YAŞAR UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

MASTER THESIS

MODELLING STRATEGIES FOR DYNAMIC IMPROVEMENT PROCEDURE ON TIME SCALE FOR INDUSTRIAL PRODUCTION PROCESS

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We certify that, as the jury we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science

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ABSTRACT

MODELING STRATEGIES FOR DYNAMIC IMPROVEMENT PROCEDURES AND CONTROL FOR INDUSTRIAL PROCESS

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The time scale is introduced by S. Hilger in order to unify the discrete and continuous analysis. Later on the studies on differential equations and difference equations are unified as a dynamic equation on time scales.

In this thesis, we deal with dynamic equations and adopt adequate measure that will ensure stability of solutions and improve the industrial sector to achieve a maximum production process and strategies. This will enhance and control, generate greater production in the entire industrial sector. Modeling strategies for dynamic improvement procedures on time scale for industrial production process can also militate in a particular specified classification of the industrial production process, which will also determine the fundamental differences of the sector and improve in the production process industries e.g. (oil and gas production, gas processing, food processing, petrochemical, chemical, power generation, and automotive industry etc) Some of which have undergo a downturn in price and reduce in their manufacturing and production process, need to be strategize and improved despite the global meltdown in the entire economical system.

Dennis Williams Okala

Signature

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January 2018

ÖZ

ENDÜSTRİYEL ÜRETİM SÜRECİİÇİN ZAMAN SKALASINDA DİNAMİK GELİŞTİRME PROSEDÜRLERİİÇİN MODELLEME STRATEJİLERİ

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Ocak 2018

Zaman skalasında kadinamik sistemler sürekli ve ayrık analizi birleştirmek üzere Stefan Hilger tarafından tanıtılmıştır. Sonrasında diferansiyel denklemler ve fark denklemleri alanındaki çalışmalar zaman skalasında dinamik sistemler olarak birleştirilmiştir.

Bu tezde biz zaman skalasında dinamik denklemler ile ilgilendik ve çözümlerin kararlı olma durumunu sağlayacak ve azami bir üretim süreci ve stratejileri elde ederek sanayi sektörünü iyileştirecek uygun ölçümleri adapte ettik. Bu, tüm endüstriyel sektörün üretimini artırır ve kontrol altına alır.Endüstriyel üretim sürecinde zaman skalasında dinamik iyileştirme prosedürleri için modelleme stratejileri, aynı zamanda sektörün temel farklılıklarını belirlerek üretim süreci endüstrilerinde (örneğin petrol yakıt üretimi, yakıt süreci, yiyecek süreci, petrokimya, kimyasal güç üretimi ve otomobil endüstrisi) geliştirecek olan endüstriyel üretim sürecinin belirli bir sınıflandırmasında da etkilidir. Fiyat düşüşüne uğramış olan bazıları, imalat ve üretim süreçlerini düşürmüştür. Tüm ekonomik sistemdeki küresel düşüşe rağmen strateji oluşturup, geliştirilmesi sağlanmıştır.

Dennis Williams Okala
İmza

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Ocak 2018

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Dennis Williams Okala

İzmir, 2018

TEXT OF OATH

I declare and honestly confirm that my study, titled modeling strategies for dynamic improvement on time scale for industrial production process and presented as a Master's Thesis, has been written without applying to any assistance inconsistent with scientific ethics and traditions.I declare, to the best of my knowledge and belief, that all content and ideasdrawn directly or indirectly from external sources, are indicated in the text and listed in the list of references.

Dennis Williams Okala

Signature

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January 23, 2018

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SYMBOLS AND ABBREVIATIONS

Symbols	**Response**
$\mathbb{T}$	Time scale
$[a, b]_{\mathbb{T}}$	$[a, b] \cap \mathbb{T}$
$\mathbb{T}^k$	the region of Δ — differentiability
$\mathbb{T}_k$	the region of ∇ — differentiability
$\mathbb{R}$	set of real numbers
$\mathbb{Z}$	set of integers
$\mathbb{C}$	set of complex numbers
$h\mathbb{Z}$	$\{ hn : n \in \mathbb{Z}, \ h > 0\}$
μ	Forward graininess function
f^{Δ}	Delta derivative of f
$\mathcal{R}$	The set of regressive functions
ρ	Backward jump operator
σ	Forward jump operator
ν	Forward graininess function
$C(\lvert a, b\rvert)$	the set of continuous function defined on $\lvert a, b\rvert$
Max $\mathbb{T}$	maximum element of time scale
Min $\mathbb{T}$	minimum element of time scale
A	the closure of A
C_{rd}	the set of right dense continuous functions
$e_p(t, s)$	generalized $\Delta -$ exponential functions
$\mathbb{C}_h$	the set of Hilger's complex numbers

$C^1[a, b]$	the set of continuouslı differentiable functions on (a,b)
$\|x\|$	Norm of x
u_δ (t)	δ neighborhood of a point t

CHAPTER 1
INTRODUCTION

The aim of my research is to improve in the production process and to implement time change pattern that can be used in the system and to show immediate response to the situation affecting the industrial sector and its application process.

1.1. History

In the litrature, there are many studies on dynamic equations on time scales. The theory of time scale was introduce by Stefan Hilger in hes PhD thesis [159] in 1988 (supervise by Bernd Aulbach) in order to unify continuous and discrete analysis. This thesis is an introduction to the study of above principle by Stefan Hilger analysis, modelling strategies for dynamic improvement procedure on time scales for industrial production process.

Some important results concerning Banach, Picard, Bolzano,Peano, Bellman Growell, and Lipschitz theorems, was used to illustrate continuous and discrete process which will serve as problem solving over a long period of time.

The general idea of Stefan Hilger 1988 is used to prove result for a dynamic equation which will eventually improve the industries.

1.2. Content

The aim of my research is to introduce time change pattern that will improve the production process in the industries, and to show immediate response to the situation affecting the industrial sector and its application process.

In the first chapter, we concentrate on the concept of time scale, with the introduction of Hilger 1988. We give the definition of forward and backward jump operators, Δ derivative and Δ integration on time scale. We illustrate these definition by examples and we showed how these derivative and integration coincide with ordinary derivative if $\mathbb{T} = \mathbb{R}$, and the difference derivative and summation if $\mathbb{T} = \mathbb{Z}$.

The n th order dynamical system on time scale can be expressed as follows

$$f(t, x_0^{\Delta}, x_0^{\Delta^2}, \ldots x^{\Delta^n}, x_1^{\Delta}, x_1^{\Delta^2} \ldots x^{\Delta^n}, x_2^{\Delta}, x_2^{\Delta^2} \ldots x^{\Delta^n}) = 0$$

where $t \in \mathbb{T}$ and $x_1, x_2, x_3 \ldots \ldots x_n$ are dependent variable and $x_i : \mathbb{T} \longrightarrow \mathbb{R}$ for $i = 1, 2, 3 \ldots \ldots n$.

This above equation produces first, second and high order dynamic equation, which is use to solved first and second order dynamic system for continuous and discrete equation respectively. We will concentrate on the dynamic equation on time scale $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$, and to see how we can obtain improvement procedure in the industrial production process. We will also apply the method of integration on time scale.

The second chapter we concentrate on the use of some important principle that will be useful in solving the problem affecting the sector. We introduced the method of Picard, Banach, Schauders, peano and Bolzano theorems to determine the continuous, existence and uniqueness of a fixed point, which is an important factor to the industrial sector. The Picard specifically illustrate about a dynamic equation with an initial value resulting to a continuous function with existence and uniqueness of the process that leads to Lipschitz continuity. While Banach fixed point theorem deals with the existence and uniqueness of fixed points of a mappings, and it gives constructive procedure for obtaining better and better approximations to a fixed point.

Meanwhile, Schauder's theorem stated that, for every open covering is reducible to a finite sub-covering. In addition he said about an open covering been the collection of open subset. Bolzano theorem illustrate about a function as continuous between an intervals (a, b) respectively. Whereby, peano theorem is a fundamental theorem that guaranties the existence of a solution to a certain initial value problem with a better estimate that will improve the industrial sector. The local existence theorem also stated about the continuous dynamic function where the dynamic equation can form an existence of a solution,

In the third chapter we use the method of a transfer function with the method of Laplace equation to solved first and second order equation, that leads to a step response approach where steady state gain emerge in the process. The second order system resulted to a damped response due to the unstableness in the process, then we used the method of Critical damped system which eventually grow positively with time. Where, the un-damped is equal to zero, meanwhile the under damped takes values below one and above one respectively, the over damped coefficient is negative.

The last chapter deals with the homogenous linear with constant coefficient which is use to determine the stability with change in the time, and maintain a standard system approach for the production process, the second order graph show the inefficient unbalance of production available, Whereby the production process is unstable at some points by increasing and decreasing the system.

CHAPTER 2
PRELIMINARIES

In this chapter, we will introduce the basic concepts of time scale, which is introduces by Stefan Hilger in his PhD thesis (Hilger, 1988).Derivative, Integration, chain rule on time scale and exponential function on time scale, which we are going to used throughout this thesis.

2.1 The Concept of Time Scale

Definition2.1A time scale is an arbitrary nonempty closed subset of real numbers.

Natural numbers, integers, are examples of time scales. While the rational numbers, irrational numbers, and complex numbers, contain an open interval between 0 and 1 are not a time scale (Bohner and Peterson 2001, Bohner and Peterson 2003).

For instance, we introduce the schematic classification of operators as follows. This is due to our analysis and collection from Martin Bohner and Allan Peterson (see pages 1-4 of above references)

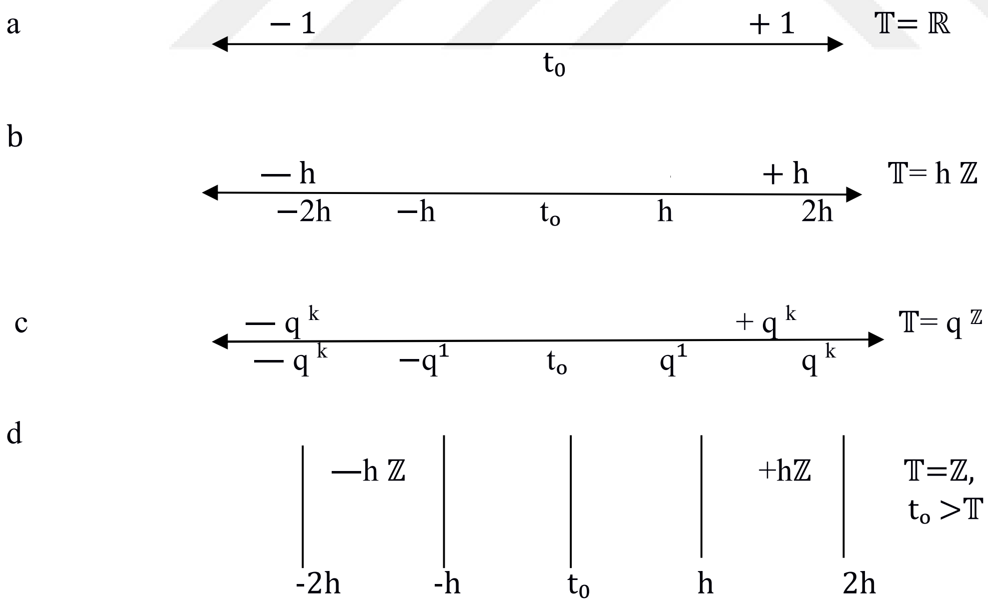


Figure1.1 Classifications of points

Definition 2.2 The forward jump operator σ: $\mathbb{T} \longrightarrow \mathbb{T}$ is define by the following

$$\sigma\ (t) = \text{in f}\ \{x \epsilon \mathbb{T},\ x > t\}.$$

Definition 2.3 Backward jump operator ρ: $\mathbb{T} \longrightarrow \mathbb{T}$ is define as follows

$$\rho \text{ (t)} = \sup \{x\epsilon\mathbb{T}, x< t\}.$$

We improve the process and lead to the forward and backward difference operators. Then by classifying the operator where inf ϕ = sup $\mathbb{T}$ represent the maximum point with change in t, and the sup ϕ = inf $\mathbb{T}$ is the minimum point for time t and $\mathbb{T}$ represent the time scales.

σ (t) > t represent right scattered

ρ (t) < t left scattered

σ (t) = t right dense

ρ (t) = t left dense

Example2.1 We can illustrate r=the above definitions by the following examples for $\mathbb{T} = \mathbb{R}$, $\mathbb{T} = \mathbb{Z}$ and $\mathbb{T} = q,^{\mathbb{Z}}$ $\mathbb{T}$ = h $\mathbb{Z}$

1. If $\mathbb{T} = \mathbb{R}$ then we will have t ϵ $\mathbb{R}$

$$\sigma \text{ (t)} = \inf \{x \,\epsilon\mathbb{R}, x> t\} = t,$$

and similarly

$$\rho \text{ (t)} = t.$$

Hence, for any t ϵ $\mathbb{R}$, t is a dense point.

2.If $\mathbb{T} = h\mathbb{Z}$ then we will have t ϵ h$\mathbb{Z}$, for h > 0

$$\sigma \text{ (t)} = \inf \{x \,\epsilon\, \mathbb{Z}, x > t\} = \{t+h, t+2h, t+3h \ \ldots\ \} = t + h$$

and similarly

$$\rho \text{ (t)} = t - h.$$

Hence, t ϵh $\mathbb{Z}$ is isolated at every point.

3, If $\mathbb{T} = q^{\mathbb{Z}}$ then for any t ϵ $q^{\mathbb{Z}}$ we have t=q^{n} for some n$\epsilon\mathbb{Z}$. In this case

$$\sigma \text{ (t)} = \inf \{x\epsilon\, q^{\mathbb{Z}}: x > t\} = \{q\, q^2, q^3 \ldots.\} = t\, q$$

$$\sigma \text{ (t)} = t\, q$$

Similarly ρ (t) =t q hence t ϵ $\mathbb{Z}$ is dense

2.1.1Hilger's △ Derivative on Time Scale

Definition 2.4 Let f: $\mathbb{T} \longrightarrow \mathbb{R}$ is continuous at point t ϵ $\mathbb{R}$ such that f^{Δ} (t) be a derivative there exist a neighborhood of a point t – δ and t + δ such that the union

$$U = \{t-\delta, t + \delta\} \cap \mathbb{T} \text{for all } \delta > 0,$$

$$|f(\sigma(t) - f(s) - f^{\wedge}(t)\sigma(t) - s)| \leq \boldsymbol{\varepsilon}\, |\sigma(t) - s|$$

for all s ϵ U

We will defined $f^{\Delta}(t)$ as a differentiable function or delta (Hilger) derivative on the timescale $\mathbb{T}$, provided there exist $t \in \mathbb{T}$, the function $f^{\Delta}: \mathbb{T}^k \longrightarrow \mathbb{R}$ is known as delta derivative off on$\mathbb{T}^k$.

If $f : \mathbb{T} \longrightarrow \mathbb{R}$ and $f : \mathbb{T} \longrightarrow \mathbb{Z}$ be a continuous and discrete function over real numbers and integer both satisfies the delta derivative ofx^{Δ}at point $t \in \mathbb{T}$. Then

The delta derivative$x' = x^{\Delta}$which is equal to Δx such that $\boldsymbol{\varepsilon} > 0$

The definition can be re-written as follows.

$$f^{\Delta}(t) = \lim_{s \to t} \frac{f\,(\sigma(s) - f(t)}{\sigma(s) - t}$$

If $\mathbb{T} = \mathbb{R}$ then $\sigma(t) = t \ \forall\, t \in \mathbb{T}$ hence the above definition is reduces to

$$f^{\Delta}(t) = \lim_{s \to t} \frac{f\,(s) - f(t)}{s - t} = f'(t)$$

If $\mathbb{T} = \mathbb{Z}$, then $\sigma(t) = t$, so

$$f^{\Delta}(t) = \lim_{s \to t} \frac{f\,(s+t) - f\,(t)}{s+t - t} = f\,(t+1) - f\,(t) = \Delta f\,(t)$$

The time scale system can best be explaining and understood, as the continuum bridge. Between discrete time and continuous time system, some of the dual properties of continuous and discrete time system are list in **Table 2.1** below. This is also a collection and simulation from different text by Martin Bohner and Allan Peterson page [5-9] Dynamic equation on time scale, By Murray. R. Spiegel Shaum’s outline page [4, 5] Calculus of finite differences and difference equation

Table 2.1

$\mathbb{T}=\mathbb{R}$ differentiation continuous (μ(t)=0)	$\mathbb{T}=\mathbb{Z}$ difference equation discrete (μ(t) = 1)	Arbitrary $\mathbb{T}$ delta derivative on time scale hybrid ($\mu = \mu$(t))
1, $x''(t)+x'(t)+kx=0$	$x(t+2)+x(t+1)+kx=0$	$x^{\Delta\Delta}(t)+x^{\Delta}(t)+kx=0$
2, $(Kx(t))' = kx'(t)$	$\Delta(kx(t)) = K\Delta x(t)$	$(kx(t))^{\Delta} = kx^{\Delta}(t)$
3, $(x+y)' = x' + y'$	$\Delta(x + y) = \Delta x + \Delta y$	$(x + y)^{\Delta} = x^{\Delta} + y^{\Delta}$
4, $(xy)' = x'y + xy'$	$\Delta(xy) = \Delta x(y(t) + x(t)\Delta y$	$(xy)^{\Delta} = x^{\Delta}y(t) + x(\sigma(t)y^{\Delta}(t)$
5, $x'(t)= \lim_{t\to 0} \frac{x(t)-x(s)}{t-s}$	$\Delta x(t) = x(t+1) - x(t)$	$x^{\Delta}(t)= \lim_{s\to t} \frac{\sigma(s)-x(t)}{\sigma(s)-t} = \frac{x(\sigma(t)-x(t)}{\sigma(t)-t}$
6, $(\frac{1}{x})'(t) = \frac{x'(t)}{x(t)x(t)}$	$\Delta\left(\frac{1}{x}\right)(t) = \frac{\Delta x(t)}{x(t)x(t+h)}$	$(1/x)^{\Delta}(t)= \frac{x^{\Delta}(t)}{x(t)x(\sigma(t))}$
7, $\left(\frac{y}{x}\right)'(t)= \frac{y'(t)x(t)-y(t)x'(t)}{(x(t))^2}$	$\Delta\left(\frac{y}{x}\right)(t)= \frac{x(t)\Delta y(t)-y(t)\Delta x(t)}{(x(t)x(t+h)}$	. $\left(\frac{y}{x}\right)^{\Delta}(t)= \frac{y^{\Delta}(t)x(t)-y(t)x^{\Delta}(t)}{x(t)x(\sigma(t))}$
8, $y^1=\lambda y$, $y(t_0)=1$, $y(t) =e^{\lambda(t-t^0)}$	$\Delta y=\lambda y$, $y(t_0)=1$ $y(t) = (1+\lambda)^{t-t_0}$	$y^{\Delta}=\lambda y$, $y(t_0)=1$, $y(t)=e\lambda^{(t,t^0)}$
9, Riemann integral $\int_a^b f(t)dt$	Discrete summation $\sum_{t-a}^{b-1} f(t)$	Delta integral $\int_a^b f(t)\Delta t$
10, Laplace transform	Z − transform	Generalized Laplace transform
11, Continuous time convolution	Discrete time convolution	Time scale convolution
12,Stability in the S − plane	Stability in the Z−plane	Stability in hilger complex plane

