YAŞAR UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

MASTER THESIS

DERIVATIONS ON *TM*-ALGEBRAS

Zeynep KABALAK

Thesis Advisor: Asist. Prof. Dr. Şule AYAR ÖZBAL

Department of Mathematics

Bornova-İZMİR
2016

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of master of science.

Asist.Prof.Dr. Şule AYAR ÖZBAL (Supervisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of master of science.

Prof. Dr. Alev FIRAT

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of master of science.

Asist.Prof.Dr. Esra DALAN YILDIRIM

Prof. Dr. Cüneyt GÜZELİŞ
Director of the Graduate School

ABSTRACT

DERIVATIONS ON *TM*-ALGEBRAS

KABALAK, Zeynep
MSc. in Department of Mathematics

Supervisor: Asist. Prof. Dr. Şule AYAR ÖZBAL
January 2016, 28 pages

This thesis consists of three parts. In the first part, preliminaries and related properties about the *TM*-Algebras are given.

In the second part, the notions of derivation, symmetric bi-derivation and generalized derivation of *TM*-Algebras are given and the related properties are listed.

In the third part, the notion of the symmetric f bi-derivation of the *TM*-algebras is defined. Then some important properties are given and proved. In addition, the notion of the generalized f-derivation is defined and related properties are studied.

Keywords: Derivation, symmetric f bi-derivation, symmetric mapping, trace, generalized f derivation.

ÖZET

TM- CEBİRLERİNDE TÜREV

Zeynep KABALAK
Yüksek Lisans Tezi, Matematik Bölümü
Tez Danışmanı: Yrd. Doç. Dr. Şule AYAR ÖZBAL
Ocak 2016, 28 sayfa

Bu tez esas olarak üç bölümden oluşmaktadır. Birinci bölümde *TM* cebirleri ile ilgili ön bilgiler ve özellikler verilmiştir.

İkinci bölümde, *TM* cebirlerinde türev, simetrik ikili türev ve genelleştirilmiş türev tanımı verilmiş ve ilgili özellikler listelenmiştir.

Üçüncü bölümde, *TM* cebirlerinde simetrik ikili f türev tanımı verilmiştir. Daha sonra önemli özellikler verilmiş ve ispatlanmıştır. Bunun yanında *TM* cebirlerinde genelleştirilmiş f türev tanımı verilmiştir ve ilgili özellikleri çalışılmıştır.

ACKNOWWLEDGEMENTS

I am very thankful to my supervisor Assist. Prof. Dr. Şule AYAR ÖZBAL for her continious support, patience, motivation and immense knowledge. Her guidance helped me all the time of my study and writing this thesis. And also thanks to my family for their patience and support.

Zeynep KABALAK

İzmir, 2016

TEXT OF OATH

I declare and honestly confirm that my study, titled " DERIVATIONS ON *TM*-ALGEBRAS " and presented as a Master's Thesis, has been written without applying to any assistance inconsistent with scientific ethics and traditions, that all sources from which I have benefited are listed in the bibliography, and that I have benefited from these sources by means of making references.

29/01/2016

Zeynep KABALAK

TABLE OF CONTENTS

Page

1. INTRODUCTION

Imai and Iseki introduced BCK and BCI Algebras and many researchers investigated extensively these algebras. It is known that the class of BCK-Algebras is a proper sub class of the class of BCI-Algebras. Tamilarasi and Mekalai introduced another algebra based on propositional calculi named with *TM*-Algebras. The notion of derivation on a ring has an important role for the characterization of rings. As generalizations of derivations, α-derivations, symmetric bi-derivations, permuting tri-derivations on prime and semi-prime rings are studied by a lot of researchers. M. Chandramouleeswaran, T. Ganeshkumar introduced the notion of derivation on *TM*-Algebras in ''Derivations on *TM*-Algebras'' and studied some simple but elegant results. Later, they studied the properties of symmetric bi-derivations on *TM*-Algebras and proved that the set of all symmetric bi-derivations on *TM*-Algebras form a semigroup under a suitably defined binary compositon in ''Symmetric Bi-Derivations On *TM*-Algenras''. Additionally, they introduced the notion of generalized derivation and some of its properties on *TM*-Algebras in ''Generalized Derivation on *TM*-Algebras''. In this paper, we introduced the notion of symmetric f bi-derivations and generalized f-derivation and some of its properties on *TM*-algebras.

2. PRELIMINARIES

In this chapter, in order to facilitate the readability of the thesis some basic definitions and properties of *TM*-Algebra that are used in proofs are given.

Definition 2.1: A *TM*-Algebra (X,*,0) is a non-empty set X with a constant 0 and a binary operation $*$ that satisfying the following axioms for all x, y, z in X: (Tamilarasi and Megalai, 2010)

1. $x * 0 = x$,
2. $(x * y) * (x * z) = z * y$.

Definition 2.2: A *TM*-Algebra X is said to be associative if $(x * y) * z = x * (y * z)$ for all $x, y, z \in X$. (Chandramouleeswaran and Ganeshkumar, 2013)

Definition 2.3: For any *TM*-Algebra (X,*,0) we define the set $G(X) = \{x \in X \mid 0 * x = x\}$ (Chandramouleeswaran and Ganeshkumar, 2013).

Remark 2.4: In a *TM*-Algebra *X*, by definition, $x \wedge y = y * (y * x)$. However, in a *TM*-Algebra, $x = y * (y * x)$. Hence, in a *TM*-Algebra $x \wedge y = x$ for all $x, y \in X$. (Chandramouleeswaran and Ganeshkumar, 2013)

Definition 2.5: Let X be a *TM*-Algebra. If we define an operation +, called addition, as $x + y = x * (0 * y)$ for all $x, y \in X$, then (X ,+) is an Abelian group with identity 0 and the additive inverse $-x = 0 * x$ for all $x \in X$. (Chandramouleeswaran and Ganeshkumar, 2013)

Remark 2.6: If we have a *TM*-Algebra (X,∗ ,0) it follows from the above definition that $(X, +)$ is an Abelian group with $-y = 0 * y$, $\forall y \in X$. Then we have $x - y = x * y$, $\forall x, y \in X$. On the other hand if we choose an Abelian group (X, +) with an identity 0 and define $x * y = x - y$, we get a *TM*-Algebra (X,∗ ,0) where $x + y = x * (0 * y)$, $\forall x, y \in X$. (Chandramouleeswaran and Ganeshkumar, 2013)

Definition 2.7: Let X be a *TM*-Algebra. A mapping $D(.,.) : X \times X \to X$ is called symmetric if $D(x, y) = D(y, x)$ holds for all x, y in *X*. (Chandramouleeswaran and Ganeshkumar, 2013)

Definition 2.8: Let *X* be a *TM*-Algebra. A mapping $d : X \to X$ defined by $d(x) = D(x, x)$ is called the trace of $D(.,.)$, where $D(.,.) : X \times X \to X$ is a symmetric mapping. (Chandramouleeswaran and Ganeshkumar, 2013)

