square test value in the sense that the differences between observed and expected values are scaled. The residual calculation is shown in Equation 31.

$$r_{i.j} = \frac{\left(C_{i.j} - C_{i.j-1}\right) - \left(\hat{C}_{i.j} - \hat{C}_{i.j-1}\right)}{\sqrt{\hat{C}_{i.j} - \hat{C}_{i.j-1}}}$$

Equation 31 Residual calculation

4. The fourth step of the algorithm resamples (with replacement) the residuals ($r_{i.j}$).
5. Now we bootstrap over the diagonal values. The "Difference of Bootstrap" is added to all cells in the upper triangle except for the diagonal values. The difference formula is shown in Equation 32.

$$X_{i.j}^{*} = r_{i.j}^{*}\sqrt{\hat{C}_{i.j} - \hat{C}_{i.j-1}} + \left(\hat{C}_{i.j} - \hat{C}_{i.j-1}\right)$$

Equation 32 Bootstrap Difference

6. Finally. the data set is quivered with using bootstrap difference by means of Equation 33.

$$C_{i.j}^{*} = C_{i.j-1}^{*} + X_{i.j}^{*}$$

Equation 33 Quiver of data set

7. The Chain Ladder is applied to calculate IBNR values.

The algorithm is run 1000 times. So we will have 1000 IBNR reserve values. Then the confidence intervals are selected by using the confidence limit. For instance. if the confidence limit of 95% is chosen then the algorithm reports 2.5th and 97.5th percentile of the iterated IBNR reserve values.

3.2 Parametric Approach

The standard error calculation of IBNR values is developed by Mack (1993). Mack showed that the Chain Ladder estimations (values in the lower triangle) follow a normal distribution. The algorithm that uses this distribution to generate confidence intervals involves the following steps:

1. The first step of the parametric approach calculates column squared standard errors (σ_k^2) by using the following equation.

$$\hat{\sigma}_k^2 = \frac{1}{I-k-1}\sum_{i=1}^{I-k} C_{ik}\left(\frac{C_{i.k+1}}{C_{ik}} - \hat{f}_k\right)^2$$

Equation 34 Error Calculation

We calculate the (σ_k^2) of the values reported in Table 157 and Table 158. For instance the squared standard error of the first column is calculated in the following way.

$\sum_{i=1}^{I-k} C_{ik}\left(\frac{C_{i.k+1}}{C_{ik}} - \hat{f}_k\right)^2 = 357848\left(\frac{11247788}{357848} - 3.490607\right)^2 + 352118\left(\frac{1236139}{352118} - 3.490607\right)^2 + ... + 376686\left(\frac{1363294}{376686} - 3.490607\right)^2 = 1282242.62$

$\hat{\sigma}_1^2 = \frac{1}{I-k-1} 1282242.62,$

where I =10 and k=1, and

$$\hat{\sigma}_1^2 = \frac{1}{10-1-1} 1282242.62$$

2. Using these columns of the squared standard errors cell-wise standard errors are calculated by using Equation 35.

$$Var(C_{i.k+1}) = C_{ik}\sigma_k^2$$

Equation 35 Cell Variance

For instance. $Var(C_{10.2}) = C_{10.1}\sigma_1^2 = 344014 \times 160280 = 55{,}138{,}676{,}578$

In order to obtain the standard errors, we take the square roots.

$$SE(C_{10.2}) = \sqrt{55{,}138{,}676{,}578} = 234{,}816.261$$

Equation 36 Standard Error

3. The third step of the parametric approach is to predict IBNR values using by Equation 37.

$$\bar{\bar{C}}_{i.j} = \hat{C}_{i.j} + z_\alpha \times SE\left(C_{i.j}\right)$$

Equation 37 Prediction of IBNR

All parameters are known in Equation 37, expect z_α and we have used the following two sampling sub-algorithms to generate those values.

3.2.1 Method I

In order to develop an error propagation strategy, we focus on the third row and (n-1)st column of the claim matrix (second row is clear: propagate 95% error). This is the place where only one estimation is required to calculate IBNR. The aim is to generate a 95% C.I. for the estimate and thus for the IBNR value. The question is to understand the relationship between confidence intervals generated in each Chain Ladder estimation. In other words. when a cell in the lower triangle is estimated through Chain Ladder. what should be the magnitude of the standard error propagated into that cell so that the accumulated errors will make a 95% confidence interval at the nth column.

Let us denote by $\breve{C}_n$, a 95% C.I. for the claim amount in the nth year. Construction of $\breve{C}_n$ also determines $P\left(\breve{C}_n\right)$. For example. setting $P\left(\breve{C}_n\right) = 0.95$ means propagating standard errors from the [2.5%. 97.5%] percentiles of the standard error distribution. The question then becomes. for $P\left(\breve{C}_n | \breve{C}_{n-1}\right) = 0.95$ what should $P\left(\breve{C}_n\right)$ be given $P\left(\breve{C}_{n-1}\right)$?

Clearly $P\left(\breve{C}_n\right)$ depends on $P\left(\breve{C}_{n-1}\right)$. The dependence of $P\left(\breve{C}_n\right)$ on $P\left(\breve{C}_{n-1}\right)$ is assumed to be linear (Chain Ladder assumption) and positive. This means that:

1. $P\left(\breve{C}_n | \breve{C}_{n-1}\right) > P\left(\breve{C}_n\right)$
2. $P\left(\breve{C}_n \cap \breve{C}_{n-1}\right) > P\left(\breve{C}_n\right).P\left(\breve{C}_{n-1}\right)$

From the definition of conditional probability, we also have

3. $P(\breve{C}_n|\breve{C}_{n-1}) = \frac{P(\breve{C}_n \cap \breve{C}_{n-1})}{P(\breve{C}_{n-1})}$

In order to see what $P(\breve{C}_n)$ should be for $P(\breve{C}_n|\breve{C}_{n-1}) = 0.95$ with a given value for $P(\breve{C}_{n-1})$, we would need additional information that relates $P(\breve{C}_n)$ to $P(\breve{C}_n \cap \breve{C}_{n-1})$, Frechét Inequality to be exact.

Frechét Inequality: The value of $P(\breve{C}_n \cap \breve{C}_{n-1})$ has the following lower and upper boundaries:

$$P(\breve{C}_n \cap \breve{C}_{n-1}) \in \left[\max\{0. P(\breve{C}_n) + P(\breve{C}_{n-1}) - 1\}. \min\{P(\breve{C}_n). P(\breve{C}_{n-1})\}\right]$$

We define $x = P(\breve{C}_{n-1})$ to obtain a parametric solution. Since the aim is $P(\breve{C}_n|\breve{C}_{n-1}) = 0.95$ by (3) we have.

$$P(\breve{C}_n \cap \breve{C}_{n-1}) = 0.95x$$

Then, by (2) we have

$$P(\breve{C}_n) < 0.95$$

independent from the value of $P(\breve{C}_{n-1})$. The first check what happens if lower bound of the Frechét inequality is satisfied.

$$P(\breve{C}_n) + P(\breve{C}_{n-1}) - 1 = P(\breve{C}_n \cap \breve{C}_{n-1}) = 0.95x$$
$$P(\breve{C}_n) = 1 - 0.05x$$

So $x > 1$ is impossible. The upper boundary clearly holds when $P(\breve{C}_n \cap \breve{C}_{n-1})$ is set to be equal to $P(\breve{C}_n)$.

As an example, if $P(\breve{C}_{n-1}) = 30\%$ then

$$P(\breve{C}_n) = 0.95 \times 0.30 = 0.285 = P(\breve{C}_n \cap \breve{C}_{n-1})$$

But, in this case co-occurring of the intervals would be only 28.5% although$P(\breve{C}_n|\breve{C}_{n-1}) = 95\%$. A proper choice then would be $P(\breve{C}_{n-1}) = 95\%$ and $P(\breve{C}_n) = 90.25\%$. Then by induction for the third row we obtain

$$P(\breve{C}_{n-2}) = 95.00\%$$
$$P(\breve{C}_{n-1}) = 90.25\%$$
$$P(\breve{C}_n) = 85.74\%$$

and similarly for the fourth row. We have,

$$P(\breve{C}_{n-3}) = 95.00\%$$
$$P(\breve{C}_{n-2}) = 90.25\%$$
$$P(\breve{C}_{n-1}) = 85.74\%$$
$$P(\breve{C}_n) = 81.45\%$$

In fact, this argument can be carried through rows as well. That means in the second row (first IBNR calculation), it is fair to expect a 95% C.I. hence $P(\breve{C}_n) = 95\%$. But for the row below the aim should be to capture only 90.25% C.I. for IBNR. Thus.

$$P(\breve{C}_{n-1}) = 90.25\%$$
$$P(\breve{C}_n) = 85.74\%$$

For the third row on the other hand.

$$P(\breve{C}_{n-2}) = 85.74\%$$
$$P(\breve{C}_{n-1}) = 81.45\%$$
$$P(\breve{C}_n) = 77.38\%$$

Then for the fourth row.

$$P(\breve{C}_{n-3}) = 81.45\%$$
$$P(\breve{C}_{n-2}) = 77.38\%$$
$$P(\breve{C}_{n-1}) = 73.51\%$$
$$P(\breve{C}_n) = 69.83\%$$

and so on.

3.2.2 Method II

The second method proposed in this thesis distributes the errors column wise in the sense that when uncertainty is shared equally among developing years a required confidence limit would be reached at the end. So if developed years were independent of each other (clearly this is not the case for the Chain Ladder Algorithm), a 95% confidence interval can be shared by two independent consecutive years as $\frac{95\%}{2} = 47.5\%$ for each. Then the question is whether development years are clearly related to each other through $\hat{f}s$, how should one distribute the uncertainty among years?

Let us take an arbitrary row say *i* in the claim matrix into consideration. In this matrix, there are *i-1* many estimations. which are located if the cells corresponding to lower triangle. So, for the third row, we need to make two estimations in order to calculate IBNR value for that particular year. And, for the fourth row, we need three estimations and so on. Let $C_{i.n-i}$ be the diagonal value corresponding to that row. Then it has already shown that

$$\hat{C}_{i.n-i+1} = \hat{f}_{n-i+1} \times C_{i.n-i}$$

In this manner, $\hat{C}_{i.n} = \hat{f}_{n-i+1} \times \ldots \times \hat{f}_{n-1} \times C_{i.n-i}$. We want to add a 95% uncertainty into that particular value in order to obtain the requested confidence interval. In order to accomplish this task, assume that we considered the distribution of the uncertainty equally among years. So.

$$\hat{f}_{n-1}\left(\hat{f}_{n-2}\left(\hat{f}_{n-3}(\ldots) + e\right) + e\right) + e = \hat{f}_{n-1} \times \hat{f}_{n-2} \times \ldots \times \hat{f}_{n-i+1} + z_\alpha$$

The right hand side of the equation basically adds a required amount of (say. 95%) uncertainty to the estimated value. Check that $C_{i.n-i}$ values are omitted in both sides of the equation. On the left hand side. each year an equal unknown amount of uncertainty are added that accumulates to the uncertainty on the right hand side. If left hand side is opened.

$$e\left(\hat{f}_{n-1} \times \ldots \times \hat{f}_{n-i+1} + \hat{f}_{n-1} \times \ldots \times \hat{f}_{n-i+2} + \cdots + \hat{f}_{n-1} \times \hat{f}_{n-2} + \hat{f}_{n-1} + 1\right)$$
$$= \hat{f}_{n-1} \times \hat{f}_{n-2} \times \ldots \times \hat{f}_{n-i+1} + z_\alpha$$

Let us define x as $x = \hat{f}_{n-1} \times \ldots \times \hat{f}_{n-i+1} + \hat{f}_{n-1} \times \ldots \times \hat{f}_{n-i+2} + \cdots + \hat{f}_{n-1} \times \hat{f}_{n-2} + \hat{f}_{n-1}$. Then.

$$e = \frac{z_\alpha}{1 + x}$$

Equation 38 Calculaation of error

gives the unknown uncertainty distributed equally to each development year. Method 2 computes this value for each row and uses Monte Carlo Simulation to sample from the Standard Normal Distribution. Sampled values are within $(-e.e)$ range.

Now we can predict to IBNR value reported in Table 157. We will focus on the first three column.