Theorem 2.5 (Rolle's Theorem on Time Scale)

If f is continuous function on a closed interval [a, b] and differentiable on the open interval (a, b) then if f (a) = f (b), if and only if there exist at least one point c such (a, b) where the

$$f^{\Delta}(c).f^{\Delta}(\sigma(c)) \leq 0$$

Proof. We assume that f (x) is not identically zero since in this case the result is immediate, suppose that f (x) > 0 for some value between a and b, then it follows since f (t) is continuous that it attains its maximum value some were between a and b say t, we consider. See Bohner and Peterson 2003 page [4-5]

$$\text{f}\,(x) = \frac{f(t+h) - f(t)}{h} \leq 0$$

$$= \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} \leq 0$$

For h $>$ 0 then taking the limit h$\longrightarrow$ 0 through positive value of h, we have $f^{\vartriangle}$ (t), $f^{\vartriangle}(\sigma\,(t)) \leq 0$

Suppose if f $(x) <$ 0 so small that t+ h is continuous between a and b, since f (t) is a maximum value it follows that

$$\text{f}(x) = \frac{f(t+h) - f(t)}{h} \geq 0$$

$$= \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} \geq 0$$

While if the limit is taken through negative values of h we have

$$f^{\vartriangle}(t), f^{\vartriangle}(\sigma\,(t)) \geq 0$$

2, showing that the mean value theorem for a continuous derivatives function of f (x) is a $\leqq x \leqq$ b and has a derivative a$<$ x $<$b then there is at least one value between a and b such that

$$f^{\vartriangle}(t) = \frac{f(\sigma(b)) - f(a)}{\sigma(b) - a}$$

That is if for any value of x such that a $\leqq$ $x \leqq$ b,

$$\text{f}\,(x) = \text{f}\,(a) + (x - a)\, f^{\vartriangle}(\sigma\,(t)).$$

Where t is between a and x, considering the function

$$\text{f}\,(x) = \text{f}\,(x) - \text{f}\,(a) - (x - a)\frac{f(\sigma(b)) - f(a)}{\sigma(b) - a}$$

From this we see that f (a)= 0, f (b) = 0 and that f (x) satisfies the condition of rolle′s theorem then there is at least one point t is between A and B such that $F^{\vartriangle}$ (t) = 0. But

$$f^{\vartriangle}(x) = f^{\vartriangle}(x) - \frac{f(\sigma(b)) - f(a)}{\sigma(b) - a}$$

So that $f^{\vartriangle}(t) = f^{\vartriangle}(t) - \frac{f(\sigma\,(b)) - f\,(a)}{\sigma(b) - a} = 0$

$$f^{\vartriangle}(t) = \frac{f(\sigma\,(b)) - f(a)}{(\sigma(b) - a}$$

Where b in the above function will be replace by x and we will obtain

$$\text{f}\,(x) = \text{f}\,(a) - (x - a)\, f^{\vartriangle}(\sigma(t))$$

Where t is between x and a respectively

Theorem 2.6 Let b be constant and M ϵ □

1: If f is defined such that for f (t) = (t + b) ^{m}

Then we will have the function to be

$$f^{\Delta}(t) = \sum_{k=0}^{m-1} (\sigma(t) + b)^{m} (t + b)^{m-1-k}$$

$$= f^{\Delta}(t) = (t + b)^{\Delta} = 1$$

2, for g defined by g (t)= $\frac{1}{(t+b)^{m}}$

We have $g^{\Delta}(t) = \sum_{k=0}^{m-1} \frac{1}{(\sigma(t)+ b)^{-k+1}(t+b)^{-m-k}}$

From the above theorem, we compute delta derivative on arbitrary time scale $\mathbb{T}$ for the element functions

$$P(t) = t \quad P(t) = (t + b)^{m}$$

And p (t) = $(t + a)^{-m}$

Where a' is constant and M ϵ □ this will illustrate the difference in derivatives from the continuous case, due to the presence of discrete points in the time scale available.

1, if p (t) = t it is clear that for any $\mathcal{E} > 0$ there exist a neighborhood**u** of t such that, where

$$p^{\Delta}(t) = 1$$

For any t ϵ $\mathbb{T}$, Hence the delta derivative of t is $t^{\Delta} = 1$ for

$$f_{k}(t) = (t + b)^{k}$$

For each k ϵ □ then using the product rule

$$f_{k}^{\Delta}(t) = [(t + b)(t + b)^{k-1}]^{\Delta}$$

$$= I \times (t + b)^{k-1} + (\sigma(t) + b) f_{k-1}^{\Delta}(t)$$

Then we will find that $p^{\Delta}(t) = f_{m}^{\Delta}(t)$ we will apply the recursive rule repeatedly to get

$$p^{\Delta}(t) = (t + b)^{m-1} + (\sigma(t) + b)^{1} (t + b)^{m-2} + (\sigma(t) + b)^{2} (t + b)^{m-3} + \ldots$$

$$= \sum_{k=0}^{m-1} (\sigma(t) + b)^{K} (t + b)^{m-1-K}$$

$$= f^{\Delta}(t) = (t + b)^{\Delta} = 1$$

2, we introduce $g_{k}(t) = (t + b)^{-K}$ for each k ϵ □ then similarly to (2) we use the recursive rule

$$g_{k}^{\Delta}(t) = -(\sigma(t) + b)(t + b)^{-k} + (\sigma(t) + b)^{-1} g^{\Delta}_{k}(t)$$

Similarly, we obtain

$$\mathrm{P}\,(\mathrm{t}) = g^{\Delta}_{m}\,(\mathrm{t}) = -\sum_{k=0}^{m-1}(\sigma\,(t) +\ b)^{-\mathrm{K}+1}\,(\mathrm{t}+\mathrm{b})^{-\mathrm{m}-\mathrm{K}}.$$

$$= \ g^{\Delta}_{m}(\mathrm{t}) = -\sum_{k=0}^{m-1}\frac{1}{(\sigma(t)^{-\mathrm{K}+1}(t+b)^{-\mathrm{m}-\mathrm{K}}}$$

If we consider the standard time scale $\mathbb{T}=\ \mathbb{R}$ then $\sigma(\mathrm{t})\ =\ \mathrm{t}$ for all t and hence the delta derivative gives the expected result for the derivative from the standard differential calculus hence the forward jump operator σ(t) encapsulates difference between delta derivatives in the continuous and in the discrete case

2.1.2 Integration on Time Scales

Definition 2.7 A function f : $\mathbb{T}\ \longrightarrow\ \mathbb{R}$ is regulated provided the right side limit exist (finite) to all right dense point in $\mathbb{T}$, then the left side limit exist (finite) to all left dense point in $\mathbb{T}$. See Advanced dynamic equation on time scale By Martin Bohner and Allan Peterson page [7, 8] and also dynamic equation on time scale page [22, 23] respectively.

$$f_{\mathrm{rt}\to\mathrm{n}} = f^{d}_{rd}\,(\mathbb{T}) = f^{d}_{rd}\,(\mathbb{T},\mathbb{R})$$

$$f_{\mathrm{Lf}\to\mathrm{n}} \ = f^{d}_{L\,f}(\mathbb{T}) = f^{d}_{L\,d}\,(\mathbb{T},\mathbb{R})$$

Definition 2.8A function f: $\mathbb{T}\ \longrightarrow\ \mathbb{R}$ is a rd –continuous provided it is continuous at right dense point in $\mathbb{T}$, and its left side limit exist finite at left dense point in $\mathbb{T}$. The sets f: $\mathbb{T}\longrightarrow\mathbb{R}$ rd– continuous is dense by,

$$c_{\mathrm{rd}} \ = c^{d}_{rd}\,(\mathbb{T}) = c^{d}_{rd}\,(\mathbb{T},\mathbb{R})$$

$$c_{\mathrm{Lf}\to\mathrm{n}} = c^{d}_{Ld}\,(\mathbb{T}) = c^{d}_{Ld}\,(\mathbb{T},\mathbb{R})$$

Theorem 2.9 Assume f: $\mathbb{T}\longrightarrow\mathbb{R}$

1: if f is rd – continuous. Then f is defined to be a continuous function

2: if f is regulated then is known as a rd – continuous function.

3: the jump operator σ is rd – continuous function.

4: if f is regulated or rd – continuous, then so is f^{σ}

5. Assume f is continuous, if g: $\mathbb{T}\longrightarrow\mathbb{R}$ is regulated or rd– continuous then f o g has the property.

Definition 2.10 Given a function f: $\mathbb{T} \longrightarrow \mathbb{R}$, where f (t) is an anti – derivative of f (t) is any function f (t) such that $F^{\Delta}(t) = f(t)$

If f(t) is any anti – derivative of f (t) then the most general anti – derivative of f (t) is called an indefinite integral and is denoted by

$$\int f\ (t) \vartriangle t = F\ (t) + c \qquad \text{c is constant}$$

Where $\int$ is the integral symbol and f (t) is the integrant function of t, and c is the constant of the integration.

Theorem 2.11 (Existence of anti−derivation) indefinite integral are rd – continuous function has an anti – derivative that is continuous at every point in the domain, in particular if t $\in \mathbb{T}$ then t is said to be continuous such that

$$\int f\ (t) \vartriangle t = \text{F (t)} + \text{c}$$

Then the integration is a continuous function on t

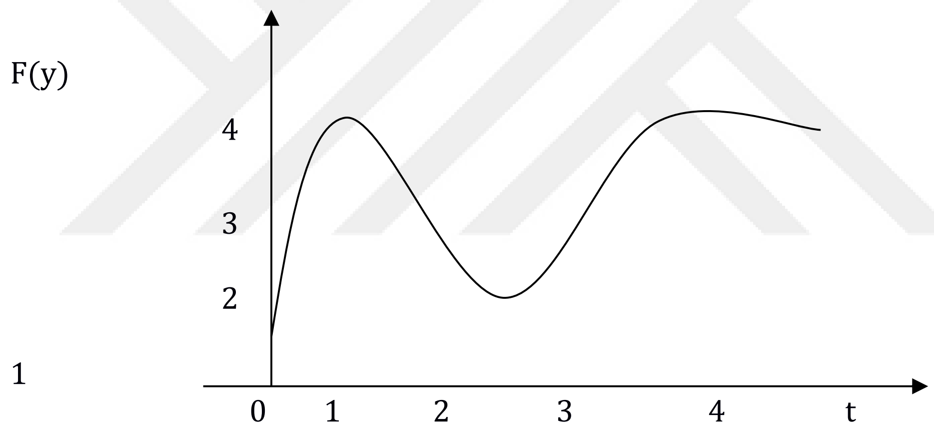


Figure 2.2 Diagram for continuous function

Theorem 2.12 if a, b, c $\in \mathbb{T}$ and $\sigma \in \mathbb{R}$ f, g $\in C_{rd}$ then

$$1,\ \int_a^b |f\ (t) + \ g(t)| \vartriangle \text{t} = \int_a^b f\ (t) \vartriangle t + \int_a^b g\ (t) \vartriangle t$$

$$2,\ \int_a^b f\ (t) \vartriangle t = -\int_b^a f\ (t) \vartriangle \text{t}$$

$$3,\ \int_a^b (\sigma f)(t) \vartriangle t = \sigma \int_a^b f\ (t) \vartriangle \text{t}$$

$$4,\ \int_a^b f\ (t) \vartriangle \text{t} = \int_c^b f\ (t) \vartriangle \text{t} + \int_a^c f\ (t) \vartriangle \text{t}$$

$$5,\ \int_a^b f(\sigma\ (t)\ g^{\Delta}\ (\text{t}) \vartriangle \text{t} = (\text{fg})\ (\text{b}) - (\text{fg})\ (\text{a}) - \int_a^b f^{\Delta}\ (\text{t})\ \text{g}\ (\text{t}) \vartriangle \text{t}$$

$$6, \int_a^b f(t) g^{\Delta}(t)\, \Delta t = (fg)(b) - (fg)(a) - \int_a^b f^{\Delta}(t)\, g(\sigma(t))\, \Delta t$$

$$7, \int_a^a f(t) \Delta t = 0$$

$$8, \text{ if } f(t) \geq 0 \text{ for all } a \leq t < b \text{ then } \int_a^b f(t)\, \Delta t \geq 0$$

$$9, \text{ if } |f(t)| \leq g(t) \text{ on } [a, b) \text{ then } \left|\int_a^b f(t) \Delta t\right| \leq \int_a^b g(t) \Delta t$$

$$10, \int f(at) \Delta t = \frac{1}{a} f(at) + c$$

$$11, \int f(at + b) \Delta t = \frac{1}{a} F(at + b) + c$$

Theorem 2.13 If f: $\mathbb{T} \longrightarrow \mathbb{R}$ is an arbitrary function and t ϵ $\mathbb{T}$then

$$\int_t^{\alpha(t)} f(r) \Delta r = \boldsymbol{\mu}(t) f(t)$$

Proof.

$$\int_t^{\alpha(t)} f(r) \Delta t = \int_t^{\alpha(t)} g(\boldsymbol{\tau})\, \boldsymbol{\Delta\tau}$$

$$= G\sigma(t) - G(t) = \boldsymbol{\mu}(t)\, G^{\Delta}(t) = \boldsymbol{\mu}(t)\, g(t)$$

The proof of this result however let f: $\mathbb{T} \longrightarrow \mathbb{R}$ an arbitrary and let t ϵ $\mathbb{T}$ if $\sigma(t) = t$ Then g: $\mathbb{T} \longrightarrow \mathbb{R}$ is the continuous function.

Theorem 2.14 Let a, b ϵ $\mathbb{T}$ and f ϵ C_{rd}

1, if $\mathbb{T} = \mathbb{R}$ then

If $$\int_a^b f(t) \Delta t = \int_a^b f(t) dt$$

Where the integral on the right is the usual Riemann n integral

2, If $|a, b|$consider of only isolated points

Then $$\int_a^b f(t) \Delta t = \begin{cases} \sum_{t \,\epsilon\, [a,b]} \mu(t) f(t) & \text{if } a<b \\ 0 & \text{if } a = b \\ \sum_{t \,\epsilon\, [b,a]} \mu(t) f(t) & \text{if } a> b \end{cases}$$

3, If $\mathbb{T}$ = h □ = {h k: k $\epsilon\mathbb{Z}$} where h> 0 then

$$\int_a^b f(t)\Delta t = \begin{cases} \sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} f(kh)h & \text{if } a<b \\ 0 & \text{if } a=b \\ \sum_{k=\frac{b}{h}}^{\frac{a}{h}-1} f(kh)h & \text{if } a>b \end{cases}$$

4, if $\mathbb{T} = \mathbb{Z}$ then

$$\int_a^b f(t)\Delta t = \begin{cases} \sum_{t=a}^{b-1} f(t) & \text{if } a<b \\ 0 & \text{if } a=b \\ -\sum_{t=b}^{a-1} f(t) & \text{if } a>b \end{cases}$$

Proof.

1: $\int_a^b f(t)\Delta t = f(b) — f(a)$

by applying the triangle inequality we obtain

$$f|b-c+c-a| = f(b)-f(c)+f(c)-f(a)$$

$$=|f(b) — f(c)|\ \Delta t + |f(c) — f(a)|\ \Delta t$$

$$= \int_c^b f(t)\Delta t + \int_a^c f(t)\Delta t = \int_a^b f(t)\Delta t$$

2, if $|a, b|$consider of only isolated points then

$$\int_a^b f(t)\Delta t = \begin{cases} \sum_{t\in[a,b]} \mu(t)f(t) & \text{if } a<b \\ 0 & \text{if } a=b \\ \sum_{t\epsilon[b,a]} \mu(t)f(t) & \text{if } a>b \end{cases}$$

Solution

$$\int_a^b f(t)\Delta t = \sum_{t=0}^{n-1} \int_{t_0}^{t_1+1} f(t)\Delta t$$

$$= \sum_{t=0}^{n-1} \int_t^{\sigma(t)} f(t)\Delta t = \sum_{t=0}^{n-1} \mu(t)f(t_1)$$

$$= \sum_{t<|a,b|} \mu(t)f(t)$$

3, if $\mathbb{T} = h\,\mathbb{Z} = \{h\,k: k\,\epsilon\mathbb{Z}\}$ where $h>0$ then

$$\int_a^b f(t) = \begin{cases} \sum_{\frac{a}{h}-1}^{\frac{b}{h}-1} f(k\,h)\,h & \text{if } a<b \\ 0 & \text{if } a=b \\ —\sum_{\frac{b}{h}-1}^{\frac{a}{h}-1} f(k\,h)h & \text{if } a>b \end{cases}$$

Solution

$$\sum_{t=0}^{n-1} \int f|b-1+1-a|\Delta \mathrm{t}$$

$$= \int_{1}^{\frac{b}{h}} f(kh)h \vartriangle t + \int_{\frac{a}{h}}^{1} f(kh)h \vartriangle t$$

$$= \sum \int_{a}^{b} f(kh)h \vartriangle t$$

4, If $\mathbb{T} = \mathbb{Z}$ then

$$\int_{a}^{b} f(t) \vartriangle t = \begin{cases} \sum_{t=a}^{b-1} f(t) & \text{if } a< b \\ 0 & \text{if } a = b \\ \sum_{t=b}^{a-1} f(t) & \text{if } a > b \end{cases}$$

Solution

$$\sum_{t=0}^{n-1} \int [b-1+1-a] f(t)\Delta \mathrm{t}$$

$$= \int_{1}^{b} f(t)\Delta \mathrm{t} + \int_{a}^{1} f(t)\Delta \mathrm{t}$$

$$= \sum \int_{a}^{b} f(t) \vartriangle t$$

2.1.3. Chain Rule

Definition 2.15 If f, g: $\mathbb{R} \longrightarrow \mathbb{R}$ be pairs of two functions by the application of chain rule if g is differentiable at t and f is differentiable at g(t) then the function becomes. See page [31- 36] of Martin Bohner and Allan Peterson for more details

$$(f \circ g)^{I} (t) = f^{I} ((t)\ g^{I} (t))$$

Theorem 2.16 Assume f: $\mathbb{R} \longrightarrow \mathbb{R}$ is continuous g: $\mathbb{T} \longrightarrow \mathbb{R}$ is differentiable on $\mathbb{T}^k$ then the function f: $\mathbb{R} \longrightarrow \mathbb{R}$ is continuously differentiable if and only if there exists a constant c in the interval [t, σ(t)]

$$(f \circ g)^{\Delta} (t) = f^{\Delta} (g (c)\ g^{\Delta} (t)$$

Proof. Putting t ϵ $\mathbb{T}^k$ by considering the process of the right scattered in the case

$$(f \circ g)^{\Delta} (t) = \frac{f\ (g(o(t) - f(g(t))}{\mu(t)}$$

Then if $(f \circ g)^{\Delta}(t) = 0$ and $g^{\Delta}(t) = 0$ we have $(\sigma (t)) = g\ (t)$ and so the above hold for any c in the real interval (t, σ(t)),hence we can say g(σ(t)) ≠ g(t)

$$(f \circ g)^{\Delta} (t) = \frac{f\ (g(\sigma(t) - f(g(t))}{g(\sigma(t) - g(t))} \bullet \frac{g(\sigma(t) - g(t)}{\mu(t)} = f^{I} (\xi) g^{\Delta}(t))$$

Where ξ is between g(t) and g(σ (t)) then since g: $\mathbb{R} \longrightarrow \mathbb{R}$ is continuous, there is c ϵ [t, σ (t)] where g(c) = ξ gives us the desired result

Example 1.1

We considered the following result g: $\mathbb{Z} \longrightarrow \mathbb{R}$ and f: $\mathbb{R} \longrightarrow \mathbb{R}$ by

$$g(t) = t^2 \text{and } f(x) = \exp(x)$$

Then $g^{\Delta}(t) = (t+1)^2 - t^2 = 2t + 1$

and $f^{I}(x) = \exp(x) = t^2 + t + t + 1 - t^2 = 2t + 1$

Hence by referring to **theorem 1.12** above

$$(f \circ g)^{\Delta}(t) = \{ \int_0^1 f(g(t+1) - f(g(t)) \Delta t\} g^{\Delta}(t)$$

$$= \int_0^1 \exp(t+1)^2 - \exp(t^2)) \Delta t\} (2t+1)$$

$$= (2t+1)\int_0^1 (\exp(t^2 + 2t + 1) - \exp(t^2)) \Delta t$$

$$= (2t+1) \exp(t^2) \int_0^1 (\exp(2t+1) -) \Delta t$$

$$= (2t+1)\exp(t^2) \left|\frac{1}{2t+1} \exp(2t+1) - t\right|_1^2$$

$$= (2t+1) \exp(t^2) \frac{1}{2(1)+1} \exp(2(1)+1) - 1 = 0$$

$$= (2t+1) \exp(t^2) \frac{1}{3} \exp 3 - 1 = \exp(t^2) (\exp(2t+1) - 1)$$

$$= \exp(t^2) \exp(2t+1) - 1)$$

Let $\mathbb{T}$ be a time scale and V: $\mathbb{T} \longrightarrow$ □, be a strictly increased in function and improve in the possibilities of the industrial production process within the system of the industry such that $\mathbb{T}$ = V ($\mathbb{T}$) is the time scale of the system.