3. DERIVATIONS, SYMMETRIC *BI*-DERIVATIONS AND GENERALIZED DERIVATION ON *TM*-ALGEBRAS

3.1 Derivations on *TM*-Algebras

Definition 3.1.1: Let $(X,*,0)$ be a *TM*-Algebra. A self map $d : X \to X$ is said to be a *(l,r)*-derivation on *X* if

$$d(x * y) = (d(x) * y) \wedge (x * d(y)), \text{ for all } x, y \in X.$$

Let $(X,*,0)$ be a *TM*-Algebra. A self map $d : X \to X$ is said to be a *(r,l)*-derivation on *X* if

$$d(x * y) = (x * d(y)) \wedge (d(x) * y), \text{ for all } x, y \in X.$$

(Chandramouleeswaran and Ganeshkumar, 2012)

Definition 3.1.2: Let $d : X \to X$ be a self map on *TM*-Algebra $(X,*,0)$. The map *d* is said to be a derivation on *X* if *d* is both a *(l,r)*-derivation and a *(r,l)*-derivation on *X*. (Chandramouleeswaran and Ganeshkumar, 2012)

Example 3.1.3: Let $(X,*,0)$ be a *TM*-Algebra with the following Cayley table:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

The self map $d : X \to X$ be defined by $d(0) = 3, d(1) = 2, d(2) = 1, d(3) = 0$ is a derivation. (Chandramouleeswaran and Ganeshkumar, 2012)

Remark 3.1.4: If *d* is a derivation on *X*, then for all $x, y \in X$

$$d(x * y) = d(x) * y = x * d(y)$$

(Chandramouleeswaran and Ganeshkumar, 2012)

Definition 3.1.5: Let *X* be a *TM*-Algebra. A self map $d : X \to X$ is said to be regular if $d(0) = 0$. (Chandramouleeswaran and Ganeshkumar, 2012

Definition 3.1.6: If *X* is a *TM*-Algebra then we define a partial ordering $\leq$ such that $x \leq y$, whenever $x * y = 0$. (Chandramouleeswaran and Ganeshkumar, 2012)

Proposition 3.1.7: Let $(X,*,0)$ be a *TM*-Algebra. If $d : X \to X$ is a regular *(r,l)*-derivation on *X* then $x \leq d(x)$ for all $x \in X$. (Chandramouleeswaran and Ganeshkumar, 2012)

Proposition 3.1.8: Let $(X,*,0)$ be a *TM*-Algebra. Let d be a derivation on X.

1. If $x * d(x) = 0$ for all $x \in X$, then d is regular.
2. If $d(x) * x = 0$ for all $x \in X$, then d is regular.

(Chandramouleeswaran and Ganeshkumar, 2012)

Proposition 3.1.9: Let d be a self map of a TM-Algebra X.

1. If *d* is regular *(l,r)*-derivation on *X*, then $d(x) = d(x) \wedge x$ for all $x \in X$.
2. If *d* is regular *(r,l)*-derivation on *X*, then $d(x) = x \wedge d(x)$ for all $x \in X$.

(Chandramouleeswaran and Ganeshkumar, 2012)

Definition 3.1.10: Let d_1, d_2 be self maps on a *TM*-Algebra *X*. We define $d_1 o d_2$ as follows: $(d_1 o d_2)(x) = d_1(d_2(x))$, for all $x, y \in X$.(Chandramouleeswaran and Ganeshkumar, 2012)

Lemma 3.1.11: Let (X,*,0) be a *TM*-Algebra. Let d_1, d_2 be two *(l,r)*-derivation on *X*. Then $(d_1 o d_2)$ is also a *(l,r)*-derivation on *X*. (Chandramouleeswaran and Ganeshkumar, 2012)

Lemma 3.1.12: Let (X,*,0) be a *TM*-Algebra. Let d_1, d_2 be two *(r,l)*-derivation on *X*. Then $(d_1 o\, d_2)$ is also a *(r,l)*-derivation on *X*. (Chandramouleeswaran and Ganeshkumar, 2012)

Theorem 3.1.13: Let (X,*,0) be a *TM*-Algebra. Let d_1, d_2 be two derivations on *X*, then $(d_1 o d_2)$ is also a derivation on *X*. (Chandramouleeswaran and Ganeshkumar, 2012)

Theorem 3.1.14: Let (X,*,0) be a *TM*-Algebra. Let d_1, d_2 be two derivations on *X*, then $(d_1 o d_2) = (d_2 o d_1)$. (Chandramouleeswaran and Ganeshkumar, 201

Definition 3.1.15: Let (X,*,0) be a TM-Algebra. Let d_1, d_2 be two self maps on X. We define $(d_1 * d_2): X \to: X$ as $(d_1 * d_2)(x) = d_1(x) * d_2(x)$ for all $x \in X$.

Theorem 3.1.16: Let (X,*,0) be a *TM*-Algebra and d_1, d_2 be two derivations of X, then $d_1 * d_2 = d_2 * d_1$. (Chandramouleeswaran and Ganeshkumar, 2012)

Lemma 3.1.17: In a *TM*-Algebra both right and left cancellation law hold good. (Chandramouleeswaran and Ganeshkumar, 2012)

Theorem 3.1.18: Let *d* be a *(l,r)*-derivation of *TM*-Algebra *X*. Then the followings hold:

1. $d(0) = d(x) * x$ for all $x \in X$.
2. d is 1-1.
3. If *d* is regular, then *d* is the identity map.
4. If there is an element $x \epsilon X$ such that $d(x) = x$, then *d* is the identity map.
5. If there is an element $x \epsilon X$ such that $d(y) * x = 0$ or $x * d(y) = 0$ for all $y \in X$, then $d(y) = x$, (ie) *d* is a constant map.

(Chandramouleeswaran and Ganeshkumar, 2012)

Theorem 3.1.19: Let *d* be a *(r,l)*-derivation of *TM*-Algebra *X*. Then the followings hold:

1. $d(0) = x * d(x)$ for all $x \in X$.
2. $d(x) = d(x) \wedge x$ for all $x \epsilon X$.
3. d is 1-1.
4. If *d* is regular, then *d* is the identity map.
5. If there is an element $x \epsilon X$ such that $d(x) = x$, then *d* is the identity map.
6. If there is an element $x \epsilon X$ such that $d(y) * x = 0$ or $x * d(y) = 0$ for all $y \in X$, then $d(y) = x$, (ie) *d* is a constant map.

(Chandramouleeswaran and Ganeshkumar, 2012)

Theorem 3.1.20: Let *X* be a *TM*-Algebra and $d_1, d_2, \ldots d_n$ be derivations on *X*, then $d_n(d_{n-1}\left(d_{n-2}\left(d_{n-3} \ldots \left(d_2(d_1(x))\right)\right)\right) \leq x$ for all $x \in X$.