	1	2	3
1	357848	1124788	1735330
2	352118	1236139	2170033
3	290507	1292306	2218525
4	310608	1418858	2195047
5	443160	1136350	2128333
6	396132	1333217	2180715
7	440832	1288463	2419861
8	359480	1421128	2864498
9	376686	1363294	
10	344014	**?**	

Table 159 First three column reported in Table 157

First iteration:

$$\bar{\bar{C}}_{10.2} = \hat{C}_{10.2} + z_\alpha \times SE(C_{10.2})$$

$\hat{C}_{i.j}$ calculation is explained in section 2.3.1 by following equation. This is a classical chain ladder prediction.

$$\hat{C}_{i.j} = C_{i.(j-1)} \times f_i$$

$$\hat{C}_{10.2} = C_{10.1} \times f_1$$

$$\hat{C}_{10.2} = 344014 \times 3.490607 = 1200818$$

$$\bar{\bar{C}}_{10.2} = 1200818 + z_\alpha \times 234816$$

z_α is randomly select with in confidence limit. If the α is 0.05. z score will be with in - 1.96 and 1.96. If the z score is -1.35.

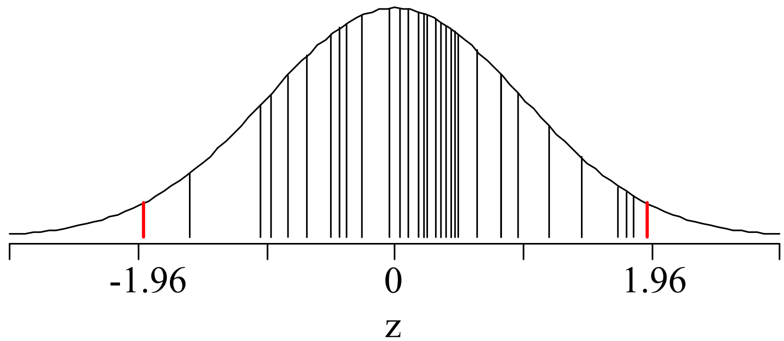


Figure 1 Variates sampled from Standard Normal Distribution Curve

$$\bar{\bar{C}}_{10.2} = 1200818 - 1.35 \times 234816.2613$$

$$\bar{\bar{C}}_{10.2} = 883816.3711$$

	1	2	3
1	357848	1124788	1735330
2	352118	1236139	2170033
3	290507	1292306	2218525
4	310608	1418858	2195047
5	443160	1136350	2128333
6	396132	1333217	2180715
7	440832	1288463	2419861
8	359480	1421128	2864498
9	376686	1363294	
10	344014	**883816.3711**	

Table 160 First iteration

The second iteration and other iterations are similar to the first iteration. But the main difference between the iterations is that the first iteration use chain ladder prediction ($\hat{C}_{i.j}$) and the second iteration (and others) uses approach prediction ($\bar{\bar{C}}_{i.j}$).

$$\bar{\bar{C}}_{i.j} = \bar{\bar{C}}_{i.j} + z_\alpha \times SE(C_{i.j}), \text{where } j > 2$$

The approach is run one thousand times. Then, the confidence intervals are selected by using the confidence limit.

For instance. the confidence limit is %95. So the 25th and 975th rows is selected from simulation matrix to determine the confidence interval.

3.3 Point Estimator

We need to point estimator compare chain ladder result with others approach result in the APPLICATION section. The point estimator is selected row from simulation matrix by means of Equation 39.

$$row\ index = \frac{risk\ free\ rate}{risk\ free\ rate + short\ term\ credis} \times run\ times$$

Equation 39 Point estimators calculation.

Let us take the risk free rate as 0.08. the credit rate as 0.12 and let the run time be 1000. Then,

$$pth = \frac{0.08}{0.08 + 0.12} \times 1000$$

$$pth = 400$$

So the 400th row is selected as the point estimator to be reported.

4 IBNR: A Software for IBNR Calculation

The calculation method for thesis has been formed with ASP.NET. This is not a software but a frame. Having the command of either C# or VB.NET means developing a Project with ASP.NET. Microsoft Visual Studio is the vital software in order to work with ASP.NET.

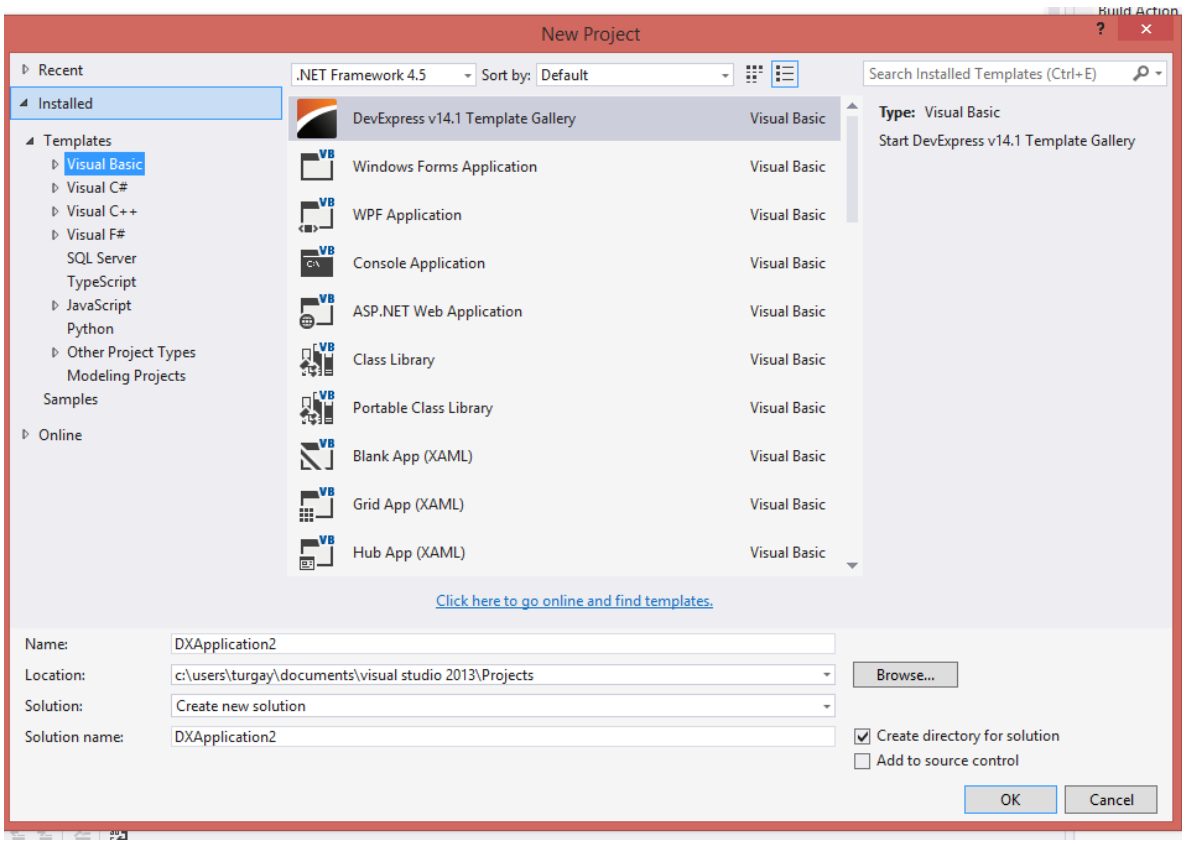


Figure 2 ASP.NET framework

4.1 Microsoft Visual Studio

Our code contains a number of powerful programming tools. Therefore in order to be able to work with ASP.Net. it is enough to have the command of one of those programming software. One of the disadvantages of the software is that it works only with Microsoft softwares family.

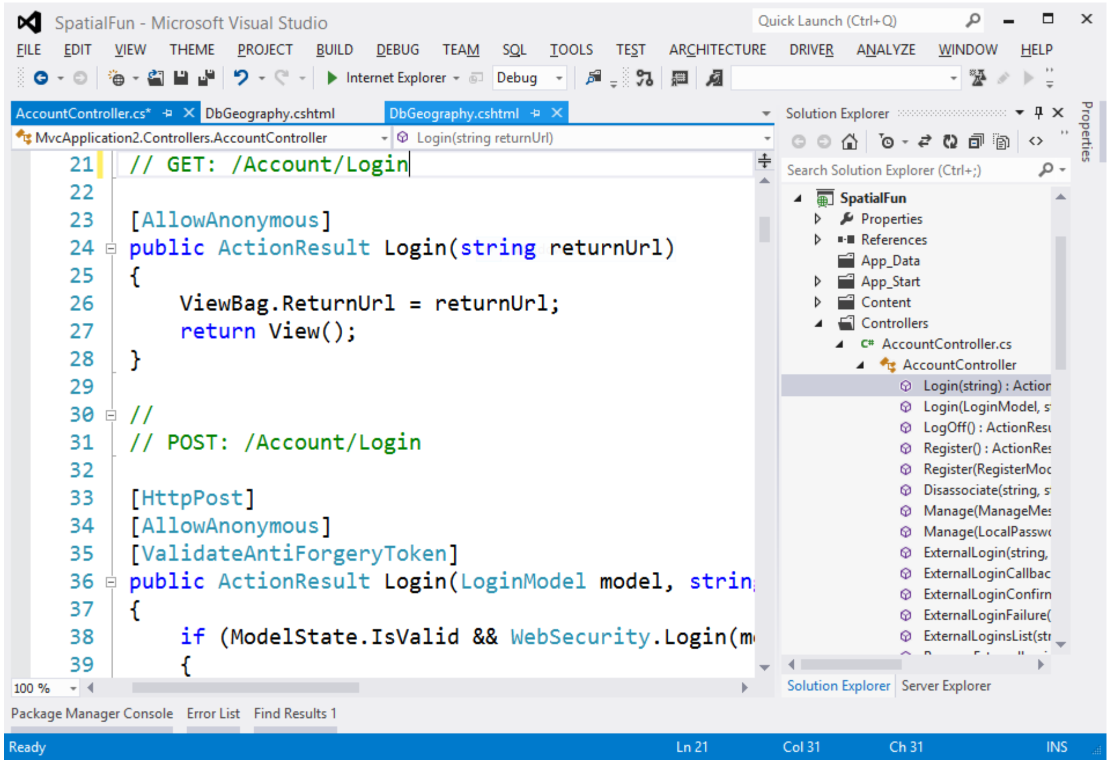


Figure 3 Microsoft Visual Studio code editor

In the software developed. components of Devexpress was used to create a user-friendly menu since the capability of Microsoft Visual Studio components were insufficient and visually bad.

4.2 Devexpress

Devexpress (http://www.devexpress.com/) is component created for ASP.NET platform and it contains many controls for ASP.NET applications. It also provides visual convenience as well as content coding.

When compared with similar components (i.e. Telerik. Ajax etc.). it is superior to its competitors for it number of controls and the features of those controls.

4.3 Web Based Software

As stated before. ASP. NET has the support of many software languages. The software developed for the thesis has been coded by C#.

The software developed is a web-based programme. With usage of internet becoming more common. the use of web-based softwares have pervaded. Web-based software means a programme runs in web browsers (i.e. Internet Explorer. Firefox. Safari. Google Chrome). The advantages of the web-based softwares are that they

- Use in various locations (branch. plant. warehouse. field and on trip).
- Internet. being the cheapest network on earth.
- No need for download. installation etc and use in standard web browsers.
- Very limited need of an assistant.
- Easy maintenance and developing.
- Being independent from operation system and hardware.
- Use in the mobile devices.