Theorem 2.17 (Substitution) let y: $\mathbb{T} \longrightarrow$ □ is strictly increasing and $\mathbb{T}$ = y ($\mathbb{T}$) is a time scale if f: $\mathbb{T} \longrightarrow$ □ is a rd – continuous function and y is differentiable with rd – continuous derivative then for a, b $\epsilon \mathbb{T}$

$$= \int_a^b f(t) y^{\Delta}(t) \Delta t = \int_{y(a)}^{y(b)} (f \circ y^{-1})(s) \Delta s$$

Proof. Since f y $^{\Delta}$ is a rd –continuous function it possesses an anti −derivative f by I.e $f^{\Delta} = f y^{\Delta}$ and

$$= \int_a^b f(t) y^{\Delta}(t) \Delta t = \int_a^b f^{\Delta}(t) \Delta t = F(b) - F(a)$$

$$= (F \circ y^{-1})(y(b)) - (F \circ y^{-1})(y(a))$$

$$= \int_{y(a)}^{y(b)} (F \circ y^{-1})^{\Delta}(s)\, \Delta s = \int_{y(a)}^{y(b)} (F^{\Delta} \circ y^{-1})(s)\, (y^{-1})^{\Delta}(s)\, \Delta$$

$$= \int_{y(a)}^{y(b)} ((f y^{\Delta})\, \sigma\, y^{-1})(s)\, (y^{-1})^{\Delta}(s) \Delta s$$

$$= \int_{y(a)}^{y(b)} (f \circ y^{-1})(s)\, |(y^{\Delta} \circ y^{-1})\, (y^{-1})|\,(s)\, \Delta s$$

$$= \int_{y(a)}^{y(b)} (f \circ y^{-1})(s)\, \Delta s$$

The proof since f y $^{\Delta}$ is a rd – continuous function it possesses an anti – derivative F by the above theorem I,e F $^{\Delta}$ = f y$^{\Delta}$ and the above equation show that it is strictly increasing in the time and there is a continuous process in the time scale

.

2.1.4 Exponential Function on Time Scales

A transformation method with the use of a cylinder will be introduced to explain and generalized the exponential function for a time scale $\mathbb{T}$, and the application of Hilger's complex plane on $\mathbb{T}$ = □ and $\mathbb{T}$ = □ specifically, which will involved the use of Hilger's Δ in this section. Contact the following book By Martin Bohner and Allan Peterson page [58 - 74] for detail information.

Definition 2.18 If the function P: $\mathbb{T} \rightarrow$ □ is regressive then the equation holds on

$$1 + \mu(t)\, p(t) \neq 0 \quad \text{for all} \quad t \in \mathbb{T}^k$$

Then the regressive set of all right dense continuous function f: $\mathbb{T} \rightarrow$ □ will denoted as

$$R = R(\mathbb{T}) = R(\mathbb{T}, \mathbb{R})$$

Definition 2.19 The Hilger's complex number is defined on a real line and integer if h >0 to be $\quad \mathbb{Z}_h = \{z \in \mathbb{C}: \frac{-\pi}{h} < \text{Im}(z) < \frac{\pi}{h}\}$ and $\quad \mathbb{C}_h = \{z \in \mathbb{C}: \; z \neq \frac{1}{h}\}$

If it is introduce and transformed on a v–cylinder then the function ξ_h: □ $_h \rightarrow$ □h will satisfy $\xi_h(z) = -\frac{1}{h}\log(1 - zh)$

If h = 0 then the logarithmic function of the equation will be

$\xi_0(z) = \log 1 = z \qquad\qquad$ for all $z \in \mathbb{C}_0 = \mathbb{C}$

Imaginary axis

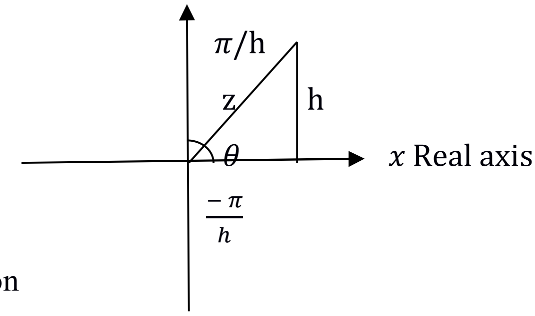
x = Re (z)

Y = lm (z)

$\mathbb{Z}_h$ = Re (z) + lm (z)

$\mathbb{Z}_h = -\pi/h + \text{lm}\ \pi/h$

Refer to the above complex solution



Definition 2.20 The first order linear dynamic equation p $\epsilon\mathcal{R}$

$$y^{\Delta} = p(t)y$$

Is called a regression

Theorem 2.21 If theorem (2.19) is regression, and then the only solution for the next theorem will be $e_{p\,(.\ \ ,t\square)}$

Proof If y is an equation then the below solution will be quotient rule y/$e_{p\,(.\ \ ,t\square)}$ and can also be used as product rule y $e_{p\,(\ .,t\square)}$ we will have the following

Quotient rule $\left(\frac{y}{e_{p\,(\ .,t_0)}}\right)^{\Delta}(t) = \frac{y^{\wedge}(t)e_p\,(t,t_0) - y\,(t)e^{\wedge}{}_p\,(t,t_0)}{e_p\,(t,t_0)e_p\,(\sigma(t,t_0)}$

$$= \frac{p(t)y(t)e_p\,(t,t_0) - y(t)p(t)e_p\,(t,t_0)}{e_p\,(t,t_0)e_p\,(\sigma(t,t_0)}$$

$$= 0$$

So that y/ e_p (t, t_0) is constant and continuous hence

$$\frac{y\,(t)}{e_p\,(t,t_0)} \equiv \frac{y\,(t_0)}{e_p\,(t_0,t_0)} = \frac{1}{1} = 1$$

Therefore y = $e_p\,(.\ ,t_0)$

Product rule $(\,ye_p(.\ ,t_0))^{\Delta}(t) = y^{\Delta}e_p(t\,,t_0) + y\,(t)e^{\wedge}{}_p(t\,,t_0)$

$$= p\,(t)\,y\,(t)e_p(t\,,t_0) + y(t)\,p(t)e_p(t\,,t_0)$$

$$= 2\,p\,(t)\,y\,(t)e_p(t\,,t_0)$$

Therefore it implies that y e_p (t, t_0) is constant let us introduce some important principle that will be useful in further cases.

Theorem 2.22 Let us consider a situation where p, q $\epsilon\mathcal{R}$ then we will have the following properties.

$$1.\ e_p\,(t,s) \equiv 1 \text{ and } e_p(t,t) \equiv 1;$$

$$2.\ e_p(\sigma(t, s) = (1 + \mu(t)\, p(t))\, e_p(t, s);$$

$$3.\ \frac{1}{e_p(t, s)} = e_{\ominus p}(t, s);$$

$$4.\ e_p(t, s) = \frac{1}{e_p(s, t)} = e_{\ominus p}(s, t);$$

$$5.\ e_p(t, s)\, e_p(s, r) = e_p(t, r);$$

$$6.\ e_p(t, s)\, e_q(t, s) = e_{p \oplus q}(t, s);$$

$$7.\ \frac{e_p(t, s)}{e_q(t, s)} = e_{p \ominus q}(t, s);$$

$$8.\ \left(\frac{1}{e_p(\cdot, s)}\right)^{\Delta} = -\frac{p(t)}{e_p^{\sigma}(\cdot, s)};$$

Definition 2.23 If f: $\mathbb{T} \longrightarrow$ □ is rd – continuous then p $\epsilon \boldsymbol{\mathcal{R}}$ is a regressive, we define the exponential function by

$$e_p(t, s) = \exp\left(\int_r^t \xi_{\mu(\tau)}\ (p(\tau))\Delta\tau\right) \text{for} \quad s, t \in \mathbb{T}$$

Lemma 1.2 If p $\epsilon \boldsymbol{\mathcal{R}}$ then the semigroup property will be

$$e_p(t, r) e_p(r, s) = e_p(t, s) \qquad \text{for all} \quad r, s, t \in \mathbb{T}$$

Is satisfied

Proof. If p $\epsilon\ \boldsymbol{\mathcal{R}}$ then r, s, t $\epsilon\ \mathbb{T}$ by using the **definition 2.23** we will obtain

$$e_p(t, r) e_p(r, s) = \exp\left(\int_r^t \xi_{\mu(\tau)}\ (p(\tau))\Delta\tau\right) \exp\left(\int_s^r \xi_{\mu(\tau)}\ (p(\tau))\Delta\tau\right)$$

$$= \exp\left(\int_r^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau + \int_s^r \xi_{\mu(\tau)}\ (p(\tau))\Delta\tau\right)$$

$$= \exp\left(\int_s^t \xi_{\mu(\tau)}\ (p(\tau))\Delta\tau\right)$$

$$= e_{\boldsymbol{p}}(t, s);$$

Example 1.3 Let $\mathbb{T} = \mathbb{Z}$ and let $\alpha \in \boldsymbol{\mathcal{R}}$ be a constant

Thene_α (t, t0) = $(1 + \alpha)^{t - t_0}$ for all t, $t_0 \in \mathbb{T}$

To show this we will note that y defined by the right hand side of the above equation

$$y^{\Delta}(t) = (1 + \alpha)^{t - t_0} - (1 + \alpha)^{t}$$

$$= (1 + \alpha)^{t}(1 + \alpha)^{-t_0} - (1 + \alpha)^{t}$$

$$= (1 + \alpha)^{t} = (1 + \alpha)^{t - t_0}$$

For all t, $t_0 \in \mathbb{T}$

Example 1.4 Let $\mathbb{T} = h\,\mathbb{Z}$ for $h > 0$ and let α $\boldsymbol{\mathcal{R}}$ be a constant i,e

$$\alpha \in \mathbb{C} \setminus \left\{\frac{1}{h}\right\}.$$

Then $e_\alpha(t, t_o) = (1 + \alpha)^{t - t_o/h}$ for all $t, t_o \in \mathbb{T}$

Showing that y defined by the right hand side satisfying the above equation

$$y^{\Delta}(t) = \left(\frac{(1+\alpha h)^{\frac{t-t_o}{h}} - (1+\alpha h)^{\frac{t}{h}}}{h}\right)$$

$$= \frac{(1+\alpha h)^{\frac{t}{h}}(1+\alpha h)^{\frac{-t_o}{h}} - (1+\alpha h)^{\frac{t}{h}}}{h}$$

$$=(1+\alpha h)^{\frac{t}{h}}(1+\alpha h)^{\frac{-t_o}{h}}$$

$$=(1+\alpha h)^{\frac{t-t_o}{h}}$$

for all $t, t_o \in \mathbb{T}$

Then the exponential function for a constant $\mathbb{T}$ in $h > 0$ is satisfies.

2.1.5Forward Difference Scheme Approach

We will introduce the scheming approach for forward difference method where we have $x^{\Delta}(t) = \Delta x_n(t)$ where $n = 0, 1, 2, 3. \ldots\ldots\ldots. n$. See Carl Erik Froberg Introduction to Numerical Analysis page [173]

t_o						
	Δx_o					
t_1		$\Delta^2 x_o$				
	Δx_1		$\Delta^3 x_o$			
t_2		$\Delta^2 x_1$		$\Delta^4 x_o$		
	Δx_2		$\Delta^3 x_1$		$\Delta^5 x_o$	
t_3		$\Delta^2 x_2$		$\Delta^4 x_1$		$\Delta^6 x_o$
	Δx_3		$\Delta^3 x_2$		$\Delta^5 x_1$	
t_4		$\Delta^2 x_3$		$\Delta^4 x_2$		
	Δx_4		$\Delta^3 x_3$			
t_5		$\Delta^2 x_4$				
	Δx_5					
t_6						

Figure 2.3forward difference approach

In general we can see directly that the quantity $\Delta^{k}x_{o}$ is a straight line sloping down to the right on the other hand since $\Delta = E\nabla$ we say for instance $\Delta x_{\square} = \nabla x_{\square}$, $\Delta^{2}x_{\square} = \nabla^{2}x_{\square}$, $\Delta^{3}x_{\square} = \nabla^{3}x_{\square}$, $\Delta^{\square}x_{\square} = \nabla^{\square}x_{\square}$ and $\Delta^{\square}x_{\square} = \nabla^{\square}x_{\square}$and so on and infer that the quantities. $\nabla^{k}x_{n}$ lies on the straight line sloping upward to the right, finally we also have also have $\Delta = E^{1/2}\delta$ and hence for instance$\Delta^{2}x_{\square} = E\delta^{2}x_{\square} = \delta^{2}x_{\square}$, $\Delta^{\square}x_{o} = \delta^{\square}x_{\square}$ and so on in this way we will find the quantities $\delta^{1/2}x_{n}$ lie on horizontal line.

CHAPTER 3

APPLICATION OF CONTRACTION MAPPING PRINCIPLE

A contraction mapping principle is one of the most fundamental and important theorem of the fixed point theory in functional analysis, due to its usefulness in the theory of ordinary differential equation especially in existence and uniqueness theory. The existences or construction of a solution of an ordinary differential equation is often reduces to an existence of a fixed point for an operator corresponding to a given O,D,E The following well-known result called Picard theorem illustrates this fact. See Element of Abstract Analysis by Michael .O. Searcoid page [251] and Applied Functional Analysis and Variational Methods in Engineering by J.N. Reddy page [202] and Charles .E. Chidume, Functional Analysis and Introduction to Metric Space [166].

3.1. Picard Theorem

Theorem 3.1 (Picard Theorem)

Consider the dynamic equation with an initial value problem

$$y^{\Delta}(x) = f(x, y), \; y(x_0) = y_0 (3.1)$$

Suppose that f satisfies the following conditions:

1, $f(x, y)$ is continuous in a closed rectangle $R = \{(x, y) -a \leq x \leq a , -b \leq y \leq b \}$ and that (x_0 , y_0) is in the interior of R,

2, $f^{\Delta}(x, y)$ is continuous in R, then the IVP (3.1) has a unique solution, which passes through (x_0, y_0). In the above theorem, the second condition can be replaces by Lipschitz continuity.

A function $f(x, y)$ is said to satisfy Lipschitz condition in the second variable if there exist a constant $M > 0$ such that

$$|f(x , y^1) - f(x , y^2)| \leq M|y^1 - y^2|$$

for all (x, y_1) and (x, y_2) in R

We observe that the rectangle is a closed bounded subset of the plane and so it is compact, Since $f(x , y)$ and $f^{\Delta}(x, y)$ are continuous on a compact set of R they are bounded , and there exist a constant K and M such that

$$|f(x , y_1) - f(x , y_2)| \leq K \text{ and} |f^{\wedge}(y)(x , y)| \leq M. . \; . . . . 2$$

for all x, y in R observe also that if $(x, y□)$ and $(x, y□)$ are in R then by mean value Theorem

$$|f(x , y_1) - f(x , y_2)| |f^{\wedge}(y)(x , y_1) + t(y_2 - , y_1)| \bullet |y_1 - y_2| . . . . 3$$

for some $t \in (0, 1)$, Then we obtain from (2) & (3) that

$$|f(x , y_1) - f(x , y_2)| \leq M|y_1 - y_2|$$

for all $(x , y□)$ and $(x , y□)$ in R thus f satisfies a Lipschitz condition in y.