(Chandramouleeswaran and Ganeshkumar, 2012)

Definition 3.1.21: Let L Der(X) denote the set of all *(l,r)*-derivations on *X*. Define the binary operation $\wedge$ on L Der(X) as follows. For $d_1, d_2 \epsilon$ L Der(X), define $(d_1 \wedge d_2)(x) = d_1(x) \wedge d_2(x)$ for all $x \epsilon X$. (Chandramouleeswaran and Ganeshkumar, 2012)

Lemma 3.1.22: If d_1 and d_2 are *(l,r)*-derivations on *X*, then $(d_1 \wedge d_2)$ is also a *(l,r)*-derivation. (Chandramouleeswaran and Ganeshkumar, 2012)

Lemma 3.1.23: The binary composition $\wedge$ defined on L Der(X) is associative. (Chandramouleeswaran and Ganeshkumar, 2012)

Theorem 3.1.24: L Der(X) is a semi-group under the binary composition $\wedge$ defined by $(d_1 \wedge d_2)(x) = d_1(x) \wedge d_2(x)$ for all $x \epsilon X$ and $d_1, d_2 \epsilon$ L Der(X). (Chandramouleeswaran and Ganeshkumar, 2012)

Theorem 3.1.25: R Der(X) is a semi-group under the binary operation $\wedge$ defined by $(d_1 \wedge d_2)(x) = d_1(x) \wedge d_2(x)$, for all $x \epsilon X$ and $d_1, d_2 \epsilon$ R Der(X). (Chandramouleeswaran and Ganeshkumar, 2012)

Theorem 3.1.26: If Der(X) denotes the set of all derivations on X, it is a semi-group under the binary operation $\wedge$ defined by $(d_1 \wedge d_2)(x) = d_1(x) \wedge d_2(x)$, for all $x \epsilon X$ and $d_1, d_2 \epsilon$ Der(X). (Chandramouleeswaran and Ganeshkumar, 2012)

Definition 3.1.27: A TM-Algebra (X,*,0) is said to be *0*-commutative if $x * (0 * y) = y * (0 * x)$ for all $x, y \epsilon X$. (Chandramouleeswaran and Ganeshkumar, 2012)

Lemma 3.1.28: If (X,*,0) is a *0*-commutative TM-Algebra. Then for all $x, y, z, t \epsilon X$:

1. $(0 * x) * (0 * y) = y * x$.
2. $(z * y) * (z * x) = x * y$.
3. $(x * y) * z = (x * z) * y$.
4. $(x * (x * y)) * y = 0$.
5. $(x * z) * (y * t) = (t * y) * (z * x)$.
6. $x * (x * y) = y$.

(Chandramouleeswaran and Ganeshkumar, 2012)

Theorem 3.1.29: Let (X, *, 0) be a *0*-commutative *TM*-Algebra and *d* be a derivation on *X*. Then $d(x) * d(y) = x * y$ for all $x, y \epsilon X$. (Chandramouleeswaran and Ganeshkumar, 2012)

3.2 Symmetric Bi-Derivations on *TM*-Algebras

Definition 3.2.1: Let X be a *TM*-Algebra and $D : X\, x\, X \rightarrow X$ be symmetric mapping. If D satisfies the identity $D(x * y, z) = (D(x, z) * y) \wedge (x * D(y, z))$ for all x, y, z in X, then D is called left-right symmetric bi-derivation. (*(l,r)* symmetric bi-derivation)

If D satisfies the identity $D(x * y, z) = (x * D(y, z)) \wedge (D(x, z) * y)$ for all x, y, z in X, then D is called right-left symmetric bi-derivation. (*(r,l)* symmetric bi-derivation)

If D is both an *(l,r)* symmetric bi-derivation and *(r,l)* symmetric bi-derivation, then D is called a symmetric bi-derivation. (Chandramouleeswaran and Ganeshkumar, 2013)

Example 3.2.2: Let (X,*,0) be a *TM*-Algebra with the following Cayley table.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Define the symmetric map $D : X\, x\, X \rightarrow X$ such that

$D(x, x) = 3$ if $x = 0,1,2,3$.

$D(0,3) = D(3,0) = D(1,2) = D(2,1) = 0$.

$D(0,2) = D(2,0) = D(1,3) = D(3,1) = 1$.

$D(0,1) = D(1,0) = D(2,3) = D(3,2) = 2$.

Then D is symmetric bi-derivation. (Chandramouleeswaran and Ganeshkumar, 2013)

Proposition 3.2.3: Let X be a *TM*-Algebra. Define a symmetric map $D : X \, x \, X \rightarrow X$ by $D(x, y) = x + y$ for all $x, y \in X$. Then D is a *(l,r)* symmetric bi-derivation. (Chandramouleeswaran and Ganeshkumar, 2013)

Theorem 3.2.4: Let X be an associative *TM*-Algebra. Then the symmetric map $D : X \, x \, X \rightarrow X$ defined by $D(x, y) = x + y$ for all $x, y \in X$ is a symmetric bi-derivation. (Chandramouleeswaran and Ganeshkumar, 2013)

Proposition 3.2.5: Let X be a *TM*-Algebra and $D : X \, x \, X \rightarrow X$ be a symmetric map. Then the followings hold:

1. If D is a *(l,r)*- symmetric bi-derivation, then $D(x, y) = D(x, y) \wedge x$ for all $x, y \in X$.
2. If D is a *(r,l)*-symmetric bi-derivation, then $D(x, y) = x \wedge D(x, y)$ for all $x, y \in X$ if and only if $D(0, y) = 0$ for all $y \in X$.

(Chandramouleeswaran and Ganeshkumar, 2013)

Proposition 3.2.6: Let X be a *TM*-Algebra and $D : X \, x \, X \rightarrow X$ be a *(l,r)*-symmetric bi-derivation. Then the followings hold:

1. $D(a, y) = D(0, y) * (0 * a) = D(0, y) + a$ for all $a, y \in X$.
2. $D(a + b, y) = D(a, y) + D(b, y) - D(0, y)$ for all $a, b, y \in X$.
3. $D(a, y) = a$ for all $a, y \in X$ if and only if $D(0, y) = 0$.

(Chandramouleeswaran and Ganeshkumar, 2013)

Proposition 3.2.7: Let X be a *TM*-Algebra and $D : X \, x \, x \rightarrow X$ be a *(r,l)*-symmetric bi-derivation. Then the followings hold:

1. $D(a, y) \in G(X)$ for all $a \in G(X)$.
2. $D(a, y) = a * D(0, y) = a + D(0, y)$ for all $a, y \in X$.
3. $D(a + b, y) = D(a, y) + D(b, y) - D(0, y)$ for all $a, b, y \in X$.
4. $D(a, y) = a$ for all $a, y \in X$ if and only if $D(0, y) = 0$.

(Chandramouleeswaran and Ganeshkumar, 2013)

Definition 3.2.8: Let D_L denote the set of all *(l,r)*-symmetric bi-derivation on *X*. Define the binary operation $\wedge$ on D_L as follows: For D_1, $D_2 \in D_L$ define $(D_1 \wedge D_2)(x,y) = D_1(x,y) \wedge D_2(x,y)$ for all $x, y \in X$. (Chandramouleeswaran and Ganeshkumar, 2013)

Proposition 3.2.9: Let D_1 and D_2 are *(l,r)*-symmetric bi-derivation on *X*, then $(D_1 \wedge D_2)$ is also a *(l,r)*-symmetric bi-derivation. (Chandramouleeswaran and Ganeshkumar, 2013)

Proposition 3.2.10: The binary composition $\wedge$ defined on D_L is associative. (Chandramouleeswaran and Ganeshkumar, 2013)

Theorem 3.2.11: D_L is a semi-group under the binary composition $\wedge$ defined by $(D_1 \wedge D_2)(x,y) = D_1(x,y) \wedge D_2(x,y)$ for all $x, y \in X$ and D_1, $D_2 \in D_L$. (Chandramouleeswaran and Ganeshkumar, 2013)

Theorem 3.2.12: D_R is a semi-group under the binary operation $\wedge$ defined by $(D_1 \wedge D_2)(x,y) = D_1(x,y) \wedge D_2(x,y)$ for all $x, y \in X$ and D_1, $D_2 \in D_R$ where D_R is the set of all *(r,l)*-symmetric bi-derivation. (Chandramouleeswaran and Ganeshkumar, 2013)

Theorem 3.2.13: If *D* denotes the set of all symmetric bi-derivation on *X*, it is a semi-group under the binary operation $\wedge$ defined by $(D_1 \wedge D_2)(x,y) = D_1(x,y) \wedge D_2(x,y)$ for all $x, y \in X$ and D_1, $D_2 \in D$. (Chandramouleeswaran and Ganeshkumar, 2013)

3.3 Generalized Derivation on *TM*-Algebras

Definition 3.3.1: Let *X* be a *TM*-Algebra. A mapping $D : X \rightarrow X$ is called a generalized *(l,r)*-derivation of *X* if there exist an *(l,r)*-derivation $d : X \rightarrow X$ such that $D(x * y) = (D(x) * y) \wedge (x * d(y))$ for all $x, y \in X$.