4.4 Object Oriented Programming

The software has been developed via Object Oriented Programming technique. As it can be understood by name. it can also be called a technique of programming that is object based. While developing a software. most of the time the same code might be used for another function. Before OOP technique was developed. the same code had to be repeated several times. OOP technique provided the user with a common pool by using classified structure. In those common pools exist repeated methods. However it is also possible to reduce the mistakes to a minimum by having the necessary calculations done within the classes without interfering with the main coding. If we take a bicycle and its different kinds as example:

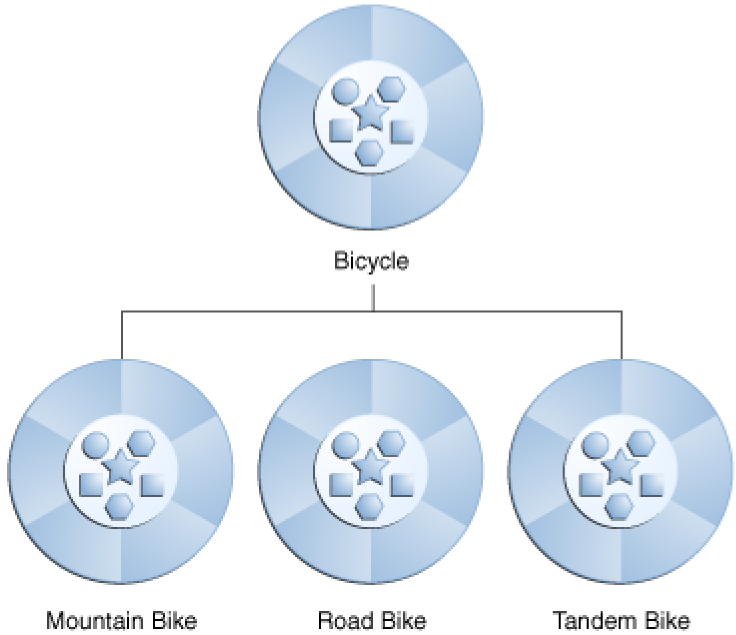


Figure 4 Structure of Object Oriented Programming

All bicycles have the same basic features. which are put into the same class and there is no need to mention those basic features repeatedly.

4.5 Object Oriented Programming in Software

The OOP technique that is used for the development of the software of the thesis was applied in different fields as well. Different controls can be used while designing a web-page whereby the existence of repeated controls is possible. In order to avoid this. controls of web user and master page were used. The layout of the pages was formed by calling the necessary control dynamically. The developed software is formed of single page and fourteen web-user controls. Each web-user control represents an IBNR method.

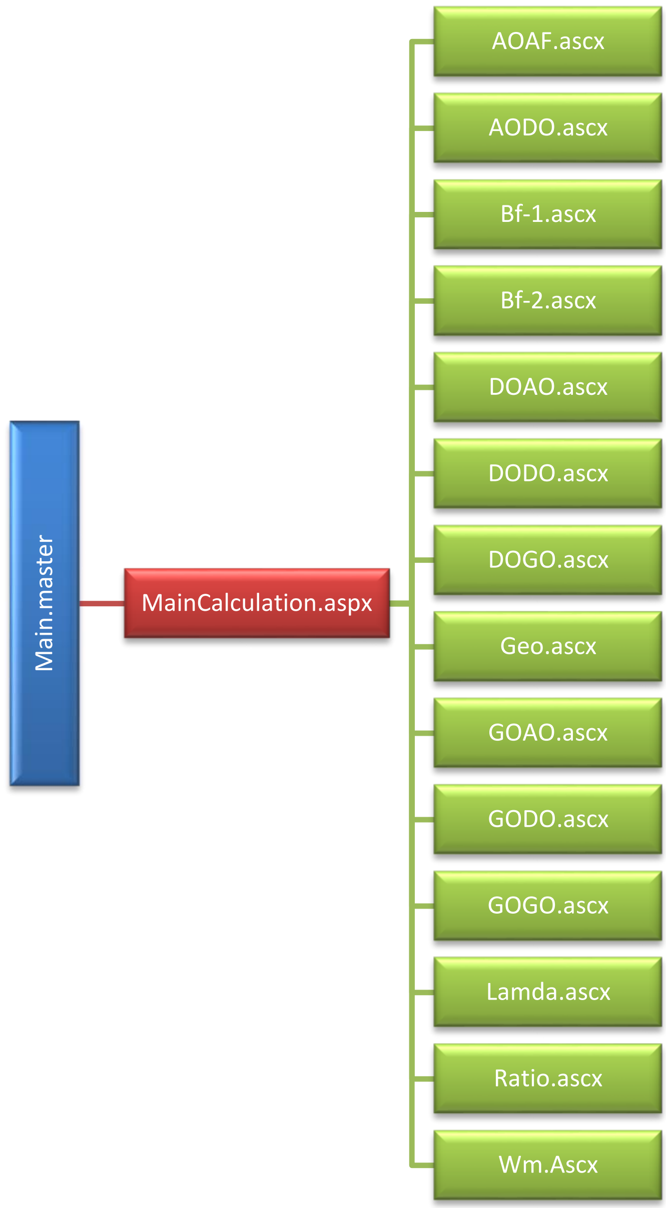


Figure 5 User control list of software

4.6 Class Architecture of the Software

Various classes have been formed in order to make related calculations in the software. In these classes. OOP technique was used to classify the repeating codes into the same pools. As seen in Figure 6 the arrows represents the relation among the classes. For instance the arrow initiating from Lambda Class reaches to Geometric Mean. Ratio Matrix and CreateBlankDataTable Class. This means Geometric Mean. Ratio Matrix and CreateBlankDataTable Class were used within the Lambda Class.

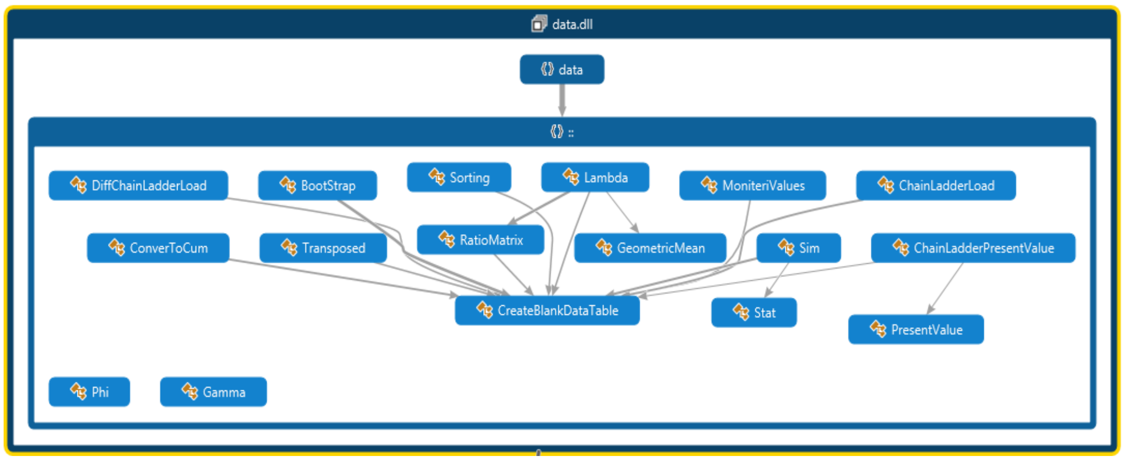


Figure 6 Architecture of class

Below, we examine the classes used in the software:

4.6.1 ChainLadderLoad

The role of this class is to complete the missing elements within the chain of ladder by variables obtained according to the method. It works with two variables. The first variable contains the raw data in data table type (in matrix structure) while the second variable is the data type to be used for estimation. The estimation variable is a vector. Once this class works. the estimations on the report screen are made.

4.6.2 BootStrapp

This class does the confidence interval calculation with Bootstrap method for estimated IBNR calculations.

4.6.3 ConverToCum

If the raw data does not form the sum of the successive years. this class is used to form the cumulative matrix.

4.6.4 CreateBlankData

The function of this class is to form a data table. While calculating. there is need for mid matrixes and this is the reason why this form was created.

4.6.5 DiffChainLadderLoad

If we have a cumulative matrix and there is need for working with a non-cumulative structure. this class fractionates the cumulative matrix into time based fragments.

4.6.6 Gamma

The “gamma” variable that is one of the estimated variables used for chain ladder method is calculated here.

4.6.7 GeometricMean

Due to the nature of data (data table) we have used. we did not apply available library averages. Therefore a class to calculate the geometric average was formed.

4.6.8 Lambda

“Lambda” variable that is one of the estimation variables of chain ladder method is calculated here.

4.6.9 Phi

"Phi" variable that is one of the estimation variables of chain ladder method is calculated here.

4.6.10 RatioMatrix

This is a class developed for calculation of the ratios in chain ladder method that is one the calculation methods of IBNR method within the matrix structure.

4.6.11 Sim

This is a class developed to find the confidence interval by parametric method. In this class Monte Carlo was used.

4.6.12 Sorting

This is a class developed to put the data in the columns of the matrix (data table) in order.

4.6.13 Transposed

This is the class to transpose the data in data table.

4.6.14 Stat

This is a class developed to calculate the probability. The stat package in R statistical language was transformed into a class in C# language.

4.7 Working Principle

If we were to examine the working principle of the software. it has quite a plain and easy interface. The following subtitles will explain the working principle of the software and how data to be used is prepared.

4.7.1 Menu

In **Error! Reference source not found.** menus and the sub-menus are presented. As seen in Figure 7. The method from the IBNR Methods menu is chosen. The detailed information on methods are gives in Chapter 3.

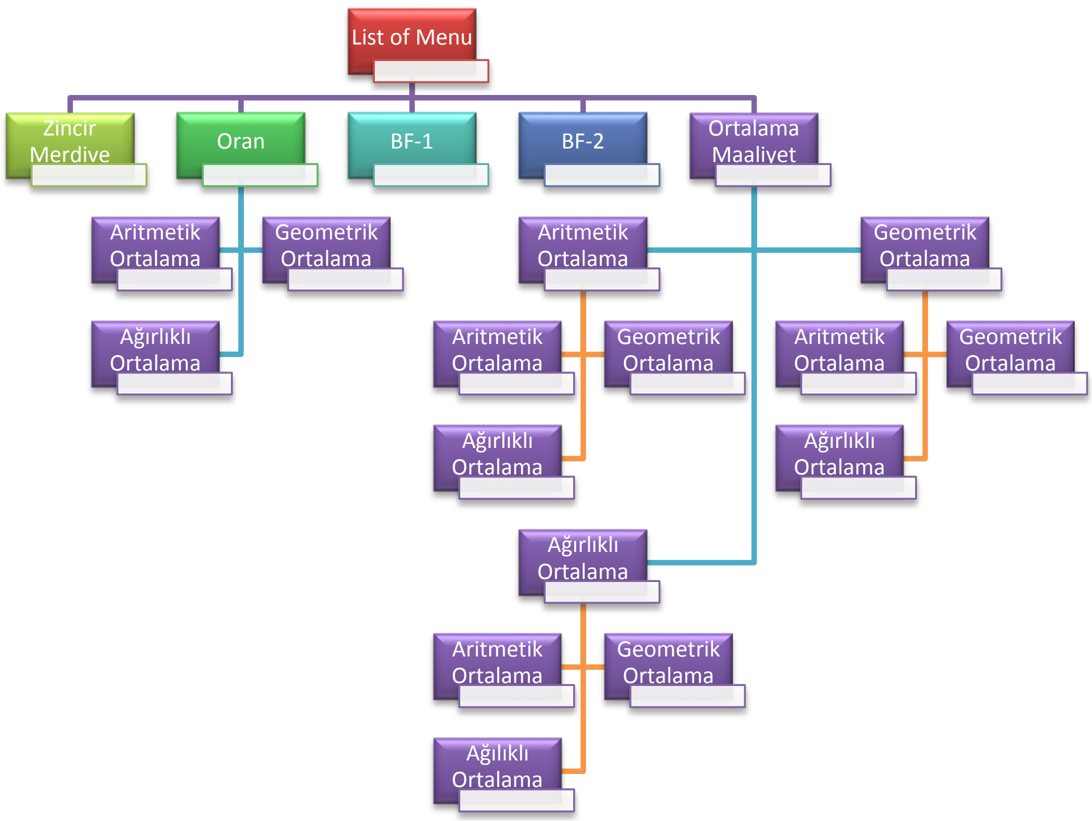


Figure 7 Hierarchy of Menu List

Figure 8 Selection of IBNR Method

4.7.2 Data Upload

Once the method is chosen. the excel file appropriate to the data structure is taken into the system. In Figure 9. this application is shown. There is a data structure for each method. In section 4.7.5. the relation between the method and data structure is given.