2: The initial value problem can be replaced by an equivalent problem to an integral equation for if h (x) is a solution of with h (x_0) = y_0 then integrating x_o to x.

$$h(x) = h(x_o) + \int_{x_0}^{x} f(s, h(s))\Delta s = y_0 + \int_{x_0}^{x} f(S, h(s))\Delta s \ldots\ldots\ldots\ldots 5$$

on the other hand if h (x) satisfies the integral equation above then replacing

h (x) by y (x)

$$y(x) = y_o + \int_{x_o}^{x} f(s, y(s))\, \Delta s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

And putting $x = x_o$ we obtain that y (x_o) = y_o further more differentiating with respect to x we obtain

$$y^{\Delta}(x) = f(x, y(x))$$

where the solution is equivalent to the solution of equation (5)

Theorem 3.2 (Picard theorem)

Let $y^{\Delta}(x) = f(x, y)$ 1

Be a given dynamic equation with the initial condition y (x_o) = y_o suppose

1, f $(x$, y) is defined and continuous in both variables x and y in some open interval in a domain D in the plane which contains the point (x_o, y_o)

2, f $(x$, y) satisfies the Lipschitz condition with respect to y i .e

$$|f(x, y_1) - f(x, y_2)| \leq M\, |y_1 - y_2|$$

for some constant M > 0 and arbitrary (x□, y□), (x□, y□) in D, Then on some closed interval $|x - x_o| \leq a$ there is a unique solution y = φ (x) of (1) satisfying the initial condition $y(x_o) = y_o$

3.2. Applications of Picard Theorem

Introducing the application of Picard theorem and consider the continuous function$x$ (t) = 0 ≤ t ≤ 1 and differentiating the integral equation

$$\boldsymbol{\delta}\, x^{\Delta}(t) + \sin x + \int_{\frac{t}{2}}^{t} |1 + x^2(s)|\, sins\, \Delta s = 0$$

By proving that there is unique continuous function, x (t) on [0, 1] with $x(0) = 0$. Moreover, $|x(t) \leq 1|$which is the solution of the above?

$$x^{\Delta}(t) = -\frac{1}{8}(\sin x(t) + \int_{\frac{t}{2}}^{t} |1 + x^2(s)| \sin s \, \Delta s \quad x(0) = 0 \quad |x(t)| \leq 1$$

Which can be written as $x^{\Delta}(t) = f(t, x)$, $x(0) = 0$ and $|x(t)| \leq 1$

To prove that the differential equation of the above has a unique solution satisfying $x(0)=0$ and $|x(t)| \leq 1$ by the above theorem, to show that f(t , x) is define as a jointly continuous in t and x and that it satisfies the Lipschitz condition in the second variable.

$$h(t, x) = \int_{\frac{t}{2}}^{t} |1 + x^2(s)| \sin s \, \Delta s$$

$1 + x^2(s)$ is continuous in t,$\sin t$ is continuous in t hence

$$= \int_{\frac{t}{2}}^{t} |1 + x^2(s)| \sin s \, \Delta s$$

is a continuous function of t, since $\sin x$ is a continuous function of x it follows that

$$f(t, x) = -\frac{1}{8}(\sin x + h(t, x))$$

is continuous in t and x in some neighborhood of f (0, 0) solving that f (t, x) is Lipschitzian in x(t) for all x(t) such that $|x'(t)| \leq 1$ so

$$|f(t, x(t) - f(t, y(t)| \leq \frac{1}{8} |\sin y(t) - \sin x(t)| + \int_{\frac{t}{2}}^{t} |y^2(t) - x^2(t)| \Delta s$$

$$\leq \frac{1}{8} |\sin y(t) - \sin x(t)| + \frac{1}{8} \int_{\frac{t}{2}}^{t} |y(s) - x(t)| \, |y(s) - x(s)| \, \Delta s$$

$$\leq \frac{1}{8} |\sin y(t) - \sin x(t)| + \frac{1}{8} \int_{\frac{t}{2}}^{t} |y(s) - x(s)| \Delta s$$

$$= |y(s) + x(s)| \leq |y(s)| + |x(s)| \leq 2$$

$$|f(t, x(t) - f(t, y(t)|$$

$$\leq \frac{1}{8} |\sin y(t) - \sin x(t)| + \frac{1}{4} \sup_{s \in (\frac{t}{2}, t)} |y(s) - x(s)| \left| t - \frac{t}{2} \right|$$

$$\leq \frac{1}{8} |\sin y(t) - \sin x(t)| + \frac{1}{4} \sup_{s \in (0,1)} |y(s) - x(s)|$$

By the mean value theorem if f $(x) = \sin x$then $|g(x) - g(y)| = |g^{\wedge}(\xi)| \bullet |x - y|$

for some $\xi \in (x, y)$ so$|\sin x - \sin y| = |\cos \xi| \bullet |x - y| \leq |x - y|$

In addition, we obtain$|f(t, x) - f(t, y)|$

$$\leq \frac{1}{8}|\sin y(t) - \sin x(t)| + \frac{1}{4} \sup_{s\epsilon(o\,,1)}|y(s) - x(s)|$$

$$\leq \frac{1}{8} \sup_{s\epsilon(0,1)}|x(s) - y(s)| + \frac{1}{4} \sup_{s\epsilon(0\,,1)}|y(s) - x(s)|$$

$$= \frac{3}{8} \sup_{s\epsilon(0\,,1)}|x(s) - y(s)|$$

$$= \frac{3}{8}\rho(x, \mathrm{y})$$

The f (t, x) satisfies a Lipschitz condition in the second variable x then the differential equation has a unique solution x(t) satisfying $x(0) = 0$ and $|x(t)| \leq 1$. See Charles .E. Chidume. Functional Analysis and introduction to Metric Space page [172]

3.3 Fixed Points of Dynamical Systems

A dynamical system describes the evolution in time of the state of some system, dynamical system arise as models in many different disciplines, they also arise as an auxiliary too for solving other problem mathematically, and the properties of dynamic systems usually are of different intrinsic with a lot of mathematical interest involved .

A dynamical system is defined by a state space x whose elements are the different state of the system can be in a prescription that relate to the system x_t ϵ x at time t to the state at previous time. We call the dynamical system continuous or discrete depending on whether the time variable is continuous or discrete. For the continuous dynamical system the time t belong to the interval $\mathbb{R}$ and the dynamic of the system is of the first order ordinary differential equation (O, D, E) of the form

$$x^{\Delta} = f\,(t)x$$

where the delta denotes a time derivative and f is a function on x there is little less of generality in assuming this form of equation, for instance the second order none autonomous ordinary differential equation (O,D,E)

$$y^{\Delta\Delta} = (t, y, y^{\Delta})$$

Discrete time consist of a sequence of points t = n to be an integer and the dynamic is defined by $\mathbb{T}$: x = x and relate to the state x_{n+1} at time t = n+.1.

Then the continuation and behavior of the dynamic time usually described by equations relating the value of a variable at one time to the values at adjacent time such equation called difference equation. See Applied Analysis by James .k. Hunter and Bruno Nachtergaele page [63]

$$x_{n+1} = \sigma\,(x_n)$$

.Approaches this state as time tends to infinity starting from the initial state in this case, we say that the fixed point is globally asymptotically stable.

Example 3.1 Dynamic of chemical engineering system process

$$x^{\triangle}\,(x) = -\,(1 + A^2B^2\,X^2) + ABX = 0$$

where $0 < x < 1 \qquad A, B \longrightarrow (0, \infty)$

A, B are perimeter belong to $(0, \infty)$ Standard state

$$= -\,x\,(1 + A^2B^2X^2) + ABX = 0$$

$$x = \frac{ABx}{1+A^2B^2X^2} \;= f\,(x)$$

$$x \in (0, 1) \longrightarrow f\,(x) \in (0, 1)$$

X = f (x) is a proper choice and there is a potential chance that this map can be treated as contraction map and we can identify the condition. If the map is a contraction map then we uses the formula.

$$F\,(u) - f\,(v) = (u - v)\,f^{\triangle}\,(w)$$

where W lies between u and v then the Taylors series expansion

$$\triangle\,(f\,(u) - f\,(v)) = Z \qquad \triangle\,(u, v)$$

$$Z = f^{\triangle}\,(w) = \Delta f\,(x) \qquad \text{where } X = W$$

$$= \frac{AB}{1+A^2B^2X^2} \;-\; \frac{ABx \bullet A^2B^2\; \bullet 2X}{(1+A^2B^2X^2)^2}$$

$$= \frac{AB}{(1+A^2B^2X^2)^2}\,|1 - A^2B^2X^2| \text{ where X=W}$$

$$z = \frac{AB(1 - A^2B^2W^2)}{(1 + A^2B^2W^2)^2}$$

By differentiating again put as max

$$Z^{\triangle}\,(W) = AB\,\left[\,\frac{-\,2A^2B^2W^2}{(1+B^2A^2W^2)} - \frac{(1 - A^2B^2W^2)(-2A^2B^2 2W)}{(1+B^2A^2W^2)^3}\,\right] = 0$$

$$= 1 = \frac{2(1 - A^2B^2W^2)}{1+A^2B^2W^2} \Rightarrow \mathrm{W} = \frac{1}{\sqrt{3AB}}$$

$$\mathrm{Z}_{\max} = \mathrm{AB}\frac{\left(1-\frac{1}{3}\right)}{\left(1+\frac{1}{3}\right)} = \frac{3AB}{8}$$

for f to be a contraction map the condition should be $\mathrm{Z} < 1,\ \mathrm{A},\ \mathrm{B} < \frac{8}{3},$ this is the condition for the existence of unique steady state.

3.3. Fundamental Principle for Industrial Systems

The industrial can be improve with the application of contraction mapping which deals with an effective method using the application of existence and uniqueness of a solution of ordinary differential equation (O, D, E)we will introduce the system of a first order ordinary differential equation with an initial value condition of the form

$$u^{\Delta}(t) = f(t, u(t))$$
$$u(t_0) = u_0$$

The function f (t, x(t)) is used to illustrate the continuous process in time t, where the function will resulted to the picard theorem by integrating the function it will eventually lead to the process of Lipschitz continuous function

$$u(t) = u_0 + \int_{to}^{t} f(t, u(t)) \Delta t$$

by the fundamental theorem of calculus, a continuous solution is an integral equation that is continuously differentiable, and lead to a fixed point equation of the form u = T (u) for the map T defined by

$$T u(t) = u_0 + \int_{to}^{t} f(s, u(s)) \Delta s$$

This condition gives guarantee the uniqueness of a solution T, to be a contraction of a suitable space of continuous functions that satisfies the process that leads to Lipschitz continuity. See Applied Analysis by James .K. Hunter and Bruno Nachtergaele page [61]

Definition 3.2 Suppose that the function f: M X □ ^{n}⟶ □ ^{n}, where f is an interval in □, then we can say that f (t, u (t)) is a globally Lipschitz continuous function of u uniformly in t. If there is a constant c< 0 such that

$$\|f(t\,,u){-}f(t\,,v)\| \leq c\ \|u{-}v\|$$

for all u, v ϵ □ ^{n} and all t ϵ M

Theorem 3.3 Suppose that f: M x □ ^{n}⟶ □ ^{n} where M is an interval in □, and as a point in the interior of M, if f (t, u(t)) is a continuous function of f(t, u(t)) and a globally Lipschitz continuous function of u uniformly in t on M x □^{n}. Then there is a unique continuously differentiable function u •M⟶ □ ^{n} that satisfies.

$$u^{\Delta}(t) = f\,(t, u\,(t))$$

We will show that T is a contraction on the space of continuous functions defined on the time interval $t_0 \leq t \leq t_0 + \boldsymbol{\delta}$ for sufficiently $\boldsymbol{\delta}$. Suppose that

$$U, v: [t_0, t_0 + \boldsymbol{\delta}] \longrightarrow \mathbb{R}^n$$

are two continuous function, then from the above we obtain we estimate

$$\|T(u){-}T(v)\|_\infty = \sup_{t_0\leq t\leq t_0\leq\delta}\|Tu(t){-}Tv(t)\|$$

$$= \sup_{t^0\leq t\leq t_0+\delta}\left\|\int_{t_0}^{t} f(s\;u(s)) -\; f(s\;v(s)\right\|\Delta s$$

$$\leq \sup_{t_0\leq t\leq t_0+\delta}\left\|\int_{t_0}^{t} f(s,u(s)){-}f(s,v(s))\right\|\Delta s$$

$$\leq \sup_{t_0\leq t\leq t_0+\delta}\int_{t_0}^{t}\|f(s,u(s)){-}f(s,v(s)\|\Delta s$$

$$\leq \sup_{t_0\leq t\leq t_0+\delta}\int_{t_0}^{t} c\|u(s){-}v(s)\|\Delta s$$

$$\leq \quad c\boldsymbol{\delta}\|u{-}v\|$$

It follows that if $\boldsymbol{\delta} < \frac{1}{c}$ then T is a contraction on c [t□, t□+$\boldsymbol{\delta}$] therefore there is a unique solution u: [t□, t□+$\boldsymbol{\delta}$] ⟶ □ ^{n} the argument holds for any t□ ϵ M. Moreover, by

covering M with over lapping intervals of length less then $\frac{1}{c}$ we see that the first order (O, D,Es) has a unique continuous solution defined on all of M

3.4. Banach Fixed Point Theorem

A Banach fixed point or Banach contraction theorem is a mapping T: X⟶X of a set X onto itself such that$x\boldsymbol{\epsilon}$ X, such that the image Tx coincides with x.

$$Tx = x$$

The Banach fixed-point theorem is an existence and uniqueness theorem for fixed point of a certain mappings and it gives constructive procedure for obtaining better and better approximations to the fixed point (solution for practical problem). The procedure are called iteration by definition this is a method such that we choose an arbitrary x_0 in a given set and calculated recursively a sequence x□, x□, x□, x□, x□. . . . .From a relative of the form

$$x_{n+1} = Tx_n \quad n = 0, 1, 2, 3, 4.....$$

That if we choose the arbitrary x□ and determine successively x□ = Tx□, x□ = Tx□ x□ = Tx□ and x□ = Tx□ and so on. Iteration procedures can also be use in nearly every Banach of applied mathematics, and convergence proof and error estimates are very often obtain by an application of Banach fixed-point theorem or more other difficult fixed point solutions.

The Banach's theorem gives sufficient condition for the existence and uniqueness of a fixed point for a class of mappings called contraction. See Erwin Kreyszig page [299] Introduction to Functional Analysis with Application.

Definition 3.5 (Contraction)

Let a be mapping T: X ⟶ X of a metric space X = (x, d) be a contraction on X, if there is a positive real number **α** ≤ 1 such that for x, y ϵ X.

$$d(Tx, Ty) \leq \alpha d(x, y) \quad (a<1),$$

Geometrically this means that any point x and y have images that are closer, together than those point x and y more precisely, the ratio

$$d(Tx, Ty)/d(x, y)$$

does not exceed a constant **α**, which is strictly less than 1

3.5. Banach Fixed Point Theorem for Differential Equations

The most applicable form of Banach fixed point theorem yield and existence and uniqueness theorem for the construction of solution of differential equations where the differential equation is often reduce to the existence or location of a fixed point for an operator defined on a set of a space functions. The fixed-point theorem have also been use to determine the existence of periodic solution for functional differential equations when solutions are already known to exist.

apart from the involvement in the theory differential equations, fixed point theorems have also been extremely usefully in such problems as finding zero of nonlinear equations and proving subjectivity theorems partly as a sequence of the importance of it applications fixed point theory has developed in to an area of independent research.

See Introduction to Functional Analysis by Erwin Kreyszig page [314].

Theorem 3.6 (Banach contraction mapping principle) let (x, ρ) be a contraction metric space and let T: X ⟶ X be a contraction map, the T has a unique fixed point in X moreover, for any x□ ϵ X.

3.6. Schauder's Fixed Point Theorem

The Schauders theorem of compactness or relative compactness some time referred to as sequentially compactness and sequentially relative compactness the word compactness or compact set is a subset k of a metric space (x, ρ) said to be compact. If every open covering G= {G$_{\alpha,}\alpha$ ϵ Δ} of K is reducible to a finite sub covering, an open cover of K is a collection of open sets $G_{\alpha,}\alpha \epsilon \Delta$, whose union contain K as a subsets.

A finite sub-cover is a finite subset of $\{G\alpha\}_{\epsilon\,\Delta}$whose union also contains the set K it can be shown that for metric space X the notions sequentially compact and compact are equivalent sequence in K has a subsequence that converges in X. See Eberhard Zeidler Applied Functional Analysis Application to Mathematical Physics page [61].

Theorem 3.7 Every metric space (x, ρ) with a finite number of points is compact.

Proof. Let X $= \{x_1, x_2, x_3. \; . \; . \; . \; . \; x_K\}$ • let {G$_{\alpha,}$ $\alpha\epsilon\Delta$} be an open cover for X then X $=_{\alpha\epsilon\Delta}^{\cup} G\alpha$. However, x_1 ϵ X ⇒ x_1 ϵ Gα, for some α ϵ Δ, Let x_1 ϵ G_{α_1} similarly, let

$x\epsilon$ G_{α_1} 1≤ $\boldsymbol{i}$ ≤ k then X = G_{α_1}, ∪ G_{α_2} , ∪ G_{α_3} ....∪ $G_{\alpha k}$ so that the finite collection $G_{\alpha i}$ $\boldsymbol{i}$ = 1 2 3.............K covers X and so X is compact.

Theorem 3.8 Every closed subset of a compact metric space is compact.

Proof. Let (x, ρ) be a compact metric space and let y be a closed subset of X we want to prove that y is compact we shall show that every open cover of y is reducible to a finite sub- cover. So let {G_α, α ϵΔ, G_α open in X} be an arbitrary open cover for y, then y $\subset_{\alpha\epsilon\Delta}^{\cup} G_\alpha$since y is closed we have y^c is open and X = y ∪ y^c implies

$$X = (_{\alpha\epsilon}{}^{\cup}G\alpha) \cup y^{c}$$

And so {G, $\alpha \epsilon \Delta$} ∪ y ^{c} is an open cover for X and since X is compact then there exist some k such that

$$X = (\overset{\cup}{_{i=1}}G\alpha i) \cup y^{c}$$

Hence, x, y $\subset \overset{\cup}{_{i=1}}\alpha i$, $\alpha \in \Delta$ and this implies that y is compact

Proposition 3.9 The following statement are true

1, A set K is relatively compact if and only if for every **ε**>0 there exist a finite **ε**–net.

2, A set k is compact if and only if it is closed and for every ε>0 there exist a finite ε — net

3, the subset F is the same as the set of all convex combination of finitely many element in F

4, 𝕜 compact set implies that subset K is compact

Theorem 3.10 (Schauder's Fixed Point Theorem) Assume that T : K ⟶ K is a continuous mapping then T has a fixed point that is K is a convex compact set in Banach space X .

Theorem 3.11(Concept of Schauder's Fixed Point) Assume that T: f ⟶ f is a continuous mapping such that T (f) is a relatively compact subset of f, then T has a fixed point that is f is a closed convex set in a Banach space X.

Theorem 3.12 (Krasnoselskiis Fixed Point Theorem). Assuming that S□ and S□ are mapping from F into X and F is a closed bounded convex subset of a Banach space X such that

1, $S_1(x) + S_2(y) \in F$

For all x, y ϵ F

2, S□ is a contraction

3, S□ is continuous and compact then S□ + S□ has a fixed point in F.

Proof.

Assume that the mappings S□, S□ satisfies the theorem in particular there exists a C ϵ (0, 1) such that

$$\|S_1(x) - S_2(y)\| \le c\|x - y\|$$

x, y ϵ F this yield

$$\|(1 - S_1)(x) - (1 - S_1)(z)\|$$

$$\ge \|x - z\| - \|S_1(x) - S_1(z)\|$$

$$\therefore \geq (1-c)\|x-z\|$$

and $\|(1-S_1)(x)-(1-S_1)(z)\|$

$$\leq \|x-z\| + \|S_1(x)-S_1(z)\|$$

$$\therefore \leq (1+c)\|x-z\|$$

Consequently $1-S_1$, F ⟶ $(1-S_1)$F is a homeomorphism and $(1-S_1)^{-1}$ exist as a continuous mapping from $(1-S_1)$(F). Furthermore we note that for each y∊ F the equation x = S□(x) + S□(y) has a unique solution x ∊ F according to Banach fixed point theorem. From this we conclude that S□(y) ∊ $(1-S_1)$(F) for every y ∊ F and also that $(1-S_1)^{-1}$ S□: F⟶F is a well defined continuous mapping. Since S□ is a compact mapping it follows that $(1-S_1)^{-1}$S□: F ⟶F is a compact mapping.

Theorem 3.13 (Schauder's Fixed Point Theorem) Let E, be a real Banach space and let c be a closed convex bounded in a nonempty subset of E, let α: C ⟶ C be a compact map then α has at least one fixed point.

3.7. Local Existence Theorem for Dynamical Systems

The local existence theorem on dynamic system is a continuous dynamic function where the dynamic equation can form an existence of solution, that is

Continuous in an interval (a, b), where a ≠0 the solution y (a) = b exist as a continuous function. Refer to Charles .E. Chidume Applicable Functional Analysis Fundamental Theorems with Applications page [174].

Definition 3.14 Let K denote an interval of the real line ℝ, D be an open set in □ x □^{n} such that n ≥1 then x: K ⟶ □ ^{n} F: D ⊆ K **X** □ ⟶ □ ^{n} are maps such that

$$x^{\Delta}(t) = f(t, x(t)) \quad x(t_0) = x_0 \in \mathbb{R}^n$$

The equation above exist a solution of the form

$$\left.\begin{aligned} x(t) &= x_n(t) \\ x^{\Delta}(t) &= x_n^{\Delta}(t) \end{aligned}\right\} \quad n = 1, 2, 3, \ldots\ldots\ldots.n$$

From the first derivative, it is clear that f (t, x (t)) ∊ $\mathbb{R}^n$, that we understand that the first derivative is in $\mathbb{R}^n$ and the solution is continuous at t to $\mathbb{R}^n$.