Let *X* be a *TM*-Algebra. A mapping $D : X \rightarrow X$ is called a generalized *(r,l)*-derivation of *X* if there exist an *(r,l)*-derivation $d : X \rightarrow X$ such that $D(x * y) = (x * D(y)) \wedge (d(x) * y)$ for all $x, y \in X$. (Chandramouleeswaran and Ganeshkumar, 2013)

Definition 3.3.2: Let X be a *TM*-Algebra. A mapping $D : X \to X$ is called a generalized derivation of X, if there exist a derivation $d : X \to X$ such that D is both a *(l,r)*-generalized derivation and a *(r,l)*-generalized derivation. (Chandramouleeswaran and Ganeshkumar, 2013)

Example 3.3.3: Let (X,*,0) be a *TM*-Algebra with the following Cayley table:

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

$d : X \to X$ be defined by $d(0) = 3, d(1) = 2, d(2) = 1, d(3) = 0$ is a derivation on X. The map $D : X \to X$ defined below is a generalized derivation on X.

$$D(0) = 2, D(1) = 3, D(2) = 0, D(3) = 1.$$

(Chandramouleeswaran and Ganeshkumar, 2013)

Lemma 3.3.4: Let D be a sel map o a *TM*-Algebra X. Then the followings hold:

1. If D is a generalized *(l,r)*-derivation of X, then $D(x) = D(x) \wedge x$ for all $x \in X$.
2. If D is a generalized *(r,l)*-derivation of X, then $D(0) = 0$ if and only if $D(x) = x \wedge d(x)$ for all $x \in X$ and for some *(r,l)*-derivation d of X.

(Chandramouleeswaran and Ganeshkumar, 2013)

Lemma 3.3.5: Let D be a generalized *(l,r)*-derivation of a *TM*-Algebra X. Then the followings hold:

1. $D(a) = D(0) + a, \forall\, x \in X.$
2. $D(a + x) = D(a) + x, \forall\, x \in X.$
3. $D(a + b) = D(a) + b = a + D(b), \forall\, a, b \in X.$

(Chandramouleeswaran and Ganeshkumar, 2013)

Theorem 3.3.6: Let D be a generalized *(r,l)*-derivation of a *TM*-Algebra X. Then the followings hold:

1. $D(a) \in G(X), \ \forall a \in G(X)$.
2. $D(a) = a * D(0) = a + D(0), \ \forall a \in X$.
3. $D(a+b) = D(a) + D(b) - D(0), \ \forall a, b \in X$.
4. D is the identity map on X if and only if $D(0) = 0$.

(Chandramouleeswaran and Ganeshkumar, 2013)

Definition 3.3.7: A *TM*-algebra X is said to be *torsion free* if it satisfies for all $x, y \in X$.

$$x + x = 0 \Rightarrow x = 0.$$

If there exist a non-zero element $x \in X$ such that $x + x = 0$, then X is not *torsion free*. (Chandramouleeswaran and Ganeshkumar, 2013)

Theorem 3.3.8: Let X be a *torsion free TM*-Algebra and let D_1 and D_2 be two generalized derivations. If $D_1 D_2 = 0$ on X, then $D_2 = 0$ on X. (Chandramouleeswaran and Ganeshkumar, 2013)

Corollary 3.3.9: Let X be a *torsion free TM*-Algebra and D be a generalized derivation. If $D^2 = 0$ on X, then $D = 0$ on X. (Chandramouleeswaran and Ganeshkumar,2013)

4. SYMMETRIC *f* BI-DERIVATIONS AND GENERALIZED *f*-DERIVATION ON *TM*-ALGEBRAS

In this chapter, the definitions of symmetric *f* bi-derivations and generalized *f*-derivation on *TM*-Algebras are given. Then the main theorems and related properties are investigated.

4.1 Symmetric *f* Bi-Derivation on *TM*-Algebras

Definition 4.1.1: Let *X* be a *TM*-Algebra and $D : X\ x\ X \rightarrow X$ be a symmetric mapping. We call *D* is a *(l,r)* symmetric *f* bi-derivation on *X* if there exists a function $f: X \rightarrow X$ such that $D(x * y, z) = (D(x,z) * f(y)) \wedge (f(x) * D(y,z))$, for all *x, y, z* in *X*.

. We call *D* is a *(r,l)* symmetric *f* bi-derivation on *X* if there exists a function $f: X \rightarrow X$ such that $D(x * y, z) = (f(x) * D(y,z)) \wedge (D(x,z) * f(y))$, for all *x, y, z* in *X*.

If *D* is both a *(l,r)* symmetric *f* bi-derivation and *(r,l)* symmetric *f* bi-derivation, then *D* is called a symmetric *f* bi-derivation.

Example 4.1.2: Let $X = \{0, 1, 2, 3\}$, and we define the binary operation $'' * ''$ on X as follows:

*	0	1	2	3
0	0	2	1	3
1	1	0	3	2
2	2	3	0	1
3	3	1	2	0

Then (X,*,0) is a *TM*-Algebra and a symmetric map $D : X\ x\ X \rightarrow X$ by

$$D(x,y) = \begin{cases} 0, & if\ x = y, x = 0,1,2,3 \\ 0, & if\ (x = 0\ and\ y = 3)\ or\ (x = 1\ and\ y = 2) \\ 3, & if\ (x = 0\ and\ y = 1)\ or\ (x = 0\ and\ y = 2) \\ 3, & if\ (x = 1\ and\ y = 3)\ or\ (x = 2\ and\ y = 3) \end{cases}$$

Then D is not a symmetric bi-derivation.

$D(1 * 2,3) = D(3,2) = 0$. On the other hand, $(D(1,3) * 2) \wedge (1 * D(2,3)) = (3 * 2) \wedge (1 * 3) = 2 \wedge 2 = 2 * (2 * 2) = 2 * 0 = 2$. Since $0 \neq 2$ and then $D(1 * 2,3) \neq (D(1,3) * 2) \wedge (1 * D(2,3))$.

Define a new function $f : X \to X$ by

$$f(x) = \begin{cases} 0, & x = 0, 3 \\ 3, & x = 1, 2 \end{cases}$$

Then we can see that D is a symmetric f bi-derivation on X.