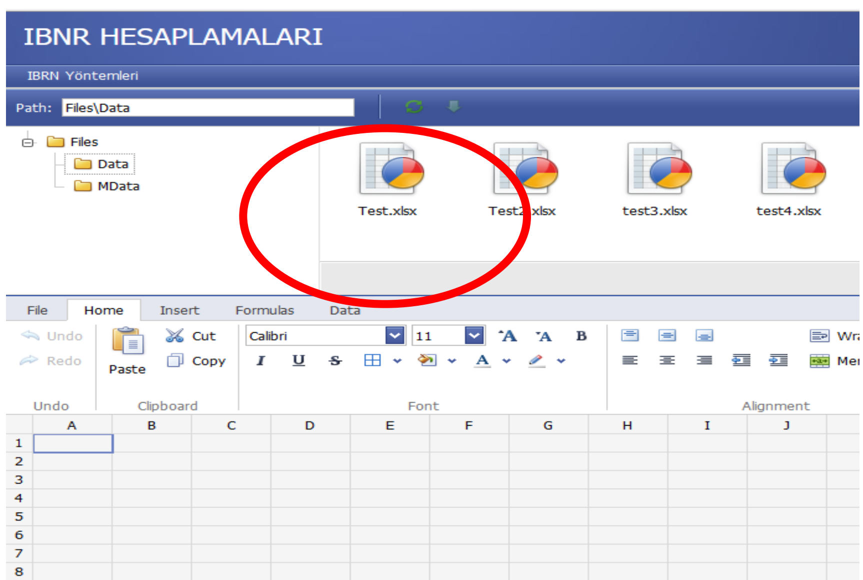


Figure 9 Data selection of IBNR calculation

If there is no data in the system or for a set of data to be calculated the file manager is opened as seen in Figure 10. Once the file is chosen. it is uploaded by clicking the upload button.

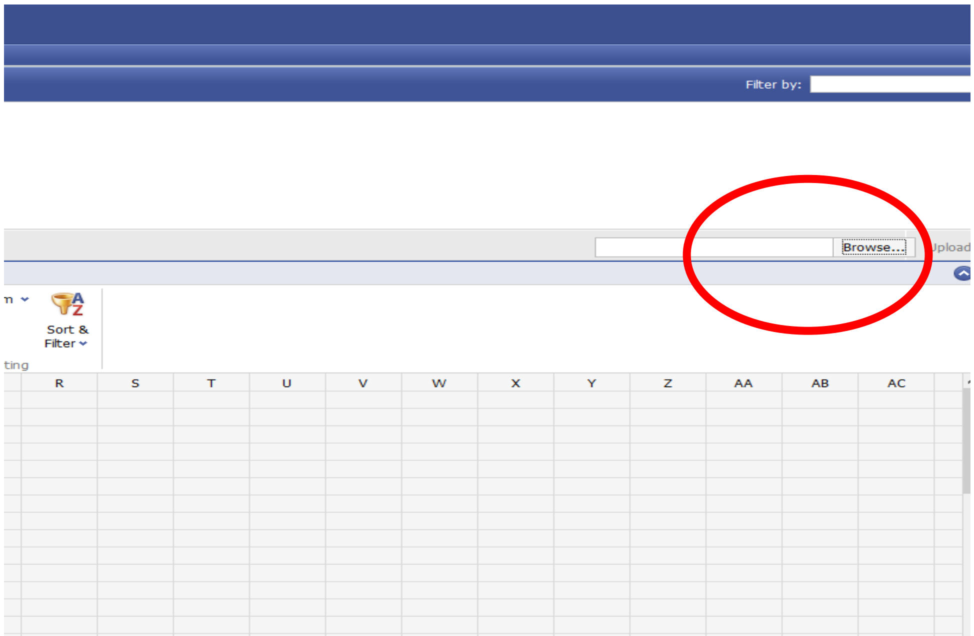


Figure 10 Data Upload

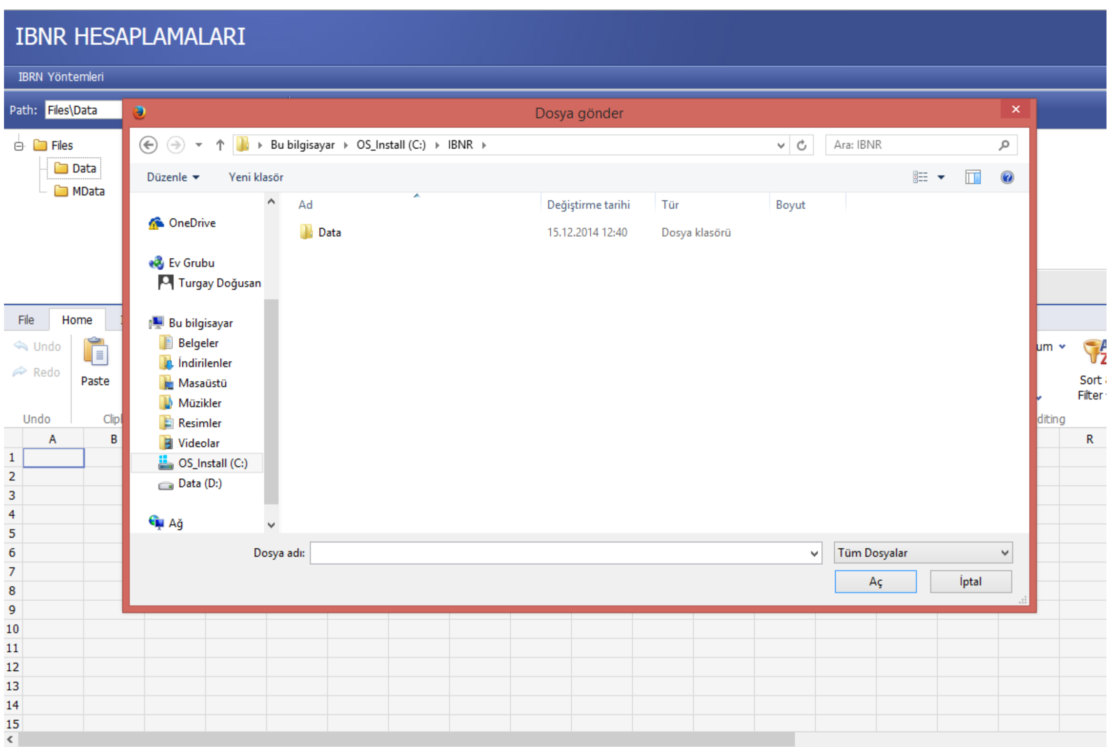


Figure 11 Uploading the chosen files

4.7.3 Calculation

Once the determined method is chosen from the menu. the IBNR calculation is materialized via “Hesapla” button at the bottom of the page as seen in Figure 12. The process of calculation may take a couple minutes depending on the method and the size of data chosen.

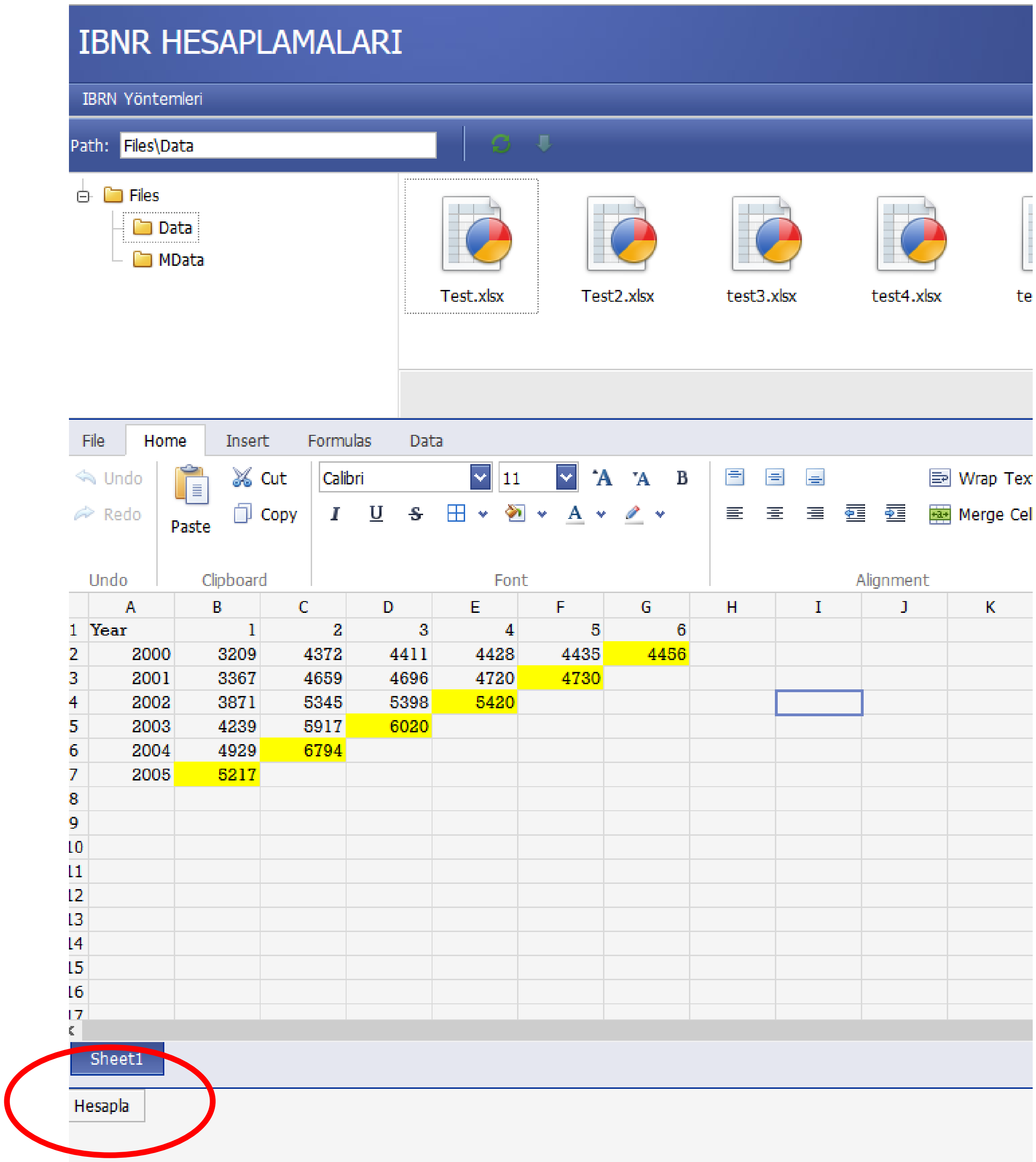


Figure 12 IBNR calculation

4.7.4 Reporting

In the reporting module of the software. we have used ASPXSpreadSheet component of DevExpress. This module contains almost all of the features of Excel. It also offers the facility of recording the report form different file types. It is also possible to draw graphics like Excel. As seen in Figure 13. the module screen is of potential of multi graphic presentation.

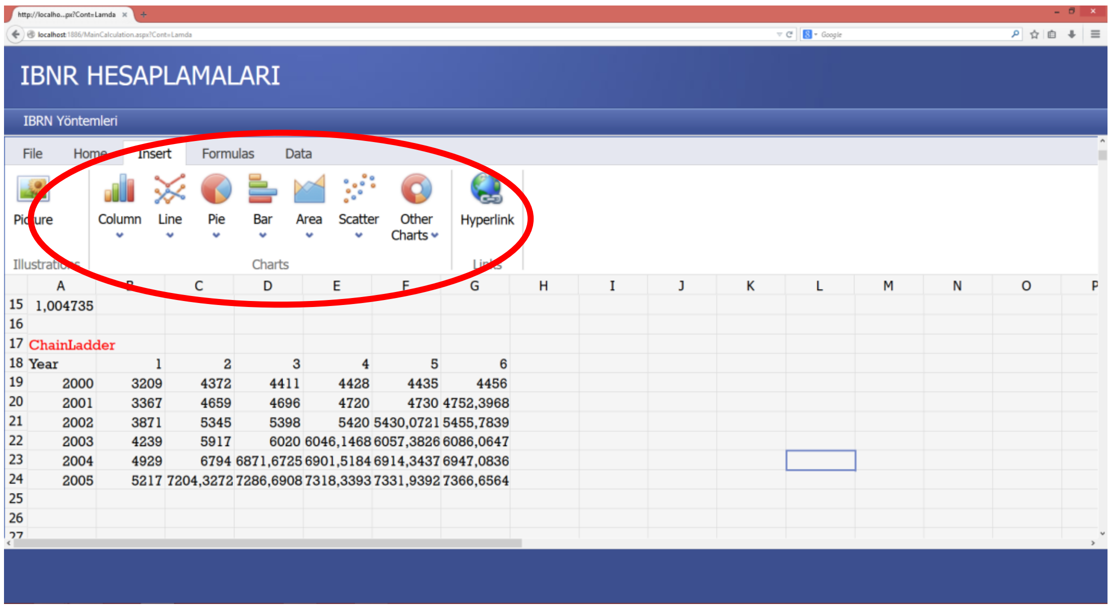


Figure 13 Reporting screen

4.7.5 Data Structure of the Methods

Under this title. we have discussed the standards of data that the software for IBNR methods should have in the following sections.