Definition 3.15 A solution x(t) of the IVP xxx is a function x: K ⟶ □ ^{n} such that1.x is differentiable.

2. f (t, (t)) ϵ D for all t ϵ K.

3. x^{Δ} (t) = f (t, (t)) for all t ϵ K

4. x (t_0) = $x_0 \in \mathbb{R}^n$

Definition 3.16 Let M be an open subset of □ ⁿ. A map F: M ⟶ □ ⁿ is called a local Lipschitzian if and only if for each ρ ϵ M there exist a neighborhood u of p such that

$$\|f(x_1) - f(x_2)\| \leq L\|x_1 - x_2\| ,$$

For all x□, x□ ϵU_ρ and for some L ≥ 0, then we can say that the local Lipschitzian is in f continuous.

Theorem 3.17 If f ϵ C^1 (M) then f is local Lipschitzian

Proof. f ϵ c^1(M) its implies that for each value of the locally Lipschitz$x$□, x□ ϵ M

$$f(x_1) - f(x_2) = \int_0^1 f^{\Delta}[\lambda x_1 + (1 - \lambda) x_2] \cdot [x_1 - x_2] \Delta\lambda\}$$

Where of course f^{Δ} $[\lambda x_1 + (1-\lambda)\, x_2]$

Is the Jacobian matrix and λ ϵ (0, 1) then if we take p ϵ M arbitrary since M is open choose r > 0 such that B(p, r), it follows that.

$$\|f(x_1) - f(x_2)\| \leq \int_0^1 {}_{y \epsilon B(p,r)}^{max} \|f^{\wedge}(y)\| \cdot \ \|x_1 - x_2\| \Delta\lambda$$

Then since B (p, r) is compact and f^{Δ} is continuous the maximum exist set.

$$L = {}_{y \epsilon B(p,r)}^{max} \|f^{\wedge}(y)\|$$

In addition, the result follows. Considering the following situation, M is an open subset of □ ⁿ n≥1x: K ⟶ □ ⁿ F: M ⟶ □ ⁿare maps such that for t_0 ϵ □, x_0 ϵM

$$x^{\Delta} = f(x), \quad x(t_0) = x_0$$

The solution of the first derivative are embodied in the following two classical theorems

Theorem 3.18 (Cauchy, Lipschitz, Picard local existence theorem and uniqueness theorem). If f is local Lipschitzian then there exist **ε** > 0 and a unique solution x of

$$x^{\Delta} = f(x), \quad x(t_0) = x_0$$

Is defined for all t ϵ K_ε

Where $K_\varepsilon = \{t_0 - \varepsilon, t_0 + \varepsilon\}$.

3.9 Peano's Existence Theore

Assuming if f(x) is to be a continuous function of x then the solution of

$$x^{\Delta}(t) = f(t, x(t)) \qquad x(t_0) = x_0$$

Always exist Peano existence theorem although the solution may not be unique one and the same initial value may give rise to many different solution, in future we will assume that f is a smooth function. If the function f is arbitrarily smooth, then the solution of a nonlinear may fail to exist for all time then if f(x) will grows continuously and fast in nature the linear function f(x) is a global existence.

Theorem 3.19 (Peano local existence theorem) If f is continuous then there existε>0, and a solution x of $x^{\Delta} = f(x)$, $x(t_0) = x_0$, Is defined for all t ϵ $K_\varepsilon$where

$$K_\varepsilon = \{t_0 - \varepsilon, t_0 + \varepsilon\}.$$

Theorem 3.20 (Cauchy Peano) suppose that the function f: $[t_0 - \alpha, t_0 + \alpha]$ x such that the B $(\alpha, \beta) \longrightarrow \mathbb{R}^n$ is continuous and bounded by M $>$ 0 then there exist

$$x(t) = x_0 + \int_{t_0}^{t} f\ (s, x(t))\Delta s$$

Has a solution that defined on $[t_0 - K.\ t_0 + K]$ where k = min$\{t_0 - \alpha\ ,\ t_0 + a]$

Definition 3.21 A sequence of a function g_k: u $\subseteq$ $\mathbb{R}$ $\longrightarrow \mathbb{R}^n$ is uniformly bounded if and only if there exist M $>$ 0 such that $|gk(t)| \leq$ M for every t ϵU.

Definition 3.22 A sequence of a functions g_k: u $\subseteq \mathbb{R} \longrightarrow \mathbb{R}^n$ is a uniform equi- continuous if for every function $\mathcal{E} > 0$ there exist a number $\delta > 0$, such that $|gk(t_1) - g(t_2)| < \mathcal{E}$ for all k $\epsilon\mathbb{N}$ and every t_1, $t_2 \epsilon$ u satisfying $|t_1 - t_2| < \delta$.

See John .K. Hunter, Introduction to Dynamical System page [3-7]

3.8. Bolzano Theorem for Continuous Functions

Bernard Bolzano carried out an experiment on continuous function in the year (1781 – 1848), that became an important contribution to mathematics in the 19th century, he was one of the first to recognize that many obvious statements about

continuous function, his observation concerning continuity were published posthumously in 1850. Bolzano theorem illustrate the function f as continuation between two interval of a function, and show a function that lies below the x –axis at x = a, and above the axis at x = b. Bolzano's theorem illustrate that they must pass true the x – axis between a and b. This property was first publishes by Bolzano in 1817. See Tom .M. Apostol one variable calculus with introduction to linear algebra page [142-143]

Theorem 3.23 A function f said to be continuous at each point of a closed interval [a, b] if and only if there exist a point c, such that f (a) and f (b) have opposite sign in the open interval (a, b) such that f (c)=0.

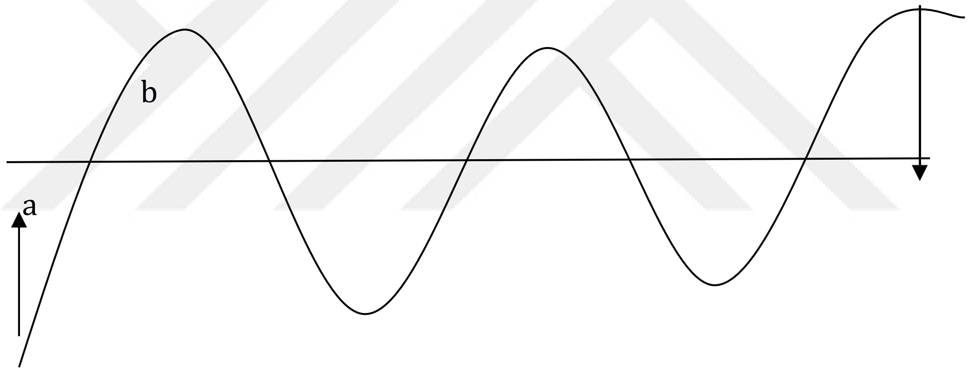
Theorem 3.24 A function f is continuous at point c such that f(c) $\neq$ 0, if and only if there exist an interval (c−δ, c+δ) in which the function f has the same sign as f(c).

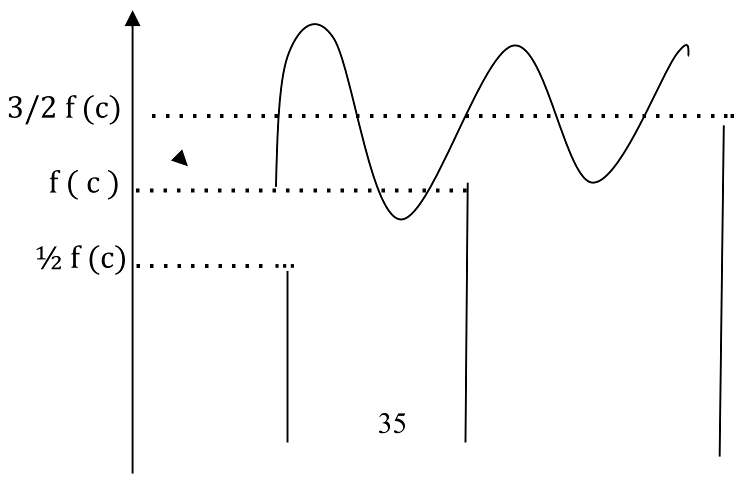
Proof theorem 2.21 suppose f(c) >0 by continuity for each $\boldsymbol{\varepsilon}$ >0 there is an increment δ >0 such that

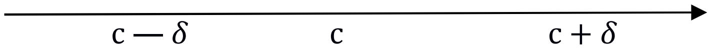
$$f\,(c) - \boldsymbol{\varepsilon} < f\,(x) < f(c) + \boldsymbol{\varepsilon}$$

Whenever $c - \delta < x < c + \delta$

Taking δ to corresponding with the value of the equation $\boldsymbol{\varepsilon} = \frac{f(c)}{2}$ This is positive $\frac{1}{2}f\,(c) < f(x) < \frac{3}{2}f\,(c)$







Here f (x)>0 for x near t because f (c)>0

Figure 3.1 Bolzano continuity theorems

Secondly by finding the small value of x where the f (x) = 0, then we will denote s such that of all those points x in the interval [a, b], for which f (x) ≤0. There exist one point in s such that f(a) < 0 therefore s is a nonempty set also s is bounded above, since there are possibilities that s lies within [a, b] so s has a supremum Let c = sup s there are only three possibilities.

$$f(c) > 0,\ f(c) < 0, \text{ and } f(c) = 0\ ,$$

but we will concentrate on f (c)>0 which has an interval

$$(c-\delta, c+\delta) \text{ or } (c-\delta, c]$$

if c = b then we can say f is positive therefore no point s can be at the right hand side of c−δ and hence c −δ is upper bound for the set S , but c −δ<c and c is the least upper bound of S. Therefore, the inequality for f (c)>0 is impossible. If f (c) <0 there is an interval

$$(c-\delta, c+\delta) \text{ or if } [c, c+\delta)$$

If c= a then we can at the same time say the function f is negative. Where f (c) <0 for some x>c contradicting the fact that c is an upper bound for s. Therefore f (c) < 0 is an impossible factor, and the only remaining possibility is f (c) = 0 also a<c<b because f (a) <0 and f (b)>0 this prove Bolzano's Theorem.

3.9. Numerical Approach and Applications of Picard Lipschitz Continuity for Industries

The numerical method to a forward difference approach which take an estimate of improvement and application to the Picard and Lipschitz continuity. Which analyses the progress of the forward difference approach to the industrial sector, we consider the behavior of Delta in numerical form of equation where the constant does change with change in the input system, the numerical equation involved the computing of discrete set of Δy_k value of argument using the forward difference approach to determine the argument.

$$\Delta y_k = y_{k+1} - y_k$$

Where there is a change in the input

$$\| y_{k+1} - y_k \|$$

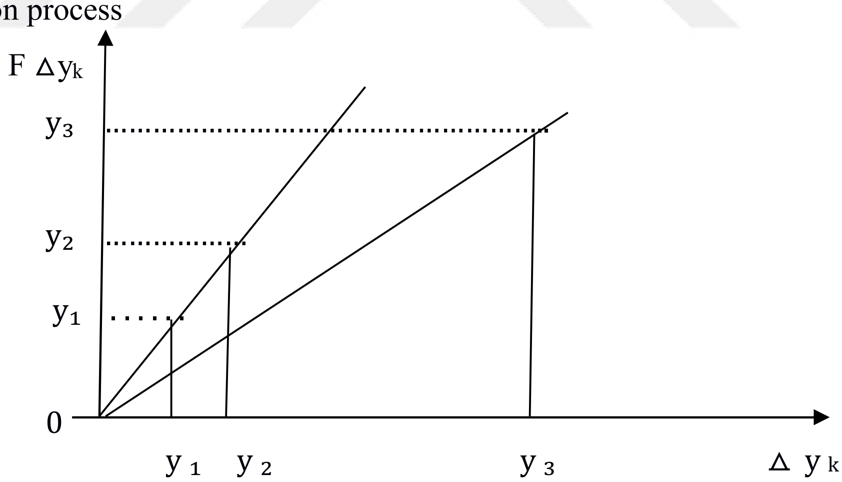
Which correspond to the change in the output?

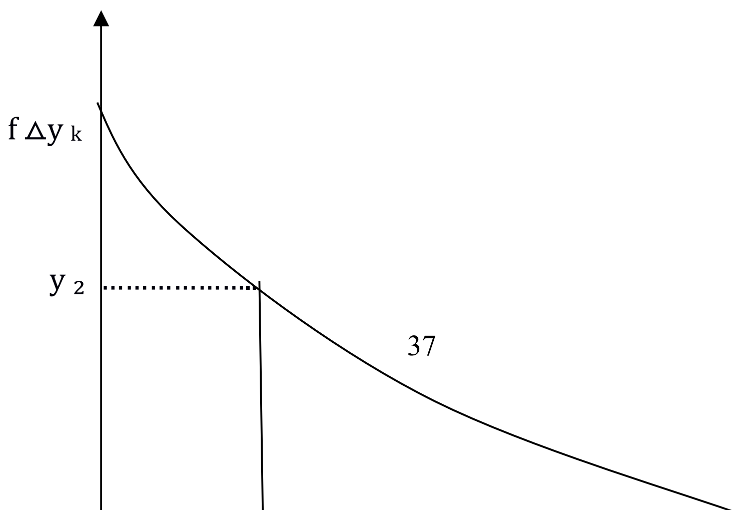
$$\| x_{k+1} - x_k \|$$

With an absolute change in the constant

Furthermore, the absolute value of the change in input is proportional to the absolute value of the change in output with the proportionality in constant equal to the slope m. If we reduce the change and make it small in input then the output will also be smaller and M will reduce and satisfies the proportionality in the input and output, which satisfies the Lipschitz condition for continuity

Which illustrate the various processes needed to apply in the industrial sector that will improve the system, if there is an increase or change in input it will also increases the output and there will be slight change in m, which will increase the global production process





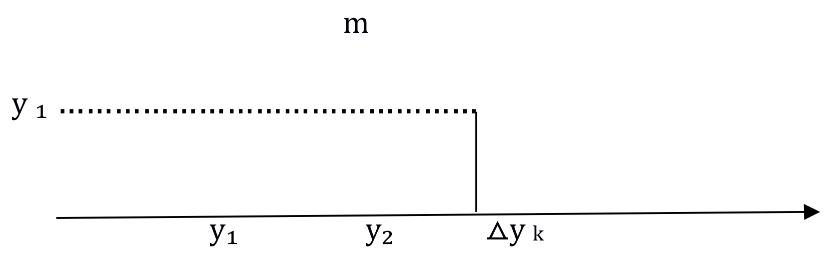


Figure 3.2 Lipschitz continuity theorems

$$\| f \Delta y_k \| = M \| \Delta y_k \|$$

$$\| f (y_{k+1} - y_k) \| = M \| y_{k+1} - y_k \|$$

$$\| f (y_{k+1}) - f (y_k)\| = M \| y_{k+1} - y_k \|$$

The picard iteration y_{k+1} take one estimate of a solution to a better estimate of solution for the k^{th} iteration the desired solution will satisfies $f(y_{k+1}) = y_k$ is a fixed point, the Lipschitz condition, help to prove convergence and uniqueness of a fixed point in a complete space.

Theorem 3.25 Consider the dynamic equation

$$y_{k+1}^{\Delta} = f (t, y_k), \quad y_{k+1} (x_o) = y_o$$

Suppose that $f (t, y_k)$ and y_{k+1}^{Δ} are continuous in some region around the point (x_o, y_o) then there is a unique solution to the initial value problem.

$$y_{k+1}^{\Delta} = f (t , y_k) \;\; y_{k+1}(x_o) = y_o \text{ where } k = 1,2,3.............n$$

$$= \int_{x_o}^{x} \frac{\Delta y k+1}{\Delta t} \Delta t \;=\; \int_{x_o}^{x} f(t , y_k) \Delta t$$

$$= \int_{x_o}^{x} \Delta y_{k+1} = \int_{x_o}^{x} f(t , y_k) \Delta t$$

$$= y_{k+1}(x) - y_k (x_0) = \int_{x_o}^{x} f \;(t , y_k) \Delta t$$

$$= y_{k+1} = y_0 + \int_{x_o}^{x} f \;(t , y_k) \Delta t$$

Putting $k = 0$

$$= y_{0+1} = y_0 + \int_{x_o}^{x} f\ (t\ , y_0)\ \Delta t$$

$$= y_1 = y_0 + \int_{x_o}^{x} f\ (t\ , y_0)\ \Delta t$$

$$= y_{k+1} = y_0 + \int_{x_o}^{x} f\ (t\ , y_k\ )\ \Delta t$$

Putting k = 0

$$= y_{0+1} = y_0 + \int_{x_o}^{x} f\ (t\ , y_0)\ \Delta t$$

$$= y_1 = y_0 + \int_{x_o}^{x} f\ (t\ , y_0)\ \Delta t$$

Putting k = 1

$$= y_{1+1} = y_0 + \int_{x_o}^{x} f\ (t\ , y_1\ )\ \Delta t$$

$$= y_2 = y_0 + \int_{x_o}^{x} f\ (t\ , y_1\ )\ \Delta t$$

Putting k = 2

$$= y_{2+1} = y_0 + \int_{x_o}^{x} f\ (t\ , y_2\ )\ \Delta t$$

- -

$$= y_{n+1} = y_0 + \int_{x_o}^{x} f\ (t\ , y_n\ )\ \Delta t$$

$$\sum_{i=1}^{n} y_{i+1} = y_0 + \sum_{i=1}^{n} \int_{x_0}^{x} f(t, y_i)\ \Delta t$$

Lipschitz condition

$$|y_{k+1} - y_k\ | \leq \int_{x_o}^{x} f\ (t\ , y_k)\ \Delta t$$

$$\leq \int_{x_o}^{x}\ |f(t,\ y_{k+1}(t)) - f(t,\ y_k(t)\ )\ |\Delta t$$

$$\leq \int_{x_o}^{x} M\ |\ y_{k+1} - y_k\ \ |\Delta t$$

$$\leq M\ |y_{k+1} - y_k\ |$$

$$= \|f(y_{k+1}(x)) - f(y_k(x_o)\ \| \leq M\|\ y_{k+1} - y_k\|$$

3.12 Bellman Gronwall Inequality for Industies

Suppose a non negative continuous function V_o : □⟶□ satisfies

$$V_{k+1}(t) = V_0 + \int_{t_o}^{t} f(x\,, y_k)\,\Delta x \quad \text{where} \quad V_{k+1}(t) = V_o e^{MT}$$

Then the nonnegative function bounded from above by a (constant time) then the area covered so far will have exponential growth. Introduced by Carmen Chicone page [128] Ordinary Differential Equations with Applications. And Basic theory of Ordinary Differential Equations by Po-fang Hsieh and Yasutaka Sibuya **Lemma 3.2** page [398]

Proof.