Example 4.1.3: Consider the *TM*-Algebra defined in Example 4.1.2 and a symmetric map $D : X \, x \, X \to X$ by

$$D(x,y) = \begin{cases} 0, & if\ (x = 0\ and\ y = 0)\ or\ (x = 1\ and\ y = 2)\ or\ (x = 3\ and\ y = 3) \\ 1, & if\ (x = 1\ and\ y = 3)\ or\ (x = 0\ and\ y = 2) \\ 2, & if\ (x = 0\ and\ y = 1)\ or\ (x = 3\ and\ y = 2) \\ 3. & if\ (x = 0\ and\ y = 3)\ or\ (x = 1\ and\ y = 1)\ or\ (x = 2\ and\ y = 2) \end{cases}$$

Then D is not a symmetric bi-derivation. $D(3 * 1,0) = D(1,0) = 2$. On the other hand, $(D(3,0) * 1) \wedge (3 * D(1,0)) = (3 * 1) \wedge (3 * 2) = 1 \wedge 2 = 2 * (2 * 1) = 2 * 3 = 1$. Since $2 \neq 1$ and then $D(3 * 1,0) \neq (D(3,0) * 1) \wedge (3 * D(1,0))$.

Define a new function $f : X \to X$ by

$$f(x) = \begin{cases} 0, & x = 0 \\ 2, & x = 1 \\ 1, & x = 2 \\ 3, & x = 3 \end{cases}$$

We can obtain that D is a *(l,r)* symmetric f bi-derivation.

Proposition 4.1.4: Let X be a *TM*-Algebra. Then the symmetric map $D : X \, x \, X \to X$ defined by $D(x,y) = f(x) + f(y)$ for all $x, y \in X$ where the function $f : X \to X$ sucht that $f(x * y) = f(x) * f(y)$, for all $x, y \in X$ is a *(l,r)* symmetric f bi-derivation.

Proof: Let $x, y, z \in X$. Then

$$D(x * y, z) = f(x * y) + f(z)$$

$$= f(x * y) * (0 * f(z))$$

$$= \big(f(x) * f(y)\big) * (0 * f(z))$$

$$= [f(x) * (0 * f(z))] * f(y) \quad \text{by} \quad ((x * y) * z = (x * z) * y)$$

$$= [f(x) + f(z)] * f(y)$$

$$= \big[f(x) * \big(f(y) + f(z)\big)\big] * [\big(f(x) * \big(f(y) + f(z)\big)\big) * \big(f(x) + f(z)\big) * f(y))]$$

$$= \big[\big(f(x) + f(z)\big) * f(y)\big] \wedge [f(x) * (f(y) + f(z))]$$

$$= (D(x, z) * f(y)) \wedge (f(x) * D(y, z))$$

This shows that D is a *(l,r)* symmetric f bi-derivation.

Theorem 4.1.5: Let X be an associative *TM*-Algebra. Then the symmetric map $D : X \, x \, X \rightarrow X$ defined by $D(x, y) = f(x) + f(y)$ for all $x, y \in X$ where the function $f : X \rightarrow X$ such that $f(x * y) = f(x) * f(y)$ for all $x, y \in X$ is a symmetric f bi-derivation.

Proof: By the Proposition 4.1.4 we have D is a *(l,r)* symmetric f bi-derivation. Then

$$D(x * y, z) = f(x * y) + f(z)$$

$$= f(x * y) * \big(0 * f(z)\big)$$

$$= \big(f(x) * f(y)\big) * \big(0 * f(z)\big)$$

$$= \Big(f(x) * \big(0 * f(z)\big)\Big) * f(y)$$

$$= (f(x) * 0) * f(z)) * f(y) \qquad \text{by} \quad (X \text{ is associative})$$

$$= \big(f(x) * f(z)\big) * f(y) = \big(f(x) * f(y)\big) * f(z) \;\dots (1)$$

From the right hand side,

$\big(f(x) * D(y,z)\big) \wedge \big(D(x,z) * f(y)\big) = f(x) * D(y,z)$ by $(x \wedge y = y * (y * x) = x)$

$$= f(x) * (f(y) + f(z))$$

$$= f(x) * (f(y) * (0 * f(z)))$$

$= f(x) * \big((f(y) * 0) * f(z)\big)$ by (X is associative)

$$= f(x) * (f(y) * f(z)) = \big(f(x) * f(y)\big) * f(z) \;\dots\; (2)$$

by (X is associative)

From (1) and (2), $D(x * y, z) = (f(x) * D(y,z)) \wedge (D(x,z) * f(y))$ for all $x, y, z \in X$. This proves that D is a *(r,l)* symmetric f bi-derivation and hence a symmetric f bi-derivation.

Proposition 4.1.6: Let X be a *TM*-Algebra and D be a symmetric map on X. Then the followings hold:

i) If D is a *(l,r)* symmetric f bi-derivation where $f(0)=0$, then $D(x,y) = D(x,y) \wedge f(x)$ for all $x, y \in X$.
ii) If D is a *(r,l)* symmetric f bi-derivation where $f(0)=0$, then $D(x,y) = f(x) \wedge D(x,y)$ for all $x, y \in X$ if and only if $D(0,y)=0$ for all $y \in X$.

Proof: (i) Let D be a *(l,r)* symmetric f bi-derivation where $f(0)=0$. Then

$$\begin{aligned} D(x,y) = D(x * 0, y) \\ &= (D(x,y) * f(0)) \wedge (f(x) * D(0,y)) \\ &= (D(x,y) * 0) \wedge (f(x) * D(0,y)) \\ &= D(x,y) \wedge (f(x) * D(0,y)) \\ &= \big(f(x) * D(0,y)\big) * ((f(x) * D(0,y)) * D(x,y)) \\ &= \big(f(x) * D(0,y)\big) * \big((f(x) * D(x,y)) * D(0,y)\big) \end{aligned}$$

by $((x * y) * z = (x * z) * y)$

$$= [(f(x) * D(0,y)) * D(0,y)] * (f(x) * D(x,y))$$
by $((x * y) * z = (x * z) * y)$
$$= [f(x) * (D(0,y) * D(0,y))] * (f(x) * D(x,y))$$
$$= (f(x) * 0) * (f(x) * D(x,y))$$
$$= f(x) * (f(x) * D(x,y))$$
$$= D(x,y) \wedge f(x)$$

(ii) Let D be a *(r,l)* symmetric *f* bi-ferivation where *f(0)=0* and *D(0,y)=0* for all $x, y \in X$. Then

$$D(x,y) = D(x * 0, y)$$

$$= (f(x) * D(0,y)) \wedge (D(x,y) * f(0))$$

$$= (f(x) * 0) \wedge (D(x,y) * f(0))$$

$$= (f(x) * 0) \wedge (D(x,y) * 0) = f(x) \wedge D(x,y)$$

Conversely, if $D(x,y) = f(x) \wedge D(x,y)$ for all $x, y \in X$, then we have

$$D(0,y) = f(0) \wedge D(0,y)$$

$$= D(0,y) * (D(0,y) * f(0))$$

$$= D(0,y) * (D(0,y) * 0)$$

$$= D(0,y) * D(0,y)$$

$$= 0$$

Proposition 4.1.7: Let X be a *TM*-Algebra and D be a *(l,r)* symmetric *f* bi-derivation where *f(0)=0*, $f(x * y) = f(x) * f(y)$ and $f(x + y) = f(x) + f(y)$ for all $x, y \in X$. Then the followings hold:

(i) $D(a,y) = D(0,y) * (0 * f(a)) = D(0,y) + f(a)$ for all $x, y \in X$.
(ii) $D(a+b,y) = D(a,y) + D(b,y) - D(0,y)$ for all $a, b, y \in X$.
(iii) $D(a,y) = f(a)$ for all $a, y \in X$ if and only if *D(0,y)=0*.