4.7.5.1 Lambda and Ratio Methods

There is a need for NxN data structure plus a year column are needed in order to use Lambda and Ratio methods for IBNR calculations. Data should be formed from A1 cell. In Figure 14, a sample of excel structure is shown.

	A	B	C	D	E	F	G
1	Year	1	2	3	4	5	6
2	2000	3209	4372	4411	4428	4435	4456
3	2001	3367	4659	4696	4720	4730	
4	2002	3871	5345	5398	5420		
5	2003	4239	5917	6020			
6	2004	4929	6794				
7	2005	5217					

Figure 14 Sample data structure for Lambda and Ratio Method

4.7.5.2 BF-1 and BF-2

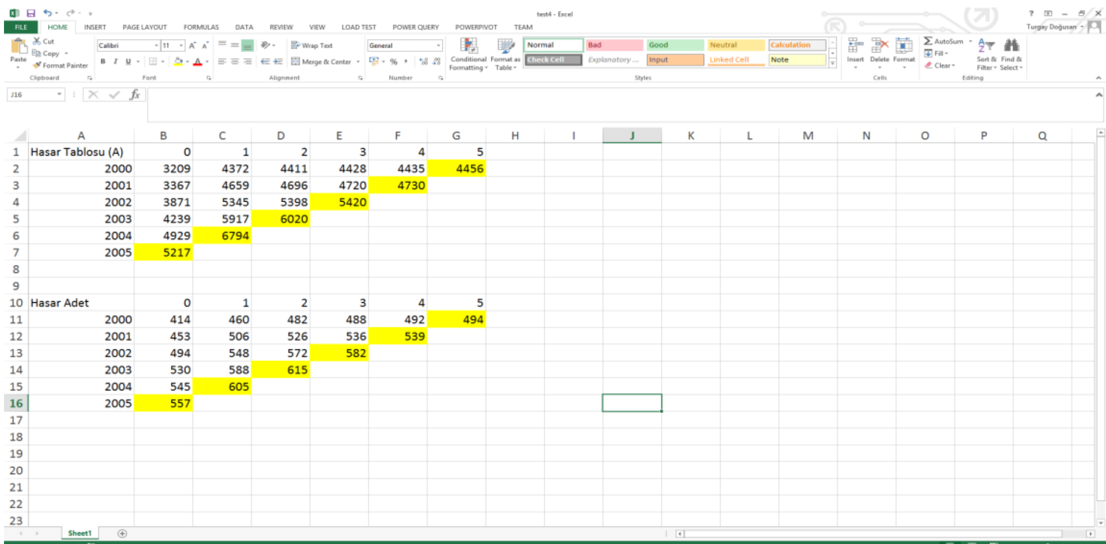
For IBNR calculations for these two methods. there is need for NxN claim table and year column and NxN premium table. The premium table should be of year column. Between claim table and premium table. there should be two rows left empty. There is a sample excel structure reported in Figure 15.

	A	B	C	D	E	F	G
1	Hasar Tablosu (A)	0	1	2	3	4	5
2	2000	3209	4372	4411	4428	4435	4456
3	2001	3367	4659	4696	4720	4730	
4	2002	3871	5345	5398	5420		
5	2003	4239	5917	6020			
6	2004	4929	6794				
7	2005	5217					
8							
9							
10	Prim Tablosu (B)	0	1	2	3	4	5
11	2000	4563	4589	4590	4591	4591	4591
12	2001	4618	4664	4671	4672	4672	
13	2002	4836	4861	4861	4863		
14	2003	5140	5168	5173			
15	2004	5633	5668				
16	2005	6389					

Figure 15 Sample data for BF methods

4.7.5.3 Average Cost Per Claim

In order to calculate IBNR via Average Cost method (including all sub-methods). there is a need for NxN Claim Table and the year column. NxN number of claim and the year column. Two rows must be left empty in between. Data in Claim Table should start from the cell A1. In Figure 16. a sample excel structure is given.



	A	B	C	D	E	F	G
1	Hasar Tablosu (A)	0	1	2	3	4	5
2	2000	3209	4372	4411	4428	4435	4456
3	2001	3367	4659	4696	4720	4730	
4	2002	3871	5345	5398	5420		
5	2003	4239	5917	6020			
6	2004	4929	6794				
7	2005	5217					
8							
9							
10	Hasar Adet	0	1	2	3	4	5
11	2000	414	460	482	488	492	494
12	2001	453	506	526	536	539	
13	2002	494	548	572	582		
14	2003	530	588	615			
15	2004	545	605				
16	2005	557					

Figure 16 Sample data structure for Average Cost Per Claim Method

5 APPLICATION

This chapter reports confidence intervals obtained using parametric and bootstrap approaches on the real data sets. There are 9 data sets used in this study in total. First. the treatment of the data is explained. The biggest square submatrix is chosen and traingularized. The result of the approaches are summarized in a table for each data set. Of particular interest is whether or not the actual IBNR value is indeed within the calculated confidence interval and if so the length of the interval, smaller the better.

5.1 Preparation of Data Set

The claims data comes from Schedule P – Analysis of Losses and Loss Expenses in the National Association of Insurance Commissioners (NAIC) database. The data was downloaded from the Casualty Actuarial Society web site as a .csv format. The data was updated in September 2011. The data shown in Table 161.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1988	31447	87470	146807	220502	256564	282111	300903	308636	314566	317889
1989	36601	99778	178262	238864	289540	318666	331488	343442	350684	NA
1990	41035	108655	190403	262674	302957	334770	352726	361103	NA	NA
1991	39031	126807	227783	316632	368941	402043	426085	NA	NA	NA
1992	42801	126926	222655	304301	350992	389250	NA	NA	NA	NA
1993	44930	173066	293039	378512	434995	NA	NA	NA	NA	NA
1994	54253	169202	296394	402244	NA	NA	NA	NA	NA	NA
1995	39780	167908	294332	NA	NA	NA	NA	NA	NA	NA
1996	62630	191258	NA	NA	NA	NA	NA	NA	NA	NA
1997	54130	NA	NA	NA	NA	NA	NA	NA	NA	NA

Table 161 Liability Data Set

First. we slice the data for compare actual loss with prediction loss. So, we focus on the biggest square matrix of the data which are shown in Table 162.

	1	2	3	4	5	6	7	8	9	10
1988	31447	87470	146807	220502	256564	282111	300903	308636	314566	317889
1989	36601	99778	178262	238864	289540	318666	331488	343442	350684	NA
1990	41035	108655	190403	262674	302957	334770	352726	361103	NA	NA
1991	39031	126807	227783	316632	368941	402043	426085	NA	NA	NA
1992	42801	126926	222655	304301	350992	389250	NA	NA	NA	NA
1993	44930	173066	293039	378512	434995	NA	NA	NA	NA	NA
1994	54253	169202	296394	402244	NA	NA	NA	NA	NA	NA
1995	39780	167908	294332	NA	NA	NA	NA	NA	NA	NA
1996	62630	191258	NA	NA	NA	NA	NA	NA	NA	NA
1997	54130	NA	NA	NA	NA	NA	NA	NA	NA	NA

Table 162 The biggest square matrix

The second step of the preparation of data set involves removing values under the diagonal from the original data. A sample data is shown in Table 163.

	1	2	3	4	5
1988	31447	87470	146807	220502	256564
1989	36601	99778	178262	238864	
1990	41035	108655	190403		
1991	39031	126807			
1992	42801				

Table 163 Input data

5.2 Analysis

For each data set we have conducted three confidence intervals each of which are calculated using the parametric approach developed in this thesis with the confidence levels as %95. In practice. if an insurance company decides to set its IBNR amount with %95 confidence then it will reserve with the amount calculated as the upper value of the confidence interval in each case.

If this predicted amount higher or lower than the actual amount. this situation introduces (asymmetrical) losses for the company as discussed in Section 2. Correspondingly the following performance criteria has been developed:

- If estimator is less than actual IBNR value. the difference should be composeted with short term credits and therefore punished with credit interest rate.

- If estimator is greater than actual IBNR value. the difference is the opportunity cost of risk free rate of return multiplied by the corresponding amount.

When these values are calculated for all years and for all methods, then ‘percentage gap’ values are calculated in the following way:

- For each year. the smallest error is found. This value is substracted from each value in that particular row and the result is divided to the smallest error. So if the error of method is 1500. while the minimum error in that year of all methods is 1000. the percentage gap is $\frac{1500-1000}{1000} = 50\%$.

After percentage gap values are found for all years. they are averaged out method-wise. Figure 17 displays one such outcome. Medical Malpractice Dataset. The first column gives the years of the data prepared with the procedure explained above. On the right of the years. for each approach, the lower and upper confidence interval limits are given. For example. corresponding to 1990 actual IBNR value of 71427. Bootstrap approach predicts the confidence interval as (101117. 134778). Bootsrap approach confidence interval fails in all of the years. Method II is successful in years 1989 and 1990. Method I approach is successful in years 1989. On the rightmost section of the table. The performances of Bootstrap. Method I and Method II with naive chain ladder estimators are displayed.

For years 1989. 1992 method I performed better compered to other methods while for 1990 and 1991 Method II is the best predictor. On the average, Method II is the best method for this data set. followed by Method I. The performance of Bootstrap is considerably low.

Tables of other datasets are given in the Appendix. When all tables are analyzed. if loss amounts follows a relatively stable pattern the methods introduced. In this thesis we do not provide a significant improvement. on the contrary a decline in the performance in IBNR estimations are reported. In those cases using the Chain Ladder provides enough precision.

Yet. it is not possible to know wheather future claim amounts will follow a relatively few volatile pattern a`priori.

Medical Malpractice Data Set	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			Method I			Method II						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
	1988	0	0	0.00	0.00	0.00	0	0	0	0	0	0	0	0	0	0
	1989	26419	37968.87	39798.80	55458.85	45945.46	31886.68	44292.52	37530.34	35773.55	40144.91	37882.20	0.76	0.00	0.03	0.04
	1990	71427	97011.10	101117.82	134778.07	116105.15	88761.20	105466.22	96792.48	93021.74	101124.96	96781.14	0.76	0.00	0.00	0.01
	1991	171567	197740.80	190990.01	261313.85	219617.80	188802.41	206774.46	197254.17	190980.74	204106.67	196963.78	0.89	0.01	0.00	0.03
	1992	231539	271543.70	238569.32	398258.00	293665.08	250566.38	293148.65	269231.83	254175.70	289610.10	269483.52	0.65	0.00	0.01	0.06
												Average Gap	**0.76**	**0.00**	**0.01**	**0.04**
												Max Gap	**0.89**	**0.01**	**0.03**	**0.06**
												Best	0	2	2	0

Figure 17 Outcome

6 CONCLUSION

The aim of this thesis is two folds. The first is to develop a national IBNR software and second is to embed a couple of confidence interval estimation methods into that software so that actuaries would have the opportunity to focus on issues related to IBNR calculations other than routine technicalities. Actuaries are faced with considering different types of risk associated with IBNR calculations. Using their expert knowledge, they can adjust a corresponding level of uncertainty fed in the calculations. The software and confidence interval estimation techniques, one of which is merely coded but other two are developed in this thesis, handle the task of providing a user friendly interface for actuaries to work and experiment with IBNR calculations with the inclusion of dimension of risk.

Due to absence of a national software that is compatible with the Turkish Legislations. actuaries has to calculate IBNR values using simple Excel Spreadsheets. These spreadsheets have to be prepared for each insurance product and for each company every year. In this regard a software that can handle a claim matrix and providing solutions for approved IBNR methods has been demanded. In our interviews with the actuaries we oserved that the available software are not compatible with the Turkish legislations. Any update request in those softwares takes more than six months. Moreover, the price of the software exceeds affordable values. The software developed in this thesis provides 15 different IBNR solutions, using Chain Ladder. Borhhuetter-Ferguson. Expected Loss Ratio and Average Cost per Claim methods permuted with three different averaging techniques (simple arithmetic average. geometric average and weighted mean) where available.