Then by applying the Bellman Gromwell inequality to determined the existence of local Lipschitz for uniqueness of the solution

$$x_{k+1}(t) = x_o + \int_{t_o}^{t} f(x(t))\,\Delta t$$

$$y_{k+1}(t) = y_0 + \int_{t_o}^{t} f(y(t))\,\Delta t$$

$$= \| x_{k+1}(t) - y_{k+1}(t) \|_2 \le \| x_0 - y_0 \| + \int_{t_o}^{t} \| f(x(t) - f(y(t) \| \,\Delta t$$

Applying Lipschitz condition

$$= \| x_{k+1}(t) - y_{k+1}(t) \|_2 \le \| x_0 - y_0 \|_2 + M \int_{t_o}^{t} \| f(x(t) - f(y(t)) \| \,\Delta t$$

$$= \| x_{k+1}(t) - y_{k+1}(t) \|_2 \le \| x_0 - y_0 \|_2 + M \int_{t_o}^{t} \| x(t) - y(t) \|_2 \,\Delta t$$

Applying Bellman Gromwell inequality

$$= \| x_{k+1}(t) - y_{k+1}(t) \|_2 \le \| x_0 - y_0 \|_2 \, e^{MT}$$

If$x_0 = y_0$

Then $\| x_{k+1}(t) - y_{k+1}(t) \|_2 = 0$

for t interval guaranty the existence and locally Lipschitz neighborhood suppose

$$\| x_0 - y_0 \|_2 = 0.1 \Rightarrow \| x_{k+1}(t) - y_{k+1}(t) \|_2$$

Is similarly small so

$$\| x_{k+1}(t) - y_{k+1}(t) \|_2 \le 0.1\, e^{MT}$$

It means that the locally Lipschitz grow gradually and guaranty the stability of the system and boundedness, global existence and uniqueness of the locally Lipschitz also assist the sector to improve in production, which requires anincrease in thelabour force as well as time increase and reduce the global challenges in the economical sector.

CHAPTER 4

DYNAMIC BEHAVIOUR FOR FIRST ORDER SYSTEM

We consider the behavior of first order equation from the initial condition to the transfer function and finally to the dynamic time response of the system. We will consider the process of a step response to determine the terms of the system.

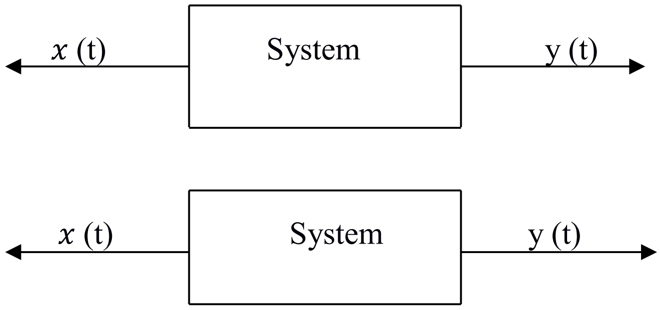


Figure 4.1 Transfer function

4.1. Transfer Function Representation

The term T is the time constant while K is the steady state gain, the characteristic for a first order system depend on both parameters and the condition of the process, and that the transfer function does not contain the initial condition explicitly. See Karl Johan Astrom and Richard .M. Murray Feedback system an introduction for scientists and engineer page [235]

4.2. Classification of Transfer Function

The process whereby time is measured does not adequately have any effect in the adjustment of the change in the input. Karl Johan Astrom and Richard .M. Murray page [235]

4.2.1. Steady State Gain in Transfer Function

This is the process of the steady change output divided by the sustained change in the input; this eventually characterized the sensitivity of the output system to the change in the input. Obviously the important aspect of the transfer function is that it is used

to determined the process of calculating the steady state of the change in the output To the steady state in the input system, the steady state input is denote by X (t), X(s) and the output to Y (t), Y (s) respectively. Then the steady state gain k calculated as follows.

$$k = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

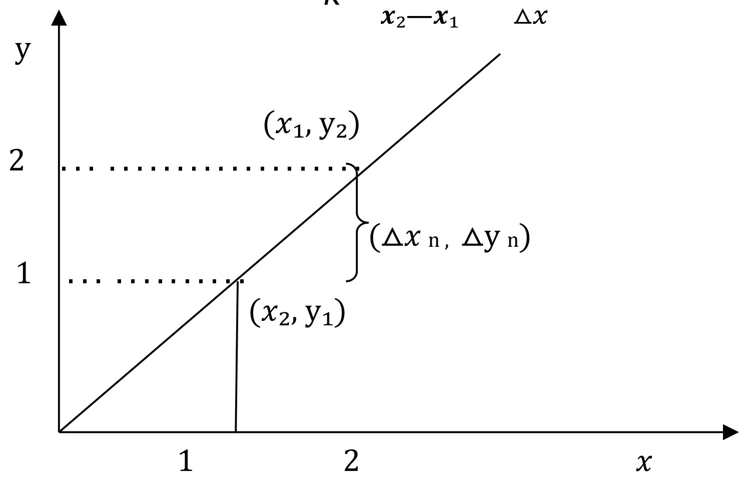


Figure 4.2 Steady state gains

Where the steady state function can be, classify as the change in Δy to change in Δx. The output properties of the transfer function depend on the order in the polynomial for the differential equation of the form.

$$a_n Y^{\Delta n}(t) + a_{n-1} Y^{\Delta n-1}(t) + a_{n-2} Y^{\Delta n-2}(t) + \text{- - - -} + a_1 Y^{\Delta}(t) + a_0 Y =$$

$$b_m X^{\Delta m}(t) + b_{m-1} X^{\Delta m-1}(t) + b_{m-2} X^{\Delta m-2}(t) + \text{- - - - -} + b_1 X^{\Delta}(t) + b_0 X.$$

Where the X and $Y$are input and output variables in the derivative respectively then the transfer function obtained becomes

$$G(s) = \frac{Y(S)}{X(s)} = \frac{\sum_{i=1}^{\infty} bis^i}{\sum_{i=1}^{\infty} ais^i}$$

$$= \frac{bmS^m + bm-1\ \ ^{m-1} + bm-2S^{m-2} + \ldots\ldots\ldots + b0}{anS^n + an-1S^{n-1} + an-2\ \ ^{n-2} + \ldots\ldots\ldots\ldots + a0}$$

4.2.2. Characteristic of Dynamic System

1, the first order process is a self regulating state which reaches a new steady state in the transfer function.

2, the ultimate value of the output system is K, then for a unit step change the input System is KM for a step of a size M, the steady state or static gain in a given parameter K, since for any change in the input there is a resulting change in the output system the steady state illustrated as follows

$$\triangle \text{ (output)} = K \triangle \text{ (input)}.$$

The condition enable us to understand that by how much the value can be change in the input in order to achieve a desired change in the output for a process with given constant K. In order to effect a change in the output we need a small change in the input if K is small then the system will grow faster.

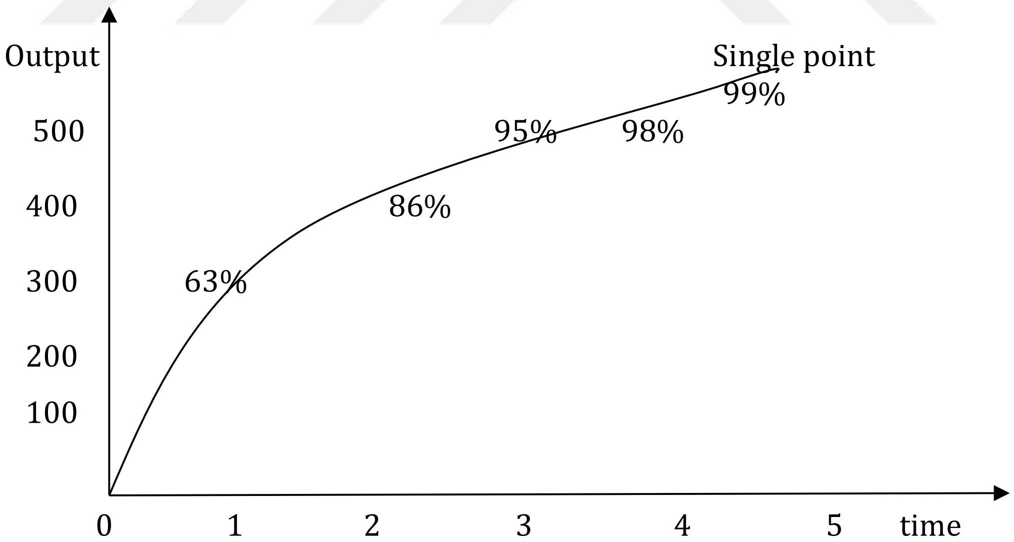
3, the slope is considered between t =0 and t = 1, which indicate steady state change in time

$$y^{\triangle}(t)/XP \,\Big|_{t=o} = (e^{\frac{-t}{T}})_{t=0} = 1$$

The condition implies that there is initial value rate of change y(t) were it can be maintained and the response will increase to reach it final value in one time constant.

4.2.3. Transient Function

Transient function can be characterize as the rise in time and how quickly the response in the time input is implemented in terms of getting closer to the estimated time expected. The usual choice for rise time, at least for first order system is the time it takes the output to increase from 64% of the initial value to its 99% of its final value. The rise in time is determining by the step response.



Fiure 4.3 Transient Response always one row

4.1.6 Step Response

This is the response in the dependent variable y, to a step change (persistent change) in the independent variable u, the change in u is

$$U(t) = \begin{cases} U_o & t \leq t_o \\ U_\infty = u_o + \triangle u & t > t_o \end{cases}$$

Where $\triangle u$ is the magnitude of the step, the static gain will be finding by dividing the difference between the steady state values of the output by the difference between the steady state values of the input. It is also classify as the change from one transfer function to another transfer function. See Chemical and Energy Process Engineering by Sigurd Skogestad page [285]

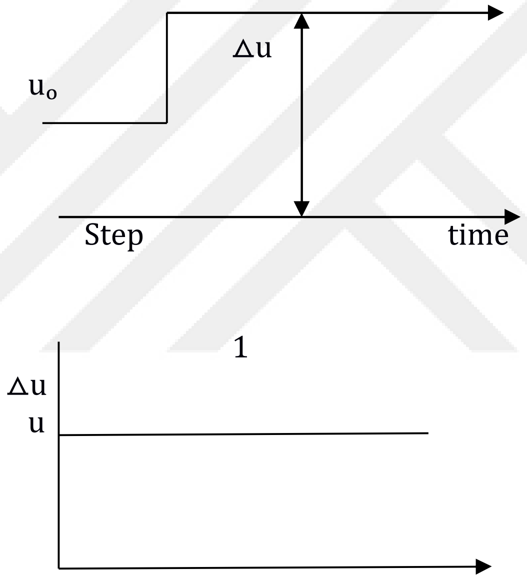


Figure 4.4 Step response Time

4.2.4. Frequency Response

(Sine wave response or sinusoidal response) this is the resulting response in y, to a persistence sinusoidal variable in the independent variable u.

$$U(t) = u_0 + \triangle u.\sin(\boldsymbol{\omega} t + \theta)$$

for small change, we can assume that the system is linear, and the output sign are sinusoidal with the frequency w

$$Y(t) = y_0 + \triangle y.\ \text{Sin}(\boldsymbol{\omega} t + \theta)$$

The frequency response where characterized into two parameters. The gain $\frac{\Delta y}{\Delta u}$ and the phase shift θ. However, depending on the frequency [rad/s], and by varying the frequency $\boldsymbol{\omega}$, we can get information on how the system reacts to quick ($\boldsymbol{\omega}$ large) and slow ($\boldsymbol{\omega}$ small) the input variation frequency analysis is an important tool in controlling the process. See Sigurd Skogestad page [285]

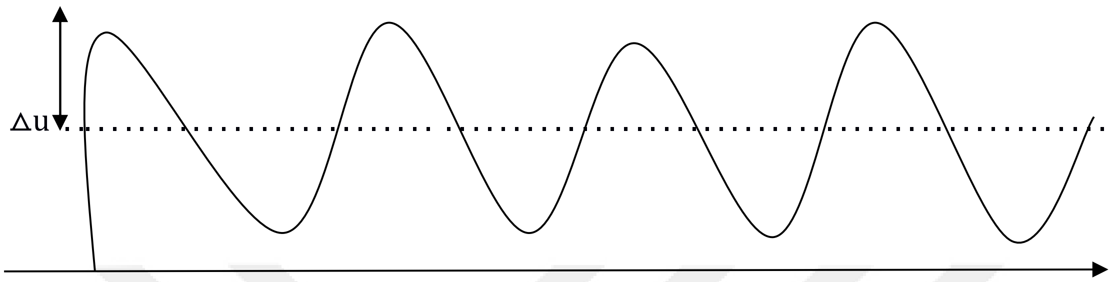


Figure 4.5 Sinusoidal Step Response Time

4.2.5. Prbs Response (Pseudo Random Binary Sequence)

The pseudo random binary sequence use the response in y when the independent variable u change at random time between two given values, which will give a good dynamic distribution and is sometimes an effective method for obtaining experimental data that can be used for estimating better production system identify and control the process. See Chemical and Energy Process Engineering by Sigurd Skogestad page [285]

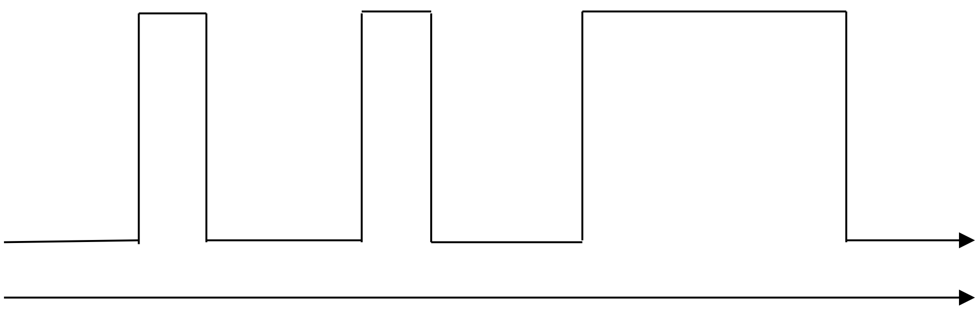


Figure 4.6 PRBS Response time

4.2.6. Impulse Response

The impulse function is defined as the limit of pulse of duration in T, or the change of the independent variable T, if the duration is very short and a amplitude $\frac{1}{T} as$T approaches zero, and is use to characterized the response of the system to brief transient inputs the impulse may be considered as the derivative of the step function. If the transfer function of the system is given by H(s) then the impulse response of a system is given by h(t) where h(t) is the inverse Laplace transfer of H(s). Then it is also characterized as the derivative of the step response h(t). Where the characteristic response

$$h\,(t) = \frac{\Delta h}{\Delta t} = (1 - e^{-\frac{t}{T}})$$

$$= \frac{1}{T} e^{-\frac{t}{T}}$$

Karl Johan Astrom and Richard .M. Murray Feedback System an Introduction for Scientists and Engineer page [235] and Chemical and Energy Process Engineering by Sigurd Skogestad page [285]

The characteristic response is an exponential decay

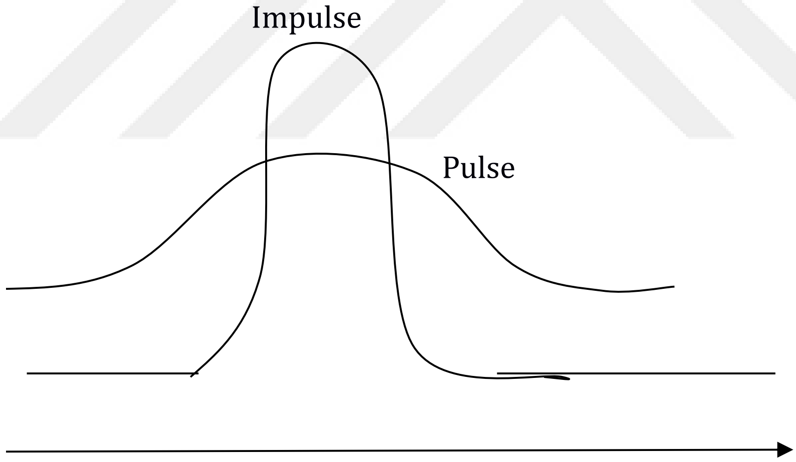


Figure 4.7 Pulse / impulse response Time

4.2.7. Ramp Response

The ramp response $\frac{1}{S^2}$ in a transfer function that grows with time without limit, such

system that grows without limit for a sustained change in input will said to have no regulation. Where the characteristic response is

$$C\,(s) = \frac{1}{S^2\,(1+ST\,)}$$

Where the partial fraction taking the inverse Laplace transform of the equation

$$\mathrm{C(t)} = 1 - \mathrm{T} + \mathrm{T}e^{-\frac{t}{T}}$$

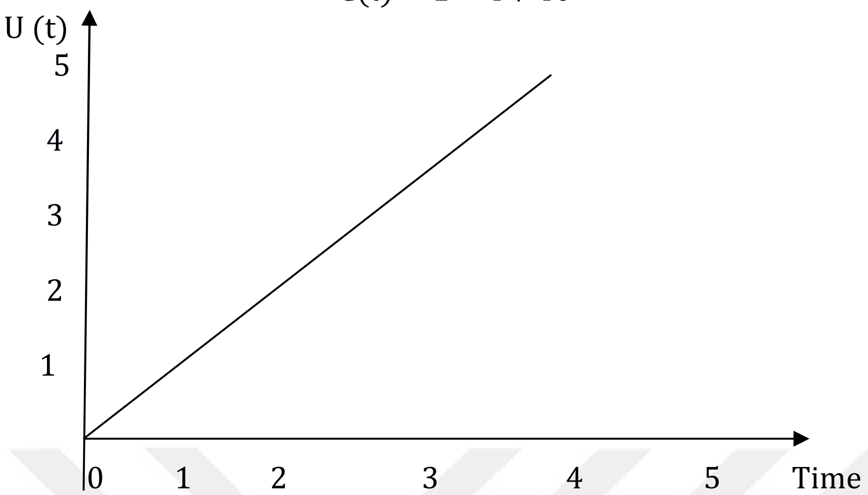


Figure 4.8 Ramp Response

4.3. First Order Systems

The first order system occurs as the result of time steady state of production in the process of the dynamic equation and transfer function for a first order system are

$$\mathrm{Ty}^{\triangle}\,(\mathrm{t}) + \mathrm{y}\,(\mathrm{t}) = \mathrm{k\,p}\,x\,(\mathrm{t})$$

$$\mathrm{G}\,(\mathrm{s}) = \frac{Y(s)}{X(s)} = \frac{KP}{1+ST}$$

The step response is monotonic with it maximum slope at the time of the step and the Time reach an initial state of a change is one time constant then the final steady state change, is equal to KP(Δx).

Step response $\mathrm{y}^{\triangle}\,(\mathrm{t}) = \mathrm{K\,p}\,(\Delta \mathrm{x})\,(1 - e^{-\frac{t}{T}})$

An impulse input occurs over a negligible time and transfer a finite amount into the system. For instance rapidly introducing a small amount of time into the industrial sector will emulates a perfect impulse, the impulse response will shows an immediate increase at time of impulse and change the output system.

4.4. Second Order for Damped System

To determined a second order ODEs of a damped system with an introduction of a characteristic roots approach system

$$M\, y^{\Delta\Delta}(t) + C\, y^{\Delta}(t) + K\, y = 0$$

Assuming the damping force is proportional to the change in time, this is generally a good approximation for a small change in time, where c represents the damping constant. The roots of the characteristic polynomial which is used to determined the exponents of the time domain output function will be

$$\lambda^2 + \frac{C}{M}\lambda + \frac{K}{M} = 0$$

By introducing the formula of the roots of a quadratic equation, we obtain the system to be

$$\lambda_1 = -\alpha + \beta \qquad \lambda_2 = -\alpha - \beta$$

where$\alpha = \frac{C}{2M}$ and $\beta = \frac{1}{2M}\sqrt{C^2 - 4km}$

We can now be interesting that depending on the small amount of damping or little damping system the following are involved.