Proof: Let *X* be a *TM*-Algebra and *D* be a *(l,r)* symmetric *f* bi-derivation where *f(0)=0*, $f(x*y)=f(x)*f(y)$ and $f(x+y)=f(x)+f(y)$ for all $x,y \in X$.

(i) Let $a=0*(0*a)$. Then

$$\begin{aligned} D(a,y) &= D(0*(0*a),y) \\ &= (D(0,y)*f(0*a)) \wedge (f(0)*D(0*a,y)) \\ &= D(0,y)*f(0*a) \\ &= D(0,y)*(f(0)*f(a)) \\ &= D(0,y)*\big(0*f(a)\big) \\ &= D(0,y)+f(a) \end{aligned}$$

(ii) By (i)

$$\begin{aligned} D(a+b,y) &= D(0,y)+f(a+b) \\ &= D(0,y)+f(a)+f(b) \\ &= D(0,y)+f(a)+D(0,y)+f(b)-D(0,y) \\ &= D(a,y)+D(b,y)-D(0,y) \end{aligned}$$

(iii) Let *D(a,y)=f(a)* for all $x,y \in X$. Put *a=0*, $D(0,y)=f(0)=0$ for all $y \in X$. Conversely, if *D(0,y)=0*, then $D(a,y)=D(0,y)+f(a)=0+f(a)=0*\big(0*f(a)\big)=f(a)$.

Proposition 4.1.8: Let *X* be a *TM*-Algebra and *D* be a *(r,l)* symmetric *f* bi-dervation where *f(0)=0 and* $f(x+y)=f(x)+f(y)$ for all $x,y \in X$. Then the followings hold:

(i) $D(a,y) \in G(X)$ for all $a \in G(X)$.
(ii) $D(a,y)=f(a)*D(0,y)=f(a)+D(0,y)$ for all $a,y \in X$.
(iii) $D(a+b,y)=D(a,y)+D(b,y)-D(0,y)$ for all $a,b,y \in X$.
(iv) $D(a,y)=f(a)$ for all $a,y \in X$ if and only if *D(0,y)=0*.

Proof: Let *X* be a *TM*-Algebra and *D* be a *(r,l)* symmetric *f* bi-dervation where *f(0)=0 and* $f(x+y)=f(x)+f(y)$ for all $x,y \in X$. Now, lets prove that given conditions.

(i) Let $a \in G(x)$, then we have $0*a=a$ such that

$$\begin{aligned} D(a,y) &= D(0*a,y) \\ &= (f(0)*D(a,y)) \wedge (D(0,y)*f(a)) \\ &= \big(0*D(a,y)\big) \wedge \big(D(0,y)*f(a)\big) \end{aligned}$$

$= (0 * D(a,y))$

This shows that $D(a,y) \in G(X)$.

(ii) By the definition of the *TM*-Algebra we know that $a * 0 = a$.

$$D(a,y) = D(a * 0, y)$$

$$= (f(a) * D(0,y)) \wedge (D(a,y) * f(0))$$

$$= (f(a) * D(0,y)) \wedge (D(a,y) * 0)$$

$$= (f(a) * D(0,y)) \wedge D(a,y)$$

$$= D(a,y) * (D(a,y) * (f(a) * D(0,y)))$$

$$= f(a) * D(0,y)$$

Again

$$D(a,y) = f(a) * D(0,y)$$

$$= f(a) * D(0 * 0, y)$$

$$= f(a) * [(f(0) * D(0,y)) \wedge (D(0,y) * f(0))]$$

$$= f(a) * ((f(0) * D(0,y))$$

$$= f(a) * (0 * D(0,y))$$

$$= f(a) + D(0,y)$$

(iii) By using (ii) we get,
$D(a+b,y) = f(a+b) + D(0,y) = f(a) + f(b) + D(0,y) = f(a) + D(0,y) + f(b) + D(0,y) - D(0,y) = D(a,y) + D(b,y) - D(0,y)$.

(iv) If $D(0,y) = 0$, then $D(a,y) = f(a) * D(0,y) = f(a) * 0 = f(a)$. Conversely, by (ii) if $D(a,y) = f(a)$ for all $x, y \in X$, then $D(0,y) = f(0) = 0$.

Semigroup of Symmetric *f* Bi-Derivations on TM-Algebras

Definition 4.1.9 : Let D_L denote the set of all *(l,r)* symmetric *f* bi-derivations on *X*. Define the binary operation $\wedge$ on D_L as follows: For D_1, $D_2 \in D_L$ define $(D_1 \wedge D_2)(x, y) = D_1(x, y) \wedge D_2(x, y)$ for all $x, y \in X$.

Proposition 4.1.10: Let D_1 and D_2 are *(l,r)* symmetric *f* bi-derivations on *X*, then $(D_1 \wedge D_2)$ is also a *(l,r)* symmetric *f* bi-derivation.

Proof: To show that $(D_1 \wedge D_2)$ is a *(l,r)* symmetric *f* bi-derivation we must prove following implication.

$$(D_1 \wedge D_2)(x * y, z) = ((D_1 \wedge D_2)(x, z) * f(y)) \wedge (f(x) * (D_1 \wedge D_2)(y, z))$$

$$\begin{aligned}(D_1 \wedge D_2)(x * y, z) &= D_1(x * y, z) \wedge D_2(x * y, z) \\ &= D_2(x * y, z) * (D_2(x * y, z) * D_1(x * y, z)) \\ &= D_1(x * y, z) \\ &= (D_1(x, z) * f(y)) \wedge (f(x) * D_1(y, z)) \\ &= D_1(x, z) * f(y) \qquad \ldots \qquad (1)\end{aligned}$$

From the right hand side,

$$(D_1 \wedge D_2)(x, z) * f(y) \wedge f(x) * (D_1 \wedge D_2)(y, z)$$

$$= (f(x) * (D_1 \wedge D_2)(y, z)) \wedge \big((f(x) * (D_1 \wedge D_2)(y, z)\big) * ((D_1 \wedge D_2)(x, z) * f(y)))$$

$$= (D_1 \wedge D_2)(x, z) * f(y)$$

$$= \big(D_1(x, z) \wedge D_2(x, z)\big) * f(y)$$

$$= D_1(x, z) * f(y) \qquad \ldots \qquad (2)$$

From (1) and (2), we get $(D_1 \wedge D_2)$ is a *(l,r)* symmetric *f* bi-derivation.

Proposition 4.1.11: The binary composition $\wedge$ defined on D_L is associative.