The Turkish legislation limits the techniques that can be used in IBNR calculations. These techniques provide only point estimations for IBNR reserve calculations. Still. Actuaries demand confidence intervals associated with those techniques. In a number of countries (e.g.. Australia. Singapore. the United Kingdom. and South Africa). insurers are required to hold provisions (i.e.. the estimate of unpaid claims) at a specified confidence level (75%). The probability of the confidence interval may vary due to several reasons. This thesis uses Bootstrap Algorithm (Lowe. 1994) to develop confidence intervals associated with approved IBNR techniques. Two additional techniques are developed by using Mack's (1993) standard error formulation. We have used Monte Carlo simulation methodology to overcome both dependency of estimations in the Chain Ladder estimations and the dimensionality problem of the resulting joint probability distribution.

The performance of all confidence interval generation techniques are compared. Results indicate that the two techniques developed in this thesis provide equivalent performances that are better than the Bootstrap`s both in terms of confidence interval length and precision in estimation. All of these methods are coded into a software so that actuaries can experiment and have the opportunity to choose from a list of legitimate solutions considering the risks associated with data. company and other macro uncertainties.

This thesis suffers from a couple of shortcomings. First, we had the opportunity compare the performance of confidence interval generation techniques on ten international datasets. Unfortunately, we have been unable to experiment with any national real data set. Therefore performance comparisons should be extended over national datasets as well.

Second. the software developed in this thesis aimed to be user-friendly but we have not tested whether or not it came out to be a user friendly one. One of our primary focus has been to make the software be able to run on multiple platforms. For this purpose. risking performance issues (related to software computation speed) the software is developed to run in web platform. Further. using brand new object oriented programming technology. required upgrades. bug fixes and sustainability of the software can be handled easily from abroad. However. the coding is still substantially complicated and cannot be straightforwardly comprehended by other programmers. an issue which should not be underestimated.

Third. the confidence interval estimation techniques developed by Lowe (1994) and two techniques introduced in this thesis are only coded to work with simple Chain Ladder. Although the software offers 15 different IBNR solution options only for four of them confidence levels can be calculated. This obvious short coming of the thesis stems from the fact that we could not find an anchor method for other IBNR techniques. like Bootstrap Algorithm is for Chain Ladder.

Fourth. the performance measure that we have used has at least two weaknesses. First. it does not take into consideration whether or not actual values are included in the generated confidence intervals in performance evaluation. It only focuses on the minimum distance between the actual value and the interval. Second. the performance measure is inherently relative. In this regard it is not possible compare performance results obtained in this thesis with results that can be obtained in other studies. Several other performance measures that are prone to aforementioned weaknesses can be tried in further studies. Yet. the coding and the analysis of these measures requires time and personal resources.

In sum. this thesis introduces first the first time a national IBNR reserve calculation software. Further it introduces two novel parametric confidence level generation techniques into the literature that perform better than orthodox algorithms.

7 BIBLIOGRAPHY

Barnett. G. and B. Zehnwirth (1998). "Best Estimates for Reserves."

Bornhuetter. R. (1972). " The actuary and ibnr." Proc. Cas. Act. Soc: 181-195.

Christofides. S. (1990). "Regression models based on log-incremental payments." Claims Reserving Manual **2**.

De Jong. P. and B. Zehnwirth (1983). "Claims reserving. state-space models and the Kalman filter." Journal of the Institute of Actuaries **110**(01): 157-181.

Friedland. J. (2009). Estimating unpaid claims using basic techniques. Casualty Actuarial Society.

Haberman. S. and A. E. Renshaw (1996). "Generalized linear models and actuarial science." The Statistician: 407-436.

Hess. K. T. and K. D. Schmidt (2002). "A comparison of models for the chain–ladder method." Insurance: mathematics and economics **31**(3): 351-364.

Institute of Actuaries. L. (1989). "Claims Reserving Manual."

Jewell. W. S. (1989). "Predicting ibnr events and delays." Astin Bulletin **19**(1): 25-55.

Kaas. R.. et al. (2001). Modern actuarial risk theory. Springer.

Mack. T. (1993). "Distribution-free calculation of the standard error of chain ladder reserve estimates." Astin Bulletin **23**(2): 213-225.

Mack. T. (1993). "Distribution-free calculation of the standard error of chain ladder reserve estimates." Astin Bulletin **23**(2): 213-225.

Makov. U. E.. et al. (1996). "Bayesian methods in actuarial science." The Statistician: 503-515.

Murphy. D. M. (1994). Unbiased loss development factors. CAS Forum. Citeseer.

Renshaw. A. E. (1989). "Chain ladder and interactive modelling.(Claims reserving and GLIM)." Journal of the Institute of Actuaries **116**(03): 559-587.

Renshaw. A. E. (1994). On the second moment properties and the implementation of certain GLIM based stochastic claims reserving models. Department of Actuarial Science & Statistics. City University.

Renshaw. A. E. and R. J. Verrall (1998). "A stochastic model underlying the chain-ladder technique." British Actuarial Journal **4**(04): 903-923.

Schnieper. R. (1991). "Separating true IBNR and IBNER claims." Astin Bulletin **21**(1): 111-127.

Scollnik. D. P. (2001). "Actuarial modeling with MCMC and BUGS." North American Actuarial Journal **5**(2): 96-124.

Tarbell. I. M. (1934). New Dealers of the Seventies. The Forum.

Taylor. G. C. and F. Ashe (1983). "Second moments of estimates of outstanding claims." Journal of Econometrics **23**(1): 37-61.

Verrall. R. (1989). "Modelling claims runoff triangles with two-dimensional time series." Scandinavian Actuarial Journal **1989**(3): 129-138.

Verrall. R. (1994). "A method for modelling varying run-off evolutions in claims reserving." Astin Bulletin **24**(2): 325-332.

Verrall. R. (1996). "Claims reserving and generalised additive models." Insurance: mathematics and economics **19**(1): 31-43.

Verrall. R. and Z. Li (1993). "Negative incremental claims: chain ladder and linear models." Journal of the Institute of Actuaries **120**(01): 171-183.

Verrall. R. J. (1990). "Bayes and empirical Bayes estimation for the chain ladder model." Astin Bulletin **20**(2): 217-243.

Verrall. R. J. (1991). "On the estimation of reserves from loglinear models." Insurance: mathematics and economics **10**(1): 75-80.

Zehnwirth. B. (1985). "Interactive Claims Reserving Forecasting System." Benhar Nominees Pty Ltd. Tunuwurra. NSW. Australia.

Wright, T. S. (1990). "A stochastic method for claims reserving in general insurance." Journal of the Institute of Actuaries **117**(03): 677-731.

Solvency II. http://ec.europa.eu/finance/insurance/solvency/solvency2/index_en.htm

APPENDIX A OUTCOMES

Risk Free Rate =1.00 and Short term credit = 1.2

Liability Data Set	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	50676	39065,01	21027,59	38493,14	23644,31	33575,68	44610,81	38945,55	36778,22	41365,73	38771,31	1,33	0,01	0,03	0,00
	1990	112554	122666,14	85584,91	120786,13	100983,62	109347,50	135925,89	121355,39	115343,84	129900,91	121960,23	0,58	0,00	0,07	0,15
	1991	242134	236409,37	196935,51	270622,61	228447,20	224265,24	248574,10	235442,00	227582,18	245347,04	235469,68	1,39	0,17	0,16	0,00
	1992	308191	307081,74	286519,56	334468,11	308503,78	294560,04	319550,51	305645,99	293554,07	320934,76	306014,07	0,00	8,76	7,35	3,26
												Average Gap	**0,82**	**2,24**	**1,90**	**0,85**
												Max Gap	**1,39**	**8,76**	**7,35**	**3,26**
												Best	1	1	0	2

Commercial Auto Data Set	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	55284	49916,32	38302,24	48441,86	45354,39	47196,44	52540,38	49596,25	48463,07	51389,80	49832,09	0,85	0,06	0,02	0,00
	1990	140297	144969,12	124310,35	142301,95	135656,35	141589,11	148558,83	144804,83	142237,90	147576,60	144783,48	0,24	0,00	0,00	0,04
	1991	290479	280470,76	256622,49	282180,45	270955,22	276957,97	283851,97	280201,37	276936,79	283984,67	280390,47	0,95	0,03	0,01	0,00
	1992	525493	483054,43	457222,13	496371,69	477911,60	478518,38	487334,54	482867,26	476841,44	489439,71	482545,64	0,12	0,00	0,01	0,00
												Average Gap	**0,54**	**0,02**	**0,01**	**0,01**
												Max Gap	**0,95**	**0,06**	**0,02**	**0,04**
												Best	0	0	1	3

PP Auto Data Set

YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
			Bootstrap			RSA Approach			D&A Approach						
			Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
1989	423554	442021,96	361647,62	500489,22	408408,13	436796,61	447180,37	441294,07	438911,68	445083,92	441557,97	0,02	0,00	0,01	0,04
1990	1276226	1365224,26	1231700,86	1460620,16	1314019,20	1358213,44	1372285,82	1364892,14	1359336,52	1371499,18	1364841,12	0,00	1,35	1,34	1,35
1991	2855617	3023881,63	2838624,86	3136962,55	2952943,25	3017614,76	3029772,51	3023425,63	3016326,21	3031089,58	3023258,30	0,00	0,72	0,72	0,73
1992	6785186	7446043,51	7134942,26	7596697,97	7331378,10	7391760,58	7501465,66	7437908,44	7380977,82	7514570,18	7438899,56	0,00	0,20	0,20	0,21
											Average Gap	**0,01**	**0,57**	**0,57**	**0,58**
											Max Gap	**0,02**	**1,35**	**1,34**	**1,35**
											Best	3	1	0	0

Workers Compensation Data Set

YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
			Bootstrap			RSA Approach			D&A Approach						
			Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
1989	88398	94108,80	79818,81	113373,25	98697,93	89888,79	98468,30	93858,47	91722,68	96506,56	94020,55	0,89	0,00	0,03	0,05
1990	241831	263711,37	246017,49	299167,09	273392,12	258068,17	269268,66	262961,97	259286,98	267954,36	263353,16	0,49	0,00	0,02	0,04
1991	513415	551324,81	516644,55	592987,27	556006,60	546242,86	556039,40	550821,71	546024,52	556870,86	551077,32	0,14	0,00	0,01	0,01
1992	938882	981568,23	936749,41	1034422,36	989587,47	969096,70	992762,82	979165,50	967224,33	996151,98	980112,05	0,26	0,00	0,02	0,06
											Average Gap	**0,44**	**0,00**	**0,02**	**0,04**
											Max Gap	**0,89**	**0,00**	**0,03**	**0,06**
											Best	0	6	0	0

Medical Malpractice Data Set	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	26419	37968,87	39798,80	55458,85	45945,46	31886,68	44292,52	37530,34	35773,55	40144,91	37882,20	0,76	0,00	0,03	0,04
	1990	71427	97011,10	101117,82	134778,07	116105,15	88761,20	105466,22	96792,48	93021,74	101124,96	96781,14	0,76	0,00	0,00	0,01
	1991	171567	197740,80	190990,01	261313,85	219617,80	188802,41	206774,46	197254,17	190980,74	204106,67	196963,78	0,89	0,01	0,00	0,03
	1992	231539	271543,70	238569,32	398258,00	293665,08	250566,38	293148,65	269231,83	254175,70	289610,10	269483,52	0,65	0,00	0,01	0,06
												Average Gap	0,76	0,00	0,01	0,04
												Max Gap	0,89	0,01	0,03	0,06
												Best	0	2	2	0

Product Liability Data Set	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	23569	19919,76	16503,63	21512,59	18676,10	18248,59	21641,38	19792,59	19393,31	20443,88	19908,49	0,34	0,03	0,00	0,00
	1990	50863	46228,22	40190,60	49321,34	44065,14	43671,74	48902,69	46074,67	45102,06	47471,03	46147,79	0,47	0,03	0,02	0,00
	1991	57070	74154,13	65559,11	87274,78	75130,35	71677,51	76720,12	73789,68	72645,30	75710,20	74057,21	0,08	0,00	0,02	0,02
	1992	64388	82093,95	72599,95	100041,17	85215,31	77648,36	86485,69	81343,26	79077,63	85262,67	81701,11	0,23	0,00	0,02	0,04
												Average Gap	0,28	0,02	0,01	0,02
												Max Gap	0,47	0,03	0,02	0,04
												Best	0	2	0	2