Table 4.1 Damped system

case	$\lambda_{1,2}$	Natural	Response
1: $C^2 > 4km$	$-\alpha \pm \frac{1}{2M}\sqrt{C^2 - 4km}$	Real and distinct	Over damping
2: $C^2 = 4km$	$-\frac{C}{2m}$	Real and response	Critical damping
3: $0 < C^2 < 4km$	$-\alpha \pm j\beta$	Complex conjugate	Under damping
4: $C^2 = 0$	$.\pm \frac{c}{2m}$	indefinitely	Un-damping

When the damping coefficient is equal to zero the system is un-damped, and when the damping coefficient is less than zero and greater than one, it means that the term is under damped, and also when the damped coefficient is equal to 4km, it is said to be critical damped. In addition, when the damped coefficient is greater than one, then the system will be over Damped, the roots of the characteristic polynomial are indefinitely complex conjugate, real and distinct finally the system has a damping

coefficient of about 4km, which indicates real repeated roots this type of system is term as critical damped.

Two entries are given above for second order system four is for an over damped system and the other is for an under damped system. The step response for an over damped system initially at steady state is monotonic with an initial slope of zero and an inflection point note that the under damped system experiences periodic behavior even for this simple input system.

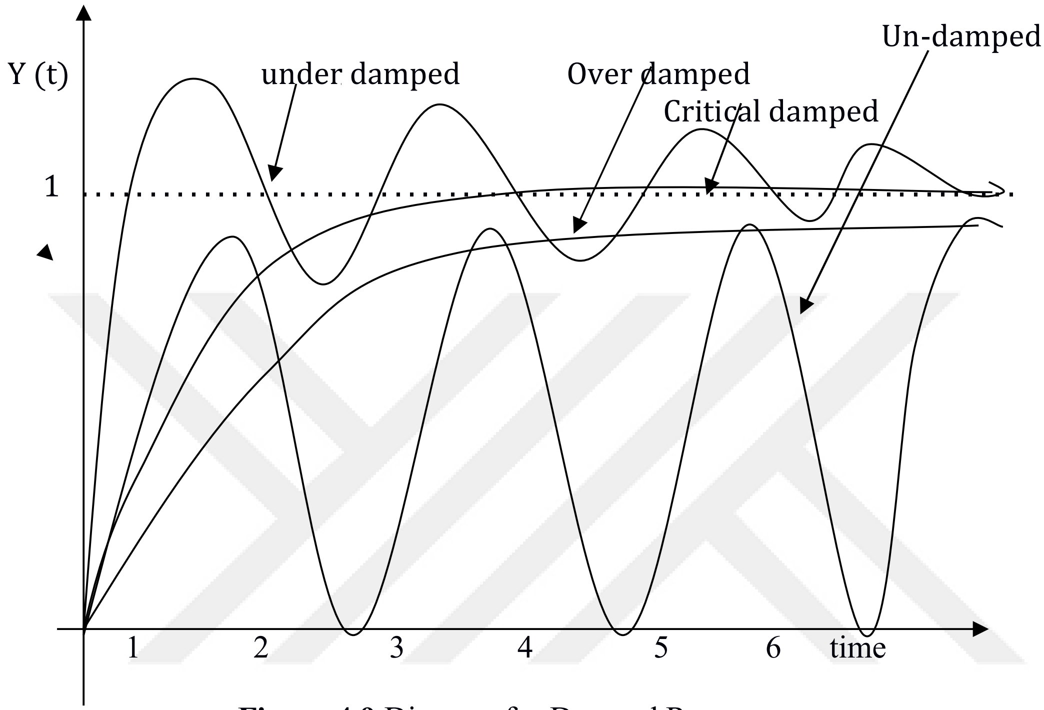


Figure 4.9 Diagram for Damped Response

The critical damped system is positive with time, while for the un-damped is equal to zero, also the over damped coefficient is negative, and the under damped below one and above one respectively. As we can see from the above diagram we understand that the critical damped obtain a steady state gain and grow gradually from zero satisfying the necessary condition, while the over damped grow more than expected.

CHAPTER 5

DYNAMIC MODLLING FOR IMPROVEMENT IN THE INDUSTRIAL SECTOR

We use the mathematical modeling to solve the physical system. Such as improving the dynamic procedure in production process in the industrial sector, specifically we will also introduce the dynamic equation on time scale and to uses the Laplace transform to solve the dynamic equation, where eventually we will introduce an alternative modeling system as follows.

1: the transfer function model.

2: introducing the block diagram.

3: introducing the time response analysis of the system.

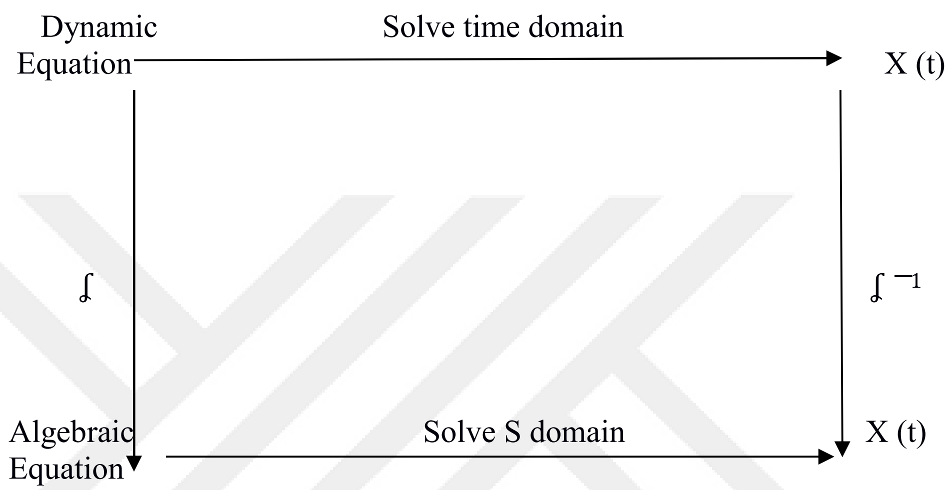


Figure 5.1 Block diagram

The non homogeneous linear system with constant coefficient of first and second order equations will be used to determine the stability and continuous in time and maintain a standard system approach for the production process, we shall defined the state to be those sets of variables that summarize the past of the system for the purpose of predicting its future. Systems described in this manner we will referred to dynamic equation on time scale.

4.5. First and Second Order Stability System

The evolution of a dynamic time system written in the form

$$x^{\Delta}(t) + x(t) = f(t) \qquad \text{where } t = 0,\ x = 0 \text{ initial condition}$$

Refer to Advance Engineering Mathematics, by K.A. Stroud page [70 - 80] by reducing the dynamic time system into Laplace transform we will then introduce transfer function.

$$L(x) = x_1 \quad , \quad L(x^{\triangle}) = \mathrm{p}x^{\triangle} - x_{o}$$

We implies $(\mathrm{p}x^{\triangle} - x_{o}) + x^{\triangle} = \frac{1}{p}$

By introducing the initial condition we obtain

$$(\mathrm{p}x^{\triangle} - 0) + x^{\triangle} = \frac{1}{p} \quad , \quad \mathrm{p}x^{\triangle} + x^{\triangle} = \frac{1}{p}$$

$$\frac{(p+1)}{p+1} x^{\triangle} = \frac{1}{p\,(p+1)} = \frac{1}{p\,(p+1)}$$

$$x^{\triangle} = \mathcal{L}^{-1}\left\{\frac{1}{p\,(p+1)}\right\} = \frac{A}{p} + \frac{B}{p+1}$$

$$1 = A(p + 1)(p) + B(p)(p + 1)$$

$$1 = A(p + 1)(p), \quad 1 = A(0 + 1)(0)$$

$$1 = A \quad \therefore A = 1$$

$$1 = B(p)(p + 1)$$

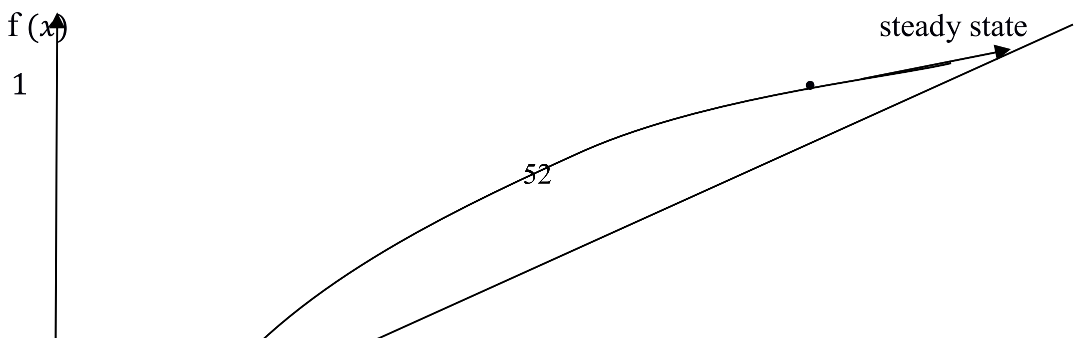
$$1 = B(-1)(-1 + 1) \quad 1 = -B \quad \therefore B = -1$$

$$x^{\triangle} = \left\{\frac{1}{p} + \frac{-1}{p+1}\right\} = \left\{\frac{1}{p} - \frac{1}{p+1}\right\}$$

$$x^{\triangle} = 1 - e^{-t}$$

Table 5.1 Date for first order system

t	e^{-t}	f (x)
0	1	0
1	0.36	0.63
2	0.14	0.86
3	0.05	0.95
4	0.02	0.98
5	0,007	0.99
∞	0	1



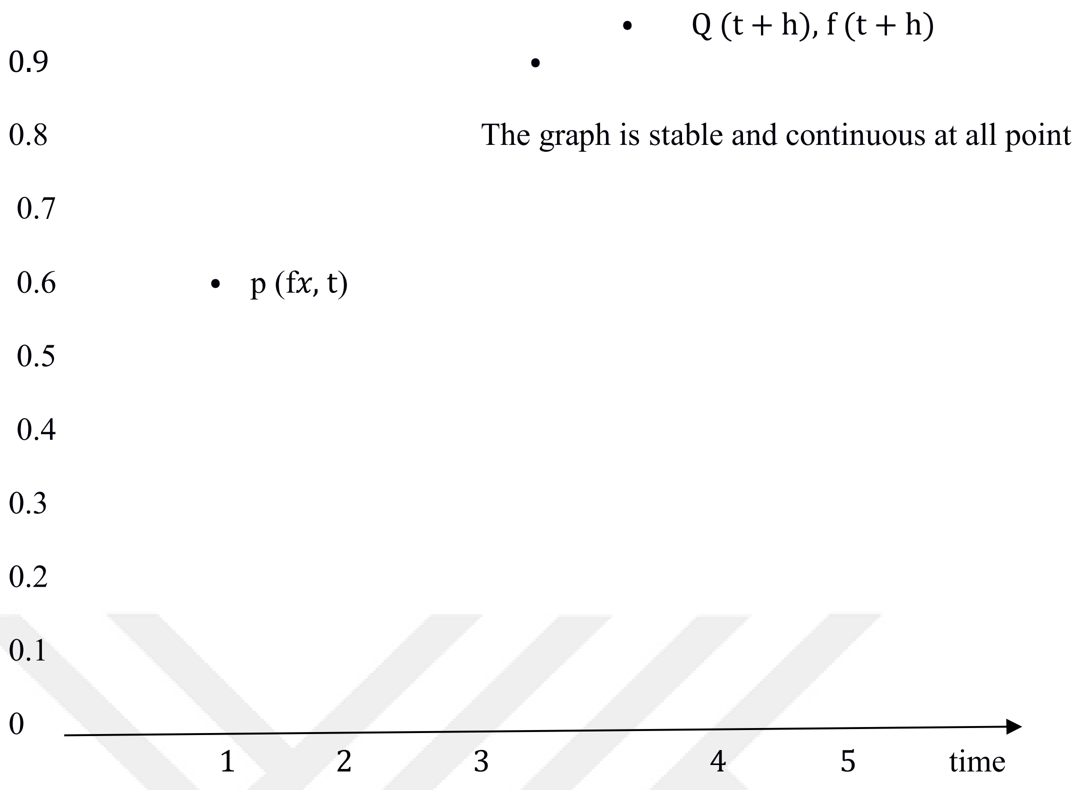


Figure 5.2 Stability diagram for first order system

Then since t –coordinate of Q is t + h then f (x) – coordinate of Q must be f (t + h) by the definition of slope then the line p Q will have slope

$$\frac{f\,(t+h)-f\,(t)}{t+h-t} = \frac{f\,(t+h)-f\,(t)}{h}$$

Since the coordinate of the point p (t, f (t)) then f is continuously passing through p has a slope $\lim_{h\to 0}\frac{f\,(t+h)-f\,(t)}{h}$

Therefore there exist a steady state of production process where there is a change in time, eventually increase the production process and also enhance more labour force which will increase and solve the global problem of the industries.

Second Order

$$x^{\triangle\triangle}(t) + x^{\triangle}(t) + x\,(t) = f\,(t)$$

Where the initial condition of the system t = 0, x_o =0 and x□ = 0. Then the transfer function will be

$$L\,(x) = x_1 \qquad L\,(x^{\triangle}) = px^{\triangle} - x_o \qquad L\,(x^{\triangle\triangle}) = p^2x^{\triangle} - px_o - x_1$$

$$(p^2x^{\triangle} - px_o - x_1) + (px^{\triangle} - x_o) + x^{\triangle} = \frac{1}{p}$$

$$p^2 x^{\Delta} - 0 - 0 + p x^{\Delta} - 0 + x^{\Delta} = \frac{1}{p}$$

$$p^2 x^{\Delta} + p x^{\Delta} + x^{\Delta} = \frac{1}{p}$$

$$(p^2 + p + 1) x^{\Delta} = \frac{1}{p}$$

$$\frac{(p^2 + p + 1)}{p^2 + p + 1} x^{\Delta} = \frac{1}{p(p^2 + p + 1)}$$

$$= x^{\Delta} = \mathcal{L}^{-1}\left\{\frac{1}{p\,(p^2 + p + 1)}\right\}$$

$$= \frac{A}{p} + \frac{BS + C}{p^2 + p + 1}$$

By equating the coefficient of like power of S we obtain

$$1 = A\,(p^2 + p + 1) + BS + C\,(p)$$

$$1 = Ap^2 + A\,p + A + Bp^2 + Cp$$

$$0 = Ap^2 + Bp^2 \quad \therefore 0 = A + B \ldots \ldots\ldots \ldots eq\,1$$

$$0 = A\,p + Cp, \quad \therefore 0 = A + C \ldots\ldots\ldots\ldots eq\,2$$

$$1 = A, \quad \therefore 1 = A \ldots\ldots\ \ldots\ldots\ldots eq3$$

Putting A = 1 into equation 2 we obtain

$$0 = A + C0 = 1 + C \quad \therefore C = -1$$

Putting A = 1 into equation 1 we have

$$0 = A + B, \quad 0 = 1 + B \quad \therefore B = -1.$$

Therefore $A = 1, \ B = -1, \ C = -1$

Since $$\frac{1}{p} + \frac{-1p - 1}{p^2 + p + 1} = \frac{1}{p} - \frac{p - 1}{p^2 + p + 1}$$

By introducing the completing, the square in the second term gives

$$p^2 + p + 1 = (p + \tfrac{1}{2})^2 + (\tfrac{\sqrt{3}}{2})^2$$

$$= \frac{p - 1}{p^2 + p + 1} = \frac{p - 1}{\left(p + \frac{1}{2}\right)^2 + (\frac{\sqrt{3}}{2})^2}$$

$$\frac{p - 1}{\left(p + \frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{p + \frac{1}{2} - \frac{3}{2}}{\left(p + \frac{1}{2}\right)^2 + (\frac{\sqrt{3}}{2})^2}$$

$$= \frac{1}{p} - \frac{p + \frac{1}{2}}{\left(p + \frac{1}{2}\right)^2 + (\frac{\sqrt{3}}{2})^2} - \frac{\sqrt{3}\ X \frac{\sqrt{3}}{2}}{\left(p + \frac{1}{2}\right)^2 + (\frac{\sqrt{3}}{2})^2}$$

$$= \frac{1}{p} - \frac{p + \frac{1}{2}}{\left(p + \frac{1}{2}\right)^2 + (\frac{\sqrt{3}}{2})^2} - \frac{\sqrt{3}\ X \frac{\sqrt{3}}{2}}{\left(p + \frac{1}{2}\right)^2 + (\frac{\sqrt{3}}{2})^2}$$

$$x^{\Delta} = 1 - e^{-\frac{t}{2}} \cos\frac{\sqrt{3}t}{2} - \sqrt{3}e^{-\frac{t}{2}} \sin\frac{\sqrt{3}t}{2}$$

Table 5.2 Date for second order system

t	$e^{-t/2}\cos\frac{\sqrt{3}}{2}$	$\sqrt{3}e^{-\frac{t}{2}}\sin\frac{\sqrt{3}}{2}$	Result
0	1	0	0
1	0.18	0.05	0.87
2	0.37	0.013	0.64
3	0.02	0.0012	0.98
4	0.14	0.007	0.87
5	0.003	0.0002	1.00

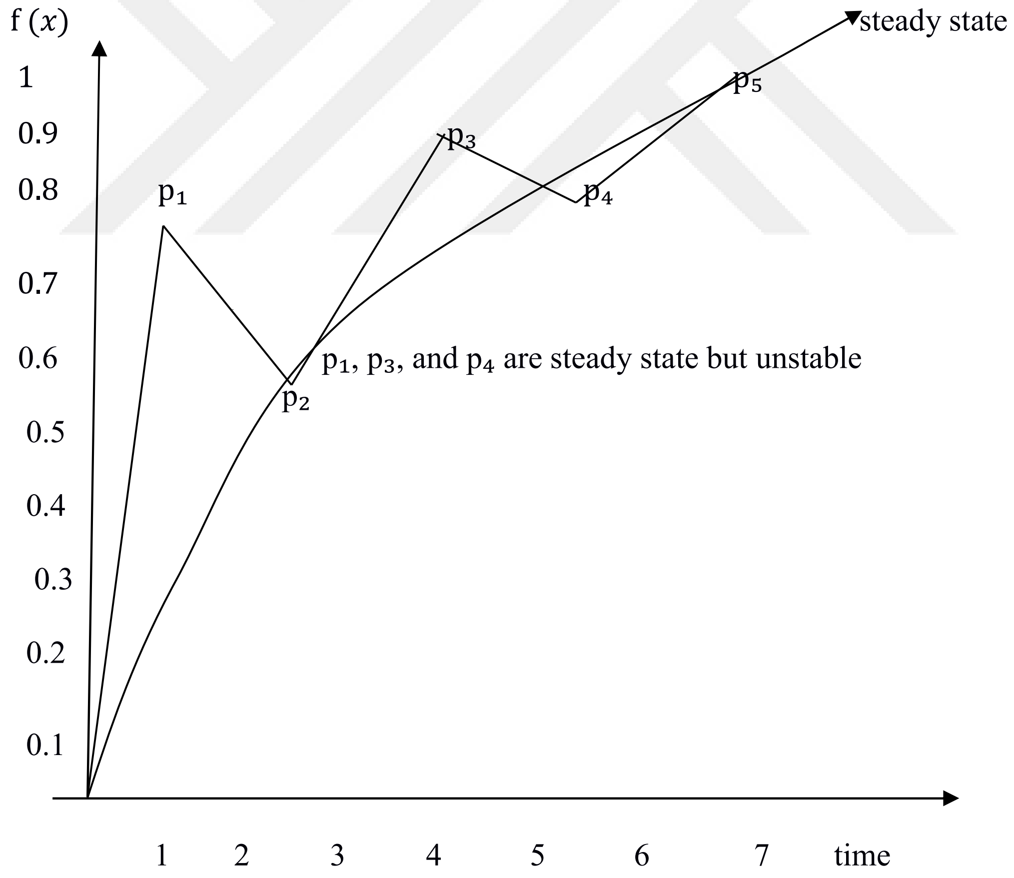


Figure 5.3 Diagram for Second Order System

For instance, the second order occurrence of latent variation let us consider a

Leontief model for an economy in which the production process is unstable at some points we are interested in describing the evaluation in time of the total utility in the production system in the economy.