Proof: Let *X* be a *TM*-Algebra. Let D_1, D_2, D_3 are *(l,r)* symmetric *f* bi-derivation. By the Proposition 4.1.10, we will prove the associativity.

$$\big((D_1 \wedge D_2) \wedge D_3\big)(x * y, z) = ((D_1 \wedge D_2)(x * y, z)) \wedge D_3(x * y, z)$$

$$= (D_1(x, z) * f(y)) \wedge D_3(x * y, z) \quad \text{by} \quad \text{(Proposition 4.1.10 (1))}$$

$$= D_3(x * y, z) * [D_3(x * y, z) * \big(D_1(x, z) * f(y)\big)]$$

$$= D_1(x, z) * f(y) \quad \ldots \quad (1)$$

From the right hand side

$$\big(D_1 \wedge (D_2 \wedge D_3)\big)(x * y, z) = D_1(x * y, z) \wedge (D_2 \wedge D_3)(x * y, z)$$

$$= D_1(x * y, z) \wedge (D_2(x, z) * f(y)) \quad \text{by} \quad \text{(Proposition 4.1.10 (1))}$$

$$= \big(D_2(x, z) * f(y)\big) * \big(\big(D_2(x, z) * f(y)\big) * D_1(x * y, z)\big)$$

$$= D_1(x * y, z)$$

$$= (D_1(x, z) * f(y)) \wedge (f(x) * D_1(y, z))$$

$$= \big(f(x) * D_1(y, z)\big) * \big(\big(f(x) * D_1(y, z)\big) * (D_1(x, z) * f(y))\big)$$

$$= D_1(x, z) * f(y) \quad \ldots \quad (2)$$

Combining (1) and (2) we get,

$$(D_1 \wedge D_2) \wedge D_3 = D_1 \wedge (D_2 \wedge D_3)$$

This proves that $\wedge$ is associative.

Combining the above two proposition, we get the following theorem.

Theorem 4.1.12: D_L is a semigroup under the binary composition $\wedge$ defined by $(D_1 \wedge D_2)(x, y) = D_1(x, y) \wedge D_2(x, y)$ for all $x, y \in X$ and D_1, $D_2 \in D_L$. Analogously we can prove that.

Theorem 4.1.13: D_R is a semigroup under the binary composition $\wedge$ defined by $(D_1 \wedge D_2)(x, y) = D_1(x, y) \wedge D_2(x, y)$ for all $x, y \in X$ and D_1, $D_2 \in D_R$ where D_R is the set of all *(r,l)* symmetric *f* bi-derivations.

Combinig the above two theorem we get the following theorem.

Theorem 4.1.14: If D denotes the set of all symmetric f bi- derivations on X, it is a semigroup under the binary operation $\wedge$ defined by $(D_1 \wedge D_2)(x,y) = D_1(x,y) \wedge D_2(x,y)$ for all $x, y \in X$ and $D_1, D_2 \in D$.

4.2 Generalized *f*-Derivation on *TM*-Algebras

Definition 4.2.1: Let X be a *TM*-Algebra. A mapping $D : X \to X$ is called a *(l,r)* generalized *f*-derivation of X, if there exists a derivation d and a function f of X such that

$$D(x * y) = D(x) * f(y) \wedge f(x) * d(y),$$

for all $x, y \in X$.

Let X be a *TM*-Algebra. A mapping $D : X \to X$ is called a *(r,l)* generalized *f*-derivation of X, if there exists a derivation d and a function f of X such that

$$D(x * y) = f(x) * D(y) \wedge d(x) * f(y),$$

for all $x, y \in X$.

Example 4.2.2: Consider the *TM*-Algebra defined in Example 4.1.2. We define a mapping $D : X \to X$ by for all $x \in X$ as follows:

$$D(x) = \begin{cases} 2, & x = 0 \\ 3, & x = 1 \\ 0, & x = 2 \\ 1, & x = 3 \end{cases}$$

And let $d : X \to X$ be a derivation of X which is defined as

$$d(x) = \begin{cases} 3, & x = 0 \\ 2, & x = 1 \\ 1, & x = 2 \\ 0, & x = 3 \end{cases}$$

Then D is not a generalized derivation of X. $D(0*1) = D(2) = 0$. On the other hand $(D(0)*1) \wedge (0*d(1)) = (2*1) \wedge (0*2) = 3 \wedge 1 = 1*(1*3) = 1*2 = 3$. Since $0 \neq 3$, then $D(0*1) \neq (D(0)*1) \wedge (0*d(1))$.

But if we define *f* as a function of *X* as

$$f(x) = \begin{cases} 0, & x = 0 \\ 2, & x = 1 \\ 1, & x = 2 \\ 3, & x = 3 \end{cases}$$

Then *D* becomes *(l,r)* generalized *f*-derivation of *X*.

Example 4.2.3: Consider the *TM*-Algebra defined in Example 4.1.2. We define a mapping $D : X \to X$ by for all $x \in X$ as follows:

$$D(x) = \begin{cases} 1, & x = 1,2 \\ 2, & x = 0,3 \end{cases}$$

And let $d : X \to X$ be a derivation of *X* which is defined as

$$d(x) = \begin{cases} 3, & x = 0 \\ 2, & x = 1 \\ 1, & x = 2 \\ 0, & x = 3 \end{cases}$$

Then *D* is not a generalized derivation of *X*. $D(0*0) = D(0) = 2$, On the other hand, $(0*D(0)) \wedge (d(0)*0) = (0*2) \wedge (3*0) = 1 \wedge 3 = 3*(3*1) = 3*1 = 1$. Since $2 \neq 1$, then $D(0*0) \neq (0*D(0)) \wedge (d(0)*0)$.

But if we define *f* as a function of *X* as:

$$f(x) = \begin{cases} 0, & x = 1,2 \\ 3, & x = 0,3 \end{cases}$$

Then *D* becomes *(r,l)* generalized *f*-derivation of *X*.

Remark 4.2.4: In a *TM*-Algebra, $x \wedge y = y*(y*x)$ for all $x, y \in X$. If we take *D* as a *(l,r)* generalized *f*-derivation of *X*, then $D(x*y) = D(x)*f(y)$ for all $x, y \in X$.

Lemma 4.2.5: Let D be a self map of a *TM*-Algebra X. Then the followings hold:

1. If D is a *(l,r)* generalized *f*-derivation on X and $f(0)=0$, then $D(x) = D(x) \wedge f(x)$, for all $x \in X$.
2. If D is a *(r,l)* generalized *f*-derivation on X and $f(0)=0$, then $D(0)=0$ if and only if $D(x) = f(x) \wedge d(x)$, for all $x \in X$ and for some *(r,l)* derivation d on X.

Proof:

1. If D be a *(l,r)* generalized *f*-derivation on X, then there exists a *(l,r)* derivation d on X such that

$$D(x * y) = (D(x) * f(y)) \wedge (f(x) * d(y))$$

for all $x, y \in X$.

$$\begin{aligned} D(x) &= D(x * 0) \\ &= (D(x) * f(0)) \wedge (f(x) * d(0)) \\ &= (D(x) * 0) \wedge (f(x) * d(0)) \\ &= \big(f(x) * d(0)\big) * \big(\big(f(x) * d(0)\big) * D(x)\big) \\ &= \big(f(x) * d(0)\big) * \big(\big(f(x) * D(x)\big) * d(0)\big) \\ &= f(x) * (f(x) * D(x)) \quad \ldots \quad ((x * z) * (y * z) = x * y) \\ &= D(x) \wedge f(x) \end{aligned}$$

2. Let D be a *(r,l)* generalized *f*-derivation on X such that $f(0)=0$ and $D(0)=0$. Then

$$D(x * y) = (f(x) * D(y)) \wedge (d(x) * f(y))$$

for all *(r,l)* derivation d ... (1)

Putting $y=0$ in (1), we get

$$\begin{aligned} D(x * 0) &= (f(x) * D(0)) \wedge d(x) \\ &= (f(x) * 0) \wedge d(x) \\ &= f(x) \wedge d(x) \end{aligned}$$

for all $x \in X$.