Reinsurance

YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
			Bootstrap			RSA Approach			D&A Approach						
			Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
1996	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
1997	3061	3448,50	0,00	8101,54	852,66	3362,30	3535,98	3446,31	3397,86	3499,16	3444,40	5,91	0,00	0,00	0,01
1998	5409	8043,24	253,19	14030,52	5444,08	7908,14	8181,88	8033,17	7926,14	8156,44	8037,44	0,00	73,80	73,92	74,09
1999	12842	17760,89	8192,85	24770,61	14377,32	17574,48	17941,53	17740,13	17555,25	17976,72	17751,80	0,00	2,19	2,20	2,20
2000	32963	39935,73	28756,96	47967,18	36193,68	39332,43	40565,59	39878,62	39124,83	40758,01	39860,26	0,00	1,14	1,13	1,16
2001	99639	113678,6788	101936,52	121224,42	109765,17	113036,13	114381,27	113670,14	112170,37	115250,78	113579,84	0,00	0,39	0,38	0,39
											Average Gap	1,48	19,28	19,31	19,37
											Max Gap	5,91	73,80	73,92	74,09
											Best	4	0	1	0

Individual loss reserving using paid Data

YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
			Bootstrap			RSA Approach			D&A Approach						
			Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
1997	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
1998	311055	468310,41	408841,82	504319,22	431961,17	429539,01	508249,96	464773,90	452219,37	483997,01	466923,4432	0,00	0,27	0,29	0,30
1999	618041	1155226,71	1008185,63	1247552,81	1106438,97	1091161,92	1219266,58	1147743,00	1118578,15	1191431,59	1152828,67	0,00	0,08	0,09	0,10
2000	1603003	1586298,01	1462561,28	1813834,63	1615626,52	1523774,98	1647229,36	1582753,91	1539922,66	1630392,62	1582778,20	0,00	0,92	0,92	0,59
2001	2843946	3822292,78	3595504,58	4108260,32	3825613,04	3754865,98	3880655,06	3814382,73	3752690,00	3886984,24	3817046,71	0,01	0,00	0,00	0,01
											Average Gap	0,00	0,32	0,33	0,25
											Max Gap	0,01	0,92	0,92	0,59
											Best	3	1	0	0

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
Glen Barnett and Ben Zehnwirth	1974	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1975	13745	50872,52	26452,89	40239,73	33157,04	50411,26	51343,26	50854,34	50811,96	50933,64	50865,42	0,00	0,91	0,91	0,91
	1976	31230	69224,22	29972,92	50094,19	39641,71	65200,22	73433,26	68981,99	68543,40	69915,01	69140,17	0,00	3,49	3,51	3,52
	1977	49718	22580,48	13592,65	23808,67	17810,90	19240,02	25884,53	22153,02	21779,84	23447,11	22541,22	0,18	0,02	0,00	0,00
	1978	53268	61842,29	26365,24	72463,40	41653,46	47069,23	77262,05	59651,23	57498,95	66254,64	61339,74	1,18	0,00	0,26	0,34
												Average Gap	**0,34**	**1,10**	**1,17**	**1,19**
												Max Gap	**1,18**	**3,49**	**3,51**	**3,52**
												Best	2	1	0	1

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
TAYLOR/ASHE Data Set	1972	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1973	445745	797149,98	1078902,35	1367035,17	1158452,36	653646,28	941041,38	780522,81	736935,88	856827,21	793532,22	1,13	0,00	0,04	0,05
	1974	1767470	1698933,94	2215976,19	2778295,85	2392378,87	1421512,66	1985272,30	1692439,91	1542752,40	1862805,19	1687880,73	6,60	0,09	0,16	0,00
	1975	2611071	2780945,07	2968028,56	3848975,74	3274237,85	2548539,45	3016891,96	2751118,34	2587070,34	2970221,93	2762591,98	3,74	0,00	0,08	0,21
	1976	2959512	4631512,97	4348878,28	6871309,32	5182345,34	4339389,45	4944126,67	4617088,91	4304523,50	4939488,88	4597148,89	0,36	0,01	0,00	0,02
												Average Gap	**2,95**	**0,03**	**0,07**	**0,07**
												Max Gap	**6,60**	**0,09**	**0,16**	**0,21**
												Best	0	2	1	1

Risk Free Rate =1.00 and Short term credit = 1.4

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
Liability Data Set	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	50676	39065,01	21027,59	38493,14	23644,31	33575,68	44610,81	38945,55	36778,22	41365,73	38771,31	1,33	0,01	0,03	0,00
	1990	112554	122666,14	85584,91	120786,13	100983,62	109347,50	135925,89	121355,39	115343,84	129900,91	121960,23	0,84	0,00	0,07	0,15
	1991	242134	236409,37	196935,51	270622,61	228447,20	224265,24	248574,10	235442,00	227582,18	245347,04	235469,68	1,39	0,17	0,16	0,00
	1992	308191	307081,74	286519,56	334468,11	308503,78	294560,04	319550,51	305645,99	293554,07	320934,76	306014,07	0,00	10,39	8,74	3,97
												Average Gap	0,89	2,64	2,25	1,03
												Max Gap	1,39	10,39	8,74	3,97
												Best	1	1	0	2

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
Commercial Auto Data Set	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	55284	49916,32	38302,24	48441,86	45354,39	47196,44	52540,38	49596,25	48463,07	51389,80	49832,09	0,85	0,06	0,02	0,00
	1990	140297	144969,12	124310,35	142301,95	135656,35	141589,11	148558,83	144804,83	142237,90	147576,60	144783,48	0,45	0,00	0,00	0,04
	1991	290479	280470,76	256622,49	282180,45	270955,22	276957,97	283851,97	280201,37	276936,79	283984,67	280390,47	0,95	0,03	0,01	0,00
	1992	525493	483054,43	457222,13	496371,69	477911,60	478518,38	487334,54	482867,26	476841,44	489439,71	482545,64	0,12	0,00	0,01	0,00
												Average Gap	0,59	0,02	0,01	0,01
												Max Gap	0,95	0,06	0,02	0,04
												Best	0	0	1	3

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
PP Auto Data Set	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	423554	442021,96	361647,62	500489,22	408408,13	437131,94	447044,02	441359,13	438950,48	445073,86	441538,34	0,19	0,00	0,01	0,04
	1990	1276226	1365224,26	1231700,86	1460620,16	1313727,65	1358338,96	1372245,28	1364348,10	1359093,63	1371241,29	1364198,21	0,00	1,35	1,35	1,37
	1991	2855617	3023881,63	2838624,86	3127997,42	2943397,34	3017847,31	3030602,62	3022802,10	3016776,91	3031096,95	3023117,44	0,00	0,90	0,91	0,92
	1992	6785186	7446043,51	7137981,73	7599931,38	7318229,59	7395373,06	7498644,01	7436950,69	7378952,78	7511909,81	7433448,45	0,00	0,22	0,22	0,24
												Average Gap	**0,05**	**0,62**	**0,62**	**0,64**
												Max Gap	**0,19**	**1,35**	**1,35**	**1,37**
												Best	3	1	0	0

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
Workers Compensation Data Set	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	88398	94108,80	79818,81	113373,25	98697,93	89722,86	98371,13	93155,21	91721,96	96486,82	93602,90	1,17	0,00	0,09	0,20
	1990	241831	263711,37	237750,89	299167,09	272798,16	258218,76	269548,45	263115,73	259351,98	268165,61	263130,35	0,45	0,00	0,00	0,03
	1991	513415	551324,81	517481,93	593230,35	553932,03	546404,14	556003,80	550285,24	546382,60	556778,74	550767,18	0,10	0,00	0,01	0,03
	1992	938882	981568,23	935423,08	1034935,06	986387,40	969931,99	993253,57	978991,69	966193,64	996100,45	978521,22	0,20	0,01	0,00	0,08
												Average Gap	**0,48**	**0,00**	**0,03**	**0,08**
												Max Gap	**1,17**	**0,01**	**0,09**	**0,20**
												Best	0	3	1	0

Medical Malpractice Data Set	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	26419	37968,87	39798,80	55458,85	45945,46	32242,93	44094,21	36640,99	35805,21	40150,01	37593,11	0,91	0,00	0,09	0,13
	1990	71427	97011,10	103236,68	138560,66	116105,15	89257,94	105309,29	95844,16	92973,30	101226,35	96477,53	0,83	0,00	0,03	0,05
	1991	171567	197740,80	190990,01	263974,06	219855,36	188606,25	207009,52	196214,02	191312,24	204404,35	197001,17	0,96	0,00	0,03	0,06
	1992	231539	271543,70	242192,23	398642,53	290081,69	250746,15	293167,01	268146,95	253373,18	289891,59	268516,39	0,60	0,00	0,01	0,09
												Average Gap	**0,82**	**0,00**	**0,04**	**0,08**
												Max Gap	**0,96**	**0,00**	**0,09**	**0,13**
												Best	0	6	0	0

Product Liability Data Set	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	23569	19919,76	16503,63	21512,59	18676,10	18208,02	21573,92	19640,61	19402,71	20447,99	19818,39	0,34	0,08	0,03	0,00
	1990	50863	46228,22	40190,60	50017,76	43888,63	43619,63	48915,45	45817,77	45046,19	47383,04	46033,02	0,50	0,09	0,04	0,00
	1991	57070	74154,13	66292,99	87274,78	74646,93	71559,04	76612,65	73806,35	72700,19	75730,04	73944,57	0,05	0,00	0,01	0,02
	1992	64388	82093,95	72337,46	100508,57	84198,83	77729,46	86474,29	81368,71	79034,81	85144,82	81559,25	0,17	0,00	0,01	0,04
												Average Gap	**0,27**	**0,04**	**0,02**	**0,02**
												Max Gap	**0,50**	**0,09**	**0,04**	**0,04**
												Best	0	2	0	2

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
Reinsurance	1996	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1997	3061	3448,50	0,00	8101,54	715,47	3363,74	3535,72	3432,50	3397,63	3499,65	3442,27	7,84	0,00	0,03	0,04
	1998	5409	8043,24	269,63	13878,08	4885,21	7900,88	8181,83	8012,66	7929,66	8155,30	8025,72	0,00	2,55	2,57	2,59
	1999	12842	17760,89	8136,11	23620,60	13804,09	17587,81	17941,22	17733,59	17561,49	17978,23	17728,88	0,00	4,08	4,08	4,11
	2000	32963	39935,73	28929,23	47119,52	35747,65	39341,77	40557,82	39827,61	39077,41	40770,80	39776,00	0,00	1,47	1,45	1,50
	2001	99639	113678,6788	102008,98	121922,39	109072,42	112959,11	114353,80	113548,15	112174,13	115221,21	113504,79	0,00	0,47	0,47	0,49
												Average Gap	**1,96**	**2,03**	**2,03**	**2,06**
												Max Gap	**7,84**	**4,08**	**4,08**	**4,11**
												Best	4	1	0	0

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
Individual loss reserving using paid Data	1997	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1998	311055	468310,41	408841,82	504319,22	431961,17	428413,69	507780,11	463073,28	452504,13	484225,64	465276,4809	0,00	0,26	0,28	0,30
	1999	618041	1155226,71	1016290,89	1247552,81	1106438,97	1085813,74	1223668,63	1143359,42	1117070,51	1193491,18	1149017,42	0,00	0,08	0,09	0,10
	2000	1603003	1586298,01	1462561,28	1815367,07	1599492,64	1524912,15	1645609,19	1577251,66	1538087,33	1635734,54	1579116,15	0,00	6,34	5,80	3,76
	2001	2843946	3822292,78	3605242,11	4105741,55	3807189,25	3764614,62	3885228,94	3816875,11	3761128,86	3886632,88	3814312,66	0,00	0,01	0,01	0,02
												Average Gap	**0,00**	**1,67**	**1,54**	**1,04**
												Max Gap	**0,00**	**6,34**	**5,80**	**3,76**
												Best	6	0	0	0

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
Glen Barnett and Ben Zehnwirth	1974	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1975	13745	50872,52	26452,89	40239,73	32415,67	50421,08	51335,03	50811,80	50812,25	50932,66	50861,42	0,00	0,99	0,99	0,99
	1976	31230	69224,22	29972,92	50094,19	38927,26	65039,30	73178,56	68487,75	68545,74	69917,48	69102,19	0,00	3,84	3,92	3,94
	1977	49718	22580,48	13455,05	23614,77	17656,24	19269,55	26030,98	21856,05	21771,63	23392,29	22475,90	0,18	0,03	0,00	0,00
	1978	53268	61842,29	26365,24	73195,04	40335,52	46928,49	76145,68	58673,77	57414,12	66275,47	61020,95	2,35	0,00	0,43	0,59
												Average Gap	**0,63**	**1,21**	**1,34**	**1,38**
												Max Gap	**2,35**	**3,84**	**3,92**	**3,94**
												Best	2	1	0	1