Assume that there are N production processes in which n economic of goods are transformed into goods of the same kind and that in order to produce one unit of good j by means of the Kth production process we need at least $a^{k}ij$ units of good i the real number.

$$a^{k}ij,\ k \in N = \{1, 2, 3, 4, \ldots .N\}$$

$$i\,j \in n = \{1, 2, 3, \ldots\ldots .n\}$$

Which where called the technology coefficients of the dynamical system, we assume that in each unit one production cycle takes place and denoted by.

$Q_i(t)$ the quantity of production i available at time t

$Ui^{k}(t)$ the quantity of production i assigned the production process k at time t.

$Yi^{k}(t)$ the quantity of product i acquire from the production process k at time t.

Then to satisfy the improvement in the production process the following condition hold.

$$\sum_{k=1}^{n} ui^{k}(t) \leq q_i(t)\ \forall i \in n$$

$$\sum aij^{k} u\,i^{k}(t) \geq y\,j^{k}(t+1)\ \ \forall k \in N\ \ i \in n$$

$$Q\,i(t) \leq \sum_{k=1}^{n} yi^{k}(t)\quad \forall\, i \in n$$

The underlying structure of the economy in the second order graph show the inefficient production unbalance of the available product

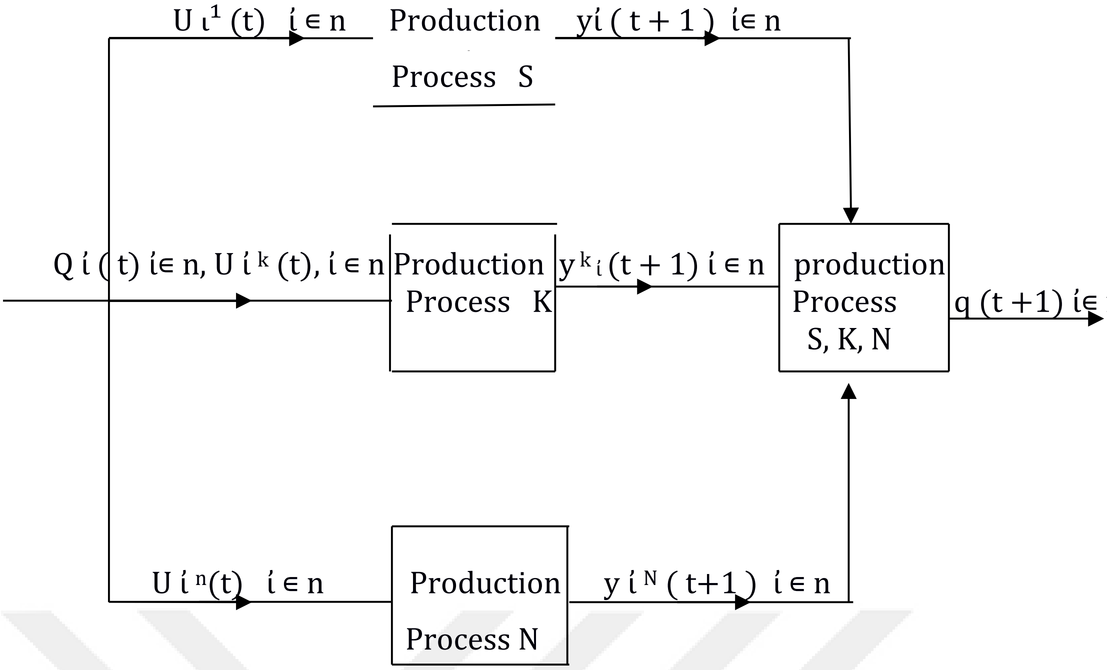


Figure 5.4 Production Process

Total utility in the economy is a function of the available amount of production process in the system, where q□, q□, q□ . . . . . .q n are the quantity of product produce. Where J: □ ⟶ □ is given by J (t) = α { q□(t) , q□(t) , q□(t) . . . . .q n} with α : □ ^{n}⟶ □ is a given function, if we identify utility with production value which will improve the amount with time then we will say

$$\alpha\,(q_1,\ q_2, q_3 \ldots\ldots q\ m) = \sum_{k=1}^{n} piqi$$

With p ι per unit production, See Jan Willem Polderman and Jan .C. Willems Introduction to Mathematical Theory of Systems and Control page [14-15]

4.6. Step Response and Time Constant

U (t)

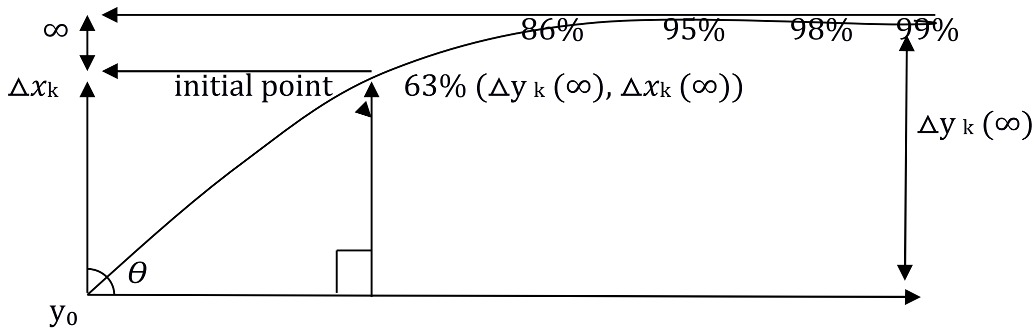


Figure 5.5 Time constant approach

Taking the step increase from the initial condition at a steady state with $\frac{\Delta y_K}{\Delta t_K} = 0$, then a step change will occur at the independent variation x(t) which eventually takes the system away from the initial steady state. The first and second order equations are stable and continuous such that it eventually approaches a new steady state at each point in time. The resulting step response in y(t) depends on change in the time and it is characterize by the following properties.

1, Steady state gain $m = \frac{\Delta y_K(\infty)}{\Delta x_K}$

2, $\tan\theta = \frac{\Delta y_K}{\Delta x_K} \Rightarrow \theta = \tan^{-1} \frac{\Delta y_K}{\Delta x_K}$

The angle at which the delay in time takes before y takes off in the direction that Δy_k change from rest at point 0, to the step response.

3: Time constant T ⟶ additional time it takes to reach 63% of the change in y that is

$$\Delta y_k (T + \theta) = 0.63 \Delta y(\infty)$$

4, $\Delta y_k(\infty) = y_{k+1}(\infty) - y_k(t_0)$

Resulting in the change in the step Response in y,

5, $\Delta x_k(\infty) = x_{k+1}(\infty) - x_k(t_0)$

The magnitude of then step Response

6, $y_k(t_0) = y_0$

The initial steady state

7, $y_k(\infty)$

The maximum steady state

8, t_0⟶the time when the change in the step occur in x, where $t_0 = 0$ is

chosen

The angle at which the delay in time occur takes an additional time before it get to the initial steady state, then after the time delay it reach a variable at which the variable takes an initial condition of about 63% before it get to the next variable, which gradually proceed continuously with an occurrence of the deferent steady state. More precisely a fraction of about

$$= \ 1 - e^{-t} \ = 1 - 0.3679 = 0.6321 \text{ or } 0.63$$

Approximately occur before it get to the next variable steady state, the step response takes it to a new steady state with the change in time, the process proceed continuously with a time constant till it get to reach 100% of its change. Because it generally takes an infinitely long time for the system to reach exactly its final state, so this will not give us a meaningful value since the time is continuous. See Chemical and Energy Process Engineering by Sigurd Skogestad page [286-287]

4.7. Criteria for Critical Point Stability in Industries

The homogeneous linear system with constant coefficient is use to determine the stability and to maintain a standard system approach for production where we have the form. View Erwin Kreyszig page [148] Advanced Engineering Mathematics.

$$y^{\triangle} = Ay \ = \ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ in Component}$$

$$x_1{}^{\triangle} = a_{11}x_1 + a_{12}x_2$$

$$x_2{}^{\triangle} = a_{21}x_1 + a_{22}x_2$$

We can obtain an overview of families of solution if we represent them parametrically as $x\,(\mathrm{t}) = \ [\, x_1(t), x_2(t)]^{\mathrm{T}}$

Moreover, x□ x□ will increase the system of the solution, and change will occur due to the change in x□ x□ and their totality output will increase rapidly.

$$x\,(\mathrm{t}) = xe^{\lambda t}$$

Substitute into the above equation

$$x^{\triangle}\,(\mathrm{t}) = \mathrm{A}xe^{\lambda t} = \mathrm{Ay} = \mathrm{A}xe^{\lambda x}$$

Dropping the Common factor $e^{\lambda x}$ we have Ax = λx. Hence, y(t) is a nonzero solution

of the above equation if λ is an eigenvalue of A and$x$a corresponding eigenvector.

We show that the general form of the phase portrait is determine largely by the type of critical point of the system in the above equation, and it can be defined as the point at which

$$\frac{\Delta x_2}{\Delta x_1} = \frac{x^{\wedge}{}_2 \Delta t}{x^{\wedge}{}_1 \Delta t} = \frac{b_{21}x_1 + b_{22}x_2}{b_{11}x_1 + b_{12}x_2}$$

We shall see how these type of critical point are related to the eigenvalue then λ = λ□ and λ□ of the characteristic equation

$$\det (A - \lambda_1) = \begin{bmatrix} b_{11} - \lambda & b_{12} \\ b_{21} & b_{22} - \lambda \end{bmatrix}$$

$$= \lambda^2 - (b_{11} + b_{22})\,\lambda + \det A = 0$$

This is a quadratic equation$\lambda^2 - p\,\lambda + q = 0$

With coefficient p q and discriminant Δ given by

$$p = b_{11} + b_{22} \qquad q = \det A = b_{11}b_{22} - b_{12}b_{21}, \qquad \Delta = p^2 - q$$

From algebra, we know that the solutions of this equation are

$$\lambda_1 = \frac{1}{2}(p + \sqrt{\Delta}), \qquad \lambda_2 = \frac{1}{2}(p - \sqrt{\Delta})$$

Furthermore the product representation of the equation gives

$$\lambda^2 - p\,\lambda + q = (\lambda - \lambda_1)(\lambda - \lambda_2)$$

$$= \lambda^2 - (\lambda_1 + \lambda_2)\,\lambda + \lambda_1\lambda_2.$$

Hence, p is the sum and q the product of the eigenvalue, also

$$\lambda_1 - \lambda_2 = \sqrt{\Delta}, \qquad p = \lambda_1 + \lambda_2, \qquad q = \lambda_1\lambda_2,\ \Delta = (\lambda_1 - \lambda_2)^2.$$

The eigenvalue criteria for critical points (Derivative Table)

Table 5.3 critical point stability

Name	$P = \lambda_1 + \lambda_2$	$q = \lambda_1\lambda_2$	$\Delta = (\lambda_1 - \lambda_2)$	$\lambda_1\lambda_2$
1: Node		$q > 0$	$\Delta \geqq 0$	Real same sign
2:saddle point		$q < 0$		Real opposite sign
3: centre	$p = 0$	$q > 0$		Pure imaginary
4: spiral point	$p \neq 0$		$\Delta < 0$	Complex, not pure imaginary

4.8. Stability of the System

Critical points may be classify in terms of their stability these concepts are basic in engineering and other applications, stability means roughly speaking that a small change of physical system at the same instance changes the behavior of the system only slightly at all future time t for critical points. Which stabilize the system and the increase the production process where p□, p□, p□, p□ and p□ are stable and to maintain a steady state with rise and fall in the system. Whereby we introduce the eigenvalue to illustrate and solve the problem mathematically and maintain a regular steady state with little difference in the second order system. See Erwin Kreyszig page [149-150] for more detail on stability of a system

Example1.1 We will introduce the second order equation, and to solve the critical points of the system.

$$\mathrm{m}x^{\Delta\Delta}(\mathrm{t}) + \mathrm{c}x^{\Delta}(\mathrm{t}) + \mathrm{k}x = 0$$

Dividing by m gives $x^{\Delta\Delta} = -\left(\frac{k}{m}\right)x - \left(\frac{c}{m}\right)x^{\Delta}$

To get a system set $x_1 = x$, $x_2 = x^{\Delta}$

then $$x_2{}^{\Delta} = x^{\Delta\Delta} = -\left(\frac{k}{m}\right)x_1 - \left(\frac{c}{m}\right)x_2.$$

hence $$x^{\Delta} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} x,$$

By introducing the method of eigenvector the solution becomes

$$\det(\mathrm{A} - \lambda\,\mathrm{I}) = \begin{bmatrix} -\lambda & 1 \\ -k/m & -c/m \end{bmatrix}$$

$$x^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0$$

We say that $\mathrm{p}_1 = -\mathrm{c}/\mathrm{m}$, $\mathrm{p}_2 = \text{-}\mathrm{k}/\mathrm{m}$, $\Delta = (\mathrm{c}/\mathrm{m})^2 - 4\mathrm{k}/\mathrm{m}$

From the equation we will obtain the following result, note that in the discriminant Δ play an essential rule note that and p□, = q

Table 5.4 Stability System

1: No damping	C=0 , p=0	q>0 ,	A centre
2: Under damping	c^2-4km , p<0	q>0 , Δ< 0	A stable and attractive spiral point
3:critical damping	$c^2=4km$, p<0	q>0 , Δ=0	A stable and attractive node
4: over damping	$c^2>4km$, p<0	q>0 , Δ>0	A stable and attractive node

From the table it shows that there is a steady and gradual increase in the production system. Where q indicate the change in time of production in the industries

4.9. Dynamic Equation with n Dimensional Continuous Time Process

We will concentrate on linear differential equation on a dynamic form $x^{\Delta} = Ax$ where A is an n × n matrix and to classify base on stability, unstability and center subspace. Let denote by λ□, λ□, λ □, ... λ_n as the eigenvalue and let u□, u□, u□, ...Un be the basis in □ ⁿthat yields the real Jordan canonical form of matrix. If the eigenvalue are real and different, then the basis vectors are the corresponding eigenvectors, if there are complex conjugate pairs of eigenvalues, then the imaginary part of the corresponding complex eigenvectors should put into the basis. This is always true if k is the complex field □ if the characteristic and minimal polynomials do factor into linear factors then the Jordan canonical form is a complex field k. For instance if λ is a double eigenvalue with a one dimensional eigenvalue space, then the eigenvector V is determined by the equation

$$AV = \lambda V + U$$

Where U is the unique eigenvector, we note that in eigenvector space the linear independent U satisfying

$$(A - \lambda I)^2 V = 0$$

Which is the basis for stable, unstable and center subspaces refer to Peter .L. Simon Differential equations and Dynamical Systems page [19]

Definition 5.1 Let {u□, u□, u□. . . . . un} ⊂ □ ⁿ be the basis determining the real

Jordan canonical form of the matrix A, let λ_k be the eigenvalue corresponding to

u_k the subspaces.

$$E_s(A) = \langle \{u_k : \mathbb{R}e\,\lambda_k < 0\} \rangle \longrightarrow \text{stable.}$$

$$E_u(A) = \langle \{u_k : \mathbb{R}e\,\lambda_k > 0\} \rangle \longrightarrow \text{unstable.}$$

$$E_c(A) = \langle \{u_k : \mathbb{R}e\,\lambda_k = 0\} \rangle \longrightarrow \text{center subspace}$$

Which satisfy the linear dynamic equation $x^{\Delta} = Ax$ where the spanned is a vectors field.

Theorem 5.2 The hyperbolic matrices A,B ϵ L(□ ⁿ) are C□ conjugate , and at the same time C□ equivalent if and only if S(A) = S(B) in case U(A) = U(B) holds as well since the center subspaces are zero dimensional.

Theorem 5.3 (Kuiper) let A, B ϵ L (□ ⁿ) be matrices with C (A) = C (B) = n these C□ equivalent if and only if they are linearly equivalent.

Theorem 5.4 The matrices A, B ϵ L (□ ⁿ) are □□ equivalent if and only if S (A) = S (B), U(A) = U (B) and their restriction to their center subspaces are linear equivalent.

4.10. Dynamic Equation with n Dimensional Discrete Time

A linear differential equation on a discrete dynamic form $x_{n+1} = Ax_n$ where A is an n× n matrix is a discrete time case in N dimension where C□ is classified as stable, unstable, and center subspaces, then let us denote the eigenvalue of the matrix With multiplicity by λ□, λ□, λ□. . . . . λ_n let u□, u□, u□ . . . . . . u_n denote the basis in □ ⁿ in which the matrix takes it Jordan canonical form using the basis the stable, unstable and center subspaces can be define as follows. See Peter .L. Simon Differential equations and Dynamical Systems page [26]

Theorem 5.5 The subspaces $E_s(A)$, $E_u(A)$, and $E_c(A)$ have the following properties

1, $E_S(A) \oplus E_U(A) \oplus E_S(A) = \mathbb{R}^n$

2, they are invariant under A, that is A (Ei (A), i = (s, u, c)

3, for any p ϵ E_S (A) we have $A^n p \longrightarrow 0$ if $n \longrightarrow +\infty$

4, for any p ϵ E_U (A) we have $A^{-n} p \longrightarrow 0$ if $n \longrightarrow -\infty$

The dimension of the stable, unstable and center subspace will play an important row in the □□ classification of linear system.

CONCLUSION

We introduced the schematic classification of operators to determine the Backward and Forward jump system where we concentrate more on the forward jump operator to determine the continuous and discrete approach, which eventually improve process.

We also talk about the contraction mapping principle with an illustration to Picard theorem that resulted to a fixed point theorem related to the Banach, Schauder's, Krasnoselkiis, Peano Bolzano and Lipschitz to determined the existence and uniqueness and fixed point, with a continuous and discrete process. We also illustrate the process with numerical approach and Picard Lipschitz continuity to estimate a better and better approximation and we analyses the improvement of the forward difference approach into the industrial sector, with a guarantee of stability.

The first and second order system is use to illustrate the system stability, with a graph that shows the steady state process, the first order system grows gradually with a step response given rise to a steady state that is continuous. While the second order system is also a steady state but unstable at some point with large value differences within the process, as indicated in the graph. An eigenvalue method was introduce to determine an improvement the second order system, and how to obtain less conservative bound for λ that can guarantee system stability. The second order system will require more additional mathematical knowledge due to its unstableness and the large variables in the system, which we will required some additional procedure in order to stabilize the process.

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