Conversely, if $D(x) = f(x) \wedge d(x)$, then

$$\begin{aligned} D(0) &= f(0) \wedge d(0) \\ &= 0 \wedge d(0) \\ &= d(0) * (d(0) * 0) \\ &= d(0) * d(0) \\ &= 0 \end{aligned}$$

Lemma 4.2.6: Let D be a *(l,r)* generalized *f*-derivation of a *TM*-Algebra *X*, *f(0)=0* and $f(x * y) = f(x) * f(y),\ f(x + y) = f(x) + f(y)$ for all $x \epsilon X$. Then the followings hold:

1. $D(a) = D(0) + f(a)$ for all $a \epsilon X$.
2. $D(a + x) = D(a) + f(x)$ for all $a, x \epsilon X$.
3. $D(a + b) = D(a) + f(b) = f(a) + D(b)$ for all $a, b \epsilon X$.

Proof:

1. By the definition of *TM*-Algebra we know that $a = 0 * (0 * a)$.

$$\begin{aligned} D(a) &= D(0 * (0 * a)) \\ &= (D(0) * f(0 * a)) \wedge (f(0) * d(0 * a)) \\ &= D(0) * f(0 * a) \\ &= D(0) * (f(0) * f(a)) \\ &= D(0) * (0 * f(a)) \\ &= D(0) + f(a) \end{aligned}$$

$D(a) = D(0) + f(a)$ for all $a \epsilon X$.

2. By using Remark 4.2.4 such that

$$\begin{aligned} D(a + x) &= D(a * (0 * x)) \\ &= D(a) * f(0 * x) \\ &= D(a) * (f(0) * f(x)) \\ &= D(a) * (0 * f(x)) \\ &= D(a) + f(x) \end{aligned}$$

$D(a + x) = D(a) + f(x)$ for all $a \epsilon X$.

3. Since *(X,+)* is an Abelian group, the result follows from:

$$D(a) + f(b) = D(a + b) = D(b + a) = D(b) + f(a)$$

Theorem 4.2.7: Let D be a *(r,l)* generalized *f*-derivation of a *TM*-Algebra *X* and *f(0)=0* where $f(x + y) = f(x) + f(y)$. Then the followings hold:

1. $D(a) \epsilon G(X)$, for all $a \epsilon G(X)$.
2. $D(a) = f(a) * D(0) = f(a) + D(0)$, for all $a \epsilon G(X)$.
3. $D(a + b) = D(a) + D(b) - D(0)$, for all $a, b \epsilon G(X)$.
4. D is the identity map on X if and only if $D(0) = 0$.

Proof:

1. For $a \epsilon G(X)$, $D(a) \epsilon G(X)$. For;
$D(a) = D(0 * a) = \big(f(0) * D(a)\big) \wedge \big(d(0) * f(a)\big) = 0 * D(a)$
$D(a) \epsilon G(X)$.

2. Since we know that $a = a * 0$.
$D(a) = D(a * 0)$
$= \big(f(a) * D(0)\big) \wedge (d(a) * f(0))$
$= \big(f(a) * D(0)\big) \wedge (d(a) * 0)$
$= \big(f(a) * D(0)\big) \wedge d(a)$
$= f(a) * D(0)$
$= f(a) * D(0 * 0)$
$= f(a) * [(f(0) * D(0) \wedge (d(0) * f(0))]$
$= f(a) * (f(0) * D(0))$
$= f(a) * (0 * D(0))$
$= f(a) + D(0)$

3. By result (2) above, we have
$D(a + b) = f(a + b) + D(0)$
$D(a + b) = f(a) + f(b) + D(0)$
Since (X,+) is an Abelian group, on satisfying the right hand side, we get

$$D(a + b) = f(a) + D(0) + f(b) + D(0) - D(0) = D(a) + D(b) - D(0)$$

4. If $D(0) = 0$, then D is the identity map. For;
$D(a)$=$D(a * 0)$
=$f(a) * D(0)$
=$f(a) * 0$
=$f(a)$
for all $a \epsilon X$.

Conversely, if *D* is the identity map on *X*, then *D(a)=f(a)=a* for all $a \epsilon X$. In particular D(0)=f(0)=0.

Remark 4.2.8: Example 4.2.3 satisfies all the given conditions in Theorem 4.2.7.

5. CONCLUSION

The aim of this work was to study derivations, symmetric bi-derivations and generalized derivation which are defined on TM-Algebra and to define new types of derivations that are symmetric f bi-derivations and generalized f derivation in this algebraic structure. In the first part, in order to clarify the reading of the thesis, some basic definitions are given about the TM-Algebra. Then, in the second part, the notion of derivations, symmetric bi-derivations and generalized derivation introduced by Chandramouleeswaran and Ganeshkumar and the main properties of them are also listed. In the last part, after giving necessary knowledge, the notion of symmetric f bi-derivation and generalized derivation are defined, examples are satisfying its properties are given. Then, related theorems and properties are investigated.

REFERENCES

Chandramouleeswaran M., Ganeshkumar T., 2012, Derivations on TM-algebras, International Journal o Mathematical Archive, 3(11), 3967-3974p.

Chandramouleeswaran M., Ganeshkumar T., 2013, Symmetric bi-derivations on TM-algebras, International Journal of Pure and Applied Mathematics, 87(1), 9-18p.

Chandramouleeswaran M., Ganeshkumar T., 2013, Generalized derivations on TM-algebras, International Journal of Algebra, 7(6), 251-258p.

Iseki K., Tanaka S., 1978, An introduction to theory of BCK-algebras, Math.Japan, 23, 1-26p.

Iseki K., 1980, On BCI-algebras, Math.Seminar Notes, 11, 313-320p.

Kaya K., 1987/1988, Prime rings with α-derivations, Hacettepe Bull. Natural Sci. And Eng., 16-17, 63-71p.

Posner E., 1957, Derivations in prime rings, Proc. Amer. Math Soc. 8, 1093-1100p.

Tamilarasi A., Megalai K., 2010, TM-Algebra an introduction, CASCT.

Vukman J., 1989, Symmetric bi-derivations on prime and semi-prime rings, Aequationes Mathematicae, 38, 252-254p.

Vukman J., 1990, Two results concerning symmetric bi-derivations on prime rings, Aequationes Mathematicae, 40, 181-189p.

CURRICULUM VITEA

Zeynep Kabalak who was born in 1990 Aliağa/İzmir, she completed primary and highschool in Aliağa. She graduated fom Deparment of Mathematics at Yaşar University. In the same time, she studied for master degree and started formation education at Dokuz Eylül University. Then she worked as a teacher at schools that connected to Ministry of National Education. Now, she is working at private teaching institution.

1990 yılında Aliağa İzmir'de doğan Zeynep Kabalak, ilköğretim ve ortaöğretimini Aliağa'da tamamladı. Yaşar Üniversitesi Matematik Bölümünden mezun oldu. Aynı yıl okulunda yüksek lisansa ve Dokuz Eylül Üniversitesinde formasyon eğitimine başladı. Milli Eğitim Bakanlığı'na bağlı okullarda öğretmen olarak görev yaptı. Şuanda özel bir eğitim kurumunda çalışmaktadır.