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
TAYLOR/ASHE Data Set	1972	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1973	445745	797149,98	1078902,35	1367035,17	1158452,36	649724,83	949397,11	777213,96	738277,37	856845,99	789995,11	1,15	0,00	0,04	0,06
	1974	1767470	1698933,94	2215976,19	2788127,64	2392378,87	1428056,39	1977309,57	1650101,46	1540851,12	1855557,09	1673796,97	5,51	0,71	0,37	0,00
	1975	2611071	2780945,07	2972961,20	3848975,74	3252189,01	2544511,01	3021515,98	2733764,42	2599751,71	2974485,26	2757318,04	4,23	0,00	0,19	0,38
	1976	2959512	4631512,97	4342745,62	6915168,97	5146947,58	4311747,95	4935544,70	4559748,79	4318876,36	4949593,65	4584733,56	0,37	0,00	0,02	0,04
												Average Gap	**2,81**	**0,18**	**0,15**	**0,12**
												Max Gap	**5,51**	**0,71**	**0,37**	**0,38**
												Best	0	3	0	1

Risk Free Rate =1.00 and Short term credit = 1.6

Liability Data Set

YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
			Bootstrap			RSA Approach			D&A Approach						
			Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
1989	50676	39065,01	21027,59	38493,14	23644,31	33513,43	44521,65	37752,51	36792,83	41369,41	38505,90	1,33	0,11	0,05	0,00
1990	112554	122666,14	85584,91	121441,09	98507,72	109421,82	136140,51	119632,24	115255,33	130766,09	121274,53	2,18	0,00	0,23	0,43
1991	242134	236409,37	196078,55	271185,85	220445,92	225135,83	248051,37	233855,36	227550,65	245274,55	234520,06	2,79	0,45	0,33	0,00
1992	308191	307081,74	288240,96	335225,67	304860,19	294253,41	319925,13	303800,39	294229,92	321256,20	305115,43	2,00	2,96	1,77	0,00
											Average Gap	**2,07**	**0,88**	**0,60**	**0,11**
											Max Gap	**2,79**	**2,96**	**1,77**	**0,43**
											Best	0	1	0	3

Commercial Auto Data Set

YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
			Bootstrap			RSA Approach			D&A Approach						
			Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
1989	55284	49916,32	38302,24	48441,86	45354,39	47268,08	52568,32	49108,97	48438,01	51379,26	49661,52	0,85	0,15	0,05	0,00
1990	140297	144969,12	124310,35	142301,95	132454,40	141386,70	148561,48	144342,78	142212,05	147781,00	144486,41	2,10	0,00	0,04	0,15
1991	290479	280470,76	256779,60	282180,45	269294,55	277091,03	284026,05	279728,56	276829,29	284369,54	279815,26	1,12	0,07	0,07	0,00
1992	525493	483054,43	457157,08	496524,23	475164,13	478397,83	487735,64	481672,76	476602,86	489456,21	482104,28	0,19	0,03	0,02	0,00
											Average Gap	**1,06**	**0,06**	**0,04**	**0,04**
											Max Gap	**2,10**	**0,15**	**0,07**	**0,15**
											Best	0	1	0	3

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
PP Auto Data Set	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	423554	442021,96	361647,62	500489,22	408408,13	436941,48	446988,44	440840,02	438943,15	445044,34	441205,58	0,40	0,00	0,02	0,07
	1990	1276226	1365224,26	1231700,86	1460620,16	1307165,39	1358349,80	1371919,93	1363778,21	1358966,50	1371590,11	1364162,61	0,00	1,83	1,84	1,88
	1991	2855617	3023881,63	2838624,86	3130613,76	2937322,26	3018303,19	3030006,79	3022702,76	3017013,62	3031468,97	3022779,71	0,00	1,04	1,05	1,06
	1992	6785186	7446043,51	7139789,72	7595023,74	7310125,43	7392866,08	7501762,58	7433561,77	7378679,43	7513223,81	7431656,18	0,00	0,24	0,23	0,26
												Average Gap	**0,10**	**0,78**	**0,79**	**0,82**
												Max Gap	**0,40**	**1,83**	**1,84**	**1,88**
												Best	3	1	0	0

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
Workers Compensation Data Set	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	88398	94108,80	79818,81	113373,25	98697,93	89879,27	98372,09	93068,49	91700,11	96542,45	93469,92	1,21	0,00	0,09	0,22
	1990	241831	263711,37	246017,49	299167,09	269699,85	258184,10	269204,42	262568,31	259529,49	267989,02	263181,21	0,34	0,00	0,03	0,06
	1991	513415	551324,81	519510,03	592987,27	549885,62	546473,90	556579,18	550251,44	545755,07	556511,30	550434,54	0,00	0,01	0,02	0,04
	1992	938882	981568,23	936749,41	1034314,98	981179,39	970236,22	993816,97	978750,71	967003,56	996603,31	979120,24	0,06	0,00	0,01	0,07
												Average Gap	**0,40**	**0,00**	**0,03**	**0,10**
												Max Gap	**1,21**	**0,01**	**0,09**	**0,22**
												Best	1	3	0	0

Medical Malpractice Data Set	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	26419	37968,87	39798,80	55458,85	45945,46	31951,61	44131,68	36644,41	35798,94	40206,88	37337,75	0,91	0,00	0,07	0,13
	1990	71427	97011,10	101117,82	134778,07	114628,42	89251,72	104744,84	95236,21	92700,86	101319,56	96329,51	0,81	0,00	0,05	0,07
	1991	171567	197740,80	191524,90	260507,19	216864,58	188456,10	207390,86	195881,83	191616,90	204121,00	196577,98	0,86	0,00	0,03	0,08
	1992	231539	271543,70	238855,78	388816,71	287278,83	248964,04	293745,04	264679,01	253984,89	289037,01	268099,80	0,68	0,00	0,10	0,21
												Average Gap	**0,82**	**0,00**	**0,06**	**0,12**
												Max Gap	**0,91**	**0,00**	**0,10**	**0,21**
												Best	0	6	0	0

Product Liability Data Set	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
	1988	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1989	23569	19919,76	16503,63	21512,59	18676,10	18224,39	21672,98	19473,92	19402,05	20436,91	19803,94	0,34	0,12	0,03	0,00
	1990	50863	46228,22	38558,97	50017,76	43803,33	43513,87	48741,40	45629,56	45134,02	47362,64	46027,04	0,52	0,13	0,04	0,00
	1991	57070	74154,13	66292,99	87784,43	74576,64	71579,17	76724,40	73539,68	72673,80	75759,34	73862,10	0,06	0,00	0,02	0,04
	1992	64388	82093,95	71923,21	100041,17	83473,19	77803,49	86372,77	80788,02	79019,21	85309,23	81507,18	0,16	0,00	0,04	0,08
												Average Gap	**0,27**	**0,06**	**0,03**	**0,03**
												Max Gap	**0,52**	**0,13**	**0,04**	**0,08**
												Best	0	2	0	2

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
Reinsurance	1996	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1997	3061	3448,50	0,00	8101,54	668,93	3361,77	3534,04	3430,78	3397,88	3498,89	3438,20	9,35	0,00	0,02	0,05
	1998	5409	8043,24	269,63	13878,08	3975,78	7907,23	8178,26	8005,65	7931,51	8162,27	8025,09	0,00	0,13	0,14	0,15
	1999	12842	17760,89	8132,81	23548,60	13439,30	17576,13	17956,10	17724,36	17568,02	17973,97	17718,72	0,00	7,17	7,16	7,24
	2000	32963	39935,73	28691,02	47070,96	35158,40	39356,55	40538,67	39776,35	39108,54	40767,59	39756,98	0,00	2,10	2,09	2,18
	2001	99639	113678,6788	101788,06	121440,17	108695,57	112988,03	114404,50	113509,70	112143,26	115254,14	113411,56	0,00	0,53	0,52	0,55
												Average Gap	**2,34**	**2,35**	**2,36**	**2,40**
												Max Gap	**9,35**	**7,17**	**7,16**	**7,24**
												Best	4	1	0	0

	YEARS	Actual IBNR	Chain Ladder	PREDICTED IBNR CONFIDENCE INTERVALS									Performance Index			
				Bootstrap			RSA Approach			D&A Approach						
				Lower	Upper	Point	Lower	Upper	Point	Lower	Upper	Point	P.I.B	P.I.R	P.ID	P.I.C
Individual loss reserving using paid Data	1997	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
	1998	311055	468310,41	408841,82	504319,22	431961,17	429366,77	507310,38	458777,83	452298,46	484296,95	464453,6673	0,00	0,22	0,27	0,30
	1999	618041	1155226,71	1008185,63	1233478,74	1092729,67	1089519,56	1218977,15	1142383,75	1118488,82	1192791,66	1149334,40	0,00	0,10	0,12	0,13
	2000	1603003	1586298,01	1458441,33	1810718,21	1586121,28	1523847,28	1650462,82	1573231,73	1536433,56	1633793,09	1578045,97	0,01	0,78	0,49	0,00
	2001	2843946	3822292,78	3595504,58	4095777,09	3791888,28	3754908,91	3884080,20	3803268,83	3755739,93	3885170,94	3810321,02	0,00	0,01	0,02	0,03
												Average Gap	**0,00**	**0,28**	**0,23**	**0,12**
												Max Gap	**0,01**	**0,78**	**0,49**	**0,30**
												Best	3	0	0	1

Glen Barnett and Ben Zehnwirth

PREDICTED IBNR CONFIDENCE INTERVALS

YEARS	Actual IBNR	Chain Ladder	Bootstrap Lower	Bootstrap Upper	Bootstrap Point	RSA Approach Lower	RSA Approach Upper	RSA Approach Point	D&A Approach Lower	D&A Approach Upper	D&A Approach Point	Performance Index P.I.B	Performance Index P.I.R	Performance Index P.ID	Performance Index P.I.C
1974	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
1975	13745	50872,52	27194,26	40239,73	32415,67	50414,13	51336,55	50758,53	50812,88	50932,11	50855,55	0,00	0,98	0,99	0,99
1976	31230	69224,22	31788,23	50094,19	38141,46	65151,76	73355,59	68086,67	68532,36	69918,33	69063,73	0,00	4,33	4,47	4,50
1977	49718	22580,48	13908,16	24285,53	17492,28	19276,96	26123,35	21853,82	21775,05	23431,81	22438,21	0,19	0,03	0,01	0,00
1978	53268	61842,29	26084,37	73994,22	39991,28	46407,63	77631,02	57375,22	57348,79	66336,62	60864,55	4,17	0,00	0,85	1,09
											Average Gap	**1,09**	**1,34**	**1,58**	**1,64**
											Max Gap	**4,17**	**4,33**	**4,47**	**4,50**
											Best	2	1	0	1

TAYLOR/ASHE Data Set

PREDICTED IBNR CONFIDENCE INTERVALS

YEARS	Actual IBNR	Chain Ladder	Bootstrap Lower	Bootstrap Upper	Bootstrap Point	RSA Approach Lower	RSA Approach Upper	RSA Approach Point	D&A Approach Lower	D&A Approach Upper	D&A Approach Point	Performance Index P.I.B	Performance Index P.I.R	Performance Index P.ID	Performance Index P.I.C
1972	0	0	0,00	0,00	0,00	0	0	0	0	0	0	0	0	0	0
1973	445745	797149,98	1078902,35	1367035,17	1158452,36	642853,98	941584,32	761138,57	736813,02	857264,22	781365,88	1,26	0,00	0,06	0,11
1974	1767470	1698933,94	2215976,19	2788127,64	2377976,61	1427928,38	1964031,25	1636682,40	1542831,06	1855425,39	1661164,30	4,57	0,91	0,55	0,00
1975	2611071	2780945,07	2976230,88	3853412,71	3221205,29	2547486,70	3017660,39	2725803,87	2598736,10	2973785,59	2754189,73	4,32	0,00	0,25	0,48
1976	2959512	4631512,97	4355383,74	6911027,85	5117473,28	4284490,69	4929768,19	4531556,06	4313952,30	4958596,66	4586710,12	0,37	0,00	0,04	0,06
											Average Gap	**2,63**	**0,23**	**0,22**	**0,16**
											Max Gap	**4,57**	**0,91**	**0,55**	**0,48**
											Best	0	3	0	1